



باسمه تعالی



جزوه درس استاتیک

استاد : دکتر شکریه

تهیه و تنظیم: انجمن علمی دانشکده مهندسی مکانیک دانشگاه

علم و صنعت ایران

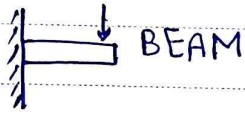
تابستان ۱۳۹۵

تعاریف در استاتیک:

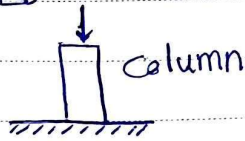
Solid = جامد \Rightarrow Deformable: قابل تغییر

Rigid = صلب

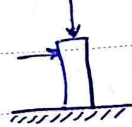
Equilibrium = تعادل



BEAM



Column



BEAM - Column

Scalar: کمیت‌های عددی - اسکالر

Vector: بردار

Tensor: کمیت‌هایی که از چند متغیر درست آمده اند - کمیت‌هایی وابسته به چند پارامتر

Engineering Mechanics, statics

W.F. Riley & L.D. Sturges

John Wiley Inc 1996.

moment

ممان (گشتاور): تمایل نیرو به چرخاندن یک جسم \leftarrow

centroid: مرکز جرم

 \leftarrow پایه‌ای

Fundamental Quantities of Mechanics:

- space
- Time
- Matter (mass - Inertia)
- Force
- Particle
- Rigid Body

عکس العمل: Interaction

* تمرین: طراحی آزمایش: جسم زمین و قطران

* قانون جاذبه (Law of gravitation):

$$F = G \frac{m_1 m_2}{r^2}$$

universal gravitational constant.

$G = 6.674 \times 10^{-11} \frac{m^3}{kg \cdot s^2} \Rightarrow SI$

$G = 3.217 \times 10^{-8} \frac{ft^3}{slug \cdot s^2} \Rightarrow U.S. \text{ Customary system}$

	Mass	Radius	Distance from Earth
Earth	$5.972 \times 10^{24} \text{ kg}$ $1.316 \times 10^{22} \text{ slug}$	$6.371 \times 10^6 \text{ m}$ $2.09 \times 10^7 \text{ ft}$	—
Moon	$7.342 \times 10^{22} \text{ kg}$ $1.071 \times 10^{21} \text{ slug}$	$1.737 \times 10^6 \text{ m}$ $5.67 \times 10^6 \text{ ft}$	$3.844 \times 10^8 \text{ m}$ $1.251 \times 10^9 \text{ ft}$
Sun	$1.989 \times 10^{30} \text{ kg}$ $1.476 \times 10^{29} \text{ slug}$	$6.957 \times 10^8 \text{ m}$ $2.28 \times 10^9 \text{ ft}$	$1.496 \times 10^{11} \text{ m}$ $4.9 \times 10^{11} \text{ ft}$

* تدرین: ۱ متر از کجا آمد (اهتمالی: طبع معج - سیر)

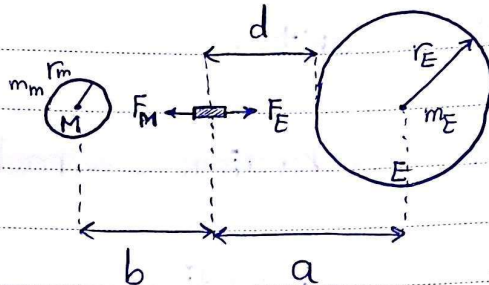
$g = G \frac{M_E}{R_E^2} \Rightarrow g: \text{gravitational acceleration}$

$g = 9.80665 \text{ m/s}^2 = 32.174 \frac{ft}{s^2}$

* سؤال: فرق ایندو کجاست؟

- $W = 200 \text{ lb}$ (وزن) @ earth surface
 © weight on the Moon surface
- a) Mass: ?
 b) 500 mi weight: = ?

If: $F_E = \gamma F_m$
 $d = ?$



اسات *

units of measurement:

Quantities: length, Area, volume, velocity, acceleration, pressure, force, moment, Energy, etc.

fundamental: length, time, mass

derived: velocity, Area, volume, acceleration
 مشتق

Units:

Quantity	Name of unit	symbol
----------	--------------	--------

* Length	meter	m
----------	-------	---

* Mass	kilogram	kg
--------	----------	----

* Time	second	s
--------	--------	---

suplimentaly = ?

* Supplementary units : مشتق نشده اما کمیت اصلی هم نیست (واحد ندارد)

plane angle → radian → rad

solid angle → steradian → Sr ^{کروچک}

* تمرین : اندازه گیری چگالی دقیق

پایان جلسه ۲

Quantity	Dirived SI units	Symbol	special Name
Area	square meter	m ²	—
Volume	Cubic meter	m ³	—
Linear velocity	meter per second	m/s	—
Angular velocity	radian per second	rad/s	—
Linear acceleration	meter per second squire	m/s ²	—
Frequency	(cycle) per second	Hz	Hertz
Density	kilogram per cubic meter	kg/m ³	—
Force	kilogram meter per second squire	N	Newton
Moment of force	Newton meter	N.m	—
pressure: (scalar)	Newton per meter square	Pa	Pascal
stress (tensor)	" " "	Pa	"
Work	Newton meter	J	Joule
Energy	" "	"	"
Power	Joule per second	W	Watt

انرژی نیست
خارجی
درونی

تقریبی می خورد...

⊛ Multiples of SI units:

Factor by which unit is multiplied	Prefix Name	Symbol
------------------------------------	-------------	--------

10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto *	h
10^1	deca *	da
10^{-1}	deci	d
10^{-2}	centi	c → کسٹ
10^{-3}	mili	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

b, a, f, yard, ...

units ^{واحد} "Dimensional consideration"

Physical Quantity	Dimension	SI (units)	U.S (units)
* Length	L	m, mm	in, ft
Area	L ²	m ² , mm ²	in ² , ft ²
Volume	L ³	m ³ , mm ³	in ³ , ft ³
Angle	1 (L/L)	rad, degree	rad, degree
* Time	T	s	s
Linear velocity	L/T	m/s	ft/s
Angular velocity	1/T	rad/s	rad/s
Linear acceleration	L/T ²	m/s ²	ft/s ²
Angular acceleration	1/T ²	rad/s ²	rad/s ²
* Mass	M	kg	slug
force	MLT ⁻²	N	lb
moment of force	ML ² T ⁻²	N.m	ft.lb
pressure	ML ⁻¹ T ⁻²	Pa, kPa	psi, ksi
stress	ML ⁻¹ T ⁻²	"	"
Energy	ML ² T ⁻²	J	ft.lb
Work	"	"	"
Power	ML ² T ⁻³	W	hp ← اسب بخار

برای حل تمرین:

① روز دوشنبه ۱۳ → ۱۱/۲

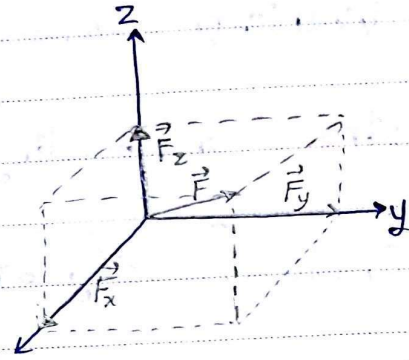
② ماشین حساب

③ جزوه - کتاب

④ تمرین هر هفته به من (احمد شکریه) تحویل داده شود.

$$\begin{aligned}
 F_x &= F \cos \theta \\
 F_y &= F \sin \theta \\
 F &= \sqrt{F_x^2 + F_y^2} \\
 \theta &= \tan^{-1} \left(\frac{F_y}{F_x} \right)
 \end{aligned}$$

→ 2D



$$\vec{F} = F \cos \theta_x \hat{i} + F \cos \theta_y \hat{j} + F \cos \theta_z \hat{k}$$

$$\cos \theta_x = \frac{F_x}{F}, \quad \cos \theta_y = \frac{F_y}{F}, \quad \cos \theta_z = \frac{F_z}{F}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad ; \quad \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

↳ orthonormality

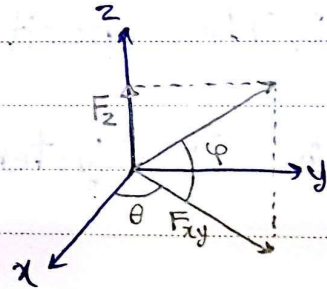
θ در محقات کروی (azimuth) φ (elevation angle) نامیده می‌شوند.

$$F_{xy} = F \cos \varphi$$

$$F_x = F \sin \varphi$$

$$F_x = F_{xy} \cos \theta = F \cos \varphi \cos \theta$$

$$F_y = F_{xy} \sin \theta = F \cos \varphi \sin \theta$$



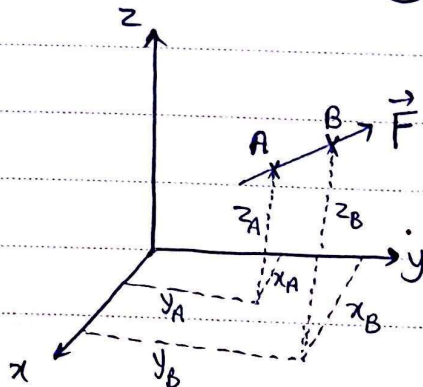
اثبات مربع

⊗ حاله که دو نقطه از بردار \vec{F} داشته باشیم:

$$\cos \theta_x = \frac{x_B - x_A}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

مربع برسد:

$$\cos \theta_z = \frac{z_B - z_A}{\dots}$$



1, varifay conversion factor the power, : قوة *

stress and bending moment from SI system to us.

2, The viscosity of crude oil is $7,13E-3 \frac{N \cdot s}{m^2}$ Express

it in us.

~~N.m~~ \Rightarrow (Torque T) \Rightarrow عزم

$$T = \mu \frac{V}{h}$$

(bending moment M) \Rightarrow کشاورختی

$$\text{psi} = \frac{lb}{in^2}, \quad \frac{lb}{ft^2} = ? \quad : \text{قوة} *$$

3, The specific heat of air under standard condition

in SI System is $1003 \frac{Nm}{kg \cdot K}$ Express it in us.

بیان حلہ ۳

$$EIy = Rx'' - P(x-a)'' - wx'' + Mx' + C$$

$$I, R, w, M, a, C = ?$$

which: $\left\{ \begin{array}{l} P = \text{force} \\ E = \text{force/area} \\ x, y = \text{length} \end{array} \right.$

$$\textcircled{1} \sigma = \frac{P}{A} + \frac{My}{I} \quad \text{which} \quad \left\{ \begin{array}{l} \sigma = \text{stress} \\ A = \text{Area} \\ M = \text{moment of force} \\ y = \text{length} \end{array} \right.$$

مثال *

P, I = ?



1,50 - 1,52 - 1,54 - 1,54 - 1,56 - 1,58

Torque

تورق

moment of force?

Concurrent force System.

۲ die ⊗

Sources of force

- 1, Contact
- 2, magnetic feild
- 3, gravitation
- 4, Impact

Effects of forces

Concurrent
تورق

- } Rigid body a, Reaction
- } solid body deformation

Balanced forces: نیروهای در تعادل قدر گرفته
 $\vec{F}_1 + \vec{F}_2 + \dots = 0$

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots = \vec{R} \Rightarrow$ un-Balanced forces

characteristics of forces:

vectorial Quantity \rightarrow کمیت برداری

① magnitude

② direction

③ point of action نقطه اعمال نیرو

Resolution: بیان (قطعه‌نامه) تجزیه

Resultant: برآیند

principle: اصل

principle: اصلی - (مدیر)

principle of Transmissibility:

می‌توان نیرو را در راستای خط اثرش نیرو را جابجا کنیم. (البته بشرط دار بودن ویرگی)

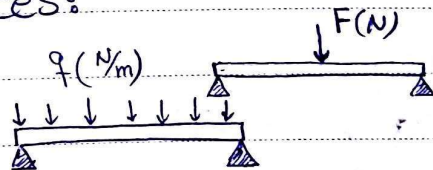
(Rigid body)

classification of forces:

① $W_{\text{snow}} = 20000 \text{ N}$

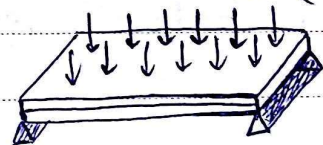
② $q = \frac{W_{\text{snow}}}{l} \quad \text{N/m}$

③ $p = \frac{W_{\text{snow}}}{l \times w} \quad \text{N/m}^2$

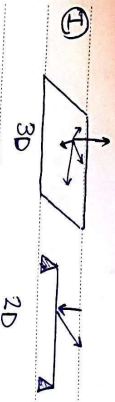


Concentrated force

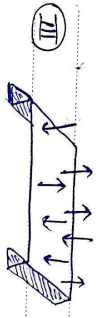
Pressure (N/m^2)



Concurrent:

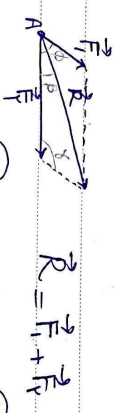


II Co-planar force sys.



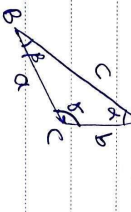
III Parallel force sys.

Resultant of two Concurrent forces:



$$\vec{R} = \vec{F}_1 + \vec{F}_2$$

قانون المثلثات: قانون جيب المثلثات $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$



$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$c' = a' + b' - kab \cos \gamma$$

$$R' = F_1' + F_2' - \gamma F_1 F_2 \cos \gamma = F_1' + F_2' + \gamma F_1 F_2 \cos \gamma$$

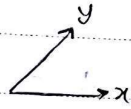
$$\frac{F_1}{\sin \beta} = \frac{R}{\sin \gamma} \Rightarrow \beta = \sin^{-1} \left(\frac{F_1 \sin \gamma}{R} \right)$$

2,6 - 2,7 - 2,10 - 2,17 ... : قسمة *

RAPCO

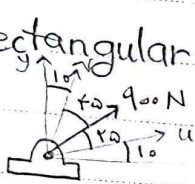
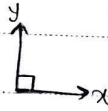
* Resolution of a force into components:

Oblique coordinate system



دک

Rectangular coordinate system

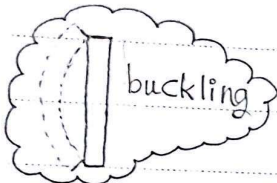


قانون سینوس

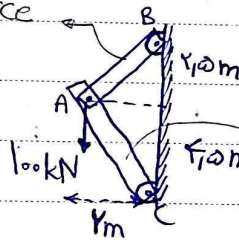
$$\frac{F_u}{\sin 90^\circ} = \frac{F_v}{\sin \alpha} = \frac{900}{\sin 110^\circ}$$

مثال ۱

→ ... Fv و Fu هر دو بردار هستند



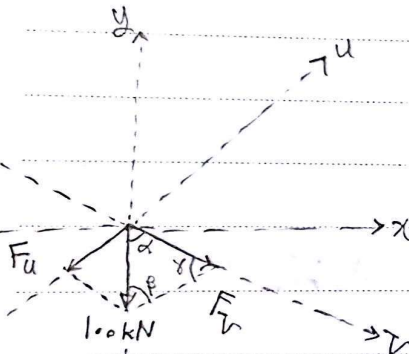
Tensile force



مثال ۲

Compressive force

در این مثال دو ضلع و بی جرم فرض شده اند

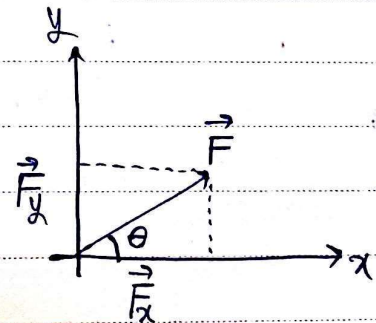


2,36 - 2,38 - 2,40 - 2,42

مثال ۳

Rectangular Components of a force:

orthogonality
و بی اثری }
orthonormality



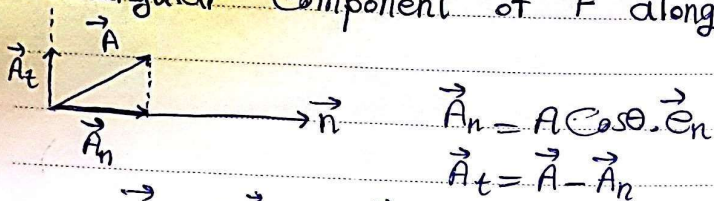
$$\vec{F} = \vec{F}_x + \vec{F}_y = F_x \vec{i} + F_y \vec{j}$$

unit vector

۱۰

بیان جاسم

Rectangular component of \vec{F} along



$$\vec{A}_n = A \cos \theta \cdot \vec{e}_n$$

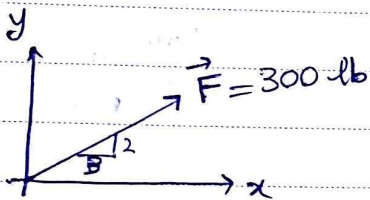
$$\vec{A}_t = \vec{A} - \vec{A}_n$$

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

$$\vec{e}_n = e_{nx} \vec{i} + e_{ny} \vec{j} + e_{nz} \vec{k} \Rightarrow F_n = \vec{F} \cdot \vec{e}_n = (F_x e_{nx} + F_y e_{ny} + F_z e_{nz})$$

$$\Rightarrow \alpha = \cos^{-1} \frac{\vec{F} \cdot \vec{e}_n}{F} = \cos^{-1} \frac{F_n}{F}$$

$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k} \Rightarrow$ vector presentation of a force.



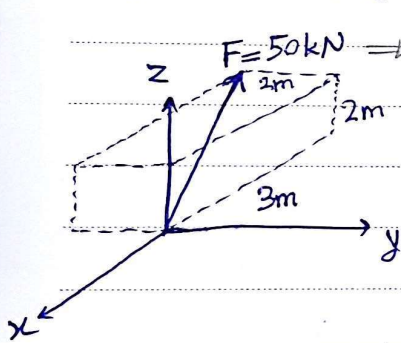
$$\theta = \tan^{-1} \frac{2}{3} = 33.69^\circ$$

$\therefore \vec{F} = 300 \text{ lb}$

$$F_x = F \cos \theta = 249.60 \text{ lb}$$

$$F_y = F \sin \theta = 166.41 \text{ lb}$$

$$\vec{F} = 50 \text{ kN} \Rightarrow \vec{F} = 249.60 \vec{i} + 166.41 \vec{j} \text{ (lb)}$$



$$d = \sqrt{(-3)^2 + (-2)^2 + (2)^2} = 4.123 \text{ m}$$

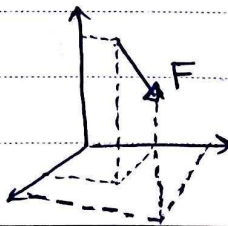
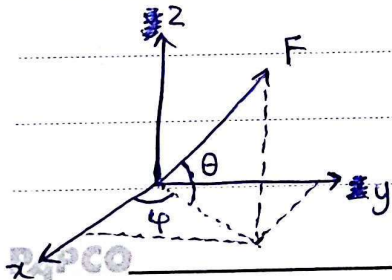
$\therefore \vec{F} = 50 \text{ kN}$

$$\theta_x = \cos^{-1} \frac{x}{d} = 136.69^\circ \Rightarrow F_x = F \cos \theta_x = -36.38 \text{ kN}$$

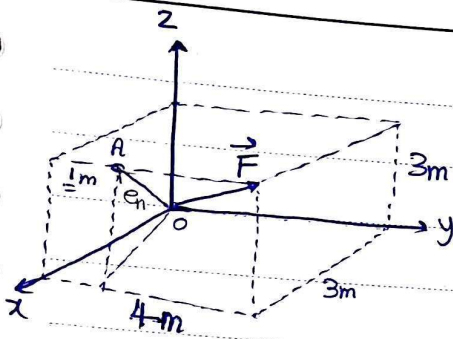
$$\theta_y = \cos^{-1} \frac{y}{d} = 119.02^\circ \Rightarrow F_y = F \cos \theta_y = -24.26 \text{ kN}$$

$$\theta_z = \cos^{-1} \frac{z}{d} = 60.98^\circ \Rightarrow F_z = F \cos \theta_z = 24.26 \text{ kN}$$

$$\Rightarrow \vec{F} = -36.38 \vec{i} - 24.26 \vec{j} + 24.26 \vec{k} \text{ (kN)}$$



||



$$\vec{F} = 12.862 \vec{i} + 17.150 \vec{j} + 12.862 \vec{k} \text{ (kN)}$$

$$OA = d = \sqrt{1^2 + 3^2 + 3^2} = 4.359 \text{ m}$$

$$\vec{e}_n = .6882 \vec{i} + .2294 \vec{j} + .6882 \vec{k}$$

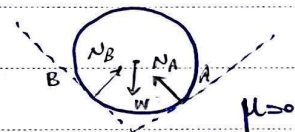
$$\vec{F}_n = \vec{F} \cdot \vec{e}_n = 21.63 \text{ kN}$$

2.73, 2.75, 2.34, 2.93 + 2.63, 2.64, 2.65, 2.66 : * تمرین *

بیان جلسه ۹

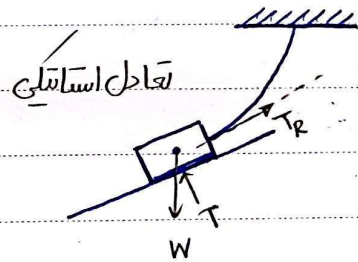
chapter 3 : statics of particles

Free body diagram



* چون نیروها همساز هستند، می توان ابعاد آن را در نظر نگیریم و زوایا بررسی کنیم.

statically Determinate system.



$$\textcircled{2D} \vec{R} = \vec{R}_x + \vec{R}_y = \vec{R}_n + \vec{R}_t = \dots$$

$$\vec{R} = 0 \Rightarrow \left. \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{array} \right\}$$

الاشکل دو بعدی باشد، دو معادله رو به رو باید حل شود.

$$\textcircled{3-D} \vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z = \dots \Rightarrow \left. \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{array} \right\}$$