Instructor's Solutions Manual

to accompany

Fundamentals of Aerodynamics

Third Edition

John D. Anderson, Jr.
Curator of Aerodynamics
National Air and Space Museum
and
Professor Emeritus
University of Maryland

McGraw Hill

Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis
Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City
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CHAPTER 1

1.1 (a) $\rho = \dfrac{p}{RT} = \dfrac{1.9 \times 10^4}{(287)(203)} = \boxed{0.326\ \text{kg/m}^3}$

(b) $T = \dfrac{p}{\rho R} = \dfrac{1058}{(1.23 \times 10^{-3})(1716)} = \boxed{501\ ^\circ R}$

1.2 $N' = -\int_{LE}^{TE} (p_u \cos\theta + \tau_u \sin\theta)\, ds_u$

$$+ \int_{LE}^{TE} (p_\ell \cos\theta - \tau_\ell \sin\theta)\, ds_\ell \qquad (1.7)$$

$ds \cos\theta = dx$

$ds \sin\theta = -dy$

Hence,

$$N' = -\int_{LE}^{TE} (p_u - p_\ell)\, dx + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, dy$$

$$N' = -\int_{LE}^{TE} [(p_u - p_\infty) - (p_\ell - p_\infty)]\, dx + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, dy$$

Divide by $q_\infty S = q_\infty c(1)$

$$\frac{N'}{q_\infty c} = -\frac{1}{c}\int_{LE}^{TE} \left[\left(\frac{p_u - p_\infty}{q_\infty}\right) - \left(\frac{p_\ell - p_\infty}{q_\infty}\right)\right] dx + \frac{1}{c}\int_{LE}^{TE} \left(\frac{\tau_u}{q_\infty} + \frac{\tau_\ell}{q_\infty}\right) dy$$

$$\boxed{c_n = \frac{1}{c}\int_0^c \left(c_{p_\ell} - c_{p_u}\right) dx + \frac{1}{c}\int_{LE}^{TE} \left(c_{f_u} + c_{f_\ell}\right) dy}$$

This is Eq. (1.15).

$$A' = \int_{LE}^{TE} (-p_u \sin\theta + \tau_u \cos\theta)\, ds_u$$

$$+ \int_{LE}^{TE} (p_\ell \sin\theta + \tau_\ell \cos\theta)\, ds_\ell \qquad (1.8)$$

$$A' = \int_{LE}^{TE} (p_u - p_\ell)\, dy + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, dx$$

$$A' = \int_{LE}^{TE} [(p_u - p_\infty) - (p_\ell - p_\infty)]\, dy + \int_0^c (\tau_u + \tau_\ell)\, dx$$

$$\frac{A'}{q_\infty c} = \frac{1}{c}\int_{LE}^{TE} \left[\left(\frac{p_u - p_\infty}{q_\infty}\right) - \left(\frac{p_\ell - p_\infty}{q_\infty}\right)\right] dy + \frac{1}{c}\int_0^c \left(\frac{\tau_u}{q_\infty} + \frac{\tau_\ell}{q_\infty}\right) dx$$

$$\boxed{c_a = \frac{1}{c}\int_{LE}^{TE} (c_{p_u} - c_{p_\ell})\, dy + \frac{1}{c}\int_0^c (c_{f_u} - c_{f_\ell})\, dx}$$

This is Eq. (1.16).

$$M'_{LE} = \int_{LE}^{TE} [(p_u \cos\theta + \tau_u \sin\theta)x - (p_u \sin\theta - \tau_u \cos\theta)y]\, ds_u$$

$$+ \int_{LE}^{TE} [-p_\ell \cos\theta + \tau_\ell \sin\theta)x + (p_\ell \sin\theta + \tau_\ell \cos\theta)y]\, ds_\ell$$

$$M'_{LE} = \int_{LE}^{TE} [p_u - p_\ell]\, x\, dx - \int_{LE}^{TE} (\tau_u + \tau_\ell)\, x\, dy$$

$$+ \int_{LE}^{TE} [p_u - p_\ell]\, y\, dy + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, y\, dx$$

$$M'_{LE} = \int_{LE}^{TE} [(p_u - p_\infty) - (p_\ell - p_\infty)]\, x\, dx - \int_{LE}^{TE} (\tau_u + \tau_\ell)\, x\, dy$$

$$+ \int_{LE}^{TE} [p_u - p_\infty) - (p_\ell - p_\infty)]\, y\, dy + \int_{LE}^{TE} (\tau_u + \tau_\ell)\, y\, dx$$

Divide by $q_\infty c^2$:

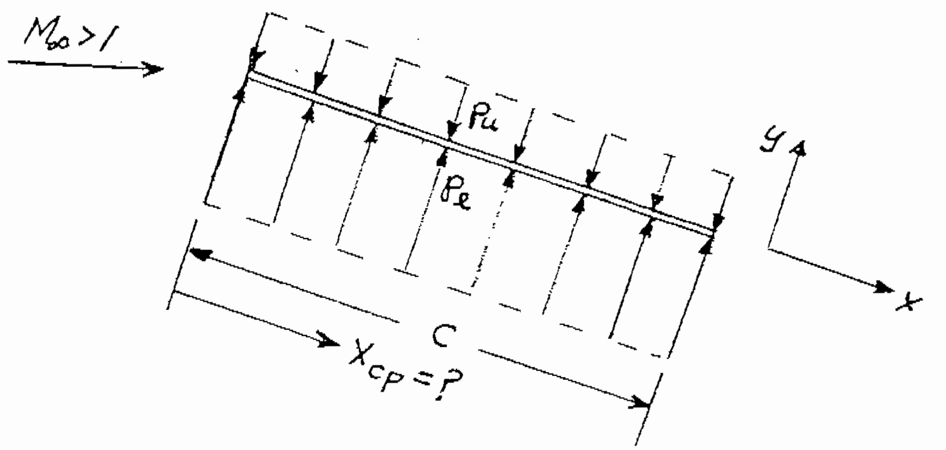
$$\frac{M_{LE'}}{q_\infty c^2} = \frac{1}{c^2}\int_{LE}^{TE} \left[\left(\frac{p_u - p_\infty}{q_\infty}\right) - \left(\frac{p_\ell - p_\infty}{q_\infty}\right)\right] x\, dx - \frac{1}{c^2}\int_{LE}^{TE} \left(\frac{\tau_u}{q_\infty} + \frac{\tau_\ell}{q_\infty}\right) x\, dy$$

$$+\frac{1}{c^2}\int_{LE}^{TE}\left[\left(\frac{p_u-p_\infty}{q_\infty}\right)-\left(\frac{p_\ell-p_\infty}{q_\infty}\right)\right] y\,dy+\frac{1}{c^2}\int_{LE}^{TE}\left(\frac{\tau_u}{q_\infty}+\frac{\tau_\ell}{q_\infty}\right) y\,dx$$

$$\boxed{\begin{aligned} c_{m_{LE}} = \frac{1}{c^2}\, [\int_0^c (C_{p_u}-C_{p_\ell})\,x\,dx - \int_{LE}^{TE} (C_{f_u}+C_{f_\ell})\,x\,dy \\ + \int_{LE}^{TE} (C_{p_u}-C_{p_\ell})\,y\,dy + \int_0^c (C_{f_u}+C_{f_\ell})\,y\,dx] \end{aligned}}$$

This is Eq. (1.17).

1.3



$$M'_{LE} = -\int_0^c (p_\ell - p_u)\,(dx)\,(1)\,x - (p_\ell - p_u)\int_0^c x\,dx$$

$$M'_{LE} = -(p_\ell - p_u)\,\frac{c^2}{2}$$

$$N' = \int_0^c (p_\ell - p_u)\,dx = (p_\ell - p_u)\,c$$

$$x_{cp} = -\frac{M'_{LE}}{N'} = -\frac{\left[-(p_\ell - p_u)\frac{c^2}{2}\right]}{(p_\ell - p_u)\ c}$$

$$\boxed{x_{cp} = c/2}$$

1.5 For a flat plate, $\theta = 0$ in Eqs. (1.7) – (1.11). Hence,

$$N' = \int_0^c (p_\ell - p_u)\, dx = \int_0^1 [-2 \times 10^4 (x-1)^2 + 1.19 \times 10^5]\, dx$$

$$N' = -2 \times 10^4 \left[\frac{x^3}{3} - x^2 + x\right]_0^1 + [1.19 \times 10^5 x]_0^1 = \boxed{1.12 \times 10^5\ N}$$

$$A' = \int_0^c (\tau_\ell - \tau_u)\, dx = \int_0^1 (731\ x^{-0.2} + 288\ x^{-0.2})\, dx$$

$$A' = [1274\ x^{0.8}]_0^1 = \boxed{1274\ N}$$

$$L' = N' \cos\alpha - A' \sin\alpha = 1.12 \times 10^5 \cos 10^\circ - 1274 \sin 10^\circ$$

$$= \boxed{1.105 \times 10^5 N}$$

$$D' = N' \sin\alpha + A' \cos\alpha = 1.12 \times 10^5 \sin 10^\circ + 1274 \cos\alpha$$

$$= \boxed{2.07 \times 10^4 N}$$

$$M'_{LE} = \int_0^c [p_u - p_\ell]\, x\, dx = \int_0^1 [2 \times 10^4 (x-1)^2 - 1.19 \times 10^5]\, x\, dx$$

$$+ 2 \times 10^4 \left[\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1 - [0.595 \times 10^5 x^2]_0^1 = \boxed{-5.78 \times 10^4\ Nm}$$

$$M'_{c/4} = M'_{LE} + L'\,(c/4) = -5.78 \times 10^4 + 1.105 \times 10^5\ (0.25)$$

$$= \boxed{-3.02 \times 10^4\ N/m}$$

$$x_{cp} = -\frac{M'_{LE}}{N'} = -\frac{(-5.78\ \times\ 10^4)}{1.12\ \times\ 10^5} = \boxed{0.516\ m}$$

1.5

$c = c_n \cos\alpha - c_a \sin\alpha$

$= (1.2) \cos 12^\circ - (0.3) \sin\alpha = \boxed{1.18}$

$c_d = c_n \sin\alpha + c_a \cos\alpha$

$= (1.2) \sin 12^\circ + (0.3) \cos\alpha = \boxed{0.279}$

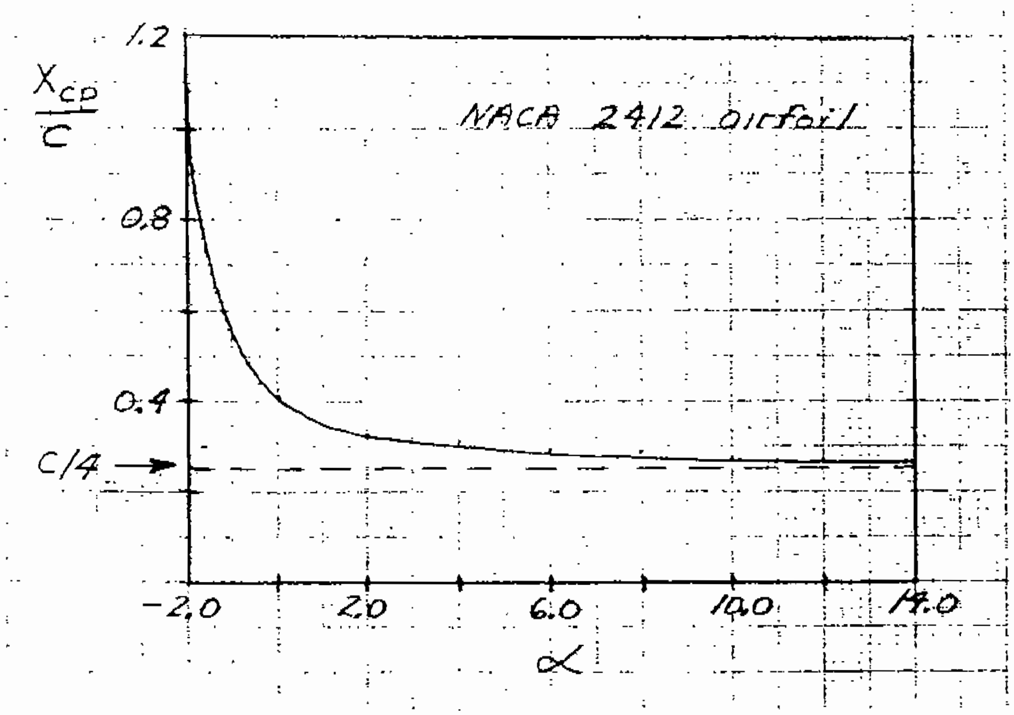
1.6 $c_n = c_\ell \cos\alpha + c_d \sin\alpha$

Also, using the more accurate N′ rather than L′ in Eq. (1.22), we have

$$x_{cp} = \frac{c}{4} - \frac{M'_{c/4}}{N'} = \frac{c}{4} - c\left(\frac{c_{m_{c/4}}}{c_n}\right)$$

<u>Hence:</u>

$\alpha(^\circ)$	c_n	x_{cp}/c
-2.0	0.0498	1.09
0	0.25	0.41
2.0	0.44	0.336
4.0	0.639	0.306
6.0	0.846	0.293
8.0	1.07	0.284
10.0	1.243	0.277
12.0	1.402	0.271
14.0	1.52	0.266



Note that x_{cp} moves forward as α is increased, and that it closely approaches the quarter-chord point in the range of α of 10° to 14°. At higher angles-of-attack, beyond the stall ($\alpha > 16^\circ$), x_{cp} will reverse its movement and move rearward as α continues to increase. Compare the above variation with the center-of-pressure measurements of the Wright Brothers on one of their airfoils, shown in Fig. 1.28.

1.7 $K = 3$ (mass, length, and time)

$$f_1 (D, \rho_\infty, V_\infty, c, g) = 0 \qquad \text{Hence } N = 5$$

We can write this expression in terms of $N - K = 5 - 3 = 2$ dimensionless Pi products:

$$f_2 (\Pi_1, \Pi_2)$$

where

$$\Pi_1 = f_3 (\rho_\infty, V_\infty, c, D)$$

$$\Pi_2 = f_4 (\rho_\infty, V_\infty, c, g)$$

Let $\Pi_1 = \rho_\infty^{a} \, V_\infty^{b} \, c^{d} \, D$

$$1 = (m\,\ell^{-3})^a\,(\ell\,t^{-1})^b\,\ell^c\,(m\,\ell\,t^{-2}) = 0$$

$$\left.\begin{array}{ll} \text{mass:} & a + 1 = 0 \\ \text{length:} & -3a + b + c + 1 = 0 \\ \text{time:} & -b - 2 = 0 \end{array}\right\} \quad \begin{array}{l} a = -1 \\ b = -2 \\ c = -2 \end{array}$$

Hence:

$$\Pi_1 = \frac{D}{\rho_\infty V_\infty^{\,2} c^2}, \text{ or } \Pi_1 = \frac{D}{\frac{1}{2}\rho_\infty V_\infty^{\,2} c^2}$$

$$\Pi_1 = \frac{D}{q_\infty c^2}$$

Let $\Pi_2 = \rho_\infty^{\,2}\ V_\infty\, c^b\, g^d$

$$1 = (m\,\ell^{-3})^a\,(\ell\,t^{-1})\,\ell^b\,(\ell\,t^{-2})^d = 0$$

$$\begin{array}{lll} \text{mass:} & a = 0 & a = 0 \\ \text{length:} & -3a + 1 + b + d = 0 & d = -1/2 \\ \text{time:} & -1 - 2d = 0 & b = -1/2 \end{array}$$

Hence:

$$\Pi_2 = \frac{V_\infty}{\sqrt{cg}}$$

Thus:

$$f_2\,(\Pi_1, \Pi_2) = f_2\left(\frac{D}{q_\infty c^2}, \frac{V_\infty}{\sqrt{cg}}\right) = 0$$

or:

$$\boxed{C_D = f(F_r)}$$

1.8 $\quad D_w = f_1\,(\rho_\infty, V_\infty, c, a_\infty, c_p, c_v)$

$K = 4$ (mass, length, time, degrees)

$$f_2(D_w, \rho_\infty, V_\infty, c, a_\infty, c_p, c_v) = 0$$

Hence, N = 7. This can be written as a function of N – K = 7 – 4 = 3 pi products:

$$f_3 = (\Pi_1, \Pi_2, \Pi_3) = 0$$

where:

$$\Pi_1 = f_4(\rho_\infty, V_\infty, c, c_p, D)$$

$$\Pi_2 = f_5(\rho_\infty, V_\infty, c, c_p, a_\infty)$$

$$\Pi_3 = f_6(\rho_\infty, V_\infty, c, c_p, c_v)$$

The dimensions of c_p and c_v are

$$[c_p] = \frac{\text{energy}}{\text{mass}(^\circ)} = \frac{(\text{force})(\text{distance})}{\text{mass}(^\circ)} = \frac{(m\ell t^{-2})(\ell)}{m(^\circ)}$$

$[c_p] = \ell^2 t^{-2} (^\circ)^{-1}$ where $(^\circ)$ degrees.

For Π_1:

$$\rho_\infty^{\,i} V_\infty^{\,j} c^k c_p^{\,n} D = \Pi_1$$

$$(m\,\ell^{-3})^i (\ell\, t^{-1})^j (\ell)^k (\ell^2 t^{-2})^n (^\circ)^{-n} (m\,\ell\, t^{-2}) = 1$$

mass: $i + 1 = 0$ $\qquad$ $i = -1$

length: $-3i + j + k + 2n + 1 = 0$ $\qquad$ $n = 0$

time: $-j - 2n - 2 = 0$ $\qquad$ $j = -2$

degrees: $-n = 0$ $\qquad$ $k = -2$

Hence:

$$\Pi_1 = \frac{D}{\rho_\infty V_\infty^2 c^2}, \text{ or } \Pi_1 = \frac{D}{q_\infty c}$$

For Π_2:

$$\Pi_2 = \rho_\infty^{\,i} V_\infty c^j c_p^{\,k} a_\infty^{\,n}$$

$$1 = (m\ \ell^{-3})^i\ (\ell\ t^{-1})\ (\ell)^j\ (\ell^2 t^{-2})^k\ (^\circ)^{-k}\ (\ell\ t^{-1})^n$$

$$\left.\begin{array}{ll} \text{mass:} & i = 0 \\ \text{length:} & -3i + 1 + j + 2k + n = 0 \\ \text{time:} & -1 - 2k - n = 0 \\ \text{degrees:} & -k = 0 \end{array}\right\} \quad \begin{array}{l} i = 0 \\ k = 0 \\ n = -1 \\ j = 0 \end{array}$$

Hence:

$$\Pi_2 = \frac{V_\infty}{a_\infty}$$

For Π_3:

$$\Pi_3 = \rho_\infty^{\ i}\ V_\infty^{\ j}\ c^k\ c_p^{\ n}\ c_v$$

$$1 = (m\ \ell^{-3})^i\ (\ell\ t^{-1})^j\ \ell^k\ (\ell^2 t^{-2})^n\ (^\circ)^{-n}\ (\ell\ t^{-2})\ (^\circ)^{-1}$$

$$\left.\begin{array}{ll} \text{mass:} & i = 0 \\ \text{length:} & -3i + j + k + 2n + 2 = 0 \\ \text{time:} & -j - 2n - 2 = 0 \\ \text{degrees:} & -n - 1 = 0 \end{array}\right\} \quad \begin{array}{l} i = 0 \\ n = -1 \\ j = 0 \\ k = 0 \end{array}$$

Hence:

$\Pi_3 = \dfrac{c_v}{c_p}$. We can take the reciprocal, and still have a dimensionless product.

Hence,

$$\Pi_3 = \frac{c_v}{c_p} = \gamma$$

Thus,

$$f_3\left(\frac{D}{q_\infty S}, \frac{V_\infty}{a_\infty}, \frac{c_p}{c_v}\right)$$

or,

$\boxed{C_D = f(M_\infty, \gamma)}$

1.9 $$\frac{M_1}{M_2} = \frac{V_1}{V_2}\frac{a_2}{a_1} = \frac{V_1}{V_2}\sqrt{\frac{T_2}{T_1}} = \frac{100}{200}\sqrt{\frac{800}{200}} = 1$$

Hence, the Mach numbers of the two flows are the same.

$$\frac{Re_1}{Re_2} = \frac{\rho_1 V_1 c_1}{\rho\ V_2 c_2}\left(\frac{\mu_2}{\mu_1}\right) = \frac{\rho_1 V_1 c_1}{\rho\ V_2 c_2}\sqrt{\frac{T_2}{T_1}} = \left(\frac{1.23}{1.739}\right)\left(\frac{100}{200}\right)\left(\frac{1}{2}\right)\sqrt{\frac{800}{200}} = 0.354$$

The Reynold's numbers are <u>different</u>. Hence, the two flows are <u>not</u> dynamically similar.

1.10 Denote free flight by subscript 1, and the wind tunnel by subscript 2. For the lift and drag coefficients to be the same in both cases, the flows must be <u>dynamically similar</u>. Hence

$$M_1 = M_2$$

and

$$Re_1 = Re_2$$

For Mach number:

$$\frac{V_1}{a_1} = \frac{V_2}{a_2}$$

Since $a \propto \sqrt{T}$, we have

$$\frac{V_2}{\sqrt{T_2}} = \frac{V_1}{\sqrt{T_1}} = \frac{250}{\sqrt{223}} = 16.7 \qquad (1)$$

For Reynolds number: $\dfrac{\rho_1 V_1 c_1}{\mu_1} = \dfrac{\rho_2 V_2 c}{\mu_2}$

Assume, as before, that $\mu \propto \sqrt{T}$. Hence

$$\frac{\rho_2 V_2 c_2}{\sqrt{T_2}} = \frac{\rho_1 V_1 c_1}{\sqrt{T_1}}$$

or,

$$\frac{\rho_2 V_2}{\sqrt{T_1}} = \frac{\rho_1 V_1}{\sqrt{T_1}}\left(\frac{c_1}{c_2}\right) = \frac{(0.414)(250)}{223}\left(\frac{5}{1}\right)$$

or,

$$\frac{\rho_2 V_2}{\sqrt{T_2}} = 34.65 \qquad (2)$$

Finally, from the equation of state:

$$\rho_2 T_2 = \frac{p_2}{R} = \frac{1.01 \times 10^5}{287} = 351.9 \qquad (3)$$

Eqs. (1) – (3) represent three equations for the three unknowns, ρ_2, V_2, and T_2. They are summarized below:

$$\frac{V_2}{\sqrt{T_2}} = 1.67 \qquad (1)$$

$$\frac{\rho_2 V_2}{\sqrt{T_2}} = 34.65 \qquad (2)$$

$$\rho_2 T_2 = 351.9 \qquad (3)$$

From Eq. (3):

$$\rho_2 = 351.9/T_2 \qquad (4)$$

Subst. (4) into (2):

$$\frac{351.9}{T_2}\left(\frac{V_2}{\sqrt{T_2}}\right) = 34.65 \ (5)$$

Subst. (1) into (5): $\frac{351.9}{T_2}(16.7) = 34.65$

Hence,

$$T_2 = \frac{(351.9)(16.7)}{(34.65)} = \boxed{169.6^{\circ}K}$$

From Eq. (1): $V_2 = 16.7\sqrt{T_2} = 16.7\ \sqrt{169.6} = \boxed{217.5\ \frac{m}{sec}}$

From Eq. (3): $\rho_2 = \frac{351.9}{T_2} = \frac{351.9}{169.6} = \boxed{2.07\frac{kg}{m^3}}$

1.11 $p_b = p_a - \rho\ g\ \Delta\ h$

$= 1.01 \times 10^5 - (1.36 \times 10^4)(9.8)(0.2)$

$p_b = \boxed{7.43 \times 10^4\ N/m^2}$

1.12 Weight = Buoyancy force + lift

$W = B + L$

$$B = \underbrace{(15{,}000)}_{\substack{\text{volume}\\(m^3)}}\ \underbrace{(1.1117)}_{\substack{\text{air density}\\\text{at 1000m}\\(kg/m^3)}}\ \underbrace{(9.8)}_{\substack{\text{acceleration}\\\text{of gravity}\\(m/sec)}} = 1.634 \times 10^5 N$$

$$q_\infty = \frac{1}{2}\ \rho_\infty\ V_\infty{}^2 = \frac{1}{2}\ (1.1117)\ (30)^2 = 500\ N/m^2$$

$S = \pi d^2/4 = \pi(14)^2/4 = 153.9\ m^2$

$L = q_\infty\ S\ C_L = (500)(153.9)(0.05) = 3487\ N$

Hence:

$W = 1.634 \times 10^5 + 3847 = \boxed{1.67 \times 10^5 N}$

1.13 Let us use the formalism surrounding Eq. (1.16) in the text. In this case, $c_d = c_a$, and from Eq. (1.16), neglecting skin friction

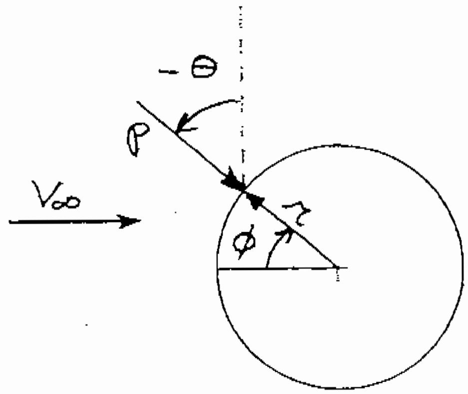
$$c_d = \frac{1}{c} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) dy \tag{1}$$

From Eq. (1.13) in the text, Eq. (1) above can be written as

$$c_d = \frac{1}{c} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) (-\sin\theta\, ds) \tag{2}$$

Draw a picture:

Following our sign convention, note that θ is drawn counterclockwise in this sketch, hence it is a <u>negative</u> angle, -θ.



From the geometry:

$$-\theta = \pi - \phi$$

Hence, $\sin(-\theta) = -\sin\theta = \sin(\pi - \theta) = \cos\phi$

Substitute this into Eq. (2), noting also that $ds = r d\phi$ and the chord c is twice the radius, $c = 2r$. From Eq. (2),

$$c_d = \frac{1}{2r} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) \cos\phi\, r\, d\phi$$

$$c_d = \frac{1}{2} \int_{LE}^{TE} \left(C_{p_u} - C_{p_\ell}\right) \cos\phi\, d\phi$$

$$c_d = \frac{1}{2} \int_{LE}^{TE} C_{p_u} \cos\phi\, d\phi - \frac{1}{2} \int_{LE}^{TE} C_{p_\ell} \cos\phi\, d\phi \tag{3}$$

Consider the limits of integration for the above integrals. The first integral is evaluated from the leading edge to the trailing edge along the <u>upper</u> surface. Hence, $\phi = 0$ at LE and π at TE.

The second integral is evaluated from the leading edge to the trailing edge along the <u>bottom</u> surface. Hence, $\phi = 2\pi$ at LE and π at the TE. Thus, Eq. (3) becomes

$$c_d = \frac{1}{2} \int_0^{\pi} C_{p_u} \cos\phi \, d\phi \quad - \frac{1}{2} \int_{2\pi}^{\pi} C_{p_\ell} \cos\phi \, d\phi \qquad (4)$$

In Eq. (4),

$$C_{p_u} = 2\cos^2\phi \qquad \text{for } 0 \le \phi \le \pi/2$$

$$C_{p_u} = 0 \qquad \text{for } \frac{\pi}{2} \le \phi \le \pi$$

$$C_{p_\ell} = 2\cos^2\phi \qquad \text{for } \frac{3\pi}{2} \le \phi \le 2\pi$$

$$C_{p_\ell} = 0 \qquad \text{for } \pi \le \phi \le \frac{3\pi}{2}$$

Thus, Eq. (4) becomes

$$c_d = \int_0^{\pi/2} \cos^3\phi \, d\phi - \int_{2\pi}^{3\pi/2} \cos^3\phi \, d\phi$$

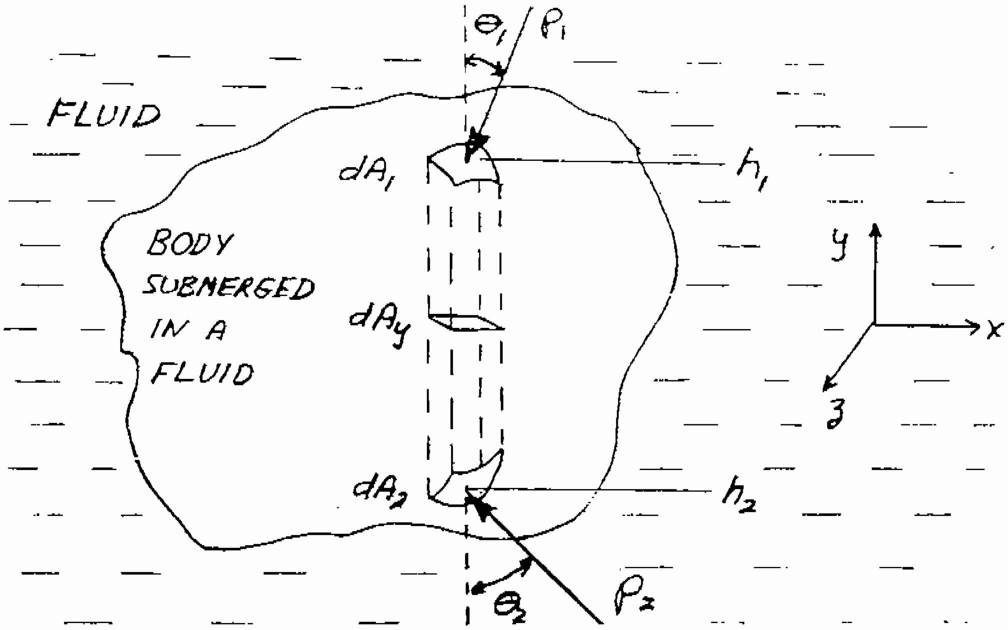
Since $\cos^3\phi \, d\phi = (\frac{1}{3} \sin\phi)(\cos^2\phi + 2)$, Eq. (5) becomes

$$c_d = [(\frac{1}{3} \sin\phi)(\cos^2\phi + 2)]_0^{\pi/2} - [(\frac{1}{3} \sin\phi)(\cos^2\phi + 2]_{2\pi}^{3\pi/2}$$

$$c_d = (\frac{1}{3})(1)(2) - (\frac{1}{3})(-1)(2)$$

$$\boxed{c_d = 4/3}$$

1.14



Consider the arbitrary body sketched above. Consider also the vertical cylinder element inside the body which intercepts the surface area dA_1 near the top of the body, and dA_2 near the bottom of the body. The pressures on dA_1 and dA_2 are p_1 and p_2 respectively, and makes angles θ_1 and θ_2 respectively with respect to the vertical line through the middle of dA_1 and dA_2. The net pressure force in the y-direction on this cylinder is:

$$dF_y = -\, p_1 \cos\theta_1\, dA_1 + p_2 \cos\theta_2\, dA_2 \tag{1}$$

Let dA_y be the projection of dA_1 and dA_2 on a plane perpendicular to the y axis.

$$dA_y = \cos\theta_1\, dA_1 = \cos\theta_2\, dA_2$$

Thus, Eq. (1) becomes

$$dF_y = (p_2 - p_1)\, dA_y \tag{2}$$

From the hydrostatic equation

$$p_2 - p_1 = \int_{h_1}^{h_2} \rho\, g\, dy \tag{3}$$

Combining Eqs. (2) and (3),

$$dF_y = \int_{h_1}^{h_2} \rho\, g\, dy\, dA_y \tag{4}$$

However, $dy\, dA_y = d\mathcal{V}$ = element of volume of the body. Thus, the total force in the y direction, F_y, is given by Eq. (4) integrated over the volume of the body

$$\underbrace{F_y}_{\text{Force on body}} = \underbrace{\oiiint \rho g \, dV}_{\text{Weight of fluid displaced by body.}}$$

1.15 From Eq. (1.45)

$$C_L = \frac{L}{q_\infty S} = \frac{2W}{\rho_\infty V_\infty^{\,2} S} = \frac{2(2950)}{(0.002377)V_\infty^{\,2}(174)}$$

$$C_L = \frac{14265}{V_\infty^{\,2}} \qquad (1)$$

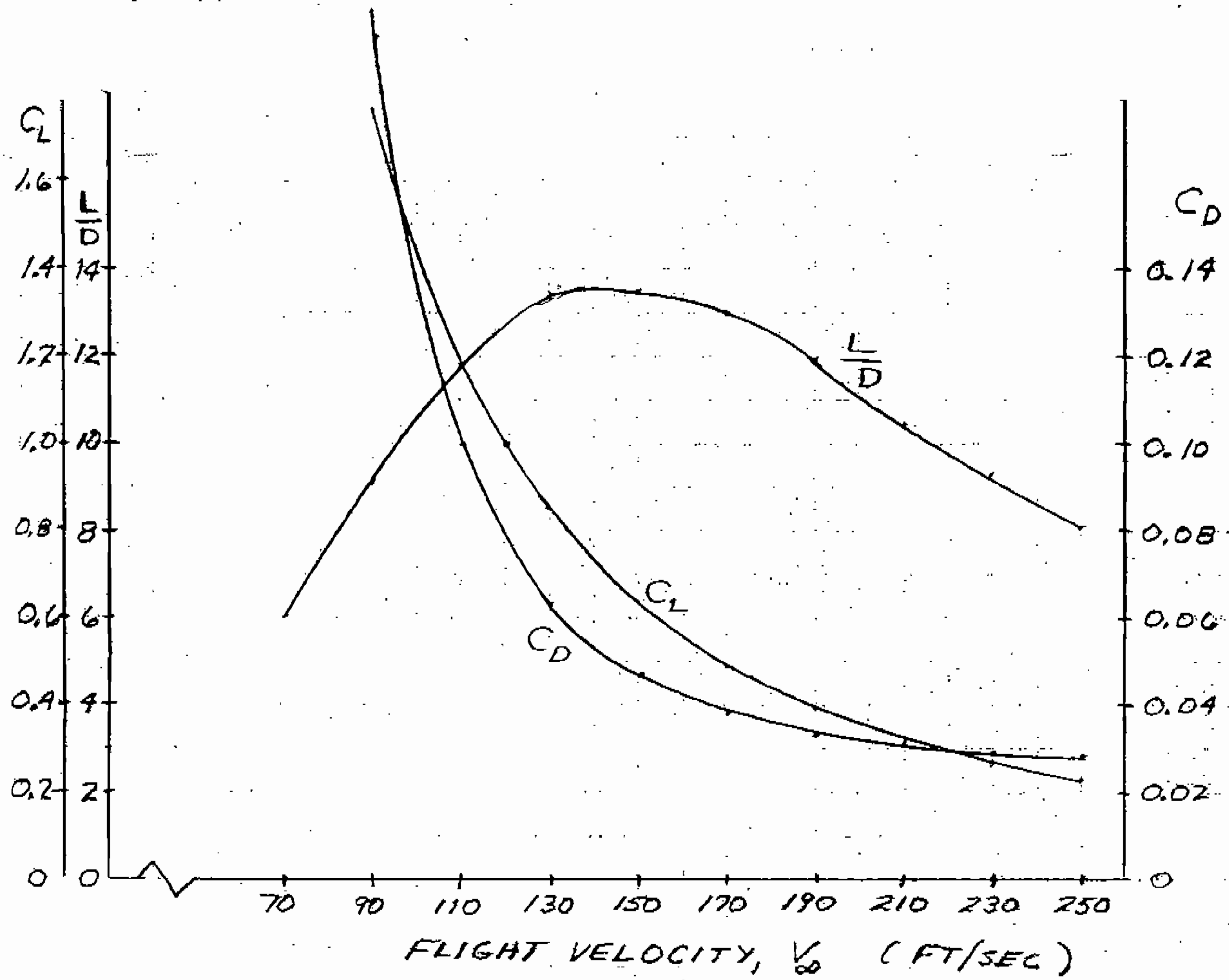
Also,

$$C_D = 0.025 + 0.054\, C_L^{\,2} \qquad (2)$$

Tabulate Eqs. (1) and (2) versus velocity.

V_∞ (ft/sec)	C_L	C_D	$\frac{L}{D} = \frac{C_L}{C_D}$
70	2.911	0.483	6.03
90	1.761	0.192	9.17
110	1.179	0.100	11.79
130	0.844	0.063	13.40
150	0.634	0.047	13.49
170	0.494	0.038	13.0
190	0.395	0.033	11.97
210	0.323	0.031	10.42
230	0.270	0.029	9.31
250	0.228	0.028	8.14

These results are plotted on the next page.



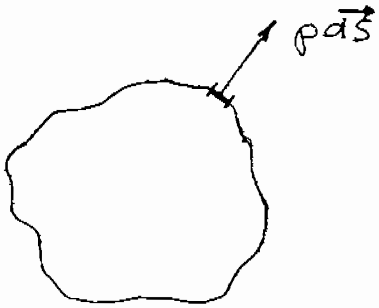
Examining this graph, we note, for steady, level flight:

1. The lift coefficient decreases as V_∞ increases.
2. At lower velocity range, the drag coefficient decreases even faster than the lift coefficient with velocity. (Note that on the graph the scale for C_D is one-tenth that for C_L.)
3. As a result, the lift-to-drag ratio first increases, goes through a maximum, and then gradually decreases as velocity increases.

It can be shown that the maximum velocity for this airplane is about 265 ft/sec at sea level. As seen in the graph, the maximum value of L/D occurs around V_∞ = 140 ft/sec, which is much lower than the maximum velocity. However, at higher velocity the value of L/D decreases only gradually as V_∞ increases. This has the practical implication that at higher speeds, even though the value of L/D is less than its maximum, it is still a reasonably high value. The range of the aircraft is proportional to L/D (see for example, Anderson, <u>Aircraft Performance and Design</u>, McGraw-Hill, 1999, or Anderson, <u>Introduction to Flight</u>, 4th ed., McGraw-Hill, 2000). To obtain maximum range, the airplane should fly at the velocity for maximum L/D, which for this case is 140 ft/sec. However, one reason to fly in an airplane is to get from one place to another in a reasonably short time. By flying at the low velocity of $V_\infty = 140$ ft/sec, the flight time may be unacceptably long. By cruising at a higher speed, say 200 ft/sec, the flight time will be cut by 30%, with only an 18% decrease in L/D.

CHAPTER 2

2.1



$$\vec{F} = -\oiint_S p\,d\vec{S}$$

If p = constant = p_∞

$$\vec{F} = -p_\infty \oiint_S p\,d\vec{S} \quad (1)$$

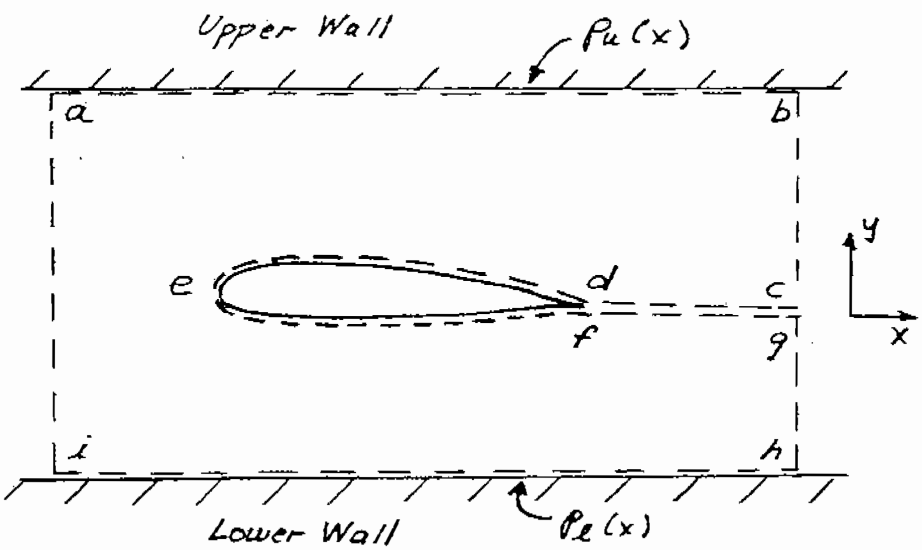
However, the integral of the surface vector over a closed surface is zero, i.e.,

$$\oiint_S d\vec{S} = 0$$

Hence, combining Eqs. (1) and (2), we have

$$\boxed{\vec{F} = 0}$$

2.2



Denote the pressure distributions on the upper and lower walls by $p_u(x)$ and $p_\ell(x)$ respectively. The walls are close enough to the model such that p_u and p_ℓ are not necessarily equal to p_∞. Assume that faces ai and bh are far enough upstream and downstream of the model such that

$$p = p_\infty \qquad \text{and } v = 0 \qquad \text{and ai and bh.}$$

Take the y-component of Eq. (2.66)

$$L = -\oiint_S (\rho\vec{V}\cdot\vec{dS})\, v - \iint_{abhi} (p\vec{dS})y$$

The first integral = 0 over all surfaces, either because $\vec{V}\cdot\vec{ds} = 0$ or because $v = 0$. Hence

$$L' = -\iint_{abhi} (p\vec{dS})y = -\left[\int_a^b p_u\,dx - \int_i^h p_\ell\,dx\right]$$

Minus sign because y-component is in downward Direction.

Note: In the above, the integrals over ia and bh cancel because $p = p_\infty$ on both faces. Hence

$$\boxed{L' = \int_i^h p_\ell\,dx - \int_a^b p_u\,dx}$$

2.3 $$\frac{dy}{dx} = \frac{v}{u} = \frac{cy/(x^2+y^2)}{cx/(x^2+y^2)} = \frac{y}{x}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$\ell n\, y = \ell n\, x + c_1 = \ell n\,(c_2\, x)$$

$$\boxed{y = c_2\, x}$$

The streamlines are straight lines emanating from the origin. (This is the velocity field and streamline pattern for a source, to be discussed in Chapter 3.)

2.4 $$\frac{dy}{dx} = \frac{v}{u} = -\frac{x}{y}$$

$$y\,dy = -\,x\,dx$$

$$y^2 = -x^2 + \text{const}$$

$$x^2 + y^2 = \text{const.}$$

The streamlines are concentric with their centers at the origin. (This is the velocity field and streamline pattern for a vortex, to be discussed in Chapter 3.)

2.5 From inspection, since there is no radial component of velocity, the streamlines must be circular, with centers at the origin. To show this more precisely,

$$u = - V_\theta \sin = - cr \frac{y}{r} = - cy$$

$$v = V_\theta \cos \theta = cr \frac{x}{r} = cx$$

$$\frac{dy}{dx} = \frac{v}{u} = -\frac{x}{y}$$

$$\boxed{y^2 + x^2 = \text{const.}}$$

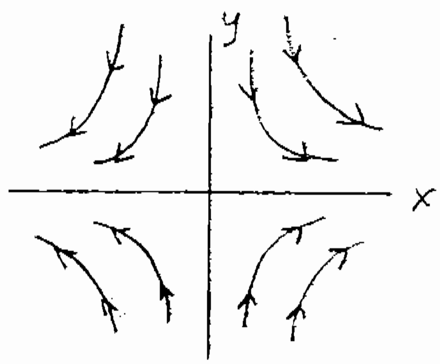
This is the equation of a circle with the center at the origin. (This velocity field corresponds to solid body rotation.)

2.6 $$\frac{dy}{dx} = \frac{v}{u} = -\frac{y}{x}$$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$\ell n\, y = x\ \ell n\, x + c_1$$

$$y = c_2/x$$



The streamlines are hyperbolas.

2.7 (a) $\frac{1}{\delta v}\frac{D(\delta v)}{Dt} = \nabla \cdot \vec{V}$

In polar coordinates: $\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(r\, V_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$

Transformation: $x = r \cos \theta$

$y = r \sin \theta$

$V_r = u \cos \theta + v \sin \theta$

$V_\theta = - u \sin \theta + v \cos \theta$

$$u = \frac{cx}{(x^2+y^2)} = \frac{cr\ \cos\theta}{r^2} = \frac{c\ \cos\theta}{r}$$

$$v = \frac{cy}{(x^2+y^2)} = \frac{cr\ \sin\theta}{r^2} = \frac{c\ \sin\theta}{r}$$

$$V_r = \frac{c}{r}\ \cos^2\theta + \frac{c}{r}\ \sin^2\theta = \frac{c}{r}$$

$$V_\theta = -\frac{c}{r}\ \cos\theta\ \sin\theta + \frac{c}{r}\ \cos\theta\ \sin\theta = 0$$

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}\ (c) + \frac{1}{r}\frac{\partial(0)}{\partial\theta} = 0$$

(b) From Eq. (2.23)

$$\nabla \times \vec{V} = e_z \left[\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} - \frac{1}{r}\frac{\partial V_r}{\partial\theta}\right]$$

$$\nabla \times V = e_z\ [0 + 0 - 0] = \boxed{0}$$

The flowfield is irrotational.

2.8 $$u = \frac{cy}{(x^2+y^2)} = \frac{cr\ \sin\theta}{r^2} = \frac{c\ \sin\theta}{r}$$

$$v = \frac{-cx}{(x^2+y^2)} = \frac{cr\ \cos\theta}{r^2} = -\frac{c\ \cos\theta}{r}$$

$$V_r = \frac{c}{r}\ \cos\theta\ \sin\theta - \frac{c}{r}\ \cos\theta\ \sin\theta = 0$$

$$V_\theta = -\frac{c}{r}\ \sin^2\theta - \frac{c}{r}\ \cos^2\theta = -\frac{c}{r}$$

(a) $$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}\ (0) + \frac{1}{r}\frac{\partial(-c/r)}{\partial\theta} = 0 + 0 = \boxed{0}$$

(b) $$\nabla \times \vec{V} = \vec{e}_z \left[\frac{\partial(-c/r)}{\partial r} - \frac{c}{r^2} - \frac{1}{r}\frac{\partial(0)}{\partial \theta} \right]$$

$$= \vec{e}_z \left[\frac{c}{r^2} - \frac{c}{r_2} - 0 \right]$$

$\nabla \times \vec{V} = \boxed{0}$ <u>except</u> at the origin, where r = 0. The flowfield is singular at the origin.

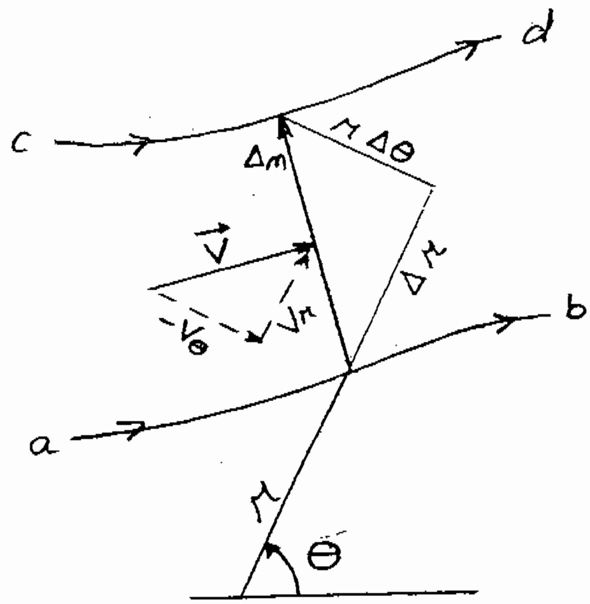
2.9 $V_r = 0.$ $V_\theta = c\,r$

$$\nabla \times \vec{V} = \vec{e}_z \left[\frac{\partial(c/r)}{\partial r} + \frac{cr}{r} - \frac{1}{r}\frac{\partial(0)}{\partial \theta} \right]$$

$$= \vec{e}_z \ (c + c - 0) = 2c\ \vec{e}_z$$

The vorticity is <u>finite</u>. The flow is <u>not</u> irrotational; it is <u>rotational</u>.

2.10



Mass flow between streamlines = $\Delta \bar{\psi}$

$$\Delta \bar{\psi} = \rho V \Delta n$$

$$\Delta \bar{\psi} = (-\rho V_\theta) \Delta r + \rho V_r (r\theta)$$

Let cd approach ab

$$d\psi = - \rho V_\theta dr + \rho r V_r d\theta \qquad (1)$$

Also, since $\bar{\psi} = \bar{\psi}(r,\theta)$, from calculus

$$d\bar{\psi} = \frac{\partial \bar{\psi}}{\partial r} dr + \frac{\partial \bar{\psi}}{\partial \theta} d\theta \qquad (2)$$

Comparing Eqs. (1) and (2)

$$-\rho V_\theta = \frac{\partial \bar{\psi}}{\partial r}$$

and

$$\rho r V_r = \frac{\partial \bar{\psi}}{\partial \theta}$$

or:

$$\rho V_r = \frac{1}{r} \frac{\partial \bar{\psi}}{\partial \theta}$$

$$\rho V_\theta = - \frac{\partial \bar{\psi}}{\partial r}$$

2.11 $u = cx = \dfrac{\partial \psi}{\partial y} : \psi = cxy + f(x)$ (1)

$$v = - cy = - \frac{\partial \psi}{\partial x} : \psi = cxy + f(y) \qquad (2)$$

Comparing Eqs. (1) and (2), f(x) and f(y) = constant

$$\boxed{\psi = c\,x\,y + \text{const.}} \tag{3}$$

$$u = cx = \frac{\partial \psi}{\partial x}\ :\ \phi = cx^2 + f(y) \tag{4}$$

$$v = -\,cy = \frac{\partial \psi}{\partial y}\ :\ \phi = -\,cy^2 + f(x) \tag{5}$$

Comparing Eqs. (4) and (5), $f(y) = -\,cy^2$ and $f(x) = cx^2$

$$\boxed{\phi = c\,(x^2 - y^2)} \tag{6}$$

Differentiating Eq. (3) with respect to x, holding ψ = const.

$$0 = cx\,\frac{dy}{dx} + cy$$

or,

$$\left(\frac{dy}{dx}\right)_{\psi=\text{const}} = -\,y/x \tag{7}$$

Differentiating Eq. (6) with respect to x, holding ϕ = const.

$$0 = 2\,c\,x - 2\,c\,y\,\frac{dy}{dx}$$

or,

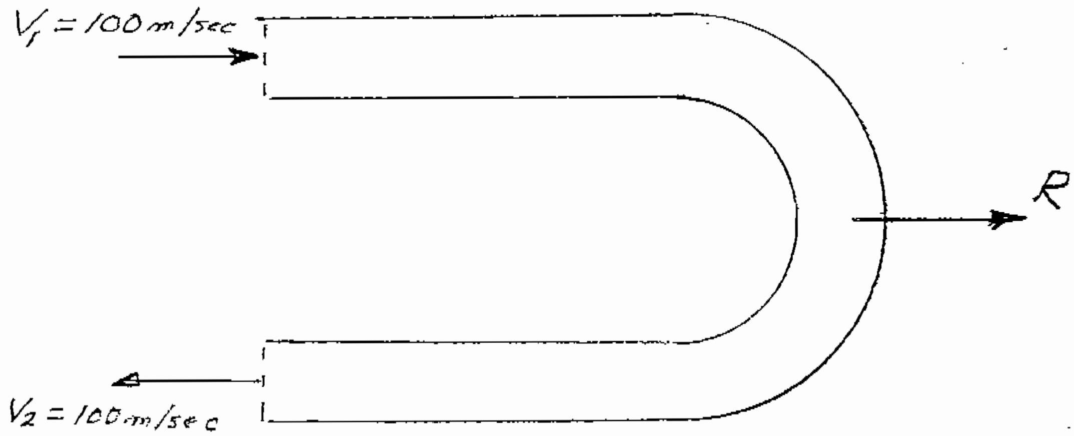
$$\left(\frac{dy}{dx}\right)_{\phi=\text{const}} = x/y \tag{8}$$

Comparing Eqs. (7) and (8), we see that

$$\left(\frac{dy}{dx}\right)_{\psi=\text{const}} = -\,\frac{1}{\left(\frac{dy}{dx}\right)_{\phi=\text{const}}}$$

Hence, lines of constant ψ are perpendicular to lines of constant ϕ.

2.12. The geometry of the pipe is shown below.



As the flow goes through the U-shape bend and is turned, it exerts a net force R on the internal surface of the pipe. From the symmetric geometry, R is in the horizontal direction, as shown, acting to the right. The equal and opposite force, -R, exerted by the pipe on the flow is the mechanism that reverses the flow velocity. The cross-sectional area of the pipe inlet is $\pi d^2/4$ where d is the inside pipe diameter. Hence, $A = \pi d^2/4 = \pi(0.5)^2/4 = 0.196 m^2$. The mass flow entering the pipe is

$$\dot{m} = \rho_1 A V_1 = (1.23)(0.196)(100) = 24.11 \text{ kg/sec.}$$

Applying the momentum equation, Eq. (2.64) to this geometry, we obtain a result similar to Eq. (2.75), namely

$$R = -\oiint (\rho \mathbf{V} \cdot \mathbf{dS}) \mathbf{V} \qquad (1)$$

Where the pressure term in Eq. (2.75) is zero because the pressure at the inlet and exit are the same values. In Eq. (1), the product (ρ **V** · **dS**) is negative at the inlet (**V** and **dS** are in opposite directions), and is positive at the exit (**V** and **dS**) are in the same direction). The magnitude of ρ **V** · **dS** is simply the mass flow, $\dot{m}$. Finally, at the inlet V_1 is to the right, hence it is in the positive x-direction. At the exit, V_2 is to the left, hence it is in the negative x-direction. Thus, $V_2 = -V_1$. With this, Eq. (1) is written as

$$R = -[-\dot{m} V_1 + \dot{m} V_2] = \dot{m} (V_1 - V_2)$$

$$= \dot{m} [V_1 - (-V_1)] = \dot{m} (2V_1)$$

$$R = (24.11)(2)(100) = \boxed{4822 \text{ N}}$$

CHAPTER 3

3.1 Consider steady, inviscid flow.

x-momentum: $$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = - \frac{\partial p}{\partial x} \tag{1}$$

y-momentum: $$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} = - \frac{\partial p}{\partial y} \tag{2}$$

z-momentum: $$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = - \frac{\partial p}{\partial z} \tag{3}$$

Multiply (1), (2), and (3) by dx, dy, and dz respectively:

$$u \frac{\partial u}{\partial x} dx + v \frac{\partial u}{\partial y} dx + w \frac{\partial u}{\partial z} dx = - \frac{1}{\rho} \frac{\partial p}{\partial x} dx \tag{4}$$

$$u \frac{\partial v}{\partial x} dy + v \frac{\partial v}{\partial y} dy + w \frac{\partial v}{\partial z} dy = - \frac{1}{\rho} \frac{\partial p}{\partial y} dy \tag{5}$$

$$u \frac{\partial w}{\partial x} dz + v \frac{\partial w}{\partial y} dz + w \frac{\partial w}{\partial z} dz = - \frac{1}{\rho} \frac{\partial p}{\partial y} dz \tag{6}$$

Add (4) + (5) + (6):

$$u\left(\frac{\partial u}{\partial x} dx + \frac{\partial v}{\partial x} dy + \frac{\partial w}{\partial x} dz\right) + v\left(\frac{\partial u}{\partial y} dx + \frac{\partial v}{\partial y} dy + \frac{\partial w}{\partial y} dz\right)$$

$$+ w\left(\frac{\partial u}{\partial z} dx + \frac{\partial v}{\partial z} dy + \frac{\partial w}{\partial z} dz\right) = - \frac{1}{\rho}\left(\frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz\right) \tag{7}$$

For irrotational flow (see Eq. (2.119)): $\nabla \times V = 0$

Hence:

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}; \quad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}; \quad \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \tag{8}$$

Subt. Eqs. (8) into (7):

$$u\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz\right) + v\left(\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy + \frac{\partial v}{\partial z}dz\right)$$

$$+ w\left(\frac{\partial w}{\partial z}dx + \frac{\partial w}{\partial z}dy + \frac{\partial w}{\partial z}dz\right) = -\frac{1}{\rho}\left(\frac{\partial p}{\partial z}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz\right)$$

$$u\,du + v\,dv + w\,dw = -\frac{1}{\rho}\,dp$$

$$\frac{1}{2}d\,(u^2 + v^2 + w^2) = \frac{1}{2}d\,(V^2) = VdV = -\frac{1}{\rho}\,dp$$

$dp = -\rho V\,dV$ which integrates to

$$\boxed{p + \frac{1}{2}\rho V^2 = \text{const.}}$$

for incompressible flow.

3.2 $$V_1 = \sqrt{\frac{(2)(p_1 - p_2)}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

$p_1 = 2116$ lb/ft^2; $p_2 = 2100$ lb/ft^2, $A_2/A_1 = 0.8$

$$V_1 = \sqrt{\frac{2(2116 - 2100)}{(0.002377)\left[\left(\frac{1}{0.8}\right)^2 - 1\right]}} = \boxed{154.7 \text{ ft/sec}}$$

3.3 $$p_1 - p_2 = \tfrac{1}{2}\rho V_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] = \tfrac{1}{2}(1.23)(90)^2\left[(1/0.85)^2 - 1\right] = \boxed{1913 \text{ N/m}^2}$$

3.4 $$V_2 = \sqrt{\frac{2\,w\,\Delta h}{\rho\left[1 - (A_2/A_1)^2\right]}}$$

$$w = \rho_m g = (1.36 \times 10^4)\,(9.8\ \frac{m}{sec^2}) = 1.33 \times 10^5\ N/m^2$$

$$\Delta h = 10\ cm = 0.1m;\ \rho = 1.23\ kg/m^3,\ \frac{A_2}{A_1} = \frac{1}{12}$$

$$V_2 = \sqrt{\frac{2(1.33\ \times\ 10^5)(0.1)}{(1.23)\left[1-\left(\frac{1}{12}\right)^2\right]}} = \boxed{147\ m/sec}$$

3.5 $\quad p_1 - p_2 = w\,\Delta h = (1.33 \times 10^5)(0.1) = 1.33 \times 10^4\ N/m^2$

$$p_2 = p_1 - 1.33 \times 10^4 = 1.01 \times 10^5 - 1.33 \times 10^4 = 8.77 \times 10^4\ N/m^2$$

$$p_o = p_2 + \frac{1}{2}\,\rho\ V_2^2 = 8.77 \times 10^4 + \frac{1}{2}\ (1.23)(147)^2 = \boxed{1.01 \times 10^5\ N/m^2}$$

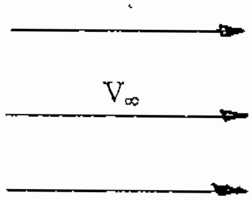
Note: It makes sense that the total pressure in the test section would equal one atmosphere, because the flow in the tunnel is drawn directly from the open ambient surroundings, and for an inviscid flow, we have no losses between the inlet and the test section.

3.6 $\quad p_o = p_\infty + \frac{1}{2}\,\rho\, V_\infty^{\ 2}$

$$V_\infty = \sqrt{\frac{2(p_o - p_\infty)}{\rho}} = \sqrt{\frac{2(1.07 - 1.01)\ \times\ 10^5}{1.23}} = \boxed{98.8\ \frac{m}{sec}}$$

3.7 $\quad C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{130}{98.8}\right)^2 = \boxed{-0.73}$

3.8



$\vec{V} = V_\infty\,\vec{i}$ $\qquad$ $V_\infty = u = \text{constant}$

$$\nabla \cdot \vec{V} = \overset{0}{\cancel{\frac{\partial u}{\partial x}}} + \overset{0}{\cancel{\frac{\partial v}{\partial y}}} + \overset{0}{\cancel{\frac{\partial w}{\partial z}}} = \boxed{0}$$

It is a physically possible incompressible flow.

$$\nabla \times V = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & \cancelto{0}{v} & \cancelto{0}{w} \end{vmatrix} = \vec{i}\ (0\text{-}0) - \vec{j}\ (0 - \cancelto{0}{\frac{\partial u}{\partial x}}) + \vec{k}\ (0 - \cancelto{0}{\frac{\partial u}{\partial y}})$$

$\boxed{\nabla \times \vec{V} = 0}$ The flow is irrotational.

3.9 For a source flow,

$$\vec{V} = V_r\ \vec{e}_r = \frac{\Lambda}{2\pi r}\ \vec{e}_r$$

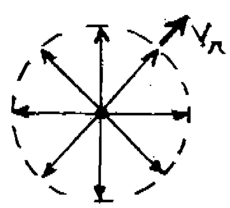
In polar coordinates:

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(r\ V_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta}$$

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}\ [r\ \frac{\Lambda}{2\pi r}] + \frac{1}{r}\frac{\partial (0)}{\partial \theta}$$

$$\nabla \cdot \vec{V} = \frac{1}{r}\cancel{\frac{\partial}{\partial r}}\cancelto{0}{(\frac{\Lambda}{2\pi})} + 0 = \boxed{0}$$

Hence, the flow is a physical possible incompressible flow, except at the origin where $r = 0$.



What happens at the origin? Visualize a cylinder of radius r wrapped around the line source per unit depth perpendicular to the page. The <u>volume flow</u> across this cylindrical surface is

$$\oiint_S \vec{V} \cdot \vec{dS} \tag{1}$$

Since we are considering a unit depth, then we have the volume flow per unit depth. This is precisely the definition of source strength, Λ. Hence, from (1),

$$\Lambda = \text{constant} = \oiint_S \vec{V} \cdot \vec{dS} \tag{2}$$

From the divergence theorem:

$$\oiint_S \vec{V} \cdot \vec{dS} = \oiiint_V (\nabla \cdot \vec{V})\ dV \tag{3}$$

Combining Eqs. (2) and (3)

$$\oiiint_V (\nabla \cdot \vec{V})\ dV = \Lambda = \text{constant} \tag{4}$$

Shrink the volume to an infinitesimal value, ΔV, around the origin. Eq. (4) becomes

$$(\nabla \cdot \vec{V})\ \Delta V = \Lambda$$

Taking the limit as $\Delta V \rightarrow 0$

$$(\nabla \cdot \vec{V}) = \lim \frac{\Lambda}{\Delta V} = \infty. \qquad \boxed{\text{Hence } \nabla \cdot \vec{V} = \infty \text{ at origin}}$$

To show that the flow is irrotational, calculate $\nabla \times \vec{V}$.

$$\nabla \times \vec{V} = \frac{1}{r} \begin{vmatrix} \vec{e}_r & r\vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ V_r & rV_\theta & V_z \end{vmatrix} = \frac{1}{r} \begin{vmatrix} \vec{e}_r & r\vec{e}_\theta & \vec{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ \frac{\Lambda}{2\pi r} & 0 & 0 \end{vmatrix}$$

$$\nabla \times \vec{V} = -r\ \vec{e}_\theta \left(\overset{0}{\cancel{\frac{\partial 0}{\partial r}}} - \overset{0}{\cancel{\frac{\partial \Lambda / 2\pi r}{\partial z}}} \right) + \vec{e}_z \left(\overset{0}{\cancel{\frac{\partial 0}{\partial r}}} - \overset{0}{\cancel{\frac{\partial \Lambda / 2\pi r}{\partial \theta}}} \right) = \boxed{0}$$

Hence,

$$\boxed{\nabla \times \vec{V} = 0 \text{ everywhere.}}$$

3.10

$$\phi = V_\infty x; \qquad \frac{\partial \phi}{\partial x} = V_\infty\,; \quad \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial \phi}{\partial y} = 0\;; \quad \frac{\partial^2 \phi}{\partial y^2} = 0$$

Hence, Laplaces equation:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 + 0 = 0 \text{ is identically satisfied.}$$

Similarly, for $\psi = V\,y$; $\dfrac{\partial \psi}{\partial x} = 0,\ \dfrac{\partial^2 \psi}{\partial x^2} = 0$

$$\frac{\partial \psi}{\partial y} = V,\ \frac{\partial^2 \psi}{\partial y^2} = 0$$

Hence, Laplaces equation:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 + 0 = 0 \text{ is identically satisfied.}$$

3.11 $\phi = \dfrac{\Lambda}{2\pi}\ \ell n\ r;\ \dfrac{\partial \phi}{\partial r} = \dfrac{\Lambda}{2\pi}\dfrac{1}{r},\ \dfrac{\partial^2 \phi}{\partial r^2} = -\dfrac{\Lambda}{2\pi}\dfrac{1}{r^2}$

$$\frac{\partial \phi}{\partial \theta} = 0\,,\ \frac{\partial^2 \phi}{\partial \theta^2} = 0$$

Hence, Laplace's equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = \cancelto{0}{\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\Lambda}{\partial \pi}\right]} + 0 = 0$$

is identically satisfied.

$$\psi = \frac{\Lambda}{2} = \theta \ ; \quad \frac{\partial \psi}{\partial r} = 0 \ . \quad \frac{\partial^2 \psi}{\partial r^2} = 0$$

$$\frac{\partial \psi}{\partial \theta} = \frac{\Lambda}{2\pi} \ , \ \frac{\partial^2 \psi}{\partial \phi^2} = 0$$

Hence, Laplaces equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = \frac{1}{r}\frac{\partial}{\partial r}(0) + \frac{1}{r^2}(0) = 0$$

is identically satisfied.

3.12 The stagnation point is a distance $\Lambda/2\pi V_\infty$ upstream of the source. Hence,

$$\frac{\Lambda}{2\pi V_\infty} = 1 \ , \ \text{or} \quad \Lambda = 2\pi V_\infty$$

The shape of the body is given by

$$\psi = V_\infty r \sin\theta + \frac{\Lambda}{2\pi}\,\theta = \frac{\Lambda}{2}$$

or,

$$r \sin\theta + \frac{\Lambda}{2\pi V_\infty}\theta = \frac{\Lambda}{2V_\infty}$$

or,

$$r \sin\theta + \frac{2\pi V_\infty}{2\pi V}\theta = \frac{2\pi V_\infty}{2V_\infty}$$

or,

$\boxed{r \sin\theta + \theta = \pi}$ Equation of the semi-infinite body.

$$r = \frac{\pi - \theta}{\sin\theta}$$

θ(rad)	r	x = r cosθ	y = r sinθ
π	1	-1	0
3	1.0033	-0.990	0.1416
2.8	1.02	-0.961	0.3416
2.5	1.072	-0.859	0.6416
2.0	1.255	-0.522	1.142
π/2	1.57	0	1.57
1.3	1.91	0.511	1.84
1.0	2.54	1.372	2.14
0.75	3.509	2.57	2.39
0.5	5.51	4.84	2.64

Cartesian Coordinates of Body

To plot the pressure coefficient:

$$V_r = V_\infty \cos\theta + \frac{\Lambda}{2\pi r} = V_\infty \cos\theta + \frac{2\pi V_\infty}{2\pi r} = V_\infty \cos\theta + \frac{V_\infty}{r}$$

$$V_\theta = -V_\infty \sin\theta$$

or,

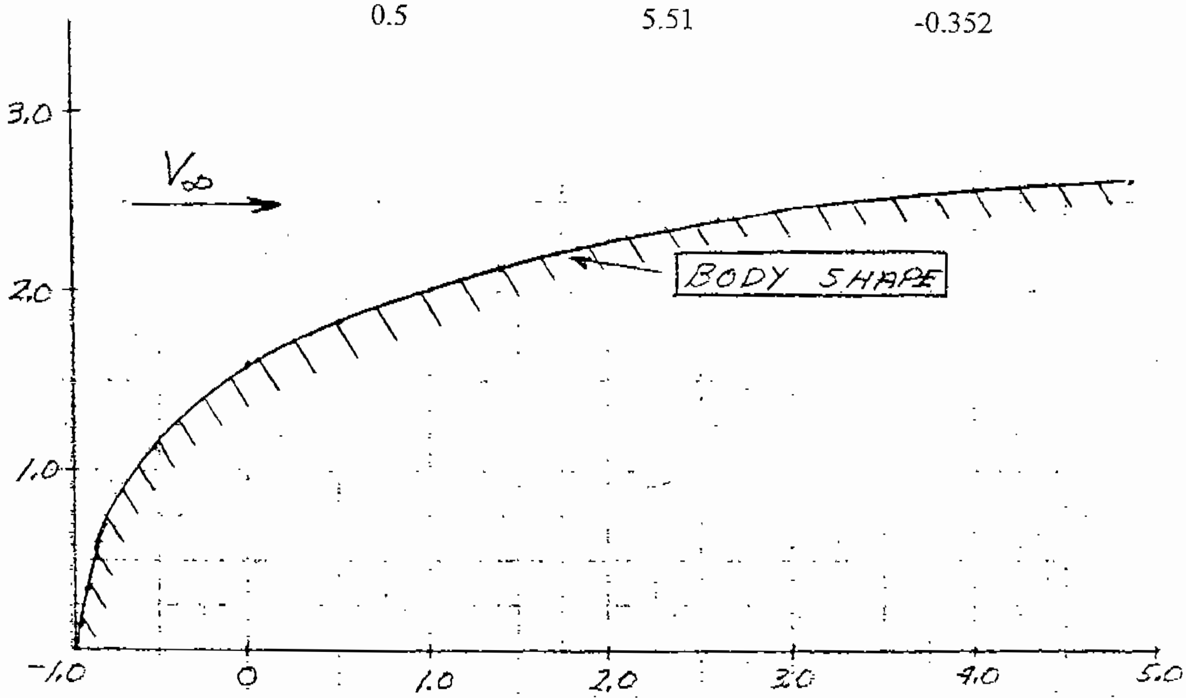
$$\frac{V_r}{V_\infty} = \cos\theta + \frac{1}{r}$$

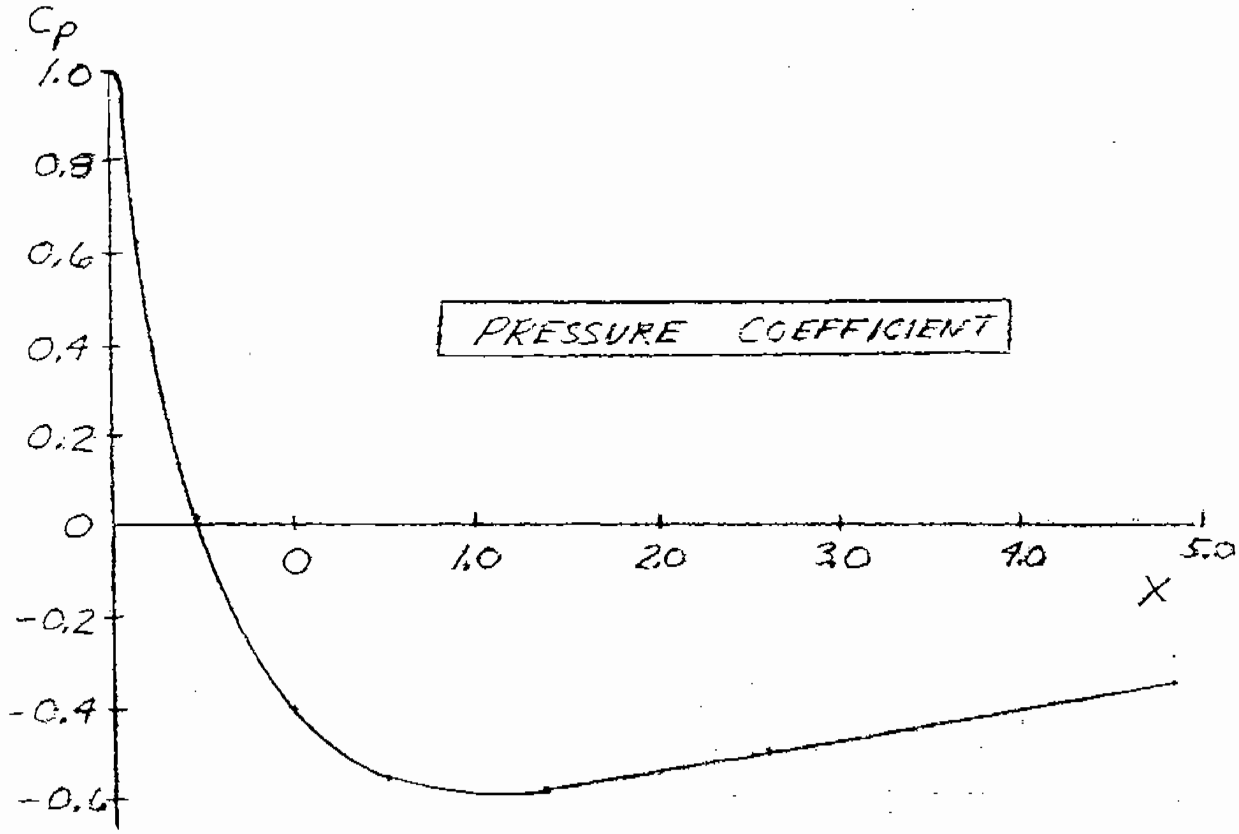
$$\frac{V_\theta}{V_\infty} = -\sin\theta$$

$$\left(\frac{V}{V_\infty}\right)^2 = \left(\frac{V_r}{V_\infty}\right)^2 + \left(\frac{V_\theta}{V_\infty}\right)^2 = \cos\theta + \frac{2}{r}\cos\theta + \frac{1}{r^2} + \sin^2\theta = 1 + \frac{2}{r}\cos\theta + \frac{1}{r^2}$$

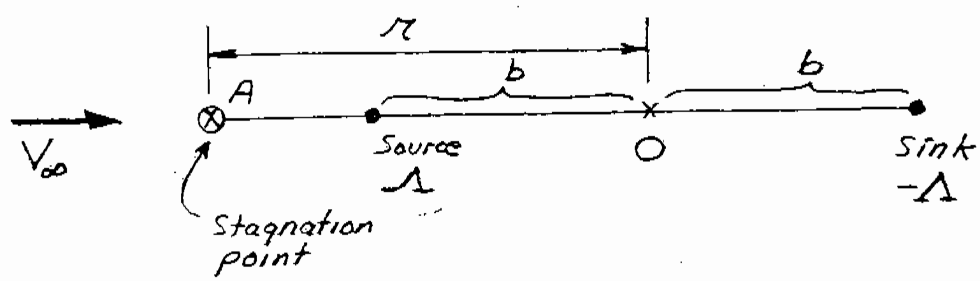
$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = -\frac{2}{r}\cos\theta - \frac{1}{r^2}$$

θ(rad)	r	C_p
π	1	1.0
3	1.00	0.98
2.8	1.02	0.886
2.5	1.072	0.624
2.0	1.255	0.0283
π/2	1.57	-0.4057
1.3	1.91	-0.554
1.0	2.54	-0.580
0.75	3.509	-0.4982
0.5	5.51	-0.352




C_p
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
PRESSURE COEFFICIENT
0
1.0
2.0
3.0
4.0
5.0
X

3.13


r
A
b
b
V_∞
Source
Λ
O
Sink
$-\Lambda$
Stagnation point

At point A: Velocity due to freestream = V_∞

Velocity due to source = $\dfrac{-\Lambda}{2\pi(r+b)}$

(note that it is in the negative x-direction)

Velocity due to sink = $\dfrac{(+\Lambda)}{2\pi(r+b)}$

(Note that it is in the positive x-direction)

Total velocity at Point A:

$$V_A = V_\infty - \frac{\Lambda}{2\pi}\frac{1}{(r-b)} + \frac{\Lambda}{2\pi}\frac{1}{(r+b)}$$

From point A to be a stagnation point, $V_A = 0$.

$$0 = V_\infty + \frac{\Lambda}{2\pi}\left[\frac{1}{(r+b)} + \frac{1}{(r-b)}\right].$$

$$0 = V_\infty + \frac{\Lambda}{2\pi}\left[\frac{r-b-(r+b)}{(r+b)(r-b)}\right] = V_\infty + \frac{\Lambda}{2\pi}\frac{(-2b)}{r^2-b^2}$$

$$V_\infty (r^2 - b^2) = \frac{\Lambda}{2\pi}(2b) = \frac{\Lambda b}{\pi}$$

$$r^2 = \frac{\Lambda b}{\pi V_\infty} + b^2$$

$$\boxed{r = \sqrt{\frac{\Lambda b}{\pi V_\infty} + b^2}}$$

3.14 $\quad V_r = \dfrac{\partial \phi}{\partial r} = \dfrac{1}{r}\dfrac{\partial \psi}{\partial \theta}$ (1)

For a doublet: $\psi = -\dfrac{k}{2\pi}\dfrac{\sin\theta}{r}$

$$\frac{\partial \psi}{\partial \theta} = -\frac{\kappa}{2\pi}\frac{\cos\theta}{r} \qquad (2)$$

Substitute (2) into (1)

$$\frac{\partial\phi}{\partial r} = \frac{1}{r}\left(-\frac{\kappa}{2\pi}\frac{\cos\theta}{r}\right) = -\frac{\kappa}{2\pi}\frac{\cos\theta}{r^2}$$

Integrating with respect to r

$$\phi = \left(-\frac{\kappa}{2\pi}\cos\theta\right)\left(-\frac{1}{r}\right)$$

or,

$$\phi = \frac{\kappa}{2\pi}\frac{\cos\theta}{r}$$

3.15 $\psi = (V_\infty r \sin\theta)\left(1 - \frac{R^2}{r^2}\right)$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = (V_\infty \cos\theta)\left(1 - \frac{R^2}{r^2}\right)$$

$$V_\theta = -\frac{\partial\psi}{\partial r} = -\left(1 + \frac{R^2}{r^2}\right) V_\infty \sin\theta$$

$$V^2 = V_r^2 + V_\theta^2 = \left(1 - \frac{R^2}{r^2}\right)^2 V_\infty^2 \cos^2\theta + \left(1 + \frac{R^2}{r^2}\right)^2 V_\infty \sin^2\theta$$

$$C_p = 1 - \frac{V^2}{V_\infty^2} = 1 - \left(1 - \frac{R^2}{r^2}\right)^2 \cos^2\theta - \left(1 - \frac{R^2}{r^2}\right)^2 \sin^2\theta$$

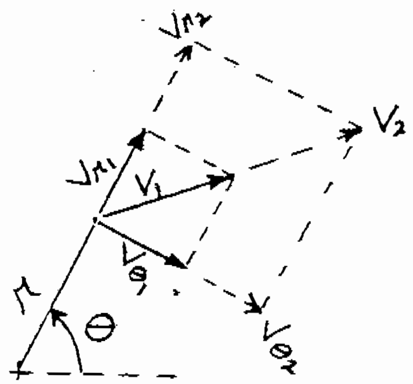
At the surface, r = R

$$C_p = 1 - 4\sin^2\theta$$

3.16 From Eq. (3.93): $\frac{V_r}{V_\infty} = \left(1 - \frac{R^2}{r^2}\right)\cos\theta$

From Eq. (3.94): $\dfrac{V_\theta}{V_\infty} = -\left(1+\dfrac{R^2}{r^2}\right)\sin\theta$

At any given point (r,θ), V_r and V_θ are both directly proportional to V_∞. Hence, the direction of the resultant, $\vec{V}$, is the same, no matter what the value of V_∞ may be. Thus, the shape of the streamlines remains the same.



3.17 From Eq. (3.119): $\dfrac{V_r}{V_\infty} = \left(1-\dfrac{R^2}{r^2}\right)\cos\theta$

From Eq. (3.94): $\dfrac{V_\theta}{V_\infty} = -\left(1+\dfrac{R^2}{r^2}\right)\sin\theta - \dfrac{\Gamma}{2\pi r V_\infty}$

Note that V_θ/V_∞ is itself a function of V_∞ via the second term. Hence, as V_∞ changes, the direction of the resultant velocity at a given point will also change. The shape of the streamlines changes when V_∞ changes.

3.18 $L' = \rho_\infty V_\infty \Gamma$

$$\Gamma = \frac{L'}{\rho_\infty V_\infty} = \frac{6}{(1.23)(30)} = \boxed{0.163\ m^2/sec}$$

3.19 At standard sea level conditions,

$$\rho_\infty = 0.002377 \ \frac{\text{slug}}{\text{ft}^3}$$

$$\mu_\infty = 3.737 \times 10^{-7} \ \frac{\text{slug}}{(\text{ft})(\text{sec})}$$

Also:

$$V = 120 \text{ mph} = 120 \left(\frac{88}{60}\right) \text{ ft/sec} = 176 \ \frac{\text{ft}}{\text{sec}}$$

$$q_\infty = \frac{1}{2} \rho_\infty V_\infty^{\ 2} = \frac{1}{2} (0.002377)(176)^2 = 36.8 \text{ lb/ft}^2$$

For the struts: D = 2 in = 0.167 ft.

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(0.002377)(187.7)(0.167)}{3.737 \ \times \ 10^{-7}} = 199{,}382$$

From Fig. 3.39, $C_D = 1$. The total frontal surface area of the struts is (25) (0.167) = 4.175 ft^2. Hence,

Drag due to struts:

$$D_S = q_\infty \, S \, C_D = (36.8)(4.175)(1) = 153 \text{ lb}$$

For the bracing wires: $D = \dfrac{3}{32}$ in = 0.0078 ft

$$\text{Re} = 199382 \left(\frac{0.0078}{0.167}\right) = 9312$$

From Fig. 3.39, $C_D = 1$. The total frontal surface area of the wires is (80) (0.0078) = 0.624 ft^2. Hence,

Drag due to wires:

$$D_w = q_\infty \, S \, C_D = (36.8)(0.624)(1) = 23 \text{ lb}$$

Total drag due to struts and wires = $D_S + D_W$ =

$$153 + 23 = \boxed{176}$$

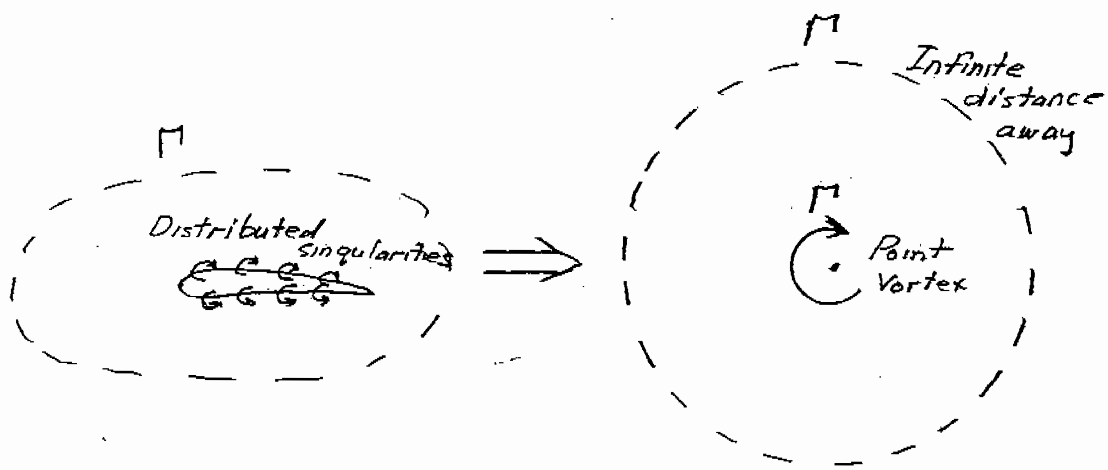
The total zero-lift drag for the airplane is (including struts and wires)

$$C_{D_o} = q_\infty \, S \; C_{D_o} = (36.8)(230)(0.036) = \boxed{304.8}$$

Note that, for this example, the drag due to the struts and wires is $\frac{176}{304.8} = 0.58$ of the total drag – i.e., 58 percent of the total drag. This clearly points out the drag reduction that was achieved in the early 1930's when airplane designers started using internally braced wings with one or more central spars, thus eliminating struts and wires completely.

3.20 The flow over the airfoil in Figure 3.37 can be syntheized by a proper distribution of singularities, i.e., point sources and vortices. The strength of the vortices, added together, gives the total circulation, Γ, around the airfoil. This value of Γ is the same along all closed curves around the airfoil, even if the closed curve is drawn a very large distance away from the airfoil. In this case, the airfoil becomes a speck on the page, and the distributed point vortices appear as one stronger point vortex with strength Γ. This is exactly equivalent to the single point vortex in Figure 3.27 for the circular cylinder, and the lift on the airfoil where the circulation is taken as the total Γ is the same as for a circular cylinder, namely Eq. (3.140),

$$L' = \rho_\infty V_\infty \Gamma$$



CHAPTER 4

4.1 $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(50)^2 = 2.97 \text{ lb/ft}^2$

$c_\ell = 0.64$ and $c_{m_{c/4}} = -0.036$

$L' = q_\infty s\ c_\ell = (2.97)(2)(1)(0.64) = \boxed{3.80 \text{ lb per unit span}}$

$M'_{c/4} = q_\infty S\ c\ c_{m_{c/4}} = (2.97)(2)(1)(2)(-0.36) = \boxed{-0.428 \text{ ft/lb per unit span}}$

4.2 $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.23)(50)^2 = 1538 \text{ N/m}^2$

$$c_\ell = \frac{L'}{q_\infty S} = \frac{1353}{(1538)(2)} = 0.44$$

From Fig. 4.5,

$\boxed{\alpha = 2^\circ}$

4.3 $\Gamma = \oint_c \vec{V} \cdot \vec{ds}$

$$\frac{D\Gamma}{Dt} = \oint_c \frac{D\vec{V}}{Dt} \cdot \vec{ds} + \oint_c \vec{V} \cdot \vec{ds}$$

$$\frac{D\vec{ds}}{Dt} = d\vec{V}$$

Hence, the second term in Eq. (1) becomes

$$\oint_c \vec{V} \cdot d\vec{V} = \oint_c d\left(\frac{V^2}{2}\right) = 0$$

From the momentum equation,

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho}\nabla p \text{ (neglecting body forces)}$$

Hence, the first term in Eq. (1) becomes

$$\oint_c \frac{D\vec{V}}{Dt}\cdot \vec{ds} = -\oint_c \frac{1}{\rho}\nabla p\cdot \vec{ds} = -\oint_c \frac{dp}{\rho}$$

When ρ = const, or $\rho = \rho(p)$, then

$-\oint_c \frac{dp}{\rho} = 0$. Hence, from Eq. (3)

$$\oint_c \frac{D\vec{V}}{Dt}\cdot \vec{ds} = 0 \qquad (4)$$

Substituting Eqs. (2) and (4) into (1), we obtain

$$\boxed{\frac{D\Gamma}{Dt} = 0}$$

Note: See Karamcheti, Ideal-Fluid Aerodynamics, for more details (pp. 239-242).

4.4 $M'_{LE} = -\rho_\infty V_\infty \int_o^c \xi\,\gamma(\xi)\,d\xi$

$$= -\rho_\infty V_\infty \int_o^\pi \frac{c}{2}(1-\cos\theta)\,\theta\,(\gamma)\frac{c}{2}\sin\theta\,d\theta$$

$$= -\rho_\infty V_\infty \frac{c^2}{4} 2\,\alpha\,V_\infty \int_o^\pi (1-\cos^2\theta)\,d\theta$$

$$= -\rho_\infty V_\infty \frac{c^2}{2}\alpha\left[\frac{\pi}{2}\right] = -\left(\frac{1}{2}\rho_\infty V_\infty^2\right) c^2\,\frac{\pi\alpha}{2}$$

$$= -q_\infty c^2\,\frac{\pi\alpha}{2}$$ This is Eq. (4.36).

4.5 $c_\ell = 2\,\pi\,\alpha$ where α is in radians. Hence

$$c_\ell = 2\pi\left(\frac{1.5}{57.3}\right) = \boxed{0.164}$$

$$c_{m_{c/4}} = -\ c_\ell / r = \boxed{-0.041}$$

4.6 (a)

For $0 \le \frac{x}{c} \le 0.4$: $\left(\frac{dz}{dx}\right)_1 = 0.2 - 0.5\left(\frac{x}{c}\right)$

For $0.4 \le \frac{x}{c} \le 1$: $\left(\frac{dz}{dx}\right)_2 = 0.0888 - 0.2222\left(\frac{x}{c}\right)$

Since $x = \frac{c}{2}(1 - \cos\theta)$, then

$$\left(\frac{dz}{dx}\right)_1 = -0.05 + 0.25\cos\theta, \text{ for } 0 \le \theta \le 1.3694$$

$$\left(\frac{dz}{dx}\right)_2 = -0.0223 + 0.1111\cos\theta, \text{ for } 1.3694 \le \theta \le \pi$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta - 1)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{1.3694} (-0.05 + 0.25\cos\theta)(\cos\theta - 1)\, d\theta - \frac{1}{\pi}\int_{1.3694}^{\pi}$$

$$\cdot (-0.0223 + 0.1111\cos\theta)(\cos\theta - 1)\, d\theta$$

$$= -\frac{1}{\pi}\int_0^{1.3694} (0.05 - 0.3\cos\theta + 0.25\cos^2\theta)\, d\theta$$

$$-\frac{1}{\pi}\int_{1.3694}^{\pi} (0.0223 - 0.13334\cos\theta + 0.1111\cos^2\theta)\, d\theta$$

$$= -\frac{1}{\pi}\left[0.05\theta - 0.3\sin\theta + 0.25\left(\frac{\theta}{2} + \frac{1}{4}\sin 2\theta\right)\right]_0^{1.3694}$$

$$- \frac{1}{\pi} \left[0.0223\ \theta - 0.1334 \sin\theta + 0.111 \left(\frac{\theta}{2} + \frac{1}{4} \sin 2\theta\right)\right]_{1.3694}^{\pi}$$

$$= - \frac{1}{\pi} [0.06847 - 0.2939 + 0.1712 + 0.0245] - \frac{1}{\pi} [0.0701 + 0.1745]$$

$$+ \frac{1}{\pi} [0.0305 - 0.1307 + 0.0761 + 0.0109]$$

$$= \frac{-0.2281}{\pi} = -0.0726 \text{ rad} = \boxed{-4.16^\circ}$$

(b)

$c_\ell = 2\pi (\alpha + \alpha_{L=0})$ where α is in radians

$$c_\ell = \frac{2\pi}{57.3} [3 - (-4.16)] = \boxed{0.782}$$

4.7 $\quad A_1 = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos\theta\, d\theta$

$$= \frac{2}{\pi} \int_0^{1.3694} (0.05 + 0.25 \cos\theta) \cos\theta\, d\theta$$

$$+ \frac{2}{\pi} \int_{1.3694}^{\pi} (-0.0223 + 0.1111 \cos\theta) \cos\theta\, d\theta$$

$$= \frac{2}{\pi} \int_0^{1.3694} (-0.05 \cos\theta + 0.25 \cos^2\theta)\, d\theta +$$

$$\frac{2}{\pi} \int_{1.3694}^{\pi} (-0.0223 \cos\theta + 0.1111 \cos^2\theta)\, d\theta$$

$$= \frac{2}{\pi} \left[-0.05 \sin\theta + 0.25 \left(\frac{\theta}{2} + \frac{1}{4} \sin 2\theta\right)\right]_0^{1.3694}$$

$$+ \frac{2}{\pi} \left[(-0.0233) \sin\theta + 0.1111 \left(\frac{\theta}{2} + \frac{1}{4} \sin 2\theta\right)\right]_{1.3694}^{\pi}$$

$$= \frac{2}{\pi} [-0.04899 + 0.25\,(0.6847 + 0.09800) + 0.1745$$

$$+ 0.02185 - 0.1111\,(0.6847 + 0.09800)]$$

$$A_1 = (0.2561)\,\frac{2}{\pi} = 0.1630$$

$$A_2 = \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos 2\theta \, d\theta$$

$$= \frac{2}{\pi} \int_0^{1.3694} (-0.05 + 0.25 \cos\theta) \cos 2\theta d\theta + \frac{2}{\pi} \int_{1.3694}^{\pi} (-0.0223$$

$$+ 0.1111 \cos\theta) \cos\theta \, d\theta$$

$$= \frac{2}{\pi} [\frac{1}{2}(-0.05) \sin 2\theta + 0.25\,(\frac{\sin\theta}{2} + \frac{\sin 3\theta}{6})]_0^{1.3694}$$

$$+ \frac{2}{\pi} [\frac{1}{2}\,(-0.0223) \sin 2\theta + 0.1111\,(\frac{\sin\theta}{2} + \frac{\sin 3\theta}{6})]_{1.3694}^{\pi}$$

$$= \frac{2}{\pi}[-0.009800 + 0.25\,(0.4899 - 0.1372) + 0.004371$$

$$- \quad 0.1111\,(0.4899 - 0.1372)]$$

$$= (0.0436)\,\frac{2}{\pi} - 0.0277$$

$$c_{m_{c/4}} = \frac{\pi}{4}\,(A_2 - A_1) = \frac{\pi}{4}\,(0.0277 - 0.1630) = \boxed{-0.1063}$$

$$\frac{x_{cp}}{c} = \frac{1}{4}\,[1 + \frac{\pi}{c_\ell}\,(A_1 - A_2)] = \frac{1}{4}\,[1 + \frac{\pi}{0.782}\,(0.1630 - 0.0277)] = \boxed{0.386}$$

4.8

	Experiment (Ref. 11)	Theory	% Difference
$\alpha_{L=0}$	-3.9°	-4.16°	6.25%
c_ℓ	0.76	0.782	2.8%
$c_{m_{c/4}}$	-0.095	-0.1063	10.6%

4.9 $M'_{LE} = -\rho_\infty V_\infty \int_0^c \xi\,\gamma(\xi)\,d\xi$

$$c_{m_{c/4}} = \frac{M'_{LE}}{\frac{1}{2}\rho_\infty V_\infty^2 c^2} = \frac{-2}{V_\infty c^2} \int_0^c \xi\,\gamma(\xi)\,d\xi \qquad (1)$$

$$\xi = \frac{c}{2}(1 - \cos\theta)$$

$$d\xi = \frac{c}{2}\sin\theta\,d\theta$$

$$\gamma(\theta) = 2V_\infty\left[A_0\frac{(1+\cos\theta)}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta\right]$$

With the above, Eq. (1) becomes

$$c_{m_{c/4}} = -\int_0^\pi A_0(1-\cos^2\theta)\,d\theta - \sum_{n=1}^{\infty}\int_0^\pi A_n(1-\cos\theta)\sin\theta\sin n\theta\,d\theta \qquad (2)$$

Note the following definite integrals:

$$\int_0^\pi \cos^2\theta\,d\theta = \frac{\pi}{2}$$

$$\int_0^\pi \sin^2\theta\,d\theta = \frac{\pi}{2}$$

$$\int_0^\pi \cos\theta\sin^2\theta\,d\theta = 0$$

$$\int_0^{\pi} \sin\theta \sin n\theta \, d\theta = 0 \qquad \text{for } n = 2, 3, \ldots$$

$$\int_0^{\pi} \cos\theta \sin\theta \sin 2\theta \, d\theta = \frac{\pi}{4}$$

$$\int_0^{\pi} \cos\theta \sin\theta \sin n\theta \, d\theta = 0 \qquad \text{for } n = 3, 4, \ldots$$

Hence, Eq. (2) becomes:

$$c_{m_{c/4}} = -\left[\pi A_0 - \frac{\pi}{2} A_0 + \frac{\pi}{2} A_1 - \frac{\pi}{4} A_2\right]$$

$$\boxed{c_{m_{c/4}} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)}$$

4.10 The slope of the lift curve is

$$a_0 = \frac{0.65 - (-0.39)}{4 - (-6)} = 0.104 \text{ per degree}$$

The slope of the moment coefficient curve is

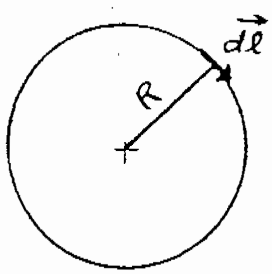
$$m_0 = \frac{-0.037 - (-0.045)}{4 - (-6)} = 8 \times 10^{-4} \text{ per degree}$$

From Eq. (4.71),

$$\bar{x}_{ac} = -\frac{m_0}{a_0} + 0.25 = -\frac{8 \times 10^{-4}}{0.104} + 0.25 = \boxed{0.242}$$

CHAPTER 5

5.1

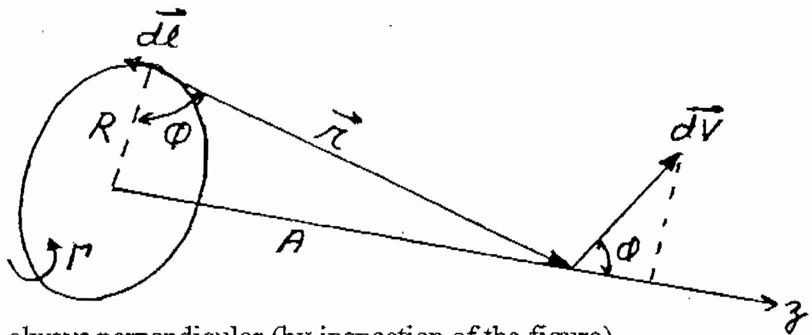


$\vec{d\ell} \times \vec{r} = (R\ d\ell)\ \vec{e}$

where $\vec{e}$ is a unit vector perpendicular to the plane of the loop, directed into the page.

$$\vec{V} = \frac{\Gamma}{4\pi}\int \frac{\vec{d\ell} \times \vec{r}}{\left|\vec{r}\right|^3} = \frac{\Gamma}{4\pi}\int_0^{2\pi R} \frac{(R\ d\ell)\vec{e}}{R^3} = \frac{\Gamma}{4\pi R^2}(2\pi R)\ \vec{e} = \boxed{\frac{\Gamma}{2R}\vec{e}}$$

5.2



Since $\vec{d\ell}$ and $\vec{r}$ are always perpendicular (by inspection of the figure),

$$\left|d\vec{V}\right| = \left|\frac{\Gamma}{4\pi}\frac{\vec{d\ell} \times \vec{r}}{\left|\vec{r}\right|^3}\right| = \frac{\Gamma}{4\pi}\frac{d\ell}{r^2}$$

By symmetry, the resultant velocity due to the entire loop must be along the x-axis. Hence,

$$\left|\vec{V}\right| = \int \left|d\vec{V}\right| \cos\theta = \left(\frac{\Gamma}{4\pi}\int_0^{2\pi R}\frac{d\ell}{r^2}\right)\cos\theta =$$

$$\frac{\Gamma}{4\pi}\frac{1}{(A^2+R^2)}(2\pi R)\cos\theta =$$

$$\frac{\Gamma}{2}\frac{R}{(A^2+R^2)}\frac{R}{\sqrt{R^2+A^2}}=\frac{\Gamma\ R^2}{2(A^2+\ R^2)^{3/2}}$$

5.3 $$a=\frac{a_o}{1+\frac{a_o}{\pi AR}(1+\tau)},$$ where $a_o = 0.1080$ per degree = 6.188 per radian

From Fig. 5.18: $\delta = \tau = 0.054$.

$$a=\frac{6.188}{1+\frac{6.188}{\pi(8)}(1+0.054)}=4.91 \text{ per rad.}$$

$$= 0.0857 \text{ per degree}$$

$$C_L = a\,(\alpha - \alpha_{L=0}) = 0.0857\ [7-(-1.3) = \boxed{0.712}$$

$$C_{D_i} = \frac{C_L^2}{\pi a R}\ (1+\delta) = \frac{(0.712)^2}{\pi(8)}\ (1.054) = \boxed{0.0212}$$

5.4 $$AR=\frac{b^2}{S}=\frac{(32)^2}{170}=6.02$$

At standard sea level, $\rho_\infty = 0.002377$ slug/ft^3

$$V_\infty = 120 \text{ mph}\left(\frac{88\ \text{ft/sec}}{60\text{mph}}\right) = 176 \text{ ft/sec}$$

$$q_\infty = \frac{1}{2}\ \rho_\infty V^2_\infty = \frac{1}{2}\ (0.002377)\ (176)^2 = 36.8 \text{ lb/ft}^2$$

$a_o = 0.1033$ per degree

$= 5.92$ per rad

$$C_L = \frac{L}{q_\infty S}=\frac{W}{q_\infty S}=\frac{2450}{(36.8)(170)}=0.3916$$

$$a = \frac{a_o}{1 + \frac{a_o}{\pi AR}(1+\tau)} = \frac{5.92}{1 + \frac{5.92}{\pi(6.02)}(1+0.12)} = 4.38 \text{ per rad}$$

$$= 0.0764 \text{ per deg}$$

$C_L = a\,(\alpha - \alpha_{L=0})$

$$\alpha = \frac{C_L}{a} + \alpha_{L=0} = \frac{0.3916}{0.0764} - 3^\circ = \boxed{2.12^\circ}$$

5.5 $$C_{D_i} = \frac{C_L^2}{\pi e AR} = \frac{(0.3916)^2}{\pi(.64)(6.02)} = 0.01267$$

$$D_i = q_\infty\, S\, C_{D_i} = (36.8)(170)(0.01267) = \boxed{79.3 \text{ lb}}$$

5.6 To be consistent, we will use Helmbold's equations for both the straight and swept wings.

(a) $a_o = 0.1$ per degree $= 0.1\,(57.3) = 5.73$ per radian

$$\frac{a_o}{\pi AR} = \frac{5.73}{\pi(6)} = 0.304$$

From Helmbold's equation for a straight wing, Eq. (5.81),

$$a = \frac{a_o}{\sqrt{1 + [a_o/(\pi AR)]^2} + a_o/(\pi AR)}$$

$$= \frac{5.73}{\sqrt{1 + (0.304)^2} + 0.304} = \frac{5.73}{1.349} = \boxed{4.247 \text{ per radian}}$$

(b) From Helmbold's equation for a swept wing, Eq. (5.82), where

$a_o \cos\Lambda = 5.73 \cos 45^\circ = 4.05$ per radian

and

$$\frac{a_o \cos\Lambda}{\pi AR} = \frac{4.05}{\pi(6)} = 0.215$$

we have

$$a = \frac{a_o \cos\Lambda}{\sqrt{1+[a_o \cos\Lambda/(\pi AR)]^2} + a_o \cos\Lambda/(\pi AR)}$$

$$= \frac{4.05}{\sqrt{1+(0.215)^2} + 0.215} = \frac{4.05}{1.23785} = \boxed{3.27 \text{ per radian}}$$

Comparing the results of parts (a) and (b), we readily conclude that the effect of wing sweep is to reduce the lift slope. Moreover, the reduction is substantial.

5.7 Again, we use Helmbold's equations.

(a) $a_o = 5.73$ per radian

$$\frac{a_o}{\pi AR} = \frac{5.73}{\pi(3)} = 0.608$$

$$a = \frac{a_o}{\sqrt{1+[a_o/(\pi AR)]^2} + a_o/(\pi AR)}$$

$$= \frac{5.73}{\sqrt{1+(0.608)^2} + 0.608} = \frac{5.73}{1.778} = \boxed{3.222 \text{ per radian}}$$

(b) $a_o \cos\Lambda = 4.05$

$$\frac{a_o \cos\Lambda}{\pi AR} = \frac{4.05}{\pi(3)} = 0.43$$

$$a = \frac{a_o \cos\Lambda}{\sqrt{1+[a_o \cos\Lambda/(\pi AR)]^2} + a_o \cos\Lambda/(\pi AR)}$$

$$= \frac{4.05}{\sqrt{1+(0.43)^2} + 0.43} = \frac{4.05}{1.5185} = \boxed{2.667}$$

In Problem 5.6, with an aspect ratio of 6, we had

$$\frac{a_{swept}}{a_{straight}} = \frac{3.27}{4.247} = 0.77$$

The lift slope for the swept wing is only 77% of that for the straight wing when the aspect ratio of both wings is 6.

In Problem 5.7, with aspect ratio 3, we have

$$\frac{a_{swept}}{a_{straight}} = \frac{2.667}{3.222} = 0.83$$

The lift slope for the swept wing is 83% of that for the straight wing.

Conclusion: Wing sweep decreases the lift slope. Moreover, wing sweep affects the lift slope to a greater degree for higher aspect ratio wings than for lower aspect ratio wings. This makes some sense, because the lift slope for low aspect ratio wings is already considerably reduced just due to the aspect ratio effect.

CHAPTER 6

6.1 $V_r = \dfrac{c}{r^2}$, $V_\theta = 0$, $V_\phi = 0$

$$\nabla \times \vec{V} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{e}_r & r\,\vec{e}_\theta & (r\sin\theta)\,\vec{e}_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ \dfrac{c}{r^2} & 0 & 0 \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \left\{ \vec{e}_r (0-0) - r\,\vec{e}_\theta \left(\frac{\partial c / r^2}{\partial \phi} - 0 \right) + r\sin\theta\ \vec{e}_\phi \left(0 - \frac{\partial c / r^2}{\partial \theta} \right) \right\}$$

$$= \frac{1}{r^2 \sin\theta} \ \{0 - 0 + 0\} = \boxed{0}$$

Flow is irrotational.

6.2 $V_r = \dfrac{c}{r^2}$, $V_\theta = 0$, $V_\phi = 0$

$$\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \left(\frac{c}{r^2}\right)\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(0) + \frac{1}{r\sin\theta}\frac{\partial (0)}{\partial \phi}$$

$$\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial c}{\partial r} + 0 + 0 = 0 + 0 + 0 = \boxed{0}$$

The flow is a physical possible incompressible flow.

6.3

For the sphere: $(C_p) = 1 - \dfrac{9}{4}\sin^2\theta$

For the cylinder: $(C_p)_{cyl.} = 1 - 4\sin^2\theta$

At the top of the sphere: $\theta = \pi/2$, hence

$$(C_p)_{sphere} = -5/4 = -1.25$$

For no manometer deflection, $(C_p)_{sphere} = (C_p)_{cyl.}$

$$-1.25 = 1 - 4\sin^2\theta$$

$$\sin^2\theta = 0.5625$$

$$\sin\theta = 0.75$$

Hence:

$$\boxed{\theta = 48.6^\circ}$$

The pressure tap on the cylinder must be located at an angular position 48.6° above or below the stagnation point.

CHAPTER 7

7.1 $p = \rho RT$

$$\rho = \frac{p}{RT} = \frac{(7.8)(2116)}{(1716)(934)} = \boxed{0.0103 \text{ slug/ft}^3}$$

7.2 (a)

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{0.4} = \boxed{6006 \frac{\text{ft lb}}{\text{slug } ^\circ\text{R}}}$$

$$c_v = \frac{R}{\gamma - 1} = \frac{1716}{0.4} = \boxed{4290 \frac{\text{ft lb}}{\text{slug } ^\circ\text{R}}}$$

$$e = c_v T = 4290\ (934) = \boxed{4.007 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

$$h = c_p T = 6006\ (934) = \boxed{5.610 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

(b) For a calorically perfect gas, c_p and c_v are constants, independent of temperature. Hence, we have again

$$\boxed{\begin{array}{l} c_p = 6006 \dfrac{\text{ft lb}}{\text{slug } ^\circ\text{R}} \\[2ex] c_v = 4290 \dfrac{\text{ft lb}}{\text{slug } ^\circ\text{R}} \end{array}}$$

Also, at standard sea level, $R = 519^\circ$R. Hence

$$E = 4290\ (519) = 2.227 \times 10^6 \frac{\text{ft lb}}{\text{slug}}$$

$$h = 6006\ (519) = \boxed{3.117 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

7.3 $c_p = \frac{R}{\gamma - 1} = \frac{(1.4)(287)}{0.4} = 1004.5\ \frac{\text{joule}}{\text{kg}\ {}^\circ\text{K}}$

$c_v = \frac{R}{\gamma - 1} = \frac{287}{0.4} = 717.5\ \frac{\text{joule}}{\text{kg}\ {}^\circ\text{K}}$

$h_2 - h_1 = c_p\,(T_2 - T_1) = (1004.5)(690\text{-}288) = \boxed{4.038 \times 10^5\ \frac{\text{joule}}{\text{kg}}}$

$e_2 - e_1 = c_v\,(T_2 - T_1) = (718.5)(690\text{-}288) = \boxed{2.884 \times 10^5\ \frac{\text{joule}}{\text{kg}}}$

$s_2 - s_1 = c_p\ \ell n \frac{T_2}{T_1} - R\ \ell n \frac{p_2}{p_1} = (1004.5)\ \ell n \frac{690}{288} - (287)\ \ell n\ 8.656 = \boxed{258.2\ \frac{\text{joule}}{\text{kg}\ {}^\circ\text{K}}}$

7.4 $\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{4.35 \times 10^4}{(287)(245)} = 0.6186\ \text{kg/m}^3$

$\frac{\rho}{\rho_\infty} = \left(\frac{p}{p_\infty}\right)^{1/\gamma}$

$\rho = \rho_\infty \left(\frac{p}{p_\infty}\right)^{1/\gamma} = 0.6186 \left(\frac{3.6 \times 10^4}{4.35 \times 10^4}\right)^{1/1.4} = \boxed{0.5404\ \frac{\text{kg}}{\text{m}^3}}$

7.5 $\frac{p}{p_o} = \left(\frac{T}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$

$T = T_o \left(\frac{p}{p_o}\right)^{\frac{\gamma}{\gamma-1}} = 500\left(\frac{1}{10}\right)^{0.2857} = \boxed{259^\circ\text{K}}$

$\rho = \frac{p}{RT} = \frac{1.01 \times 10^5}{(287)(259)} = \boxed{1.359\ \text{kg/m}^3}$

7.6 $pv = RT$, hence $v = \frac{RT}{p}$

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2} = -\frac{v}{p}$$

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = \frac{1}{p}$$

Note: $1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2$

$$\tau_T = \frac{1}{p} = \frac{1}{(0.2)(1.01 \times 10^5)} = \boxed{4.95 \times 10^{-5} \frac{m^2}{N}}$$

For an isentropic process: $\dfrac{p_1}{p_2} = \left(\dfrac{\rho_1}{\rho_2}\right)^\gamma = \left(\dfrac{v_2}{v_1}\right)^\gamma$

I.e., $p_1 v_1^\gamma = p_2 v_2^\gamma$ or $pv^\gamma = \text{constant} = c_1$

$$v = \left(\frac{c_1}{p}\right)^{1/\gamma}$$

$$\left(\frac{\partial v}{\partial p}\right)_s = \frac{1}{\gamma}(c_1)^{1/\gamma}(p)^{-(1/\gamma)-1} = -\frac{1}{\gamma}\left(pv^\gamma\right)^{1/\gamma}(p)^{(-1-\gamma)/\gamma} = -\frac{1}{\gamma} v p^{-1}$$

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s = -\frac{1}{v}\left(-\frac{v}{\gamma p}\right) = \frac{1}{\gamma p}$$

$$\tau_s = \frac{1}{(1.4)(0.2)(1.01 \times 10^5)} = \boxed{3.536 \times 10^{-5} \frac{m^2}{N}}$$

7.7 $\quad c_p = \dfrac{\gamma R}{\gamma - 1} = \dfrac{(1.4)(1716)}{(0.4)} = 6006 \dfrac{\text{ft lb}}{\text{slug } °R}$

$$h_o = h + \frac{V^2}{2} = c_p T + \frac{V^2}{2} = (6006)(480) + \frac{(1300)^2}{2} = \boxed{3.728 \times 10^6 \frac{\text{ft lb}}{\text{slug}}}$$

7.8 $\quad$ Let $(h_o)_{res}$ = total enthalpy of the reservoir = $c_p\,(T_o)_{res}$

$(h_o)_e$ = total enthalpy at the exit $= c_p\, T_e + \dfrac{V_e^2}{2}$

For an adiabatic flow, h_o = constant. Hence

$$(h_o)_{res} = (h_o)_e$$

$$c_p(T_o)_{res} = c_p\, T_e + \frac{V_e^2}{2}$$

$$V_e = \sqrt{2\ c_p\left[(T_o)_{res} - T_e\right]} = \sqrt{2(1004.5)(1000-600)} = \boxed{896.4 \text{ m/sec}}$$

7.9 $$T_\infty = \frac{p_\infty}{\rho_\infty R} = \frac{(0.61)(1.01\ \text{x}\ 10^5)}{(0.819)(287)} = 262.1\ ^\circ K$$

$$\frac{T}{T_\infty} = \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma}; \ T = T_\infty \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma} = 262.1 \left(\frac{0.5}{0.61}\right)^{0.2857} = 247.6\ ^\circ K$$

Since the flow is isentropic, it is also adiabatic. Hence, h_o = constant

$$h_\infty + \frac{V_\infty^2}{2} = h + \frac{V^2}{2}$$

$$V = \sqrt{2(h_\infty - h) + V_\infty^2} = \sqrt{2\ c_p(T_\infty - T) + V_\infty^2} = \sqrt{2(1004.5)(262.1-247.6)+(300)^2}$$

$$= \boxed{345 \text{ m/sec}}$$

7.10 $$p_\infty + \rho\,\frac{V_\infty^2}{2} = p + \rho\,\frac{V^2}{2}$$

$$V = \sqrt{\frac{2(p_\infty - p)}{\rho} + V_\infty^{\ 2}} = \sqrt{\frac{2(1.01\ \text{x}\ 10^5)(0.61-0.5)}{0.819} + (300)^2} = 342.2\ \text{m/sec}$$

$$\%\ \text{error} = \left(\frac{345-342.2}{345}\right) \text{x}\ 100 = \boxed{0.81\%}$$

7.11 $T = T_\infty \left(\frac{p}{p_\infty}\right)^{(\gamma-1)/\gamma} = 262.1 \left(\frac{0.3}{0.61}\right)^{0.2857} = 214\ ^\circ K$

$$V = \sqrt{2(1004.5)(262.1-214)+(300)^2} = 432 \text{ m/sec}$$

7.12 $V = \sqrt{\frac{2(1.01 \text{ x } 10^5)(0.61-0.3)}{0.819}+(300)^2} = 408 \text{ m/sec}$

$$\% \text{ error} = \left(\frac{432-408.7}{432}\right) \text{x } 100 = \boxed{5.55\%}$$

7.13 From Eq. (7.53)

$$h + \frac{V^2}{2} = \text{constant}$$

From Eqs. (7.6b) and (7.9),

$$h = c_p T = \frac{\gamma RT}{\gamma - 1} \tag{1}$$

From the equation of state,

$$RT = p/\rho \tag{2}$$

Combining Eqs. (1) and (2),

$$h = \frac{\gamma}{\gamma-1}\left(\frac{p}{\rho}\right) \tag{3}$$

Hence, Eq. (7.53) can be written as

$$\frac{\gamma}{\gamma-1}\left(\frac{p}{\rho}\right) + \frac{V^2}{2} = \text{constant} \tag{4}$$

In the limit of $\gamma \to \infty$, Eq. (4) becomes

$$\frac{p}{\rho} + \frac{V^2}{2} = \text{constant}$$

or,

$$p + \tfrac{1}{2} \rho V^2 = \text{constant}$$

which is Bernoulli's equation. Hence, the energy equation for compressible flow can be reduced to Bernoulli's equation for the case of $\gamma \to \infty$. Hence, the ratio of specific heats for incompressible flow is infinite, which of course does not exist in nature. This is just another example of the special inconsistencies associated with the assumption of incompressible flow, i.e., constant density flow, which of course does not exist in nature. This is why we have stated earlier in this book that incompressible flow is a myth.

As to the question whether Bernoulli's equation is a statement of Newton's second law or an energy equation, we now see that it is both. For an incompressible flow, the application of the fundamental principles of Newton's second law and the conservation of energy are redundant, both leading to the same equation, namely Bernoulli's equation. However, philosophically this author feels strongly that Bernoulli's equation is fundamentally a statement of Newton's second law – it is a mechanical equation. This is how we derived Bernoulli's equation in a very straightforward manner in Chapter 3. For the study of inviscid incompressible flow, we need only to apply the fundamental principles of mass conservation and Newton's second law. The principle of conservation of energy is redundant and is not needed.

CHAPTER 8

8.1 $a = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(230)} = \boxed{304 \text{ m/sec}}$

8.2 $c_p T_o = c_p T_e + \dfrac{V_e^2}{2}$

$$T_e = T_o - \frac{V_e^2}{2c_p} = 519 - \frac{(1385)^2}{2(6006)} = 359.3\ ^oR$$

$$a_e = \sqrt{\gamma RT_e} = \sqrt{(1.4)(1716)(359.3)} = 929.1\ ^oR$$

$$M_e = \frac{V_e}{a_e} = \frac{1385}{929.1} = \boxed{1.49}$$

8.3 $a = \sqrt{\gamma RT_e} = \sqrt{(1.4)(287)(300)} = 347.2 \text{ m/sec}$

$$M = \frac{V}{a} = \frac{250}{347.2} = 0.72$$

From Tables: $\dfrac{T_o}{T} = 1.104$ and $\dfrac{p_o}{p} = 1.412$

$$T_o = 1.104\ T = 1.104\ (300) = \boxed{331.2\ ^oK}$$

$$p_o = 1.412\ p = 1.412\ (1.2) = \boxed{1.694 \text{ atm}}$$

$$\frac{p^*}{p} = \frac{p^*}{p_o}\frac{p_o}{p} = (0.528)(1.412) = 0.7455$$

$$p^* = 0.7455\ p = 0.455\ (1.2) = \boxed{0.8946 \text{ atm}}$$

$$\frac{T^*}{T} = \frac{T^*}{T_o}\frac{T_o}{T} = 0.8333\ (1.104) = 0.92$$

$$T^* = 0.92\ (300) = \boxed{276\ ^oK}$$

$$a^* = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(276)} = 333 \text{ m/sec}$$

$$M^* = \frac{V}{a^*} = \frac{250}{333} = \boxed{0.75}$$

8.4 $a = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(700)} = 1297$ ft/sec

$$M = \frac{v}{a} = \frac{2983}{1297} = 2.3$$

From Tables: $\frac{T_o}{T} = 2.058$ and $\frac{p_o}{p} = 12.5$

$$T_o = 2.058\ T = 2.058\ (700) = \boxed{1441\ ^oR}$$

$$p_o = 12.5\ p = 12.5\ (1.6) = \boxed{20\ atm}$$

$$\frac{T^*}{T} = \frac{T^*}{T_o}\frac{T_o}{T} = (0.8333)\ (2.058) = 1.715$$

$$T^* = 1.715\ T = 1.715\ (700) = \boxed{1200\ ^oR}$$

$$\frac{p^*}{p} = \frac{p^*}{p_o}\frac{p_o}{p} = (0.528)(12.5) = 6.6$$

$$p^* = 6.6\ p = 6.6\ (1.6) = \boxed{10.56\ atm}$$

$$a^* = \sqrt{\gamma RT^*} = \sqrt{(1.4)(1716)(1200)} = 1698 \text{ ft/sec}$$

$$M^* = \frac{V}{a^*} = \frac{2983}{1698} = \boxed{1.757}$$

8.5 From Tables: $\frac{p_o}{p} = 7.824$ and $\frac{T_o}{T} = 1.8$

Hence, for the test section flow,

$$p_o = 7.824\ p = 7.824\ (1) = 7.824 \text{ atm}$$

$$T_o = 1.8\ T = 1.8\ (230) = 414\ ^oK$$

Since the flow is isentropic, both p_o and T_o are constant throughout the flow. Also, in the reservoir, $M \approx 0$. Hence, the reservoir pressure and temperature are

$$\boxed{\begin{aligned} p_o &= 7.824 \text{ atm} \\ T_o &= 414 \text{ °K} \end{aligned}}$$

8.6 From the Standard Altitude Tables, at 10,000 ft.,

$p_\infty = 1455.6 \text{ lb/ft}^2$ and $T_\infty = 483.04$ °R

From Table A.1: For $M_\infty = 0.82$; $\dfrac{p_o}{p_\infty} = 1.555$, $\dfrac{T_o}{T_\infty} = 1.134$

For $M = 1$; $\dfrac{p_o}{p} = 1.893$, $\dfrac{T_o}{T} = 1.2$

Since the flow is isentropic, p_o = constant and T_o = constant.

$$p = \frac{p}{p_o}\frac{p_o}{p_\infty} p_\infty = \frac{1}{1.893}(1.555)(1455.6) = \boxed{1196 \text{ lb/ft}^2}$$

$$T = \frac{T}{T_o}\frac{T_o}{T_\infty} T_\infty = \frac{1}{1.2}(1.134)(483.04) = \boxed{456.5 \text{ °R}}$$

8.7 From Table A.2: $\dfrac{p_2}{p_1} = 7.72$, $\dfrac{\rho_2}{\rho_1} = 3.449$, $\dfrac{T_2}{T_1} = 2.238$,

$$\frac{p_{o_2}}{p_1} = 9.181, \ \boxed{M_2 = 0.5039}, \ \frac{p_{o_2}}{p_{o_1}} = 0.4601$$

Hence,

$$p_2 = \frac{p_2}{p_1} p_1 = 7.72\,(1) = \boxed{7.72 \text{ atm}}$$

$$T_2 = \frac{T_2}{T_1} T_1 = 2.238\,(288) = \boxed{644.5 \text{ °K}}$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(1)(1.01 \ x \ 10^5)}{(287)(288)} = 1.222 \text{ kg/m}^3$$

$$\rho_2 = \frac{\rho_2}{\rho_1} \ \rho_1 = 3.449\ (1.222) = \boxed{4.21 \text{ kg/m}^3}$$

$$p_{o_2} = \frac{p_{o_2}}{p_1} p_1 = 9.181\ (1) = \boxed{9.181 \text{ atm}}$$

$$T_{o_2} = T_{o_1} = \frac{T_{o_1}}{T_1} \ T_1 = (2.352)(288) = \boxed{677.4\ ^\circ K} \text{ (using Table A.1)}$$

$$s_2 = s_1 = - R \ \ell n \ \frac{p_{o_2}}{p_{o_1}} = (287) \ \ell n \ 0.4601 = 222.8 \ \frac{\text{joule}}{\text{kg} \ ^\circ K}$$

8.8 $\quad \frac{\rho_2}{\rho_1} = 10.33$. From Table A.2, $\boxed{M_1 = 3.0}$, $\frac{T_2}{T_1} = 2.679$, $\frac{p_{o_2}}{p_1} = 12.06$

Thus,

$$T_1 = \frac{T_1}{T_2} T_2 = \frac{1}{2.679} \ (1390) = \boxed{518.9^\circ R}$$

From Table A.1, for $M_1 = 3.0$, $\frac{T_{o_1}}{T_1} = 2.8$

$$T_{o\,2} = T_{o_1} = \frac{T_{o_1}}{T_1} \ T_1 = 2.8\ (518.9) = \boxed{1453^\circ R}$$

$$p_{o\,2} = \frac{p_{o_2}}{p_1} \ p_1 = (12.06)\ (1) = \boxed{12.06 \text{ atm}}$$

8.9 $\quad \frac{p_{o_2}}{p_{o_1}} = e^{-(s_2 - s_1)/R} = e^{-(199.5)/287} = 0.499$

From Table A.2: $\boxed{M_1 = 2.5}$

8.10 From Table A.2: $\frac{T_2}{T_1} = 2.799$ and $M_2 = 0.4695$

Hence,

$$T_2 = \frac{T_2}{T_1} T_1 = 2.799\,(480) = 1343.5^{\circ}R$$

$$a_2 = \sqrt{(1.4)(1716)(1343.5)} = 1796.6 \text{ ft/sec}$$

$$V_2 = M_2\, a_2 = (0.4695)(1796.6) = \boxed{843.5 \text{ ft/sec}}$$

From Table A.1, for $M_2 = 0.4695$, $\frac{T_{o_2}}{T_2} = 1.044$

$$T_2{}^* = \frac{T_2{}^*}{T_{o_2}} \frac{T_{o_2}}{T_2} T_2 = (0.8333)(1.044)(1343.5) = 1169^{\circ}R$$

$$A_2{}^* = \sqrt{\gamma R T_2^*} = \sqrt{(1.4)(1716)(1169)} = 1676 \text{ ft/sec}$$

$$M_2{}^* = \frac{V_2}{a_2^*} = \frac{843.5}{1676} = \boxed{0.503}$$

8.11 Is the flow subsonic or supersonic? For sonic flow, $\frac{p_o}{p} = \frac{1}{0.528} = 1.893$, which is higher than 1.555. Hence, the flow is <u>subsonic</u>. From Table A.1, for

$$\frac{p_o}{p} = 1.555, \; M = 0.82.$$

$$a = \sqrt{\gamma R T} = \sqrt{(1.4)(287)(288)} = 340.2 \text{ m/sec}$$

$$V = Ma = (0.82)(340.2) = 278.9 \text{ m/sec}$$

8.12 The ratio $\frac{7712.8}{2116} = 3.645$ is larger than 1.893. Hence, the flow is <u>supersonic</u>. This means that a normal shock wave exists in front of the nose of the Pitot tube. From Table A.2, for

$$\frac{p_{o_2}}{p_1} = \frac{7712.8}{2116} = 3.645, M_1 = 1.56$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{(1.4)(1716)(519)} = 1116.6 \text{ ft/sec}$$

$$V_1 = M_1 a_1 = (1.56)(1116.6) = \boxed{1742 \text{ ft/sec}}$$

8.13 (a) $$\rho = \frac{p}{RT} = \frac{1.01 \times 10^5}{(287)(288)} = 1.22 \text{ kg/m}^3$$

$$V = \sqrt{\frac{2(p_o - p)}{\rho}} = \sqrt{\frac{2(1.555 - 1.0)(1.01 \times 10^5)}{1.22}} = 303 \text{ m/sec} \quad \underline{\text{INCORRECT}}$$

$$\% \text{ error} = \frac{303 - 278.9}{278.9} = \boxed{8.69\%}$$

(b) $$\rho = \frac{p}{RT} = \frac{2116}{(1716)(519)} = 0.002376 \text{ slug/ft}^3$$

$$V = \sqrt{\frac{2(p_o - p)}{\rho}} = \sqrt{\frac{2(7712.8 - 2116)}{0.002376}} = 2170.5 \text{ ft/sec} \quad \underline{\text{INCORRECT}}$$

$$\% \text{ error} = \frac{2170.5 - 1742}{1742} = \boxed{24.6\%}$$

8.14 $$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right) = \frac{\gamma + 1 + 2\gamma\ M_1^2 - 2\gamma}{\gamma + 1} = \frac{1 - \gamma + 2\gamma\ M_1^2}{\gamma + 1} \qquad (1)$$

$$\frac{p_{o_2}}{p_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad (2)$$

$$M_2^2 = \frac{1 + [(\gamma - 1)/2] M_1^2}{\gamma\ M_1^2 - (\gamma - 1)/2}$$

Working with the expression inside the parenthesis of Eq. (2):

$$1+\frac{\gamma-1}{2}M_2^2 = 1+\frac{\gamma-1}{2}\left[\frac{1+\left(\frac{\gamma-1}{2}\right)\ M_1^2}{\gamma\ M_1^2-(\gamma-1)/2}\right] = 1+(\gamma\text{ - }1)\left[\frac{1+\left(\frac{\gamma-1}{2}\right)\ M_1^2}{2\gamma\ M_1^2-(\gamma-1)}\right]$$

$$= 1+(\gamma\text{ - }1)\left[\frac{2+(\gamma-1)\ M_1^2}{4\gamma\ M_1^2-2(\gamma-1)}\right] = \frac{4\gamma\ M_1^2-2(\gamma-1)+2(\gamma-1)+(\gamma-1)^2M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)}$$

$$= \frac{4\gamma\ M_1^2+(\gamma^2-2\gamma+1)\ M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)} = \frac{(\gamma^2+2\gamma+1)\ M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)}$$

$$= \frac{(\gamma+1)^2M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)} \qquad (4)$$

Combining Eqs. (4), (2), and (1), we have:

$$\frac{p_{o_2}}{p_1} = \frac{p_{o_2}}{p_2}\frac{p_2}{p_1} = \left[\frac{(\gamma+1)^2M_1^2}{4\gamma\ M_1^2-2\ (\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1-\gamma+2\gamma\ M_1^2}{\gamma+1}\right]$$ which is Eq. (8.80)

8.15 At 80,000 ft., $T_\infty = 389.99^\circ R$

$$V_\infty = 2112\left(\frac{88}{60}\right) = 3097.6 \text{ ft/sec}$$

$$a_\infty = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(389.99)} = 967.9 \text{ ft/sec}$$

$$M_\infty = \frac{3097.6}{967.9} = 3.2$$

From Appendix A:

For $M_\infty = 3.2$, $\frac{T_o}{T_\infty} = 3.048$

$T_o = 3.048\ T_\infty = 3.048\ (389.99) = 1188.7\ ^\circ R$

Since $0°F = 460°R$, the

$$\boxed{T_o = 728.7°F}$$

8.16 $\dfrac{p_{o_2}}{p_1} = \dfrac{1.13}{0.1} = 11.3.$

From Appendix B, $M_\infty = \boxed{2.9}$

8.17 The temperature at the stagnation point is the total temperature in the freestream, because the total temperature is constant across the normal shock. From Eq. (8.40),

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 = 1 + \frac{1.4 - 1}{2}(36)^2 = 260.2$$

Since $T_\infty = 300$ K, we have

$$T_o = (260.2)(300) = \boxed{78{,}060K}$$

This is an ungodly high temperature. It is also <u>incorrect</u>, because long before the air would reach this temperature, it would chemically dissociate and ionize. In such a chemically reacting gas, the specific heats are <u>not constant</u>, which means that Eq. (8.40) is not valid for such a chemically reacting flow. In reality, the temperature at the stagnation point on the Apollo was close to 11,000 K, much lower than our estimate above, but still plenty high. Air at 11,000 K is a partially ionized plasma. For the analysis of high temperature, chemically reacting flows, techniques much different than those discussed in this book must be used. See for example Anderson, <u>Modern Compressible Flow</u>, 2nd ed., McGraw-Hill, 1990, or Anderson, <u>Hypersonic and High Temperature Gas Dynamics</u>, McGraw-Hill, 1989, reprinted by the American Institute of Aeronautics and Astronautics, 2000.

8.18 Use Eq. (8.40)

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2$$

For $T_o = 11{,}000$ K, $T_\infty = 300$ K, and $M_\infty = 36$, this equation becomes:

$$\frac{11{,}000}{300} = 1 + \frac{\gamma - 1}{2}(36)^2$$

$$35.67 = 648\,\gamma - 648$$

or,

$$\gamma = \frac{683.7}{648} = \boxed{1.055}$$

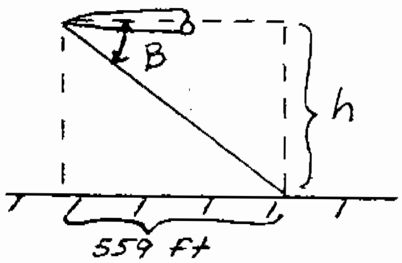
In order to use Eq. (8.40) to estimate a reasonably valid stagnation temperature for the Apollo, we have to use an "effective gamma" of 1.055. To double check this, return to Eq. (8.40), insert $\gamma = 1.055$, and calculate T_o.

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 = 1 + \frac{1.055 - 1}{2} (36)^2 = 36.64$$

or,

$$T_o = 36.64 \ T_\infty = 36.64 \ (300) = \boxed{11{,}000 \text{ K}}$$

9.1



$$\beta = \mathrm{Sin}^{-1}\left(\frac{1}{1.5}\right) = 41.8°$$

$$h = 559 \,\mathrm{Tan}\,\beta = 559 \,\mathrm{Tan}\, 41.8°$$

$$\boxed{h = 500 \text{ ft}}$$

9.2 $M_{n_1} = M_1 \sin\beta = (4.0)\sin 30° = 2$

From Table A.2, for $M_{n_1} = 2$: $\frac{p_2}{p_1} = 4.5$; $\frac{T_2}{T_1} = 1.687$, $\frac{p_{o_2}}{p_{o_1}} = 0.7209$, $M_{n_2} = 0.5774$

$$p_2 = \frac{p_2}{p_1}\, p_1 = (4.5)\,(2.65 \times 10^4) = \boxed{1.193 \times 10^5 \text{ N/m}^2}$$

$$T_2 = \frac{T_2}{T_1}\, T_1 = (1.687)(223.3) = \boxed{376.7°K}$$

From the θ-β-M diagram: $\theta = 17.7°$

$$M_2 = \frac{M_{n_2}}{\sin(\beta-\theta)} = \frac{0.5774}{\sin(30-17.7)} = \boxed{2.71}$$

From Table A.1, for $M_1 = 4$: $\frac{p_{o_1}}{p_1} = 151.8$, $\frac{T_{o_1}}{T_1} = 4.2$

$$p_{o_2} = \frac{p_{o_2}}{p_{o_1}}\,\frac{p_{o_1}}{p_1}\, p_1 = (0.7209)(151.8)(2.65 \times 10^4) = \boxed{2.9 \times 10^6 \text{ N/m}^2}$$

$$T_{o_2} = T_{o_1} = \frac{T_{o_1}}{T_1}\, T_1 = (4.2)(223.3) = \boxed{937.9°K}$$

$$s_2 - s_1 = -R\ \ell n\ \frac{p_{o_2}}{p_{o_1}} = -(287)\ \ell n\ 0.7209 = \boxed{93.9\ \frac{\text{joule}}{\text{kgm}^\circ\text{K}}}$$

9.3 Consider an oblique shock. For such a case,

$$\frac{p_{o_2}}{p_{o_1}} = \underbrace{\left(\frac{p_{o_2}}{p_2}\right)}_{\substack{\text{Depends on \underline{actual} Mach} \\ \text{number behind the shock} \\ M_2,\ \underline{\text{not}}\ M_{n_2}}} \times \underbrace{\left(\frac{p_2}{p_1}\right)}_{\substack{\text{Depends on \underline{normal}} \\ \text{Mach number upstream} \\ \text{of the shock, } M_{n_1}.}} \qquad (1)$$

In the derivation of Eq. (8.80), we related M_2 directly to M_1 through Eq. (8.78). This holds only for a <u>normal</u> shock. If we wish to use Eq. (8.78) for an oblique shock, then <u>both</u> M_2 and M_1 in Eq. (8.78) are replaced by M_{n_2} and M_{n_1}. However, in Eq. (1) above, p_{o_2}/p_2 Depends on M_2, not M_{n_2}. Because Eq. (8.78) does <u>not</u> relate M_2 to M_1 for an oblique shock (it relates M_{n_2} to M_{n_1}), then Eq. (8.78) cannot be used for the derivation of p_{o_2}/p_1 for an oblique shock. Therefore, the derivation of Eq. (8.80) holds only for a <u>normal</u> shock. It can <u>not</u> be used for an oblique shock, even with M_1 replaced by M_{n_1}. On the other hand,

$$s_2 - s_1 = c_p\ \ell n\ \frac{p_2}{p_1} - R\ \ell n\ \frac{T_2}{T_1}$$

where p_2/p_1 and T_2/T_1 for an oblique shock depend <u>only</u> on M_{n_1}. Since $\frac{p_{o_2}}{p_{o_1}} = e^{-(s_2 - s_1)/R}$ then clearly $\frac{p_{o_2}}{p_{o_1}}$ depends only on M_{n_1}. For these reasons, when using Table A.2 to determine changes across an <u>oblique</u> shock, using M_{n_1}, the total pressure ratio $\frac{p_{o_2}}{p_{o_1}}$ is a valid column, but the column giving $\frac{p_{o_2}}{p_1}$ is not valid.

9.4 To <u>CORRECTLY</u> calculate p_{o_2}:

$$M_{n_1} = M_1 \sin\beta = 3 \sin 36.87^\circ = 1.8$$

From Table A.2, for $M_{n_1} = 1.8$: $\quad \dfrac{p_{o_2}}{p_{o_1}} = 0.8127$

From Table A.1, for $M_1 = 3$: $\quad \dfrac{p_{o_2}}{p_1} = 36.73$

$$p_{o_2} \ \frac{p_{o_2}}{p_{o_1}} \ \frac{p_{o_1}}{p_1} \ p_1 = (0.8127)(36.73)(1) = \boxed{29.85 \text{ atm}}$$

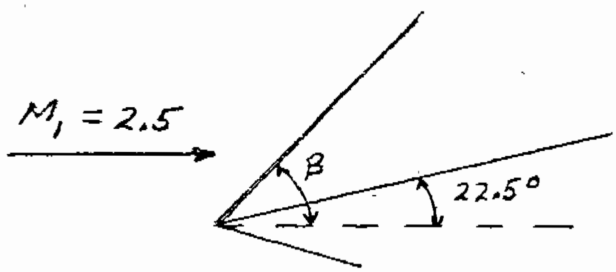
(b) The INCORRECT calculation of p_{o_2} would be as follows:

From Table A.2, for $M_{n_1} = 1.8$: $\quad \dfrac{p_{o_2}}{p_1} = 4.67$

$$p_{o_2} \ \frac{p_{o_2}}{p_1} \ p_1 = 4.67 \text{ (1 atm)} = 4.67 \text{ atm.} \quad \text{Totally } \underline{\text{WRONG}}$$

$$\% \text{ error} = \frac{29.85 - 4.67}{4.67} \times 100 = 539\% \quad \text{-- a terribly large error.}$$

9.5



From the θ-β-M diagram: $\boxed{\beta = 46°}$

$$M_{n_1} = M_1 \sin\beta = 2.5 \sin 46° = 1.8$$

From Table A.2, for $M_{n_1} = 1.8$, $\dfrac{p_2}{p_1} = 3.613$, $\dfrac{T_2}{T_1} = 1.532$, $M_{n_2} = 0.6165$

$$p_2 = \frac{p_2}{p_1} \ p_1 = 3.613 \text{ (1 atm)} = \boxed{3.613 \text{ atm}}$$

$$T_2 = \frac{T_2}{T_1}\, T_1 = (1.532)(300) = \boxed{459.6°K}$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.6165}{\sin(46 - 22.5)} = \boxed{1.546}$$

9.6 From the θ-β-M diagram, shock detachment occurs when $\alpha > 28.7°$. When $\alpha = \theta = 28.7°$, $\beta = 64.5°$.

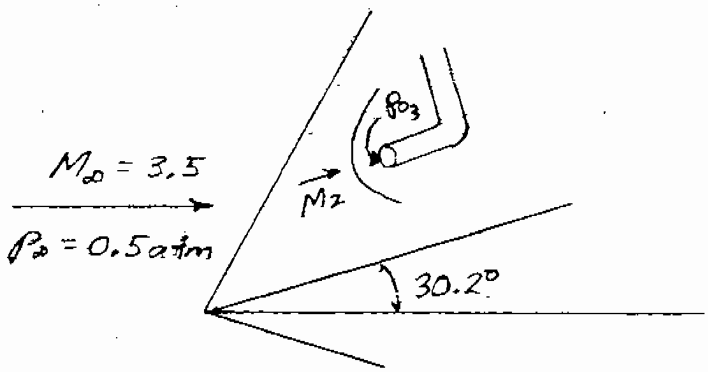
$$M_{n_1} = M_1 \sin\beta = 2.4 \ \sin 64.5° = 2.17$$

From Table A.2, for $M_{n_1} = 2.17$: $\frac{p_2}{p_1} = 5.327$

$$p_{max} = \frac{p_2}{p_1}\, p_1 = 5.327\ (1\ \text{atm}) = \boxed{5.327\ \text{atm}}$$

and the maximum pressure occurs when $\alpha = \boxed{28.7°}$

9.7



From the θ-β-M diagram: $\beta = 48°$

$$M_{n_1} = M_1 \sin\beta = 3.5 \ \sin 48° = 2.60$$

From Table A.2: $\frac{p_{o_2}}{p_{o_\infty}} = 0.4601$, $M_{n_2} = 0.5039$,

$$M_2 = \frac{M_{n_2}}{\sin(\beta-\theta)} = \frac{0.5039}{\sin(48-30.2)} = 1.648$$

From Table A.2, for $M_2 = 1.648$; $\frac{p_{o_3}}{p_{o_2}} = 0.876$

From Table A.1, for $M = 3.5$: $\frac{p_{o_\infty}}{p_\infty} = 76.27$

$$p_{o_3} = \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_{o_\infty}} \frac{p_{o_\infty}}{p_\infty} p_\infty = (0.876)(0.4601)(76.27)(0.5) = \boxed{15.37 \text{ atm}}$$

9.8 From Table A.1, for $M_1 = 4$, $\frac{p_{o_1}}{p_1} = 151.8$

Hence, $p_{o_1} = \frac{p_{o_1}}{p_1} p_1 = 151.8$ (1 atm) = 151.8 atm.

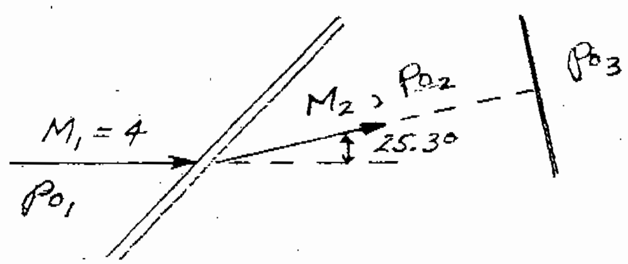
a) $M_1 = 4$ $\longrightarrow$ p_{o_1} | p_{o_2}

From Table A.2, for $M_1 = 4$: $\frac{p_{o_2}}{p_{o_1}} = 0.1388$

$$p_{o_2} = \frac{p_{o_2}}{p_{o_1}} p_{o_1} = 0.1388\,(151.8) = 21.07 \text{ atm}$$

Loss in total pressure = $p_{o_1} - p_{o_2} = 151.8 - 21.07 = \boxed{130.7 \text{ atm}}$

b)



From the θ-β-M diagram,

$$\beta = 38.7°$$

$$M_{n_1} = M_1 \sin\beta = 4 \sin 38.7° = 2.5$$

From Table A.2, for $M_{n_1} = 2.5$: $\frac{p_{o_2}}{p_{o_1}} = 0.499$, $M_{n_2} = 0.513$

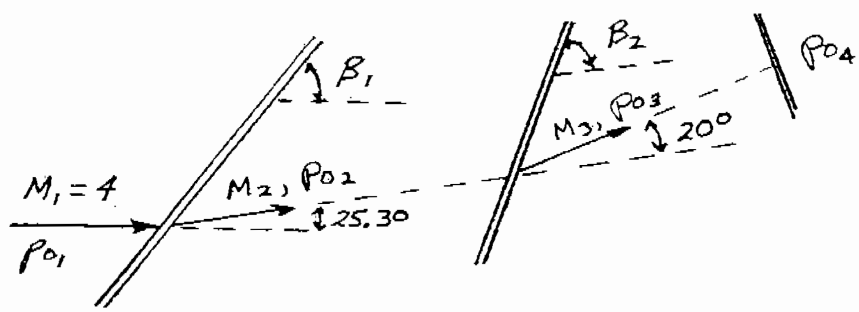
$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.513}{\sin(38.7 - 25.3)} = 2.21$$

From Table A.2, for $M_2 = 2.21$: $\frac{p_{o_3}}{p_{o_2}} = 0.6236$

$$p_{o_3} = \frac{p_{o_3}}{p_{o_1}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = (0.6236)(0.499)(151.8)(1 \text{ atm}) = 47.24 \text{ atm}$$

Loss in total pressure = $p_{o_1} - p_{o_3} = 151.8 - 47.24 = \boxed{104.6 \text{ atm}}$

c)



From part (b) above, $M_2 = 2.21$, $\frac{p_{o_2}}{p_{o_1}} = 0.499$.

From the β-θ-M diagram: $\beta_2 = 47.3°$

For the second shock: $M_{n_2} = M_2 \sin \beta_2 = 2.21 \sin 47.3° = 1.624$

From Table A.2, for $M_{n_2} = 1.624$: $\frac{p_{o_3}}{p_{o_2}} = 0.8877$, $M_{n_3} = 0.6625$

$$M_3 = \frac{M_{n_3}}{\sin(\beta_2 - \theta_2)} = \frac{0.6625}{\sin(47.3 - 20)} = 1.444$$

From Table A.2, for $M_3 = 1.444$: $\frac{p_{o_4}}{p_{o_3}}$ - 0.947

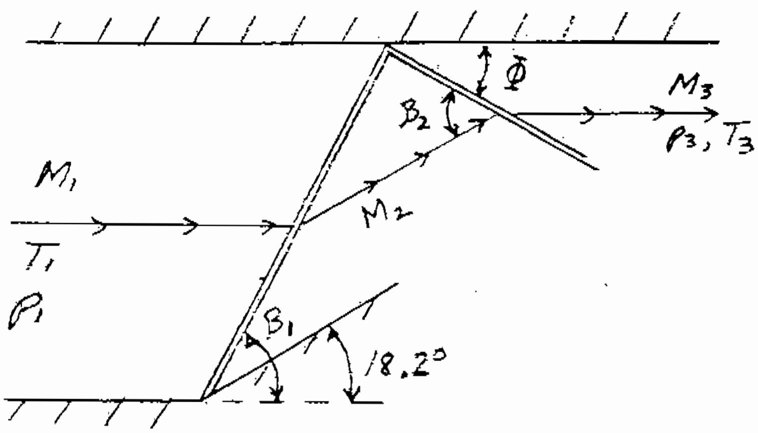
$$p_{o_4} = \frac{p_{o_4}}{p_{o_3}} \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = (0.947)(0.8877)(0.499)(151.8)$$

$$p_{o_4} = 63.68 \text{ atm}$$

Loss in total pressure = $p_{o_1} - p_{o_4} = 151.8 - 63.68 = \boxed{88.1 \text{ atm}}$

CONCLUSION: To decrease a supersonic flow to subsonic speeds via a shock system, a series of oblique shocks followed by a normal shock yields a smaller total pressure loss than a normal shock by itself. Hence, a system of oblique shocks, followed by a normal shock is a more efficient means of slowing a supersonic flow to subsonic speeds than a single normal shock itself.

9.9



From the θ-β-M diagram, $\beta_1 = 34.2°$

$$M_{n_1} = M_1 \sin \beta_1$$

$$= (3.2) \sin 34.2° = 1.8$$

From Table A.2; for $M_{n_1} = 1.8$: $\frac{p_2}{p_1} = 3.613$, $\frac{T_2}{T_1} = 1.532$,

$$M_{n_2} = 0.6165$$

$$M_2 = \frac{M_{n_2}}{\sin(\beta_1 - \theta_1)} = \frac{0.6165}{\sin(34.2 - 8.2)} = 2.24$$

For the <u>Reflected Shock</u>:

From the θ-β-M diagram, for $M_2 = 2.24$ and $\theta = 18.2°$: $\beta_2 = 44°$

$$M_{n_2} = M_2 \sin \beta_2 = 2.24 \sin 44° = 1.56$$

From Table A.2, for $M_{n_2} = 1.56$: $\frac{p_3}{p_2} = 2.673$, $\frac{T_3}{T_2} = 1.361$, $M_{n_3} = 0.6809$

$$M_3 = \frac{M_{n_3}}{\sin(\beta_2 - \theta)} = \frac{0.6809}{\sin(44 - 18.2)} = \boxed{1.56}$$

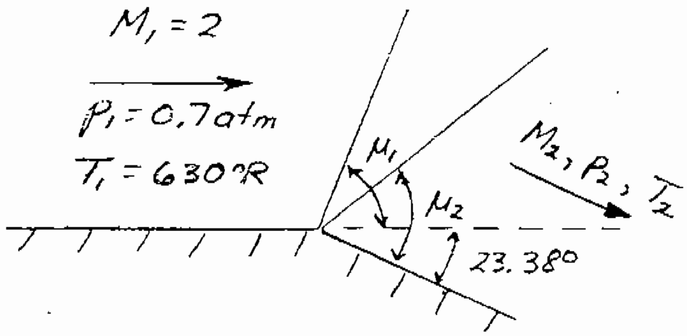
<u>Note</u>: The fact that M_3 and M_{n_2} are equal is just a coincidence.

$$\Phi = \beta_2 - \theta = 44 - 18.2 = \boxed{25.8°}$$

$$p_3 = \frac{p_3}{p_2}\frac{p_2}{p_1} p_1 = (2.673)(3.613)(1 \text{ atm}) = \boxed{9.66 \text{ atm}}$$

$$T_3 = \frac{T_3}{T_2}\frac{T_2}{T_1} T_1 = (1.361)(1.532)(520) = \boxed{1084°R}$$

9.10



From Table A.3: For $M_1 = 2$, $\nu_1 = 26.38°$

$$\nu_2 = \theta + \nu_1 = 23.38° + 26.38° = 49.76°$$

Hence,

$$\boxed{M_2 = 3.0}$$

From Table A.1, for $M_1 = 2$: $\dfrac{p_{o_1}}{p_1} = 7.824$, $\dfrac{T_{o_1}}{T_1} == 1.8$

For $M_2 = 3$: $\dfrac{p_{o_2}}{p_2} = 36.73$, $\dfrac{T_{o_2}}{T_2} = 2.8$

However: $p_{o_1} = p_{o_2}$ and $T_{o_1} = T_{o_2}$. Thus

$$p_2 = \frac{p_2}{p_{o_2}} \frac{p_{o_1}}{p_1} p_1 = \left(\frac{1}{36.73}\right)(7.824)(0.7) = \boxed{0.149 \text{ atm}}$$

$$T_2 = \frac{T_2}{T_{o_2}} \frac{T_{o_1}}{T_1} T_1 = \left(\frac{1}{2.8}\right)(1.8)(630) = \boxed{405°R}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{(0.149)(2116)}{(1716)(405)} = \boxed{4.537 \times 10^{-4} \text{ slug/ft}^3}$$

$$p_{o_2} = p_{o_1} = \frac{p_{o_1}}{p_1} p_1 = (7.824)(0.7) = \boxed{5.477 \text{ atm}}$$

$$T_{o_2} = T_{o_1} = \frac{T_{o_1}}{T_1} T_1 = (1.8)(630) = \boxed{1134°R}$$

From Table A.3: for $M_1 = 2$, $\mu_1 = 30°$

For $M_2 = 3$, $\mu_2 = 19.47$

Referenced to the upstream direction:

Angle of forward Mach line = $\mu_1 = \boxed{30°}$

Angle of rearward Mach line = $\mu_2 - \theta = 19.47 - 23.38° = \boxed{-3.91°}$

Note: The rearward Mach line is <u>below</u> the upstream direction for this problem.

9.11 From Table A.1, for $M_1 = 1.58$: $\frac{p_{o_1}}{p_1} = 4.127$

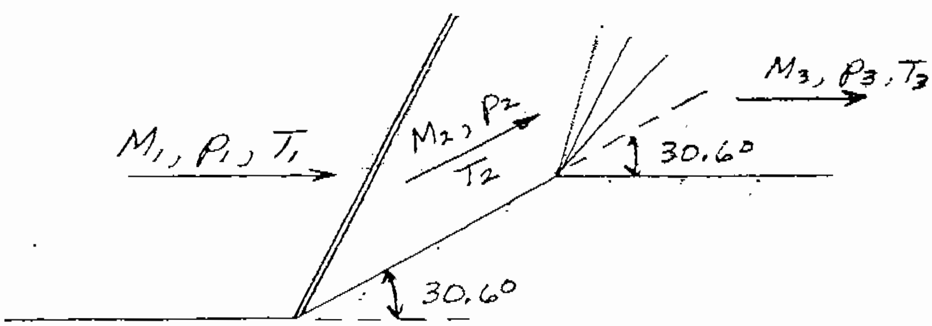
$$\frac{p_{o_2}}{p_2} = \frac{p_{o_1}}{p_2} = \frac{p_{o_1}}{p_1}\frac{p_1}{p_2} = (4.127)\left(\frac{1}{0.1306}\right) = 31.6$$

From Table A.1, for $\frac{p_{o_2}}{p_2} = 31.6$, $M_2 = 2.9$

From Table A.3, for $M_1 = 1.58$; $\nu_1 = 14.27$, for $M_2 = 2.9$: $\nu_2 = 47.79$

$$\theta = \nu_2 - \nu_1 = 47.79 - 14.27 = \boxed{33.52°}$$

9.12



From the θ-β-M diagram:

For $M_1 = 3$ and $\theta = 30.6°$, $\beta = 53.1°$

$$M_{n_1} = M_1 \sin\ \beta = 3 \sin 53.1 = 2.4$$

From Table A.2, for $M_{n_1} = 2.4$: $\frac{p_2}{p_1} = 6.553$, $\frac{T_2}{T_1} = 2.04$, $\frac{p_{o_2}}{p_{o_1}} = 0.541$, $M_{n_2} = 0.531$

$$M_2 = \frac{M_{n_2}}{\sin(\beta - \theta)} = \frac{0.5231}{\sin(53.1 - 30.6)} = 1.37$$

From Table A.3: For $M_2 = 1.37$, $\nu_2 = 8.128$

$$\nu_3 = 8.128 + 30.6 = 38.73°$$

From Table A.3: For $\nu_3 = 38.73°$, $M_3 = \boxed{2.48}$

From Table A.1: For $M_1 = 3$, $\frac{p_{o_1}}{p_1} = 36.73$, $\frac{T_{o_1}}{T_1} = 2.8$

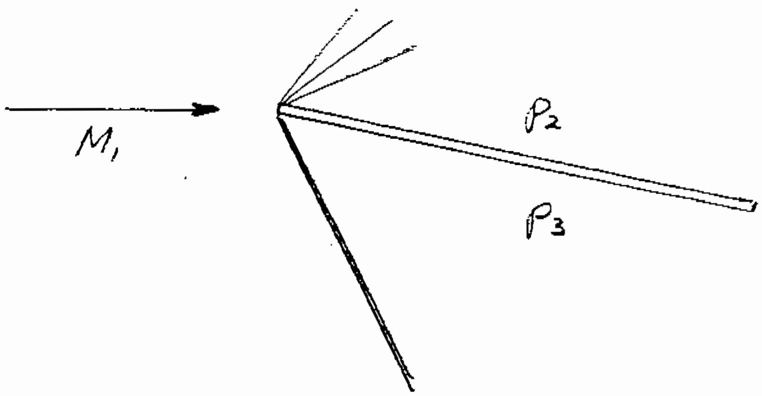
For $M_3 = 2.48$, : $\frac{p_{o_3}}{p_3} = 16.56$, $\frac{T_{o_3}}{T_3} = 2.23$

$$p_3 = \frac{p_3}{p_{o_3}} \frac{p_{o_3}}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} p_1 = \left(\frac{1}{16.56}\right) (1)(0.5401)(36.73)(1 \text{ atm}) = \boxed{120 \text{ atm}}$$

$$T_3 = \frac{T_3}{T_{o_3}} \frac{T_{o_3}}{T_{o_2}} \frac{T_{o_2}}{T_{o_1}} \frac{T_{o_1}}{T_1} T_1 = \left(\frac{1}{2.23}\right) (1)(1)(2.8)(285) = \boxed{357.8°K}$$

Clearly, $p_3 \neq p_1$, $T_3 \neq T_1$, and $M_3 \neq M$. Why? Because there is an entropy increase across the shock wave, which permanently alters the thermodynamic state of the original flow, even after it is brought back to its original direction.

9.13



(a) For $M_1 = 2.6$ and $\theta = 5°$, $\beta = 26.5°$

$$M_{n_1} = M_1 \sin\beta = 2.6 \sin 26.5° = 1.16$$

From Table A.2: $\dfrac{p_3}{p_1} = 1.403$

From Table A.1, for $M_1 = 2.6$: $\dfrac{p_{o_1}}{p_1} = 19.95$

From Table A.3, for $M_1 = 2.6$: $\nu_1 = 41.41°$

$$\nu_2 = \nu_1 + \theta = 41.41 + 5° = 46.41° \rightarrow M_2 = 2.83$$

From Table A.1, for $M_2 = 2.83$: $\dfrac{p_{o_2}}{p_2} = 28.4$

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} = (0.0352)(1)(19.95) = 0.7022$$

$$c_\ell = \frac{L'}{q_\infty S} = \frac{(p_3 - p_2)c\ \cos\alpha}{q_\infty\ c\ (1)} = \frac{(p_3 - p_2)}{q_\infty} \cos\alpha$$

$$q_\infty = q_1 = ½\ \rho_1\ V_1^2 = \frac{\gamma\ p_1 \rho_1 V_1^2}{2\ \gamma\ p_1} = \frac{\gamma\ p_1 V_1^2}{2\ a_1^2} = \frac{\gamma\ p_1 M_1^2}{2}$$

$$c_\ell = \frac{2(p_3 - p_2)}{\gamma\ p_1\ M_1^2} \cos\alpha = \frac{2}{\gamma\ M_1^2} \left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right) \cos\alpha$$

$$c_\ell = \frac{2}{(1.4)(2.6)^2} (1.403 - 0.7022) \cos 5^\circ = \boxed{0.148}$$

$$c_d = \frac{2}{\gamma\ M_1^2} \left(\frac{p_3}{p_2} - \frac{p_2}{p_1}\right) \sin\alpha = c_\ell \frac{\sin\alpha}{\cos\alpha} = 0.148 \frac{\sin 5^\circ}{\cos 5^\circ} = \boxed{0.0129}$$

(b) For $M_1 = 2.6$ and $\theta = 15^\circ$, $\beta = 35.9^\circ$

$$M_{n_1} = M_1 \sin\beta = 2.6 \sin 35.9^\circ = 1.525$$

From Table A.2: $\frac{p_3}{p_1} = 2.529$

From Table A.1, for $M_1 = 2.6$: $\frac{p_{o_1}}{p_1} = 19.95$

From Table A.3, for $M_1 = 2.6$: $\nu_1 = 41.41^\circ$

$$\nu_2 = \nu_1 + \theta = 41.41 + 15 = 56.41^\circ \rightarrow M_2 = 3.37$$

From Table A.1, for $M_2 = 3.37$: $\frac{p_{o_2}}{p_2} = 63.33$

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} \frac{p_{o_1}}{p_1} = \left(\frac{1}{63.33}\right)(1)(19.95) = 0.315$$

$$c_\ell = \frac{2}{\gamma\ M_1^2} \left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right) \cos\alpha = \frac{2}{(1.4)(2.6)^2} (2.529 - 0.315) \cos 15^\circ = \boxed{0.452}$$

$$c_d = c_\ell \frac{\sin\alpha}{\cos\alpha} = 0.452 \frac{\sin 15^\circ}{\cos 15^\circ} = \boxed{0.121}$$

(c) For $M_1 = 2.6$ and $\theta = 30^\circ$, $\beta = 59.3^\circ$

$$M_{n_1} = M_1 \sin\beta = 2.6 \sin 59.3^\circ = 2.24$$

$$\frac{p_3}{p_1} = 5.687,\ \frac{p_{o_1}}{p_1} = 19.95,\ \nu_1 = 41.41°$$

$$\nu_2 = \nu_1 + \theta = 41.41 + 30 = 71.41° \rightarrow M_2 = 4.46$$

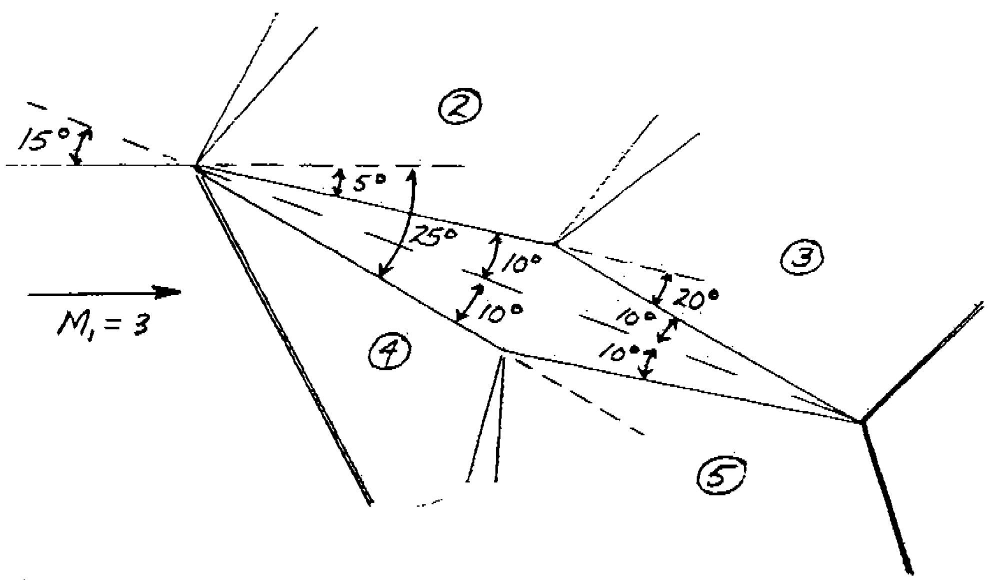
$$\frac{p_{o_2}}{p_2} = 275.25$$

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}}\frac{p_{o_2}}{p_{o_1}}\frac{p_{o_1}}{p_1} = \left(\frac{1}{275.25}\right)(1)(19.95) = 0.0725$$

$$c_\ell = \frac{2}{\gamma M_1^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\cos\alpha = \frac{2}{(1.4)(2.6)^2}(5.687 - 0.0725) = \boxed{1.19}$$

$$c_d = 1.19\,\frac{\sin 30°}{\cos 30°} = \boxed{0.687}$$

9.14



For region 2:

$\nu_1 = 49.76°$

$\nu_2 = \nu_1 + \theta = 49.76° + 5° = 54.76° \rightarrow M_2 = 3.27$

For $M_1 = 3$: $\dfrac{p_{o_1}}{p_1} = 36.73$:

For $M_2 = 3.27$, $\dfrac{p_{o_2}}{p_2} = 54.76$

For region 3:

$\nu_3 = \nu_2 + \theta = 54.76° + 20° = 74.76° \rightarrow M_3 = 4.78$

For $M_3 = 4.78$: $\dfrac{p_{o_3}}{p_3} = 407.83$

For region 4:

$M_1 = 3$ and $\theta = 25° \rightarrow \beta = 44°$

$M_{n_1} = M_1 \sin\beta = 3 \sin 44 = 2.08$

$\dfrac{p_4}{p_1} = 4.881$, $M_{n_4} = 0.5643$, and $\dfrac{p_{o_4}}{p_{o_1}} = 0.6835$

$$M_4 = \frac{M_{n_4}}{\sin(\beta - \theta)} = \frac{0.5643}{\sin(44 - 25)} = 1.733.$$

Thus,

$\nu_5 = 18.69$, $\dfrac{p_{o_4}}{p_4} = 5.165$

For region 5:

$\nu_5 = \nu_4 + \theta = 18.69° + 20° = 38.69° \rightarrow M_5 = 2.48$

$$\frac{p_{o_5}}{p_5} = 16.56$$

Pressure ratios

$$\frac{p_2}{p_1} = \frac{p_2}{p_{o_2}}\frac{p_{o_1}}{p_{o_1}}\frac{p_{o_1}}{p_1} = \left(\frac{1}{54.76}\right)(1)(36.73) = 0.6707$$

$$\frac{p_3}{p_1} = \frac{p_2}{p_1}\frac{p_3}{p_2} = \frac{p_2}{p_1}\frac{p_3}{p_{o_3}}\frac{p_{o_3}}{p_{o_2}}\frac{p_{o_2}}{p_2} = (0.6707)\left(\frac{1}{407.83}\right)(1)(54.76) = 0.09$$

$$\frac{p_4}{p_1} = 4.881$$

$$\frac{p_5}{p_1} = \frac{p_5}{p_{o_5}}\frac{p_{o_5}}{p_{o_4}}\frac{p_{o_4}}{p_{o_1}}\frac{p_{o_1}}{p_1} = \left(\frac{1}{16.56}\right)(1)(0.6835)(36.73) = 1.516$$

Let ℓ = length of each face of the diamond wedge.

$$L' = p_4\ \ell\ \cos 25° + p_5\ \ell\ \cos 5° - p_2\ \ell\ \cos 5° - p_3 \cos 25°$$

$$L' = (p_4 - p_3)\ \ell\ \cos 25° + (p_5 - p_2)\ \ell\ \cos 5°$$

$$c_\ell = \frac{L'}{q_\infty S} = \frac{L'}{\frac{\gamma}{2} p_1 M_1^2 c} = \frac{2}{\gamma\ M_1^2}\frac{\ell}{c}\left[\left(\frac{p_4}{p_1} - \frac{p_3}{p_1}\right)\cos 25° + \left(\frac{p_5}{p_1} - \frac{p_2}{p_1}\right)\cos 5°\right]$$

$$c_\ell = \frac{2}{(1.4)(3)^2}\ \frac{\ell}{c}\ [(4.881 - 0.09)\cos 25° + (1.516 - 0.6707)\cos 5°]$$

$$c_\ell = 0.823\ \frac{\ell}{c}.$$

However,

$$\frac{c/2}{\ell} = \cos 10° \qquad \frac{\ell}{c} = \frac{1}{2\ \cos\ 10°} = 0.5077$$

$$c_\ell = (0.823)(0.5077) = \boxed{0.418}$$

$$D' = p_4\ \ell\ \sin 25° + p_5\ \ell\ \sin 5° - p_2\ \ell\ \sin 5° - p_3\ \ell\ \sin 25°$$

$$D' = (p_4 - p_3)\ \ell\ \sin 25° + (p_5 - p_2)\ \ell\ \sin 5°$$

$$c_d = \frac{D'}{q_\infty S} = \frac{D'}{\frac{\gamma}{2} p_1 M_1^2 c} = \frac{2}{\gamma M_1^2} \frac{\ell}{c} \left[\left(\frac{p_4}{p_1} - \frac{p_3}{p_1}\right)\sin 25° + \left(\frac{p_5}{p_1} - \frac{p_2}{p_1}\right)\sin 5°\right]$$

$$c_d = \frac{2}{(1.4)(3)^2} \frac{\ell}{c} [(4.881 - 0.09) \sin 25° + (1.516 - 0.6707) \sin 5°]$$

$$c_d = 0.333 \frac{\ell}{c} = 0.333\ (0.5077) = \boxed{0.169}$$

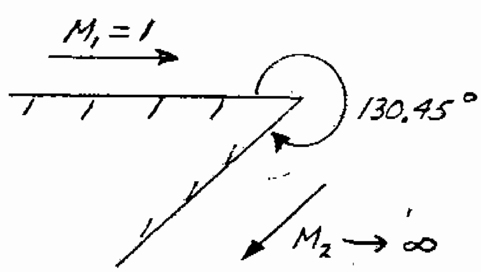
9.15 The maximum expansion would correspond to $M_2 \to \infty$. From Eq. (9.42) in the text,

$$\lim_{M_2 \to \infty} \nu_2 = \lim_{M_2 \to \infty} \left\{\sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+11}(M_2^2 - 1)} - \tan^{-1}\sqrt{M^2 - 1}\right\}$$

$$= \sqrt{\frac{\gamma+1}{\gamma-1}}\ \frac{\pi}{2} - \frac{\pi}{2} - \left(\sqrt{\frac{\gamma+1}{\gamma-1}} - 1\right) \frac{\pi}{2} = 2.277 \text{ rad} = 130.45°$$

Since, for $M_1 = 1$, $\nu_1 = 0$, then

$$\theta_{max} = \nu_2 - \nu_1 = 130.45 - 0 = \boxed{130.45°}$$



9.16 For the cylinder, with c_d based on frontal area,

$$(D')_{cyl} = q_\infty S c_d = q_\infty d(1)/(4/3) = \frac{4}{3} (d) q_\infty$$

For the dimensional wedge airfoil, referring to Figure 9.27.

$$(D')_w = (p_2 - p_3) t$$

Hence,

$$\frac{(D')_{cyl}}{(D')_w} = \frac{\frac{4}{3}(d)\ q_\infty}{(p_2 - p_3)\ t}$$

However, $t = d$ and $q_\infty = \frac{\gamma}{2} p_1 M_1^2$

Thus,

$$\frac{(D')_{cyl}}{(D')_w} = \frac{\frac{4}{3}\left(\frac{\gamma}{2}\right) M_1^2}{\left(\frac{p_2}{p_1} - \frac{p_3}{p_1}\right)} = \frac{\frac{2}{3}\ \gamma\ M_1^2}{\left(\frac{p_2}{p_1} - \frac{p_3}{p_1}\right)}$$

To calculate p_2/p_1, we have, for $M_1 = 5$ and $\theta = 5°$, $\beta = 15.1°$.

$$M_{n,1} = M_1 \sin\beta = 5 \sin(15.1°) = 1.303$$

From Appendix B, for $M_{n,1} = 1.302$, $\frac{p_2}{p_1} = 1.805$. Also,

$$M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)} = \frac{0.786}{\sin(15.1 - 5)} = 4.48.$$

To calculate $\frac{p_3}{p_1}$, the flow is expanded through an angle of 10°. From Table C, for $M_2 =$ 4.48, $\nu_2 = 71.83$ (nearest entry).

$$\nu_3 = \nu_2 + \theta = 71.83 + 10 = 81.38°$$

Hence, $M_3 = 5.6$ (nearest entry)

From Appendix A: For $M_1 = 5$, $\dfrac{p_{o_1}}{p_3} = 529.1$

For $M_3 = 5.6$, $\dfrac{p_{o_3}}{p_3} = 1037$

From Appendix B: For $M_{n_1} = 1.303$, $\dfrac{p_{o_2}}{p_{o_1}} = 0.9794$

Thus,

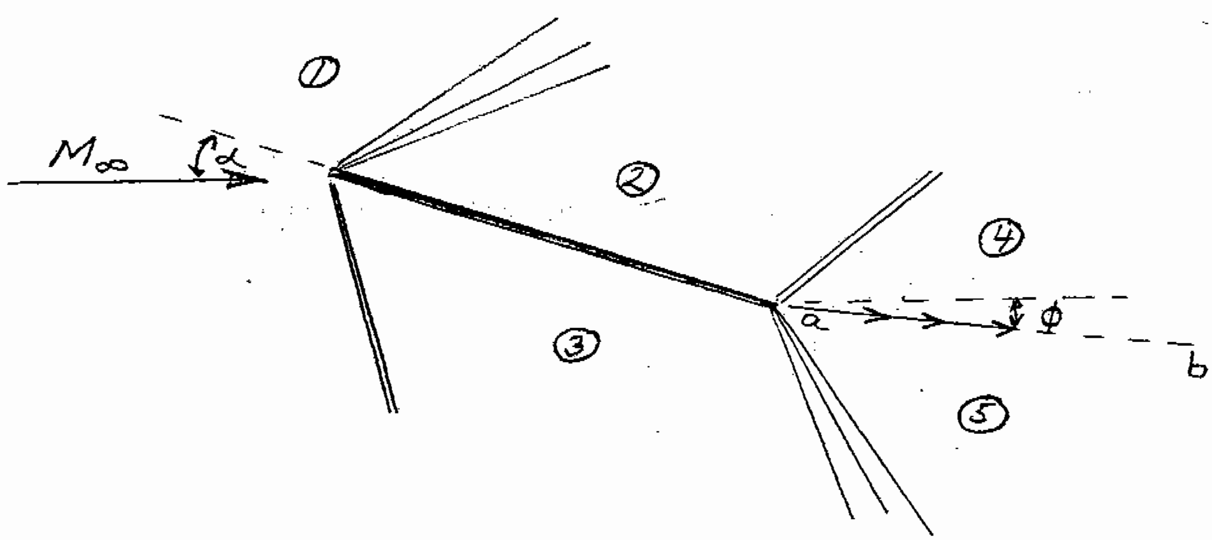
$$\frac{p_3}{p_1} = \frac{p_3}{p_{o_3}}\frac{p_{o_3}}{p_{o_2}}\frac{p_{o_2}}{p_{o_1}}\frac{p_{o_1}}{p_1} = \left(\frac{1}{1037}\right)(1)(0.9794)(529.1) = 0.5$$

Hence,

$$\frac{(D')_{cyl}}{(D')_w} = \frac{\frac{2}{3}\gamma\, M_1^2}{\left(\frac{p_2}{p_1} - \frac{p_3}{p_1}\right)} = \frac{\frac{2}{3}(1.4)(5)^2}{(1.805 - 0.5)} = \boxed{17.9}$$

Note: This is why we try to avoid blunt leading edges on supersonic vehicles. (However, at hypersonic speeds, blunt leading edges are necessary to reduce the aerodynamic heating.)

9.17 The supersonic flow over a flat plate at a given angle of attack in a freestream with a given Mach number, M_∞, is sketched below.



The flow direction downstream of the leading edge is given by line ab. The flow direction is below the horizontal (below the direction of M_∞) because lift is produced on the flat plate, and due to overall momentum considerations, the downstream flow must be inclined slightly downward. Also, line ab is a slip line; the entropy in region 4 is different than in region 5 because the flows over the top and bottom of the plate have gone through shock waves of different strengths. The boundary condition that must hold across the slip line is constant pressure, i.e., $p_4 = p_5$. It is this boundary condition that fixes the strengths of the expansion wave and the shock wave at the trailing edge.

To calculate the trailing edge shock and expansion waves, and the flow direction downstream, use the following iterative approach:

1. Assume a value for ϕ.
2. Calculate the strength of the trailing edge shock for the local deflection angle $(\alpha-\phi)$. This gives, among other quantities, a value of p_4.
3. Calculate the strength of the trailing edge expansion wave for a local expansion angle of $(\alpha-\phi)$. This gives a value for p_5.
4. Compare p_4 and p_5 from steps 3 and 4. If they are different, assume a new value of ϕ.
5. Repeat steps 2-4 until $p_4 = p_5$. When this condition is satisfied, the iteration has converged, and the trailing edge flow is now determined.

CHAPTER 10

10.1 From Table A.1, for $A_e/A^* = 2.193$, $\boxed{M_e = 2.3}$

$$\frac{p_{o_e}}{p_e} = 12.5, \quad \frac{T_{o_e}}{T_e} = 2.058.$$

For isentropic flow, T_o = constant and p_o = constant. Hence,

$$p_{o_e} = p_o = \boxed{5 \text{ atm}}, \text{ and } T_{o_e} = T_o = \boxed{520°R}$$

$$p_e = \frac{p_e}{p_{o_e}} p_o = \left(\frac{1}{12.5}\right)(5 \text{ atm}) = \boxed{0.4 \text{ atm}}$$

$$T_e = \frac{T_e}{T_{o_e}} T_o = \left(\frac{1}{2.058}\right)(520) = \boxed{252.7°R}$$

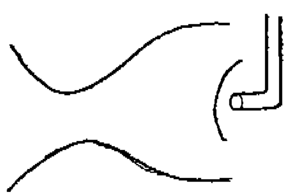
$$\rho_e = \frac{p_e}{RT_e} = \frac{(0.4)(2116)}{(1716)(252.7)} = \boxed{0.00195 \text{ slug/ft}^3}$$

$$a_e = \sqrt{\gamma RT_e} = \sqrt{(1.4)(1716)(252.7)} = 779.2 \text{ ft/sec}$$

$$u_e = M_e a_e = (2.3)(779.2) = \boxed{1792 \text{ ft/sec}}$$

10.2 $\frac{p_o}{p_e} = \frac{1}{0.3143} = 3.182$. From Table A.1, we see that $\boxed{M_e = 1.4}$, and $A_e/A^* = \boxed{1.115}$.

10.3 Ahead of the normal shock in front of the Pitot tube,



$$p_{o_1} = p_o = 2.02 \times 10^5 \text{ N/m}^2$$

$$\frac{p_{o_2}}{p_{o_1}} = \frac{8.92 \times 10^4}{2.02 \times 10^5} = 0.4416$$

From Table A.2: $M_e = 2.65$

From Table A.1: $A_e/A^* = \boxed{3.036}$

10.4 $\dot{m} = \rho^* u^* A^*; \; \rho_o = \dfrac{p_o}{RT_o} = \dfrac{(5)(2116)}{(1716)(520)} = 0.01186 \dfrac{\text{slug}}{\text{ft}^3}$

$$\rho^* = \frac{\rho^*}{\rho_o}\rho_o = (0.634)(0.01186) = 0.007519 \text{ slug/ft}^3$$

$$T^* = \frac{T^*}{T_o} T_o = (0.833)(520) = 433.2°\text{R}$$

$$u^* = a^* = \sqrt{(1.4)(1716)(433.2)} = 1020 \text{ ft/sec}$$

$$\dot{m} = \rho^* u^* A^* = (0.007519)(1020)\left(\frac{4}{144}\right) = \boxed{0.213 \ \frac{\text{slug}}{\text{sec}}}$$

10.5 $\dot{m} = \rho^* u^* A^*$

$$u^* = \sqrt{\gamma R T^*} \text{ and } \rho^* = \frac{p^*}{RT^*}$$

Hence,

$$\dot{m} = \frac{p^*}{RT^*} A^* \sqrt{\gamma R T^*} = \frac{p^* A^*}{RT^*} \sqrt{\gamma}$$

Since, $M^* = 1$, then

$$\frac{T_o}{T^*} = 1 + \frac{\gamma - 1}{2} M^{*2} = \frac{\gamma + 1}{2}$$

$$\frac{p_o}{p^*} = \left(\frac{\gamma + 1}{2}\right)^{\gamma/(\gamma - 1)}$$

Thus,

$$\dot{m} = \sqrt{\frac{\gamma}{R}}\, A^* \left(\frac{\gamma+1}{2}\right)^{-/(\gamma+1)/2(\gamma-1)} \frac{p_o}{\sqrt{T_o}}$$

or,

$$\boxed{\dot{m} = \frac{p_o A^*}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+!)/(\gamma-1)}}}$$

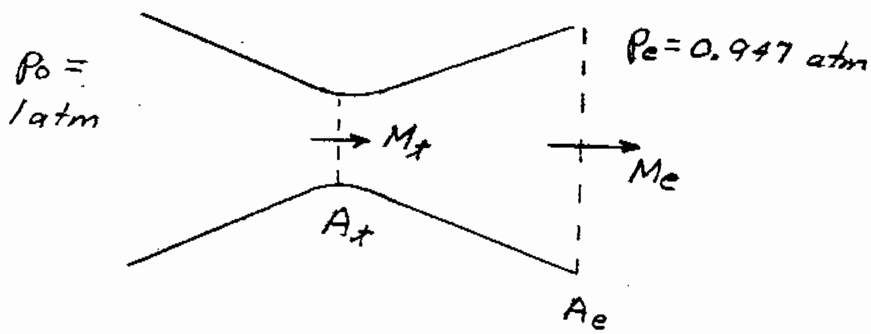
10.6 $p_o = 5 \text{ atm} = 5(2116) = 10580 \text{ lb/ft}^2$

$A^* = 4/144 = 0.02778 \text{ ft}^2$

$$\dot{m} = \frac{(105809)(0.02778)}{\sqrt{520}} \sqrt{\frac{(1.4)}{(1716)}\left(\frac{2}{2.4}\right)^6} = \boxed{0.213 \ \frac{\text{slug}}{\text{sec}}}$$

which is the same as obtained in Problem 10.4

10.7



First, check to see if the flow is sonic at the throat.

$$\frac{p_o}{p_e} = \frac{1}{0.947} = 1.056$$

From Table A., for $\frac{p_o}{p_e} = 1.056$: $M_e = 0.28$ and $A_e/A^* = 2.166$

Since $\frac{A_e}{A_t} = 1.616 < \frac{A_e}{A^*} = 2.166$, then $A_t > A^*$. The throat size is larger than that for sonic flow, hence the throat Mach number, M_t, is subsonic.

$$\frac{A_t}{A^*} = \frac{A_t}{A_e}\frac{A_e}{A^*} = \frac{1}{1.616}(2.166) = 1.34$$

From Table A.1, for $\frac{A_t}{A^*} = 1.34$; $\boxed{M_t = 0.5}$, $\frac{p_o}{p_t} = 1.186$

$$p_t = \frac{p_t}{p_o}\frac{p_o}{p_e}p_e = \left(\frac{1}{1.186}\right)(1.056)(0.947) = \boxed{0.843 \text{ atm}}$$

10.8 Note: The equation for $\dot{m}$ given in Problem 10.5 can not be used here because the flow is not choked, i.e., the throat Mach number is not sonic.

$$\dot{m} = \rho_e A_e u_e$$

From Table A.1, for $\frac{p_o}{p_e} = 1.056$: $M_e = 0.28$, $\frac{T_o}{T_e} = 1.016$

$$T_e = T_o/1.016 = 288/1.016 = 283.5°K$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{(0.947)(1.01 \times 10^5)}{(287)(283.5)} = 1.176 \text{ kg/m}^3$$

$$a_e = \sqrt{\gamma RT_e} = \sqrt{(1.4)(287)(283.5)} = 337.5 \text{ m/sec}$$

$$u_e = M_e a_e = (0.28)(337.5) = 94.5 \text{ m/sec}$$

$$A_e = A_t \left(\frac{A_e}{A_t}\right) = (0.3)(1.616) = 0.4848 \text{ m}^2$$

$$\dot{m} = \rho_e A_e u_e = (1.176)(0.4848)(94.5) = \boxed{53.88 \text{ kg/sec}}$$

10.9 (a) $\dfrac{p_o}{p_e} = \dfrac{1}{0.94} = 1.064.$

From Table A.1: $M_e = 0.3$ and $A_e/A^* = 2.035$. $\dfrac{A_t}{A^*} \dfrac{A_t}{A_e} \dfrac{A_e}{A^*} = \left(\dfrac{1}{1.53}\right)(2.035) = 1.33.$

Since $A_t > A^*$, then the flow is completely subsonic. No shock wave exists. Hence, from Table A.1, for $\dfrac{p_o}{p_e} = 1.064$, $\boxed{M_e = 0.3}$.

(b) $\dfrac{p_o}{p_e} = \dfrac{1}{0.886} = 1.129.$

From Table A.1, for $\dfrac{p_o}{p_e} = 1.129$: $M_e = 0.42$ and $\dfrac{A_e}{A^*} = 1.539.$

$$\frac{A_t}{A^*} = \frac{A_t}{A_e}\frac{A_e}{A^*} = \left(\frac{1}{1.53}\right)(1.529) = 0.999 \approx 1.0.$$

Hence, $A_t = A^*$, and the flow is precisely sonic at the throat. It is subsonic everywhere else. Hence, from the above $\boxed{M_e = 0.42}$.

(c) From the above results, clearly when p_e is reduced below 0.866 atm, sonic flow will occur at the throat, and the nozzle will be choked. Since $p_e = 0.75$ atm is far above the supersonic exit pressure, we suspect that a normal shock wave exists within the nozzle. Note that, if we run the same calculation as in parts (a) and (b) above, we find:

$$\frac{p_o}{p_e} = \left(\frac{1}{0.75}\right) = 1.333.$$

From Table A.1, for $\dfrac{p_o}{p_e} = 1.333$, we have

$$\frac{A_e}{A^*} = 1.127$$

$\frac{A_t}{A^*} = \frac{A_t}{A_e}\frac{A_e}{A^*} = \left(\frac{1}{1.53}\right)(1.127) = 0.7366$. Since it is impossible for $A_t < A^*$, then clearly the flow can not be completely isentropic. There must be a shock wave inside the nozzle, with a consequent change in both p_o and A^* across the shock. Hence, the above calculation is meaningless. Instead, set up the following trial-and-error process as follows:

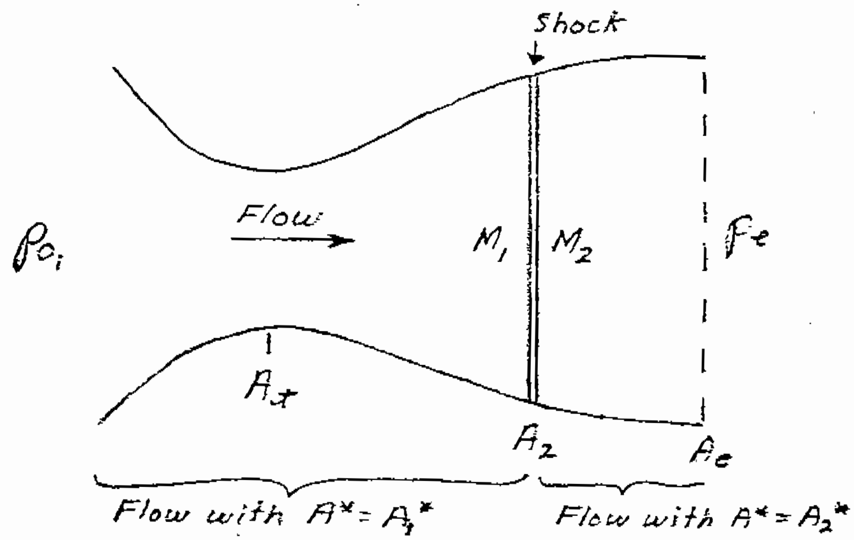
Assume a normal shock exists inside the nozzle, say at a location where $A_2/A_t = 1.024$. Let:

A_1^* = sonic throat area for the flow ahead of the shock.

A_2^* = sonic throat area for the flow behind the shock.

p_{o_1} = total pressure for the flow ahead of shock.

p_{o_2} = total pressure for the flow behind shock.



Note that $p_{o_2} < p_{o_1}$ $A_2^* > A_1^*$

which comes from the shock wave theory discussed in the text.

Key equation:

$$p_e = \frac{p_e}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} p_{o_1} \tag{1}$$

To find the values of the ratios in Eq. (1):

From Table A.1 for $A_2/A_1{}^* = 1.204$: $M_1 = 1.54$

From Table A.2 for $M_1 = 1.54$: $M_2 = 0.6874$, $\frac{p_{o_2}}{p_{o_1}} = 0.9166$

From Table A.1, for $M_2 = 0.6874$: $\frac{A_2}{A^*} = 1.1018$

$$\frac{A_e}{A_2{}^*} = \frac{A_e}{A_t} \frac{A_t}{A_2} \frac{A_2}{A_2{}^*} = (1.53)\left(\frac{1}{1.204}\right)(1.1018) = 1.4$$

From Table A.1, for $\frac{A_e}{A_2{}^*} = 1.4$: $M_e = 0.47$, $\frac{p_{o_2}}{p_e} = 1.163$

Returning to Eq. (1):

$$p_e = \frac{p_e}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} p_{o_1} = \left(\frac{1}{1.163}\right)(0.9166)(1\ \text{atm}) = 0.788\ \text{atm.}$$

This is slightly higher than the given $p_e = 0.75$. Hence, move the shock wave slightly downstream.

<u>Assume</u> $A_2/A_t = 1.301$

From Table A.1: $M_1 = 1.66$

From Table A.1, for $M_1 = 1.66$: $\frac{p_{o_2}}{p_{o_1}} = 0.872$, $M_2 = 0.6512$

From Table A.1, for $M_2 = 0.6512$: $\frac{A_2}{A_2{}^*} = 1.1356$

$$\frac{A_e}{A_2{}^*} = \frac{A_e}{A_t} \frac{A_t}{A_2} \frac{A_2}{A_2{}^*} = (1.53)\left(\frac{1}{1.301}\right)(1.1356) = 1.335$$

From Table A.1, for $\frac{A_e}{A_2^*} = 1.335$: $M_e = 0.50$, $\frac{p_{o_2}}{p_e} = 1.1862$

From Eq. (1):

$$p_e = \frac{p_e}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} p_{o_1} = \left(\frac{1}{1.1862}\right)(0.872)(1 \text{ atm}) = 0.735 \text{ atm.}$$

Interpolate: $\frac{A_2}{A_t} = 1.301 - (1.301 - 1.204)\,\frac{0.75 - 0.735}{0.788 - 0.735} = 1.274$

Thus, <u>Assume</u> $A_2/A_t = 1.274$

From Table A.1: $M_1 = 1.63$

From Table A.2: $M_2 = 0.6596$, $\frac{p_{o_2}}{p_{o_1}} = 0.8838$

From Table A.1: $\frac{A_2}{A_2^*} = 1.1265$

$$\frac{A_e}{A_2^*} = \frac{A_e}{A_t}\frac{A_t}{A_2}\frac{A_2}{A_2^*} = (1.53)\left(\frac{1}{1.274}\right)(1.1265) = 1.353$$

From Table A.1: $M_e = 0.49$, $\frac{p_{o_2}}{p_e} = 1.178$

$$p_e = \frac{p_e}{p_{o_2}} \frac{p_{o_2}}{p_{o_1}} p_{o_1} = \left(\frac{1}{1.178}\right)(0.8838)(1 \text{ atm}) = 0.75 \text{ atm}$$

Hence, p_e calculated agrees with p_e given. Thus,

$\boxed{M_e = 0.49}$

(d) $\frac{p_{o_1}}{p_e} = \frac{1 \text{ atm}}{0.154 \text{ atm}} = 6.49$

From Table A.1: $\frac{A_e}{A^*} = 1.53$, which is precisely the given area ratio of the nozzle. Hence, for this case, we have a completely isentropic expansion, where,

$\boxed{M_e = 1.88}$

10.10 From the θ-β-M diagram, for $\theta = 20°$ and $\beta = 41.8°$, we have $M_1 = 2.6$. From Table A.1,

$$\frac{A_e}{A^*} = \boxed{2.896}$$

10.11 From Table A.1, for $\frac{A_e}{A^*} = 6.79$, $M_e = 3.5$

From Table A.2, for $M_e = 3.5$: $\frac{p_{o_2}}{p_{o_1}} = 0.2129$

$$p_{o_1} = \frac{p_{o_1}}{p_{o_2}}\, p_{o_2} = \left(\frac{1}{0.2129}\right)(1.448) = \boxed{6.8 \text{ atm}}$$

10.12 From Table A.1, for $M_e = 2.8$: $\frac{p_{o_e}}{p_e} = 27.14$, $\frac{T_{o_e}}{T_e} = 2.568$

At standard sea level: $p = 2116$ lb/ft^2, $T = 519°R$

$$p_o = \frac{p_{o_e}}{p_e}\, p_e = (27.14)(2116) = \boxed{57{,}430 \text{ lb/ft}^2 = 27.14 \text{ atm}}$$

$$T_o = \frac{T_{o_e}}{T_e}\, T_e = (2.568)(519) = \boxed{1333°R}$$

$$\rho_o = \frac{p_o}{RT_o} = \frac{57430}{(1716)(1333)} = 0.251 \text{ slug/ft}^3$$

$$\rho^* = (0.6339)(0.0251) = 0.0159 \text{ slug/ft}^3$$

$$T^* = 0.833\ (1333) = 1110°R$$

$$a^* = \sqrt{\gamma RT^*} = \sqrt{(1.4)(1716)(1110)} = 1633 \text{ ft/sec} = u^*$$

$$\dot{m} = \rho^* u^* A_1^*$$

$$A_1^* = \frac{\dot{m}}{\rho^* u^*} = \frac{1}{(0.0159)(1633)} = \boxed{0.0385 \text{ ft}^3}$$

From Table A.1: $A_e/A^* = 3.5$

$$A_e = \frac{A_e}{A^*} A^* = (3.5)(0.0385) = \boxed{0.1348 \text{ ft}^2}$$

From Eq. (10.38) in text: $\dfrac{A_{t_2}}{A_{t_1}} = \dfrac{A_2^*}{A_1^*} = \dfrac{p_{o_1}}{p_{o_2}}$

From Table A.2: for $M_e = 2.8$: $\dfrac{p_{o_2}}{p_{o_1}} = 0.3895$

$$A_{t_2} = A_{t_1}\left(\frac{p_{o_1}}{p_{o_2}}\right) = A_1^*\left(\frac{p_{o_1}}{p_{o_2}}\right) = (0.0385)\left(\frac{1}{0.3895}\right) = \boxed{0.0988 \text{ ft}^2}$$

10.13 $\dot{m} = \rho^* u^* A^*$ (1)

Also, $R = \mathcal{R}/\mathcal{M} = \dfrac{8314}{16} = 519.6 \dfrac{\text{joule}}{\text{kg K}}$

$$\rho^* = \frac{\rho^*}{\rho_o}\rho_o = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \frac{p_o}{RT_o} = \left(\frac{2}{2.2}\right)^{\frac{1}{0.2}} \frac{p_o}{(519.6)(3600)} = 3.319 \times 10^{-7}\, p_o$$

$$T^* = \frac{T^*}{T_o} T_o = \left(\frac{2}{\gamma+1}\right)(3600) = 3273 \text{ K}$$

$$u^* = a^* = \sqrt{\gamma R T^*} = \sqrt{(1.2)(519.6)(3273)} = 1428.6 \text{ m/sec}$$

Hence, from Eq. (1), with $\dot{m} = 287.2 \dfrac{\text{kg}}{\text{sec}}$,

$$287.2 = (3.319 \times 10^{-7}\, p_o)(1428.6)(0.2)$$

or,

$$p_o = \frac{287.2}{(3.319 \times 10^{-7})(1428.6)(0.2)} = 3.029 \times 10^6 \ \frac{N}{m^2}$$

or,

$$p_o = \frac{3.029 \ x \ 10^6}{1.01 \ x \ 10^5} = \boxed{30 \text{ atm}}$$

10.14 We assume the flow velocity is low at the diffuser exit; hence the total pressure at the exit is 1 atm. From Appendix B, for M = 3, $\frac{p_{o_2}}{p_{o_1}} = 0.3283$.

$$\eta_D = \frac{p_B / p_o}{p_{o_2} / p_{o_1}} = 1.2$$

$$\frac{p_B}{p_o} = 1.2 \ \frac{p_{o_2}}{p_{o_1}} = 1.2 \ (0.3283) = 0.394$$

$$p_o = \frac{p_B}{0.394} = \frac{1}{0.394} = \boxed{2.54 \text{ atm}}$$

CHAPTER 11

11.1 $$u = \frac{\partial\phi}{\partial x} = V_\infty + \frac{2\pi\ (70)}{\sqrt{1-M_\infty^2}}\ e^{-2\pi\sqrt{1-M_\infty^2}y}\ \cos(2\pi x)$$

$$v = \frac{\partial\phi}{\partial y} = -\ \frac{70}{\sqrt{1-M_\infty^2}}\ \left(2\pi\sqrt{1-M_\infty^2}\right)\ e^{-2\pi\sqrt{1-M_\infty^2}y}\ \sin(2\pi x)$$

$$= -\ 140\ \pi\ e^{-2\pi\sqrt{1-M_\infty^2}y}\ \sin(2\pi x)$$

$$a_\infty = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(519)} = 1116.6\ \text{ft/sec}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{700}{1116.6} = 0.6269$$

Thus, at (x,y) = (0.2, 0.2)

$$u = 700 + \frac{2\pi(70)}{0.779}\ e^{-2\pi(0.779)(0.2)}\ \cos[2\pi(.2)] = 765.6\ \text{ft/sec}$$

$$v = -140\ \pi\ e^{-2\pi(.779)(.2)}\ \sin[2\pi(.2)] = -157.2\ \text{ft/sec}$$

$$V = \sqrt{u^2 + v^2} = \sqrt{(765.6)^2 - (157.2)^2} = 781.6\ \text{ft/sec}$$

From Table A.1, for $M_\infty = 0.6269$, $\frac{T_o}{T_\infty} = 1.079$

$$T_o = 1.079\ T_\infty = 0.079\ (519) = 560°R$$

$$a_o = \sqrt{\gamma RT_o} = \sqrt{(1.4)(1716)(560)} = 1160\ \text{ft/sec}$$

$$a^2 = a_o^2 + \frac{\gamma - 1}{2}\ (V^2) = 1.345 \times 10^6 - (.2)(781.6)^2 = 1.223 \times 10^6 \left(\frac{\text{ft}}{\text{sec}}\right)^2$$

$$a = 1106\ \text{ft/sec}$$

$$M = \frac{V}{a} = \frac{781.6}{1106} = \boxed{0.7067}$$

From Table A.1, for $M = 0.6269$: $\frac{p_o}{p_\infty} = 1.3065$, $\frac{T_o}{T_\infty} = 1.079$

$$\text{For } M = 0.7067 = \frac{p_o}{p} = 1.400,\ \frac{T_o}{T} = 1.101$$

$$p = \frac{p}{p_o}\,\frac{p_o}{p_\infty}\,p_\infty = \left(\frac{1}{1.4}\right)(1.3065)(1\text{ atm}) = \boxed{0.933\text{ atm}}$$

$$T = \frac{T}{T_o}\,\frac{T_o}{T_\infty}\,T_\infty = \left(\frac{1}{1.101}\right)(1.079)(519) = \boxed{508.6°R}$$

11.2 The results of Fig. 4.5 are for low-speed, incompressible flow. Hence, from Fig. 4.5, at $\alpha = 5°$, at $\alpha = 5°$,

$$c_{\ell_o} = 0.75$$

$$c_\ell = \frac{c_{\ell_o}}{\sqrt{1-M_\infty^2}} = \frac{0.75}{\sqrt{1-(0.6)^2}} = \boxed{0.938}$$

11.3 $C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2}} = \frac{-0.54}{\sqrt{1-(.58)^2}} = \frac{-0.54}{0.8146} = \boxed{-0.663}$

$$\text{(b) } C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2} + \left(\frac{M_\infty^2}{1+\sqrt{1-M_\infty^2}}\right)\frac{C_{p_o}}{2}} = \frac{-0.54}{0.8146 + \left[\frac{0.3364}{1+0.8146}\right]\frac{(-0.54)}{2}}$$

$$C_p = \boxed{-0.7063}$$

$$\text{(c) } C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2} + \left[M_\infty^2\left(1+\frac{\gamma-1}{2}M_\infty^2\right)/2\sqrt{1-M_\infty^2}\right]C_{p_o}}$$

$$C_p = \frac{-0.54}{0.8146 + [0.3364(1.067)/1.6292](-0.54)}$$

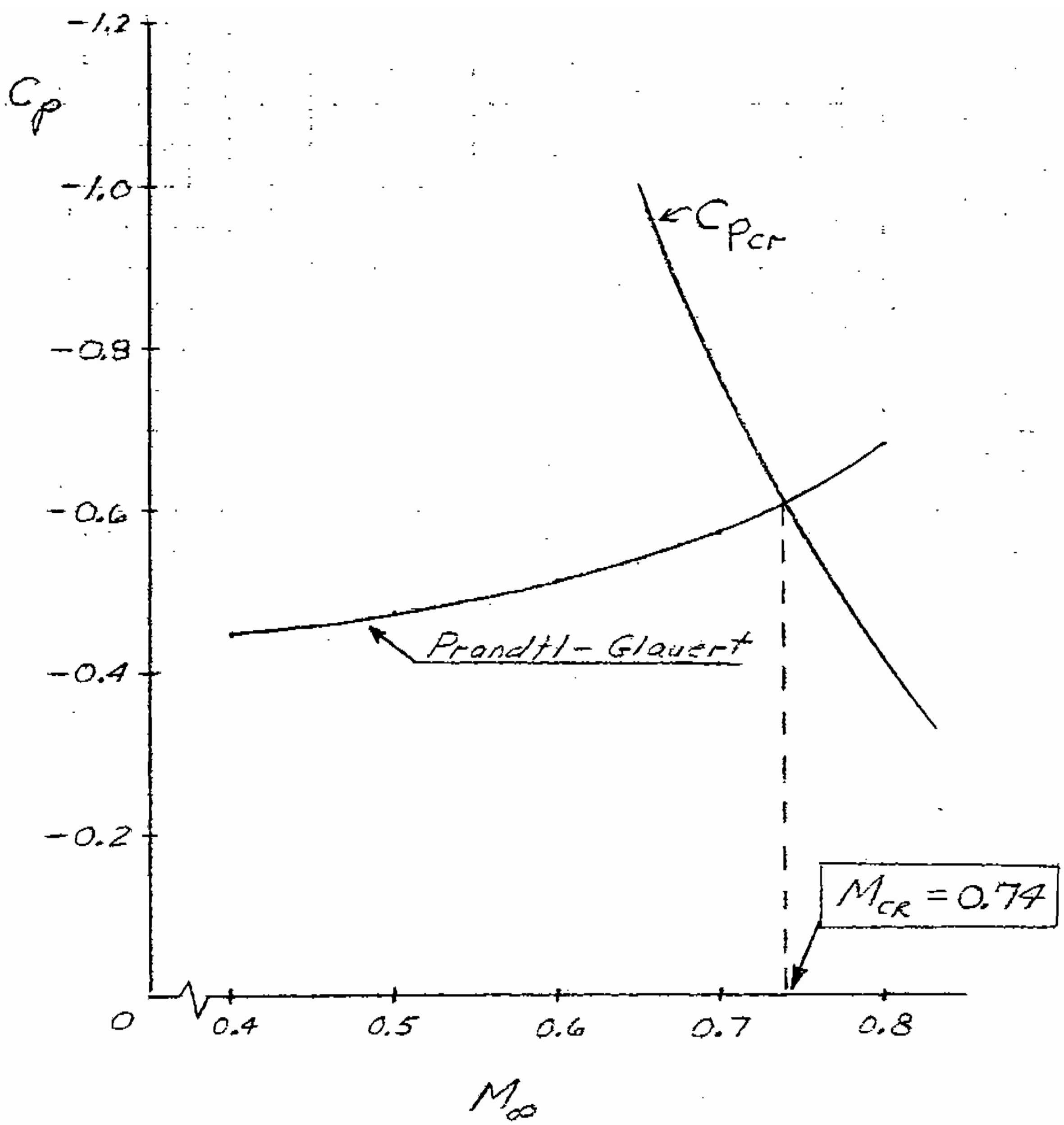
$$C_p = \boxed{-0.7763}$$

Note the differences: There is a 17% discrepancy between the three compressibility corrections. Of the three, experience has shown the Karman-Tsien rule to be more accurate.

11.4 For the pressure coefficient on the airfoil:

$$C_p = \frac{C_{p_o}}{\sqrt{1-M_\infty^2}} = \frac{-0.41}{\sqrt{1-M_\infty^2}}$$

M_∞	0.3	0.4	0.5	0.6	0.7	0.8
C_p	-0.43	-0.447	-0.473	-0.513	-0.574	-0.683



C_p
-1.2
-1.0
-0.8
-0.6
-0.4
-0.2
0
0.4
0.5
0.6
0.7
0.8
M_∞
$C_{p_{cr}}$
Prandtl-Glauert
$M_{CR} = 0.74$

11.5 When $M = M_{cr}$, then p at the minimum pressure point is clearly p_{cr}.

$$\frac{p}{p_\infty} = \frac{p_{cr}}{p_\infty} = \underbrace{\left(\frac{p_{cr}}{p_o}\right)}_{\substack{\text{Evaluated}\\ \text{at } M=1}}\underbrace{\left(\frac{p_o}{p_\infty}\right)}_{\substack{\text{Evaluated}\\ \text{at } M=0.8}} = (0.528)(1.524) = \boxed{0.805}$$

11.6 From Appendix A:

For $M_\infty = 0.5$, $\frac{p_o}{p_\infty} = 1.186$

For $M = 0.86$, $\frac{p_o}{p} = 1.621$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{\gamma}{2} p_\infty M_\infty^2} = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p}{p_\infty} - 1\right)$$

$$\frac{p}{p_\infty} = \frac{p_o / p_\infty}{p_o / p} = \frac{1.186}{1.621} = 0.7316$$

$$C_p = \frac{2}{(1.4)(0.5)^2} = (0.7316 - 1) = \boxed{-1.53}$$

Check: Using Eq. (11.58)

$$C_p = \frac{2}{\gamma\, M_\infty^2}\left[\left(\frac{1 + \frac{\gamma - 1}{2} M_\infty^2}{1 + \frac{\gamma - 1}{2} M^2}\right)^{\frac{\gamma}{\gamma - 1}} - 1\right]$$

$$= \frac{2}{(1.4)(0.5)^2}\left[\left(\frac{1 + 0.2(0.5)^2}{1 + 0.2(0.86)^2}\right)^{3.5} - 1\right] = \boxed{-1.53}$$

It checks!

11.7 First, calculate $C_{p,o}$ at point A from the information in Figure 11.5(a). The <u>actual</u> pressure coefficient is

$$C_{p,A} = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p_A}{p_\infty} - 1\right)$$

where

$$\frac{p_A}{p_\infty} = \frac{p_A}{p_o}\,\frac{p_o}{p_\infty}$$

From Appendix A (interpolating between entries for more accuracy for this problem),

$$\text{For } M_\infty = 0.3: \qquad \frac{p_o}{p_\infty} = 1.064$$

$$\text{For } M_A = 0.435: \qquad \frac{p_o}{p_A} = 1.139$$

Thus,

$$C_{p,A} = \frac{2}{(1.4)(0.3)^2}\left(\frac{1.064}{1.139} - 1\right) = -\,1.045$$

From the Prandtl-Glauert rule,

$$C_{p,o} = C_{p,A}\ \sqrt{1 - M_\infty^2} = (-1.045)\sqrt{1-(0.3)^2} = -\,0.9969$$

For the case of part (c) where $M_\infty = 0.61$, again using the Prandtl-Glauert rule,

$$C_{p,A} = \frac{C_{p,o}}{\sqrt{1 - M_\infty^2}} = \frac{-0.9969}{\sqrt{1-(0.61)^2}} = -\,1.258$$

To find the local Mach number, M_a, which corresponds to this value of $C_{p,A}$, note that

$$C_{p,A} = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p_A}{p_\infty} - 1\right)$$

or,

$$\frac{p_A}{p_\infty} = \frac{\gamma\, M_\infty^2 C_{p,A}}{2} + 1 = \frac{(1.4)(0.61)^2(-1.258)}{2} + 1 = 0.6723$$

However,

$$\frac{p_A}{p_\infty} = \frac{p_A}{p_o}\,\frac{p_o}{p_\infty} \text{ where } \frac{p_o}{p_\infty} \text{ for } M_\infty = 0.61 \text{ is } 1.286$$

Thus,

$$\frac{p_A}{p_o} = \frac{p_A / p_\infty}{p_o / p_\infty} = \frac{0.6723}{1.286} = 0.523$$

Hence,

$$\frac{p_o}{p_A} = 1.912.$$

From Appendix A, for $\frac{p_o}{p_A} = 1.912,\ M_A = \boxed{1.01}$

This is close enough. Hence, given the numbers in Figure 11.5(a), the numbers in Figure 11.15(c) are consistent with the laws of physics.

11.8 There is a three-dimensional relieving effect for the flow over a sphere. The flow over a cylinder is two-dimensional – in order to get out of the way of the cylinder, the flow can move only upwards or downwards. This means it must greatly accelerate to get out of the way of the cylinder. In contrast, the flow over a sphere is three-dimensional – it can move not only upward or downward but also sideways. This extra degree of freedom means that the flow does not have to speed up so much in flowing over the sphere. Hence, the freestream Mach number of the sphere is higher in order to achieve sonic flow on the sphere – i.e., the critical Mach number is higher.

CHAPTER 12

12.1 Consider $\alpha = 5° = 0.0873$ rad.

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.0873)}{\sqrt{(2.6)^2 - 1}} = \boxed{0.1455}$$

From exact theory (Prob. 9.13): $c_\ell = 0.148$

$$\%\ \text{error} = \frac{0.148 - 0.1455}{0.148} \times 100 = 1.69\%$$

$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}} = c_\ell\ \alpha = (0.1455)(0.0873) = \boxed{0.0127}$$

From exact theory (Prob. 9.13): $c_d = 0.0129$

$$\%\ \text{error} = \frac{0.0129 - 0.0127}{0.0129} \times 100 = 1.53\%$$

(b) $\alpha = 15° = 0.2618$ rad

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \boxed{0.436}$$

From exact theory (Prob. 9.13): $c_\ell = 0.452$

$$\%\ \text{error} = \frac{0.452 - 0.426}{0.452} \times 100 = 3.47\%$$

$$c_d = c_\ell\ \alpha = (0.436)(0.2618) = 0.114$$

From exact theory (Prob. 9.13): $c_d = 0.121$

$$\%\ \text{error} = \frac{0.121 - 0.114}{0.121} \times 100 = 5.7\%$$

(c) $\alpha = 30° = 0.5236$ rad

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.5236)}{\sqrt{(2.6)^2 - 1}} = \boxed{0.873}$$

From exact theory (Prob. 9.13): $c_\ell = 1.19$

$$\%\ \text{error} = \frac{1.19 - 0.873}{1.19} \times 100 = 26.7\%$$

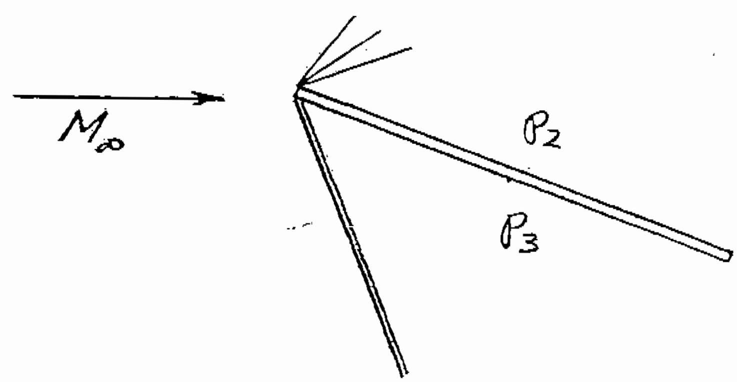
$$c_d = c_\ell\ \alpha = (0.873)(0.5236) = \boxed{0.457}$$

From exact theory (Prob. 9.13): $c_d = 0.687$

$$\%\ \text{error} = \frac{0.687 - 0.457}{0.687} = 33.5\%$$

Conclusion: At low α, linear theory is reasonably accurate. However, its accuracy deteriorates rapidly at high α. This is no surprise; we do not expect linear theory to hold for large perturbations. It appears that linear theory is reasonable to at least 5°, and that it is acceptable as high as 15°. At 30° it is unacceptable. Keep in mind that the above comments pertain to the lift and wave drag coefficients only. They say nothing about the accuracy of the pressure distributions themselves.

12.2



(a) $C_p = \dfrac{p-p_\infty}{q_\infty} = \dfrac{p-p_\infty}{\dfrac{\gamma}{2}p_\infty M_\infty^2} = \dfrac{2}{\gamma\ M_\infty^2}\left(\dfrac{p}{p_\infty}-1\right)$

$$\frac{p}{p_\infty} = \frac{\gamma\ M_\infty^2\ C_p}{2} + 1$$

$$C_p = \pm\frac{2\theta}{\sqrt{M_\infty^2-1}} = \pm\frac{2\theta}{\sqrt{(2.6)^2-1}} = \pm\frac{2\theta}{2.4}$$

or,

$$C_p = \pm\, 0.83333\theta$$

$$\frac{p}{p_\infty} = \frac{\gamma\ M_\infty^2\ C_p}{2} + 1 = \pm\frac{(1.4)(2.6)^2(0.8333)}{2} + 1$$

$$\frac{p}{p_\infty} = \pm\, 3.943\theta + 1$$

<u>Hence</u>: Examining the physical picture: recalling $\alpha = 5° = 0.873$ rad.

$$\frac{p_2}{p_\infty} = -3.943\ (.0873) + 1 = \boxed{0.6558}$$

From exact theory (Prob. 9.13): $\dfrac{p_2}{p_\infty} = 0.7022$

$$\%\ \text{error} = \frac{0.7022-0.6558}{0.7022} \times 100 = 6.6\%$$

$$\frac{p_3}{p_\infty} + 3.943\theta + 1 = 3.943\ (.0873) + 1 = \boxed{1.344}$$

From exact theory (Prob. 9.13): $\dfrac{p_3}{p_\infty} = 1.403$

$$\%\ \text{error} = \frac{1.403-1.344}{1.403} \times 100 = 4.2\%$$

(b) For $\alpha = 15° = 0.2618$ rad:

$$\frac{p_2}{p_\infty} = -3.943\theta + 1 = -3.943\ (.2618) + 1 = -0.0322 \text{ (physically impossible)}$$

The result from exact theory (Prob. 9.13) is $\frac{p_2}{p_\infty} = 0.315$

$$\frac{p_3}{p_\infty} = 3.943\theta + 1 = 3.943\ (.2618) + 1 = \boxed{2.032}$$

From exact theory (Prob. 9.13): $\frac{p_3}{p_\infty} = 2.529$

$$\%\ \text{error} = \frac{2.529 - 2.032}{2.529} \times 100 = 19.7\%$$

(c) For $\alpha = 30° = 0.5236$ rad

$$\frac{p_2}{p_\infty} = -3.943\theta + 1 = -3.943\ (0.5236) + 1 = -1.064 \text{ (physically impossible)}$$

The result from exact theory (Prob. 9.13) is $\frac{p_2}{p_\infty} = 0.0725$

$$\frac{p_3}{p_\infty} = 3.943\theta + 1 = 3.943\ (0.5236) + 1 = \boxed{3.065}$$

From exact theory (Prob. 9.13): $\frac{p_3}{p_\infty}$ 5.687

$$\%\ \text{error} = \frac{5.687 - 3.065}{5.687} \times 100 = 46\%$$

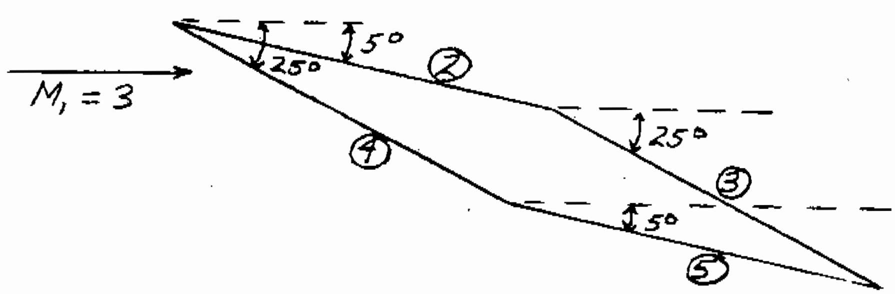
Conclusions: (1) Pressures predicted by linear theory rapidly become inaccurate as α increases. (2) Pressures predicted by linear theory are reasonable only at low values of α, say below 5°. (3) At each value of α, the % error is much greater for pressure than for lift and wave drag coefficients. (See Prob. 12.1). Hence, linear theory works better for c_ℓ and c_d than it does for p. What happens is that the inaccuracies in p on the top and bottom surfaces tend to compensate, yielding a more accurate aerodynamic force coefficient.

12.3 $\frac{p}{p_\infty} = \frac{\gamma\ M_\infty^2\ C_p}{2} + 1$ where $C_p = \pm \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$

$$C_p = \pm \frac{2\theta}{\sqrt{(3)^2 - 1}} = \pm 0.7071\theta$$

$$\frac{p}{p_\infty} = \pm \frac{(1.4)(3)^2(0.7071)\theta}{2} + 1$$

$$\frac{p}{p_\infty} = \pm 4.455\theta + 1$$



Surface 2: $\theta = 5° = 0.08727$ rad.

$$\frac{p_2}{p_\infty} = -4.455\ (.08727) + 1 = 0.6112$$

Surface 3: $\theta = 25° = 0.4663$ rad

$$\frac{p_3}{p_\infty} = -4.455\ (.4363) + 1 = -0.9439$$

Note: Although a negative pressure is not physically possible, in order to calculate the net force, we must carry it as such.

Surface 4: $\theta = 25° = 0.4363$ rad

$$\frac{p_4}{p_\infty} = 4.455\ (.4363) + 1 = 2.944$$

Surface 5: $\theta = 5° = 0.08727$ rad

$$\frac{p_5}{p_\infty} = 4.455\,(.08727) + 1 = 1.3888$$

$$c_\ell = \frac{2}{\gamma\ M_1^2}\ \frac{\ell}{c}\left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty}\right)\cos 25° + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty}\right)\cos 5°\right] \text{ (From Prob. 9.14)}$$

$$c_\ell = \frac{2}{(1.4)(3)^2}\ \frac{\ell}{c}\ [(2.944 + 0.9439)\cos 25° + (1.3888 - 0.6112)\cos 5°]$$

$$c_\ell = 0.682\ \frac{\ell}{c}.\ \text{However, } \frac{\ell}{c} = 0.5077 \text{ (From Prob. 9.14)}$$

$$c_\ell = (0.682)(.5077) = \boxed{0.346}$$

$$c_d = \frac{2}{\gamma\ M_1^2}\ \frac{\ell}{c}\left[\left(\frac{p_4}{p_\infty} - \frac{p_3}{p_\infty}\right)\sin 25° + \left(\frac{p_5}{p_\infty} - \frac{p_2}{p_\infty}\right)\sin 5°\right]$$

$$c_d = \frac{2}{(1.4)(3)^2}\ (.5077)\ [(2.944 + 0.9439)\sin 25° + (1.3888 - 0.6112)\sin 5°]$$

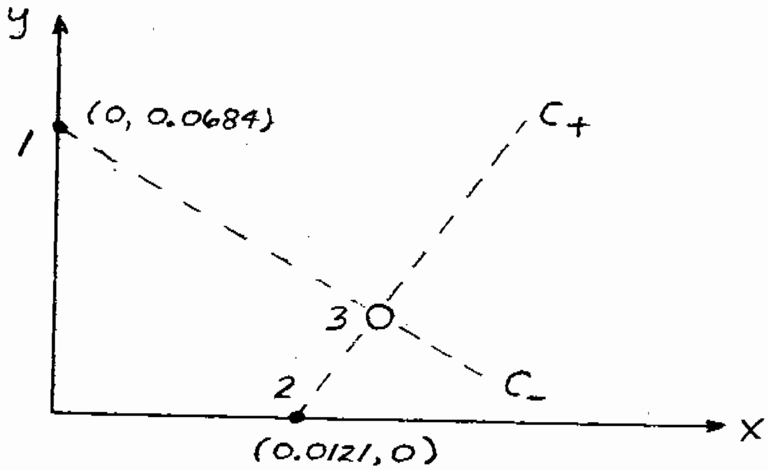
$$c_d = \boxed{0.1089}$$

Comparison

	Exact (Prob. 9.14)	Linear Theory	% Error
c_ℓ	0.418	0.346	17.2%
c_d	0.169	0.1089	35.6%

<u>**CHAPTER 13**</u>

13.1



At point 1:

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{(1.4)(287)(288)} = 340 \text{ m/sec}$$

$$V_1 = \sqrt{u_1^2 + v_1^2} = \sqrt{(639)^2 + (232.6)^2} = 680 \text{ m/sec}$$

$$M_1 = \frac{V_1}{a_1} = \frac{680}{340} = 2$$

$$\theta_1 = \text{Tan}^{-1} \frac{v_1}{u_1} = \text{Tan}^{-1}\left(\frac{232.6}{639}\right) = 20°$$

$$\nu_1 = (M_1) = 26.38°$$

$$K\_ = \theta + \nu = 20 + 26.38 = 46.38°$$

At point 2:

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{(1.4)(287)(288)} = 340 \text{ m/sec}$$

$$V_2 = 680 \text{ m/sec}$$

$$M_2 = \frac{V_1}{a_1} = \frac{680}{340} = 2$$

$$\theta_2 = 0°$$

$$\nu_2 = 26.38°$$

$$K_+ = \theta - \nu = -26.38°$$

At point 3:

$$\theta_3 = ½\,[(K_-)_1 + (K_+)_2] = ½\,(46.38 - 26.38) = 10°$$

$$\nu_3 = ½\,[(K_-)_1 + (K_+)_2] = ½\,(46.38 - 26.38) = 36.38°$$

$$M_3 = 2.4$$

To obtain the other flow variables at point 3, note that:

$$\frac{p_{o_1}}{p_1} = 7.824 \text{ and } \frac{p_{o_3}}{p_3} = 14.62$$

$$p_3 = \frac{p_3}{p_{o_3}}\,\frac{p_{o_3}}{p_{o_1}}\,\frac{p_{o_1}}{p_1}\,p_1 = \left(\frac{1}{14.62}\right)(1)(7.824)(1 \text{ atm}) = \boxed{0.535 \text{ atm}}$$

$$\frac{T_{o_1}}{T_1} = 1.8 \text{ and } \frac{T_{o_3}}{T_3} = 2.152$$

$$T_3 = \frac{T_3}{T_{o_3}}\,\frac{T_{o_3}}{T_{o_1}}\,\frac{T_{o_1}}{T_1}\,T_1 = \left(\frac{1}{2.152}\right)(1)(1.8)(288) = \boxed{240.9°K}$$

$$a_3 = \sqrt{\gamma R T_3} = \sqrt{(1.4)(287)(240.9)} = 211.1 \text{ m/sec}$$

$$V_3 = M_3 a_3 = 2.4\,(311.1) = 746.6 \text{ m/sec}$$

$$u_3 = V_3 \cos\theta_3 = 746.6 \cos 10° = \boxed{735.3 \text{ m/sec}}$$

$$v_3 = V_3 \sin\theta_3 = 746.6 \sin 10° = \boxed{129.6 \text{ m/sec}}$$

To locate point 3:

Along the C_+ Characteristic:

$$\theta_{ave} = \tfrac{1}{2}(\theta_2 + \theta_3) = \tfrac{1}{2}(0 + 10) = 5°$$

$$\mu_{ave} = \tfrac{1}{2}(\mu_2 + \mu_3) = \tfrac{1}{2}(30° + 24.62°) = 27.31°$$

$$\frac{dy}{dx} = \text{Tan}(\theta_{ave} + \mu_{ave}) = \text{Tan}(5° + 27.31°) = 0.6324$$

Thus:

$$y = 0.6324\,x - 0.00765 \qquad (1)$$

Along the C_- characteristic:

$$\theta_{ave} = \tfrac{1}{2}(\theta_1 + \theta_3) = \tfrac{1}{2}(20° + 10°) = 15°$$

$$\mu_{ave} = \tfrac{1}{2}(\mu_1 + \mu_3) = \tfrac{1}{2}(30 + 24.62) = 27.31°$$

$$\frac{dy}{dx} = \text{Tan}(\theta_{ave} - \mu_{ave}) = \text{Tan}(15° - 27.31°) = -0.2182$$

$$y = -0.2182\,x + 0.0684 \qquad (2)$$

Point 3 lies at the intersection of Eqs. (1) and (2)

$$y = 0.6324\,x - 0.00765$$

$$y = -0.2182\,x + 0.0684$$

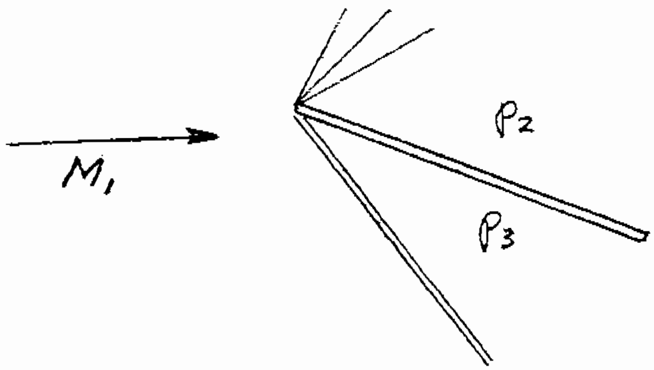
Solving simultaneously: $x = 0.0894$

$y = 0.0489$

Thus: $\boxed{(x_3, y_3) = (0.0894, 0.0489)}$

CHAPTER 14

14.1



$$c_\ell = \left(C_{p_3} - C_{p_2}\right)\cos\alpha$$

$$c_d = \left(C_{p_3} - C_{p_2}\right)\sin\alpha$$

(a) Using straight Newtonian theory:

$$C_p = 2\sin^2\alpha$$

For $\alpha = 5°$:

$$C_{p_3} = 2\sin^2 5° = 0.0152$$

$$C_{p_2} = 0$$

$$c_\ell = 0.0152\cos 5° = \boxed{0.0151}$$

$$c_d = 0.0152\sin 5° = \boxed{0.00132}$$

For $\alpha = 15°$:

$$C_{p_3} = 2\sin^2 15° = 0.1340,\ C_{p_2} = 0$$

$$c_\ell = 0.1340\cos 15° = \boxed{0.129}$$

$$c_d = 0.1340\sin 15° = \boxed{0.0347}$$

For $\alpha = 30°$:

$$C_{p_3} = 2 \sin^2 30° = 0.5$$

$$c_\ell = 0.5 \cos 30° = \boxed{0.433}$$

$$c_d = 0.5 \sin 35° = \boxed{0.25}$$

(b) Using modified Newtonian:

$$C_p = C_{p_{max}} \sin^2 \alpha$$

$$C_{p_{max}} = \frac{p_o - p_\infty}{q_\infty} = \frac{p_o - p_\infty}{\frac{\gamma}{2} M_\infty^2 p_\infty} = \frac{2}{\gamma\ M_\infty^2}\left(\frac{p_o}{p_\infty} - 1\right)$$

$$C_{p_{max}} = \frac{2}{(1.4)(2.6)^2}\ (9.181 - 1) = 1.729$$

For $\alpha = 15°$

$$C_{p_3} = 1.729 \sin^2 5° = 0.0131$$

$$c_\ell = 0.0131\ \cos 5° = \boxed{0.0131}$$

$$c_d = 0.0131 \sin 5° = \boxed{0.00114}$$

For $\alpha = 15°$

$$C_{p_3} = 1.729 \sin^2 15° = 0.1158$$

$$c_\ell = 0.1158\ \cos 15° = \boxed{0.1119}$$

$$c_d = 0.1158 \sin 15° = \boxed{0.030}$$

For $\alpha = 30°$

$$C_{p_3} = 1.729 \sin^2 30° = 0.4323$$

$$c_\ell = 0.4323\ \cos 30° = \boxed{0.374}$$

$$c_d = 0.4323 \sin 30° = \boxed{0.216}$$

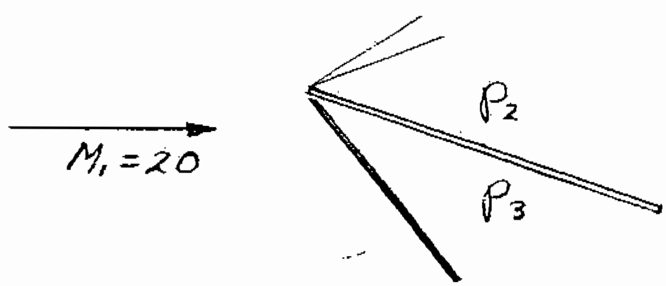
Comparison:

α	Exact c_ℓ (Prob. 9.13)	Newtonian c_ℓ	% error	Mod. Newtonian c_ℓ	% error
5°	0.148	0.0151	90	0.0131	91
15°	0.452	0.129	71	0.1119	75.2
30°	1.19	0.433	63.6	0.374	68.6

α	Exact c_d (Prob. 9.13)	Newtonian c_ℓ	% error	Mod. Newtonian c_d	% error
5°	0.0129	0.00132	90	0.00114	91
15°	0.121	0.0347	71	0.03	75.2
30°	0.687	0.25	63.6	0.216	68.6

Conclusion: Newtonian theory gives terrible results for a flat plate a moderate α at low Supersonic Mach numbers.

14.2



From Newtonian theory:

$$C_p = 2\sin^2\alpha = 2\sin^2 20° = 0.234$$

$c_\ell = 0.234 \cos\alpha = \boxed{0.220}$

$c_d = 0.234 \sin\alpha = \boxed{0.08}$

From shock-expansion theory:

On the top surface: $\nu_2 = \nu_1 + \theta = 116.2 + 20 = 136.20$

This is beyond the maximum expansion angle. Hence, a "void" exists on the top surface, i.e., $p_2 = 0$.

On the bottom surface: From the θ-β-M diagram,

$\beta = 24.9°$

$M_{n_1} = M_1 \sin\beta = 20 \sin 24.9° = 8.4$

$$\frac{p_3}{p_1} = 82.15$$

From Prob. 9.13:

$$c_\ell = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p_3}{p_1} - \frac{p_2}{p_1}\right)\cos\alpha$$

and

$$c_d = c_\ell\,\frac{\sin\alpha}{\cos\alpha}$$

$$c_\ell = \frac{2}{(1.4)(20)^2}\,(82.15 - 0)\cos 20° = 0.2757$$

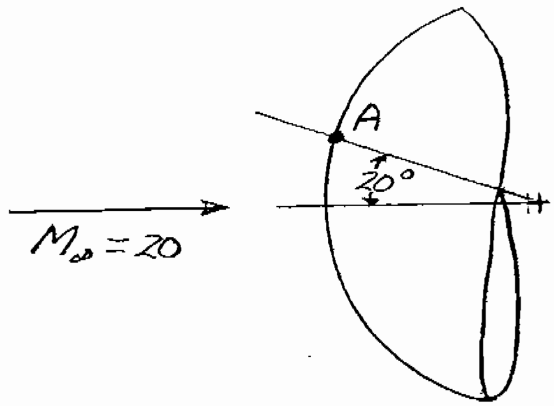
$c_d = 0.2757 \text{ Tan } 20° = 0.100$

For c_ℓ: $\%\ \text{error} = \dfrac{0.2757 - 0.220}{0.2757} = 20\%$

For c_d: $\%\ \text{error} = \dfrac{0.100 - 0.08}{0.10} = 20\%$

Note: Newtonian theory works much better for blunt bodies, i.e., for large values of θ.

14.3



(a) Use Eq. (14.7) to estimate the pressure at point A. We first need to obtain $C_{p,max}$, which is a function of $p_{o,2}/p_\infty$. From Appendix B for $M_\infty = 20$, $p_{o,2}/p_\infty = 0.5155 \times 10^3$. Hence,

$$C_{p,max} = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p_{o,2}}{p_\infty} - 1\right) = \frac{2}{(1.4)(20)^2}\,(515.3 - 1) = 1.837$$

From Eq. (14.7), at point A on the surface

$$C_{p_A} = C_{p,max}\sin^2\theta = (1.837)\sin^2 20° = 0.2149$$

Since

$$C_{p_A} = \frac{2}{\gamma\, M_\infty^2}\left(\frac{p_A}{p_\infty} - 1\right)$$

then,

$$\frac{p_A}{p_\infty} = \frac{\gamma\, M_\infty^2 C_{p_A}}{2} + 1 = \frac{(1.4)(20)^2\,(0.2149)}{2} + 1 = 61.17$$

Hence,

$$P_A = 61.17\,(3.06) = \boxed{187.2 \text{ lb/ft}^2}$$

(b) The stagnation temperature is found from Eq. (8.40)

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2}\,M_\infty^{\,2} = 1 + 0.2\,(20)^2 = 81$$

Assuming an isentropic flow from the stagnation point to point A,

$$\frac{p_A}{p_{o,2}} = \frac{p_A / p_\infty}{p_{o,2} / p_\infty} = \left(\frac{T_A}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$$

or,

$$\frac{T_A}{T_o} = \left(\frac{61.17}{515.5}\right)^{\frac{\gamma-1}{\gamma}} = (0.1187)^{0.2857} = 0.5439$$

$$T_A = \frac{T_A}{T_o}\left(\frac{T_o}{T_\infty}\right) T_\infty = (0.5439)(81)(500) = \boxed{22{,}028°R}$$

(Please note. Relative to our discussion in Problems 8.17 and 8.18, we know this estimate of T_A to be too large because we are not taking into account the effect of chemically reacting flow.)

(c) At point A, for an isentropic flow, $p_{o,A} = p_{o,2}$

$$\frac{p_{o,A}}{p_A} = \left(1 + \frac{\gamma-1}{2} M_A^2\right)^{\frac{\gamma}{\gamma-1}} = \frac{p_{o,2} / p_\infty}{p_A / p_\infty} = \frac{515.5}{61.17} = 8.427$$

$$1 + \frac{\gamma-1}{2} M_A^2 = (8.427)^{\frac{\gamma-1}{\gamma}} = (8.427)^{0.2857} = 1.8385$$

$$M_A^2 = (1.8385 - 1)\ \frac{2}{\gamma - 1} = (0.8385)(5) = 4.1925$$

$$\boxed{M_A = 2.05}$$

(d) $a_A = \sqrt{\gamma R T_A} = \sqrt{(1.4)(1716)(22{,}028)} = 7275$ ft/sec

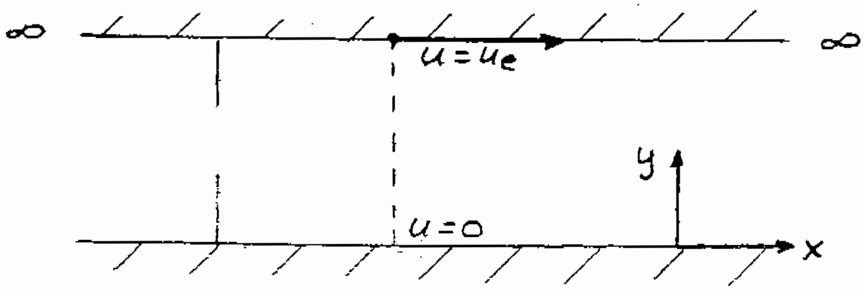
$$V_A = a_A M_A = (7275)(2.05) = \boxed{1.49 \times 10^4 \text{ ft/sec}}$$

<u>Note</u>: Once again, this estimate of V_A is too high because T_A, hence a_A, is too high.

<u>Also note</u>: The purpose of this problem is to illustrate that, from the Newtonian sine-squared law for pressure variations, the other flow field quantities can also be obtained.

CHAPTER 15

15.1



(a) Since the plates are infinite in length, u = u(y) only. Also, v = 0, i.e., the flow is in the x-direction only. The governing equation is Eq. (15.18a), which reduces to the following u = u(y), v = 0 and p = const.

$$0 = \frac{d}{dy}\left(\mu \frac{du}{dy}\right)$$

Integrating:

$$\mu \frac{du}{dy} = \text{const} = c_1$$

$$\mu u = c_1 y + c_2$$

At y = 0, u = 0: $c_2 = 0$

At y = h, u = u_e: $\mu u_e = c_1 h$

$$c_1 = \frac{\mu u_e}{h}$$

Thus:

$$\mu u = \frac{\mu u_e}{h} y, \text{ or } \boxed{u = u_e \left(\frac{y}{h}\right)}$$

The velocity variation is <u>linear</u> between the plates.

$$\text{(b) } \frac{du}{dy} = \frac{u_e}{h}$$

$$\tau = \mu \frac{du}{dy} = \mu \frac{u_e}{h}$$

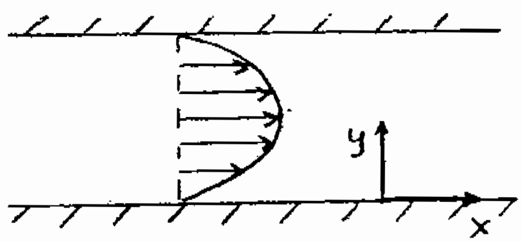
$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} \frac{T_o + 110}{T + 110} = \left(\frac{320}{288.16}\right)^{3/2} \frac{288.16 + 110}{320 + 110} = 1.084$$

$$\mu = 1.084\ \mu_o = 1.084\ (1.7894 \times 10^{-5}) = 1.94 \times 10^{-5}\ \frac{\text{kg}}{\text{m sec}}$$

$$\tau = (1.94 \times 10^{-5}) \left(\frac{30}{0.01}\right) = \boxed{5.82 \times 10^{-2}\ \text{N/m}^2}$$

The shear stress is constant, and hence is the same on the top and bottom walls.

15.2



$$u = u(y),\ v = 0,\ p = p(x)$$

$$0 = \frac{dp}{dx} + \frac{d}{dy}\left(\mu \frac{du}{dy}\right)$$

$$\mu \frac{du}{dy} = -\left(\frac{dp}{dx}\right) y + c_1$$

$$\mu u = -\left(\frac{dp}{dx}\right) \frac{y^2}{2} + c_1\, y + c_2$$

$$u = -\left(\frac{dp}{dx}\right) \frac{y^2}{2\mu} + \frac{c_1}{\mu}\, y + \frac{c_2}{\mu}$$

At $y = 0$, $u = 0$. Thus $c_2 = 0$

At y = h, u = 0. Thus,

$$0 = -\left(\frac{dp}{dx}\right)\frac{h^2}{2\mu} + \frac{c_1}{\mu}h \qquad\qquad c_1 = \left(\frac{dp}{dx}\right)\frac{h}{2}$$

$$u = -\left(\frac{dp}{dx}\right)\frac{y^2}{2\mu} + \left(\frac{dp}{dx}\right)\frac{h}{2\mu}y$$

$$\boxed{u = \frac{1}{2\mu}\left(\frac{dp}{dx}\right)(h\,y - y^2)}$$

The velocity profile is parabolic.

$$\frac{du}{dy} = -\left(\frac{dp}{dx}\right)\frac{y}{\mu} + \left(\frac{dp}{dx}\right)\frac{h}{2\mu}$$

On the bottom plate, y = 0: $\tau = \mu\,\frac{du}{dy}$

$$\tau = [-\left(\frac{dp}{dx}\right)\frac{0}{\mu} + \left(\frac{dp}{dx}\right)\frac{h}{2\mu}]\,\mu = \frac{h}{2}\left(\frac{dp}{dx}\right)$$

On the top plate, y = h: $\tau = \mu\,(-\frac{du}{dy})$ since dy is negative, i.e., the distance away from the top plate is in the downward (negative direction)

$$\tau = \mu\,[+\left(\frac{dp}{dx}\right)\frac{h}{\mu} - \left(\frac{dp}{dx}\right)\frac{h}{2\mu}]$$

$$\tau = \frac{h}{2}\left(\frac{dp}{dx}\right)$$

For both the top and bottom walls,

$$\tau = \frac{h}{2}\left(\frac{dp}{dx}\right)$$

Shear stress varies linearly with the magnitude of the pressure gradient.

Note: Due to the content of chapters 16, 17, and 18, no homework problems are required.

CHAPTER 19

19.1 1 mi/hr = 0.4471 m/sec

$$V_\infty = \left(141\frac{\text{mi}}{\text{hr}}\right)\left(\frac{0.4471\ \text{m/sec}}{1\ \text{mi/hr}}\right) = 63.04\ \text{m/sec}$$

$$\text{Re}_c = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(1.23)(63.04)(1.6)}{1.7894\ \text{x}\ 10^{-5}} = 6.93\ \text{x}\ 10^6$$

$$\text{(a)}\ C_f = \frac{1.328}{\sqrt{\text{Re}_c}} = \frac{1.328}{\sqrt{6.93\ \text{x}\ 10^6}} = 5.04\ \text{x}\ 10^{-4}$$

Noting that drag exists on both the bottom and top surfaces, we have

$$D_f = 2\, q_\infty\, S\, C_f = 2(½)(1.23)(63.04)^2(9.75)(1.6)(5.04\ \text{x}\ 10^{-4}) = \boxed{38.4\ \text{N}}$$

$$\text{(b)}\ C_f = \frac{0.074}{\text{Re}_c^{\ 1/5}} = \frac{0.074}{(6.93\text{x}10^6)^{1/5}} = 3.17\ \text{x}\ 10^{-3}$$

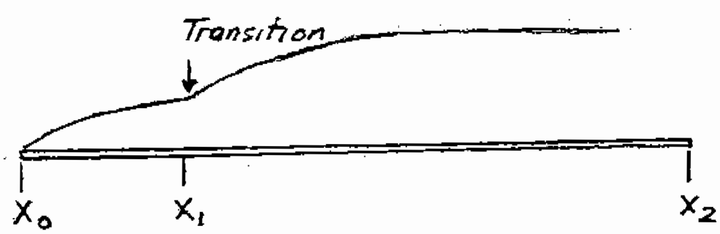
$$D_f = \frac{(C_f)_{\text{turb}}}{(C_f)_{\text{lam}}}(38.4) = \frac{3.17\ \text{x}\ 10^{-3}}{5.04\ \text{x}\ 10^{-4}}\ (38.4) = \boxed{241.5\text{N}}$$

Note that turbulent skin friction is 6.28 times larger than the laminar value.

19.2 (a) $= \dfrac{5.0x}{\sqrt{\text{Re}_x}} = \dfrac{(5.0)(1.6)}{\sqrt{6.93\ \text{x}\ 10^6}} = 3.04\ \text{x}\ 10^{-3}\ \text{m} = \boxed{0.304\ \text{cm}}$

(b) $= \dfrac{0.37x}{\text{Re}_x^{\ 1/5}} = \dfrac{(0.37)(1.6)}{(6.93\text{x}10^6)^{1/5}} = 2.54\ \text{x}\ 10^{-2}\ \text{m} = \boxed{2.54\ \text{cm}}$

19.3



$q_\infty = \frac{1}{2}\,(1.23)(63.04)^2 = 2444\ \text{N/m}^2$

$$Re_c = 5 \times 10^5 \; \frac{\rho_\infty V_\infty (x_1 - x_o)}{\mu_\infty}$$

$$(x_1 - x_o) = \frac{5 \times 10^5 \mu_\infty}{\rho_\infty V_\infty} = \frac{(5\text{x}10^5)(1.7894 \times 10^{-5})}{(1.23)(63.04)} = 0.1154\ \text{m}$$

Laminar drag on $(x_1 - x_o)$:

$$C_f = \frac{1.328}{\sqrt{5 \times 10^5}} = 1.878 \times 10^{-3}$$

$$D_f = q_\infty\, S\, C_f = (2444)(0.1154)(9.75)(1.878 \times 10^{-3}) = 5.16\ \text{N}$$

Turbulent drag on $(x_1 - x_o)$:

$$C_f = \frac{0.074}{(5 \times 10^5)^{1/5}} = 5.36 \times 10^{-3}$$

$$D_f = \left(\frac{5.36 \times 10^{-3}}{1.878 \times 10^{-3}}\right) 5.16 = 14.73\ \text{N}$$

From Prob. 19.1, the turbulent drag on $(x_2 - x_o)$ was 241.5 N. Hence,

Turbulent drag on $(x_2 - x_1) = 241.5 - 14.73 = 226.8$ N

Total skin friction drag = [Laminar drag on $(x_1 - x_o)$] + [Turbulent drag on $(x_2 - x_1)$]

$$= 5.16 + 226.8 = \boxed{232\ \text{N}}$$

19.4 At standard sea level: $\rho_\infty = 0.002377$ slug/ft

$$T_\infty = 519°R$$

$$a_\infty = \sqrt{\gamma RT} = \sqrt{(1.4)(1716)(519)} = 1117\ \text{ft/sec}$$

$$V_\infty = M_\infty\, a_\infty = 4\,(1117) = 4468\ \text{ft/sec}$$

$$\mathrm{Re}_c = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(.002377)(4468)(5/12)}{3.7373 \times 10^{-7}} = 1.18 \times 10^7$$

$$\text{Incompressible } C_f \equiv C_{f_o} = \frac{1.328}{\sqrt{\mathrm{Re}_c}} = \frac{1.328}{\sqrt{1.18 \times 10^7}} = 3.866 \times 10^{-4}$$

From Fig. 18.8:

$C_f/C_{f_o} \approx 0.85$; $C_f = 3.286 \times 10^{-4}$

$D_f = q_\infty S C_f = (½)(.002377)(4468)^2(5/12)(3.286 \times 10^{-4})$

$D_f = \boxed{3.248 \text{ lb}}$ on one side of the plate.

19.5 For incompressible flow:

$$C_{f_o} = \frac{0.074}{\mathrm{Re}_c^{1/5}} = \frac{0.074}{(1.18 \times 10^7)^{1/5}} = 2.85 \times 10^{-3}$$

From Fig. 19.1: $C_f \approx 1.6 \times 10^{-3}$

(The effect of Mach number is to reduce C_f by about 44% in this case.)

From Prob. 19.4, the laminar value of D_f is 3.248 for a value of $C_f = 3.286 \times 10^{-4}$. Hence, the turbulent value is

$$D_f = \left(\frac{2.85 \times 10^{-3}}{3.286 \times 10^{-4}}\right)(3.248) = \boxed{28.2 \text{ lb}}$$

19.6 From Eq. (18.32):

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \qquad (1)$$

From Eq. (18.41) with Pr = 1:

$$\rho u \frac{\partial h_o}{\partial x} + \rho v \frac{\partial h_o}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial h_o}{\partial y}\right) \qquad (2)$$

Eqs. (1) and (2) are identical. Hence

$$h_o = c_1 + c_2\, u,$$ where c_1 and c_2 are constants.

At the wall, $u = 0$ and $h_o = h_{o_w} = h_w$. Hence,

$$h_w = c_1 + 0, \text{ or } c_1 = h_w$$

At the boundary layer edge:

$$h_{o_e} = c_1 + c_2\, u_e = h_w + c_2\, u_e$$

$$c_2 = \frac{h_{o_e} - h_w}{u_e}$$

Thus:

$$h_o = c_1 + c_2\, u = h_w + \frac{h_{o_e} - h_w}{u_e}\, u$$

Since

$h = c_p\, T$, then

$$\boxed{T_o = T_w + \left(T_{o_e} - T_w\right)\frac{u}{u_e}}$$

19.7 From Eq. (18.70),

$$\dot{q}_w = 0.763\ \mathrm{Pr}^{-0.65}\,(\rho_e \mu_e)\,\sqrt{\frac{du_e}{dx}}\,(h_{aw} - h_w) \qquad (1)$$

where, from Eq. (18.82), the velocity gradient is given by

$$\frac{du_e}{dx} = \frac{1}{R}\sqrt{\frac{2(p_e - p_\infty)}{\rho_e}} \qquad (2)$$

The subscript e denotes properties at the outer edge of the stagnation point boundary layer, i.e., p_e and ρ_e are the inviscid stagnation point values of pressure and density. The speed of sound in the ambient atmosphere is

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(287)(246.1)} = 314.5 \text{ m/sec}$$

(a) For $V_\infty = 1500$ m/sec, we have

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{1500}{314.5} = 4.77$$

From Appendix B (nearest entry),

$$\frac{p_{o,2}}{p_\infty} = 29.52$$

and from Appendix A (nearest entry),

$$\frac{T_o}{T_\infty} = 5.512$$

Hence,

$$P_{o,2} \equiv p_e = (29.52)(583.59) = 1.723 \text{ x } 10^4 \text{ N/m}^2$$

$$T_o \equiv T_e = (5.512)(246.1) = 1357 \text{ K}$$

$$\rho_e \equiv \frac{p_e}{RT_e} = \frac{1.723 \text{ x } 10^4}{(287)(1357)} = 0.044 \text{ kg/m}^3$$

From Southerland's law, Eq. (15.3), using the standard sea level value of $\mu_o = 1.7894 \text{ x } 10^{-5}$ kg/(m)(sec) at $T_o = 288$K, we have

$$\frac{\mu_e}{\mu_o} = \left(\frac{T_e}{T_o}\right)^{3/2} \frac{T_o + 110}{T_e + 110} = \left(\frac{1357}{288}\right)^{3/2} \left(\frac{288+110}{1357+110}\right) = 2.77$$

$$\mu_e = (2.77)(1.789 \text{ x } 10^{-5}) = 4.957 \text{ x } 10^{-5} \text{ kg/(m)(sec)}$$

From Eq. (2) above

$$\frac{du_e}{dx} = \frac{1}{R}\sqrt{\frac{2(p_e - p_\infty)}{\rho_e}} = \frac{1}{(0.0254)}\sqrt{\frac{2(1.723 \text{ x } 10^4 - 583.59)}{0.044}} = 3.42 \text{ x } 10^4\text{/sec}$$

Assuming a recovery factor r = 1, then $h_{aw} = h_o$.

$$h_{aw} = h_o = h_\infty + \frac{V_\infty^{\ 2}}{2} = c_p T_\infty + \frac{V_\infty^{\ 2}}{2} = (1008)(246.1) + \frac{(1500)^2}{2}$$

$$= 2.48 \text{ x } 10^5 + 11.25 \text{ x } 10^5 = 13.73 \text{ x } 10^5 \text{ joule/kg}$$

$$h_{aw} = c_p T_w = (1008)(400) = 4.032 \text{ x } 10^5 \text{ joule/kg}$$

The "rho-mu" product is

$$\rho_e\mu_e = (1.044)(4.957 \times 10^{-5}) = 2.18 \times 10^{-6} \frac{(kg)^2}{m^4 \sec}$$

From Eq. (1) above

$$\dot{q}_w = 0.763 \, Pr^{-0.65} (\rho_e\mu_e) \sqrt{\frac{du_e}{dx}} \, (h_{aw} - h_w)$$

$$= 0.763 \, (0.72)^{-0.65} (2.18 \times 10^{-6})(3.42 \times 10^4)^{1/2} (13.73 - 4.032) \times 10^5$$

$$= 369.3 \frac{\text{joules}}{\sec(m^2)} = \boxed{369.3 \frac{\text{watt}}{m^2}}$$

(b) For $V_\infty = 4500$ m/sec, we have

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{4500}{314.5} = 14.31$$

From Appendix B (interpolated)

$$\frac{p_{o,2}}{p_\infty} = 264.0$$

From Appendix A (interpolated)

$$\frac{T_o}{T_\infty} = 41.94$$

Thus:

$$p_e = (264)(583.59) = 1.54 \times 10^5 \text{ N/m}^2$$

$$T_e = 41.94 \, (246.1) = 10{,}321 \text{ K}$$

$$\rho_e = \frac{p_e}{RT_e} = \frac{1.54 \times 10^5}{(187)(10{,}321)} = 0.052 \text{ kg/m}^3$$

$$\frac{\mu_e}{\mu_o} = \left(\frac{T_e}{T_o}\right)^{3/2} \frac{T_o + 110}{T_e + 110} = \left(\frac{10321}{288}\right)^{3/2} \left(\frac{288 + 110}{10321 + 110}\right) = 8.186$$

$\mu_e = (8.186)(1.7894 \times 10^{-5}) = 1.465 \times 10^{-4}$ kg/(m)(sec)

From Eq. (2)

$$\frac{du_e}{dx} = \frac{1}{R}\sqrt{\frac{2(p_e - p_\infty)}{\rho_e}} = \frac{1}{(0.0254)}\sqrt{\frac{2(1.54 \;\; x \;\; 10^5 - 58359)}{0.052}} = 9.56 \times 10^4/\text{sec}$$

$$h_{aw} = h_\infty + \frac{V_\infty^2}{2} = 2.48 \times 10^5 + \frac{(4500)^2}{2} = 1.037 \times 10^7 \text{ joules/kg}$$

$$\rho_e\mu_e = (0.052)(1.465 \times 10^{-4}) = 7.62 \times 10^{-6} \frac{(\text{kg})^2}{\text{m}^4 \text{ sec}}$$

$$\dot{q}_w = 0.763 \, Pr^{-0.65} (\rho_e\mu_e) \sqrt{\frac{du_e}{dx}} \, (h_{aw} - h_w)$$

$$= 0.763 \, (0.72)^{-0.65} (7.62 \times 10^{-6})(9.56 \times 10^4)^{1/2} (1.037 \times 10^7 - 4.032 \times 10^5)$$

$$= \boxed{2.218 \times 10^4 \; \frac{\text{watts}}{\text{m}^2}}$$

Comparing the results from parts (a) and (b), we note

$$\frac{\left(\dot{q}_w\right)_{v=4500}}{\left(\dot{q}_w\right)_{v=1500}} = \frac{2.218 \;\; x \;\; 10^4}{369.3} = 60$$

When the velocity increased from 1500 m/sec to 4500 m/sec, a factor of 3, the heat transfer increased by a factor of 60. This illustrates the rapid growth of the importance of aerodynamic heating as vehicles fly faster, well into the hypersonic flight regime. A simple, approximate analysis for aerodynamic heating which assumes very high Mach numbers (so that $h_{aw} >> h_w$) indicates that aerodynamic heating is proportional to V_∞^3. (See for example, Anderson, Introduction of Flight, 4th ed., McGraw-Hill, 2000, page 570.) For the present example, in going from a relatively low, not quite hypersonic condition ($M_\infty = 4.77$) to a relatively high Mach number of $M_\infty = 14.31$, the increase was even faster.