Analysis of Variance & Multivariate Analysis of Variance

Presentation Highlights

Overview of Analysis of Variance (ANOVA)

Introduction and Analysis of Multivariate Analysis of Variance (MANOVA)

Comparison of ANOVA versus MANOVA

ANOVA (Analysis of Variance)

Definition:

Analysis involving the investigation of the effects of one treatment variable on an interval-scaled dependent variable.

Purpose:

To test differences in means (for groups or variables) for statistical significance

Hypothesis:

$$\text{Ho}: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \ldots = \mu_\text{k}$$

$$\text{Ha}: \text{ At least one } \mu_\text{k} \text{ is different}$$

Use when you have one or more independent variables and only ONE dependent variable.

ANOVA Assumptions

- Random sampling – subjects are randomly sampled for the purpose of significance testing.
- Interval data – assumes an interval-level dependent.
- Homogeneity of variances – dependent variables should have the same variance in each category of the independent variable.

ANOVA

One-Way ANOVA Example:

A call center manager wants to know if there is a significant difference in average handle times amongst three different call operators.

Independent Variable: Call Operator

Dependent Variable: Average Handle Time

Hypothesis:

$$\text{Ho}: \mu_1 = \mu_2 = \mu_3$$
$$\text{Ha}: \text{ At least one } \mu \text{ is different}$$

ANOVA

Call Center Example Data: Average Handle Times (seconds)

	Operator 1	Operator 2	Operator 3
	76.5	74.5	72.5
	76.0	75.0	75.2
	74.5	74.5	74.8
	73.7	74.2	76.0
	75.6	74.2	73.9
	75.4	74.5	73.8
	73.8	73.4	75.3
	76.1	74.2	75.8
	74.1	75.2	74.9
	75.1	75.6	75.1
Mean	$\overline{X}_1 =$ **75.1**	$\overline{X}_2 =$ **74.5**	$\overline{X}_3 =$ **74.7**
Grand Mean	$\overline{\overline{X}} =$ **74.8**		

ANOVA

F-Test: Used to determine whether there is more variability in the scores of one sample than in the scores of another sample.

$$F = \frac{\text{Variance between groups}}{\text{Variance within groups}} = \frac{MS_{between}}{MS_{within}}$$

Within group – variances of the observations in each group weighted for group size

Between group – variance of the set of group means from the overall mean of all observations

Operator 1	Operator 2	Operator 3
76.5	74.5	72.5
76.0	75.0	75.2
74.5	74.5	74.8
73.7	74.2	76.0
75.6	74.2	73.9
75.4	74.5	73.8
73.8	73.4	75.3
76.1	74.2	75.8
74.1	75.2	74.9
75.1	75.6	75.1

ANOVA

$$SS_{total} = SS_{within} + SS_{between}$$

SS_{total} = square the deviation of each handle time from the grand mean and sum up the squares

SS_{within} = square the deviation of each handle time from its group mean and sum up the squares

$SS_{between}$ = square the deviation of each group mean from the grand mean multiplying by the number of items in each group and sum up the totals

$SS_{within} = 22.5$ $SS_{between} = 1.9$

ANOVA

The next step involves dividing the various sums of squares by their appropriate degrees of freedom.

$$MS_{within} = \frac{SS_{within}}{cn - c} \qquad \frac{22.5}{27.0} = 0.8$$

$$MS_{between} = \frac{SS_{between}}{c - 1} \qquad \frac{1.9}{2.0} = 1.0$$

$$F = \frac{MS_{between}}{MS_{within}} \qquad \frac{1.0}{0.8} = 1.1$$

In the F Distribution Table (A.5 p. 711) , the critical value of F at the .05 level for 2 and 27 degrees of freedom indicates that an F of 3.35 would be required to reject the null hypothesis.

ANOVA

In our example…

Call Center ANOVA Table				
Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Between Groups	1.9	2	1.0	
Within Groups	22.5	27	0.8	1.1
Total	24.4	29		

We cannot reject the null hypothesis and therefore conclude that there is not a statistically significant difference between the average handle times of operators 1, 2, and 3.

Multiple Analysis of Variance (MANOVA)

Definition:

Analysis involving the investigation of the main and interaction effects of categorical (independent) variables on ***multiple*** dependent interval variables.

Purpose:

To determine if individual categorical independent variables have an effect on a group, or ***related*** set of interval dependent variables.

For example:

We may conduct a study where we try two different textbooks (independent variables), and we are interested in the students' improvements in math and physics. In that case, we have two dependent variables, and our hypothesis is that both together are affected by the difference in textbooks.

Multiple Analysis of Variance (MANOVA)

Assumptions:

- The independent variables are categorical
- There are multiple dependent variables that are continuous and interval
- There is a relationship between the dependent variables
- The number of observations for each combination of the factor are the same (balanced experiment)

Multiple Analysis of Variance (MANOVA)

Example:

A call center manager wants to know if the operator or method of answering calls makes a difference on average handle time, wait time and customer satisfaction.

Independent Variables: Call Operator and Method of Answering

***Group* of Dependent Variables**: Average Handle Time, Wait Time and Customer Satisfaction

Multiple Analysis of Variance (MANOVA)

- ***Ho***: the means of AHT, WT and CS are the same for Operator 1 & 2
- ***Ha***: the means of AHT, WT and CS are not the same for Operator 1 & 2

- ***Ho***: the means of AHT, WT and CS are the same for Method of Answering 1 & 2
- ***Ha***: the means of AHT, WT and CS are not the same for Method of Answering 1 & 2

Multiple Analysis of Variance (MANOVA)

Handle Time	Wait Time	Customer Sat.	Operator	Method of Answering
76.5	39.5	4.4	1	1
76.2	39.9	6.4	1	1
75.8	39.6	3.0	1	1
76.5	39.6	4.1	1	1
76.5	39.2	0.8	1	1
76.9	39.1	5.7	1	2
77.2	40.0	2.0	1	2
76.9	39.9	3.9	1	2
76.1	39.5	1.9	1	2
76.3	39.4	5.7	1	2
76.7	39.1	2.8	2	1
76.6	39.3	4.1	2	1
77.2	38.3	3.8	2	1
77.1	38.4	1.6	2	1
76.8	38.5	3.4	2	1
77.1	39.2	8.4	2	2
77.0	38.8	5.2	2	2
77.2	39.7	6.9	2	2
77.5	40.1	2.7	2	2
77.6	39.2	1.9	2	2

Multiple Analysis of Variance (MANOVA)

Multivariate Tests[c]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	Pillai's Trace	1.000	397502.5[b]	3.000	14.000	.000	1.000	1192507.5	1.000
	Wilks' Lambda	.000	397502.5[b]	3.000	14.000	.000	1.000	1192507.5	1.000
	Hotelling's Trace	85179.104	397502.5[b]	3.000	14.000	.000	1.000	1192507.5	1.000
	Roy's Largest Root	85179.104	397502.5[b]	3.000	14.000	.000	1.000	1192507.5	1.000
VAR00004	Pillai's Trace	.618	7.554[b]	3.000	14.000	.003	.618	22.663	.948
	Wilks' Lambda	.382	7.554[b]	3.000	14.000	.003	.618	22.663	.948
	Hotelling's Trace	1.619	7.554[b]	3.000	14.000	.003	.618	22.663	.948
	Roy's Largest Root	1.619	7.554[b]	3.000	14.000	.003	.618	22.663	.948
VAR00005	Pillai's Trace	.477	4.256[b]	3.000	14.000	.025	.477	12.767	.744
	Wilks' Lambda	.523	4.256[b]	3.000	14.000	.025	.477	12.767	.744
	Hotelling's Trace	.912	4.256[b]	3.000	14.000	.025	.477	12.767	.744
	Roy's Largest Root	.912	4.256[b]	3.000	14.000	.025	.477	12.767	.744
VAR00004 * VAR00005	Pillai's Trace	.223	1.339[b]	3.000	14.000	.302	.223	4.016	.280
	Wilks' Lambda	.777	1.339[b]	3.000	14.000	.302	.223	4.016	.280
	Hotelling's Trace	.287	1.339[b]	3.000	14.000	.302	.223	4.016	.280
	Roy's Largest Root	.287	1.339[b]	3.000	14.000	.302	.223	4.016	.280

a. Computed using alpha = .05

Entering this data into SPSS gives us the following output. Examine the p-values for Wilk's Lambda.

If the p-value for each is less than .05, then we can conclude that factor has an effect on the dependent variables.

In this example, both the Operator and Method of Answer are significant.

Multiple Analysis of Variance (MANOVA)

Between-Subjects SSCP Matrix

			VAR00001	VAR00002	VAR00003
Hypothesis	Intercept	VAR00001	117918.72	60376.045	6042.979
		VAR00002	60376.045	30913.385	3094.091
		VAR00003	6042.979	3094.091	309.684
	VAR00004	VAR00001	1.740	-1.504	.855
		VAR00002	-1.504	1.300	-.739
		VAR00003	.855	-.739	.421
	VAR00005	VAR00001	.760	.682	1.930
		VAR00002	.682	.613	1.733
		VAR00003	1.930	1.733	4.900
	VAR00004 * VAR00005	VAR00001	.000	.016	.044
		VAR00002	.016	.545	1.469
		VAR00003	.044	1.469	3.960
Error		VAR00001	1.764	.020	-3.070
		VAR00002	.020	2.628	-.552
		VAR00003	-3.070	-.552	64.924

Based on Type III Sum of Squares

These matrices allow for partitioning of the variance, just as a Sums of Squares does in a univariate ANOVA. The diagonal (1.740, 1.301 and 0.4205) are the SS for the Operator when each of these responses are analyzed as a univariate response.

The SSCP Matrix for Error is equal to the Error SS in a univariate ANOVA. The diagonal here is the Error SS when each of the responses is analyzed as a univariate response.

Multiple Analysis of Variance (MANOVA)

Residual SSCP Matrix

		VAR00001	VAR00002	VAR00003
Sum-of-Squares and Cross-Products	VAR00001	1.764	2.000E-02	-3.070
	VAR00002	2.000E-02	2.628	-.552
	VAR00003	-3.070	-.552	64.924
Covariance	VAR00001	.110	1.250E-03	-.192
	VAR00002	1.250E-03	.164	-3.450E-02
	VAR00003	-.192	-3.450E-02	4.058
Correlation	VAR00001	1.000	.009	-.287
	VAR00002	.009	1.000	-.042
	VAR00003	-.287	-.042	1.000

Based on Type III Sum of Squares

The Residual SSCP Matrix shows the degree of correlation among the dependent variables.

Because the degree of overall correlation is weak (the strongest relationship being between Handle Time and Customer Sat, but still a weak correlation at –0.29), you could possibly achieve more accurate results with three univariate ANOVA’s on these responses.

Multiple Analysis of Variance (MANOVA)

What does this mean?

As the call center manager, I have learned that the particular operator that a customer gets when calling, and the method that the operator uses to answer the call has a significant impact on the group of dependent variables.

However, I have also learned that the dependent variables are not as correlated as I thought, and therefore I could run a univariate ANOVA on each of them and possibly better understand the impact that the independent variables has on each of the dependent variables alone.

Conclusions:
ANOVA vs. MANOVA

- ANOVA uses one or more categorical independents as predictors, but only one dependent variable. In MANOVA there is more than one dependent variable.

- In ANOVA, we use the F-test to determine significance of a factor. In MANOVA, we use a multivariate F-test called Wilk's Lambda.

- The F value in ANOVA is based on a comparison of the factor variance to the error variance. In MANOVA, we compare the factor variance-covariance matrix to the error variance-covariance matrix to obtain Wilks' lambda. The "covariance" here is included because the measures are probably correlated and we must take this correlation into account when performing the significance test.