Solutions Manual Volume 1 & 2
Chapters 1-6 to Accompany + CHAPTERS 7-11

ENGINEERING MECHANICS:
STATICS SECOND EDITION

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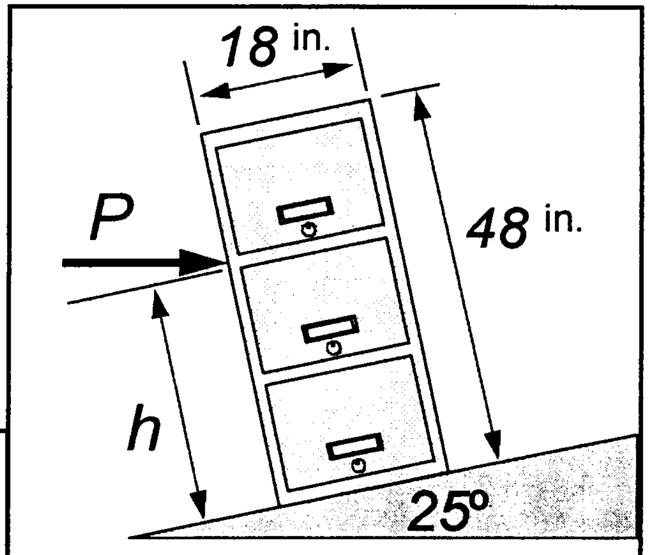

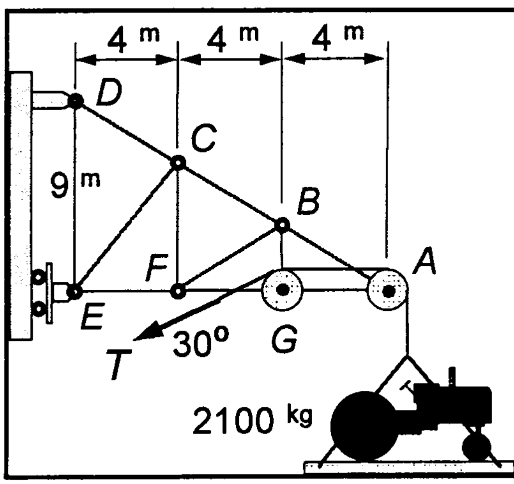

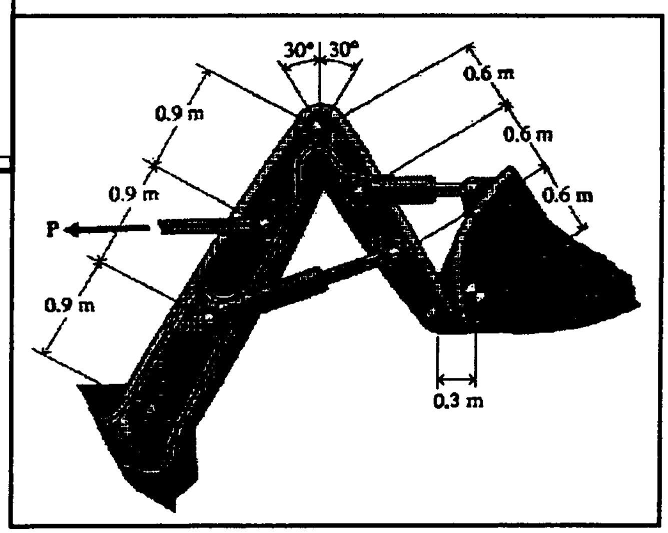

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INTRODUCTION

This manual was written with the same care and precision with which the book was written. Included is a complete solution for every problem in the book. Each solution appears with the original problem statement and, where appropriate, the problem figure. This is done for the convenience of the instructor, who no longer will have to refer to both the book and the solutions manual in preparing for class.

As a guide to the problem material, problems with answers that are included in the back of the book are marked with an asterisk [*], and problems intended to be solved with the aid of a programmable calculator or a computer are marked with a [C].

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Solutions

1-1* Calculate the mass m of a body that weighs 600 lb at the surface of the earth.

SOLUTION

$$m = \frac{W}{g} = \frac{600}{32.17} = 18.65 \text{ slug} \qquad \text{Ans.}$$

1-2* Calculate the weight W of a body at the surface of the earth if it has a mass m of 675 kg.

SOLUTION

$$W = 675(9.807) = 6.62(10^3) \text{ N} = 6.62 \text{ kN} \qquad \text{Ans.}$$

1-3 Calculate the gravitational constant g, in U.S. Customary units, for a location on the surface of the moon.

SOLUTION

$$g_m = \frac{Gm_m}{r_m^2} = \frac{33.439(10^{-8})(5.037)(10^{21})}{[5.702(10^6)]^2} = 5.328 \text{ ft/s}^2 \cong 5.33 \text{ ft/s}^2 \qquad \text{Ans.}$$

1-4 Calculate the gravitational constant g, in SI units, for a location on the surface of the sun.

SOLUTION

$$g_s = \frac{Gm_s}{r_s^2} = \frac{6.673(10^{-11})(1.990)(10^{30})}{[6.960(10^8)]^2} = 0.2741(10^3) \text{ m/s}^2 \cong 274 \text{ m/s}^2 \qquad \text{Ans.}$$

1-5* The planet Venus has a diameter of 7700 mi and a mass of $3.34(10^{23})$ slug. Determine the gravitational acceleration at the surface of the planet.

SOLUTION

$$g_V = \frac{Gm_V}{r_V^2} = \frac{3.439(10^{-8})(3.34)(10^{23})}{[7700(5280)/2]^2} = 27.8 \text{ ft/s}^2 \qquad \text{Ans.}$$

1-6* The gravitational acceleration at the surface of Mars is 3.73 m/s^2 and the mass of Mars is $6.39(10^{23})$ kg. Determine the radius of Mars.

SOLUTION

From Eq. (1-3):
$$g_M = \frac{Gm_M}{r_M^2}$$

$$r_M = \sqrt{\frac{Gm_M}{g_M}} = \sqrt{\frac{6.673(10^{-11})(6.39)(10^{23})}{3.73}} = 3.381(10^6)\ \text{m} \cong 3.38(10^6)\ \text{m} \quad \text{Ans.}$$

1-7 Determine the gravitational force, in kips, exerted by the sun on the earth.

SOLUTION

$$F = \frac{Gm_e m_s}{r^2} = \frac{3.439(10^{-8})(4.095)(10^{23})(1.364)(10^{29})}{[4.908(10^{11})]^2}$$
$$= 0.7974(10^{22})\ \text{lb} = 7.97(10^{18})\ \text{kip} \quad \text{Ans.}$$

1-8 Determine the gravitational force, in kilonewtons, exerted by the earth on the moon.

SOLUTION

$$F = \frac{Gm_e m_m}{r^2} = \frac{6.673(10^{-11})(5.976)(10^{24})(7.350)(10^{22})}{[3.844(10^8)]^2}$$
$$= 1.984(10^{20})\ \text{N} = 1.984(10^{17})\ \text{kN} \quad \text{Ans.}$$

1-9* The equatorial radius of the earth is $2.0925(10^7)$ ft and the polar radius is $2.0856(10^7)$ ft. Determine the gravitational acceleration g at the two locations.

SOLUTION

$$g_e = \frac{Gm_e}{r_e^2} = \frac{3.439(10^{-8})(4.095)(10^{23})}{[2.0925(10^7)]^2} = 32.16\ \text{ft/s}^2 \quad \text{Ans.}$$

$$g_p = \frac{Gm_e}{r_p^2} = \frac{3.439(10^{-8})(4.095)(10^{23})}{[2.0856(10^7)]^2} = 32.38\ \text{ft/s}^2 \quad \text{Ans.}$$

1-10* Two spherical bodies have masses of 60 kg and 80 kg, respectively. Determine the gravitational force of attraction between the spheres if the distance from center to center is 600 mm.

SOLUTION

$$F = G\frac{m_1m_2}{r^2} = \frac{6.673(10^{-11})(60)(80)}{(0.600)^2} = 8.90(10^{-7})\text{ N} \qquad \text{Ans.}$$

1-11 Two solid spherical bodies have 12 in. and 10 in. diameters and are made of a material that weighs 0.284 lb/in.^{3} Determine the gravitational force of attraction between the two spheres when they are touching each other.

SOLUTION

$$m = \rho V = \frac{\gamma}{g}V = \frac{0.284}{32.17}\left(\frac{4}{3}\pi R^3\right) = 0.03698\ R^3$$

$$m_1 = 0.03698(6)^3 = 7.988\text{ slug}$$

$$m_2 = 0.03698(5)^3 = 4.623\text{ slug}$$

$$F = G\frac{m_1m_2}{r^2} = \frac{3.439(10^{-18})(7.988)(4.623)}{(11/12)^2} = 1.511(10^{-6})\text{ lb} \qquad \text{Ans.}$$

1-12 A satellite is placed in orbit 1.6(10^6) m above the surface of the moon. If the mass of the satellite is 3.0(10^4) kg, determine the gravitational force exerted on the satellite by the moon.

SOLUTION

$$r = h + r_m = 1.6(10^6) + 1.738(10^6) = 3.338(10^6)\text{ m}$$

$$F = \frac{Gm_sm_m}{r^2} = \frac{6.673(10^{-11})(3.0)(10^4)(7.350)(10^{22})}{[3.338(10^6)]^2}$$

$$= 13.21(10^3)\text{ N} \cong 13.21\text{ kN} \qquad \text{Ans.}$$

1-13* Determine the weight W of a satellite when it is in orbit 8500 miles above the surface of the earth if the satellite weighs 7600 lb at the surface.

SOLUTION

From Eq. (1-3): $W = \dfrac{Gm_e m}{r^2}$ $\qquad W_0 r_0^2 = W_h r_h^2 = Gm_e m$

From Table 1-1: $r_0 = 2.090(10^7)$ ft

$$r_h = r_0 + h = 2.090(10^7) + 8500(5280) = 6.578(10^7) \text{ ft}$$

$$W_h = \frac{W_0 r_0^2}{r_h^2} = \frac{7600[2.090(10^7)]^2}{[6.578(10^7)]^2} = 767 \text{ lb} \qquad \text{Ans.}$$

1-14* Determine the weight W of a satellite when it is in orbit $20.2(10^6)$ m above the surface of the earth if the satellite weighs 8450 N at the surface.

SOLUTION

From Eq. (1-3): $W = \dfrac{Gm_e m}{r^2}$ $\qquad W_0 r_0^2 = W_h r_h^2 = Gm_e m$

From Table 1-1: $r_0 = 6.371(10^6)$ m

$$r_h = r_0 + h = 6.371(10^6) + 20.2(10^6) = 26.571(10^6) \text{ m}$$

$$W_h = \frac{W_0 r_0^2}{r_h^2} = \frac{8450[6.371(10^6)]^2}{[26.571(10^6)]^2} = 485.8 \text{ N} \cong 486 \text{ N} \qquad \text{Ans.}$$

1-15 If a woman weighs 135 lb when standing on the surface of the earth, how much would she weigh when standing on the surface of the moon?

SOLUTION

$$g_m = \frac{Gm_m}{r_m^2} = \frac{3.439(10^{-8})(5.037)(10^{21})}{[5.702(10^6)]^2} = 5.328 \text{ ft/s}^2$$

$$W_m = mg_m = \frac{W}{g_e} g_m = \frac{135}{32.17}(5.328) = 22.36 \text{ lb} \cong 22.4 \text{ lb} \qquad \text{Ans.}$$

1-16 Determine the weight W of a body that has a mass of 1000 kg
(a) At the surface of the earth.
(b) At the top of Mt. McKinley (6193 m above sea level).
(c) In a satellite at an altitude of 250 km.

SOLUTION

From Eq. (1-3): $W = \dfrac{Gm_e m}{r^2}$ $W_0 r_0^2 = W_h r_h^2 = Gm_e m$

From Table 1-1: $r_0 = 6.371(10^6)$ m

(a) $W_0 = mg = 1000(9.807) = 9807 \text{ N} \cong 9.81 \text{ kN}$ Ans.

(b) $r_h = r_0 + r_h = 6.371(10^6) + 6193 = 6.377(10^6)$ m

$$W_h = \frac{W_0 r_0^2}{r_h^2} = \frac{9807[(6.371)(10^6)]^2}{[6.377(10^6)]^2} = 9789 \text{ N} \cong 9.79 \text{ kN}$$ Ans.

(c) $r_h = r_0 + h = 6.371(10^6) + 250(10^3) = 6.621(10^6)$ m

$$W_h = \frac{W_0 r_0^2}{r_h^2} = \frac{9807[6.371(10^6)]^2}{[6.621(10^6)]^2} = 9080 \text{ N} \cong 9.08 \text{ kN}$$ Ans.

1-17* If a man weighs 210 lb at sea level, determine the weight W of the man
(a) At the top of Mt. Everest (29,028 ft above sea level).
(b) In a satellite at an altitude of 200 mi.

SOLUTION

From Eq. (1-3): $W = \dfrac{Gm_e m}{r^2}$ $W_0 r_0^2 = W_h r_h^2 = Gm_e m$

From Table 1-1: $r_0 = 2.090(10^7)$ ft

(a) $r_h = r_0 + h = 2.090(10^7) + 29028 = 2.0929028(10^7)$ ft

$$W_h = \frac{W_0 r_0^2}{r_h^2} = \frac{210[2.090(10^7)]^2}{[2.0929028(10^7)]^2} = 209.4 \text{ lb} \cong 209 \text{ lb}$$ Ans.

(b) $r_h = r_0 + h = 2.090(10^7) + 200(5280) = 2.1956(10^7)$ ft

$$W_h = \frac{W_0 r_0^2}{r_h^2} = \frac{210[2.090(10^7)]^2}{[2.1956(10^7)]^2} = 190.29 \text{ lb} \cong 190.3 \text{ lb}$$ Ans.

1-18 A space traveler weighs 800 N on earth. A planet having a mass of $5(10^{25})$ kg and a diameter of $30(10^6)$ m orbits a distant star. Determine the weight W of the traveler on the surface of this planet.

SOLUTION

$$m = \frac{W}{g} = \frac{800}{9.807} = 81.57 \text{ slug}$$

$$W = \frac{Gmm_p}{r_p^2} = \frac{6.673(10^{-11})(81.57)(5)(10^{25})}{[15(10^6)]^2} = 1209.6 \text{ N} \cong 1210 \text{ N} \qquad \text{Ans.}$$

1-19* The planet Jupiter has a mass of $1.302(10^{26})$ slug and a visible diameter (top of the cloud layers) of 88,700 mi. Determine the gravitational acceleration g

(a) At a point 100,000 miles above the top of the clouds.

(b) At the top of the cloud layers.

SOLUTION

$$r_j = \frac{1}{2} d_j = \frac{1}{2}(88{,}700)(5280) = 2.342(10^8) \text{ ft}$$

$$g = G\frac{m_j}{r_j^2} = 3.439(10^{-8})\frac{1.302(10^{26})}{[2.342(10^8)]^2} = 81.63 \text{ ft/s}^2 \cong 81.6 \text{ ft/s}^2 \qquad \text{Ans.}$$

1-20* The planet Saturn has a mass of $5.67(10^{26})$ kg and a visible diameter (top of clouds) of $12.00(10^7)$ m. The weight W of a planetary probe on earth is 4.50 kN. Determine

(a) The weight of the probe when it is $6(10^8)$ m above the top of the clouds.

(b) The weight of the probe as it begins its penetration of the cloud layers.

SOLUTION

$$m_p = \frac{W_p}{g} = \frac{4.5(10^3)}{9.807} = 458.9 \text{ kg}$$

(a) $r = r_s + h = 6.00(10^7) + 6(10^8) = 66.0(10^7)$ m

$$W = \frac{Gm_pm_s}{r^2} = \frac{6.673(10^{-11})(458.9)(5.67)(10^{26})}{[66.0(10^9)]^2} = 7.03 \text{ N} \qquad \text{Ans.}$$

(b) $$W = \frac{Gm_pm_s}{r_s^2} = \frac{6.673(10^{-11})(458.9)(5.67)(10^{26})}{[6.00(10^9)]^2} = 482 \text{ N} \qquad \text{Ans.}$$

1-21 The first U.S. satellite, Explorer I, had a mass of approximately 1 slug. Determine the force exerted on the satellite by the earth at the low and high points of its orbit which were 175 mi and 2200 mi, respectively, above the surface of the earth.

SOLUTION

$$r_L = r_e + h_L = 2.090(10^7) + 175(5280) = 2.182(10^7) \text{ ft}$$

$$r_H = r_e + h_H = 2.090(10^7) + 2200(5280) = 3.251(10^7) \text{ ft}$$

$$F_s = m_s g = 1(32.17) = 32.17 \text{ lb}$$

$$F_s = \frac{Gm_e m_s}{r^2}$$

$$F_0 r_0^2 = F_H r_H^2 = F_L r_L^2 = Gm_e m$$

$$F_L = \frac{F_0 r_0^2}{r_L^2} = \frac{32.17[(2.090)(10^7)]^2}{[2.182(10^7)]^2} = 25.51 \text{ lb} \cong 25.5 \text{ lb} \qquad \text{Ans.}$$

$$F_H = \frac{F_0 r_0^2}{r_H^2} = \frac{32.17[(2.090)(10^7)]^2}{[3.251(10^7)]^2} = 13.296 \text{ lb} \cong 13.30 \text{ lb} \qquad \text{Ans.}$$

1-22 A neutron star has a mass of $2(10^{30})$ kg and a diameter of $10(10^3)$ m. Determine the gravitational force of attraction on a 10-kg space probe

(a) When it is 10^6 m from the center of the star.

(b) At the instant of impact with the surface of the star.

SOLUTION

$$F = \frac{Gm_p m_s}{r^2}$$

$$\text{(a) } F = \frac{6.673(10^{-11})(10)(2)(10^{30})}{(10^6)^2} = 133.46(10^7) \text{ N} \cong 1335(10^6) \text{ N} \qquad \text{Ans.}$$

$$\text{(b) } F = \frac{6.673(10^{-11})(10)(2)(10^{30})}{[5(10^3)]^2} = 5.338(10^{13}) \text{ N} \cong 53.4(10^{12}) \text{ N} \qquad \text{Ans.}$$

1-23* At what distance from the surface of the earth, in miles, is the weight of a body equal to one-half of its weight on the earth's surface?

SOLUTION

$$W = \frac{Gm_e m_b}{r^2} \qquad W_0 r_0^2 = W_h r_h^2 = Gm_e m_b$$

From Table 1-1: $r_0 = 2.090(10^7)$ ft

$$\frac{W_h}{W_0} = \frac{W_0/2}{W_0} = \frac{r_0^2}{r_h^2} \qquad r_h^2 = (r_0 + h)^2 = 2r_0^2$$

$$h = (\sqrt{2} - 1)r_0 = (\sqrt{2} - 1)(2.090)(10^7) = 8.657(10^6) \text{ ft}$$

$$= 1639.6 \text{ mi} \cong 1640 \text{ mi}$$ Ans.

1-24 At what distance, in kilometers, from the surface of the earth on a line from center to center would the gravitational force of the earth on a body be exactly balanced by the gravitational force of the moon on the body?

SOLUTION

$$F = \frac{Gm_e m_b}{a^2} = \frac{Gm_m m_b}{b^2} \qquad a = \left[\frac{m_e b^2}{m_m}\right]^{1/2} = \left[\frac{5.976(10^{24})}{7.350(10^{22})}\, b^2\right]^{1/2} = 9.017b$$

From Table 1-1: $a + b = 9.017b + b = 3.844(10^8)$ m

Therefore: $b = 0.3837(10^8)$ m $\qquad a = 3.4603(10^8)$ m

$h = a - r_e = 3.4603(10^8) - 6.371(10^6) = 339.7(10^6) \text{ m} \cong 340(10^3) \text{ km}$ Ans.

1-25* Determine the weight W, in U.S. Customary units, of an 85-kg steel bar under standard conditions (sea level at a latitude of 45 degrees)

SOLUTION

$$W = mg = (85 \text{ kg})(9.807 \text{ m/s}^2) = 833.6 \frac{\text{kg}\cdot\text{m}}{\text{s}^2} = 833.6 \text{ N}$$

$$= 833.6 \text{ N}\left(\frac{1 \text{ lb}}{4.448 \text{ N}}\right) = 187.4 \text{ lb}$$ Ans.

1-26* Determine the mass m, in SI units, for a 600-lb steel beam under standard conditions (sea level at a latitude of 45 degrees).

SOLUTION

$$m = \frac{W}{g} = \frac{600 \text{ lb}}{32.17 \text{ ft/s}^2} = 18.651 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}} = 18.651 \text{ slug.}$$

$$= 18.651 \text{ slug} \left(\frac{14.59 \text{ kg}}{1 \text{ slug}}\right) = 272 \text{ kg} \qquad \text{Ans.}$$

1-27 The velocity of light in space is approximately 186,000 mi/s. The velocity of light in units of kilometers per hour is?

SOLUTION

$$v = \left(186{,}000 \frac{\text{mi}}{\text{s}}\right)\left(\frac{3600 \text{ s}}{\text{h}}\right)\left(\frac{1.609 \text{ km}}{\text{mi}}\right) = 1.077(10^9) \text{ km/h} \qquad \text{Ans.}$$

1-28 Using the fact that 1 m = 39.37 in., convert 5 m^3 of concrete to units of cubic yards of concrete.

SOLUTION

$$V = \left(5 \text{ m}^3\right)\left(\frac{39.37 \text{ in.}}{1 \text{ m}}\right)^3\left(\frac{1 \text{ yd}}{36 \text{ in.}}\right)^3 = 6.54 \text{ yd}^3 \qquad \text{Ans.}$$

1-29* Using the fact that 1 in. = 25.40 mm, convert a speed of 75 mi/h to units of meters per second.

SOLUTION

$$v = \left(75 \frac{\text{mi}}{\text{hr}}\right)\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)\left(\frac{25.40 \text{ mm}}{1 \text{ in.}}\right)\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 33.5 \text{ m/s} \qquad \text{Ans.}$$

1-30* The fuel consumption of an automobile is 14 km/L. The fuel consumption, in miles per gallon, is?

SOLUTION

$$FC = \left(14 \frac{\text{km}}{\text{L}}\right)\left(\frac{3.785 \text{ L}}{1 \text{ gal}}\right)\left(\frac{1 \text{ mi}}{1.609 \text{ km}}\right) = 32.93 \text{ mi/gal} \cong 32.9 \text{ mi/gal} \qquad \text{Ans.}$$

1-31 An automobile has a 350-in.^{3} engine displacement. The engine displacement, in liters, is?

SOLUTION

$$V = \left(350 \text{ in.}^3\right)\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right)^3\left(\frac{1 \text{ L}}{0.001 \text{ m}^3}\right) = 5.736 \text{ L} \cong 5.74 \text{ L} \qquad \text{Ans.}$$

1-32 How many barrels of oil are contained in 100 kL of oil? One barrel (petroleum) equals 42.0 gal.

SOLUTION

$$V = \left(100(10^3) \text{ L}\right)\left(\frac{0.2642 \text{ gal}}{\text{L}}\right)\left(\frac{1 \text{ barrel}}{42.0 \text{ gal}}\right) = 629 \text{ barrels} \qquad \text{Ans.}$$

1-33* Express a speed of 20 nm/h (1 nautical mile = 6076 ft) in units of kilometers per minute.

SOLUTION

$$v = \left(20 \frac{\text{nm}}{\text{h}}\right)\left(\frac{6076 \text{ ft}}{\text{nm}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right) = 0.617 \text{ km/min} \qquad \text{Ans.}$$

1-34* One acre equals 43,560 ft.^{2} One hectare equals 10^4 m.^{2} Determine the number of acres in 500 hectares.

SOLUTION

$$A = \left(500 \text{ hectare}\right)\left(\frac{10^4 \text{ m}^2}{\text{hectare}}\right)\left(\frac{10.76 \text{ ft}^2}{\text{m}^2}\right)\left(\frac{1 \text{ acre}}{43,560 \text{ ft}^2}\right) = 1235 \text{ acres} \qquad \text{Ans.}$$

1-35 Verify the conversion factors listed in Table 1-6 for converting the following quantities from U.S. Customary units to SI units by using the values listed for length as defined values:
(a) Velocity
(b) Acceleration

SOLUTION

(a) Velocity:

$$\left(1\ \frac{\text{in.}}{\text{s}}\right)\left(\frac{1\ \text{m}}{39.37\ \text{in.}}\right) = 0.0254\ \text{m/s} \qquad \text{Ans.}$$

$$\left(1\ \frac{\text{ft}}{\text{s}}\right)\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right) = 0.3048\ \text{m/s} \qquad \text{Ans.}$$

$$\left(1\ \frac{\text{mi}}{\text{h}}\right)\left(\frac{1\ \text{km}}{0.6214\ \text{mi}}\right) = 1.609\ \text{km/h} \qquad \text{Ans.}$$

(b) Acceleration:

$$\left(1\ \frac{\text{in.}}{\text{s}^2}\right)\left(\frac{1\ \text{m}}{39.37\ \text{in.}}\right) = 0.0254\ \text{m/s}^2 \qquad \text{Ans.}$$

$$\left(1\ \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right) = 0.3048\ \text{m/s}^2 \qquad \text{Ans.}$$

1-36 Verify the conversion factors listed in Table 1-6 for converting the following quantities from SI units to U.S. Customary units by using the values listed for length as defined values:
(a) Area
(b) Volume

Use 1 gal = 231 in.^3 and 1 L = 0.001 m^3.

SOLUTION

(a) Area:

$$\left(1\ \text{m}^2\right)\left(\frac{39.37\ \text{in.}}{1\ \text{m}}\right)^2 = 1550\ \text{in.}^2 \qquad \text{Ans.}$$

$$\left(1\ \text{m}^2\right)\left(\frac{3.281\ \text{ft}}{1\ \text{m}}\right)^2 = 10.76\ \text{ft}^2 \qquad \text{Ans.}$$

(b) Volume:

$$\left(1\ \text{mm}^3\right)\left(\frac{1\ \text{in.}}{25.40\ \text{mm}}\right)^3 = 61.02(10^{-6})\ \text{in.}^3 \qquad \text{Ans.}$$

$$\left(1\ \text{m}^3\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right)^3 = 35.31\ \text{ft}^3 \qquad \text{Ans.}$$

$$\left(1\ \text{L}\right)\left(\frac{0.001\ \text{m}^3}{1\ \text{L}}\right)\left(\frac{39.37\ \text{in.}}{1\ \text{m}}\right)^3\left(\frac{1\ \text{gal}}{231\ \text{in.}^3}\right) = 0.2642\ \text{gal} \qquad \text{Ans.}$$

1-37* Verify the conversion factors listed in Table 1-6 for converting the following quantities from U.S. Customary units to SI units by using the values listed for length and force as defined values:

(a) Mass

(b) Distributed load

SOLUTION

(a) Mass:

$$\left(1\ \text{slug}\right)\left(\frac{1\ \text{lb}\cdot\text{s}^2/\text{ft}}{1\ \text{slug}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right)\left(\frac{4.448\ \text{N}}{1\ \text{lb}}\right)\left(\frac{1\ \text{kg}}{\text{N}\cdot\text{s}^2/\text{m}}\right) = 14.59\ \text{kg} \qquad \text{Ans.}$$

(b) Distributed load:

$$\left(1\ \frac{\text{lb}}{\text{ft}}\right)\left(\frac{4.448\ \text{N}}{1\ \text{lb}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right) = 14.59\ \text{N/m} \qquad \text{Ans.}$$

1-38* Verify the conversion factors listed in Table 1-6 for converting the following quantities from SI units to U.S. Customary units by using the values listed for length and mass as defined values:

(a) Pressure or stress

(b) Bending moment or torque

SOLUTION

(a) Pressure or stress:

$$\left(1\ \text{Pa}\right)\left(\frac{1\ \text{N/m}^2}{1\ \text{Pa}}\right)\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)\left(\frac{1\ \text{slug}}{14.59\ \text{kg}}\right)\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)\left(\frac{1\ \text{lb}\cdot\text{s}^2/\text{ft}}{\text{slug}}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.1450(10^{-3})\ \text{lb/in.}^2$$

$1\ \text{kPa} = 10^3\ \text{Pa} = 0.1450\ \text{psi}$ Ans.

$1\ \text{MPa} = 10^6\ \text{Pa} = 145.0\ \text{psi}$ Ans.

(b) Bending moment or torque:

$$\left(1\ \text{N}\cdot\text{m}\right)\left(\frac{1\ \text{lb}}{4.448\ \text{N}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right) = 0.7376\ \text{ft}\cdot\text{lb}$$

1-39 One acre equals 43,560 ft^2. One gallon equals 231 in^3. Determine the number of liters of water in 2500 acre·ft of water.

SOLUTION

$$\left(2500 \text{ acre}\cdot\text{ft}\right)\left(\frac{43{,}560 \text{ ft}^2}{\text{acre}}\right)\left[\frac{12 \text{ in.}}{1 \text{ ft}}\right]^3\left(\frac{1 \text{ gal}}{231 \text{ in.}^3}\right)\left(\frac{3.785 \text{ L}}{1 \text{ gal}}\right)$$

$$= 3.083(10^9) \text{ L} \cong 3.08(10^9) \text{ L} \qquad \text{Ans.}$$

1-40 The viscosity of crude oil under conditions of standard temperature and pressure is $7.13(10^{-3})$ $N\cdot s/m^2$. The viscosity of crude oil in U.S. Customary units ($lb\cdot s/ft^2$) is?

SOLUTION

$$\nu = 7.13(10^{-3}) \frac{\text{N}\cdot\text{s}}{\text{m}^2}\left(\frac{1 \text{ lb}}{4.448 \text{ N}}\right)\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right)^2 = 1.489(10^{-4}) \frac{\text{lb}\cdot\text{s}}{\text{ft}^2} \qquad \text{Ans.}$$

1-41* The air pressure in an automobile tire is 35 psi. Express the pressure in appropriate SI units (kPa) by using the values listed in Table 1-6 for length and force as defined values.

SOLUTION

$$p = \frac{35 \text{ lb}}{\text{in.}^2}\left[\frac{4.448 \text{ N}}{1 \text{ lb}}\right]\left[\frac{39.37 \text{ in.}}{1 \text{ m}}\right]^2 = \left[\frac{1 \text{ Pa}}{1 \text{ N/m}^2}\right]\left[\frac{1 \text{ kPa}}{1000 \text{ Pa}}\right] = 241 \text{ kPa} \qquad \text{Ans.}$$

1-42 The stress in a steel bar is 150 MPa. Express the stress in appropriate U.S. Customary units (ksi) by using the values listed in Table 1-6 for length and force as defined values.

SOLUTION

$$\sigma = 150 \text{ MPa} = 150(10^6) \text{ Pa}\left(\frac{1 \text{ N/m}^2}{1 \text{ Pa}}\right)\left(\frac{0.2248 \text{ lb}}{1 \text{ N}}\right)\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right)^2$$

$$= 21.75(10^3) \text{ lb/in.}^2 \cong 21.8 \text{ ksi} \qquad \text{Ans.}$$

1-43* Express the density, in SI units (kg/m^3), of a specimen of material that has a specific weight of 0.284 $lb/in.^3$

SOLUTION

$$\rho = \frac{\gamma}{g} = \frac{0.284\ lb/in.^3}{32.17(12)\ in./s^2} = 735.7(10^{-6})\ \frac{lb \cdot s^2}{in.^4}$$

$$\rho = 735.7(10^{-6})\ \frac{lb \cdot s^2}{in.^4}\left(\frac{4.448\ N}{1\ lb}\right)\left(\frac{39.37\ in.}{1\ m}\right)^4\left(\frac{1\ kg \cdot m/s^2}{1\ N}\right) = 7860\ kg/m^3 \qquad \text{Ans.}$$

1-44* Express the specific weight, in U.S. Customary units ($lb/in.^3$), of a specimen of material that has a density of 4500 kg/m^3.

SOLUTION

$$\gamma = \rho g = 4500\ \frac{kg}{m^3}\left(9.807\ \frac{m}{s^2}\right)\left(\frac{1\ N}{1\ kg \cdot m/s^2}\right) = 45.13(10^3)\ N/m^3$$

$$\gamma = 45.13(10^3)\ \frac{N}{m^3}\left(\frac{1\ lb}{4.448\ N}\right)\left(\frac{1\ m}{39.37\ in.}\right)^3 = 01626\ lb/in.^3 \qquad \text{Ans.}$$

1-45 By definition, 1 hp = 33,000 ft·lb/min and 1 W = 1 N·m/s. Verify the conversion factors listed in Table 1-6 for converting power from U.S. Customary units to SI units by using the values listed for length and force as defined values.

SOLUTION

$$1\ \frac{ft \cdot lb}{s} = 1\ \frac{ft \cdot lb}{s}\left[\frac{0.3048\ m}{ft}\right]\left[\frac{4.448\ N}{1\ lb}\right]\left[\frac{1\ W}{N \cdot m/s}\right] = 1.3558\ W \cong 1.356\ W \qquad \text{Ans.}$$

$$1\ hp = 33{,}000\ \frac{ft \cdot lb}{min}\left[\frac{0.3048\ m}{1\ ft}\right]\left[\frac{4.448\ N}{1\ lb}\right]\left[\frac{1\ min}{60\ s}\right]\left[\frac{1\ W}{N \cdot m/s}\right] = 745.7\ W \qquad \text{Ans.}$$

1-46 The specific heat of air under standard atmospheric pressure, in SI units, is 1003 N·m/kg·K. The specific heat of air under standard atmospheric pressure, in U.S. Customary units (ft·lb/slug·°R), is?

SOLUTION

$$C_p = 1003\ \frac{N \cdot m}{kg \cdot K}\left(\frac{1\ lb}{4.448\ N}\right)\left(\frac{3.281\ ft}{1\ m}\right)\left(\frac{14.59\ kg}{1\ slug}\right)\left(\frac{1\ K}{1.8^\circ R}\right) = 6000\ \frac{ft \cdot lb}{slug \cdot {}^\circ R} \qquad \text{Ans.}$$

1-47* Newton's law of gravitation can be expressed in equation form as

$$F = G \frac{m_1 m_2}{r^2}$$

If F is a force, m_1 and m_2 are masses, and r is a distance, determine the dimensions of G.

SOLUTION

$$G = \frac{Fr^2}{m_1 m_2} = \frac{(ML/T^2)(L)^2}{(M)(M)} = \frac{L^3}{MT^2} \qquad \text{Ans.}$$

1-48* The elongation of a bar of uniform cross section subjected to an axial force is given by the equation

$$\delta = \frac{PL}{AE}$$

What are the dimensions of E if δ and L are lengths, P is a force, and A is an area?

SOLUTION

$$E = \frac{PL}{A\delta} = \frac{(ML/T^2)(L)}{(L^2)(L)} = \frac{M}{LT^2} \qquad \text{Ans.}$$

1-49 An important parameter in certain types of fluid flow problems when a free surface is present is the Froude number (Fr) which can be expressed in equation form as

$$Fr = \left[\frac{\rho v^2}{Lw}\right]^{1/2}$$

where ρ is the density of the fluid, v is a velocity, L is a length, and w is the specific weight of the fluid. Show that the Froude number is dimensionless.

SOLUTION

$$Fr = \left[\frac{\rho v^2}{Lw}\right]^{1/2} = \left[\frac{(M/L^3)(L/T)^2}{(L)(M/L^2T^2)}\right]^{1/2} = 1 \text{ (Dimensionless)} \qquad \text{Ans.}$$

1-50 An important parameter in fluid flow problems involving thin films is the Weber number (We) which can be expressed in equation form as

$$We = \frac{\rho v^2 L}{\sigma}$$

where ρ is the density of the fluid, v is a velocity, L is a length, and σ is the surface tension of the fluid. If the Weber number is dimensionless, what are the dimensions of the surface tension σ?

SOLUTION

$$\sigma = \frac{\rho v^2 L}{We} = \frac{(M/L^3)(L/T)^2(L)}{(1)} = \frac{M}{T^2} \qquad \text{Ans.}$$

1-51* The period of oscillation of a simple pendulum is given by the equation

$$T = k(L/g)^{1/2}$$

where T is in seconds, L is in feet, g is the acceleration due to gravity, and k is a constant. What are the dimensions of k for dimensional homogeneity?

SOLUTION

$$k = \frac{T}{\sqrt{L/g}} = \frac{T}{\sqrt{L/(L/T^2)}} = 1 \text{ (Dimensionless)} \qquad \text{Ans.}$$

1-52* In the equation

$$y = y_0 + vt + \frac{1}{2}at^2$$

y and y_0 are distances, v is a velocity, a is an acceleration, and t is time. Is the equation dimensionally homogeneous?

SOLUTION

$$y = L \qquad vt = (L/T)(T) = L$$

$$y_0 = L \qquad \frac{1}{2}at^2 = (1)(L/T^2)(T)^2 = L$$

All terms have the dimension L; therefore, the equation is dimensionally homogeneous. Ans.

1-53 The modulus k of a coil spring (force required to stretch the spring a unit distance) can be expressed in equation form as

$$k = \frac{Gr^4}{4R^3 n}$$

in which r and R are lengths and n is a dimensionless number. Determine the dimensions of G (a property of the spring material).

SOLUTION

$$G = \frac{4kR^3 n}{r^4} = \frac{(1)(ML/T^2L)(L)^3(1)}{(L)^4} = \frac{M}{LT^2} \qquad \text{Ans.}$$

1-54 In the dimensiomally homogeneous equation

$$U = Fd - \frac{Wv^2}{2g}$$

F is a force, W is a force, d is a length, and v is a linear velocity. Determine the dimensions of U and g.

SOLUTION

$$Fd = (ML/T^2)(L) = ML^2/T^2$$

All terms have the dimension ML^2/T^2: $U = ML^2/T^2$ Ans.

$$\frac{Wv^2}{2g} = ML^2/T^2 \qquad g = \frac{(ML/T^2)(L/T)^2}{(1)(ML^2/T^2)} = L/T^2 \qquad \text{Ans.}$$

1-55* In the dimensiomally homogeneous equation

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

σ is a stress, A is an area, M is a moment of a force, and c is a length. Determine the dimensions of P and I.

SOLUTION

All terms have the dimension M/LT^2:

$$P = (M/LT^2)(L^2) = ML/T^2 \qquad \text{Ans.}$$

$$I = \frac{(ML^2/T^2)(L)}{(M/LT^2)} = L^4 \qquad \text{Ans.}$$

1-56* In the dimensionally homogeneous equation

$$Pd = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

d is a length, m is a mass, v is a linear velocity, and ω is an angular velocity. Determine the dimensions of P and I.

SOLUTION

$$\frac{1}{2}mv^2 = (1)(M)(L/T)^2 = ML^2/T^2$$

All terms have the dimension ML^2/T^2:

$$P = \frac{ML^2/T^2}{(L)} = ML/T^2 \qquad I = \frac{ML^2/T^2}{(1)(1/T)^2} = ML^2$$ Ans.

1-57 In the dimensiomally homogeneous equation

$$\tau = \frac{Tr}{J} + \frac{VQ}{Ib}$$

τ is a stress, T is a torque, V is a force, r and b are lengths, and I is a second moment of an area. Determine the dimensions of J and Q.

SOLUTION

All terms have the dimension M/LT^2:

$$J = \frac{(ML^2/T^2)(L)}{(M/LT^2)} = L^4 \qquad Q = \frac{(M/LT^2)(L^4)(L)}{(ML/T^2)} = L^3$$ Ans.

1-58 In the dimensiomally homogeneous equation

$$\tau = \frac{P}{A} + \frac{Tr}{J}$$

τ is a stress, A is an area, T is a torque, and r is a length. Determine the dimensions of P and J.

SOLUTION

All terms have the dimension M/LT^2:

$$P = (M/LT^2)(L^2) = ML/T^2 \qquad J = \frac{(ML^2/T^2)(L)}{(M/LT^2)} = L^4$$ Ans.

1-59* The equation $x = Ae^{-t/b} \sin(at + \alpha)$ is dimensionally homogeneous. If A is a length and t is time, determine the dimensions of x, a, b, and α.

SOLUTION

$\frac{t}{b} = \frac{T}{b} = 1$ therefore $b = T$ Ans.

$at = a(T) = 1$ therefore $a = \frac{1}{T}$ Ans.

$\alpha = 1$ (Dimensionless) Ans.

$x = A(1) = L(1) = L$ Ans.

1-60* In the dimensionally homogeneous equation $w = x^3 + ax^2 + bx + a^2b/x$, if x is a length, what are the dimensions of a, b, and w?

SOLUTION

If $x = L$, each term has the dimension L^3: $w = L^3$ Ans.

$ax^2 = a(L^2) = L^3$ $a = L$ Ans.

$bx = b(L) = L^3$ $b = L^2$ Ans.

1-61 Determine the dimensions of a, b, c, and y in the dimensionally homogeneous equation

$$y = Ae^{-bt} \cos\left[\sqrt{1 - a^2}\, bt + c\right]$$

in which A is a length and t is time.

SOLUTION

$bt = b(T) = 1$ $b = \frac{1}{T}$ Ans.

$\sqrt{1 - a^2}\, bt = \sqrt{1 - a^2}\,(1) = 1$ $a = 1$ (dimensionless) Ans.

$c = 1$ (dimensionless) $y = A(1)(1) = L(1)(1) = L$ Ans.

1-62 Determine the dimensions of c, ω, k, and P in the differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P\cos\omega t$$

in which m is mass, x is length, and t is time.

SOLUTION

$$m\frac{d^2x}{dt^2} = M(L/T^2) = ML/T^2$$

All terms have the dimension ML/T^2:

$$c\frac{dx}{dt} = c(L/T) = ML/T^2 \qquad c = M/T \qquad \text{Ans.}$$

$$kx = k(L) = ML/T^2 \qquad k = M/T^2 \qquad \text{Ans.}$$

$$\omega t = \omega(T) = 1 \text{ (dimensionless)} \qquad \omega = 1/T \qquad \text{Ans.}$$

$$P\cos\omega t = P(1) = ML/T^2 \qquad P = ML/T^2 \qquad \text{Ans.}$$

1-63* Round off the following numbers to two significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 0.0153   (b) 0.0347   (c) 0.0566

SOLUTION

$$\%\text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 0.015: $\frac{0.015 - 0.0153}{0.0153}(100) = -1.961\ \%$ Ans.

(b) For 0.035: $\frac{0.035 - 0.0347}{0.0347}(100) = +0.865\ \%$ Ans.

(c) For 0.057: $\frac{0.057 - 0.0566}{0.0566}(100) = +0.707\ \%$ Ans.

1-64* Round off the following numbers to two significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 0.8374 (b) 0.4729 (c) 0.6644

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 0.84: $\frac{0.84 - 0.8374}{0.8374}(100) = +0.310\ \%$ Ans.

(b) For 0.47: $\frac{0.47 - 0.4729}{0.4729}(100) = -0.613\ \%$ Ans.

(c) For 0.66: $\frac{0.66 - 0.6644}{0.6644}(100) = -0.662\ \%$ Ans.

1-65 Round off the following numbers to two significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 1.8394 (b) 3.4629 (c) 6.7523

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 1.8: $\frac{1.8 - 1.8394}{1.8394}(100) = -2.14\ \%$ Ans.

(b) For 3.5: $\frac{3.5 - 3.4629}{3.4629}(100) = +1.071\ \%$ Ans.

(c) For 6.8: $\frac{6.8 - 6.7523}{6.7523}(100) = +0.706\ \%$ Ans.

1-66 Round off the following numbers to two significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 3.6544 (b) 7.5638 (c) 8.9223

SOLUTION

$$\%\ \text{Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 3.7: $\frac{3.7 - 3.6544}{3.6544}(100) = +1.248\ \%$ Ans.

(b) For 7.6: $\frac{7.6 - 7.5638}{7.5638}(100) = +0.479\ \%$ Ans.

(c) For 8.9: $\frac{8.9 - 8.9223}{8.9223}(100) = -0.250\ \%$ Ans.

1-67* Round off the following numbers to three significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 26.394 (b) 74.829 (c) 55.336

SOLUTION

$$\%\ \text{Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 26.4: $\frac{26.4 - 26.394}{26.394}(100) = +0.0227\ \%$ Ans.

(b) For 74.8: $\frac{74.8 - 74.829}{74.829}(100) = -0.0388\ \%$ Ans.

(c) For 55.3: $\frac{55.3 - 55.336}{55.336}(100) = -0.0651\ \%$ Ans.

1-68* Round off the following numbers to three significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 374.93   (b) 826.48   (c) 349.33

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 375: $\frac{375 - 374.93}{374.93}(100) = +0.01867\ \%$ Ans.

(b) For 826: $\frac{826 - 826.48}{826.48}(100) = -0.0581\ \%$ Ans.

(c) For 349: $\frac{349 - 349.33}{349.33}(100) = -0.0945\ \%$ Ans.

1-69 Round off the following numbers to three significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 6471.9   (b) 3628.7   (c) 7738.2

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 6470: $\frac{6470 - 6471.9}{6471.9}(100) = -0.0294\ \%$ Ans.

(b) For 3630: $\frac{3630 - 3628.7}{3628.7}(100) = +0.0358\ \%$ Ans.

(c) For 7740: $\frac{7740 - 7738.2}{7738.2}(100) = +0.0233\ \%$ Ans.

1-70 Round off the following numbers to three significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 8521.4   (b) 6748.3   (c) 9378.7

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 8520: $\frac{8520 - 8521.4}{8521.4}(100) = -0.01643\ \%$ Ans.

(b) For 6750: $\frac{6750 - 6748.3}{6748.3}(100) = +0.0252\ \%$ Ans.

(c) For 9380: $\frac{9380 - 9378.7}{9378.7}(100) = +0.01386\ \%$ Ans.

1-71* Round off the following numbers to four significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 63,746.2   (b) 27,382.6   (c) 55,129.9

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 63,750: $\frac{63{,}750 - 63{,}746.2}{63{,}746.2}(100) = +0.00596\ \%$ Ans.

(b) For 27,380: $\frac{27{,}380 - 27{,}382.6}{27{,}382.6}(100) = -0.00950\%$ Ans.

(c) For 55,130: $\frac{55{,}130 - 55{,}129.9}{55{,}129.9}(100) = +0.0001814\ \%$ Ans.

1-72* Round off the following numbers to four significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 937,284   (b) 274,918   (c) 339,872

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 937,300: $\frac{937,300 - 937,284}{937,284}(100) = +0.001707\ \%$ Ans.

(b) For 274,900: $\frac{274,900 - 274,918}{274,918}(100) = -0.00655\ \%$ Ans.

(c) For 339,900: $\frac{339,900 - 339,872}{339,872}(100) = +0.00824\ \%$ Ans.

1-73 Round off the following numbers to four significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 918,273   (b) 284,739   (c) 342,691

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 918,300: $\frac{918,300 - 918,273}{918,273}(100) = +0.00294\ \%$ Ans.

(b) For 284,700: $\frac{284,700 - 284,739}{284,739}(100) = -0.01370\ \%$ Ans.

(c) For 342,700: $\frac{342,700 - 342,691}{342,691}(100) = 0.00263\ \%$ Ans.

1-74 Round off the following numbers to four significant figures. Find the percent difference between each rounded-off number and the original number by using the original number as the reference.

(a) 624,373   (b) 785,239   (c) 936,491

SOLUTION

$$\% \text{ Diff} = \frac{N_R - N_T}{N_T}(100)$$

(a) For 624,400: $\frac{624{,}400 - 624{,}373}{624{,}373}(100) = +0.00432\ \%$ Ans.

(b) For 785,200: $\frac{785{,}200 - 785{,}239}{785.239}(100) = -0.00497\ \%$ Ans.

(c) For 936,500: $\frac{936{,}500 - 936{,}491}{936{,}491}(100) = +0.000961\ \%$ Ans.

C1-75 A common practice in rounding answers is to report numbers whose leading digit is 1 to an accuracy of 4 significant figures and all other numbers to an accuracy of 3 significant figures. Although this practice probably started with the accuracy with which slide rules could be read, it also reflects the fact that an accuracy of greater than 0.2 percent is seldom possible. This project will examine the error introduced by this and some other rounding schemes. For each of the rounding schemes below,

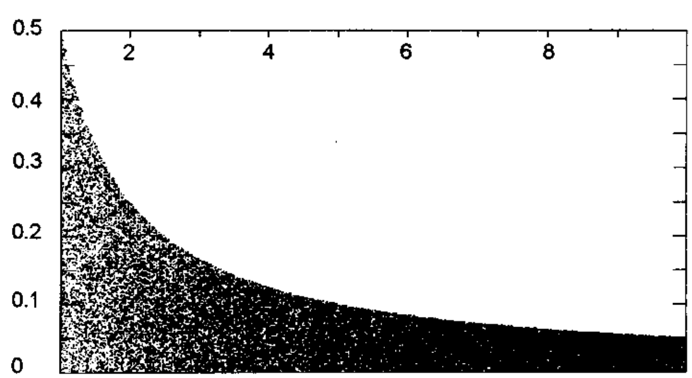
1. Generate 20,000 random numbers between 1 and 10.
2. Round each number to the specified number of significant figures. (Note that 3 significant figures is equivalent to 2 decimal places, 4 significant figures is equivalent to 3 decimal places, etc., since all numbers are between 1 and 10.)
3. Calculate the percent relative error for each number.

$$\text{PercentRelError} = \left|\frac{\text{Number} - \text{RoundNumber}}{\text{Number}}\right| * 100$$

4. Plot PercentRelError versus Number.
5. Comment on the maximum round-off error and the distribution of round-off error.

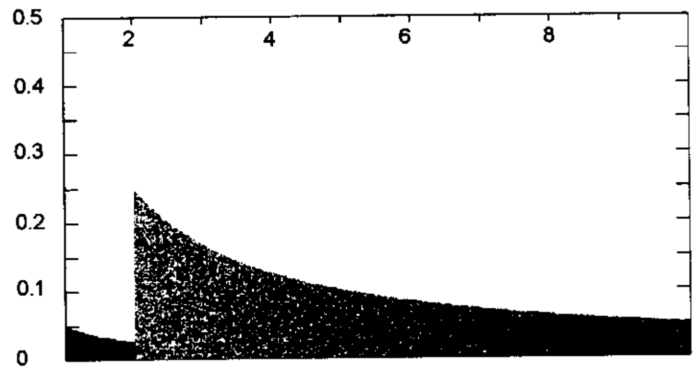
a. Round all numbers to an accuracy of 3 significant figures.
b. Round numbers less than 2 to an accuracy of 4 significant figures and numbers greater than 2 to an accuracy of 3 significant figures.
c. Round numbers less than 3 to an accuracy of 4 significant figures and numbers greater than 3 to an accuracy of 3 significant figures.
d. Round numbers less than 5 to an accuracy of 4 significant figures and numbers greater than 5 to an accuracy of 3 significant figures.

SOLUTION

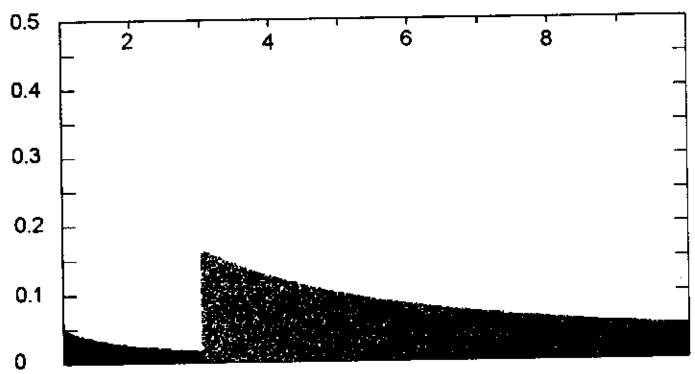


(a)

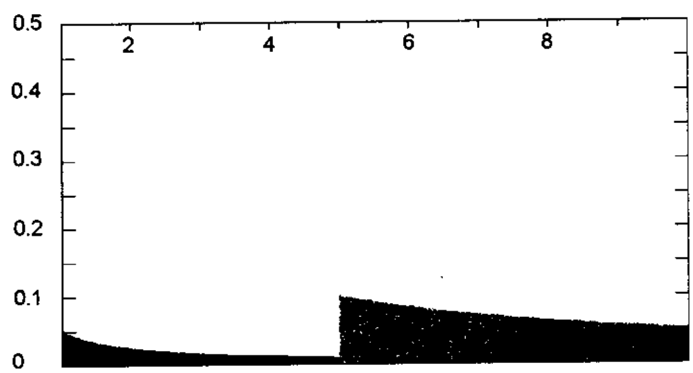
C1-75 (Continued)



(b)



(c)



(d)

C1-76 When engineers deal with angles, they are usually more interested in the sine or cosine of the angle than they are with the angle itself. Since sin 5° = cos 85° = sin 175° = sin 1085° = ... = 0.08716 the rounding of angles requires a different scheme than that described in Problem C1-75. That is, angles should be rounded to a specified number of decimal places rather than a specified number of significant figures. This project will examine the error introduced by rounding angles to various numbers of decimal places. For each of the cases below,

1. Generate 20,000 random angles between 1° and 89°. (Use a random number generator that produces decimal numbers and not just integers.) Calculate the sine and cosine of each angle.
2. Round each angle to the specified number of decimal places and calculate the sine and cosine of the rounded angle.
3. Calculate the percent relative error for each angle.

$$\text{PercentRelError} = \left|\frac{\sin(\text{Angle}) - \sin(\text{RoundAngle})}{\sin(\text{Angle})}\right| * 100$$

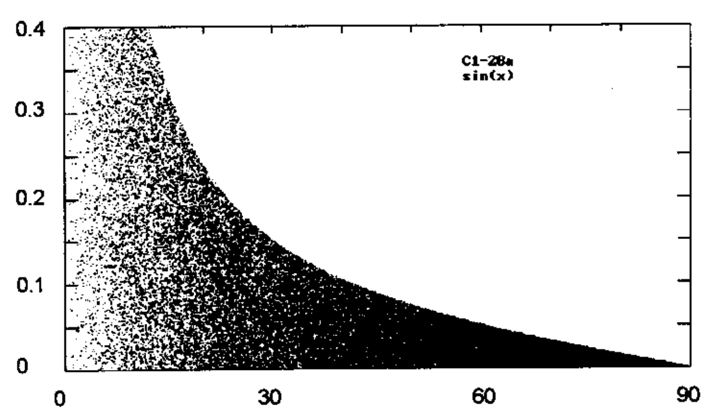
or

$$\text{PercentRelError} = \left|\frac{\cos(\text{Angle}) - \cos(\text{RoundAngle})}{\cos(\text{Angle})}\right| * 100$$

4. Plot PercentRelError versus Angle.
5. Comment on the maximum round-off error and the distribution of round-off error.

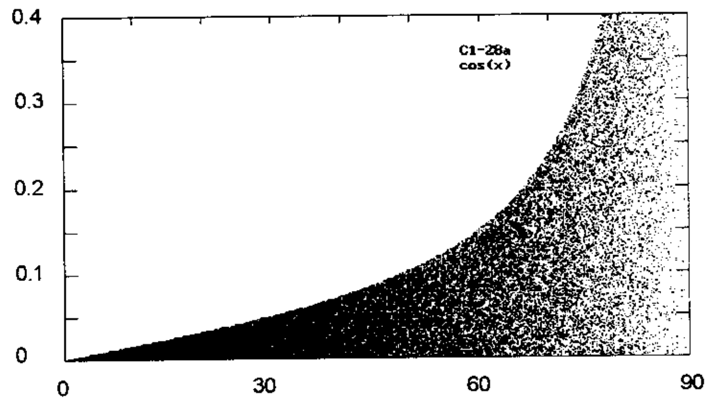
a. Round all angles to an accuracy of 1 decimal place.
b. Round all angles to an accuracy of 2 decimal places.
c. Round angles less than 10° to an accuracy of 3 decimal places and angles greater than 10° to an accuracy of 2 decimal places.

SOLUTION

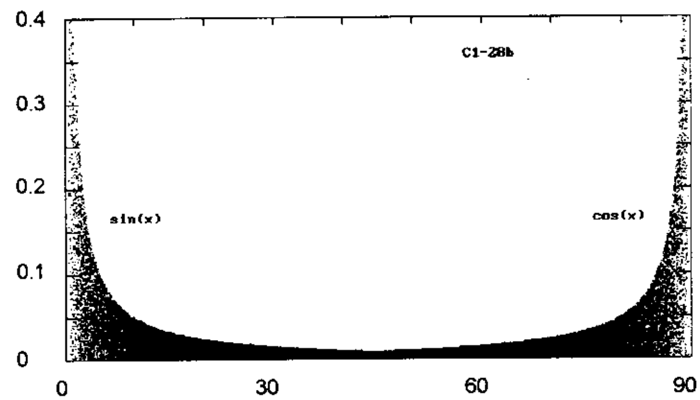


(a) SIN X

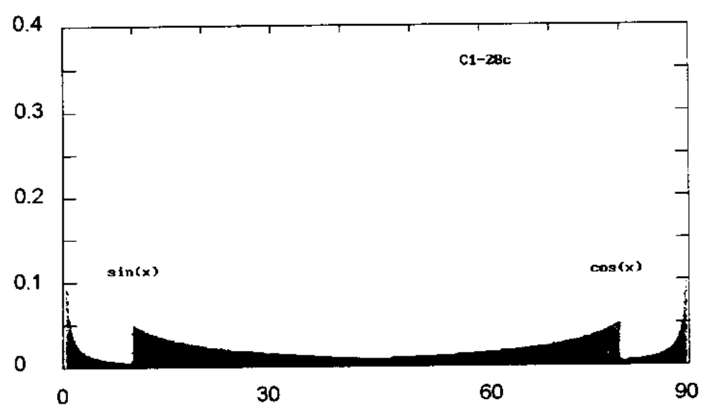
C1-76 (Continued)



(a) Cos x



(b)



(c)

C1-77 When two numbers are added or multiplied together, the result is always less accurate than the original numbers. This project will examine the error introduced by rounding two numbers before they are multiplied together.

a. Generate 80 random numbers between 4.51 and 5.49 (that is, 5 ± 0.1*RND). If any pair of these numbers are rounded to the nearest integer (5) and then multiplied together, the result will be 25. How does this result compare with the correct product obtained by multiplying the original two numbers together? Is the result accurate to the nearest integer? Is the result accurate to less than 10 percent?

b. Repeat part a for numbers between 49.51 and 50.49 (50 ± 0.01*RND). Is the result accurate to the nearest integer? Is the result accurate to less than 1 percent?

c. Generate 20,000 random integers between 1 and 49. For each integer N, generate two random numbers which will round to that integer

$$N1 = N \pm 0.5*RND \qquad N2 = N \pm 0.5*RND$$

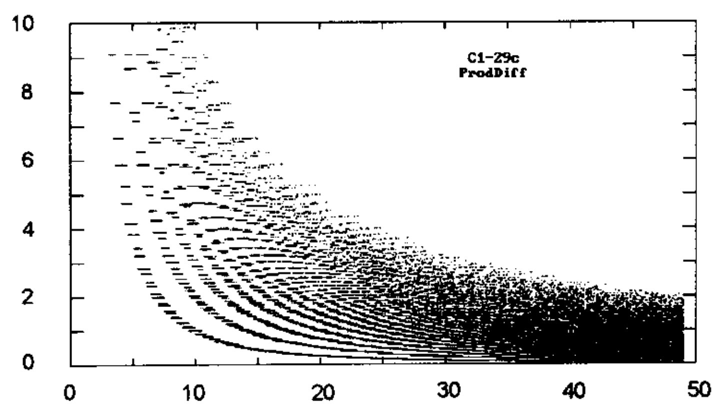
Plot the percent relative difference in the products

$$\text{ProdDiff} = \left|\frac{N1*N2 - N*N}{N*N}\right|*100$$

versus N. Compare this with the percent relative difference in the original numbers

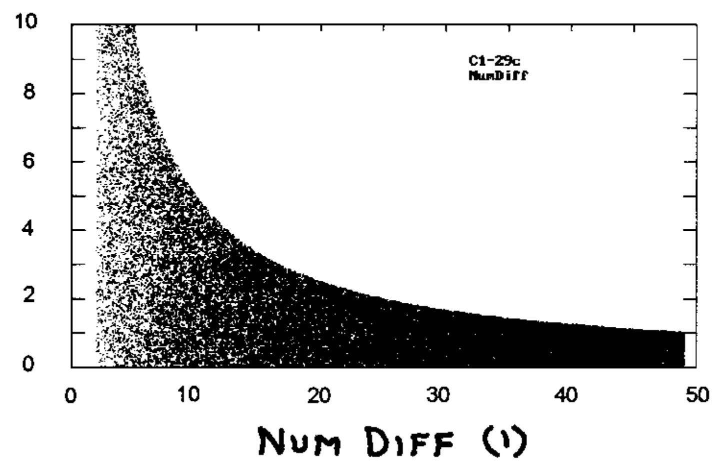
$$\text{NumDiff} = \left|\frac{N1 - N}{N}\right|*100$$

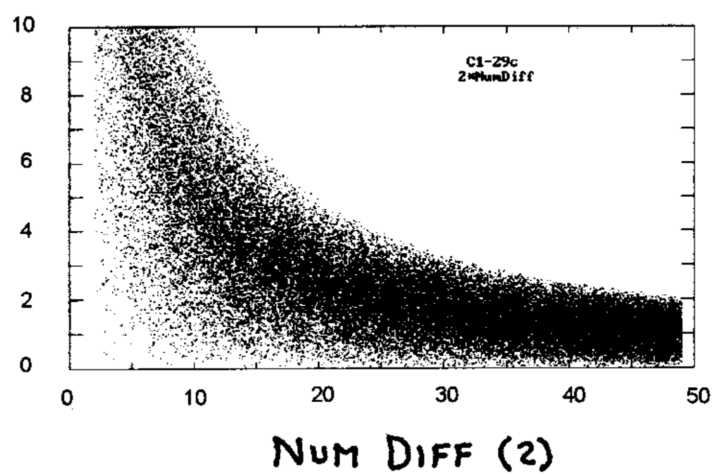
SOLUTION



PROD. DIFF.

C1-77 (Continued)





1-78* The planet Neptune has a mass of $1.03(10^{26})$ kg and a visible diameter (top of the cloud layers) of $4.86(10^{7})$ m. Determine the gravitational acceleration g at the top of the cloud layers.

SOLUTION

$$g_N = \frac{Gm_N}{r_N^2} = \frac{6.673(10^{-11})(1.03)(10^{26})}{[2.43(10^7)]^2} = 11.6398 \text{ m/s}^2 \cong 11.64 \text{ m/s}^2 \qquad \text{Ans.}$$

1-79* The weight of the first Russian satellite, Sputnik I, on the surface of the earth was 184 lb. Determine the force exerted on the satellite by the earth at the low and high points of its orbit which were 149 mi and 597 mi, respectively, above the surface of the earth.

SOLUTION

From Eq. (1-2): $F = \dfrac{Gm_e m}{r^2}$ $\qquad m = \dfrac{W}{g} = \dfrac{184}{32.17} = 5.720$ slug

From Table 1-1: $r_0 = 2.090(10^7)$ ft

$$r_L = r_0 + h_L = 2.090(10^7) + 149(5280) = 2.169(10^7) \text{ ft}$$

$$F_L = \frac{Gm_e m}{r_L^2} = \frac{3.439(10^{-8})(4.095)(10^{23})(5.720)}{[2.169(10^7)]^2} = 171.2 \text{ lb} \qquad \text{Ans.}$$

$$r_H = r_0 + h_H = 2.090(10^7) + 597(5280) = 2.405(10^7) \text{ ft}$$

$$F_H = \frac{Gm_e m}{r_H^2} = \frac{3.439(10^{-8})(4.095)(10^{23})(5.720)}{[2.405(10^7)]^2} = 139.3 \text{ lb} \qquad \text{Ans.}$$

1-80 The planet Jupiter has a mass of $1.90(10^{27})$ kg and a radius of $7.14(10^{7})$ m. Determine the force of attraction between the earth and Jupiter when the minimum distance between the two planets is $6(10^{11})$ m.

SOLUTION

$$F = \frac{Gm_e m_J}{r^2} = \frac{6.673(10^{-11})(5.976)(10^{24})(1.90)(10^{27})}{[6(10^{11})]^2} = 2.10(10^{18}) \text{ N} \qquad \text{Ans.}$$

1-81 On the surface of the earth the weight of a body is 200 lb. At what distance from the center of the earth would the weight of the body be

(a) 100 lb? (b) 50 lb?

SOLUTION

From Eqs. (1-2): $F = \dfrac{Gm_1m_2}{r^2}$ $\qquad Fr^2 = Wr_e^2 = Gm_em$

Therefore: $\qquad r = \sqrt{Wr_e^2/F} = \sqrt{\dfrac{W}{F}}\, r_e$

(a) For F = 100 lb: $\quad r = \sqrt{\dfrac{200}{100}}\,(2.090)(10^7) = 2.96(10^7)$ ft $\qquad$ Ans.

(b) For F = 50 lb: $\quad r = \sqrt{\dfrac{200}{50}}\,(2.090)(10^7) = 4.18(10^7)$ ft $\qquad$ Ans.

1-82* At what distance from the center of the earth would the force of attraction between two spheres 1 m in diameter in contact equal the force of attraction of the earth on one of the spheres? The mass of each sphere is 250 kg.

SOLUTION

From Eq. (1-2): $F = \dfrac{Gm_1m_2}{r^2}$ $\qquad \dfrac{Gm_sm_s}{(2r_s)^2} = \dfrac{Gm_em_s}{r^2}$

$$r = \sqrt{\frac{4r_s^2m_e}{m_s}} = \sqrt{\frac{4(0.5)^2(5.976)(10^{24})}{250}} = 0.1546(10^{12})\text{ m} \qquad \text{Ans.}$$

1-83* The weight of a satellite on the surface of the earth prior to launch is 250 lb. When the satellite is in orbit 5000 miles from the surface of the earth, determine the force of attraction between the earth and the satellite.

SOLUTION

From Eq. (1-2): $F = \dfrac{Gm_em}{r^2}$ $\qquad m = \dfrac{W}{g} = \dfrac{250}{32.17} = 7.771$ slug

From Table 1-1: $r_0 = 2.090(10^7)$ ft

$$r = r_0 + h = 2.090(10^7) + 5000(5280) = 4.730(10^7)\text{ ft}$$

$$F_L = \frac{Gm_em}{r_L^2} = \frac{3.439(10^{-8})(4.095)(10^{23})(7.771)}{[4.730(10^7)]^2} = 48.9\text{ lb} \qquad \text{Ans.}$$

1-84 A fluid has a dynamic viscosity of $1.2(10^{-3})$ $N{\cdot}s/m^2$. Express its dynamic viscosity in U.S. Customary units ($lb{\cdot}s/ft^2$).

SOLUTION

$$\mu = 1.2(10^{-3})\ \frac{N{\cdot}s}{m^2}\left(\frac{0.2248\ lb}{1\ N}\right)\left(\frac{0.3048\ m}{1\ ft}\right)^2 = 2.51(10^{-5})\ \frac{lb{\cdot}s}{ft^2} \qquad \text{Ans.}$$

1-85 The stress equation for eccentric loading of a short column is

$$\sigma = -\frac{P}{A} - \frac{Pey}{I}$$

If P is a force, A is an area, and e and y are lengths, what are the dimensions of stress σ and second moment of area I?

SOLUTION

$$\frac{P}{A} = \frac{ML/T^2}{L^2} = M/LT^2$$

All terms have the dimension M/LT^2:

$$\sigma = M/LT^2 \qquad I = \frac{(ML/T^2)(L)(L)}{(M/LT^2)} = L^4 \qquad \text{Ans.}$$

1-86* Determine the dimension of c in the dimensionally homogeneous equation

$$v = \frac{mg}{c}\left[1 - e^{-ct/m}\right]$$

in which v is a velocity, m is a mass, t is time, and g is the gravitational acceleration.

SOLUTION

$$\frac{ct}{m} = \frac{c(T)}{(M)} = 1 \text{ (Dimensionless)} \qquad c = M/T \qquad \text{Ans.}$$

$$v = \frac{mg}{c}\left[1 - e^{-ct/m}\right] = \frac{(M)(L/T^2)(1)}{c} = L/T \qquad c = M/T \qquad \text{Ans.}$$

1-87* In the dimensionally homogeneous equation $R = cv + ag$, R is a force, v is a velocity, and g is an acceleration. Determine the dimensions of a and c.

SOLUTION

All terms have the dimension ML/T^2:

$$cv = c(L/T) = ML/T^2 \qquad c = M/T \qquad \text{Ans.}$$

$$ag = a(L/T^2) = ML/T^2 \qquad a = M \qquad \text{Ans.}$$

1-88 When a body moves through a fluid it experiences a resistance to its motion which can be represented by the equation $F = \frac{1}{2}C_D\rho V^2A$ where F is a force, ρ is the density of the fluid, V is the velocity of the body relative to the fluid, and A is the cross-sectional area of the body. Show that the drag coefficient C_D is dimensionless.

SOLUTION

$$C_D = \frac{2F}{\rho V^2 A} = \frac{ML/T^2}{(M/L^3)(L/T)^2(L^2)} = 1 \text{ (Dimensionless)} \qquad \text{Ans.}$$

1-89 Develop an expression for the change in gravitational acceleration Δg between the surface of the earth and a height h when $h << R_e$.

SOLUTION

From Eq. (1-3): $$g = \frac{Gm_e}{r_e^2} \qquad g + \Delta g = \frac{Gm_e}{(r_e + h)^2}$$

Therefore: $$\Delta g = \frac{Gm_e}{(r_e + h)^2} - \frac{Gm_e}{r_e^2} = \frac{Gm_e[r_e^2 - (r_e + h)^2]}{r_e^2(r_e + h)^2}$$

$$= -\frac{Gm_e h(2r_e + h)}{r_e^2(r_e + h)^2}$$

For $h << r_e$:

$$\Delta g \cong -\frac{2Gm_e h}{r_e^3} = -g\left(\frac{2h}{r_e}\right) \qquad \text{Ans.}$$

2-1* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-1.

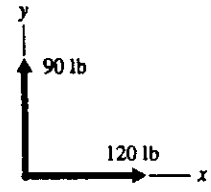
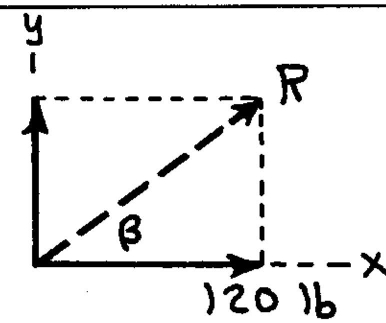


Fig. P2-1

SOLUTION



From Eq. 2-1:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$$

$$= 120^2 + 90^2 + 2(120)(90) \cos 90^\circ$$

$$R = 150.0 \text{ lb}$$

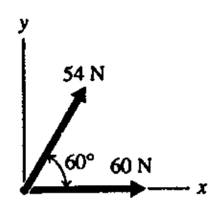
From Eq. 2-2:

$$\beta = \sin^{-1} \frac{F_2 \sin \phi}{R}$$

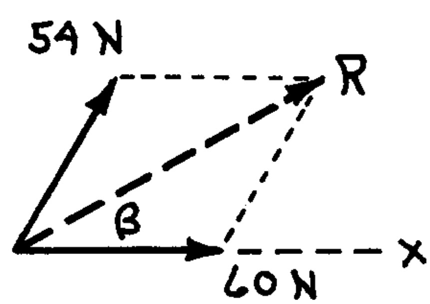
$$= \sin^{-1} \frac{90 \sin 90^\circ}{150.0} = 36.87^\circ$$

$\bar{R} = 150.0$ lb $\angle$ 36.9° Ans.

2-2* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-2



SOLUTION



From Eq. 2-1:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$$

$$= 60^2 + 54^2 + 2(60)(54) \cos 60^\circ$$

$$R = 98.77 \cong 98.8 \text{ N}$$

From Eq. 2-2:

$$\beta = \sin^{-1} \frac{F_2 \sin \phi}{R}$$

$$= \sin^{-1} \frac{54 \sin 60^\circ}{98.77} = 28.26^\circ$$

$\bar{R} = 98.8$ N $\angle$ 28.3° Ans.

2-3 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-3.

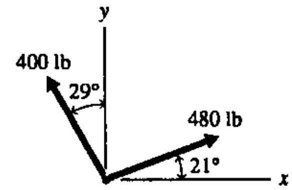
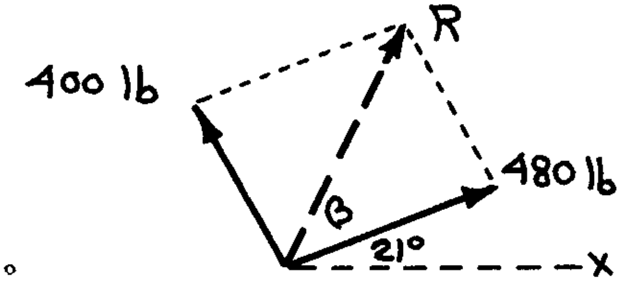


Fig. P2-3

SOLUTION



$\phi = 90^\circ + 29^\circ - 21^\circ = 98^\circ$

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$$

$$= 480^2 + 400^2 + 2(480)(400) \cos 98^\circ$$

$R = 580.48 \cong 580$ lb

$$\beta = \sin^{-1} \frac{F_2 \sin \phi}{R} = \sin^{-1} \frac{400 \sin 98^\circ}{580.48} = 43.03^\circ$$

$\theta = \beta + 21^\circ = 43.03^\circ + 21^\circ = 64.03^\circ \cong 64.0^\circ$ $\qquad \bar{R} = 580$ lb $\measuredangle$ 64.0° Ans.

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2-4 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-4.

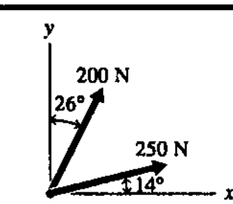
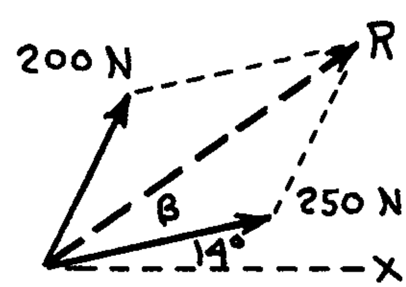


Fig. P2-4

SOLUTION



$\phi = 90^\circ - 14^\circ - 26^\circ = 50^\circ$

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$$

$$= 250^2 + 200^2 + 2(250)(200) \cos 50^\circ$$

$R = 408.39 \cong 408$ N

$$\beta = \sin^{-1} \frac{F_2 \sin \phi}{R} = \sin^{-1} \frac{200 \sin 50^\circ}{408.39} = 22.03^\circ$$

$\theta = \beta + 14^\circ = 22.03^\circ + 14^\circ = 36.03^\circ \cong 36.0^\circ$ $\qquad \bar{R} = 408$ N $\measuredangle$ 36.0° Ans.

2-5* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-5.

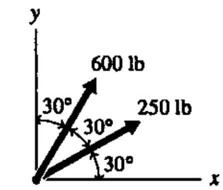


Fig. P2-5

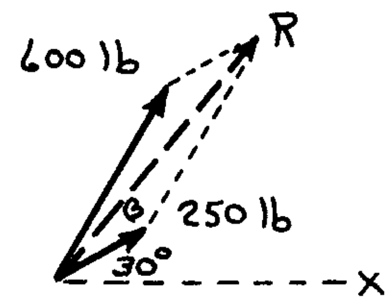
SOLUTION

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$$

$$= 250^2 + 600^2 + 2(250)(600) \cos 30^\circ$$

$$R = 826.02 \text{ lb} \cong 826 \text{ lb}$$



$$\beta = \sin^{-1} \frac{F_2 \sin \phi}{R} = \sin^{-1} \frac{600 \sin 30^\circ}{826.02} = 21.30^\circ$$

$$\theta = \beta + 30^\circ = 21.30^\circ + 30^\circ = 51.30^\circ = 51.3^\circ$$

$\bar{R} = 826$ lb ∠ 51.3° Ans.

2-6* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-6.

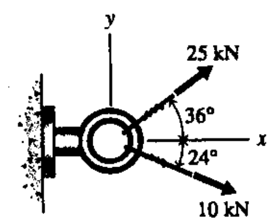


Fig. P2-6

SOLUTION

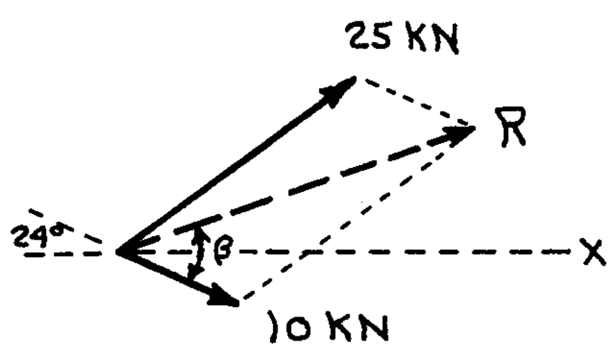
$$\phi = 36^\circ + 24^\circ = 60^\circ$$

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$$

$$= 10^2 + 25^2 + 2(10)(25) \cos 60^\circ$$

$$R = 31.22 \text{ kN} \cong 31.2 \text{ kN}$$



$$\beta = \sin^{-1} \frac{F_2 \sin \phi}{R} = \sin^{-1} \frac{25 \sin 60^\circ}{31.22} = 43.91^\circ$$

$$\theta = B - 24 = 43.91 - 24 = 19.91^\circ$$

$\bar{R} = 31.2$ kN ∠ 19.91° Ans.

2-7 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-7.

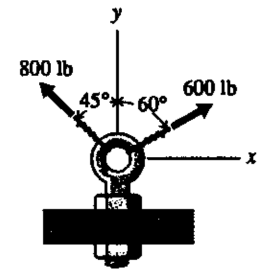
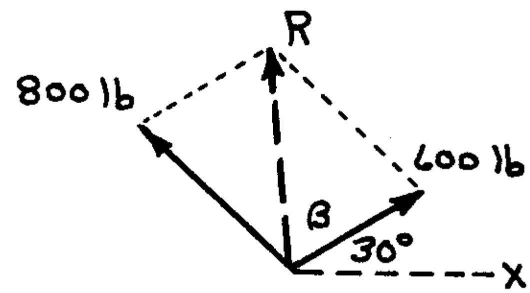


Fig. P2-7

SOLUTION



$\phi = 45^\circ + 60^\circ = 105^\circ$

$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\phi$

$= 600^2 + 800^2 + 2(600)(800) \cos 105^\circ$

$R = 866.91 \text{ lb} \cong 867 \text{ lb}$

$\beta = \sin^{-1} \dfrac{F_2 \sin\phi}{R} = \sin^{-1} \dfrac{800 \sin 105^\circ}{866.91} = 63.046^\circ$

$\theta = \beta + 30 = 63.046 + 30 = 93.046^\circ \cong 93.0^\circ$ $\qquad \bar{R} = 867 \text{ lb} \measuredangle 87.0^\circ$ Ans.

2-8 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-8.

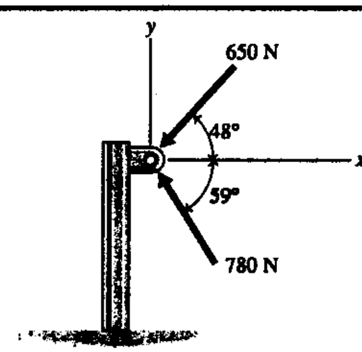
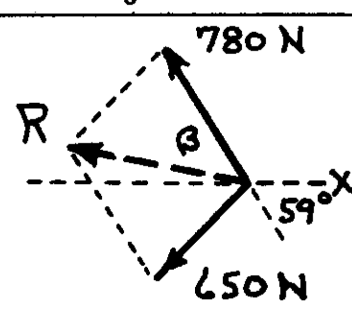


Fig. P2-8

SOLUTION



$\phi = 48^\circ + 59^\circ = 107^\circ$

$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\phi$

$= 780^2 + 650^2 + 2(780)(650) \cos 107^\circ$

$R = 856.99 \text{ N} \cong 857 \text{ N}$

$\beta = \sin^{-1} \dfrac{F_2 \sin\phi}{R} = \sin^{-1} \dfrac{650 \sin 107^\circ}{856.99} = 46.50^\circ$

$\theta = 59^\circ - \beta = 59 - 46.50 = 12.50^\circ$ $\qquad \bar{R} = 857 \text{ N} \measuredangle 12.50^\circ$ Ans.

2-9* Determine the magnitude of the resultant $\bar{\mathbf{R}}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-9.

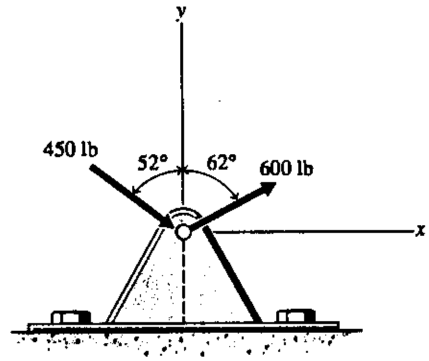


Fig. P2-9

SOLUTION

$\phi = 180^\circ - 62^\circ - 52^\circ = 66^\circ$

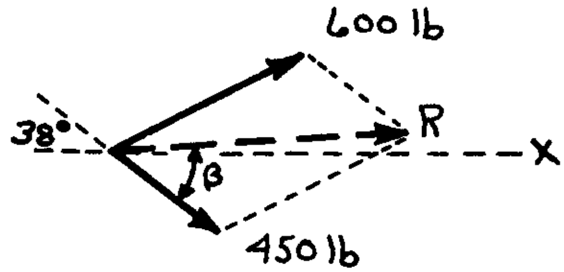
$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$

$= 450^2 + 600^2 + 2(450)(600) \cos 66^\circ$

$R = 884.39 \text{ lb} \cong 884 \text{ lb}$

$\beta = \sin^{-1} \dfrac{F_2 \sin \phi}{R} = \sin^{-1} \dfrac{600 \sin 66^\circ}{884.39} = 38.300^\circ$

$\theta = \beta - 38^\circ = 38.300^\circ - 38 = 0.300^\circ$



$\bar{\mathbf{R}} = 884 \text{ lb} \measuredangle 0.300^\circ$ Ans.

2-10* Determine the magnitude of the resultant $\bar{\mathbf{R}}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-10.

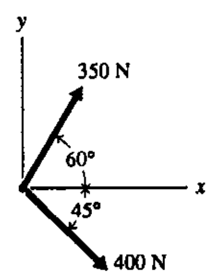


Fig. P2-10

SOLUTION

$\phi = 60^\circ + 45^\circ = 105^\circ$

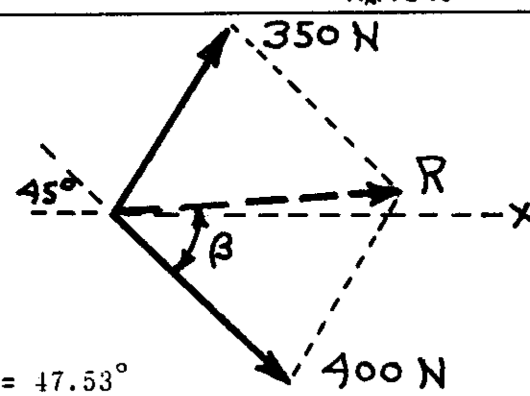
$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi$

$= 400^2 + 350^2 + 2(400)(350) \cos 105^\circ$

$R = 458.29 \text{ N} \cong 458 \text{ N}$

$\beta = \sin^{-1} \dfrac{F_2 \sin \phi}{R} = \sin^{-1} \dfrac{350 \sin 105^\circ}{458.29} = 47.53^\circ$

$\theta = \beta - 45^\circ = 47.53^\circ - 45^\circ = 2.53^\circ$



$\bar{\mathbf{R}} = 458 \text{ N} \measuredangle 2.53^\circ$ Ans.

2-11* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-11.

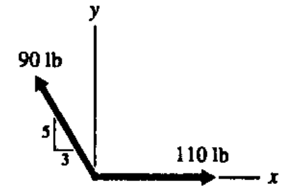
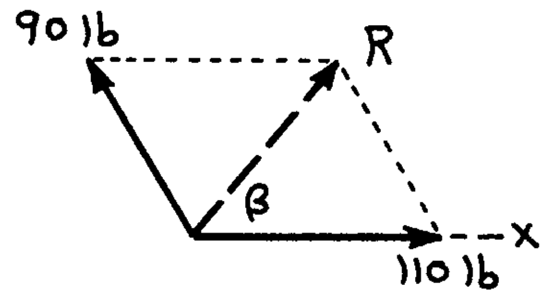


Fig. P2-11

SOLUTION



$$\phi = 180^\circ - \tan^{-1}\frac{5}{3} = 120.96^\circ$$

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\phi$$

$$= 110^2 + 90^2 + 2(110)(90)\cos 120.96^\circ$$

$$R = 100.07 \text{ lb} \cong 100.1 \text{ lb}$$

$$\beta = \sin^{-1}\frac{F_2\sin\phi}{R} = \sin^{-1}\frac{90\sin 120.96^\circ}{100.07} = 50.46^\circ$$

$\bar{R} = 100.1$ lb $\angle$ 50.5° Ans.

2-12* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-12.

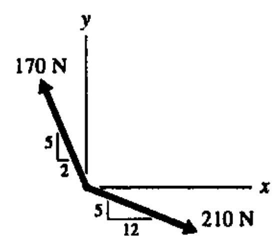
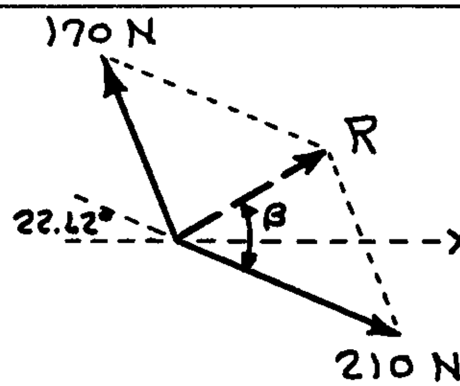


Fig. P2-12

SOLUTION



$$\phi = 180^\circ + \tan^{-1}\frac{5}{12} - \tan^{-1}\frac{5}{2} = 134.42^\circ$$

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\phi$$

$$= 210^2 + 170^2 + 2(210)(170)\cos 134.42^\circ$$

$$R = 151.74 \text{ N} \cong 151.7 \text{ N}$$

$$\beta = \sin^{-1}\frac{F_2\sin\phi}{R} = \sin^{-1}\frac{170\sin 134.42^\circ}{151.74} = 53.15^\circ$$

$$\theta = \beta - 22.62 = 53.15 - 22.62 = 30.53 \cong 30.5^\circ$$

$\bar{R} = 151.7$ N $\angle$ 30.5° Ans.

2-13 Determine the magnitude of the resultant $\bar{\mathbf{R}}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-13.

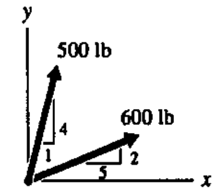
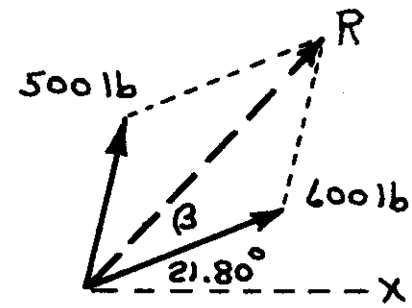


Fig. P2-13

SOLUTION



$$\phi = \tan^{-1}\frac{4}{1} - \tan^{-1}\frac{2}{5} = 54.16^\circ$$

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\phi$$

$$= 600^2 + 500^2 + 2(600)(500)\cos 54.16^\circ$$

$$R = 980.47 \text{ lb} \cong 980 \text{ lb}$$

$$\beta = \sin^{-1}\frac{F_2 \sin\phi}{R} = \sin^{-1}\frac{500 \sin 54.16^\circ}{980.47} = 24.42^\circ$$

$$\theta = \beta + 21.80 = 24.42 + 21.80 = 46.22 \cong 46.2$$ $\bar{\mathbf{R}} = 980 \text{ lb} \angle 46.2^\circ$ Ans.

2-14 Determine the magnitude of the resultant $\bar{\mathbf{R}}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-14.

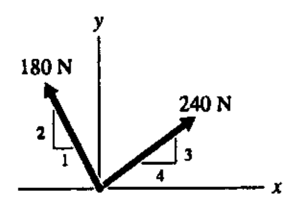
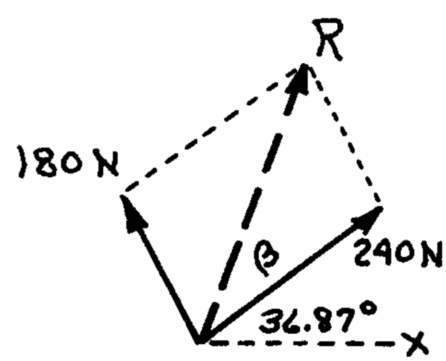


Fig. P2-14

SOLUTION



$$\phi = 180^\circ - \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{2}{1} = 79.70^\circ$$

From Eqs. 2-1 and 2-2:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\phi$$

$$= 240^2 + 180^2 + 2(240)(180)\cos 79.70^\circ$$

$$R = 324.73 \text{ N} \cong 325 \text{ N}$$

$$\beta = \sin^{-1}\frac{F_2 \sin\phi}{R} = \sin^{-1}\frac{180 \sin 79.70^\circ}{324.73} = 33.05^\circ$$

$$\theta = \beta + 36.87 = 33.05 + 36.87 = 69.92^\circ \cong 69.9^\circ$$ $\bar{\mathbf{R}} = 325 \text{ N} \angle 69.9^\circ$ Ans.

2-15* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-15.

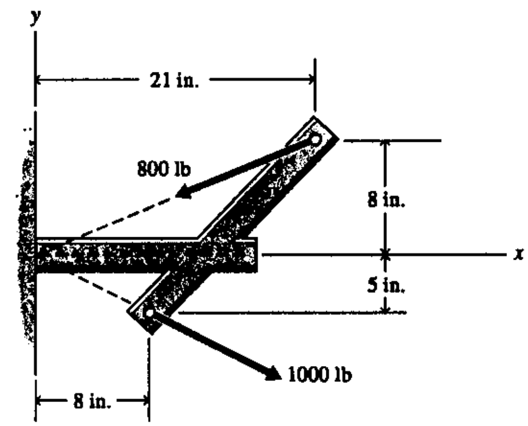
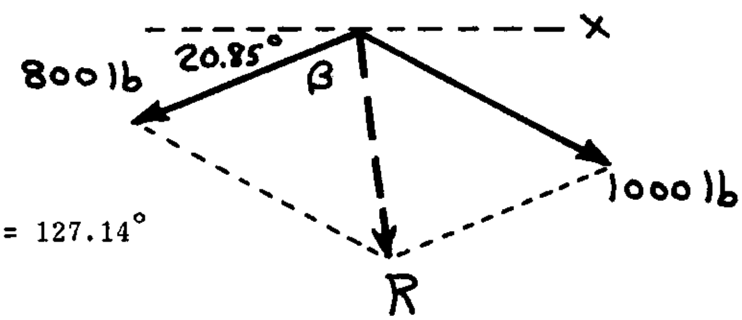


Fig. P2-15

SOLUTION



$$\theta_1 = \tan^{-1}\frac{8}{21} = 20.85^\circ$$

$$\theta_2 = \tan^{-1}\frac{5}{8} = 32.01^\circ$$

$$\phi = 180^\circ - 20.85^\circ - 32.01^\circ = 127.14^\circ$$

From Eq. 2-1:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\phi$$

$$= 800^2 + 1000^2 + 2(800)(1000)\cos 127.14^\circ$$

$$R = 820.96 \text{ lb} \cong 821 \text{ lb}$$

From Eq. 2-2:

$$\beta = \sin^{-1}\frac{F_2 \sin\phi}{R} = \sin^{-1}\frac{1000 \sin 127.14^\circ}{820.96} = 76.17^\circ$$

$$\theta = \beta + 20.85^\circ = 76.17 + 20.85 = 97.02^\circ \cong 97.0^\circ$$

$\bar{R} = 821 \text{ lb} \searrow 83.0^\circ$ Ans.

2-16 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the two forces shown in Fig. P2-16.

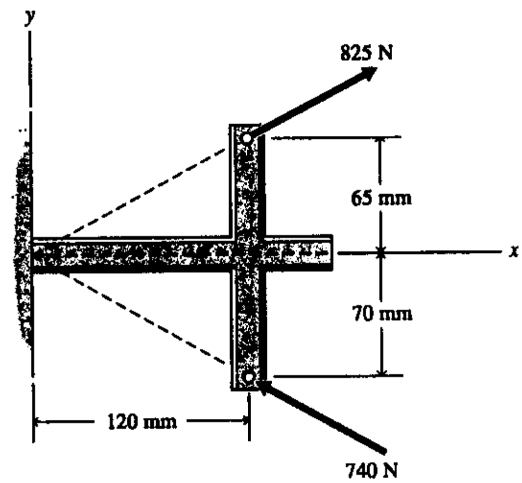


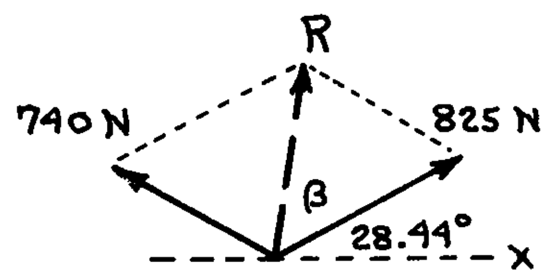
Fig. P2-16

SOLUTION

$$\theta_1 = \tan^{-1}\frac{65}{120} = 28.44^\circ$$

$$\theta_2 = \tan^{-1}\frac{70}{120} = 30.26^\circ$$

$$\phi = 180^\circ - 28.44^\circ - 30.26^\circ = 121.30^\circ$$



From Eq. 2-1:

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\phi$$

$$= 825^2 + 740^2 + 2(825)(740)\cos 121.30^\circ$$

$$R = 770.64 \text{ N} \cong 771 \text{ N}$$

From Eq. 2-2:

$$\beta = \sin^{-1}\frac{F_2\sin\phi}{R} = \sin^{-1}\frac{740\sin 121.30^\circ}{770.64} = 55.14^\circ$$

$$\theta = \beta + 28.44 = 55.14 + 28.44 = 83.58^\circ \cong 83.6^\circ$$

$\bar{R} = 771$ N ∠ 83.6° Ans.

2-17* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-17.

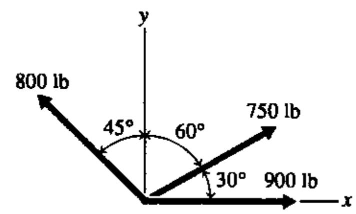
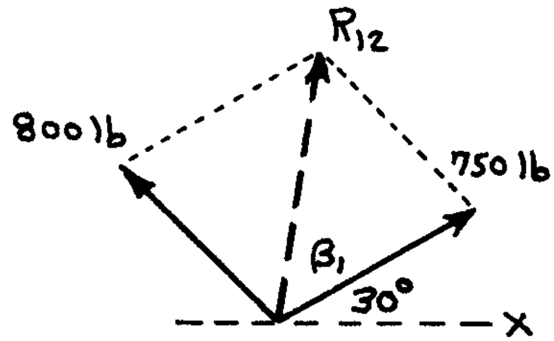


Fig. P2-17

SOLUTION



$\phi_1 = 60° + 45° = 105°$

From Eq. 2-1:

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$

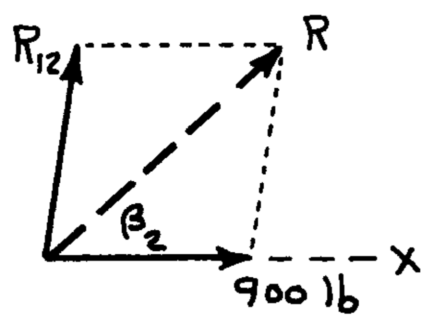
$\quad = 750^2 + 800^2 + 2(750)(800) \cos 105°$

$R_{12} = 944.41 \text{ lb} \cong 944 \text{ lb}$

From Eq. 2-2:

$\beta_1 = \sin^{-1} \dfrac{F_2 \sin \phi_1}{R_{12}}$

$\quad = \sin^{-1} \dfrac{800 \sin 105°}{944.41} = 54.91°$



Similarly:

$\phi_2 = 30° + \beta_1 = 30° + 54.91° = 84.91°$

$R^2 = F_3^2 + R_{12}^2 + 2F_3R_{12} \cos \phi_2$

$\quad = 900^2 + 944.41^2 + 2(900)(944.41) \cos 84.91°$

$R = 1361.15 \text{ lb} \cong 1361 \text{ lb}$

$\beta_2 = \sin^{-1} \dfrac{R_{12} \sin \phi_2}{R}$

$\quad = \sin^{-1} \dfrac{944.41 \sin 84.91°}{1361.15} = 43.72°$

$\bar{R} = 1361 \text{ lb} \measuredangle 43.7°$ Ans.

2-18* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-18.

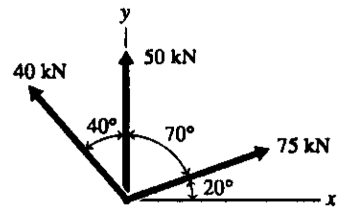
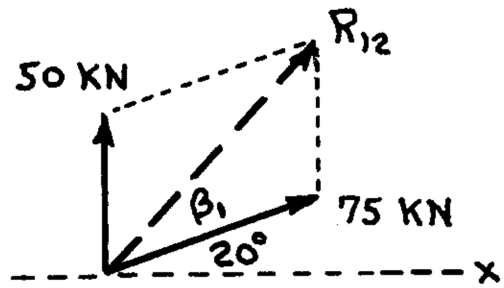


Fig. P2-18

SOLUTION



$$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$$

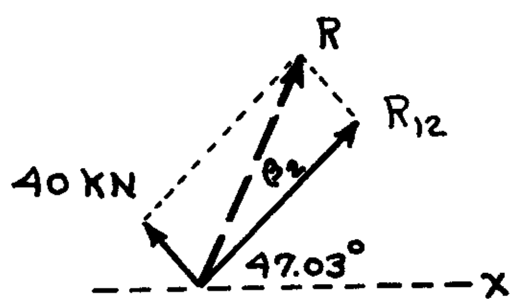
$$= 75^2 + 50^2 + 2(75)(50) \cos 70^\circ$$

$$R_{12} = 103.39 \text{ kN} \cong 103.4 \text{ kN}$$

$$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$$

$$= \sin^{-1} \frac{50 \sin 70^\circ}{103.39} = 27.03^\circ$$

Similarly:



$$\phi_2 = 40^\circ + (70^\circ - \beta_1)$$

$$= 40^\circ + (70^\circ - 27.03^\circ) = 82.97^\circ$$

$$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$$

$$= 103.39^2 + 40^2 + 2(103.39)(40) \cos 82.97^\circ$$

$$R = 115.33 \text{ kN} \cong 115.3 \text{ kN}$$

$$\beta_2 = \sin^{-1} \frac{F_3 \sin \phi_2}{R}$$

$$= \sin^{-1} \frac{40 \sin 82.97^\circ}{115.33} = 20.13^\circ$$

$$\theta = 20^\circ + \beta_1 + \beta_2 = 20^\circ + 27.03^\circ + 20.13^\circ = 67.16^\circ \cong 67.2^\circ$$

$\bar{R} = 115.3 \text{ kN} \measuredangle 67.2^\circ$ Ans.

2-19 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-19.

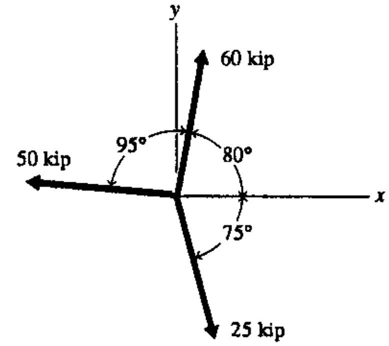
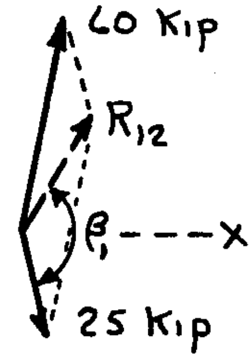


Fig. P2-19

SOLUTION



$\phi_1 = 75^\circ + 80^\circ = 155^\circ$

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$

$= 25^2 + 60^2 + 2(25)(60) \cos 155^\circ$

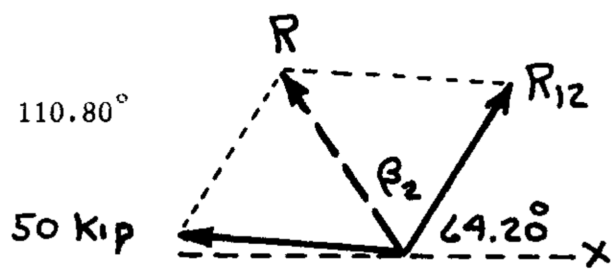
$R_{12} = 38.81 \text{ kip} \cong 38.8 \text{ kip}$

$\beta_1 = \sin^{-1} \dfrac{F_2 \sin \phi_1}{R_{12}}$

$= \sin^{-1} \dfrac{60 \sin 155^\circ}{38.81} = 139.20^\circ$

Similarly:

$\phi_2 = 250^\circ - \beta_1 = 250^\circ - 139.20^\circ = 110.80^\circ$



$R^2 = F_3^2 + R_{12}^2 + 2F_3R_{12} \cos \phi_2$

$= 50^2 + 38.81^2 + 2(50)(38.11) \cos 110.80^\circ$

$R = 51.27 \text{ kip} \cong 51.3 \text{ kip}$

$\beta_2 = \sin^{-1} \dfrac{F_3 \sin \phi_2}{R}$

$= \sin^{-1} \dfrac{50 \sin 110.80^\circ}{51.27} = 65.74^\circ$

$\theta = \beta_1 - 75 + \beta_2 = 139.20 - 75 + 65.74 = 129.94^\circ \cong 129.9^\circ$

$\bar{R} = 51.3$ kip ⦪ 50.1° Ans.

2-20 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-20.

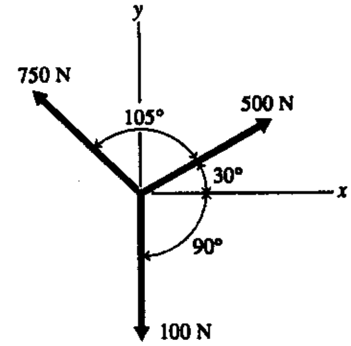
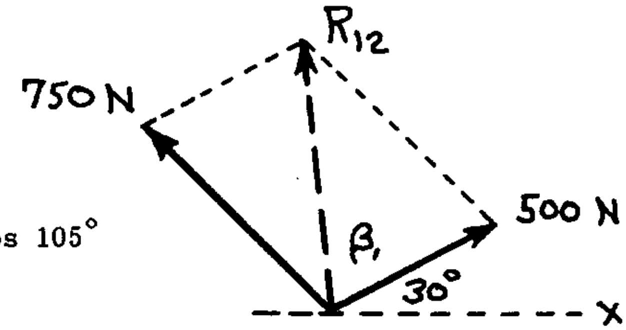


Fig. P2-20

SOLUTION



From Eq. 2-1:

$$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$$

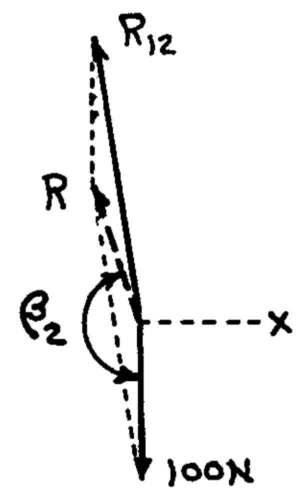
$$= 500^2 + 750^2 + 2(500)(750) \cos 105^\circ$$

$$R_{12} = 786.38 \text{ N} \cong 786 \text{ N}$$

From Eq. 2-2:

$$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$$

$$= \sin^{-1} \frac{750 \sin 105^\circ}{786.38} = 67.11^\circ$$



Similarly:

$$\phi_2 = 240^\circ - \beta_1 = 240^\circ - 67.11^\circ = 172.89^\circ$$

$$R^2 = F_3^2 + R_{12}^2 + 2F_3R_{12} \cos \phi_2$$

$$= 100^2 + 786.38^2 + 2(100)(786.38) \cos 172.89^\circ$$

$$R = 687.26 \text{ N} \cong 687 \text{ N}$$

$$\beta_2 = \sin^{-1} \frac{R_{12} \sin \phi_2}{R}$$

$$= \sin^{-1} \frac{786.38 \sin 172.89^\circ}{687.26} = 171.86^\circ$$

$$\theta = \beta_2 - 90^\circ = 171.86^\circ - 90^\circ = 81.86^\circ \cong 81.9^\circ$$

$\bar{R}$ = 687 N ∡ 81.9° Ans.

2-21* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-21.

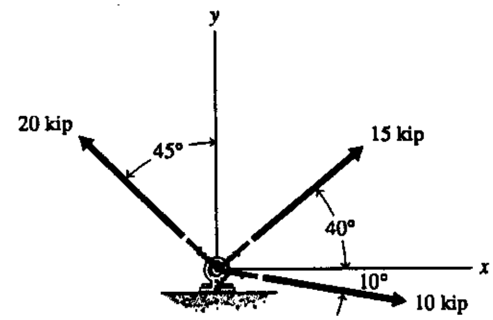
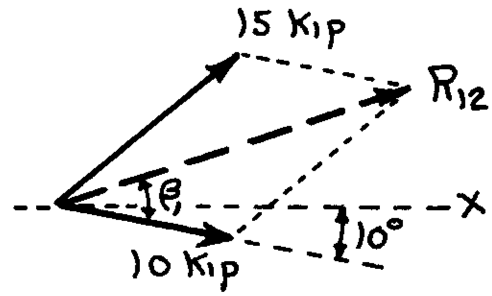


Fig. P2-21

SOLUTION



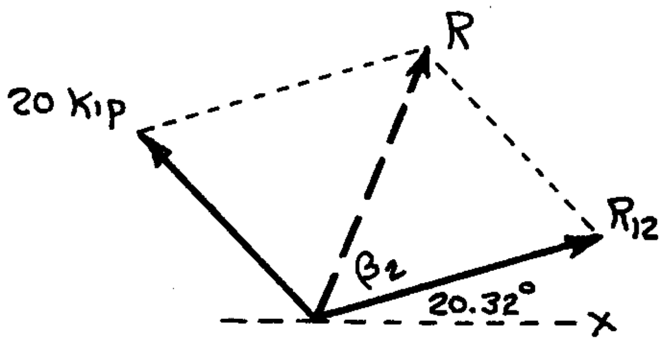
$\phi_1 = 40^\circ + 10^\circ = 50^\circ$

From Eq. 2-1:

$$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$$

$$= 10^2 + 15^2 + 2(10)(15) \cos 50^\circ$$

$R_{12} = 22.76 \text{ kip} \cong 22.8 \text{ kip}$



From Eq. 2-2:

$$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$$

$$= \sin^{-1} \frac{15 \sin 50^\circ}{22.76} = 30.32^\circ$$

Similarly:

$\phi_2 = 145^\circ - \beta_1 = 145^\circ - 30.32^\circ = 114.68^\circ \cong 114.7^\circ$

$$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$$

$$= 22.76^2 + 20^2 + 2(22.76)(20) \cos 114.68^\circ$$

$R = 23.19 \text{ kip} \cong 23.2 \text{ kip}$

$$\beta_2 = \sin^{-1} \frac{F_3 \sin \phi_2}{R}$$

$$= \sin^{-1} \frac{20 \sin 114.68^\circ}{23.19} = 51.60^\circ$$

$\theta = \beta_1 + \beta_2 - 10^\circ = 30.32^\circ + 51.60^\circ - 10^\circ = 71.92^\circ \cong 71.9^\circ$

$\bar{R} = 23.2 \text{ kip} \angle 71.9^\circ$ Ans.

2-22* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-22.

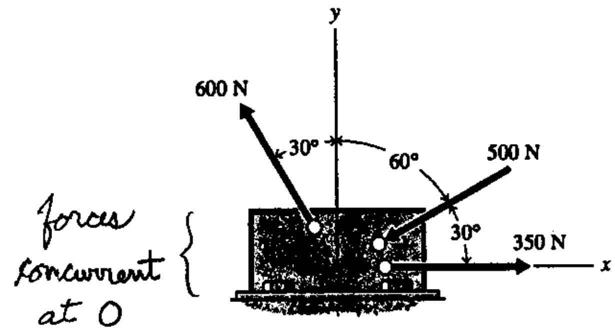
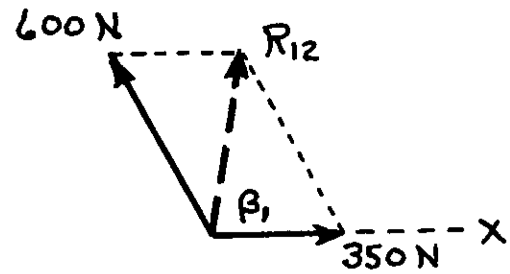


Fig. P2-22

SOLUTION



$\phi_1 = 30^\circ + 60^\circ + 30^\circ = 120^\circ$

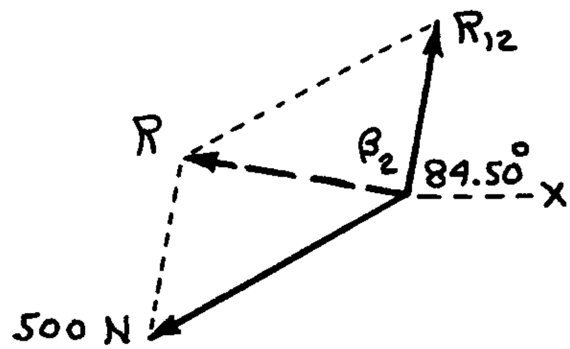
From Eq. 2-1:

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$

$= 350^2 + 600^2 + 2(350)(600) \cos 120^\circ$

$R_{12} = 522.02 \text{ N} \cong 522 \text{ N}$

From Eq. 2-2:



$$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$$

$$= \sin^{-1} \frac{600 \sin 120^\circ}{522.02} = 84.50^\circ$$

Similarly:

$\phi_2 = 180^\circ - (\beta_1 - 30^\circ) = 180^\circ - (84.50^\circ - 30^\circ) = 125.50^\circ$

$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$

$= 522.02^2 + 500^2 + 2(522.02)(500) \cos 125.50^\circ$

$R = 468.37 \text{ N} \cong 468 \text{ N}$

$$\beta_2 = \sin^{-1} \frac{F_3 \sin \phi_2}{R}$$

$$= \sin^{-1} \frac{500 \sin 125.50^\circ}{468.37} = 60.35^\circ$$

$\theta = \beta_1 + \beta_2 = 84.50^\circ + 60.35^\circ = 144.85^\circ \cong 144.9^\circ$

$\bar{R} = 468 \text{ N} \measuredangle 35.1^\circ$ Ans.

2-23 Determine the magnitude of the resultant $\vec{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-23.

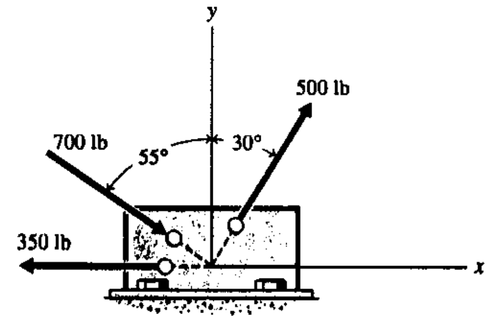
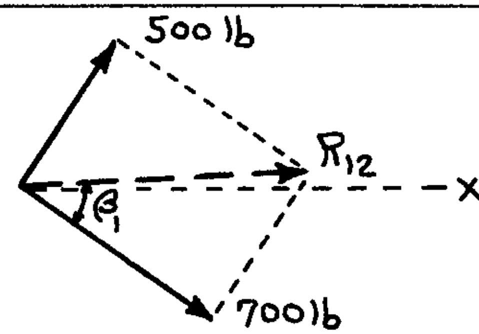


Fig. P2-23

SOLUTION



$\phi_1 = 90^\circ - 55^\circ + 60^\circ = 95^\circ$

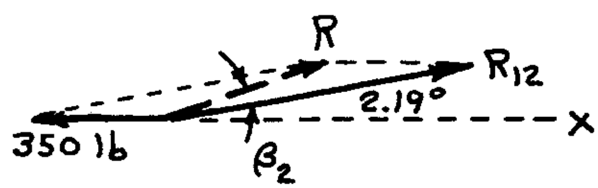
From Eq. 2-1:

$$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$$

$$= 700^2 + 500^2 + 2(700)(500) \cos 95^\circ$$

$R_{12} = 824.01 \text{ lb} \cong 824 \text{ lb}$

From Eq. 2-2:



$$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$$

$$= \sin^{-1} \frac{500 \sin 95^\circ}{824.01} = 37.19^\circ$$

Similarly:

$$\phi_2 = 180^\circ - (\beta_1 - 35^\circ) = 180^\circ - (37.19^\circ - 35^\circ) = 177.81^\circ \cong 177.8^\circ$$

$$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$$

$$= 824.01^2 + 350^2 + 2(824.01)(350) \cos 177.81^\circ$$

$R = 474.45 \text{ lb} \cong 474 \text{ lb}$

$$\beta_2 = \sin^{-1} \frac{F_3 \sin \phi_2}{R}$$

$$= \sin^{-1} \frac{350 \sin 177.81^\circ}{474.45} = 1.615^\circ$$

$$\theta = \beta_1 + \beta_2 - 10^\circ = 37.19^\circ - 35^\circ + 1.615^\circ = 3.805^\circ \cong 3.81^\circ$$

$\vec{R} = 474 \text{ lb} \measuredangle 3.81^\circ$ Ans.

2-24 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-24.

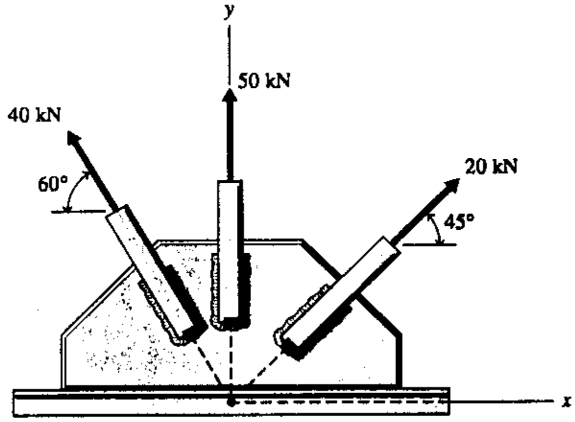
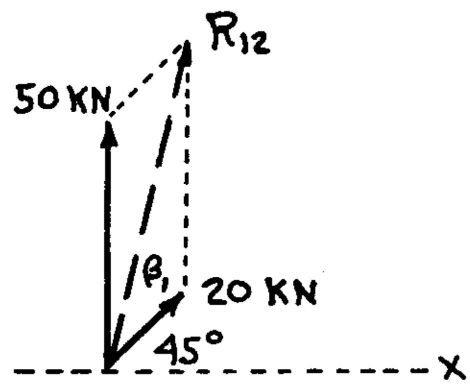


Fig. P2-24

SOLUTION



$\phi_1 = 90° - 45° = 45°$

From Eq. 2-1:

$$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$$

$$= 20^2 + 50^2 + 2(20)(50) \cos 45°$$

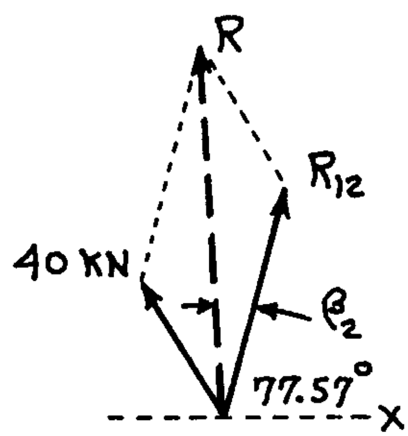
$R_{12} = 65.68 \text{ kN} \cong 65.7 \text{ kN}$

From Eq. 2-2:

$$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$$

$$= \sin^{-1} \frac{50 \sin 45°}{65.68} = 32.57°$$

Similarly:



$\phi_2 = 75° - \beta_1 = 75° - 32.57° = 42.43°$

$$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$$

$$= 65.68^2 + 40^2 + 2(65.68)(40) \cos 42.43°$$

$R = 98.96 \text{ kN} \cong 99.0 \text{ kN}$

$$\beta_2 = \sin^{-1} \frac{F_3 \sin \phi_2}{R}$$

$$= \sin^{-1} \frac{40 \sin 42.43°}{98.96} = 15.83°$$

$\theta = 45° + \beta_1 + \beta_2 = 45° + 32.57° + 15.83° = 93.40°$

$\bar{R} = 99.0 \text{ kN} ⦪ 86.6°$ Ans.

2-25* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-25.

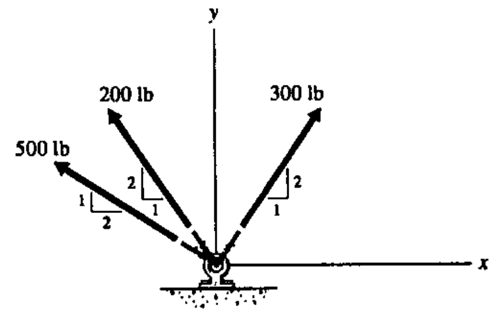
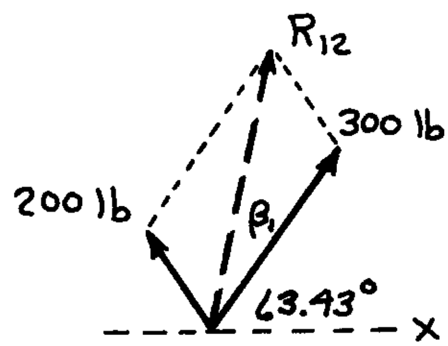


Fig. P2-25

SOLUTION



$\theta_1 = \tan^{-1} \frac{2}{1} = 63.43°$

$\theta_2 = \tan^{-1} \frac{2}{1} = 63.43°$

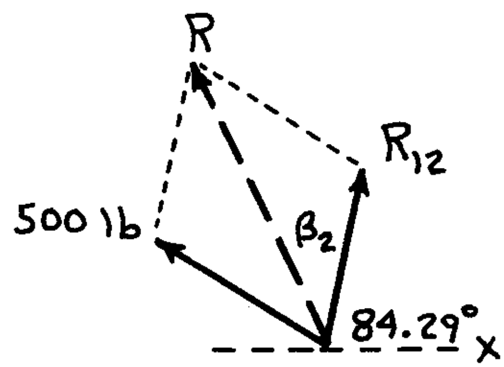
$\theta_3 = \tan^{-1} \frac{1}{2} = 26.57°$

$\phi_1 = 180° - 63.43° - 63.43° = 53.14°$

From Eq. 2-1:

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1 = 300^2 + 200^2 + 2(300)(200) \cos 53.14°$

$R_{12} = 449.43 \text{ lb} \cong 449 \text{ lb}$



From Eq. 2-2:

$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$

$= \sin^{-1} \frac{200 \sin 53.14°}{449.43} = 20.86°$

Similarly:

$\phi_2 = 90° - \beta_1 = 90° - 20.86° = 69.14°$

$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$

$= 449.43^2 + 500^2 + 2(449.43)(500) \cos 69.14°$

$R = 782.3 \text{ lb} \cong 782 \text{ lb}$

$\beta_2 = \sin^{-1} \frac{F_3 \sin \phi_2}{R} = \sin^{-1} \frac{500 \sin 69.14°}{782.3} = 36.67°$

$\theta = \theta_1 + \beta_1 + \beta_2 = 63.43° + 20.86° + 36.67° = 120.96° \cong 121.0°$

$\bar{R} = 782 \text{ lb} \measuredangle 59.0°$ Ans.

2-26* Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-26.

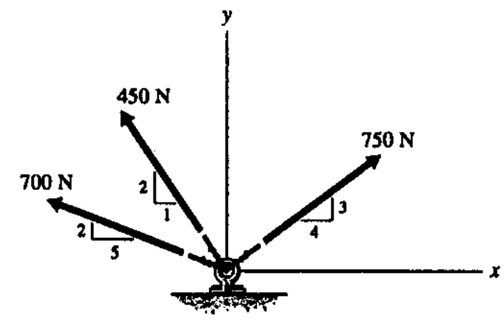
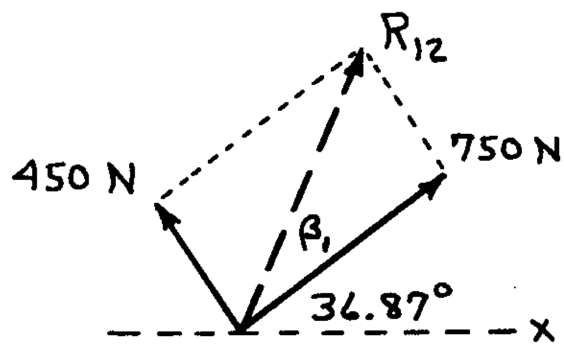


Fig. P2-26

SOLUTION



$\theta_1 = \tan^{-1}\frac{3}{4} = 36.87^\circ$

$\theta_2 = \tan^{-1}\frac{2}{1} = 63.43^\circ$

$\theta_3 = \tan^{-1}\frac{2}{5} = 21.80^\circ$

$\phi_1 = 180^\circ - 36.87^\circ - 63.43^\circ = 79.70^\circ$

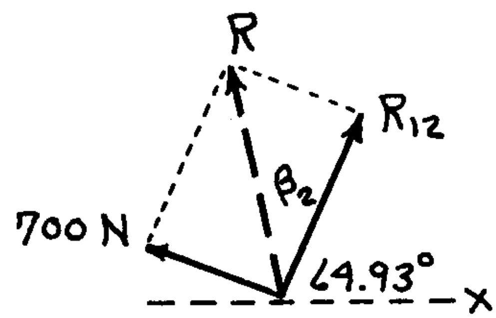
From Eqs. 2-1 and 2-2:

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1 = 750^2 + 450^2 + 2(750)(450) \cos 79.70^\circ$

$R_{12} = 941.11 \text{ N} \cong 941 \text{ N}$

$\beta_1 = \sin^{-1}\frac{F_2 \sin \phi_1}{R_{12}}$

$= \sin^{-1}\frac{450 \sin 79.70^\circ}{941.11} = 28.06^\circ$



Similarly:

$\phi_2 = 180^\circ - \theta_1 - \theta_3 - \beta_1 = 180^\circ - 36.87^\circ - 21.80^\circ - 28.06^\circ = 93.27^\circ$

$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$

$= 941.11^2 + 700^2 + 2(941.11)(700) \cos 93.27^\circ$

$R = 1140.4 \text{ N} \cong 1140 \text{ N}$

$\beta_2 = \sin^{-1}\frac{F_3 \sin \phi_2}{R}$

$= \sin^{-1}\frac{700 \sin 93.27^\circ}{1140.4} = 37.79^\circ$

$\theta = \theta_1 + \beta_1 + \beta_2 = 36.87^\circ + 28.06^\circ + 37.79^\circ = 102.72^\circ \cong 102.7^\circ$

$\bar{R} = 1140 \text{ N} \measuredangle 77.3^\circ$ Ans.

2-27 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-27.

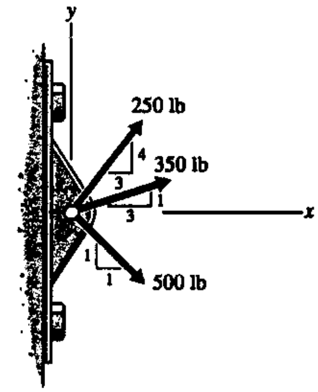


Fig. P2-27

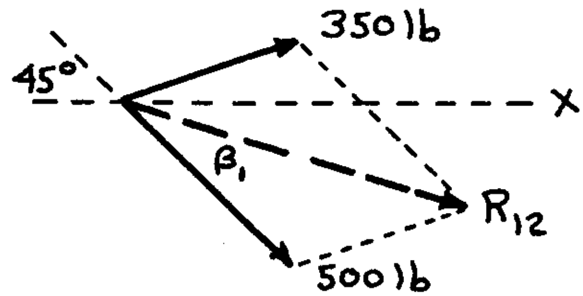
SOLUTION

$\theta_1 = \tan^{-1}\frac{1}{1} = 45.00^\circ$

$\theta_2 = \tan^{-1}\frac{1}{3} = 18.43^\circ$

$\theta_3 = \tan^{-1}\frac{4}{3} = 53.13^\circ$

$\phi_1 = 45.00^\circ + 18.43^\circ = 63.43^\circ$



From Eq. 2-1:

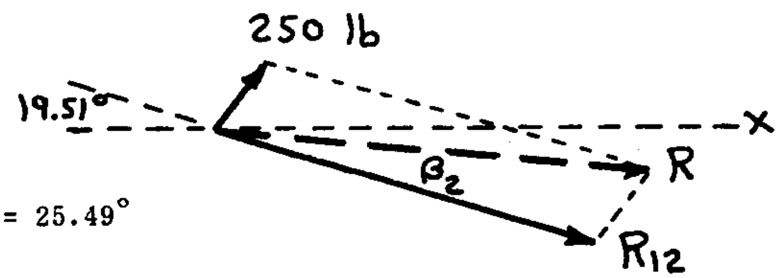
$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\phi_1 = 500^2 + 350^2 + 2(500)(350)\cos 63.43^\circ$

$R_{12} = 727.36 \text{ lb} \cong 727 \text{ lb}$

From Eq. 2-2:

$\beta_1 = \sin^{-1}\frac{F_2 \sin\phi_1}{R_{12}}$

$= \sin^{-1}\frac{350 \sin 63.43^\circ}{727.36} = 25.49^\circ$



Similarly:

$\phi_2 = \theta_3 + 45^\circ - \beta_1 = 53.13^\circ + 45^\circ - 25.49^\circ = 72.64^\circ$

$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos\phi_2$

$= 727.36^2 + 250^2 + 2(727.36)(250)\cos 72.64^\circ$

$R = 836.70 \text{ lb} \cong 837 \text{ lb}$

$\beta_2 = \sin^{-1}\frac{F_3 \sin\phi_2}{R} = \sin^{-1}\frac{250 \sin 72.64^\circ}{836.70} = 16.570^\circ$

$\theta = \beta_1 + \beta_2 - 45^\circ = 25.49^\circ + 16.57^\circ - 45.00^\circ = -2.94^\circ$

$\bar{R} = 837 \text{ lb} \searrow 2.94^\circ$ Ans.

2-28 Determine the magnitude of the resultant $\bar{R}$ and the angle θ between the x axis and the line of action of the resultant for the three forces shown in Fig. P2-28.

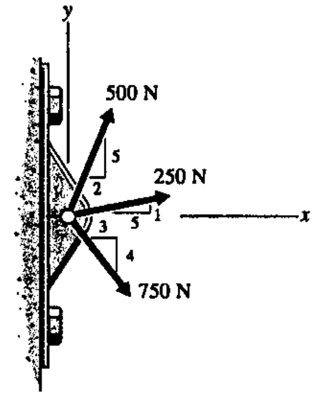
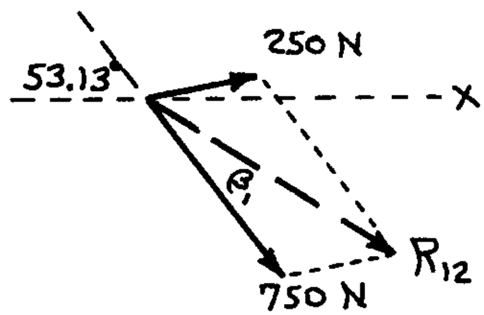


Fig. P2-28

SOLUTION



$\theta_1 = \tan^{-1}\frac{4}{3} = 53.13°$

$\theta_2 = \tan^{-1}\frac{1}{5} = 11.31°$

$\theta_3 = \tan^{-1}\frac{5}{2} = 68.20°$

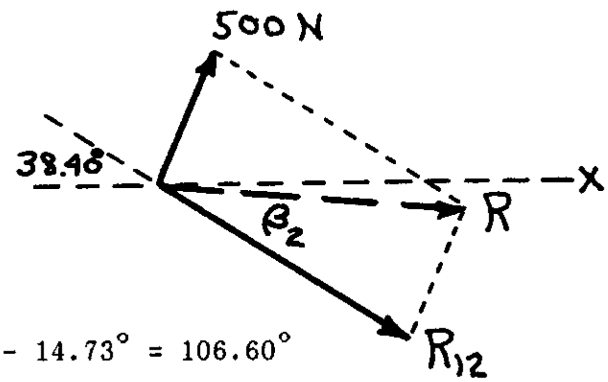
$\phi_1 = 53.13° + 11.31° = 64.44°$

From Eq. 2-1:

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1 = 750^2 + 250^2 + 2(750)(250) \cos 64.44°$

$R_{12} = 887.02 \text{ N} \cong 887 \text{ N}$

From Eq. 2-2:



$\beta_1 = \sin^{-1}\frac{F_2 \sin \phi_1}{R_{12}}$

$= \sin^{-1}\frac{250 \sin 64.44°}{887.02} = 14.730°$

Similarly:

$\phi_2 = \theta_3 + \theta_1 - \beta_1 = 68.20° + 53.13° - 14.73° = 106.60°$

$R^2 = R_{12}^2 + F_3^2 + 2R_{12}F_3 \cos \phi_2$

$= 887.02^2 + 500^2 + 2(887.02)(500) \cos 106.60°$

$R = 885.10 \text{ N} \cong 885 \text{ N}$

$\beta_2 = \sin^{-1}\frac{F_3 \sin \phi_2}{R} = \sin^{-1}\frac{500 \sin 106.60°}{885.10} = 32.78°$

$\theta = \beta_1 + \beta_2 - \theta_1 = 14.730° + 32.78° - 53.13° = -5.62°$

$\bar{R} = 885 \text{ N} \searrow 5.62°$ Ans.

2-29* Determine the magnitude of the resultant $\vec{R}$ and the angle θ between the x axis and the line of action of the resultant for the four forces shown in Fig. P2-29.

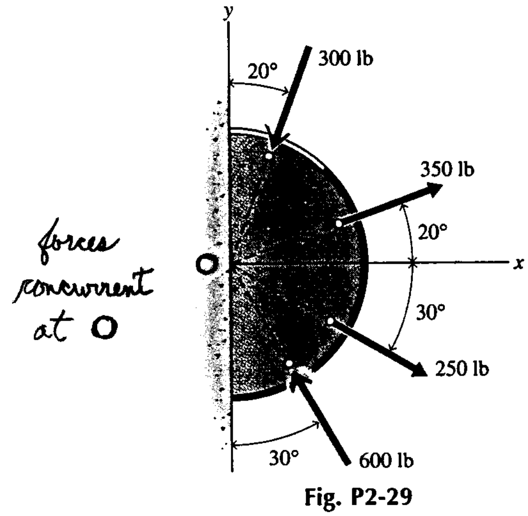


Fig. P2-29

SOLUTION

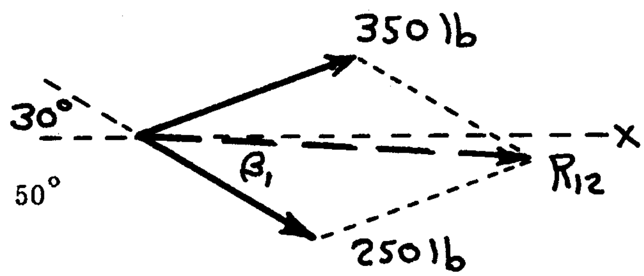
$\phi_1 = 30^\circ + 20^\circ = 50^\circ$

From Eq. 2-1:

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$

$= 250^2 + 350^2 + 2(250)(350) \cos 50^\circ$

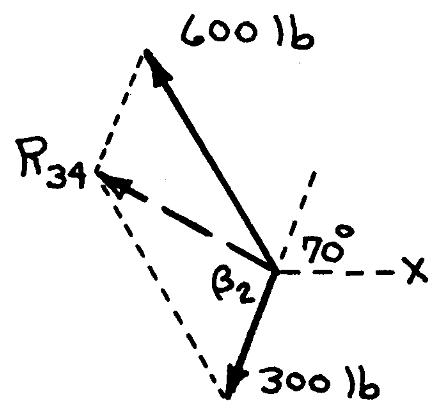
$R_{12} = 545.42$ lb $\cong 545$ lb



From Eq. 2-2:

$\beta_1 = \sin^{-1} \dfrac{F_2 \sin \phi_1}{R}$

$= \sin^{-1} \dfrac{350 \sin 50^\circ}{545.42} = 29.44^\circ$



Similarly:

$\phi_2 = 180^\circ - 20^\circ - 30^\circ = 130^\circ$

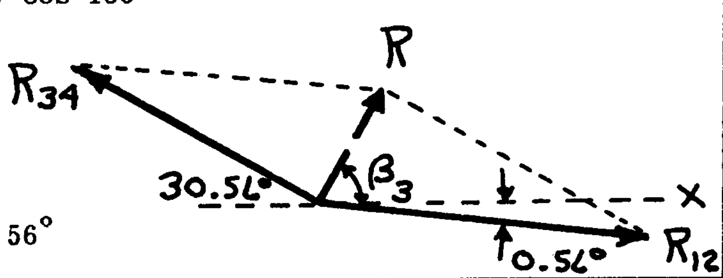
$R_{34}^2 = F_3^2 + F_4^2 + 2F_3F_4 \cos \phi_2$

$= 300^2 + 600^2 + 2(300)(600) \cos 130^\circ$

$R_{34} = 467.54$ lb $\cong 468$ lb

$\beta_2 = \sin^{-1} \dfrac{F_4 \sin \phi_2}{R_{34}}$

$= \sin^{-1} \dfrac{600 \sin 130^\circ}{467.54} = 100.56^\circ$



2-29* (Continued)

$$\phi_3 = 360^\circ - 80^\circ - \beta_1 - \beta_2$$
$$= 360^\circ - 80^\circ - 29.44^\circ - 100.56^\circ = 150.00^\circ$$

$$R^2 = R_{12}^2 + R_{34}^2 + 2R_{12}R_{34}\cos\phi_3$$
$$= 545.42^2 + 467.54^2 + 2(545.42)(467.54)\cos 150^\circ$$

$$R = 272.75 \text{ lb} \cong 273 \text{ lb}$$

$$\beta_3 = \sin^{-1}\frac{F_{34}\sin\phi_3}{R}$$
$$= \sin^{-1}\frac{467.54\sin 150^\circ}{272.75} = 58.99^\circ$$

$$\theta = \beta_1 + \beta_3 - 30^\circ$$
$$= 29.44^\circ + 58.99^\circ - 30^\circ$$
$$= 58.43^\circ \cong 58.4^\circ$$

$$\vec{R} = 273 \text{ lb} \measuredangle 58.4^\circ \quad \text{Ans.}$$

2-30 Determine the magnitude of the resultant $\overline{\mathbf{R}}$ and the angle θ between the x axis and the line of action of the resultant for the four forces shown in Fig. P2-30.

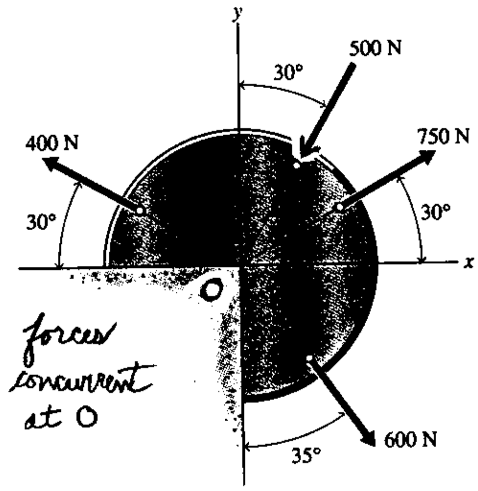
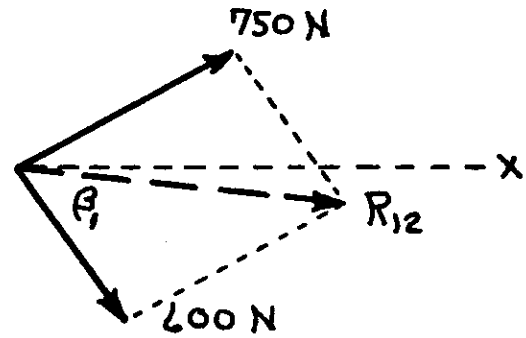


Fig. P2-30

SOLUTION



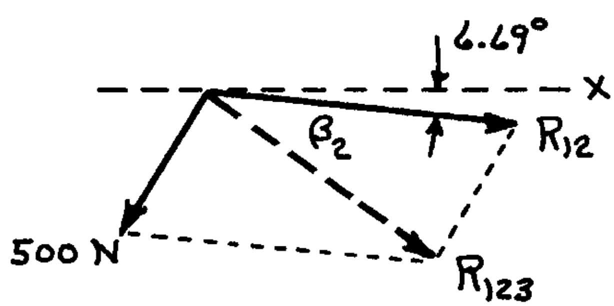
$\phi_1 = 90^\circ + 30^\circ - 35^\circ = 85^\circ$

From Eq. 2-1:

$R_{12}^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \phi_1$

$= 600^2 + 750^2 + 2(600)(750) \cos 85^\circ$

$R_{12} = 1000.47 \text{ N} \cong 1000 \text{ N}$



From Eq. 2-2:

$$\beta_1 = \sin^{-1} \frac{F_2 \sin \phi_1}{R_{12}}$$

$$= \sin^{-1} \frac{750 \sin 85^\circ}{1000.47} = 48.31^\circ$$

Similarly:

$\phi_2 = \beta_1 + 35^\circ + 30^\circ = 48.31 + 35^\circ + 30^\circ = 113.31^\circ$

$R_{123}^2 = R_{12}^2 + F_3^2 + 2R_{13}F_3 \cos \phi_2$

$= 1000.47^2 + 500^2 + 2(1000.47)(500) \cos 113.31^\circ$

$R_{123} = 924.69 \text{ N} \cong 925 \text{ N}$

$$\beta_2 = \sin^{-1} \frac{F_3 \sin \phi_2}{R_{123}} = \sin^{-1} \frac{500 \sin 113.31^\circ}{924.69} = 29.77^\circ$$

2-30 (Continued)

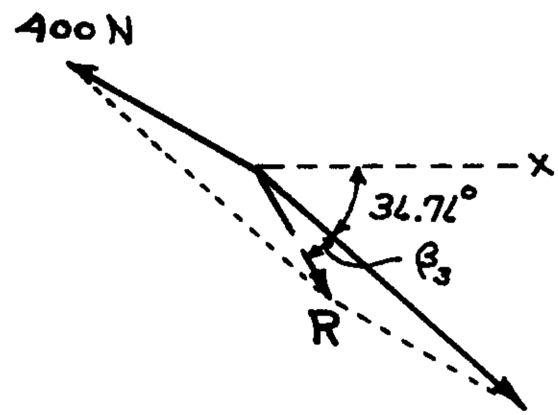
$$\phi_3 = 90^\circ + 30^\circ + 35^\circ + \beta_1 - \beta_2$$

$$= 155^\circ + 48.31^\circ - 29.77^\circ = 173.54^\circ$$

$$R^2 = R_{123}^2 + F_4^2 + 2R_{123}F_4 \cos \phi_3$$

$$= 924.69^2 + 400^2 + 2(924.69)(400) \cos 173.54^\circ$$

$$R = 529.15 \text{ N} \cong 529 \text{ N}$$



$$\beta_3 = \sin^{-1} \frac{F_4 \sin \phi_3}{R}$$

$$= \sin^{-1} \frac{400 \sin 173.54^\circ}{529.15} = 4.88^\circ$$

$$\theta = -55^\circ + \beta_1 - \beta_2 - \beta_3$$

$$= -55^\circ + 48.31^\circ - 29.77^\circ - 4.88^\circ$$

$$= -41.34^\circ \cong -41.3^\circ$$

$\mathbf{R} = 529 \text{ N} \searrow 41.3^\circ$ Ans.

2-31* Determine the magnitudes of the u and v components of the 750-lb force shown in Fig. P2-31.

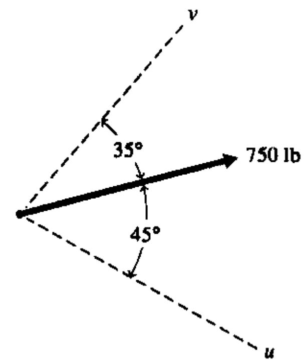


Fig. P2-31

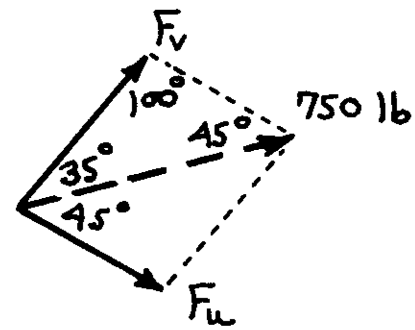
SOLUTION

From the law of sines:

$$\frac{F_u}{\sin 35^\circ} = \frac{F_v}{\sin 45^\circ} = \frac{750}{\sin 100^\circ}$$

$$F_u = \frac{750}{\sin 100^\circ} \sin 35^\circ = 437 \text{ lb}$$

$$F_v = \frac{750}{\sin 100^\circ} \sin 45^\circ = 539 \text{ lb}$$



2-32* Determine the magnitudes of the u and v components of the 1000-N force shown in Fig. P2-32.

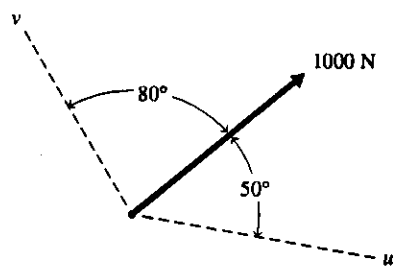


Fig. P2-32

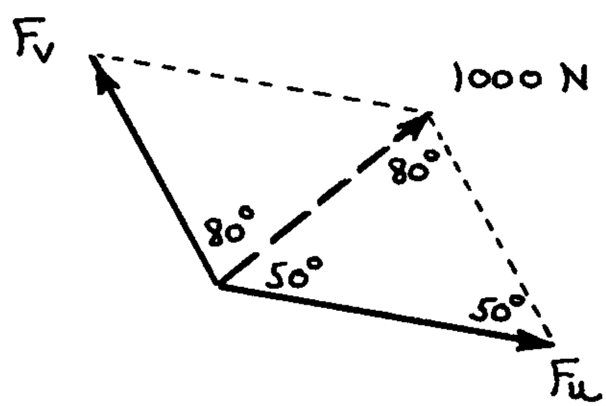
SOLUTION

From the law of sines:

$$\frac{F_u}{\sin 80^\circ} = \frac{F_v}{\sin 50^\circ} = \frac{1000}{\sin 50^\circ}$$

$$F_u = \frac{1000}{\sin 50^\circ} \sin 80^\circ = 1286 \text{ N}$$

$$F_v = \frac{1000}{\sin 50^\circ} \sin 50^\circ = 1000 \text{ N}$$



2-33 Determine the magnitudes of the u and v components of the 850-lb force shown in Fig. P2-33.

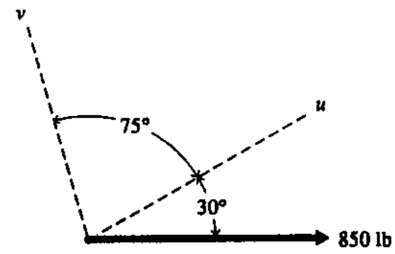


Fig. P2-33

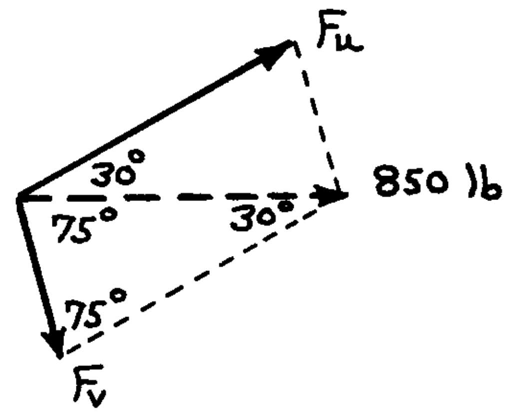
SOLUTION

From the law of sines:

$$\frac{F_u}{\sin 75^\circ} = \frac{F_v}{\sin 30^\circ} = \frac{850}{\sin 75^\circ}$$

$$F_u = \frac{850}{\sin 75^\circ} \sin 75^\circ = 850 \text{ lb}$$

$$F_v = \frac{850}{\sin 75^\circ} \sin 30^\circ = 440 \text{ lb}$$



2-34 Determine the magnitudes of the u and v components of the 1500-N force shown in Fig. P2-34.

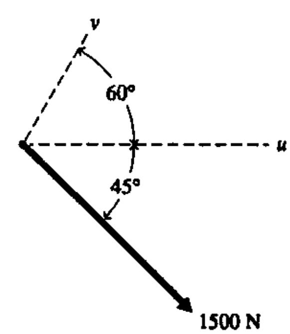


Fig. P2-34

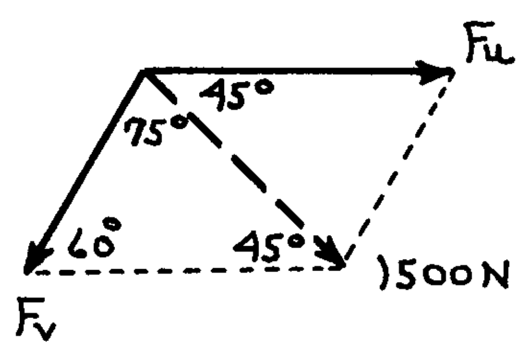
SOLUTION

From the law of sines:

$$\frac{F_u}{\sin 75^\circ} = \frac{F_v}{\sin 45^\circ} = \frac{1500}{\sin 60^\circ}$$

$$F_u = \frac{1500}{\sin 60^\circ} \sin 75^\circ = 1673 \text{ N}$$

$$F_v = \frac{1500}{\sin 60^\circ} \sin 45^\circ = 1225 \text{ N}$$



2-35* Two forces $\bar{F}_u$ and $\bar{F}_v$ are applied to a bracket as shown in Fig. P2-35. The resultant $\bar{R}$ of the two forces has a magnitude of 600 lb, and the angle between the line of action of the resultant and the y-axis is 28°. Determine the magnitudes of forces $\bar{F}_u$ and $\bar{F}_v$.

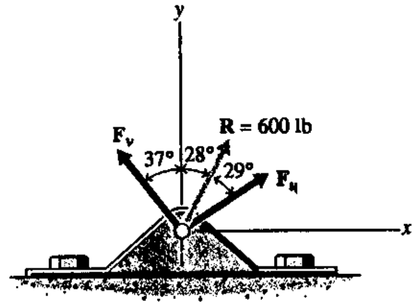


Fig. P2-35

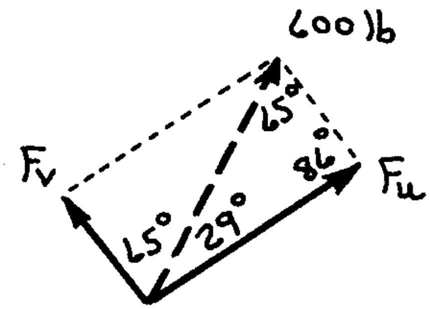
SOLUTION

From the law of sines:

$$\frac{F_u}{\sin 65^\circ} = \frac{F_v}{\sin 29^\circ} = \frac{600}{\sin 86^\circ}$$

$$F_u = \frac{600}{\sin 86^\circ} \sin 37^\circ = 545 \text{ lb}$$

$$F_v = \frac{600}{\sin 86^\circ} \sin 29^\circ = 292 \text{ lb}$$



2-36* Two forces $\bar{F}_u$ and $\bar{F}_v$ are applied to a circular plate as shown in Fig. P2-36. The resultant $\bar{R}$ of the two forces has a magnitude of 900 N, and the angle between the line of action of the resultant and the x-axis is 30°. Determine the magnitudes of forces $\bar{F}_u$ and $\bar{F}_v$.

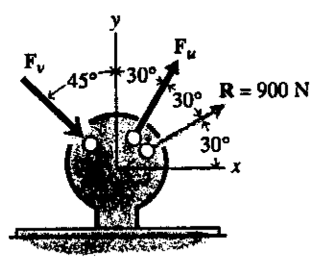


Fig. P2-36

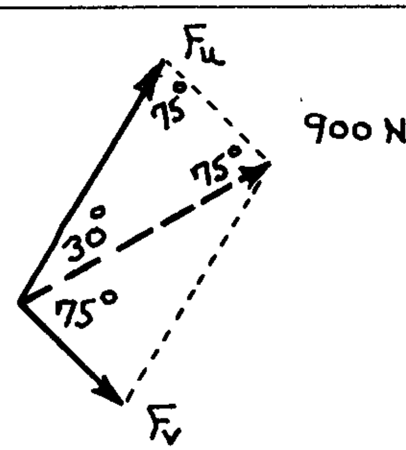
SOLUTION

From the law of sines:

$$\frac{F_u}{\sin 75^\circ} = \frac{F_v}{\sin 30^\circ} = \frac{900}{\sin 75^\circ}$$

$$F_u = \frac{900}{\sin 75^\circ} \sin 75^\circ = 900 \text{ N}$$

$$F_v = \frac{900}{\sin 75^\circ} \sin 30^\circ = 466 \text{ N}$$



2-37 Two ropes are used to tow a boat upstream as shown in Fig. P2-37. The resultant $\bar{R}$ of the rope forces $\bar{F}_u$ and $\bar{F}_v$ has a magnitude of 400 lb and its line of action is directed along the axis of the boat. Determine the magnitudes of forces $\bar{F}_u$ and $\bar{F}_v$.

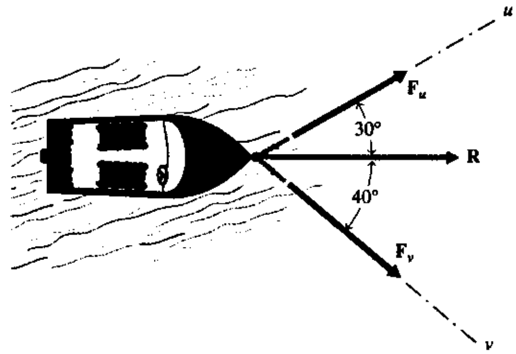


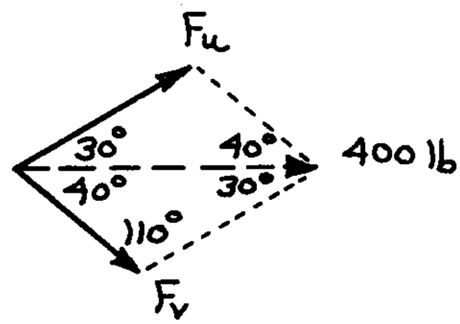
Fig. P2-37

SOLUTION

$$\frac{F_u}{\sin 40^\circ} = \frac{F_v}{\sin 30^\circ} = \frac{400}{\sin 110^\circ}$$

$$F_u = \frac{400}{\sin 110^\circ} \sin 40^\circ = 274 \text{ lb}$$

$$F_v = \frac{400}{\sin 110^\circ} \sin 30^\circ = 213 \text{ lb}$$



2-38 Two cables are used to support a stoplight as shown in Fig. P2-38. The resultant $\bar{R}$ of the cable forces $\bar{F}_u$ and $\bar{F}_v$ has a magnitude of 1350 N and its line of action is vertical. Determine the magnitudes of forces $\bar{F}_u$ and $\bar{F}_v$.

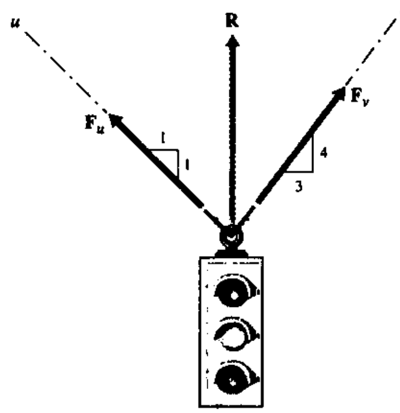


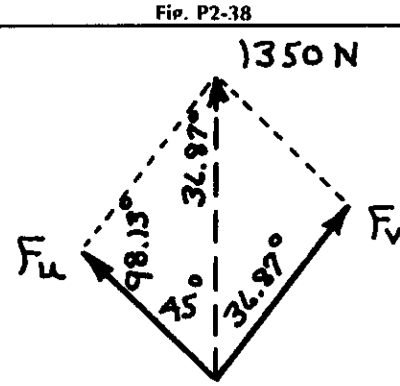
Fig. P2-38

SOLUTION

$$\frac{F_u}{\sin 36.87^\circ} = \frac{F_v}{\sin 45^\circ} = \frac{1350}{\sin 98.13^\circ}$$

$$F_u = \frac{1350}{\sin 98.13^\circ} \sin 36.87^\circ = 818 \text{ N}$$

$$F_v = \frac{1350}{\sin 98.13^\circ} \sin 45^\circ = 964 \text{ N}$$



2-39* Two forces $\bar{F}_u$ and $\bar{F}_v$ are applied to a bracket as shown in Fig. P2-39. If the resultant $\bar{R}$ of the two forces has a magnitude of 725 lb and a direction as shown on the figure, determine the magnitudes of forces $\bar{F}_u$ and $\bar{F}_v$.

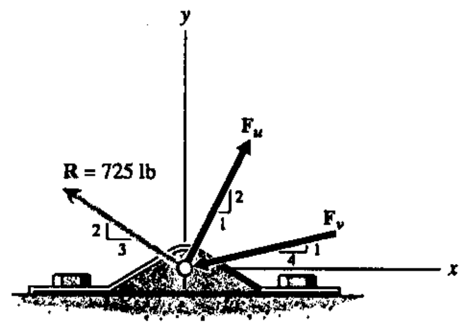


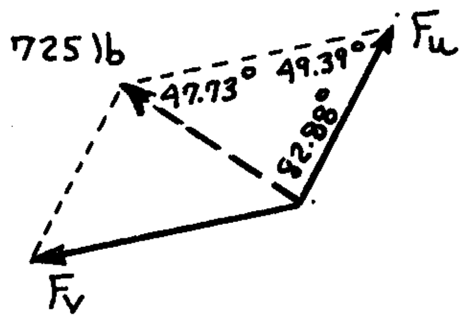
Fig. P2-39

SOLUTION

$$\frac{F_u}{\sin 47.73^\circ} = \frac{F_v}{\sin 82.88^\circ} = \frac{725}{\sin 49.39^\circ}$$

$$F_u = \frac{725}{\sin 49.39^\circ} \sin 47.73^\circ = 707 \text{ lb}$$

$$F_v = \frac{725}{\sin 49.39^\circ} \sin 82.88^\circ = 948 \text{ lb}$$



2-40* Two forces $\bar{F}_u$ and $\bar{F}_v$ are applied to a bracket as shown in Fig. P2-40. If the resultant $\bar{R}$ of the two forces has a magnitude of 375 N and a direction as shown on the figure, determine the magnitudes of forces $\bar{F}_u$ and $\bar{F}_v$.

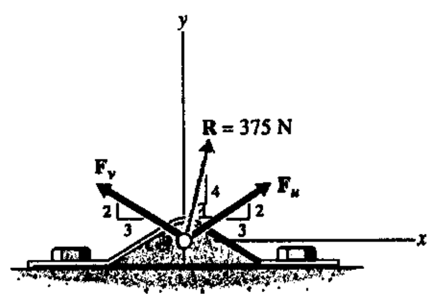


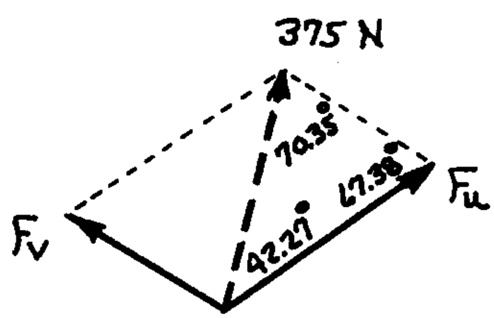
Fig. P2-40

SOLUTION

$$\frac{F_u}{\sin 70.35^\circ} = \frac{F_v}{\sin 42.27^\circ} = \frac{375}{\sin 67.38^\circ}$$

$$F_u = \frac{375}{\sin 67.38^\circ} \sin 70.35^\circ = 383 \text{ N}$$

$$F_v = \frac{375}{\sin 67.38^\circ} \sin 42.27^\circ = 273 \text{ N}$$



2-41 A 2000-lb force is resisted by two pipe struts as shown in Fig. P2-41. Determine the component $\mathbf{F}_u$ of the force along the axis of strut AB and the component $\mathbf{F}_v$ of the force along the axis of strut BC.

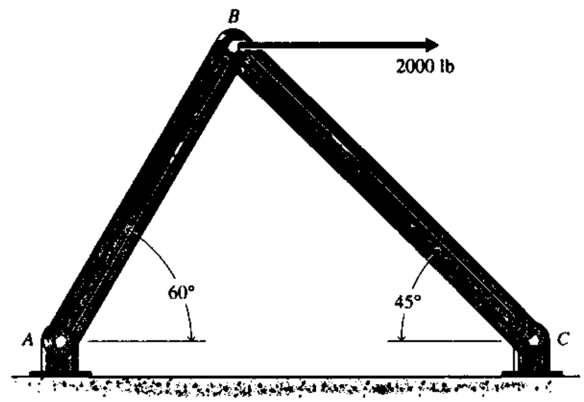
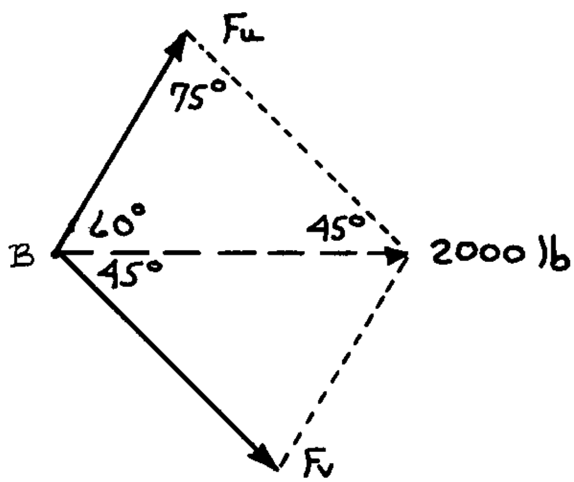


Fig. P2-41

SOLUTION

From the law of sines:



$$\frac{F_u}{\sin 45^\circ} = \frac{F_v}{\sin 60^\circ} = \frac{2000}{\sin 75^\circ}$$

$$F_u = \frac{2000}{\sin 75^\circ} \sin 45^\circ = 1464 \text{ lb}$$

$\mathbf{F}_u$ = 1464 lb ∠ 60° Ans.

$$F_v = \frac{2000}{\sin 75^\circ} \sin 60^\circ = 1793 \text{ lb}$$

$\mathbf{F}_v$ = 1793 lb ⦩ 45° Ans.

2-42 A 750-N force is resisted by two pipe struts as shown in Fig. P2-42. Determine the component $\bar{F}_u$ of the force along the axis of strut AC and the component $\bar{F}_v$ of the force along the axis of strut BC.

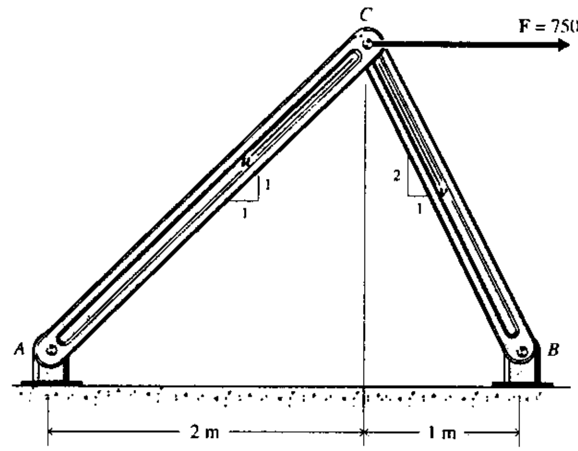


Fig. P2-42

SOLUTION

From the law of sines:

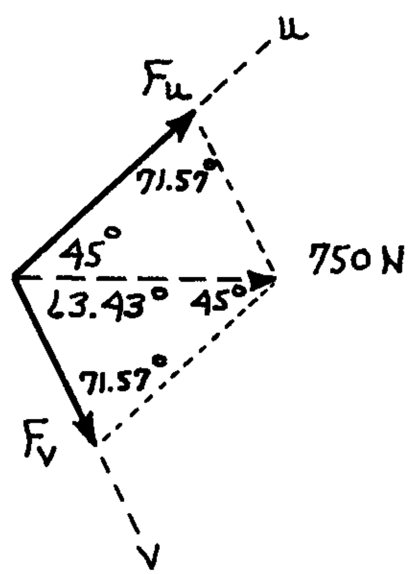
$$\frac{F_u}{\sin 63.43^\circ} = \frac{F_v}{\sin 45^\circ} = \frac{750}{\sin 71.57^\circ}$$

$$F_u = \frac{750}{\sin 71.57^\circ} \sin 63.43^\circ = 707 \text{ N}$$

$\bar{F}_u$ = 707 N ∠ 45° Ans.

$$F_v = \frac{750}{\sin 71.57^\circ} \sin 45^\circ = 559 \text{ N}$$

$\bar{F}_v$ = 559 N ↘ 63.4° Ans.



2-43* Three forces are applied to a bracket as shown in Fig. P2-43. The magnitude of the resultant $\bar{R}$ of the three forces is 5000 lb. If the force $\bar{F}_1$ has a magnitude of 3000 lb, determine the magnitudes of forces $\bar{F}_2$ and $\bar{F}_3$.

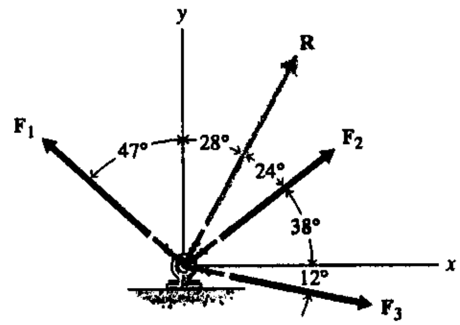
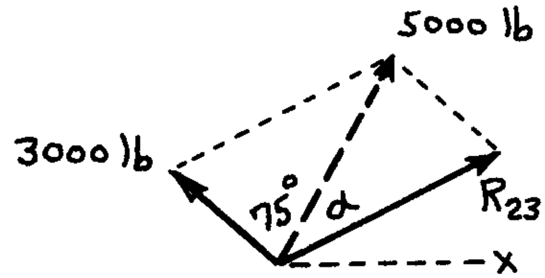


Fig. P2-43

SOLUTION

For the system of forces:

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = \bar{F}_1 + \bar{R}_{23}$$

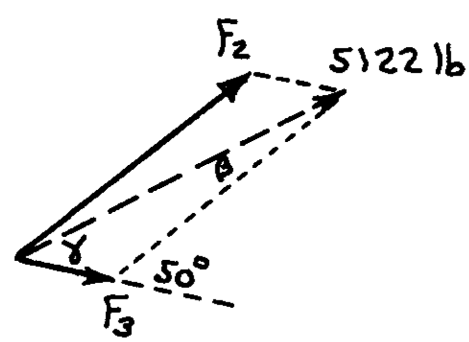


From the parallelogram of forces constructed using $\bar{R}$ and $\bar{F}_1$:

$$R_{23} = \sqrt{5000^2 + 3000^2 - 2(5000)(3000)\cos 75^\circ} = 5122 \text{ lb}$$

$$\frac{3000}{\sin\alpha} = \frac{R_{23}}{\sin 75^\circ} = \frac{5122}{\sin 75^\circ}$$

$$\alpha = \sin^{-1}\frac{3000 \sin 75^\circ}{5122} = 34.45^\circ$$



For the second parallelogram:

$$\beta = \alpha - 24^\circ = 34.45 - 24 = 10.45^\circ$$

$$\gamma = 74^\circ - \alpha = 74 - 34.45 = 39.55^\circ$$

$$\frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\beta} = \frac{5122}{\sin 130^\circ}$$

$$F_2 = \frac{5122}{\sin 130^\circ}\sin\gamma = \frac{5122}{\sin 130^\circ}\sin 39.55^\circ = 4257.5 \text{ lb} \cong 4260 \text{ lb}$$ Ans.

$$F_3 = \frac{5122}{\sin 130^\circ}\sin\beta = \frac{5122}{\sin 130^\circ}\sin 10.45^\circ = 1212.7 \text{ lb} \cong 1213 \text{ lb}$$ Ans.

2-44 A gusset plate is used to transfer forces from three bars to a beam as shown in Fig. P2-44. The magnitude of the resultant $\bar{R}$ of the three forces is 5000 N. If the force $\bar{F}_1$ has a magnitude of 1000 N, determine the magnitudes of forces $\bar{F}_2$ and $\bar{F}_3$.

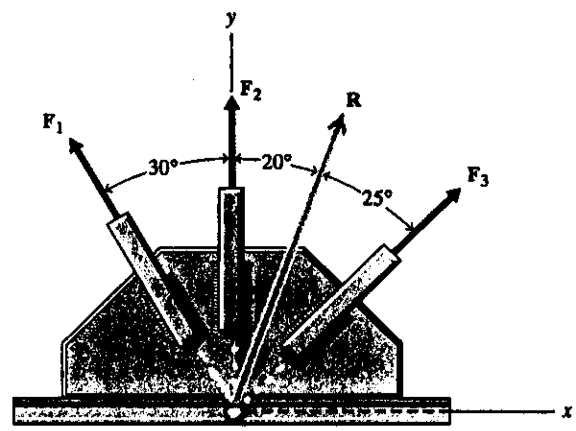
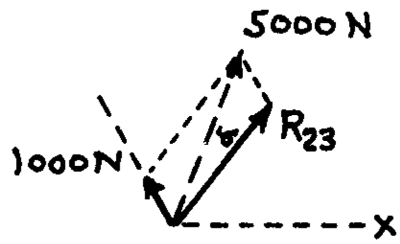


Fig. P2-44

SOLUTION

For the system of forces:



$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = \bar{F}_1 + \bar{R}_{23}$$

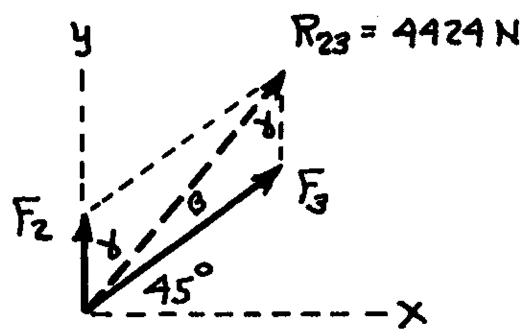
From the parallelogram of forces constructed using $\bar{R}$ and $\bar{F}_1$:

$$R_{23} = \sqrt{1000^2 + 5000^2 - 2(1000)(5000)\cos 50^\circ} = 4424 \text{ N}$$

$$\frac{1000}{\sin\alpha} = \frac{R_{23}}{\sin 50^\circ} = \frac{4424}{\sin 50^\circ}$$

$$\alpha = \sin^{-1}\frac{1000 \sin 50^\circ}{4424} = 9.97^\circ$$

For the second parallelogram:



$$\beta = 25^\circ - \alpha = 25 - 9.97 = 15.03^\circ$$

$$\gamma = 20^\circ + \alpha = 20 + 9.97 = 29.97^\circ$$

$$\frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} = \frac{4424}{\sin 135^\circ}$$

$$F_2 = \frac{4424}{\sin 135^\circ}\sin\beta = \frac{4424}{\sin 135^\circ}\sin 15.03^\circ = 1622 \text{ N}$$ Ans.

$$F_3 = \frac{4424}{\sin 135^\circ}\sin\gamma = \frac{4424}{\sin 135^\circ}\sin 29.97^\circ = 3125 \text{ N}$$ Ans.

2-45* Determine the x and y scalar components of the force shown in Fig. P2-45.

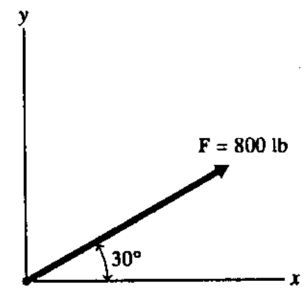


Fig. P2-45

SOLUTION

$F_x = F \cos \theta_x = 800 \cos 30^\circ = 692.8 \text{ lb} \cong 693 \text{ lb}$ Ans.

$F_y = F \sin \theta_x = 800 \sin 30^\circ = 400 \text{ lb}$ Ans.

2-46* Determine the x and y scalar components of the force shown in Fig. P2-46.

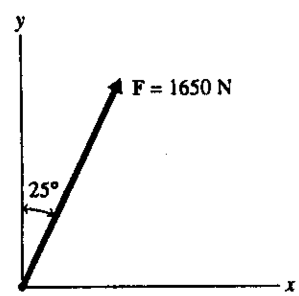


Fig. P2-46

SOLUTION

$F_x = F \cos \theta_x = 1650 \cos (90^\circ - 25^\circ)$

$= 1650 \cos 65^\circ = 697.3 \text{ N} \cong 697 \text{ N}$ Ans.

$F_y = F \sin \theta_x = 1650 \sin (90^\circ - 25^\circ)$

$= 1650 \sin 65^\circ = 1495.4 \text{ N} \cong 1495 \text{ N}$ Ans.

2-47 Determine the x and y scalar components of the force shown in Fig. P2-47.

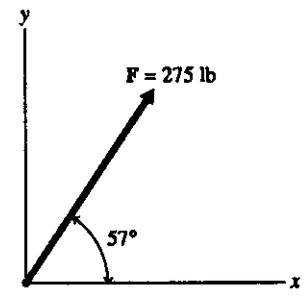


Fig. P2-47

SOLUTION

$F_x = F \cos \theta_x = 275 \cos 57^\circ = 149.98 \text{ lb} \cong 150.0 \text{ lb}$ Ans.

$F_y = F \sin \theta_x = 275 \sin 57^\circ = 230.6 \text{ lb} \cong 231 \text{ lb}$ Ans.

2-48 Determine the x and y scalar components of the force shown in Fig. P2-48.

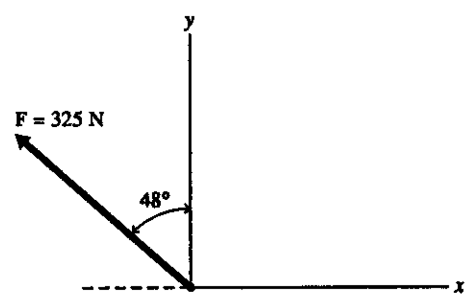


Fig. P2-48

SOLUTION

$F_x = F \cos \theta_x = 325 \cos (90 + 48^\circ)$

$= -325 \cos 42^\circ = 241.5 \text{ N} \cong 242 \text{ N}$ Ans.

$F_y = F \sin \theta_x = 325 \sin (90 + 48^\circ)$

$= 325 \sin 42^\circ = 217.4 \text{ N} \cong 217 \text{ N}$ Ans.

2-49* Determine the x and y scalar components of the force shown in Fig. P2-49.

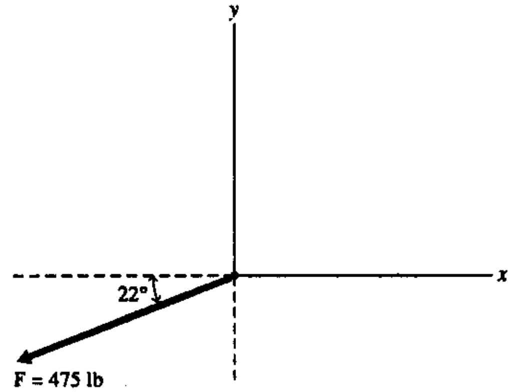


Fig. P2-49

SOLUTION

$F_x = F \cos \theta_x = 475 \cos (180^\circ + 22^\circ)$

$= -475 \cos 22^\circ = -440.4 \text{ lb} \cong -440 \text{ lb}$ Ans.

$F_y = F \sin \theta_x = 475 \sin (180^\circ + 22^\circ)$

$= -475 \sin 22^\circ = -177.94 \text{ lb} \cong -177.9 \text{ lb}$ Ans.

2-50* Determine the x and y scalar components of the force shown in Fig. P2-50.

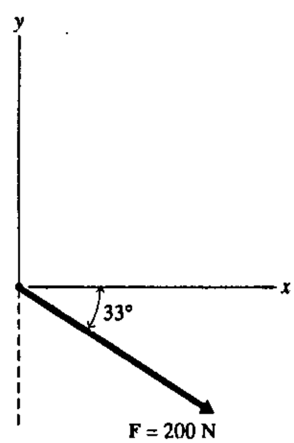


Fig. P2-50

SOLUTION

$F_x = F \cos \theta_x = 200 \cos (-33^\circ)$

$= 200 \cos 33^\circ = 167.73 \text{ N} \cong 167.7 \text{ N}$ Ans.

$F_y = F \sin \theta_x = 200 \sin (-33^\circ)$

$= -200 \sin 33^\circ = -108.93 \text{ N} \cong -108.9 \text{ N}$ Ans.

2-51 Determine the x and y scalar components of the force shown in Fig. P2-51.

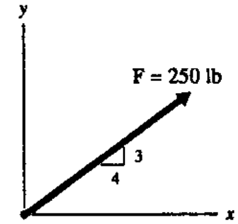


Fig. P2-51

SOLUTION

$F_x = F \cos \theta_x = 250(4/5) = 200$ lb   Ans.

$F_y = F \sin \theta_x = 250(3/5) = 150$ lb   Ans.

2-52 Determine the x and y scalar components of the force shown in Fig. P2-52.

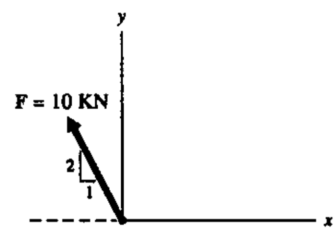


Fig. P2-52

SOLUTION

$F_x = F \cos \theta_x = 10(-1/\sqrt{5}) = -4.472$ kN $\cong$ -4.47 kN   Ans.

$F_y = F \sin \theta_x = 10(2/\sqrt{5}) = 8.944$ kN $\cong$ 8.94 kN   Ans.

2-53* Determine the x and y scalar components of the force shown in Fig. P2-53.

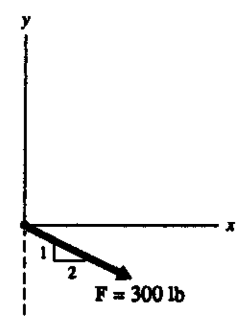


Fig. P2-53

SOLUTION

$F_x = F \cos \theta_x = 300(2/\sqrt{5}) = 268.3$ lb $\cong$ 268 lb   Ans.

$F_y = F \sin \theta_x = 300(-1/\sqrt{5}) = -134.16$ lb $\cong$ -134.2 lb   Ans.

2-54* Determine the x and y scalar components of the force shown in Fig. P2-54.

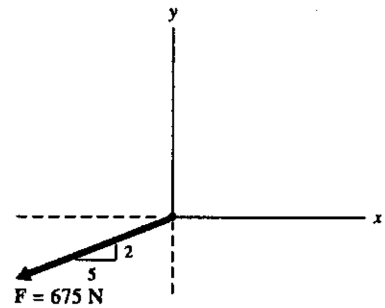


Fig. P2-54

SOLUTION

$F_x = F \cos \theta_x = 675(-5/\sqrt{29}) = -626.7 \text{ N} \cong -627 \text{ N}$ Ans.

$F_y = F \sin \theta_x = 675(-2/\sqrt{29}) = -250.7 \text{ N} \cong -251 \text{ N}$ Ans.

2-55* For the force shown in Fig. P2-55
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

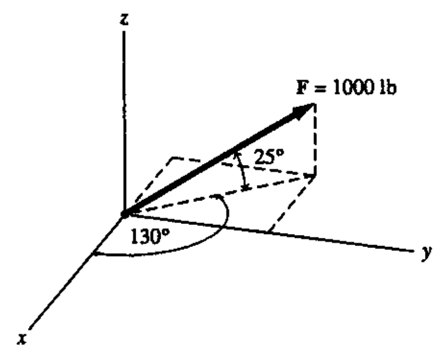


Fig. P2-55

SOLUTION

(a) $F_{xy} = F \cos \phi = 1000 \cos 25^\circ = 906.3 \text{ lb}$

$F_z = F \sin \phi = 1000 \sin 25^\circ = 422.6 \text{ lb} \cong 423 \text{ lb}$ Ans.

$F_x = F_{xy} \cos \theta = 906.3 \cos 130^\circ = -582.6 \text{ lb} \cong -583 \text{ lb}$ Ans.

$F_y = F_{xy} \sin \theta = 906.3 \sin 130^\circ = 694.2 \text{ lb} \cong 694 \text{ lb}$ Ans.

(b) $\mathbf{F} = -583\,\hat{i} + 694\,\hat{j} + 423\,\hat{k}$ Ans.

2-56* For the force shown in Fig. P2-56
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

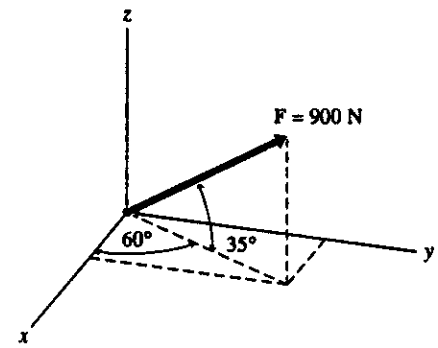


Fig. P2-56

SOLUTION

(a) $F_{xy} = F \cos \phi = 900 \cos 35^\circ = 737.2$ N

$F_z = F \sin \phi = 900 \sin 35^\circ = 516.2 \text{ N} \cong 516 \text{ N}$ Ans.

$F_x = F_{xy} \cos \theta = 737.2 \cos 60^\circ = 368.6 \text{ N} \cong 369 \text{ N}$ Ans.

$F_y = F_{xy} \sin \theta = 737.2 \sin 60^\circ = 638.4 \text{ N} \cong 638 \text{ N}$ Ans.

(b) $\mathbf{F} = 369\,\hat{i} + 638\,\hat{j} + 516\,\hat{k}$ N Ans.

2-57 For the force shown in Fig. P2-57
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

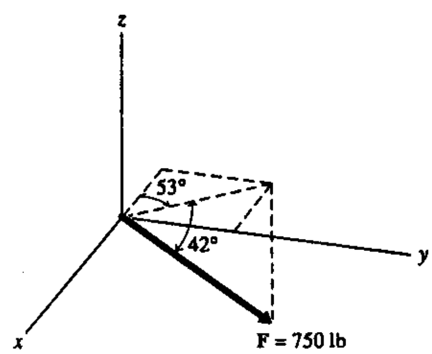


Fig. P2-57

SOLUTION

(a) $F_{xy} = F \cos \phi = 750 \cos (-42^\circ) = 557.36$ lb

$F_z = F \sin \phi = 750 \sin (-42^\circ) = -501.8 \text{ lb} \cong -502 \text{ lb}$ Ans.

$F_x = F_{xy} \cos \theta = 557.36 \cos (180^\circ - 53^\circ) = -335.4 \text{ lb} \cong -335 \text{ lb}$ Ans.

$F_y = F_{xy} \sin \theta = 557.36 \sin (180^\circ - 53^\circ) = 445.1 \text{ lb} \cong 445 \text{ lb}$ Ans.

(b) $\mathbf{F} = -335\,\hat{i} + 445\,\hat{j} - 502\,\hat{k}$ lb Ans.

2-58 For the force shown in Fig. P2-58
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

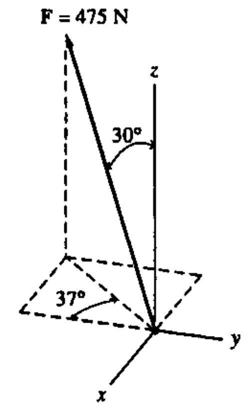


Fig. P2-58

SOLUTION

(a) $F_{xy} = F \cos \phi = 475 \cos 60^\circ = 237.5$ N

$F_z = F \sin \phi = 475 \sin 60^\circ = 411.4 \text{ N} \cong 411 \text{ N}$ Ans.

$F_x = F_{xy} \cos \theta = 237.5 \cos (-127^\circ) = -142.93 \text{ N} \cong -142.9 \text{ N}$ Ans.

$F_y = F_{xy} \sin \theta = 237.5 \sin (-127^\circ) = -189.68 \text{ N} \cong -189.7 \text{ N}$ Ans.

(b) $\mathbf{F} = -142.9\, \hat{i} - 189.7\, \hat{j} + 411\, \hat{k}$ N Ans.

2-59* For the force shown in Fig. P2-59
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

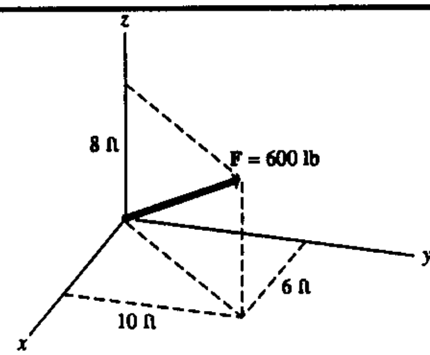


Fig. P2-59

SOLUTION

(a) $d = \sqrt{x^2 + y^2 + z^2} = \sqrt{(6)^2 + (10)^2 + (8)^2} = 14.142$

$F_x = F \cos \theta_x = 600 \left(\frac{6}{14.142}\right) = 254.6 \text{ lb} \cong 255 \text{ lb}$ Ans.

$F_y = F \cos \theta_y = 600 \left(\frac{10}{14.142}\right) = 424.3 \text{ lb} \cong 424 \text{ lb}$ Ans.

$F_x = F \cos \theta_z = 600 \left(\frac{8}{14.142}\right) = 339.4 \text{ lb} \cong 339 \text{ lb}$ Ans.

(b) $\mathbf{F} = 255\, \hat{i} + 424\, \hat{j} + 339\, \hat{k}$ Ans.

2-60* For the force shown in Fig. P2-60
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

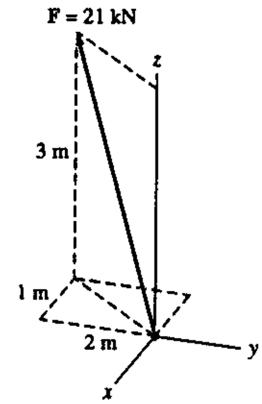


Fig. P2-60

SOLUTION

(a) $d = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-1)^2 + (-2)^2 + (3)^2} = 3.742$ m

$F_x = F \cos \theta_x = 21 \left(\frac{-1}{3.742}\right) = -5.612$ kN $\cong -5.61$ kN Ans.

$F_y = F \cos \theta_y = 21 \left(\frac{-2}{3.742}\right) = -11.224$ kN $\cong -11.22$ kN Ans.

$F_x = F \cos \theta_z = 21 \left(\frac{3}{3.742}\right) = 16.836$ kN $\cong 16.84$ kN Ans.

(b) $\mathbf{F} = -5.61\,\hat{i} - 11.22\,\hat{j} + 16.84\,\hat{k}$ kN Ans.

2-61 For the force shown in Fig. P2-61
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

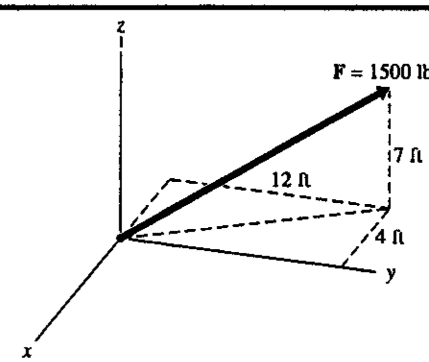


Fig. P2-61

SOLUTION

(a) $d = \sqrt{(-4)^2 + (12)^2 + (7)^2} = \sqrt{209} = 14.457$

$F_x = F \cos \theta_x = 1500(-4/\sqrt{209}) = -415.03$ lb $\cong -415$ lb Ans.

$F_y = F \cos \theta_y = 1500(12/\sqrt{209}) = 1245.09$ lb $\cong 1245$ lb Ans.

$F_x = F \cos \theta_z = 1500(7/\sqrt{209}) = 726.30$ lb $\cong 726$ lb Ans.

(b) $\mathbf{F} = -415\,\hat{i} + 1245\,\hat{j} + 726\,\hat{k}$ lb Ans.

2-62 For the force shown in Fig. P2-62
(a) Determine the x, y, and z scalar components of the force.
(b) Express the force in Cartesian vector form.

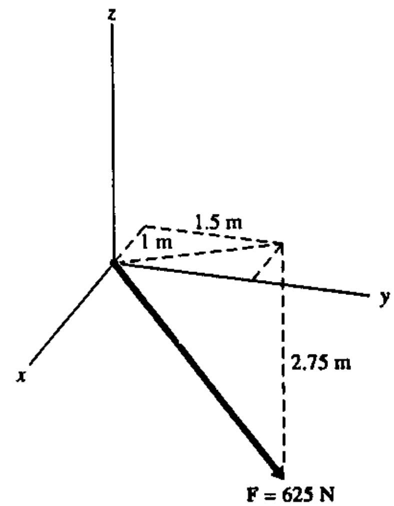


Fig. P2-62

SOLUTION

(a) $d = \sqrt{(-1)^2 + (1.5)^2 + (-2.75)^2} = 3.2882$

$F_x = F \cos \theta_x = 625(-1/3.2882) = -190.07 \text{ N} \cong -190.1 \text{ N}$ Ans.

$F_y = F \cos \theta_y = 625(1.5/3.2882) = 285.1 \text{ N} \cong 285 \text{ N}$ Ans.

$F_z = F \cos \theta_z = 625(-2.75/3.2882) = -192.82 \text{ N} \cong -192.8 \text{ N}$ Ans.

(b) $\mathbf{F} = -190.1\,\hat{\imath} + 285\,\hat{\jmath} - 192.8\,\hat{k}$ N Ans.

2-63* Two forces are applied to an eyebolt as shown in Fig. P2-63.

(a) Determine the x, y, and z scalar components of force $\bar{F}_1$.

(b) Express force $\bar{F}_1$ in Cartesian vector form.

(c) Determine the angle α between forces $\bar{F}_1$ and $\bar{F}_2$.

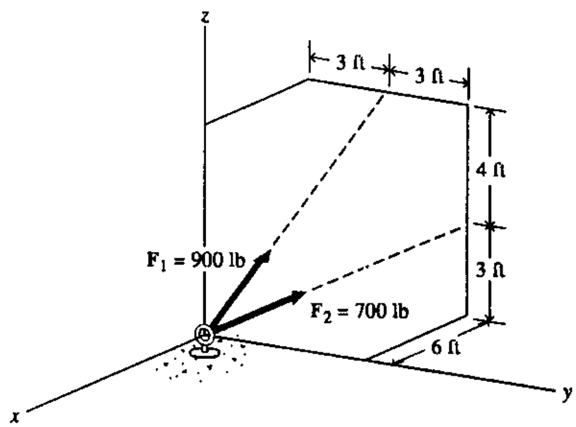


Fig. P2-63

SOLUTION

(a) $d_1 = \sqrt{x_2^2 + y_1^2 + z_1^2} = \sqrt{(-6)^2 + (3)^2 + (7)^2} = 9.695$ ft

$F_{1x} = F_1 \cos\theta_{1x} = 900\left(\frac{-6}{9.695}\right) = -556.99 \text{ lb} \cong -557 \text{ lb}$ Ans.

$F_{1y} = F_1 \cos\theta_{1y} = 900\left(\frac{3}{9.695}\right) = 278.49 \text{ lb} \cong 278 \text{ lb}$ Ans.

$F_{1z} = F_1 \cos\theta_{1z} = 900\left(\frac{7}{9.695}\right) = 649.82 \text{ lb} \cong 650 \text{ lb}$ Ans.

(b) $F_1 = F_{1x}\,\hat{i} + F_{1y}\,\hat{j} + F_{1z}\,\hat{k} = -557\,\hat{i} + 278\,\hat{j} + 650\,\hat{k}$ lb Ans.

(c) $d_2 = \sqrt{x_2^2 + y_2^2 + z_2^2} = \sqrt{(-6)^2 + (6)^2 + (3)^2} = 9.00$ ft

$\hat{e}_2 = \frac{-6}{9}\,\hat{i} + \frac{6}{9}\,\hat{j} + \frac{3}{9}\,\hat{k} = -\frac{2}{3}\,\hat{i} + \frac{2}{3}\,\hat{j} + \frac{1}{3}\,\hat{k}$

$\hat{e}_1 = \frac{-6}{9.695}\,\hat{i} + \frac{3}{9.695}\,\hat{j} + \frac{7}{9.695}\,\hat{k}$

$\cos\alpha = \hat{e}_1 \cdot \hat{e}_2 = \left(-\frac{6}{9.695}\right)\left(-\frac{2}{3}\right) + \left(\frac{3}{9.695}\right)\left(\frac{2}{3}\right) + \left(\frac{7}{9.695}\right)\left(\frac{1}{3}\right) = 0.8595$

$\alpha = 30.73 \cong 30.7^\circ$ Ans.

2-64* Two forces are applied to an eyebolt as shown in Fig. P2-64.

(a) Determine the x, y, and z scalar components of force $\bar{F}_1$.

(b) Express force $\bar{F}_1$ in Cartesian vector form.

(c) Determine the angle α between forces $\bar{F}_1$ and $\bar{F}_2$.

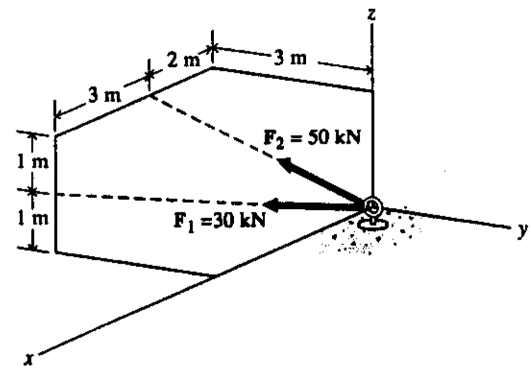


Fig. P2-64

SOLUTION

(a) $d_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} = \sqrt{(5)^2 + (-3)^2 + (1)^2} = 5.916$ m

$F_{1x} = F_1 \cos\theta_{1x} = 30\left(\frac{5}{5.916}\right) = 25.35 \text{ kN} \cong 25.4 \text{ kN}$ Ans.

$F_{1y} = F_1 \cos\theta_{1y} = 30\left(\frac{-3}{5.916}\right) = -15.213 \text{ kN} \cong -15.21 \text{ kN}$ Ans.

$F_{1z} = F_1 \cos\theta_{1z} = 30\left(\frac{1}{5.916}\right) = 5.071 \text{ kN} \cong 5.07 \text{ kN}$ Ans.

(b) $\bar{F}_1 = F_{1x}\,\hat{i} + F_{1y}\,\hat{j} + F_{1z}\,\hat{k} = 25.4\,\hat{i} - 15.21\,\hat{j} + 5.07\,\hat{k}$ kN Ans.

(c) $d_2 = \sqrt{x_2^2 + y_2^2 + z_2^2} = \sqrt{(2)^2 + (-3)^2 + (2)^2} = 4.123$ m

$\hat{e}_2 = \frac{2}{4.123}\,\hat{i} + \frac{-3}{4.123}\,\hat{j} + \frac{2}{4.123}\,\hat{k}$

$\hat{e}_1 = \frac{5}{5.916}\,\hat{i} + \frac{-3}{5.916}\,\hat{j} + \frac{1}{5.916}\,\hat{k}$

$\cos\alpha = \hat{e}_1 \cdot \hat{e}_2 = \left(\frac{5}{5.916}\right)\left(\frac{2}{4.123}\right) + \left(\frac{-3}{5.916}\right)\left(\frac{-3}{4.123}\right) + \left(\frac{1}{5.916}\right)\left(\frac{2}{4.123}\right)$

$= 0.8609$ $\alpha = 30.58 \cong 30.6^\circ$ Ans.

2-65 Two forces are applied to an eyebolt as shown in Fig. P2-65.

(a) Determine the x, y, and z scalar components of force $\bar{F}_2$.

(b) Express force $\bar{F}_2$ in Cartesian vector form.

(c) Determine the magnitude of the rectangular component of force $\bar{F}_2$ along the line of action of force $\bar{F}_1$.

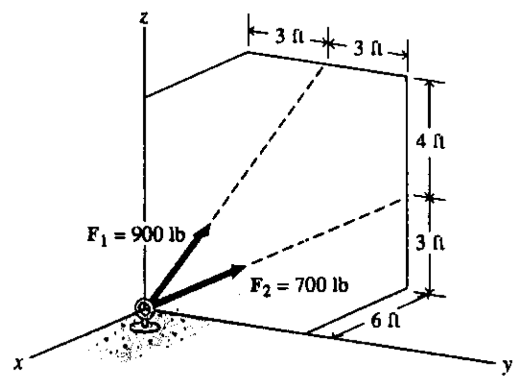


Fig. P2-65

SOLUTION

(a) $d_2 = \sqrt{x_2^2 + y_2^2 + z_2^2} = \sqrt{(-6)^2 + (6)^2 + (3)^2} = 9.00$ ft

$F_{2x} = F_2 \cos\theta_{2x} = 700\left(\frac{-6}{9}\right) = -466.7 \text{ lb} \cong -467 \text{ lb}$ Ans.

$F_{2y} = F_2 \cos\theta_{2y} = 700\left(\frac{6}{9}\right) = 466.7 \text{ lb} \cong 467 \text{ lb}$ Ans.

$F_{2z} = F_2 \cos\theta_{2z} = 700\left(\frac{3}{9}\right) = 233.3 \text{ lb} \cong 233 \text{ lb}$ Ans.

(b) $\bar{F}_2 = F_{2x}\,\hat{i} + F_{2y}\,\hat{j} + F_{2z}\,\hat{k} = -466.7\,\hat{i} + 466.7\,\hat{j} + 233.3\,\hat{k}$ lb

$\cong -467\,\hat{i} + 467\,\hat{j} + 233\,\hat{k}$ lb   Ans.

(c) $d_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} = \sqrt{(-6)^2 + (3)^2 + (7)^2} = 9.695$ ft

$\hat{e}_1 = \frac{-6}{9.695}\,\hat{i} + \frac{3}{9.695}\,\hat{j} + \frac{7}{9.695}\,\hat{k}$

$F_n = \bar{F}_2 \cdot \hat{e}_1 = -466.7\left(\frac{-6}{9.695}\right) + 466.7\left(\frac{3}{9.695}\right) + 233.3\left(\frac{7}{9.695}\right)$

$= 601.7 \text{ lb} \cong 602 \text{ lb}$ Ans.

2-66 Two forces are applied to an eyebolt as shown in Fig. P2-66.

(a) Determine the x, y, and z scalar components of force $\vec{F}_2$.

(b) Express force $\vec{F}_2$ in Cartesian vector form.

(c) Determine the magnitude of the rectangular component of force $\vec{F}_2$ along the line of action of force $\vec{F}_1$.

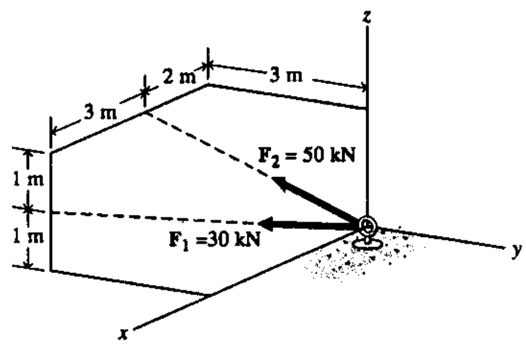


Fig. P2-66

SOLUTION

(a) $d_2 = \sqrt{x_2^2 + y_2^2 + z_2^2} = \sqrt{(2)^2 + (-3)^2 + (2)^2} = 4.123$ m

$F_{2x} = F_2 \cos \theta_{2x} = 50\left(\frac{2}{4.123}\right) = 24.25$ kN $\cong 24.3$ kN Ans.

$F_{2y} = F_2 \cos \theta_{2y} = 50\left(\frac{-3}{4.123}\right) = -36.38$ kN $\cong -36.4$ kN Ans.

$F_{2z} = F_2 \cos \theta_{2z} = 50\left(\frac{2}{4.123}\right) = 24.25$ kN $\cong 24.3$ kN Ans.

(b) $F_2 = F_{2x}\,\hat{i} + F_{2y}\,\hat{j} + F_{2z}\,\hat{k} = 24.25\,\hat{i} - 36.38\,\hat{j} + 24.25\,\hat{k}$ kN

$\cong 24.3\,\hat{i} - 36.4\,\hat{j} + 24.3\,\hat{k}$ kN Ans.

(c) $d_1 = \sqrt{x_1^2 + y_1^2 + z_1^2} = \sqrt{(5)^2 + (-3)^2 + (1)^2} = 5.916$ m

$\hat{e}_1 = \frac{5}{5.916}\,\hat{i} + \frac{-3}{5.916}\,\hat{j} + \frac{1}{5.916}\,\hat{k}$

$F_n = \vec{F}_2 \cdot \hat{e}_1 = 24.25\left(\frac{5}{5.916}\right) - 36.38\left(\frac{-3}{5.916}\right) + 24.25\left(\frac{1}{5.916}\right)$

$= 43.04$ kN $\cong 43.0$ kN Ans.

2-67* Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the two forces shown in Fig. P2-67.

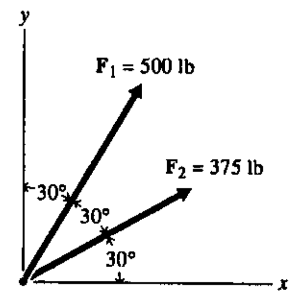


Fig. P2-67

SOLUTION

$$R_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 = 500 \cos 60^\circ + 375 \cos 30^\circ = 574.8 \text{ lb}$$

$$R_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 = 500 \sin 60^\circ + 375 \sin 30^\circ = 620.5 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(574.8)^2 + (620.5)^2} = 845.8 \text{ lb} \cong 846 \text{ lb} \qquad \text{Ans.}$$

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{620.5}{574.8} = 47.19^\circ \cong 47.2^\circ \qquad \text{Ans.}$$

2-68* Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the two forces shown in Fig. P2-68.

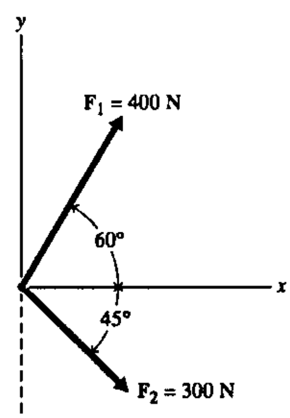


Fig. P2-68

SOLUTION

$$R_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 = 400 \cos 60^\circ + 300 \cos 45^\circ = 412.1 \text{ N}$$

$$R_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 = 400 \sin 60^\circ + 300 \sin 45^\circ = 134.28 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(412.1)^2 + (134.28)^2} = 433.4 \text{ N} \cong 433 \text{ N} \qquad \text{Ans.}$$

$$\theta_x = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{134.28}{412.1} = 18.048^\circ \cong 18.05^\circ \qquad \text{Ans.}$$

2-69 Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the two forces shown in Fig. P2-69.

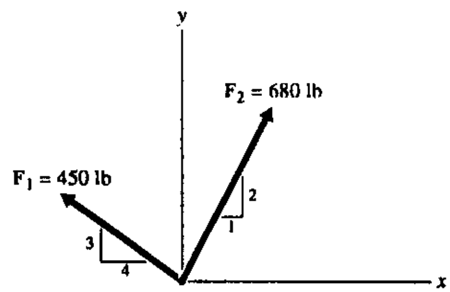


Fig. P2-69

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 = 450(-4/5) + 680(1/\sqrt{5}) = -55.89 \text{ lb}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 = 450(3/5) + 680(2/\sqrt{5}) = 878.21 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-55.89)^2 + (878.21)^2} = 879.99 \text{ lb} \cong 880 \text{ lb}$$ Ans.

$$\theta_x = \tan^{-1}\frac{R_y}{R} = \tan^{-1}\frac{878.21}{-55.89} = 93.64^\circ \cong 93.6^\circ$$ Ans.

$\bar{R}$ = 880 lb ↖ 86.4° Ans.

2-70 Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the two forces shown in Fig. P2-70.

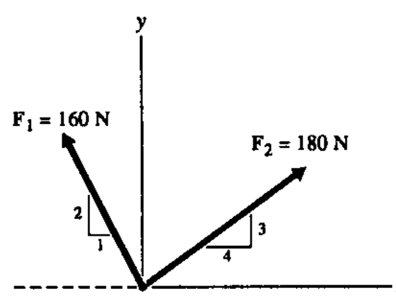


Fig. P2-70

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 = 160(-1/\sqrt{5}) + 180(4/5) = 72.45 \text{ N}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 = 160(2/\sqrt{5}) + 180(3/5) = 251.11 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(72.45)^2 + (251.11)^2} = 261.35 \text{ N} \cong 261 \text{ N}$$ Ans.

$$\theta_x = \tan^{-1}\frac{R_y}{R} = \tan^{-1}\frac{251.11}{72.45} = 73.91^\circ \cong 73.9^\circ$$ Ans.

$\bar{R}$ = 261 N ↗ 73.9° Ans.

2-71* Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the three forces shown in Fig. P2-71.

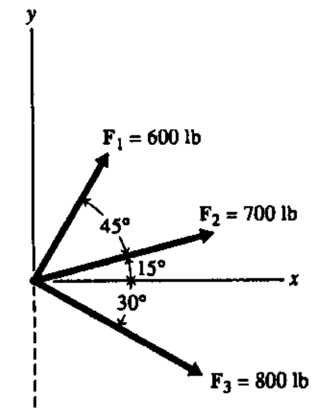


Fig. P2-71

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3$$

$$= 600 \cos 60^\circ + 700 \cos 15^\circ + 800 \cos 30^\circ = 1669.0 \text{ lb}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3$$

$$= 600 \sin 60^\circ + 700 \sin 15^\circ - 800 \sin 30^\circ = 300.8 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1669.0)^2 + (300.8)^2} = 1695.9 \text{ lb} \cong 1696 \text{ lb}$$ Ans.

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{300.8}{1669.0} = 10.217^\circ \cong 10.22^\circ$$ Ans.

$$\mathbf{R} = 1696 \text{ lb} \measuredangle 10.22^\circ$$ Ans.

2-72* Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the three forces shown in Fig. P2-72.

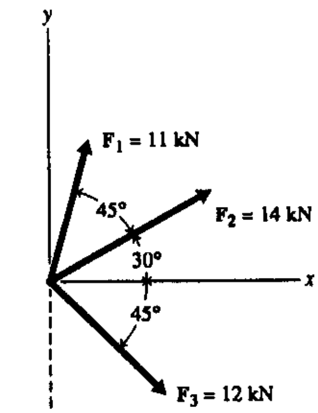


Fig. P2-72

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3$$
$$= 11 \cos 75^\circ + 14 \cos 30^\circ + 12 \cos 45^\circ = 23.46 \text{ kN}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3$$
$$= 11 \sin 75^\circ + 14 \sin 30^\circ - 12 \sin 45^\circ = 9.140 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(23.46)^2 + (9.140)^2} = 25.18 \text{ kN} \cong 25.2 \text{ kN}$$ Ans.

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{9.140}{23.46} = 21.29^\circ \cong 21.3^\circ$$ Ans.

$\bar{R} = 25.2$ kN ∠ 21.3° Ans.

2-73 Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the three forces shown in Fig. P2-73.

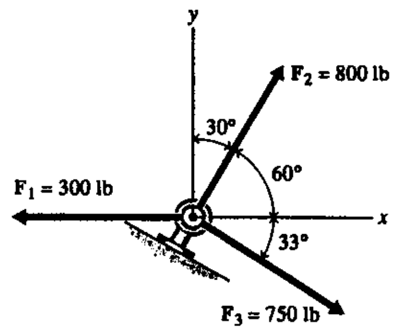


Fig. P2-73

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3$$

$$= 300(-1) + 800 \cos 60^\circ + 750 \cos 33^\circ = 729.00 \text{ lb}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3$$

$$= 300(0) + 800 \sin 60^\circ - 750 \sin 33^\circ = 284.34 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(729.00)^2 + (284.34)^2} = 782.49 \text{ lb} \cong 782 \text{ lb}$$ Ans.

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{284.34}{729.00} = 21.31^\circ \cong 21.3^\circ$$ Ans.

$$\mathbf{R} = 782 \text{ lb} \measuredangle 21.3^\circ$$ Ans.

2-74* Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the three forces shown in Fig. P2-74.

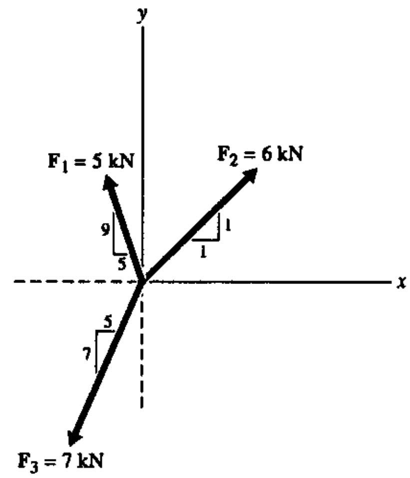


Fig. P2-74

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3$$

$$= 5(-5/\sqrt{106}) + 6(1/\sqrt{2}) + 7(-5/\sqrt{74}) = -2.254 \text{ kN}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3$$

$$= 5(9/\sqrt{106}) + 6(1/\sqrt{2}) + 7(-7/\sqrt{74}) = 2.917 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.254)^2 + (2.917)^2} = 3.686 \text{ kN} \cong 3.69 \text{ kN}$$ Ans.

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{2.917}{-2.254} = 127.69^\circ \cong 127.7^\circ$$ Ans.

$\mathbf{R} = 3.69$ kN ⦣ 52.3° Ans.

2-75 Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the three forces shown in Fig. P2-75.

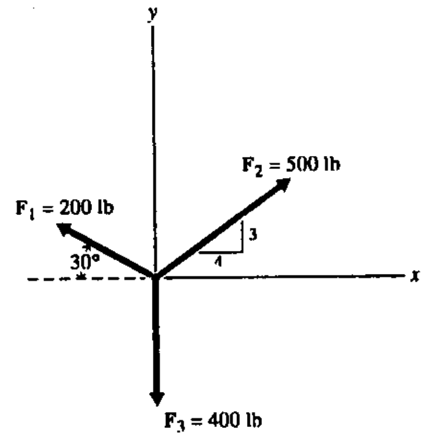


Fig. P2-75

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3$$
$$= -200 \cos 30^\circ + 500(4/5) + 400(0) = 226.79 \text{ lb}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3$$
$$= 200 \sin 30^\circ + 500(3/5) + 400(-1) = 0$$

$$\bar{R} = R_x = 226.79 \text{ lb} \cong 227 \text{ lb}$$ Ans.

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{0}{226.19} = 0^\circ$$ Ans.

$\bar{R} = 227$ lb $\longrightarrow$ Ans.

2-76* Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x axis for the four forces shown in Fig. P2-76.

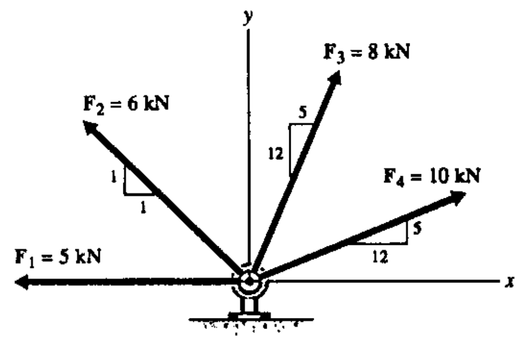


Fig. P2-76

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4$$
$$= 5(-1) + 6(-1/\sqrt{2} + 8(5/13) + 10(12/13) = 3.065 \text{ kN}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$$
$$= 5(0) + 6(1/\sqrt{2}) + 8(12/13) + 10(5/13) = 15.473 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(3.065)^2 + (15.473)^2} = 15.774 \text{ kN} \cong 15.77 \text{ kN} \qquad \text{Ans.}$$

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{15.473}{3.065} = 78.795^\circ \cong 78.8^\circ \qquad \text{Ans.}$$

$$\bar{\mathbf{R}} = 15.77 \text{ kN} \measuredangle 78.8^\circ \qquad \text{Ans.}$$

2-77 Determine the magnitude R of the resultant and the angle θ_x between the line of action of the resultant and the x-axis for the four forces shown in Fig. P2-77.

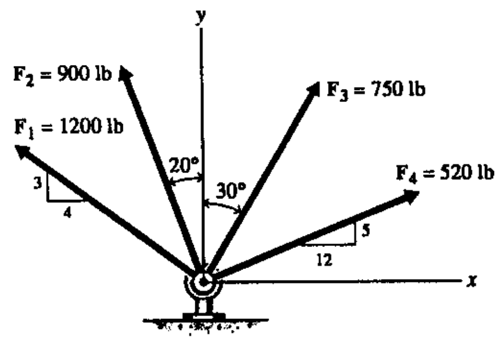


Fig. P2-77

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4$$

$$= 1200(-4/5) - 900 \cos 70^\circ + 750 \cos 30^\circ + 520(12/13) = -412.82 \text{ lb}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$$

$$= 1200(3/5) + 900 \sin 70^\circ + 750 \sin 60^\circ + 520(5/13) = 2415.24 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-412.82)^2 + (2415.24)^2} = 2450.27 \text{ lb} \cong 2450 \text{ lb} \qquad \text{Ans.}$$

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{2415.24}{-412.82} = 99.699^\circ \cong 99.7^\circ \qquad \text{Ans.}$$

$$\bar{\mathbf{R}} = 2450 \text{ lb} \measuredangle 80.3^\circ \qquad \text{Ans.}$$

2-78* Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the two forces shown in Fig. P2-78.

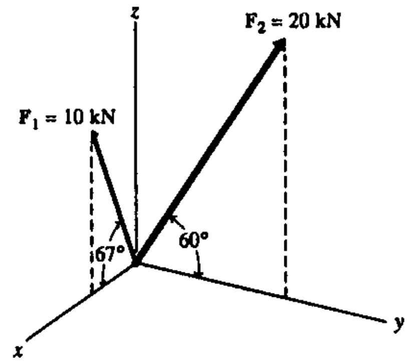


Fig. P2-78

SOLUTION

$$R_x = F_1 \cos\theta_{1x} + F_2 \cos\theta_{2x} = 10 \cos 67^\circ + 20 \cos 90^\circ = 3.907 \text{ kN}$$

$$R_y = F_1 \cos\theta_{1y} + F_2 \cos\theta_{2y} = 10 \cos 90^\circ + 20 \cos 60^\circ = 10.000 \text{ kN}$$

$$R_z = F_1 \cos\theta_{1z} + F_2 \cos\theta_{2z} = 10 \cos 23^\circ + 20 \cos 30^\circ = 26.526 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(3.907)^2 + (10.000)^2 + (26.526)^2}$$

$$= 28.62 \text{ kN} \cong 28.6 \text{ kN}$$ Ans.

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{3.907}{28.62} = 82.15^\circ \cong 82.2^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{10.00}{28.62} = 69.55^\circ \cong 69.6^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{26.53}{28.62} = 22.03^\circ \cong 22.0^\circ$$ Ans.

2-79* Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the two forces shown in Fig. P2-79.

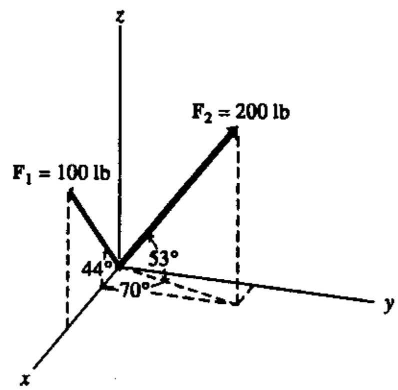


Fig. P2-79

SOLUTION

$$R_x = F_1 \cos\theta_{1x} + F_2 \cos\phi_2 \cos\theta_2$$
$$= 100 \cos 44^\circ + 200 \cos 53^\circ \cos 70^\circ = 113.10 \text{ lb}$$

$$R_y = F_1 \cos\theta_{1y} + F_2 \cos\phi_2 \sin\theta_2$$
$$= 100 \cos 90^\circ + 200 \cos 53^\circ \sin 70^\circ = 113.10 \text{ lb}$$

$$R_z = F_1 \cos\theta_{1z} + F_2 \sin\phi_2$$
$$= 100 \cos 46^\circ + 200 \sin 53^\circ = 229.19 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(113.10)^2 + (113.10)^2 + (229.19)^2}$$
$$= 279.48 \text{ lb} \cong 279 \text{ lb} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{113.10}{279.48} = 66.13^\circ \cong 66.1^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{113.10}{279.48} = 66.13^\circ \cong 66.1^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{229.19}{279.48} = 34.91^\circ \cong 34.9^\circ \qquad \text{Ans.}$$

2-80 Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the three forces shown in Fig. P2-80.

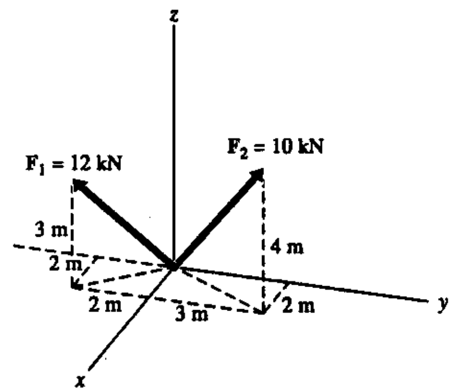


Fig. P2-80

SOLUTION

$$d_1 = \sqrt{(2)^2 + (-2)^2 + (3)^2} = \sqrt{17}$$

$$d_2 = \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$$

$$R_x = F_1 \cos\theta_{1x} + F_2 \cos\theta_{2x} = 12(2/\sqrt{17}) + 10(2/\sqrt{29}) = 9.535 \text{ kN}$$

$$R_y = F_1 \cos\theta_{1y} + F_2 \cos\theta_{2y} = 12(-2/\sqrt{17}) + 10(3/\sqrt{29}) = -0.250 \text{ kN}$$

$$R_z = F_1 \cos\theta_{1z} + F_2 \cos\theta_{2z} = 12(3/\sqrt{17}) + 10(4/\sqrt{29}) = 16.159 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(9.535)^2 + (-0.250)^2 + (16.159)^2}$$

$$= 18.764 \text{ kN} \cong 18.76 \text{ kN}$$ Ans.

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{9.535}{18.764} = 59.46^\circ \cong 59.4^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{-0.250}{18.764} = 90.76^\circ \cong 90.8^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{16.159}{18.764} = 30.55^\circ \cong 30.6^\circ$$ Ans.

2-81 Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the three forces shown in Fig. P2-81.

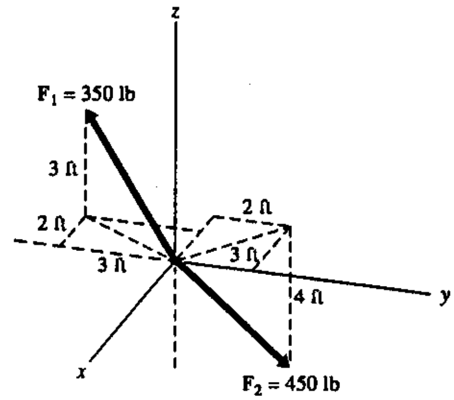


Fig. P2-81

SOLUTION

$$d_1 = \sqrt{(-2)^2 + (-3)^2 + (3)^2} = \sqrt{22}$$

$$d_2 = \sqrt{(-3)^2 + (2)^2 + (-4)^2} = \sqrt{29}$$

$$R_x = F_1 \cos\theta_{1x} + F_2 \cos\theta_{2x} = 350(-2/\sqrt{22}) + 450(-3/\sqrt{29}) = -399.93 \text{ lb}$$

$$R_y = F_1 \cos\theta_{1y} + F_2 \cos\theta_{2y} = 350(-3/\sqrt{22}) + 450(2/\sqrt{29}) = -56.73 \text{ lb}$$

$$R_z = F_1 \cos\theta_{1z} + F_2 \cos\theta_{2z} = 350(3/\sqrt{22}) + 450(-4/\sqrt{29}) = -110.39 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(-399.93)^2 + (-56.73)^2 + (-110.39)^2}$$

$$= 418.75 \text{ lb} \cong 419 \text{ lb}$$ Ans.

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{-399.93}{418.75} = 162.76^\circ \cong 162.8^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{-56.73}{418.75} = 97.79^\circ \cong 97.8^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{-110.39}{418.75} = 105.28^\circ \cong 105.3^\circ$$ Ans.

2-82* Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the three forces shown in Fig. P2-82.

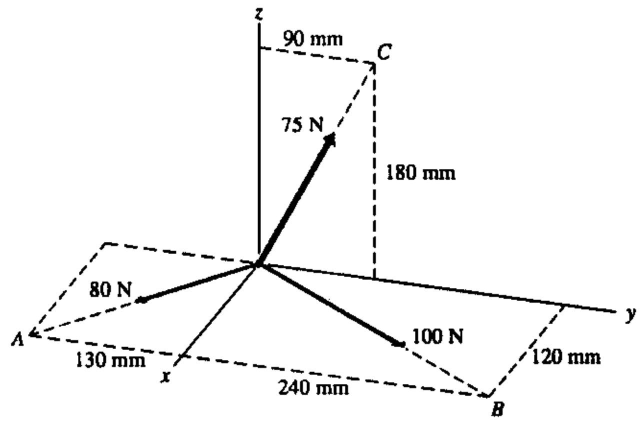


Fig. P2-82

SOLUTION

$$\theta_{Ax} = \tan^{-1}\frac{130}{120} = 47.29^\circ$$

$$\theta_{Bx} = \tan^{-1}\frac{240}{120} = 63.43^\circ$$

$$\theta_{Cy} = \tan^{-1}\frac{180}{90} = 63.43^\circ$$

$$R_x = F_A \cos\theta_{Ax} + F_B \cos\theta_{Bx} + F_C \cos\theta_{Cx}$$
$$= 80 \cos 47.29^\circ + 100 \cos 63.43^\circ + 75 \cos 90^\circ = 98.99 \text{ N}$$

$$R_y = F_A \cos\theta_{Ay} + F_B \cos\theta_{By} + F_C \cos\theta_{Cy}$$
$$= -80 \cos 42.71^\circ + 100 \cos 26.57^\circ + 75 \cos 63.43^\circ = 64.20 \text{ N}$$

$$R_z = F_A \cos\theta_{Az} + F_B \cos\theta_{Bz} + F_C \cos\theta_{Cz}$$
$$= 80 \cos 90^\circ + 100 \cos 90^\circ + 75 \cos 26.57^\circ = 67.08 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(98.99)^2 + (64.20)^2 + (67.08)^2}$$
$$= 135.72 \text{ N} \cong 135.7 \text{ N} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{98.99}{135.72} = 43.17^\circ \cong 43.2^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{64.20}{135.72} = 61.77^\circ \cong 61.8^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{67.08}{135.72} = 60.38^\circ \cong 60.4^\circ \qquad \text{Ans.}$$

2-83* Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the three forces shown in Fig. P2-83.

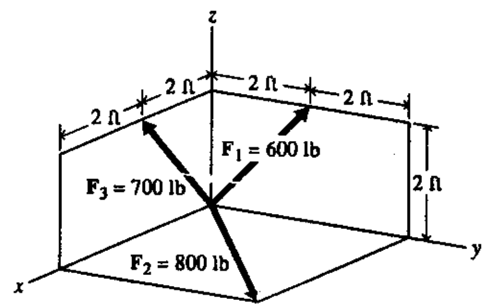


Fig. P2-83

SOLUTION

$$\hat{e}_1 = \frac{0\,\hat{\imath} + 2\,\hat{\jmath} + 2\,\hat{k}}{\sqrt{(0)^2 + (2)^2 + (2)^2}} = 0\,\hat{\imath} + 0.7071\,\hat{\jmath} + 0.7071\,\hat{k}$$

$$\hat{e}_2 = \frac{4\,\hat{\imath} + 4\,\hat{\jmath} + 0\,\hat{k}}{\sqrt{(4)^2 + (4)^2 + (0)^2}} = 0.7071\,\hat{\imath} + 0.7071\,\hat{\jmath} + 0\,\hat{k}$$

$$\hat{e}_3 = \frac{2\,\hat{\imath} + 0\,\hat{\jmath} + 2\,\hat{k}}{\sqrt{(0)^2 + (2)^2 + (2)^2}} = 0.7071\,\hat{\imath} + 0\,\hat{\jmath} + 0.7071\,\hat{k}$$

$$\bar{F}_1 = F_1\hat{e}_1 = 600(0\,\hat{\imath} + 0.7071\,\hat{\jmath} + 0.7071\,\hat{k}) = 424.26\,\hat{\jmath} + 424.26\,\hat{k} \text{ lb}$$

$$\bar{F}_2 = F_2\hat{e}_2 = 800(0.7071\,\hat{\imath} + 0.7071\,\hat{\jmath} + 0\,\hat{k}) = 565.68\,\hat{\imath} + 565.68\,\hat{\jmath} \text{ lb}$$

$$\bar{F}_3 = F_3\hat{e}_3 = 700(0.7071\,\hat{\imath} + 0\,\hat{\jmath} + 0.7071\,\hat{k}) = 494.97\,\hat{\imath} + 494.97\,\hat{k} \text{ lb}$$

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 1060.65\,\hat{\imath} + 989.94\,\hat{\jmath} + 919.23\,\hat{k} \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(1060.65)^2 + (989.94)^2 + (919.23)^2}$$

$$= 1717.54 \text{ lb} \cong 1718 \text{ lb} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{1060.65}{1717.54} = 51.86° \cong 51.9° \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{989.94}{1717.54} = 54.80° = 54.8° \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{919.23}{1717.54} = 57.64° \cong 57.6° \qquad \text{Ans.}$$

2-84 Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the three forces shown in Fig. P2-84.

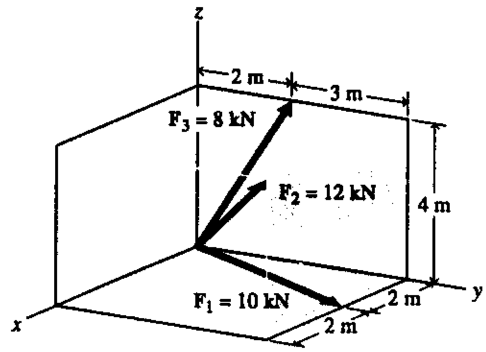


Fig. P2-84

SOLUTION

$$\hat{e}_1 = \frac{2\,\hat{\imath} + 5\,\hat{\jmath} + 0\,\hat{k}}{\sqrt{(2)^2 + (5)^2 + (0)^2}} = 0.3714\,\hat{\imath} + 0.9285\,\hat{\jmath} + 0\,\hat{k}$$

$$\hat{e}_2 = \frac{4\,\hat{\imath} + 5\,\hat{\jmath} + 4\,\hat{k}}{\sqrt{(4)^2 + (5)^2 + (4)^2}} = 0.5298\,\hat{\imath} + 0.6623\,\hat{\jmath} + 0.5298\,\hat{k}$$

$$\hat{e}_3 = \frac{0\,\hat{\imath} + 2\,\hat{\jmath} + 4\,\hat{k}}{\sqrt{(0)^2 + (2)^2 + (4)^2}} = 0\,\hat{\imath} + 0.4472\,\hat{\jmath} + 0.8944\,\hat{k}$$

$$\bar{F}_1 = F_1\hat{e}_1 = 10(0.3714\,\hat{\imath} + 0.9285\,\hat{\jmath} + 0\,\hat{k}) = 3.714\,\hat{\imath} + 9.285\,\hat{\jmath}\text{ kN}$$

$$\bar{F}_2 = F_2\hat{e}_2 = 12(0.5298\,\hat{\imath} + 0.6623\,\hat{\jmath} + 0.5298\,\hat{k})$$

$$= 6.358\,\hat{\imath} + 7.948\,\hat{\jmath} + 6.358\,\hat{k}\text{ kN}$$

$$\bar{F}_3 = F_3\hat{e}_3 = 8(0\,\hat{\imath} + 0.4472\,\hat{\jmath} + 0.8944\,\hat{k}) = 3.578\,\hat{\jmath} + 7.155\,\hat{k}\text{ kN}$$

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 10.072\,\hat{\imath} + 20.811\,\hat{\jmath} + 13.513\,\hat{k}\text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(10.072)^2 + (20.811)^2 + (13.513)^2}$$

$$= 26.780\text{ kN} \cong 26.8\text{ kN}$$ Ans.

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{10.072}{26.780} = 67.91^\circ \cong 67.9^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{20.811}{26.780} = 39.00^\circ = 39.0^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{13.513}{26.780} = 59.70^\circ = 59.7^\circ$$ Ans.

2-85* Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the three forces shown in Fig. P2-85.

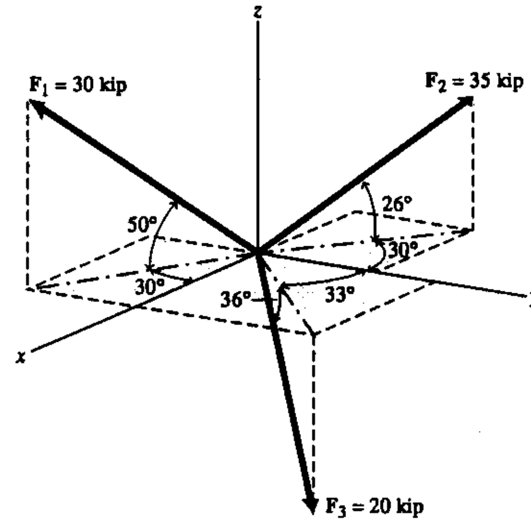


Fig. P2-85

SOLUTION

$F_{1x} = 30 \cos 50^\circ \cos 30^\circ = 16.700$ kip $\qquad F_{1z} = 30 \sin 50^\circ = 22.981$ kip

$F_{1y} = -30 \cos 50^\circ \sin 30^\circ = -9.642$ kip

$F_{2x} = -35 \cos 26^\circ \sin 30^\circ = -15.729$ kip $\qquad F_{2z} = 35 \sin 26^\circ = 15.343$ kip

$F_{2y} = 35 \cos 26^\circ \cos 30^\circ = 27.243$ kip

$F_{3x} = 20 \cos 36^\circ \sin 33^\circ = 8.812$ kip $\qquad F_{3z} = -20 \sin 36^\circ = -11.756$ kip

$F_{3y} = 20 \cos 36^\circ \cos 33^\circ = 13.570$ kip

$R_x = F_{1x} + F_{2x} + F_{3x} = 16.700 - 15.729 + 8.812 = 9.783$ kip

$R_y = F_{1y} + F_{2y} + F_{3y} = -9.642 + 27.243 + 13.570 = 31.171$ kip

$R_z = F_{1z} + F_{2z} + F_{3z} = 22.981 + 15.343 - 11.756 = 26.568$ kip

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(9.783)^2 + (31.171)^2 + (26.568)^2}$$

$$= 42.109 \text{ kip} \cong 42.1 \text{ kip} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{9.783}{42.109} = 76.57^\circ \cong 76.6^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1} \frac{R_y}{R} = \cos^{-1} \frac{31.171}{42.109} = 42.24^\circ \cong 42.2^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1} \frac{R_z}{R} = \cos^{-1} \frac{26.568}{42.109} = 50.88^\circ \cong 50.9^\circ \qquad \text{Ans.}$$

2-86 Determine the magnitude R of the resultant and the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the positive x-, y-, and z-coordinate axes for the three forces shown in Fig. P2-86.

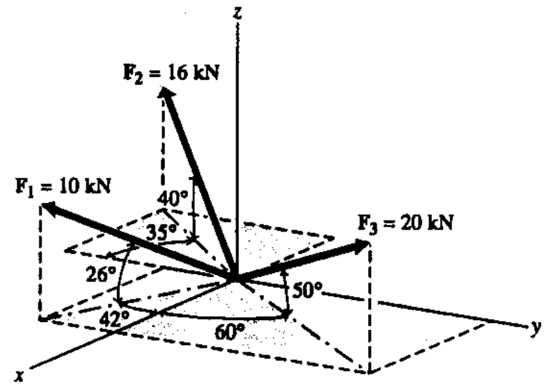


Fig. P2-86

SOLUTION

$$R_x = 10 \cos 26^\circ \cos 42^\circ - 16 \cos 40^\circ \sin 35^\circ + 20 \cos 50^\circ \cos 60^\circ = 6.077 \text{ kN}$$

$$R_y = -10 \cos 26^\circ \sin 42^\circ - 16 \cos 40^\circ \cos 35^\circ + 20 \cos 50^\circ \sin 60^\circ = -4.921 \text{ kN}$$

$$R_z = 10 \sin 26^\circ + 16 \sin \sin 40^\circ + 20 \sin 50^\circ = 29.989 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(6.077)^2 + (-4.921)^2 + (29.989)^2}$$

$$= 30.992 \text{ kN} \cong 31.0 \text{ kN} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{6.077}{30.992} = 78.69^\circ \cong 78.7^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{-4.921}{30.992} = 99.14^\circ \cong 99.1^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{29.989}{30.992} = 44.06^\circ \cong 44.1^\circ \qquad \text{Ans.}$$

2-87* Two forces are applied to a bracket as shown in Fig. P2-87. Determine the magnitude of the resultant $\bar{R}$ of the two forces and the angle θ between the x-axis and the line of action of the resultant.

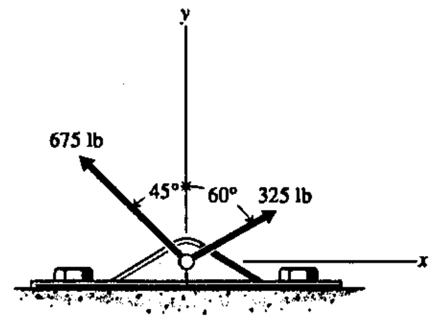


Fig. P2-87

SOLUTION

$$R_x = -675 \sin 45^\circ + 325 \sin 60^\circ = -195.84 \text{ lb}$$

$$R_y = 675 \cos 45^\circ + 325 \cos 60^\circ = 639.80 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-195.84)^2 + (639.80)^2} = 669.10 \text{ lb} \cong 669 \text{ lb}$$ Ans.

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{639.80}{-195.84} = 107.02^\circ \cong 107.0^\circ$$ Ans.

$$\bar{R} = 669 \text{ lb} \angle 73.0^\circ$$ Ans.

2-88* Two forces are applied to a bracket as shown in Fig. P2-88. Determine the magnitude of the resultant $\bar{R}$ of the two forces and the angle θ between the x-axis and the line of action of the resultant.

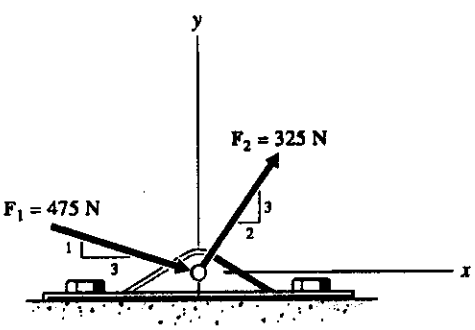


Fig. P2-88

SOLUTION

$$R_x = 475(3/\sqrt{10}) + 325(2/\sqrt{13}) = 630.90 \text{ N}$$

$$R_y = 475(-1/\sqrt{10}) + 325(3/\sqrt{13}) = 120.21 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(630.90)^2 + (120.21)^2} = 642.25 \text{ N} \cong 642 \text{ N}$$ Ans.

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{120.21}{630.90} = 10.788^\circ \cong 10.79^\circ$$ Ans.

$$\bar{R} = 642 \text{ N} \angle 10.79^\circ$$ Ans.

2-89 An 800-lb force is applied to the post shown in Fig. P2-89. Determine
(a) The magnitudes of the x- and y-components of the force.
(b) The magnitudes of the u- and v-components of the force.

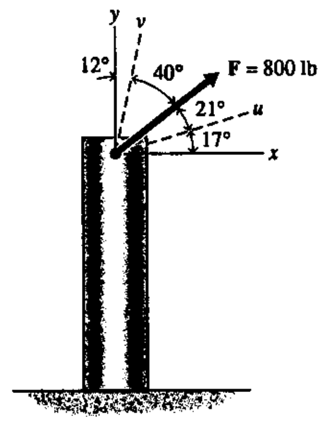
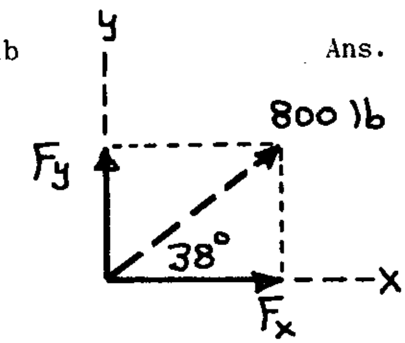


Fig. P2-89

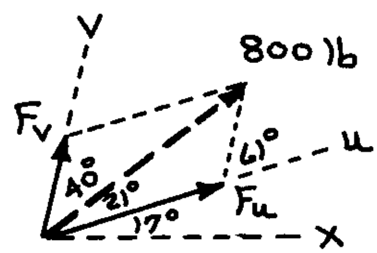
SOLUTION

(a) $F_x = F \cos \theta_x = 800 \cos 38^\circ = 630.4 \text{ lb} \cong 630 \text{ lb}$ Ans.

$F_y = F \sin \theta_x = 800 \sin 38^\circ = 492.5 \text{ lb} \cong 493 \text{ lb}$ Ans.



(b) From the parallelogram of forces constructed using the force $\bar{F}$ and lines parallel to the u and v axes and the law of sines:



$$\frac{F_u}{\sin 40^\circ} = \frac{F_v}{\sin 21^\circ} = \frac{F}{\sin 119^\circ}$$

$$F_u = \frac{F \sin 40^\circ}{\sin 119^\circ} = \frac{800 \sin 40^\circ}{\sin 119^\circ} = 587.9 \text{ lb} \cong 588 \text{ lb}$$ Ans.

$$F_v = \frac{F \sin 21^\circ}{\sin 119^\circ} = \frac{800 \sin 21^\circ}{\sin 119^\circ} = 327.8 \text{ lb} \cong 328 \text{ lb}$$ Ans.

2-90 Two forces $\bar{F}_1$ and $\bar{F}_2$ are applied to an eyebolt as shown in Fig. P2-90. Determine

(a) The magnitude and direction (angle θ_x) of the resultant $\bar{R}$ of the two forces.

(b) The magnitudes of two other forces $\bar{F}_u$ and $\bar{F}_v$ that would have the same resultant.

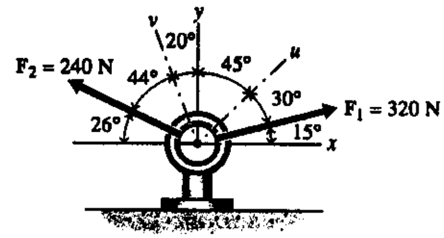
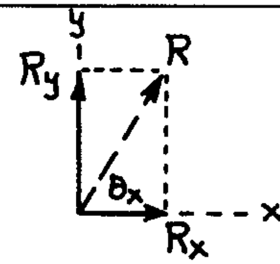


Fig. P2-90

SOLUTION



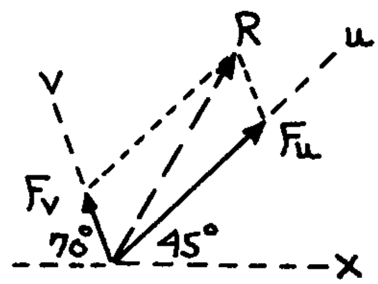
(a) $R_x = 320 \cos 15^\circ - 240 \cos 26^\circ = 93.39$ N

$R_y = 320 \sin 15^\circ + 240 \sin 26^\circ = 188.03$ N

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(93.39)^2 + (188.03)^2} = 209.95 \cong 210$ N Ans.

$\theta_x = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{188.03}{93.39} = 63.59^\circ \cong 63.6^\circ$ Ans.

(b) From the parallelogram of forces constructed using the resultant $\bar{R}$ and lines parallel to the u and v axes and the law of sines:



$$\frac{F_u}{\sin 46.41^\circ} = \frac{F_v}{\sin 18.59^\circ} = \frac{R}{\sin 115^\circ}$$

$F_u = \frac{R \sin 46.41^\circ}{\sin 115^\circ} = \frac{209.95 \sin 46.41^\circ}{\sin 115^\circ} = 167.03 \text{ N} \cong 167.0$ N Ans.

$F_v = \frac{R \sin 18.59^\circ}{\sin 115^\circ} = \frac{209.95 \sin 18.59^\circ}{\sin 115^\circ} = 73.52 \text{ N} \cong 73.5$ N Ans.

2-91* Three forces are applied to a bracket mounted on a post as shown in Fig. P2-91. Determine

(a) The magnitude and direction (angle θ_x) of the resultant $\mathbf{R}$ of the three forces.

(b) The magnitudes of two other forces $\mathbf{F}_x$ and $\mathbf{F}_y$ that would have the same resultant.

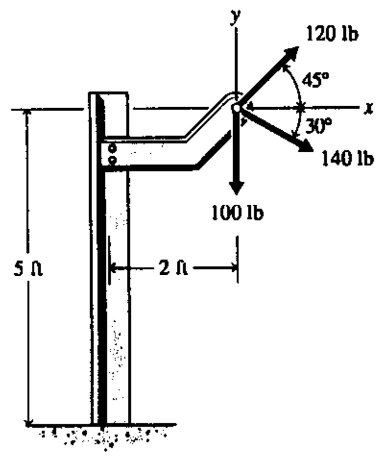


Fig. P2-91

SOLUTION

(a) $R_x = 120 \cos 45^\circ + 140 \cos 30^\circ = 206.10$ lb

$R_y = 120 \sin 45^\circ - 140 \sin 30^\circ - 100 = -85.15$ lb

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(206.10)^2 + (-85.15)^2} = 223.0 \text{ N} = 223 \text{ N}$ Ans.

$\theta_x = \tan^{-1} \dfrac{R_y}{R_x} = \tan^{-1} \dfrac{-85.15}{206.10} = 22.45^\circ \text{ lb} \cong 22.5 \text{ lb}^\circ$ Ans.

(b) $F_x = R_x = 206.10 \text{ lb} \cong 206 \text{ lb}$ $\mathbf{F}_x = 206 \text{ lb} \rightarrow$ Ans.

$F_y = R_y = -85.15 \text{ lb} \cong -85.2 \text{ lb}$ $\mathbf{F}_y = 85.2 \text{ lb} \downarrow$ Ans.

2-92 Four forces act on a small airplane in flight, as shown in Fig. P2-92; its weight, the thrust provided by the engine, the lift provided by the wings, and the drag resulting from its motion through the air. Determine the resultant of the four forces and its line of action with respect to the axis of the plane.

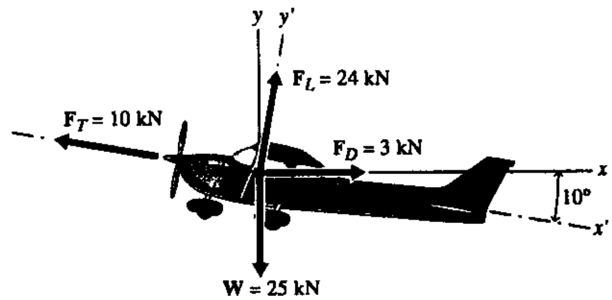


Fig. P2-92

SOLUTION

$R_{x'} = -10 + 25 \sin 10^\circ + 3 \cos 10^\circ = -2.7044$ kN

$R_{y'} = 24 - 25 \cos 10^\circ + 3 \sin 10^\circ = -0.0992$ kN

$R = \sqrt{R_{x'}^2 + R_{y'}^2} = \sqrt{(-2.7044)^2 + (-0.0992)^2} = 2.7062 \text{ kN} \cong 2.71 \text{ kN}$ Ans.

$\theta_{x'} = \tan^{-1} \dfrac{R_{x'}}{R} = \tan^{-1} \dfrac{-0.0992}{-2.7062} = -177.901^\circ \cong -177.9^\circ$ Ans.

$\bar{R} = 2.71$ kN ↗ 2.10° Ans.

2-93 Three forces are applied to a stalled automobile as shown in Fig. P2-93. Determine the magnitude of the force $\bar{F}$ and the magnitude of the resultant $\bar{R}$ if the line of action of the resultant is along the x-axis.

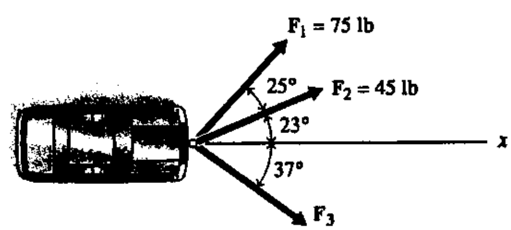


Fig. P2-93

SOLUTION

$R_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3$

$= 75 \sin 48^\circ + 45 \sin 23^\circ - F_3 \sin 37^\circ = 0$

$F_3 = 121.83 \text{ lb} \cong 121.8 \text{ lb}$ Ans.

$R = R_x = F_1 \cos \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3$

$= 75 \cos 48^\circ + 45 \cos 23^\circ + 121.83 \cos 37^\circ$

$= 188.91 \text{ lb} \cong 188.9 \text{ lb}$ Ans.

2-94* Four forces are applied at a point in a body as shown in Fig. P2-94. Determine the magnitude of the resultant $\bar{R}$ of the four forces and the angle θ between the x-axis and the line of action of the resultant.

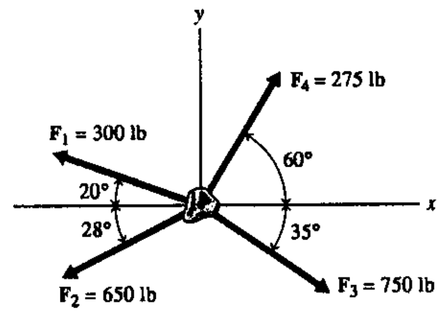


Fig. P2-94

SOLUTION

$$R_x = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4$$

$$= -300 \cos 20^\circ - 650 \cos 28^\circ + 750 \cos 35^\circ + 275 \cos 60^\circ$$

$$= -103.96 \text{ lb} \cong -104.0 \text{ lb}$$

$$R_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$$

$$= 300 \sin 20^\circ - 650 \sin 28^\circ - 750 \sin 35^\circ + 275 \sin 60^\circ$$

$$= -394.58 \text{ lb} \cong -395 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-103.96)^2 + (-394.58)^2} = 408.04 \text{ lb} \cong 408 \text{ lb}$$ Ans.

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-394.58}{-103.96} = -104.76^\circ \cong -104.8^\circ$$ Ans.

$\bar{R} = 408$ lb ↙ 75.2° Ans.

2-95 Four forces are applied at a point in a body as shown in Fig. P2-95. Determine the magnitude and direction (angles θ_x, θ_y, and θ_z) of the resultant $\vec{R}$ of the four forces.

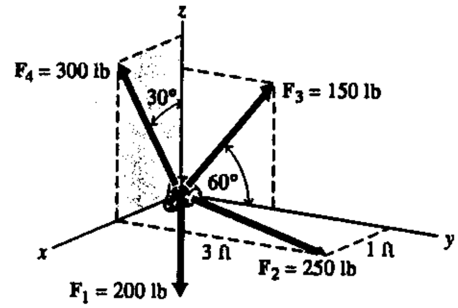


Fig. P2-95

SOLUTION

$$R_x = F_1 \cos\theta_{1x} + F_2 \cos\theta_{2x} + F_3 \cos\theta_{3x} + F_4 \cos\theta_{4x}$$
$$= 200(0) + 250(1/\sqrt{10}) + 150(0) + 300 \cos 60^\circ = 229.06 \text{ lb}$$

$$R_y = F_1 \cos\theta_{1y} + F_2 \cos\theta_{2y} + F_3 \cos\theta_{3y} + F_4 \cos\theta_{4y}$$
$$= 200(0) + 250(3/\sqrt{10}) + 150 \cos 60^\circ + 300(0) = 312.17 \text{ lb}$$

$$R_z = F_1 \cos\theta_{1z} + F_2 \cos\theta_{2z} + F_3 \cos\theta_{3z} + F_4 \cos\theta_{4z}$$
$$= 200(-1) + 250(0) + 150 \cos 30^\circ + 300 \cos 30^\circ = 189.71 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(229.06)^2 + (312.17)^2 + (189.71)^2}$$
$$= 431.17 \text{ lb} \cong 431 \text{ lb}$$ Ans.

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{229.06}{431.17} = 57.91^\circ \cong 57.9^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{312.17}{431.17} = 43.61^\circ \cong 43.6^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{189.71}{431.17} = 63.89^\circ \cong 63.9^\circ$$ Ans.

2-96* Four forces are applied at a point in a body as shown in Fig. P2-96. Determine the magnitude and direction (angles θ_x, θ_y, and θ_z) of the resultant $\mathbf{R}$ of the four forces.

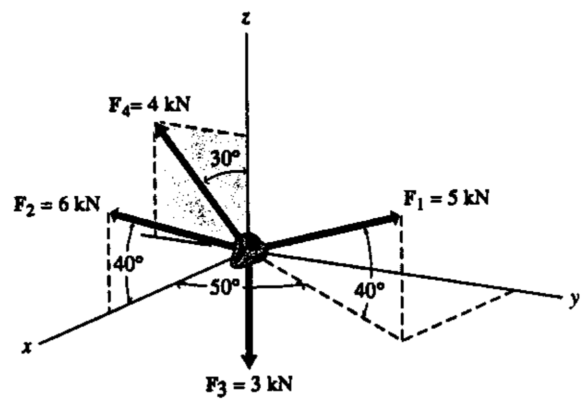


Fig. P2-96

SOLUTION

$F_{1x} = 5 \cos 40^\circ \cos 50^\circ = 2.462$ kN

$F_{1y} = 5 \cos 40^\circ \sin 50^\circ = 2.934$ kN

$F_{1z} = 5 \sin 40^\circ = 3.214$ kN

$F_{3x} = 0$

$F_{3y} = 0$

$F_{3z} = -3.000$ kN

$F_{2x} = 6 \cos 40^\circ = 4.596$ kN

$F_{2y} = 0$

$F_{2z} = 6 \sin 40^\circ = 3.857$ kN

$F_{4x} = 0$

$F_{4y} = -4 \sin 30^\circ = -2.000$ kN

$F_{4z} = 4 \cos 30^\circ = 3.464$ kN

$R_x = 2.462 + 4.596 = 7.058$ kN

$R_y = 2.934 - 2.000 = 0.934$ kN

$R_z = 3.214 + 3.857 - 3.000 + 3.464 = 7.535$ kN

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(7.058)^2 + (0.934)^2 + (7.535)^2}$$

$$= 10.366 \text{ kN} \cong 10.37 \text{ kN}$$ Ans.

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{7.058}{10.366} = 47.09^\circ \cong 47.1^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{0.939}{10.366} = 84.83^\circ \cong 84.8^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{7.535}{10.366} = 43.37^\circ \cong 43.4$$ Ans.

2-97 Two forces are applied at a point in a body as shown in Fig. P2-97. Determine

(a) The magnitude and direction (angles θ_x, θ_y, and θ_z) of the resultant $\mathbf{R}$ of the two forces.

(b) The magnitude of the rectangular component of force $\mathbf{F}_1$ along the line of action of force $\mathbf{F}_2$.

(c) The angle α between forces $\mathbf{F}_1$ and $\mathbf{F}_2$.

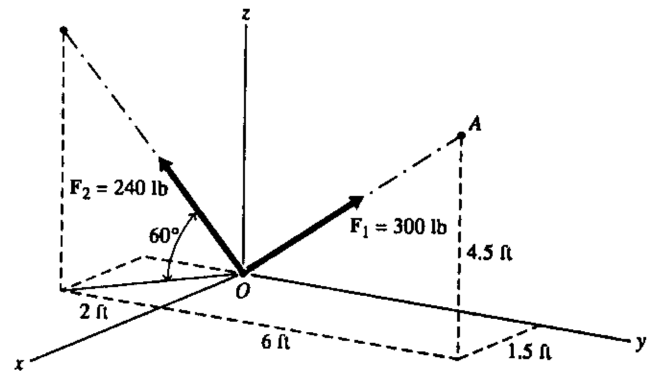


Fig. P2-97

SOLUTION

(a) $$\hat{e}_1 = \frac{1.5\,\hat{i} + 6\,\hat{j} + 4.5\,\hat{k}}{\sqrt{(1.5)^2 + (6)^2 + (4.5)^2}} = 0.1961\,\hat{i} + 0.7845\,\hat{j} + 0.5883\,\hat{k}$$

$$\hat{e}_2 = \frac{1.5\,\hat{i} - 2\,\hat{j} + 4.33\,\hat{k}}{\sqrt{(1.5)^2 + (-2)^2 + (4.33)^2}} = 0.3000\,\hat{i} - 0.4000\,\hat{j} + 0.8660\,\hat{k}$$

$$\mathbf{F}_1 = F_1\hat{e}_1 = 300(0.1961\,\hat{i} + 0.7845\,\hat{j} + 0.5883\,\hat{k}) = 58.83\,\hat{i} + 235.35\,\hat{j} + 176.49\,\hat{k} \text{ lb}$$

$$\mathbf{F}_2 = F_2\hat{e}_2 = 240(0.3000\,\hat{i} - 0.4000\,\hat{j} + 0.8660\,\hat{k}) = 72.00\,\hat{i} - 96.00\,\hat{j} + 207.84\,\hat{k} \text{ lb}$$

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = 130.83\,\hat{i} + 139.35\,\hat{j} + 384.33\,\hat{k} \text{ lb}$$

$$R = \sqrt{(130.83)^2 + (139.35)^2 + (384.33)^2} = 429.23 \cong 429 \text{ lb}$$ Ans.

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{130.83}{429.23} = 72.25^\circ \cong 72.3^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{139.35}{429.23} = 71.06^\circ \cong 71.1^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{384.35}{429.23} = 26.43^\circ \cong 26.4^\circ$$ Ans.

(b) $$\mathbf{F}_n = \mathbf{F}_1\cdot\hat{e}_2 = [29.42\,\hat{i} + 117.68\,\hat{j} + 88.25\,\hat{k}]\cdot[0.3000\,\hat{i} - 0.4000\,\hat{j} + 0.8660\,\hat{k}] = 38.2 \text{ lb}$$ Ans.

(c) $$\alpha = \cos^{-1}(\hat{e}_1\cdot\hat{e}_2) = \cos^{-1}[(0.1961\,\hat{i} + 0.7845\,\hat{j} + 0.5883\,\hat{k})\cdot(0.3000\,\hat{i} - 0.4000\,\hat{j} + 0.8660\,\hat{k})] = \cos^{-1}(0.2545) = 75.3^\circ$$ Ans.

2-98 Three forces are applied with cables to the anchor block shown in Fig. P2-98. Determine
(a) The magnitude and direction (angles θ_x, θ_y, and θ_z) of the resultant $\bar{R}$ of the three forces.
(b) The magnitude of the rectangular component of force $\bar{F}_1$ along the line of action of force $\bar{F}_2$.
(c) The angle α between forces $\bar{F}_1$ and $\bar{F}_3$.

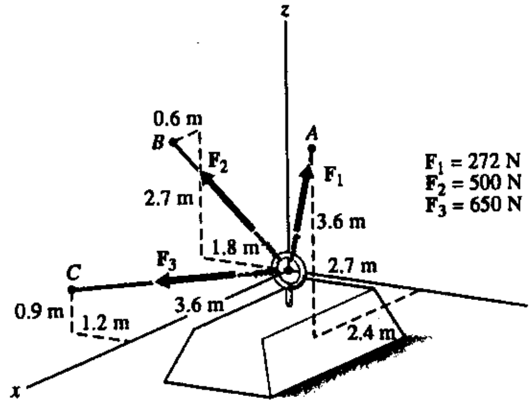


Fig. P2-98

SOLUTION

(a) $$\hat{e}_1 = \frac{2.4\,\hat{i} + 2.7\,\hat{j} + 3.6\,\hat{k}}{\sqrt{(2.4)^2 + (2.7)^2 + (3.6)^2}} = 0.4706\,\hat{i} + 0.5294\,\hat{j} + 0.7059\,\hat{k}$$

$$\hat{e}_2 = \frac{0.6\,\hat{i} - 1.8\,\hat{j} + 2.7\,\hat{k}}{\sqrt{(0.6)^2 + (-1.8)^2 + (2.7)^2}} = 0.1818\,\hat{i} - 0.5455\,\hat{j} + 0.8182\,\hat{k}$$

$$\hat{e}_3 = \frac{3.6\,\hat{i} - 1.2\,\hat{j} + 0.9\,\hat{k}}{\sqrt{(3.6)^2 + (-1.2)^2 + (0.9)^2}} = 0.9231\,\hat{i} - 0.3077\,\hat{j} + 0.2308\,\hat{k}$$

$$\bar{F}_1 = F_1\hat{e}_1 = 272(0.4706\,\hat{i} + 0.5294\,\hat{j} + 0.7059\,\hat{k})$$
$$= 128.00\,\hat{i} + 144.00\,\hat{j} + 192.00\,\hat{k}\text{ N}$$

$$\bar{F}_2 = F_2\hat{e}_2 = 500(0.1818\,\hat{i} - 0.5455\,\hat{j} + 0.8182\,\hat{k})$$
$$= 90.90\,\hat{i} - 272.75\,\hat{j} + 409.10\,\hat{k}\text{ N}$$

$$\bar{F}_3 = F_3\hat{e}_3 = 650(0.9231\,\hat{i} - 0.3077\,\hat{j} + 0.2308\,\hat{k})$$
$$= 600.00\,\hat{i} - 200.00\,\hat{j} + 150.00\,\hat{k}\text{ N}$$

$$\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 818.90\,\hat{i} - 328.75\,\hat{j} + 751.10\,\hat{k}\text{ N}$$

$$R = \sqrt{(818.90)^2 + (-328.75)^2 + (751.10)^2} = 1158.80 \cong 1159\text{ N} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{818.90}{1158.80} = 45.03^\circ \cong 45.0^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{-328.75}{1158.80} = 106.48^\circ \cong 106.5^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{751.10}{1158.80} = 49.596 \cong 49.6^\circ \qquad \text{Ans.}$$

2-98 (Continued)

(b) $\mathbf{F}_n = \mathbf{F}_1 \cdot \hat{\mathbf{e}}_2 = [64.00\,\hat{\mathbf{i}} + 72.00\,\hat{\mathbf{j}} + 96.00\,\hat{\mathbf{k}}] \cdot$

$[0.1818\,\hat{\mathbf{i}} - 0.5455\,\hat{\mathbf{j}} + 0.8182\,\hat{\mathbf{k}}] = 50.9$ N Ans.

(c) $\alpha = \cos^{-1}(\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_3) = \cos^{-1}[(0.4706\,\hat{\mathbf{i}} + 0.5294\,\hat{\mathbf{j}} + 0.7059\,\hat{\mathbf{k}}) \cdot$

$(0.9231\,\hat{\mathbf{i}} - 0.3077\,\hat{\mathbf{j}} + 0.2308\,\hat{\mathbf{k}})]$

$= \cos^{-1}(0.4344) = 64.3°$ Ans.

C2-99 Two forces $\bar{A}$ and $\bar{B}$ are applied to an eye bolt as shown in Fig. P2-99. If the magnitudes of the two forces are A = 50 lb and B = 100 lb, calculate and plot the magnitude of the resultant R as a function of the angle θ_A ($0° < \theta_A < 180°$). Also calculate and plot the angle θ_R that the resultant makes with the force $\bar{B}$ as a function of the angle θ_A. When is the resultant a maximum? When is the resultant a minimum? When is the angle θ_R a maximum? Repeat for A = 100 lb and B = 50 lb.

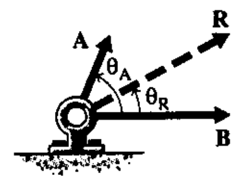
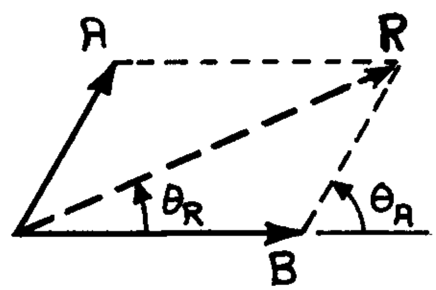


Fig. P2-99

SOLUTION



From the law of cosines:

$$R = \sqrt{A^2 + B^2 - 2AB\cos(180° - \theta_A)}$$
$$= \sqrt{A^2 + B^2 + 2AB\cos\theta_A}$$

From the law of sines:

$$\frac{\sin\theta_R}{A} = \frac{\sin(180 - \theta_A)}{R} \qquad \theta_R = \sin^{-1}\frac{A\sin\theta_A}{R}$$

For A = 50 lb and B = 100 lb:

$$R = \sqrt{(50)^2 + (100)^2 + 2(50)(100)\cos\theta_A} = 100\sqrt{1.25 + \cos\theta_A} \qquad \text{Ans.}$$

$$\theta_R = \sin^{-1}\frac{50\sin\theta_A}{100\sqrt{1.25 + \cos\theta_A}} = \sin^{-1}\frac{\sin\theta_A}{2\sqrt{1.25 + \cos\theta_A}} \qquad \text{Ans.}$$

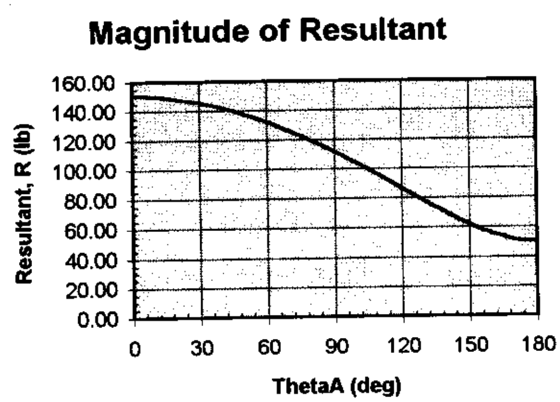
For A = 100 lb and B = 50 lb:

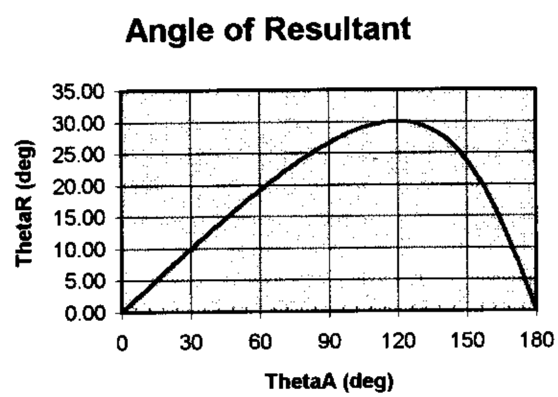
$$R = \sqrt{(100)^2 + (50)^2 + 2(100)(50)\cos\theta_A} = 100\sqrt{1.25 + \cos\theta_A} \qquad \text{Ans.}$$

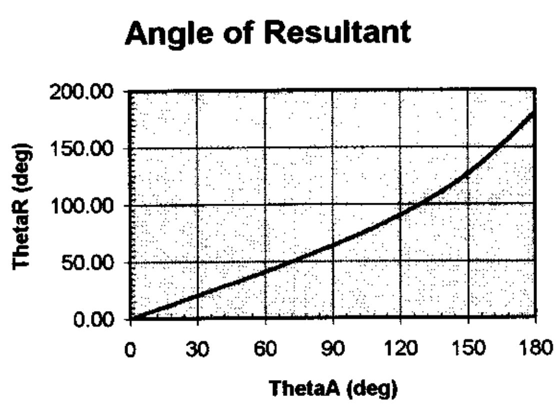
$$\theta_R = \sin^{-1}\frac{100\sin\theta_A}{100\sqrt{1.25 + \cos\theta_A}} = \sin^{-1}\frac{\sin\theta_A}{\sqrt{1.25 + \cos\theta_A}} \qquad \text{Ans>}$$

R_{max} = 150 lb when $\theta = 0°$ R_{min} = 50 lb when $\theta = 180°$

C2-99 (Continued)







C2-100 Two forces $\bar{A}$ and $\bar{B}$ are applied to an eye bolt using ropes as shown in Fig. P2-100. The resultant of the two forces has a magnitude R = 4 kN and makes an angle of 30° with the force $\bar{A}$ as shown. If both of the forces pull on the eye bolt as shown (ropes cannot push on the eye bolt), what is the range of angles ($\theta_{min} < \theta_B < \theta_{max}$) for which this problem has a solution? Calculate and plot the required magnitudes A and B as functions of the angle θ_B ($\theta_{min} < \theta_B < \theta_{max}$). Why is the magnitude of $\bar{B}$ a minimum when $\theta_B = 90°$?

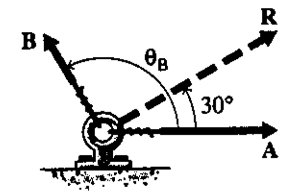
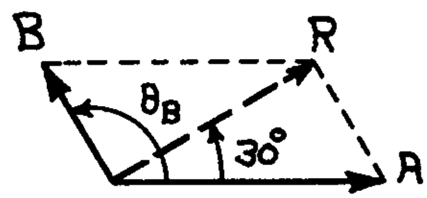


Fig. P2-100

SOLUTION



From the law of sines:

$$\frac{R}{\sin(180° - \theta_B)} = \frac{A}{\sin(\theta_B - 30°)} = \frac{B}{\sin \theta_R}$$

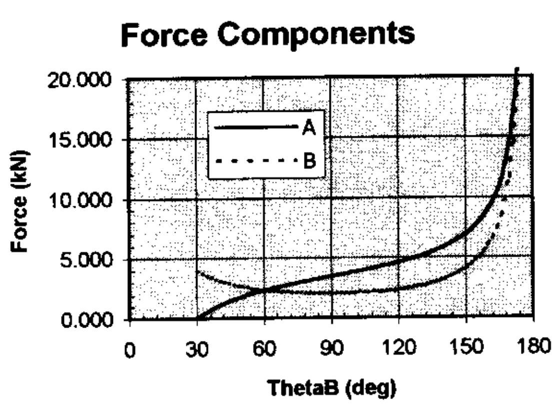
$$A = \frac{R \sin(\theta_B - 30°)}{\sin(180° - \theta_B)} = \frac{4 \sin(\theta_B - 30°)}{\sin \theta_B}$$ Ans.

$$B = \frac{R \sin \theta_R}{\sin(180° - \theta_B)} = \frac{4 \sin 30°}{\sin \theta_B} = \frac{2}{\sin \theta_B}$$ Ans.

When $\theta_B = 90°$, $B = B_{min} = 2$

When $A = 0$, $\sin(\theta_B - 30°) = 0$ $\quad\quad \theta_B = 30° = \theta_{min}$

When $B < 0$, $\sin \theta_B < 0$ $\quad\quad \theta_B > 180° = \theta_{max}$



3-1* Block A of Fig. P3-1 rests on a smooth (frictionless) surface. If the block weighs 25 lb, determine the force exerted on the block by the surface and the force $\bar{P}$ required to prevent motion of the block.

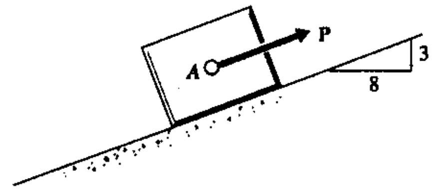
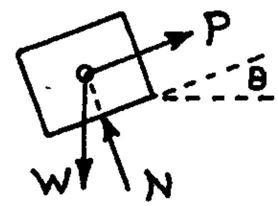


Fig. P3-1

SOLUTION

$$\theta = \tan^{-1}\frac{3}{8} = 20.56^\circ$$

From a free-body diagram for the block:



$+\nearrow \Sigma F_x = P - W\sin\theta = P - 25\sin 20.56^\circ = 0$ $P = 8.78$ lb   Ans.

$+\nwarrow \Sigma F_y = N - W\cos\theta = N - 25\cos 20.56^\circ = 0$ $N = 23.4$ lb   Ans.

3-2* Block A of Fig. P3-2 rests on a smooth (frictionless) surface. If the mass of the block is 100 kg, determine the force exerted on the block by the surface and the force $\bar{P}$ required to prevent motion of the block.

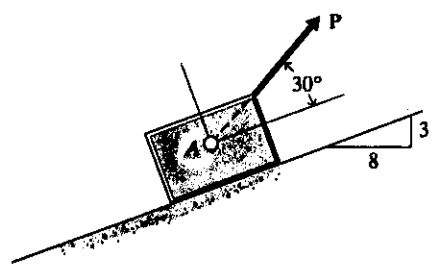


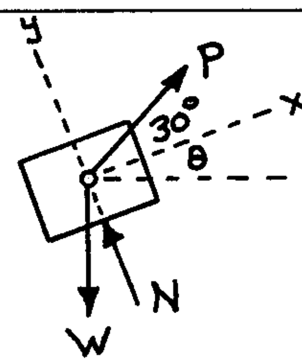
Fig. P3-2

SOLUTION

$$\theta = \tan^{-1}\frac{3}{8} = 20.56^\circ$$

$$W = mg = 100(9.807) = 980.7\text{ N}$$

From a free-body diagram for the block:



$$+\nearrow \Sigma F_x = P\cos 30^\circ - W\sin\theta$$
$$= P\cos 30^\circ - 980.7\sin 20.56^\circ = 0$$

$P = 397.7\text{ N} \cong 398\text{ N}$ Ans.

$$+\nwarrow \Sigma F_y = P\sin 30^\circ + N - W\cos\theta$$
$$= 397.7\sin 30^\circ + N - 980.7\cos 20.56^\circ = 0$$

$N = 719.4\text{ N} \cong 719\text{ N}$ Ans.

3-3 A homogeneous cylinder weighing 500 lb rests against two smooth planes that form a trough as shown in Fig. P3-3. Determine the forces exerted on the cylinder by the planes at contact points A and B.

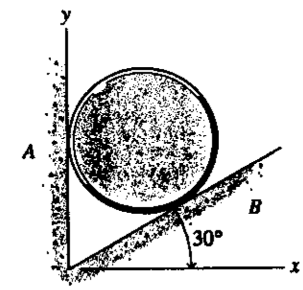
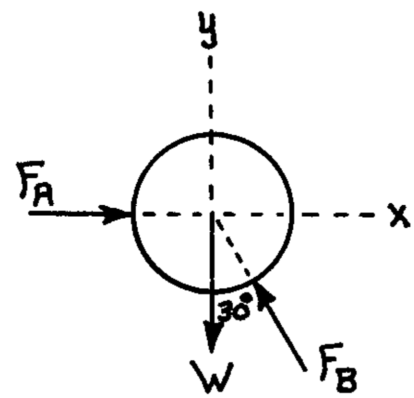


Fig. P3-3

SOLUTION

From a free-body diagram for the cylinder:



$$+\uparrow \Sigma F_y = F_B \cos 30^\circ - W$$

$$= F_B \cos 30^\circ - 500 = 0$$

$$F_B = 577.35 \text{ lb} \cong 577 \text{ lb}$$

$$\bar{F}_B = 577 \text{ lb} \measuredangle 60^\circ \qquad \text{Ans.}$$

$$+\longrightarrow \Sigma F_x = F_A - F_B \sin 30^\circ$$

$$= F_A - 577.35 \sin 30^\circ = 0$$

$$F_A = 288.68 \text{ lb} \cong 289 \text{ lb}$$

$$\bar{F}_A = 289 \text{ lb} \longrightarrow \qquad \text{Ans.}$$

3-4 A homogeneous cylinder with a mass of 250 kg is supported against a smooth surface by a cable as shown in Fig. P3-4. Determine the forces exerted on the cylinder by the cable and by the smooth surface at contact point C.

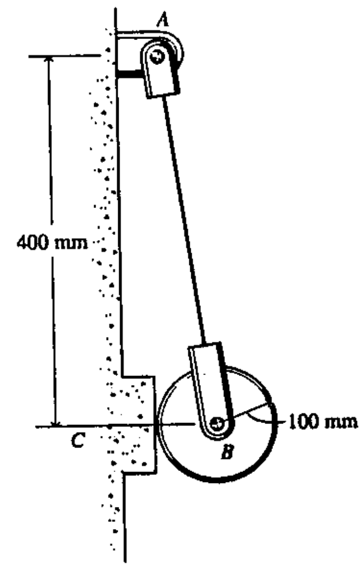


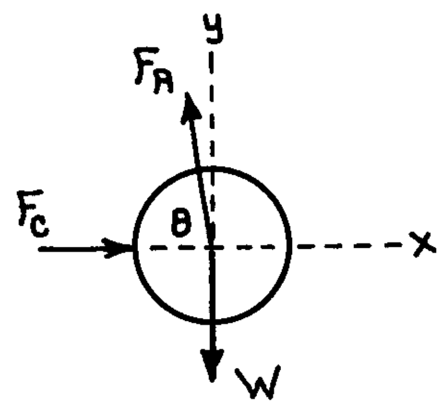
Fig. P3-4

SOLUTION

$$\theta = \tan^{-1}\frac{400}{100} = 75.96^\circ$$

$$W = mg = 250(9.807) = 2452 \text{ N}$$

From a free-body diagram for the cylinder:



$$+\uparrow \Sigma F_y = F_A \sin\theta - W$$

$$= F_A \sin 75.96^\circ - 2452 = 0$$

$$F_A = 2527.5 \text{ N} \cong 2530 \text{ N}$$

$$\vec{F}_A = 2530 \text{ N} \measuredangle 76.0^\circ$$ Ans.

$$+\longrightarrow \Sigma F_x = F_C - F_A \cos\theta$$

$$= F_C - 2527.5 \cos 75.96^\circ = 0$$

$$F_C = 613.17 \text{ N} \cong 613 \text{ N}$$

$$F_C = 613 \text{ N} \longrightarrow$$ Ans.

3-5* A 1000-lb block is supported by a strut and two cables as shown in Fig. P3-5. Determine the forces exerted on the pin at C by the strut and by the two cables.

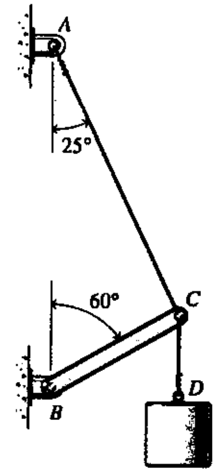
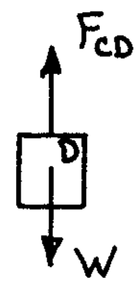


Fig. P3-5

SOLUTION

From a free-body diagram for block D:



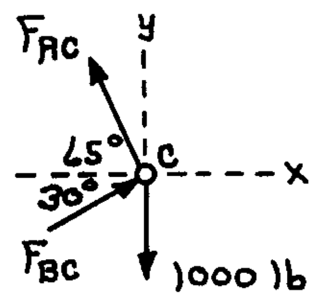
$$+\uparrow \Sigma F_y = F_{CD} - W$$

$$= F_{CD} - 1000 = 0$$

$$F_{CD} = 1000 \text{ lb} \uparrow$$

$$\bar{F}_{CD} = 1000 \text{ lb} \downarrow \text{ (on pin C)}$$ Ans.

From a free-body diagram for pin C:



$$+\longrightarrow \Sigma F_x = F_{BC} \cos 30^\circ - F_{AC} \cos 65^\circ = 0$$

$$F_{AC} = 2.049\ F_{BC}$$

$$+\uparrow \Sigma F_y = F_{BC} \sin 30^\circ + F_{AC} \sin 65^\circ - 1000$$

$$= F_{BC} \sin 30^\circ + 2.049\ F_{BC} \sin 65^\circ - 1000 = 0$$

$$F_{BC} = 424.3 \text{ lb} \cong 424 \text{ lb}$$

$$\bar{F}_{BC} = 424 \text{ lb} \measuredangle 30.0^\circ$$ Ans.

$$F_{AC} = 2.049\ F_{BC} = 2.049(424.3) = 869.4 \text{ lb} \cong 869 \text{ lb}$$

$$\bar{F}_{AC} = 869 \text{ lb} \measuredangle 65.0^\circ$$ Ans.

3-6* A body with a mass of 750 kg is supported by the flexible cable system shown in Fig. P3-6. Determine the tensions in cables AC, BC, and CD.

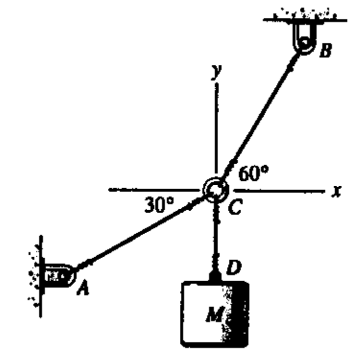
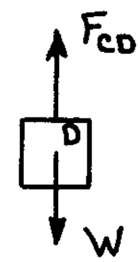


Fig. P3-6

SOLUTION

$W = mg = 750(9.807) = 7355$ N

From a free-body diagram for block D:



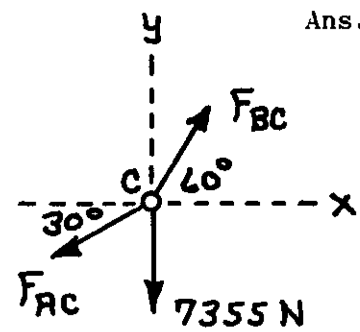
$+\uparrow \Sigma F_y = F_{CD} - W$

$= F_{CD} - 7355 = 0$

$F_{CD} = 7355 \text{ N} \cong 7.36 \text{ kN} \uparrow$

$\mathbf{F}_{CD} = 7.36 \text{ kN} \downarrow$ (on ring C) Ans.

From a free-body diagram for ring C:



$+\rightarrow \Sigma F_x = F_{BC} \cos 60^\circ - F_{AC} \cos 30^\circ = 0$

$F_{BC} = 1.7321\ F_{AC}$

$+\uparrow \Sigma F_y = F_{BC} \sin 60^\circ - F_{AC} \sin 30^\circ - 7355$

$= 1.7321\ F_{AC} \sin 60^\circ - F_{AC} \sin 30^\circ - 7355 = 0$

$F_{AC} = 7355 \text{ N} \cong 7.36 \text{ kN}$

$\mathbf{F}_{AC} = 7.36 \text{ kN} \swarrow 30.0^\circ$ Ans.

$F_{BC} = 1.7321\ F_{AC} = 1.7321(7355) = 12{,}740 \text{ N} \cong 12.74 \text{ kN}$

$\mathbf{F}_{BC} = 12.74 \text{ kN} \nearrow 60.0^\circ$ Ans.

3-7 The collar A shown in Fig. P3-7 is free to slide on the smooth rod BC. Determine the forces exerted on the collar by the cable and by the rod when the force $\bar{F}$ = 900 lb is applied to the collar.

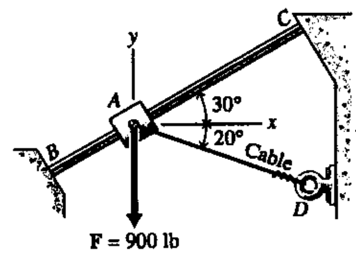
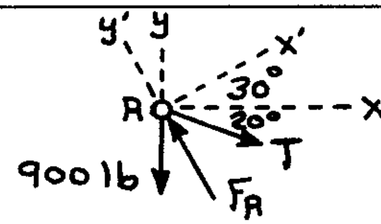


Fig. P3-7

SOLUTION

From a free-body diagram for the collar:



$+\nearrow \Sigma F_{x'} = T \cos 50^\circ - 900 \cos 60^\circ = 0$

$T = 700.08 \text{ lb} \cong 700 \text{ lb}$ $\bar{T} = 700 \text{ lb} \searrow 20.0^\circ$ Ans.

$+\nwarrow \Sigma F_{y'} = F_A - 900 \sin 60^\circ - T \sin 50^\circ$

$= F_A - 900 \sin 60^\circ - 700.08 \sin 50^\circ = 0$

$F_A = 1315.71 \text{ lb} \cong 1316 \text{ lb}$ $\bar{F}_A = 1316 \text{ lb} \nwarrow 60.0^\circ$ Ans.

3-8* An automobile stuck in a muddy field is being moved by using a cable fastened to a tree as shown in Fig. P3-8. When the cable is in the position shown and force $\bar{P}$ = 500 N, determine the x- and y-components of the cable force being applied to the automobile.

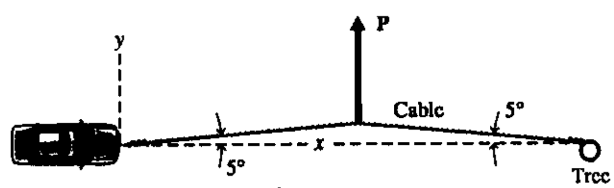
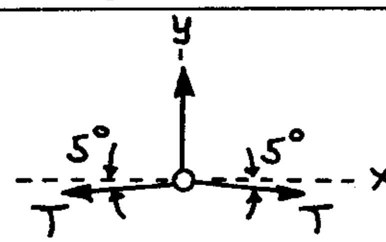


Fig. P3-8

SOLUTION

From a free-body diagram of a point on the cable where load P is applied:



$+\uparrow \Sigma F_y = P - 2T \sin 5^\circ$

$= 500 - 2T \sin 5^\circ = 0$ $T = 2868 \text{ N}$

$F_x = T \cos 5^\circ = 2868 \cos 5^\circ = 2857 \text{ N} \cong 2.86 \text{ kN}$ Ans.

$F_y = T \sin 5^\circ = 2868 \sin 5^\circ = 249.96 \text{ N} \cong 250 \text{ N}$ Ans.

3-9 Determine the magnitudes of forces $\bar{F}_2$ and $\bar{F}_3$ so that the particle shown in Fig. P3-9 is in equilibrium.

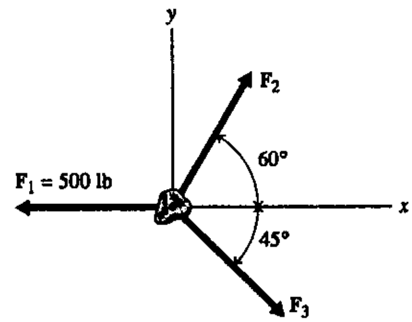
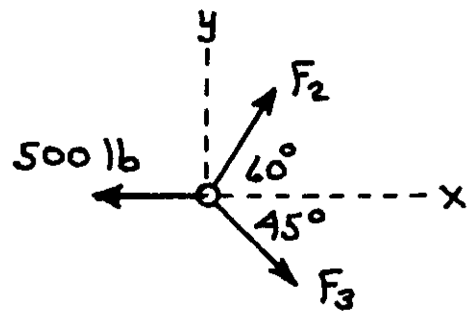


Fig. P3-9

SOLUTION

From the free-body diagram for the particle:



$$+\uparrow \Sigma F_y = F_2 \sin 60^\circ - F_3 \sin 45^\circ$$

$$F_2 = 0.8165\ F_3$$

$$+\rightarrow \Sigma F_x = F_2 \cos 60^\circ + F_3 \cos 45^\circ - 500$$

$$= 0.8165\ F_3 \cos 60^\circ + F_3 \cos 45^\circ - 500 = 0$$

$$F_3 = 448.29 \text{ lb} \cong 448 \text{ lb}$$

$$\bar{F}_3 = 448 \text{ lb} \searrow 45.0^\circ$$ Ans.

$$F_2 = 0.8165\ F_3 = 0.8165(448.29) = 366.03 \text{ lb} \cong 366 \text{ lb}$$

$$\bar{F}_2 = 366 \text{ lb} \angle 60.0^\circ$$ Ans.

3-10* Determine the magnitude and direction angle θ of force $\bar{F}_4$ so that the particle shown in Fig. P3-10 is in equilibrium.

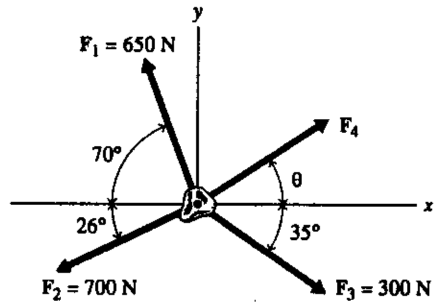
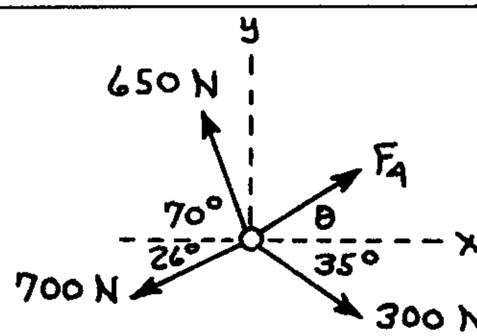


Fig. P3-10

SOLUTION

From the free-body diagram for the particle:



$$+\rightarrow \Sigma F_x = F_4 \cos\theta - 650 \cos 70^\circ - 700 \cos 26^\circ + 300 \cos 35^\circ = 0$$

$$F_4 \cos\theta = 605.72 \text{ N}$$

$$+\uparrow \Sigma F_y = F_4 \sin\theta + 650 \sin 70^\circ - 700 \sin 26^\circ - 300 \sin 35^\circ = 0$$

$$F_4 \sin\theta = -131.87 \text{ N}$$

Solving for force F_4 and angle θ yields:

$$\theta = \tan^{-1} \frac{-131.87}{605.72} = -12.282^\circ \cong -12.28^\circ$$ Ans.

$$F_4 = \frac{2.544}{\sin\theta} = \frac{-131.87}{\sin(-12.282^\circ)} = 619.91\text{N} \cong 620 \text{ N}$$ Ans.

$$\bar{F}_4 = 620 \text{ N} \searrow 12.28^\circ$$ Ans.

3-11 Determine the forces in cables A and B if block W of Fig. P3-11 weighs 350 lb.

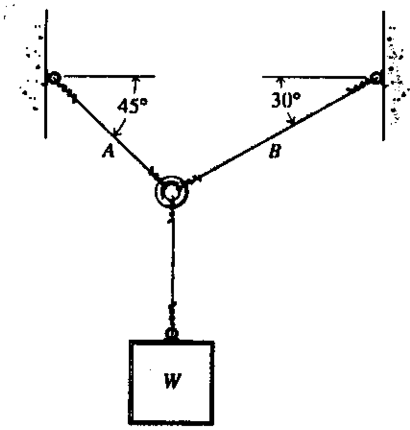
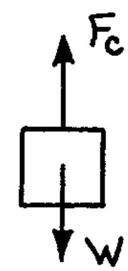


Fig. P3-11

SOLUTION

From a free-body diagram for the block W:



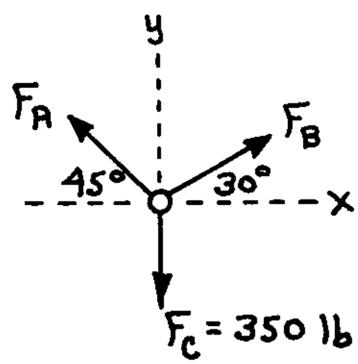
$+\uparrow \Sigma F_y = F_C - W$

$= F_C - 350 = 0$

$F_C = 350 \text{ lb} \uparrow$

$\bar{F}_C = 350 \text{ lb} \downarrow$ (on the ring) Ans.

From a free-body diagram for the ring:



$+\rightarrow \Sigma F_x = F_B \cos 30^\circ - F_A \cos 45^\circ = 0$

$F_B = 0.8165\, F_A$

$+\uparrow \Sigma F_y = F_B \sin 30^\circ + F_A \sin 45^\circ - 350$

$= 0.8165\, F_A \sin 30^\circ + F_A \sin 45^\circ - 350 = 0$

$F_A = 313.80 \text{ lb} \cong 314 \text{ lb}$

$\bar{F}_A = 314 \text{ lb} \measuredangle 45.0^\circ$ Ans.

$F_B = 0.8165\, F_A = 0.8165(313.80) = 256.22 \text{ lb} \cong 256 \text{ lb}$

$\bar{F}_B = 256 \text{ lb} \measuredangle 30.0^\circ$ Ans.

3-12 A body with a mass of 300 kg is supported by the flexible cable system shown in Fig. P3-12. Determine the tensions in cables A, B, C, D, and E.

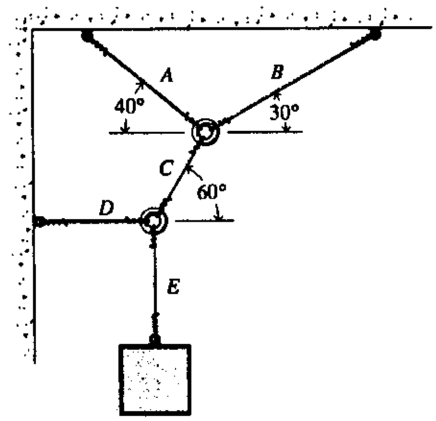
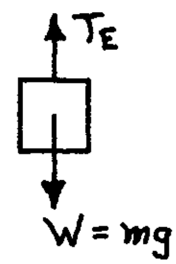


Fig. P3-12

SOLUTION

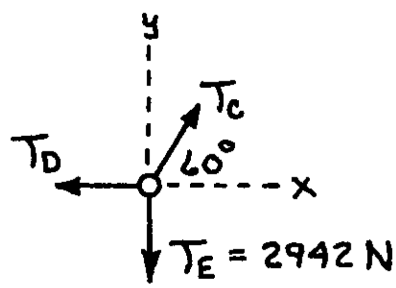
From a free-body diagram for the body:



$+\uparrow \Sigma F_y = T_E - mg = 0$

$T_E = mg = 300(9.807) = 2942 \text{ N} \cong 2940 \text{ N}$ Ans.

From a free-body diagram for the lower cable joint:



$+\uparrow \Sigma F_y = T_C \sin 60^\circ - 2942 = 0$

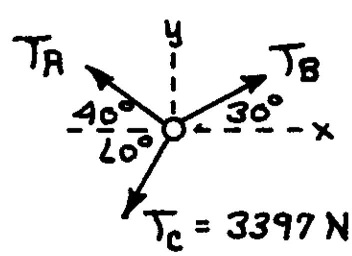
$T_C = 3397 \text{ N} \cong 3400 \text{ N}$ Ans.

$+\rightarrow \Sigma F_x = T_C \cos 60^\circ - T_D$

$= 3397 \cos 60^\circ - T_D = 0$

$T_D = 1698.5 \text{ N} \cong 1699 \text{ N}$ Ans.

From a free-body diagram for the upper cable joint:



$+\uparrow \Sigma F_y = T_B \sin 30^\circ + T_A \sin 40^\circ - 3397 \sin 60^\circ = 0$

$+\rightarrow \Sigma F_x = T_B \cos 30^\circ - T_A \cos 40^\circ - 3397 \cos 60^\circ = 0$

Solving yields: $T_A = 1807.53 \text{ N} \cong 1808 \text{ N}$ Ans.

$T_B = 3560.03 \text{ N} \cong 3560 \text{ N}$ Ans.

3-13* Two flower pots are supported with cables as shown in Fig. P3-13. If pot A weighs 10 lb and pot B weighs 8 lb, determine the tension in each of the cables and the slope of cable BC.

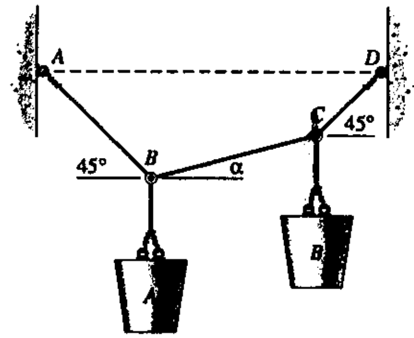
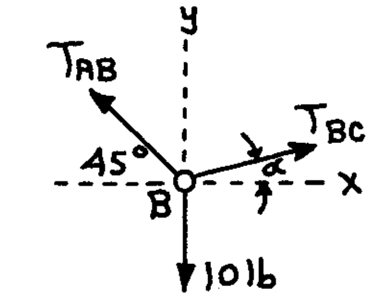


Fig. P3-13

SOLUTION

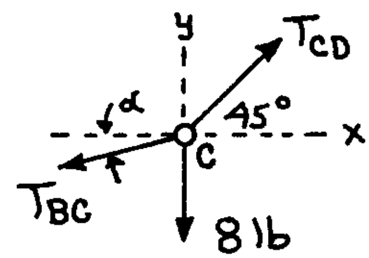
From a free-body diagram for joint B:



(1) $+\rightarrow \Sigma F_x = -T_{AB}\cos 45^\circ + T_{BC}\cos\alpha = 0$

(2) $+\uparrow \Sigma F_y = T_{AB}\sin 45^\circ + T_{BC}\sin\alpha - 10 = 0$

From a free-body diagram for joint C:



(3) $+\rightarrow \Sigma F_x = T_{CD}\cos 45^\circ - T_{BC}\cos\alpha = 0$

(4) $+\uparrow \Sigma F_y = T_{CD}\sin 45^\circ - T_{BC}\sin\alpha - 8 = 0$

From (1) and (3): $-T_{AB}\cos 45^\circ + T_{CD}\cos 45^\circ = 0$

$T_{AB} = T_{CD}$

From (2), and (4): $T_{AB}\sin 45^\circ + T_{CD}\sin 45^\circ = 18$

$2T_{AB}\sin 45^\circ = 18$

$T_{AB} = T_{CD} = 12.728 \text{ lb} \cong 12.73 \text{ lb}$ Ans.

From (1): $T_{BC}\cos\alpha = 12.728\cos 45^\circ = 9.000$

From (2): $T_{BC}\sin\alpha = 10 - 12.728\sin 45^\circ = 1.000$

$\alpha = \tan^{-1}\frac{1}{9} = 6.3402^\circ \cong 6.34^\circ$ Ans.

$T_{BC} = \dfrac{9.000}{\sin 6.3402^\circ} = -9.055 \text{ lb} \cong 9.06 \text{ lb}$ Ans.

3-14* Three smooth homogeneous cylinders A, B, and C are stacked in a V-shaped trough as shown in Fig. P3-14. Each cylinder has a diameter of 500 mm and a mass of 100 kg. Determine the forces exerted on cylinder A by the inclined surfaces.

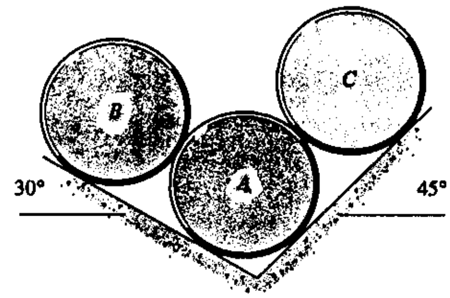
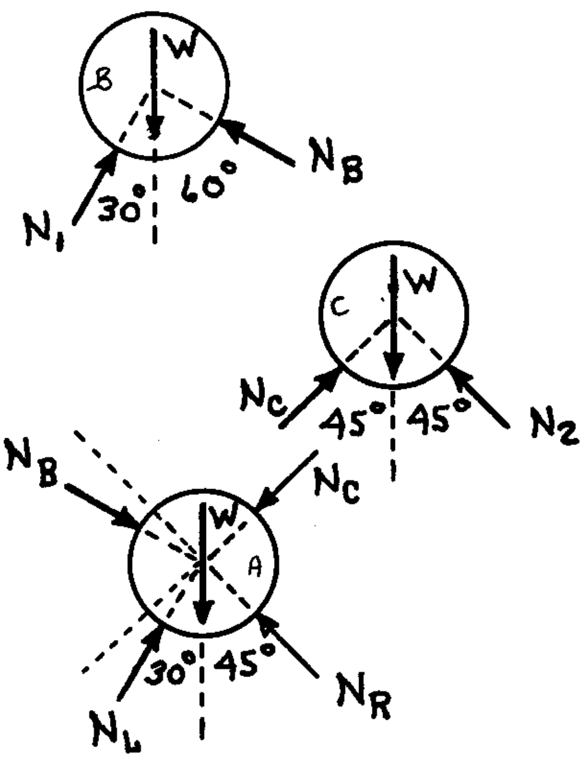


Fig. P3-14

SOLUTION

$W = mg = 100(9.807) = 980.7$ N



From a free-body diagram for cylinder B:

$+\searrow\ \Sigma F_x = -N_B + W \sin 30^\circ$

$= -N_B + 980.7 \sin 30^\circ = 0$

$N_B = 490.35$ N

From a free-body diagram for cylinder C:

$+\nearrow\ \Sigma F_y = N_C - W \sin 45^\circ$

$= N_C - 980.7 \sin 45^\circ = 0$

$N_C = 693.46$ N

From a free-body diagram for cylinder A:

$+\nearrow\ \Sigma F_x = N_L \cos 15^\circ + 490.35 \sin 15^\circ - 980.7 \cos 45^\circ - 693.46 = 0$

$N_L = 1304.46 \text{ N} \cong 1304 \text{ N}$ Ans.

$+\nwarrow\ \Sigma F_y = N_R - 980.7 \sin 45^\circ + 1304.46 \sin 15^\circ - 490.35 \cos 15^\circ = 0$

$N_R = 829.48 \text{ N} \cong 829 \text{ N}$ Ans.

3-15 Three smooth homogeneous cylinders A, B, and C are stacked in a V-shaped trough as shown in Fig. P3-15. Cylinder A weighs 100 lb; cylinders B and C each weigh 200 lb. All cylinders have a 5-in. diameter. Determine the minimum angle θ for equilibrium.

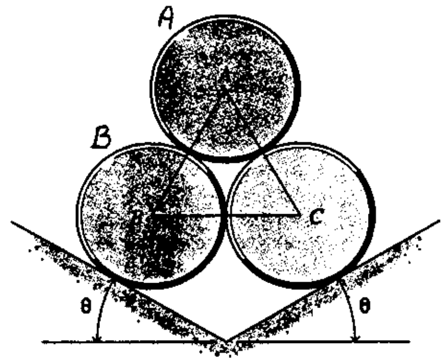
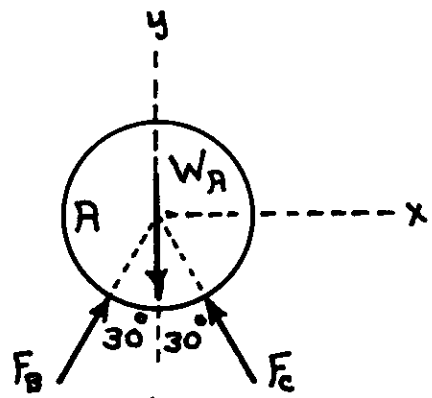


Fig. P3-15

SOLUTION

From a free-body diagram for cylinder A:



$$+\rightarrow \Sigma F_x = F_B \sin 30^\circ - F_C \sin 30^\circ = 0$$

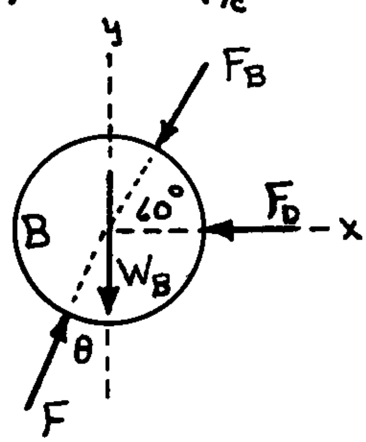
$$F_B = F_C$$

$$+\uparrow \Sigma F_y = F_B \cos 30^\circ + F_C \cos 30^\circ - W_A$$

$$= F_B \cos 30^\circ + F_B \cos 30^\circ - 100 = 0$$

$$F_B = F_C = 57.74 \text{ lb}$$

From a free-body diagram for cylinder B:



$$+\rightarrow \Sigma F_x = F \sin\theta - F_B \cos 60^\circ - F_D$$

$$= F \sin\theta - 57.74 \cos 60^\circ - F_D = 0$$

At the minimum angle θ: $F_D = 0$

Therefore: $F \sin\theta = 28.87$ lb

$$+\uparrow \Sigma F_y = F \cos\theta - W_B - F_B \sin 60^\circ = 0$$

$$= F \cos\theta - 200 - 57.74 \sin 60^\circ = 0$$

$$F \cos\theta = 250.00 \text{ lb}$$

$$\theta_{min} = \tan^{-1} \frac{28.87}{250.0} = 6.587^\circ \cong 6.59^\circ$$ Ans.

3-16* The masses of cylinders A and B of Fig. P3-16 are 40 kg and 75 kg, respectively. Determine the forces exerted on the cylinders by the inclined surfaces and the magnitude and direction of the force exerted by cylinder A on cylinder B when the cylinders are in equilibrium. Assume that all surfaces are smooth.

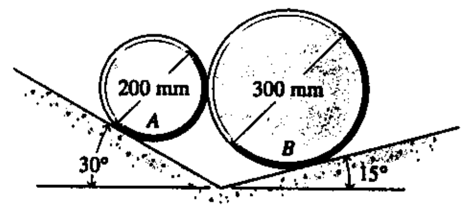
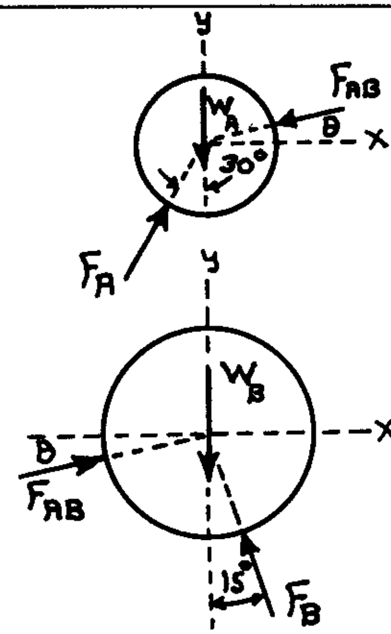


Fig. P3-16

SOLUTION



From a free-body diagram for cylinder A:

(a) $+\rightarrow \Sigma F_x = F_A \sin 30^\circ - F_{AB} \cos\theta = 0$

(b) $+\uparrow \Sigma F_y = F_A \cos 30^\circ - F_{AB} \sin\theta - W_A = 0$

$$W_A = m_A g = 40(9.807) = 392.28 \text{ N}$$

From a free-body diagram for cylinder B:

(c) $+\rightarrow \Sigma F_x = F_{AB} \cos\theta - F_B \sin 15^\circ = 0$

(d) $+\uparrow \Sigma F_y = F_{AB} \sin\theta + F_B \cos 15^\circ - W_B = 0$

$$W_B = m_B g = 75(9.807) = 735.53 \text{ N}$$

From Eqs. (a) and (c): $F_A \sin 30^\circ - F_B \sin 15^\circ = 0 \qquad F_B = 1.9319\, F_A$

From Eqs. (b) and (d): $F_A \cos 30^\circ + F_B \cos 15^\circ = W_A + W_B = 1127.81$

Solving yields: $F_A = 412.80 \text{ N} \cong 413 \text{ N}$ Ans.

$F_B = 797.49 \text{ N} \cong 797 \text{ N}$ Ans.

With $F_A = 412.80$ N and $F_B = 797.49$ N:

Eqs. (c) and (d) yield: $F_{AB} \sin\theta = -34.78$ N

$F_{AB} \cos\theta = 206.41$ N

Therefore: $\theta = \tan^{-1} \dfrac{-34.78}{206.41} = -9.567^\circ \cong -9.57^\circ$ Ans.

$F_{AB} = \dfrac{206.41}{\cos(-9.567^\circ)} = 209.32 \text{ N} \cong 209 \text{ N}$ Ans.

3-17 A continuous cable is used to support blocks A and B as shown in Fig. P3-17. Block A is supported by a small wheel that is free to roll on the cable. Determine the displacement y of block A for equilibrium if the weights of blocks A and B are 50 lb and 75 lb, respectively.

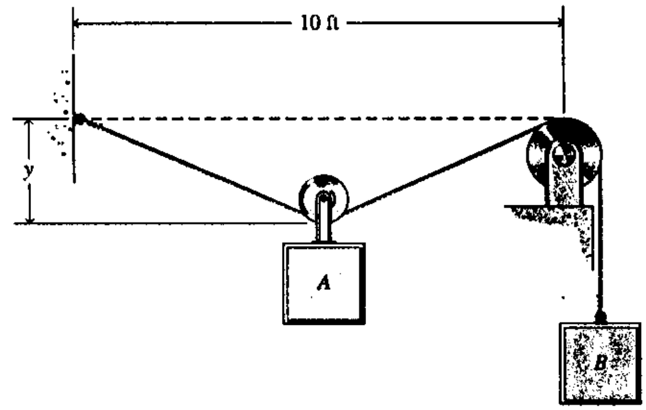
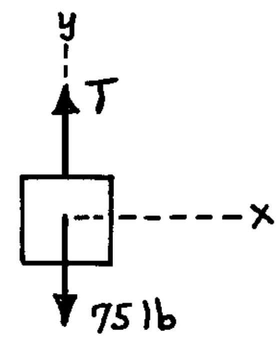


Fig. P3-17

SOLUTION

From a free-body diagram for block B:

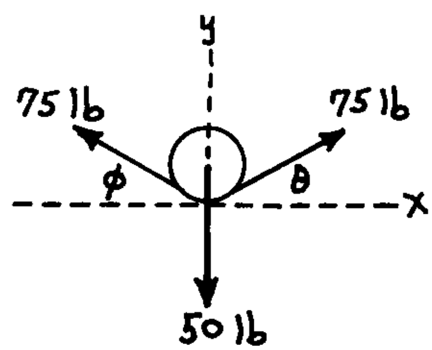


$+\uparrow \Sigma F_y = T - 75 = 0$

$T = 75$ lb

The cable is continuous; therefore, the tension is constant.

From a free-body diagram for block A:



$+\rightarrow \Sigma F_x = 75 \cos\theta - 75 \cos\phi = 0$

$\cos\theta = \cos\phi \qquad \theta = \phi$

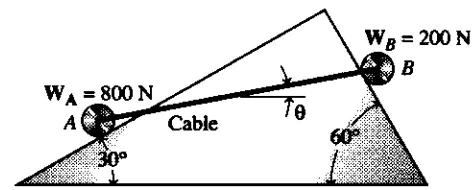
$+\uparrow \Sigma F_y = 75 \sin\theta + 75 \sin\phi - 50$

$= 2(75) \sin\phi - 50 = 0$

$\phi = \sin^{-1} \dfrac{50}{2(75)} = 19.47^\circ$

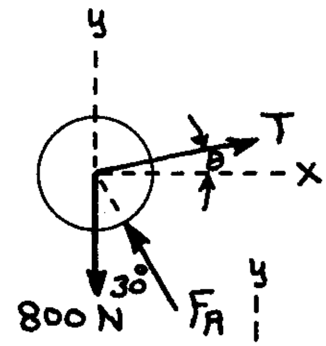
$y = 5 \tan\phi = 5 \tan 19.47^\circ = 1.768$ ft   Ans.

3-18 Two bodies A and B weighing 800 N and 200 N, respectively, are held in equilibrium on perpendicular surfaces by a connecting flexible cable that makes an angle θ with the horizontal as shown in Fig. P3-18. Find the reactions of the surfaces on the bodies, the tension in the cable, and the angle θ. Assume all surfaces to be smooth.



SOLUTION

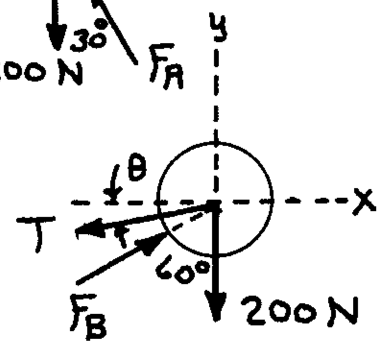
From a free-body diagram for body A:



(a) $+\rightarrow \Sigma F_x = T\cos\theta - F_A \sin 30^\circ = 0$

(b) $+\uparrow \Sigma F_y = T\sin\theta + F_A \cos 30^\circ - 800 = 0$

From a free-body diagram for body B:



(c) $+\rightarrow \Sigma F_x = -T\cos\theta + F_B \sin 60^\circ = 0$

(d) $+\uparrow \Sigma F_y = -T\sin\theta + F_B \cos 60^\circ - 200 = 0$

From Eqs. (a) and (c): $0.8660F_B - 0.5000F_A = 0$

From Eqs. (b) and (d): $0.5000F_B + 0.8660F_A = 1000$

Solving yields: $F_A = 866$ N Ans.

$F_B = 500$ N Ans.

With $F_A = 866$ N and $F_B = 500$ N:

Eqs. (a) and (b) yield: $T\sin\theta = 50$ N

$T\cos\theta = 433$ N

Therefore: $\theta = \tan^{-1}\dfrac{50.0}{433} = 6.587^\circ \cong 6.59^\circ$ Ans.

$T = \dfrac{433}{\cos 6.587^\circ} = 435.9 \text{ N} \cong 436 \text{ N}$ Ans.

3-19* The particle shown in Fig. P3-19 is in equilibrium under the action of the four forces shown on the free-body diagram. Determine the magnitude and the coordinate direction angles of the unknown force $\mathbf{F}_4$.

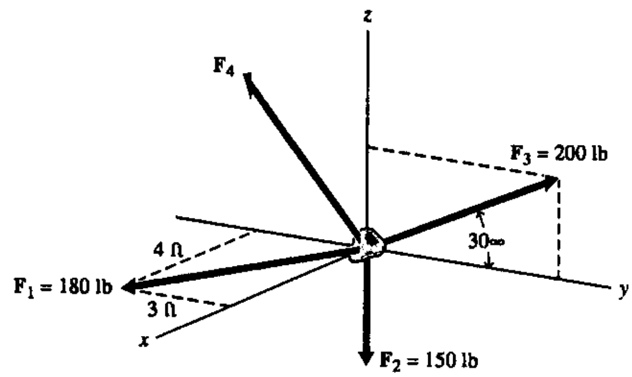
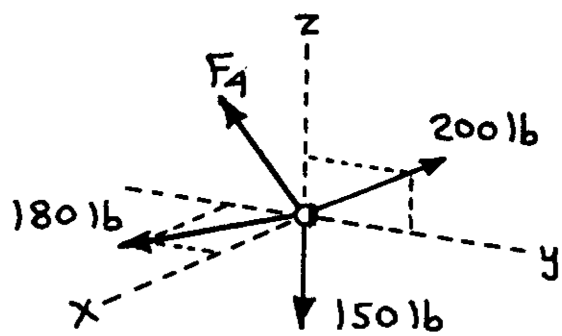


Fig. P3-19

SOLUTION

From the free-body diagram for the particle:



$$R = \Sigma F_x\, \hat{i} + \Sigma F_y\, \hat{j} = \Sigma F_z\, \hat{k} = 0$$

$$\Sigma F_x = F_{4x} + \frac{4}{5}(180) = 0$$

$$F_{4x} = -144.00 \text{ lb}$$

$$\Sigma F_y = F_{4y} - \frac{3}{5}(180) + 200 \cos 30^\circ = 0$$

$$F_{4y} = -65.21 \text{ lb}$$

$$\Sigma F_z = F_{4z} + 200 \sin 30^\circ - 150 = 0$$

$$F_{4z} = 50.00 \text{ lb}$$

$$F_4 = \sqrt{(-144.00)^2 + (-65.21)^2 + (50.00)^2} = 165.80 \text{ lb} = 165.8 \text{ lb} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{F_{4x}}{F_4} = \cos^{-1}\frac{-144.00}{165.80} = 150.29^\circ \cong 150.3^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{F_{4y}}{F_4} = \cos^{-1}\frac{-65.21}{165.80} = 113.16^\circ \cong 113.2^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{F_{4z}}{F_4} = \cos^{-1}\frac{50.00}{165.80} = 74.44^\circ \cong 74.4^\circ \qquad \text{Ans.}$$

3-20* The particle shown in Fig. P3-20 is in equilibrium under the action of the four forces shown on the free-body diagram. Determine the magnitude and the coordinate direction angles of the unknown force $\bar{F}_4$.

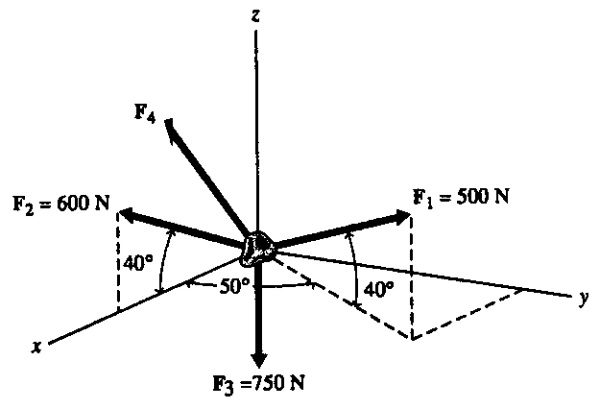
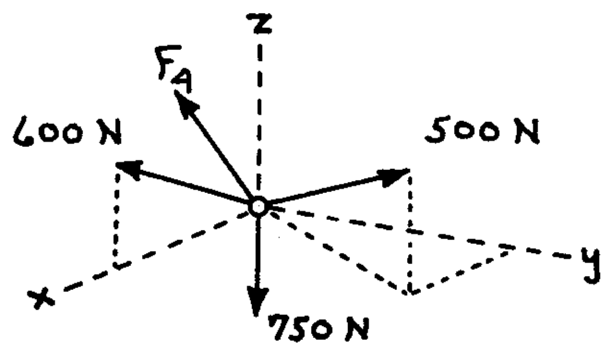


Fig. P3-20

SOLUTION

From the free-body diagram for the particle:



$$\bar{R} = \Sigma F_x\,\hat{i} + \Sigma F_y\,\hat{j} + \Sigma F_z\,\hat{k} = \bar{0}$$

$\Sigma F_x = F_{4x} + 500\cos 40^\circ \cos 50^\circ + 600\cos 40^\circ = 0$

$F_{4x} = -705.83$ N

$\Sigma F_y = F_{4y} + 500\cos 40^\circ \sin 50^\circ = 0$

$F_{4y} = -293.41$ N

$\Sigma F_z = F_{4z} + 500\sin 40^\circ + 600\sin 40^\circ - 750 = 0$

$F_{4z} = 42.93$ N

$$F_4 = \sqrt{(-705.83)^2 + (-293.41)^2 + (42.93)^2} = 765.59\text{ N} \cong 766\text{ N} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{F_{4x}}{F_4} = \cos^{-1}\frac{-705.83}{765.59} = 157.21^\circ \cong 157.2^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{F_{4y}}{F_4} = \cos^{-1}\frac{-293.41}{765.59} = 112.53^\circ \cong 112.5^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{F_{4z}}{F_4} = \cos^{-1}\frac{42.93}{765.59} = 86.79^\circ \cong 86.8^\circ \qquad \text{Ans.}$$

3-21 The block shown in Fig. P3-21 weighs 500 lb. Determine the tensions in cables AB, AC, and AD.

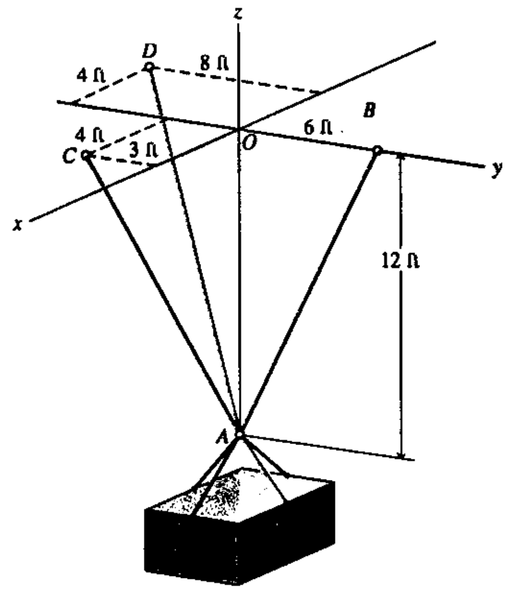


Fig. P3-21

SOLUTION

$$\mathbf{T}_{AB} = T_{AB}\left[\frac{6\,\hat{j} + 12\,\hat{k}}{\sqrt{(6)^2 + (12)^2}}\right] = (0.4472\,\hat{j} + 0.8944\,\hat{k})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\left[\frac{4\,\hat{i} - 3\,\hat{j} + 12\,\hat{k}}{\sqrt{(4)^2 + (-3)^2 + (12)^2}}\right] = (0.3077\,\hat{i} - 0.2308\,\hat{j} + 0.9231\,\hat{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\left[\frac{-4\,\hat{i} - 8\,\hat{j} + 12\,\hat{k}}{\sqrt{(-4)^2 + (-8)^2 + (12)^2}}\right] = (-0.2673\,\hat{i} - 0.5345\,\hat{j} + 0.8018\,\hat{k})T_{AD}$$

$$\mathbf{W} = -500\,\hat{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W}$$

$$= (0.3077T_{AC} - 0.2673T_{AD})\,\hat{i} + (0.4472T_{AB} - 0.2308T_{AC} - 0.5345T_{AD})\,\hat{j}$$

$$+ (0.8944T_{AB} + 0.9231T_{AC} + 0.8018T_{AD} - 500)\,\hat{k} = 0$$

Thus:

$$0.3077T_{AC} - 0.2673T_{AD} = 0$$

$$0.4472T_{AB} - 0.2308T_{AC} - 0.5345T_{AD} = 0$$

$$0.8944T_{AB} + 0.9231T_{AC} + 0.8018T_{AD} = 500$$

Solving yields:

$T_{AB} = 267.36 \text{ lb} \cong 267 \text{ lb}$ Ans.

$T_{AC} = 141.31 \text{ lb} \cong 141.3 \text{ lb}$ Ans.

$T_{AD} = 162.67 \text{ lb} \cong 162.7 \text{ lb}$ Ans.

3-22 Struts AB and AC of Fig. P3-22 can transmit only axial tensile or compressive forces. Determine the forces in struts AB and AC and the tension in cable AD when force $\bar{F}$ = 1250 N.

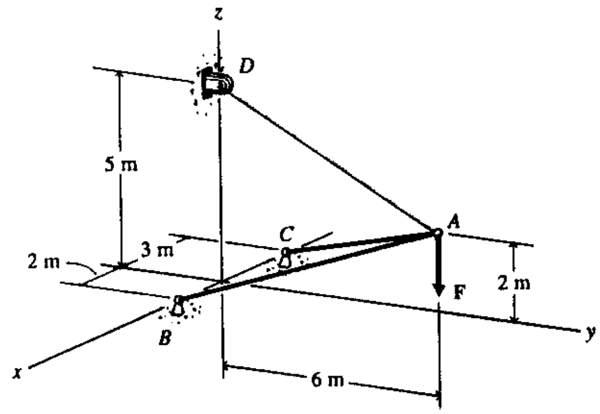


Fig. P3-22

SOLUTION

$$\bar{T}_{AD} = T_{AD}\left[\frac{-6\,\hat{j} + 3\,\hat{k}}{\sqrt{(-6)^2 + (3)^2}}\right] = (-0.8944\,\hat{j} + 0.4472\,\hat{k})T_{AD}$$

$$\bar{F}_{AB} = F_{AB}\left[\frac{2\,\hat{i} - 6\,\hat{j} - 2\,\hat{k}}{\sqrt{(2)^2 + (-6)^2 + (-2)^2}}\right] = (0.3015\,\hat{i} - 0.9045\,\hat{j} - 0.3015\,\hat{k})F_{AB}$$

$$\bar{F}_{AC} = F_{AC}\left[\frac{-3\,\hat{i} - 6\,\hat{j} - 2\,\hat{k}}{\sqrt{(-3)^2 + (-6)^2 + (-2)^2}}\right] = (-0.4286\,\hat{i} - 0.8571\,\hat{j} - 0.2857\,\hat{k})F_{AC}$$

$$\bar{F} = -\,1250\,\hat{k}\ \text{N}$$

$$\bar{R} = \bar{F}_{AB} + \bar{F}_{AC} + \bar{T}_{AD} + \bar{F}$$

$$= (0.3015F_{AB} - 0.4286F_{AC})\,\hat{i} + (-0.8944T_{AD} - 0.9045F_{AB} - 0.8571F_{AC})\,\hat{j}$$
$$+ (0.4472T_{AD} - 0.3015F_{AB} - 0.2857F_{AC} - 1250)\,\hat{k} = 0$$

Thus:

$$0.3015F_{AB} - 0.4286F_{AC} = 0$$

$$-0.8944T_{AD} - 0.9045F_{AB} - 0.8571F_{AC} = 0$$

$$0.4472T_{AD} - 0.3015F_{AB} - 0.2857F_{AC} = 1250$$

Solving yields:

T_{AD} = 1667.10 N = 1677 N (T)   Ans.

F_{AB} = -995.07 N = 995 N (C)   Ans.

F_{AC} = -699.99 N = 700 N (C)   Ans.

3-23* Determine the forces in legs AB, AC, and AD of the tripod shown in Fig. P3-23 when force $\bar{F}$ = 75 lb. The legs can transmit only axial forces.

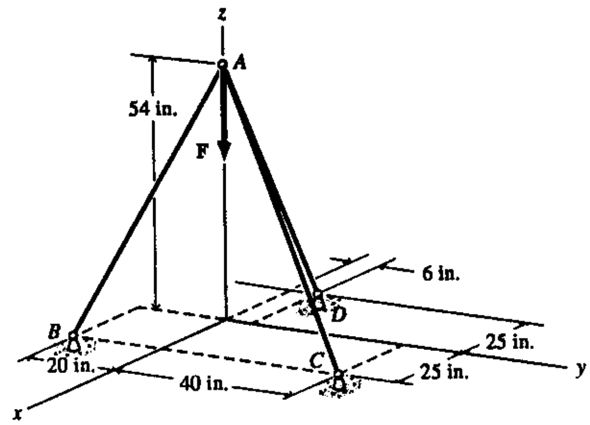


Fig. P3-23

SOLUTION

$$\bar{F}_B = F_B\left[\frac{25\,\hat{i} - 20\,\hat{j} - 54\,\hat{k}}{\sqrt{(25)^2 + (-20)^2 + (-54)^2}}\right] = (0.3982\,\hat{i} - 0.3186\,\hat{j} - 0.8602\,\hat{k})F_B$$

$$\bar{F}_C = F_C\left[\frac{25\,\hat{i} + 40\,\hat{j} - 54\,\hat{k}}{\sqrt{(25)^2 + (40)^2 + (-54)^2}}\right] = (0.3487\,\hat{i} + 0.5579\,\hat{j} - 0.7531\,\hat{k})F_C$$

$$\bar{F}_D = F_D\left[\frac{-25\,\hat{i} + 6\,\hat{j} - 54\,\hat{k}}{\sqrt{(-25)^2 + (6)^2 + (-54)^2}}\right] = (-0.4180\,\hat{i} + 0.1003\,\hat{j} - 0.9029\,\hat{k})F_D$$

$$\bar{F} = -75\,\hat{k}\text{ lb}$$

$$\bar{R} = \bar{F}_B + \bar{F}_C + \bar{F}_D + \bar{F}$$

$$= (0.3982F_B + 0.3487F_C - 0.4180F_D)\,\hat{i}$$
$$+ (-0.3186F_B + 0.5579F_C + 0.1003F_D)$$
$$+ (-0.8602F_B - 0.7531F_C - 0.9029F_D - 75)\,\hat{k} = \bar{0}$$

Thus:

$$0.3982F_B + 0.3487F_C - 0.4180F_D = 0$$
$$-0.3186F_B + 0.5579F_C + 0.1003F_D = 0$$
$$-0.8602F_B - 0.7531F_C - 0.9029F_D = 75$$

Solving yields:

F_B = -33.42 lb ≅ 33.4 lb (C) Ans.

F_C = -11.62 lb = 11.62 lb (C) Ans.

F_D = -41.53 lb ≅ 41.5 lb (C) Ans.

3-24* Column AB of Fig. P3-24 can support a maximum axial compressive force of 7500 N. Determine the tensions in each of the cables when this level of load exists in the column.

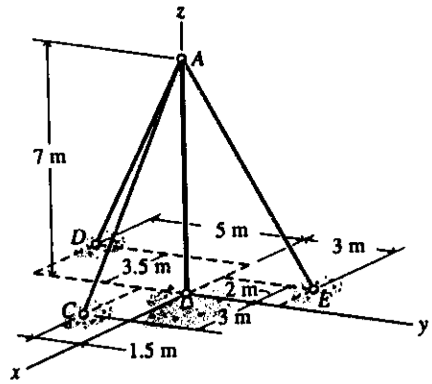


Fig. P3-24

SOLUTION

$$\bar{F}_C = F_C\left[\frac{3\,\hat{i} - 1.5\,\hat{j} - 7\,\hat{k}}{\sqrt{(3)^2 + (-1.5)^2 + (-7)^2}}\right] = (0.3865\,\hat{i} - 0.1932\,\hat{j} - 0.9018\,\hat{k})F_C$$

$$\bar{F}_D = F_D\left[\frac{-3.5\,\hat{i} - 5\,\hat{j} - 7\,\hat{k}}{\sqrt{(-3.5)^2 + (-5)^2 + (-7)^2}}\right] = (-0.3769\,\hat{i} - 0.5384\,\hat{j} - 0.7537\,\hat{k})F_D$$

$$\bar{F}_E = F_E\left[\frac{-2\,\hat{i} + 3\,\hat{j} - 7\,\hat{k}}{\sqrt{(-2)^2 + (3)^2 + (-7)^2}}\right] = (-0.2540\,\hat{i} + 0.3810\,\hat{j} - 0.8890\,\hat{k})F_E$$

$$\bar{F}_B = 7500\,\hat{k}$$

$$\bar{R} = \bar{F}_C + \bar{F}_D + \bar{F}_E + \bar{F}_B$$

$$= (0.3865F_C - 0.3769F_D - 0.2540F_E)\,\hat{i}$$
$$+ (-0.1932F_C - 0.5384F_D + 0.3810F_E)\,\hat{j}$$
$$+ (-0.9018F_C - 0.7537F_D - 0.8890F_E + 7500)\,\hat{k} = \bar{0}$$

Thus:

$$0.3865F_C + 0.3769F_D - 0.2540F_E = 0$$

$$-0.1932F_C + 0.5384F_D - 0.3810F_E = 0$$

$$-0.9018F_C - 0.7537F_D - 0.8890F_E = -7500$$

Solving yields:

$F_C = 3647$ N ≅ 3.65 kN (T) Ans.

$F_D = 1277.2$ N ≅ 1.277 kN (T) Ans.

$F_E = 3654$ N ≅ 3.65 kN (T) Ans.

3-25 The circular disk shown in Fig. P3-25 weighs 750 lb. Determine the tensions in cables A, B, and C.

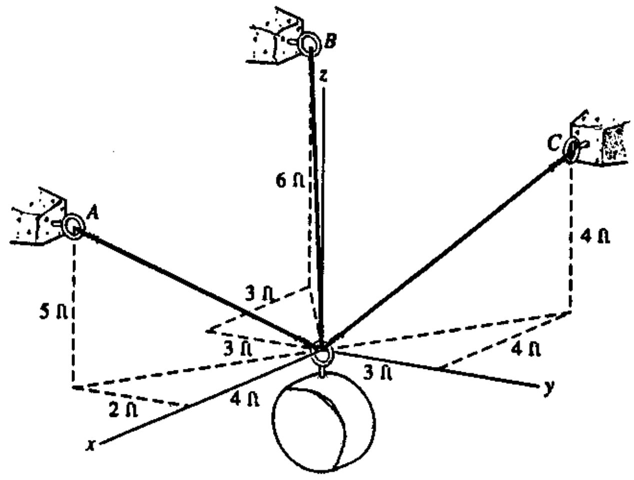


Fig. P3-25

SOLUTION

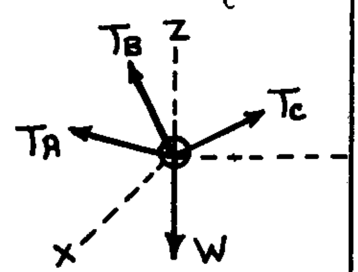
$$\mathbf{T}_A = T_A\left[\frac{4\,\hat{i} - 2\,\hat{j} + 5\,\hat{k}}{\sqrt{(4)^2 + (-3)^2 + (4)^2}}\right]$$

$$= (0.5963\,\hat{i} - 0.2981\,\hat{j} + 0.7454\,\hat{k})T_A$$

$$\mathbf{T}_B = T_B\left[\frac{-3\,\hat{i} - 3\,\hat{j} + 6\,\hat{k}}{\sqrt{(-3)^2 + (-3)^2 + (6)^2}}\right] = (-0.4082\,\hat{i} - 0.4082\,\hat{j} + 0.8165\,\hat{k})T_B$$

$$\mathbf{T}_C = T_C\left[\frac{-4\,\hat{i} + 3\,\hat{j} + 4\,\hat{k}}{\sqrt{(-4)^2 + (3)^2 + (4)^2}}\right] = (-0.6247\,\hat{i} + 0.4685\,\hat{j} + 0.6247\,\hat{k})T_C$$

$$\mathbf{W} = -750\,\hat{k}$$



$$\mathbf{R} = \mathbf{T}_A + \mathbf{T}_B + \mathbf{T}_C + \mathbf{W}$$

$$= (0.5963T_A - 0.4082T_B - 0.6247T_C)\,\hat{i}$$

$$+ (-0.2981T_A - 0.4082T_B + 0.4685T_C)\,\hat{j}$$

$$+ (0.7454T_A + 0.8165T_B + 0.6247T_C - 750)\,\hat{k} = \mathbf{0}$$

Thus:

$$0.5963T_A - 0.4082T_B - 0.6247T_C = 0$$

$$-0.2981T_A - 0.4082T_B + 0.4685T_C = 0$$

$$0.7454T_A + 0.8165T_B + 0.6247T_C = 750$$

Solving yields:

$F_A = 525.61$ lb $\cong$ 526 lb (T) Ans.

$F_B = 109.71$ lb $\cong$ 109.7 lb (T) Ans.

$F_C = 430.02$ lb $\cong$ 430 lb (T) Ans.

3-26 Two forces are applied in a horizontal plane to a loading ring at the top of a post as shown in Fig. P3-26. The post can transmit only an axial compressive force. Two guy wires AC and BC are used to hold the loading ring in equilibrium. Determine the force transmitted by the post and the forces in the two guy wires.

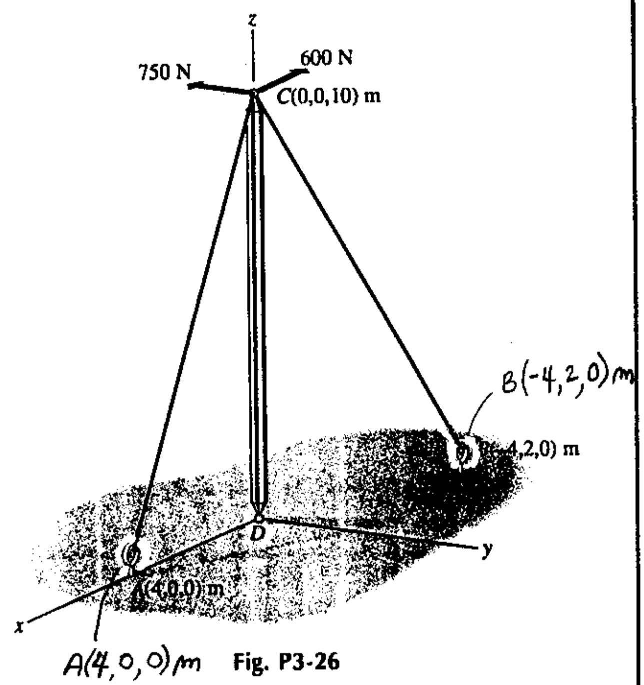


Fig. P3-26

SOLUTION

$$\mathbf{T}_A = T_A\left[\frac{4\,\hat{i} - 10\,\hat{k}}{\sqrt{(4)^2 + (-10)^2}}\right]$$

$$= (0.3714\,\hat{i} - 0.9285\,\hat{k})T_A$$

$$\mathbf{T}_B = T_B\left[\frac{-4\,\hat{i} + 2\,\hat{j} - 10\,\hat{k}}{\sqrt{(-4)^2 + (2)^2 + (-10)^2}}\right] = (-0.3651\,\hat{i} + 0.1826\,\hat{j} - 0.9129\,\hat{k})T_B$$

$$\mathbf{F}_C = -600\,\hat{i} - 750\,\hat{j} \qquad \mathbf{F}_D = F_D\,\hat{k}$$

Thus:

$$\mathbf{R} = \Sigma\mathbf{F} = \mathbf{T}_A + \mathbf{T}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$= (0.3714T_A - 0.3651T_B - 600)\,\hat{i} + (0.1826T_B - 750)\,\hat{j} + (-0.9285T_A - 0.9129T_B + 1.0000F_D)\,\hat{k} = \mathbf{0}$$

Thus:

$$0.3714T_A - 0.3651T_B = 600$$

$$0.1826T_B = 750$$

$$-0.9285T_A - 0.9129T_B + 1.0000F_D = 0$$

Solving yields:

$T_A = 5653$ N ≅ 5.65 kN (T)   Ans.

$T_B = 4107$ N ≅ 4.11 kN (T)   Ans.

$F_D = 8999$ N ≅ 9.00 kN (T)   Ans.

C3-27 A 75-lb stop light is suspended between two poles as shown in Fig. P3-27. Neglect the weight of the flexible cables and plot the tension in both cables as a function of the sag distance d ($0 \le d \le 8$ ft.) Determine the minimum sag d for which both tensions are less than:
(a) 100 lb.
(b) 250 lb.
(c) 500 lb.

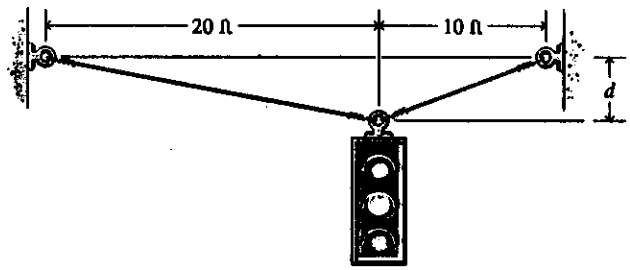


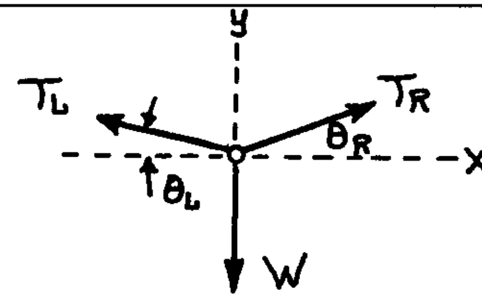
Fig. P3-27

SOLUTION

$$\tan \theta_L = d/20$$

$$\tan \theta_R = d/10$$

From a free-body diagram for the cable joint:



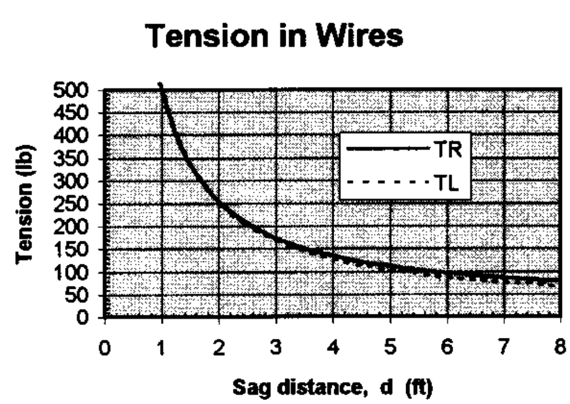
$$+ \longrightarrow \Sigma F_x = T_R \cos \theta_R - T_L \cos \theta_L = 0$$

$$+ \uparrow \Sigma F_y = T_R \sin \theta_R + T_L \sin \theta_L - W = 0$$

Solving yields:

$$T_R = \frac{W \cos \theta_L}{\sin \theta_R \cos \theta_L + \cos \theta_R \sin \theta_L}$$

$$T_L = \frac{W \cos \theta_R}{\sin \theta_L \cos \theta_R + \cos \theta_L \sin \theta_R}$$



C3-28 A 50-kg load is suspended from a pulley as shown in Fig. P3-28. The tension in the flexible cable does not change as it passes around the small frictionless pulleys, and the weight of the cable may be neglected. Plot the force P required for equilibrium as a function of the sag distance d ($0 \le d \le 1$ m). Determine the minimum sag d for which P is less than:

(a) Twice the weight of the load.
(b) Four times the weight of the load.
(c) Eight times the weight of the load.

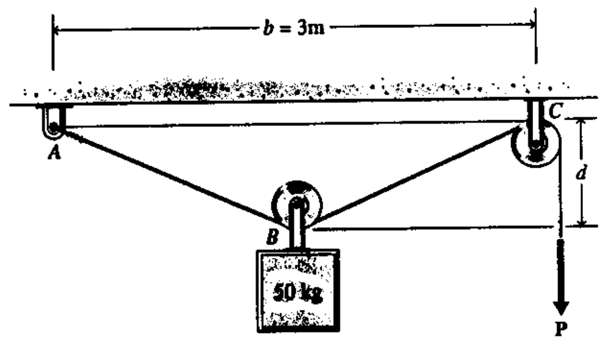
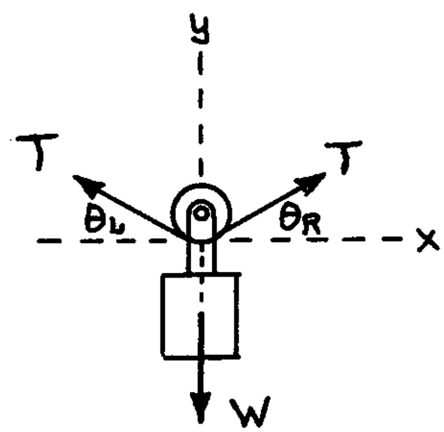


Fig. P3-28

SOLUTION

$W = mg = 50(9.807) = 490.35$ N

From a free-body diagram for the 50-kg load:



The cable is continuous; therefore, the tension is constant.

$$+\rightarrow \Sigma F_x = T\cos\theta_R - T\cos\theta_L = 0$$

$$\text{Therefore: } \theta_L = \theta_R = \theta = \tan^{-1}\frac{d}{1.5}$$

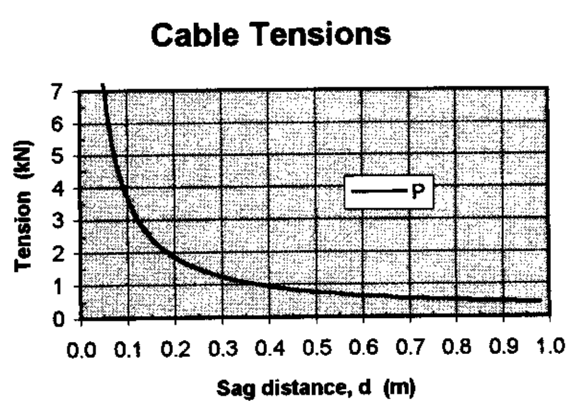
$$+\uparrow \Sigma F_y = T\sin\theta + T\sin\theta - W = 0$$

$$T = P = \frac{W}{2\sin\theta}$$

(a) $P < 2W$, $d > 0.38730$ m $\cong$ 387 mm

(b) $P < 4W$, $d > 0.18898$ m $\cong$ 189 mm

(c) $P < 8W$, $d > 0.09393$ m $\cong$ 94 mm



C3-29 Three identical steel disks are stacked in a box a shown in Fig. P3-29. The weight and diameter of the smooth disks are 50 lb and 12 in., respectively. Plot the three forces exerted on disk C (by disk A, by the side wall, and by the floor) as a function of the distance b between the walls of the box (24 in. ≤ b ≤ 36 in.) Determine the range of b for which:

(a) The force at the floor is larger than the other two forces.
(b) None of the three forces exceeds 100 lb.
(c) None of the three forces exceeds 200 lb.

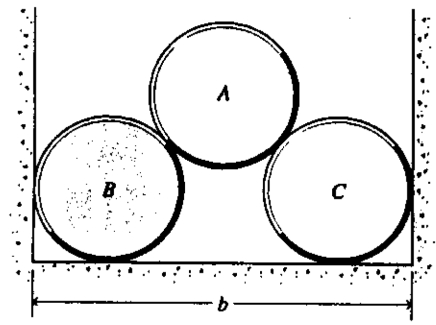
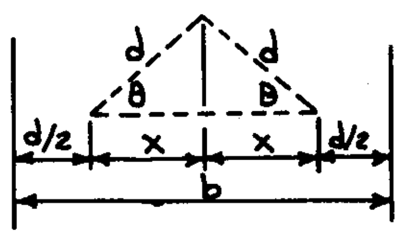


Fig. P3-29

SOLUTION

$$\cos\theta = \frac{x}{d} = \frac{(b-d)/2}{d} = \frac{(b-12)/2}{12}$$



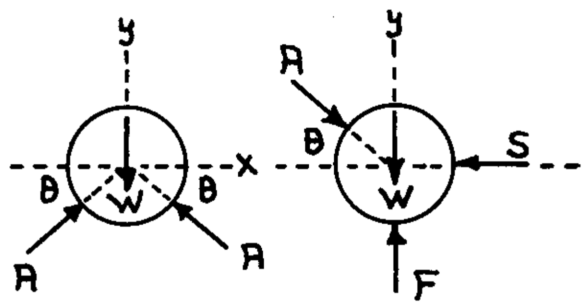
From a free-body diagram for cylinder A:

$$+\uparrow \Sigma F_y = 2A\sin\theta - W$$

$$= 2A\sin\theta - 50 = 0$$

$$A = \frac{25}{\sin\theta}$$

From a free-body diagram for cylinder C:



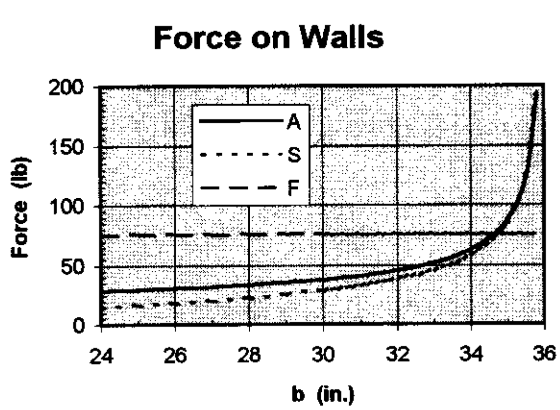
$$+\longrightarrow \Sigma F_x = A\cos\theta - S = 0$$

$$S = A\cos\theta \qquad A > S$$

$$+\uparrow \Sigma F_y = F - A\sin\theta - W$$

$$= F - A\sin\theta - 50 = 0$$

$$F = 50 + \frac{25}{\sin\theta}\sin\theta = 75 \text{ lb}$$



(a) A < 75 lb, b < 34.6 in.

(b) A < 100 lb, b < 35.2 in.

(c) A < 200 lb, b < 35.8 in.

C3-30 Two small wheels are connected by a light-weight rigid rod as shown in Fig. P3-30. Plot the angle θ (between the rod and the horizontal) as a function of the weight W_1 ($W_1 \le 10\ W_2$). Determine the weight W_1 for which:

(a) $\theta = -50°$.

(b) $\theta = 10°$.

(c) $\theta = 25°$.

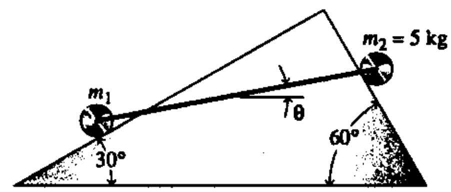
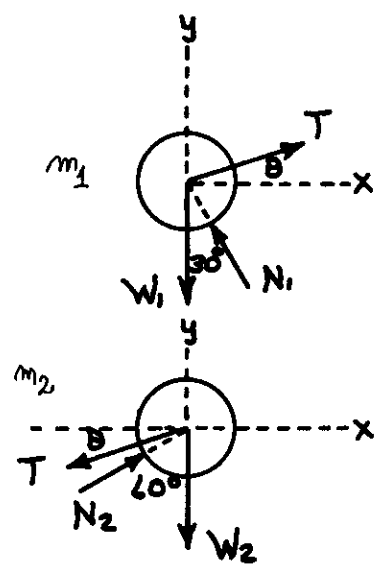


Fig. P3-30

Do you think the equilibrium positions of parts a, b, and c are stable? (That is, if the wheels were disturbed slightly, do you think they would return to the equilibrium position or do you think they would slide off of the triangular surface?)

SOLUTION



From a free-body diagram for wheel 1:

$$+\nearrow \Sigma F = T\cos(30° - \theta) - W_1 \cos 60° = 0$$

From a free-body diagram for wheel 2:

$$+\nwarrow \Sigma F = T\sin(30° - \theta) - W_2 \cos 30° = 0$$

$$T = \frac{W_1 \cos 60°}{\cos(30° - \theta)} = \frac{W_2 \cos 30°}{\sin(30° - \theta)}$$

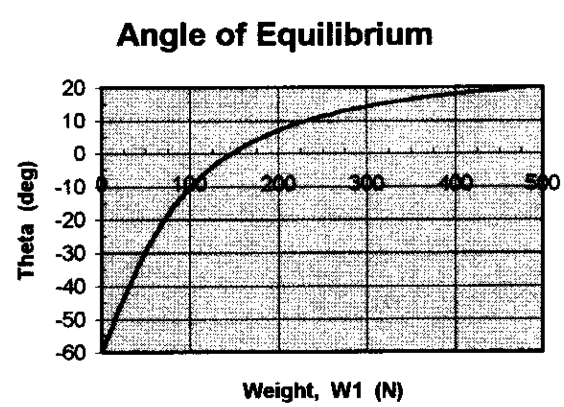
$$\tan(30° - \theta) = \frac{W_2 \cos 30°}{W_1 \cos 60°}$$

$$W_1 = \frac{W_2 \cos 30°}{\tan(30° - \theta)\cos 60°}$$

(a) For $\theta = -50°$: $W_1 = 14.98$ N

(b) For $\theta = 10°$: $W_1 = 233$ N

(c) For $\theta = 25°$: $W_1 = 971$ N



3-31* Determine the magnitude and direction angle θ of force $\mathbf{F}_4$ so that the particle shown in Fig. P3-31 is in equilibrium.

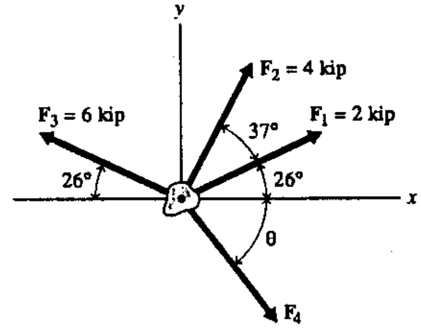
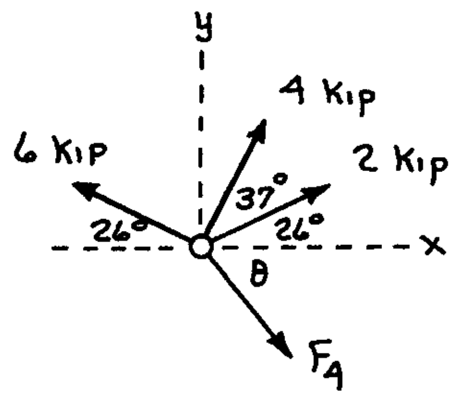


Fig. P3-31

SOLUTION

From the free-body diagram for the particle:



$$+\rightarrow \Sigma F_x = F_4 \cos\theta - 6\cos 26^\circ + 4\cos 63^\circ + 2\cos 26^\circ = F_4\cos\theta - 1.7792 = 0$$

$$+\uparrow \Sigma F_y = -F_4 \sin\theta + 6\sin 26^\circ + 4\sin 63^\circ + 2\sin 26^\circ = -F_4\sin\theta + 7.0710 = 0$$

Solving yields:

$$\theta = \tan^{-1}\frac{7.0710}{1.7792} = 75.88^\circ \cong 75.9^\circ$$ Ans.

$$F_4 = \frac{7.0710}{\sin\theta} = \frac{7.0710}{\sin 75.88^\circ} = 7.291 \text{ kip} \cong 7.29 \text{ kip}$$ Ans.

3-32* Determine the magnitude and direction angle θ of force $\mathbf{F}_4$ so that the particle shown in Fig. P3-32 is in equilibrium.

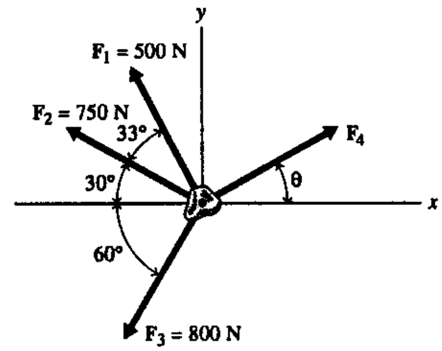
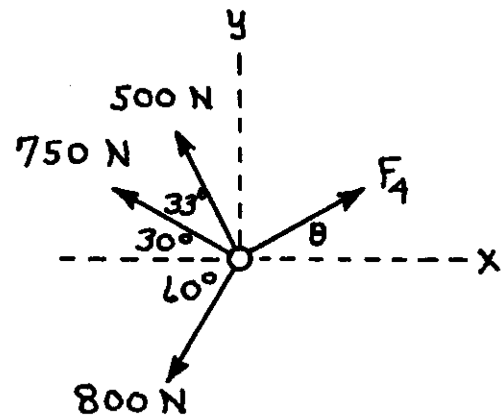


Fig. P3-32

SOLUTION

From the free-body diagram for the particle:



$$+\rightarrow \Sigma F_x = F_4 \cos\theta - 500 \cos 63^\circ - 750 \cos 30^\circ - 800 \cos 60^\circ = F_4 \cos\theta - 1276.5 = 0$$

$$+\uparrow \Sigma F_y = F_4 \sin\theta + 500 \sin 63^\circ + 750 \sin 30^\circ - 800 \sin 60^\circ = F_4 \sin\theta + 127.68 = 0$$

Solving yields:

$$\theta = \tan^{-1}\frac{-127.68}{1276.5} = -5.712^\circ \cong -5.71^\circ$$ Ans.

$$F_4 = \frac{1276.5}{\cos\theta} = \frac{1276.5}{\cos(-5.712^\circ)} = 1282.9 \text{ N} \cong 1283 \text{ N}$$ Ans.

3-33 Two 10-in. diameter pipes and a 6-in. diameter pipe are supported in a pipe rack as shown in Fig. P3-33. The 10-in. diameter pipes each weigh 350 lb and the 6-in. diameter pipe weighs 225 lb. Determine the forces exerted on the pipes by the supports at contact surfaces A, B, and C. Assume all surfaces to be smooth.

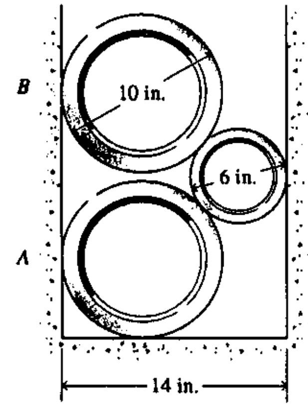
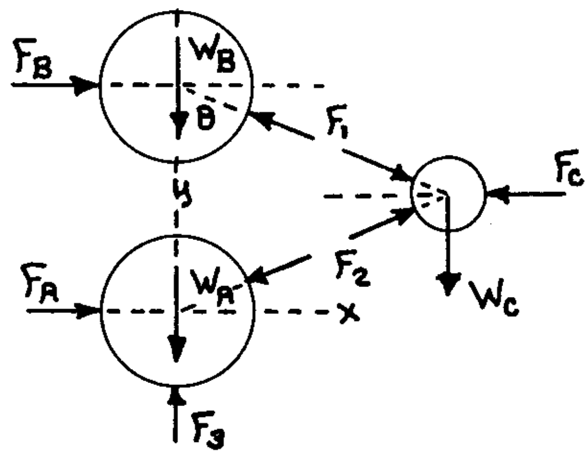


Fig. P3-33

SOLUTION

$$\theta = \sin^{-1}\frac{6}{8} = 48.59^\circ$$



From a free-body diagram for the upper pipe:

$$+\uparrow \Sigma F_y = F_1 \cos\theta - W_B$$
$$= F_1 \cos 48.59^\circ - 350 = 0$$

$$F_1 = 529.15 \text{ lb}$$

$$+\rightarrow \Sigma F_x = F_B - F_1 \sin\theta = 0$$

$$F_B = F_1 \sin\theta = 529.15 \sin 48.59^\circ = 396.86 \text{ lb} \cong 397 \text{ lb}$$ Ans.

From a free-body diagram for the middle pipe:

$$+\uparrow \Sigma F_y = F_2 \cos\theta - F_1 \cos\theta - W_C$$
$$= F_2 \cos 48.59^\circ - 529.15 \cos 48.59^\circ - 225 = 0$$

$$F_2 = 869.32 \text{ lb}$$

$$+\rightarrow \Sigma F_x = F_1 \sin\theta + F_2 \sin\theta - F_C$$
$$= 529.15 \sin 48.59^\circ + 869.32 \sin 48.59^\circ - F_C = 0$$

$$F_C = 1048.8 \text{ lb} \cong 1049 \text{ lb}$$ Ans.

From a free-body diagram for the lower pipe:

$$+\rightarrow \Sigma F_x = F_A - F_2 \sin\theta = F_A - 869.32 \sin 48.59^\circ = 0$$

$$F_A = 651.99 \text{ lb} \cong 652 \text{ lb}$$ Ans.

3-34 The mass of block A in Fig. P3-34 is 250 kg. Block A is supported by a small wheel that is free to roll on the continuous cable between supports B and C. The length of the cable is 42 m. Determine the distance x and the tension T in the cable when the system is in equilibrium.

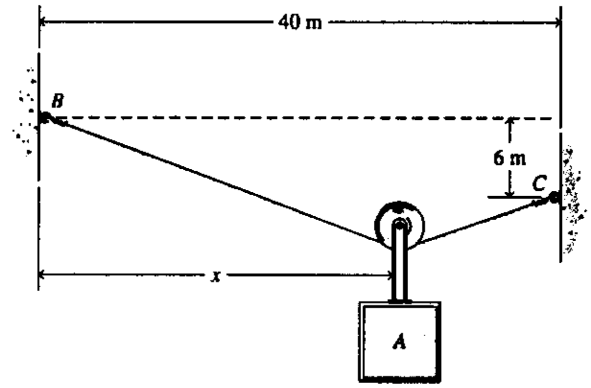
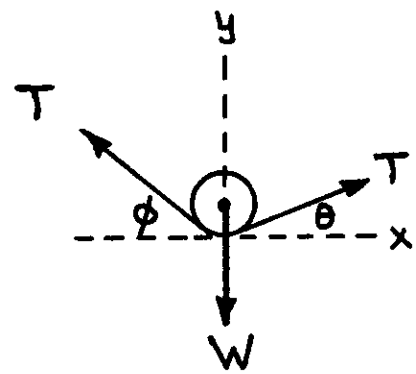


Fig. P3-34

SOLUTION

From a free-body diagram for the wheel:



$+ \rightarrow \Sigma F_x = T\cos\theta - T\cos\phi = 0$

$\cos\theta = \cos\phi \qquad \theta = \phi$

From the span and length of the cable:

$L\cos\phi = 42\cos\phi = 40$ m

$\phi = \cos^{-1}\dfrac{40}{42} = 17.753^\circ$

$x\tan\phi = (40 - x)\tan\phi + 6$

$2x\tan\phi = 40\tan\phi + 6$

$x = \dfrac{40\tan\phi + 6}{2\tan\phi} = \dfrac{40\tan 17.753^\circ + 6}{2\tan 17.753^\circ} = 29.37\text{ m} \cong 29.4\text{ m}$ Ans.

$+ \uparrow \Sigma F_y = 2T\sin\phi - mg = 0$

$T = \dfrac{mg}{2\sin\phi} = \dfrac{250(9.807)}{2\sin 17.753^\circ} = 4020\text{ N} = 4.02\text{ kN}$ Ans.

3-35* Two bodies weighing 150 lb and 250 lb, respectively, rest on a cylinder and are connected by a rope as shown in Fig. P3-35. Find the reactions of the cylinder on the bodies, the tension in the rope, and the angle θ. Assume all surfaces to be smooth.

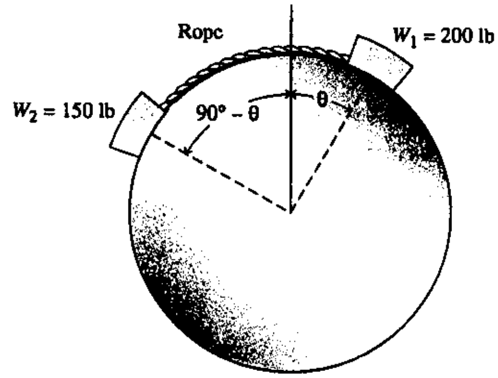
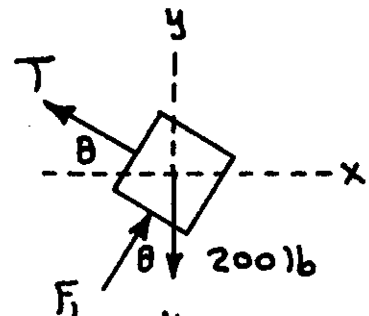


Fig. P3-35

SOLUTION

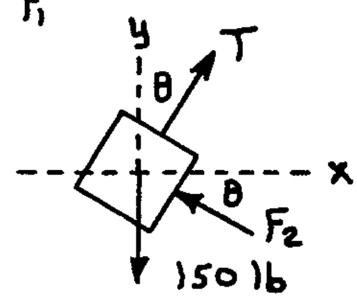
From a free-body diagram for body 1:



(a) $+\rightarrow \Sigma F_x = F_1 \sin\theta - T\cos\theta = 0$

(b) $+\uparrow \Sigma F_y = F_1 \cos\theta + T\sin\theta - 200 = 0$

From a free-body diagram for body 2:



(c) $+\rightarrow \Sigma F_x = T\sin\theta - F_2\cos\theta = 0$

(d) $+\uparrow \Sigma F_y = T\cos\theta + F_2\sin\theta - 150 = 0$

From Eqs. (a) and (d): $(F_1 + F_2)\sin\theta = 150$ lb

From Eqs. (b) and (c): $(F_1 + F_2)\cos\theta = 200$ lb

Therefore: $\theta = \tan^{-1}\dfrac{150}{200} = 36.87^\circ \cong 36.9^\circ$ Ans.

With $\theta = 36.87^\circ$:

Eqs. (a) and (b) yield: $T = 120.0$ lb Ans.

$F_1 = 160.0$ lb Ans.

Eqs. (c) and (d) yield: $F_2 = 90.0$ lb Ans.

3-36* The particle shown in Fig. P3-36 is in equilibrium under the action of the four forces shown on the free-body diagram. Determine the tensions in cables A, B, and C.

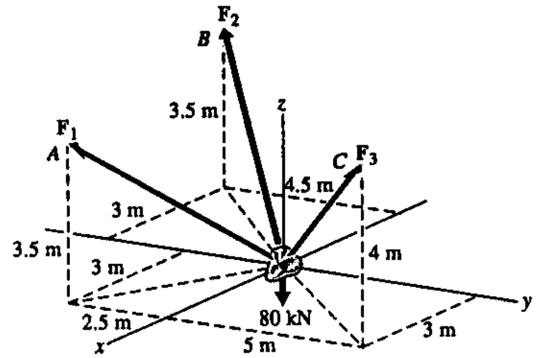


Fig. P3-36

SOLUTION

$$\bar{F}_1 = F_1\left[\frac{3\,\hat{i} - 2.5\,\hat{j} + 3.5\,\hat{k}}{\sqrt{(3)^2 + (-2.5)^2 + (3.5)^2}}\right] = (0.5721\,\hat{i} - 0.4767\,\hat{j} + 0.6674\,\hat{k})F_1$$

$$\bar{F}_2 = F_2\left[\frac{-3\,\hat{i} - 4.5\,\hat{j} + 3.5\,\hat{k}}{\sqrt{(-3)^2 + (-4.5)^2 + (3.5)^2}}\right] = (-0.4657\,\hat{i} - 0.6985\,\hat{j} + 0.5433\,\hat{k})F_2$$

$$\bar{F}_3 = F_3\left[\frac{3\,\hat{i} + 5\,\hat{j} + 4\,\hat{k}}{\sqrt{(3)^2 + (5)^2 + (4)^2}}\right] = (0.4243\,\hat{i} + 0.7071\,\hat{j} + 0.5657\,\hat{k})F_3$$

$$\bar{W} = -80\,\hat{k}$$

$$\bar{R} = \Sigma\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{W}$$

$$= (0.5721F_1 - 0.4657F_2 + 0.4243F_3)\,\hat{i}$$
$$+ (-0.4767F_1 - 0.6985F_2 + 0.7071F_3)\,\hat{j}$$
$$+ (0.6674F_1 + 0.5433F_2 + 0.5657F_3 - 80)\,\hat{k} = \bar{0}$$

Thus:

$$0.5721F_1 - 0.4657F_2 + 0.4243F_3 = 0$$
$$-0.4767F_1 - 0.6985F_2 + 0.7071F_3 = 0$$
$$0.6674F_1 + 0.5433F_2 + 0.5657F_3 = 80$$

Solving yields:

$T_A = F_1 = 3.745 \text{ kN} \cong 3.75 \text{ kN}$ Ans.

$T_B = F_2 = 69.82 \text{ kN} \cong 69.8 \text{ kN}$ Ans.

$T_C = F_3 = 70.71 \text{ kN} \cong 70.7 \text{ kN}$ Ans.

3-37 The hot-air balloon shown in Fig. P3-37 is tethered with three mooring cables. If the net lift of the balloon is 900 lb, determine the force exerted on the balloon by each of the cables.

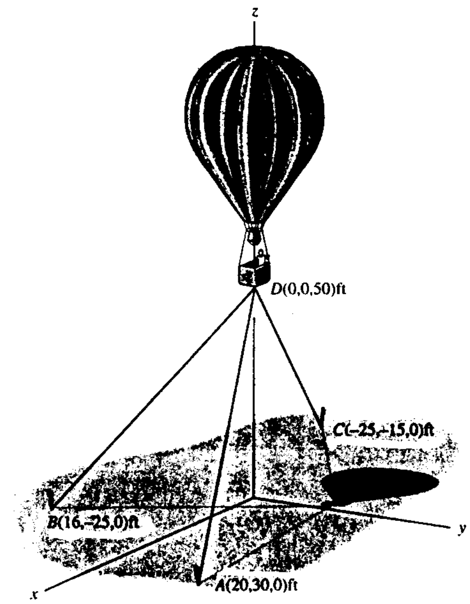


Fig. P3-37

SOLUTION

$$\mathbf{T}_A = T_A\left[\frac{20\,\hat{i} + 30\,\hat{j} - 50\,\hat{k}}{\sqrt{(20)^2 + (30)^2 + (-50)^2}}\right]$$

$$= (0.3244\,\hat{i} + 0.4867\,\hat{j} - 0.8111\,\hat{k})T_A$$

$$\mathbf{T}_B = T_B\left[\frac{16\,\hat{i} - 25\,\hat{j} - 50\,\hat{k}}{\sqrt{(16)^2 + (-25)^2 + (-50)^2}}\right] = (0.2752\,\hat{i} - 0.4299\,\hat{j} - 0.8599\,\hat{k})T_B$$

$$\mathbf{T}_C = T_C\left[\frac{-25\,\hat{i} - 15\,\hat{j} - 50\,\hat{k}}{\sqrt{(-25)^2 + (-15)^2 + (-50)^2}}\right] = (-0.4319\,\hat{i} - 0.2592\,\hat{j} - 0.8639\,\hat{k})T_C$$

$$\mathbf{L} = 900\,\hat{k}$$

$$\mathbf{R} = \Sigma\mathbf{F} = \mathbf{T}_A + \mathbf{T}_B + \mathbf{T}_C + \mathbf{L}$$

$$= (0.3244T_A + 0.2752T_B - 0.4319T_C)\,\hat{i}$$
$$+ (0.4867T_A - 0.4299T_B - 0.2592T_C)\,\hat{j}$$
$$+ (-0.8111T_A - 0.8599T_B - 0.8639T_C + 900)\,\hat{k} = \mathbf{0}$$

Thus:

$$0.3244T_A + 0.2752T_B - 0.4319T_C = 0$$
$$0.4867T_A - 0.4299T_B - 0.2592T_C = 0$$
$$-0.8111T_A - 0.8599T_B - 0.8639T_C = -900$$

Solving yields:

$T_A = 418.2$ lb $\cong 418$ lb Ans.

$T_B = 205.2$ lb $\cong 205$ lb Ans.

$T_C = 444.9$ lb $\cong 445$ lb Ans.

3-38 The traffic light shown in Fig. P3-38 is supported by a system of cables. Determine the tensions in cables A, B, and C if the traffic light has a mass of 100 kg.

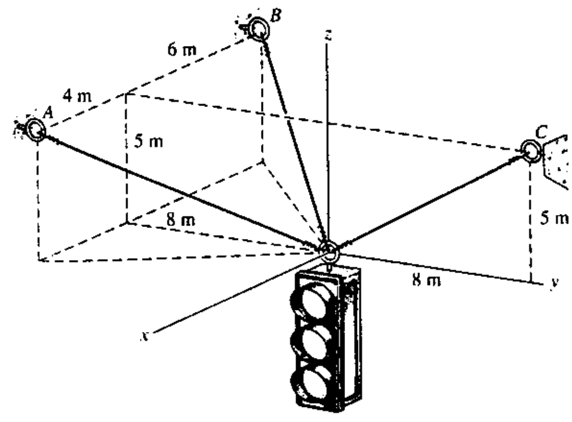


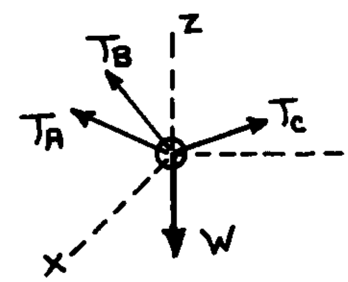
Fig. P3-38

SOLUTION

$$\bar{T}_A = T_A\left[\frac{4\,\hat{i} - 8\,\hat{j} + 5\,\hat{k}}{\sqrt{(4)^2 + (-8)^2 + (5)^2}}\right] = (0.3904\,\hat{i} - 0.7807\,\hat{j} + 0.4880\,\hat{k})T_A$$

$$\bar{T}_B = T_B\left[\frac{-6\,\hat{i} - 8\,\hat{j} + 5\,\hat{k}}{\sqrt{(-6)^2 + (-8)^2 + (5)^2}}\right] = (-0.5367\,\hat{i} - 0.7155\,\hat{j} + 0.4472\,\hat{k})T_B$$

$$\bar{T}_C = T_C\left[\frac{8\,\hat{j} + 5\,\hat{k}}{\sqrt{(8)^2 + (5)^2}}\right] = (0.8480\,\hat{j} + 0.5300\,\hat{k})T_C$$



$$\bar{W} = m\bar{g} = -(100)(9.807)\,\hat{k} = -980.7\,\hat{k}$$

$$\bar{R} = \bar{T}_A + \bar{T}_B + \bar{T}_C + \bar{W}$$

$$= (0.3904T_A - 0.5367T_B)\,\hat{i} + (-0.7807T_A - 0.7155T_B + 0.8480T_C)\,\hat{j} + (0.4880T_A + 0.4472T_B + 0.5300T_C - 980.7)\,\hat{k} = \bar{0}$$

Thus:

$$0.3904T_A - 0.5367T_B = 0$$

$$-0.7807T_A - 0.7155T_B + 0.8480T_C = 0$$

$$0.4880T_A + 0.4472T_B + 0.5300T_C = 980.7$$

Solving yields:

$T_A = 602.9$ N $\cong 603$ N   Ans.

$T_B = 438.6$ N $\cong 439$ N   Ans.

$T_C = 925.1$ N $\cong 925$ N   Ans.

C3-39 A 1000-lb load is securely fastened to a hoisting rope as shown in Fig. P3-39. The tension in the flexible cable does not change as it passes around the small frictionless pulley at the right support. The weight of the cable may be neglected. Plot the force P required for equilibrium as a function of the sag distance d ($0 \le d \le 10$ ft). Determine the minimum sag d for which P is less than:

(a) Twice the weight of the load.

(b) Four times the weight of the load.

(c) Eight times the weight of the load.

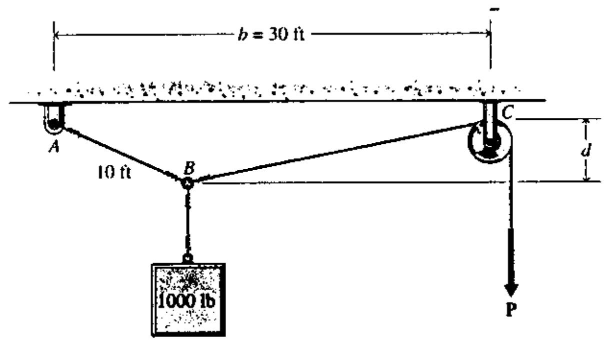
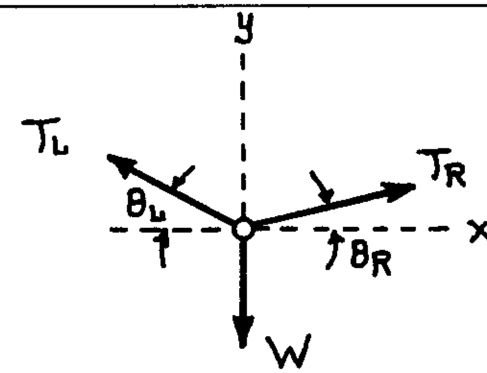


Fig. P3-39

Plot both the force P and the tension T_{AB} in segment AB of the cable on the same graph.

SOLUTION

From a free-body diagram for joint B:



$$\sin \theta_L = \frac{d}{10}$$

$$\tan \theta_R = \frac{d}{30 - 10 \cos \theta_L}$$

$$+ \rightarrow \Sigma F_x = T_R \cos \theta_R - T_L \cos \theta_L = 0$$

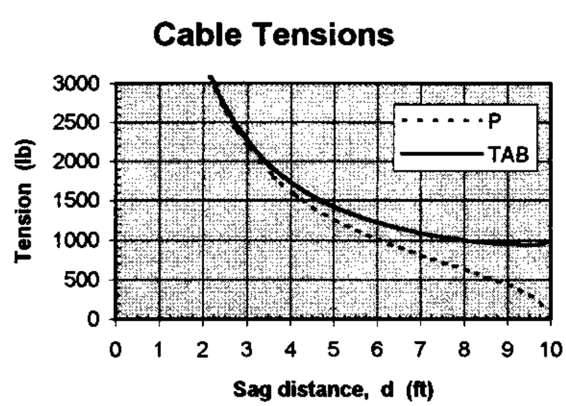
$$+ \uparrow \Sigma F_y = T_R \sin \theta_R + T_L \sin \theta_L - W = 0$$

Solving yields:

$$T_R = \frac{W \cos \theta_L}{\sin \theta_R \cos \theta_L + \cos \theta_R \sin \theta_L}$$

$$T_L = \frac{W \cos \theta_R}{\sin \theta_L \cos \theta_R + \cos \theta_L \sin \theta_R}$$

$$P = T_R \qquad T_{AB} = T_L$$



C3-40 A pair of steel disks are stacked in a box as shown in Fig. P-40. The masses and diameters of the smooth disks are $m_A = 5$ kg, $m_B = 25$ kg, $d_A = 100$ mm, and $d_B = 200$ mm. Plot the two forces exerted on disk A (by disk B and by the side wall) as a function of the distance b between the walls of the box ($200 \text{ mm} \le b \le 300 \text{ mm}$). Determine the range of b for which:

(a) The force at the side wall is less than W_A, the weight of disk A.

(b) Neither of the two forces exceeds $2W_A$.

(c) Neither of the two forces exceeds $4W_A$.

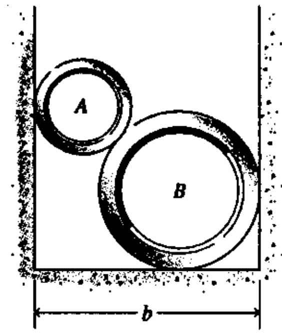
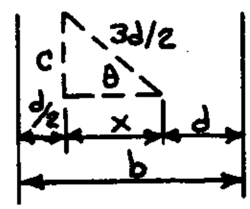


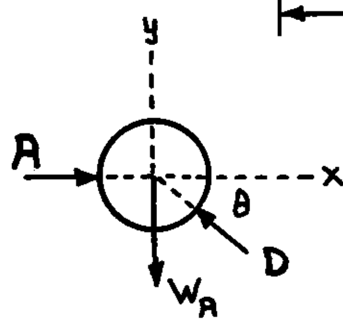
Fig. P3-40

SOLUTION



$$\cos\theta = \frac{b - 150}{150}$$

From a free-body diagram for pipe A:



$$+\uparrow \Sigma F_y = D\sin\theta - W_A = 0$$

$$D = \frac{W_A}{\sin\theta} = \frac{5(9.807)}{\sin\theta} = \frac{49.04}{\sin\theta}$$

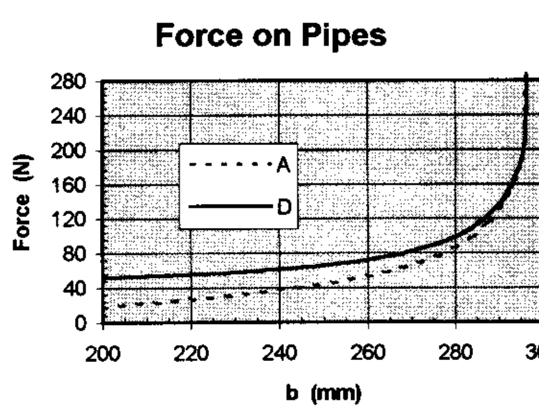
$$+\longrightarrow \Sigma F_x = A - D\cos\theta = 0$$

$$A = D\cos\theta \qquad\qquad D > A$$

(a) $A < 49.04$ N, $\theta > 45.0°$, $b < 256$ mm

(b) $D < 98.08$ N, $\theta > 26.6°$, $b < 284$ mm

(c) $D < 196.2$ N, $\theta > 14.5°$, $b < 295$ mm



4-1* Determine the moment of the 250-lb force shown in Fig. P4-1 about point A.

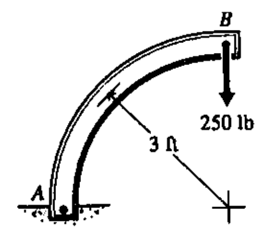


Fig. P4-1

SOLUTION

$M_A = |F|d = 250(3) = 750$ ft·lb $\qquad \mathbf{M}_A = 750$ ft·lb ↻ Ans.

4-2* Determine the moments of the 225-N force shown in Fig. P4-2 about points A, B, and C.

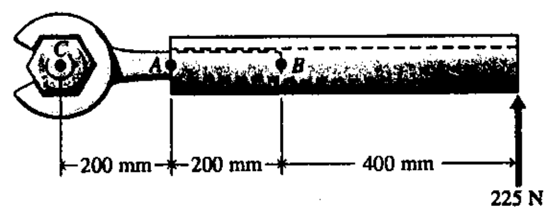


Fig. P4-2

SOLUTION

$M_A = |F|d_A = 225(0.600) = 135.0$ N·m $\qquad \mathbf{M}_A = 135.0$ N·m ↺ Ans.

$M_B = |F|d_B = 225(0.400) = 90.0$ N·m $\qquad \mathbf{M}_B = 90.0$ N·m ↺ Ans.

$M_C = |F|d_C = 225(0.800) = 180.0$ N·m $\qquad \mathbf{M}_C = 180.0$ N·m ↺ Ans.

4-3 Two forces are applied at a point in a plane as shown in Fig. P4-3 Determine

(a) The moments of force $\mathbf{F}_1$ about points A and B

(b) The moments of force $\mathbf{F}_2$ about points B and C.

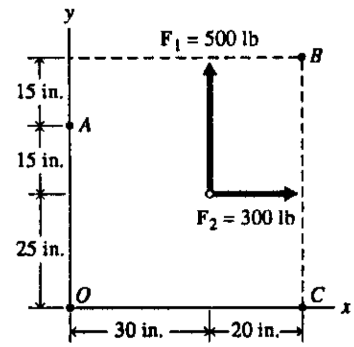


Fig. P4-3

SOLUTION

(a) $M_A = |F_1| d_A = 500(30) = 15{,}000 \text{ in.·lb} = 15.00 \text{ in.·kip}$

$\mathbf{M}_A = 15.00$ in.·kip ↺ Ans.

$M_B = |F_1| d_B = 500(20) = 10{,}000 \text{ in.·lb} = 10.00 \text{ in.·kip}$

$\mathbf{M}_B = 10.00$ in.·kip ↻ Ans.

(b) $M_B = |F_2| d_B = 300(30) = 9000 \text{ in.·lb} = 9.00 \text{ in.·kip}$

$\mathbf{M}_B = 9.00$ in.·kip ↺ Ans.

$M_C = |F_2| d_C = 500(25) = 7500 \text{ in.·lb} = 7.50 \text{ in.·kip}$

$\mathbf{M}_C = 7.50$ in.·kip ↻ Ans.

4-4 Two forces are applied at a point in a plane as shown in Fig. P4-4. Determine

(a) The moments of force $\mathbf{F}_1$ about points B and C.

(b) The moments of force $\mathbf{F}_2$ about points A and C.

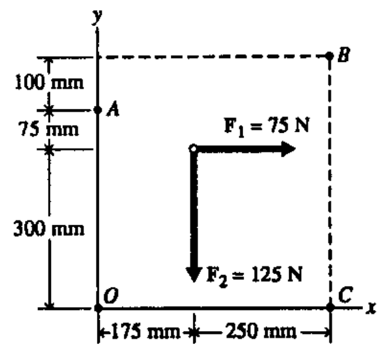


Fig. P4-4

SOLUTION

(a) $M_B = |F_1|d_B = 75(0.175) = 13.125 \text{ N·m} = 13.13 \text{ N·m}$

$\mathbf{M}_B = 13.13 \text{ N·m}$ ↺   Ans.

$M_C = |F_1|d_C = 75(0.300) = 22.500 \text{ N·m} = 22.5 \text{ N·m}$

$\mathbf{M}_C = 22.5 \text{ N·m}$ ↻   Ans.

(b) $M_A = |F_2|d_A = 125(0.175) = 21.875 \text{ N·m} = 21.9 \text{ N·m}$

$\mathbf{M}_A = 21.9 \text{ N·m}$ ↻   Ans.

$M_C = |F_2|d_C = 125(0.250) = 31.25 \text{ N·m} = 31.3 \text{ N·m}$

$\mathbf{M}_C = 31.3 \text{ N·m}$ ↺   Ans.

4-5* Two forces are applied to a bracket as shown in Fig. P4-5. Determine

(a) The moment of force $\mathbf{F}_1$ about point A.

(b) The moment of force $\mathbf{F}_2$ about point B.

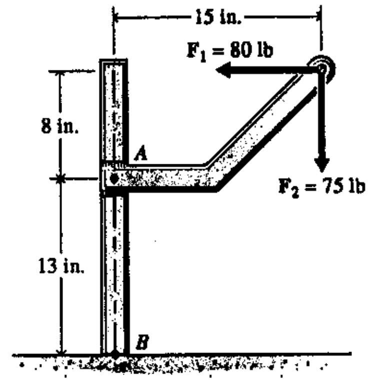


Fig. P4-5

SOLUTION

(a) $M_A = |F_1|d_A = 80(8) = 640$ in.·lb

$\mathbf{M}_A = 640$ in.·lb ↺ Ans.

(b) $M_B = |F_2|d_B = 75(15) = 1125$ in.·lb

$\mathbf{M}_B = 1125$ in.·lb ↻ Ans.

4-6* Two forces are applied to a bracket as shown in Fig. P4-6. Determine

(a) The moment of force $\mathbf{F}_1$ about point A.

(b) The moment of force $\mathbf{F}_2$ about point B.

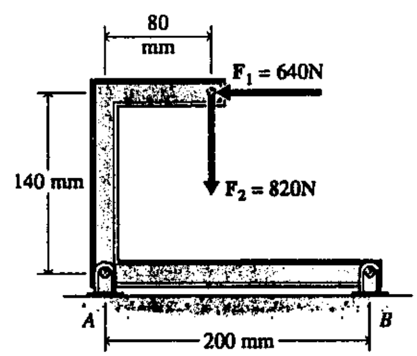


Fig. P4-6

SOLUTION

(a) $M_A = |F_1|d_A = 640(0.140) = 89.6$ N·m

$\mathbf{M}_A = 89.6$ N·m ↺ Ans.

(b) $M_B = |F_2|d_B = 820(0.200 - 0.080) = 98.4$ N·m

$\mathbf{M}_B = 98.4$ N·m ↺ Ans.

4-7 Two forces are applied to a beam as shown in Fig. P4-7. Determine the moments of forces $\bar{F}_1$ and $\bar{F}_2$ about point A.

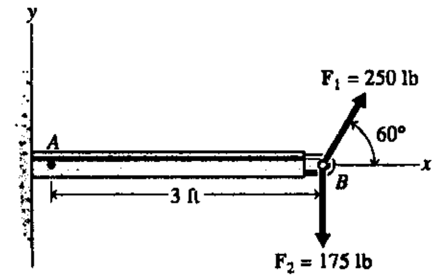


Fig. P4-7

SOLUTION

(a) $M_{A1} = |F_1| d_{A1} = 250(3 \sin 60^\circ) = 649.5$ ft·lb ≅ 650 ft·lb

$\bar{M}_{A1} = 650$ ft·lb ↺ Ans.

(b) $M_{A2} = |F_2| d_{A2} = 175(3) = 525$ ft·lb ≅ 525 ft·lb

$\bar{M}_{A2} = 525$ ft·lb ↻ Ans.

4-8 Two forces are applied to a beam as shown in Fig. P4-8. Determine the moments of forces $\bar{F}_1$ and $\bar{F}_2$ about point A.

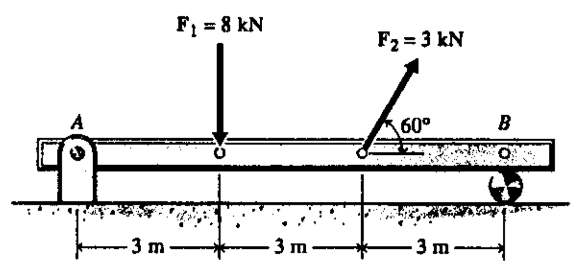


Fig. P4-8

SOLUTION

$M_{A1} = |F_1| d_{A1} = 8(3) = 24\ 0$ kN·m

$\bar{M}_{A1} = 24\ 0$ kN·m ↻ Ans.

$M_{A2} = |F_2| d_{A2} = 3(6 \sin 60^\circ) = 15.588$ kN·m ≅ 15.59 kN·m

$\bar{M}_{A2} = 15.59$ kN·m ↺ Ans.

4-9* Three forces are applied to a circular plate as shown in Fig. P4-9. Determine

(a) The moment of force $\mathbf{F}_1$ about point O.

(b) The moment of force $\mathbf{F}_3$ about point O.

(c) The moment of force $\mathbf{F}_2$ about point A.

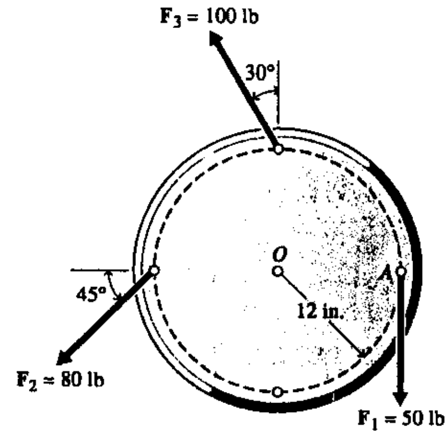


Fig. P4-9

SOLUTION

(a) $M_O = |F_1|d_1 = 50(12) = 600$ in.·lb

$\mathbf{M}_O = 600$ in.·lb ↻ Ans.

(b) $M_O = |F_3|d_3 = 100(12 \sin 30^\circ) = 600$ in.·lb

$\mathbf{M}_O = 600$ in.·lb ↺ Ans.

(c) $M_A = |F_2|d_2 = 80(24 \sin 45^\circ) = 1358$ in.·lb

$\mathbf{M}_A = 1358$ in.·lb ↺ Ans.

4-10* Determine the moment of the 500-N force shown in Fig. P4-10 about points A and B.

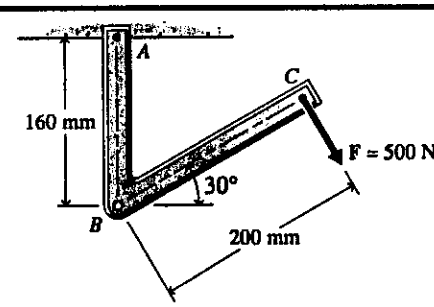


Fig. P4-10

SOLUTION

$\mathbf{M}_B = |F|d_B = 500(0.200) = 100$ N·m

$\mathbf{M}_B = 100$ N·m ↻ Ans.

$M_A = |F|d_A = 500(0.200 - 0.160 \sin 30^\circ) = 60$ N·m

$M_A = 60$ N·m ↻ Ans.

4-11 Determine the moment of the 350-lb force shown in Fig. P4-11 about points A and B.

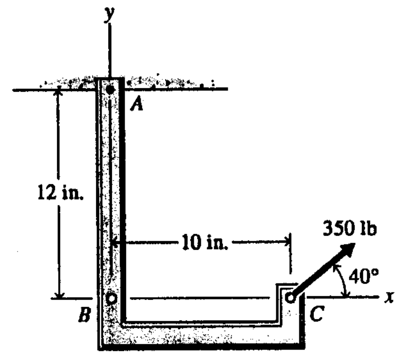


Fig. P4-11

SOLUTION

$M_A = |F|d_A = 350(10 \sin 40^\circ + 12 \cos 40^\circ)$

$= 5467 \text{ in.}\cdot\text{lb} \cong 5.47 \text{ in.}\cdot\text{kip}$ $\qquad \mathbf{M}_A = 5.47 \text{ in.}\cdot\text{kip}$ ↺ Ans.

$M_B = |F|d_B = 350(10 \sin 40^\circ)$

$= 2250 \text{ in.}\cdot\text{lb} \cong 2.25 \text{ in.}\cdot\text{kip}$ $\qquad \mathbf{M}_B = 2.25 \text{ in.}\cdot\text{kip}$ ↺ Ans.

4-12 Determine the moment of the 750-N force shown in Fig. P4-12 about points A, B, and C.

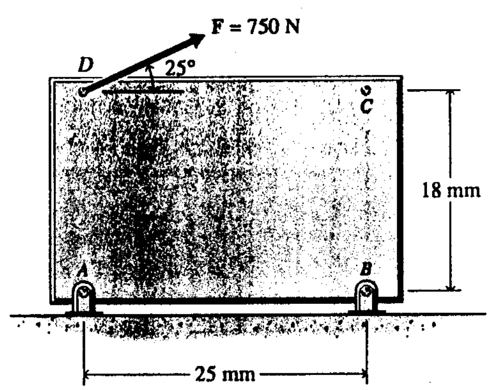


Fig. P4-12

SOLUTION

$M_A = |F|d_A = 750(0.018 \cos 25^\circ)$

$= 12.235 \text{ N}\cdot\text{m} \cong 12.24 \text{ N}\cdot\text{m}$ $\qquad \mathbf{M}_A = 12.24 \text{ N}\cdot\text{m}$ ↻ Ans.

$M_B = |F|d_B = 750(0.018 \cos 25^\circ + 0.025 \sin 25^\circ)$

$= 20.159 \text{ N}\cdot\text{m} \cong 20.2 \text{ N}\cdot\text{m}$ $\qquad \mathbf{M}_B = 20.2 \text{ N}\cdot\text{m}$ ↻ Ans.

$M_C = |F|d_C = 750(0.025 \sin 25^\circ)$

$= 7.924 \text{ N}\cdot\text{m} \cong 7.92 \text{ N}\cdot\text{m}$ $\qquad \mathbf{M}_C = 7.92 \text{ N}\cdot\text{m}$ ↻ Ans.

4-13* Two forces are applied to an angle bracket as shown in Fig. P4-13. Determine the moments of forces $\bar{F}_1$ and $\bar{F}_2$ about points A and B.

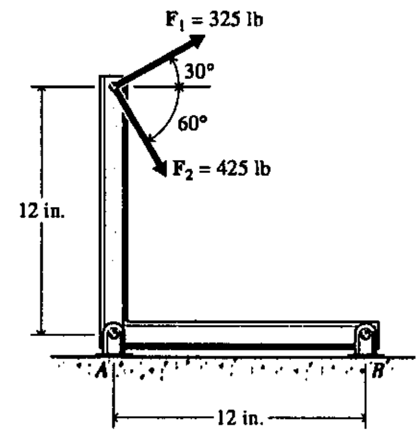


Fig. P4-13

SOLUTION

$M_{A1} = |F_1|d_{A1} = 325(12 \cos 30^\circ) = 3377$ in.·lb

$\bar{M}_{A1} \cong 3380$ in.·lb ↻ Ans.

$M_{A2} = |F_2|d_{A2} = 425(12 \cos 60^\circ) = 2550$ in.·lb

$\bar{M}_{A2} \cong 2550$ in.·lb ↻ Ans.

$M_{B1} = |F_1|d_{B1} = 325(12 \cos 30^\circ + 12 \sin 30^\circ) = 5327$ in.·lb

$\bar{M}_{B1} \cong 5330$ in.·lb ↻ Ans.

$M_{B2} = |F_2|d_{B2} = 425(12 \sin 60^\circ - 12 \cos 60^\circ) = 1866.7$ in.·lb

$\bar{M}_{B2} \cong 1867$ in.·lb ↺ Ans.

4-14* Two forces are applied to an eye bracket as shown in Fig. P4-14. Determine the moments of forces $\mathbf{F}_1$ and $\mathbf{F}_2$ about points A and B.

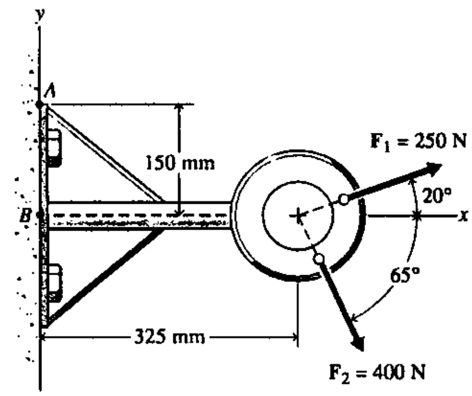


Fig. P4-14

SOLUTION

$M_{A1} = |F_1|d_{A1} = 250\ (0.150 \cos 20^\circ + 0.325 \sin 20^\circ) = 63.03\ \text{N·m}$

$\mathbf{M}_{A1} \cong 63.0\ \text{N·m}$ ↺   Ans.

$M_{A2} = |F_2|d_{A2} = 400(0.325 \sin 65^\circ - 0.150 \cos 65^\circ) = 92.46\ \text{N·m}$

$\mathbf{M}_{A2} \cong 92.5\ \text{N·m}$ ↻   Ans.

$M_{B1} = |F_1|d_{B1} = 250(0.325 \sin 20^\circ) = 81.25\ \text{N·m}$

$\mathbf{M}_{B1} \cong 81.3\ \text{N·m}$ ↺   Ans.

$M_{B2} = |F_2|d_{B2} = 400(0.325 \sin 65^\circ) = 117.82\ \text{N·m}$

$\mathbf{M}_{B2} \cong 117.8\ \text{N·m}$ ↻   Ans.

4-15 Two forces are applied to a bracket as shown in Fig. P4-15. Determine the moments of forces $\mathbf{F}_1$ and $\mathbf{F}_2$ about points A and B.

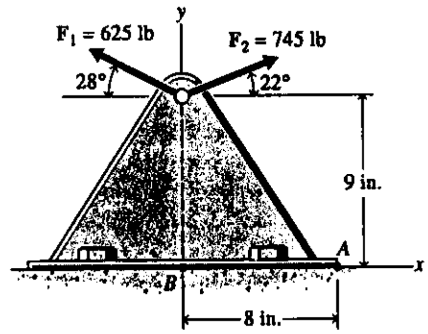


Fig. P4-15

SOLUTION

$M_{A1} = |F_1| d_{A1} = 625\ (9 \cos 28^\circ - 8 \sin 28^\circ)$

$= 2619 \text{ in.·lb} \cong 2.62 \text{ in.·kip}$

$\mathbf{M}_{A1} = 2.62 \text{ in.·kip}$ ↺ Ans.

$M_{A2} = |F_2| d_{A2} = 745(9 \cos 22^\circ + 8 \sin 22^\circ)$

$= 8449 \text{ in.·lb} \cong 8.45 \text{ in.·kip}$

$\mathbf{M}_{A2} = 8.45 \text{ in.·kip}$ ↻ Ans.

$M_{B1} = |F_1| d_{B1} = 625(9 \cos 28^\circ)$

$= 4967 \text{ in.·lb} \cong 4.97 \text{ in.·kip}$

$\mathbf{M}_{B1} = 4.97 \text{ in.·kip}$ ↺ Ans.

$M_{B2} = |F_2| d_{B2} = 745(9 \cos 22^\circ)$

$= 6217 \text{ in.·lb} \cong 6.22 \text{ in.·kip}$

$\mathbf{M}_{B2} = 6.22 \text{ in.·kip}$ ↻ Ans.

4-16 Two forces are applied to a beam as shown in Fig. P4-16. Determine the moments of forces $\mathbf{F}_1$ and $\mathbf{F}_2$ about points A and B.

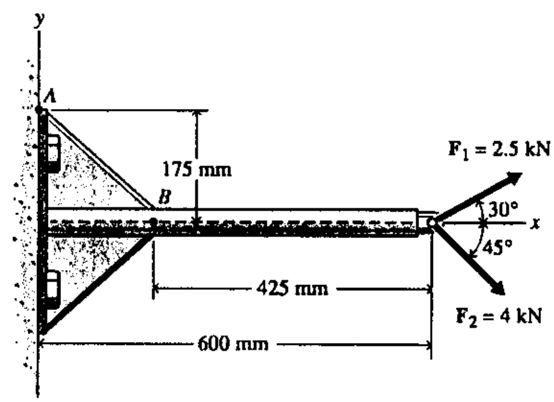


Fig. P4-16

SOLUTION

$$M_{A1} = |F_1| d_{A1} = 2.5\,(0.600 \sin 30^\circ + 0.175 \cos 30^\circ)$$

$$= 1.1289 \text{ kN}\cdot\text{m} \cong 1.129 \text{ kN}\cdot\text{m}$$

$$\mathbf{M}_{A1} = 1.129 \text{ kN}\cdot\text{m} \circlearrowleft \qquad \text{Ans.}$$

$$M_{A2} = |F_2| d_{A2} = 4(0.600 \sin 45^\circ - 0.175 \cos 45^\circ)$$

$$= 1.2021 \text{ kN}\cdot\text{m} \cong 1.202 \text{ kN}\cdot\text{m}$$

$$\mathbf{M}_{A2} = 1.202 \text{ kN}\cdot\text{m} \circlearrowright \qquad \text{Ans.}$$

$$M_{B1} = |F_1| d_{B1} = 2.5(0.425 \sin 30^\circ)$$

$$= 0.5313 \text{ kN}\cdot\text{m} \cong 0.531 \text{ kN}\cdot\text{m}$$

$$\mathbf{M}_{B1} = 0.531 \text{ kN}\cdot\text{m} \circlearrowleft \qquad \text{Ans.}$$

$$M_{B2} = |F_2| d_{B2} = 4(0.425 \sin 45^\circ)$$

$$= 1.2021 \text{ kN}\cdot\text{m} \cong 1.202 \text{ kN}\cdot\text{m}$$

$$\mathbf{M}_{B2} = 1.202 \text{ kN}\cdot\text{m} \circlearrowright \qquad \text{Ans.}$$

4-17* Determine the moment of the 300-lb force shown in Fig. P4-17 about point A.

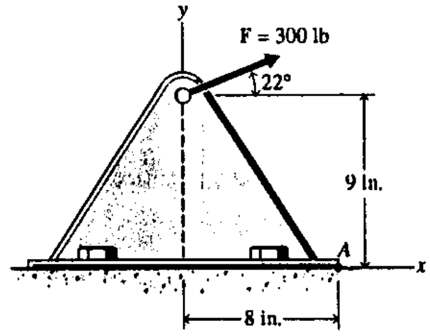


Fig. P4-17

SOLUTION

$+\circlearrowleft M_A = -F \cos 22^\circ (9) - F \sin 22^\circ (8)$

$= -300 \cos 22^\circ (9) - 300 \sin 22^\circ (8)$

$= -3402 \text{ in.}\cdot\text{lb} \cong -3.40 \text{ in.}\cdot\text{kip}$ $\quad \mathbf{M}_A = 3.40 \text{ in.}\cdot\text{kip} \circlearrowright$ Ans.

4-18* Determine the moment of the 250-N force shown in Fig. P4-18 about point A.

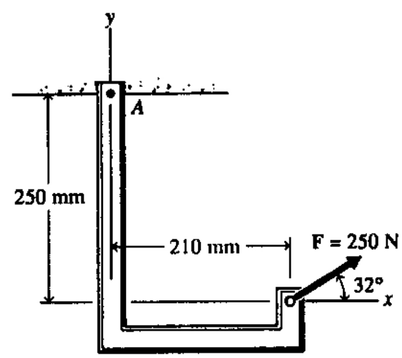


Fig. P4-18

SOLUTION

$+\circlearrowleft M_A = F \cos 32^\circ (250) + F \sin 32^\circ (210)$

$= 250 \cos 32^\circ (250) + 250 \sin 32^\circ (210)$

$= 80{,}824 \text{ N}\cdot\text{mm} = 80.8 \text{ N}\cdot\text{m}$ $\quad \mathbf{M}_A = 80.8 \text{ N}\cdot\text{m} \circlearrowleft$ Ans.

4-19 Determine the moment of the 750-lb force shown in Fig. P4-19 about point O.

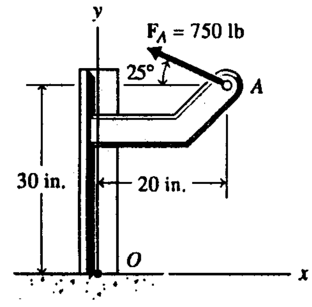


Fig. P4-19

SOLUTION

$$+\circlearrowleft M_O = 750 \cos 25^\circ (30) + 750 \sin 25^\circ (20)$$

$$= 26{,}731 \text{ in.·lb} \cong 26.7 \text{ in.·kip} \qquad \mathbf{M}_O = 26.7 \text{ in.·kip} \circlearrowleft \qquad \text{Ans.}$$

4-20 A 160-N force is applied to the handle of a door as shown in Fig. P4-20. Determine the moments of the force about hinges A and B.

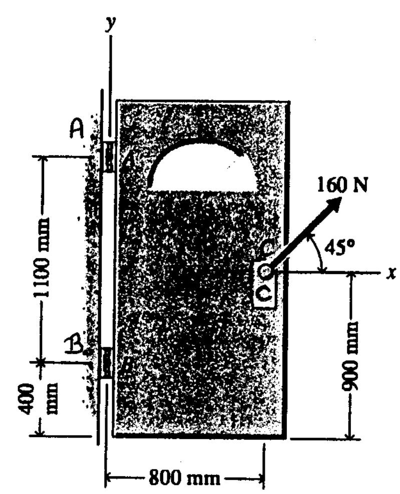


Fig. P4-20

SOLUTION

$$+\circlearrowleft M_A = 160 \cos 45^\circ (1.500 - 0.900) + 160 \sin 45^\circ (0.800)$$

$$= 158.39 \text{ N·m} \cong 158.4 \text{ N·m} \qquad \mathbf{M}_A = 158.4 \text{ N·m} \circlearrowleft \qquad \text{Ans.}$$

$$+\circlearrowleft M_B = -160 \cos 45^\circ (0.900 - 0.400) + 160 \sin 45^\circ (0.800)$$

$$= 33.94 \text{ N·m} \cong 33.9 \text{ N·m} \qquad \mathbf{M}_B = 33.9 \text{ N·m} \circlearrowleft \qquad \text{Ans.}$$

4-21* Determine the moment of the 425-lb force shown in Fig. P4-21 about point B.

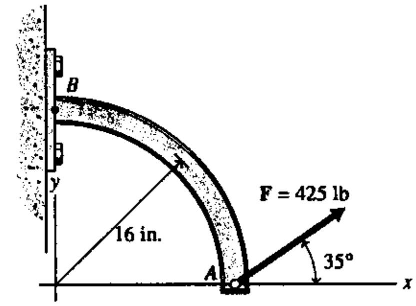


Fig. P4-21

SOLUTION

$+\circlearrowleft M_B = 425 \cos 35^\circ(16) + 425 \sin 35^\circ (16) = 9470$ in.·lb

$\vec{M}_B = 9470$ in.·lb $\circlearrowleft$ Ans.

4-22 Determine the moments of the 16-kN force shown in Fig. P4-22 about points A and B.

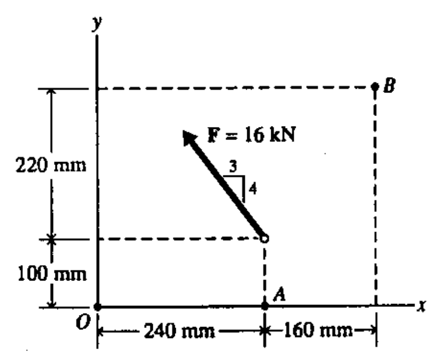


Fig. P4-22

SOLUTION

$+\circlearrowleft M_A = 16(3/5)(0.100) = 0.960$ N·m $= 0.960$ N·m

$\vec{M}_A = 0.960$ N·m $\circlearrowleft$ Ans.

$+\circlearrowleft M_B = -16(3/5)(0.220) - 16(4/5)(0.160) = -4.160$ N·m $= -4.16$ N·m

$\vec{M}_B = 4.16$ N·m $\circlearrowright$ Ans.

4-23* A 50-lb force is applied to the handle of a lug wrench which is being used to tighten the nuts on the rim of an automobile tire as shown in Fig. P4-23. The diameter of the bolt circle is 5 1/2 in. Determine the moments of the force about the axle for the wheel (point O) and about the point of contact of the wheel with the pavement (point A).

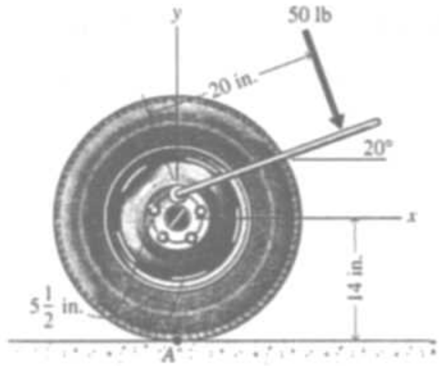


Fig. P4-23

SOLUTION

$+\circlearrowleft M_O = -50(20 + 2.75 \sin 20^\circ)$

$= -1047.03$ in.·lb ≅ -1047 in.·lb $\qquad \mathbf{M}_O = 1047$ in.·lb ↻ Ans.

$+\circlearrowleft M_A = -50(20 + 16.75 \sin 20^\circ)$

$= -1286.44$ in.·lb ≅ -1286 in.·lb $\qquad \mathbf{M}_A = 1286$ in.·lb ↻ Ans.

4-24* Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to a beam as shown in Fig. P4-24. Determine

(a) The moment of force $\mathbf{F}_1$ about point A.

(b) The moment of force $\mathbf{F}_2$ about point B.

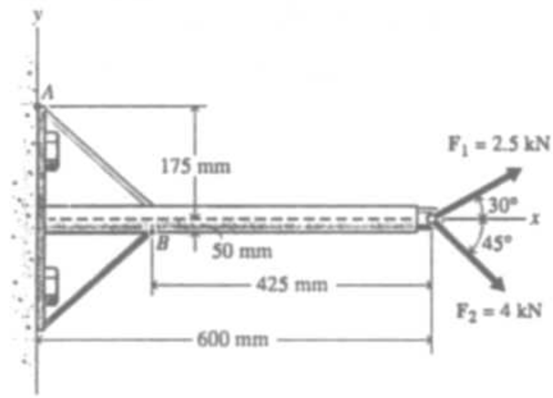


Fig. P4-24

SOLUTION

(a) $+\circlearrowleft M_A = F_1 \cos 30^\circ (175) + F_1 \sin 30^\circ (600)$

$= 2.5 \cos 30^\circ (175) + 2.5 \sin 30^\circ (600)$

$= 1129$ kN·mm $= 1.129$ kN·m $\qquad \mathbf{M}_A = 1.129$ kN·m ↺ Ans.

(b) $+\circlearrowleft M_B = -F_2 \cos 45^\circ (50) - F_2 \sin 45^\circ (425)$

$= -4 \cos 45^\circ (50) - 4 \sin 45^\circ (425)$

$= -1344$ kN·mm $= -1.344$ kN·m $\qquad \mathbf{M}_B = 1.344$ kN·m ↻ Ans.

4-25 Three forces $\overline{F}_A$, $\overline{F}_B$, and $\overline{F}_C$ are applied to a beam as shown in Fig. P4-25. Determine

(a) The moments of forces $\overline{F}_A$ and $\overline{F}_C$ about point O.

(b) The moment of force $\overline{F}_B$ about point D.

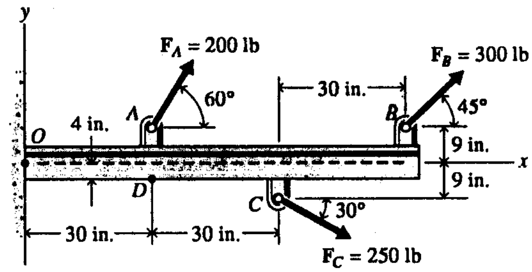


Fig. P4-25

SOLUTION

(a) $+\circlearrowleft M_{OA} = -F_A \cos 60^\circ (9) + F_A \sin 60^\circ (30)$

$= -200 \cos 60^\circ (9) + 200 \sin 60^\circ (30)$

$= 4296$ in.•lb $\cong$ 4.30 in.•kip

$\overline{M}_{OA} = 4.30$ in.•kip ↺ Ans.

$+\circlearrowleft M_{OC} = F_C \cos 30^\circ (9) - F_C \sin 30^\circ (60)$

$= 250 \cos 30^\circ (9) - 250 \sin 30^\circ (60)$

$= -5551$ in.•lb $= -5.55$ in.•kip

$\overline{M}_{OC} = 5.55$ in.•kip ↻ Ans.

(b) $+\circlearrowleft M_D = -F_B \cos 45^\circ (9 + 4) + F_B \sin 45^\circ (60)$

$= -300 \cos 45^\circ (13) + 300 \sin 45^\circ (60)$

$= 9970$ in.•lb $= 9.97$ in.•kip

$\overline{M}_D = 9.97$ in.•kip ↺ Ans.

4-26 Three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are applied to a bracket as shown in Fig. P4-26. Determine the moments of each of the forces about point B.

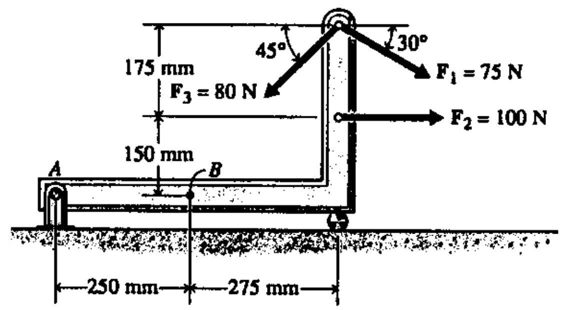


Fig. P4-26

SOLUTION

$$+\circlearrowleft M_{B1} = -F_1 \cos 30^\circ (175 + 150) - F_1 \sin 30^\circ (275)$$

$$= -75 \cos 30^\circ (325) - 75 \sin 30^\circ (275)$$

$$= -31{,}422 \text{ N·mm} \cong -31.4 \text{ N·m}$$

$$\mathbf{M}_{B1} = 31.4 \text{ N·m} \circlearrowright$$ Ans.

$$+\circlearrowleft M_{B2} = -F_2 (150) = -100(150)$$

$$= -15{,}000 \text{ N·mm} = -15.00 \text{ N·m}$$

$$\mathbf{M}_{B2} = 15.00 \text{ N·m} \circlearrowright$$ Ans.

$$+\circlearrowleft M_{B3} = F_3 \cos 45^\circ (150 + 175) - F_3 \sin 45^\circ (275)$$

$$= 80 \cos 45^\circ (325) - 80 \sin 45^\circ (275)$$

$$= 2828 \text{ N·mm} \cong 2.83 \text{ N·m}$$

$$\mathbf{M}_{B3} = 2.83 \text{ N·m} \circlearrowleft$$ Ans.

4-27* Determine the moments of the 50-lb force shown in Fig. P4-27 about points A and B.

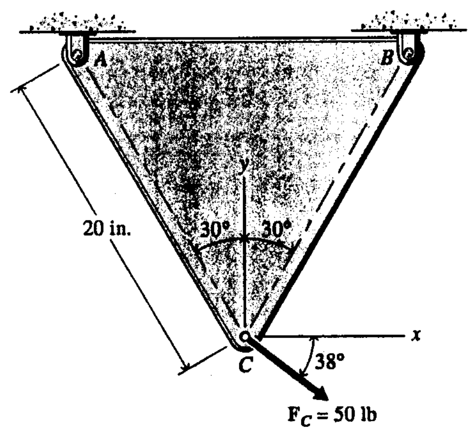


Fig. P4-27

SOLUTION

$+ \circlearrowleft M_A = 50 \cos 38^\circ(20 \cos 30^\circ) - 50 \sin 38^\circ(20 \sin 30^\circ)$

$= 374.6 \text{ in.}\cdot\text{lb} \cong 375 \text{ in.}\cdot\text{lb}$ $\quad \bar{M}_A \cong 375 \text{ in.}\cdot\text{lb} \circlearrowleft$ Ans.

$+ \circlearrowleft M_B = 50 \cos 38^\circ(20 \cos 30^\circ) + 50 \sin 38^\circ(20 \sin 30^\circ)$

$= 990.3 \text{ in.}\cdot\text{lb} \cong 990 \text{ in.}\cdot\text{lb}$ $\quad \bar{M}_B \cong 990 \text{ in.}\cdot\text{lb} \circlearrowleft$ Ans.

4-28* Determine the moments of the 450-N force shown in Fig. P4-28 about points A and B.

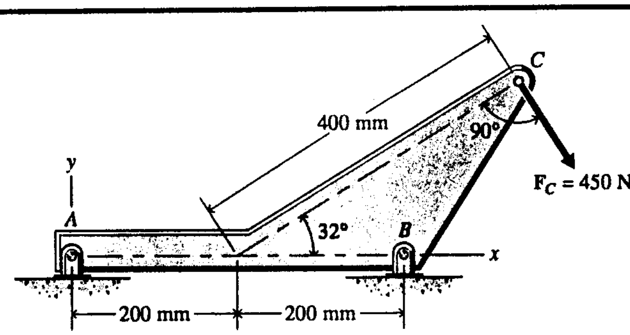


Fig. P4-28

SOLUTION

$+ \circlearrowleft M_A = -450(0.400 + 0.200 \cos 32^\circ)$

$= -256.3 \text{ N}\cdot\text{m} \cong -256 \text{ N}\cdot\text{m}$ $\quad \bar{M}_A \cong 256 \text{ N}\cdot\text{m} \circlearrowright$ Ans.

$+ \circlearrowleft M_B = -450(0.400.- 0.200 \cos 32^\circ)$

$= -103.68 \text{ N}\cdot\text{m} \cong -103.7 \text{ N}\cdot\text{m}$ $\quad \bar{M}_B \cong 103.7 \text{ N}\cdot\text{m} \circlearrowright$ Ans.

4-29 Determine the moments of the 300-lb force shown in Fig. P4-29 about points A and B.

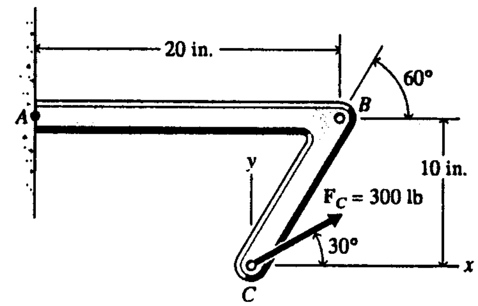


Fig. P4-29

SOLUTION

+↺ $M_A = 300 \cos 30^\circ(10) + 300 \sin 30^\circ(20 - 10/\tan 60^\circ)$

$= 4732.6$ in.·lb $\cong 4.73$ in.·kip $\qquad \mathbf{M}_A \cong 4.73$ in.·kip ↺ Ans.

+↺ $M_B = 300 \cos 30^\circ(10) - 300 \sin 30^\circ(10/\tan 60^\circ)$

$= 1732.1$ in.·lb $\cong 1.732$ in.·kip $\qquad \mathbf{M}_B \cong 1.732$ in.·kip ↺ Ans.

4-30 Determine the moments of the 300-N force shown in Fig. P4-30 about points A and B.

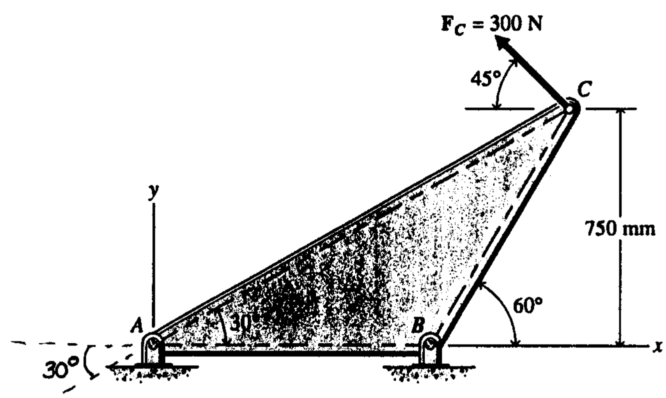


Fig. P4-30

SOLUTION

+↺ $M_A = 300 \cos 45^\circ (0.750) + 300 \sin 45^\circ (0.750/\tan 30^\circ)$

$= 434.7$ N·m $\cong 435$ N·m $\qquad \mathbf{M}_A \cong 435$ N·m ↺ Ans.

+↺ $M_B = 300 \cos 45^\circ(0.750) - 300 \sin 45^\circ(0.750/\tan 60^\circ)$

$= 250.95$ N·m $\cong 251$ N·m $\qquad \mathbf{M}_B \cong 251$ N·m ↺ Ans.

4-31* Determine the moment of the 375-lb force shown in Fig. P4-31 about point O.

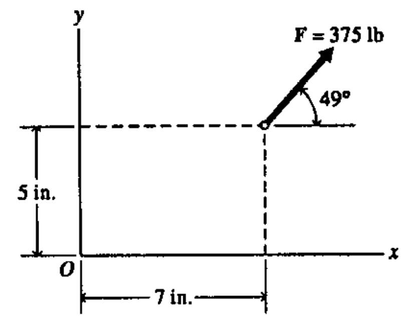


Fig. P4-31

SOLUTION

$\mathbf{F} = 375(\cos 49^\circ\ \hat{i} + \sin 49^\circ\ \hat{j}) = 246.0\ \hat{i} + 283.0\ \hat{j}$ lb

$\bar{r} = 7\ \hat{i} + 5\ \hat{j}$ in.

$\mathbf{M}_O = \bar{r} \times \mathbf{F} = (7\ \hat{i} + 5\ \hat{j}) \times (246.0\ \hat{i} + 283.0\ \hat{j})$

$= 1981\ \hat{k} - 1230\ \hat{k} = 751\ \hat{k}$ in.·lb $= 751$ in.·lb ↺   Ans.

4-32* Determine the moment of the 675-N force shown in Fig. P4-32 about point O.

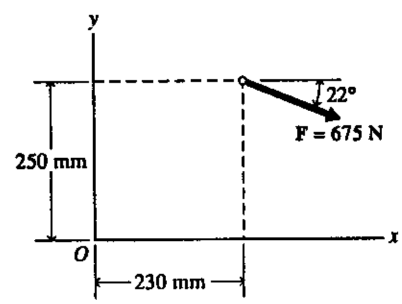


Fig. P4-32

SOLUTION

$\mathbf{F} = 675(\cos 22^\circ\ \hat{i} - \sin 22^\circ\ \hat{j}) = 625.8\ \hat{i} - 252.9\ \hat{j}$ N

$\bar{r} = 0.230\ \hat{i} + 0.250\ \hat{j}$ m

$\mathbf{M}_O = \bar{r} \times \mathbf{F} = (0.230\ \hat{i} + 0.250\ \hat{j}) \times (625.8\ \hat{i} - 252.9\ \hat{j})$

$= -58.17\ \hat{k} - 156.45\ \hat{k}$

$= -214.62\ \hat{k}$ N·m $\cong -215\ \hat{k}$ N·m $\cong 215$ N·m ↻   Ans.

4-33 Determine the moment of the 760-lb force shown in Fig. P4-33 about point A.

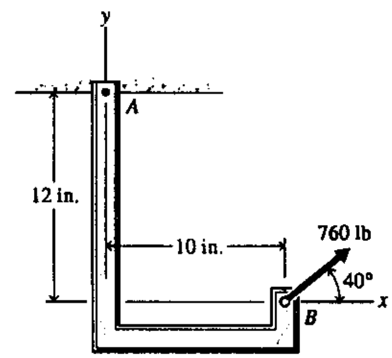


Fig. P4-33

SOLUTION

$\vec{F} = 760(\cos 40^\circ\ \hat{i} + \sin 40^\circ\ \hat{j}) = 582.19\ \hat{i} + 488.52\ \hat{j}$ lb

$\vec{r} = 10\ \hat{i} - 12\ \hat{j}$ in.

$\vec{M}_A = \vec{r} \times \vec{F} = (10\ \hat{i} - 12\ \hat{j}) \times (582.19\ \hat{i} + 488.52\ \hat{j})$

$= 11{,}871\ \hat{k}$ in.·lb $\cong 11.87\ \hat{k}$ in.·kip $\cong$ 11.87 in.·kip ↺ Ans.

4-34 Determine the moment of the 750-N force shown in Fig. P4-34 about point B.

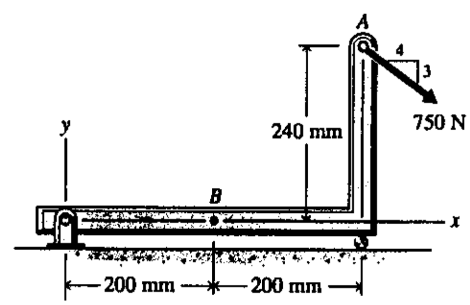


Fig. P4-34

SOLUTION

$\vec{F} = 750[(4/5)\ \hat{i} + (-3/5)\ \hat{j}] = 600\ \hat{i} - 450\ \hat{j}$ N

$\vec{r} = 200\ \hat{i} + 240\ \hat{j}$ mm

$\vec{M}_A = \vec{r} \times \vec{F} = (200\ \hat{i} + 240\ \hat{j}) \times (600\ \hat{i} - 450\ \hat{j})$

$= -234{,}000\ \hat{k}$ N·mm $= -234\ \hat{k}$ N·m = 234 N·m ↻ Ans.

4-35 A 250-lb force is applied to a beam as shown in Fig. P4-35. Determine the moment of the force about point A.

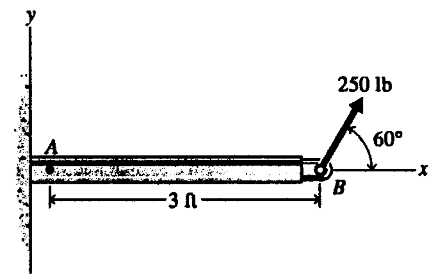


Fig. P4-35

SOLUTION

$\vec{F} = 250(\cos 60^\circ\ \hat{i} + \sin 60^\circ\ \hat{j}) = 125.0\ \hat{i} + 216.5\ \hat{j}$ lb

$\vec{r} = 3\ \hat{i}$ ft

$\vec{M}_A = \vec{r} \times \vec{F} = (3\ \hat{i}) \times (125.0\ \hat{i} + 216.5\ \hat{j})$

$= 649.5\ \hat{k}$ ft·lb $\cong 650\ \hat{k}$ ft·lb $\cong 650$ ft·lb ↺ Ans.

4-36 A 500-N force is applied to a beam as shown in Fig. P4-36. Determine
(a) The moment of the force about point B.
(b) The moment of the force about point C.

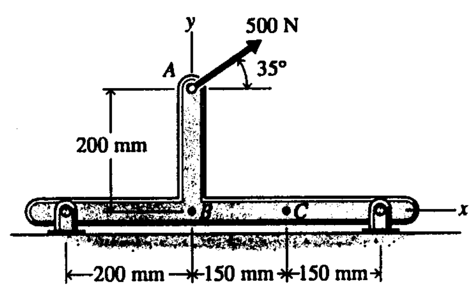


Fig. P4-36

SOLUTION

$\vec{F} = 500(\cos 35^\circ\ \hat{i} + \sin 35^\circ\ \hat{j}) = 409.6\ \hat{i} + 286.8\ \hat{j}$ N

(a) $\vec{M}_B = \vec{r} \times \vec{F} = (200\ \hat{j}) \times (409.6\ \hat{i} + 286.8\ \hat{j})$

$= -81{,}920\ \hat{k}$ N·mm $\cong -81.9\ \hat{k}$ N·m $\cong 81.9$ N·m ↻ Ans.

(b) $\vec{M}_C = \vec{r} \times \vec{F} = (-150\ \hat{i} + 200\ \hat{j}) \times (409.6\ \hat{i} + 286.8\ \hat{j})$

$= -124{,}940\ \hat{k}$ N·mm $\cong -124.9\ \hat{k}$ N·m $\cong 124.9$ N·m ↻ Ans.

4-37* Two forces $\bar{F}_1$ and $\bar{F}_2$ are applied to a triangular plate as shown in Fig. P4-37. Determine

(a) The moment of force $\bar{F}_1$ about point A.

(b) The moment of force $\bar{F}_2$ about point B.

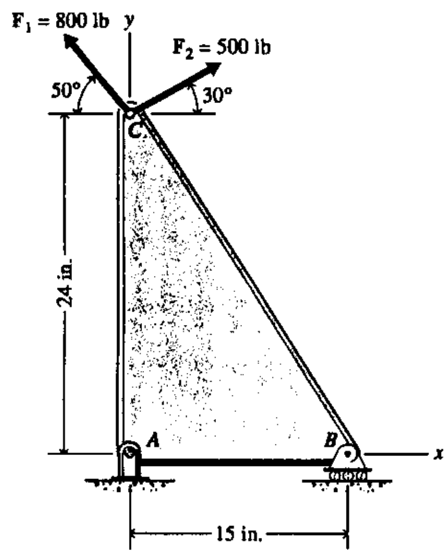


Fig. P4-37

SOLUTION

(a) $\bar{F}_1 = 800(-\cos 50^\circ\ \hat{i} + \sin 50^\circ\ \hat{j}) = -514.2\ \hat{i} + 612.8\ \hat{j}$ lb

$\bar{r}_{C/A} = 24\ \hat{j}$ in.

$\bar{M}_A = \bar{r}_{C/A} \times \bar{F}_1 = (24\ \hat{j}) \times (-514.2\ \hat{i} + 612.8\ \hat{j})$

$= 12{,}341\ \hat{k}$ in.·lb $\cong 12.34\ \hat{k}$ in.·kip

$\cong 12.34$ in.·kip ↺ Ans.

(a) $\bar{F}_2 = 500(\cos 30^\circ\ \hat{i} + \sin 30^\circ\ \hat{j}) = 433.0\ \hat{i} + 250.0\ \hat{j}$ lb

$\bar{r}_{C/B} = -15\ \hat{i} + 24\ \hat{j}$ in.

$\bar{M}_B = \bar{r}_{C/B} \times \bar{F}_2 = (-15\ \hat{i} + 24\ \hat{j}) \times (433.0\ \hat{i} + 250.0\ \hat{j})$

$= -14{,}142\ \hat{k}$ in.·lb $\cong -14.14\ \hat{k}$ in.·kip

$\cong 14.14$ in.·kip ↻ Ans.

4-38* Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied to a bracket as shown in Fig. P4-38. Determine

(a) The moment of force $\mathbf{F}_1$ about point O.

(b) The moment of force $\mathbf{F}_2$ about point A.

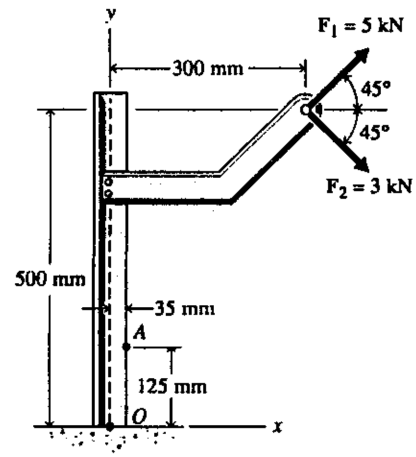


Fig. P4-38

SOLUTION

(a) $\mathbf{F}_1 = 5(\cos 45^\circ\ \hat{i} + \sin 45^\circ\ \hat{j}) = 3.536\ \hat{i} + 3.536\ \hat{j}$ kN

$r_{C/O} = 300\ \hat{i} + 500\ \hat{j}$ mm

$\mathbf{M}_O = \vec{r}_{C/O} \times \mathbf{F}_1 = (300\ \hat{i} + 500\ \hat{j}) \times (3.536\ \hat{i} + 3.536\ \hat{j})$

$= -707.2\ \hat{k}$ kN·mm $\cong -0.707\ \hat{k}$ kN·m $\cong 0.707$ kN·m ↻ Ans.

(b) $\mathbf{F}_2 = 3(\cos 45^\circ\ \hat{i} - \sin 45^\circ\ \hat{j}) = 2.121\ \hat{i} - 2.121\ \hat{j}$ kN

$r_{C/A} = (300 - 35)\ \hat{i} + (500 - 125)\ \hat{j} = 265\ \hat{i} + 375\ \hat{j}$ mm

$\mathbf{M}_A = \vec{r}_{C/A} \times \mathbf{F}_2 = (265\ \hat{i} + 375\ \hat{j}) \times (2.121\ \hat{i} - 2.121\ \hat{j})$

$= -1357.4\ \hat{k}$ kN·mm $\cong -1.357\ \hat{k}$ kN·m $\cong 1.357$ kN·m ↻ Ans.

4-39 Two forces $\bar{F}_1$ and $\bar{F}_2$ are applied to a bracket as shown in Fig. P4-39. Determine

(a) The moment of force $\bar{F}_1$ about point B.

(b) The moment of force $\bar{F}_2$ about point A.

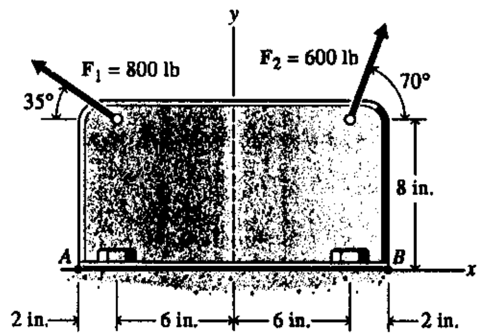


Fig. P4-39

SOLUTION

(a) $\bar{F}_1 = 800(-\cos 35^\circ\ \hat{i} + \sin 35^\circ\ \hat{j}) = -655.3\ \hat{i} + 458.9\ \hat{j}$ lb

$\bar{r}_{1/B} = -14\ \hat{i} + 8\ \hat{j}$ in.

$\bar{M}_B = \bar{r}_{1/B} \times \bar{F}_1 = (-14\ \hat{i} + 8\ \hat{j}) \times (-655.3\ \hat{i} + 458.9\ \hat{j})$

$= -1182.2\ \hat{k}$ in.·lb $\cong -1182\ \hat{k}$ in.·lb

$\cong 1182$ in.·lb ↻ Ans.

(a) $\bar{F}_2 = 600(\cos 70^\circ\ \hat{i} + \sin 70^\circ\ \hat{j}) = 205.2\ \hat{i} + 563.8\ \hat{j}$ lb

$\bar{r}_{2/A} = 14\ \hat{i} + 8\ \hat{j}$ in.

$\bar{M}_A = \bar{r}_{2/A} \times \bar{F}_2 = (14\ \hat{i} + 8\ \hat{j}) \times (205.2\ \hat{i} + 563.8\ \hat{j})$

$= 6252\ \hat{k}$ in.·lb $\cong 6250\ \hat{k}$ in.·lb

$\cong 6250$ in.·lb ↺ Ans.

4-40 A 450-N force is applied to a bracket as shown in Fig. P4-40. Determine the moment of the force
(a) About point B.
(b) About point C.

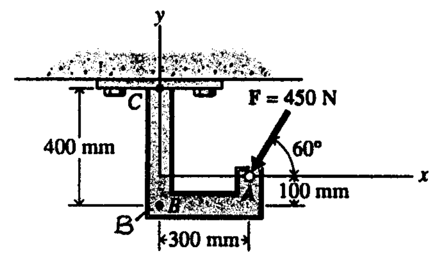


Fig. P4-40

SOLUTION

$$\vec{F} = 450(-\cos 60^\circ\ \hat{i} - \sin 60^\circ\ \hat{j}) = -225.0\ \hat{i} - 389.7\ \hat{j}\ \text{N}$$

(a) $\vec{r}_{A/B} = 300\ \hat{i} + 100\ \hat{j}$ mm

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F} = (300\ \hat{i} + 100\ \hat{j}) \times (-225.0\ \hat{i} - 389.7\ \hat{j})$$

$$= -94{,}410\ \hat{k}\ \text{N·mm} \cong -94.4\ \hat{k}\ \text{N·m} \cong 94.4\ \text{N·m} \circlearrowright$$ Ans.

(b) $\vec{r}_{A/C} = 300\ \hat{i} - 300\ \hat{j}$ mm

$$\vec{M}_C = \vec{r}_{A/C} \times \vec{F} = (300\ \hat{i} - 300\ \hat{j}) \times (-225.0\ \hat{i} - 389.7\ \hat{j})$$

$$= -184{,}410\ \hat{k}\ \text{N·mm} \cong -184.4\ \hat{k}\ \text{N·m} \cong 184.4\ \text{N·m} \circlearrowright$$ Ans.

4-41* Two forces $\vec{F}_1$ and $\vec{F}_2$ are applied to a gusset plate as shown in Fig. P4-41. Determine

(a) The moment of force $\vec{F}_1$ about point A.

(b) The moment of force $\vec{F}_2$ about point B.

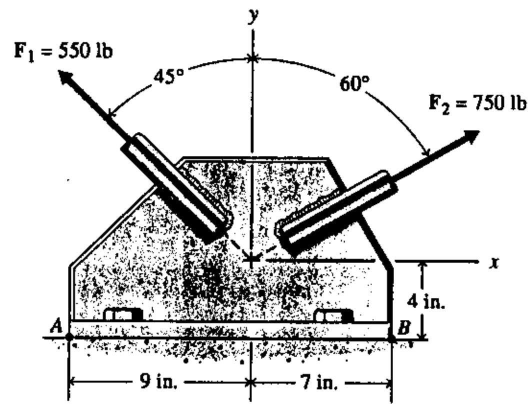


Fig. P4-41

SOLUTION

(a) $\vec{F}_1 = 550(-\cos 45^\circ\ \hat{i} + \sin 45^\circ\ \hat{j}) = -388.9\ \hat{i} + 388.9\ \hat{j}$ lb

$\vec{r}_{1/A} = 9\ \hat{i} + 4\ \hat{j}$ in.

$\vec{M}_A = \vec{r}_{1/A} \times \vec{F}_1 = (9\ \hat{i} + 4\ \hat{j}) \times (-388.9\ \hat{i} + 388.9\ \hat{j})$

$= 5056\ \hat{k}$ in.·lb $\cong 5.06\ \hat{k}$ in.·kip

$\cong 5.06$ in.·kip ↺ Ans.

(b) $\vec{F}_2 = 750(\cos 30^\circ\ \hat{i} + \sin 30^\circ\ \hat{j}) = 649.5\ \hat{i} + 375.0\ \hat{j}$ lb

$\vec{r}_{2/A} = -7\ \hat{i} + 4\ \hat{j}$ in.

$\vec{M}_B = \vec{r}_{2/B} \times \vec{F}_2 = (-7\ \hat{i} + 4\ \hat{j}) \times (649.5.2\ \hat{i} + 375.0\ \hat{j})$

$= -5223\ \hat{k}$ in.·lb $\cong -5.22\ \hat{k}$ in.·kip

$\cong 5.22$ in.·kip ↻ Ans.

4-42* Two forces are applied to an eyebar as shown in Fig. P4-42. Determine
(a) The moment of force $\bar{F}_1$ about point A.
(b) The moment of force $\bar{F}_2$ about point B.

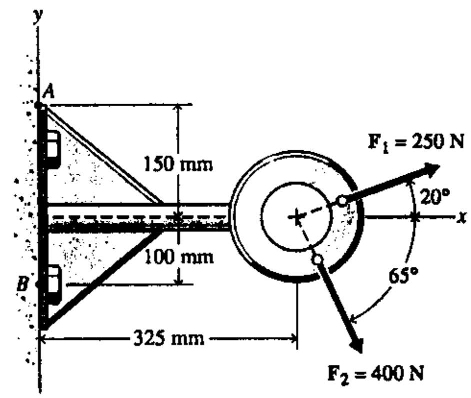


Fig. P4-42

SOLUTION

(a) $\bar{F}_1 = 250(\cos 20^\circ\ \hat{i} + \sin 20^\circ\ \hat{j}) = 234.92\ \hat{i} + 85.51\ \hat{j}$ N

$r_{1/A} = 325\ \hat{i} - 150\ \hat{j}$ mm

$\bar{M}_A = \bar{r}_{1/A} \times \bar{F}_1 = (325\ \hat{i} - 150\ \hat{j}) \times (234.92\ \hat{i} + 85.51\ \hat{j})$

$= 63{,}029\ \hat{k}$ N·mm $\cong 63.0\ \hat{k}$ N·m $\cong 63.0$ N·m ↺ Ans.

(b) $\bar{F}_2 = 400(\cos 65^\circ\ \hat{i} - \sin 65^\circ\ \hat{j}) = 169.05\ \hat{i} - 362.52\ \hat{j}$ N

$r_{2/B} = 325\ \hat{i} + 100\ \hat{j}$ mm

$\bar{M}_B = \bar{r}_{2/B} \times \bar{F}_2 = (325\ \hat{i} + 100\ \hat{j}) \times (169.05\ \hat{i} - 362.52\ \hat{j})$

$= -134{,}724\ \hat{k}$ N·mm $\cong -134.7\ \hat{k}$ N·m $\cong 134.7$ N·m ↻ Ans.

4-43 A 583-lb force is applied to a bracket as shown in Fig. P4-43. Determine the moment of the force
(a) About point D.
(b) About point E.

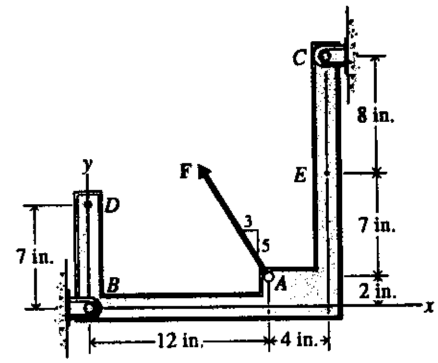


Fig. P4-43

SOLUTION

$$\mathbf{F} = 583\left[\frac{-3\,\hat{i} + 5\,\hat{j}}{\sqrt{(-3)^2 + (5)^2}}\right] = -300\,\hat{i} + 500\,\hat{j} \text{ lb}$$

(a) $\mathbf{r}_{A/D} = 12\,\hat{i} - 5\,\hat{j}$ in.

$$\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{F} = (12\,\hat{i} - 5\,\hat{j}) \times (-300\,\hat{i} + 500\,\hat{j})$$

$$= 4500\,\hat{k} \text{ in.·lb} = 4.50\,\hat{k} \text{ in.·kip}$$

$$= 4.50 \text{ in.·kip} \circlearrowleft$$ Ans.

(b) $\mathbf{r}_{A/E} = -4\,\hat{i} - 7\,\hat{j}$ in.

$$\mathbf{M}_E = \mathbf{r}_{A/E} \times \mathbf{F} = (-4\,\hat{i} - 7\,\hat{j}) \times (-300\,\hat{i} + 500\,\hat{j})$$

$$= -4100\,\hat{k} \text{ in.·lb} = -4.10\,\hat{k} \text{ in.·kip}$$

$$= 4.10 \text{ in.·kip} \circlearrowright$$ Ans.

4-44 A 650-N force is applied to a bracket as shown in Fig. P4-44. Determine the moment of the force
(a) About point D.
(b) About point E.

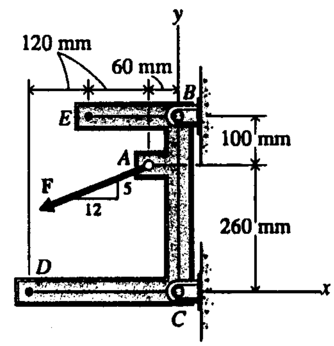


Fig. P4-44

SOLUTION

$$\vec{F} = 650\left(-\frac{12}{13}\,\hat{i} - \frac{5}{13}\,\hat{j}\right) = -600\,\hat{i} - 250\,\hat{j} \quad \text{N}$$

(a) $\vec{r}_{A/D} = 240\,\hat{i} + 260\,\hat{j}$ mm

$$\vec{M}_D = \vec{r}_{A/D} \times \vec{F} = (240\,\hat{i} + 260\,\hat{j}) \times (-600\,\hat{i} - 250\,\hat{j})$$

$$= 96{,}000\,\hat{k}\ \text{N}\cdot\text{mm} = 96.0\,\hat{k}\ \text{N}\cdot\text{m}$$

$$= 96.0\ \text{N}\cdot\text{m}\ \circlearrowleft$$ Ans.

(b) $\vec{r}_{A/E} = 120\,\hat{i} - 100\,\hat{j}$ mm

$$\vec{M}_E = \vec{r}_{A/E} \times \vec{F} = (120\,\hat{i} - 100\,\hat{j}) \times (-600\,\hat{i} - 250\,\hat{j})$$

$$= -90{,}000\,\hat{k}\ \text{N}\cdot\text{mm} = 90.0\,\hat{k}\ \text{N}\cdot\text{m}$$

$$= 90.0\ \text{N}\cdot\text{m}\ \circlearrowright$$ Ans.

4-45* A force with a magnitude of 970 lb acts at a point in a body as shown in Fig. P4-45. Determine the moment of the force about point C.

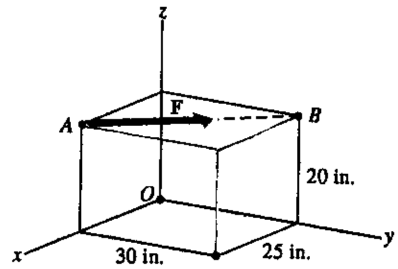


Fig. P4-45

SOLUTION

$$\vec{F} = 970\left[\frac{-25\,\hat{i} + 30\,\hat{j}}{\sqrt{(-25)^2 + (30)^2}}\right] = -621.0\,\hat{i} + 745.2\,\hat{j} \text{ lb}$$

$$\vec{r}_{B/C} = -25\,\hat{i} + 20\,\hat{k} \text{ in.}$$

$$\vec{M}_C = \vec{r}_{B/C} \times \vec{F} = (-25\,\hat{i} + 20\,\hat{k}) \times (-621.0\,\hat{i} + 745.2\,\hat{j})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -25 & 0 & 20 \\ -621.0 & 745.2 & 0 \end{vmatrix}$$

$$= -14{,}904\,\hat{i} - 12{,}420\,\hat{j} - 18{,}630\,\hat{k} \text{ in.·lb}$$

$$\cong -14.90\,\hat{i} - 12.42\,\hat{j} - 18.63\,\hat{k} \text{ in.·kip}$$ Ans.

4-46 A force with a magnitude of 890 N acts at a point in a body as shown in Fig. P4-46. Determine the moment of the force about point C.

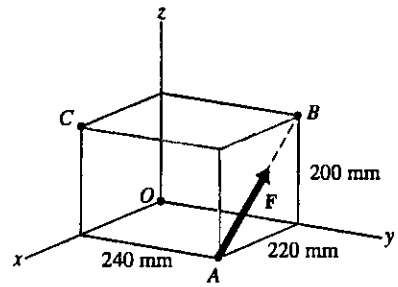


Fig. P4-46

SOLUTION

$$\bar{F} = 890\left[\frac{-220\ \hat{i} + 200\ \hat{k}}{\sqrt{(-220)^2 + (200)^2}}\right] = -658.5\ \hat{i} + 598.7\ \hat{k}\ \text{N}$$

$$\bar{r}_{A/C} = 240\ \hat{j} - 200\ \hat{k}\ \text{mm}$$

$$\bar{M}_C = \bar{r}_{A/C} \times \bar{F} = (240\ \hat{j} - 200\ \hat{k}) \times (-658.5\ \hat{i} + 598.7\ \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 240 & -200 \\ -658.5 & 0 & 598.7 \end{vmatrix}$$

$$= 143{,}688\ \hat{i} + 131{,}700\ \hat{j} + 158{,}040\ \hat{k}\ \text{N}\cdot\text{mm}$$

$$\cong 143.7\ \hat{i} + 131.7\ \hat{j} + 158.0\ \hat{k}\ \text{N}\cdot\text{m}$$ Ans.

4-47* A force with a magnitude of 928 lb acts at a point in a body as shown in Fig. P4-47. Determine the moment of the force about point O.

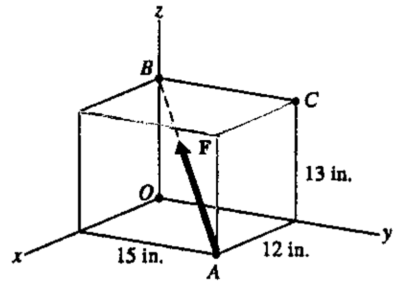


Fig. P4-47

SOLUTION

$$\vec{F} = 928\left[\frac{-12\,\hat{i} - 15\,\hat{j} + 13\,\hat{k}}{\sqrt{(-12)^2 + (-15)^2 + (13)^2}}\right] = -480.0\,\hat{i} - 600.1\,\hat{j} + 520.1\,\hat{k} \text{ lb}$$

$$\vec{r}_{B/O} = 13\,\hat{k} \text{ in.}$$

$$\vec{M}_C = \vec{r}_{B/O} \times \vec{F} = (13\,\hat{k}) \times (-480.0\,\hat{i} - 600.1\,\hat{j} + 520.1\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 13 \\ -480.0 & -600.1 & 520.1 \end{vmatrix}$$

$$= 7801\,\hat{i} - 6240\,\hat{j} \text{ in.·lb}$$

$$\cong 7.80\,\hat{i} - 6.24\,\hat{j} \text{ in.·kip}$$ Ans.

4-48 A force with a magnitude of 860 N acts at a point in a body as shown in Fig. P4-48. Determine

(a) The moment of the force about point C.

(b) The perpendicular distance from the line of action of the force to point C

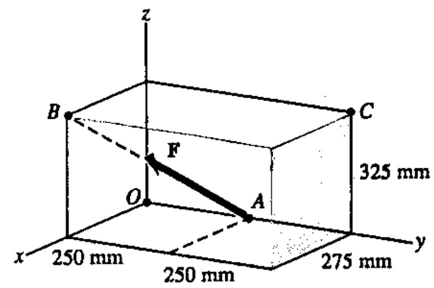


Fig. P4-48

SOLUTION

(a) $\bar{F} = 860\left[\dfrac{275\ \hat{i} - 250\ \hat{j} + 325\ \hat{k}}{\sqrt{(275)^2 + (-250)^2 + (325)^2}}\right] = 479.0\ \hat{i} - 435.5\ \hat{j} + 566.1\ \hat{k}$ N

$\bar{r}_{B/C} = 275\ \hat{i} - 500\ \hat{j}$ mm

$\bar{M}_C = \bar{r}_{B/C} \times \bar{F} = (275\ \hat{i} - 500\ \hat{j}) \times (479.0\ \hat{i} - 435.5\ \hat{j} + 566.1\ \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 275 & -500 & 0 \\ 479.0 & -435.5 & 566.1 \end{vmatrix}$$

$= -283{,}050\ \hat{i} - 155{,}678\ \hat{j} + 119{,}738\ \hat{k}$ N·mm

$\cong 283\ \hat{i} - 155.7\ \hat{j} + 119.7\ \hat{k}$ N·m Ans.

(b) $M = \sqrt{(283.05)^2 + (155.68)^2 + (119.74)^2} = 344.52$ N·m

$d = \dfrac{M}{F} = \dfrac{344.52}{860} = 0.4006 \text{ m} \cong 0.401 \text{ m}$ Ans.

4-49* A force with a magnitude of 650 lb acts at a point in a body as shown in Fig. P4-49. Determine

(a) The moment of the force about point A.

(b) The direction angles associated with the moment vector.

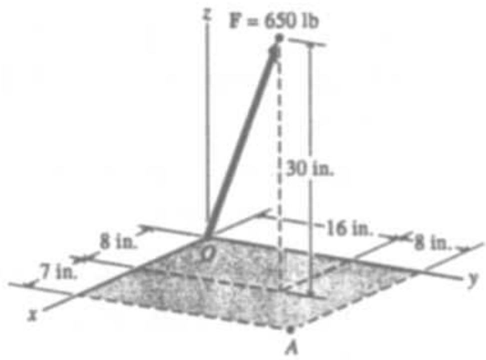


Fig. P4-49

SOLUTION

(a) $\vec{F} = 650\left[\dfrac{8\,\hat{i} + 16\,\hat{j} + 30\,\hat{k}}{\sqrt{(8)^2 + (16)^2 + (30)^2}}\right] = 148.88\,\hat{i} + 297.75\,\hat{j} + 558.28\,\hat{k}$ lb

$\vec{r}_{O/A} = -15\,\hat{i} - 24\,\hat{j}$ in.

$\vec{M}_A = \vec{r}_{O/A} \times \vec{F} = (-15\,\hat{i} - 24\,\hat{j}) \times (148.88\,\hat{i} + 297.75\,\hat{j} + 558.28\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -15 & -24 & 0 \\ 148.88 & 297.75 & 558.28 \end{vmatrix}$$

$= -13{,}399\,\hat{i} + 8374\,\hat{j} - 893\,\hat{k}$ in.·lb

$\cong -13.40\,\hat{i} + 8.37\,\hat{j} - 0.893\,\hat{k}$ in.·kip   Ans.

(b) $M_A = \sqrt{(-13{,}399)^2 + (8374)^2 + (-893)^2} = 15{,}826$ in.·lb

$\theta_x = \cos^{-1}\dfrac{M_x}{M_A} = \cos^{-1}\dfrac{-13{,}399}{15{,}826} = 147.84^\circ \cong 147.8^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_y}{M_A} = \cos^{-1}\dfrac{8374}{15{,}826} = 58.05^\circ \cong 58.1^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_z}{M_A} = \cos^{-1}\dfrac{-893}{15{,}826} = 93.23^\circ \cong 93.2^\circ$ Ans.

4-50* A force with a magnitude of 1000 N acts at a point in a body as shown in Fig. P4-50. Determine

(a) The moment of the force about point A.

(b) The direction angles associated with the moment vector.

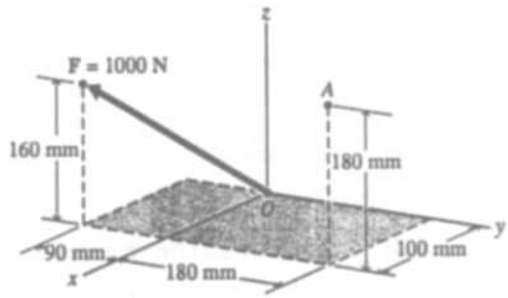


Fig. P4-50

SOLUTION

(a) $\mathbf{F} = 1000\left[\dfrac{100\ \hat{i} - 90\ \hat{j} + 160\ \hat{k}}{\sqrt{(100)^2 + (-90)^2 + (160)^2}}\right] = 478.4\ \hat{i} - 430.5\ \hat{j} + 765.4\ \hat{k}$ N

$\mathbf{r}_{O/A} = -100\ \hat{i} - 180\ \hat{j} - 180\ \hat{k}$ mm

$\mathbf{M}_A = \mathbf{r}_{O/A} \times \mathbf{F} = (-100\ \hat{i} - 180\ \hat{j} - 180\ \hat{k}) \times (478.4\ \hat{i} - 430.5\ \hat{j} + 765.4\ \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -100 & -180 & -180 \\ 478.4 & -430.5 & 765.4 \end{vmatrix}$$

$= -215{,}262\ \hat{i} - 9572\ \hat{j} + 129{,}162\ \hat{k}$ N·mm

$\cong -215\ \hat{i} - 9.57\ \hat{j} + 129.2\ \hat{k}$ N·m Ans.

(b) $M_A = \sqrt{(215.26)^2 + (9.57)^2 + (129.16)^2} = 251.22$ N·m

$\theta_x = \cos^{-1}\dfrac{M_x}{M_A} = \cos^{-1}\dfrac{-215.26}{251.22} = 148.97^\circ \cong 149.0^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_y}{M_A} = \cos^{-1}\dfrac{-9.57}{251.22} = 92.18^\circ \cong 92.2^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_z}{M_A} = \cos^{-1}\dfrac{129.16}{251.22} = 59.06^\circ \cong 59.1^\circ$ Ans.

4-51 A force with a magnitude of 400 lb acts at a point in a body as shown in Fig. P4-51. Determine
(a) The moment of the force about point O.
(b) The direction angles associated with the moment vector.

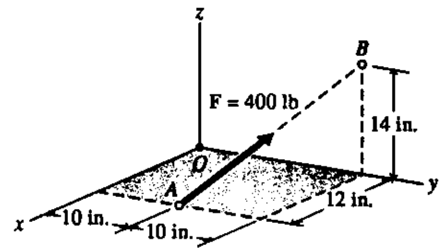


Fig. P4-51

SOLUTION

(a) $\bar{F} = 400\left[\dfrac{-12\,\hat{i} + 10\,\hat{j} + 14\,\hat{k}}{\sqrt{(-12)^2 + (10)^2 + (14)^2}}\right]$

$= -228.83\,\hat{i} + 190.69\,\hat{j} + 266.97\,\hat{k}$ lb

$\bar{r}_{A/O} = 12\,\hat{i} + 10\,\hat{j}$ in.

$\bar{M}_O = \bar{r}_{A/O} \times \bar{F} = (12\,\hat{i} + 10\,\hat{j}) \times (-228.83\,\hat{i} + 190.69\,\hat{j} + 266.97\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 12 & 10 & 0 \\ -228.83 & 190.69 & 266.97 \end{vmatrix}$$

$= 2669.7\,\hat{i} - 3203.6\,\hat{j} + 4576.6\,\hat{k}$ in.·lb

$\cong 2.67\,\hat{i} - 3.20\,\hat{j} + 4.58\,\hat{k}$ in.·kip Ans.

(b) $M_O = \sqrt{(2669.7)^2 + (-3203.6)^2 + (4576.6)^2} = 6191.6$ in.·lb

$\theta_x = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{2669.7}{6191.6} = 64.46° \cong 64.5°$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_y}{M_O} = \cos^{-1}\dfrac{-3203.6}{6191.6} = 121.16° \cong 121.2°$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_z}{M_O} = \cos^{-1}\dfrac{4576.6}{6191.6} = 42.34° \cong 42.3°$ Ans.

4-52* Determine the moment of the 760-N force shown in Fig. P4-52 about point B.

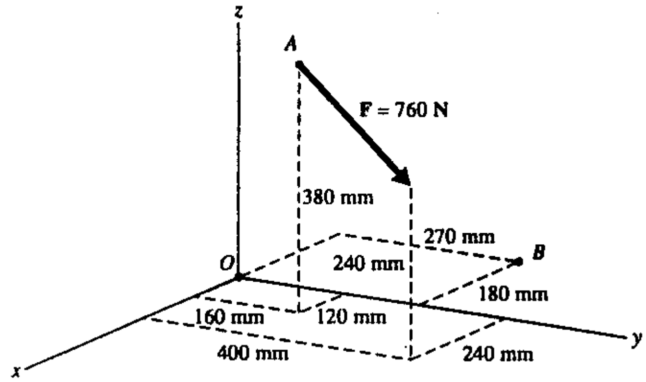


Fig. P4-52

SOLUTION

$$\vec{F} = 760\left[\frac{(240 - 120)\,\hat{i} + (400 - 160)\,\hat{j} + (240 - 380)\,\hat{k}}{\sqrt{(120)^2 + (240)^2 + (-140)^2}}\right]$$

$$= 301.3\,\hat{i} + 602.7\,\hat{j} - 351.6\,\hat{k} \text{ N}$$

$$\vec{r}_{A/B} = 300\,\hat{i} - 110\,\hat{j} + 380\,\hat{k} \text{ mm}$$

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F} = (300\,\hat{i} - 110\,\hat{j} + 380\,\hat{k}) \times (301.3\,\hat{i} + 602.7\,\hat{j} - 351.6\,\hat{k})$$

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 300 & -110 & 380 \\ 301.3 & 602.7 & -351.6 \end{vmatrix}$$

$$= -190{,}350\,\hat{i} + 219{,}974\,\hat{j} + 213{,}953\,\hat{k} \text{ N·mm}$$

$$\cong -190.4\,\hat{i} + 220\,\hat{j} + 214\,\hat{k} \text{ N·m}$$ Ans.

$$M_B = \sqrt{(-190.35)^2 + (219.97)^2 + (213.95)^2} = 361.10 \text{ N·m} \cong 361 \text{ N·m}$$ Ans.

4-53* Determine the moment of the 580-lb force shown in Fig. P4-53 about point B.

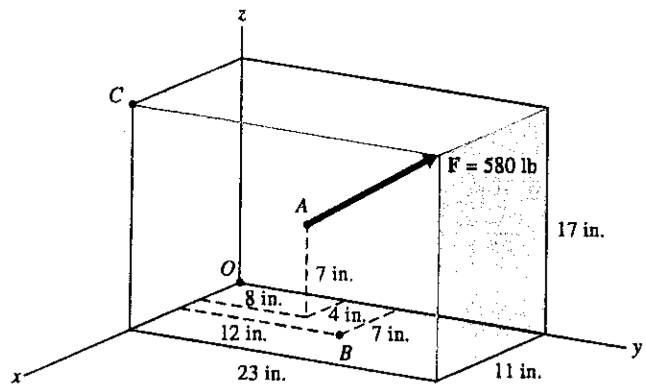


Fig. P4-53

SOLUTION

$$\bar{F} = 580\left[\frac{(11 - 4)\,\hat{i} + (23 - 8)\,\hat{j} + (17 - 7)\,\hat{k}}{\sqrt{(7)^2 + (15)^2 + (10)^2}}\right]$$

$$= 209.94\,\hat{i} + 449.87\,\hat{j} + 299.91\,\hat{k}\ \text{lb}$$

$$\bar{r}_{A/B} = -3\,\hat{i} - 4\,\hat{j} + 7\,\hat{k}\ \text{in.}$$

$$\bar{M}_B = \bar{r}_{A/B} \times \bar{F} = (-3\,\hat{i} - 4\,\hat{j} + 7\,\hat{k}) \times (209.94\,\hat{i} + 449.87\,\hat{j} + 299.91\,\hat{k})$$

$$\bar{M}_B = \bar{r}_{A/B} \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 7 \\ 209.94 & 449.87 & 299.91 \end{vmatrix}$$

$$= -4348.7\,\hat{i} + 2369.3\,\hat{j} - 509.9\,\hat{k}\ \text{in.·lb}$$

$$\cong -4.35\,\hat{i} + 2.37\,\hat{j} - 0.510\,\hat{k}\ \text{in.·kip}$$ Ans.

$$M_B = \sqrt{(-4348.7)^2 + (2369.3)^2 + (-509.9)^2}$$

$$= 4978.4\ \text{in.·lb} \cong 4.98\ \text{in.·kip}$$ Ans.

4-54 A 720-N force is applied to a T-bar as shown in Fig. P4-54. Determine
(a) The moment of the force about point O.
(b) The direction angles associated with the moment vector.

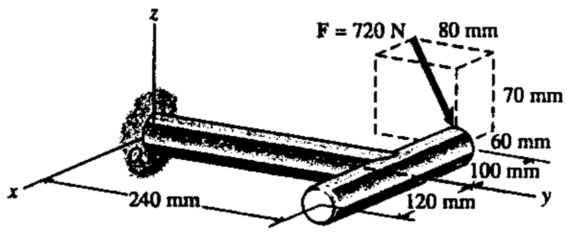


Fig. P4-54

SOLUTION

(a) $\bar{F} = 720\left[\dfrac{60\,\hat{i} + 80\,\hat{j} - 70\,\hat{k}}{\sqrt{(60)^2 + (80)^2 + (-70)^2}}\right] = 353.91\,\hat{i} + 471.88\,\hat{j} - 412.89\,\hat{k}$ N

$\bar{r}_{F/O} = -100\,\hat{i} + 240\,\hat{j}$ mm

$\bar{M}_O = \bar{r}_{F/O} \times \bar{F} = (-100\,\hat{i} + 240\,\hat{j}) \times (353.91\,\hat{i} + 471.88\,\hat{j} - 412.89\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -100 & 240 & 0 \\ 353.91 & 471.88 & -412.89 \end{vmatrix}$$

$= -99{,}094\,\hat{i} - 41{,}289\,\hat{j} - 132{,}126\,\hat{k}$ N·mm

$\cong -99.1\,\hat{i} - 41.3\,\hat{j} - 132.1\,\hat{k}$ N·m   Ans.

(b) $M_A = \sqrt{(-99.09)^2 + (-41.29)^2 + (-132.13)^2} = 170.24$ N·m

$\theta_x = \cos^{-1}\dfrac{M_x}{M_A} = \cos^{-1}\dfrac{-99.09}{170.24} = 125.60^\circ \cong 125.6^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_y}{M_A} = \cos^{-1}\dfrac{-41.29}{170.24} = 104.04^\circ \cong 104.0^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_z}{M_A} = \cos^{-1}\dfrac{-132.13}{170.24} = 140.91^\circ \cong 140.9^\circ$ Ans.

4-55 A 750-lb force is applied to a pipe bracket as shown in Fig. P4-55. Determine
(a) The moment of the force about point O.
(b) The direction angles associated with the moment vector.

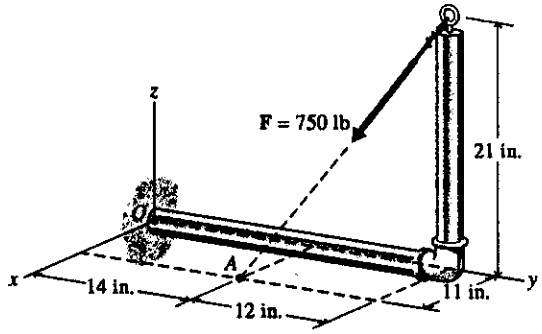


Fig. P4-55

SOLUTION

(a) $\bar{F} = 750\left[\dfrac{11\,\hat{i} - 12\,\hat{j} - 21\,\hat{k}}{\sqrt{(11)^2 + (-12)^2 + (-21)^2}}\right]$

$= 310.49\,\hat{i} - 338.72\,\hat{j} - 592.76\,\hat{k}$ lb

$\vec{r}_{A/O} = 11\,\hat{i} + 14\,\hat{j}$ in.

$\bar{M}_O = \vec{r}_{A/O} \times \bar{F} = (11\,\hat{i} + 14\,\hat{j}) \times (310.49\,\hat{i} - 338.72\,\hat{j} - 592.76\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 11 & 14 & 0 \\ 310.49 & -338.72 & -592.76 \end{vmatrix}$$

$= -8299\,\hat{i} + 6520\,\hat{j} - 8073\,\hat{k}$ in.·lb

$\cong -8.30\,\hat{i} + 6.52\,\hat{j} - 8.07\,\hat{k}$ in.·kip   Ans.

(b) $M_O = \sqrt{(-8299)^2 + (6520)^2 + (-8073)^2} = 13{,}300$ in.·lb

$\theta_x = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{-8299}{13{,}300} = 128.61^\circ \cong 128.6^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_y}{M_O} = \cos^{-1}\dfrac{6520}{13{,}300} = 60.64^\circ \cong 60.6^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_z}{M_O} = \cos^{-1}\dfrac{-8073}{13{,}300} = 127.37^\circ \cong 127.4^\circ$ Ans.

4-56* A 610-lb force is applied to a lever attached to a post as shown in Fig. P4-56. Determine
(a) The moment of the force about point O at the base of the post.
(b) The direction angles associated with the moment vector.

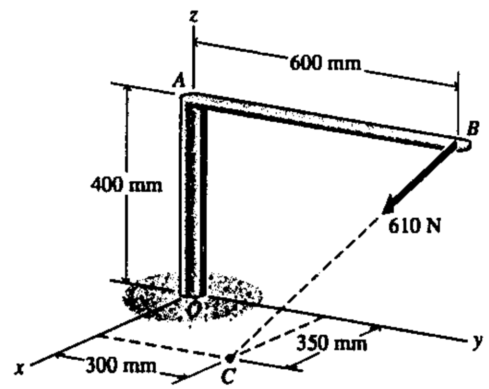


Fig. P4-56

SOLUTION

(a) $\bar{F} = 610\left[\dfrac{350\,\hat{i} - 300\,\hat{j} - 400\,\hat{k}}{\sqrt{(350)^2 + (-300)^2 + (-400)^2}}\right]$

$= 349.81\,\hat{i} - 299.84\,\hat{j} - 399.79\,\hat{k}$ N

$\bar{r}_{C/O} = 350\,\hat{i} + 300\,\hat{j}$ mm

$\bar{M}_O = \bar{r}_{C/O} \times \bar{F} = (350\,\hat{i} + 300\,\hat{j}) \times (349.81\,\hat{i} - 299.84\,\hat{j} - 399.79\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 350 & 300 & 0 \\ 349.81 & -299.84 & -399.79 \end{vmatrix}$$

$= -119{,}937\,\hat{i} + 139{,}927\,\hat{j} - 209{,}887\,\hat{k}$ N·mm

$\cong -119.9\,\hat{i} + 139.9\,\hat{j} - 210\,\hat{k}$ N·m Ans.

(b) $M_O = \sqrt{(-119.94)^2 + (139.93)^2 + (-209.89)^2} = 279.32$ N·m

$\theta_x = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{-119.94}{279.32} = 115.43^\circ \cong 115.4^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{139.93}{279.32} = 59.94^\circ \cong 59.9^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{-209.89}{279.32} = 138.71^\circ \cong 138.7^\circ$ Ans.

4-57* Determine the moment of the 1000-lb force shown in Fig. P4-57 about point O.

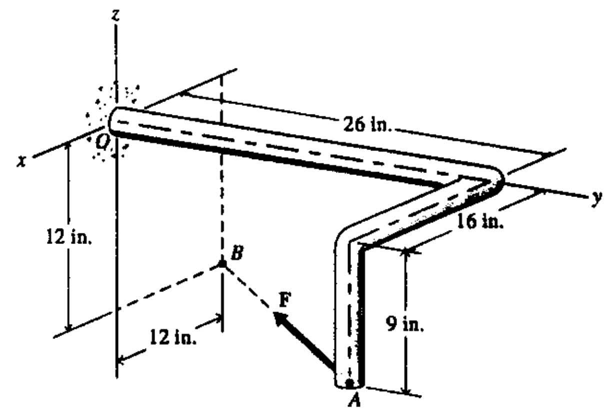


Fig. P4-57

SOLUTION

$$\bar{F} = 1000\left[\frac{-28\,\hat{i} - 26\,\hat{j} - 3\,\hat{k}}{\sqrt{(-28)^2 + (-26)^2 + (-3)^2}}\right] = -730.55\,\hat{i} - 678.36\,\hat{j} - 78.27\,\hat{k}\ \text{lb}$$

$$\bar{r}_{B/O} = -12\,\hat{i} - 12\,\hat{k}\ \text{in.}$$

$$\bar{M}_O = \bar{r}_{B/O} \times \bar{F} = (-12\,\hat{i} - 12\,\hat{j}) \times (-730.55\,\hat{i} - 678.36\,\hat{j} - 78.27\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -12 & 0 & -12 \\ -730.55 & -678.36 & -78.27 \end{vmatrix}$$

$$= -8140\,\hat{i} + 7827\,\hat{j} + 8140\,\hat{k}\ \text{in.·lb}$$

$$\cong -8.14\,\hat{i} + 7.83\,\hat{j} + 8.14\,\hat{k}\ \text{in.·kip} \qquad \text{Ans.}$$

(b) $$M_O = \sqrt{(-8140)^2 + (7827)^2 + (8140)^2} = 13{,}921\ \text{in.·lb}$$

$$\cong 13.92\ \text{in.·lb} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{M_x}{M_O} = \cos^{-1}\frac{-8140}{13{,}921} = 125.78^\circ \cong 125.8^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{M_x}{M_O} = \cos^{-1}\frac{7827}{13{,}921} = 55.79^\circ \cong 55.8^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{M_x}{M_O} = \cos^{-1}\frac{8140}{13{,}921} = 54.22^\circ \cong 54.2^\circ \qquad \text{Ans.}$$

4-58* Determine the moment of the 480-N force shown in Fig. P4-58 about point O.

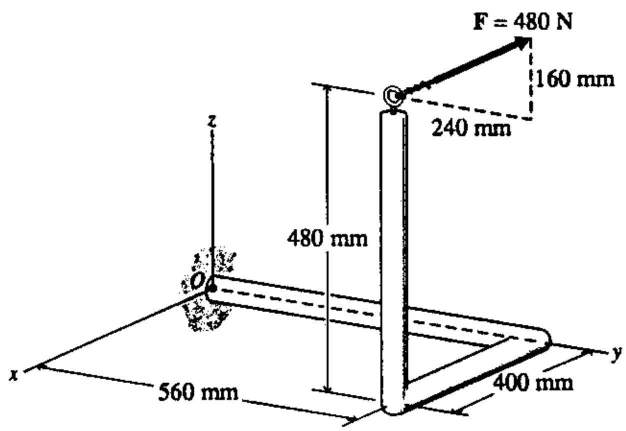


Fig. P4-58

SOLUTION

(a) $\vec{F} = 480\left[\dfrac{240\,\hat{j} + 160\,\hat{k}}{\sqrt{(240)^2 + (160)^2}}\right] = 399.38\,\hat{j} + 266.26\,\hat{k}$ N

$\vec{r}_{F/O} = 400\,\hat{i} + 560\,\hat{j} + 480\,\hat{k}$ mm

$\vec{M}_O = \vec{r}_{F/O} \times \vec{F} = (400\,\hat{i} + 560\,\hat{j} + 480\,\hat{k}) \times (399.38\,\hat{j} + 266.26\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 400 & 560 & 480 \\ 0 & 399.38 & 266.26 \end{vmatrix}$$

$= -42{,}597\,\hat{i} - 106{,}504\,\hat{j} + 159{,}752\,\hat{k}$ N·mm

$\cong -42.6\,\hat{i} - 106.5\,\hat{j} + 159.8\,\hat{k}$ N·m   Ans.

(b) $M_O = \sqrt{(-42.60)^2 + (-106.50)^2 + (159.75)^2} = 196.66$ N·m

$\cong 196.7$ N·m   Ans.

$\theta_x = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{-42.60}{196.66} = 102.51^\circ \cong 102.5^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{-106.50}{196.66} = 122.79^\circ \cong 122.8^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{159.75}{196.66} = 35.68^\circ \cong 35.7^\circ$ Ans.

4-59 A force with a magnitude of 580 lb acts at a point in a body as shown in Fig. P4-59. Determine
(a) The moment of the force about point B.
(b) The direction angles associated with the moment vector.

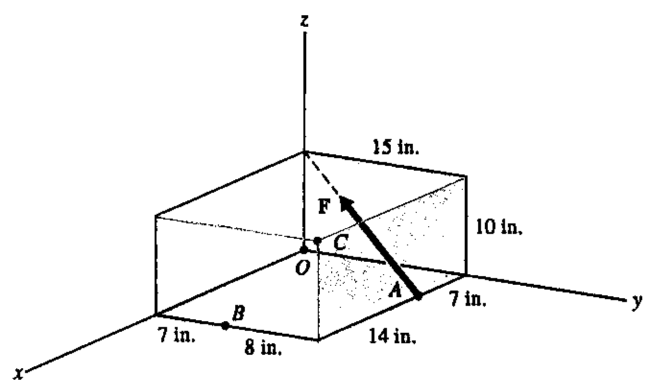


Fig. P4-59

SOLUTION

(a) $\vec{F} = 580\left[\dfrac{-7\,\hat{i} - 15\,\hat{j} + 10\,\hat{k}}{\sqrt{(-7)^2 + (15)^2 + (10)^2}}\right]$

$= -209.94\,\hat{i} - 449.87\,\hat{j} + 299.91\,\hat{k}$ lb

$\vec{r}_{A/B} = -14\,\hat{i} + 8\,\hat{j}$ in.

$\vec{M}_B = \vec{r}_{A/B} \times \vec{F} = (-14\,\hat{i} + 8\,\hat{j}) \times (-209.94\,\hat{i} - 449.87\,\hat{j} + 299.91\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -14 & 8 & 0 \\ -209.94 & -449.87 & 299.91 \end{vmatrix}$$

$= 2399\,\hat{i} + 4199\,\hat{j} + 7978\,\hat{k}$ in.·lb

$\cong 2.40\,\hat{i} + 4.20\,\hat{j} + 7.98\,\hat{k}$ in.·kip   Ans.

(b) $M_B = \sqrt{(2399)^2 + (4199)^2 + (7978)^2} = 9329$ in.·lb $\cong$ 9.33 in.·kip   Ans.

$\theta_x = \cos^{-1}\dfrac{M_x}{M_B} = \cos^{-1}\dfrac{2399}{9329} = 75.10^\circ = 75.1^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_y}{M_B} = \cos^{-1}\dfrac{4199}{9329} = 63.24^\circ \cong 63.2^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_z}{M_B} = \cos^{-1}\dfrac{7978}{9329} = 31.22^\circ \cong 31.2^\circ$ Ans.

4-60 A force with a magnitude of 585 N acts at a point in a body as shown in Fig. P4-60. Determine
(a) The moment of the force about point C.
(b) The direction angles associated with the moment vector.

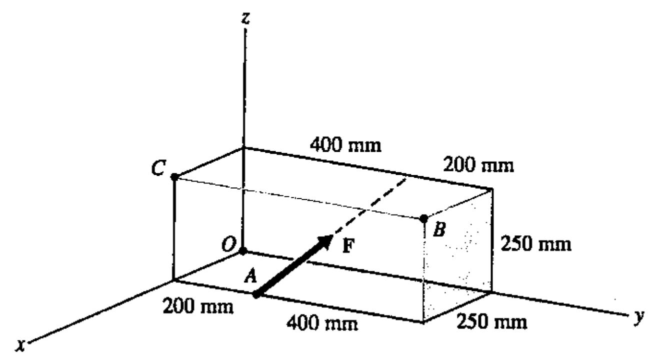


Fig. P4-60

SOLUTION

(a) $\bar{F} = 585\left[\dfrac{-250\,\hat{i} + 200\,\hat{j} + 250\,\hat{k}}{\sqrt{(-250)^2 + (200)^2 + (250)^2}}\right]$

$= -360.0\,\hat{i} + 288.0\,\hat{j} + 360.0\,\hat{k}$ N

$\bar{r}_{A/C} = 200\,\hat{j} - 250\,\hat{k}$ mm

$\bar{M}_C = \bar{r}_{A/C} \times \bar{F} = (200\,\hat{j} - 250\,\hat{k}) \times (-360.0\,\hat{i} + 288.0\,\hat{j} + 360.0\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 200 & -250 \\ -360.0 & 288.0 & 360.0 \end{vmatrix}$$

$= 144{,}000\,\hat{i} + 90{,}000\,\hat{j} + 72{,}000\,\hat{k}$ N·mm

$\cong 144.0\,\hat{i} + 90.0\,\hat{j} + 72.0\,\hat{k}$ N·m   Ans.

(b) $M_C = \sqrt{(144.0)^2 + (90.0)^2 + (72.0)^2} = 184.45$ N·m

$\theta_x = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{144.0}{184.45} = 38.68° \cong 38.7°$ Ans.

$\theta_y = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{90.0}{184.45} = 60.79° \cong 60.8°$ Ans.

$\theta_z = \cos^{-1}\dfrac{M_x}{M_O} = \cos^{-1}\dfrac{72.0}{184.45} = 67.02° \cong 67.0°$ Ans.

4-61* The magnitude of the force $\vec{F}$ in Fig. P4-61 is 450 lb. Determine the scalar component of the moment at point B about line BC.

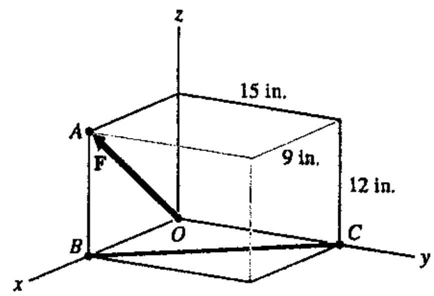


Fig. P4-61

SOLUTION

$$\vec{F} = 450\left[\frac{9\,\hat{i} + 12\,\hat{k}}{\sqrt{(9)^2 + (12)^2}}\right] = 270\,\hat{i} + 360\,\hat{k}\ \text{lb}$$

$$\vec{r}_{A/B} = 12\,\hat{k}\ \text{in.}$$

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F} = (12\,\hat{k}) \times (270\,\hat{i} + 360\,\hat{k}) = 3240\,\hat{j}\ \text{in.}\cdot\text{lb}$$

$$\hat{e}_{BC} = \frac{-9\,\hat{i} + 15\,\hat{j}}{\sqrt{(-9)^2 + (15)^2}} = -0.5145\,\hat{i} + 0.8575\,\hat{j}$$

$$M_{BC} = \vec{M}_B\cdot\hat{e}_{BC} = (3240\,\hat{j})\cdot(-0.5145\,\hat{i} + 0.8575\,\hat{j})$$

$$= 2778\ \text{in.}\cdot\text{lb} \cong 2.78\ \text{in.}\cdot\text{kip}$$ Ans.

4-62* The magnitude of the force $\vec{F}$ in Fig. P4-62 is 595 N. Determine the scalar component of the moment at point O about line OC.

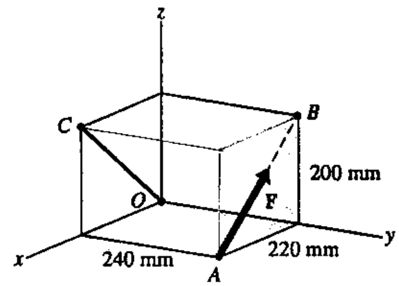


Fig. P4-62

SOLUTION

$$\vec{F} = 595\left[\frac{-220\ \hat{i} + 200\ \hat{k}}{\sqrt{(-220)^2 + (200)^2}}\right] = -440.3\ \hat{i} + 400.2\ \hat{k}\ \ \text{N}$$

$$\vec{r}_{A/O} = 0.220\ \hat{i} + 0.240\ \hat{j}\ \ \text{m}$$

$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F} = (0.220\ \hat{i} + 0.240\ \hat{j}) \times (-440.3\ \hat{i} + 400.2\ \hat{k})$$

$$= 96.05\ \hat{i} - 88.04\ \hat{j} + 105.67\ \hat{k}$$

$$\hat{e}_{OC} = \frac{220\ \hat{i} + 200\ \hat{k}}{\sqrt{(220)^2 + (200)^2}} = 0.7399\ \hat{i} + 0.6727\ \hat{k}$$

$$M_{OC} = \vec{M}_O \cdot \hat{e}_{OC} = (96.05\ \hat{i} - 88.04\ \hat{j} + 105.67\ \hat{k}) \cdot (0.7399\ \hat{i} + 0.6727\ \hat{k})$$

$$= 142.15\ \text{N·m} \cong 142.2\ \text{N·m}$$ Ans.

4-63 The magnitude of the force $\mathbf{F}$ in Fig. P4-63 is 680 lb. Determine the scalar component of the moment at point O about line OC.

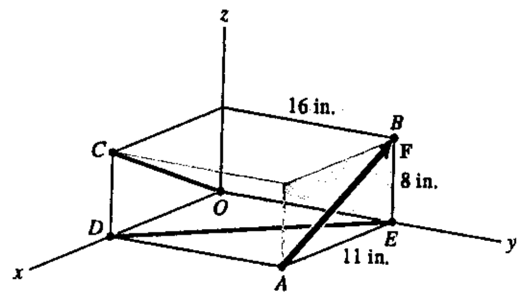


Fig. P4-63

SOLUTION

$$\mathbf{F} = 680\left[\frac{-11\,\hat{i} + 8\,\hat{k}}{\sqrt{(-11)^2 + (8)^2}}\right] = -549.9\,\hat{i} + 400.0\,\hat{k} \text{ lb}$$

$$\vec{r}_{A/O} = 11\,\hat{i} + 16\,\hat{j}$$

$$\mathbf{M}_O = \vec{r}_{A/O} \times \mathbf{F} = (11\,\hat{i} + 16\,\hat{j}) \times (-549.9\,\hat{i} + 400.0\,\hat{k})$$

$$= 6400\,\hat{i} - 4400\,\hat{j} + 8798\,\hat{k} \text{ in.}\cdot\text{lb}$$

$$\hat{e}_{OC} = \frac{11\,\hat{i} + 8\,\hat{k}}{\sqrt{(11)^2 + (8)^2}} = 0.8087\,\hat{i} + 0.5882\,\hat{k}$$

$$M_{OC} = \mathbf{M}_O \cdot \hat{e}_{OC}$$

$$= (6400\,\hat{i} - 4400\,\hat{j} + 8798\,\hat{k}) \cdot (0.8087\,\hat{i} + 0.5882\,\hat{k})$$

$$= 10{,}351 \text{ in.}\cdot\text{lb} \cong 10.35 \text{ in.}\cdot\text{kip}$$ Ans.

4-64 The magnitude of the force $\bar{F}$ in Fig. P4-64 is 635 N. Determine the scalar component of the moment at point O about line OC.

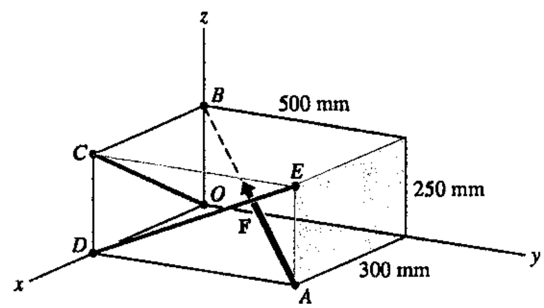


Fig. P4-64

SOLUTION

$$\bar{F} = 635\left[\frac{-300\,\hat{i} - 500\,\hat{j} + 250\,\hat{k}}{\sqrt{(-300)^2 + (-500)^2 + (250)^2}}\right]$$

$$= -300.3\,\hat{i} - 500.5\,\hat{j} + 250.2\,\hat{k}\ \text{N}$$

$$\bar{r}_{A/O} = 0.300\,\hat{i} + 0.500\,\hat{j}\ \text{m}$$

$$\bar{M}_O = \bar{r}_{A/O} \times \bar{F} = (0.300\,\hat{i} + 0.500\,\hat{j}) \times (-300.3\,\hat{i} - 500.5\,\hat{j} + 250.2\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.300 & 0.500 & 0 \\ -300.3 & -500.5 & 250.2 \end{vmatrix}$$

$$= 125.10\,\hat{i} - 75.06\,\hat{j}\ \text{N}\cdot\text{m}$$

$$\hat{e}_{OC} = \frac{300\,\hat{i} + 250\,\hat{k}}{\sqrt{(300)^2 + (250)^2}} = 0.7682\,\hat{i} + 0.6402\,\hat{k}$$

$$M_{OC} = \bar{M}_O \cdot \hat{e}_{OC}$$

$$= (125.10\,\hat{i} - 75.06\,\hat{j}) \cdot (0.7682\,\hat{i} + 0.6402\,\hat{k})$$

$$= 96.10\ \text{N}\cdot\text{m} = 96.1\ \text{N}\cdot\text{m}$$ Ans.

4-65* The magnitude of the force $\vec{F}$ in Fig. P4-65 is 680 lb. Determine the scalar component of the moment at point D about line DE.

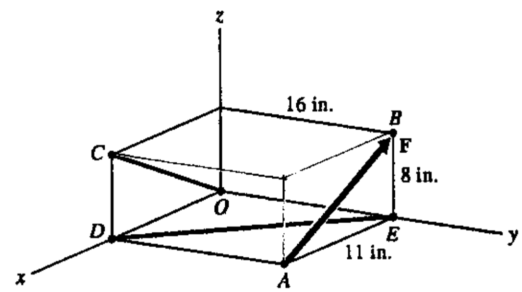


Fig. P4-65

SOLUTION

$$\vec{F} = 680\left[\frac{-11\,\hat{i} + 8\,\hat{k}}{\sqrt{(-11)^2 + (8)^2}}\right] = -549.9\,\hat{i} + 400.0\,\hat{k} \text{ lb}$$

$$\vec{r}_{A/D} = 16\,\hat{j} \text{ in.}$$

$$\vec{M}_D = \vec{r}_{A/D} \times \vec{F} = (16\,\hat{j}) \times (-549.9\,\hat{i} + 400.0\,\hat{k})$$

$$= 6400\,\hat{i} + 8798\,\hat{k} \text{ in.}\cdot\text{lb}$$

$$\hat{e}_{DE} = \frac{-11\,\hat{i} + 16\,\hat{k}}{\sqrt{(-11)^2 + (16)^2}} = -0.5665\,\hat{i} + 0.8240\,\hat{j}$$

$$M_{DE} = \vec{M}_D \cdot \hat{e}_{DE}$$

$$= (6400\,\hat{i} + 8798\,\hat{k})\cdot(-0.5665\,\hat{i} + 0.8240\,\hat{j})$$

$$= -3626 \text{ in.}\cdot\text{lb} \cong -3.63 \text{ in.}\cdot\text{kip}$$ Ans.

4-66* The magnitude of the force $\vec{F}$ in Fig. P4-66 is 635 N. Determine the scalar component of the moment at point D about line DE.

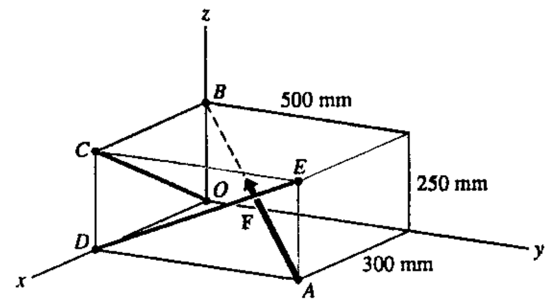


Fig. P4-66

SOLUTION

$$\vec{F} = 635\left[\frac{-300\,\hat{i} - 500\,\hat{j} + 250\,\hat{k}}{\sqrt{(-300)^2 + (-500)^2 + (250)^2}}\right]$$

$$= -300.3\,\hat{i} - 500.5\,\hat{j} + 250.2\,\hat{k} \text{ N}$$

$$\vec{r}_{A/D} = 0.500\,\hat{j} \text{ m}$$

$$\vec{M}_D = \vec{r}_{A/D} \times \vec{F} = (0.500\,\hat{j}) \times (-300.3\,\hat{i} - 500.5\,\hat{j} + 250.2\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.500 & 0 \\ -300.3 & -500.5 & 250.2 \end{vmatrix}$$

$$= 125.10\,\hat{i} + 150.15\,\hat{k} \text{ N}\cdot\text{m}$$

$$\hat{e}_{DE} = \frac{500\,\hat{i} + 250\,\hat{k}}{\sqrt{(500)^2 + (250)^2}} = 0.8945\,\hat{i} + 0.4472\,\hat{k}$$

$$M_{DE} = \vec{M}_D \cdot \hat{e}_{DE} = (125.10\,\hat{i} + 150.15\,\hat{k}) \cdot (0.8945\,\hat{j} + 0.4472\,\hat{k})$$

$$= 67.14 \text{ N}\cdot\text{m} \cong 67.1 \text{ N}\cdot\text{m}$$ Ans.

4-67 Determine the scalar component of the moment of the 800-lb force shown in Fig. P4-67 about line OA.

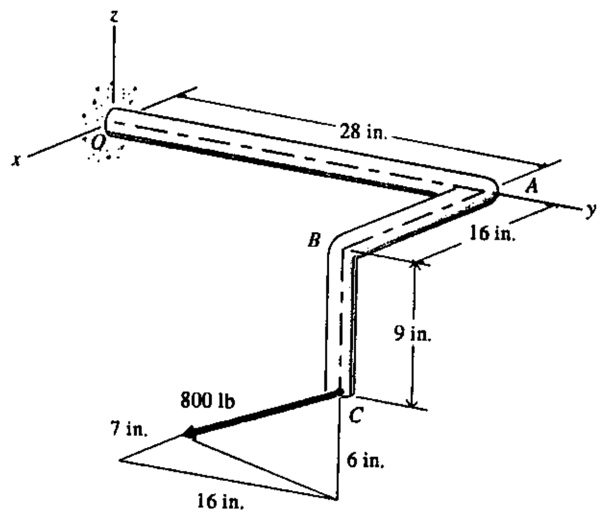


Fig. P4-67

SOLUTION

$$\mathbf{F} = 800\left[\frac{-7\,\hat{\imath} - 16\,\hat{\jmath} - 6\,\hat{k}}{\sqrt{(-7)^2 + (-16)^2 + (-6)^2}}\right] = -303.3\,\hat{\imath} - 693.2\,\hat{\jmath} - 259.9\,\hat{k} \text{ lb}$$

$$\bar{r}_{C/O} = 16\,\hat{\imath} + 28\,\hat{\jmath} - 9\,\hat{k} \text{ in.}$$

$$\bar{M}_O = \bar{r}_{C/O} \times \mathbf{F} = (16\,\hat{\imath} + 28\,\hat{\jmath} - 9\,\hat{k}) \times (-303.3\,\hat{\imath} - 693.2\,\hat{\jmath} - 259.9\,\hat{k})$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 16 & 28 & -9 \\ -300.3 & -693.2 & -259.9 \end{vmatrix}$$

$$= 13{,}516\,\hat{\imath} + 6888\,\hat{\jmath} - 2599\,\hat{k} \text{ in.}\cdot\text{lb}$$

$$\hat{e}_{OA} = 1.000\,\hat{\jmath}$$

$$M_{OA} = \bar{M}_O \cdot \hat{e}_{OC} = (13{,}516\,\hat{\imath} + 6888\,\hat{\jmath} - 2599\,\hat{k}) \cdot (1.000\,\hat{\jmath})$$

$$= 6888 \text{ in.}\cdot\text{lb} \cong 6.89 \text{ in.}\cdot\text{kip}$$ Ans.

4-68 Determine the scalar component of the moment of the 750-N force shown in Fig. P4-68 about line OC.

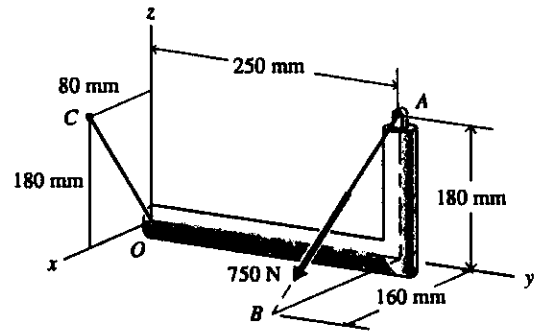


Fig. P4-68

SOLUTION

$$\vec{F} = 750\left[\frac{160\,\hat{i} - 180\,\hat{k}}{\sqrt{(160)^2 + (-180)^2}}\right] = 498.3\,\hat{i} - 560.6\,\hat{k} \text{ N}$$

$$\vec{r}_{A/O} = 0.250\,\hat{j} + 0.180\,\hat{k} \text{ m}$$

$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F} = (0.250\,\hat{j} + 0.180\,\hat{k}) \times (498.3\,\hat{i} - 560.6\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.250 & 0.180 \\ 498.3 & 0 & -560.6 \end{vmatrix}$$

$$= -140.15\,\hat{i} + 89.69\,\hat{j} - 124.58\,\hat{k} \text{ N·m}$$

$$\hat{e}_{OC} = \frac{80\,\hat{i} + 180\,\hat{k}}{\sqrt{(80)^2 + (180)^2}} = 0.4061\,\hat{i} + 0.9138\,\hat{k}$$

$$M_{OC} = \vec{M}_O \cdot \hat{e}_{OC} = (-140.15\,\hat{i} + 89.69\,\hat{j} - 124.58\,\hat{k}) \cdot (0.4061\,\hat{i} + 0.9138\,\hat{k})$$

$$= -170.76 \text{ N·m} \cong -170.8 \text{ N·m}$$ Ans.

4-69* Determine the scalar component of the moment of the 500-lb force shown in Fig. P4-69 about line AC.

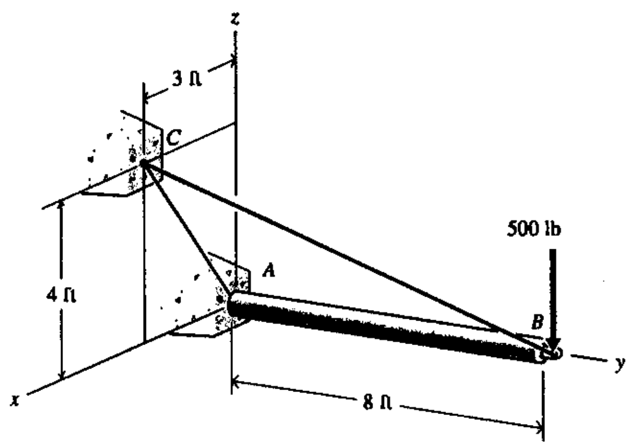


Fig. P4-69

SOLUTION

$$\vec{F} = -500\ \hat{k}$$

$$\vec{r}_{B/A} = 8\ \hat{j}$$

$$\vec{M}_A = \vec{r}_{B/A} \times \vec{F} = (8\ \hat{j}) \times (-500\ \hat{k}) = -4000\ \hat{i}\ \text{ft}\cdot\text{lb}$$

$$\hat{e}_{AC} = \frac{3\ \hat{i} + 4\ \hat{k}}{\sqrt{(3)^2 + (4)^2}} = 0.600\ \hat{i} + 0.800\ \hat{k}$$

$$M_{AC} = \vec{M}_A \cdot \hat{e}_{AC} = (-4000\ \hat{i}) \cdot (0.600\ \hat{i} + 0.800\ \hat{k})$$

$$= -2400\ \text{ft}\cdot\text{lb} = -2.40\ \text{ft}\cdot\text{kip}$$ Ans.

4-70* Determine the scalar component of the moment of the 750-N force shown in Fig. P4-70 about the axis of shaft AB.

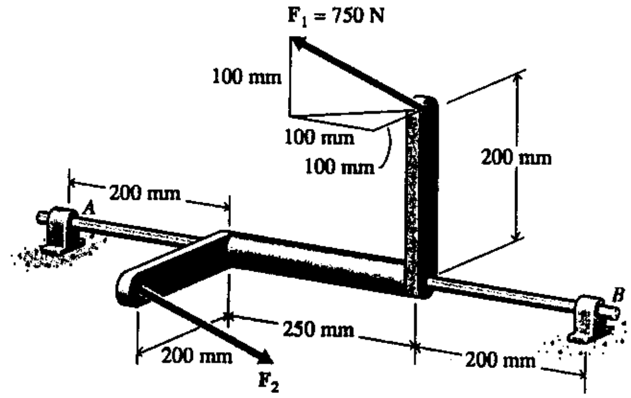


Fig. P4-70

SOLUTION

$$\mathbf{F}_1 = 750\left[\frac{100\,\hat{i} - 100\,\hat{j} + 100\,\hat{k}}{\sqrt{(100)^2 + (-100)^2 + (100)^2}}\right] = 433.0\,\hat{i} - 433.0\,\hat{j} + 433.0\,\hat{k} \text{ N}$$

$$\vec{r}_{F/A} = 0.450\,\hat{j} + 0.200\,\hat{k} \text{ m}$$

$$\mathbf{M}_A = \vec{r}_{F/A} \times \mathbf{F}_1 = (0.450\,\hat{j} + 0.200\,\hat{k}) \times (433.0\,\hat{i} - 433.0\,\hat{j} + 433.0\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.450 & 0.200 \\ 433.3 & -433.3 & 433.3 \end{vmatrix}$$

$$= 281.45\,\hat{i} + 86.60\,\hat{j} - 194.85\,\hat{k} \text{ N}\cdot\text{m}$$

$$\hat{e}_{AB} = 1.000\,\hat{j}$$

$$M_{AB} = \mathbf{M}_A \cdot \hat{e}_{AB} = (281.45\,\hat{i} + 86.60\,\hat{j} - 194.85\,\hat{k}) \cdot (1.000\,\hat{j})$$

$$= 86.60 \text{ N}\cdot\text{m} = 86.6 \text{ N}\cdot\text{m}$$ Ans.

4-71 Determine the scalar component of the moment of the 100-lb force shown in Fig. P4-71 about the axis of the hinges (line AB).

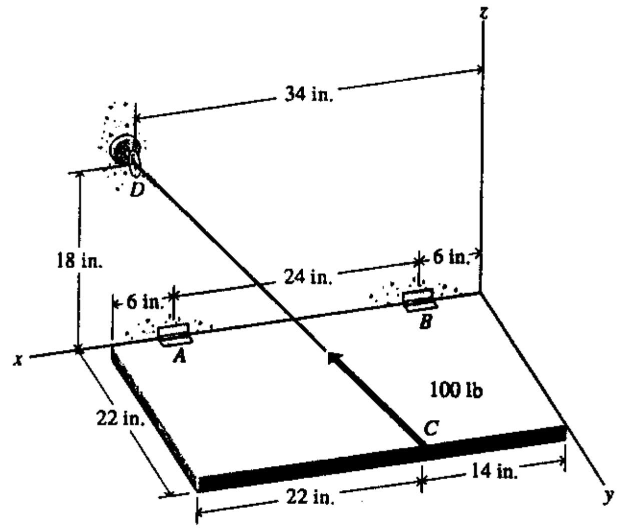


Fig. P4-71

SOLUTION

$$\bar{F} = 100\left[\frac{22\,\hat{i} - 22\,\hat{j} + 18\,\hat{k}}{\sqrt{(22)^2 + (-22)^2 + (18)^2}}\right] = 61.21\,\hat{i} - 61.21\,\hat{j} + 50.08\,\hat{k} \text{ lb}$$

$$\bar{r}_{C/A} = -16\,\hat{i} + 22\,\hat{j} \text{ in.}$$

$$\bar{M}_A = \bar{r}_{C/A} \times \bar{F} = (-16\,\hat{i} + 22\,\hat{j}) \times (61.21\,\hat{i} - 61.21\,\hat{j} + 50.08\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -16 & 22 & 0 \\ 61.21 & -61.21 & 50.08 \end{vmatrix}$$

$$= 1101.8\,\hat{i} + 801.3\,\hat{j} - 367.3\,\hat{k} \text{ in.}\cdot\text{lb}$$

$$\hat{e}_{AB} = -1.000\,\hat{i}$$

$$M_{AB} = \bar{M}_A \cdot \hat{e}_{AB} = (1101.8\,\hat{i} + 801.3\,\hat{j} - 367.3\,\hat{k}) \cdot (-1.000\,\hat{i})$$

$$= -1101.8 \text{ in.}\cdot\text{lb} \cong 1102 \text{ in.}\cdot\text{lb}$$ Ans.

4-72* Determine the moment of the 610-N force shown in Fig. P4-72 about line CD. Express the result in Cartesian vector form.

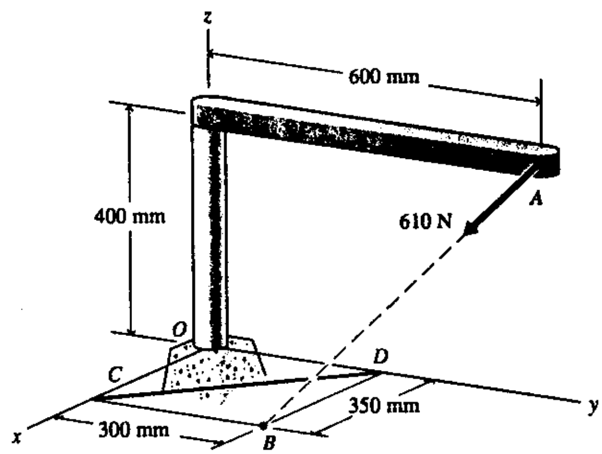


Fig. P4-72

SOLUTION

$$\mathbf{F} = 610\left[\frac{350\,\hat{i} - 300\,\hat{j} - 400\,\hat{k}}{\sqrt{(350)^2 + (-300)^2 + (-400)^2}}\right]$$

$$= 349.8\,\hat{i} - 299.8\,\hat{j} - 399.8\,\hat{k} \text{ N}$$

$$\vec{r}_{B/C} = 0.300\,\hat{j} \text{ m}$$

$$\mathbf{M}_C = \vec{r}_{B/C} \times \mathbf{F} = (0.300\,\hat{j}) \times (349.8\,\hat{i} - 299.8\,\hat{j} - 399.8\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0.300 & 0 \\ 349.8 & -299.8 & -399.8 \end{vmatrix} = -119.94\,\hat{i} - 104.94\,\hat{k} \text{ N·m}$$

$$\hat{e}_{CD} = \frac{-350\,\hat{i} + 300\,\hat{j}}{\sqrt{(-350)^2 + (300)^2}} = -0.7593\,\hat{i} + 0.6508\,\hat{j}$$

$$M_{CD} = \mathbf{M}_C \cdot \hat{e}_{CD} = (-119.94\,\hat{i} - 104.94\,\hat{k}) \cdot (-0.7593\,\hat{i} + 0.6508\,\hat{j})$$

$$= 91.07 \text{ N·m} \cong 91.1 \text{ N·m}$$

$$\mathbf{M}_{CD} = M_{CD}\hat{e}_{CD} = 91.07(-0.7593\,\hat{i} + 0.6508\,\hat{j})$$

$$= -69.1\,\hat{i} + 59.3\,\hat{j} \text{ N·m}$$ Ans.

4-73* A 200-lb force is applied to a lever-shaft assembly as shown in Fig. P4-73. Determine the moment of the force about line OC. Express the results in Cartesian vector form.

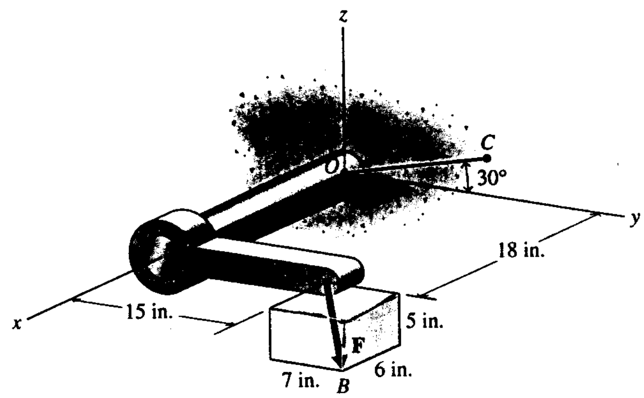


Fig. P4-73

SOLUTION

$$\vec{F} = 200\left[\frac{6\,\hat{i} + 7\,\hat{j} - 5\,\hat{k}}{\sqrt{(6)^2 + (7)^2 + (-5)^2}}\right] = 114.42\,\hat{i} + 133.48\,\hat{j} - 95.35\,\hat{k} \text{ lb}$$

$$\vec{r}_{A/O} = 18\,\hat{i} + 15\,\hat{j} \text{ in.}$$

$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F} = (18\,\hat{i} + 15\,\hat{j}) \times (114.42\,\hat{i} + 133.48\,\hat{j} - 95.35\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 18 & 15 & 0 \\ 114.42 & 133.48 & -95.35 \end{vmatrix}$$

$$= -1430.25\,\hat{i} + 1716.30\,\hat{j} + 686.34\,\hat{k} \text{ in.}\cdot\text{lb}$$

$$\hat{e}_{OC} = \cos 30^\circ\,\hat{j} + \sin 30^\circ\,\hat{k} = 0.8660\,\hat{j} + 0.5000\,\hat{k}$$

$$M_{OC} = \vec{M}_O \cdot \hat{e}_{OC}$$

$$= (-1430.25\,\hat{i} + 1716.30\,\hat{j} + 686.34\,\hat{k}) \cdot (0.8660\,\hat{j} + 0.5000\,\hat{k})$$

$$= 1829.5 \text{ in.}\cdot\text{lb}$$

$$\vec{M}_{OC} = M_{OC}\,\hat{e}_{OC}$$

$$= 1829.5(0.8660\,\hat{j} + 0.5000\,\hat{k}) = 1584\,\hat{j} + 915\,\hat{k} \text{ in.}\cdot\text{lb}$$ Ans.

4-74 A bracket is subjected to an 825-N force as shown in Fig. P4-74. Determine the moment of the force about line OB. Express the results in Cartesian vector form.

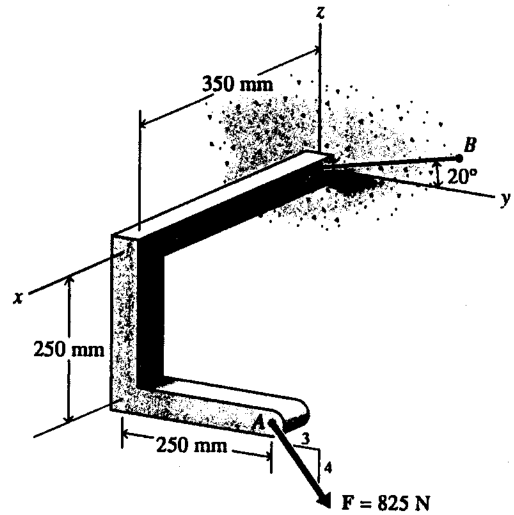


Fig. P4-74

SOLUTION

$$\vec{F} = 825\left[\frac{3\,\hat{j} - 4\,\hat{k}}{5}\right] = 495\,\hat{j} - 660\,\hat{k}\ \text{N}$$

$$\vec{r}_{A/O} = 0.350\,\hat{i} + 0.250\,\hat{j} - 0.250\,\hat{k}\ \text{m}$$

$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F} = (0.350\,\hat{i} + 0.250\,\hat{j} - 0.250\,\hat{k}) \times (495\,\hat{j} - 660\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.350 & 0.250 & -0.250 \\ 0 & 495 & -660 \end{vmatrix}$$

$$= -41.25\,\hat{i} + 231.00\,\hat{j} + 173.25\,\hat{k}\ \text{N·m}$$

$$\hat{e}_{OB} = \cos 20^\circ\,\hat{j} + \sin 20^\circ\,\hat{k} = 0.9397\,\hat{j} + 0.3420\,\hat{k}$$

$$M_{OB} = \vec{M}_O \cdot \hat{e}_{OB}$$

$$= (-41.25\,\hat{i} + 231.00\,\hat{j} + 173.25\,\hat{k}) \cdot (0.9397\,\hat{j} + 0.3420\,\hat{k})$$

$$= 276.3\ \text{N·m} \cong 276\ \text{N·m}$$

$$\vec{M}_{OB} = M_{OB}\,\hat{e}_{OB}$$

$$= 276.3(0.9397\,\hat{j} + 0.3420\,\hat{k}) = 260\,\hat{j} + 94.5\,\hat{k}\ \text{N·m}$$ Ans.

4-75 A pipe bracket is subjected to a 200-lb force as shown in Fig. P4-75. Determine the moment of the force about line BC. Express the result in Cartesian vector form.

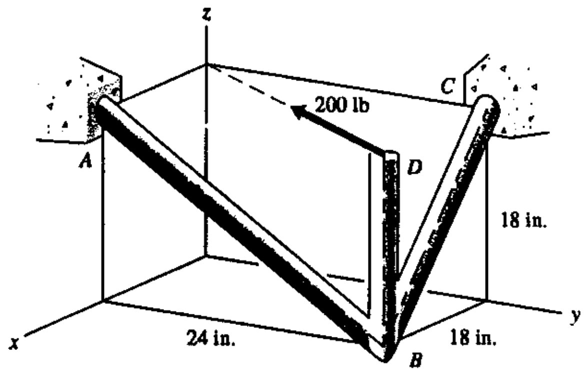


Fig. P4-75

SOLUTION

$$\mathbf{F} = 200\left[\frac{-18\,\hat{\imath} - 24\,\hat{k}}{\sqrt{(-18)^2 + (-24)^2}}\right] = -120.0\,\hat{\imath} - 160.0\,\hat{\jmath} \quad \text{lb}$$

$$\vec{r}_{D/B} = 18\,\hat{k}$$

$$\mathbf{M}_B = \vec{r}_{D/B} \times \mathbf{F} = (18\,\hat{k}) \times (-120.0\,\hat{\imath} - 160.0\,\hat{\jmath})$$

$$= 2880\,\hat{\imath} - 2160\,\hat{\jmath} \text{ in.}\cdot\text{lb}$$

$$\hat{e}_{BC} = \frac{-18\,\hat{\imath} + 18\,\hat{k}}{\sqrt{(-18)^2 + (18)^2}} = -0.7071\,\hat{\imath} + 0.7071\,\hat{k}$$

$$M_{BC} = \mathbf{M}_B \cdot \hat{e}_{BC}$$

$$= (2880\,\hat{\imath} - 2160\,\hat{\jmath})\cdot(-0.7071\,\hat{\imath} + 0.7071\,\hat{k}) = -2036 \text{ in.}\cdot\text{lb}$$

$$\mathbf{M}_{BC} = M_{BC}\hat{e}_{BC} = -2036(-0.7071\,\hat{\imath} + 0.7071\,\hat{k})$$

$$= 1440\,\hat{\imath} - 1440\,\hat{k} \text{ in.}\cdot\text{lb}$$ Ans.

4-76* A 534-N force $\bar{F}$ is applied to a lever-shaft assembly as shown in Fig. P4-76. Determine the moment of the force about line OB. Express the result in Cartesian vector form.

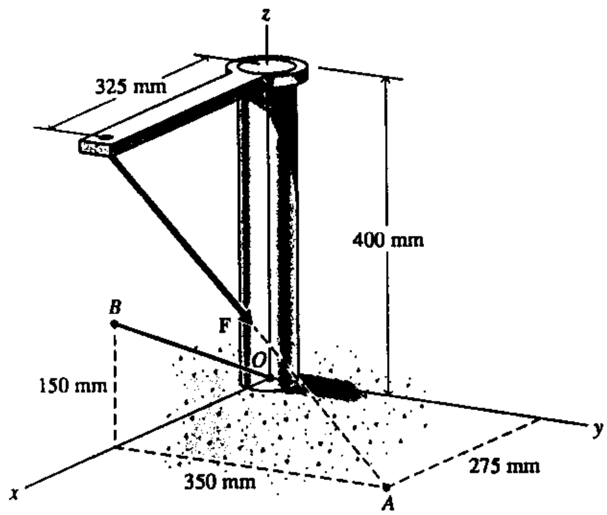


Fig. P4-76

SOLUTION

$$\bar{F} = 534\left[\frac{-50\ \hat{i} + 350\ \hat{j} - 400\ \hat{k}}{\sqrt{(-50)^2 + (350)^2 + (-400)^2}}\right] = -50.0\ \hat{i} + 350.1\ \hat{j} - 400.1\ \hat{k}\ \ \text{N}$$

$$\bar{r}_{A/O} = 0.275\ \hat{i} + 0.350\ \hat{j}\ \ \text{m}$$

$$\bar{M}_O = \bar{r}_{A/O} \times \bar{F} = (0.275\ \hat{i} + 0.350\ \hat{j}) \times (-50.0\ \hat{i} + 350.1\ \hat{j} - 400.1\ \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.275 & 0.350 & 0 \\ -50.0 & 350.1 & -400.1 \end{vmatrix}$$

$$= -140.04\ \hat{i} + 110.03\ \hat{j} + 113.78\ \hat{k}$$

$$\hat{e}_{OB} = \frac{275\ \hat{i} + 150\ \hat{k}}{\sqrt{(275)^2 + (150)^2}} = 0.8779\ \hat{i} + 0.4789\ \hat{k}$$

$$M_{OB} = \bar{M}_O \cdot \hat{e}_{OB}$$

$$= (-140.04\ \hat{i} + 110.03\ \hat{j} + 113.78\ \hat{k}) \cdot (0.8779\ \hat{i} + 0.4789\ \hat{k})$$

$$= -68.45\ \text{N}\cdot\text{m} \cong -68.5\ \text{N}\cdot\text{m}$$

$$\bar{M}_{OB} = M_{OB}\hat{e}_{OB} = -68.45(0.8779\ \hat{i} + 0.4789\ \hat{k})$$

$$= -60.09\ \hat{i} - 32.78\ \hat{k}\ \text{N}\cdot\text{m} \cong -60.1\ \hat{i} - 32.8\ \hat{k}\ \text{N}\cdot\text{m}$$ Ans.

4-77* A curved bar is subjected to a 660-lb force as shown in Fig. P4-77. Determine the moment of the force about line BC. Express the results in Cartesian vector form.

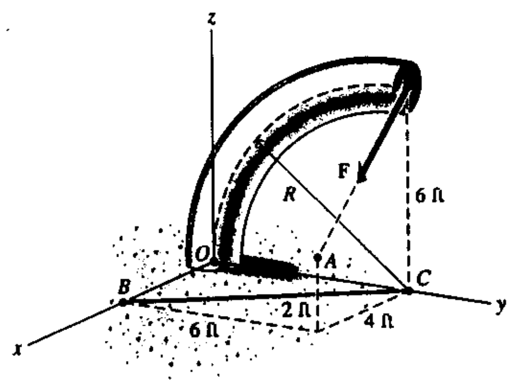


Fig. P4-77

SOLUTION

$$\mathbf{F} = 660\left[\frac{4\,\hat{\imath} - 4\,\hat{k}}{\sqrt{(4)^2 + (-4)^2}}\right] = 466.7\,\hat{\imath} - 466.7\,\hat{k} \text{ lb}$$

$$\bar{r}_{A/B} = 6\,\hat{\jmath} + 2\,\hat{k} \text{ ft}$$

$$\mathbf{M}_B = r_{A/B} \times \mathbf{F} = (6\,\hat{\jmath} + 2\,\hat{k}) \times (466.7\,\hat{\imath} - 466.7\,\hat{k})$$

$$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 6 & 2 \\ 466.7 & 0 & -466.7 \end{vmatrix}$$

$$= -2800\,\hat{\imath} + 933.4\,\hat{\jmath} - 2800\,\hat{k} \text{ ft·lb}$$

$$\hat{e}_{BC} = \frac{-4\,\hat{\imath} + 6\,\hat{\jmath}}{\sqrt{(-4)^2 + (6)^2}} = -0.5547\,\hat{\imath} + 0.8321\,\hat{\jmath}$$

$$M_{BC} = \mathbf{M}_B \cdot \hat{e}_{BC}$$

$$= (-2800\,\hat{\imath} + 933.4\,\hat{\jmath} - 2800\,\hat{k}) \cdot (-0.5547\,\hat{\imath} + 0.8321\,\hat{\jmath})$$

$$= 2329.8 \text{ ft·lb} \cong 2.33 \text{ ft·kip}$$

$$\mathbf{M}_{BC} = M_{BC}\hat{e}_{BC} = 2329.8(-0.5547\,\hat{\imath} + 0.8321\,\hat{\jmath})$$

$$= -1292.3\,\hat{\imath} + 1938.6\,\hat{\jmath} \text{ ft·lb}$$

$$\cong -1292\,\hat{\imath} + 1939\,\hat{\jmath} \text{ ft·lb}$$ Ans.

4-78* A bracket is subjected to a 384-N force as shown in Fig. P4-78. Determine the moment of the force about line OC. Express the results in Cartesian vector form.

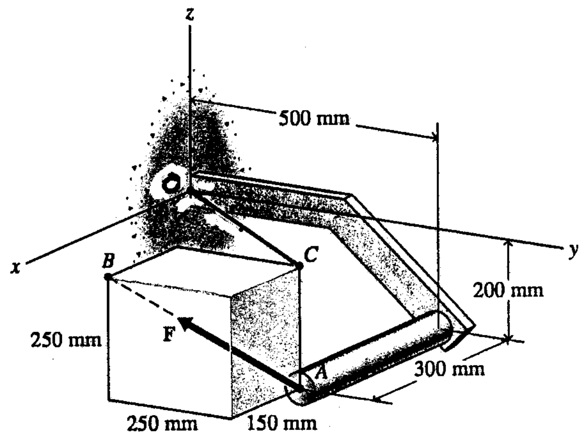


Fig. P4-78

SOLUTION

$$\vec{F} = 384\left[\frac{150\,\hat{i} - 250\,\hat{j} + 250\,\hat{k}}{\sqrt{(150)^2 + (-250)^2 + (250)^2}}\right] = 150.0\,\hat{i} - 250.0\,\hat{j} + 250.0\,\hat{k}\ \text{N}$$

$$\vec{r}_{A/O} = 0.300\,\hat{i} + 0.500\,\hat{j} - 0.200\,\hat{k}\ \text{m}$$

$$\vec{M}_O = \vec{r}_{A/O} \times \vec{F}$$

$$= (0.300\,\hat{i} + 0.500\,\hat{j} - 0.200\,\hat{k}) \times (150.0\,\hat{i} - 250.0\,\hat{j} + 250.0\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.300 & 0.500 & -0.200 \\ 150.0 & -250.0 & 250.0 \end{vmatrix}$$

$$= 75.00\,\hat{i} - 100.50\,\hat{j} - 150.00\,\hat{k}\ \text{N}\cdot\text{m}$$

$$\hat{e}_{OC} = \frac{300\,\hat{i} + 500\,\hat{j} + 50\,\hat{k}}{\sqrt{(300)^2 + (500)^2 + (50)^2}} = 0.5126\,\hat{i} + 0.8544\,\hat{j} + 0.08544\,\hat{k}$$

$$M_{OC} = \vec{M}_O \cdot \hat{e}_{OC}$$

$$= (75.00\,\hat{i} - 100.50\,\hat{j} - 150.00\,\hat{k}) \cdot (0.5126\,\hat{i} + 0.8544\,\hat{j} + 0.0854\,\hat{k})$$

$$= -64.08\ \text{N}\cdot\text{m} \cong -64.1\ \text{N}\cdot\text{m}$$

$$\vec{M}_{OC} = M_{OC}\hat{e}_{OC} = 64.08(0.5126\,\hat{i} + 0.8544\,\hat{j} + 0.8544\,\hat{k})$$

$$= -32.85\,\hat{i} - 54.75\,\hat{j} - 5.475\,\hat{k}\ \text{N}\cdot\text{m}$$

$$\cong -32.9\,\hat{i} - 54.8\,\hat{j} - 5.48\,\hat{k}\ \text{N}\cdot\text{m}$$ Ans.

4-79 The magnitude of force $\vec{F}$ in Fig. P4-79 is 781 lb. Determine
(a) The component of the moment at point C parallel to line CD.
(b) The component of the moment at point C perpendicular to line CD and the direction angles associated with this moment vector.

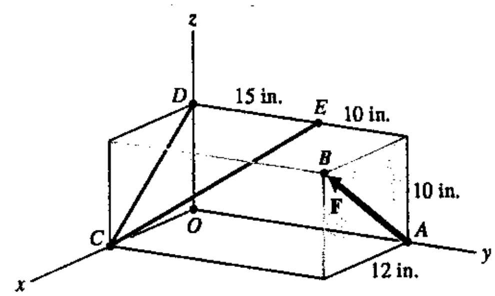


Fig. P4-79

SOLUTION

(a) $\vec{F} = 781\left[\dfrac{12\,\hat{i} + 10\,\hat{k}}{\sqrt{(12)^2 + (10)^2}}\right] = 600.0\,\hat{i} + 500.0\,\hat{k}$ lb

$\vec{r}_{A/C} = -12\,\hat{i} + 25\,\hat{j}$ in.

$$\vec{M}_C = r_{A/C} \times \vec{F} = (-12\,\hat{i} + 25\,\hat{j}) \times (600.0\,\hat{i} + 500.0\,\hat{k})$$
$$= 12{,}500\,\hat{i} + 6000\,\hat{j} - 15{,}000\,\hat{k} \text{ in.}\cdot\text{lb}$$
$$= 12.500\,\hat{i} + 6.000\,\hat{j} - 15.000\,\hat{k} \text{ in.}\cdot\text{kip}$$

$$\hat{e}_{CD} = \frac{-12\,\hat{i} + 10\,\hat{k}}{\sqrt{(-12)^2 + (10)^2}} = -0.7682\,\hat{i} + 0.6402\,\hat{k}$$

$$M_{CD} = \vec{M}_C \cdot \hat{e}_{CD} = (12.50\,\hat{i} + 6.00\,\hat{j} - 15.00\,\hat{k})\cdot(-0.7682\,\hat{i} + 0.6402\,\hat{k})$$
$$= -19.206 \text{ in.}\cdot\text{kip} \cong -19.21 \text{ in.}\cdot\text{kip}$$

$$\vec{M}_{CD} = M_{CD}\hat{e}_{CD} = -19.206(-0.7682\,\hat{i} + 0.6402\,\hat{k})$$
$$= 14.754\,\hat{i} - 12.296\,\hat{k} \text{ in.}\cdot\text{kip}$$ Ans.

(b) $\vec{M}_\perp = \vec{M}_C - \vec{M}_{CD}$
$$= (12.500\,\hat{i} + 6.000\,\hat{j} - 15.000\,\hat{k}) - (14.754\,\hat{i} - 12.296\,\hat{k})$$
$$= -2.254\,\hat{i} + 6.000\,\hat{j} - 2.704\,\hat{k} \text{ in.}\cdot\text{kip}$$

$$M_\perp = \sqrt{(-2.254)^2 + (6.000)^2 + (-2.704)^2} = 6.956 \text{ in.}\cdot\text{kip}$$

$\theta_x = \cos^{-1}\dfrac{-2.254}{6.956} = 108.91^\circ \cong 108.9^\circ$ Ans.

$\theta_y = \cos^{-1}\dfrac{6.000}{6.956} = 30.39^\circ \cong 30.4^\circ$ Ans.

$\theta_z = \cos^{-1}\dfrac{-2.704}{6.956} = 112.88^\circ \cong 112.9^\circ$ Ans.

4-80 The magnitude of force $\bar{F}$ in Fig. P4-80 is 976 N. Determine
(a) The component of the moment at point C parallel to line CE.
(b) The component of the moment at point C perpendicular to line CE and the direction angles associated with this moment vector.

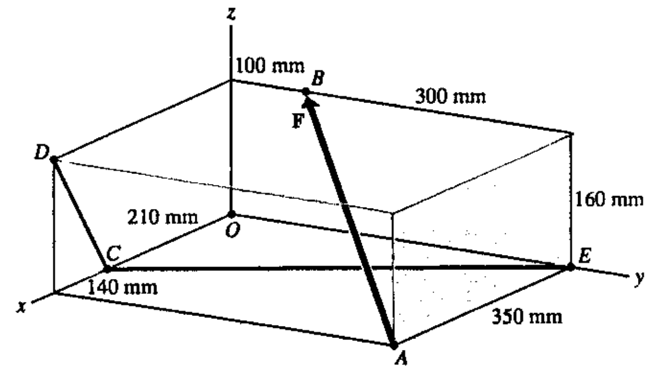


Fig. P4-80

SOLUTION

(a) $$\bar{F} = 976\left[\frac{-350\,\hat{i} - 300\,\hat{j} + 160\,\hat{k}}{\sqrt{(-350)^2 + (-300)^2 + (160)^2}}\right] = -700\,\hat{i} - 600\,\hat{j} + 320\,\hat{k} \text{ N}$$

$$\bar{r}_{A/C} = 140\,\hat{i} + 400\,\hat{j} \text{ mm}$$

$$\bar{M}_C = \bar{r}_{A/C} \times \bar{F} = (140\,\hat{i} + 400\,\hat{j}) \times (-700\,\hat{i} - 600\,\hat{j} + 320\,\hat{k})$$
$$= 128{,}000\,\hat{i} - 44{,}800\,\hat{j} + 196{,}000\,\hat{k} \text{ N·mm}$$
$$= 128.00\,\hat{i} - 44.80\,\hat{j} + 196.00\,\hat{k} \text{ N·m}$$

$$\hat{e}_{CE} = \frac{-210\,\hat{i} + 400\,\hat{j}}{\sqrt{(-210)^2 + (400)^2}} = -0.4648\,\hat{i} + 0.8854\,\hat{j}$$

$$M_{CE} = \bar{M}_C \cdot \hat{e}_{CE} = (128.00\,\hat{i} - 44.80\,\hat{j} + 196.00\,\hat{k}) \cdot (-0.4648\,\hat{i} + 0.8854\,\hat{j})$$
$$= -99.16 \text{ N·m} \cong -99.2 \text{ N·m}$$

$$\bar{M}_{CE} = M_{CE}\hat{e}_{CE} = -99.16(-0.4648\,\hat{i} + 0.8854\,\hat{j})$$
$$= 46.09\,\hat{i} - 87.80\,\hat{j} \text{ N·m}$$ Ans.

(b) $$\bar{M}_{\perp} = \bar{M}_C - \bar{M}_{CE}$$
$$= (128.00\,\hat{i} - 44.80\,\hat{j} + 196.00\,\hat{k}) - (46.09\,\hat{i} - 87.80\,\hat{j})$$
$$= 81.91\,\hat{i} + 43.00\,\hat{j} + 196.00\,\hat{k} \text{ N·m}$$

$$M_{\perp} = \sqrt{(81.91)^2 + (43.00)^2 + (196.00)^2} = 216.74 \text{ N·m} \cong 217 \text{ N·m}$$

$$\theta_x = \cos^{-1}\frac{81.91}{216.74} = 67.80^\circ = 67.8^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{43.00}{216.74} = 78.56^\circ \cong 78.6^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{196.00}{216.74} = 25.27^\circ \cong 25.3^\circ$$ Ans.

4-81* Determine the moment of the couple shown in Fig. P4-81 and the perpendicular distance between the two forces.

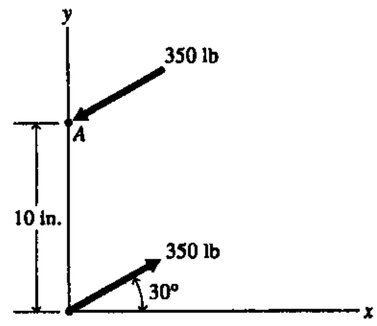


Fig. P4-81

SOLUTION

$\vec{F}_A = 350(-\cos 30^\circ\ \hat{i} - \sin 30^\circ\ \hat{j}) = -303.1\ \hat{i} - 175.0\ \hat{j}$ lb

$\vec{r}_{A/O} = 10\ \hat{j}$ in.

$\vec{M}_O = \vec{r}_{A/O} \times \vec{F}_A = (10\ \hat{j}) \times (-303.1\ \hat{i} - 175.0\ \hat{j})$

$= 3031\ \hat{k}$ in.·lb $\cong 3030\ \hat{k}$ in.·lb Ans.

$d = \dfrac{M_O}{F_A} = \dfrac{3031}{350} = 8.66$ in. Ans.

4-82* Determine the moment of the couple shown in Fig. P4-82 and the perpendicular distance between the two forces.

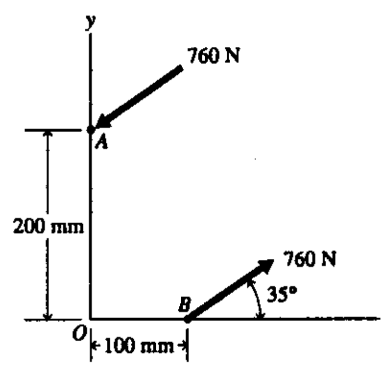


Fig. P4-82

SOLUTION

$\vec{F}_A = 760(-\cos 35^\circ\ \hat{i} - \sin 35^\circ\ \hat{j}) = -622.6\ \hat{i} - 435.9\ \hat{j}$ N

$\vec{r}_{A/B} = -0.100\ \hat{i} + 0.200\ \hat{j}$ m

$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}_A = (-0.100\ \hat{i} + 0.200\ \hat{j}) \times (-622.6\ \hat{i} - 435.9\ \hat{j})$

$= 168.11\ \hat{k}$ N·m $\cong 168.1\ \hat{k}$ N·m Ans.

$d = \dfrac{M_B}{F_A} = \dfrac{168.11}{760} = 0.221$ m $\cong 221$ mm Ans.

4-83 Two parallel forces of opposite sense $\vec{F}_1 = -70\,\hat{i} - 120\,\hat{j} - 80\,\hat{k}$ lb and $\vec{F}_2 = 70\,\hat{i} + 120\,\hat{j} + 80\,\hat{k}$ lb act at points B and A of a body as shown in Fig. P4-83. Determine the moment of the couple and the perpendicular distance between the two forces.

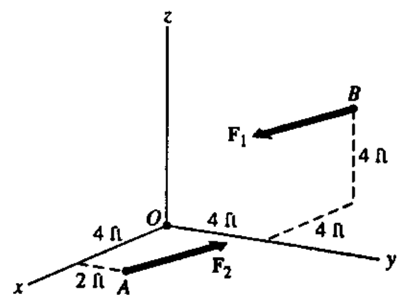


Fig. P4-83

SOLUTION

$$\vec{F}_1 = -70\,\hat{i} - 120\,\hat{j} - 80\,\hat{k}\text{ lb}$$

$$F_1 = \sqrt{(-70)^2 + (-120)^2 + (-80)^2} = 160.31\text{ lb}$$

$$\vec{r}_{B/A} = -8\,\hat{i} + 2\,\hat{j} + 4\,\hat{k}\text{ ft}$$

$$\vec{M}_A = \vec{r}_{B/A} \times \vec{F}_1 = (-8\,\hat{i} + 2\,\hat{j} + 4\,\hat{k}) \times (-70\,\hat{i} - 120\,\hat{j} - 80\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -8 & 2 & 4 \\ -70 & -120 & -80 \end{vmatrix}$$

$$= -320\,\hat{i} - 920\,\hat{j} + 1100\,\hat{k}\text{ ft·lb}$$

$$M_A = \sqrt{(-320)^2 + (-920)^2 + (1100)^2} = 1469.3\text{ ft·lb} \cong 1469\text{ ft·lb}$$ Ans.

$$d = \frac{M_A}{F_1} = \frac{1469.29}{160.31} = 9.165\text{ ft} \cong 9.17\text{ ft}$$ Ans.

4-84 Two parallel forces of opposite sense
$\vec{F}_1 = 125\,\hat{i} + 200\,\hat{j} + 250\,\hat{k}$ N and
$\vec{F}_2 = -125\,\hat{i} - 200\,\hat{j} - 250\,\hat{k}$ N
act at points A and B of a body as shown in Fig. P4-84. Determine the moment of the couple and the perpendicular distance between the two forces.

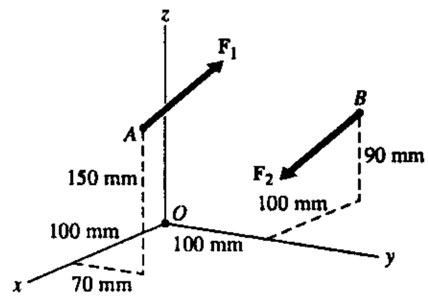


Fig. P4-84

SOLUTION

$$\vec{F}_1 = 125\,\hat{i} + 200\,\hat{j} + 250\,\hat{k} \text{ N}$$

$$F_1 = \sqrt{(125)^2 + (200)^2 + (250)^2} = 343.69 \text{ N}$$

$$\vec{r}_{A/B} = 0.200\,\hat{i} - 0.030\,\hat{j} + 0.060\,\hat{k} \text{ m}$$

$$\vec{M}_B = \vec{r}_{A/B} \times \vec{F}_1$$

$$= (0.200\,\hat{i} - 0.030\,\hat{j} + 0.060\,\hat{k}) \times (125\,\hat{i} + 200\,\hat{j} + 250\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.200 & -0.030 & 0.060 \\ 125 & 200 & 250 \end{vmatrix}$$

$$= -19.50\,\hat{i} - 42.50\,\hat{j} + 43.75\,\hat{k} \text{ N}\cdot\text{m}$$

$$M_B = \sqrt{(-19.50)^2 + (-42.50)^2 + (43.75)^2} = 64.04 \text{ N}\cdot\text{m} \cong 64.0 \text{ N}\cdot\text{m}$$ Ans.

$$d = \frac{M_B}{F_1} = \frac{64.04}{343.69} = 0.18633 \text{ m} \cong 186.3 \text{ mm}$$ Ans.

4-85* A bracket is loaded with a system of forces as shown in Fig. P4-85. Express the resultant of the force system in Cartesian vector form.

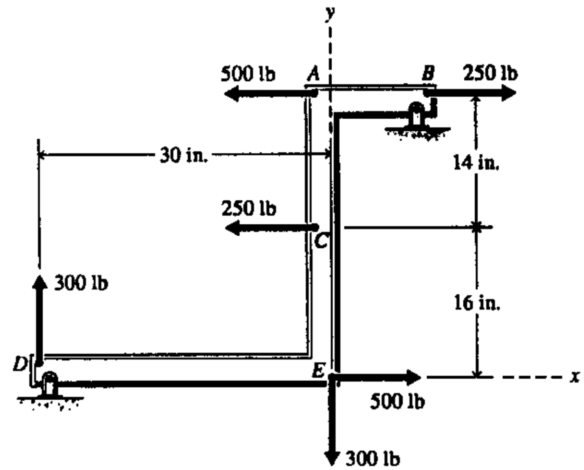


Fig. P4-85

SOLUTION

An examination of Fig. P4-85 indicates that the force system consists of a system of three couples in a plane. A scalar analysis yields:

With counterclockwise moments positive:

For forces D_y and E_y:

$$M_1 = -F_{Dy}d_1 = -300(30) = -9000 \text{ in.}\cdot\text{lb}$$

For forces C_x and B_x:

$$M_2 = -F_{Cx}d_2 = -250(14) = -3500 \text{ in.}\cdot\text{lb}$$

For forces A_x and E_x:

$$M_3 = F_{Ax}d_3 = 500(30) = 15{,}000 \text{ in.}\cdot\text{lb}$$

$$C = \Sigma M = M_1 + M_2 + M_3$$

$$= -9000 - 3500 + 15{,}000 = 2500 \text{ in.}\cdot\text{lb} = 2.50 \text{ in.}\cdot\text{kip}$$

$$\mathbf{C} = 2.50 \text{ in.}\cdot\text{kip} \circlearrowleft$$

$$= 2.50\ \mathbf{k} \text{ in.}\cdot\text{kip}$$ Ans.

4-86* A plate is loaded with a system of forces as shown in Fig. P4-86. Express the resultant of the force system in Cartesian vector form.

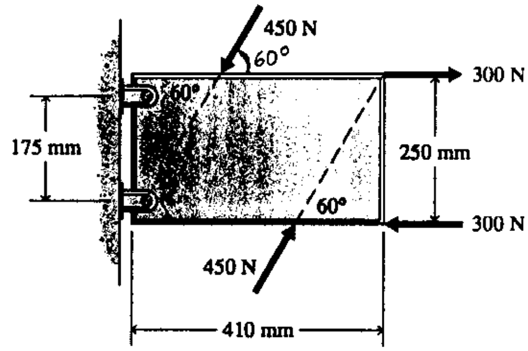


Fig. P4-86

SOLUTION

With counterclockwise moments positive:

For the 450-N forces:

$d = (410 - 250 \tan 30^\circ) \cos 30^\circ = 230.07$ mm

$M_1 = 450(0.23007) = 103.53$ N·m

$M_2 = -300(0.250) = -75.00$ N·m

$M = M_1 + M_2 = 103.53 - 75.00 = 28.53$ N·m ≅ 28.5 N·m

$\vec{C} = 28.5$ N·m ↺

$= 28.5\ \hat{k}$ N·m Ans.

4-87 A bracket is loaded with a system of forces as shown in Fig. P4-87. Express the resultant of the force system in Cartesian vector form.

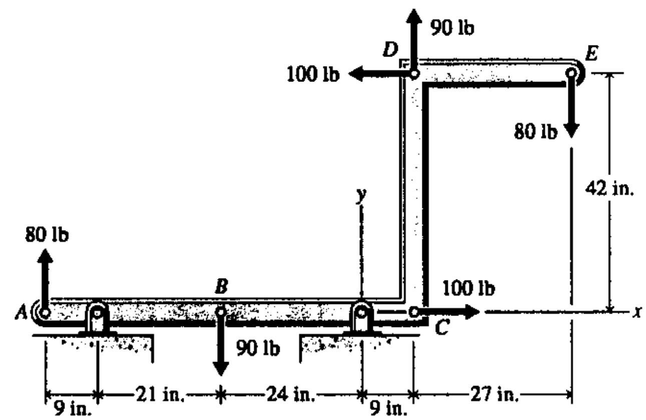


Fig. P4-87

SOLUTION

An examination of Fig. P4-87 indicates that the force system consists of a system of three couples in a plane. A scalar analysis yields:

With counterclockwise moments positive:

For forces A and E:

$$M_1 = -F_A d_1 = -80(90) = -7200 \text{ in.}\cdot\text{lb}$$

For forces B and D_y:

$$M_2 = F_B d_2 = 90(33) = 2970 \text{ in.}\cdot\text{lb}$$

For forces C and D_x:

$$M_3 = F_C d_3 = 100(42) = 4200 \text{ in.}\cdot\text{lb}$$

$$C = \Sigma M = M_1 + M_2 + M_3$$

$$= -7200 + 2970 + 4200 = -30 \text{ in.}\cdot\text{lb}$$

$$\vec{C} = 30 \text{ in.}\cdot\text{lb} \circlearrowright$$

$$= -30\,\hat{k} \text{ in.}\cdot\text{lb}$$ Ans.

4-88 A plate is loaded with a system of forces as shown in Fig. P4-88. Express the resultant of the force system in Cartesian vector form.

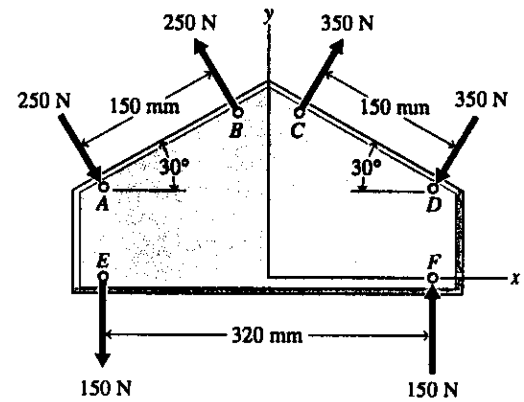


Fig. P4-88

SOLUTION

An examination of Fig. P4-88 indicates that the force system consists of a system of three couples in a plane. A scalar analysis yields:

With counterclockwise moments positive:

For forces A and B:

$$M_1 = F_A d_1 = 250(0.150) = 37.50 \text{ N·m}$$

For forces C and D:

$$M_2 = -F_C d_2 = -350(0.150) = -52.50 \text{ N·m}$$

For forces E and F:

$$M_3 = F_E d_3 = 150(0.320) = 48.00 \text{ N·m}$$

$$C = \Sigma M = M_1 + M_2 + M_3$$

$$= 37.50 - 52.50 + 48.00 = 33.00 \text{ N·m} = 33.0 \text{ N·m}$$

$$\vec{C} = 33.0 \text{ N·m} \circlearrowleft$$

$$= 33.0\, \hat{k} \text{ N·m}$$ Ans.

4-89* Determine the total moment of the two couples shown in Fig. P4-89 and the direction angles associated with the total moment vector.

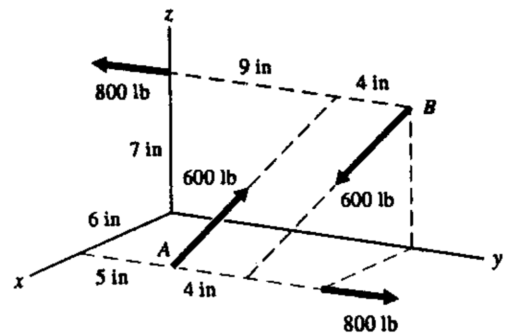


Fig. P4-89

SOLUTION

For the 600-lb forces:

$$F_1 = 600\left[\frac{6\,\hat{i} - 4\,\hat{j} - 7\,\hat{k}}{\sqrt{(6)^2 + (-4)^2 + (-7)^2}}\right] = 358.2\,\hat{i} - 238.8\,\hat{j} - 417.9\,\hat{k} \text{ lb}$$

$$\vec{r}_1 = 4\,\hat{j}$$

$$\vec{M}_1 = \vec{r}_1 \times \vec{F}_1 = (4\,\hat{j}) \times (358.2\,\hat{i} - 238.8\,\hat{j} - 417.9\,\hat{k})$$
$$= -1671.6\,\hat{i} - 1432.8\,\hat{k} \text{ in.·lb}$$

For the 800 lb forces:

$$\vec{F}_2 = -800\,\hat{j} \text{ lb}$$

$$\vec{r}_2 = -6\,\hat{i} + 7\,\hat{k} \text{ in.}$$

$$\vec{M}_2 = \vec{r}_2 \times \vec{F}_2 = (-6\,\hat{i} + 7\,\hat{k}) \times (-800\,\hat{j}) = 5600\,\hat{i} + 4800\,\hat{k} \text{ in.·lb}$$

$$\vec{C} = \vec{M}_1 + \vec{M}_2 = (-1671.6\,\hat{i} - 1432.8\,\hat{k}) + (5600\,\hat{i} + 4800\,\hat{k})$$
$$= 3928.4\,\hat{i} + 3367.2\,\hat{k} \text{ in.·lb} \qquad \text{Ans.}$$

$$C = |\vec{C}| = \sqrt{(3928.4)^2 + (3367.2)^2} = 5174.0 \text{ in.·lb}$$

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{3928.4}{5174.0} = 40.6^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{0}{5174.0} = 90.0^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{3367.2}{5174.0} = 49.4^\circ \qquad \text{Ans.}$$

4-90* Three couples are applied to a rectangular block as shown in Fig. P4-90. Determine the magnitude of the resultant couple $\vec{C}$ and the direction angles associated with the resultant couple vector.

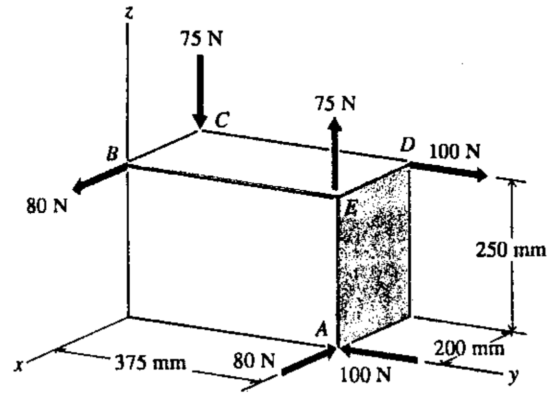


Fig. P4-90

SOLUTION

$\vec{F}_B = 80\ \hat{i}$ N $\vec{r}_{B/A} = -0.375\ \hat{j} + 0.250\ \hat{k}$ m

$\vec{F}_C = -75\ \hat{k}$ N $\vec{r}_{C/A} = -0.200\ \hat{i} - 0.375\ \hat{j} + 0.250\ \hat{k}$ m

$\vec{F}_D = 100\ \hat{j}$ N $\vec{r}_{D/A} = -0.200\ \hat{i} + 0.250\ \hat{k}$ m

Summing moments about point A yields:

$$\vec{C} = (\vec{r}_{B/A} \times \vec{F}_B) + (\vec{r}_{C/A} \times \vec{F}_C) + (\vec{r}_{D/A} \times \vec{F}_D)$$

$$= (-0.375\ \hat{j} + 0.250\ \hat{k}) \times (80\ \hat{i})$$

$$+ (-0.200\ \hat{i} - 0.375\ \hat{j} + 0.250\ \hat{k}) \times (-75\ \hat{k})$$

$$+ (-0.200\ \hat{i} + 0.250\ \hat{k}] \times [100\ \hat{j})$$

$$= 3.125\ \hat{i} + 5.000\ \hat{j} + 10.000\ \hat{k}\ \text{N}\cdot\text{m}$$

$$C = \sqrt{(3.125)^2 + (5.000)^2 + (10.000)^2} = 11.609\ \text{N}\cdot\text{m} \cong 11.61\ \text{N}\cdot\text{m}$$ Ans.

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{3.125}{11.609} = 74.4^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{5.000}{11.609} = 64.5^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{10.000}{11.609} = 30.5^\circ$$ Ans.

4-91 Three couples are applied to a bent bar as shown in Fig. P4-91. Determine the magnitude of the resultant couple $\vec{C}$ and the direction angles associated with the resultant couple vector.

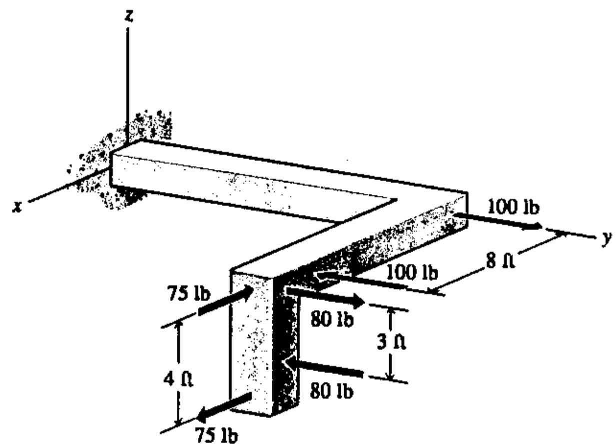


Fig. P4-91

SOLUTION

The scalar components of the resultant couple $\vec{C}$ are:

$$C_x = -80(3) = -240 \text{ ft·lb}$$

$$C_y = -75(4) = -300 \text{ ft·lb}$$

$$C_z = -100(8) = -800 \text{ ft·lb}$$

The resultant couple $\vec{C}$ expressed in Cartesian vector form is:

$$\vec{C} = -240\ \hat{i} - 300\ \hat{j} - 800\ \hat{k}$$

$$C = \sqrt{(-240)^2 + (-300)^2 + (-800)^2}$$

$$= 887.5 \text{ ft·lb} \cong 887 \text{ ft·lb}$$ Ans.

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{-240}{887.5} = 105.7°$$ Ans.

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{-300}{887.5} = 109.8°$$ Ans.

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{-800}{887.5} = 154.3°$$ Ans.

4-92 The input and output torques from a gear box are shown in Fig. P4-92. Determine the magnitude and direction of the resultant torque.

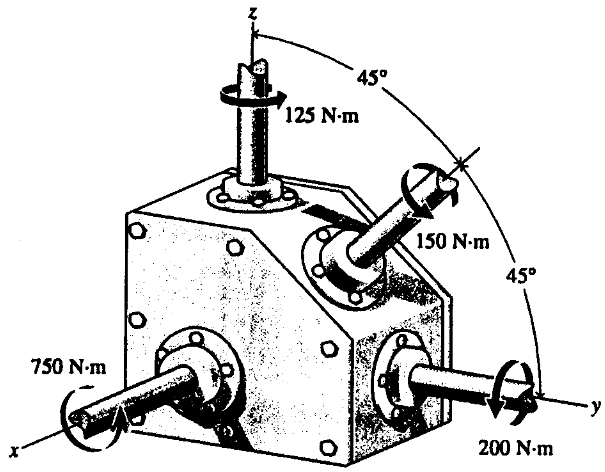


Fig. P4-92

SOLUTION

$$M_x = 750 \text{ N·m}$$

$$M_y = 200 + 150 \cos 45^\circ = 306.1 \text{ N·m}$$

$$M_z = 125 + 150 \cos 45^\circ = 231.1 \text{ N·m}$$

$$\bar{C} = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

$$= 750 \hat{i} + 306.1 \hat{j} + 231.1 \hat{k} \text{ N·m}$$

$$C = \sqrt{(750.0)^2 + (306.1)^2 + (231.1)^2}$$

$$= 842.4 \text{ N·m} \cong 842 \text{ N·m}$$ Ans.

$$\theta_x = \cos^{-1} \frac{M_x}{M} = \cos^{-1} \frac{750.0}{842.4} = 27.1^\circ$$ Ans.

$$\theta_y = \cos^{-1} \frac{M_y}{M} = \cos^{-1} \frac{306.1}{842.4} = 68.7^\circ$$ Ans.

$$\theta_z = \cos^{-1} \frac{M_z}{M} = \cos^{-1} \frac{231.1}{842.4} = 74.1^\circ$$ Ans.

4-93* Three couples are applied to a bent bar as shown in Fig. P4-93. Determine
(a) The magnitude of the resultant couple $\vec{C}$ and the direction angles associated with the resultant couple vector.
(b) The scalar component of the resultant couple $\vec{C}$ about line OA.

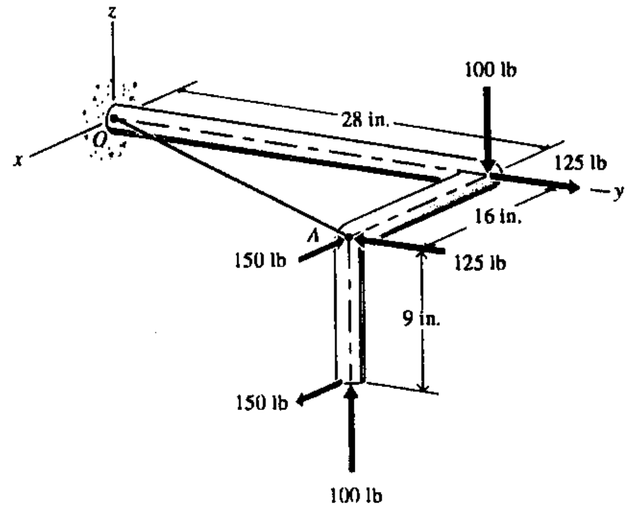


Fig. P4-93

SOLUTION

(a) For the 125-lb forces:

$$C_z = -125(16) = -2000 \text{ in.}\cdot\text{lb}$$

For the 100-lb forces:

$$C_y = -100(16) = -1600 \text{ in.}\cdot\text{lb}$$

For the 150-lb forces:

$$C_y = -150(9) = -1350 \text{ in.}\cdot\text{lb}$$

$$\vec{C} = -2950\ \hat{j} - 2000\ \hat{k} \text{ in.}\cdot\text{lb}$$

$$C = |\vec{C}| = \sqrt{(-2950)^2 + (-2000)^2} = 3564 \text{ in.}\cdot\text{lb} \cong 3.56 \text{ in.}\cdot\text{kip} \qquad \text{Ans.}$$

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{0}{3564} = 90.0^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{-2950}{3564} = 145.87^\circ \cong 145.9^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{-2000}{3564} = 124.14^\circ \cong 124.1^\circ \qquad \text{Ans.}$$

(b) $$\hat{e}_{OA} = \frac{16\ \hat{i} + 28\ \hat{j}}{\sqrt{(16)^2 + (28)^2}} = 0.4961\ \hat{i} + 0.8682\ \hat{j}$$

$$C_{OA} = \vec{C}\cdot\hat{e}_{OA} = (-2950\ \hat{j} - 2000\ \hat{k})\cdot(0.4961\ \hat{i} + 0.8682\ \hat{j})$$

$$= -2561 \text{ in.}\cdot\text{lb} \cong -2.56 \text{ in.}\cdot\text{kip} \qquad \text{Ans.}$$

4-94* Three couples are applied to a bent bar as shown in Fig. P4-94. Determine
(a) The magnitude of the resultant couple $\vec{C}$ and the direction angles associated with the resultant couple vector.
(b) The scalar component of the resultant couple $\vec{C}$ about line OA.

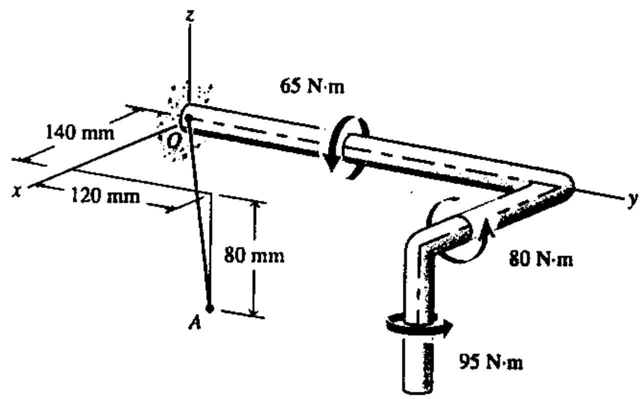


Fig. P4-94

SOLUTION

(a) $\vec{C} = 80\,\hat{i} + 65\,\hat{j} + 95\,\hat{k}$

$$C = |\vec{C}| = \sqrt{(80)^2 + (65)^2 + (95)^2} = 140.18 \text{ N·m}$$

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{80}{140.18} = 55.2^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{65}{140.18} = 62.4^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{95}{140.18} = 47.3^\circ$$ Ans.

(b) $$\hat{e}_{OA} = \frac{140\,\hat{i} + 120\,\hat{j} - 80\,\hat{k}}{\sqrt{(140)^2 + (120)^2 + (-80)^2}} = 0.6965\,\hat{i} + 0.5970\,\hat{j} - 0.3980\,\hat{k}$$

$$C_{OA} = \vec{C}\cdot\hat{e}_{OA} = (80\,\hat{i} + 65\,\hat{j} + 95\,\hat{k})\cdot(0.6965\,\hat{i} + 0.5970\,\hat{j} - 0.3980\,\hat{k})$$

$$= 56.72 \text{ N·m} \cong 56.7 \text{ N·m}$$ Ans.

4-95 Three couples are applied to a rectangular block as shown in Fig. P4-95. Determine
(a) The magnitude of the resultant couple $\vec{C}$ and the direction angles associated with the resultant couple vector.
(b) The scalar component of the resultant couple $\vec{C}$ about line OA.

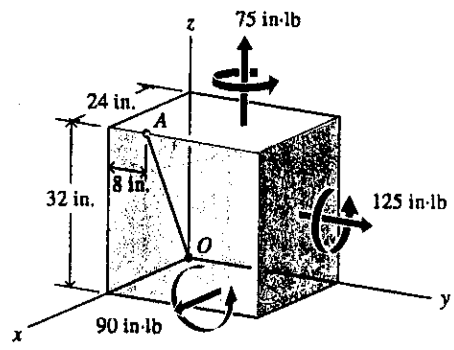


Fig. P4-95

SOLUTION

(a) $\vec{C} = 90\ \hat{i} + 125\ \hat{j} + 75\ \hat{k}$ in.·lb

$$C = \sqrt{(90)^2 + (125)^2 + (75)^2} = 171.32 \text{ in.·lb}$$

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{90}{171.32} = 58.3^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{125}{171.32} = 43.1^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{75}{171.32} = 64.0^\circ$$ Ans.

(b) $$\hat{e}_{OA} = \frac{24\ \hat{i} + 8\ \hat{j} + 32\ \hat{k}}{\sqrt{(24)^2 + (8)^2 + (32)^2}} = 0.5883\ \hat{i} + 0.1961\ \hat{j} + 0.7845\ \hat{k}$$

$$C_{OA} = \vec{C}\cdot\hat{e}_{OA}$$

$$= (90\ \hat{i} + 125\ \hat{j} + 75\ \hat{k})\cdot(0.5883\ \hat{i} + 0.1961\ \hat{j} + 0.7845\ \hat{k})$$

$$= 136.36 \text{ in.·lb} \cong 136.4 \text{ in.·lb}$$ Ans.

4-96 Two couples are applied to a rectangular block as shown in Fig. P4-96. Determine
(a) The magnitude of the resultant couple $\vec{C}$ and the direction angles associated with the resultant couple vector.
(b) The scalar component of the resultant couple $\vec{C}$ about line AB.

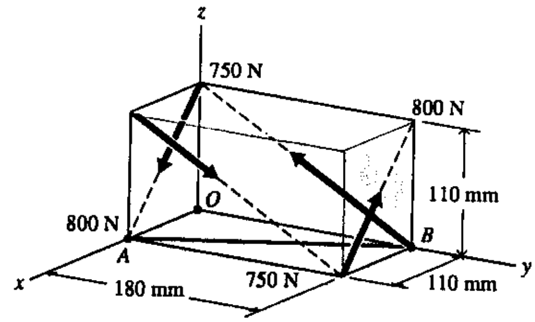


Fig. P4-96

SOLUTION

(a) $\vec{F}_1 = 800(-\cos 45^\circ\ \hat{i} + \sin 45^\circ\ \hat{k}) = -565.7\ \hat{i} + 565.7\ \hat{k}$ N

$$\vec{F}_2 = 750\left[\frac{180\ \hat{j} - 110\ \hat{k}}{\sqrt{(180)^2 + (-110)^2}}\right] = 640.0\ \hat{j} - 391.1\ \hat{k} \text{ N}$$

$$\vec{C} = (\vec{r}_1 \times \vec{F}_1) + (\vec{r}_2 \times \vec{F}_2)$$

$$= (0.180\ \hat{j}) \times (-565.7\ \hat{i} + 565.7\ \hat{k}) + (0.110\ \hat{i}) \times (640.0\ \hat{j} - 391.1\ \hat{k})$$

$$= 101.83\ \hat{i} + 43.02\ \hat{j} + 172.23\ \hat{k} \text{ N·m}$$

$$C = \sqrt{(101.83)^2 + (43.02)^2 + (172.23)^2} = 204.65 \text{ N·m}$$ Ans.

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{101.83}{204.65} = 60.2^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{43.02}{204.65} = 77.9^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{172.23}{204.65} = 32.7^\circ$$ Ans.

(b) $$\hat{e}_{AB} = \frac{-110\ \hat{i} + 180\ \hat{j}}{\sqrt{(-110)^2 + (180)^2}} = -0.5215\ \hat{i} + 0.8533\ \hat{j}$$

$$C_{AB} = \vec{C}\cdot\hat{e}_{AB}$$

$$= (101.83\ \hat{i} + 43.02\ \hat{j} + 172.23\ \hat{k})\cdot(-0.5215\ \hat{i} + 0.8533\ \hat{j})$$

$$= -16.395 \text{ N·m} \cong -16.40 \text{ N·m}$$ Ans.

4-97* Replace the 50-lb force shown in Fig. P4-97 by a force at point A and a couple. Express your answer in Cartesian vector form.

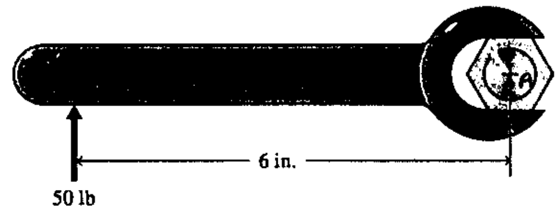


Fig. P4-97

SOLUTION

$\vec{F}_A = \vec{F} = 50\ \hat{j}$ lb   Ans.

$C_A = Fd = 50(6) = 300$ in.·lb

$\vec{C}_A = 300$ in.·lb ↻ $= -300\ \hat{k}$ in.·lb   Ans.

4-98* Replace the 300-N force shown in Fig. P4-98 by a force at point B and a couple. Express your answer in Cartesian vector form.

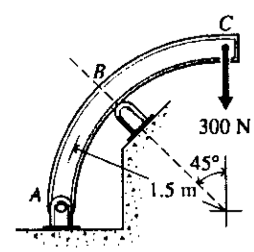


Fig. P4-98

SOLUTION

$\vec{F}_B = \vec{F} = -300\ \hat{j}$ N   Ans.

$C_B = Fd = 300(1.5 \sin 45^\circ) = 318.2$ N·m

$\vec{C}_B = 318$ N·m ↻ $= -318\ \hat{k}$ N·m   Ans.

4-99 Replace the 275-lb force shown in Fig. P4-99 by a force at point A and a couple. Express your answer in Cartesian vector form.

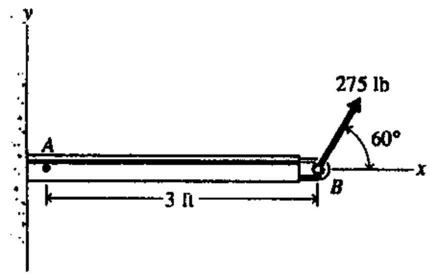


Fig. P4-99

SOLUTION

$\vec{F}_A = \vec{F} = 275[\cos 60^\circ\ \hat{i} + \sin 60^\circ\ \hat{j}]$

$= 137.5\ \hat{i} + 238.2\ \hat{j}$ lb $\cong 137.5\ \hat{i} + 238\ \hat{j}$ lb   Ans.

$\vec{C}_A = \vec{r} \times \vec{F} = (3\ \hat{i}) \times (137.5\ \hat{i} + 238.2\ \hat{j})$

$= 714.6\ \hat{k}$ ft·lb $\cong 715\ \hat{k}$ ft·lb   Ans.

4-100* Replace the 675-N force shown in Fig. P4-100 by a force at point B and a couple. Express your answer in Cartesian vector form.

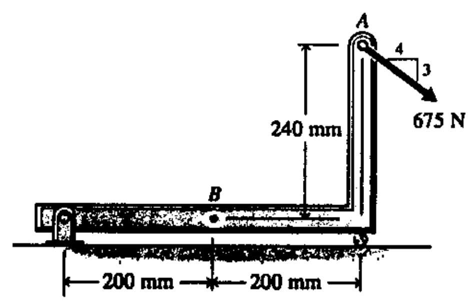


Fig. P4-100

SOLUTION

$\vec{F}_B = \vec{F} = 675(0.8\ \hat{i} - 0.6\ \hat{j}) = 540\ \hat{i} - 405\ \hat{j}$ N   Ans.

$\vec{r}_B = 0.200\ \hat{i} + 0.240\ \hat{j}$ m

$\vec{C}_B = \vec{r}_B \times \vec{F} = (0.200\ \hat{i} + 0.240\ \hat{j}) \times (540\ \hat{i} - 405\ \hat{j})$

$= -210.6\ \hat{k} \cong -211\ \hat{k}$ N·m   Ans.

4-101 Replace the 300-lb force shown in Fig. P4-101 by a force at point B and a couple. Express your answer in Cartesian vector form.

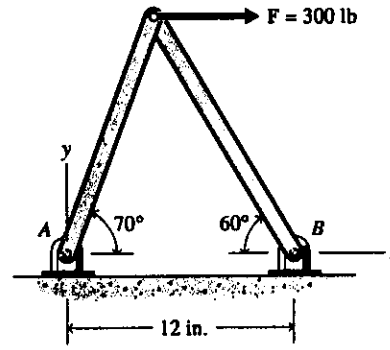


Fig. P4-101

SOLUTION

$\vec{F}_B = \vec{F} = 300\ \hat{i}$ lb = 300 lb $\longrightarrow$ Ans.

$d\ (\tan 30^\circ + \tan 20^\circ) = 12$ in. $d = 12.748$ in.

$C_B = Fd = 300(12.748) = 3824$ in.·lb

$\vec{C}_B = 3824$ in.·lb ↻ $= -3824\ \hat{k}$ in.·lb $\cong -3.82\ \hat{k}$ in.·kip Ans.

4-102 Replace the 600-N force shown in Fig. P4-102 by a force at point A and a couple. Express your answer in Cartesian vector form.

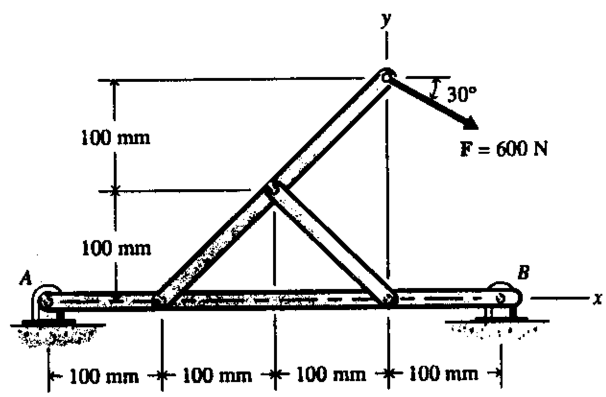


Fig. P4-102

SOLUTION

$\vec{F}_A = \vec{F} = 600(\cos 30^\circ\ \hat{i} - \sin 30^\circ\ \hat{j})$
$= 519.6\ \hat{i} - 300\ \hat{j}$ N $\cong 520\ \hat{i} - 300\ \hat{j}$ N Ans.

$\vec{C}_A = \vec{r} \times \vec{F}$
$= (0.300\ \hat{i} + 0.200\ \hat{j}) \times (519.6\ \hat{i} - 300\ \hat{j})$
$= -193.92\ \hat{k}$ N·m $\cong -193.9\ \hat{k}$ N·m Ans.

4-103* Replace the 900-lb force shown in Fig. P4-103 by a force at point B and a couple. Express your answer in Cartesian vector form.

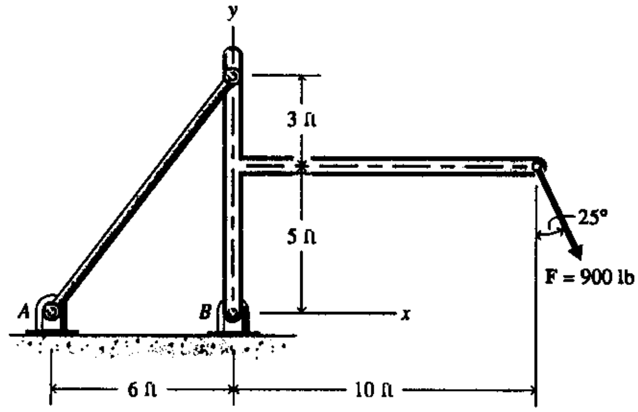


Fig. P4-103

SOLUTION

$\vec{F}_B = \vec{F} = 900(\sin 25^\circ\ \hat{i} - \cos 25^\circ\ \hat{j})$

$= 380.4\ \hat{i} - 815.7\ \hat{j}$ lb $= 380\ \hat{i} - 816\ \hat{j}$ lb   Ans.

$\vec{C}_B = \vec{r} \times \vec{F}$

$= (10\ \hat{i} + 5\ \hat{j}) \times (380.4\ \hat{i} - 815.7\ \hat{j})$

$= -10{,}059\ \hat{k}$ ft·lb $\cong -10.06\ \hat{k}$ ft·kip   Ans.

4-104 Replace the 350-N force shown in Fig. P4-104 by a force at point B and a couple. Express your answer in Cartesian vector form.

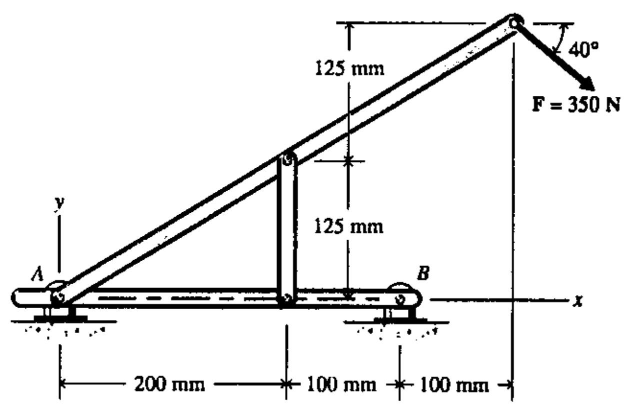


Fig. P4-104

SOLUTION

$\vec{F}_B = \vec{F} = 350(\cos 40^\circ\ \hat{i} - \sin 40^\circ\ \hat{j})$

$= 268.1\ \hat{i} - 225.0\ \hat{j}$ N $\cong 268\ \hat{i} - 225\ \hat{j}$ N   Ans.

$\vec{C}_B = \vec{r} \times \vec{F}$

$= (0.100\ \hat{i} + 0.250\ \hat{j}) \times (268.1\ \hat{i} - 225.0\ \hat{j})$

$= -89.53\ \hat{k}$ N·m $\cong -89.5\ \hat{k}$ N·m   Ans.

4-105* Replace the 50-lb force shown in Fig. P4-105 by a force at point B and a couple. Express your answer in Cartesian vector form.

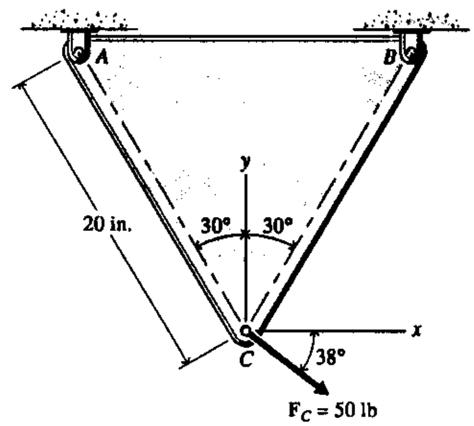


Fig. P4-105

SOLUTION

$\vec{F}_B = \vec{F} = 50(\cos 38^\circ\ \hat{i} - \sin 38^\circ\ \hat{j})$

$= 39.40\ \hat{i} - 30.78\ \hat{j}$ lb $\cong 39.4\ \hat{i} - 30.8\ \hat{j}$ lb — Ans.

$\vec{r} = -20 \sin 30^\circ\ \hat{i} - 20 \cos 30^\circ\ \hat{j} = -10.000\ \hat{i} - 17.321\ \hat{j}$

$\vec{C}_B = \vec{r} \times \vec{F} = (-10.000\ \hat{i} - 17.321\ \hat{j}) \times (39.40\ \hat{i} - 30.78\ \hat{j})$

$= 990.2\ \hat{k}$ in.·lb $\cong 990\ \hat{k}$ in.·lb — Ans.

4-106* Replace the 300-N force shown in Fig. P4-106 by a force at point B and a couple. Express your answer in Cartesian vector form.

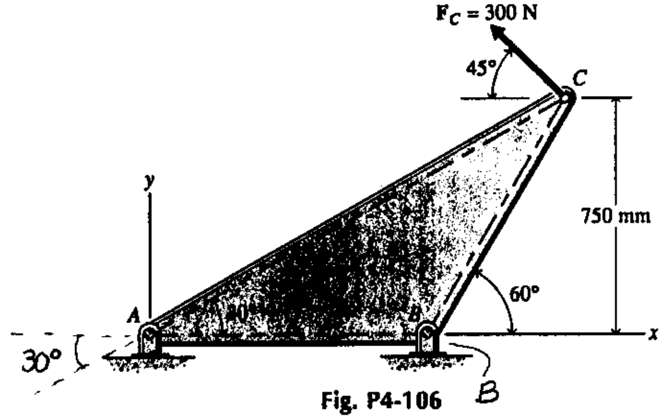


Fig. P4-106

SOLUTION

$\vec{F}_B = \vec{F} = 300(-\cos 45^\circ\ \hat{i} + \sin 45^\circ\ \hat{j})$

$= -212.1\ \hat{i} + 212.1\ \hat{j}$ N $\cong -212\ \hat{i} + 212\ \hat{j}$ N — Ans.

$\vec{r} = 0.750 \tan 30^\circ\ \hat{i} + 0.750\ \hat{j} = 0.4330\ \hat{i} + 0.750\ \hat{j}$

$\vec{C}_B = \vec{r} \times \vec{F} = (0.4330\ \hat{i} + 0.750\ \hat{j}) \times (-212.1\ \hat{i} + 212.1\ \hat{j})$

$= 250.9\ \hat{k}$ N·m $\cong 251\ \hat{k}$ N·m — Ans.

4-107 Replace the 300-lb force shown in Fig. P4-107 by a force at point A and a couple. Express your answer in Cartesian vector form.

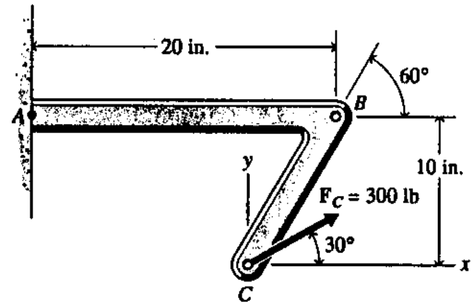


Fig. P4-107

SOLUTION

$\vec{F}_A = \vec{F} = 300(\cos 30^\circ\ \hat{\imath} + \sin 30^\circ\ \hat{\jmath})$

$= 259.8\ \hat{\imath} + 150.0\ \hat{\jmath}$ lb $\cong 260\ \hat{\imath} + 150.0\ \hat{\jmath}$ lb Ans.

$\vec{r} = (20 - 10 \tan 30^\circ)\ \hat{\imath} - 10\ \hat{\jmath}$ in. $= 14.226\ \hat{\imath} - 10\ \hat{\jmath}$ in.

$\vec{C}_A = \vec{r} \times \vec{F} = (14.226\ \hat{\imath} - 10\ \hat{\jmath}) \times (259.8\ \hat{\imath} + 150.0\ \hat{\jmath})$

$= 4732\ \hat{k}$ in.·lb $\cong 4.73\ \hat{k}$ in.·kip Ans.

4-108 Replace the 450-N force shown in Fig. P4-108 by a force at point A and a couple. Express your answer in Cartesian vector form.

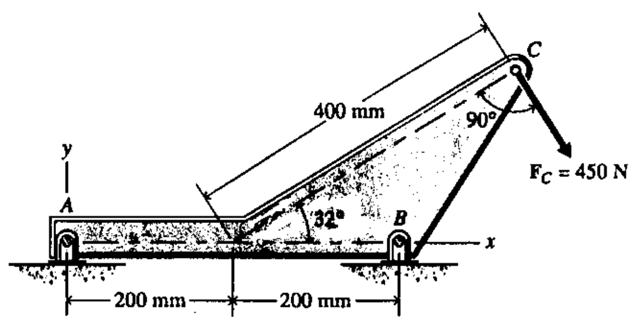


Fig. P4-108

SOLUTION

$\vec{F}_A = \vec{F} = 450(\cos 58^\circ\ \hat{\imath} - \sin 58^\circ\ \hat{\jmath})$

$= 238.5\ \hat{\imath} - 381.6\ \hat{\jmath}$ N $\cong 239\ \hat{\imath} - 382\ \hat{\jmath}$ N Ans.

$\vec{r} = (0.200 + 0.400 \cos 32^\circ)\ \hat{\imath} + 0.400 \sin 32^\circ\ \hat{\jmath}$

$= 0.5392\ \hat{\imath} + 0.2120\ \hat{\jmath}$ m

$\vec{C}_A = \vec{r} \times \vec{F} = (0.5392\ \hat{\imath} + 0.2120\ \hat{\jmath}) \times (238.5\ \hat{\imath} - 381.6\ \hat{\jmath})$

$= -256.3\ \hat{k}$ N·m $\cong -256\ \hat{k}$ N·m Ans.

4-109* The force $\bar{F}$ shown in Fig. P4-109 has a magnitude of 580 lb. Replace the force $\bar{F}$ by a force $\bar{F}_B$ at point B and a couple $\bar{C}$.
(a) Express the force $\bar{F}_B$ and the couple $\bar{C}$ in Cartesian vector form.
(b) Determine the direction angles associated with the couple vector.

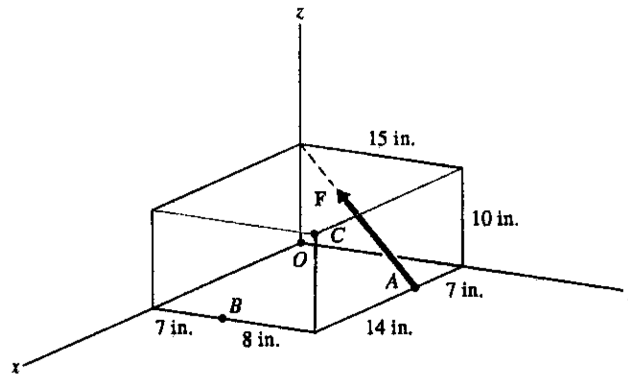


Fig. P4-109

SOLUTION

(a) $$\bar{F}_B = \bar{F} = 580\left[\frac{-7\,\hat{i} - 15\,\hat{j} + 10\,\hat{k}}{\sqrt{(-7)^2 + (-15)^2 + (10)^2}}\right]$$

$$= -209.9\,\hat{i} - 449.9\,\hat{j} + 299.9\,\hat{k} \text{ lb}$$ Ans.

$$\bar{r}_{A/B} = -14\,\hat{i} + 8\,\hat{j} \text{ in.}$$

$$\bar{C} = \bar{M}_B = \bar{r}_{A/B} \times \bar{F} = (-14\,\hat{i} + 8\,\hat{j}) \times (-209.9\,\hat{i} - 449.9\,\hat{j} + 299.9\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -14 & 8 & 0 \\ -209.9 & -449.9 & 299.9 \end{vmatrix}$$

$$= 2399.2\,\hat{i} + 4198.6\,\hat{j} + 7977.8\,\hat{k} \text{ in.}\cdot\text{lb}$$

$$\cong 2.40\,\hat{i} + 4.20\,\hat{j} + 7.98\,\hat{k} \text{ in.}\cdot\text{kip}$$ Ans.

(b) $$C = |\bar{M}_B| = \sqrt{(2399.2)^2 + (4198.6)^2 + (7977.8)^2} = 9329.0 \text{ in.}\cdot\text{lb}$$

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{2399.2}{9329.0} = 75.1^\circ$$ Ans.

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{4198.6}{9329.0} = 63.2^\circ$$ Ans.

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{7977.8}{9329.0} = 31.2^\circ$$ Ans.

4-110* The force $\vec{F}$ shown in Fig. P4-110 has a magnitude of 494 N. Replace the force $\vec{F}$ by a force $\vec{F}_C$ at point C and a couple $\vec{C}$.
(a) Express the force $\vec{F}_C$ and the couple $\vec{C}$ in Cartesian vector form.
(b) Determine the direction angles associated with the couple vector.

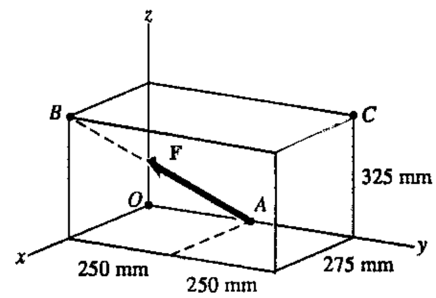


Fig. P4-110

SOLUTION

(a) $$\vec{F}_C = \vec{F} = 494\left[\frac{275\,\hat{i} - 250\,\hat{j} + 325\,\hat{k}}{\sqrt{(275)^2 + (-250)^2 + (325)^2}}\right]$$

$$= 275.2\,\hat{i} - 250.1\,\hat{j} + 325.2\,\hat{k} \text{ N}$$

$$\cong 275\,\hat{i} - 250\,\hat{j} + 325\,\hat{k} \text{ N} \qquad \text{Ans.}$$

$$\vec{r}_{A/C} = -0.250\,\hat{j} - 0.325\,\hat{k} \text{ m}$$

$$\vec{C} = \vec{M}_C = \vec{r}_{A/C} \times \vec{F} = (-0.250\,\hat{j} - 0.325\,\hat{k}) \times (275.2\,\hat{i} - 250.1\,\hat{j} + 325.2\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -0.250 & -0.325 \\ 275.2 & -250.1 & 325.2 \end{vmatrix}$$

$$= -162.58\,\hat{i} - 89.44\,\hat{j} + 68.80\,\hat{k} \text{ N}\cdot\text{m}$$

$$= -162.6\,\hat{i} - 89.4\,\hat{j} + 68.8\,\hat{k} \text{ N}\cdot\text{m} \qquad \text{Ans.}$$

(b) $$C = |\vec{M}_C| = \sqrt{(-162.58)^2 + (-89.44)^2 + (68.80)^2} = 197.90 \text{ N}\cdot\text{m}$$

$$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{-162.58}{197.90} = 145.2^\circ \qquad \text{Ans.}$$

$$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{-89.44}{197.90} = 116.9^\circ \qquad \text{Ans.}$$

$$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{68.80}{197.90} = 69.7^\circ \qquad \text{Ans.}$$

4-111 Replace the 660-lb force shown in Fig. P4-111 by a force at point B and a couple. Express your answer in Cartesian vector form.

point B is 1" above ground

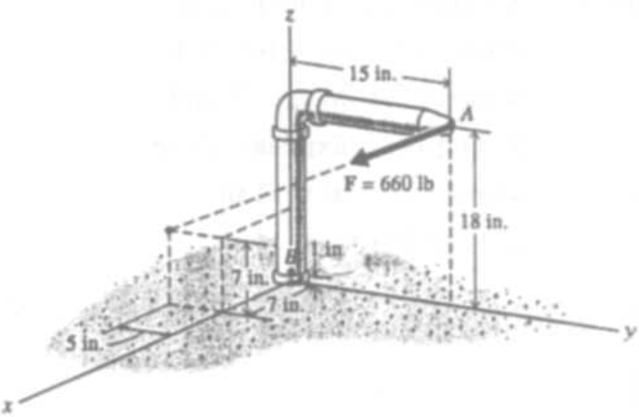


Fig. P4-111

SOLUTION

$$\bar{F}_B = \bar{F} = 660\left[\frac{7\,\hat{i} - 20\,\hat{j} - 11\,\hat{k}}{\sqrt{(7)^2 + (-20)^2 + (-11)^2}}\right]$$

$$= 193.51\,\hat{i} - 552.89\,\hat{j} - 304.09\,\hat{k}$$

$$\cong 193.5\,\hat{i} - 553\,\hat{j} - 304\,\hat{k}\ \text{lb}$$ Ans.

$$\bar{C} = \bar{M}_B = \bar{r}_{A/B} \times \bar{F} = (15\,\hat{j} + 17\,\hat{k}) \times (193.51\,\hat{i} - 552.89.1\,\hat{j} - 304.09\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 15 & 17 \\ 193.51 & -552.89 & -304.09 \end{vmatrix}$$

$$= 4838\,\hat{i} + 3290\,\hat{j} - 2903\,\hat{k}\ \text{in.}\cdot\text{lb}$$

$$\cong 4.84\,\hat{i} + 3.29\,\hat{j} - 2.90\,\hat{k}\ \text{in.}\cdot\text{kip}$$ Ans.

4-112 Replace the 500-N force shown in Fig. P4-112 by a force at hinge C and a couple. Express your answer in Cartesian vector form.

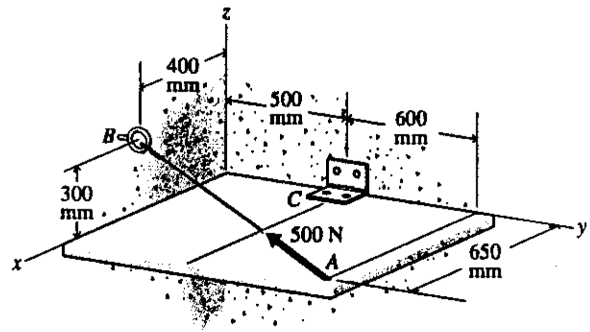


Fig. P4-112

SOLUTION

$$\bar{F}_C = \bar{F} = 500\left[\frac{-250\,\hat{i} - 1100\,\hat{j} + 300\,\hat{k}}{\sqrt{(-250)^2 + (-1100)^2 + (300)^2}}\right]$$

$$= -107.09\,\hat{i} - 471.19\,\hat{j} + 128.51\,\hat{k} \text{ N}$$

$$\cong -107.1\,\hat{i} - 471\,\hat{j} + 128.5\,\hat{k} \text{ N}$$ Ans.

$$\bar{C} = \bar{M}_C = \bar{r}_{A/C} \times \bar{F}$$

$$= (0.650\,\hat{i} + 0.600\,\hat{j}) \times (-107.09\,\hat{i} - 471.19\,\hat{j} + 128.51\,\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.650 & 0.600 & 0 \\ -107.09 & -471.19 & 128.51 \end{vmatrix}$$

$$= 77.11\,\hat{i} - 83.53\,\hat{j} - 242.02\,\hat{k} \text{ N}\cdot\text{m}$$

$$\cong 77.1\,\hat{i} - 83.5\,\hat{j} - 242\,\hat{k} \text{ N}\cdot\text{m}$$ Ans.

4-113* Determine the magnitude and direction of the resultant of the two forces shown in Fig. P4-113 and the perpendicular distance d_R from point A to the line of action of the resultant.

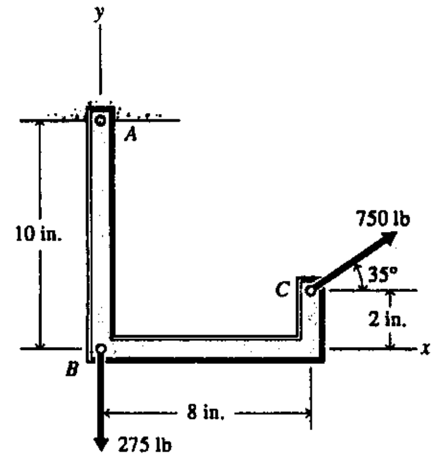


Fig. P4-113

SOLUTION

$$R_x = \Sigma F_x = 750 \cos 35^\circ = 614.36 \text{ lb}$$

$$R_y = \Sigma F_y = 750 \sin 35^\circ - 275 = 155.18 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(614.36)^2 + (155.18)^2} = 633.66 \text{ lb} \cong 634 \text{ lb}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{614.36}{633.66} = 14.177^\circ \cong 14.18^\circ$$

$$\bar{R} = 634 \text{ lb} \measuredangle 14.18^\circ$$ Ans.

$$+ \circlearrowleft M_A = 750 \cos 35^\circ (8) + 750 \sin 35^\circ (8)$$

$$= 8356.6 \text{ in.}\cdot\text{lb} = 8356.6 \text{ in.}\cdot\text{lb} \circlearrowleft$$

$$\Sigma M_A = Rd_R = 633.66 d_R = 8356.6 \text{ in.}\cdot\text{lb}$$

$$d_R = 13.188 \text{ in.} \cong 13.19 \text{ in.}$$ Ans.

4-114* Determine the magnitude and direction of the resultant of the two forces shown in Fig. P4-114 and the perpendicular distance d_R from point A to the line of action of the resultant.

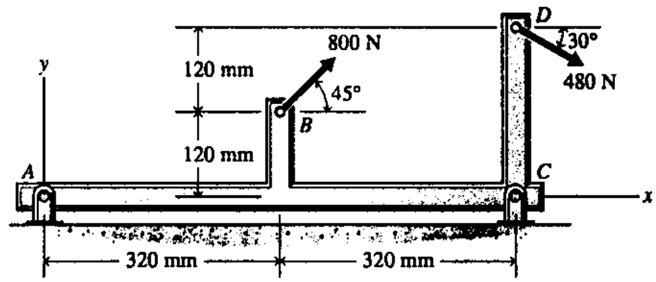


Fig. P4-114

SOLUTION

$R_x = \Sigma F_x = 800 \cos 45^\circ + 480 \cos 30^\circ = 981.38$ N

$R_y = \Sigma F_y = 800 \sin 45^\circ - 480 \sin 30^\circ = 325.69$ N

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(981.38)^2 + (325.69)^2} = 1034.01 \text{ N} \cong 1034 \text{ N}$

$\theta_x = \cos^{-1} \dfrac{R_x}{R} = \cos^{-1} \dfrac{981.38}{1034.01} = 18.359^\circ \cong 18.36^\circ$

$\mathbf{R} = 1034 \text{ N} \measuredangle 18.36^\circ$ Ans.

$+ \circlearrowleft M_A = 800 \sin 45^\circ (0.320) - 800 \cos 45^\circ (0.120)$

$- 480 \cos 30^\circ (0.240) - 480 \sin 30^\circ (0.640)$

$= -140.23 \text{ N·m} = 140.23 \text{ N·m} \circlearrowright$

$\Sigma M_A = Rd_R = 1034.0 d_R = 140.23$ N·m

$d_R = 0.13562 \text{ m} \cong 135.6 \text{ mm}$ Ans.

4-115 Replace the three forces shown in Fig. P4-115 by an equivalent force-couple system at point B.

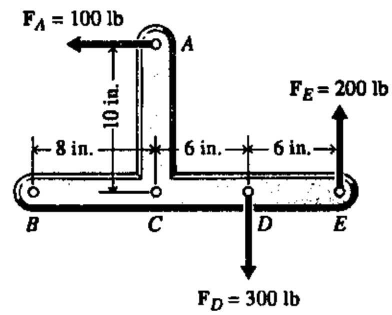


Fig. P4-115

SOLUTION

$R_x = \Sigma F_x = -100 \text{ lb}$

$R_y = \Sigma F_y = 200 - 300 = -100 \text{ lb}$

$R = \sqrt{(-100)^2 + (-100)^2} = 141.42 \text{ lb} \cong 141.4 \text{ lb}$

$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{-100}{141.42} = -135.00° = 135.0°$

$\mathbf{R} = 141.4$ lb ↙ 45.0° Ans.

$C_B = \Sigma M_B = 100(10) - 300(14) + 200(20) = 800 \text{ in.}\cdot\text{lb}$

$\mathbf{C}_B = 800$ in.·lb ↺ Ans.

4-116 Replace the three forces shown in Fig. P4-116 by an equivalent force-couple system at point O.

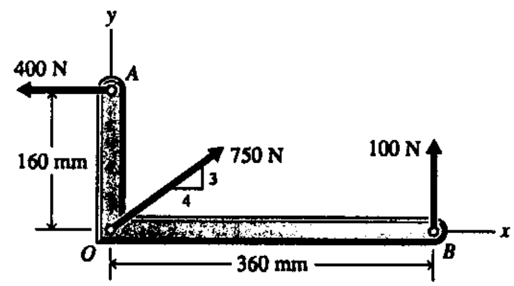


Fig. P4-116

SOLUTION

$$R_x = \Sigma F_x = -400 + \frac{4}{5}(750) = 200 \text{ N}$$

$$R_y = \Sigma F_y = \frac{3}{5}(750) + 100 = 550 \text{ N}$$

$$R = \sqrt{(200)^2 + (550)^2} = 585.2 \text{ N} \cong 585 \text{ N}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{200}{585.2} = 70.02^\circ \cong 70.0^\circ$$

$\bar{R} = 585 \text{ N} \measuredangle 70.0^\circ$ Ans.

$$C_O = \Sigma M_O = 400(0.160) + 100(0.360) = 100.0 \text{ N}\cdot\text{m}$$

$\bar{C}_O = 100.0 \text{ N}\cdot\text{m} \circlearrowleft$ Ans.

4-117* Three forces are applied to a beam as shown in Fig. P4-117. Determine the resultant $\vec{R}$ of the three forces and the location of its line of action with respect to support A.

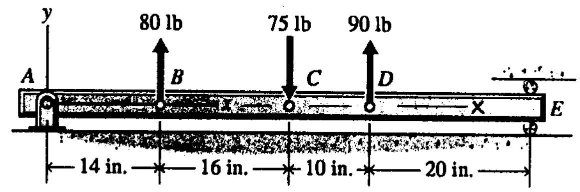


Fig. P4-117

SOLUTION

$\vec{R} = \Sigma\vec{F} = 80\,\hat{j} - 75\,\hat{j} + 90\,\hat{j} = 95\,\hat{j}$ lb = 95 lb ↑ Ans.

$+ ↺\ \Sigma M_A = 80(14) - 75(30) + 90(40) = 2470$ in.·lb = 2470 in.·lb ↺

$\Sigma M_A = Rx_R = 95x_R = 2470$ in.·lb $x_R = 26.0$ in. → Ans.

4-118* Four forces are applied to a truss as shown in Fig. P4-118. Determine the magnitude and direction of the resultant of the four forces and the perpendicular distance d_R from point A to the line of action of the resultant.

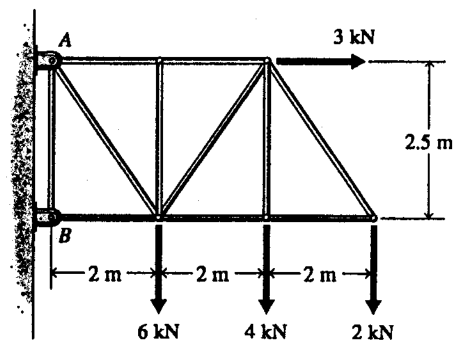


Fig. P4-118

SOLUTION

$R_x = \Sigma F_x = 3$ kN

$R_y = \Sigma F_y = -6 - 4 - 2 = -12$ kN

$R = \sqrt{(3)^2 + (-12)^2} = 12.369 \cong 12.37$ kN

$\theta_x = \cos^{-1}\dfrac{R_x}{R} = \cos^{-1}\dfrac{3}{12.369} = -75.96^\circ \cong -76.0^\circ$

$\vec{R} = 12.37$ kN ↘ 76.0° Ans.

$+ ↺\ \Sigma M_A = -6(2) - 4(4) - 2(6) = -40$ kN·m = 40 kN·m ↻

$\Sigma M_A = Rd_R = 12.369d_R = 40$ kN·m $d_R = 3.234$ m $\cong 3.23$ m Ans.

4-119* Replace the three forces shown in Fig. P4-119 by an equivalent force-couple system at point B.

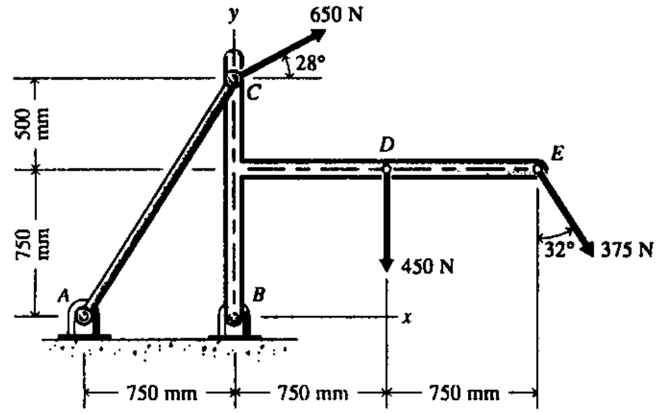


Fig. P4-119

SOLUTION

$R_x = \Sigma F_x = 650 \cos 28^\circ + 375 \sin 32^\circ = 772.64$ N

$R_y = \Sigma F_y = 650 \sin 28^\circ - 375 \cos 32^\circ - 450 = -462.86$ N

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(772.64)^2 + (-462.86)^2} = 900.67 \text{ N} \cong 901 \text{ N}$

$\theta_x = \cos^{-1} \dfrac{R_x}{R} = \cos^{-1} \dfrac{772.64}{900.67} = -30.92^\circ \cong -30.9^\circ$

$\bar{R} = 901 \text{ N} \searrow 30.9^\circ$ Ans.

$C_B = \Sigma M_B = -650 \cos 28^\circ (1.250) - 450(0.750)$

$- 375 \cos 32^\circ (1.500) - 375 \sin 32^\circ (0.750)$

$= -1680.96 \text{ N·m} \cong -1.681 \text{ kN·m}$

$\bar{C}_B \cong 1.681 \text{ kN·m}$ ↻ Ans.

4-120* Replace the three forces shown in Fig. P4-120 by an equivalent force-couple system at point A.

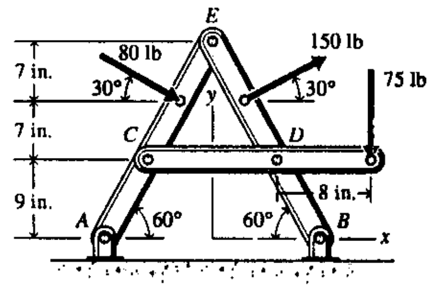


Fig. P4-120

SOLUTION

$$R_x = \Sigma F_x = 80 \cos 30^\circ + 150 \cos 30^\circ = 199.19 \text{ lb}$$

$$R_y = \Sigma F_y = -80 \sin 30^\circ + 150 \sin 30^\circ - 75 = -40.00 \text{ lb}$$

$$\vec{R} = \sqrt{R_x^2 + R_y^2} = \sqrt{(199.19)^2 + (-40.00)^2} = 203.17 \text{ lb} \cong 203 \text{ lb}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{199.19}{203.17} = -11.359^\circ \cong -11.36^\circ$$

$\vec{R} = 203$ lb ↘ 11.36° Ans.

$$C_A = \Sigma M_A = -80 \cos 30^\circ (16) - 80 \sin 30^\circ (16 \tan 30^\circ) - 150 \cos 30^\circ (16) + 150 \sin 30^\circ (30 \tan 30^\circ) - 75(8 + 37 \tan 30^\circ) = -4459.6 \text{ in.}\cdot\text{lb} \cong -4.46 \text{ in.}\cdot\text{kip}$$

$\vec{C}_A \cong 4.46$ in.·kip ↻ Ans.

4-121 Replace the three forces shown in Fig. P4-121 by an equivalent force-couple system at point A.

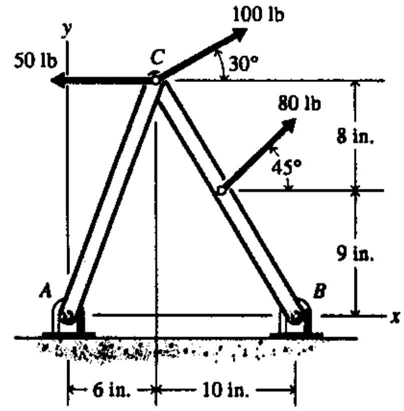


Fig. P4-121

SOLUTION

$$R_x = \Sigma F_x = -50 + 100 \cos 30^\circ + 80 \cos 45^\circ = 93.17 \text{ lb}$$

$$R_y = \Sigma F_y = 100 \sin 30^\circ + 80 \sin 45^\circ = 106.57 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(93.17)^2 + (106.57)^2} = 141.55 \cong 141.6 \text{ lb}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{93.17}{141.55} = 48.84^\circ \cong 48.8^\circ$$

$\mathbf{R} = 141.6$ lb ↗ 48.8° Ans.

$$C_A = \Sigma M_A = 50(17) + 100 \sin 30^\circ (6) - 100 \cos 30^\circ (17)$$

$$+ 80 \sin 45^\circ [6 + (8/17)(10)] - 80 \cos 45^\circ (9)$$

$$= -225.7 \text{ in.·lb} \cong -226 \text{ in.·lb}$$

$\mathbf{C}_A \cong 226$ in.·lb ↻ Ans.

4-122 Replace the three forces shown in Fig. P4-122 by an equivalent force-couple system at point C.

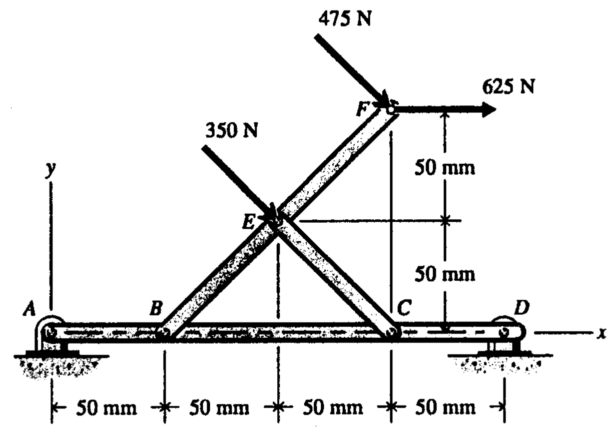


Fig. P4-122

SOLUTION

$$R_x = \Sigma F_x = 350 \cos 45^\circ + 475 \cos 45^\circ + 625 = 1208.4 \text{ N}$$

$$R_y = \Sigma F_y = -350 \sin 45^\circ - 475 \sin 45^\circ = -583.4 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1208.4)^2 + (583.4)^2} = 1341.9 \text{ N} \cong 1342 \text{ N}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{1208.4}{1341.9} = -25.77^\circ \cong -25.8^\circ$$

$\bar{R} = 1342$ N ↘ 25.8° Ans.

$$C_C = \Sigma M_C = -475 \cos 45^\circ (0.100) - 625(0.100)$$

$$= -96.09 \text{ N·m} \cong -96.1 \text{ N·m}$$

$\bar{C}_C \cong 96.1$ N·m ↻ Ans.

4-123* Replace the three forces shown in Fig. P4-123 by an equivalent force-couple system at point O.

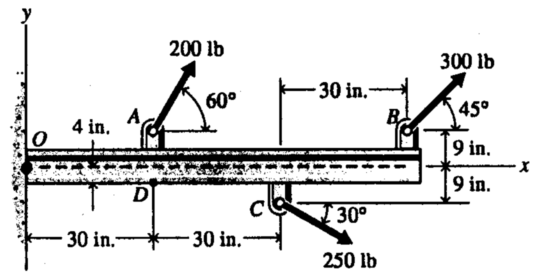


Fig. P4-123

SOLUTION

$$R_x = \Sigma F_x = 200 \cos 60^\circ + 300 \cos 45^\circ + 250 \cos 30^\circ = 528.64 \text{ lb}$$

$$R_y = \Sigma F_y = 200 \sin 60^\circ + 300 \sin 45^\circ - 250 \sin 30^\circ = 260.34 \text{ lb}$$

$$\bar{R} = \sqrt{R_x^2 + R_y^2} = \sqrt{(528.64)^2 + (260.34)^2} = 589.27 \text{ lb} \cong 589 \text{ lb}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{528.64}{589.27} = 26.22^\circ \cong 26.2^\circ$$

$$\bar{R} = 589 \text{ lb} \measuredangle 26.2^\circ$$ Ans.

$$\Sigma M_O = -200 \cos 60^\circ (9) + 200 \sin 60^\circ (30) + 250 \cos 30^\circ (9)$$

$$- 250 \sin 30^\circ (60) - 300 \cos 45^\circ (9) + 300 \sin 45^\circ (90)$$

$$= 15{,}928 \text{ in.}\cdot\text{lb} \cong 15.93 \text{ in.}\cdot\text{kip}$$

$$\bar{C}_O \cong 15.93 \text{ in.}\cdot\text{kip} \circlearrowleft$$ Ans.

4-124* Four forces are applied to a post as shown in Fig. P4-124. Determine the resultant $\mathbf{R}$ of the four forces and the location of its line of action with respect to support E.

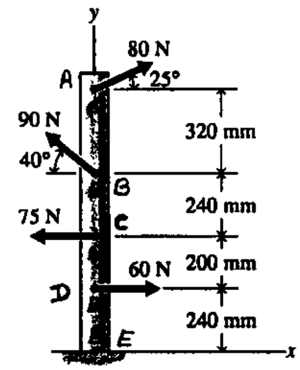


Fig. P4-124

SOLUTION

$$R_x = \Sigma F_x = 80 \cos 25^\circ - 90 \cos 40^\circ - 75 + 60 = -11.44 \text{ N}$$

$$R_y = \Sigma F_y = 80 \sin 25^\circ + 90 \sin 40^\circ = 91.66 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-11.44)^2 + (91.66)^2} = 92.37 \text{ N} = 92.4 \text{ N}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{-11.44}{92.37} = 97.11^\circ \cong 97.1^\circ$$

$$\mathbf{R} = 92.4 \text{ N} \measuredangle 82.9^\circ$$ Ans.

$$+\circlearrowleft \Sigma M_E = -80 \cos 25^\circ (1000) + 90 \cos 40^\circ (680)$$

$$+ 75(440) - 60(240) = -7023 \text{ N·mm} = 7023 \text{ N·mm} \circlearrowright$$

$$d_R = \frac{\Sigma M_E}{R} = \frac{7023}{92.37} = 76.0 \text{ mm}$$ Ans.

$$x_R = \frac{\Sigma M_E}{R_y} = \frac{7023}{91.66} = 76.6 \text{ mm} \leftarrow$$ Ans.

$$y_R = \frac{\Sigma M_E}{R_x} = \frac{7023}{11.44} = 614 \text{ mm} \downarrow$$ Ans.

4-125 Four forces and a couple are applied to a rectangular plate as shown in Fig. P4-125. Determine the magnitude and direction of the resultant of the force-couple system and the distance x_R from point O to the intercept of the line of action of the resultant with the x-axis.

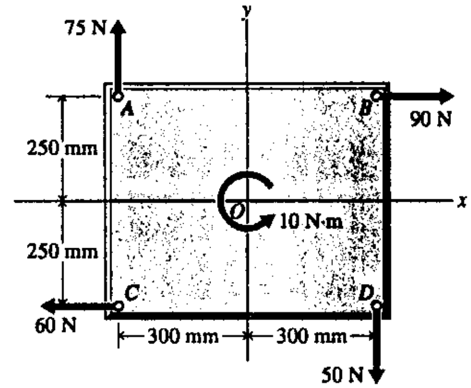


Fig. P4-125

SOLUTION

$$R_x = \Sigma F_x = 90 - 60 = 30 \text{ N}$$

$$R_y = \Sigma F_y = 75 - 50 = 25 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(30)^2 + (25)^2} = 39.05 \cong 39.1 \text{ N}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{30}{39.05} = 39.80° = 39.8°$$

$$\vec{R} = 39.1 \text{ N} \measuredangle 39.8°$$ Ans.

$$+ \circlearrowleft \Sigma M_O = 10 - 75(0.300) - 90(0.250) - 50(0.300) - 60(0.250)$$

$$= -65.0 \text{ N·m} = 65.0 \text{ N·m} \circlearrowright$$

$$x_R = \frac{\Sigma M_O}{R_y} = \frac{-65.0}{25} = -2.60 \text{ m} = 2.60 \text{ m} \leftarrow$$ Ans.

4-126 Four forces and a couple are applied to a frame as shown in Fig. P4-126. Determine the magnitude and direction of the resultant and the perpendicular distance d_R from point A to the line of action of the resultant.

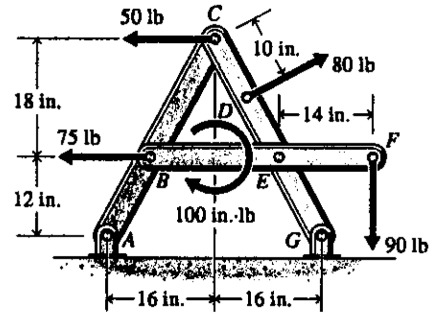


Fig. P4-126

SOLUTION

$$\theta = \tan^{-1}\frac{16}{30} = 28.07^\circ$$

$$R_x = \Sigma F_x = 80\cos 28.07^\circ - 50 - 75 = -54.41 \text{ lb}$$

$$R_y = \Sigma F_y = 80\sin 28.07^\circ - 90 = -52.36 \text{ lb}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-54.41)^2 + (-52.36)^2} = 75.51 \text{ lb} \cong 75.5 \text{ lb}$$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{-54.41}{75.51} = -136.10^\circ = -136.1^\circ$$

$$\mathbf{R} = 75.5 \text{ lb} \measuredangle 43.9^\circ$$ Ans.

$$+\circlearrowleft \Sigma M_A = 75(12) + 50(30) - 100 - 80\cos 28.07^\circ (30 - 10\cos 28.09^\circ)$$
$$+ 80\sin 28.07^\circ (16 + 10\sin 28.07^\circ)$$
$$- 90(16 + 18\tan 28.09^\circ + 14)$$
$$= -1979.3 \text{ in.}\cdot\text{lb} = 1979.3 \text{ in.}\cdot\text{lb} \circlearrowright$$

$$d_R = \frac{\Sigma M_A}{R} = \frac{1979.3}{75.51} = 26.21 \text{ in.} \cong 26.2 \text{ in.}$$ Ans.

4-127* Determine the resultant of the parallel force system shown in Fig. P4-127 and locate the intersection of the line of action of the resultant with the xy-plane.

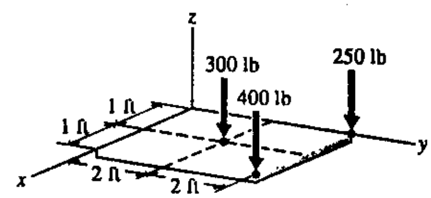


Fig. P4-127

SOLUTION

$\bar{R} = \Sigma F_z \hat{k} = (-300 - 400 - 250)\,\hat{k} = -950\,\hat{k}$ lb Ans.

$\Sigma M_x = -300(2) - 400(4) - 250(4) = -3200$ ft·lb

$\Sigma M_y = 300(1) + 400(2) = 1100$ ft·lb

$x_R = -\dfrac{\Sigma M_y}{R} = -\dfrac{1100}{-950} = 1.1579 \text{ ft} \cong 1.158 \text{ ft}$ Ans.

$y_R = \dfrac{\Sigma M_x}{R} = -\dfrac{-3200}{-950} = 3.368 \text{ ft} \cong 3.37 \text{ ft}$ Ans.

4-128* Determine the resultant of the parallel force system shown in Fig. P4-128 and locate the intersection of the line of action of the resultant with the xy-plane.

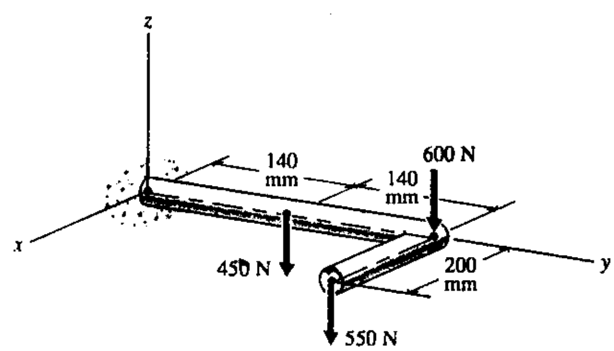


Fig. P4-128

SOLUTION

$\bar{R} = \Sigma F_z \hat{k} = (-450 - 600 - 550)\,\hat{k} = -1600\,\hat{k}$ N Ans.

$\Sigma M_x = -450(0.140) - 600(0.280) - 550(0.280) = -385$ N·m

$\Sigma M_y = 550(0.200) = 110$ N·m

$x_R = -\dfrac{\Sigma M_y}{R} = -\dfrac{110}{-1600} = 0.06875 \text{ m} \cong 68.8 \text{ mm}$ Ans.

$y_R = \dfrac{\Sigma M_x}{R} = -\dfrac{-385}{-1600} = 0.2406 \text{ m} \cong 241 \text{ mm}$ Ans.

4-129 Determine the resultant of the parallel force system shown in Fig. P4-129 and locate the intersection of the line of action of the resultant with the xy-plane.

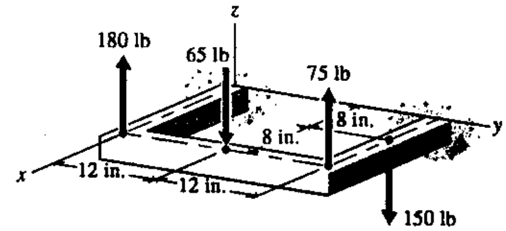


Fig. P4-129

SOLUTION

$$\bar{R} = \Sigma F_z \hat{k} = (180 - 65 + 75 - 150)\hat{k} = 40 \hat{k} \text{ lb}$$ Ans.

$$\Sigma M_x = -65(12) + 75(24) - 150(24) = -2580 \text{ in.·lb}$$

$$\Sigma M_y = -180(16) + 65(16) - 75(16) + 150(8) = -1840 \text{ in.·lb}$$

$$x_R = -\frac{\Sigma M_y}{R} = -\frac{-1840}{40} = 46.0 \text{ in.}$$ Ans.

$$y_R = \frac{\Sigma M_x}{R} = -\frac{-2580}{40} = -64.5 \text{ in.}$$ Ans.

4-130 Determine the resultant of the parallel force system shown in Fig. P4-130 and locate the intersection of the line of action of the resultant with the xy-plane.

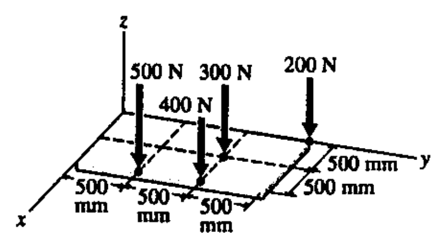


Fig. P4-130

SOLUTION

$$\bar{R} = \Sigma F_z \hat{k} = (-500 - 400 - 300 - 200)\hat{k} = -1400 \hat{k} \text{ N}$$ Ans.

$$\Sigma M_x = -500(0.500) - 400(1.000) - 300(1.000) - 200(1.500) = -1250 \text{ N·m}$$

$$\Sigma M_y = 500(1.000) + 400(1.000) + 300(0.500) = 1050 \text{ N·m}$$

$$x_R = -\frac{\Sigma M_y}{R} = -\frac{1050}{-1400} = 0.75 \text{ m} \cong 750 \text{ mm}$$ Ans.

$$y_R = \frac{\Sigma M_x}{R} = -\frac{-1250}{-1400} = 0.8929 \text{ m} \cong 893 \text{ mm}$$ Ans.

4-131* Determine the resultant of the parallel force system shown in Fig. P4-131 and locate the intersection of the line of action of the resultant with the yz-plane.

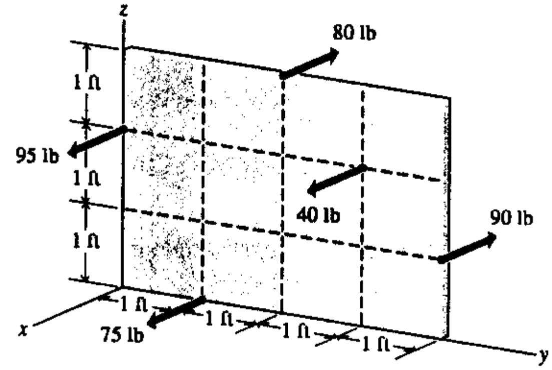


Fig. P4-131

SOLUTION

$$\bar{R} = \Sigma F_x\,\hat{i} = (95 + 75 + 40 - 80 - 90)\,\hat{i} = 40\,\hat{i}\text{ lb}$$ Ans.

$$\Sigma M_y = 95(2) + 40(2) - 80(3) - 90(1) = -60\text{ ft}\cdot\text{lb}$$

$$\Sigma M_z = 80(2) + 90(4) - 40(3) - 75(1) = 325\text{ ft}\cdot\text{lb}$$

$$y_R = -\frac{\Sigma M_z}{R} = -\frac{325}{40} = -8.13\text{ ft} \qquad z_R = \frac{\Sigma M_y}{R} = \frac{-60}{40} = -1.50\text{ ft}$$ Ans.

4-132 Determine the resultant of the parallel force system shown in Fig. P4-132 and locate the intersection of the line of action of the resultant with the xz-plane.

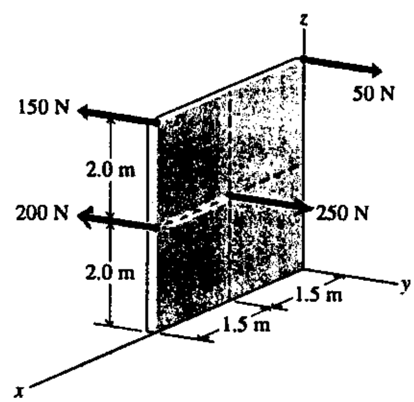


Fig. P4-132

SOLUTION

$$\bar{R} = \Sigma F_y\,\hat{j} = (250 + 50 - 200 - 150)\,\hat{j} = -50\,\hat{j}\text{ N}$$ Ans.

$$\Sigma M_x = 150(4) + 200(2) - 250(2) - 50(4) = 300\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 250(1.5) - 150(3.0) - 200(3.0) = -675\text{ N}\cdot\text{m}$$

$$x_R = \frac{\Sigma M_z}{R} = \frac{-675}{-50} = 13.5\text{ m} \qquad z_R = -\frac{\Sigma M_x}{R} = -\frac{300}{-50} = 6.00\text{ m}$$ Ans.

4-133* The resultant of the parallel force system shown in Fig. P4-133 is a couple which can be expressed in Cartesian vector form as

$\bar{C} = -1160\ \hat{j} + 2250\ \hat{k}$ in.·lb

Determine the magnitudes of forces $\bar{F}_1$, $\bar{F}_2$, and $\bar{F}_3$.

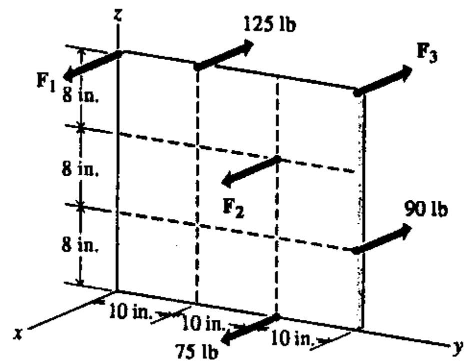


Fig. P4-133

SOLUTION

$$\bar{R} = \bar{0}$$

$$\bar{C} = -1160\ \hat{j} + 2250\ \hat{k} \text{ in.·lb}$$

$$\bar{R} = \Sigma F_x\ \hat{i} = (F_1 + F_2 + 75 - 125 - F_3 - 90)\ \hat{i} = \bar{0}$$

$$F_1 + F_2 - F_3 = 140 \qquad \text{(a)}$$

$$\Sigma M_y = F_1(24) + F_2(16) - F_3(24) - 125(24) - 90(8) = -1160$$

$$3F_1 + 2F_2 - 3F_3 = 320 \qquad \text{(b)}$$

$$\Sigma M_z = F_3(30) - F_2(20) + 125(10) - 75(20) + 90(30) = 2250$$

$$2F_2 - 3F_3 = 20 \qquad \text{(c)}$$

Solving Eqs. (a), (b), and (c) yields:

$F_1 = 100.0$ lb Ans.

$F_2 = 100.0$ lb Ans.

$F_3 = 60.0$ lb Ans.

4-134* The resultant of the parallel force system shown in Fig. P4-134 is a couple which can be expressed in Cartesian vector form as

$\bar{C} = -180\ \hat{i} + 435\ \hat{j}$ N·m

Determine the magnitudes of forces $\bar{F}_1$, $\bar{F}_2$, and $\bar{F}_3$.

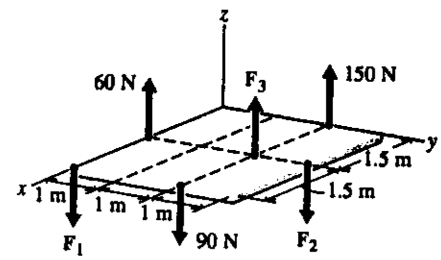


Fig. P4-134

SOLUTION

$$\bar{R} = \bar{0}$$

$$\bar{C} = -180\ \hat{i} + 435\ \hat{j}\ \text{N}\cdot\text{m}$$

$$\bar{R} = \Sigma F_z\ \hat{k} = (F_3 + 60 + 150 - F_1 - 90 - F_2)\ \hat{k} = \bar{0}$$

$$F_1 + F_2 - F_3 = 120 \qquad \text{(a)}$$

$$\Sigma M_x = F_3(2) + 150(2) - 90(2) - F_2(3) = -180$$

$$3F_2 - 2F_3 = 300 \qquad \text{(b)}$$

$$\Sigma M_y = F_1(3) + 90(3) + F_2(1.5) - F_3(1.5) - 60(1.5) = 435$$

$$2F_1 + F_2 - F_3 = 170 \qquad \text{(c)}$$

Solving Eqs. (a), (b), and (c) yields:

$F_1 = 50.0$ N Ans.

$F_2 = 160.0$ N Ans.

$F_3 = 90.0$ N Ans.

4-135 The resultant of the parallel force system shown in Fig. P4-135 is a couple which can be expressed in Cartesian vector form as

$\vec{C} = 850\,\hat{j} - 950\,\hat{k}$ in.·lb

Determine the magnitude and sense of each of the unknown forces ($\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$).

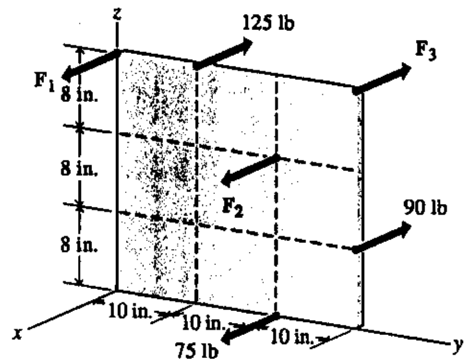


Fig. P4-135

SOLUTION

$\vec{R} = \vec{0}$

$\vec{C} = 850\,\hat{j} - 950\,\hat{k}$ in.·lb

$\vec{R} = \Sigma F_x\,\hat{i} = (F_1 + F_2 + 75 - F_3 - 125 - 90)\,\hat{i} = \vec{0}$

$$F_1 + F_2 - F_3 = 140 \quad \text{(a)}$$

$\Sigma M_y = F_1(24) - 125(24) + F_2(16) - F_3(24) - 90(8) = 850$

$$24F_1 + 16F_2 - 24F_3 = 4570 \quad \text{(b)}$$

$\Sigma M_z = 125(10) - F_2(20) - 75(20) + F_3(30) + 90(30) = -950$

$$2F_2 - 3F_3 = 340 \quad \text{(c)}$$

Solving Eqs. (a), (b), and (c) yields:

$F_1 = 77.1$ lb Ans.

$F_2 = -151.3$ lb Ans.

$F_3 = -214.2$ lb Ans.

4-136 The resultant of the parallel force system shown in Fig. P4-136 is a couple which can be expressed in Cartesian vector form as

$\vec{C} = 200\ \hat{i} + 350\ \hat{j}$ N·m

Determine the magnitude and sense of each of the unknown forces ($\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$).

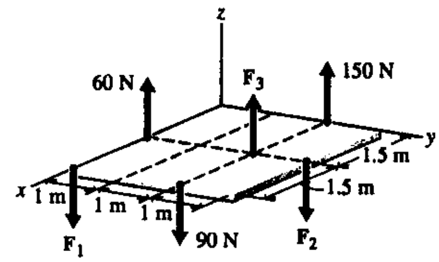


Fig. P4-136

SOLUTION

$\vec{R} = \vec{0}$

$\vec{C} = 200\ \hat{i} + 350\ \hat{j}$ N·m

$\vec{R} = \Sigma F_z\ \hat{k} = (F_3 + 60 + 150 - F_1 - 90 - F_2)\ \hat{k} = \vec{0}$

$$F_1 + F_2 - F_3 = 120 \qquad (a)$$

$\Sigma M_x = F_3(2) + 150(2) - 90(2) - F_2(3) = 200$

$$3F_2 - 2F_3 = -80 \qquad (b)$$

$\Sigma M_y = F_1(3) + 90(3) + F_2(1.5) - F_3(1.5) = 350$

$$6F_1 + 3F_2 - 3F_3 = 340 \qquad (c)$$

Solving Eqs. (a), (b), and (c) yields:

$F_1 = -6.67$ N Ans.

$F_2 = -333$ N Ans.

$F_3 = -460$ N Ans.

4-137* Replace the force system shown in Fig. P4-137 with a force $\vec{R}$ through point D and a couple $\vec{C}$.

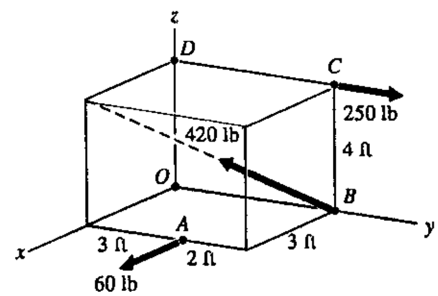


Fig. P4-137

SOLUTION

$\vec{F}_A = 60\ \hat{i}$ lb

$$\vec{F}_B = 420\left[\frac{3\ \hat{i} - 5\ \hat{j} + 4\ \hat{k}}{\sqrt{(3)^2 + (-5)^2 + (4)^2}}\right] = 178.2\ \hat{i} - 297.0\ \hat{j} + 237.6\ \hat{k}\text{ lb}$$

$\vec{F}_C = 250\ \hat{j}$ lb

$$\vec{R} = \Sigma\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C = 238.2\ \hat{i} - 47.0\ \hat{j} + 237.6\ \hat{k}\text{ lb}$$

$$\cong 238\ \hat{i} - 47.0\ \hat{j} + 238\ \hat{k}\text{ lb}$$ Ans.

$\vec{r}_{A/D} = 3\ \hat{i} + 3\ \hat{j} - 4\ \hat{k}$ ft $\qquad \vec{r}_{C/D} = 5\ \hat{j}$ ft

$\vec{r}_{B/D} = 5\ \hat{j} - 4\ \hat{k}$ ft

$$\vec{C} = \Sigma\vec{M}_D = (\vec{r}_{A/D} \times \vec{F}_A) + (\vec{r}_{B/D} \times \vec{F}_B) + (\vec{r}_{C/D} \times \vec{F}_C)$$

$$= [(3\ \hat{i} + 3\ \hat{j} - 4\ \hat{k}) \times (60\ \hat{i})]$$

$$+ [(5\ \hat{j} - 4\ \hat{k}) \times (178.2\ \hat{i} - 297\ \hat{j} + 237.6\ \hat{k})]$$

$$+ [(5\ \hat{j}) \times (250\ \hat{j})]\text{ ft·lb}$$

$$= -952.8\ \hat{j} - 1071\ \hat{k}\text{ ft·lb} \cong = -953\ \hat{j} - 1071\ \hat{k}\text{ ft·lb}$$ Ans.

For the force $\vec{R}$

$R = 339.7$ lb $\cong 340$ lb

$$\theta_x = \cos^{-1}\frac{238.2}{339.7} = 45.5°$$

$$\theta_y = \cos^{-1}\frac{-47.0}{339.7} = 98.0°$$

$$\theta_z = \cos^{-1}\frac{237.6}{339.7} = 45.6°$$

For the couple $\vec{C}$

$C = 1433.5$ ft·lb $\cong 1434$ ft·lb

$$\theta_x = \cos^{-1}\frac{0}{1433.5} = 90°$$

$$\theta_y = \cos^{-1}\frac{-952.8}{1433.5} = 131.7°$$

$$\theta_z = \cos^{-1}\frac{-1071}{1433.5} = 138.3°$$

4-138 Replace the force system shown in Fig. P4-138 with a force $\bar{R}$ through point D and a couple $\bar{C}$.

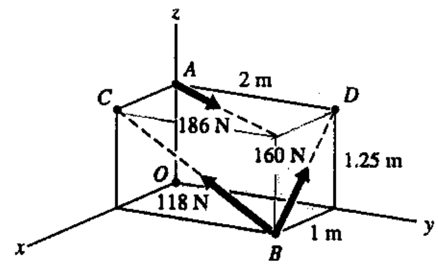


Fig. P4-138

SOLUTION

$$\bar{F}_A = 186\left[\frac{1\,\hat{i} + 2\,\hat{j}}{\sqrt{(1)^2 + (2)^2}}\right] = 83.18\,\hat{i} + 166.36\,\hat{j}\text{ N}$$

$$\bar{F}_{BC} = 118\left[\frac{-2\,\hat{j} + 1.25\,\hat{k}}{\sqrt{(-2)^2 + (1.25)^2}}\right] = -100.06\,\hat{j} + 62.54\,\hat{k}\text{ N}$$

$$\bar{F}_{BD} = 160\left[\frac{-1\,\hat{i} + 1.25\,\hat{k}}{\sqrt{(-1)^2 + (1.25)^2}}\right] = -99.95\,\hat{i} + 124.94\,\hat{k}\text{ N}$$

$$\bar{F}_B = \bar{F}_{BC} + \bar{F}_{BD} = -99.95\,\hat{i} - 100.06\,\hat{j} + 187.48\,\hat{k}\text{ N}$$

$$\bar{R} = \Sigma\bar{F} = \bar{F}_A + \bar{F}_B = -16.77\,\hat{i} + 66.30\,\hat{j} + 187.48\,\hat{k}\text{ N}$$
$$\cong -16.77\,\hat{i} + 66.3\,\hat{j} + 187.5\,\hat{k}\text{ N}$$ Ans.

$r_{A/D} = -2\,\hat{j}$ m $\qquad$ $r_{B/D} = 1\,\hat{i} - 1.25\,\hat{k}$ m

$$\bar{C} = \Sigma\bar{M}_D = (\bar{r}_{A/D} \times \bar{F}_A) + (\bar{r}_{B/D} \times \bar{F}_B)$$
$$= [(-2\,\hat{j}) \times (83.18\,\hat{i} + 166.36\,\hat{j})]$$
$$+ [(1\,\hat{i} - 1.25\,\hat{k}) \times (-99.95\,\hat{i} - 100.06\,\hat{j} + 187.48\,\hat{k})]$$
$$= -125.1\,\hat{i} - 62.55\,\hat{j} + 66.30\,\hat{k}\text{ N·m}$$
$$\cong -125.1\,\hat{i} - 62.6\,\hat{j} + 66.3\,\hat{k}\text{ N·m}$$ Ans.

For the force $\bar{R}$

$R = 199.56$ N $\cong 199.6$ N

$$\theta_x = \cos^{-1}\frac{-16.77}{199.56} = 94.8°$$

$$\theta_y = \cos^{-1}\frac{66.30}{199.56} = 70.6°$$

$$\theta_z = \cos^{-1}\frac{187.48}{199.56} = 20.0°$$

For the couple $\bar{C}$

$C = 154.78$ N·m $\cong 154.8$ N·m

$$\theta_x = \cos^{-1}\frac{-125.1}{154.78} = 143.9°$$

$$\theta_y = \cos^{-1}\frac{-62.54}{154.78} = 113.8°$$

$$\theta_z = \cos^{-1}\frac{66.30}{154.78} = 64.6°$$

4-139* Forces are applied at points A, B, and C of the bar shown in Fig. P4-139. Replace this system of forces with a force $\bar{R}$ through point O and a couple $\bar{C}$.

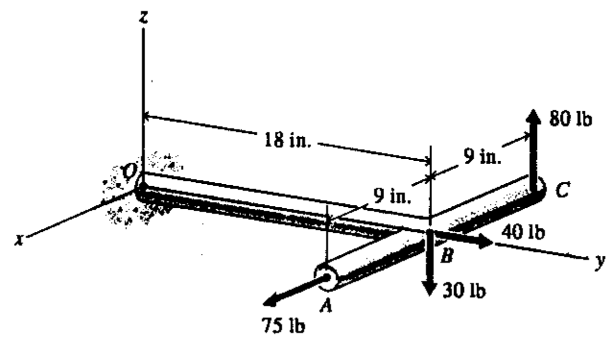


Fig. P4-139

SOLUTION

$\bar{F}_A = 75\ \hat{i}$ lb $\qquad \bar{r}_{A/O} = 9\ \hat{i} + 18\ \hat{j}$ in.

$\bar{F}_B = 40\ \hat{j} - 30\ \hat{k}$ lb $\qquad \bar{r}_{B/O} = 18\ \hat{j}$ in.

$F_C = 80\ \hat{k}$ lb $\qquad \bar{r}_{C/O} = -9\ \hat{i} + 18\ \hat{j}$ in.

$\bar{R} = \Sigma\bar{F} = \bar{F}_A + \bar{F}_B + \bar{F}_C = 75.0\ \hat{i} + 40.0\ \hat{j} + 50.0\ \hat{k}$ lb   Ans.

$\bar{C} = \Sigma\bar{M}_O = (\bar{r}_{A/O} \times \bar{F}_A) + (\bar{r}_{B/O} \times \bar{F}_B) + (\bar{r}_{C/O} \times \bar{F}_C)$

$= [(9\ \hat{i} + 18\ \hat{j}) \times (75\ \hat{i})] + [(18\ \hat{j}) \times (40\ \hat{j} - 30\ \hat{k})] + [(-9\ \hat{i} + 18\ \hat{j}) \times (80\ \hat{k})]$

$= 900\ \hat{i} + 720\ \hat{j} - 1350\ \hat{k}$ in.·lb   Ans.

$R = \sqrt{(75.0)^2 + (40.0)^2 + (50.0)^2} = 98.62 \text{ lb} \cong 98.6 \text{ lb}$

$C = \sqrt{(900)^2 + (720)^2 + (-1350)^2} = 1775.1 \text{ in.·lb} \cong 1775 \text{ in.·lb}$

For the force $\bar{R}$

$\theta_x = \cos^{-1}\dfrac{75.0}{98.62} = 40.5°$

$\theta_y = \cos^{-1}\dfrac{40.0}{98.62} = 66.1°$

$\theta_z = \cos^{-1}\dfrac{50.0}{98.62} = 59.5°$

For the couple $\bar{C}$

$\theta_x = \cos^{-1}\dfrac{900}{1775.1} = 59.5°$

$\theta_y = \cos^{-1}\dfrac{720}{1775.1} = 66.1°$

$\theta_z = \cos^{-1}\dfrac{-1350}{1775.1} = 139.5°$

4-140* Forces are applied at points A, B, and C of the bar shown in Fig. P4-140. Replace this system of forces with a force $\vec{R}$ through point O and a couple $\vec{C}$.

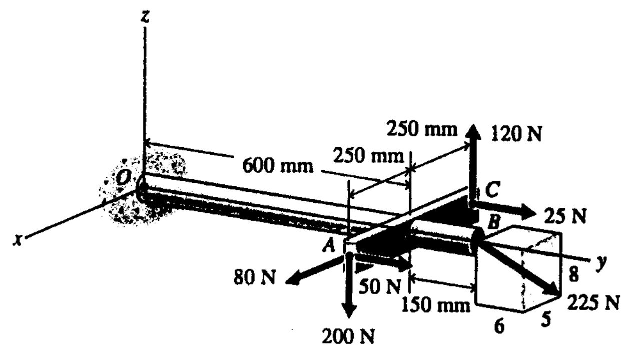


Fig. P4-140

SOLUTION

$\vec{F}_A = 80\,\hat{i} + 50\,\hat{j} - 200\,\hat{k}$ N $\qquad \vec{r}_{A/O} = 0.250\,\hat{i} + 0.600\,\hat{j}$ m

$$\vec{F}_B = 225\left[\frac{-5\,\hat{i} + 6\,\hat{j} - 8\,\hat{k}}{\sqrt{125}}\right]$$

$= (-100.6\,\hat{i} + 120.7\,\hat{j} - 161.0\,\hat{k})$ $\qquad \vec{r}_{B/O} = 0.750\,\hat{j}$ m

$F_C = 25\,\hat{j} + 120\,\hat{k}$ N $\qquad \vec{r}_{C/O} = -0.250\,\hat{i} + 0.600\,\hat{j}$ m

$\vec{R} = \Sigma\vec{F} = \vec{F}_A + \vec{F}_B + \vec{F}_C = -20.6\,\hat{i} + 195.7\,\hat{j} + -241\,\hat{k}$ N Ans.

$\vec{C} = \Sigma\vec{M}_O = (\vec{r}_{A/O} \times \vec{F}_A) + (\vec{r}_{B/O} \times \vec{F}_B) + (\vec{r}_{C/O} \times \vec{F}_C)$

$= [(0.250\,\hat{i} + 0.600\,\hat{j}) \times (80\,\hat{i} + 50\,\hat{j} - 200\,\hat{k})]$

$+ [(0.750\,\hat{j}) \times (-100.6\,\hat{i} + 120.7\,\hat{j} - 161.0\,\hat{k})]$

$+ [(-0.250\,\hat{i} + 0.600\,\hat{j}) \times (25\,\hat{j} + 120\,\hat{k})]$

$= -168.8\,\hat{i} + 80.0\,\hat{j} + 33.7\,\hat{k}$ N·m Ans.

$$R = \sqrt{(-20.6)^2 + (195.7)^2 + (-241)^2} = 311.1 \cong 311 \text{ N}$$

$$C = \sqrt{(-168.8)^2 + (80.0)^2 + (33.7)^2} = 189.81 \cong 189.8 \text{ N·m}$$

For the force $\vec{R}$

$\theta_x = \cos^{-1}\dfrac{-20.6}{311.1} = 93.8°$

$\theta_y = \cos^{-1}\dfrac{195.7}{311.1} = 51.0°$

$\theta_z = \cos^{-1}\dfrac{-241}{311.1} = 140.8°$

For the couple $\vec{C}$

$\theta_x = \cos^{-1}\dfrac{-168.8}{189.81} = 152.8°$

$\theta_y = \cos^{-1}\dfrac{80.0}{189.81} = 65.1°$

$\theta_z = \cos^{-1}\dfrac{33.7}{189.81} = 79.8°$

4-141 Forces are applied at points B, C, and D of the bar shown in Fig. P4-141. Replace this system of forces with a force $\bar{R}$ through point A and a couple $\bar{C}$.

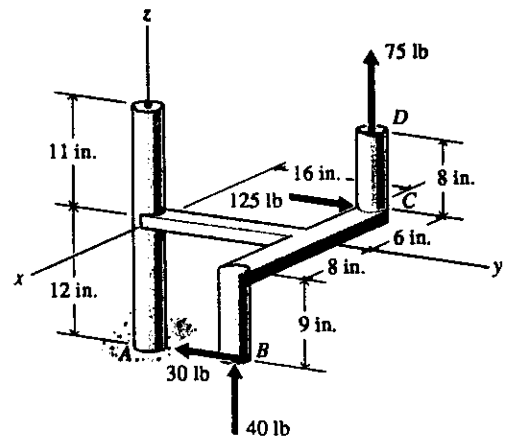


Fig. P4-141

SOLUTION

$R_x = 0$ $R_y = -30 + 125 = 95$ lb

$R_z = 40 + 75 = 115$ lb $\bar{R} = 95\,\hat{j} + 115\,\hat{k}$ lb Ans.

$R = \sqrt{(95)^2 + (115)^2} = 149.16 \text{ lb} \cong 149.2 \text{ lb}$

$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\frac{0}{149.16} = 90.0°$

$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\frac{95}{149.16} = 50.44° \cong 50.4°$

$\theta_z = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\frac{115}{149.16} = 39.56° \cong 39.6°$

$C_x = 40(16) + 30(3) + 75(16) - 125(12) = 430$ in.·lb

$C_y = -40(8) + 75(6) = 130$ in.·lb

$C_z = -30(8) - 125(6) = -990$ in.·lb

$\bar{C} = 430\,\hat{i} + 130\,\hat{j} - 990\,\hat{k}$ in.·lb

$C = \sqrt{(430)^2 + (130)^2 + (990)^2} = 1087.2$ in.·lb

$\theta_x = \cos^{-1}\frac{C_x}{C} = \cos^{-1}\frac{430}{1087.2} = 66.7°$ Ans.

$\theta_y = \cos^{-1}\frac{C_y}{C} = \cos^{-1}\frac{130}{1087.2} = 83.1°$ Ans.

$\theta_z = \cos^{-1}\frac{C_z}{C} = \cos^{-1}\frac{-990}{1087.2} = 155.6°$ Ans.

4-142 Forces are applied at points A, B, and C of the bar shown in Fig. P4-142. Replace this system of forces with a force $\bar{R}$ through point O and a couple $\bar{C}$.

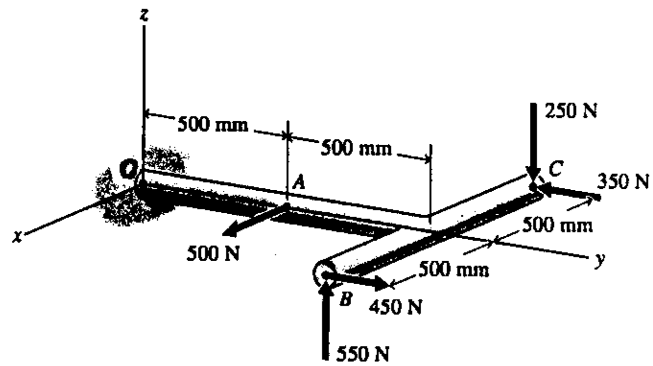


Fig. P4-142

SOLUTION

$$\bar{R} = \Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} = 500 \hat{i} + (450 - 350) \hat{j} + (550 - 250) \hat{k}$$

$$= 500 \hat{i} + 100 \hat{j} + 300 \hat{k} \text{ N}$$ Ans.

$$R = \sqrt{(500)^2 + (100)^2 + (300)^2} = 591.6 \text{ N} \cong 592 \text{ N}$$

$$\theta_x = \cos^{-1} \frac{R_x}{R} = \cos^{-1} \frac{500}{591.6} = 32.31^\circ \cong 32.3^\circ$$

$$\theta_y = \cos^{-1} \frac{R_y}{R} = \cos^{-1} \frac{100}{591.6} = 80.27^\circ \cong 80.3^\circ$$

$$\theta_z = \cos^{-1} \frac{R_z}{R} = \cos^{-1} \frac{300}{591.6} = 59.53^\circ \cong 59.5^\circ$$

$$\bar{C}_O = (\vec{r}_{A/O} \times \vec{F}_A) + (\vec{r}_{B/O} \times \vec{F}_B) + (\vec{r}_{C/O} \times \vec{F}_C)$$

$$= (0.500 \hat{j}) \times (500 \hat{i}) + (0.500 \hat{i} + 1.000 \hat{j}) \times (450 \hat{j} + 550 \hat{k})$$

$$+ (-0.500 \hat{i} + 1.000 \hat{j}) \times (-350 \hat{j} - 250 \hat{k})$$

$$= 300 \hat{i} - 400 \hat{j} + 150 \hat{k}$$ Ans.

$$C_O = \sqrt{(300)^2 + (400)^2 + (150)^2} = 522.02 \text{ N·m} \cong 522 \text{ N·m}$$

$$\theta_x = \cos^{-1} \frac{C_x}{C} = \cos^{-1} \frac{300}{522.02} = 54.92^\circ \cong 54.9^\circ$$ Ans.

$$\theta_y = \cos^{-1} \frac{C_y}{C} = \cos^{-1} \frac{-400}{522.02} = 140.01^\circ \cong 140.0^\circ$$ Ans.

$$\theta_z = \cos^{-1} \frac{C_z}{C} = \cos^{-1} \frac{150}{522.02} = 73.30^\circ \cong 73.3^\circ$$ Ans.

4-143* Forces are applied at points A, B, and C of the bar shown in Fig. P4-143. Replace this system of forces with a force $\bar{R}$ through point O and a couple $\bar{C}$.

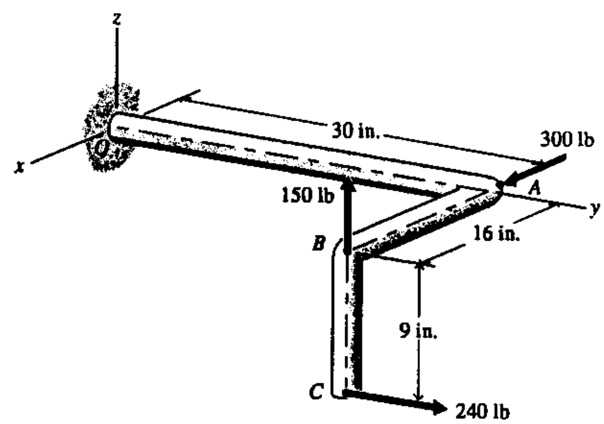


Fig. P4-143

SOLUTION

$\bar{R} = \Sigma F_x \,\hat{i} + \Sigma F_y \,\hat{j} + \Sigma F_z \,\hat{k} = 300\,\hat{i} + 240\,\hat{j} + 150\,\hat{k}$ lb Ans.

$\Sigma M_x = 150(30) + 240(9) = 6660$ in.•lb

$\Sigma M_y = -150(16) = -2400$ in.•lb

$\Sigma M_z = 240(16) - 300(30) = -5160$ in.•lb

$\bar{C}_O = \Sigma M_x \,\hat{i} + \Sigma M_y \,\hat{j} + \Sigma M_z \,\hat{k} = 6660\,\hat{i} - 2400\,\hat{j} - 5160\,\hat{k}$ in.•lb

$= 6.66\,\hat{i} - 2.40\,\hat{j} - 5.16\,\hat{k}$ in.•kip Ans.

$R = \sqrt{(300)^2 + (240)^2 + (150)^2} = 412.4 \text{ lb} \cong 412 \text{ lb}$

$C_O = \sqrt{(6660)^2 + (-2400)^2 + (-5160)^2} = 8760 \text{ in.•lb} = 8.76 \text{ in.•kip}$

For the force $\bar{R}$

$\theta_x = \cos^{-1} \dfrac{300}{412.4} = 43.3°$

$\theta_y = \cos^{-1} \dfrac{240}{412.4} = 54.4°$

$\theta_z = \cos^{-1} \dfrac{150}{412.4} = 68.7°$

For the couple $\bar{C}_O$

$\theta_x = \cos^{-1} \dfrac{6660}{8760} = 40.5°$

$\theta_y = \cos^{-1} \dfrac{-2400}{8760} = 105.9°$

$\theta_z = \cos^{-1} \dfrac{-5160}{8760} = 126.1°$

4-144* Forces are applied at points B and C of the bar shown in Fig. P4-144. Replace this system of forces with a force $\bar{\mathbf{R}}$ through point O and a couple $\bar{\mathbf{C}}$.

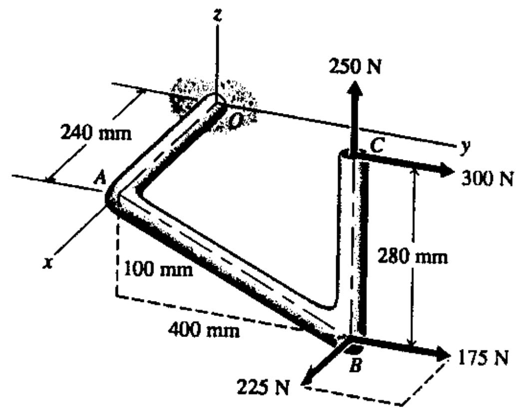


Fig. P4-144

SOLUTION

$$\bar{\mathbf{R}} = \Sigma F_x\,\hat{\imath} + \Sigma F_y\,\hat{\jmath} + \Sigma F_z\,\hat{k} = 225\,\hat{\imath} + (175 + 300)\,\hat{\jmath} + 250\,\hat{k}$$

$$= 225\,\hat{\imath} + 475\,\hat{\jmath} + 250\,\hat{k}\ \text{N} \qquad \text{Ans.}$$

$$\bar{\mathbf{C}}_O = (\vec{r}_{B/O} \times \mathbf{F}_B) + (\vec{r}_{C/O} \times \mathbf{F}_C)$$

$$= (0.240\,\hat{\imath} + 0.400\,\hat{\jmath} - 0.100\,\hat{k}) \times (225\,\hat{\imath} + 175\,\hat{\jmath})$$

$$+ (0.240\,\hat{\imath} + 0.400\,\hat{\jmath} + 0.180\,\hat{k}) \times (300\,\hat{\jmath} + 250\,\hat{k})$$

$$= 63.5\,\hat{\imath} - 82.5\,\hat{\jmath} + 24.0\,\hat{k}\ \text{N}\cdot\text{m} \qquad \text{Ans.}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{(225)^2 + (475)^2 + (250)^2} = 582.02\ \text{N} \cong 582\ \text{N}$$

$$C_O = \sqrt{C_x^2 + C_y^2 + C_z^2}$$

$$= \sqrt{(63.5)^2 + (-82.5)^2 + (24.0)^2} = 106.84\ \text{N}\cdot\text{m} \cong 106.8\ \text{N}\cdot\text{m}$$

For the force $\bar{\mathbf{R}}$

$$\theta_x = \cos^{-1}\frac{225}{582.0} = 67.3^\circ$$

$$\theta_y = \cos^{-1}\frac{475}{582.0} = 35.3^\circ$$

$$\theta_z = \cos^{-1}\frac{250}{582.0} = 64.6^\circ$$

For the couple $\bar{\mathbf{C}}_O$

$$\theta_x = \cos^{-1}\frac{63.5}{106.84} = 53.5^\circ$$

$$\theta_y = \cos^{-1}\frac{-82.5}{106.84} = 140.6^\circ$$

$$\theta_z = \cos^{-1}\frac{24.0}{106.84} = 77.0^\circ$$

4-145 Forces are applied at points B and D of the bar shown in Fig. P4-145. Replace this system of forces with a force $\bar{R}$ through point A and a couple $\bar{C}$.

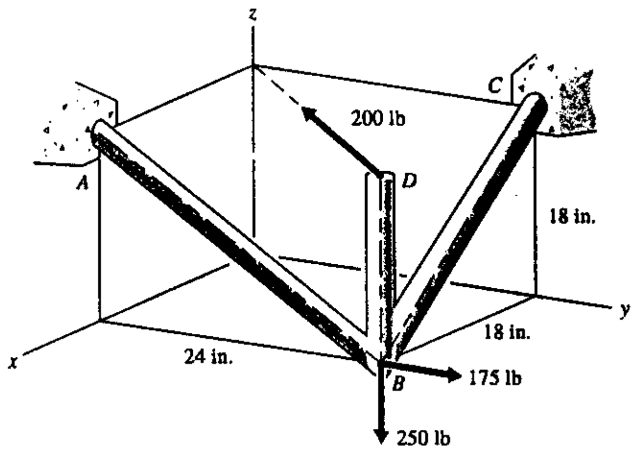


Fig. P4-145

SOLUTION

$$\theta = \tan^{-1}\frac{24}{18} = 53.13^\circ$$

$$\bar{R} = (-200\cos 53.13^\circ)\,\hat{i} + (-200\sin 53.13^\circ + 175)\,\hat{j} - 250\,\hat{k}$$

$$= -120\,\hat{i} + 15\,\hat{j} - 250\,\hat{k}\ \text{lb}$$ Ans.

$$\bar{C}_A = (\bar{r}_{B/A} \times \bar{F}_B) + (\bar{r}_{D/A} \times \bar{F}_D)$$

$$= (24\,\hat{j} - 18\,\hat{k}) \times (175\,\hat{j} - 250\,\hat{k})$$

$$+ (24\,\hat{j}) \times (-200\cos 53.13^\circ\,\hat{i} - 200\sin 53.13^\circ\,\hat{j})$$

$$= -2850\,\hat{i} + 2880\,\hat{k}\ \text{in.}\cdot\text{lb}$$ Ans.

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(-120)^2 + (15)^2 + (-250)^2} = 277.71\ \text{lb} \cong 278\ \text{lb}$$

$$C_A = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-2850)^2 + (2880)^2} = 4052\ \text{in.}\cdot\text{lb} \cong 4.05\ \text{in.}\cdot\text{kip}$$

For the force $\bar{R}$

$$\theta_x = \cos^{-1}\frac{-120}{277.71} = 115.6^\circ$$

$$\theta_y = \cos^{-1}\frac{15}{277.71} = 86.9^\circ$$

$$\theta_z = \cos^{-1}\frac{-250}{277.71} = 154.2^\circ$$

For the couple $\bar{C}_A$

$$\theta_x = \cos^{-1}\frac{-2850}{4052} = 135.3^\circ$$

$$\theta_y = \cos^{-1}\frac{0}{4052} = 90.0^\circ$$

$$\theta_z = \cos^{-1}\frac{2880}{4052} = 44.7^\circ$$

4-146 Forces are applied at points A and B of the bar shown in Fig. P4-146. Replace this system of forces with a force $\vec{R}$ through point O and a couple $\vec{C}$.

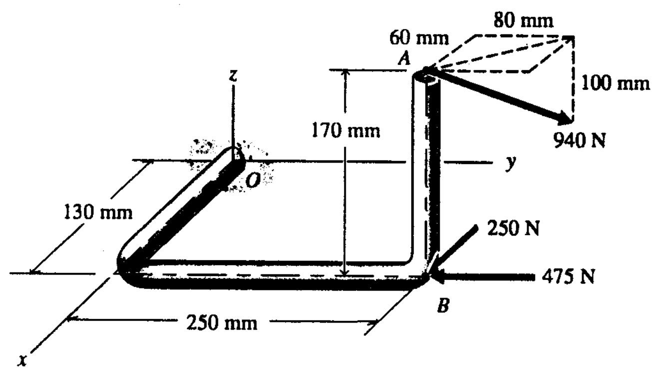


Fig. P4-146

SOLUTION

$$\vec{F}_A = 940\left[\frac{-60\ \hat{i} + 80\ \hat{j} - 100\ \hat{k}}{\sqrt{(-60)^2 + (80)^2 + (-100)^2}}\right] = -398.81\ \hat{i} + 531.74\ \hat{j} - 664.68\ \hat{k}\ \text{N}$$

$$\vec{F}_B = 250\ \hat{i} - 475\ \hat{j}\ \text{N}$$

$$\vec{R} = \vec{F}_A + \vec{F}_B = -148.81\ \hat{i} + 56.74\ \hat{j} - 664.68\ \hat{k}$$
$$\cong -148.8\ \hat{i} + 56.7\ \hat{j} - 665\ \hat{k}\ \text{N}$$ Ans.

$$\vec{C}_O = (\vec{r}_{A/O} \times \vec{F}_A) + (\vec{r}_{B/O} \times \vec{F}_B)$$
$$= (0.130\ \hat{i} + 0.250\ \hat{j} + 0.170\ \hat{k}) \times (-398.81\ \hat{i} + 531.74\ \hat{j} - 664.68\ \hat{k})$$
$$+ (0.130\ \hat{i} + 0.250\ \hat{j}) \times (250\ \hat{i} - 475\ \hat{j})$$
$$= -256.56\ \hat{i} + 18.62\ \hat{j} + 44.57\ \hat{k}\ \text{N·m}$$
$$\cong -257\ \hat{i} + 18.62\ \hat{j} + 44.6\ \hat{k}\ \text{N·m}$$ Ans.

$$R = \sqrt{(-148.81)^2 + (56.74)^2 + (-664.68)^2} = 683.49\ \text{N} \cong 684\ \text{N}$$

$$C_O = \sqrt{(-256.56)^2 + (18.62)^2 + (44.57)^2} = 261.07\ \text{N·m} \cong 261\ \text{N·m}$$

For the force $\vec{R}$

$$\theta_x = \cos^{-1}\frac{-148.81}{683.49} = 102.6^\circ$$
$$\theta_y = \cos^{-1}\frac{56.74}{683.49} = 85.2^\circ$$
$$\theta_z = \cos^{-1}\frac{-664.68}{683.49} = 166.5^\circ$$

For the couple $\vec{C}_O$

$$\theta_x = \cos^{-1}\frac{-256.56}{261.07} = 169.3^\circ$$
$$\theta_y = \cos^{-1}\frac{18.62}{261.07} = 85.9^\circ$$
$$\theta_z = \cos^{-1}\frac{44.57}{261.07} = 80.2^\circ$$

4-147* Forces are applied at points A, B, and C of the bar shown in Fig. P4-147. Replace this system of forces with a force $\vec{R}$ through point O and a couple $\vec{C}$.

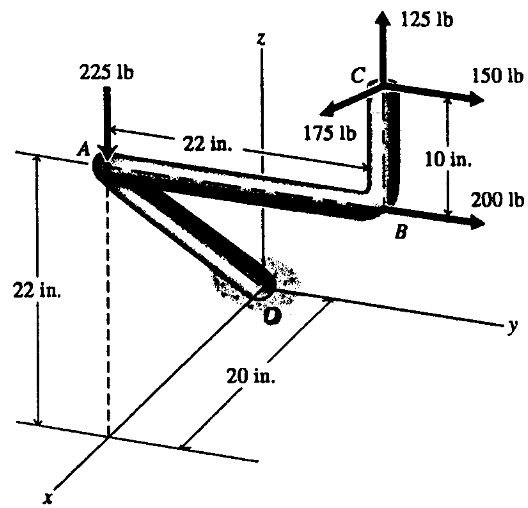


Fig. P4-147

SOLUTION

$$\vec{R} = \Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} = 175 \hat{i} + (200 + 150) \hat{j} + (-225 + 125) \hat{k}$$

$$= 175 \hat{i} + 350 \hat{j} - 100 \hat{k} \text{ lb}$$ Ans.

$$\vec{C}_O = (\vec{r}_{A/O} \times \vec{F}_A) + (\vec{r}_{B/O} \times \vec{F}_B) + (\vec{r}_{C/O} \times \vec{F}_C)$$

$$= (20 \hat{i} + 22 \hat{k}) \times (-225 \hat{k}) + (20 \hat{i} + 22 \hat{j} + 22 \hat{k}) \times (200 \hat{j})$$

$$+ (20 \hat{i} + 22 \hat{j} + 32 \hat{k}) \times (175 \hat{i} + 150 \hat{j} + 125 \hat{k})$$

$$= -6450 \hat{i} + 7600 \hat{j} + 3150 \hat{k} \text{ in.}\cdot\text{lb}$$

$$= -6.45 \hat{i} + 7.60 \hat{j} + 3.15 \hat{k} \text{ in.}\cdot\text{kip}$$ Ans.

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(175)^2 + (350)^2 + (-100)^2} = 833.7 \text{ lb}$$

$$C_A = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-6450)^2 + (7600)^2 + (3150)^2} = 10{,}454 \text{ in.}\cdot\text{lb}$$

For the force $\vec{R}$

$$\theta_x = \cos^{-1} \frac{175}{833.7} = 77.9°$$

$$\theta_y = \cos^{-1} \frac{350}{833.7} = 65.2°$$

$$\theta_z = \cos^{-1} \frac{-100}{833.7} = 96.9°$$

For the couple $\vec{C}_O$

$$\theta_x = \cos^{-1} \frac{-6450}{10{,}454} = 128.1°$$

$$\theta_y = \cos^{-1} \frac{7600}{10{,}454} = 43.4°$$

$$\theta_z = \cos^{-1} \frac{3150}{10{,}454} = 72.5°$$

4-148 Forces are applied at points A and B of the bar shown in Fig. P4-148. Replace this system of forces with a force $\bar{R}$ through point O and a couple $\bar{C}$.

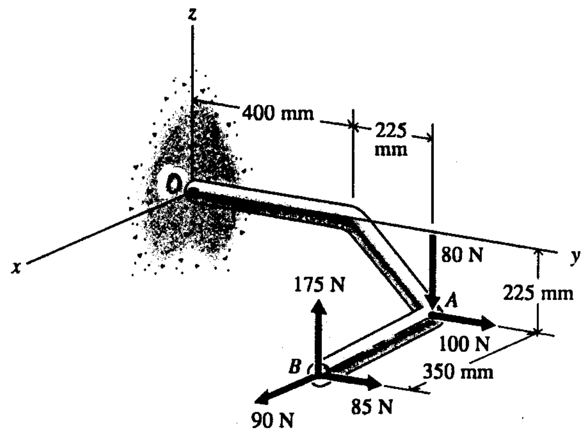


Fig. P4-148

SOLUTION

$$\bar{R} = \Sigma F_x\,\hat{i} + \Sigma F_y\,\hat{j} + \Sigma F_z\,\hat{k} = 90\,\hat{i} + (100 + 85)\,\hat{j} + (175 - 80)\,\hat{k}$$

$$= 90.0\,\hat{i} + 185.0\,\hat{j} + 95.0\,\hat{k}\ \text{N} \qquad \text{Ans.}$$

$$\bar{C}_O = (\bar{r}_{A/O} \times \bar{F}_A) + (\bar{r}_{B/O} \times \bar{F}_B)$$

$$= (0.625\,\hat{j} - 0.225\,\hat{k}) \times (100\,\hat{j} - 80\,\hat{k})$$

$$+ (0.350\,\hat{i} + 0.625\,\hat{j} - 0.225\,\hat{k}) \times (90\,\hat{i} + 85\,\hat{j} + 175\,\hat{k})$$

$$= 101.00\,\hat{i} - 81.50\,\hat{j} - 26.50\,\hat{k}\ \text{N·m}$$

$$= 101.0\,\hat{i} - 81.5\,\hat{j} - 26.5\,\hat{k}\ \text{N·m} \qquad \text{Ans.}$$

$$R = \sqrt{(90.0)^2 + (185.0)^2 + (95.0)^2} = 226.6\ \text{N} \cong 227\ \text{N}$$

$$C_O = \sqrt{(101.00)^2 + (-81.50)^2 + (-26.50)^2} = 132.46\ \text{N·m} \cong 132.5\ \text{N·m}$$

For the force $\bar{R}$

$$\theta_x = \cos^{-1}\frac{90.0}{226.6} = 66.6^\circ$$

$$\theta_y = \cos^{-1}\frac{185.0}{226.6} = 35.3^\circ$$

$$\theta_z = \cos^{-1}\frac{95.0}{226.6} = 65.2^\circ$$

For the couple $\bar{C}_O$

$$\theta_x = \cos^{-1}\frac{101.00}{132.46} = 40.3^\circ$$

$$\theta_y = \cos^{-1}\frac{-81.50}{132.46} = 128.0^\circ$$

$$\theta_z = \cos^{-1}\frac{-26.50}{132.46} = 101.5^\circ$$

4-149* Reduce the forces shown in Fig. P4-149 to a wrench and locate the intersection of the wrench with the xz-plane.

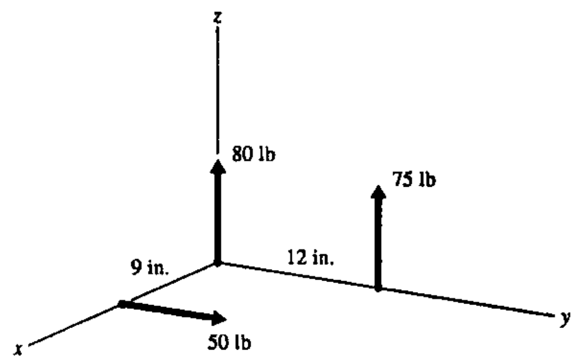


Fig. P4-149

SOLUTION

$$\vec{R} = \Sigma F_x\ \hat{i} + \Sigma F_y\ \hat{j} + \Sigma F_z\ \hat{k} = 50\ \hat{j} + (75 + 80)\ \hat{k}$$

$$= 50.0\ \hat{j} + 155.0\ \hat{k}\ \text{lb}$$ Ans.

$$\vec{C} = \Sigma \vec{M}_O = 75(12)\ \hat{i} + 50(9)\ \hat{k} = 900\ \hat{i} + 450\ \hat{k}$$

$$R = \sqrt{(50.0)^2 + (155.0)^2} = 162.86\ \text{lb}$$

$$\hat{e}_R = \frac{50.0}{162.86}\ \hat{i} + \frac{155.0}{162.86}\ \hat{k} = 0.3070\ \hat{i} + 0.9517\ \hat{k}$$

$$C = \vec{C}\cdot\hat{e}_R = (900\ \hat{i} + 450\ \hat{k})\cdot(0.3070\ \hat{i} + 0.9517\ \hat{k}) = 704.57$$

$$\vec{C}_{\parallel} = C_{\parallel}\hat{e}_R = 704.57(0.3070\ \hat{i} + 0.9517\ \hat{k})$$

$$= 216.3\ \hat{i} + 670.5\ \hat{k}\ \text{in.}\cdot\text{lb}$$ Ans.

$$\vec{C}_{\perp} = \vec{C} - \vec{C}_{\parallel} = (900\ \hat{i} + 450\ \hat{k}) - (216.3\ \hat{i} + 670.5\ \hat{k})$$

$$= 683.7\ \hat{i} - 220.5\ \hat{k}\ \text{in.}\cdot\text{lb}$$

Alternatively:

$$\vec{C}_{\perp} = \vec{r} \times \vec{R} = (x_R\ \hat{i} + z_R\ \hat{k}) \times (50.0\ \hat{j} + 155.0\ \hat{k})$$

$$= -50.0z_R\ \hat{i} - 155.0x_R\ \hat{j} + 50.0x_R\ \hat{k}\ \text{in.}\cdot\text{lb}$$

From $\hat{i}$: $-50.0z_R = 683.7$ $z_R = -13.674$ in. ≅ -13.74 in. Ans.

From $\hat{k}$: $50.0x_R = -220.5$ $x_R = -4.410$ in. ≅ -4.41 in. Ans.

4-150* Reduce the forces shown in Fig. P4-150 to a wrench and locate the intersection of the wrench with the xz-plane.

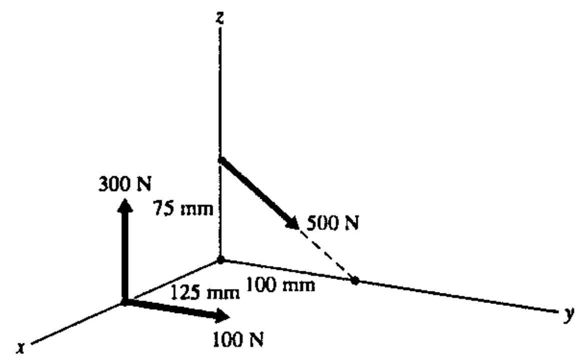


Fig. P4-150

SOLUTION

$$\theta = \tan^{-1}\frac{75}{100} = 36.87^\circ$$

$$\bar{R} = \Sigma F_x\,\hat{i} + \Sigma F_y\,\hat{j} + \Sigma F_z\,\hat{k} = (100 + 500\cos 36.87^\circ)\,\hat{j} + (300 - 500\sin 36.87^\circ)\,\hat{k} = 500\,\hat{j} \text{ N}$$ Ans.

$$\bar{C} = \Sigma\bar{M}_O = 500\cos 36.87^\circ\,(0.075)\,\hat{i} - 300(0.125)\,\hat{j} + 100(0.125)\,\hat{k}$$

$$= 30.00\,\hat{i} - 37.50\,\hat{j} + 12.50\,\hat{k}$$

$$C = \bar{C}\cdot\hat{e}_R = (30.00\,\hat{i} - 37.50\,\hat{j} + 12.50\,\hat{k})\cdot(1.000\,\hat{j}) = -37.50 \text{ N}\cdot\text{m}$$

$$\bar{C}_{\parallel} = C_{\parallel}\hat{e}_R = -37.50(1.000\,\hat{j}) = -37.50\,\hat{j} \text{ N}\cdot\text{m}$$ Ans.

$$\bar{C}_{\perp} = \bar{C} - \bar{C}_{\parallel} = (30.00\,\hat{i} - 37.50\,\hat{j} + 12.50\,\hat{k}) - (-37.50\,\hat{j})$$

$$= 30.00\,\hat{i} + 12.50\,\hat{k} \text{ N}\cdot\text{m}$$

Alternatively:

$$\bar{C}_{\perp} = \bar{r} \times \bar{R} = (x_R\,\hat{i} + z_R\,\hat{k}) \times (500\,\hat{j}) = -500 z_R\,\hat{i} + 500 x_R\,\hat{k} \text{ N}\cdot\text{m}$$

From $\hat{i}$: $-500 z_R = 30.00$ $\qquad z_R = -0.060 \text{ m} = -60.0 \text{ mm}$ Ans.

From $\hat{k}$: $500 x_R = 12.50$ $\qquad x_R = 0.0250 \text{ m} = 25.0 \text{ mm}$ Ans.

4-151 Reduce the forces shown in Fig.P4-151 to a wrench and locate the intersection of the wrench with the xy-plane.

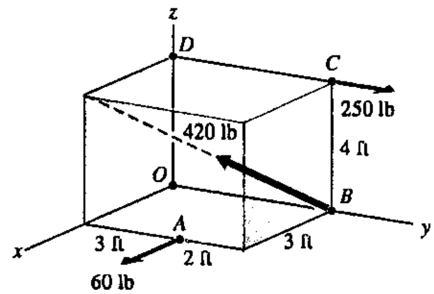


Fig. P4-151

SOLUTION

$$\mathbf{F}_B = 420\left[\frac{3\,\hat{i} - 5\,\hat{j} + 4\,\hat{k}}{\sqrt{(3)^2 + (-5)^2 + (4)^2}}\right] = 178.2\,\hat{i} - 297.0\,\hat{j} + 237.6\,\hat{k} \text{ lb}$$

$$\mathbf{R} = \Sigma F_x\,\hat{i} + \Sigma F_y\,\hat{j} + \Sigma F_z\,\hat{k} = (60 + 178.2)\,\hat{i} + (250 - 297.0)\,\hat{j} + 237.6\,\hat{k}$$
$$= 238.2\,\hat{i} - 47.0\,\hat{j} + 237.6\,\hat{k} \text{ lb} \qquad \text{Ans.}$$

$$R = \sqrt{(238.2)^2 + (-47.0)^2 + (237.6)^2} = 339.7 \text{ lb}$$

$$\mathbf{C} = \Sigma\mathbf{M}_O = (\mathbf{r}_{A/O} \times \mathbf{F}_A) + (\mathbf{r}_{B/O} \times \mathbf{F}_B) + (\mathbf{r}_{C/O} \times \mathbf{F}_C)$$
$$= [(3\,\hat{i} + 3\,\hat{j}) \times (60\,\hat{i})] + [(5\,\hat{j}) \times (178.2\,\hat{i} - 297\,\hat{j} + 237.6\,\hat{k})]$$
$$+ [(5\,\hat{j} + 4\,\hat{k}) \times (250\,\hat{j})] = 188.0\,\hat{i} - 1071\,\hat{k} \text{ ft·lb}$$

$$\hat{e}_R = \frac{238.2}{339.7}\,\hat{i} + \frac{-47.0}{339.7}\,\hat{j} + \frac{237.6}{339.7}\,\hat{k} = 0.7012\,\hat{i} - 0.1384\,\hat{j} + 0.6994\,\hat{k}$$

$$C_\parallel = \mathbf{C}\cdot\hat{e}_R = (188.0\,\hat{i} - 1071\,\hat{k})\cdot(0.7012\,\hat{i} - 0.1384\,\hat{j} + 0.6994\,\hat{k})$$
$$= -617.2 \text{ ft·lb}$$

$$\mathbf{C}_\parallel = C_\parallel\hat{e}_R = -617.2(0.7012\,\hat{i} - 0.1384\,\hat{j} + 0.6994\,\hat{k})$$
$$= -432.9\,\hat{i} + 85.4\,\hat{j} - 431.7\,\hat{k} \text{ ft·lb} \qquad \text{Ans.}$$

$$\mathbf{C}_\perp = \mathbf{C} - \mathbf{C}_\parallel = (188\,\hat{i} - 1071\,\hat{k}) - (-432.9\,\hat{i} + 85.4\,\hat{j} - 431.7\,\hat{k})$$
$$= 620.9\,\hat{i} - 85.4\,\hat{j} - 639.3\,\hat{k} \text{ ft·lb}$$

Alternatively:

$$\mathbf{C}_\perp = \mathbf{r} \times \mathbf{R} = (x_R\,\hat{i} + y_R\,\hat{j}) \times (238.2\,\hat{i} - 47.0\,\hat{j} + 237.6\,\hat{k})$$
$$= 237.6y_R\,\hat{i} - 237.6x_R\,\hat{j} + (-47.0x_R - 238\ 2y_R)\,\hat{k}$$

From $\hat{i}$: $237.6y_R = 620.9$ $\qquad y_R = 2.613 \text{ ft} \cong 31.4 \text{ in.}$ Ans.

From $\hat{j}$: $-237.6x_R = -85.4$ $\qquad x_R = 0.359 \text{ ft} \cong 4.31 \text{ in.}$ Ans.

4-152 Reduce the forces shown in Fig.P4-152 to a wrench and locate the intersection of the wrenchwith the xy-plane.

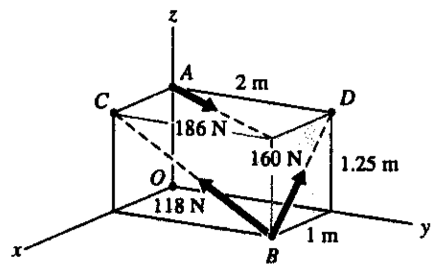


Fig. P4-152

SOLUTION

$$\bar{F}_A = 186\left[\frac{1\,\hat{i} + 2\,\hat{j}}{\sqrt{5}}\right] = 83.18\,\hat{i} + 166.36\,\hat{j}\ \text{N}$$

$$\bar{F}_B = 118\left[\frac{-2\,\hat{j} + 1.25\,\hat{k}}{\sqrt{5.5625}}\right] + 160\left[\frac{-1\,\hat{i} + 1.25\,\hat{k}}{\sqrt{2.5625}}\right]$$

$$= -99.95\,\hat{i} - 100.06\,\hat{j} + 187.48\,\hat{k}\ \text{N}$$

$$\bar{R} = \Sigma\bar{F} = \bar{F}_A + \bar{F}_B = -16.77\,\hat{i} + 66.30\,\hat{j} + 187.48\,\hat{k}\ \text{N}$$ Ans.

$$R = \sqrt{(-16.77)^2 + (66.30)^2 + (187.48)^2} = 199.56\ \text{N}$$

$$\bar{C} = \Sigma\bar{M}_O = (\bar{r}_{A/O} \times \bar{F}_A) + (\bar{r}_{B/O} \times \bar{F}_B)$$

$$= [(1.25\,\hat{k}) \times (83.18\,\hat{i} + 166.36\,\hat{j})]$$

$$+ [(1\,\hat{i} + 2\,\hat{j}) \times (-99.95\,\hat{i} - 100.06\,\hat{j} + 187.48\,\hat{k})]$$

$$= 167.01\,\hat{i} - 83.51\,\hat{j} + 99.84\,\hat{k}\ \text{N}\cdot\text{m}$$

$$\hat{e}_R = \frac{-16.77}{199.56}\,\hat{i} + \frac{66.30}{199.56}\,\hat{j} + \frac{187.48}{199.56}\,\hat{k} = -0.0840\,\hat{i} + 0.3322\,\hat{j} + 0.9395\,\hat{k}$$

$$C_{\parallel} = \bar{C}\cdot\hat{e}_R = (167.01\,\hat{i} - 83.51\,\hat{j} + 99.84\,\hat{k})\cdot(-0.0840\,\hat{i} + 0.3322\,\hat{j} + 0.9395\,\hat{k})$$

$$= 52.03\ \text{N}\cdot\text{m}$$

$$\bar{C}_{\parallel} = C_{\parallel}\hat{e}_R = 52.03(-0.0840\,\hat{i} + 0.3322\,\hat{j} + 0.9395\,\hat{k})$$

$$= -4.371\,\hat{i} + 17.28\,\hat{j} + 48.88\,\hat{k}\ \text{N}\cdot\text{m}$$ Ans.

$$\bar{C}_{\perp} = \bar{C} - \bar{C}_{\parallel} = 171.38\,\hat{i} - 100.79\,\hat{j} + 50.96\,\hat{k}\ \text{N}\cdot\text{m}$$

$$\bar{C}_{\perp} = \bar{r} \times \bar{R} = (x_R\,\hat{i} + y_R\,\hat{j}) \times (-16.77\,\hat{i} + 66.30\,\hat{j} + 187.48\,\hat{k})$$

$$= 187.48\,y_R\,\hat{i} - 187.48\,x_R\,\hat{j} + (66.3\,x_R + 16.77\,y_R)\,\hat{k}$$

From $\hat{j}$: $-187.48x_R = -100.79$ $x_R = 0.5376\ \text{m} \cong 538\ \text{mm}$ Ans.

From $\hat{i}$: $187.48y_R = 171.38$ $y_R = 0.9141\ \text{m} \cong 914\ \text{mm}$ Ans.

4-153 A 2500-lb jet engine is suspended from the wing of an airplane as shown in Fig. P4-153. Determine the moment produced by the engine at point A in the wing when the plane is
(a) On the ground with the engine not operating.
(b) In flight with the engine developing a thrust $\bar{T}$ of 15,000 lb.

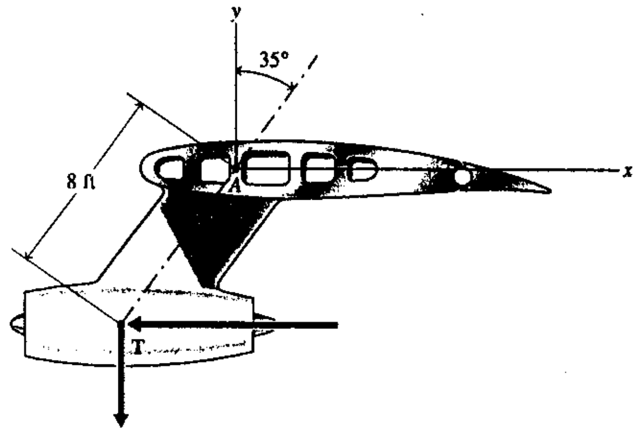


Fig. P4-153

SOLUTION

(a) $\bar{M} = M_z\hat{k} = Wd_W\,\hat{k} = 2500(8 \sin 35^\circ)\,\hat{k}$

$= 11{,}472\,\hat{k}$ ft·lb ≅ 11.47 ft·kip Ans.

(b) $\bar{M} = M_z\hat{k} = (Wd_W - Td_T)\,\hat{k} = [2500(8 \sin 35^\circ) - 15{,}000(8 \cos 35^\circ)]\,\hat{k}$

$= -86{,}827\,\hat{k}$ ft·lb ≅ -86.8 ft·kip Ans.

4-154 Determine the moment of the 1650-N force shown in Fig. P4-154 about point O.

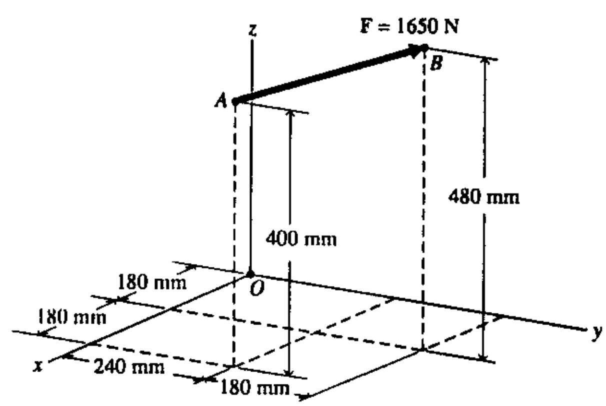


Fig. P4-154

SOLUTION

$$\bar{F} = 1650\left[\frac{-180\,\hat{i} + 180\,\hat{j} + 80\,\hat{k}}{\sqrt{(-180)^2 + (180)^2 + (80^2}}\right] = -1113.1\,\hat{i} + 1113.1\,\hat{j} + 494.7\,\hat{k}\ \text{N}$$

$\bar{M}_O = \bar{r} \times \bar{F}$

$= (0.360\,\hat{i} + 0.240\,\hat{j} + 0.400\,\hat{k}) \times (-1113.1\,\hat{i} + 1113.1\,\hat{j} + 494.7\,\hat{k})$

$= -327\,\hat{i} - 623\,\hat{j} + 668\,\hat{k}$ N·m Ans.

4-155* The driving wheel of a truck is subjected to the force-couple system shown in Fig. P4-155. Replace this system by an equivalent single force and determine the point of application of the force along the vertical diameter of the wheel.

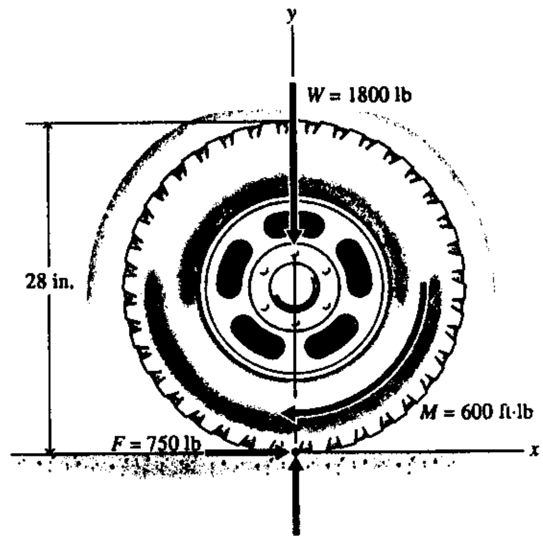


Fig. P4-155

SOLUTION

$\vec{R} = \Sigma F_x\ \hat{i} + \Sigma F_y\ \hat{j} = 750\ \hat{i} + (1800 - 1800)\ \hat{j} = 750\ \hat{i}$ lb Ans.

$\vec{C} = \Sigma M_z\ \hat{k} = -600(12)\ \hat{k} = -7200\ \hat{k}$ in.·lb

$M_z = Ry_R = 750y_R = 7200$ $y_R = 9.60$ in. Ans.

4-156* A bracket is subjected to the force-couple system shown in Fig. P4-156. Determine
(a) The magnitude and direction of the resultant force $\vec{R}$.
(b) The perpendicular distance from support O to the line of action of the resultant.

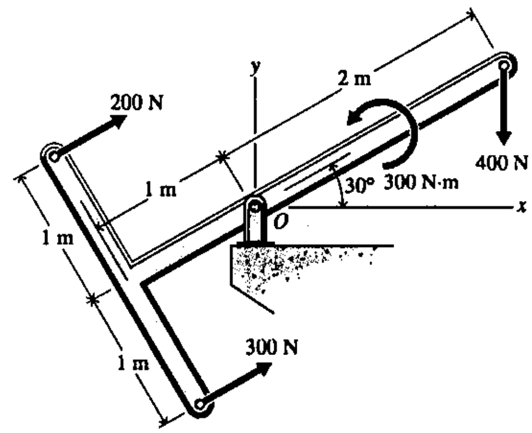


Fig. P4-156

SOLUTION

(a) $\vec{R} = \Sigma F_x \,\hat{i} + \Sigma F_y \,\hat{j}$

$= (200 \cos 30^\circ + 300 \cos 30^\circ)\,\hat{i}$

$+ (200 \sin 30^\circ + 300 \sin 30^\circ - 400)\,\hat{j} = 433.0\,\hat{i} - 150.0\,\hat{j}$ N

$R = \sqrt{(433.0)^2 + (-150.0)^2} = 458.2 \text{ N} \cong 458 \text{ N}$

$\theta_x = \tan^{-1} \dfrac{R_y}{R_x} = \dfrac{-150.0}{433.0} = -19.11^\circ$

$\vec{R} = 458 \text{ N} \searrow 19.11^\circ$ Ans.

(b) $\vec{C} = \Sigma M_O \,\hat{k} = [300(1) - 200(1) - 400(2 \cos 30^\circ) + 300]\,\hat{k}$

$= -292.8\,\hat{k} \text{ N·m} = 292.8 \text{ N·m} \curvearrowright$

$M_O = Rd = 458.2(d) = 292.8 \text{ N·m}$ $d = 0.639 \text{ m} = 639 \text{ mm}$ Ans.

4-157 Determine the resultant of the parallel force system shown in Fig. P4-157 and locate the intersection of its line of action with the xy-plane.

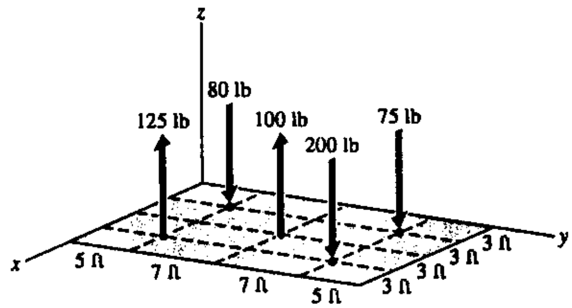


Fig. P4-157

SOLUTION

$$\mathbf{R} = \Sigma F\,\hat{\mathbf{k}} = (125 - 80 + 100 - 200 - 75)\,\hat{\mathbf{k}}$$

$$= -130\,\hat{\mathbf{k}}\text{ lb} = 130\text{ lb}\downarrow$$ Ans.

$$\Sigma M_x = 125(5) - 80(5) + 100(12) - 200(19) - 75(19)$$

$$= -3800\text{ ft}\cdot\text{lb}$$

$$\Sigma M_y = 80(3) + 75(3) - 100(6) - 125(9) + 200(9)$$

$$= 540\text{ ft}\cdot\text{lb}$$

$\Sigma M_x = Ry_R = -130y_R = -3800\text{ ft}\cdot\text{lb}$ $\quad y_R = 29.23\text{ ft} \cong 29.2\text{ ft}$ Ans.

$\Sigma M_y = -Rx_R = 130x_R = 540\text{ ft}\cdot\text{lb}$ $\quad x_R = 4.154\text{ ft} \cong 4.15\text{ ft}$ Ans.

4-158 A 200-N force is applied at corner B of a rectangular plate as shown in Fig. P4-158. Determine

(a) The moment of the force about point O.

(b) The moment of the force about line OD.

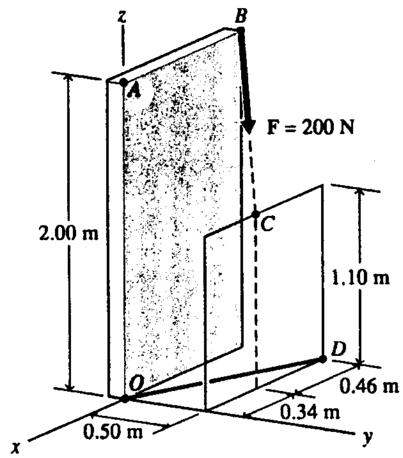


Fig. P4-158

SOLUTION

(a) $\vec{F} = 200\left[\dfrac{0.46\,\hat{i} + 0.50\,\hat{j} - 0.90\,\hat{k}}{\sqrt{(0.46)^2 + (0.50)^2 + (0.90)^2}}\right]$

$= 81.58\,\hat{i} + 88.68\,\hat{j} - 159.62\,\hat{k}$ N

$\vec{r}_{B/O} = -0.80\,\hat{i} + 2.00\,\hat{k}$ m

$\vec{M}_O = \vec{r}_{B/O} \times \vec{F} = (-0.80\,\hat{i} + 2.00\,\hat{k}) \times (81.58\,\hat{i} + 88.68\,\hat{j} - 159.62\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.80 & 0 & 2.00 \\ 81.58 & 88.68 & -159.62 \end{vmatrix}$$

$= -177.36\,\hat{i} + 35.46\,\hat{j} - 70.94\,\hat{k}$ N·m

$\cong -177.4\,\hat{i} + 35.5\,\hat{j} - 70.9\,\hat{k}$ N·m Ans.

(b) $\hat{e}_{OD} = \left[\dfrac{-0.80\,\hat{i} + 0.50\,\hat{j}}{\sqrt{(-0.80)^2 + (0.50)^2}}\right] = -0.8480\,\hat{i} + 0.5300\,\hat{j}$

$M_{OD} = M_O \cdot \hat{e}_{OD}$

$= (-177.36\,\hat{i} + 35.46\,\hat{j} - 70.95\,\hat{k}) \cdot (-0.8480\,\hat{i} + 0.5300\,\hat{j})$

$= 169.2$ N·m Ans.

4-159* A Z-section is loaded with a system of forces as shown in Fig. P4-159. Express the resultant of this force system in Cartesian vector form.

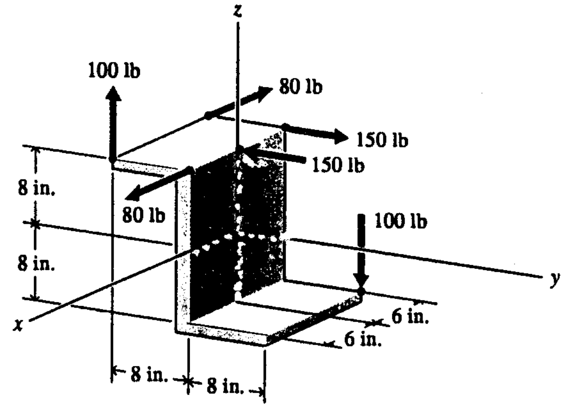


Fig. P4-159

SOLUTION

$\vec{R} = \Sigma F_x\ \hat{i} + \Sigma F_y\ \hat{j} + \Sigma F_z\ \hat{k}$

$= (80 - 80)\ \hat{i} + (150 - 150)\ \hat{j} + (100 - 100)\ \hat{k} = \vec{0}$ Ans.

$\vec{C} = \Sigma M_x\ \hat{i} + \Sigma M_y\ \hat{j} + \Sigma M_z\ \hat{k}$

$= -(100)(16)\ \hat{i} - (100)(12)\ \hat{j} - (150)(6)\ \hat{k} - (80)(8)\ \hat{k}$

$= -1600\ \hat{i} - 1200\ \hat{j} - 1540\ \hat{k}$ in.•lb Ans.

$C = \sqrt{(-1600)^2 + (-1200)^2 + (-1540)^2} = 2524.2$ in.•lb

$\theta_x = \cos^{-1}\dfrac{C_x}{C} = \cos^{-1}\dfrac{-1600}{2524.2} = 129.3°$

$\theta_y = \cos^{-1}\dfrac{C_y}{C} = \cos^{-1}\dfrac{-1200}{2524.2} = 118.4°$

$\theta_z = \cos^{-1}\dfrac{C_z}{C} = \cos^{-1}\dfrac{-1540}{2524.2} = 127.6°$

4-160* A bent rod supports two forces as shown in Fig. P4-160. Determine
(a) The moment of the two forces about point O.
(b) The moment of the two forces about line OA.

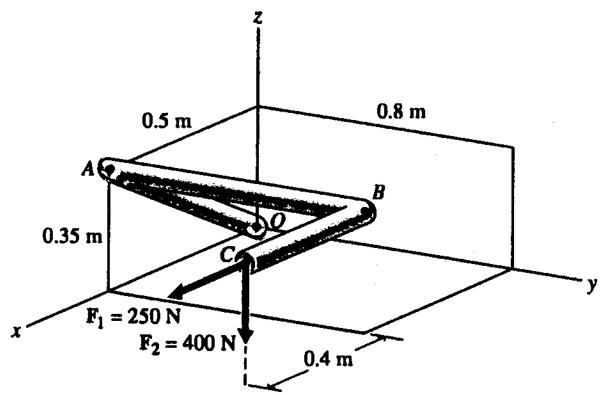


Fig. P4-160

SOLUTION

(a) $\mathbf{M}_O = \vec{r}_{C/O} \times \vec{F} = (0.90\,\hat{i} + 0.80\,\hat{j} + 0.35\,\hat{k}) \times (250\,\hat{i} - 400\,\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.90 & 0.80 & 0.35 \\ 250 & 0 & -400 \end{vmatrix}$$

$$= -320.0\,\hat{i} + 447.5\,\hat{j} - 200.0\,\hat{k} \text{ N·m}$$

$$= -320\,\hat{i} + 448\,\hat{j} - 200\,\hat{k} \text{ N·m}$$ Ans.

(b) $$\hat{e}_{OA} = \left[\frac{0.50\,\hat{i} + 0.35\,\hat{k}}{\sqrt{(-0.50)^2 + (0.35)^2}}\right] = 0.8193\,\hat{i} + 0.5735\,\hat{j}$$

$$M_{OA} = M_O \cdot \hat{e}_{OA} = (-320.0\,\hat{i} + 447.5\,\hat{j} - 200.0\,\hat{k}) \cdot (0.8193\,\hat{i} + 0.5735\,\hat{k})$$

$$= -376.9 \text{ N·m} \cong -377 \text{ N·m}$$ Ans.

4-161 The force $\bar{F}$ in Fig. P4-161 can be expressed in Cartesian vector form as

$$\bar{F} = 60\ \hat{i} + 100\ \hat{j} + 120\ \hat{k}\ \text{lb}$$

Determine the scalar component of the moment at point B about line BC.

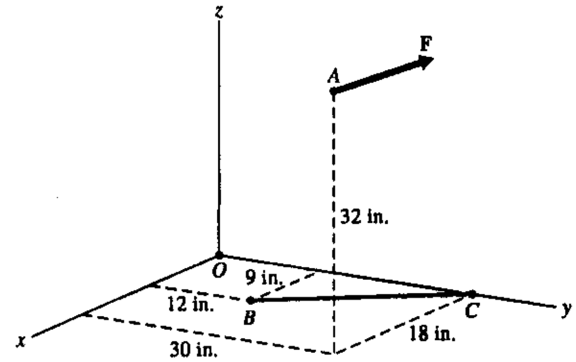


Fig. P4-161

SOLUTION

$$\bar{r}_{A/B} = (18 - 9)\ \hat{i} + (30 - 12)\ \hat{j} + (32 - 0)\ \hat{k}$$

$$= 9\ \hat{i} + 18\ \hat{j} + 32\ \hat{k}\ \text{in.}$$

$$\bar{M}_B = \bar{r}_{A/B} \times \bar{F} = (9\ \hat{i} + 18\ \hat{j} + 32\ \hat{k}) \times (60\ \hat{i} + 100\ \hat{j} + 120\ \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 9 & 18 & 32 \\ 60 & 100 & 120 \end{vmatrix}$$

$$= -1040\ \hat{i} + 840\ \hat{j} - 180\ \hat{k}\ \text{in.}\cdot\text{lb}$$

$$\hat{e}_{BC} = \left[\frac{-9\ \hat{i} + 18\ \hat{j}}{\sqrt{(-9)^2 + (18)^2}}\right] = -0.4472\ \hat{i} + 0.8944\ \hat{j}$$

$$M_{BC} = \bar{M}_B \cdot \hat{e}_{BC}$$

$$= (-1040\ \hat{i} + 840\ \hat{j} - 180\ \hat{k}) \cdot (-0.4472\ \hat{i} + 0.8944\ \hat{j})$$

$$= 1216\ \text{in.}\cdot\text{lb}$$ Ans.

4-162 A bent rod supports a 450-N force as shown in Fig. P4-162.
(a) Replace the 450-N force with a force $\vec{R}$ through point O and a couple $\vec{C}$.
(b) Determine the twisting moments produced by force $\vec{F}$ in the three different segments of the rod.

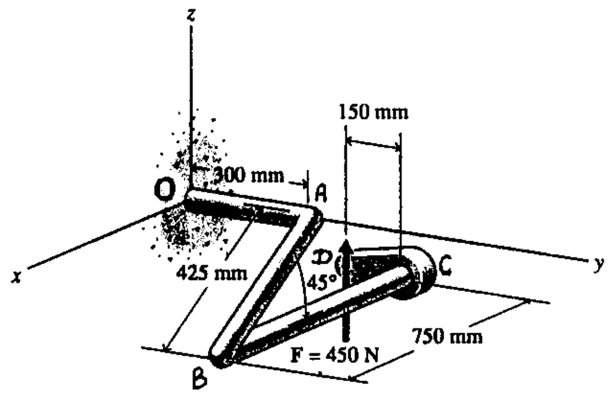


Fig. P4-162

SOLUTION

(a) $\vec{R} = \Sigma F = 450\ \hat{k}$ N Ans.

$\vec{r}_{D/O} = (0.425 \cos 45^\circ - 0.750)\ \hat{i}$

$+ (0.300 - 0.150)\ \hat{j} - 0.425 \sin 45^\circ\ \hat{k}$

$= -0.4495\ \hat{i} + 0.1500\ \hat{j} - 0.3005\ \hat{k}$ m

$\vec{C} = \vec{r}_{D/O} \times \vec{F} = (-0.4495\ \hat{i} + 0.1500\ \hat{j} - 0.3005\ \hat{k}) \times (450\ \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.4495 & 0.1500 & -0.3005 \\ 0 & 0 & 450 \end{vmatrix}$$

$= (67.50\ \hat{i} + 202.3\ \hat{j})$ N·m $\cong (67.5\ \hat{i} + 202\ \hat{j})$ N·m Ans.

(b) $\vec{r}_{D/B} = -0.750\ \hat{i} - 0.150\ \hat{j}$

$$\vec{M}_B = \vec{r}_{D/B} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.750 & -0.150 & 0 \\ 0 & 0 & 450 \end{vmatrix} = -67.5\ \hat{i} + 337.5\ \hat{j} \text{ N·m}$$

$\hat{e}_{BA} = -0.707\ \hat{i} + 0.707\ \hat{k}$

$M_{OA} = \vec{M}_O \cdot \hat{e}_{OA} = (67.50\ \hat{i} + 202.3\ \hat{j}) \cdot (1.000\ \hat{j}) = 202.3$ N·m Ans.

$M_{BA} = \vec{M}_B \cdot \hat{e}_{BA} = (-67.5\ \hat{i} + 337.5\ \hat{j}) \cdot (-0.707\ \hat{i} + 0.707\ \hat{k})$

$= 47.7$ N·m Ans.

$M_{BC} = \vec{M}_B \cdot \hat{e}_{BC} = (-67.5\ \hat{i} + 337.5\ \hat{j}) \cdot (-1.000\ \hat{i}) = 67.5$ N·m Ans.

5-1* Locate the center of gravity for the three particles shown in Fig. P5-1 if $W_A = 25$ lb, $W_B = 30$ lb, and $W_C = 45$ lb.

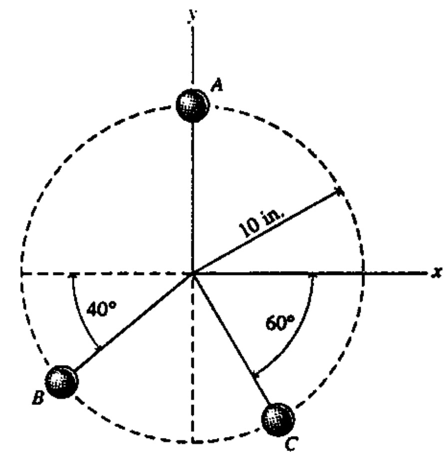


Fig. P5-1

SOLUTION

$$W = \Sigma W_i = W_A + W_B + W_C = 25 + 30 + 45 = 100 \text{ lb}$$

$$\Sigma W_i x_i = W_A x_A + W_B x_B + W_C x_C$$

$$= 25(0) + 30(-10 \cos 40^\circ) + 45(10 \cos 60^\circ) = -4.813 \text{ in.·lb}$$

$$\Sigma W_i y_i = W_A y_A + W_B y_B + W_C y_C$$

$$= 25(10) + 30(-10 \sin 40^\circ) + 45(-10 \sin 60^\circ) = -332.55 \text{ in.·lb}$$

$$W x_G = \Sigma W_i x_i$$

$$x_G = \frac{\Sigma W_i x_i}{W} = \frac{-4.813}{100} = -0.04813 \text{ in.} \cong -0.481 \text{ in.}$$ Ans.

$$W y_G = \Sigma W_i y_i$$

$$y_G = \frac{\Sigma W_i y_i}{W} = \frac{-332.55}{100} = -3.3255 \cong -3.33 \text{ in.}$$ Ans.

5-2* Locate the center of mass for the three particles shown in Fig. P5-2 if $m_A = 26$ kg, $m_B = 21$ kg, and $m_C = 36$ kg.

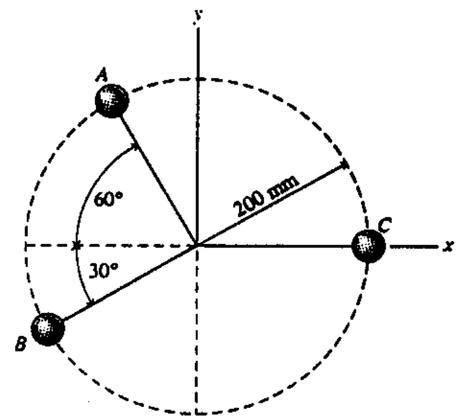


Fig. P5-2

SOLUTION

$$m = \Sigma m_i = m_A + m_B + m_C = 26 + 21 + 36 = 83 \text{ kg}$$

$$\Sigma m_i x_i = m_A x_A + m_B x_B + m_C x_C$$

$$= 26(-200 \cos 60^\circ) + 21(-200) \cos 30^\circ) + 36(200) = 962.69 \text{ kg·mm}$$

$$\Sigma m_i y_i = m_A y_A + m_B y_B + m_C y_C$$

$$= 26(200 \sin 60^\circ) + 21(-200 \sin 30^\circ) + 36(0) = 2403.33 \text{ kg·mm}$$

$$m x_G = \Sigma m_i x_i$$

$$x_G = \frac{\Sigma m_i x_i}{m} = \frac{962.69}{83} = 11.599 \text{ mm} \cong 11.60 \text{ mm}$$ Ans.

$$m y_G = \Sigma m_i y_i$$

$$y_G = \frac{\Sigma m_i y_i}{m} = \frac{2403.33}{83} = 28.956 \text{ mm} \cong 29.0 \text{ mm}$$ Ans.

5-3 Locate the center of gravity for the four particles shown in Fig. P5-3 if $W_A = 20$ lb, $W_B = 25$ lb, $W_C = 30$ lb, and $W_D = 40$ lb.

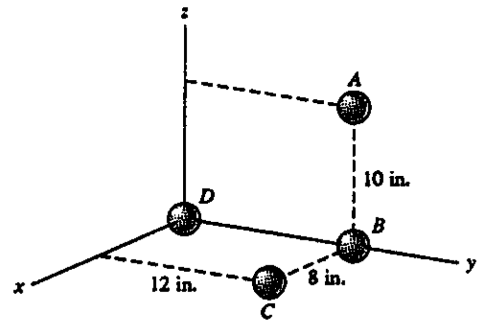


Fig. P5-3

SOLUTION

$$W = \Sigma W_i = W_A + W_B + W_C + W_D = 20 + 25 + 30 + 40 = 115 \text{ lb}$$

$$M_{yz} = \Sigma W_i x_i = W_A x_A + W_B x_B + W_C x_C + W_D x_D$$
$$= 20(0) + 25(0) + 30(8) + 40(0) = 240 \text{ in.}\cdot\text{lb}$$

$$M_{zx} = \Sigma W_i y_i = W_A y_A + W_B y_B + W_C y_C + W_D y_D$$
$$= 20(12) + 25(12) + 30(12) + 40(0) = 900 \text{ in.}\cdot\text{lb}$$

$$M_{xy} = \Sigma W_i z_i = W_A z_A + W_B z_B + W_C z_C + W_D z_D$$
$$= 20(10) + 25(0) + 30(0) + 40(0) = 200 \text{ in.}\cdot\text{lb}$$

$W x_G = \Sigma W_i x_i = M_{yz}$ $\qquad$ $x_G = \dfrac{M_{yz}}{W} = \dfrac{240}{115} = 2.09$ in. $\qquad$ Ans.

$W y_G = \Sigma W_i y_i = M_{zx}$ $\qquad$ $y_G = \dfrac{M_{zx}}{W} = \dfrac{900}{115} = 7.83$ in. $\qquad$ Ans.

$W z_G = \Sigma W_i z_i = M_{xy}$ $\qquad$ $z_G = \dfrac{M_{xy}}{W} = \dfrac{200}{115} = 1.739$ in. $\qquad$ Ans.

5-4 Locate the center of mass for the four particles shown in Fig. P5-4 if $m_A = 16$ kg, $m_B = 24$ kg, $m_C = 14$ kg, and $m_D = 36$ kg.

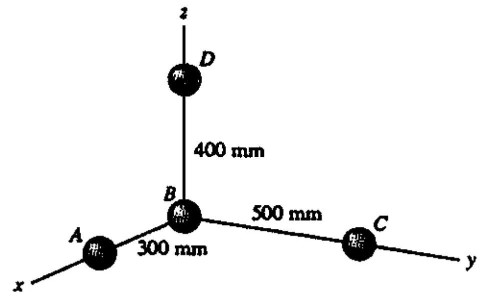


Fig. P5-4

SOLUTION

$$m = \Sigma m_i = m_A + m_B + m_C + m_D = 16 + 24 + 14 + 36 = 90 \text{ kg}$$

$$M_{yz} = \Sigma m_i x_i = m_A x_A + m_B x_B + m_C x_C + m_D x_D$$
$$= 16(300) + 24(0) + 14(0) + 36(0) = 4800 \text{ kg·mm}$$

$$M_{zx} = \Sigma m_i y_i = m_A y_A + m_B y_B + m_C y_C + m_D y_D$$
$$= 16(0) + 24(0) + 14(500) + 36(0) = 7000 \text{ kg·mm}$$

$$M_{xy} = \Sigma m_i z_i = m_A z_A + m_B z_B + m_C z_C + m_D z_D$$
$$= 16(0) + 24(0) + 14(0) + 36(400) = 14{,}400 \text{ kg·mm}$$

$mx_G = \Sigma m_i x_i = M_{yz}$ $x_G = \dfrac{M_{yz}}{m} = \dfrac{4800}{90} = 53.3$ mm Ans.

$my_G = \Sigma m_i y_i = M_{zx}$ $y_G = \dfrac{M_{zx}}{m} = \dfrac{7000}{90} = 77.8$ mm Ans.

$mz_G = \Sigma m_i z_i = M_{xy}$ $z_G = \dfrac{M_{xy}}{m} = \dfrac{14{,}400}{90} = 160.0$ mm Ans.

5-5* Locate the center of gravity for the five particles shown in Fig. P5-5 if $W_A = 25$ lb, $W_B = 35$ lb, $W_C = 15$ lb, $W_D = 28$ lb, and $W_E = 16$ lb.

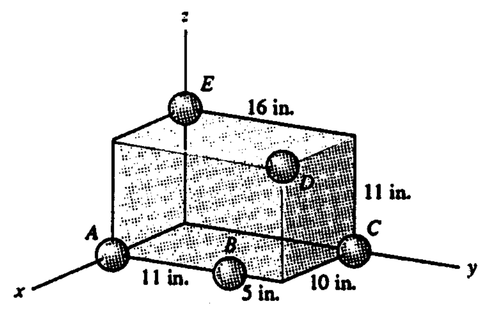


Fig. P5-5

SOLUTION

$$W = \Sigma W_i = W_A + W_B + W_C + W_D + W_E = 25 + 35 + 15 + 28 + 16 = 119 \text{ lb}$$

$$M_{yz} = \Sigma W_i x_i = W_A x_A + W_B x_B + W_C x_C + W_D x_D + W_E x_E$$
$$= 25(10) + 35(10) + 15(0) + 28(10) + 16(0) = 880 \text{ in.}\cdot\text{lb}$$

$$M_{zx} = \Sigma W_i y_i = W_A y_A + W_B y_B + W_C y_C + W_D y_D + W_E y_E$$
$$= 25(0) + 35(11) + 15(16) + 28(16) + 16(0) = 1073 \text{ in.}\cdot\text{lb}$$

$$M_{xy} = \Sigma W_i z_i = W_A z_A + W_B z_B + W_C z_C + W_D z_D + W_E z_E$$
$$= 25(0) + 35(0) + 15(0) + 28(11) + 16(11) = 484 \text{ in.}\cdot\text{lb}$$

$W x_G = \Sigma W_i x_i = M_{yz}$ $\qquad x_G = \dfrac{M_{yz}}{W} = \dfrac{880}{119} = 7.39$ in. Ans.

$W y_G = \Sigma W_i y_i = M_{zx}$ $\qquad y_G = \dfrac{M_{zx}}{W} = \dfrac{1073}{119} = 9.02$ in. Ans.

$W z_G = \Sigma W_i z_i = M_{xy}$ $\qquad z_G = \dfrac{M_{xy}}{W} = \dfrac{484}{119} = 4.07$ in. Ans.

5-6* Locate the center of mass for the five particles shown in Fig. P5-6 if $m_A = 2$ kg, $m_B = 3$ kg, $m_C = 4$ kg, $m_D = 3$ kg, and $m_E = 2$ kg.

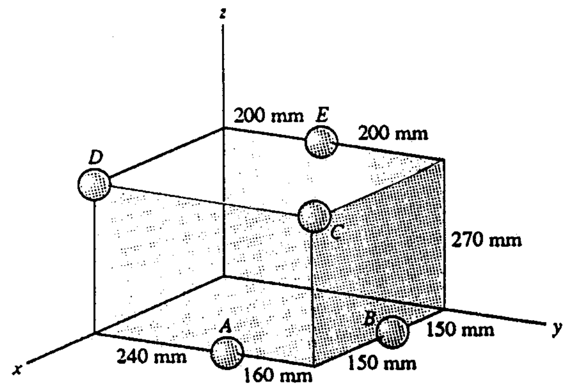


Fig. P5-6

SOLUTION

$$m = \Sigma m_i = m_A + m_B + m_C + m_D + m_E = 2 + 3 + 4 + 3 + 2 = 14 \text{ kg}$$

$$M_{yz} = \Sigma m_i x_i = m_A x_A + m_B x_B + m_C x_C + m_D x_D + m_E x_E$$
$$= 2(300) + 3(150) + 4(300) + 3(300) + 2(0) = 3150 \text{ kg·mm}$$

$$M_{zx} = \Sigma m_i y_i = m_A y_A + m_B y_B + m_C y_C + m_D y_D + m_E y_E$$
$$= 2(240) + 3(400) + 4(400) + 3(0) + 2(200) = 3680 \text{ kg·mm}$$

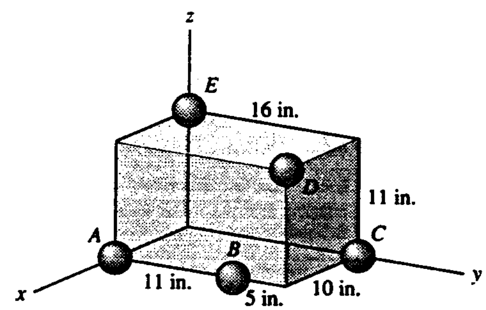
$$M_{xy} = \Sigma m_i z_i = m_A z_A + m_B z_B + m_C z_C + m_D z_D + m_E z_E$$
$$= 2(0) + 3(0) + 4(270) + 3(270) + 2(270) = 2430 \text{ kg·mm}$$

$m x_G = \Sigma m_i x_i = M_{yz}$ $\qquad x_G = \dfrac{M_{yz}}{m} = \dfrac{3150}{14} = 225$ mm   Ans.

$m y_G = \Sigma m_i y_i = M_{zx}$ $\qquad y_G = \dfrac{M_{zx}}{m} = \dfrac{3680}{14} = 263$ mm   Ans.

$m z_G = \Sigma m_i z_i = M_{xy}$ $\qquad z_G = \dfrac{M_{xy}}{m} = \dfrac{2430}{14} = 173.6$ mm   Ans.

5-7 Locate the center of gravity for the five particles shown in Fig. P5-5 if $W_A = 15$ lb, $W_B = 24$ lb, $W_C = 35$ lb, $W_D = 18$ lb, and $W_E = 26$ lb.



SOLUTION

$$W = \Sigma W_i = W_A + W_B + W_C + W_D + W_E = 15 + 24 + 35 + 18 + 26 = 118 \text{ lb}$$

$$M_{yz} = \Sigma W_i x_i = W_A x_A + W_B x_B + W_C x_C + W_D x_D + W_E x_E$$
$$= 15(10) + 24(10) + 35(0) + 18(10) + 26(0) = 570 \text{ in.}\cdot\text{lb}$$

$$M_{zx} = \Sigma W_i y_i = W_A y_A + W_B y_B + W_C y_C + W_D y_D + W_E y_E$$
$$= 15(0) + 24(11) + 35(16) + 18(16) + 26(0) = 1112 \text{ in.}\cdot\text{lb}$$

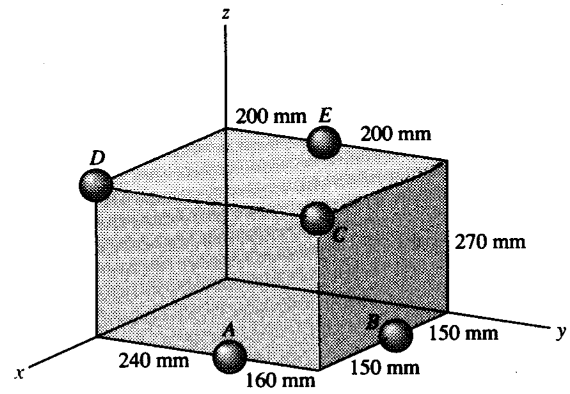
$$M_{xy} = \Sigma W_i z_i = W_A z_A + W_B z_B + W_C z_C + W_D z_D + W_E z_E$$
$$= 15(0) + 24(0) + 35(0) + 18(11) + 26(11) = 484 \text{ in.}\cdot\text{lb}$$

$W x_G = \Sigma W_i x_i = M_{yz}$ $\qquad x_G = \dfrac{M_{yz}}{W} = \dfrac{570}{118} = 4.83$ in. Ans.

$W y_G = \Sigma W_i y_i = M_{zx}$ $\qquad y_G = \dfrac{M_{zx}}{W} = \dfrac{1112}{118} = 9.42$ in. Ans.

$W z_G = \Sigma W_i z_i = M_{xy}$ $\qquad z_G = \dfrac{M_{xy}}{W} = \dfrac{484}{118} = 4.10$ in. Ans.

5-8 Locate the center of mass for the five particles shown in Fig. P5-6 if $m_A = 6$ kg, $m_B = 9$ kg, $m_C = 5$ kg, $m_D = 7$ kg, and $m_E = 4$ kg.



SOLUTION

$$m = \Sigma m_i = m_A + m_B + m_C + m_D + m_E = 6 + 9 + 5 + 7 + 4 = 31 \text{ kg}$$

$$M_{yz} = \Sigma m_i x_i = m_A x_A + m_B x_B + m_C x_C + m_D x_D + m_E x_E$$
$$= 6(300) + 9(150) + 5(300) + 7(300) + 4(0) = 6750 \text{ kg}\cdot\text{mm}$$

$$M_{zx} = \Sigma m_i y_i = m_A y_A + m_B y_B + m_C y_C + m_D y_D + m_E y_E$$
$$= 6(240) + 9(400) + 5(400) + 7(0) + 4(200) = 7840 \text{ kg}\cdot\text{mm}$$

$$M_{xy} = \Sigma m_i z_i = m_A z_A + m_B z_B + m_C z_C + m_D z_D + m_E z_E$$
$$= 6(0) + 9(0) + 5(270) + 7(270) + 4(270) = 4320 \text{ kg}\cdot\text{mm}$$

$$m x_G = \Sigma m_i x_i = M_{yz} \qquad x_G = \frac{M_{yz}}{m} = \frac{6750}{31} = 218 \text{ mm}$$ Ans.

$$m y_G = \Sigma m_i y_i = M_{zx} \qquad y_G = \frac{M_{zx}}{m} = \frac{7840}{31} = 253 \text{ mm}$$ Ans.

$$m z_G = \Sigma m_i z_i = M_{xy} \qquad z_G = \frac{M_{xy}}{m} = \frac{4320}{31} = 139.4 \text{ mm}$$ Ans.

5-9* Three bodies with masses of 2, 4, and 6 slugs are located at points (2, 3, 4), (3, -4, 5), and (-3, 4, 6), respectively. Locate the mass center of the system if the distances are measured in feet.

SOLUTION

$$m = \Sigma m_i = m_A + m_B + m_C = 2 + 4 + 6 = 12 \text{ slug}$$

$$M_{yz} = \Sigma m_i x_i = m_A x_A + m_B x_B + m_C x_C$$
$$= 2(2) + 4(3) + 6(-3) = -2 \text{ slug}\cdot\text{ft}$$

$$M_{zx} = \Sigma m_i y_i = m_A y_A + m_B y_B + m_C y_C$$
$$= 2(3) + 4(-4) + 6(4) = 14 \text{ slug}\cdot\text{ft}$$

$$M_{xy} = \Sigma m_i z_i = m_A z_A + m_B z_B + m_C z_C$$
$$= 2(4) + 4(5) + 6(6) = 64 \text{ slug}\cdot\text{ft}$$

$$m x_G = \Sigma m_i x_i = M_{yz} \qquad x_G = \frac{M_{yz}}{m} = \frac{-2}{12} = -0.1667 \text{ ft}$$ Ans.

$$m y_G = \Sigma m_i y_i = M_{zx} \qquad y_G = \frac{M_{zx}}{m} = \frac{14}{12} = 1.167 \text{ ft}$$ Ans.

$$m z_G = \Sigma m_i z_i = M_{xy} \qquad z_G = \frac{M_{xy}}{m} = \frac{64}{12} = 5.33 \text{ ft}$$ Ans.

5-10 Three bodies with masses of 3, 6, and 7 kg are located at points (4, -3, 1), (-1, 3, 2), and (2, 2, -4), respectively. Locate the mass center of the system if the distances are measured in meters.

SOLUTION

$$m = \Sigma m_i = m_A + m_B + m_C = 3 + 6 + 7 = 16 \text{ kg}$$

$$M_{yz} = \Sigma m_i x_i = m_A x_A + m_B x_B + m_C x_C$$
$$= 3(4) + 6(-1) + 7(2) = -20 \text{ kg·m}$$

$$M_{zx} = \Sigma m_i y_i = m_A x_A + m_B x_B + m_C x_C$$
$$= 3(-3) + 6(3) + 7(2) = 23 \text{ kg·m}$$

$$M_{xy} = \Sigma m_i z_i = m_A x_A + m_B x_B + m_C x_C$$
$$= 3(1) + 6(2) + 7(-4) = -13 \text{ kg·m}$$

$$m x_G = \Sigma m_i x_i = M_{yz} \qquad x_G = \frac{M_{yz}}{m} = \frac{20}{16} = 1.25 \text{ m}$$ Ans.

$$m y_G = \Sigma m_i y_i = M_{zx} \qquad y_G = \frac{M_{zx}}{m} = \frac{23}{16} = 1.438 \text{ m}$$ Ans.

$$m z_G = \Sigma m_i z_i = M_{xy} \qquad z_G = \frac{M_{xy}}{m} = \frac{-13}{16} = -0.813 \text{ m}$$ Ans.

5-11* Locate the centroid of the shaded triangular area shown in Fig. P5-11 if b = 12 in. and h = 8 in.

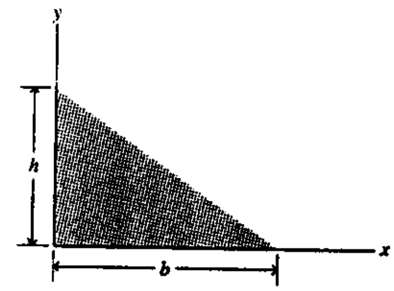
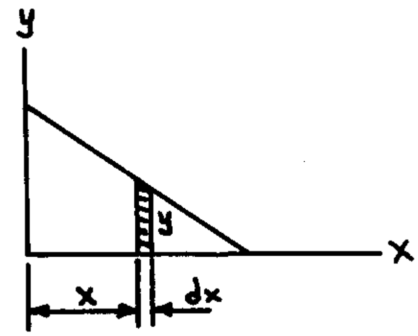


Fig. P5-11

SOLUTION

For the differential element of area shown in the sketch at the right:



$$y = \frac{h}{b}(b - x)$$

$$dA = y\,dx = \frac{h}{b}(b - x)\,dx$$

$$dM_x = \frac{y}{2}\,dA = \frac{h^2}{2b^2}(b - x)^2\,dx$$

$$dM_y = x\,dA = \frac{h}{b}(b - x)\,x\,dx$$

$$M_x = \int_0^b \frac{h^2}{2b^2}(b - x)^2\,dx = \frac{h^2}{2b^2}\int_0^b (b - x)^2\,dx$$

$$= \frac{h^2}{2b^2}\left[b^2x - bx^2 + \frac{x^3}{3}\right]_0^b = \frac{bh^2}{6}$$

$$M_y = \int_0^b \frac{h}{b}(b - x)\,x\,dx = \frac{h}{b}\int_0^b (bx - x^2)\,dx$$

$$= \frac{h}{b}\left[\frac{bx^2}{2} - \frac{x^3}{3}\right]_0^b = \frac{b^2h}{6}$$

$$A = \frac{1}{2}\,bh$$

$Ax_C = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{b^2h/6}{bh/2} = \frac{b}{3} = \frac{12}{3} = 4.00$ in. Ans.

$Ay_C = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{b^2h/6}{bh/2} = \frac{h}{3} = \frac{8}{3} = 2.67$ in. Ans.

5-12* Locate the centroid of the shaded triangular area shown in Fig. P5-12 if b = 200 mm and h = 300 mm.

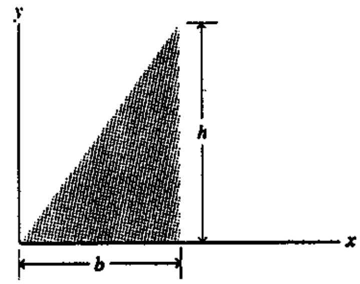
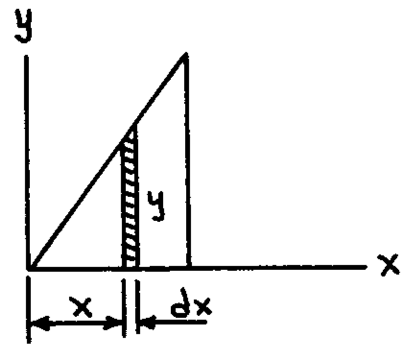


Fig. P5-12

SOLUTION

For the differential element of area shown in the sketch at the right:



$$y = \frac{h}{b}\, x$$

$$dA = y\, dx = \frac{h}{b}\, x\, dx$$

$$dM_x = \frac{y}{2}\, dA = \frac{h^2}{2b^2}\, x^2\, dx$$

$$dM_y = x\, dA = \frac{h}{b}\, x^2\, dx$$

$$M_x = \int_0^b \frac{h^2}{2b^2}\, x^2\, dx = \frac{h^2}{2b^2} \int_0^b x^2\, dx = \frac{h^2}{2b^2}\left[\frac{x^3}{3}\right]_0^b = \frac{h^2 b}{6}$$

$$M_y = \int_0^b \frac{h}{b}\, x^2\, dx = \frac{h}{b} \int_0^b x^2\, dx = \frac{h}{b}\left[\frac{x^3}{3}\right]_0^b = \frac{b^2 h}{3}$$

$$A = \frac{1}{2}\, bh$$

$Ax_C = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{b^2h/3}{bh/2} = \frac{2}{3}\, b = \frac{2}{3}(200) = 133.3 \text{ mm}$ Ans.

$Ay_C = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{h^2b/6}{bh/2} = \frac{h}{3} = \frac{300}{3} = 100.0 \text{ mm}$ Ans.

5-13 Locate the centroid of the shaded area shown in Fig. P5-13.

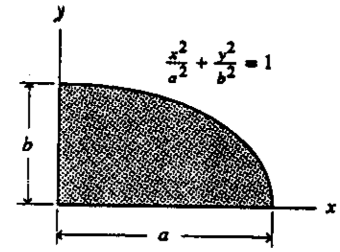
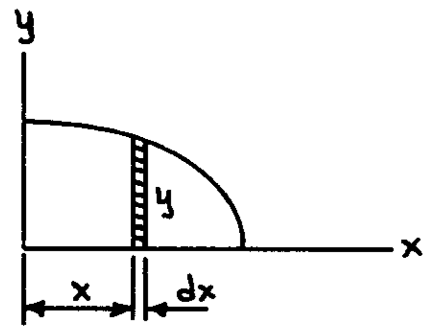


Fig. P5-13

SOLUTION

For the differential element of area shown in the sketch at the right:



$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$dA = y\,dx = \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$dM_x = \frac{y}{2}\,(y\,dx) = \frac{b^2}{2a^2}(a^2 - x^2)\,dx$$

$$M_x = \frac{b^2}{2a^2}\int_0^a x\sqrt{a^2 - x^2}\,dx = \frac{b^2}{2a^2}\left[a^2x - \frac{x^3}{3}\right]_0^a = \frac{ab^2}{3}$$

$$dM_y = x\,(y\,dx) = \frac{b}{a}\,x\sqrt{a^2 - x^2}\,dx$$

$$M_y = \frac{b}{a}\int_0^a x\sqrt{a^2 - x^2}\,dx = \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2 - x^2)^3}\,\right]_0^a = \frac{a^2b}{3}$$

$$A = \int_A y\,dx = \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= \frac{b}{2a}\left[x\sqrt{a^2 - x^2} + a^2\sin^{-1}\frac{x}{a}\right]_0^a = \frac{\pi ab}{4}$$

$Ax_C = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{a^2b/3}{\pi ab/4} = \frac{4a}{3\pi}$ Ans.

$Ay_C = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{ab^2/3}{\pi ab/4} = \frac{4b}{3\pi}$ Ans.

5-14 Locate the centroid of the shaded area shown in Fig. P5-14.

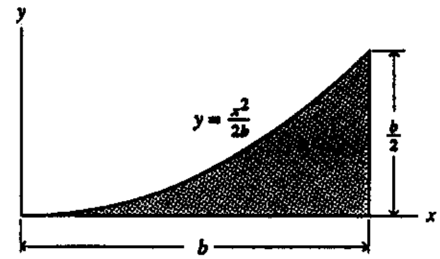
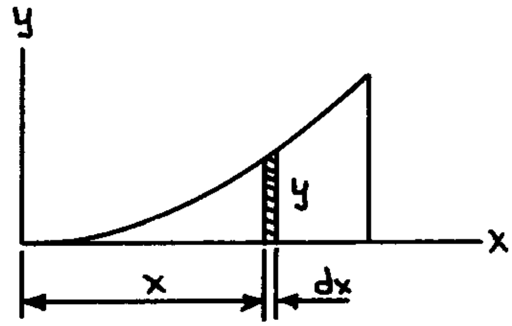


Fig. P5-14

SOLUTION

For the differential element of area shown in the sketch at the right:



$$y = \frac{x^2}{2b}$$

$$dA = y\,dx = \frac{x^2}{2b}\,dx$$

$$dM_x = \frac{y}{2}\,dA) = \frac{x^4}{8b^2}\,dx$$

$$dM_y = x\,dA = \frac{x^3}{2b}\,dx$$

$$M_x = \int_0^b \frac{x^4}{8b^2}\,dx = \frac{1}{8b^2}\int_0^b x^4\,dx = \frac{1}{8b^2}\left[\frac{x^5}{5}\right]_0^b = \frac{b^3}{40}$$

$$M_y = \int_0^b \frac{x^3}{2b}\,dx = \frac{1}{2b}\int_0^b x^3\,dx = \frac{1}{2b}\left[\frac{x^4}{4}\right]_0^b = \frac{b^3}{8}$$

$$A = \int_A y\,dx = \int_0^b \frac{x^2}{2b}\,dx = \frac{1}{2b}\int_0^b x^2\,dx = \frac{1}{2b}\left[\frac{x^3}{3}\right]_0^b = \frac{b^2}{6}$$

$Ax_C = M_y$ $x_C = \frac{M_y}{A} = \frac{b^3/8}{b^2/6} = \frac{3}{4}\,b$ Ans.

$Ay_C = M_x$ $y_C = \frac{M_x}{A} = \frac{b^3/40}{b^2/6} = \frac{3}{20}\,b$ Ans.

5-15* Locate the centroid of the shaded area shown in Fig. P5-15.

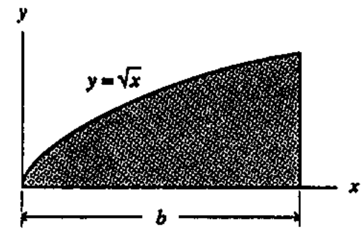
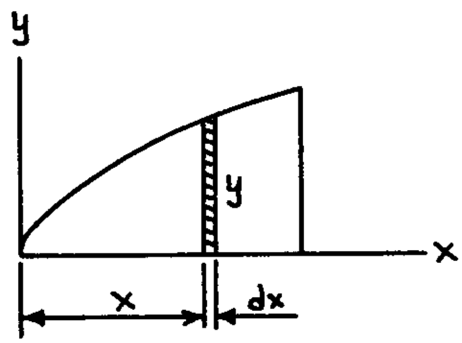


Fig. P5-15

SOLUTION

For the differential element of area shown in the sketch at the right:



$$y = \sqrt{x}$$

$$dA = y\,dx = \sqrt{x}\,dx$$

$$dM_x = \frac{y}{2}\,dA = \frac{x}{2}\,dx$$

$$dM_y = x\,dA = x^{3/2}\,dx$$

$$M_x = \int_0^b \frac{x}{2}\,dx = \frac{1}{2}\int_0^b x\,dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^b = \frac{b^2}{4}$$

$$M_y = \int_0^b x^{3/2}\,dx = \left[\frac{2}{5}x^{5/2}\right]_0^b = \frac{2}{5}b^{5/2}$$

$$A = \int_A y\,dx = \int_0^b \sqrt{x}\,dx = \left[\frac{2}{3}x^{3/2}\right]_0^b = \frac{2}{3}b^{3/2}$$

$Ax_C = M_y$ $\qquad x_C = \dfrac{M_y}{A} = \dfrac{2b^{5/2}/5}{2b^{3/2}/3} = \dfrac{3}{5}b$ Ans.

$Ay_C = M_x$ $\qquad y_C = \dfrac{M_x}{A} = \dfrac{b^2/4}{2b^{3/2}/3} = \dfrac{3}{8}b^{1/2}$ Ans.

5-16 Locate the centroid of the shaded area shown in Fig. P5-16.

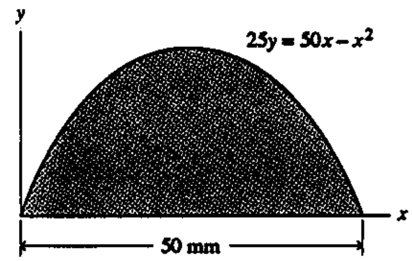
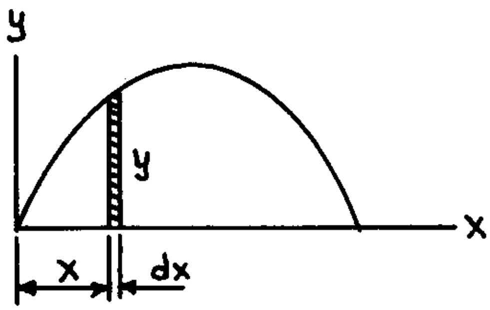


Fig. P5-16

SOLUTION

For the differential element of area shown in the sketch at the right:



$$y = 2x - 0.04\ x^2$$

$$dA = y\ dx = (2x - 0.04\ x^2)\ dx$$

$$dM_x = \frac{y}{2}\ dA = \frac{1}{2}(2x - 0.04x^2)^2\ dx = (2x^2 - 0.08x^3 + 0.0008x^4)\ dx$$

$$dM_y = x\ dA = (2x^2 - 0.04x^3)\ dx$$

$$M_x = \int_0^{50} (x^2 - 0.08x^3 + 0.0008x^4)\ dx$$

$$= \left[\frac{2x^3}{3} - \frac{0.08x^4}{4} + \frac{0.0008x^5}{5}\right]_0^{50} = 8333\ \text{mm}^3$$

$$M_y = \int_0^{50} (2x^2 - 0.04x^3)\ dx = \left[\frac{2x^3}{3} - \frac{0.04x^4}{4}\right]_0^{50} = 20{,}833\ \text{mm}^3$$

$$A = \int_A y\ dx = \int_0^{50} (2x - 0.04\ x^2)\ dx = \left[x^2 - \frac{0.04}{3}\ x^3\right]_0^{50} = 833.3\ \text{mm}^2$$

$Ax_C = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{20{,}833}{833.3} = 25$ mm Ans.

$Ay_C = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{8333}{833.3} = 10$ mm Ans.

5-17* Locate the centroid of the shaded area shown in Fig. P5-17.

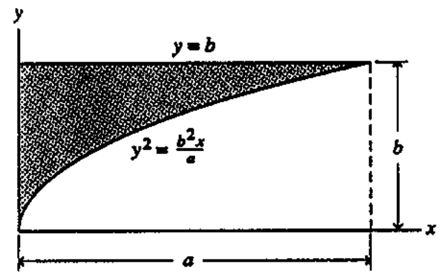
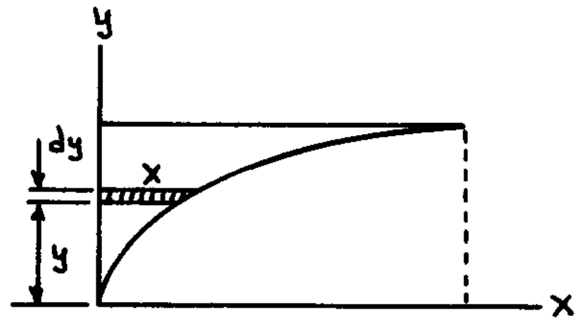


Fig. P5-17

SOLUTION

For the differential element of area shown in the sketch at the right:



$$x = \frac{ay^2}{b^2}$$

$$dA = x\ dy = \frac{ay^2}{b^2}\ dy$$

$$dM_x = y\ dA = \frac{ay^3}{b^2}\ dy$$

$$dM_y = \frac{x}{2}\ dA = \frac{a^2y^4}{2b^4}\ dy$$

$$M_x = \int_0^b \frac{ay^3}{b^4}\ dy = \left[\frac{ay^4}{4b^4}\right]_0^b = \frac{ab^2}{4}$$

$$M_y = \int_0^b \frac{a^2y^4}{2b^4}\ dy = \left[\frac{a^2y^5}{10b^4}\right]_0^b = \frac{a^2b}{10}$$

$$A = \int_A x\ dy = \int_0^b \frac{ay^2}{b^2}\ dy = \left[\frac{ay^3}{3b^2}\right]_0^b = \frac{ab}{3}$$

$Ax_C = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{a^2b/10}{ab/3} = \frac{3a}{10}$ Ans.

$Ay_C = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{ab^2/4}{ab/3} = \frac{3b}{4}$ Ans.

5-18* Locate the centroid of the shaded area shown in Fig. P5-18.

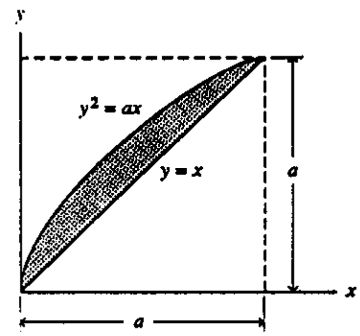
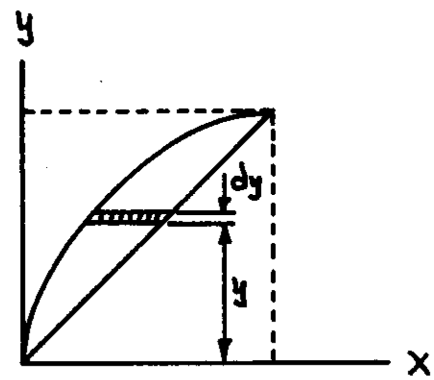


Fig. P5-18

SOLUTION

For the differential element of area shown in the sketch at the right:



$$dA = (x_2 - x_1)\ dy = \left(y - \frac{y^2}{a}\right)\ dy$$

$$dM_x = y\ dA = \left(y^2 - \frac{y^3}{a}\right)\ dy$$

$$dM_y = \frac{1}{2}(x_2 + x_1)\ dA$$

$$= \frac{1}{2}\left(x_2^2 - x_1^2\right)\ dy = \frac{1}{2}\left(y^2 - \frac{y^4}{a^2}\right)\ dy$$

$$M_x = \int_0^a \left(y^2 - \frac{y^3}{a}\right)\ dy = \left[\frac{y^3}{3} - \frac{y^4}{4a}\right]_0^a = \frac{a^3}{12}$$

$$M_y = \int_0^a \frac{1}{2}\left(y^2 - \frac{y^4}{a^2}\right)\ dy = = \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^5}{5a^2}\right]_0^a = \frac{a^3}{15}$$

$$A = \int_0^a \left(y - \frac{y^2}{a}\right)\ dy = \left[\frac{y^2}{2} - \frac{y^2}{3a}\right]_0^a = \frac{a^2}{6}$$

$Ax_C = M_y$ $\qquad x_C = \dfrac{M_y}{A} = \dfrac{a^3/15}{a^2/6} = \dfrac{2a}{5}$ Ans.

$Ay_C = M_x$ $\qquad y_C = \dfrac{M_x}{A} = \dfrac{a^3/12}{a^2/6} = \dfrac{a}{2}$ Ans.

5-19 Locate the centroid of the shaded area shown in Fig. P5-19.

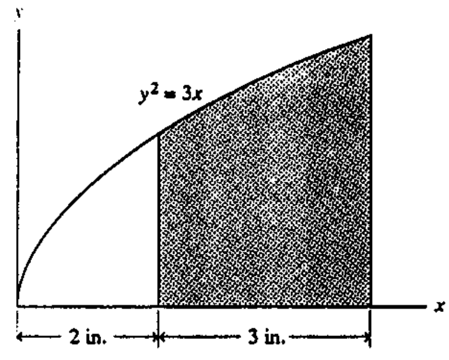
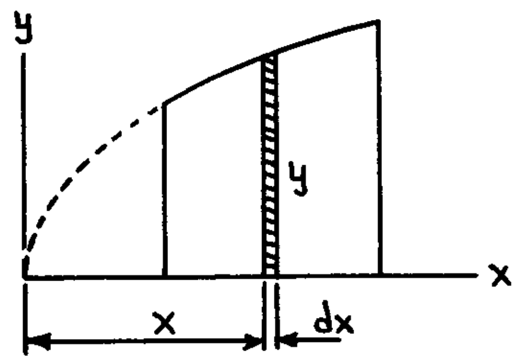


Fig. P5-19

SOLUTION



For the differential element of area shown in the sketch at the right:

$$y = \sqrt{3x}$$

$$dA = y\,dx = \sqrt{3x}\,dx$$

$$dM_x = \frac{y}{2}\,dA = \frac{3x}{2}\,dx$$

$$dM_y = x\,dA = \sqrt{3}\,x^{3/2}\,dx$$

$$M_x = \int_2^5 \frac{3x}{2}\,dx = \left[\frac{3x^2}{4}\right]_2^5 = 15.75 \text{ in.}^3$$

$$M_y = \int_2^5 \sqrt{3}\,x^{3/2}\,dx = \sqrt{3}\left[\frac{x^{5/2}}{5/2}\right]_2^5 = 34.81 \text{ in.}^3$$

$$A = \int_2^5 \sqrt{3x}\,dx = \sqrt{3}\left[\frac{x^{3/2}}{3/2}\right]_2^5 = 9.644 \text{ in.}^2$$

$Ax_C = M_y$ $\qquad x_C = \dfrac{M_y}{A} = \dfrac{34.81}{9.644} = 3.61$ in. Ans.

$Ay_C = M_x$ $\qquad y_C = \dfrac{M_x}{A} = \dfrac{15.75}{9.644} = 1.633$ in. Ans.

5-20 Locate the centroid of the shaded area shown in Fig. P5-20.

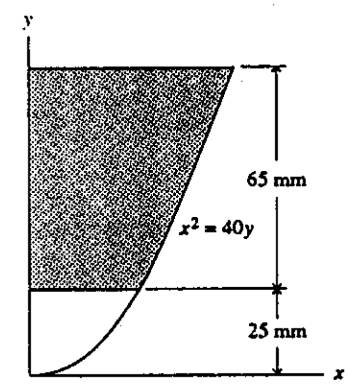
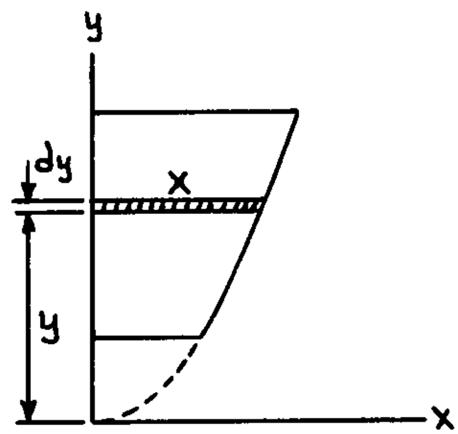


Fig. P5-20

SOLUTION

For the differential element of area shown in the sketch at the right:



$$x = \sqrt{40y}$$

$$dA = x\ dy = \sqrt{40y}\ dy$$

$$dM_x = y\ dA = \sqrt{40}\ y^{3/2}\ dy$$

$$dM_y = \frac{x}{2}\ dA = 20\ y\ dy$$

$$M_x = \int_{25}^{90} \sqrt{40}\ y^{3/2}\ dy = \sqrt{40}\ \left[\frac{y^{5/2}}{5/2}\right]_{25}^{90} = 186{,}494\ \text{mm}^3$$

$$M_y = \int_{25}^{90} 20\ y\ dy = 20\left[\frac{y^2}{2}\right]_{25}^{90} = 74{,}750\ \text{mm}^3$$

$$A = \int_{25}^{90} \sqrt{40y}\ dy = \left.\frac{\sqrt{40}\ y^{3/2}}{3/2}\right]_{25}^{90} = 3073\ \text{mm}^2$$

$Ax_C = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{186{,}494}{3073} = 24.3\ \text{mm}$ Ans.

$Ay_C = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{74{,}750}{3073} = 60.7\ \text{mm}$ Ans.

5-21 Locate the centroid of the shaded area shown in Fig. P5-21.

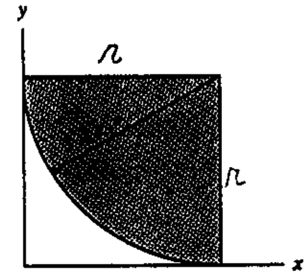
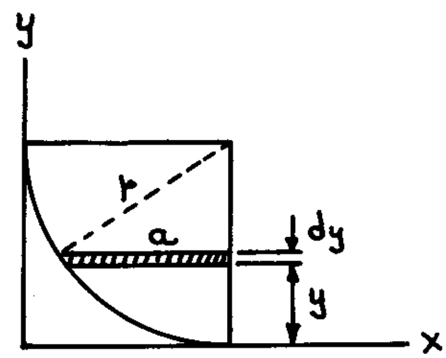


Fig. P5-21

SOLUTION

For the differential element of area shown in the sketch at the right:



$$a^2 = r^2 - (r - y)^2$$

$$a = \sqrt{2ry - y^2}$$

$$dA = a\,dy = \sqrt{2ry - y^2}\,dy$$

$$dM_x = y\,dA = y\sqrt{2ry - y^2}\,dy$$

$$M_x = \int_0^r y\sqrt{2ry - y^2}\,dy$$

$$= \left[-\frac{(2ry - y^2)^{3/2}}{3} + \frac{r(y - r)}{2}\sqrt{2ry - y^2} + \frac{r^3}{2}\sin^{-1}\frac{y - r}{r}\right]_0^r$$

$$= \frac{3\pi r^3 - 4r^3}{12}$$

$$A = \frac{1}{4}\pi r^2$$

$Ay_C = M_x$ $y_C = \dfrac{M_x}{A} = \dfrac{(3\pi r^3 - 4r^3)/12}{\pi r^2/4} = r - \dfrac{4r}{3\pi}$ Ans.

Similarly:

$Ax_C = M_y$ $x_C = r - \dfrac{4r}{3\pi}$ Ans.

5-22 Locate the centroid of the shaded area shown in Fig. P5-22.

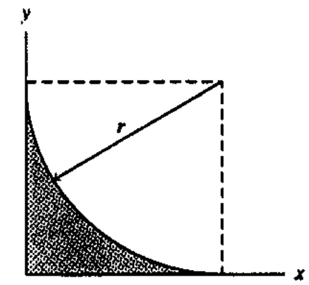
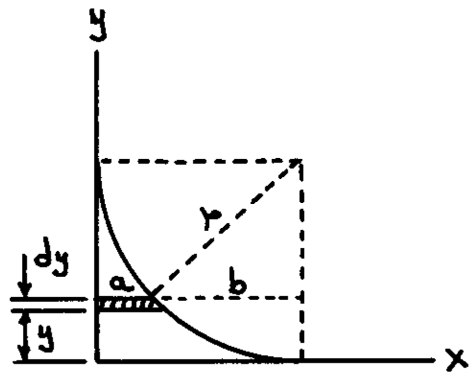


Fig. P5-22

SOLUTION

For the differential element of area shown in the sketch at the right:



$$b^2 = r^2 - (r - y)^2$$

$$b = \sqrt{2ry - y^2}$$

$$a = r - \sqrt{2ry - y^2}$$

$$dA = a\,dy = \left(r - \sqrt{2ry - y^2}\right) dy$$

$$dM_x = y\,dA = y\left(r - \sqrt{2ry - y^2}\right) dy$$

$$M_x = \int_0^r \left(ry - y\sqrt{2ry - y^2}\right) dy$$

$$= \left[\frac{ry^2}{2} + \frac{(2ry - y^2)^{3/2}}{3} - \frac{r(y - r)}{2}\sqrt{2ry - y^2} - \frac{r^3}{2}\sin^{-1}\frac{y - r}{r}\right]_0^r$$

$$= \frac{(10 - 3\pi)r^3}{12}$$

$$A = r^2 - \frac{1}{4}\pi r^2 = \frac{(4 - \pi)}{4}r^2$$

$Ay_C = M_x$ $\qquad y_C = \dfrac{M_x}{A} = \dfrac{(10 - 3\pi)r^3/12}{(4 - \pi)r^2/4} = \dfrac{(10 - 3\pi)}{(12 - 3\pi)}r$ Ans.

Similarly:

$Ax_C = M_y$ $\qquad x_C = \dfrac{(10 - 3\pi)}{(12 - 3\pi)}r$ Ans.

5-23 Locate the centroid of the volume obtained by revolving the shaded area shown in Fig. P5-23 about the x-axis.

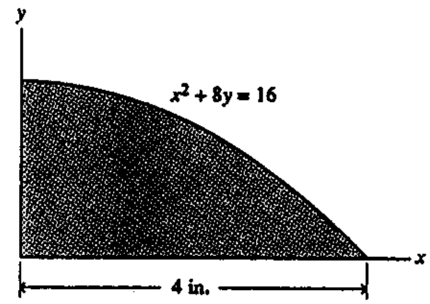
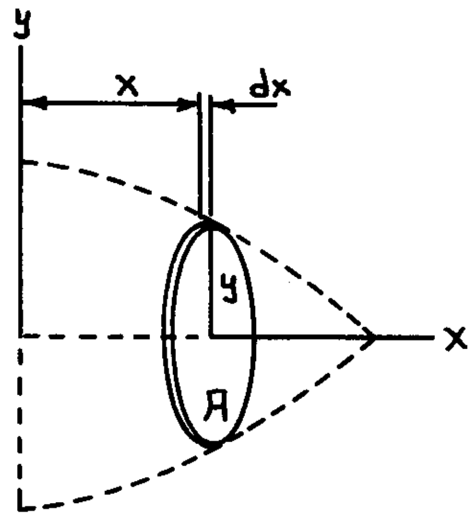


Fig. P5-23

SOLUTION

For the differential element of volume shown in the sketch at the right:



$$y = \frac{1}{8}(16 - x^2)$$

$$A = \pi y^2 = \frac{\pi}{64}(16 - x^2)^2$$

$$dV = A\,dx = \frac{\pi}{64}(16 - x^2)^2\,dx$$

$$= \frac{\pi}{64}(256 - 32x + x^4)\,dx$$

$$M_{yz} = \int_V x\,dV = \frac{\pi}{64}\int_0^4 (256x - 32x^3 + x^5)\,dx$$

$$= \frac{\pi}{64}\left[\frac{256x^2}{2} - \frac{32x^4}{4} + \frac{x^6}{6}\right]_0^4 = 33.51 \text{ in.}^4$$

$$V = \frac{\pi}{64}\int_0^4 (256 - 32x + x^4)\,dx = \frac{\pi}{64}\left[256x - \frac{32x^3}{3} + \frac{x^5}{5}\right]_0^4 = 26.81 \text{ in.}^3$$

$Vx_C = M_{yz}$ $x_C = \dfrac{M_{yz}}{V} = \dfrac{33.51}{26.81} = 1.250$ in.   Ans.

From symmetry:

$y_C = z_C = 0$ Ans.

5-24 Locate the centroid of the volume obtained by revolving the shaded area shown in Fig. P5-24 about the x-axis.

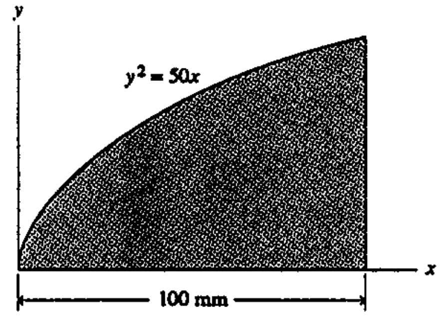
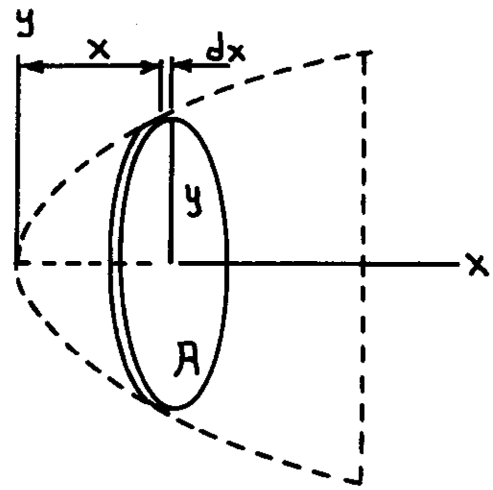


Fig. P5-24

SOLUTION

For the differential element of volume shown in the sketch at the right:



$$A = \pi y^2 = \pi(50x) = 50\pi\ x$$

$$dV = A\ dx = 50\pi\ x\ dx$$

$$M_{yz} = \int_V x\ dV = \int_0^b 50\pi\ x^2\ dx = 50\pi\left[\frac{x^3}{3}\right]_0^b = \frac{50\pi b^3}{3}$$

$$V = \int_0^b 50\pi\ x\ dx = 50\pi\left[\frac{x^2}{2}\right]_0^b = 25\pi b^2$$

$$Vx_C = M_{yz} \qquad x_C = \frac{M_{yz}}{V} = \frac{50\pi b^3/3}{25\pi b^2} = \frac{2}{3}\ b$$

With b = 100 mm:

$$x_C = \frac{2}{3}(100) = 66.7\ \text{mm}$$ Ans.

From symmetry:

$$y_C = z_C = 0$$ Ans.

5-25 Locate the centroid of the curved slender rod shown in Fig. P5-25.

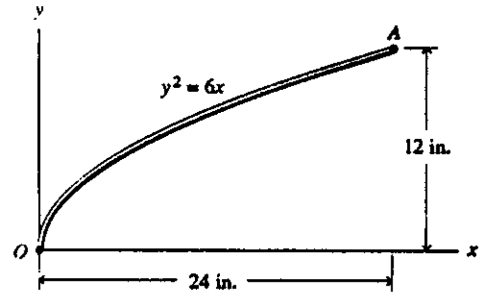
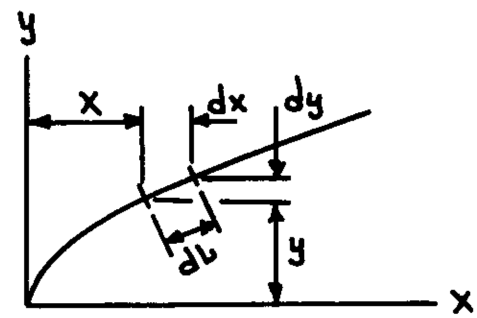


Fig. P5-25

SOLUTION

For the differential element of length shown in the sketch at the right:



$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{(dx/dy)^2 + 1}\ dy$$

$$y^2 = 6x \qquad \frac{dx}{dy} = \frac{y}{3}$$

$$dL = \sqrt{(y/3)^2 + 1}\ dy = \frac{1}{3}\sqrt{y^2 + 9}\ dy$$

$$M_x = \int_L y\ dL = \int_0^{12} \frac{1}{3}\, y\sqrt{y^2 + 9}\ dy = \frac{1}{3}\left[\frac{1}{3}\sqrt{(y^2 + 9)^3}\ \right]_0^{12} = 207.28\ \text{in}^2$$

$$M_y = \int_L x\ dL = \int_0^{12} \frac{1}{18}\, y^2\sqrt{y^2 + 9}\ dy$$

$$= \frac{1}{18}\left[\frac{y}{4}\sqrt{(y^2 + 9)^3} - \frac{9y}{8}\sqrt{y^2 + 9} - \frac{81}{8}\ln\left(y + \sqrt{y^2 + 9}\ \right)\right]_0^{12}$$

$$= 304.96\ \text{in}^2$$

$$L = \int_0^{12} \frac{1}{3}\sqrt{y^2 + 9} = \frac{1}{3}\left[\frac{y}{2}\sqrt{y^2 + 9} + \frac{9}{2}\ln\left(y + \sqrt{y^2 + 9}\ \right)\right]_0^{12} = 27.88\ \text{in.}$$

$Lx_C = M_y$ $\qquad x_C = \dfrac{M_y}{L} = \dfrac{304.96}{27.88} = 10.94$ in. Ans.

$Ly_C = M_x$ $\qquad y_C = \dfrac{M_x}{L} = \dfrac{207.28}{27.88} = 7.43$ in. Ans.

5-26 Locate the centroid of the curved slender rod shown in Fig. P5-26 if b = 50 mm.

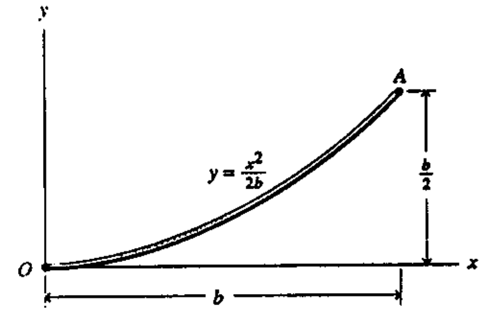
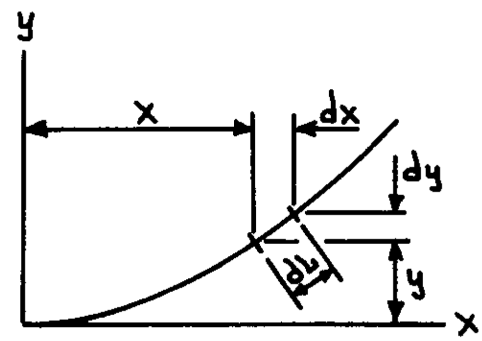


Fig. P5-26

SOLUTION

For the differential element of length shown in the sketch at the right:



$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + (dy/dx)^2}\ dx$$

$$y = \frac{x^2}{2b} \qquad \frac{dy}{dx} = \frac{x}{b}$$

$$dL = \sqrt{1 + (x/b)^2}\ dx = \frac{1}{b}\sqrt{b^2 + x^2}\ dx$$

$$M_x = \int_L y\ dL = \int_0^b \frac{x^2}{2b^2}\sqrt{b^2 + x^2}\ dx$$

$$= \frac{1}{2b^2}\left[\frac{x}{4}\sqrt{(b^2 + x^2)^3} - \frac{b^2 x}{8}\sqrt{b^2 + x^2} - \frac{b^4}{8}\ln\left(x + \sqrt{b^2 + x^2}\right)\right]_0^b$$

$$= 0.2101b^2\ \ 0.2101(50)^2 = 525.25\ \text{mm}^2$$

$$M_y = \int_L x\ dL = \int_0^b \frac{1}{b}\, x\sqrt{b^2 + x^2}\ dx = \frac{1}{b}\left[\frac{1}{3}\sqrt{(b^2 + x^2)^3}\ \right]_0^b$$

$$= 0.6095b^2 = 0.6095(50)^2 = 1523.75\ \text{mm}^2$$

$$L = \int_0^b \frac{1}{b}\sqrt{b^2 + x^2}\ dx = \frac{1}{b}\left[\frac{x}{2}\sqrt{b^2 + x^2} + \frac{b^2}{2}\ln\left(x + \sqrt{b^2 + x^2}\right)\right]_0^b$$

$$= 1.1478b = 1.1478(50) = 57.39\ \text{mm}$$

$L x_C = M_y$ $\qquad x_C = \frac{M_y}{L} = \frac{1523.75}{57.39} = 26.6\ \text{mm}$ Ans.

$L y_C = M_x$ $\qquad y_C = \frac{M_x}{L} = \frac{525.25}{57.39} = 9.15\ \text{mm}$ Ans.

5-27 Locate the centroid of the volume of the portion of a right circular cone shown in Fig. P5-27.

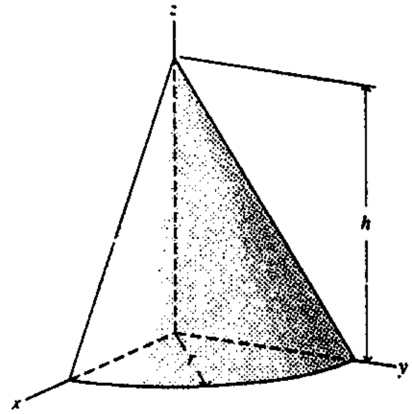
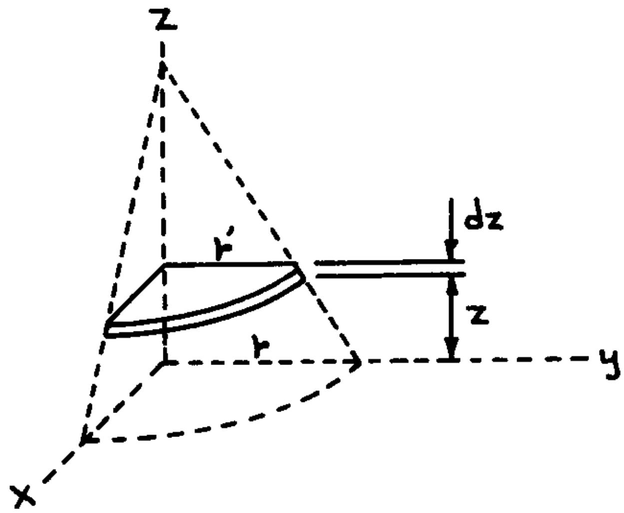


Fig. P5-27

SOLUTION

For the differential element of volume shown in the sketch at the right:



From similar triangles:

$$r' = \frac{r}{h}(h - z)$$

$$A = \frac{1}{4}\pi r'^2 = \frac{\pi r^2}{4h^2}(h - z)^2$$

$$dV = A\,dz = \frac{\pi r^2}{4h^2}(h - z)^2\,dz$$

$$M_{xy} = \int_V z\,dV = \int_0^h \frac{\pi r^2}{4h^2}\,z\,(h - z)^2\,dz = \frac{\pi r^2}{4h^2}\left[\frac{h^2z^2}{2} - \frac{2hz^3}{3} + \frac{z^4}{4}\right]_0^h = \frac{\pi r^2h^2}{48}$$

$$dM_{yz} = \frac{4r'}{3\pi}\,dV = \frac{4}{3\pi}\left(\frac{\pi r^3}{4h^3}\right)(h - z)^3\,dz = \frac{r^3}{3h^3}(h - z)^3\,dz$$

$$M_{yz} = \int_0^h \frac{r^3}{3h^3}(h - z)^3\,dz = \frac{r^3}{3h^3}\left[\frac{(h - z)^4}{4}\right]_0^h = \frac{r^3h}{12}$$

$$V = \int_0^h \frac{\pi r^2}{4h^2}(h - z)^2\,dz = \frac{\pi r^2}{4h^2}\left[\frac{(h - z)^3}{3}\right]_0^h = \frac{\pi r^2h}{12}$$

$Vx_C = M_{yz}$ $\qquad x_C = y_C = \dfrac{M_{yz}}{V} = \dfrac{r^3h/12}{\pi r^2h/12} = \dfrac{r}{\pi}$ Ans.

$Vz_C = M_{xy}$ $\qquad z_C = \dfrac{M_{xy}}{V} = \dfrac{\pi r^2h^2/48}{\pi r^2h/12} = \dfrac{h}{4}$ Ans.

5-28 Locate the mass center of the hemisphere shown in Fig. P5-28 if the density ρ at any point P is proportional to the distance from the xy-plane to point P.

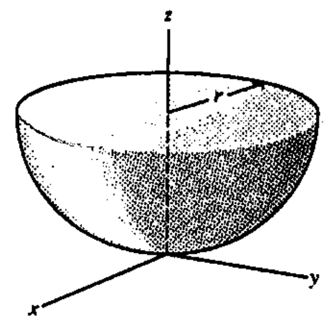
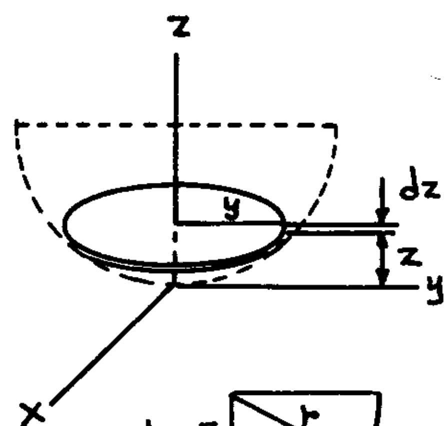
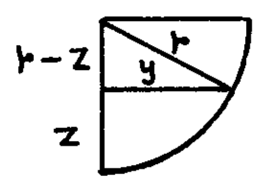


Fig. P5-28

SOLUTION

For the differential element of volume shown in the sketch at the right:





$$r^2 = y^2 + (r - z)^2$$

$$y^2 = 2rz - z^2$$

$$dm = \rho\, dV = kz\,(\pi y^2)\, dz$$

$$= k\pi(2rz^2 - z^3)\, dz$$

$$m = k\pi \int_0^r (2rz^2 - z^3)\, dz = k\pi\left[\frac{2rz^3}{3} - \frac{z^4}{4}\right]_0^r = \frac{5k\pi r^4}{12}$$

$$M_{xy} = \int_m z\, dm = k\pi \int_0^h (2rz^3 - z^4)\, dz = k\pi\left[\frac{2rz^4}{4} - \frac{z^5}{5}\right]_0^r = \frac{3k\pi r^5}{10}$$

$$mz_C = M_{xy} \qquad z_C = \frac{M_{xy}}{m} = \frac{3k\pi r^5/10}{5k\pi r^4/12} = \frac{18}{25}\, r$$ Ans.

From symmetry:

$$x_C = y_C = 0$$ Ans.

5-29 Locate the mass center of the right circular cone shown in Fig. 5-29 if the density ρ at any point P is proportional to the distance from the xy-plane to point P.

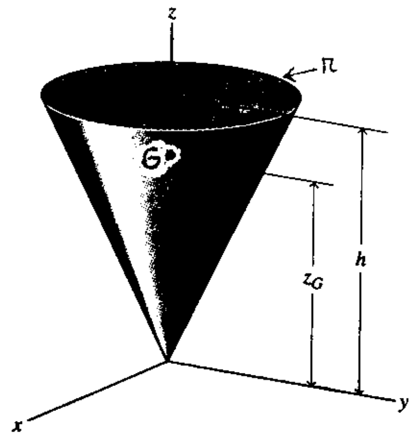
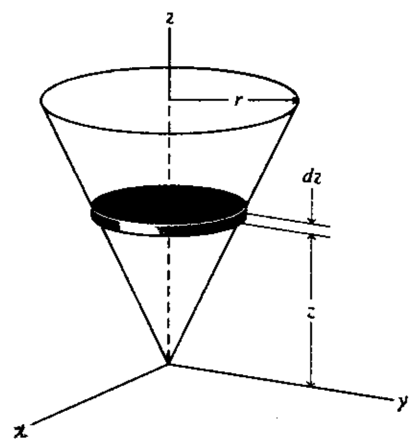


Fig. P5-29

SOLUTION

For the differential element of volume shown in the sketch at the right:



$$dm = \rho\ dV = kz\ dV = kz\ (\pi y^2\ dz)$$

$$= kz\ \pi\left(\frac{rz}{h}\right)^2 dz = \frac{k\pi r^2}{h^2}\ z^3\ dz$$

$$M_{xy} = \int_m z\ dm = \frac{k\pi r^2}{h^2}\int_0^h z^4\ dz = \frac{k\pi r^2}{h^2}\left[\frac{z^5}{5}\right]_0^h = \frac{k\pi r^2 h^3}{5}$$

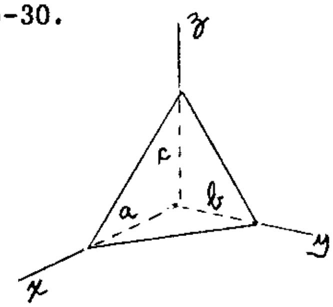
$$m = \frac{k\pi r^2}{h^2}\int_0^h z^3\ dz = \frac{k\pi r^2}{h^2}\left[\frac{z^4}{4}\right]_0^h = \frac{k\pi r^2 h^2}{4}$$

$m z_C = M_{xy}$ $\qquad z_C = \dfrac{M_{xy}}{m} = \dfrac{k\pi r^2 h^3/5}{k\pi r^2 h^2/4} = \dfrac{4}{5}\ h$ Ans.

From symmetry:

$x_C = y_C = 0$ Ans.

5-30 Locate the centroid of the volume of the tetrahedron shown in Fig. P5-30.



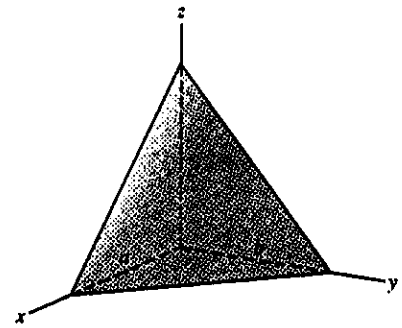
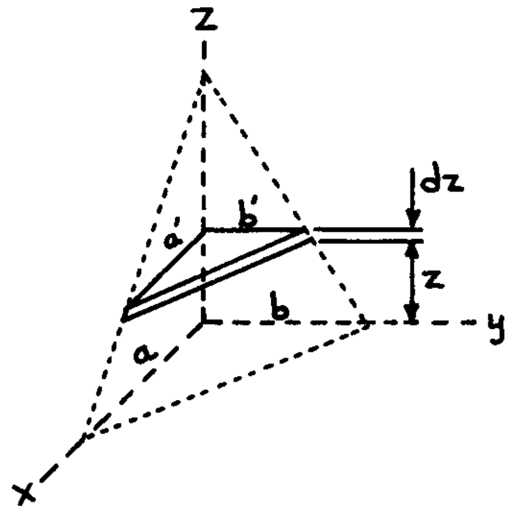


Fig. P5-30

SOLUTION

For the differential element of volume shown in the sketch at the right:



$$M_{xy} = \int_V z\,dV = \int_0^c \frac{ab}{2c^2}\, z\,(c - z)^2\,dz$$

$$= \frac{ab}{2c^2}\int_0^c (c^2 z - 2cz^2 + z^3)\,dz$$

$$= \frac{ab}{2c^2}\left[\frac{c^2 z^2}{2} - \frac{2cz^3}{3} + \frac{z^4}{4}\right]_0^c = \frac{abc^2}{24}$$

$$V = \int_0^c \frac{ab}{2c^2}(c - z)^2\,dz = \frac{ab}{2c^2}\left[c^2 z - \frac{2cz^2}{2} + \frac{z^3}{3}\right]_0^c = \frac{1}{6}\,abc$$

$Vz_C = M_{xy}$ $\qquad z_C = \frac{M_{xy}}{V} = \frac{abc^2/24}{abc/6} = \frac{c}{4}$ Ans.

Similarly: $\qquad x_C = \frac{a}{4}$ Ans.

$y_C = \frac{b}{4}$ Ans.

5-31 Locate the centroid of the slender rod shown in Fig. P5-31.

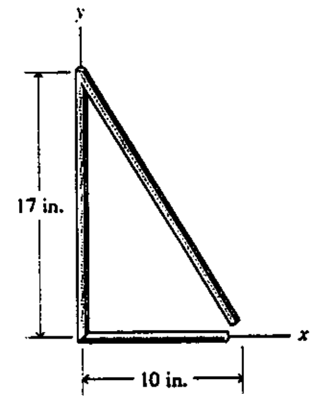
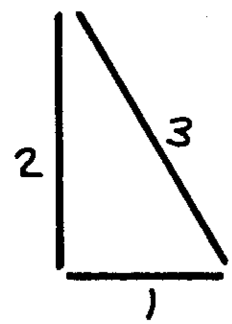


Fig. P5-31

SOLUTION

The rod can be divided into three parts. The centroid for the composite rod is determined by listing the length, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$L_3 = \sqrt{(10)^2 + (17)^2} = 19.723 \text{ in.}$$

Part	L_i (in.)	x_{Ci} (in.)	M_y (in.^{2})	y_{Ci} (in.)	M_x (in.^{2})
1	10	5	50	0	0
2	17	0	0	8.5	144.5
3	19.723	5	98.62	8.5	167.65
Σ	46.723		148.62		312.15

$Lx_C = \Sigma L_i x_{Ci} = M_y$ $\qquad x_C = \dfrac{M_y}{L} = \dfrac{148.62}{46.723} = 3.18 \text{ in.}$ Ans.

$Ly_C = \Sigma L_i y_{Ci} = M_x$ $\qquad y_C = \dfrac{M_x}{L} = \dfrac{312.15}{46.723} = 6.68 \text{ in.}$ Ans.

5-32 Locate the centroid of the slender rod shown in Fig. P5-32.

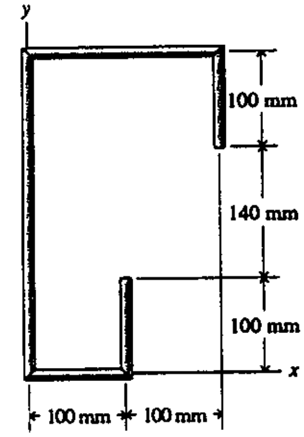
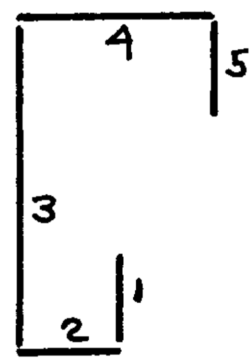


Fig. P5-32

SOLUTION



The rod can be divided into five parts. The centroid for the composite rod is determined by listing the length, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

Part	L_i (mm)	x_{Ci} (mm)	M_y (mm^2)	y_{Ci} (mm)	M_x (mm^2)
1	100	100	10,000	50	5000
2	100	50	5000	0	0
3	340	0	0	170	57,800
4	200	100	20,000	340	68,000
5	100	200	20,000	290	29,000
Σ	840		55,000		159,800

$Lx_C = \Sigma L_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{L} = \frac{55,000}{840} = 65.5$ mm Ans.

$Ly_C = \Sigma L_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{L} = \frac{159,800}{840} = 190.2$ mm Ans.

5-33 Locate the centroid of the shaded area shown in Fig. P5-33.

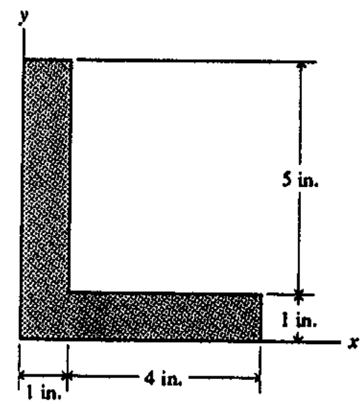
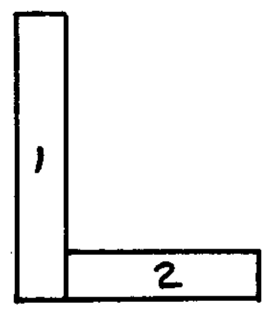


Fig. P5-33

SOLUTION

The shaded area can be divided into two rectangles. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



Part	A_i ($in.^2$)	x_{Ci} (in.)	M_y ($in.^3$)	y_{Ci} (in.)	M_x ($in.^3$)
1	6	0.5	3	3	18
2	4	3	12	0.5	2
Σ	10		15		20

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{15}{10} = 1.5$ in. Ans.

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{20}{10} = 2.0$ in. Ans.

5-34 Locate the centroid of the shaded area shown in Fig. P5-34.

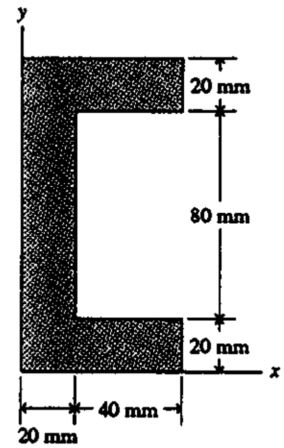
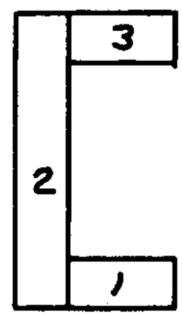


Fig. P5-34

SOLUTION



The shaded area can be divided into three rectangles. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

Part	A_i (mm^2)	x_{Ci} (mm)	M_y (mm^3)	y_{Ci} (mm)	M_x (mm^3)
1	800	40	32,000	10	8000
2	2400	10	24,000	60	144,000
3	800	40	32,000	110	88,000
Σ	4000		88,000		240,000

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{88,000}{4000} = 22.0 \text{ mm}$ Ans.

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{240,000}{4000} = 60.0 \text{ mm}$ Ans.

5-35 Locate the centroid of the shaded area shown in Fig. P5-35.

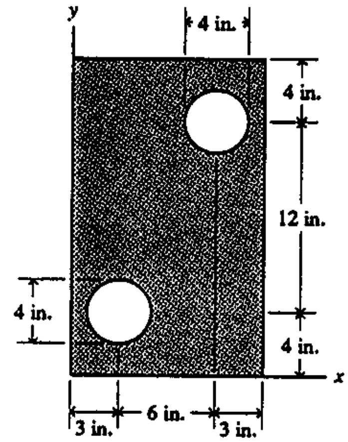
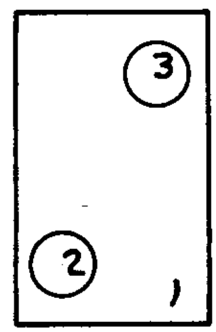


Fig. P5-35

SOLUTION



The shaded area can be divided into a rectangle with two circles removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

Part	A_i ($in.^2$)	x_{Ci} (in.)	M_y ($in.^3$)	y_{Ci} (in.)	M_x ($in.^3$)
1	240	6	1440	10	2400
2	-12.57	3	-37.71	4	-50.28
3	-12.57	9	-113.13	16	-201.12
Σ	214.86		1289.16		2148.60

$$Ax_C = \Sigma A_i x_{Ci} = M_y \qquad x_C = \frac{M_y}{A} = \frac{1289.16}{214.86} = 6.00 \text{ in.} \qquad \text{Ans.}$$

$$Ay_C = \Sigma A_i y_{Ci} = M_x \qquad y_C = \frac{M_x}{A} = \frac{2148.60}{214.86} = 10.00 \text{ in.} \qquad \text{Ans.}$$

5-36 Locate the centroid of the shaded area shown in Fig. P5-36.

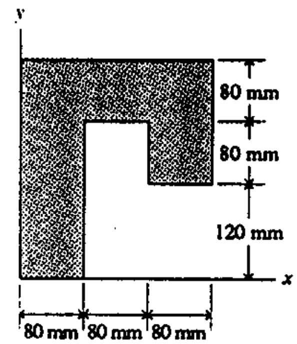
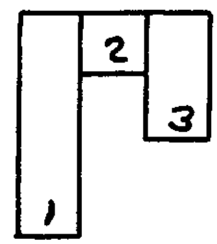


Fig. P5-36

SOLUTION



The shaded area can be divided into three rectangles. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

Part	A_i (mm^2)	x_{Ci} (mm)	M_y (mm^3)	y_{Ci} (mm)	M_x (mm^3)
1	22,400	40	896,000	140	3,136,000
2	6400	120	768,000	240	1,536,000
3	12,800	200	2,560,000	200	2,560,000
Σ	41,600		4,224,000		7,232,000

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $x_C = \dfrac{M_y}{A} = \dfrac{4,224,000}{41,600} = 101.5$ mm   Ans.

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $y_C = \dfrac{M_x}{A} = \dfrac{7,232,000}{41,600} = 173.8$ mm   Ans.

5-37 Locate the centroid of the shaded area shown in Fig. P5-37.

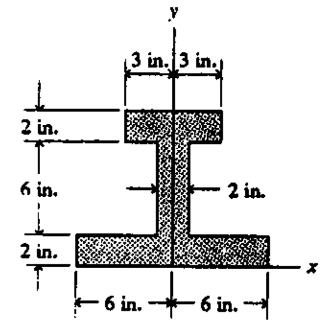
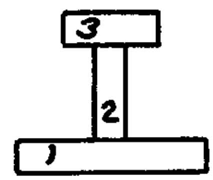


Fig. P5-37

SOLUTION



The shaded area can be divided into three rectangles. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

Part	A_i ($in.^2$)	x_{Ci} (in.)	M_y ($in.^3$)	y_{Ci} (in.)	M_x ($in.^3$)
1	24	0	0	1	24
2	12	0	0	5	60
3	12	0	0	9	108
Σ	48		0		192

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $x_C = \dfrac{M_y}{A} = \dfrac{0}{48} = 0$ Ans.

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $y_C = \dfrac{M_x}{A} = \dfrac{192}{48} = 4.00$ in.   Ans.

(Note symmetry about y axis)

5-38 Locate the centroid of the shaded area shown in Fig. P5-38.

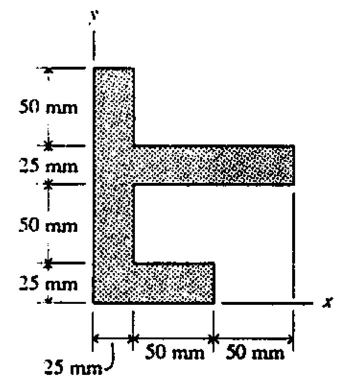
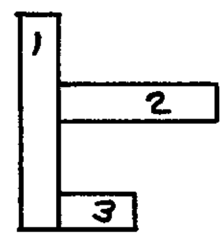


Fig. P5-38

SOLUTION

The shaded area can be divided into three rectangles. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



Part	A_i (mm^2)	x_{Ci} (mm)	M_y (mm^3)	y_{Ci} (mm)	M_x (mm^3)
1	3750	12.5	46,875	75	281,250
2	2500	75	187,500	87.5	218,750
3	1250	50	62,500	12.5	15,625
Σ	7500		296,875		515,625

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $\qquad x_C = \dfrac{M_y}{A} = \dfrac{296{,}875}{7500} = 39.6 \text{ mm}$ Ans.

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $\qquad y_C = \dfrac{M_x}{A} = \dfrac{515{,}625}{7500} = 68.8 \text{ mm}$ Ans.

5-39 Locate the centroid of the shaded area shown in Fig. P5-39.

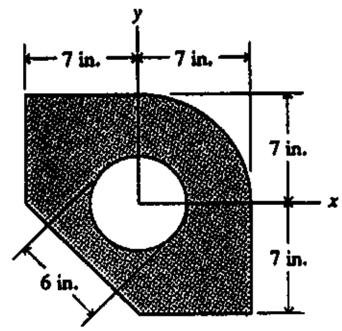
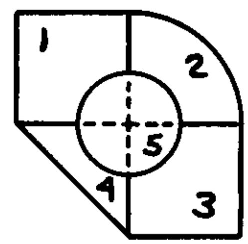


Fig. P5-39

SOLUTION



The shaded area can be divided into two squares, a quarter circle, and a triangle with a circle removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

$$A_2 = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(7)^2 = 38.48 \text{ in.}^2$$

$$x_{C2} = y_{C2} = \frac{4r}{3\pi} = \frac{4(7)}{3\pi} = 2.971 \text{ in.}$$

$$A_4 = \pi r^2 = \pi(3)^2 = 28.27 \text{ in.}^2$$

Part	A_i (in.^{2})	x_{Ci} (in.)	M_y (in.^{3})	y_{Ci} (in.)	M_x (in.^{3})
1	49	-3.5	-171.5	3.5	171.5
2	38.48	2.971	114.32	2.971	114.32
3	49	3.5	171.5	-3.5	-171.5
4	24.5	-2.333	-57.16	-2.333	-57.16
5	-28.27	0	0	0	0
Σ	132.71		57.16		57.16

$A x_C = \Sigma A_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{57.2}{132.71} = 0.431$ in. Ans.

$A y_C = \Sigma A_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{57.2}{132.71} = 0.431$ in. Ans.

5-40 Locate the centroid of the shaded area shown in Fig. P5-40.

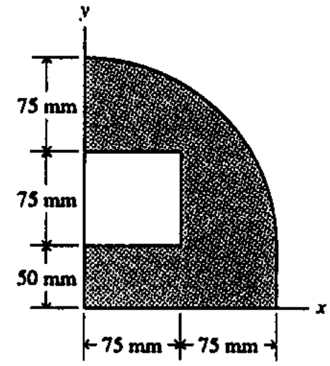
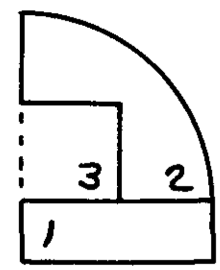


Fig. P5-40

SOLUTION



The shaded area can be divided into a rectangle, and a quarter circle, with a square removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

$$A_2 = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(150)^2 = 17,671 \text{ mm}^2$$

$$x_{C2} = \frac{4r}{3\pi} = \frac{4(150)}{3\pi} = 63.66 \text{ mm}$$

$$y_{C2} = 50 + \frac{4r}{3\pi} = 50 + \frac{4(150)}{3\pi} = 113.66 \text{ mm}$$

Part	A_i (mm^2)	x_{Ci} (mm)	M_y (mm^3)	y_{Ci} (mm)	M_x (mm^3)
1	7500	75	562,500	25	187,500
2	17,671	63.66	1,124,936	113.66	2,008,486
3	-5625	37.5	-210,938	87.5	-492,188
Σ	19,546		1,476,498		1,703,798

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $x_C = \frac{M_y}{A} = \frac{1,476,498}{19,546} = 75.5$ mm   Ans.

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $y_C = \frac{M_x}{A} = \frac{1,703,798}{19,546} = 87.2$ mm   Ans.

5-41 Locate the centroid of the slender rod shown in Fig. P5-41.

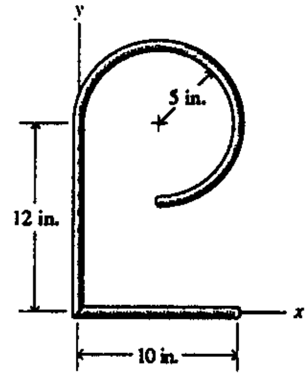
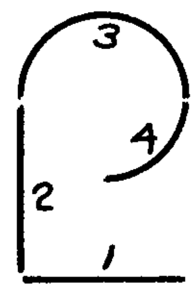


Fig. P5-41

SOLUTION

The rod can be divided into four parts. The centroid for the composite rod is determined by listing the length, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$L_3 = \pi r = \pi(5) = 15.708 \text{ in.}$$

$$L_4 = \frac{1}{2}\pi r = \frac{1}{2}\pi(5) = 7.854 \text{ in.}$$

$$y_{C3} = 12 + \frac{2r}{\pi} = 12 + \frac{2(5)}{\pi} = 15.183 \text{ in.}$$

$$y_{C4} = 12 - \frac{2r}{\pi} = 12 - \frac{2(5)}{\pi} = 8.817 \text{ in.}$$

$$x_{C4} = 5 + \frac{2r}{\pi} = 5 + \frac{2(5)}{\pi} = 8.183 \text{ in.}$$

Part	L_i (in.)	x_{Ci} (in.)	M_y (in.^{2})	y_{Ci} (in.)	M_x (in.^{2})
1	10	5	50	0	0
2	12	0	0	6	72
3	15.708	5	78.54	15.183	238.49
4	7.854	8.183	64.27	8.817	69.25
Σ	45.56		192.81		379.74

$Lx_C = \Sigma L_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{L} = \frac{192.81}{45.56} = 4.23$ in. Ans.

$Ly_C = \Sigma L_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{L} = \frac{379.74}{45.56} = 8.33$ in. Ans.

5-42 Locate the centroid of the slender rod shown in Fig. P5-42.

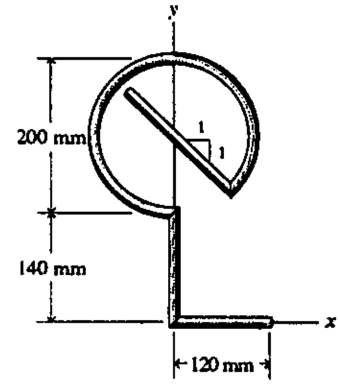
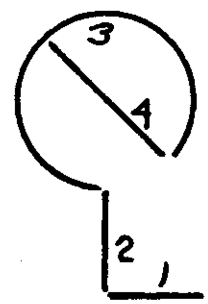


Fig. P5-42

SOLUTION

The rod can be divided into four parts. The centroid for the composite rod is determined by listing the length, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$\alpha = \frac{7}{8}\pi = 157.5^\circ$$

$$L_3 = 2r\alpha = 2(100)(7\pi/8) = 549.78 \text{ mm}$$

$$n_C = \frac{r \sin \alpha}{\alpha} = \frac{100 \sin 157.5^\circ}{7\pi/8} = 13.92 \text{ mm}$$

$$y_{C3} = 240 + 13.92 \cos 22.5^\circ = 252.86 \text{ mm}$$

$$x_{C3} = -13.92 \sin 22.5^\circ = -5.33 \text{ mm}$$

Part	L_i (mm)	x_{Ci} (mm)	M_y (mm^2)	y_{Ci} (mm)	M_x (mm^2)
1	120	60	7200	0	0
2	140	0	0	70	9800
3	549.78	-5.33	-2930	252.86	139,017
4	200	0	0	240	48,000
Σ	1009.78		4270		196,817

$Lx_C = \Sigma L_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{L} = \frac{4270}{1009.78} = 4.23 \text{ mm}$ Ans.

$Ly_C = \Sigma L_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{L} = \frac{196,817}{1009.78} = 194.9 \text{ mm}$ Ans.

5-43 Locate the centroid of the shaded area shown in Fig. P5-43.

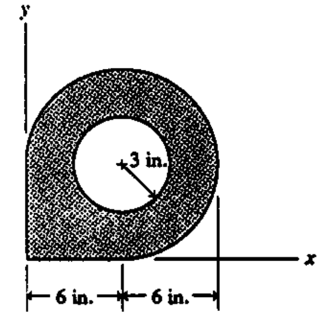
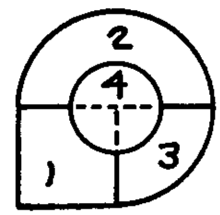


Fig. P5-43

SOLUTION

The shaded area can be divided into a square, a half circle, and a quarter circle with a circle removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$A_2 = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(6)^2 = 56.55 \text{ in.}^2$$

$$y_{C2} = 6 + \frac{4r}{3\pi} = 6 + \frac{4(6)}{3\pi} = 8.546 \text{ in.}$$

$$A_3 = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(6)^2 = 28.27 \text{ in.}^2$$

$$x_{C3} = 6 + \frac{4r}{3\pi} = 6 + \frac{4(6)}{3\pi} = 8.546 \text{ in.}$$

$$y_{C3} = 6 - \frac{4r}{3\pi} = 6 - \frac{4(6)}{3\pi} = 3.454 \text{ in.}$$

$$A_4 = \pi r^2 = \pi(3)^2 = 28.27 \text{ in.}^2$$

Part	A_i (in.^{2})	x_{Ci} (in.)	M_y (in.^{3})	y_{Ci} (in.)	M_x (in.^{3})
1	36	3	108	3	108
2	56.55	6	339.3	8.546	483.3
3	28.27	8.546	241.6	3.454	97.6
4	-28.27	6	-169.6	6	-169.6
Σ	92.55		519.3		519.3

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{519.3}{92.55} = 5.61$ in. **Ans.**

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{519.3}{92.55} = 5.61$ in. **Ans.**

5-44 Locate the centroid of the shaded area shown in Fig. P5-44.

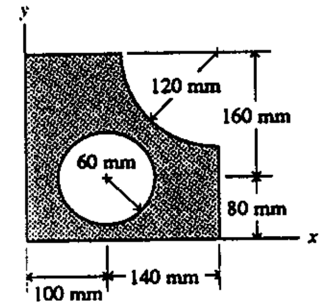
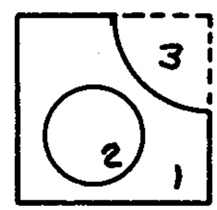


Fig. P5-44

SOLUTION

The shaded area can be divided into a square with a circle and a quarter circle removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$A_2 = \pi r^2 = \pi(60)^2 = 11,310 \text{ mm}^2$$

$$A_3 = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(120)^2 = 11,310 \text{ mm}^2$$

$$x_{C3} = 240 - \frac{4r}{3\pi} = 240 - \frac{4(120)}{3\pi} = 189.07 \text{ mm}$$

$$y_{C3} = 240 - \frac{4r}{3\pi} = 240 - \frac{4(120)}{3\pi} = 189.07 \text{ mm}$$

Part	A_i (mm^2)	x_{Ci} (mm)	M_y (mm^3)	y_{Ci} (mm)	M_x (mm^3)
1	57,600	120	6,912,000	120	6,912,000
2	-11,310	100	-1,131,000	80	-904,800
3	-11,310	189.07	-2,138,382	189.07	-2,138,382
Σ	34,980		3,642,618		3,868,818

$$Ax_C = \Sigma A_i x_{Ci} = M_y \qquad x_C = \frac{M_y}{A} = \frac{3,642,618}{34,980} = 104.1 \text{ mm} \qquad \text{Ans.}$$

$$Ay_C = \Sigma A_i y_{Ci} = M_x \qquad y_C = \frac{M_x}{A} = \frac{3,868,818}{34,980} = 110.6 \text{ mm} \qquad \text{Ans.}$$

5-45 Locate the centroid of the shaded area shown in Fig. P5-45.

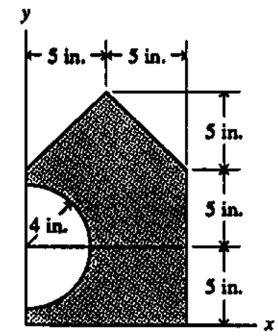
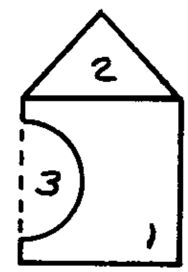


Fig. P5-45

SOLUTION

The shaded area can be divided into a triangle and a rectangle with a half circle removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$A_3 = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(4)^2 = 25.13 \text{ in.}^2$$

$$x_{C3} = \frac{4r}{3\pi} = \frac{4(4)}{3\pi} = 1.6977 \text{ in.}$$

$$y_{C2} = 10 + \frac{5}{3} = 11.667 \text{ in.}$$

Part	A_i (in.^{2})	x_{Ci} (in.)	M_y (in.^{3})	y_{Ci} (in.)	M_x (in.^{3})
1	100	5	500	5	500
2	25	5	125	11.667	291.68
3	-25.13	1.6977	-42.66	5	-125.65
Σ	99.87		582.34		666.03

$Ax_C = \Sigma A_i x_{Ci} = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{582.34}{99.87} = 5.83$ in. Ans.

$Ay_C = \Sigma A_i y_{Ci} = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{666.03}{99.87} = 6.67$ in. Ans.

5-46 Locate the centroid of the shaded area shown in Fig. P5-46.

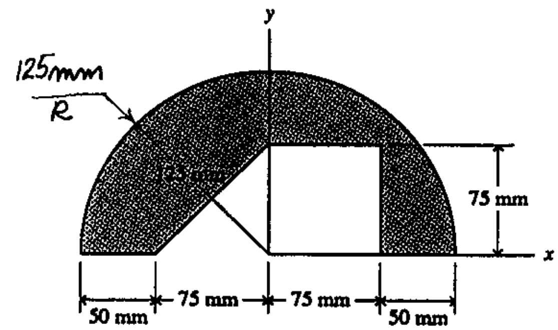
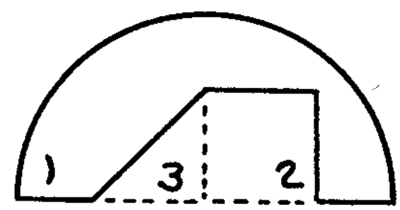


Fig. P5-46

SOLUTION

The shaded area can be divided into a half circle with a square and a triangle removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$A_1 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(125)^2 = 24,545 \text{ mm}^2$$

$$y_{C1} = \frac{4r}{3\pi} = \frac{4(125)}{3\pi} = 53.05 \text{ mm}$$

Part	A_i (mm^2)	x_{Ci} (mm)	M_y (mm^3)	y_{Ci} (mm)	M_x (mm^3)
1	24,545	0	0	53.05	1,302,112
2	-5625	37.50	-210,938	37.50	-210,938
3	-2813	-25.00	70,325	25.00	-70,325
Σ	16,107		-140,613		1,020,849

$$Ax_C = \Sigma A_i x_{Ci} = M_y \qquad x_C = \frac{M_y}{A} = \frac{-140,613}{16,107} = -8.73 \text{ mm}$$ Ans.

$$Ay_C = \Sigma A_i y_{Ci} = M_x \qquad y_C = \frac{M_x}{A} = \frac{1,020,849}{16,107} = 63.4 \text{ mm}$$ Ans.

5-47* Locate the centroid of the slender rod shown in Fig. P5-47.

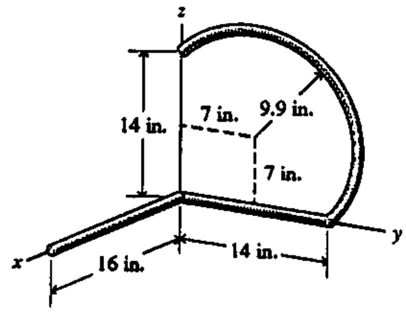
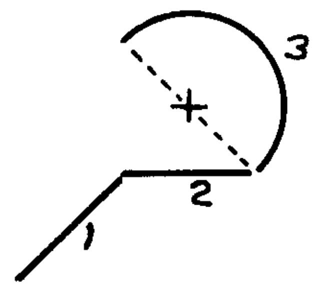


Fig. P5-47

SOLUTION



The centroid for the composite rod can be determined by listing the length, centroid location, and first moments for the individual parts in a table and applying Eqs. 5-13. Thus,

$$L_3 = \pi r = \pi(9.90) = 31.1 \text{ in.}$$

$$y_{C3} = 7 + \frac{r \sin \alpha}{\alpha} \cos 45^\circ = 7 + \frac{9.90 \sin (\pi/2)}{\pi/2} \cos 45^\circ = 11.457 \text{ in.}$$

$$z_{C3} = 7 + \frac{r \sin \alpha}{\alpha} \sin 45^\circ = 7 + \frac{9.90 \sin (\pi/2)}{\pi/2} \sin 45^\circ = 11.457 \text{ in.}$$

Part	L_i (in.)	x_{Ci} (in.)	M_{yz} (in.^{2})	y_i (in.)	M_{zx} (in.^{2})	z_i (in.)	M_{xy} (in.^{2})
1	16.0	8	128	0	0	0	0
2	14.0	0	0	7	98	0	0
3	31.1	0	0	11.457	356.3	11.457	356.3
	61.1		128		454.3		356.3

$Lx_C = \Sigma L_i x_{Ci} = M_{yz}$ $\qquad x_C = \frac{M_{yz}}{L} = \frac{128}{61.1} = 2.09$ in. Ans.

$Ly_C = \Sigma L_i y_{Ci} = M_{zx}$ $\qquad y_C = \frac{M_{zx}}{L} = \frac{454.3}{61.1} = 7.44$ in. Ans.

$Lz_C = \Sigma L_i z_{Ci} = M_{xy}$ $\qquad z_C = \frac{M_{xy}}{L} = \frac{356.3}{61.1} = 5.83$ in. Ans.

5-48 Locate the centroid of the slender rod shown in Fig. P5-48.

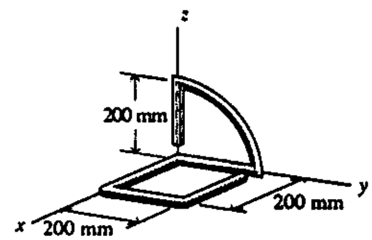
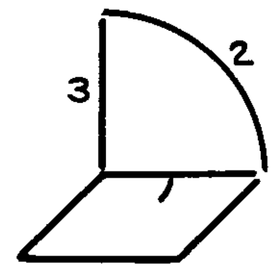


Fig. P5-48

SOLUTION

The rod can be divided into three parts. The centroid for the composite rod is determined by listing the length, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,



$$L_2 = \frac{1}{2}\pi r = \frac{1}{2}\pi(200) = 314.16 \text{ mm}$$

$$y_{C2} = z_{C2} = \frac{2r}{\pi} = \frac{2(200)}{\pi} = 127.32 \text{ mm}$$

Part	L_i (mm)	x_{Ci} (mm)	M_{yz} (mm^2)	y_{Ci} (mm)	M_{zx} (mm^2)	z_{Ci} (mm)	M_{xy} (mm^2)
1	800	100	80,000	100	80,000	0	0
2	314.16	0	0	127.32	40,000	127.32	40,000
3	200	0	0	0	0	100	20,000
Σ	1314.16		80,000		120,000		60,000

$$Lx_C = \Sigma L_i x_{Ci} = M_{yz} \qquad x_C = \frac{M_{yz}}{L} = \frac{80,000}{1314.16} = 60.9 \text{ mm} \qquad \text{Ans.}$$

$$Ly_C = \Sigma L_i y_{Ci} = M_{zx} \qquad y_C = \frac{M_{zx}}{L} = \frac{120,000}{1314.16} = 91.3 \text{ mm} \qquad \text{Ans.}$$

$$Lz_C = \Sigma L_i z_{Ci} = M_{xy} \qquad z_C = \frac{M_{xy}}{L} = \frac{60,000}{1314.16} = 45.7 \text{ mm} \qquad \text{Ans.}$$

5-49 A bracket is made of brass (γ = 0.316 lb/in^3) and aluminum (γ = 0.100 lb/in^3) plates as shown in Fig. P5-49.
(a) Locate the centroid of the bracket.
(b) Locate the center of gravity of the bracket.

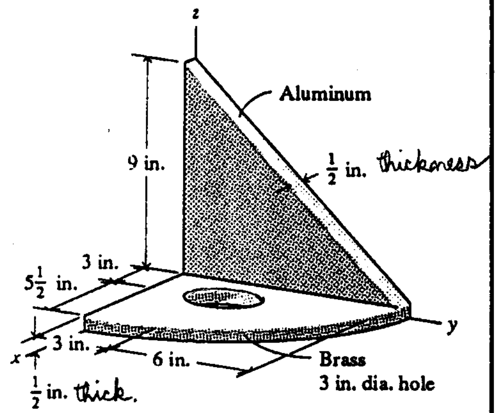


Fig. P5-49

SOLUTION

For the aluminum plate:

$$V_A = \frac{1}{2}(9)(9)(0.50) = 20.25 \text{ in}^3$$

$$W_A = \gamma_A V_A = 0.100(20.25) = 2.025 \text{ lb}$$

$$x_{CA} = 0.25 \text{ in.} \qquad y_{CA} = 3.00 \text{ in.} \qquad z_{CA} = \frac{1}{2} + \frac{9}{3} = 3.50 \text{ in.}$$

For the brass plate:

$$V_B = \frac{1}{4}\pi(9)^2(0.5) - (\pi)(1.5)^2(0.5) = 31.809 - 3.534 = 28.275 \text{ in}^3$$

$$W_B = \gamma_B V_B = 0.316(28.275) = 8.935 \text{ lb} \qquad \frac{4r}{3\pi} = \frac{4(9)}{3\pi} = 3.820 \text{ in.}$$

$$x_{CB} = \frac{31.809(3.820) - 3.534(3.5)}{28.275} = 3.860 \text{ in.}$$

$$y_{CB} = \frac{31.809(3.820) - 3.534(3)}{28.275} = 3.922 \text{ in.} \qquad z_{CB} = 0.25 \text{ in.}$$

(a) $$x_C = \frac{20.25(0.25) + 28.275(3.860)}{20.25 + 28.275} = 2.35 \text{ in.}$$ Ans.

$$y_C = \frac{20.25(3.00) + 28.275(3.923)}{20.25 + 28.275} = 3.54 \text{ in.}$$ Ans.

$$z_C = \frac{20.25(3.50) + 28.275(0.250)}{20.25 + 28.275} = 1.606 \text{ in.}$$ Ans.

(b) $$x_G = \frac{2.025(0.25) + 8.935(3.860)}{2.025 + 8.935} = 3.19 \text{ in.}$$ Ans.

$$y_G = \frac{2.025(3.00) + 8.935(3.923)}{2.025 + 8.935} = 3.75 \text{ in.}$$ Ans.

$$z_G = \frac{2.025(3.50) + 8.935(0.250)}{2.025 + 8.953} = 0.850 \text{ in.}$$ Ans.

5-50 A cylinder with a hemispherical cavity and a conical cap is shown in Fig. P5-50.
(a) Locate the centroid of the composite volume if R = 140 mm, L = 250 mm and h = 300 mm.
(b) Locate the center of mass of the composite volume if the cylinder is made of steel (ρ = 7870 kg/m^3) and the cap is made of aluminum (ρ = 2770 kg/m^3).

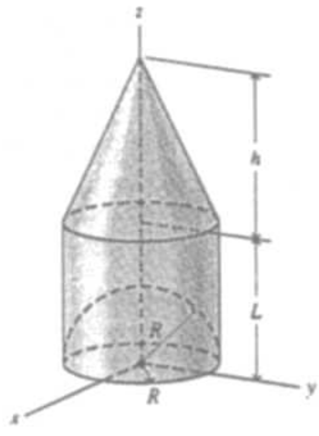


Fig. P5-50

SOLUTION

For the cone:

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(0.140)^2(0.300) = 0.006,158 \text{ m}^3$$

$$m = \gamma V = 2770(0.006,158) = 17.058 \text{ kg}$$

$$x_C = y_C = 0 \qquad z_C = L + \frac{h}{4} = 250 + \frac{300}{4} = 325 \text{ mm}$$

For the cylinder:

$$V = \pi r^2 L = \pi(0.140)^2(0.250) = 0.015,394 \text{ m}^3$$

$$m = \gamma V = 7870(0.015,394) = 121.15 \text{ kg}$$

$$x_C = y_C = 0, \qquad z_C = \frac{L}{2} = \frac{250}{2} = 125 \text{ mm}$$

For the hemisphere:

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3}(\pi)(0.140)^3 = 0.005,747 \text{ m}^3$$

$$m = \gamma V = 7870(0.005,747) = 45.23 \text{ kg}$$

$$x_C = y_C = 0, \qquad z_C = \frac{3}{8}r = \frac{3}{8}(140) = 52.5 \text{ mm}$$

(a) $$z_C = \frac{0.006,158(325) + 0.015,394(125) - 0.005,747(525)}{0.006,158 + 0.015,394 - 0.005,747} = 229 \text{ mm} \qquad \text{Ans.}$$

$$x_C = y_C = 0 \qquad \text{Ans.}$$

(b) $$z_G = \frac{17.058(325) + 121.15(125) - 45.23(52.5)}{17.058 + 121.15 - 45.23} = 197.0 \text{ mm} \qquad \text{Ans.}$$

$$x_G = y_G = 0 \qquad \text{Ans.}$$

5-51 A bracket is made of steel (γ = 0.284 lb/in.^{3}) and aluminum (γ = 0.100 lb/in.^{3}) plates as shown in Fig. P5-51.
(a) Locate the centroid of the bracket.
(b) Locate the center of gravity of the bracket.

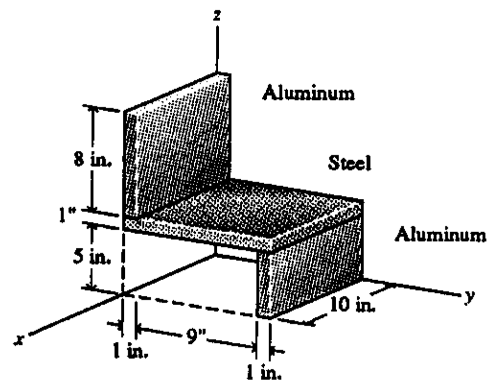
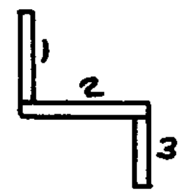


Fig. P5-51

SOLUTION



For plate 1: $V_1 = 8(1)(10) = 80 \text{ in}^3$

$W_1 = \gamma_1 V_1 = 0.100(80) = 8.00 \text{ lb}$

$x_{C1} = 5.00$ in. $y_{C1} = 0.50$ in. $z_{C1} = 10.00$ in.

For plate 2: $V_2 = 11(10)(1) = 110 \text{ in}^3$

$W_2 = \gamma_2 V_2 = 0.284(110) = 31.24 \text{ lb}$

$x_{C2} = 5.00$ in. $y_{C2} = 5.50$ in. $z_{C2} = 5.50$ in.

For plate 3: $V_3 = 5(1)(10) = 50 \text{ in}^3$

$W_3 = \gamma_3 V_3 = 0.100(50) = 5.00 \text{ lb}$

$x_{C3} = 5.00$ in. $y_{C3} = 10.50$ in. $z_{C3} = 2.50$ in.

(a) $$x_C = \frac{80(5.00) + 110(5.00) + 50(5.00)}{80 + 110 + 50} = 5.00 \text{ in.}$$ Ans.

$$y_C = \frac{80(0.50) + 110(5.50) + 50(10.50)}{80 + 110 + 50} = 4.875 \text{ in.}$$ Ans.

$$z_C = \frac{80(10.00) + 110(5.50) + 50(2.50)}{80 + 110 + 50} = 6.375 \text{ in.}$$ Ans.

(b) $$x_G = \frac{8.00(5.00) + 31.24(5.00) + 5.00(5.00)}{8.00 + 31.24 + 5.00} = 5.00 \text{ in.}$$ Ans.

$$y_G = \frac{8.00(0.50) + 31.24(5.50) + 5.00(10.50)}{8.00 + 31.24 + 5.00} = 5.16 \text{ in.}$$ Ans.

$$z_G = \frac{8.00(10.00) + 31.24(5.50) + 5.00(2.50)}{8.00 + 31.24 + 5.00} = 5.97 \text{ in.}$$ Ans.

5-52 A cylinder with a conical cavity and a hemispherical cap is shown in Fig. P5-52.
(a) Locate the centroid of the composite volume if R = 200 mm, and h = 250 mm.
(b) Locate the center of mass of the composite volume if the cylinder is made of brass (ρ = 8750 kg/m^3) and the cap is made of aluminum (ρ = 2770 kg/m^3).

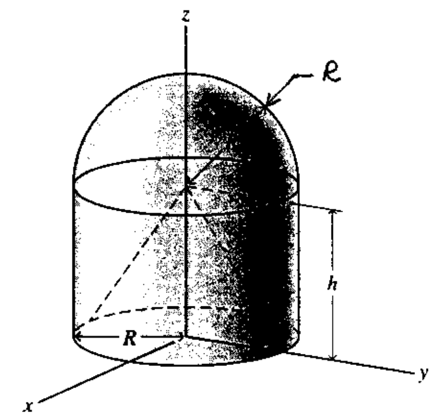


Fig. P5-52

SOLUTION

For the hemisphere:

$$V = \frac{2}{3}\pi r^3 = \frac{2}{3}(\pi)(0.200)^3 = 0.016,755\ \text{m}^3$$

$$m = \gamma V = 2770(0.016,755 = 46.41\ \text{kg}$$

$$x_C = y_C = 0 \qquad z_C = h + \frac{3}{8}r = 250 + \frac{3}{8}(200) = 325\ \text{mm}$$

For the cylinder:

$$V = \pi r^2 L = \pi(0.200)^2(0.250) = 0.031,416\ \text{m}^3$$

$$m = \gamma V = 8750(0.031,416 = 274.89\ \text{kg}$$

$$x_C = y_C = 0 \qquad z_C = \frac{L}{2} = \frac{250}{2} = 125\ \text{mm}$$

For the cone:

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(0.200)^2(0.250) = 0.010,472\ \text{m}^3$$

$$m = \gamma V = 8750(0.010,472) = 91.63\ \text{kg}$$

$$x_C = y_C = 0 \qquad z_C = \frac{h}{4} = \frac{250}{4} = 62.5\ \text{mm}$$

(a) $$z_C = \frac{0.016,755(325) + 0.031,416(125) - 0.010,472(62.5)}{0.016,755 + 0.031,416 - 0.010,472} = 231\ \text{mm} \qquad \text{Ans.}$$

$$x_C = y_C = 0 \qquad \text{Ans.}$$

(b) $$z_G = \frac{46.41(325) + 274.89(125) - 91.63(62.5)}{46.41 + 274.89 - 91.63} = 190.4\ \text{mm} \qquad \text{Ans.}$$

$$x_G = y_G = 0 \qquad \text{Ans.}$$

5-53 Locate the center of gravity of the bracket shown in Fig. P5-53 if the holes have 6-in. diameters.

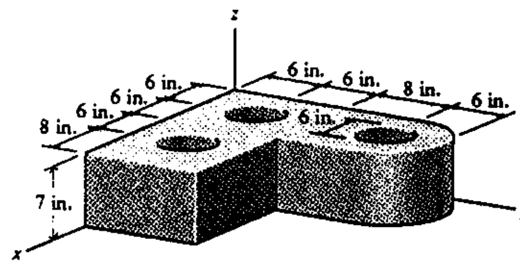
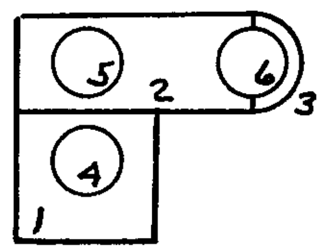


Fig. P5-53

SOLUTION



$$V_3 = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(6)^2(7) = 395.8 \text{ in.}^3$$

$$y_{C3} = 20 + \frac{4r}{3\pi} = 20 + \frac{4(6)}{3\pi} = 22.55 \text{ in.}$$

$$V_4 = V_5 = V_6 = \pi r^2 h = \pi(3)^2(7) = 197.9 \text{ in.}^3$$

Part	V_i (in.^{2})	x_{Gi} (in.)	M_{yz} (in.^{3})	y_{Gi} (in.)	M_{zx} (in.^{3})	z_{Gi} (in.)	M_{xy} (in.^{3})
1	1176	19	22,344	6	7056	3.5	4115
2	1680	6	10,080	10	16,800	3.5	5880
3	395.8	6	2375	22.55	8925	3.5	1385
4	-197.9	18	-3563	6	-1188	3.5	-693
5	-197.9	6	-1188	6	-1188	3.5	-693
6	-197.9	6	-1188	20	-3958	3.5	-693
Σ	2658		28,860		26,447		9302

$Vx_G = \Sigma V_i x_{Gi} = M_{yz}$ $\qquad x_G = \frac{M_{yz}}{V} = \frac{28,860}{2658} = 10.86$ in. Ans.

$Vy_G = \Sigma V_i y_{Gi} = M_{zx}$ $\qquad y_G = \frac{M_{zx}}{V} = \frac{26,447}{2658} = 9.95$ in. Ans.

$Vz_G = \Sigma V_i z_{Gi} = M_{xy}$ $\qquad z_G = \frac{M_{xy}}{V} = \frac{9302}{2658} = 3.50$ in. Ans.

(Note that the plane z = 3.50 in. is a plane of symmetry)

5-54 Locate the center of mass of the machine component shown in Fig. P5-54. The brass (ρ = 8750 kg/m^3) disk C is mounted on the steel (ρ = 7870 kg/m^3) shaft B.

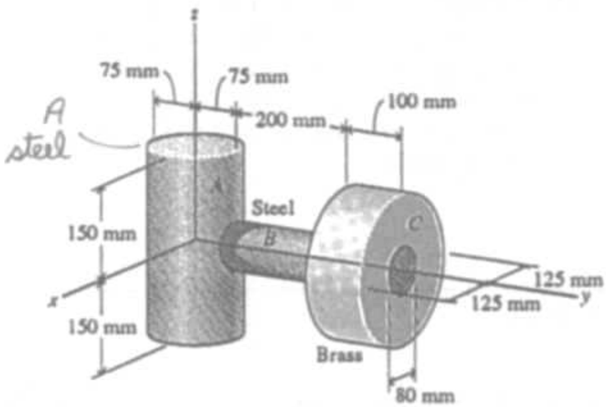
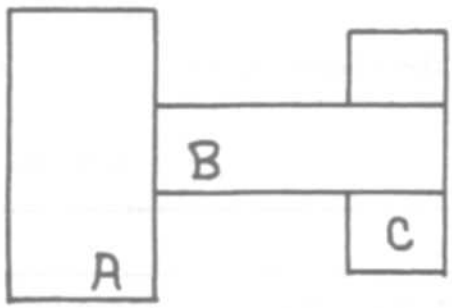


Fig. P5-54

SOLUTION

The machine component consists of two solid circular cylinders and a hollow disk.



For circular cylinder A:

$$V_1 = \pi r^2 h = \pi(0.075)^2(0.300) = 0.005301 \text{ m}^3$$

$$m_1 = \rho_1 V_1 = 7870(0.005301) = 41.72 \text{ kg} \qquad y_{G1} = 0$$

For circular cylinder B:

$$V_2 = \pi r^2 h = \pi(0.040)^2(0.300) = 0.001508 \text{ m}^3$$

$$m_2 = \rho_2 V_2 = 7870(0.001508) = 11.87 \text{ kg} \qquad y_{G2} = 225 \text{ mm}$$

For the hollow disk C:

$$V_3 = \pi(r_o^2 - r_i^2)h = \pi[(0.125)^2 - (0.040)^2](0.100) = 0.004406 \text{ m}^3$$

$$m_3 = \rho_3 V_3 = 8750(0.004406) = 38.55 \text{ kg} \qquad y_{G3} = 325 \text{ mm}$$

$$y_G = \frac{41.72(0) + 11.87(225) + 38.55(325)}{41.72 + 11.87 + 38.55} = 165.0 \text{ mm} \qquad \text{Ans.}$$

From symmetry: $x_G = z_G = 0$ Ans.

5-55* Determine the surface area A and volume V of the solid body generated by revolving the shaded area of Fig. P5-55 through an angle of 360° about the y-axis.

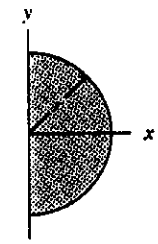


Fig. P5-55

SOLUTION

For the boundary line: $x_{CL} = \frac{2R}{\pi}$ $L = \pi R$

$$A = 2\pi x_{CL} L = 2\pi\left(\frac{2R}{\pi}\right)(\pi R) = 4\pi R^2 \qquad \text{Ans.}$$

For the shaded area: $x_{CA} = \frac{4R}{3\pi}$ $A = \frac{1}{2}\pi R^2$

$$V = 2\pi x_{CA} A = 2\pi\left(\frac{4R}{3\pi}\right)\left(\frac{1}{2}\pi R^2\right) = \frac{4}{3}\pi R^3 \qquad \text{Ans.}$$

5-56* Determine the surface area A and volume V of the solid body generated by revolving the shaded area of Fig. P5-56 through an angle of 360° about the y-axis.

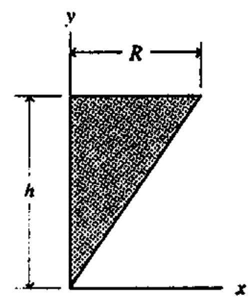


Fig. P5-56

SOLUTION

For the boundary line: $x_{CL} = \frac{R}{2}$ $L = R + \sqrt{R^2 + h^2}$

$$A = 2\pi(x_{CL})L = 2\pi\left(\frac{R}{2}\right)\left(R + \sqrt{R^2 + h^2}\right) = \pi R\left(R + \sqrt{R^2 + h^2}\right) \qquad \text{Ans.}$$

For the shaded area: $x_{CA} = \frac{R}{3}$ $A = \frac{1}{2}Rh$

$$V = 2\pi(x_{CA})A = 2\pi\left(\frac{R}{3}\right)\left(\frac{Rh}{2}\right) = \frac{1}{3}\pi R^2 h \qquad \text{Ans.}$$

5-57 Determine the surface area A and volume V of the solid body generated by revolving the shaded area of Fig. P5-57 through an angle of 360° about the x-axis.

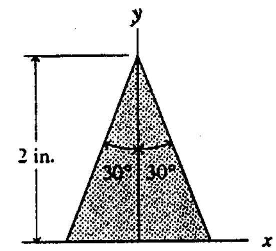


Fig. P5-57

SOLUTION

For the boundary line: $y_{CL} = 2$ in. $L = 2\left(\dfrac{2}{\cos 30^\circ}\right) = 4.619$ in.

$$A = 2\pi y_{CL} L = 2\pi(1)(4.619) = 29.02 \text{ in}^2 \cong 29.0 \text{ in}^2 \quad \text{Ans.}$$

For the shaded area: $y_{CA} = \dfrac{2}{3}$ in. $A = 2(2 \tan 30^\circ) = 2.309$ in^2

$$V = 2\pi y_{CA} A = 2\pi\left(\frac{2}{3}\right)(2.309) = 9.672 \text{ in}^3 \cong 9.67 \text{ in}^3 \quad \text{Ans.}$$

5-58 Determine the surface area A and volume V of the solid body generated by revolving the shaded area of Fig. P5-58 through an angle of 360° about the y-axis.

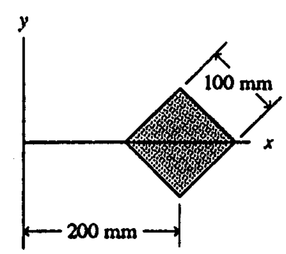


Fig. P5-58

SOLUTION

For the boundary line: $x_{CL} = 200$ mm $L = 4(100) = 400$ mm

$$A = 2\pi x_{CL} L = 2\pi(200)(400) = 502.7(10^3) \text{ mm}^2 \cong 503(10^3) \text{ mm}^2 \quad \text{Ans.}$$

For the shaded area: $y_{CA} = 200$ mm $A = (100)^2 = 10{,}000$ mm^2

$$V = 2\pi x_{CA} A = 2\pi(200)(10{,}000) = 12.566(10^6) \text{ mm}^3 \cong 12.57(10^6) \text{ mm}^3 \quad \text{Ans.}$$

5-59 Determine the surface area A and volume V of the solid body generated by revolving the shaded area shown in Fig. P5-59 through an angle of 360° about the y-axis.

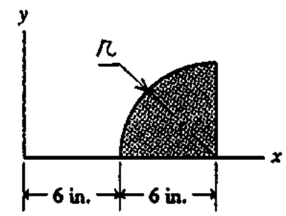
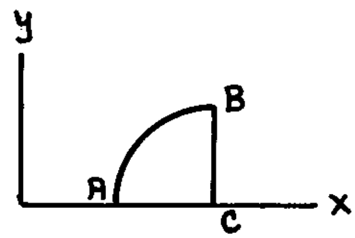


Fig. P5-59

SOLUTION



For the boundary line:

$$L_{AB} = \frac{1}{2}\pi r = \frac{1}{2}\pi(6) = 9.425 \text{ in.}$$

$$(x_C)_{AB} = 12 - \frac{2r}{\pi} = 12 - \frac{2(6)}{\pi} = 8.180 \text{ in.}$$

$$A_{AB} = 2\pi(x_C)_{AB}L_{AB} = 2\pi(8.180)(9.425) = 484.4 \text{ in.}^2$$

$$A_{BC} = 2\pi(x_C)_{BC}L_{BC} = 2\pi(12)(6) = 452.4 \text{ in.}^2$$

$$A_{AC} = 2\pi(x_C)_{AC}L_{AC} = 2\pi(9)(6) = 339.3 \text{ in.}^2$$

$$A_S = A_{AB} + A_{BC} + A_{AC} = 1276.1 = 1276 \text{ in.}^2$$ Ans.

For the shaded area:

$$A = \frac{\pi}{4}r^2 = \frac{\pi}{4}(6)^2 = 28.27 \text{ in.}^2$$

$$(x_C)_A = 12 - \frac{4r}{3\pi} = 12 - \frac{4(6)}{3\pi} = 9.454 \text{ in.}$$

$$V_S = 2\pi(x_C)_A A = 2\pi(9.454)(28.27) = 1679 \text{ in.}^3$$ Ans.

5-60 Determine the surface area A and volume V of the solid body generated by revolving the shaded area shown in Fig. P5-60 through an angle of 360° about the y-axis.

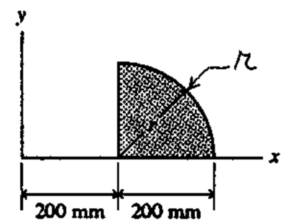
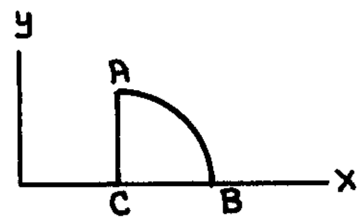


Fig. P5-60

SOLUTION



For the boundary line:

$$L_{AB} = \frac{1}{2}\pi r = \frac{1}{2}\pi(200) = 314.2 \text{ mm}$$

$$(x_C)_{AB} = 200 + \frac{2r}{\pi} = 200 + \frac{2(200)}{\pi} = 327.3 \text{ mm}$$

$$A_{AB} = 2\pi(x_C)_{AB}L_{AB} = 2\pi(327.3)(314.2) = 646.1(10^3) \text{ mm}^2$$

$$A_{AC} = 2\pi(x_C)_{AC}L_{AC} = 2\pi(200)(200) = 251.3(10^3) \text{ mm}^2$$

$$A_{BC} = 2\pi(x_C)_{BC}L_{BC} = 2\pi(300)(200) = 377.0(10^3) \text{ mm}^2$$

$$A_S = A_{AB} + A_{AC} + A_{BC} = 1274(10^3) \text{ mm}^2 = 1.274(10^6) \text{ mm}^2$$ Ans.

For the shaded area:

$$A = \frac{\pi}{4}r^2 = \frac{\pi}{4}(200)^2 = 31.42(10^3) \text{ mm}^2$$

$$x_{CA} = 200 + \frac{4r}{3\pi} = 200 + \frac{4(200)}{3\pi} = 284.9 \text{ mm}$$

$$V_S = 2\pi(x_{CA})A = 2\pi(284.9)(31.42(10^3) = 56.2(10^6) \text{ mm}^3$$ Ans.

5-61 Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-61 through an angle of 360° about the x-axis.

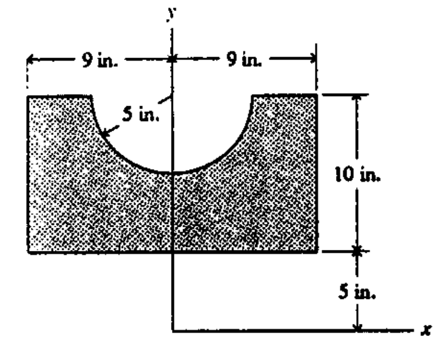
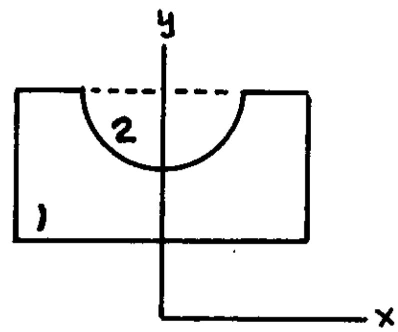


Fig. P5-61

SOLUTION



For the rectangle:

$$A_1 = 18(10) = 180 \text{ in}^2$$

$$y_{C1} = 10 \text{ in.}$$

For the half circle:

$$A_2 = \frac{\pi}{2}(5)^2 = 39.27 \text{ in}^2$$

$$x_{C2} = 15 - \frac{4r}{3\pi} = 15 - \frac{4(5)}{3\pi} = 12.878 \text{ in.}$$

$$V = \Sigma 2\pi y_{Ci} A_i = 2\pi y_{C1} A_1 - 2\pi y_{C2} A_2$$
$$= 2\pi(10)(180) - 2\pi(12.878)(39.27) = 8132 \text{ in}^3 \qquad \text{Ans.}$$

5-62 Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-62 through an angle of 360° about the x-axis.

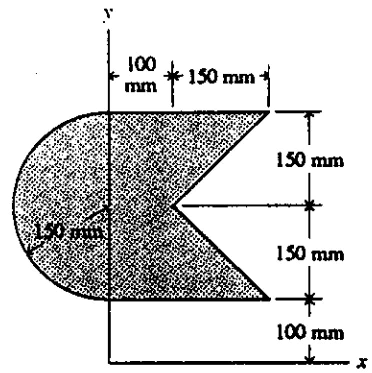
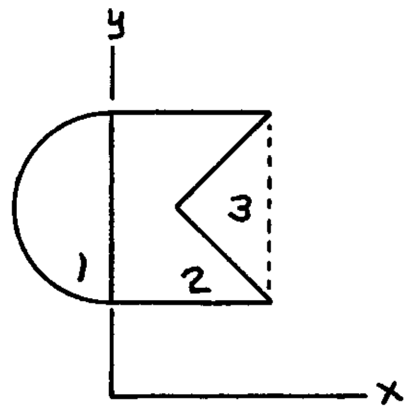


Fig. P5-62

SOLUTION



For the half circle:

$$A_1 = \frac{\pi}{2}(150)^2 = 35{,}343 \text{ mm}^2$$

$$y_{C1} = 250 \text{ mm}$$

For the rectangle:

$$A_2 = 250(300) = 75{,}000 \text{ mm}^2$$

$$y_{C2} = 250 \text{ mm}$$

For the triangle:

$$A_3 = \frac{1}{2}(300)(150) = 22{,}500 \text{ mm}^2$$

$$y_{C3} = 250 \text{ mm}$$

$$V = \Sigma 2\pi y_{Ci} A_i = 2\pi y_{C1} A_1 + 2\pi y_{C2} A_2 - 2\pi y_{C3} A_3$$

$$= 2\pi(250)(35{,}343) + 2\pi(250)(75{,}000) - 2\pi(250)(22{,}500)$$

$$= 137.98(10^6) \text{ mm}^3 \cong 138.0(10^6) \text{ mm}^3$$ Ans.

5-63 Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-63 through an angle of 180° about the y-axis.

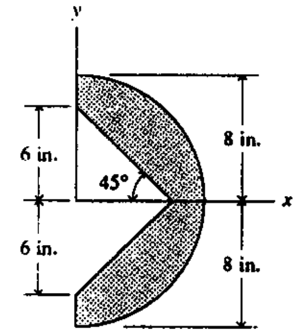
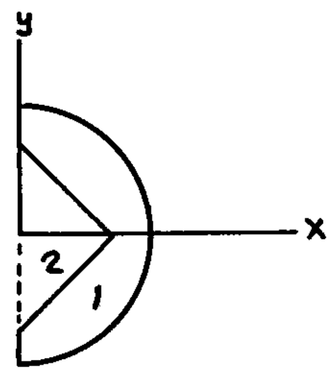


Fig. P5-63

SOLUTION



For the half circle:

$$A_1 = \frac{\pi}{2}(8)^2 = 100.53 \text{ in.}^2$$

$$x_{C1} = \frac{4r}{3\pi} = \frac{4(8)}{3\pi} = 3.395 \text{ in.}$$

For the triangle:

$$A_2 = \frac{1}{2}(12)(6) = 36 \text{ in.}^2$$

$$x_{C2} = \frac{1}{3}(6) = 2.00 \text{ in.}$$

$$V = \Sigma\theta x_{Ci}A_i = \pi x_{C1}A_1 - \pi x_{C2}A_2$$

$$= \pi(3.395)(100.53) - \pi(2.00)(36) = 846 \text{ in.}^3 \qquad \text{Ans.}$$

5-64 Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-64 through an angle of 270° about the y-axis.

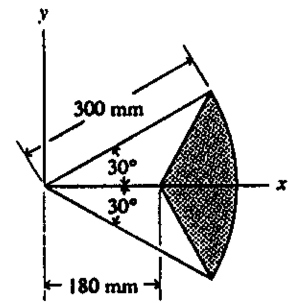
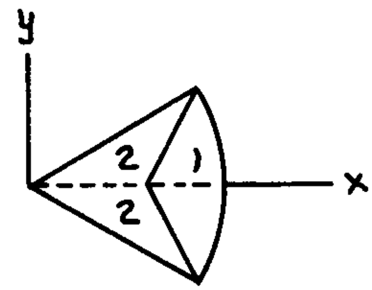


Fig. P5-64

SOLUTION



For the circular sector:

$$A_1 = r^2\alpha = (300)^2\left(\frac{\pi}{6}\right) = 47{,}124 \text{ mm}^2$$

$$x_{C1} = \frac{2r\sin\alpha}{3\alpha} = \frac{2(300)\sin 30^\circ}{3(\pi/6)} = 190.99 \text{ mm}$$

For the two triangles:

$$A_2 = 2\left(\frac{1}{2}bh\right) = 2\left(\frac{1}{2}\right)(180)(300\sin 30^\circ) = 27{,}000 \text{ mm}^2$$

$$x_{C2} = \frac{a+b}{3} = \frac{300\cos 30^\circ + 180}{3} = 146.60 \text{ mm}$$

$$V = \Sigma\theta x_{Ci}A_i = \frac{3}{2}\pi x_{C1}A_1 - \frac{3}{2}\pi x_{C2}A_2$$

$$= \frac{3}{2}\pi[190.99(47{,}124) - 146.60(27{,}000)]$$

$$= 23.76(10^6) \text{ mm}^3 \cong 23.8(10^6) \text{ mm}^3$$ Ans.

5-65* Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-65 through an angle of 360° about the x-axis.

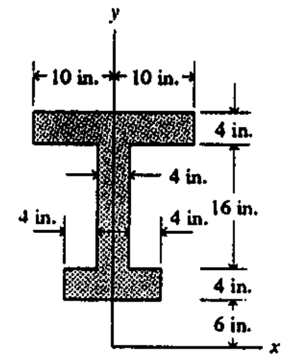
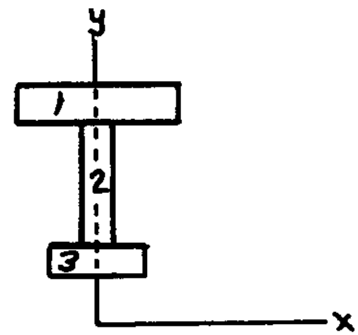


Fig. P5-65

SOLUTION



$$V = \Sigma 2\pi y_{Ci} A_i = 2\pi y_{C1} A_1 + 2\pi y_{C2} A_2 + 2\pi y_{C3} A_3$$

$$= 2\pi(28)(80) + 2\pi(18)(64) + 2\pi(8)(48)]$$

$$= 23{,}725 \text{ in}^3 \cong 23{,}700 \text{ in}^3$$ Ans.

5-66 Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-66 through an angle of 360° about the y-axis.

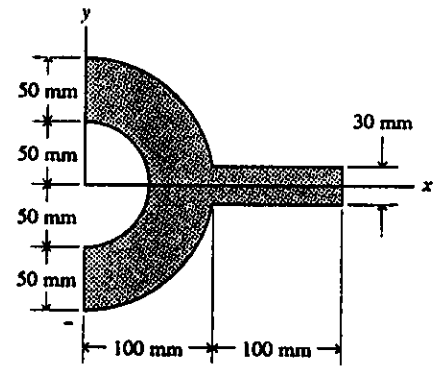
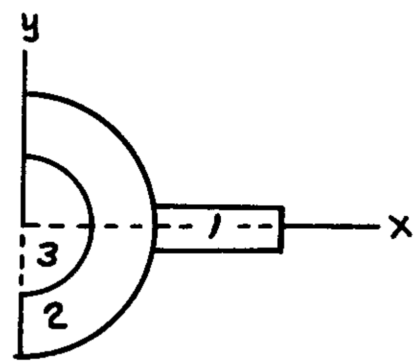


Fig. P5-66

SOLUTION



For the rectangle:

$A_1 = bh = 100(30) = 3000 \text{ mm}^2$

$x_{C1} = 150 \text{ mm}$

For the half circle:

$A_2 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(100)^2 = 15{,}708 \text{ mm}^2$

$x_{C2} = \frac{4r}{3\pi} = \frac{4(100)}{3\pi} = 42.44 \text{ mm}$

For the half circle removed:

$A_3 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(50)^2 = 3927 \text{ mm}^2$

$x_{C3} = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.22 \text{ mm}$

$$V = \Sigma 2\pi x_{Ci}A_i = 2\pi x_{C1}A_1 + 2\pi x_{C2}A_2 + 2\pi x_{C3}A_3$$

$$= 2\pi(150)(3000) + 2\pi(42.44)(15{,}708) - 2\pi(21.22)(3927)$$

$$= 6.493(10^6) \text{ mm}^3 \cong 6.49(10^6) \text{ mm}^3$$ Ans.

5-67 Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-67 through an angle of 360° about the y-axis.

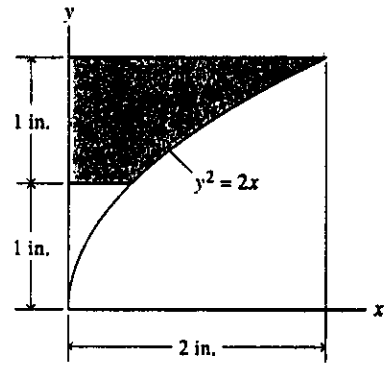
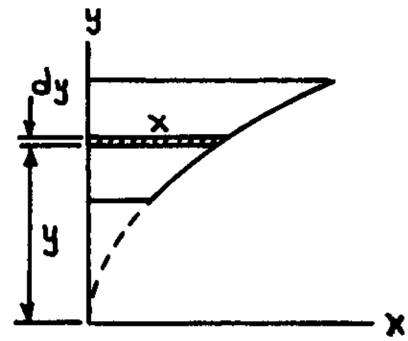


Fig. P5-67

SOLUTION

The area and centroid location for the shaded area are not generally known or listed in tables. Thus, they must be determined. For the element shown in the sketch:



$$dA = x\ dy = \frac{1}{2}y^2\ dy$$

$$A = \int_A dA = \int_1^2 \frac{1}{2}y^2\ dy = \left[\frac{y^3}{6}\right]_1^2 = 1.1667\ \text{in}^2$$

$$M_y = \int_A \frac{x}{2}\ dA = \int_1^2 \frac{y^4}{8}\ dy = \left[\frac{y^5}{40}\right]_1^2 = 0.775\ \text{in}^3$$

$$Ax_C = M_y \qquad x_C = \frac{M_y}{A} = \frac{0.775}{1.1667} = 0.6643\ \text{in.}$$

$$V = 2\pi x_C A = 2\pi(0.6643)(1.1667) = 4.87\ \text{in}^3$$ Ans.

When the area and centroid location are not known, the volume is usually determined directly by integration. Thus,

$$V = \int_1^2 \pi x^2\ dy = \pi\int_1^2 \frac{y^4}{4}\ dy = \left[\frac{\pi y^5}{20}\right]_1^2 = 4.87\ \text{in}^3$$

5-68 Determine the volume V of the solid body generated by revolving the shaded area shown in Fig. P5-68 through an angle of 360° about the y-axis.

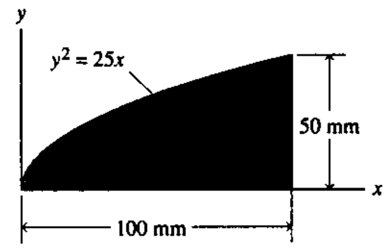
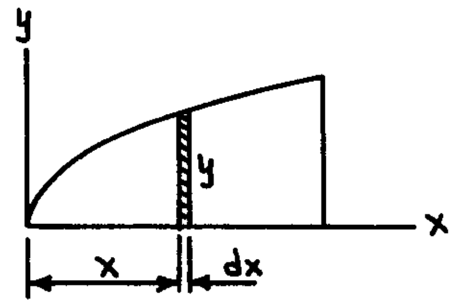


Fig. P5-68

SOLUTION

The area and centroid location for the shaded area are not generally known or listed in tables. Thus, they must be determined. For the element shown in the sketch:



$$dA = y\,dx = 5\sqrt{x}\,dx$$

$$A = \int_0^{100} 5x^{1/2}\,dx = \left[\frac{10}{3}x^{3/2}\right]_0^{100} = 3333 \text{ mm}^2$$

$$M_y = \int_A x\,dA = \int_0^{100} 5x^{3/2}\,dx = \left[\frac{10^{5/2}}{5}\right]_0^{100} = 200{,}000 \text{ mm}^3$$

$$Ax_C = M_y \qquad x_C = \frac{M_y}{A} = \frac{200{,}000}{3333} = 60.0 \text{ mm}$$

$$V = 2\pi x_C A = 2\pi(60.0)(3333) = 1.257(10^6) \text{ mm}^3$$ Ans.

When the area and centroid location are not known the volume is usually determined directly by integration. Thus,

$$V = \int_0^{100} 2\pi x\,(y\,dx) = \int_0^{100} 10\pi x^{3/2}\,dx$$

$$= \left[4\pi x^{5/2}\right]_0^{100} = 1.257(10^6) \text{ mm}^3$$

5-69 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-69.

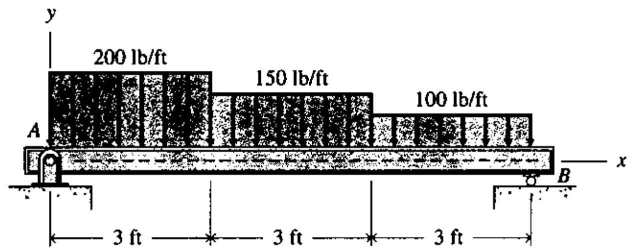
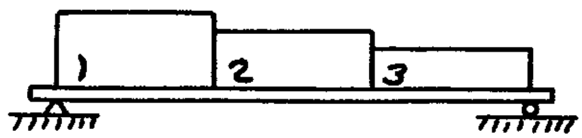


Fig. P5-69

SOLUTION



$A_1 = 200(3) = 600$ lb

$x_{C1} = 1.5$ ft.

$A_2 = 150(3) = 450$ lb

$x_{C2} = 4.5$ ft.

$A_3 = 100(3) = 300$ lb

$x_{C3} = 7.5$ ft.

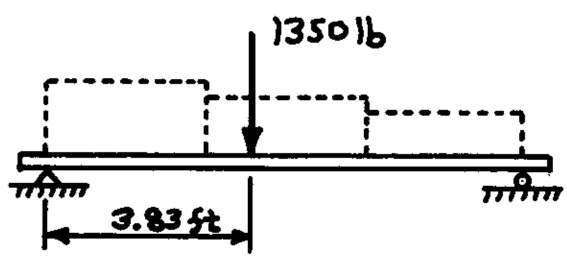
$R = \Sigma F = A_1 + A_2 + A_3 = 600 + 450 + 300 = 1350$ lb

$\bar{R} = 1350$ lb ↓ Ans.

$+\circlearrowright M_A = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3}$

$= 600(1.5) + 4.50(4.5) + 300(7.5) = 5175$ ft·lb

$Rd = M_A$ $\qquad d = \frac{M_A}{R} = \frac{5175}{1350} = 3.83$ ft ⟶ Ans.



5-70 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-70.

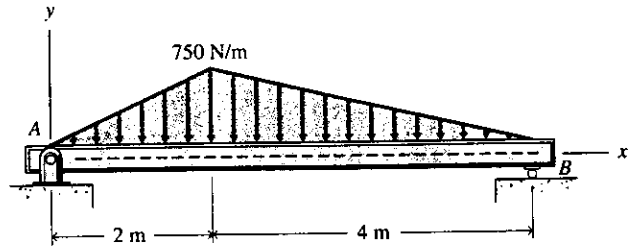


Fig. P5-70

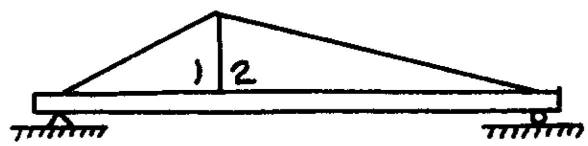
SOLUTION

$A_1 = \frac{1}{2}(750)(2) = 750$ N

$x_{C1} = \frac{2}{3}(2) = 1.333$ m

$A_2 = \frac{1}{2}(750)(4) = 1500$ N

$x_{C2} = 2 + \frac{1}{3}(4) = 3.333$ m



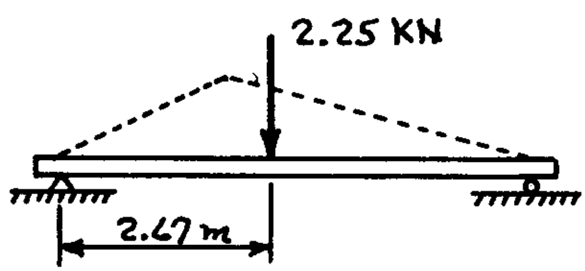
$R = \Sigma F = A_1 + A_2 = 750 + 1500 = 2250$ N

$\bar{R} = 2.25$ kN   Ans.

$+\circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2}$

$= 750(1.333) + 1500(3.333) = 6000$ N·m

$Rd = M_A$ $d = \frac{M_A}{R} = \frac{6000}{2500} = 2.67$ m $\longrightarrow$ Ans.



5-71 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-71.

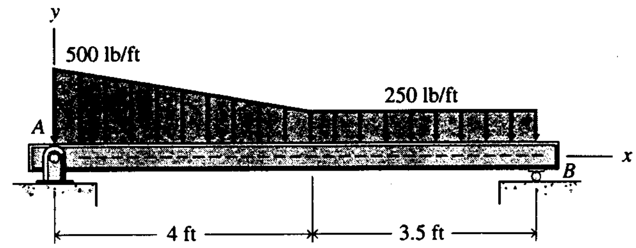
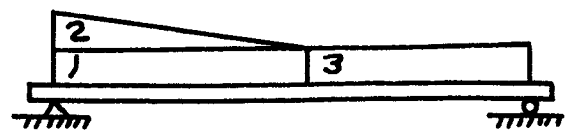


Fig. P5-71

SOLUTION



$A_1 = 250(4) = 1000$ lb

$x_{C1} = 2$ ft

$A_2 = \frac{1}{2}(250)(4) = 500$ lb

$x_{C2} = \frac{1}{3}(4) = 1.333$ ft

$A_3 = 250(3.5) = 875$ lb

$x_{C3} = 5.75$ ft

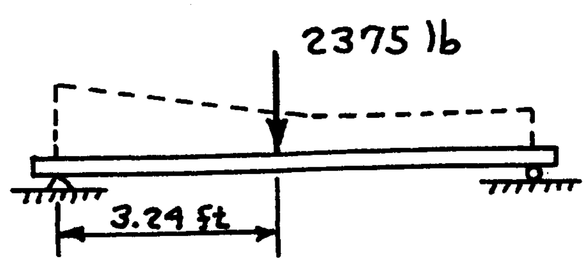
$R = \Sigma F = A_1 + A_2 + A_3 = 1000 + 500 + 875 = 2375$ lb

$\bar{R} = 2375$ lb ↓ Ans.

$+ \circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3}$

$= 1000(2) + 500(1.333) + 875(5.75) = 7698$ ft·lb

$Rd = M_A$ $d = \frac{M_A}{R} = \frac{7698}{2375} = 3.24$ ft → Ans.



5-72 Determine the resultant $\bar{\mathbf{R}}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-72.

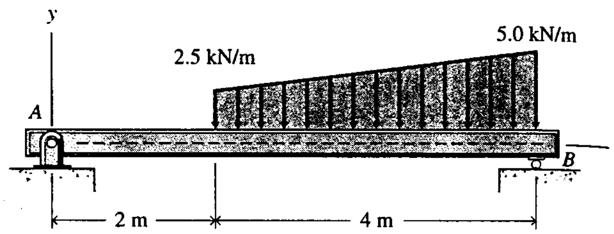
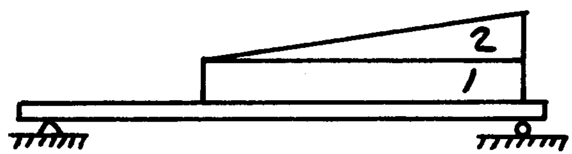


Fig. P5-72

SOLUTION



$A_1 = 2.5(4) = 10$ kN

$x_{C1} = 4$ m

$A_2 = \frac{1}{2}(2.5)(4) = 5$ kN

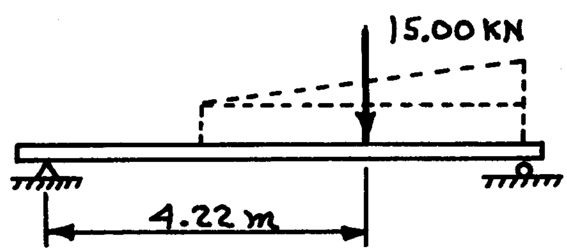
$x_{C2} = 2 + \frac{2}{3}(4) = 4.667$ m

$R = \Sigma F = A_1 + A_2 = 10 + 5 = 15$ kN

$\bar{\mathbf{R}} = 15.00$ kN ↓ Ans.

$+\circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} = 10(4) + 5(4.667) = 63.335$ kN·m

$Rd = M_A \qquad d = \frac{M_A}{R} = \frac{63.335}{15.00} = 4.22$ m → Ans.



5-73 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-73.

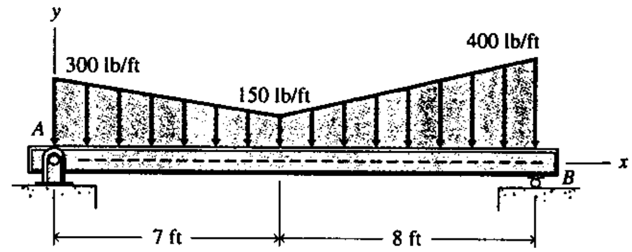
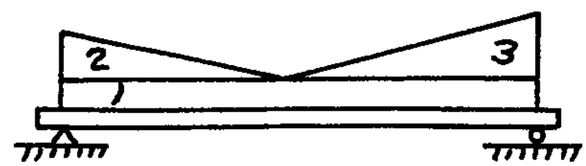


Fig. P5-73

SOLUTION



$$A_1 = 150(7 + 8) = 2250 \text{ lb}$$

$$x_{C1} = \frac{1}{2}(7 + 8) = 7.50 \text{ ft.}$$

$$A_2 = \frac{1}{2}(150)(7) = 525 \text{ lb}$$

$$x_{C2} = \frac{1}{3}(7) = 2.333 \text{ ft.}$$

$$A_3 = \frac{1}{2}(250)(8) = 1000 \text{ lb}$$

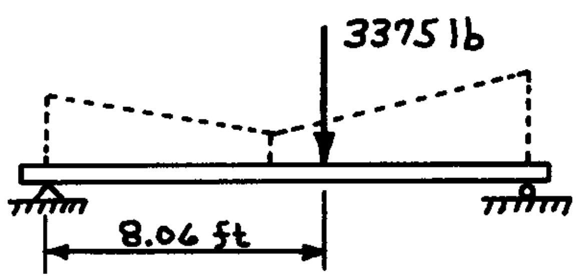
$$x_{C3} = 7 + \frac{2}{3}(8) = 12.333 \text{ ft.}$$

$$R = \Sigma F = A_1 + A_2 + A_3 = 2250 + 525 + 1000 = 3775 \text{ lb}$$

$$\bar{R} = 3775 \text{ lb} \downarrow$$ Ans.

$$+\circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3}$$
$$= 2250(7.50) + 525(2.333) + 1000(12.333) = 30{,}433 \text{ ft·lb}$$

$$Rd = M_A \qquad d = \frac{M_A}{R} = \frac{30{,}433}{3775} = 8.06 \text{ ft}$$ Ans.



5-74 Determine the resultant $\overline{\mathbf{R}}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-74.

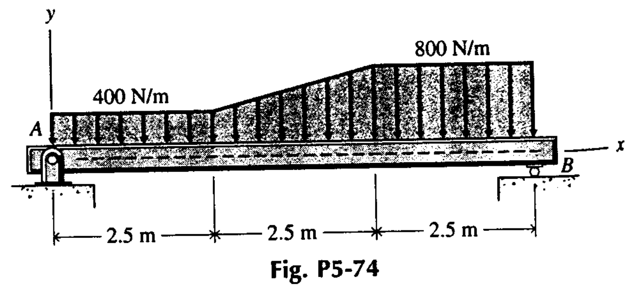
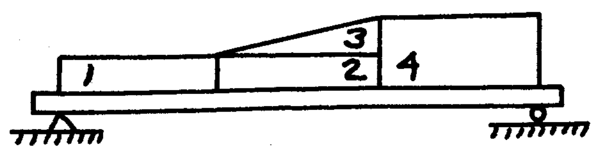


Fig. P5-74

SOLUTION

$A_1 = 400(2.5) = 1000$ N

$x_{C1} = 1.25$ m



$A_2 = 400(2.5) = 1000$ N

$x_{C2} = 3.75$ m

$A_3 = \frac{1}{2}(400)(2.5) = 500$ N

$x_{C3} = 2.5 + \frac{2}{3}(2.5) = 4.167$ m

$A_4 = 800(2.5) = 2000$ N

$x_{C4} = 6.25$ m

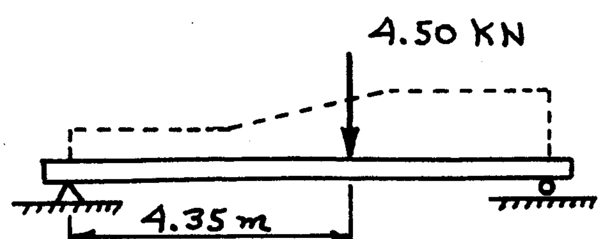
$R = \Sigma F = A_1 + A_2 + A_3 + A_4 = 1000 + 1000 + 500 + 2000 = 4500$ N

$\overline{\mathbf{R}} = 4500 \text{ N} \downarrow = 4.50 \text{ kN} \downarrow$ Ans.

$+\circlearrowleft M_A = A_1x_{C1} + A_2x_{C2} + A_3x_{C3} + A_4x_{C4}$

$= 1000(1.25) + 1000(3.75) + 500(4.167) + 2000(6.25) = 19{,}834$ N·m

$Rd = M_A$ $d = \frac{M_A}{R} = \frac{19{,}584}{4500} = 4.35 \text{ m} \longrightarrow$ Ans.



5-75 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-75.

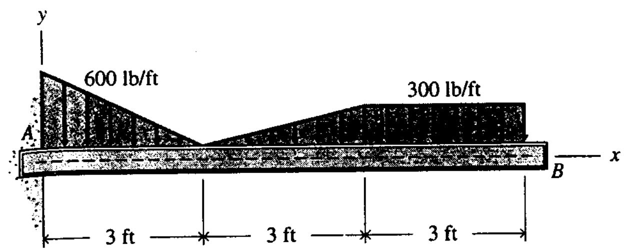
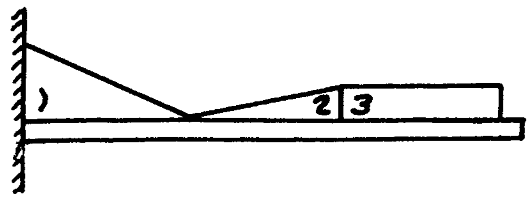


Fig. P5-75

SOLUTION



$$A_1 = \frac{1}{2}(600)(3) = 900 \text{ lb}$$

$$x_{C1} = 1.0 \text{ ft}$$

$$A_2 = \frac{1}{2}(300)(3) = 450 \text{ lb}$$

$$x_{C2} = 5.0 \text{ ft}$$

$$A_3 = 300(3) = 900 \text{ lb}$$

$$x_{C3} = 7.5 \text{ ft}$$

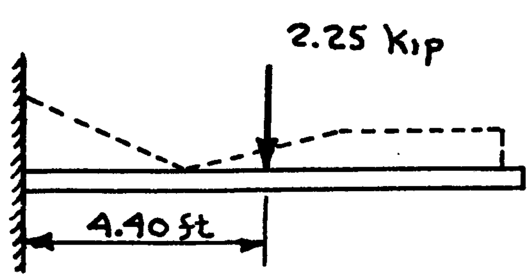
$$R = \Sigma F = A_1 + A_2 + A_3 + A_4 = 900 + 450 + 900 = 2250 \text{ lb}$$

$$\bar{R} = 2250 \text{ lb} = 2.25 \text{ kip} \downarrow$$ Ans.

$$+\circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3} + A_4 x_{C4}$$

$$= 900(1.0) + 450(5.0) + 900(7.5) = 9900 \text{ ft}\cdot\text{lb}$$

$$Rd = M_A \qquad d = \frac{M_A}{R} = \frac{9900}{2250} = 4.40 \text{ ft}$$ Ans.



5-76 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-76.

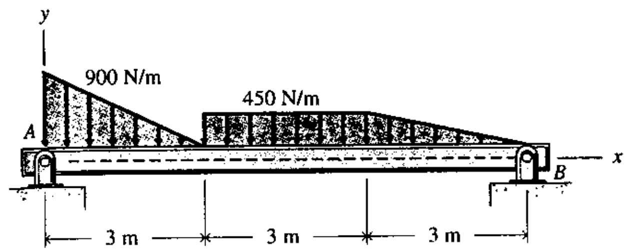
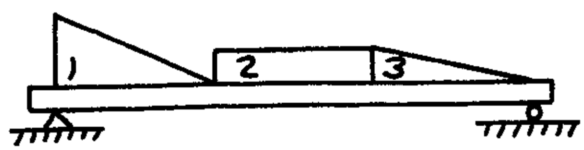


Fig. P5-76

SOLUTION



$A_1 = \frac{1}{2}(900)(3) = 1350$ N

$x_{C1} = 1.0$ m

$A_2 = 450(3) = 1350$ N

$x_{C2} = 4.5$ m

$A_3 = \frac{1}{2}(450)(3) = 675$ N

$x_{C3} = 7.0$ m

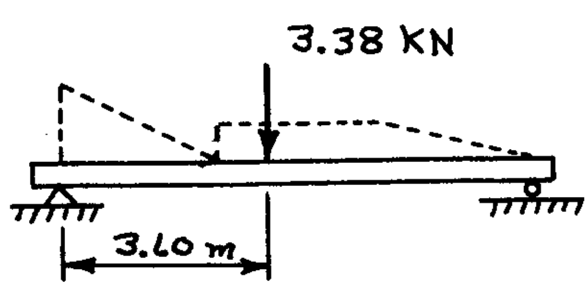
$R = \Sigma F = A_1 + A_2 + A_3 + A_4 = 1350 + 1350 + 675 = 3375$ N

$\bar{R} = 3375 \text{ N} \downarrow \cong 3.38 \text{ kN} \downarrow$ Ans.

$+\circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3} + A_4 x_{C4}$

$= 1350(1.0) + 1350(4.5) + 675(7.0) = 12{,}150$ N·m

$Rd = M_A$ $\quad d = \frac{M_A}{R} = \frac{12{,}150}{3375} = 3.60 \text{ m} \rightarrow$ Ans.



5-77 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-77.

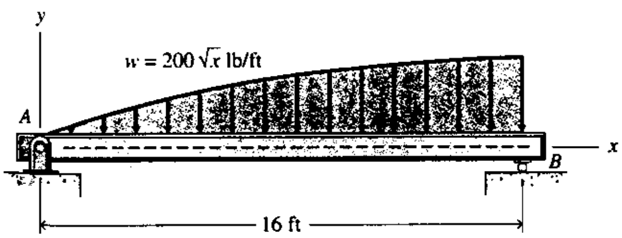
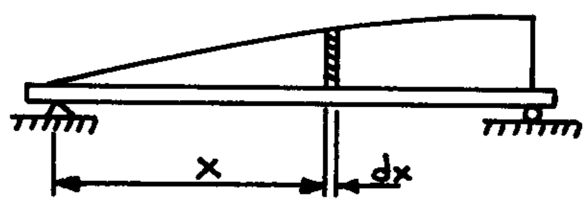


Fig. P5-77

SOLUTION



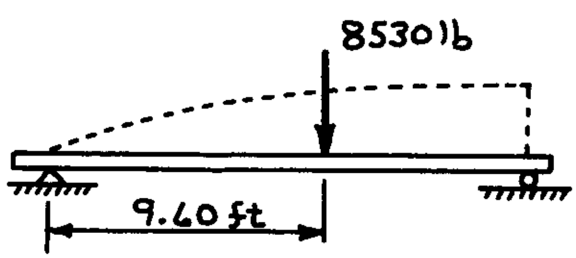
$$R = A = \int_A w\,dx = \int_0^{16} 200(x)^{1/2}\,dx$$

$$= \frac{2}{3}(200)(x)^{3/2}\Big]_0^{16} = 8533 \text{ lb} \cong 8530 \text{ lb}$$

$\bar{R}$ = 8530 lb ↓   Ans.

$$M_A = \int_0^{16} 200(x)^{3/2}\,dx = \frac{2}{5}(200)(x)^{5/2}\Big]_0^{16} = 81{,}920 \text{ ft·lb}$$

$Rd = M_A$ $d = \dfrac{M_A}{R} = \dfrac{81{,}920}{8533} = 9.60$ ft   Ans.



5-78 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-78.

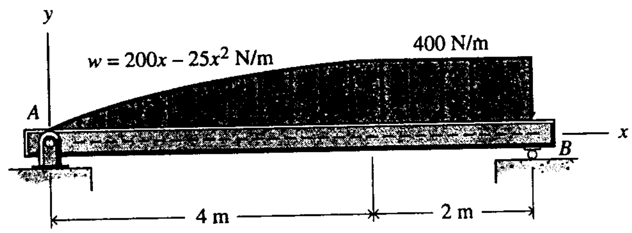
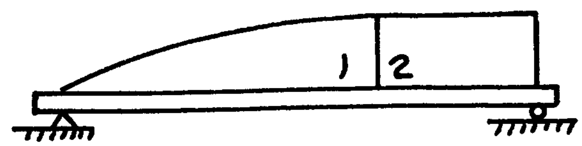


Fig. P5-78

SOLUTION



$$A_1 = \int_A w\,dx = \int_0^4 (200x - 25x^2)\,dx = \left[\frac{200x^2}{2} - \frac{25x^3}{3}\right]_0^4 = 1066.7 \text{ N}$$

$$M_A = \int_A x\,dA = \int_0^4 (200x^2 - 25x^3)\,dx = \left[\frac{200x^3}{3} - \frac{25x^4}{4}\right]_0^4 = 2666.7 \text{ N·m}$$

$$x_{C1} = \frac{M_A}{A_1} = \frac{2666.7}{1066.7} = 2.500 \text{ m}$$

$$A_2 = 400(2) = 800 \text{ N}$$

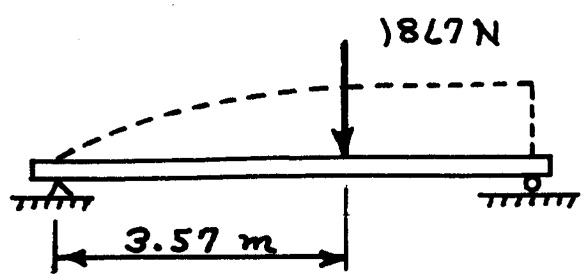
$$x_{C2} = 5 \text{ m}$$

$$R = \Sigma F = A_1 + A_2 = 1066.7 + 400(2) = 1866.7 \cong 1867 \text{ N}$$

$\bar{R} = 1867$ N ↓ Ans.

$$+\circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} = 1066.7(2.500) + 800(5) = 6666.7 \text{ N·m}$$

$Rd = M_A$ $d = \dfrac{M_A}{R} = \dfrac{6666.7}{1866.7} = 3.57$ m ⟶ Ans.



5-79 Determine the resultant $\bar{\mathbf{R}}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-79.

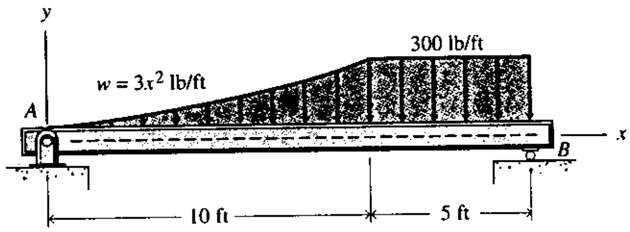
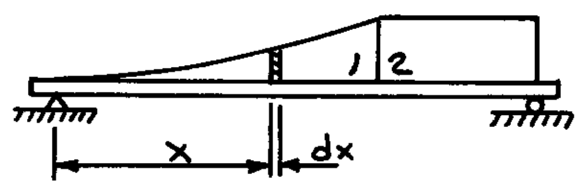


Fig. P5-79

SOLUTION



$$A_1 = \int_A w\,dx = \int_0^{10} 3x^2\,dx = \left[x^3\right]_0^{10} = 1000 \text{ lb}$$

$$M_A = \int_A x\,dA = \int_0^{10} 3x^3\,dx = \left[\frac{3x^4}{4}\right]_0^{4} = 7500 \text{ ft}\cdot\text{lb}$$

$$x_{C1} = \frac{M_A}{A_1} = \frac{7500}{1000} = 7.50 \text{ ft}$$

$$A_2 = 300(5) = 1500 \text{ lb}$$

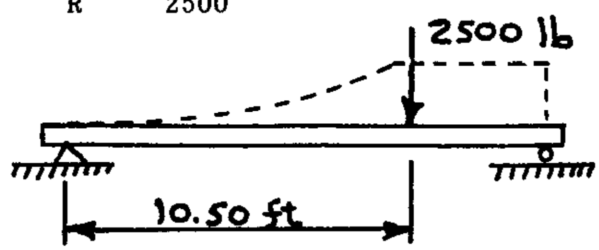
$$x_{C2} = 10 + \frac{1}{2}(5) = 12.5 \text{ ft}$$

$$R = \Sigma F = A_1 + A_2 = 1000 + 1500 = 2500 \text{ lb}$$

$\bar{\mathbf{R}} = 2500$ lb ↓   Ans.

$$+\circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} = 1000(7.50) + 1500(12.5) = 26{,}250 \text{ ft}\cdot\text{lb}$$

$Rd = M_A$ $d = \dfrac{M_A}{R} = \dfrac{26{,}250}{2500} = 10.50$ ft   Ans.



5-80 Determine the resultant $\bar{R}$ of the system of distributed loads and locate its line of action with respect to the left support for the beam shown in Fig. P5-80.

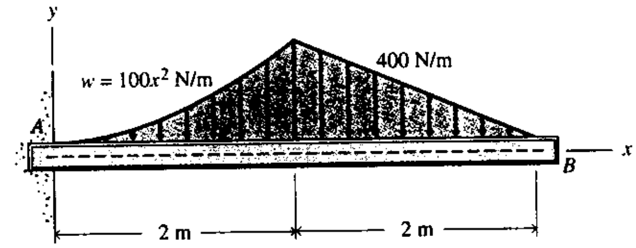
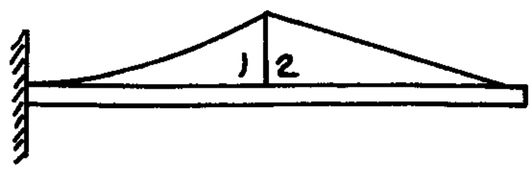


Fig. P5-80

SOLUTION



$$A_1 = \int_A w\,dx = \int_0^2 100\,x^2\,dx = \frac{100x^3}{3}\Big]_0^2 = 266.67\text{ N}$$

$$M_A = \int_A x\,dA = \int_0^2 100\,x^3\,dx = \frac{100x^4}{4}\Big]_0^2 = 400\text{ N}\cdot\text{m}$$

$$x_{C1} = \frac{M_A}{A_1} = \frac{400}{266.67} = 1.500\text{ m}$$

$$A_2 = \frac{1}{2}(400)(2) = 400\text{ N}$$

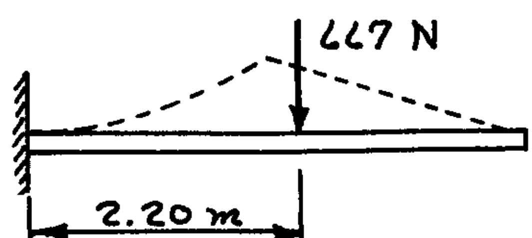
$$x_{C2} = 2 + \frac{1}{3}(2) = 2.667\text{ m}$$

$$R = \Sigma F = A_1 + A_2 = 266.67 + 400 = 666.67\text{ N} \cong 667\text{ N}$$

$\bar{R} = 667\text{ N} \downarrow$ Ans.

$$+ \circlearrowleft M_A = A_1 x_{C1} + A_2 x_{C2} = 266.67(1.500) + 400(2.667) = 1466.8\text{ N}\cdot\text{m}$$

$Rd = M_A$ $d = \frac{M_A}{R} = \frac{1466.8}{667.67} = 2.20\text{ m} \longrightarrow$ Ans.



5-81* If the dam shown in Fig. P5-81 is 200 ft wide, determine the magnitude of the resultant force $\vec{R}$ exerted on the dam by the water ($\gamma = 62.4$ lb/ft^3) pressure.

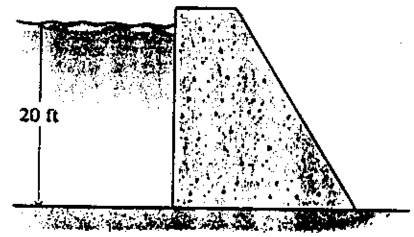
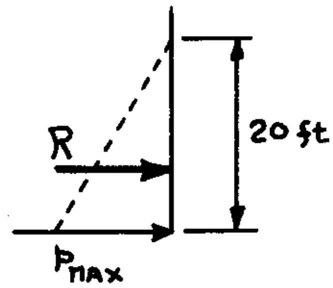


Fig. P5-81

SOLUTION

$$p_{max} = \gamma d = 62.4(20) = 1248 \text{ lb/ft}^2$$



$$R = V_{ps} = \frac{1}{2} p_{max} hw$$

$$= \frac{1}{2}(1248)(20)(200)$$

$$= 2.496(10^6) \cong 2.50(10^6) \text{ lb}$$ Ans.

5-82* If the dam shown in Fig. P5-82 is 50 m wide, determine the magnitude of the resultant force $\vec{R}$ exerted on the dam by the water ($\rho = 1000$ kg/m^3) pressure.

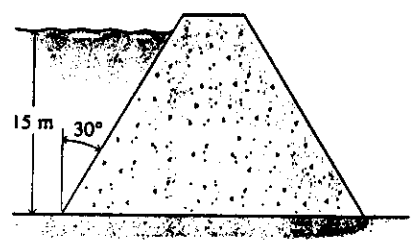
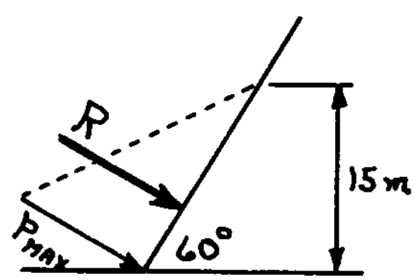


Fig. P5-82

SOLUTION

$$p_{max} = \rho g d = 1000(9.81)(15)$$

$$= 147.15(10^3) \text{ N/m}^2$$

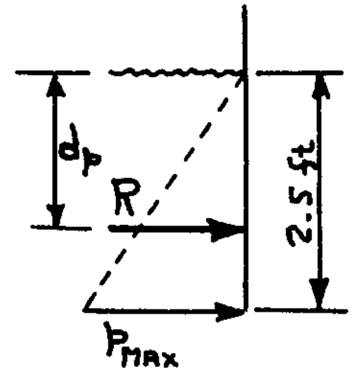


$$R = V_{ps} = \frac{1}{2} p_{max} \left(\frac{hw}{\cos 30^\circ} \right)$$

$$= \frac{1}{2}(147.15)(10^3) \left[\frac{15(50)}{\cos 30^\circ} \right] = 63.7(10^6) \text{ N} = 63.7 \text{ MN}$$ Ans.

5-83 A glass walled fish tank is 2 ft wide by 6 ft long by 3 ft deep. When the water (γ = 62.4 lb/ft^3) in the tank is 2.5 ft deep, determine the magnitude of the resultant force $\bar{R}$ exerted on a 2 by 3-ft end plate by the water pressure and the distance from the water surface to the center of pressure.

SOLUTION



$p_{max} = \gamma d$

$= 62.4(2.5) = 156$ lb/ft^2

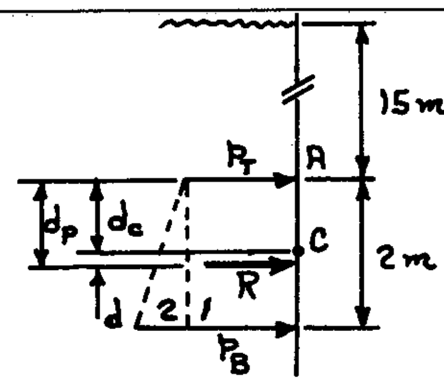
$R = V_{ps} = \frac{1}{2}p_{max}hw$

$= \frac{1}{2}(156)(2.5)(2) = 390$ lb Ans.

$d_p = \frac{2}{3}d = \frac{2}{3}(2.5) = 1.667$ ft Ans.

5-84 A flat steel plate is used to seal an opening 1 m wide by 2 m high in the vertical wall of a large water (ρ = 1000 kg/m^3) tank. When the water level in the tank is 15 m above the top of the opening, determine the magnitude of the resultant force $\bar{R}$ exerted on the plate by the water pressure and the distance from the centroid of the area of the plate to the center of pressure.

SOLUTION



$p_T = \rho g d_T = 1000(9.81)(15)$

$= 147,150$ N/m^2

$p_B = \rho g d_B = 1000(9.81)(17)$

$= 166,770$ N/m^2

$V_{ps1} = p_T hw = 147,150(2)(1) = 294,300$ N

$V_{ps2} = \frac{1}{2}(p_B - p_T)hw = \frac{1}{2}(166,770 - 147,150)(2)(1) = 19,620$ N

$R = \Sigma F = V_{ps1} + V_{ps2} = 294,300 + 19,620$

$= 313,920 \text{ N} \cong 314(10^3) \text{ N} \cong 314$ kN Ans.

$M_A = V_{ps1}d_{C1} + V_{ps2}d_{C2} = 294,300(1) + 19,620(1.3333) = 320,460$ N·m

$Rd_p = M_A$ $\qquad d_p = \frac{M_A}{R} = \frac{320,460}{313,920} = 1.0208$ ft

$d = d_p - d_C = 1.0208 - 1 = 0.0208$ ft Ans.

5-85* The width of the rectangular gate shown in Fig. P5-85 is 8 ft. Determine the magnitude of the resultant force $\bar{R}$ exerted on the gate by the water (γ = 62.4 lb/ft^3) pressure and the location of the center of pressure with respect to the hinge at the bottom of the gate.

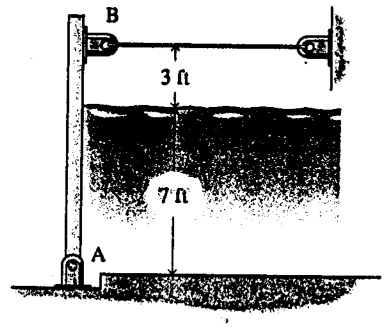


Fig. P5-85

SOLUTION

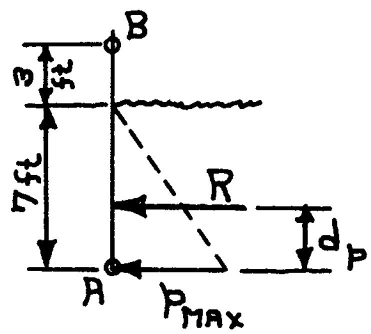
$$p_{max} = \gamma d = 62.4(7) = 436.8 \text{ lb/ft}^2$$

$$R = V_{ps} = \frac{1}{2}p_{max}hw$$

$$= \frac{1}{2}(436.8)(7)(8)$$

$$= 12230.4 \text{ lb} \cong 12.23 \text{ kip} \qquad \text{Ans.}$$

$$d_p = \frac{1}{3}h = \frac{1}{3}(7) = 2.33 \text{ ft} \qquad \text{Ans.}$$



5-86 The width of the rectangular gate shown in Fig. P5-86 is 2 m. Determine the magnitude of the resultant force $\bar{R}$ exerted on the gate by the water (ρ = 1000 kg/m^3) pressure and the location of the center of pressure with respect to the hinge at the top of the gate.

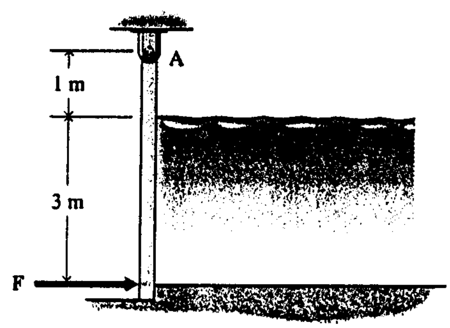


Fig. P5-86

SOLUTION

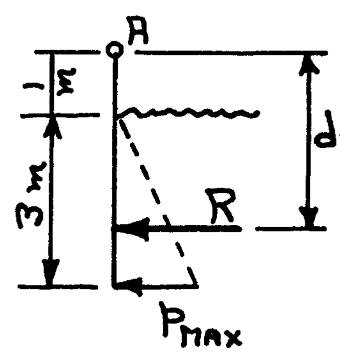
$$p_{max} = \rho gh = 1000(9.807)(3) = 29,421 \text{ N/m}^2$$

$$R = V_{ps} = \frac{1}{2}p_{max}hw$$

$$= \frac{1}{2}(29.421)(3)(2)$$

$$= 88,263 \text{ N} \cong 88.3 \text{ kN} \qquad \text{Ans.}$$

$$d_p = 1 + \frac{2}{3}(h) = 1 + \frac{2}{3}(3) = 3\text{m} \qquad \text{Ans.}$$



5-87* The width of the rectangular gate shown in Fig. P5-87 is 4 ft. Determine the magnitude of the resultant force $\bar{R}$ exerted on the gate by the water (γ = 62.4 lb/ft^3) pressure and the location of the center of pressure with respect to the hinge at the bottom of the gate.

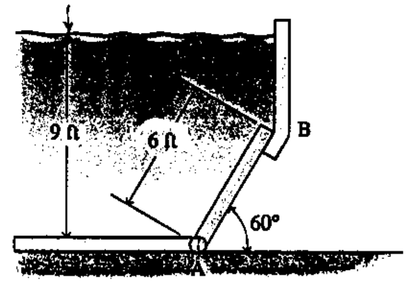
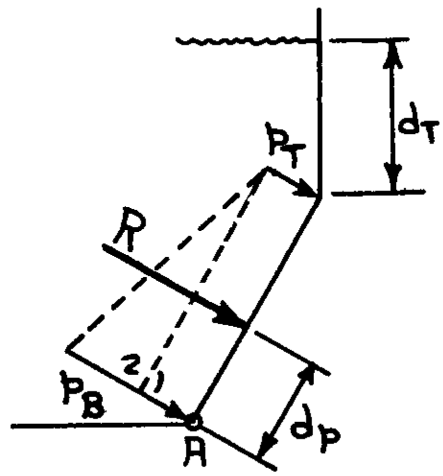


Fig. P5-87

SOLUTION



$d_T = 9 - 6 \sin 60° = 3.804$ ft

$p_T = \gamma d_T = 62.4(3.804) = 237.4$ ft

$p_B = \gamma d_B = 62.4(9) = 561.6$ ft

$V_{ps1} = p_T a w = 237.4(6)(4) = 5698$ lb

$V_{ps2} = \frac{1}{2}(p_B - p_T)aw = \frac{1}{2}(561.6 - 237.4)(6)(4) = 3890$ lb

$R = \Sigma F = V_{ps1} + V_{ps2} = 5698 + 3890 = 9588 \text{ lb} \cong 9.59$ kip   Ans.

$M_A = V_{ps1}d_{C1} + V_{ps2}d_{C2} = 5698(3) + 3890(2) = 24{,}874$ ft·lb

$Rd_p = M_A \qquad d_p = \frac{M_A}{R} = \frac{24874}{9588} = 2.594 \text{ ft} \cong 2.59$ ft   Ans.

5-88* The width of the rectangular gate shown in Fig. P5-88 is 4 m. Determine the magnitude of the resultant force $\vec{R}$ exerted on the gate by the water (ρ = 1000 kg/m^3) pressure and the location of the center of pressure with respect to the hinge at the bottom of the gate.

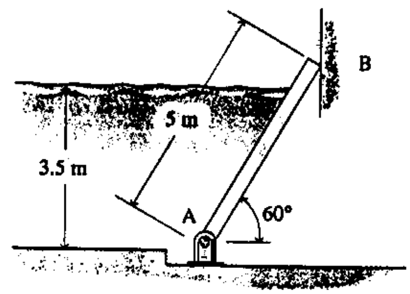


Fig. P5-88

SOLUTION

$$a = 3.5/\sin 60^\circ = 4.041 \text{ m}$$

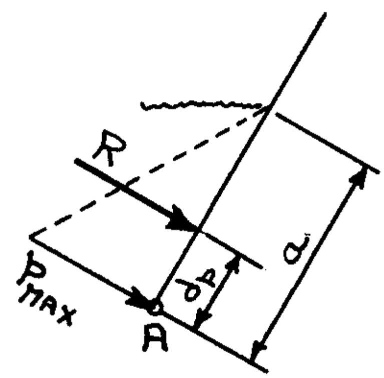
$$p_{max} = \rho g d = 1000(9.807)(3.5) = 34{,}325 \text{ N/m}^2$$

$$R = V_{ps} = \frac{1}{2} p_{max} a w$$

$$= \frac{1}{2}(34{,}325)(4.041)(4)$$

$$= 277{,}415 \text{ N} \cong 277 \text{ kN} \qquad \text{Ans.}$$

$$d_p = \frac{1}{3} a = \frac{1}{3}(4.041) = 1.347 \text{ m} \qquad \text{Ans.}$$



5-89 The width of the rectangular gate shown in Fig. P5-89 is 3 ft. Determine the magnitude of the resultant force $\vec{R}$ exerted on the gate by the water (γ = 62.4 lb/ft^3) pressure and the location of the center of pressure with respect to the hinge at the bottom of the gate.

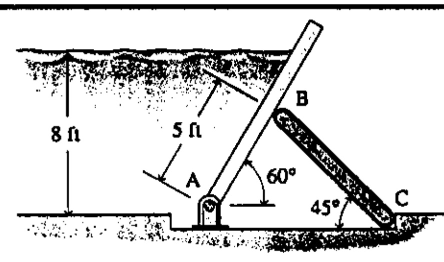


Fig. P5-89

SOLUTION

$$a = 8/\sin 60^\circ = 9.238 \text{ ft}$$

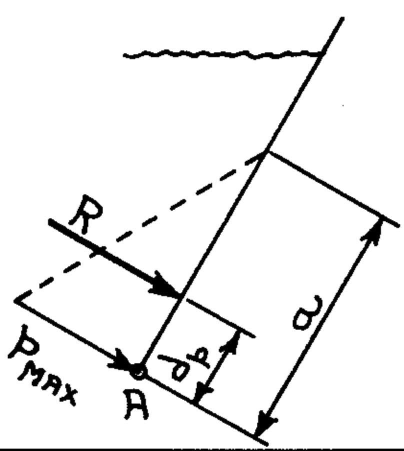
$$p_{max} = \gamma h = 62.4(8) = 499.2 \text{ lb/ft}^2$$

$$R = V_{ps} = \frac{1}{2} p_{max} a w$$

$$= \frac{1}{2}(499.2)(9.238)(3)$$

$$= 6917 \text{ lb} \cong 6.92 \text{ kip} \qquad \text{Ans.}$$

$$d_p = \frac{1}{3} a = \frac{1}{3}(9.238) = 3.08 \text{ ft} \qquad \text{Ans.}$$



5-90 The width of the cylindrical gate shown in Fig. P5-90 is 4 ft. Determine the magnitude and the slope of the line of action of the resultant force $\vec{R}$ exerted on the gate by the water (ρ = 1000 kg/m^3) pressure if d = 10 ft and a = 4 ft.

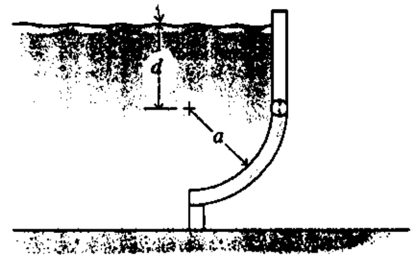
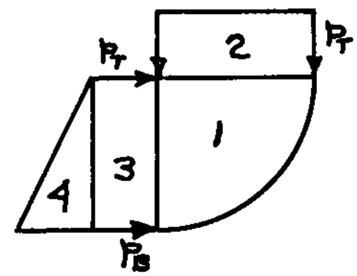


Fig. P5-90

SOLUTION



$$W = \gamma V = \gamma \, \frac{1}{4}\pi a^2 w$$

$$= 62.4\left(\frac{1}{4}\right)(\pi)(4)^2(4) = 3137 \text{ lb}$$

$$p_T = \gamma d_T = 62.4(10) = 624 \text{ lb/ft}^2$$

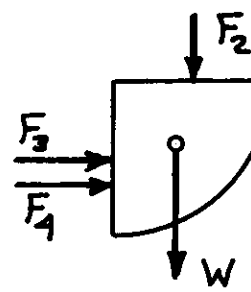
$$p_B = \gamma d_B = 62.4(14) = 873.6 \text{ lb/ft}^2$$

$$F_2 = V_{ps2} = p_T a w = 624(4)(4) = 9984 \text{ lb}$$

$$F_3 = V_{ps3} = p_T a w = 624(4)(4) = 9984 \text{ lb}$$

$$F_4 = V_{ps4} = \frac{1}{2}(p_B - p_T)aw$$

$$= \frac{1}{2}(873.6 - 624)(4)(4) = 1997 \text{ lb}$$



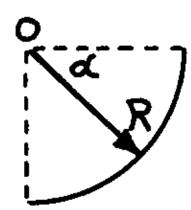
$$F_V = F_2 + W = 9984 + 3137 = 13{,}121 \text{ lb} \downarrow$$

$$F_H = F_3 + F_4 = 9984 + 1997 = 11{,}981 \text{ lb} \longrightarrow$$

$$R = \sqrt{F_V^2 + F_H^2} = \sqrt{(13121)^2 + (11981)^2}$$

$$= 17{,}768 \text{ lb} \cong 17.77 \text{ kips}$$ Ans.

$$\alpha = \tan^{-1}\frac{F_V}{F_H} = \tan^{-1}\frac{13{,}121}{11{,}981} = 47.6^\circ \searrow$$



$$M_O = F_2(a/2) + W(4a/3\pi) - F_3(a/2) - F_4(2a/3)$$

$$= 9984(2) + 3137(1.6977) - 9984(2) - 1997(2.667) = 0$$

$$Rd_p = M_O \qquad d_p = \frac{M_O}{R} = \frac{0}{17{,}768} = 0 \text{ (R passes through O)}$$

5-91 Determine the magnitude and locate the line of action of the resultant force $\bar{R}$ exerted by the water (γ = 62.4 lb/ft^3) pressure on a 5-ft length of the dam shown in Fig. P5-91.

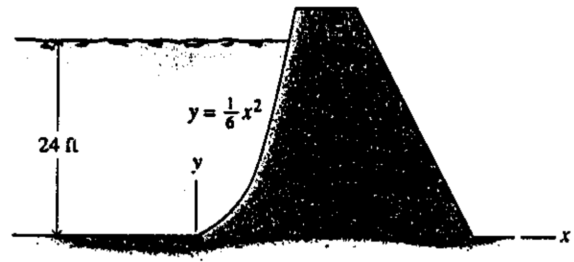
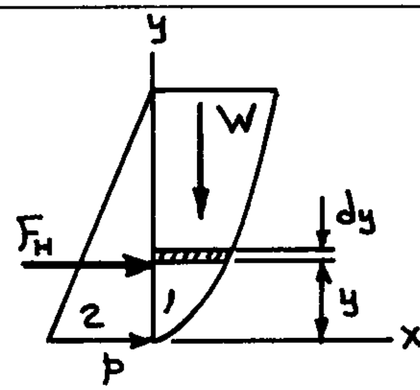


Fig. P5-91

SOLUTION



$$x = \sqrt{6}\,(y)^{1/2}$$

$$dV = Lx\,dy = 5\sqrt{6}\,(y)^{1/2}\,dy$$

$$V_1 = \int_0^{24} 5\sqrt{6}\,(y)^{1/2}\,dy$$

$$= \left[\frac{2}{3}(5\sqrt{6})(y^{3/2})\right]_0^{24} = 960 \text{ ft}^3$$

$$M_{O1} = \int_V \frac{x}{2}\,dV = \int_0^{24} 15y\,dy = \left[\frac{15y^2}{2}\right]_0^{24} = 4320 \text{ ft}^4$$

$$x_{C1} = \frac{M_{O1}}{V_1} = \frac{4320}{960} = 4.5 \text{ ft}$$

$$p = \gamma d = 62.4(24) = 1497.6 \text{ lb/ft}^2$$

$$F_H = V_{ps2} = \frac{1}{2}(1497.6)(24)(5) = 89{,}856 \text{ lb}$$

$$W = \gamma V = 62.4(960) = 59{,}904 \text{ lb}$$

$$\alpha = \tan^{-1}\frac{F_V}{F_H} = \tan^{-1}\frac{59{,}904}{89{,}856} = 33.7° \;\searrow$$ Ans.

$$R = \sqrt{F_H^2 + F_V^2} = \sqrt{(89{,}856)^2 + (59{,}904)^2} = 107{,}993 \text{ lb} \cong 108.0 \text{ kip}$$ Ans.

$$M_O = Rd = F_H d_H = W d_W = 89{,}856(8) + 59{,}904(4.5) = 988{,}416 \text{ ft}\cdot\text{lb}$$

$$d = \frac{M_O}{R} = \frac{988{,}416}{107{,}993} = 9.15 \text{ ft} \nearrow$$ Ans.

On the curved surface: $y = \frac{1}{6}x^2$

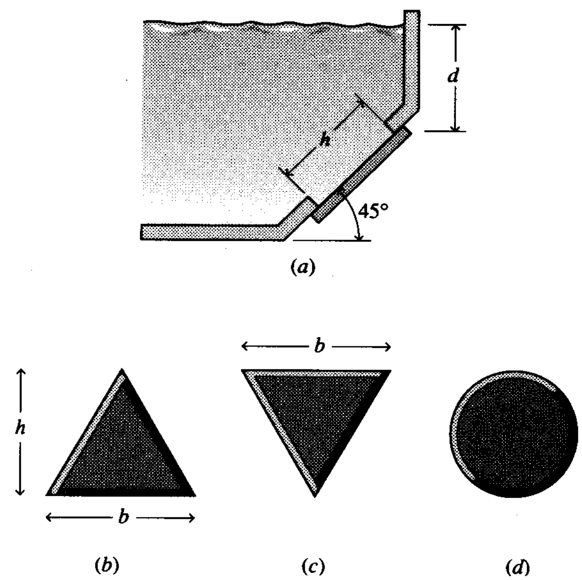
Therefore:

$$M_O = R_x y + R_y x$$

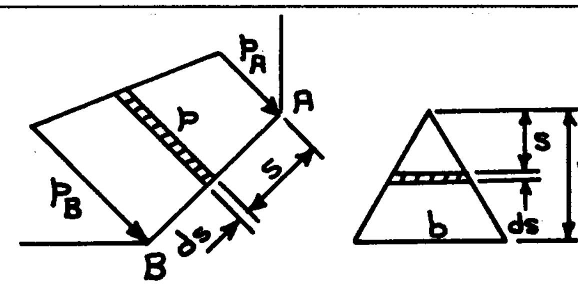
$$= 89{,}856(x^2/6) + 59{,}904x = 988{,}416$$

$x = 6.37$ ft $y = 6.76$ ft Ans.

5-92* A flat plate is used to seal an opening in a large water (ρ = 1000 kg/m^3) tank as shown in Fig. P5-92. If the opening has the cross section shown in Fig. P5-92b, determine the magnitude of the resultant force $\vec{R}$ exerted on the plate by the water pressure and the location of the center of pressure with respect to the bottom of the opening if d = 5 m, h = 2 m, and b = 2 m.



SOLUTION



$$p = \rho g(d + s \sin 45^\circ)$$
$$= 1000(9.81)(5 + 0.7071s)$$
$$= 49{,}050 + 6937s$$

$$dA = \frac{b}{h}\, s\, ds = \frac{2}{2}\, s\, ds = s\, ds$$

$$dR = p\, dA = (49{,}050 + 6937s)(s\, ds) = (49{,}050s + 6937s^2)\, ds$$

$$R = \int_0^2 (49{,}050s + 6937s^2)\, ds$$

$$= \Big[24{,}525s^2 + 2312s^3\Big]_0^2 = 116{,}596 \text{ N} \cong 116.6 \text{ kN}$$ Ans.

$$dM_A = s\, dR = (49{,}050s^2 + 6937s^3)\, ds$$

$$M_A = \int_0^2 (49{,}050s^2 + 6937s^3)\, ds = \Big[16{,}350s^3 + 1734s^4\Big]_0^2 = 158{,}544 \text{ N}\cdot\text{m}$$

$$d_{P/A} = \frac{M_A}{R} = \frac{158{,}544}{116{,}596} = 1.3598 \text{ m}$$

With respect to the bottom of the opening:

$$d_{P/B} = h - d_{P/A} = 2 - 1.3598 = 0.6402 \text{ m} \cong 0.640 \text{ m}$$ Ans.

5-93* A flat plate is used to seal an opening in a large water (γ = 62.4 lb/ft^3) tank as shown in Fig. P5-92. If the opening has the cross section shown in Fig. P5-92c, determine the magnitude of the resultant force $\vec{R}$ exerted on the plate by the water pressure and the location of the center of pressure with respect to the bottom of the opening if d = 15 ft, h = 5 ft, and b = 5 ft.

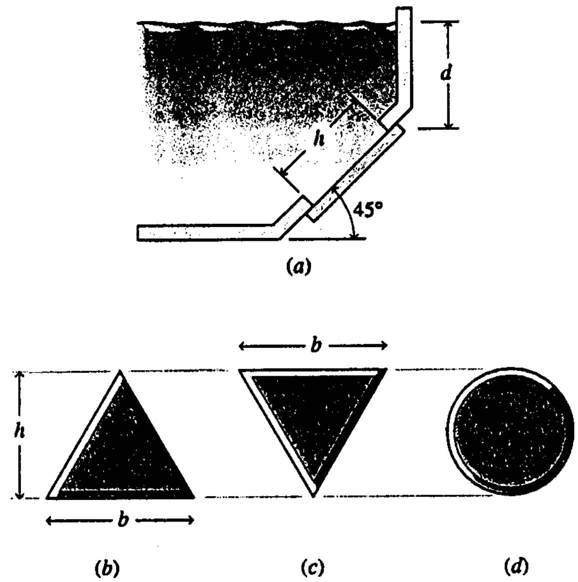
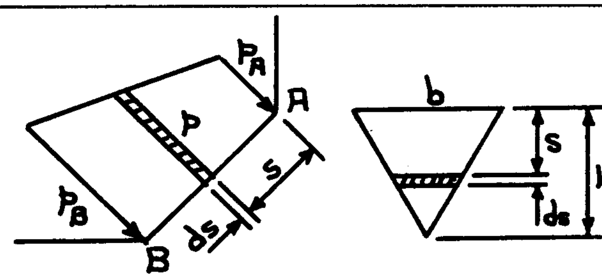


Fig. P5-92

SOLUTION



$$p = \gamma(d + s \sin 45^\circ)$$
$$= 62.4(15 + 0.7071s)$$
$$= 936 + 44.12s$$

$$dA = \frac{b}{h}(h - s)\ ds$$
$$= \frac{5}{5}(5 - s)\ ds = (5 - s)\ ds$$

$$dR = p\ dA = (936 + 44.1s)(5 - s)\ ds = (4680 - 715.4s - 44.12s^2)\ ds$$

$$R = \int_0^5 (4680 - 715.4s - 44.12s^2)\ ds$$
$$= \Big[4680s - 357.7s^2 - 14.707s^3\Big]_0^5 = 12{,}619 \text{ lb} \cong 12.62 \text{ kip} \qquad \text{Ans.}$$

$$dM_A = s\ dR = (4680s - 715.4s^2 - 44.12s^3)\ ds$$

$$M_A = \int_0^5 (4680s - 715.4s^2 - 44.12s^3)\ ds$$
$$= \Big[2340s^2 - 238.5s^3 - 11.03s^4\Big]_0^5 = 21{,}794 \text{ ft}\cdot\text{lb}$$

$$d_{P/A} = \frac{M_A}{R} = \frac{21{,}794}{12{,}619} = 1.7271 \text{ ft}$$

With respect to the bottom of the opening:

$$d_{P/B} = h - d_{P/A} = 5 - 1.7271 = 3.2729 \text{ ft} \cong 3.27 \text{ ft} \qquad \text{Ans.}$$

5-94 A flat plate is used to seal an opening in a large water (ρ = 1000 kg/m^3) tank as shown in Fig. P5-92. If the opening has the cross section shown in Fig. P5-92d, determine the magnitude of the resultant force $\bar{R}$ exerted on the plate by the water pressure and the location of the center of pressure with respect to the bottom of the opening if d = 10 m, and h = 1 m.

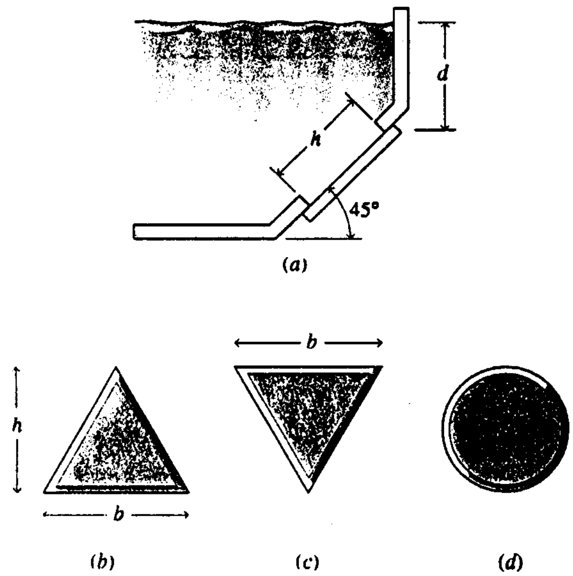
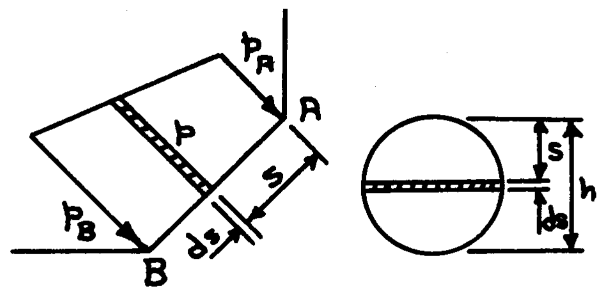


Fig. P5-92

SOLUTION



$$p = \rho g(d + s \sin 45^\circ)$$
$$= 1000(9.81)(10 + 0.7071s)$$
$$= 98{,}100 + 6937s$$

$$a = \sqrt{(0.5)^2 - (0.5 - s)^2}$$

$$dA = 2a\ ds = 2(s - s^2)^{1/2}\ ds$$

$$dR = p\ dA = [(196{,}200(s - s^2)^{1/2} + 13{,}874s(s - s^2)^{1/2}]\ ds$$

$$R = 196{,}200 \int_0^1 \sqrt{s - s^2}\ ds + 13{,}874 \int_0^1 s\sqrt{s - s^2}\ ds$$

$$= 79.77(10^3)\ \text{N} \cong 79.8\ \text{kN}$$ Ans.

$$dM_A = s\ dR = [(196{,}200s(s - s^2)^{1/2} + 13{,}874s^2(s - s^2)^{1/2}]\ ds$$

$$M_A = 196{,}200 \int_0^1 s(s - s^2)^{1/2}\ ds + 13{,}874 \int_0^1 s^2(s - s^2)^{1/2}\ ds$$

$$= 40{,}226\ \text{N}\cdot\text{m}$$

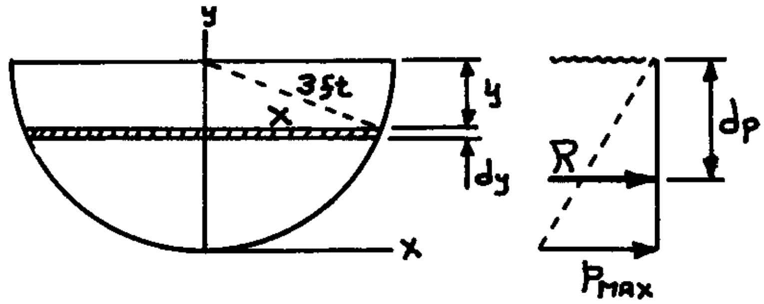
$$d_{P/A} = \frac{M_A}{R} = \frac{40{,}226}{79{,}770} = 0.5043\ \text{m}$$

With respect to the bottom of the opening:

$$d_{P/B} = h - d_{P/A} = 1 - 0.5043 = 0.4957\ \text{m} \cong 0.496\ \text{m}$$ Ans.

5-95 A water (γ = 62.4 lb/ft^3) trough 6 ft wide by 10 ft long by 3 ft deep has semicircular ends. When the trough is full of water, determine the magnitude of the resultant force $\bar{R}$ exerted on an end of the trough by the water pressure and the location of the center of pressure with respect to the water surface.

SOLUTION



$$p = \gamma y$$

$$dA = 2x\ dy = 2\sqrt{9 - y^2}\ dy$$

$$dR = p\ dA = 2\gamma y\sqrt{9 - y^2}\ dy$$

$$R = 2\gamma \int_0^3 y\sqrt{9 - y^2}\ dy = 2\gamma\left[-\frac{1}{3}\left(9 - y^2\right)^{3/2}\right]_0^3$$

$$= 18\gamma = 18(62.4) = 1123.2 \text{ lb} = 1123 \text{ lb}$$ Ans.

$$M_T = 2\gamma \int_0^3 y^2\sqrt{9 - y^2}\ dy$$

$$= 2\gamma\left[-\frac{y}{4}\left(9 - y^2\right)^{3/2} + \frac{9y}{8}\sqrt{9 - y^2} + \frac{81}{8}\sin^{-1}\frac{y}{3}\right]_0^3 = \frac{81\pi\gamma}{8}$$

$Rd_p = M_T$ $\qquad d_p = \frac{M_T}{R} = \frac{81\pi\gamma/8}{18\gamma} = 1.767$ ft Ans.

5-96 A 4-m diameter water (ρ = 1000 kg/m^3) pipe is filled to the level shown in Fig. P5-96. Determine the magnitude of the resultant force $\vec{R}$ exerted on a 2-m length of the curved section AB of the pipe by the water pressure and locate the line of action of the resultant.

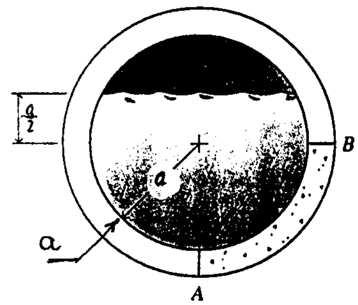
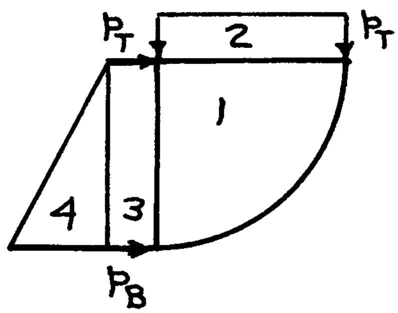
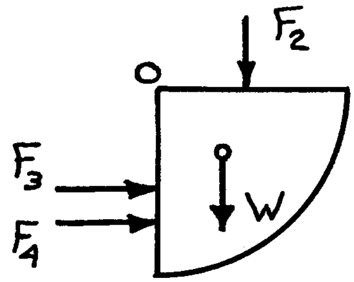


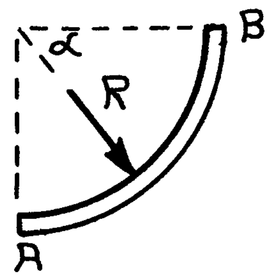
Fig. P5-96

SOLUTION





$$W = \rho g V = 1000(9.81)\left(\frac{\pi}{4}\right)(2)^2(2) = 61{,}638 \text{ N}$$



$$p_T = \rho g d_T = 1000(9.81)(1) = 9810 \text{ N/m}^2$$

$$p_B = \rho g d_B = 1000(9.81)(3) = 29{,}430 \text{ N/m}^2$$

$$F_2 = V_{ps2} = p_T rL = 9810(2)(2) = 39{,}240 \text{ N}$$

$$F_3 = V_{ps3} = p_T rL = 9810(2)(2) = 39{,}240 \text{ N}$$

$$F_4 = V_{ps4} = \frac{1}{2}(p_B - p_T)rL = \frac{1}{2}(29{,}430 - 9810)(2)(2) = 39{,}240 \text{ N}$$

$$F_V = W + F_2 = 61{,}638 + 39{,}240 = 100{,}878 \text{ N}$$

$$F_H = F_3 + F_4 = 39{,}240 + 39{,}240 = 78{,}480 \text{ N}$$

$$R = \sqrt{F_V^2 + F_H^2} = \sqrt{(100{,}878)^2 + (78{,}480)^2} = 127{,}810 \text{ N} = 127.8 \text{ kN} \qquad \text{Ans.}$$

$$\alpha = \tan^{-1}\frac{F_V}{F_H} = \tan^{-1}\frac{100{,}878}{78{,}480} = 52.1^\circ \qquad \text{Ans.}$$

$$M_O = Rd = F_2(r/2) + W(4r/3\pi) - F_3(r/2) - F_4(2r/3)$$
$$= 39{,}240(1) + 61{,}638(0.8488) - 39{,}240(1) - 39{,}240(1.3333) = 0$$

$$Rd_p = M_O \qquad d_p = \frac{M_O}{R} = \frac{0}{127{,}810} = 0 \text{ (R passes through O)} \qquad \text{Ans.}$$

C5-97 A rectangular gate holds back water (γ = 62.4 lb/ft^3) as shown in Fig. P5-97. The gate is 10 ft high and 8 ft wide and is pivoted at a point 4.5 ft from its bottom edge. When the water level is sufficiently low, the gate rests against the stop at C and does not touch the stop at A. When the water level is sufficiently high, the gate presses against the stop a A and does not touch the stop at C.

(a) Plot F_A, F_{Bx}, and F_C, the horizontal components of the forces at the two stops and the pivot, as a function of the water depth h ($5 \le h \le 35$ ft).

(b) Plot d, the location of the center of pressure relative to the pivot B as a function of the water depth h ($5 \le h \le 35$ ft).

(c) If the stop a A were removed, at what depth of water would the gate rotate and allow the water to drain out? Where is the location of the center of pressure for this depth?

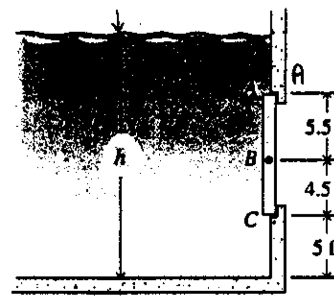
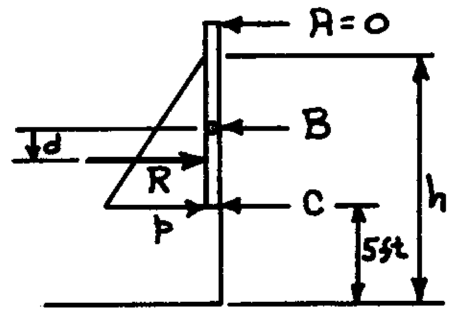


Fig. P5-97

SOLUTION



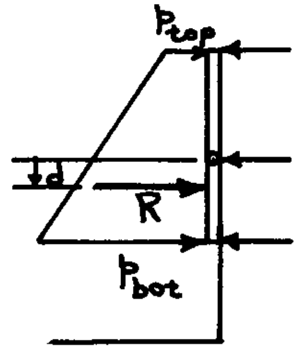
For $h \le 15$:

$$R = \frac{1}{2}[\gamma(h - 5)](h - 5)(8)$$

$$d = 4.5 - \frac{1}{3}(h - 5)$$

$$4.5C = Rd \qquad C = Rd/4.5$$

$$B + C = R \qquad B = R - C$$



For $h > 15$:

$$p_{top} = \gamma(h - 15)$$

$$p_{bot} = \gamma(h - 5)$$

$$R = \gamma(h - 15)(10)(8) + \frac{1}{2}[\gamma(10)](10)(8)$$

$$+\circlearrowleft \Sigma M_A = R(5.5 + d)$$

$$= [\gamma(h - 15)(10)(8)](5) + [(1/2)(\gamma)(10)(10)(8)](2/3)(10)$$

$$d = \frac{\gamma(h - 15)(10)(8)(5) + (1/2)(\gamma)(10)(10)(8)(2/3)(10)}{R} - 5.5$$

C5-97 (Continued)

If $d > 0$: $A = 0$

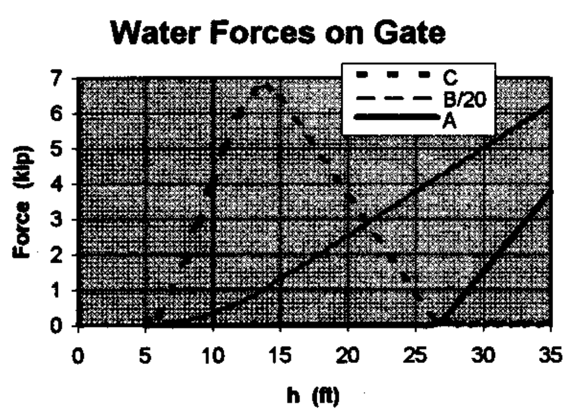
$C = Rd/4.5$

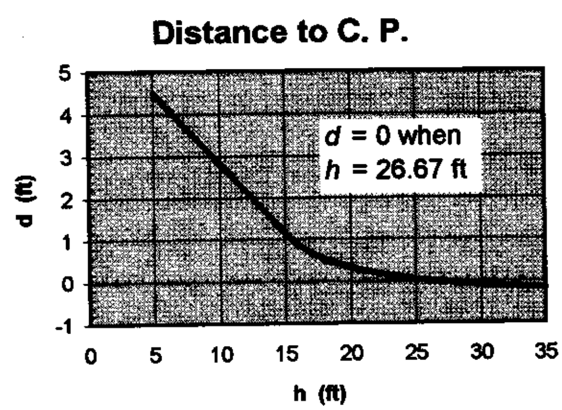
$B = R - C$

If $d < 0$: $C = 0$

$A = |Rd/5.5|$

$B = R - A$





C5-98 A water tank at the aquarium has a circular window 2 m in diameter in a vertical wall as shown in Fig. P5-98.

(a) Plot R, the resultant of the water (ρ = 1000 kg/m^3) pressure on the glass window, as a function of the water depth h ($0.5 \le h \le 5$ m).

(b) Plot d, the location of the center of pressure relative to the center of the circular window, as a function of the water depth h ($0.5 \le h \le 5$ m).

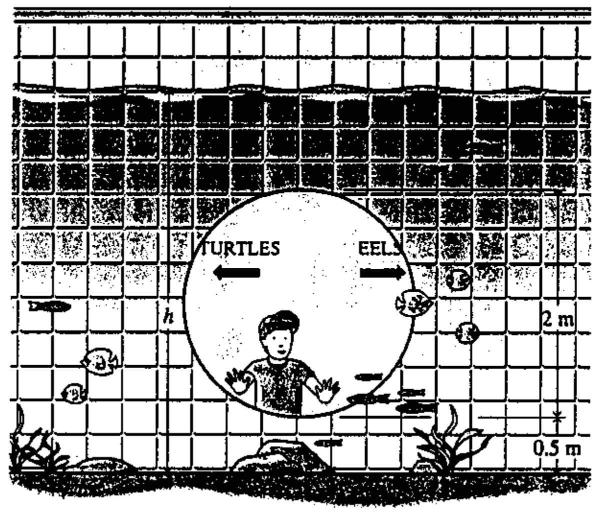


Fig. P5-98

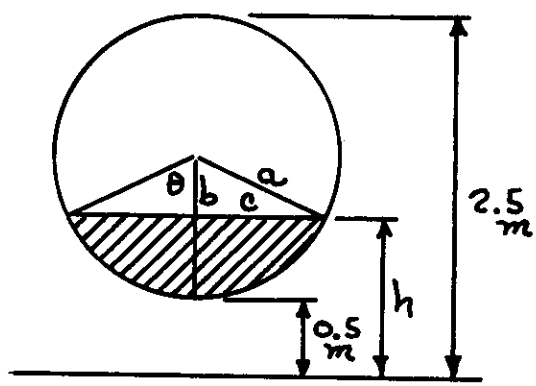
SOLUTION

For $h \le 2.5$ m:

$$b = a + 0.5 - h$$

$$c = \sqrt{a^2 - b^2}$$

$$\tan \theta = \frac{c}{b}$$



The shaded area A of water at the window is obtained by using the data in Table 5-1 for a triangle and a sector of a circle. Thus:

$$A = A_s - A_t = \theta a^2 - bc$$

Location of the centroid of area A with respect to the center of the circle:

$$Ad_C = A_s d_s - A_t d_t = \theta a^2\left(\frac{2}{3}\,\frac{a \sin \theta}{\theta}\right) - bc\left(\frac{2}{3}\,b\right) = \frac{2}{3}(a^3 \sin \theta - b^2 c)$$

$$d_C = \frac{A_s d_s - A_t d_t}{A_s - A_t} = \frac{2(a^3 \sin \theta - b^2 c)}{3(\theta a^2 - bc)}$$

$$R = p_C A = \gamma(d_C - b)A$$

C5-98 (Continued)

$$d = \frac{1}{R}\int_A \rho(p\ dA)$$

$$= \frac{1}{R}\int_b^a \rho[\gamma(\rho - b)](2)\sqrt{a^2 - \rho^2}\ d\rho$$

$$= \frac{2\gamma}{R}\left[-\frac{\rho}{4}\left(a^2 - \rho^2\right)^{3/2} + \frac{a^2}{8}\left(\rho\sqrt{a^2 - \rho^2} + a^2 \sin^{-1}\frac{\rho}{a}\right) + \frac{b}{3}\left(a^2 - \rho^2\right)^{3/2}\right]_b^a$$

$$= \frac{2\gamma}{R}\left[\frac{\pi a^4}{16} + \frac{b}{4}\left(a^2 - b^2\right)^{3/2} - \frac{a^2 b}{8}\sqrt{a^2 - b^2} - \frac{a^4}{8}\sin^{-1}\frac{b}{a} - \frac{b}{3}\left(a^2 - b^2\right)^{3/2}\right]$$

For $h > 2.5$:

$$p_C = \gamma(h - 1.5)$$

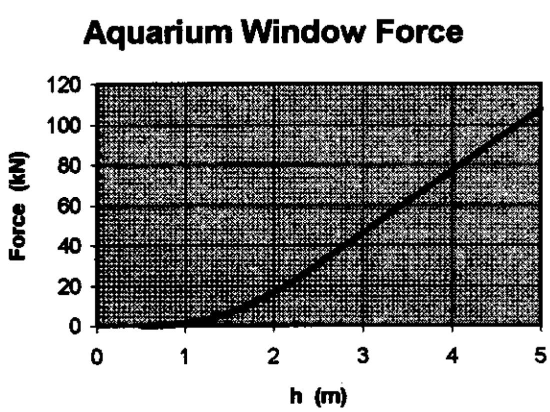
$$R = p_C A = p_C(\pi a^2) = \gamma(h - 1.5)(\pi a^2)$$

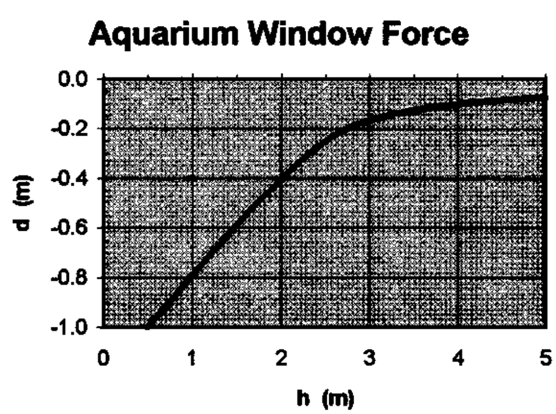
$$d = \frac{1}{R}\int_A \rho(p\ dA)$$

$$= \frac{1}{R}\int_{-a}^{+a} \rho[\gamma(h - 1.5 + \rho)](2)\sqrt{a^2 - \rho^2}\ d\rho$$

$$= \frac{2\gamma}{R}\left[-\frac{\rho}{4}\left(a^2 - \rho^2\right)^{3/2} + \frac{a^2}{8}\left(\rho\sqrt{a^2 - \rho^2} + a^2 \sin^{-1}\frac{\rho}{8}\right) + \frac{1}{3}(h - 1.5)\left(a^2 - \rho^2\right)^{3/2}\right]_{-a}^{+a}$$

$$= \frac{\gamma\pi a^4}{4R} = \frac{\gamma\pi a^4}{4[\gamma(h - 1.5)(\pi a^2)]} = \frac{a^2}{4(h - 1.5)}$$





C5-99 A wine barrel is modeled as a circular cylinder 4 ft in diameter and 6 ft long on its side as shown in Fig. P5-99.

(a) Plot R, the resultant of the wine (γ = 56 lb/ft^3) pressure on the end of the barrel, as a function of the wine depth h ($0 \le h \le 4$ ft).

(b) Plot d, the location of the center of pressure relative to the center of the circle, as a function of the wine depth h ($0 \le h \le 4$ ft).

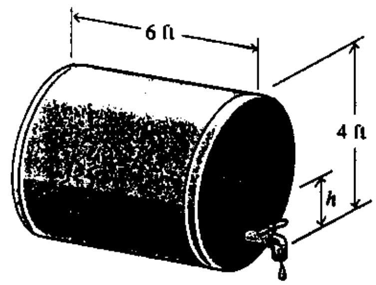
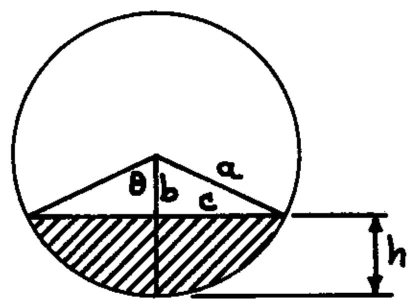


Fig. P5-99

SOLUTION

From the geometry of a cross section through the cylinder:



$$b = a - h$$

$$c = \sqrt{a^2 - b^2}$$

$$\tan \theta = \frac{c}{b}$$

The cross sectional area A of the wine is obtained by using the data in Table 5-1 for a triangle and a sector of a circle:

$$A = A_s - A_t = \theta a^2 - bc$$

Location of the centroid of area A with respect to the center of the circle:

$$Ad_C = A_s d_s - A_t d_t$$

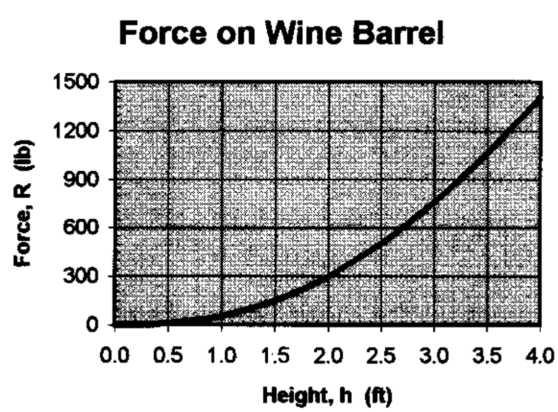
$$= \theta a^2\left(\frac{2}{3}\,\frac{a \sin \theta}{\theta}\right) - bc\left(\frac{2}{3}\,b\right)$$

$$= \frac{2}{3}(a^3 \sin \theta - b^2 c)$$

$$d_C = \frac{A_s d_s - A_t d_t}{A_s - A_t}$$

$$= \frac{2(a^3 \sin \theta - b^2 c)}{3(\theta a^2 - bc)}$$

$$R = p_C A = \gamma(d_C - b)A$$



C5-99 (Continued)

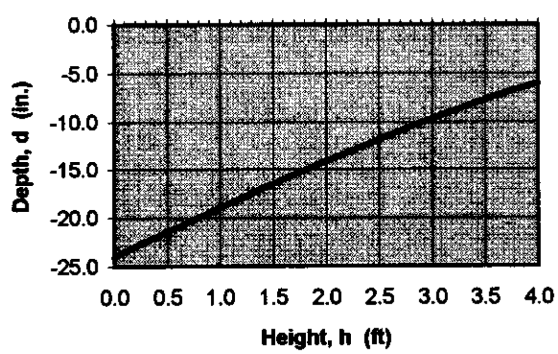
$$d = \frac{1}{R}\int_A \rho(p\ dA)$$

$$= \frac{1}{R}\int_b^a \rho[\gamma(\rho - b)](2)\sqrt{a^2 - \rho^2}\ d\rho$$

$$= \frac{2\gamma}{R}\left[-\frac{\rho}{4}\left(a^2 - \rho^2\right)^{3/2} + \frac{a^2}{8}\left(\rho\sqrt{a^2 - \rho^2} + a^2 \sin^{-1}\frac{\rho}{a}\right) + \frac{b}{3}\left(a^2 - \rho^2\right)^{3/2}\right]_b^a$$

$$= \frac{2\gamma}{R}\left[\frac{\pi a^4}{16} + \frac{b}{4}\left(a^2 - b^2\right)^{3/2} - \frac{a^2 b}{8}\sqrt{a^2 - b^2} - \frac{a^4}{8}\sin^{-1}\frac{b}{a} - \frac{b}{3}\left(a^2 - b^2\right)^{3/2}\right]$$

Force on Wine Barrel



C5-100 The tank shown in Fig. P5-100 is basically rectangular (1 m square by 2 m long). It is being raised from its side to its end. If the tank is one-fourth full of oil ($\rho = 850\ kg/m^3$):

(a) Plot h, the depth of oil in the tank, as a function of the angle θ ($0^\circ \le \theta \le 90^\circ$).

(b) Plot R, the resultant of the oil pressure on the end of the tank, as a function of θ ($0^\circ \le \theta \le 90^\circ$).

(c) Plot d, the location of the center of pressure relative to the center of the square, as a function of the angle θ ($0^\circ \le \theta \le 90^\circ$).

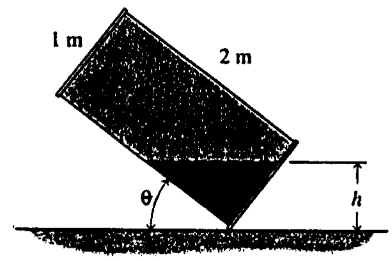
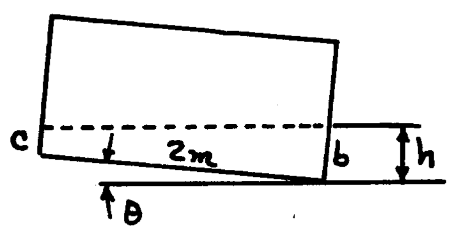


Fig. P5-100

SOLUTION



For $\theta = 0^\circ$:

$$V = \left(\frac{1}{4}\right)(2)(1)(1)$$

$$= 0.5 = \text{constant}$$

For $0 \le \theta \le \theta_1$:

$$V = 0.5 = \frac{1}{2}(b + c)(2)(1)$$

$$\cos\theta = \frac{h}{b} \qquad b = \frac{h}{\cos\theta}$$

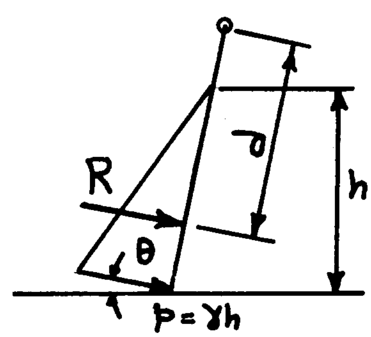
$$\tan\theta = \frac{b - c}{2} \qquad c = b - 2\tan\theta$$

$$b + c = 2b - 2\tan\theta = \frac{2h}{\cos\theta} - 2\tan\theta = 0.5$$

$$h = 0.25\cos\theta + \sin\theta$$

$$R = \frac{1}{2}(\gamma h)(b)(1) \qquad d = \frac{1}{2} - \frac{1}{3}b$$

θ_1 = angle at which c = 0

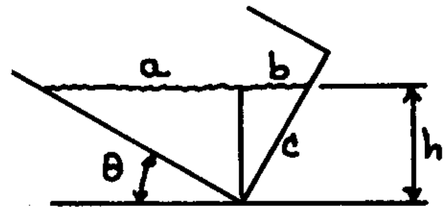


For $\theta_1 \le \theta \le \theta_2$:

$$V = \frac{1}{2}(a + b)h = 0.5$$

$$a = \frac{h}{\tan\theta} \qquad b = h\tan\theta$$

C5-100 (Continued)

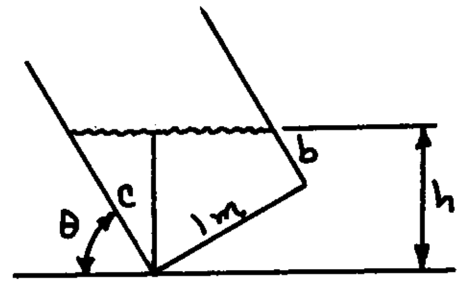


$$h = \left[\frac{\tan\theta}{1+\tan^2\theta}\right]^{1/2}$$

$$c = \frac{h}{\cos\theta}$$

$$R = \frac{1}{2}(\gamma h)(c)(1) \qquad d = \frac{1}{2} - \frac{1}{3}c$$

θ_2 = angle at which c = 1



For $\theta_2 \leq \theta \leq 90^\circ$:

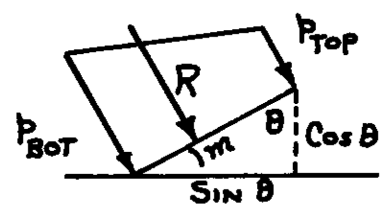
$$V = \frac{1}{2}(b + c)(1)(1) = 0.5$$

$$\sin\theta = \frac{h}{c} \qquad c = \frac{h}{\sin\theta}$$

$$\tan\theta = \frac{1}{c - b} \qquad b = c - \frac{1}{\tan\theta}$$

$$b + c = 2c - \frac{1}{\tan\theta} = \frac{2h}{\sin\theta} - \frac{1}{\tan\theta} = 1$$

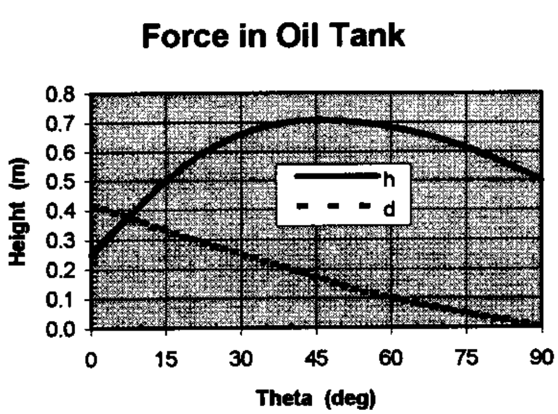
$$h = \frac{\sin\theta + \cos\theta}{2}$$

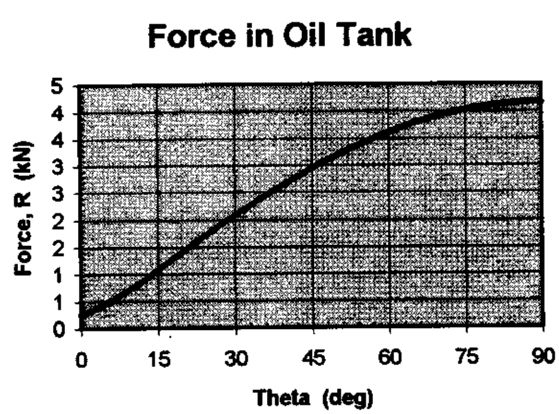


$$p_{top} = \gamma(h - \cos\theta) \qquad p_{bot} = \gamma h$$

$$R = [\gamma(h - \cos\theta)](1)(1) + \frac{1}{2}[\gamma\cos\theta]\ (1)(1)$$

$$d = \frac{1}{R}[\gamma(h - \cos\theta)](0) + \left(\frac{1}{2}\gamma\cos\theta\right)\left(\frac{1}{2} - \frac{1}{3}\right)$$





5-101 Locate the centroid of the shaded area shown in Fig. P5-101.

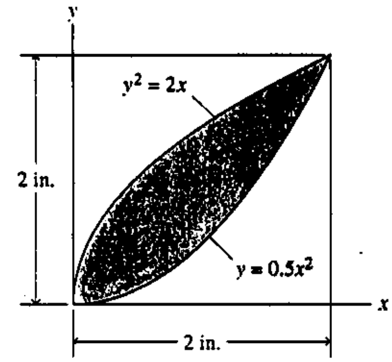
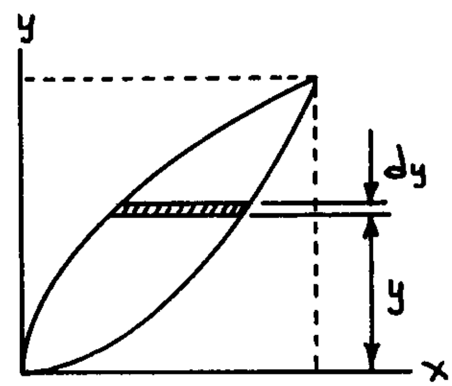


Fig. P5-101

SOLUTION

For the differential element of area shown in the sketch at the right:



$$dA = (x_2 - x_1)\,dy = \left[\sqrt{2}y^{1/2} - \frac{1}{2}y^2\right] dy$$

$$dM_x = y\,dA = \left[\sqrt{2}y^{3/2} - \frac{1}{2}y^3\right] dy$$

$$dM_y = \frac{1}{2}(x_2 + x_1)\,dA = \frac{1}{2}\left[2y - \frac{1}{4}y^4\right] dy$$

$$M_x = \int_0^2 \left(\sqrt{2}y^{3/2} - \frac{1}{2}y^3\right) dy = \left[\frac{2\sqrt{2}}{5}y^{5/2} - \frac{y^4}{8}\right]_0^2 = 1.200 \text{ in.}^3$$

$$M_y = \int_0^2 \frac{1}{2}\left(2y - \frac{1}{4}y^4\right) dy = \frac{1}{2}\left[\frac{2y^2}{2} - \frac{y^5}{20}\right]_0^2 = 1.200 \text{ in.}^3$$

$$A = \int_0^2 \left(\sqrt{2}y^{1/2} - \frac{1}{2}y^2\right) dy = \left[\frac{2\sqrt{2}}{3}y^{3/2} - \frac{y^3}{6}\right]_0^2 = 1.3333 \text{ in.}^2$$

$Ax_C = M_y$ $\qquad x_C = \frac{M_y}{A} = \frac{1.200}{1.3333} = 0.900$ in. Ans.

$Ay_C = M_x$ $\qquad y_C = \frac{M_x}{A} = \frac{1.200}{1.3333} = 0.900$ in. Ans.

5-102 Locate the centroid of the shaded area shown in Fig. P5-102.

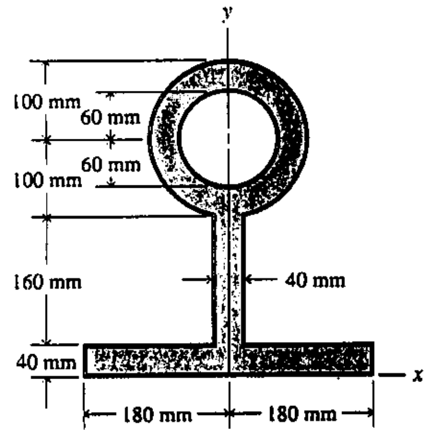
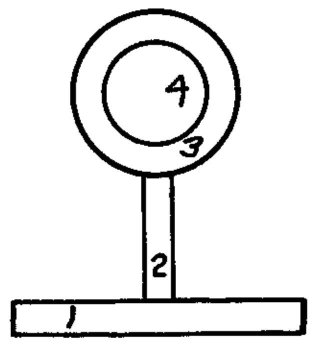


Fig. P5-102

SOLUTION



The shaded area can be divided into two rectangles and a circle with a circle removed. The centroid for the composite area is determined by listing the area, the centroid location, and the first moment for the individual parts in a table and applying Eqs. 5-13. Thus,

Part	A_i (mm^2)	x_{Ci} (mm)	M_y (mm^3)	y_{Ci} (mm)	M_x (mm^3)
1	14,400	0	0	20	288,000
2	6400	0	0	120	768,000
3	31,416	0	0	300	9,424,800
4	-11,310	0	0	300	-3,393,000
Σ	40,906		0		7,087,800

$Ax_C = M_y$ $\qquad x_C = \dfrac{M_y}{A} = \dfrac{0}{40,906} = 0$ Ans.

$Ay_C = M_x$ $\qquad y_C = \dfrac{M_x}{A} = \dfrac{7,087,800}{40,906} = 173.3$ mm Ans.

(Note that the composite area is symmetric about the y-axis; therefore, $x_C = 0$)

5-103 Locate the centroid of the volume shown in Fig. P5-103 if R = 10 in. and h = 32 in.

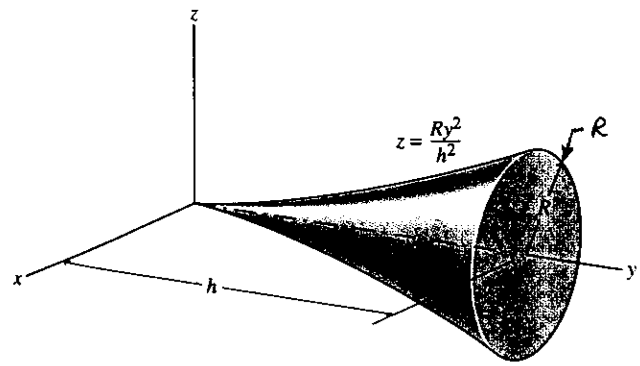
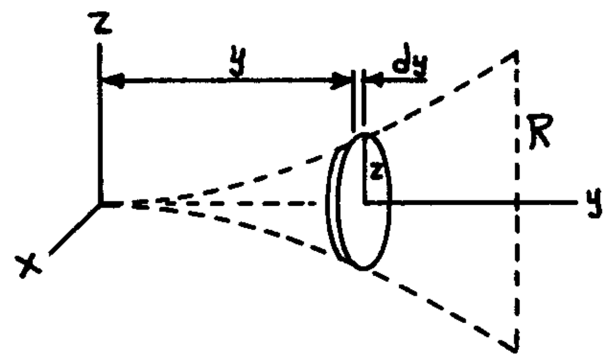


Fig. P5-103

SOLUTION

For the differential element of volume shown in the sketch at the right:



$$dV = \pi z^2\ dy = \frac{\pi R^2 y^4}{h^4}\ dy$$

$$M_{xz} = \int_V y\ dV = \frac{\pi R^2}{h^4}\int_0^h y^5\ dy = \frac{\pi R^2}{h^4}\left[\frac{y^6}{6}\right]_0^h = \frac{\pi R^2 h^2}{6}$$

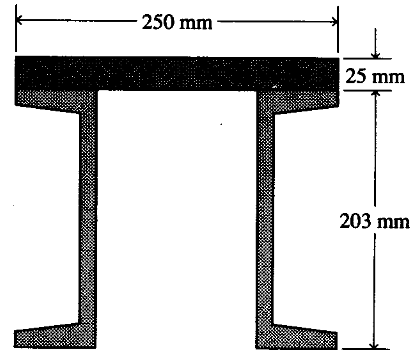
$$V = \int_V dV = \frac{\pi R^2}{h^4}\int_0^h y^4\ dy = \frac{\pi R^2}{h^4}\left[\frac{y^5}{5}\right]_0^h = \frac{\pi R^2 h}{5}$$

$Vy_C = M_{xz}$ $\qquad y_C = \frac{M_{xz}}{V} = \frac{\pi R^2 h^2/6}{\pi R^2 h/5} = \frac{5}{6}\ h$

With h = 32 in.: $\qquad y_C = \frac{5}{6}\ h = \frac{5}{6}(32) = 26.7$ in. $\qquad$ Ans.

From symmetry: $\qquad x_C = z_C = 0$ $\qquad$ Ans.

5-104 Two channel sections and a plate are used to form the cross section shown in Fig. P5-104. Each of the channels has a cross-sectional area of 2605 mm^2. Locate the y-coordinate of the centroid of the composite section with respect to the top surface of the plate.



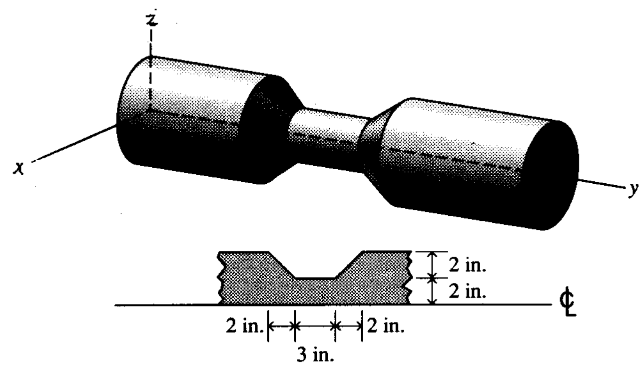
SOLUTION

$$A = 250(25) + 2(2605) = 11{,}460 \text{ mm}^2$$

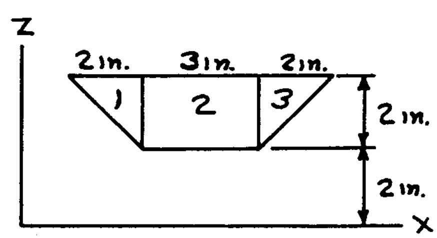
$$M_x = 250(25)(-12.5) + 2(2605)(-126.5) = -737{,}190 \text{ mm}^3$$

$$Ay_C = M_x \qquad y_C = \frac{M_x}{A} = \frac{-737{,}190}{11{,}460} = -64.3 \text{ mm}$$ Ans.

5-105 Determine the volume of material removed when the groove is cut in the circular shaft shown in Fig. P5-105.



SOLUTION



$$z_{C1} = 2 + \frac{2}{3}(2) = 3.333 \text{ in.}$$

$$z_{C2} = 2 + 1 = 3 \text{ in.}$$

$$z_{C3} = 2 + \frac{2}{3}(2) = 3.333 \text{ in.}$$

$$A = A_1 + A_2 + A_3 = 2 + 6 + 2 = 10 \text{ in.}^2$$

$$M_y = A_1 z_{C1} + A_2 z_{C2} + A_2 z_{C3} = 2(3.333) + 6(3) + 2(3.333) = 31.33 \text{ in.}^3$$

$$Az_C = M_y \qquad z_C = \frac{M_y}{A} = \frac{31.33}{10} = 3.133 \text{ in.}$$

$$V = 2\pi z_C A = 2\pi(3.133)(10) = 196.9 \text{ in.}^3$$ Ans.

5-106 Locate the centroid and the mass center of the volume shown in Fig. P5-106 which consists of an aluminum cylinder (ρ = 2770 kg/m^3) and a steel (ρ = 7870 kg/m^3) cylinder and sphere.

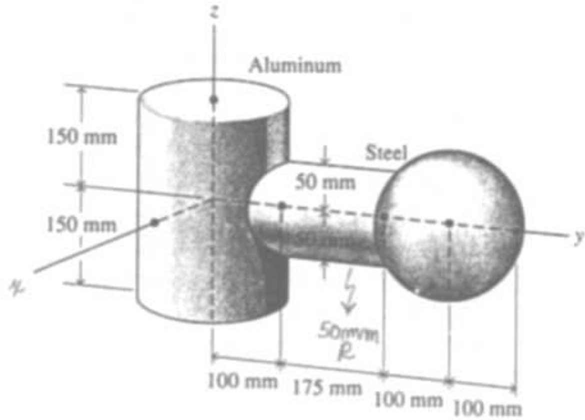


Fig. P5-106

SOLUTION

For the aluminum cylinder:

$V_1 = \pi r^2 h = \pi(100)^2(300) = 9.425(10^6)\text{ mm}^3$ $x_{C1} = z_{C1} = 0$

$m_1 = \rho_1 V_1 = 2770(9.425)(10^{-3}) = 26.11\text{ kg}$ $y_{C1} = 0$

For the steel cylinder:

$V_2 = \pi r^2 L = \pi(50)^2(175) = 1.374(10^6)\text{ mm}^3$ $x_{C2} = z_{C2} = 0$

$m_2 = \rho_2 V_2 = 7870(1.374)(10^{-3}) = 10.81\text{ kg}$ $y_{C2} = 187.5\text{ mm}$

For the steel sphere:

$V_3 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(100)^3 = 4.189(10^6)\text{ mm}^3$ $x_{C3} = z_{C3} = 0$

$m_3 = \rho_3 V_3 = 7870(4.189)(10^{-3}) = 32.97\text{ kg}$ $y_{C3} = 375\text{ mm}$

$$y_C = \frac{9.425(0) + 1.374(187.5) + 4.189(375)}{9.425 + 1.374 + 4.189} = 122.0\text{ mm}$$ Ans.

From symmetry: $x_C = z_C = 0$ Ans.

$$y_G = \frac{26.11(0) + 10.81(187.5) + 32.97(375)}{26.11 + 10.81 + 32.97} = 206\text{ mm}$$ Ans.

From symmetry: $x_G = z_G = 0$ Ans.

5-107 Determine the resultant $\bar{\mathbf{R}}$ of the system of distributed loads on the beam of Fig. P5-107 and locate its line of action with respect to the left support of the beam.

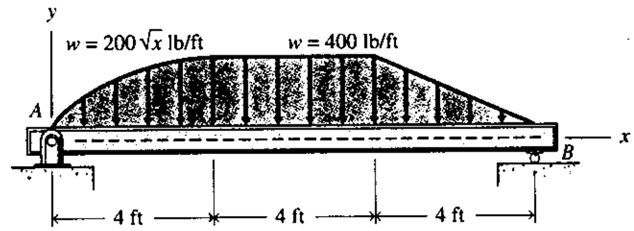
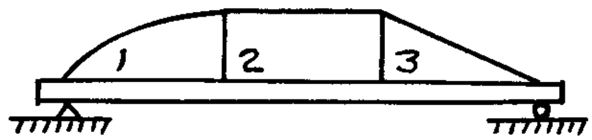


Fig. P5-107

SOLUTION



$$A_1 = \int_A w\,dx = \int_0^4 200(x)^{1/2}\,dx = \left[\frac{2}{3}(200)x^{3/2}\right]_0^4 = 1066.7 \text{ lb}$$

$$M_A = \int_A x\,dA = \int_0^4 200(x)^{3/2}\,dx = \left[\frac{2}{5}(200)x^{5/2}\right]_0^4 = 2560 \text{ ft}\cdot\text{lb}$$

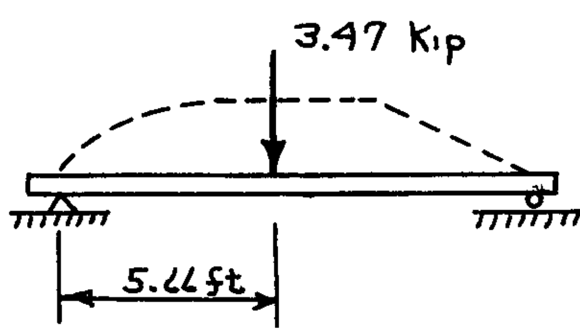
$$Ax_{C1} = M_A \qquad x_{C1} = \frac{M_A}{A_1} = \frac{2560}{1066.7} = 2.400 \text{ ft}$$

$$A_2 = 400(4) = 1600 \text{ lb} \qquad x_{C2} = 6 \text{ ft}$$

$$A_3 = \frac{1}{2}(400)(4) = 800 \text{ lb} \qquad x_{C3} = 9.333 \text{ ft}$$

$$R = \Sigma F = A_1 + A_2 + A_3 = 1066.7 + 1600 + 800 = 3466.7 \cong 3.47 \text{ kip}$$

$\bar{\mathbf{R}} = 3.47$ kip ↓   Ans.



$$M_A = A_1x_{C1} + A_2x_{C2} + A_3x_{C3}$$
$$= 1066.7(2.400) + 1600(6)$$
$$+ 800(9.333) = 19{,}626 \text{ ft}\cdot\text{lb}$$

$$Rd = M_A$$

$$d = \frac{M_A}{R} = \frac{19{,}626}{3466.7} = 5.66 \text{ ft} \longrightarrow$$ Ans.

5-108 Determine the resultant $\bar{R}$ of the system of distributed loads on the beam of Fig. P5-108 and locate its line of action with respect to the left support of the beam.

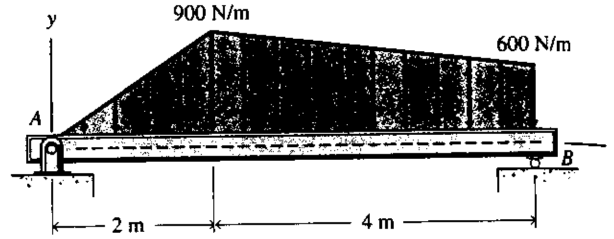
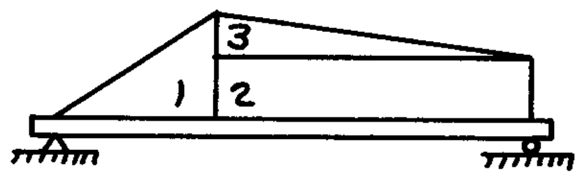


Fig. P5-108

SOLUTION



$A_1 = \frac{1}{2}(900)(2) = 900$ N

$x_{C1} = \frac{2}{3}(2) = 1.333$ m

$A_2 = 600(4) = 2400$ N

$x_{C2} = 2 + \frac{1}{2}(4) = 4.00$ m

$A_3 = \frac{1}{2}(900 - 600)(4) = 600$ N

$x_{C3} = 2 + \frac{1}{3}(4) = 3.333$ m

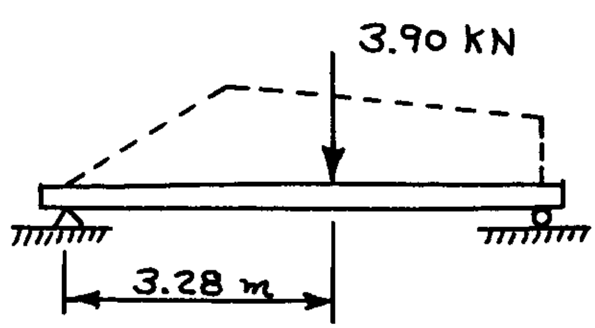
$R = \Sigma F = A_1 + A_2 + A_3 = 900 + 2400 + 600 = 3900 \text{ N} = 3.90 \text{ kN}$

$\bar{R} = 3.90 \text{ kN} \downarrow$ Ans.

$M_A = A_1 x_{C1} + A_2 x_{C2} + A_3 x_{C3}$

$= 900(1.333) + 2400(4.00) + 600(3.333) = 12{,}800 \text{ N·m}$

$Rd = M_A$ $d = \frac{M_A}{R} = \frac{12{,}800}{3900} = 3.28 \text{ m} \rightarrow$ Ans.



5-109 A square viewing window is located in a large water tank as shown in Fig. P5-109. Determine the resultant force $\bar{R}$ exerted by the water pressure on the window and locate its line of action with respect to point A at the top of the window if d = 35 ft and h = 25 ft. Use 62.4 lb/ft^3 for the specific weight γ of water.

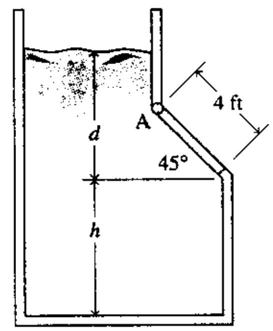
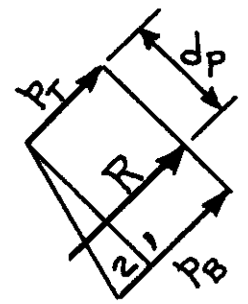


Fig. P5-109

SOLUTION



$d_T = 35 - 4 \sin 45^\circ = 32.17$ ft

$p_T = \gamma d_T = 62.4(32.17) = 2008$ lb/ft^2

$p_B = \gamma d_B = 62.4(35) = 2184$ lb/ft^2

$V_{ps1} = p_T a^2 = 2008(4)^2 = 32,128$ lb

$V_{ps2} = \frac{1}{2}(p_B - p_T)a^2 = \frac{1}{2}(2184 - 2008)(4)^2 = 1408$ lb

$R = \Sigma F = V_{ps1} + V_{ps2} = 32,128 + 1408 = 33,536 \text{ lb} \cong 33.5 \text{ kip}$ Ans.

$M_A = V_{ps1}d_{C1} + V_{ps2}d_{C2} = 32,128(2) + 1408(2.667) = 68,011 \text{ ft}\cdot\text{lb}$

$Rd_p = M_A \qquad d_p = \frac{M_A}{R} = \frac{68,011}{33,536} = 2.027 \text{ ft} \cong 2.03 \text{ ft}$ Ans.

6-1 Draw a free-body diagram for the cantilever beam shown in Fig. P6-1 which has a weight $\bar{W}$.

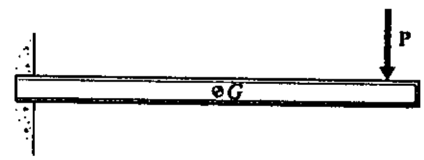
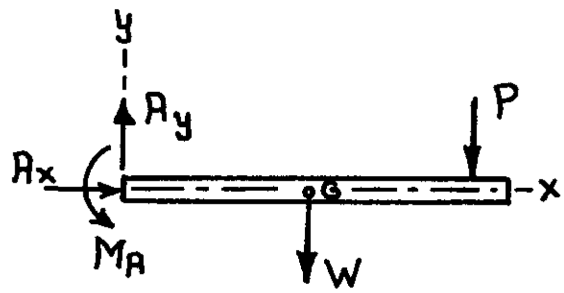


Fig. P6-1

SOLUTION

The action of the fixed support at the left end of the beam is represented by force components $\bar{A}_x$ and $\bar{A}_y$ and a moment $\bar{M}_A$. The weight $\bar{W}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



6-2 Draw a free-body diagram for the beam shown in Fig. P6-2 which has a mass m.

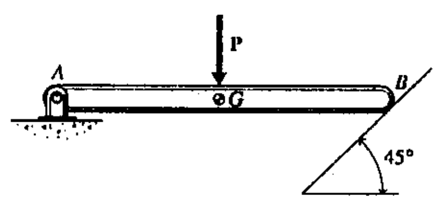
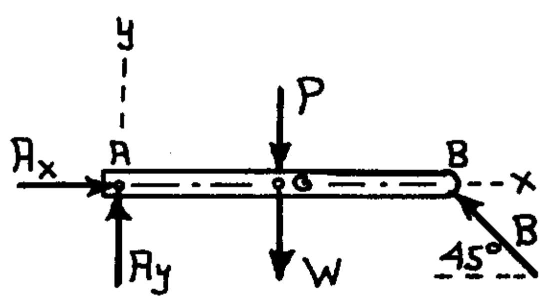


Fig. P6-2

SOLUTION

The action of the pin at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. Force $\bar{B}$ acts normal to the supporting surface at B. The weight $\bar{W} = m\bar{g}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



6-3 Draw a free-body diagram for the cylinder shown in Fig. P6-3 which has a weight $\mathbf{W}$.

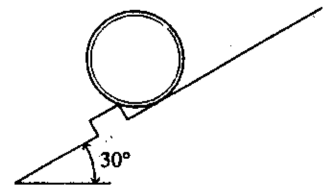
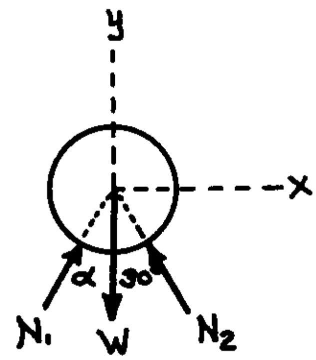


Fig. P6-3

SOLUTION

Forces $\mathbf{N}_1$ and $\mathbf{N}_2$ act normal to the surface of the cylinder at points of contact with the supporting surfaces. The weight $\mathbf{W}$ of the cylinder acts through the center of gravity G of the cylinder and is directed toward the center of the earth.



6-4 Draw a free-body diagram for the curved bar shown in Fig. P6-4.

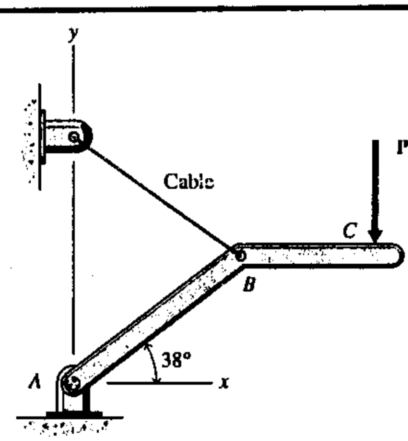
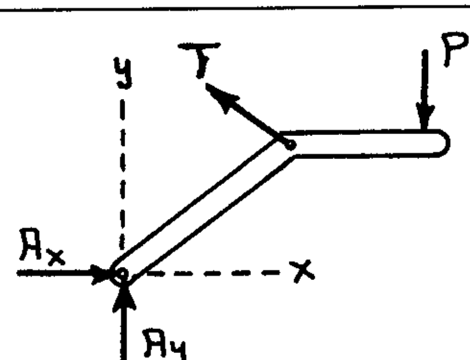


Fig. P6-4

SOLUTION

The action of the pin at support A is represented by force components $\mathbf{A}_x$ and $\mathbf{A}_y$. The cable at B exerts a tensile force $\mathbf{T}$ on the bar that is tangent to the cable at point B.



6-5 Draw a free-body diagram for the curved bar shown in Fig. P6-5.

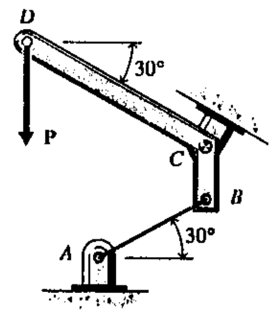
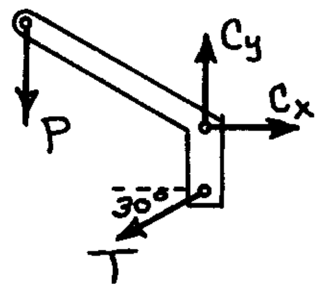


Fig. P6-5

SOLUTION

The cable at B exerts a tensile force $\bar{T}$ on the bar that is tangent to the cable at point B. The action of the pin at support C is represented by force components $\bar{C}_x$ and $\bar{C}_y$.



6-6 Draw a free-body diagram for the angle bracket shown in Fig. P6-6.

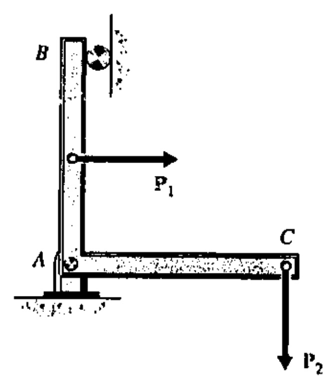
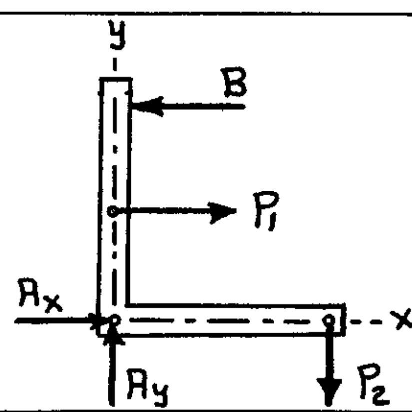


Fig. P6-6

SOLUTION

The action of the pin at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The roller at B exerts a compressive force $\bar{B}$ normal to the surface of the bracket.



6-7 Draw a free-body diagram for the curved bar shown in Fig. P6-7.

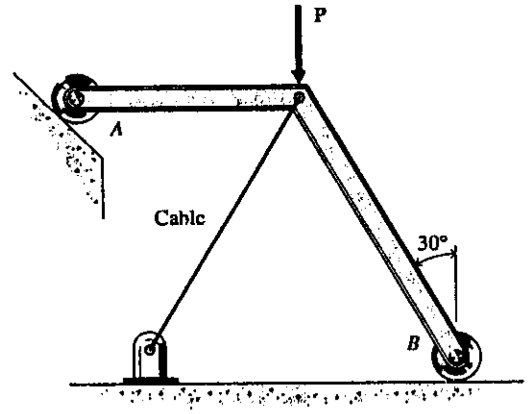
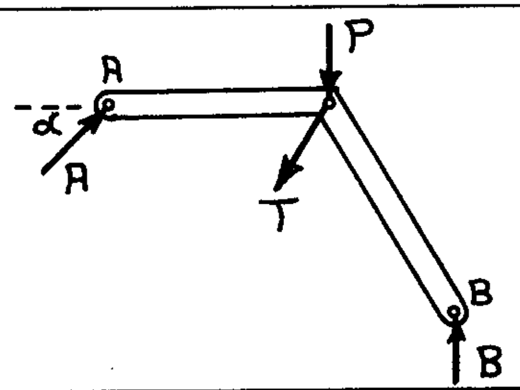


Fig. P6-7

SOLUTION

The cable exerts a tensile force $\bar{T}$ on the bar that is tangent to the cable at the point of attachment. The wheels at A and B exert forces $\bar{A}$ and $\bar{B}$ on the bar that are normal to the surfaces supporting the wheels.



6-8 Draw a free-body diagram for the beam shown in Fig. P6-8.

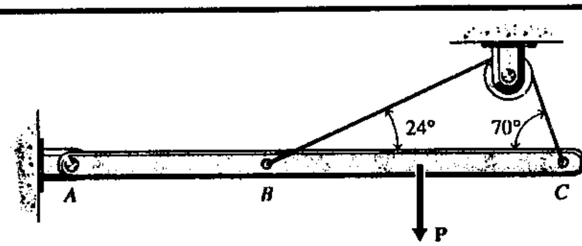
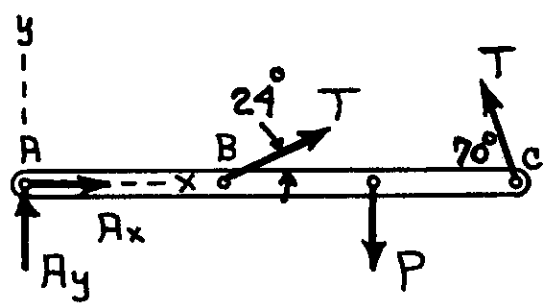


Fig. P6-8

SOLUTION

The action of the pin at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The cable is continuous over the pulley; therefore, the force in the cable is constant. At points B and C the cable exerts tensile forces $\bar{T}$ on the beam that are tangent to the cable.



6-9 Draw a free-body diagram for the sled shown in Fig. P6-9.

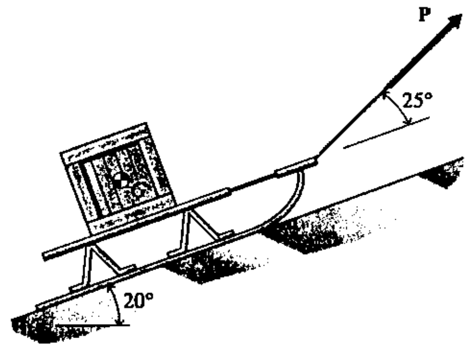
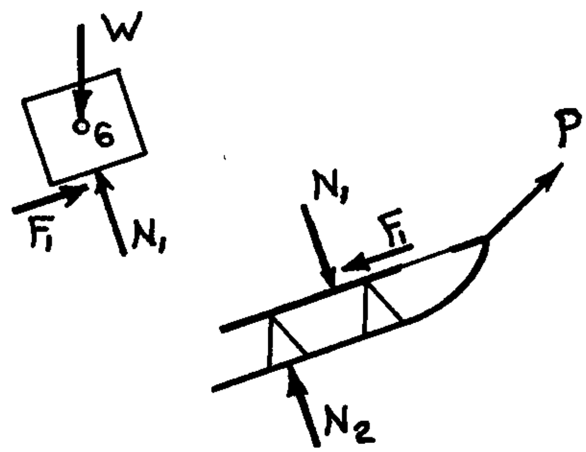


Fig. P6-9

SOLUTION

The tow rope exerts a force $\bar{P}$ on the sled that is tangent to the rope at the point of attachment. The box exerts normal and friction forces $\bar{N}_1$ and $\bar{F}_1$ on the sled. The support surface would exert only a normal forces $\bar{N}_2$ on the sled since it is assumed to be smooth.



6-10 Draw a free-body diagram for the diving board shown in Fig. P6-10.

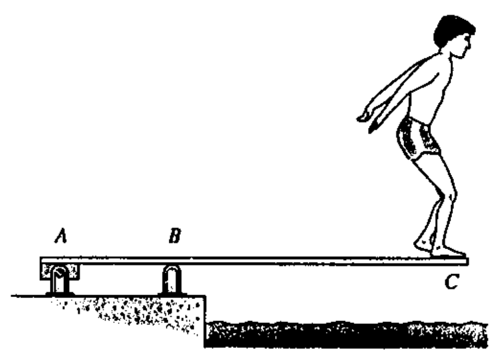
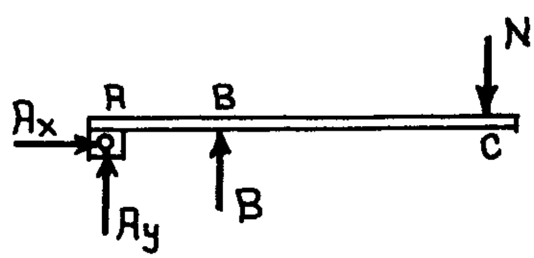


Fig. P6-10

SOLUTION

The action of the pin at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. Support B exerts a normal force $\bar{B}$ on the board. The diver exerts a normal force $\bar{N}$ on the board that is equal to his weight.



6-11 Draw a free-body diagram for the cart shown in Fig. P6-11 which has a weight **W**.

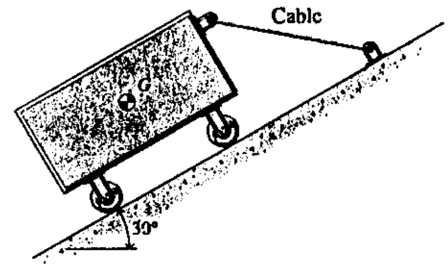
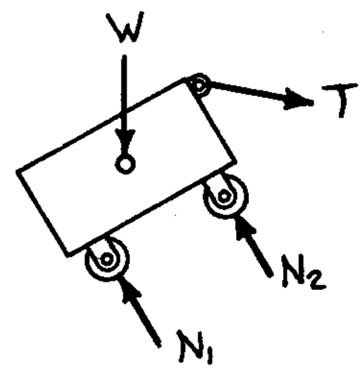


Fig. P6-11

SOLUTION

The cable exerts a force **T** on the cart that is tangent to the cable at the point of attachment. The weight **W** of the cart acts through the center of gravity G of the cart and is directed toward the center of the earth. The support surface exerts normal forces $\mathbf{N}_1$ and $\mathbf{N}_2$ on the wheels since the surface is assumed to be smooth.



6-12 Draw a free-body diagram for the lawn mower shown in Fig. P6-12 which has a weight **W** and is resting on a rough surface.

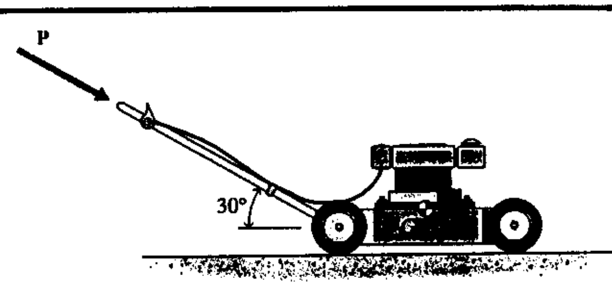
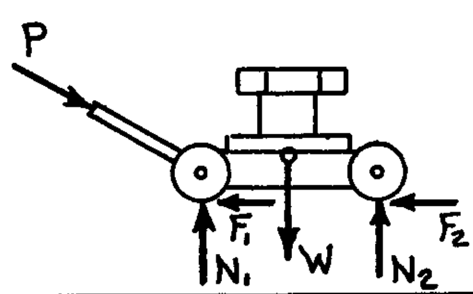


Fig. P6-12

SOLUTION

The weight **W** of the mower acts through the center of gravity G of the mower and is directed toward the center of the earth. The support surface exerts normal forces $\mathbf{N}_1$ and $\mathbf{N}_2$ and frictional forces $\mathbf{F}_1$ and $\mathbf{F}_2$ on the wheels since the surface is assumed to be rough.



6-13 Draw a free-body diagram for
(a) the cylinder shown in Fig. P6-13 which has a weight **W**,
(b) bar ACE shown in Fig. P6-13.
(c) bar BCD shown in Fig. P6-13.

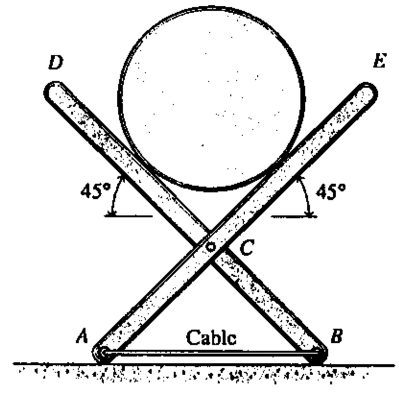
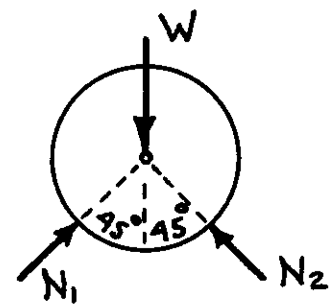


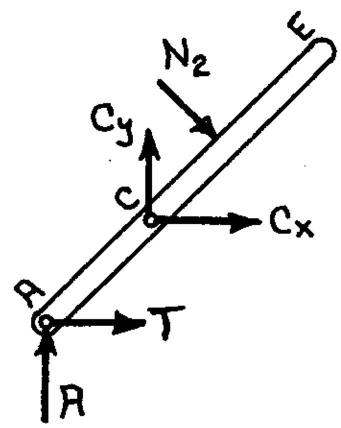
Fig. P6-13

SOLUTION

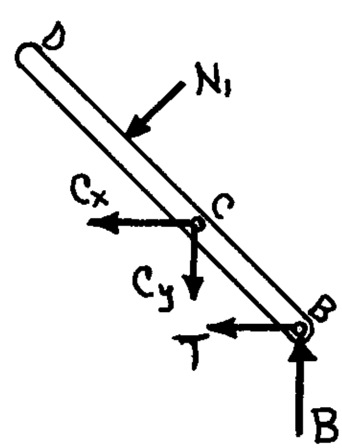
(a) The weight **W** of the cylinder acts through the center of gravity G of the cylinder and is directed toward the center of the earth. Forces $\mathbf{N}_1$ and $\mathbf{N}_2$ act normal to the surface of the cylinder at points of contact with the supporting bars.



(b) Force **A** acts normal to the smooth supporting surface at A. The cable at A exerts a force **T** on bar ACE that is tangent to the cable at A. The action of the pin at C is represented by force components $\mathbf{C}_x$ and $\mathbf{C}_y$. Force $\mathbf{N}_2$ acts normal to the surface of bar ACE at the point of contact with the cylinder.



(c) Force **B** acts normal to the smooth supporting surface at B. The cable at B exerts a force **T** on bar BCD that is tangent to the cable at B. The action of the pin at C is represented by force components $\mathbf{C}_x$ and $\mathbf{C}_y$. Force $\mathbf{N}_1$ acts normal to the surface of bar BCD at the point of contact with the cylinder.



6-14 Draw a free-body diagram for

(a) the cylinder shown in Fig. P6-14 which has a mass m.

(b) the frame shown in Fig. P6-14. Neglect the weight of the frame.

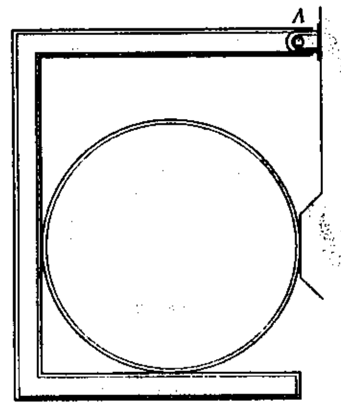
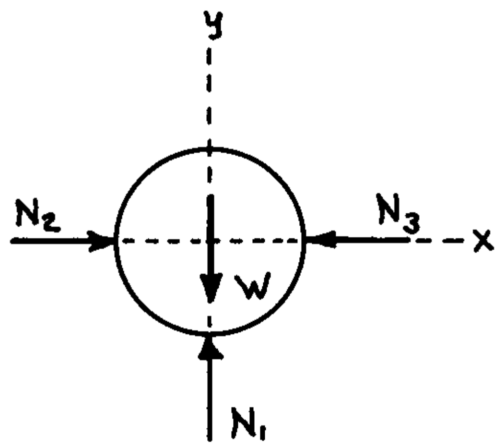


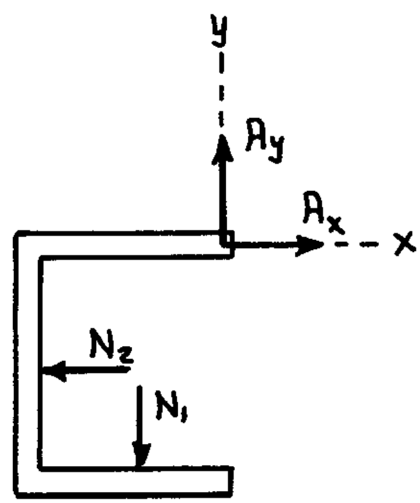
Fig. P6-14

SOLUTION

(a) Forces $\mathbf{N}_1$, $\mathbf{N}_2$, and $\mathbf{N}_3$ act normal to the surface of the cylinder at points of contact with the frame and the wall. The weight $\mathbf{W}$ of the cylinder acts through the center of gravity G of the cylinder and is directed toward the center of the earth.



(b) The action of the pin at support A of the frame is represented by force components $\bar{A}_x$ and $\bar{A}_y$. Forces $\mathbf{N}_1$ and $\mathbf{N}_2$ act normal to the surface of the frame at points of contact with the cylinder.



6-15 Draw a free-body diagram for bracket AB shown in Fig. P6-15.

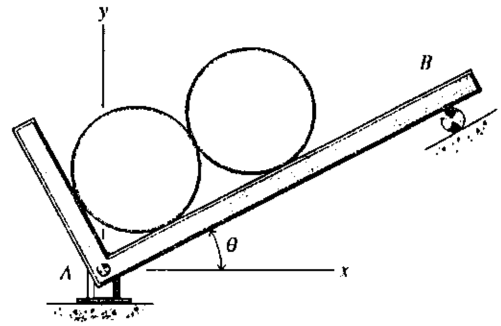
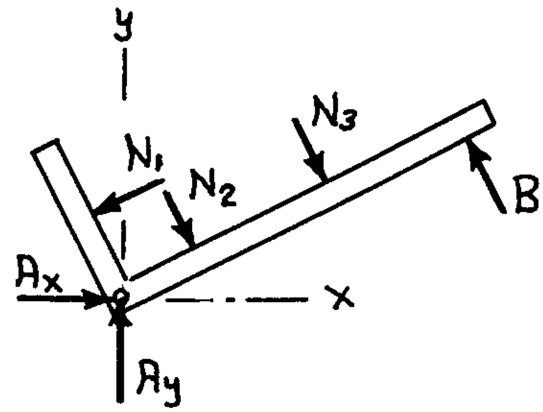


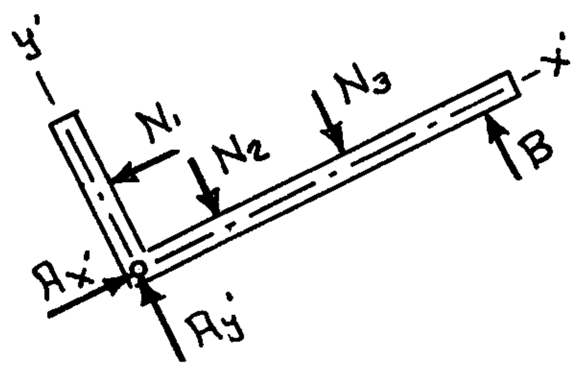
Fig. P6-15

SOLUTION

The action of the pin at support A of the bracket is represented by force components $\bar{A}_x$ and $\bar{A}_y$. Forces $\bar{N}_1$, $\bar{N}_2$, and $\bar{N}_3$ act normal to the surface of the bracket at points of contact with the cylinders. Support B exerts a normal force $\bar{B}$ on the bracket.



In a similar manner, the free-body diagram can be drawn using x′ and y′ axes as:



6-16 Draw a free-body diagram for

(a) bar AC shown in Fig. P6-16.

(b) bar DE shown in Fig. P6-16.

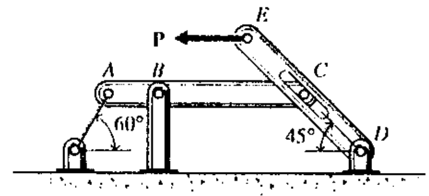
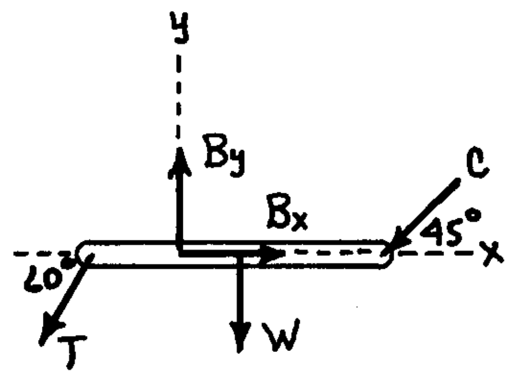


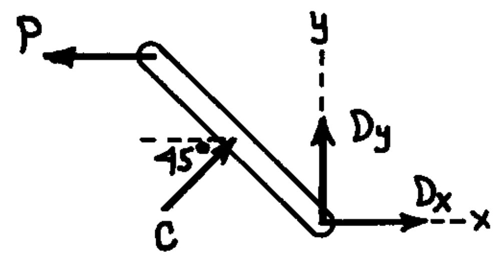
Fig. P6-16

SOLUTION

(a) The cable exerts a force $\bar{T}$ on bar AC that is tangent to the cable at point A. The action of the pin at support B is represented by force components $\bar{B}_x$ and $\bar{B}_y$. The pin at C exerts a force $\bar{C}$ on bar AC that is normal to the surface of the slot in bar DE.



(b) The action of the pin at support D is represented by force components $\bar{D}_x$ and $\bar{D}_y$. The pin at C exerts a force $\bar{C}$ on bar DE that is normal to the surface of the slot in the bar.



6-17 Draw a free-body diagram for
(a) the cylinder shown in Fig. P6-17 which has a weight $\mathbf{W}$.
(b) bar AC shown in Fig. P6-17.
(c) bar BCD shown in Fig. P6-17.

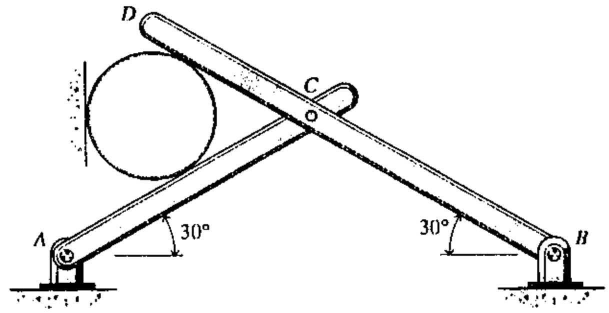
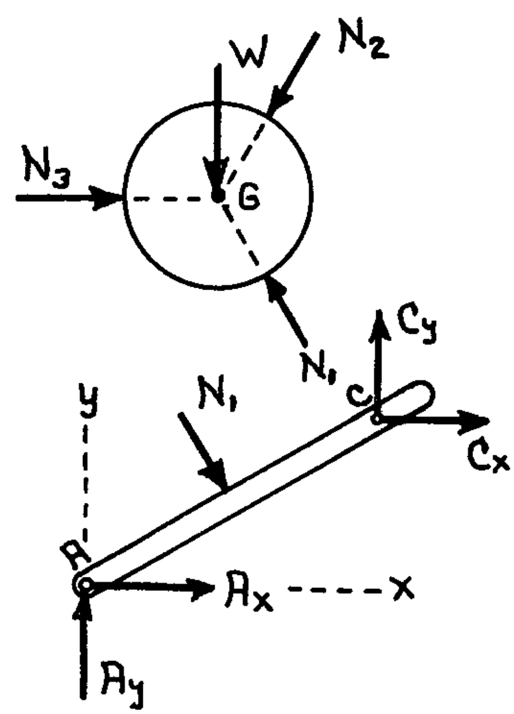


Fig. P6-17

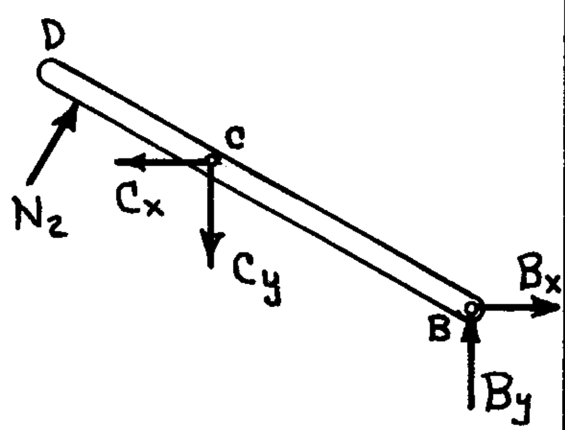
SOLUTION

(a) Forces $\mathbf{N}_1$, $\mathbf{N}_2$, and $\mathbf{N}_3$ act normal to the surface of the cylinder at points of contact with the the bars and the wall. The weight $\mathbf{W}$ of the cylinder acts through the center of gravity G of the cylinder and is directed toward the center of the earth.



(b) The action of the pin at support A of bar AC is represented by force components $\mathbf{A}_x$ and $\mathbf{A}_y$. A force $\mathbf{N}_1$ acts normal to the surface of bar AC at the point of contact with the cylinder. The action of the pin at C is represented by force components $\mathbf{C}_x$ and $\mathbf{C}_y$.

(c) A force $\mathbf{N}_2$ acts normal to the surface of bar BD at the point of contact with the cylinder. The action of the pin at C is represented by force components $\mathbf{C}_x$ and $\mathbf{C}_y$. The roller at B exerts a force $\mathbf{B}$ on bar BD that is normal to the supporting surface.



6-18 Draw a free-body diagram for
(a) bar AB shown in Fig. P6-18.
(b) bar CB shown in Fig. P6-18.

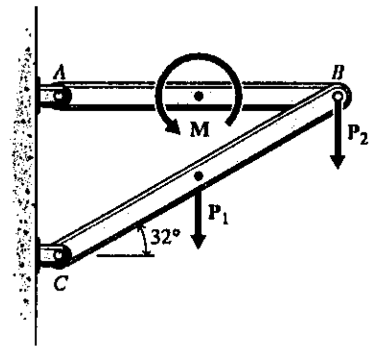
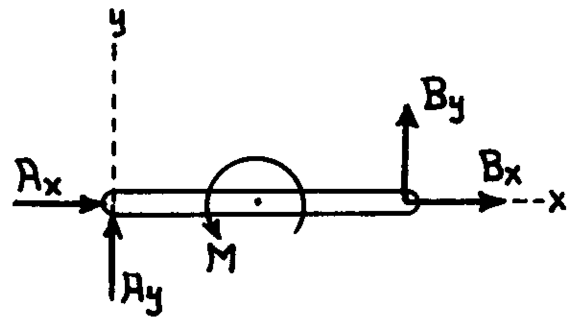


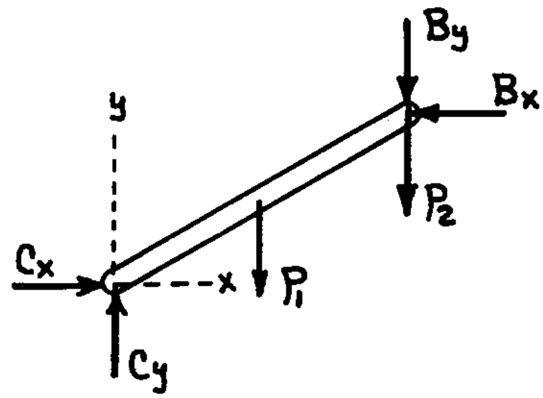
Fig. P6-18

SOLUTION

(a) The action of the pin at support A of bar AB is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The action of the pin at support B of bar AB is represented by force components $\bar{B}_x$ and $\bar{B}_y$.



(b) The action of the pin at support C of bar CB is represented by force components $\bar{C}_x$ and $\bar{C}_y$. The pin at the right end of bar BC transmits forces $\bar{B}_x$ and $\bar{B}_y$ from bar AB to bar CB and supports load $\bar{P}_2$. The free-body diagram for bar CB shows pin B as part of the bar.



6-19 Draw a free-body diagram for
(a) bar BE shown in Fig. P6-19.
(b) bar DF shown in Fig. P6-19.

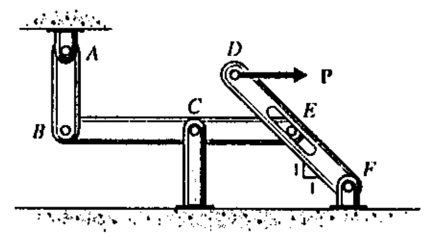
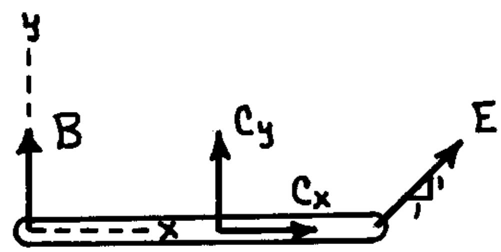


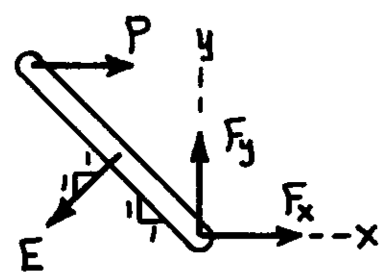
Fig. P6-19

SOLUTION

(a) The link at the left end of bar BE exerts a force $\bar{B}$ on the bar that is directed along the axis of the link. The action of the pin support at point C of bar BE is represented by forces $\bar{C}_x$ and $\bar{C}_y$. The pin at E exerts a force $\bar{E}$ on bar BE that is normal to the surface of the slot in bar DF.



(b) The action of the pin support at the right end of bar DF is represented by force components $\bar{F}_x$ and $\bar{F}_y$. The pin at E exerts a force $\bar{E}$ on bar DF that is normal to the surface of the slot in the bar.



6-20 Draw a free-body diagram for

(a) bar AC shown in Fig. P6-20.

(b) bar DF shown in Fig. P6-20.

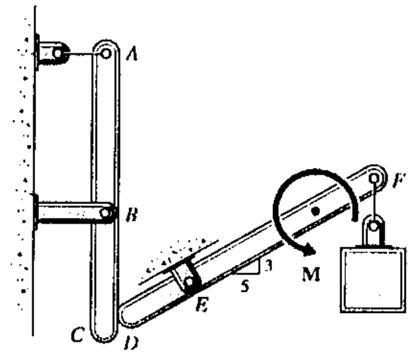
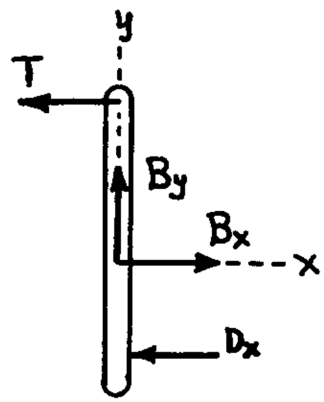


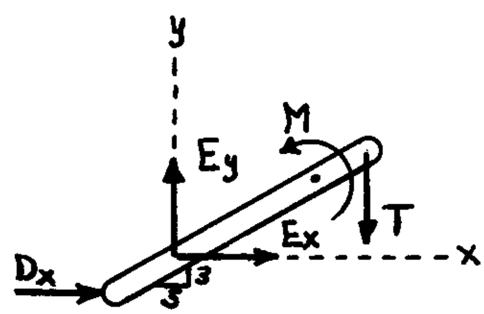
Fig. P6-20

SOLUTION

(a) The cable at A exerts a force $\bar{T}$ on bar AC that is tangent to the cable at the point of attachment. The action of the pin at support B is represented by force components $\bar{B}_x$ and $\bar{B}_y$. Bar DF exerts a force D_x at point D that is normal to bar AC at the point of contact.



(b) The cable at F exerts a force $\bar{T}$ on bar DF that is tangent to the cable at the point of attachment. The action of the pin at support E is represented by force components $\bar{E}_x$ and $\bar{E}_y$. Bar AC exerts a force $\bar{D}_x$ at point D that is normal to bar AC at the point of contact.



6-21 Draw a free-body diagram for the bent bar shown in Fig. P6-21 which is fixed at the wall at A.

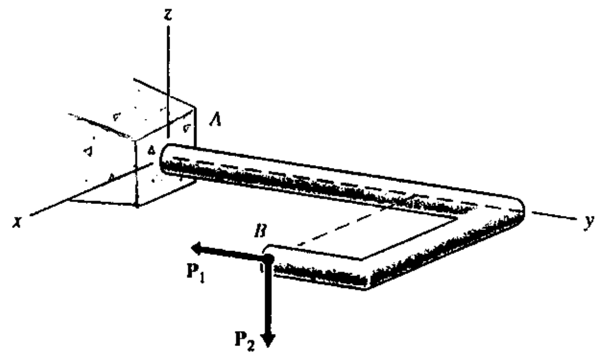
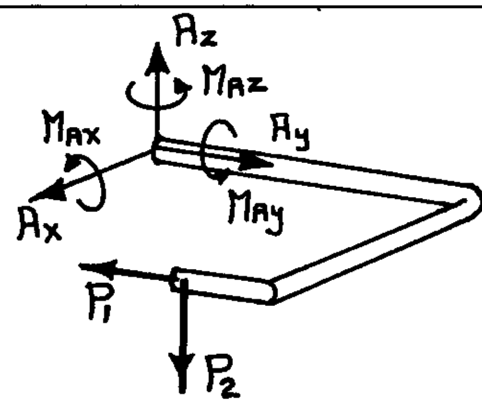


Fig. P6-21

SOLUTION

The action of the fixed support at the left end A of the bar is represented by force components $\bar{A}_x$, $\bar{A}_y$, and $\bar{A}_z$ and by moment components $\bar{M}_{Ax}$, $\bar{M}_{Ay}$, and $\bar{M}_{Az}$.



6-22 Draw a free-body diagram for the bent bar shown in Fig. P6-22 which is fixed at the wall at A.

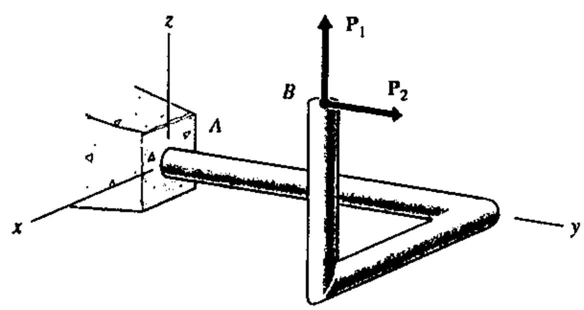
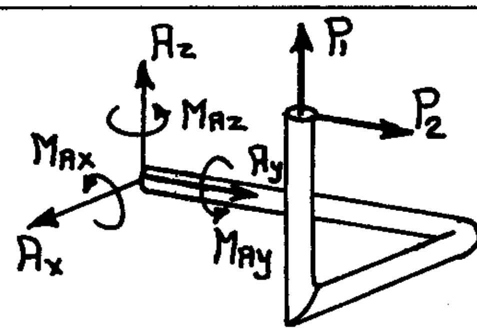


Fig. P6-22

SOLUTION

The action of the fixed support at the left end A of the bar is represented by force components $\bar{A}_x$, $\bar{A}_y$, and $\bar{A}_z$ and by moment components $\bar{M}_{Ax}$, $\bar{M}_{Ay}$, and $\bar{M}_{Az}$.



6-23 Draw a free-body diagram for the shaft shown in Fig. P6-23. The bearing at A is a thrust bearing and the bearing at D is a ball bearing. Neglect the weights of the shaft and the levers.

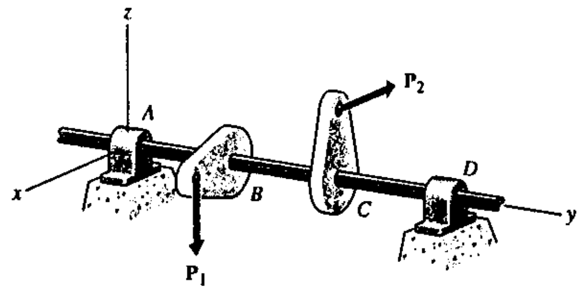
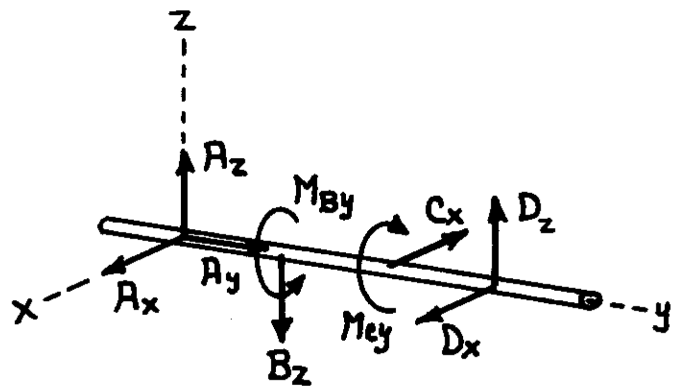


Fig. P6-23

SOLUTION



The action of the thrust bearing at support A is represented by force components $\bar{A}_x$, $\bar{A}_y$, and $\bar{A}_z$. The action of the lever at B is represented by a force component $\bar{B}_z$ and a moment component $\bar{M}_{By}$. The action of the lever at C is represented by a force component $\bar{C}_x$ and a moment component $\bar{M}_{Cy}$. The action of the ball bearing at support D is represented by force components $\bar{D}_x$ and $\bar{D}_z$.

6-24 Draw a free-body diagram for the block shown in Fig. P6-24 which has a mass m. The support at A is a ball and socket. The support at B is a pin and bracket.

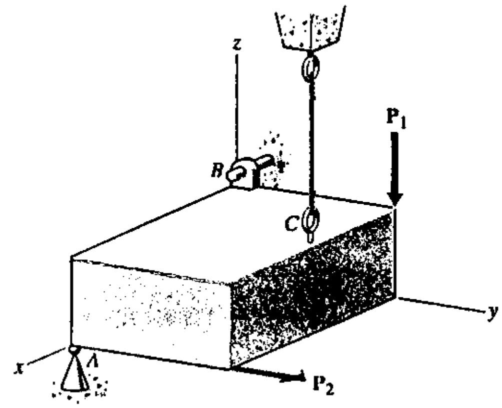
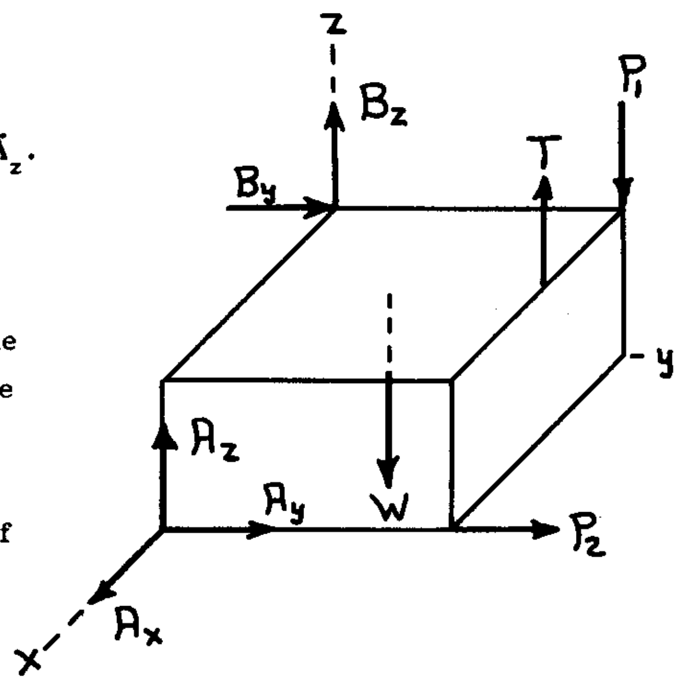


Fig. P6-24

SOLUTION

The action of the ball and socket joint at support A is represented by force components $\bar{A}_x$, $\bar{A}_y$, and $\bar{A}_z$. The action of the pin and bracket at support B is represented by force components $\bar{B}_y$ and $\bar{B}_z$. The cable at C exerts a force $\bar{T}$ on the block that is tangent to the cable at the point of attachment. The weight $\bar{W} = m\bar{g}$ of the block acts through the center of gravity G of the block and is directed toward the center of the earth.



6-25 Draw a free-body diagram for the bent bar shown in Fig. P6-25. The support at A is a journal bearing and the supports at B and C are ball bearings.

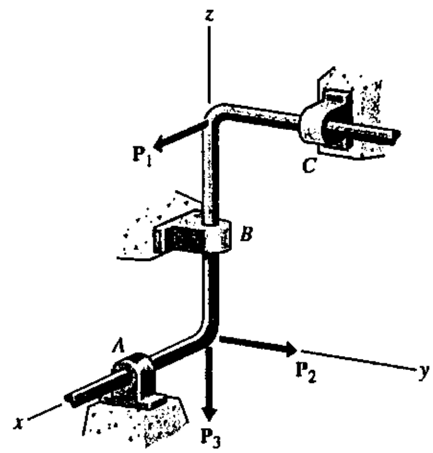
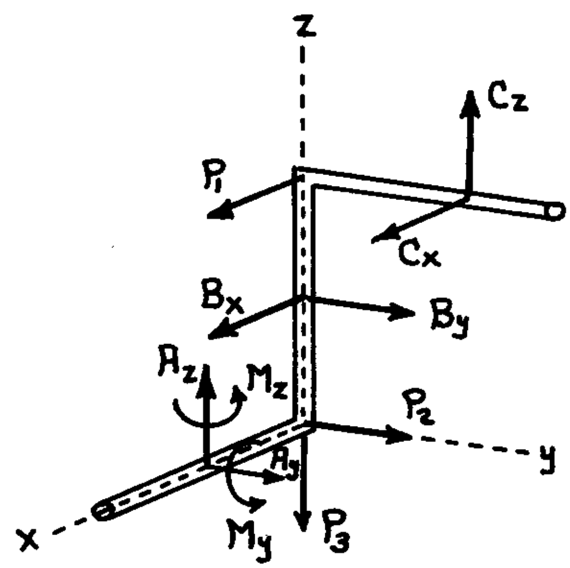


Fig. P6-25

SOLUTION

The action of the journal bearing at support A is represented by force components $\bar{A}_y$ and $\bar{A}_z$ and moment components $\bar{M}_y$ and $\bar{M}_z$. The ball bearings at supports B and C are represented by force components $\bar{B}_x$ and $\bar{B}_y$ at bearing B and force components $\bar{C}_x$ and $\bar{C}_z$ at bearing C.



6-26 Draw a free-body diagram for the bar bracket shown in Fig. P6-26. The support at B is a ball and socket joint. The ends of the bars at A and C rest against smooth surfaces.

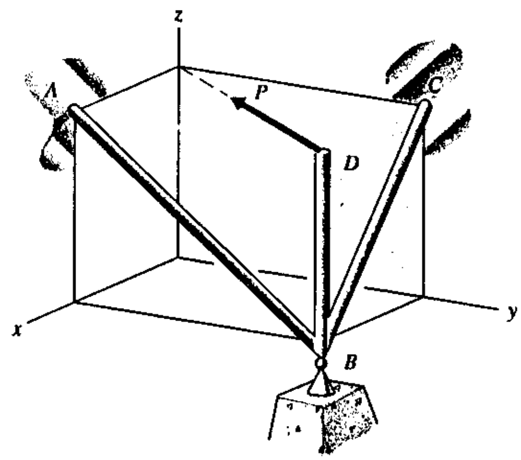
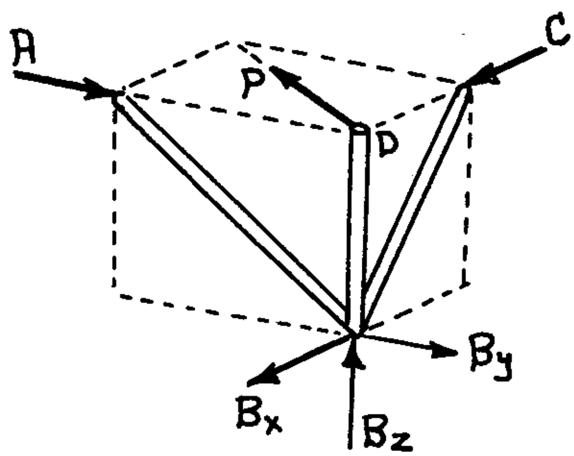


Fig. P6-26

SOLUTION

The action of the ball-and-socket joint at support B is represented by force components $\bar{B}_x$, $\bar{B}_y$, and $\bar{B}_z$. The smooth surfaces at A and C exert forces $\bar{A}$ and $\bar{C}$ on the bar bracket that are nornal to the surfaces.



6-27 Draw a free-body diagram for the bar shown in Fig. P6-27. The bar rests against a smooth vertical wall at end D. The support at A is a ball and socket. The cable is not continuous at B.

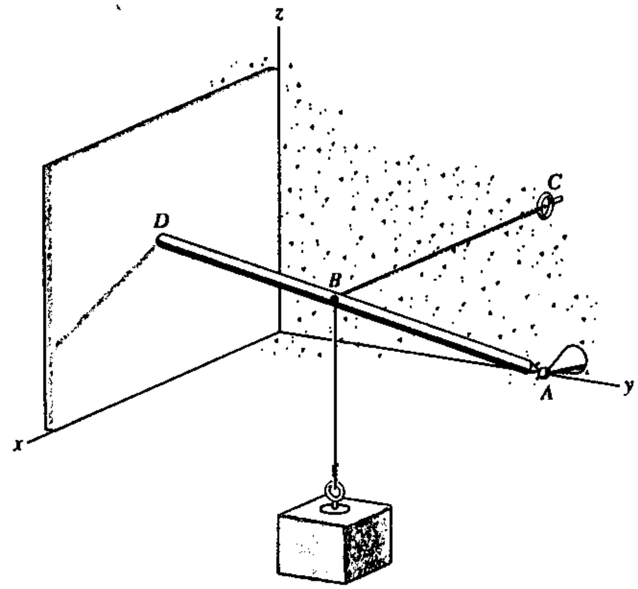
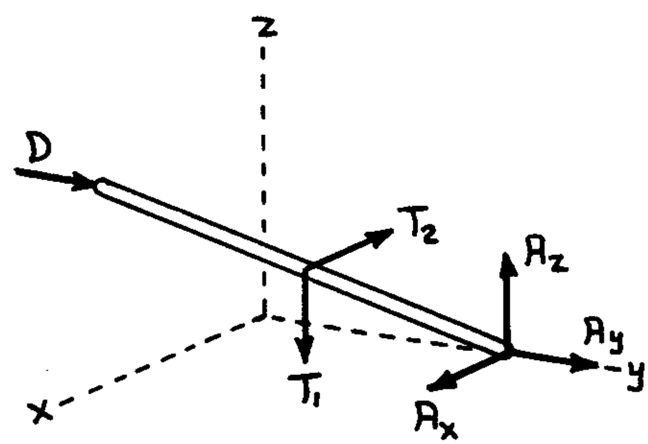


Fig. P6-27

SOLUTION

The action of the ball and socket joint at support A is represented by force components $\bar{A}_x$, $\bar{A}_y$, and $\bar{A}_z$. The smooth surface at D exerts a force $\bar{D}$ normal to the surface. The cables at B exert forces $\bar{T}_1$ and $\bar{T}_2$ on the bar that are tangent to the cables at the points of attachment.



6-28 Draw a free-body diagram for the door shown in Fig. P6-28 which has a weight **W**.

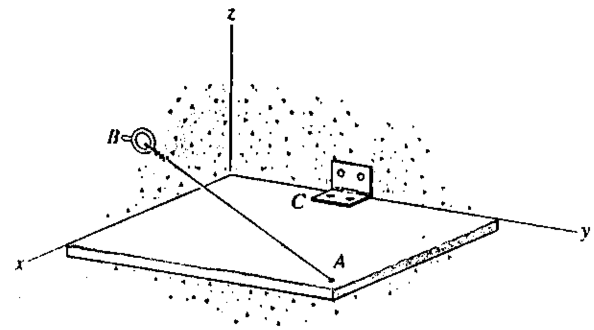
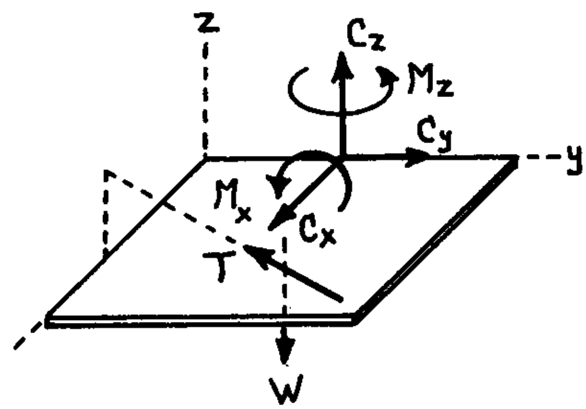


Fig. P6-28

SOLUTION

The cable at A exerts a force **T** on the door that is tangent to the cable at the point of attachment. The action of the hinge at C is represented by force components $\mathbf{C}_x$, $\mathbf{C}_y$, and $\mathbf{C}_z$ and moment components $\mathbf{M}_x$ and $\mathbf{M}_z$. The weight **W** of the door acts through the center of gravity G of the door and is directed toward the center of the earth.



6-29 Draw a free-body diagram for the bent bar shown in Fig. P6-29. The support at A is a ball and socket joint, the supports at B are a cable and a link, and the support at C is a link.

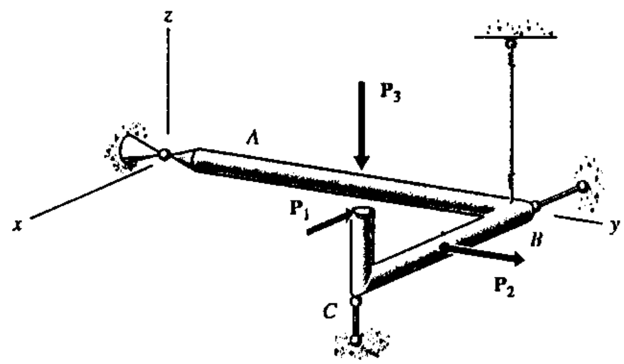
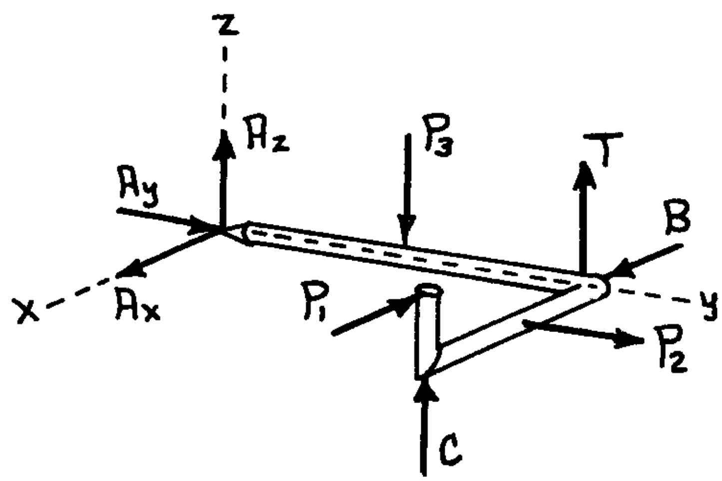


Fig. P6-29

SOLUTION

The action of the ball and socket joint at support A is represented by force components $\bar{A}_x$, $\bar{A}_y$, and $\bar{A}_z$. The cable at B exerts a force $\bar{T}$ on the bar that is tangent to the cable at the point of attachment. The links at B and C exert forces $\bar{B}$ and $\bar{C}$ on the bar that are directed along the axes of the links.



6-30 Draw a free-body diagram for the bent bar shown in Fig. P6-30. Supports at B and C are ball bearings. The horizontal and vertical surfaces at A are smooth.

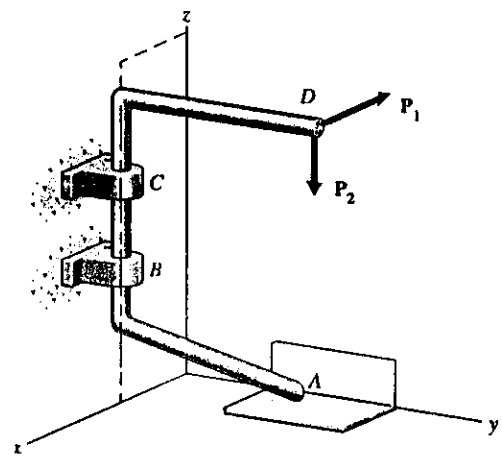
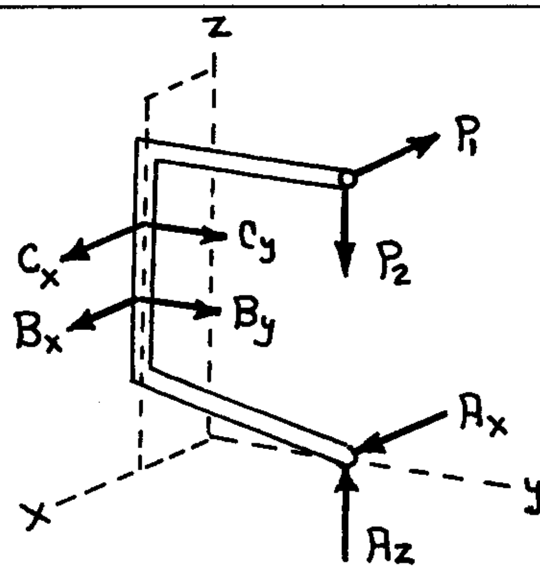


Fig. P6-30

SOLUTION

The smooth horizontal and vertical surfaces at A exert forces $\bar{A}_x$ and $\bar{A}_z$ on the bar that are normal to the two surfaces. The ball bearings at supports B and C are represented by force components $\bar{B}_x$ and $\bar{B}_y$ at bearing B and force components $\bar{C}_x$ and $\bar{C}_y$ at bearing C.



6-31* A curved slender bar is loaded and supported as shown in Fig. P6-31. Determine the reaction at support A.

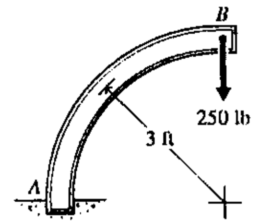
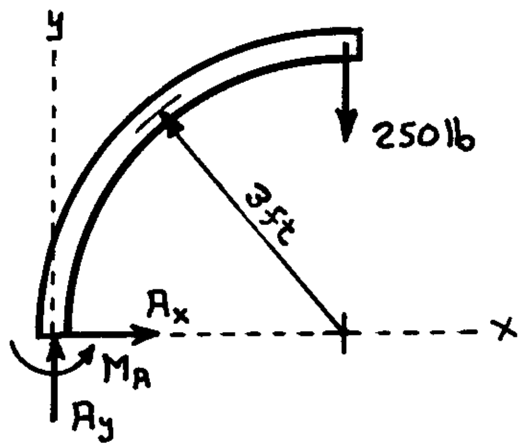


Fig. P6-31

SOLUTION

A free-body diagram of the bar is shown at the right. The reaction at support A is represented by force components A_x and A_y and a moment M_A.



$+\rightarrow \Sigma F_x = A_x = 0$

$A_x = 0$

$+\uparrow \Sigma F_y = A_y - 250 = 0$

$A_y = 250 \text{ lb}$

$\bar{A} = A_x \hat{i} + A_y \hat{j}$

$= 250 \hat{j} \text{ lb} = 250 \text{ lb} \uparrow$ Ans.

$+ ↺ \Sigma M_A = M_A - 250(3) = 0$

$M_A = 750 \text{ ft}\cdot\text{lb}$

$= 750 \hat{k} \text{ ft}\cdot\text{lb} = 750 \text{ ft}\cdot\text{lb}$ ↺ Ans.

6-32* A curved slender bar is loaded and supported as shown in Fig. P6-32. Determine the reactions at supports A and B.

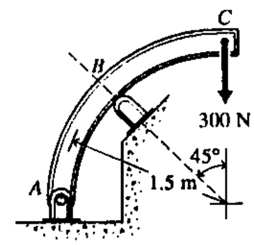
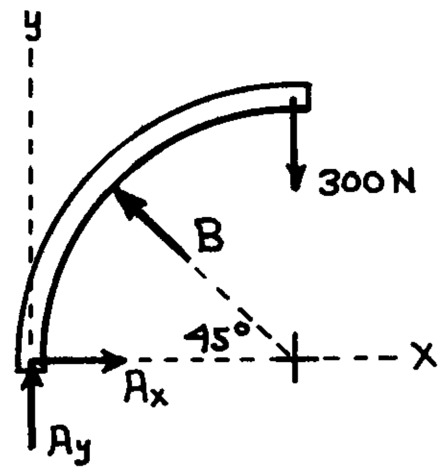


Fig. P6-32

SOLUTION

A free-body diagram of the bar is shown at the right. The reaction at support A is represented by force components A_x and A_y. The reaction at support B is represented by a force $\bar{B}$ normal to the surface of the bar.



$+\circlearrowleft \Sigma M_A = B(1.5 \sin 45^\circ) - 300(1.5) = 0$

$B = 424.3 \text{ N} \cong 424 \text{ N}$

$\bar{B} = 424.3(-\cos 45^\circ\ \hat{i} + \sin 45^\circ\ \hat{j})$

$= -300\ \hat{i} + 300\ \hat{j} \text{ N} = 424 \text{ N} \measuredangle 45^\circ$ Ans.

$+\rightarrow \Sigma F_x = A_x - B \cos 45^\circ$

$= A_x - 424.3 \cos 45^\circ = 0$ $A_x = 300$ N

$+\uparrow \Sigma F_y = A_y + B \sin 45^\circ - 300$

$= A_y + 424.3 \sin 45^\circ - 300 = 0$ $A_y = 0$

$\bar{A} = A_x\ \hat{i} + A_y\ \hat{j}$

$= 300\ \hat{i} \text{ N} = 300 \text{ N} \rightarrow$ Ans.

6-33 A beam is loaded and supported as shown in Fig. P6-33. The beam has a uniform cross section and weighs 250 lb. Determine the reactions at supports A and B.

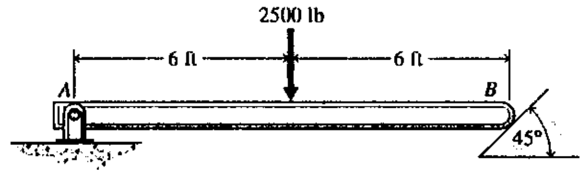
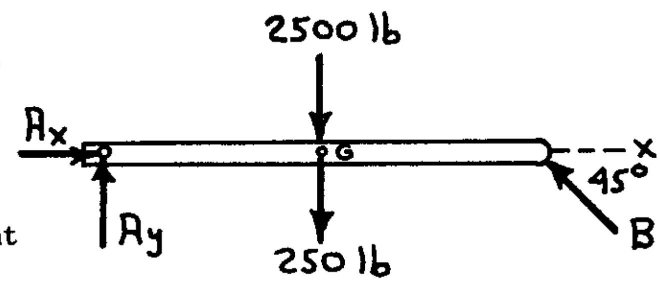


Fig. P6-33

SOLUTION

A free-body diagram of the beam is shown at the right. The reaction at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The reaction at B is a force $\bar{B}$ normal to the support surface. The weight $\bar{W}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



$$+\circlearrowleft \Sigma M_A = B(12 \sin 45^\circ) - 2500(6) - 250(6) = 0$$

$$B = 1944.5 \text{ lb} \cong 1945 \text{ lb}$$

$$\bar{B} = 1944.5(-\cos 45^\circ\ \hat{i} + \sin 45^\circ\ \hat{j})$$

$$= -1375\ \hat{i} + 1375\ \hat{j} \text{ lb} = 1945 \text{ lb} \measuredangle 45^\circ \qquad \text{Ans.}$$

$$+\rightarrow \Sigma F_x = A_x - B \cos 45^\circ = A_x - 1944.5 \cos 45^\circ = 0$$

$$A_x = 1375 \text{ lb}$$

$$+\uparrow \Sigma F_y = A_y - 2500 - 250 + B \sin 45^\circ$$

$$= A_y - 2500 - 250 + 1944.5 \sin 45^\circ = 0$$

$$A_y = 1375 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(1375)^2 + (1375)^2} = 1944.5 \text{ lb} \cong 1945 \text{ lb}$$

$$\theta = \tan^{-1} \frac{A_y}{A_x} = \tan^{-1} \frac{1375}{1375} = 45.0^\circ$$

$$\bar{A} = 1375\ \hat{i} + 1375\ \hat{j} \text{ lb} = 1945 \text{ lb} \measuredangle 45.0^\circ \qquad \text{Ans.}$$

6-34 A beam is loaded and supported as shown in Fig. P6-34. The beam has a uniform cross section and a mass of 180 kg. Determine the reaction at support A.

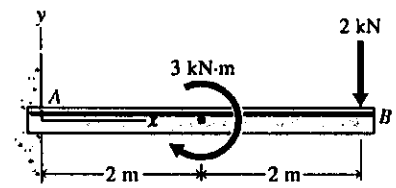
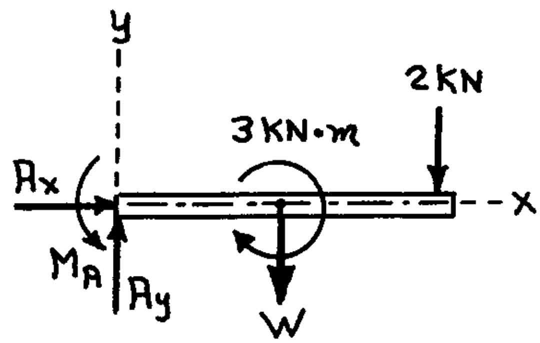


Fig. P6-34

SOLUTION

A free-body diagram of the beam is shown at the right. The reaction at support A is represented by force components $\vec{A}_x$ and $\vec{A}_y$ and a moment $\vec{M}$. The weight $\vec{W} = m\vec{g}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



$W = mg = 180(9.807)$

$= 1765.3 \text{ N} \cong 1.765 \text{ kN}$

$+ \rightarrow \Sigma F_x = A_x = 0 \qquad A_x = 0$

$+ \uparrow \Sigma F_y = A_y - W - 2$

$= A_y - 1.7653 - 2 = 0$

$A_y = 3.7653 \text{ kN} \cong 3.77 \text{ kN}$

$\vec{A} = 3.77\ \hat{j} \text{ kN} = 3.77 \text{ kN} \uparrow$ Ans.

$+ \circlearrowleft \Sigma M_A = M_A - 3 - 1.7653(2) - 2(4) = 0$

$M_A = 14.531 \text{ kN·m} \cong 14.53 \text{ kN·m}$

$\vec{M}_A = 14.53\ \hat{k} \text{ kN·m} = 14.53 \text{ kN·m} \circlearrowleft$ Ans.

6-35* A beam is loaded and supported as shown in Fig. P6-35. The beam has a uniform cross section and weighs 425 lb. Determine the reactions at supports A and E.

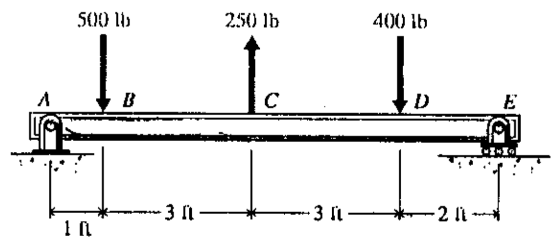
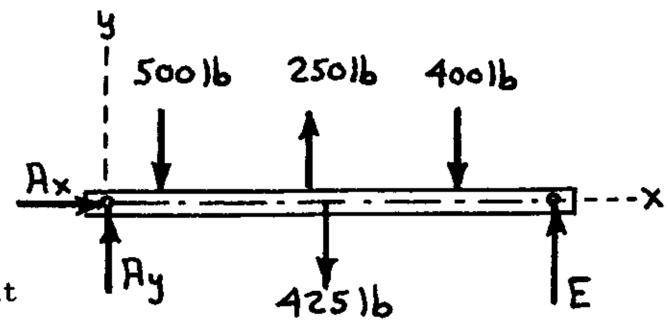


Fig. P6-35

SOLUTION

A free-body diagram of the beam is shown at the right. The reaction at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The reaction at E is a force $\bar{E}$ normal to the support surface. The weight $\bar{W}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



$+\circlearrowleft \Sigma M_A = E(9) - 500(1) + 250(4) - 425(4.5) - 400(7) = 0$

$E = 468.1 \text{ lb} \cong 468 \text{ lb}$

$\bar{E} = 468\ \hat{j} \text{ lb} = 468 \text{ lb} \uparrow$ Ans.

$+\rightarrow \Sigma F_x = A_x = 0 \qquad A_x = 0$

$+\circlearrowleft \Sigma M_E = -A_y(9) + 500(8) - 250(5) + 425(4.5) + 400(2) = 0$

$A_y = 606.9 \text{ lb} \cong 607 \text{ lb}$

$\bar{A} = 607\ \hat{j} \text{ lb} = 607 \text{ lb} \uparrow$ Ans.

6-36* A beam is loaded and supported as shown in Fig. P6-36. The beam has a uniform cross section and a mass of 120 kg. Determine the reactions at supports A and B.

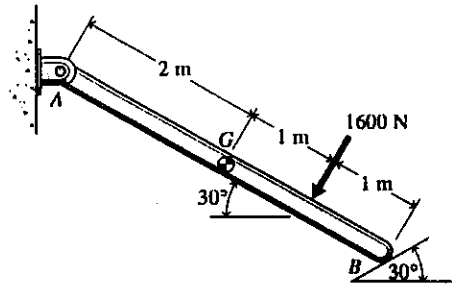
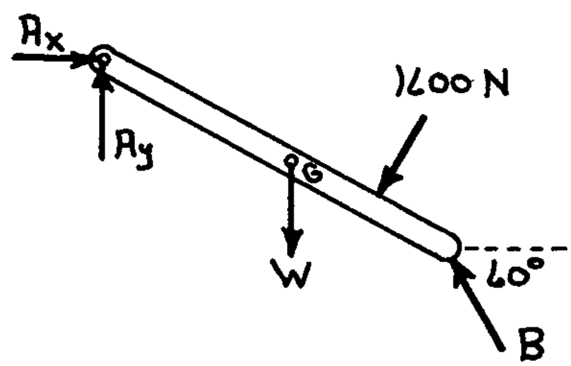


Fig. P6-36

SOLUTION

A free-body diagram of the beam is shown at the right. The reaction at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The reaction at B is a force $\bar{B}$ normal to the support surface. The weight $\bar{W}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



$W = mg = 120(9.807) = 1176.8 \text{ N}$

$+ \circlearrowleft \Sigma M_A = -1176.8(2 \cos 30^\circ) - 1600(3) + B(4 \sin 30^\circ) = 0$

$B = 3419 \text{ N} \cong 3.42 \text{ kN}$

$\bar{B} = 3.419(-\cos 60^\circ\ \hat{i} + \sin 60^\circ\ \hat{j})$

$= -1.710\ \hat{i} + 2.96\ \hat{j} \text{ kN} = 3.42 \text{ kN} \measuredangle 60^\circ$ Ans.

$+ \rightarrow \Sigma F_x = A_x - 1600 \cos 60^\circ - 3419 \cos 60^\circ = 0$

$A_x = 2510 \text{ N} = 2.510 \text{ kN}$

$+ \uparrow \Sigma F_y = A_y - 1176.8 - 1600 \sin 60^\circ + 3419 \sin 60^\circ = 0$

$A_y = -398.5 \text{ N} \cong -0.399 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2510)^2 + (-398.5)^2} = 2541 \text{ N} \cong 2.54 \text{ kN}$

$\theta = \tan^{-1} \dfrac{A_y}{A_x} = \tan^{-1} \dfrac{-398.5}{2510} = -9.02^\circ$

$\bar{A} = 2.51\ \hat{i} - 0.398\ \hat{j} \text{ kN} = 2.54 \text{ kN} \measuredangle 9.02^\circ$ Ans.

6-37 A structural member is loaded and supported as shown in Fig. P6-37. The member has a uniform cross section and weighs 208 lb. Determine the reactions at supports A and B.

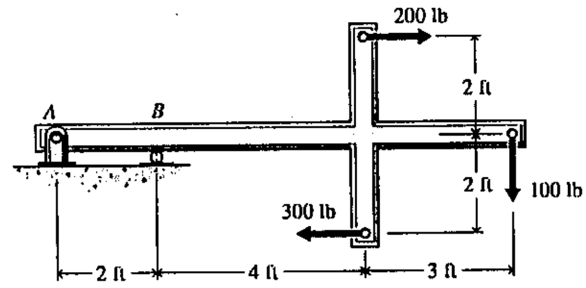
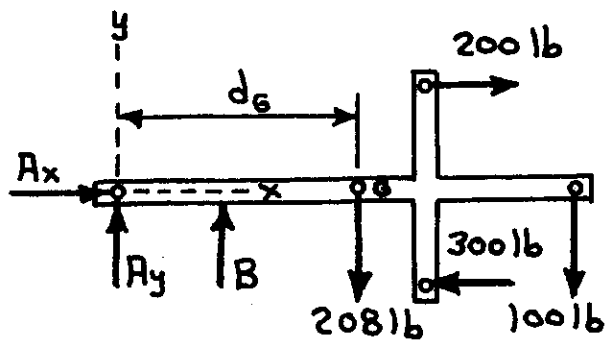


Fig. P6-37

SOLUTION

A free-body diagram of the beam is shown at the right. The reaction at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The reaction at B is a vertical force $\bar{B}$. The weight $\bar{W}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



$$Ld_G = L_1d_1 + L_2d_2 + L_3d_3$$

$$13d_G = 6(3) + 4(6) + 3(7.5) \qquad d_G = 4.962 \text{ ft}$$

$$+\rightarrow \Sigma F_x = A_x + 200 - 300 = 0$$

$$A_x = 100 \text{ lb}$$

$$+\circlearrowleft \Sigma M_B = -A_y(2) - 208(2.962) - 200(2) - 100(7) - 300(2) = 0$$

$$A_y = -1158.0 \text{ lb} = -1158 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(100)^2 + (-1158)^2} = 1162.3 \text{ lb} \cong 1162 \text{ lb}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} = \tan^{-1}\frac{-1162}{100} = -85.08^\circ \cong -85.1^\circ$$

$$\bar{A} = 100\,\hat{i} - 1158\,\hat{j} \text{ lb} = 1162 \text{ lb} \searrow 85.1^\circ \qquad \text{Ans.}$$

$$+\circlearrowleft \Sigma M_A = B(2) - 208(4.962) - 200(2) - 100(9) - 300(2) = 0$$

$$B = 1466.0 \text{ lb} = 1466 \text{ lb}$$

$$\bar{B} = 1466\,\hat{j} \text{ lb} = 1466 \text{ lb} \uparrow \qquad \text{Ans.}$$

6-38 A beam is loaded and supported as shown in Fig. P6-38. The beam has a uniform cross section and a mass of 20 kg. Determine the reaction at support A and the tension T in the cable.

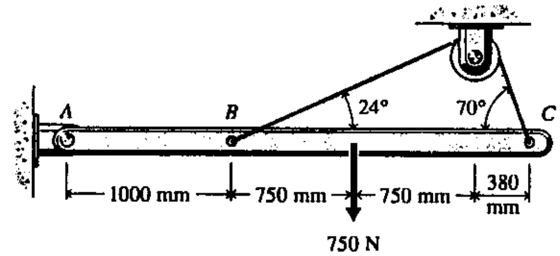
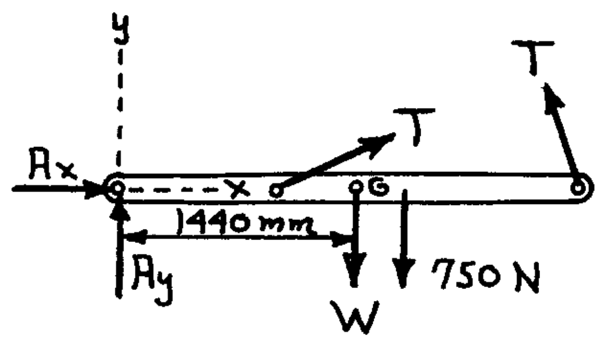


Fig. P6-38

SOLUTION

The action of the pin at support A is represented by force components $\bar{A}_x$ and $\bar{A}_y$. The cable is continuous over the pulley; therefore, the force in the cable is constant. At points B and C the cable exerts tensile forces $\bar{T}$ on the beam that are tangent to the cable. The weight $\bar{W}$ of the beam acts through the center of gravity G of the beam and is directed toward the center of the earth.



$$W = mg = 20(9.807) = 196.14 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = T \sin 24^\circ(1.000) - 196.14(1.440) - 750(1.750) + T \sin 70^\circ (2880) = 0$$

$$T = 512.3 \text{ N} \cong 512 \text{ N}$$ Ans.

$$+\rightarrow \Sigma F_x = A_x + T\cos 24^\circ - T\cos 70^\circ$$

$$= A_x + 512.3 \cos 24^\circ - 512.3 \cos 70^\circ = 0$$

$$A_x = -292.8 \text{ N} \cong -293 \text{ N}$$

$$+\uparrow \Sigma F_y = A_y + T\sin 24^\circ - W - 750 + T\sin 70^\circ$$

$$= A_y + 512.3 \sin 24^\circ - 196.14 - 750 + 512.3 \sin 70^\circ = 0$$

$$A_y = 256.4 \text{ N} \cong 256 \text{ N}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-292.8)^2 + (256.4)^2} = 389.2 \text{ N} \cong 389 \text{ N}$$ Ans.

$$\theta_x = \tan^{-1}\frac{256.4}{-292.8} = 138.79^\circ \cong 138.8^\circ$$

$$\bar{A} = -293\,\hat{i} + 256\,\hat{j} \text{ lb} = 389 \text{ lb} \measuredangle 41.2^\circ$$ Ans.

6-39* A beam is loaded and supported as shown in Fig. P6-39. The beam has a uniform cross section and weighs 40 lb. The pulley at B weighs 50 lb. Determine the reaction at support A and the tension T in the cable.

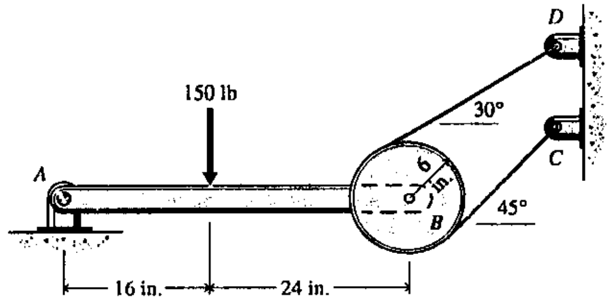
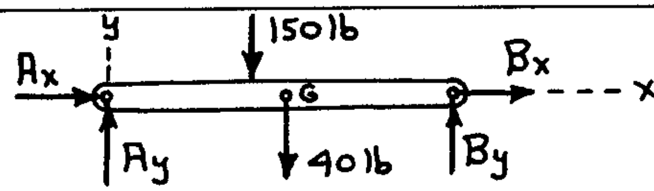


Fig. P6-39

SOLUTION

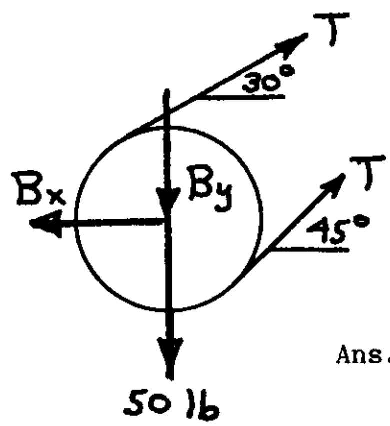
From a free-body diagram for the beam:



$+\circlearrowleft \Sigma M_A = B_y(40) - 150(16) - 40(20) = 0$ $B_y = 80$ lb

$+\circlearrowleft \Sigma M_B = -A_y(40) + 150(24) + 40(20) = 0$ $A_y = 110$ lb

From a free-body diagram for the pulley:



$+\circlearrowleft \Sigma M_B = T_2(6) - T_1(6) = 0$

$T_1 = T_2 = T$

$+\uparrow \Sigma F_y = T \sin 30^\circ + T \sin 45^\circ - B_y - 50$

$= T \sin 30^\circ + T \sin 45^\circ - 80 - 50 = 0$

$T = 107.70$ lb $= 107.7$ lb Ans.

$+\rightarrow \Sigma F_x = T \cos 30^\circ + T \cos 45^\circ - B_x$

$= 107.70 \cos 30^\circ + 107.70 \cos 45^\circ - B_x = 0$

$B_x = 169.43$ lb $\cong 169.4$ lb

From the free-body diagram for the beam:

$+\rightarrow \Sigma F_x = B_x + A_x = 169.43 + A_x = 0$

$A_x = -169.43$ lb $\cong -169.4$ lb

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-169.43)^2 + (110)^2} = 202.0$ lb $= 202$ lb

$\theta = \tan^{-1} \frac{A_y}{A_x} = \frac{110}{-169.43} = 147.01^\circ \cong 147.0^\circ$

$\mathbf{A} = -169.4\,\hat{i} + 110\,\hat{j}$ lb $= 202$ lb ∡ 33.0° Ans.

6-40* An angle bracket is loaded and supported as shown in Fig. P6-40. Determine the reactions at supports A and B.

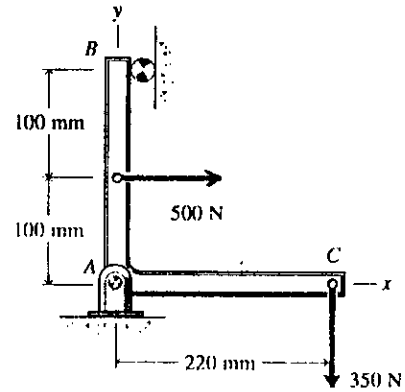
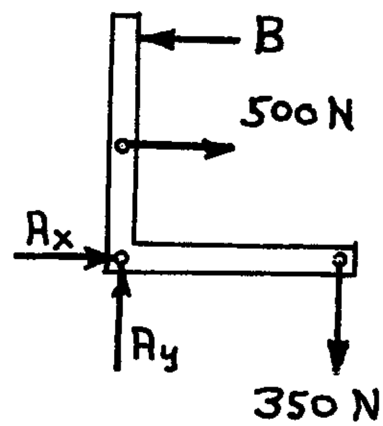


Fig. P6-40

SOLUTION

From a free-body diagram for the angle bracket:



$+\circlearrowleft \Sigma M_A = B(0.200) - 500(0.100)$

$- 350(0.220) = 0$

$B = 635 \text{ N}$

$\bar{B} = -635\,\hat{j}\text{ N} = 636 \text{ N} \leftarrow$ Ans.

$\rightarrow + \Sigma F_x = A_x + 500 - B$

$= A_x + 500 - 635 = 0$

$A_x = 135 \text{ N}$

$+\uparrow \Sigma F_y = A_y - 350 = 0$

$A_y = 350 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(135)^2 + (350)^2} = 375.1 \text{ lb} \cong 375 \text{ lb}$

$\theta = \tan^{-1}\dfrac{A_y}{A_x} = \tan^{-1}\dfrac{350}{135} = 68.9^\circ$

$\bar{A} = 135\,\hat{i} + 350\,\hat{j}\text{ lb} = 375 \text{ lb} \measuredangle 68.9^\circ$ Ans.

6-41 A 30-lb force $\bar{P}$ is applied to the brake petal of an automobile as shown in Fig. P6-41. Determine the force $\bar{Q}$ applied to the brake cylinder and the reaction at support A.

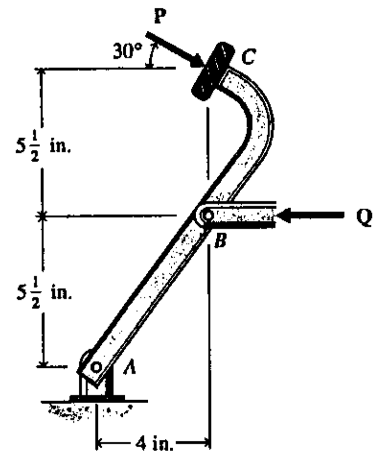
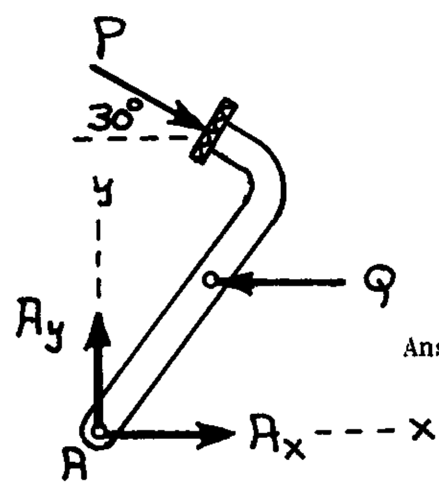


Fig. P6-41

SOLUTION

From a free-body diagram for the brake pedal:



$$+\circlearrowleft \Sigma M_A = -30 \cos 30^\circ (11)$$
$$- 30 \sin 30^\circ (4) + Q(5.5) = 0$$
$$Q = 62.87 \text{ lb} \cong 62.9 \text{ lb}$$
$$\bar{Q} = 62.9\ \hat{i} \text{ lb} = 62.9 \text{ lb} \leftarrow$$

Ans.

$$+\rightarrow \Sigma F_x = A_x + P \cos 30^\circ - Q$$
$$= A_x + 30 \cos 30^\circ - 62.87 = 0$$
$$A_x = 36.89 \text{ lb} \cong 36.9 \text{ lb}$$

$$+\uparrow \Sigma F_y = A_y - P \sin 30^\circ$$
$$= A_y - 30 \sin 30^\circ = 0$$
$$A_y = 15.00 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(36.89)^2 + (15.00)^2} = 39.82 \text{ lb} \cong 39.8 \text{ lb}$$

$$\theta = \tan^{-1} \frac{A_y}{A_x} = \tan^{-1} \frac{15.00}{36.89} = 22.13^\circ \cong 22.1^\circ$$

$$\bar{A} = 36.9\ \hat{i} + 15.0\ \hat{j} \text{ lb} = 39.8 \text{ lb} \measuredangle\ 22.1^\circ$$

Ans.

6-42 An angle bracket is loaded and supported as shown in Fig. P6-42. Determine the force exerted by the cable at A and the reaction at support B.

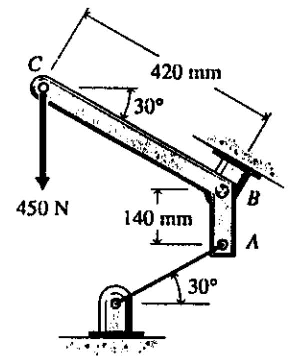
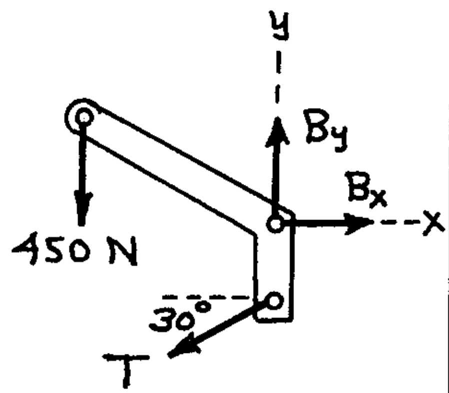


Fig. P6-42

SOLUTION

From a free-body diagram for the angle bracket:



$+\circlearrowleft \Sigma M_B = 450(420 \cos 30^\circ)$

$\quad - T \cos 30^\circ(140) = 0$

$T = 1350 \text{ N}$ Ans.

$+ \rightarrow \Sigma F_x = B_x - T \cos 30^\circ$

$\quad = B_x - 1350 \cos 30^\circ = 0$

$B_x = 1169.1 \text{ N} \cong 1169 \text{ N}$

$+ \uparrow \Sigma F_y = B_y - 450 - T \sin 30^\circ$

$\quad = B_y - 450 - 1350 \sin 30^\circ = 0$

$B_y = 1125 \text{ N}$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(1169.1)^2 + (1125)^2} = 1622.4 \text{ N} \cong 1622 \text{ N}$$

$$\theta = \tan^{-1} \frac{B_y}{B_x} = \tan^{-1} \frac{1125}{1169.1} = 43.90^\circ \cong 43.9^\circ$$

$$\mathbf{B} = 1169\, \hat{i} + 1125\, \hat{j} \text{ N} = 1622 \text{ N} \measuredangle\ 43.9^\circ$$ Ans.

6-43* Determine the reactions at supports A and B of the curved bar shown in Fig. P6-43.

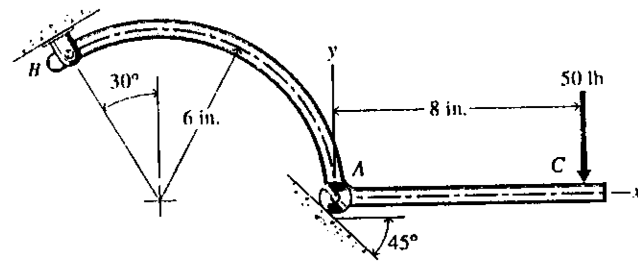
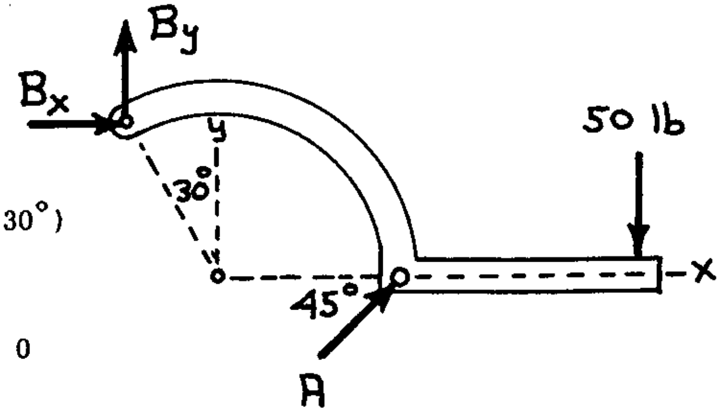


Fig. P6-43

SOLUTION

From a free-body diagram for the curved bar:



$+ \circlearrowleft \Sigma M_B = A \sin 45^\circ(6 + 6 \sin 30^\circ)$

$+ A \cos 45^\circ(6 \cos 30^\circ)$

$- 50(14 + 6 \sin 30^\circ) = 0$

$A = 84.68 \text{ lb} \cong 84.7 \text{ lb}$

$\bar{A} = 84.7 \text{ lb} \measuredangle 45.0^\circ$ Ans.

$+ \rightarrow \Sigma F_x = B_x + A \cos 45^\circ$

$= B_x + 84.68 \cos 45^\circ = 0$

$B_x = -59.88 \text{ lb} \cong -59.9 \text{ lb}$

$+ \uparrow \Sigma F_y = B_y + A \sin 45^\circ - 50$

$= B_y + 84.68 \sin 45^\circ - 50 = 0$

$B_y = -9.878 \text{ lb} \cong -9.88 \text{ lb}$

$B = \sqrt{B_x^2 + B_x^2} = \sqrt{(59.88)^2 + (9.88)^2} = 60.69 \text{ lb} \cong 60.7 \text{ lb}$

$\theta = \tan^{-1} \frac{B_y}{B_x} = \tan^{-1} \frac{-9.88}{-59.88} = -170.63^\circ \cong -170.6^\circ$

$\bar{B} = -59.9\ \hat{i} - 9.88\ \hat{j} \text{ N} = 60.7 \text{ N} \measuredangle\ 9.37^\circ$ Ans.

6-44* Determine the force exerted by the cable at B and the reaction at support A of the curved bar shown in Fig. P6-44.

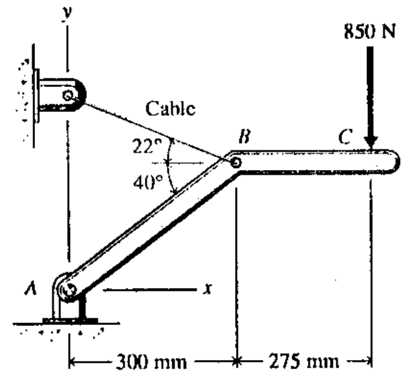
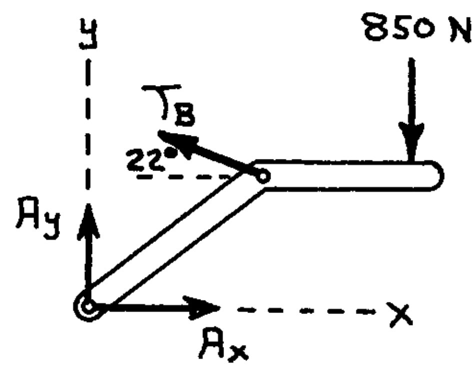


Fig. P6-44

SOLUTION

From a free-body diagram for the curved bar:



$$+\circlearrowleft \Sigma M_A = T_B \cos 22^\circ(0.300 \tan 40^\circ) + T_B \sin 22^\circ(0.300) - 850(0.575) = 0$$

$$T_B = 1413.46 \text{ N} \cong 1413 \text{ N}$$

$$+\rightarrow \Sigma F_x = A_x - T_B \cos 22^\circ = A_x - 1413.46 \cos 22^\circ = 0$$

$$A_x = 1310.5 \text{ N} \cong 1311 \text{ N}$$

$$+\uparrow \Sigma F_y = A_y + T_B \sin 22^\circ - 850 = A_y + 1413.46 \sin 22^\circ - 850 = 0$$

$$A_y = 320.5 \text{ N} \cong 321 \text{ N}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(1310.5)^2 + (320.5)^2} = 1349.1 \text{ N} \cong 1349 \text{ N}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} = \tan^{-1}\frac{320.5}{1310.5} = 13.743^\circ \cong 13.74^\circ$$

$$\bar{A} = 1311\,\hat{i} + 321\,\hat{j} \text{ N} = 1349 \text{ N} \measuredangle 13.74^\circ$$ Ans.

6-45 A rope and pulley system is used to support a body as shown in Fig. P6-45. Each pulley is free to rotate and the ropes are continuous over the pulleys. Determine the force $\bar{P}$ required to hold the body in equilibrium if the weight $\bar{W}$ of the body is 400 lb.

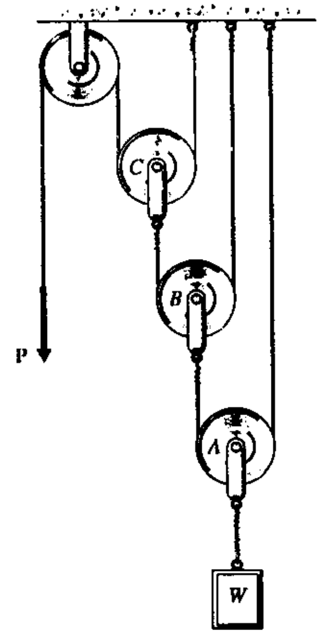


Fig. P6-45

SOLUTION

From a free-body diagram for pulley A:

$$+\uparrow \Sigma F_y = 2T_1 - 400 = 0$$

$$T_1 = 200 \text{ lb}$$

From a free-body diagram for pulley B:

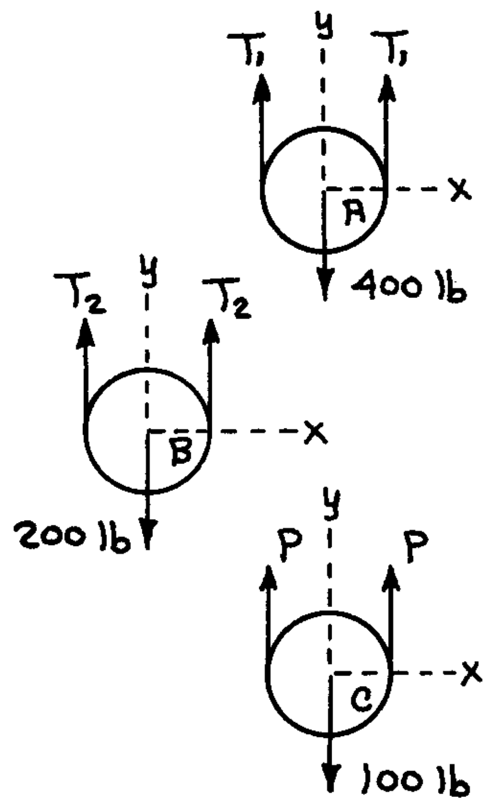
$$+\uparrow \Sigma F_y = 2T_2 - 200 = 0$$

$$T_2 = 100 \text{ lb}$$

From a free-body diagram for pulley C:

$$+\uparrow \Sigma F_y = 2P - 100 = 0$$

$$P = 50 \text{ lb} \qquad \text{Ans.}$$



6-46 A rope and pulley system is used to support a body as shown in Fig. P6-46. Each pulley is free to rotate and the ropes are continuous over the pulleys. Determine the force $\bar{P}$ required to hold the body in equilibrium if the mass m of the body is 250 kg.

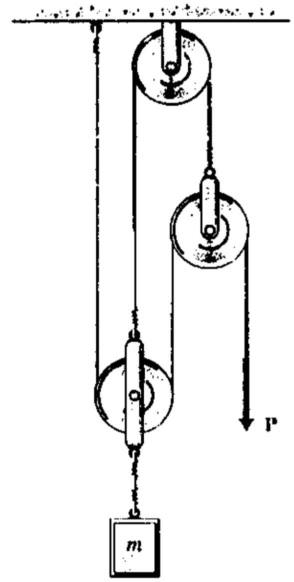


Fig. P6-46

SOLUTION

$W = mg = 250(9.807) = 2452$ N

From a free-body diagram for pulley A:

$+\uparrow \Sigma F_y = T_1 - 2P = 0$

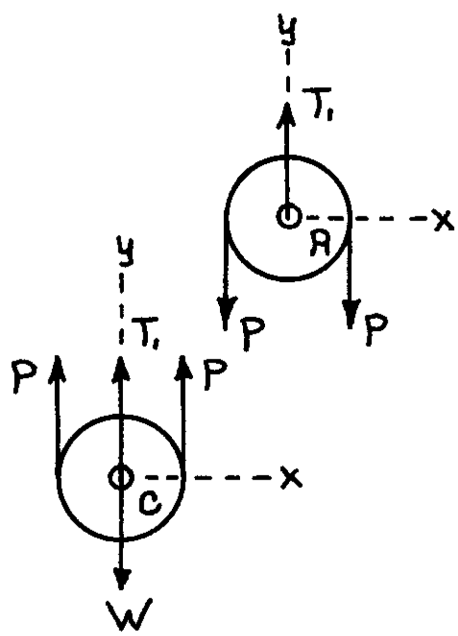
$T_1 = 2P$

From a free-body diagram for pulley C:

$+\uparrow \Sigma F_y = 2P + T_1 - W = 0$

$= 2P + 2P - 2452 = 0$

$P = 613$ N Ans.



6-47* A beam is loaded and supported as shown in Fig. P6-47. The beam has a uniform cross section and weighs 975 lb. Determine the reactions at supports A and B.

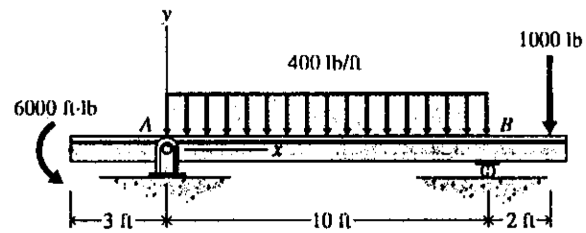


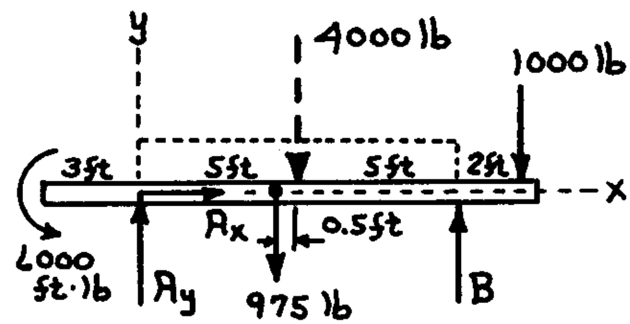
Fig. P6-47

SOLUTION

The distributed load can be represented on the free-body diagram by a resultant $\bar{R}$ at a distance d_x from support A.

$$R = wL_w = 400(10) = 4000 \text{ lb}$$

$$d_x = \frac{1}{2}L_w = \frac{1}{2}(10) = 5 \text{ ft}$$



From a free-body diagram for the beam:

$$+\rightarrow \Sigma F_x = A_x = 0$$

$$A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 6000 - A_y(10) + 975(5.5) + 4000(5) - 1000(2) = 0$$

$$A_y = 2936 \text{ lb} \cong 2940 \text{ lb}$$

$$\bar{A} = 2940\ \hat{j} \text{ lb} = 2940 \text{ lb} \uparrow$$ Ans.

$$+\circlearrowleft \Sigma M_A = 6000 - 975(4.5) - 4000(5) + B(10) - 1000(12) = 0$$

$$B = 3039 \text{ lb} \cong 3040 \text{ lb}$$

$$\bar{B} = 3040\ \hat{j} \text{ lb} = 3040 \text{ lb} \uparrow$$ Ans.

6-48* A beam is loaded and supported as shown in Fig. P6-48. The beam has a uniform cross section and a mass of 275 kg. Determine the reaction at support A.

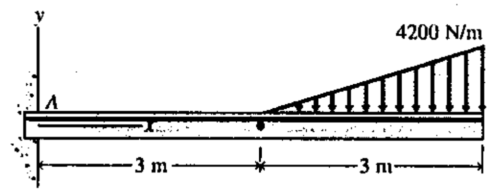
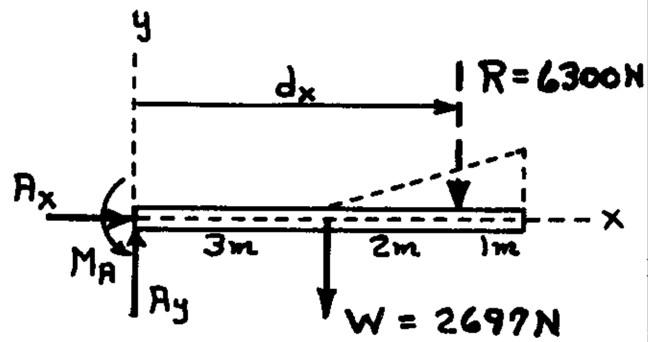


Fig. P6-48

SOLUTION

The distributed load can be represented on the free-body diagram by a resultant $\bar{R}$ at a distance d_x from support A.



$$R = \frac{1}{2}wL_w = \frac{1}{2}(4200)(3) = 6300\text{ N}$$

$$d_x = 3 + \frac{2}{3}L_w = 3 + \frac{2}{3}(3) = 5\text{ m}$$

$$W = mg = 275(9.807) = 2697\text{ N}$$

From a free-body diagram for the beam:

$+ \rightarrow \Sigma F_x = A_x = 0 \qquad A_x = 0$

$+ \uparrow \Sigma F_y = A_y - 2697 - 6300 = 0$

$A_y = 8997\text{ N} \cong 9000\text{ N}$

$\bar{A} = 9000\,\hat{j}\text{ N} = 9000\text{ N}\uparrow$ Ans.

$+ \circlearrowleft \Sigma M_A = M_A - 2697(3) - 6300(5) = 0$

$M_A = 39{,}591\text{ N}\cdot\text{m} \cong 39.6\text{ kN}\cdot\text{m}$

$\bar{M}_A = 39.6\,\hat{k}\text{ kN}\cdot\text{m} = 39.6\text{ kN}\cdot\text{m}\circlearrowleft$ Ans.

6-49 Determine the force $\bar{P}$ required to push the 300-lb cylinder over the small block shown in Fig. P6-49.

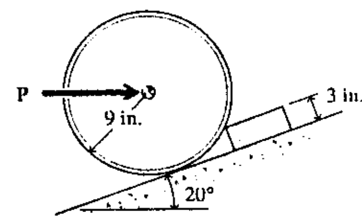
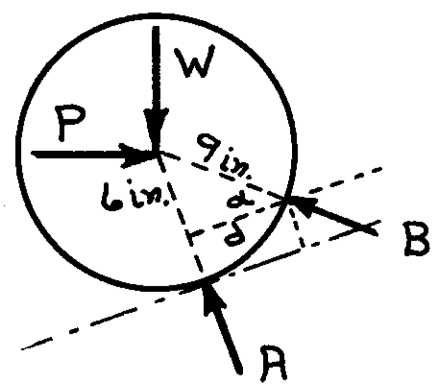


Fig. P6-49

SOLUTION

From a free-body diagram for the cylinder:



$\alpha = \sin^{-1}\frac{6}{9} = 41.81°$

$d = 9\cos 41.81° = 6.708$ in.

For impending motion: A = 0

$+\circlearrowleft \Sigma M_B = P\sin 20°\,(d) - P\cos 20°\,(6) + W\sin 20°\,(6) + W\cos 20°\,(d)$

$= P\sin 20°\,(6.708) - P\cos 20°\,(6) + 300\sin 20°\,(6) + 300\cos 20°\,(6.708) = 0$

$P = 750$ lb   Ans.

6-50 The mass of the cylinder shown in Fig. P6-50 is 100 kg. Determine the reactions at contact points A and B. All surfaces are smooth.

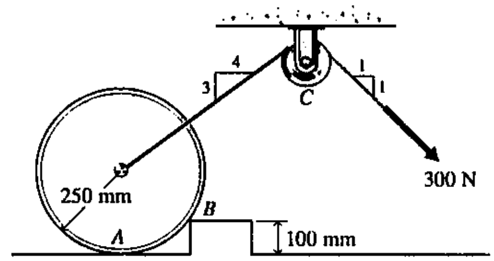
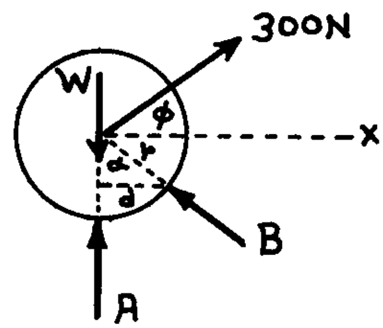


Fig. P6-50

SOLUTION

From a free-body diagram for the cylinder:



$\phi = \tan^{-1}\frac{3}{4} = 36.87^\circ$

$\alpha = \cos^{-1}\frac{150}{250} = 53.13^\circ$

$d = 250 \sin 53.13^\circ = 200$ mm

$W = mg = 100(9.807) = 980.7$ N

$+ \circlearrowleft \Sigma M_B = 980.7(200) - A(200) - 300 \cos 36.87^\circ (150)$
$- 300 \sin 36.87^\circ (200) = 0$

$A = 621.0$ N

$\bar{A} = 621\ \hat{j}$ N $= 621$ N $\uparrow$ Ans.

$+ \rightarrow \Sigma F_x = 300 \cos 36.87^\circ - B \sin 53.13^\circ = 0$

$B = 300$ N

$\bar{B} = 300(-\cos 36.87^\circ\ \hat{i} + \sin 36.87^\circ\ \hat{j})$

$= -240\ \hat{i} + 180.0\ \hat{j}$ N $= 300$ N ↖ 36.9° Ans.

6-51* A lever is loaded and supported as shown in Fig. P6-51. Determine the force exerted on the lever by link CD and the reaction at support A.

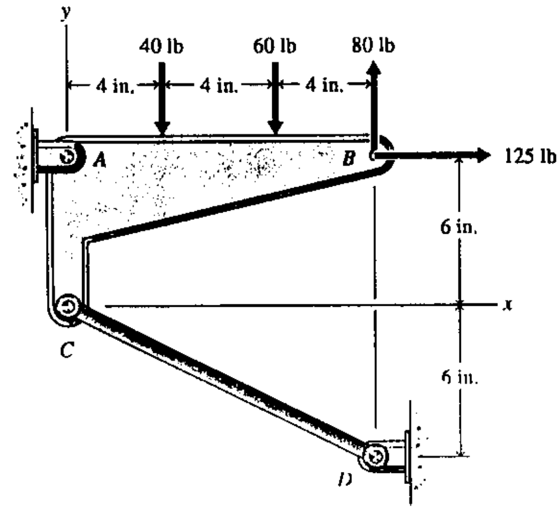
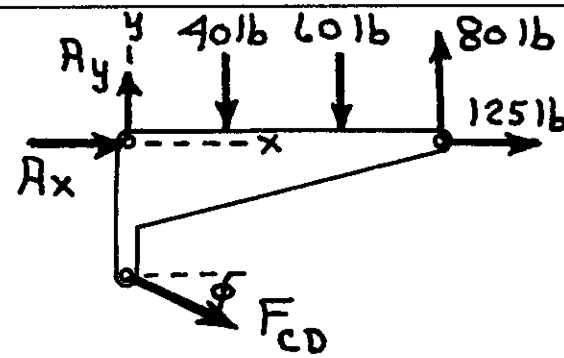


Fig. P6-51

SOLUTION

From a free-body diagram for the lever:



$$\phi = \tan^{-1}\frac{1}{2} = 26.57^\circ$$

$$+\circlearrowleft \Sigma M_A = F_{CD}\cos 26.57^\circ\,(6) + 80(12) - 60(8) - 40(4) = 0$$

$$F_{CD} = -59.63 \text{ lb} \cong 59.6 \text{ lb (C)}$$

$$+\rightarrow \Sigma F_x = A_x + 125 + F_{CD}\cos 26.57^\circ$$

$$= A_x + 125 + (-59.63)\cos 26.57^\circ = 0$$

$$A_x = -71.67 \text{ lb} \cong -71.7 \text{ lb}$$

$$+\uparrow \Sigma F_y = A_y - 40 - 60 + 80 - F_{CD}\sin 26.57^\circ$$

$$= A_y - 40 - 60 + 80 - (-59.63)\sin 26.57^\circ = 0$$

$$A_y = -6.672 \text{ lb} \cong -6.67 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-71.67)^2 + (-6.672)^2} = 71.98 \text{ lb} \cong 72.0 \text{ lb}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} = \tan^{-1}\frac{-6.672}{-71.67} = -174.68^\circ \cong -174.7^\circ$$

$$\bar{A} = -71.7\,\hat{i} - 6.67\,\hat{j} \text{ N} = 72.0 \text{ N} \measuredangle 5.32^\circ$$ Ans.

6-52* Determine the reactions at supports A and B of the curved bar shown in Fig. P6-52.

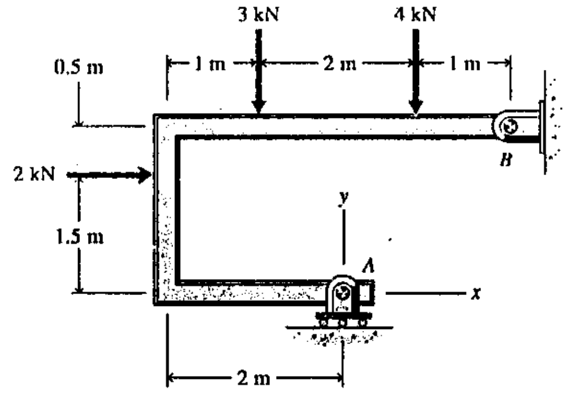
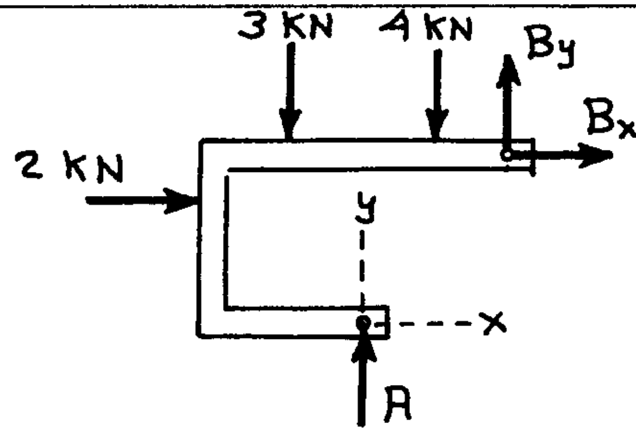


Fig. P6-52

SOLUTION

From a free-body diagram for the curved bar:



$+\circlearrowleft \Sigma M_B = 2(0.5) + 3(3) + 4(1) - A(2) = 0$

$A = 7.00$ kN

$\bar{A} = 7.00\ \hat{j}$ kN $= 7.00$ kN ↑ Ans.

$+\rightarrow \Sigma F_x = B_x + 2 = 0$

$B_x = -2.00$ kN

$+\uparrow \Sigma F_y = B_y - 4 - 3 + A$

$= B_y - 4 - 3 + 7 = 0$

$B_y = 0$

$\bar{B} = -2.00\ \hat{i}$ kN $= 2.00$ kN ← Ans.

6-53 Determine the force exerted by the cable at B and the reaction at support A of the bar shown in Fig. P6-53.

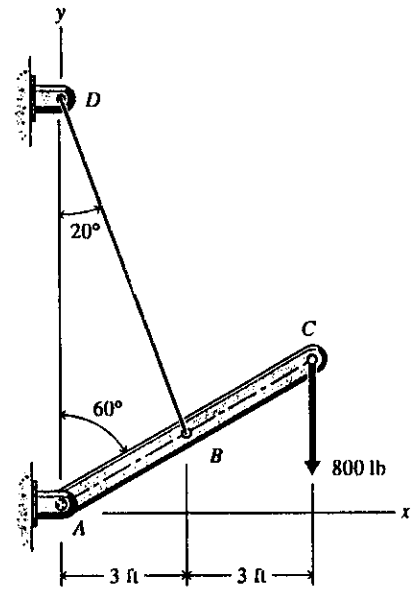
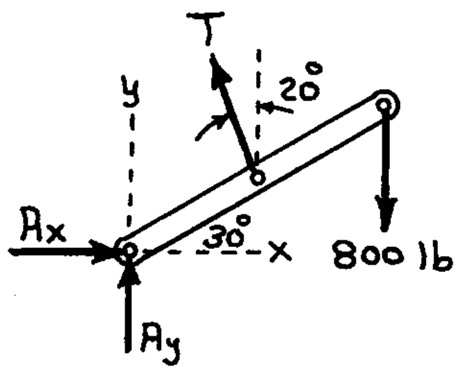


Fig. P6-53

SOLUTION

From a free-body diagram for the bar:



$+\circlearrowleft \Sigma M_A = T\cos 20^\circ(3)$

$\quad + T\sin 20^\circ(3\tan 30^\circ)$

$\quad - 800(6) = 0$

$T = 1407.02 \text{ lb} \cong 1407 \text{ lb}$ Ans.

$+\rightarrow \Sigma F_x = A_x - T\sin 20^\circ$

$\quad = A_x - 1407.02\sin 20^\circ = 0$

$A_x = 481.2 \text{ lb} \cong 481 \text{ lb}$

$+\uparrow \Sigma F_y = A_y + T\cos 20^\circ - 800$

$\quad = A_y + 1407.02\cos 20^\circ - 800 = 0$

$A_y = -522.1 \text{ lb} \cong -522 \text{ lb}$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(481.2)^2 + (-522.1)^2} = 710.03 \text{ lb} \cong 710 \text{ lb}$

$\theta = \tan^{-1}\frac{A_y}{A_x} = \tan^{-1}\frac{-522.1}{481.2} = -47.33^\circ \cong -47.3^\circ$

$\bar{A} = 481\,\hat{i} - 522\,\hat{j} \text{ lb} = 710 \text{ lb} \searrow 47.3^\circ$ Ans.

6-54 The man shown in Fig. P6-54 has a mass of 75 kg; the beam has a mass of 40 kg. The beam is in equilibrium with the man standing at the end and pulling on the cable. Determine the force exerted on the cable by the man and the reaction at support C.

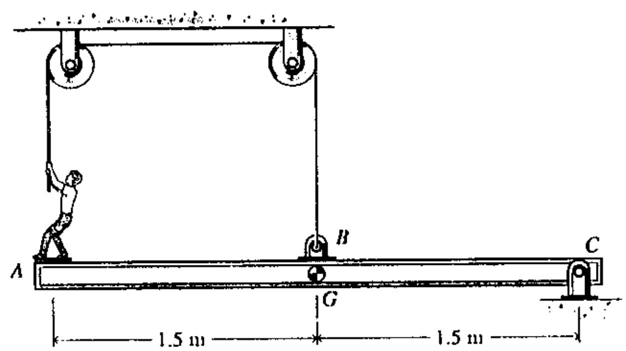
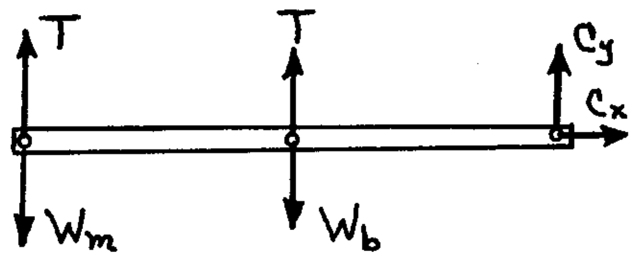


Fig. P6-54

SOLUTION

From a free-body diagram for the beam and the man:



$W_m = m_m g = 75(9.807) = 735.53$ N

$W_b = m_b g = 40(9.807) = 392.28$ N

$+\circlearrowleft \Sigma M_C = W_m(3) - T(3) + W_b(1.5) - T(1.5)$

$= 735.53(3) - T(3) + 392.28(1.5) - T(1.5) = 0$

$T = 621.11 \text{ N} \cong 621 \text{ N}$ Ans.

$+\rightarrow \Sigma F_x = C_x = 0 \qquad C_x = 0$

$+\uparrow \Sigma F_y = C_y + 2T - W_m - W_b$

$= C_y + 2(621.11) - 735.53 - 392.28 = 0$

$C_y = -114.41 \text{ N} \cong -114.4 \text{ N}$

$\vec{C} = -114.4\,\hat{j} \text{ N} = 114.4 \text{ N} \downarrow$ Ans.

6-55* A rope and pulley system is used to support a body as shown in Fig. P6-55. Each pulley is free to rotate, and the ropes are continuous over the pulleys. Determine the force $\bar{P}$ required to hold the body in equilibrium if the weight $\bar{W}$ of the body is 250 lb.

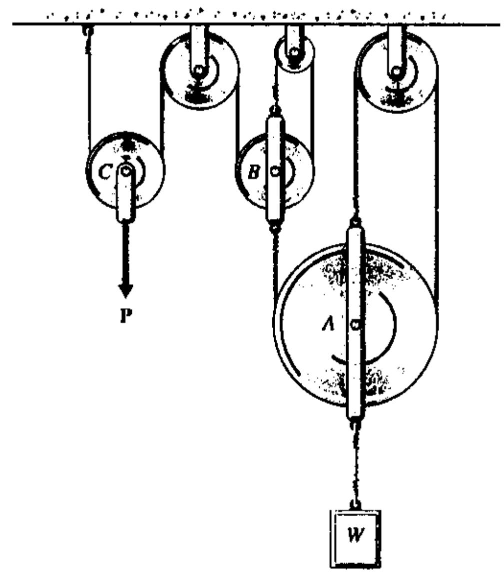


Fig. P6-55

SOLUTION

From a free-body diagram for pulley A:

$$\Sigma F_y = 3T_1 - W$$

$$= 3T_1 - 250 = 0$$

$$T_1 = 83.33 \text{ lb}$$

From a free-body diagram for pulley B:

$$\Sigma F_y = 3T_2 - T_1$$

$$= 3T_2 - 83.33 = 0$$

$$T_2 = 27.78 \text{ lb}$$

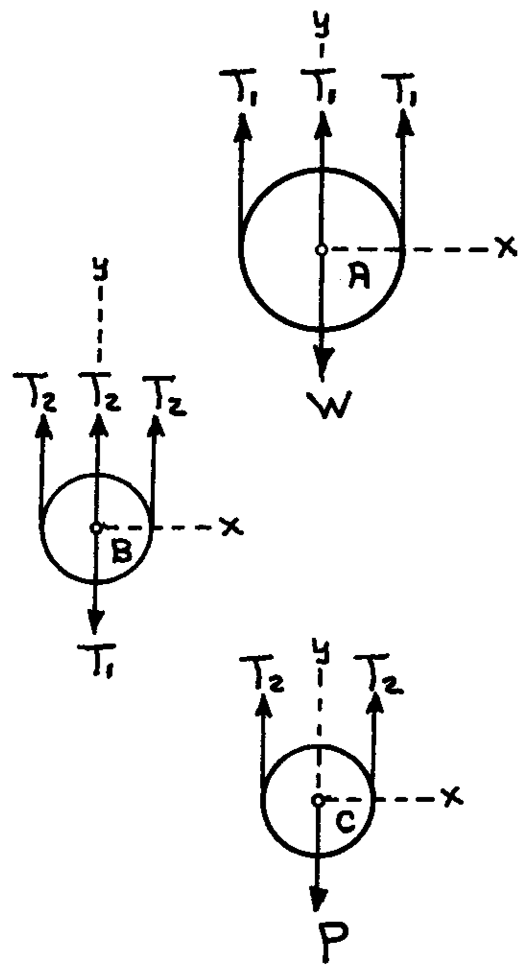
From a free-body diagram for pulley C:

$$\Sigma F_y = 2T_2 - P$$

$$= 2(27.78) - P = 0$$

$$P = 55.56 \text{ lb} \cong 55.6 \text{ lb}$$

$$\bar{P} = -55.6\ \hat{j} \text{ lb} = 55.6 \text{ lb} \downarrow$$ Ans.



6-56* Pulleys 1 and 2 of the rope and pulley system shown in Fig. P6-56 are connected and rotate as a unit. The radii of the pulleys are 100 mm and 300 mm, respectively. Rope A is fastened to pulley 1 at point A′. Rope B is fastened to pulley 2 at point B′. Rope C is continuous over pulleys 3 and 4. Determine the tension **T** in rope C required to hold body W in equilibrium if the mass of body W is 225 kg.

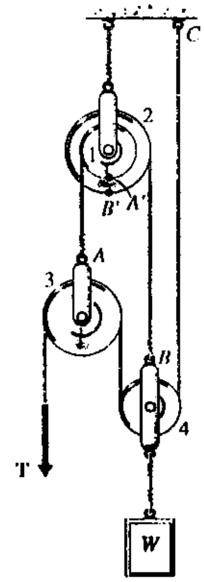
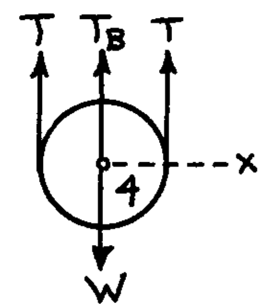


Fig. P6-56

SOLUTION

From a free-body diagram of pulley 4:

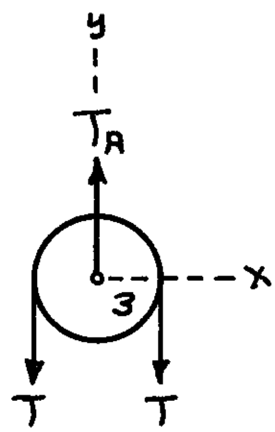


$W = mg = 225(9.807) = 2206.6$ N

$+\uparrow \Sigma F_y = 2T + T_B - 2206.6 = 0$

$T_B = 2206.6 - 2T$

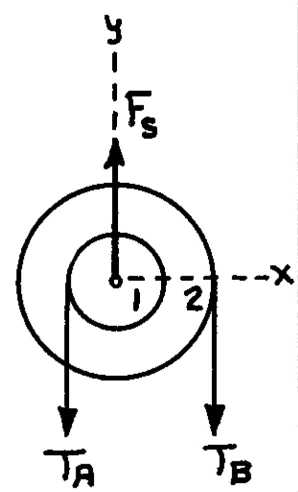
From a free-body diagram of pulley 3:



$+\uparrow \Sigma F_y = T_A - 2T = 0$

$T_A = 2T$

From a free-body diagram of compound pulleys 1 and 2:



$\circlearrowleft \Sigma M_O = T_A(100) - T_B(300)$

$= 2T(100) - (2206.6 - 2T)(300) = 0$

$T = 827.5 \text{ N} \cong 828 \text{ N}$

$\mathbf{T} = -828\,\hat{j} \text{ N} = 828 \text{ N} \downarrow$ Ans.

6-57 Pulleys A and B of the chain hoist shown in Fig. P6-57 are connected and rotate as a unit. The chain is continuous and each of the pulleys contain slots that prevent the chain from slipping. Determine the force **F** required to hold a 1000-lb block W in equilibrium if the radii of pulleys A and B are 3.5 and 4.0 in., respectively.

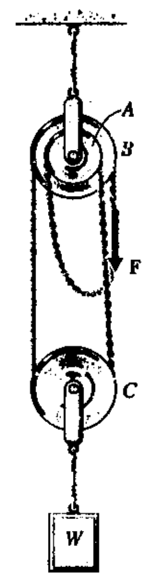
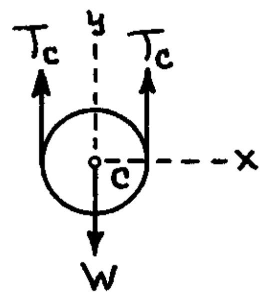


Fig. P6-57

SOLUTION

From a free-body diagram of pulley C:

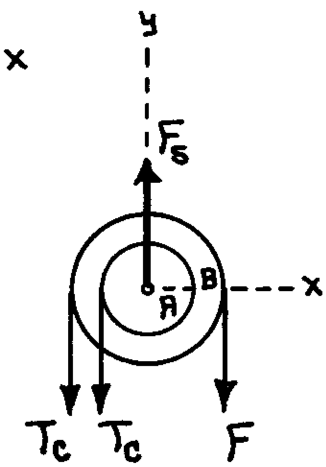


$$+\uparrow \Sigma F_y = 2T_C - W$$

$$= 2T_C - 1000 = 0$$

$$T_C = 500 \text{ lb}$$

From a free-body diagram of compound pulleys A and B:



$$+\circlearrowleft \Sigma M_O = T_C(4.0) - T_C(3.5) - F(4.0)$$

$$= 500(4.0) - 500(3.5) - F(4.0) = 0$$

$$F = 62.5 \text{ lb}$$

$$\mathbf{F} = -62.5\,\hat{\mathbf{j}} \text{ lb} = 62.5 \text{ lb} \downarrow \qquad \text{Ans.}$$

6-58 A wagon with a mass of 3500 kg is held in equilibrium on an inclined surface by using a cable as shown in Fig. P6-58. Determine the force in the cable and the forces exerted on the wheels at A and B by the inclined surface.

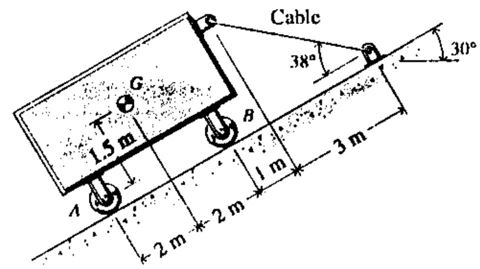
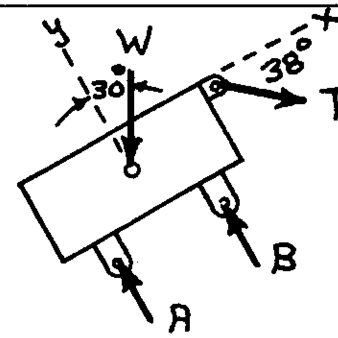


Fig. P6-58

SOLUTION



$W = mg = 3500(9.807) = 34{,}325$ N

From a free-body diagram of the cart:

$+\nearrow \Sigma F_x = T\cos 38^\circ - W\sin 30^\circ$

$= T\cos 38^\circ - 34{,}325\sin 30^\circ = 0$

$T = 21{,}780 \text{ N} \cong 21.8 \text{ kN}$ Ans.

$+\circlearrowleft \Sigma M_A = B(4) + W\sin 30^\circ(1.5) - W\cos 30^\circ(2)$

$- T\cos 38^\circ(3\tan 38^\circ) - T\sin 38^\circ(5)$

$= B(4) + 34{,}325\sin 30^\circ(1.5) - 34{,}325\cos 30^\circ(2)$

$- 21{,}780\cos 38^\circ(3\tan 38^\circ) - 21{,}780\sin 38^\circ(5) = 0$

$B = 35{,}245 \text{ N} \cong 35.2 \text{ kN}$

$\bar{B} = 35.2 \text{ kN} \measuredangle 60^\circ$ Ans.

$+\nwarrow \Sigma F_y = A + B - W\cos 30^\circ - T\sin 38^\circ$

$= A + 35{,}245 - 34{,}325\cos 30^\circ - 21{,}780\sin 38^\circ = 0$

$A = 7890 \text{ N} \cong 7.89 \text{ kN}$

$\bar{A} = 7.89 \text{ kN} \measuredangle 60^\circ$ Ans.

6-59* The lawn mower shown in Fig. P6-59 weighs 35 lb. Determine the force $\bar{P}$ required to move the mower at a constant velocity and the forces exerted on the front and rear wheels by the inclined surface.

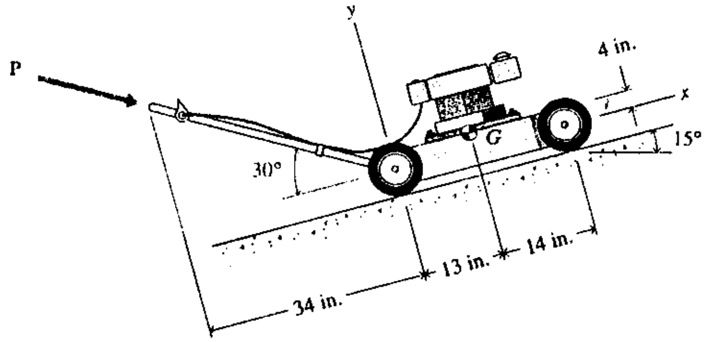
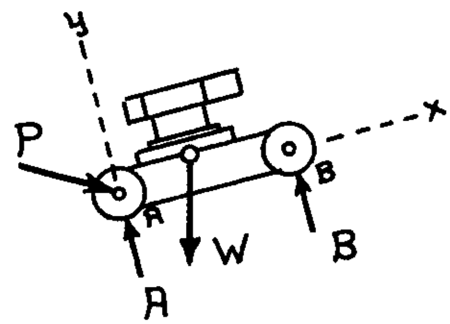


Fig. P6-59

SOLUTION

From a free-body diagram of the lawn mower:



$+\nearrow \Sigma F_x = P\cos 30^\circ - W\sin 15^\circ$

$= P\cos 30^\circ - 35\sin 15^\circ = 0$

$P = 10.460 \text{ lb} = 10.46 \text{ lb}$

$\bar{P} = 10.46 \text{ lb} \searrow 15^\circ$ Ans.

$+\circlearrowleft \Sigma M_A = B(27) + W\sin 15^\circ (4) - W\cos 15^\circ (13)$

$= B(27) + 35\sin 15^\circ (4) - 35\cos 15^\circ (13) = 0$

$B = 14.936 \text{ lb} \cong 14.94 \text{ lb}$

$\bar{B} = 14.94 \text{ lb} \nwarrow 75^\circ$ Ans.

$+\nwarrow \Sigma F_y = A + B - P\sin 30^\circ - W\cos 15^\circ$

$= A + 14.936 - 10.460\sin 30^\circ - 35\cos 15^\circ = 0$

$A = 24.10 \text{ lb}$

$\bar{A} = 24.1 \text{ lb} \nwarrow 75^\circ$ Ans.

6-60* The coal wagon shown in Fig. P6-60 is used to haul coal from a mine. If the mass of the coal and wagon is 2000 kg, determine the force $\bar{P}$ required to move the wagon at a constant velocity and the forces exerted on the front and rear wheels by the inclined surface.

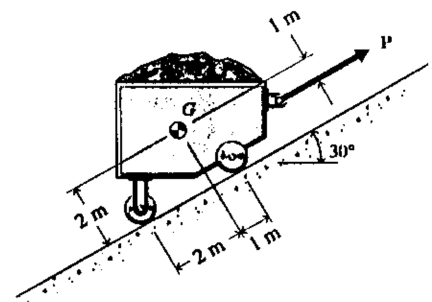
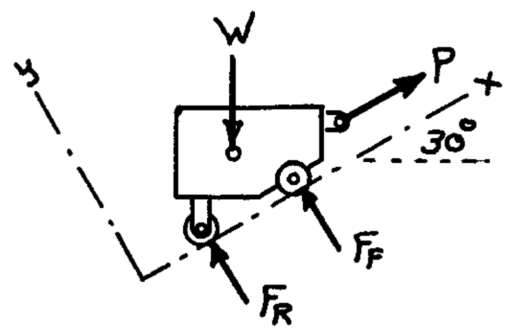


Fig. P6-60

SOLUTION

$W = 2000(9.807) = 19{,}614$ N

From a free-body diagram of the coal wagon:



$+\nearrow \Sigma F_x = P - W \sin 30^\circ$

$= P - 19{,}614 \sin 30^\circ = 0$

$P = 9807 \text{ N} \cong 9.81 \text{ kN}$

$\bar{P} = 9.81 \text{ kN} \measuredangle 30^\circ$ Ans.

$+\circlearrowleft \Sigma M_A = W \cos 30^\circ (1) + W \sin 30^\circ (1) - F_R (3)$

$= 19{,}614 \cos 30^\circ (1) + 19{,}614 \sin 30^\circ (1) - F_R(3) = 0$

$F_R = 8931 \text{ N} \cong 8.93 \text{ kN}$

$\bar{F}_R = 8.93 \text{ kN} \measuredangle 60^\circ$ Ans.

$+\nwarrow \Sigma F_y = F_F + F_R - W \cos 30^\circ$

$= F_F + 8931 - 19{,}614 \cos 30^\circ = 0$

$F_F = 8055 \text{ N} \cong 8.06 \text{ kN}$

$\bar{F}_F = 8.06 \text{ kN} \measuredangle 60^\circ$ Ans.

6-61 Bar AB is supported in a horizontal position by two cables as shown in Fig. P6-61. Determine the magnitude of force $\bar{P}$ and the force in each of the cables.

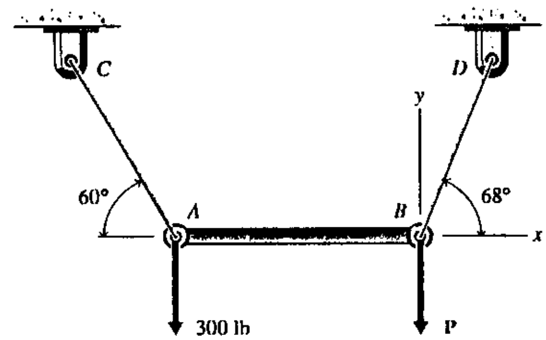
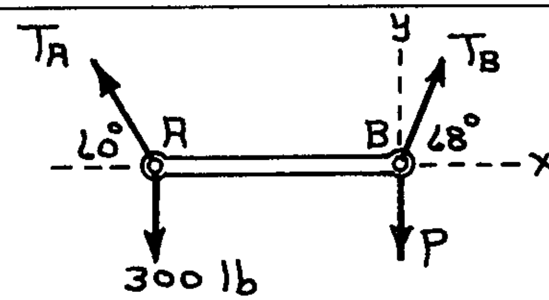


Fig. P6-61

SOLUTION

From a free-body diagram of bar AB:



$$+\circlearrowleft \Sigma M_B = 300\,(L) - T_A \sin 60^\circ\,(L) = 0$$

$$T_A = 346.4 \text{ lb} \cong 346 \text{ lb} \qquad \text{Ans.}$$

$$+\rightarrow \Sigma F_x = T_B \cos 68^\circ - T_A \cos 60^\circ$$

$$= T_B \cos 68^\circ - 346.4 \cos 60^\circ = 0$$

$$T_B = 462.4 \text{ lb} \cong 462 \text{ lb} \qquad \text{Ans.}$$

$$+\circlearrowleft \Sigma M_A = T_B \sin 68^\circ\,(L) - P(L)$$

$$= 462.4 \sin 68^\circ\,(L) - P(L) = 0$$

$$P = 428.7 \text{ lb} \cong 429 \text{ lb} \qquad \text{Ans.}$$

6-62* A frame is loaded and supported as shown in Fig. P6-62. Determine the reactions at supports A and E.

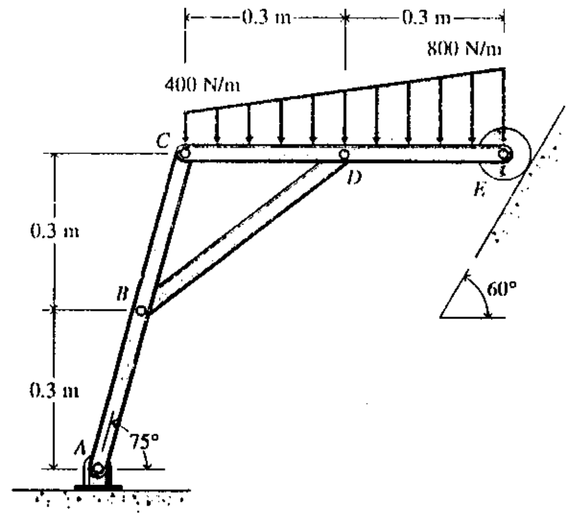
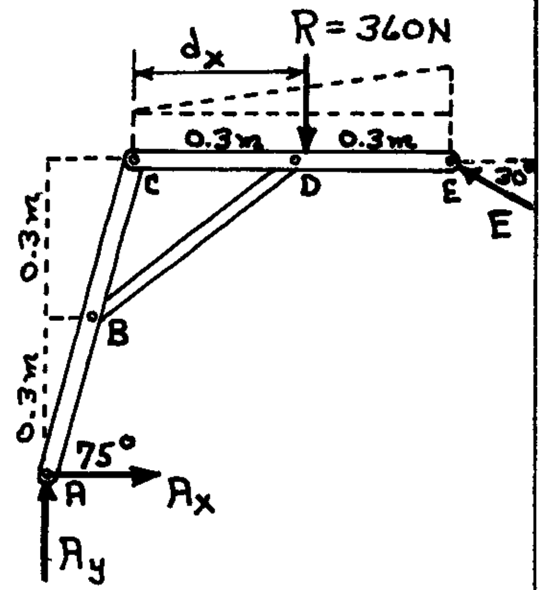


Fig. 6-62

SOLUTION

The distributed load can be replaced on the free-body diagram by a resultant force R at a distance d_x from joint C.



$$R = 400(0.6) + \frac{1}{2}(400)(0.6)$$

$$= 240 + 120 = 360 \text{ N}$$

$$Rd_x = A_1d_1 + A_2d_2$$

$$360d_x = 400(0.6)(0.3) + \frac{1}{2}(400)(0.6)(0.4)$$

$$d_x = 0.3333 \text{ m}$$

From a free-body diagram for the complete frame:

$$+\circlearrowleft \Sigma M_A = E\cos 30^\circ\,(0.6) + E\sin 30^\circ\,(0.6 + 0.6\tan 15^\circ) - 360(0.3333 + 0.6\tan 15^\circ) = 0$$

$E = 197.63 \text{ N} \cong 197.6 \text{ N}$ $\qquad$ $\bar{E} = 197.6 \text{ N} \angle 150.0^\circ$ Ans.

$+\rightarrow \Sigma F_x = A_x - 197.63\cos 30^\circ = 0$ $\qquad$ $A_x = 171.15 \text{ N} \cong 171.2 \text{ N}$

$+\uparrow \Sigma F_y = A_y + 197.63\sin 30^\circ - 360 = 0$ $\qquad$ $A_y = 261.19 \text{ N} \cong 261 \text{ N}$

$$A = \sqrt{(A_x)^2 + (A_y)^2} = \sqrt{(171.15)^2 + (261.19)^2} = 312.3 \text{ N} \cong 312 \text{ N}$$

$\theta_x = \tan^{-1}\dfrac{261.19}{171.15} = 56.76^\circ \cong 56.8^\circ$ $\qquad$ $\bar{A} \cong 312 \text{ N} \angle 56.8^\circ$ Ans.

6-63 The wrecker truck of Fig. P6-63 has a weight of 15,000 lb and a center of gravity at G. The force exerted on the rear (drive) wheels by the ground consists of both a normal component B_y and a tangential component B_x while the force exerted on the front wheel consists of a normal force A_y only. Determine the maximum pull P the wrecker can exert when $\theta = 30^\circ$ if B_x cannot exceed $0.8B_y$ (because of friction considerations) and the wrecker does not tip over backwards (the front wheels remain in contact with the ground).

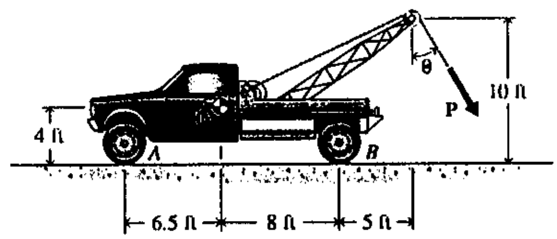
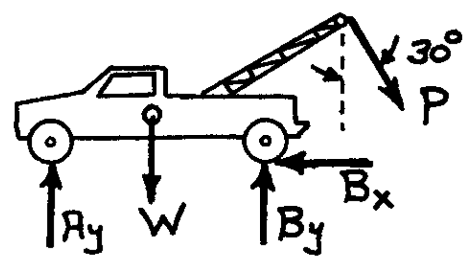


Fig. P6-63

SOLUTION

From a free-body diagram for the wrecker truck:



For impending tipping: $A_y = 0$

$$+\circlearrowleft \Sigma M_B = W(8) - P\sin 30^\circ(10) - P\cos 30^\circ(5)$$
$$= 15{,}000(8) - P\sin 30^\circ(10) - P\cos 30^\circ(5) = 0$$

$$P = 12{,}862 \text{ lb} \cong 12.86 \text{ kip}$$

$$+\uparrow \Sigma F_y = B_y - W - P\cos 30^\circ$$
$$= B_y - 15{,}000 - 12{,}862\cos 30^\circ = 0$$

$$B_y = 26{,}139 \text{ lb} \cong 26.1 \text{ kip}$$

$$+\rightarrow \Sigma F_x = -B_x + P\sin 30^\circ$$
$$= -B_x + 12{,}862\sin 30^\circ = 0$$

$$B_x = 6431 \text{ lb} \cong 6.43 \text{ kip}$$

$$B_x(\max) = 0.8B_y = 0.8(26{,}139) = 20{,}911 \text{ lb} > 6431 \text{ lb}$$

Therefore: $P_{max} = 12.86$ kip Ans.

6-64 Bar AB of Fig. P6-64 has a uniform cross section, a mass of 25 kg, and a length of 1 m long. Determine the angle θ for equilibrium.

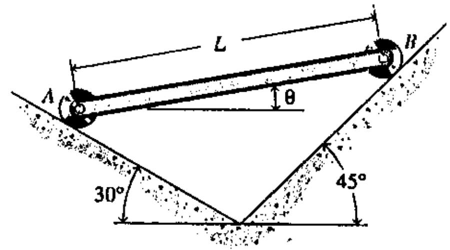
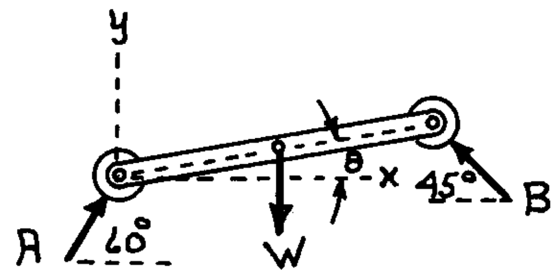


Fig. P6-64

SOLUTION

From a free-body diagram for the bar:



$W = mg = 25(9.807) = 245.2$ N

$+\rightarrow \Sigma F_x = A \cos 60^\circ - B \cos 45^\circ = 0$

$+\uparrow \Sigma F_y = A \sin 60^\circ + B \sin 45^\circ - W$

$= A \sin 60^\circ + B \sin 45^\circ - 245.2 = 0$

Solving yields:

$A = 179.50$ N $B = 126.92$ N

$+\circlearrowleft \Sigma M_A = B \sin 45^\circ (L \cos \theta) + B \cos 45^\circ (L \sin \theta) - W(0.5L \cos \theta)$

$= 126.92 \sin 45^\circ (1 \cos \theta) + 126.92 \cos 45^\circ (1 \sin \theta)$

$- 245.2(0.5 \cos \theta) = 0$

$89.75 \sin \theta = 32.85 \cos \theta$

$\theta = \tan^{-1} \dfrac{32.85}{89.75} = 20.1^\circ$ Ans.

6-65* The crane and boom, shown in Fig. P6-65, weigh 12,000 lb and 600 lb, respectively. When the boom is in the position shown, determine
(a) The maximum load that can be lifted by the crane.
(b) The tension in the cable used to raise and lower the boom when the load being lifted is 3600 lb.
(c) The pin reaction at boom support A when the load being lifted is 3600 lb.

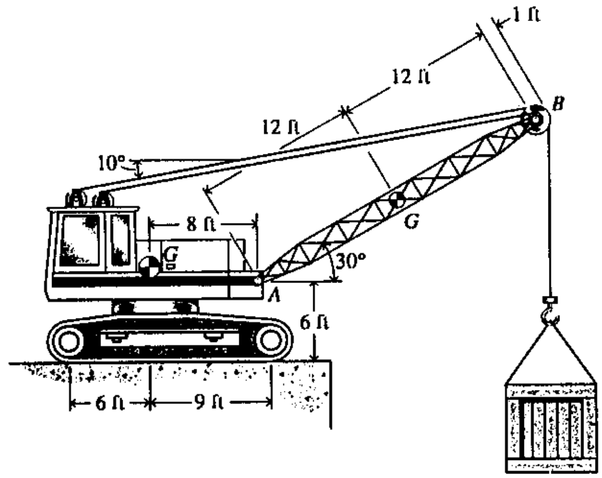
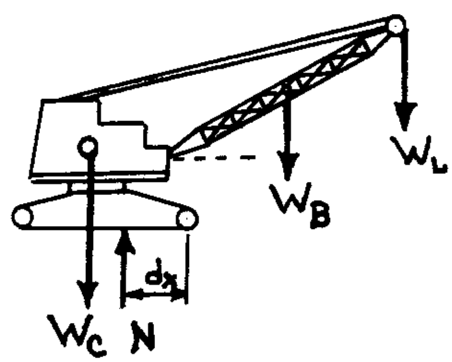


Fig. P6-65

SOLUTION

(a) From a free-body diagram for the crane:



$$+\uparrow \Sigma F_y = N - W_C - W_B - W_L$$

$$= N - 12{,}000 - 600 - W_L = 0$$

$$N = 12{,}600 + W_L \text{ lb}$$

$$+\circlearrowleft \Sigma M_C = W_C(9) - W_B(12\cos 30^\circ - 1) - W_L(24\cos 30^\circ + 1 - 1) - Nd_x$$

$$= 12{,}000(9) - 600(12\cos 30^\circ - 1)$$

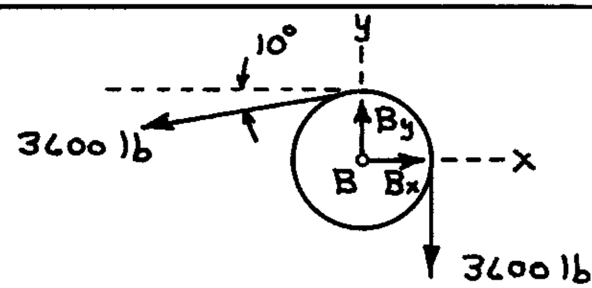
$$- W_L(24\cos 30^\circ + 1 - 1) - (12{,}600 + W_L)d_x$$

$$= 102{,}365 - 20.785\, W_L - (12{,}600 + W_L)d_x = 0$$

For impending tipping: $d_x = 0$

$$W_L(\max) = \frac{102{,}365}{20.785} = 4925 \text{ lb}$$ Ans.

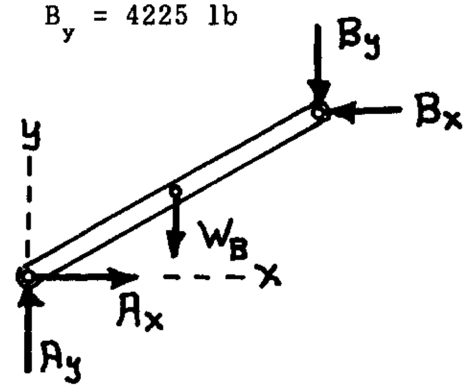
6-65 (Continued)



(b) From a free-body diagram for the pulley at B:

$$+\rightarrow \Sigma F_x = B_x - 3600 \cos 10^\circ = 0 \qquad B_x = 3545 \text{ lb}$$

$$+\uparrow \Sigma F_y = B_y - 3600 - 3600 \sin \Phi = 0 \qquad B_y = 4225 \text{ lb}$$



From a free-body diagram for the boom:

$$+\circlearrowleft \Sigma M_A = B_x(24 \sin 30^\circ) - B_y(24 \cos 30^\circ) - W_B(12 \cos 30^\circ) + T(24 \sin 20^\circ) = 0$$

$$= 3545(24 \sin 30^\circ) - 4225(24 \cos 30^\circ) - 600(12 \cos 30^\circ) + T(24 \sin 20^\circ) = 0$$

$$T = 6275 \text{ lb} \cong 6.28 \text{ kip}$$ Ans.

(c) $+\rightarrow \Sigma F_x = A_x - B_x - T \cos 10^\circ$

$$= A_x - 3545 - 6275 \cos 10^\circ = 0 \qquad A_x = 9725 \text{ lb}$$

$+\uparrow \Sigma F_y = A_y - B_y - T \sin 10^\circ - 600$

$$= A_y - 4225 - 6275 \sin 10^\circ - 600 = 0 \qquad A_y = 5915 \text{ lb}$$

$$A = \sqrt{(A_x)^2 + (A_y)^2} = \sqrt{(9725)^2 + (5915)^2} = 11{,}383 \text{ lb} \cong 11.38 \text{ kip}$$

$$\theta_x = \tan^{-1} \frac{5915}{9725} = 31.31^\circ \cong 31.3^\circ$$

$$\mathbf{A} = 9.73\,\hat{i} + 5.92\,\hat{j} \text{ kip} = 11.38 \text{ kip} \measuredangle 31.3^\circ$$ Ans.

6-66 The garage door ABCE shown in Fig. P6-66 is being raised by a cable DE. The one-piece door is a homogeneous rectangular slab weighing 225 lb. Frictionless rollers B and C run in tracks at each side of the door as shown. Determine the tension T in the cable and the forces B and C on the frictionless rollers when d = 75 in.

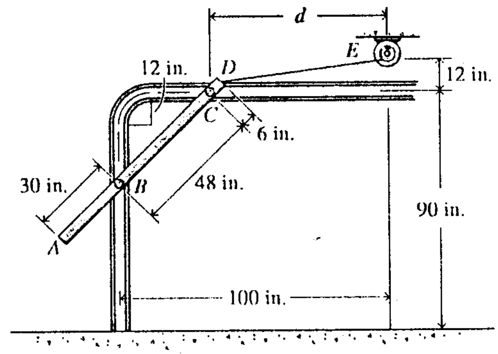
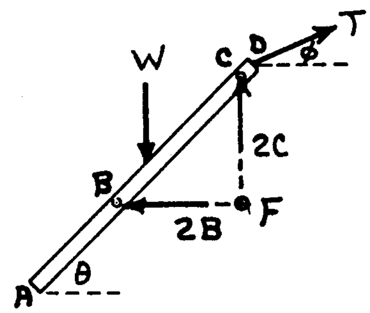


Fig. P6-66

SOLUTION

From a free-body diagram for the door:



$$\theta = \cos^{-1}\frac{100 - 75}{48} = 58.61^\circ$$

$$\phi = \tan^{-1}\frac{12 - 6\sin\theta}{75 - 6\cos\theta}$$

$$= \tan^{-1}\frac{12 - 6\sin 58.61^\circ}{75 - 6\cos 58.61^\circ} = 5.466^\circ$$

$$+\circlearrowleft \Sigma M_F = 225(36\cos 58.61^\circ) + T\sin 5.466^\circ(6\cos 58.61^\circ)$$
$$-T\cos 5.466^\circ(48\sin 58.61^\circ + 6\sin 58.61^\circ) = 0$$

$T = 92.54$ lb $\cong 93.0$ lb **Ans.**

$$+\rightarrow \Sigma F_x = T\cos\phi - 2B = 92.54\cos 5.466^\circ - 2B = 0$$

$B = 46.06$ lb $\cong 46.1$ lb

$\mathbf{B} = -46.1\,\hat{\imath}$ lb $= 46.1$ lb $\leftarrow$ **Ans.**

$$+\uparrow \Sigma F_y = 2C + T\sin\phi - 225 = 2C + 92.54\sin 5.466^\circ - 225 = 0$$

$C = 108.09$ lb $\cong 108.1$ lb

$\mathbf{C} = 108.1\,\hat{\jmath}$ lb $= 108.1$ lb $\uparrow$ **Ans.**

C6-67 The lever shown in Fig. P6-67 is formed in a quarter circular arc of radius 20 in.. Plot A and B, the magnitude of the pin force at A and the force on the smooth support B, as a function of θ ($0^\circ \le \theta \le 85^\circ$), the angle at which the support is located.

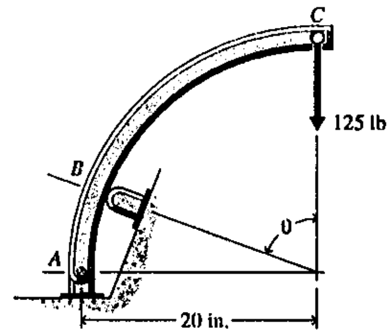
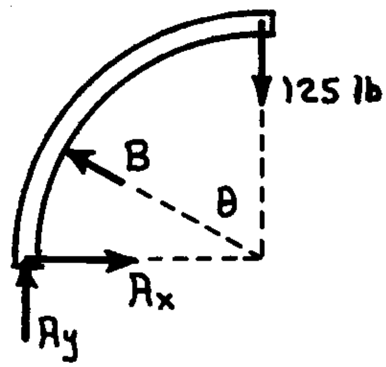


Fig. P6-67

SOLUTION

From a free-body diagram for the lever:



$$+\circlearrowleft \Sigma M_A = B(20\cos\theta) - 125(20) = 0$$

$$B = \frac{125}{\cos\theta}$$

$$+\rightarrow \Sigma F_x = A_x - B\sin\theta = 0$$

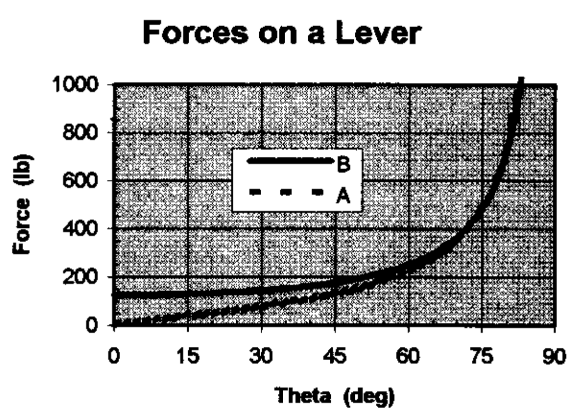
$$= A_x - (125/\cos\theta)\sin\theta = 0 \qquad A_x = 125\tan\theta$$

$$+\uparrow \Sigma F_y = A_y + B\cos\theta - 125$$

$$= A_y - (125/\cos\theta)\cos\theta - 125 = 0 \qquad A_y = 0$$

Therefore:

$$A = A_x = 125\tan\theta$$



C6-68 A lightweight pipe is slipped over the handle of a wrench to give extra leverage (Fig. P6-68). If the inside of the pipe is smooth and fits loosely about the handle of the wrench, plot the forces exerted on the wrench at A and B as a function of the overlap distance d ($0 \le d \le 200$ mm).

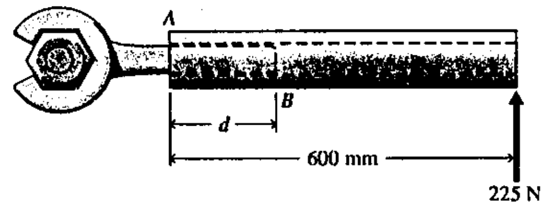
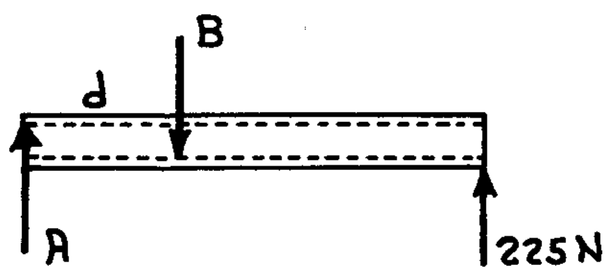


Fig. P6-68

SOLUTION

From a free-body diagram of the pipe:

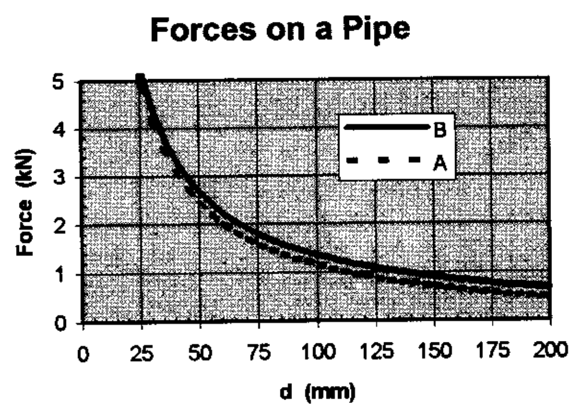


$$+\circlearrowleft \Sigma M_A = 225(600) - Bd = 0$$

$$B = \frac{135,000}{d} \text{ N}$$

$$+\uparrow \Sigma F_y = A - B + 225 = 0$$

$$A = B - 225 \text{ N}$$



C6-69 The crane and boom shown in Fig. P6-69 weigh 12,000 lb and 600 lb, respectively. The pulleys at D and E are small and the cables attached to them are essentially parallel.

(a) Plot d, the location of the resultant force relative to point C, as a function of the boom angle θ ($0^\circ \le \theta \le 80^\circ$), when the crane is lifting a 3600-lb load.

(b) Plot A, the magnitude of the reaction force on the pin at A, as a function of θ ($0^\circ \le \theta \le 80^\circ$) when the crane is lifting a 3600-lb load.

(c) It is desired that the resultant force on the tread always be at least 1 ft behind C to ensure that the crane is never in danger of tipping over. Plot W_{max}, the maximum load that may be lifted, as a function of θ ($0^\circ \le \theta \le 80^\circ$).

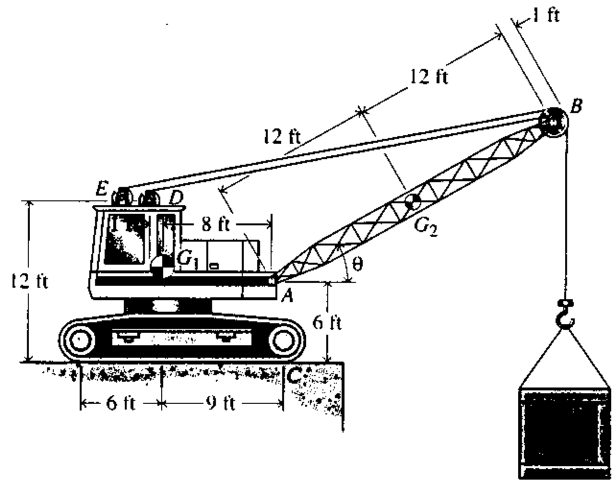
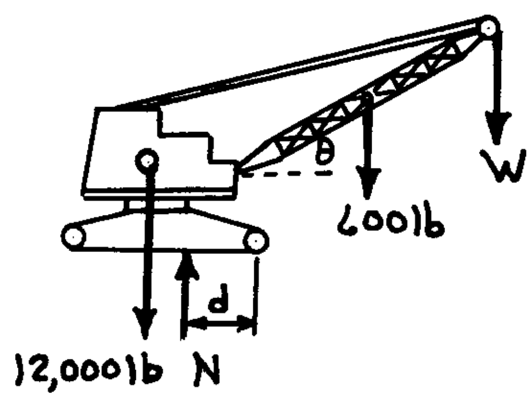


Fig. P6-69

SOLUTION

(a) From a free-body diagram for the crane:



$$+\circlearrowleft \Sigma M_C = 12{,}000(9) - (600)(12\cos\theta - 1) - W(24\cos\theta + 1 - 1) - Nd = 0$$

$$+\uparrow \Sigma F_y = N - 12{,}000 - 600 - W = 0$$

$$N = 12{,}600 + W$$

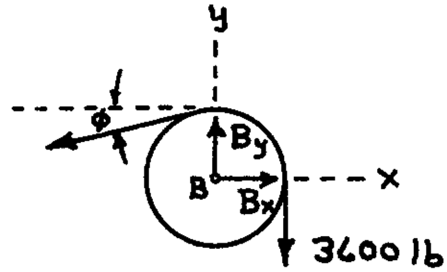
$$d = \frac{108{,}600 - 7200\cos\theta - 24W\cos\theta}{N}$$

For W = 3600 lb: $N = 12{,}600 + 3600 = 16{,}200$ lb

$$d = \frac{108{,}600 - 93{,}600\cos\theta}{16{,}200}$$

C6-69 (Continued)

(b) From a free-body diagram of the pulley at B:



$$\tan \phi = \frac{24 \sin \theta - 6}{24 \cos \theta + 9}$$

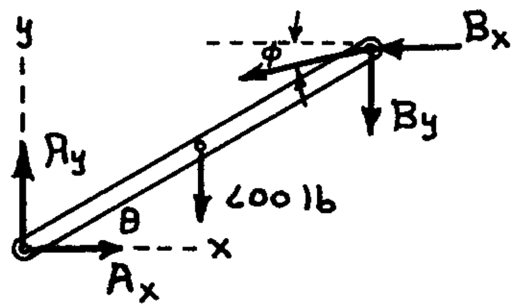
$$+ \rightarrow \Sigma F_x = B_x - 3600 \cos \phi = 0$$

$$B_x = 3600 \cos \phi$$

$$+ \uparrow \Sigma F_y = B_y - 3600 - 3600 \sin \phi = 0$$

$$B_y = 3600(1 + \sin \phi)$$

From a free-body diagram of the boom:



$$+ \circlearrowleft \Sigma M_A = B_x(24 \sin \theta) - B_y(24 \cos \theta) - 600(12 \cos \theta) + T[24 \sin (\theta - \phi)] = 0$$

$$T = \frac{7200 \cos \theta + 24B_y \cos \theta - 24B_x \sin \theta}{24 \sin (\theta - \phi)}$$

$$+ \rightarrow \Sigma F_x = A_x - B_x - T \cos \phi = 0 \qquad A_x = B_x + T \cos \phi$$

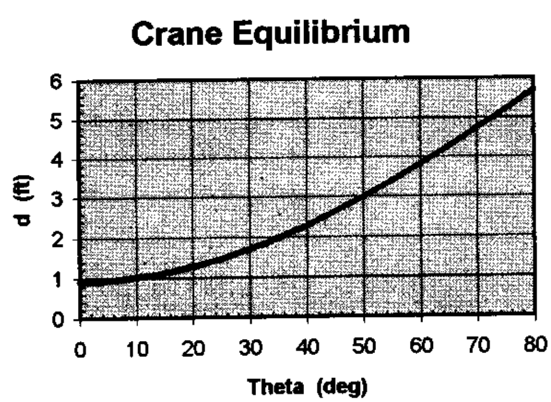
$$+ \uparrow \Sigma F_y = A_y - B_y - T \sin \phi - 600 = 0 \qquad A_y = B_y + T \sin \phi + 600$$

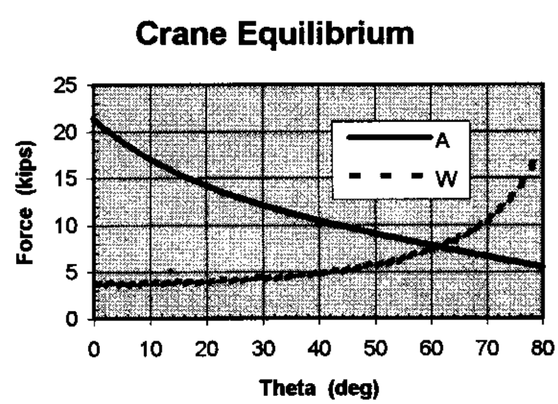
$$A = \sqrt{(A_x)^2 + (A_y)^2}$$

(c) For d = 1:

$$N = 12600 + W = 108,600 - 7200 \cos \theta - 24W \cos \theta$$

$$W = \frac{96,000 - 7200 \cos \theta}{1 + 24 \cos \theta}$$





C6-70 The tower crane shown in Fig. P6-70 is used to lift construction materials. The counterweight C weighs 31,000 N; the motor M weighs 4500 N; the weight of the boom AB is 36,000 N and can be considered as acting at point G; and the weight of the tower is 23,000 N and can be considered as acting at its midpoint.

(a) If the tower is lifting a 9000-N load, calculate and plot the reaction forces on the feet at D and E as a function of the distance x at which the weight is being lifted.

(b) It is desired that the reaction forces at the feet D and E always be greater than 4500 N to ensure that the tower is never in danger of tipping over. Plot W_{max}, the maximum load that can be lifted as a function of the distance x ($0 \le x \le 36$ m).

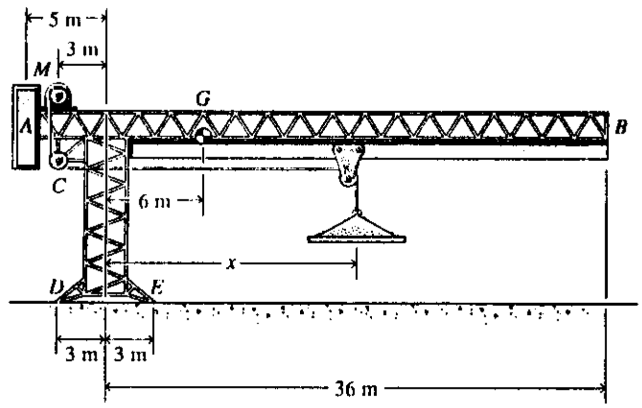
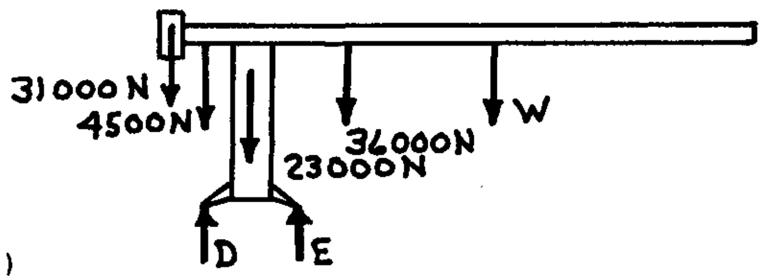


Fig. P6-70

SOLUTION

From a free-body diagram for the crane:



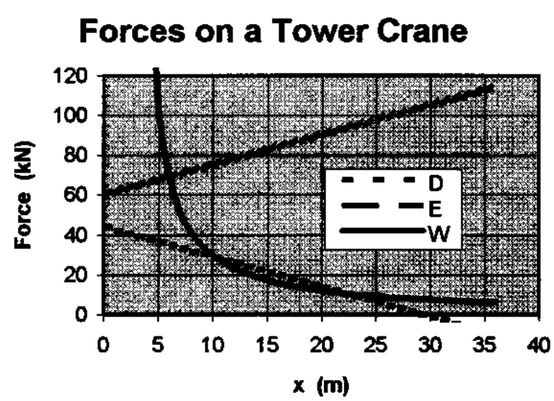
$$+\circlearrowleft \Sigma M_H = 3E - 3D - Wx + 31{,}000(5) + 4500(3) - 36{,}000(6) = 0$$

$$+\uparrow \Sigma F_y = D + E - W - 94{,}500 = 0$$

Solving yields:

$$D = 39{,}333.3 + \frac{W}{6}(3 - x)$$

$$E = 55{,}166.6 + \frac{W}{6}(x + 3)$$



(a) If W = 9000 N:

$$D = 43833.3 - 1500x$$

$$E = 59666.6 + 1500x$$

D = 0 and the crane will tip if: $x > \dfrac{43833.3}{1500} = 29.22$ m

(b) If D = 4500 N: $W_{max} = \dfrac{209{,}000}{x - 3}$

C6-71 The hydraulic cylinder BC is used to tip the box of the dump truck shown in Fig. P6-71. If the combined weight of the box and the load is 22,000 lb and acts through the center of gravity G

(a) Plot C, the force in the hydraulic cylinder, as a function of the angle θ $(0 \le \theta \le 80^\circ)$.

(b) Plot A, the magnitude of the reaction force on the pin at A, as a function of θ $(0^\circ \le \theta \le 80^\circ)$.

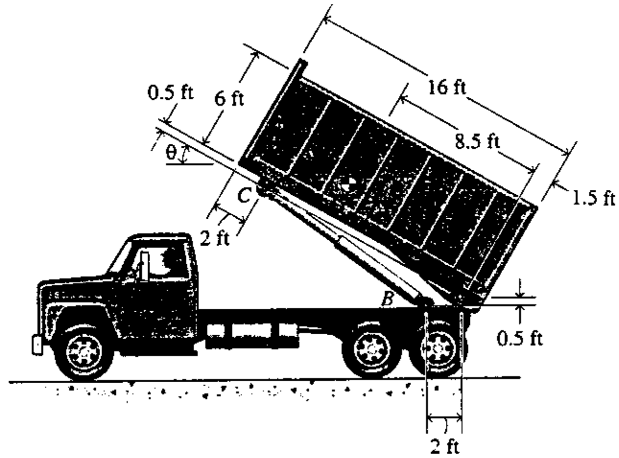


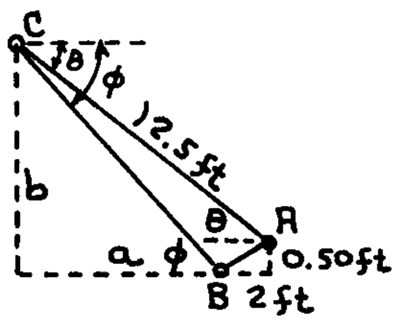
Fig. P6-71

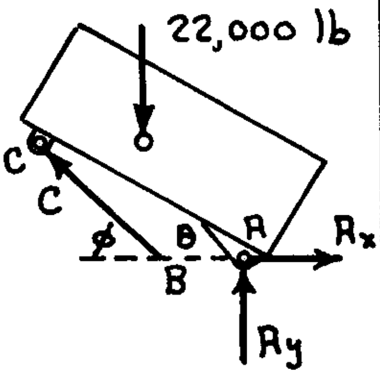
SOLUTION

$a = 12.5 \cos \theta - 2$

$b = 12.5 \sin \theta + 0.5$

$\phi = \tan^{-1} \dfrac{b}{a}$





From a free-body diagram for the truck box:

$+\rightarrow \Sigma F_x = A_x - C \cos \phi = 0$

$A_x = C \cos \phi$

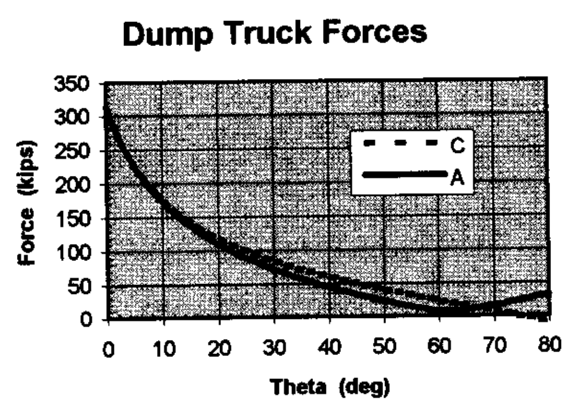
$+\uparrow \Sigma F_y = A_y + C \sin \phi - 22{,}000 = 0$

$A_y = 22{,}000 - C \sin \phi$

$A = \sqrt{(A_x)^2 + (A_y)^2}$

$+\circlearrowleft \Sigma M_A = 8.5(22{,}000 \cos \theta) - 2.5(22{,}000 \sin \theta) - 12.5[C \sin(\phi - \theta)] = 0$

$$C = \frac{187{,}000 \cos \theta - 55{,}000 \sin \theta}{12.5 \sin (\phi - \theta)}$$



C6-72 An extension ladder is being raised into position by a hydraulic cylinder as shown in Fig. P6-72. If the ladder has a mass of 500 kg and center of gravity at G

(a) Plot B, the force in the hydraulic cylinder, as a function of the angle θ ($10^\circ \le \theta \le 90^\circ$).

(b) Plot A, the magnitude of the reaction force on the pin at A, as a function of θ ($10^\circ \le \theta \le 90^\circ$).

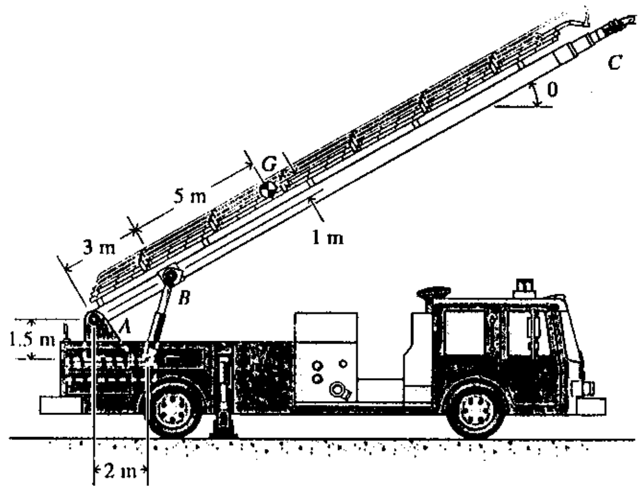


Fig. P6-72

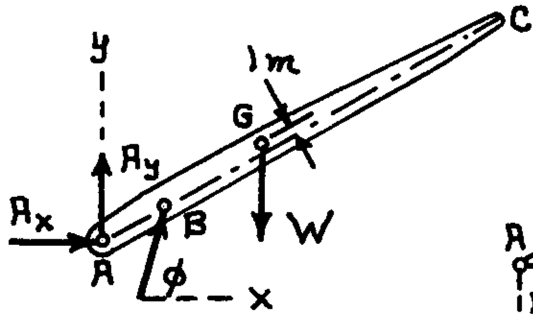
SOLUTION

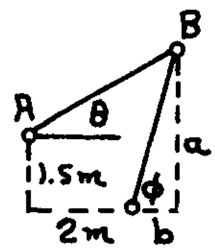
$W = 500(9.807) = 4904$ N

$a = 1.5 + 3 \sin \theta$

$b = 3 \cos \theta - 2$

$\phi = \tan^{-1} \dfrac{a}{b}$





From a free-body diagram for the extension ladder:

$+\rightarrow \Sigma F_x = A_x + B \cos \phi = 0$

$+\uparrow \Sigma F = A_y + B \sin \phi - W = 0$

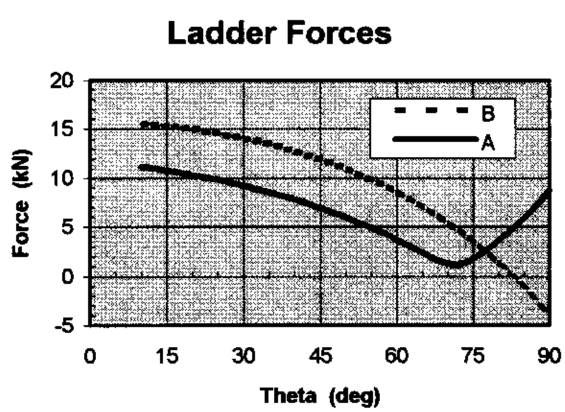
$A_x = -B \cos \phi$

$A_y = W - B \sin \phi$

$A = \sqrt{(A_x)^2 + (A_y)^2}$

$+\circlearrowleft \Sigma M_A = (B \sin \phi)(3 \cos \theta) - (B \cos \phi)(3 \sin \theta) - (W \cos \theta)(8) + (W \sin \theta)(1) = 0$

$$B = \frac{8W \cos \theta - W \sin \theta}{3 \cos \theta \sin \phi - 3 \sin \theta \cos \phi}$$



6-73* Determine the reaction at support A of the pipe system shown in Fig. P6-73 when the force applied to the pipe wrench is 50 lb.

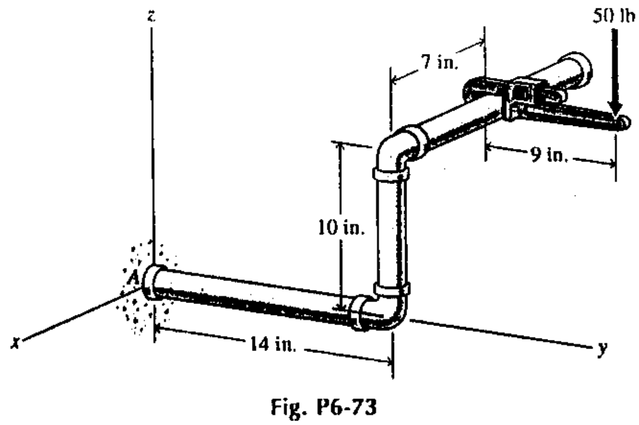
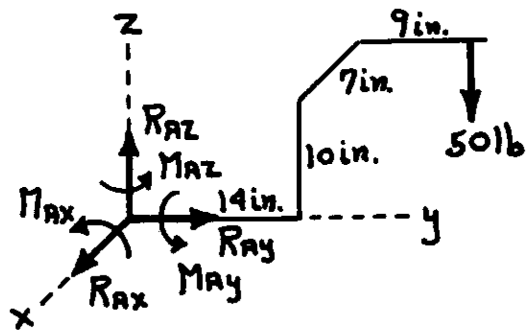


Fig. P6-73

SOLUTION

From a free-body diagram for the pipe system:



For force equilibrium:

$$\Sigma\vec{F} = \vec{R}_A + \vec{P}$$

$$= R_{Ax}\,\hat{i} + R_{Ay}\,\hat{j} + (R_{Az} - 50)\,\hat{k} = \vec{0}$$

$$\vec{R}_A = R_{Ax}\,\hat{i} + R_{Ay}\,\hat{j} + R_{Az}\,\hat{k}$$

$$= 0\,\hat{i} + 0\,\hat{j} + 50\,\hat{k} = 50\,\hat{k}\text{ lb} = 50\text{ lb}\uparrow$$ Ans.

For the load P:

$$\vec{r}_{B/A} = -7\,\hat{i} + 23\,\hat{j} + 10\,\hat{k}$$

$$\vec{M}_{AP} = \vec{r}_{B/A} \times \vec{P} = (-7\,\hat{i} + 23\,\hat{j} + 10\,\hat{k}) \times (-50\,\hat{k})$$

$$= -1150\,\hat{i} - 350\,\hat{j}\text{ in.}\cdot\text{lb}$$

For moment equilibrium:

$$\Sigma\vec{M}_A = \vec{C}_A + \vec{M}_{AP} = (M_{Ax} - 1150)\,\hat{i} + (M_{Ay} - 350)\,\hat{j} + M_{Az}\,\hat{k} = \vec{0}$$

$$\vec{C}_A = M_{Ax}\,\hat{i} + M_{Ay}\,\hat{j} + M_{Az}\,\hat{k} = 1150\,\hat{i} + 350\,\hat{j}\text{ in.}\cdot\text{lb}$$ Ans.

$$C_A = \sqrt{(M_{Ax})^2 + (M_{Ay})^2} = \sqrt{(1150)^2 + (350)^2} = 1202\text{ in.}\cdot\text{lb}$$

6-74* The rectangular plate of uniform thickness shown in Fig. P6-74 has a mass of 500 kg. Determine the tensions in the three cables supporting the plate.

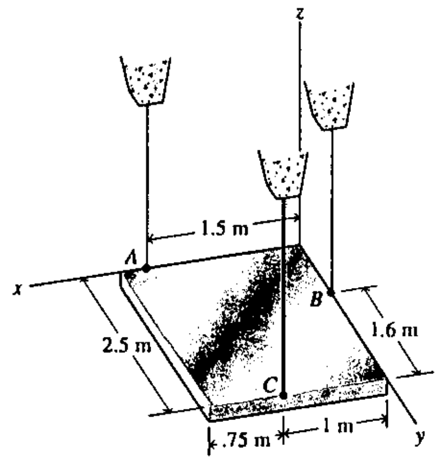
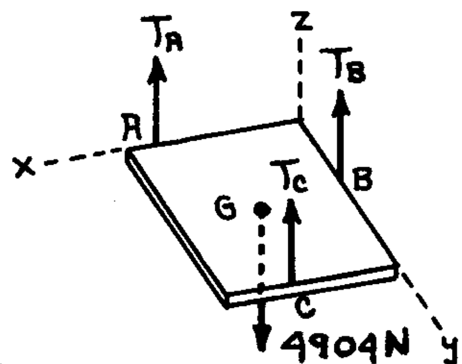


Fig. P6-74

SOLUTION

From a free-body diagram for the plate:



$$\mathbf{W} = m\vec{g} = 500(9.807)(-\hat{k}) = -4904\ \hat{k}\ \text{N}$$

For moment equilibrium:

$$\Sigma\mathbf{M}_A = (\vec{r}_{B/A} \times \mathbf{T}_B) + (\vec{r}_{C/A} \times \mathbf{T}_C) + (\vec{r}_{G/A} \times \mathbf{W})$$

$$= [(-1.5\ \hat{i} + 0.9\ \hat{j}) \times (T_B\ \hat{k})] + [(-0.5\ \hat{i} + 2.5\ \hat{j}) \times (T_C\ \hat{k})] + [(-0.625\ \hat{i} + 1.25\ \hat{j}) \times (-4904\ \hat{k})]$$

$$= (0.9T_B + 2.5T_C - 6130)\ \hat{i} + (1.5T_B + 0.5T_C - 3065)\ \hat{j} = \vec{0}$$

Solving yields:

$T_B = 1393.2$ N $\mathbf{T}_B \cong 1393\ \hat{k}$ N = 1393 N ↑   Ans.

$T_C = 1950.5$ N $\mathbf{T}_C \cong 1951\ \hat{k}$ N = 1951 N ↑   Ans.

For force equilibrium:

$$\Sigma\mathbf{F} = \mathbf{T}_A + \mathbf{T}_B + \mathbf{T}_C + \mathbf{W}$$

$$= (T_A + T_B + T_C - 4904)\ \hat{k}$$

$$= (T_A + 1393.2 + 1950.5 - 4904)\ \hat{k} = \vec{0}$$

$T_A = 1560.3$ N $\mathbf{T}_A \cong 1560\ \hat{k}$ N = 1560 N ↑   Ans.

6-75 A 500-lb homogeneous circular plate is supported by three cables as shown in Fig. P6-75. Determine the tensions in the three cables.

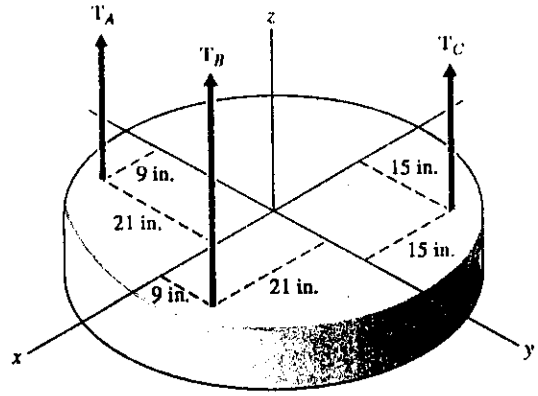
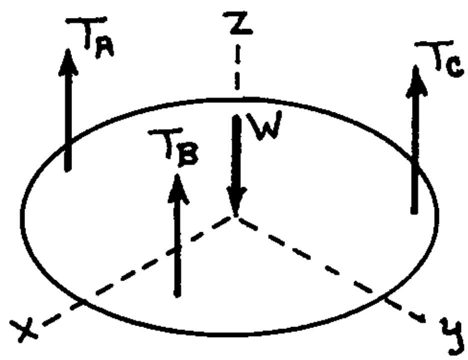


Fig. P6-75

SOLUTION

From a free-body diagram for the plate:



For moment equilibrium:

$$\Sigma \mathbf{M}_A = (\vec{r}_{B/A} \times \mathbf{T}_B) + (\vec{r}_{C/A} \times \mathbf{T}_C) + (\vec{r}_{G/A} \times \mathbf{W})$$

$$= [(12\ \hat{i} + 30\ \hat{j}) \times (T_B\ \hat{k})] + [(-24\ \hat{i} + 36\ \hat{j}) \times (T_C\ \hat{k})]$$

$$+ [(-9\ \hat{i} + 21\ \hat{j}) \times (-500\ \hat{k})]$$

$$= (30T_B + 36T_C - 10{,}500)\ \hat{i} + (-12T_B + 24T_C - 4500)\ \hat{j} = \bar{0}$$

Solving yields:

$T_B = 78.13$ lb $\cong$ 78.1 lb $\qquad \mathbf{T}_B = 78.1\ \hat{k}$ lb = 78.1 lb ↑ $\qquad$ Ans.

$T_C = 226.56$ lb $\cong$ 227 lb $\qquad \mathbf{T}_C = 227\ \hat{k}$ lb = 227 lb ↑ $\qquad$ Ans.

For force equilibrium:

$$\Sigma \mathbf{F} = \mathbf{T}_A + \mathbf{T}_B + \mathbf{T}_C + \mathbf{W}$$

$$= (T_A + T_B + T_C - 500)\ \hat{k}$$

$$= (T_A + 78.13 + 226.56 - 500)\ \hat{k} = \bar{0}$$

$T_A = 195.31$ lb $\cong$ 195.3 lb $\qquad \mathbf{T}_A = 195.3\ \hat{k}$ lb 195.3 lb ↑ $\qquad$ Ans.

6-76 The triangular plate of uniform thickness shown in Fig. P6-76 has a mass of 400 kg. Determine the tensions in the two cables supporting the plate and the reaction at the ball support.

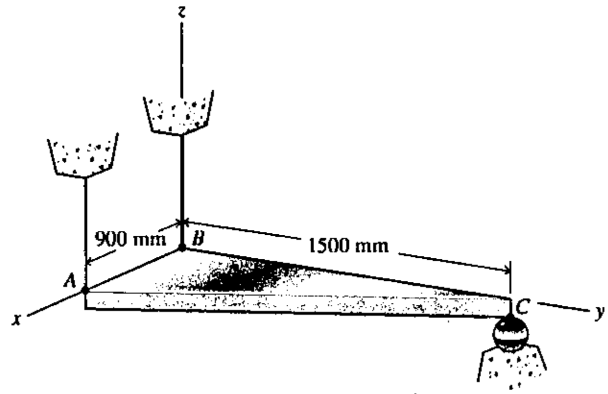
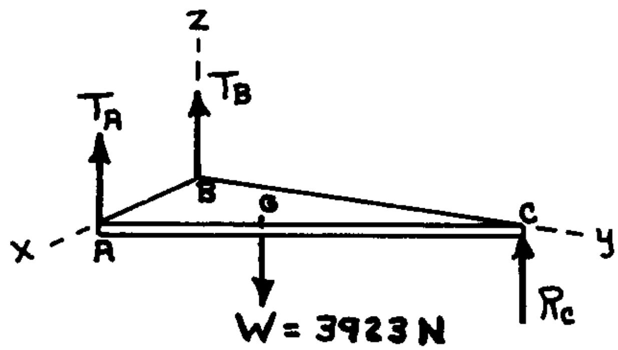


Fig. P6-76

SOLUTION

$W = mg = 400(9.807) = 3923$ N

From a free-body diagram for the plate:



For moment equilibrium:

$$\Sigma \mathbf{M}_C = (\vec{\mathbf{r}}_{A/C} \times \mathbf{T}_A) + (\vec{\mathbf{r}}_{B/C} \times \mathbf{T}_B) + (\vec{\mathbf{r}}_{C/C} \times \mathbf{W})$$

$$= [(0.900\ \hat{\imath} - 1.500\ \hat{\jmath}) \times (T_A\ \hat{k})] + [(-1.500\ \hat{\jmath}) \times (T_B\ \hat{k})] + [(0.300\ \hat{\imath} - 1.000\ \hat{\jmath}) \times (-3923\ \hat{k})]$$

$$= (-1.500T_A - 1.500T_B + 3923)\ \hat{\imath} + (-0.900T_A + 1176.9)\ \hat{\jmath} = \vec{0}$$

Solving yields:

$T_A = 1307.7$ N $\cong 1308$ N $\mathbf{T}_A = 1308\ \hat{k}$ N $= 1308$ N ↑   Ans.

$T_B = 1307.7$ N $\cong 1308$ N $\mathbf{T}_B = 1308\ \hat{k}$ N $= 1308$ N ↑   Ans.

For force equilibrium:

$$\Sigma \mathbf{F} = \mathbf{R}_C + \mathbf{T}_A + \mathbf{T}_B + \mathbf{W}$$

$$= (R_C + 1307.7 + 1307.7 - 3923)\ \hat{k} = \vec{0}$$

$R_C = 1307.6$ N $\cong 1308$ N $\mathbf{R}_C = 1308\ \hat{k}$ N $= 1308$ N ↑   Ans.

6-77* The bent bar shown in Fig. P6-77 is supported with three brackets that exert only force reactions on the bar. Determine the reactions at supports A, B, and C.

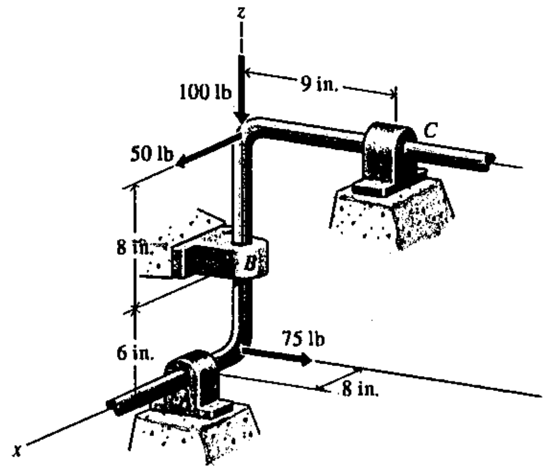
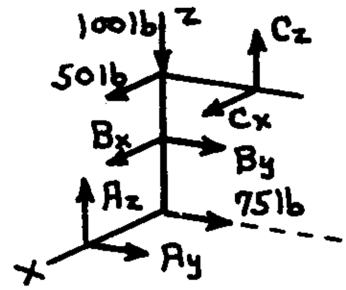


Fig. P6-77

SOLUTION

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma \vec{M}_A = (\vec{r}_{B/A} \times \vec{B}) + (\vec{r}_{C/A} \times \vec{C}) + (\vec{r}_{D/A} \times \vec{D}) + (\vec{r}_{E/A} \times \vec{E})$$

$$= [(-8\,\hat{i} + 6\,\hat{k}) \times (B_x\,\hat{i} + B_y\,\hat{j})]$$

$$+ [(-8\,\hat{i} + 9\,\hat{j} + 14\,\hat{k}) \times (C_x\,\hat{i} + C_z\,\hat{k})]$$

$$+ [(-8\,\hat{i}) \times (75\,\hat{j})] + [(-8\,\hat{i} + 14\,\hat{k}) \times (50\,\hat{i} - 100\,\hat{k})]$$

$$= (-6B_y + 9C_z)\,\hat{i} + (6B_x + 14C_x + 8C_z - 100)\,\hat{j}$$
$$+ (-8B_y - 9C_x - 600)\,\hat{k} = \vec{0}$$

For force equilibrium:

$$\Sigma \vec{F} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

$$= A_y\,\hat{j} + A_z\,\hat{k} + B_x\,\hat{i} + B_y\,\hat{j} + C_x\,\hat{i} + C_z\,\hat{k} + 75\,\hat{j} + 50\,\hat{i} - 100\,\hat{k}$$

$$= (B_x + C_x + 50)\,\hat{i} + (A_y + B_y + 75)\,\hat{j} + (A_z + C_z - 100)\,\hat{k} = \vec{0}$$

Solving yields:

$A_y = 450$ lb	$B_x = -450$ lb	$C_x = 400$ lb
$A_z = 450$ lb	$B_y = -525$ lb	$C_z = -350$ lb

$\vec{A} = 450\,\hat{j} + 450\,\hat{k}$ lb   Ans.

$\vec{B} = -450\,\hat{i} - 525\,\hat{j}$ lb   Ans. $\vec{C} = 400\,\hat{i} - 350\,\hat{k}$ lb   Ans.

6-78* The bent bar shown in Fig. P6-78 is supported with two brackets that exert only force reactions on the bar. End C of the bar rests against smooth horizontal and vertical surfaces. Determine the reactions at supports A, B, and C.

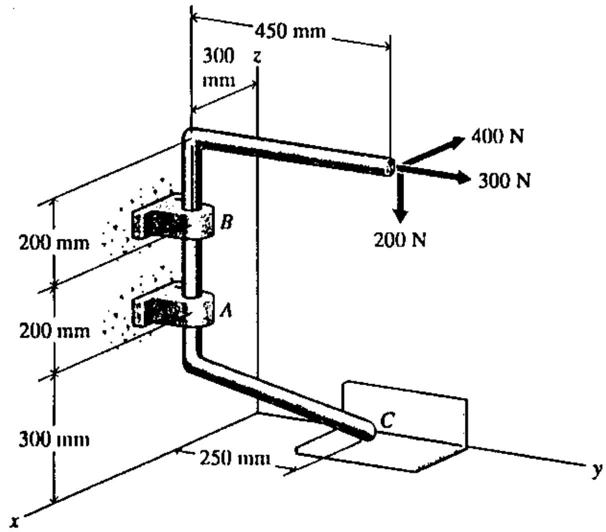
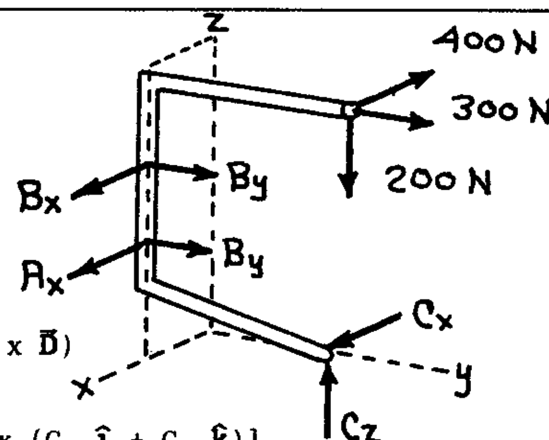


Fig. P6-78

SOLUTION

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma \vec{M}_A = (\vec{r}_{B/A} \times \vec{B}) + (\vec{r}_{C/A} \times \vec{C}) + (\vec{r}_{D/A} \times \vec{D})$$
$$= [(0.200\,\hat{k}) \times (B_x\,\hat{i} + B_y\,\hat{j})]$$
$$+ [(-0.300\,\hat{i} + 0.250\,\hat{j} - 0.300\,\hat{k}) \times (C_x\,\hat{i} + C_z\,\hat{k})]$$
$$+ [(0.450\,\hat{j} + 0.400\,\hat{k}) \times (-400\,\hat{i} + 300\,\hat{j} - 200\,\hat{k})]$$

$$= (-0.200B_y + 0.250C_z - 210)\,\hat{i}$$
$$+ (0.200B_x - 0.300C_x + 0.300C_z - 160)\,\hat{j} + (-0.250C_x + 180)\,\hat{k} = \vec{0}$$

For force equilibrium:

$$\Sigma \vec{F} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$
$$= A_x\,\hat{i} + A_y\,\hat{j} + B_x\,\hat{i} + B_y\,\hat{j} + C_x\,\hat{i} + C_z\,\hat{k} - 400\,\hat{i} + 300\,\hat{j} - 200\,\hat{k}$$
$$= (A_x + B_x + C_x - 400)\,\hat{i} + (A_y + B_y + 300)\,\hat{j} + (C_z - 200)\,\hat{k} = \vec{0}$$

Solving yields:

$A_x = -1900$ N	$B_x = 1580$ N	$C_x = 720$ N
$A_y = 500$ N	$B_y = -800$ N	$C_z = 200$ N

$\vec{A} = -1900\,\hat{i} + 500\,\hat{j}$ N Ans.

$\vec{B} = 1580\,\hat{i} - 800\,\hat{j}$ N Ans. $\vec{C} = 720\,\hat{i} + 200\,\hat{k}$ N Ans.

6-79 Bar AB is used to support an 850-lb load as shown in Fig. P6-79. End A of the bar is supported with a ball and socket joint. End B of the bar is supported with two cables. Determine the components of the reaction at support A and the tensions in the two cables.

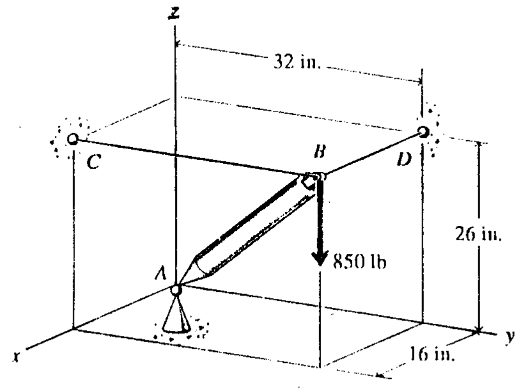
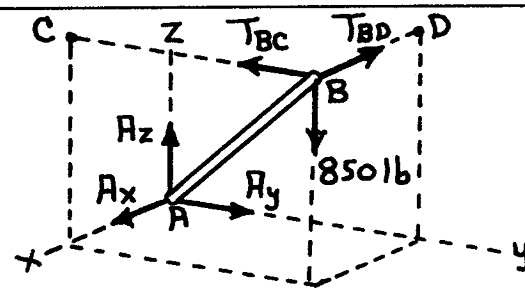


Fig. P6-79

SOLUTION



From a free-body diagram for the bar:

For moment equilibrium:

$$\Sigma \mathbf{M}_A = (16\,\hat{i} + 26\,\hat{k}) \times (-T_{BC}\,\hat{j}) + (32\,\hat{j} + 26\,\hat{k}) \times (-T_{BD}\,\hat{i}) + (16\,\hat{i} + 32\,\hat{j}) \times (-850\,\hat{k})$$

$$= (26\,T_{BC} - 27{,}200)\,\hat{i} + (-26\,T_{BD} + 13{,}600)\,\hat{j} + (-16\,T_{BC} + 32\,T_{BD})\,\hat{k} = \bar{0}$$

From which:

$T_{BC} = 1046.2$ lb $\cong 1046$ lb $\qquad \mathbf{T}_{BC} = -1046\,\hat{j}$ lb $\qquad$ Ans.

$T_{BD} = 523.1$ lb $\cong 523$ lb $\qquad \mathbf{T}_{BD} = -523\,\hat{i}$ lb $\qquad$ Ans.

For force equilibrium:

$$\Sigma \mathbf{F} = \bar{A} + \mathbf{T}_{BC} + \mathbf{T}_{BD} + \mathbf{W}$$

$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} - T_{BC}\,\hat{j} - T_{BD}\hat{i} - 850\,\hat{k}$$

$$= (A_x - 523.1)\,\hat{i} + (A_y - 1046.2)\,\hat{j} + (A_z - 850)\,\hat{k} = 0$$

From which:

$A_x = 523.1$ lb $\cong 523$ lb

$A_y = 1046.2$ lb $\cong 1046$ lb

$A_z = 850$ lb $\qquad \bar{A} = 523\,\hat{i} + 1046\,\hat{j} + 950\,\hat{k}$ lb $\qquad$ Ans.

6-80 Bar AC of Fig. P6-80 rests against a smooth surface at end C and is supported at end A with a ball-and-socket joint. The cable at B is attached midway between the ends of the bar. Determine the reactions at supports A and C and the tension in the cable at B.

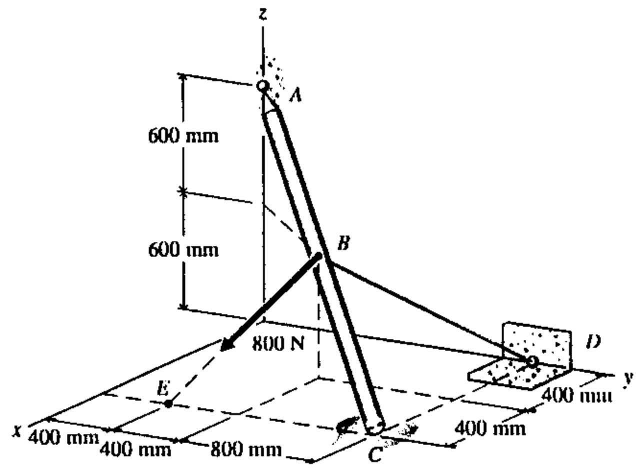


Fig. P6-80

SOLUTION

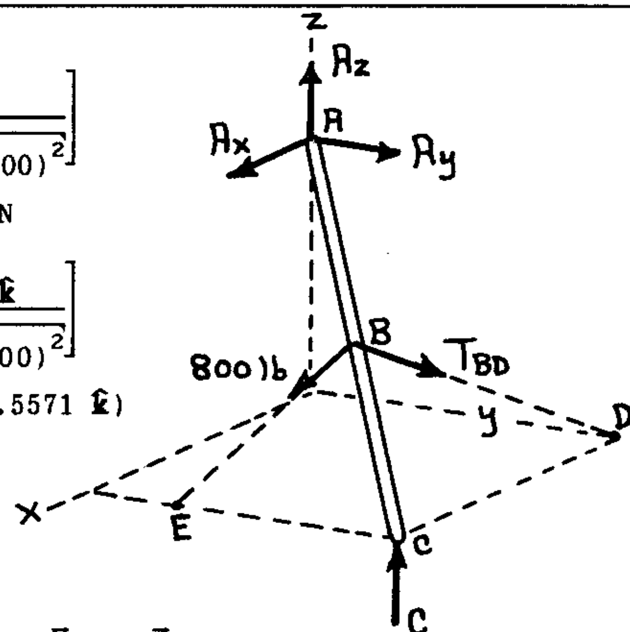
$$\mathbf{F}_{BE} = 800\left[\frac{400\,\hat{i} - 400\,\hat{j} - 600\,\hat{k}}{\sqrt{(400)^2 + (-400)^2 + (-600)^2}}\right]$$

$$= 388.1\,\hat{i} - 388.1\,\hat{j} - 582.1\,\hat{k}\ \text{N}$$

$$\mathbf{T}_{BD} = T_{BD}\left[\frac{-400\,\hat{i} + 800\,\hat{j} - 600\,\hat{k}}{\sqrt{(-400)^2 + (800)^2 + (-600)^2}}\right]$$

$$= T_{BD}(-0.3714\,\hat{i} + 0.7428\,\hat{j} - 0.5571\,\hat{k})$$

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma \mathbf{M}_A = (\vec{r}_{E/A} \times \mathbf{F}_{BE}) + (\vec{r}_{D/A} \times \mathbf{T}_{BD}) + (\vec{r}_{C/A} \times \vec{C})$$

$$= (0.800\,\hat{i} + 0.400\,\hat{j} - 1.200\,\hat{k}) \times (388.1\,\hat{i} - 388.1\,\hat{j} - 582.1\,\hat{k})$$

$$+ (1.600\,\hat{j} - 1.200\,\hat{k}) \times (-0.3714T_{BD}\,\hat{i} + 0.7428T_{BD}\,\hat{j} - 0.5571T_{BD}\,\hat{k})$$

$$+ (0.800\,\hat{i} + 1.600\,\hat{j} - 1.200\,\hat{k}) \times (C_z\,\hat{k})$$

$$= (1.600C_z - 698.56)\,\hat{i} + (0.4457T_{BD} - 0.800C_z)\,\hat{j}$$

$$+ (0.5942T_{BD} - 465.72)\,\hat{k} = 0$$

From which:

$C_z = 436.6\ \text{N} \cong 437\ \text{N}$ $\qquad \vec{C} = 436.6\,\hat{k}\ \text{N} \cong 437\,\hat{k}\ \text{N} \cong 437\ \text{N}\uparrow$ Ans.

$T_{BD} = 783.7\ \text{N} \cong 784\ \text{N}$ $\qquad \mathbf{T}_{BD} = -291.1\,\hat{i} + 582.2\,\hat{j} - 436.7\,\hat{k}\ \text{N}$

$\cong -291\,\hat{i} + 582\,\hat{j} - 437\,\hat{k}\ \text{N}$ Ans.

6-80 (Continued)

For force equilibrium:

$$\Sigma \vec{F} = \vec{A} + \vec{F}_{BE} + \vec{T}_{BD} + \vec{C}$$

$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} + 388.1\ \hat{i} - 388.1\ \hat{j} - 582.1\ \hat{k}$$

$$- 291.1\ \hat{i} + 582.2\ \hat{j} - 436.7\ \hat{k} + 436.6\ \hat{k} = 0$$

$$= (A_x + 97.0)\ \hat{i} + (A_y + 194.1)\ \hat{j} + (A_z\ )\ \hat{k}$$

From which:

$A_x = -97.0$ N

$A_y = -194.1$ N

$A_z = 582.2$ N $\qquad \vec{A} = -97.0\ \hat{i} - 194.1\ \hat{j} + 582\ \hat{k}$ N Ans.

6-81* A shaft is loaded through a pulley and a lever (see Fig. P6-81) that are fixed to the shaft. Friction between the belt and pulley prevents slipping of the belt. Determine the force $\bar{P}$ required for equilibrium and the reactions at supports A and B. The support at A is a ball bearing and the support at B is a thrust bearing. The bearings exert only force reactions on the shaft.

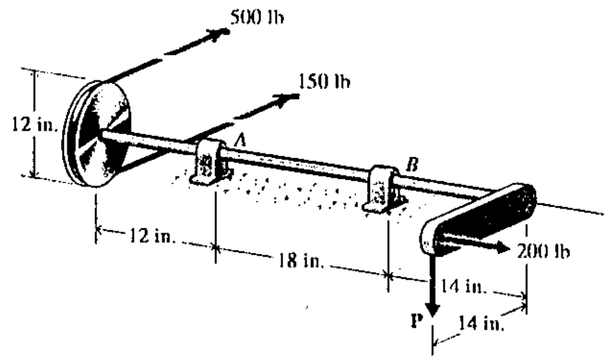
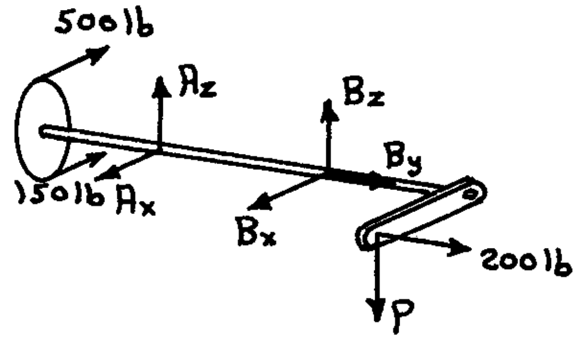


Fig. P6-81

SOLUTION

From a free-body diagram for the system:



$$\Sigma M_y = P(14) + 150(6) - 500(6) = 0$$

$\bar{P} = 150$ lb ↓ $= -150\,\hat{k}$ lb   Ans.

For moment equilibrium:

$$\Sigma \bar{M}_B = [(-18\,\hat{j}) \times (A_x\,\hat{i} + A_z\,\hat{k})] + [(-30\,\hat{j} + 6\,\hat{k}) \times (-500\,\hat{i})]$$
$$+ [(-30\,\hat{j} - 6\,\hat{k}) \times (-150\,\hat{i})] + [(14\,\hat{i} + 14\,\hat{j}) \times (200\,\hat{j} - 150\,\hat{k})]$$
$$= (-18A_z - 2100)\,\hat{i} + (18A_x - 16{,}700)\,\hat{k} = \bar{0}$$

$\bar{A} = 927.8\,\hat{i} - 116.67\,\hat{k}$ lb $\cong 928\,\hat{i} - 116.7\,\hat{k}$ lb   Ans.

$$A = \sqrt{(927.8)^2 + (-116.67)^2} = 935.1 \text{ lb} \cong 935 \text{ lb}$$

For force equilibrium:

$$\Sigma \bar{F} = 927.8\,\hat{i} - 116.67\,\hat{k} + B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$
$$- 500\,\hat{i} - 150\,\hat{i} + 200\,\hat{j} - 150\,\hat{k}$$
$$= (B_x + 277.8)\,\hat{i} + (B_y + 200)\,\hat{j} + (B_z - 266.67)\,\hat{k} = \bar{0}$$

$\bar{B} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k} = -277.8\,\hat{i} - 200\,\hat{j} + 266.67\,\hat{k}$ lb

$\cong -278\,\hat{i} - 200\,\hat{j} + 267\,\hat{k}$ lb   Ans.

$$B = \sqrt{(-277.8)^2 + (-200)^2 + (266.67)^2} = 433.9 \text{ lb} \cong 434 \text{ lb}$$

6-82* The shaft with two levers shown in Fig. P6-82 is used to change the direction of a force. Determine the force $\vec{P}$ required for equilibrium and the reactions at supports A and B. The support at A is a ball bearing and the support at B is a thrust bearing. The bearings exert only force reactions on the shaft.

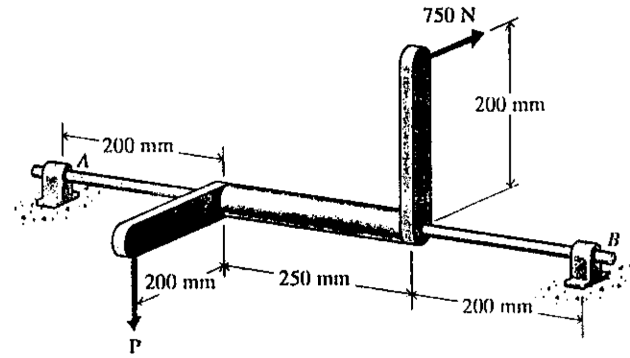
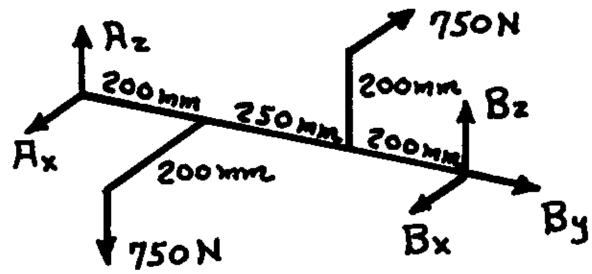


Fig. P6-82

SOLUTION

From a free-body diagram for the system:



$$\Sigma M_y = P(200) - 750(200) = 0$$

$$\vec{P} = 750 \text{ N} \downarrow = -750\ \hat{k} \text{ N} \qquad \text{Ans.}$$

For moment equilibrium:

$$\Sigma \vec{M}_B = [(-650\ \hat{j}) \times (A_x\ \hat{i} + A_z\ \hat{k})]$$

$$+ [(-200\ \hat{j} + 200\ \hat{k}) \times (-750\ \hat{i})] + [(200\ \hat{i} - 450\ \hat{j}) \times (-750\ \hat{k})]$$

$$= (-650A_z + 337{,}500)\ \hat{i} + (650A_x - 150{,}000)\ \hat{k} = \vec{0}$$

$$\vec{A} = 230.8\ \hat{i} + 0\ \hat{j} + 519.2\ \hat{k} \text{ N} \cong 231\ \hat{i} + 519\ \hat{k} \text{ N} \qquad \text{Ans.}$$

$$A = \sqrt{(230.8)^2 + (519.2)^2} = 568.2 \text{ N} \cong 568 \text{ N}$$

For force equilibrium:

$$\Sigma \vec{F} = 230.8\ \hat{i} + 519.2\ \hat{k} + B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k} - 750\ \hat{i} - 750\ \hat{k}$$

$$= (B_x - 519.2)\ \hat{i} + B_y\ \hat{j} + (B_z - 230.8)\ \hat{k} = \vec{0}$$

$$\vec{B} = B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k}$$

$$= 519.2\ \hat{i} + 0\ \hat{j} + 230.8\ \hat{k} \text{ N} \cong 519\ \hat{i} + 231\ \hat{k} \text{ N} \qquad \text{Ans.}$$

$$B = \sqrt{(519.2)^2 + (230.8)^2} = 568.2.0 \text{ N} \cong 568 \text{ N}$$

6-83 The plate shown in Fig. P6-83 is supported in a horizontal position by two hinges and a cable. The hinges have been properly aligned; therefore, they exert only force reactions on the plate. Assume that the hinge at B resists any force along the axis of the hinge pins. Determine the reactions at supports A and B and the tension in the cable.

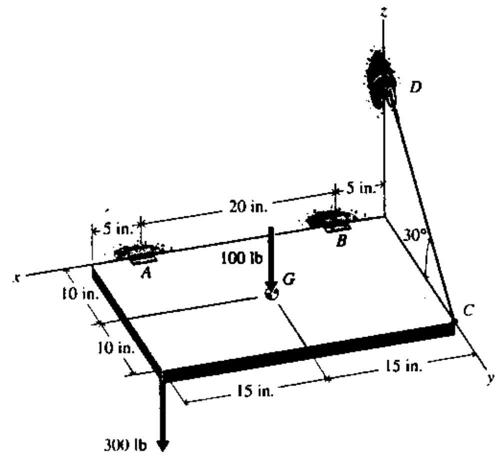
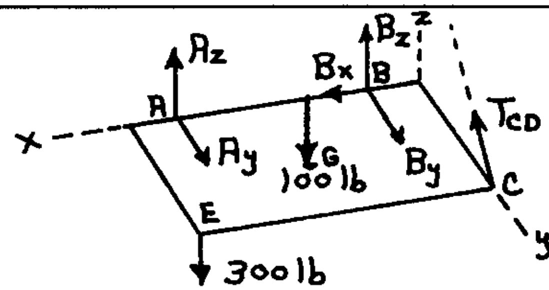


Fig. P6-83

SOLUTION

From a free-body diagram for the plate:



For moment equilibrium:

$$\Sigma \mathbf{M}_B = (\vec{r}_{A/B} \times \mathbf{A}) + (\vec{r}_{C/B} \times \mathbf{T}_{CD}) + (\vec{r}_{G/B} \times \mathbf{W}) + (\vec{r}_{E/B} \times \mathbf{P})$$

$$= (20\ \hat{i}) \times (A_y \hat{j} + A_z \hat{k}) + (-5\ \hat{i} + 20\ \hat{j}) \times (-0.8660 T_{CD}\ \hat{j} + 0.5000 T_{CD} \hat{k})$$
$$+ (10\ \hat{i} + 10\ \hat{j}) \times (-100\ \hat{k}) + (25\ \hat{i} + 20\ \hat{j}) \times (-300\ \hat{k})$$

$$= (10T_{CD} - 7000)\ \hat{i} + (2.500T_{CD} - 20A_z + 8500)\ \hat{j}$$
$$+ (4.330T_{CD} + 20A_y)\ \hat{k} = \vec{0}$$

From which:

$T_{CD} = 700$ lb $\qquad \mathbf{T}_{CD} = -606.2\ \hat{i} + 350.0\ \hat{j}$ lb

$A_z = 512.5$ lb $\cong 513$ lb $\qquad \cong -606\ \hat{i} + 350\ \hat{j}$ lb Ans.

$A_y = -151.55$ lb $\cong -151.6$ lb $\qquad \mathbf{A} = -151.6\ \hat{j} + 513\ \hat{k}$ lb Ans.

For force equilibrium:

$$\Sigma \mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{T}_{CD} - \mathbf{W} - \mathbf{P}$$
$$= A_y\ \hat{j} + A_z\ \hat{k} + B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k} - 606.2\ \hat{j} + 350.0\ \hat{k} - 100\ \hat{k} - 300\ \hat{k}$$
$$= B_x\ \hat{i} + (-151.55 + B_y - 606.2)\ \hat{j} + (512.5 + B_z - 50)\ \hat{k} = 0$$

From which: $\quad B_x = 0 \qquad B_y = 757.75 \cong 758$ lb

$B_z = -462.5 \cong -463$ lb $\qquad \mathbf{B} = 758\ \hat{j} - 463\ \hat{k}$ lb Ans.

6-84 The block W shown in Fig. P6-84 has a mass of 250 kg. Bar AB rests against a smooth vertical wall at end B and is supported at end A with a ball and socket joint. The two cables are attached to a point on the bar midway between the ends. Determine the reactions at supports A and B and the tensions in the two cables.

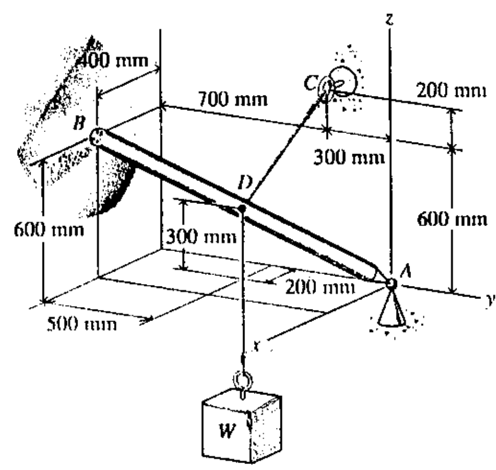


Fig. P6-84

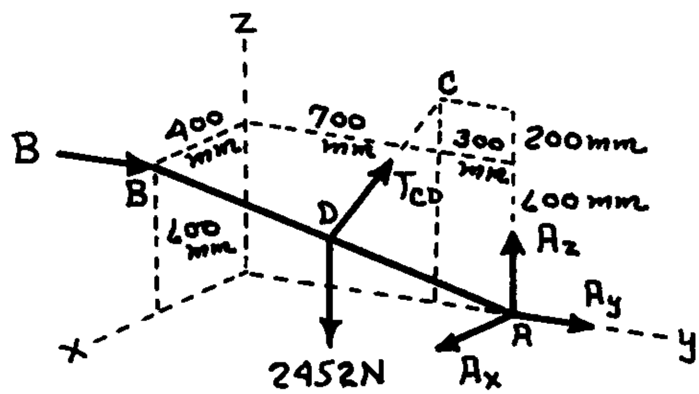
SOLUTION

$$\mathbf{T}_{CD} = T_{CD}\hat{\mathbf{e}}_{C/D} = T_{CD}\left[\frac{-0.200\ \hat{\mathbf{i}} + 0.200\ \hat{\mathbf{j}} + 0.500\ \hat{\mathbf{k}}}{\sqrt{(-0.200)^2 + (0.200)^2 + (0.500)^2}}\right]$$

$$= T_{CD}(-0.3482\ \hat{\mathbf{i}} + 0.3482\ \hat{\mathbf{j}} + 0.8704\ \hat{\mathbf{k}})$$

$$\mathbf{W} = m\mathbf{g} = 250(9.807)(-\hat{\mathbf{k}}) = -2452\ \hat{\mathbf{k}}\ \text{N}$$

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma\mathbf{M}_A = (\mathbf{r}_{C/A} \times \mathbf{T}_{CD}) + (\mathbf{r}_{B/A} \times \mathbf{B}) + (\mathbf{r}_{D/A} \times \mathbf{W})$$

$$= [(-0.300\ \hat{\mathbf{j}} + 0.800\ \hat{\mathbf{k}}) \times (-0.3482T_{CD}\ \hat{\mathbf{i}} + 0.3482T_{CD}\ \hat{\mathbf{j}} + 0.8704T_{CD}\ \hat{\mathbf{k}})]$$

$$+ [(0.400\ \hat{\mathbf{i}} - 1.000\ \hat{\mathbf{j}} + 0.600\ \hat{\mathbf{k}}) \times (B\ \hat{\mathbf{j}})]$$

$$+ [(0.200\ \hat{\mathbf{i}} - 0.500\ \hat{\mathbf{j}} + 0.300\ \hat{\mathbf{k}}) \times (-2452\ \hat{\mathbf{k}})]$$

$$= (-0.5397T_{CD} - 0.600B + 1226.0)\ \hat{\mathbf{i}} + (-0.2786T_{CD} + 490.4)\ \hat{\mathbf{j}}$$

$$+ (-0.10446T_{CD} + 0.400B)\ \hat{\mathbf{k}} = \bar{\mathbf{0}}$$

6-84 (Continued)

Solving yields:

$T_{CD} = 1760.2 \text{ N} \cong 1760 \text{ N}$

$\mathbf{T}_{CD} = 1760.2(-0.3482\,\hat{i} + 0.3482\,\hat{j} + 0.8704\,\hat{k})$

$= -612.9\,\hat{i} + 612.9\,\hat{j} + 1532.1\,\hat{k} \text{ N}$

$\cong -613\,\hat{i} + 613\,\hat{j} + 1532\,\hat{k} \text{ N}$ Ans.

$\bar{B} = 459.7\,\hat{j} \text{ N} \cong 460\,\hat{j} \text{ N}$ Ans.

For force equilibrium:

$\Sigma\mathbf{F} = \bar{A} + \bar{B} + \mathbf{T}_{CD} + \bar{W}$

$= A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k} + 459.7\,\hat{j} - 612.9\,\hat{i} + 612.9\,\hat{j} + 1532.1\,\hat{k} - 2452\,\hat{k}$

$= (A_x - 612.9)\,\hat{i} + (A_y + 1072.6)\,\hat{j} + (A_z - 919.9)\,\hat{k} = \bar{0}$

$\bar{A} = 612.9\,\hat{i} - 1072.6\,\hat{j} + 919.9\,\hat{k}$

$\cong 613\,\hat{i} - 1073\,\hat{j} + 920\,\hat{k} \text{ N}$ Ans.

$A = \sqrt{(613.1)^2 + (-1073.0)^2 + (920.3)^2} = 1540.8 \text{ N} \cong 1541 \text{ N}$

6-85* The plate shown in Fig. P6-85 weighs 200 lb and is supported in a horizontal position by a hinge and a cable. Determine the reactions at the hinge and the tension in the cable.

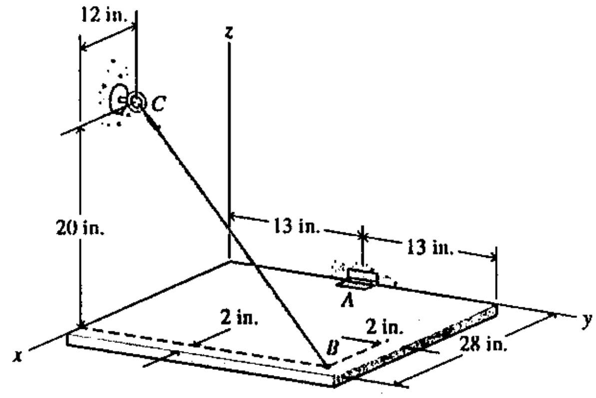


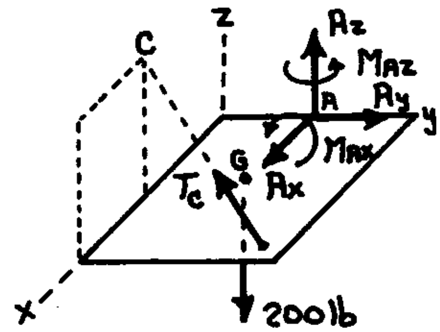
Fig. P6-85

SOLUTION

$$\mathbf{T}_C = T_C \hat{e}_{C/B}$$

$$= T_C \left[\frac{-12\,\hat{i} - 24\,\hat{j} + 20\,\hat{k}}{\sqrt{(-12)^2 + (-24)^2 + (20)^2}} \right]$$

$$= -0.3586T_C\,\hat{i} - 0.7171T_C\,\hat{j} + 0.5976T_C\,\hat{k}$$



From a free-body diagram for the plate:

$$\Sigma\mathbf{M}_A = \mathbf{C}_A + (\mathbf{r}_{G/A} \times \mathbf{W}) + (\mathbf{r}_{B/A} \times \mathbf{T}_C)$$

$$= M_{Ax}\,\hat{i} + M_{Az}\,\hat{k} + [(14\,\hat{i}) \times (-200\,\hat{k})]$$

$$+ [(26\,\hat{i} + 11\,\hat{j}) \times (-0.3586T_C\,\hat{i} - 0.7171T_C\,\hat{j} + 0.5976T_C\,\hat{k})]$$

$$= (M_{Ax} + 6.5736T_C)\,\hat{i} + (2800 - 15.5376T_C)\,\hat{j} + (M_{Az} - 14.7000T_C)\,\hat{k} = \mathbf{0}$$

Solving yields:

$$\mathbf{C}_A = M_{Ax}\,\hat{i} + M_{Az}\,\hat{k} = -1185\,\hat{i} + 2649\,\hat{k} \text{ in.·lb}$$ Ans.

$$T_C = 180.21 \text{ lb} \cong 180.2 \text{ lb}$$ Ans.

$$\mathbf{T}_C = 180.21(-0.3586\,\hat{i} - 0.7171\,\hat{j} + 0.5976\,\hat{k})$$

$$= -64.62\,\hat{i} - 129.22\,\hat{j} + 107.69\,\hat{k} \text{ lb}$$

$$\Sigma\mathbf{F} = \mathbf{A} + \mathbf{T}_C + \mathbf{W}$$

$$= A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k} - 64.62\,\hat{i} - 129.22\,\hat{j} + 107.69\,\hat{k} - 200\,\hat{k} = \mathbf{0}$$

$$= (A_x - 64.62)\,\hat{i} + (A_y - 129.22)\,\hat{j} + (A_z - 92.31)\,\hat{k} = \mathbf{0}$$

$$\mathbf{A} = 64.62\,\hat{i} + 129.22\,\hat{j} + 92.31\,\hat{k}$$

$$\cong 64.6\,\hat{i} + 129.2\,\hat{j} + 92.3\,\hat{k} \text{ lb}$$ Ans.

$$A = \sqrt{(64.62)^2 + (129.22)^2 + (92.31)^2} = 171.45 \text{ lb} \cong 171.5 \text{ lb}$$

6-86 Beam CD of Fig. P6-86 is supported at the left end C by a smooth pin and bracket and at the right end D by a continuous cable that passes around a frictionless pulley. The lines of action of the force in the cable pass through point D. Determine the components of the reaction at support C and the force in the cable when a 5-kN load W is being supported by the beam.

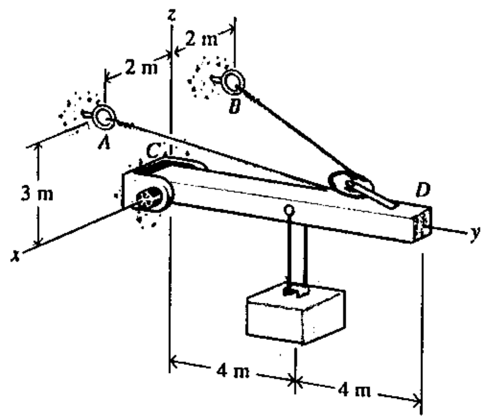
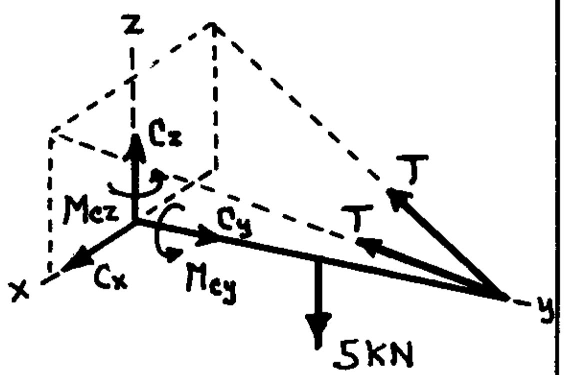


Fig. P6-86

SOLUTION



Since the cable is continuous:

$T_{AD} = T_{BD} = T$

$$\mathbf{T}_{AD} = T\hat{\mathbf{e}}_{A/D} = T\left[\frac{2\ \hat{\imath} - 8\ \hat{\jmath} + 3\ \hat{\mathbf{k}}}{\sqrt{(2)^2 + (-8)^2 + (3)^2}}\right]$$

$$= 0.2279T\ \hat{\imath} - 0.9117T\ \hat{\jmath} + 0.3419T\ \hat{\mathbf{k}}$$

$$\mathbf{T}_{BD} = T\hat{\mathbf{e}}_{B/D} = T\left[\frac{-2\ \hat{\imath} - 8\ \hat{\jmath} + 3\ \hat{\mathbf{k}}}{\sqrt{(-2)^2 + (-8)^2 + (3)^2}}\right]$$

$$= -0.2279T\ \hat{\imath} - 0.9117T\ \hat{\jmath} + 0.3419T\ \hat{\mathbf{k}}$$

From a free-body diagram for the beam:

For moment equilibrium:

$$\Sigma\mathbf{M}_C = \mathbf{C}_C + [(\bar{\mathbf{r}}_{D/C}) \times (\mathbf{T}_{AD} + T_{BD}) + (\bar{\mathbf{r}}_{E/C} \times \mathbf{W})$$

$$= M_{Cy}\ \hat{\jmath} + M_{Cz}\ \hat{\mathbf{k}} + [(8\ \hat{\jmath}) \times (-1.8234T\ \hat{\jmath} + 0.6838T\ \hat{\mathbf{k}})]$$

$$+ [(4\ \hat{\jmath}) \times (-5\ \hat{\mathbf{k}})]$$

$$= (5.4704T - 20)\ \hat{\imath} + (M_{Cy})\ \hat{\jmath} + (M_{Cz})\ \hat{\mathbf{k}} = \bar{\mathbf{0}}$$

Solving yields:

$$\mathbf{C}_C = M_{Cy}\ \hat{\jmath} + M_{Cz}\ \hat{\mathbf{k}} = \bar{\mathbf{0}}$$ Ans.

$$T = 3.656\ \text{kN} \cong 3.66\ \text{kN}$$ Ans.

6-86 (Continued)

$$\mathbf{T}_{AD} = 3.656(0.2279\,\hat{\imath} - 0.9117\,\hat{\jmath} + 0.3419\,\hat{k})$$
$$= 0.8332\,\hat{\imath} - 3.3332\,\hat{\jmath} + 1.2500\,\hat{k}\ \text{kN}$$

$$\mathbf{T}_{BD} = 3.656(-0.2279\,\hat{\imath} - 0.9117\,\hat{\jmath} + 0.3419\,\hat{k})$$
$$= -0.8332\,\hat{\imath} - 3.3332\,\hat{\jmath} + 1.2500\,\hat{k}\ \text{kN}$$

For force equilibrium:

$$\Sigma\mathbf{F} = \mathbf{R}_C + \mathbf{T}_{AD} + \mathbf{T}_{BD} + \mathbf{W}$$
$$= R_{Cx}\,\hat{\imath} + R_{Cy}\,\hat{\jmath} + R_{Cz}\,\hat{k} + 0.8332\,\hat{\imath} - 3.3332\,\hat{\jmath} + 1.2500\,\hat{k}$$
$$- 0.8332\,\hat{\imath} - 3.3332\,\hat{\jmath} + 1.2500\,\hat{k} - 2.5\,\hat{k}$$
$$= (R_{Cx})\,\hat{\imath} + (R_{Cy} - 6.6664)\,\hat{\jmath} + (R_{Cz} - 2.500)\,\hat{k} = \mathbf{0}$$

$$\mathbf{R}_C = R_{Cx}\,\hat{\imath} + R_{Cy}\,\hat{\jmath} + R_{Cz}\,\hat{k}$$
$$= 6.666\,\hat{\jmath} + 2.500\,\hat{k}\ \text{kN}$$
$$\cong 6.67\,\hat{\jmath} + 2.50\,\hat{k}\ \text{kN}$$ Ans.

$$R_C = \sqrt{(6.666)^2 + (2.500)^2} = 7.119\ \text{kN} \cong 7.12\ \text{kN}$$

6-87* The plate shown in Fig. P6-87 weighs 150 lb and is supported in a horizontal position by two hinges and a cable. The hinges have been properly aligned; therefore, they exert only force reactions on the plate. Assume that the hinge at B resists any force along the axis of the hinge pins. Determine the reactions at supports A and B and the tension in the cable.

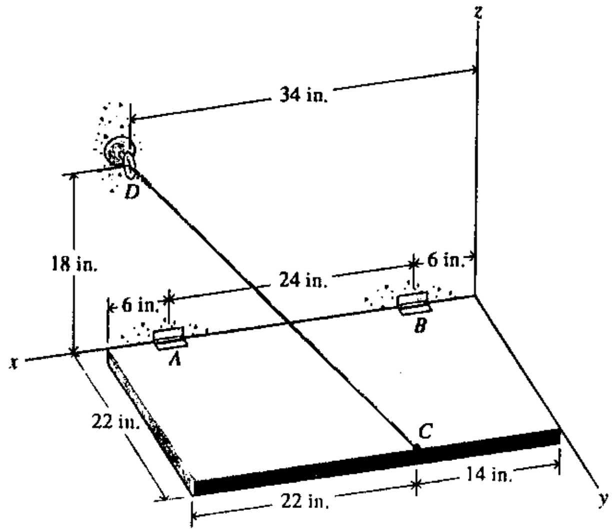


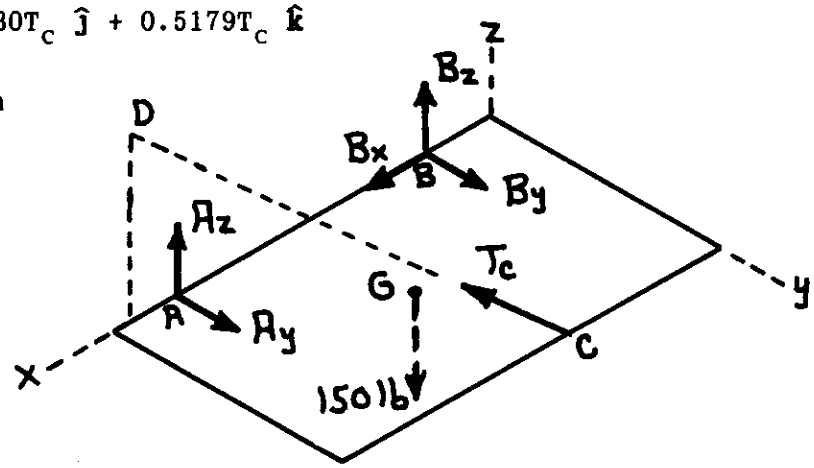
Fig. P6-87

SOLUTION

$$\vec{T}_C = T_C\hat{e}_{D/C} = T_C\left[\frac{20\ \hat{i} - 22\ \hat{j} + 18\ \hat{k}}{\sqrt{(20)^2 + (-22)^2 + (18)^2}}\right]$$

$$= 0.5754T_C\ \hat{i} - 0.6330T_C\ \hat{j} + 0.5179T_C\ \hat{k}$$

From a free-body diagram for the plate:



For moment equilibrium:

$$\Sigma\vec{M}_B = (\vec{r}_{C/B} \times \vec{T}_C) + (\vec{r}_{A/B} \times \vec{A}) + (\vec{r}_{G/B} \times \vec{W})$$

$$= [(8\ \hat{i} + 22\ \hat{j}) \times (0.5754T_C\ \hat{i} - 0.6330T_C\ \hat{j} + 0.5179T_C\ \hat{k})]$$

$$+ [(24\ \hat{i}) \times (A_y\ \hat{j} + A_z\ \hat{k})] + [(12\ \hat{i} + 11\ \hat{j}) \times (-150\ \hat{k})]$$

$$= (11.3938T_C - 1650)\ \hat{i} + (-4.1432T_C - 24A_z + 1800)\ \hat{j}$$

$$+ (-17.7228T_C + 24A_y)\ \hat{k} = \vec{0}$$

6-87 (Continued)

Solving yields:

$T_C = 144.82$ lb $\cong 144.8$ lb Ans.

$\mathbf{T}_C = 144.82(0.5754\,\hat{i} - 0.6330\,\hat{j} + 0.5179\,\hat{k})$

$= 83.33\,\hat{i} - 91.67\,\hat{j} + 75.00\,\hat{k}$ lb

$A_y = 106.98$ lb

$A_z = 50.00$ lb

$\bar{\mathbf{A}} = 106.98\,\hat{j} + 50.00\,\hat{k}$ lb $\cong 107.0\,\hat{j} + 50.0\,\hat{k}$ lb Ans.

$A = \sqrt{(106.98)^2 + (50.00)^2} = 118.09$ lb $\cong 118.1$ lb

For force equilibrium:

$$\Sigma\bar{\mathbf{F}} = \bar{\mathbf{A}} + \bar{\mathbf{B}} + \mathbf{T}_C + \bar{\mathbf{W}} = 106.98\,\hat{j} + 50.00\,\hat{k} + B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$$

$$+ 83.33\,\hat{i} - 91.67\,\hat{j} + 75.00\,\hat{k} - 150\,\hat{k}$$

$$= (B_x + 83.33)\,\hat{i} + (B_y + 106.98 - 91.67)\,\hat{j}$$

$$+ (B_z + 50.00 + 75.00 - 150.00)\,\hat{k} = \bar{0}$$

$\bar{\mathbf{B}} = B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k}$

$= -83.33\,\hat{i} - 15.31\,\hat{j} + 25.00\,\hat{k}$ lb

$\cong -83.3\,\hat{i} - 15.31\,\hat{j} + 25.0\,\hat{k}$ lb Ans.

$B = \sqrt{(-83.33)^2 + (-15.31)^2 + (25.00)^2} = 88.34$ lb $\cong 88.3$ lb

6-88* A bar is supported by a ball-and-socket joint, link, and cable as shown in Fig. P6-88. Determine the reactions at supports A (ball-and-socket joint) and B (link) and the tension in the cable.

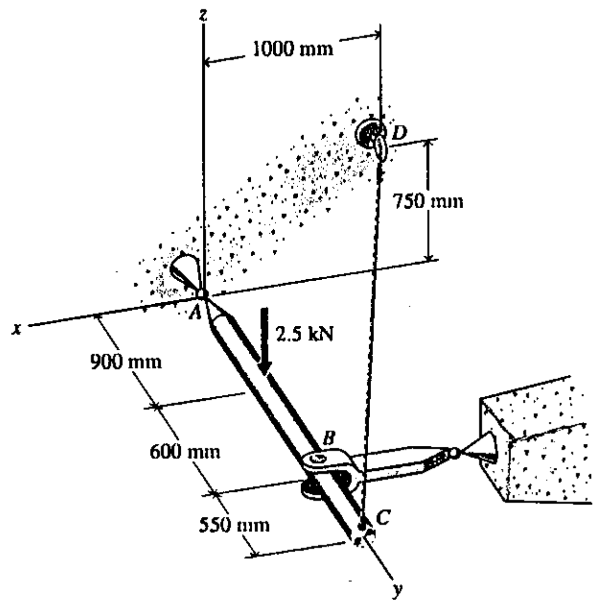


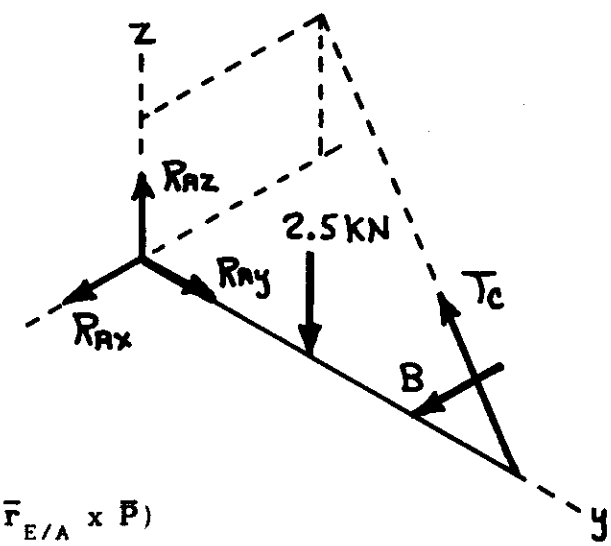
Fig. P6-88

SOLUTION

$$T_C = T_C \hat{e}_{D/C} = T_C\left[\frac{-1.000\ \hat{i} - 2.050\ \hat{j} + 0.750\ \hat{k}}{\sqrt{(-1.000)^2 + (-2.050)^2 + (0.750)^2}}\right]$$

$$= -0.4165T_C\ \hat{i} - 0.8538T_C\ \hat{j} + 0.3124T_C\ \hat{k}$$

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma\vec{M}_A = (\vec{r}_{C/A} \times \vec{T}_C) + (\vec{r}_{B/A} \times \vec{B}) + (\vec{r}_{E/A} \times \vec{P})$$

$$= [(2.050\ \hat{j}) \times (-0.4165T_C\ \hat{i} - 0.8538T_C\ \hat{j} + 0.3124T_C\ \hat{k})]$$

$$+ [(1.500\ \hat{j}) \times (B\ \hat{i})] + [(0.900\ \hat{j}) \times (-2.50\ \hat{k})]$$

$$= (0.6404T_C - 2.250)\ \hat{i} + (0.8538T_C - 1.500B)\ \hat{k} = \vec{0}$$

6-88 (Continued)

Solving yields:

$B = 2.000 \text{ kN} = 2.00 \text{ kN}$ $\quad\quad$ $\bar{B} = 2.00\ \hat{i}\ \text{kN}$ Ans.

$T_C = 3.513 \text{ kN} \cong 3.51 \text{ kN}$

$\bar{T}_C = 3.513(-0.4165\ \hat{i} - 0.8538\ \hat{j} + 0.3124\ \hat{k})$

$= -1.4632\ \hat{i} - 3.000\ \hat{j} + 1.0975\ \hat{k}\ \text{kN}$

$\cong -1.463\ \hat{i} - 3.00\ \hat{j} + 1.098\ \hat{k}\ \text{kN}$ Ans.

For force equilibrium:

$\Sigma\bar{F} = \bar{R}_A + \bar{T}_C + \bar{B} + \bar{P}$

$= (R_{Ax} - 1.4632 + 2.000)\ \hat{i} + (R_{Ay} - 3.000)\ \hat{j}$

$+ (R_{Az} + 1.0975 - 2.50)\ \hat{k} = \bar{0}$

$\bar{R}_A = R_{Ax}\ \hat{i} + R_{Ay}\ \hat{j} + R_{Az}\ \hat{k}$

$= -0.5368\ \hat{i} + 3.000\ \hat{j} + 1.4025\ \hat{k}\ \text{kN}$

$\cong -0.537\ \hat{i} + 3.00\ \hat{j} + 1.403\ \hat{k}\ \text{kN}$ Ans.

$R_A = \sqrt{(-0.5368)^2 + (3.000)^2 + (1.4025)^2} = 3.355 \text{ kN} \cong 3.36 \text{ kN}$

6-89 A beam is supported by a ball-and-socket joint and two cables as shown in Fig. P6-89. Determine the reaction at support A (the ball-and-socket joint) and the tensions in the two cables.

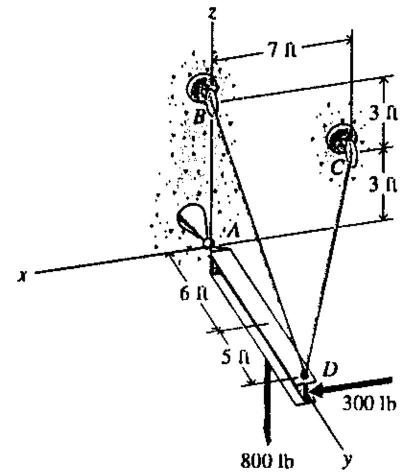


Fig. P6-89

SOLUTION

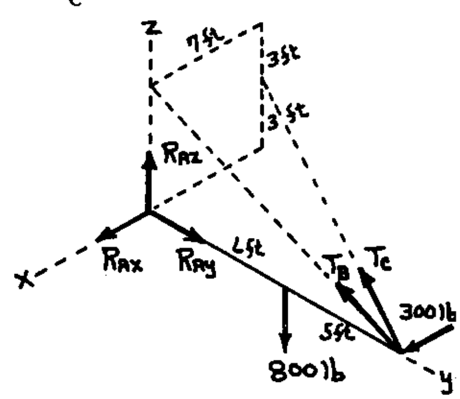
$$\mathbf{T}_B = T_B\,\hat{\mathbf{e}}_{B/D} = T_B\left[\frac{0\,\hat{\imath} - 11\,\hat{\jmath} + 6\,\hat{k}}{\sqrt{0^2 + (-11)^2 + (6)^2}}\right]$$

$$= -0.8779T_B\,\hat{\jmath} + 0.4789T_B\,\hat{k}$$

$$\mathbf{T}_C = T_C\,\hat{\mathbf{e}}_{C/D} = T_C\left[\frac{-7\,\hat{\imath} - 11\,\hat{\jmath} + 3\,\hat{k}}{\sqrt{(-7)^2 + (-11)^2 + (3)^2}}\right]$$

$$= -0.5232T_C\,\hat{\imath} - 0.8222T_C\,\hat{\jmath} + 0.2242T_C\,\hat{k}$$

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma\overline{\mathbf{M}}_A = (\overline{\mathbf{r}}_{D/A} \times \mathbf{T}_B) + (\overline{\mathbf{r}}_{D/A} \times \mathbf{T}_C) + (\overline{\mathbf{r}}_{D/A} \times \overline{\mathbf{P}}_1) + (\overline{\mathbf{r}}_{E/A} \times \overline{\mathbf{P}}_2)$$

$$= [(11\,\hat{\jmath}) \times (-0.8779T_B\,\hat{\jmath} + 0.4789T_B\,\hat{k})]$$

$$+ [(11\,\hat{\jmath}) \times (-0.5232T_C\,\hat{\imath} - 0.8222T_C\,\hat{\jmath} + 0.2242T_C\,\hat{k})]$$

$$+ [(11\,\hat{\jmath}) \times (300\,\hat{\imath})] + [(6\,\hat{\jmath}) \times (-800\,\hat{k})]$$

$$= (5.2679T_B + 2.4662T_C - 4800)\,\hat{\imath} + (5.7552T_C - 3300)\,\hat{k} = \overline{0}$$

6-89 (Continued)

Solving yields:

$T_B = 642.7 \text{ lb} \cong 643 \text{ lb}$ Ans.

$T_C = 573.4 \text{ lb} \cong 573 \text{ lb}$ Ans.

$$\mathbf{T}_B = 642.7(-0.8779\,\hat{j} + 0.4789\,\hat{k})$$
$$= -564.2\,\hat{j} + 307.8\,\hat{k} \text{ lb}$$

$$\mathbf{T}_C = 573.4(-0.5232\,\hat{i} - 0.8222\,\hat{j} + 0.2242\,\hat{k})$$
$$= -300.0\,\hat{i} - 471.4\,\hat{j} + 128.6\,\hat{k} \text{ lb}$$

For force equilibrium:

$$\Sigma\mathbf{F} = \mathbf{R}_A + \mathbf{T}_B + \mathbf{T}_C + \mathbf{P}_1 + \mathbf{P}_2$$
$$= (R_{Ax} - 300.0 + 300.0)\,\hat{i}$$
$$+ (R_{Ay} - 564.2 - 471.4)\,\hat{j}$$
$$+ (R_{Az} + 307.8 + 128.6 - 800)\,\hat{k} = \mathbf{0}$$

$$\mathbf{R}_A = R_{Ax}\,\hat{i} + R_{Ay}\,\hat{j} + R_{Az}\,\hat{k}$$
$$= 1035.6\,\hat{j} + 363.6\,\hat{k} \text{ lb}$$
$$\cong 1036\,\hat{j} + 364\,\hat{k} \text{ lb}$$ Ans.

$$R_A = \sqrt{(1035.6)^2 + (363.6)^2} = 1097.6 \text{ lb} \cong 1098 \text{ lb}$$

6-90 The plate shown in Fig. P6-90 has a mass of 75 kg. The brackets at supports A and B exert only force reactions on the plate. Each of the brackets can resist a force along the axis of pins in one direction only. Determine the reactions at supports A and B and the tension in the cable.

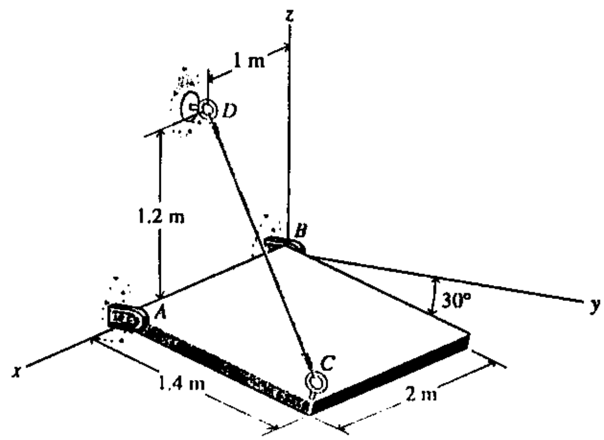


Fig. P6-90

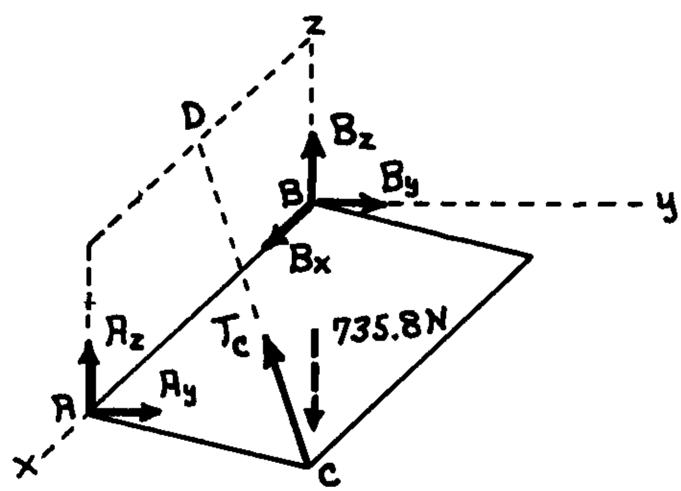
SOLUTION

$$\vec{T}_C = T_C\hat{e}_{D/C} = T_C\left[\frac{-1.00\ \hat{i} - 1.40\cos 30^\circ\ \hat{j} + (1.20 + 1.40\sin 30^\circ)\ \hat{k}}{\sqrt{(-1.00)^2 + (-1.40\cos 30^\circ)^2 + (1.20 + 1.40\sin 30^\circ)^2}}\right]$$

$$= -0.4056T_C\ \hat{i} - 0.4917T_C\ \hat{j} + 0.7706T_C\ \hat{k}$$

$$\vec{W} = m\vec{g} = 75(9.81)(-\hat{k}) = -735.75\ \hat{k}\ \text{N} \cong -735.8\ \hat{k}\ \text{N}$$

From a free-body diagram for the plate:



For moment equilibrium:

$$\Sigma\vec{M}_B = (\vec{r}_{C/B} \times \vec{T}_C) + (\vec{r}_{A/B} \times \vec{A}) + (\vec{r}_{G/B} \times \vec{W})$$

$$= [(1\ \hat{i} + 1.2\ \hat{k}) \times (-0.4056T_C\ \hat{i} - 0.4917T_C\ \hat{j} + 0.7706T_C\ \hat{k})]$$

$$+ [(2\ \hat{i}) \times (A_y\ \hat{j} + A_z\ \hat{k})]$$

$$+ [(1\ \hat{i} + 0.70\cos 30^\circ\ \hat{j} - 0.70\sin 30^\circ\ \hat{k}) \times (-735.8\ \hat{k})]$$

$$= (0.5900T_C - 446.0)\ \hat{i} + (-1.2573T_C - 2A_z + 735.8)\ \hat{j}$$

$$+ (-0.4917T_C + 2A_y)\ \hat{k} = \vec{0}$$

6-90 (Continued)

Solving yields:

$T_C = 755.9 \text{ N} \cong 756 \text{ N}$ Ans.

$\mathbf{T}_C = 755.9(-0.4056\ \hat{\imath} - 0.4917\ \hat{\jmath} + 0.7706\ \hat{k})$

$= -306.6\ \hat{\imath} - 371.7\ \hat{\jmath} + 582.5\ \hat{k} \text{ N}$

$A_y = 185.84 \text{ N}$

$A_z = -107.30 \text{ N}$

$\bar{A} = 185.84\ \hat{\jmath} - 107.30\ \hat{k} \text{ N} \cong 185.8\ \hat{\jmath} - 107.3\ \hat{k} \text{ N}$ Ans.

$A = \sqrt{(185.84)^2 + (-107.30)^2} = 214.59 \text{ N} \cong 215 \text{ N}$

For force equilibrium:

$\Sigma\bar{F} = \bar{A} + \bar{B} + \bar{T}_C + \bar{W} = 185.84\ \hat{\jmath} - 107.30\ \hat{k} + B_x\ \hat{\imath} + B_y\ \hat{\jmath} + B_z\ \hat{k}$

$- 306.6\ \hat{\imath} - 371.7\ \hat{\jmath} + 582.5\ \hat{k} - 735.8\ \hat{k}$

$= (B_x - 306.6)\ \hat{\imath} + (B_y + 185.84 - 371.7)\ \hat{\jmath}$

$+ (B_z - 107.30 + 582.5 - 735.8)\ \hat{k} = \bar{0}$

$\bar{B} = B_x\ \hat{\imath} + B_y\ \hat{\jmath} + B_z\ \hat{k}$

$= 306.6\ \hat{\imath} + 185.86\ \hat{\jmath} + 260.6\ \hat{k} \text{ N}$

$\cong 307\ \hat{\imath} + 185.9\ \hat{\jmath} + 261\ \hat{k} \text{ N}$ Ans.

$B = \sqrt{(306.6)^2 + (185.86)^2 + (260.6)^2} = 443.2 \text{ N} \cong 443 \text{ N}$

6-91* A bar is supported by a ball-and-socket joint and two cables as shown in Fig. P6-91. Determine the reaction at support A (the ball-and-socket joint) and the tensions in the two cables.

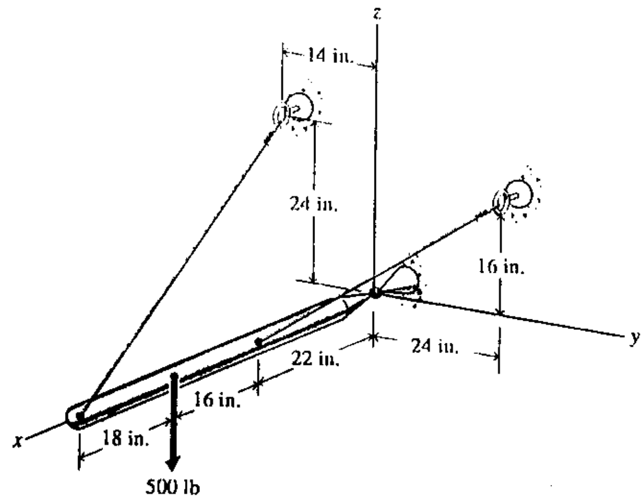


Fig. P6-91

SOLUTION

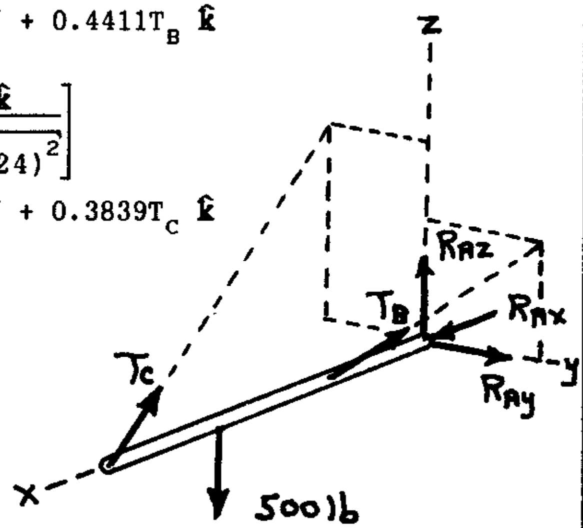
$$\mathbf{T}_B = T_B\,\hat{\mathbf{e}}_B = T_B\left[\frac{-22\,\hat{\mathbf{i}} + 24\,\hat{\mathbf{j}} + 16\,\hat{\mathbf{k}}}{\sqrt{(-22)^2 + (24)^2 + (16)^2}}\right]$$

$$= -0.6064T_B\,\hat{\mathbf{i}} + 0.6616T_B\,\hat{\mathbf{j}} + 0.4411T_B\,\hat{\mathbf{k}}$$

$$\mathbf{T}_C = T_C\,\hat{\mathbf{e}}_C = T_C\left[\frac{-56\,\hat{\mathbf{i}} - 14\,\hat{\mathbf{j}} + 24\,\hat{\mathbf{k}}}{\sqrt{(-56)^2 + (-14)^2 + (24)^2}}\right]$$

$$= -0.8958T_C\,\hat{\mathbf{i}} - 0.2239T_C\,\hat{\mathbf{j}} + 0.3839T_C\,\hat{\mathbf{k}}$$

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma\mathbf{M}_A = (\vec{\mathbf{r}}_{B/A} \times \mathbf{T}_B) + (\vec{\mathbf{r}}_{C/A} \times \mathbf{T}_C) + (\vec{\mathbf{r}}_{E/A} \times \mathbf{P})$$

$$= [(24\,\hat{\mathbf{j}} + 16\,\hat{\mathbf{k}}) \times (-0.6064T_B\,\hat{\mathbf{i}} + 0.6616T_B\,\hat{\mathbf{j}} + 0.4411T_B\,\hat{\mathbf{k}})]$$

$$+ [(-14\,\hat{\mathbf{j}} + 24\,\hat{\mathbf{k}}) \times (-0.8958T_C\,\hat{\mathbf{i}} - 0.2239T_C\,\hat{\mathbf{j}} + 0.3839T_C\,\hat{\mathbf{k}})]$$

$$+ [(38\,\hat{\mathbf{i}}) \times (-500\,\hat{\mathbf{k}})]$$

$$= (-9.7024T_B - 21.4992T_C + 19{,}000)\,\hat{\mathbf{j}} + (14.5536T_B - 12.5412T_C)\,\hat{\mathbf{k}} = \vec{\mathbf{0}}$$

6-91 (Continued)

Solving yields:

$$T_B = 548.31 \text{ lb} \cong 548 \text{ lb} \qquad \text{Ans.}$$

$$T_C = 636.31 \text{ lb} \cong 636 \text{ lb} \qquad \text{Ans.}$$

$$\mathbf{T}_B = 548.31(-0.6064\,\hat{i} + 0.6616\,\hat{j} + 0.4411\,\hat{k})$$

$$= -332.49\,\hat{i} + 362.76\,\hat{j} + 241.86\,\hat{k} \text{ lb}$$

$$\mathbf{T}_C = 636.31(-0.8958\,\hat{i} - 0.2239\,\hat{j} + 0.3839\,\hat{k})$$

$$= -570.00\,\hat{i} - 142.47\,\hat{j} + 244.28\,\hat{k} \text{ lb}$$

For force equilibrium:

$$\Sigma\mathbf{F} = \mathbf{R}_A + \mathbf{T}_B + \mathbf{T}_C + \mathbf{P} = (R_{Ax} - 332.49 - 570.00)\,\hat{i}$$

$$+ (R_{Ay} + 362.76 - 142.47)\,\hat{j}$$

$$+ (R_{Az} + 241.86 + 244.28 - 500)\,\hat{k} = \mathbf{0}$$

$$\mathbf{R}_A = R_{Ax}\,\hat{i} + R_{Ay}\,\hat{j} + R_{Az}\,\hat{k}$$

$$= 902.49\,\hat{i} - 220.29\,\hat{j} + 13.86\,\hat{k} \text{ lb}$$

$$\cong 902\,\hat{i} - 220\,\hat{j} + 13.86\,\hat{k} \text{ lb} \qquad \text{Ans.}$$

$$R_A = \sqrt{(902.49)^2 + (-220.29)^2 + (13.86)^2} = 929.09 \text{ lb} \cong 929 \text{ lb}$$

6-92 The plate shown in Fig. P6-92 has a mass of 100 kg. The hinges at supports A and B exert only force reactions on the plate. Assume that the hinge at B resists any force along the axis of the hinge pins. Determine the reactions at supports A and B and the tension in the cable.

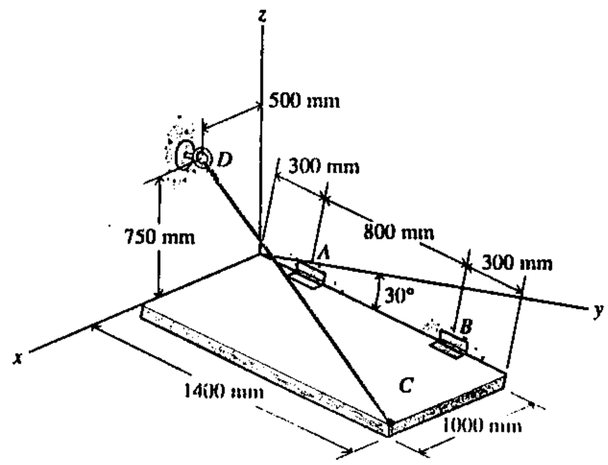


Fig. P6-92

SOLUTION

$$\hat{e}_{D/C} = \frac{-0.500\ \hat{i} - 1.400 \cos 30^\circ\ \hat{j} + (0.750 + 1.400 \sin 30^\circ)\ \hat{k}}{\sqrt{(-0.500)^2 + (-1.400 \cos 30^\circ)^2 + (0.750 + 1.400 \sin 30^\circ)^2}}$$

$$= -0.2557\ \hat{i} - 0.6201\ \hat{j} + 0.7417\ \hat{k}$$

$$\mathbf{T}_C = T_C \hat{e}_{D/C} = -0.2557 T_C\ \hat{i} - 0.6201 T_C\ \hat{j} + 0.7417 T_C\ \hat{k}$$

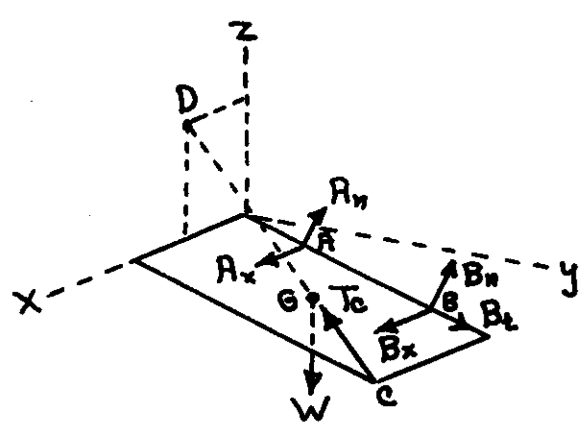
$$\mathbf{W} = m\vec{g} = 100(9.807)(-\hat{k}) = -980.7\ \hat{k}\ \text{N}$$

$$\vec{r}_{C/B} = 1.000\ \hat{i} + 0.300 \cos 30^\circ\ \hat{j} - 0.300 \sin 30^\circ\ \hat{k}$$
$$= 1.000\ \hat{i} + 0.2598\ \hat{j} - 0.1500\ \hat{k}$$

$$\vec{r}_{A/B} = -0.800 \cos 30^\circ\ \hat{j} + 0.800 \sin 30^\circ\ \hat{k}$$
$$= -0.6928\ \hat{j} + 0.400\ \hat{k}$$

$$\vec{r}_{G/B} = 0.500\ \hat{i} - 0.400 \cos 30^\circ\ \hat{j} + 0.400 \sin 30^\circ\ \hat{k}$$
$$= 0.500\ \hat{i} - 0.3464\ \hat{j} + 0.200\ \hat{k}$$

From a free-body diagram for the plate:



6-92 (Continued)

For moment equilibrium:

$$\Sigma \mathbf{M}_B = (\vec{r}_{C/B} \times \mathbf{T}_C) + (\vec{r}_{A/B} \times \mathbf{A}) + (\vec{r}_{G/B} \times \mathbf{W})$$

$$= [(1.000\ \hat{i} + 0.2598\ \hat{j} - 0.1500\ \hat{k}) \times (-0.2557T_C\ \hat{i} - 0.6201T_C\ \hat{j} + 0.7417T_C\ \hat{k})]$$

$$+ [(-0.6928\ \hat{j} + 0.400\ \hat{k}) \times (A_x\ \hat{i} + A_n \sin 30^\circ\ \hat{j} + A_n \cos 30^\circ\ \hat{k})]$$

$$+ [(0.500\ \hat{i} - 0.3464\ \hat{j} + 0.200\ \hat{k}) \times (-980.7\ \hat{k})]$$

$$= (-0.8000A_n + 0.0997T_C + 339.7)\ \hat{i} + (-0.7033T_C + 0.4000A_x + 490.4)\ \hat{j} + (-0.5537T_C + 0.6928A_x)\ \hat{k} = \vec{0}$$

Solving yields:

$$T_C = 1278.35 \text{ N} \cong 1278 \text{ N}$$ Ans.

$$\mathbf{T}_C = 1278.35(-0.2557\ \hat{i} - 0.6201\ \hat{j} + 0.7417\ \hat{k}) = -326.87\ \hat{i} - 792.70\ \hat{j} + 948.15\ \hat{k} \text{ N}$$

$A_x = 1021.68$ N $A_n = 583.94$ N

$$\mathbf{A} = A_x\ \hat{i} + A_y\ \hat{j} + A_z\ \hat{k} = A_x\ \hat{i} + A_n \sin 30^\circ\ \hat{j} + A_n \cos 30^\circ\ \hat{k} \text{ N}$$

$$= 1021.68\ \hat{i} + 291.97\ \hat{j} + 505.71\ \hat{k} \text{ N}$$

$$\cong 1022\ \hat{i} + 292\ \hat{j} + 506\ \hat{k} \text{ N}$$ Ans.

$$A = \sqrt{(1021.68)^2 + (291.97)^2 + (505.71)^2} = 1176.78 \text{ N} \cong 1177 \text{ N}$$

For force equilibrium:

$$\Sigma \mathbf{F} = \mathbf{A} + \mathbf{B} + \mathbf{T}_c + \mathbf{W}$$

$$= 1021.68\ \hat{i} + 291.97\ \hat{j} + 505.71\ \hat{k} + B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k}$$

$$- 326.87\ \hat{i} - 792.70\ \hat{j} + 948.15\ \hat{k} - 980.7\ \hat{k}$$

$$= (B_x + 694.81)\ \hat{i} + (B_y - 500.73)\ \hat{j} + (B_z + 473.16)\ \hat{k} = \vec{0}$$

$$\mathbf{B} = B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k} = -694.81\ \hat{i} + 500.73\ \hat{j} - 473.16\ \hat{k} \text{ N}$$

$$\cong -695\ \hat{i} + 501\ \hat{j} - 473\ \hat{k} \text{ N}$$ Ans.

$$B = \sqrt{(-694.81)^2 + (500.73)^2 + (-473.16)^2} = 978.45 \text{ N} \cong 978 \text{ N}$$

For Hinge B:

$$B_n = B_y \sin 30^\circ + B_z \cos 30^\circ = 500.73 \sin 30^\circ - 473.16 \cos 30^\circ = -159.40 \text{ N}$$

$$B_t = B_y \cos 30^\circ - B_z \sin 30^\circ = 500.73 \cos 30^\circ + 473.16 \sin 30^\circ = 670.22 \text{ N}$$

C6-93 An I-beam is supported by a ball-and-socket joint at A, by a rope BC, and by a horizontal force P as shown in Fig. P6-93. In addition, a 560-lb load is suspended from a movable support at D. If the uniform beam is 80 in. long and weights 225 lb
(a) Plot the force P required to keep the beam aligned with the y-axis as a function of the distance d ($0 \le d \le 80$ in.).
(b) On the same graph plot T_{BC}, the tension in the rope, and A, the magnitude of the force exerted on the ball-and-socket joint.

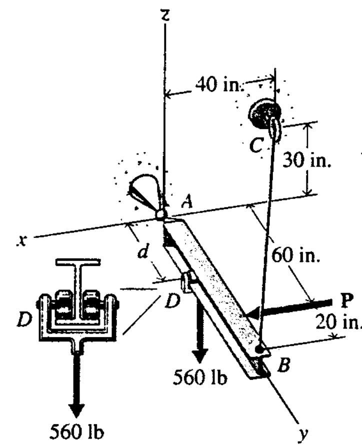


Fig. P6-93

SOLUTION

$$\mathbf{T}_{BC} = T_{BC}\left[\frac{-40\,\hat{i} - 80\,\hat{j} + 30\,\hat{k}}{\sqrt{(-40)^2 + (-80)^2 + (30)^2}}\right]$$

$$= -0.4240T_{BC}\,\hat{i} - 0.8480T_{BC}\,\hat{j} + 0.3180T_{BC}\,\hat{k}$$

$$\Sigma\mathbf{M}_A = [(40\,\hat{j}) \times (-225\,\hat{k})] + [(d\,\hat{j}) \times (-560\,\hat{k})] + [(60\,\hat{j}) \times (P\,\hat{i})]$$

$$+ [(80\,\hat{j}) \times (-0.4240T_{BC}\,\hat{i} - 0.8480T_{BC}\,\hat{j} + 0.3180T_{BC}\,\hat{k})] = \mathbf{0}$$

$\hat{i}$: $-9000 - 560d + 25.4399T_{BC} = 0$ $\qquad T_{BC} = \dfrac{9000 + 560d}{25.4399}$

$\hat{j}$: $0 = 0$

$\hat{k}$: $-60P + 33.9199T_{BC} = 0$ $\qquad P = \dfrac{33.9199T_{BC}}{60}$

$$\Sigma F_x = A_x + P - 0.4240T_{BC} = 0$$

$$\Sigma F_y = A_y - 0.8480T_{BC} = 0$$

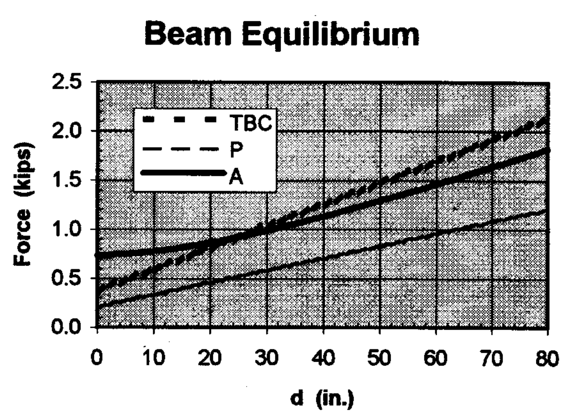
$$\Sigma F_z = A_z + 0.3180T_{BC} - 785 = 0$$

$$A_x = 0.4240T_{BC} - P$$

$$A_y = 0.8480T_{BC}$$

$$A_z = 785 - 0.3180T_{BC}$$

$$A = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$



C6-94 A hand winch is used to raise a 75-kg load as shown in Fig. P6-94. If the force P is always perpendicular to both the handle DE and the arm CD, plot A and B, the magnitudes of the bearing forces, as a function of the angle θ ($0 \le \theta \le 360^\circ$).

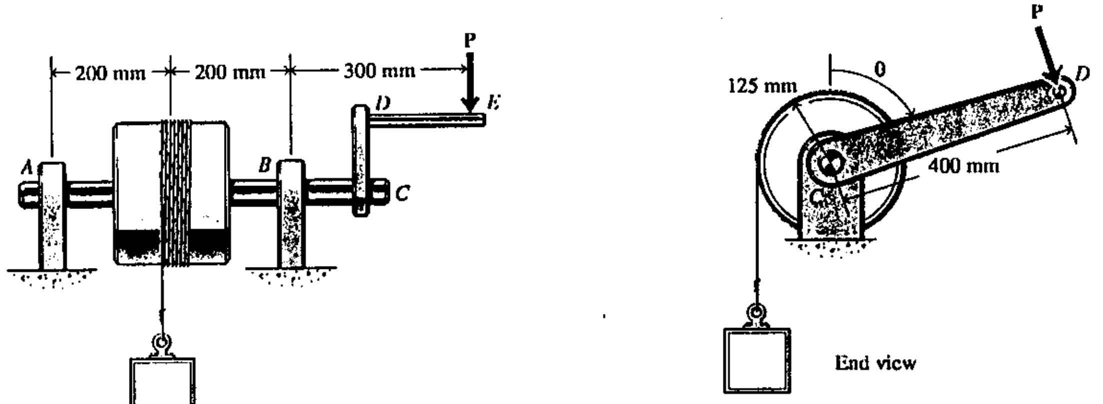
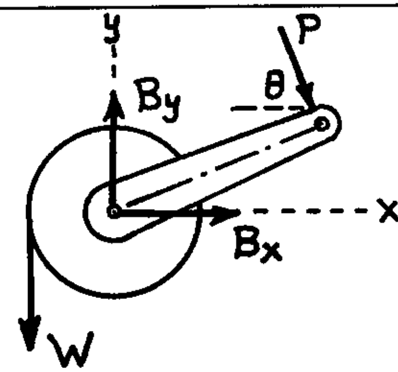


Fig. P6-94

SOLUTION



From a free-body diagram of the system with the axes centered in bearing A:

$W = mg = 75(9.807) = 735.53$ N

$$\Sigma \vec{F} = (A_x + B_x + P\cos\theta)\,\hat{i} + (A_y + B_y - P\sin\theta - 735.53)\,\hat{j} = \vec{0}$$

$$\Sigma \vec{M}_A = (400\,\hat{k}) \times (B_x\,\hat{i} + B_y\,\hat{j}) + (-125\,\hat{i} + 200\,\hat{k}) \times (-735.53\,\hat{j}) + (400\sin\theta\,\hat{i} + 400\cos\theta\,\hat{j} + 700\,\hat{k}) \times (P\cos\theta\,\hat{i} - P\sin\theta\,\hat{j}) = \vec{0}$$

From force equilibrium:

$\hat{i}$: $A_x + B_x + P\cos\theta = 0$

$\hat{j}$: $A_y + B_y - P\sin\theta - 735.53 = 0$

From moment equilibrium:

$\hat{i}$: $-400\,B_y + 700\,P\sin\theta + 147{,}106 = 0$

$\hat{j}$: $400\,B_x + 700\,P\cos\theta = 0$

$\hat{k}$: $91{,}941.25 - 400P\cos^2\theta - 400P\sin^2\theta = 0$

C6-94 (Continued)

Solving yields:

$$P = 229.853 \text{ N}$$

$$A_x = 172.390 \cos\theta$$

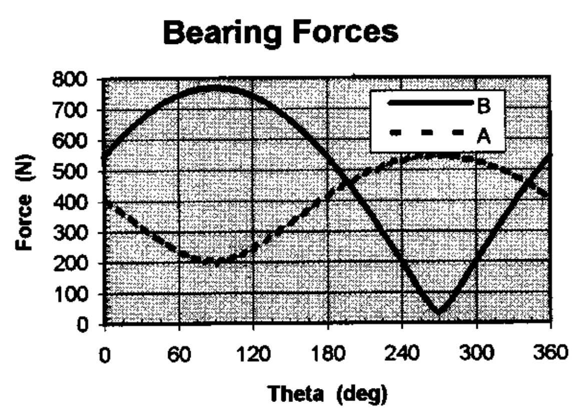
$$A_y = 367.765 - 172.390 \sin\theta$$

$$A = \sqrt{(A_x)^2 + (A_y)^2}$$

$$B_x = -402.243 \cos\theta$$

$$B_y = 367.765 + 402.243 \sin\theta$$

$$B = \sqrt{(B_x)^2 + (B_y)^2}$$



6-95* A curved slender bar is loaded and supported as shown in Fig. P6-95. Determine the reactions at supports A and B and the tension **T** in the cable.

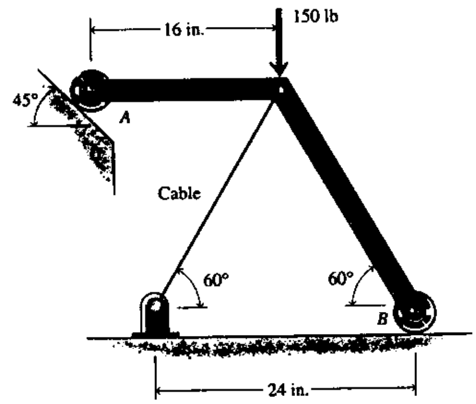
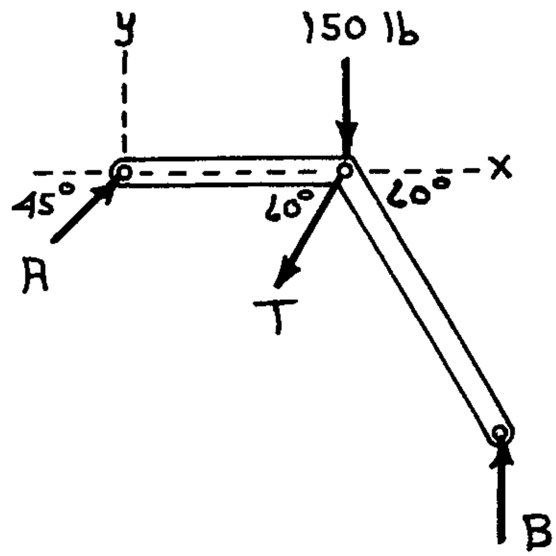


Fig. P6-95

SOLUTION

From a free-body diagram for the curved bar:



$$+\circlearrowleft \Sigma M_A = B(28) - 150(16) - T \sin 60^\circ (16) = 0$$

(a) $28B - 13.856T = 2400$

$$+\rightarrow \Sigma F_x = A \cos 45^\circ - T \cos 60^\circ = 0$$

(b) $A = 0.7071\ T$

$$+\uparrow \Sigma F_y = B + A \sin 45^\circ - T \sin 60^\circ - 150 = 0$$

(c) $B + 0.7071A - 0.8660T = 150$

Equations (a), (b), and (c) yield:

$T = 498.9 \text{ lb} \cong 499 \text{ lb}$ $\quad$ $\mathbf{T} = 499 \text{ lb} \measuredangle 60^\circ$ Ans.

$A = 352.77 \text{ lb} \cong 353 \text{ lb}$ $\quad$ $\mathbf{A} = 353 \text{ lb} \measuredangle 45^\circ$ Ans.

$B = 332.60 \text{ lb} \cong 333 \text{ lb}$ $\quad$ $\mathbf{B} = 333 \text{ lb} \uparrow$ Ans.

6-96* Determine the reactions at supports A and B of the truss shown in Fig. P6-96.

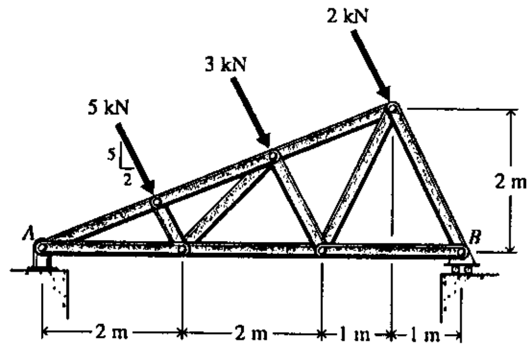
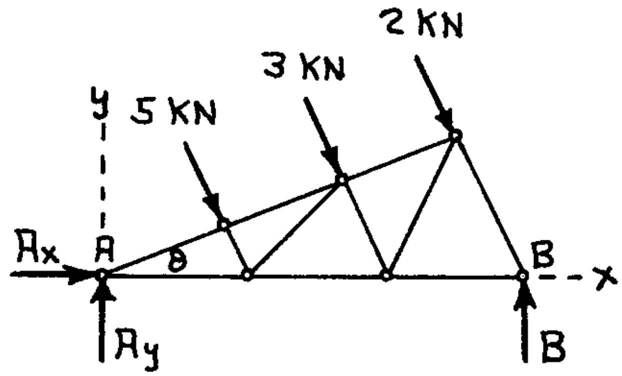


Fig. P6-96

SOLUTION

From a free-body diagram for the truss:

$$\theta = \tan^{-1} \frac{2}{5} = 21.80^\circ$$



$$+ \rightarrow \Sigma F_x = A_x + 5 \sin 21.80^\circ + 3 \sin 21.80^\circ + 2 \sin 21.80^\circ = 0$$

$$A_x = -3.714 \text{ kN} \cong -3.71 \text{ kN}$$

$$+ \circlearrowleft \Sigma M_B = -A_y(6) + 5(4 \cos 21.80^\circ) + 3(2 \cos 21.80^\circ) = 0$$

$$A_y = 4.023 \text{ kN} \cong 4.02 \text{ kN}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.714)^2 + (4.023)^2} = 5.475 \text{ kN} \cong 5.48 \text{ kN}$$

$$\theta = \tan^{-1} \frac{A_y}{A_x} = \tan^{-1} \frac{4.023}{-3.714} = 132.7^\circ$$

$$\mathbf{A} = A_x \hat{i} + A_y \hat{j}$$

$$= -3.71 \hat{i} + 4.02 \hat{j} \text{ kN} = 5.48 \text{ kN} ⦪ 47.3^\circ$$ Ans.

$$+ \circlearrowleft \Sigma M_A = B(6) - 5(2 \cos 21.80^\circ) - 3(4 \cos 21.80^\circ) - 2(6 \cos 21.80^\circ) = 0$$

$B = 5.261$ kN $\qquad \mathbf{B} = 5.26 \hat{j}$ kN = 5.26 kN ↑ Ans.

6-97 A cylinder is supported by a bar and cable as shown in Fig. P6-97. The weight of the cylinder is 150 lb and the weight of the bar is 20 lb. If all surfaces are smooth, determine the reaction at support C of the bar and the tension $\vec{T}$ in the cable.

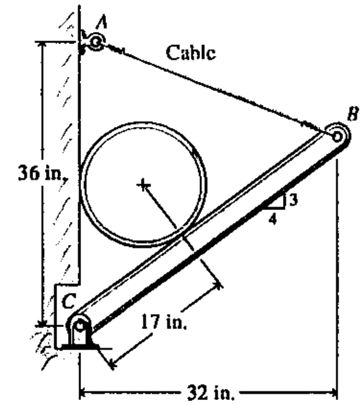
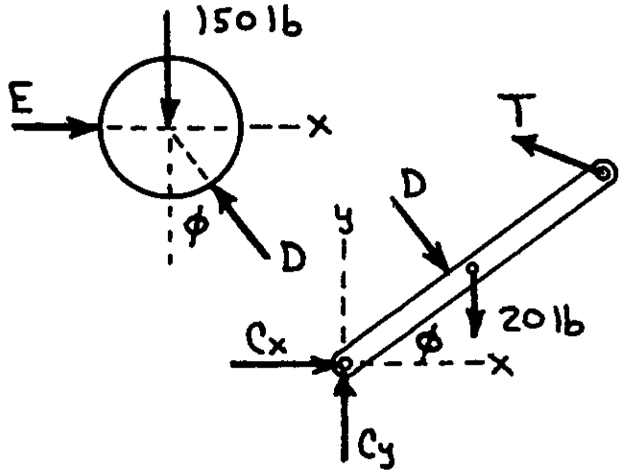


Fig. P6-97

SOLUTION



From a free-body diagram for the cylinder:

$$\phi = \tan^{-1}\frac{3}{4} = 36.87^\circ$$

$$+\uparrow \Sigma F_y = D\cos 36.87^\circ - 150 = 0$$

$$D = 187.50 \text{ lb}$$

From a free-body diagram of the bar:

$$\alpha = \tan^{-1}\frac{12}{32} = 20.56^\circ$$

$$+\circlearrowleft \Sigma M_C = T\sin 20.56^\circ\,(32) + T\cos 20.56^\circ\,(24) - 187.5(17) - 20(16) = 0$$

$$T = 104.06 \text{ lb} \cong 104.1 \text{ lb}$$

$$\vec{T} = 104.06(-\cos 20.56^\circ\,\hat{i} + \sin 20.56^\circ\,\hat{j})$$

$$= -97.4\,\hat{i} + 36.5\,\hat{j} \text{ lb} = 104.1 \text{ lb} \measuredangle 20.6^\circ$$ Ans.

$$+\rightarrow \Sigma F_x = C_x + 187.50\sin 36.87^\circ - 104.06\cos 20.56^\circ = 0$$

$$C_x = -15.068 \text{ lb} \cong -15.07 \text{ lb}$$

$$+\uparrow \Sigma F_y = C_y - 187.50\cos 36.87^\circ + 104.06\sin 20.56^\circ - 20 = 0$$

$$C_y = 133.46 \text{ lb} \cong 133.5 \text{ lb}$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-15.068)^2 + (133.46)^2} = 134.31 \text{ lb} \cong 134.3 \text{ lb}$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{133.46}{-15.068} = 96.4^\circ$$

$$\vec{C} = -15.07\,\hat{i} + 133.5\,\hat{j} \text{ lb} = 134.3 \text{ lb} \measuredangle 83.6^\circ$$ Ans.

6-98* A cylinder is supported by a bar and cable as shown in Fig. P6-98. The mass of the 200-mm diameter cylinder is 75 kg. Determine the reaction at support A and the tension $\bar{T}$ in the cable.

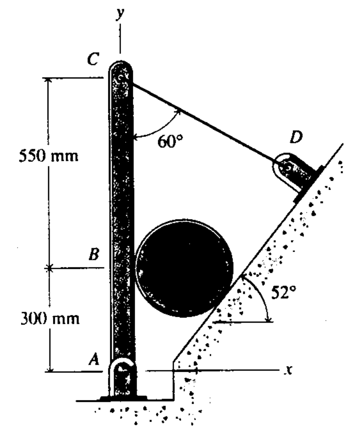
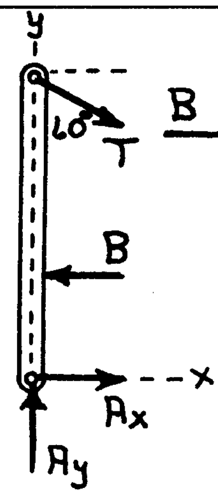
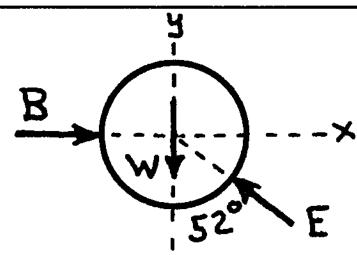


Fig. P6-98

SOLUTION





$W = mg = 75(9.807) = 735.53$ N

From a free-body diagram for the cylinder:

$+\uparrow \Sigma F_y = E \cos 52^\circ - W$

$= E \cos 52^\circ - 735.53 = 0$ $\qquad E = 1194.7$ N

$+\rightarrow \Sigma F_x = B - E \sin 52^\circ$

$= B - 1194.7 \sin 52^\circ = 0$ $\qquad B = 941.4$ N

From a free-body diagram for the bar:

$+\circlearrowleft \Sigma M_A = 941.4(0.300) - T \sin 60^\circ (0.850) = 0$

$T = 383.7 \text{ N} \cong 384 \text{ N}$

$+\rightarrow \Sigma F_x = A_x + T \sin 60^\circ - B$

$= A_x + 383.7 \sin 60^\circ - 941.4 = 0$ $\qquad A_x = 609.1 \text{ N} \cong 609 \text{ N}$

$+\uparrow \Sigma F_y = A_y - T \cos 60^\circ$

$= A_y - 383.7 \cos 60^\circ = 0$ $\qquad A_y = 191.91 \text{ N} \cong 191.9 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(609.1)^2 + (191.91)^2} = 638.6 \text{ N} \cong 639 \text{ N}$

$\theta = \tan^{-1} \dfrac{A_y}{A_x} = \tan^{-1} \dfrac{191.91}{609.39} = 17.48^\circ$

$\bar{A} = 609\ \hat{i} + 191.9\ \hat{j}$ N $= 639$ N $\measuredangle\ 17.48^\circ$ Ans.

6-99 Two beams are loaded and supported as shown in Fig. P6-99. Determine the reactions at supports A, B, and C. Neglect the weights of the beams.

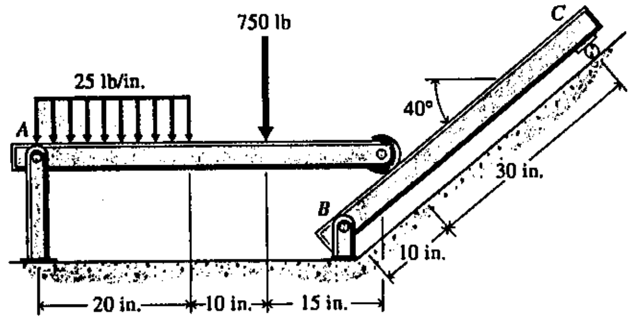
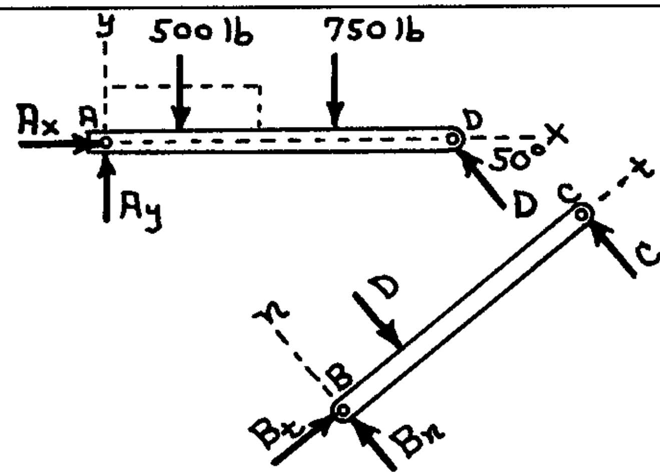


Fig. P6-99

SOLUTION

From a free-body diagram of the horizontal beam:

The distributed load is replaced by a resultant force $\bar{R}$ at a distance d_x from the left support.



$R = wL_w = 25(20) = 500$ lb

$d_x = \frac{1}{2}L_w = \frac{1}{2}(20) = 10$ in.

$+ \circlearrowleft \Sigma M_A = -500(10) - 750(30) + D\cos 40^\circ (45) = 0$ $D = 797.7 \text{ lb} \cong 798 \text{ lb}$

$+ \rightarrow \Sigma F_x = A_x - 797.7 \sin 40^\circ = 0$ $A_x = 512.8 \text{ lb} \cong 513 \text{ lb}$

$+ \uparrow \Sigma F_y = A_y - 500 + 750 + 797.7 \sin 50^\circ = 0$ $A_y = 638.9 \text{ lb} \cong 639 \text{ lb}$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(512.8)^2 + (638.9)^2} = 819.2 \text{ lb} \cong 819 \text{ lb}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} = \tan^{-1}\frac{638.9}{512.8} = 51.24^\circ \cong 51.2^\circ$$

$\bar{A} = 513\,\hat{i} + 639\,\hat{j}$ lb $= 819$ lb ∠ 51.2° Ans.

$+ \circlearrowleft \Sigma M_B = -D(10) + C(40) = -797.7(10) + C(40) = 0$

$C = 199.43 \text{ lb} \cong 199.4 \text{ lb}$ $\bar{C} = 199.4$ lb ∠ 50.0° Ans.

$+ \nwarrow \Sigma F_n = B_n - D + C = B_n - 797.7 + 199.4 = 0$ $B_n = 598.3 \text{ lb} \cong 598 \text{ lb}$

$+ \nearrow \Sigma F_t = B_t = 0$ $B_t = 0$

$\bar{B} = 598$ lb ∠ 50.0° Ans.

6-100 A bracket is loaded and supported as shown in Fig. P6-100. Determine the reactions at supports A and B. Neglect the weight of the bracket.

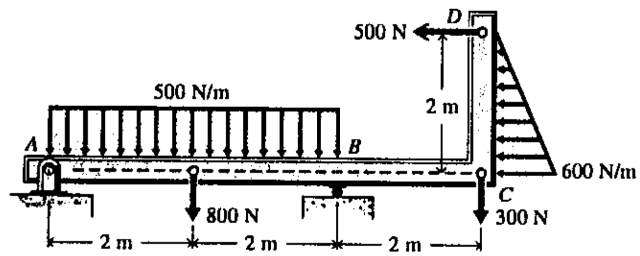
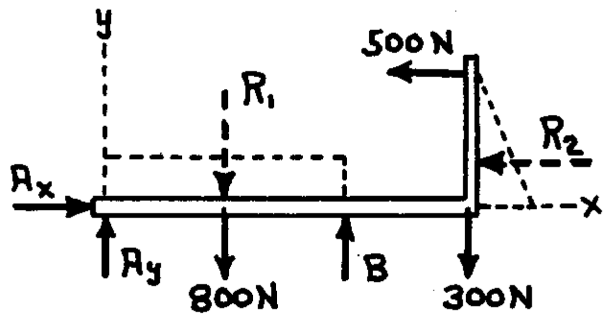


Fig. P6-100

SOLUTION

From a free-body diagram of the bracket:



The distributed loads are replaced by resultants $\bar{R}_1$ and $\bar{R}_2$ at distances d_x and d_y from the left support, respectively.

$R_1 = w_1 L_{w1} = 500(4) = 2000$ N

$d_x = \frac{1}{2} L_{w1} = \frac{1}{2}(4) = 2$ m

$R_2 = \frac{1}{2} w_2 L_{w2} = \frac{1}{2}(600)(2) = 600$ N

$d_y = \frac{1}{3} L_{w2} = \frac{1}{3}(2) = 0.6667$ m

$+ \rightarrow \Sigma F_x = A_x - R_2 - 500$

$= A_x - 600 - 500 = 0$ $\quad A_x = 1100$ N

$+ \circlearrowleft \Sigma M_B = -A_y(4) + 800(2) + R_1 d_x - 300(2) + R_2 d_y + 500(2)$

$= -A_y(4) + 800(2) + 2000(2) - 300(2) + 600(0.6667) + 500(2) = 0$

$A_y = 1600$ N

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(1100)^2 + (1600)^2} = 1941.6 \text{ N} \cong 1942$ N

$\theta_x = \tan^{-1} \frac{A_y}{A_x} = \tan^{-1} \frac{1600}{1100} = 55.49° \cong 55.5°$

$\bar{A} = 1100\,\hat{i} + 1600\,\hat{j}$ N $= 1942$ N ∠ $55.5°$ Ans.

$+ \circlearrowleft \Sigma M_A = B(4) - 800(2) - 2000(2) - 300(6) + 600(0.6667) + 500(2) = 0$

$B = 1500$ N $\quad \bar{B} = 1500\,\hat{j}$ N $= 1500$ N ↑ Ans.

6-101* The shaft shown in Fig. P6-101 is part of a drive system in a factory. Friction between the belts and pulleys prevents slipping of the belts. Determine the torque $\vec{T}$ required for equilibrium and the reactions at supports A and B. The support at A is a journal bearing and the support at B is a thrust bearing. The bearings exert only force reactions on the shaft.

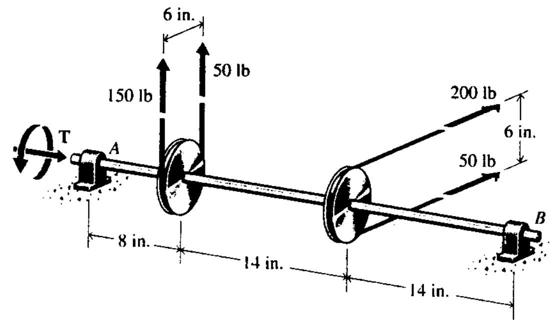
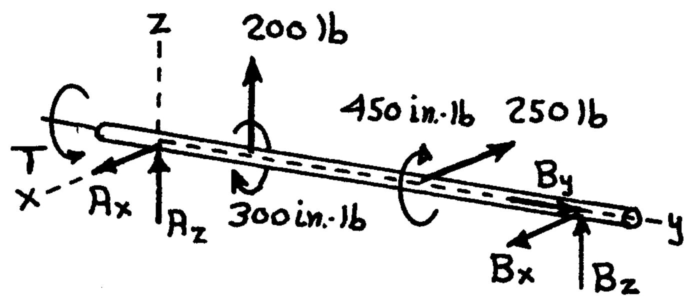


Fig. P6-101

SOLUTION

From a free-body diagram for the shaft:



$$\Sigma M_y = T + 50(3) - 150(3) + 50(3) - 200(3) = 0$$

$\vec{T} = 750\ \hat{j}$ in.·lb Ans.

For moment equilibrium:

$$\Sigma \vec{M}_B = [(-36\ \hat{j}) \times (A_x\ \hat{i} + A_z\ \hat{k})] + [(-14\ \hat{j} + 3\ \hat{k}) \times (-200\ \hat{i})] + [(-14\ \hat{j} - 3\ \hat{k}) \times (-50\ \hat{i})] + [(3\ \hat{i} - 28\ \hat{j}) \times (150\ \hat{k})] + [(-3\ \hat{i} - 28\ \hat{j}) \times (50\ \hat{k})] + 750\ \hat{j}$$

$$= (-36A_z - 5600)\ \hat{i} + (36A_x - 3500)\ \hat{k} = \vec{0}$$

$\vec{A} = 97.22\ \hat{i} - 155.56\ \hat{k}$ lb $\cong 97.2\ \hat{i} - 155.6\ \hat{k}$ lb Ans.

$$A = \sqrt{(97.22)^2 + (-155.56)^2} = 183.44 \text{ lb} \cong 183.4 \text{ lb}$$

For force equilibrium:

$$\Sigma \vec{F} = 97.22\ \hat{i} - 155.56\ \hat{k} + B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k} - 200\ \hat{i} - 50\ \hat{i} + 150\ \hat{k} + 50\ \hat{k}$$

$$= (B_x - 152.78)\ \hat{i} + B_y\ \hat{j} + (B_z + 44.44)\ \hat{k} = \vec{0}$$

$\vec{B} = B_x\ \hat{i} + B_y\ \hat{j} + B_z\ \hat{k} = 152.78\ \hat{i} + 0\ \hat{j} - 44.44\ \hat{k}$ lb

$\cong 152.8\ \hat{i} - 44.4\ \hat{k}$ lb Ans.

$$B = \sqrt{(152.78)^2 + (-44.44)^2} = 159.11 \text{ lb} \cong 159.1 \text{ lb}$$

6-102* A beam is loaded and supported as shown in Fig. P6-102. Determine the reactions at supports A and B when m_1 = 75 kg and m_2 = 225 kg.

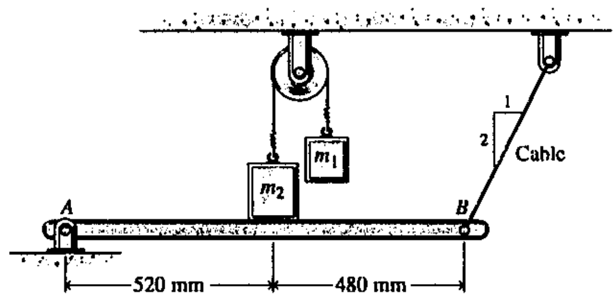
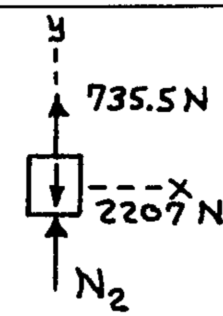


Fig. P6-102

SOLUTION

$W_1 = m_1 g = 75(9.807) = 735.5$ N

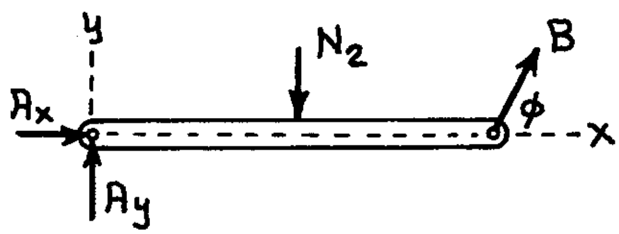
$W_2 = m_2 g = 225(9.807) = 2207$ N



From a free-body diagram of mass m_2:

$+\uparrow \Sigma F_y = N_2 + 735.5 - 2207 = 0$

$N_2 = 1471.5$ N



From a free-body diagram of the beam:

$\phi = \tan^{-1} \frac{2}{1} = 63.43^\circ$

$+ \circlearrowleft \Sigma M_A = B \sin 63.43^\circ (1000) - 1471.5(520) = 0$ B = 855.5 N ≅ 856 N

$\vec{B} = 855.5(\cos 63.43^\circ \hat{i} + \sin 63.43^\circ \hat{j})$

$= 383 \hat{i} + 765 \hat{j}$ N = 856 N ∠ 63.4° Ans.

$+ \rightarrow \Sigma F_x = A_x + B \cos 63.43^\circ = A_x + 855.5 \cos 63.43^\circ = 0$

$A_x = -382.7$ N ≅ -383 N

$+\uparrow \Sigma F_y = A_y - 1471.5 + B \sin 63.43^\circ = A_y - 1471.5 + 855.5 \sin 63.43^\circ = 0$

$A_y = 706.4$ N ≅ 706 N

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-382.7)^2 + (706.4)^2} = 803.41$ N ≅ 803 N

$\theta_x = \tan^{-1} \frac{A_y}{A_x} = \tan^{-1} \frac{706.4}{-382.7} = 118.44^\circ \cong 118.4^\circ$

$\vec{A} = -382 \hat{i} + 706 \hat{j}$ N = 803 N ∠ 61.6° Ans.

6-103 A bar is loaded and supported as shown in Fig. P6-103. End A of the bar is supported with a ball-and-socket joint. Determine the components of the reaction at support A and the tension $\vec{T}$ in the cable.

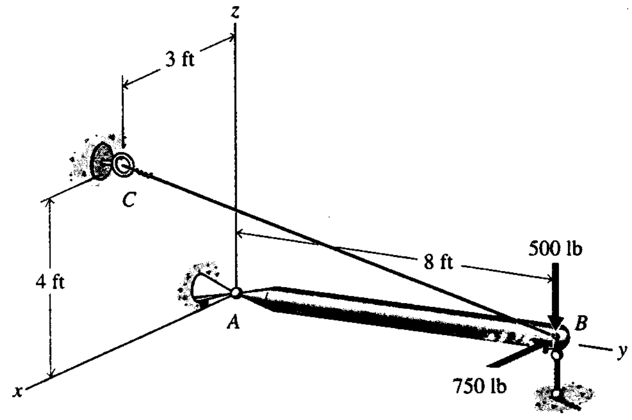
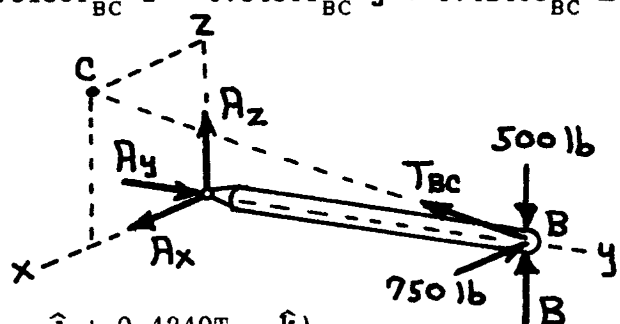


Fig. P6-103

SOLUTION

$$\vec{T}_{BC} = T_{BC}\left[\frac{3\,\hat{i} - 8\,\hat{j} + 4\,\hat{k}}{\sqrt{(3)^2 + (-8)^2 + (4)^2}}\right] = 0.3180T_{BC}\,\hat{i} - 0.8480T_{BC}\,\hat{j} + 0.4240T_{BC}\,\hat{k}$$

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma\vec{M}_A = (\vec{r}_{B/A} \times \vec{T}_{BC}) + (\vec{r}_{B/A} \times \vec{B})$$

$$= (8\,\hat{j}) \times (0.3180T_{BC}\,\hat{i} - 0.8480T_{BC}\,\hat{j} + 0.4240T_{BC}\,\hat{k})$$
$$+ (8\,\hat{j}) \times (-750\,\hat{i} - 500\,\hat{k} + B\,\hat{k})$$

$$= (3.392T_{BC} - 4000 + 8B)\,\hat{i} + (-2.544T_{BC} + 6000)\,\hat{k} = \vec{0}$$

From which:

$$T_{BC} = 2358.5 \text{ lb} \cong 2.36 \text{ kip}$$

$$\vec{T}_{BC} = 2358.5(0.3180\,\hat{i} - 0.8480\,\hat{j} + 0.4240\,\hat{k})$$
$$= 750\,\hat{i} - 2000\,\hat{j} + 1000\,\hat{k} \text{ lb}$$ Ans.

$B = -500$ lb $\vec{B} = -500\,\hat{k}$ lb $= 500$ lb ↓   Ans.

For force equilibrium:

$$\Sigma\vec{F} = \vec{A} + \vec{B} + \vec{T}_{BC} - 750\,\hat{i} - 500\,\hat{k}$$

$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} - 500\,\hat{k}$$
$$+ 750\,\hat{i} - 2000\,\hat{j} + 1000\,\hat{k} - 750\,\hat{i} - 500\,\hat{k} = \vec{0}$$

$$= (A_x)\,\hat{i} + (A_y - 2000)\,\hat{j} + (A_z)\,\hat{k} = 0$$ $\vec{A} = 2000\,\hat{j}$ lb   Ans.

6-104 A bar is loaded and supported as shown in Fig. P6-104. End A of the bar is supported with a ball-and-socket joint. Determine the components of the reaction at support A and the tensions $\mathbf{T}_C$ and $\mathbf{T}_D$ in the two cables.

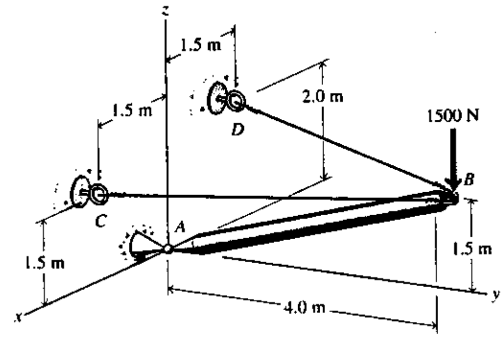


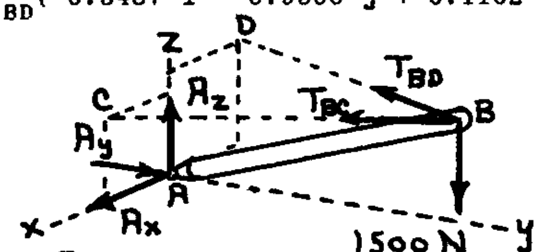
Fig. P6-104

SOLUTION

$$\mathbf{T}_{BC} = T_{BC}\left[\frac{1.5\,\hat{\imath} - 4.0\,\hat{\jmath}}{\sqrt{(1.5)^2 + (-4.0)^2}}\right] = T_{BC}(0.3511\,\hat{\imath} - 0.9363\,\hat{\jmath})$$

$$\mathbf{T}_{BD} = T_{BD}\left[\frac{-1.5\,\hat{\imath} - 4.0\,\hat{\jmath} + 0.5\,\hat{k}}{\sqrt{(-1.5)^2 + (-4.0)^2 + (0.5)^2}}\right] = T_{BD}(-0.3487\,\hat{\imath} - 0.9300\,\hat{\jmath} + 0.1162\,\hat{k})$$

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma \mathbf{M}_A = (\vec{r}_{B/A} \times \mathbf{T}_{BC}) + (\vec{r}_{B/A} \times \mathbf{T}_{BD}) + (\vec{r}_{B/A} \times \vec{B})$$

$$= (4.0\,\hat{\jmath} + 1.5\,\hat{k}) \times (0.3511T_{BC}\,\hat{\imath} - 0.9363T_{BC}\,\hat{\jmath})$$

$$+ (4.0\,\hat{\jmath} + 1.5\,\hat{k}) \times (-0.3487\,T_{BD}\,\hat{\imath} - 0.9300\,T_{BD}\,\hat{\jmath} + 0.1162\,T_{BD}\,\hat{k})$$

$$+ (4.0\,\hat{\jmath} + 1.5\,\hat{k}) \times (-1500\,\hat{k})$$

$$= (1.4045T_{BC} + 1.8598T_{BD} - 6000)\,\hat{\imath} + (0.5267T_{BC} - 0.5231T_{BD})\,\hat{\jmath} + (-1.4045T_{BC} + 1.3948T_{BD})\,\hat{k} = \vec{0}$$

$T_{BC} = 1831.01 \text{ N} \cong 1831 \text{ N}$ $\qquad$ $T_{BD} = 1843.54 \text{ N} \cong 1844 \text{ N}$

$\mathbf{T}_{BC} = 1831.01(0.3511\,\hat{\imath} - 0.9363\,\hat{\jmath}) = 643\,\hat{\imath} - 1714\,\hat{\jmath}$ N $\qquad$ Ans.

$\mathbf{T}_{BD} = 1843.54(-0.3487\,\hat{\imath} - 0.9300\,\hat{\jmath} + 0.1162\,\hat{k}$

$= -643\,\hat{\imath} - 1714\,\hat{\jmath} + 214\,\hat{k}$ N $\qquad$ Ans.

For force equilibrium:

$$\Sigma \mathbf{F} = \vec{A} + \mathbf{T}_{BC} + \mathbf{T}_{BD} - 1500\,\hat{k}$$

$$= A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k} + 643\,\hat{\imath} - 1714\,\hat{\jmath} - 643\,\hat{\imath} - 1714\,\hat{\jmath} + 214\,\hat{k}\text{ N} - 1500\,\hat{k} = \vec{0}$$

$= (A_x)\,\hat{\imath} + (A_y)\,\hat{\jmath} + (A_z - 1286)\,\hat{k} = 0$ $\qquad$ $\vec{A} = 1286\,\hat{k}$ N $\qquad$ Ans.

6-105* The 200-lb plate shown in Fig. P6-105 is supported in a horizontal position by two hinges and a continuous cable. The hinges have been properly aligned; therefore, they exert only force reactions on the plate. Assume that the hinge at B resists any force along the axis of the hinge pins. Determine the reactions at supports A and B and the tension $\vec{T}$ in the cable.

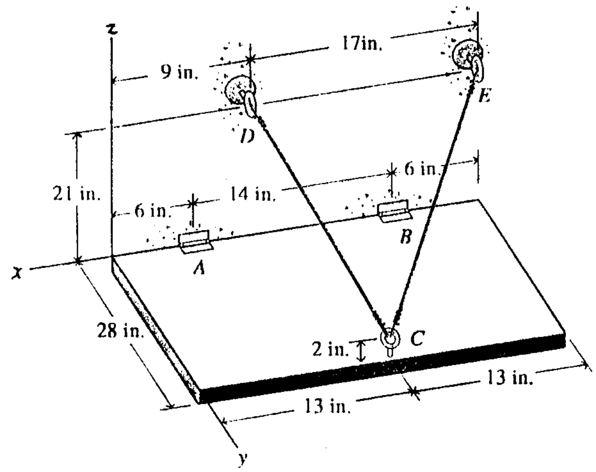
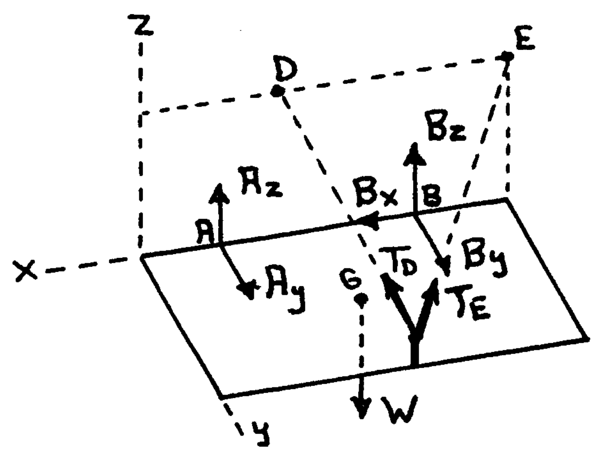


Fig. P6-105

SOLUTION

$$\vec{T}_D = T\left[\frac{4\,\hat{i} - 28\,\hat{j} + 19\,\hat{k}}{\sqrt{(4)^2 + (-28)^2 + (19)^2}}\right] = T(0.1174\,\hat{i} - 0.8218\,\hat{j} + 0.5576\,\hat{k})\text{ lb}$$

$$\vec{T}_E = T\left[\frac{-13\,\hat{i} - 28\,\hat{j} + 19\,\hat{k}}{\sqrt{(-13)^2 + (-28)^2 + (19)^2}}\right] = T(-0.3586 - 0.7724\,\hat{j} + 0.5242\,\hat{k})\text{ lb}$$



From a free-body diagram for the plate:

For moment equilibrium:

$$\Sigma\vec{M}_A = (\vec{r}_{B/A} \times \vec{B}) + (\vec{r}_{D/A} \times \vec{T}_D) + (\vec{r}_{E/A} \times \vec{T}_E) + (\vec{r}_{G/A} \times \vec{W})$$

$$= (-14\,\hat{i}) \times (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k})$$

$$+ (-3\,\hat{i} + 21\,\hat{k}) \times (0.1174\,T\,\hat{i} - 0.8218\,T\,\hat{j} + 0.5576\,\hat{k})$$

$$+ (-20\,\hat{i} + 21\,\hat{k}) \times (-0.3586\,T\,\hat{i} - 0.7724\,T\,\hat{j} + 0.5242\,\hat{k})$$

$$+ (-7\,\hat{i} + 14\,\hat{j}) \times (-200\,\hat{k})$$

$$= (33.4782\,T - 2800)\,\hat{i} + (14\,B_z + 7.0916\,T - 1400)\,\hat{j}$$

$$+ (-14\,B_y + 17.9134\,T)\,\hat{k} = \vec{0}$$

$T = 83.64$ lb $\cong 83.6$ lb Ans.

6-105 (Continued)

$\mathbf{T}_D = 83.64(0.1174\ \hat{i} - 0.8218\ \hat{j} + 0.5576\ \hat{k})$

$= 9.82\ \hat{i} - 68.7\ \hat{j} + 46.6\ \hat{k}$ lb

$\mathbf{T}_E = 83.64(-0.3586 - 0.7724\ \hat{j} + 0.5242\ \hat{k})$

$= -30.0\ \hat{i} - 64.6\ \hat{j} + 43.8\ \hat{k}$ lb

$B_y = 107.02$ lb $\cong 107.0$ lb $\qquad B_z = 57.63$ lb $\cong 57.6$ lb

For force equilibrium:

$\Sigma\mathbf{F} = \bar{A} + \bar{B} + \mathbf{T}_D + \mathbf{T}_E + \bar{W}$

$= (A_y\ \hat{j} + A_z\ \hat{k}) + (B_x\ \hat{i} + 107.0\ \hat{j} + 57.6\ \hat{k})$

$+ (9.82\ \hat{i} - 68.7\ \hat{j} + 46.6\ \hat{k}) + (-30.0\ \hat{i} - 64.6\ \hat{j} + 43.8\ \hat{k}) - 200\ \hat{k}$

$= (B_x - 20.174)\ \hat{i} + (A_y - 26.319)\ \hat{j} + (A_z - 51.888)\ \hat{k} = \bar{0}$

$B_x = 20.174$ lb $\cong 20.2$ lb $\qquad \bar{B} = 20.2\ \hat{i} + 107.0\ \hat{j} + 57.6\ \hat{k}$ lb Ans.

$A_y = 26.319$ lb $\cong 26.3$ lb

$A_z = 51.888$ lb $\cong 51.9$ lb $\qquad \bar{A} = 26.3\ \hat{j} + 51.9\ \hat{k}$ lb Ans.

6-106 A bent bar is loaded and supported as shown in Fig. P6-106. End A of the bar is supported with a ball-and-socket joint, point B with a cable and link, and point C with a link. Determine the components of the reaction at support A and the forces in the cable and links.

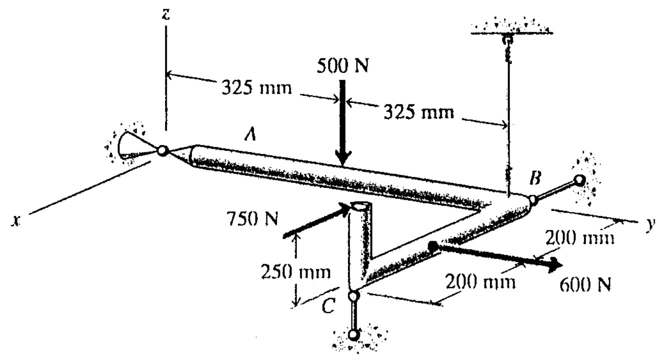
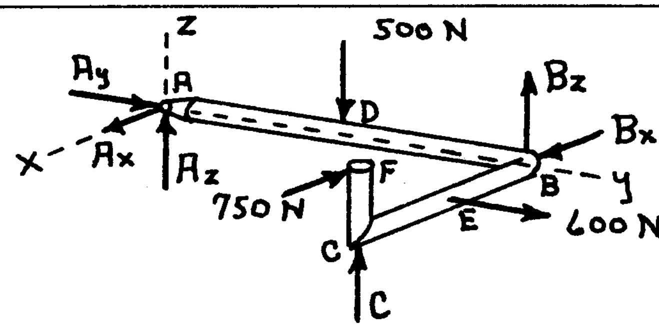


Fig. P6-106

SOLUTION

From a free-body diagram for the bar:



For moment equilibrium:

$$\Sigma \bar{M}_A = (\vec{r}_{D/A} \times \bar{F}_1) + (\vec{r}_{B/A} \times \bar{B}) + (\vec{r}_{E/A} \times \bar{F}_2) + (\vec{r}_{C/A} \times \bar{C}) + (\vec{r}_{F/A} \times \bar{F}_3)$$

$$= (0.325\ \hat{j}) \times (-500\ \hat{k}) + (0.650\ \hat{j}) \times (B_x\ \hat{i} + B_z\ \hat{k})$$

$$+ (0.200\ \hat{i} + 0.650\ \hat{j}) \times (600\ \hat{j}) + (0.650\ \hat{j} + 0.400\ \hat{i}) \times (-C_z\ \hat{k})$$

$$+ (0.400\ \hat{i} + 0.650\ \hat{j} + 0.250\ \hat{k}) \times (-750\ \hat{i})$$

$$= (0.650\ B_z - 0.650\ C_z - 162.5)\ \hat{i} + (-0.400\ C_z - 187.5)\ \hat{j} + (-0.650\ B_x + 607.5)\ \hat{k} = \bar{0}$$

$B_x = 934.6\ N \cong 935\ N$

$B_z = 718.8\ N \cong 719\ N$ $\qquad \bar{B} = 935\ \hat{i} + 719\ \hat{k}\ N$ Ans.

$C_z = -468.8\ N \cong -469\ N$ $\qquad \bar{C} = -469\ \hat{k}\ N$ Ans.

For force equilibrium:

$$+\Sigma \bar{F} = \bar{A} + \bar{B} + \bar{C} + \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k} + 934.6\ \hat{i} + 718.8\ \hat{k} - 468.8\ \hat{k} - 500\ \hat{k} + 600\ \hat{j} - 750\ \hat{i}$$

$$= (A_x + 184.6)\ \hat{i} + (A_y + 600)\ \hat{j} + (A_z - 250)\ \hat{k} = 0$$

$A_x = -184.62 \cong -184.6\ N$ $\qquad A_y = -600 \cong -600\ N$

$A_z = 250\ N$ $\qquad \bar{A} = -184.6\ \hat{i} - 600\ \hat{j} + 250\ \hat{k}\ N$ Ans.