

Review of Heat Transfer

熱傳回顧

Reference:

1. Incropera, DeWitt, Bergmann, and Lavine, Fundamentals of Heat and Mass Transfer, 6th Ed., 2007.
2. Çengel, Y.A., Heat Transfer, 2nd Ed., 2003

Three modes of heat transfer: conduction, convection and radiation

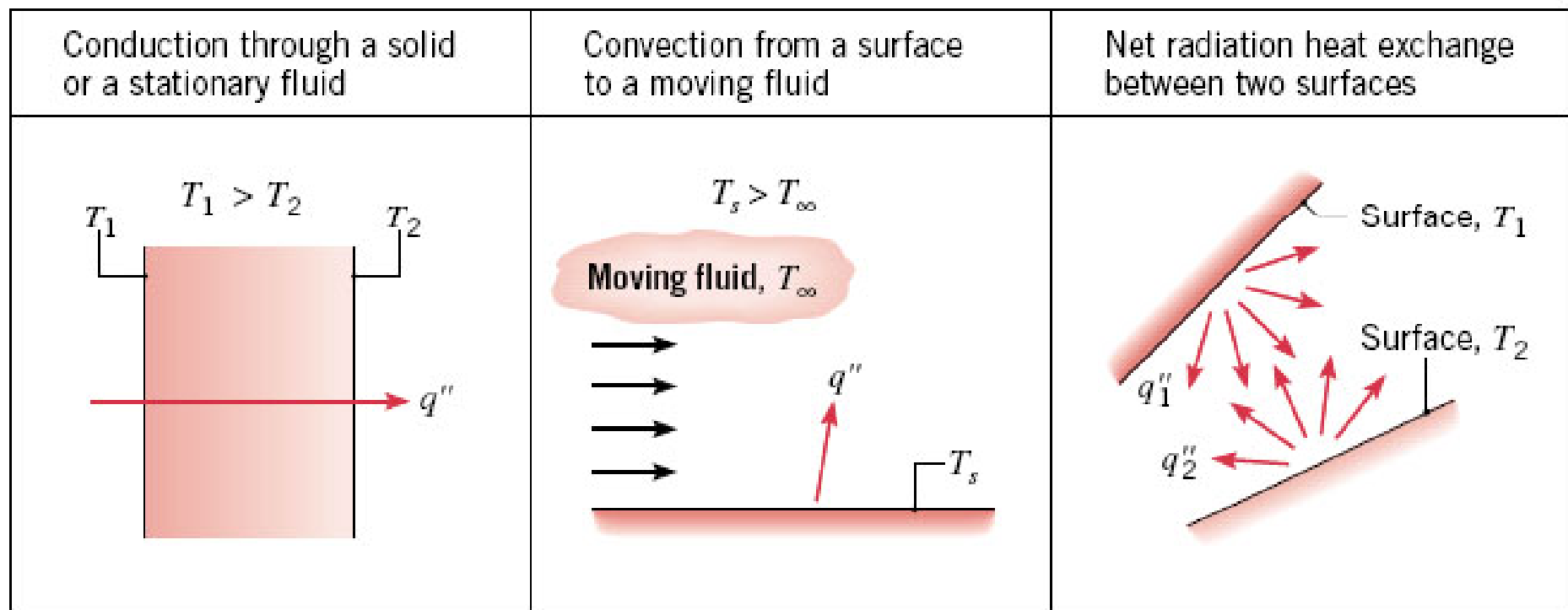
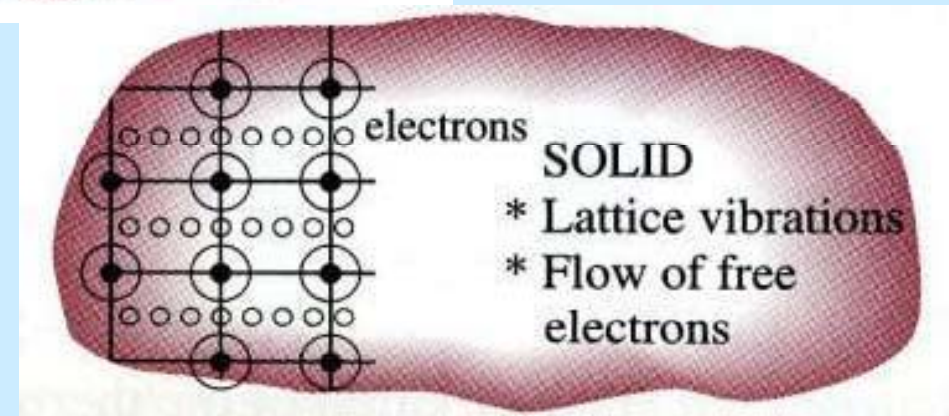
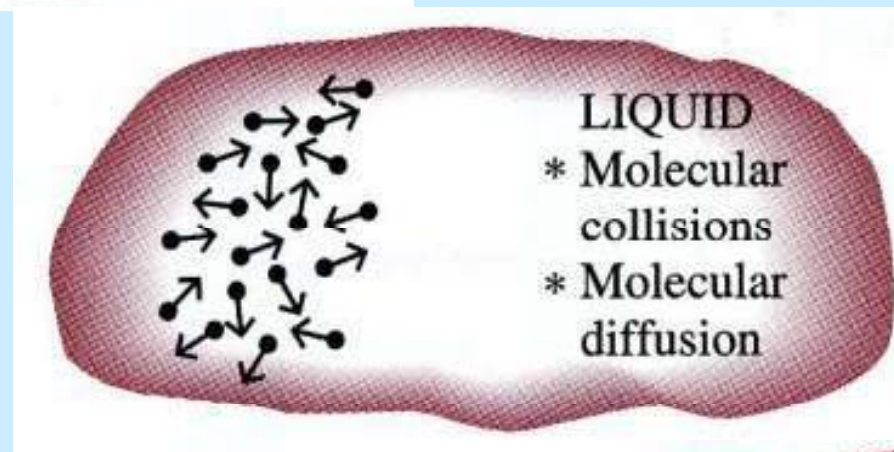
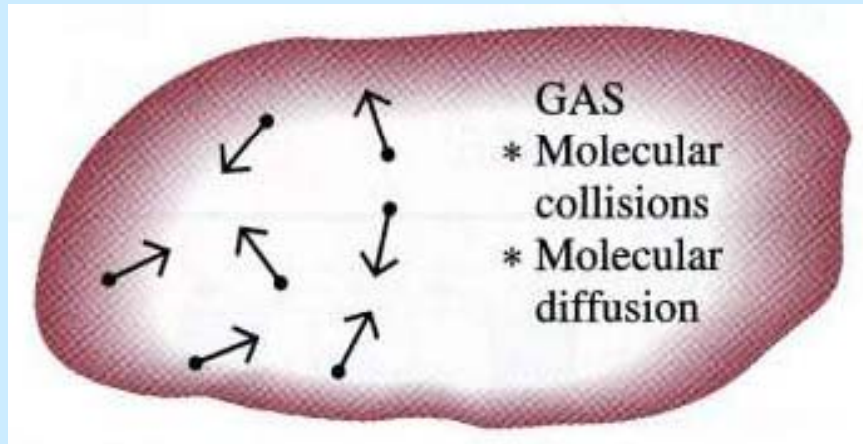


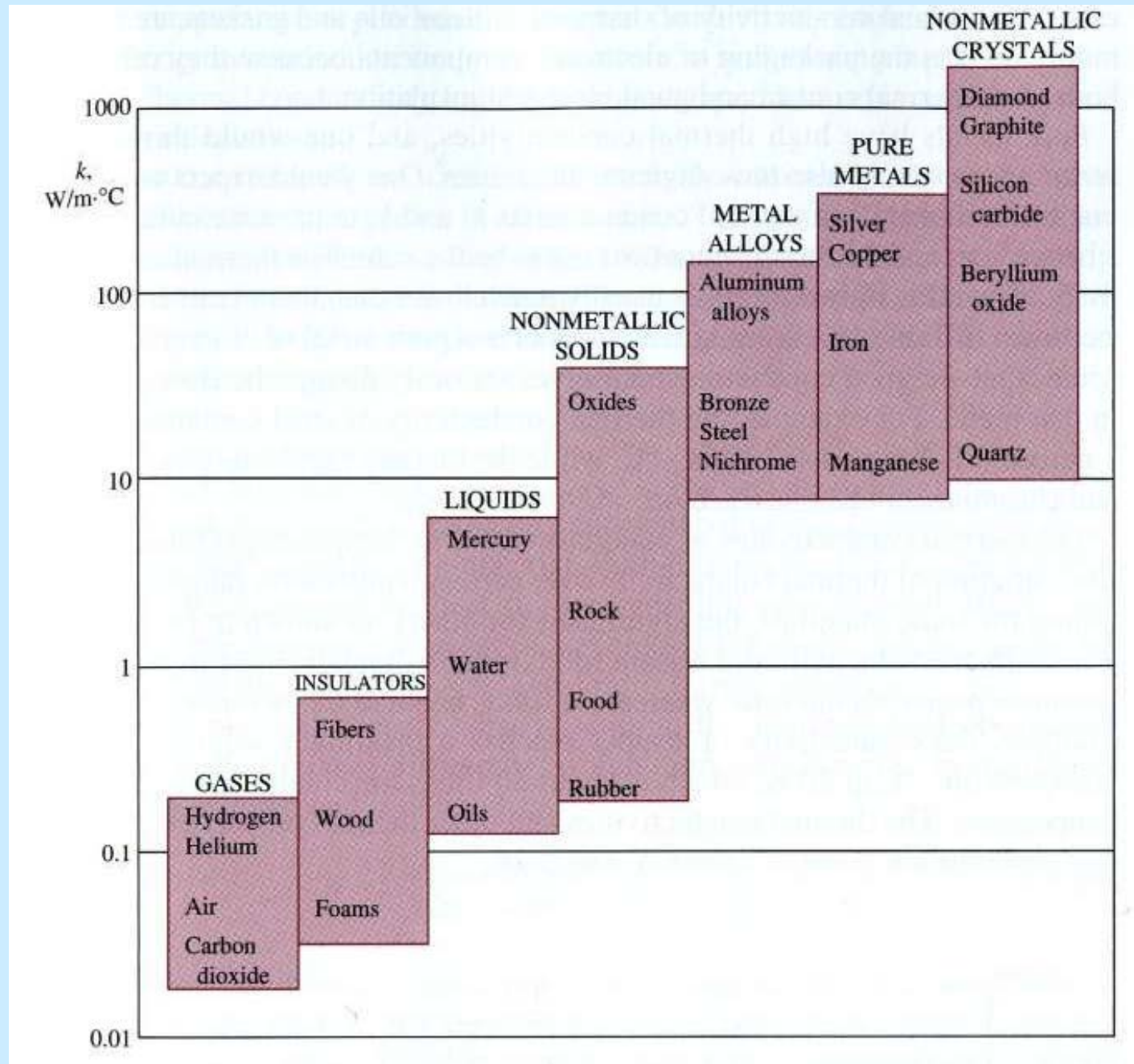
FIGURE 1.1 Conduction, convection, and radiation heat transfer modes.

Conduction Mechanisms in Different Phases



Source: Y.A. Cengel

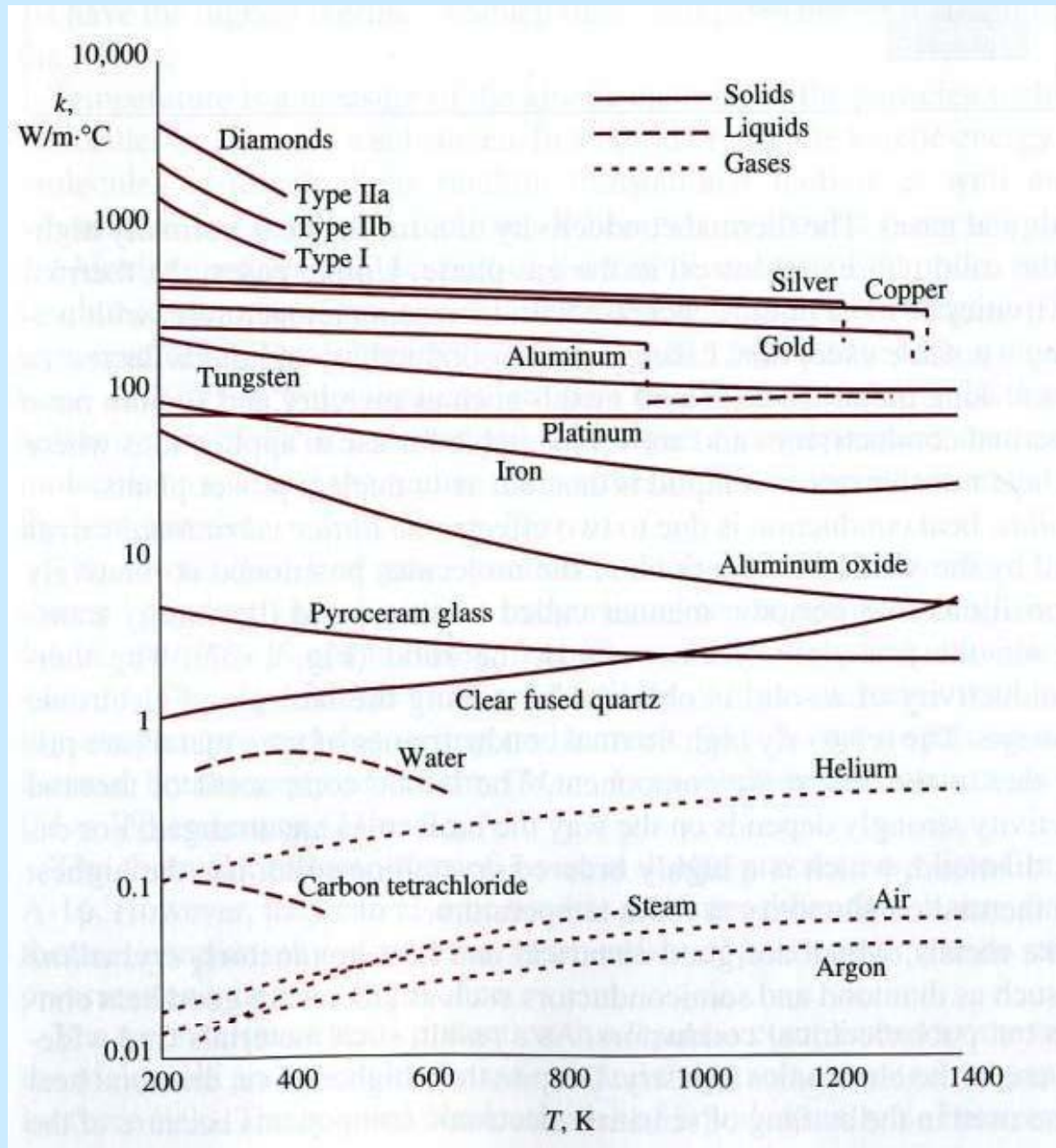
Thermal Conductivities of Various Materials



Under room temp.

Source: Y.A. Cengel

Temperature Effect on the Thermal Conductivities of Various Materials



Source: Y.A. Cengel

Thermal Conductivities of Some Materials

TABLE 1-1

The thermal conductivities of some materials at room temperature

Material	$k, \text{W/m} \cdot ^\circ\text{C}$
Diamond	2300
Silver	429
Copper	401
Gold	317
Aluminum	237
Iron	80.2
Mercury (l)	8.54
Glass	0.78
Brick	0.72
Water (l)	0.613
Human skin	0.37
Wood (oak)	0.17
Helium (g)	0.152
Soft rubber	0.13
Glass fiber	0.043
Air (g)	0.026
Urethane, rigid foam	0.026

Source: Y.A. Cengel

The Heat Diffusion Equation of Conduction

In Cartesian coordinates:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

If $k = \text{constant}$,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

If steady-state,

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + \dot{q} = 0$$

If one-dimensional with no energy generation,

$$\frac{d}{dx} \left(k \frac{dT}{dx} \right) = 0$$

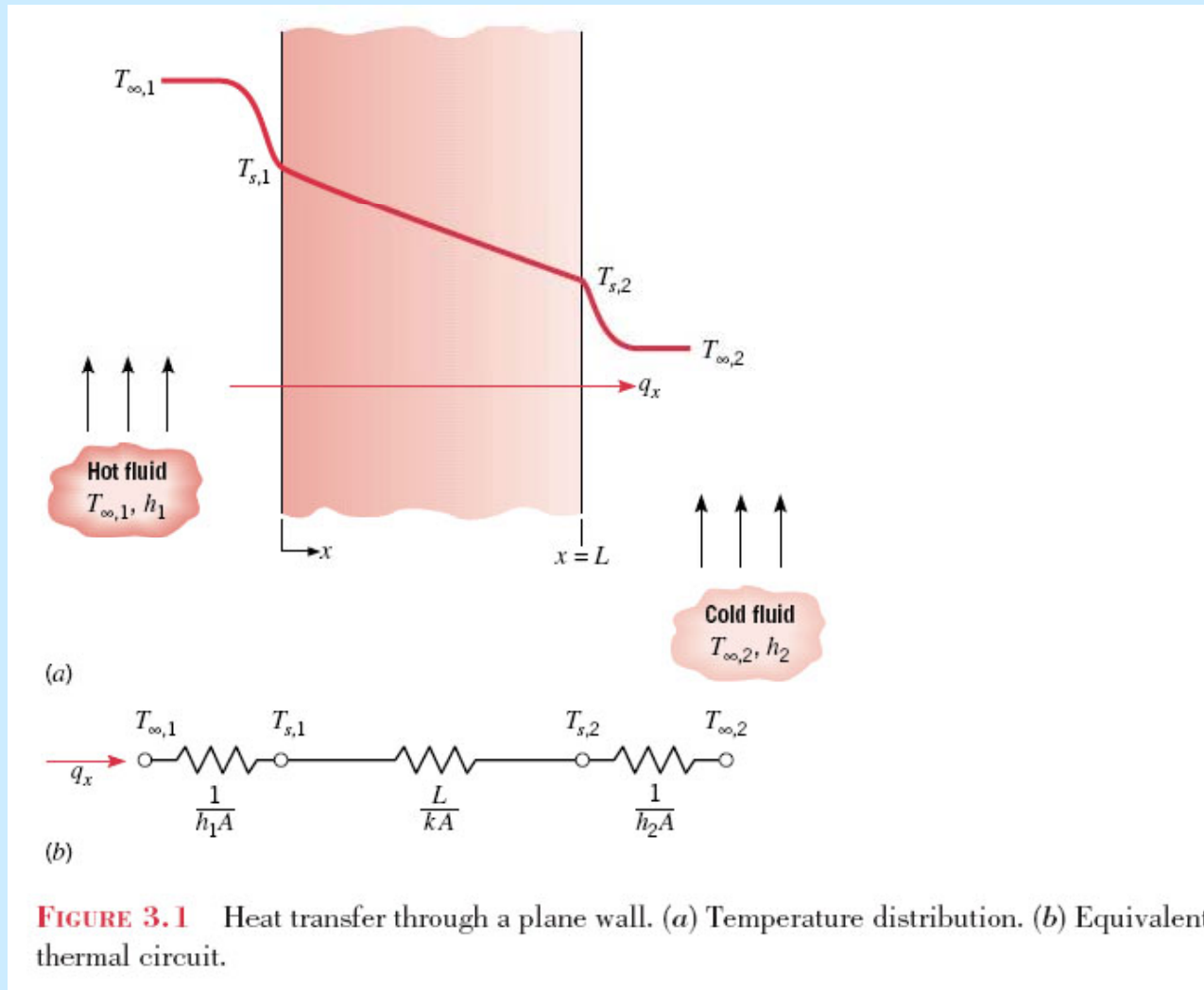
Under steady-state, 1-D conditions with no energy generation,

$$dq_x''/dx = 0$$

Steady-State, 1-D Conduction with No Heat Source

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are to be determined by boundary conditions.



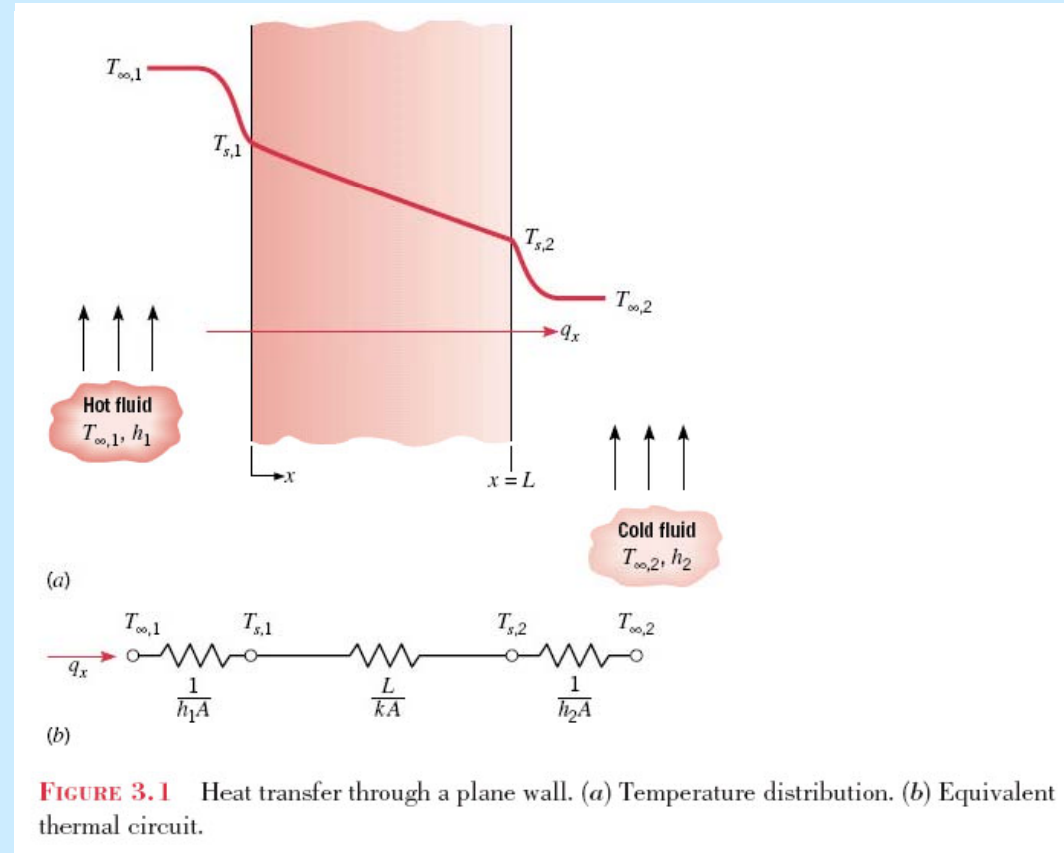
Thermal Resistance

With the analogy between the diffusion of heat and electrical charge, the **thermal resistance** for conduction is

$$R_{t,\text{cond}} = \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA}$$

Similar for convection

$$R_{t,\text{conv}} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$$



In Fig. 3.1, the **total thermal resistance**, R_{tot} , is

$$R_{\text{tot}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

The Composite Wall (Fig. 3.2; Fig. 3.3)

$$R_{\text{tot}} = \sum R_t = \frac{\Delta T}{q} = \frac{1}{UA}$$

where U is the **overall heat transfer coefficient**, defined by analogy to Newton's law of cooling as

$$q_x \equiv UA\Delta T$$

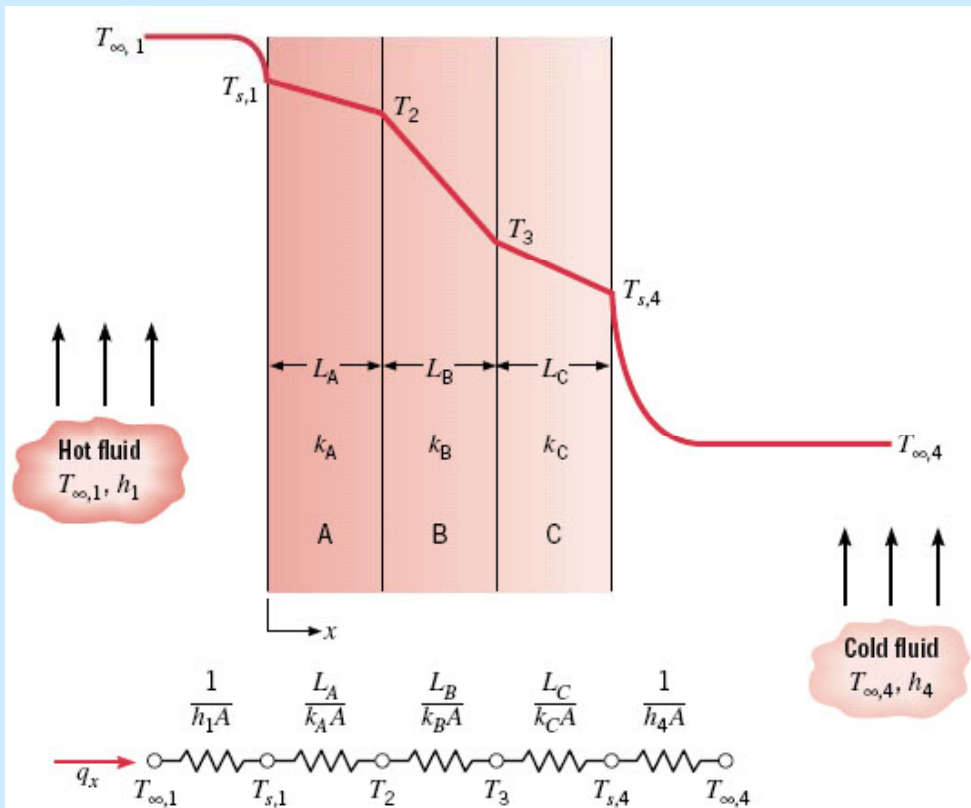


FIGURE 3.2 Equivalent thermal circuit for a series composite wall.

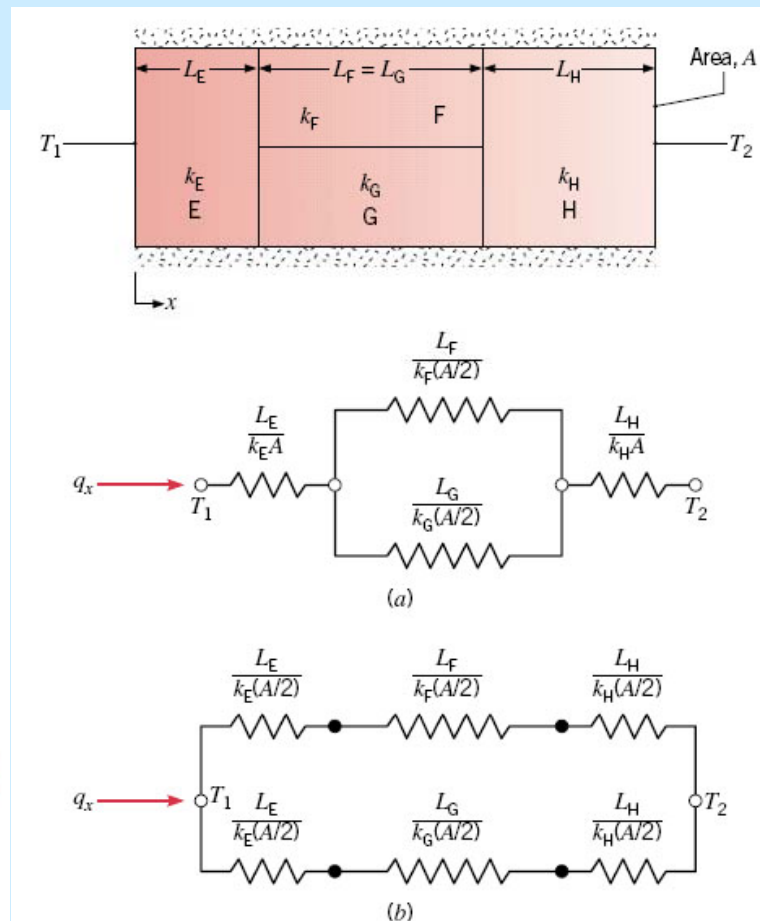


FIGURE 3.3 Equivalent thermal circuits for a series-parallel composite wall.

Examples of The Composite Wall

$$R_1 = 0.5^\circ\text{C/W}, R_2 = 1.0^\circ\text{C/W}$$

Series composite wall

$$R = R_1 + R_2 = 0.5 + 1.0 = 1.5^\circ\text{C/W}$$

Parallel composite wall

$$R = 1/(1/R_1 + 1/R_2) = 1/3^\circ\text{C/W}$$

If R_1 is halved to become 0.25°C/W

$$\rightarrow R = R_1 + R_2 = 0.25 + 1.0 = 1.25^\circ\text{C/W} \quad R = 1/(1/R_1 + 1/R_2) = 1/5^\circ\text{C/W}$$

If R_2 is halved to become 0.5°C/W

$$\rightarrow R = R_1 + R_2 = 0.5 + 0.5 = 1.0^\circ\text{C/W} \quad R = 1/(1/R_1 + 1/R_2) = 1/4^\circ\text{C/W}$$

The more efficient way is to: improve the largest R in series but improve the smallest R in parallel.

The Problem of Convection

Determination of heat convection coefficients (local h and average \bar{h}) is viewed as *the problem of convection*.

However, the problem is not a simple one, as

$$h \text{ or } \bar{h} = f(\text{fluid properties, surface geometry, flow conditions})$$

↑ i.e., $\rho, \mu, k_f, c_{p,f}$

Once h or \bar{h} is known, then

Local Convection

$$q'' = h(T_s - T_\infty)$$

Global Convection

$$q = \bar{h}A(T_s - T_\infty)$$

Nusselt No.—Dimensionless heat transfer coeff.

$$Nu_x \equiv \frac{h_x x}{k}, \quad \overline{Nu}_L \equiv \frac{\bar{h}L}{k}$$

In general, $Nu_x = CRe_x^m Pr^n$ or $\overline{Nu}_L = CRe_L^m Pr^n$

Representative Ranges of Convection Thermal Resistance

	h (W/m ² K)	Areal $R_{th,conv}$ (Kcm ² /W)
Natural Convection		
Air	2~25	5,000~400
Oils	20~200	500~50
Water	100~1,000	100~10
Forced Convection		
Air	20~200	5,000~500
Oils	200~2,000	500~50
Water	1,000~10,000	10~1
Microchannel Cooling	40,000	0.25
Impinging Jet Cooling	2,400~49,300	4.1~0.20
Heat Pipe		0.049*

* based on THERMACORE 5 mm heat pipe having capacity of 20W with $\Delta T=5K$.

Laminar and Turbulent Flow

Critical Reynolds no. where **transition** from laminar boundary layer to turbulent occurs:

$$Re_{x,c} = \frac{\rho u_{\infty} x_c}{\mu} = 5 \times 10^5$$

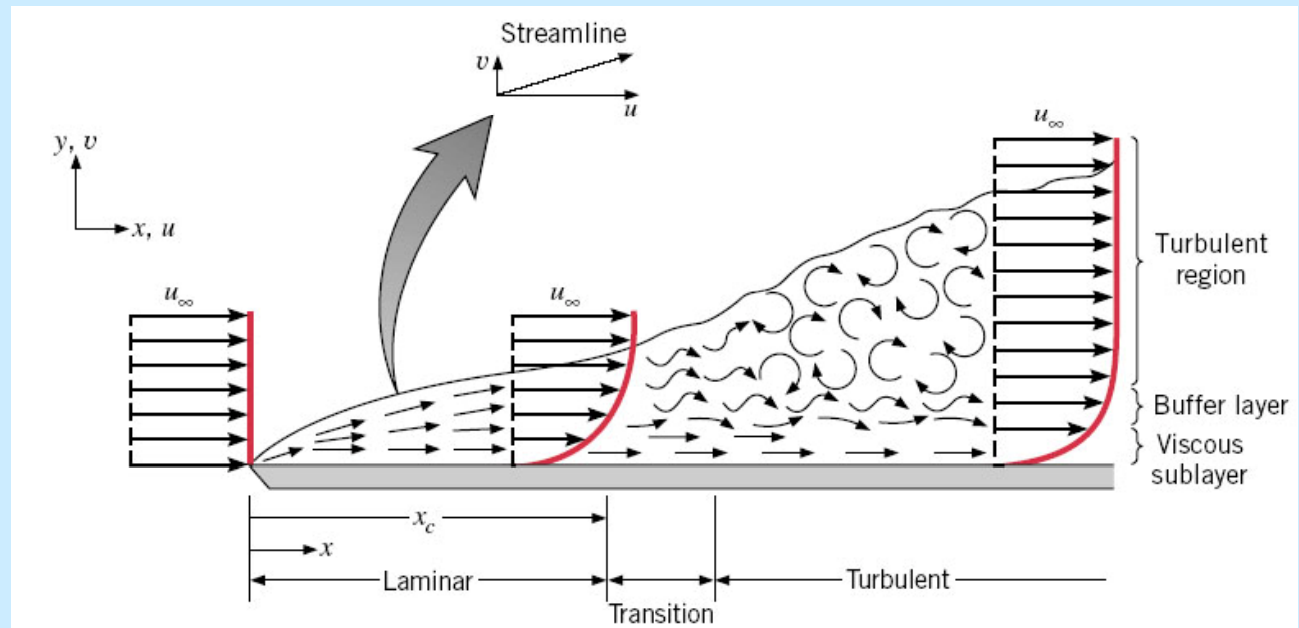
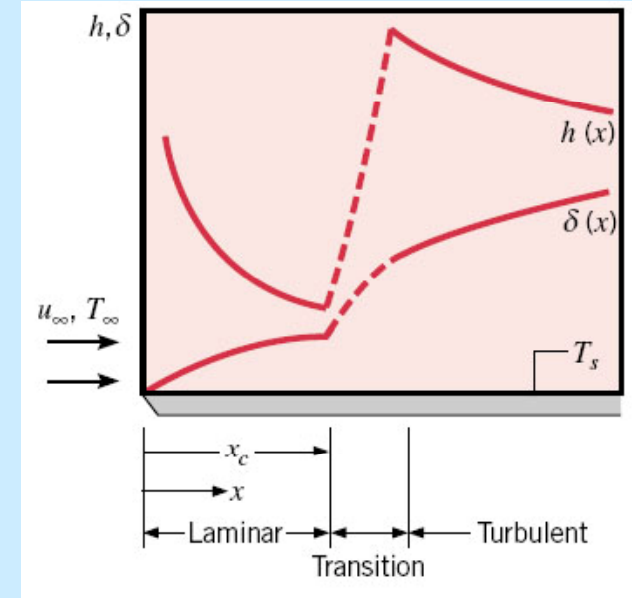
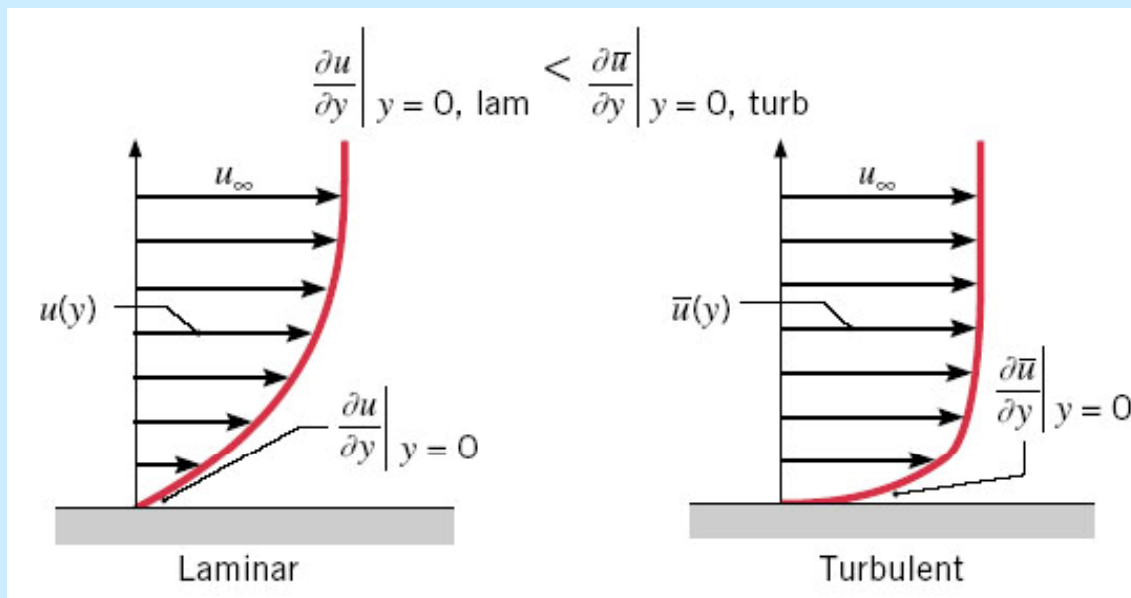


FIGURE 6.6 Velocity boundary layer development on a flat plate.



2-D Governing Equations for Heat Convection

- Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

- momentum eq. (incompressible):

$$\text{x-dir.} \quad \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + X$$

$$\text{y-dir.} \quad \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Y$$

- energy equation:

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \cancel{\frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2}$$

usually negligible

energy transport
thru convection

heat transport thru
conduction

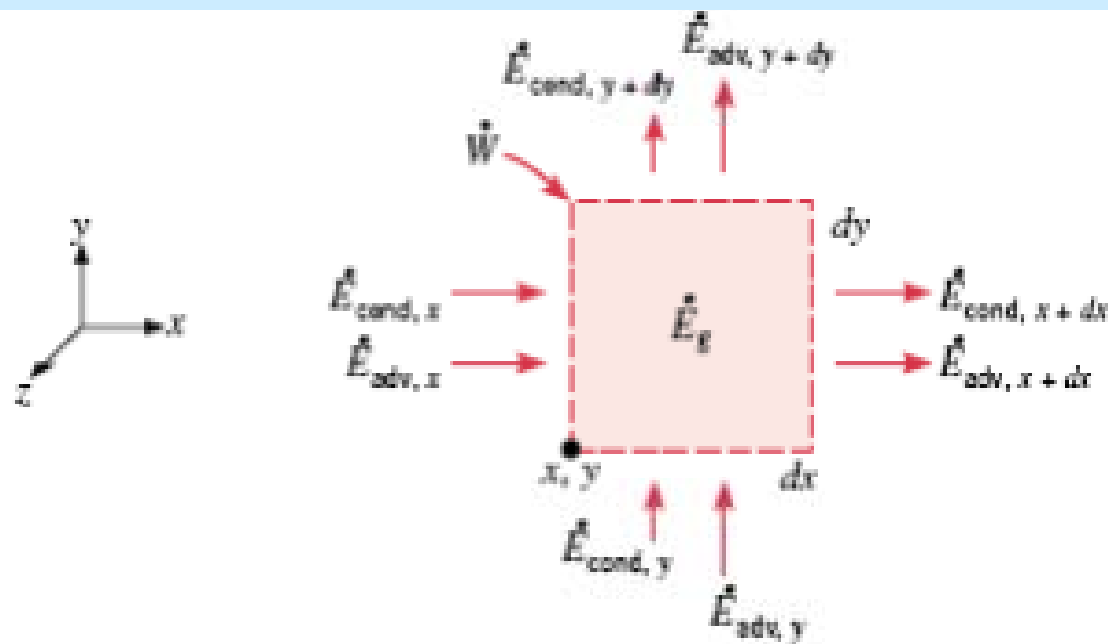


FIGURE 6S.5

Differential control volume ($dx \cdot dy \cdot 1$) for energy conservation in two-dimensional flow of a viscous fluid with heat transfer.

$$\begin{aligned}
 \dot{E}_{adv,x} - \dot{E}_{adv,x+dx} &\equiv \rho u \left(e + \frac{V^2}{2} \right) dy - \left\{ \rho u \left(e + \frac{V^2}{2} \right) \right. \\
 &\quad \left. + \frac{\partial}{\partial x} \left[\rho u \left(e + \frac{V^2}{2} \right) \right] dx \right\} dy \\
 &= - \frac{\partial}{\partial x} \left[\rho u \left(e + \frac{V^2}{2} \right) \right] dx dy
 \end{aligned} \tag{6S.15}$$

$$\begin{aligned}
 \dot{E}_{cond,x} - \dot{E}_{cond,x+dx} &= - \left(k \frac{\partial T}{\partial x} \right) dy - \left[- k \frac{\partial T}{\partial x} - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx \right] dy \\
 &= - \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) dx dy
 \end{aligned} \tag{6S.16}$$

Boundary Layer Equations for Heat Convection

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp_{\infty}}{dx} + \nu \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

energy transport thru convection heat transport thru conduction

In non-dimensional forms,

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{\text{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Re}_L \text{Pr}} \frac{\partial^2 T^*}{\partial y^{*2}}$$

Functional Form of the Solutions

From the non-dimensional momentum equation

$$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right),$$

where dp^*/dx^* depends on the surface geometry.

For a prescribed geometry, the shear stress and the friction coefficient at the surface are

$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \left(\frac{\mu V}{L} \right) \left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=0}$$

$$C_f = \frac{\tau_s}{\rho V^2 / 2} = \frac{2}{Re_L} \left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=0} = \frac{2}{Re_L} f(x^*, Re_L)$$

From the non-dimensional energy equation

$$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$$
$$h = -\frac{k_f}{L} \frac{(T_\infty - T_s)}{(T_s - T_\infty)} \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0} = +\frac{k_f}{L} \frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

Nusselt number can be defined as

$$Nu \equiv \frac{hL}{k_f} = +\frac{\partial T^*}{\partial y^*} \Big|_{y^*=0}$$

For a prescribed geometry,

$$Nu = f(x^*, Re_L, Pr)$$

The spatially average Nusselt number is then

$$\overline{Nu} = \frac{\overline{h}L}{k_f} = f(Re_L, Pr)$$

Forced Convection over a Flat Plate

- Constant-Temperature Plate

Local laminar convection (analytic solution)

$$Nu_x \equiv \frac{h_x x}{k} = 0.332 Re_x^{1/2} Pr^{1/3} \quad Pr \geq 0.6$$

Local turbulent convection

$$Nu_x = St Re_x Pr = 0.0296 Re_x^{4/5} Pr^{1/3}, \quad 0.6 < Pr < 60$$

- Constant-Heat-Flux Plate

Local laminar convection (analytic solution)

$$Nu_x \equiv \frac{h_x x}{k} = 0.453 Re_x^{1/2} Pr^{1/3} \quad Pr \geq 0.6$$

Local turbulent convection

$$Nu_x = St Re_x Pr = 0.0308 Re_x^{4/5} Pr^{1/3}, \quad 0.6 < Pr < 60$$

Global convection of a mixed boundary layer

$$\bar{h}_L = \frac{1}{L} \left(\int_0^{x_c} h_{\text{lam}} dx + \int_{x_c}^L h_{\text{turb}} dx \right)$$

$$\rightarrow \overline{Nu}_L = (0.037 Re_L^{4/5} - A) Pr^{1/3}, \quad A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$

Average plate temperature with constant heat flux

$$\overline{(T_s - T_\infty)} = \frac{1}{L} \int_0^L (T_s - T_\infty) dx = \frac{q_s''}{L} \int_0^L \frac{x}{k Nu_x} dx = \frac{q_s'' L}{k \overline{Nu}_L}$$

where \overline{Nu}_L can be relevant correlations for constant-temperature plates.

Limitations on Use of Convection Coefficients

Errors as large as 25% may be incurred by using the expressions due to varying free stream turbulence and surface roughness.

Boundary Layer Flow over a Flat Plate

Global Nu (h) and C_f (τ_s)

Laminar $Nu_L \equiv \frac{\bar{h}L}{k} = 0.664 Re_L^{1/2} Pr^{1/3}$ $C_f = 1.328 / Re_L^{1/2}$

Turbulent $Nu_L \equiv \frac{\bar{h}L}{k} = 0.037 Re_L^{4/5} Pr^{1/3}$ $C_f = 0.074 / Re_L^{1/5}$

$$\tau_s = C_f \frac{\rho V^2}{2}$$

Chilton-Colburn analogy:

$$\frac{C_f}{2} = \frac{Nu}{Re Pr^{1/3}}, \quad 0.6 < Pr < 60$$

V	Laminar		Turbulent	
1	$h = 1$	$\Delta P = 1$	$h = 1$	$\Delta P = 1$
2	1.414	2.828	1.741	3.480
3	1.732	5.196	2.408	7.225

◆ Increasing flow velocity would increase both heat transfer and drag, with a stronger effect for drag.

Internal Flow Conditions

$$\text{Re}_D \equiv \frac{\rho u_m D}{\mu} \quad \text{Re}_{D,c} \approx 2300$$

where u_m is the *mean fluid velocity* over the tube cross section.

Entrance length

- For laminar flow is $\left(\frac{x_{fd,h}}{D}\right)_{\text{lam}} \approx 0.05 \text{Re}_D$
- For turbulent flow $10 \leq \left(\frac{x_{fd,h}}{D}\right)_{\text{turb}} \leq 60$

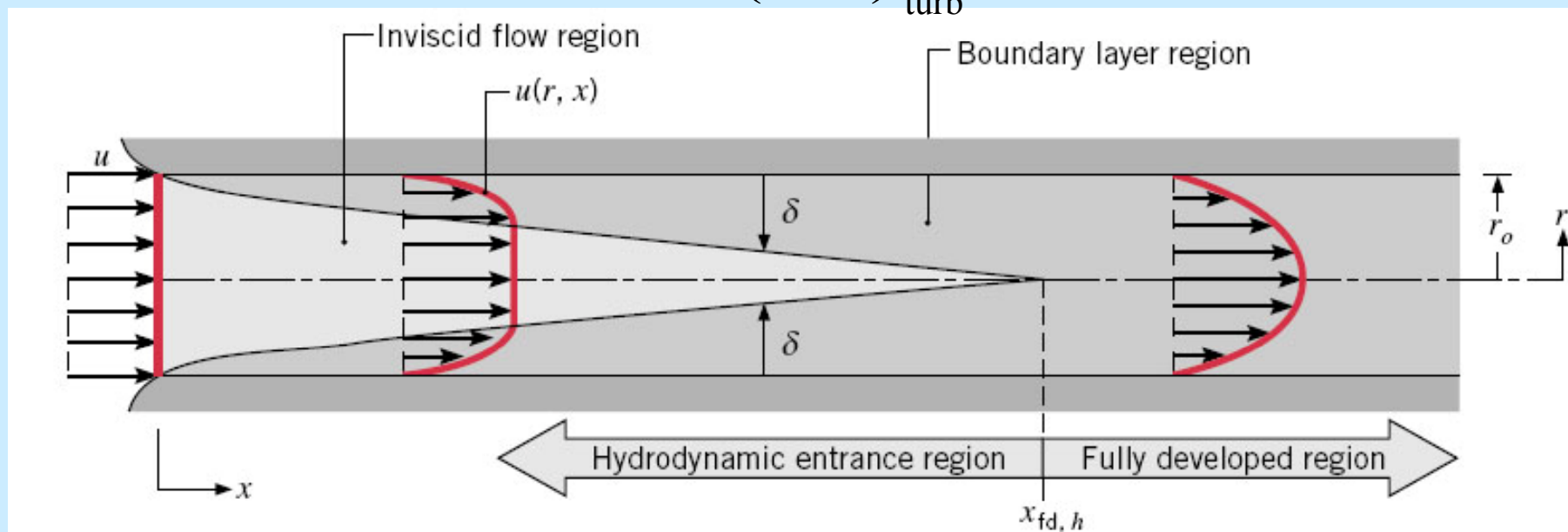


FIGURE 8.1 Laminar, hydrodynamic boundary layer development in a circular tube.

Thermal entrance length

- For laminar flow: $\left(\frac{x_{fd,t}}{D}\right)_{\text{lam}} \approx 0.05 \text{Re}_D \text{Pr}$
- For turbulent flow: $\left(\frac{x_{fd,t}}{D}\right)_{\text{turb}} \approx 10$

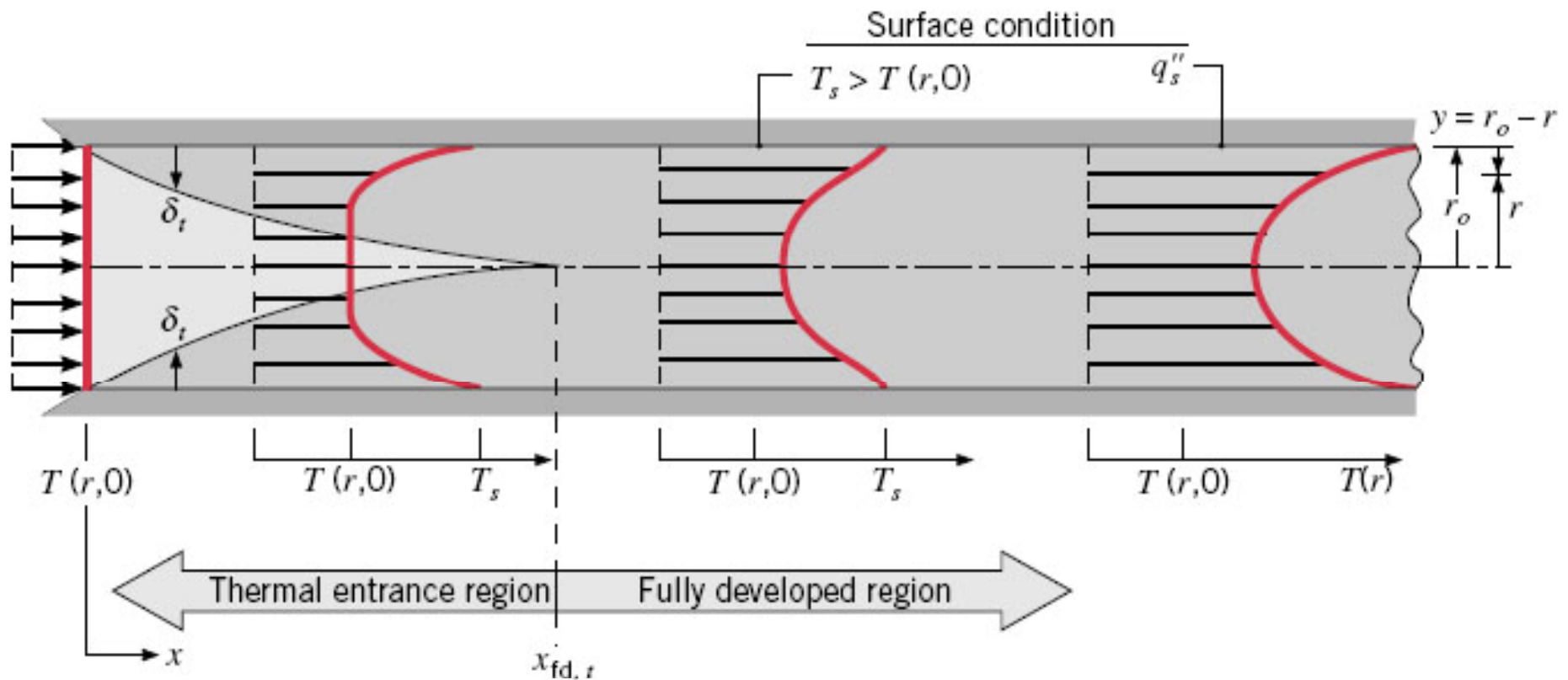


FIGURE 8.4 Thermal boundary layer development in a heated circular tube.

Newton's Law of Cooling

$$q_s'' = h(T_s - T_m)$$

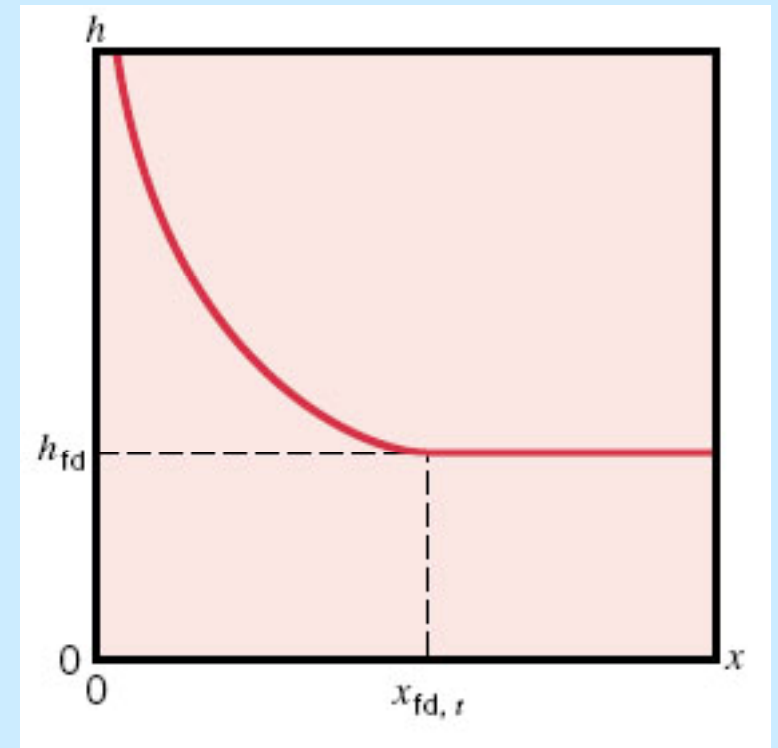
Fully Developed Conditions

The *thermally fully developed* condition is when the *relative shape* of the profile no longer changes and is stated as

$$\frac{\partial}{\partial x} \left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)} \right]_{\text{fd},t} = 0$$

for cases with either a uniform surface heat flux or a uniform surface temperature.

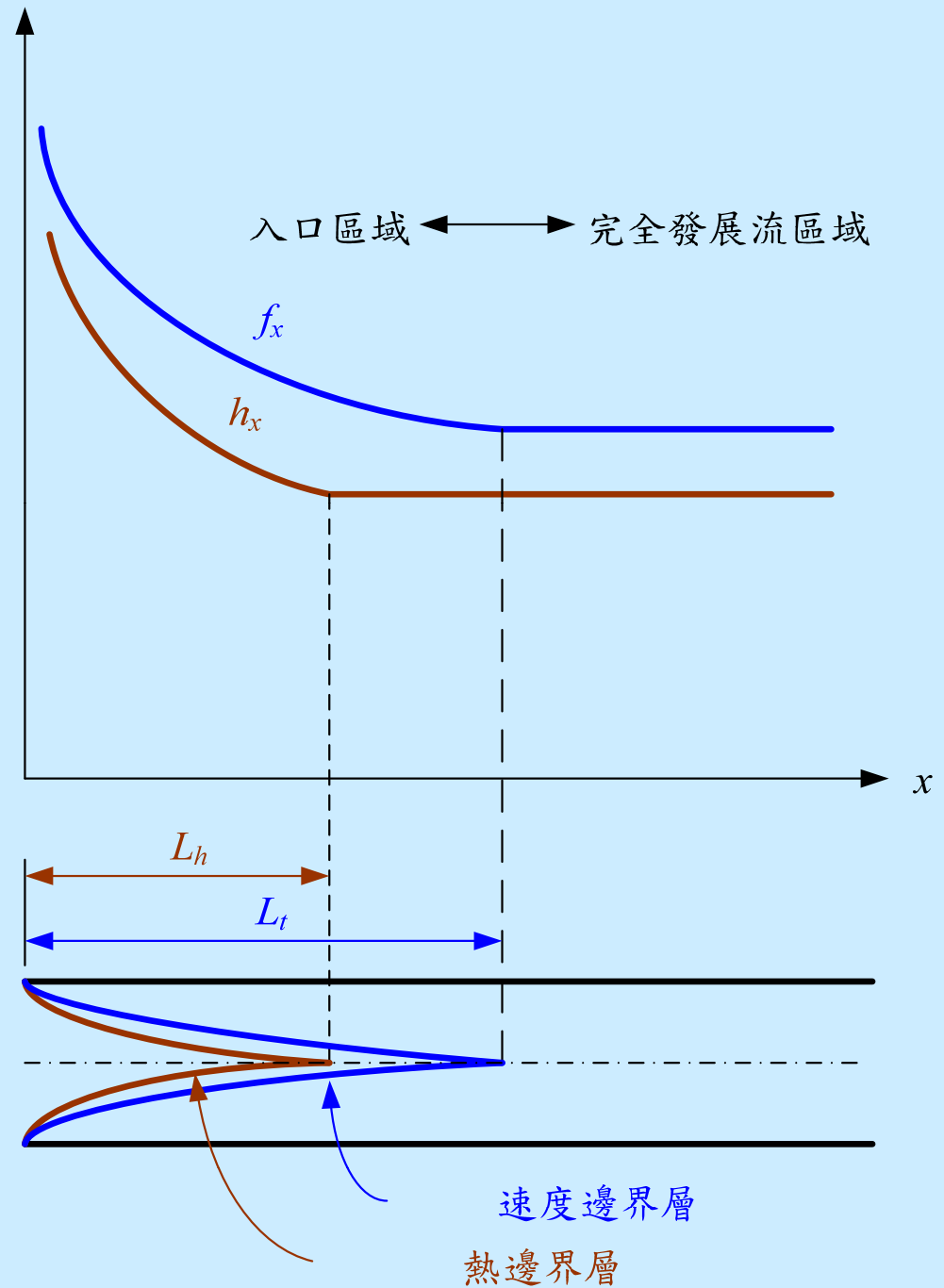
$$\frac{\partial}{\partial r} \left(\frac{T_s - T}{T_s - T_m} \right) \Big|_{r=r_0} = \frac{-\partial T / \partial r \Big|_{r=r_0}}{T_s - T_m} = -\frac{h}{k} \neq f(x)$$



Hence, in the *thermally fully developed flow* of a fluid with constant properties, the *local convection coefficient is a constant*.

Fully Developed Pipe Flow

For $Pr < 1$



The Energy Balance

General Considerations

Energy conservation for a differential control volume leads to

$$dq_{\text{conv}} = \dot{m} c_p dT_m$$

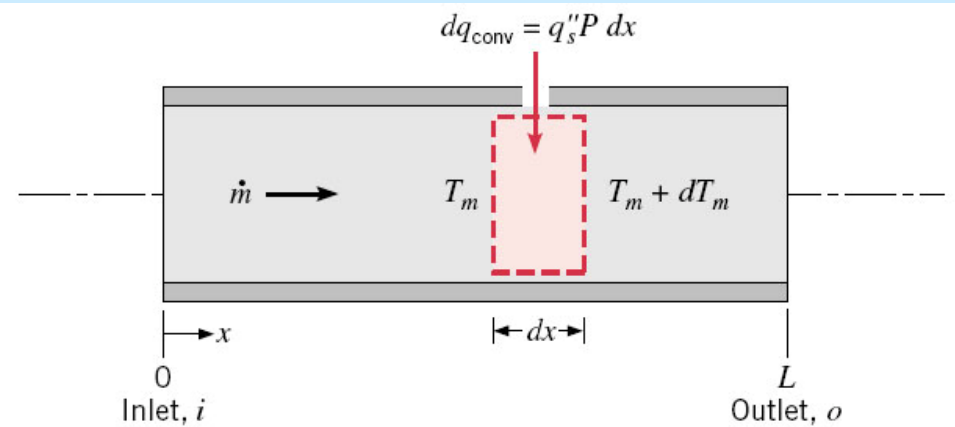


FIGURE 8.6 Control volume for internal flow in a tube.

• Constant surface heat flux

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} h(T_s - T_m)$$

$$\rightarrow T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x$$

• Constant surface temperature

$$q_{\text{conv}} = \bar{h} A_s \Delta T_{\text{lm}}$$

$$\text{where } \Delta T_{\text{lm}} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}$$

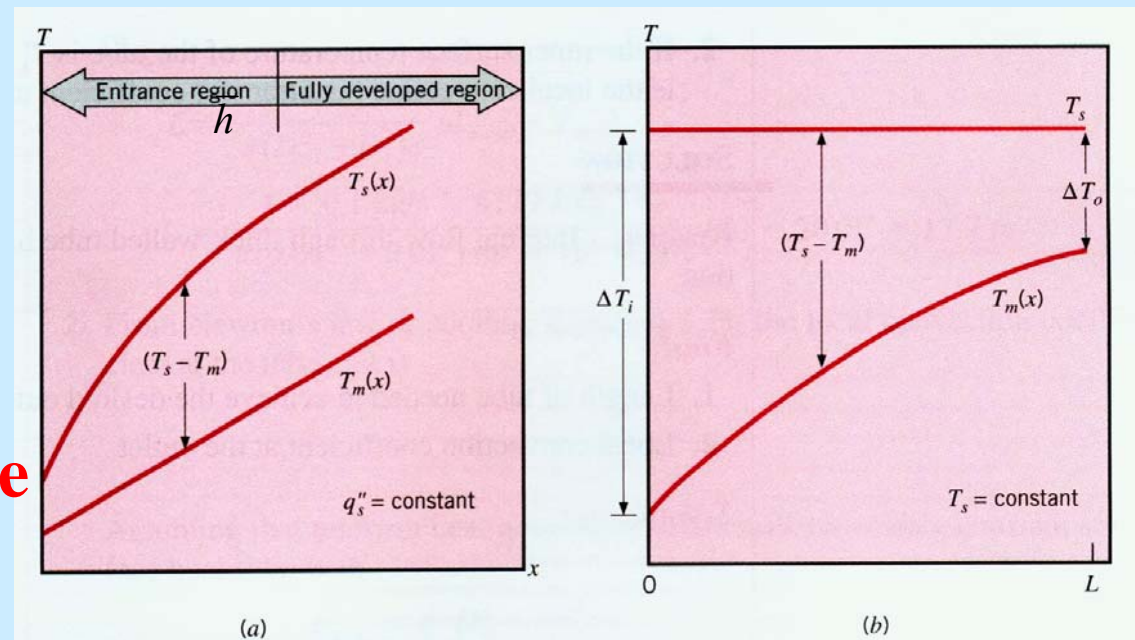


FIGURE 8.7 Axial temperature variations for heat transfer in a tube. (a) Constant surface heat flux. (b) Constant surface temperature.

This means for constant T_s , the total q_{conv} is proportional to \bar{h} and the log mean temperature difference ΔT_{lm} .

Fully Developed Laminar Flow in Noncircular Tubes

Hydraulic diameter:

$$D_h \equiv \frac{4A_c}{P}$$

$$Nu = \frac{hD_h}{k} = \text{constant}$$



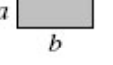

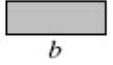
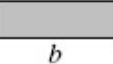
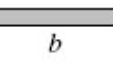

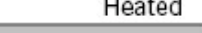


$$f Re_{D_h} = \text{constant}$$

$$\begin{aligned} f Re_D &\equiv \frac{-(dp/dx)D}{\rho u_m^2 / 2} \frac{\rho u_m D}{\mu} \\ &= \frac{-2(dp/dx)D^2}{\mu u_m} = C \end{aligned}$$

$$\rightarrow h \propto 1/D_h$$

$$\Delta P \propto 1/D_h^2$$

TABLE 8.1 Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

Cross Section	$\frac{b}{a}$	$Nu_D \equiv \frac{hD_h}{k}$		$f Re_{D_h}$
		(Uniform q_s'')	(Uniform T_s)	
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	∞	8.23	7.54	96
	∞	5.39	4.86	96
	∞	5.39	4.86	96
	—	3.11	2.49	53

Used with permission from W. M. Kays and M. E. Crawford, *Convection Heat and Mass Transfer*, 3rd ed. McGraw-Hill, New York, 1993.

Heat Transfer Enhancement

To induce secondary flow or turbulence

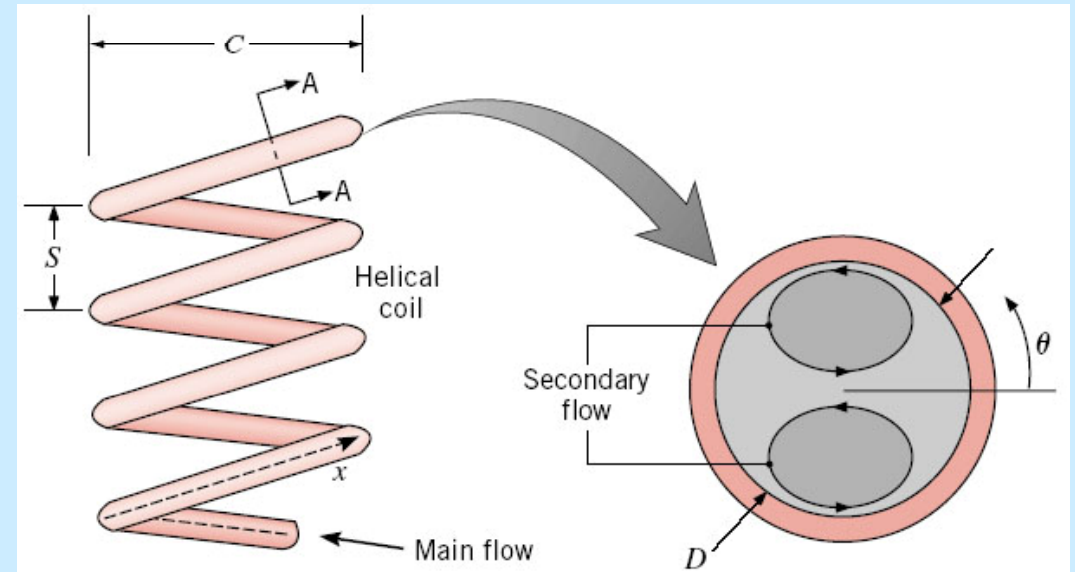
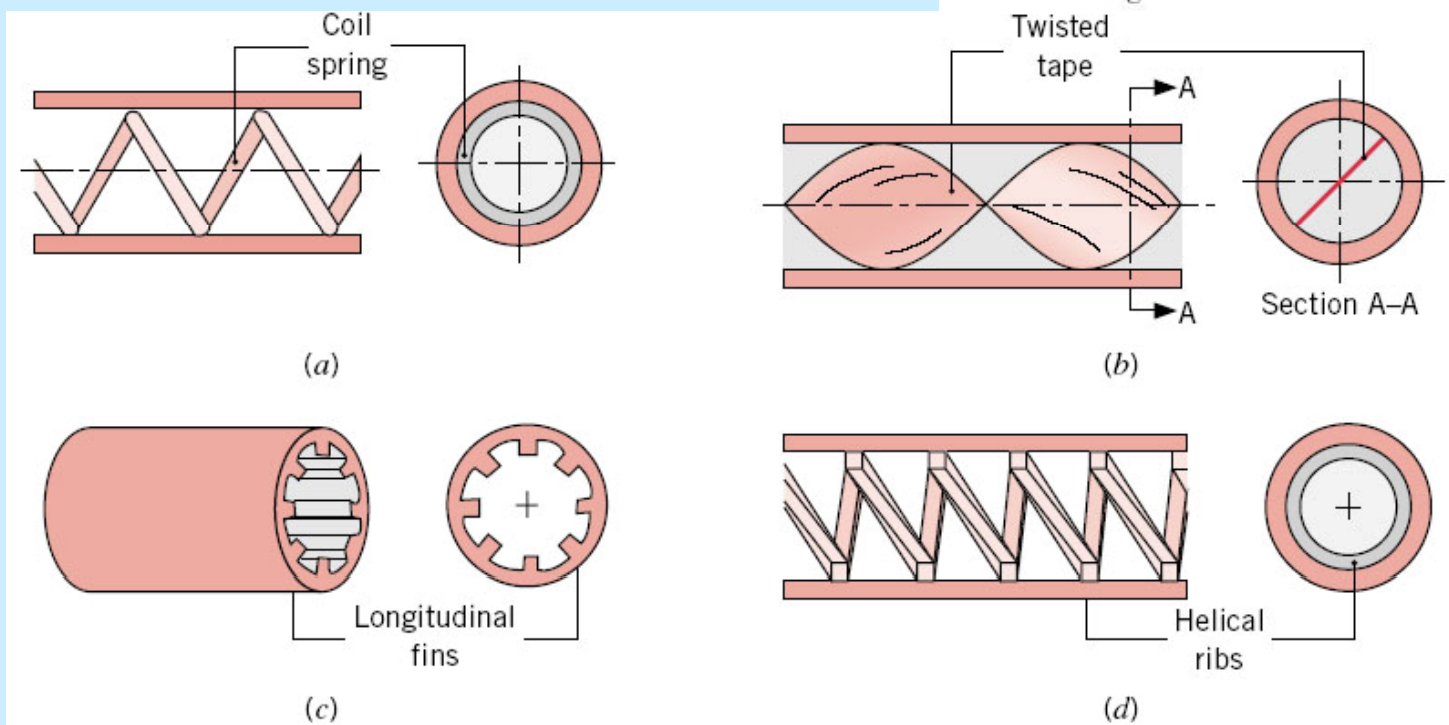
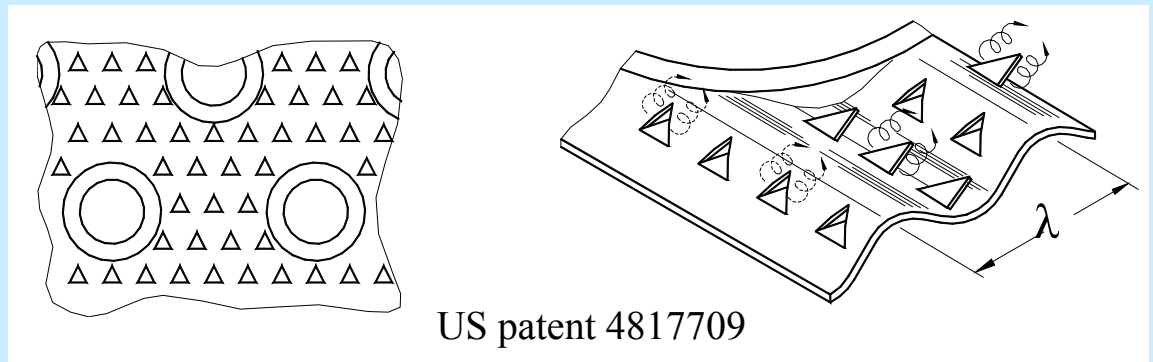
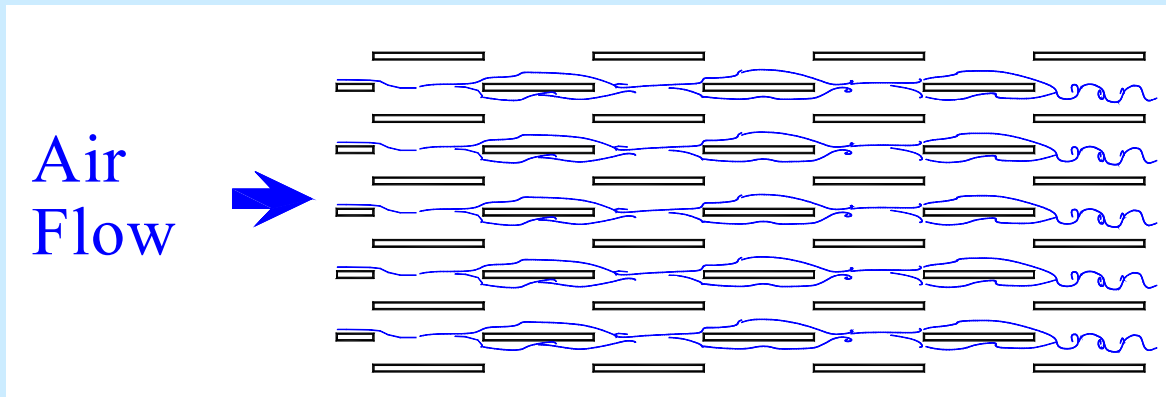


FIGURE 8.13 Schematic of helically coiled tube and secondary flow in enlarged cross-sectional view.

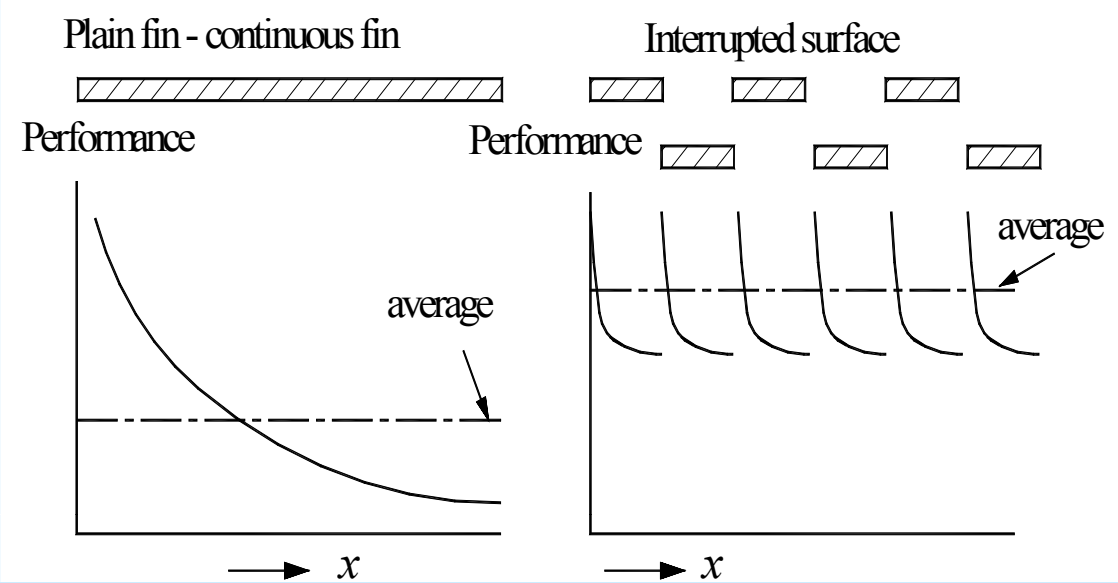
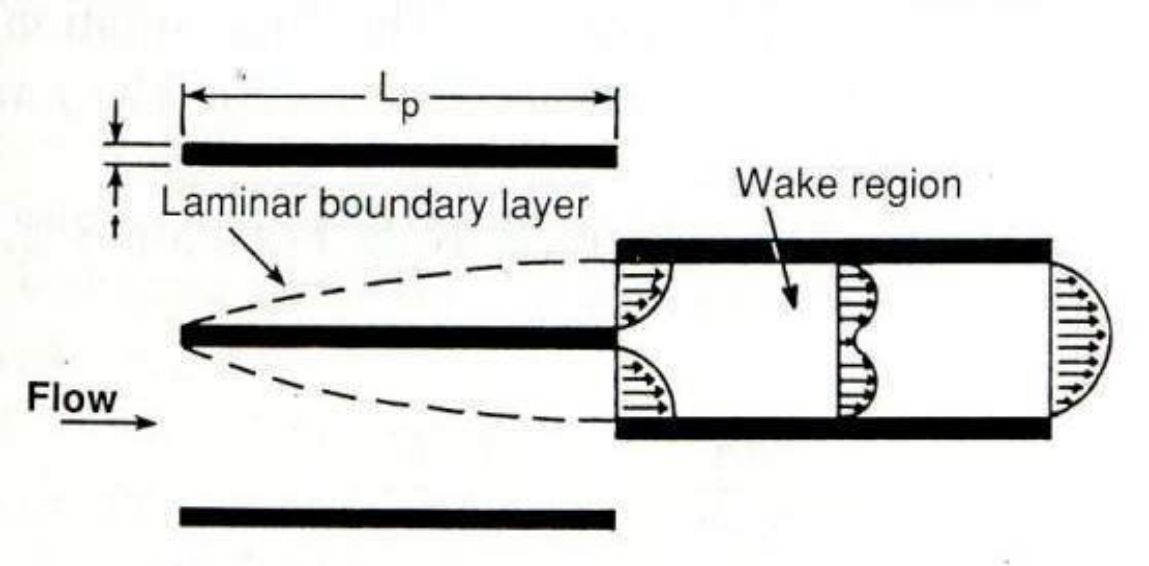


Heat Transfer Enhancement

- Boundary layer regrowth
- Flow transition or turbulence

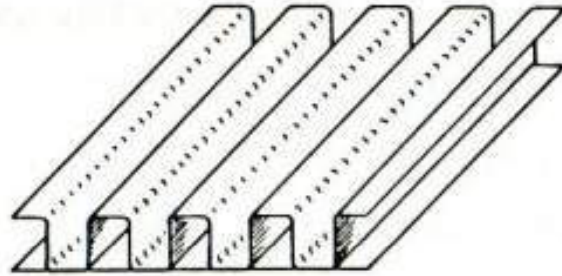


Heat Transfer Enhancement

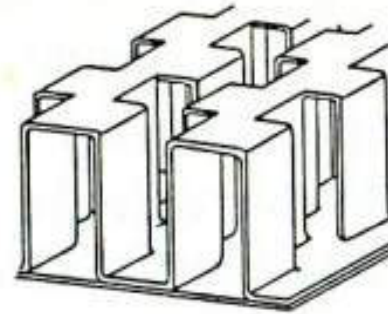


Heat Transfer Enhancement

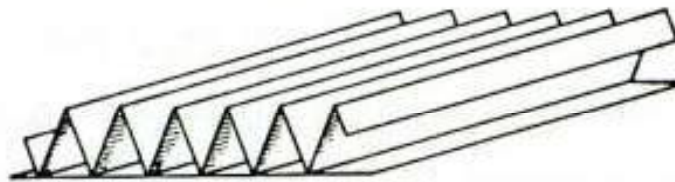
a. Rectangular



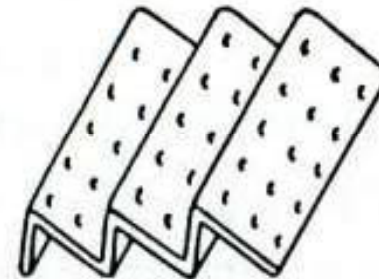
d. Offset Strip Fin



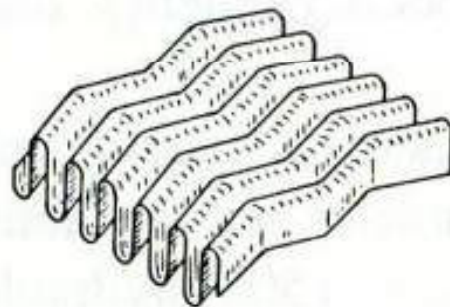
b. Triangular



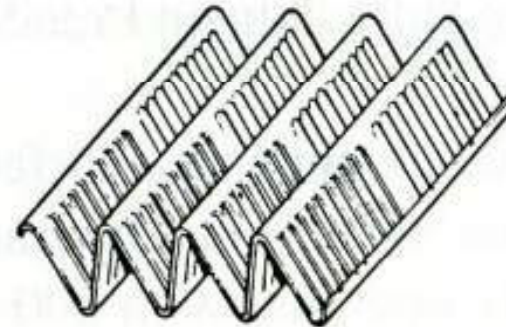
e. Perforated



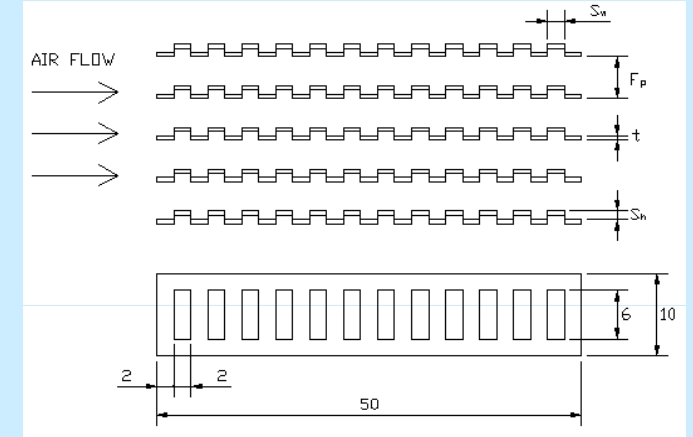
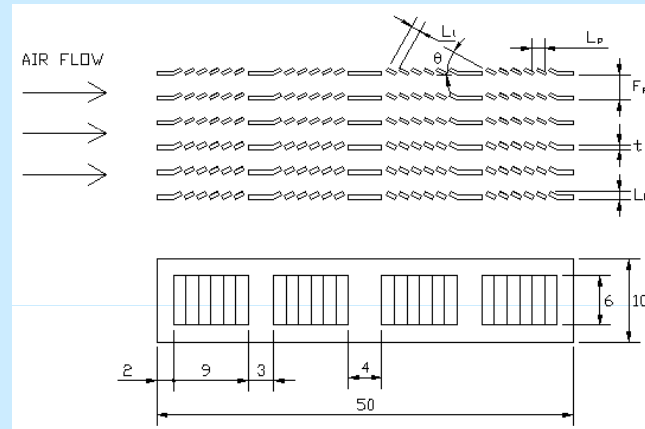
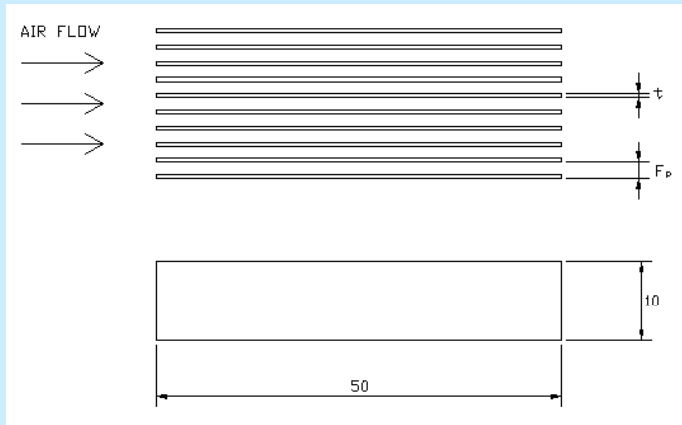
c. Wavy



f. Louvered



Heat Transfer Enhancement



平板型



百葉窗型



裂口型

Natural Convection

Natural convection is due to buoyancy from density difference as a result of temperature and/or concentration variations.

$Gr_L^{1/2}$ plays the same role in free convection that the Re plays in forced convection.

$$Gr_L \equiv \frac{g\beta(T_s - T_\infty)L}{u_0^2} \left(\frac{u_0 L}{\nu} \right)^2 = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

In general,

$$Nu_L = f(Re_L, Gr_L, Pr)$$

If $(Gr_L / Re_L^2) \approx 1$ both free & forced convection to be considered

If $(Gr_L / Re_L^2) \ll 1 \rightarrow Nu_L = f(Re_L, Pr)$ forced convection

If $(Gr_L / Re_L^2) \gg 1 \rightarrow Nu_L = f(Gr_L, Pr)$ free convection

The Effects of Turbulence

For vertical plates the transition occurs at

$$Ra_{x,c} = Gr_{x,c} Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} \approx 10^9$$

Empirical Correlations

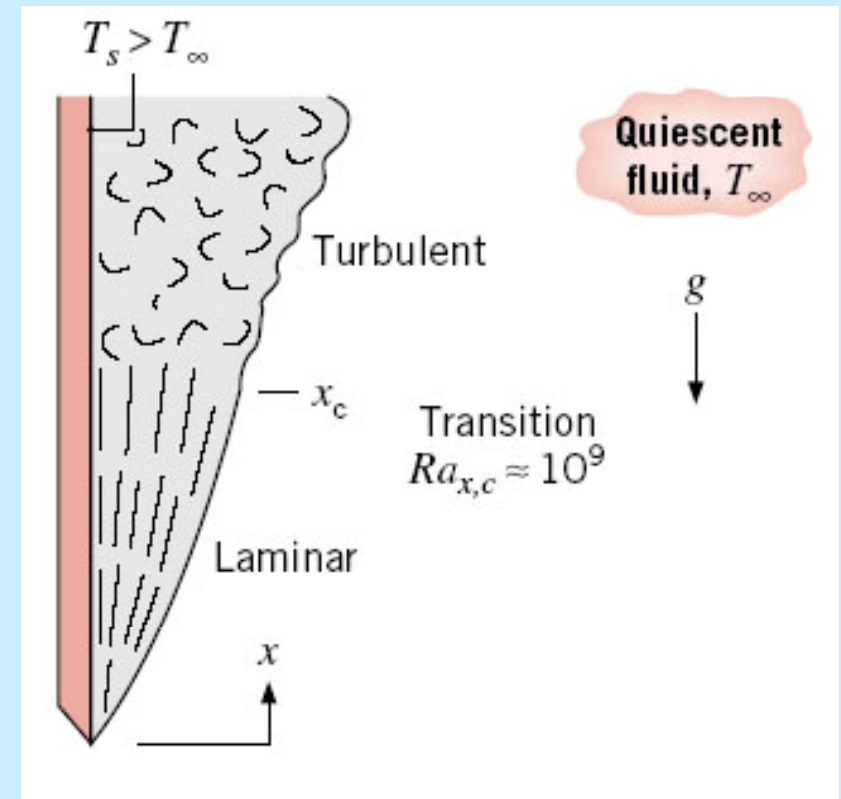
External Free Convection Flows

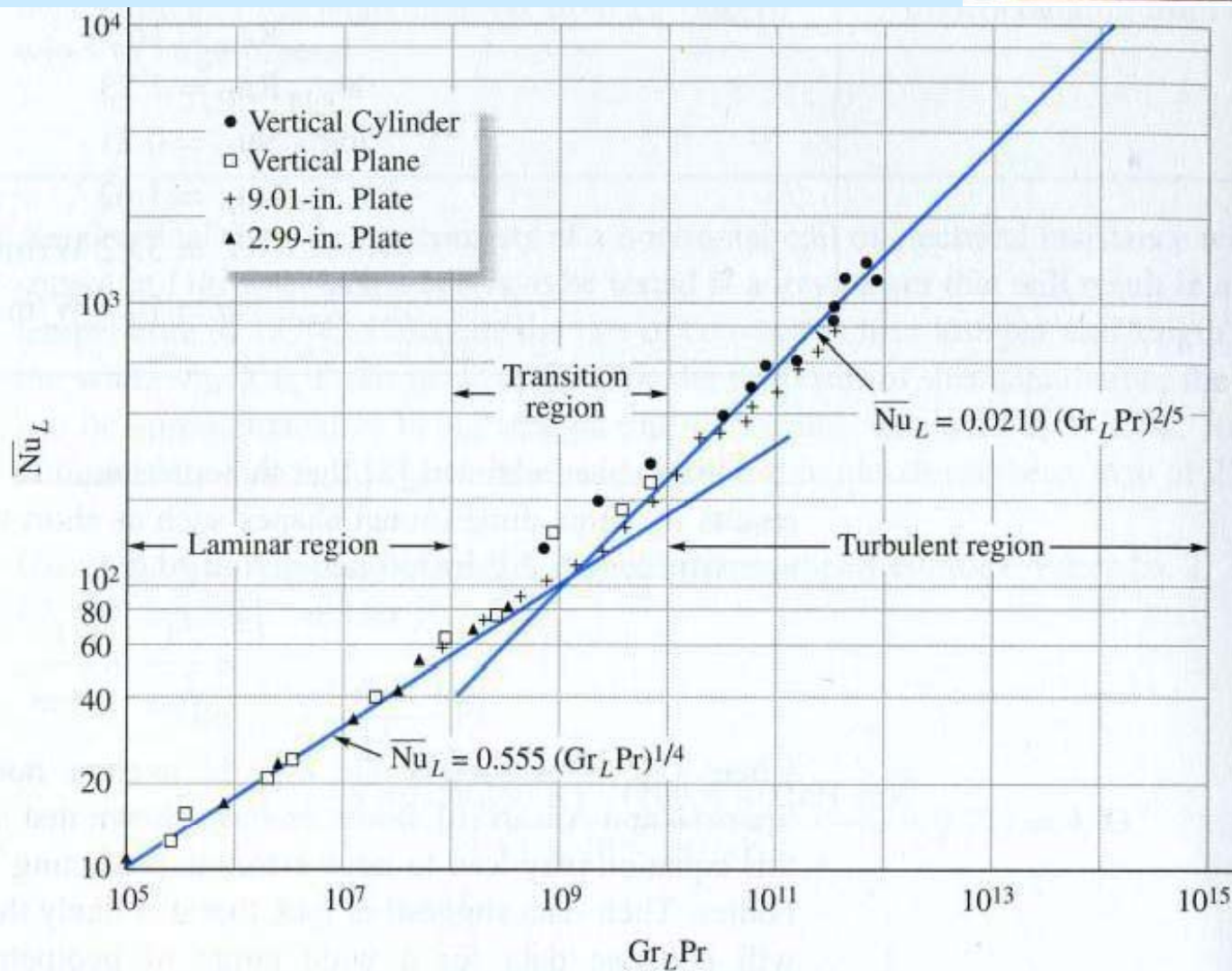
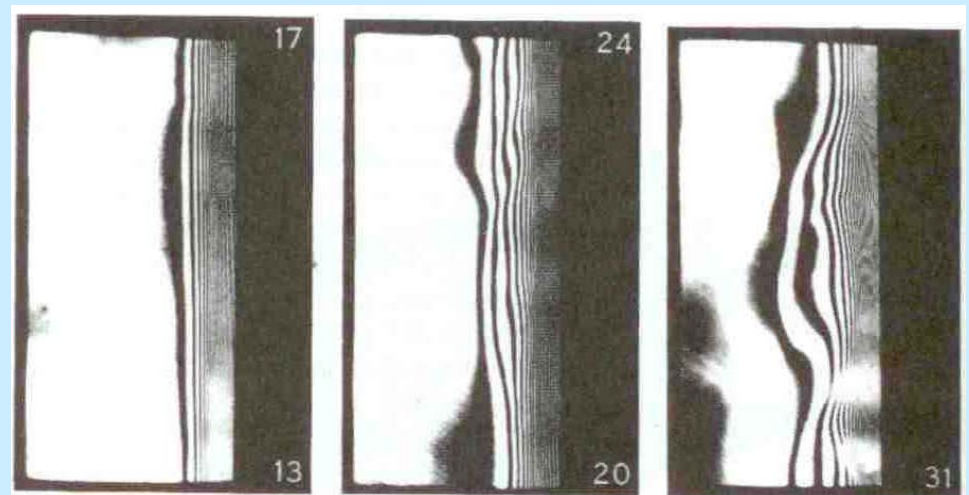
Generally,

$$\overline{Nu}_L = \frac{\overline{h}L}{k} = C Ra_L^n$$

$n = 1/4$, for laminar flow

$n = 1/3$, for turbulent flow





F Kreith & MS Bohn,
Principles of Heat
Transfer, 2001

Flow Pattern

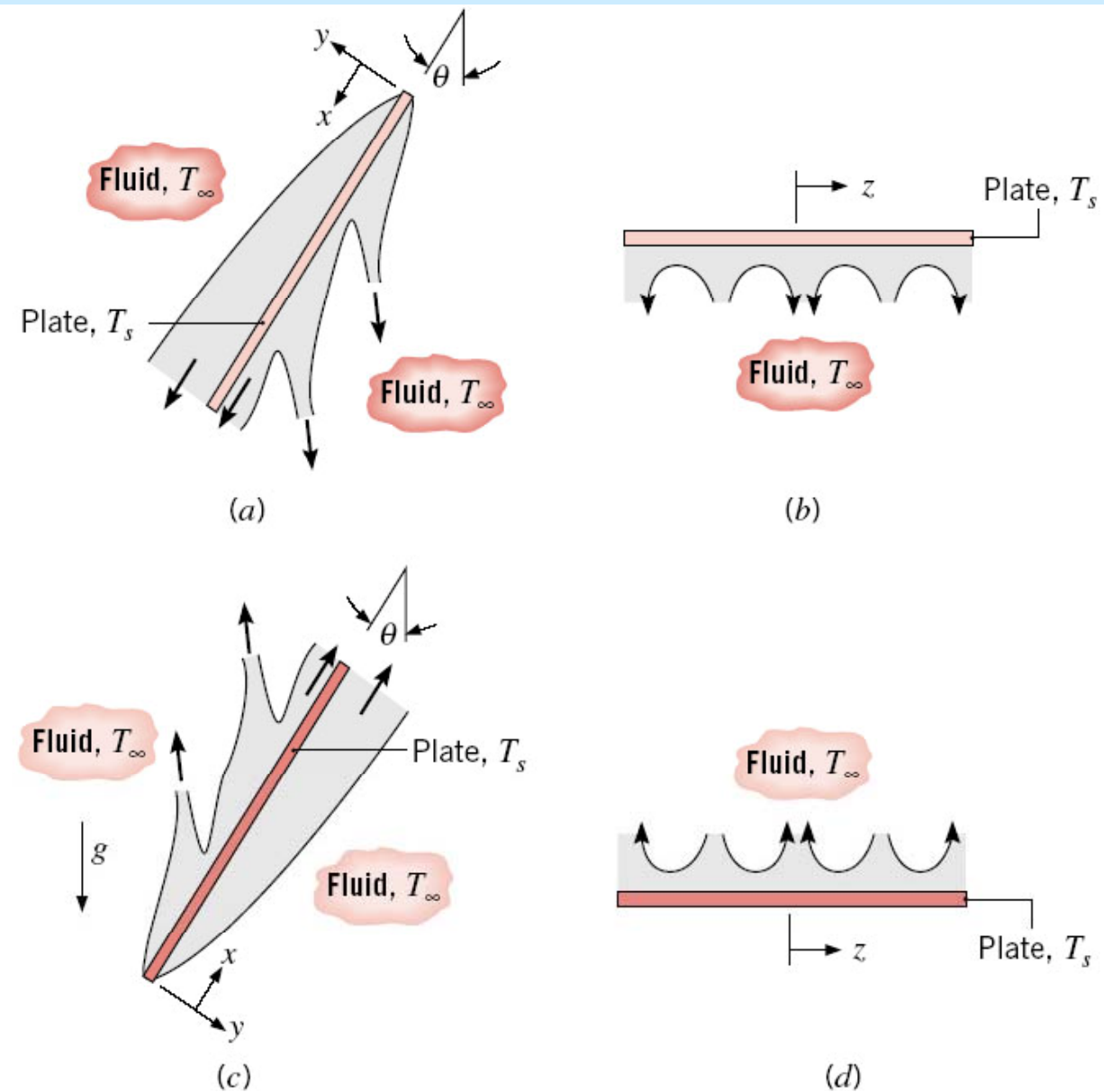


FIGURE 9.6 Buoyancy-driven flows on an inclined plate: (a) side view of flows at top and bottom surfaces of a cold plate ($T_s < T_\infty$), (b) end view of flow at bottom surface of cold plate, (c) side view of flows at top and bottom surfaces of a hot plate ($T_s > T_\infty$), and (d) end view of flow at top surface of hot plate.

Flow Pattern

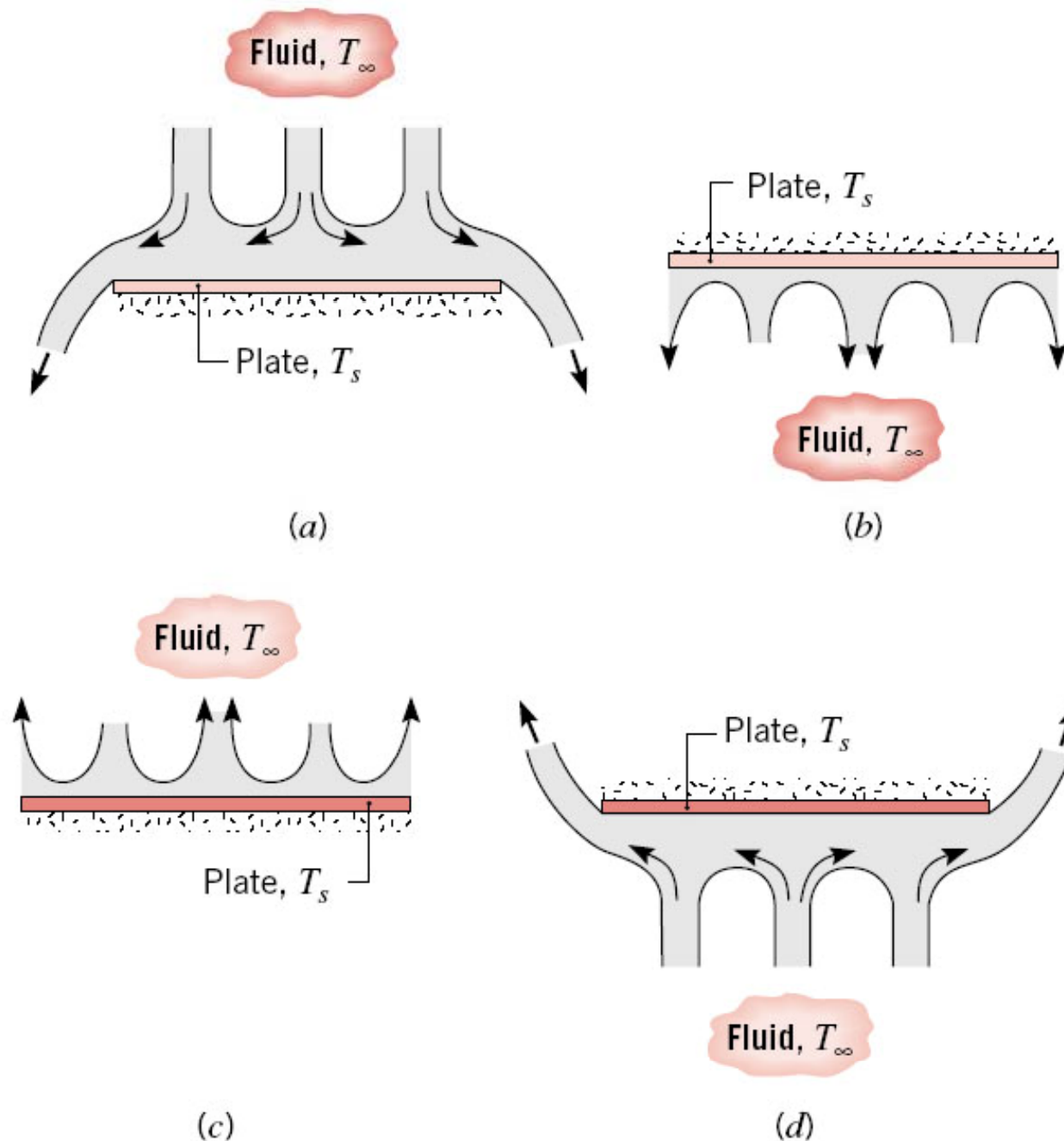
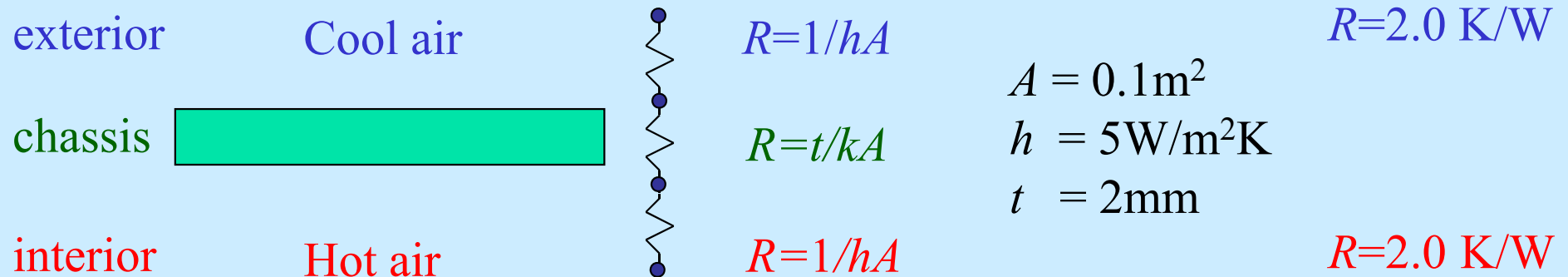


FIGURE 9.7

Buoyancy-driven flows on horizontal cold ($T_s < T_\infty$) and hot ($T_s > T_\infty$) plates: (a) top surface of cold plate, (b) bottom surface of cold plate, (c) top surface of hot plate, and (d) bottom surface of hot plate.

Example of Natural Convection

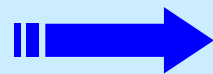


Chassis Material 1 $k=0.2\text{W/mK}$, $R=0.1\text{K/W}$

Chassis Material 2 $k=100\text{W/Mk}$, $R=0.0002\text{K/W}$

$$\rightarrow R_{1_total} = 4.1\text{K/W}$$

$$\rightarrow R_{2_total} = 4.0002\text{K/W}$$



The chassis material is not critical when there is only natural convection.

Thermal Radiation

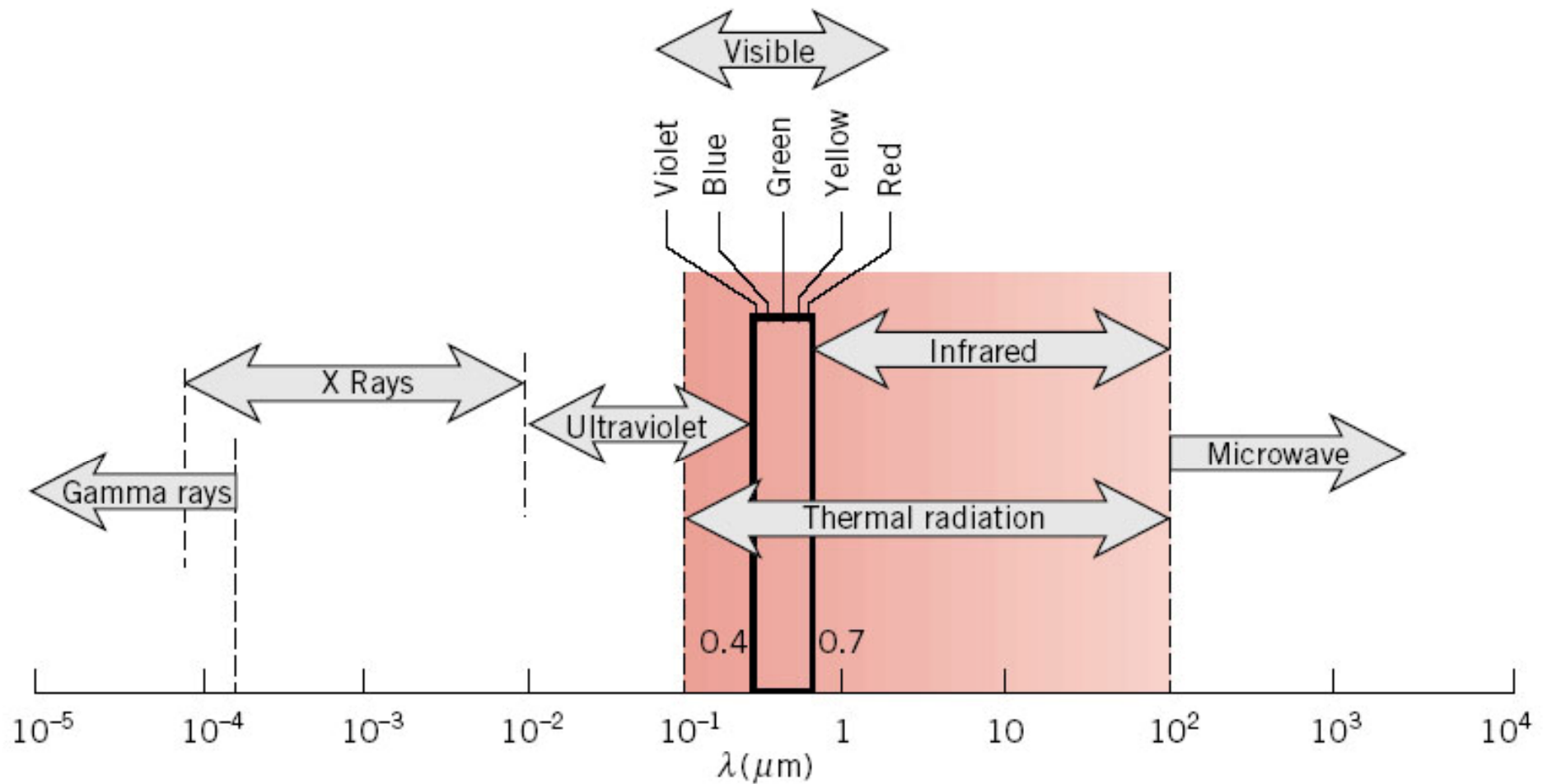
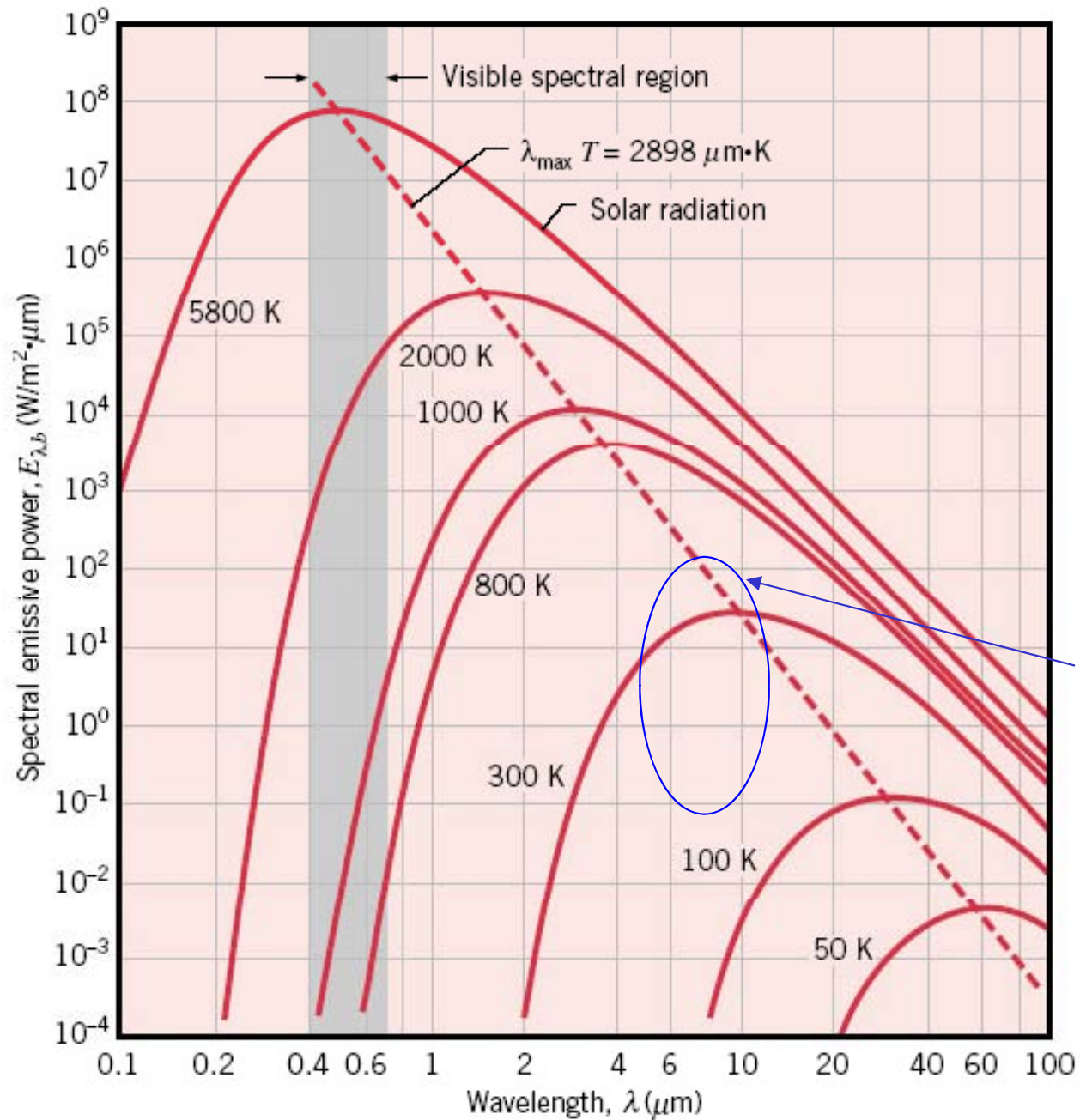


FIGURE 12.3 Spectrum of electromagnetic radiation.

Black-Body Emission



For electronic equipment, ~300-350K

FIGURE 12.12 Spectral blackbody emissive power.

Real Surface vs Black Surface

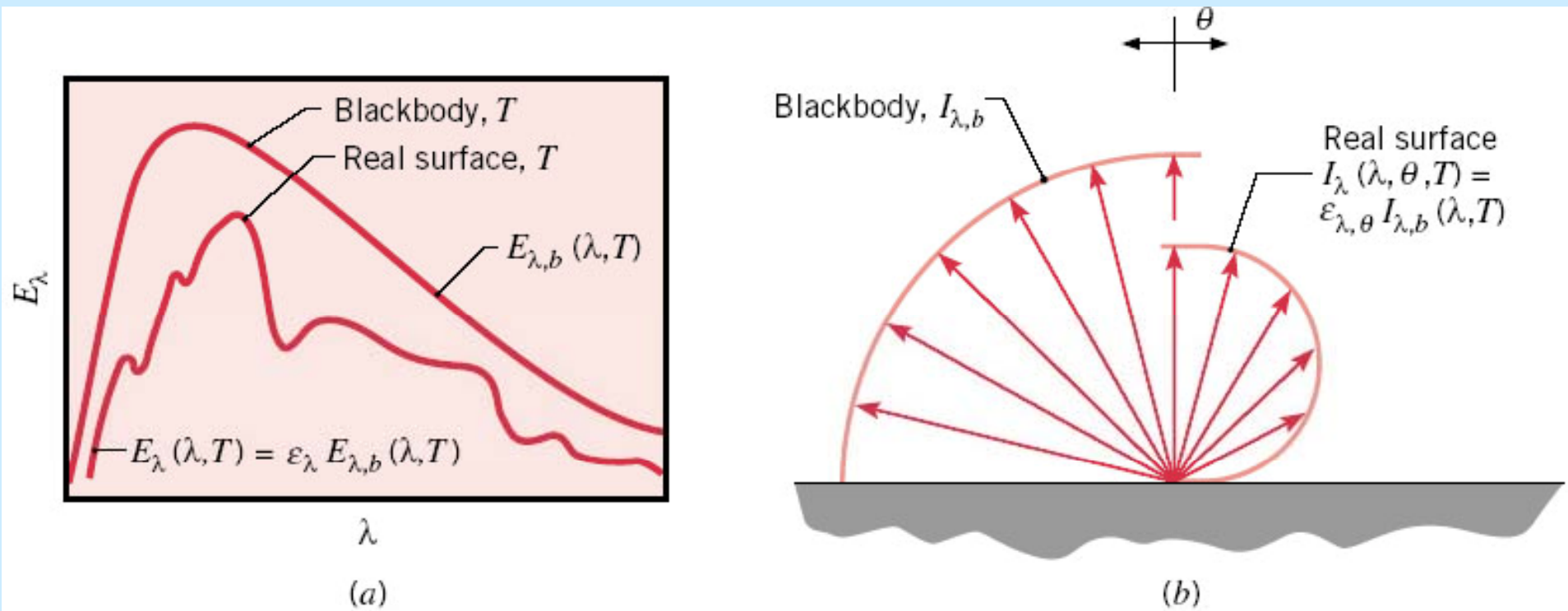


FIGURE 12.15 Comparison of blackbody and real surface emission. (a) Spectral distribution. (b) Directional distribution.

Emissivities of Some Real Surfaces

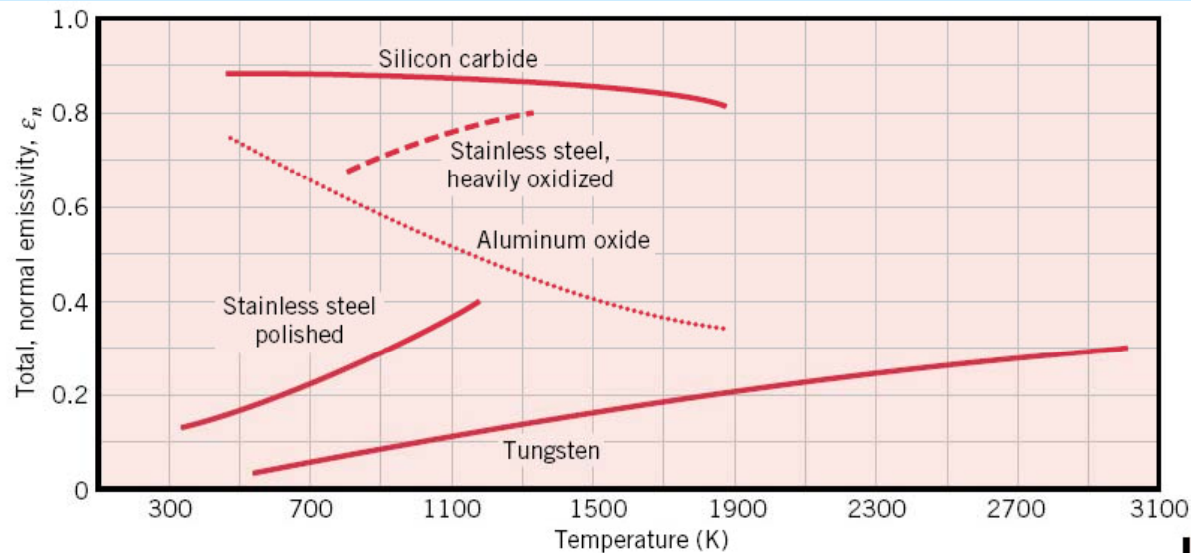


FIGURE 12.18 Temperature dependence of the total, normal emissivity ϵ_n of selected materials.

The Kirchhoff's Law

For a gray and diffuse surface, the surface emissivity and absorptivity are equal at a given T ,

$$\epsilon(T) = \alpha(T).$$

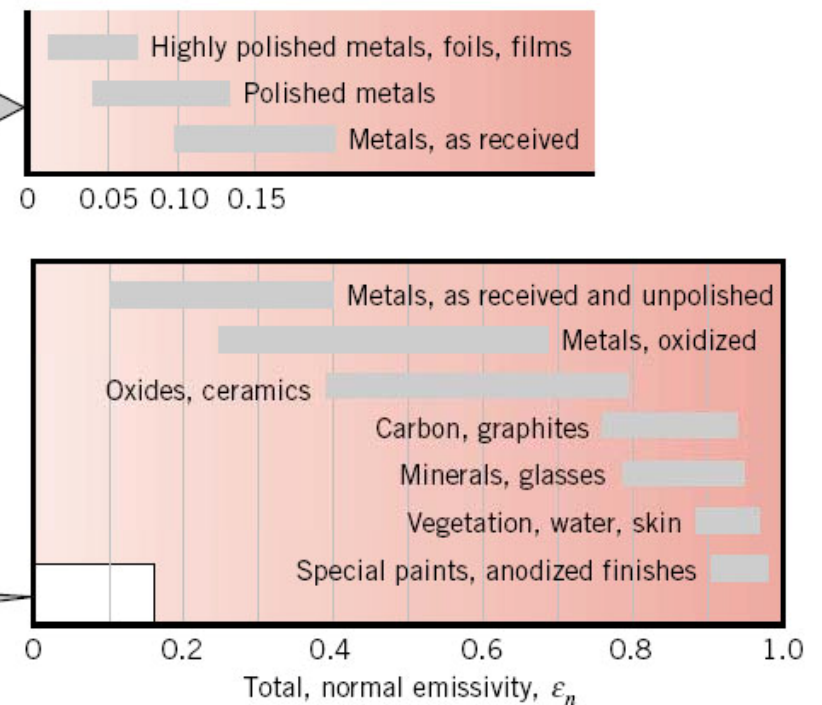


FIGURE 12.19 Representative values of the total, normal emissivity ϵ_n .

Radiation Heat Transfer

The radiative heat transfer rate of an object in a very large environment can be calculated with

Stefan-Boltzmann constant = $5.67 \times 10^{-8} \text{W/m}^2\text{K}^4$

$$Q_{\text{rad}} = \underset{\substack{\downarrow \\ \text{Emissivity}}}{\varepsilon} \overset{\substack{\uparrow \\ \text{Stefan-Boltzmann constant}}}{\sigma} A_s \underset{\substack{\downarrow \\ \text{View Factor} \\ \text{(Shape Factor)}}}{F_{\text{s-sur}}} (T_s^4 - T_{\text{sur}}^4)$$

Emissivity ε

Al, Polished 0.05

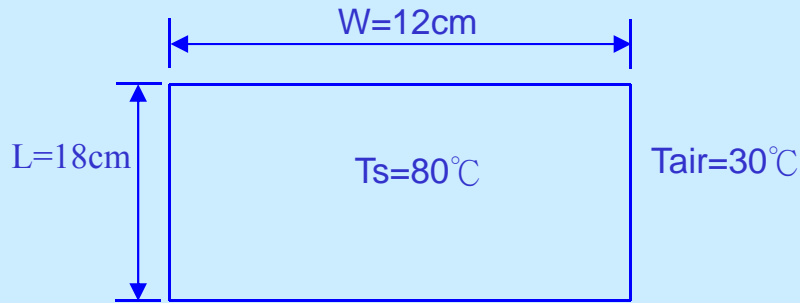
Al, Anodized 0.7-0.95

Cu, Polished 0.06

Cu, Oxidized 0.78

Example of Natural Convection and Radiation

Calculate Q under natural convection and radiation



$$T_f = \frac{T_s + T_\infty}{2} = 55^\circ\text{C}$$

$$K = 0.0277\text{W/Mk}, Pr=0.72, \nu=1.846 \times 10^{-5}\text{ m}^2/\text{s}$$

$$\beta = 1/T_f = 1/328\text{K}, \rho = 1.07\text{ kg/m}^3, \varepsilon = 0.95$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr$$

$$= \frac{(9.81\text{m}^2/\text{s})[1/(328\text{K})][(80-30)\text{K}](0.18\text{m})^3}{(1.846 \times 10^{-5}\text{m}^2/\text{s})^2} (0.72) = 1.8 \times 10^7 < 10^9, \text{ laminar}$$

$$Nu = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$$

cf. Incropera et al. (9.26)

$$= \left\{ 0.825 + \frac{0.387(1.846 \times 10^7)^{1/6}}{[1 + (0.492/0.7)^{9/16}]^{8/27}} \right\}^2 = 37.1$$

$$h = \frac{Nu \times k}{L} = 5.7 \text{ W/m}^2\text{ }^\circ\text{C}$$

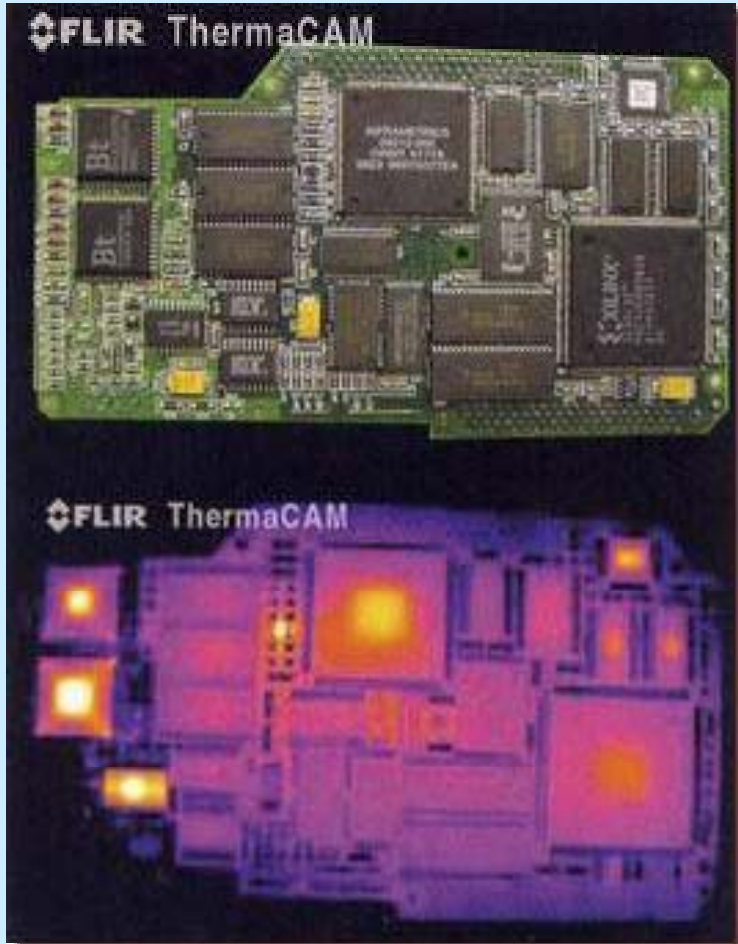
$$Q = hA_s(T_s - T_\infty)$$

$$= (5.7 \text{ W/m}^2\text{ }^\circ\text{C})[(0.18 \text{ m})(0.12 \text{ m})](80 - 30)^\circ\text{C} = \boxed{6.156 \text{ W}}$$

$$Q_{\text{rad}} = \varepsilon \sigma A(T_s^4 - T_{\text{sur}}^4)$$

$$= 0.95 \times 5.67 \times 10^{-8} \times (0.18 \times 0.12) \times (353^4 - 300^4) = \boxed{8.64 \text{ W}}$$

IR Thermal Imager or Thermometer



◆ Air invisible

◆ Not for covered objects

◆ For relative temperature measurement only, because the surface emissivities are different for different materials. To obtain the precise temperature, the surface emissivity is required.