



# APPLIED HYDRAULICS

CHAPTER 5:

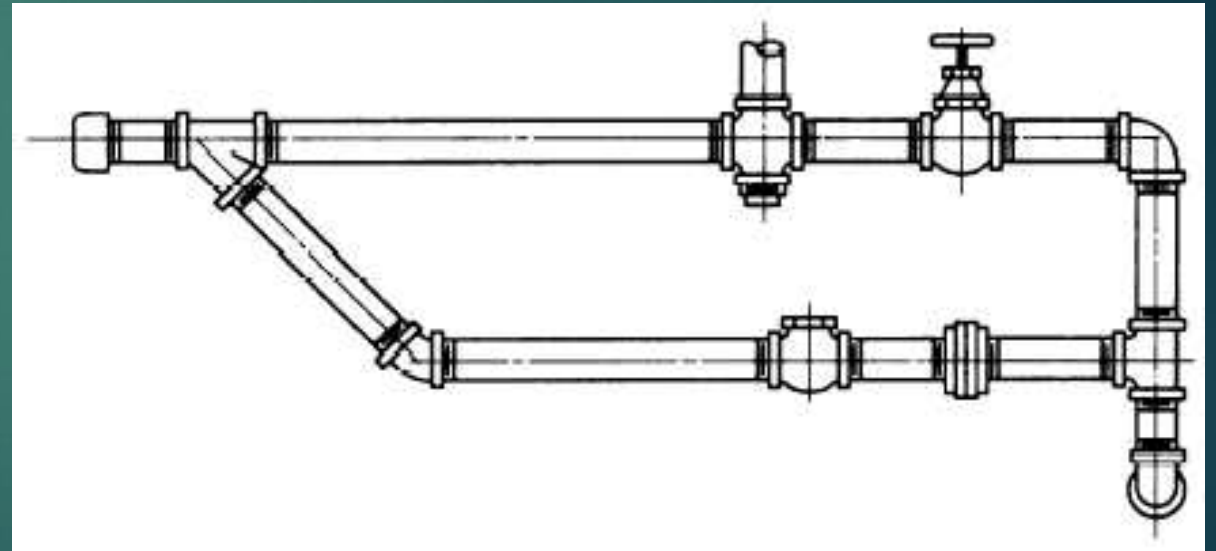
PIPE FLOW

# Pipe Flow

- Introduction
- Reynolds number
- Velocity Profile
- Pressure Drop and Head Loss
- Horizontal and Inclined Pipes
- Moody Chart
- Energy Equation

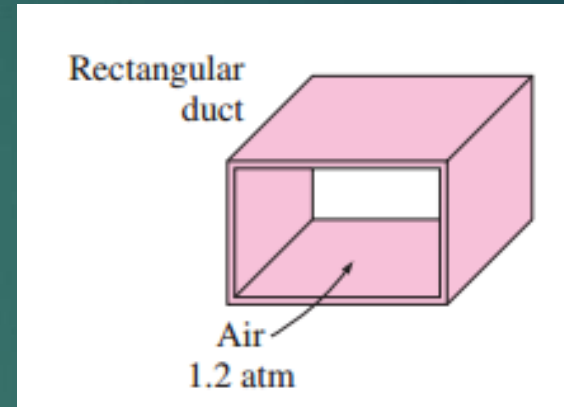
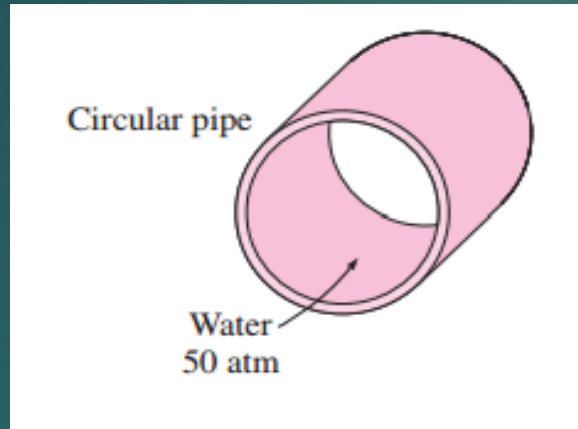
## Pipe Flow | Introduction

- In the pipe flow, there is no direct atmospheric flow and there would be **hydraulic pressure** only.
- Pipe flow **has pressure** (above or below atmospheric), while, open channel flow is always at **atmospheric** pressure.
- The fluid is usually forced to flow by a **fan** or **pump** through a flow section.
- This should not be confused with open-channel flow where flow is driven by **gravity alone**.



# Pipe Flow | Introduction

- The terms **pipe**, **duct**, and **conduit** are usually used interchangeably for flow sections.



- In general, flow sections of **circular** cross section are referred to as pipes.
- Flow sections of **noncircular cross** section as ducts.
- Small diameter pipes** are usually referred to as tubes.

# Pipe Flow | Reynolds Number

## Reynolds Number

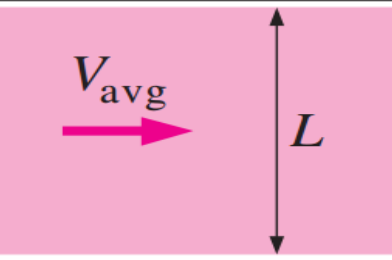
- The ratio of inertial forces to viscous forces
- In a circular pipe is:

$$Re = \frac{V \times L}{\nu}$$

$V$  is average flow velocity

$L$  is characteristic length of the geometry or hydraulic diameter  $D_h$ .

( $L = D$  or diameter in this case)



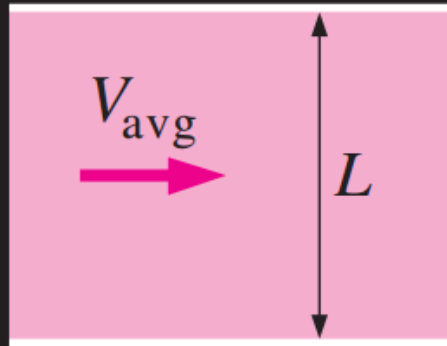
The diagram shows a pink rectangular channel. A pink arrow labeled  $V_{avg}$  points to the right inside the channel. A vertical double-headed arrow on the right side of the channel is labeled  $L$ , representing the characteristic length.

$$\begin{aligned} Re &= \frac{\text{Inertial forces}}{\text{Viscous forces}} \\ &= \frac{\rho V_{avg}^2 L^2}{\mu V_{avg} L} \\ &= \frac{\rho V_{avg} L}{\mu} \\ &= \frac{V_{avg} L}{\nu} \end{aligned}$$

$\nu$  is kinematic viscosity  
*dynamic viscosity*  $\mu$   
 $= \frac{\text{the density of the fluid } \rho}{\text{dynamic viscosity } \mu}$

# Pipe Flow | Reynolds Number

- At **large Reynolds numbers**, the inertial forces are large relative to the viscous forces, and thus the viscous forces cannot prevent the rapid fluctuations of the fluid (*Turbulent flow*).
- At **small or moderate Reynolds numbers**, however, the viscous forces are large enough to suppress these fluctuations and to keep the fluid “in line.” (*Laminar flow*)



The diagram shows a pink rectangular channel. A pink arrow labeled  $V_{avg}$  points to the right, representing the average velocity. A vertical double-headed arrow on the right side of the channel is labeled  $L$ , representing the characteristic length (width) of the channel.

$$\begin{aligned} \text{Re} &= \frac{\text{Inertial forces}}{\text{Viscous forces}} \\ &= \frac{\rho V_{avg}^2 L^2}{\mu V_{avg} L} \\ &= \frac{\rho V_{avg} L}{\mu} \\ &= \frac{V_{avg} L}{\nu} \end{aligned}$$

# Pipe Flow | Reynolds Number

- Hydraulic Diameter

$$Re = \frac{V \times D_h}{\nu}$$

$$D_h = \frac{4A}{P_w}$$

- Where **A** is the *cross-sectional area* of the pipe and **p** is its *wetted perimeter*.

|                  |                                  |                          |
|------------------|----------------------------------|--------------------------|
| <b>Pipe Flow</b> | $Re \lesssim 2300$               | <b>laminar flow</b>      |
|                  | $2300 \lesssim Re \lesssim 4000$ | <b>transitional flow</b> |
|                  | $Re \gtrsim 4000$                | <b>turbulent flow</b>    |

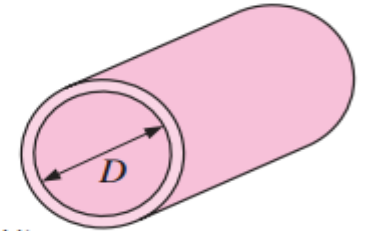
Laminar flow :  $Re < 500$  (viscous > inertia)

Open Channel Flow

Transitional flow:  $500 < Re < 1300$

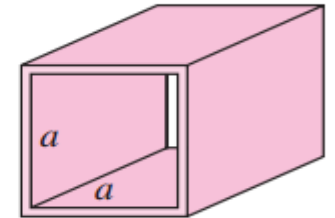
Turbulent flow:  $Re > 1300$  (inertia > viscous)

Circular tube:



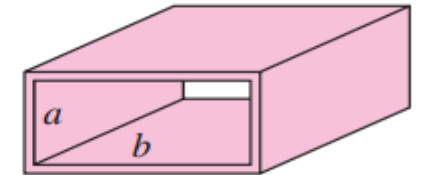
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



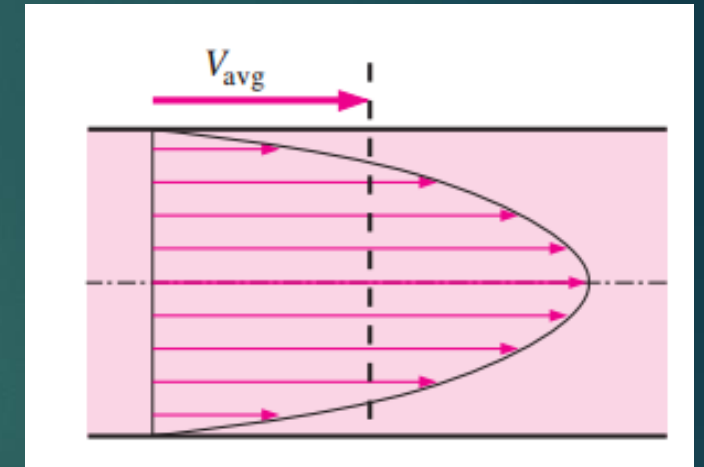
$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

## Pipe Flow | Velocity Profile

- Consider a fluid entering a circular pipe at a uniform velocity. Because of the **no-slip condition**, the fluid particles in the layer in contact with the surface of the pipe come to a complete stop.

The **no-slip condition** assumes that at a solid boundary, the fluid will have zero velocity relative to the boundary.

- This layer also causes the fluid particles in the adjacent layers to **slow down gradually** as a result of friction.
- To make up for this velocity reduction, the velocity of the fluid at the midsection of the pipe **has to increase** to keep the flow rate through the pipe **constant**.
- As a result, a velocity gradient *develops along the pipe*.

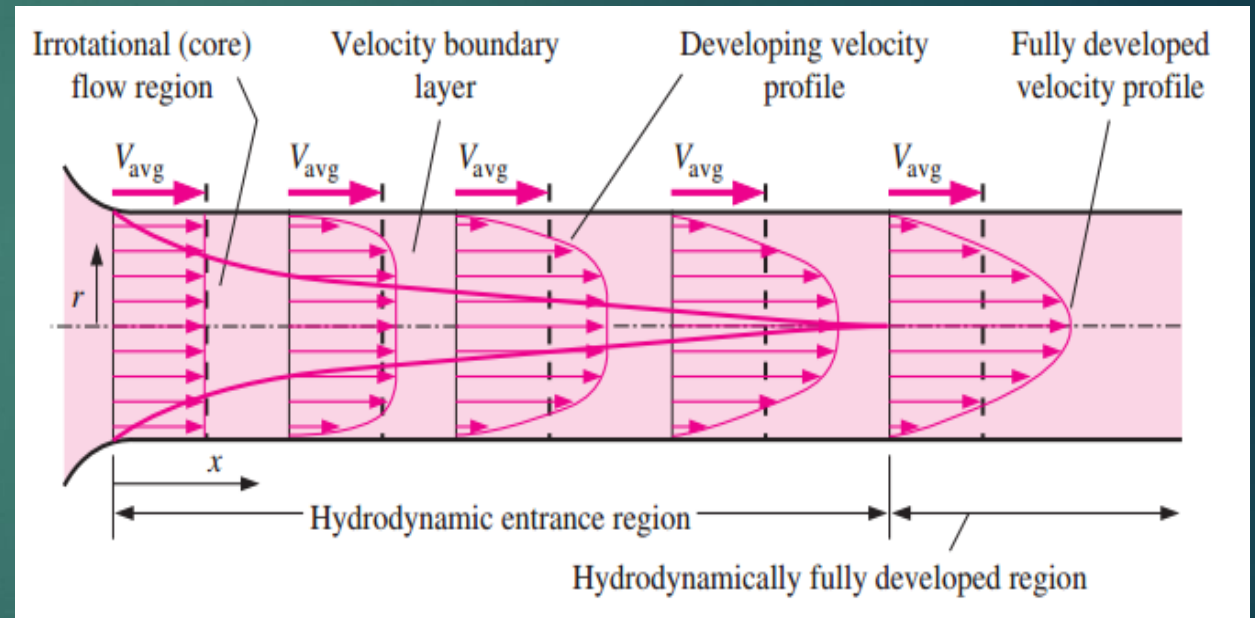




## Pipe Flow | Velocity Profile

- The region of the flow in which the effects of boundary are felt is called the **velocity boundary layer** or just the **boundary layer**.

- The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the **pipe center** and thus fills the entire pipe.

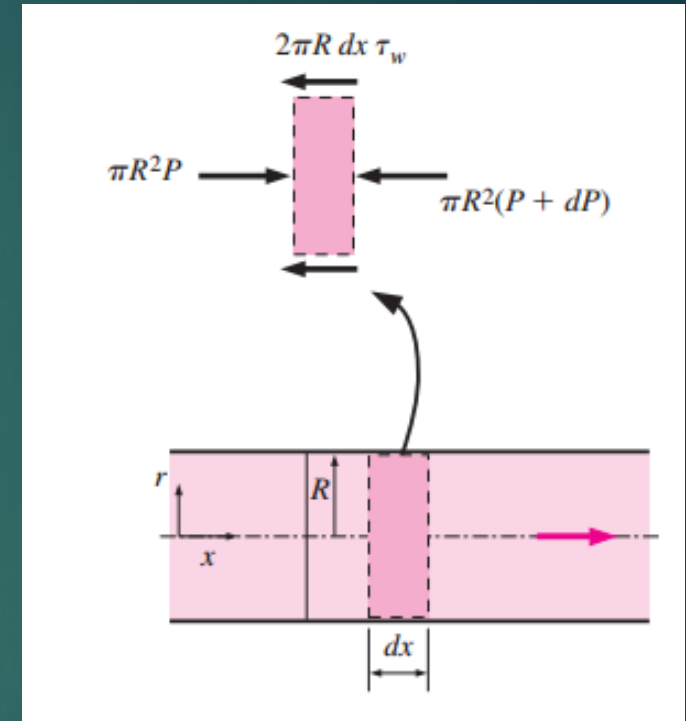
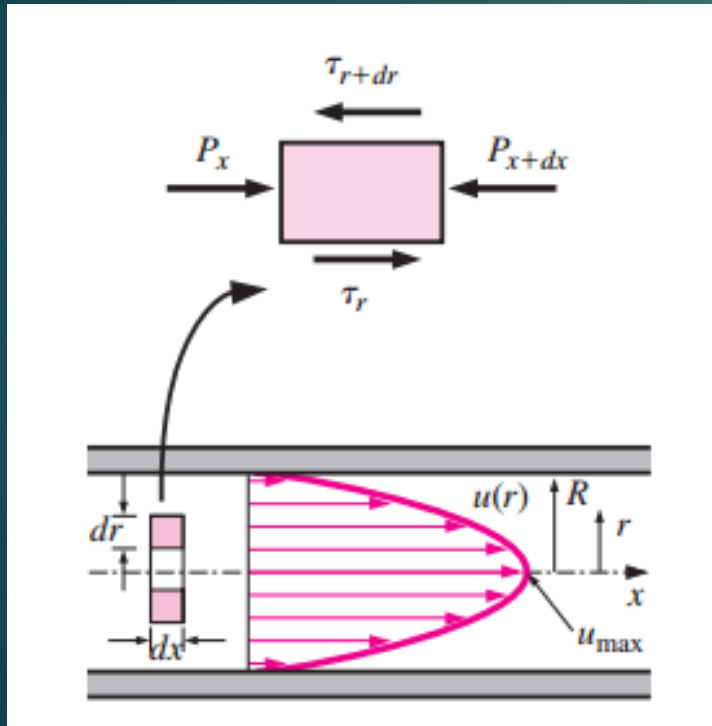


- The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the **hydrodynamically fully developed** region.

# Pipe Flow | Velocity Profile

- The velocity profile can be rewritten as:

$$u_r = 2V_{avg} \left( 1 - \frac{r^2}{R^2} \right)$$



- The maximum velocity occurs at the centerline and is determined by substituting  $r = 0$

$$u_{max} = 2V_{avg}$$

# Pipe Flow | Velocity Profile

## Example 1

Plot the velocity profile if the average velocity is **10 m/s** and the pipe diameter is **0.5 m**.

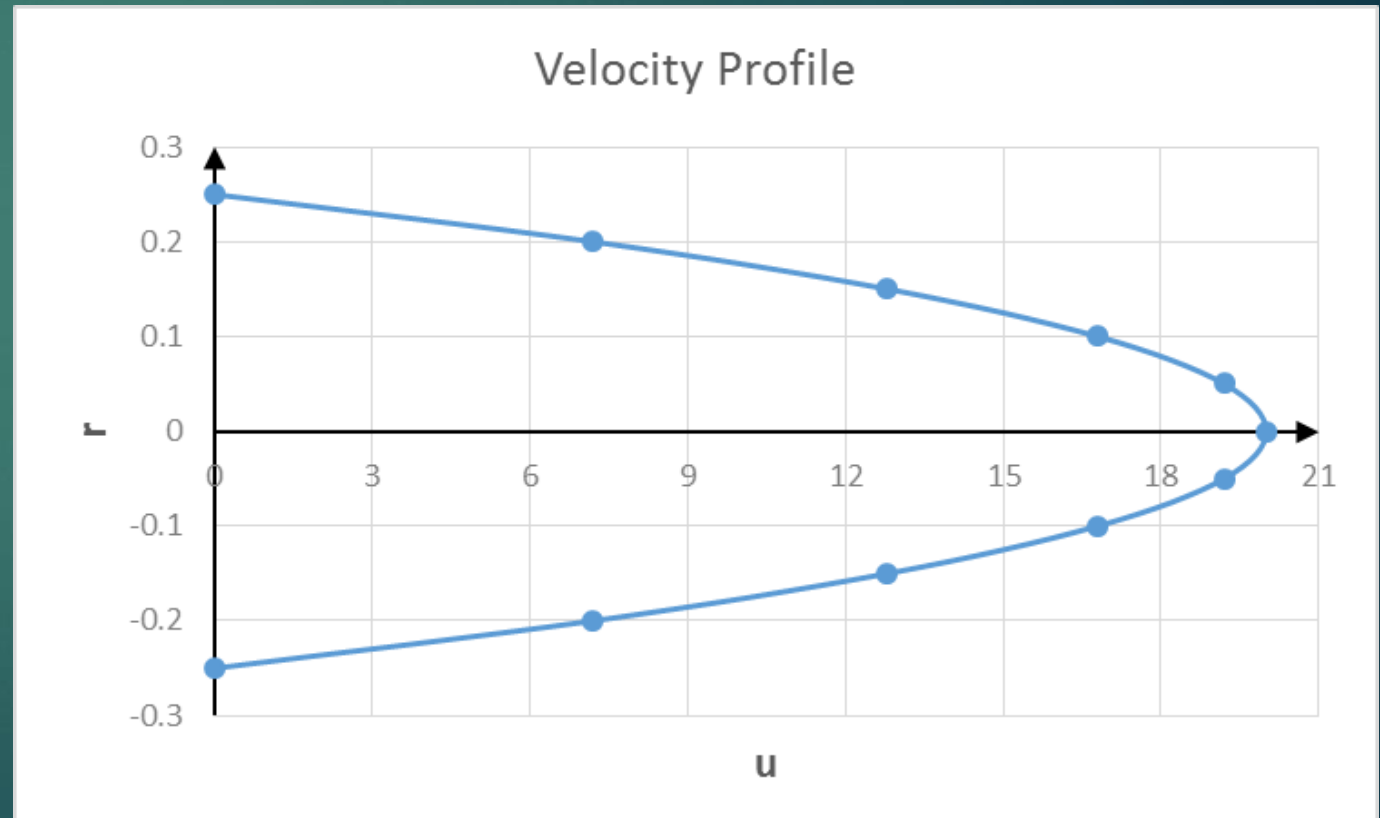
$$u_r = 2V_{avg} \left( 1 - \frac{r^2}{R^2} \right)$$

# Pipe Flow

## Solution

| $V_{ave}$ | R    | r     | u    |
|-----------|------|-------|------|
| 10        | 0.25 | 0.25  | 0    |
|           |      | 0.2   | 7.2  |
|           |      | 0.15  | 12.8 |
|           |      | 0.1   | 16.8 |
|           |      | 0.05  | 19.2 |
|           |      | 0     | 20   |
|           |      | -0.05 | 19.2 |
|           |      | -0.1  | 16.8 |
|           |      | -0.15 | 12.8 |
|           |      | -0.2  | 7.2  |
|           |      | -0.25 | 0    |

$$u_r = 2V_{avg} \left( 1 - \frac{r^2}{R^2} \right)$$

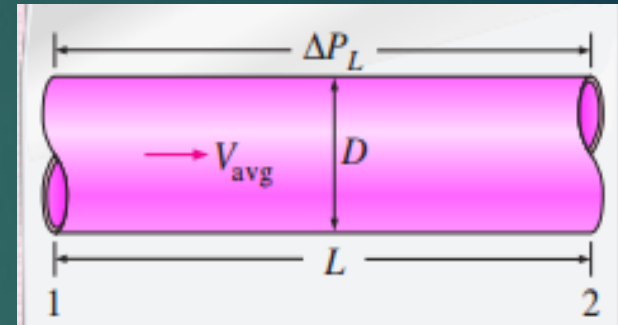


# Pipe Flow | Pressure Drop and Head Loss

- The pressure drop  $\Delta P$  is directly related to the power requirements of the fan or pump to maintain flow.

$$V_{avg} = -\frac{R^2}{8\mu} \left( \frac{dp}{dx} \right)$$

$\mu$  Dynamic viscosity



- The **pressure drop** in a laminar flow by integrating from the  $\frac{dp}{dx}$  on the length  $L$ :

$$\frac{dp}{dx} = \frac{P_2 - P_1}{L} = \text{Constant}$$

- Then substituting it into  $V_{avg}$  expression, will be resulted in:

The SI unit for pressure is the **pascal** (Pa), equal to one newton per square metre (**N/m<sup>2</sup>** or **kg·m<sup>-1</sup>·s<sup>-2</sup>**)

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

## Pipe Flow | Pressure Drop and Head Loss

- The pressure drop is function of the **viscosity** of the fluid, and  $\Delta P$  would be zero if there were no friction.

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

- In practice, it is found convenient to express the pressure loss for **all types of flows** (**laminar** or **turbulent** flows, circular or noncircular pipes, smooth or rough surfaces, horizontal or inclined pipes) as:

$$\text{Pressure loss per unit length } \frac{\Delta P}{L} = f \frac{\rho V_{avg}^2}{D}$$

$$\text{Pressure loss: } \Delta P_L = f \frac{L \rho V_{avg}^2}{D}$$

## Pipe Flow | Pressure Drop and Head Loss

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{avg}^2}{2}$$

$\frac{\rho V_{avg}^2}{2}$  is the dynamic pressure and  $f$  is the dimensionless **Darcy** friction factor.

- Friction factor ( $f$ ) for fully developed laminar flow in a circular pipe is:

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

- This equation shows that in laminar flow, the friction factor is a function of the **Reynolds number only** and is **independent of the roughness of the pipe surface**

## Pipe Flow | Pressure Drop and Head Loss

- **Pressure loss** (usually caused by friction during fluid flow) is **non-recoverable**. As it will be transformed to thermal energy (heat).
- **Pressure drop** due to the difference of static pressure (different elevations) between two points is **recoverable** (gravitational potential energy).
- Therefore, the pressure drop and pressure loss are **equivalent** if:
  - (1) The flow section is *horizontal*
  - (2) The flow section *does not involve* any pump or turbine,
  - (3) The cross-sectional area of the flow section is *constant*, and
  - (4) The velocity profiles at sections 1 and 2 are the *same shape*.



# Pipe Flow | Pressure Drop and Head Loss

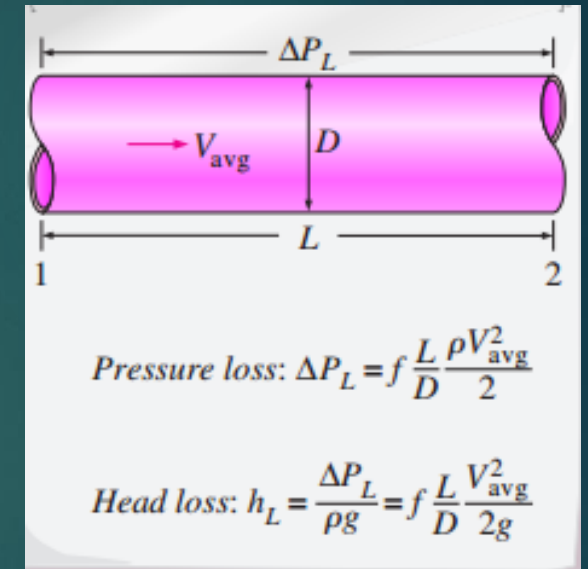
- The **head loss** ( $h_L$ ) for both turbulent and laminar flows can be calculated as:

$$\text{Head loss: } h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

- The head loss  $h_L$  represents the **additional height** that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe.
- The required **pumping power** to overcome the pressure loss is:

$$P_w = \underbrace{Q}_{V_{avg} \times A} \Delta P_L = Q \left[ \underbrace{\rho g}_{\gamma} h_L \right] = Q \gamma h_L$$

$$\frac{m^3}{s} \times \frac{N}{m^3} \times m = \frac{Nm}{s}$$




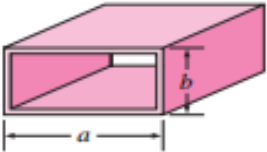
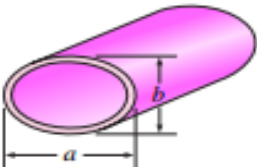
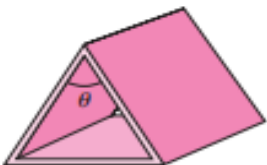
- The power lost by hydraulic jump is:

$$P = \gamma Q \Delta E \rightarrow \begin{cases} \gamma = \text{Specific weight of water} \\ Q = \text{Discharge} \end{cases}$$

# Pipe Flow | Pressure Drop and Head Loss

- The friction factor  $f$  relations are given in the table for fully developed laminar flow in pipes of various cross sections.
- The Reynolds number for flow in these pipes is based on the hydraulic diameter  $D_h = \frac{4A}{P_w}$ , where  $A$  is the cross-sectional area of the pipe and  $p$  is its wetted perimeter.

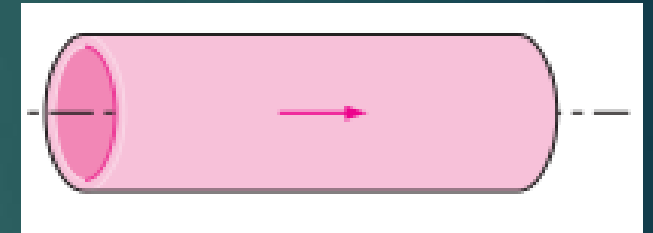
Friction factor for fully developed *laminar flow* in pipes of various cross sections ( $D_h = 4A_c/p$  and  $Re = V_{avg} D_h/\nu$ )

| Tube Geometry   | $a/b$<br>or $\theta^\circ$                              | Friction Factor<br>$f$   |
|---|---|--|
| Circle<br>               | —   | 64.00/Re   |
| Rectangle<br>            | $\frac{a}{b}$<br>1<br>2<br>3<br>4<br>6<br>8<br>$\infty$ | 56.92/Re<br>62.20/Re<br>68.36/Re<br>72.92/Re<br>78.80/Re<br>82.32/Re<br>96.00/Re |
| Ellipse<br>             | $\frac{a}{b}$<br>1<br>2<br>4<br>8<br>16                 | 64.00/Re<br>67.28/Re<br>72.96/Re<br>76.60/Re<br>78.16/Re                         |
| Isosceles triangle<br> | $\theta$<br>10°<br>30°<br>60°<br>90°<br>120°            | 50.80/Re<br>52.28/Re<br>53.32/Re<br>52.60/Re<br>50.96/Re                         |

# Pipe Flow | Horizontal Pipes

- The average velocity for **laminar** flow in a **horizontal** pipe is:

$$\Delta P = \frac{32\mu L V_{avg}}{D^2} \rightarrow V_{avg} = \frac{\Delta P \cdot D^2}{32\mu L}$$



- Then the flow rate for **laminar flow** through a **horizontal** pipe of diameter  $\underline{D}$  and length  $\underline{L}$  becomes:

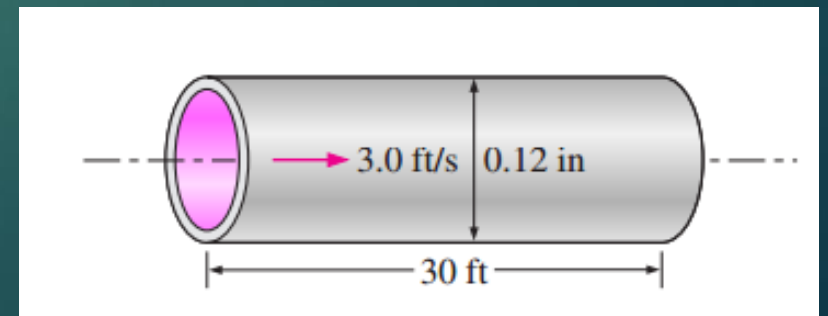
$$Q = \underbrace{V_{avg}}_{\frac{\Delta P D^2}{32\mu L}} \times \underbrace{A}_{\pi \left(\frac{D^2}{4}\right)} = \frac{\Delta P \pi D^4}{128\mu L}$$

## Pipe Flow | Horizontal Pipes

### Example 2

Water at 40°F ( $\rho = 62.42 \text{ lb/ft}^3$  and  $\mu = 1.038 \times 10^{-3} \text{ lb/ft}$ ) is flowing through a 0.12-in- (or 0.010 ft) diameter 30-ft-long horizontal pipe steadily at an average velocity of 3.0 ft/s. Determine:

- (a) The head loss
- (b) The pressure drop, and
- (c) The pumping power requirement to overcome this pressure drop.

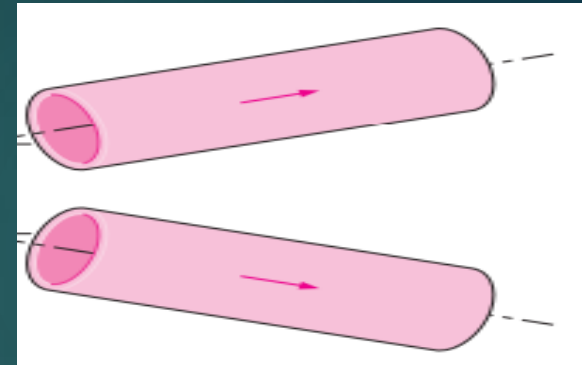


# Pipe Flow | Horizontal and Inclined Pipes

- The **average velocity** and the **flow rate** relations for laminar flow through pipes are, respectively:

$$V_{avg} = \frac{(\Delta P - \rho g L \sin \theta) D^2}{32 \mu L}$$

$$Q = \frac{(\Delta P - \rho g L \sin \theta) \pi D^4}{128 \mu L}$$



Horizontal pipe:  $\dot{V} = \frac{\Delta P \pi D^4}{128 \mu L}$

Inclined pipe:  $\dot{V} = \frac{(\Delta P - \rho g L \sin \theta) \pi D^4}{128 \mu L}$

Uphill flow:  $\theta > 0$  and  $\sin \theta > 0$

Downhill flow:  $\theta < 0$  and  $\sin \theta < 0$

- In inclined pipes, the **combined effect** of pressure difference and gravity drives the flow.
- Gravity** helps downhill flow but opposes uphill flow.
- Therefore, **much greater pressure differences** need to be applied to maintain a specified flow rate in uphill flow

### Example 3

Oil at 20°C ( $\rho = 888 \text{ kg/m}^3$  and  $\mu = 0.800 \text{ kg/m} \cdot \text{s}$ ) is flowing steadily through a 5-cm-diameter 40-m-long pipe. The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively. Determine the flow rate of oil through the pipe assuming the pipe is:

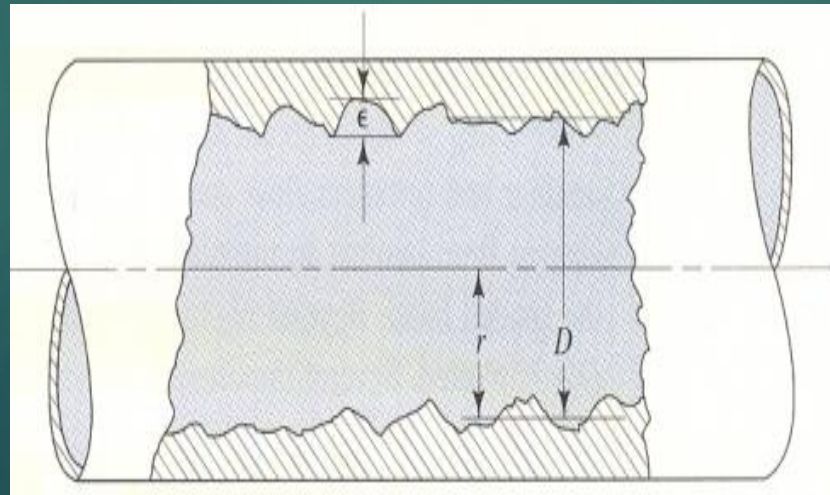
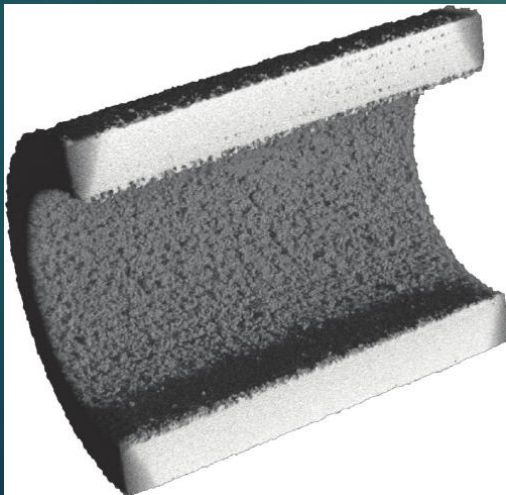
- (a) Horizontal
- (b) Inclined 15° upward, and
- (c) Inclined 15° downward.
- (d) Also verify that the flow through the pipe is laminar.

# Pipe Flow | Moody Chart

- Friction factor ( $f$ ) for fully developed laminar flow in a circular pipe is:

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}$$

- The friction factor in fully developed turbulent pipe flow depends on the **Reynolds number** and the **relative roughness**  $K_e = e/D$ , which is the ratio of the mean height of roughness of the pipe to the pipe diameter.



# Pipe Flow | Moody Chart

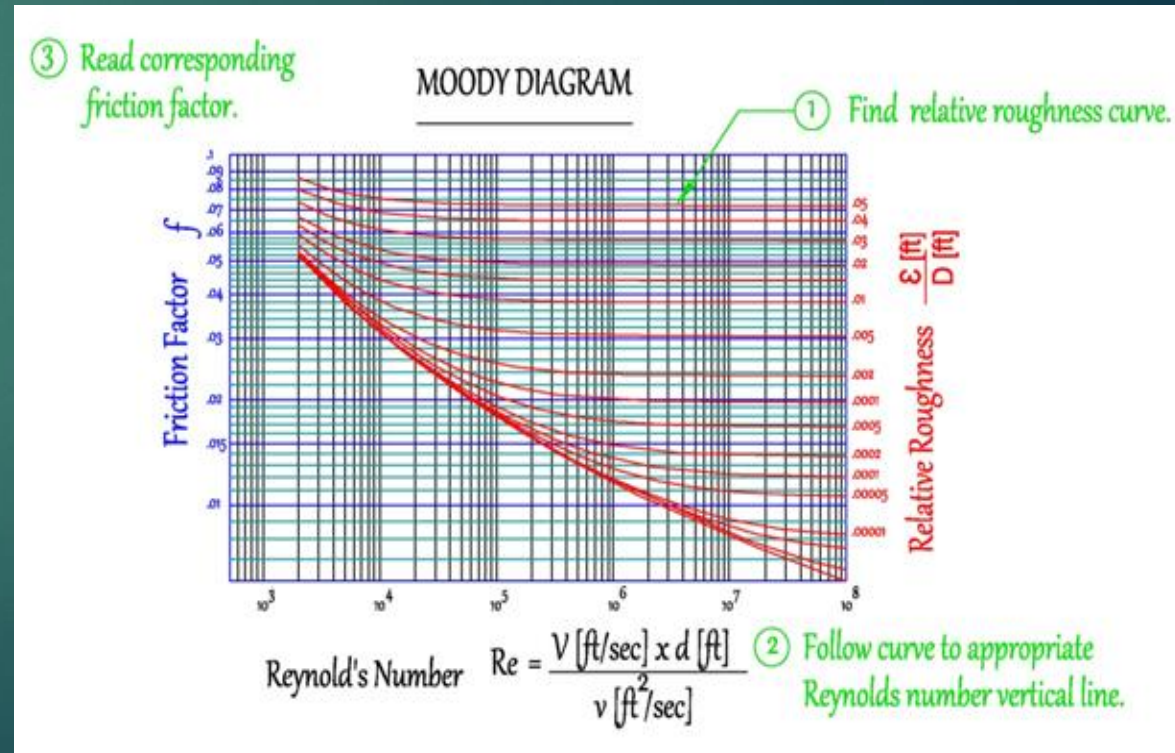
- All results are obtained from **experiments** using artificially roughened surfaces.

$$\frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{\epsilon/D}{3.7} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \quad (\text{turbulent flow})$$

- Although it is developed for **circular pipes**, it can also be used for **noncircular pipes** by replacing the diameter by the hydraulic diameter.

| Relative Roughness, $\epsilon/D$ | Friction Factor, $f$ |
|----------------------------------|----------------------|
| 0.0*                             | 0.0119               |
| 0.00001                          | 0.0119               |
| 0.0001                           | 0.0134               |
| 0.0005                           | 0.0172               |
| 0.001                            | 0.0199               |
| 0.005                            | 0.0305               |
| 0.01                             | 0.0380               |
| 0.05                             | 0.0716               |

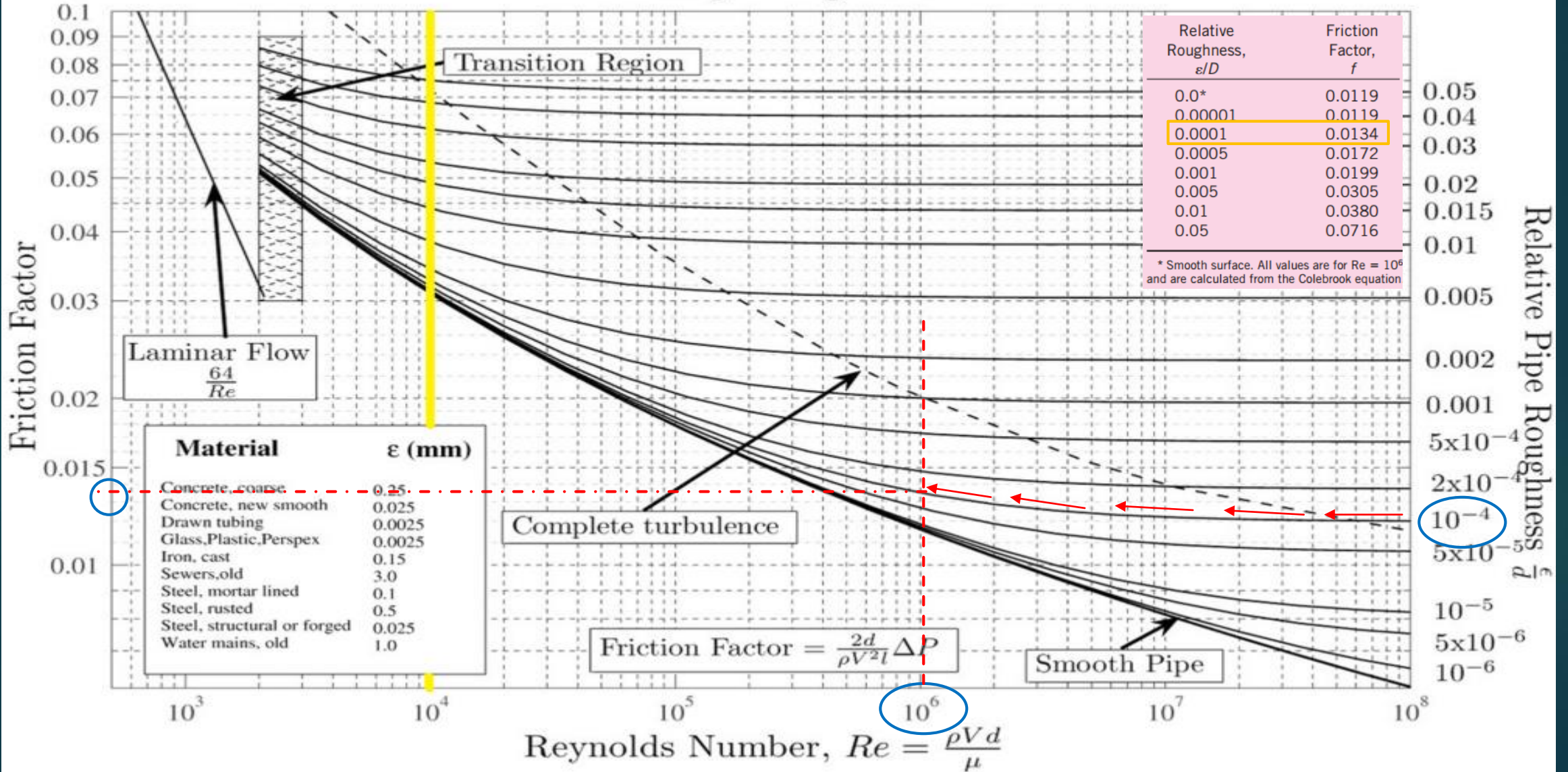
\* Smooth surface. All values are for  $\text{Re} = 10^6$  and are calculated from the Colebrook equation





# Pipe Flow | Moody Chart

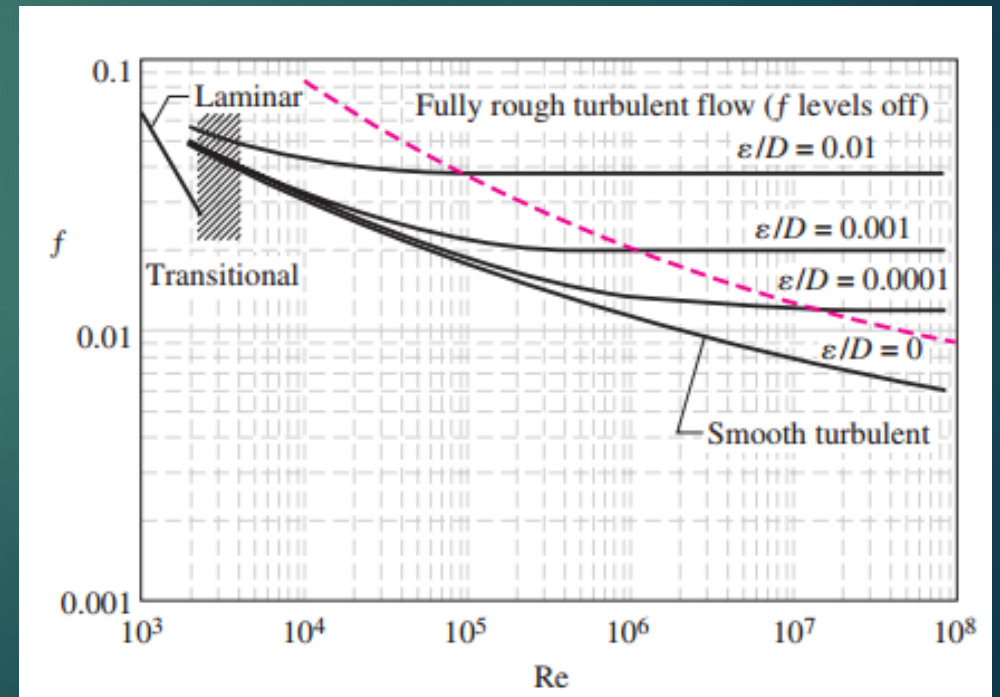
## Moody Diagram



# Pipe Flow | Moody Chart

Observations from the Moody chart:

- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is **independent of surface roughness**.
- The friction factor is a **minimum** for a smooth pipe (but still not zero because of the no-slip condition) and increases with roughness.
- At very large Reynolds numbers the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus **the friction factors are independent of the Reynolds number**



# Pipe Flow | Energy Equation

- The energy equation (**Bernoulli Equation**) in the pipe systems can be written as:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_{pump} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{turbine} + \sum h_L$$

- $h_{pump}$  is the pump head delivered to the fluid,
- $h_{turbine}$  is the turbine head extracted from the fluid,
- $\sum h_L$  is the total head loss between sections 1 and 2,
- $V_1$  and  $V_2$  are the average velocities at sections 1 and 2, respectively.

Pressure is defined as the force over area

$$P = \frac{F}{A} \left( \frac{N}{m^2} \right) \equiv \text{Pascal (Pa)}$$

For incompressible liquids, we have:

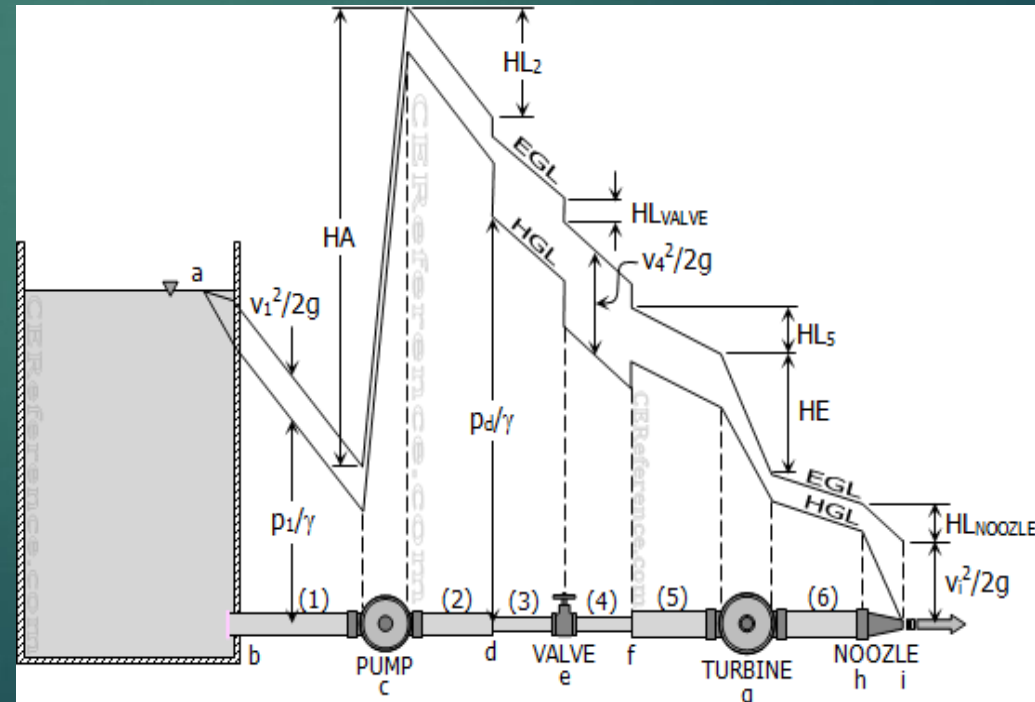
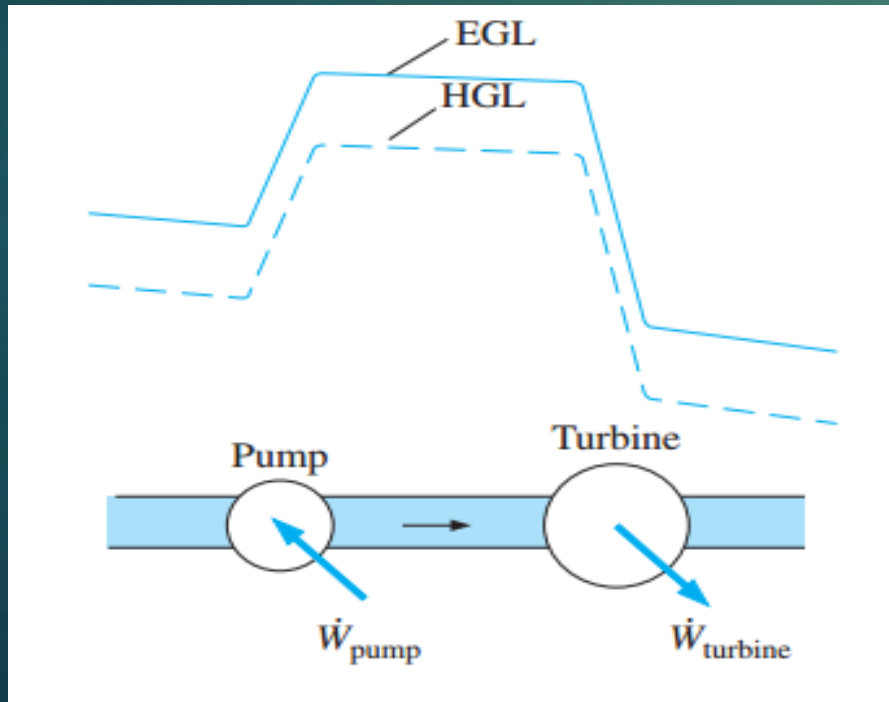
$$P = \rho g h = \gamma h$$

$h$  is the height of the liquid column.

# Pipe Flow | Energy Equation

- The total head loss is:

$$\sum h_L = \left( \frac{fL}{D} + K_{bend} + K_{expansion} + K_{outlet} + K_{turbine} \right) \times \frac{V^2}{2g}$$



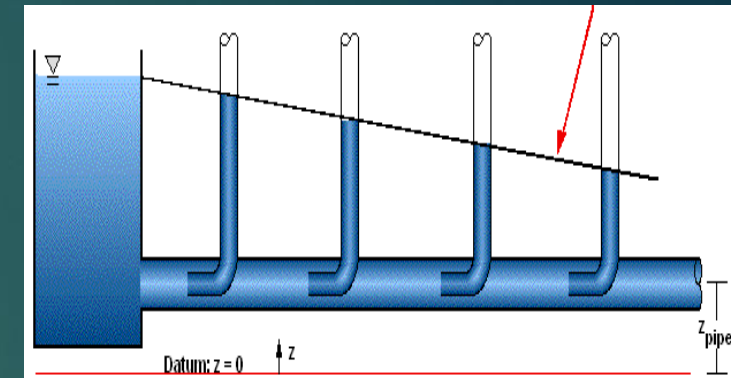
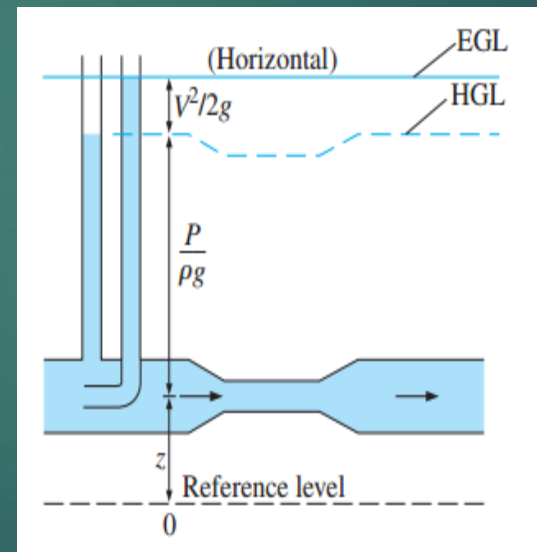
# Pipe Flow | Energy Equation

- EGL (Energy Grade Line) is the total head and measured by pitot tube.

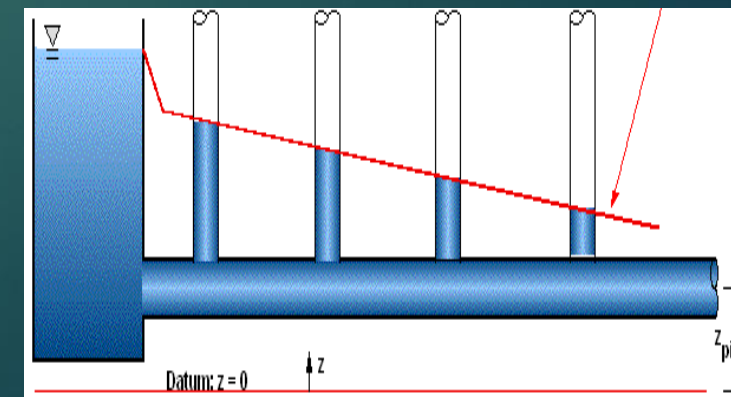
$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

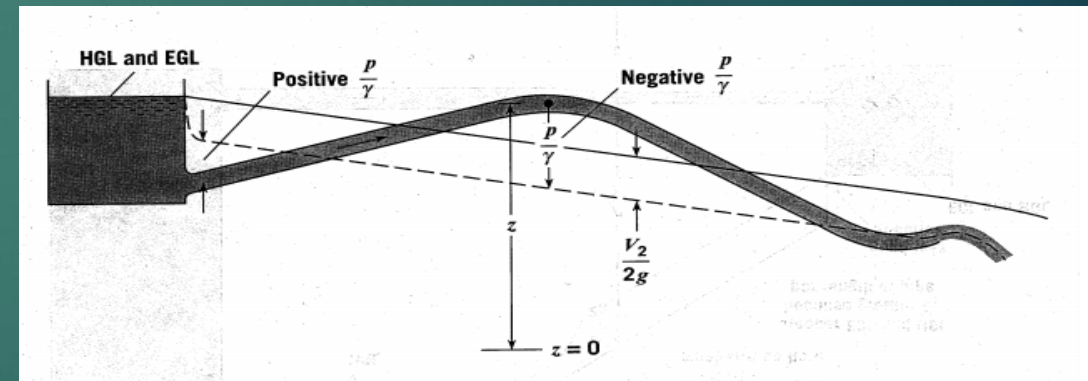
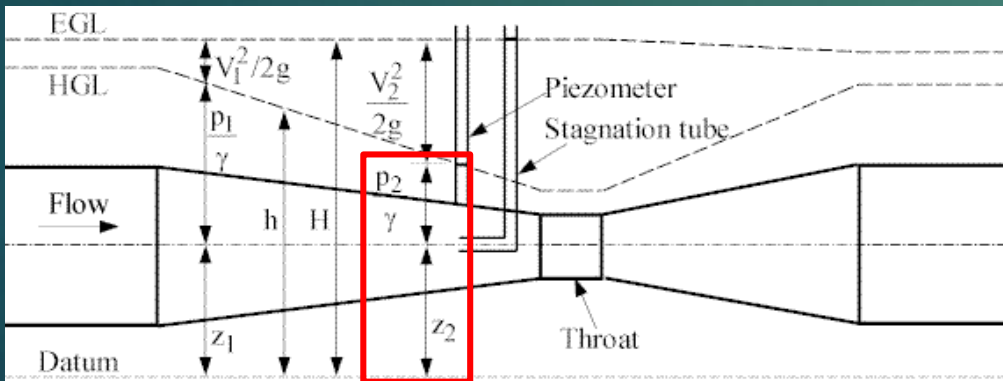
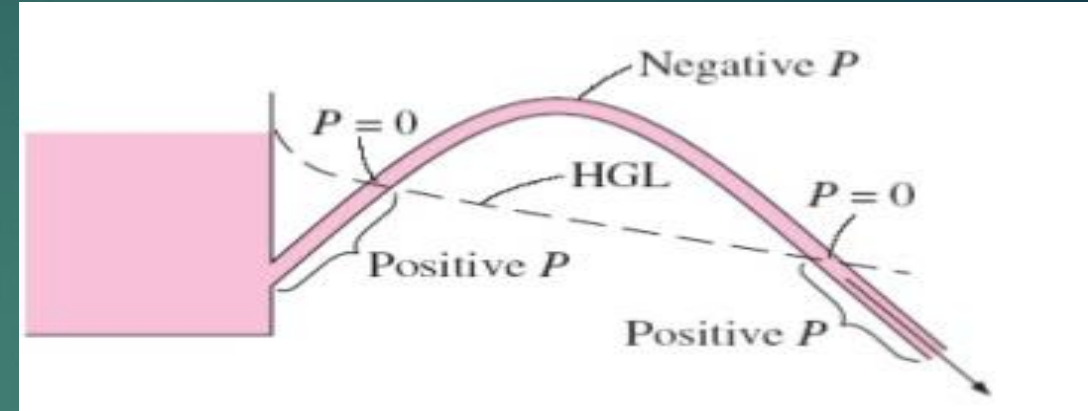
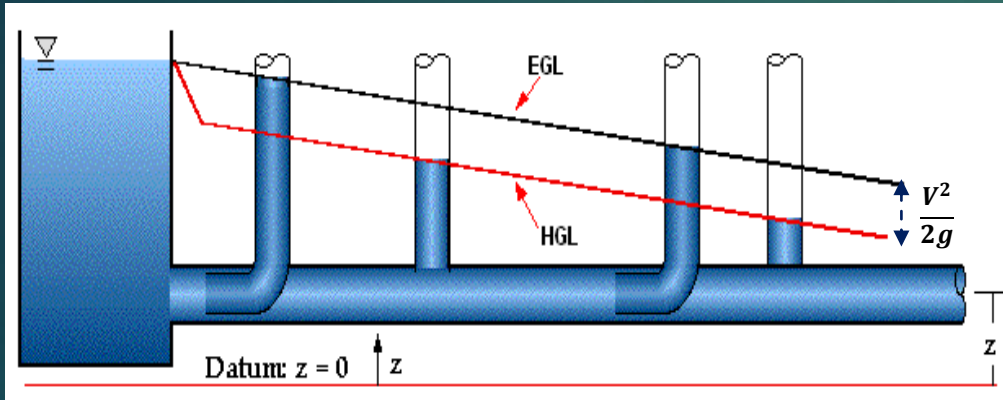
- If the velocity **increases**, then the piezometric head must decrease regardless of any changes in elevation.
- HGL (Hydraulic Grade Line) is a line representing the total head available to the fluid - **minus the velocity head** and measured by **piezometers**.



$$\frac{P}{\rho g} + Z$$



# Pipe Flow | Energy Equation



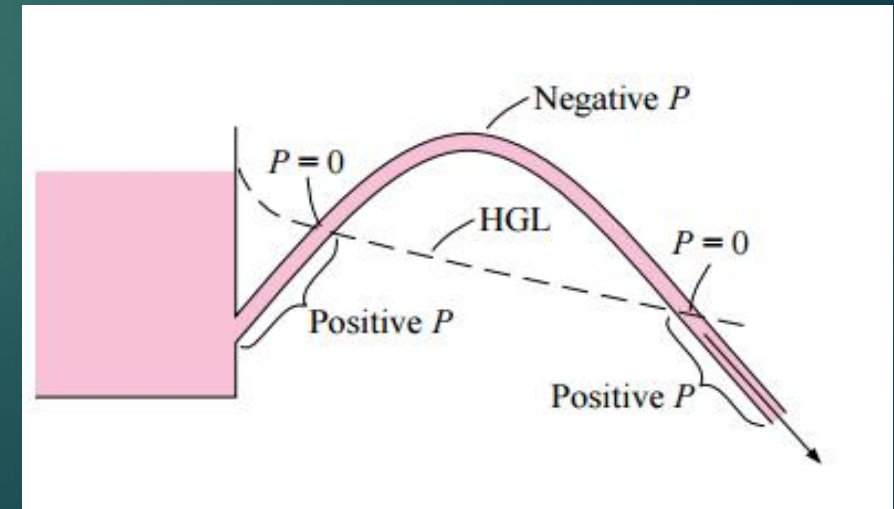
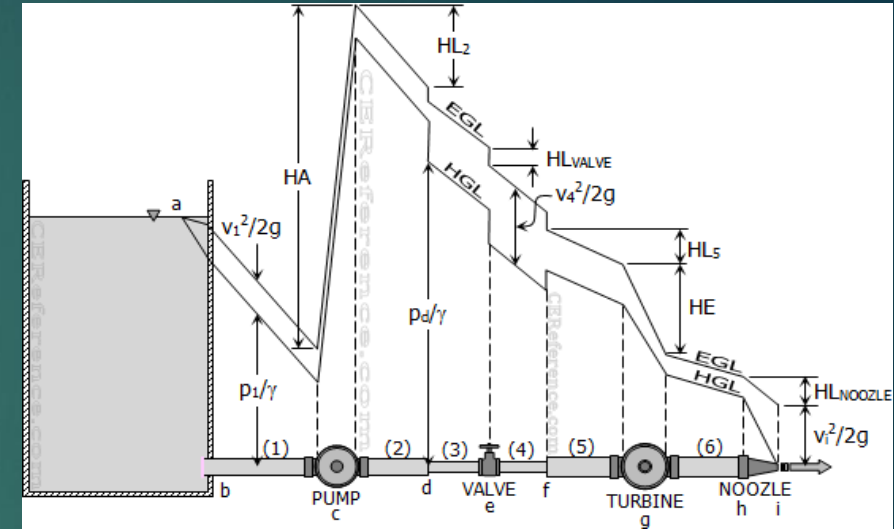
- $\gamma$  is always positive, so pressure is negative  $\rightarrow$  vacuum or pressure is below atmospheric

## Pipe Flow | Energy Equation

- The EGL is always a distance  $\frac{V_2^2}{2g}$  above the HGL.
- These two lines approach each other as the **velocity decreases**, and they diverge as the velocity *increases*.
- The height of the HGL decreases as the velocity increases, and vice versa.
- For open channel flow, the HGL coincides with the **free surface** of the liquid, and the EGL is a distance  $\frac{V_2^2}{2g}$  above the free surface.
- At a pipe exit, the pressure head (gage pressure) is **zero** (atmospheric pressure) and thus the HGL coincides with the pipe exit (Gage pressure is measured **relative** to the local atmospheric pressure).

# Pipe Flow | Energy Equation

- Frictional effects causes the EGL and HGL to **slope downward** in the direction of flow.
- A component that generates significant frictional effects causes a **sudden drop** in both EGL and HGL at that location.
- The pressure of a fluid is zero at locations where **the HGL intersects the fluid**.
- The pressure in a flow section that lies **above** the HGL is **negative**, and the pressure in a section that lies **below** the HGL is **positive**.

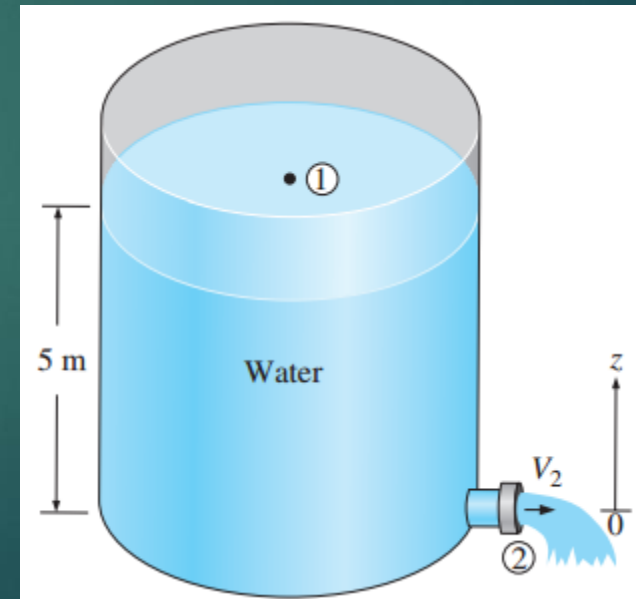




# Pipe Flow | Energy Equation

## Example 4

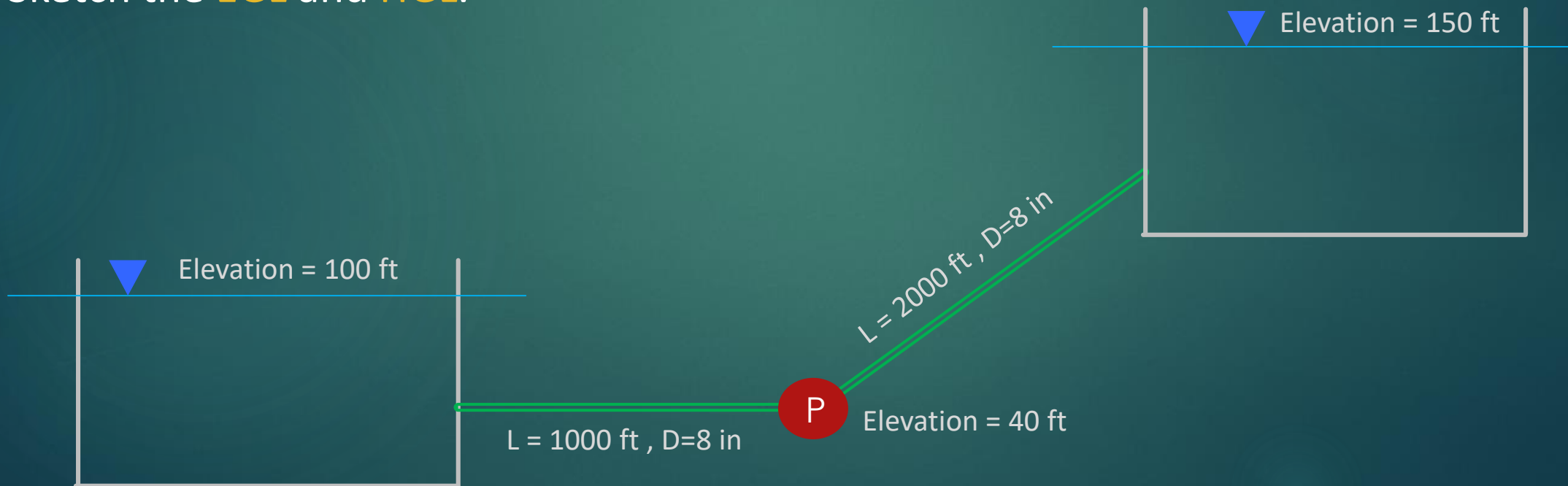
A large tank open to the atmosphere is filled with water to a height of **5 m** from the outlet tap. A tap near the bottom of the tank is now opened, and water flows out from the smooth and rounded outlet. Determine the maximum water velocity at the outlet (no loss).



# Pipe Flow | Energy Equation

## Example 5

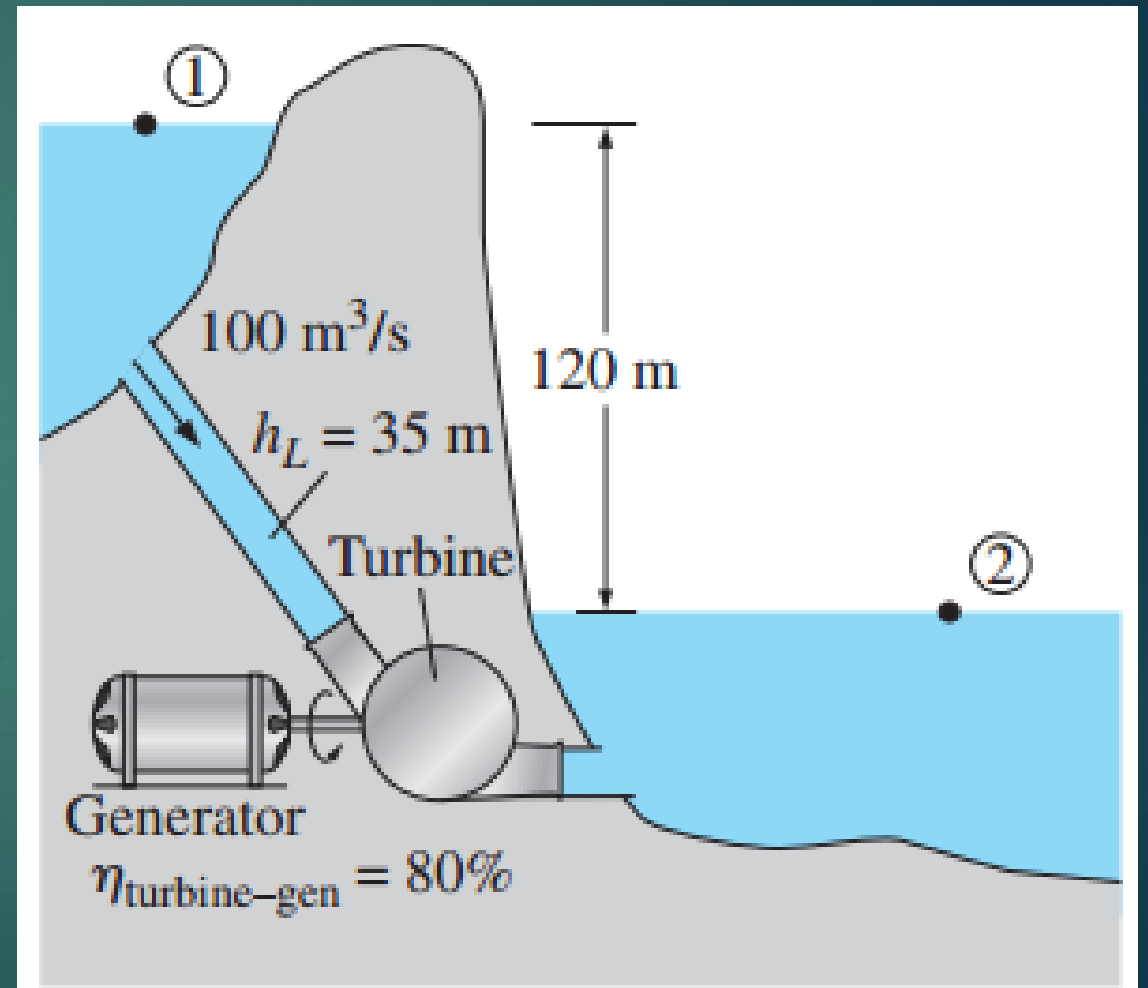
What horsepower must be supplied to the water to pump **2.5 cfs** at **68°F** from the lower to the upper reservoir? Assume the pipe is **steel** (Assume entrance loss coefficient is **0.5**, exit loss coefficient is **1.0**, and the relative roughness is **0.0002**). Sketch the **EGL** and **HGL**.



## Pipe Flow | Energy Equation

### Example 6

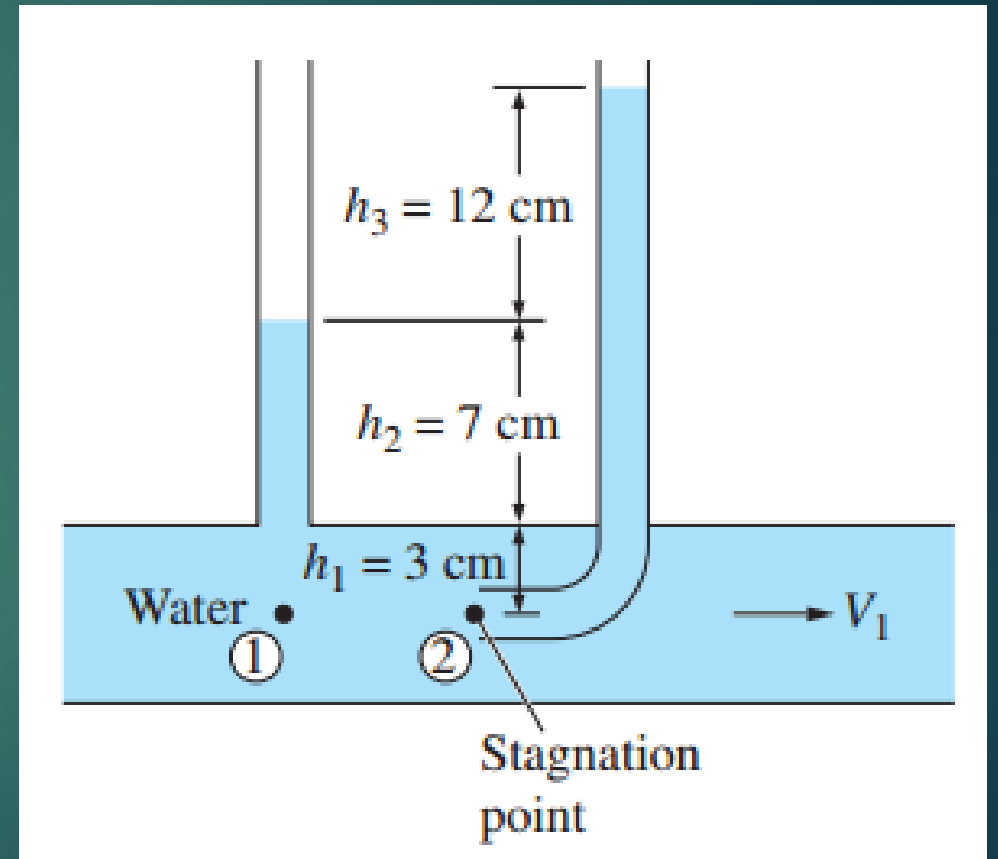
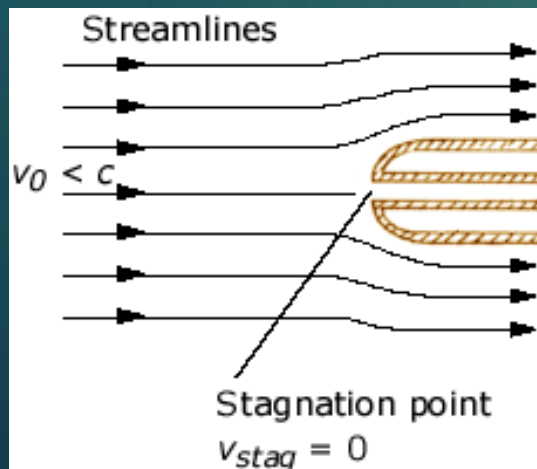
In a hydroelectric power plant,  $100 \text{ m}^3/\text{s}$  of water flows from an elevation of  $120 \text{ m}$  to a turbine, where electric power is generated. The total irreversible head loss in the piping system from point 1 to point 2 (excluding the turbine unit) is determined to be  $35 \text{ m}$ . If the overall efficiency of the turbine–generator is  $80 \text{ percent}$ , estimate the electric power output.



# Pipe Flow | Energy Equation

## Example 7

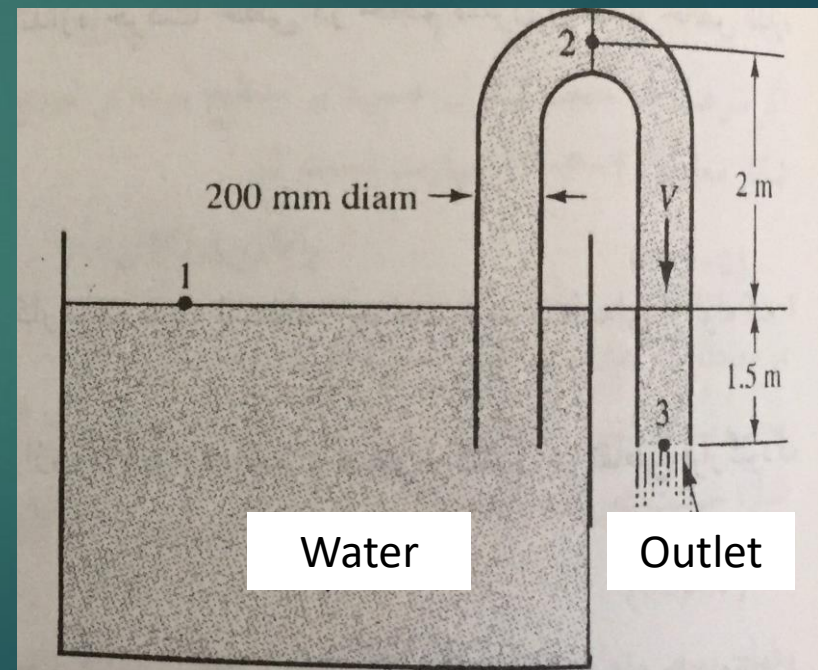
A piezometer and a Pitot tube are tapped into a horizontal water pipe, to measure static and stagnation (static dynamic) pressures. For the indicated water column heights, determine the *velocity at the center of the pipe* (Hint:  $V_2 = 0$ ).



# Pipe Flow | Energy Equation

## Example 8

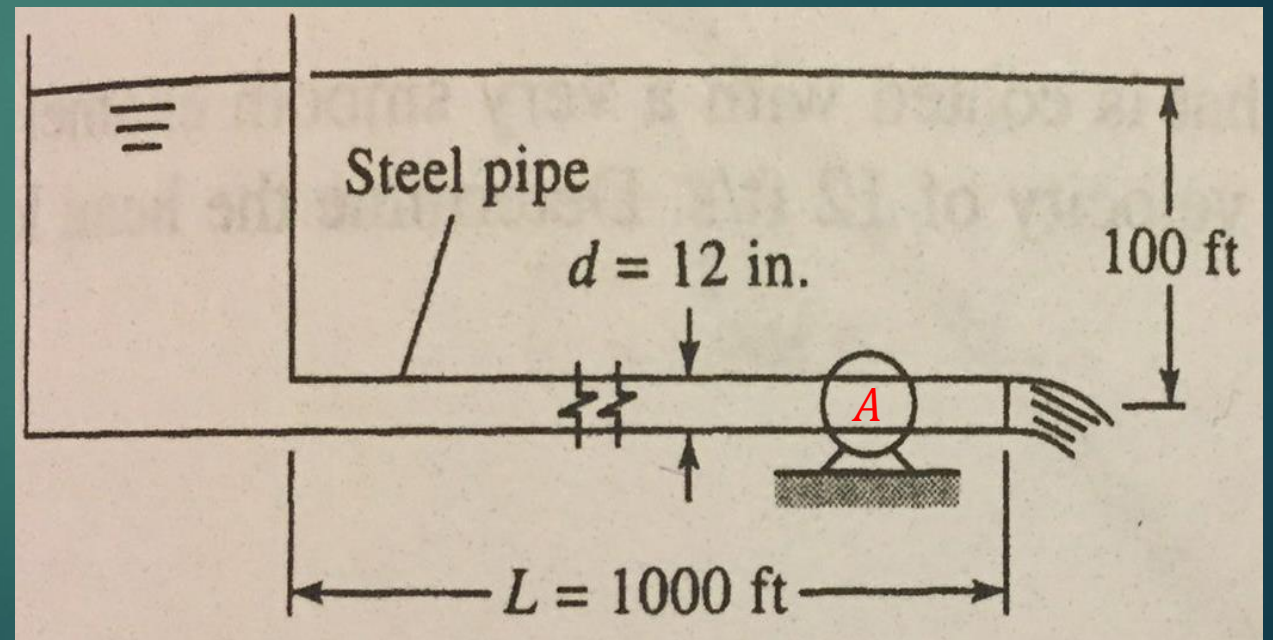
The flow rate in a Siphon is **150 L/s**. What is the head loss between points 1 and 3? And if **67%** of the head loss happens between points 1 to 2, what would be the pressure head at point 2?



## Homework 5

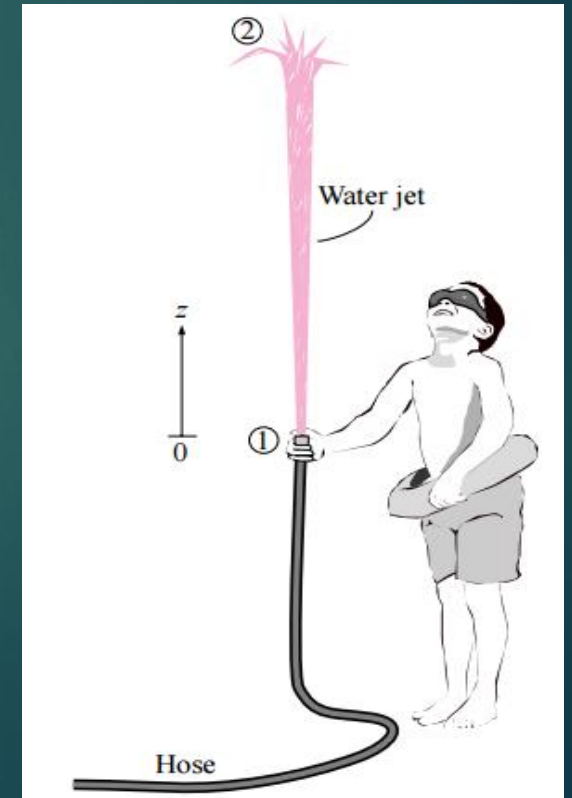
**Q1.** The discharge of water in this system is 20 cfs. Is the machine at A a pump or turbine and what is its horsepower ( $f = 0.0135$ ,  $K_{entrance} = 0.5$ )?

Hint: assume A is a pump. If  $hp < 0$  then, the assumption is not right.



## Homework 5

**Q2.** What is the maximum height that the jet could achieve if water is flowing from a hose (attached to a water main) at **400 kPa** gage (no head loss).





# APPLIED HYDRAULICS

CHAPTER 6:

PIPE FLOW



## Pipe Flow

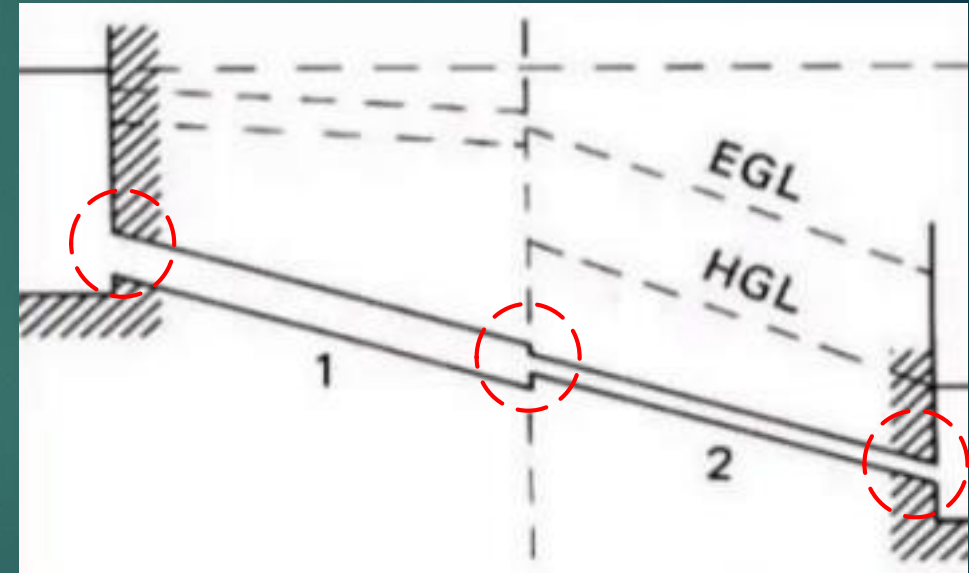
- Pipes in series
- Branching pipes

## Pipe Flow | Pipes in series

- When **two** or **more** pipes of different diameters or roughness are connected in such a way that the fluid follows a **single flow path** throughout the system, the system represents a **series** pipeline.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \sum h_L$$

$$Q = V_1 A_1 = V_2 A_2 \quad \text{as } A_1 \neq A_2 \rightarrow V_1 \neq V_2$$



$$\sum h_L = h_{L-in} + h_{L1} + h_{L-join} + h_{L2} + h_{L-out}$$

## Pipe Flow | Pipes in series

$$\sum h_L = h_{L-in} + h_{L1} + h_{L-join} + h_{L2} + h_{L-out}$$

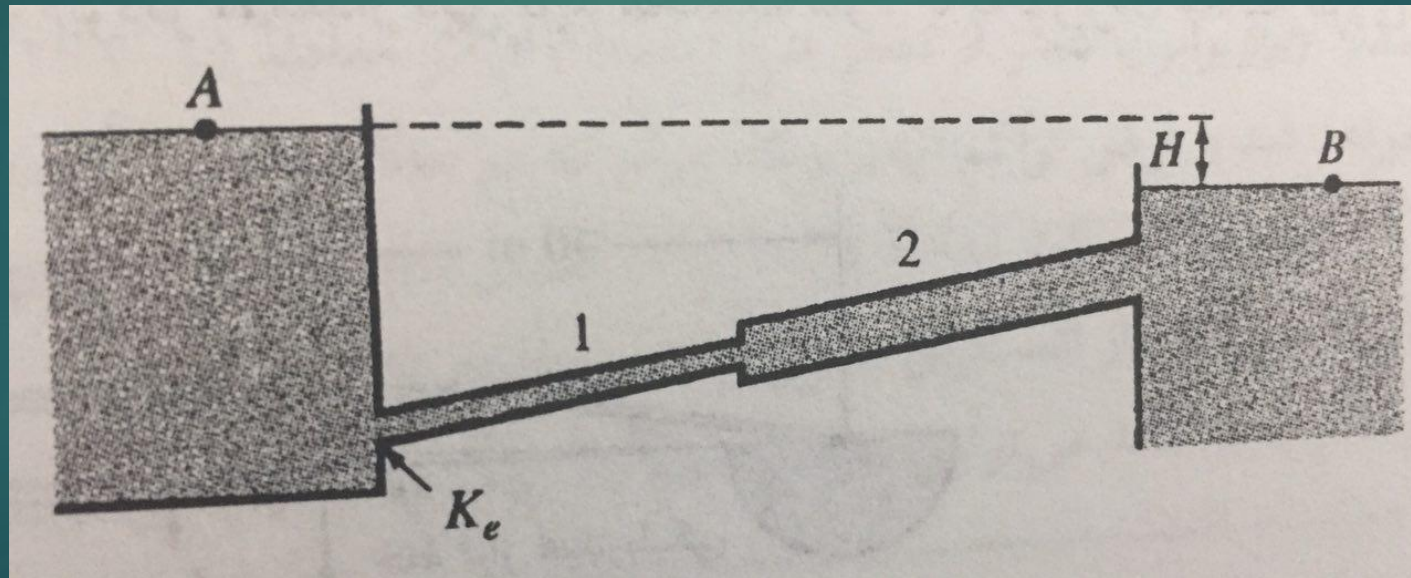
$$h_{L-join} = \frac{(V_1 - V_2)^2}{2g}$$

$$h_{L-join} = \frac{\left(\frac{Q}{A_1} - \frac{Q}{A_2}\right)^2}{2g} = \frac{Q^2 \left(\frac{4}{\pi D_1^2} - \frac{4}{\pi D_2^2}\right)^2}{2g} = \underbrace{\left[\frac{4^2}{2g\pi^2}\right]}_{\substack{SI: 0.0827 \\ ES: 0.0251}} Q^2 \times \left(\frac{1}{D_1^2} - \frac{1}{D_2^2}\right)^2$$

## Pipe Flow | Pipes in series

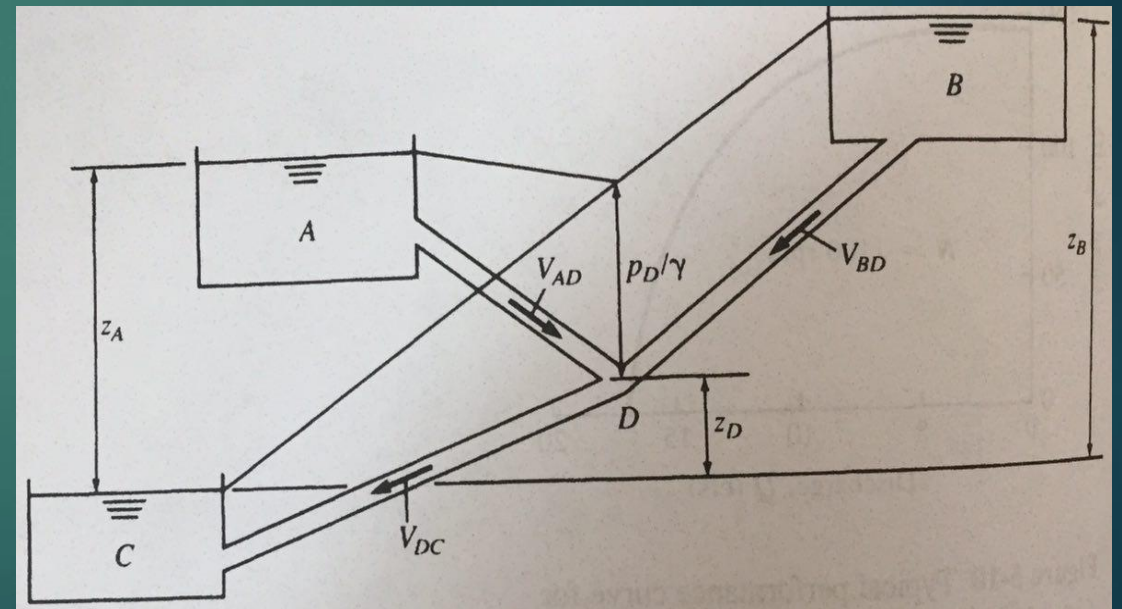
### Example 1

Calculate the flow rate if  $D_1 = 0.6 \text{ m}$ ,  $L_1 = 300 \text{ m}$ ,  $K_{in} = 0.5$ ,  $K_{out} = 1.0$ ,  $f_1 = 0.026$  and  $D_2 = 1 \text{ m}$ ,  $L_2 = 240 \text{ m}$ ,  $f_2 = 0.016$ ,  $H = 6 \text{ m}$ .



## Pipe Flow | Branching pipes

- Consider the case shown in the following figure, where three reservoirs are connected by a **branched-pipe** system.
- The problem here is to determine the **discharge in each pipe** and the **Total Head** (or Total Energy) **at the junction point** (here point D).
- The solution will be obtained by solving the **energy** equation and **continuity** equation.
- Flow goes into junction = Flow out of junction (Continuity)



# Pipe Flow | Branching pipes

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D + h_{L,A-D}$$

$$Z_A = \underbrace{\frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D}_{H_{T,junction}} + h_{L,A-D} \rightarrow Z_A - H_{T,d} = f_{AD} \frac{L_{AD}}{D_{AD}} \times \frac{V_{AD}^2}{2g}$$

$$\frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B = \frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D + h_{L,B-D}$$

$$Z_B = \underbrace{\frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D}_{H_{T,junction}} + h_{L,B-D} \rightarrow Z_B - H_{T,d} = f_{BD} \frac{L_{BD}}{D_{BD}} \times \frac{V_{BD}^2}{2g}$$

$$\frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C = \frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D + h_{L,C-D}$$

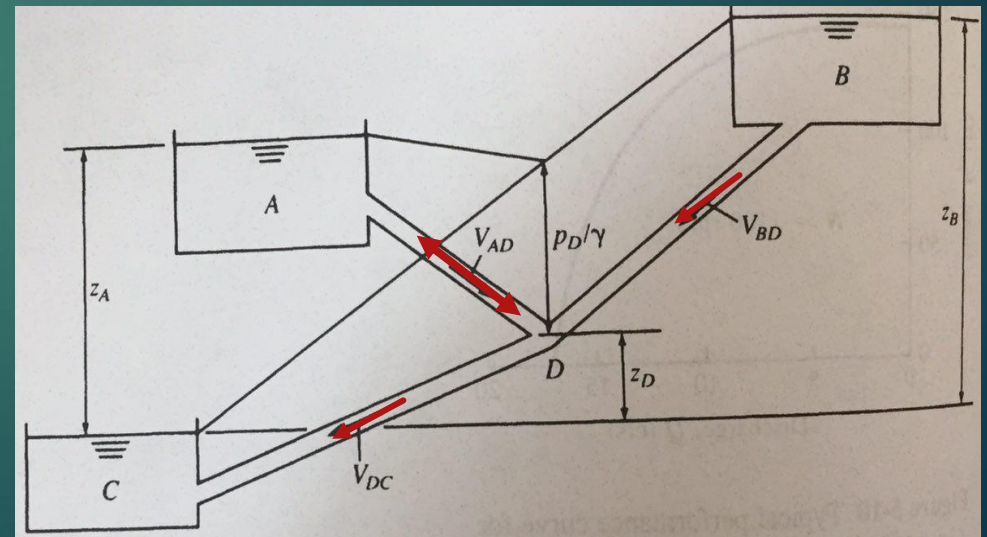
$$Z_C = \underbrace{\frac{P_D}{\gamma} + \frac{V_D^2}{2g} + Z_D}_{H_{T,junction}} + h_{L,C-D} \rightarrow Z_C - H_{T,d} = f_{CD} \frac{L_{CD}}{D_{CD}} \times \frac{V_{CD}^2}{2g}$$

## Pipe Flow | Branching pipes

- It is usual to **ignore minor losses** (entry and exit losses) as practical hand calculations become impossible (fortunately they are often negligible).
- One of the problems is that it is sometimes difficult to decide which **direction** fluid will flow.
- If the direction of flow is not obvious, a direction has to be **assumed**.
- If the **wrong assumption** is made (no physically possible solution will be obtained), then make another assumption.

$$Q_B = Q_A + Q_C$$

$$Q_B + Q_A = Q_C$$



## Pipe Flow | Branching pipes

### Steps:

1. Assume a value of the **Total Head**  $\left(\frac{P}{\gamma} + \frac{V^2}{2g} + Z\right)$  **at the junction**,
2. Assume value for friction factor  $f$
3. Find velocity  $V$
4. Compute Flow rate  $Q$  and Check to see if **continuity** is (or is not) satisfied.
5. If  $Q_{out} > Q_{in}$  then **lower** guess of head, and if  $Q_{out} < Q_{in}$  then **larger** guess of head.



# Pipe Flow | Branching pipes

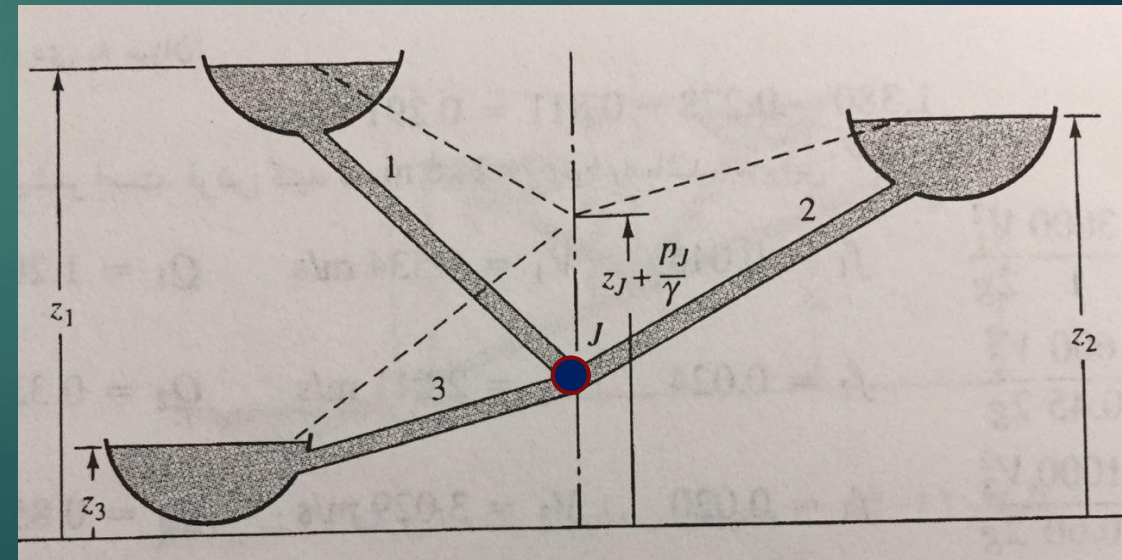
## Example 2

Calculate the flow rate if:

$$L_1 = 3000 \text{ m}, e_1/D_1 = 0.0002, D_1 = 1 \text{ m}, Z_1 = 30 \text{ m}$$

$$L_2 = 600 \text{ m}, e_2/D_2 = 0.002, D_2 = 0.45 \text{ m}, Z_2 = 18 \text{ m}$$

$$L_3 = 1000 \text{ m}, e_3/D_3 = 0.001, D_3 = 0.6 \text{ m}, Z_3 = 9 \text{ m}$$



# Pipe Flow | Branching pipes



| Try   | Line   | e/D    | L    | D    | Area    | Guess HT,J | Guess f  | Z  | Z-HT,J | Flow Direction  | V      | Q      | Q(out)-Q(in) | Check Point   | Re          | f        |
|-------|--------|--------|------|------|---------|------------|----------|----|--------|-----------------|--------|--------|--------------|---------------|-------------|----------|
| Try 1 | 1 to J | 0.0002 | 3000 | 1    | 0.785   | 23         | 0.014    | 30 | 7      | Into Junction   | 1.8083 | 1.4195 | 0.330        | Not close yet | 1808314.132 | 0.01429  |
|       | 2 to J | 0.002  | 600  | 0.45 | 0.15896 | 23         | 0.024    | 18 | -5     | Out of Junction | 1.7509 | 0.2783 |              |               | 787901.6833 | 0.02365  |
|       | J to 3 | 0.001  | 1000 | 0.6  | 0.2826  | 23         | 0.02     | 9  | -14    | Out of Junction | 2.8706 | 0.8112 |              |               | 1722365.815 | 0.0198   |
| Try 2 | 1 to J | 0.0002 | 3000 | 1    | 0.785   | 24         | 0.01429  | 30 | 6      | Into Junction   | 1.6571 | 1.3008 | 0.150        | Not close yet | 1657098.732 | 0.014336 |
|       | 2 to J | 0.002  | 600  | 0.45 | 0.15896 | 24         | 0.02365  | 18 | -6     | Out of Junction | 1.9321 | 0.3071 |              |               | 869466.1919 | 0.02363  |
|       | J to 3 | 0.001  | 1000 | 0.6  | 0.2826  | 24         | 0.0198   | 9  | -15    | Out of Junction | 2.9863 | 0.8439 |              |               | 1791799.502 | 0.0198   |
| Try 3 | 1 to J | 0.0002 | 3000 | 1    | 0.785   | 24.85      | 0.014336 | 30 | 5.15   | Into Junction   | 1.5328 | 1.2032 | 0.007        | Close enough  | 1532775.297 | 0.014336 |
|       | 2 to J | 0.002  | 600  | 0.45 | 0.15896 | 24.85      | 0.02363  | 18 | -6.85  | Out of Junction | 2.0653 | 0.3283 |              |               | 929407.2842 | 0.02363  |
|       | J to 3 | 0.001  | 1000 | 0.6  | 0.2826  | 24.85      | 0.0198   | 9  | -15.85 | Out of Junction | 3.0698 | 0.8675 |              |               | 1841867.629 | 0.0198   |

$$V_1 = 1.532 \text{ m/s}$$

$$Q_1 = 1.2 \text{ m}^3/\text{s}$$

$$V_2 = 2.065 \text{ m/s}$$

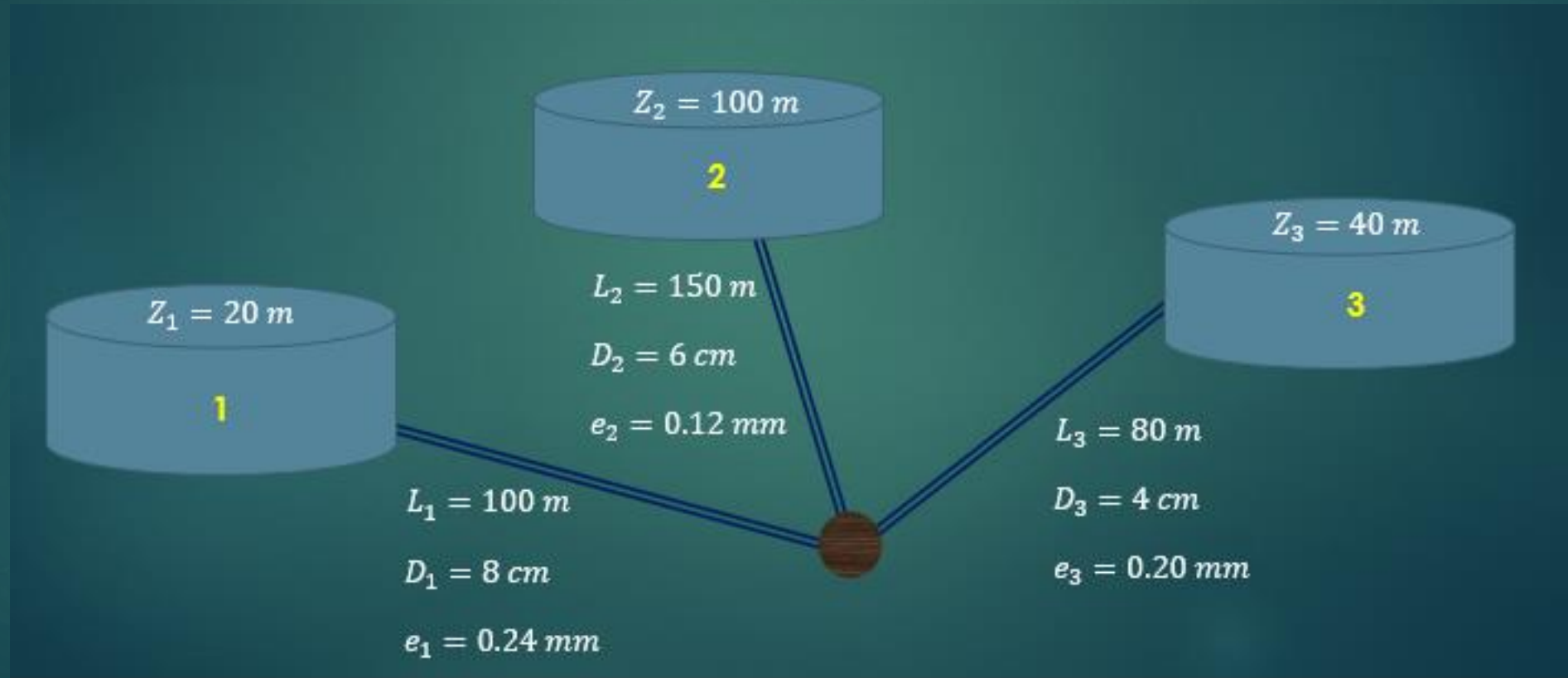
$$Q_2 = 0.32 \text{ m}^3/\text{s}$$

$$V_3 = 3.069 \text{ m/s}$$

$$Q_3 = 0.86 \text{ m}^3/\text{s}$$

## Homework 6

**Q1.** Determine the discharge in the pipes. Neglect minor losses.





# APPLIED HYDRAULICS

CHPATER 7:

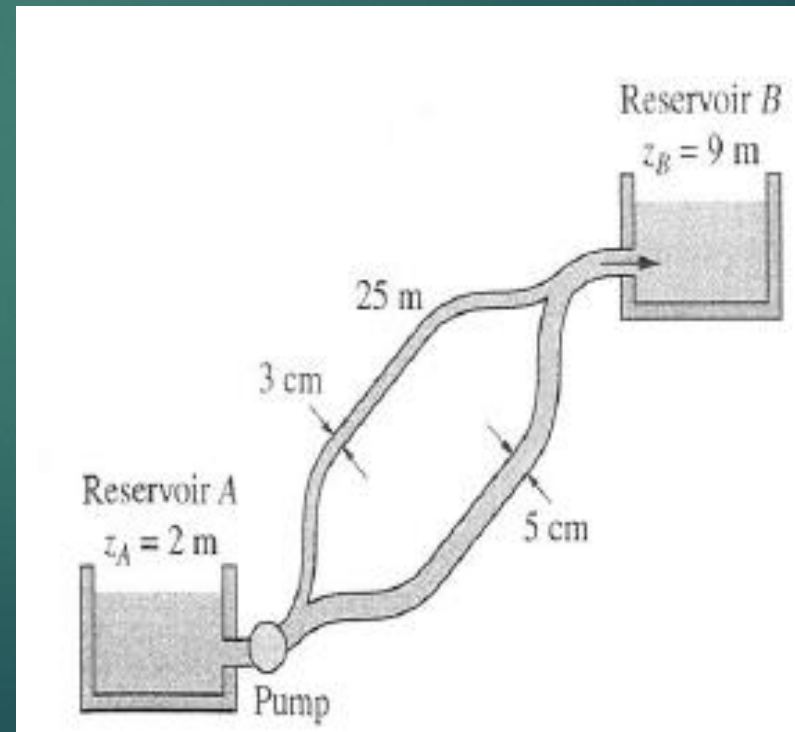
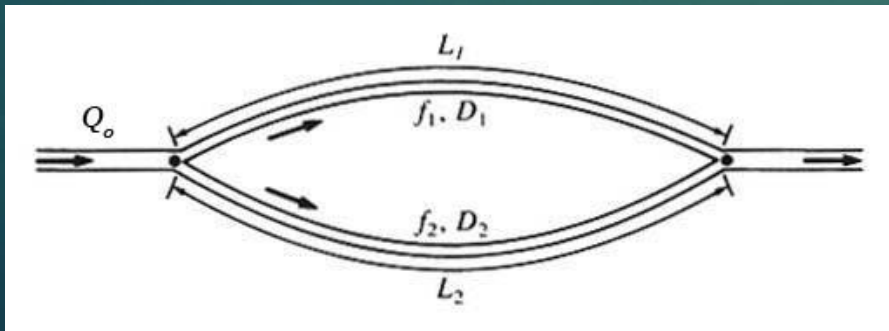
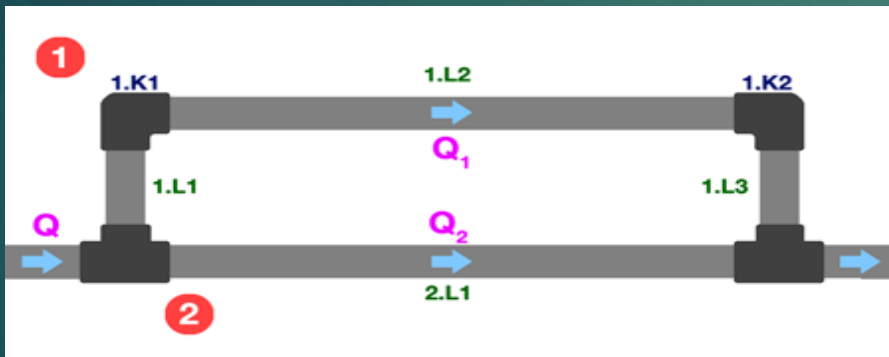
PIPE FLOW

## Pipe Flow

- Parallel pipes
- Pipe networks

# Pipe Flow | Parallel pipes

- A combination of **two or more pipes** connected between two points and then **rejoins**.
- So that the discharge divides at the first junction and rejoins at the next is known as **pipes in parallel**.



## Pipe Flow | Parallel pipes

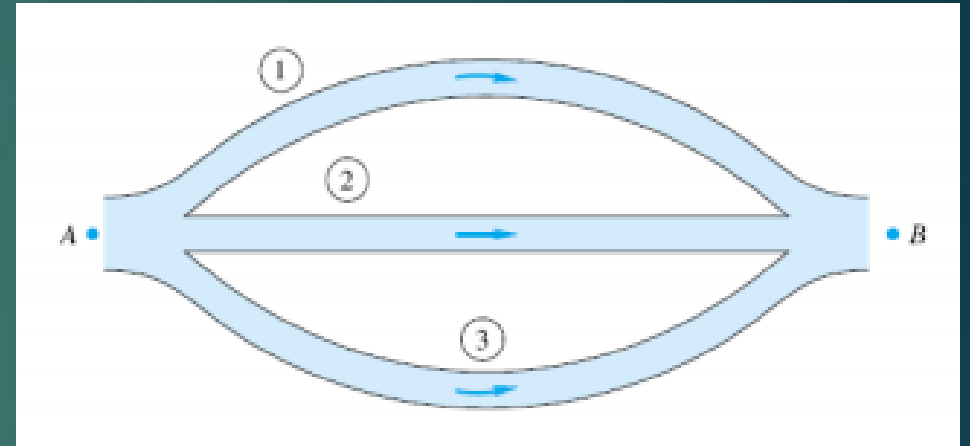
- The energy equation between point 1 and 2 can be written as:

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_L$$

- Applying the continuity equation to the system

$$Q_A = Q_1 + Q_2 + Q_3 = Q_B$$

- Here the head loss between the two junctions is the **same for all pipes**.



$$\underbrace{f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g}}_{h_{L-1}} = \underbrace{f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}}_{h_{L-2}} = \underbrace{f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g}}_{h_{L-3}}$$

## Pipe Flow | Parallel pipes

$$f_1 \frac{L_1 V_1^2}{D_1 2g} = f_2 \frac{L_2 V_2^2}{D_2 2g} \rightarrow \left( \frac{V_1}{V_2} \right)^2 = \frac{f_2}{f_1} \times \frac{L_2}{L_1} \times \frac{D_1}{D_2}$$

$$f_1 \frac{L_1 V_1^2}{D_1 2g} = f_3 \frac{L_3 V_3^2}{D_3 2g} \rightarrow \left( \frac{V_1}{V_3} \right)^2 = \frac{f_3}{f_1} \times \frac{L_3}{L_1} \times \frac{D_1}{D_3}$$

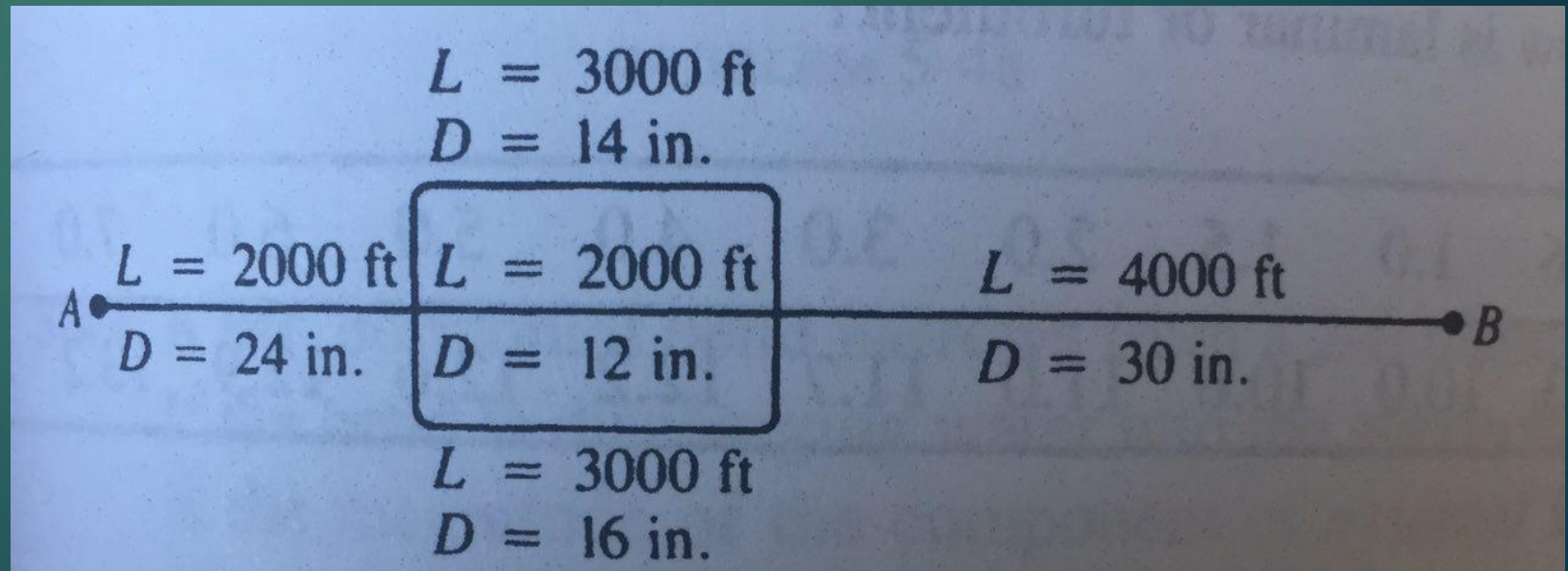
$$f_2 \frac{L_2 V_2^2}{D_2 2g} = f_3 \frac{L_3 V_3^2}{D_3 2g} \rightarrow \left( \frac{V_2}{V_3} \right)^2 = \frac{f_3}{f_2} \times \frac{L_3}{L_2} \times \frac{D_2}{D_3}$$



## Pipe Flow | Parallel pipes

### Example 1

With a flow of  $20 \text{ cfs}$  of water, find the head loss and the division of flow in the pipe from A to B. Assume  $f = 0.030$  for all pipes.



## Pipe Flow | Parallel pipes

- Advantages of a parallel system over a single pipe is **continuous operating of the system** and **cost of maintenance**.
- The parallel piping system can be kept in continuous operation **without failures** unless all of the parallel pipes in a system would fail at the same time.
- The initial cost of a parallel pipeline maybe **higher** than a single one, however, the maintenance and operational costs is much **less** than the series systems.
- The engineering decision about weather many small pipes are better than a large one depends on the **conditions of application** in which the underlying fluid mechanics is a major player.

## Pipe Flow | Parallel pipes

- Number of small pipes with radius  $r$  which are equivalent to one large pipe with radius  $R$ :
- For example, the total flow rate in five pipes of radius  $1 \text{ in}$  is the same as in one pipe of radius  $1.495 \text{ in}$  in laminar flow and  $1.495 \text{ in}$  in turbulent flow.

$$N = \left(\frac{R}{r}\right)^\alpha$$

$$\alpha = 4 \quad \textit{Laminar flow}$$

$$\alpha = \frac{19}{7} \quad \textit{Turbulent flow}$$

$$5 = \left(\frac{R}{1}\right)^\alpha$$

$$\alpha = 4 \quad \textit{Laminar flow}$$

$$R = 1.495$$

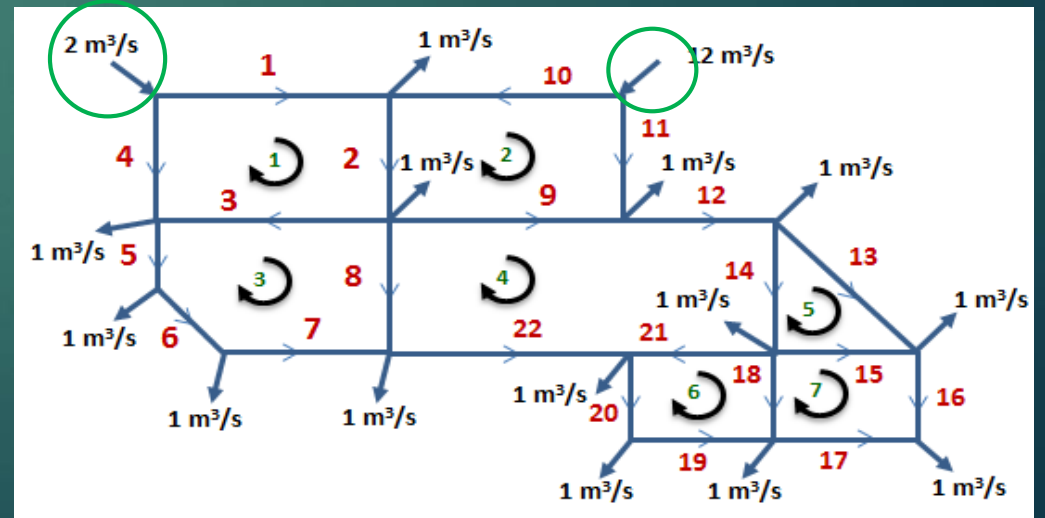
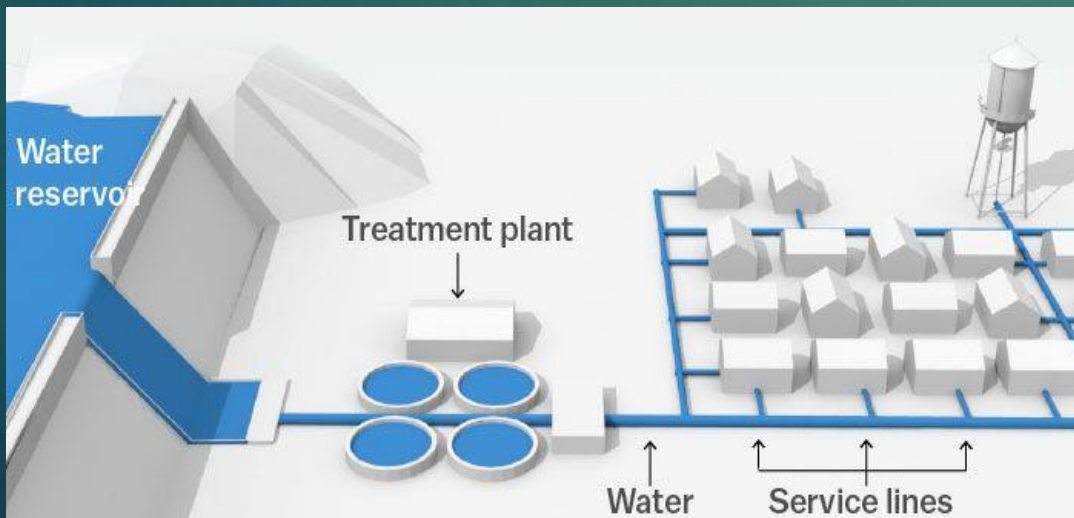
$$\alpha = \frac{19}{7}$$

$$\textit{Turbulent flow}$$

$$R = 1.809$$

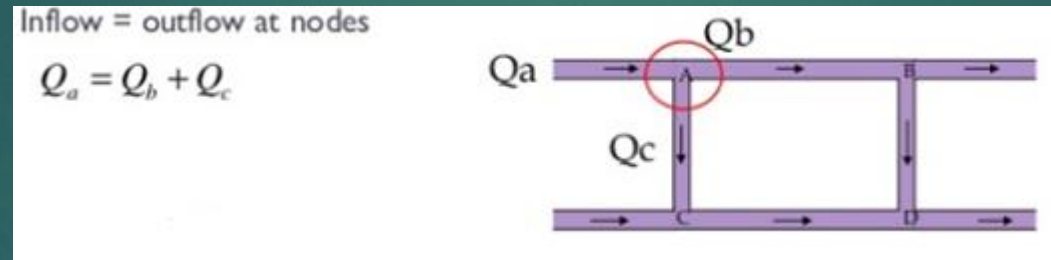
# Pipe Flow | Pipe Networks

- The most common pipe networks are the **water distribution systems**.
- These systems have one or more **sources** (discharge of water into the system) and a number of **loads** such as household and commercial establishment.
- The engineers is often engaged to **design the original system** or to recommend an **expansion to the network**.



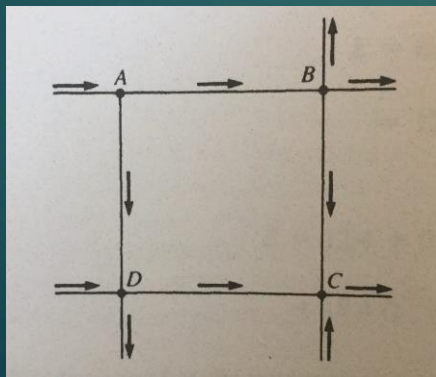
# Pipe Flow | Pipe Networks

- The solution of pipe network problems must satisfy three basic requirements:
- Continuity must be satisfied. The **inflow at each node = the outflow at each node**.



$$\sum Q = 0 \quad (Q_{in} = Q_{out})$$

- Based on energy equation, **summation of head loss** in a close loop is zero.



$$h_{L,A-B} + h_{L,B-C} = h_{L,A-D} + h_{L,D-C}$$

$$\sum h_{L-Loop} = 0$$

$$\sum h_{L-Loop} = \underbrace{h_{L,A-B} + h_{L,B-C}}_{+} \underbrace{h_{L,A-D} + h_{L,D-C}}_{-} = 0$$

## Pipe Flow | Pipe Networks

$$h_L = \frac{f \cdot L V^2}{D \cdot 2g} = \frac{f \cdot L Q^2}{D \cdot 2g A^2} = \left[ \frac{8 \cdot f \cdot L}{g \cdot D^5 \cdot \pi^2} \right] Q^2 = KQ^2$$

$$\sum h_{L-Loop} = 0 \rightarrow \sum KQ^2 = 0$$

- More general form of this equation is:

$$\sum KQ^n = 0$$

- Based on the Darcy-Weisbach equation  $n = 2$  and based on the Hazen-Williams equation  $n = 1.85$

## Hardy Cross Method

- Assume values of  $Q_a$  for each pipe
- Calculate  $\sum h_L = KQ^2$
- If  $\sum h_L$  is **zero**, then the solution is correct. If not, the **correction factor  $\Delta$**  should be applied.

- Calculate the correction factor  $\Delta$

$$\Delta = -\frac{\sum KQ_a^n}{\sum |nKQ_a^{n-1}|} \Rightarrow n = 2 \rightarrow \Delta = -\frac{\sum KQ_a^2}{\sum |2KQ_a|} = -\frac{1}{2} \frac{\sum h_L}{\sum \left| \frac{h_L}{Q_a} \right|}$$

- Then,  $Q_{a,new} = Q_a + \Delta$

- Repeat steps until  $\Delta$  becomes small and  $\sum h_L \cong 0$

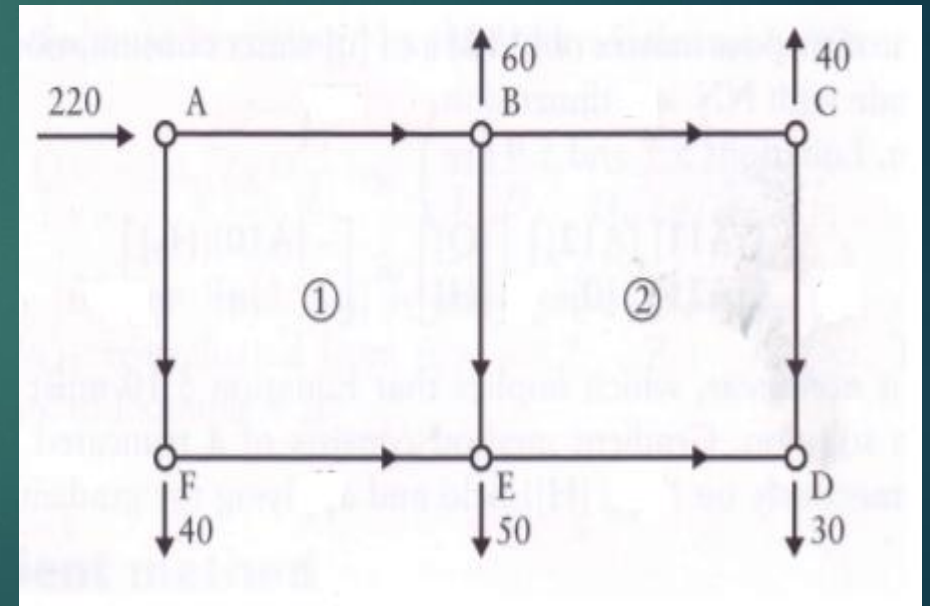
# Pipe Flow | Pipe Networks

## Example 2

Neglecting minor losses in the pipe, determine the flows in the pipes.

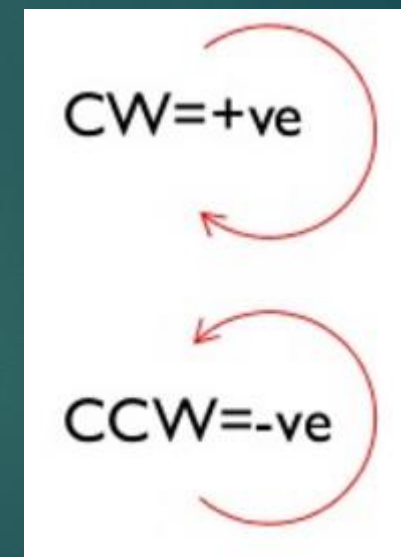
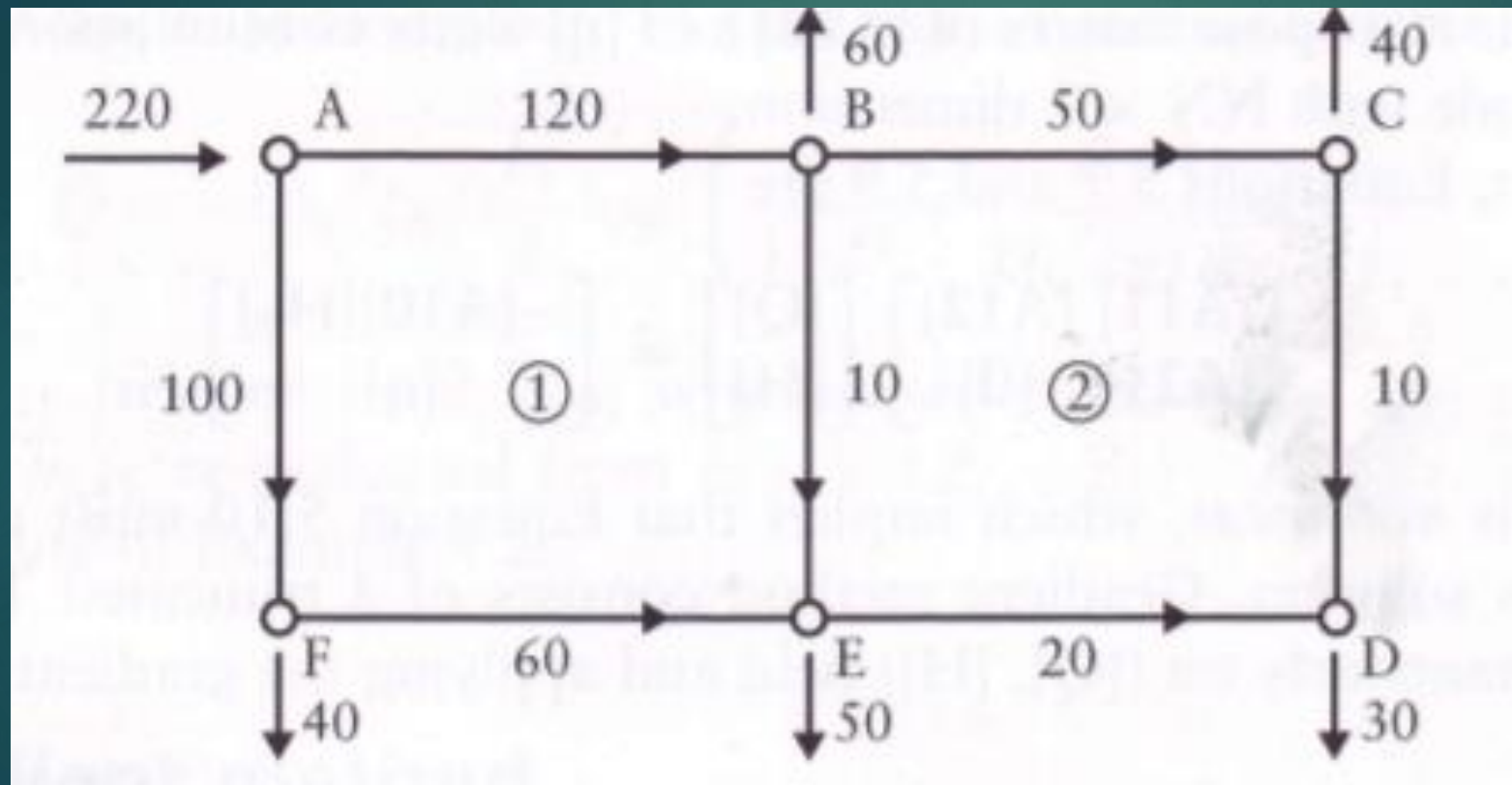
| Pipe          | AB  | BC  | CD  | DE  | EF  | AF  | BE  |
|---------------|-----|-----|-----|-----|-----|-----|-----|
| Length (m)    | 600 | 600 | 200 | 600 | 600 | 200 | 200 |
| Diameter (mm) | 250 | 150 | 100 | 150 | 150 | 200 | 100 |

Roughness size of all pipes = 0.06 mm





# Pipe Flow | Pipe Networks



# Pipe Flow | Pipe Networks

## Loop 1

| Loop 1 | Path | D (mm) | D (m) | e       | e/D     | L      | Qa (Lit/Sec) | Qa (m <sup>3</sup> /s) | A         | Re*10 <sup>5</sup> | f       | k        | hL     | hL/Qa   | Delta (Lit/S) |
|--------|------|--------|-------|---------|---------|--------|--------------|------------------------|-----------|--------------------|---------|----------|--------|---------|---------------|
| Try_1  | A-B  | 250.00 | 0.25  | 0.00006 | 0.00024 | 600.00 | 120.00       | 0.1200                 | 0.0490625 | 6.1146             | 0.0157  | 797.83   | 11.49  | 95.74   | 14.23280      |
|        | B-E  | 100.00 | 0.10  | 0.00006 | 0.0006  | 200.00 | 10.00        | 0.0100                 | 0.00785   | 1.2739             | 0.0205  | 33911.39 | 3.39   | 339.11  |               |
|        | F-E  | 150.00 | 0.15  | 0.00006 | 0.0004  | 600.00 | -60.00       | 0.0600                 | 0.0176625 | 5.0955             | 0.0172  | 11240.49 | -40.47 | 674.43  |               |
|        | A-F  | 200.00 | 0.20  | 0.00006 | 0.0003  | 200.00 | -100.00      | 0.1000                 | 0.0314    | 6.3694             | 0.0162  | 837.45   | -8.37  | 83.74   |               |
|        |      |        |       |         |         |        |              |                        |           |                    |         |          | -33.96 | 1193.03 |               |
| Loop 1 |      |        |       |         |         |        |              |                        |           |                    |         |          |        |         |               |
| Try_2  | A-B  | 250.00 | 0.25  | 0.00006 | 0.00024 | 600.00 | 134.23       | 0.1342                 | 0.0490625 | 6.8399             | 0.0156  | 792.75   | 14.28  | 106.41  | -0.80857      |
|        | B-E  | 100.00 | 0.10  | 0.00006 | 0.0006  | 200.00 | 24.23        | 0.0242                 | 0.00785   | 3.0870             | 0.0188  | 31099.22 | 18.26  | 753.62  |               |
|        | F-E  | 150.00 | 0.15  | 0.00006 | 0.0004  | 600.00 | -45.77       | 0.0458                 | 0.0176625 | 3.8868             | 0.0175  | 11436.54 | -23.96 | 523.42  |               |
|        | A-F  | 200.00 | 0.20  | 0.00006 | 0.0003  | 200.00 | -85.77       | 0.0858                 | 0.0314    | 5.4629             | 0.0164  | 847.78   | -6.24  | 72.71   |               |
|        |      |        |       |         |         |        |              |                        |           |                    |         |          | 2.35   | 1456.17 |               |
| Loop 1 |      |        |       |         |         |        |              |                        |           |                    |         |          |        |         |               |
| Try_3  | A-B  | 250.00 | 0.25  | 0.00006 | 0.00024 | 600.00 | 133.42       | 0.1334                 | 0.0490625 | 6.7987             | 0.01543 | 784.11   | 13.96  | 104.62  | -0.08639      |
|        | B-E  | 100.00 | 0.10  | 0.00006 | 0.0006  | 200.00 | 23.42        | 0.0234                 | 0.00785   | 2.9840             | 0.01875 | 31016.51 | 17.02  | 726.54  |               |
|        | F-E  | 150.00 | 0.15  | 0.00006 | 0.0004  | 600.00 | -46.58       | 0.0466                 | 0.0176625 | 3.9555             | 0.01726 | 11279.70 | -24.47 | 525.36  |               |
|        | A-F  | 200.00 | 0.20  | 0.00006 | 0.0003  | 200.00 | -86.58       | 0.0866                 | 0.0314    | 5.5144             | 0.01616 | 835.38   | -6.26  | 72.32   |               |
|        |      |        |       |         |         |        |              |                        |           |                    |         |          | 0.25   | 1428.84 |               |
| Loop 1 |      |        |       |         |         |        |              |                        |           |                    |         |          |        |         |               |
| Try_4  | A-B  | 250.00 | 0.25  | 0.00006 | 0.00024 | 600.00 | 133.34       | 0.1333                 | 0.0490625 | 6.7943             | 0.01543 | 784.11   | 13.94  | 104.55  | -0.00005      |
|        | B-E  | 100.00 | 0.10  | 0.00006 | 0.0006  | 200.00 | 23.34        | 0.0233                 | 0.00785   | 2.9730             | 0.01875 | 31016.51 | 16.89  | 723.86  |               |
|        | F-E  | 150.00 | 0.15  | 0.00006 | 0.0004  | 600.00 | -46.66       | 0.0467                 | 0.0176625 | 3.9628             | 0.01726 | 11279.70 | -24.56 | 526.33  |               |
|        | A-F  | 200.00 | 0.20  | 0.00006 | 0.0003  | 200.00 | -86.66       | 0.0867                 | 0.0314    | 5.5199             | 0.01616 | 835.38   | -6.27  | 72.40   |               |
|        |      |        |       |         |         |        |              |                        |           |                    |         |          | 0.00   | 1427.14 |               |

# Pipe Flow | Pipe Networks

## Loop 2

| Loop 2 | Path | D (mm) | D (m) | e       | e/D    | L      | Qa (Lit/Sec) | Qa (m <sup>3</sup> /s) | A         | Re*10 <sup>5</sup> | f      | k        | hf     | hL/Qa   | Delta (Lit/S) |
|--------|------|--------|-------|---------|--------|--------|--------------|------------------------|-----------|--------------------|--------|----------|--------|---------|---------------|
| Try_1  | B-C  | 150.00 | 0.15  | 0.00006 | 0.0004 | 600.00 | 50.00        | 0.0500                 | 0.0176625 | 4.2463             | 0.0174 | 11371.19 | 28.43  | 568.56  | -2.57827      |
|        | C-D  | 100.00 | 0.10  | 0.00006 | 0.0006 | 200.00 | 10.00        | 0.0100                 | 0.00785   | 1.2739             | 0.0205 | 33911.39 | 3.39   | 339.11  |               |
|        | E-D  | 150.00 | 0.15  | 0.00006 | 0.0004 | 600.00 | -20.00       | 0.0200                 | 0.0176625 | 1.6985             | 0.0189 | 12351.46 | -4.94  | 247.03  |               |
|        | B-E  | 100.00 | 0.10  | 0.00006 | 0.0006 | 200.00 | -23.42       | 0.0234                 | 0.00785   | 2.9834             | 0.0189 | 31264.64 | -17.15 | 732.22  |               |
|        |      |        |       |         |        |        |              |                        |           |                    |        |          | 9.73   | 1886.92 |               |
| Loop 2 |      |        |       |         |        |        |              |                        |           |                    |        |          |        |         |               |
| Try_2  | B-C  | 150.00 | 0.15  | 0.00006 | 0.0004 | 600.00 | 47.42        | 0.0474                 | 0.0176625 | 4.0273             | 0.0174 | 11371.19 | 25.57  | 539.24  | -0.00294      |
|        | C-D  | 100.00 | 0.10  | 0.00006 | 0.0006 | 200.00 | 7.42         | 0.0074                 | 0.00785   | 0.9454             | 0.0205 | 33911.39 | 1.87   | 251.68  |               |
|        | D-E  | 150.00 | 0.15  | 0.00006 | 0.0004 | 600.00 | -22.58       | 0.0226                 | 0.0176625 | 1.9175             | 0.0189 | 12351.46 | -6.30  | 278.87  |               |
|        | E-B  | 100.00 | 0.10  | 0.00006 | 0.0006 | 200.00 | -26.00       | 0.0260                 | 0.00785   | 3.3119             | 0.0189 | 31264.64 | -21.13 | 812.83  |               |
|        |      |        |       |         |        |        |              |                        |           |                    |        |          | 0.01   | 1882.62 |               |
| Loop 2 |      |        |       |         |        |        |              |                        |           |                    |        |          |        |         |               |
| Try_3  | B-C  | 150.00 | 0.15  | 0.00006 | 0.0004 | 600.00 | 47.42        | 0.0474                 | 0.0176625 | 4.0271             | 0.0174 | 11371.19 | 25.57  | 539.21  | 0.00000       |
|        | C-D  | 100.00 | 0.10  | 0.00006 | 0.0006 | 200.00 | 7.42         | 0.0074                 | 0.00785   | 0.9451             | 0.0205 | 33911.39 | 1.87   | 251.58  |               |
|        | D-E  | 150.00 | 0.15  | 0.00006 | 0.0004 | 600.00 | -22.58       | 0.0226                 | 0.0176625 | 1.9177             | 0.0189 | 12351.46 | -6.30  | 278.91  |               |
|        | E-B  | 100.00 | 0.10  | 0.00006 | 0.0006 | 200.00 | -26.00       | 0.0260                 | 0.00785   | 3.3123             | 0.0189 | 31264.64 | -21.14 | 812.92  |               |
|        |      |        |       |         |        |        |              |                        |           |                    |        |          | 0.00   | 1882.62 |               |

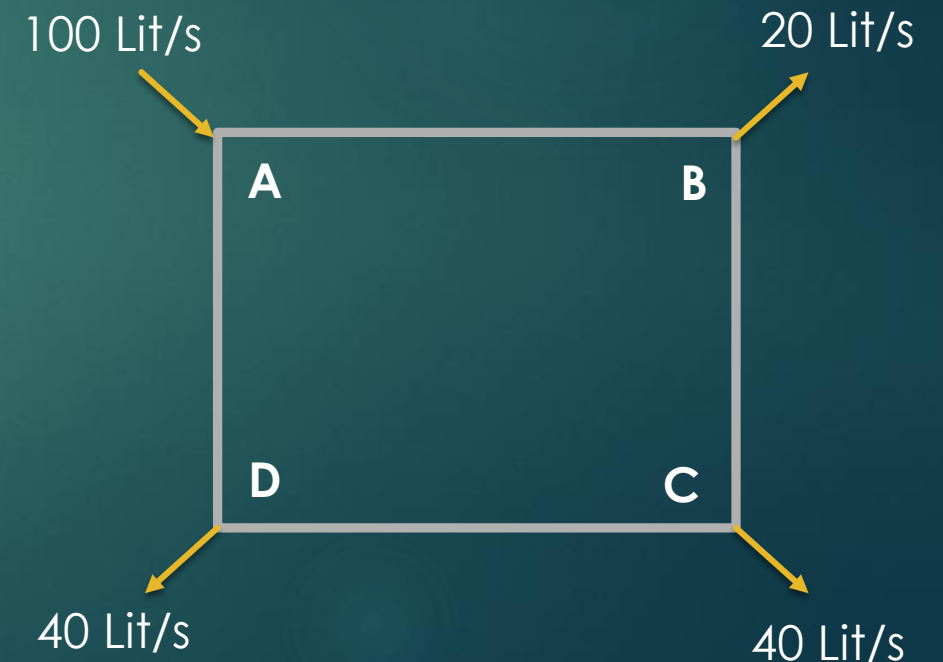
## Final Results

| Pipe | Q (L/s) | hf (m) |
|------|---------|--------|
| A-B  | 133.34  | 13.96  |
| B-E  | 26      | 21.13  |
| F-E  | 46.66   | 24.47  |
| A-F  | 86.66   | 6.26   |
| B-C  | 47.42   | 25.57  |
| C-D  | 7.42    | 1.87   |
| E-D  | 22.58   | 6.3    |

## Homework 7

**Q1.** For the following loop shown, all pipes are **1 km** long and **300 mm** in diameter, with a friction factor of **0.0163**. if minor losses can be neglected, find the discharge in all the pipes.

Due: First session after spring break (Tuesday, April 4<sup>th</sup>)





# APPLIED HYDRAULICS

## CHAPTER 8:

# DISCHARGE MEASUREMENT

## Pipe Flow

- Instruments for Discharge Measurement

## Pipe Flow | Instruments

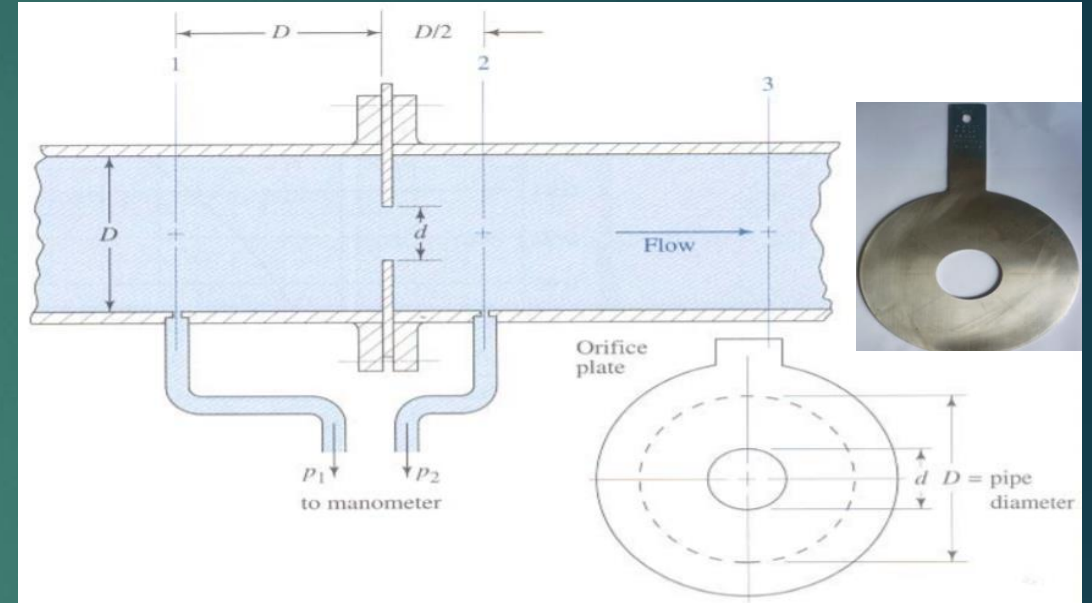
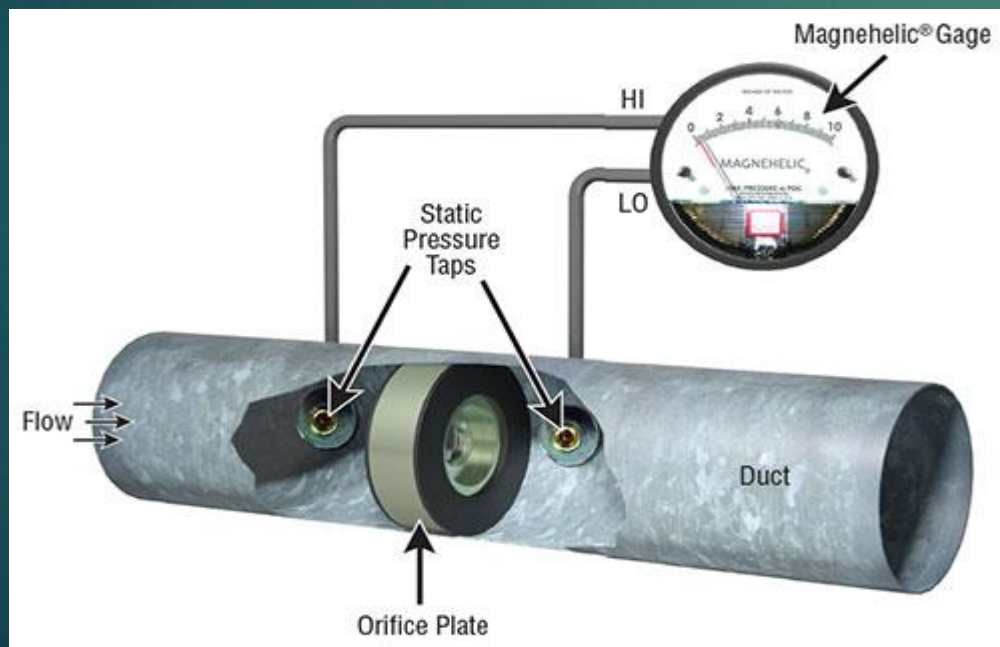
- The methods of flow measurement can broadly be classified as either **Direct** or **Indirect** methods.
- Direct methods involve the **actual measurement of the quantity of flow** for a given time interval (Velocity-Area integration method).
- Indirect methods involve the measurement of **pressure change** (or some other variables) which in turn is directly related to the rate of flow.
- Flow through **orifice**, **venturi meters**, and **flow nozzles** are all devices which employ **indirect method** to measure the rate of flow in ***closed conduits***.



# Pipe Flow | Instruments

## Orifice

- An orifice plate is fundamentally a **plate** with a **hole** machined through it which is inserted into a pipe.

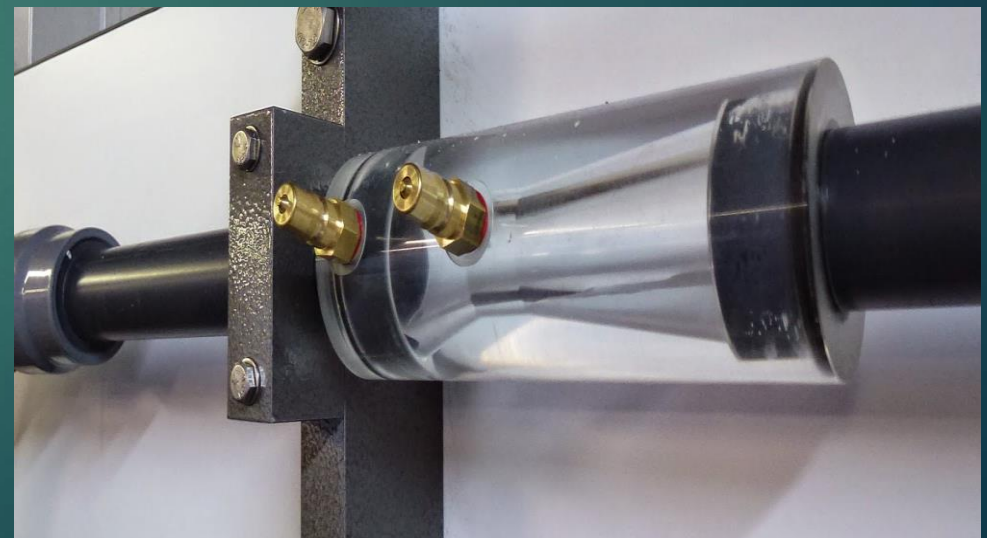
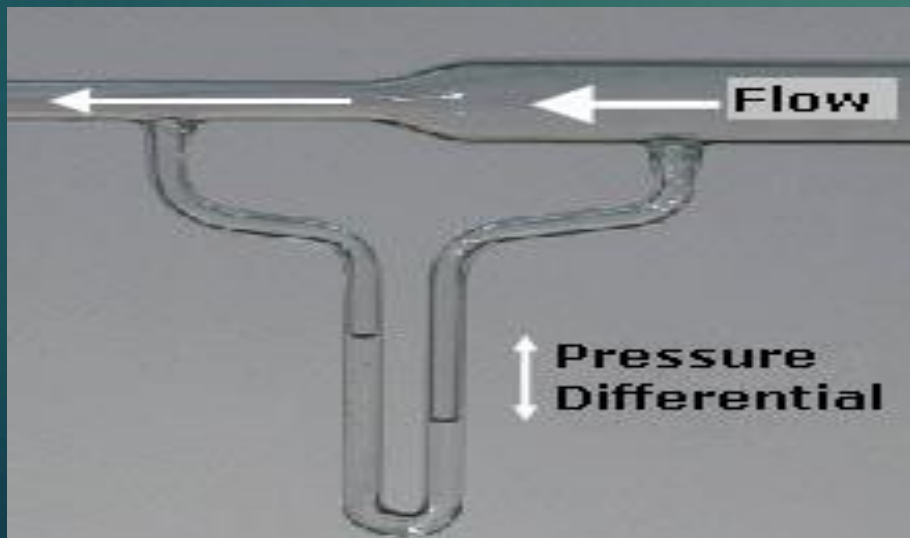
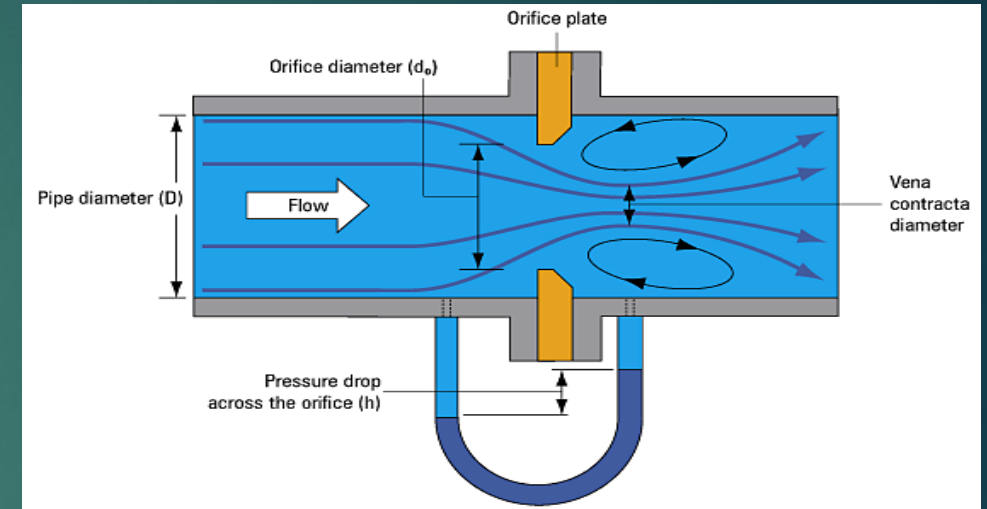


- As flow passes through the hole it produces a **pressure difference** across the hole (some of which is recovered).

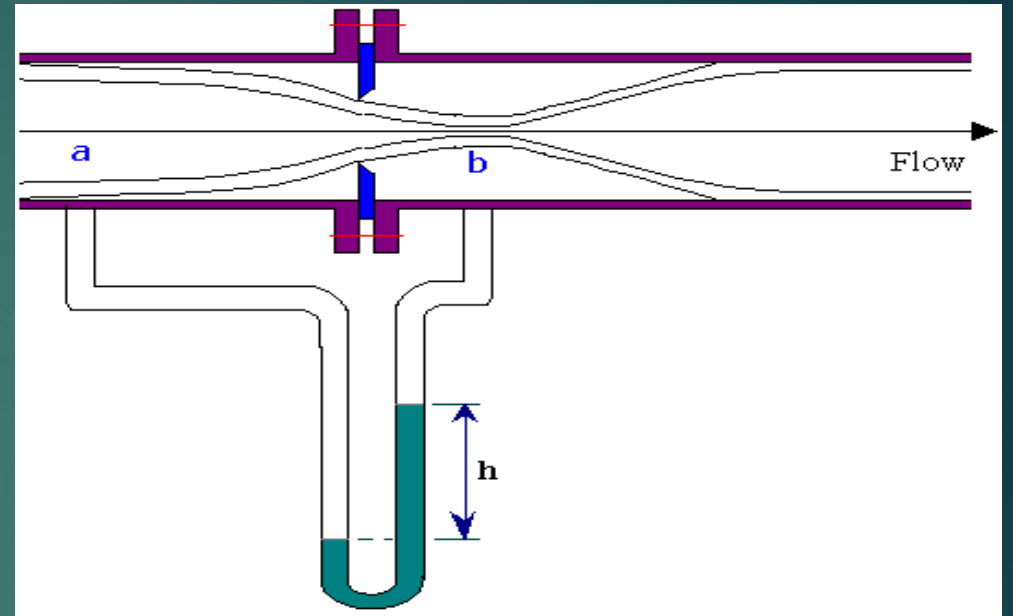
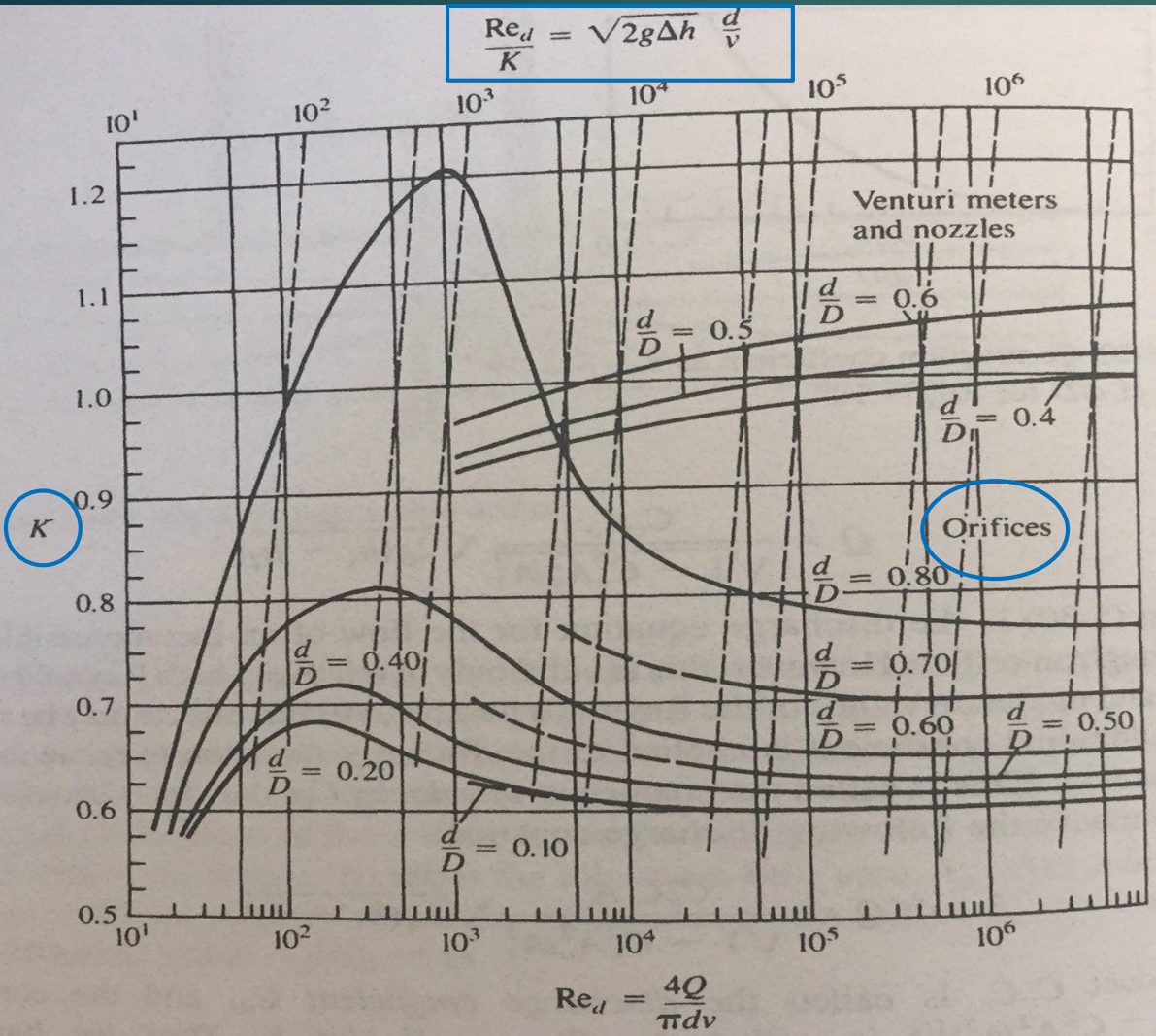
# Pipe Flow | Instruments

## How it works

- As the fluid flows through the orifice plate the **velocity increases**, at the **expense of pressure head**.
- The **pressure drops** suddenly as the orifice is passed.



# Pipe Flow | Instruments



$$Q = K A_0 \sqrt{2g\Delta h}$$

$d$ : orifice diameter  
 $D$ : Pipe diameter  
 $h$  or  $\Delta h$  is the pressure drop

$$\frac{Re_d}{K} = \sqrt{2g\Delta h} \frac{d}{v}$$

And

$$\frac{d}{D}$$

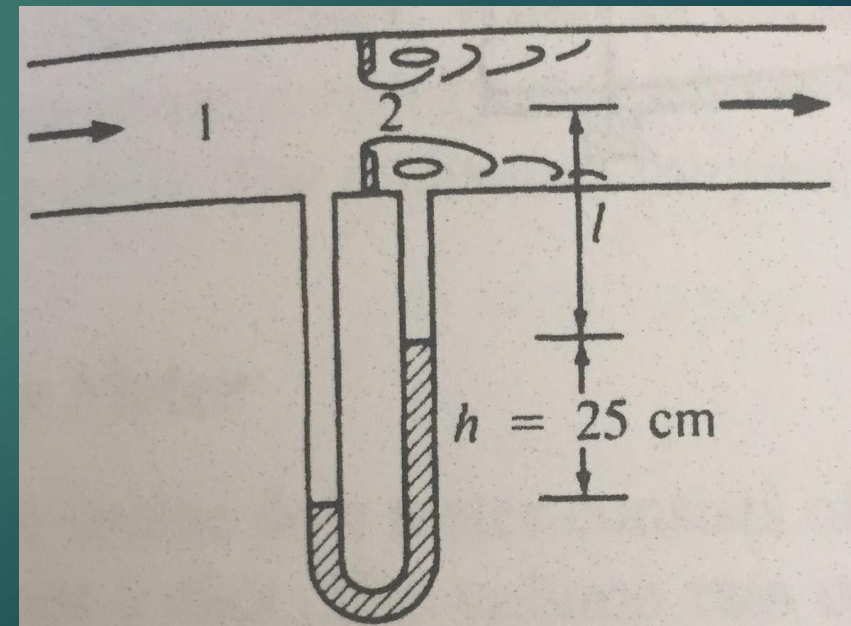
And

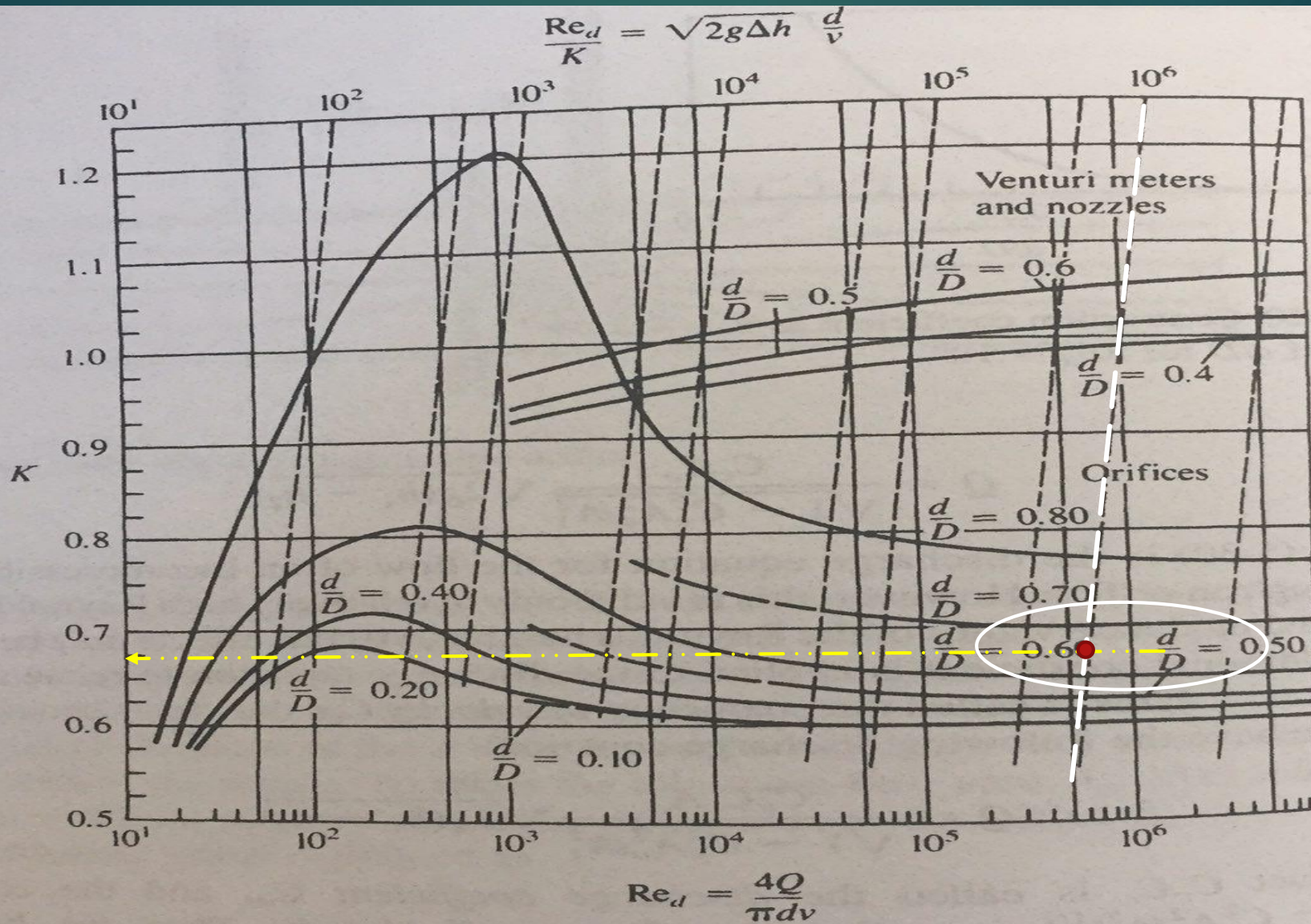
$$A_0 = \pi \frac{d^2}{4}$$

# Pipe Flow | Instruments

## Example 1

A **15 cm** orifice is located in a horizontal **24 cm** water pipe, and a water-mercury manometer is connected to either side of the orifice. When the deflection on the manometer is **25 cm**, what is the discharge in the system? Assume the water temperature is **20°C**.

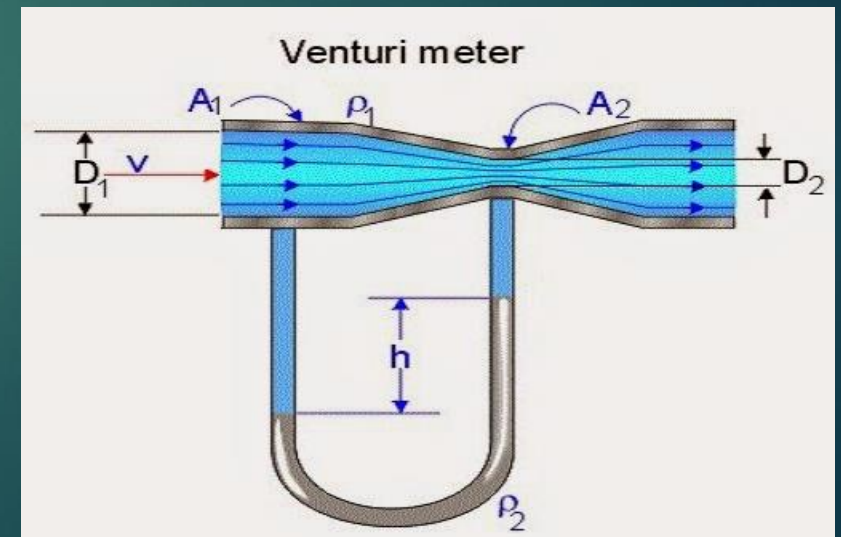
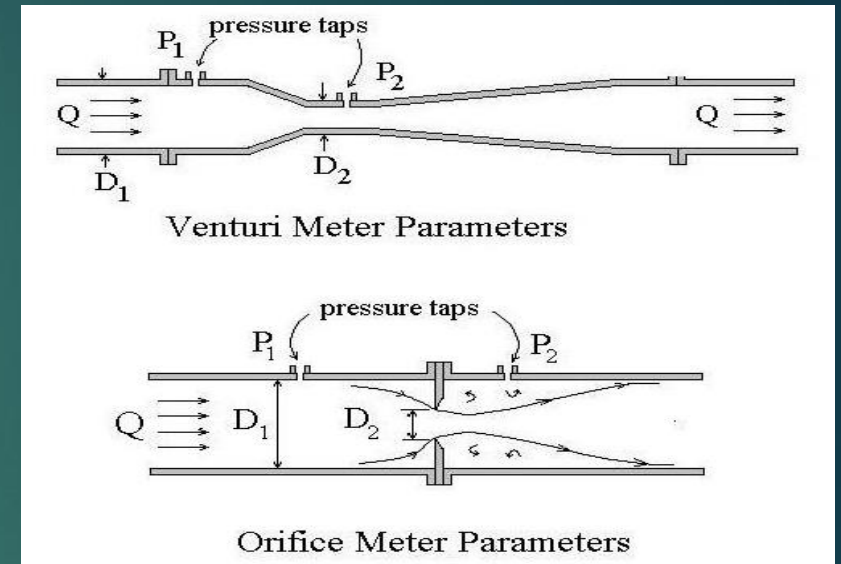




# Pipe Flow | Instruments

## Venturi Meter

- Although the Orifice is a simple and accurate device for the measurement for flow, however the **head loss** for the orifice is **quite large**.
- This device operates on the same principles as the Orifice but with a **much smaller head loss**.
- Inside of the venturimeter pressure difference is created by **reducing the cross-sectional** area of the flow passage.
- As the inlet area of the venturi is large than at the throat, **the velocity at the throat increases** resulting in **decrease of pressure**.



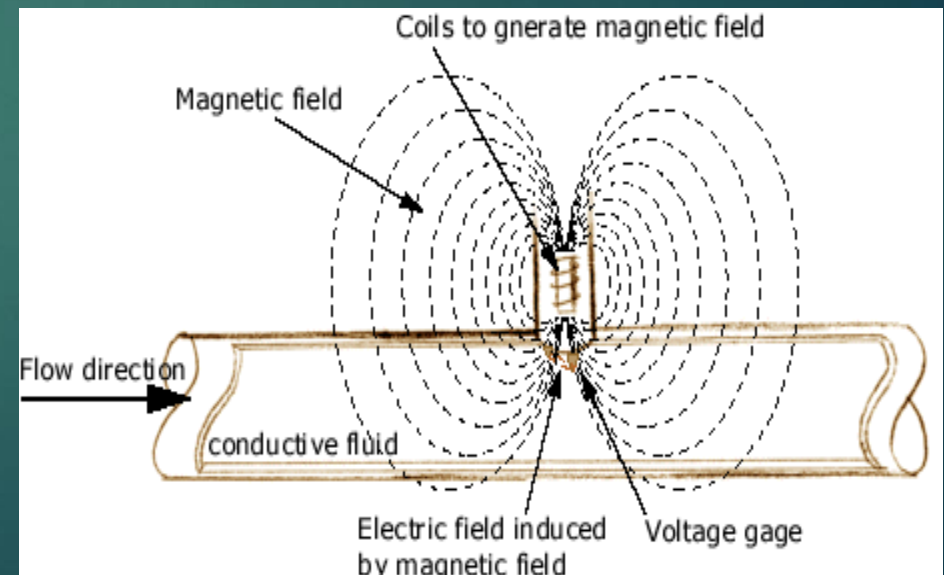
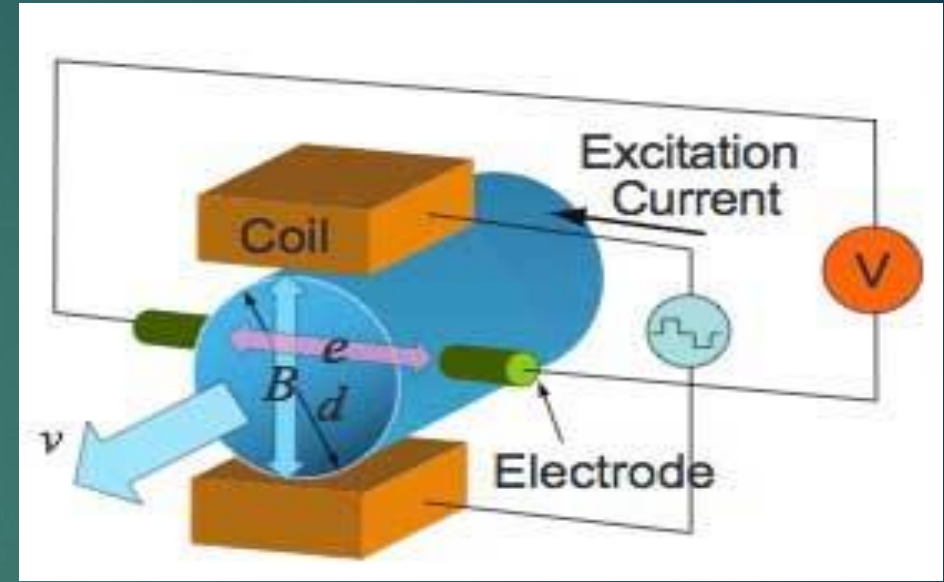
## Orifice-Venturi



# Pipe Flow | Instruments

## Electromagnetic Flow Meter

- It's basic principle is that a conductor that moves in a **magnetic field** produces an **electromotive force**.
- Hence, liquids having a degree of conductivity will **generate a voltage between the electrodes** in which it is proportional to the flow velocity.
- The major disadvantage of this device is its **high cost**.





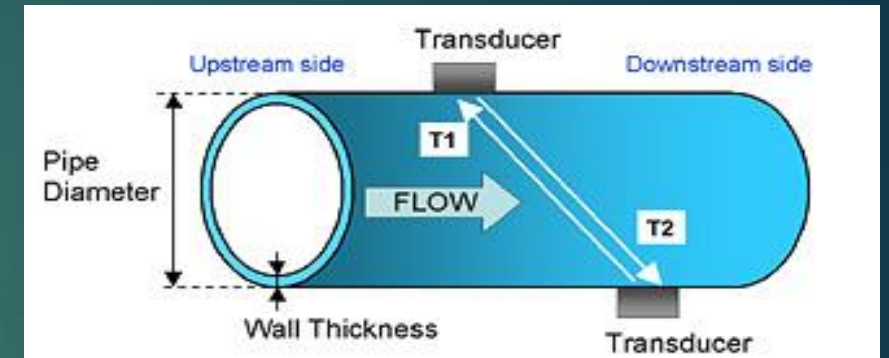
## Electromagnetic Flow Meter



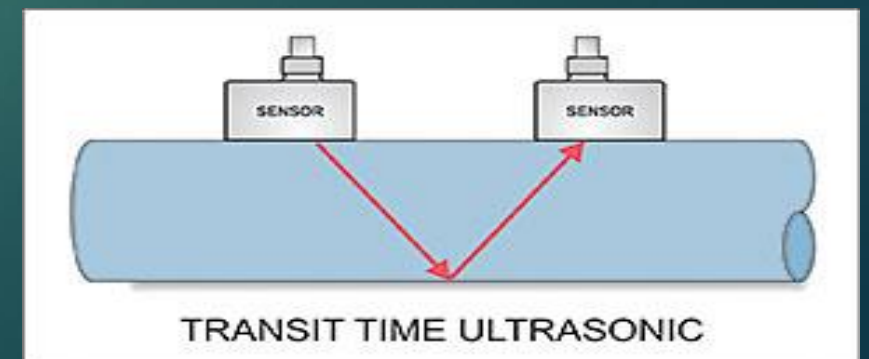
# Pipe Flow | Instruments

## Ultrasonic Flow Meter

- Ultrasonic flowmeters use **sound waves** to determine the velocity of a fluid flowing in a pipe.
- At no flow conditions, the **frequencies of an ultrasonic wave** transmitted into a pipe and its **reflections** from the fluid are the **same**.
- Under flowing conditions, the frequency of the reflected wave is different due to the **Doppler effect**.
- The transmitter processes signals from the **transmitted wave and its reflections** to determine the flow rate.



The **Doppler effect** is the change in frequency or wavelength of a wave for an observer moving relative to its source.



## Ultrasonic Flow Meter



# Open Channel Flow | Instruments

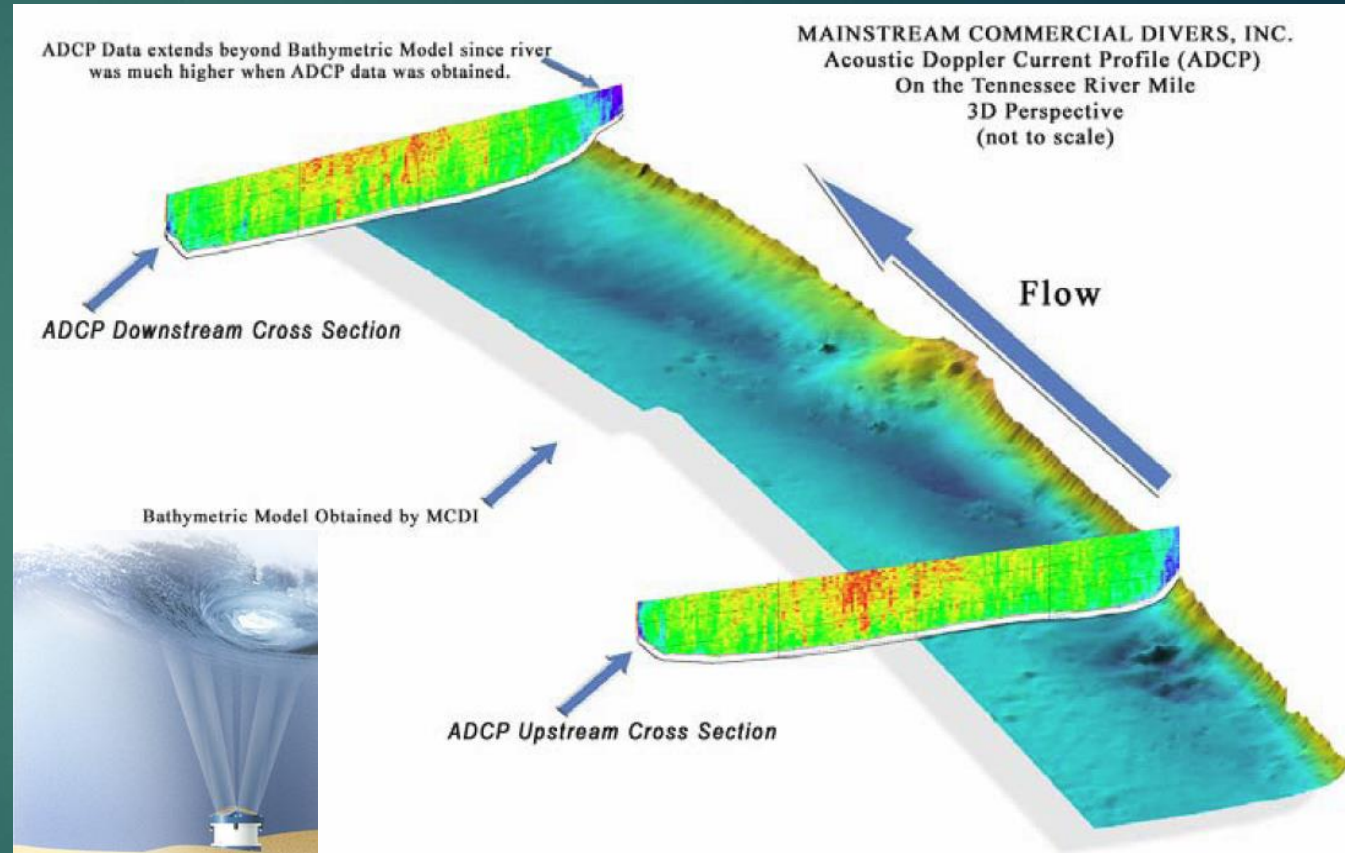
- The **Acoustic Doppler Current Profiler (ADCP)** is a device that uses sound and the Doppler principle.
- The ADCP is commonly used to measure water velocity and discharge in streams as shallow as **1.0 ft** deep.



- Measurements of depth, velocity profiles, distance, and direction travelled are then **combined to calculate discharge**.

# Open Channel Flow | Instruments

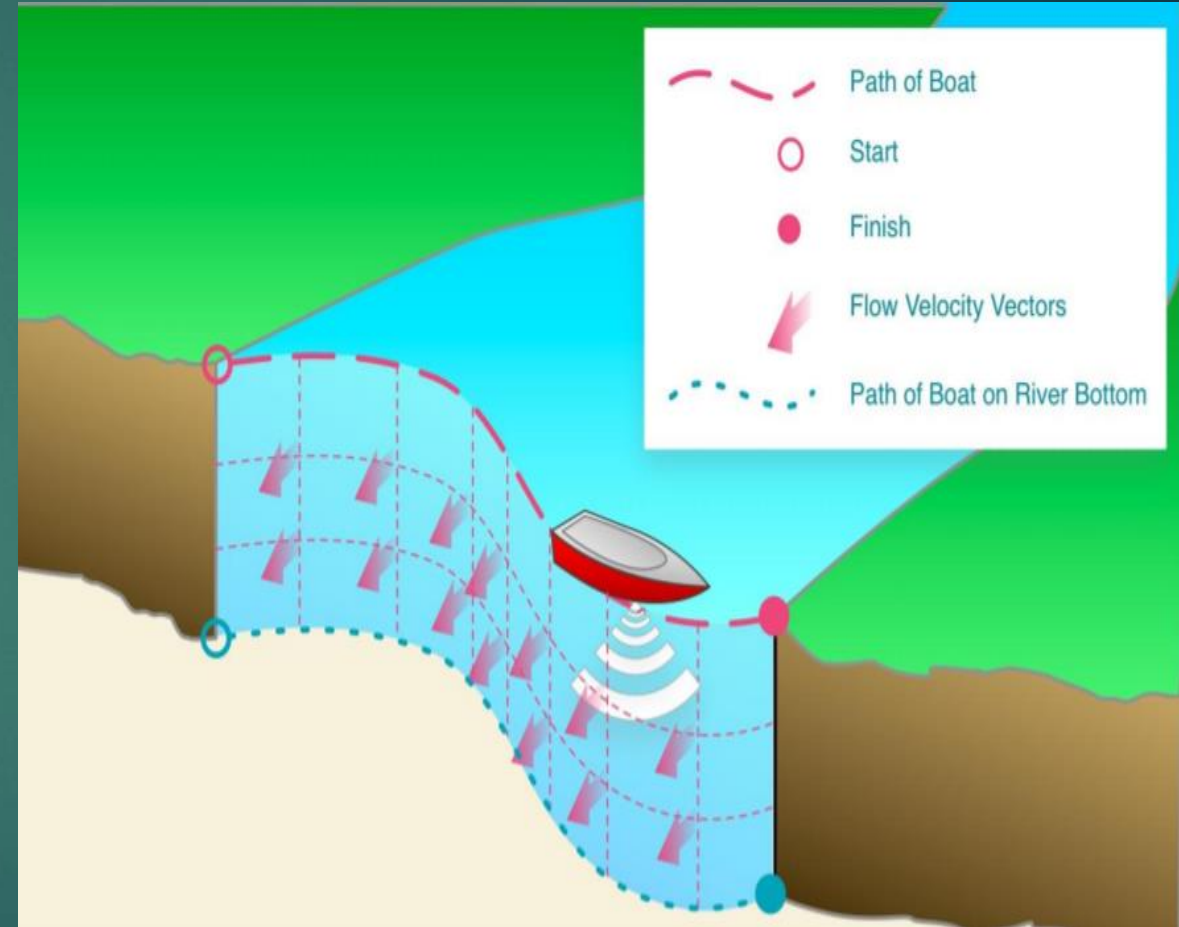
- Complex calculations are involved in the discharge calculation requiring **manufacturer software and a computer**.
- *Calculation details are in Mueller and Wagner (2009).*
- This tool can measure water velocities at a **spatial** and **temporal** scale.



# Open Channel Flow | Instruments

## It works by

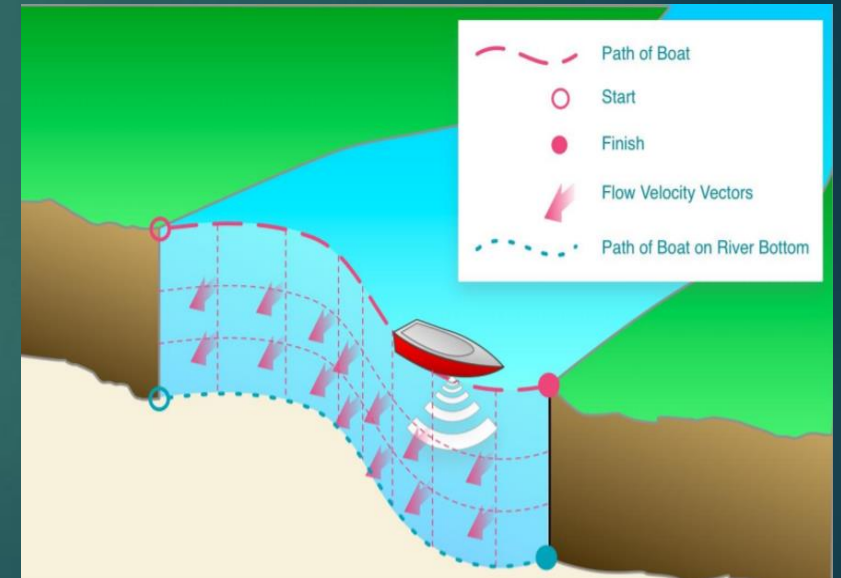
- **Boat mounting** an ADCP with transducers beneath the water surface and moving the boat across the river channel.
- Converting the measured Doppler to velocities.
- Many data points can be measured across the river as ADCPs measure velocities in **large parts of the water column**, and **depths at many points**



# Open Channel Flow | Instruments

## It works by

- Measuring velocities over a **large part of the water column** beneath the ADCP continuously.
- The velocity calculation is **directly** related to the **speed of sound** in the water, which varies with changes in: water temperature, salinity, pressure, and, sediment concentration.
- A temperature change of **5°C**, or a salinity change of **12 parts per thousand**, results in a speed of sound change of **1%**.





## Acoustic Doppler Current Profiler

