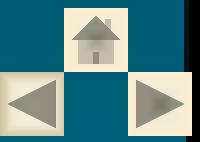
In the name of God

Director of the course

Mohammad Javad Ashrafi

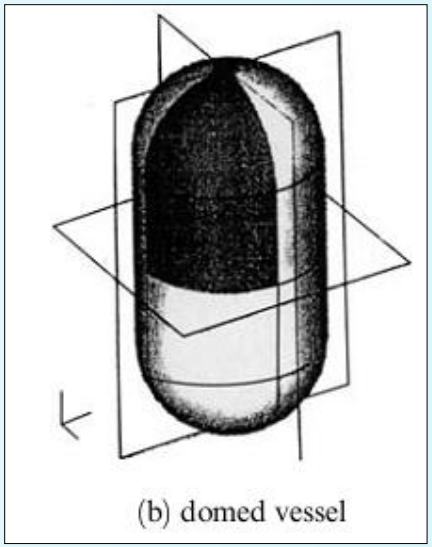


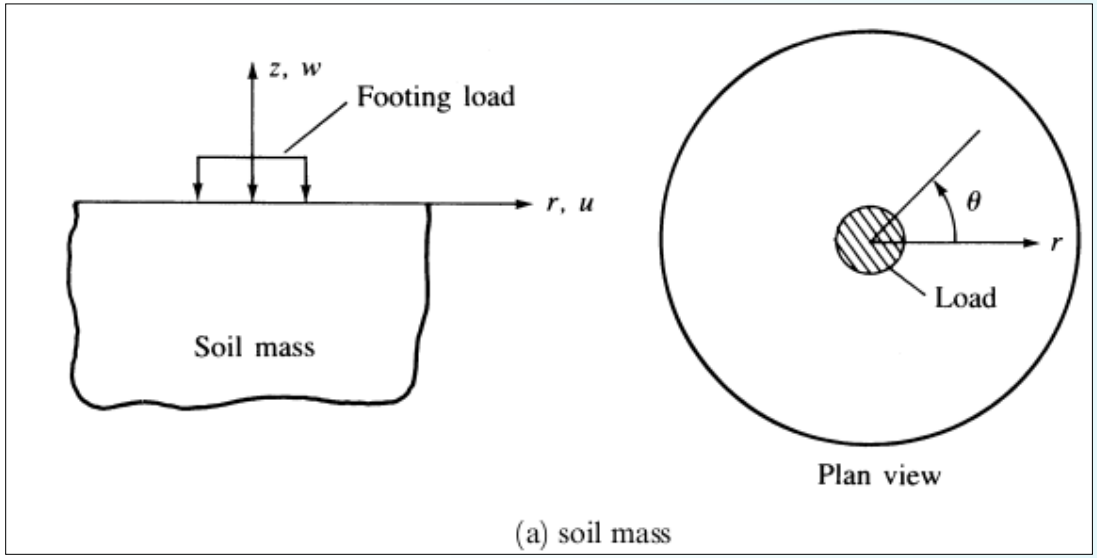
Session Title

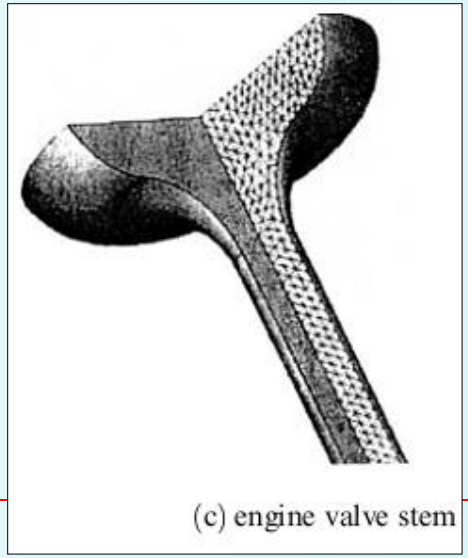
University of Golpayegan
Mechanical Engineering

2-D Elements: Axisymmetric Elements

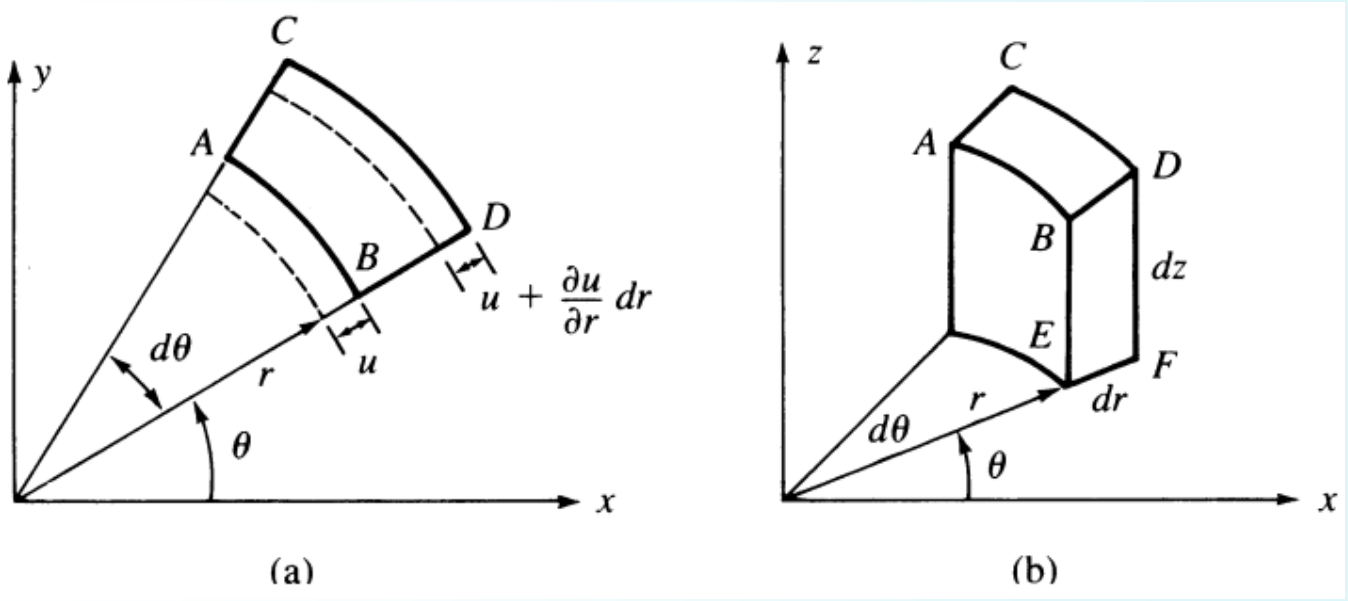
- Assumption: symmetry with respect to geometry and loading about an axis of the body







- the stresses are independent of the θ coordinate



$$\varepsilon_r = \frac{\partial u}{\partial r} \qquad \varepsilon_\theta = \frac{u}{r} \qquad \varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}$$

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{Bmatrix} = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & v & 0 \\ v & 1-v & v & 0 \\ v & v & 1-v & 0 \\ 0 & 0 & 0 & \frac{1-2v}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix}$$

$$N_i = \frac{1}{2A}(\alpha_i + \beta_i r + \gamma_i z)$$

$$N_j = \frac{1}{2A}(\alpha_j + \beta_j r + \gamma_j z)$$

$$N_m = \frac{1}{2A}(\alpha_m + \beta_m r + \gamma_m z)$$

$$\{\psi\} = \begin{Bmatrix} u(r,z) \\ w(r,z) \end{Bmatrix} = \begin{bmatrix} N_i & 0 & N_j & 0 & N_m & 0 \\ 0 & N_i & 0 & N_j & 0 & N_m \end{bmatrix} \begin{Bmatrix} u_i \\ w_i \\ u_j \\ w_j \\ u_m \\ w_m \end{Bmatrix}$$

$$\{\psi\} = [N]\{d\}$$

$$\{\varepsilon\} = [\underline{B}_i \quad \underline{B}_j \quad \underline{B}_m] \begin{Bmatrix} u_i \\ w_i \\ u_j \\ w_j \\ u_m \\ w_m \end{Bmatrix}$$

$$[B_i] = \frac{1}{2A} \begin{bmatrix} \beta_i & 0 \\ 0 & \gamma_i \\ \frac{\alpha_i}{r} + \beta_i + \frac{\gamma_i z}{r} & 0 \\ \gamma_i & \beta_i \end{bmatrix}$$

$$[k] = \iiint_V [B]^T [D][B]\, dV$$

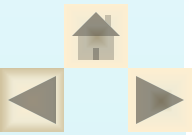
$$[k] = 2\pi \iint_A [B]^T [D][B] r\, dr\, dz$$

- Evaluate [B] for a centroidal point (r,z) of the element

$$r = \bar{r} = \frac{r_i + r_j + r_m}{3} \qquad z = \bar{z} = \frac{z_i + z_j + z_m}{3}$$

$$[B(\bar{r}, \bar{z})] = [\bar{B}]$$

$$[k] = 2\pi \bar{r} A [\bar{B}]^T [D][\bar{B}]$$



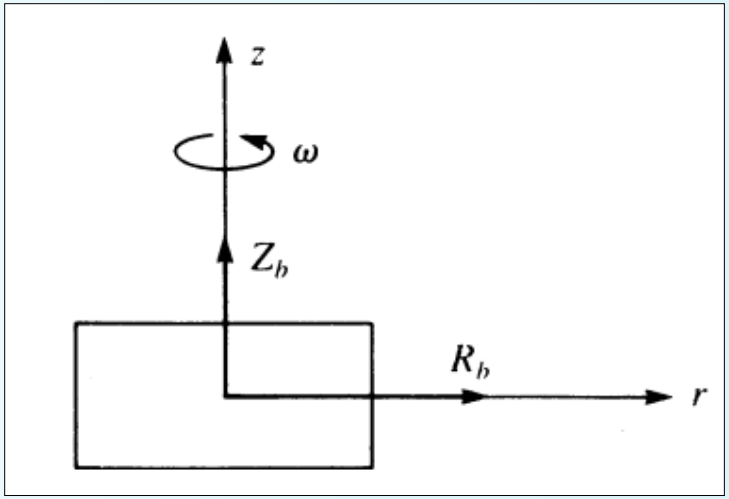
Body Forces

- Gravity (in the direction of the *z* axis)
- centrifugal forces in rotating machine parts (in the direction of the *r* axis)

$$\{f_b\} = 2\pi \iint_A [N]^T \begin{Bmatrix} R_b \\ Z_b \end{Bmatrix} r\, dr\, dz$$

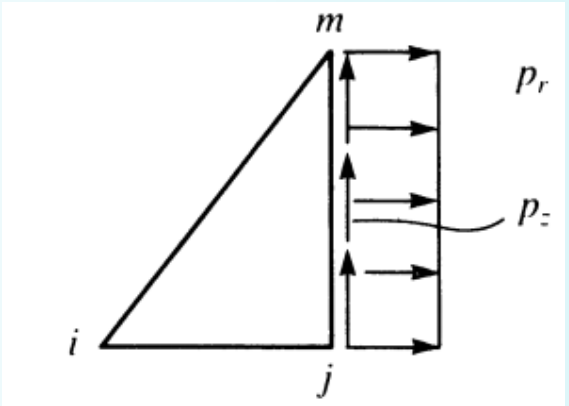
$$\{f_b\} = \frac{2\pi \bar{r} A}{3} \begin{Bmatrix} \bar{R}_b \\ Z_b \\ \bar{R}_b \\ Z_b \\ \bar{R}_b \\ Z_b \end{Bmatrix}$$

$$\bar{R}_b = \omega^2 \rho \bar{r}$$



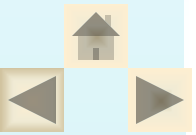
Surface Forces

$$\{f_s\} = \iint_S [N_s]^T \begin{Bmatrix} p_r \\ p_z \end{Bmatrix} dS$$



$$\{f_s\} = \frac{2\pi r_j (z_m - z_j)}{2} \begin{Bmatrix} 0 \\ 0 \\ p_r \\ p_z \\ p_r \\ p_z \end{Bmatrix}$$

Example 9.1: p.421



Example 9.2: p.422