Statics

Systems of Forces

Statics problems involve a system of balanced forces.

$$\sum F_x = 0 \qquad 10.35$$

$$\sum F_y = 0 \qquad 10.36$$

$$\sum F_z = 0 \qquad 10.37$$

Sample from the *NCEES Handbook:*

STATICS

FORCE

A *force* is a *vector* quantity. It is defined when its **(1)** magnitude, **(2)** point of application, and **(3)** direction are known.

RESULTANT (TWO DIMENSIONS)

The *resultant*, *F*, of *n* forces with components $F_{x,i}$ and $F_{y,i}$ has the magnitude of

$$F = \left[\left(\sum_{i=1}^{n} F_{x,i}\right)^2 + \left(\sum_{i=1}^{n} F_{y,i}\right)^2\right]^{1/2}$$

The resultant direction with respect to the *x*-axis using four-quadrant angle functions is

$$\theta = \arctan\left(\sum_{i=1}^{n} F_{y,i} \Big/ \sum_{i=1}^{n} F_{x,i}\right)$$

The vector form of a force is

$$\boldsymbol{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

RESOLUTION OF A FORCE

$F_x = F \cos \theta_x$; $F_y = F \cos \theta_y$; $F_z = F \cos \theta_z$

$\cos \theta_x = F_x / F$; $\cos \theta_y = F_y / F$; $\cos \theta_z = F_z / F$

Separating a force into components (geometry of force is known $R = \sqrt{x^2 + y^2 + z^2}$)

$F_x = (x/R)F$; $F_y = (y/R)F$; $F_z = (z/R)F$

MOMENTS (COUPLES)

CENTROIDS OF MASSES, AREAS, LENGTHS, AND VOLUMES

Formulas for centroids, moments of inertia, and first moment of areas are presented in the **MATHEMATICS** section for continuous functions. The following discrete formulas are for defined regular masses, areas, lengths, and volumes:

$\boldsymbol{r}_c = \Sigma\, m_n \boldsymbol{r}_n / \Sigma\, m_n$, where

m_n = the *mass of each particle* making up the system,

$\boldsymbol{r}_n$ = the *radius vector* to each particle from a selected reference point, and

$\boldsymbol{r}_c$ = the *radius vector* to the *center of the total mass* from the selected reference point.

The *moment of area* (M_a) is defined as

$$M_{ay} = \Sigma\, x_n a_n$$

$$M_{ax} = \Sigma\, y_n a_n$$

$$M_{az} = \Sigma\, z_n a_n$$

The *centroid of area* is defined as

$$\left.\begin{aligned} x_{ac} &= M_{ay}/A \\ y_{ac} &= M_{ax}/A \\ z_{ac} &= M_{az}/A \end{aligned}\right\} \text{with respect to center of the coordinate system}$$

where $A = \Sigma\, a_n$

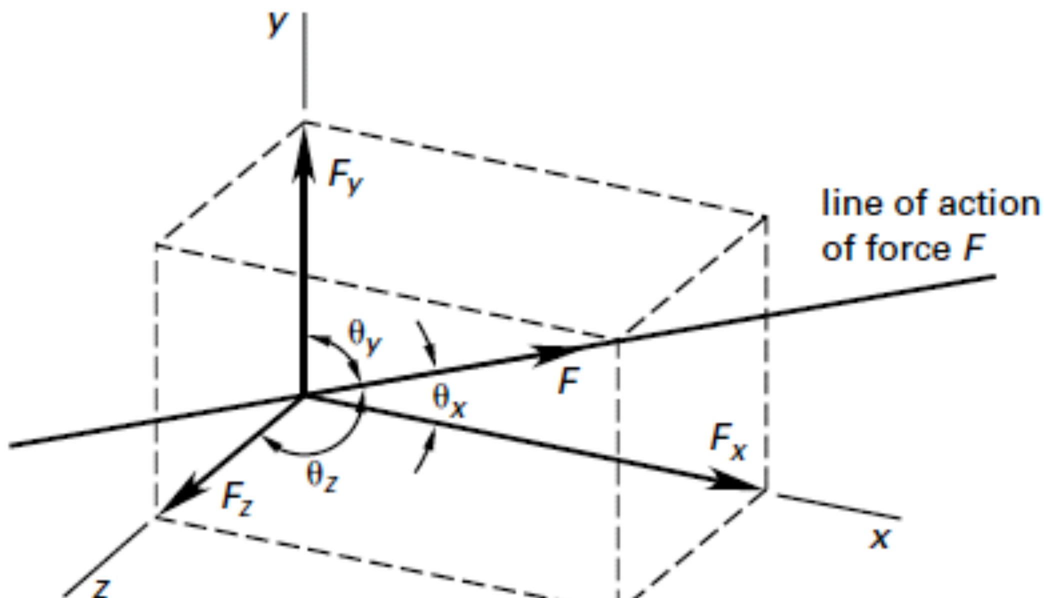
The *centroid of a line* is defined as

$x_{lc} = (\Sigma\, x_n l_n)/L$, where $L = \Sigma\, l_n$

www.ncees.org

Forces

Figure 10.1 Components and Direction Angles of a Force



Resultant Force

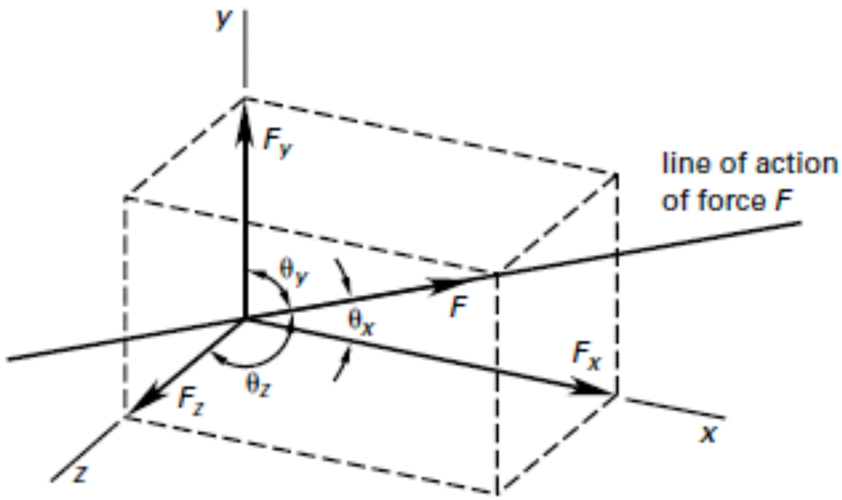
$$\mathbf{F} = \mathbf{i}\sum_{i=1}^{n} F_{x,i} + \mathbf{j}\sum_{i=1}^{n} F_{y,i} \quad \begin{bmatrix}\text{two}\\ \text{dimensional}\end{bmatrix} \qquad 10.2$$

$$R = \sqrt{\left(\sum_{i=1}^{n} F_{x,i}\right)^2 + \left(\sum_{i=1}^{n} F_{y,i}\right)^2} \qquad 10.3$$

$$\theta = \tan^{-1}\left(\frac{\sum_{i=1}^{n} F_{y,i}}{\sum_{i=1}^{n} F_{x,i}}\right) \qquad 10.4$$

Resolution of a Force

Figure 10.1 Components and Direction Angles of a Force



$$F_x = F \cos\theta_x \qquad 10.5$$

$$F_y = F \cos\theta_y \qquad 10.6$$

$$F_z = F \cos\theta_z \qquad 10.7$$

Example Statics Problems

(FESP)

Problem–1

A rigid body in static equilibrium experiences

(a) only small forces.
(b) only large forces.
(c) no balanced forces.
(d) no unbalanced forces.

The answer

Problem–4

All of the following attributes characterize a force except

(a) magnitude.
(b) direction.
(c) line of action.
(d) center of rotation.

The answer

Example Statics Problems

(FESP)

Problem–1

A rigid body in static equilibrium experiences

(a) only small forces.
(b) only large forces.
(c) no balanced forces.
(d) no unbalanced forces.

The answer is (d)

Problem–4

All of the following attributes characterize a force except

(a) magnitude.
(b) direction.
(c) line of action.
(d) center of rotation.

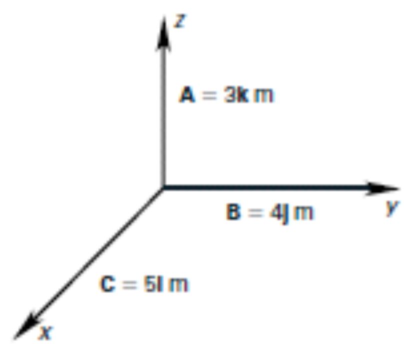
The answer is (d)

Example Statics Problems

(EFPRB)

STATICS–1

What is the length of the vector $\mathbf{A}+\mathbf{B}+\mathbf{C}$, the sum of three orthogonal vectors?



(A) 3.5 m (B) 4.3 m (C) 7.1 m (D) 10 m

$$\begin{aligned} |\mathbf{A}+\mathbf{B}+\mathbf{C}| &= \sqrt{\mathrm{A}^2+\mathrm{B}^2+\mathrm{C}^2} \\ &= \sqrt{(3\text{ m})^2+(4\text{ m})^2+(5\text{ m})^2} \\ &= 7.07\text{ m} \quad (7.1\text{ m}) \end{aligned}$$

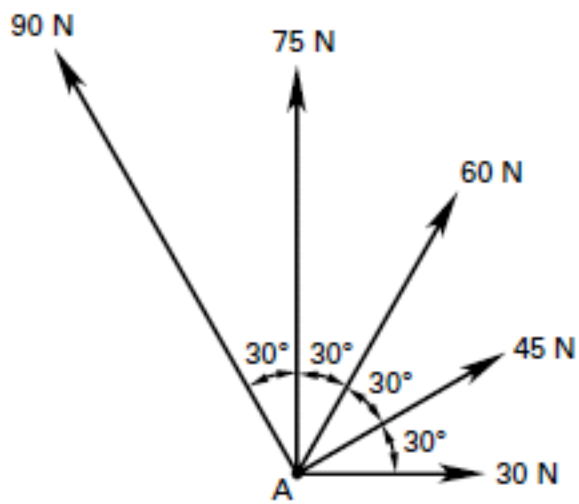
The answer is (C).

Example Statics Problems

FERM prob. 1, p. 10-6

Problem 1

The five forces shown act at point A. What is the magnitude of the resultant force?



(A) 32 N
(B) 156 N
(C) 182 N
(D) 234 N

Solution

$$\sum F_x = 30\text{ N} + (45\text{ N})\cos 30° + (60\text{ N})\cos 60° + (75\text{ N})\cos 90° + (90\text{ N})\cos 120° = 54\text{ N}$$

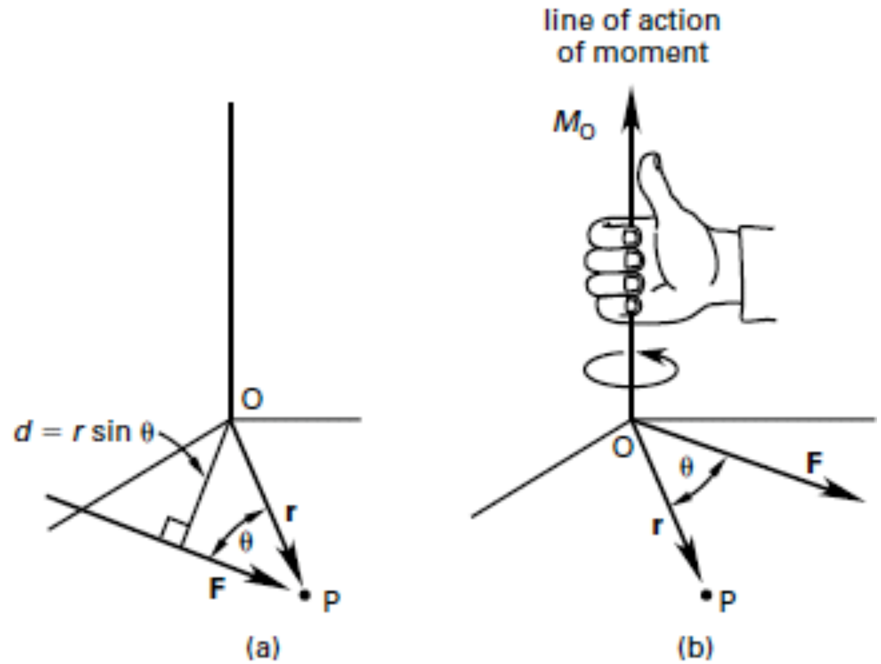
$$\sum F_y = (30\text{ N})\sin 0° + (45\text{ N})\sin 30° + (60\text{ N})\sin 60° + 75\text{ N} + (90\text{ N})\sin 120° = 227.4\text{ N}$$

$$R = \sqrt{(54\text{ N})^2 + (227.4\text{ N})^2} = 233.7\text{ N} \quad (234\text{ N})$$

Answer is D.

Moments

Figure 10.2 Right-Hand Rule



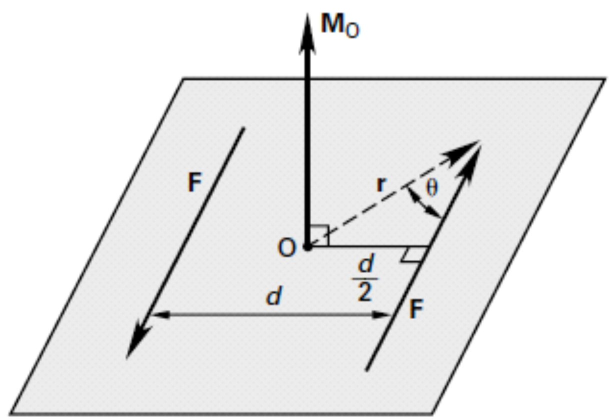
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \quad 10.8$$

$$M_O = |\mathbf{M}_O| = |\mathbf{r}||\mathbf{F}| \sin\theta = d|\mathbf{F}| \quad [\theta \le 180°] \quad 10.9$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2} \quad 10.16$$

Couples

Figure 10.3 Couple



$$M_O = 2rF \sin\theta = Fd \qquad 10.17$$

Equilibrium Requirements

$$\mathbf{R} = 0 \qquad 10.23$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 0 \qquad 10.24$$

$$\mathbf{M} = 0 \qquad 10.25$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = 0 \qquad 10.26$$

$$R_x = 0 \qquad 10.27$$

$$R_y = 0 \qquad 10.28$$

$$R_z = 0 \qquad 10.29$$

$$M_x = 0 \qquad 10.30$$

$$M_y = 0 \qquad 10.31$$

$$M_z = 0 \qquad 10.32$$

Example Moment Problems

(FESP)

Problem–5

The moment due to an applied force on a body is zero only when

(a) the force is negative.
(b) the force is through the origin.
(c) the line of action passes through the center of rotation.
(d) the force is a function of time.

The answer is

Example Moment Problems

(FESP)

Problem–5

The moment due to an applied force on a body is zero only when

(a) the force is negative.
(b) the force is through the origin.
(c) the line of action passes through the center of rotation.
(d) the force is a function of time.

The answer is (c)

Example Moment Problems

(FESP)

Problem–6

The moment of a force **F** applied at a distance **r** from a point O is equal to what quantity?

(a) $\mathbf{M}_\mathrm{O} = \mathbf{r} \cdot \mathbf{F}$
(b) $\mathbf{M}_\mathrm{O} = \nabla \cdot \mathbf{F}$
(c) $\mathbf{M}_\mathrm{O} = \mathbf{r} \times \mathbf{F}$
(d) $\mathbf{M}_\mathrm{O} = \nabla \times \mathbf{F}$

The answer is

Example Moment Problems

(FESP)

Problem–6

The moment of a force $\mathbf{F}$ applied at a distance $\mathbf{r}$ from a point O is equal to what quantity?

(a) $\mathbf{M}_O = \mathbf{r} \cdot \mathbf{F}$
(b) $\mathbf{M}_O = \nabla \cdot \mathbf{F}$
(c) $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$
(d) $\mathbf{M}_O = \nabla \times \mathbf{F}$

The answer is (c)

Example Moment Problems

(FESP)

Problem–8

A couple is composed of a pair of forces that are

(a) unequal, opposite, and nonparallel.
(b) unequal, opposite, and parallel.
(c) equal, opposite, and parallel.
(d) equal and parallel forces.

The answer is

Example Moment Problems

(FESP)

Problem–8

A couple is composed of a pair of forces that are

(a) unequal, opposite, and nonparallel.
(b) unequal, opposite, and parallel.
(c) equal, opposite, and parallel.
(d) equal and parallel forces.

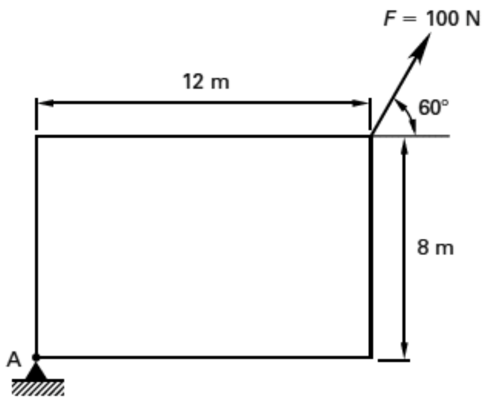
The answer is (c)

Example Moment Problems

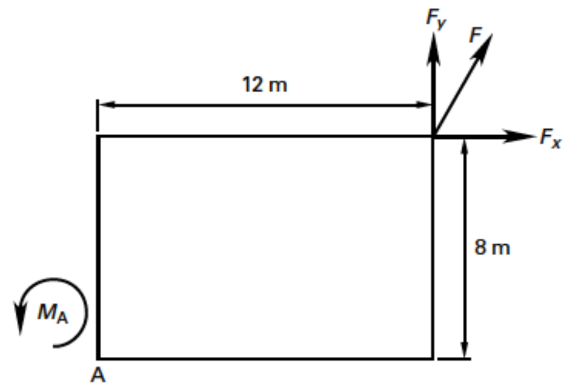
(EFPRB)

STATICS–2

Determine the magnitude of the moment of the force F about the corner A.



(A) 120 N·m (B) 240 N·m (C) 320 N·m (D) 640 N·m



$$F_x = (100\text{ N})\cos 60° = 50.0\text{ N}$$

$$F_y = (100\text{ N})\sin 60° = 86.6\text{ N}$$

Taking counterclockwise moments as positive,

$$\begin{aligned}\sum M_A &= -yF_x + xF_y \\ &= -(8\text{ m})(50.0\text{ N}) + (12\text{ m})(86.6\text{ N}) \\ &= 640\text{ N·m}\end{aligned}$$

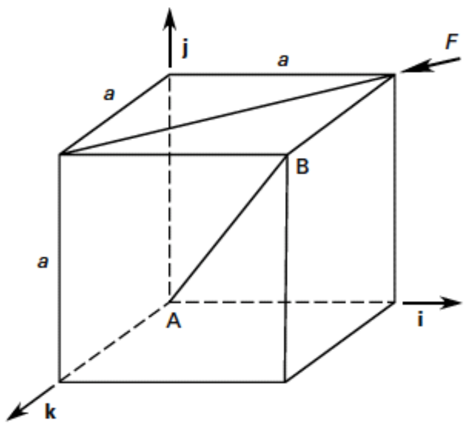
The answer is (D).

Example Moment Problems

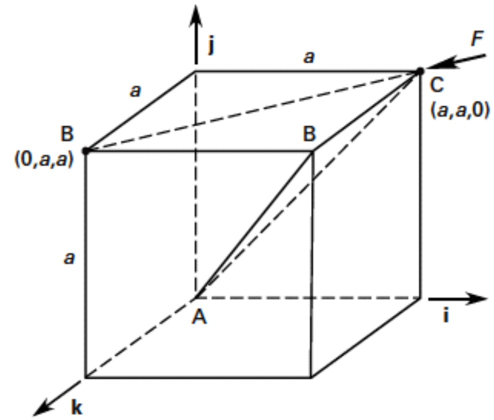
(EFPRB)

STATICS–3

A cube of side length a is acted upon by a force F as shown. Determine the magnitude of the moment of F about the diagonal AB.



(A) $\frac{aF}{\sqrt{8}}$ (B) $\frac{aF}{\sqrt{6}}$ (C) $\frac{aF}{\sqrt{4}}$ (D) $\frac{aF}{\sqrt{3}}$



$$M_A = \mathbf{r}_{AC} \times \mathbf{F}$$
$$= a(\mathbf{i}+\mathbf{j}) \times \frac{F}{\sqrt{2}}(-\mathbf{i}+\mathbf{k})$$
$$= \frac{aF}{\sqrt{2}}(\mathbf{i}-\mathbf{j}+\mathbf{k})$$
$$U_{AB} = \frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$$
$$M_{AB} = U_{AB} \cdot M_A$$
$$= \left(\frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})\right) \cdot \left(\frac{aF}{\sqrt{2}}(\mathbf{i}+\mathbf{j}+\mathbf{k})\right)$$
$$= \frac{aF}{\sqrt{6}}$$

The answer is (B).

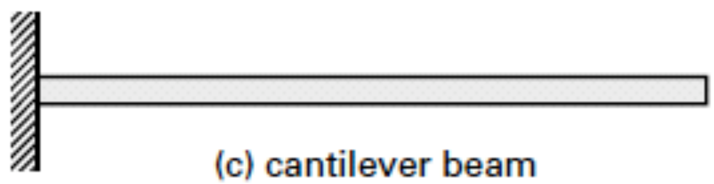
Determinacy

Determinate Systems

Figure 10.4 Types of Determinate Systems


(a) simply supported beam

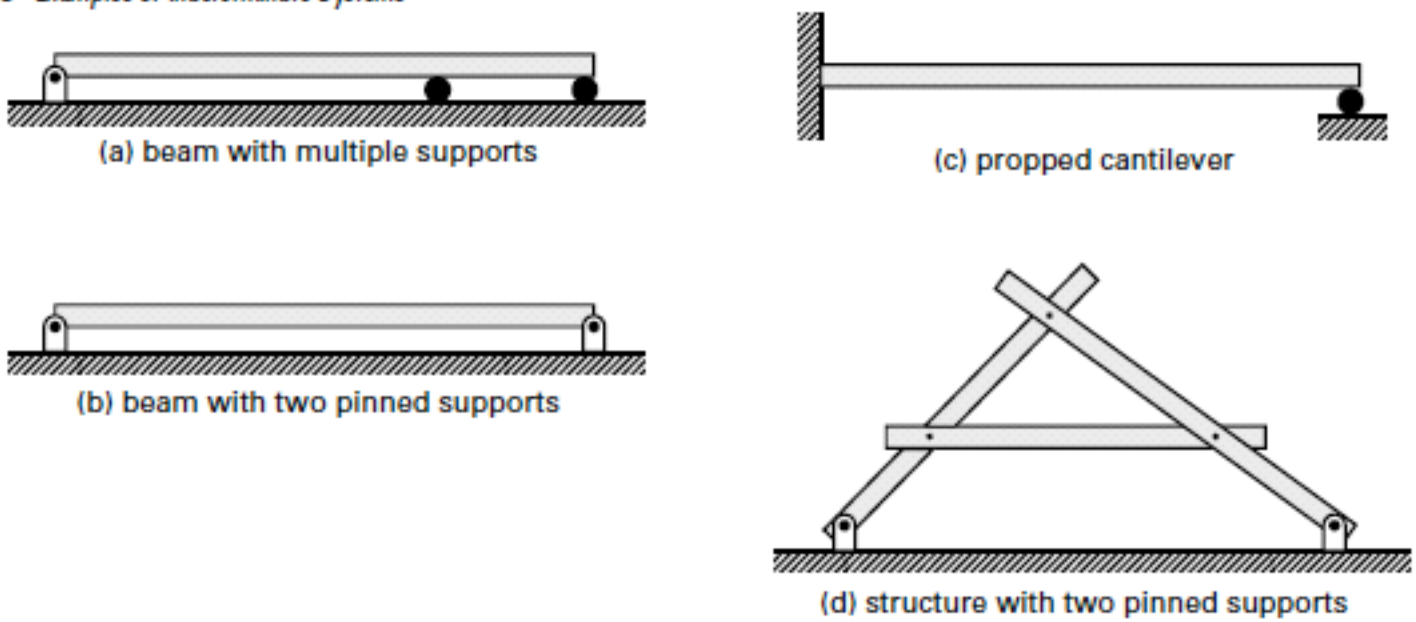

(b) overhanging beam


(c) cantilever beam

Determinacy

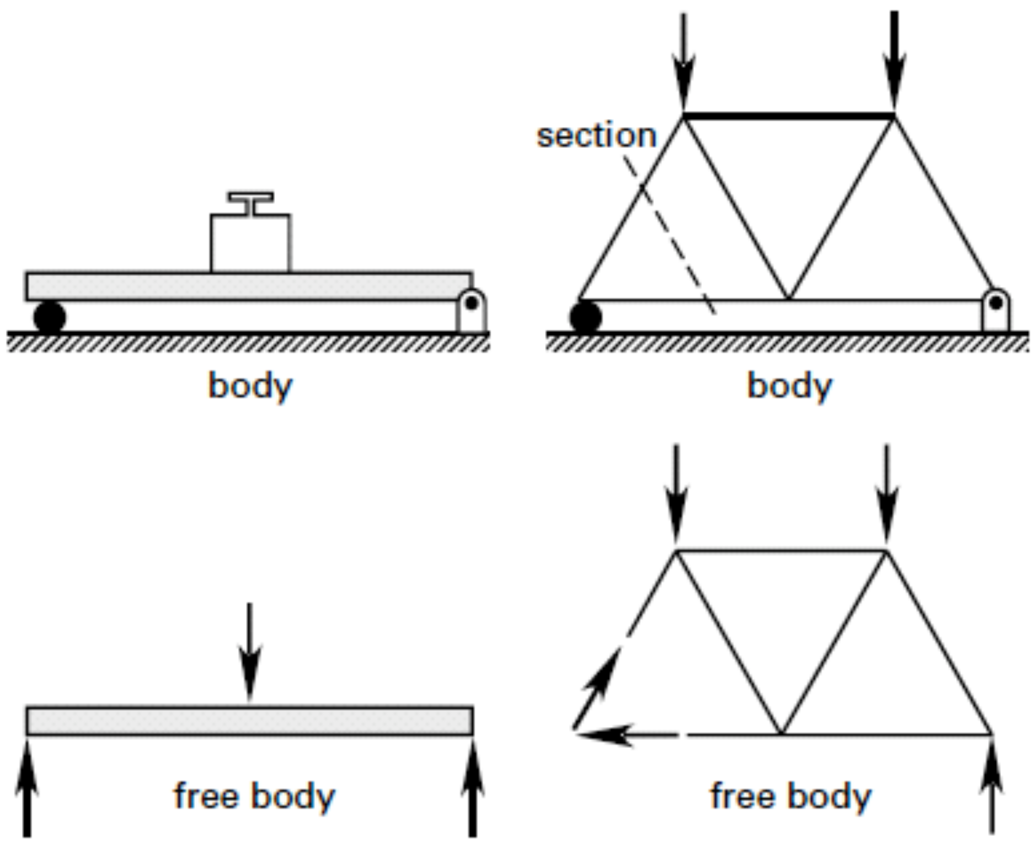
Indeterminate Systems

Figure 10.5 Examples of Indeterminate Systems



Free-Body Diagrams

Figure 10.6 Bodies and Free Bodies

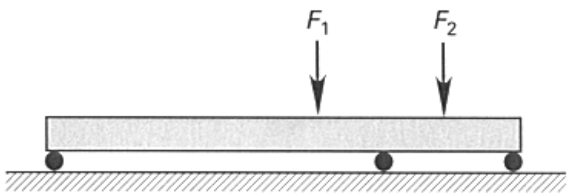


Example Determinacy Problems

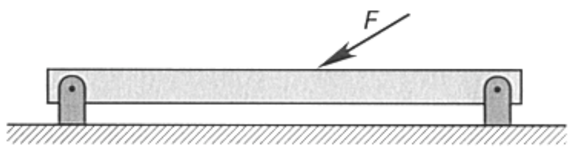
Indeterminate vs. Determinate Problem

(FESP)

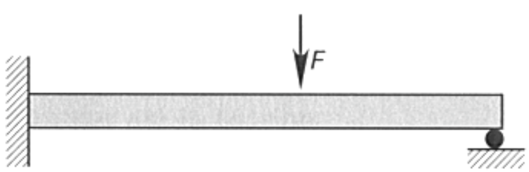
Problem–26



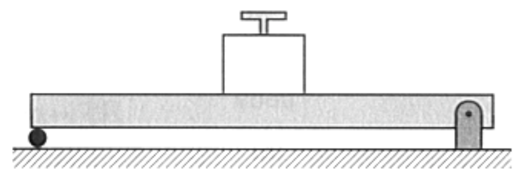
(a)



(b)



(c)



(d)

In the illustrations shown, all of the structures are statically indeterminant except which of the following?

(a) a
(b) b
(c) c
(d) d

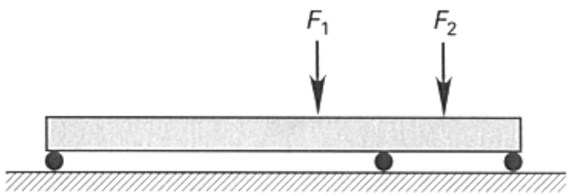
The answer is

Example Determinacy Problems

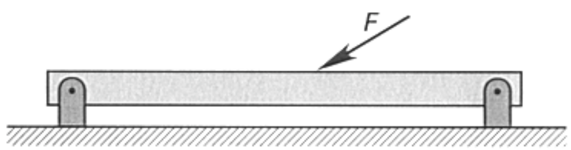
Indeterminate vs. Determinate Problem

(FESP)

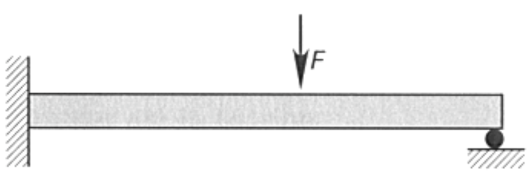
Problem–26



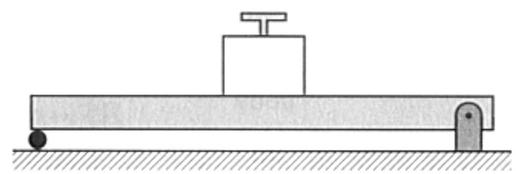
(a)



(b)



(c)



(d)

In the illustrations shown, all of the structures are statically indeterminant except which of the following?

(a) a
(b) b
(c) c
(d) d

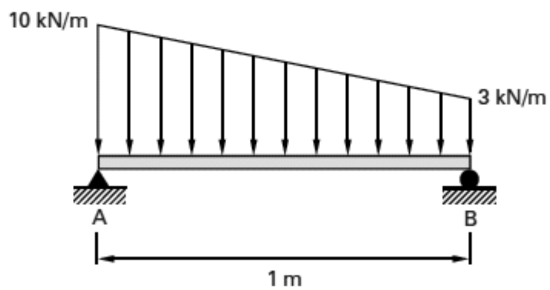
The answer is (d)

Example Determinacy Problems

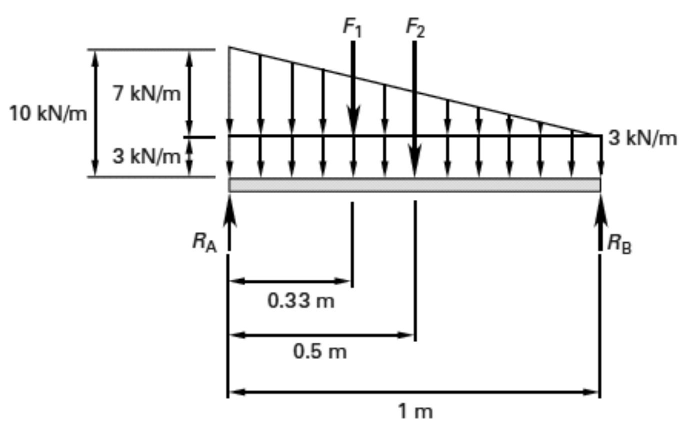
Linear Force System Problem (EFPRB)

STATICS–25

What is most nearly the reaction force at support B on the simply supported beam with a linearly varying load?



(A) 1.5 kN (B) 2.3 kN (C) 2.6 kN (D) 3.5 kN



$$F_1 = \tfrac{1}{2}Lh = \left(\frac{1}{2}\right)(1\ \text{m})\left(7\ \frac{\text{kN}}{\text{m}}\right) = 3.5\ \text{kN}$$

$$F_2 = Lh = (1\ \text{m})\left(3\ \frac{\text{kN}}{\text{m}}\right) = 3\ \text{kN}$$

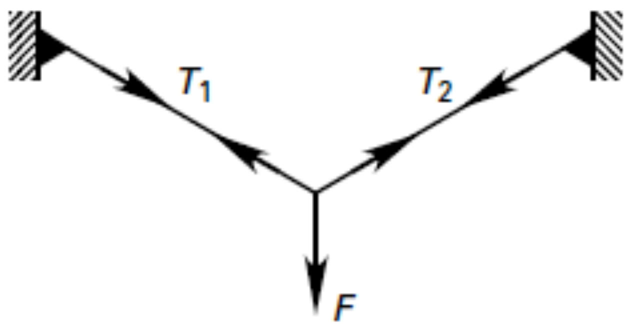
Sum the moments around support A.

$$\sum M_\text{A} = 0 = R_\text{B}(1\ \text{m}) - F_1(0.3\ \text{m}) - F_2(0.5\ \text{m})$$
$$= R_\text{B}(1\ \text{m}) - (3.5\ \text{kN})(0.3\ \text{m}) - (3\ \text{kN})(0.5\ \text{m})$$
$$R_\text{B} = 2.55\ \text{kN} \quad (2.6\ \text{kN})$$

The answer is (C).

Cables

Figure 12.2 Cable with Concentrated Load

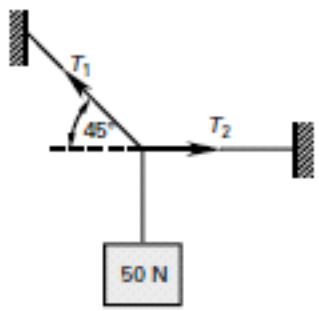


Cables

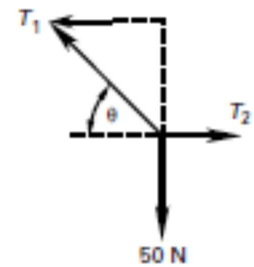
Example (EFPRB):

STATICS–5

Find the tensions, T_1 and T_2, in the ropes shown so that the system is in equilibrium.



(A) $T_1 = 50.0$ N, $T_2 = 0.0$ N
(B) $T_1 = 50.0$ N, $T_2 = 50.0$ N
(C) $T_1 = 70.7$ N, $T_2 = 50.0$ N
(D) $T_1 = 70.7$ N, $T_2 = 70.7$ N



$$\sum F_y = 0 = T_1 \sin 45° - 50\text{ N} = 0$$

$$T_1 \sin 45° = 50\text{ N}$$

$$T_1 = 70.7\text{ N}$$

$$\sum F_x = 0$$

$$T_1 \cos 45° - T_2 = 0$$

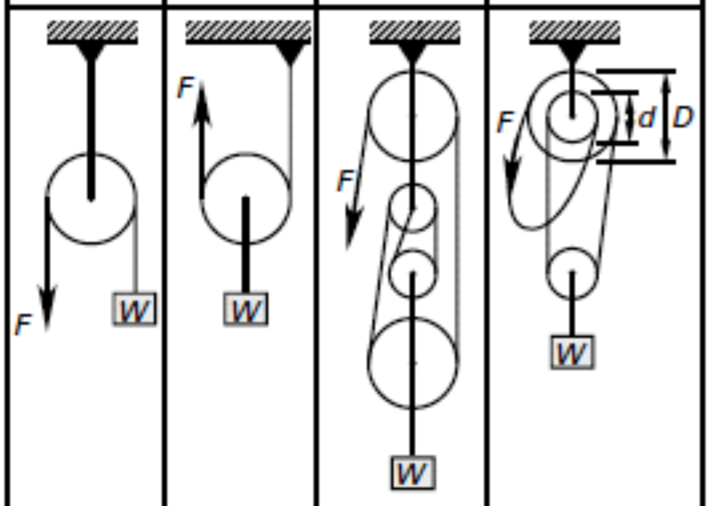
$$T_2 = T_1 \cos 45°$$

$$= 50\text{ N}$$

The answer is (C).

Pulleys

Figure 12.1 Mechanical Advantage of Rope-Operated Machines



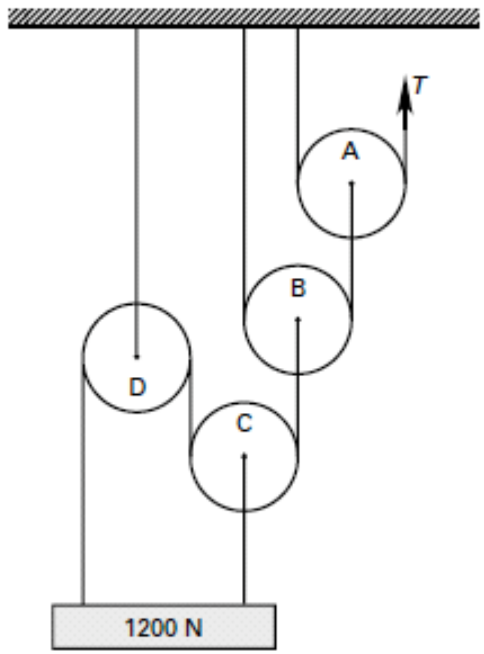
	fixed sheave	free sheave	ordinary pulley block (n sheaves)	differential pulley block
F_{ideal}	W	$\frac{W}{2}$	$\frac{W}{n}$	$\frac{W}{2}\left(1-\frac{d}{D}\right)$

Pulleys

Example (FERM prob. p. 12-3):

Problem 1

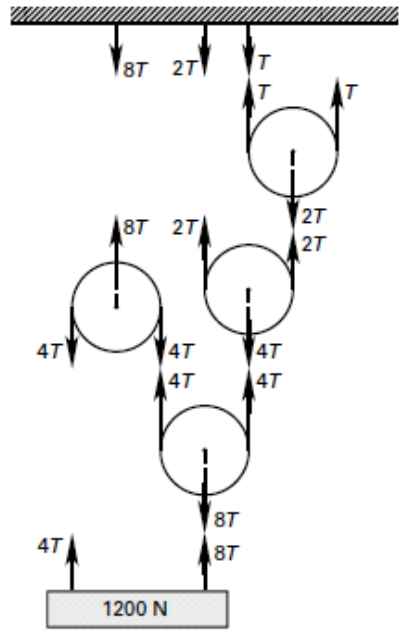
Find the tension, T, that must be applied to pulley A to lift the 1200 N weight.



(A) 100 N
(B) 300 N
(C) 400 N
(D) 600 N

Solution

The free bodies of the system are shown.



$$\sum F_y = 0$$
$$= -1200\text{ N} + 4T + 8T$$

$$12T = 1200\text{ N}$$

$$T = 100\text{ N}$$

Answer is A.

Friction

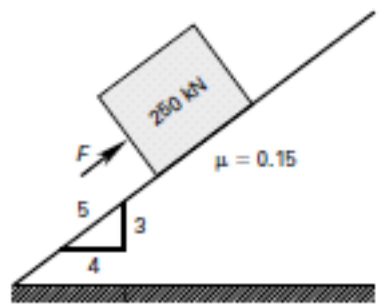
$$F = \mu N \qquad 12.1$$

Friction

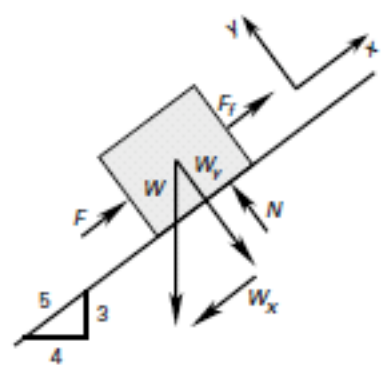
Example (EFPRB):

STATICS–12

Determine the force, F, required to keep the package from sliding down the plane shown.



(A) 15 kN (B) 35 kN (C) 65 kN (D) 120 kN



$$\sum F_y = 0$$
$$W_y - N = 0$$
$$W_y = \tfrac{4}{5}W$$
$$N = \tfrac{4}{5}W$$
$$= 200 \text{ kN}$$
$$F_f = \mu N$$
$$= (0.15)(200 \text{ kN})$$
$$= 30 \text{ kN}$$
$$\sum F_x = 0$$
$$F - W_x + F_f = 0$$
$$F = W_x - F_f$$
$$W_x = \tfrac{3}{5}W$$
$$= 150 \text{ kN}$$
$$F = 150 \text{ kN} - 30 \text{ kN}$$
$$= 120 \text{ kN}$$

The answer is (D).

Statics 7-17a

Trusses

Figure 11.1 Parts of a Bridge Truss

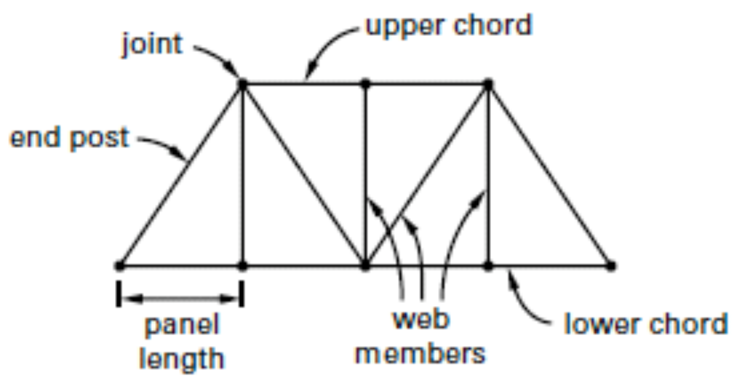


Figure 11.2 Special Types of Trusses

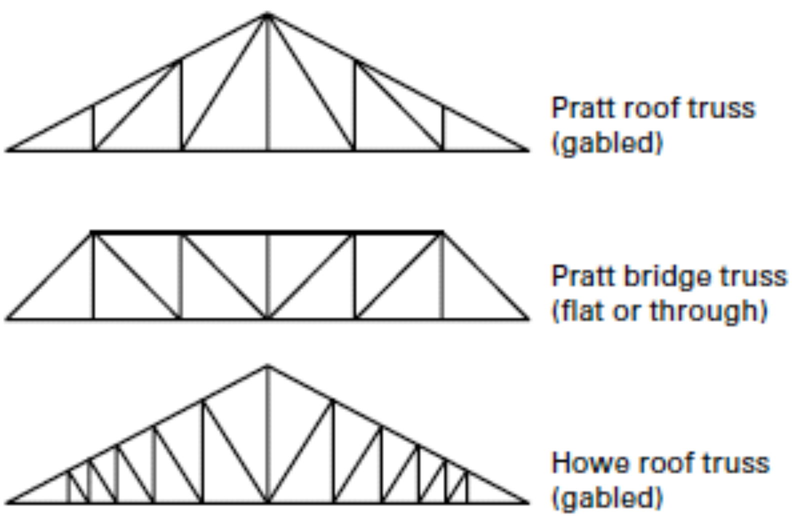
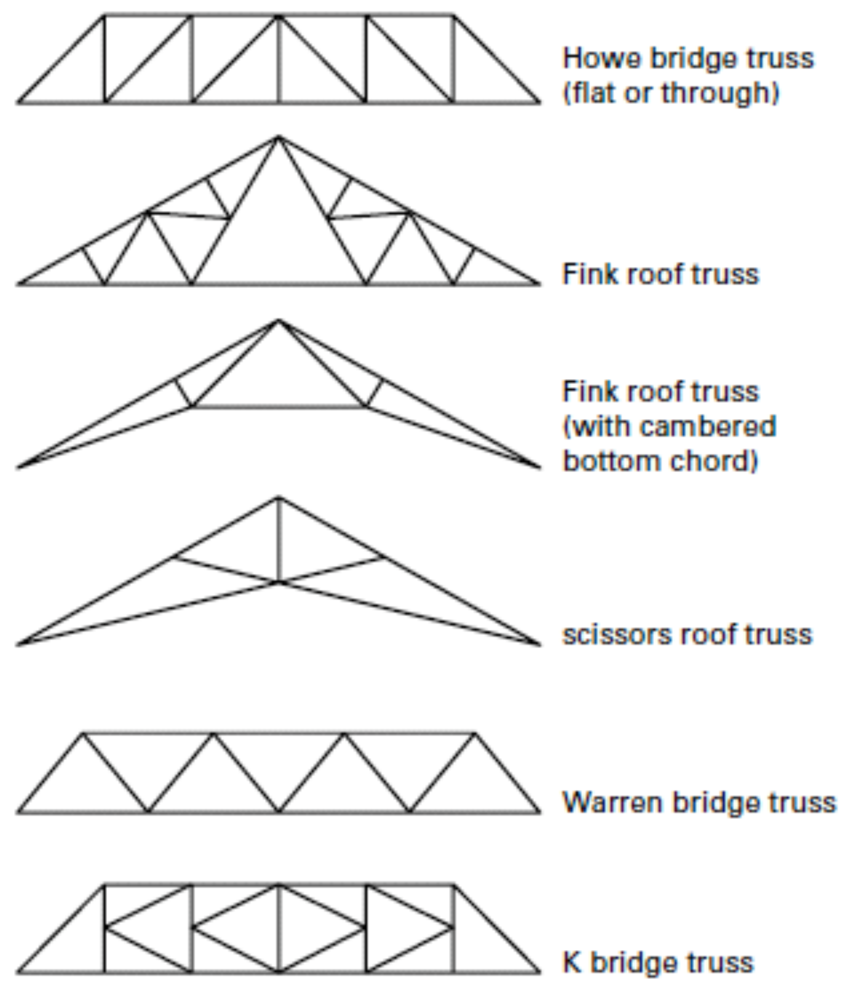


Figure 11.2 Special Types of Trusses *(continued)*

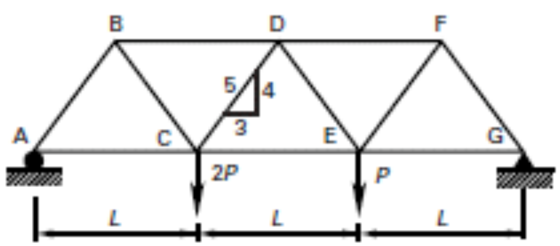


Trusses

Example (EFPRB):

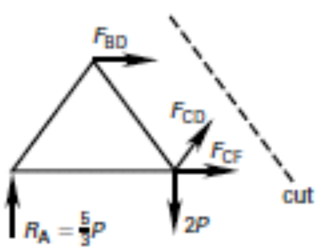
STATICS–17

Determine the force in member CD.



(A) $\frac{1}{12}P$ (B) $\frac{1}{3}P$ (C) $\frac{5}{12}P$ (D) P

Use the method of sections.



Only CD can support a vertical force.

$$\sum F_y = 0$$

$$0 = R_A - 2P + CD_y$$

$$CD_y = \frac{P}{3}$$

$$CD = \tfrac{5}{4}CD_y$$

$$= \left(\frac{5}{4}\right)\left(\frac{P}{3}\right)$$

$$= \frac{5P}{12}$$

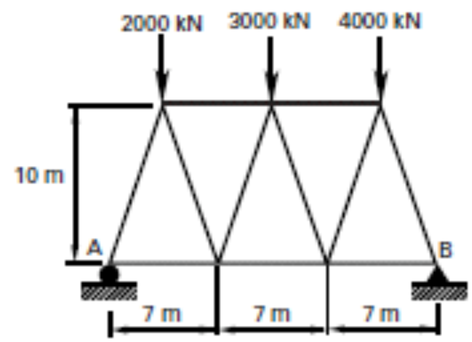
The answer is (C).

Trusses

Example (EFPRB):

STATICS–18

A truss is subjected to three loads. The truss is supported by a roller at A and by a pin joint at B. What is most nearly the reaction force at A?



(A) 3800 kN (B) 4400 kN (C) 4900 kN (D) 5000 kN

The rolling support at A can only support a vertical reaction force. R_A is the reaction force at A.

$$\sum M_B = 0$$

$$0 = -R_A(21 \text{ m}) + (2000 \text{ kN})(17.5 \text{ m}) + (3000 \text{ kN})(10.5 \text{ m}) + (4000 \text{ kN})(3.5 \text{ m})$$

$$R_A = 3833 \text{ kN} \quad (3800 \text{ kN})$$

The answer is (A).