Fracture Mechanics

basic assumptions:

$$\sigma_{ii} = \sum_{i=1}^{3} \sigma_{ii} = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

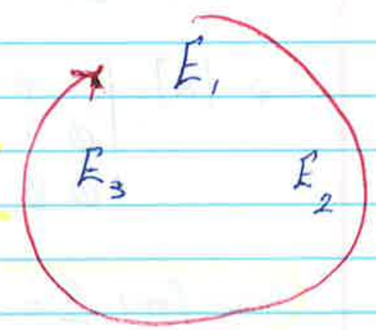
$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ \phi & \text{if } i \neq j \end{cases}$$

$$e_i \cdot e_j\ \delta_{ij} = (e_1 e_1 + e_1 e_2 + e_1 e_3 + e_2 e_1 + e_2 e_2 + e_2 e_3 + e_3 e_1 + e_3 e_2 + e_3 e_3)$$

$$\Rightarrow \delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3$$

$$E_i \times E_j = e_{ijk} E_k$$

$$\begin{cases} +1 & 123,\ 231,\ 312 \\ -1 & 132,\ 321,\ 213 \\ \phi & \text{any other} \end{cases}$$



$$\underline{a} \times \underline{b} = (a_i \underline{e}_i) \times (b_j \underline{e}_j) = a_i b_j (\underline{e}_i \times \underline{e}_j) = a_i b_j\ e_{ijk}\ \underline{e}_k$$

$$e_{ijk}\ e_{lmn} = \overbrace{\begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}}^{\det}$$

$$\underline{a} \times \underline{b} = \det \begin{vmatrix} e_1 & e_2 & e_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \underline{e}_1 (a_2 b_3 - a_3 b_2) - \underline{e}_2 (a_1 b_3 - a_3 b_1) + \underline{e}_3 (a_1 b_2 - a_2 b_1)$$

Tensors:

* $[\underline{a} \otimes \underline{b}]\underline{v} = \underline{a}(\underline{b} \cdot \underline{v}) \quad \forall \underline{v} \in \mathcal{E}^n$

* $\underline{L} = L_{ij}\, \underline{e}_i \otimes \underline{e}_j = \sum_i^3 \sum_j^3 L_{ij}\, \underline{e}_i \otimes \underline{e}_j$

exp: $f(\underline{e}_1) = -\underline{e}_1$ $\quad f(\underline{v}) = \underline{A}\,\underline{v}$

$f(\underline{e}_2) = \underline{e}_2$

$f(\underline{e}_3) = \underline{e}_3$ $\quad \underline{A} = \underbrace{A_{ij}}_{\underline{e}_i \cdot \underline{A}\,\underline{e}_j} \underline{e}_i \otimes \underline{e}_j$

$\Rightarrow A_{i1} = \underline{e}_i \cdot \underline{A}\,\underline{e}_1 = -\underline{e}_1 \rightarrow \underline{e}_1 \otimes \underline{e}_1$

$A_{i2} = \underline{e}_i \cdot \underline{A}\,\underline{e}_2 = \underline{e}_2 \rightarrow \underline{e}_2 \otimes \underline{e}_2$

$A_{i3} = \underline{e}_i \cdot \underline{A}\,\underline{e}_3 = \underline{e}_3 \rightarrow \underline{e}_3 \otimes \underline{e}_3$

$$\Rightarrow [A] = \begin{bmatrix} -1 & \phi & \phi \\ \phi & 1 & \phi \\ \phi & \phi & 1 \end{bmatrix}$$

Tensor form: $\underline{A} = -\underline{e}_1 \otimes \underline{e}_1 + \underline{e}_2 \otimes \underline{e}_2 + \underline{e}_3 \otimes \underline{e}_3$

exp: Prove $\underline{I}\,\underline{u} = \underline{v}$

$(T_{ij}\, \underline{e}_i \otimes \underline{e}_j)(u_m\, \underline{e}_m)$

$= T_{ij}\, u_m \underbrace{(\underline{e}_i \otimes \underline{e}_j)\,\underline{e}_m}_{\underline{e}_i (\underline{e}_j \cdot \underline{e}_m)}$

$(\underline{e}_j \cdot \underline{e}_m) \rightarrow \delta_{jm}$

$\Rightarrow T_{ij}\, u_m\, \delta_{ji} \cdot \underline{e}_i \quad \Rightarrow T_{ij}\, u_j \cdot \underline{e}_i = \underline{v} = v_i\, \underline{e}_j$

$m = j$

* $(\underline{T} + \underline{V})\underline{v} = \underline{T}\underline{v} + \underline{V}\underline{v}$
* $\underline{T}(\underline{S} + \underline{V}) = \underline{T}\underline{S} + \underline{T}\underline{V}$
* $\alpha(\underline{T}\underline{v}) = \underline{T}(\alpha\underline{v})$
* Tensor transposo $T_{ij} = T_{ji}$, $\underline{T} = \underline{T}^T$

  $\underline{u} \cdot \underline{T}\underline{v} = \underline{T}^T\underline{u} \cdot \underline{v}$
* $(\underline{T}\underline{V})\underline{v} = \underline{T}(\underline{V}\underline{v})$
* $\underline{Q}\underline{u} \cdot \underline{Q}\underline{v} = \underline{u} \cdot \underline{v}$
* $\underline{T}\underline{V} \neq \underline{V}\underline{T}$
* $\underline{Q}^T\underline{Q} = \underline{I}$
* $(\underline{T}\underline{V})\underline{S} = \underline{T}(\underline{V}\underline{S})$
* $\underline{Q}^T = \underline{Q}^{-1} \Rightarrow \det(\underline{Q}) = \pm 1$
* $\det(\underline{Q}) = \det(\underline{Q}^T)$

trace (tr):

$\rightarrow I_1 = tr(\underline{T}) = T_{11} + T_{22} + T_{33}$

$\rightarrow I_{II} = \frac{1}{2}\left[(tr\,\underline{T})^2 - tr(\underline{T}^2)\right]$

$\rightarrow I_{III} = \det \underline{T}$

exp: $\underline{S} = \underline{E} + \underline{W}$ $\quad \underline{S} = \underline{S}^T$ $\quad \underline{S} = -\underline{S}^T$

$E = \frac{1}{2}(\underline{S} + \underline{S}^T)$

$W = \frac{1}{2}(\underline{S} - \underline{S}^T)$

$\Rightarrow E + W = \frac{1}{2}[\underline{S} + \underline{S}^T + \underline{S} - \underline{S}^T]$

$= \underline{S}$

Exp: $\Psi = T_{ii} S_{jj}$

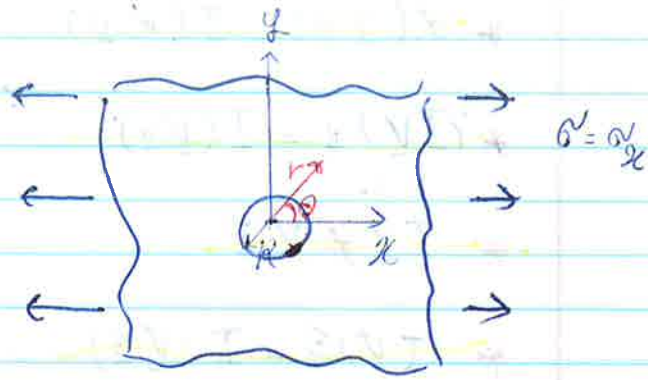
$$= \sum_{i=1}^{3} T_{ii} S_{jj} = (T_{11} + T_{22} + T_{33}) S_{jj}$$

or

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} = T_{11} S_{11} + T_{22} S_{11} + T_{33} S_{11} + T_{11} S_{22} + T_{11} S_{22}$$

$$\sigma_r = \sigma \cos^2\theta = \frac{\sigma}{2}(1 + \cos 2\theta)$$

$$\sigma_{r\theta} = \frac{-\sigma}{2} \sin(2\theta)$$



$$\sigma_r = \frac{\sigma}{2}\left(1 + \cos(2\theta)\right)$$

$$F(r, \theta) = f(r) \cos 2\theta \text{ and } \begin{cases} f(r) = Ar^2 + \cancel{Br^4} + Cr^{-2} + D \\ A = \frac{-\sigma}{4} \quad C = \frac{-\sigma R^4}{4} \quad D = \frac{\sigma R^2}{2} \end{cases}$$

$$\Rightarrow \sigma_r = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial r^2}$$

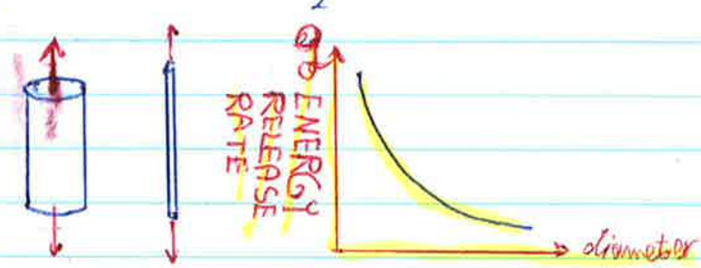
$$\sigma_\theta = \frac{\partial^2 F}{\partial r^2}$$

$$\Rightarrow \sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \theta}\right)$$

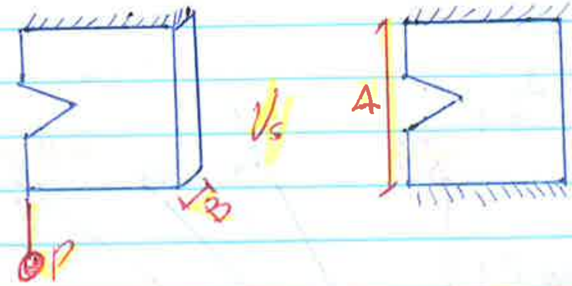
$$\sigma_\theta = \frac{\sigma}{2}\left[1 + \frac{R^2}{r^2} - \left(1 + 3\frac{R^4}{r^4}\right)\cos 2\theta\right] \quad \text{Exp} \begin{cases} r = R \\ \theta = \frac{\pi}{2} = 90^\circ \end{cases}$$

Kirsch, 1898

$$\Rightarrow \sigma_\theta = 1 + 1 - (1 + 3)(-1) = \frac{60}{2} = 30$$



$$\mathcal{G} = \text{Energy Release Rate} = \frac{d\Pi}{dA}$$



Strain Energy $U = \int_0^{\Delta} P\, d\Delta = \frac{P\Delta}{2}$

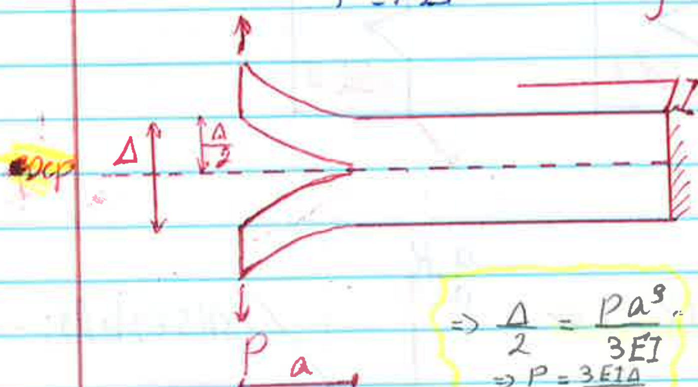
$F = P\Delta$

$$\mathcal{G} = \frac{P}{2B}\left(\frac{d\Delta}{da}\right) \text{ and } C = \frac{\Delta}{P} \Rightarrow \Delta = CP$$

(Compliance)

$$\Rightarrow \boxed{\mathcal{G} = \frac{P^2}{2B}\left(\frac{dC}{da}\right)}$$

Energy release rate



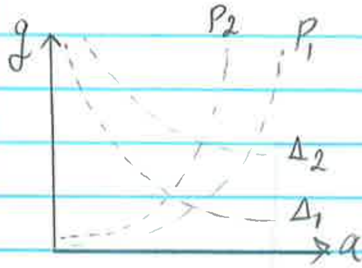
$$\boxed{\Rightarrow \frac{\Delta}{2} = \frac{Pa^3}{3EI} \quad \Rightarrow P = \frac{3EI\Delta}{2a^3}}$$

$$\boxed{I = \frac{Bh^3}{12}}$$

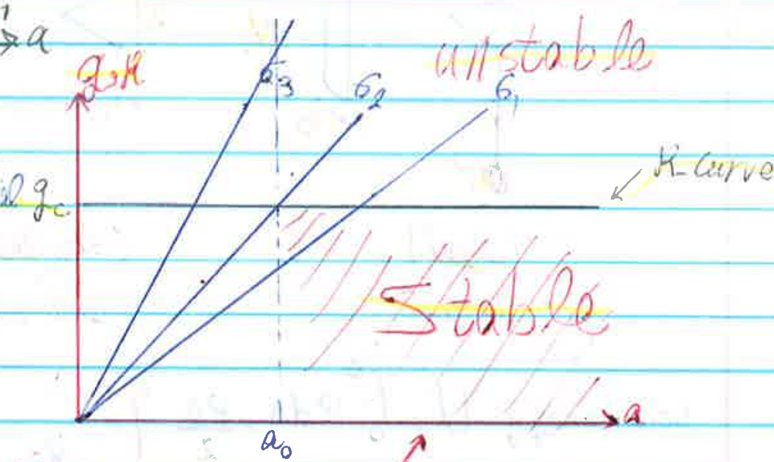
$$C = \frac{\Delta}{P} \text{ and } \Delta = \frac{2Pa^3}{3EI} \Rightarrow C = \frac{2a^3}{3EI} = \frac{12 \times 2a^3}{3EBh^3} = \frac{8a^3}{EBh^3}$$

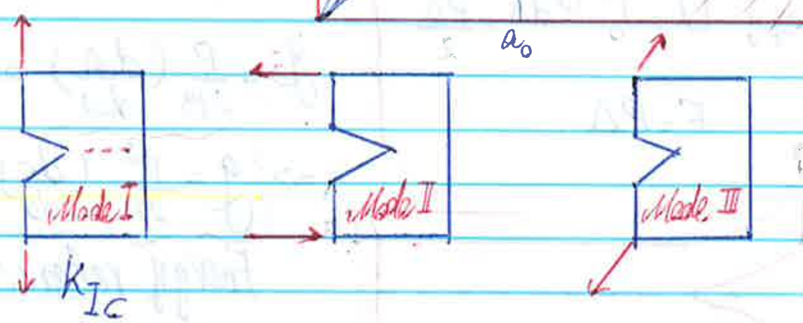
$$\Rightarrow \frac{P^2}{2B}\frac{dC}{da} = \frac{24a^2}{EBh^3} \Rightarrow \mathcal{G}(P,a) = \frac{12P^2a^2}{B^2h^3E} \quad \text{(load) control}$$

$$\mathcal{G}(\Delta,a) = \frac{9\Delta^2 EI}{4Ba^4} \quad \text{(displacement) control}$$



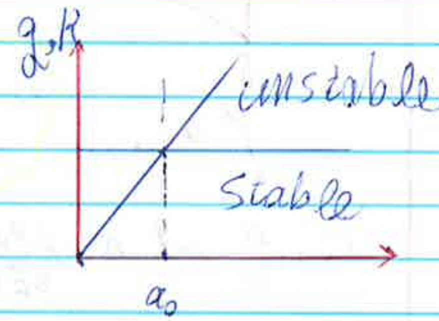
R-curve:
for brittle material g_c





stable crack: $g = R$ or

$$\frac{dg}{da} \leq \frac{dR}{da}$$



unstable crack: $\frac{dg}{da} > \frac{dR}{da}$

Stress Intensity Factor "K" (Irwin 1950's)

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{r\theta} \end{Bmatrix} \quad \operatorname{div} \underline{\underline{\sigma}} = 0$$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{E}}}} \, \underline{\underline{\varepsilon}}$$

$$\begin{cases} \dfrac{\partial \sigma_{rr}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \theta_{r\theta}}{\partial \theta} + \dfrac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \\ \dfrac{1}{r}\dfrac{\partial \sigma_{\theta\theta}}{\partial \theta} + \dfrac{\partial \sigma_{r\theta}}{\partial r} + \dfrac{2\sigma_{r\theta}}{r} = 0 \end{cases} \quad (1)$$

$$\Delta(\sigma_{rr} + \sigma_{\theta\theta}) = 0$$

$$\begin{cases} \sigma_{rr}(r,\theta) = \sigma_{rr}(r,-\theta) = r^{\lambda} f(\theta) \\ \sigma_{\theta\theta}(r,\theta) = \sigma_{\theta\theta}(r,\theta) = r^{\lambda} g(\theta) \\ \sigma_{r\theta}(r,\theta) = -\sigma_{r\theta}(r,\theta) = r^{\lambda} h(\theta) \end{cases} \quad (2)$$

Sub. (2) in (1):

$$\begin{cases} (\lambda+1) f(\theta) + h'(\theta) - g(\theta) = 0 & 1 \\ (\lambda+2) h(\theta) = -g'(\theta) & 2 \\ \lambda^2 q(\theta) + q''(\theta) = 0 & 3 \end{cases}$$

$$q(\theta) = f(\theta) + g(\theta)$$

$$s^2 + \lambda^2 = 0$$

$$A_1 \cos\lambda\theta + \cancel{B_1 \sin\lambda\theta} = q(\theta)$$

$$* \begin{cases} f(\theta) = -g(\theta) + A_1 \cos\lambda\theta \\ h(\theta) = \dfrac{1}{\lambda+2} g'(\theta) \end{cases}$$

Sub. * in 1:

$$g''(\theta) + (\lambda+2)^2 g(\theta) = A_1(\lambda+1)(\lambda+2)\cos\lambda\theta$$

$$\frac{A_1}{4}(\lambda+2)\cos\lambda\theta$$

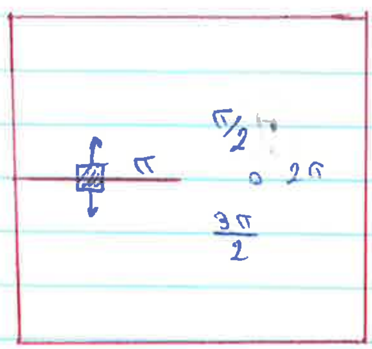
$$* \; g(\theta) = A_2 \cos(\lambda+2)\theta + \cancel{B_2 \sin(\lambda+2)} + g_{\text{particulier}}$$

$$P(\theta) = C\cos\lambda\theta$$

$$C = \frac{A_1(\lambda+2)}{4}$$

$*\ f(\theta) = A_2 \cos(\lambda+2)\theta + \frac{A_1}{4}(2-\lambda)\cos\lambda\theta$

$*\ h(\theta) = A_2 \sin(\lambda+2)\theta + \frac{A_1}{4}\lambda\sin\lambda\theta$.



$\sigma_{\theta\theta} = 0 = g(\pm\pi) = 0$

$\sigma_{r\theta} = 0 = h(\pm\pi) = 0$

$= \cos\lambda\pi$

$g(\pm\pi) = A_2\cos[(\lambda+2)\pi] + \frac{A_1}{4}(\lambda+2)\cos\lambda\pi = 0$

$= [A_2 + \frac{A_1}{4}(\lambda+2)]\cos\lambda\pi = 0$

$\cos\lambda\pi = 0$ @ $\pi/2, 3\pi/2, \ldots, \frac{(2n+1)\pi}{2} \Rightarrow \lambda = \frac{(2n+1)}{2}$

$h(\pi) = A_2 \sin(\lambda+2)\pi + \frac{A_1}{4}\lambda\sin\lambda\pi = 0 = [A_2 + \frac{A_1}{4}\lambda]\sin\lambda\pi = 0$ $\quad \lambda = n$

$\sin\lambda\pi$

$\sigma \sim r^{\lambda} \Longleftrightarrow \varepsilon \sim r^{\lambda}$

$\varepsilon_{rr} = \frac{\partial u_r}{\partial r} \Rightarrow u \sim r^{\lambda+1}$

$\lambda + 1 > 0 \Rightarrow \boxed{\lambda > 1}$

$$\sigma_{rr} = r^{\lambda} f(\theta) = \frac{A_1}{\sqrt{r}}\left[\frac{-1}{8}\cos\frac{3\theta}{2} + \frac{5}{8}\cos\frac{\theta}{2}\right]$$

$$\sigma_{\theta\theta} = r^{\lambda}\, g(\theta) = \frac{A_1}{\sqrt{r}}\left[\frac{1}{8}\cos\frac{3\theta}{2} + \frac{3}{8}\cos\frac{\theta}{2}\right]$$

$$\sigma_{r\theta} = r^{\lambda}\, h(\theta) = \frac{A_1}{\sqrt{r}}\left[\frac{1}{8}\sin\frac{3\theta}{2} + \frac{1}{8}\sin\frac{\theta}{2}\right]$$

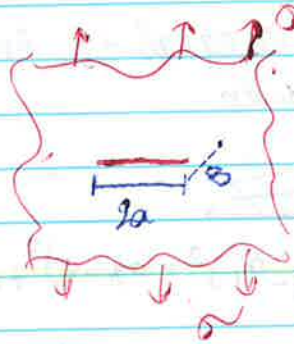
$$K_I = \sqrt{2\pi}\, A_1 \Rightarrow A_1 = \frac{K_I}{\sqrt{2\pi}}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left[\frac{5}{4}\cos\frac{\theta}{2} - \frac{1}{4}\cos\frac{3\theta}{2}\right] \Rightarrow \sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}} \Rightarrow K_I = \sqrt{2\pi r}\,\sigma_{rr}$$

$$\sigma_{\theta\theta}\ \frac{K_I}{\sqrt{2\pi r}}\left[\frac{3}{4}\cos\frac{\theta}{2} + \frac{1}{4}\cos\frac{3\theta}{2}\right]$$

$$\sigma_{r\theta}\ \frac{K_I}{\sqrt{2\pi r}}\left[\frac{1}{4}\sin\frac{\theta}{2} + \frac{1}{4}\sin\frac{3\theta}{2}\right]$$

$$\mathcal{G} \Leftrightarrow K$$



$$\mathcal{G} = \frac{-d\Pi}{dA} \qquad \Pi = \Pi_0 - \frac{\pi\sigma^2 a^2 B}{E}$$

$$A = aB$$

$$\Pi_0 - \frac{\pi\sigma^2 a A}{E}$$

$$\Rightarrow \mathcal{G} = \frac{\pi\sigma^2 a}{E}$$

$$\Rightarrow \sigma^2 = \frac{\mathcal{G}E}{\pi a}$$

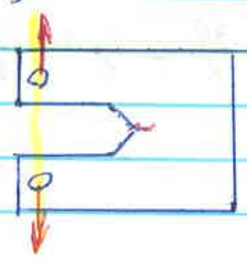
$$K = \sigma\sqrt{\pi a} \Rightarrow \sigma^2 = \frac{K^2}{(\sqrt{\pi a})^2} = \frac{K^2}{\pi a}$$

$$\frac{\mathcal{G}E}{\pi a} = \frac{K^2}{\pi a} \Rightarrow \mathcal{G} = \frac{K^2}{E}\ \Big\}\ \text{Plane stress} \qquad \mathcal{G} = \frac{1-\nu^2}{E}\,\pi\sigma^2 a\ \Big\}\ \text{Plane strain}$$

K_{Ic}: is a material property.

* One way to find K_{Ic} of a material is Compact Tension Specimen (CTS)

CTS

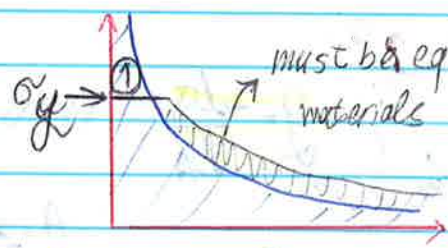


K:

$\sigma_{zz} = \phi$

$\sigma_{xz} = \phi$

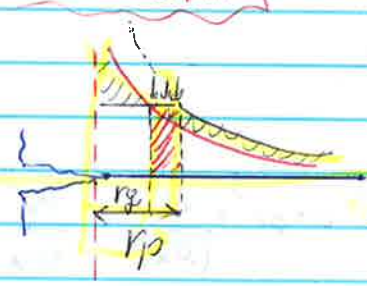
$\sigma_{xy} = \phi$



$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2=(\sigma_x-0)^2+(0-0)^2+(0-\sigma_x)^2=2\sigma_x^2=2\sigma_{yield}^2$$

$$\sigma_{xx}(r,\theta)=\frac{K}{\sqrt{2\pi r}} \qquad \sigma_{yy}(r,\theta)=\frac{K_I}{2\sqrt{\pi r}}$$

$$r_y=\frac{K_I^2}{2\pi\sigma_y^2}$$

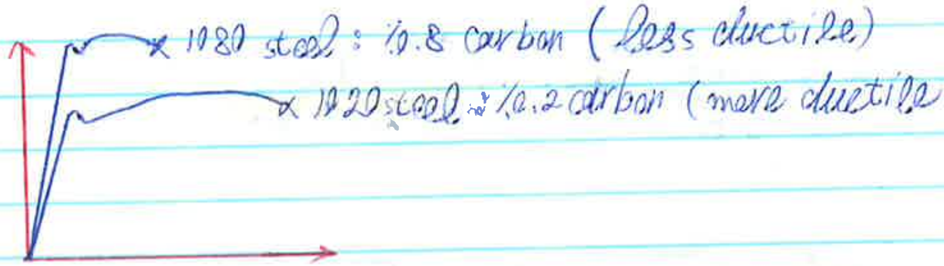


$$r_p=\frac{1}{\sigma_y}\int_0^{r_y}\sigma\,dx$$

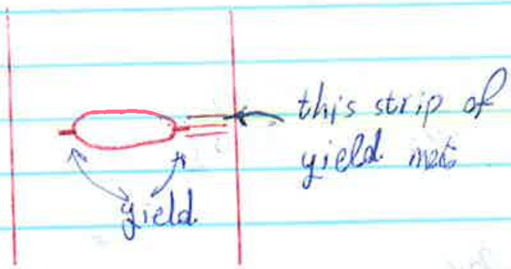
$$\sigma_y=\int_0^{r_y}\sigma_x\,dx-\sigma_y r_y=(r_p-r_y)\sigma_y$$

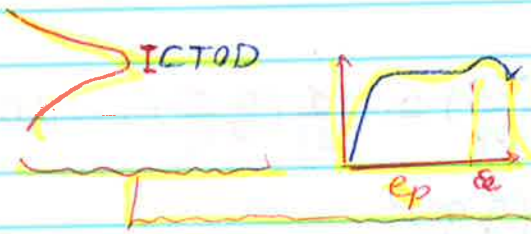
$$\Downarrow$$

$$r_p=\frac{1}{\pi}\left(\frac{K_I}{\sigma_y}\right)^2$$



Dugdale 1960s

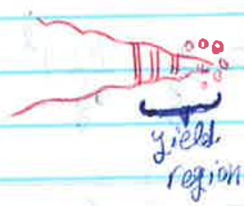




Thermo set → Highly ...
linear elastic

Thermo plastic → chain ...

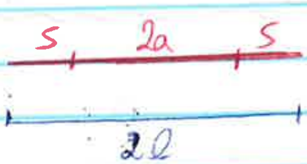
Elastomers →
linear elastic



$$SIF_{\text{crack tip}} \Rightarrow \Phi = K^{e} + K^{p}$$

elastic: $K_e = \sigma\sqrt{\pi(a+s)}$

plastic: $K^{p} = -K^{e}$



By force P on crack we have:

$$K^{a} = \frac{P}{\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}$$

$$K^{(p)} = \frac{-\sigma_y}{\sqrt{\pi(a+s)}}\int_{a}^{a+x}\left(\sqrt{\frac{a+s+x}{a+s-x}}+\sqrt{\frac{a+s-x}{a+s+x}}\right)dx$$

$$\Rightarrow K^{p} = \frac{-\sigma x^2}{\sqrt{\pi}} \sqrt{a+s} \int_a^{a+x} \frac{dx}{(a+s)^2 - x^2}$$

$$\Rightarrow K^{e} = \sigma\sqrt{\pi(a+s)} = \frac{2\sigma x}{\sqrt{\pi}} \sqrt{a+s}\ \cos^{-1}\left(\frac{a}{a+s}\right)$$

$$\cos^{-1}\left(\frac{a}{a+s}\right) = \frac{\pi}{2}\frac{\sigma}{\sigma_y}$$

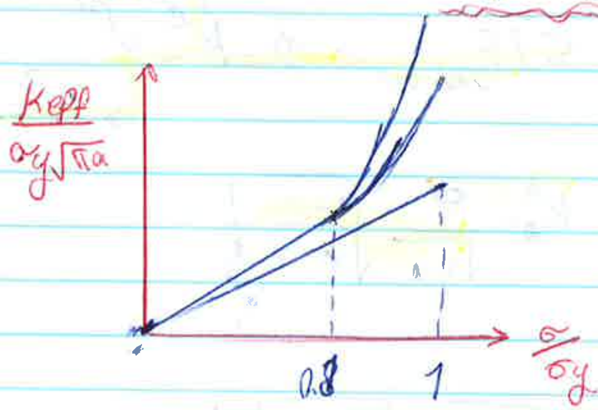
Taylor

$$\frac{a}{a+s} = \cos\left(\frac{\pi}{2}\frac{\sigma}{\sigma_y}\right) \simeq 1 - \frac{1}{2!}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_y}\right)^2 + \dots H.O.T$$

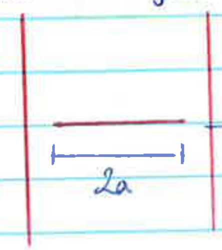
$$\frac{a}{a+s} \approx 1 - \frac{1}{2}\left(\frac{\pi}{2}\frac{\sigma}{\sigma_y}\right)^2$$

$$S = \frac{\pi^2\sigma^2 a}{8\sigma_y^2} = \frac{\pi}{8}\left(\frac{K_I(a)}{\sigma_y}\right)^2 \approx \frac{3}{8}\left(\frac{K_I}{\sigma_y}\right)^2$$

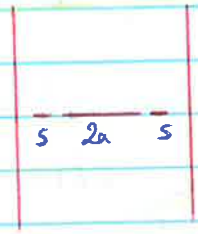
$$\Rightarrow a_{effective} = a + s \quad \Rightarrow K_{effective} = \sigma\sqrt{\pi a \cdot \sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_y}\right)}$$



small scale yielding

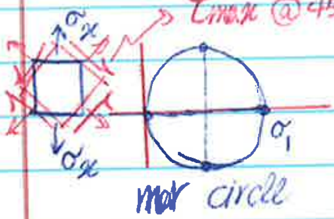


→ LEFM



$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_4)^2=2(\sigma_{yield})^2$$

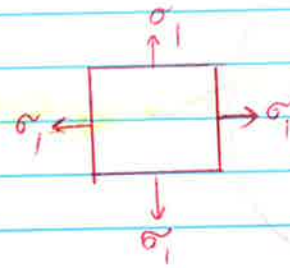
τ_{max} @ 45^0



$\sigma_1=\sigma_x$

$\sigma_2=\phi$

$\sigma_3=\phi$



$$\Rightarrow \sigma_{1,2}=\sigma_{xy}=\frac{\sigma_{xx}+\sigma_{yy}}{2}\pm\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\sigma_{xy}^2}$$

$$\Rightarrow \frac{\sigma_{xx}+\sigma_{yy}}{2}=\frac{K_I}{\sqrt{2\pi r}}\cos\theta$$

r → radi of plastic zone

$$\sigma_{xy}=\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_y=\frac{1}{4\pi}\left(\frac{K_I}{\sigma_y}\right)^2\left(1+\cos\frac{\theta}{2}+\frac{3}{2}\sin^2\theta\right)$$

Plane stress

$$\sigma_y^2=\frac{K_I^2}{2\pi r}\cos^2\theta/2\left[(1-2\nu)^2+3\sin^2\frac{\theta}{2}\right]$$

Plane strain

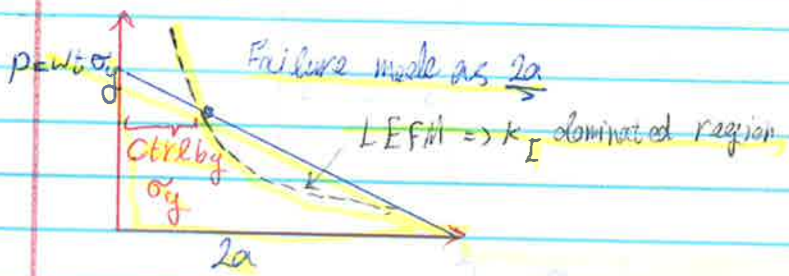
$$\Rightarrow r(\theta)=\frac{3}{2\pi}\left(\frac{K_I}{\sigma_y}\right)^2\cos^2\frac{\theta}{2}\left[\sin^2\frac{\theta}{2}+\frac{(1-2\nu)^2}{3}\right];\ \theta=0\Rightarrow r(\theta)=\left(\frac{K_I}{\sigma_y}\right)^2\frac{1}{54}$$

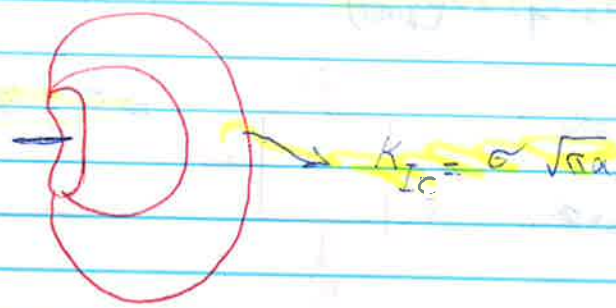
→ plane stress gives much bigger yielding zone at crack tip.

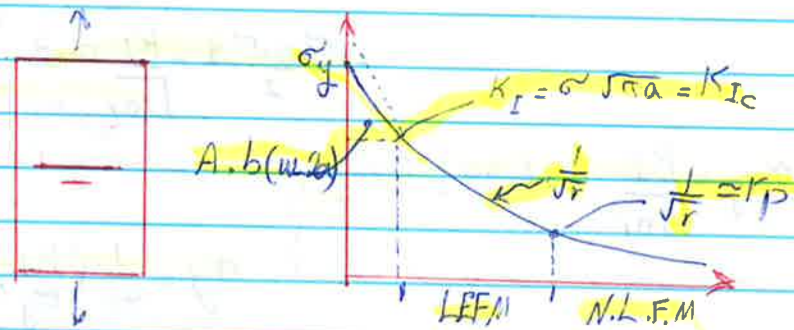
→ Element size control the plastic zone.

types of failure → LEFM
→ Plastic yielding = Full scale yielding
→ collapse $P=[(w-2a)t]\,\sigma_y$

→ Plastic yielding occures when crack is too small.

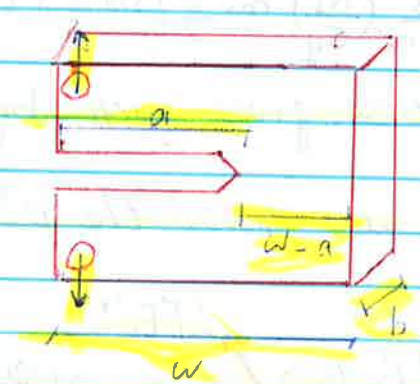






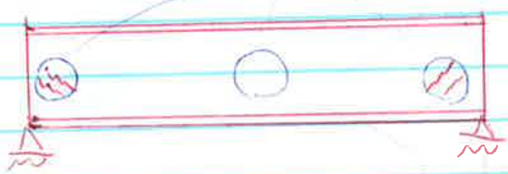
CTOD: is controlled by ductility of the material which trapped in crack tip

* for calculating K_{Ic} use Compact Tension Test (CT):

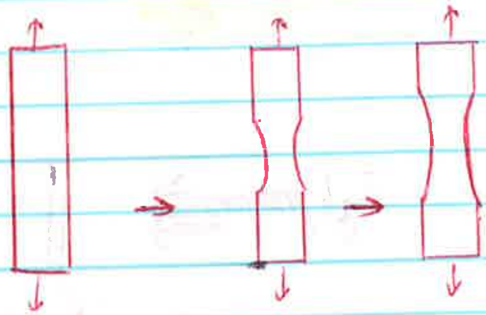


* Bridge girder:

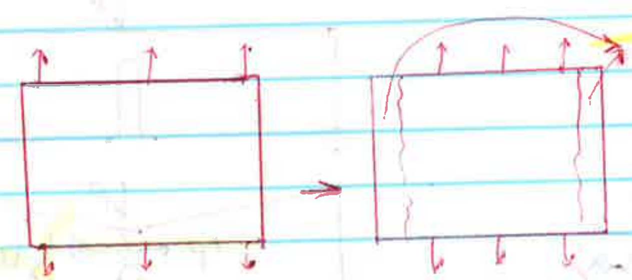
Crack growth areas



materials are more ductile when under shear loads.

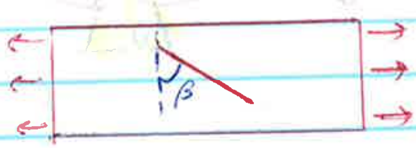


necking



shear stresses

when reach K_{Ic} crack growth vertically



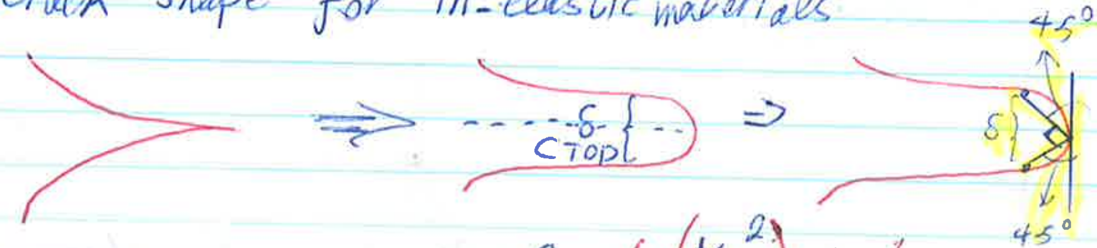
$$K_{IA} = K_{I0} \cos^2\beta$$

$$K_{IIA} = K_{I0} \cos\beta \sin\beta$$

when reach K_{IIc} crack growth horizontally

$\Rightarrow \sigma_{rr} \sigma_{\theta\theta} \tau_{r\theta}$

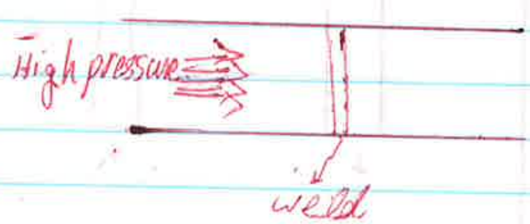
* Crack shape for in-elastic materials

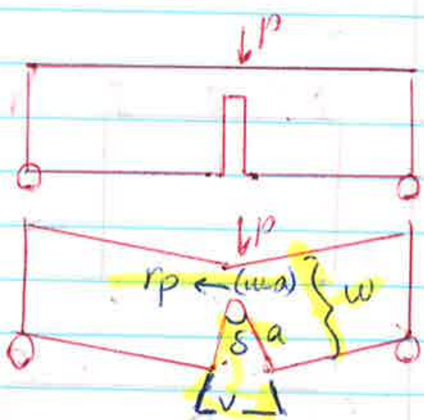


$$\delta = \frac{4}{\pi}\left(\frac{K_I}{\sigma_{ys}}\right)^2 \frac{1}{E}$$

$$\delta = \frac{4}{\pi}\frac{G}{\sigma_{ys}}$$

* in welding point of pipes, we have plane strain.





$$\frac{V}{r(w-a)+a} = \frac{\delta}{r(w-a)}$$

$$\delta = \frac{r(w-a)V}{r(w-a)+a}$$

$$\delta = \delta_w + \delta_p = \frac{K_I^2}{m\,\sigma_{ys}E} + \frac{r(w-a)V}{r(w-a)+a}$$ for plane strain situation

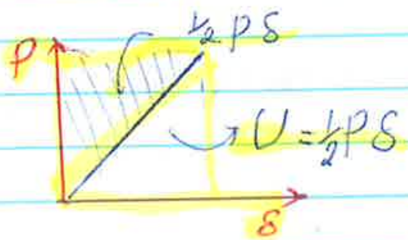
* J-Integral

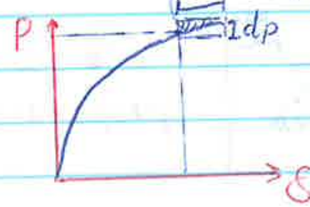
$$J = -\frac{d\Pi}{dA}$$

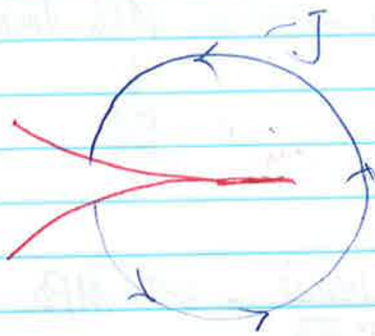
potential energy

Area

$\Pi = U - F \qquad \Pi = U - \Delta P = V^{*} \qquad d\Delta \Rightarrow (d\Delta - dp)\frac{1}{2}$







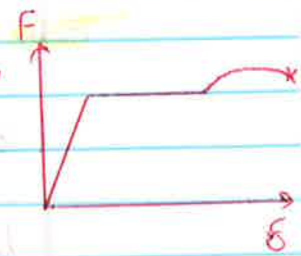
$$J = \int_{\Gamma} \left(w\,dy - T_i \frac{\partial u}{\partial x}\, ds \right)$$

w: strain energy density

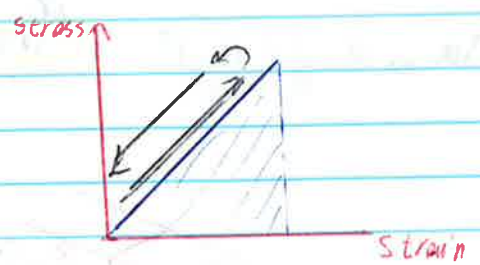
T_i: Transition on surface "S"

$\frac{\partial u}{\partial x}$: distribution vector component

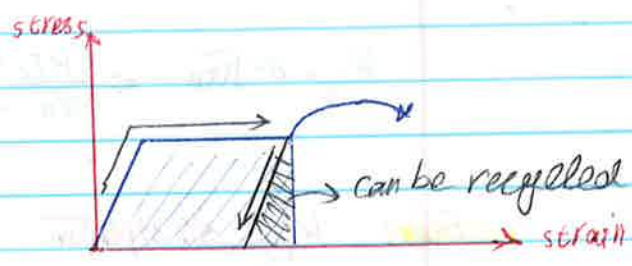
ductile steel under tention



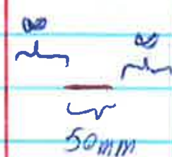
$\Pi = U - \underset{w = F.\delta}{F}$



elastic



in_elastic

Exp: 

Fracture @ 500Mpa

what load can it carry without crack propagate?

$$K_{Ic} = \sigma\sqrt{\pi a} = 500\sqrt{\pi(25)} = 4431\ MPa\sqrt{mm}$$

$\downarrow$ $\frac{KN}{m^2}$

$$\Rightarrow K_{Ic} = 140.12\ MPa\sqrt{m}\ \checkmark$$

$$K_{IC} = 140.12 = \sigma\sqrt{\pi(50)} \Rightarrow \sigma = \frac{140.12}{\pi\sqrt{50}} = 354\ MPa$$

growth? crack

Exp: Code: $\sigma_0 = \frac{f_y}{1.5}$ which material produce more

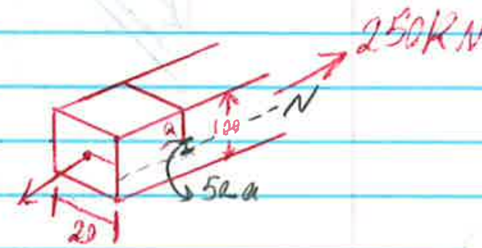
		σ_0	$a_{critical}$
a) $f_y = 1.2\,GPa$	$K_{Ic} = 70\ MPa\sqrt{m}$	$0.8GPa$	$a = \frac{1}{\pi}\left(\frac{70}{800}\right)^2 = 2.44mm$ crack size = 4.9mm
b) $f_y = 1.8\,GPa$	$K_{Ic} = 50\ MPa\sqrt{m}$	$1.2GPa$	$a = \frac{1}{\pi}\left(\frac{50}{1200}\right)^2 = 0.55mm$ crack size = 1.1 mm

both are rocket motorcasing

$$K_{Ic} = \sigma\sqrt{\pi a} \Rightarrow \left(\frac{K_{Ic}}{\sigma_0}\right)^2 = \pi a \Rightarrow a = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_0}\right)^2$$

$$a_{critical} = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_0}\right)^2$$

Exp: $K_{Ic} = 50\ MPa\sqrt{m}$



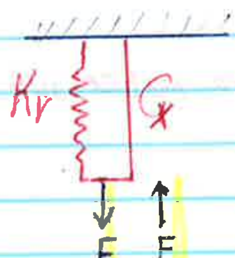
$$\sigma_A = \frac{250KN}{(100-a)(20)} = \frac{250KN}{2000-20a}$$

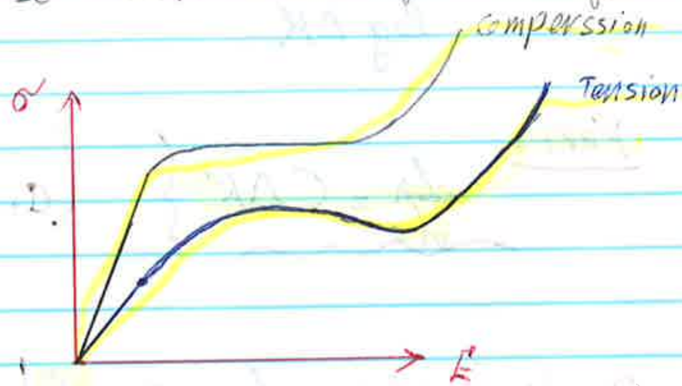
$$\sigma_F = \frac{Mc}{I} = \frac{250\,(10\,mm)\,(100-a)/2}{\left(\frac{20(100-a)^3}{12}\right)} = \frac{(2.5\,kN.m)\,(50-a/2)}{\frac{20(100-a)^3}{12}}$$

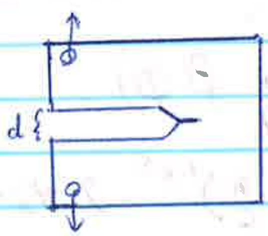
$$\Rightarrow K_{I_C} = \left\{\sigma\sqrt{\pi a}\right\} = \left(\frac{(2.5\,kN.m)\,(50-a/2)}{\frac{20(100-a)^3}{12}}\right)\sqrt{\pi a}$$

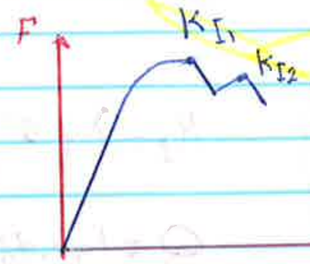
$\sigma \rightarrow \frac{250\,kN}{2000-20a}$

Try: $a = 14\,mm \Rightarrow K_{I_C} = 30.5$ ... = ... right or wrong



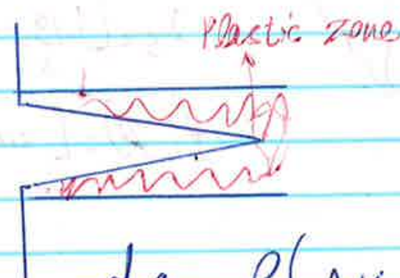


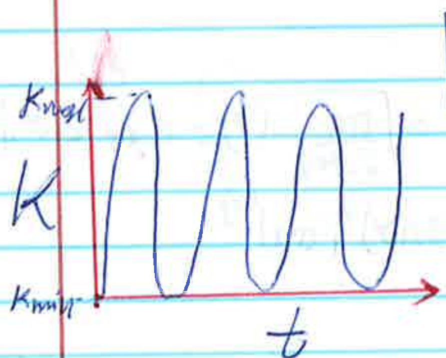




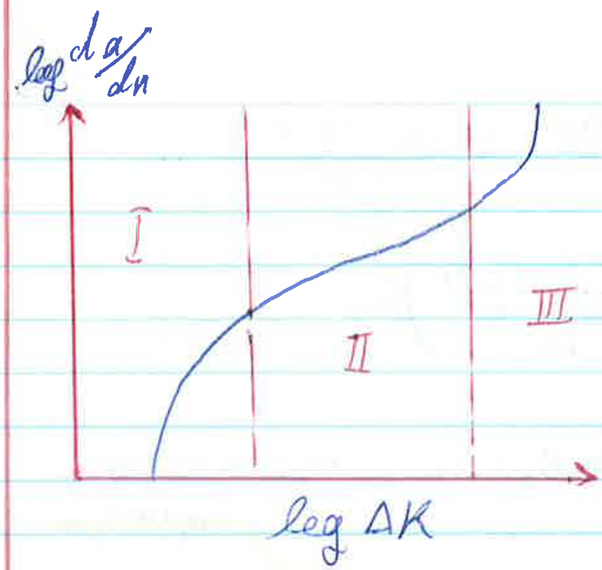
$K_{I_1} > K_{I_2}$

* fatigue:





$$\frac{da}{dN} = f(\Delta K, R) \quad \begin{cases} \Delta K = K_{max} - K_{min} \\ R = \frac{K_{min}}{K_{max}} \end{cases}$$



Forman

$$\text{II}: \frac{da}{dN} = \frac{C\Delta K^m}{(1-R)K_c - \Delta K}$$

Paris

$$\frac{da}{dN} = C\Delta K^m$$

(C, m are material constant)

Dowling, Begley $\dfrac{da}{dN} = C\Delta J^m$

Exp8

$$K_I = \lambda_s \sigma \sqrt{\frac{\pi a}{Q}} f(\phi)$$

$$Q = 1 + 1.464 \left(\frac{a}{c}\right)^{1.65} = 2.464$$

$$\lambda_s = \left[1.13 - 0.09\left(\frac{a}{c}\right)\right]\left(1 + 0.1(1 - \sin\phi)^2\right)$$

$$f(\phi) = \left[\sin^2(\phi) + \left(\frac{a}{c}\right)^2 \cos^2\phi\right]^{1/4}$$

$$K_{max} = 1.04\, \sigma \sqrt{\frac{\pi a}{2.464}}\, (1) = 0.663 \Delta\sigma \sqrt{\pi a} = \Delta K$$

$$\frac{da}{dN} = C(0.663\Delta\sigma)^m (\pi a)^{m/2}$$

$$\Rightarrow N = \frac{1}{C\left(0.663\sqrt{\pi\Delta\sigma}\right)^m}\int_{a_0}^{a_f} a^{-m/2}\,da$$

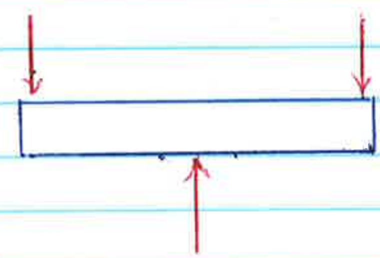
$$\Rightarrow N = \frac{a_0^{1-m/2} - a_f^{1-m/2}}{C\left(\frac{m}{2}-1\right)\left(0.663\sqrt{\pi\Delta\sigma}\right)^m}, \quad m \neq 2$$

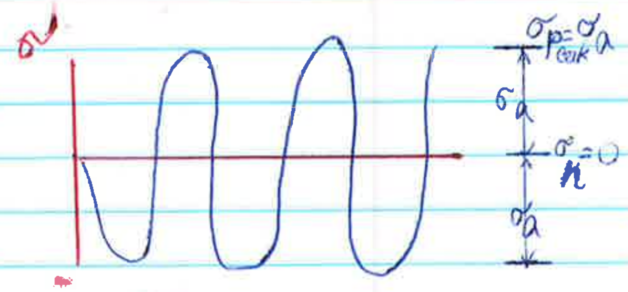
K_{max}

K_{min}

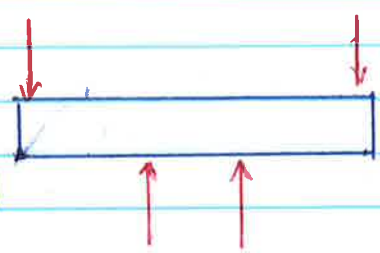
$$\frac{da}{dN} = C\Delta K^n$$

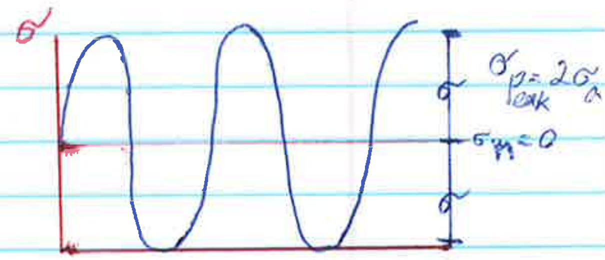
TPBT





FPBT
(Breaks first under fatigue)





$$\sigma_a = C + D\log(N)$$

Exp: A1015 Steel: $\sigma_y = 33\,Ksi$

$\sigma_u = 60\,Ksi$

$N = 10^6$

$C = 79.1$

$D = -10.1$

$$\sigma_a = 79.1 - 10.1\,\big(\underbrace{\log(10^6)}_{6}\big) = 18.5\,Ksi$$

Low cycle fatigue

High Cycle fatigue