



APPLIED HYDRAULICS

CHAPTER 1:

OPEN CHANNEL FLOW

Open Channel Flow

- Differences between flow in pipes and open channels
- Channel classification
- Flow classification
- Effective forces in flow analysis
- Velocity distribution

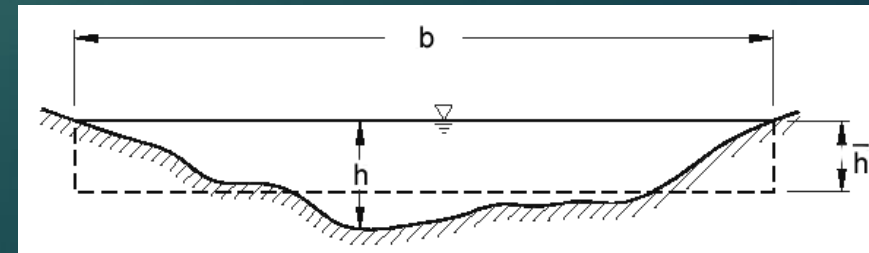
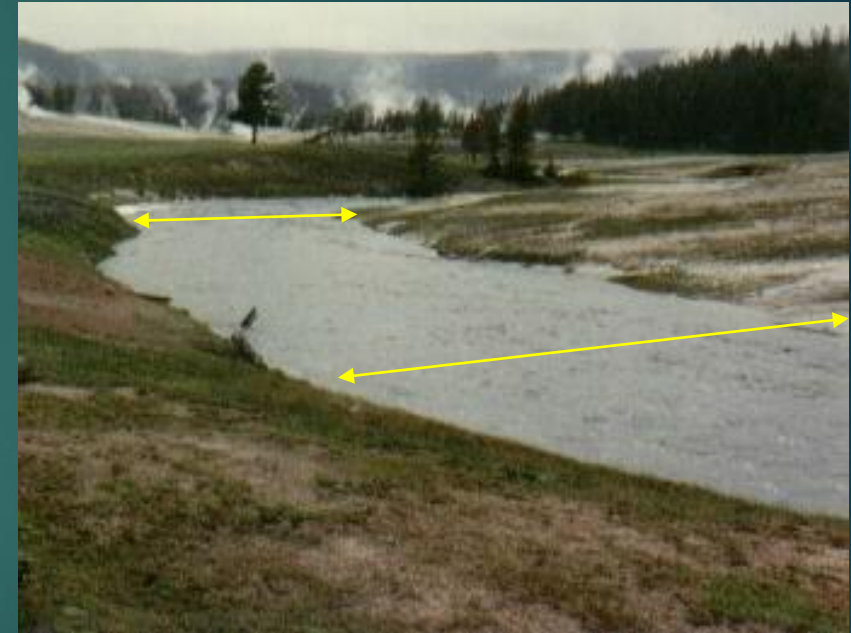
Basic Concepts

- An open channel is one that has its **top surface open** and so, water having pressure equal to the atmospheric pressure.
- Open-channel flows are not entirely included within *rigid boundaries*; a part of the flow is in contact with nothing at all, just empty space.
- Because the flow boundary is freely deformable, in contrast to the solid boundaries, the flow surface is called a **free surface**.



OPEN CHANNEL FLOW | Channel vs. Pipe

- In the pipe flow, there is no direct atmospheric flow and there would be **hydraulic pressure** only.
- The flow in open channel is due to **gravity** while in pipe flow **pressure works** (e.g., pumping water)
- In open channels, flow conditions are greatly influenced by **slope** of the channel.
- In pipes the flow cross section is known and fixed while it is unknown in advance.



Types of Open-Channels

Examples of open channels flow are **river**, **streams**, **flumes**, **sewers**, **ditches** and **lakes**.



Types of Open-Channels



Classification of Open-Channel

Natural Channels: Very irregular in shape.

Rivers, tidal estuaries.



Artificial Channels: Developed by men and usually designed with regular geometric shapes.

Irrigation canals, laboratory flumes.



Prismatic Channels: unvarying cross-section and constant bottom slope.

Artificial channels like Rectangular, Trapezoid



Non-Prismatic Channels varying cross-section and bottom slope.

The natural channels are usually prismatic



Classification of Open-Channel

Rigid Boundary Channels: Non-changeable boundaries (bed and sides). Lined canals, sewers and non-erodible unlined canals



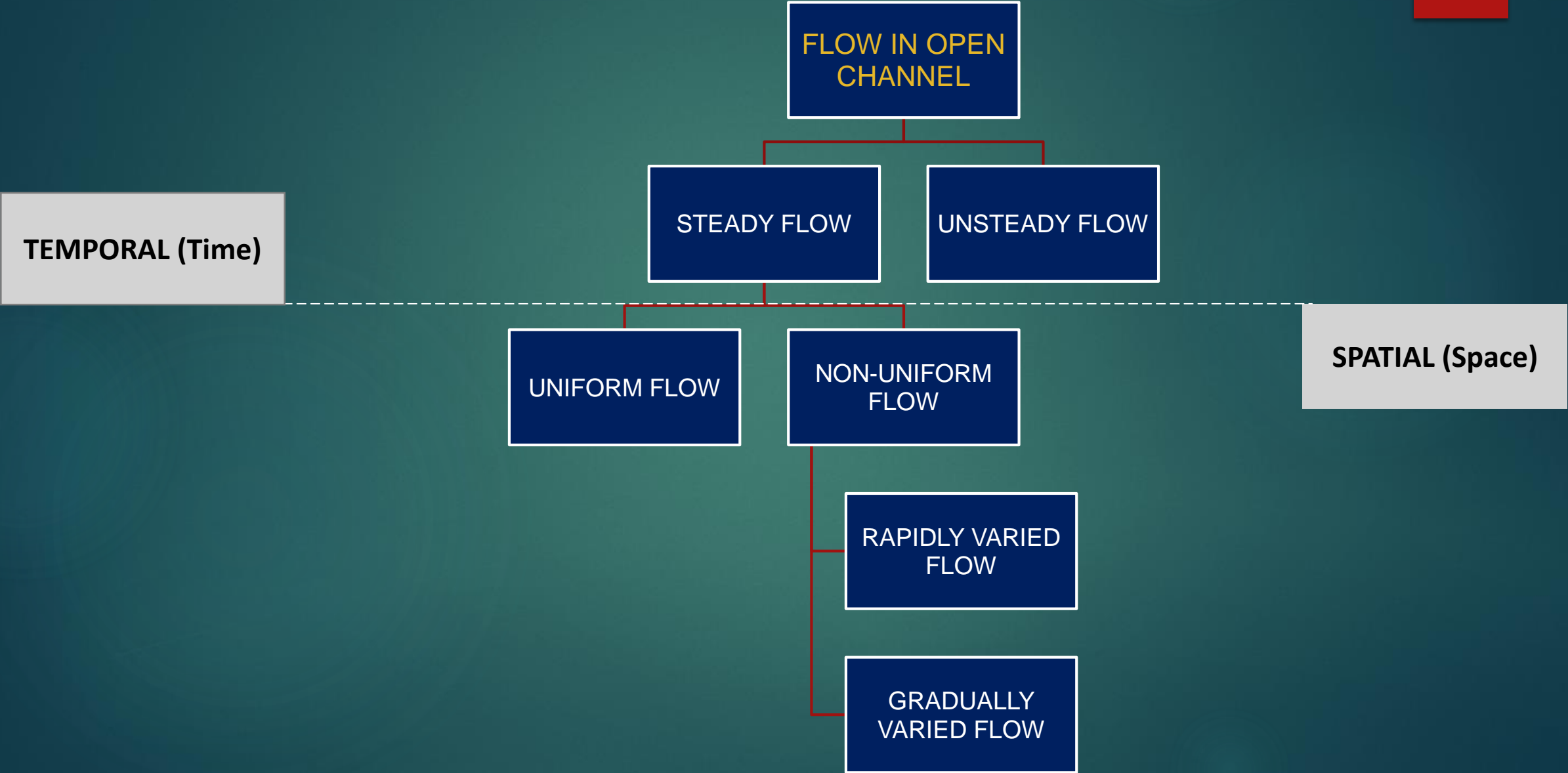
Small Slope Channels: having a bottom slope less than 1 in 10 (10%).

Mobile Boundary Channels: Boundary is composed of loose sedimentary particles moving under the action of flowing water. An alluvial channel



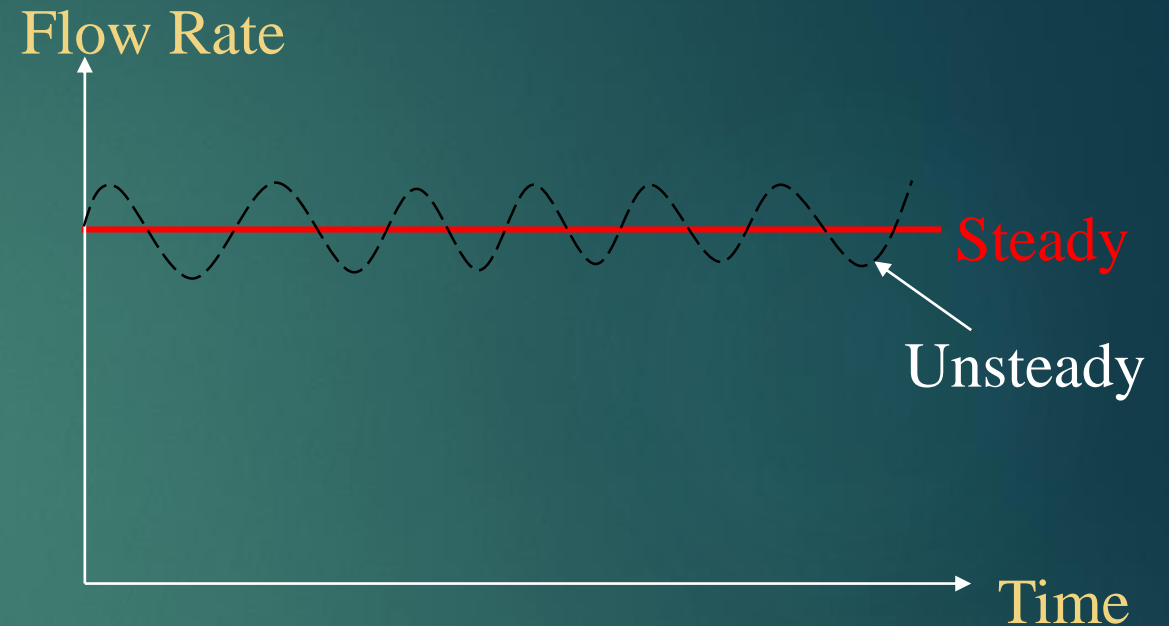
Large Slope Channels: having a bottom slope greater than 1 in 10 (10%).

Flow classification



Flow classification (Time Criteria)

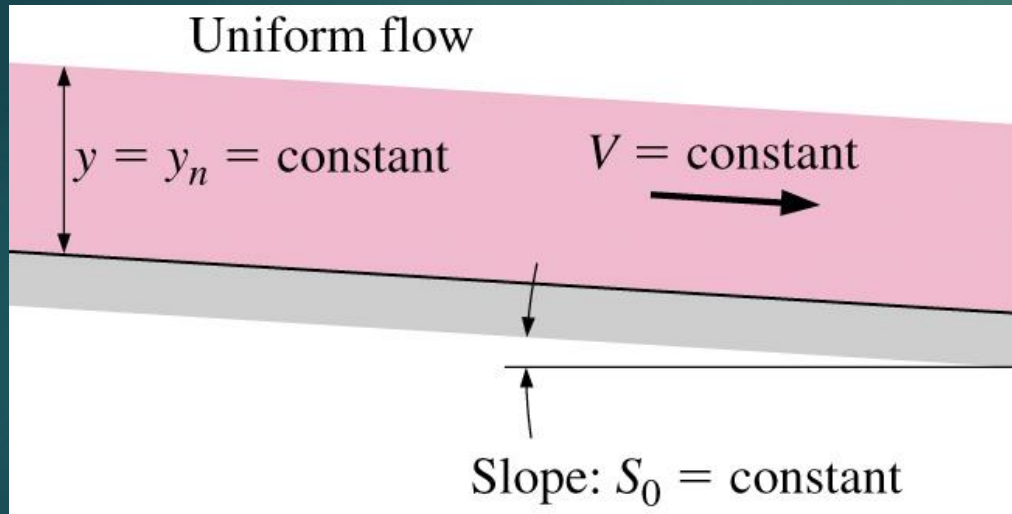
- If the flow parameters, such as velocity, pressure, density and flow rate do not vary with **time** then the flow is **steady**.
- **Steady flow** ($dy/dt = 0$).
Water depth at one point is **same all the time**. (Flow constant with time)
- If the flow parameters vary with time then the flow is categorized as **unsteady**.



- **Unsteady flow** ($dy/dt \neq 0$)
Water **depth changes all the time**. (Flow variation with time)

Flow classification (Space Criteria - Uniform Flow)

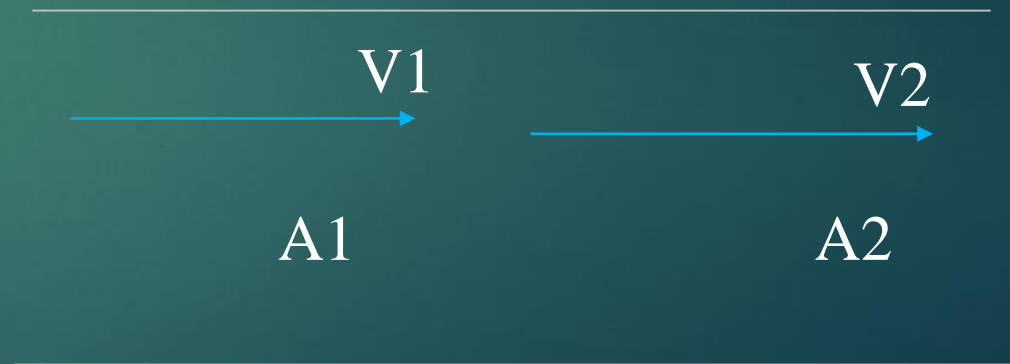
- If the flow parameters do not vary with **distance** along the flow path, then the flow is **uniform**.



- Depth of water in a uniform flow is called **normal depth** or y_n

- Uniform flow** ($dy/dx = 0$)
Water depth same along the whole length of flow.

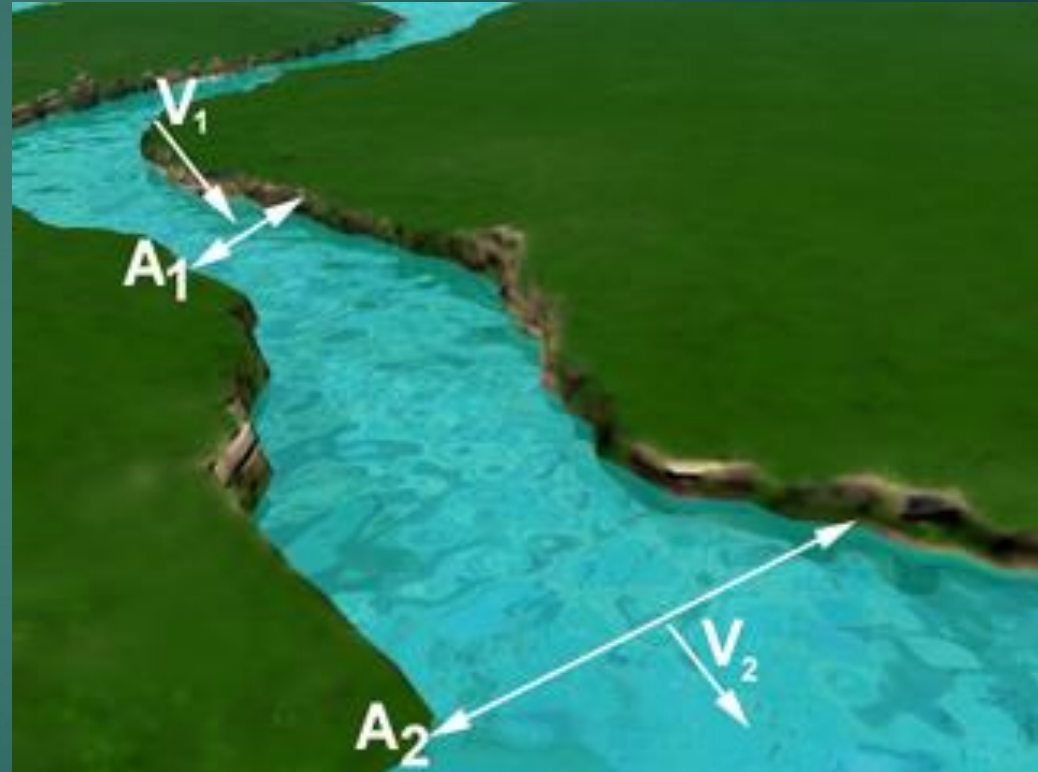
$$V_1 = V_2$$
$$A_1 = A_2$$



Uniform Flow

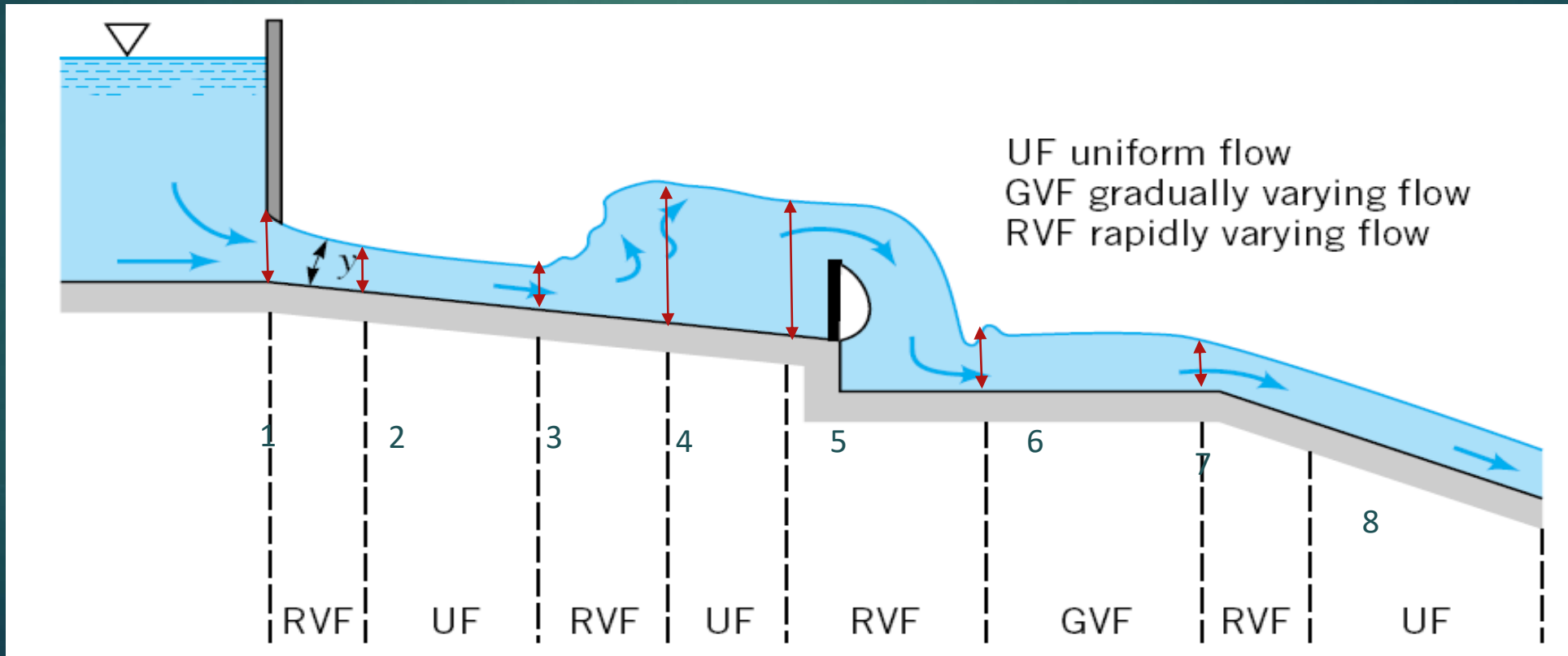
Flow classification (Space Criteria - Non uniform Flow)

- If the flow parameters **vary** with **distance** along the flow path, then the flow is non-uniform.
- **Non-uniform** flow ($dy/dx \neq 0$)
Water **depth changes** either rapidly or gradually.



Flow classification (Space Criteria – Non Uniform Flow)

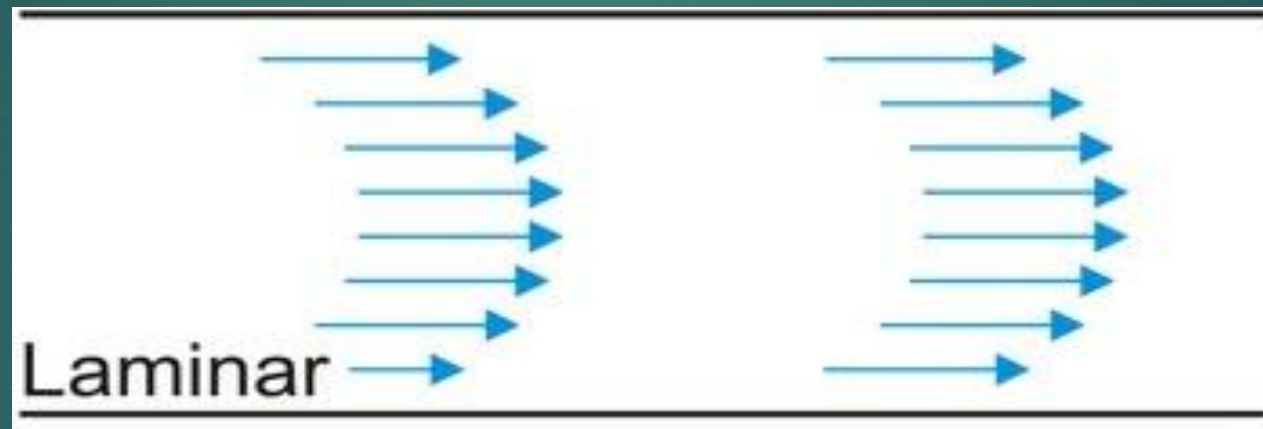
- Rapidly varying flows (**RVF**): flow depths that vary considerably over a short distance
- Gradually varying flow (**GVF**): flow depths that vary slowly with distance.



Flow classification (flow particles motion)

Laminar

- Type of fluid flow in which the fluid travels smoothly or in regular paths.

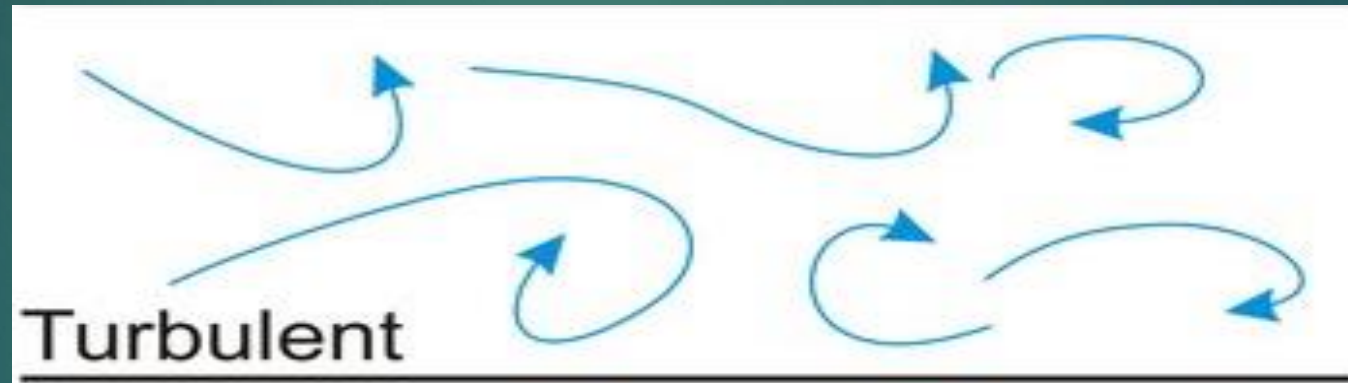


- The flow channel is relatively small, the fluid is **moving slowly**, and its viscosity is relatively high.

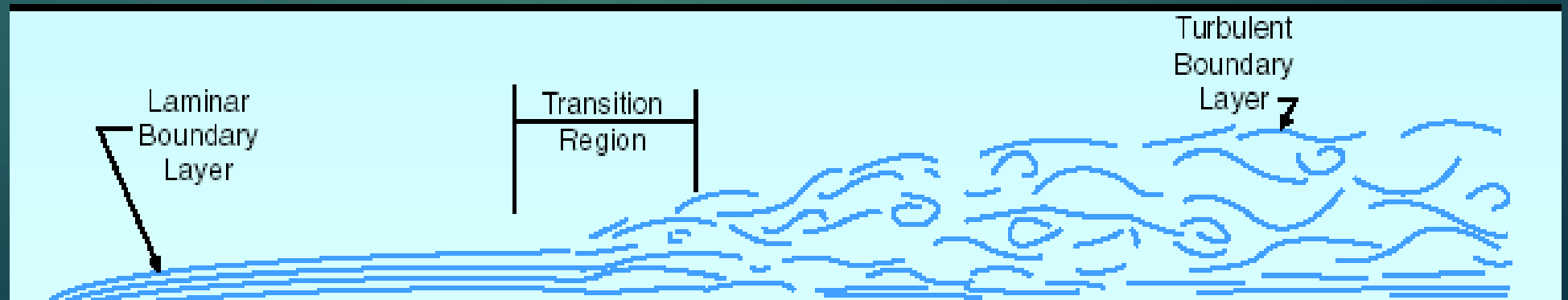
Flow classification (flow particles motion)

Turbulent

- The fluid undergoes **irregular** fluctuations and mixing.
- Most kinds of fluid flow are turbulent except near solid boundaries



Transitional



Effective Forces In Flow Analysis (Reynolds number)

Viscous
Inertia
Gravity

A non-dimensional number

$$\text{Reynolds number} = \frac{\text{Inertia Forces}}{\text{Viscous Forces}}$$

$$R_e = \frac{V \times D}{\nu}$$

where V = Average velocity of flow, D = pipe diameter, and ν = Kinematic viscosity of the fluid.

$$R_e = \frac{V \times R_h}{\nu}$$

where V = Average velocity of flow, R_h = is hydraulic radius, and ν = Kinematic viscosity of the fluid.

Laminar flow : **Re < 500** (viscous > inertia)

Transitional flow: **500 < Re < 1300**

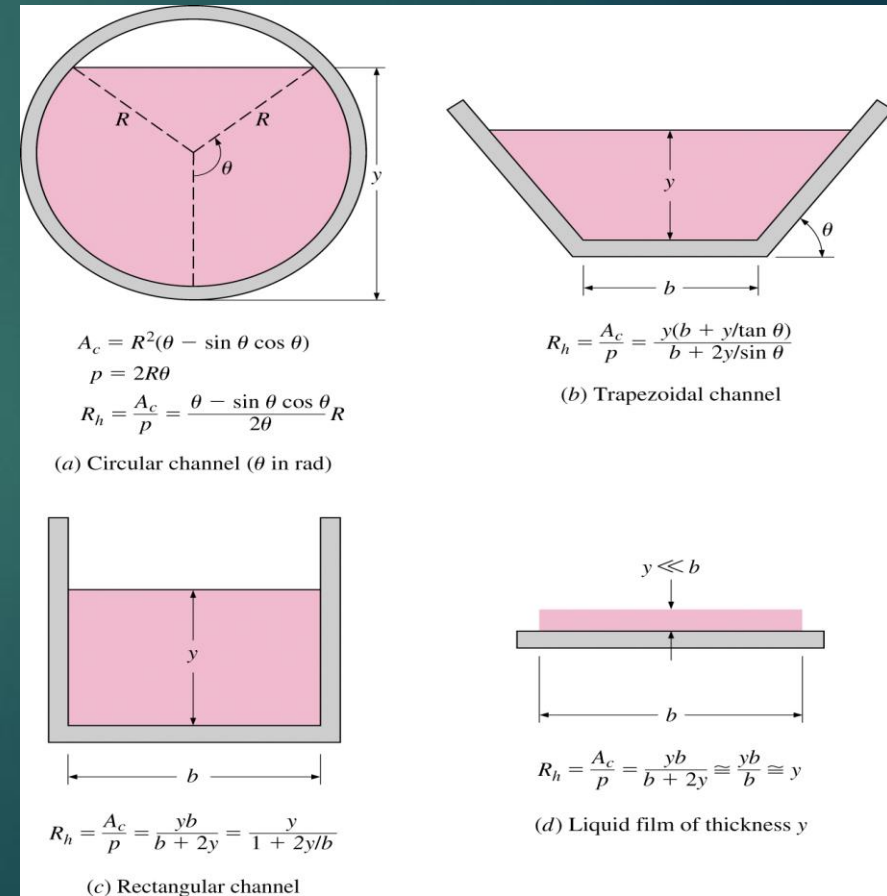
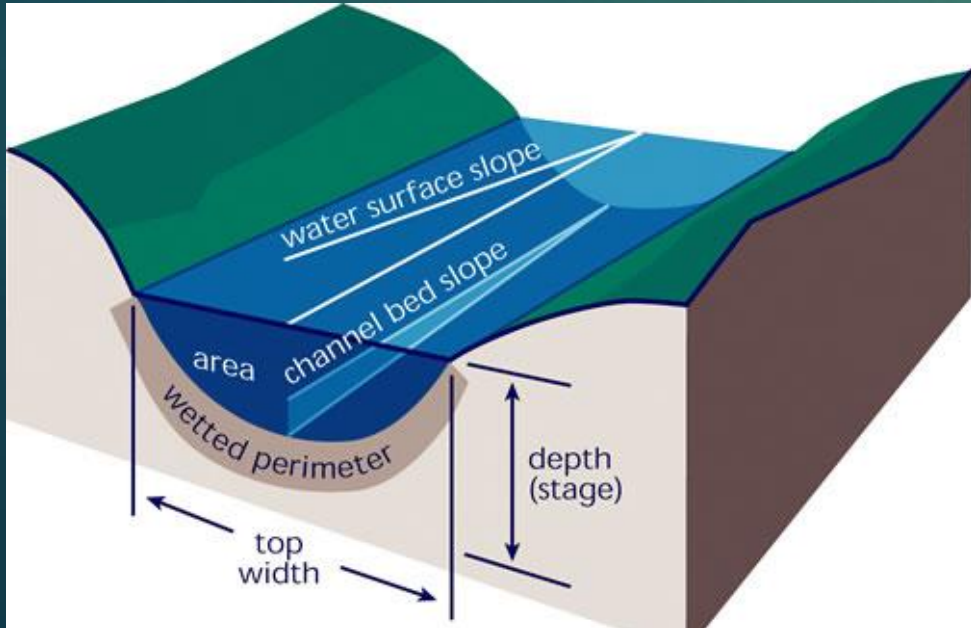
Turbulent flow: **Re > 1300** (inertia > viscous)

Effective Forces In Flow Analysis (hydraulic radius)

- The **hydraulic radius** is the term used to describe the shape of a channel.

$$R_h = \frac{\text{Cross sectional area}}{\text{wetted perimeter}} = \frac{A}{P}$$

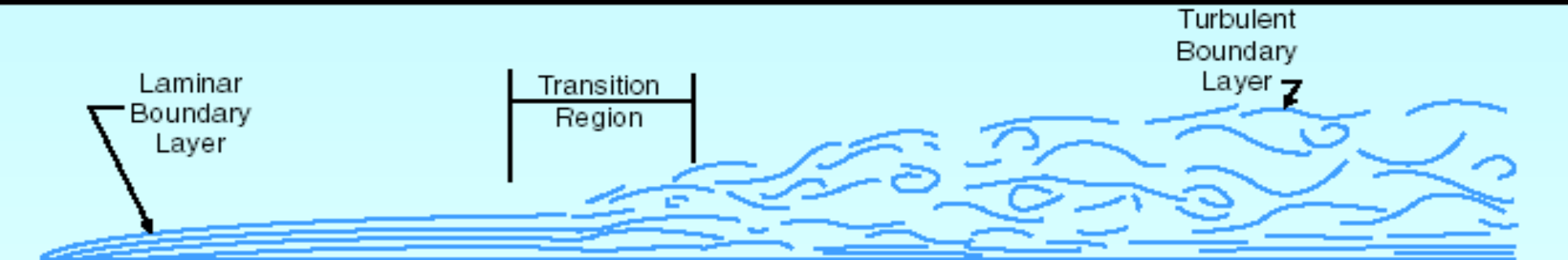
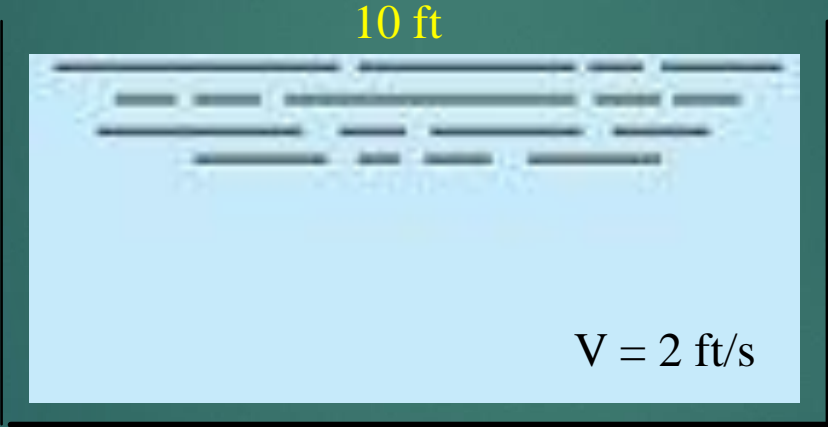
- The wetted perimeter *does not* include the free surface.



Effective Forces In Flow Analysis

Example 1

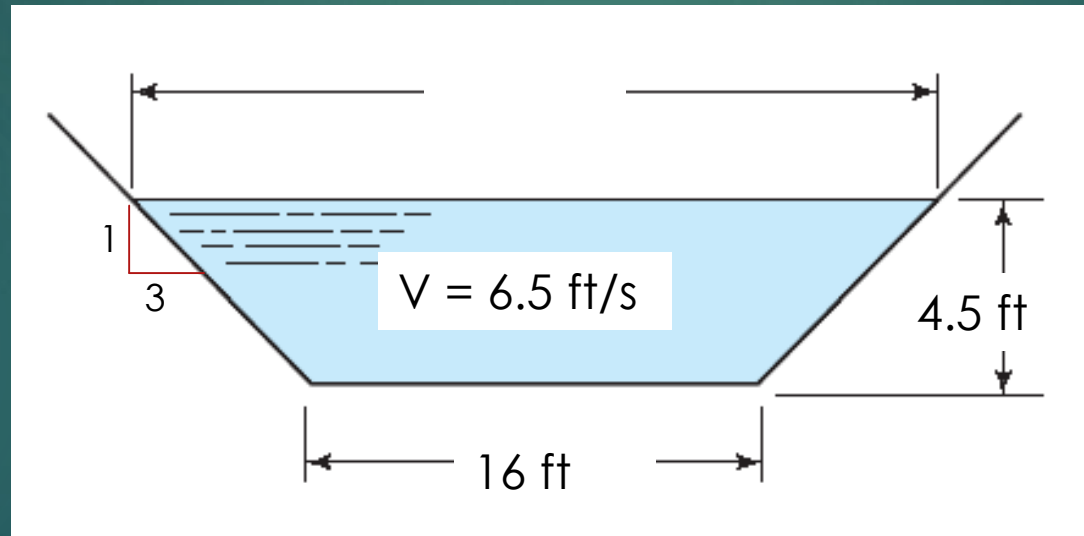
Determine the type of flow (laminar or turbulent) for the following rectangular channel.



Effective Forces In Flow Analysis

Example 2

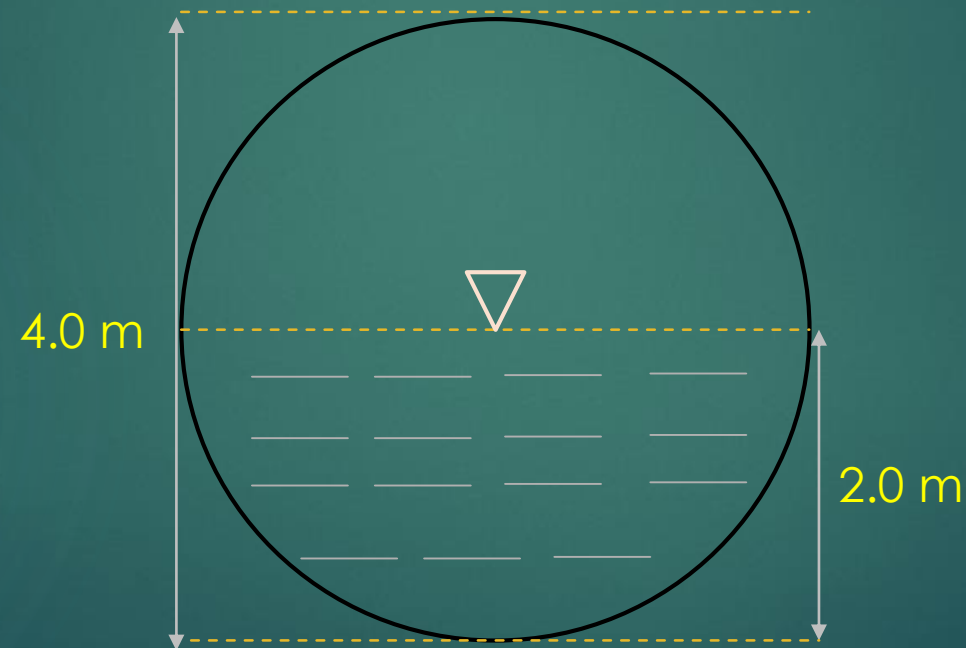
Determine the type of flow (laminar or turbulent) for the following trapezoidal channel.



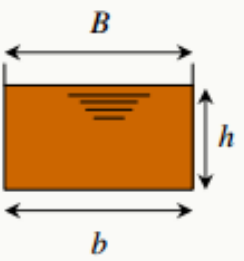
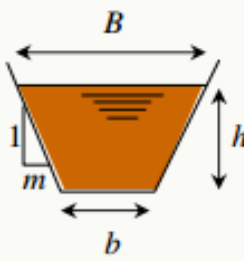
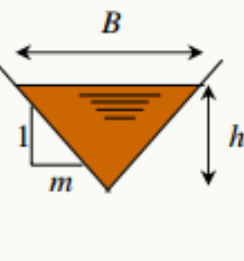
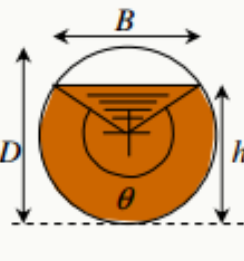
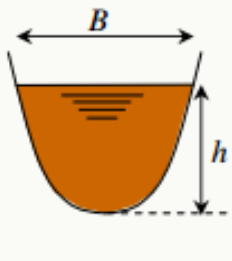
Effective Forces In Flow Analysis

Example 3

Determine the hydraulic radius for the following circular channel.



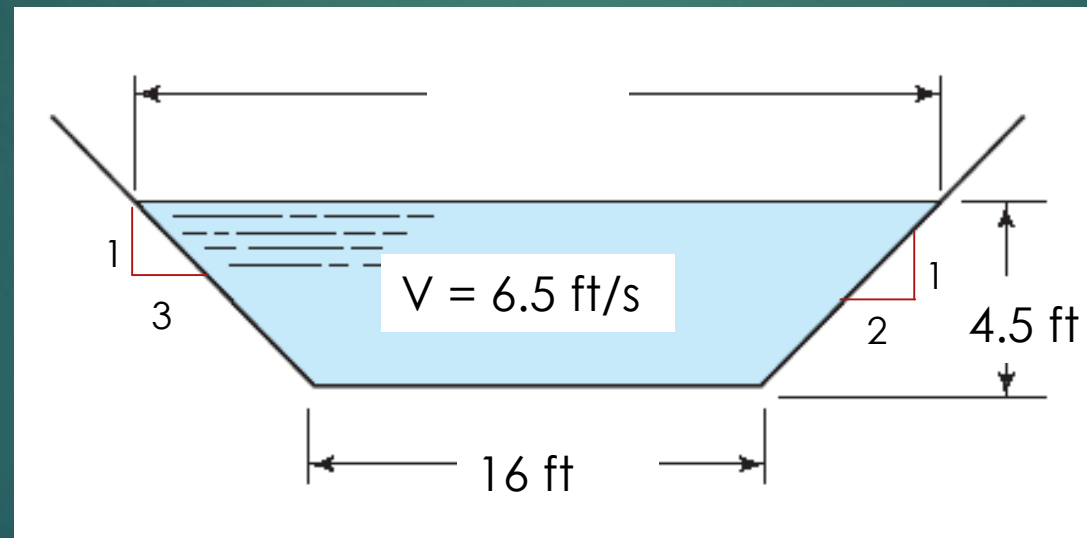
Effective Forces In Flow Analysis

	<i>rectangular</i>	<i>trapezoidal</i>	<i>triangular</i>	<i>circular</i>	<i>parabolic</i>
					
<i>flow area</i> A	bh	$(b + mh)h$	mh^2	$\frac{1}{8}(\theta - \sin \theta)D^2$	$\frac{2}{3}Bh$
<i>wetted perimeter</i> P	$b + 2h$	$b + 2h\sqrt{1 + m^2}$	$2h\sqrt{1 + m^2}$	$\frac{1}{2}\theta D$	$B + \frac{8}{3}\frac{h^2}{B}$ *
<i>hydraulic radius</i> R_h	$\frac{bh}{b + 2h}$	$\frac{(b + mh)h}{b + 2h\sqrt{1 + m^2}}$	$\frac{mh}{2\sqrt{1 + m^2}}$	$\frac{1}{4}\left[1 - \frac{\sin \theta}{\theta}\right]D$	$\frac{2B^2h}{3B^2 + 8h^2}$ *
<i>top width</i> B	b	$b + 2mh$	$2mh$	$(\sin \theta / 2)D$ or $2\sqrt{h(D - h)}$	$\frac{3}{2}Ah$
<i>hydraulic depth</i> D_h	h	$\frac{(b + mh)h}{b + 2mh}$	$\frac{1}{2}h$	$\left[\frac{\theta - \sin \theta}{\sin \theta / 2}\right]\frac{D}{8}$	$\frac{2}{3}h$

Open Channel Flow

Quiz 1

Determine the type of flow (laminar or turbulent) for the following trapezoidal channel.



Effective Forces In Flow Analysis (Froude number)

Viscous
Inertia
Gravity

Dimensionless number

$$Fr = \frac{\text{Inertia Forces}}{\text{Gravity Forces}} = \frac{V}{\sqrt{gR_h}}$$

$Fr < 1$: Flow is subcritical

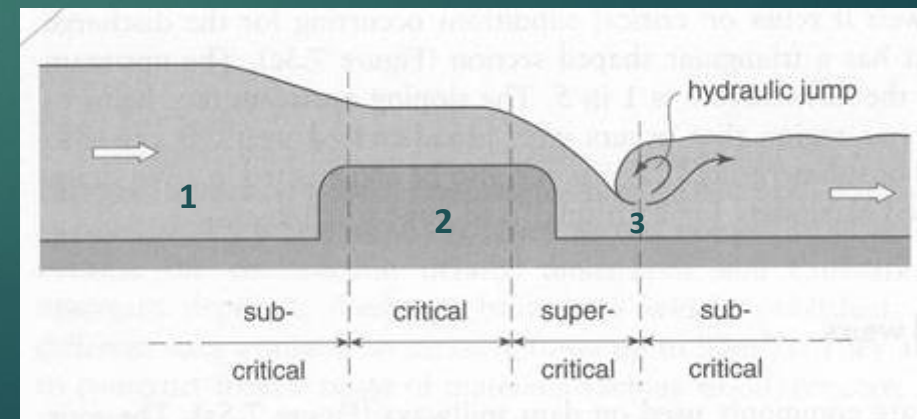
Flow is **deep**, **slow** with a **low energy** state

$Fr = 1$: Flow is critical

There is a perfect **balance** between the gravitational and inertial forces.

$Fr > 1$: Flow is supercritical

Flow is **fast flow** with a **high energy** state

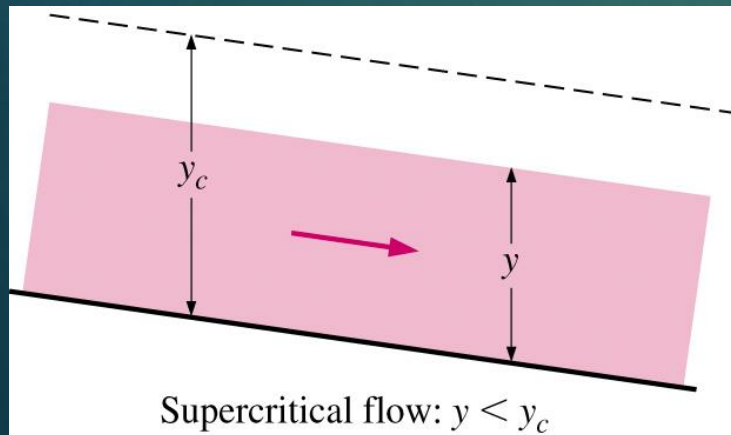
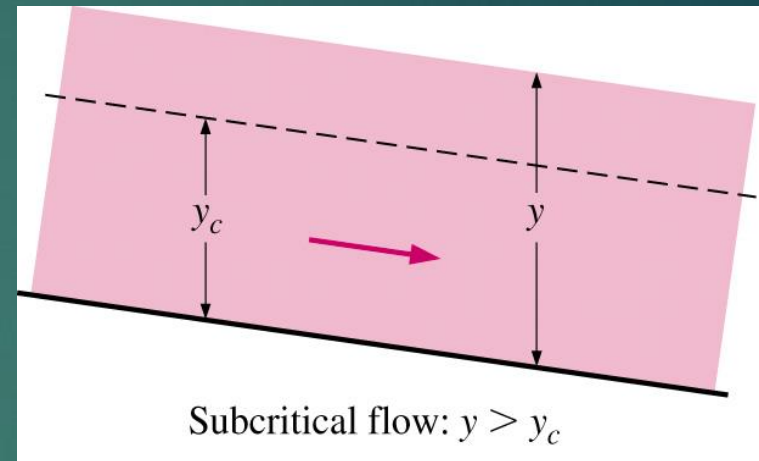


Effective Forces In Flow Analysis (Critical Depth)

Critical depth y_c occurs at $Fr = 1$

$$\left. \begin{aligned} Fr &= 1 \\ R_h &= y \\ Fr^2 &= \frac{V^2}{gy} \end{aligned} \right\} y = y_c = \frac{V^2}{g}$$

At low flow velocities ($Fr < 1$), So: $y > y_c$

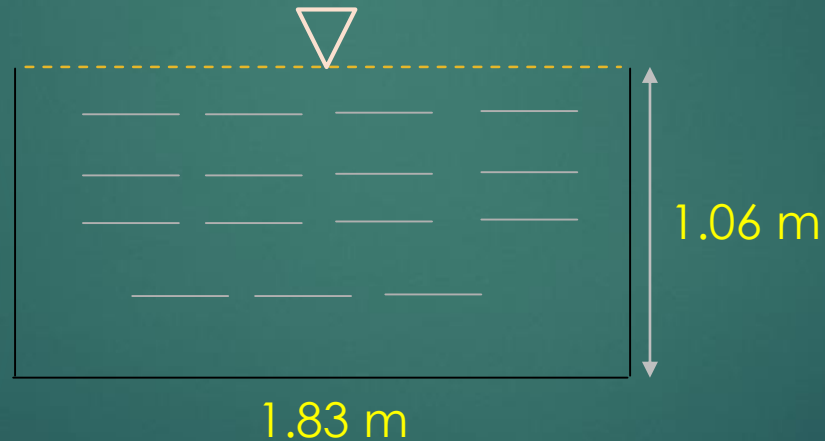


At high flow velocities ($Fr > 1$), So: $y < y_c$

Effective Forces In Flow Analysis

Example 4

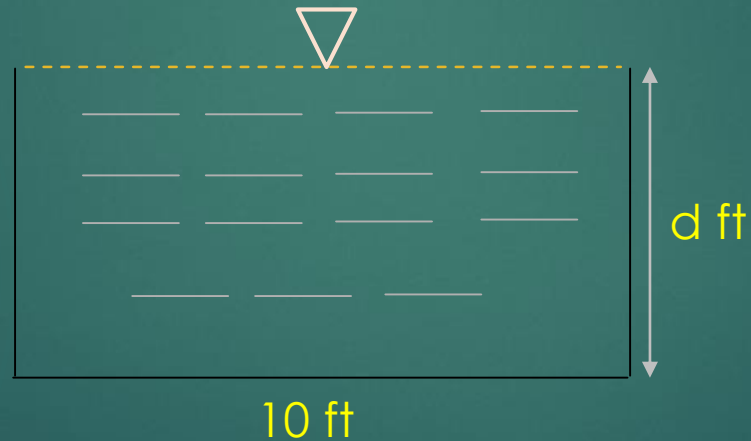
Determine the flow condition (subcritical, critical or supercritical) for a rectangular channel with the flow velocity of 2.32 m/s.



Effective Forces In Flow Analysis

Example 5

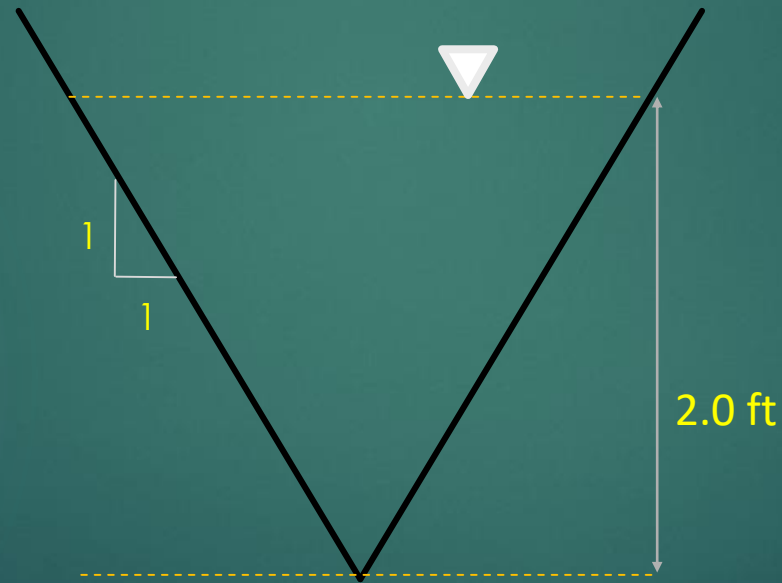
Flow in a rectangular channel with the width of 10 ft is critical with the velocity of 11.35 ft/s. Find the depth of flow.



Effective Forces In Flow Analysis

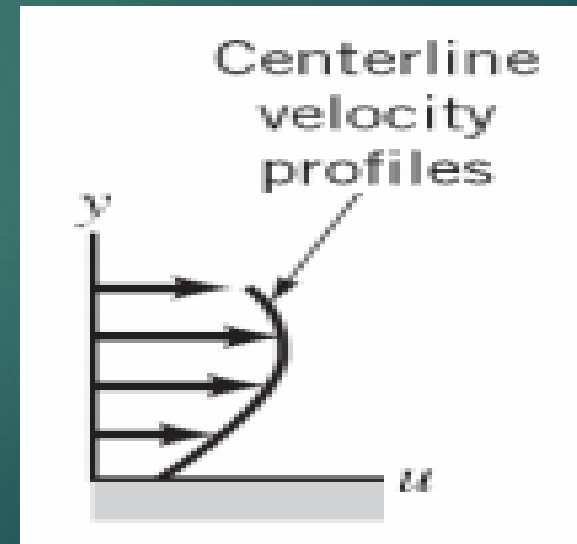
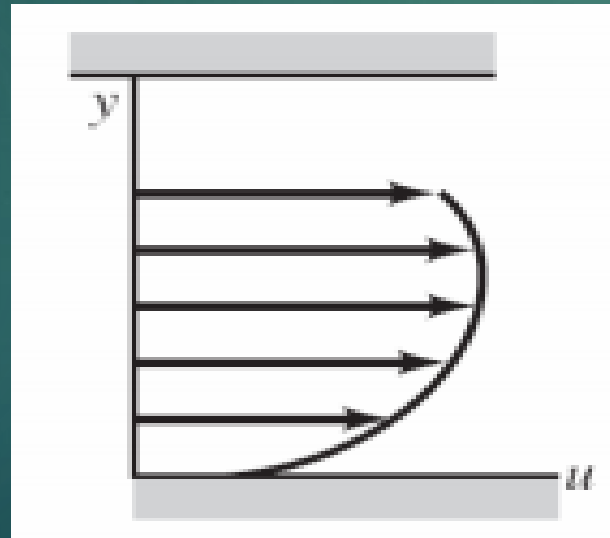
Example 6

Determine the critical depth and flow condition (sub or super critical) in a triangle channel with the side slope of 1:1, depth of 2 ft, and $V = \underline{5.28}$ ft/s.



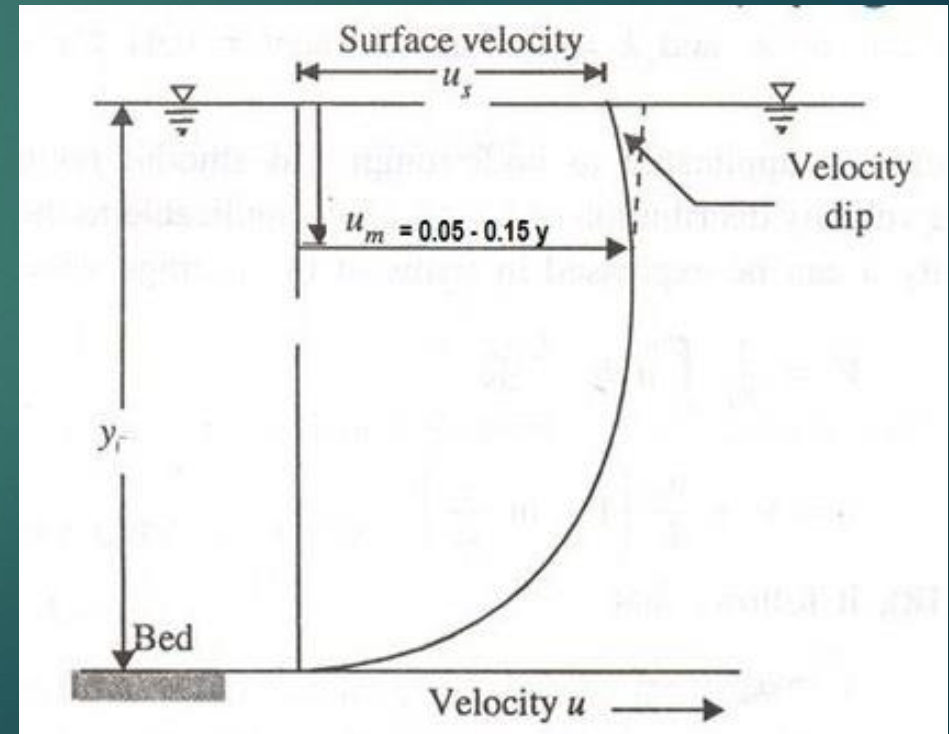
Velocity Distribution on Open Channels

- The velocities in channel are **not uniformly distributed** (usually axisymmetric) in channel section because of presence of a **free surface** and **friction** along the channel wall.
- It might be expected to find the maximum velocity at the free surface where the **shear force** is **zero** but this is not the case.



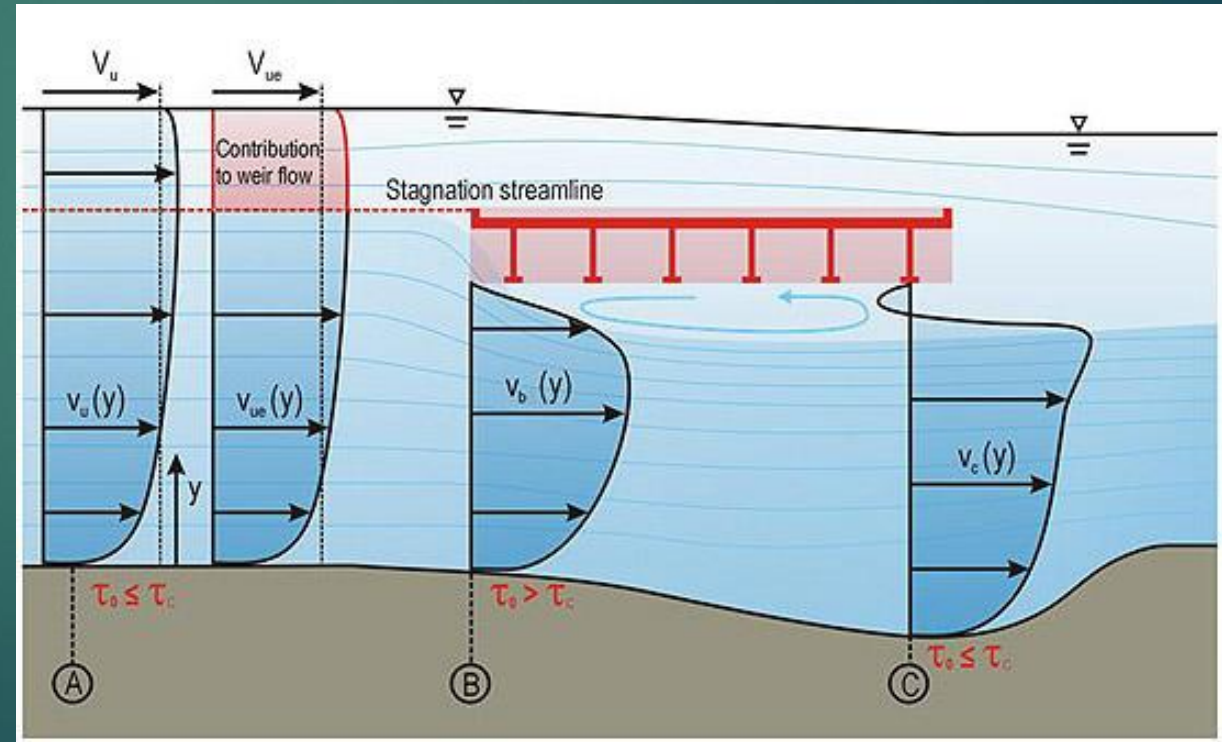
Velocity Distribution on Open Channels

- The maximum velocity is usually found just below the surface.
- The reason is the presence of **secondary currents** which are circulating from the boundaries towards the section center and **resistance at the air/water interface**.
- The measured maximum velocity usually appears to occur below the free surface at a distance **0.05** to **0.15** (some references say **0.25**) of the depth.



Velocity Distribution on Open Channels

- The velocity distribution at each section of a channel depends on many factors including **shape** of the channel at the section, the channel **roughness** and the presence of **bends**.
- The **roughness** causes the **curvature** of vertical velocity distribution **increases**.



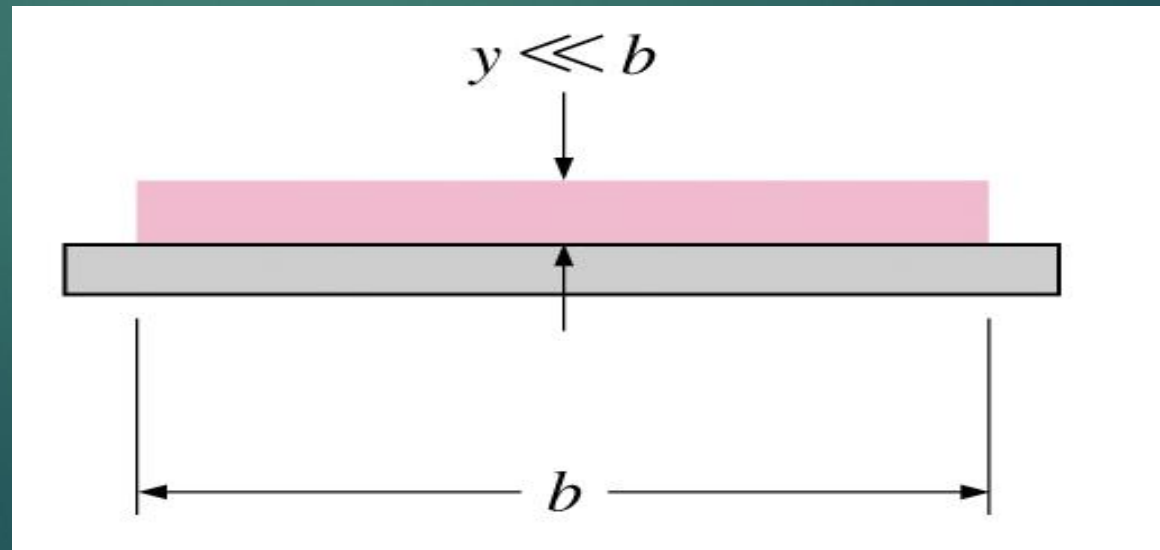
Velocity Distribution on Open Channels

- In a **wide** open channel, the sides of the channel have no influence on the velocity distribution in the central region.
- A wide channel can be defined as rectangular channel that width is greater than 10 times the depth of flow.
- Hydraulic radius for a wide channel will approximate the depth.

$$R_h = \frac{b \cdot y}{b + 2y}$$

$$b \gg y$$

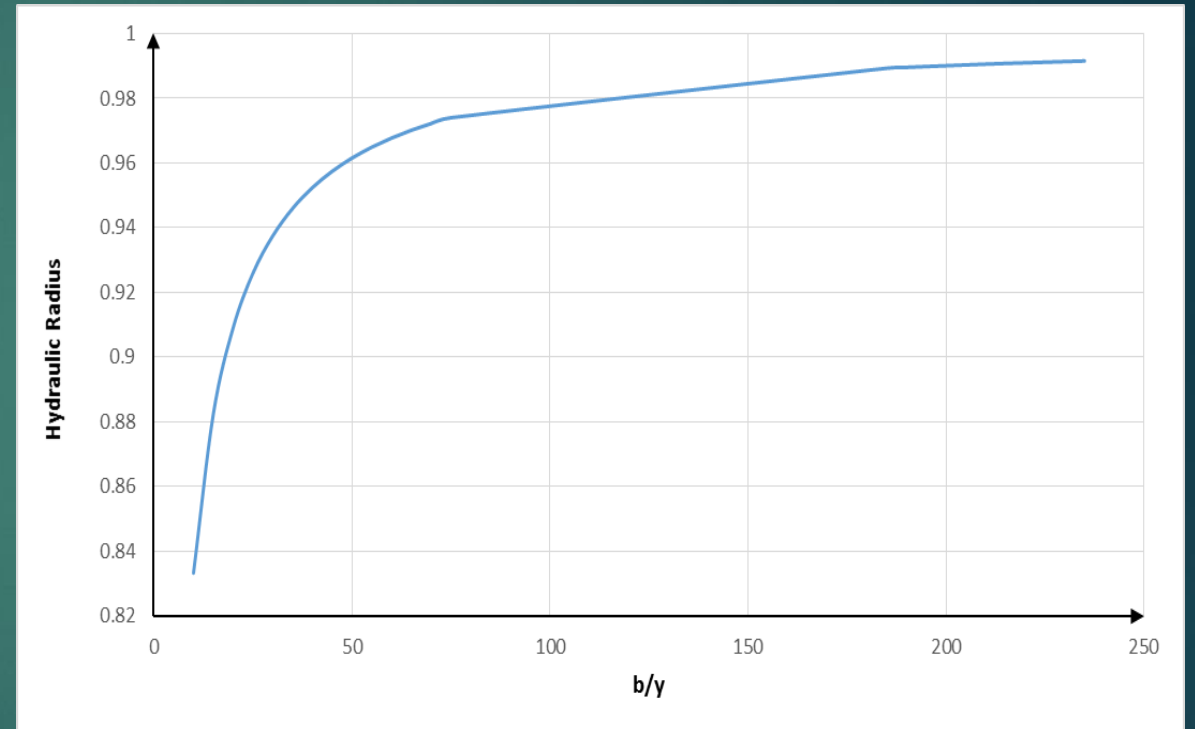
$$\frac{b}{y} > 10 \rightarrow R_h \approx y$$



Velocity Distribution on Open Channels

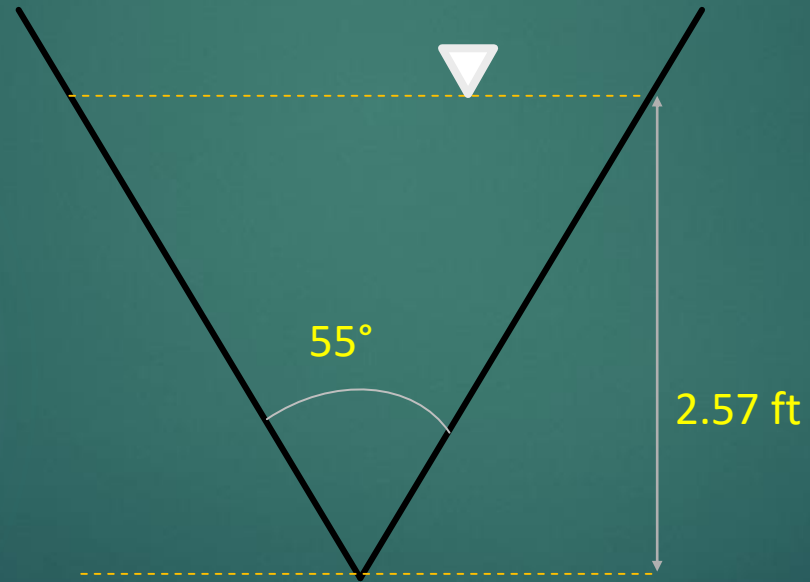
$$\frac{b}{y} = 10 \rightarrow R_h = \frac{10y^2}{12y} = 0.83y$$

$$\frac{b}{y} = 200 \rightarrow R_h = \frac{200y^2}{202y} = 0.99y$$



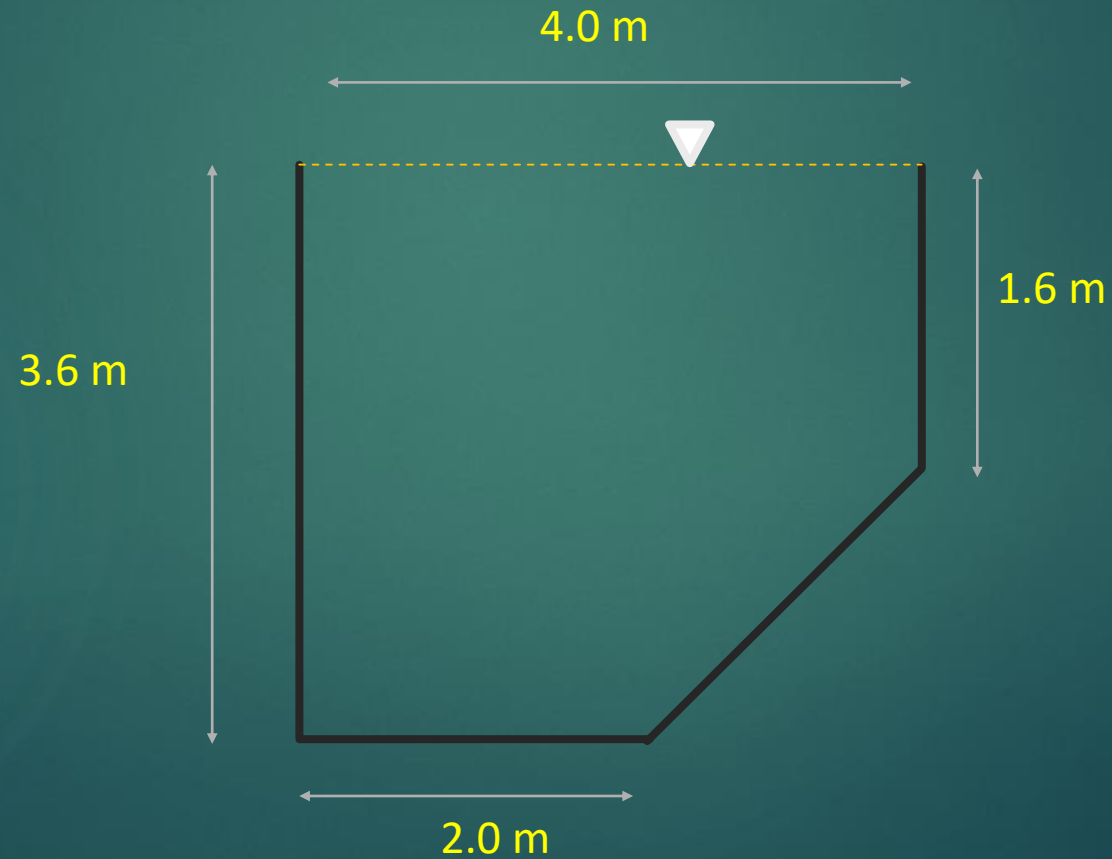
Homework 1

Q1. Determine the flow regime (Laminar, Turbulent or Transitional) in the following triangle channel which the flow velocity is 2.9 ft/s.



Homework 1

Q2. Calculate the hydraulic radius for the following channel.



Homework 1

Q3. Determine the flow condition (sub or supercritical) in a rectangular flume with the flow velocity of 1.52 m/s and width of 1.36 m, if:

- a. Ratio of flume width to flow depth is 4.0
- b. Ratio flume width to flow depth is 0.5
- c. Flume is very wide ($\frac{b}{y} \geq 10$)



APPLIED HYDRAULICS

CHAPTER 2:

OPEN CHANNEL FLOW

Open Channel Flow

- Fundamental equations
- Best hydraulic cross sections

Equations in Open Channel Flow (Continuity equation)

Continuity equation

$$Q = AV$$

Q is typically called the discharge

A The cross sectional area of flow

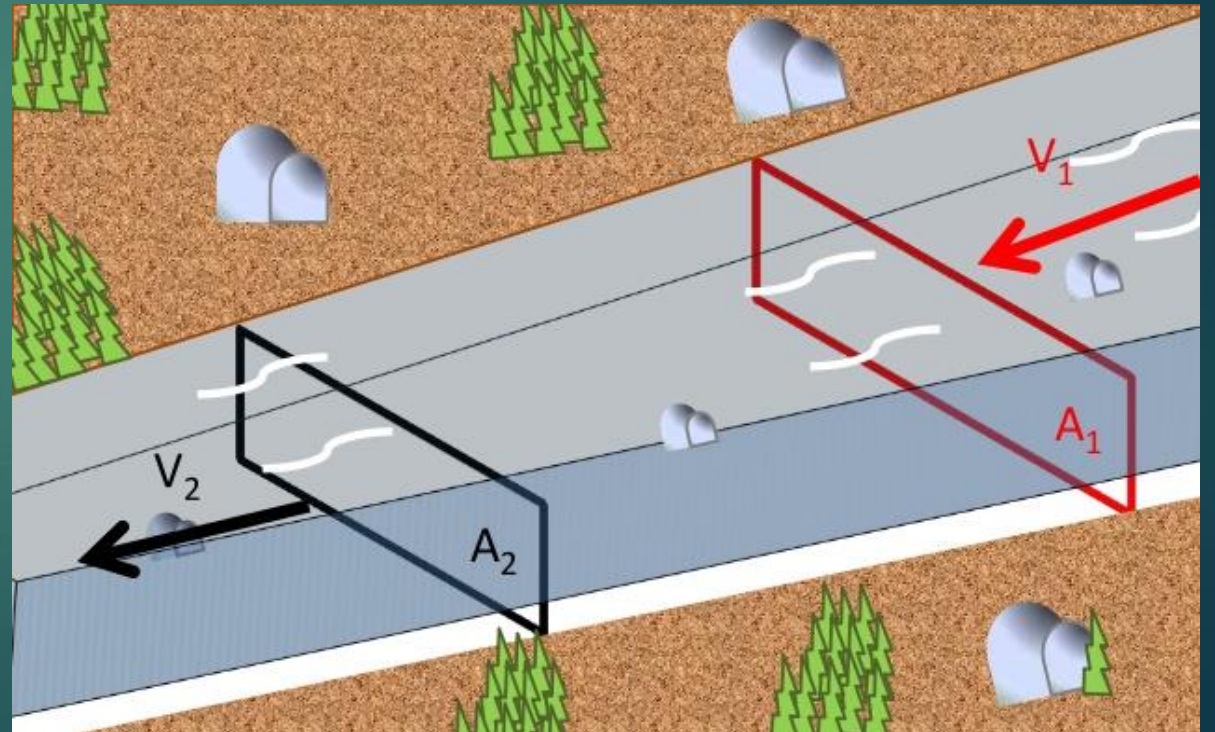
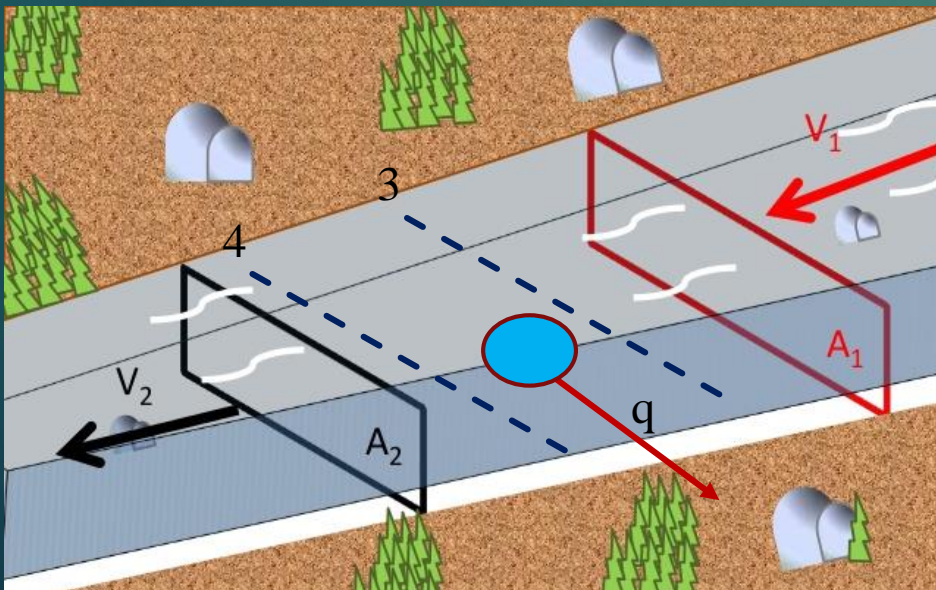
V The mean velocity

If the flow is steady, inflow is equal to the outflow.

$$Q_{entering} = Q_{leaving}$$

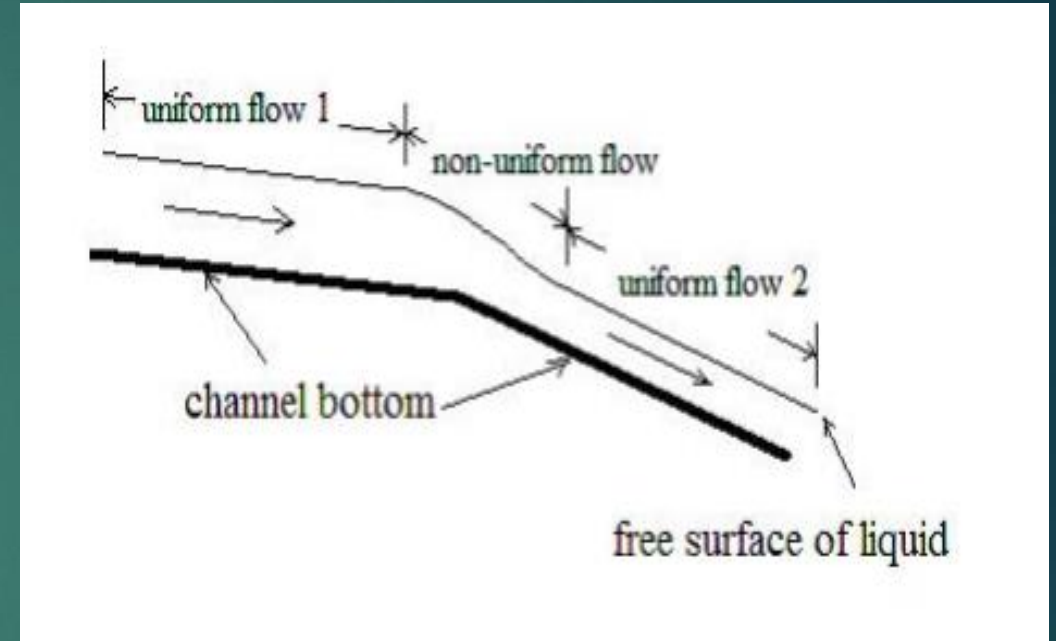


$$A_1V_1 = A_2V_2$$



Uniform Flow

- **Uniform** flow: there is a **constant flow rate** of liquid passing through it, average velocity, bottom slope, and cross-section shape & size.
- Such a channel is called prismatic channel.
- The depth in the channel with uniform flow is **normal depth** (y_n).



- For reaches of channel where the bottom slope, cross-section shape, and/or cross-section size change, **non-uniform** flow will occur.

Uniform Flow

- To have uniform flow, the channel must be **straight** and without change in slope and **cross section** along the length of the channel.
- In a uniform flow we can show that:

$$V = C\sqrt{R_h S_0}$$

C is called the Chezy C

R_h the hydraulic radius

- The Chezy coefficient **C** are function of the **roughness** of the channel bottom and wall; and the depth of flow.

Description of Channel	Chezy Coefficient
Many grove heights of flood waters	7 - 12.5
Many weeds as high as water	12.5 - 20
Base of channel is clean with a little to moderate grove on the cliff wall channel	20 - 30
Channel with a bit of short grassy weeds	30 - 45
Channel is clean and not a new channel, it has been decaying	40 - 55

Equations in Open Channel Flow (Manning equation)

Manning equation

One the most commonly empirical equations governing Open Channel Flow

$$V = \frac{1}{n} R_h^{2/3} S_0^{1/2}$$

SI system

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2}$$

$$C = \frac{R_h^{1/6}}{n}$$

n is Manning roughness coefficient

S_0 is channel slope

$$Q = \frac{1.49}{n} A R_h^{2/3} S_0^{1/2}$$

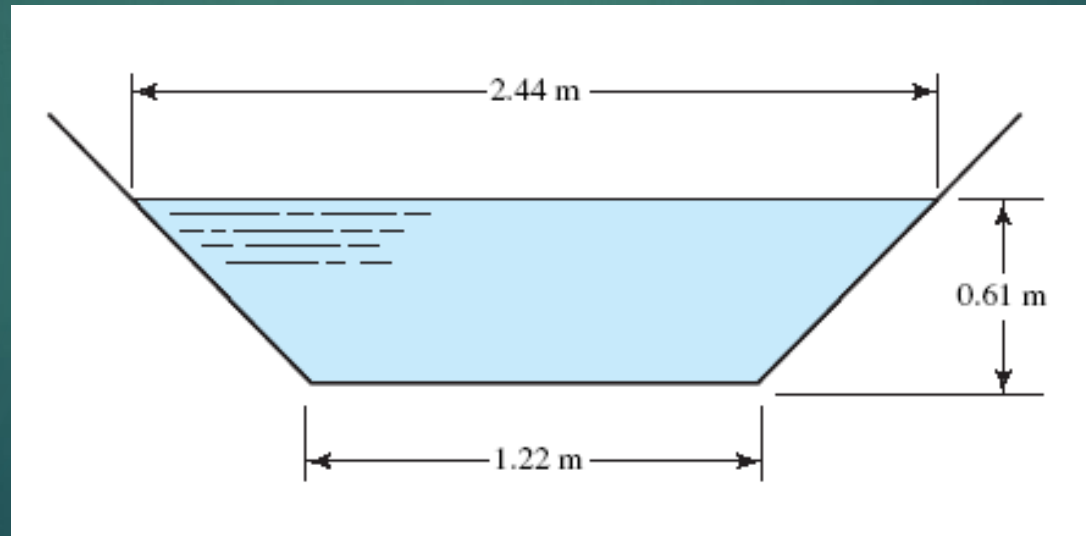
English system

Channel type	Surface material and form	Manning's n range
River	earth, straight	0.02-0.025
	earth, meandering	0.03-0.05
unlined canal	gravel (75-150mm), straight	0.03-0.04
	gravel (75-150mm), winding	0.04-0.08
lined canal	earth, straight	0.018-0.025
	rock, straight	0.025-0.045
lab. models	concrete	0.012-0.017
	mortar	0.011-0.013
	Perspex	0.009

Equations in Open Channel Flow (Manning equation)

Example 1

Calculate the **slope** on which the channel shown in the following figure must be laid if it is to carry 1.416 m³/s of water with a depth of 0.61 m. The sides and bottom of the channel are made of formed, unfinished concrete.

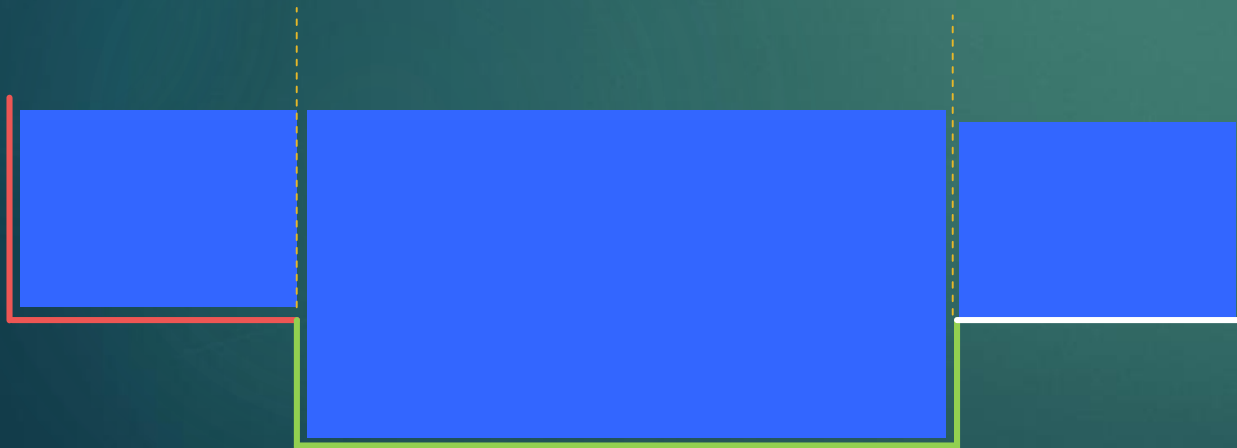
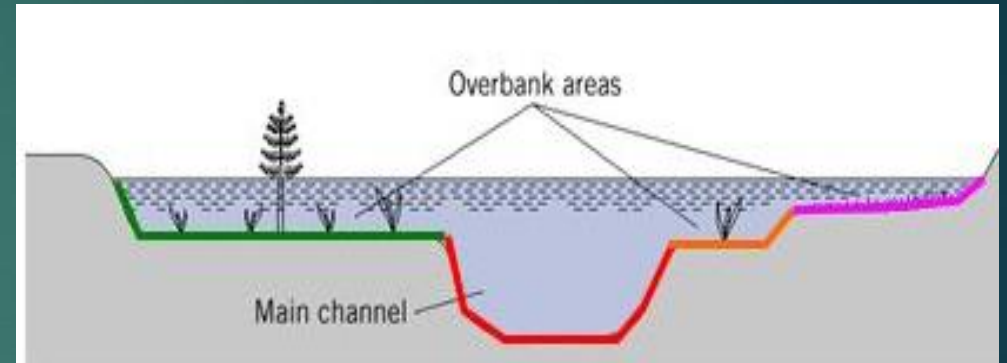
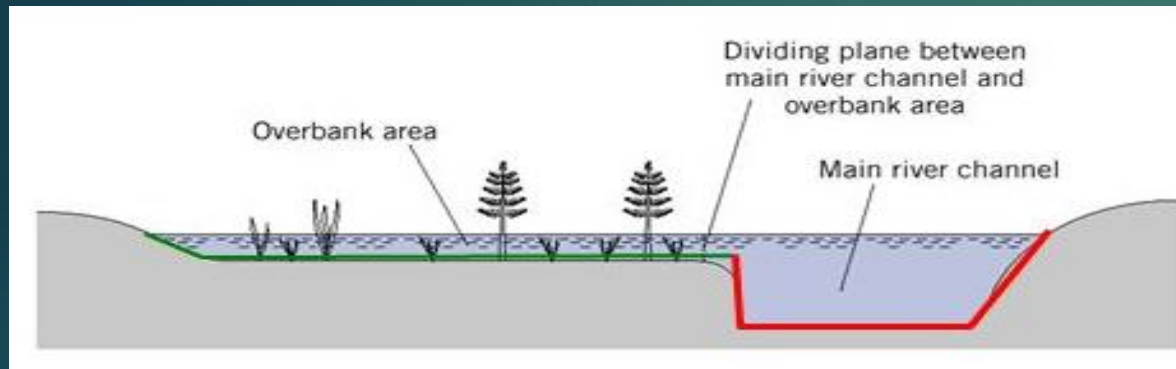


Equations in Open Channel Flow (Manning equation)

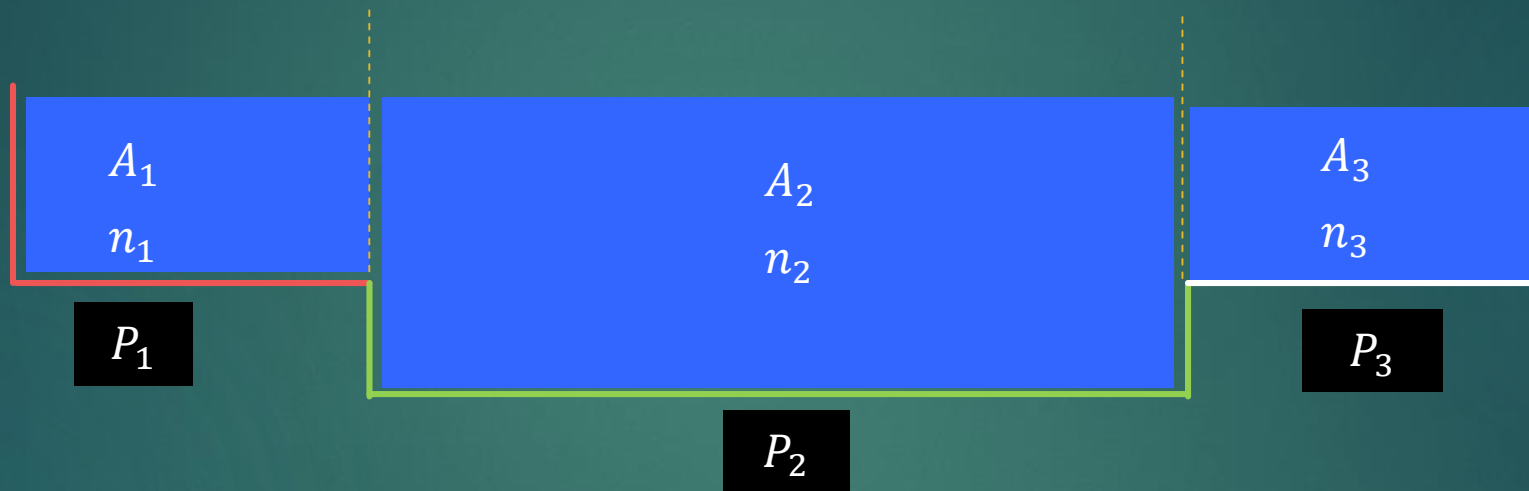
Example 2

Determine the depth of the water in a rectangular channel that is made of **unfinished concrete** with the width of **2 m** to carry **12 m³/s** of water when laid on a **1.2-percent** slope.

Equations in Open Channel Flow (Compound Channels)



Equations in Open Channel Flow (Compound Channels)

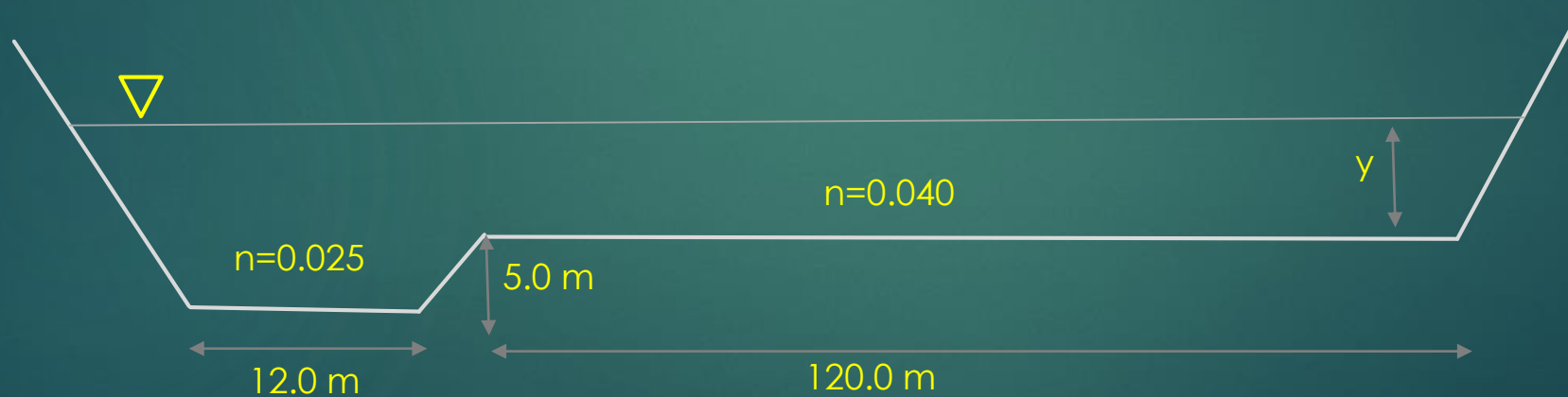


$$Q = \left[\frac{1}{n_1} A_1 R_{h1}^{2/3} S_0^{1/2} \right] + \left[\frac{1}{n_2} A_2 R_{h2}^{2/3} S_0^{1/2} \right] + \left[\frac{1}{n_3} A_3 R_{h3}^{2/3} S_0^{1/2} \right]$$

Equations in Open Channel Flow (Compound Channels)

Example 3

Determine the discharge in the following compound channel as bed slope is **0.0009**, the depth is **8.0** ft, and side slope **1:1**.



Best Hydraulic Cross Sections

- The quantity of $AR_h^{2/3}$ in the manning equation is called the section factor.
- In another words, the section factor relating to uniform flow is given by $A \left(\frac{A}{P} \right)^{2/3}$

$$Q = \left[\frac{1}{n} AR_h^{2/3} S_0^{1/2} \right] = \left[\frac{1}{n} A \left(\frac{A}{P} \right)^{2/3} S_0^{1/2} \right] = \left[\frac{1}{n} A^{5/3} \left(\frac{1}{P} \right)^{2/3} S_0^{1/2} \right]$$

- For a given roughness and slope, the **discharge** will **increase** with **increasing cross-sectional area** while **decrease** with **increasing wetted perimeter**.

Best Hydraulic Cross Sections

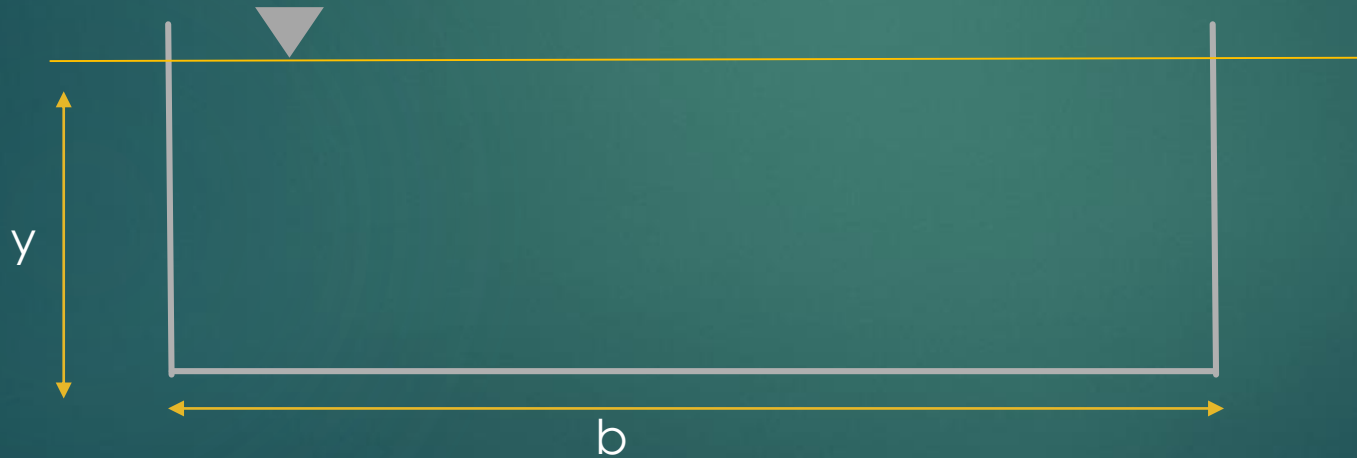
- The best hydraulic cross-section for a given \underline{A} , \underline{n} , and \underline{S}_0 is the cross-section that conveys maximum discharge.
- The minimum lining area will reduce **construction expenses** and therefore that cross-section is economically the most efficient one.

$$Q_{\max} = \underbrace{\left[\frac{1}{n} S_0^{1/2} A^{5/3} \right]}_{\text{Constant}} \left[\left(\frac{1}{P_{\min}} \right)^{2/3} \right]$$

Best Hydraulic Cross Sections

Example 4

Determine the best cross-sectional area for a rectangular channel with $Q=10 \text{ m}^3/\text{s}$, $n=0.02$, and $S_0=0.0009$.



Best Hydraulic Cross Sections

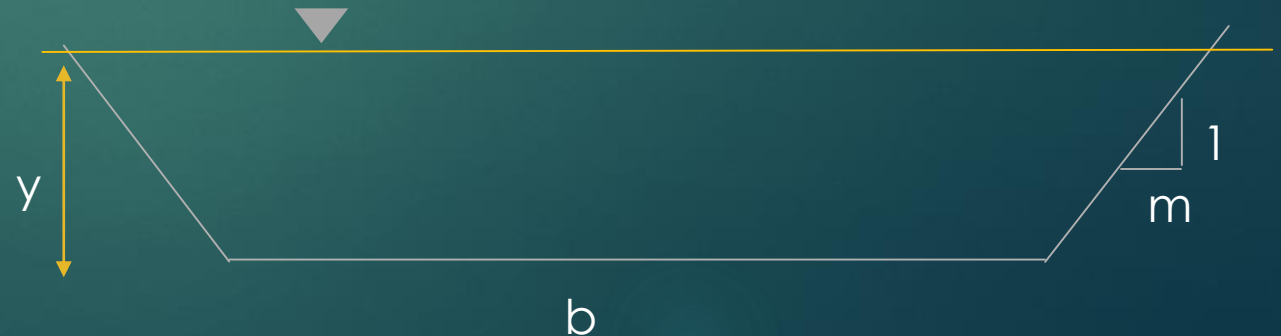
- To find the best cross-sectional area for a Trapezoidal channel, the following requirements should be met:

$$\begin{cases} P = 4y\sqrt{1+m^2} - 2my \\ m = \frac{1}{\sqrt{3}} \end{cases} \rightarrow \begin{cases} P = 2\sqrt{3}y \\ b = \frac{2\sqrt{3}}{3}y \\ A = \sqrt{3}y^2 \end{cases}$$

$$R_h = \frac{y}{2}$$

- Or, if there is a predetermined value for m , just **the first** requirement needs to be met.

$$P = 4y\sqrt{1+m^2} - 2my$$



Best Hydraulic Cross Sections

Example 5

Determine the best cross-sectional area for a trapezoidal channel with $Q=200$ m^3/s , $n=0.016$, and $S_0=0.0004$.

Best Hydraulic Cross Sections

Example 6

Determine the best cross-sectional area for a trapezoidal channel with $m=2$, $Q=20$ m^3/s , $n=0.025$, and $S_0=0.0009$.

Homework 2

Q1. Water flows in a rectangular channel which are made of concrete with the width of 12 m and depth of 2.5 m. The bottom slope is 0.0028 m/m. Find the velocity and flow rate.

Homework 2

Q2. Water flows in a circular channel which are made of unfinished concrete with the diameter of 500 mm and the bottom slope of 0.005 m/m. Find the velocity and flow rate if the channel is half full.

Homework 2

Q3. Water flows in a rectangular channel with the width of 18 m, flow rate of 35 m³/s, $n = 0.011$ and the bottom slope of 0.00078 m/m. Find the depth.



APPLIED HYDRAULICS

CHAPTER 3:

OPEN CHANNEL FLOW

Open Channel Flow

- Energy Equation
- Critical flow and Velocity
- Specific Energy

Equations in Open Channel Flow (Energy equation)

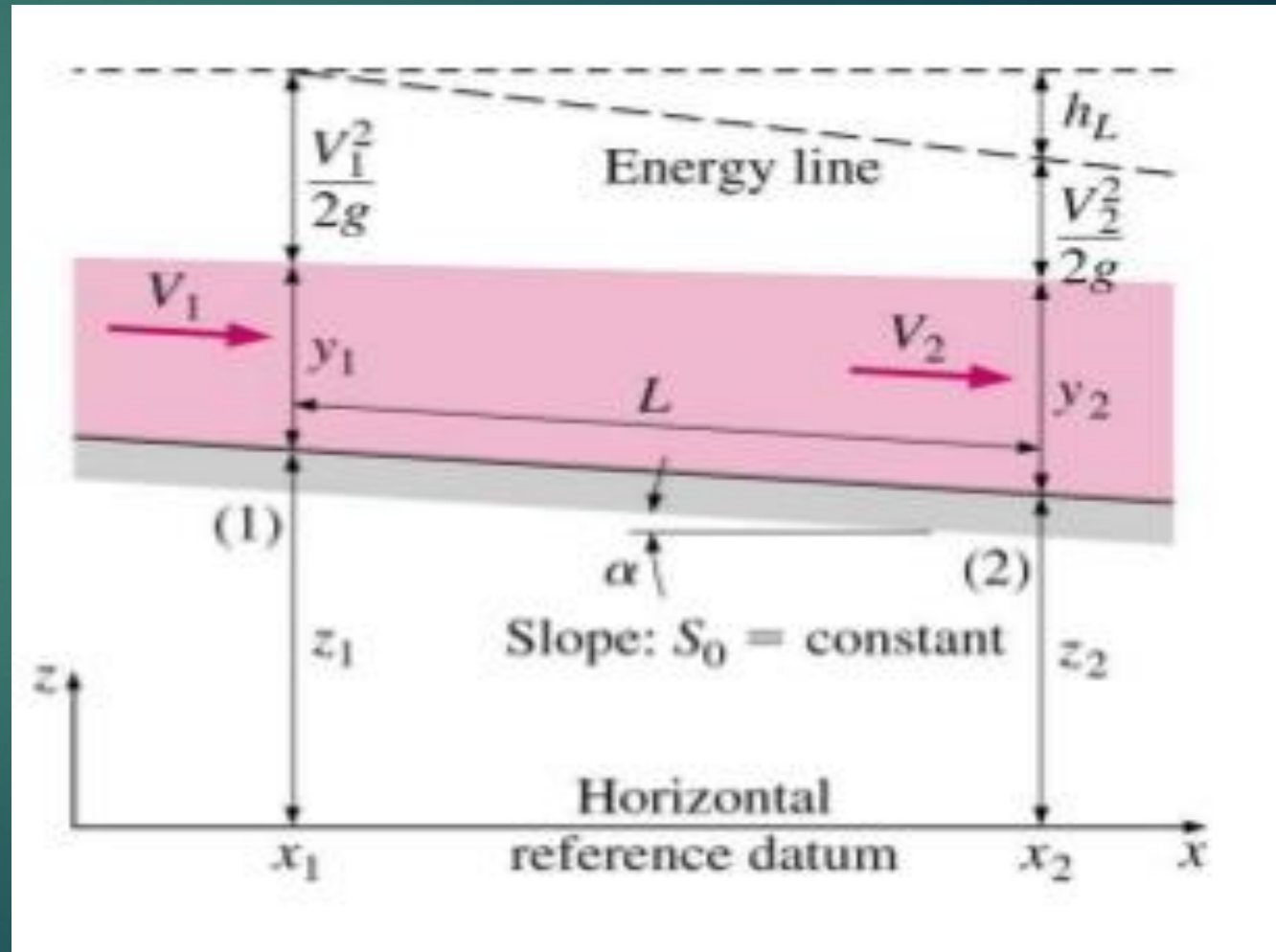
Energy equation

$$\frac{V_1^2}{2g} + y_1 + Z_1 = \frac{V_2^2}{2g} + y_2 + Z_2 + h_L$$

$\frac{V^2}{2g}$ is the velocity head

h_L is the head loss

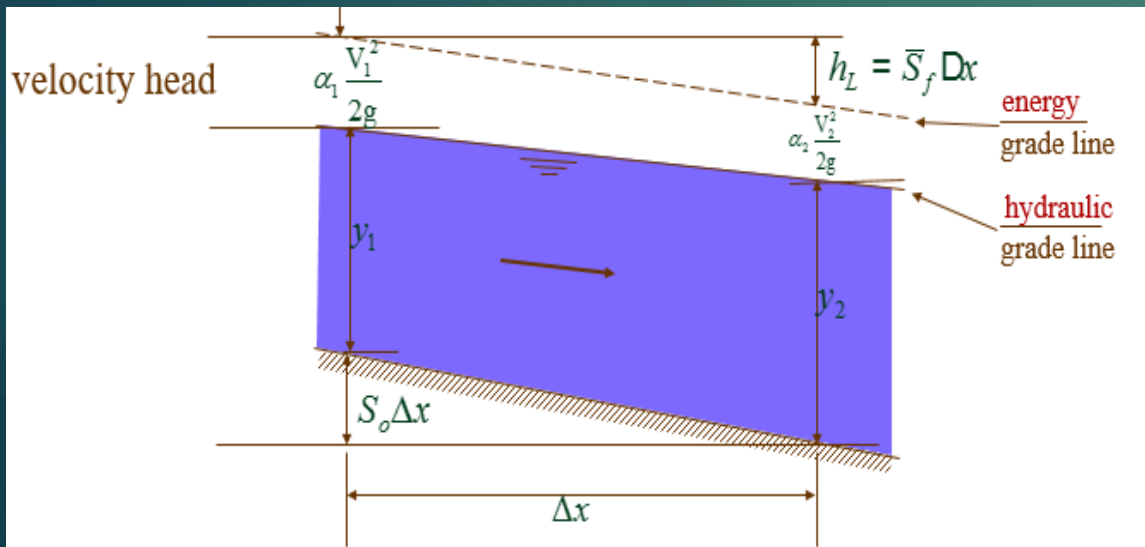
Z is the static head



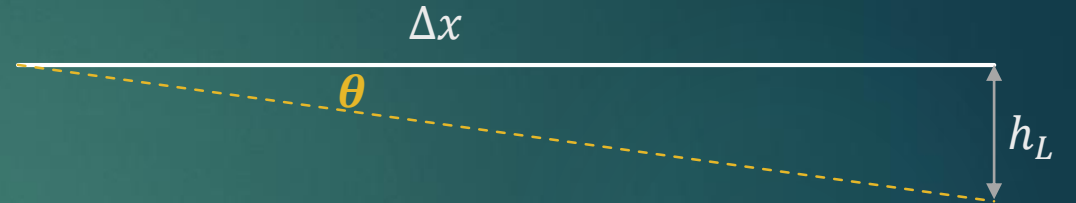
Equations in Open Channel Flow (Energy equation)

Energy equation

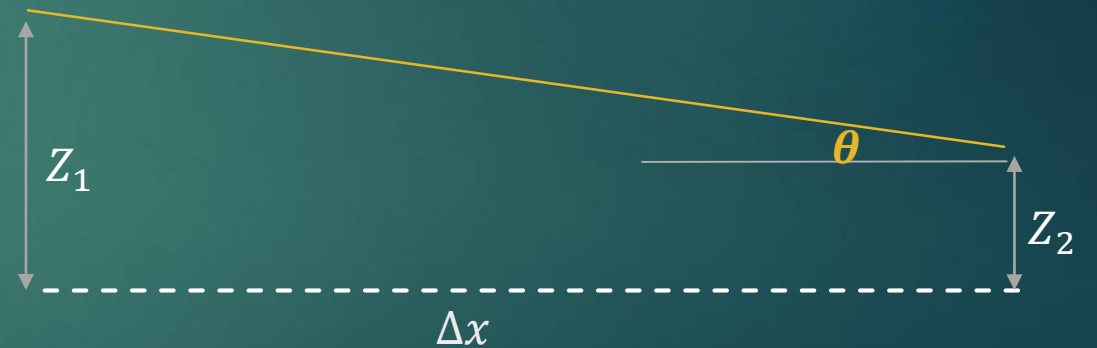
$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + (S_f - S_0)\Delta x$$



Bottom slope (S_0) not necessarily equal to EGL slope (S_f)



$$S_f = \tan \theta = \frac{h_L}{\Delta x} \rightarrow h_L = S_f \Delta x$$

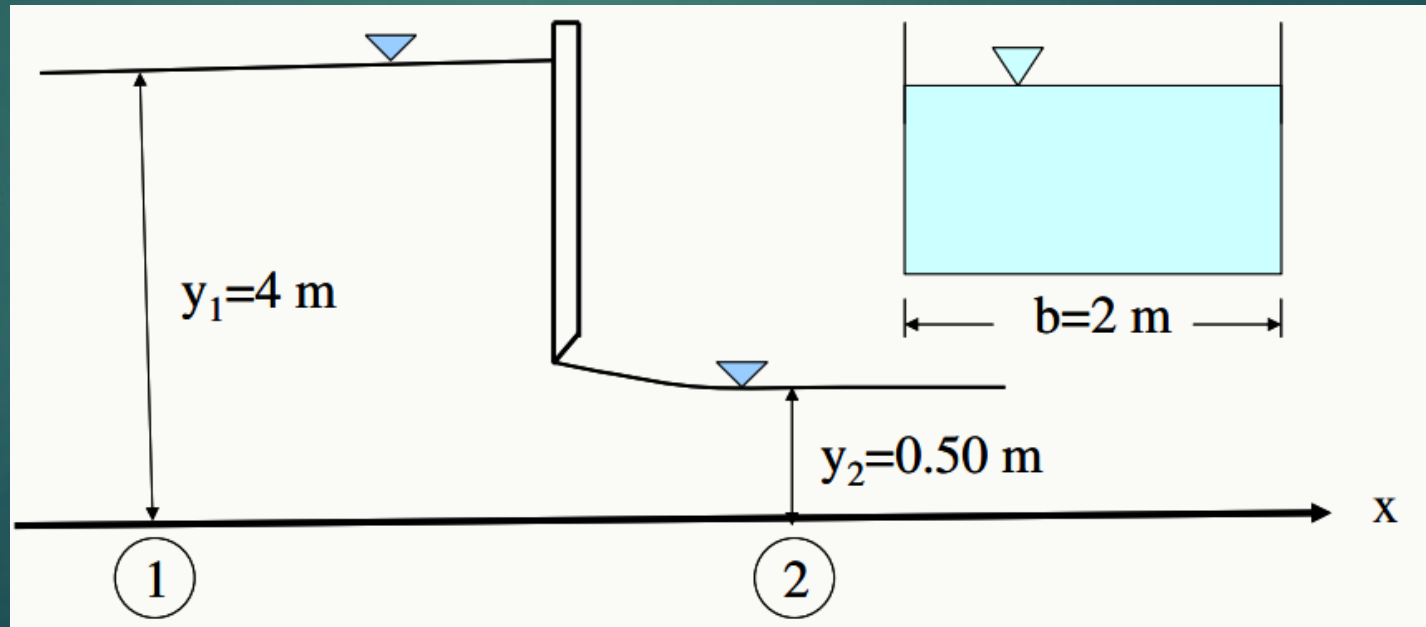


$$\tan \theta = \frac{Z_1 - Z_2}{\Delta x} = S_0 \rightarrow Z_1 - Z_2 = S_0 \Delta x$$

Equations in Open Channel Flow (Energy equation)

Example 7

Water flows under a sluice gate in a horizontal rectangular channel of **2 m** wide. If the depths of flow before and after the gate are **4 m**, and **0.50 m**, compute the discharge in the channel (no head loss).

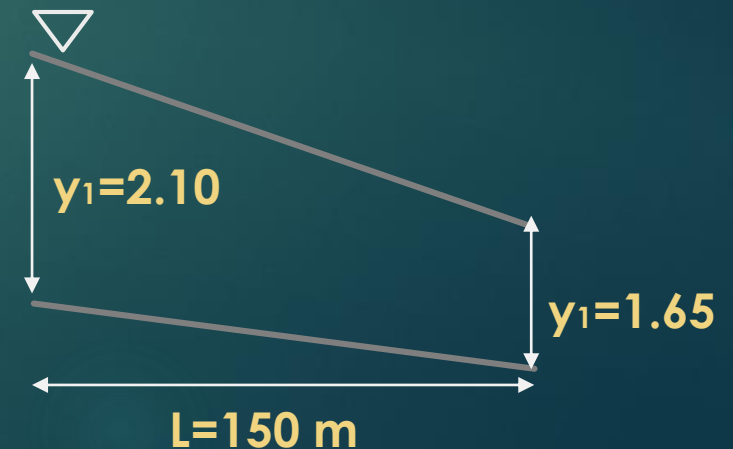
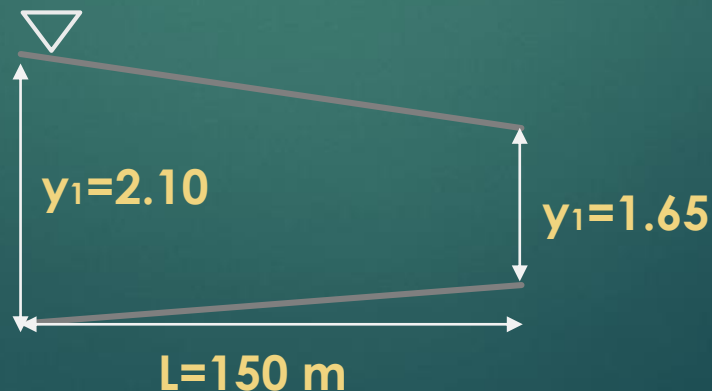
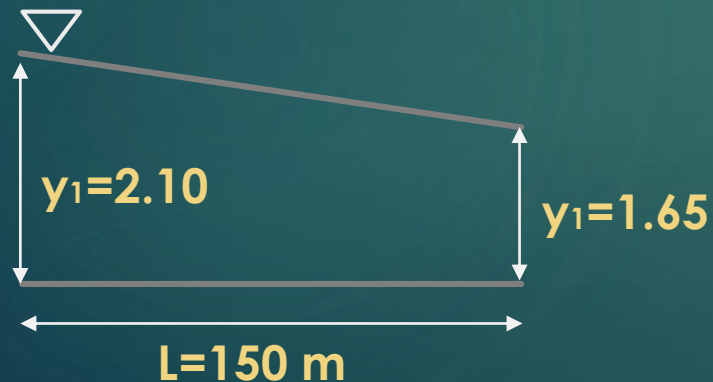


Equations in Open Channel Flow (Energy equation)

Example 8

Determine the head loss in a rectangular open channel with the width of **3 (m)** and flow rate of **8.5 (m³/s)** if:

- The bed slope is **0.0**
- The bed slope increasing in flow direction and it is **0.2 m** in **100 m**
- The bed slope decreasing in flow direction and it is **0.2 m** in **100 m**

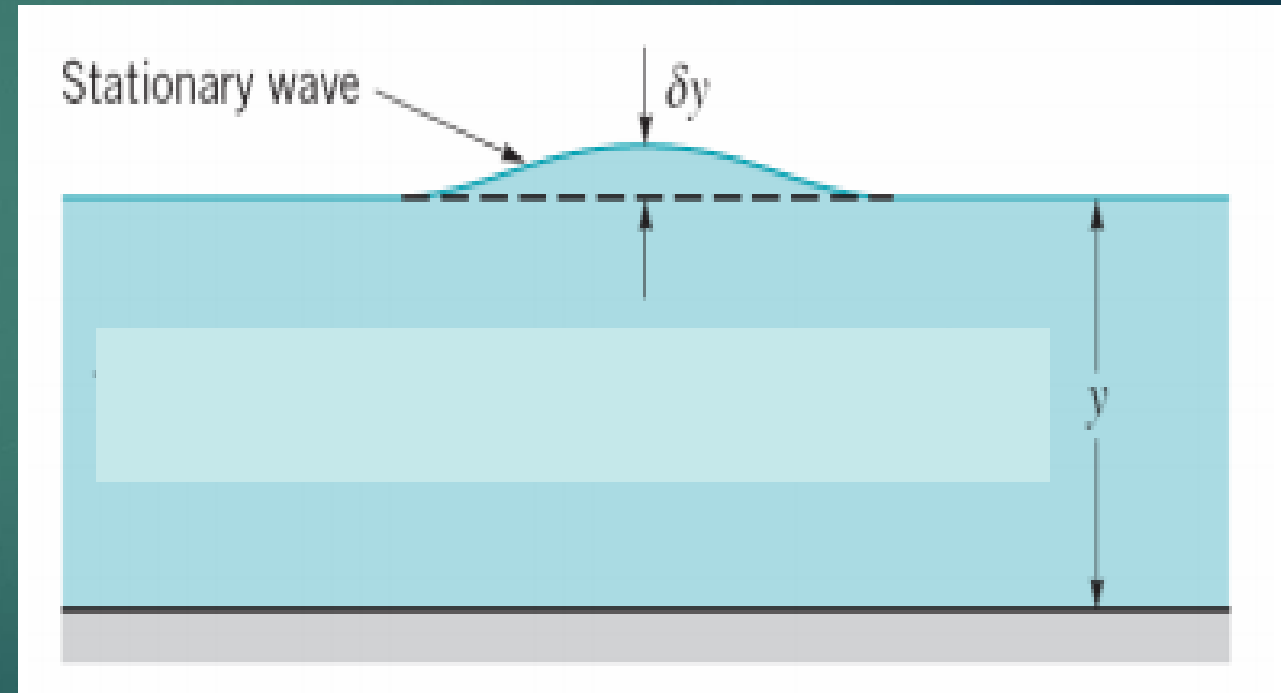
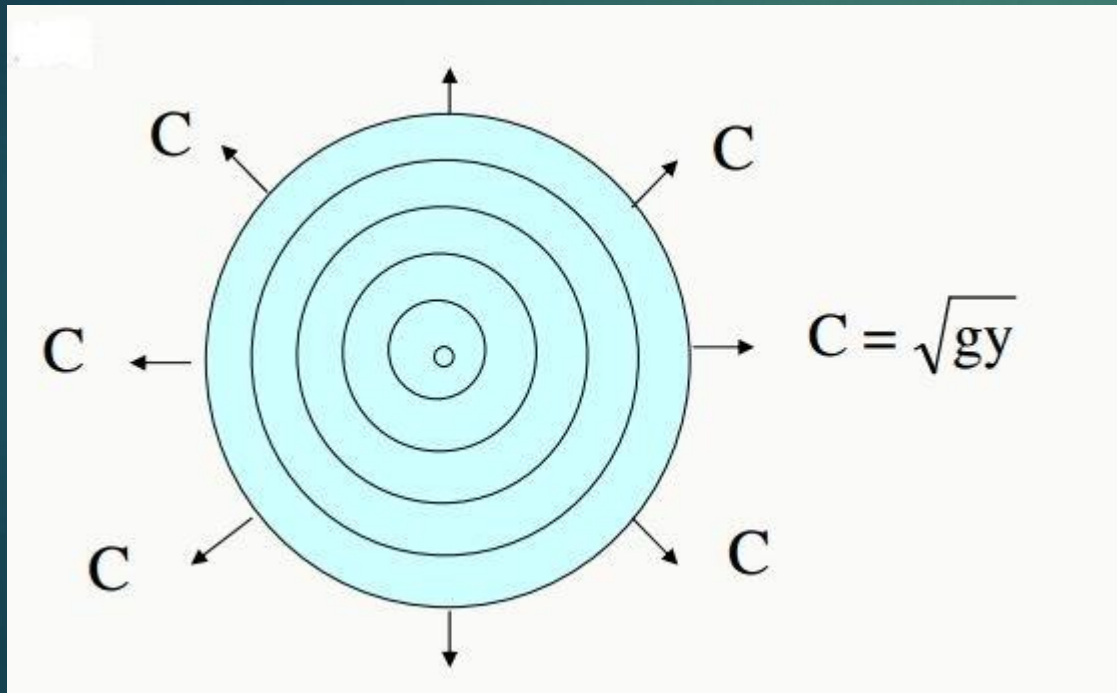


Critical Flow and Velocity

- Critical flow is the dividing point between the **subcritical** flow regime, where normal depth is greater than critical depth, and the **supercritical** flow regime (rapid flow), where normal depth is less than critical depth.
- Critical depth is the **depth of water at critical flow**, a very **unstable** condition.
- A characteristic of critical depth flow is often a **series of surface undulations** over a very short stretch of channel.
- Before finalizing a channel design, the designer must verify that the **normal depth** of a channel is either greater than or less than the **critical depth**.

Critical Flow and Velocity

- The wave speed (C) is: $C = \sqrt{gy}$



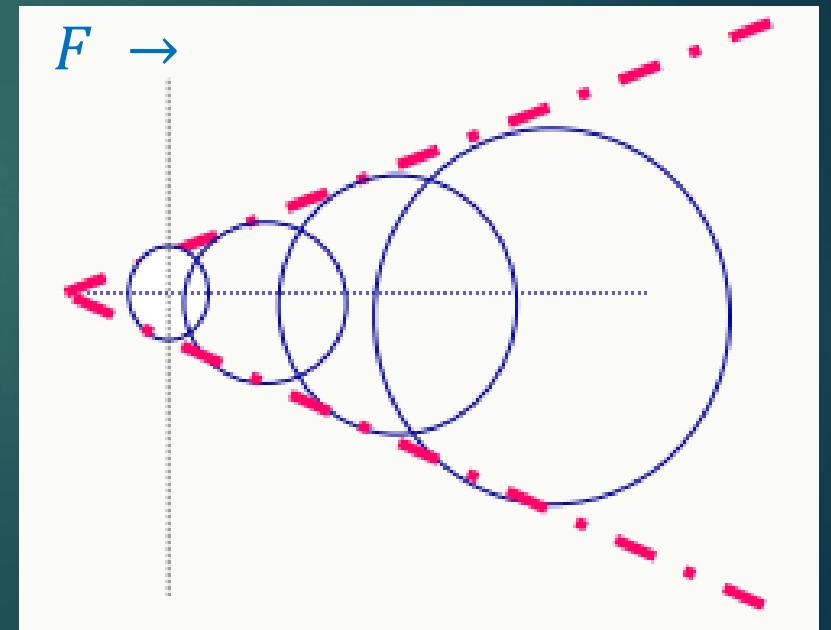
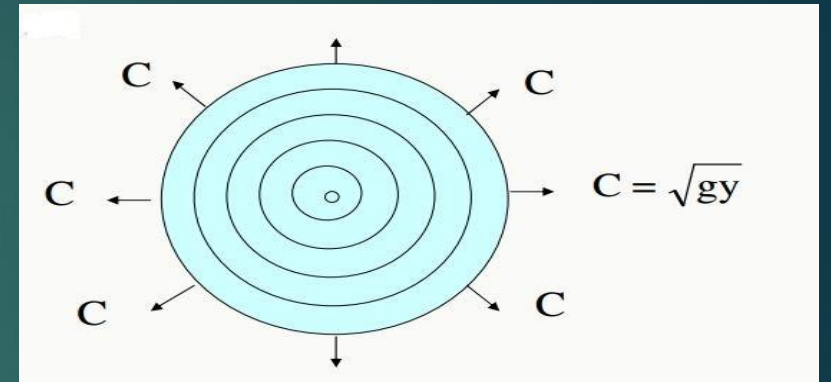
Critical Flow and Velocity

- For a wide rectangular channel, the hydraulic depth, $R_h=y$. Therefore, Froude number becomes:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{V}{C}$$

Super Critical $\rightarrow Fr > 1 \rightarrow V > C$

- Since $V > C$, it CANNOT propagate upstream it can propagate only **towards downstream**.
- This means the flow at **upstream will not be affected**.
- In other words, there is no hydraulic communication between upstream and downstream flow.



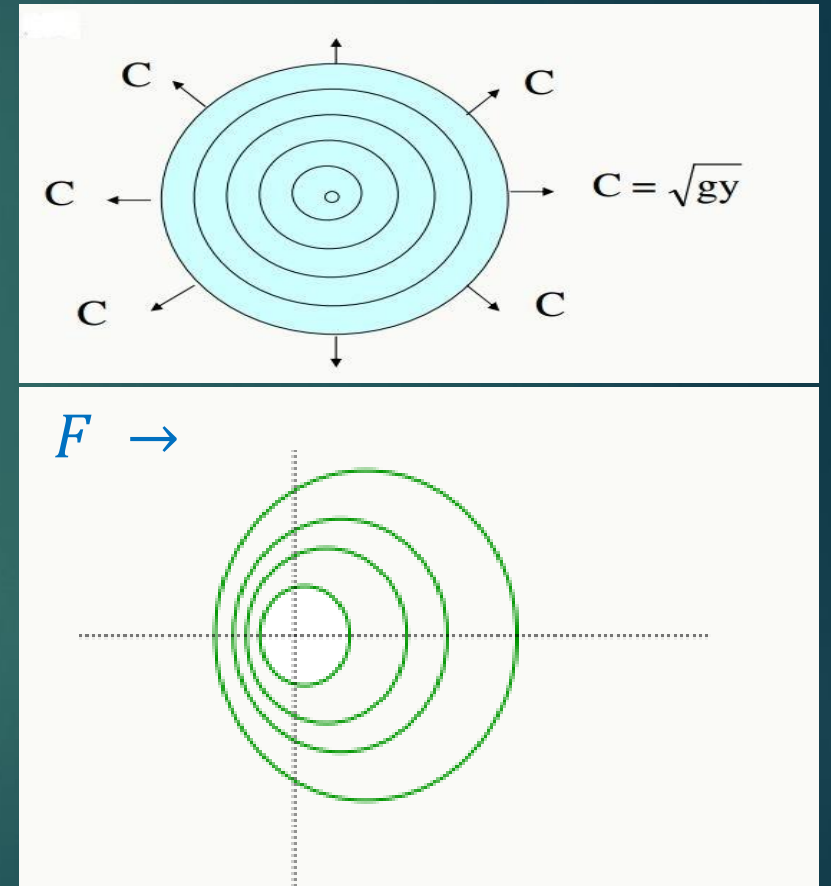
Critical Flow and Velocity

- For a wide rectangular channel:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{V}{C}$$

Subcritical \rightarrow $Fr < 1 \rightarrow V < C$

- Since $V < C$, it CAN propagate **both upstream and downstream**.
- This means the flow at upstream and downstream will **both be affected**.
- In other words, there is hydraulic communication between upstream and downstream flow.



Critical Flow and Velocity

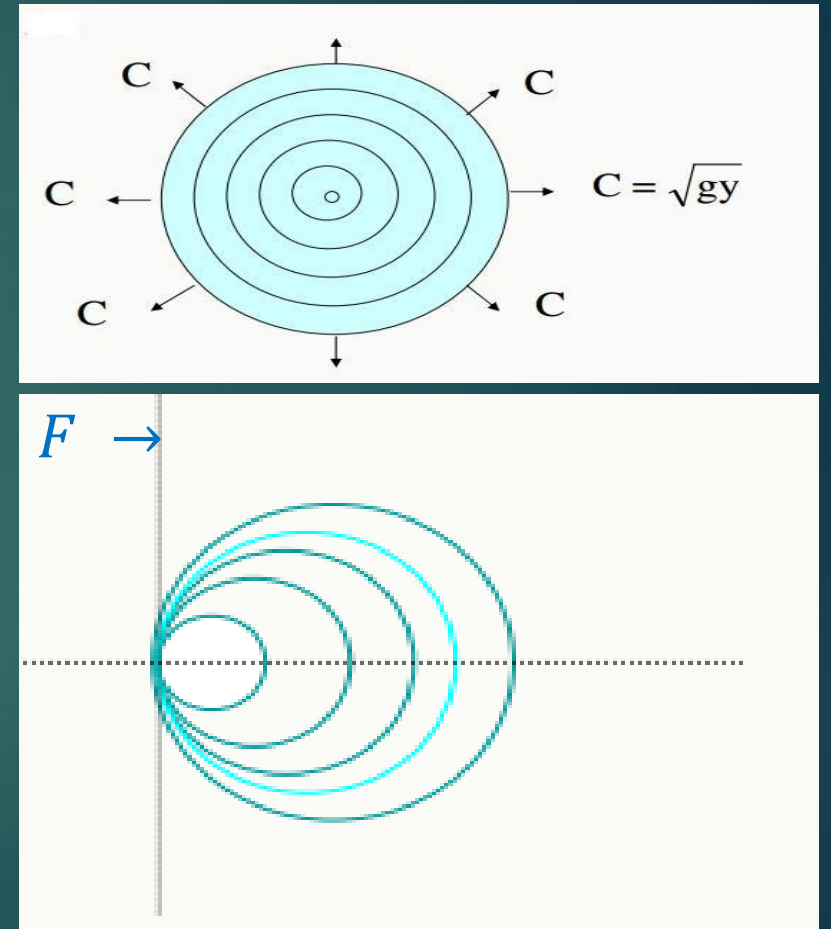
- For a wide rectangular channel:

$$Fr = \frac{V}{\sqrt{gy}} = \frac{V}{C}$$

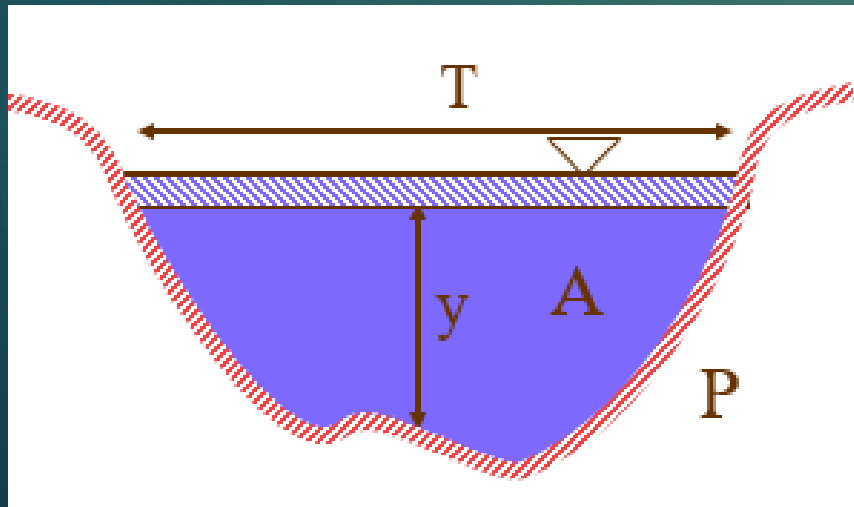
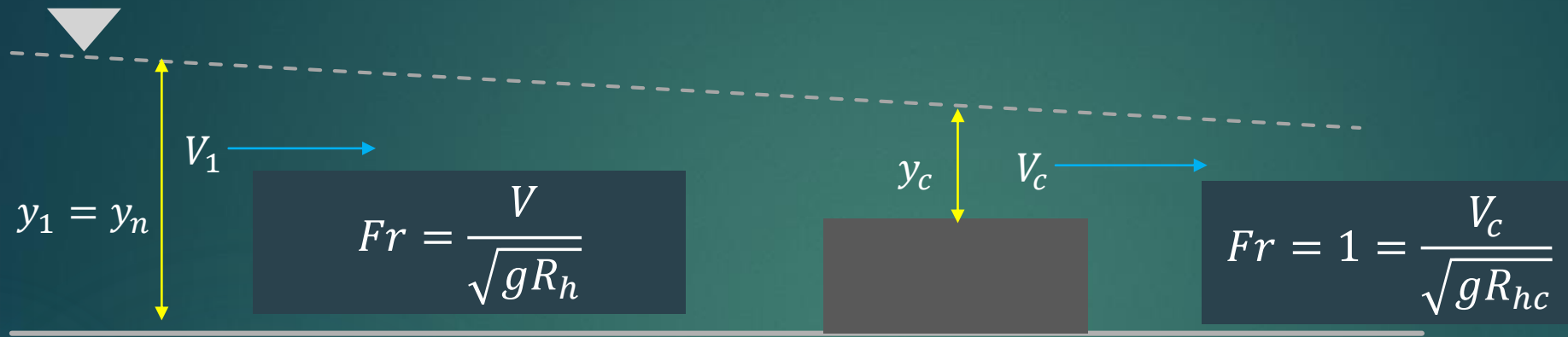
Critical \rightarrow $Fr = 1 \rightarrow V = C$

- Since $V = C$, it CAN propagate **only downstream**.

- This means the flow at downstream will **both be affected**.



Critical Flow and Velocity



$$\frac{Q^2 T}{g A^3} = Fr^2$$

$$\frac{Q^2 T_c}{g A^3} = 1$$

T =surface width

Critical Flow and Velocity

- Critical flow characteristics:

Unstable surface

Series of stationary or standing waves

Observed standing waves on the surface of a liquid in a vibrating container

- Occurrence

Broad crested weir (and other weirs)

Channel Controls (rapid changes in cross-section)

Over falls

Changes in channel slope from mild to steep

- Used for flow measurements

Unique relationship between depth and discharge

Critical Flow and Velocity

In a **rectangular** channel, we have:

$$\frac{Q^2 T_c}{g A^3} = 1$$



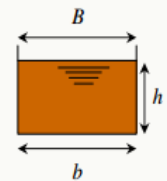
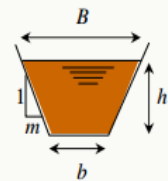
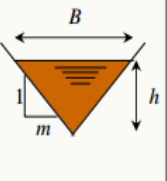
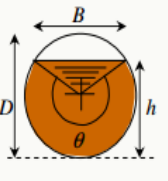
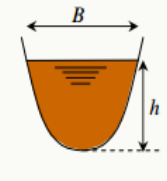
$$y_c = \left(\frac{q^2}{g} \right)^{1/3}$$

$$q \text{ (m}^2\text{/s)} = \frac{Q}{b = T}$$

Discharge per unit width

$$q = \sqrt{g y_c^3}$$

$$\frac{y_c}{2} = \frac{V_c^2}{2g}$$

	<i>rectangular</i>	<i>trapezoidal</i>	<i>triangular</i>	<i>circular</i>	<i>parabolic</i>
					
<i>flow area</i> <i>A</i>	bh	$(b + mh)h$	mh^2	$\frac{1}{8}(\theta - \sin \theta)D^2$	$\frac{2}{3}Bh$
<i>wetted perimeter</i> <i>P</i>	$b + 2h$	$b + 2h\sqrt{1 + m^2}$	$2h\sqrt{1 + m^2}$	$\frac{1}{2}\theta D$	$B + \frac{8}{3}\frac{h^2}{B}$ *
<i>hydraulic radius</i> <i>R_h</i>	$\frac{bh}{b + 2h}$	$\frac{(b + mh)h}{b + 2h\sqrt{1 + m^2}}$	$\frac{mh}{2\sqrt{1 + m^2}}$	$\frac{1}{4}\left[1 - \frac{\sin \theta}{\theta}\right]D$	$\frac{2B^2 h}{3B^2 + 8h^2}$ *
<i>top width</i> <i>B</i>	b	$b + 2mh$	$2mh$	$\frac{(\sin \theta / 2)D}{\text{or } 2\sqrt{h(D - h)}}$	$\frac{3}{2}Ah$
<i>hydraulic depth</i> <i>D_h</i>	h	$\frac{(b + mh)h}{b + 2mh}$	$\frac{1}{2}h$	$\left[\frac{\theta - \sin \theta}{\sin \theta / 2}\right]\frac{D}{8}$	$\frac{2}{3}h$

Energy Equation and Critical Flow

Example 9

Determine the critical depth and critical velocity of flow in a rectangular channel with the width of **2 m** and flow rate of **8 m³/s**.

Energy Equation and Critical Flow

Example 10

Determine the critical depth and critical velocity of flow in a trapezoidal channel with the width of 2 m, side slope 1:2, and flow rate of 8 m³/s.

Specific Energy

$$E = \frac{V^2}{2g} + y$$

E known as **specific energy**, (total energy per unit weight measured above bed level)

For a given discharge Q , the velocity is Q/A . Then:

$$E = \frac{Q^2}{2gA^2} + y$$

For a given specific energy, we have:

$$\underbrace{\frac{V_1^2}{2g} + y_1}_{E_1} + Z_1 = \underbrace{\frac{V_2^2}{2g} + y_2}_{E_2} + Z_2$$



$$E_1 - E_2 = \underbrace{Z_2 - Z_1}_{\Delta Z}$$

Specific Energy

If there would be critical depth, then

$$E_1 - E_c = \Delta Z_c$$

In a rectangular channel, we would have $Q = bq$ and $A = by$

$$E = \frac{q^2}{2gy^2} + y$$

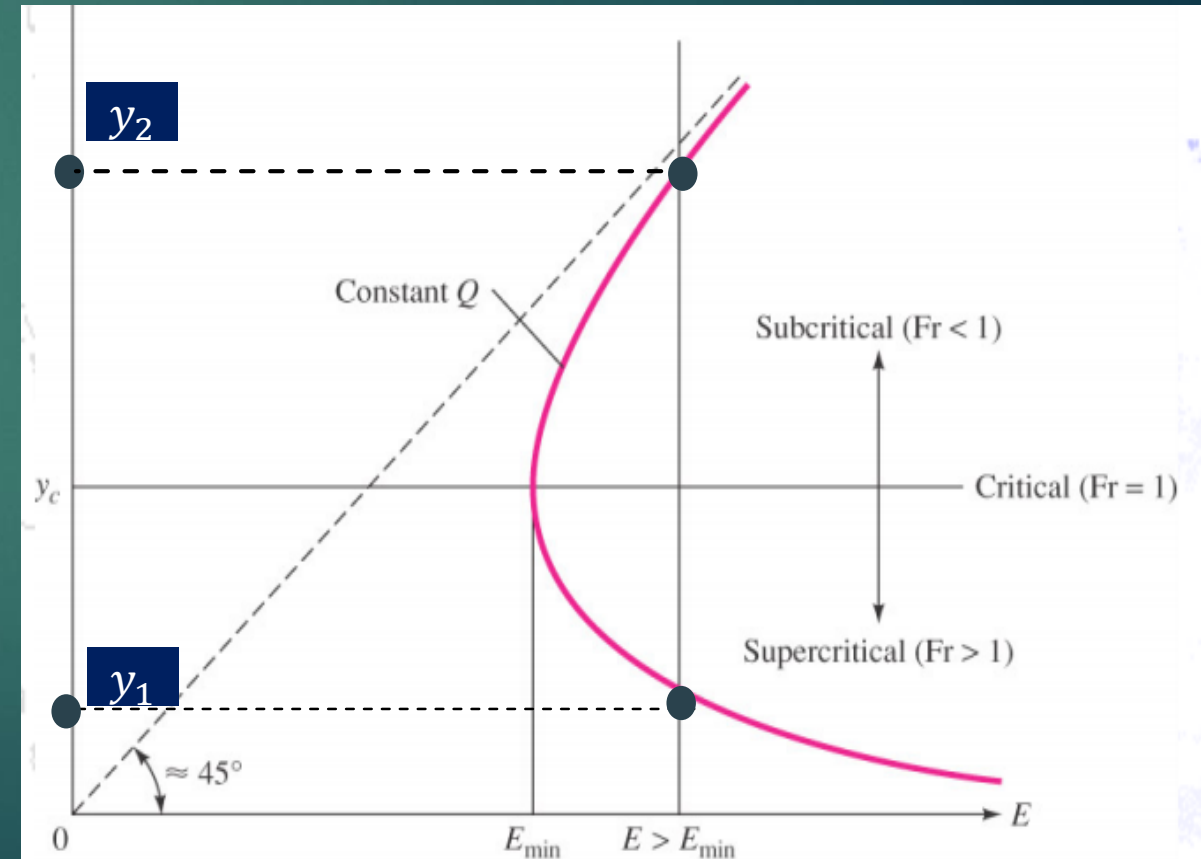
In a rectangular channel when there is critical flow, we have:

$$y_c = \frac{2}{3}E_c = \frac{2}{3}E_{min}$$

Specific Energy

- How many depths given a specific energy?
3 (in which one of them is **negative**)
- How many possible depths given a specific energy? **2**
- The specific energy reaches a **minimum** value E_s , called the critical point, characterized by the critical depth y_c and critical velocity V_c .
- y_1 and y_2 are **Alternate** depths (same specific energy)

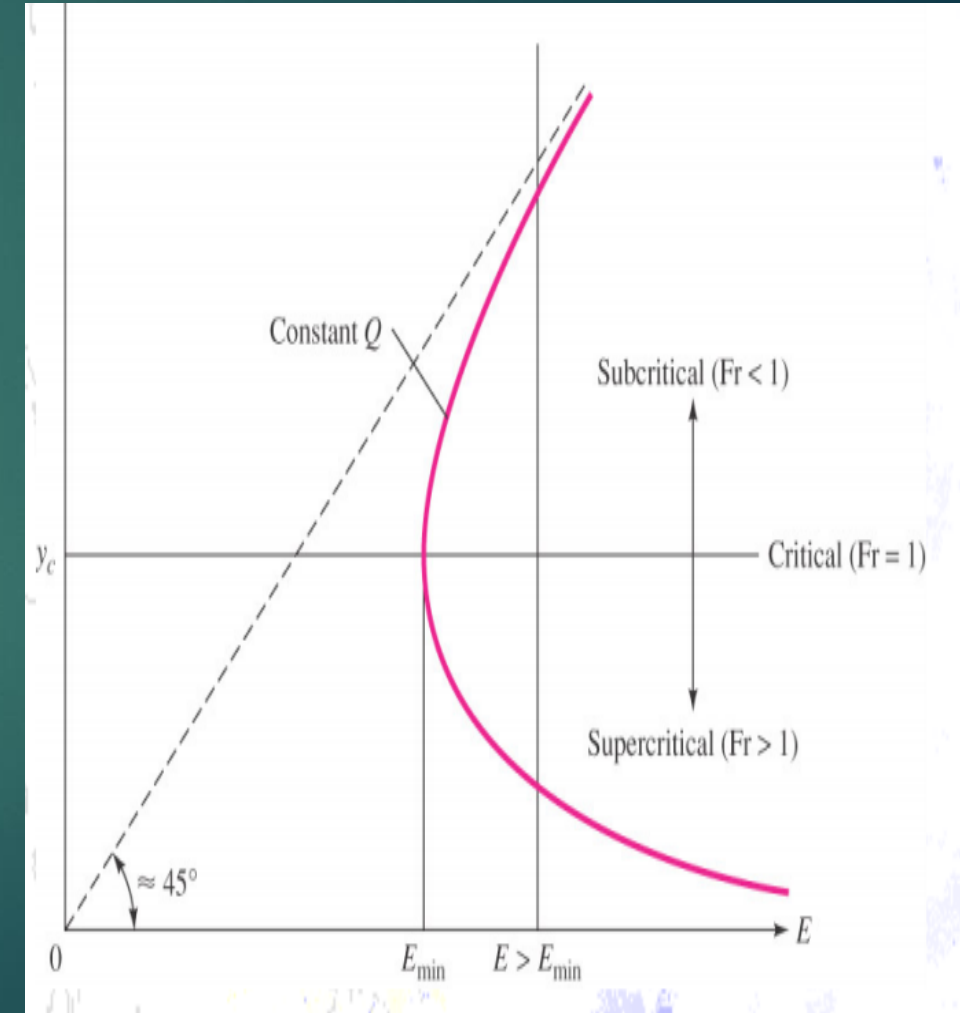
$$E = \frac{Q^2}{2gA^2} + y$$



Specific Energy

Upper part of the curve

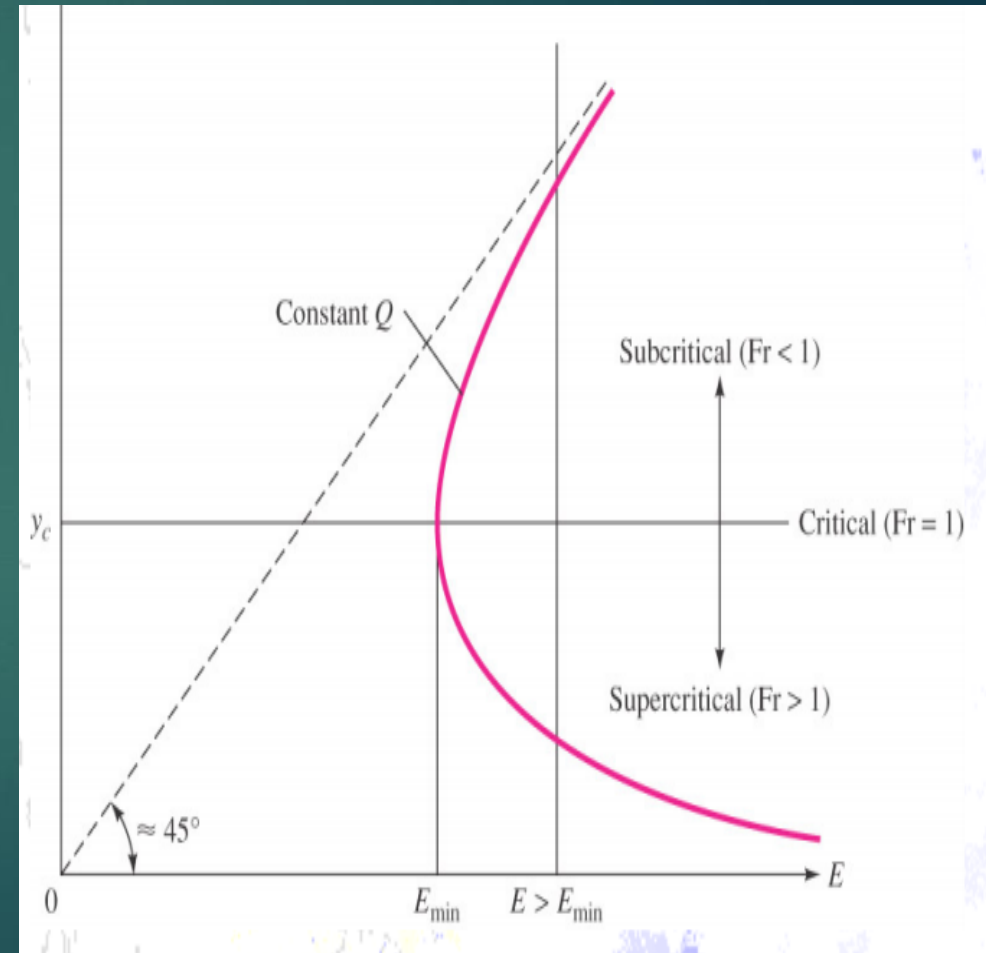
- If the value of “ E ” increases on the upper part of the curve, then “ y ” increases.
- For upper part, “ y ” is greater than “ y_c ”.
- For upper part of the curve, velocity is **less** than critical velocity.
- The flow in this portion is termed as *sub-critical flow*.
- The channel is called as deep channel for sub-critical flow.



Specific Energy

Lower part of the curve

- If the value of “ E ” **increases**, we can see that the value of “ y ” **decreases** in lower part of the curve.
- For lower part of the curve, “ y ” is less than “ y_c ”.
- For lower part of the curve, velocity is **greater** than critical velocity.
- For lower part of the curve, the flow is termed as *super critical flow*.
- The channel is called as shallow channel for super critical flow.

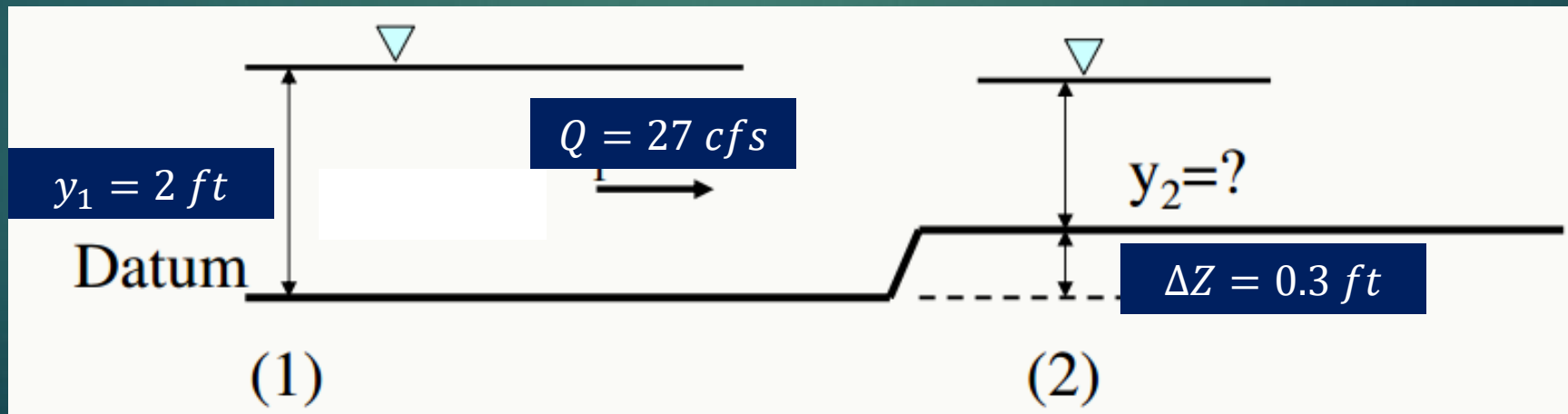


Energy Equation and Critical Flow

Example 11

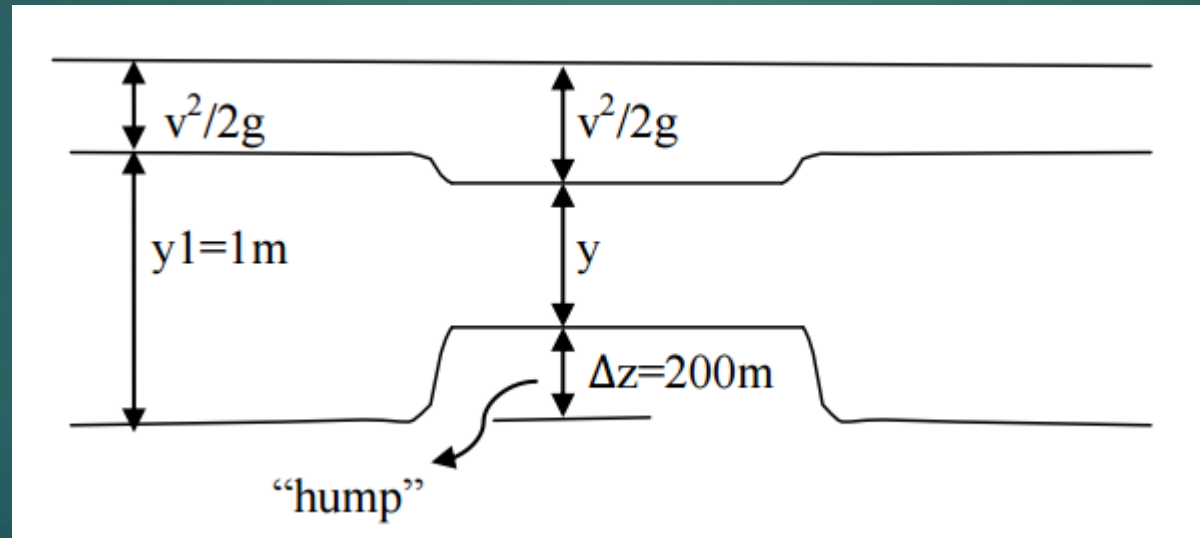
Water flow in a rectangular channel with the flow rate of $27 \text{ ft}^3/\text{s}$, depth of 2 ft , and width of 4 ft .

- Determine the flow condition (sub or super critical flow)
- If there is an upward step of 0.3 ft (as shown in figure below), what would be the water depth on the upward step (assume there is no head loss)?



Homework 3

Q1. A rectangular channel $b = 1.5\text{m}$, $Q = 900\text{L/s}$, the depth of flow before the hump is 1m and $\Delta z = 200\text{mm}$, compute the depth of flow above the hump.



Homework 3

Q2. A rectangular channel $b = \underline{2.0 \text{ m}}$, $Q = \underline{2 \text{ m}^3/\text{s}}$, the depth of uniform flow before the hump is 0.8 m. What should be the height of the hump (Δz) to have critical flow over it (no head loss).

Homework 3

Q3. What is the critical depth and critical velocity in trapezoidal channel with the side slope of 1:2, width of 3 m, and flow capacity of 20 m³/s.



APPLIED HYDRAULICS

CHAPTER 4:

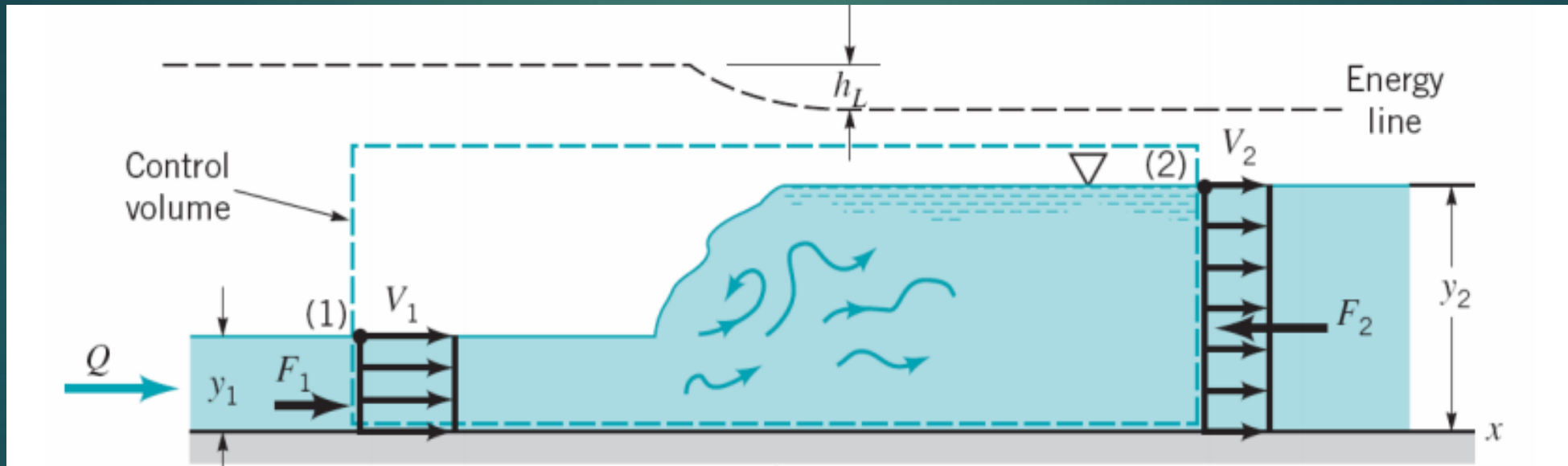
OPEN CHANNEL FLOW

Open Channel Flow

- Hydraulic jump
- Design of lined and unlined channels
- Flow Control and Measurement

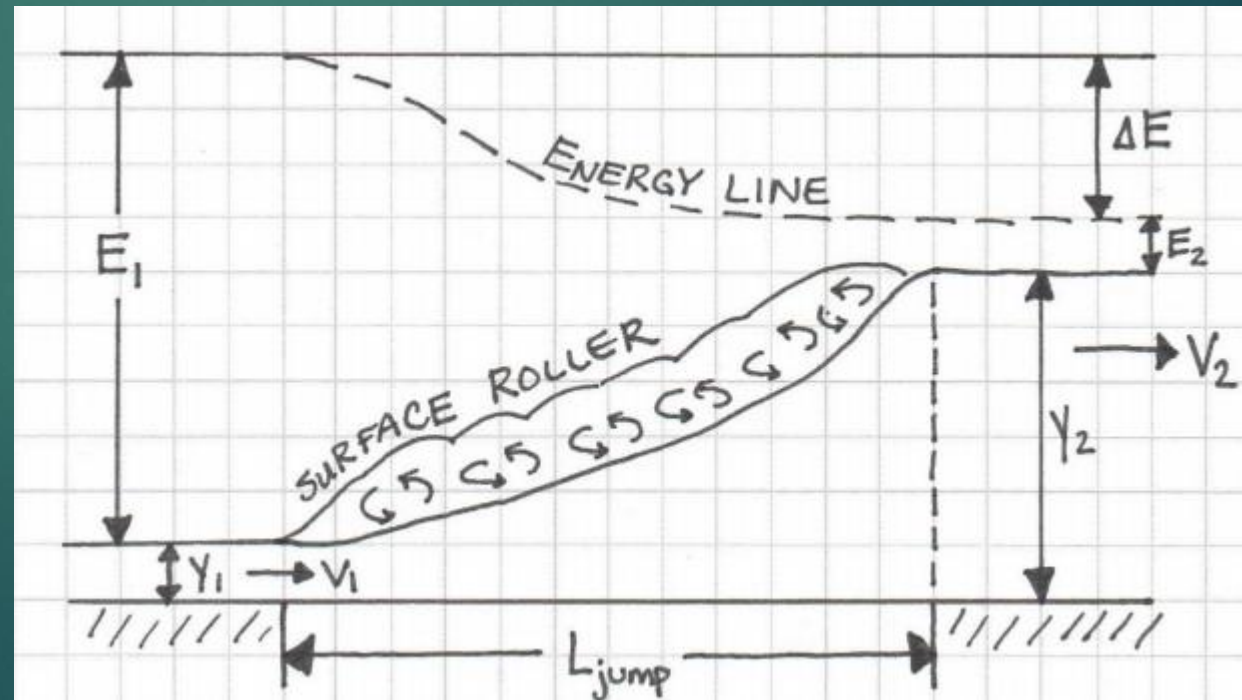
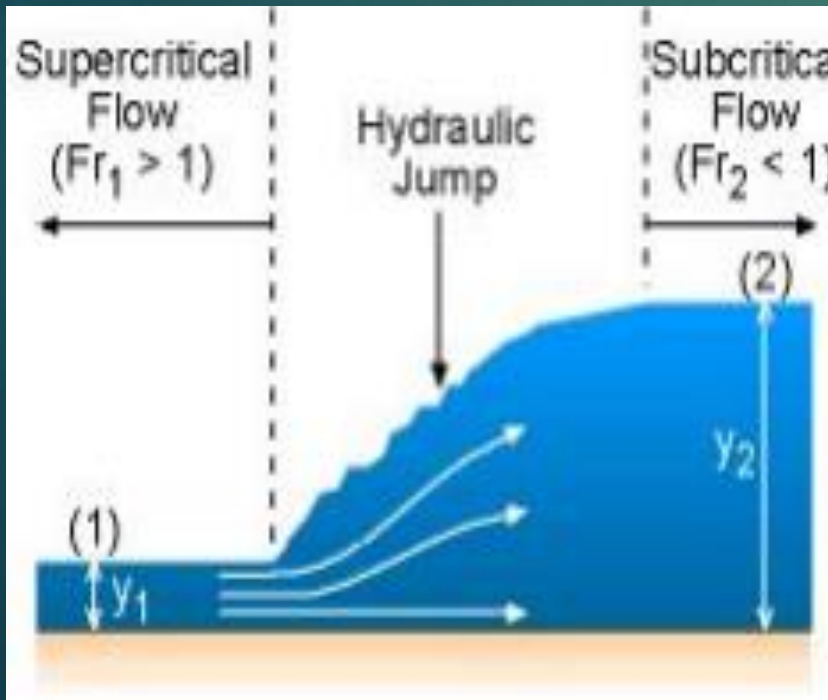
Hydraulic Jump

- When flow is supercritical in a upstream section of a channel and is then forced to become subcritical in a downstream section, the **Hydraulic Jump** occurs.



- Conjugate depths** refer to the depth (y_1) upstream and the depth (y_2) downstream of the hydraulic jump.

Hydraulic Jump



Hydraulic Jump

- In a rectangular channel, the relation between depth of flow before and after the jump is:

$$\frac{y_2}{y_1} = \frac{1}{2} \left(\sqrt{1 + 8Fr_1^2} - 1 \right)$$

- The height of hydraulic jump is:

$$h_j = y_2 - y_1$$

- The power lost by hydraulic jump is:

$$P = \gamma Q \Delta E$$

$$\rightarrow \begin{cases} \gamma = \text{Specific weight of water} \\ Q = \text{Discharge} \end{cases}$$

- The head loss or energy dissipation during a jump is:

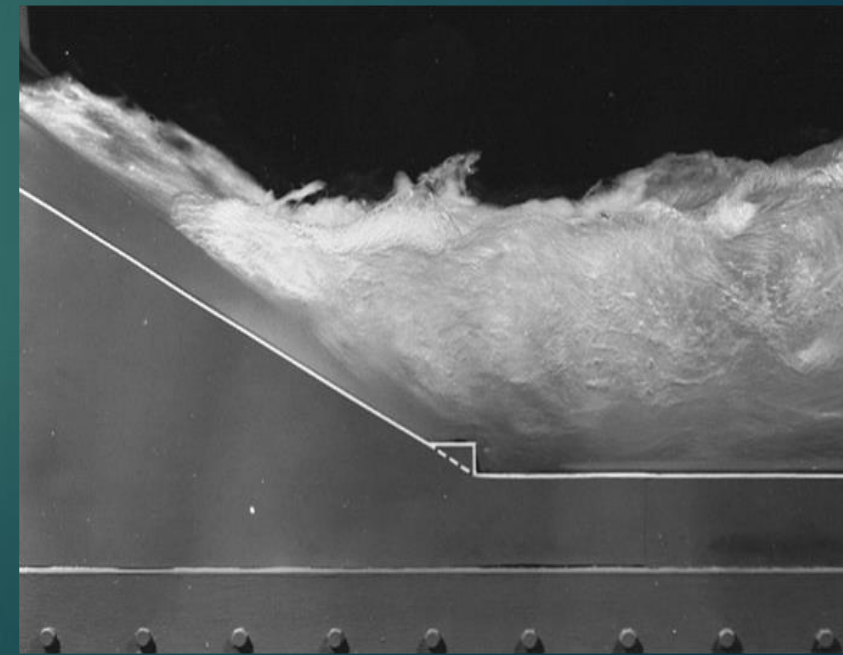
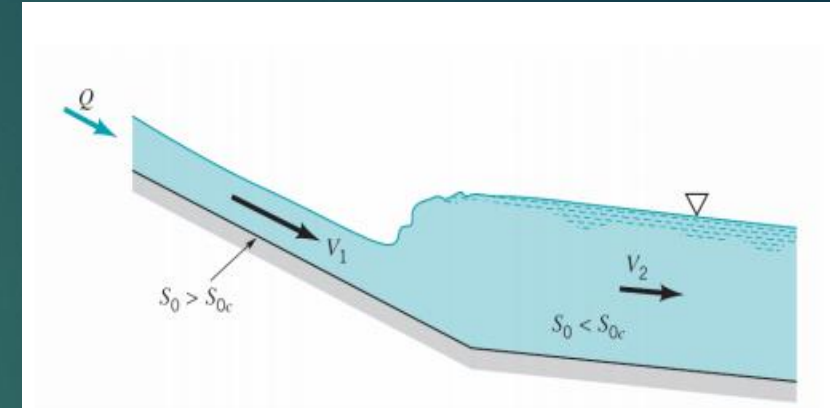
$$\Delta E = \frac{(y_2 - y_1)^3}{4y_2y_1}$$

Hydraulic Jump



Hydraulic Jump

- Jump caused by a change in channel slope.
- Jump caused by a hydraulic structure



Hydraulic Jump

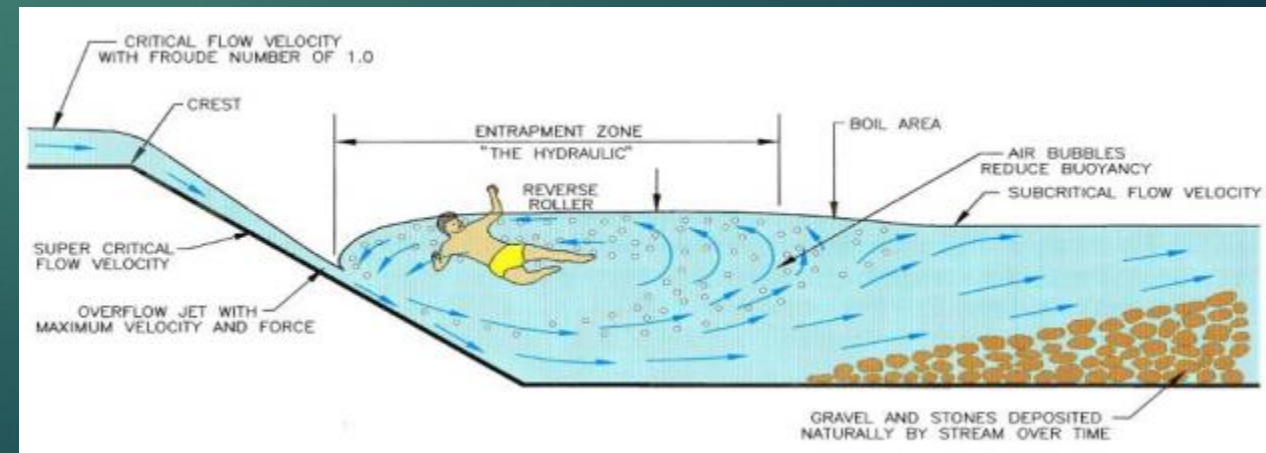
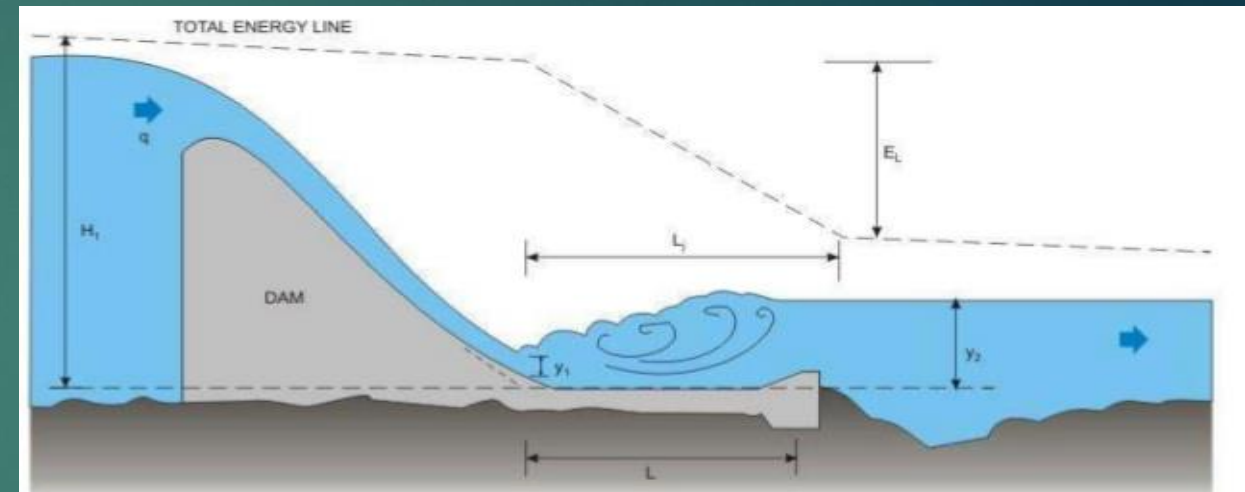
- Jump classification

Fr_1	y_2/y_1	Classification	Sketch
<1	1	Jump impossible	
1 to 1.7	1 to 2.0	Standing wave or undulant jump	
1.7 to 2.5	2.0 to 3.1	Weak jump	
2.5 to 4.5	3.1 to 5.9	Oscillating jump	
4.5 to 9.0	5.9 to 12	Stable, well-balanced steady jump; insensitive to downstream conditions	
>9.0	>12	Rough, somewhat intermittent strong jump	

Hydraulic Jump

Facts

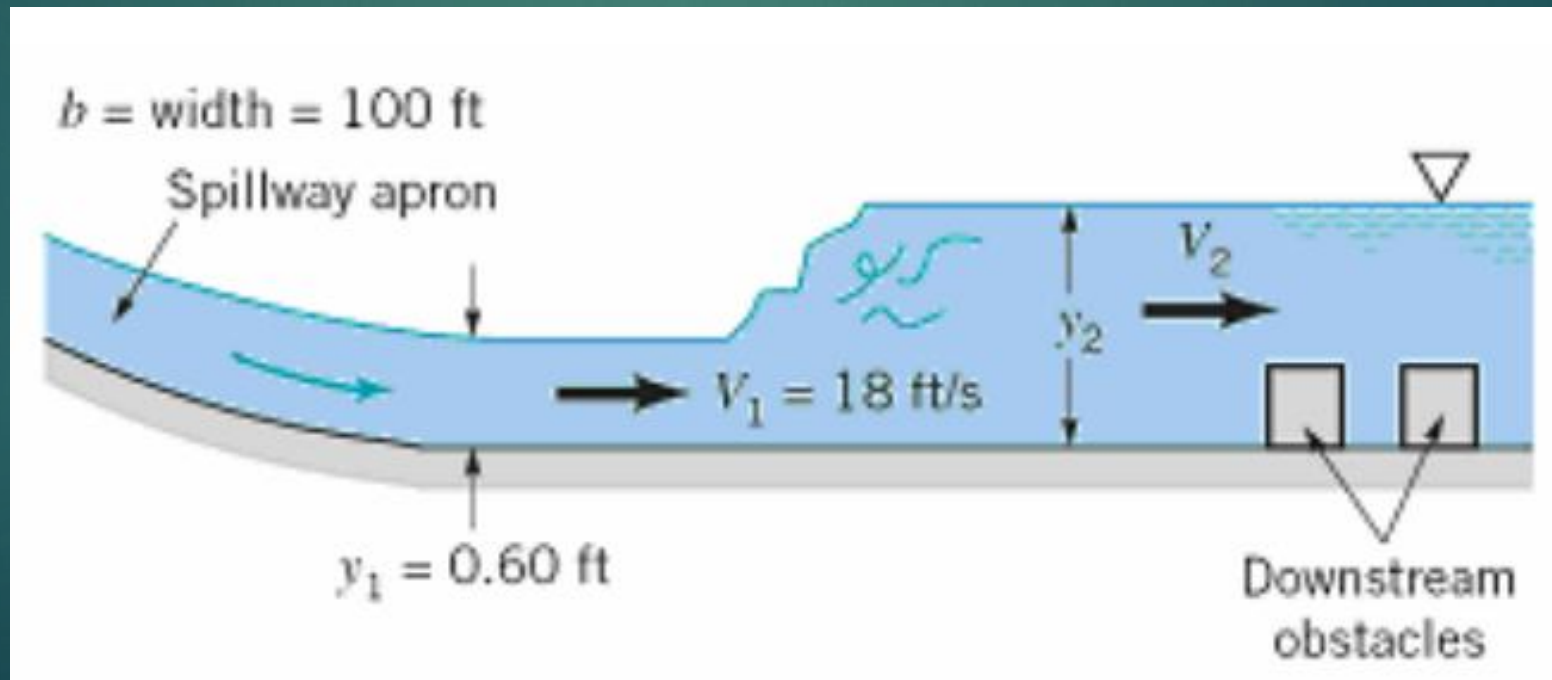
- **Dissipates the energy** of water over a spillway to reduce the erosion issue.
- **Traps air in the water** that could be useful for removing wastes and pollution in the water
- **Reverses the flow** of water, it can be used to mix chemicals for water purification
- **Maintains a high water level** on the downstream side that is useful for irrigation purposes



Hydraulic Jump

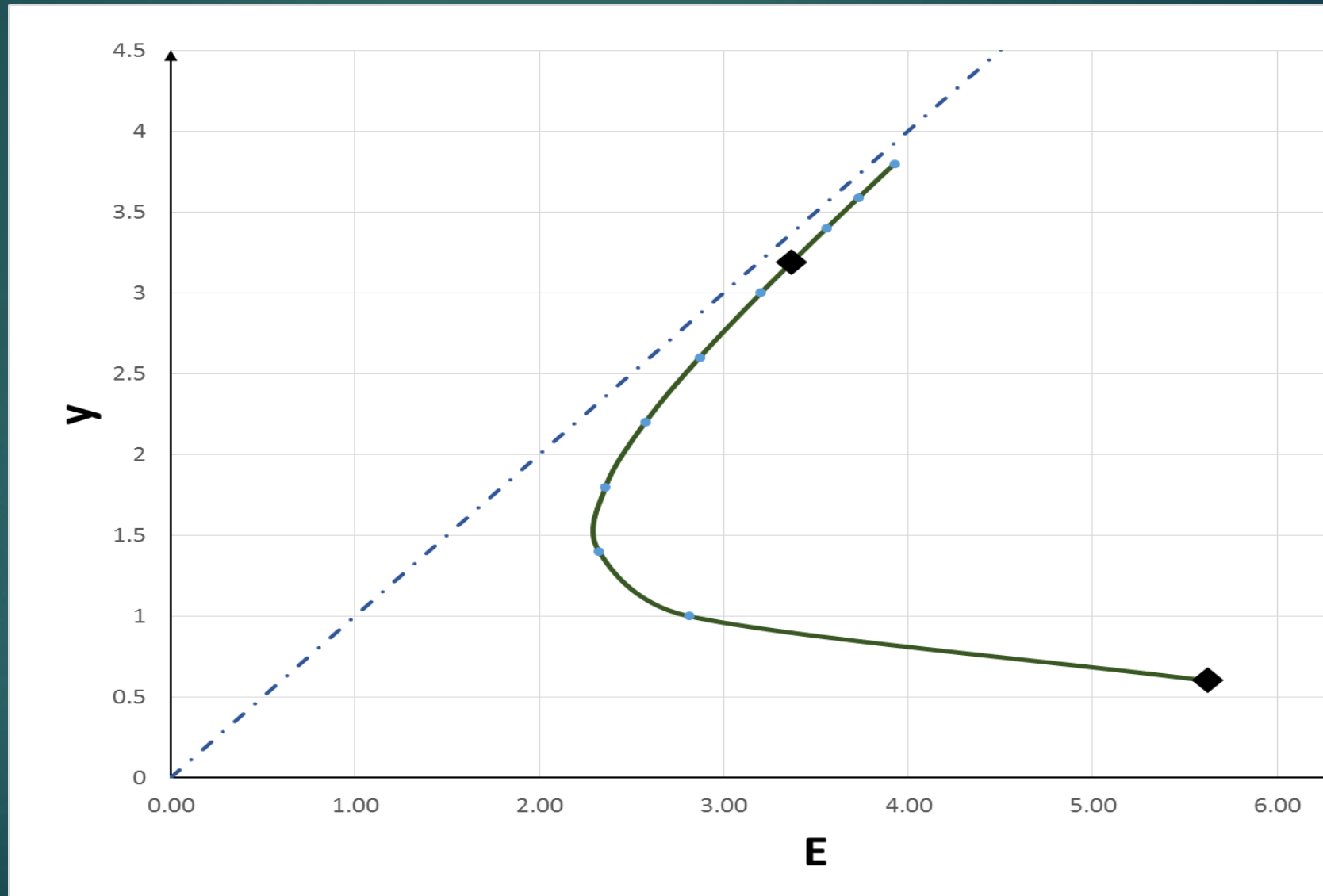
Example 1

Water downstream of an spillway flows in a **100 ft** wide rectangular channel with the depth of **0.6 ft** and velocity of **18 ft/s**. Determine the depth after the jump, the Froude numbers before and after the jump, height of the jump, the head loss and power dissipated during the jump. And plot the y vs specific energy.



Hydraulic Jump

y	E
0.6	5.63
1	2.81
1.4	2.32
1.8	2.36
2.2	2.57
2.6	2.87
3	3.20
3.19	3.37
3.4	3.56
3.59	3.73
3.8	3.93



Design of unlined and lined channels

- In general, a natural channel system continually changes its position and shape as a result of **hydraulic forces** acting on its bed and banks.
- The design of open channels should be based on **maximum permissible velocities**.



Design of unlined and lined channels

In the unlined channels:

- The stability of channels is more dependent on the **physical** and **chemical** properties of the soil than hydraulic properties.
- **Stable hydraulic section** is the most important factor.
- If a higher velocity is desired, a **geotechnical report** should be provided to identify the soil material classification for the maximum permissible velocity determination.

Material	Side slopes (vert:hor)
Hard rock	Vertical
Weathered, cracked or soft rock	Vertical
Clay and hard gravel	1:0,5
Clay loam and gravel loam	1:1
Sandy loam	1:1,5
Sandy soil	1:2

Design of unlined and lined channels

In the lined channels:

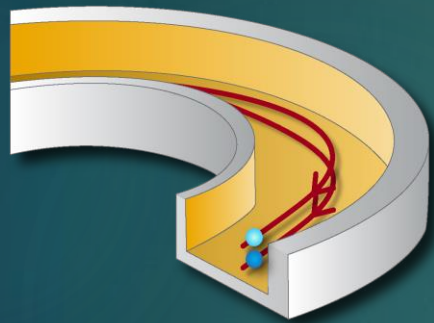
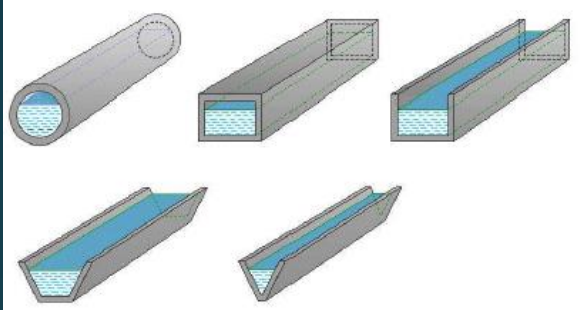
- These channels are lined with materials that **do not erode easily**, e.g. concrete, stone pitching, steel, wood, glass, plastic, etc.
- The choice of material depends on **availability and cost** of respective materials.
- The aspect that need to be taken into consideration is the **quantity of lining material** (or finding the best hydraulic cross section).
- **Minimizing lining material costs** is a factor.

Design of erodible or unlined channels

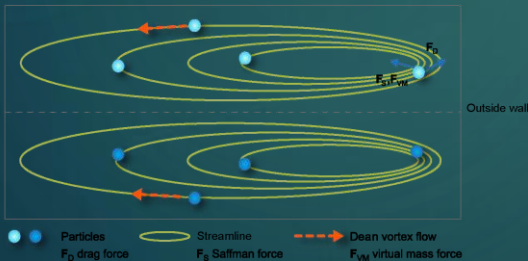
Design procedure

- Estimate Manning Coefficient (n)
- Compute the value of the section factor ($AR_h^{2/3}$)
- Compute Normal Depth (y_n)
- Compute channel properties (y, Q, V)
- Check minimum permissible velocity

Design of erodible or unlined channels



Channel cross section



Maximum Permissible Velocities

TABLE 7.5 Maximum permissible velocities as recommended by Fortier and Scobey (1926) for straight channels of small slope and after aging

Material (1)	n (2)	Clear water				Water transporting colloidal silts			
		\bar{u} , ft/s (3)	τ_o , lb/ft ² (4)	\bar{u} , m/s (5)	τ_o , N/m ² (6)	\bar{u} , ft/s (7)	τ_o , lb/ft ² (8)	\bar{u} , m/s (9)	τ_o , N/m ² (10)
Fine sand, noncolloidal	0.020	1.50	0.027	0.457	1.29	2.50	0.075	0.762	3.59
Sandy loam, noncolloidal	0.020	1.75	0.037	0.533	1.77	2.50	0.075	0.762	3.59
Silt loam, noncolloidal	0.020	2.00	0.048	0.610	2.30	3.00	0.11	0.914	5.27
Alluvial silts, noncolloidal	0.020	2.00	0.048	0.610	2.30	3.50	0.15	1.07	7.18
Ordinary firm loam	0.020	2.50	0.075	0.762	3.59	3.50	0.15	1.07	7.18
Volcanic ash	0.020	2.50	0.075	0.762	3.59	3.50	0.15	1.07	7.18
Stiff clay, very colloidal	0.025	3.75	0.26	1.14	12.4	5.00	0.46	1.52	22.0
Alluvial silts, colloidal	0.025	3.75	0.26	1.14	12.4	5.00	0.46	1.52	22.0
Shales and hardpans	0.025	6.00	0.67	1.83	32.1	6.00	0.67	1.83	32.1
Fine gravel	0.020	2.50	0.075	0.762	3.59	5.00	0.32	1.52	15.3
Graded loam to cobbles when noncolloidal	0.030	3.75	0.38	1.14	18.2	5.00	0.66	1.52	31.6
Graded silts to cobbles when colloidal	0.030	4.00	0.43	1.22	20.6	5.50	0.80	1.68	38.3
Coarse gravel noncolloidal	0.025	4.00	0.30	1.22	14.4	6.00	0.67	1.83	32.1
Cobbles and shingles	0.035	5.00	0.91	1.52	43.6	5.50	1.10	1.68	52.7

Small Slope Channels: having a bottom slope less than 1 in 10 (10%).

Large Slope Channels: having a bottom slope greater than 1 in 10 (10%).

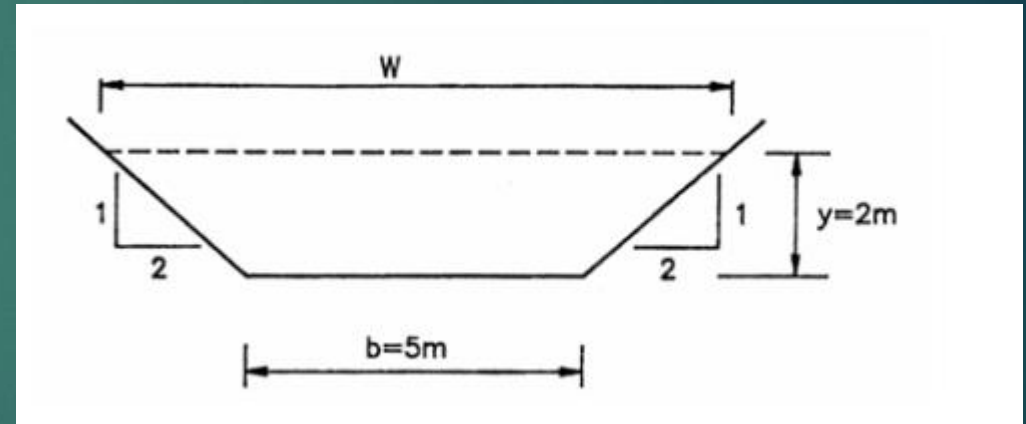
Design of erodible or unlined channels

Material	Average flow velocity [m/s]
Very light flowing sand	0,2 – 0,3
Very light loose sand	0,3 – 0,4
Coarse sand or light sandy soil	0,4 – 0,6
Normal sandy soil	0,6 – 0,7
Sandy loam soil	0,7 – 0,8
Loamy alluvial soil	0,8 – 1,0
Firm loam, clay loam	1,0 – 1,2
Stiff clay and gravelly soil	1,2 – 1,5
Coarse and rocky gravel	2,0 – 2,5
Conglomerate, soft shale, soft rock formation	2,0 – 2,5
Hard rock	3,0 – 4,5
Concrete	4,5 – 6,0

Design of erodible or unlined channels

Example 2

The normal flow depth in a trapezoidal concrete channel is **2 m**. The base width is **5 m** with side slopes **1:2**. The channel slope is **0.001** and Manning's **$n = 0,015$** . Determine the *flow rate*, and *average flow velocity*.



Design of erodible or unlined channels

Example 3

Determine the width (b) and safe flow depth (y) of a trapezoidal spillway with a slope of 0.0016 , side slope $1:1.5$, and a flow rate of $7750 \text{ m}^3/\text{h}$. The spillway is built in sandy loam soil.

Design of erodible or unlined channels

Example 4

A trapezoidal open channel (Stiff-Clay, $n=0.035$) with the side slope 1:3 ($m=3$) is to be constructed with the following conditions: $Q_{100} = 191$ cfs, Upstream elevation 4,918 ft, Downstream elevation 4,917 ft, Channel length 900 ft, Bottom width 10 ft.

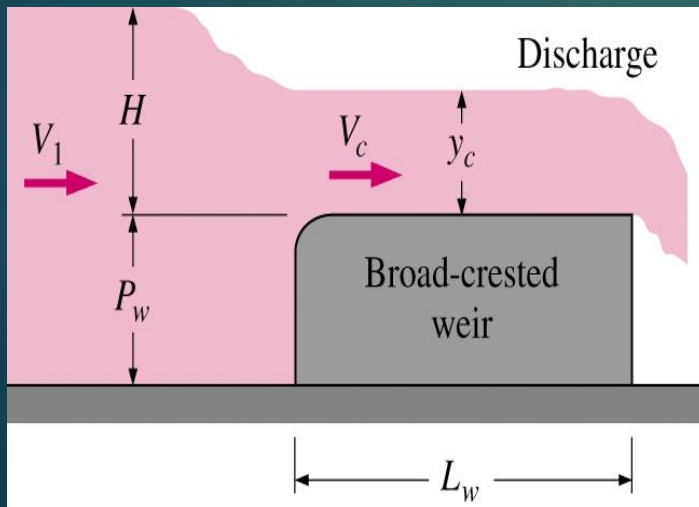
* 100-year design flow or Q_{100} means a flow with the return period of 100 years. A return period, is an estimate of the likelihood of an event, such as an earthquake, flood or a river discharge flow to occur.

This does not mean that a 100-year flood will happen regularly every 100 years, or only once in 100 years. In any given 100-year period, a 100-year event may occur once, twice, more, or not at all, and each outcome has a probability that can be computed as below.

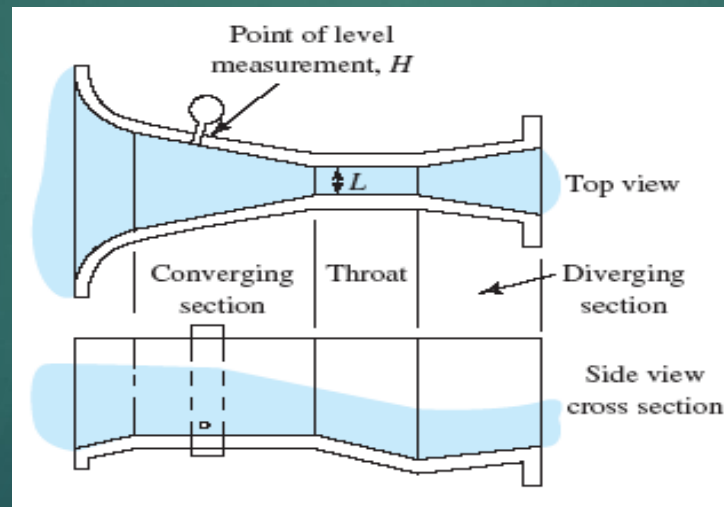
Flow Control and Measurement

- In open channel flows, flow rate is controlled by **partially blocking** the channel.
- The way this works is, **blocking** the channel to change the **shape** and **velocity** of the flow (e.g., critical flow).

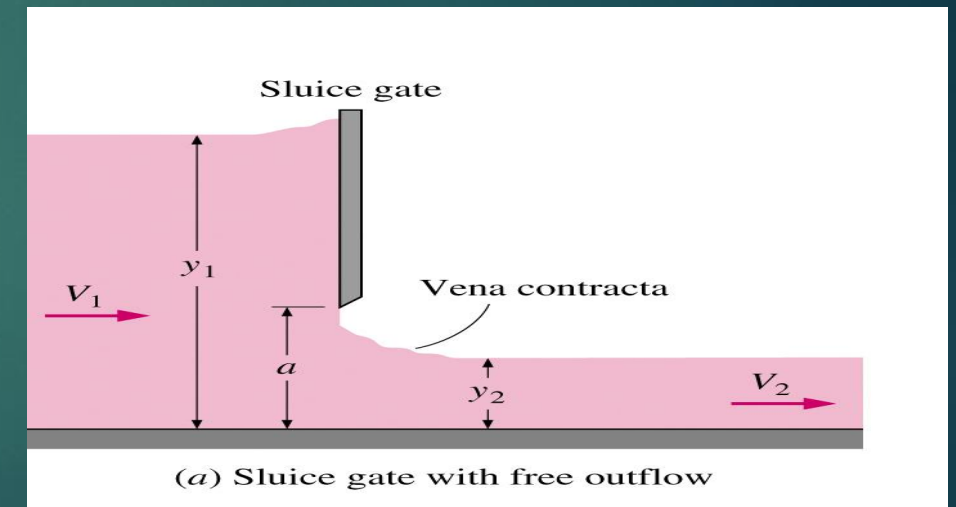
Weir : Flows over a device



Flume: Flows through a device



Underflow gate : Flows under a device



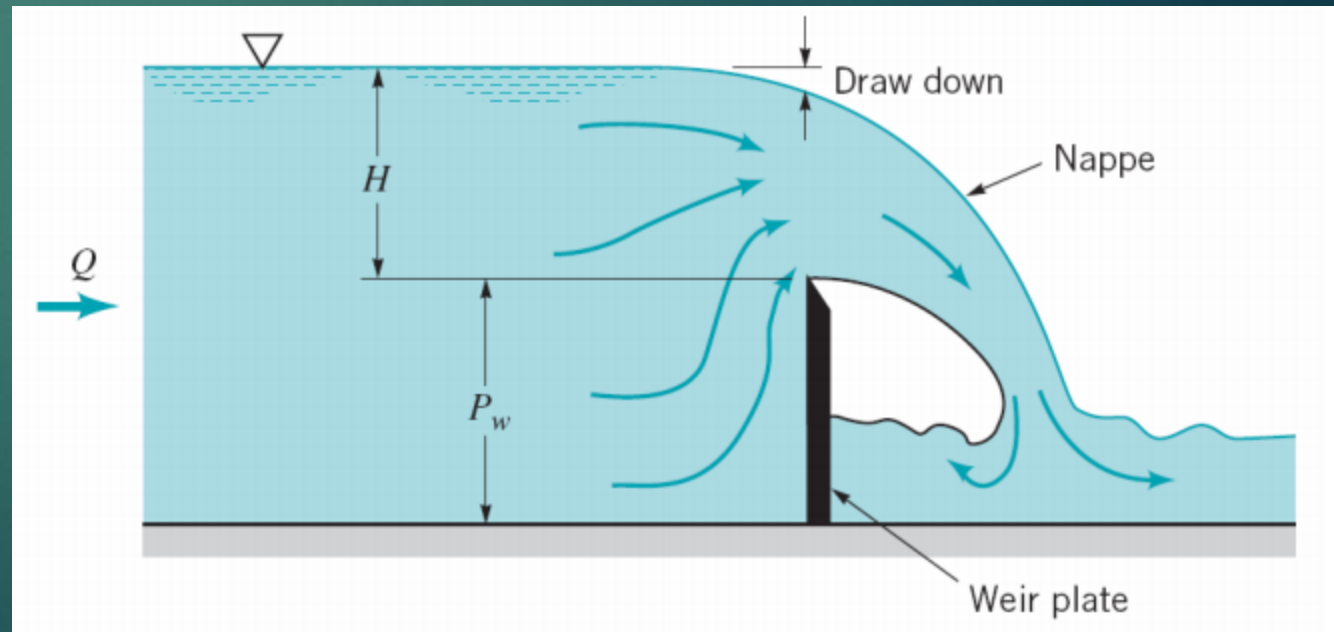
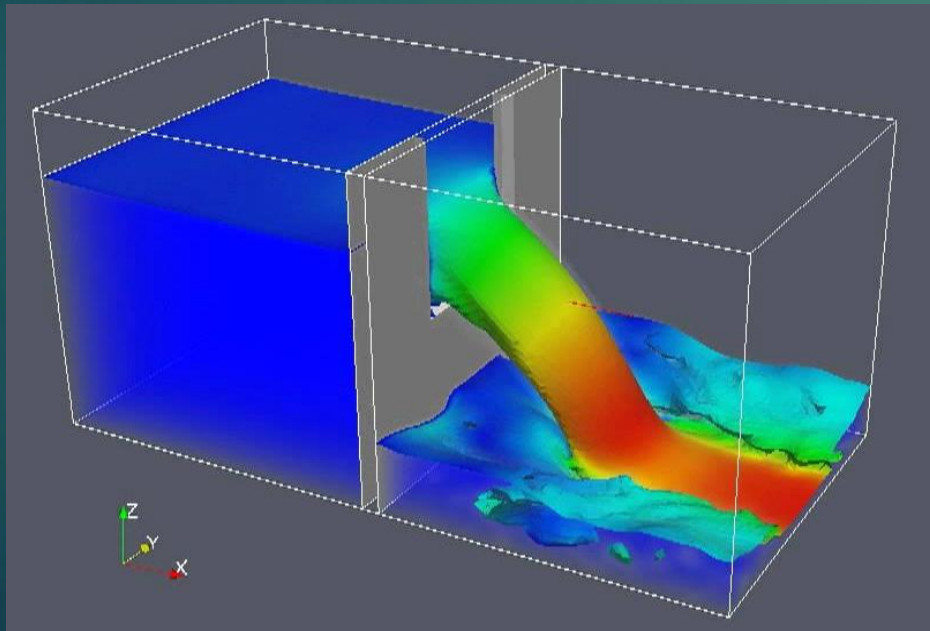
Flow Control and Measurement (weirs)

Weir provides a convenient method of determining the flowrate in an open channel in terms of a single depth measurement.



Flow Control and Measurement

- A **sharp-Crested** weir is essentially a vertical-edged flat plate placed across the channel.
- The fluid must flow across the sharp edge and **drop into the pool** downstream of the weir plate.



Flow Control and Measurement

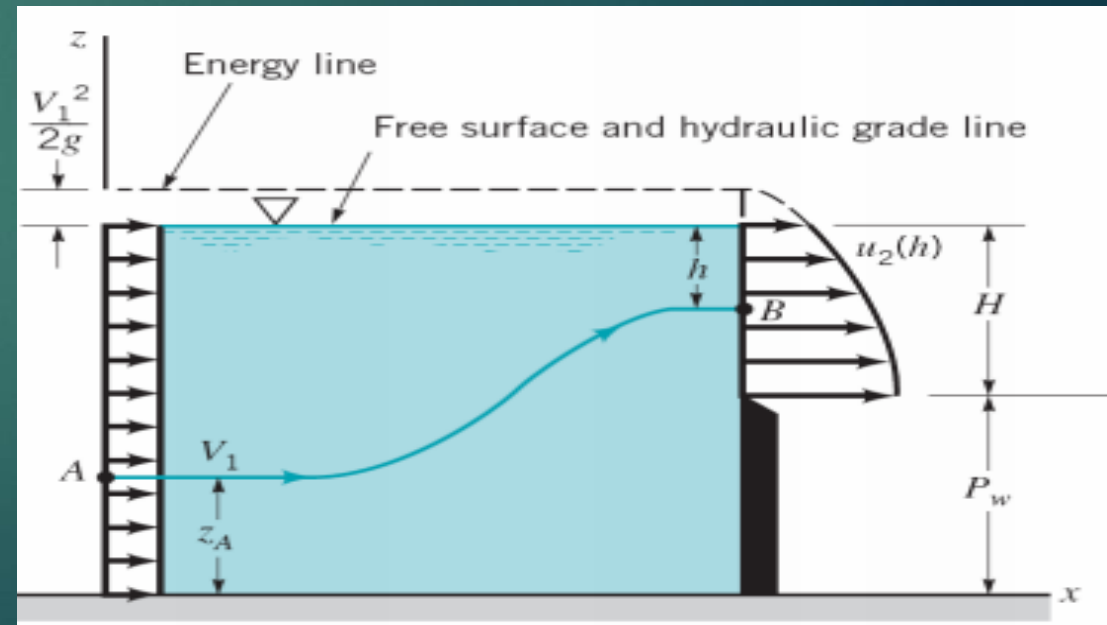
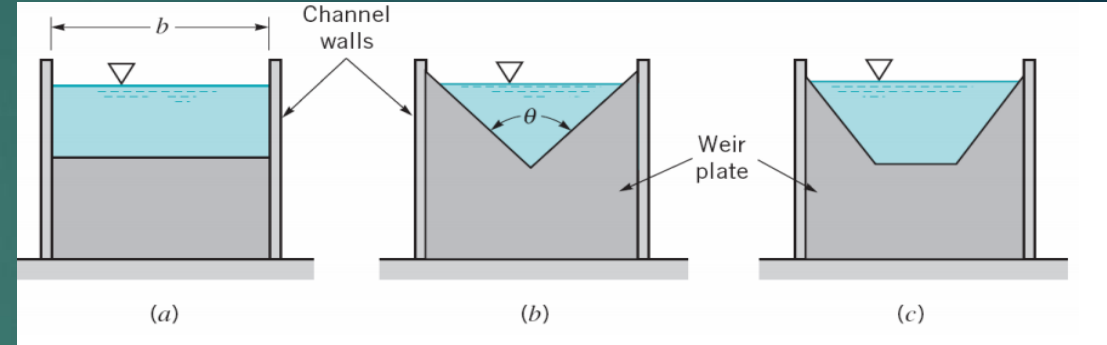
- **Sharp-crested** weir plate geometry: (a) rectangular, (b) triangular, (c) trapezoidal.

Rectangular sharp crested weir

$$Q = C_{wr} \frac{2}{3} \sqrt{2gb} H^{3/2}$$

$$C_{wr} = 0.611 + 0.075 \left(\frac{H}{P_w} \right)$$

C_{wr} is the rectangular weir coefficient.

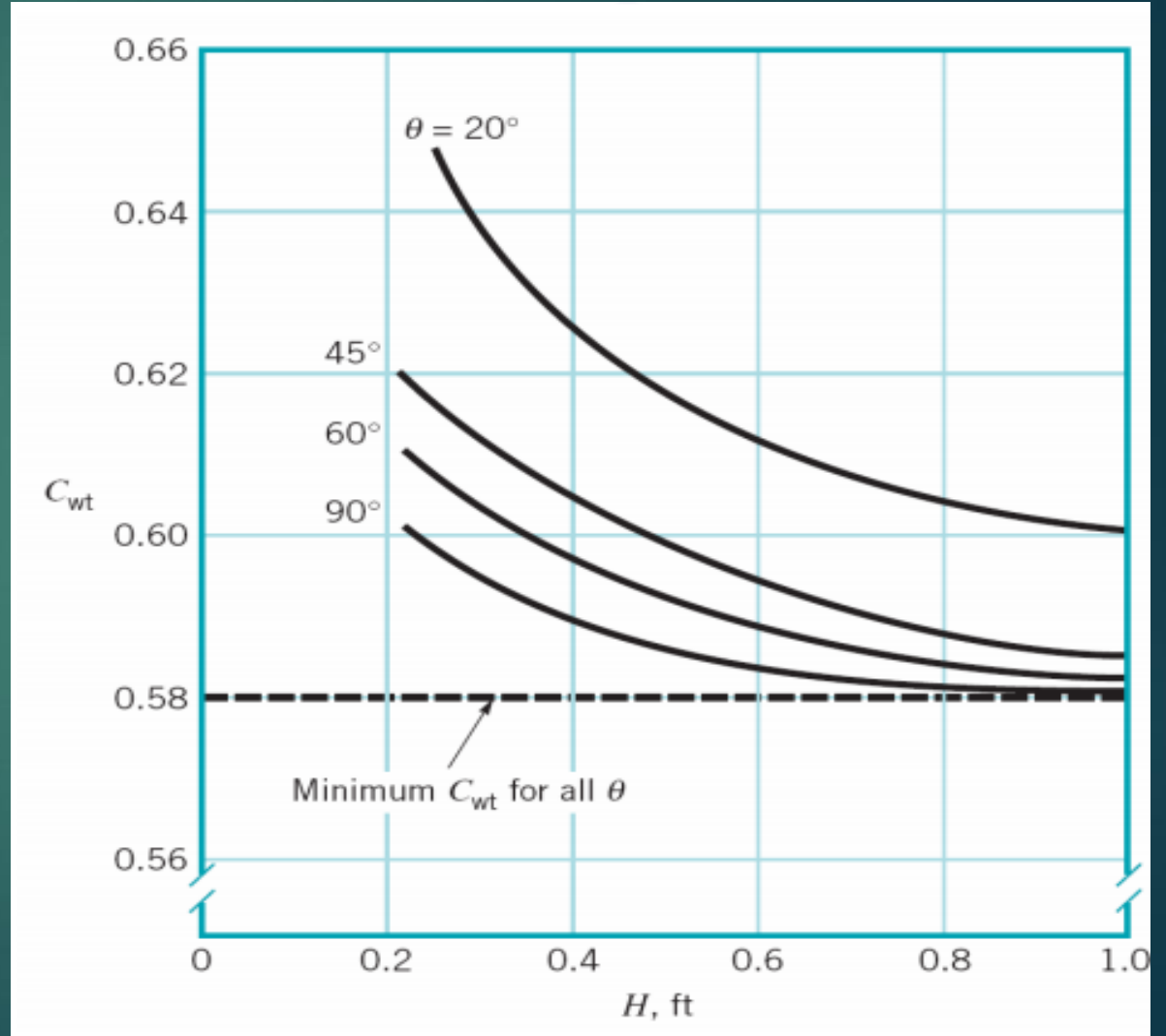
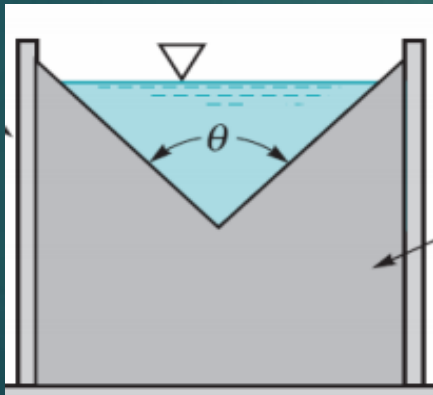


Flow Control and Measurement

Triangle sharp crested weir

$$Q = C_{wt} \frac{8}{15} \tan\left(\frac{\theta}{2}\right) \sqrt{2gb} H^{5/2}$$

C_{wt} is the triangle weir coefficient.



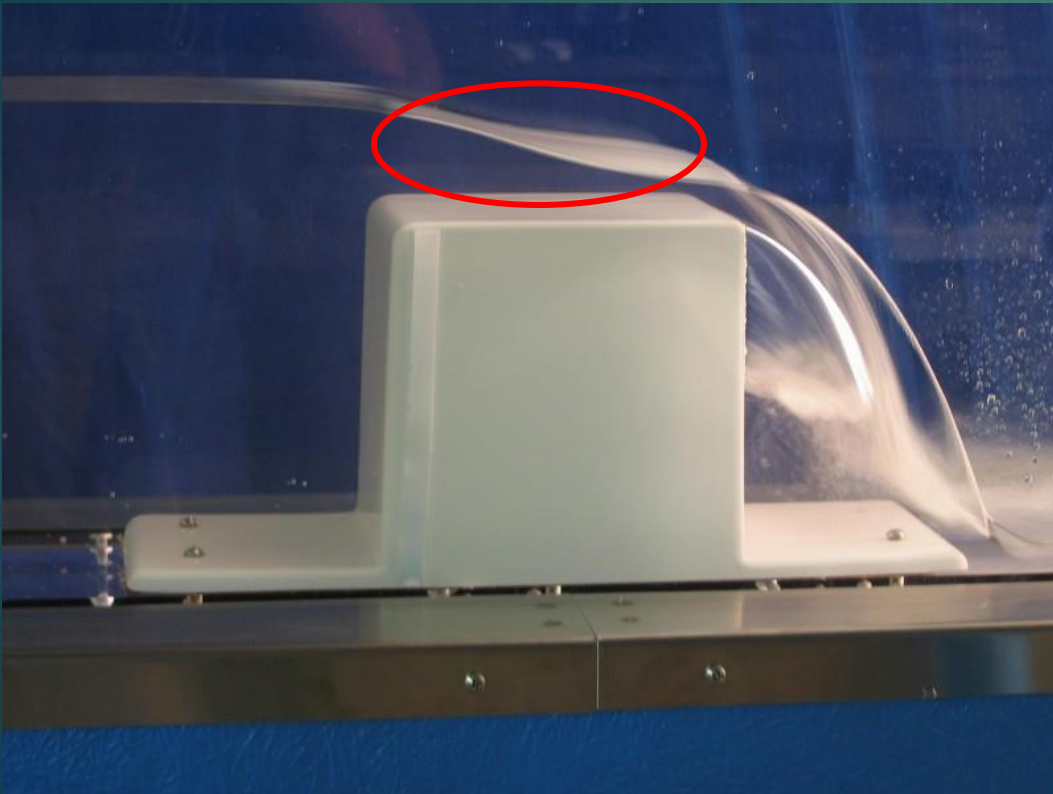
Flow Control and Measurement (Broad Crested weir)

- This is the **simplest** device for flow measurement.
- The **width** of the weir is taken as the width of the waterway.

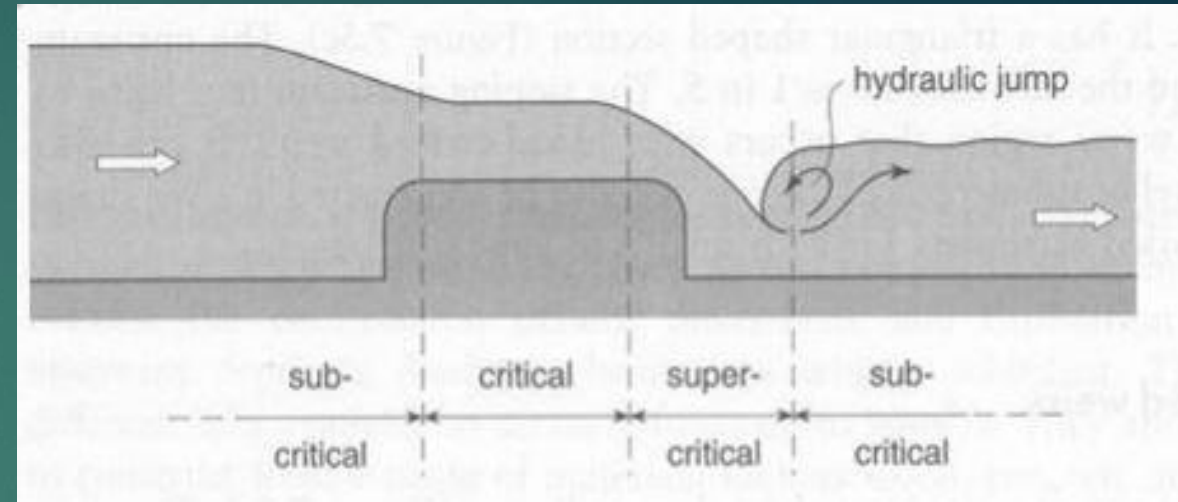
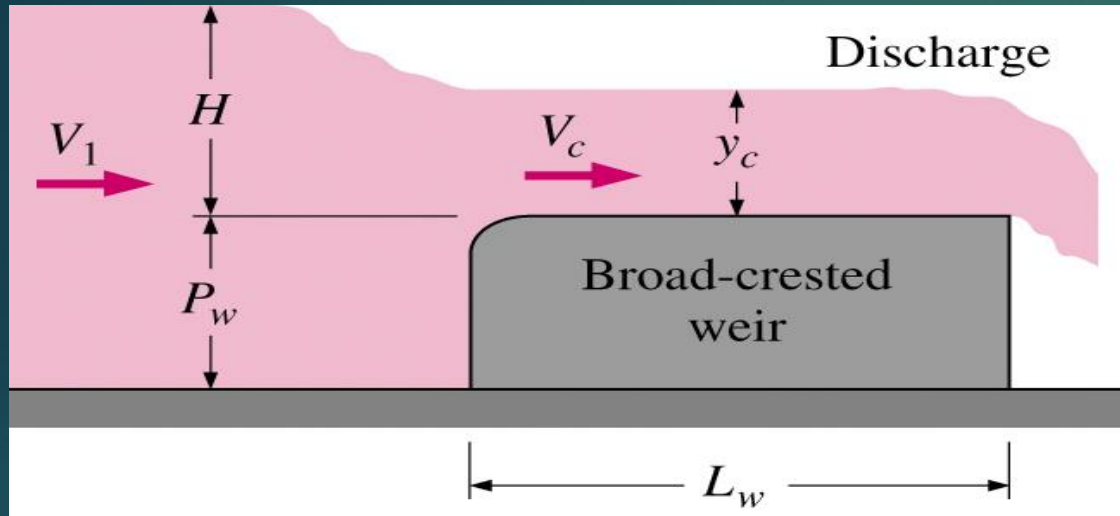


Flow Control and Measurement (Broad Crested weir)

- A key feature of a properly operating broad crested weir is **critical flow** over the weir crest.



Flow Control and Measurement (Broad Crested weir)



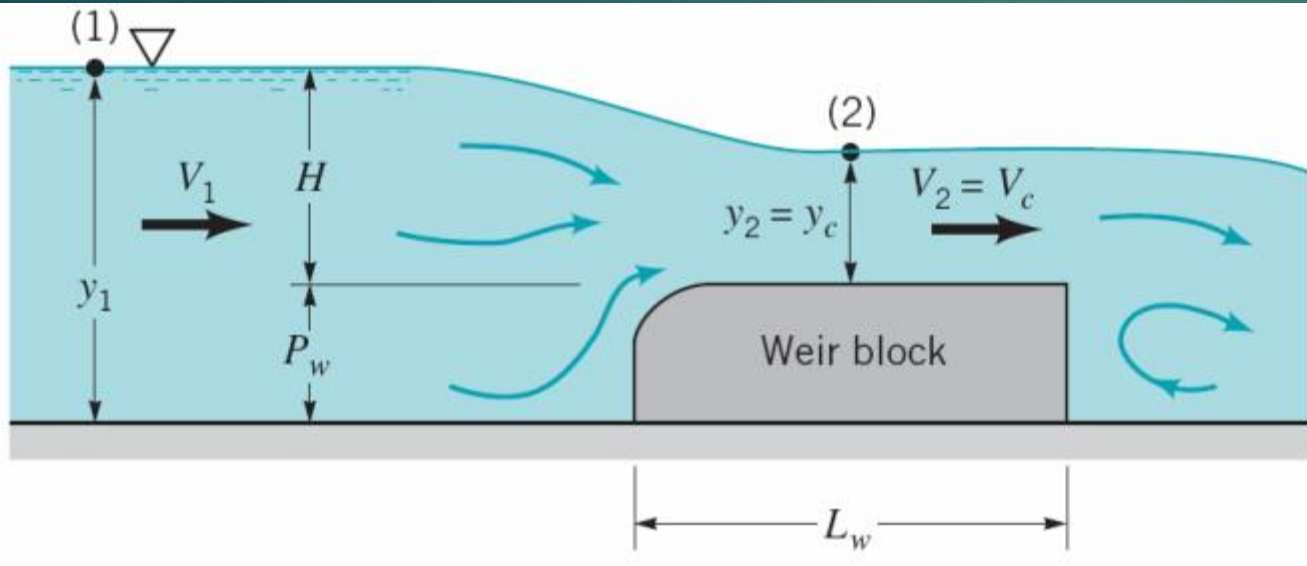
Advantages:

- Cost effective installation due to ease of design and construction.
- Relatively small head loss across the structure
- Capable of measuring discharge in small to medium channels

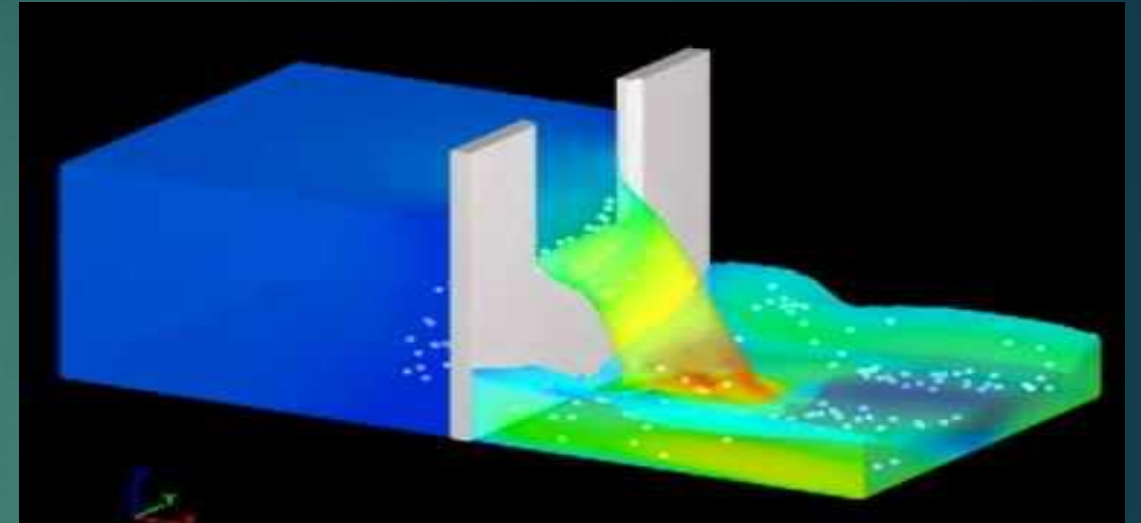
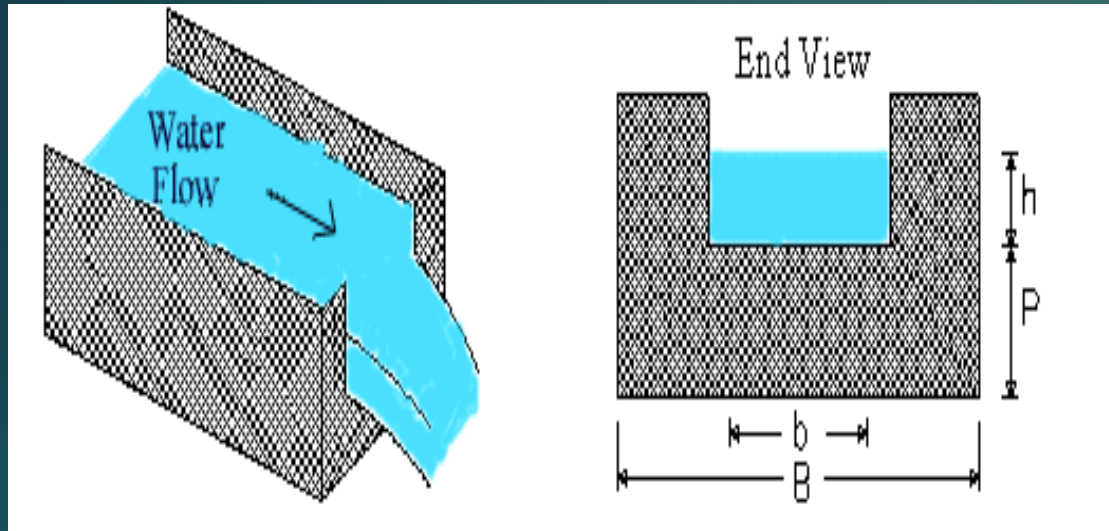
Flow Control and Measurement (Rectangular weir)

$$Q = C_{wb} \sqrt{2gb} \left(\frac{2}{3}\right)^{3/2} H^{3/2}$$

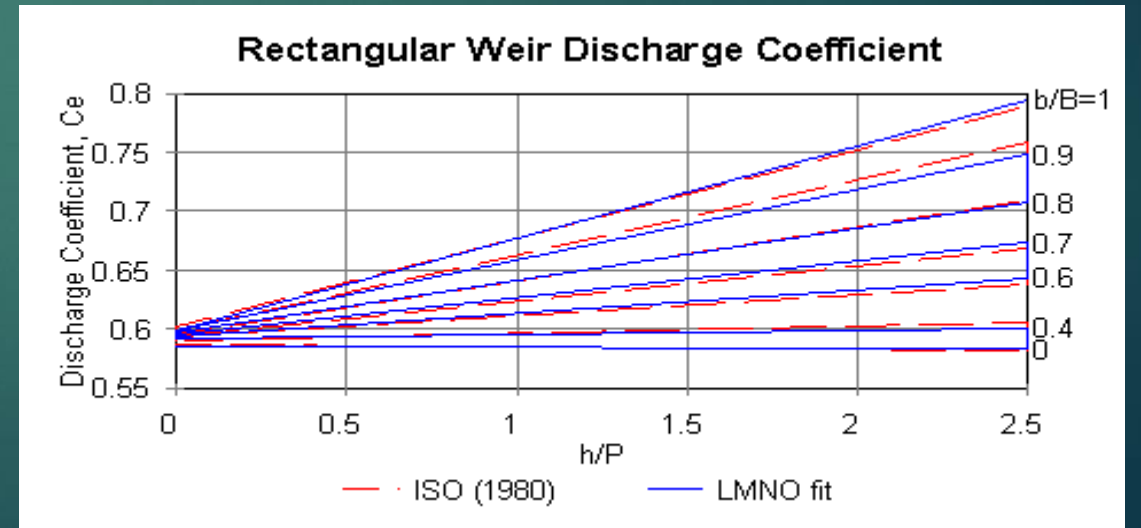
$$C_{wb} = \frac{0.65}{\left[1 + \frac{H}{P_w}\right]^{1/2}}$$



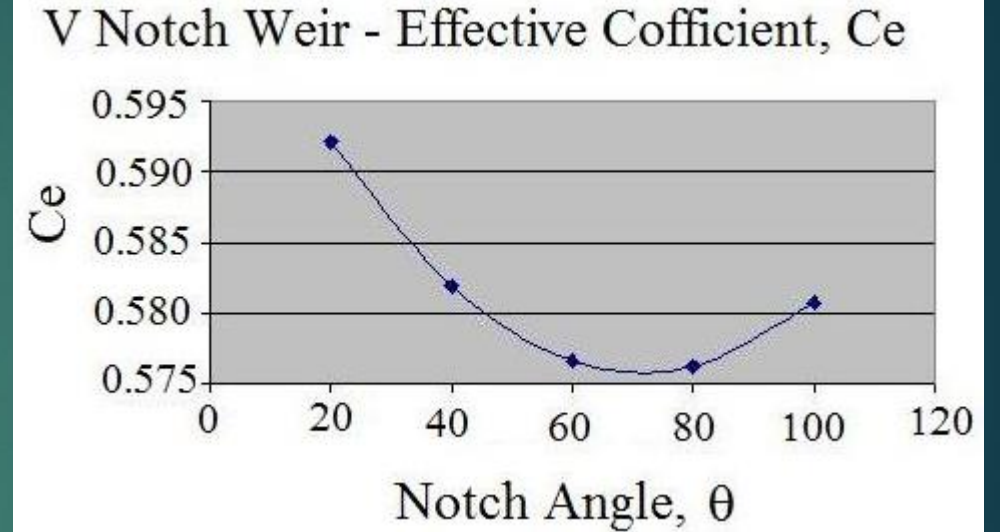
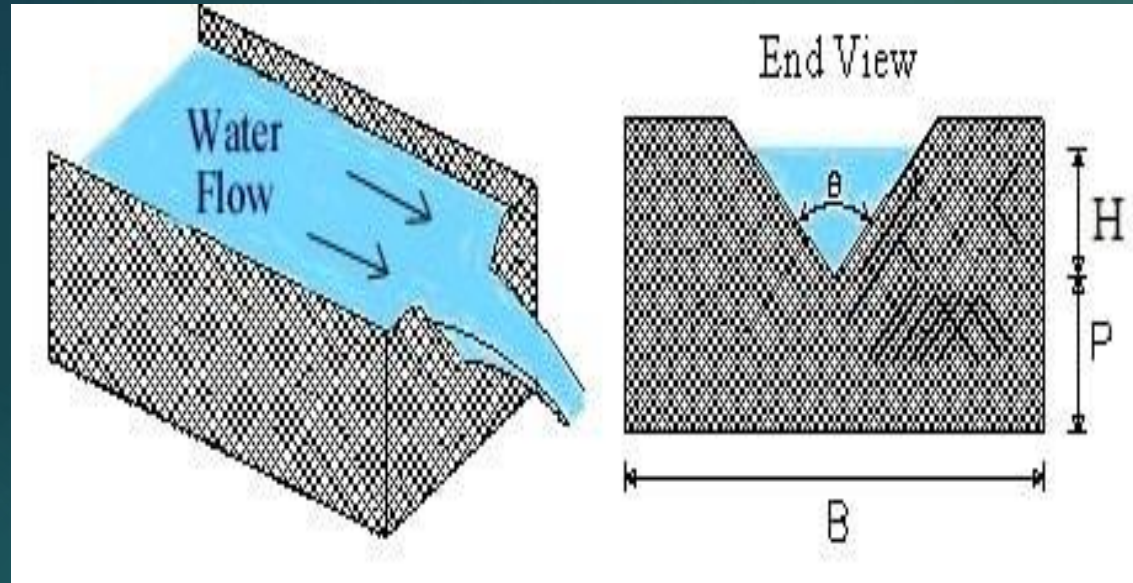
Flow Control and Measurement (Rectangular weir)



$$Q = \frac{2}{3} C_e \times b \times \sqrt{2g} h^{3/2}$$



Flow Control and Measurement (Triangular or V-notch weirs)



$$Q = \frac{8}{15} C_e \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

- One of the best for relatively small flows
- C_e is a function of θ



Flow Control and Measurement (Broad Crested weir)

Example 5

Water flows in a rectangular channel with the width of **2 m** with **H=0.5 m**. This flow rate is to be measured by using a:

- a. Rectangular sharp-crested weir
- b. Triangular sharp-crested weir with $\theta = 90^\circ$
- c. Broad crested-weir

If the weir height is **1 m**, calculate the flow rate.

Homework 4

Q1. Hydraulic hump occurs in a rectangular channel with the flow rate of $500 \text{ ft}^3/\text{s}$ and width of 10 ft . If the depth of flow before the jump is 3.1 ft , what would be the depth after the jump, head loss during the jump, and velocity after and before the jump.

Homework 4

Q2. Hydraulic hump occurs in a rectangular channel with the width of 9 m. If the depths of flow before and after the jump are 1.55 m and 3.08 m, respectively, what would be the flow rate in the channel (Assume $R_h=y$)?