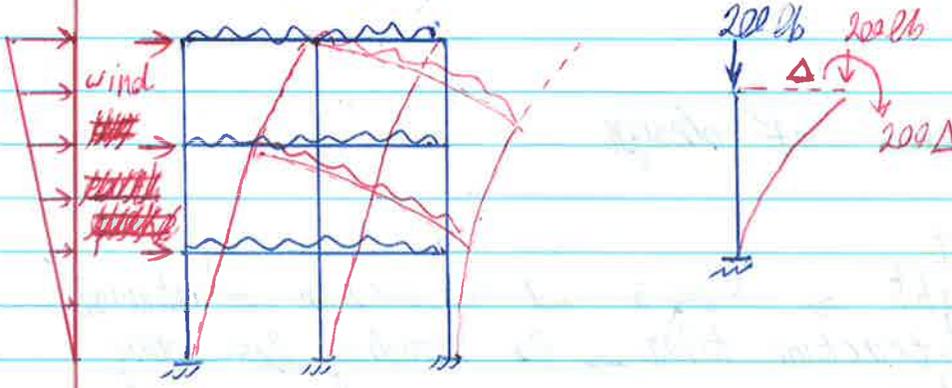
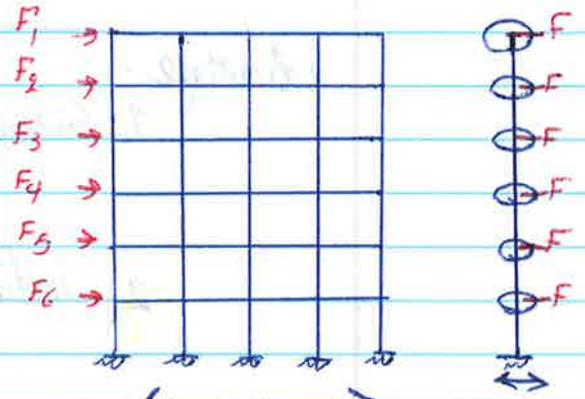
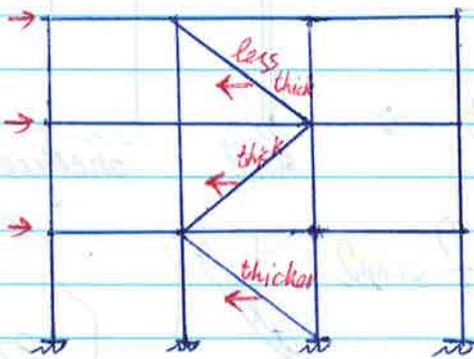


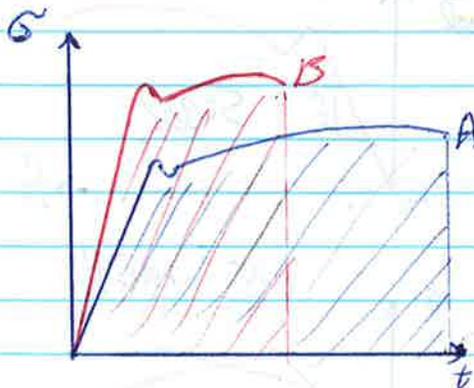
Advance Structural Steel



*Controlling the excessive deflection key to structural stability



$$F = \frac{w}{g} \ddot{a} = F_1 + F_2 + F_3 + F_4 + F_5 + F_6$$



$$\text{Ductility} = \frac{l - l_0}{l_0}$$

Toughness = area under chart (σ - t)
= Energy Absorbed.

redundant \rightarrow to provide added safety against unknown situations

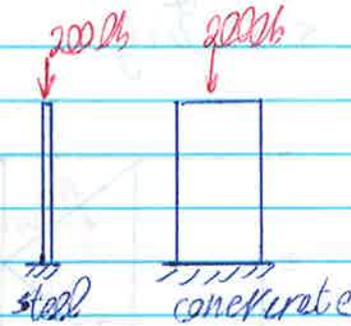
* Principles of design

1. Cost
2. Weight $\rightarrow F = \frac{w}{g} \ddot{a}$ and Transportation of material
3. Construction time \rightarrow for spending less money
4. Labor

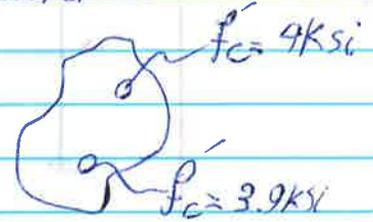
* Steel Vs. Concrete

advantage

1. Strength/weight \Rightarrow



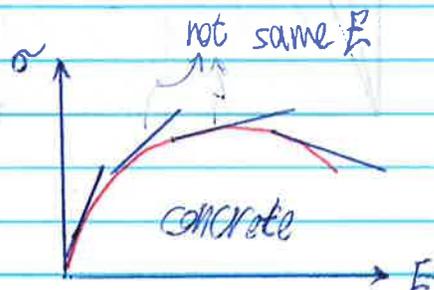
2. Uniformity of steel



3. Elasticity of steel

$$\delta = \frac{PL^3}{EI}$$

\rightarrow always $\approx 30,000 \text{ KSI}$



4. Permanence of steel because concrete has corrosion problem.

5. Ductile → warning (concrete has no Ductile)

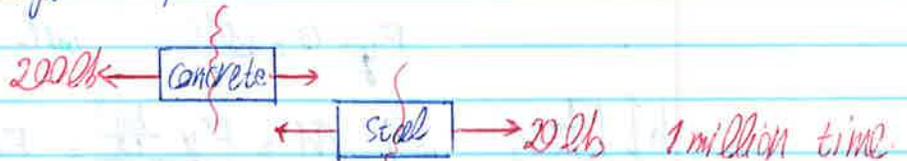
6. Toughness → steel is better in toughness means can handle loads in a period of time. not just one force.

* Problems of steel

1. Maintenance → Paint → corrosion

2. Performance in high Temp.

3. Fatigue



4. buckling of steel



* Skeleton of steel structure consist of Tension member, columns and beams.

* Tension members: (more area is better) (shape doesn't matter)



How we choose: $P = F_y A_g \Rightarrow A_g = \frac{P}{F_y}$

Cable *

* Compression member (column):



$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EAr^2}{L^2} \Rightarrow \text{only } r \text{ is not constant}$$

r = radius of gyration
= How much it spread out



same Area but II is better in compression

Compression \rightarrow more spread out better

* bending member (beam):

$$F_y = \sigma = \frac{Mc}{I} \quad M = \frac{F_y I}{c}$$

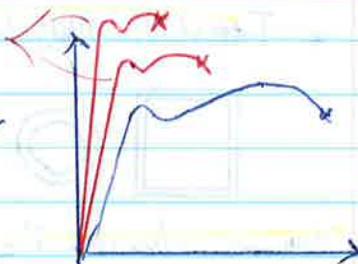
$\Rightarrow M = \frac{F_y \frac{bh^3}{12}}{h/2} = F_y \frac{bh^2}{6}$

$\Rightarrow F_y \frac{bh^2}{6} = F_y \frac{b \frac{b^2}{16}}{6} = F_y \frac{b^3}{96}$



$$\Rightarrow F_y \frac{bh^2}{6} = F_y \frac{b/4 h^2}{6} = F_y \frac{bb^2}{4 \times 6} = F_y \frac{b^3}{24}$$

Steel \rightarrow Fe + C
more carbon \rightarrow stronger
 \rightarrow brittle



* with increasing 0.01% carbon \rightarrow 5 Ksi increase in Steel

Cast Iron = $C > 2\%$

Steel = 0.5% - 1.7% of C

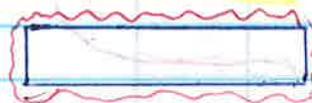
Wrought Iron = $C < 0.15\%$

types of steel

\rightarrow Structural Steel:

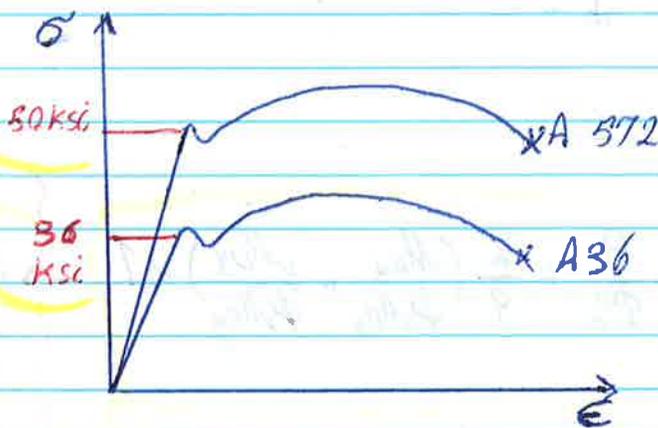
1. Regular Steel (A36, A53, A500, A501, A529)
2. High strength / Low Alloy (A572, A618, A913, A992)

\rightarrow Corrosion resistant steel (A242, A588, A897)



\rightarrow Cu vapor

* Adding low Alloys (Silicon, Manganese, Copper) makes steel High-Strength

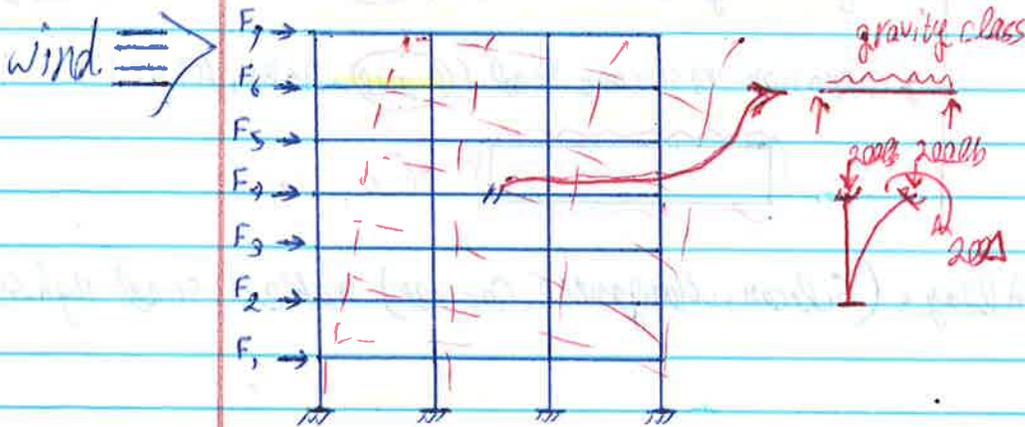
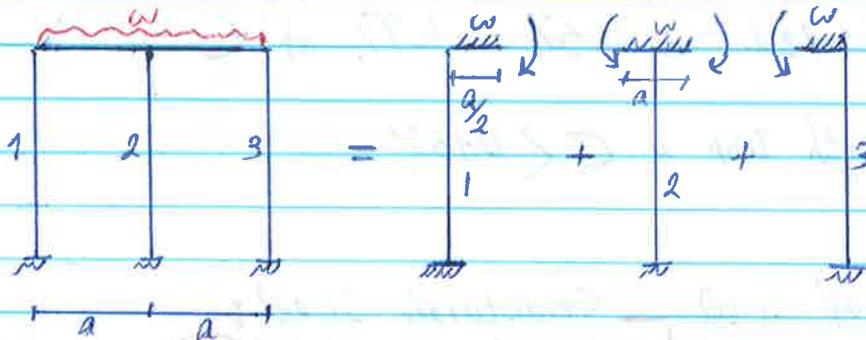


Now days:

- structures are of A572
- structural joints and connections are of A36

Chapter 11

Beam-Column



← Earthquake

$$\frac{P_u}{\phi P_n} \geq 0.2 \rightarrow \frac{P_u}{\phi P_n} + \frac{8}{9} \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1$$

like a column or tension member

$$\frac{P_u}{\phi P_n} \leq 0.2 \rightarrow \frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1$$

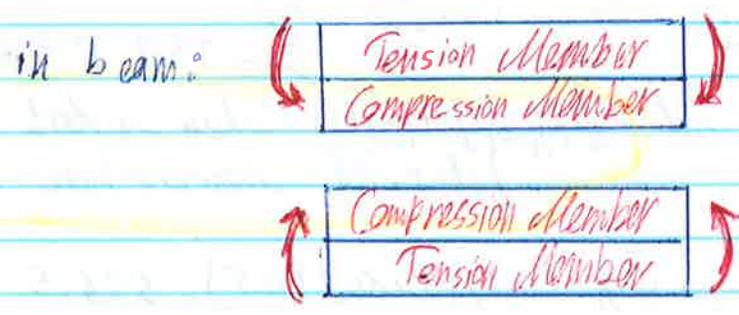
like a beam

P_u : Calculated Factor Load

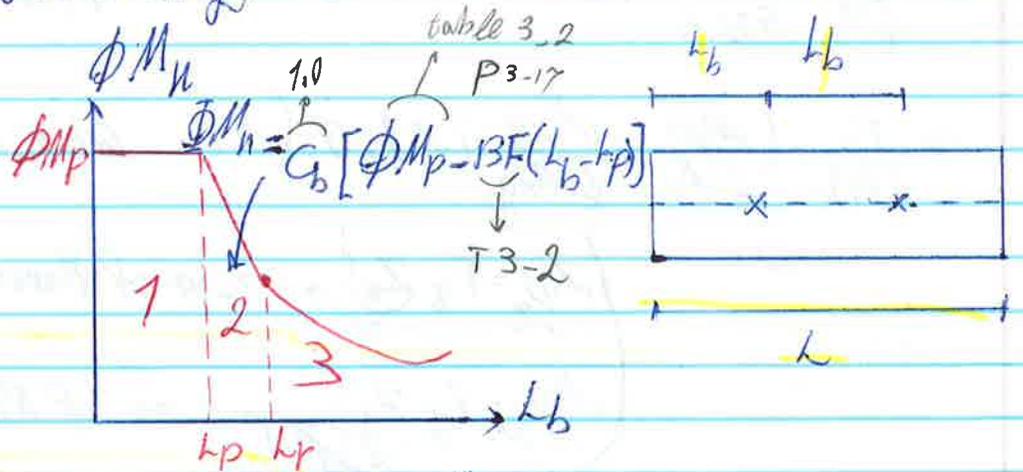
ΦP_n : Capacity of column

← Under Tension → $T = \Phi F_y A_g$ (independent of length)

→ Under Compression ← $C = \Phi \frac{\sigma^2 E}{(\frac{KL}{r})^2}$ (dependent to length)



* for bracing



Zone ① Plastic hinge develops by entire section reach yielding.

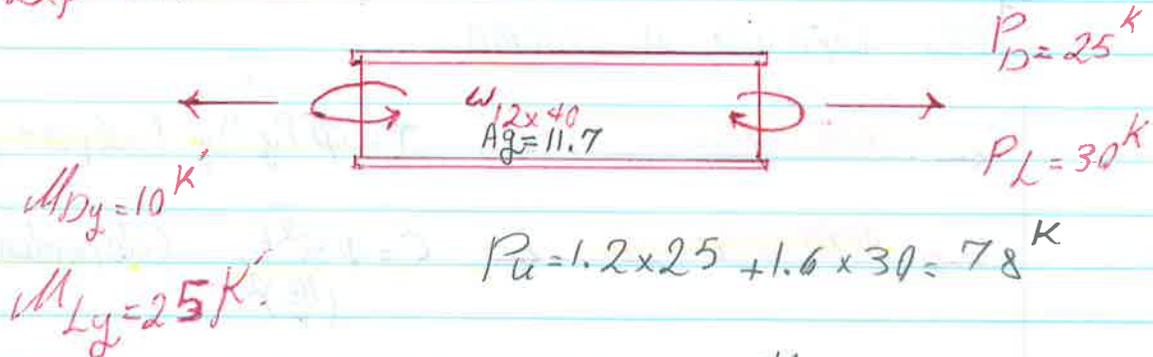
Zone ② some portion of α -section reach yielding.

Zone ③ Torsional Buckling

homework: problem 11-1 next Thursday

Beam Tension Member

Exp 11-1:



$$M_{uy} = 1.2 \times 10 + 1.6 \times 25 = 52 \text{ k}$$

$$\frac{P_u}{\phi P_n} = \frac{78}{\phi P_n}$$

and $\phi P_n = \begin{cases} \phi F_y A_g & \text{when we don't have holes} \\ \phi F_u A_e & \text{when we have holes} \end{cases}$ T5-1 P 5-12

$$\Rightarrow \phi P_n = (0.9)(50 \text{ ksi})(11.7) = 526.5$$

$$\Rightarrow \frac{P_u}{\phi P_n} = \frac{78}{526.5} < 0.2 \Rightarrow \text{formula H1-1b applies}$$

$$\Rightarrow \frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1.0 \text{ (acts like a beam)}$$

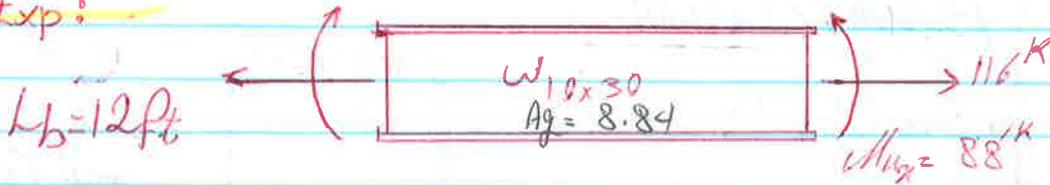
$M_{px} = F_y Z_x$ P 3-19 of Manual

$M_{py} = F_y Z_y$ P 3-30 of Manual = 16.8

$$\frac{78}{2 \times 526.5} + \left(\frac{52}{63} \right) = 0.899 < 1 \checkmark \text{ Design is good}$$

$0.9 \leq \dots \leq 1$ is always a good design

Exp:



$$\phi P_n = \phi F_y A_g = (0.9)(50 \text{ ksi})(8.84) = 397.8$$

$$\frac{P_u}{\phi P_n} = \frac{116}{397.8} = 0.292 > 0.2 \Rightarrow (\text{act like tension member})$$

\Rightarrow we use H1-7a formula

$$\frac{P_u}{\phi P_n} + \frac{8}{9} \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1.0$$

$$\Rightarrow \frac{116}{397.8} + \frac{8}{9} \left(\frac{88}{0} \right)$$

Hint: in the "y" direction there is no zoning and the formula is:

$$\phi M_{ny} = \phi F_y Z_y \quad (\text{for exp last page})$$

P 3-19: we have $W_{10 \times 30} \xrightarrow{P3-20}$ $\begin{cases} L_p = 4.84 \text{ ft} \\ L_r = 16.1 \text{ ft} \end{cases} \Rightarrow L_p \leq 12 \leq L_r$
Zone 2

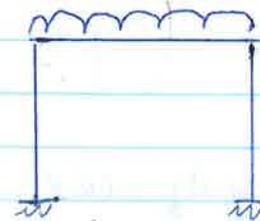
$$\text{Zone 2} \Rightarrow \phi M_n = \phi [\phi M_p - 13F(L_b - L_p)]$$

$$= 1 [137 - 4.61(12 - 4.84)] = 104 \text{ k}$$

$$\Rightarrow \frac{116}{397.8} + \frac{8}{9} \left(\frac{88}{104} + 0 \right) = 1.044 > 1 \quad \text{N.G.}$$

Beam-column

* Scenario I:



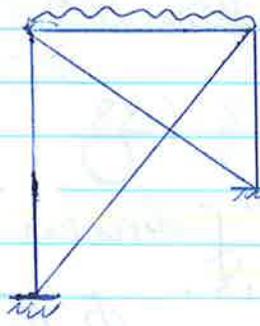
load and frame are symmetric

we call this situation:

- Non-translational
- Non-sway
- Non-lateral displacement
- No horizontal load

Pnt
Mnt

* Scenario II:

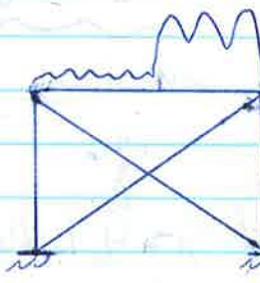


Asymmetrical structure but symmetrical loading

Mt

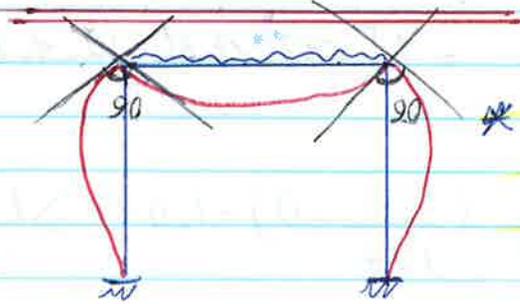
because we fixed the structure from swaying

* Scenario III:

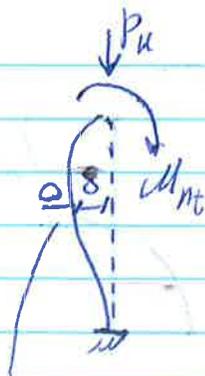


Mt

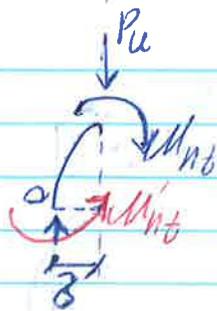
symmetric structure but non symmetric loading



* degree btw. tangent of column and beam is always 90°



fail or equilibrium



$$\sum M_0 = 0 \Rightarrow M_{nt}' - M_{nt} - P_u \delta = 0$$

$$M_{nt}' = M_{nt} + P_u \delta$$

secondary moment

- solutions
1. non linear FE analysis
 2. AISC empirical method (use this) $\Rightarrow M_{nt}' = B_1 M_{nt}$

$$B_1 = \frac{1}{1 - \frac{P_u}{P_{e1}}}$$

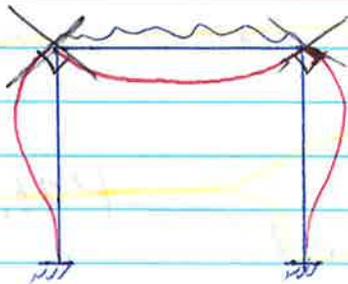
for single-curvature

$$P_{e1} = \text{buckling load} = \frac{\pi^2 EI}{(KL)^2}$$

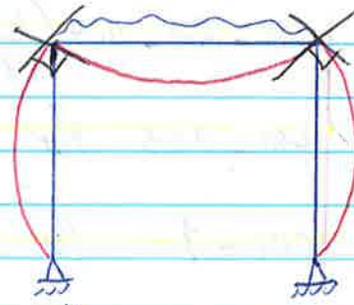
braced column / Braced frame

- K
- P 16.1 - 511 \Rightarrow single column
 - P 16.1 - 512 \Rightarrow braced frame
 - NT $\Rightarrow K=1$ (conservative)

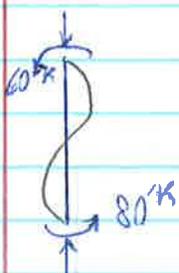
T 1-1 p 1.27



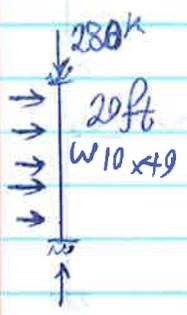
equal and opposite and single-curvature



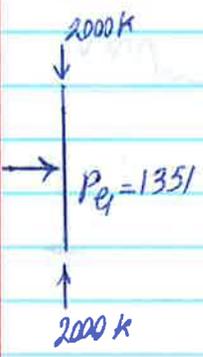
$$M_{nt}' = M_{nt} + P_u \delta = B_1 M_{nt} = \frac{1}{1 - \frac{P_u}{P_{e1}}} M_{nt}$$



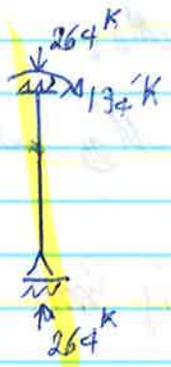
$$C_m = 0.6 - 0.4 \left(+ \frac{60}{80} \right) = 0.3$$



$$C_m = 1 - 0.4 \frac{P_u}{P_{e1}} = 1 - 0.4 \frac{280}{\frac{\sigma^2 \times 29000 \times 272}{(1 \times 20 \times 12)^2}} = 0.92$$



$$C_m = 1 - 0.2 \frac{P_u}{P_{e1}} = 1 - 0.2 \frac{200}{1351} = 0.97$$



$$C_m = 0.6 - 0.4 \left(\frac{\phi}{\psi_2} \right) = 0.6$$

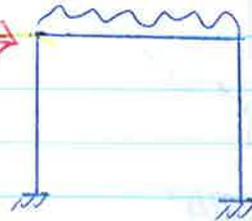
in this case we dont have horizontal load so that we cant use p161-525

2nd Beam-Columns

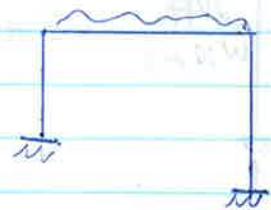
LT = laterally translational

horizontal load \rightarrow

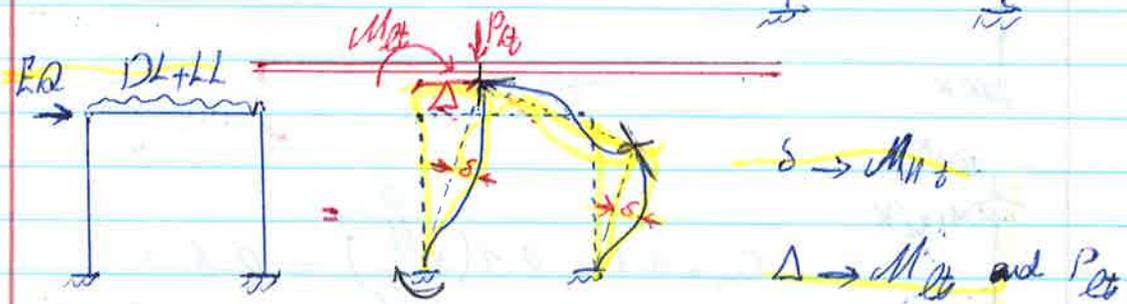
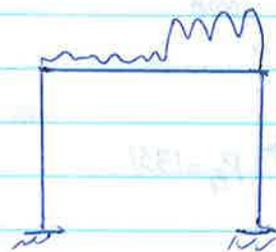
* Scenario I:



* Scenario II:

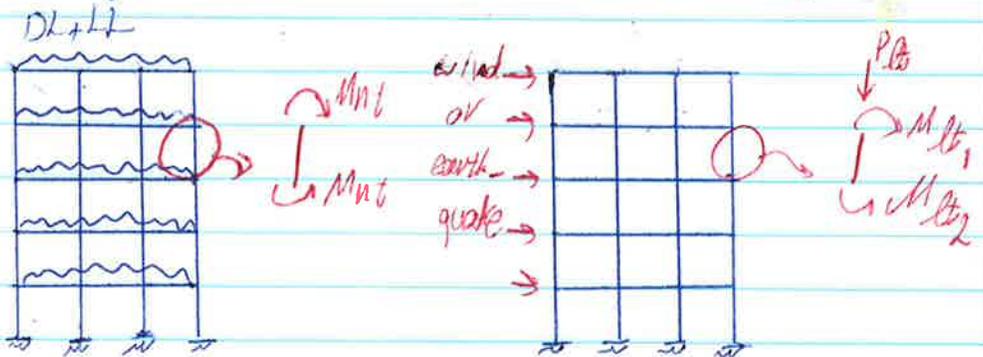


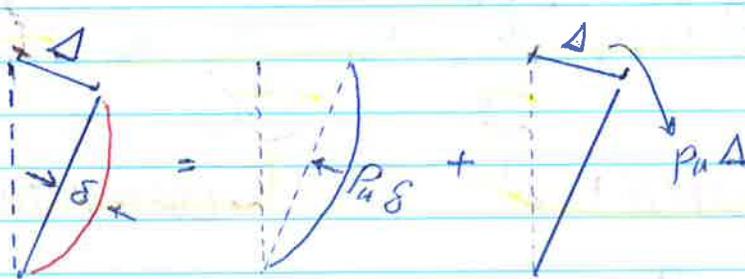
* Scenario III:



$$\Rightarrow M_u = \beta_1 M_{u1} + \beta_2 M_{u2}$$

coming from DL+LL \rightarrow coming from EQ





$$M_u = (M_{nt} + P_u \delta) + (M_{lt} + P_u \Delta)$$

$$= B_1 M_{nt} + B_2 M_{lt}$$

$P_u = P_{nt} + B_2 P_{lt}$

DL + LL → wind or earthquake

$$B_2 = \frac{1}{1 - \frac{P_{story}}{P_{e, story}}}$$

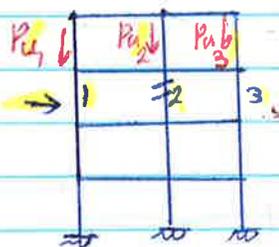
$P_{story} = \sum P_u$ for Entire Floor

$P_{e, story} = \sum P_{e2}$ for Entire Floor

$$P_{e1} = \frac{\sigma^2 EI}{(KL)^2} \rightarrow K \approx 1.0 \text{ or braced frame}$$

$$P_{e2} = \frac{\sigma^2 EI}{(KL)^2} \rightarrow K \text{ for unbraced frame}$$

P 16.1.3/3 > 1.0



$$P_{story} = P_{u1} + P_{u2} + P_{u3}$$

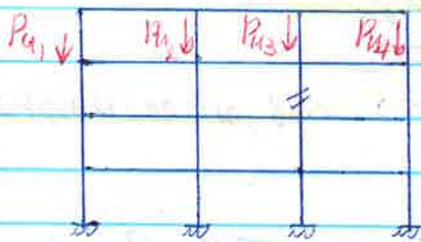
$$P_{e, story} = P_{e2,1} + P_{e2,2} + P_{e2,3}$$

$$B_2 = \frac{1}{1 - \frac{P_{story}}{P_{e, story}}}$$

$$\sum P_{e2} = P_{e, \text{Story}} = \frac{\text{height of the floor} \times H \times R_M}{\Delta_H} = \frac{H}{\left(\frac{\Delta_H}{h}\right)} \times R_M$$

horizontal load on the frame $\{ \text{IMP} \}$
 DRIFT $\left\{ \begin{array}{l} \text{DRIFT INDEX} \end{array} \right.$

$$R_M = 1 - 0.15 \frac{P_{mp}}{P_{\text{story}}}$$



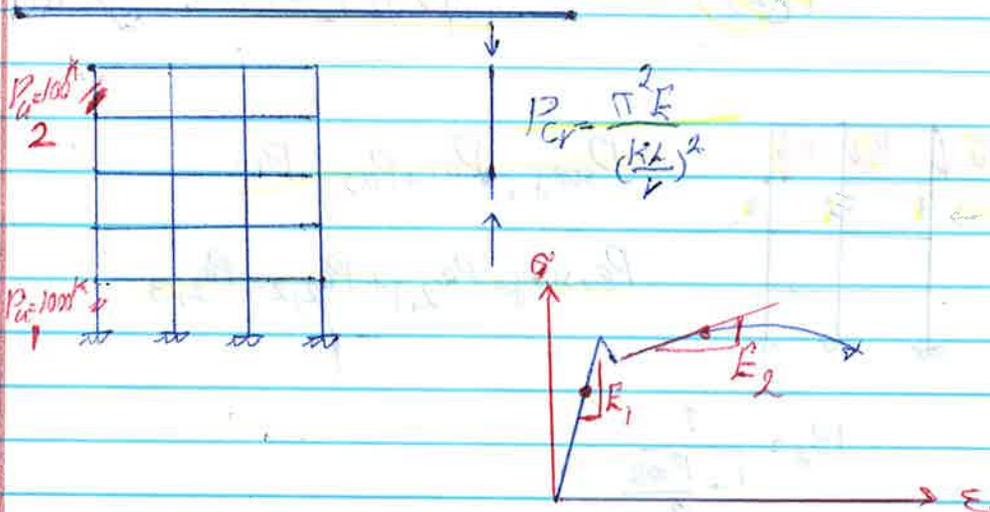
$$P_{\text{story}} = P_{u1} + P_{u2} + P_{u3} + P_{u4}$$

$P_{mp} = P_u$ @ columns design to take transverse load

code: $P_{mp} = \frac{1}{3} \sum P_u$

$$\frac{\Delta_H}{L} = \begin{cases} 0.0015 - 0.003 & \text{(without load factor)} \\ 0.004 & \text{(Factored load)} \end{cases}$$

Direct Analysis Method



$$\frac{P_u}{P_y} \leq 0.5 \rightarrow C_b = 1.0$$

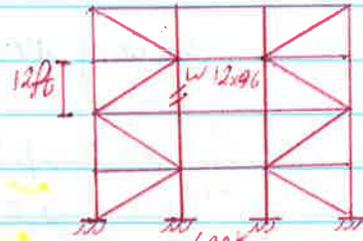
$$\frac{P_u}{P_y} > 0.5 \rightarrow C_b = 4 \left(\frac{P_u}{P_y} \right) \left(1 - \frac{P_u}{P_y} \right)$$

$$\Rightarrow \text{FK} \rightarrow 0.8 C_b \text{FK}$$

Exp II-3:

$$P_D = 175 \text{ k} \quad P_L = 300 \text{ k}$$

$$M_{Dx} = 60 \text{ k} \quad M_{Lx} = 60 \text{ k}$$



Not case \Rightarrow

$$P_u = 1.2 \times 175 + 1.6 \times 300 = 690 \text{ k} = P_{nt}$$

$$M_{ux} = 1.2 \times 60 + 1.6 \times 60 = 168 = M_{ntx}$$

\hookrightarrow From now on we don't use it anymore because $M_{ux} = \beta_1 M_{ntx}$ So we only use "M_{ntx}"

Single Curvature $\hookrightarrow C_m = 1.0$

$$\beta_{1x} = \frac{C_m}{1 - \frac{P_u}{P_{e1}}} \quad \textcircled{1} \quad C_m = 0.6 - 0.4 \left(-\frac{M_1}{M_2} \right) = 1.0 \quad \textcircled{2}$$

$$P_{e1} = \frac{\pi^2 EI_x}{(KL)^2} = \frac{\pi^2 \times 29000 \times 833}{(1 \times 12 \times 12)^2} = 11498 \quad \textcircled{3}$$

$$\textcircled{1} \text{ and } \textcircled{2} \text{ and } \textcircled{3} \Rightarrow \beta_{1x} = \frac{1}{1 - \frac{690}{11498}} = 1.064$$

$$\Rightarrow M_{ux} = \beta_{1x} M_{ntx} = 1.064 \times 168 = 178.8 \text{ kft}$$

$$\frac{P_u}{\phi P_n} = \frac{690}{1080} = 0.639 > 0.2 \Rightarrow \text{act more like a column}$$

$$\hookrightarrow P4-19 \Rightarrow KL = 12' \Rightarrow \phi P_n = 1080$$

$$\Rightarrow \frac{P_u}{\phi P_n} + \frac{8}{9} \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}} \right) \leq 1.0 \quad \text{and } L_p = 10.9 \quad L_r = 46.6$$

$$\text{table 3-2} \quad L'_b = 12 \rightarrow \text{Zone 2}$$

$$\Rightarrow \phi M_{nx} = 1 [551 - 5.7 (12 - 10.9)] = 544.6$$

$$\Rightarrow 0.639 + \frac{8}{9} \left(\frac{178.8}{544.6} \right) = 0.9... < 1 \quad \checkmark \text{ design is ok}$$

$$\left. \begin{aligned} & \phi P_n + b_x M_{ux} + b_y M_{uy} \leq 1.0 \quad \frac{P_u}{\phi P_n} > 0.2 \\ & \frac{1}{2} \phi P_n + \frac{9}{8} (b_x M_{ux} + b_y M_{uy}) \leq 1.0 \quad \frac{P_u}{\phi P_n} < 0.2 \end{aligned} \right\}$$

solving problem using Table 6-1 and above formula:

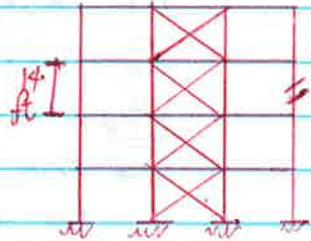
$$\frac{P_u}{\phi P_n} > 0.2 \Rightarrow \phi P_n + b_x M_{ux} + b_y M_{uy} \leq 1.0$$

$$\Rightarrow 0.924 \times 10^{-3} \times 690 + 1.63 \times 10^{-3} \times 178.8 = 0.929 < 1.0 \quad \checkmark \text{ design is ok}$$

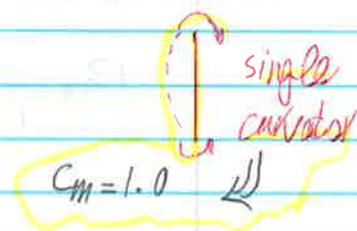
Exp 11-5: EI method: don't change "E"

$$P_u = 1.2 P_D + 1.6 P_L = 244 \text{ k}$$

$$\begin{aligned} M_{Dx} &= 60 \text{ k} \\ M_{Lx} &= 80 \text{ k} \\ M_{Dy} &= 40 \text{ k} \\ M_{Ly} &= 60 \text{ k} \end{aligned}$$



$$\begin{aligned} M_{ntx} &= 1.2 \times 60 + 1.6 \times 80 = 200 \text{ k} \\ M_{nty} &= 1.2 \times 40 + 1.6 \times 60 = 144 \text{ k} \end{aligned}$$



$$KL = 14' \Rightarrow \phi P_n = 1370 \text{ (table 4-1)}$$

$$\frac{P_u}{\phi P_n} = \frac{244}{1370} = 0.178 < 0.2 \Rightarrow \text{act like a beam } \star$$

$$M_{ux} = B_{1x} \times M_{ntx} \text{ and } M_{uy} = B_{1y} \times M_{nty}$$

$$B_{1x} = \frac{1}{1 - \frac{P_u}{P_{e1x}}} \quad P_{e1x} = \frac{\pi^2 EI_x}{(KL)^2} \approx 1.0 \quad = \frac{\pi^2 \times 29000 \times 1380}{(1 \times 14 \times 12)^2} = 13995 \text{ k}$$

$$\Rightarrow B_{1x} = \frac{1}{1 - \frac{244}{13995}} = 1.018$$

hw:
11-12 / 11-11

due: next thurs day

$$B_{1y} = \frac{1}{1 - \frac{P_u}{P_{ey}}} \quad P_{ey} = \frac{\pi^2 \times 29000 \times 495}{(1 \times 14 \times 12)^2} = 5020$$

$$\Rightarrow B_{1y} = \frac{1}{1 - \frac{244}{5020}} = 1.051$$

$$\Rightarrow M_{ux} = 1.08 \times 200 = 203.6 \text{ kft} \quad M_{uy} = 1.051 \times 144 = 151.3 \text{ kft}$$

table 6-18: $KL=14$ $p = 0.73 \times 10^{-3}$ $b_x = 1.13 \times 10^{-3}$
 $b_y = 2.32 \times 10^{-3}$

$$\Rightarrow \text{Second formula: } \frac{1}{2} p P_u + \frac{2}{8} (b_x M_{ux} + b_y M_{uy})$$

$$= \frac{1}{2} \times 0.73 \times 10^{-3} \times 244 + \frac{2}{8} (1.13 \times 10^{-3} \times 203.6 + 2.32 \times 10^{-3} \times 151.3)$$

$$= 0.743 < 1 \checkmark \text{ OK but not good}$$

Solve with direct method: From \star will change \circ

$$B_{1x} = \frac{C_{m_x}}{1 - \frac{P_u}{P_{ex}}} = \frac{1}{1 - \frac{244}{11,196}} = 1.022$$

$$P_{ex} = \frac{\pi^2 \cdot 0.8 (C_b) E I}{(K_1 L_x)^2} = \frac{\pi^2 \times 0.8 \times 1 \times 29000 \times 1380}{(1 \times 12 \times 14)^2} = 11,196$$

$$\frac{P_u}{P_y} = \frac{244}{(35.3)(50)} = 0.138 < 0.5 \Rightarrow C_b = 1.0$$

$\hookrightarrow A \cdot F_y$

$$M_{ux} = B_{1x} M_{1x} = 1.022 \times 200 = 204.4 \text{ kft}$$

$$C_{m_y} = 0.6 - 0.4 \left(-\frac{M_1}{M_2} \right) = 1.0 \Rightarrow B_{1y} = \frac{C_{m_y}}{1 - \frac{P_u}{P_{ey}}}$$

$$P_{ey} = \frac{\sigma^2 0.8 C_b E I_y}{(K_y L_y)^2} = \frac{\sigma^2 0.8 \times 1 \times 29000 \times 495}{(1 \times 14 \times 12)^2} = 4016$$

$$\Rightarrow B_{1y} = \frac{C_{m1y}}{1 - \frac{P_u}{P_{ey}}} = \frac{1}{1 - \frac{249}{4016}} = 1.065 \Rightarrow M_{1y} = B_{1y} M_{nt1y} = 1.065 \times 141 = 153.3 \text{ kft}$$

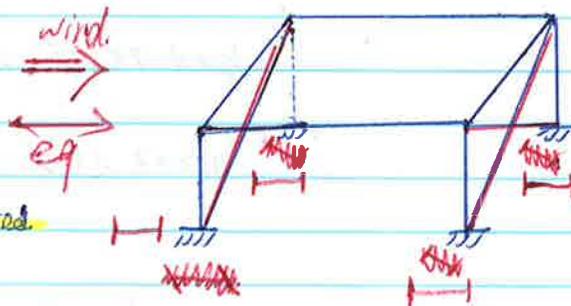
AISC T 6-1 $\Rightarrow K_h = 14$: $P = 1 \times 10^{-3}$, $b_x = 1.13 \times 10^{-3}$, $b_y = 2.33 \times 10^{-3}$

$$\frac{1}{2} P P_u + \frac{9}{8} (b_x M_{ux} + b_y M_{uy}) \leq 1.0 \Rightarrow 0.749 \leq 1 \checkmark \text{ OK but not good}$$

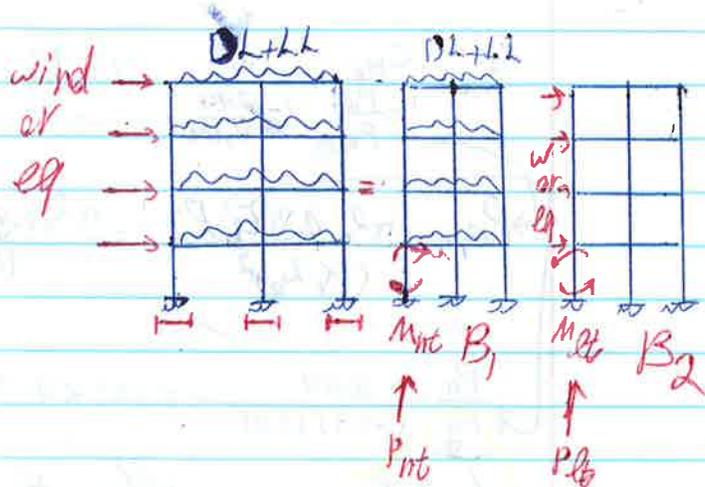
Unbraced Frame

Scenario I

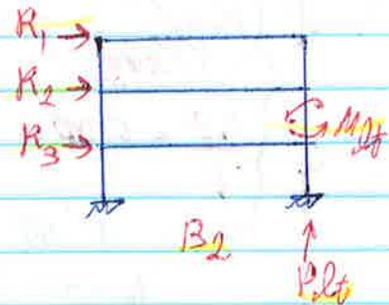
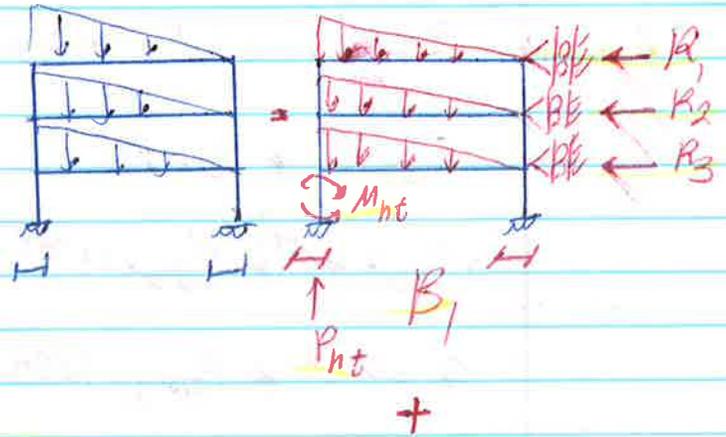
* weak axis is always braced



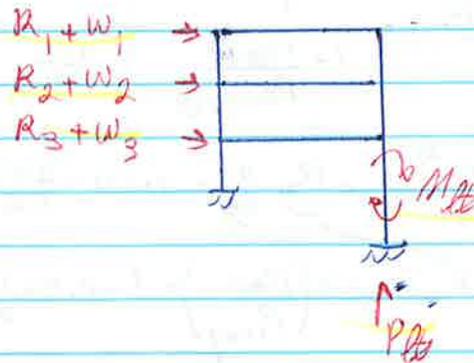
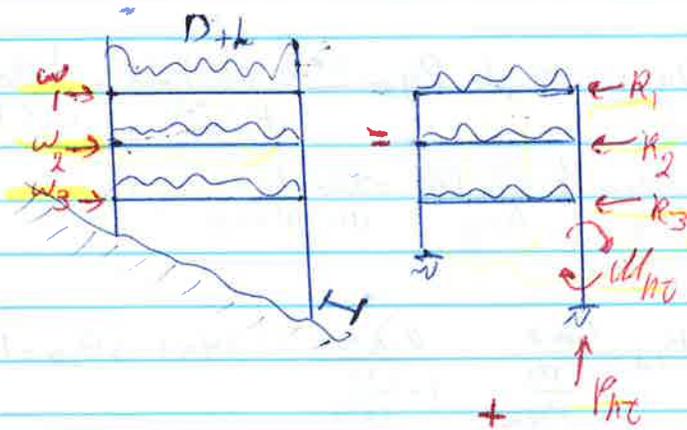
Scenario II



Scenario III



Scenario IV

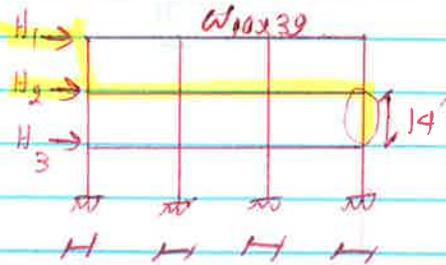


Ex 11.2: Direct Method Analysis

$$P_{nt} = 130^k \quad * \quad P_{et} = 25^k$$

$$M_{nt} = 45^k\text{-ft} \quad * \quad M_{et} = 15^k\text{-ft}$$

$$C_m = 0.85 \quad * \quad P_{story} = 160^k$$

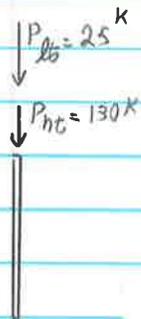


$$P_{mf}/P_{story} = 0.333 \quad * \quad \text{Story Shear} = H_1 + H_2 = 33.4^k$$

$$\frac{\Delta f}{L} = 0.0025$$

L

→ 1/3 of all columns in this floor are designed for horizontal loads



$$\text{Direct Method } P_{ex} = \frac{\pi^2 \cdot 0.8 \cdot C_b \cdot E \cdot I_x}{(K_1 L_x)^2} = \frac{\pi^2 \cdot 0.8 \cdot 1 \cdot 29000 \cdot 209}{(1 \times 12 \times 14)^2} = 1696^k$$

$$P_u = \frac{P_{nt} + B_2 P_{et}}{A F_y} = \frac{130 + 25}{(1.5 \text{ in}^2)(50)} = 0.27 < 0.5 \Rightarrow C_b = 1.0$$

$$B_{1x} = \frac{C_m \alpha}{1 - \frac{P_u}{P_{ex}}} = \frac{0.85}{1 - \frac{155}{1696}} = 0.94 < 1 \Rightarrow B_{1x} = 1.0$$

$$B_{2x} = \frac{1}{1 - \frac{P_{story}}{P_{story,x}}} = \frac{1}{1 - \frac{160}{1269.2}} = 1.15$$

$$P_{story,x} = R_m \frac{H}{\Delta f} = 0.95 \times \frac{33.4}{0.0025} = 1269.2$$

$$R_m = 1 - 0.15 \left(\frac{P_{mf}}{P_{story}} \right) = 1 - 0.15 \times \frac{1}{3} = 0.95$$

$$P_u = P_{nt} + B_2 P_{et} = 130 + 1.15(25) = 158.8^k$$

Homework 11-14 / 11-15
due Thursday

$$M_{ux} = \beta_{1x} M_{10} + \beta_{2x} M_{20} = 1(45) + 1.15(15) = 62.3 \text{ kft}$$

$KL \Rightarrow K_y L_y$ EI Method direct method
 equilibrium. $K_y L_y = K_{2x} L_x$ ~~(K_{2x} L_x)~~ } $KL = 1.0 \times 14$

$$\frac{P_u}{\phi P_n} = \frac{158.8}{306} = 0.52 > 0.2 \text{ act like a column}$$

$$\hookrightarrow KL = 14 \quad T4-1; \phi P_n = 306$$

Table 6-1: $p = 3.27 \times 10^{-3}$ $b_{px} = 5.96 \times 10^{-3}$

$$\Rightarrow 3.27 \times 10^{-3} \times (158.8) + 5.96 \times 10^{-3} (62.3) = 0.891 < 1 \quad \checkmark \text{ OK}$$

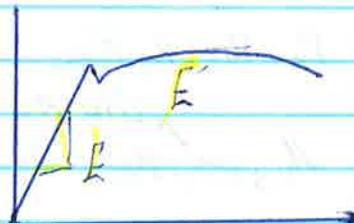
EI Method: $P_{e1x} = 2120$ $\beta_{1x} = 0.92$ $\beta_{2x} = 1.11$
 $P_{u1x} = 157.8 \text{ k}$ $M_{u1x} = 61.7 \text{ kft}$ Restory, $\alpha = 15805$

$\Rightarrow K_y L_y = 1 \times 14 = 14 \quad \checkmark$ controlling one
 Equivalen: $K_y L_y = \frac{K_{2x} L_x}{r_{xy}} = \frac{1.2 \times 14}{2.16} = 18$

$$\Rightarrow 0.884 < 1 \quad \checkmark \text{ OK}$$

Direct: $\frac{\Delta H}{L} = 0.0025$ E'

EI: $\frac{\Delta H}{L} = 0.002$ E



Design of beam - column

shapes:



Formula: $H_1 - 1(a)$

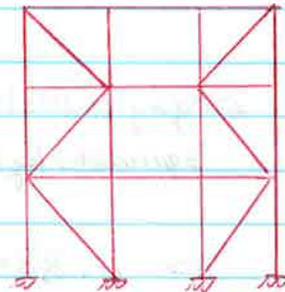
$$\begin{array}{rcc}
 P_u & = & M_{ux} & , & M_{uy} \\
 \uparrow & & \uparrow & & \uparrow \\
 P_{D1} & & M_{D1x} & & M_{D1y} \\
 P_{L1} & & M_{L1x} & & M_{L1y} \\
 B_2 & & B_{1x} & & B_{1y}
 \end{array}$$

$$P_{u, \text{equi}} = P_u + m M_{ux} + m u \frac{2.0}{H} M_{uy}$$

Ex 11.13

$$(KL)_{ex} = (KL)_{ey} = 12 \text{ ft}$$

$$\begin{array}{l}
 P_{D1} = 250 \text{ k} \\
 M_{D1x} = 180 \text{ kft} \\
 M_{D1y} = 70 \text{ kft}
 \end{array}$$



$$P_{u, \text{equi}} = P_u + m M_{ux} + m u M_{uy}$$

$$P_u = P_{D1} + B_2 P_{L1} = 250 \text{ k}$$

$$M_{ux} = B_1 M_{D1x} + B_2 M_{L1x} = 180 \text{ kft}$$

$$M_{uy} = B_1 M_{D1y} + B_2 M_{L1y} = 70 \text{ kft}$$

W_{12} and using the chart: $m=1.6$

$$P_{u, \text{equiv}} = 250 + 1.6(180) + 1.6 \times 2(70) = 762 \text{ K}$$

$$KL = 12 \text{ ft} \text{ Table 4-1} \rightarrow W_{12 \times 72} \quad (\phi P_n = 806 \text{ K})$$

$$\frac{P_u}{\phi P_n} = \frac{762}{806} = 0.31 > 0.2 \quad \text{Table 6-1} \Rightarrow KL = 12': \quad L_b = 12', \quad K=1 \text{ for Braced}$$
$$p = 1.26 \times 10^{-3}, \quad b_x = 2.23 \times 10^{-3}, \quad b_y = 4.82 \times 10^{-3}$$

$$(1.26 \times 10^{-3})(250) + 2.23 \times 10^{-3}(180) + 4.82 \times 10^{-3}(70) = 1.049 > 1 \text{ N.G.}$$

$$\Rightarrow \text{Take } 12 \times 79 \quad \circ \quad KL = 12: \quad p = 1.43 \times 10^{-3} \quad b_x = 2.02 \times 10^{-3} \quad b_y = 4.37 \times 10^{-3}$$

$$\frac{p P_u}{250} + b_x \frac{M_{ux}}{100} + b_y \frac{M_{uy}}{70} = 0.252 < 1 \text{ OK} \checkmark$$

ANALYSIS by EI Method:

$$P_{e1x} = \frac{\pi^2 EI_x}{(KL)_x^2} = \frac{\pi^2 \times 29000 \times 662}{(1 \times 12 \times 12)^2} = 9138 \text{ K}$$

$$\beta_{1,x} = \frac{C_{m,x}}{1 - \frac{P_u}{P_{e1x}}} = \frac{0.85}{1 - \frac{250}{9138}} = 0.87 < 1 \Rightarrow \beta_{1,x} = 1$$

$$P_{e1y} = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2 \times 29000 \times 216}{(1 \times 12 \times 12)^2} = 2981 \text{ K}$$

$$\beta_{1,y} = \frac{C_{m,y}}{1 - \frac{P_u}{P_{e1y}}} = \frac{0.85}{1 - \frac{254}{2981}} = 0.93 > 1 \Rightarrow \beta_{1,y} = 1$$

Table 6-1: $W_{12 \times 79}$: $KL = 12 \checkmark$

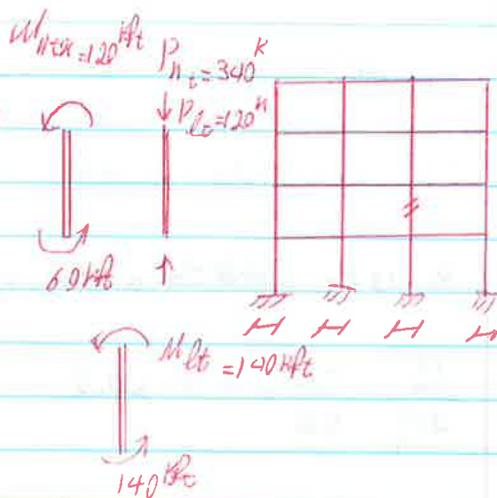
H.W: 11-19
11-22

Exp 11-11

$L_x = L_y = 12 \text{ ft}$
 $K_x = 1.72$
 $K_y = 1.0 \checkmark$

$P_{\text{story}} = 2400 \text{ K}$

$P_{e, \text{story}} = 50,000 \text{ K}$



Assume $\beta_{1x} = 1$

$$\beta_{2x} = \frac{1}{1 - \frac{P_{\text{story}}}{P_{e, \text{story}}}} = \frac{1}{1 - \frac{2400}{50,000}} = 1.05$$

$$P_u = P_{nt} + \beta_2 P_{dt} = 340 + 1.05(120) = 466 \text{ K}$$

$$M_{ux} = \beta_1 M_{ntx} + \beta_2 M_{dtx} = 1(120) + (1.05)(140) = 267 \text{ Kft}$$

$$P_{u, \text{equiv}} = P_u + m M_{ux} = 466 + 1.6 \times 267 = 894$$

First try \Rightarrow Table 4-1: $(KL)_y = 12 \text{ ft}$ \rightarrow controlling one $\rightarrow W_{12 \times 87}$

$$\text{equiv. } (KL)_y = (KL)_x / r_x / r_y = \frac{1.72 \times 12}{1.75} = 11.79 \rightarrow W_{12 \times 87}$$

Table 6-1 and $KL = 12$, $L_b = 12$: $\phi = 1.02 \times 10^{-3}$, $b_x = 1.82 \times 10^{-3}$

$$\frac{P_u}{\phi P_n} = \frac{466}{0.81} = 0.475 > 0.2$$

$$\Rightarrow 1.02 \times 10^{-3} (466) + 1.82 \times 10^{-3} (267) = 0.961 < 1 \text{ OK} \checkmark$$

check $\beta_{1x} = 1.0$: $P_{e1x} = \frac{\pi^2 EI_x}{(KL)^2} = \frac{\pi^2 \times 29000 \times 740}{(1 \times 12 \times 12)^2} = 10,214 \text{ K}$
Braced condition \leftarrow

$$C_{mx} = 0.6 - 0.4 \left(\frac{M_1}{M_2} \right) = 0.6 - 0.4 \left(+ \frac{60}{120} \right) = 0.4$$

$$\Rightarrow \beta_{1x} = \frac{C_{mx}}{1 - P_u / P_{e1x}} = \frac{0.4}{1 - 466 / 10214} = 0.42 < 1 \Rightarrow \beta_{1x} = 1 \checkmark$$

Bolt Connections

- A 307: → Ordinary bolt or Common bolt
→ made out of A36 steel
→ Available from $\frac{5}{8}$ inch to 1.5 inch
→ Only static load
+ Secondary members (like purlins)

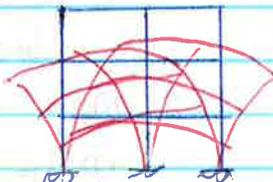
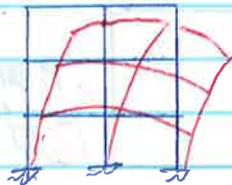
High Strength:

- A 325 (Heat treated medium carbon) for High rise building, large bridges
A 490 (Alloy steel) [Expensive] use for nuclear reactor
1312 ohms
Air crafts

Bolt installation:

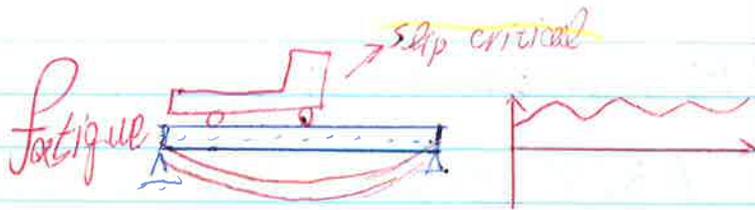
1. snug tight not common  wrench applied a to to tight form
2. fully tensioned more common
3. slip critical

Scenario I

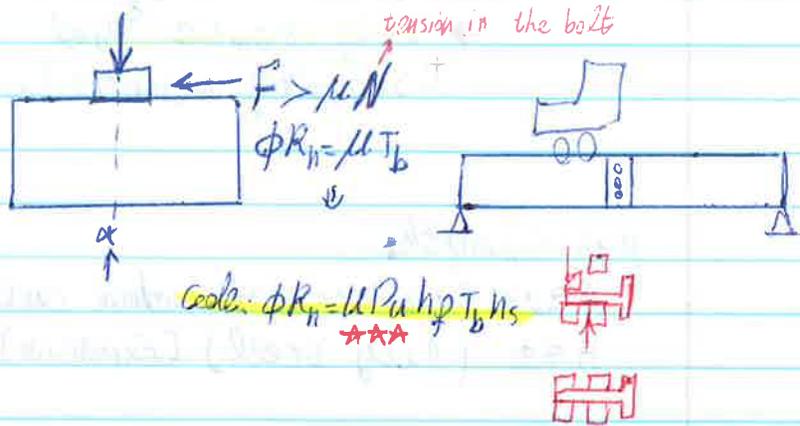


fully tensioned 27

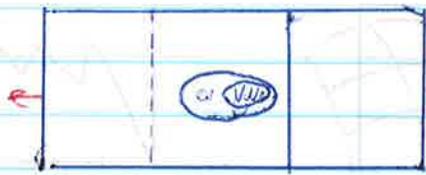
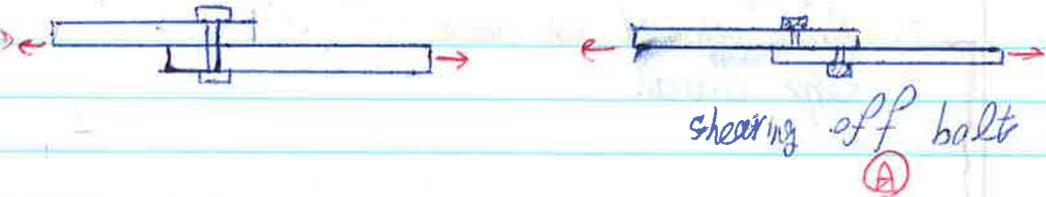
Scenario II:



3. slip critical
→ NO SLIP OCCURS



Bearing → strength failure
slip critical → slip occurs



$\phi R_n = \phi F_v \times A_b$

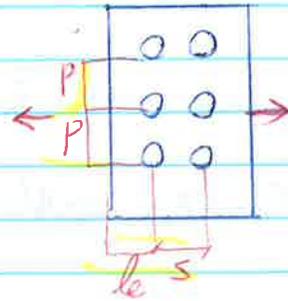
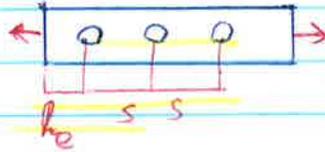
$\phi R_n = \phi 1.2 l_e t F_u \leq 2.4 d t F_u$

l_e → plate thickness d → bolt diameter

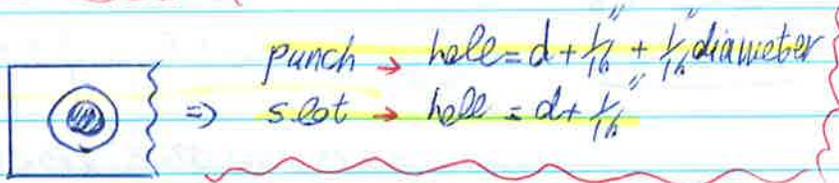
(B)_s
 $\phi R_n = \phi 1.0 l_e t F_u \leq 2 d t F_u$

bearing for seismic areas

$L_c = ?$



$$L_c = \min \left\{ \begin{array}{l} l_e - \frac{1}{2}d \\ s - \text{hole} \end{array} \right.$$

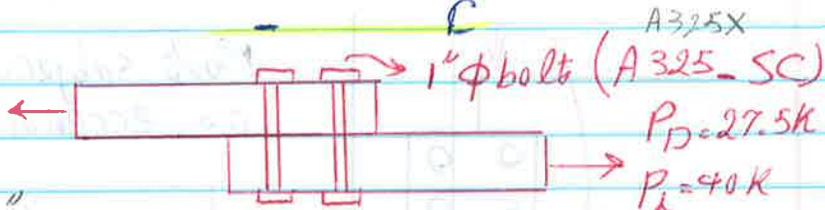


AAA lat page:

$$\phi R_n = u \cdot P_u \cdot h_f \cdot T_b \cdot n_s$$

Annotations: $u = 0.3$ surface A, $h_f = 1.13$, $T_b = 1.0$, $n_s = 1$ or 2

Exp 12.4:

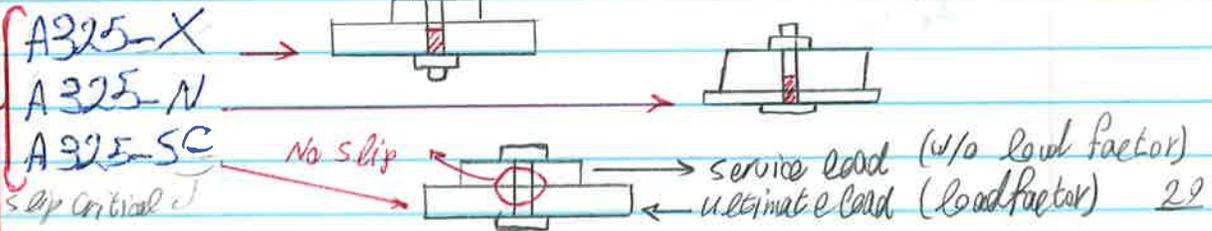


$$h = 1 + \frac{1}{8} = \frac{9}{8}$$

$$\Rightarrow L_c = \min \left\{ \begin{array}{l} 1.75 - \frac{1}{2}(\frac{9}{8}) = 1.1875 \approx 1.19 \\ 3 - \frac{9}{8} = 1.875 \end{array} \right. \quad (\text{most of the time } 3")$$

$$P_u = 1.2(27.5) + 1.6(40) = 97K$$

Hint: there are always minimum 2 bolt in one line



A325 - SC $\phi R_n / \text{bolt} = 1.0 P_u h_f T_b N_s$

1" ϕ bolt and A325 $\Rightarrow T_b = 51 \Rightarrow -1(0.3)(1.13)(1.0) \times 51 \times 1 = 17.29 \text{ K/bolt}$

$P_u = 97 \text{ K} \Rightarrow N = \frac{97}{17.29} = 5.61 \approx 6 \text{ bolt}$

A325 - X ϕ

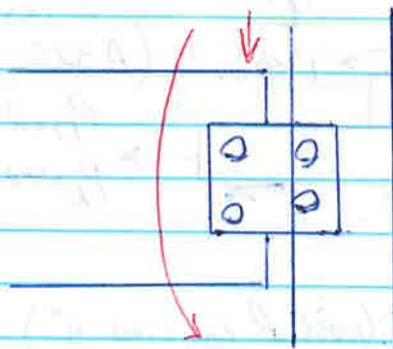
Bearing type $\Rightarrow \phi R_n / \text{bolt} = \phi 1.5 L_c t F_u \leq \phi 2.4 d t F_u$

$= 0.75 \times 1.5 \times 1.187 \times \frac{5}{8} \times 65 \leq 0.75 \times 2.4 \times 1 \times \frac{5}{8} \times 65$

$\Rightarrow 54.25 \leq 73.1 \text{ K}$

Bolt Shearing $\Rightarrow \phi R_n / \text{bolt} = 0.75 (68 \text{ ksi}) (\pi d^2) = 40.03$

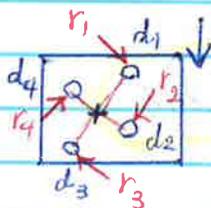
$\Rightarrow \phi R_n / \text{bolt} = 40.03 \text{ K} \quad N = \frac{97}{40.03} = 2.43 \approx 3 \text{ bolt}$



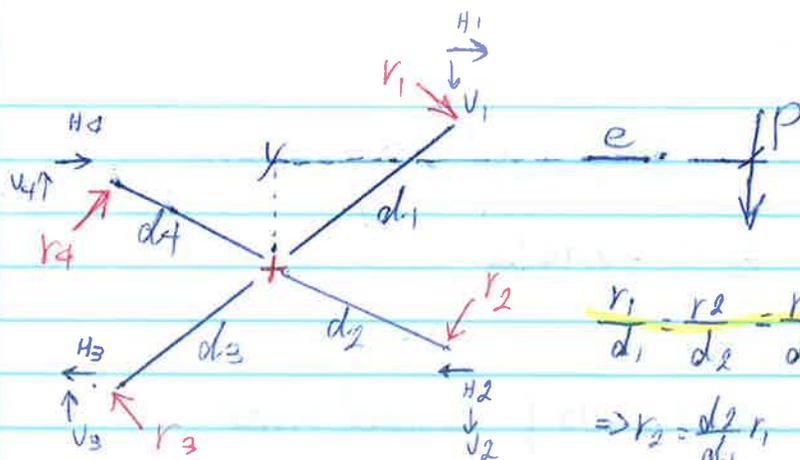
Bolt subject to eccentric shear

vertical shear

Torsional shear



- bolts (x/y direction)
- draw center between c/g to bolt



$$\frac{r_1}{d_1} = \frac{r_2}{d_2} = \frac{r_3}{d_3} = \frac{r_4}{d_4}$$

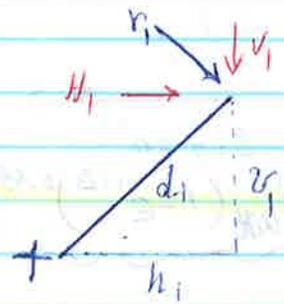
$$\Rightarrow r_2 = \frac{d_2}{d_1} r_1 \quad r_3 = \frac{d_3}{d_1} r_1 \quad r_4 = \frac{d_4}{d_1} r_1$$

$$P \cdot e = r_1 d_1 + r_2 d_2 + r_3 d_3 + r_4 d_4$$

$$M = r_1 \frac{d_1^2}{d_1} + r_2 \frac{d_2^2}{d_1} + r_3 \frac{d_3^2}{d_1} + r_4 \frac{d_4^2}{d_1}$$

$$= \frac{r_1}{d_1} (d_1^2 + d_2^2 + d_3^2 + d_4^2) = \frac{r_1}{d_1} \sum d_i^2$$

$$r_1 = \frac{M d_1}{\sum d_i^2} \quad r_2 = \frac{M d_2}{\sum d_i^2} \quad r_3 = \frac{M d_3}{\sum d_i^2}$$



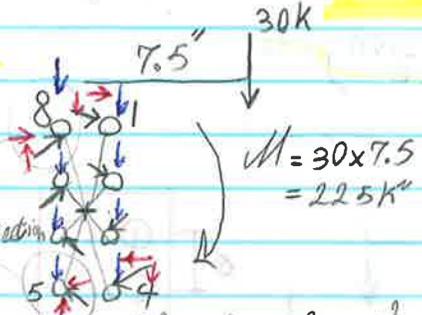
$$\frac{r_1}{d_1} = \frac{H_1}{d_1} = \frac{V_1}{d_1}$$

$$H_1 = \frac{r_1}{d_1} V_1 = \frac{M d_1}{\sum d_i^2} \frac{V_1}{d_1} = \frac{M V_1}{\sum d_i^2}$$

$$V_1 = \frac{H_1 \sum d_i^2}{M}$$

$$\begin{cases} H_1 = \frac{M V_1}{\sum d_i^2} \\ V_1 = \frac{M H_1}{\sum d_i^2} \end{cases}$$

Exp 13.10



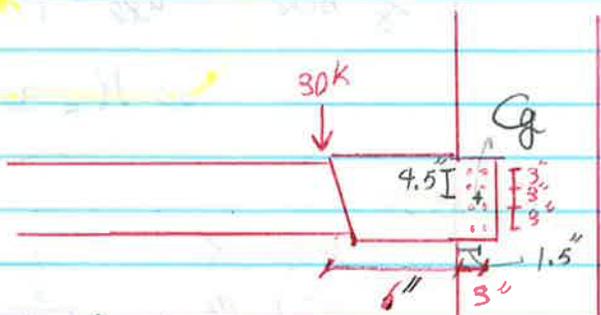
Eliminate bc y-direction is opposite

$$\sum d_i^2 = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2 + d_7^2 + d_8^2$$

$$= h_1^2 + v_1^2 + h_2^2 + v_2^2 + \dots$$

$$= (h_1^2 + h_2^2 + \dots + h_8^2) + (v_1^2 + v_2^2 + \dots + v_8^2)$$

$$= [4(4.5^2) + 4(1.5^2)] + [8 \times 1.5^2] = 108 \text{ in}^2$$



HW 12-20
 12-33
 13-4

⇒ only 1 and 4 are critical and also symmetric

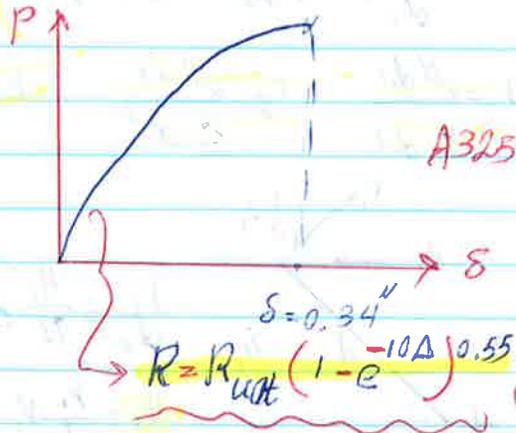
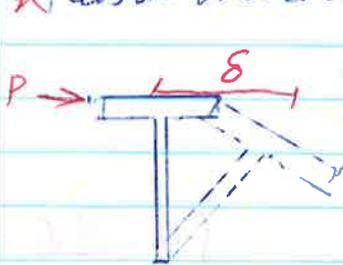
$$H_1 = \frac{M U_1}{E d^2} = \frac{225 \times 4.5}{108} = 9.38 \rightarrow$$

$$U_1 = \frac{M h_1}{E d^2} = \frac{225 \times 1.5}{108} = 3.13 \downarrow \text{torsional shear}$$

$$\text{vertical shear} = P/8 = \frac{30}{8} = 3.75 \text{ k} \downarrow \Rightarrow$$

$$R_u = \sqrt{(3.13 + 3.75)^2 + 9.38^2} = 11.63 \text{ k}$$

* Plastic Methods



A325-X

7/8" bolt

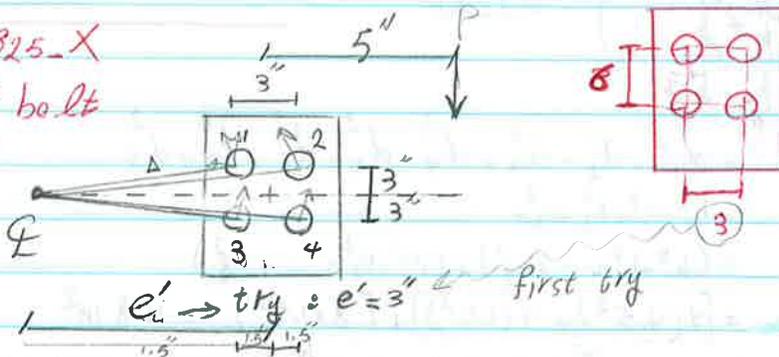
$$R_{ult} = \phi F_u A_b = 0.75 \times 68 \left[\frac{\pi}{4} \left(\frac{7}{8} \right)^2 \right] = 30.6$$

$$\Rightarrow R = 30.6 (1 - e^{-10\Delta})^{0.55}$$

Exp 13.2:

A325-X

7/8" bolt



HW: $\begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \begin{matrix} 3'' \\ 3'' \\ 4'' \end{matrix} \begin{matrix} 5'' \\ 5'' \\ 5'' \end{matrix}$

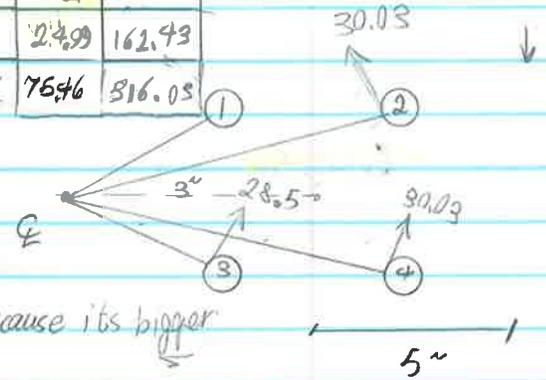
$R_{ult} = 50.5$

try: $e' = 3$
ans: $e' = 2.06''$ ✓

13-10 Elastic Method
13-12 AISC Table
Plastic

	Δ	h	v	$d = \sqrt{h^2 + v^2}$	R	R_v	$R_{x,d}$
1	←	1.5	3	3.9541	28.5	12.74	95.48
2	0.34	4.5	3	5.4083	30.03	24.99	162.43
3	←	1.5	3	3.9541	28.5	12.74	95.58
4	0.34	4.5	3	5.4083	30.03	24.99	162.43
					Σ	76.46	516.05

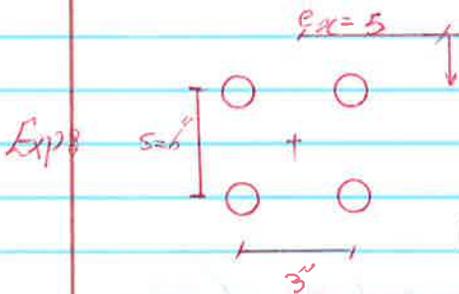
$\Delta = \frac{\Delta_4}{d_4} \times \frac{d_3}{d_3} = \frac{0.34}{5.4083} \times \frac{3}{3.9541} = 0.211$



$M_{rd} \Rightarrow P = \frac{516.05}{3+5} = 64.5 \neq 75.46 \Rightarrow e' \times$ because its bigger

we have to use smaller $e' \Rightarrow e' = 2.5$

$e' = 2.4''$



base on the chart in the book

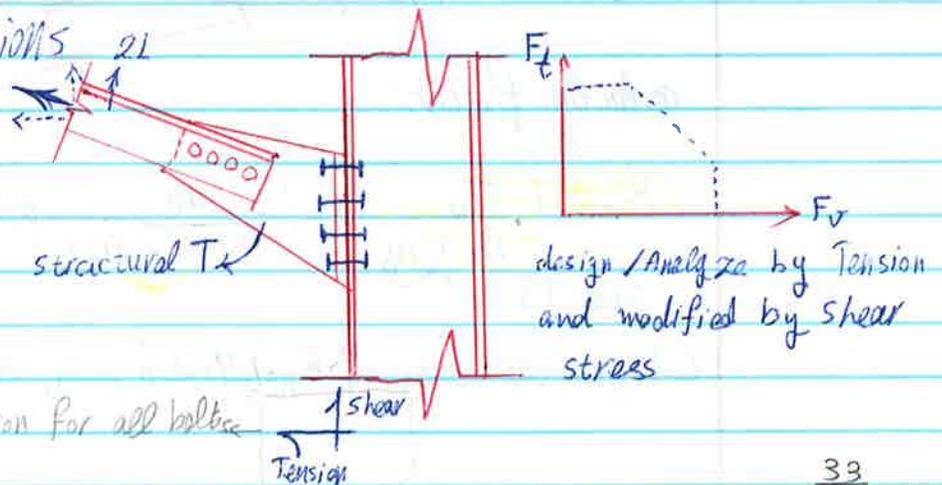
$n = 2$ $C = 2.19$ table

$R_n = F_v \times A_b = 68 \left(\frac{5}{8} \left(\frac{7}{8} \right)^2 \right) = 40.8$

$R_n = C R_n = 2.1 \times 40.8 = 91.4$

$\phi R_n = 0.75 \times 91.4 = 68.55 K = P$

Braeing connections



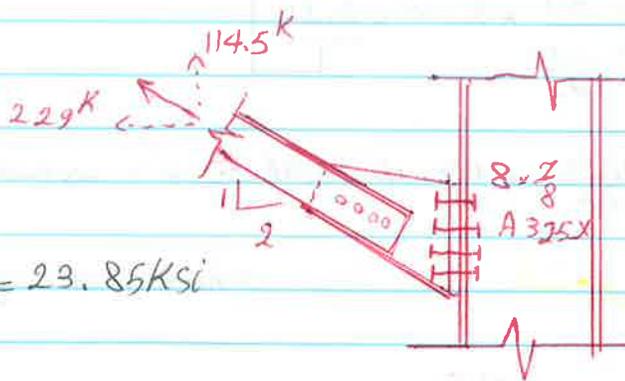
hw: 13-17

So instead of $F_{nt} \rightarrow \phi$ shear

we use:

$$F'_{nt} = 1.3 F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v < F_{nt}$$

Exp 13-5:



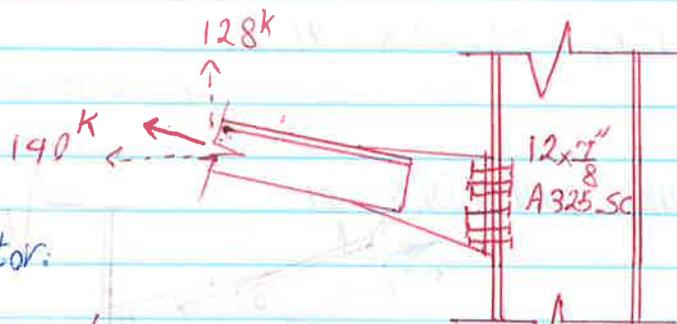
$$f_v = \frac{114.5 \text{ k}}{8(0.6 \text{ in}^2)} = 23.85 \text{ Ksci}$$

$$f_t = \frac{229 \text{ k}}{8(0.6)} = 47.7 \text{ Ksci}$$

chart: $F_{nt} \rightarrow 90$; $F_{nv} = 68 \text{ Ksci}$

$$F'_{nt} = 1.3(90) - \frac{90}{0.75 \times 68} \times 23.85 = 74.9 < 90 \quad \checkmark \text{ OK}$$

Exp



reduction factor:

$$R_{sc} = 1 - \frac{T_u}{D_u T_b n_b} = 1 - \frac{140}{(1.13)(39)(12)} = 0.735 \quad \text{class B}$$

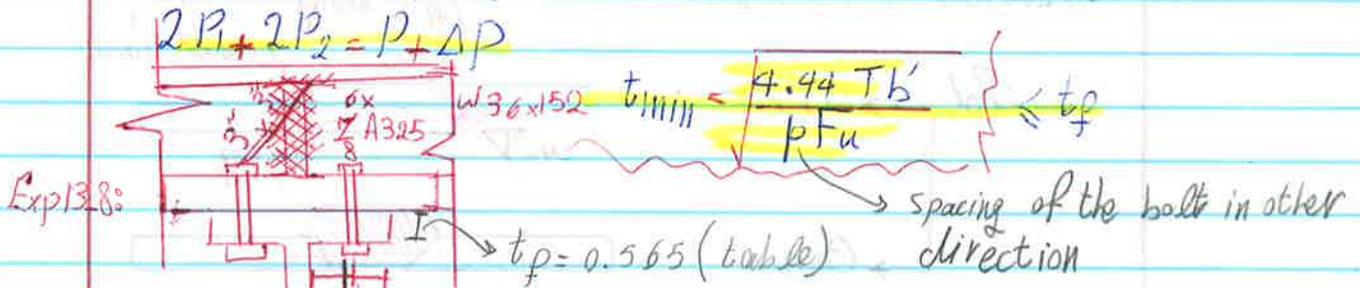
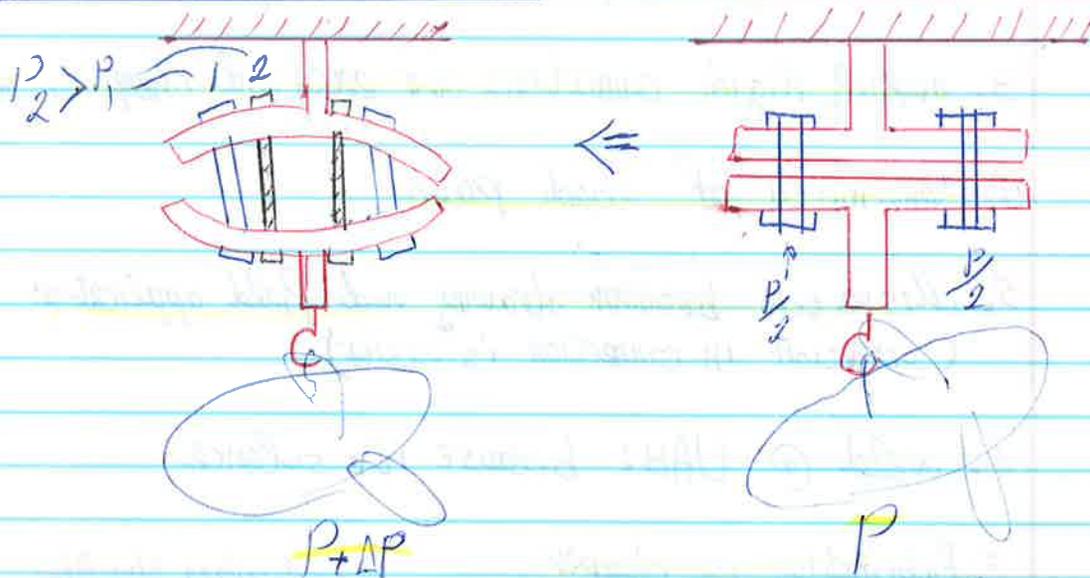
Table J3-1

Original capacity: $\phi R_n = 1/11 D_u n_p T_b n_s = (0.5)(1.13)(1.0)(39)(1) = 22.03$

$$\phi R'_h = 0.795 \times 22.03 = 16.2 \frac{\text{K}}{\text{bolt}} \times 12 = 194.4 \text{ K}$$

$$\Rightarrow 194.4 > 128 \checkmark \text{ OK}$$

PRYING ACTION



$$b' = 2 - \frac{0.345}{2} - \frac{7/8}{2} = 1.39''$$

$$T_u = 1.2P + 1.6L = 100 \text{ K}$$

$$T = \frac{T_u}{6} = \frac{100}{6} = 16.67 \frac{\text{K}}{\text{bolt}}$$

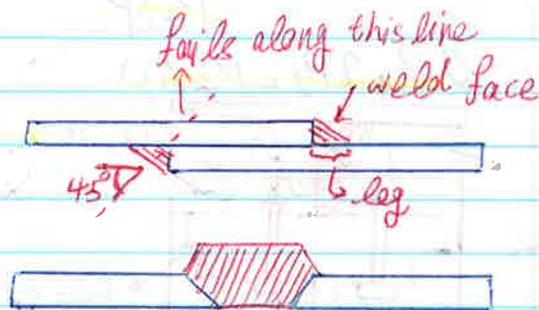
A36 $\rightarrow F_u = 58$

$$\Rightarrow t_{min} = \sqrt{\frac{4.44 \times 16.67}{3 \times 58}} = 0.769 > t_f = 0.565 \times \text{N.G}$$

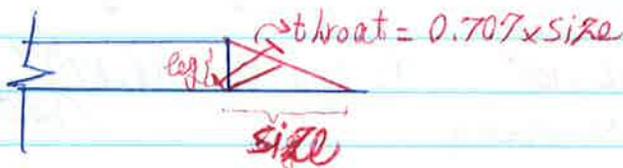
Weld Connection

1. Over 15% saving \$ (nuts, bolts, washers...)
2. Its pretty easy for complex shapes
3. making rigid connections are easy and cheap or moment connection
4. Continuity of load path.
5. Mismatch between drawing and field application (correction in connection is easier)
6. Weld @ UAB: because its silence
7. Fabrication is cheaper.

weld → Fillet
 → Groove



50-100% more expensive than fillet connection



$$\text{weld capacity/inch} = (0.707 \times \text{size}) \times \phi F_w$$

0.75 (0.6 x 70)

Shear Capacity

most of the times use vol E 70ksi

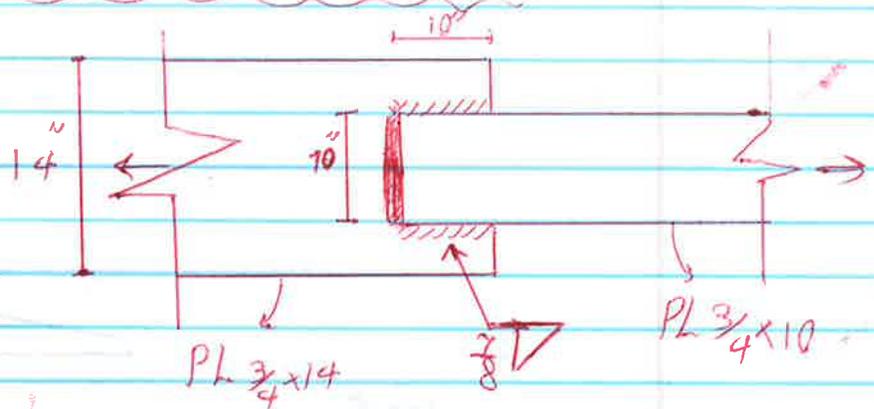
$$\beta = 1.2 - 0.002 \left(\frac{l}{w} \right) \leq 1.0 \quad \left\{ \begin{array}{l} \text{length of the weld (maximum length side)} \\ \frac{l}{w} \leq 100 \rightarrow \text{don't need } \beta \end{array} \right.$$

the minimum size and maximum size of fillet weld \rightarrow P16.1-111

$$\begin{array}{l} \text{maximum size} \left\{ \begin{array}{l} \leftarrow \frac{1}{4} \text{ max weld} = t_{\min} \\ \rightarrow \frac{1}{4} \text{ max weld} = t_{\min} \frac{1}{16} \end{array} \right. \end{array}$$

Exp 14.2:

$$\phi R_n = ?$$



$$\text{weld capacity/inch} = \left(0.707 \times \frac{7}{8} \right) \times (0.75) \times (0.6 \times 70)$$

$$\phi R_n = \text{weld capacity/in} \times 30 = 194.9 \text{ K}$$

$$\beta = 1.2 - 0.002 \left(\frac{l}{w} \right) = 1.2 - 0.002 \left(\frac{10}{\frac{7}{8}} \right) = 1.2 - 0.002 \times 22.88 = 1.18 > 1$$

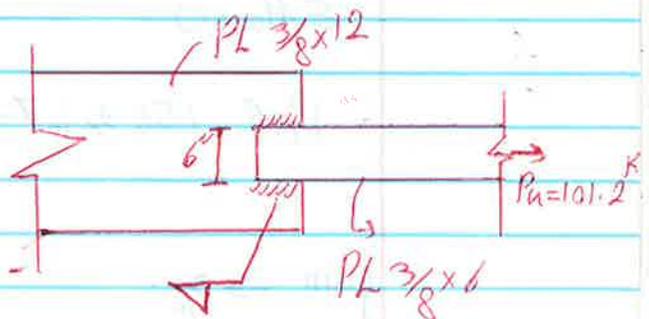
$$\Rightarrow \beta = 1.0$$

$$\Rightarrow \phi R_n = 194.9 \text{ K} \checkmark$$

Exp 14.3:

$$\text{Min thickness} \rightarrow \frac{3}{16} \text{ (table)}$$

$$\text{max thickness} \rightarrow t_{\min} - \frac{1}{16} = \frac{3}{8} - \frac{1}{16} = \frac{5}{16}$$

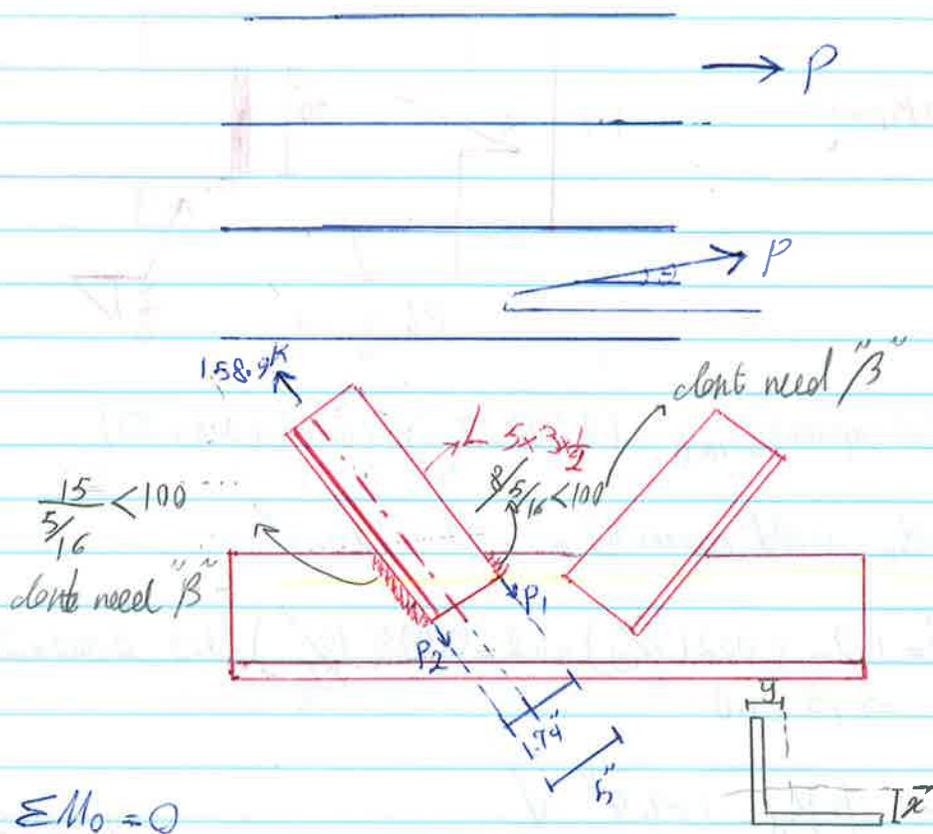


$$\text{weld capacity/inch} = (0.707 \times \frac{5}{16}) \times 0.75 \times (0.6 \times 70) = 9.28 \text{ k/in}$$

$$\text{weld length} : L = \frac{101.2}{9.28} = 14.59'' \sim 15''$$

Symmetric : 7.5" on top and 7.5" in the bottom

$$\frac{L}{w} = \frac{7.5}{\frac{5}{16}} = 24 < 100 : \text{don't need to calculate } \beta$$



$$\sum M_0 = 0$$

$$P_1 \times 5 - 158.9 \times 1.74 = 0 \Rightarrow P_1 = 55.3 \text{ k}$$

$$P_2 = 158.9 - 55.3 = 103.6 \text{ k}$$

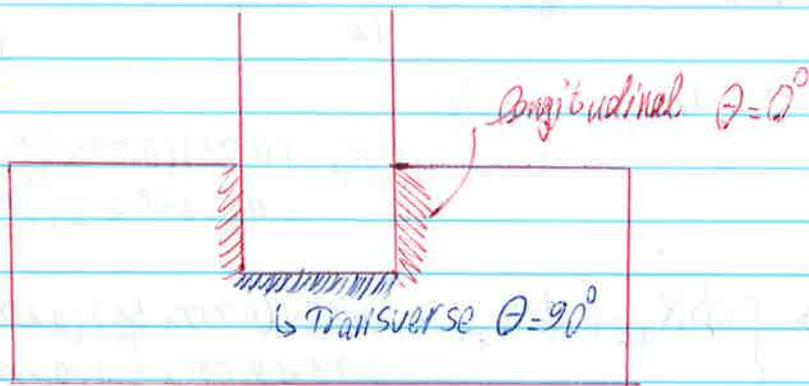
$$\left. \begin{array}{l} \text{min} \\ \text{max} \end{array} \right\} \rightarrow \frac{3}{16}$$

$$\rightarrow \frac{1}{2} - \frac{1}{16} = \frac{7}{16}$$

\Rightarrow we wanna be in the middle $\left(\frac{5}{16}\right)$

$$\text{weld capacity / in} = (0.707 \times \frac{5}{16}) (0.75) (0.6 \times 70) = 6.96 \text{ k/in}$$

$$P_1 \rightarrow L_1 = \frac{55.3}{6.96} = 7.95'' \approx 8''$$

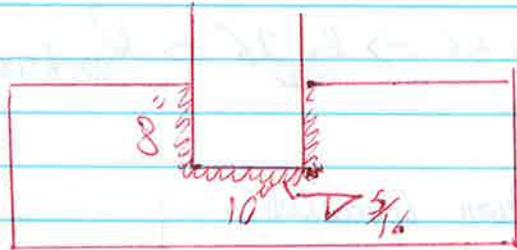


$$(1 + 0.5 \sin^2 \theta) A_w$$

Case I (AISC): $R_n = R_{nw} + R_{nt}$ } choose larger one

Case II: $R_n = 0.85R_{nw} + 1.5R_{nt}$

Exp 14.5:



$$R_{nw} = (0.707 \times \frac{5}{16}) \times (0.6 \times 70) \times 16 = 148.5$$

$$R_{nt} = (0.707 \times \frac{5}{16}) \times (0.6 \times 70) \times 10 = 92.8$$

$$\Rightarrow \int R_n = R_{nw} + R_{nt} = 148.5 + 92.8 = 241.3$$

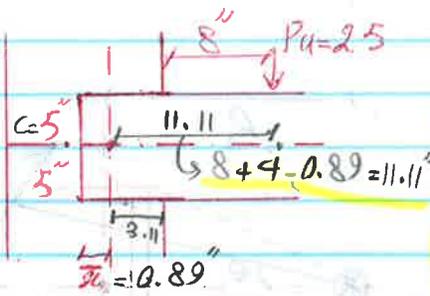
$$\int R_n = 0.85 \times 148.5 + 1.5 \times 92.8 = 265.4 \Rightarrow R_n = 265.4 \checkmark$$

1. Direct shear $\tau_D = \frac{P_u}{A_w}$

2. Torsional shear $\tau_t = \frac{T_u}{J}$; $J = I_x + I_y$

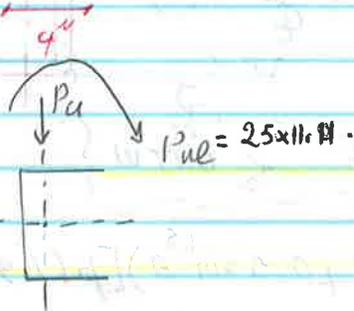
$$\tau = \sqrt{\tau_D^2 + \tau_t^2} = \sqrt{\left(\tau_D + \frac{T_u h}{J}\right)^2 + \left(\frac{T_u v}{J}\right)^2}$$

Exp 14.8:



$$A = (4 \times 1) \times 2 + 10 \times 1 = 18 \text{ in}^2$$

$$\bar{x} = \frac{[(4 \times 1) \times 2] \times 2}{18} = 0.89''$$



$$I_{x-x} = 283.3'' \text{ and } I_{y-y} = 28.4$$

$$J = 283.3 + 28.4 = 311.7$$

$$\tau_D = \frac{P_u}{A_w} = \frac{25}{18} = 1.39 \text{ K/in}$$

$$T_{th,1} = \frac{T_u v}{J} = \frac{(25 \times 11.11) \times 5}{311.7} = 4.46 \text{ K/in}$$

$$T_{th,2} = \frac{T_u h}{J} = \frac{(25 \times 11.11) \times 3.11}{311.7} = 2.77 \text{ K/in}$$

$$\Rightarrow \tau = \sqrt{(2.77 + 1.39)^2 + (4.46)^2} = 6.10 \text{ K/in}$$

$$\Rightarrow 6.1 = (0.75)(0.707 W)(0.6 \times 70) \Rightarrow W = 0.274'' \approx \frac{5}{16}$$

* Solve by Table 8-8:

$$a_0 = 11.11 \Rightarrow a = \frac{11.11}{2.10} \Rightarrow a = 1.11$$

HW: 14-14 / 14-31

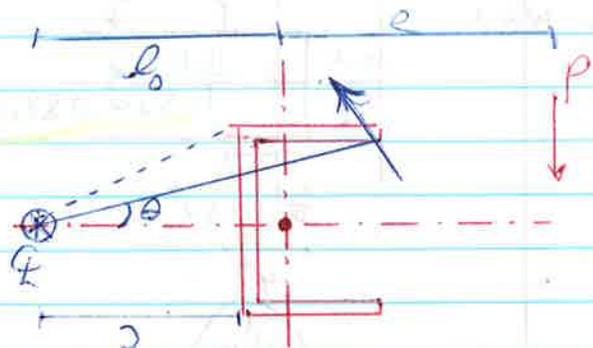
$$KL = 4'' \Rightarrow K = \frac{4}{l} = 0.4$$

Table: Coefficient = 1.31

$$D = \frac{2.5}{0.75 \times 1.31 \times 1 \times 10} = \frac{2.54}{16} \approx \frac{3}{16} \quad \checkmark$$

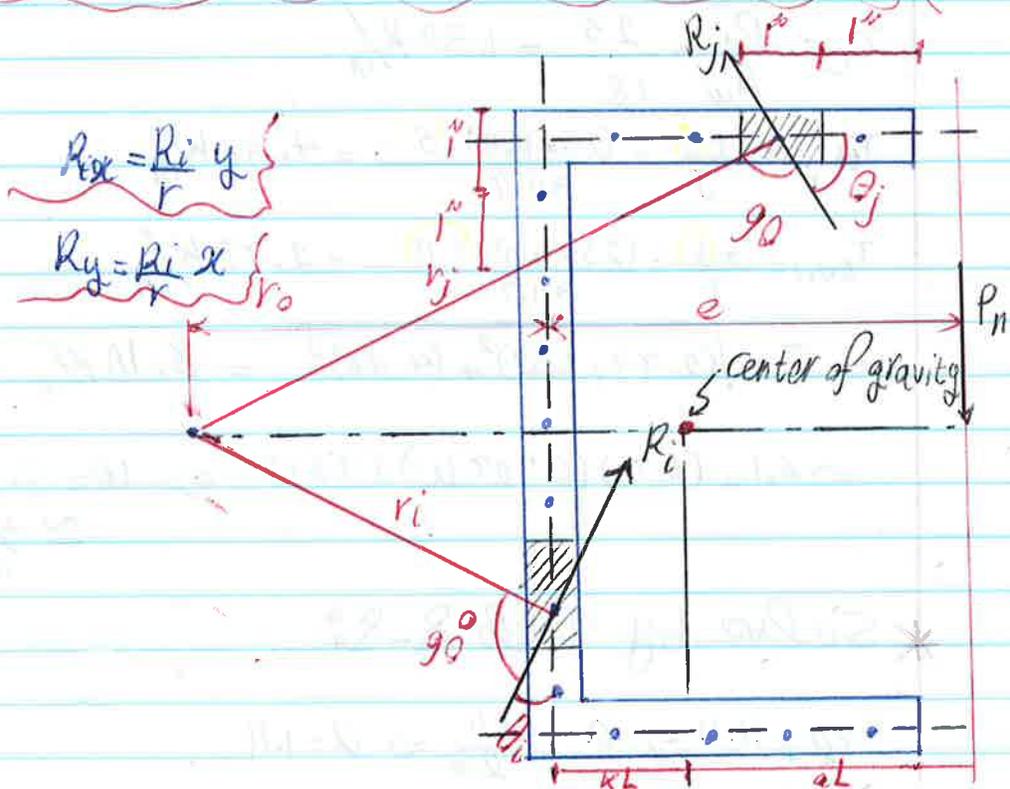
Table 3: $C_1 = 1$ (F70)

* Plastic method



$$\Delta_{max} = 1.087 w (\theta + \phi)^{0.65} \leftarrow 0.17 w$$

$$R = 0.6 F_{E70} \times A_w (1 + 0.5 \sin^{1.5} \theta) [p (1.9 - 0.9 \phi)]^{0.3}$$



step 1) Split weld into 1" section

step 2) Calculate the θ for each element

$$R_i = 0.6 F_{Exx} t_e (1 + 0.5 \sin^{1.5} \theta) \left[\frac{\Delta_i}{\Delta_m} \left(1.9 - 0.9 \frac{\Delta_i}{\Delta_m} \right) \right]^{0.3}$$

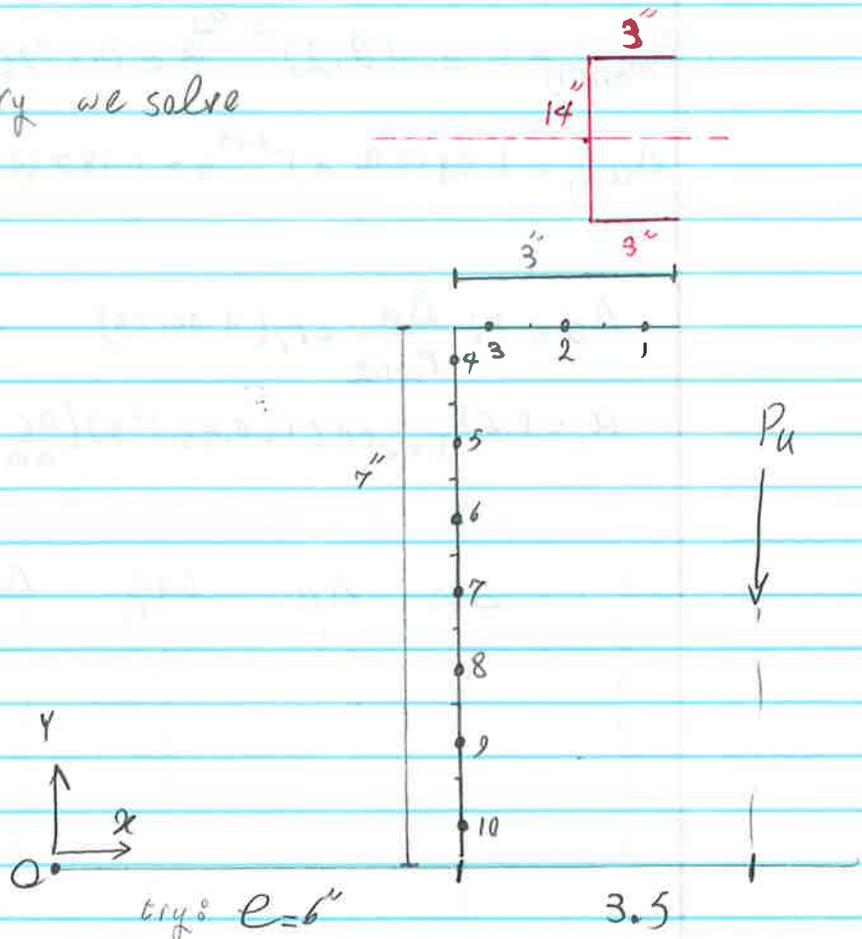
$$= 0.5 \times 70 \times (0.707 \times \frac{1}{4}) (1 + 0.5 \sin^{1.5} \theta) \left[\frac{\Delta_i}{\Delta_m} \left(1.9 - 0.9 \frac{\Delta_i}{\Delta_m} \right) \right]^{0.3}$$

Δ_i = deformation of element = $r_i \frac{\Delta u}{r_{crit2}}$

$$\Delta_m = 0.209(\theta)$$

exp:

because of symmetry we solve for $\frac{1}{2}$



$$r_i = \sqrt{x^2 + y^2}$$

$$\sqrt{x^2 + y^2}$$

$$\frac{d r_i}{d \theta}$$

Horizontal	length	x	y	r_i	rad	(degree)
					θ_j	θ_j
1	1	8.5	7	11.011	0.882	50.5
2	1	7.5	7	10.259	0.820	47
3	1	6.5	7	9.552	0.748	42.9
vertical						
4	1	6	6.5	8.846	0.825	47.3
5	1	6	5.5	8.139	0.742	42.5
6	1	6	4.5	7.5	0.644	36.9
7	1	6	3.5	6.946	0.528	30.3
8	1	6	2.5	6.5	0.395	22.6
9	1	6	1.5	6.185	0.245	14
Point 10	1	6	0.5	6.021	0.083	4.8

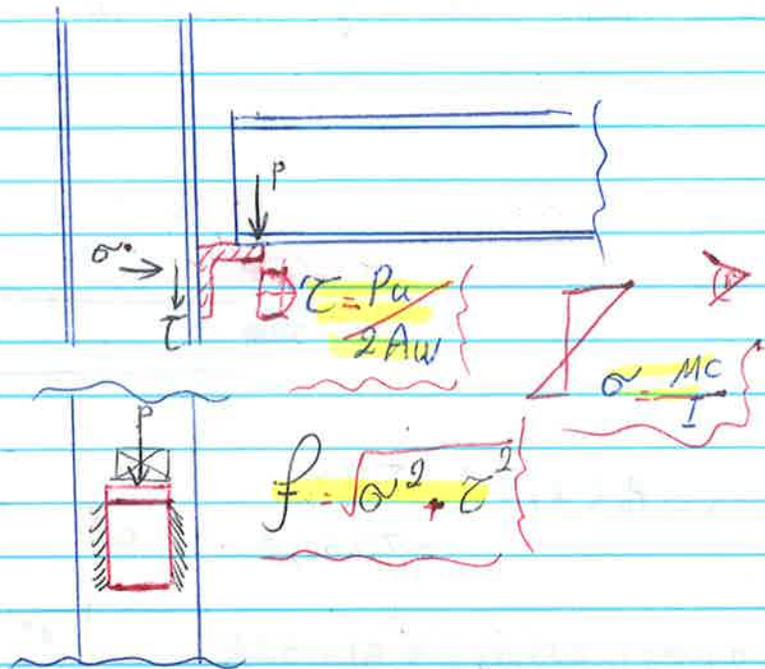
$$\Delta u_{max}(1) = 0.209(\theta + 2)^{-0.32} \quad a = 0.209(50.5 + 2)^{0.32} (0.25) = 0.0143$$

$$\Delta u_{(1)} = 1.087(\theta + 6)^{-0.65} \quad a = 1.087(50.5 + 6)^{0.65} (0.25) = 0.01973$$

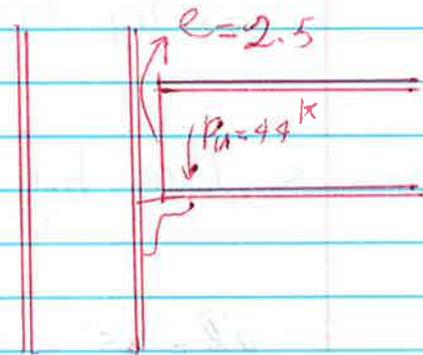
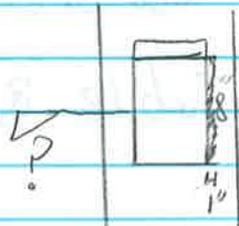
$$\Delta c_i = r_i \frac{\Delta u}{r_i \sin^2} = r_i (0.00179)$$

$$R_i = 2.6 F_{Ex} te (1 + 0.5 \sin^{1.5} \theta) \left[\frac{\Delta c_i}{\Delta m} (1.9 - 0.9 \frac{\Delta c_i}{\Delta m}) \right]^{0.3}$$

	Δm	Δu	$\Delta u / r_i$	Δc_i	$\Delta c_i / \Delta m$	R_i
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						



Expo 14-11 what's the weld size?



$$2A_w = 8 \times 4 \times 2 = 16$$

$$I = 2 \times \frac{bh^3}{12} = 2 \times \frac{4 \times 8^3}{12} = 85.3$$

$$\Rightarrow \tau = \frac{P_u}{2A_w} = \frac{44}{16} = 2.75 \text{ k/in}$$

$$\sigma = \frac{Mc}{I} = \frac{(44 \times 2.5)(4)}{85.3} = 5.16 \text{ k/in}$$

$$\Rightarrow f = \sqrt{(5.16)^2 + (2.75)^2} = 5.85 \text{ k/in}$$

$$\text{weld capacity/in} = (0.707 \times 3) (0.7 \times 70) (0.75) = 5.85$$

46

$$\Rightarrow \beta = 0.263 = \frac{5}{16}$$

Solve by tables:

$$al = 2.5 \Rightarrow a = 0.31$$

$$Kl = 0 \Rightarrow K = \checkmark$$

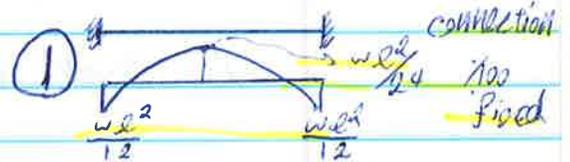
$$P_{min} = \frac{44}{0.75 \times 1 \times 3.09 \times 8} = 2.3 \approx 3$$

→ i/c's out of plane condition

$$\hookrightarrow \frac{3}{16}$$

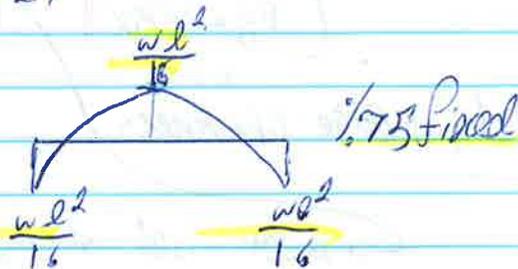
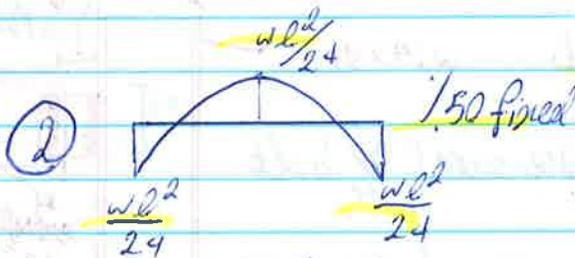
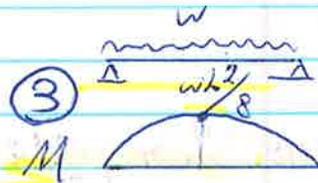
* Building Connections (Chapter 15)

1. FR - Fully Restrained - Rigid connection - Take moment - Fixed



2. PR - Partially Restrained

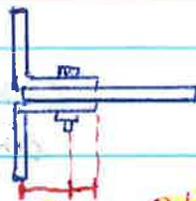
3. Simple connection - shear connection



Hint: in 1 we always use $\frac{3}{4}'' \phi$ A325-N @ 3" unless posted
Exp 15.1

1 bolts $R_u = 172K$

$$\frac{5}{16} + \frac{3}{8} + \frac{1}{4} + \frac{1}{4} = 3 \approx 4$$



$$2 \frac{1}{2} = 3 \frac{1}{2}$$

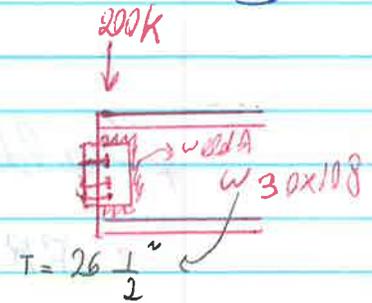
whole plate table 10-1 (No. of bolts = 6)

⇒ $4 \times \frac{3 \frac{1}{2} \times 5}{2 \times 16}$ (web of beam)
(check the length of angle) +7

in welding } welding of angle to beam is always called weld A
 ~ ~ ~ ~ ~ column ~ ~ ~ ~ ~ weld B

exp: weld A and bolt to column

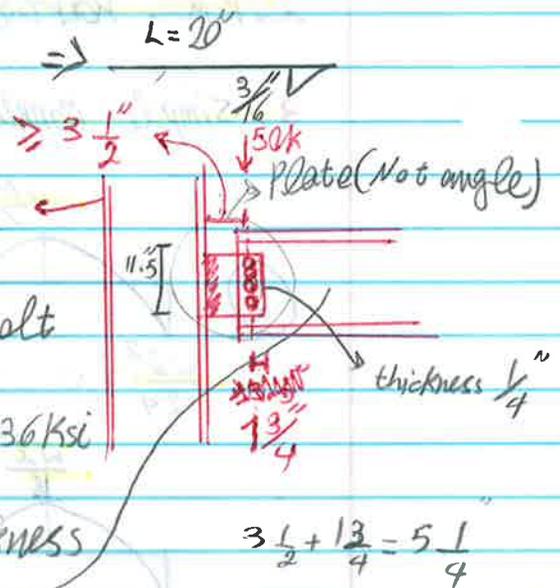
USE T 10-3 & only left part cause we only have weld A



use lower size of weld $\Rightarrow L = 20"$ and $\phi R_n = 223K$

exp: Shear plate/shear tab
 15.19

we go T 10-9a & make sure $\left\{ \begin{array}{l} 3/4" \text{ bolt} \\ F_y = 36 \text{ Ksi} \end{array} \right.$



$\Rightarrow L = 11 \frac{1}{2}"$ and $\frac{1}{4}"$ plate thickness

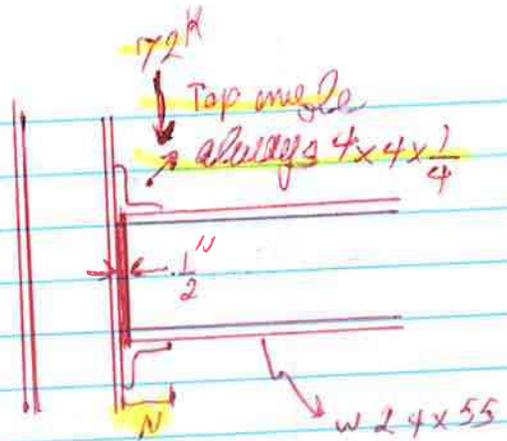
$$3 \frac{1}{2} + 1 \frac{3}{4} = 5 \frac{1}{4}$$

\rightarrow beam web strength T 10.1 unoped
 $351 \text{ K/in} \Rightarrow 381 \times (0.38) = 133.9 \text{ K}$
 $133.9 \text{ K} > 50 \text{ K}$
 beam can handle the shear force.

web thickness of the beam

Exp
155

seated beam connection

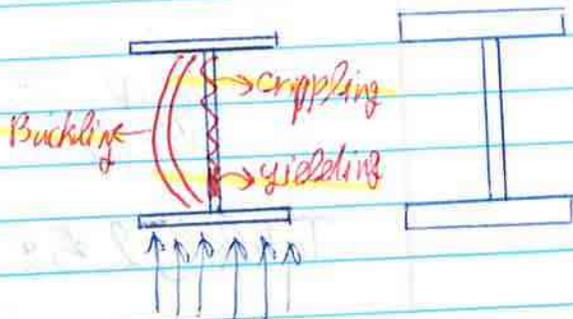


for yielding:

$$\phi R_n = \phi R_{n1} + N(\phi R_{n2})$$

web crippling

$$\phi R_n = \phi R_3 + N(\phi R_4)$$



use T9-4: pick $\phi R_1 / \phi R_2 / \phi R_3 / \phi R_4$

$$\Rightarrow \text{yielding} \Rightarrow 72 = 49.6 + N(19.8) \Rightarrow N = 1.13''$$

$$\text{crippling} \Rightarrow 72 = 63.7 + N(5.61) \Rightarrow N = 1.48''$$

AISC: must be $3\frac{1}{2}''$ or $4''$

\Rightarrow So we use $3\frac{1}{2}''$ ✓

use T10-6 (unstiffened):

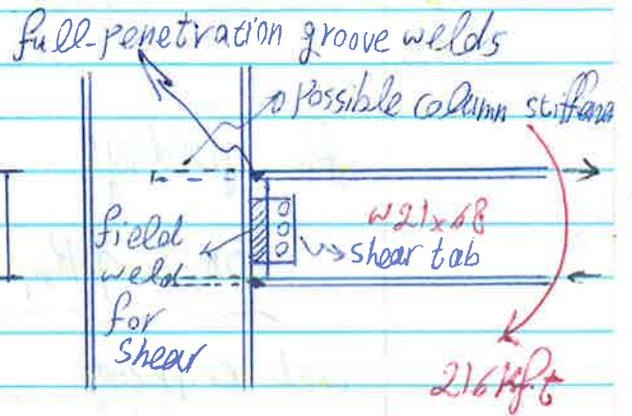
base on $1\frac{1}{2}'' \Rightarrow$ angle thickness = $\frac{3}{4}$

$$\Rightarrow \text{Angle} = 8 \times 3\frac{1}{2} \times \frac{3}{4}$$

Moment-Resistant connection

Exp 5-6:

effective: $21.1 \cdot 0.685$



$$T = C = \frac{216 \times R}{21.1 \cdot 0.685} = 127^k$$

Table J2.5: Strength is controlled by the base Metal

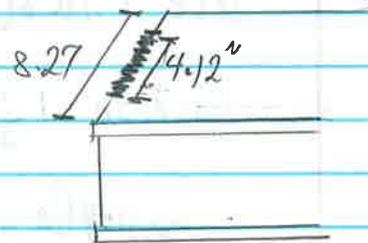
$$\left(\underbrace{F_y}_{\text{metal}} = 50 \text{ ksi and } F_y_{\text{weld}} = 70 \text{ ksi} \right)$$

controlling

$$A_w = \frac{127^k}{\phi F_y} = \frac{127}{0.9 \times 50} = 2.82 \text{ in}^2$$

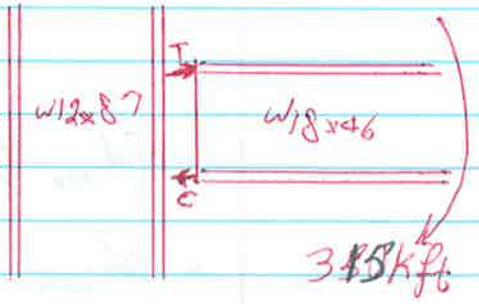
$$\Rightarrow b_w \times 0.685 = 2.82 \Rightarrow b_w = 4.12''$$

$$W_{21 \times 68}: b_f = 8.27''$$



Exp use sum of 15.7
(modified.)

$$d - t_f = 18.1 - 0.605$$



$$T = C = \frac{315 \times 12}{18.1 - 0.605} = 216 \text{ k}$$

$$A_w = \frac{216}{\phi F_y} = \frac{216}{0.9 \times 50} = 4.8 \text{ in}^2 \text{ and } b_w = \frac{4.8}{0.605} = 8 \text{ in}$$

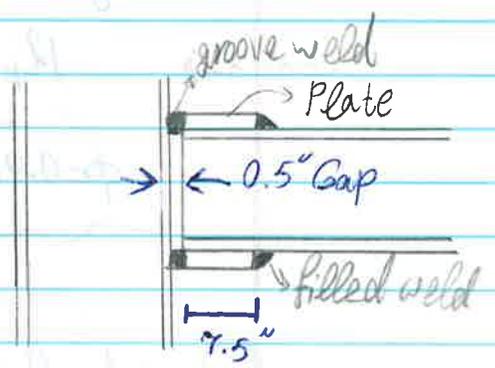
$b_w > b_f \times$

$W18x46: b_f = 6.06 \text{ in}$

$$b_f = \frac{4.8}{t_f} \Rightarrow t_f = \frac{4.8}{6.06} \approx 1.338 \text{ in}$$

$$b_w = \frac{4.8}{1.338} = 3.6 < 5$$

\Rightarrow Plates $1 \frac{1}{3} \times 5 \times 13$



* check the plates to see if they can handle $T = C = 216 \text{ k}$

Tension: $\phi F_y A_g = (5 \times 1.338)(0.9(50)) = 216 \text{ k} \checkmark$

Compression: $C = \phi F_c \times A = 32(5 \times 1.338) = \dots > 216 \text{ k} \checkmark$

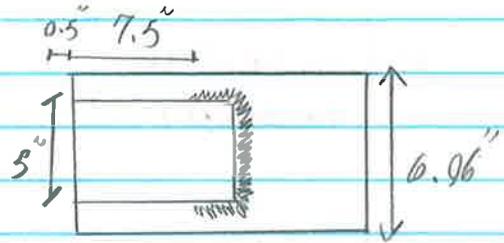
$$r = \frac{t}{\sqrt{12}} = \frac{1.338}{\sqrt{12}} = 0.384 \text{ in}$$

$$K L / r = \frac{0.65 \times 8}{0.384} = 13.5$$

Table 4-22 $\Rightarrow \phi = 32 \text{ ksi}$ (original)

{ 15-16
15-17

check the filled welds:



weld_{min} = $\frac{3}{16}$ "

weld_{max} = $1.33 - \frac{1}{16} = 1.27$ " } we have to use the maximum size

weld capacity = $(0.707 \times 1.27)(0.6 \times 70) 0.85 L > 216^k \Rightarrow L = 5.7"$

① Flange local bending of the column:

Exp 5-7a $R_n = 6.25 \alpha_f^2 F_{yf}$ } = $6.25 \times 0.81^2 \times 30 = 205$

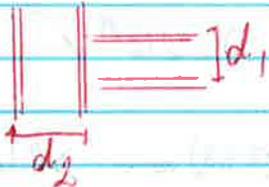
$\phi = 0.9$ (LRFD) } $\phi R_n = 0.9 \times 205 = 184.5^k < 216^k \times$

column will bend

② web of the column yielding:

$d_1 > d_2$ } $R_n = (5k + N) F_{yw} t_w$ } = $5 \times 1.41 + 6.0 \times (50)(0.33) = 337.6^k$

$\phi = 1$ (LRFD) } $\Rightarrow \phi R_n = 1 \times 337.6 > 216^k \checkmark$



③ web crippling of the column

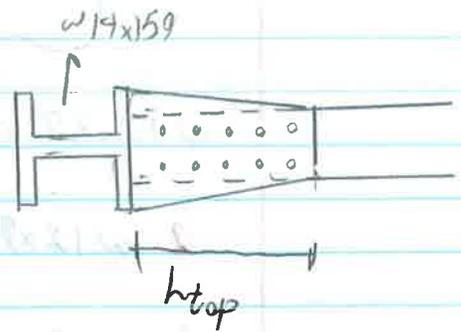
$d_{beam} - t_b > \frac{d_{col}}{2}$ } $R_n = 0.8 t_w^2 \left(1 + 9 \left(\frac{N}{d} \right) \left(\frac{t_w}{t_f} \right)^{1.5} \right) \sqrt{\frac{E F_{yw} t_f}{t_w}}$

* Connection that make plastic hinge in high loads:

review 16.1-64 important

Exp: $\frac{3}{4}$ bolts

$$\left. \begin{array}{l} F_y = 50 \text{ ksi} \\ F_u = 65 \text{ ksi} \end{array} \right\} \frac{F_y}{F_u} = \frac{65}{50} = 0.83 > 0.8$$



hole diameter = $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ beam flange

$$F_u A_{Pn} = 65 (6.45 - 2 \times \frac{7}{8} \times 0.645) = 346 \text{ K} \quad (1)$$

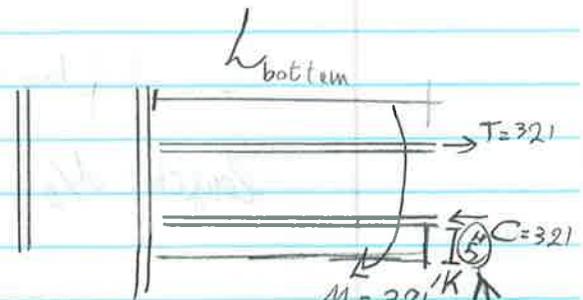
$$\Rightarrow (1) < (2)$$

$$Y_t F_y A_{Pg} = 1.1 \times 50 \times 6.45 = 355 \text{ K} \quad (2)$$

$$\Rightarrow M_n = \frac{F_u A_{Pn}}{A_{Pg}} S_x = \left(\frac{346}{645} \right) (12) = 412$$

$$\Rightarrow \phi M_n = 0.9 \times 412 = 371 \text{ Kft}$$

$$T = C = \frac{371}{13.89} \times 12 = 321 \text{ K}$$



$$\frac{3}{4} \text{ bolts: } \phi F_n = 0.75 F_u \times \frac{\pi}{4} \left(\frac{3}{4} \right)^2 = 0.75 \times 90 \times 0.442 = 29.8 \text{ K/bolt}$$

$$\Rightarrow \text{max bolt} = 8 : 8 \times 29.8 = 238 \text{ K} < 321 \text{ K} \quad \text{Not good}$$

$$\text{Required Moment} = \frac{371 \times 12}{238} = 18.7 \quad (\text{T and C must} < 238)$$

$$\text{Extra arms} = \frac{-13.89}{9.8} \approx 5$$

$$\Rightarrow T_{\text{new}} = \frac{371 \times 12}{13.89 + 5} = 236^k$$

$$\Rightarrow \text{each bolt: } \frac{236^k}{8} = 29.5^k < 29.8^k \checkmark$$

* extra beam lengths (WT5x24.5)

$$\phi V_n = \phi (0.6 F_y) (t_w \times l) \Rightarrow 236 = \phi (0.6 \times 50) \times 0.34 \times l \Rightarrow l = 23.1 \approx 24''$$

bottom

* bolts on top of the beam connected to T:

$$\phi R_n = 0.75 \times F_u \times 0.4418 = 0.75 \times 48 \times \frac{A_{\text{bolts}}}{0.4418} = 15.9^k$$

$$\Rightarrow \# \text{ of bolts} = \frac{236}{15.9} = 14.8 \approx 16 \Rightarrow 8 @ \text{ each row and } 3'' \text{ spacing}$$

$$\Rightarrow L_{\text{top}} = (7 \times 3) + (2 \times 1) + (1) = 24''$$

* T stub design (top T):

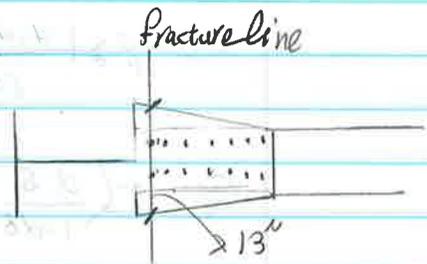
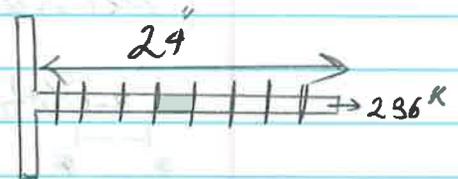
$$\text{yielding} \Rightarrow T_u = \phi F_y A_g$$

$$A_g = \frac{236}{0.9 \times 50} = 5.2 \text{ in}^2 \Rightarrow 13 t_w = 5.2 \Rightarrow t_w = 0.4''$$

$$\text{Fracture} \Rightarrow T_u = \phi F_u A_n$$

$$A_n = \frac{236}{0.75 \times 65} = 4.8 \text{ in}^2$$

$$\Rightarrow (13 - 2 \times \frac{7}{8}) t_w = 4.8 \Rightarrow t_w = 0.43''$$



Hw: repeat whole thing with column: W16x100

→ design top T without any prying Action:

$$t_f \geq \frac{4.44 T b'}{\sqrt{w F_u (1 + \alpha \delta)}} \geq 0.85 \left(T = \frac{236}{2} = 118 \text{ K} / \alpha = 0 \text{ (w/o prying)} \right)$$

$$b' = b - \frac{d_b}{2} = \frac{9.99}{2} - \frac{t_w}{2} - \frac{d_b}{2} = \frac{4}{2} - \frac{t_w}{2} - \frac{3/4}{2} \approx 1.25 \text{ (Assume)}$$

$$\Rightarrow t_f \geq \frac{4.44(118)(1.25)}{14 \times 65 \times 1} \geq 0.85 \checkmark \text{ OK}$$

→ design top T with prying Action:

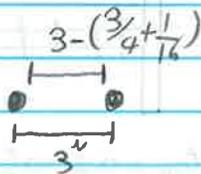
$$t_f \geq \frac{4.44 T b'}{\sqrt{w F_u (1 + \alpha \delta)}} \quad \begin{cases} \alpha = \rho \\ \delta = \rho \end{cases}$$

$$\beta = \left(\frac{B}{T} - 1 \right) \frac{a'}{b'} \approx 1.25$$

bolt capacity

$$\beta > 1 \Rightarrow \alpha = 1$$

$$\beta < 1 \Rightarrow \alpha = \min \left(1, \frac{1}{8} \left(\frac{\beta}{1 - \beta} \right) \right) = 0.018$$



$$\delta = \frac{3 - \left(\frac{3}{4} + \frac{1}{16} \right)}{3} = 0.73$$

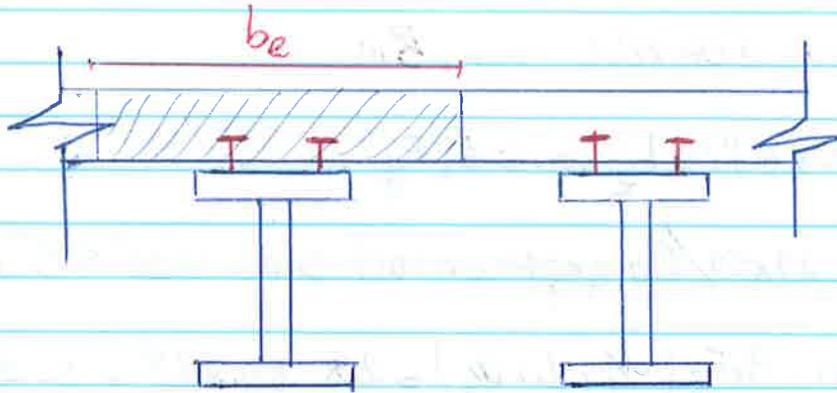
$$\Rightarrow t_f \geq \frac{4.44 \times 118 \times 1.25}{65(1 + 0.018 \times 0.73)} = 0.84 \checkmark \geq 0.84 \text{ OK} \checkmark$$

$$Q = T \left(\frac{\alpha \delta}{1 + \alpha \delta} \right) \left(\frac{b'}{a'} \right) = 118 \left(\frac{0.018 \times 0.73}{1 + 0.018 \times 0.73} \right) \left(\frac{1.375}{1.875} \right) = 0.01 T$$

$$a' = a + \frac{d_b}{2} = \frac{7.4}{2} + \frac{d_b}{2} = \frac{7.4}{2} + \frac{3/4}{2} = 1.875$$

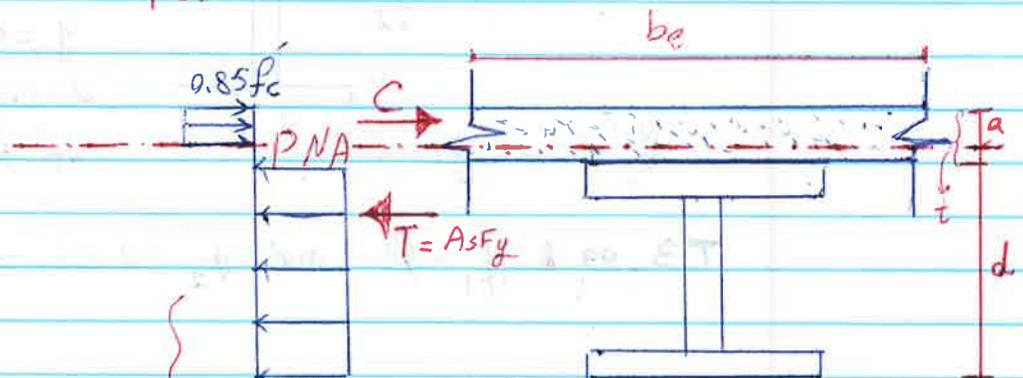
$$b' = b - \frac{d_b}{2} = \frac{9}{2} - \frac{t_w}{2} - \frac{d_b}{2} = \frac{4}{2} - \frac{0.5}{2} - \frac{3/4}{2} = 1.375$$

* Composite structures



- AISC: $\frac{b_e}{2} \geq$
1. $\frac{1}{8}$ beam span
 2. $\frac{1}{2}$ x beam ϕ to ϕ
 3. Distance from beam ϕ to slab edge

- AASHTO: $b_e \geq$
1. $\frac{1}{4}$ beam span
 2. 12 x slab
 - 3.



$$\phi M_n = \phi M_p$$

$$= A_s F_y \left(\frac{d}{2} + t - \frac{a}{2} \right)$$

$$\frac{h}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}}$$

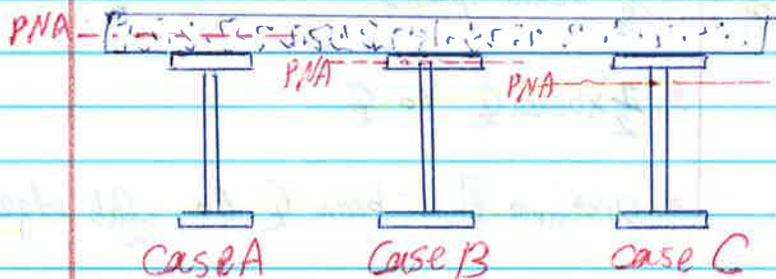
Ex 5.16.1 $b_e = 100$ in beam W₃₀ × 99 ($d = 30$ in)

Thickness of concrete deck = 5 in

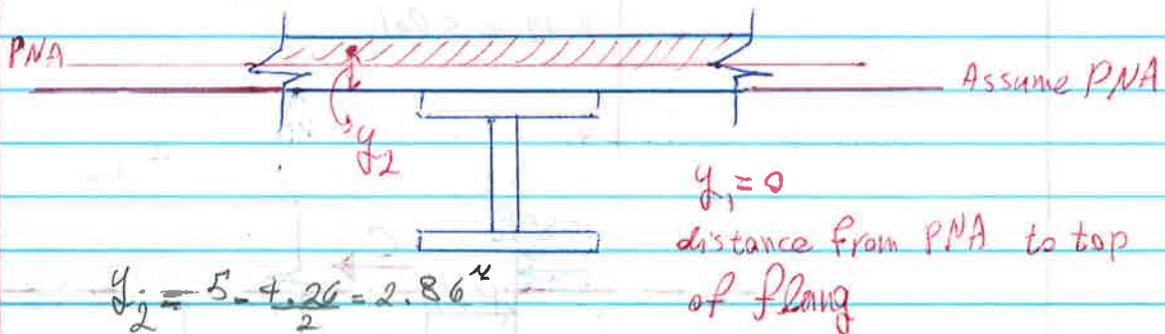
$$C = T \Rightarrow 0.85 f'_c \times b_e \times a = A_s F_y$$

$$\Rightarrow a = 29 \times 50 / (0.85 \times 4 \text{ Ksi} \times 100) \Rightarrow a = 4.26'' < 5'' \text{ OK (CASE A)}$$

$$\phi M_p = A_s F_y \left(\frac{d}{2} + t - \frac{a}{2} \right) = 29 \times 50 \left(\frac{29.7}{2} + 5 - \frac{4.26}{2} \right) = 25694 \text{ Kft}$$

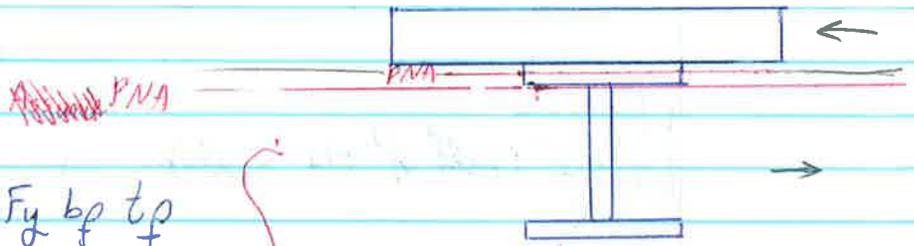


*in case A, AISC assume that PNA is at the bottom of concrete slab



$$y_2 = 5 - \frac{4.26}{2} = 2.86''$$

$$T 3-99 : y_1 = \phi \text{ and } y_2 = 2.86 \Rightarrow \phi M_p = 1926 \checkmark$$



$$C = 0.85 f'_c b_e t + F_y b_f t_f$$

$$T = F_y (A_s - b_f t_f)$$

$C > T \rightarrow$ PNA within flange

$C < T \rightarrow$ PNA within web

Exp beam = $\sqrt{30 \times 116}$ $b_e = 80$ $t = 4$

$$\Rightarrow C = 0.85 \times 4 \times 80 \times 4 + 50 \times 10.5 \times 0.85 = 1534 \text{ K}$$

$$T = 50(34.2 - 10.5 \times 0.85) = 1264 \text{ K}$$

$C > T \Rightarrow$ Case B

$$\Rightarrow \text{Case B: } 0.85 f'_c b_e t + F_y b_f \bar{y} = F_y (A_s - b_f \bar{y})$$

$$\Rightarrow \bar{y} = \frac{F_y A_s - 0.85 f'_c b_e t}{2 F_y b_f} = \frac{50 \times 34.2 - 0.85 \times 4 \times 80 \times 4}{2 \times 50 \times 10.5} = 0.592$$

$$\phi M_n = \phi M_p = 0.85 f'_c b_e t \left(\frac{t}{2} + \bar{y} \right) + 2 F_y b_f \bar{y} \left(\frac{\bar{y}}{2} \right) + F_y A_s \left(\frac{d}{2} - \bar{y} \right)$$

$$= 0.85 \times 4 \times 80 \times 4 \left(\frac{4}{2} + 0.592 \right) + 2 \times 50 \times 80 \times 0.592 \left(\frac{0.592}{2} \right) + 50 \times 33.2 \left(\frac{80}{2} - 0.592 \right)$$

$$= 2074 \text{ kft}$$

HW: 16-1

$$\begin{cases} y_1 = 0.592 \\ y_2 = 2'' \text{ (half of the slab)} \end{cases}$$

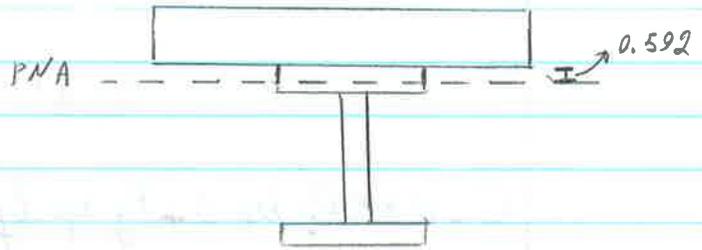
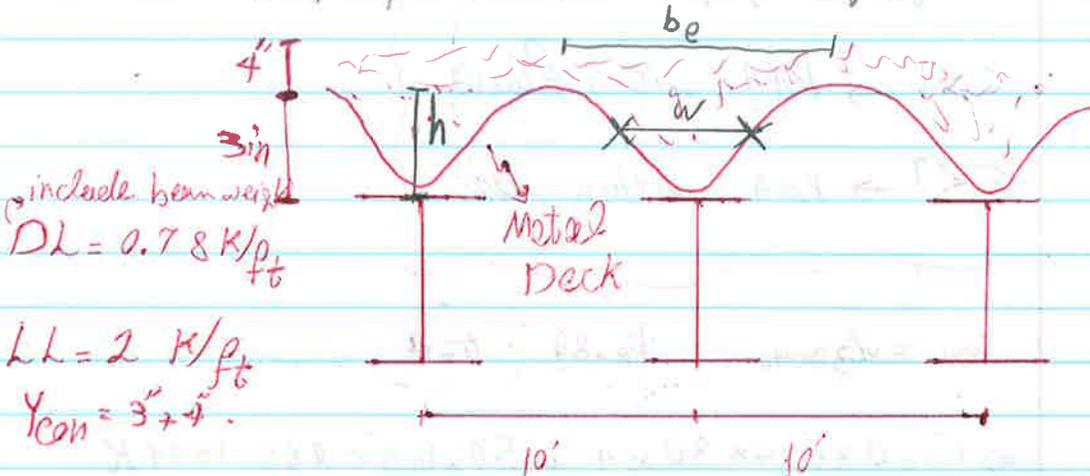


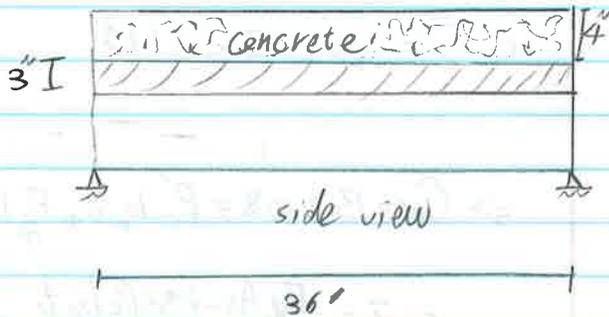
table 3-19 $\Rightarrow \phi M_p = (\text{interpolate}) = 2070 \text{ kft}$

Exp 16-3



$$W_u = 1.2D + 1.6L = 1.2 \times 0.78 + 1.6 \times 2 = 4.14 \text{ K/pft}$$

$$M_u = W_u \frac{l^2}{8} = 670.7$$



$$\frac{be}{2} \rightarrow \frac{1}{8} \text{ span} = \frac{1}{8} (36 \times 12) = 54''$$

$$\Rightarrow be = 54 \times 2 = 108''$$

$$\rightarrow 0.5 (10 \times 12) = 60''$$

$Y_{con} = \text{distance of top of slab to top of beam} = 7''$

$$\begin{aligned} a &\approx \frac{t_{slab}}{2} \rightarrow 15'' \text{ try} \\ &= \frac{4}{2} = 2'' \end{aligned}$$

$$Y_1 = 0; \quad Y_2 = Y_{con} - \frac{a}{2} = 7 - \frac{2}{2} = 6''$$

T 3-19 ($Y_1 = \emptyset, Y_2 = 6''$) . \circ W₁₈₋₄₆

} Shored \rightarrow use some bamboos or ... to handle the load before concrete get dry
} Unshored \rightarrow

①

$$W = \frac{4}{12} \times 110 \times 10 + \underbrace{46}_{\substack{\text{W}_{18 \times 46}}} = 413 \text{ lb/ft}$$

$$M = \frac{0.413 \times 36^2}{8} = 66.9 \text{ kft} < \phi M_p (\text{W}_{18 \times 46}) \checkmark$$

② check deflection

$$\Delta = \frac{M L^2}{(C_1) I_x} = \frac{66.9 \times 36^2}{161 \times 712} = 0.76''$$

$\left. \begin{array}{l} \Delta_{allow} = \frac{L}{180} = \frac{36 \times 12}{180} = 2.5'' \\ 0.76'' < 2.5'' \checkmark \text{OK} \end{array} \right\}$

first try: $\Sigma Q_n = A_s F_y \Rightarrow W_{18 \times 46}$

Total shear resistance by stirrups

$$\Sigma Q_n = 13 \times 5 \times 50 = 675 \text{ K}$$

$$\Rightarrow C = 675 \text{ K}$$

$$a = \frac{C}{0.85 f_c b_e} = \frac{675}{0.85 \times 4 \times 108} = 1.84 < 4'' \text{ within slab } \swarrow \text{NA}$$

$$Y_2 = Y_{con} - \alpha_1 = 7 - \frac{1.84}{2} = 6.08''$$

T3-19 ($Y_1 = 0, Y_2 = 6.08''$):

(TFL)

$$Y_1 = 0 \rightarrow \Sigma Q_n = 675^k$$

$$\textcircled{3} Y_1 = 0.303 \rightarrow \Sigma Q_n = 494^k \star \rightarrow \phi M_n = 677 \text{ kft} > 671 \text{ kft} \downarrow \text{required}$$

$$a = \frac{\Sigma Q_n}{0.85 f_c' b e} = \frac{494}{0.85 \times 4 \times 108} = 1.35''$$

$$\text{Actual } Y_2 = 7 - \frac{1.35}{2} = 6.33$$

$$Y_1 = 0.303$$

$$\phi M_n = 678 + \frac{0.33}{0.5} (677 - 678)$$

$$= 690.5 > 670.7 \text{ OK} \checkmark$$

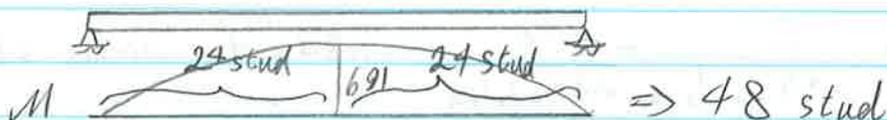
USE $\omega_{18 \times 46}$

Design of studs (stirrups)

T3-21 (lightweight concrete, $f_c' = 4 \text{ ksi}$): $\frac{\omega}{h} > 1.5$

$$\text{capacity / stud} = 21.2^k$$

$$\Sigma Q_n = 494^k \Rightarrow N = \frac{494}{21.2} = 23.3 \approx 24 \text{ } \frac{3}{4}'' \text{ stud}$$



hw: 16-8

check live load Deflection

$$\Delta \not> \frac{\text{span}}{360} = \frac{36 \times 12}{360} = 1.2$$

$$M_L = L \times \frac{l^4}{8} = \frac{2 \times 36^4}{8} = 324 K'$$

$$C_1 = 161$$

$$\Delta_L = \frac{M_L l^2}{C_1 I} = \frac{324 \times 36^2}{161 \times}$$

T3-20:

$$I = 200 + \frac{0.33}{0.5} (2090 - 2000) = 2059$$

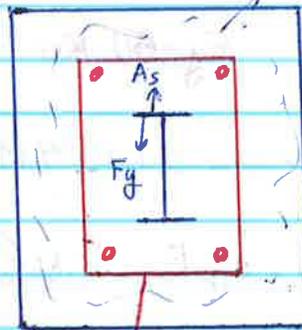
$$\Delta = \frac{M_L l^2}{C_1 I} = \frac{324 \times 36^2}{161 \times 2059} = 1.27'' > 1.2$$

$$CK: V_u = \frac{4.14 \times 36}{2} = 74.5$$

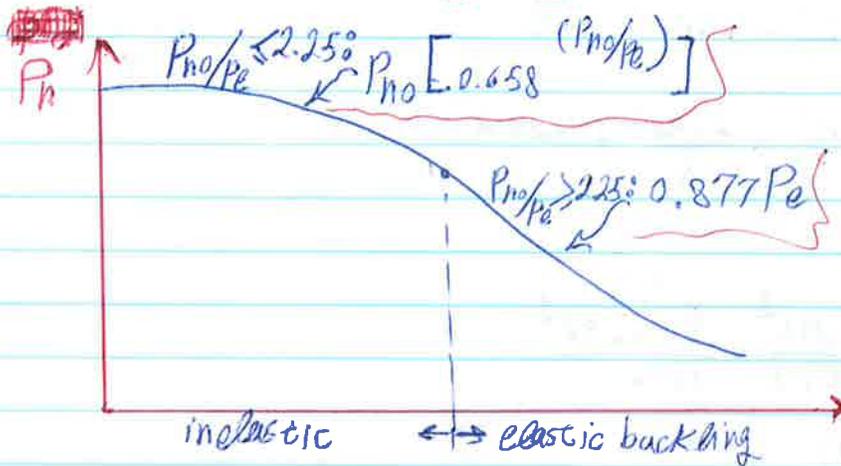
ck $\rightarrow \phi V_u \Rightarrow$ Table 3-2 (195 K) for steel section
w 18x46

Composite column:

1. Small size column.
55-60% cost saving
2. Corrosion + Fire protection
3. Very efficient construction



$$P_{no} = A_s F_y + A_{sr} F_{yr} + 0.85 f'_c A_c$$



$$P_e = \frac{\pi^2 E I_{\text{effective}}}{(KL)^2}$$

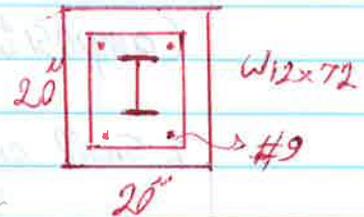
$E I_{\text{effective}}$ = effective stiffness + composite

$$= E_s I_s + 0.5 E_s I_r + C_1 E_c I_c$$

$$C_1 = 0.1 + 2 \left(\frac{A_s}{A_c + A_s} \right) \leq 0.3$$

EXPT 18: $KL = 12'$ $F_y = 50 \text{ Ksi}$ $f'_c = 3.5 \text{ Ksi}$

$$A_c = 20 \times 20 - 21.1 - 4(1 \text{ in}^2) = 374.9 \text{ in}^2$$



$$P_{no} = A_s F_y + A_{sr} F_{yr} + 0.85 f'_c A_c = 21.1 \times 50 + 4(1) \times 60 + 0.85 \times 3.5 \times 374.9 = 2410 \text{ K}$$

$$C_1 = 0.1 + 2 \left(\frac{21.1}{374.9 + 21.1} \right) \leq 0.3 \Rightarrow 0.2066 \leq 0.3 \checkmark \text{ OK}$$

$$E I_{\text{eff}} = E_s I_{sr} + 0.5 E_s I_r + C_1 E_c I_c = 29000 \times 195 + 0.5(29000)(4 \times 1 \times 7.5^2) + 0.2066 \times 326.7 \times 13138 = 17.785 \times 10^6 \text{ K in}^2$$

(weak axis)

$$P_e = \frac{\pi^2 E I_{eff}}{(kL)^2} = \frac{\pi^2 \times 29000 \times 17.785 \times 10^6}{(12 \times 12)^2} = 8465$$

$$\frac{P_{no}}{P_e} = \frac{2410}{8465} = 0.28 < 2.25$$

$$\Rightarrow P_{no} \left[0.658^{(P_{no}/P_e)} \right] = 2410 \left[0.658^{\frac{2410}{8465}} \right] = 2139$$

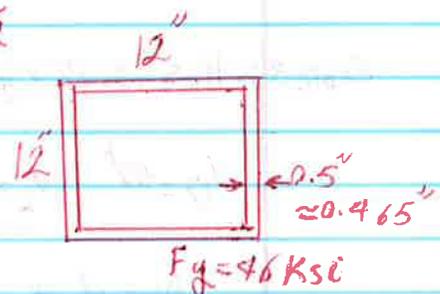
$$\Rightarrow \phi_c P_n = 0.75 \times 2139 = 1604 \text{ K}$$

In case column is steel is inside

Concrete for column steel is outside

Exp 112: HSS (High Strength Structure) 12x12x1/2

$$(KL)_x = (KL)_y = 16' \quad \phi P_n = ?$$



check the steel thickness $\frac{1}{16} \leq 2.26 \sqrt{\frac{E}{F_y}}$

$$\Rightarrow \frac{12}{0.465} = 25.81 < 2.26 \sqrt{\frac{29000}{50}} \quad \checkmark \text{ OK}$$

$$P_{no} = A_s F_y + C_2 f'_c (A_c + A_{sr} \frac{E_s}{E_c}) = (20.9)(46) + 0.85 \times 4 (123.1 + 0) = 1380 \text{ K}$$

$$\begin{cases} C_2 = 0.85 & \text{rectangular} \\ C_2 = 0.95 & \text{circular} \end{cases}$$

$$A_c = 12 \times 12 - 20.9 = 123.1 \text{ in}^2$$

$$I_{eff} = E_s I_s + E_{sr} I_{sr} + C_3 F_c I_c = 29000 \times 457 + 0 + 0.85 \times 3492 \times 1271 = 17.026 \times 10^6$$

$$C_3 = 0.6 + 2 \left(\frac{A_s}{A_c + A_s} \right) \leq 0.9 \Rightarrow 0.6 + 2 \left(\frac{20.9}{123.1 + 20.9} \right) = 0.85 < 0.9 \quad \checkmark$$

$$E_c = \omega_c \sqrt[15]{P'_c} = 145 \sqrt[15]{4} = 3492$$

$$I_c = \frac{12 \times 12^3}{12} - 453 = 1271 \quad \rightarrow I_s$$

$$P_e = \frac{\pi^2 EI_{eff}}{(KL)^2} = \frac{\pi^2 \times 29000 \times 17.026 \times 10^6}{(12)^2} = 4558 \text{ K}$$

$$P_{no}/P_e = \frac{1380}{4558} = 0.3 < 2.65 \Rightarrow P_n = P_{no} \left[0.65 \frac{P_{no}/P_e}{4558} \right] = 1380 \left[0.658 \frac{1380}{4558} \right] = 1216 \text{ K}$$

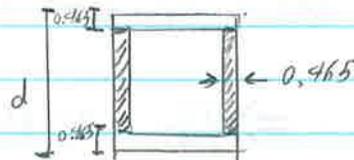
$$\phi P_n = 0.75 \times 1216 = 912 \text{ K}$$

Exp 17.3: HSS 12x12x $\frac{3}{8}$ $(KL)_x = (KL)_y = 16'$

$$V_D = 50 \text{ K} \quad V_L = 100 \text{ K} \Rightarrow V_u = 1.2 \times 50 + 1.6 \times 100 = 220 \text{ K}$$

$$h_w = d_{3e} = 12 - 3(0.465) = 10.605$$

$$A_w = 2h_w t_w = 2 \times 10.605 \times 0.465 = 9.86$$



$$V_n = 0.6 F_y A_w = 0.6 \times 46 \times 9.86 = 272 \text{ K}$$

$$\phi V_n = 0.9 \times 272 = 244.8 > 220 \text{ K} \quad \text{OK} \checkmark$$

Exp 17.4: $(KL)_x = (KL)_y = 15'$ HSS 10x10x $\frac{3}{8}$ Filled with $f'_c = 4 \text{ Ksi}$

$$F_y = 46.6 \text{ Ksi}$$

$$\phi P_n = ?$$

$$T4-15 (KL=15, \text{HSS}) \Rightarrow 575 \text{ K} \approx \phi P_n$$

HW: 17-1
17-4

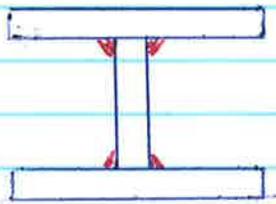
Exp 17-5: HSS 20x20 x 5/8 $(KL)_x = 24$ $(KL)_y = 12$

$(KL)_y = 12 \Rightarrow$ equivalent $(KL)_y = \frac{(KL)_x}{r_x/r_y} = \frac{24}{1.54} = 15.58 \Rightarrow KL = 15.58$

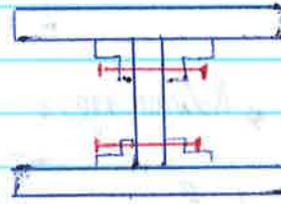
T 4-15: $\phi P_n = 1653k$

PLATE GIRDER

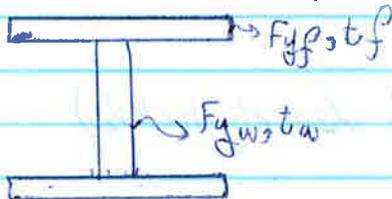
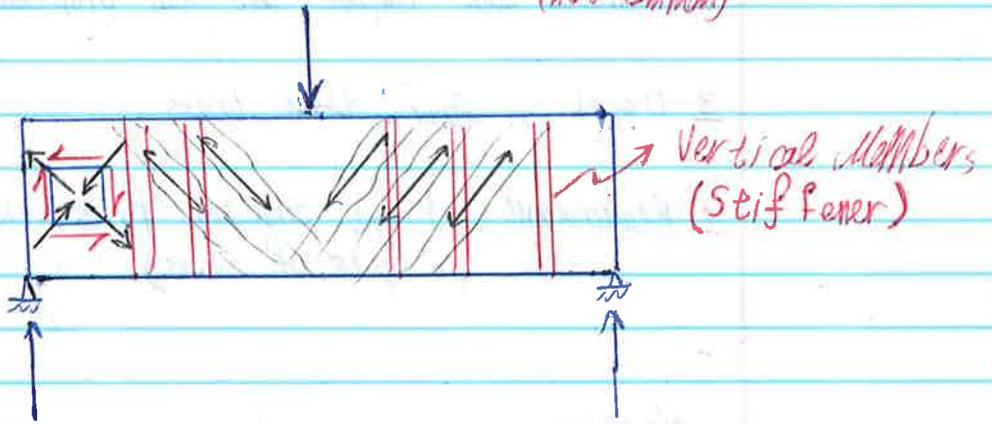
1. Steel bridges
2. beams with very large span



welding plates



use angle and bolts
(not common)



Bridges : $80' - 150' \Rightarrow > 200'$
 (Highway)
 Railway Bridges: $50' - 180' \Rightarrow > 400'$

1 depth of the girder = $\frac{1}{6}$ to $\frac{1}{15}$ of span.

very high load.

concentrated load

very low load

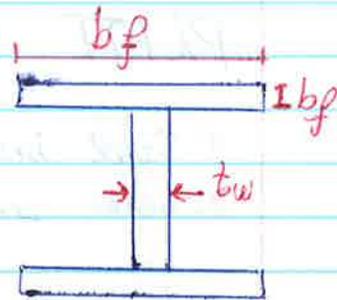
concentrated load

($\frac{1}{10}$ to $\frac{1}{12}$)

2. t_f : 1" to 2"

3. $b_f \approx \frac{\text{depth of the girder}}{4}$

4. $t_w \approx \frac{3}{4}$ flange thickness



* Advantage :

1 Faster and cheaper erection

2 Vibration and Impact are not problems anymore

3 Depth is lower than truss

4 Redundant: if one connection fails, the beam is not going to fail. (in spite of truss)

* Names :

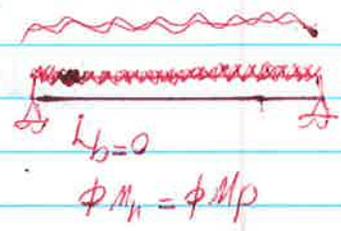
1 Built up I-section (web doesn't buckle)

2 Built up plate section (make sure web must buckle)

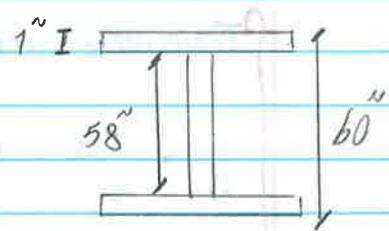
Ex 18.2: depth of built up I-section = 60"

span = 70' $\left\{ \begin{array}{l} \text{DL (without beam)} = 1.1 \text{ K/ft} \\ \text{LL} = 3 \text{ K/ft} \end{array} \right.$
 A36 steel \rightarrow full lateral bracing in compression flange

$d_{web} = \frac{t_o L}{15} = \frac{70 \times 12}{15} \approx 60'' \checkmark \text{ OK}$
 $t_f = 1'' \text{ to } 2'' = 1''$



$b_f = d_f = \frac{b_f}{4} = 15''$
 $t_w = \frac{3}{4} t_f = 0.75''$



DL of beam: flanges: $2(15 \times 1) = 30 \text{ in}^2$
 $0.75 \times 58 = 43.5 \text{ in}^2$
 $\underline{73.5 \text{ in}^2}$

steel weight = 490 lb/ft \Rightarrow I-beam = $\frac{490}{73.5} = 250.1 \text{ lb/ft}$
 $= 0.25 \text{ K/ft}$

$w_u = 1.2(1.1 + 0.25) + 1.6(3) = 6.42 \text{ K/ft}$

shear $V_u = \frac{w_u L}{2} = 6.42 \times \frac{70}{2} = 224.7 \text{ K}$

moment $M_u = \frac{w_u L^2}{8} = 6.42 \times \frac{70^2}{8} = 3932 \text{ K ft}$

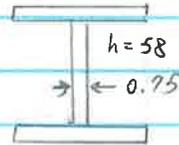
I-section \rightarrow web must not buckle

Table B4.1:

$$\frac{h}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}} = 3.76 \sqrt{\frac{29000}{36}} = 106.7$$

$$t_w = \frac{106.7}{h} = \frac{106.7}{58} = 0.544''$$

⇒ Web is $\frac{9}{16}'' \times 58''$



check for shear:

$$\phi V_n = \phi 0.6 F_y A_w C_v$$

section G
16.1-68
16.1-69

Transverse stiffeners are not required where $\frac{h}{t_w} \leq 2.46 \sqrt{\frac{E}{F_y}}$

$$\frac{h}{t_w} = \frac{58}{\frac{9}{16}} = 103.1 < 2.4 \sqrt{\frac{29000}{36}} = 69.82 \Rightarrow \text{stiffener is needed}$$

$$V_u < \phi V_n \quad \text{Equ 92.1 with } K_v = 5$$

$$\frac{h}{t_w} = 103.1 < 260 \Rightarrow K_v = 5 \text{ without stiffener}$$

$$\text{Try the one in the middle: } \frac{h}{t_w} = 103.1 > 1.37 \sqrt{\frac{K_v E}{F_y}} = 1.37 \sqrt{\frac{5 \times 29000}{36}} = 86.94$$

$$\Rightarrow C_v = \frac{1.51 E K_v}{\left(\frac{h}{t_w}\right)^2 F_y} = \frac{1.51 \times 29000 \times 5}{(103.1)^2 \times 36} = 0.572$$

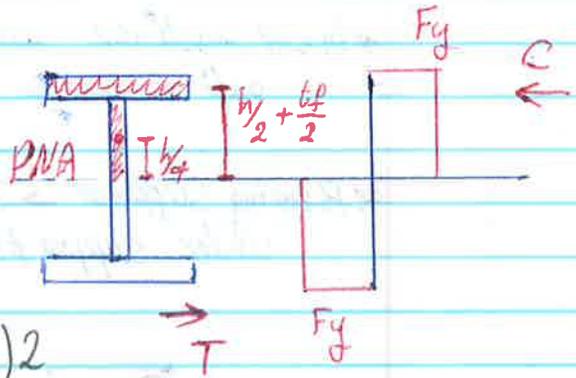
$$\phi V_n = (0.9) 0.6 F_y A_w C_v = (0.9) (0.6 \times 36) \left(58 \times \frac{9}{16}\right) (0.572) = 403.1^k$$

$$403.1^k > 224.7^k = V_u \quad \checkmark \text{ OK}$$

* Web handles the shear and flange handle the moment

Check Moment Capacity:

$$M_p = F_y Z$$



Z = Plastic Section Modulus

= Moment of area about PNA

$$= (A_f \times (h/2 + t_f/2) + (h/2 \times t_w) \times h/4) \times 2$$

$$= A_f (h + t_f) + 2 (h/2) t_w (h/4) \quad (1)$$

$$M_u = \phi M_p = \phi F_y Z \Rightarrow Z_{\text{required}} = \frac{M_u}{\phi F_y} \quad (2)$$

$$(1) \text{ and } (2) \Rightarrow \frac{M_u}{\phi F_y} \Rightarrow A_f = \dots$$

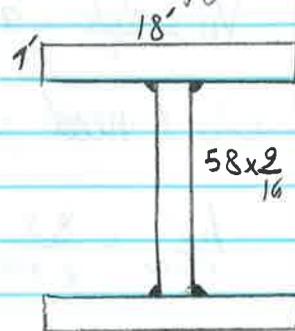
$$\Rightarrow A_f = \frac{M_u}{\phi F_y (h + t_f)} - \frac{t_w h^2}{4 (h + t_f)} = \frac{3937 \times 12}{0.9 \times 58 \times 16} - \frac{9/16 \times 58^2}{4 (58 + 1)} = 16.66 \text{ in}^2$$

\rightarrow PL 18x1 for flange

$$\frac{b_f}{2 t_f} < 0.38 \sqrt{\frac{E}{F_y}} \Rightarrow \frac{18}{2 \times 1} < 10.79 \text{ OK}$$

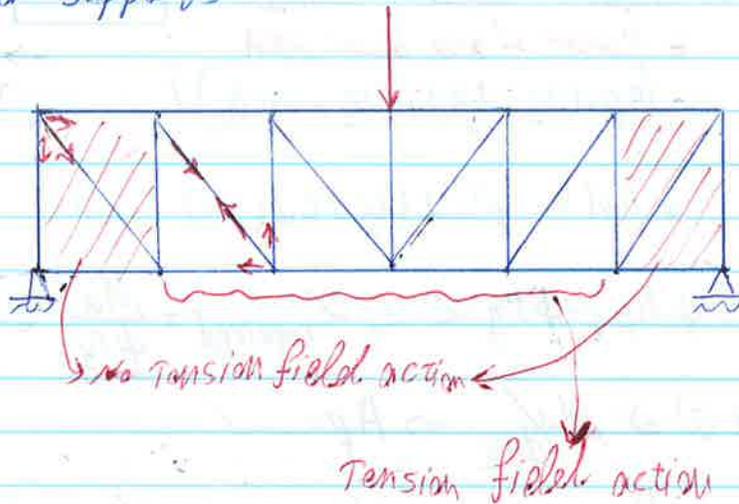
check the girder weight:

$$\frac{9 \times 58 + 2(18 \times 1)}{16} \times 490 \text{ lb/ft} = 233.5 \text{ lb/ft} < 250 \text{ lb/ft} \text{ OK}$$



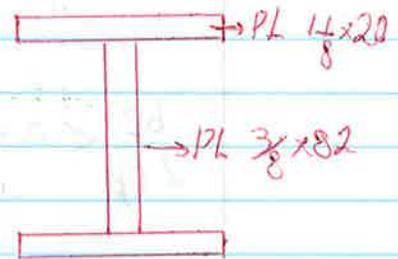
* Intermediate stiffener \rightarrow stiffeners at middle that have a gap at bottom

* Bearing Stiffener \rightarrow stiffeners at first and end of the beam or under supports



Exp 18.5

$L = 65'$ $D = 1.1 \frac{K}{f_0}$ without beam
 $L = 2 \frac{K}{f_0}$



$$A = 2 \left(\frac{1}{8} \times 20 + \frac{3}{8} \times 82 \right) = 75.75$$

$$\frac{w}{ft} = \frac{75.75}{144} \times 490 \rightarrow 258 \text{ K/ft}$$

$$w_u = 1.2(1.1 + 0.258) + 1.6(2) = 4.83 \text{ K/ft}$$

$$V_u = \frac{w_u h}{2} = \frac{4.83 \times 65}{2} = 156.89 \text{ K}$$

Do I need stiffener:

$$\frac{h}{t_w} = \frac{82}{0.375} = 219 < 260 \Rightarrow K_v = 5$$

$$1.37 \sqrt{\frac{K_v E}{F_y}} = 1.37 \sqrt{\frac{5 \times 29000}{36}} = 86.95 < \frac{h}{t_w}$$

$$G 2.5 \rightarrow C_v = \frac{1.51 E K_v}{\left(\frac{h}{t_w}\right)^2 F_y} = \frac{1.51 \times 29000 \times 5}{219 \times 36} = 0.1268$$

$$\Rightarrow \phi V_n = 0.9 (0.6 F_y) A_w C_v = 0.9 \times (0.6 \times 36) \times (82 \times 0.375) \times (0.1268) = 77.87^k$$

$$\Rightarrow 77.87^k < V_u = 150.98^k$$

* T 3-16a (for End parts) right side is $\frac{V_u}{A_w}$

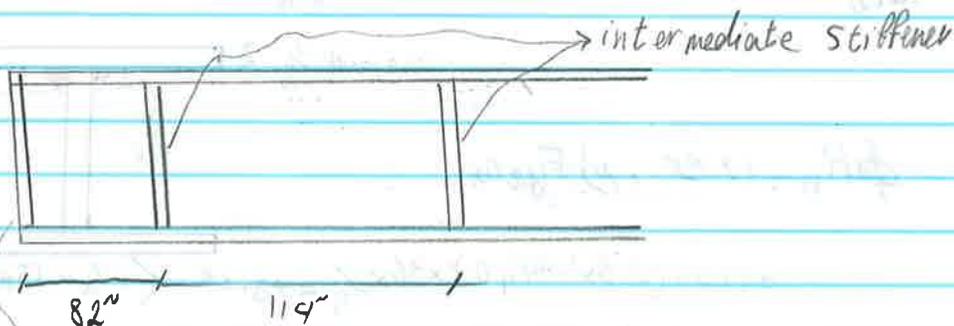
* T 3-16b (for middle parts)

$$V_u @ 82 \text{ in edge} = 123.97$$

$$T 3-16b: \frac{V_u}{A_w} = \frac{123.97}{31.59} = 3.92 \text{ ksi}$$

$$\phi V_n = 77.87 < 123.97 \Rightarrow \text{need stiffener}$$

$$\text{hence use table} \Rightarrow \frac{a}{h} = 1.4 \Rightarrow a = 1.4 \times 82 = 114.8''$$



→ bearing stiffener (only needed at the point of reaction or concentrated load)

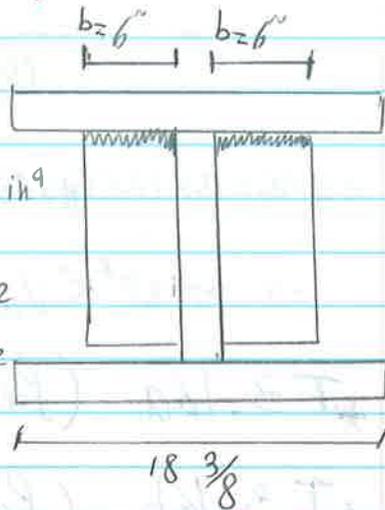
$$J = \frac{2.5}{\left(\frac{82}{h}\right)^2} - 2 = \frac{2.5}{\left(\frac{82}{82}\right)^2} - 2 = 0.5$$

Transverse: $I_{st} = b \frac{t_w^3}{12} j = 82 \times \left(\frac{3}{8}\right)^3 \times 0.5 = 2.16 \text{ in}^4$

$\leftarrow \text{min}(w=82, h=82)$

$$I_{st} = \frac{bh^3}{12} = \frac{1/4 \times 6^3}{12} \times 2 = 9 \text{ in}^4 \gg 2.16 \text{ in}^4$$

so we can use just one plate at one side, doesn't have to be at both sides



Sec G2.2 and J10.8:

bearing stiffener must connected to the top

for bearings



$$I_k = 1 \frac{1}{8} I + \frac{5}{16} I = 1.44 I$$

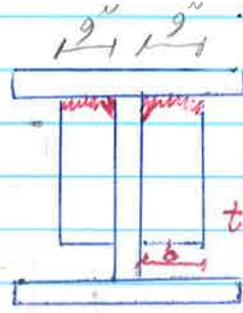
$$\phi R_n = (2.5K + N) F_y t_w$$

$$= (1.0)(2.5 \times 1.44 + 0) \times 36 \times \frac{3}{8} = 48.6 \text{ K} < V_u = 156.98 \text{ K}$$

\Rightarrow need bearing stiffener

hw 18-5
18-7

TB4-1:



$$b/t \leq 0.56 \sqrt{E/F_y}$$

$$\Rightarrow \frac{9}{t} \leq 0.56 \sqrt{\frac{29000}{36}}$$

Plates:

$$I = \frac{5}{8} \times (9 \times 2 + \frac{3}{8})^3 / 12 = 323 \text{ in}^4$$

$$\Rightarrow t \approx \frac{5}{8}$$

$$A = 2(9 \times \frac{5}{8}) + (12 \text{ tw}) \times \frac{3}{8} = 12.94$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{323}{12.94}} = 5 \frac{3}{8}$$

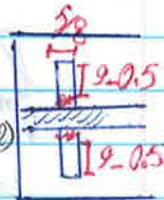
$$Kl = 0.75 \times h = 0.75 \times 82 = 61.5$$

$$\Rightarrow \frac{Kl}{r} = \frac{61.5}{5} = 12.9 \quad \text{and} \quad T4-22 : \phi F_y = 32.17$$

$$\phi P_n = 32.17 \times 12.94 = 416 > V_u = 156.98 \quad \checkmark \text{OK}$$

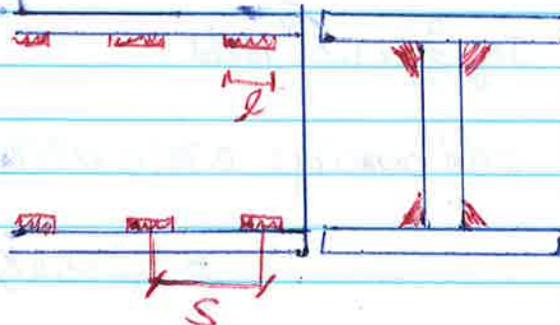
shear area of plate

$$AISC \rightarrow 1.8 F_y A_{pd} > V_u \Rightarrow 1.8 \times 36 \times 0.75 (9 - 0.5) (\frac{5}{8}) (2) = 516 \text{ K} > 156.98 \quad \checkmark \text{OK}$$

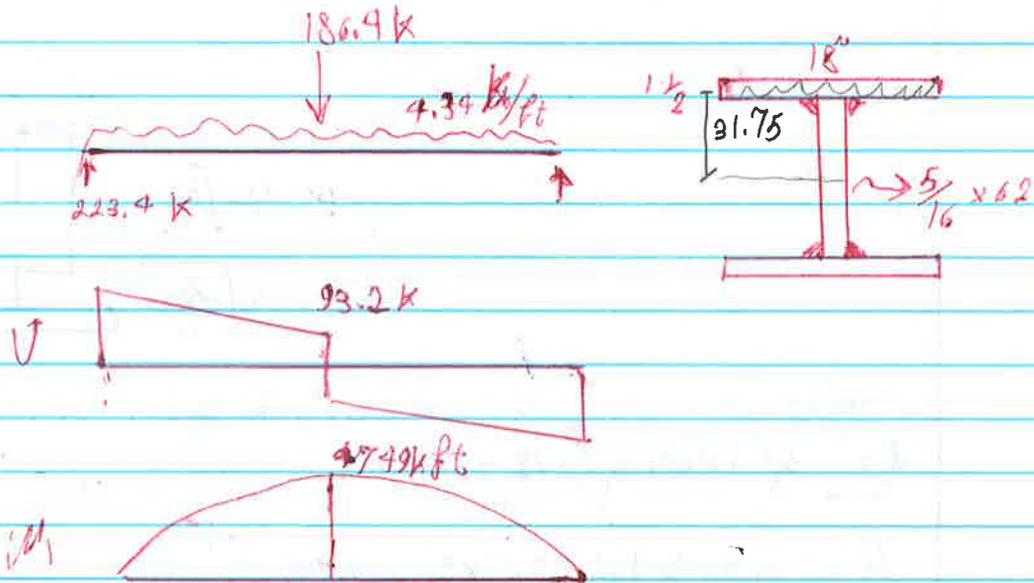


$$A_{pd} = (9 - 0.5) (\frac{5}{8}) \times 2$$

Connection Design



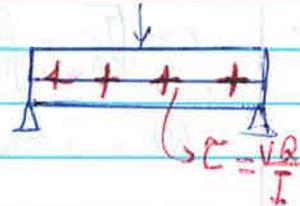
Exp:



$$\tau = \frac{VQ}{I} = 3.158 \text{ k/in}$$

$$I = 60642 \text{ in}^4$$

$$Q = (18 \times 1.5) \times 31.75 = 857.2$$

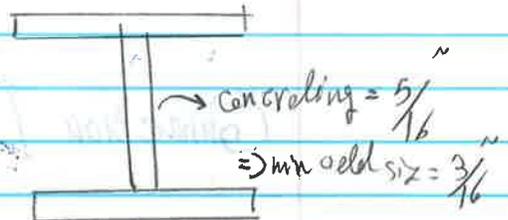
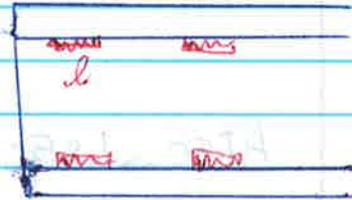


weld size:

$$l_{\text{min}} = 4w \geq 1.5''$$

$$= 4\left(\frac{3}{16}\right) \geq 1.5 \Rightarrow 0.75 \geq 1.5$$

$$\Rightarrow l = 1.5''$$



Try $\frac{3}{16} \times 1.5''$ weld

weld capacity: $0.75(0.6 \times 70)(0.707 \times \frac{3}{16}) \times 2$ both side

$$= 8.352 \text{ k/in}$$

web capacity & (1) yielding: $0.6 F_y \times \overset{\text{weld size}}{5/16} = 0.6 \times 36 \times 5/16 = 6.75 \text{ K/in}$ our capacity

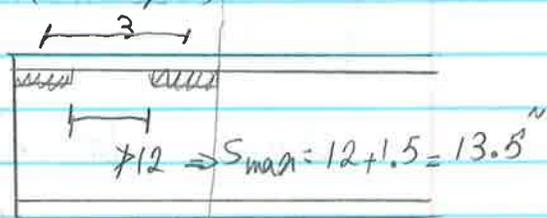
2) Fracture: $0.75 (0.6 \times \overset{F_u}{58}) 5/16 = 8.156 \text{ K/in}$

⇒ each weld can handle: $6.75 \times 1.5 = 10.13 \text{ K}$

$$T = 3.158 = \frac{10.13}{S} \Rightarrow S = 3.21'' \approx 3''$$

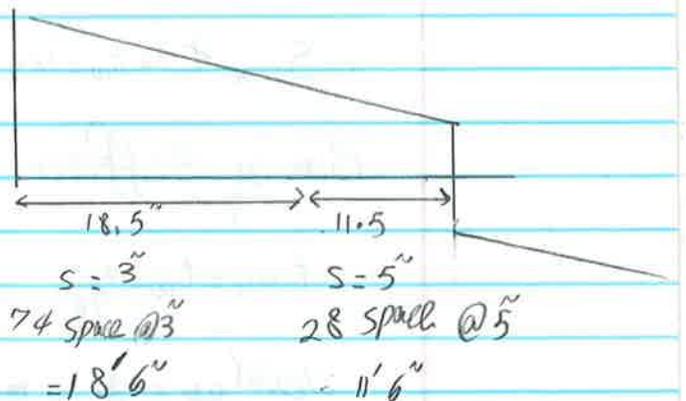
Code: $S_{max} \leq 0.75 \sqrt{\frac{F_u}{F_y}} t_p \times 12''$

$$\Rightarrow S_{max} \leq 0.75 \sqrt{\frac{29000}{96}} 1.5 \times 12'' \Rightarrow 31.9'' \times 12''$$



$$\frac{V_u R}{I} = \frac{10.13}{5''} \Rightarrow V_u = \frac{10.13}{5''} \times \frac{60640}{857.2} = 143.3 \text{ K}$$

$$\Rightarrow V_u = 223.454.34 \text{ K} = 143.3 \Rightarrow \alpha = 18.5''$$

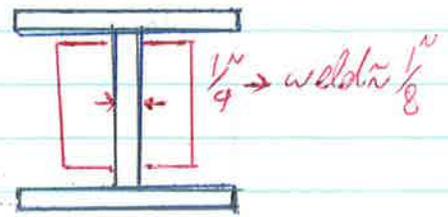


Transverse Stiffener

weld length $4w \times 1.5''$

$$\Rightarrow 4 \times \frac{1}{8} \times 1.5 \Rightarrow 5 \times 1.5''$$

\Rightarrow weld length = $1.5''$



Weld: $\phi F_w / in \Rightarrow 0.75(0.6 \times 70)(0.707) \frac{1}{8} \times 2 \times 2 = 5.568 k/in$

Web: 1. yielding $\Rightarrow 1 \times (0.6 \times 36) (\frac{1}{4}) = 5.4 k/in$

2. Fracture $\Rightarrow 0.75(0.6 \times 58) (\frac{1}{4}) = 6.525 k/in$

2 place $\phi F_w = 5.4 \times 2 = 10.8$

AISC: $f = 0.945 \sqrt{\frac{F_y^3}{E}}$ $h = 0.045 \sqrt{\frac{36^3}{29000}} \times 62 = 3.536$

$\Rightarrow \tau = 3.536 = \frac{10.8 k/in \times 1.5''}{S} \Rightarrow S = 4.5''$

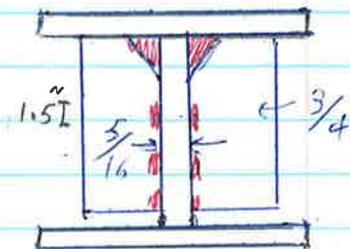
$S_{max} \times 16 t_w = 16 \times \frac{5}{16} = 5'' \Rightarrow S = 4.5'' \checkmark$

Bearing Stiffener

$t_{min} = t_w = \frac{5}{16}$

weld size = $\frac{3}{16} \rightarrow min$

min length $\Rightarrow 4w \times 1.5 \Rightarrow 4 \times \frac{3}{16} \times 1.5 \Rightarrow 0.75 \times 1.5 \Rightarrow l = 1.3''$



$$\text{weld} \rightarrow 0.75(0.6 \times 70) (0.709 \times \frac{3}{16}) \times 2 = 8.352 \text{ K/in}$$

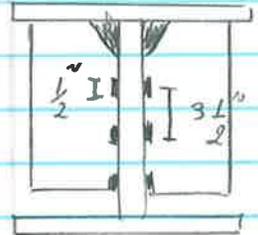
$$\text{web} \rightarrow \begin{cases} 1. \text{yielding: } 1(0.6 \times 36 \times \frac{5}{16}) = 6.75 \text{ K/in} \end{cases}$$

$$\begin{cases} 2. \text{fracture: } 0.75(0.6 \times 58 \times \frac{5}{16}) = 8.156 \text{ K/in} \end{cases}$$

$$2 \text{ plate} \rightarrow 2 \times 6.75 = 13.5 \text{ K/in}$$

$$\text{shear flow} = \frac{R_{max}}{\text{length available } h} = \frac{223.4}{62 - 2 \times \frac{5}{16}} = 3.662$$

$$\tau = \frac{13.5 \text{ K/in} \times 1.5}{S} = 3.662 \Rightarrow S = 5.53 \approx 5 \frac{1}{2}$$



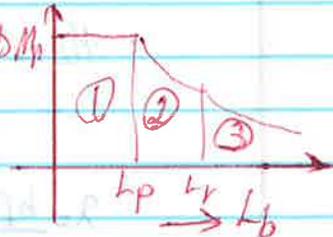
chapter F eq 1-4 to 1-15

ϕM_n for plate girder

1. Lateral Torsional Buckling (LTB) (2) ϕM_n

ϕM_n if $L_b = 0$

second check



2. Flange Local Buckling (FLB) (3)
gonna happen if the flange is non compact

third check



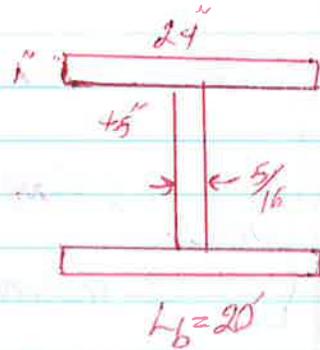
3. (TFY) unsymmetric section around $x-x$

4. Compression Flange Yielding (CFY) (1)
(ψ)

first check

Exp 18.4

① is the web compact? $\Rightarrow \lambda_{tw} > 3.76 \sqrt{\frac{E}{F_y}}$
 $\Rightarrow 144 > 106.72 \Rightarrow$ non compact web
 (create girder)



② is the flange compact? $\Rightarrow \lambda_{fp} > 0.38 \sqrt{\frac{E}{F_y}}$
 $\frac{24/2}{1} > 0.38 \sqrt{\frac{29000}{36}} \Rightarrow 12 > 10.79$
 \Rightarrow the flange is non compact

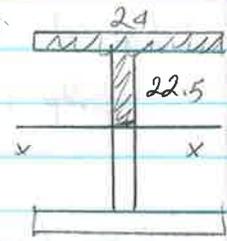
checking steps : 1. Y

2. LTB

3. FLB

1. $\phi M_n = R_{pc} F_y S_{xx}$ $R_{pc} = \left[\frac{M_p}{M_{yc}} - \left(\frac{M_p - 1}{M_{yc}} \right) \left(\frac{\lambda - \lambda_{pw}}{\lambda_w - \lambda_{pw}} \right) \right] \leq \frac{M_p}{M_{yc}}$

$\frac{M_p}{M_{yc}} = \frac{F_y Z}{F_y S} = \frac{24 \times 12 (22.5 + 0.5)}{22.5 \times 0.7125 \times \frac{22.5}{2}} = 1.068$



$\lambda = \frac{b_f}{2t_f} = 12$ $\lambda_{fp} = 0.38 \sqrt{\frac{E}{F_y}} = 10.79$

$\lambda_{fp} = 0.95 \sqrt{\frac{E}{F_y}} = 15.58$

$\Rightarrow R_{pc} = 1.022$ $\phi M_n = 1.022 \times 36 \times \frac{1132}{12} = 3624$ (Y)

2. LTB Zone 2 or 3?

$$L_b = 20' \quad L_p = 1.1 \sqrt{\frac{EI}{F_y}} = 17.42 \quad L_r = 63.67'$$

$$\Rightarrow \text{Zone 2: } M_n = C_b \left[R_{pc} M_{yc} - (R_{pc} M_{yc} - F_L S_{xc}) \left[\frac{L_b - L_p}{L_r - L_p} \right] \right] \leq R_{pc} M_{yc}$$

↳ table 3-1

now we use $C_b = 1.0$

$$\Rightarrow M_n = 1.0 [\dots] = 3560 \text{ (LTB)}$$

3. FLB $M_n = \left[R_{pc} M_{yc} - (R_{pc} M_{yc} - \frac{F_L S_{xc}}{L_{xc}}) \left[\frac{\lambda - \lambda_{FE}}{\lambda_{LF} - \lambda_{FE}} \right] \right]$

eq F.4.6a $\Rightarrow F_L \checkmark$ $\frac{S_{xc}^{\text{tension}}}{S_{xc}^{\text{compression}}} = 1 \rightarrow$ (because of symmetry)

$$\Rightarrow F_L = 0.7 F_y = 25.2$$

$$\Rightarrow M_n = 3334$$

$$\phi M_n = 0.9 \times 3334 = 3001 \text{ Kft}$$

$\frac{d}{dt} \left[\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right] = \frac{d}{dt} \left[\frac{1}{2} m v^2 \right] + \frac{d}{dt} \left[\frac{1}{2} k x^2 \right]$

$\frac{d}{dt} \left[\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right] = \frac{d}{dt} \left[\frac{1}{2} m v^2 \right] + \frac{d}{dt} \left[\frac{1}{2} k x^2 \right]$

$\frac{d}{dt} \left[\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right] = \frac{d}{dt} \left[\frac{1}{2} m v^2 \right] + \frac{d}{dt} \left[\frac{1}{2} k x^2 \right]$

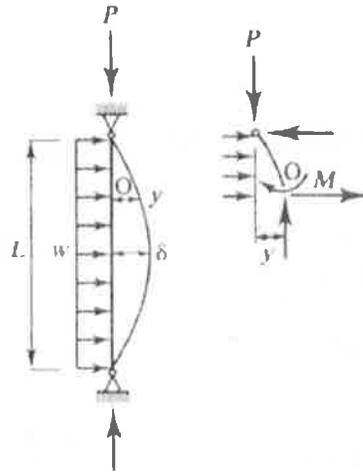
$\frac{d}{dt} \left[\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right] = \frac{d}{dt} \left[\frac{1}{2} m v^2 \right] + \frac{d}{dt} \left[\frac{1}{2} k x^2 \right]$

$\frac{d}{dt} \left[\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right] = \frac{d}{dt} \left[\frac{1}{2} m v^2 \right] + \frac{d}{dt} \left[\frac{1}{2} k x^2 \right]$

$\frac{d}{dt} \left[\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right] = \frac{d}{dt} \left[\frac{1}{2} m v^2 \right] + \frac{d}{dt} \left[\frac{1}{2} k x^2 \right]$

$\frac{d}{dt} \left[\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right] = \frac{d}{dt} \left[\frac{1}{2} m v^2 \right] + \frac{d}{dt} \left[\frac{1}{2} k x^2 \right]$

■ FIGURE 6.3



Figure

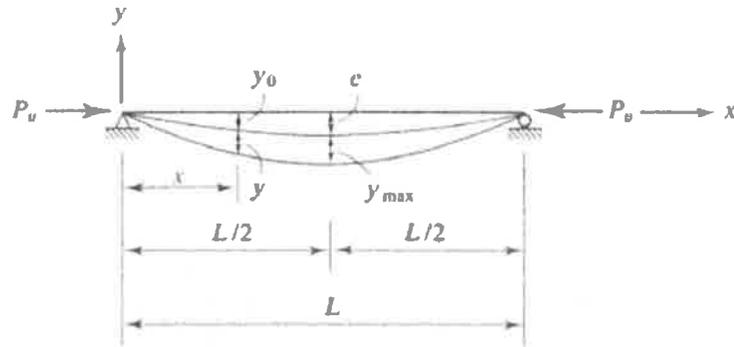
shows a beam-column with an axial load and a transverse uniform load. At an arbitrary point O , there is a bending moment caused by the uniform load and an additional moment Py , caused by the axial load acting at an eccentricity from the longitudinal axis of the member. This secondary moment is largest where the deflection is largest — in this case, at the centerline, where the total moment is $wL^2/8 + P\delta$. Of course, the additional moment causes an additional deflection over and above that resulting from the transverse load. Because the total deflection cannot be found directly, this problem is non-linear; and without knowing the deflection, we cannot compute the moment.

Ordinary structural analysis methods that do not take the displaced geometry into account are referred to as *first-order* methods. Iterative numerical techniques, called *second-order* methods, can be used to find the deflections and secondary moments, but these methods are impractical for manual calculations and are usually implemented with a computer program. Most current design codes and specifications, including the AISC Specification, permit the use of either a second-order analysis or the *moment amplification method*. This method entails computing the maximum bending moment resulting from flexural loading (transverse loads or member end moments) by a first-order analysis, then multiplying by a *moment amplification factor* to account for the secondary moment. An expression for this factor will now be developed.

Figure 6.4 shows a simply supported member with an axial load and an initial out-of-straightness. This initial crookedness can be approximated by

$$y_0 = e \sin \frac{\pi x}{L}$$

■ FIGURE 6.4



where e is the maximum initial displacement, occurring at midspan. For the coordinate system shown, the moment - curvature relationship can be written as

$$\frac{d^2 y}{dx^2} = -\frac{M}{EI}$$

The bending moment M is caused by the eccentricity of the axial load P_u with respect to the axis of the member. This eccentricity consists of the initial crookedness y_0 plus additional deflection y resulting from bending. At any location, the moment is

$$M = P_u(y_0 + y)$$

Substituting this equation into the differential equation, we obtain

$$\frac{d^2 y}{dx^2} = -\frac{P_u}{EI} \left(e \sin \frac{\pi x}{L} + y \right)$$

Rearranging gives

$$\frac{d^2 y}{dx^2} + \frac{P_u}{EI} y = -\frac{P_u e}{EI} \sin \frac{\pi x}{L}$$

which is an ordinary, nonhomogeneous differential equation. Because it is a second-order equation, there are two boundary conditions. For the support conditions shown, the boundary conditions are

$$\text{At } x = 0, \quad y = 0 \quad \text{and at } x = L, \quad y = 0$$

That is, the displacement is zero at each end. A function that satisfies both the differential equation and the boundary conditions is

$$y = B \sin \frac{\pi x}{L}$$

where B is a constant. Substituting into the differential equation, we get

$$-\frac{\pi^2}{L^2} B \sin \frac{\pi x}{L} + \frac{P_u}{EI} B \sin \frac{\pi x}{L} = -\frac{P_u e}{EI} \sin \frac{\pi x}{L}$$

Solving for the constant gives

$$B = \frac{-\frac{P_u e}{EI}}{\frac{P_u}{EI} - \frac{\pi^2}{L^2}} = \frac{-e}{1 - \frac{\pi^2 EI}{P_u L^2}} = \frac{e}{\frac{P_c}{P_u} - 1}$$

where

$$P_c = \frac{\pi^2 EI}{L^2} = \text{the Euler buckling load}$$

$$\therefore y = B \sin \frac{\pi x}{L} = \left[\frac{e}{(P_c/P_u) - 1} \right] \sin \frac{\pi x}{L}$$

$$\begin{aligned} M &= P_u(y_0 + y) \\ &= P_u \left\{ e \sin \frac{\pi x}{L} + \left[\frac{e}{(P_c/P_u) - 1} \right] \sin \frac{\pi x}{L} \right\} \end{aligned}$$

The maximum moment occurs at $x = L/2$:

$$\begin{aligned} M_{\max} &= P_u \left[e + \frac{e}{(P_c/P_u) - 1} \right] \\ &= P_u e \left[\frac{(P_c/P_u) - 1 + 1}{(P_c/P_u) - 1} \right] \\ &= M_0 \left[\frac{1}{1 - (P_u/P_c)} \right] \end{aligned}$$

where M_0 is the unamplified maximum moment. In this case, it results from initial crookedness, but in general it can be the result of transverse loads or end moments. The moment amplification factor is therefore

$$\frac{1}{1 - (P_u/P_c)} \quad (6.4)$$

As we describe later, the exact form of the AISC moment amplification factor can be slightly different from that shown in Expression 6.4.

Table 1.
Revised Table of m for Steel Beam-Column

Values of m														
F_y	36 ksi						50 ksi							
	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
KL (ft)	10	12	14	16	18	20	22 and over	10	12	14	16	18	20	22 and over
1st Approximation														
All Shapes	2.0	1.9	1.8	1.7	1.6	1.5	1.3	1.9	1.8	1.7	1.6	1.4	1.3	1.2
Subsequent Approximation														
S4,5,6	1.3	1.0	0.8	0.7	0.6	0.5	0.5	1.1	0.9	0.8	0.7	0.6	0.5	0.5
W,M4	3.1	2.3	1.7	1.4	1.1	1.0	0.8	2.4	1.8	1.4	1.1	1.0	0.9	0.8
W,M5	3.2	2.7	2.1	1.7	1.4	1.2	1.0	2.8	2.2	1.7	1.4	1.1	1.0	0.9
W,M6	2.8	2.5	2.1	1.8	1.5	1.3	1.1	2.5	2.2	1.8	1.5	1.3	1.2	1.1
W8	2.5	2.3	2.2	2.0	1.8	1.6	1.4	2.4	2.2	2.0	1.7	1.5	1.3	1.2
W10	2.1	2.0	1.9	1.8	1.7	1.6	1.4	2.0	1.9	1.8	1.7	1.5	1.4	1.3
W12	1.7	1.7	1.6	1.5	1.5	1.4	1.3	1.7	1.6	1.5	1.5	1.4	1.3	1.2
W14	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.5	1.4	1.4	1.3	1.3	1.2	1.2