Text book : Reservior Engineering Handbook by: Tarek Ahmed.

در امتحان final ، دو سئوال از این کتاب داده شده.

Refreces: ① Applied petroleum Reservior engineering by:
B. C. Craft &
M. Hawkins.

② Petroleum Reservior engineering Methods by: H. C "slip" slider

③ Fundamental of Reservior engineering by: L. P. Dake

Grade policy:

4 Quizzes : 10%

4 Homework : 10%

midterm : 30%

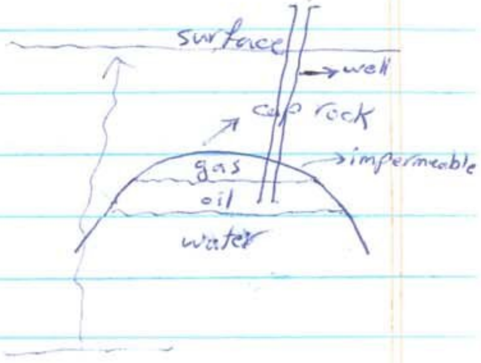
Final exam: 50%

Attendance : extra mark

Fluid Flow in Porous Media:

fluid in moving to surface from high depth (primary migration)

1) Faces no impermeable layer
   eg: gases in Masjid soliman, oil sands in canada
2) Faces a impermeable lyayer
   fluid traps below the layer



- secondary migration: due to gravity difference, gas, oil & water will be separated.

- Drive Mechanisms (primary recovery)
  1) gas cap drive — فشار گاز بالای نفت
  2) Natural water drive
  3) solution gas drive (expansion of gas & oil)
  4) compaction of formation — وقتی نفت از مخزن خارج می شود ← فشار لایه های بالای زیاد شده و نفت بیشتری خارج می شود.

Recovery Mechanisms
1) primary Recovery (oil is produced by reservior energy)
2) (*) suplementary recovery (energy is given to oil)

(*)
1- secondary recovery, water flooding & polymer flooding
2- tertiary recovery, surfactant, gas injection (natural gas, N_2, CO_2)
   → SURFace ACTAtive ageNT
3- Thermal recovery, Hot water injection, Hot water, steam, Insitu combustion, High Pressure / Air Injection
   SAGD
   VAPEX

سوال - ۱۵ پارامتر را نام ببرید که یک مخزن را با آن می توان مشخص کرد.

امتحان سال گذشته

Important Parameters in Reservior Engineering:

① Reservior specific :

1) pressure

2) Temprature

3) depth

4) porosity

5) permeability

6) bubble point pressure 7) dew point pressure

② Fluids

1) Density $\begin{cases} \rho_g \\ \rho_w \\ \rho_o \end{cases}$

2) viscosity $\begin{cases} \mu_g \\ \mu_w \\ \mu_o \end{cases}$

3) FVF (formation volume factor) (B_o) $\begin{cases} B_g \\ B_w \\ B_o \end{cases}$

4) R_s

5) compressibility $\begin{cases} C_g \\ C_w \\ C_o \\ C_f \end{cases}$

6) compressibility factor Z

7) lithology

8) pay zone thickness ضخامت لایه ی مخزن

9) Geometry $\begin{cases} \text{Radius} \\ \text{Dimensions} \end{cases}$

10) saturations $\begin{cases} S_g \\ S_w \\ S_o \end{cases}$

11) Interfacial tension

12) water oil contact

13) gas oil contact

chapter 6: Fundamental of Reservior Fluid flow

flow in a porous media is difficult $\overset{\text{due}}{\vee}$ to complicated fluid path.

objective: to develope mathematical relations to describe fluid flow (fluid behaivior) in porous media.

shape of a reservior:

Irregular boundries: Analytical solutions do not exist → Numerical simulation

Mathematical relations
- 1- Kinds of fluids
- 2- Flow Regime
- 3- Reservior geometry
- 4- Number of fluids

(*) Kinds of fluids: Isothermal compressibility coefficient.

1- Incompressible fluids

2- Slightly compressible fluids حفظ کنیم ✓

3- compressible fluids.

$$\boxed{\begin{aligned} C &= -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \\ C &= \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T \end{aligned}} \quad psi^{-1}$$

- Incompressible fluids $C = 0$, no liquid

we use this assumption to simplify derivation of equations.

- Slightly compressible fluids, all liquids

$$C = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \xrightarrow{\text{isothermal}} \int_{P_{ref}}^{P} C\,dp = \int_{V_{ref}}^{V} -\frac{dV}{V} \xrightarrow[\text{ثابت است C}]{\text{فرض می کنیم}} C(P-P_{ref}) = -\ln\frac{V}{V_{ref}}$$

① → $\boxed{V = V_{ref}\, e^{C(P_{ref}-P)}}$, $C_{oil} = (5 \underset{\text{تا}}{-} 100) \times 10^{-6}\ psi^{-1}$

می‌دانیم: $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

for small $x_s \longrightarrow e^{x} = 1 + x$ برای slightly compressible ها

حفظ کنیم

① $\boxed{V = V_{ref} \left(1 + c(P_{ref} - P)\right)}$ → از اینجا به بعد از این فرمول

$\boxed{\rho = \rho_{ref} \left(1 - c(P_{ref} - P)\right)}$ استفاده می کنیم.

- compressible fluids: ② $c = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$, all gases.

real gases: $PV = ZnRT \longrightarrow V = \frac{ZnRT}{P} \longrightarrow$

③ $\left(\frac{\partial V}{\partial P}\right)_T = nRT\left(\frac{\left(\frac{\partial Z}{\partial P}\right)_T P - Z}{P^2}\right)$

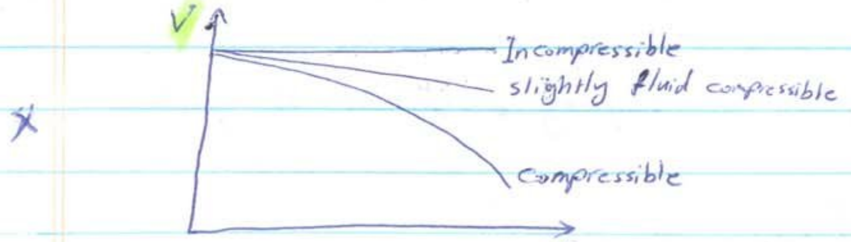
②,③ $\longrightarrow c = -\frac{1}{\frac{ZnRT}{P}} nRT\left(\frac{\left(\frac{\partial Z}{\partial P}\right)_T P - Z}{P^2}\right)$

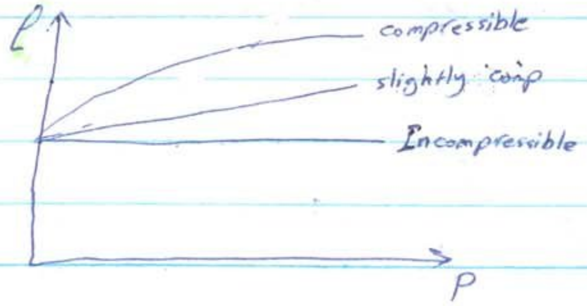
حفظ کنیم

$\boxed{c = \frac{1}{P} - \frac{1}{Z}\left(\frac{\partial Z}{\partial P}\right)_T}$

برای compressible fluid ها
از این فرمول استفاده می کنیم

for ideal gases, $Z = 1 \Rightarrow \boxed{c = \frac{1}{P}}$





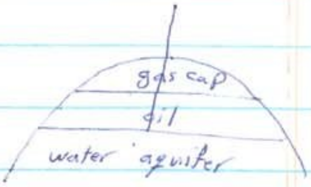
Ⓐ Flow regimes
- 1- Steady state flow
- 2- un steady state flow
- 3- pseudo steady state flow

این حالت بصورت real نیست ولی می توان ایجاد کرد.

- steady state flow: $\left(\frac{\partial P}{\partial t}\right)_i = 0$, $P_i =$ constant

cases that can be assumed steady state:

1- stronge aquifer: چون حجم water خیلی زیاد است، وقتی نفت خارج می شود، عملاً فشار ثابت می ماند

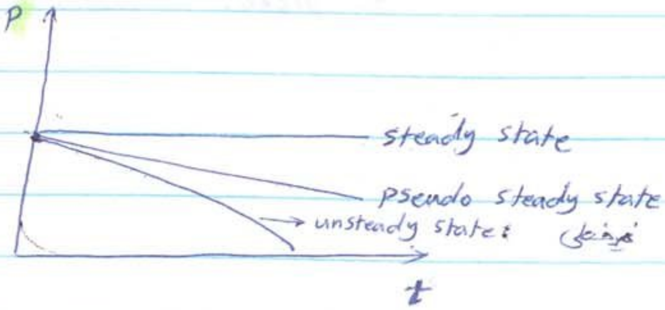


2- pressure maintenace
- 1) gas injection in gas cap
- 2) water injection in aquifer

- unsteady state flow: $\left(\frac{\partial P}{\partial t}\right)_i = f(\underset{\text{location}}{\underset{\downarrow}{(i)}}, t)$

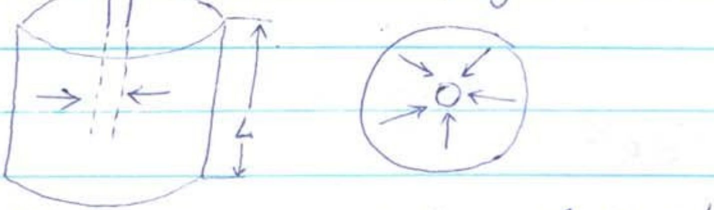
- Pseudo steady state flow: (semi-steady state flow)
  quasi-steady state flow

$$\left(\frac{\partial P}{\partial t}\right)_i = \text{constant}$$



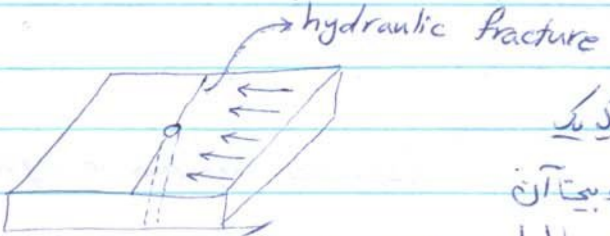
Ⓐ Reservoir geometry
- 1- Linear
- 2- spherical & hemispherical
- 3- cylindrical (radial)

- Radial flow : در مخازن تقریباً شبیه این است.
(very close to real cases)



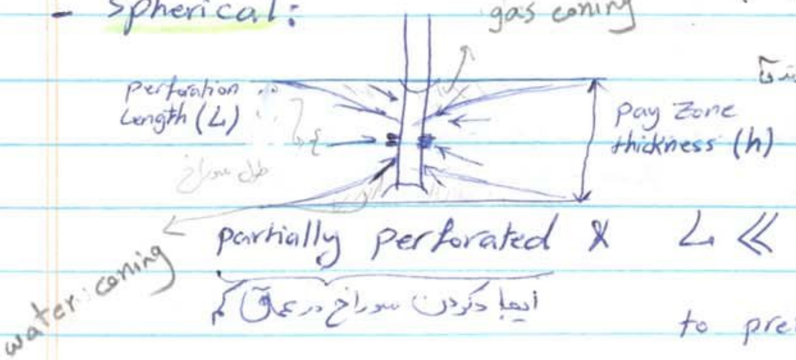
flow area is not constant $A = 2\pi r L$

- Linear flow :



به طور عملی نیست مگر اینکه یک
fracture ایجاد کنیم و بعد آن
ماسه سنگ‌هایی گذارند تا سیال را به well هدایت کند.

- spherical:



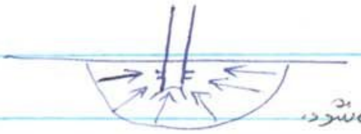
partially perforated & $L \ll h$

ایجاد کردن سوراخ در عمق کم

- partially perforated را در وسط چاه ایجاد می‌کنند تا
gas و water، همراه oil خارج نشود.
وقتی سیال از مسافت های دور به چاه می‌رسد افت
فشار در درون چاه ایجاد شده و از پایین water و
از بالا gas می‌خواهند که وارد چاه شوند و یک حالت
coning در پایین و بالا ایجاد می‌شود.

to prevent water & gas coning

- Hemispherical



برای جلوگیری از water coning در حالت hemispherical، perforated را در عمق کم ایجاد کرده تا water وارد چاه نشود.

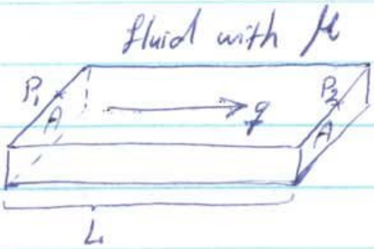
The well is partially penetrated in the pay Zone
to pervent water coning

- چون سیال از مسافت های دور به چاه می‌رسد و از یک سوراخ کوچک عبور می‌کند، سرعتش زیاد است
و فشار کمتر می‌شود و باعث به وجود آمدن یک coning می‌شود. برای جلوگیری از این کار
عملیات cementing پس از گذشت مدتی از عمر چاه، در اعماق پایین و water coning انجام
می‌دهند تا partially perforate به وجود آمده در طول زمان را پر کند.

⊛ Number of fluids
- 1- single phase - (water, oil or gas)
- 2- Two-phase (water-oil & oil-gas & water-gas)
- 3- Three phase (oil + water + gas)

Fluid flow Equations:

Darcy (1856)



$$\left.\begin{array}{l} q \propto A \\ q \propto (P_1 - P_2) \\ q \propto \frac{1}{L} \\ q \propto \frac{1}{\mu} \end{array}\right\} \begin{array}{l} \longrightarrow q \propto \frac{A(P_1-P_2)}{\mu L} \\ \longrightarrow \boxed{q = \frac{KA(P_1-P_2)}{\mu L}} \end{array}$$

that K: permeability

$$V_a = \frac{q}{A} = \frac{K(P_1-P_2)}{\mu L} \Rightarrow V_a = \frac{q}{A} = \frac{-K}{\mu}\frac{\partial P}{\partial x}$$

that: $\boxed{V_a = \text{apparent velocity}} = \frac{q}{A}$ ✓ حفظ لزم

✓ حفظ لزم

actual velocity: $V_{ac} = \boxed{\frac{V_a}{\underset{\to \text{prosity}}{\phi}}}$

$$\longrightarrow \text{in } \underset{(\frac{m}{s})}{V} = \frac{q^{(\frac{m^3}{s})}}{A_{(m^2)}} = -\frac{K^{(m^2)}}{\mu_{(pa.s)}}\frac{dP_{(pa)}}{dx_{(m)}}$$

$$\longrightarrow \underset{\frac{cm}{s}}{V} = \frac{\overset{\frac{cm^3}{s}}{q}}{\underset{cm^2}{A}} = \frac{-\overset{\to \text{Darcy}}{K}}{\underset{cp}{\mu}}\frac{\overset{\to \text{atm}}{dP}}{\underset{cm}{dx}}$$

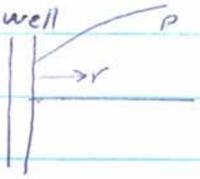
$\longrightarrow 1\ m^2 = 1.01325 \times 10^{12}\ D$

$\boxed{1\ D = 1\ \mu m^2}$

$1\ D = 10^{-12}\ m^2$

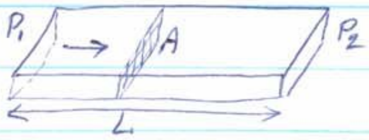
$$\boxed{q = \frac{-KA}{\mu}\frac{dP}{dx}} \longrightarrow \text{for Linear flow}$$

for radial & spherical flow :



$$q = \frac{KA}{\mu}\frac{dP}{dr}$$

Linear flow of incompressible fluids in Steady State flow.



$$q = \frac{-KA}{\mu}\frac{dP}{dx} \longrightarrow \frac{q\mu}{KA}\int_{0}^{L} dx = -\int_{P_1}^{P_2} dP \longrightarrow \boxed{\underset{\frac{\text{cm}^3}{\text{s}}}{q} = \frac{\overset{\text{Darcy}}{K}\overset{\text{cm}^2}{A}}{\underset{\text{cp}}{\mu}\underset{\text{cm}}{L}}\underset{\text{atm}}{(P_1-P_2)}}$$

$$\longrightarrow \underset{\frac{\text{bbl}}{\text{day}}}{q} = \frac{0.001127\,\overset{\text{md}}{K}\overset{\text{ft}^2}{A}}{\underset{\text{cp}}{\mu}\underset{\text{ft}}{L}}\underset{\text{psi}}{(P_1-P_2)}$$

$$\longrightarrow \boxed{q = \underset{\text{حذف شود}}{\textcircled{-}}0.001127\frac{KA}{\mu}\frac{dP}{dx}}$$

$$1\ \text{bbl} = 5.615\ \text{ft}^3$$

$$\boxed{q = 0.001127 \frac{KA(P_1 - P_2)}{\mu L}}$$ ✓ حفظ بالتم

for linear system (steady state & incomp)

مخزن مايل مسطحياً.

Tilted reservoirs:

1 & 2 are not in a horizontal surface.

we should consider the effect of gravity.

$$\underset{pa}{P} = \underset{\frac{kg}{m^3}}{\rho}\ \underset{\frac{m}{s^2}}{g}\ \underset{m}{h}$$

$$\underset{psi}{P} = \boxed{\underset{\frac{lb}{ft^3}}{\rho g}}\ \underset{ft}{h} \left(\frac{1\,ft^2}{144\,in^2} \right) = \frac{\overset{\frac{lb}{ft^3}}{\gamma}\ \overset{ft}{h}}{144} \quad \frac{lb}{in^2}$$

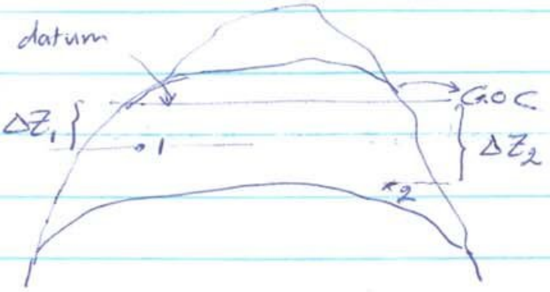
in your book $P = \frac{\rho \Delta Z}{144}$

$$S.G = \frac{\rho g}{\rho g_{water}} \longrightarrow \overset{\frac{lb}{ft^3}}{\rho g} = S.G\,(62.4)^{\frac{lb}{ft^3}}$$

$$\frac{lb}{in^2} \leftarrow psi \leftarrow P = \rho g \frac{h}{144} = \underbrace{S.G}_{\gamma}\,(62.\overset{\frac{lb}{ft^3}}{4})\ h \left(\frac{1\,ft^2}{144\,in^2} \right)$$ 1 ft = 12 in

IN your book : $\boxed{\underset{psi}{P} = 0.433\ \underset{\text{specific gravity or } \frac{gr}{cm^3}}{\gamma}\ \overset{ft}{\Delta Z}}$

1- Define a datum (Gas oil contact, water oil contact



2- Define fluid potential ϕ_i

$\rho \rightarrow \frac{lb}{ft^3}$ $\quad$ $\gamma \rightarrow \frac{gr}{cm^3}$

$$\boxed{\phi_i = P_i - \frac{\rho \Delta Z_i}{144}} \checkmark \text{ or } \boxed{\phi_i = P_i - 0.433\, \gamma\, \Delta Z_i} \checkmark$$ حفظ کنیم

$\frac{\rho}{144} \rightarrow \frac{psi}{ft}$ ✓

عکس تمام قراردادهای قبلی: اگر ΔZ_i زیر خط datum باشد مثبت است ولی اگر
بالا خط datum باشد منفی است.

ΔZ_i: positive if below datum
Negative if above datum.

$$\boxed{q = \frac{0.001127 KA (\phi_1 - \phi_2)}{\mu L}}$$ ✓ حفظ باشیم

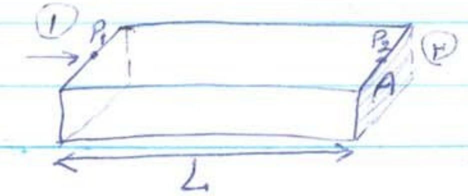
Steady state Linear slightly compressible flow:

① $q = -0.001127 \frac{KA}{\mu} \frac{dP}{dx}$

$V = V_{ref} (1 + C (P_{ref} - P))$, $q = \frac{dV}{dt} = \frac{V}{t}$

$\xrightarrow{②} q = q_{ref} (1 + C (P_{ref} - P))$

$\xrightarrow{① = ②} q_{ref} (1 + C (P_{ref} - P)) = -0.001127 \frac{KA}{\mu} \frac{dP}{dx}$



$\mu\, q_{ref} \int_{0}^{L} dx = -0.001127 KA \int_{P_1}^{P_2} \frac{dP}{1 + C(P_{ref} - P)}$

$$\mu q_{ref} L = -0.001127\, KA \left(\frac{-1}{c}\right) Ln\left(1 + c\left(P_{ref} - P\right)\right)\Big]_{P_1}^{P_2}$$

$$\rightarrow \boxed{q_{ref} = 0.001127\, \frac{KA}{\mu c L}\, Ln\left[\frac{1 + c\left(P_{ref} - P_2\right)}{1 + c\left(P_{ref} - P_1\right)}\right]}\ \checkmark$$

حفظ کنیم

if ref = 1, upstream $\Rightarrow$

$$q_1 = 0.001127\, \frac{KA}{\mu c L}\, Ln\left(1 + c\left(P_1 - P_2\right)\right)$$

$$\rightarrow q_2 = 0.001127\, \frac{KA}{\mu c L}\, Ln\, \frac{1}{1 + c\left(P_2 - P_1\right)}$$

$$q_2 > q_1 \quad (\text{if } P_1 > P_2)$$

جرم ثابت است ولی جریان حجمی متغیر است.

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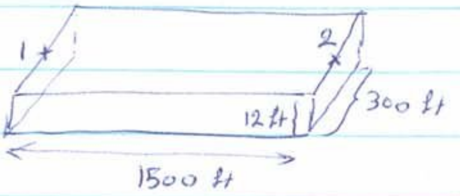
ex: A porous media has a uniform permeability of 245 md to oil at 17% connat water saturation. The Porosity is 32%. A sample of this media with 1500 ft length, 300 ft with, and 12 ft thickness is chosen for experimental investigation. The oil has 3.2 cp viscosity & 1.25 $\frac{bbl}{STB}$ $\Rightarrow$ formation volume factor

a) what pressure drop will cause 100 $\frac{bbl}{day}$ of oil to flow.

sol: $\underset{md}{K = 245}$, $\underbrace{Sw_i \text{ or } Sw_c}_{\text{never displaces}} = 0.17$, $\phi = 0.32$

آبی که اصلاً حرکت نمی کند و به دیواره چسبیده است.



$\mu_o = 3.2$

$B_o = 1.25$

$$\longleftrightarrow \quad q = 0.001127 \frac{kA}{\mu L} (P_1 - P_2)$$

$$100 = 0.001127 \frac{(245)(300(12))}{3.2(1500)} (P_1 - P_2) \Longrightarrow \boxed{(P_1 - P_2) = 482.9 \text{ psi}}$$

b) what about 200 $\frac{bbl}{day}$:

$$\longrightarrow \Delta P_{new} = 2 \, (P_1 - P_2)_{100} = 2(482.9)_{psi}$$

$$\frac{q_1}{\Delta P_1} = \frac{\overset{2q_1}{q_2}}{\Delta P_2} \longrightarrow \Delta P_2 = 2\Delta P_1$$

c) what is the apparent velocity of oil in ft/day for 100 bbl/day

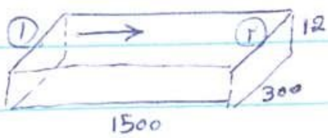
$$V_a = \frac{q}{A} = \frac{100 \text{ bbl/day}}{12\,(300)\text{ ft}^2} \left(\frac{5.615 \text{ ft}^3}{1 \text{ bbl}} \right) = 0.156 \text{ ft/day}$$

pore pressure gradient:

$$p = \frac{\rho g h}{144} \rightarrow \frac{p}{h} = \frac{\rho g}{144} \qquad \rho g = S.G\,(62.4)$$

$$\Rightarrow \frac{p}{h} = \frac{(0.9)(62.4)}{144} = 0.39 \frac{\text{psi}}{\text{ft}}$$

ادامه سؤال جلسه قبل :



$K = 245\ md$ $\mu_o = 3.2$

$S_{wc} = 0.17$

$\phi = 0.32$ $B_o = 1.25\ \frac{bbl}{STB}$

$q = 100\ \frac{bbl}{day} \Longrightarrow \Delta p = 482.9\ psi$

c) what is the apparent velocity of oil in $\frac{ft}{day}$ at $100\ \frac{bbl}{day}$?

sol: $V_{app} = \frac{q}{A} = \frac{100\ \frac{bbl}{day}}{300(12)\ ft^2}\left(\frac{5.615\ ft^3}{1\ bbl}\right) = 0.156\ \frac{ft}{day}$

d) what is the actual velocity of oil

sol: $V_{act} = \frac{q}{A_{pors}} = \frac{q}{\phi_o A} = \frac{V_{app}}{\phi_o}$

$$\checkmark\ \boxed{V_{act}} = \frac{V_{app}}{\phi_o} = \frac{V_{app}}{\phi S_o} = \frac{V_{app}}{\phi(1 - S_{wc})} = \frac{0.156}{0.32(1-0.17)}$$

porosity of oil

$= 0.587\ \frac{ft}{day}$

we assumed the path of pores are parallel

V_{act}

نکته: در این قسمت فرض کردیم که تمام pore ها به صورت لوله های موازی باشد. ← $V_{act} > V_{app}$

e) what time will be required for compelete oil displacement by water injection assuming plug (piston) flow & $100\ \frac{bbl}{day}$ production.

sol: روش ①

$\checkmark\ \boxed{V_{act} = \frac{L}{t}} \Longrightarrow t = \frac{L}{V_{act}} = \frac{1500}{0.587} = 2555\ days \simeq 7\ years$

روش ②

$\checkmark\ \boxed{q = \frac{pore\ volume}{t}} \Rightarrow t = \frac{pore\ volume\ (oil\ volume)}{q}$

$$= \frac{1500\,(300)\,(12)\,(0.32)\,(1-0.17)\ ft^3}{100\ \frac{bbl}{day}} \times \left(\frac{5.617\ ft^3}{1\ bbl}\right)^{-1}$$

$= 2555\ days.$

f) what is pressure gradient for $100\ \frac{bbl}{day}$.

sol: $\frac{\Delta P}{L} = \frac{482.9\ psi}{1500\ ft} = 0.322\ \frac{psi}{ft}$

✓ g) if $C_o = 65 \times 10^{-6}\ psi^{-1}$, how much is the flow rate greater at downstream than upstream flow rate of $100\ \frac{bbl}{day}$.

incompressible درحالت $\rightarrow$ $q_1 = q_2$

$\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 v_1 A_1 = \rho_2 v_2 A_2 \rightarrow \rho_1 q_1 = \rho_2 q_2 \xrightarrow{\rho_2 < \rho_1} q_2 > q_1$ اما درحالت slightly comp. چون

↳ due to expantion

$$q_1 = 0.001127 \frac{KA}{\mu c L} \ln\left[1 + c(P_1 - P_2)\right]$$

در قسمت های قبل ΔP بدست آمده برای حالت incompressible بود ولی در اینجا فرض کردیم که c داریم و سیال slightly compressible است. پس ΔP متفاوت است و باید بدست آوریم:

upstream

$$100 = 0.001127 \frac{245(12)(300)}{3.2(65\times10^{-6})1500} \ln\left[1 + 65\times10^{-6}(P_1 - P_2)\right]$$

$$\longrightarrow P_1 - P_2 = 490.5 \text{ psi}$$

downstream

$$\longrightarrow q_2 = 0.001127 \frac{KA}{\mu c L} \ln\left[\frac{1}{1 + c(P_2 - P_1)}\right]$$

$$\longrightarrow \boxed{q_2 = 103.23 \ \frac{bbl}{day}}$$

Two observation wells are located 2500 ft apart. The static well pressure at the top of perforations (9332 ft subsea) in well A is 4365 psia and at the top of perforations (9672 ft subsea) in well B is 4372 psia,

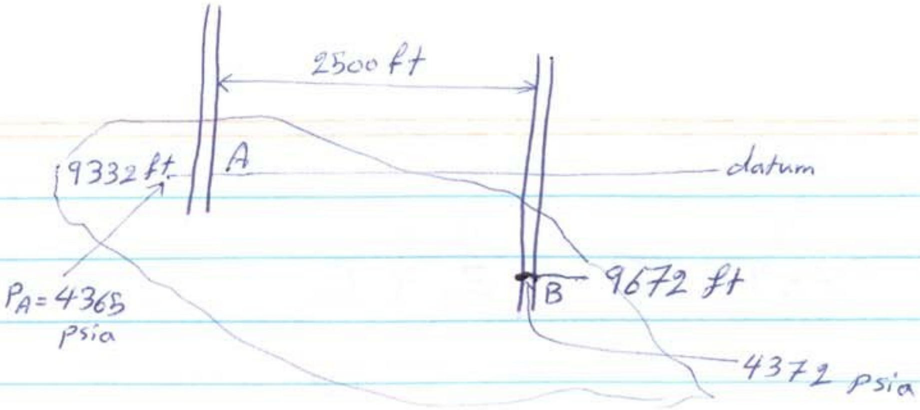
The reservoir fluid gradient is $0.25 \frac{psi}{ft}$. Reservoir permeability is 245 md & reservoir viscosity is 0.63 cp.

a) In what direction is the fluid flowing between wells.

b) what is the fluid velocity in $\frac{ft}{s}$

c) what is the flow rate.

(۳)



$K = 245$ md

$\mu = 0.63$ cp

gradient $= 0.25 \frac{psi}{ft}$

a)

- نقطه‌ی datum بهتر است یکی از location ها گرفته شود، (ما A را می‌گیریم.)

$$\phi_i = P_i - \frac{\rho}{144}\Delta Z_i \longrightarrow \phi_A = 4365 + 0 = 4365 \text{ psi}$$

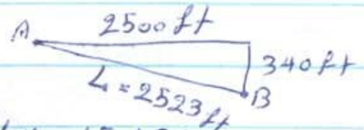
$$\& \ \phi_B = P_B - \frac{\rho}{144}\Delta Z_i = 4372 - \boxed{0.25}\left[9672 - 9332\right] = 4287 \text{ psi}$$

$\frac{\rho}{144}$ ← از گرادیان هستند.

پتانسیل B از A کمتر است ← جریان از A به B می‌رود.

If datum at B $\longrightarrow \begin{cases} \phi'_B = 4372 \text{ psi} \\ \phi'_A = 4450 \text{ psi} \end{cases} \Longrightarrow \Delta\phi' = 78 \text{ psi}$

b) $q = 0.001127 \frac{KA}{\mu L}(\phi_A - \phi_B)$



$$\longrightarrow V = \frac{q}{A} = \frac{0.001127(245)}{0.63 \times (2523)}(78)\ \frac{bbl/day}{ft^2}\left(\frac{5.615 ft^3}{1 \text{ bbl}}\right)\left(\frac{24(3600) s}{1 day}\right)$$

$$= 8.8 \times 10^{-7} \frac{ft}{s}$$

c) غیرقابل حل است چون ابعاد را نداده

نداده ولی L را داریم.

Steady state linear compressible flow: (gases)

$$n = \frac{PV}{ZRT} \Rightarrow n = \frac{P_{sc} V_{sc}}{Z_{sc} R T_{sc}}$$

$$\frac{PV}{ZRT} = \frac{P_{sc} V_{sc}}{Z_{sc} R T_{sc}} \xrightarrow{Z_{sc}=1} \frac{PV}{ZT} = \frac{P_{sc} V_{sc}}{T_{sc}}$$

$$\overset{\div t}{\Rightarrow} \frac{\overset{psi}{P}\, \overset{bbl/day}{q}}{Z \underset{°R}{T}} = \frac{\overset{psi}{P_{sc}}\, \overset{SCF/day}{Q_{sc}}}{\underset{°R}{T_{sc}}} \left(\frac{1 bbl}{5.615 SCF} \right)$$

$$\Rightarrow ① \; q = \frac{P_{sc}}{T_{sc}} \left(\frac{ZT}{P} \right) \left(\frac{Q_{sc}}{5.615} \right)$$

$$② \; q = -0.001127 \frac{KA}{\mu} \frac{dP}{dx}$$

$$\Rightarrow \frac{P_{sc}}{T_{sc}} \left(\frac{ZT}{P} \right) \frac{Q_{sc}}{5.615} = -0.001127 \frac{KA}{\mu} \frac{dP}{dx}$$

$$\rightarrow \frac{P_{sc} Q_{sc}}{T_{sc} (0.006328) AK} \int_{x_0}^{L} dx = -\frac{1}{TZ\mu} \int_{P_1}^{P_2} P\, dp$$

if T, Z & μ to be constant

$$Q_{sc} = \frac{0.003164 \, T_{sc} AK (P_1^2 - P_2^2)}{P_{sc} T L Z \mu}$$

$T_{sc} = 520 °R$
$P_{sc} = 14.7 psia$

$$\rightarrow \boxed{\overset{SCF/day}{Q_{sc}} = \frac{0.111924 \, AK (P_1^2 - P_2^2)}{T L Z \mu}}$$

✓ مهمة كتير

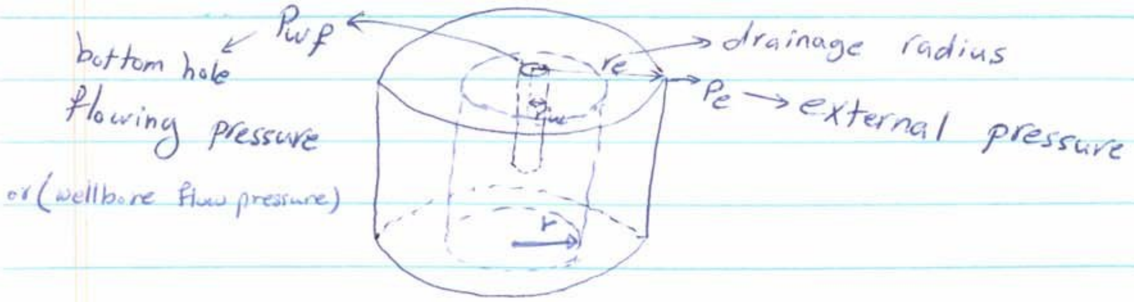
Valid for $P < 2000$ psi

gas properties (μ, Z) at $\bar{P} = \sqrt{\frac{P_1^2 + P_2^2}{2}}$

⑤

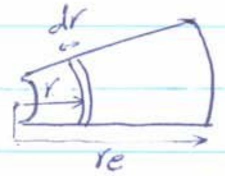
تا اینجا تقریباً سیال به صورت واقعی به صورت خطی جریان نمی یابد ولی به طور طبیعی به صورت Radial است.

steady state (Radial flow of Incompressible fluids)



$q =$ constant

در حالت steady state، q در نقاط اطراف چاه یکسان است.



$$q = 0.001127\frac{KA}{\mu}\frac{dP}{dr}$$

$$A = 2\pi r h$$

$$\longrightarrow q = 0.001127\frac{K2\pi rh}{\mu}\frac{dP}{dr}$$

h = pay zone thickness

ارتفاع لایه نفتی

$$\longrightarrow q\int_{r_w}^{r_e}\frac{dr}{r} = 0.001127\frac{K2\pi h}{\mu}\int_{P_{wf}}^{P_e}dP$$

① $$\boxed{\underset{\frac{bbl}{day}}{q} = \frac{0.00708\,\overset{md}{K}\,\overset{ft}{h}}{\underset{cp}{\mu}\,Ln\frac{r_e}{r_w}}\,\overset{psi}{(P_e - P_{wf})}}$$

ft

② $$\boxed{q\ \frac{bbl}{day} = Q_o\ \frac{STB}{day} \times B_o\ \frac{bbl}{STB}}$$ ✓

حفظ کنید ✓

$$\dashrightarrow \boxed{\underset{\frac{STB}{day}}{Q_o} = \frac{0.00708\,kh}{\mu_o B_o\,Ln\frac{r_e}{r_w}}(P_e - P_{wf})}$$

Hw #1

$\dot{m}_{in} = \dot{m}_{out} + \text{accumulation}$

$$\Delta t\left[4\pi r^2 \rho_r V_r - 4\pi (r+\Delta r)^2 \rho_{r+\Delta r} V_{r+\Delta r}\right] = m(\Delta t + t) - m(t)$$

$$4\pi \left(r^2 \rho_r V_r - (r+\Delta r)^2 \rho_{r+\Delta r} V_{r+\Delta r}\right) = \frac{dm}{dt}$$

For the left hand side of the above eq. we have:

$$\frac{dm}{dt} = \frac{d(\rho V)}{dt} = \frac{d(\rho \phi 4\pi r^2 \Delta r)}{dt} = 4\pi r^2 dr \frac{d(\rho\phi)}{dt}$$

$$4\pi\left[r^2 \rho_r V_r - (r+\Delta r)^2 \rho_{r+\Delta r} V_{r+\Delta r}\right] = 4\pi r^2 \Delta r \frac{d(\rho\phi)}{dt}$$

$$\lim_{\Delta r \to 0} \frac{r^2 \rho_r V_r - (r+\Delta r)^2 \rho_{r+\Delta r} V_{r+\Delta r}}{\Delta r} = r^2 \frac{d(\rho\phi)}{dt}$$

$$\frac{d(\rho V r^2)}{dr} = r^2 \frac{d(\rho\phi)}{dt}$$

For the right hand side $\longrightarrow r^2\left[\rho\phi(c_f + c_\phi)\frac{dP}{dt}\right] = r^2 \frac{d(\rho\phi)}{dt}$

&

$$V = \frac{6.328k}{\mu}\frac{dP}{dr}$$

$$\frac{6.328k}{\mu}\frac{d}{dr}\left[\rho r^2 \frac{dP}{dr}\right] = r^2\left[\rho\phi(c_f + c_\phi)\frac{dP}{dt}\right)$$

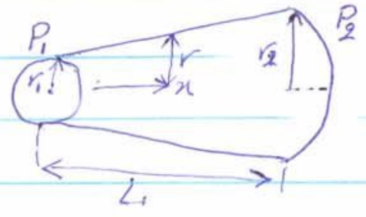
10/10

$$\frac{6.328k}{\mu}\left[\left(\frac{d\rho}{dr}\right) r^2 \frac{dP}{dr} + 2r\rho\frac{dP}{dr} + \rho r^2 \frac{d^2P}{dr^2}\right] = r^2\left[\rho\phi(c_f + c_\phi)\frac{dP}{dt}\right]$$

But we have; $\frac{d\rho}{dr} = \frac{d\rho}{dP} \cdot \frac{dP}{dr}$

$$\rightarrow \frac{6.328k}{\mu}\left[\underbrace{\frac{d\rho}{dP} r^2 \left(\frac{dP}{dr}\right)^2}_{\to 0} + 2r\rho\frac{dP}{dr} + \rho r^2 \frac{d^2P}{dr^2}\right] = r^2\left[\rho\phi(c_t)\frac{dP}{dt}\right]$$

$$\rightarrow \boxed{\frac{d^2P}{dr^2} + \frac{2}{r}\frac{dP}{dr} = \frac{\mu c_t \phi}{6.328k}\frac{dP}{dt}}$$



3.75/10

حل quiz:

- steady state
- slightly compressible

flow area با x متغیر است.

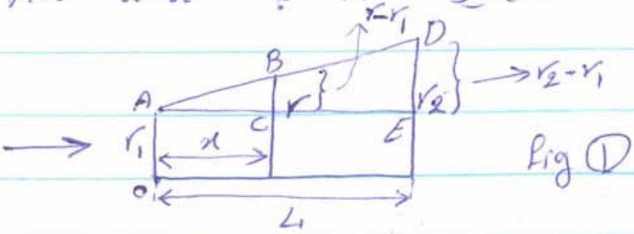


fig ①

$$A = \pi r^2 \qquad \xrightarrow{①} \quad q = q_{ref}\left(1 + c(P_{ref} - P)\right)$$

$$② \quad q = -0.001127 \frac{kA}{\mu}\frac{dP}{dx}$$

$$\xrightarrow{②,①} \quad q_{ref}\left(1 + c(P_{ref} - P)\right) = -0.001127 \frac{k\pi r^2}{\mu}\frac{dp}{dx}$$

& from fig① $\rightarrow \frac{r-r_1}{x} = \frac{r_2 - r_1}{L} \Longrightarrow r = r_1 + x\left(\frac{r_2 - r_1}{L_1}\right)$

$$\longrightarrow q_{ref}\int_0^{L_1} \frac{dx}{\left(r_1 + x\frac{r_2 - r_1}{L_1}\right)^2} = -0.001127 \frac{k\pi}{\mu}\int_{P_1}^{P_2} \frac{dP}{1 + c(P_{ref} - P)}$$

$$\longrightarrow q_{ref} = \frac{0.001127\, k\pi r_1 r_2}{\mu c L_1} Ln\left[\frac{1 + c(P_{ref} - P_2)}{1 + c(P_{ref} - P_1)}\right]$$

at $P_{ref} = P_2 \longrightarrow q_{ref} = q_2$

$$\longrightarrow \boxed{q_2 = 0.001127 \frac{k\pi r_1 r_2}{\mu c L} Ln\left[\frac{1}{1 + c(P_2 - P_1)}\right]}$$

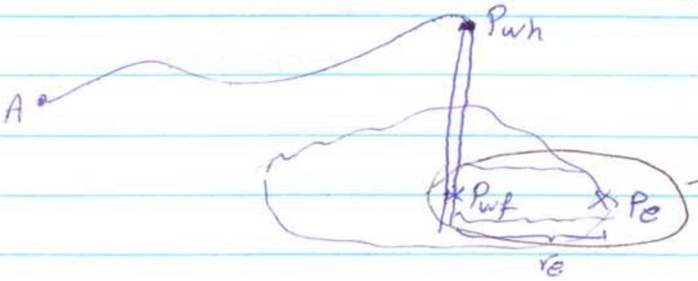
وقتی فشار مخزن (boundary) و well را داشتیم ، Q_o را می یابیم.

②

ادامه جلسه قبل

$$\boxed{Q_o = \frac{0.00708\,kh\,(P_e - P_{wf})}{\mu_o B_o \ln\frac{r_e}{r_w}}}$$ ✓

→ این رابطه را حفظ باشیم



A : drainage area (acres)

$$\pi \underset{ft}{r_e^2} = A\,(acres)\left(\frac{43560\,ft^2}{1\,acres}\right) \longrightarrow r_e = \sqrt{\frac{43560\,A}{\pi}}$$

برحسب فشاری که در r_e وجود دارد.

$$P = P_{wf} + \left[\frac{Q_o B_o \mu_o}{0.00708\,kh}\right] \ln\left(\frac{r}{r_w}\right)$$

Homework:

Craft & Hawkins have shown that average pressure in the reservoirs happens at $\overset{(*)}{r_{ave}} = 0.61\,r_e$:

در کتاب craft اثباتش را پیدا کنیم. (فشار متوسط مخزن)

$$\xrightarrow{(*)} Q_o = \frac{0.00708\,kh\,(\bar{P}_r - P_{wf})}{\mu_o B_o \ln\left(\frac{0.61 r_e}{r_w}\right)} \quad ①$$

$$\longrightarrow \ln\left(\frac{0.61 r_e}{r_w}\right) = \ln 0.61 + \ln\frac{r_e}{r_w} = \ln\frac{r_e}{r_w} - 0.5 \quad ②$$

$$\xrightarrow{①,②} \boxed{\underset{\frac{STB}{day}}{Q_o} = \frac{0.000708\,kh\,(\bar{P}_r - P_{wf})}{\mu_o B_o \underbrace{\left[\ln\frac{r_e}{r_w} - 0.5\right]}_{\ln\left(\frac{0.61 r_e}{r_w}\right)}}}$$ ✓

وقتی که $\bar{P}_r$ (فشار متوسط مخزن) را داشتیم و Q_o را محاسبه می کنیم.

③

A_T = Totall Area of field

q_T = Totall production flow rate of field

$A_T \quad q_T$

$A_w \quad q_w \rightarrow$ well flowrate

$$\rightarrow A_w = A_T \frac{q_w}{q_T}$$

drainage area

Steady state slightly compressible for radial flow :

① $q = q_{ref}\left(1 + c(P_{ref} - P)\right)$

② $q = 0.001127 \dfrac{K 2\pi r h}{\mu} \dfrac{dP}{dr}$

$$\xrightarrow{②,①} q_{ref}\int_{r_w}^{r_e}\frac{dr}{r} = 0.001127\left(\frac{2\pi K h}{\mu}\right)\int_{P_{wf}}^{P_e}\frac{dP}{1+c(P_{ref}-P)}$$

$$\rightarrow q_{ref} = \frac{0.00708\, Kh}{\mu\, c\, Ln\left(\frac{r_e}{r_w}\right)} Ln\left[\frac{1+c\,(P_{ref}-P_{wf})}{1+c\,(P_{ref}-P_e)}\right] \checkmark$$

حفظ لازم

if $P_{ref} = P_{wf} \rightarrow q_{ref} = q_o$, $\overset{\frac{bbl}{day}}{q_o} = \overset{\frac{STB}{day}}{Q_o}\, B_o \rightarrow \frac{bbl}{STB}$

$$\rightarrow Q_o = \frac{0.00708\, Kh}{B_o \mu_o c_o Ln\left(\frac{r_e}{r_w}\right)} Ln\left[\frac{1}{1+c(P_{wf}-P_e)}\right] \checkmark$$

حفظ لازم

$Q_o \rightarrow \frac{STB}{day}$

example:

An oil well is producing from a 45 acres reservoir with permeability of 35 md. The initial pressure & the bottom hole flowing pressures are 2100 & 1800 psi, respectively.

A well of 0.1 m radius has been drilled. The pay zone thickness is 50 ft. $B_o = 1.21 \frac{bbl}{STB}$, $\mu_o = 3\ cp$.

a) Calculate the amount of oil produced in 5 years.

b) if $C_o = 20 \times 10^{-6}\ psi^{-1}$, redo the calculation.

← دوباره انجام دادن

sol: initial pressure = boundry pressure = P_e

a)

$$\boxed{Q_o = \frac{0.000708\ Kh}{\mu_o B_o \ln \frac{r_e}{r_w}} (P_e - P_{wf})}$$

$$\pi r_e^2 = 43560 \overset{\text{acres}}{A} \Rightarrow 43560(45) = \pi r_e^2$$

$$\rightarrow \boxed{r_e = 790.1\ ft}$$

دقت شود، در امتحان اگر قطر را داده باید به شعاع تبدیل کرد. ✓

$$\rightarrow r_w = 0.1\ m \left(\frac{100\ cm}{1\ m}\right)\left(\frac{1\ in}{2.54\ cm}\right)\left(\frac{1\ ft}{12\ in}\right) = 0.328\ ft$$

$$\rightarrow Q_o = \frac{0.00708(35)(50)(2100-1800)}{3(1.21) \ln \frac{790.1}{0.328}}$$

$$\rightarrow Q_o = 131 \frac{STB}{day}$$

$$\rightarrow Q_{total} = 131 \left(\frac{365}{1\ year}\right)(5) = 239,985\ STB$$

b)

$$Q_o = \frac{0.00708\ Kh}{\mu B_o c \ln \frac{r_e}{r_w}} \ln\left[\frac{1}{1 + c(P_{wf} - P_e)}\right] \Rightarrow$$

$$Q_o = \frac{0.00708(35)(50)}{3(1.21)(20\times10^{-6}) \ln \frac{790.1}{0.328}} \ln\left[\frac{1}{1+20\times10^{-6}(1800-2100)}\right]$$

نکته سؤال: با دو فرمول حساب کرده و متوجه می شویم که دو فرمول جواب یکسان (۱) می دهد ← در محاسبات از فرمول اول استفاده می شود.

(۵)

Steady state radial flow of compressible fluids: (gases)

① $q = 0.001127 \frac{K_g \overset{\to 2\pi r h}{A}}{\mu_g} \frac{dP}{dr}$

$q = \frac{P_{sc}}{T_{sc}} \left(\frac{ZT}{P}\right) \frac{Q_{sc}}{5.615} \Longrightarrow$ (۲) $\overset{\nearrow \frac{bbl}{day}}{q_g} = \frac{P_{sc}}{5.615\, T_{sc}} \left(\frac{ZT}{P}\right) \overset{\nearrow \frac{SCF}{day}}{Q_g}$

$\xrightarrow{①, (۲)}$, $P_{sc} = 14.7$ psi , $T_{sc} = 520°R \Longrightarrow$

$$\frac{T Q_g}{Kh} \int_{r_w}^{r} \frac{dr}{r} = 0.703 \int_{P_{wf}}^{P} \left(\frac{2P}{\mu_g Z}\right) dP$$

$$\longrightarrow \frac{T Q_g}{Kh} \ln \frac{r}{r_w} = 0.703 \int_{P_{wf}}^{P} \frac{2P}{\mu_g Z} dP$$

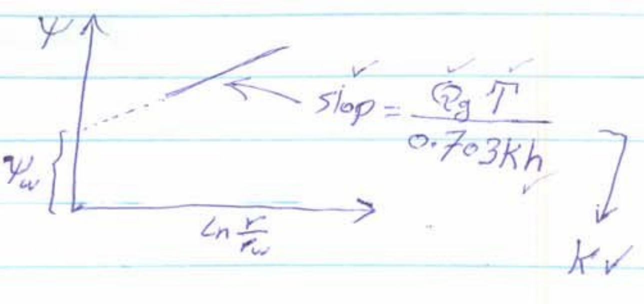
$$\longrightarrow \frac{T Q_g}{Kh} \ln \frac{r}{r_w} = 0.703 \left[\int_{0}^{P} \left(\frac{2P}{\mu_g Z}\right) dP - \int_{0}^{P_{wf}} \left(\frac{2P}{\mu_g Z}\right) dP \right]$$

$m(p) = \psi = \int_{0}^{P} \left(\frac{2P}{\mu_g Z}\right) dP$: real gas potential or real gas pseudo pressure

$\frac{Psi^2}{c.p}$ ← می‌توان سؤال داد که واحد ψ چیست؟

$$Q_g = \frac{0.703\, Kh (\psi - \psi_w)}{T \ln(\frac{r}{r_w})}$$

$$\psi = \psi_w + \frac{Q_g T}{0.703\, Kh} \ln \frac{r}{r_w}$$



$$Q_g = \frac{0.703\, Kh (\psi - \psi_w)}{T \ln(\frac{r}{r_w})}$$

$$\longrightarrow \underset{\swarrow \frac{SCF}{day}}{Q_g} = \frac{0.703\, Kh (\psi_e - \psi_w)}{T \ln(\frac{r_e}{r_w})}$$

برای بدست آوردن ψ_w باید به صورت مجازی فرض کنیم که سیال از boundary (e) به سمت چاه می آید و یک Q_g دارد.

$$\longrightarrow \quad Q_g = \frac{kh(\psi_e - \psi_w)}{1422\,T\,\ln\frac{r_e}{r_w}} \quad \checkmark \quad \text{or} \quad Q_g = \frac{kh(\bar{\psi}_r - \psi_w)}{1422\,T\left[\ln\frac{r_e}{r_w} - 0.5\right]}$$

1000 هزار ← $\frac{MSCF}{day}$

T → °R

$$\psi' = \int_{0}^{P} \frac{2P}{\mu_g Z}\,dP \quad \checkmark$$

P	μ_g	Z	$\frac{2P}{\mu_g Z}$
⋮	⋮	⋮	⋮

$\longrightarrow$

ψ = Area under curve

Approximation of the gas flow:

$$\frac{T Q_g}{kh} \ln\frac{r}{r_w} = 0.703 \int_{P_{wf}}^{P} \frac{2P}{\mu_g Z}\,dP$$

if μ_g & Z = constant

$$\Longrightarrow \quad Q_g = \frac{kh(P_e^2 - P_{wf}^2)}{1422\,T(\mu_g Z)\ln\frac{r_e}{r_w}} \quad \checkmark$$

$\frac{MSCF}{day}$

μ_g & Z at $\bar{P} = \sqrt{\frac{P_{wf}^2 + P_e^2}{2}}$ valid if $P < 2000$ psi

$\frac{M\,scf}{day} = 10^3\,\frac{scf}{day}$

$\boxed{\text{Hw \# 1}}$

columetric average pressure $\frac{9}{10}$

$dV = 2\pi r dr h \phi$ ϕ !

$$P_{av} = \frac{\int_{r_w}^{r_e} P\, dV}{\int_{r_w}^{r_e} dV} \quad , \quad P_{av} = \frac{\int_0^L p(2\pi r h \phi dr)}{\pi(r_e^2 - r_w^2) h\phi}$$

$r_w \rightarrow 0$

$$P_{av} = \frac{2}{r_e^2}\int_{r_w}^{r_e} P r dr \; , \; P = P_w + \frac{q\mu\bar{B}}{kh} \ln\frac{r}{r_w}$$

0.00708

$$P_{av} = \frac{2}{r_e^2}\int_{r_w}^{r_e}\left[P_w + \frac{q\bar{\mu}\bar{B}}{\underset{0.00708}{\cancel{1.02}}kh}\ln\frac{r}{r_w}\right]dr \quad \times r$$

$$P_{av} = \frac{2}{r_e^2}\left[P_w\frac{r_e^2}{2}\int_{r_w}^{r_e} + \frac{q\mu B}{\underset{0.00708}{}kh}\left(\frac{r^2}{2}\ln\frac{r}{r_w} - \frac{r^2}{4}\right)\Bigg|_{r_w}^{r_e}\right]$$

$$P_{av} = \frac{2}{r_e^2}\left[P_w\left(\frac{r_e^2}{2} - \frac{r_w^2}{2}\right) + \frac{q\bar{\mu}\bar{B}}{\underset{0.00708}{}kh}\left(\frac{r_e^2}{2}\ln\frac{r_e}{r_w} - \frac{r_e^2}{4} \cancel{- \frac{r_w^2}{4} - \frac{r_w^2}{2}\ln\frac{r_e}{r_w} + \frac{r_w^2}{4}}\right)\right]$$

$$P_{av_0} = P_w + \frac{q\bar{\mu}\bar{B}}{\underset{0.00708}{}kh}\left(\ln\frac{r_e}{r_w} - \frac{1}{2}\right)$$

$$P_{av} = P_w + \frac{q\bar{\mu}\bar{B}}{\underset{0.00708}{}kh}\left(\ln\frac{r_{av}}{r_w}\right)$$

$$\ln\frac{r_e}{r_w} - \frac{1}{2} = \ln\frac{r_{av}}{r_w} \longrightarrow \ln\frac{r_{av}}{r_w} - \ln\frac{r_e}{r_w} = -\frac{1}{2}$$

$$\ln\frac{r_{av}}{r_e} = -\frac{1}{2} \longrightarrow \frac{r_{av}}{r_e} = e^{-0.5} \longrightarrow r_{av} = 0.61\, r_e$$

①

جریان چند فازی

Horizontal Multiphase flow in radial direction:

$$q_o = 0.001127 \frac{K_o (2\pi r h)}{\mu_o} \frac{dP}{dr}$$

$$q_w = 0.001127 \frac{K_w (2\pi r h)}{\mu_w} \frac{dP}{dr} \longleftrightarrow \frac{bbl}{day}$$

$$q_g = 0.001127 \frac{K_g (2\pi r h)}{\mu_g} \frac{dP}{dr}$$

K_g, K_o, K_w : effective permeability

* Relative permeability

$$\underset{\text{relative permeability for oil}}{K_{ro}} = \frac{K_o}{\underset{\text{absolute permeability}}{K}} \Longrightarrow \begin{cases} K_o = K_{ro} \times K \\ K_w = K_{rw} \times K \\ K_g = K_{rg} \times K \end{cases}$$

$$\underset{\frac{STB}{day}}{Q_o} = \frac{\overset{\frac{bbl}{day}}{q_o}}{\underset{\frac{bbl}{STB}}{B_o}}$$

$$Q_w = \frac{q_w}{B_w}$$

$$\underset{\frac{SCF}{day}}{Q_g} = \frac{q_g}{B_g} \longrightarrow \boxed{\frac{bbl}{SCF}}$$

در اصل $\frac{ft^3}{SCF}$ است ولی در
اینجا B_g را برحسب $\frac{bbl}{SCF}$ می نویسیم.

$$\Longrightarrow Q_o = 0.00708 (rhK) \frac{K_{ro}}{\mu_o B_o} \frac{dP}{dr} \quad ①$$

$$Q_w = 0.00708 (rhK) \frac{K_{rw}}{\mu_w B_w} \frac{dP}{dr} \quad ②$$

$$\frac{SCF}{day} \longleftarrow Q_g = 0.00708 (rhK) \frac{K_{rg}}{\mu_g B_g} \frac{dP}{dr} \quad ③$$

$$B_g = \frac{V_{T,P}\,(ft^3)}{V_{sc}\,(SCF)} = \frac{\frac{ZnRT}{P}}{\frac{Z_{sc}\,nR\,T_{sc}}{P_{sc}}} = 0.028772\,\frac{ZT}{P}\;\frac{ft^3}{scf} \times \left(\frac{1\,bbl}{5.615\,ft^3}\right)$$

$$\Rightarrow \boxed{B_g = 0.00503\,\frac{ZT}{P}\;\left(\frac{bbl}{scf}\right)}$$

(1) $$Q_o = \frac{0.00708\,K\,K_{ro}\,h}{\mu_o\,B_o\,Ln\frac{r_e}{r_w}}\,(P_e - P_{wf})$$

(2) $$Q_w = \frac{0.00708\,K\,K_{rw}\,h}{\mu_w\,B_w\,Ln\frac{r_e}{r_w}}\,(P_e - P_{wf})$$

(3) $\frac{MSCF}{day}$

$$Q_g = \frac{K\,K_{rg}\,h\,(\psi_e - \psi_w)}{1422\,T\,Ln\frac{r_e}{r_w}}$$

$$Q_g = \frac{k\,K_{rg}\,h\,(P_e^2 - P_{wf}^2)}{1422\,(\mu_g Z)_{av}\,T\,Ln\frac{r_e}{r_w}}$$

* پ‌ها دارای توان ۲ هستند

water oil Ratio = WOR
(water cut)

$$WOR = \frac{Q_w}{Q_o} = \frac{②}{①} \Rightarrow WOR = \frac{K_{rw}}{K_{ro}}\left(\frac{\mu_o\,B_o}{\mu_w\,B_w}\right)$$

GOR ≡ Gas oil Ratio

$$GOR = \frac{\text{Total gas flow rate}}{\text{oil flow rate}} = \frac{\text{free gas + solution gas}}{\text{oil flow rate}}$$

$$= \frac{\text{Free gas}}{Q_o} + \frac{\text{solution gas}}{Q_o} = \frac{③}{①} + R_s \longrightarrow \text{gas solubility}\;\frac{SCF}{STB}$$

$$\boxed{GOR = R_s + \frac{K_{rg}}{K_{ro}}\left(\frac{\mu_o\,B_o}{\mu_g\,B_g}\right)}$$

B_o: $\frac{bbl}{STB}$

B_g: $\frac{bbl}{SCF}$

GOR: $\frac{SCF}{STB}$

R_s: $\frac{scf}{STB}$

①

Example: An oil well with radius of 0.4 ft is flowing 370 $\frac{STB}{day}$ from a uniform sand of 30 ft thickness & 250 md permeability to oil. oil viscosity is 0.85 cp & formation volume factor is 1.45 $\frac{bbl}{STB}$. The flowing bottom hole pressure is 2800 psia & average prosity 18% & average connate water 22%.

a) what is the reservoir pressure at 10 ft radius.

b) what is the pressure gradient at 2 ft radius.

$r_w = 0.4$ ft

Sol: $Q = 370 \frac{STB}{day}$, $h = 30$ ft, $K = 250$ md,

$\mu_o = 0.85$ cp, $B_o = 1.45 \frac{bbl}{STB}$,

$P_{wf} = 2800$ psia, $\phi = 0.18$, $S_{wi} = 0.22$

- compressibility را با ← incompressible درنظر بگیریم.

$$Q_o = \frac{0.00708\,Kh}{\mu_o\,B_o}\,\frac{P_e - P_{wf}}{Ln\frac{r_e}{r_w}}$$

$$\rightarrow ① \; Q_o = \frac{0.00708\,Kh}{\mu_o\,B_o}\,\frac{P - P_{wf}}{Ln\frac{r}{r_w}} \quad \text{or} \quad Q_o = \frac{0.00708\,Kh}{\mu_o\,B_o}\,\frac{P_e - P}{Ln\frac{r_e}{r}}$$

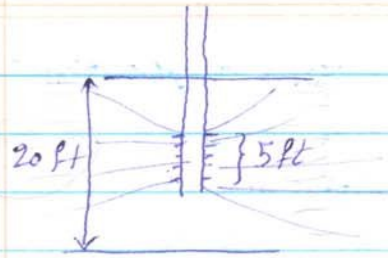
$$\xrightarrow{①} (a) \quad 370 = \frac{0.00708(250)(30)}{0.85(1.45)}\,\frac{P - 2800}{Ln\frac{r}{0.4}} \quad ②$$

$\longrightarrow$ at $r = 10$ ft $\longrightarrow P = 2827.6$ psia

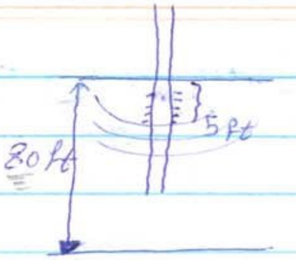
or ② can be writen: $P = 2800 + 8.588\,Ln\frac{r}{0.4}$

$$(b) \quad \frac{dP}{dr} = 8.588\,\frac{\frac{1}{0.4}}{\frac{r}{0.4}} \Rightarrow \left.\frac{dP}{dr}\right|_{r=2ft} = 4.294\,\frac{psi}{ft}$$

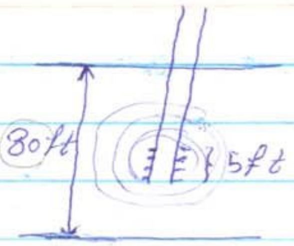
Radial flow

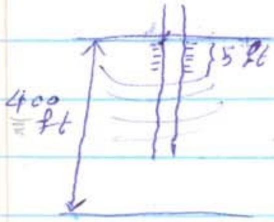


initially hemispherical
finally radial

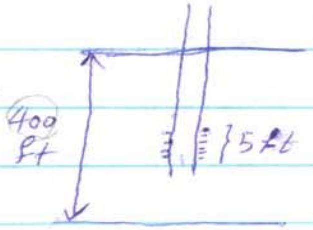


initially spherical
finally radial





hemispherical



spherical

Steady state spherical flow for incompressible fluid:

$$q = 0.001127\,\frac{KA}{\mu}\,\frac{dP}{dr} \quad , \quad A = 4\pi r^2$$

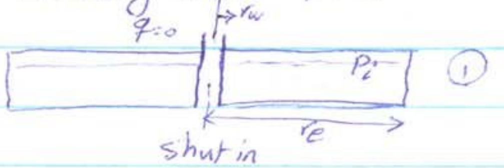
$$\longrightarrow q = 0.001127\,\frac{K\,4\pi r^2}{\mu}\,\frac{dP}{dr} \quad , \Longrightarrow \int_{r_1}^{r_2}\frac{dr}{r^2} = 2\times\frac{0.00708K}{q\mu}\int_{P_1}^{P_2} dP$$

$$① \Longrightarrow \boxed{q = \frac{2(0.00708)K(P_2 - P_1)}{\mu\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}} \quad ✓$$

① for hemispherical flow $A = 2\pi r^2 \Longrightarrow q = \frac{0.00708K(P_2 - P_1)}{\mu\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$ ✓

→ برای spherical در ② ضرب می شود.

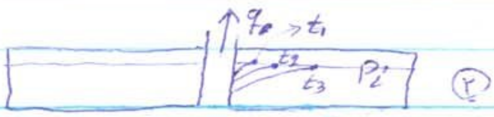
Unsteady state flow -



shut in

Production at constant Pwf

r_1, r_2, r_3 is r_{inv}



production at constant q

Transient flow: the time period that the drainage radius does not sense the production.

تا زمانی که فشار تحول به شعاع drainage نرسیده است.

r_{inv} : investigation radius.

if $r_{inv} < r_e$, we have transient flow.
in this case it seems that reservoir is infinite in size or <u>infinite acting reservoir</u>.

①

Basic Transient flow equation

Transient state : Reservoir behaves as infinite in size

Basic Equations:

continuity equation	°F = 1.8 °C + 32
Momentum equation	°R = °F + 459.69
Navier-stockes equation	°K = °C + 273.16
Bernouli equation	°R = 1.8 °K
Energy equation	$1 \text{ bbl} = 5.615 \text{ ft}^3$
→ Darcy equation	
Conservation of mass	
Newton's equation	
1st law of thermodynamics.	

Equations in fluid flow

1- $\underbrace{\text{Continuity equation}}_{\text{differential equation}}$ $\underbrace{\text{(conservation of mass)}}_{\text{algebric equation}}$: 1 equation (scaller است)

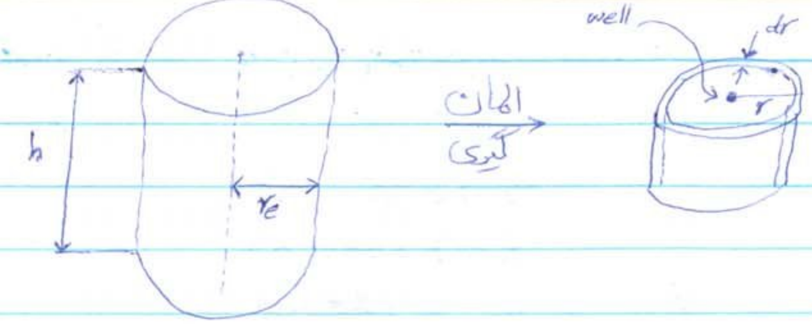
2- Momentum equation : 3 equations (برداری است)
(Navier-Stockes, Darcy equation (in porous media), Newton's equation)

3- Energy equation : 1 equation (چون انرژی scaller است)
(Bernuli's equation & 1st law of thermodynamics)

1- continuity equation or conservation of mass

2- Mementum equation (Darcy's Law) valid for laminar flow.

3- compressibility equations.



① $$\begin{bmatrix}\text{Mass entering volume}\\ \text{element during } \Delta t\end{bmatrix} - \begin{bmatrix}\text{Mass leaving volume}\\ \text{element during } \Delta t\end{bmatrix} = \begin{bmatrix}\text{rate of mass}\\ \text{accumulation}\\ \text{during } \Delta t\end{bmatrix}$$

$$\text{mass flow rate} = \frac{dm}{dt} = \rho V A \Rightarrow dm = \rho V A\, dt$$

② $(Mass)_{in} = \Delta t\,(\rho V A)_{r+dr}$, $A_{r+dr} = 2\pi(r+dr)h$

$$(Mass)_{in} = 2\pi \Delta t\, h\,(r+dr)\,(\rho V)_{r+dr}$$

③ $(Mass)_{out} = \Delta t\,(\rho V A)_r$, $A_r = 2\pi r h$

این سرعت apprent velocity است

$$(Mass)_{out} = 2\pi \Delta t\, h\, r\,(\rho V)_r$$

$\rho = \frac{m}{V} \Rightarrow m = \rho V$

$dv = 2\pi r dr\, h$

$(mass)_{accum} = V_p \rho = V_T \phi \rho$

④ $$\underbrace{(Mass)_{accum}}_{m} \underset{=}{=} \underbrace{2\pi r\, dr\, h}_{V_T} \underset{x}{} \underbrace{\left[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t\right]}_{\rho}$$

④③② در ① $\xrightarrow{}$ $2\pi h \Delta t\,(r+dr)(\rho V)_{r+dr} - 2\pi \Delta t\, h\, r\,(\rho V)_r =$

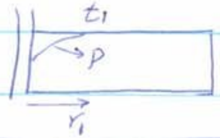
$2\pi r h dr\left[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t\right]$

$\xrightarrow[2\pi r h dr \Delta t]{\text{Divided by}}$

$$\frac{1}{r\,dr}(r+dr)(\rho V)_{r+dr}-\frac{(\rho V)_r}{dr}=\frac{1}{\Delta t}\left[(\phi\rho)_{t+\Delta t}-(\phi\rho)_t\right]$$

$$\lim_{\substack{dr\to 0\\ dt\to 0}}\left[\frac{1}{r\,dr}\left[(r+dr)(\rho V)_{r+dr}-r(\rho V)_r\right]=\frac{1}{\Delta t}\left[(\phi\rho)_{t+\Delta t}-(\phi\rho)_t\right]\right]$$

$$④ \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left[\overset{ft}{r}\,\overset{\frac{lb}{ft^3}}{\rho}\,\overset{\frac{ft}{day}}{V}\right]=\frac{\partial}{\partial t}(\phi\rho)$$ continuity equation



$$\Rightarrow P=f(r,t)$$

حالا از فرمول بالا رابطه‌ای بنویسیم که طبق
مطلب جلو و اول مبحث P تابعی از r و t باشد.

$$q=0.001127\,\frac{kA_r}{\mu}\frac{dP}{dr}$$

$$\overset{\frac{ft}{day}}{V}=\frac{\overset{\frac{bbl}{day}}{q}}{\underset{ft^2}{A_r}}=0.001127\frac{k}{\mu}\frac{dP}{dr}\left(\frac{5.615\ ft^3}{1\ bbl}\right) \quad ⑤$$

$$④,⑤ \Longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left[r\rho\left(5.615(0.001127)\frac{k}{\mu}\frac{\partial P}{\partial r}\right]=\frac{\partial}{\partial t}(\phi\rho)$$

$$\xrightarrow{⑥}\frac{0.006328}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}(\rho r)\frac{\partial P}{\partial r}\right]=\frac{\partial}{\partial t}(\phi\rho)$$

$$⑦\ \frac{\partial}{\partial t}(\phi\rho)=\phi\frac{\partial\rho}{\partial t}+\rho\frac{\partial\phi}{\partial t}$$

formation compressibility : $c_f=-\frac{1}{V_P}\left(\frac{\partial V_P}{\partial P}\right)_T \Rightarrow c_f=\frac{1}{\phi}\frac{\partial\phi}{\partial P}$

$$⑧\ \frac{\partial\phi}{\partial t}=\frac{\partial\phi}{\partial P}\frac{\partial P}{\partial t}=c_f\,\phi\frac{\partial P}{\partial t}$$

$$⑥,⑦,⑧ \Rightarrow \frac{0.006328}{r}\frac{\partial}{\partial r}\left[\frac{k}{\mu}(\rho r)\frac{\partial P}{\partial r}\right]=\rho\phi c_f\frac{\partial P}{\partial t}+\phi\frac{\partial\rho}{\partial t}$$

$$⑨\ \frac{0.006328\,k}{\mu}\left[\frac{\rho}{r}\frac{\partial P}{\partial r}+\frac{\partial\rho}{\partial r}\frac{\partial P}{\partial r}+\rho\frac{\partial^2 P}{\partial r^2}\right]=\rho\phi c_f\frac{\partial P}{\partial t}+\phi\frac{\partial\rho}{\partial t}$$

Radial flow of slightly compressible flow

$$⑩\left\{\begin{aligned} &\frac{\partial \rho}{\partial r} = \frac{\partial \rho}{\partial P}\frac{\partial P}{\partial r} \\ &\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial P}\frac{\partial P}{\partial t} \\ &\underset{\swarrow}{C} = \frac{1}{\rho}\frac{\partial \rho}{\partial P} \Longrightarrow \frac{\partial \rho}{\partial P} = \rho C \end{aligned}\right.$$

fluid compressibility

$$\xrightarrow{⑩,⑨} 0.006328\,\frac{K}{\mu}\left[\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \underbrace{C\left(\frac{\partial P}{\partial r}\right)^2}_{0}\right] \quad ⑪$$

$$= \phi c_f \frac{\partial P}{\partial t} + \phi c \frac{\partial P}{\partial t}$$

total compressibility $c_t = c_f + c$

In general: $c_t = c_f + c_o S_o + c_w S_w + c_g S_g$

Equation is valid for single phase flow (oil flow). However, the other phases can exist as immobile.

$$\xrightarrow[c_f + c = c_t]{⑪} 0.006328\,\frac{K}{\mu}\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right) = \phi c_t \frac{\partial P}{\partial t}$$

$$\longrightarrow \boxed{\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\overset{\to psi}{\partial P}}{\underset{\to ft}{\partial r}} = \frac{\phi \overset{\to cp}{\mu} \overset{\to psi^{-1}}{c_t}}{0.006328 \underset{md}{K}}\,\frac{\partial P}{\underset{day}{\partial t}\left(\frac{1\,day}{24\,hrs}\right)}}$$ Diffusivity equation

many applications in well testing

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi \mu c_t}{0.000264 K}\frac{\partial P}{\partial t}$$

✓ Diffusivity constant: $\eta = \dfrac{0.000264 K}{\phi \mu c_t}$

$$\left.\right\} \frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t}$$

$$\Downarrow$$

$$P = f(r, t, K, \phi, \mu, c_t)$$

Importance of diffusivity constant:
It involves both rock & fluid properties.

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t}$$: Diffusivity equation

✓ Valid for
- 1- laminar flow
- 2- Homogneous reservoir
- 3- single phase flow
- 4- uniform reservoir thickness
- 5- fluid properties independent of pressure

A special case : steady state

$$\frac{\partial P}{\partial t} = 0 \Rightarrow \frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = 0$$

$$\frac{d^2 P}{dr^2} + \frac{1}{r}\frac{dP}{dr} = 0 \quad , \quad M = \frac{dP}{dr}$$

$$\frac{dM}{dr} + \frac{1}{r} M = 0 \Rightarrow \frac{dM}{M} = \frac{-dr}{r}$$

$$\rightarrow Ln\, M = -Ln\, r + C_1 \Rightarrow Ln(Mr) = Ln\, C$$

$$\rightarrow M = \frac{C}{r} \Rightarrow \frac{dP}{dr} = \frac{C}{r} \Rightarrow \int_{P_{wf}}^{P} dP = \int_{r_w}^{r} C \frac{dr}{r}$$

①
$$\Rightarrow \boxed{P = P_{wf} + C\, Ln \frac{r}{r_w}}$$

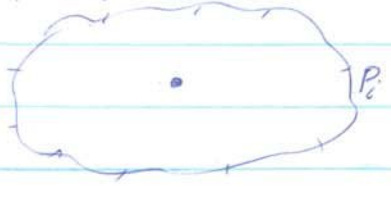
$$Q = \frac{0.00708 Kh (P - P_{wf})}{\mu_o B_o Ln(\frac{r}{r_w})}$$ ① ≃ ②

② $$P = P_{wf} + \underbrace{\frac{\mu_o B_o Q}{0.00708\ Kh}}_{C} Ln \frac{r}{r_w}$$

✓ solutions for constant terminal Pressure in Transient Flow :

روش اول حل
معادله diffusivity

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t} \nearrow P = P(r,t)$$



Ei-Function solution :

Initial condition at $t = 0$, $P = P_i$

Strong aquifer

Boundary conditions $\begin{cases} \text{at } r=\infty ,\ P=P_i \text{ or } P(\infty,t)=P_i \\ \text{(reservoir is acting as infinite)} \\ \\ \text{at } r=r_w \Rightarrow q=0.001127\dfrac{KA}{\mu}\dfrac{dP}{dr}\Big|_{r=r_w} \\ \Rightarrow \dfrac{dP}{dr}\Big|_{r=r_w}=\dfrac{q\mu}{0.001127KA} \end{cases}$

Mathews & Russel solution

$$P(r,t)=P_i+\left[\frac{70.6\,Q_o\mu B_o}{kh}\right]E_i\left[\overbrace{-\frac{948\,\phi\mu c_t r^2}{kt}}^{x}\right]$$

t → hr

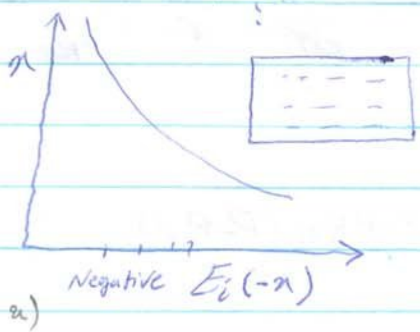
E_i = The E_i Function   well is in the center of reservoir

$$E_i(-x)=-\int_x^{\infty}\frac{e^{-u}du}{u}=\ln x-\frac{x}{1!}+\frac{x^2}{2(2!)}-\frac{x^3}{3(3!)}+\cdots$$

Table 6-1 x $-\boxed{E_i(-x)}$ → negative

positive

Figure 6-19



corresponded:

$x<0.02$

$E_i(-x)=\ln x+0.577=\ln(1.781x)$

$$E_i(-x) = a_1 + a_2 \ln(x) + a_3 (\ln x)^2 + a_4 (\ln x)^3 + a_5 x + a_6 x^2 + a_7 x^3 + \frac{a_8}{x}$$

$a_1 = -0.33153973$

$a_2 = -0.81512322$

$a_3 = 5.22123384 \times 10^{-2}$

$a_4 = 5.9849819 \times 10^{-3}$ $\longrightarrow$ for $0.02 < x < 3.0$

$a_5 = 0.662318450$

$a_6 = \ldots\ldots$

$a_7 = \ldots\ldots$

$a_8 = \ldots\ldots$ Note: for $x > 10.9 \Rightarrow E_i(-x) \approx 0$

ex: Calculate the time needed to obtain a pressure drop of 227.5 psi 600 ft away from a well with 15,000 $\frac{STB}{day}$, $K = 0.15D$, $\phi = 0.09$, $r_w = 0.4$ ft, $P_i = 3{,}000$ psi, $B_o = 1.39 \frac{bbl}{STB}$, $c_t = 6 \times 10^{-6}$ psi^{-1}, $\mu = 0.4$ cp, $h = 20$ ft

sol:

① $$\underset{P_i - dP}{\underset{\downarrow}{P(r,t)}} = P_i + \frac{70.6\, Q_o \mu B_o}{Kh} E_i\left(\frac{-948 \phi \mu c_t r^2}{K \underset{\searrow \text{hour}}{t}}\right)$$

$$3000 - 227.5 = 3000 + \frac{70.6(15000)(0.4)(1.39)}{(0.15 \times 10^3)(20} E_i\left(\frac{-948(0.09)(0.4)\overbrace{}^{6\times10^{-6}(600)^2}}{150t}\right)$$

$$\longrightarrow E_i\left(\frac{-0.4914}{t}\right) = -1.159$$

Table 6.1:

x	$-E_i(-x)$
0.2	1.22265 $\longleftarrow$ interpolate
0.3	0.90568

$$\longrightarrow \overset{x}{\boxed{\frac{0.4914}{t}}} = 0.22 \longrightarrow t = 2.23 \text{ hrs}$$

ex) Calculate the time needed to obtain a pressure drop of 227.5 psi 600 ft away from a well with 18,000 $\frac{STB}{day}$, $\phi = 0.0695$, $k_o = 0.1 D$ (= 100 md), $r_w = 0.5 ft$, $P_i = 3000 psi$, $B_o = 1.39 \frac{bbl}{STB}$, $C_t = 6\times10^{-6} psi^{-1}$, $\mu = 0.4 cp$, $h = 141 ft$.

شبیه مسئله قبلی

$$① \Longrightarrow 3000 - 227.5 = 3000 + \frac{70.6(18000)(0.4)(1.39)}{100(141)} \times E_i\left(\frac{-948(0.0695)(0.4)6\times10^{-6}(600)^2}{100t}\right)$$

$$\longrightarrow -4.539 = E_i\left(\frac{-0.569}{t}\right)$$

From Table 6.1 & Figure 6-19, $E_i = -4.539$ corresponds to $x < 0.02$

$$\xrightarrow{②} E_i(-x) = \ln x + 0.577$$

$$\xrightarrow{②,①} -4.539 = \ln\left(\frac{0.569}{t}\right) + 0.577 \longrightarrow t = 94.8 \text{ hrs}$$

✓ The Dimensionless Pressure Drop solution:

روش دوم حل معادله diffusivity

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t}$$

$$Q_o = \frac{0.00708\, kh(P_e - P_{wf})}{\mu B_o \ln\frac{r_e}{r_w}} \Longrightarrow \underbrace{\ln\frac{r_e}{r_w}}_{\text{dimensionless}} = \overbrace{\frac{P_e - P_{wf}}{\frac{Q_o \mu B_o}{0.00708\, kh}}}^{\text{dimensionless}}$$

$P_e - P_{wf}$ → دارای دیمانسیون psi

$\frac{Q_o \mu B_o}{0.00708\, kh}$ → دارای هم psi است.

Dimensionless external radious : $r_{eD} = \frac{r_e}{r_w}$

" radial distance : $r_D = \frac{r}{r_w}$ ①

" pressure : $P_D = \frac{P_i - P_{(r,t)}}{\frac{Q_o \mu_o B_o}{0.00708\, kh}}$ ②

" time : $t_D = \frac{0.000264\, kt}{\phi \mu c_t r_w^2}$ ③ ✓

← ⊛ روش بدست آمدن

(۳)

(*) $$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi \mu c_t}{0.000264\, k}\frac{\partial P}{\partial t}$$

solution the $$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t}$$

①، ②

$$-\frac{Q_o \mu B_o}{0.00708\, k h r_w^2}\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_w r_D}\left(\frac{-Q_o \mu B_o}{0.00708\, kh}\frac{\partial P_D}{r_w \partial r_D}\right) = \frac{1}{\eta}\left(\frac{-Q_o \mu B_o}{0.00708\, kh}\frac{\partial P_D}{\frac{\phi \mu c_t r_w^2}{0.000264 k}\partial t_D}\right)$$

$$\frac{\partial^2 P}{\partial r^2} = \frac{\partial}{\partial r}\left(\frac{\partial P}{\partial (r_D r_w)}\right) = \frac{\partial}{\partial (r_D r_w)}\left(\frac{\partial P}{\partial r_D r_w}\right)$$

$$\boxed{\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D}}$$ → این رابطه در well testing استفاده می شود

solved by ⟹ Everdingen & Hurst } ص ۳۹۶ توضیحات

① for infinite acting reservoirs ⟹ Table 6.2

$r_D \approx \infty \Longrightarrow P_D = f(t_D)$

$P_D = f(r_D, t_D)$

for $t_D > 100$ $\quad P_D = 0.5\left[\ln t_D + 0.80907\right]$

for $t_D < 0.01$ $\quad P_D = 2\sqrt{\frac{t_D}{\pi}}$ — در کتاب

for $0.02 < t_D < 1000$

اگر باشد $0.00 < t_D < 1000 \Longrightarrow$ from Table 6.2

quiz #2 $\frac{10}{10}$

For a reservoir at 3000 psi & 100°c, absoulut permeability is 200md. Calculate WOR & GOR Knowing that gas solubility is 1200 $\frac{scf}{STB}$. $Z = 0.85$

Parametrs	oil	water	gas
Effective permeability K_r (md)	130	70	30
μ viscosity	2.2	0.9	0.017
B Formation volume Factor $\left(\frac{bbl}{STB}\right)$	1.23	1.1	—

WOR = 1.4717943

GOR = 39554.987 $\frac{scf}{STB}$

$B_g = 0.00503 \frac{ZT}{P}$

①

when $x < 0.02$ $\quad E_i(-x) = \ln x + 0.577$

$$P(r,t) = P_i + \left[\frac{70.6\,Q_o B_o \mu}{Kh}\right] E_i\left(\frac{-948\,\phi \mu c_t r^2}{Kt}\right)$$

$$① \Rightarrow E_i(-x) = \ln x + \ln(1.781) = \ln(1.781x)$$

when $x < 0.02 \Rightarrow P(r,t) = P_i + \left[\frac{70.6\,Q_o B_o \mu}{Kh}\right] \ln\left[1.781\left(\frac{948\,\phi \mu c_t r^2}{Kt}\right)\right]$

$$\rightarrow P(r,t) = P_i + \frac{70.6\,Q_o B_o \mu}{Kh} \ln\left[\frac{Kt}{1688.09\,\phi \mu c_t r^2}\right]^{-1}$$

$$P(r,t) = P_i - \frac{70.6\,Q_o B_o \mu}{Kh}\left(2.3026 \log\left[\frac{Kt}{1688.09\,\phi \mu c_t r^2}\right]\right.$$

$$= P_i - \frac{162.6\,Q_o B_o \mu}{Kh}\left[\log \frac{K\overset{\text{hours}}{t}}{\phi \mu c_t r^2} - 3.23\right]$$

$$P_{wf} = P_i - \frac{162.6\,Q_o B_o \mu}{Kh}\left[\log \frac{Kt}{\phi \mu c_t r_w^2} - 3.23\right]$$

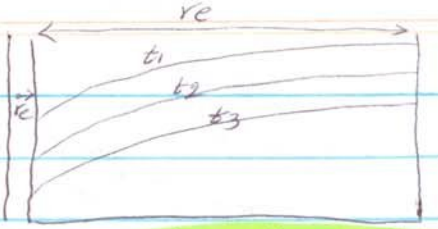
$\hookrightarrow$ for $t > 9.48 \times 10^4 \frac{\phi \mu_o c_t r^2}{K}$ $\quad r = r_w$

ادامه بالای صفحه ۲ جلسه قبل

④ we have finite reservoirs.

(re) when the effect of production (or the pressure profile) reachs the external radious, Transient Period ends and Psuedo steady state starts.

It is belived that there is a short time between transient & pseudo steady state which is called late Transient : short time state. Due to complexity & shortness, it is ignored.



$$\left.\frac{dP}{dt}\right|_r = \text{constant.}$$

جالب ← example 6-12 صـ ۴۰۱ و بالای آن

این قسمت در کتاب Craft & Hawkins است:

Time when the system reaches Pseudo steady state:

Hw# 2

$$\boxed{t_{pss} = \frac{1200\,\phi\,\mu\,c_t\,r_e^2}{k}}$$

$t_{pss} \rightarrow$ hr

oil reservoir

ex) $\mu = 1.5$ cp , $c_t = 15\times10^{-6}$ psi^{-1}, $r_e = 1000$ ft, $K = 100$ md,
$\phi = 20\%$ بعد از چه مدتی حالت pseudo steady state آغاز می شود؟

$$t_{pss} = \frac{1200(0.2)(1.5)(15\times10^{-6})(1000)^2}{100} = 54 \text{ hrs} = 2.25 \text{ days}$$

gas reservoir

شیل است ← tight gas

ex) $\mu = 0.015$ cp , $c_t = 400\times10^{-6}$ psi^{-1}, $r_e = 2500$ ft, $K = 1$ md
$\phi = 20\%$

$$t_{pss} = \frac{1200\times(0.2)(0.015)(400\times10^{-6})(2500)^2}{1} = \boxed{9{,}00_0 \text{hrs}}$$

Prove Hw:

$$c = -\frac{1}{V}\frac{\partial V}{\partial P} \rightarrow dV = -cVdP \rightarrow dV = -c\,\frac{\pi r_e^2\phi h}{5.615}(P_i - \bar{P}_r) \quad ①$$

$$② \begin{cases} \bar{P}_r = P_{wf} + \dfrac{Q\mu B}{0.00708Kh}\left[\ln\dfrac{r_e}{r_w} - 0.5\right) \\ P_i = P_{wf} + \dfrac{Q\mu B}{0.00708Kh}\ln\dfrac{r_e}{r_w} \end{cases}$$

$$\xrightarrow[\text{① در}]{②} dV = \frac{c\pi r_e^2\phi h}{5.615}\,\frac{Q\mu B}{0.00708Kh}(0.5) \quad ③$$

$$t = \frac{dV}{q} = \frac{dV}{QB} \rightarrow t = \frac{c\pi r_e^2\phi}{5.615}\,\frac{\mu}{0.00708K}(0.5)\times\frac{24\text{ hr}}{1\text{ day}}$$

$$t = \frac{\overset{50}{\widetilde{39.51}}\,c r_e^2\phi\mu}{k}\times\frac{24\text{ hr}}{1\text{ day}} = \boxed{\frac{1200\,\phi\,\mu\,c_t r_e^2}{k}}$$

Saturday 7th of ordibehesht - Quiz # 3

Pseudo steady state

$$\frac{\partial P}{\partial t}\Big|_r = \text{constant}$$

$$c_t = -\frac{1}{V}\frac{dV}{dP} \Longrightarrow dV = -cV\,dP$$

$$\frac{dV}{dt} = -cV\frac{dP}{dt} \Longrightarrow \underset{①}{q} = -cV\frac{dP}{dt}$$

$$\overset{\text{bbl/day}}{q} = Q_o B_o \quad ②$$

$$①, ② \Longrightarrow \underset{\text{hrs}}{\frac{dP}{dt}} = \frac{-q}{C_t V} = -\frac{Q_o B_o}{C_t V} \longrightarrow \frac{dP}{dt} = \frac{-q}{24 C_t V} \quad ③$$

$$\underset{\text{bbl}}{V} = \pi r_e^2 h\phi\left(\frac{1\ \text{bbl}}{5.615\ ft^3}\right) \longrightarrow \underset{④}{V} = \frac{\pi r_e^2 h\phi}{5.615} = \frac{Ah\phi}{5.615}$$

$$③, ④ \Longrightarrow \frac{dP}{dt} = \frac{-q}{24 C_t \frac{Ah\phi}{5.615}} \Longrightarrow \underset{\text{psi/hr}}{\frac{dP}{dt}} = -\frac{0.2339\,\overset{}{\underset{\text{bbl/day}}{q}}}{C_t A h\phi} = \frac{-0.2339\,q}{C_t \pi r_e^2 h\phi}$$

$$\frac{dP}{dt}\uparrow \longleftarrow q\uparrow$$

$$\frac{dP}{dt}\downarrow \longleftarrow C_t\uparrow$$

decline : $-\frac{dP}{dt}$

کاهش یافتن

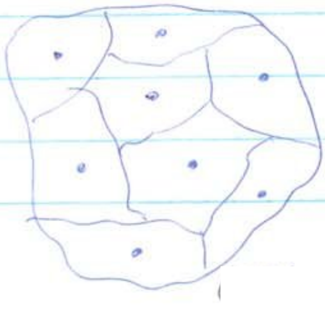
⟹ Gas reservoir depletion occures later than oil reservoirs.

تقلیل - نقصان

Ahwaz oil field

60 Km - 7 Km

415



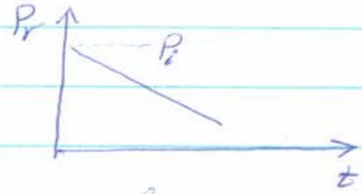
[illegible]

→ the whole reservoir

$$\frac{d\bar{P}}{dt} = \frac{dP}{dt}$$

فشار متوسط مخزن

$$\frac{\bar{P}_r - P_i}{t} = \frac{dP}{dt} \Longrightarrow \boxed{P_i - \bar{P}_r = \frac{0.23397}{c_t A h \phi} t}$$



$$\bar{P}_r = \frac{\int P dV}{\int dV} \Longrightarrow \bar{P}_r = \frac{\sum \bar{P}_{r_i} V_i}{\sum V_i} \Longrightarrow \bar{P}_r = \frac{\sum \bar{P}_{r_i} q_i}{\sum q_i}$$

Radial flow of Slightly compressible fluids:

for Pseudo steady state:

برداشت از مخزن

* هر چقدر q بیشتر افت فشار بیشتر

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi \mu c_t}{0.000264 K}\frac{dP}{dt}$$

کمتری

* مخازنی که دارای compressibility زیاد می‌باشند

افت فشار کمتری نسبت به مخازن دارای که

c_t پایین می‌باشند دارند.

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi \mu c_t}{0.000264 K}\left(\frac{-0.23397}{c_t A h \phi}\right)$$

$$⑥ \quad \frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{-887.22 q \mu}{A h K} \xrightarrow{\text{solution}}$$

$$X = \frac{\partial P}{\partial r} \Longrightarrow \frac{\partial X}{\partial r} + \frac{1}{r}X = \frac{-887.22 q \mu}{A h K}$$

$$⑥ \Longrightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P}{\partial r}\right) = \frac{-887.22 \, q \mu}{A h K}$$

No Flow at outer boundary

$$\Longrightarrow r\frac{dP}{dr} = \frac{-887.22 q \mu}{A h K}\frac{r^2}{2} + C_1$$

boundry condition $\left.\frac{dP}{dr}\right|_{r=r_e} = 0$

at $r = r_e$, $q = 0$, $q = 0.001127 \frac{KA}{\mu}\frac{dp}{dr} \Rightarrow \frac{dp}{dr} = 0$

چون در drainage area r دبی نداریم.

(because $q|_{r=r_e} = 0$)

$$\Longrightarrow C_1 = \frac{887.22 q \mu}{A h K} \times \frac{r_e^2}{2} = \frac{887.22 q \mu r_e^2}{\pi r_e^2 h K \, 2} = \frac{141.2 q \mu}{h K}$$

$$\Rightarrow \frac{dP}{dr} = -\frac{141.2\, q\mu}{hk}\,\frac{r}{r_e^2} + \frac{141.2\, q\mu}{hk}\,\frac{1}{r}$$

$$\int dp = \int \left(\frac{-141.2\, q\mu}{hk}\,\frac{r}{r_e^2} + \frac{141.2\, q\mu}{hk}\,\frac{1}{r}\right) dr$$

$$P = P_{wf} + \frac{141.2\, q\mu}{hk}\left[\ln\frac{r}{r_w} - \frac{r^2}{2r_e^2} + \cancelto{0}{\frac{r_w^2}{2r_e^2}}\right]$$

$$\Rightarrow \boxed{P = P_{wf} + \frac{141.2\, q\mu}{hk}\left[\ln\frac{r}{r_w} - \frac{r^2}{2r_e^2}\right]}$$

at $r = r_e$, $P = P_i \rightarrow$ boundary فشار

$$P_i = P_{wf} + \frac{141.2\, q\mu}{hk}\left[\ln\frac{r_e}{r_w} - \frac{1}{2}\right]$$

$$q = \frac{0.00708\, Kh\,(P_i - P_{wf})}{\mu\left[\ln\frac{r_e}{r_w} - 0.5\right]}$$

$$\boxed{Q_o = \frac{0.00708\, Kh\,(P_i - P_{wf})}{\mu B_o\left[\ln\frac{r_e}{r_w} - 0.5\right]}} \longrightarrow$$ اگر از فشار boundary استفاده کنیم

در شرایط pseudo steady state :

اگر از فشار boundary استفاده کنیم در فرمول 0.5 خواهیم داشت.

$$\bar{P}_r = \frac{\int P\,dv}{\int dv}$$

$$\bar{P}_r = \frac{\int_{r_w}^{r_e}\left(P_{wf} + \frac{141.2\, q\mu}{hK}\left[\ln\frac{r}{r_w} - \frac{r^2}{2r_e^2}\right]\right) 2\pi r\,dr\,h\phi}{\underbrace{\int_{r_w}^{r_e} 2\pi r\,dr\,h\phi}_{\pi h\phi\left[r_e^2 - \cancelto{0}{r_w^2}\right]}}$$

در شرایط pseudo steady stat :

$$\bar{P}_r = P_{wf} + \frac{141.2\, q\mu}{Kh}\left(\ln\frac{r_e}{r_w} - 0.75\right)$$

اگر از فشار متوسط استفاده کنیم در فرمول 0.75 خواهیم داشت.

$$\boxed{Q_o = \frac{0.00708\, Kh\,(\bar{P}_r - P_{wf})}{\mu B_o\left[\ln\frac{r_e}{r_w} - 0.75\right]}} \longrightarrow$$ اگر از فشار متوسط استفاده کنیم.

$$\ln \frac{a r_e}{r_w} = \ln \frac{r_e}{r_w} - 0.75$$

$$\rightarrow \quad \ln a + \ln \frac{r_e}{r_w} = \ln \frac{r_e}{r_w} - 0.75$$

$$\rightarrow \ln a = -0.75 \Rightarrow a = 0.471$$

Average pressure occures at 47% of drainage radius.

$$\boxed{r_{ar} = 0.471\, r_e}$$

Shape factor : C_A

1- for drainage areas different from a circular

2- for different locations of the well.

Table 6-4 , shape factors are given

In terms of average reservoir pressure.

$$\boxed{Q = \frac{Kh(\bar{P}_r - P_{wf})}{162.6\, B \mu \operatorname{Log}\left[\frac{4A}{1.781 C_A r_w^2}\right]}}$$

〃 C_A

اگر از فشار متوسط استفاده کنیم همراه با shape factor

hr

In terms of initial reservoir pressure :

$$P_{wf} = \left[P_i - \frac{0.23396\, Q B t}{A h \phi C_t}\right] - \frac{162.6\, Q B \mu}{Kh} \operatorname{Log}\left[\frac{4A}{1.781 C_A r_w^2}\right]$$

$\frac{bbl}{day}$

اگر بخواهیم از فشار boundary استفاده کنیم. همراه با shape factor

Table 6.4 for a circle : $C_A = 31.62$

$$\operatorname{Log} \frac{4A}{1.781 C_A r_w^2} = \operatorname{Log} \frac{4\pi r_e^2}{1.781\,(31.62)\, r_w^2}$$

حالت Pseudo steady state

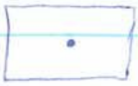
$$= \operatorname{Log}\left[0.223 \frac{r_e^2}{r_w^2}\right] = \frac{\ln\left[0.223 \left(\frac{r_e}{r_w}\right)^2\right]}{2.3} = \frac{1}{2.3} \ln\left[0.223^{\frac{1}{2}} \left(\frac{r_e}{r_w}\right)\right]^2$$

$$= \frac{2}{2.3}\left[\ln \frac{r_e}{r_w} - 0.75\right] \Rightarrow \boxed{Q = \frac{0.00708\, Kh(\bar{P}_r - P_{wf})}{\mu B\left[\ln \frac{r_e}{r_w} - 0.75\right]}}$$

همان فرمول صفحه قبل بدست آمد

Problem (10) دارای C_A می باشد
(in book)

center of a 40 acres square drilling pattern:



$\phi = 20\%$, $h = 15\ ft$, $K = 60\ md$

$\mu_o = 1.5\ cp$, $B_o = 1.4\ \frac{bbl}{STB}$, $r_w = 0.25\ ft$

$P_i = 2000\ psi$, $P_{wf} = 1500\ psi$, $Q = ?$, $\boxed{C_t = 15 \times 10^{-6}\ psi^{-1}} \rightarrow$ این مقدار را در کتاب نداده است.

چون شرایط boundary را داده پس از فرمول $P_{wf} =$ استفاده می‌کنیم

Table 6-4 $\Longrightarrow C_A = 30.8828$

shape factor

sol:

$$P_{wf} = \left[P_i - \frac{0.23396\, QBt}{Ah\phi C_t}\right] - \frac{162.6\, QB\mu}{Kh} \log\left[\frac{4A}{1.781\, \boxed{C_A}\, r_w^2}\right]$$

$$1500 = \left[2000 - \frac{0.23396\, Q(1.4)t}{40(43560)(15)(0.2)(15\times10^{-6})}\right] - \frac{162.6\, Q(1.4)(1.5)}{60(15)} \log\left[\frac{4(40)(43560)}{1.781(30.8828)(0.25)^2}\right]$$

$$\longrightarrow \boxed{Q = \frac{500}{2.3928 + 0.004177\, t}} \longrightarrow hrs$$

در اینجا احتیاج به C_t نداریم

If $\bar{P}_r = 2000\ psi$

$$Q = \frac{Kh(\bar{P}_r - P_{wf})}{162.6\, B\mu \log\left[\frac{4A}{1.781\, C_A r_w^2}\right]} = \frac{60(15)(2000-1500)}{162.6(1.4)(1.5)\log\left[\frac{4(40)(43560)}{1.781(30.8828)(0.25)^2}\right]}$$

$$= 209\ \frac{STB}{day}$$

Radial flow for compressible fluids in

Radial flow of compressible fluids in pseudo-steady state.

$$m(p) = \psi = \int_{0}^{p} \left(\frac{2p}{\mu_g Z}\right) dp$$

$m(p) = \psi$: pressure pseudo or gas potential.

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{1}{\eta}\frac{\partial p}{\partial t} \longrightarrow \frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{1}{\eta}\frac{\partial m(p)}{\partial t}$$

$$\underbrace{Q_g}_{\frac{Mscf}{day}} = \frac{kh\left(m(\bar{p}_r) - m(p_{wf})\right)}{1422\, \underbrace{T}_{°R} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]} \quad ①$$

two approximations to calculate $m(p)$:

1- pressure squared approximation.

$$m(\bar{p}_r) - m(p_{wf}) = \int_{p_{wf}}^{\bar{p}_r} \frac{2p}{\mu_g Z}\, dp = \frac{\bar{p}_r^{\,2} - p_{wf}^2}{\bar{\mu}_g\, \bar{Z}}$$

$$Q_g = \frac{kh\left(\bar{p}_r^{\,2} - p_{wf}^2\right)}{1422\, T\, \bar{\mu}\, \bar{Z} \left[\ln \frac{r_e}{r_w} - 0.75\right]}$$

$\bar{\mu}_g$ & $\bar{Z}$ at $\bar{p} = \sqrt{\dfrac{\bar{p}_r^{\,2} + p_{wf}^2}{2}}$ $\quad p < 2000$ psi

2- pressure approximation Method

$$\underbrace{q}_{\frac{bbl}{day}} = \underbrace{\frac{p_{sc}}{5.615\, T_{sc}} \frac{ZT}{p}}_{B_g\,\left(\frac{bbl}{scf}\right)} \underbrace{Q_{sc}}_{\frac{scf}{day}}$$

$$B_g = \frac{p_{sc}}{5.615\, T_{sc}} \frac{ZT}{p} \Rightarrow \frac{p}{Z} = \frac{p_{sc}\, T}{5.615\, T_{sc}} \left(\frac{1}{B_g}\right)$$

$$m(\bar{P}_r) - m(P_{wf}) = \int_{P_{wf}}^{\bar{P}_r} \frac{2P}{\mu_g Z}\, dp = \int_{P_{wf}}^{\bar{P}_r} \frac{2}{\mu_g} \frac{P_{sc} T}{5.615\, T_{sc}} \frac{1}{B_g}\, dp$$

$$m(\bar{P}_r) - m(P_{wf}) = \frac{2 T P_{sc}}{5.615\, T_{sc}} \int_{P_{wf}}^{\bar{P}_r} \frac{dP}{\mu_g B_g} \xrightarrow[P > 3000\, psi]{for}$$

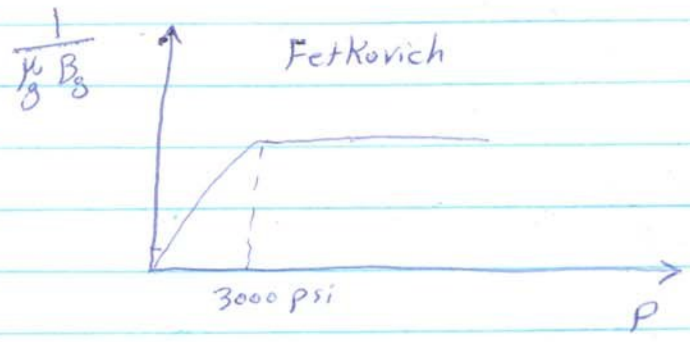
$$m(\bar{P}_r) - m(P_{wf}) = \frac{2 T P_{sc}}{5.615\, T_{sc} \bar{\mu}_g \bar{B}_g} (\bar{P}_r - P_{wf}) \quad ②$$

$$①, ② \longrightarrow Q_g = \frac{kh \left(\frac{2 T P_{sc}}{5.615\, T_{sc} \bar{\mu}_g \bar{B}_g} (\bar{P}_r - P_{wf}) \right)}{1422\, T \left(\ln \frac{r_e}{r_w} - 0.75 \right)}$$

$$Q_g = \frac{kh (\bar{P}_r - P_{wf})}{142200\, \bar{\mu}_g \bar{B}_g \left[\ln \frac{r_e}{r_w} - 0.75 \right]}$$

1422

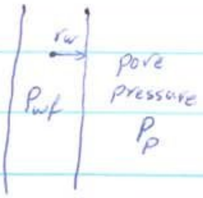
properties at $\bar{P} = \frac{\bar{P}_r + P_{wf}}{2}$

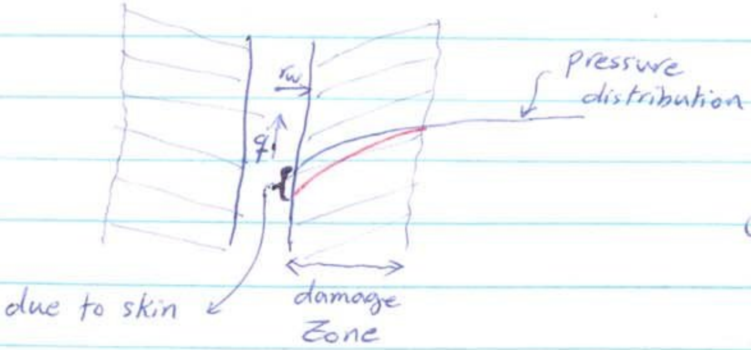


Drilling Techiniques

1) convectional drilling : $P_{wf} > P_p$, invasion of drilling fluid into formation : Formation damage

2) Underbalanced drilling : $P_{wf} < P_p$, It seems no formation damage.
Due to operational problems, we have formation damage.





مثل : $I = \frac{\Delta V}{R}$

$q = \frac{\Delta P}{R}$

Formation damige :

1) reduce s production (ΔP = constant)

2) reduces P_{wf} (q = constant) ⟶ GOR↑

Methods to reduce formation damage :

1) UBD

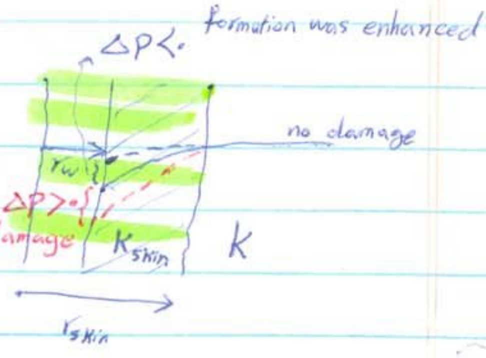
2) Stimulation
   - 1- acidizing
   - 2- Hydraulic fracturing

To express formation damage :

1) depth of $\underbrace{\text{invation}}_{\text{نفوذ}}$, r_{skin}

2) to use parameter S

parameters : K_{skin} , K , $(\Delta P)_{due\ to\ skin}$



we have 3 cases

1- $\Delta P_{skin} > 0$, we have formation damage.

$K_{skin} < K$

2- $\Delta P_{skin} < 0$, wellbore improvement

$K_{skin} > K$

3- $\Delta P_{skin} = 0$, No damage ,

$K = K_{skin}$

Hawkins Assumption

$$\Delta P_{skin} = \begin{bmatrix} \Delta P \text{ in skin Zone} \\ \text{due to } K_{skin} \end{bmatrix} - \begin{bmatrix} \Delta P \text{ in skin Zone} \\ \text{due to } K \end{bmatrix}$$

$$Q_o = \frac{0.00708 Kh \Delta P}{\mu_o B_o \ln(\frac{r_e}{r_w})} \Rightarrow \Delta P = \frac{Q_o B_o \mu_o \ln \frac{r_e}{r_w}}{0.00708 \ Kh}$$

$$\Delta P_{skin} = \frac{Q_o \mu_o B_o \ln \frac{r_{skin}}{r_w}}{0.00708 K_{skin} h} - \frac{Q_o \mu_o B_o \ln \frac{r_{skin}}{r_w}}{0.00708 K h}$$

$$\Delta P_{skin} = \frac{Q_o \mu_o B_o}{0.00708 Kh} \left[\frac{K}{K_{skin}} - 1\right] \ln \frac{r_{skin}}{r_w}$$

Definition

$$\boxed{\Delta P_{skin} = \frac{Q_o B_o \mu_o}{0.00708 Kh} S} \longrightarrow \boxed{S = \left(\frac{K}{K_{skin}} - 1\right) \ln \frac{r_{skin}}{r_w}}$$

skin factor

ⓐ

1- positive skin, $S > 0$, $\begin{cases} K > K_{skin} \\ r_{skin} > r_w \end{cases}$

we have formation damage.

2- Negative skin, $S < 0$, $\begin{cases} K < K_{skin} \\ \checkmark \end{cases}$

wellbore improvement

3- No skin, $S = 0$, $K = K_{skin}$

$$\Delta P_{skin} = \frac{Q_o \mu_o B_o}{0.00708\, Kh} S \quad ①$$

for steady state : $Q_o = \dfrac{0.00708\, Kh}{\mu_o B_o \ln \frac{r_e}{r_w}} (P_i - P_{wf})$ ②

$$(\Delta P)_{actual} = (\Delta P)_{ideal} + (\Delta P)_{skin}$$

→ یعنی حالتی invade Zone نداریم

$$(P_i - P_{wf})_{actual} = (P_i - P_{wf})_{ideal} + (\Delta P)_{skin}$$

$$①, ② \Longrightarrow (P_i - P_{wf}) = \frac{Q_o \mu_o B_o \ln \frac{r_e}{r_w}}{0.00708\ Kh} + \frac{Q_o \mu_o B_o}{0.00708\ Kh} S$$

$$\Longrightarrow Q_o = \frac{0.00708\, Kh (P_i - P_{wf})_{actual}}{\mu_o B_o \left[\ln \frac{r_e}{r_w} + S \right]} \quad ③$$ steady state(incomperssible)

By replacing $\ln \frac{r_e}{r_w}$ by $\ln\left(\frac{r_e}{r_w}\right) + S$, all the pervious relations can be used for considering skin factor.

- pseudo steady state radial flow:

$$Q_o = \frac{0.00708\, Kh (\bar{P}_r - P_{wf})}{\mu_o B_o \left[\ln \frac{r_e}{r_w} - 0.75 + S \right]}$$

for compressible flow:

$$Q_g = \frac{Kh\left[m(\bar{P}_r) - m(P_{wf}) \right]}{1422\, T \left[\ln \frac{r_e}{r_w} - 0.75 + S \right]}$$

pressure-squared approximation:

$$Q_g = \frac{kh\left(\bar{P}_r^2 - P_{wf}^2\right)}{1422 T \bar{\mu} \bar{Z}\left[\ln \frac{r_e}{r_w} - 0.75 + S\right]}$$

pressure approximation

$$Q_g = \frac{kh\left[\bar{P}_r - P_{wf}\right]}{142200 \; \bar{\mu} \bar{B}_g\left[\ln \frac{r_e}{r_w} - 0.75 + S\right]}$$

ex: A well is drilled by conventional drilling. Given
$P_i = 3000$ psi , $P_{wf} = 2300$ psi, drainage area $= 70$ acres ,
$r_w = 0.15$ ft , $K = 30$ md , $h = 200$ ft , $\mu_o = 2$ cp , $B_o = 1.15 \frac{bbl}{STB}$,
$Q_o = 1000 \frac{STB}{day}$.
Assuming the depth of invasion to be 1 ft.
- بنطلع ال depth عادة r_{skin} ع باشن

a) calculate permeability of skin Zone.

$$Q_o = \frac{0.00708\, kh\left(P_i - P_{wf}\right)}{\mu_o B_o\left[\ln \frac{r_e}{r_w} + S\right]} \quad , \quad A = \pi r_e^2 \Rightarrow 70 \times 43560 = \pi r_e^2$$

$$\Rightarrow r_e = 985 \text{ ft}$$

$$1000 = \frac{0.00708 \times 30 \times 200\,(3000 - 2300)}{2(1.15)\left[\ln \frac{985}{0.15} + S\right]} \Rightarrow S = 4.138$$

$$S = \left(\frac{K}{K_{skin}} - 1\right) \ln \frac{r_{skin}}{r_w}$$

$$4.138 = \left(\frac{30}{K_{skin}} - 1\right) \ln \frac{1}{0.15} \Rightarrow \boxed{K_{skin} = 9.429 \text{ md}}$$

b) If the well is drilled under balanced, How much the flow rate could be?

چون under balance گفته ← $S=0$ فرض می کنیم

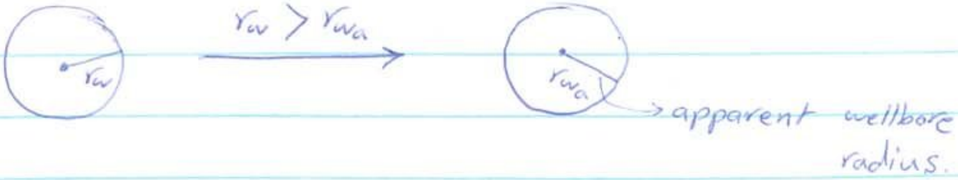
$$Q_o = \frac{0.00708\,Kh\,(P_i - P_{wf})}{\mu_o B_o \left[\ln \frac{r_e}{r_w} + \underset{0}{S}\right]}$$

$$Q_o = \frac{0.00708\,(30)(200)(3000-2300)}{2(1.15)\ \ln \frac{985.4}{0.15}} \Longrightarrow \boxed{Q_o = 1470\ \frac{STB}{day}}$$

c) It is required the production to be increased by 123 percent. which stimulation (S) can achieved this objective?

$$Q_{new} = Q_{old}\,(2.23) = 1000\,(2.23) = 2230\ \frac{STB}{day}$$

$$2230 = \frac{0.00708\,(30)(200)(3000-2300)}{2(0.15)\left[\ln \frac{985.4}{0.15} + S\right]} \Longrightarrow \boxed{S=-3}$$

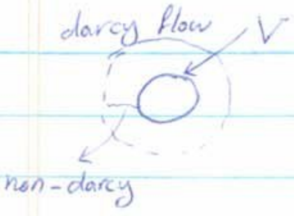


$$\ln \frac{r_e}{r_w} + S = \ln \frac{r_e}{r_{wa}} \Longrightarrow \ln \frac{r_w}{r_{wa}} = S = \ln e^{S}$$

$$\Longrightarrow \frac{r_w}{r_{wa}} = e^{S} \Longrightarrow r_{wa} = r_w\, e^{-S}$$

- به خاطر وجود skin، چاه را با یک شعاع کوچکتر r_{wa} فرض می کنیم.

- Turbulent flow factor: (r_w) turbulant flow near wellbore



turbulent flow around wellbore, especially in gas reservoir. (non-Darcian flow)

non-Darcy effect is considered as an extera pressure drop around wellbore (the same as skin factor)

$$(\Delta\psi)_{actual} = (\Delta\psi)_{ideal} + (\Delta\psi)_{skin} + (\Delta\psi)_{non-Darcy\ flow}$$: For gases

$$\Rightarrow (\Delta\psi)_{non-darcy} = 3.161 \times 10^{-12} \left[\frac{\beta T \gamma_g}{\mu_{gw} h^2 r_w} \right] Q_g^2 = F\, Q_g^2$$

رانکین OR

Q_g → $\frac{Mscf}{day}$

$$F = 3.161 \times 10^{-12} \left[\frac{\beta T \gamma_g}{\mu_{gw} h^2 . r_w} \right]$$

γ_g → s.p Gravity

h → ft, ارتفاع زون

r_w → ft

μ_{gw} → gas viscosity at wellbore (cp)

β → Turbulance parameter

F → non darcy flow coefficient

- Non-darcy flow is intepreted as a rate dependent skin: –

$$\beta = 1.88 \times 10^{-10}\ K^{-1.47}\ \phi^{-0.53}$$

K → Permeability (md)

ϕ → porosity

$$D = \frac{FKh}{1422\ T}$$

D → Inertial or turbulent flow factor

$$Re = \frac{\rho V D}{\mu} = \frac{\text{Inertia force}}{\text{viscouse force}}$$

laminar flow: viscous force $\gg$ Inertia force (far away from wellbore)

Turbulent flow: viscouse force $\ll$ Inertia force (near well bore)

replace S by $s' = S + DQ_g$ to consider non-Darcy effect.

Pseudo steady state: ← Turbulent flow بسبب ال

$$Q_g = \frac{kh\left[m(\bar{P}_r) - m(P_{wf})\right]}{1422T\left[\ln\frac{r_e}{r_w} - 0.75 + S + DQ_g\right]}$$

pressure squared approach

$$Q_g = \frac{kh\left[\bar{P}_r^2 - P_{wf}^2\right]}{1422\,T\,\bar{\mu}\,\bar{Z}\left[\ln\frac{r_e}{r_w} - 0.75 + S + DQ_g\right]}$$

pressure approximation

$$Q_g = \frac{kh\left(\bar{P}_r - P_{wf}\right)}{142200\,\bar{\mu}\,\bar{B}_g\left[\ln\frac{r_e}{r_w} - 0.75 + S + DQ_g\right]}$$

Equations are implicit in Q_g, iterative method.

Steady state flow:

$$Q_g = \frac{kh\left(\bar{P}_r^2 - P_{wf}^2\right)}{1422\,T\,\bar{\mu}\,\bar{Z}\left[\ln\frac{r_e}{r_w} - 0.5 + S + DQ_g\right]}$$

$$Q_g = \frac{kh\left[P_e^2 - P_{wf}^2\right]}{1422\,T\,\bar{\mu}\,\bar{Z}\left[\ln\frac{r_e}{r_w} + S + DQ_g\right]}$$

→ any sum of individual solutions to the diffusivity equation is also a solution to that equation. (in transient)

principle of super position:

sum of individual solutions is a solution of the problem.

F_1: solution number one
F_2: 〃 〃 2
F_3: 〃 〃 3

$$F = F_1 + F_2 + F_3 + \cdots$$
↳ is a solution

we can use principle of superposition if the governing partial differential equation is linear.

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t}$$

فرض می کنیم F_1 و F_2 دو جواب معادله روبرو باشند.

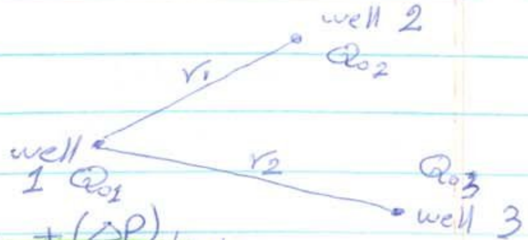
$$\left.\begin{aligned} \frac{\partial^2 F_1}{\partial r^2} + \frac{1}{r}\frac{\partial F_1}{\partial r} &= \frac{1}{\eta}\frac{\partial F_1}{\partial t} \\ \frac{\partial^2 F_2}{\partial r^2} + \frac{1}{r}\frac{\partial F_2}{\partial r} &= \frac{1}{\eta}\frac{\partial F_2}{\partial t} \end{aligned}\right] \frac{\partial^2}{\partial r^2}[F_1+F_2] + \frac{1}{r}\frac{\partial}{\partial r}[F_1+F_2] = \frac{1}{\eta}\frac{\partial}{\partial t}[F_1+F_2]$$

معادله‌ای non linear می‌باشد که در آن متغیر وابسته در یک ترم بیش از یک بار تکرار شود، مثل:

$$\frac{\partial^2 P}{\partial^2 r} + \frac{P}{r}\frac{\partial P}{\partial r} = \frac{1}{\eta}\frac{\partial P}{\partial t} \quad \text{or} \quad \frac{\partial^2 P}{\partial^2 r} + \left(\frac{\partial P}{\partial r}\right)^2 = \frac{1}{\eta}\frac{\partial P}{\partial r}$$

1- Effect of multiple wells → constant flow rate
2- Effect of variable flow rate
3- Effect of boundaries
4- Effect of pressure change

Effect of multiple wells:



$P_i - P_{\text{wellbore at well 1}}$
↑

$$(\Delta P)_{\text{totall pressure at well 1}} = (\Delta P)_{\text{due to well 1}} + (\Delta P)_{\text{due to well 2}} + (\Delta P)_{\text{due to well 3}}$$

$$P(r,t) = P_i + \frac{70.6\, Q_o B_o \mu}{K h} E_i\left(\frac{-948\, \phi \mu c_t r^2}{K t}\right)$$

$$p(r,t) = P_i + \frac{70.6\, Q_{o2} B_o \mu}{Kh} E_i\left(\frac{-948\, \phi \mu C_t r_1^2}{Kt}\right)$$ فشار چاه 1 در اثر برداشت از well 2 Q_{o2}

$$\Rightarrow \Delta P_{\substack{\text{due to}\\ \text{well 2}}} = \frac{-70.6\, Q_{o2} B_o \mu}{Kh} E_i\left(\frac{-948\, \phi \mu C_t r_1^2}{Kt}\right)$$

$$\Delta P_{\substack{\text{due to}\\ \text{well 3}}} = \frac{-70.6\, Q_{o3} B_o \mu}{Kh} E_i\left(\frac{-948\, \phi \mu C_t r_2^2}{Kt}\right)$$

$$\Delta P_{\text{due to well 2}} = ?$$

when $x < 0.02 \Rightarrow E_i(-x) = \ln x + 0.577 = \ln x + \ln 1.781 = \ln (1.781 x)$

$$P(r,t) = P_i + \frac{70.6\, Q_o B_o \mu}{Kh} E_i\left[\frac{-948\, \phi \mu C_t r^2}{Kt}\right]$$

$$= P_i + \frac{70.6\, Q_o B_o \mu}{Kh} \ln\left(1.781\left(\frac{+948\, \phi \mu C_t r^2}{Kt}\right)\right)$$

$$= P_i + \frac{70.6\, Q_o B_o \mu}{Kh} \ln\left(\frac{Kt}{1688.09\, \phi \mu C_t r^2}\right)^{-1}$$

$$\Rightarrow P(r,t) = P_i - \frac{70.6\, Q_o B_o \mu}{Kh}\left[\log \frac{Kt}{\phi \mu C_t r^2} - 3.23\right] 2.3$$

$$① \quad P(r_w, t) = P_{wf} = P_i - \frac{162.6\, Q_o B_o \mu}{Kh}\left[\log \frac{Kt}{\phi \mu C_t r_w^2} - 3.23\right]$$

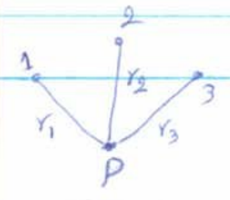
$$\Delta P_{skin} = \frac{Q_o B_o \mu}{0.00708\, Kh} S = \frac{141.2\, Q_o B_o \mu}{Kh} S = 162.6\,(0.87) \frac{Q_o B_o \mu}{Kh} S$$

$$P_i - P_{wf} = \frac{162.6\, Q_o B_o \mu}{Kh}\left[\log \frac{Kt}{\phi \mu C_t r_w^2} - 3.23\right]$$

$$\Delta P_{skin} = 162.6 \frac{Q_o B_o \mu}{Kh}(0.87 S)$$

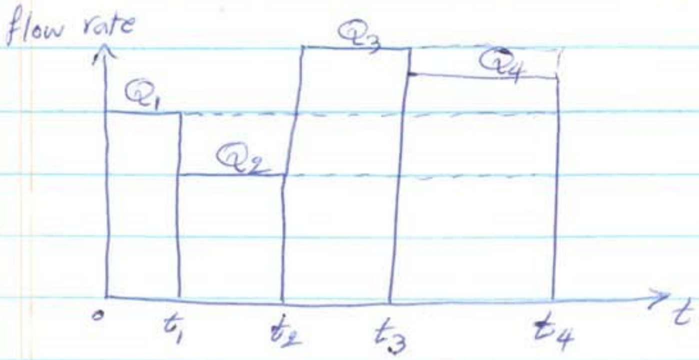
$$\left(P_i - P(r_w, t)\right)_{\text{at well 1}} = \frac{162.6\, Q_o B_o \mu}{Kh}\left[\log \frac{Kt}{\phi \mu C_t r_w^2} - 3.23 + 0.87 S\right]$$

$$\underbrace{(\Delta P)_{\substack{\text{totall pressure}\\ \text{at well 1}}}} = \frac{162.6\, Q_o B_o \mu}{Kh}\left[\log \frac{Kt}{\phi \mu C_t r_w^2} - 3.23 + 0.87 S\right]$$
$$- \frac{70.6\, Q_{o2} B_o \mu}{Kh} E_i\left[\frac{-948\, \phi \mu C_t r_1^2}{Kt}\right]$$
$$- \frac{70.6\, Q_{o3} B_o \mu}{Kh} E_i\left[\frac{-948\, \phi \mu C_t r_2^2}{Kt}\right]$$



t: برحسب ساعت می باشد

Effect of variable flow rate



one well

$$(\Delta P)_{totall} = \Delta P_{due\ to\ Q_1 - 0} + \Delta P_{due\ to\ Q_2 - Q_1} + \Delta P_{due\ to\ Q_3 - Q_2} + \Delta P_{due\ to\ Q_4 - Q_3}$$

we are looking for pressure drop at sand face

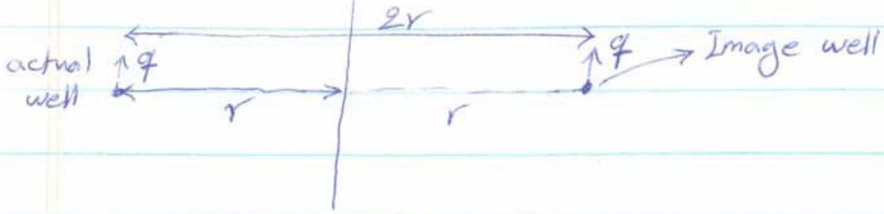
$$\Delta P_{totall} = \frac{162.6 (Q_1 - 0) B \mu}{Kh} \left[Log \frac{Kt4}{\phi \mu c_t r_w^2} - 3.23 + 0.87S \right]$$

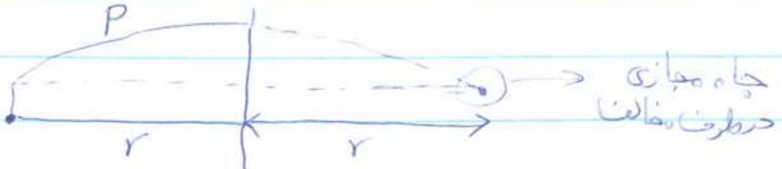
$$+ \frac{162.6 (Q_2 - Q_1) B \mu}{Kh} \left[Log \frac{K(t_4 - t_1)}{\phi \mu c_t r_w^2} - 3.23 + 0.87S \right]$$

$$+ \frac{162.6 (Q_3 - Q_2) B \mu}{Kh} \left[Log \frac{K(t_4 - t_2)}{\phi \mu c_t r_w^2} - 3.23 + 0.87S \right]$$

$$+ \frac{162.6 (Q_4 - Q_3) B \mu}{Kh} \left[Log \frac{K(t_4 - t_3)}{\phi \mu c_t r_w^2} - 3.23 + 0.87S \right]$$

Effect of reservoir Boundary.





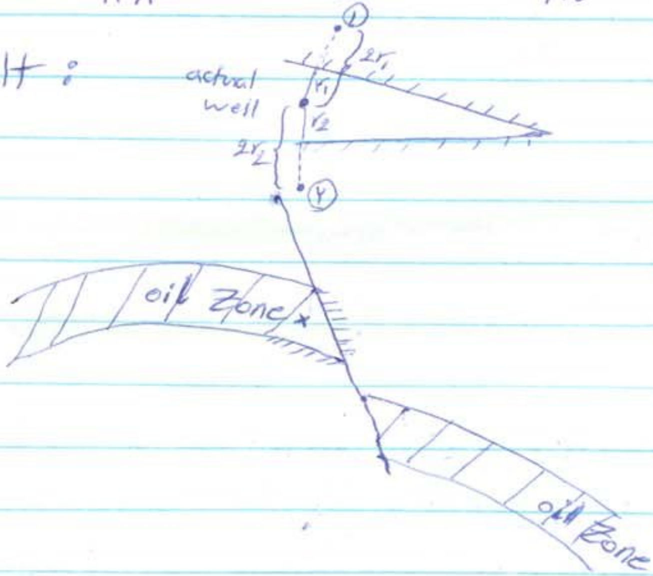
at boundary, there is no flow $q\Big|_{at\ boundry} = 0 \Longrightarrow \frac{KA}{\mu} \frac{\partial P}{\partial r}\Big|_{at\ boundry} = 0 \Longrightarrow \frac{\partial P}{\partial r}\Big|_{at\ boundry} = 0$

$$\Delta P_{totall} = \Delta P_{actual\ well} + \Delta P_{due\ to\ image\ well}$$

$$\Delta P_{totall} = \frac{162.6\, Q\, B\, \mu}{kh}\left[\log \frac{kt}{\phi \mu c_t r_w^2} - 3.23 + 0.87S\right]$$

$$-\frac{70.6\, Q B \mu}{kh} E_i\left[\frac{-948 \phi \mu c_t (2r)^2}{kt}\right]$$

Fault :



$$\Delta P_{totall} = \Delta P_{actual\ well} + \Delta P_{due\ to\ ①} + \Delta P_{due\ to\ Ⓨ}$$

$$\Delta P_{total} = \frac{162.6\, Q_o B \mu}{kh}\left[\log \frac{kt}{\phi \mu c_t r_w^2} - 3.23 + 0.87S\right]$$

$$-\frac{70.6\, Q_o B \mu}{kh} E_i\left(\frac{-948 \phi \mu c_t (2r_1)^2}{kt}\right)$$

$$-\frac{70.6\, Q_o B \mu}{kh} E_i\left[\frac{-948 \phi \mu c_t (2r_2)^2}{kt}\right]$$

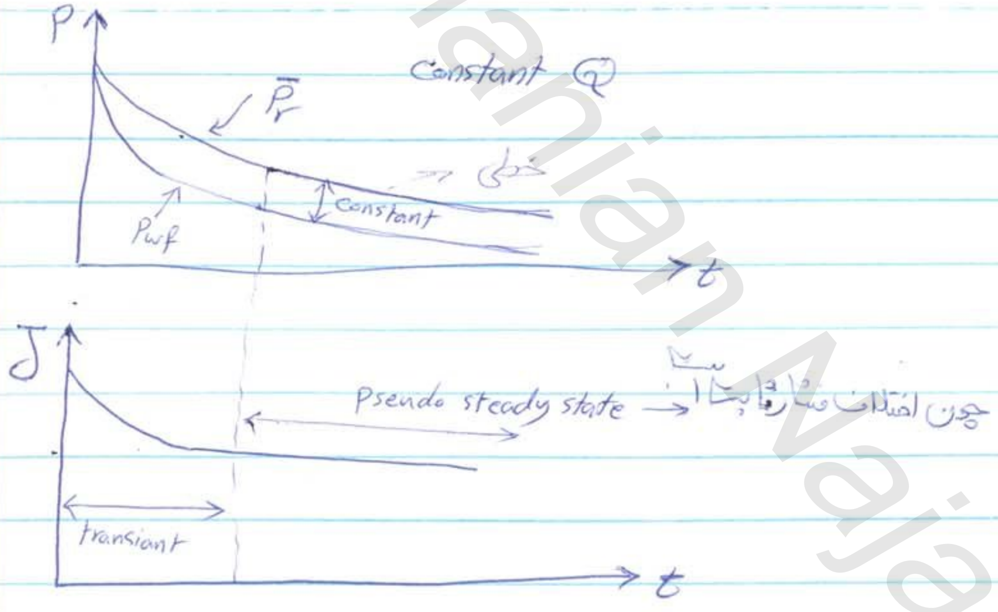
Chapter 7: oil well performance:

vertical oil well performance.

productivity Index : PI, J

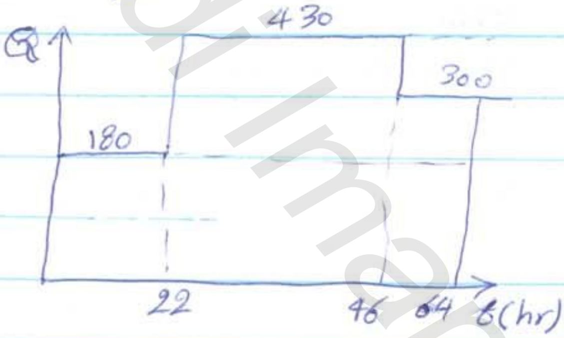
$$J: \frac{\text{total flow rate}}{\underbrace{\text{Pressure drawdown}}_{\text{اختلاف فشار}}} \Longrightarrow \underbrace{J}_{\frac{STB}{day.psi}} = \frac{Q_o}{\bar{P}_r - P_{wf}} = \frac{Q_o}{\Delta P}$$

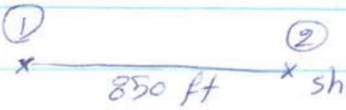
A Test is run on the well, the well is shut in $(Q=0)$ well the bottom hole pressure stabilizes, the recorded pressure is $\bar{P}_r$



J is valid for Psendo steady state.

EX: A well is opened to flow at 180 $\frac{STB}{day}$ for 22 hours. The flow rate is then increased to 430 $\frac{STB}{day}$ and lasted for 24 hours. The well flow rate is then reduced to 300 $\frac{STB}{day}$ for 18 hours. Calculate the pressure drop in a shut in well 850 ft away from the well. Given prosity = 18%, viscosity = 2cp, $B_o = 1.25 \frac{bbl}{STB}$, $r_w = 0.3$ ft, $h = 35$ ft, $K = 120$ md, $P_i = 3200$ psi, $C_t = 10^{-5}$ psi^{-1}.





sol: $\Delta P = \frac{-70.6 Q_o \mu B_o}{Kh} E_i\left[\frac{-948 \phi \mu C_t r^2}{Kt}\right]$

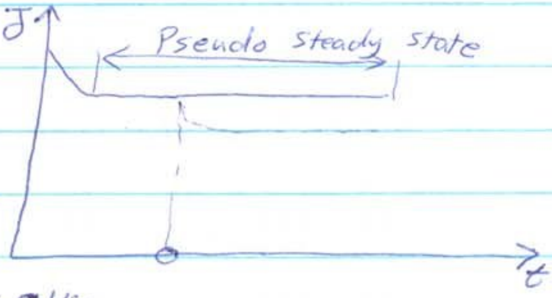
$$\Delta P_{totall} = \frac{-70.6(180)(2)(1.25)}{120(35)} E_i\left[\frac{-948(0.18)(2)(10^{-5})(850)^2}{120(64)}\right]$$

$$- \frac{70.6\ (430-180)\ (2)(1.25)}{120\ (35)} E_i\left[\frac{-948(0.18)(2)(10^{-5})\ (850)^2}{120\ (64-22)}\right]$$

$$- \frac{70.6\ (300-430)(2)(1.25)}{120(35)} E_i\left[\frac{-948(0.18)(2)(10^{-5})(850)^2}{120(64-46)}\right]$$

By monitoring J, we can distinguish the reason of drop in J.

$$PI = J = \frac{Q_o}{\bar{P}_r - P_{wf}}$$



well has became damaged due to:
- Skin
- sand production
- workover
- completion.
- injection operation
- mechanical problems.

Pseudo steady state.

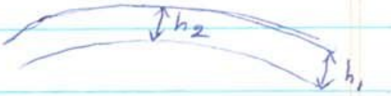
$$Q_o = \frac{0.00708\, k_o h (\bar{P}_r - P_{wf})}{\mu_o B_o \left[\ln \frac{r_e}{r_w} - 0.75 + S\right]}$$

$$K_{ro} = \frac{K_o}{K} \Longrightarrow K_o = K K_{ro}$$

$$PI = J = \frac{Q_o}{\bar{P}_r - P_{wf}}$$

$\frac{STB}{day \cdot psi}$

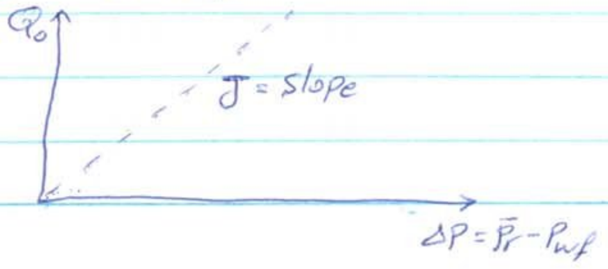
$$\longrightarrow J = \frac{0.00708\, k h}{\left[\ln \frac{r_e}{r_w} - 0.75 + S\right]} \left(\frac{k_{ro}}{\mu_o B_o}\right)$$



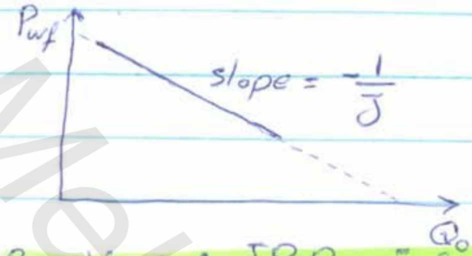
For comparison of different wells in a reservoir specific productivity Index: $J_s = \frac{J}{h}$

$$J_s = \frac{0.00708\, K}{\left[\ln \frac{r_e}{r_w} - 0.75 + S\right]} \left(\frac{k_{ro}}{\mu_o B_o}\right) = \frac{J}{h}$$

$$J = \frac{Q_o}{\bar{P}_r - P_{wf}} \Longrightarrow Q_o = J(\bar{P}_r - P_{wf})$$



$$P_{wf} = \bar{P}_r - \frac{1}{J} Q_o$$

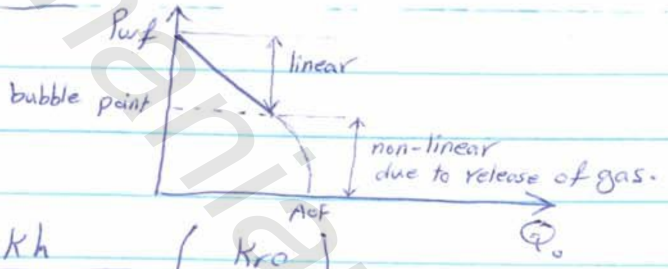


P_{wf} Vs Q_o : IPR : Inflow Performance Relasionship:

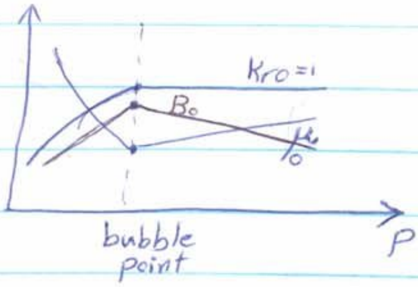
Maximum flow rate at $P_{wf}=0$. Q_o at $P_{wf}=0$ is called AOF (Absolute open Flow)

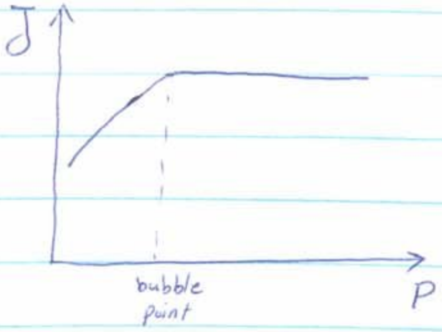
$$AOF = Q_o = J \bar{P}_r$$

$$Q_o = J \Delta P$$



$$J = \frac{0.00708\, Kh}{\left[Ln \frac{r_e}{r_w} - 0.75 + S \right]} \left(\frac{K_{ro}}{\mu_o B_o} \right)$$





→ in

ex) A well as a diameter of 12" & a drainage radious of 660 ft. The sand which is penetrated is 23 ft thick & contains a crude oil with viscosity of 1.6 cp & FVF is 1.623, $k_o = 53$ md.

$r_w = \frac{12}{2}" = \boxed{0.5\ ft} = 6"$

a) what is $J = ?$

$$J = \frac{0.00708\, k_o h}{\mu_o B_o \left[\ln \frac{r_e}{r_w} - 0.75 + S\right]} = \frac{0.00708(53)(23)}{1.6(1.623)\left[\ln \frac{660}{0.5} - 0.75 + 0\right]}$$

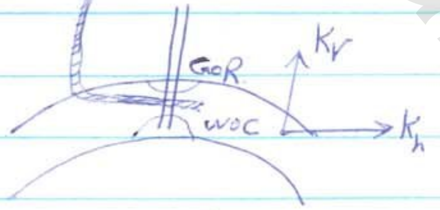
$$J = 0.516 \frac{STB}{day \cdot psi}$$

b) what is specific productivity Index.

$$J_s = \frac{J}{h} = \frac{0.516}{23} = 0.0224 \frac{STB}{day \cdot psi \cdot ft}$$

c) what is production rate under 100 psi drawdown.

$$Q_o = J \Delta P = 0.516(100) = 51.6 \frac{STB}{day}$$

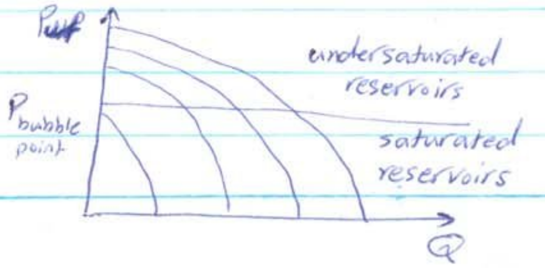


Drawdown ↑ ⟶ Q ↑ $\xrightarrow{problems}$
- 1. Two phase: GOR ↑
- 2. Water & gas Coning

In horizontal wells: drawdown is low.

a) $K_v \gg K_h$: Horizontal

b) $K_v \ll K_h$: vertical



Empirical Methods to predict non-linearity behavior of IPR for solution gas drive reservoirs.

1- Vojel's Method. → مورد توجه قرار دهیم

2- Wiggin's Method.

3- Standing's 〃 .

4- Fetkovich 〃 .

5- The Klins-clark .

To draw the IPR, we need one stabilized $(P_{wf} \text{ vs } Q_o)$ & $\bar{P}_r$ & P_b

dimensionless pressure $= \dfrac{P_{wf}}{\bar{P}_r}$

〃 flow rate $= \dfrac{Q_o}{\underset{AOF}{Q_{o\,max}}} \rightarrow$ at $P_{wf}=0$

$(Q_o)_{max} = AOF$

$$\boxed{\frac{Q_o}{Q_{o\,max}} = 1 - 0.2\left(\frac{P_{wf}}{\bar{P}_r}\right) - 0.8\left(\frac{P_{wf}}{\bar{P}_r}\right)^2}$$

Pressures in $\boxed{\text{Psig}}$ ✓ مهم

Can be used for water production $\dfrac{Q_o}{(Q_o)_{max}} = \dfrac{Q_L}{(Q_L)_{max}}$, $Q_L = Q_o + Q_w$

water cut ⩽ 97%

$\bar{P}_r > P_b$, Q_o at P_{wf}

Vogel's method for method for saturated oil reservoirs.

$\bar{P}_r \leqslant P_b$

1. using the stabilized flow data (P_{wf} vs Q_o), calculate $Q_{o\,max}$:

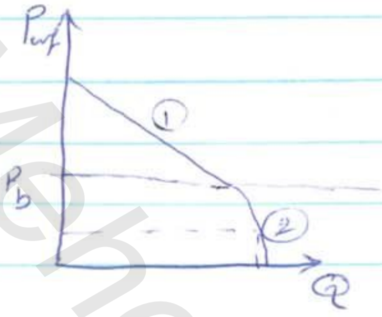
$$(Q_o)_{max} = \frac{Q_o}{\left[1-0.2\left(\frac{P_{wf}}{\bar{P}_r}\right)-0.8\left(\frac{P_{wf}}{\bar{P}_r}\right)^2\right]}$$

2- construct the IPR curve by assumming various values of P_{wf} & calculating Q_o.

$$Q_o = (Q_o)_{max}\left[1-0.2\left(\frac{P_{wf}}{\bar{P}_r}\right)-0.8\left(\frac{P_{wf}}{\bar{P}_r}\right)^2\right]$$

Vojel's method for undersaturated oil reservoirs.

$\bar{P}_r > P_b$



two cases $\begin{cases} 1- P_{wf} \geq P_b \\ 2- P_{wf} < P_b \end{cases}$

Data require : $\begin{cases} \bar{P}_r \\ P_b \\ \text{or } Q \,\&\, P_{wf} \end{cases}$

Case 1 : $P_{wf} \geq P_b$

Linear parts: 2 points

1- using the stabilized test data (Q_o at P_{wf})

$$J = \frac{Q_o}{\bar{P}_r - P_{wf}} \rightarrow Q_o = J(\bar{P}_r - P_{wf}) \rightarrow P_{wf} > P_b$$

2- calculate oil flow at the bubble point.

$$Q_{ob} = J(\bar{P}_r - P_b) \quad ①$$

non-linear parts: 5 points ← 3- Generate IRR values for below P_b by assuming different values of $P_{wf} < P_b$

$$Q_o = Q_{ob} + \frac{J P_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_b}\right) - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right] \quad ②$$

$P_{wf} = 0 \rightarrow$ & $AOF = Q_{omax} = Q_{ob} + \frac{J P_b}{1.8}$

Case 2 : $P_{wf} < P_b$

$$①,② \Rightarrow Q_o = J(\bar{P}_r - P_b) + \frac{J P_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_b}\right) - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right]$$

$$1- \quad J = \frac{Q_o}{(\bar{P}_r - P_b) + \frac{P_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_b}\right) - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right]}$$

linear part : 2 points

2- $Q_{ob} = J(\bar{P}_r - P_b)$

3- for $P_{wf} > P_b \Rightarrow Q_o = J(\bar{P}_r - P_{wf})$

non-linear parts: 5 points ← 4- for $P_{wf} < P_b \Rightarrow Q_o = Q_{ob} + \frac{J P_b}{1.8}\left[1 - 0.2\left(\frac{P_{wf}}{P_b}\right) - 0.8\left(\frac{P_{wf}}{P_b}\right)^2\right]$

Immiscible Displacement Processes (water flooding)

$$\text{Recovery} = \underset{\text{original oil in place (OOIP)}}{V_{oi}} - \underset{\text{oil volume at any time}}{V_o}$$

pore volume = PV

$$\text{Recovery} = PV\, S_{oi} - PV\, \bar{S}_o = PV(S_{oi} - \bar{S}_o)$$

No gas in the reservoir.

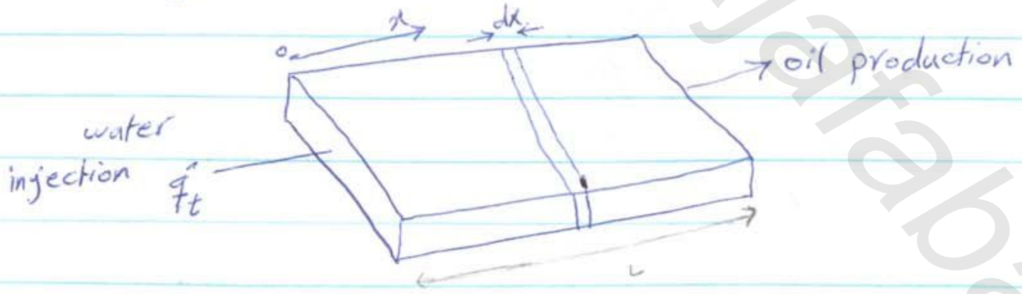
$$\text{Recovery} = PV(1 - S_{wi} - (1 - \bar{S}_w)) = PV(\bar{S}_w - S_{wi})$$

$$RF = \text{Recovery factor} = \frac{\text{Recovery}}{OOIP}$$

$$RF = \frac{PV(S_{oi} - \bar{S}_o)}{PV\, S_{oi}} \Rightarrow RF = \frac{S_{oi} - \bar{S}_o}{S_{oi}}$$

$$RF = \frac{PV(\bar{S}_w - S_{wi})}{PV(S_{oi})} \Rightarrow RF = \frac{\bar{S}_w - S_{wi}}{1 - S_{wi}}$$

The Bulkley - Leverett Displacement mechanism



Total through put $q_t = q_w + q_o = cte \left(\frac{bbl}{day}\right)$

at x: water saturation S_w

at $x+dx$: " " $S_w \pm dS_w \simeq S_w - dS_w$ → oil res

چون به oil res نزدیکتر شویم S_w کمتر می‌شود.

at t: water saturation : S_w

at t+dt: " " : $S_w + dS_w$

ϕ: prosity , A_c: cross-section (ft^2) , dx (ft) , t (days)

Amount of water in the element reservoir : w

bbl ↖

$$w = \frac{A_c \cdot dx \cdot \phi \cdot S_w}{5.615}$$

$$\frac{dw}{dt} = \frac{A_c \cdot dx \cdot \phi}{5.615} \left(\frac{\partial S_w}{\partial t}\right)_x \quad ①$$

f_w: fraction of water in the total flow of q'_t

$f_w = f_w(x, t)$

$$\frac{dw}{dt} = (f_w - df_w)\, q'_t - f_w\, q'_t = d f_w\, q'_w \quad ②$$

$$①, ② \Longrightarrow \frac{A_c \cdot dx \cdot \phi}{5.615} \left(\frac{\partial S_w}{\partial t}\right)_x = d f_w \cdot q'_w \quad ③$$

$$\left(\frac{\partial S_w}{\partial t}\right)_x = \frac{-5.615\, q'_t}{A_c\, \phi} \left(\frac{\partial f_w}{\partial x}\right)_t$$

$$f_w = \frac{q_w}{q'_t} = \frac{q_w}{q_o + q_w}$$

$$q_o = \frac{0.00708\, Kh\, (P_e - P_{wf})}{\mu \ \ln \frac{r_e}{r_w}} \qquad \text{radial}$$

$$q_o = \frac{0.001127\, K_o\, A_c\, (P_e - P_{wf})}{\mu_o\, L} \qquad \text{linear}$$

$$q_w = \frac{0.001127\, K_w\, A_c\, (P_e - P_{wf})}{\mu_w\, L}$$

$$q_w = \frac{0.00708\, K_w h\, (P_e - P_{wf})}{\mu_w \ln \frac{r_e}{r_w}}$$ radial

$$\Rightarrow f_w = \frac{\frac{K_w}{\mu_w}}{\frac{K_w}{\mu_w} + \frac{K_o}{\mu_o}} \Longrightarrow f_w = \frac{1}{1 + \frac{K_o}{\mu_o} \cdot \frac{\mu_w}{K_w}} = \frac{1}{1 + \frac{K_{ro}}{K_{rw}} \cdot \frac{\mu_w}{\mu_o}}$$

Mobility $= \frac{K}{\mu}$, $M_w = \frac{K_w}{\mu_w}$, $M_o = \frac{K_o}{\mu_o}$

$f_w = f_w(S_w)$, $S_w = S_w(x, t)$

$$dS_w = \left(\frac{\partial S_w}{\partial x}\right)_t dx + \left(\frac{\partial S_w}{\partial t}\right)_x dt$$

Rate of advance of a $\overset{dS_w = 0}{\text{constant}}$ saturation plane or front.

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{-\left(\frac{\partial S_w}{\partial t}\right)_t}{\left(\frac{\partial S_w}{\partial x}\right)_t} \xRightarrow{③} \left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{\frac{+5.615\, q_t}{\phi \cdot A_c} \left(\frac{\partial f_w}{\partial x}\right)_t}{\left(\frac{\partial S_w}{\partial x}\right)_t}$$

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{5.615\, q_t'}{\phi\, A_c} \left(\frac{\partial f_w}{\partial S_w}\right)_t$$

$$\Longrightarrow x = \frac{5.615\, q_t' \cdot t}{\phi \cdot A_c} \left(\frac{\partial f_w}{\partial S_w}\right)_{t, S_w}$$

$$\Longrightarrow \frac{x}{L} = \frac{5.615\, q_t' \cdot t}{\phi\, A_c \cdot L} \left(\frac{\partial f_w}{\partial S_w}\right)_{t, S_w}$$

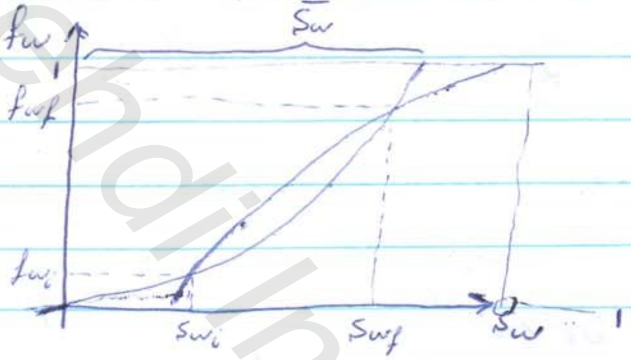
$\phi A_c L$ = pore volume (ft^3)

$5.615\, q_t' \cdot t$ = total injected volume (ft^3)

$\frac{5.615\, q_t \cdot t}{\phi \cdot A_c \cdot L}$ = number of pore volume injected $= PV_{inj}$

$$\frac{x}{L} = PV_{inj} \left(\frac{\partial f_w}{\partial S_w} \right)_{S_w, t} \qquad ④$$

Welg's solution: $\frac{\partial f_w}{\partial S_w} = \frac{f_{wf} - f_{wi}}{\underbrace{S_{wf}}_{front} - S_{wi}}$



از نقطه S_{wi} و f_{wi} یک مماس بر منحنی رسم می‌کنیم.

time required the water to reach the production well:
Break through at break through: $x = L$

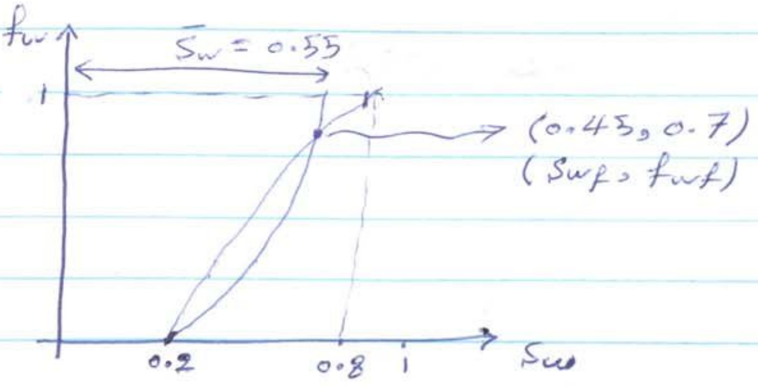
$$④ \Rightarrow \left(\frac{\partial f_w}{\partial S_w} \right)_{\substack{at \\ break\ through}} = \frac{1}{PV_{inj}} \quad \text{or}$$

$$PV_{inj} = \frac{1}{\left(\frac{\partial f_w}{\partial S_w} \right)_{at\ break\ through}}$$

Ex: For a water flooding process that has reached breakthrough, the following data are recorevered: calculate water saturation at break through & recovery factor. $\mu_{oil} = 5\ cp$, $\mu_w = 0.5\ cp$, $B_o = 1.3$, $B_w = 1$

$$f_w = \frac{1}{1 + \frac{K_{ro}}{K_{rw}} + \frac{\mu_w}{\mu_o}}$$

S_w	K_{rw}	K_{ro}	f_w
0.2	0	0.8	0
0.25	0.002	0.61	0.032
0.3	0.009	0.47	0.161
0.35	0.02	0.37	0.351
0.4	0.033	0.285	0.537
0.45	0.051	0.22	0.699
0.5	0.075	0.163	0.821
0.55	0.1	0.12	0.893
0.6	0.132	0.081	0.942
0.65	0.17	0.05	0.971
0.7	0.208	0.027	0.988
0.75	0.251	0.01	0.996
0.8	0.3	1	1



saturation at break through = 0.45

$$\text{Recovery Factor} = \frac{\bar{S}_w - S_{wi}}{1 - S_{wi}} = \frac{0.55 - 0.2}{1 - 0.2} \times 100$$

$$= 43.75\%$$