

$$\vec{F} = 12.862 \vec{i} + 17.150 \vec{j} + 12.862 \vec{k} \text{ (kN)}$$

$$OA = d = \sqrt{1^2 + 3^2 + 3^2} = 4.359 \text{ m}$$

$$\vec{e}_n = .6882 \vec{i} + .2294 \vec{j} + .6882 \vec{k}$$

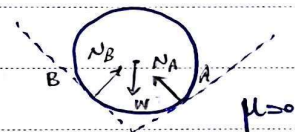
$$\vec{F}_n = \vec{F} \cdot \vec{e}_n = 21.63 \text{ kN}$$

2.73, 2.75, 2.34, 2.93 + 2.63, 2.64, 2.65, 2.66 : * تمرین *

بیان جلسه ۹

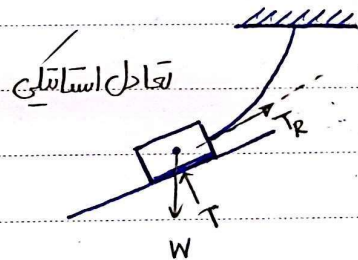
chapter 3 : statics of particles

Free body diagram



* چون نیروها همساز هستند، می توان ابعاد آن را در نظر نگیریم و زوایا بررسی کنیم.

statically Determinate system.

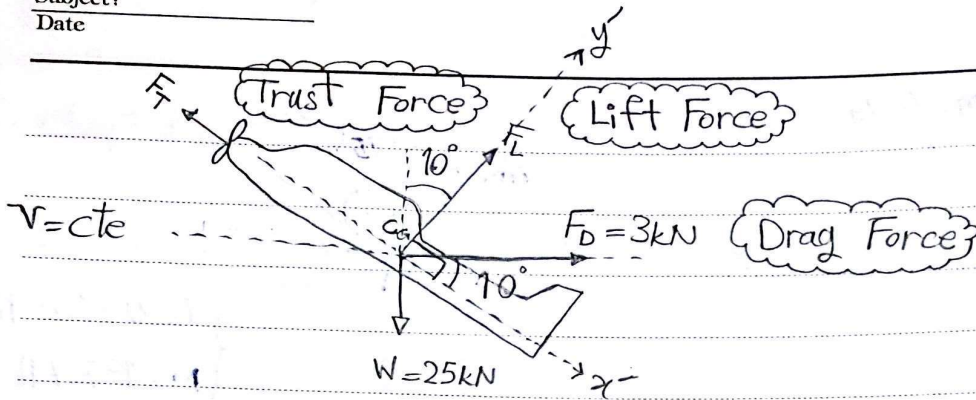


$$\textcircled{2D} \vec{R} = \vec{R}_x + \vec{R}_y = \vec{R}_n + \vec{R}_t = \dots$$

$$\vec{R} = 0 \Rightarrow \left. \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{array} \right\}$$

الاشکل دو بعدی باشد، دو معادله رو به رو باید حل شود.

$$\textcircled{3-D} \vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z = \dots \Rightarrow \left. \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma F_z = 0 \end{array} \right\}$$

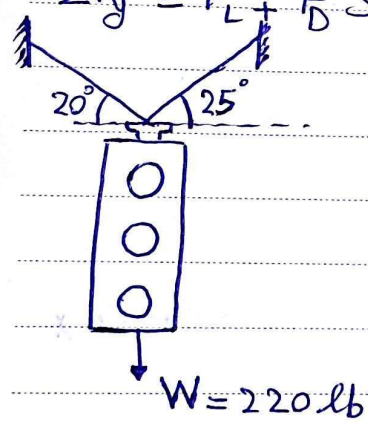


* هوایما:

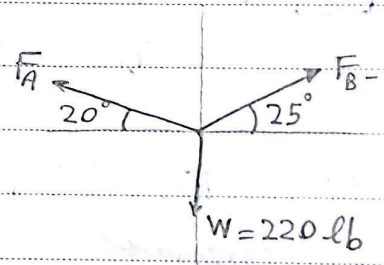
* تمرین: چیراتویب کطف سوراخ هایی روی بدنی خود دارد

$$\sum F_{x'} = -F_T + F_D \cos 10^\circ + W \cos 80^\circ = 0 \Rightarrow F_T = 7.296 \text{ kN}$$

$$\sum F_{y'} = F_L + F_D \sin 10^\circ - W \cos 10^\circ = 0 \Rightarrow F_L = 24.099 \text{ kN}$$



(F.B.D)



* مثال:

$$\sum F_x = 0 \Rightarrow F_B \cos 25^\circ - F_A \cos 20^\circ = 0$$

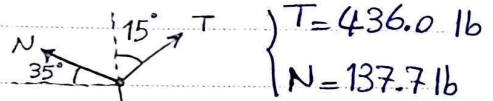
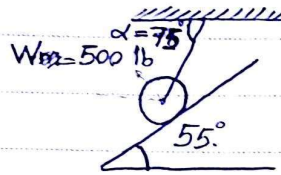
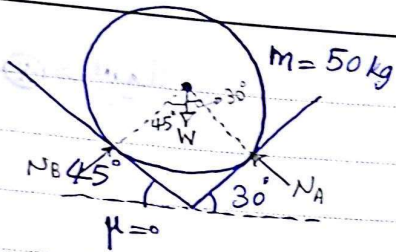
$$\sum F_y = 0 \Rightarrow F_B \sin 25^\circ + F_A \sin 20^\circ - W = 0$$

$$\Rightarrow \begin{cases} F_A = 281.981 \text{ lb} \\ F_B = 292.361 \text{ lb} \end{cases}$$

* تمرین: چیراتویب نووسی برای تمرین قبل، θ را از 0° تا 90° تغییر دهید. چه تغییری در

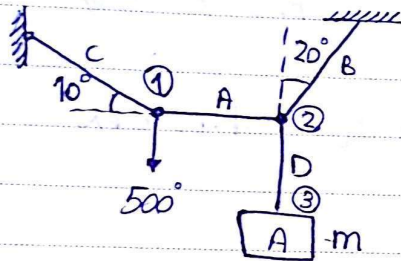
اندازه نیروی طناب ها ایجاد می شود.

Subject:
Date:



$$\left. \begin{array}{l} T = 436.0 \text{ lb} \\ N = 137.7 \text{ lb} \end{array} \right\}$$

if: $N = T$ (ب)
 $\alpha = ?$



$$T_A, T_B, T_C, T_D, m = ?$$

$$T_C = 2879 \text{ N}$$

$$T_A = 2835 \text{ N}$$

3.3, 3.4, 3.6, 3.14, 3.15, 3.23, 3.35

← حساب ←

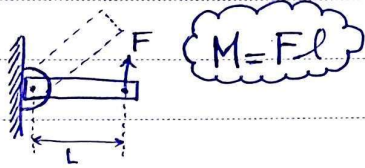
* تمرین :

Chapter 4

Rigid Bodies: Equivalent force/moment systems.

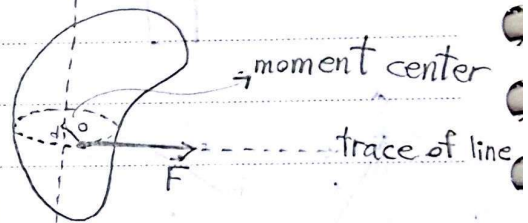
essential: لازم
sufficient: کافی

Moment (گشتاور - ممان) ^{شیر} تمایل برای چرخاندن یک جسم حول یک نقطه یا محور:

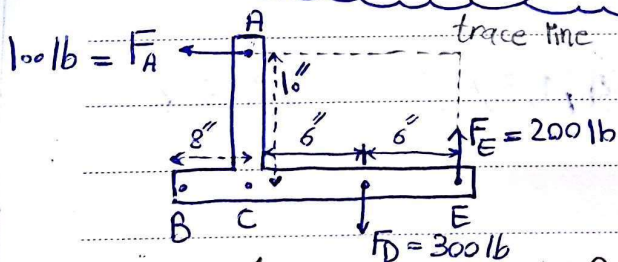


trace of line: خط اثر

moment arm: (d) بازوی ممان



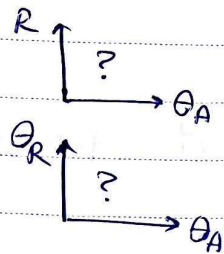
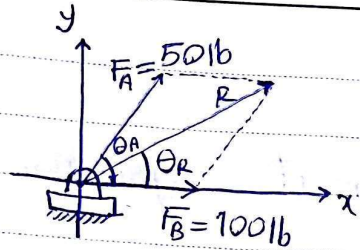
* تمرین: سرعت حرکت انتقالی زمین به دور خورشید چقدر است؟



* مثال:

$$\frac{F_A \text{ on } E}{F_A @ E} M = 10 \times 100 = 1000 \text{ in}\cdot\text{lb} \quad (+)$$

$$\frac{F_D \text{ on } B}{F_D @ B} M = (8 + 6) \times 300 = 4200 \text{ in}\cdot\text{lb} \quad (-)$$

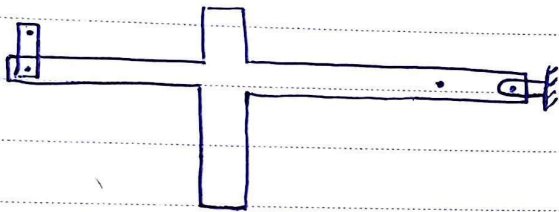


* پروژه کا مسویری :

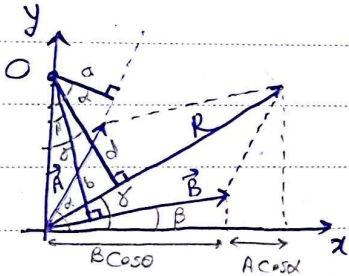
$$R = F_B \times \cos \theta_R$$

Varignon Theorem (قضیہ وارینون) مثال ۲ :
principle of moments

ممان برائید، برابر است با برائید ممان ہا۔



شکل از تختہ بہ سرعت پاک شد!



$$M_A = Aa = A(h \cos \alpha)$$

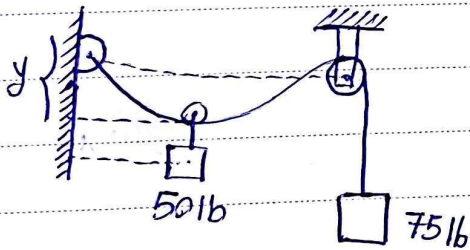
مثال ۳ :

$$M_B = Bb = B(h \cos \beta)$$

$$M_R = Rd = R(h \cos \delta)$$

$$* h \times R \cos \delta = h \times B \cos \beta + h \times A \cos \alpha$$

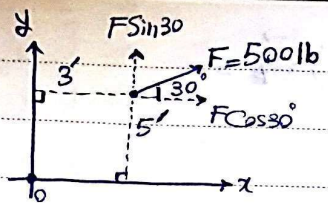
$$\Rightarrow M_R = M_B + M_A$$



y = ?

{4, 1, 3, 5}

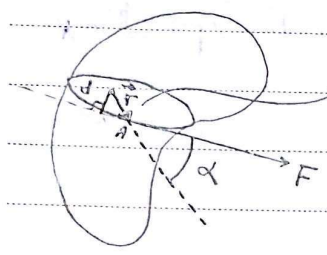
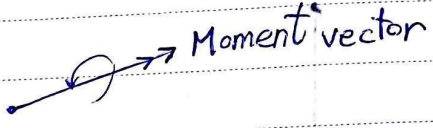
* تقریب :



$$M_o = -F \cos 30^\circ * 5 + F \sin 30^\circ * 3 = -1415 \text{ ft.lb}$$

4.18, 21, 23, 28, 30 : تمرین *

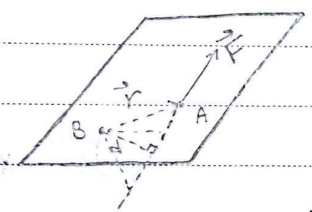
* Vector presentation of Moment



position vector

$$\vec{M}_o = \vec{r} \times \vec{F} = |\vec{r}| |\vec{F}| \sin \alpha \vec{e} = |\vec{M}_o| \vec{e} ; |\vec{r}| \sin \alpha = d$$

$$\vec{M}_o = \vec{M}_{ox} + \vec{M}_{oy} + \vec{M}_{oz}$$

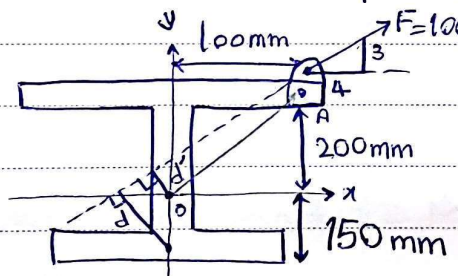


$$\vec{r} = \vec{r}_A - \vec{r}_B = (x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_A - z_B)\hat{k}$$

منحصراً یفرد unique

$$M_o = ?$$

$$d = ?$$



$$\vec{F} = 1000 \left(\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j} \right) \text{ (N)}$$

$$\vec{r} = 0.1\hat{i} + 0.2\hat{j}$$

$$\vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.1 & 0.2 & 0 \\ 400 & 300 & 0 \end{vmatrix} = -100 \hat{k} \text{ (N.m)}$$

$$\theta_x = \cos^{-1} \frac{M_{ox}}{M_o} ; M_o = \sqrt{M_{ox}^2 + M_{oy}^2 + M_{oz}^2}$$

$$\theta_y = \cos^{-1} \frac{M_{oy}}{M_o}$$

$$\theta_z = \cos^{-1} \frac{M_{oz}}{M_o}$$

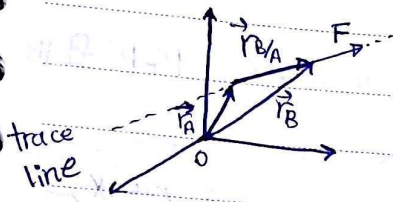
$$d' = \frac{100}{1000} = 0.1 \text{ (m)}$$

4.36, 40, 42, 43 : تمرین *

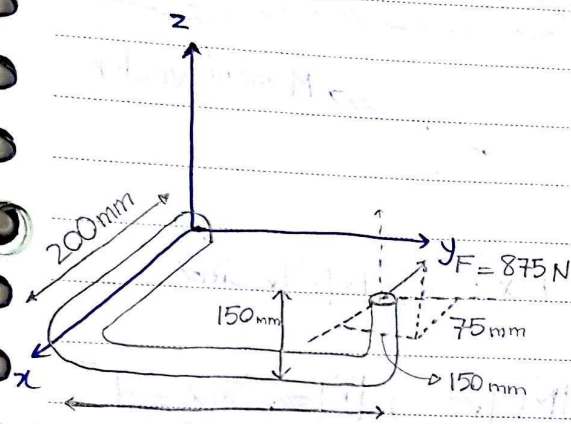
PAPCO در استادیوم مسلمانان و در سایر مراکز مستحقین این و علامت ها رو تا آخر سال طبق اون

↓

مستحقین کن.



$$M_o = \vec{r}_B \times \vec{F} = (\vec{r}_A + \vec{r}_{B/A}) \times \vec{F} = \vec{r}_A \times \vec{F} + \vec{r}_{B/A} \times \vec{F} = \vec{r}_A \times \vec{F}$$

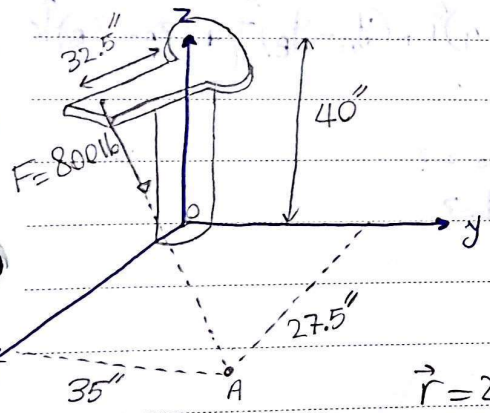


$$\vec{F} = 875 \times \left[\frac{75\hat{i} + 150\hat{j} + 140\hat{k}}{\sqrt{75^2 + 150^2 + 140^2}} \right] \text{ N}$$

$$\vec{r} = 0.2\hat{i} + 0.25\hat{j} + 0.15\hat{k} \text{ m}$$

$$\Rightarrow \vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.2 & 0.25 & 0.15 \\ 300.4 & 600.8 & 560.7 \end{vmatrix} = 50.06\hat{i} - 67.08\hat{j} + 45.06\hat{k} \text{ N.m}$$

باساعتگرد ← ساعتگرد

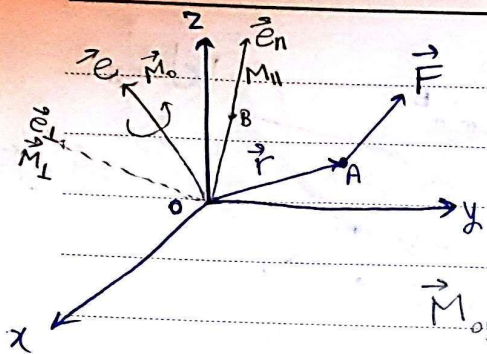


$$\vec{F} = 800 \times \left[\frac{(-5)\hat{i} + (35)\hat{j} + (-40)\hat{k}}{(5^2 + 35^2 + 40^2)^{\frac{1}{2}}} \right] = -74.93\hat{i} + 524.5\hat{j} - 599.4\hat{k} \text{ (lb)}$$

$$\vec{r} = 27.5\hat{i} + 35\hat{j} \text{ (in)} \Rightarrow \vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 27.5 & 35 & 0 \\ -74.93 & 524.5 & -599.4 \end{vmatrix} =$$

$$= -20979\hat{i} + 16484\hat{j} + 17046\hat{k} \text{ (in.lb)}$$

$$\Rightarrow d = \frac{M_o}{F} = \sqrt{\quad} \text{ (in)}$$



∠ : cross

∴ : dot

$$\vec{M}_O = (\vec{r} \times \vec{F})$$

$$\vec{M}_{OB} = \vec{M}_{||} = (\vec{M}_O \cdot \vec{e}_n) \vec{e}_n =$$

$$= \underbrace{(\vec{r} \times \vec{F}) \cdot \vec{e}_n}_{\vec{M}_O} \vec{e}_n = |M_{OB}| \vec{e}_n \Rightarrow M_{OB} = (\vec{r} \times \vec{F}) \cdot \vec{e}_n = \begin{vmatrix} e_{nx} & e_{ny} & e_{nz} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

triple product

$$\vec{M}_{\perp} = \vec{M}_O - \vec{M}_{||}$$

4.50, 55, 56, 58

* تمرین *

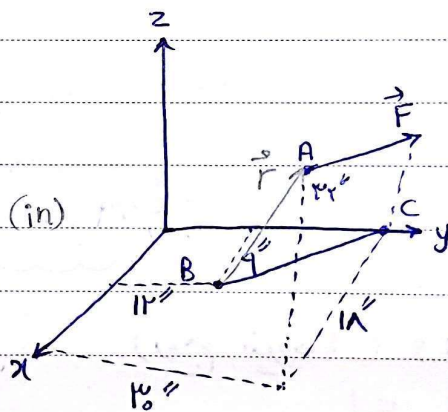
$\vec{M}_{BC} = ?$

بعد از عدد

$$\vec{F} = 40\vec{i} + 100\vec{j} + 110\vec{k} \quad (1b)$$

$$\vec{r} = (11-9)\vec{i} + (10-11)\vec{j} + 12\vec{k} \quad (in)$$

$$\vec{M}_B = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & 11 & 12 \\ 40 & 100 & 110 \end{vmatrix} =$$



* مثال *

$$= -1050\vec{i} + 1100\vec{j} - 110\vec{k} \quad (in. 1b)$$

$$\Rightarrow \vec{e}_{BC} = \frac{-9\vec{i} + (10-11)\vec{j}}{\sqrt{(-9)^2 + (1)^2}} = -0.9947\vec{i} + 0.1118\vec{j} \quad (\text{واحد ندارد})$$

$$\vec{M}_{BC} = \vec{M}_B \cdot \vec{e}_{BC} = 1114.718 \quad (in. 1b)$$

$$\Rightarrow \vec{M}_{BC} = \vec{M}_B \cdot \vec{e}_{BC} = -1044.97\vec{i} + 1018.93\vec{j} \quad (in. 1b)$$

$\vec{M}_{Oc} = ?$ $\vec{M}_o = ?$

$\vec{F} = (-150)\hat{i} + (-100)\hat{j} + (750-100 \cdot \tan 40^\circ)\hat{k}$
 $\sqrt{(-150)^2 + (-100)^2 + (750-100 \cdot \tan 40^\circ)^2}$

$= -239,2\hat{i} - 91,27\hat{j} + 271,09\hat{k}$

$\vec{r} = -0,15\hat{i} + 0,75\hat{k}$ (m)

$\Rightarrow \vec{M}_o = \vec{r} \times \vec{F} = 239,2\hat{i} - 91,27\hat{j} + 271,09\hat{k}$ (N.m)

$\vec{e}_{oc} = -\hat{i}$

$\Rightarrow M_{oc} = \vec{M}_o \cdot \vec{e}_{oc} = -239,2$ N.m

$\Rightarrow \vec{M}_{oc} = M_{oc} \vec{e}_{oc} = (-239,2)(-\hat{i}) = +239,2\hat{i}$

$F_1, 49, 71, 74, 77$ x تمرین :

⊗ Couples : (زوج نیروها)

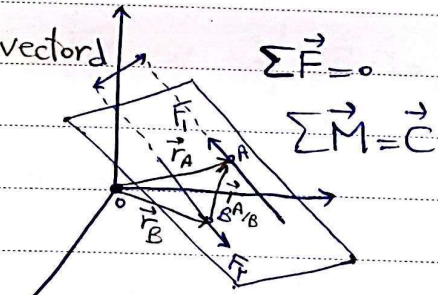
free vector

$M_A = F_1 \times d = Fd$

$M_B = F_1 \times d = Fd$

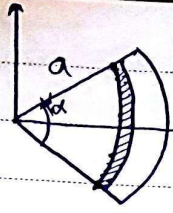
$\vec{M}_o = \vec{r}_A \times \vec{F}_1 + \vec{r}_B \times \vec{F}_2 = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) =$

$= (\vec{r}_A - \vec{r}_B) \times \vec{F} = \vec{r}_{AB} \times \vec{F} =$



شرط : مختلف الجفت وهم انداز

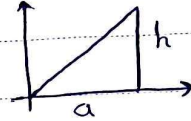
$= |\vec{r}_{AB}| \times |\vec{F}| \times \sin \alpha \times \vec{e} = Fd\vec{e} = \vec{C}$ (کویل)



$\int da = ?$

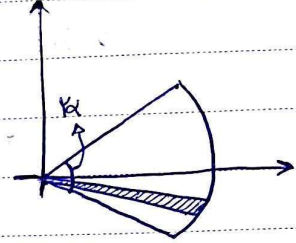
۲

$\int da = ?$



۱

$\int da = ?$



۳

google.com

برو صفحه ای اینترنت

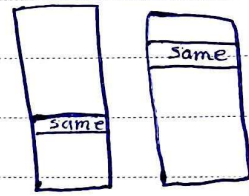
لغت

۷

تمودار با درجه ای دلخواه من در دیانه؟

۴

مثل هم جا را بیالند.



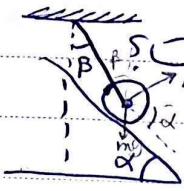
۵

Hyper

برو جایی که من میخوام

لغت

۶

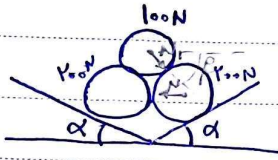


* سؤال ۱: نیروی کابل و نیروی تماس بین کره و سطح شیب را چقدر است؟

$$T \cos \beta = mg \rightarrow T = \frac{mg}{\cos \beta}$$

$$T \sin \beta = N \sin \alpha \Rightarrow N = \frac{mg}{\cos \beta} \times \sin \beta \times \frac{1}{\sin \alpha}$$

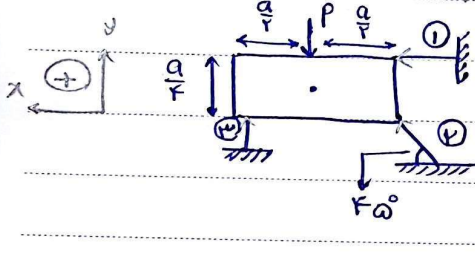
حل شد



$$\alpha_{min} = \alpha$$

* سؤال ۲

$$N = N_1 \cos \beta \quad (\text{شرط})$$

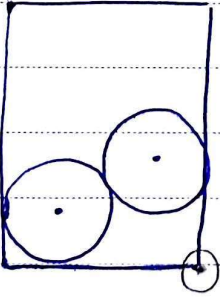


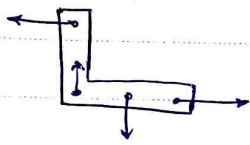
$$\left. \begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_A &= 0 \end{aligned} \right\} \begin{aligned} &N_1, N_2, N_3 \\ &N_1, N_2, N_3 \end{aligned}$$

* سؤال ۳

* نکته: برآیند میان ها را همیشه در نقطه ای بنویسید که محمولات بیشتری را دارد.

* سؤال ۴: نیروی Q چقدر باشد که ظرف واژگون نشود؟





شکل های دایره کویلی: (برای مثال)

* مثال فیزیکی:

$$\vec{C} = \sum \vec{C}_x + \sum \vec{C}_y + \sum \vec{C}_z =$$

$$= \sum C_x \hat{i} + \sum C_y \hat{j} + \sum C_z \hat{k} \Rightarrow |\vec{C}| = \sqrt{(\sum C_x)^2 + (\sum C_y)^2 + (\sum C_z)^2}$$

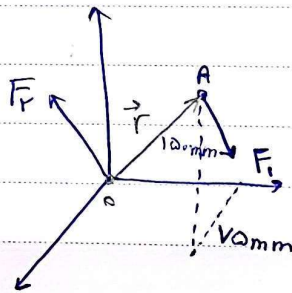
$$\Rightarrow \vec{C} = \cos \theta_x \hat{i} + \cos \theta_y \hat{j} + \cos \theta_z \hat{k}$$

$$\theta_x = \cos^{-1} \frac{\sum C_x}{|\vec{C}|}, \theta_y = \cos^{-1} \frac{\sum C_y}{|\vec{C}|}, \theta_z = \cos^{-1} \frac{\sum C_z}{|\vec{C}|}$$

$\vec{C} = ?$

$$F = -700 \hat{i} + 1000 \hat{j} - 2000 \hat{k}$$

$$\Rightarrow \vec{r} = 0.70 \hat{i} + 0.90 \hat{j} + 0.10 \hat{k} \text{ (m)}$$



* مثال 8

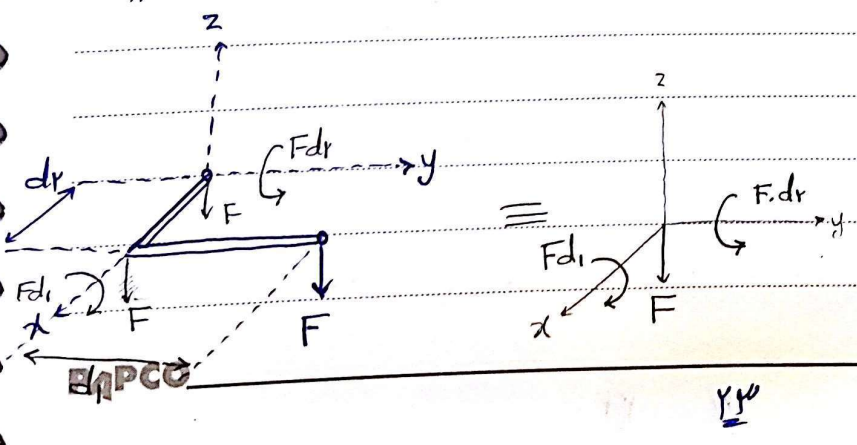
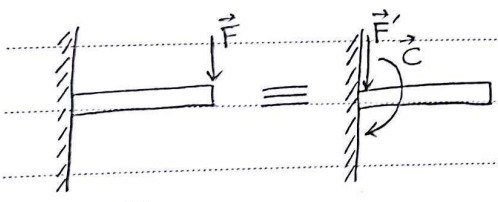
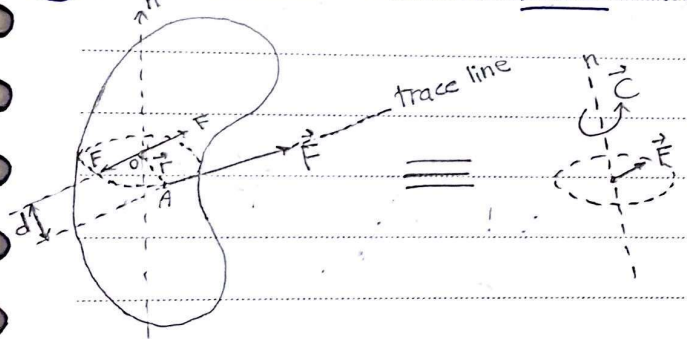
$$\Rightarrow \vec{C} = \vec{M}_o = \vec{r} \times \vec{F} = -111 \hat{i} + 170 \hat{j} + 1420 \hat{k} \text{ (N.m)}$$

$$\Rightarrow |\vec{C}| = |\vec{M}_o| = 1711 \text{ N.m} \xrightarrow{\text{solid}} d = \frac{C}{F} = \frac{M_o}{F}$$

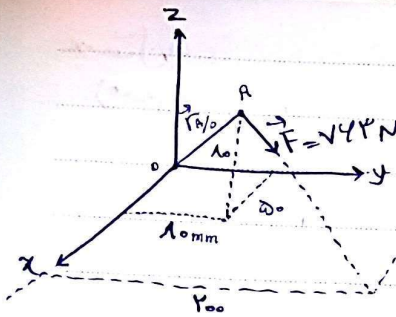
$C_x = 40 \text{ N.m}$
 $C_y = 90 \text{ N.m}$
 $C_z = 100 \text{ N.m}$
 $\vec{C} = ?$
 $\vec{C} = \sum \vec{C}_x + \sum \vec{C}_y + \sum \vec{C}_z$
 $\sum \vec{C}_x = 40 \text{ N.m}$
 $\sum \vec{C}_y = 90 + 90 \cos 40^\circ = 127.9 \text{ N.m}$
 $\sum \vec{C}_z = 40 + 90 \sin 40^\circ = 100 \text{ N.m}$
 $\Rightarrow \vec{C} = 40\hat{i} + 127.9\hat{j} + 100\hat{k} \rightarrow C = \dots \rightarrow \theta_x, \theta_y, \theta_z = \dots$

$F_1, A_1, 11^\circ, 11^\circ, 91, 95, 95$
 $\Sigma F = 0 \rightarrow$ (مستقيم) (مستقيم)

*** Resolution of a force into a force and a couple:**



$\Sigma F = 0$



$$\vec{F} = \frac{(+)i + (+)j + (+)k}{\sqrt{(\quad)^2 + (\quad)^2 + (\quad)^2}} \times 442 \Rightarrow \dots$$

$$\Rightarrow \vec{F} = 200\vec{i} + 200\vec{j} - 200\vec{k} \text{ (N)}$$

$$\vec{r}_{A/O} = 0.1\vec{i} + 0.1\vec{j} + 0.1\vec{k} \text{ (m)} \Rightarrow \vec{M}_O = \vec{C} = \vec{r} \times \vec{F} = -0.1\vec{i} + 0.1\vec{j} + 0.1\vec{k} \text{ (N.m)}$$

$$\Rightarrow |\vec{C}| = 90 \text{ (N.m)} \rightarrow \theta_x, \theta_y, \theta_z = \dots$$

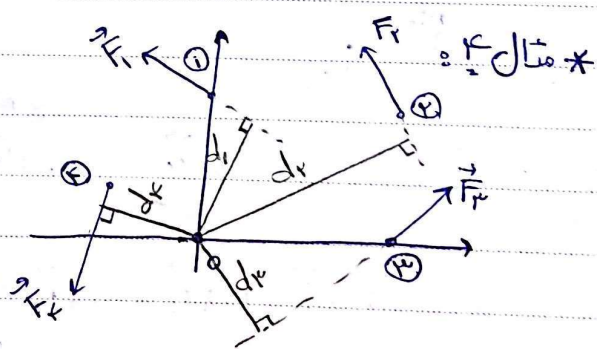
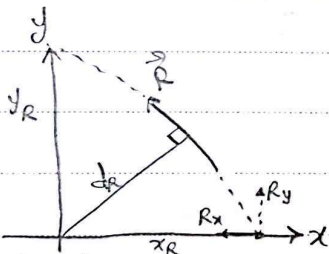
$$\Rightarrow \begin{cases} \theta_x = \cos^{-1} \frac{|\vec{C}_x|}{|\vec{C}|} = 100.5^\circ \\ \theta_y = 100.5^\circ \\ \theta_z = 100.5^\circ \end{cases}$$

شبهه جوب دادند؟
 $(97, 91, 102, 105, 112)$

* تمرین *

*** Simplification of a force system (Resultant):**

A Coplanar force system:

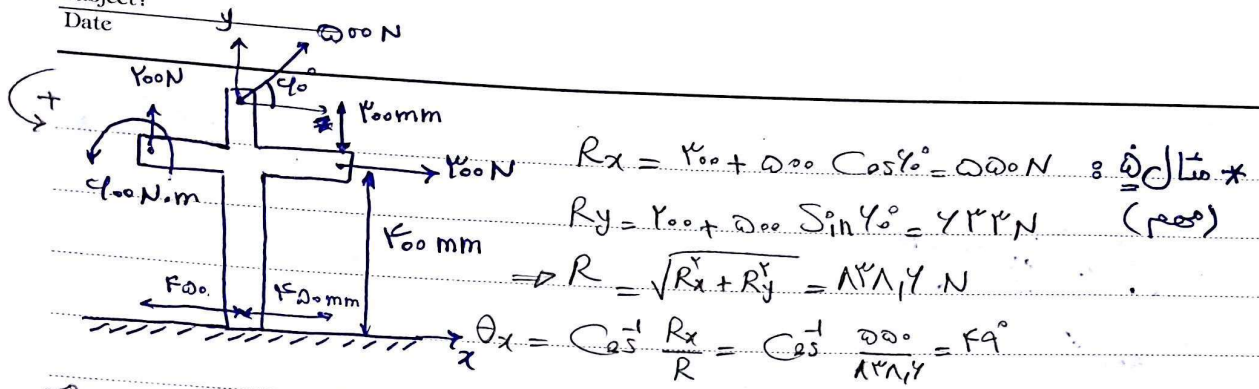


$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

$$\vec{C} = \vec{R} \cdot d\vec{r} = \vec{F}_1 d_1 + \vec{F}_2 d_2 + \dots + \vec{F}_n d_n$$

$$x_R = \frac{R dx}{R_y}, \quad y_R = \frac{R dy}{R_x} \quad (R dx = x_R x + R_y x^2 = x_R x + R_y)$$

Subject:
Date:



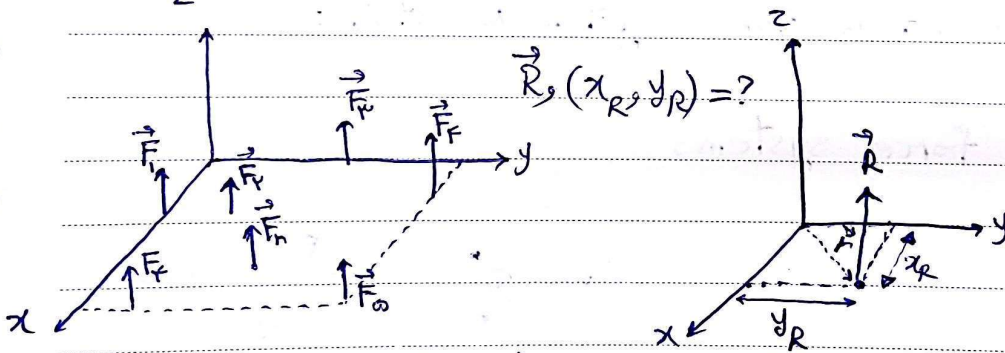
$\Rightarrow M_o = C = R d_R = 400 \times 0.4 + 400 - 200 \times \cos 40^\circ \times 0.4 - 400 \times 0.4 =$
 $= +110 \text{ N.m} \Rightarrow d_R = \frac{M_o}{R} = \frac{C}{R} = 237 \text{ mm}$

$\Rightarrow \left. \begin{aligned} R d_R &= x_R \times R_y \rightarrow x_R = \dots \\ R d_R &= y_R \times R_x \rightarrow y_R = \dots \end{aligned} \right\}$

113, 110, 121, 124

○ مركز

Non Coplanar, Parallel force systems

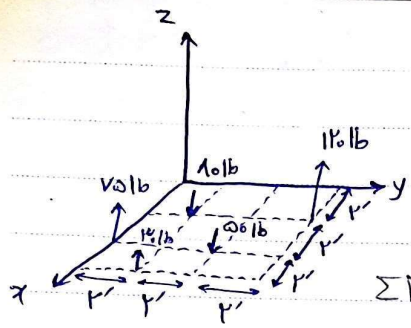


$\vec{R} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = R \vec{k} = (\sum F) \vec{k}$

$\vec{M}_o = \vec{r} \times \vec{R} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \dots + \vec{r}_n \times \vec{F}_n$

$-M_y = R x_R = F_1 x_1 + F_2 x_2 + \dots + F_n x_n = -\sum M_y$

$+M_x = R y_R = F_1 y_1 + F_2 y_2 + \dots + F_n y_n = +\sum M_x$



: مثال ٤

$$\vec{R} = \sum \vec{F}_i = (10 + 10 - 10 - 10 + 10)\vec{k} = 10\vec{k} \quad (1b)$$

$$x_R = \frac{-\sum M_y}{R}, \quad y_R = \frac{\sum M_x}{R}$$

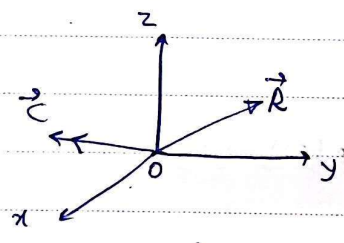
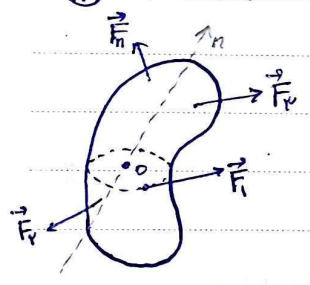
$$\sum M_y = -10 \times 1 - 10 \times 1 + 10 \times 1 + 10 \times 1 - 10 \times 1 = -10 \text{ (ft. lb)}$$

$$\sum M_x = 10 \times 0 + 10 \times 1 - 10 \times 1 - 10 \times 1 + 10 \times 1 = 10 \text{ (ft. lb)}$$

$$\rightarrow x_R = \frac{-\sum M_y}{R}, \quad y_R = \frac{\sum M_x}{R} \Rightarrow \dots \Rightarrow \left. \begin{array}{l} x_R = 1 \text{ ft.} \\ y_R = 1 \text{ ft.} \end{array} \right\}$$

$F_1, 12, 13, 14, 15, 16, 17, 18, 19, 20$ sense: ?

⊗ General force systems:



$$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2 + (\sum F_z)^2}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R}, \dots$$

$$\vec{C} = \sum \vec{C}_x + \sum \vec{C}_y + \sum \vec{C}_z \Rightarrow C = \sqrt{(\sum C_x)^2 + (\sum C_y)^2 + (\sum C_z)^2}$$

$$\Rightarrow \theta_x = \cos^{-1} \frac{C_x}{C}, \theta_y = \dots, \theta_z = \dots$$

مثال در سنجی بعد ...