CIVIL ENGINEERING SYSTEMS ANALYSIS

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Today's Lecture

1. Tree Diagrams
2. Decision Criteria
3. Decision Trees

How to make a "good" decision?

- Sometime, in the course of a project, an engineer or team leader needs to select a path between two (or more than two) choices.
- Is there a **quantitative** methodology to rationally make that decision to **reduce the risk**?
- The purpose of **decision analysis** is to assist **decision makers** in making **better** decisions in complex situations, usually under uncertainty.
- Decision analysis is modeling procedure based on the techniques of "Statistics" and "Operation Research" to find quantitative insight into the consequence(s) of each option in decision making.

Make Decision Under Uncertainty

Newsweek recalls 125,000 Madam

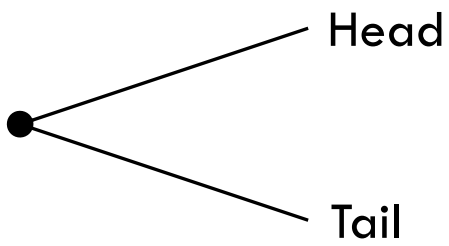
Hillary Clinton signs a copy of Newsweek's 'Madam President' commemorative magazine on November 7 CREDIT: JUSTIN SULLIVAN/GETTY IMAGES

Revisit some concepts of probability

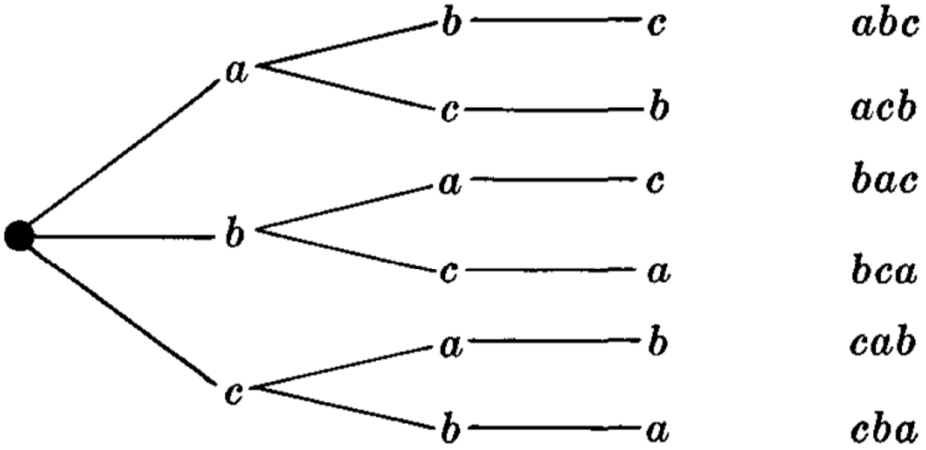
- **Expected value:** Suppose random variable X can take value x_1 with probability P_1, value x_2 with probability P_2, and so on, up to value x_k with probability P_k. Then the expectation of this random variable is defined as:
  - $E[X] = P_1 x_1 + P_2 x_2 + \cdots + P_k x_x$
- **Bayes' rule**
  - $Pr[A|B] = \frac{Pr[B|A]\,Pr[A]}{Pr[B]}$ *(Probability of event A given that event B happened)*
  - Bayes' theorem is used to update the probability for a hypothesis as more evidence or information becomes available. In simple words you reduce the number of possible events based on given information.

Tree diagrams

- A tree diagram is a device to calculate all the logical possibilities of a sequence of events when each event can occur in a finite number of ways.
- Tree diagram for toss of a coin:



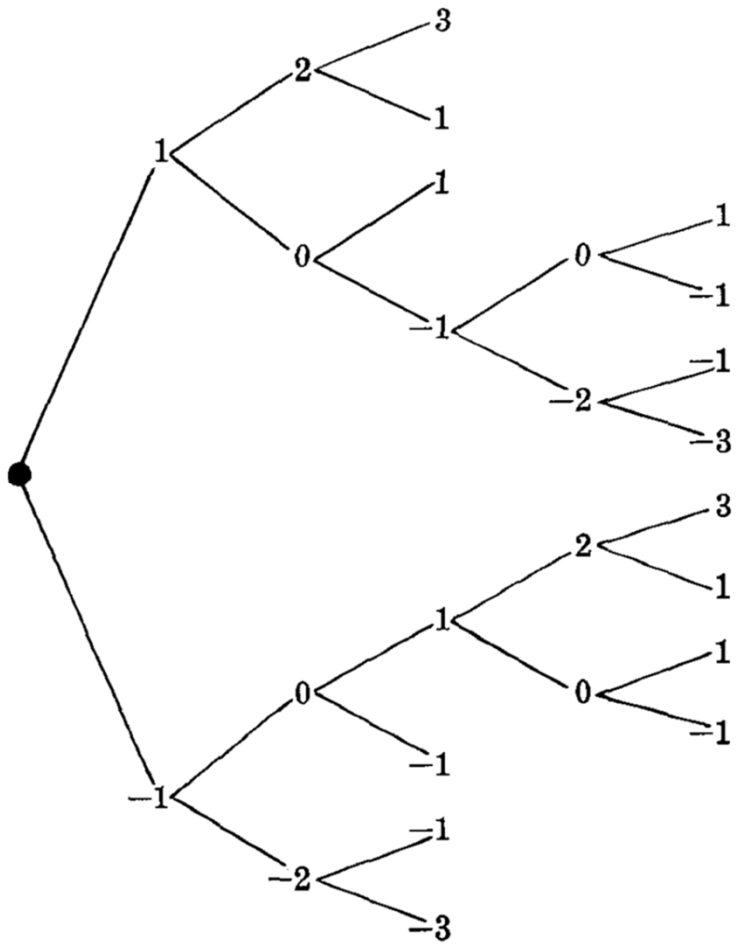
- All permutations of {a, b , c}



The six permutations are listed on the right of the diagram.

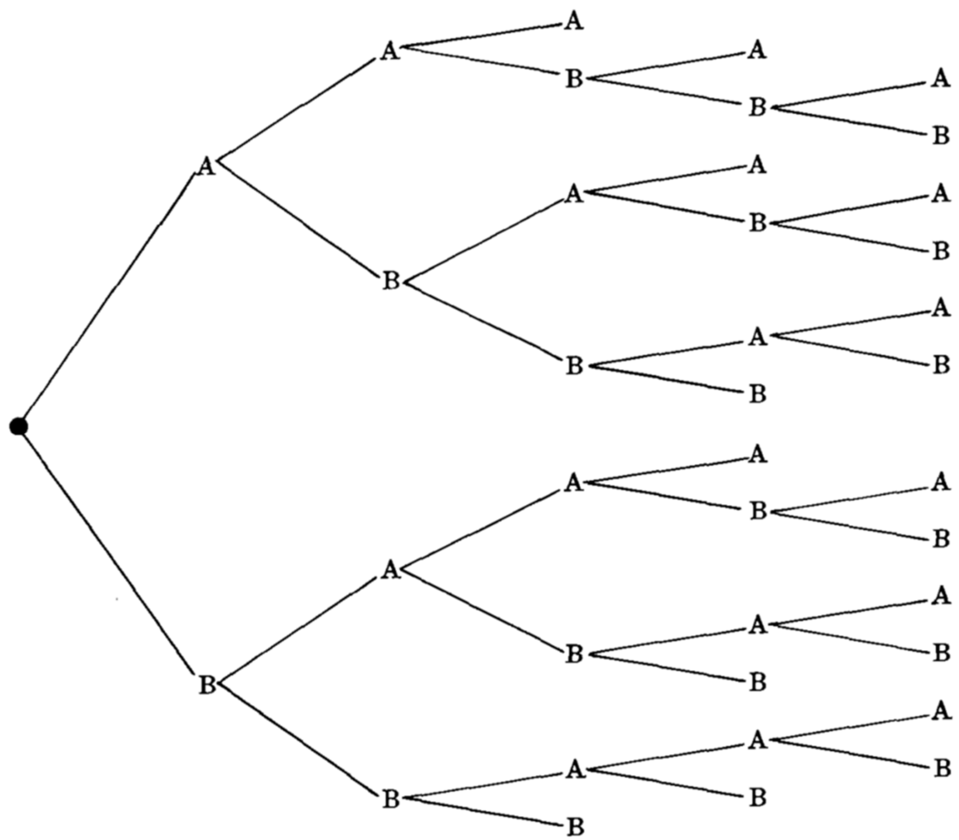
Example 1

- A man is at the origin on the x-axis and takes a one unit step either to the left or to the right. He stops if he reaches 3 or -3, or if he occupies any position – other than origin – more than once. Find the number of different paths he can travel.
  - There are 14 different paths, each pass associated with an end point of the tree branches



Example 2

- Teams A and B play in a tournament. The team that first wins 3 games wins the tournament. Find all number of possible ways which may occur.
  - Symmetry



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Decision Analysis

Decision making under uncertainty

- These are the kinds of decision making in the face of great uncertainty that decision analysis is designed to address. Decision analysis provides a framework and methodology for rational decision making when the outcomes are uncertain.
  - An oil company deciding whether to drill for oil in a particular location. How likely is oil there? How much? How deep will they need to drill?
- Should we make the decision immediately or first do some *testing* (at some expense) to reduce the level of uncertainty? decision analysis divides.
- Decision making between the cases of without experimentation and with experimentation. Example?

Decision making

- In general terms, the decision maker must choose an **action** from a set of possible actions. The set contains all the *feasible alternatives* under consideration for how to proceed with the problem of concern.
- These random factors determine what situation will be found at the time that the action is executed. Each of these possible situations is referred to as a possible **state of nature**.
- For each combination of an action and a state of nature, the decision maker knows what the resulting payoff would be. The **payoff** is a quantitative measure of the value to the decision maker of the consequences of the outcome.

Example 3

- A company owns a land that may contain oil. A geologist has reported that there is 1 chance in 4 of oil. Because of this prospect, someone has offered to purchase the land for $90,000. However, the company is considering holding the land in order to drill for oil itself. The cost of drilling is $100,000. If oil is found, the revenue will be $800,000.

TABLE 15.1 Prospective profits for the Goferbroke Company

Alternative \ Status of Land	Payoff	
	Oil	**Dry**
Drill for oil	$700,000	−$100,000
Sell the land	$ 90,000	$ 90,000
Chance of status	1 in 4	3 in 4

Procedure

1. The *decision maker* needs to choose one of the *alternative actions*.

2. *Nature* then would choose one of the possible *states of nature*.

3. Each combination of an **action** and **state of nature** would result in a ***payoff***, which is given as one of the entries in a *payoff table*.

4. This payoff table should be used to find an ***optimal action*** for the decision maker according to an appropriate criterion.

- One additional element needs to be added to the decision analysis framework. The decision maker generally will have some information that should be taken into account about the relative likelihood of the possible states of nature. Such information can usually be translated to a probability.

Decision making criterion

- **Maximin payoff criterion:** For each possible action, find the *minimum payoff* over all possible states of nature. Next, find the *maximum* of these minimum payoffs. Choose the action whose minimum payoff gives this maximum.
- **Maximum likelihood criterion:** Identify the most likely state of nature (the one with largest probability). For this state of nature, find the action with the maximum payoff. Choose this action.
- **Bayes' decision rule:** Using the best available estimates of the probabilities of the respective states of nature (currently the prior probabilities), calculate the expected value of the payoff for each of the possible actions. Choose the action with the maximum expected payoff.

Example 3-1: Maximin payoff criterion

- Maximin payoff criterion: For each possible action, find the *minimum payoff* over all possible states of nature. Next, find the *maximum* of these minimum payoffs. Choose the action whose minimum payoff gives this maximum.

	A	B	C	D	E	F	G	H	I
1	Maximin Payoff Criterion for the Goferbroke Co. Problem								
2									
3				State of Nature				Minimum	
4		Alternative	Oil	Dry				in Row	
5		Drill	700	-100				-100	
6		Sell	90	90				90	Maximin
7									
8									
9									

Note: this criterion is not often used in decision making against nature because it is an extremely conservative criterion. this criterion normally is of interest only to a very cautious decision maker!

Example 3-2: Maximum likelihood criterion

Maximum likelihood criterion: Identify the most likely state of nature (the one with the largest prior probability). For this state of nature, find the action with the maximum payoff. Choose this action.

	A	B	C	D	E	F	G	H
1	Maximum Likelihood Criterion for the Goferbroke Co. Problem							
2								
3				State of Nature				
4		Alternative	Oil	Dry				
5		Drill	700	-100				
6		Sell	90	90				Maximum
7								
8								
9								
10		Prior Probability	0.25	0.75				
11			Maximum					

The drawback of the criterion is that it ignores much relevant information. No state of nature is considered other than the most likely one. In a problem with many possible states of nature, the probability of the most likely one may be quite small, so focusing on just this one state of nature is quite unwarranted.

Example 3-3: Bayes' decision rule

Bayes' decision rule: Using the best available estimates of the probabilities of the respective states of nature (currently the prior probabilities), calculate the expected value of the payoff for each of the possible actions. Choose the action with the maximum expected payoff.

	A	B	C	D	E	F	G	H	I
1	Bayes' Decision Rule for the Goferbroke Co. Problem								
2									
3			State of Nature					Expected	
4		Alternative	Oil	Dry				Payoff	
5		Drill	700	-100				100	Maximum
6		Sell	90	90				90	
7									
8									
9									
10		Prior Probability	0.25	0.75					

- Advantage of Bayes' decision rule is that it incorporates all the available information!
- Disadvantage of Bayes' decision rule: sometimes the estimates of the probabilities necessarily are largely subjective and shaky to be trusted!

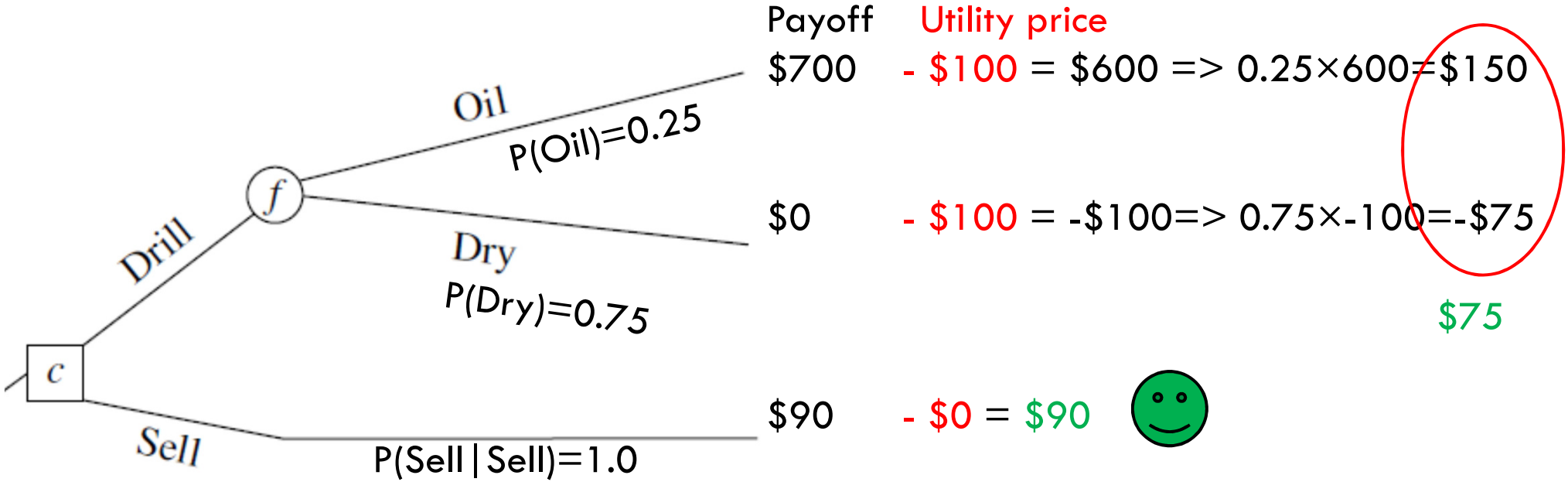
Decision Tree

Decision Tree

- Decision trees provide a way of *visually displaying* the problem and then *organizing the computational work of decision making*.
- The nodes of the decision tree are referred to as **forks,** and the arcs are called **branches.** A **decision fork,** represented by a square, indicates that a decision needs to be made at that point in the process. A **chance fork,** represented by a circle, indicates that a random event occurs at that point.

Example 3: Cont.

- Which action (drill for oil or sell the land) should be chosen in the example 1?
  - **Decision fork,** represented by a square (Sell or not sell?)
  - **Chance fork,** represented by a circle (Is there an oil reservoir?)



Example 4: (Textbook, Hillier and Lieberman)

- You are given the following payoff table (in units of thousands of dollars) for a decision analysis problem, **(a)** Which alternative should be chosen under the maximin payoff criterion? **(b)** Which alternative should be chosen under the maximum likelihood criterion? **(c)** Which alternative should be chosen under Bayes' decision rule? **(d)** Using Bayes' decision rule analysis graphically with respect to the prior probabilities of states S1 and S2 (without changing the prior probability of state S3) to determine the decision

	State of Nature		
Alternative	S_1	S_2	S_3
A_1	220	170	110
A_2	200	180	150
Prior probability	0.6	0.3	0.1

Example 4: Cont.

- a) A_2 must be chosen

	State of Nature			
Alternative	S_1	S_2	S_3	Min
A_1	220	170	110	110
A_2	200	180	150	150
Prior Probability	0.6	0.3	0.1	

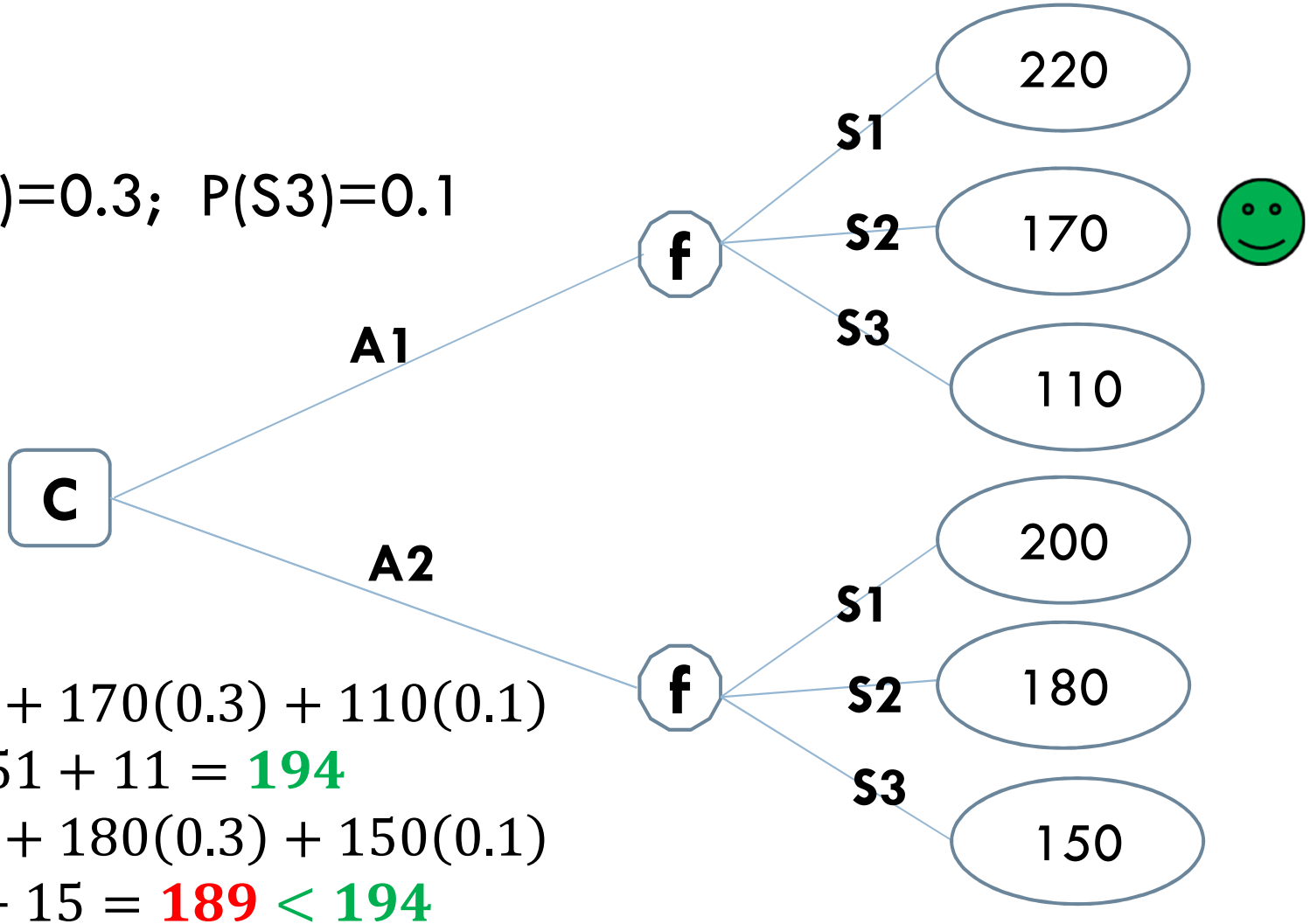
- b) The most likely state of nature is S_1 and the alternative with highest profit in this state is A_1.
- c) A_1 must be chosen

	State of Nature			Exp.
Alternative	S_1	S_2	S_3	Payoff
A_1	220	170	110	194
A_2	200	180	150	189
Prior Probability	0.6	0.3	0.1	

Example 4: Cont.

Part d)

P(S1)=0.6; P(S2)=0.3; P(S3)=0.1



$$E[A1] = 220(0.6) + 170(0.3) + 110(0.1)$$
$$= 132 + 51 + 11 = \mathbf{194}$$
$$E[A2] = 200(0.6) + 180(0.3) + 150(0.1)$$
$$= 120 + 54 + 15 = \mathbf{189} < \mathbf{194}$$