In the name of God

Director of the course

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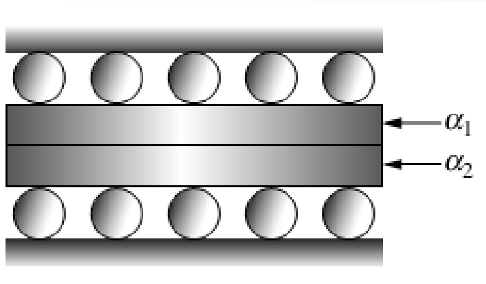
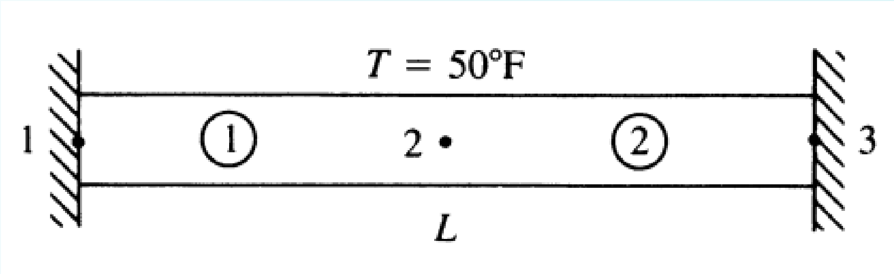
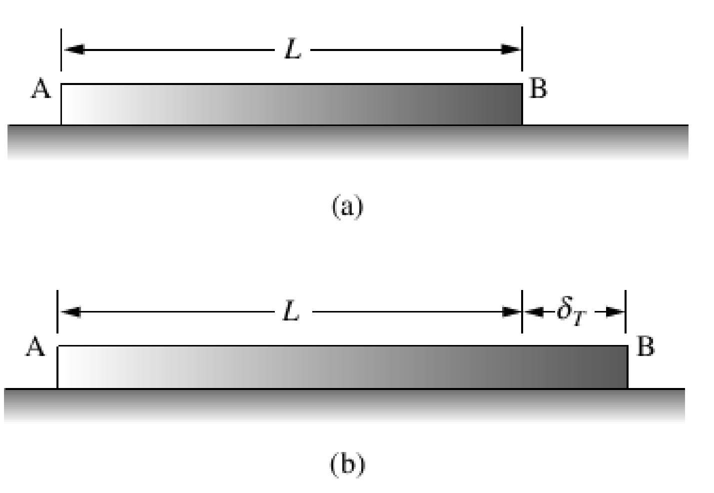
Session Title

Thermal Stress







$$\delta_T = \alpha T L$$

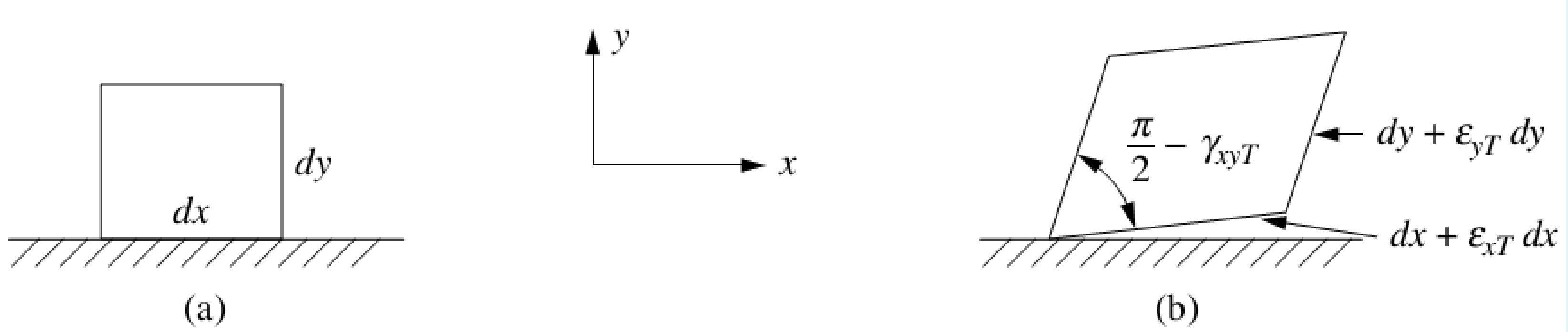
$$\varepsilon_T = \alpha T$$

- For 1-D problem:

$$\varepsilon_T = \alpha T$$

- For 2-D problem:
  - Plane stress
  - Plane strain
  - Axisymmetric

$$\{\varepsilon_T\} = \begin{Bmatrix} \varepsilon_{xT} \\ \varepsilon_{yT} \\ \gamma_{xyT} \end{Bmatrix}$$



- Isotropic:
  - Plane stress:

$$\{\varepsilon_T\} = \begin{Bmatrix} \alpha T \\ \alpha T \\ 0 \end{Bmatrix}$$

  - Plane strain:

$$\{\varepsilon_T\} = (1+\nu)\begin{Bmatrix} \alpha T \\ \alpha T \\ 0 \end{Bmatrix}$$

  - Axisymmetric:

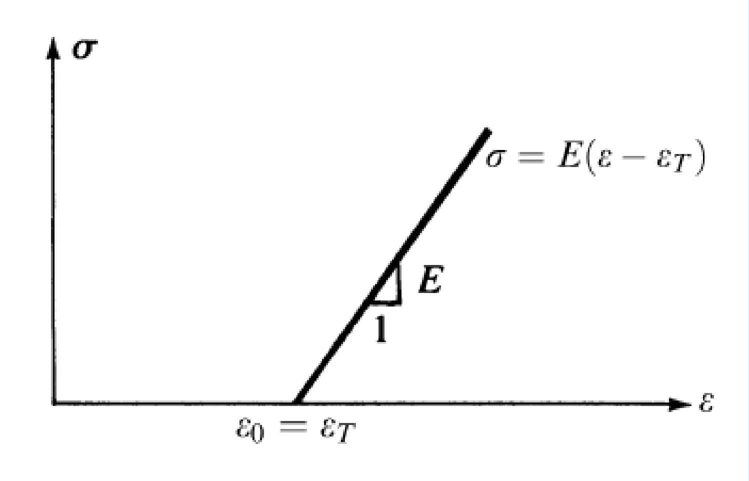
$$\{\varepsilon_T\} = \begin{Bmatrix} \varepsilon_{\tau T} \\ \varepsilon_{zT} \\ \varepsilon_{\theta T} \\ \gamma_{rzT} \end{Bmatrix} = \begin{Bmatrix} \alpha T \\ \alpha T \\ \alpha T \\ 0 \end{Bmatrix}$$

- Temperature change:
  - Strain without stress
  - Stress without strain

$$\varepsilon_x = \frac{\sigma_x}{E} + \varepsilon_T$$

$$\underline{\varepsilon} = [D]^{-1}\underline{\sigma} + \underline{\varepsilon}_T$$

$$\boxed{\underline{\sigma} = \underline{D}(\underline{\varepsilon} - \underline{\varepsilon}_T)}$$



- Strain energy density:

$$\boxed{u_0 = \tfrac{1}{2}\underline{\sigma}(\underline{\varepsilon} - \underline{\varepsilon}_T)} \Rightarrow \boxed{u_0 = \tfrac{1}{2}(\underline{\varepsilon} - \underline{\varepsilon}_T)^T \underline{D}(\underline{\varepsilon} - \underline{\varepsilon}_T)}$$

$$U = \int_V u_0 \, dV$$

$$U = \int_V \frac{1}{2} (\underline{\varepsilon} - \underline{\varepsilon}_T)^T \underline{D} (\underline{\varepsilon} - \underline{\varepsilon}_T) \, dV$$

$$U = \frac{1}{2} \int_V (\underline{B}\underline{d} - \underline{\varepsilon}_T)^T \underline{D} (\underline{B}\underline{d} - \underline{\varepsilon}_T) \, dV$$

$$U = \frac{1}{2} \int_V (\underline{d}^T \underline{B}^T \underline{D}\underline{B}\underline{d} - \underline{d}^T \underline{B}^T \underline{D}\underline{\varepsilon}_T - \underline{\varepsilon}_T^T \underline{D}\underline{B}\underline{d} + \underline{\varepsilon}_T^T \underline{D}\underline{\varepsilon}_T) \, dV$$

$$U_L = \frac{1}{2} \int_V \underline{d}^T \underline{B}^T \underline{D}\underline{B}\underline{d} \, dV$$

$$U = U_L + U_T$$

$$U_T = \int_V \underline{d}^T \underline{B}^T \underline{D}\underline{\varepsilon}_T \, dV$$

$$\frac{\partial U}{\partial \underline{d}} = 0$$

$$\frac{\partial U_L}{\partial \underline{d}} = \int_V \underline{B}^T \underline{D}\,\underline{B}\, dV\, \underline{d}$$

$$\frac{\partial U_T}{\partial \underline{d}} = \int_V \underline{B}^T \underline{D} \underline{\varepsilon}_T\, dV = \{f_T\}$$

- For 1-D bar element

$$\{\varepsilon_T\} = \{\varepsilon_{xT}\} = \{\alpha T\}$$

$$[D] = [E] \qquad [B] = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix}$$

$$\{f_T\} = A\int_0^L [B]^T [D]\{\alpha T\}\, dx$$

$$\{f_T\} = \begin{Bmatrix} f_{T1} \\ f_{T2} \end{Bmatrix} = \begin{Bmatrix} -E\alpha TA \\ E\alpha TA \end{Bmatrix}$$

- For 2-D CST element:

$$\{f_T\} = [B]^T[D]\{\varepsilon_T\}tA$$

Plane stress $\Rightarrow$

$$\{f_T\} = \begin{Bmatrix} f_{Tix} \\ f_{Tiy} \\ \vdots \\ f_{Tmy} \end{Bmatrix} = \frac{\alpha E t T}{2(1-\nu)} \begin{Bmatrix} \beta_i \\ \gamma_i \\ \beta_j \\ \gamma_j \\ \beta_m \\ \gamma_m \end{Bmatrix}$$

Example