به‌نام خدا

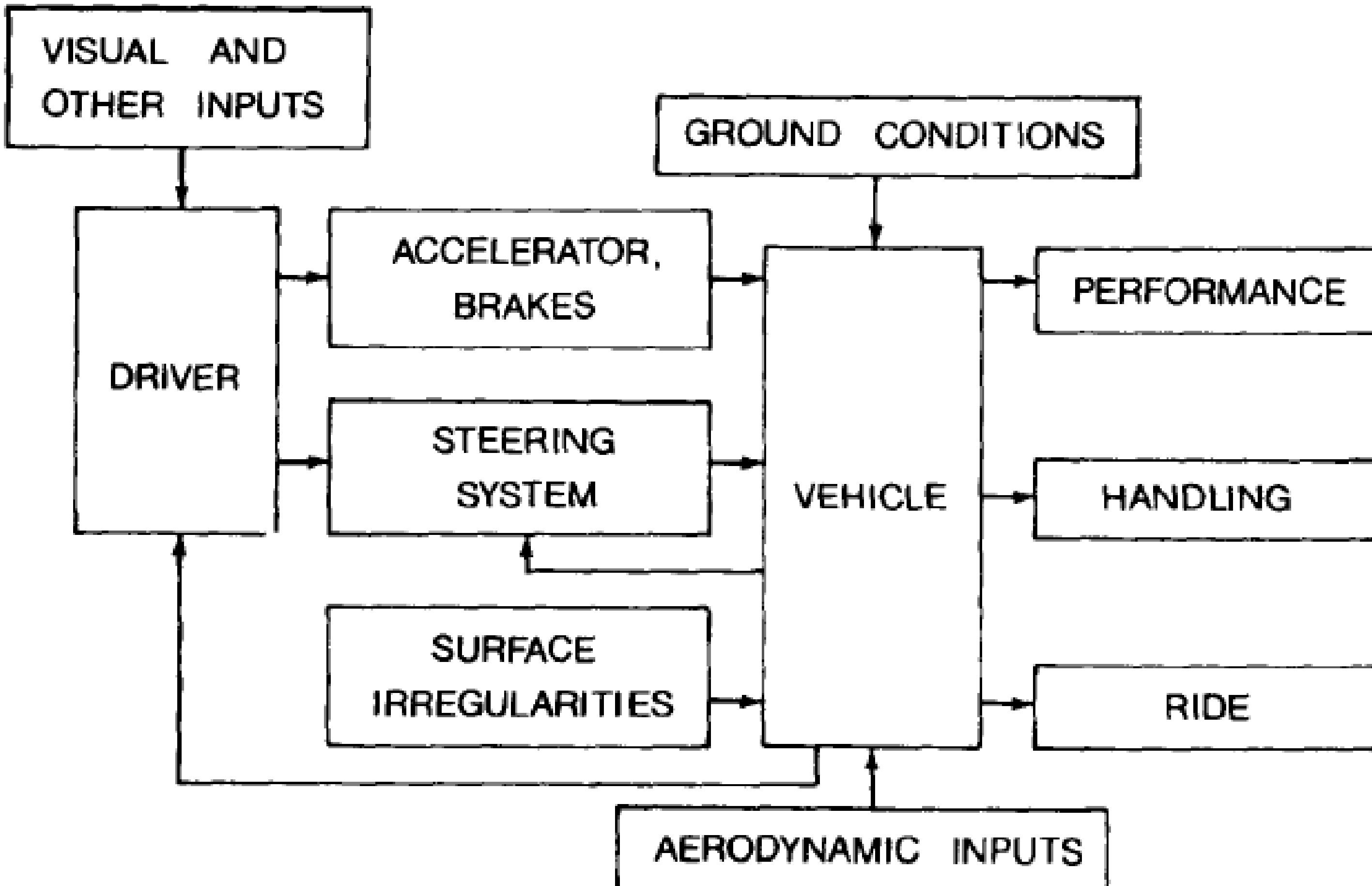
سیستم‌های شاسی و بدنه خودرو

چرخ و تایر

دوره کارشناسی ارشد مهندسی خودرو
دانشگاه علم و صنعت ایران

پاییز ۹۵

Wheels and Tires



Wheels and Tires

Road vehicle wheels include three element

- Rim
- Tire
- Pressurized air

When a tire is installed on a rim and is inflated, it is called a wheel.

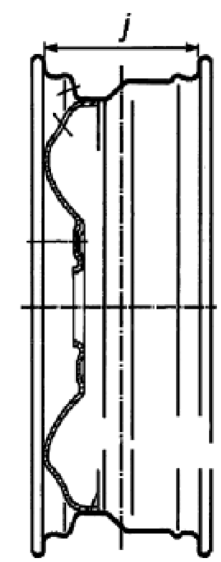
Vehicle wheels have two functions:

- *Support the weight of the vehicle*
- *Exchange longitudinal and side forces with the road surface,* to move the vehicle and control its path

Rim Characteristics

- A rim has two main parts: flange and spider. The flange (hub) is the ring or shell on which the tire is mounted. The spider (center section) is the disc section that is attached to the hub.
- Rim shape and dimensions are standardized to be exchangeable with those of other manufacturers.
- Rim size may be written as:

$$5\frac{1}{2}\,J \times 15$$

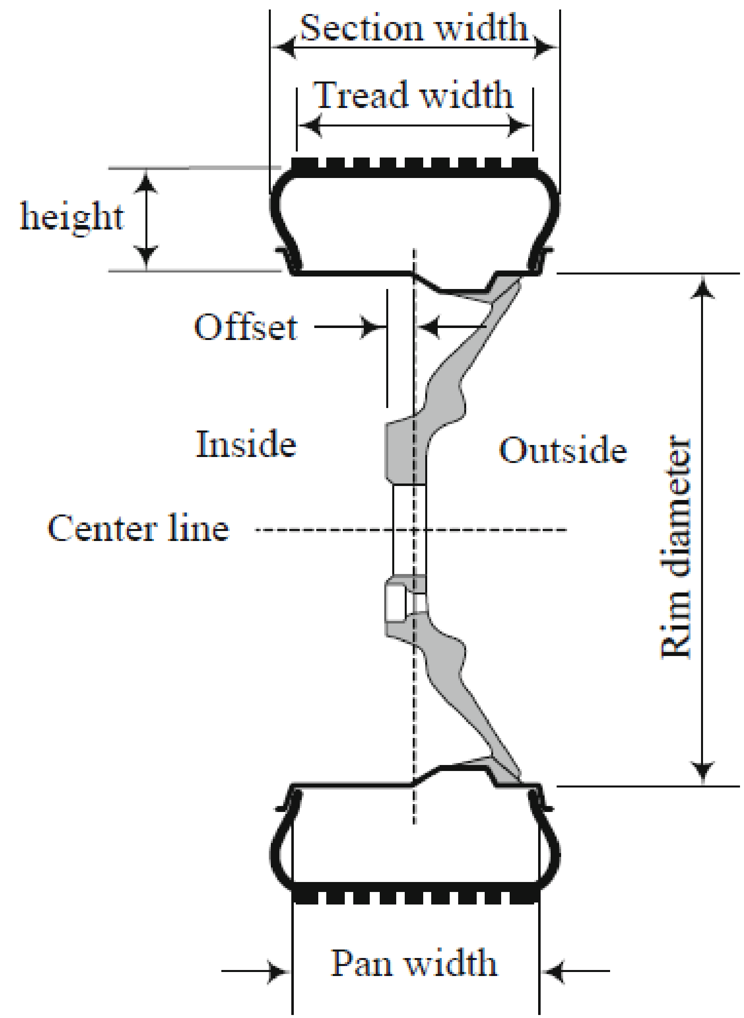


Rim Characteristics

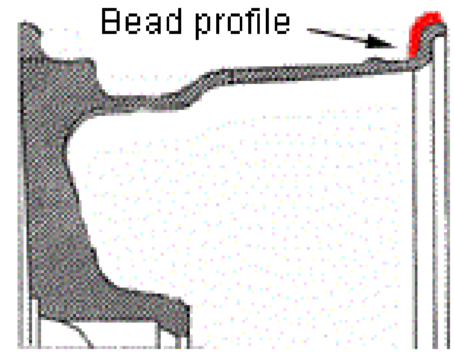
- Another sample of rim numbering and its meaning is shown below:

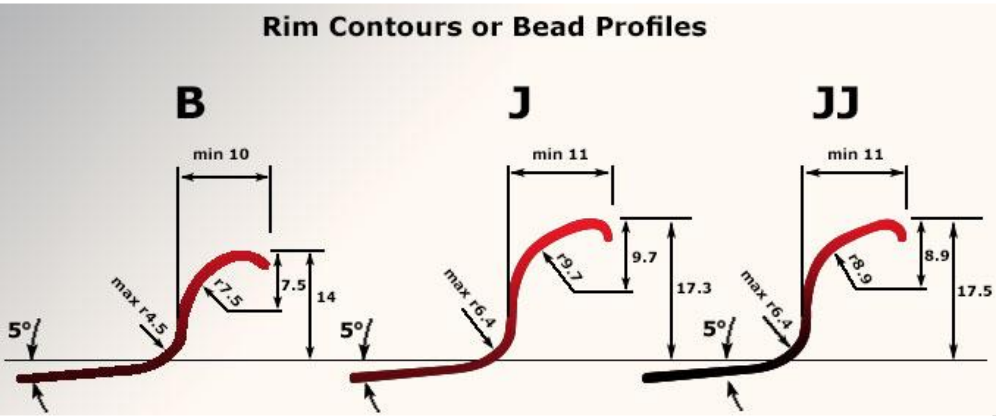
7 ½ – JJ × 15 55 5 – 114.3

7 1/2	*Rim width* [in]
JJ	*Flange shape code*
15	*Rim diameter* [in]
55	*Offset* [mm]
5	*Number of bolts*
114.3	*Pitch circle diameter*

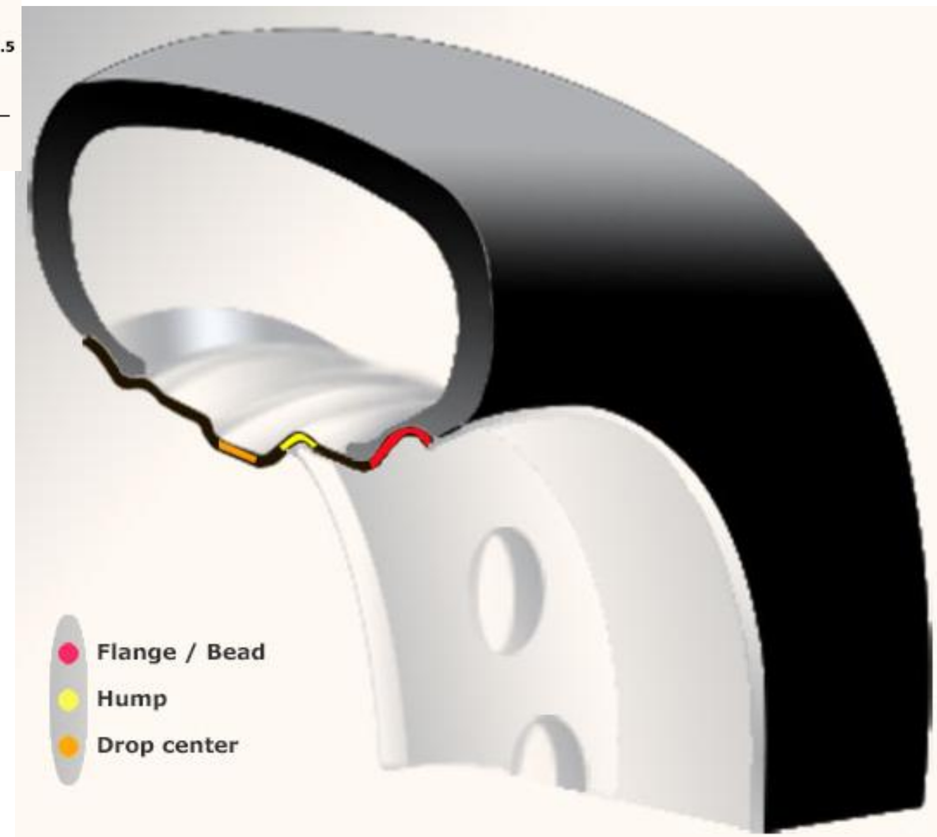


Rim Characteristics



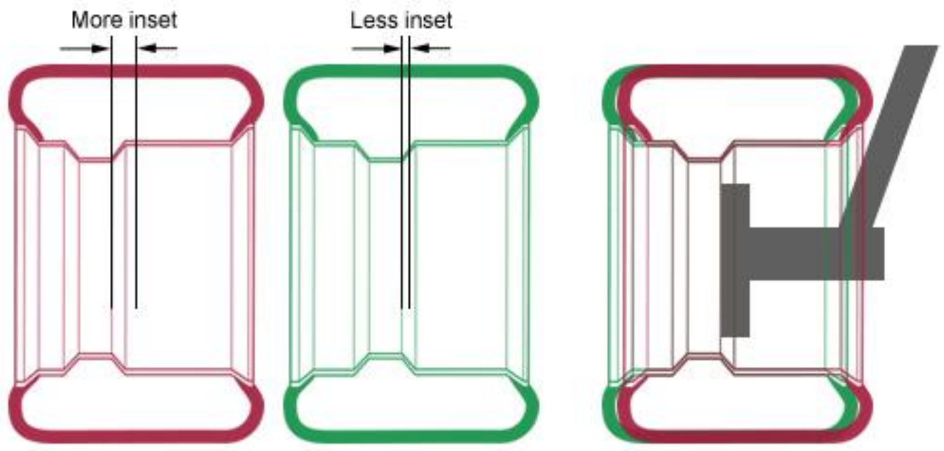


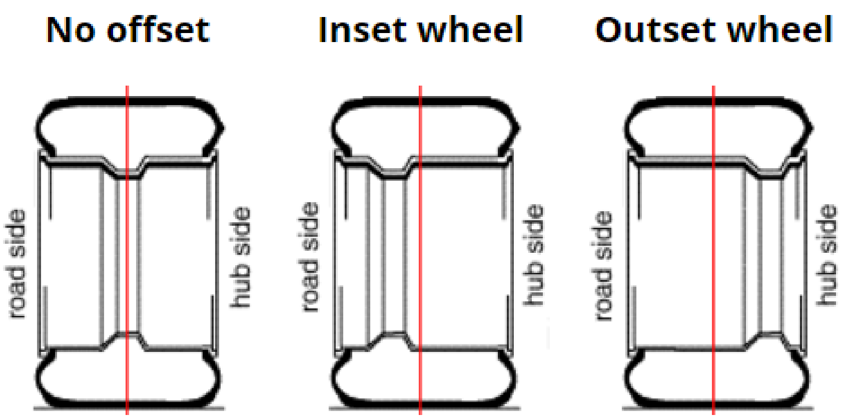
- The flange shape (bead profile) code signifies the tire-side profile of the rim and can be B, C, D, E, F, G, J, JJ, and K.
- J is the most common shape of a rim flange on passenger cars.
- B was used for smaller wheel sizes primarily for older car models
- JJ designation is most common on 4x4 and SUV vehicles.



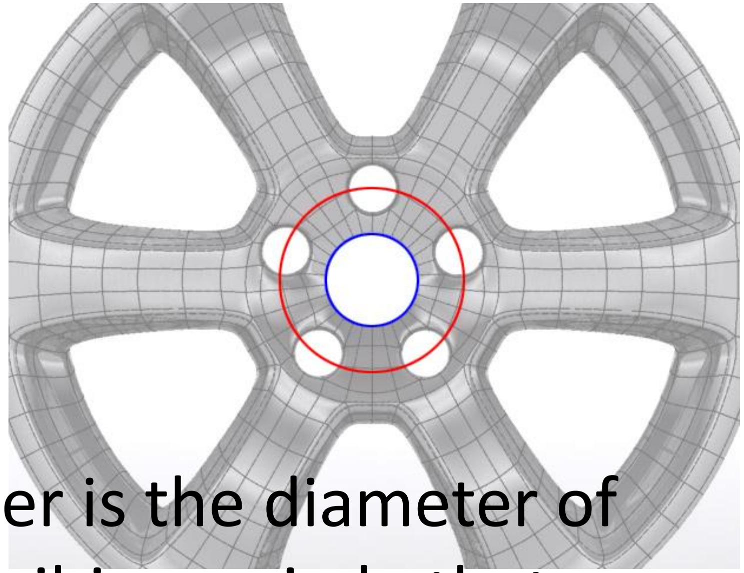
Rim Characteristics

- Offset is the distance between the inner plane and the center plane of the rim. A rim may be designed with a negative, zero, or positive offset.
- A rim has a positive offset if the spider is outward from the center plane (inset wheel), the wheel is tucked into the car.
- A rim has a negative offset if the spider is inward from the center plane toward the hub (outset wheel), the wheel sticks out.
- Increasing the inset of a wheel, decreases the clearance between the inner edge of the wheel and the suspension components!





Rim Characteristics

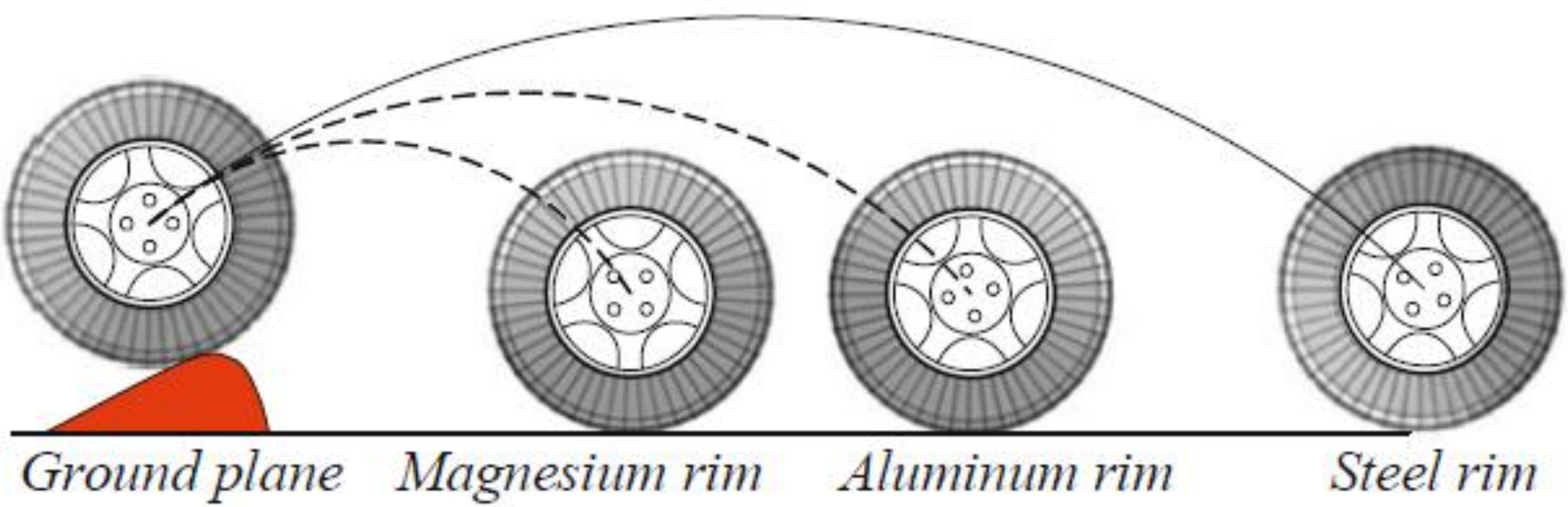


Pitch Circle Diameter

- The PCD, or Pitch Circle Diameter is the diameter of the invisible circle formed by scribing a circle that passes through the center point of each mounting hole (red circle in the image).
- If you've got the right number of bolts, but they're the wrong spacing, again the wheel won't fit.

Alloy Rims

- Steel is the main material for manufacturing rims. Other than steel, composite materials and light alloys such as aluminum, magnesium, and titanium are also used for manufacturing rims.
- Weight, cost, corrosion resistance, thermal conductivity, cast-ability, machinability, recycling, and resilience are important factors in selecting rim materials.



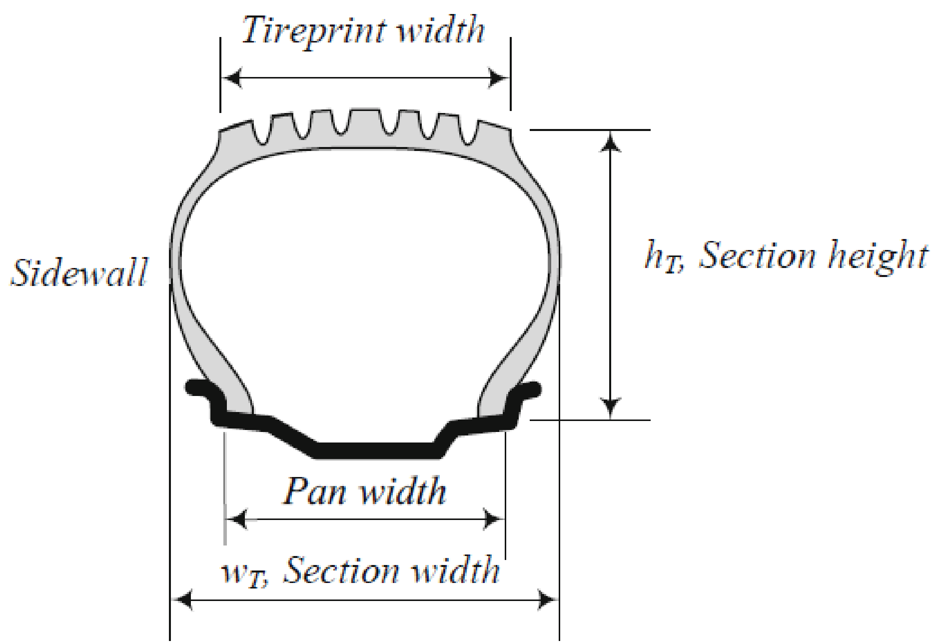
The difference between aluminum, magnesium, and steel rims in resilience results in the different behaviors in regaining road contact after a jump.

Alloy Rims

- Aluminum is very good for its weight, thermal conductivity, corrosion resistance, easy casting, low temperature, machinability, and recycling.
- Magnesium is about 30% lighter than aluminum, and is excellent for size stability and impact resistance. However, magnesium is more expensive and it is used mainly for luxury or racing cars. The corrosion resistance of magnesium is not as good as aluminum.
- Titanium is much stronger than aluminum and magnesium with excellent corrosion resistance. However, titanium is expensive and hard to be machine processed.

Tire Characteristics

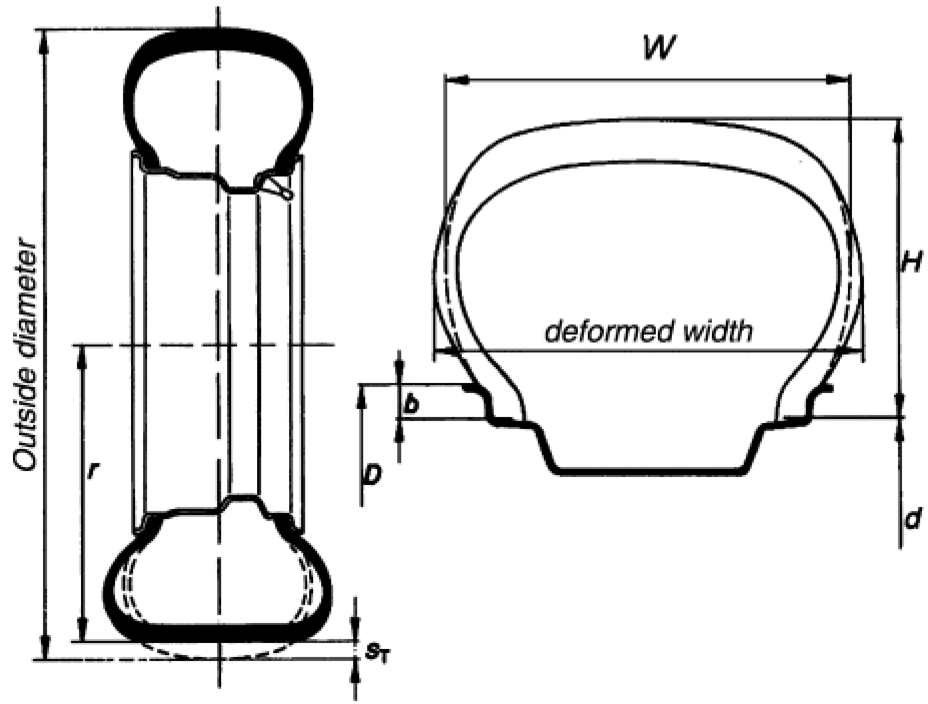
- Average tire life is between 30,000 and 160,000 km
- Tires can be punctured and require immediate substitution.
- Tire parameters such as dimensions, maximum load carrying capacity, and maximum speed index are usually indicated on its sidewall.
- The cross section view, below, of a tire on a rim shows the dimension parameters.



Tire Characteristics

Each tire is designated by a group of letters and numbers:

185/*65 R 14 82 T*



- non-deformed width *W*
- *aspect ratio (H/W)*
- type of tire plies
- rim diameter
- *load factor*
- maximum speed

(speed that the tire can sustain for ten minutes without failure)

Speed (km/h)	80	130	150	160	170	180	190	210	240	270
Letter	F	M	P	Q	R	S	T	H	V	W

Tire Characteristics

- The load index is a representation of the maximum load each tire is designed to support.

Index	Maximum load
0	45 kg ≈ 99 lbf
...	...
71	345 kg ≈ 761 lbf
72	355 kg ≈ 783 lbf
73	365 kg ≈ 805 lbf
74	375 kg ≈ 827 lbf
75	387 kg ≈ 853 lbf
76	400 kg ≈ 882 lbf
77	412 kg ≈ 908 lbf
78	425 kg ≈ 937 lbf
79	437 kg ≈ 963 lbf
80	450 kg ≈ 992 lbf
81	462 kg ≈ 1019 lbf
82	475 kg ≈ 1047 lbf
83	487 kg ≈ 1074 lbf
84	500 kg ≈ 1102 lbf
85	515 kg ≈ 1135 lbf
86	530 kg ≈ 1163 lbf
87	545 kg ≈ 1201 lbf
88	560 kg ≈ 1235 lbf
89	580 kg ≈ 1279 lbf
90	600 kg ≈ 1323 lbf
91	615 kg ≈ 1356 lbf
92	630 kg ≈ 1389 lbf
93	650 kg ≈ 1433 lbf
94	670 kg ≈ 1477 lbf
95	690 kg ≈ 1521 lbf
96	710 kg ≈ 1565 lbf
97	730 kg ≈ 1609 lbf
98	750 kg ≈ 1653 lbf
99	775 kg ≈ 1709 lbf
100	800 kg ≈ 1764 lbf
101	825 kg ≈ 1819 lbf
102	850 kg ≈ 1874 lbf
103	875 kg ≈ 1929 lbf
104	900 kg ≈ 1984 lbf
105	925 kg ≈ 2039 lbf
106	950 kg ≈ 2094 lbf
107	975 kg ≈ 2149 lbf
108	1000 kg ≈ 2205 lbf
109	1030 kg ≈ 2271 lbf
110	1060 kg ≈ 2337 lbf
111	1090 kg ≈ 2403 lbf
113	1120 kg ≈ 2469 lbf
113	1150 kg ≈ 2581 lbf
114	1180 kg ≈ 2601 lbf
115	1215 kg ≈ 2679 lbf
116	1250 kg ≈ 2806 lbf
117	1285 kg ≈ 2833 lbf
118	1320 kg ≈ 2910 lbf
119	1360 kg ≈ 3074 lbf
120	1400 kg ≈ 3086 lbf
121	1450 kg ≈ 3197 lbf
122	1500 kg ≈ 3368 lbf
123	1550 kg ≈ 3417 lbf
124	1600 kg ≈ 3527 lbf
125	1650 kg ≈ 3690 lbf
126	1700 kg ≈ 3748 lbf
127	1750 kg ≈ 3858 lbf
128	1800 kg ≈ 3968 lbf
...	...
199	13600 kg ≈ 30000 lbf

Tire Characteristics

- The manufacturing date of a tire is indicated on the tire sidewall using four digits representing the week and year the tire was built, e.g. 1612 represents week #16 of year 2012.

Uniform Tire Quality Grading (UTQG)

Tire manufacturers also rate their products for wear, wet traction, and heat resistance.

- Tread wear rating index: The higher the wear number, the longer the tire lifetime. An index of 100 is equivalent to approximately 20000 miles or 32000km.

Index	*Life (Approximate)*	
100	32000 km	20000 mi
150	48000 km	30000 mi
200	64000 km	40000 mi
250	80000 km	50000 mi
300	96000 km	60000 mi
400	129000 km	80000 mi
500	161000 km	100000 mi

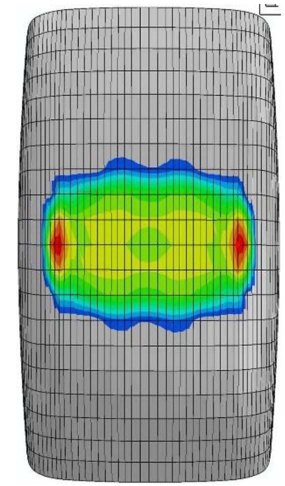
Uniform Tire Quality Grading (UTQG)

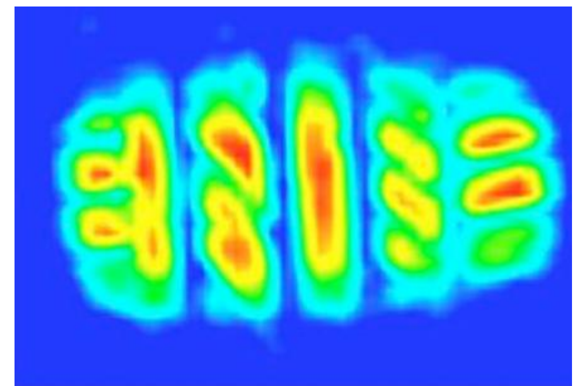
- Wet traction: wet traction is rated in letters between "A" to "C" where A is the best, B is intermediate and C is acceptable. An A wet traction rating is typically an indication that the tire has a deep open tread pattern with lots of fine lines in the tread blocks.
- Heat resistance: heat resistance is rated in letters between "A" to "C", where A is the best. An A heat resistance rating indicates two things: first, low rolling resistance due to stiffer tread belts, stiffer sidewalls, or harder compounds; second, thinner sidewalls and more stable blocks in the tread pattern.

Uniform Tire Quality Grading (UTQG)

- Temperature rating: is indicated by a letter between “A" to “C“, where A is the best.
- Traction rating: indicates how well a tire grips the road surface. This is an overall rating for both dry and wet conditions. Tires are rated as: “AA" for the best, “A" for better, “B" for good, and “C" for acceptable.

Tireprint
(tire footprint, contact patch)





- The contact area between a tire and the road is called the tireprint. At any point of a tireprint, the normal and friction forces are transmitted between the road and tire.
- The effect of the contact forces can be described by a resulting force system including force and torque vectors applied at the center of the tireprint.
- The area of the tireprint is inversely proportional to the tire pressure.
- Lowering the tire pressure is a technique used for off-road vehicles in sandy, muddy, or snowy areas, and for drag racing.

Proper Inflation Pressure

- In a properly-inflated tire, approximately 95% of the vehicle weight is supported by the air pressure in the tire and 5% is supported by the tire wall.
- An under-inflated tire will support less of the vehicle weight with the air pressure in the tire; therefore, more weight will be supported by the tire wall. This tire load increase causes the tire to have a larger tireprint that creates more friction and more heat.
- In an over-inflated tire, too much of the vehicle weight is supported by the tire air pressure. The vehicle will be bouncy and hard to steer because the tireprint is small and only the center of the tireprint is contacting the road.

Plus one (+1) concept

- Speedometer is calibrated to tell the speed by how many times the wheel spins around.
- If you make your wheel diameter smaller, it's going to spin more times to go the same distance.
- Your car doesn't know you changed the tire and wheel size, so it will give you an inaccurate reading.
- Plus one (+1) concept is used to put wider, bigger wheels and tires on cars without sacrificing speedometer accuracy.

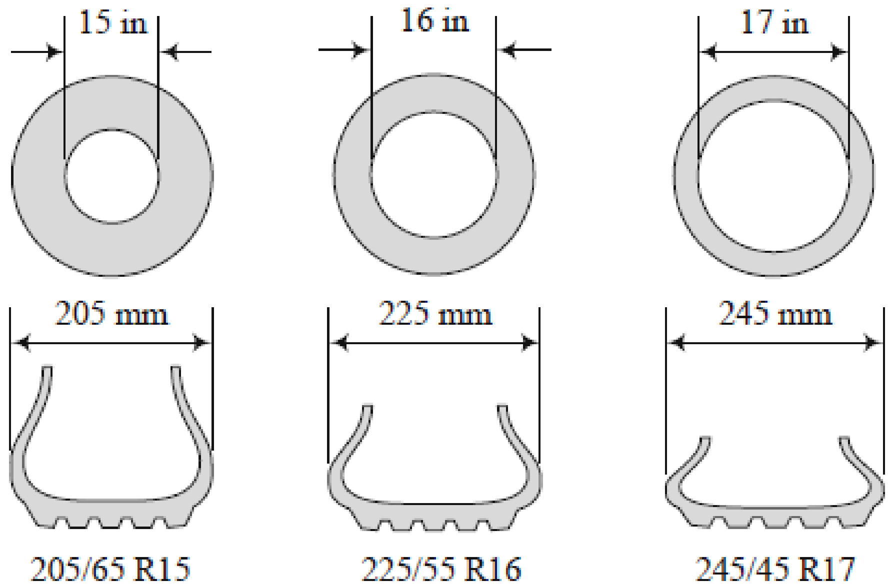
Plus one (+1) concept

Use online calculators (may give different results)

OR

Use the following procedure

Plus one (+1) concept



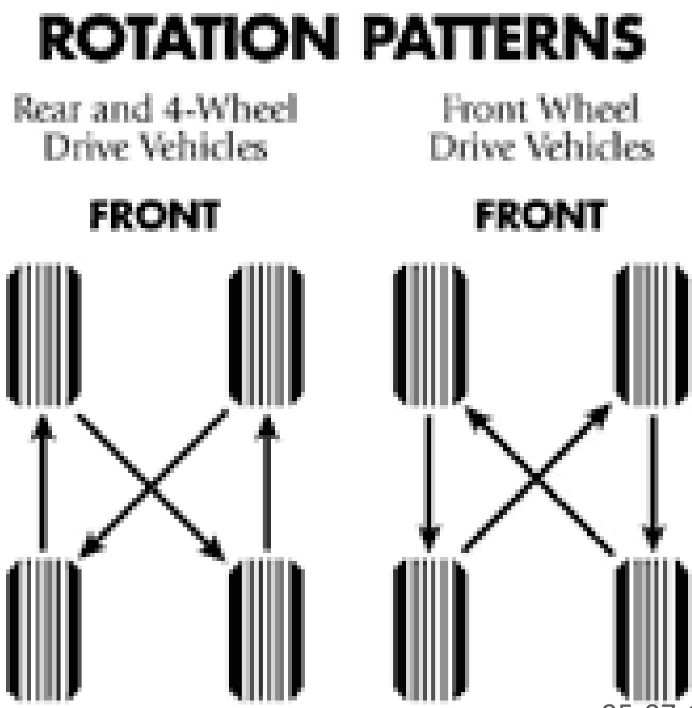
- add 20mm to the tire width
- subtract 10% from the aspect ratio.
- add 1 in to the rim diameter,

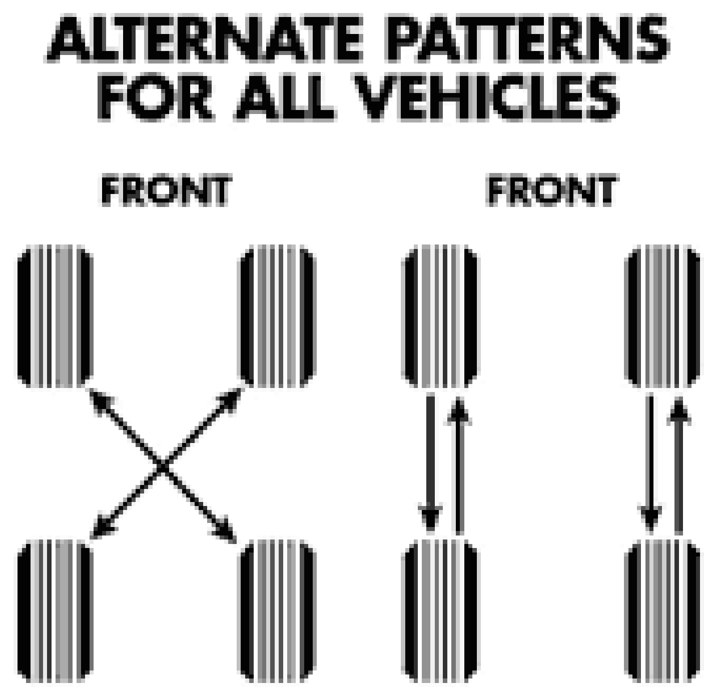
Rim width

- aspect ratio of 50 and above
- 70% of the tire’s width, rounded to the nearest 0.5 in
- 255/50R16 tire - width of 255mm = 10.04 in * 70% = 7.028 ≈ 7 in [7 x 16 rim] (up to 8.5 in wide).
- aspect ratio of 45 and below
- 85% of the tire’s width, rounded to the nearest 0.5 in
- 255/45R17 tire - width of 255mm = 10.04 in * 85% = 8.534 in≈ 8.5in [8 ½ x 17 rim] (up to 10 in wide).
- A rim width of up to 1.5 inch wider can also be used.

Rotating the tires

- In most vehicles, the front and rear tires will wear at different rates. So, it is advised to swap the front and rear tires as they wear down to even out the wear patterns.
- Front tires, especially on front-wheel drive vehicles, wear out more quickly than rear tires.

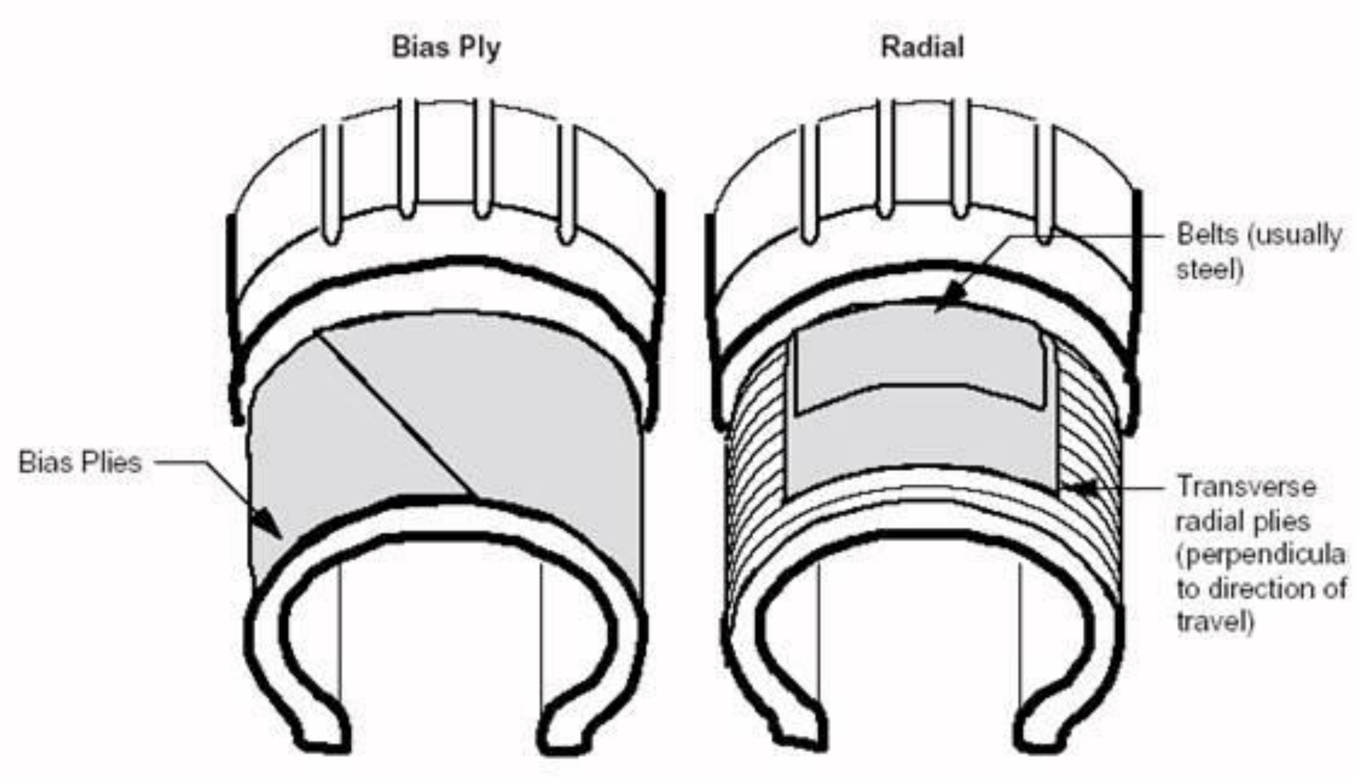




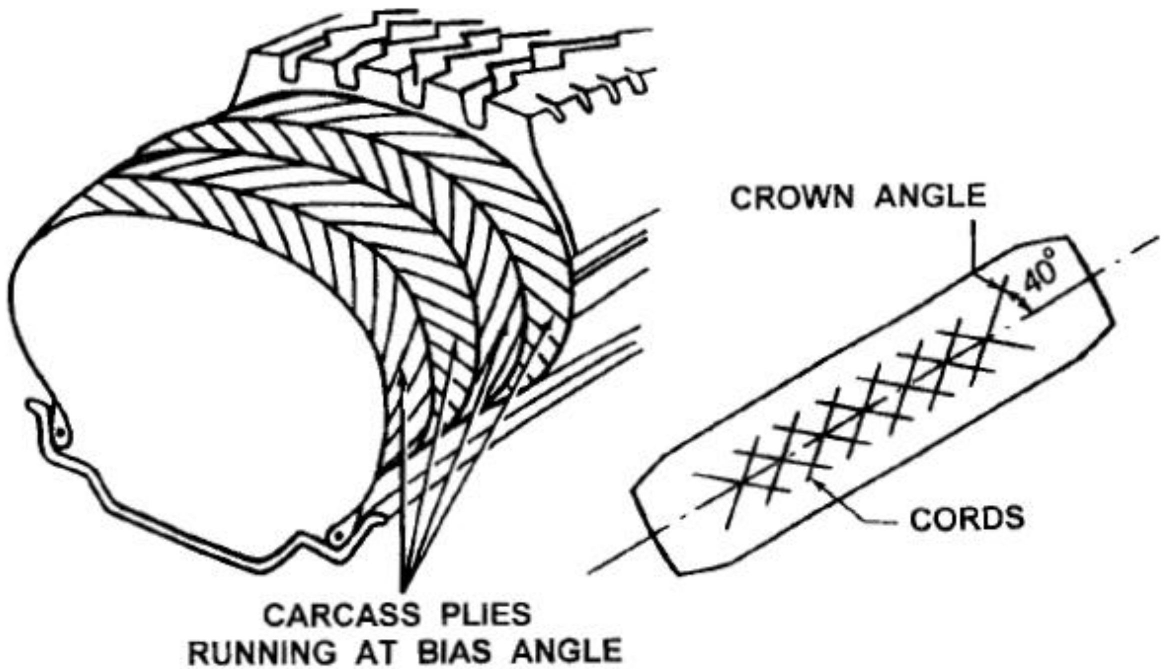
Wheels and Tires

The average weight of a passenger car tire is 10-12 kg.
The average weight of a light truck tire is 14-16 kg.
The average weight of commercial truck tire is 135-180 kg.

Tire Structure

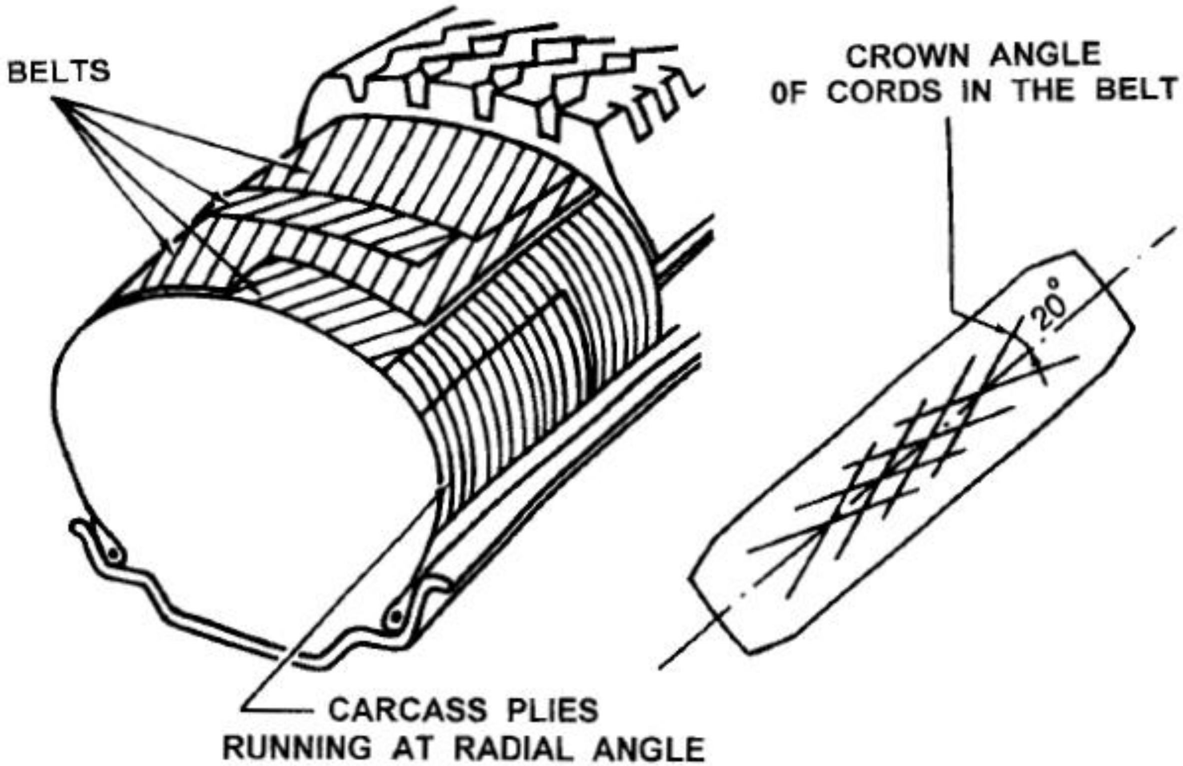


Cross-ply (bias) tires



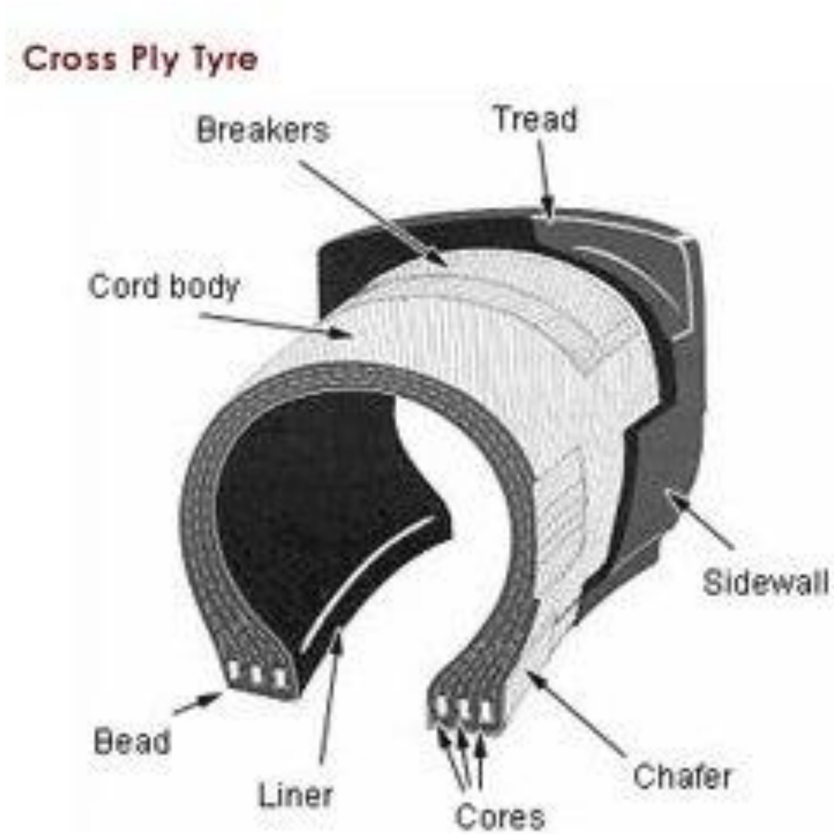
A bias-ply tire has two plies (for light-load tires) or more (up to 20 plies for heavy-load tires). The cords in adjacent plies overlap in a diamond-shaped (criss-cross) pattern.

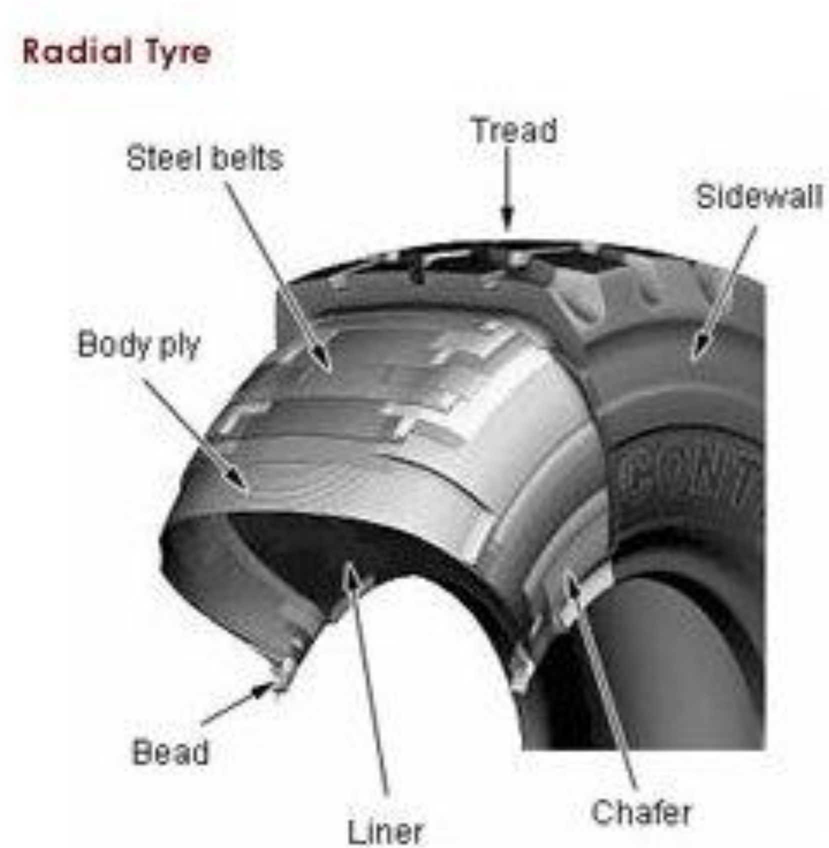
Radial-ply tires



A radial-ply tire has one or more layers of cords in the carcass extending radially from bead to bead, resulting in a crown angle of 90°. A belt of several layers of cords (usually steel or other high-modulus materials) is fitted under the tread. The cords in the belt are laid at a low crown angle of approximately 20°.
For passenger car tires, usually there are two radial plies in the carcass and two plies of steel cords and two plies of synthetic material cords in the belt. For truck tires, usually there is one radial steel ply in the carcass and four steel plies in the belt.

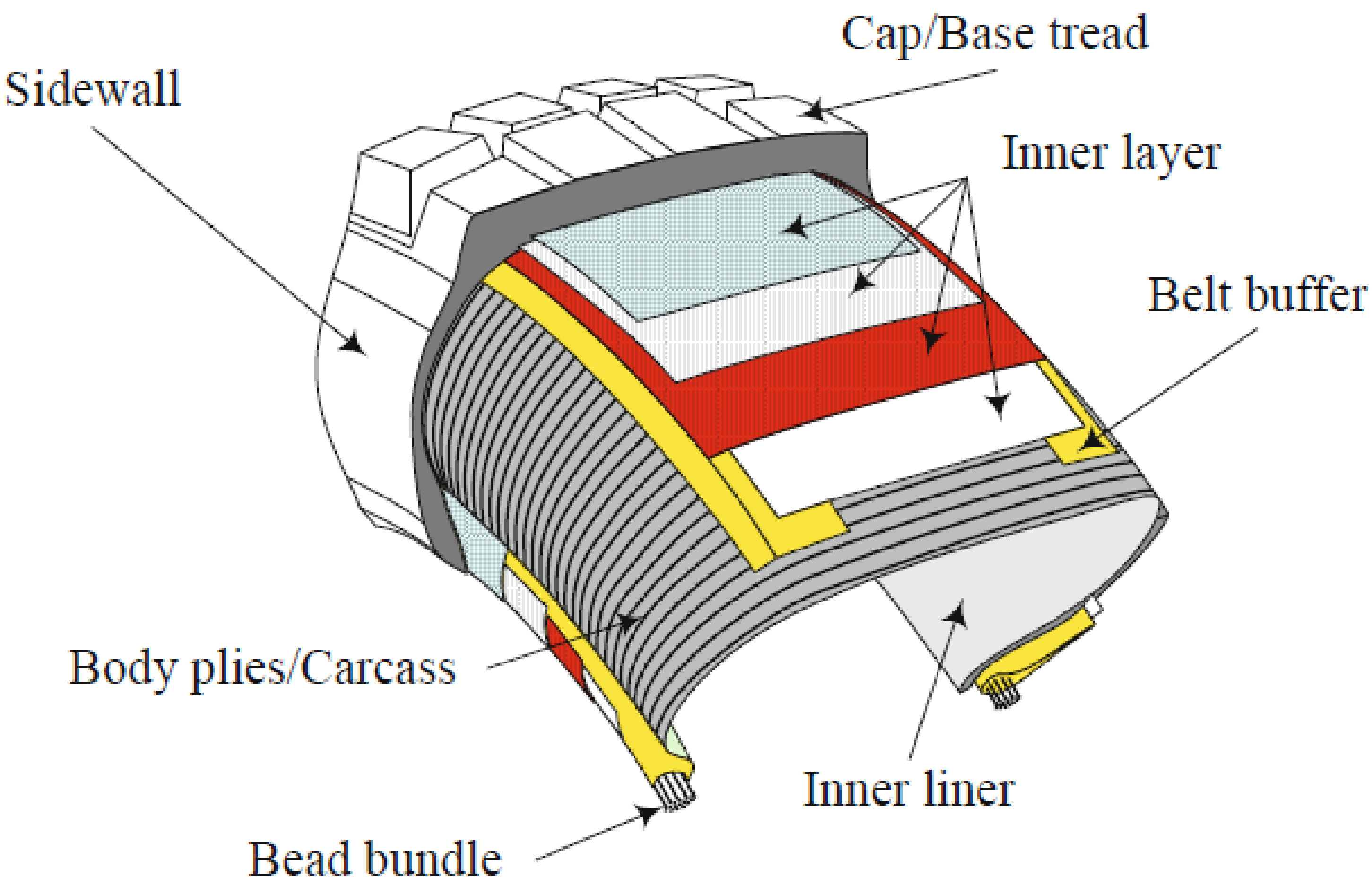
Tire Structure





The strength of bias-ply tires increases by increasing the number of plies and bead wires. However, more plies means more mass, which increases heat and reduces tire life. To increase a radial tire's strength, larger diameter steel cables are used in the tire's carcass.

Tire Components



Tire Components

طوقه

- Bead or bead bundle is a loop of high strength steel cable coated with rubber. It gives the tire the strength it needs to stay seated on the wheel rim and to transfer the tire forces to the rim.

Tire Components

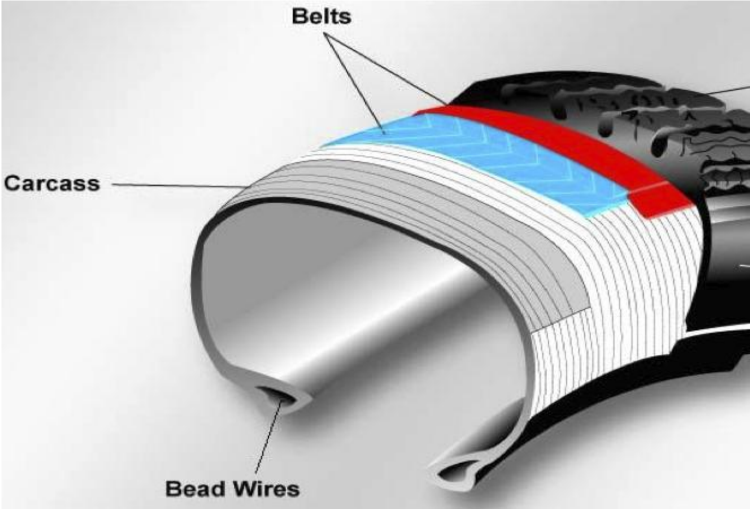
لایه ها

- Inner layers also called plies are made up of different fabrics. The most common ply cords are cotton, polyester, rayon, steel, fiberglass, and aramid.

منجید

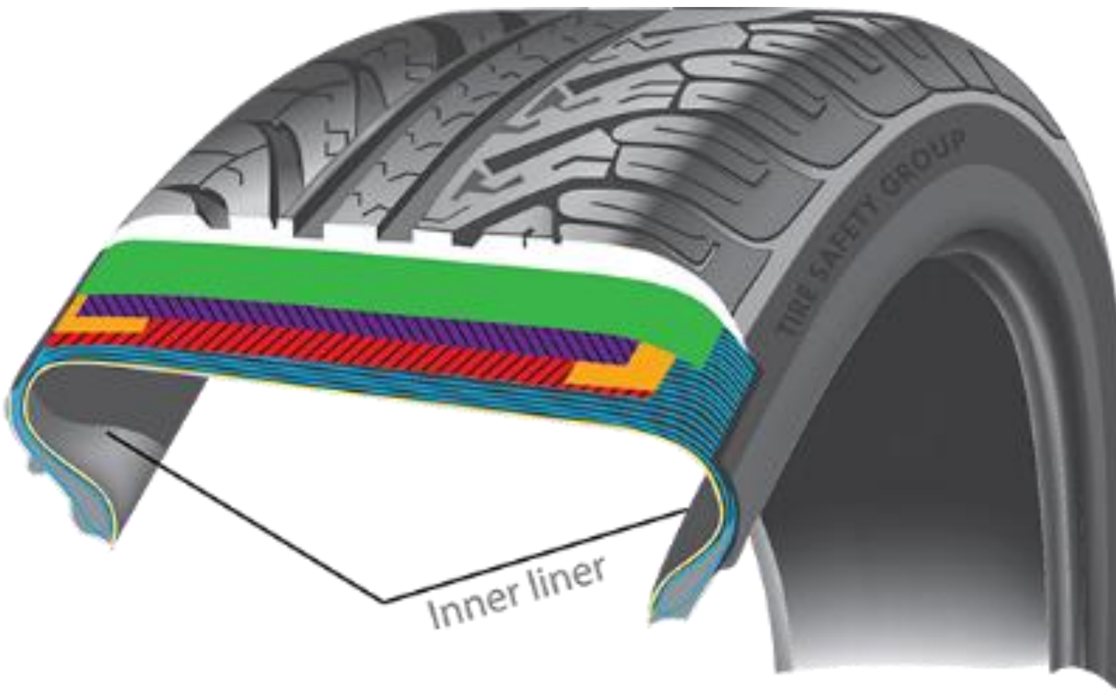
- The carcass composed of the body plies, keep the air in the tire and are the main part in supporting the tension forces generated by tire air pressure.

- A tire's strength is often described by the number of carcass plies or the thickness of the ply cords.



Tire Components

آستر داخلی



- An inner liner is a specially compounded rubber that forms the inside of a tubeless tire, which prevents loss of air pressure.

Tire Components

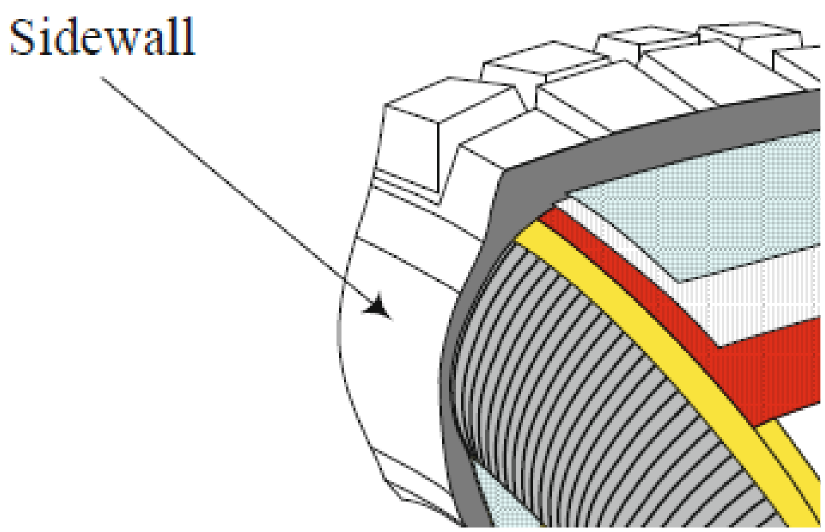
تسمه

- Belts are rubber-coated layers of steel, polyester, nylon, Kevlar or other materials running around the tire circumference, under the tread. They are designed to reinforce body plies to hold the tread flat on the road and make the best contact with the road.

Tire Components

دیواره تایر

- The sidewall provides lateral stability for the tire and protects the body plies. It may contain additional components to help increase the lateral stability.

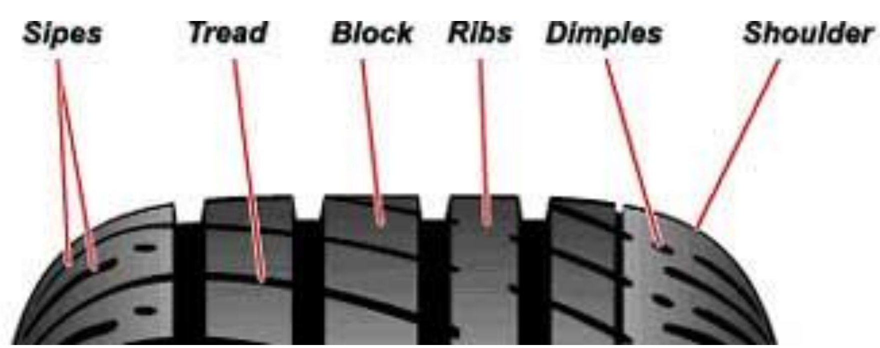


Tire Components

آج تایر

- The tread is the portion of the tire that comes in contact with the road. The tread is made from a mixture of different kinds of natural and synthetic rubbers.
- Tread designs vary widely depending on the specific purpose of the tire. The tread groove is the space or area between two tread rows or blocks. The tread groove gives the tire traction and is especially useful during rain or snow.

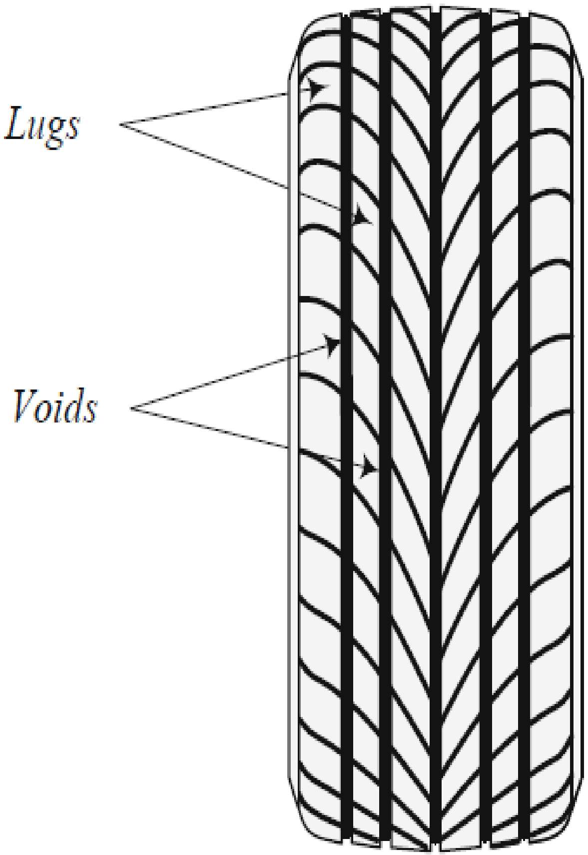
شیارهای آج



Tire Components

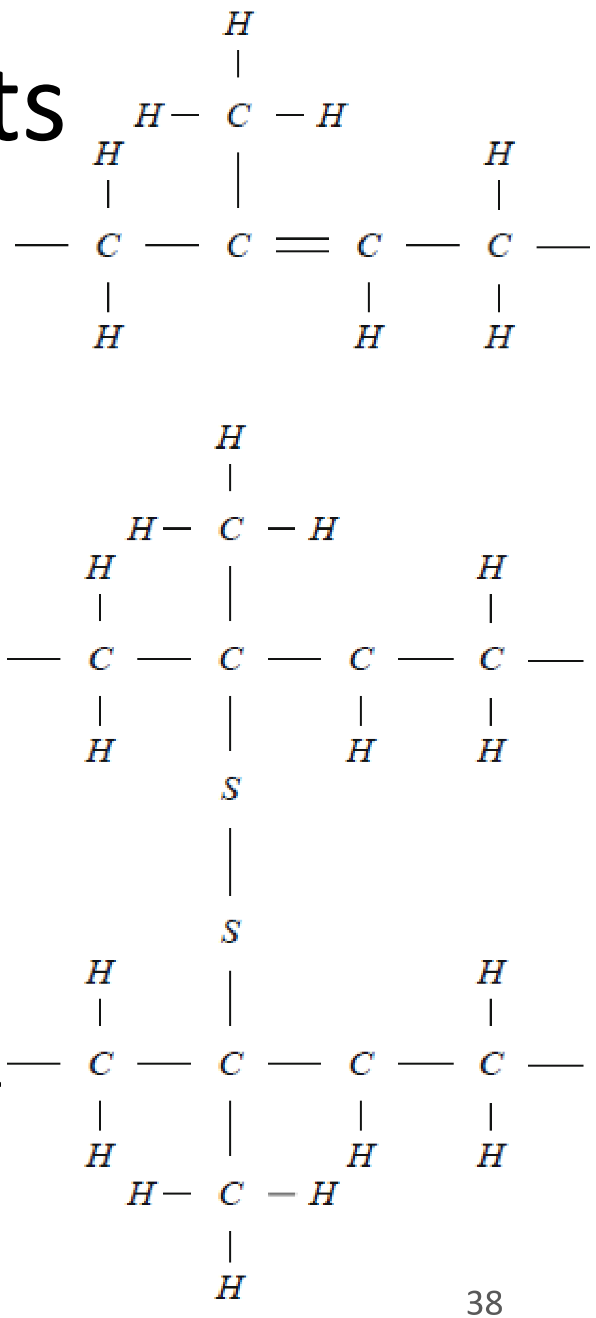
شیارها

بلوک آج



- The tread pattern is made up of lugs (blocks) and voids (grooves). The lugs are the sections of rubber that make contact with the road and voids are the spaces that are located between the lugs.
- Wide and straight grooves running circumferentially have a lower noise level and high lateral friction.
- Lateral grooves running from side to side increase traction and noise levels.
- Tires need both circumferential and lateral grooves. The water on the road is compressed into the grooves by the vehicle's weight and is evacuated from the tireprint region, out to the sides of the wheel.
- Self-cleaning is the ability of a tire's tread pattern to release mud or material from the voids. A better mud tire releases the mud or material easily from the voids.

Tire Components

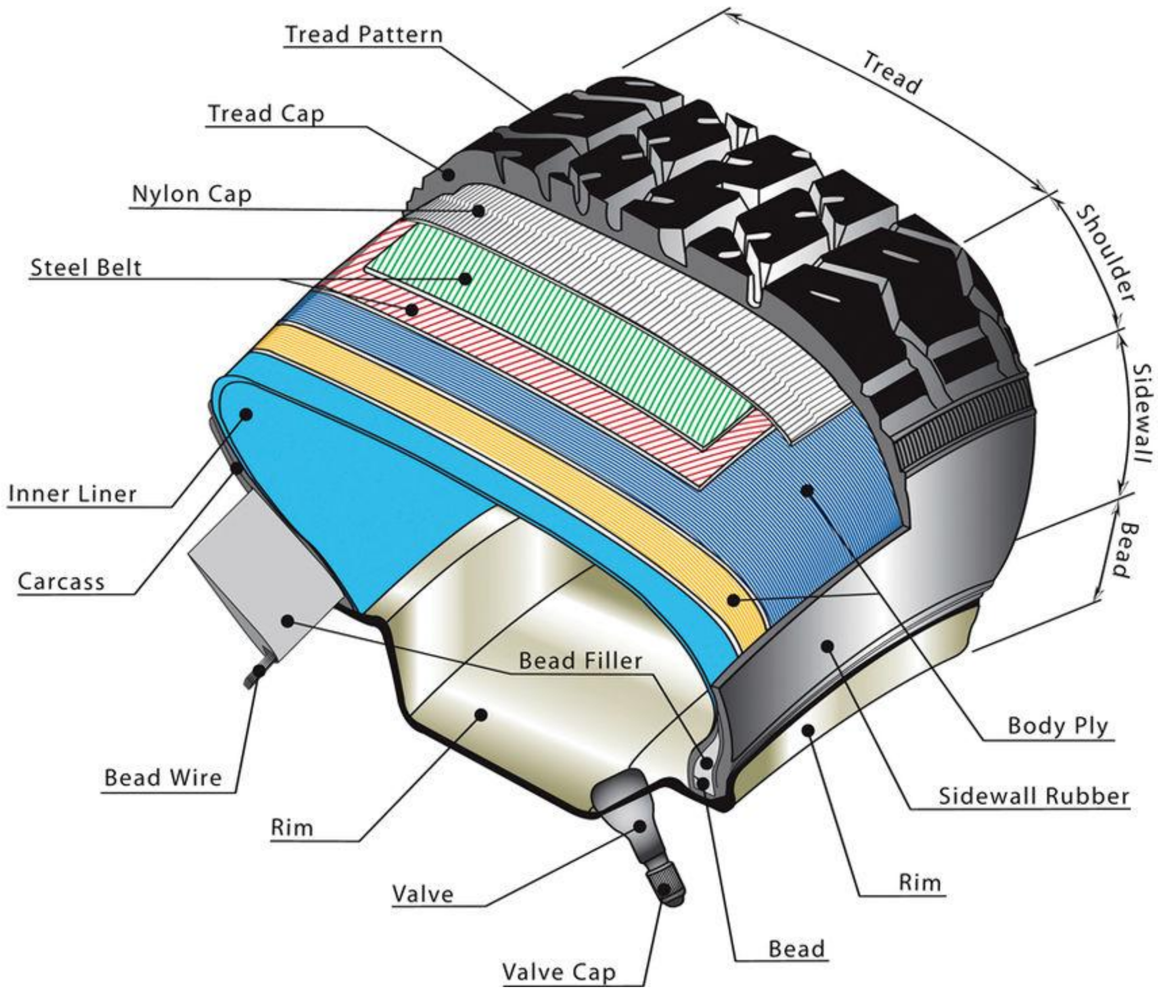


There are five major rubbers used in tire production:

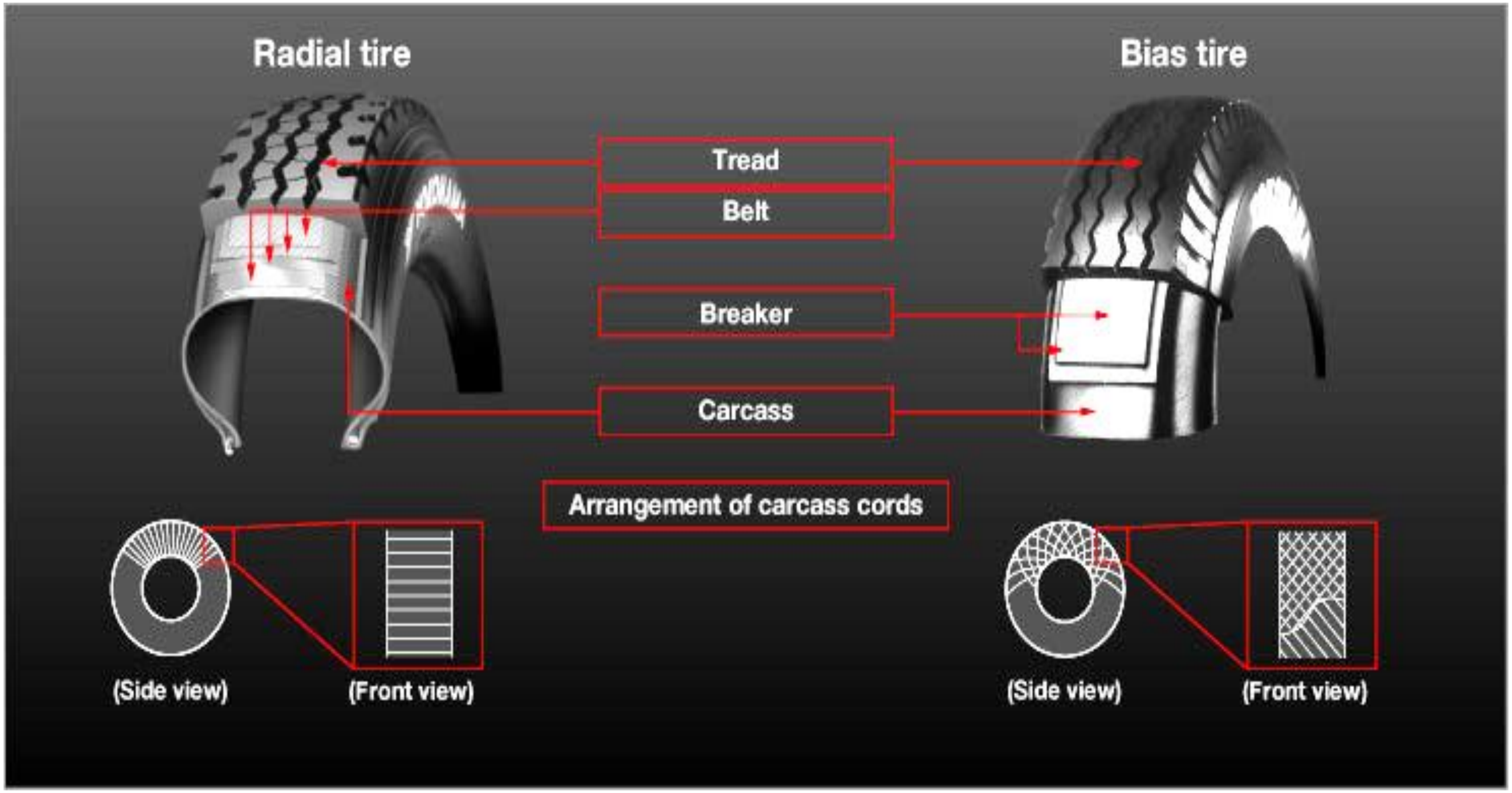
- natural rubber,
- stirene-butadiene rubber (SBR),
- butadiene rubber (BR),
- butyl rubber,
- halogenated butyl rubber.

The first three are primarily used for tread and sidewall compounds, while butyl rubber and halogenated butyl rubber are primarily used for the inner liner and the inside portion that holds the compressed air inside the tire.

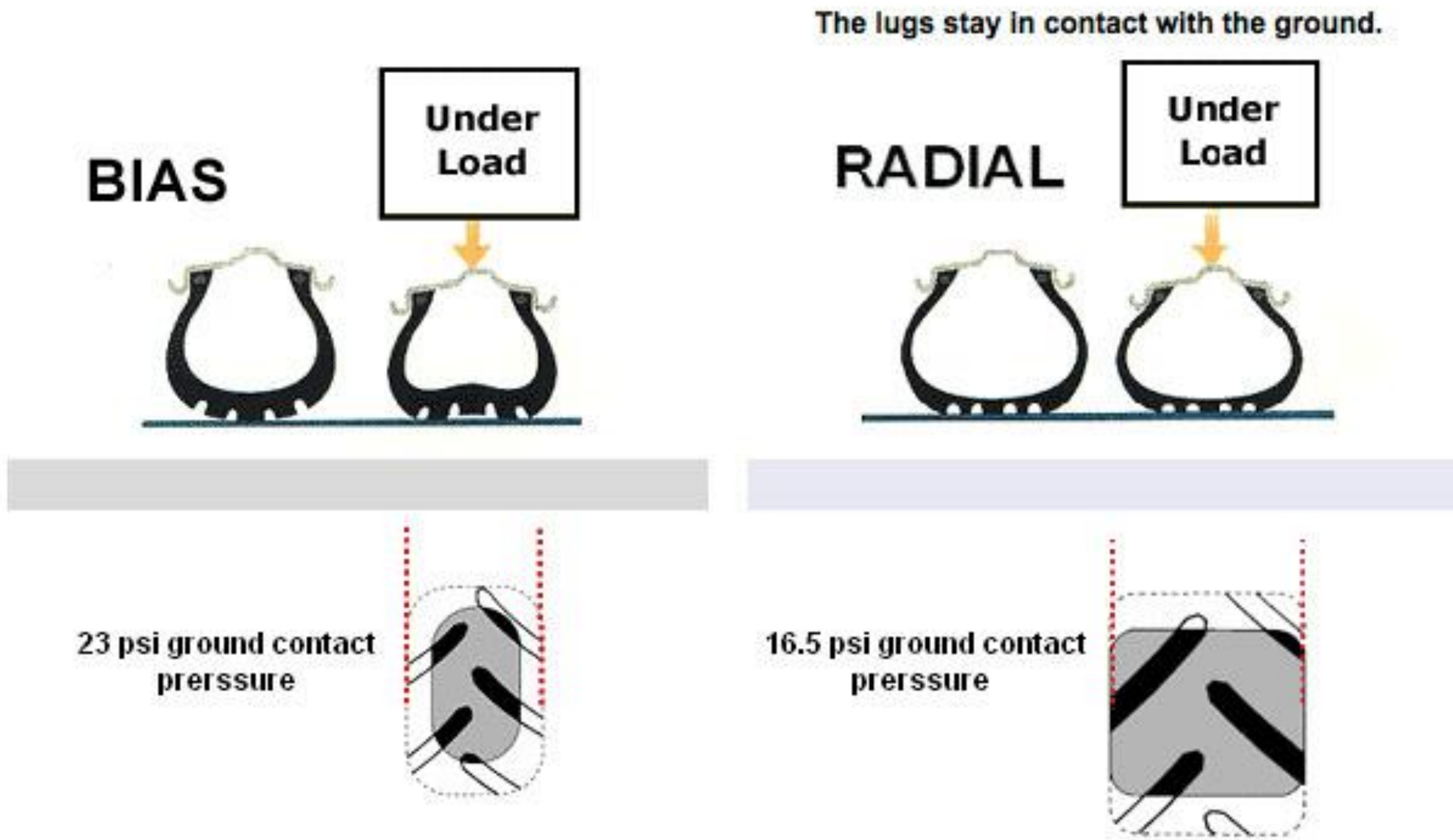
Tire Components



Tire Structure

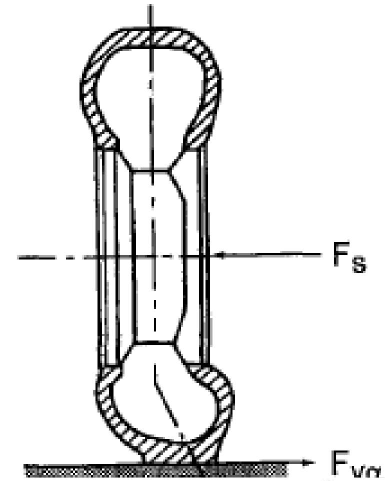


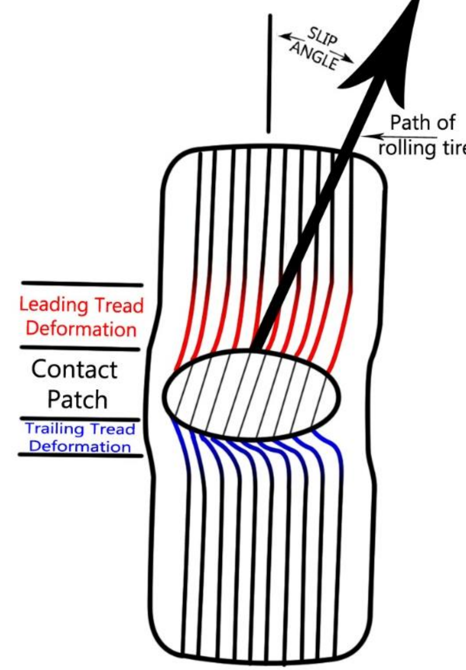
Tire Structure



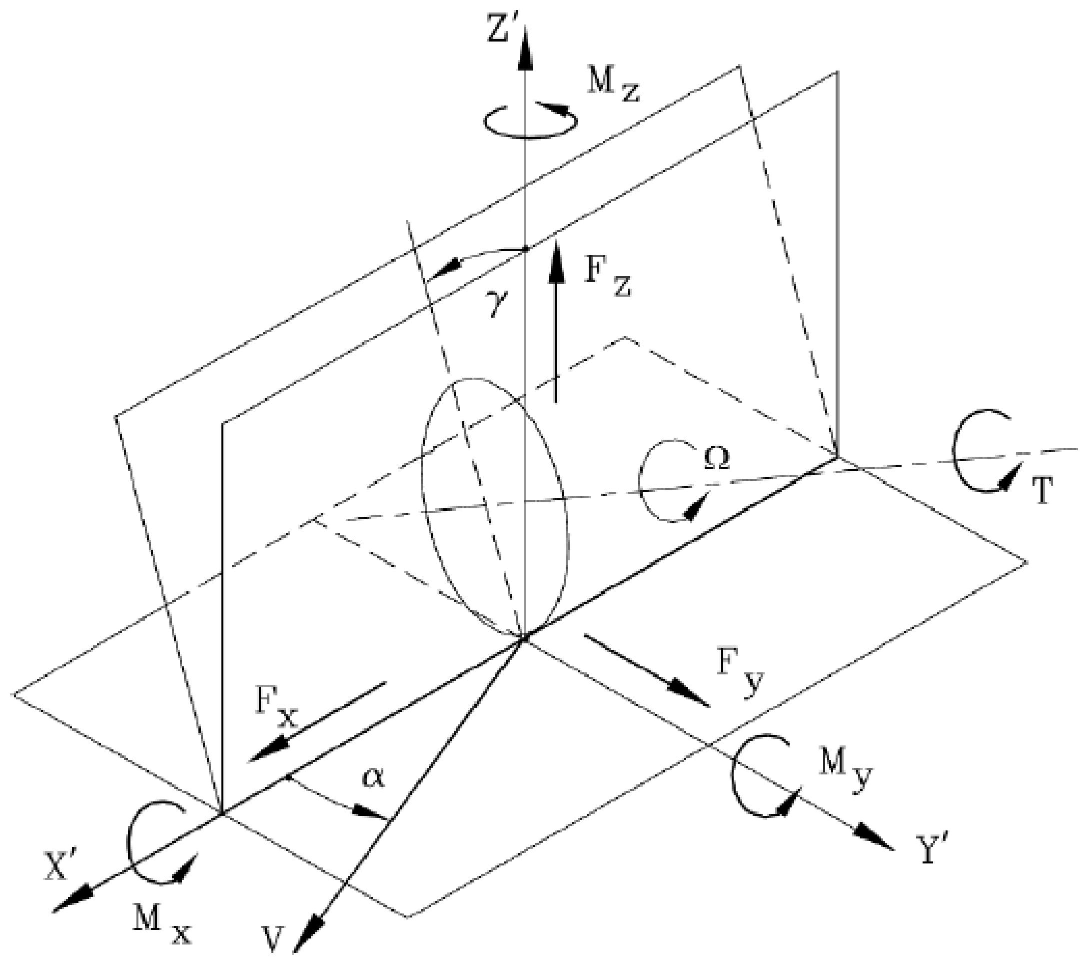
Wheels and Tires

- Rubber is the main material used to make a tire compliant. The elastic characteristic of a tire allows the tire to be pointed in a direction different than the direction the car is pointed.
- There is no way for a vehicle to turn without rubber tires, unless it moves at a very low speed.





Wheels and Tires



Wheels and Tires

- *Wheel sideslip angle α*

  angle between the *X'Z' plane and the direction of the wheel hub*
- *Inclination or camber angle γ*

  angle between the *X'Z' plane and the* wheel equatorial plane
- While the inclination angle is stated with reference to the road, the camber angle is usually stated with reference to the car
- We will assume that inclination and camber angle coincide

Tire Operation

- Prepared ground, when the tire is in contact with paved or concrete surfaces (on-road driving)
- *Unprepared ground, when the tire is in contact with natural surfaces or* dirt roads (off-road driving)

The physical phenomenon discriminating the two:

- Ground deformation is neglected on dry paved roads.
- Ground deformations should be considered in unprepared roads.

on-road driving

Two different aspects of on-road driving are considered:

- The *adhesion between rubber and ground; because of this phenomenon,* tires can exchange forces with the ground.
- *The elasticity of the tire structure; gives the tire the* capabilities to *absorb certain* road irregularities.

These are the primary reasons that tires slip in two directions of the area of contact when longitudinal and lateral forces are applied.

on-road driving

Rubber-ground adhesion

- the result of physical phenomena that allow a specimen of rubber set on the ground and pressed with a certain vertical force to withstand forces contained in the ground plane, without any relative motion.

Adhesion is caused by two phenomena:

- *Physical adhesion*
- *Local deformation*

on-road driving

Physical adhesion

- Attraction forces between rubber molecules and ground molecules (between adhesive sites)
- Impurities present between the two contact surfaces
- Lateral forces are balanced by the adhesion force

Adhesion force is controlled by:

- Surface energy of contacting materials.
- *Damping properties of those materials (especially rubber);* controlled by temperatures and relative speed.
- *Deformation of contacting surfaces, because of lateral forces which can also* cause instability as in the case of *stick and slip.*

Local deformations

- caused by road irregularities

on-road driving

- Adhesion force comprises about 70% of the total friction force when rubber is on dry paved road.
- These phenomena change their mechanism radically on wet surfaces;

Three fundamental cases:

- *Water layer thickness is high enough to establish a permanent lubricated* opening between tire and ground (*aquaplaning); (*tangential forces can be calculated from liquid viscosity).
- *Water layer thickness is insufficient to establish permanent lubrication, but* sufficient to preclude adhesion forces; local deformation forces can still occur if the ground is rough.
- *Water is completely removed from the contact area. The behavior of rubber* is then as was explained (by means of tire grooves and draining pavement).

off-road driving

- Ground deformations should also be taken into account
- Ground deformations could predominate in certain situations, which can interfere with the mechanical parts of the chassis.

Shape of the surface has a purely geometric impact on chassis design, concerning:

- *Wheelbase*
- *Track*
- *Wheel diameter*
- *Available suspension stroke*
- *Clearance of the chassis from the ground and*
- *Attack angles*

off-road driving

- The mechanical properties of soil are determined by the solid incoherent particles that are its main component;

The primary characteristics of these particles are:

- granulometry,
- apparent density (*apparent* vs *real* density)
- water content (*humidity*) expressed in percent of water to solids

liquid limit: cohesive effect between particles and therefore the shear resistance of the soil is eliminated

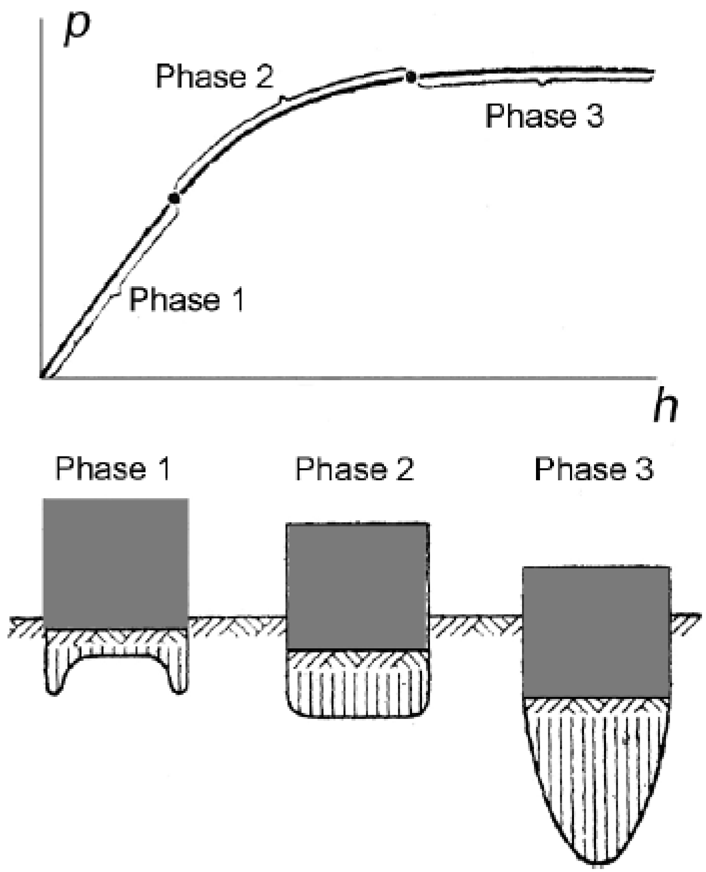
plastic limit: the soil looses its capability of being shaped, being too brittle

plasticity index: difference between the two limits

relative content of water of the soil: ratio between the actual water content of a given soil and the quantity related to the plastic limit

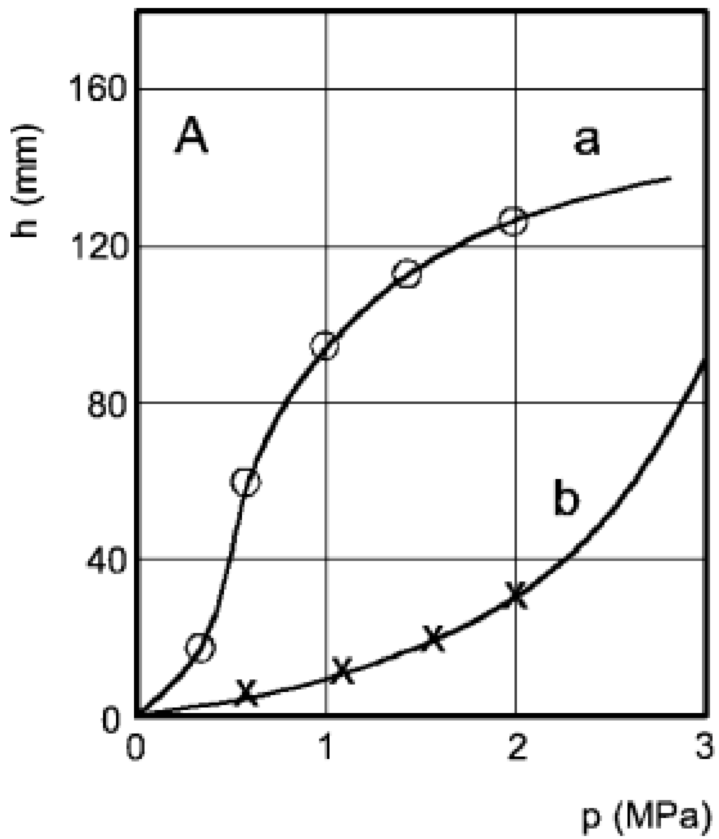
off-road driving

sinking *h as a function of applied pressure p, for various kinds of soil*



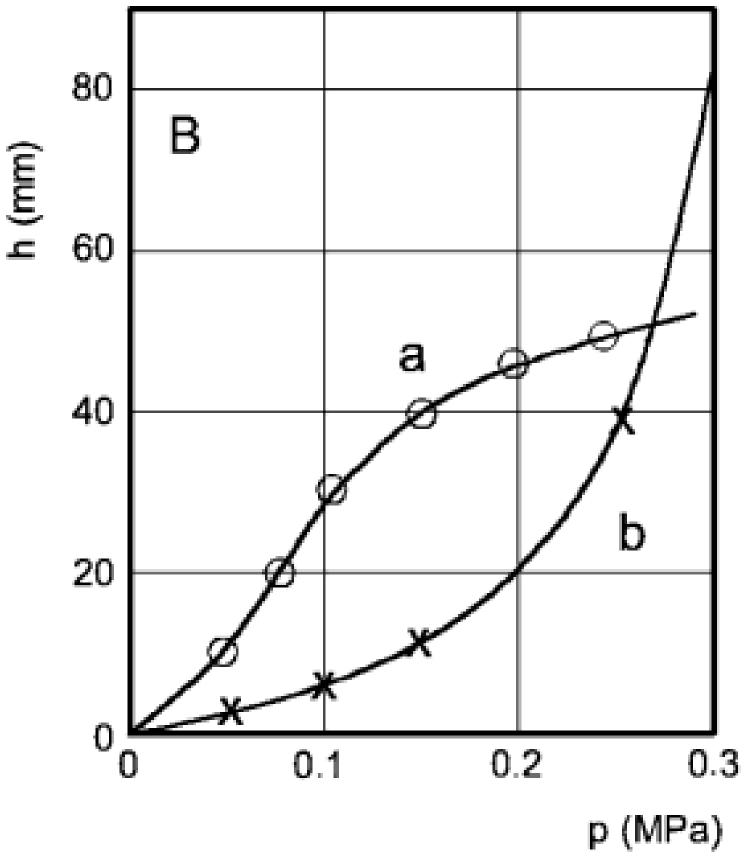
off-road driving

- Cohesive soil (a: humid soil, b: plastic soil);



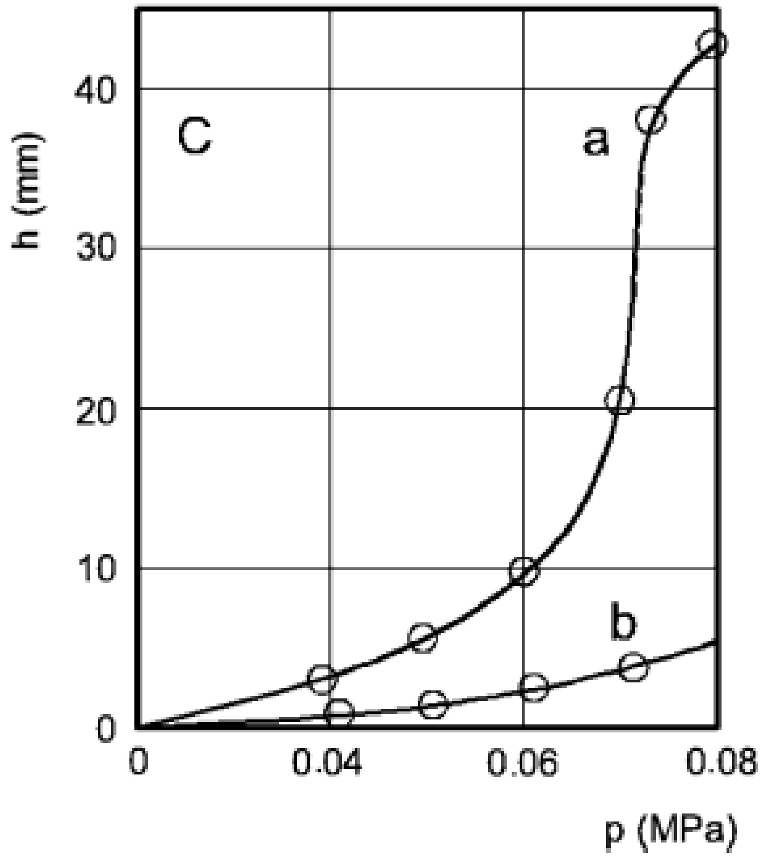
off-road driving

- Sandy soil (a: loose sand, about 200 mm thick; b: a compacted layer);



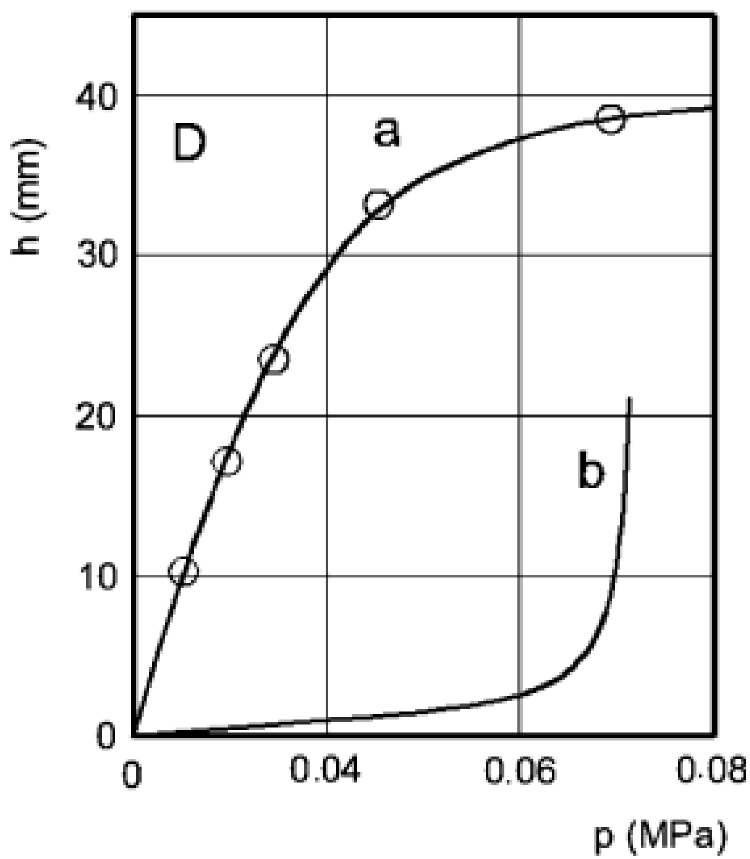
off-road driving

- Layer of peat (a: pressing pad of 4 m^2 of surface; b: pressing pad of about 0.4 m^2);



off-road driving

- Snowy ground (a: fresh snow, density of 0.15 g/cm², b: compacted snow, density of 0.20 g/cm²).



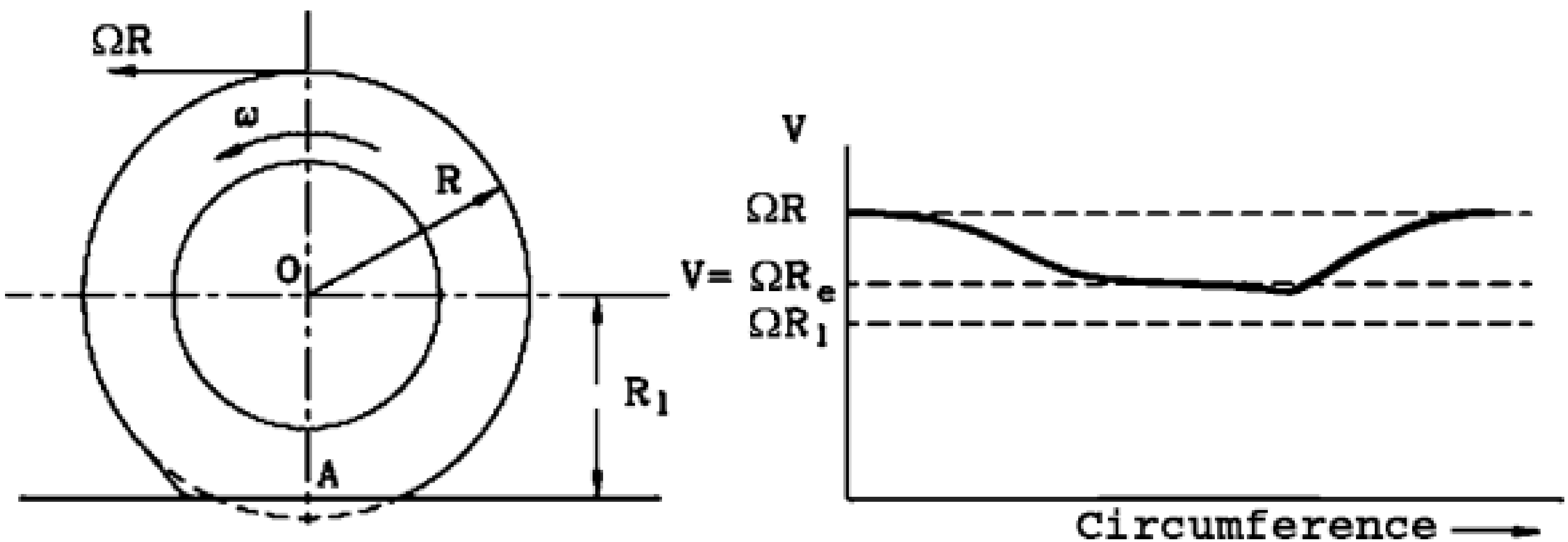
Rolling Radius

- Relationship between the angular velocity Ω and the forward speed *V of a rolling rigid wheel* of radius *R is simply:*

$$V = \Omega R$$

- for a pneumatic tire an effective rolling radius *Re can be defined:*

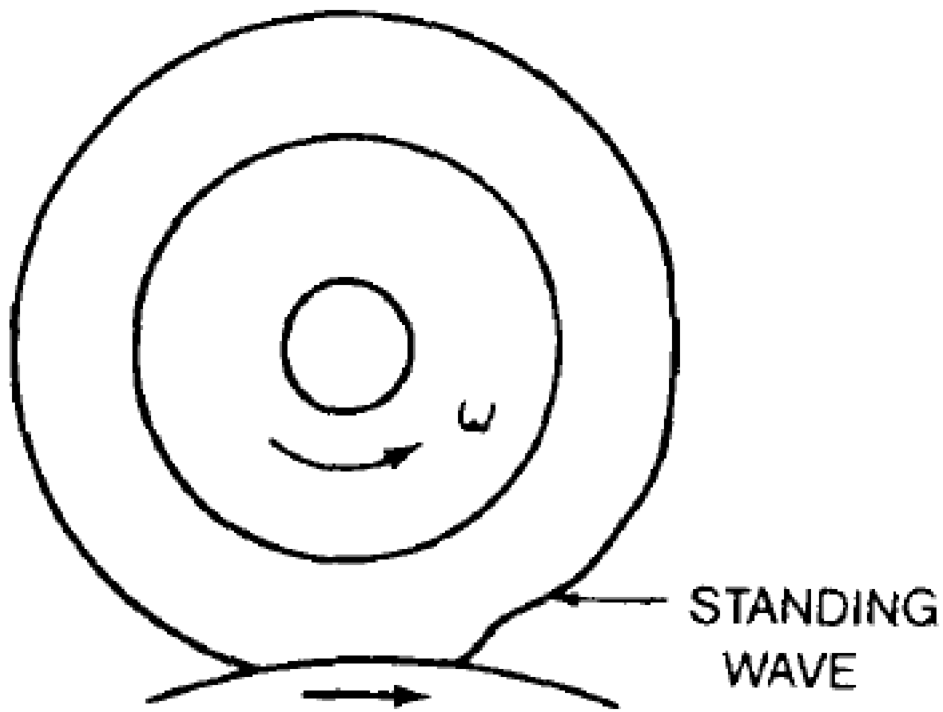
$$Re = V/\Omega$$



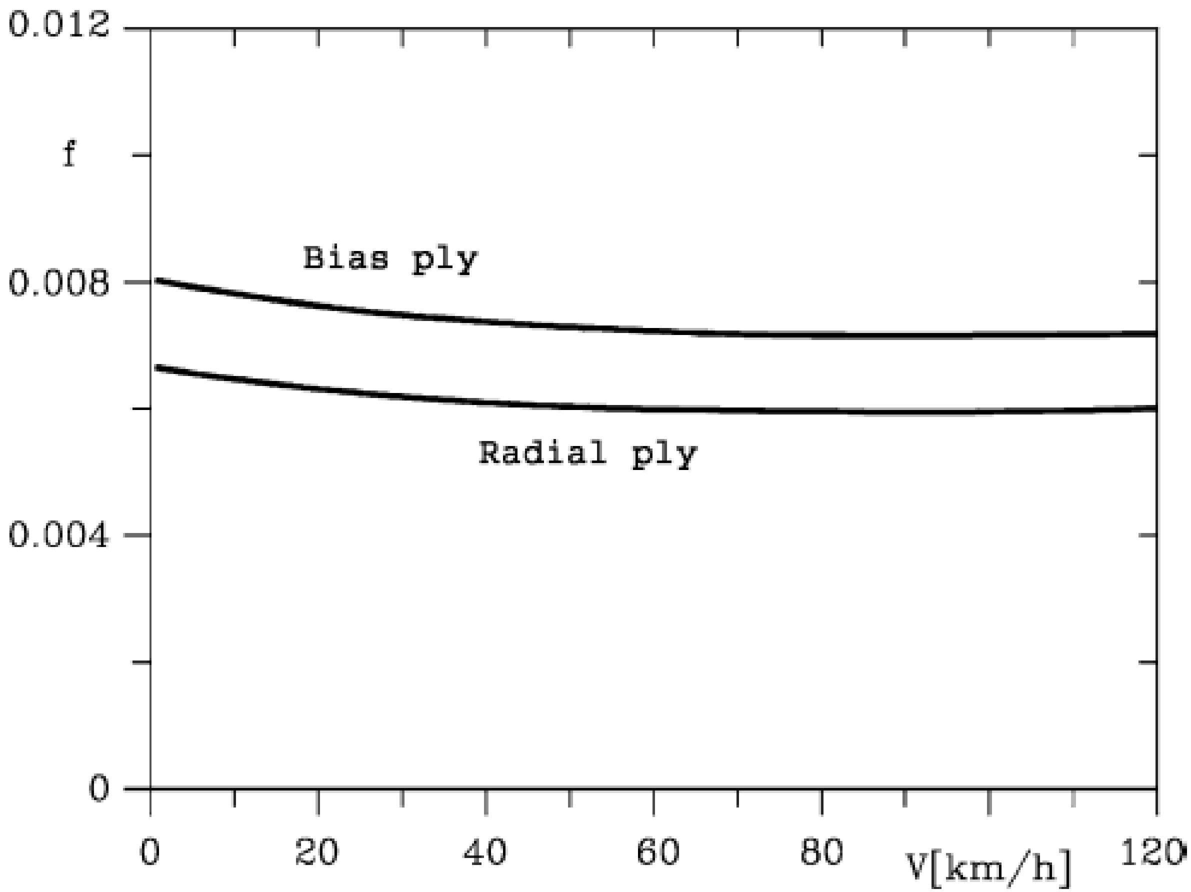
Standing Waves

Formation of Standing Waves at Threshold Speed

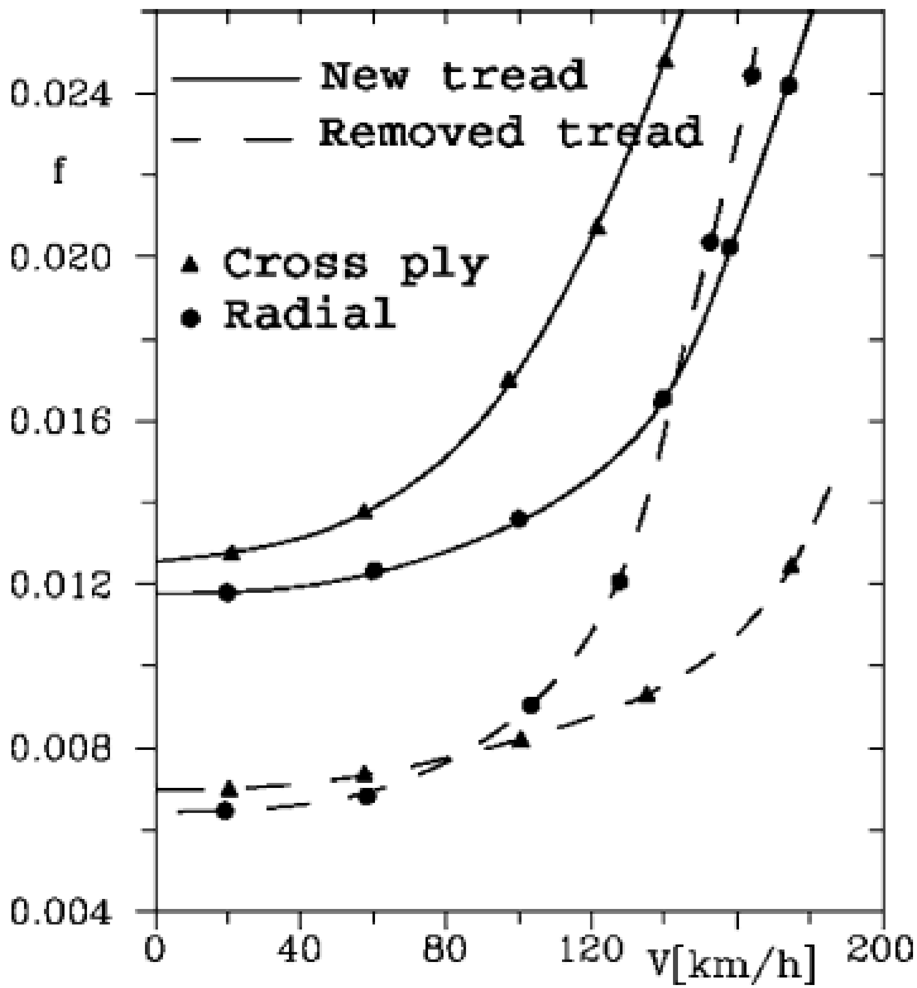
$$\sqrt{F_t / \rho_t}$$



Friction

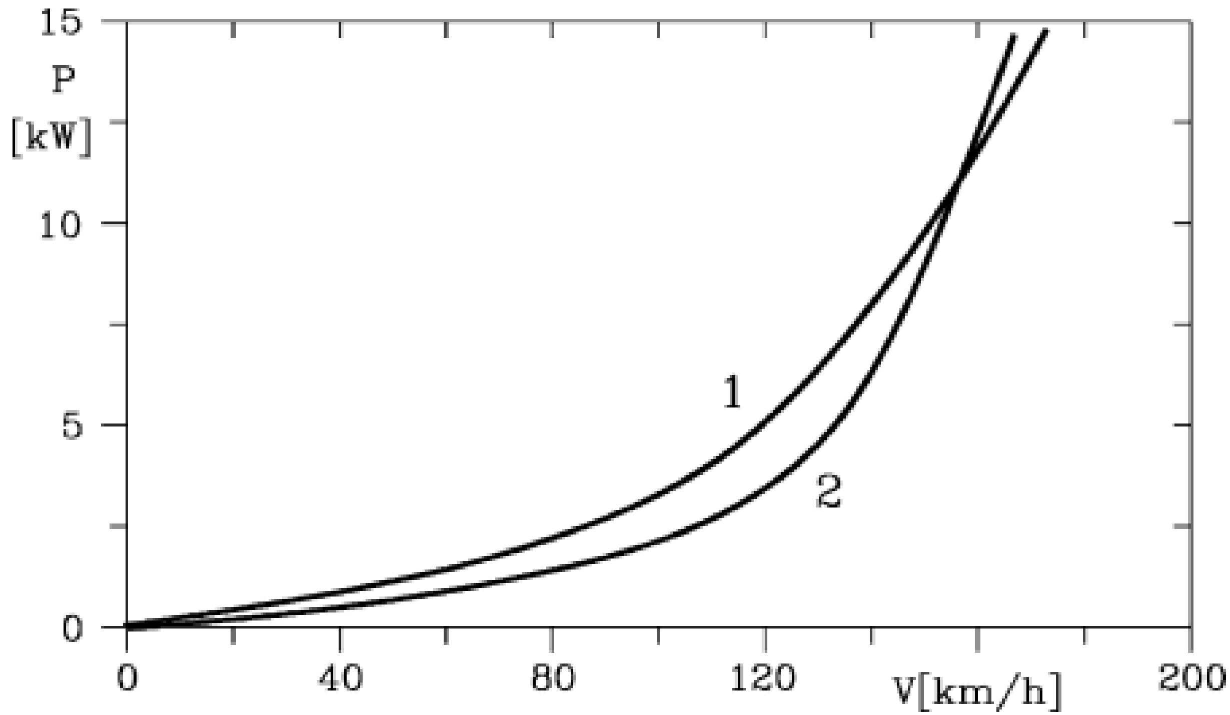


Effect of wear on friction



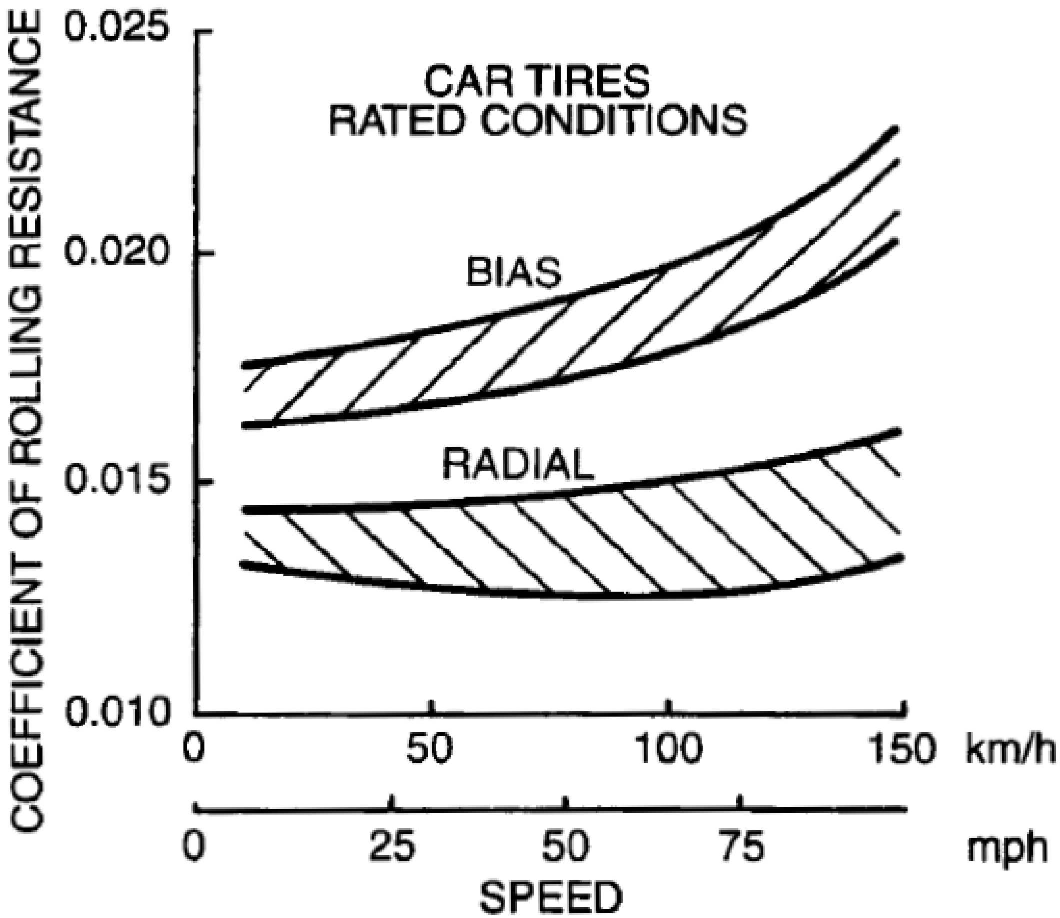
Rolling Power

Rolling power as a function of speed.
Curve 1: synthetic rubber; curve 2: natural rubber.



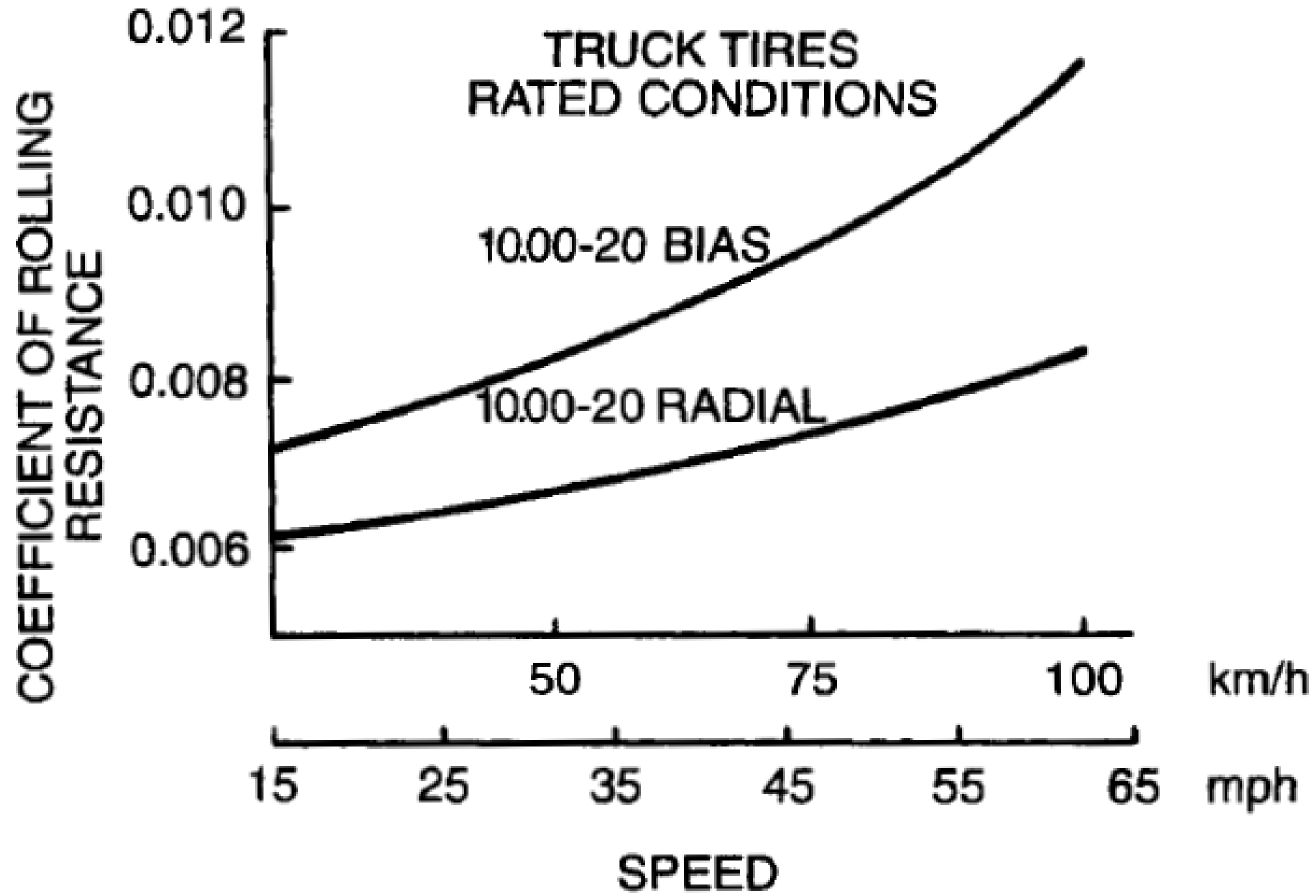
Coefficient of Rolling Resistance

Smooth, flat road surface under rated load and inflation pressure

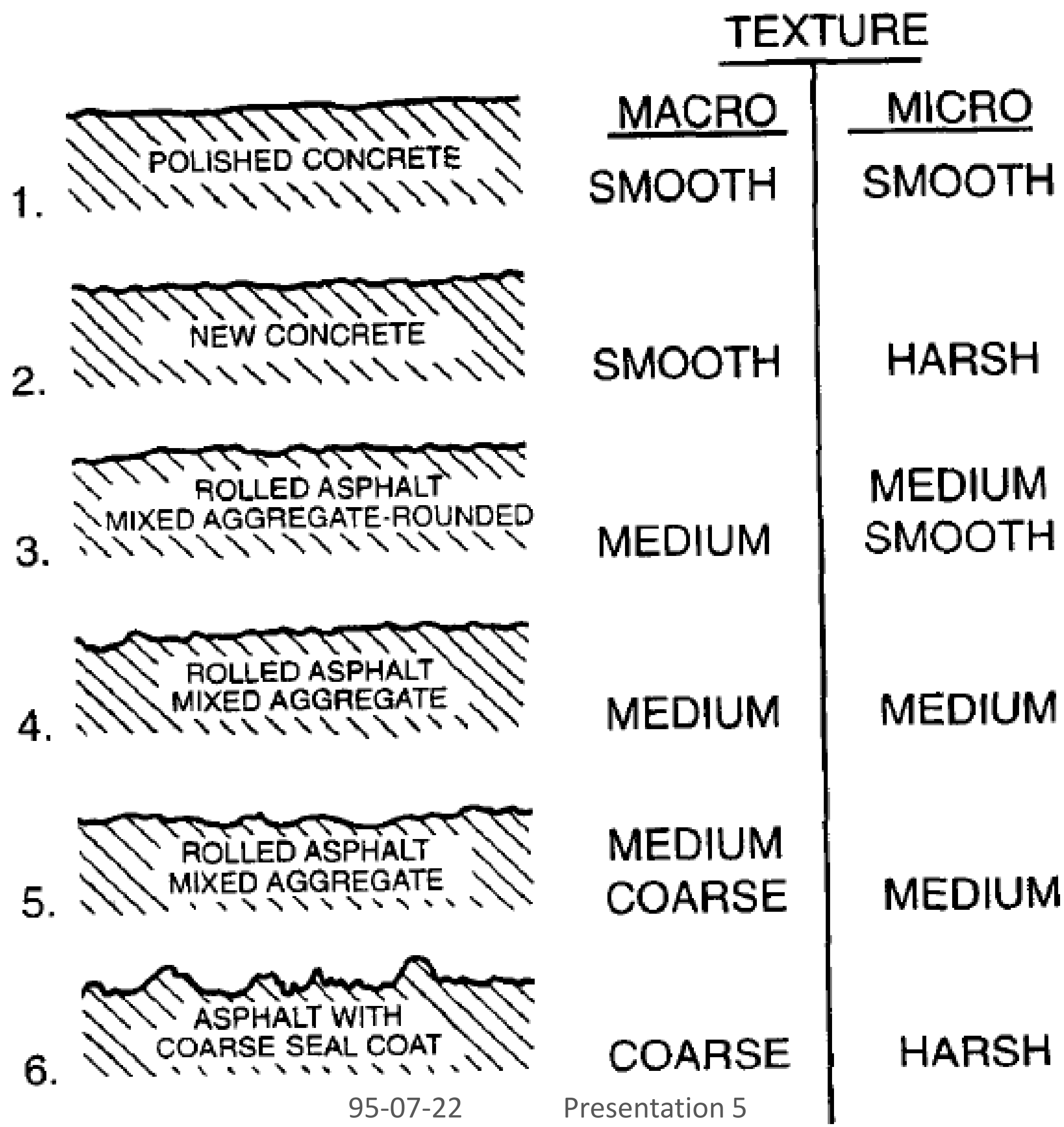


Coefficient of Rolling Resistance

Tires of same size under rated load and inflation pressure

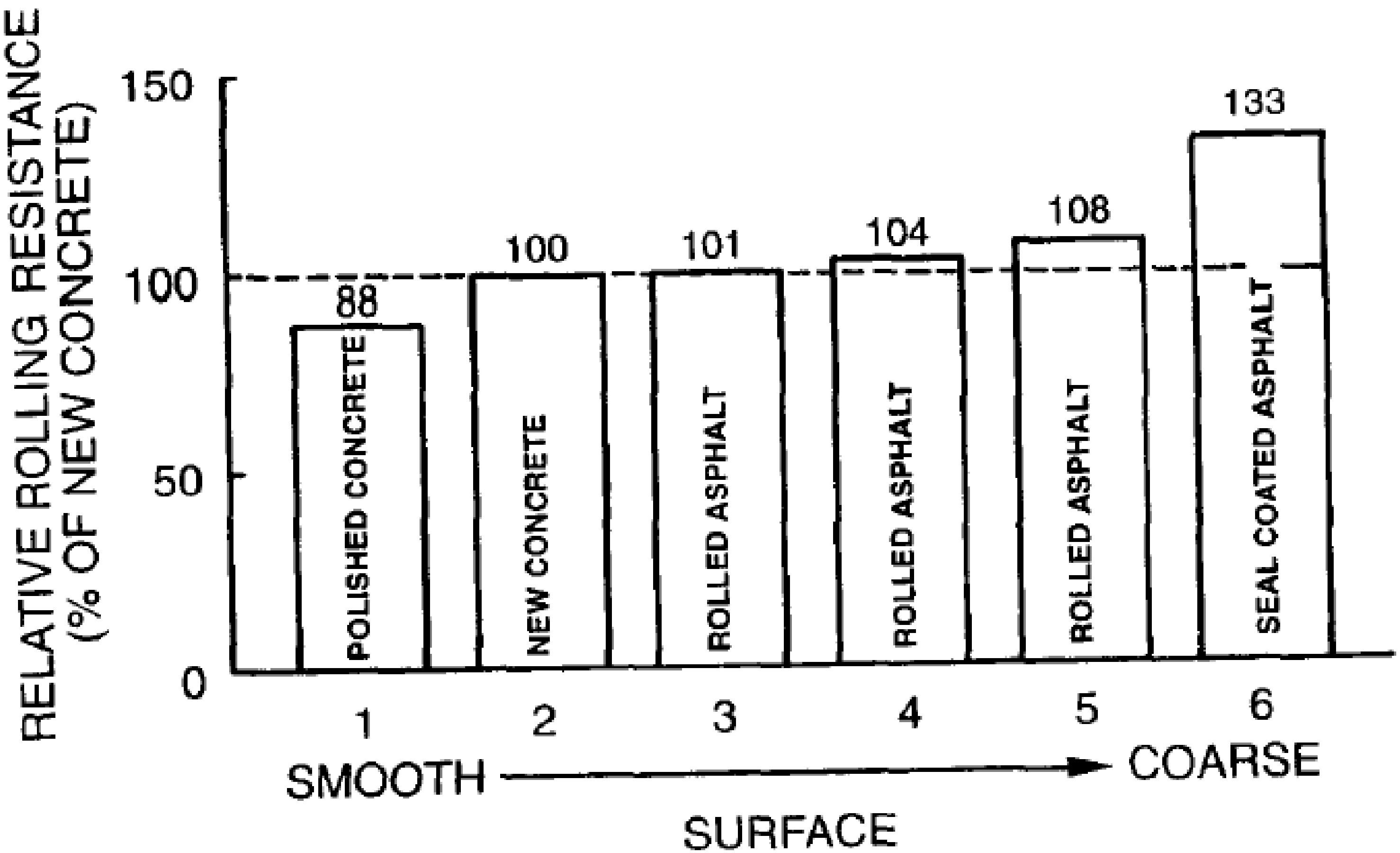


Coefficient of Rolling Resistance



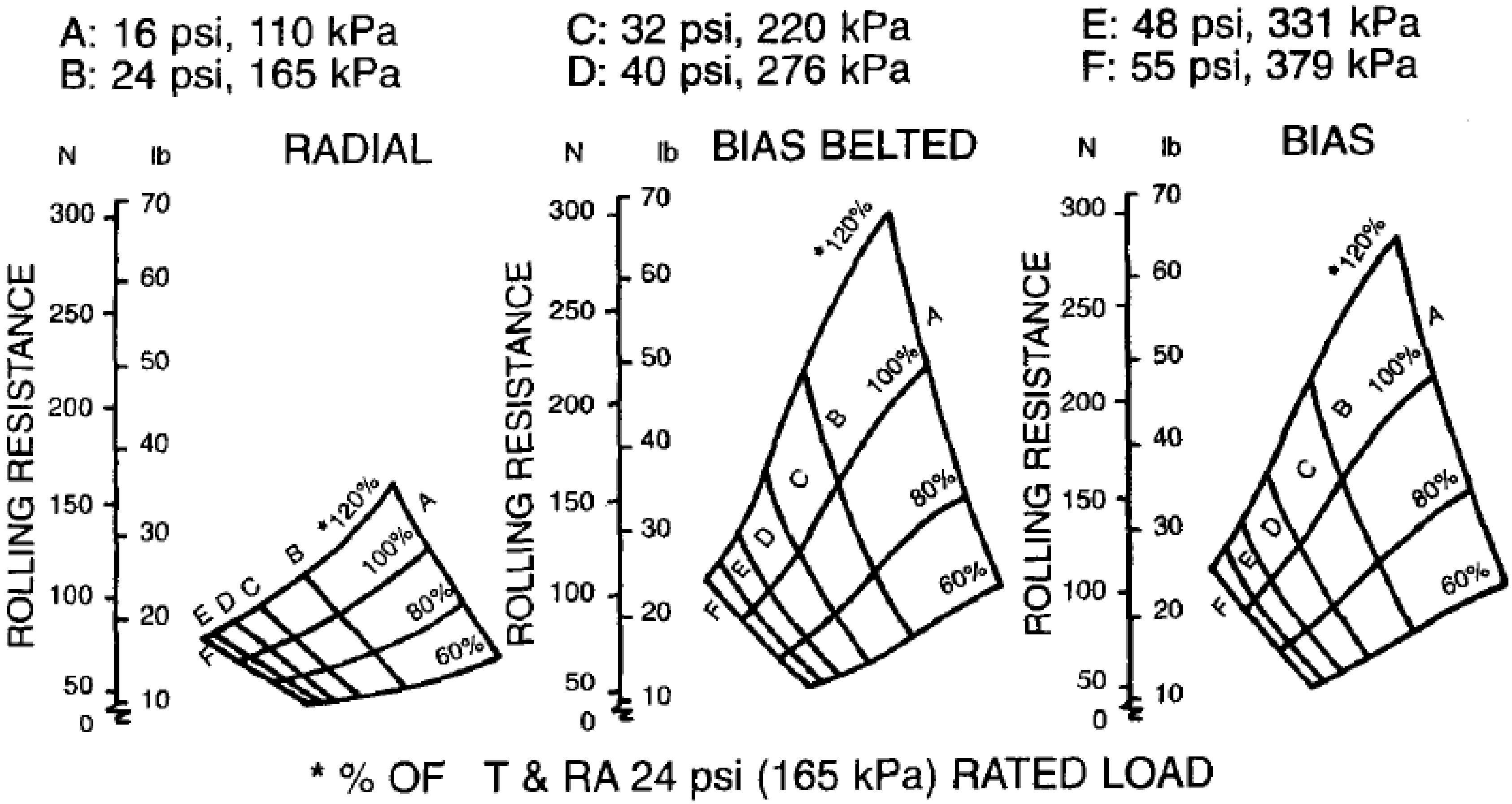
Coefficient of Rolling Resistance

Surface Texture of Hard Road Pavements



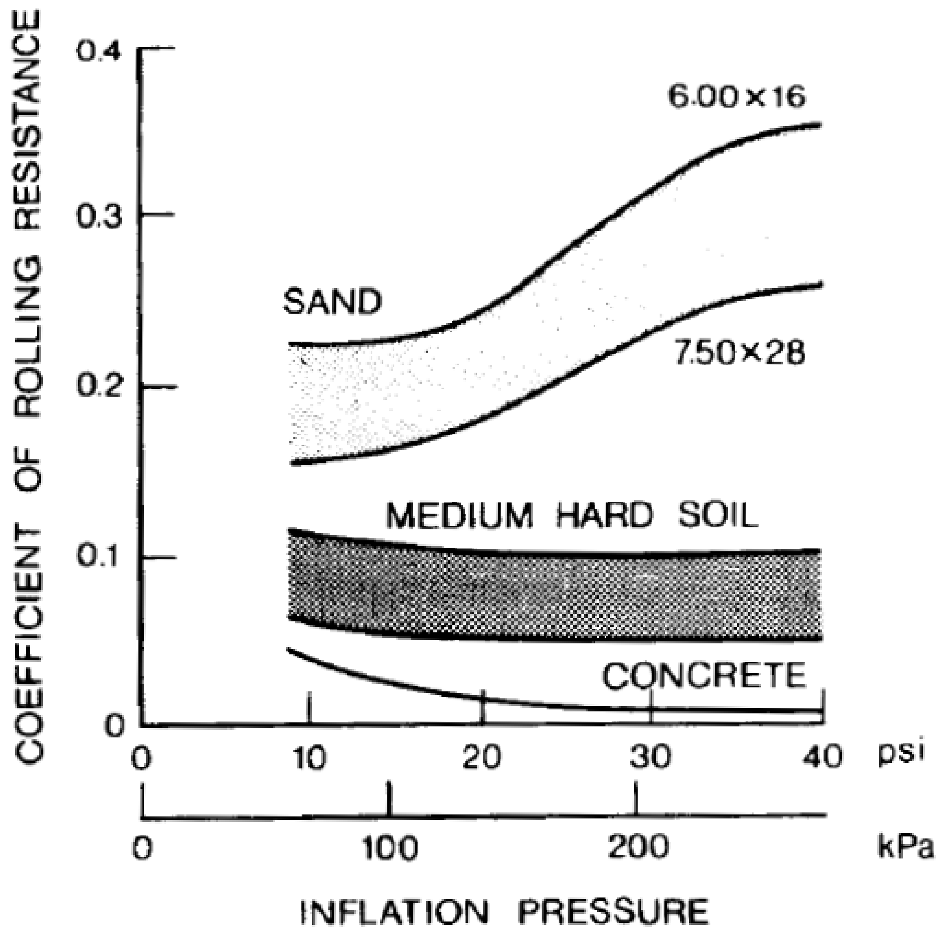
Coefficient of Rolling Resistance

Variation of rolling resistance of radial-ply, bias-belted, and bias-ply car tires with load and inflation pressure



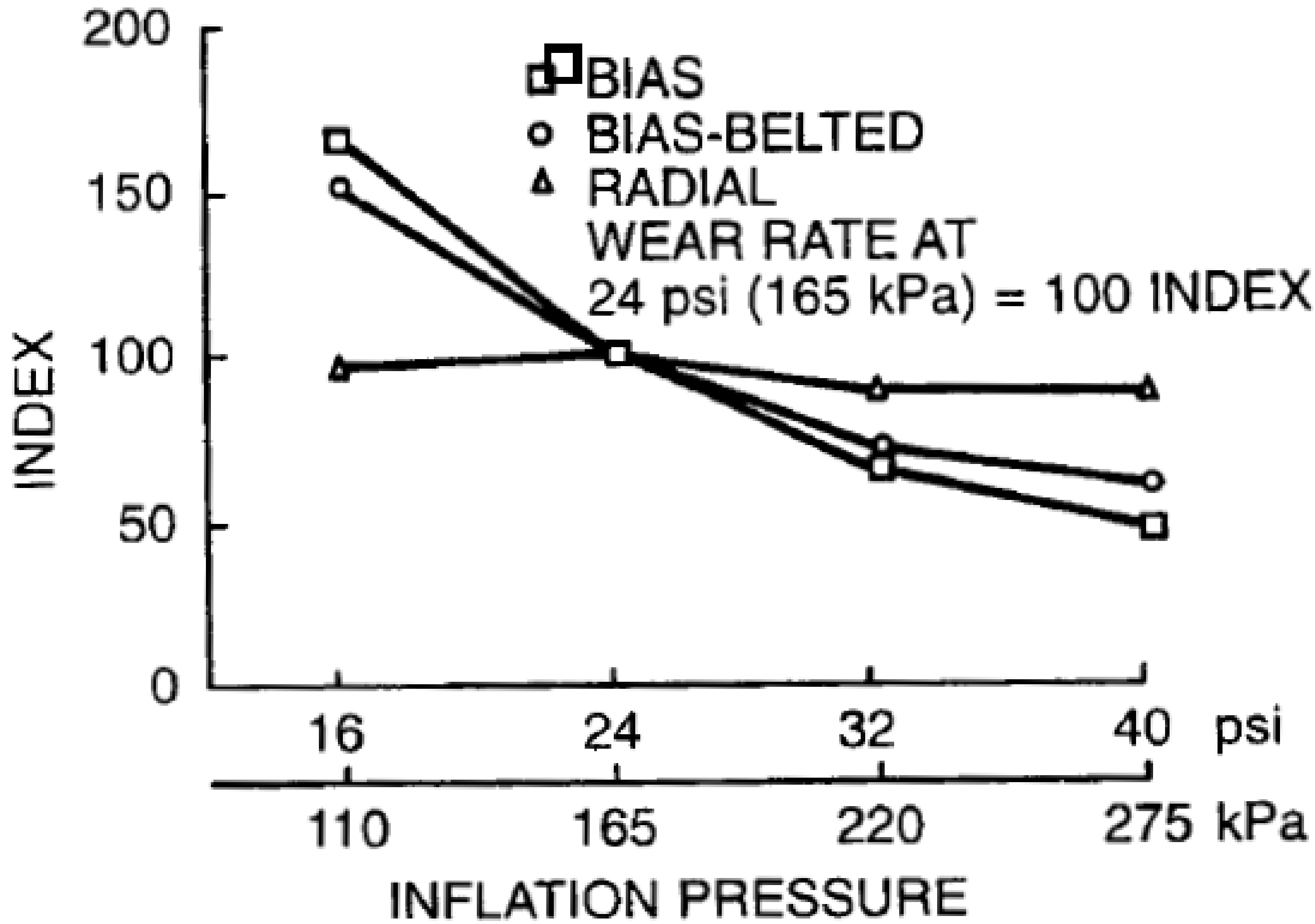
Coefficient of Rolling Resistance

Variation of rolling resistance of types of deformable ground with inflation pressure



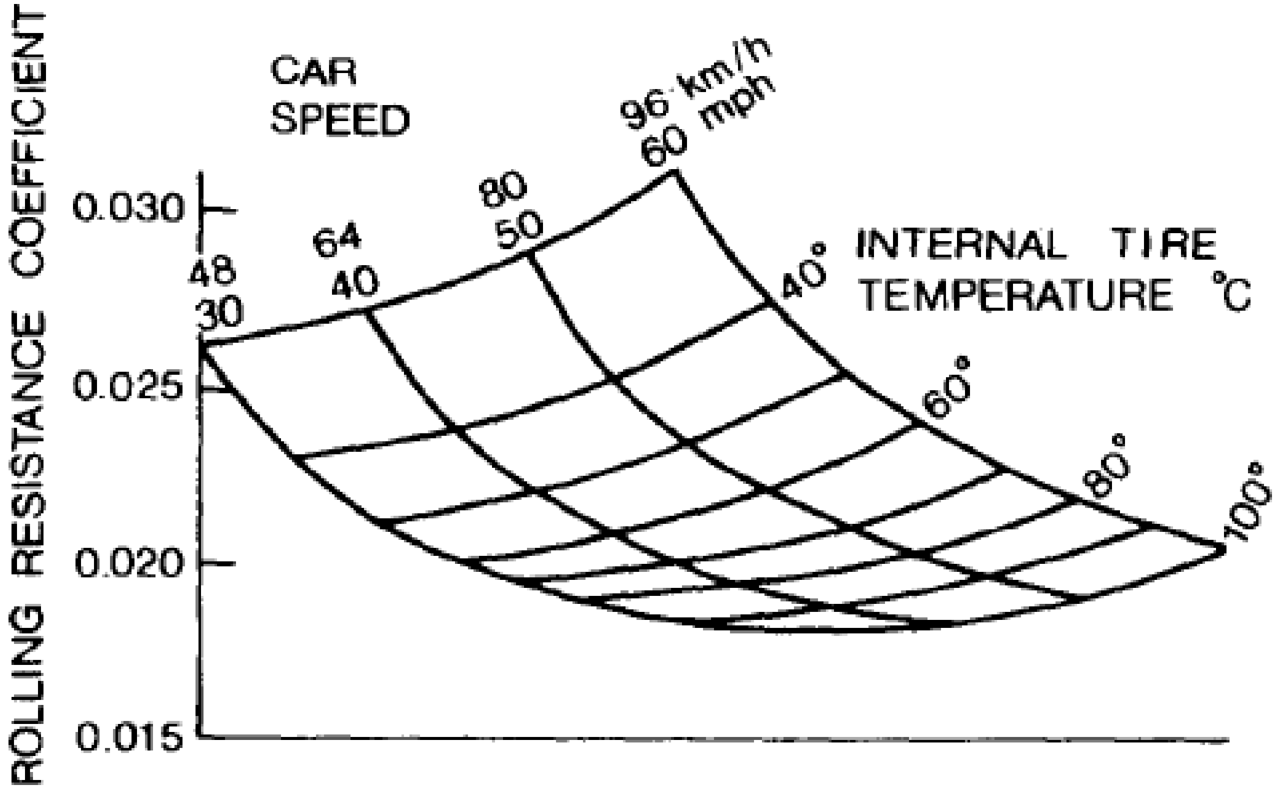
Coefficient of Rolling Resistance

Tread Wear



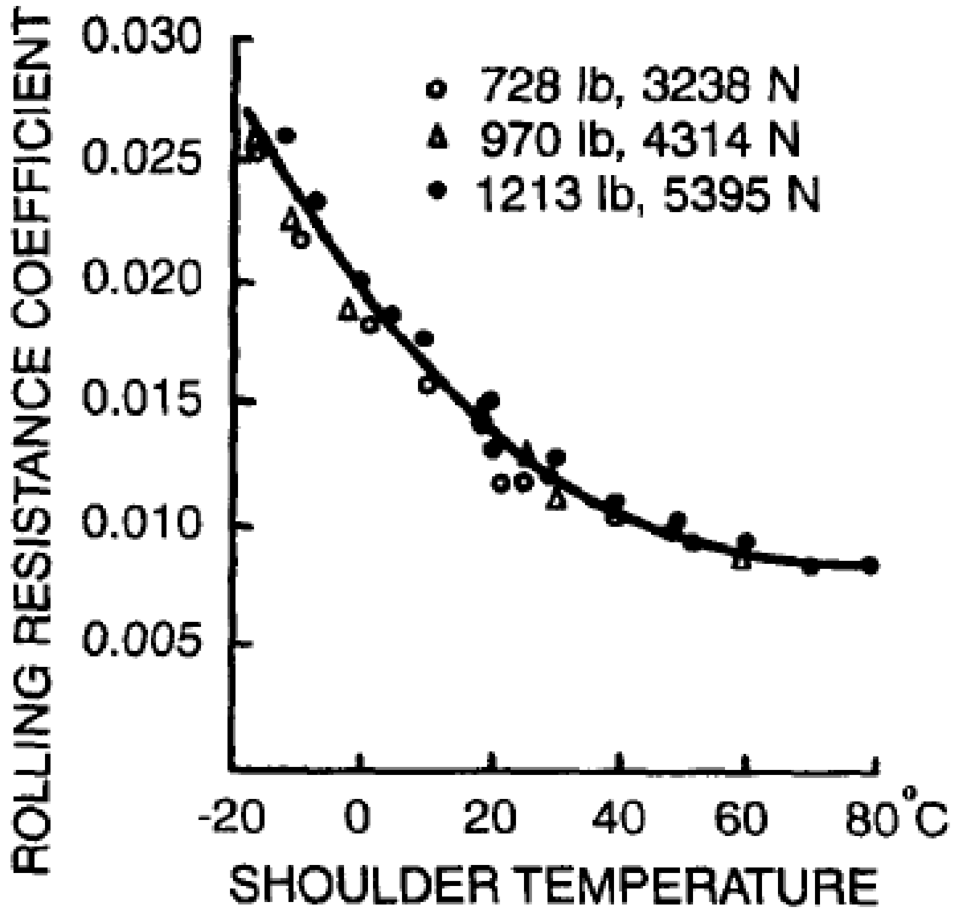
Coefficient of Rolling Resistance

Car tire



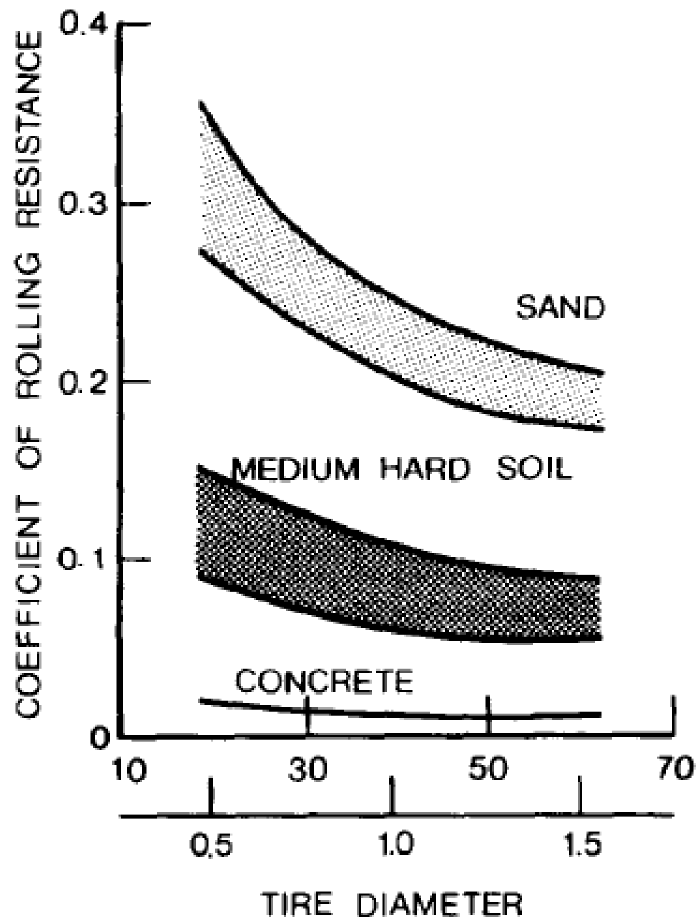
Coefficient of Rolling Resistance

Car tire



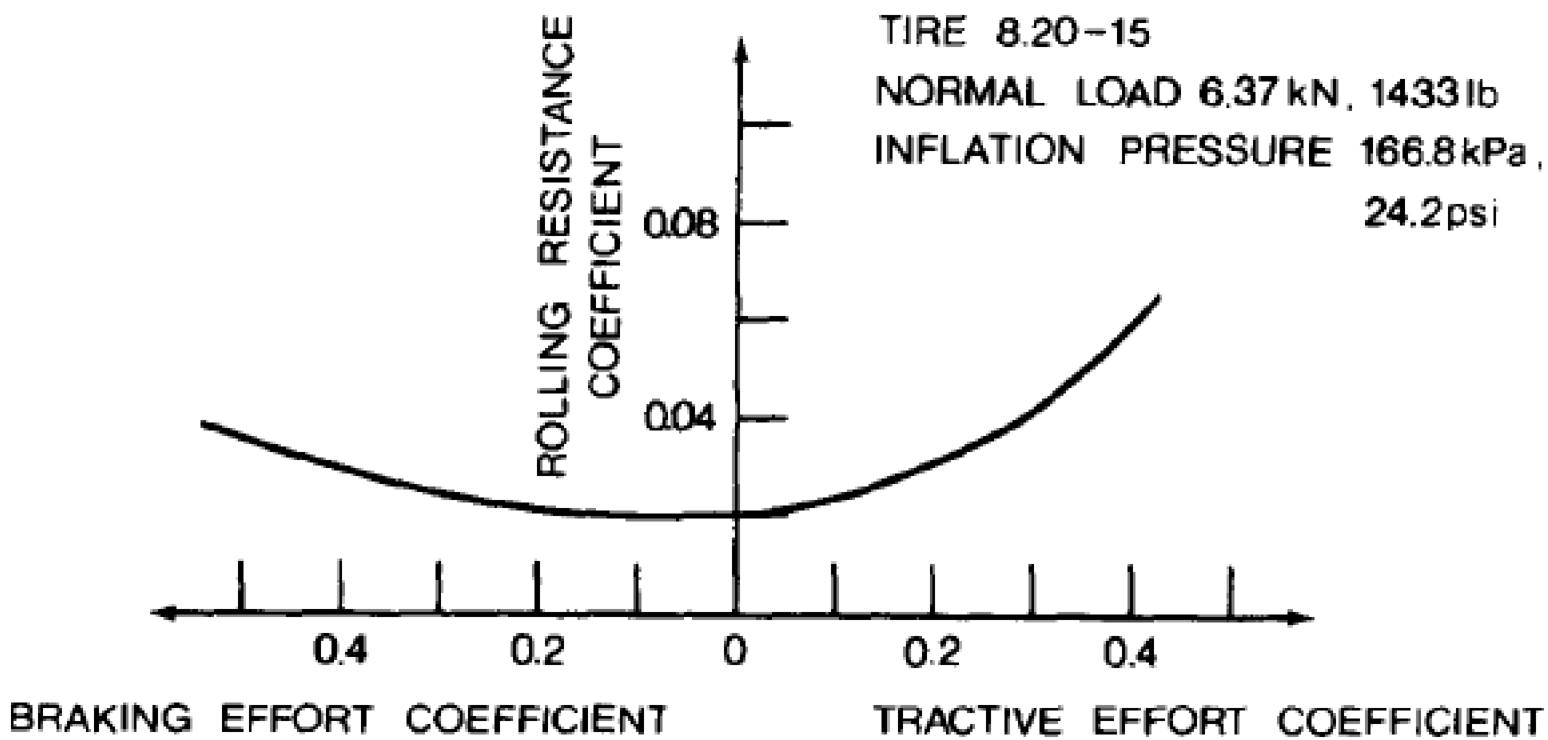
Coefficient of Rolling Resistance

Car tire



Coefficient of Rolling Resistance

Truck tire



Coefficient of Rolling Resistance

Empirical formulas

radial-ply passenger car tires

$$f_r = 0.0136 + 0.40 \times 10^{-7}\, V^2$$

bias-ply passenger car tires

$$f_r = 0.0169 + 0.19 \times 10^{-6}\, V^2$$

radial-ply truck tires

$$f_r = 0.006 + 0.23 \times 10^{-6}\, V^2$$

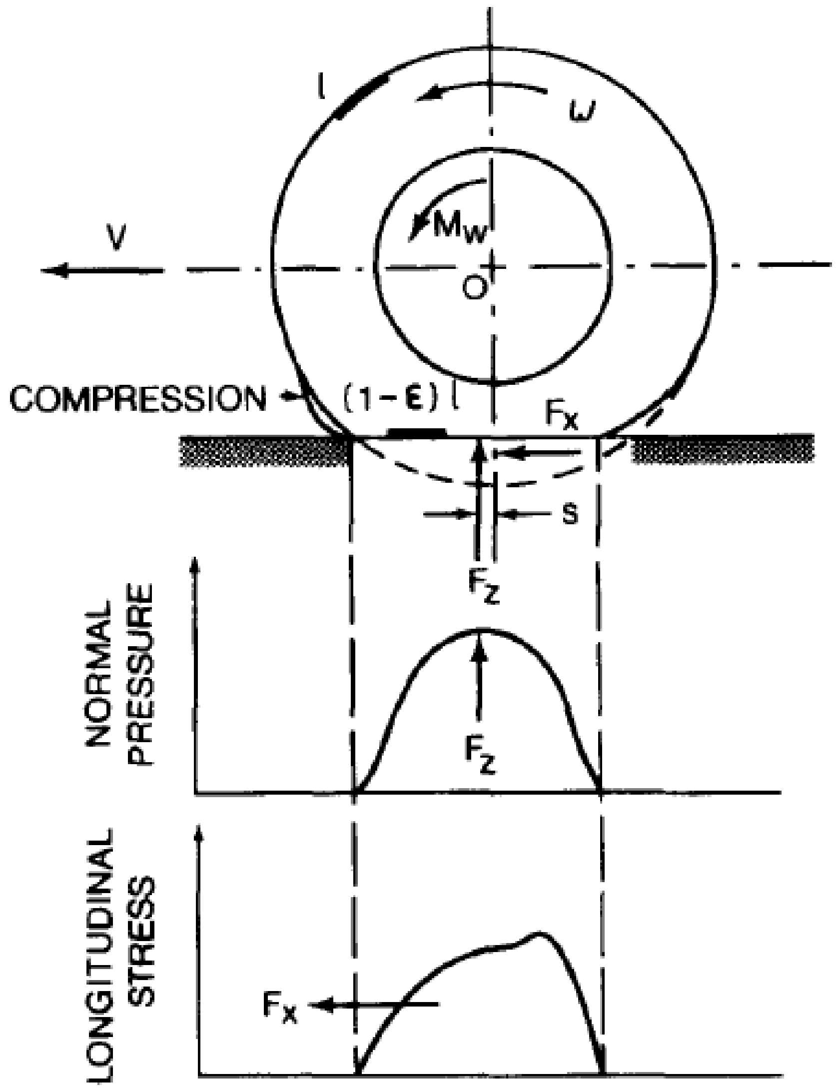
bias-ply truck tires

$$f_r = 0.007 + 0.45 \times 10^{-6}\, V^2$$

V is in km/h

Tractive Effort

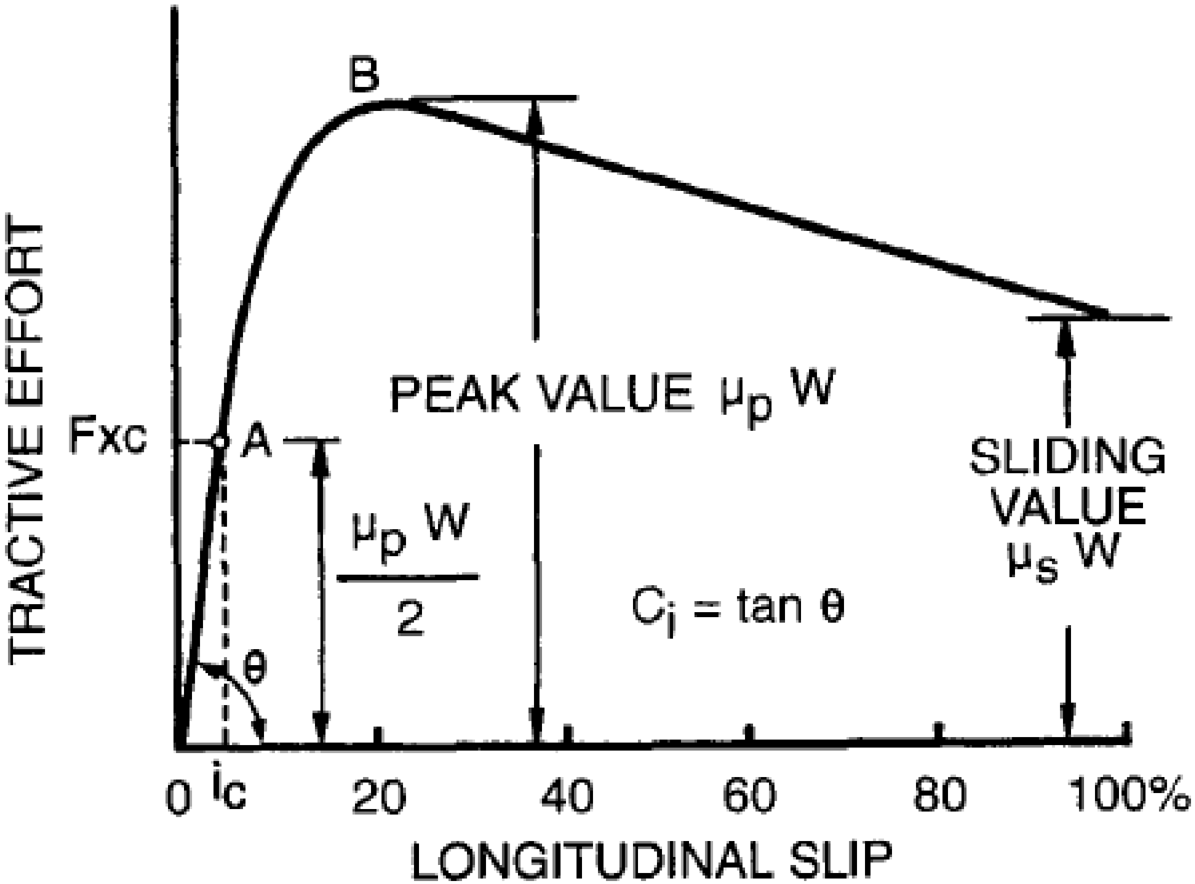
تایر در حال شتابگیری



Tractive Effort

$$i = \left(1 - \frac{V}{r\omega}\right) \times 100\% = \left(1 - \frac{r_e}{r}\right) \times 100\%$$

لغزش طولی



Tractive Effort

تئوری اصلی (Julien)

تغییر شکل طولی در طول سطح تماس

$$e = e_0 + x\epsilon$$

تغییر شکل طولی در ابتدای سطح تماس

$$e_0 = \lambda\epsilon$$

$$e = (\lambda + x)\epsilon$$

نیروی طولی دیفرانسیلی بر واحد طول المان

$$\frac{dF_x}{dx} = k_t e = k_t(\lambda + x)\epsilon$$

برآیند نیروی طولی

$$F_x = \int_0^x k_t(\lambda + x)\epsilon\, dx = k_t \lambda x \epsilon \left(1 + \frac{x}{2\lambda}\right)$$

Tractive Effort

تئوری اصلی (Julien)

نیروی رانش طولی (بدون لغزش)

$$F_x = \int_0^x k_t(\lambda + x)\epsilon \, dx = k_t \lambda x \epsilon \left(1 + \frac{x}{2\lambda}\right)$$

$$F_x = k_t \lambda l_t \epsilon \left(1 + \frac{l_t}{2\lambda}\right) = K_t \epsilon$$

$$x \le l_c = \frac{\mu_p p b}{k_t \epsilon} - \lambda = \frac{\mu_p W}{l_t k_t \epsilon} - \lambda$$

لبه انتهایی سطح تماس در آستانه لغزش

$$l_t = l_c = \frac{\mu_p W}{l_t k_t i} - \lambda$$

$$i_c = \frac{\mu_p W}{l_t k_t (l_t + \lambda)}$$

$$F_{xc} = \frac{\mu_p W[1 + (l_t/2\lambda)]}{1 + (l_t/\lambda)}$$

Tractive Effort

تئوری اصلی (Julien)

در ناحیه غیرخطی

$$F_{xs} = \mu_p W(1 - l_c/l_t)$$

$$F_{xa} = k_t \lambda i l_c \left(1 + \frac{l_c}{2\lambda}\right)$$

$$F_x = F_{xs} + F_{xa} = \mu_p W - \frac{\lambda(\mu_p W - K'i)^2}{2l_t K'i} \qquad K' = l_t k_t \lambda$$

مشخص کردن مقدار پارامتر λ در این تئوری نیاز، زحمت زیادی دارد و باید با انجام اندازه گیری های دقیقی صورت گیرد.

Tractive Effort

تئوری ساده سازی شده با صرف نظر از λ

در ناحیه خطی

نیروی طولی دیفرانسیلی
بر واحد طول المان

$$\frac{dF_x}{dx} = k_t x \epsilon = k_t x i$$

$$F_x = \int_0^{l_t} k_t i x \, dx = (k_t l_t^2/2) i$$

$$\frac{k_t l_t^2}{2} = C_i = \tan\,\theta = \left.\frac{\partial F_x}{\partial i}\right|_{i=0}$$

$$F_x = C_i i$$

Tractive Effort

تئوری ساده سازی شده با صرف نظر از λ

نیروی طولی دیفرانسیلی بر واحد طول المان

$$\frac{dF_x}{dx} = k_t l_t i = \mu_p p b = \frac{\mu_p W}{l_t}$$

آستانه لغزش

$$i_c = \frac{\mu_p W}{k_t l_t^2} = \frac{\mu_p W}{2C_i}$$

$$F_{xc} = C_i i_c = \frac{\mu_p W}{2}$$

Tractive Effort

تئوری ساده سازی شده با صرف نظر از λ

ناحیه غیرخطی

$$F_{xs} = \mu_p W \left(1 - \frac{l_c}{l_t}\right) = \mu_p W \left(1 - \frac{\mu_p W}{2C_i i}\right)$$

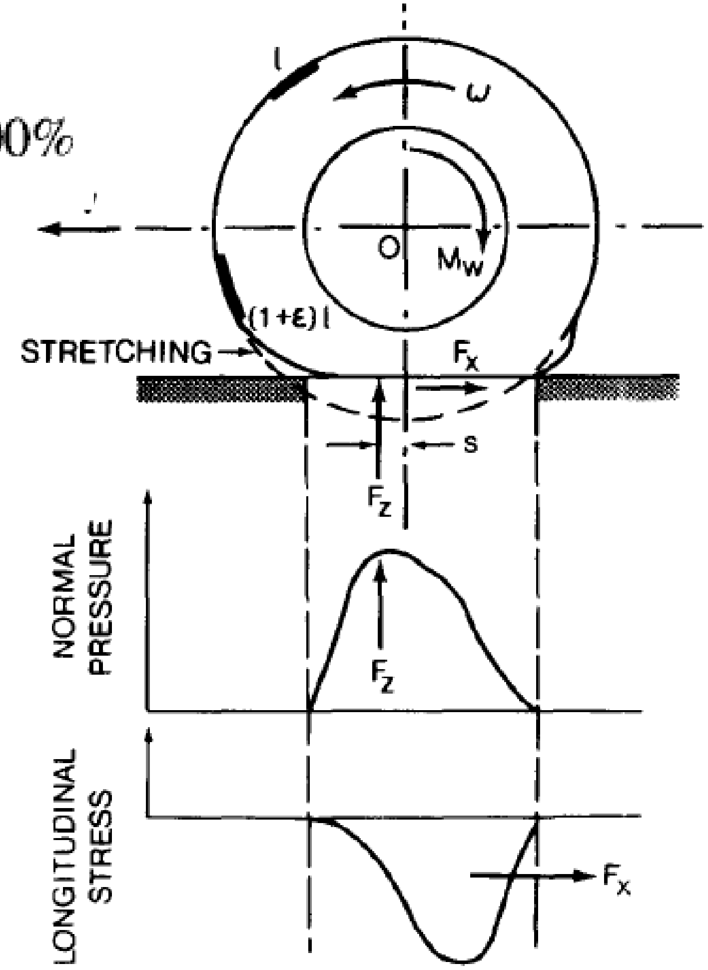
$$F_{xa} = \frac{1}{2}\frac{\mu_p W l_c}{l_t} = \frac{\mu_p^2 W^2}{4C_i i}$$

$$F_x = F_{xs} + F_{xa} = \mu_p W \left(1 - \frac{\mu_p W}{4C_i i}\right)$$

Braking Effort

تئوری ساده سازی شده (برای تایر در ترمزگیری)

$$i_s = \left(1 - \frac{r\omega}{V}\right) \times 100\% = \left(1 - \frac{r}{r_e}\right) \times 100\%$$



Braking Effort

تئوری ساده سازی شده (برای تایر در ترمزگیری)

$$|i| = |i_s/(1 - i_s)|$$

بدون لغزش

$$F_x = C_s i_s/(1 - i_s) \qquad C_s = \left.\frac{\partial F_x}{\partial i_s}\right|_{i_s=0}$$

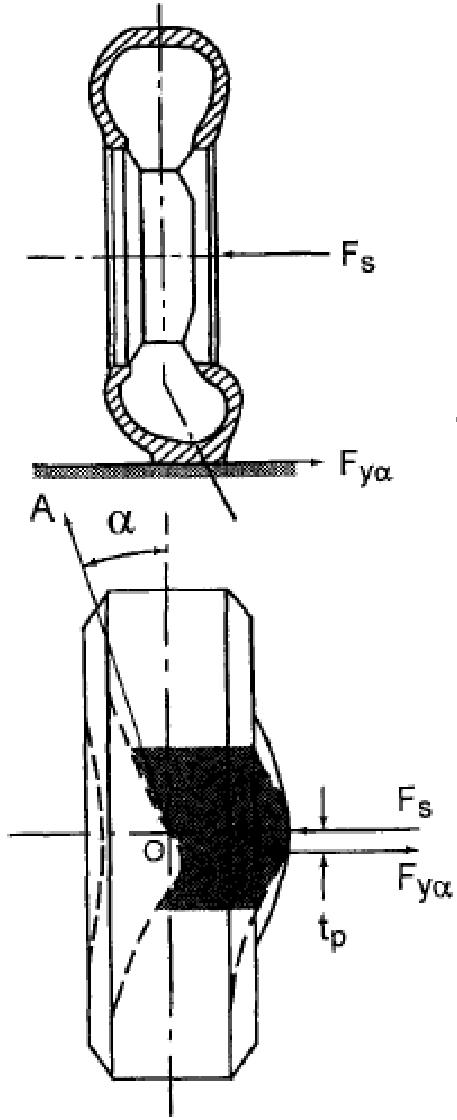
آستانه لغزش

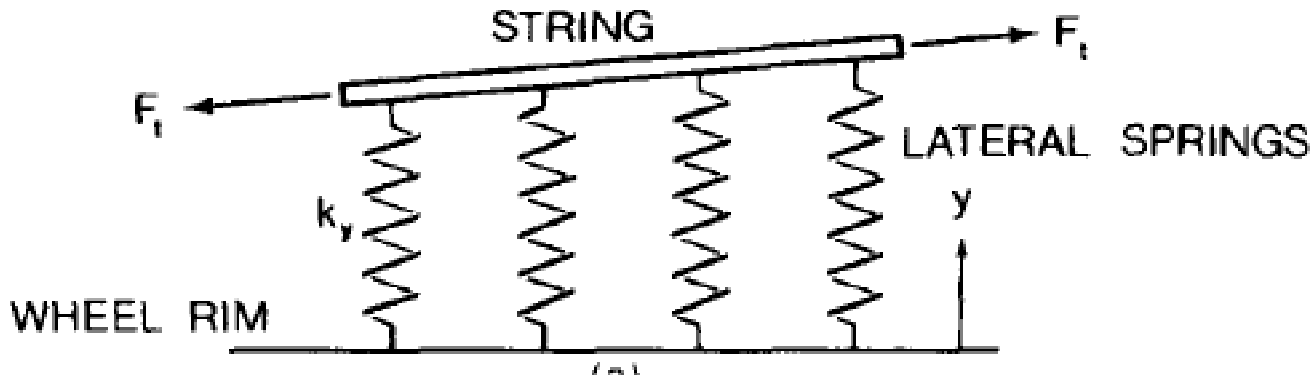
$$i_{sc} = \frac{\mu_p W}{2C_s + \mu_p W} \qquad F_{xc} = \frac{C_s i_{sc}}{1 - i_{sc}} = \frac{\mu_p W}{2}$$

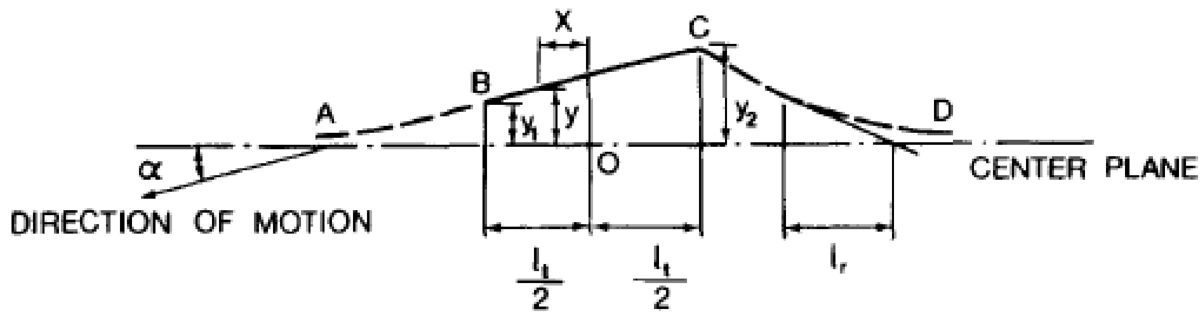
با لغزش

$$F_x = \mu_p W\left[1 - \frac{\mu_p W(1 - i_s)}{4C_s i_s}\right]$$

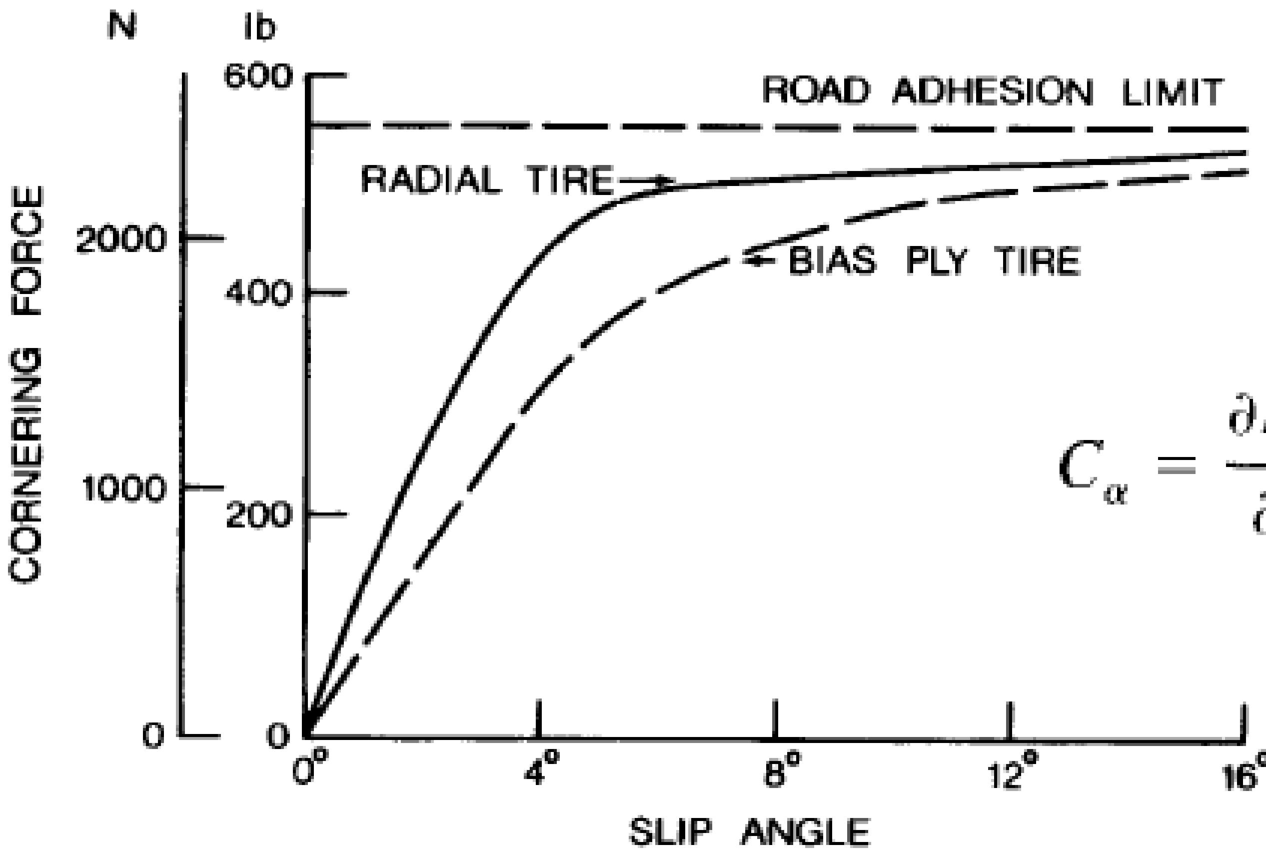
Cornering Properties





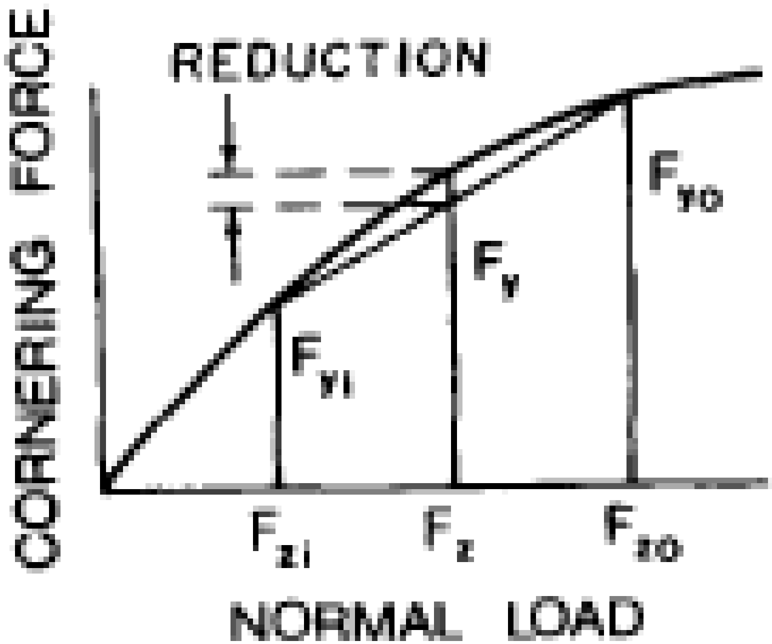


Cornering Properties

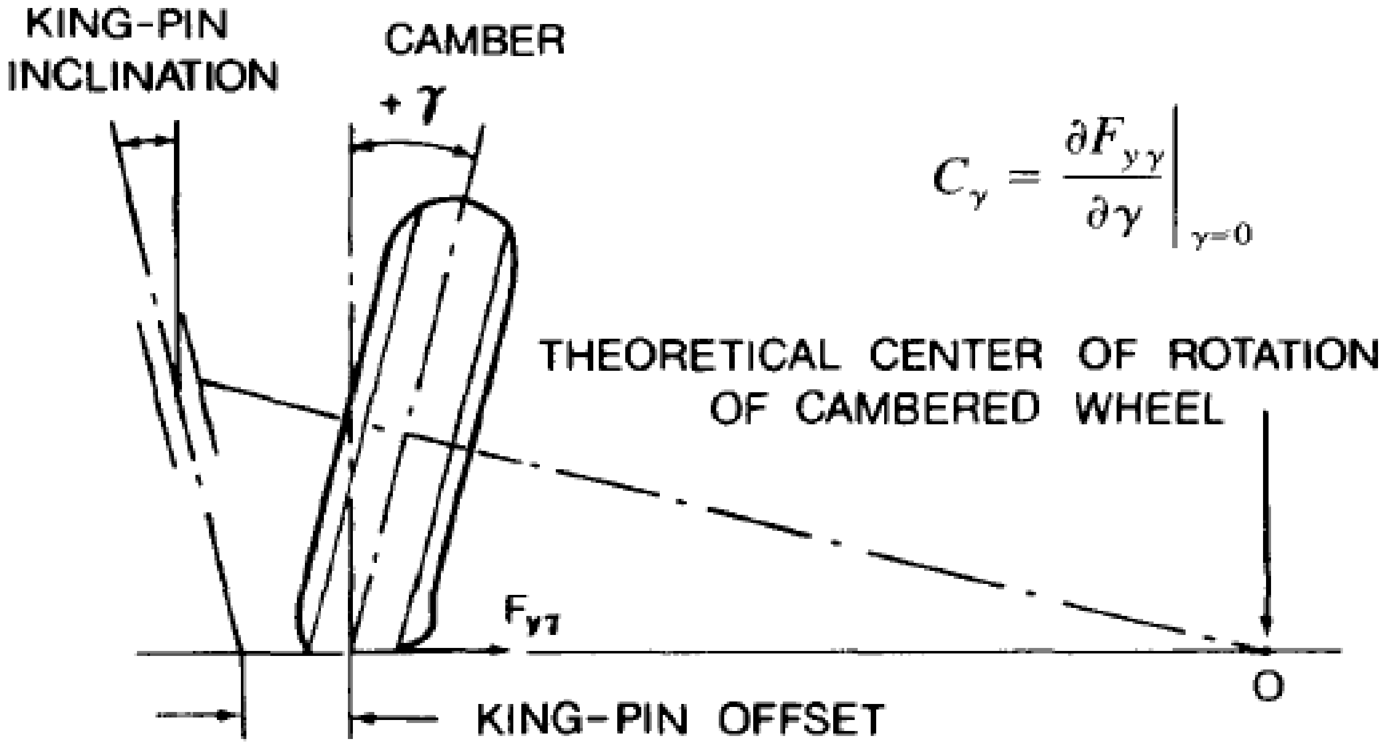


$$C_\alpha = \left.\frac{\partial F_{y\alpha}}{\partial \alpha}\right|_{\alpha=0}$$

Cornering Properties



Camber Thrust



$$C_\gamma = \left.\frac{\partial F_{y\gamma}}{\partial \gamma}\right|_{\gamma=0}$$

$$F_y = F_{y\alpha} \pm F_{y\gamma}$$

$$F_y = C_\alpha \alpha \pm C_\gamma \gamma$$

Cornering Properties

مولفه ناشی از تغییر شکل جانبی المان سطح تماس

$$dF_{y1} = k_y y \, dx$$

مولفه ناشی از کشش محیطی سطح تماس (مدل طناب)

$$dF_{y2} = -F_t \frac{d^2 y}{dx^2} dx$$

$$F_t = k_y l_r^2$$

برآیند نیرو باید در خارج سطح تماس با زمین صفر باشد

$$k_y \left(y - l_r^2 \frac{d^2 y}{dx^2} \right) = 0$$

حل معادله دیفرانسیل فوق:

$$y = \frac{y_2 \sinh [(x - l_t/2)/l_r] + y_1 \sinh [(l_t/2 + l_h - x)/l_r]}{\sinh (l_h/l_r)}$$

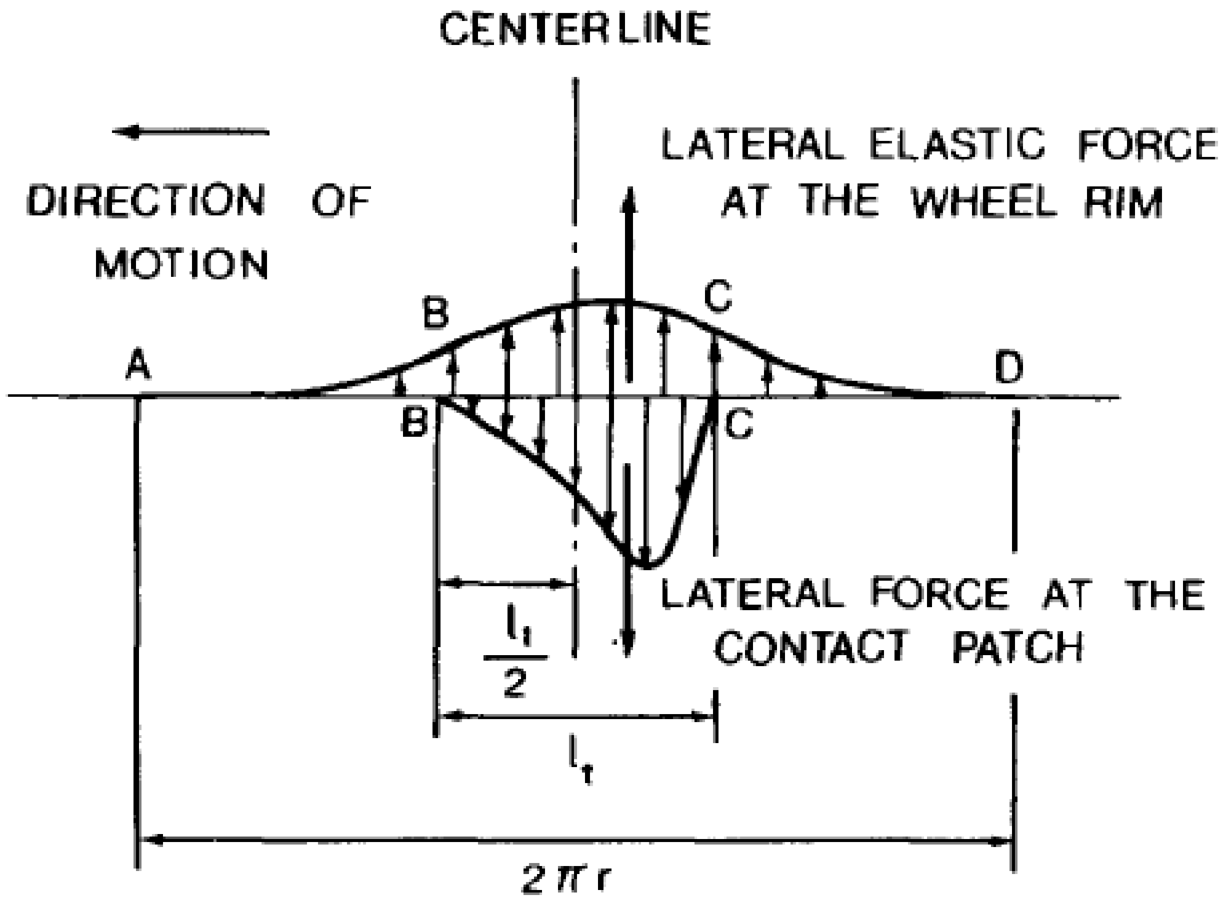
پروفیل ناحیه جلو سطح تماس

$$y = y_1 \exp \left[\frac{-(x - l_t/2)}{l_r} \right] \qquad x > l_t/2$$

پروفیل ناحیه عقب سطح تماس

$$y = y_2 \exp \left[\frac{-(l_t/2 + l_h - x)}{l_r} \right] \qquad x < l_t/2 + l_h$$

Cornering Properties



Cornering Properties

تئوری Temple

مولفه های نیروی عرضی تایر در سطح تماس

$$k_y\left(y - l_r^2 \frac{d^2y}{dx^2}\right) \begin{cases} dF_{y1} = k_y y\, dx \\ dF_{y2} = -F_t \dfrac{d^2y}{dx^2}\, dx \end{cases}$$

نیروی عرضی تایر در سطح تماس

$$F_y = k_y \int_{-l_t/2}^{l_t/2} \left(y - l_r^2 \frac{d^2y}{dx^2}\right) dx$$

$$= k_y \int_{-l_t/2}^{l_t/2} y\, dx - k_y l_r^2 \left(\frac{dy}{dx}\right)\Bigg]_{-l_t/2}^{l_t/2}$$

$$= k_y (y_1 + y_2) l_t/2 + k_y l_r (y_1 + y_2)$$

$$= k_y (y_1 + y_2)(l_r + l_t/2)$$

Cornering Properties

تئوری Temple

مولفه های نیروی عرضی تایر در سطح تماس

$$k_y\left(y - l_r^2\frac{d^2y}{dx^2}\right)\begin{cases} dF_{y1} = k_y y\, dx \\ dF_{y2} = -F_t\dfrac{d^2y}{dx^2}\, dx \end{cases}$$

گشتاور نیروی عرضی تایر حول محور قائم در مرکز سطح تماس (گشتاور بازگرداننده)

$$M_z = k_y\int_{-l_t/2}^{l_t/2} x\left(y - l_r^2\frac{d^2y}{dx^2}\right)dx$$

$$= k_y\int_{-l_t/2}^{l_t/2} xy\, dx - k_y l_r^2\left(x\frac{dy}{dx} - y\right)\Bigg]_{-l_t/2}^{l_t/2}$$

$$= k_y\frac{(l_t/2)^2}{3}(y_1 - y_2) + k_y l_r\left(l_r + \frac{l_t}{2}\right)(y_1 - y_2)$$

$$= k_y(y_1 - y_2)\left[\frac{(l_t/2)^2}{3} + l_r\left(l_r + \frac{l_t}{2}\right)\right]$$

Cornering Properties

تئوری Temple

در غیاب لغزش در سطح تماس

$$\alpha \simeq \tan\alpha = \frac{y_1 - y_2}{l_t} = -\frac{y_1}{l_r}$$

با جایگذاری این مقدار در رابطه نیروی عرضی تایر و گشتاور بازگرداننده:

$$\frac{F_y}{\alpha} = 2k_y\left(l_r + \frac{l_t}{2}\right)^2$$

$$\frac{M_z}{\alpha} = k_y l_t\left[\frac{(l_t/2)^2}{3} + l_r\left(l_r + \frac{l_t}{2}\right)\right]$$

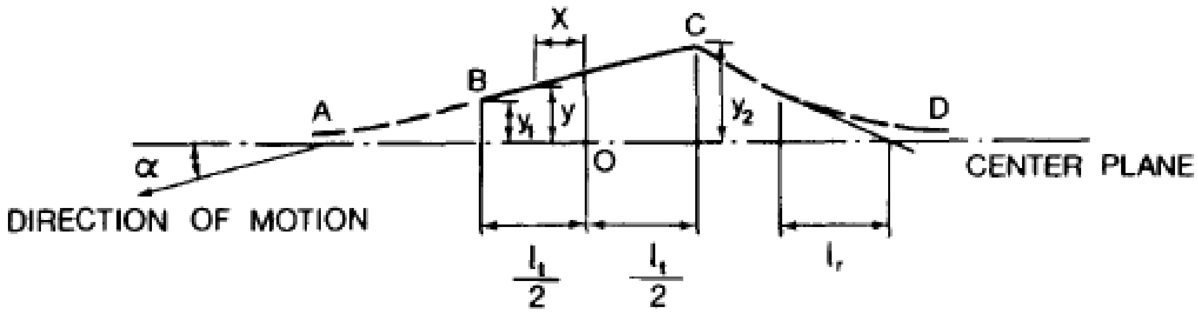
Pneumatic trail:

$$t_p = \frac{M_z}{F_y} = \frac{(l_t/2)\,[(l_t/2)^2/3 + l_r\,(l_r + l_t/2)]}{(l_r + l_t/2)^2}$$

Cornering Properties

تئوری ساده سازی شده (در غیاب لغزش)

$$y' = x \tan \alpha$$



نیروی عرضی دیفرانسیلی
بر واحد طول المان

$$\frac{dF_{y\alpha}}{dx} = k_y' x \tan \alpha$$

$$F_{y\alpha} = \int_0^{l_t} k_y' x \tan \alpha \, dx$$

$$= (k_y' l_t^2 / 2) \tan \alpha$$

$$F_{y\alpha} = C_\alpha \tan \alpha \approx C_\alpha \alpha \qquad \frac{k_y' l_t^2}{2} = C_\alpha = \left.\frac{\partial F_{y\alpha}}{\partial \alpha}\right|_{\alpha=0}$$

Cornering Properties

تئوری ساده سازی شده (آستانه لغزش)

$$\alpha_c = \frac{\mu_p W}{2C_\alpha} \qquad F_{y\alpha c} = \frac{\mu_p W}{2}$$

با وجود لغزش

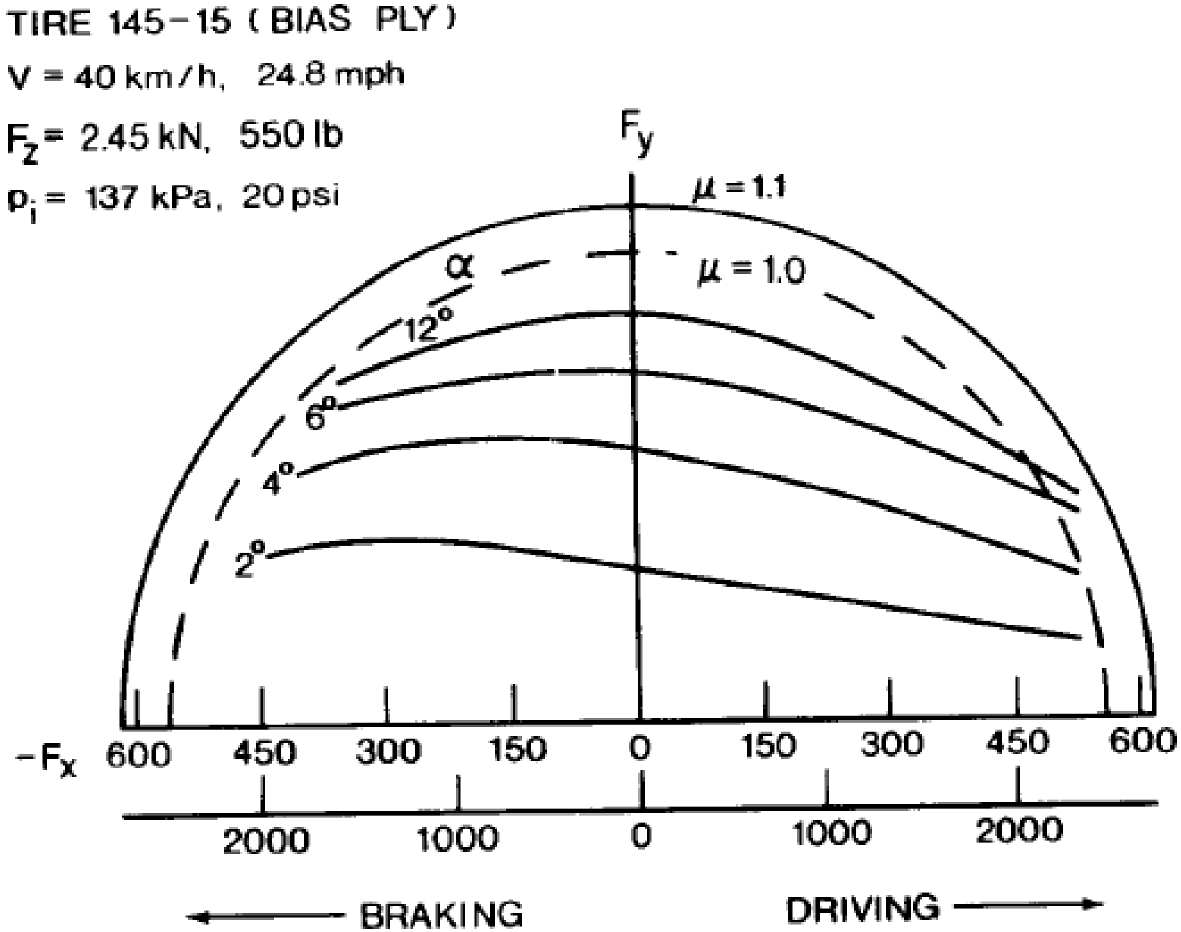
$$F_{y\alpha} = \mu_p W\left(1 - \frac{\mu_p W}{4C_\alpha \tan\alpha}\right) = \mu_p W\left(1 - \frac{\mu_p W}{4C_\alpha \alpha}\right)$$

نمونه ای از روابط تجربی برای تخمین نیروهای عرضی تایر

$$F_{y\alpha} = c_1\alpha + c_2\alpha^2 + c_3\alpha^3$$

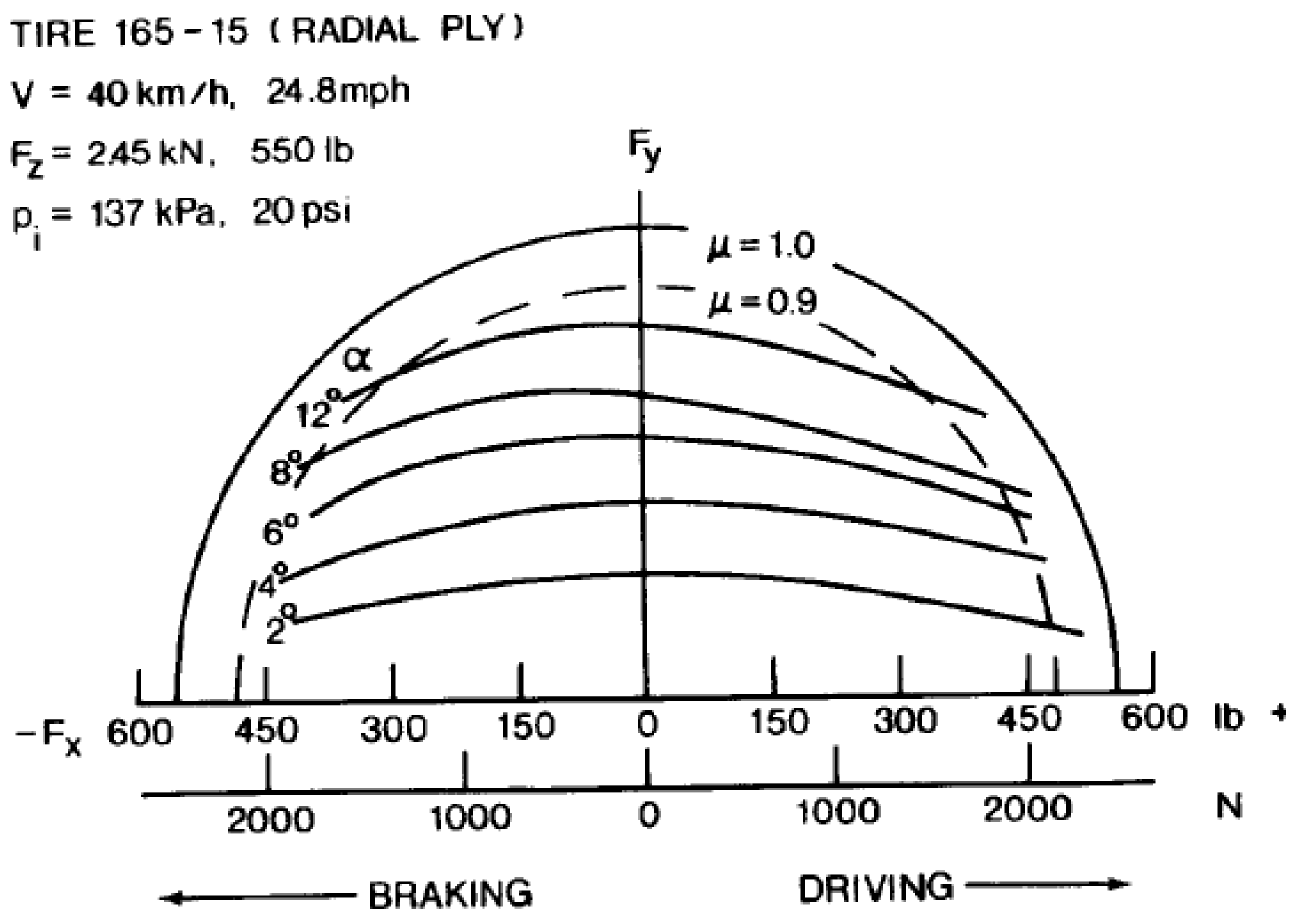
Friction Ellipse

تفاوت قابلیت ایجاد نیروی عرضی تایر بایاس در شتابگیری و ترمزگیری

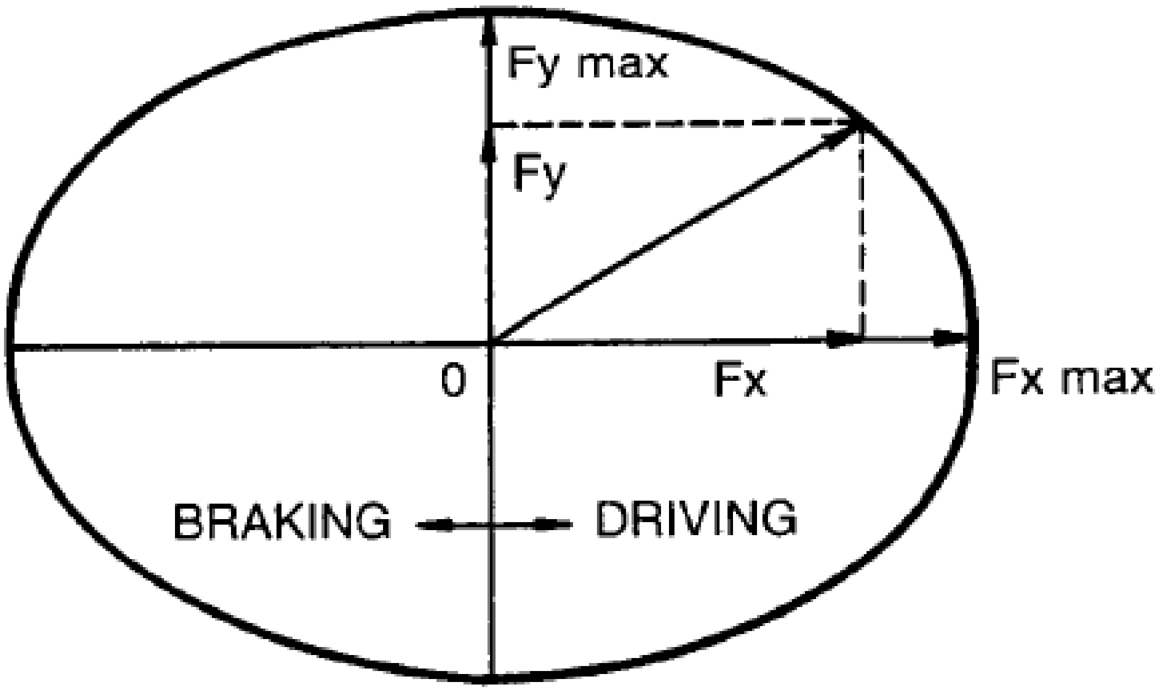


Friction Ellipse

تفاوت نامحسوس قابلیت ایجاد نیروی عرضی تایر رادیال در شتابگیری و ترمزگیری

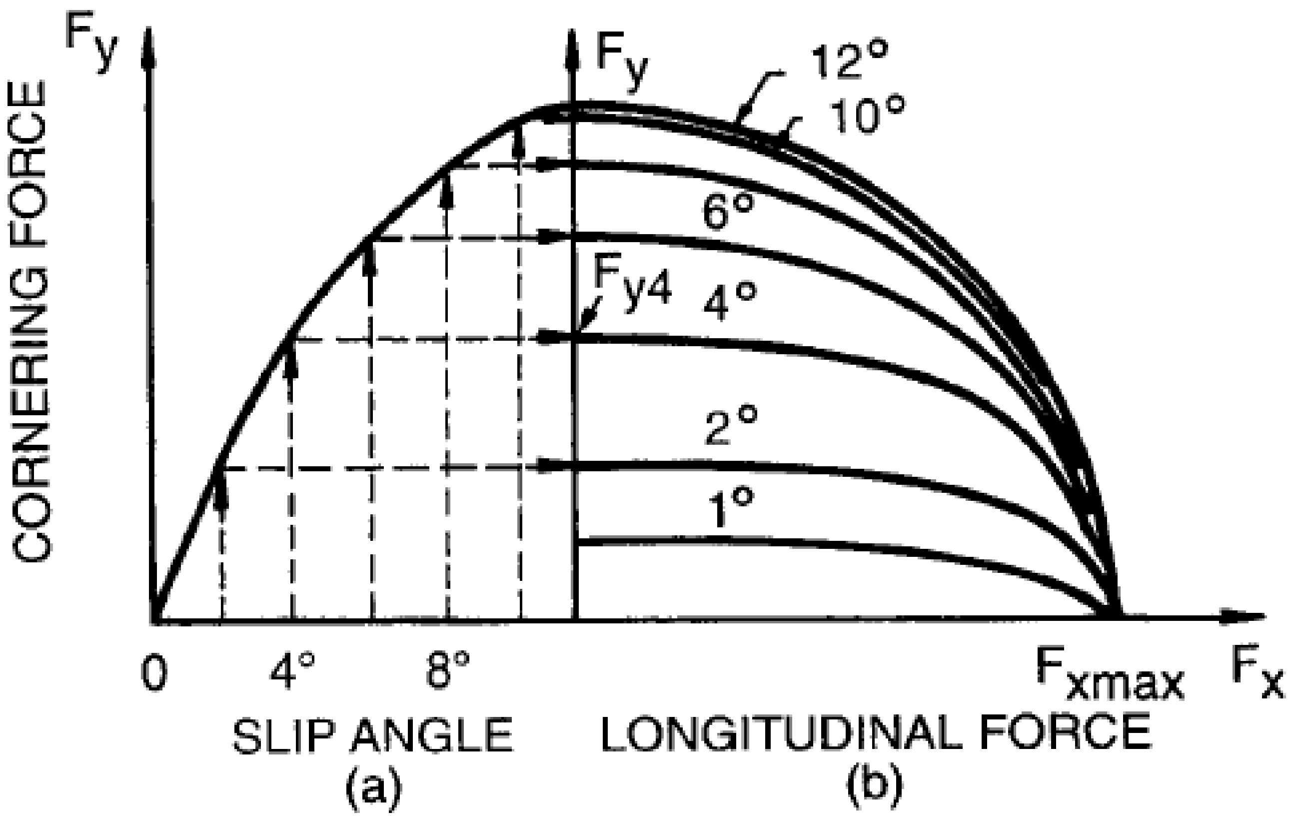


Friction Ellipse



روابطی که تا اینجا نوشته شد، مربوط به حرکت طولی خالص یا حرکت عرضی خالص بود و از مفهوم بیضی اصطکاک برای تعریف حرکت طولی و عرضی ترکیبی استفاده می شود.

Friction Ellipse



$$(F_y/F_{y4})^2 + (F_x/F_{xmax})^2 = 1$$

Combined Cornering and Braking

ترکیب تئوری های ساده سازی شده حرکت طولی خالص و حرکت عرضی خالص

نیروی طولی دیفرانسیلی بر واحد طول المان $$\frac{dF_x}{dx} = k_t x i_s/(1 - i_s)$$

میزان انحراف جانبی المان $$y' = x \tan \alpha/(1 - i_s)$$

نیروی عرضی دیفرانسیلی بر واحد طول المان $$\frac{dF_{y\alpha}}{dx} = k'_y x \tan \alpha/(1 - i_s)$$

بر اساس بیضی اصطکاک، شرط عدم لغزش نقطه ای در فاصله x از جلوی سطح تماس، این است که برآیند نیروهای طولی و عرضی از اصطکاک کمتر باشند:

$$\sqrt{[k_t x i_s/(1 - i_s)]^2 + [k'_y x \tan \alpha/(1 - i_s)]^2} = \mu p b = \frac{\mu W}{l_t}$$

b عرض سطح تماس

l_t طول سطح تماس

p فشار یکنواخت وارده در سطح تماس

μ ضریب چسبندگی تایر و جاده

W بار تایر

Combined Cornering and Braking

ترکیب تئوری های ساده سازی شده حرکت طولی خالص و حرکت عرضی خالص

$$\frac{l_c}{l_t} = \frac{\mu W(1 - i_s)}{2\sqrt{(k_t l_t^2 i_s/2)^2 + (k_y' l_t^2 \tan \alpha/2)^2}}$$

$$k_t l_t^2/2 = C_s \qquad k_y' l_t^2/2 = C_{\alpha'}$$

$$= \frac{\mu W(1 - i_s)}{2\sqrt{(C_s i_s)^2 + (C_\alpha \tan \alpha)^2}}$$

If $l_c / l_t \geq 1$, the entire contact patch is an adhesion region:

$$F_x = \int_0^{l_t} [k_t x i_s/(1 - i_s)]\, dx = k_t l_t^2 i_s/2(1 - i_s)$$

$$= C_s i_s/(1 - i_s)$$

$$F_{y\alpha} = \int_0^{l_t} [k_y' x \tan \alpha/(1 - i_s)]\, dx$$

$$= k_y' l_t^2 \tan \alpha/2(1 - i_s)$$

$$= C_\alpha \tan \alpha/(1 - i_s)$$

Combined Cornering and Braking

ترکیب تئوری های ساده سازی شده حرکت طولی خالص و حرکت عرضی خالص

If $l_c / l_t < 1$, sliding between the tread and the ground will take place:

نیروی طولی در ناحیه چسبندگی $$F_{xa} = \int_0^{l_c} [k_t x i_s/(1 - i_s)]\, dx = \frac{\mu^2 W^2 C_s i_s (1 - i_s)}{4\,[(C_s i_s)^2 + (C_\alpha \tan\alpha)^2]}$$

نیروی طولی در ناحیه لغزش $$F_{xs} = \frac{\mu W C_s i_s}{\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}} \left[1 - \frac{\mu W (1 - i_s)}{2\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}}\right]$$

$$F_x = F_{xa} + F_{xs} = \frac{\mu W C_s i_s}{\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}} \left[1 - \frac{\mu W (1 - i_s)}{4\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}}\right]$$

نیروی عرضی در ناحیه چسبندگی $$F_{y\alpha a} = \int_0^{l_c} [k'_y x \tan\alpha/(1 - i_s)]\, dx = \frac{\mu^2 W^2 C_\alpha \tan\alpha\, (1 - i_s)}{4[(C_s i_s)^2 + (C_\alpha \tan\alpha)^2]}$$

نیروی عرضی در ناحیه لغزش $$F_{y\alpha s} = \frac{\mu W C_\alpha \tan\alpha}{\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}} \left[1 - \frac{\mu W (1 - i_s)}{2\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}}\right]$$

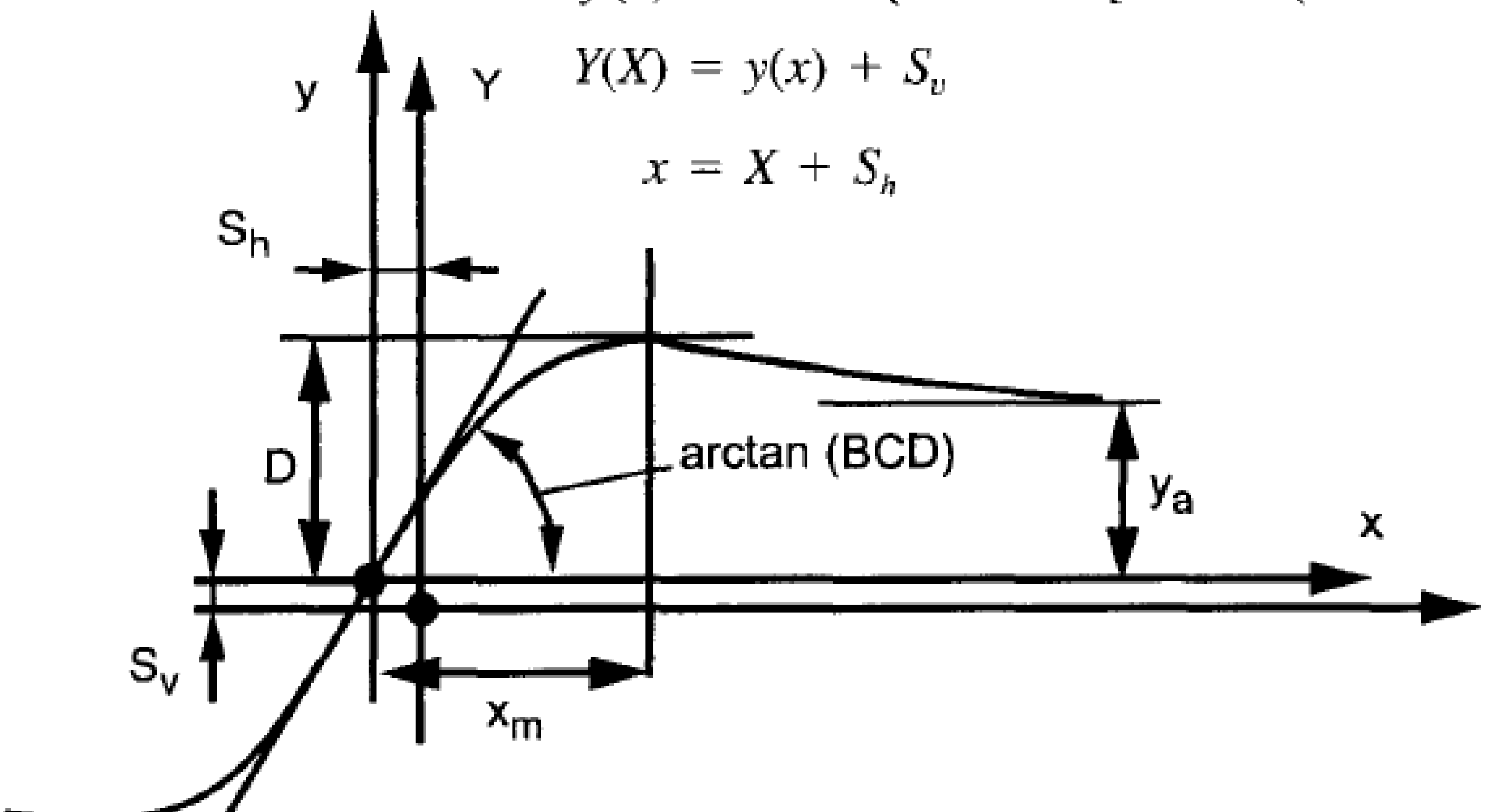
$$F_{y\alpha} = F_{y\alpha a} + F_{y\alpha s} = \frac{\mu W C_\alpha \tan\alpha}{\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}} \left[1 - \frac{\mu W (1 - i_s)}{4\sqrt{(C_s i_s)^2 + (C_\alpha \tan\alpha)^2}}\right]$$

Magic Formula

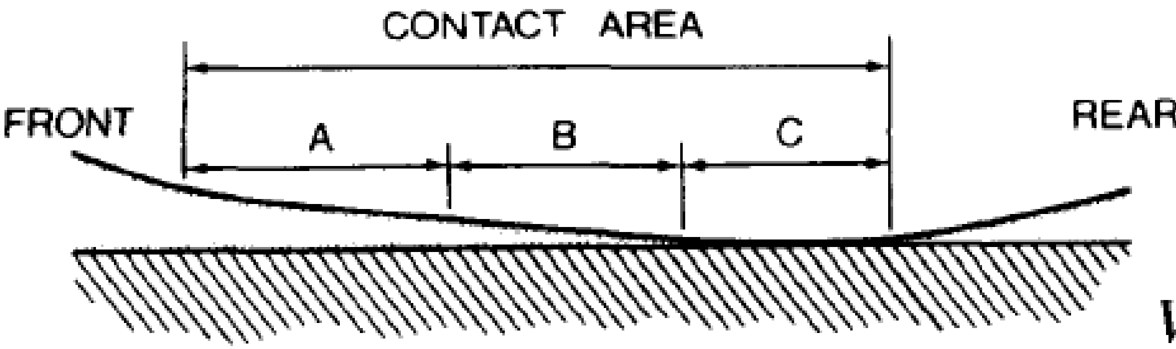
$$y(x) = D \sin \{C \arctan [Bx - E(Bx - \arctan Bx)]\}$$

$$Y(X) = y(x) + S_v$$

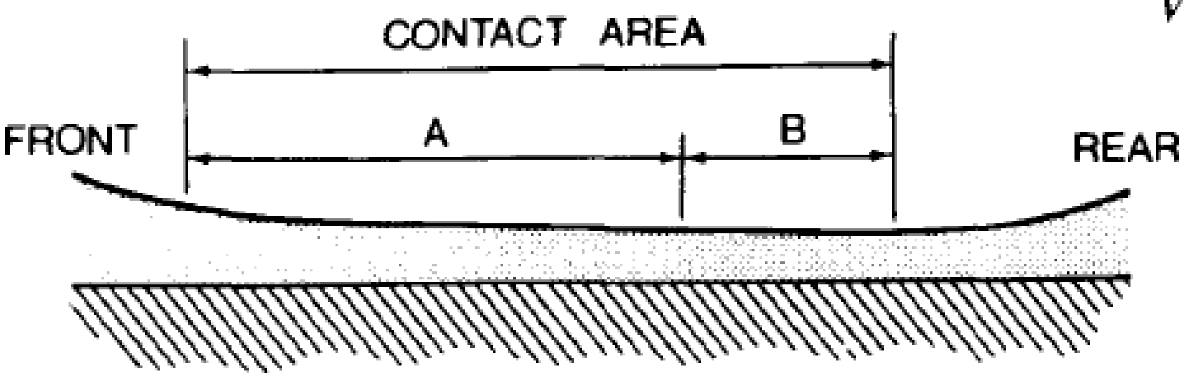
$$x = X + S_h$$



Hydroplaning



(a) PARTIAL HYDROPLANING



(b) COMPLETE HYDROPLANING

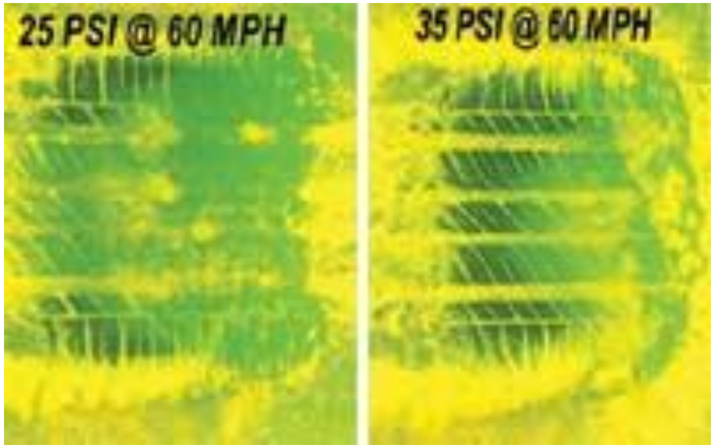
psi

$$V_p = 10.35\sqrt{p_i}\ \text{mph}$$

$$V_p = 6.34\sqrt{p_i}\ \text{km/h}$$

kPa

Hydroplaning



25 PSI @ 60 MPH

30 PSI @ 60 MPH