



APPLIED HYDRAULICS

CHAPTER 12:

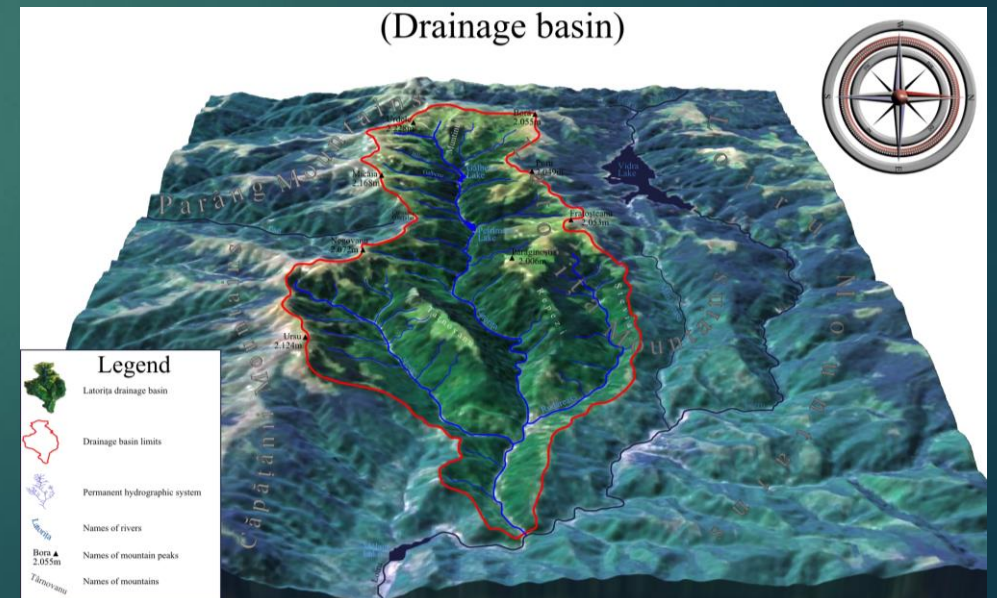
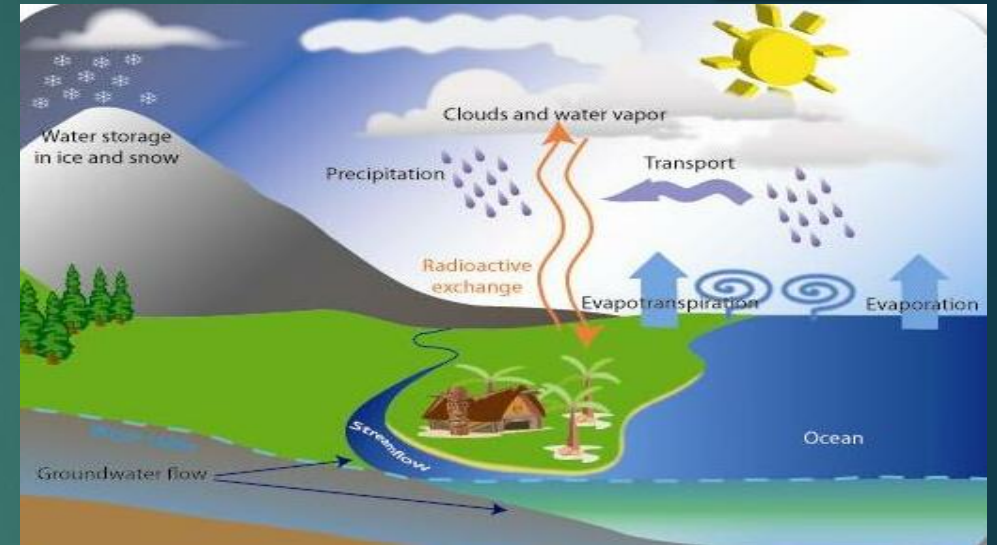
HYDROLOGY

Hydrology

- Introduction
- Hydrologic cycle
- Statistical methods in Hydrology
- Frequency analysis

Hydrology

- Hydrology is the scientific study of the **movement, distribution, and quality of water** on Earth and other planets.
- A practitioner of hydrology is a hydrologist, working within the fields of **earth or environmental science, physical geography, geology or civil and environmental engineering.**
- Hydrology is subdivided into:
 - Surface water hydrology,
 - Groundwater hydrology (hydrogeology), and
 - Marine hydrology.



Hydrology and Hydraulic Differences

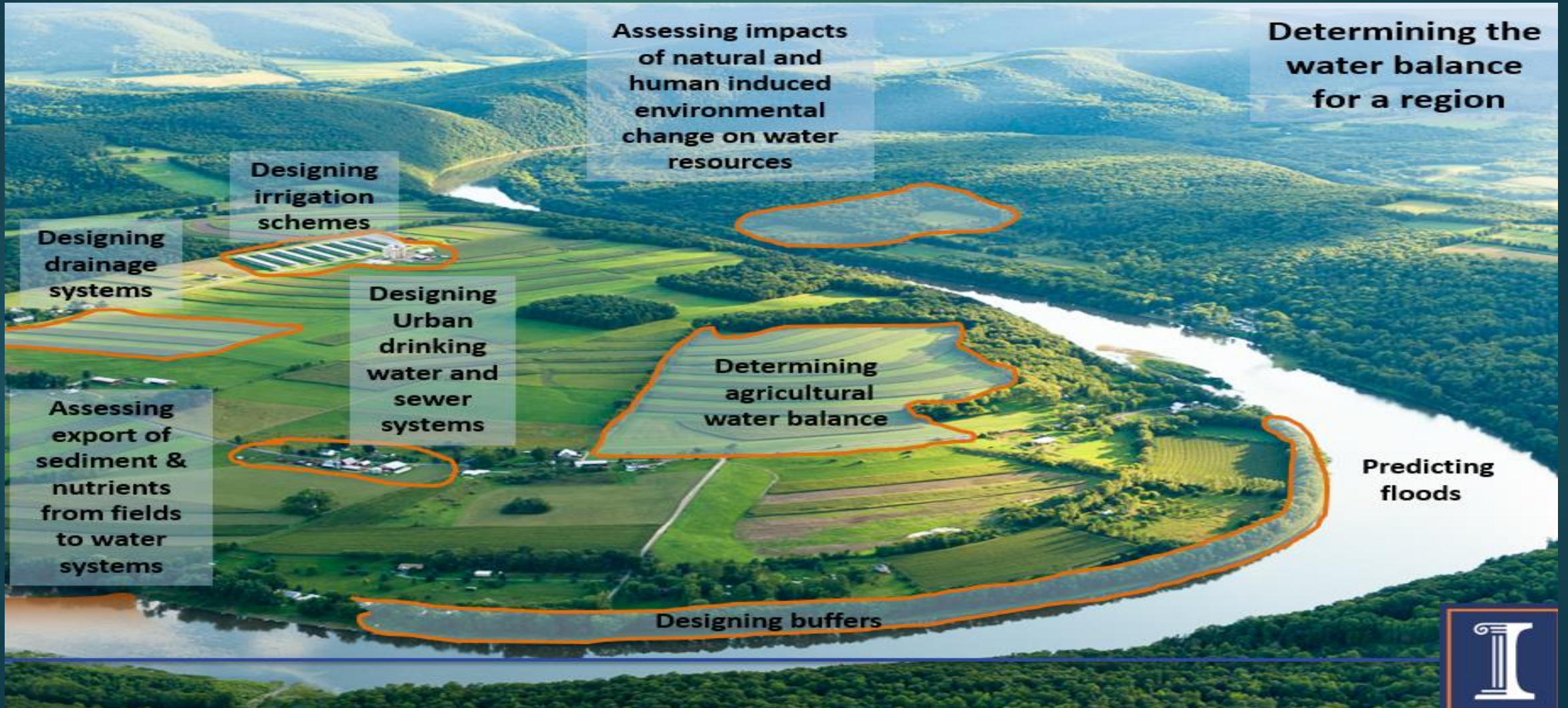
- **Hydrology** is generally related to the study of rainfall and water in connection to geography and geology .
- Hydrology deals with precipitation (rain, snow), evaporation, infiltration, groundwater flow, surface runoff, streamflow.
- **Hydraulics** is defined as the study of the mechanical behavior of water in physical systems.
- **Hydraulic** is the analysis of how surface, and/or subsurface flows move from one point to the next.
- **Hydraulic** analysis is used to evaluate flow in rivers, streams, storm drain networks, water aqueducts, water lines, sewers, etc.

Hydrology

Hydrology Branches

<p>Chemical Hydrology</p> <p>Study of chemical characteristics of water</p> <p>Water Quality</p> <p>Chemistry of water in rivers and lakes, both of pollutants and natural solutes</p>	<p>Eco Hydrology</p> <p>Study of interactions of living organisms and the hydrologic cycle</p>	<p>Hydrogeology</p> <p>Study of the distribution and movement of groundwater in the soils and rocks of the Earth's crust</p>	<p>Hydrometeorology</p> <p>Study of the transfer of water and energy between land and water body surfaces and the lower atmosphere</p>	<p>Surface Hydrology</p> <p>Study of hydrologic processes that operate at or near Earth's surface</p>	<p>Drainage Basin Management</p> <p>Covers water-storage, in the form of reservoirs, and flood-protection</p>
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Hydrology Applications



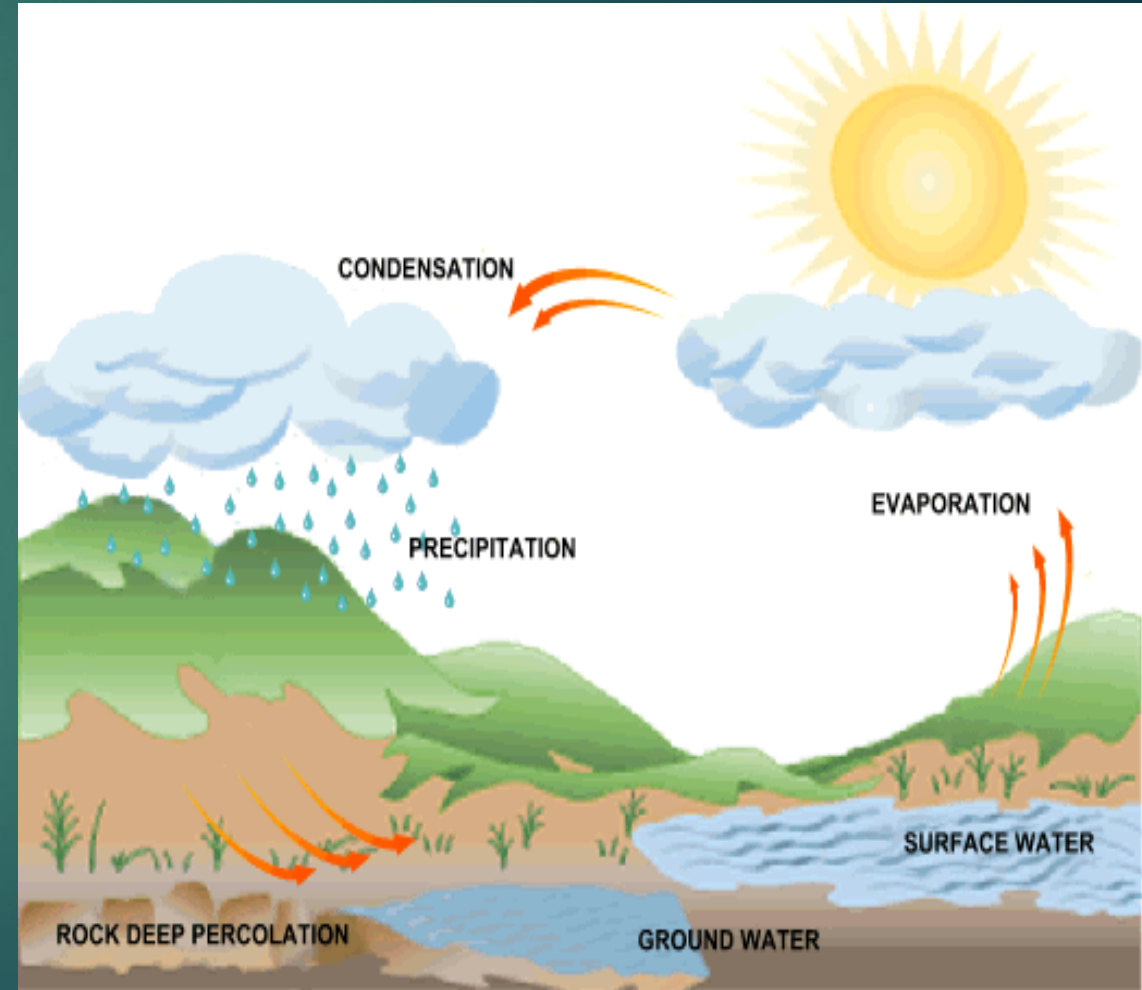
Hydrology

Hydrologic Cycle

- The illustration shows the **hydrologic cycle** in which water leaves the atmosphere and **falls to earth** as precipitation where it enters surface waters or infiltrates into the water table and groundwater.
- Then, it is taken **back into the atmosphere** by transpiration and evaporation to begin the cycle again.

Evaporation

- As water is heated by the sun, and then **evaporate** and rise as invisible vapour in the atmosphere.



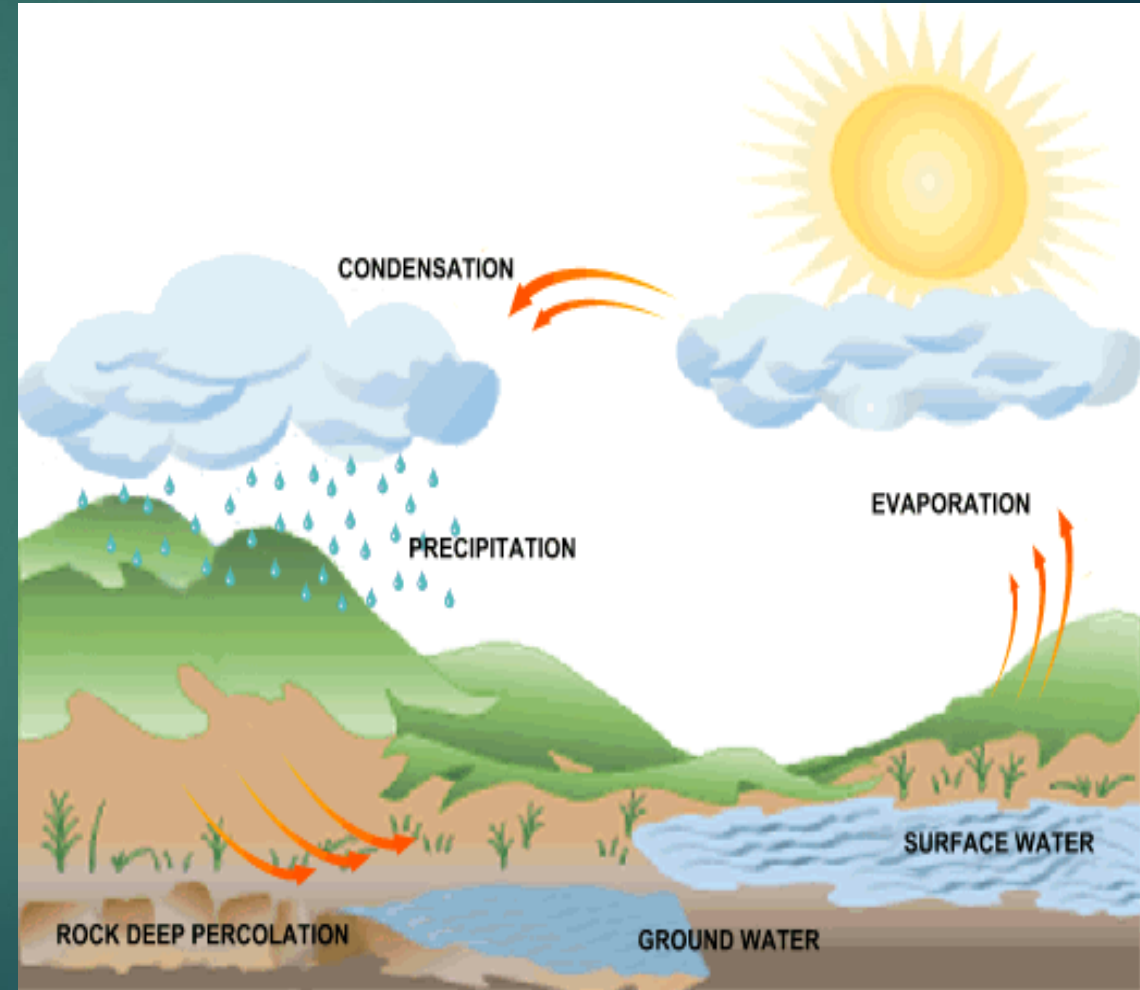
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Condensation

- As water vapour rises, it cools and eventually **condenses**, usually **on tiny particles of dust** in the air.
- These water particles then collect and **form clouds**.

Precipitation

- Precipitation in the form of **rain, snow** comes from clouds.
- It is the **primary** connection in the water cycle that provides for the delivery of atmospheric water to the Earth. Most precipitation falls as **rain**.



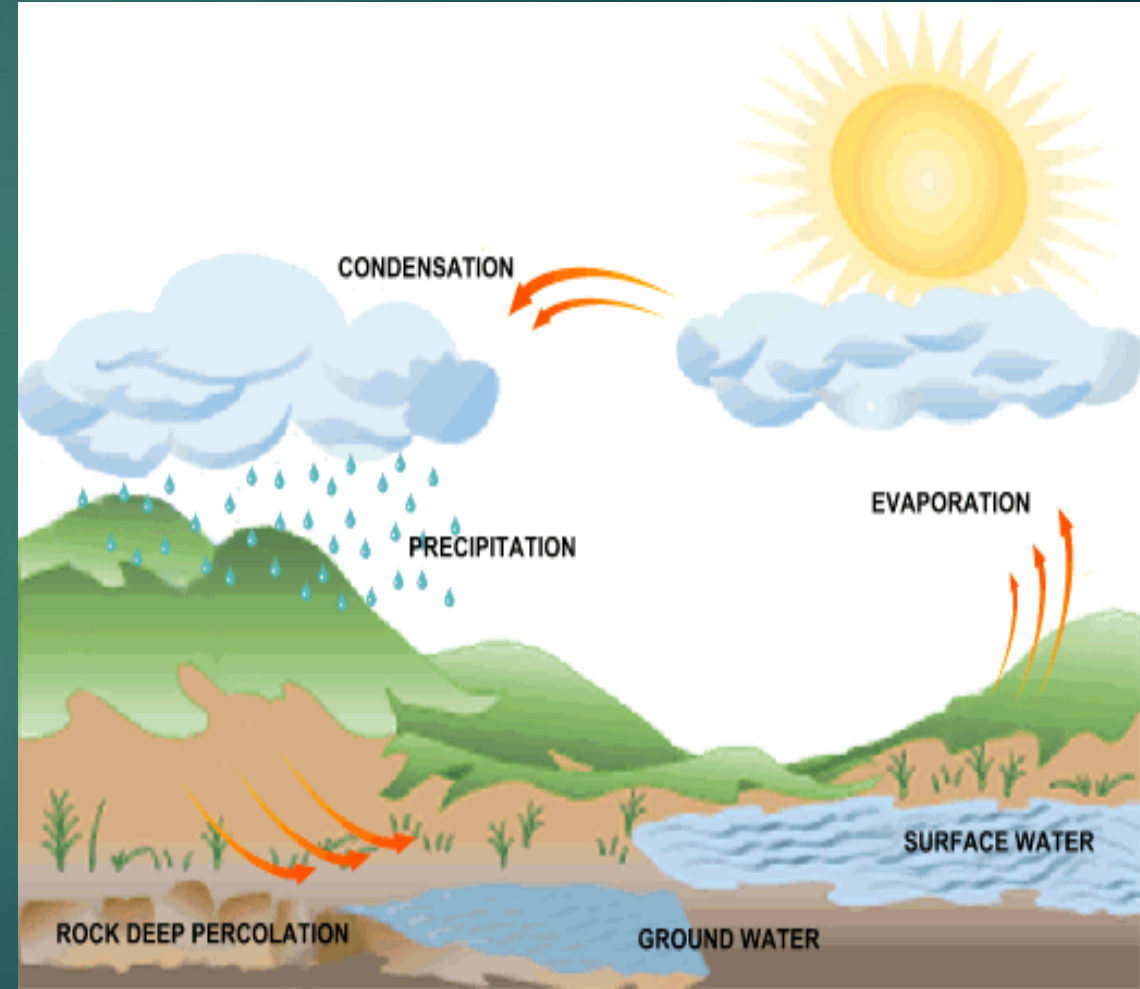
Hydrology

Runoff

- Excessive rain or snowmelt can produce **overland flow** to creeks and ditches.
- Runoff is **visible flow of water** in rivers, creeks and lakes as the water stored in the basin drains out.

Infiltration

- Some of the precipitation and snow melt moves downwards, **infiltrates** through cracks, joints and pores in soil and rocks until it reaches the water table where it becomes **groundwater**.



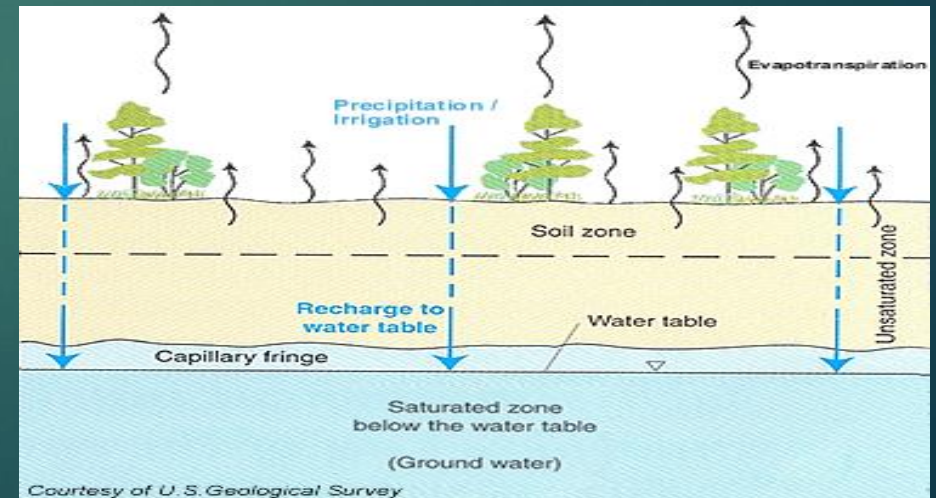
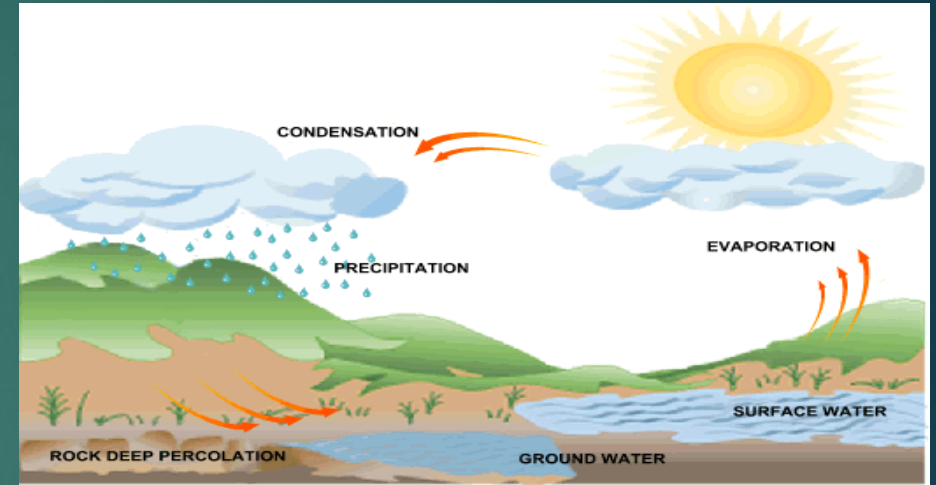
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Groundwater

- Subterranean water is **held** in cracks and pore spaces.
- Depending on the geology, the groundwater can **flow** to support streams.
- It can also be **tapped** by wells. Some groundwater is very old and may have been there for thousands of years.

Water table

- The water table is the level at which water **stands in a shallow well**.



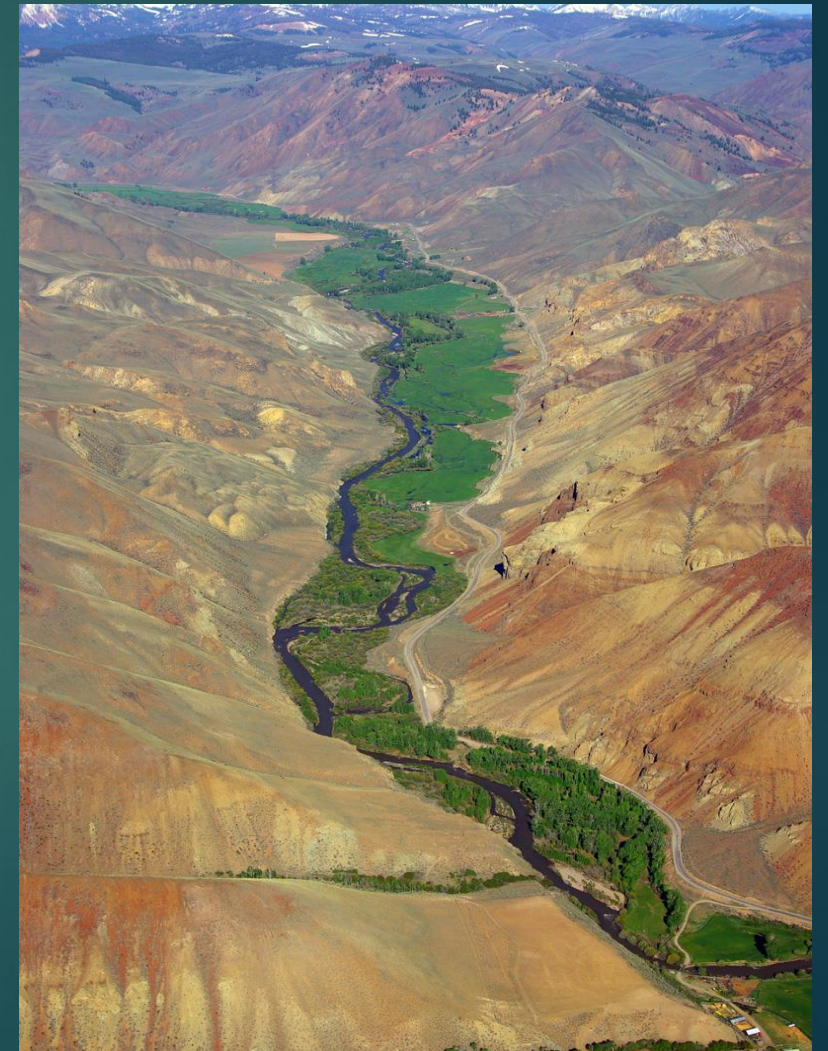
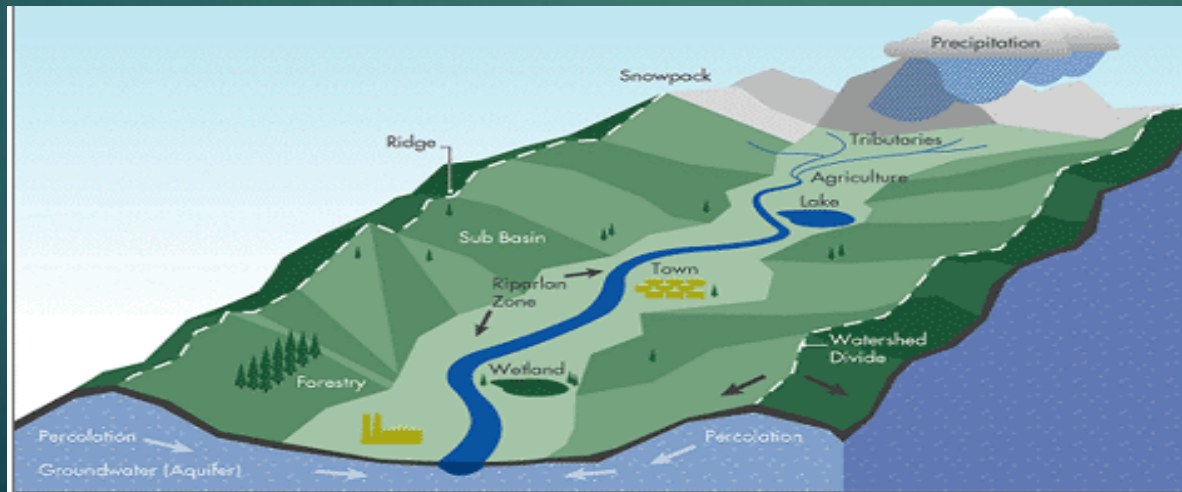
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Hydrology

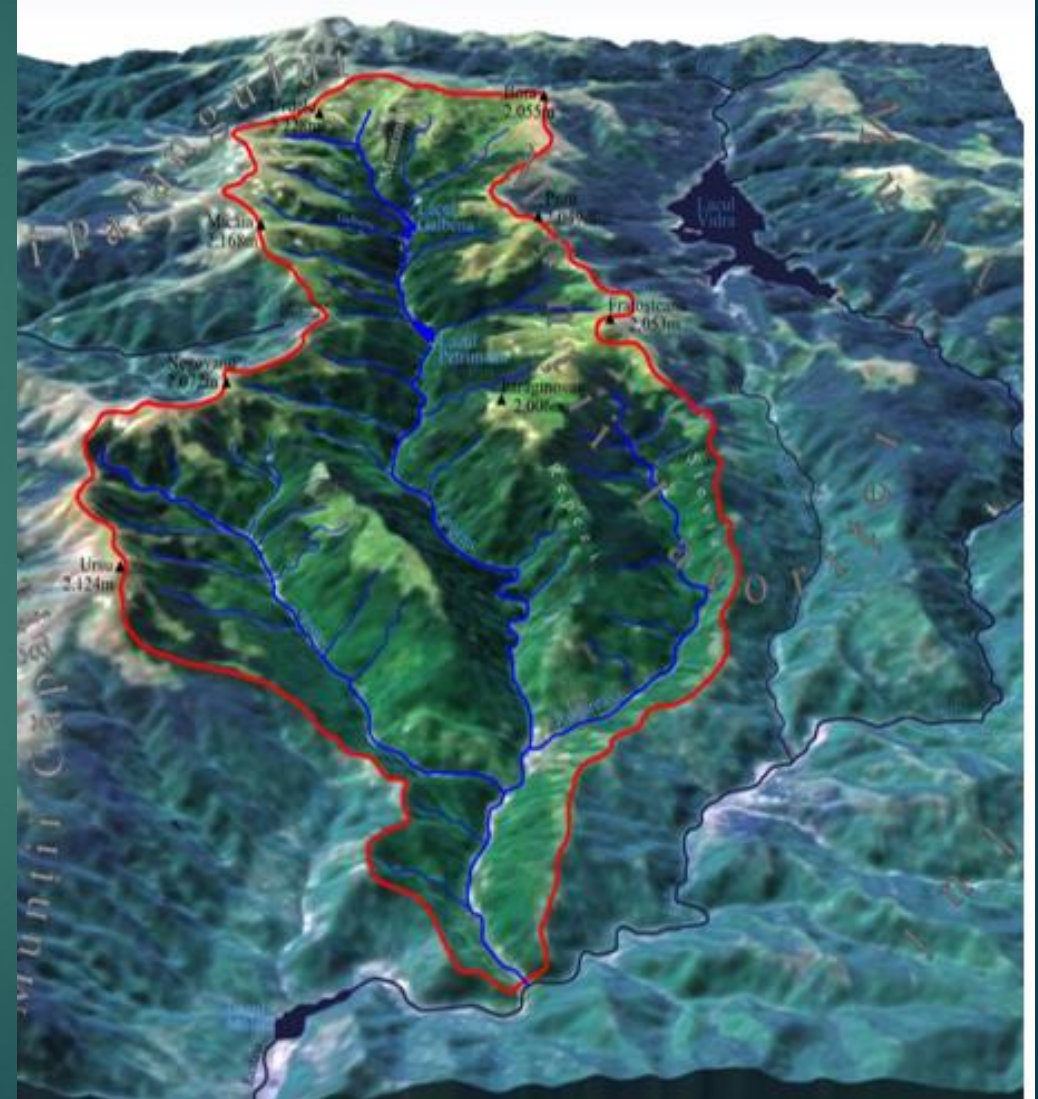
Watershed

- A watershed is an area of land that **drains all the streams** and rainfall to a **common outlet** such as the outflow of a reservoir, mouth of a bay, or any point along a stream channel.
- Water is **channeled** into soils, ground-waters, creeks, and streams, making its way to larger rivers and eventually the sea.



Hydrology

- Watershed also known as: **Catchment**, **Catchment area**, **Catchment basin**, **Drainage area**, **River basin**, and **Water basin**.
- Watershed characteristics factor include:
 - i. Topography
 - ii. Shape
 - iii. Size
 - iv. Soil type
 - v. Land use



Data Analysis in Hydrology

- The development of almost all hydrologic design methods uses some form of **data analysis**.
- In other words, the design method usually is made operational only after analyzing measured **hydrologic data**.
- The necessary methods in hydrology are:
 1. Probability concepts,
 2. Statistical moments,
 3. Hypothesis tests,
 4. Regression analysis,
 5. Frequency analysis.

Probability

- Probability is a scale of measurement that is used to describe the **likelihood** of an event.
- The **scale** on which probability is measured extends from **0 to 1**.
- A value of **1** indicates a **certainty of occurrence** of the event and a value of **0** indicates a **certainty of failure** to occur or nonoccurrence of the event.
- Probability is specified as a percentage.
- For example, when the weatherperson indicates that there is a **30%** chance of rain, it means that based on past experience and under similar meteorological conditions it has rained **3 out of 10 times**.

Random Variables

- There are two types of random variables:
 1. A **discrete** random variable is one that may only **take on distinct**, usually integer, values.
- For example, the outcome of a roll of a dice may only take on the **integer values** from 1 to 6 and is therefore a discrete random variable.
- Or, if we define a random variable to be whether or not a river has overflowed its banks in any one year, then the variable is **discrete** because **only discrete outcomes are possible**; the out-of-bank flood occurred or it did not occur in any one year.

2. The outcome of an event may take on **any value within a continuum of values**; such a random variable is called **continuous**.
 - For example, the volume of rainfall occurring during storm events is an example of a continuous random variable because it can be any value on a **continuous scale** from **0** inches to **some very large value**.
 - A distinction is made between discrete and continuous random variables, because the **computation of probabilities** is different for the two types.
 - A probability density function (**PDF**) is used to define the likelihood of occurrence of a **continuous random variable**.

Hydrology

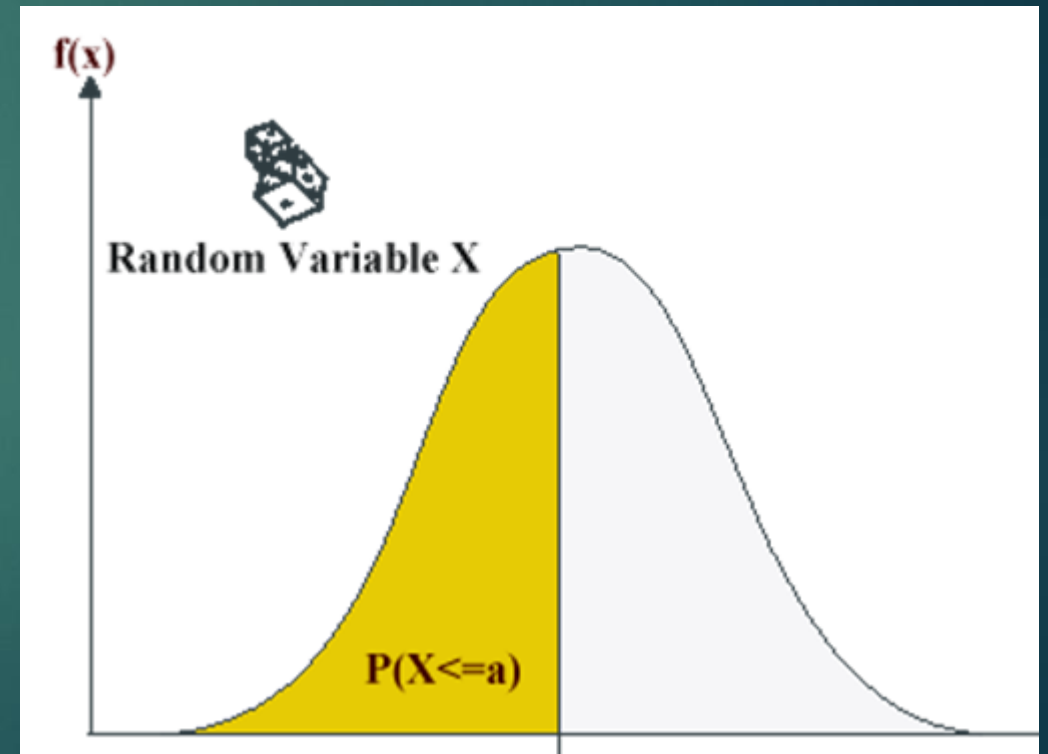
- The probability that the random variable x lies within the interval from x_1 , to x_2 is given by the **integral** of the density function from x_1 , to x_2 , over all values within the interval:

$$p(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f(x) dx$$

$f(x)$ is the density function

- It is also noted that the integral of the PDF from $-\infty$ to $+\infty$ equals 1.0.

$$p(x_1 \leq x \leq x_2) = \int_{-\infty}^{+\infty} f(x) dx = 1$$



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- The **Cumulative Distribution Function** (CDF) of a continuous random variable is defined by:

$$F(X) = p(x \leq x_k) = \int_{-\infty}^{x_k} f(x) dx$$

$f(x)$ is the density function

- It is also noted that the integral of the PDF from $-\infty$ to $+\infty$ equals 1.0.

$$p(x \geq x_k) = \int_{x_k}^{+\infty} f(x) dx = 1 - p(x \leq x_k) = 1 - F(X)$$

Example 1

Assume that the probability distribution of evaporation E on any day during the year is given by:

$$F(E) = \begin{cases} 4 & 0 \leq E \leq 0.25 \text{ in/day} \\ 0 & \text{otherwise} \end{cases}$$

What is the probability of E between 0.1 to 0.2? And what is the probability of E less than 0.15 in/day? What is the probability that E is greater than 0.05 in/day?

Population

- The total set of observation (x_1, x_2, \dots, x_n) with finite or infinite sample data length is referred to the **population**.
- Whether summarizing a data set or attempting to find the population, one must **characterize** the sample.
- **Moments** are useful descriptors of data.
- For example, the mean (or average), which is a moment, is an important **characteristic of a set of observations** on a random variable, such as rainfall volume or the concentration of a water pollutant.
- There are three important moments: (1) **Mean**, (2) **Variance**, (3) **Skew**

Hydrology

- The **mean** is the **first moment** measured about the origin, it is also the **average** of all observations on a random variable.
- For a **continuous** random variable, it is computed as:

$$E(X) = \bar{x} \text{ or } \mu = \int_{-\infty}^{+\infty} xf(x)dx$$

- For a **discrete** random variable, the mean is given by

$$E(X) = \bar{x} \text{ or } \mu = \frac{1}{n} \sum_{i=1}^n x_i$$

Hydrology

- The **variance** is the **second moment** about the mean. The variances of the population and sample are denoted by $Var (X)$ or σ^2 . For a **continuous** random variable, the variance is computed by:

$$Var (X) = \sigma^2 = \int_{-\infty}^{+\infty} (x - \mu)^2 f(x) dx$$

- For a discrete random variable, the variance is given by:

$$Var (X) = \sigma^2 = \frac{1}{n - 1} \sum_{i=1}^n (x_i - \mu)^2$$

$$Var (X) = \sigma^2 = \frac{1}{n - 1} \left[\left(\sum_{i=1}^n x_i^2 \right) - n\bar{x}^2 \right]$$

$$\text{Standard Deviation} = \sigma = \sqrt{Var (X)}$$

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- The **skew** is the **third moment** measured about the mean and it is denoted by $Var(X)$ or σ^2 .
- For a **continuous** random variable, the variance is computed by:

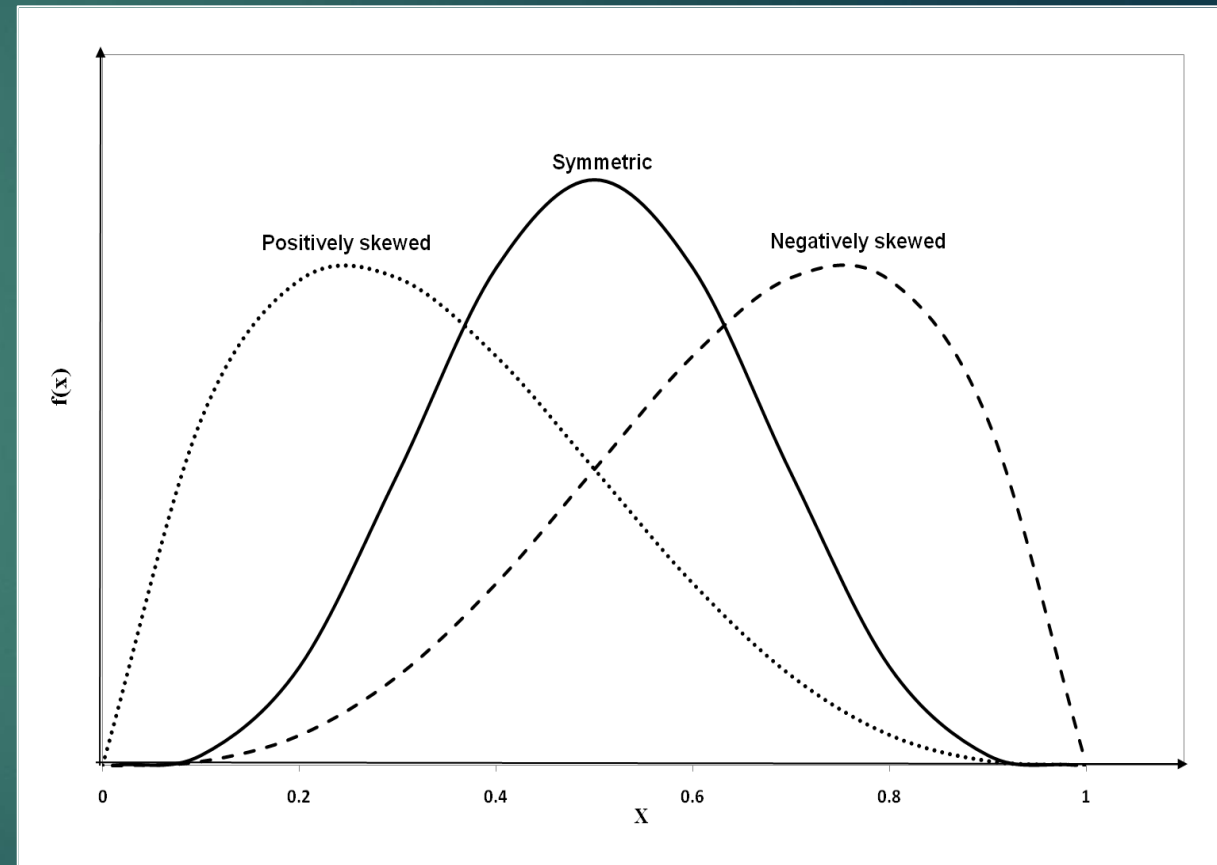
$$g = \gamma = \int_{-\infty}^{+\infty} (x - \mu)^3 f(x) dx$$

- For a **discrete** random variable, the variance is given by:

$$Var(X) = \sigma^2 = \frac{n}{(n-1)(n-2)\sigma^3} \sum_{i=1}^n (x_i - \mu)^3$$

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- The skewness coefficient is a dimensionless coefficient of which sign shows the degree of PDF **symmetry**.
- If $\gamma < 0$ distribution is **skewed to the left** (negatively skewed);
- If $\gamma > 0$ the distribution is **skewed to the right** and the PDF tail will be heavier there (positively skewed).
- The skewness coefficient for **symmetric** distribution, is **zero**.



Hydrology

Example 2

Observed flow data of a river is available from 1960 to 1989 and presented in the following table. Determine the **mean**, **variance**, **standard deviation** and **skewness**.

Introduction to Risk and Uncertainty in Hydrosystem Engineering

Year	Flow (Q , cfs)	Year	Flow (Q , cfs)
1960	5759.278	1975	8742.511
1961	1113.213	1976	5984.455
1962	2531.58	1977	8685.196
1963	1716.61	1978	8752.519
1964	4718.362	1979	7366.032
1965	9248.881	1980	868.9257
1966	399.9453	1981	6121.915
1967	2306.414	1982	8277.696
1968	2947.42	1983	4654.374
1969	5472.883	1984	6820.767
1970	6096.213	1985	2575.634
1971	2589.628	1986	7508.591
1972	2111.904	1987	8238.025
1973	3945.78	1988	29.47444
1974	9789.477	1989	7021.186

$$E(a) = a$$

$$\text{Var}(a) = 0$$

$$E(bx) = b(Ex)$$

$$\text{Var}(bx) = b^2\text{Var}(x)$$

$$E(a + bx) = a + bE(x)$$

$$\text{Var}(a + bx) = \text{Var}(a) + \text{Var}(bx) = b^2\text{Var}(x)$$

Example 3

Assume the following density function over the interval $[0,1]$: $f(x) = x + 3$

Calculate $E[x]$, $Var[X]$, $E[4x]$, $Var[4X]$, and $E[4x + 5]$ and $Var[4x + 5]$.



APPLIED HYDRAULICS

CHAPTER 13:

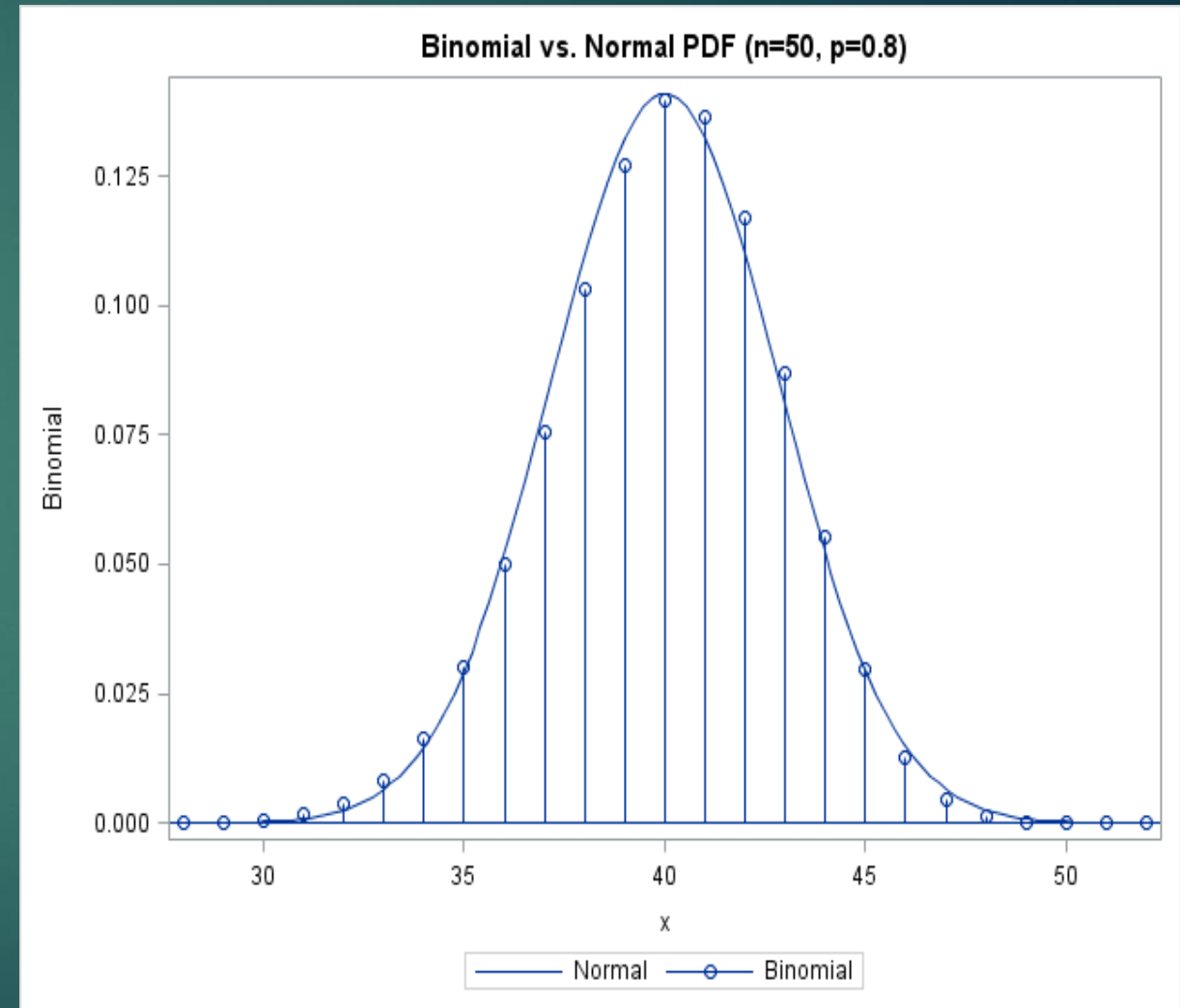
HYDROLOGY

Hydrology

- Binomial Distribution
- Normal Distribution
- Hydrographs

Hydrology

- Random variables can be measured either at **discrete** values or over a **continuous** scale.
- **Probability functions** is used to represent a random variable and to determine the probability of occurrence.
- Although there are many probability functions, for our purposes, only two will be discussed in this section:
 1. **Binomial**
 2. **Normal**



Binomial Distribution

- The binomial distribution is used to define probabilities of **discrete events**. It is applicable to random variables that satisfy the following four assumptions:
 - i. There are **n** occurrences, or trials, of the random variable.
 - ii. The **n** trials are independent.
 - iii. There are only **two possible outcomes** for each trial.
 - iv. The probability of each outcome is **constant** from trial to trial.
- The probabilities of occurrence of any random variable **satisfying these four assumptions** can be computed using the binomial distribution.

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- For example, in flipping a coin n times, the n trials are **independent**, there are only **two possible outcomes**, and the **probability of a head remains 0.5**.
- One **could not** use the binomial distribution to compute probabilities for the **toss of a dice** because there are six possible outcomes.
- However, if one defines the two outcomes as either an **even** or an **odd** number of dots, then the four assumptions would apply.



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- As this distribution has a binary base, the outcomes can be either **success** or **fail**.
- If the probability of *success* and *fail* occurrence is denoted by **p** and **q**, respectively, the binomial probability mass function (**PMF**) can be written as:

$$p(x) = {}^1_nC_x p^x q^{n-x} = B(n, p) \quad , \quad x = 0, 1, 2, \dots, n$$

- where $q = 1 - p$ and 1_nC_x is a Binomial coefficient:

$${}^1_nC_x = \binom{n}{x} = \frac{n!}{x! (n-x)!}$$

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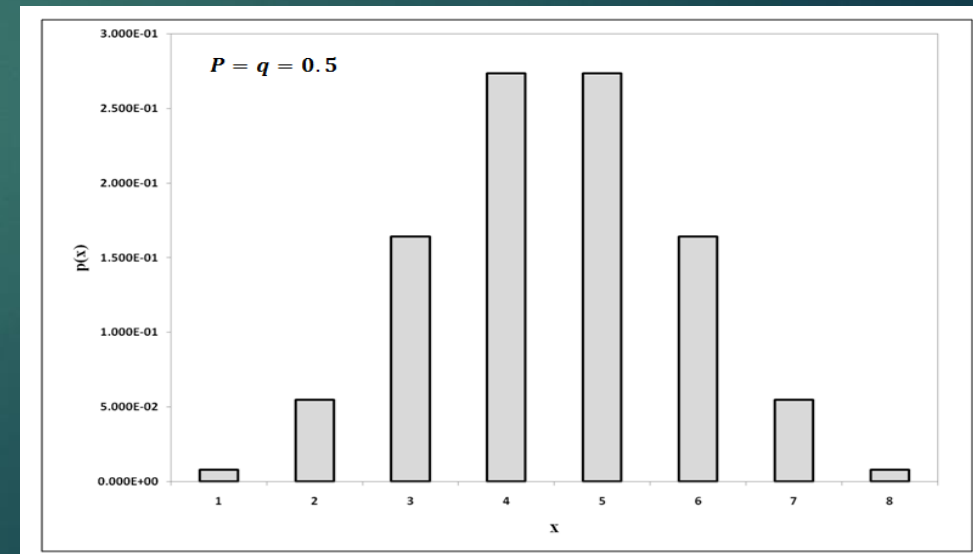
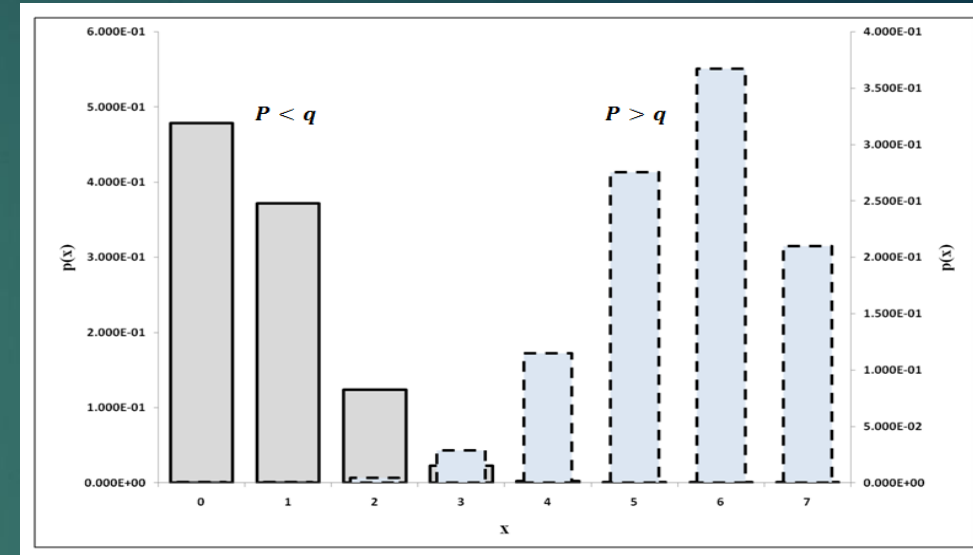
- If random variable X follows a Binomial distribution, then **mean**, **variance** and **skewness** of X can be estimated as follows:

$$E[X] = np$$

$$Var(X) = npq$$

$$\gamma_x = \frac{1 - 2p}{\sqrt{np(1 - p)}}$$

- The values p and q produce the shape of probability mass function (PMF) of a Binomial random variable;
- When $p < q$ the PMF is **positively** skewed,
- If $p > q$ the PMF is **negatively** skewed, and
- For $p = q = 0.5$ it is **symmetric**.



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Example 1

Calculate the probability of having x successes in seven trials ($n = 7$) for different p values such as $p = 0.1$, $p = 0.5$, and $p = 0.8$.

n	p	x	$p(x)$	n	p	x	$p(x)$	n	p	x	$p(x)$
7	0.1	0	4.783E-01	7	0.5	0	7.813E-03	7	0.8	0	1.280E-05
		1	3.720E-01			1	5.469E-02			1	3.584E-04
		2	1.240E-01			2	1.641E-01			2	4.301E-03
		3	2.296E-02			3	2.734E-01			3	2.867E-02
		4	2.552E-03			4	2.734E-01			4	1.147E-01
		5	1.701E-04			5	1.641E-01			5	2.753E-01
		6	6.300E-06			6	5.469E-02			6	3.670E-01
		7	1.000E-07			7	7.813E-03			7	2.097E-01

Normal Distribution

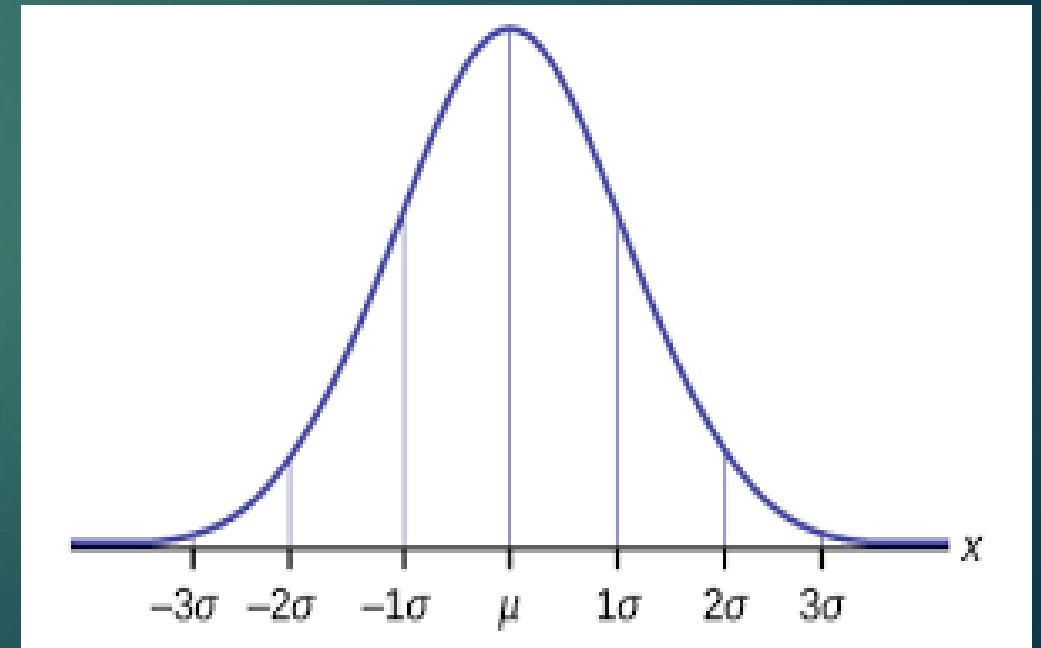
- Normal distribution is one of the **most commonly used** probability distributions by engineers.
- It is also known as **Gaussian** distribution in honor of *Carl Friedrich Gauss*.
- Normal distribution can be identified from the mean (μ_x) which shows the **center of distribution**, and variance (σ_x^2) which determines the distribution **spread** or the **height** and **width** of the normal curve.
- Hence, the normal random variable x with mean (μ_x) and variance (σ_x^2) can be shown as $N \sim (\mu_x, \sigma_x^2)$. The variance is always positive while the mean can have **negative** or **positive** value.

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- The PDF of a normal distribution is defined as follows:

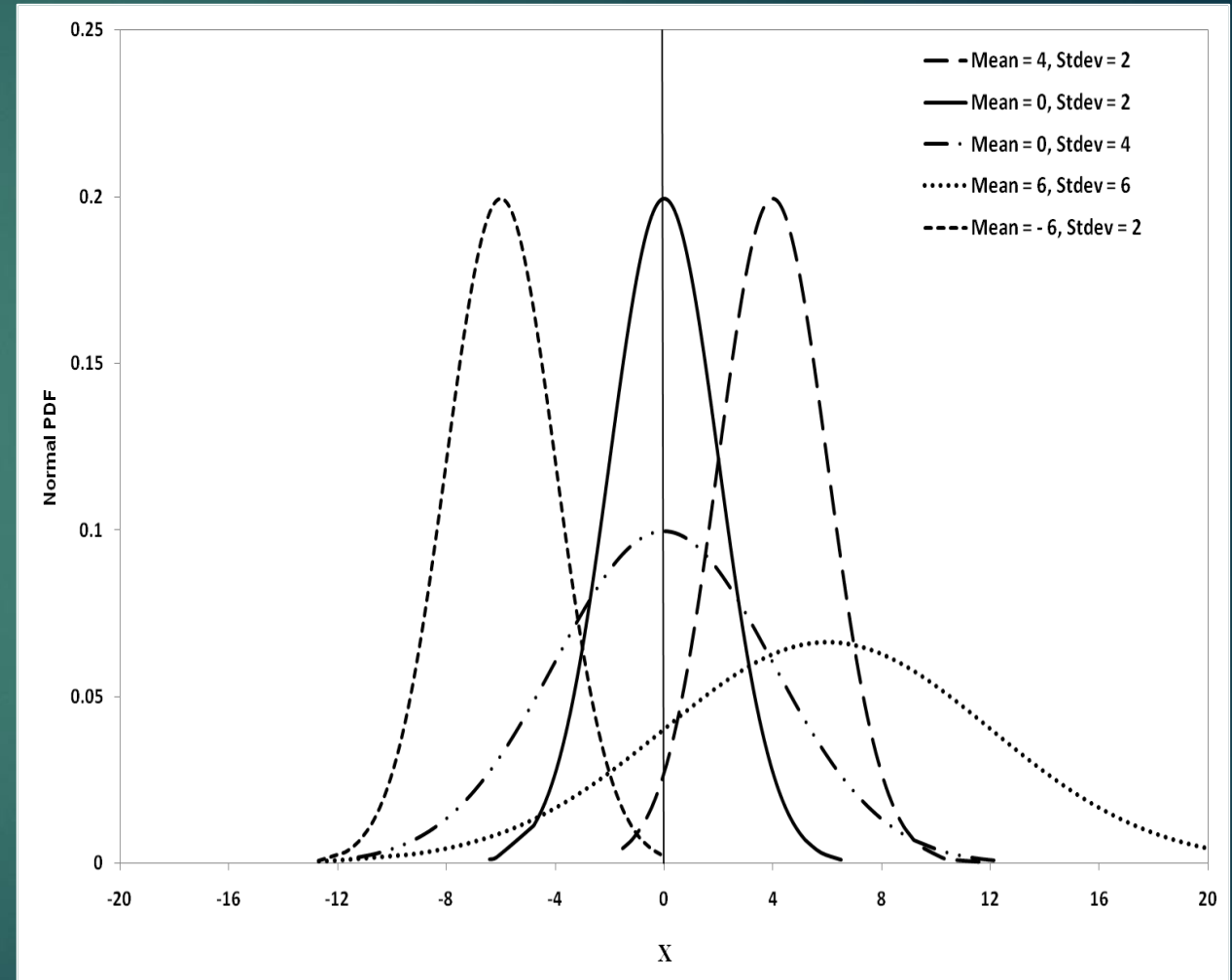
$$f(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp \left[-\frac{1}{2} \left(\frac{x - \mu_x}{\sigma_x} \right)^2 \right] \quad \text{for } -\infty < x < \infty$$

- The normal PDF is a **bell-shaped** curve with a **peak at the mean**, and extends to $\pm\infty$.
- The biggest distribution concentration is located in the **center** and it decreases along the x -axis.



Hydrology

- Mean, variance, and skewness of normal distribution are μ_x , σ_x^2 , and 0, respectively.
- Normal distribution can be used as a simple model to explain complex events in various science fields when there is sufficiently large number of independent random variables.



Hydrology

- The probability that the random variable x lies within the interval from x_1 , to x_2 is given by the **integral** of the density function from x_1 , to x_2 , over all values within the interval:

$$p(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f(x) dx$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp \left[-\frac{1}{2} \left(\frac{x - \mu_x}{\sigma_x} \right)^2 \right] \quad \text{for } -\infty < x < \infty$$

- Standard normal distribution* is a normal distribution with **mean and standard deviation** of 0 and 1, respectively.

Hydrology

- The **standard** type uses a transferring factor (z) in the following form:
- in which x is the normal random variable with mean μ_x and standard deviation σ_x .
- On the other hand, for a known Z value, the normal random variable x with mean μ_x and standard deviation σ_x can be computed as:
- The probability density function (**PDF**) of standard normal distribution z is:

$$Z = \frac{x - \mu_x}{\sigma_x}$$

$$x = \sigma_x Z + \mu_x$$

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \quad \text{for } -\infty < z < \infty$$

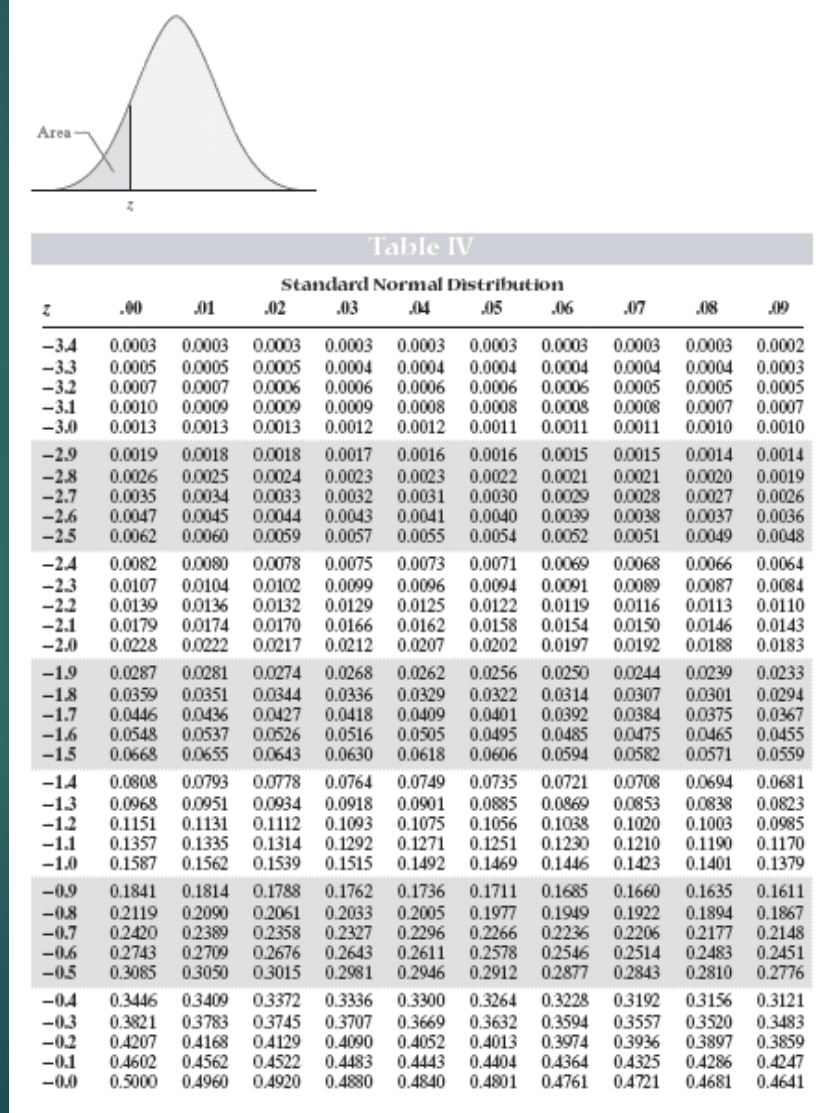
Hydrology

- The probability that the random variable z lies within the interval from z_1 , to z_2 is given by the **integral** of the density function from z_1 , to z_2 , over all values within the interval:

$$p(z_1 \leq z \leq z_2) = \int_{z_1}^{z_2} f(z) dz$$

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) \quad \text{for } -\infty < z < \infty$$

- Because probabilities for the standard normal distribution are frequently required, integrals, have been computed and placed in **tabular form**.



Hydrology

- Probabilities for cases in which the **lower limit** is different from $-\infty$ can be determined using the following identity:

$$p(z_1 \leq z \leq z_2) = \underbrace{p(-\infty \leq z \leq z_2)}_{\text{From z Table}} - \underbrace{p(-\infty \leq z \leq z_1)}_{\text{From z Table}} = \\ p(z \leq z_2) - p(z \leq z_1)$$

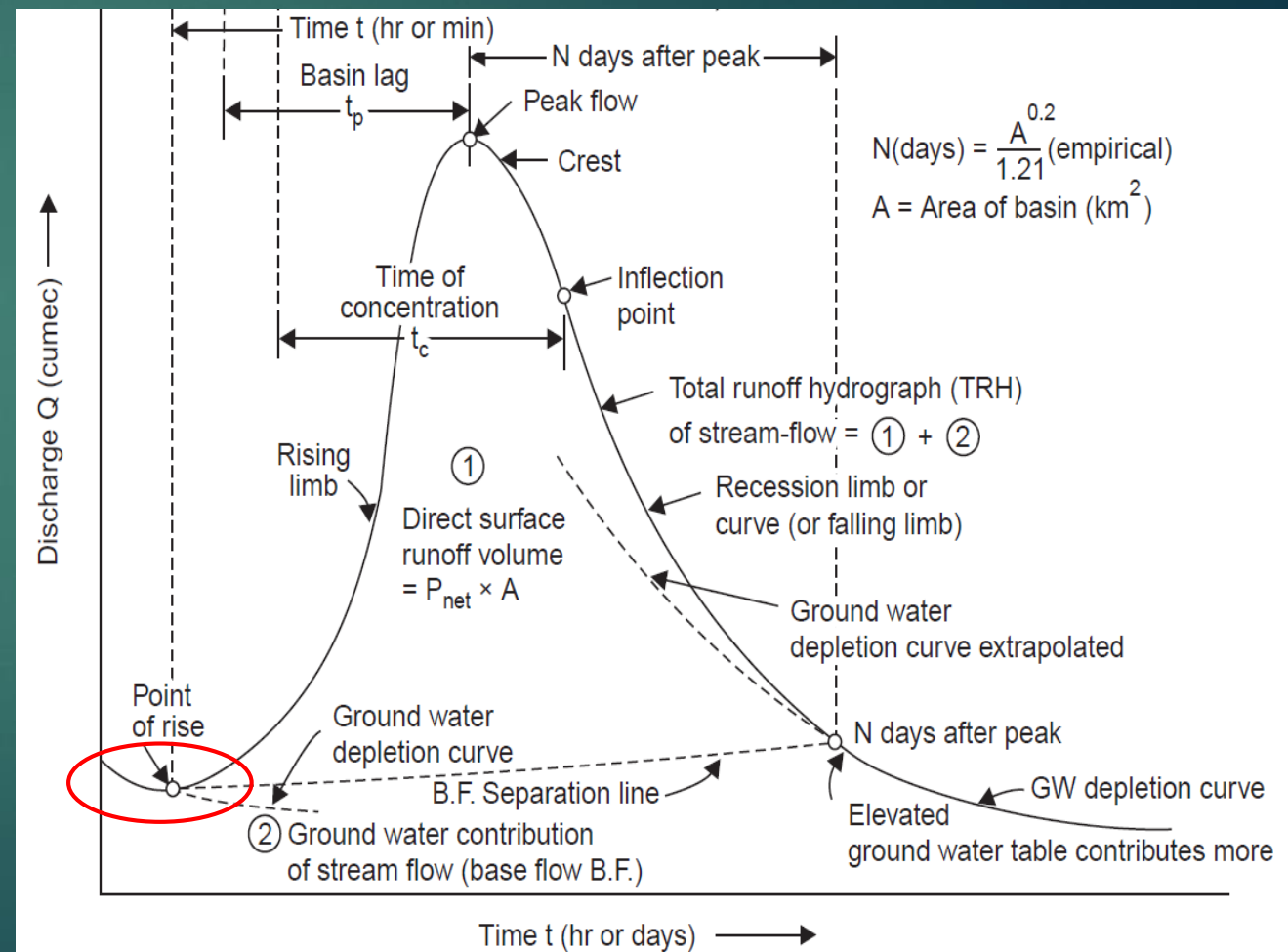
- For example, the probability of $p(0.55 \leq z \leq 2.25)$ would be:

$$p(0.55 \leq z \leq 2.25) = p(z \leq 2.25) - p(z \leq 0.55)$$

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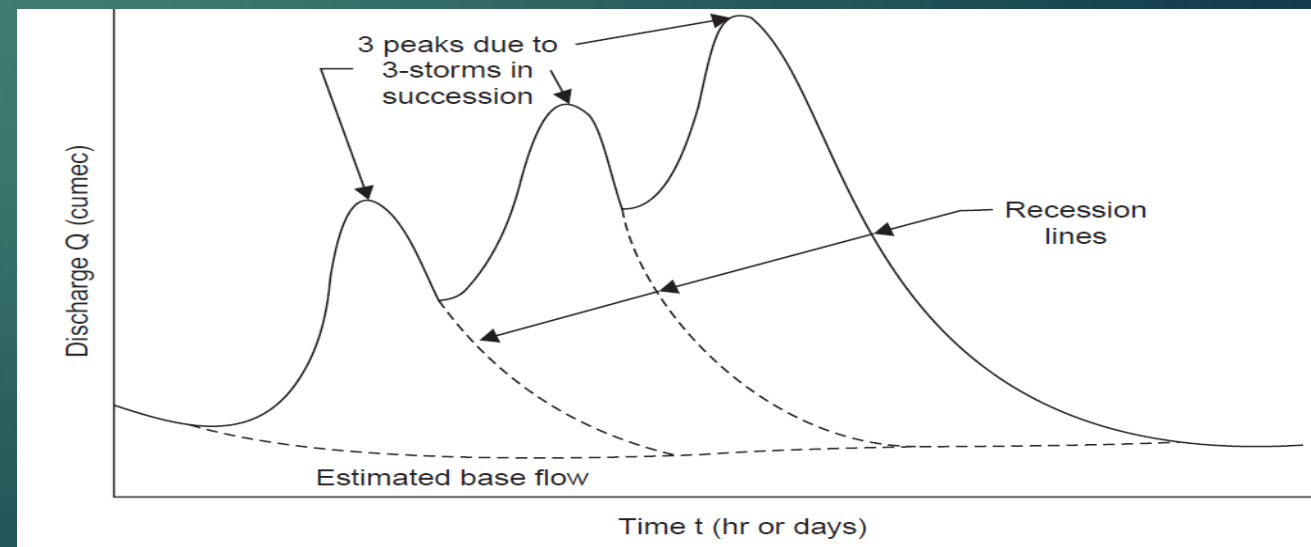
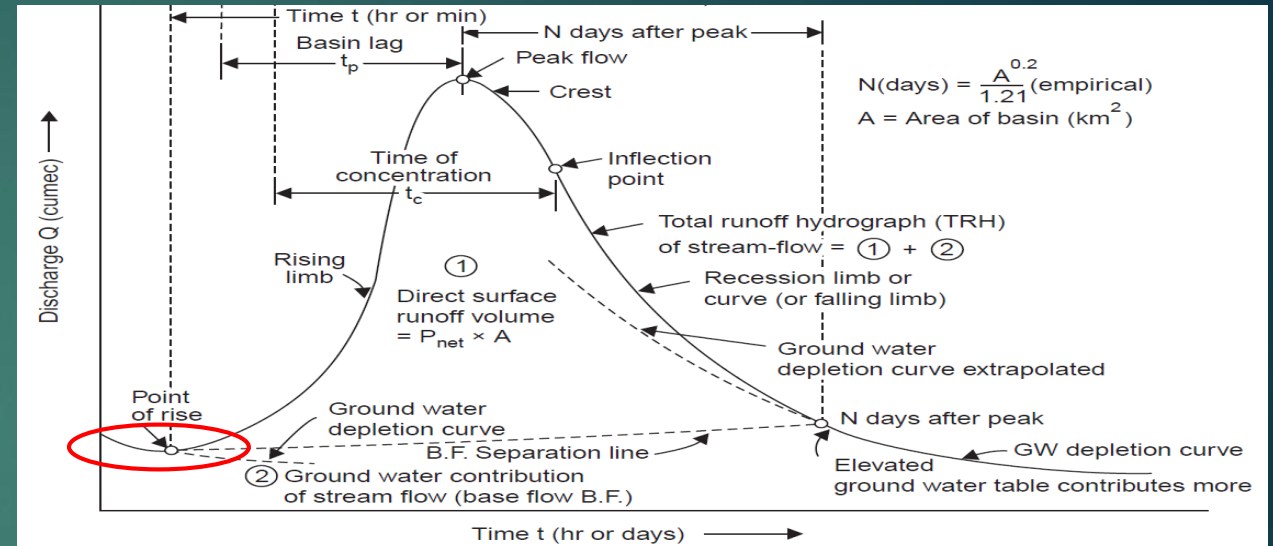
Hydrographs

- A hydrograph is a graph showing **discharge** (i.e., stream flow at the concentration point) **versus time**.
- At the beginning, there is only **base flow**.
- *Baseflow* is the portion of streamflow that comes from "the sum of deep **subsurface flow** and delayed shallow subsurface flow".



Hydrology

- The hydrograph gradually **rises** and reaches its **peak value** after a time t_p (called *lag time* or *basin lag*).
- Thereafter it **declines** and there is a change of slope at the inflection point.
- in actual streams gauged, the hydrograph may have a **single peak** or **multiple peaks** according to the complexity of storms.



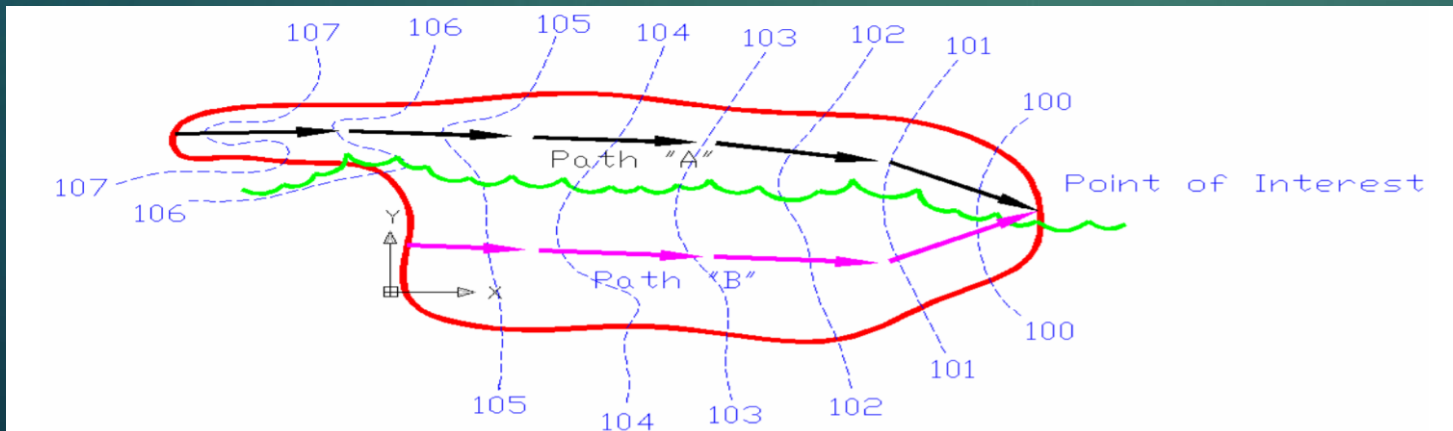
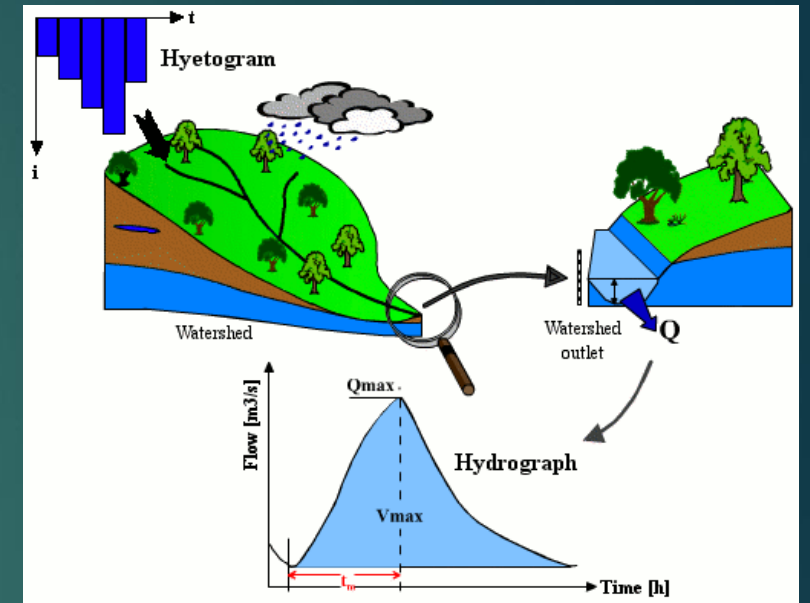
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- Time of concentration (t_c) is defined as the time needed for water to flow from the **most remote point** in a watershed to the **watershed outlet**.

$$t_c(\text{hr}) = 0.06628 L^{0.77} S^{-0.385}$$

L is length (km) and S is slope

- It is a function of the **topography, geology, and land use** within the watershed.



Path "A" is 1000 ft. long with a Time of Concentration = **1.00 Hours**

Path "B" is 750 ft. long with a Time of Concentration = **1.25 Hours**

Hydrology

Floods

- A flood is an **unusual** high stage of a river due to runoff from rainfall and/or melting of snow in quantities **too great** to be **confined** in the normal water surface elevations of the river or stream.
- The maximum flood that any structure can **safely pass** is called the '**design flood**' and is selected after consideration of *economic* and *hydrologic* factors.
- The design flood is related to the **project feature**; for example, the spillway design flood may be **much higher** than the flood control reservoir design flood.

Hydrology

- When the structure is designed for a flood **less than the maximum** probable, there exists a certain amount of **flood risk** to the structure.
- However, sometimes we have to do this as it is not **economical** to design for 100% flood protection.
- Protection against the highest rare floods is uneconomical because of the **large investment** and **infrequent flood occurrence**.
- In the design flood estimates, reference is usually made to three classes:
 1. Standard Project Flood (**SPF**),
 2. Maximum Probable Flood (**MPF**)
 3. Probable Maximum Precipitation (**PMP**).

Hydrology

- **Standard Project Flood (SPF):** This is the estimate of the flood likely to occur from the most severe combination of the **meteorological and hydrological** conditions, but excluding extremely rare combination.
- **Maximum Probable Flood (MPF):** This differs from the SPF in that it includes the **extremely rare and catastrophic floods** and is usually confined to spillway design of very high dams.
- **Probable Maximum Precipitation (PMP):** the **greatest depth of precipitation** for a given duration that is physically possible over a given storm area at a particular geographical location at a certain time of the year.

Estimation of Peak Flood

- The maximum flood discharge (**peak flood**) in a river may be determined by the following methods:
 1. Physical indications of past floods—flood marks and local enquiry
 2. Empirical formulae and curves
 3. Concentration time method
 4. Overland flow hydrograph
 5. **Rational method**
 6. Unit hydrograph
 7. Flood frequency studies

Rational Method

- The rational method is based on the application of the following formula:

$$Q = CiA$$

- where $C\left(\frac{ft^3}{s}\right)$ is a coefficient depending on the runoff qualities of the catchment called the runoff coefficient (0.2 to 0.8), the intensity of rainfall $i\left(\frac{inch}{hr}\right)$ is equal to the design intensity or critical intensity of rainfall i_c , and $A(acres)$ is the catchment area.

Type of terrain	Value of C
<i>Flat residential areas</i>	0.4
<i>Moderately steep residential area</i>	0.6
<i>Built up areas—impervious</i>	0.8
<i>Rolling lands and clay-loam soils</i>	0.5
<i>Hilly areas, forests, clay and loamy soils</i>	0.5
<i>Flat cultivated lands and sandy soils</i>	0.2

Hydrology

Example 3

Consider the design problem where a peak discharge is required to size a storm drain inlet for a 2.4-acre parking area in Baltimore, with a time of concentration of 0.1 hr and a slope of 1.5%. The rainfall intensity is 8.6 in./hr and the runoff coefficient is 0.95. What is the design discharge?

Flood Frequency

- Flood frequency analyses are used to **predict design floods** for sites along a river.
- The technique involves using observed **annual peak flow discharge** data to calculate statistical information such as mean values, standard deviations, and skewness.
- Flood frequency plays a vital role in providing estimates of floods return periods.
- In order to evaluate the **optimum design** specification for hydraulic structures, and to **prevent over-designing** or *under designing*, it is imperative to apply statistical tools to create flood frequency estimates.

Hydrology

- Along with hydraulic design, flood frequency estimates are also useful in **flood insurance** and **flood zoning activities**.
- Accurate estimation of flood frequency not only helps engineers in designing safe structures but also in **protection against economic losses** due to **maintenance** of structures.
- In order to understand how flood frequency analysis works, it is essential to understand the concept of **return period**.
- The theoretical definition of return period is the **inverse of the probability** that an event will be exceeded in **a given year**.

Hydrology

- In general, **return period** provides an estimate of the likelihood of any event in **one year**.
- These events include natural disasters such as floods or earthquakes.
- For example, a **10-year** return period corresponds to a flood that an exceedance probability of 0.10 or a **10%** chance that the flow will **exceed** in one year.
- This **does not** mean that a 10-year flood will happen regularly every 10 years, or only once in 100 years.
- In any given 100-year period, a 100-year event may occur **once, twice**, more, or not at all.

Hydrology

- The probability of occurrence of a flood (having a recurrence interval T-yr) in any year, is:

$$P = \frac{1}{T}$$

- The probability that it will not occur in a given year

$$q = 1 - P = 1 - \frac{1}{T}$$