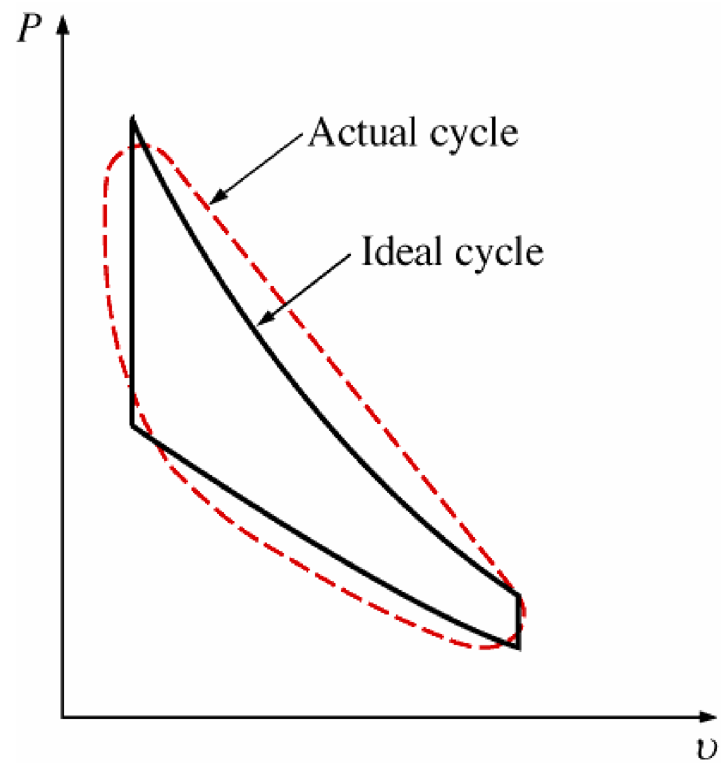
Thermodynamic Cycles

- Look at different cycles that approximate real processes
- You can categorize these processes several different ways
  - Power Cycles vs. Refrigeration
  - Gas vs. Vapor
  - Closed vs. open
  - Internal Combustion vs. External Combustion

Power Cycles

- Otto Cycle
  - Spark Ignition
- Diesel Cycle
- Brayton Cycle
  - Gas Turbine
- Rankine Cycle

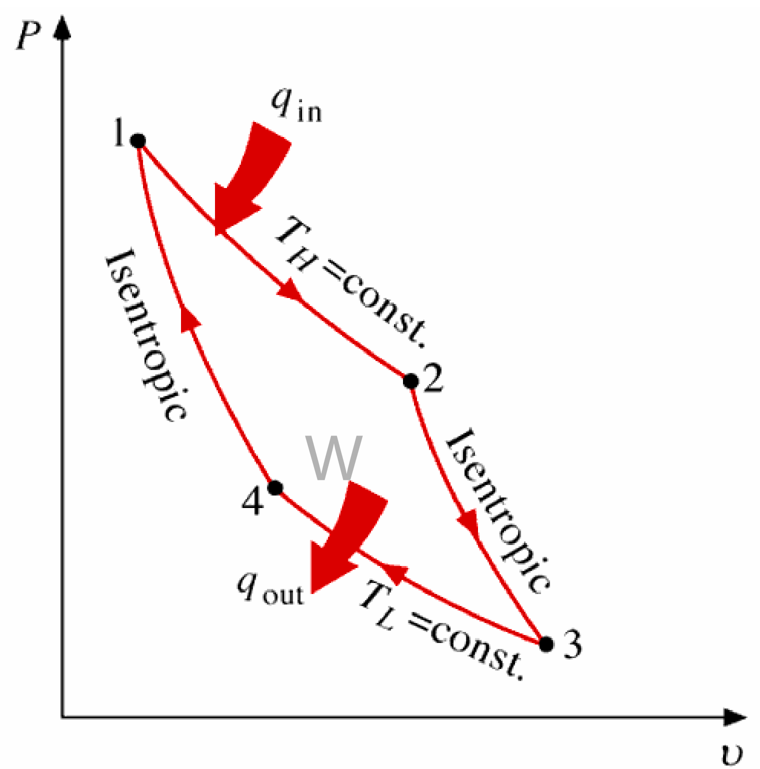


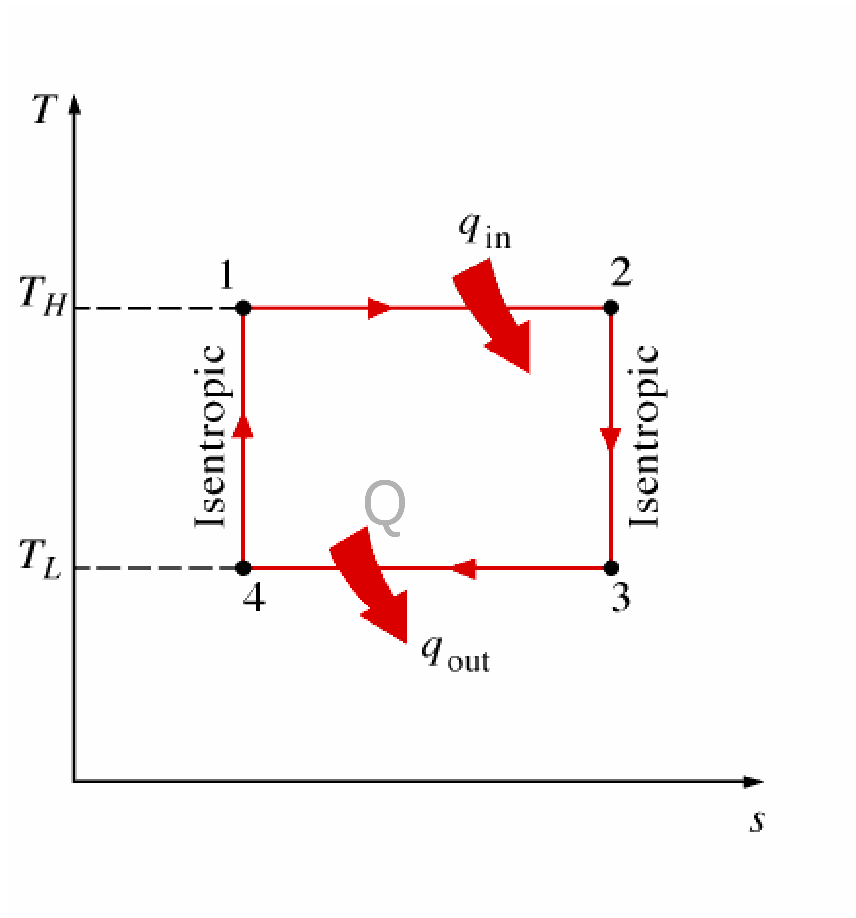
These are all heat engines. They convert heat to work, so the efficiency is:

$$\eta_{th} = \frac{W_{net}}{Q_{in}}$$

- We’ll be using ideal cycles to analyze real systems, so lets start with the only ideal cycle we’ve studied so far

Carnot Cycle





Q-W=0 → Q=W

In addition, we know that the efficiency for a Carnot Cycle is:

$$\eta_{th,\,Carnot} = 1 - \frac{T_L}{T_H}$$

Carnot Cycle is not a good model for most real processes

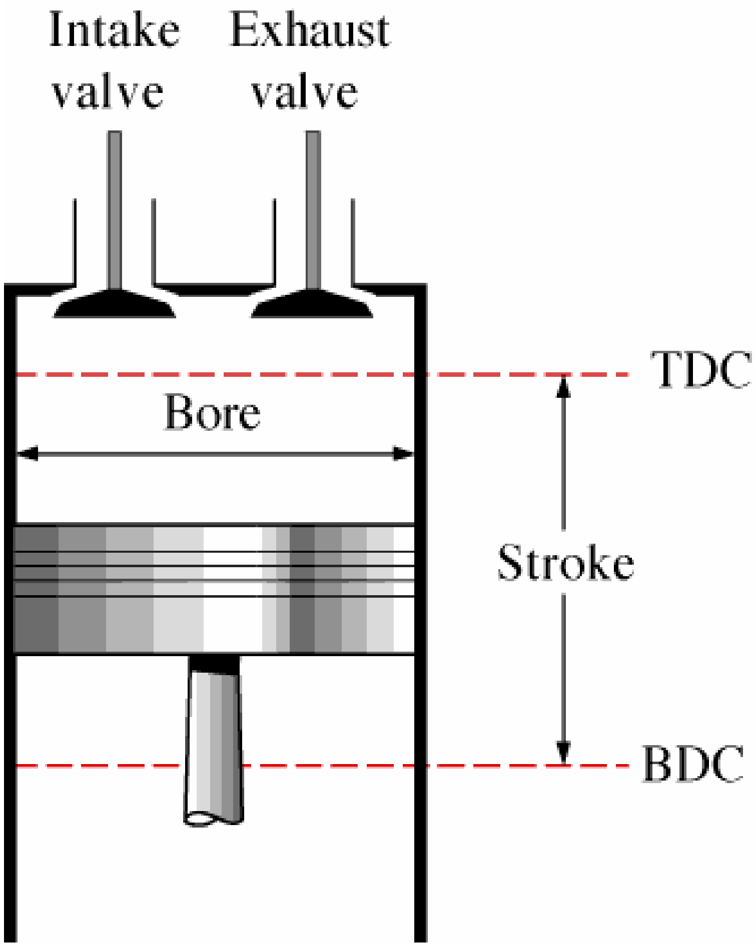
- For example
  - Internal combustion engine
  - Gas turbine
- We need to develop a new model, that is still ideal

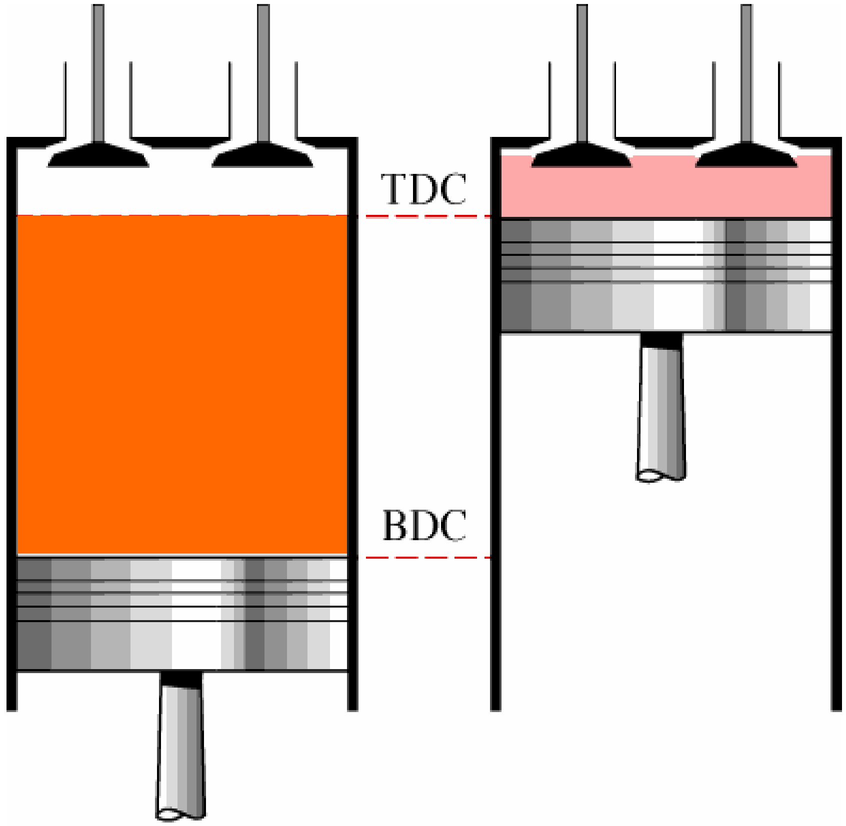
Air-Standard Assumptions

- Air continuously circulates in a closed loop and behaves as an ideal gas
- All the processes are internally reversible
- Combustion is replaced by a heat-addition process from the outside
- Heat rejection replaces the exhaust process

- Also assume a constant value for C_p, evaluated at room temperature

Terminology for Reciprocating Devices





(*a*) Displacement volume (*b*) Clearance volume

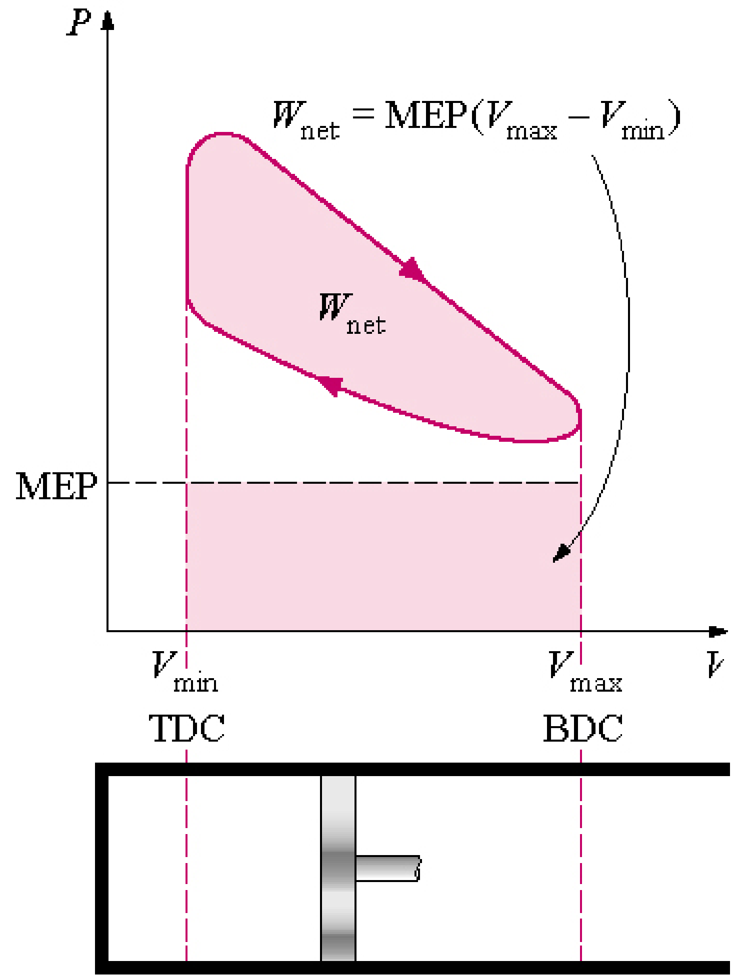
Compression Ratio

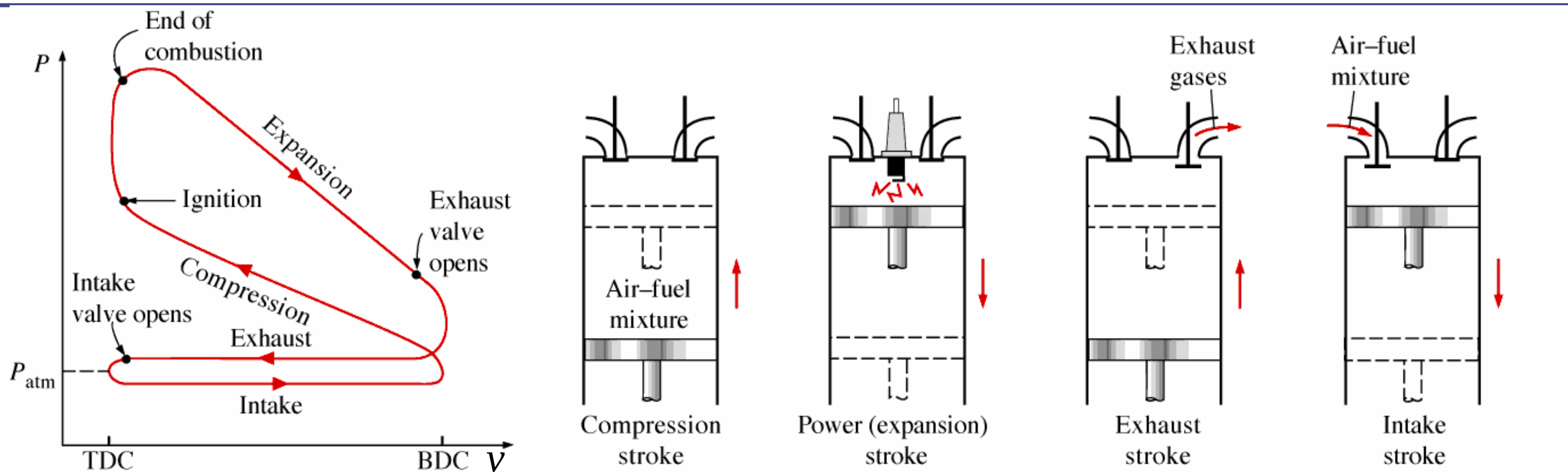
$$r = \frac{V\max}{V\min} = \frac{V_{BDC}}{V_{TDC}}$$

Mean Effective Pressure

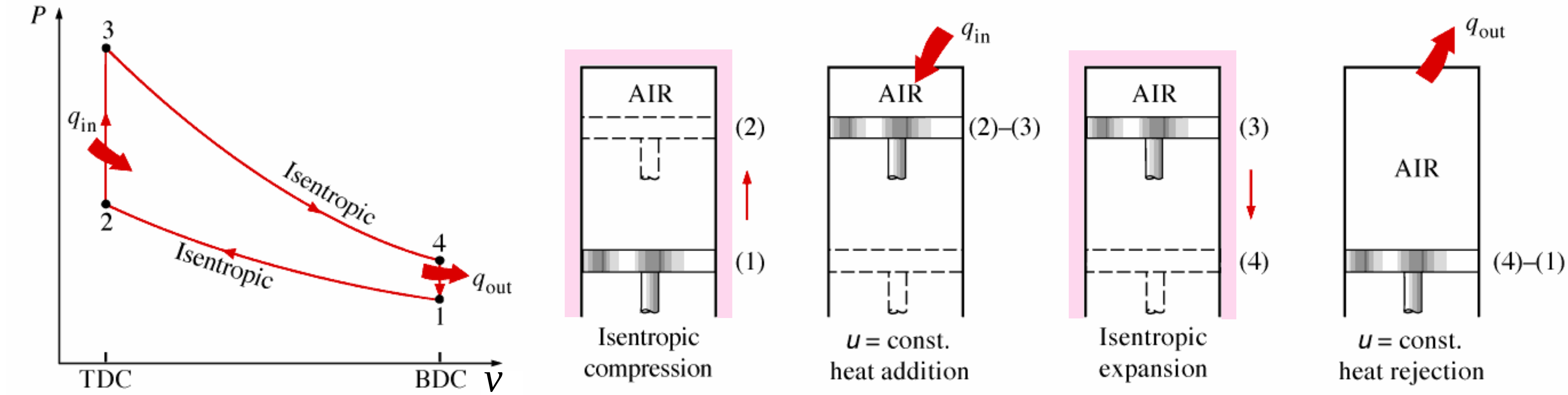
$$W = \int_1^2 P dV$$

$$W = P \Delta V$$

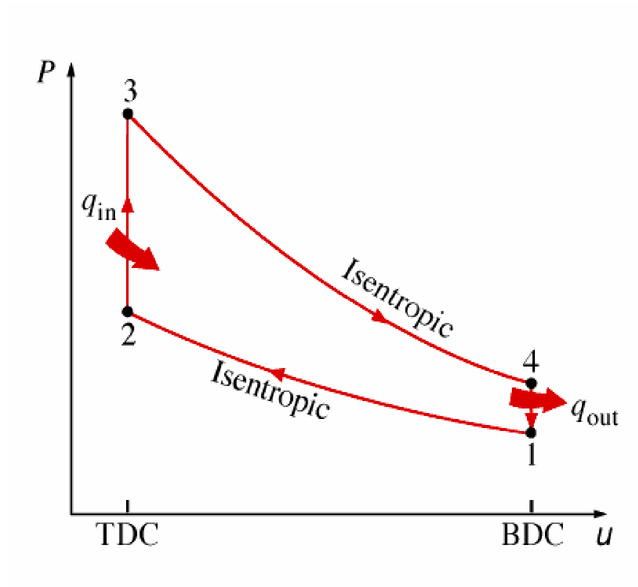


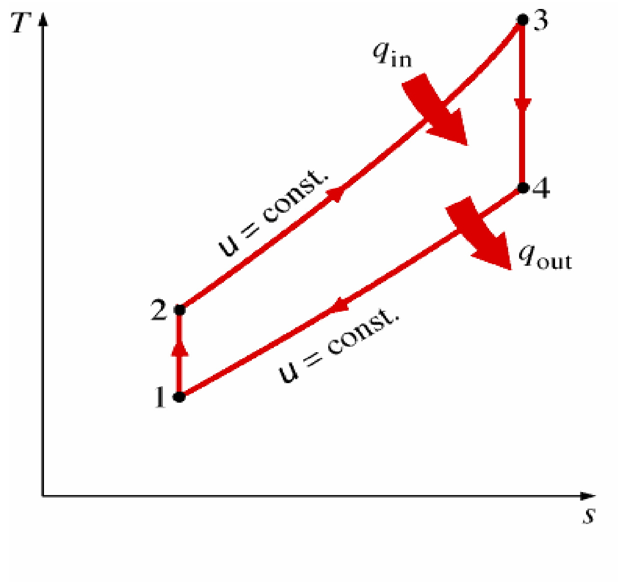


(*a*) Actual four-stroke spark-ignition engine



(*b*) Ideal Otto cycle





1-2 Isentropic Compression

2-3 Constant Volume Heat Addition

3-4 Isentropic Expansion

4-1 Constant Volume Heat Rejection

Thermal Efficiency of the Otto Cycle

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{Q_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

Apply First Law Closed System to Process 2-3, V = Constant

$$Q_{net,23} - W_{net,23} = \Delta U_{23}$$

$$W_{net,23} = W_{other,23} + W_{b,23} = 0 + \int_2^3 PdV = 0$$

$$Q_{net,\,23} = \Delta U_{23}$$

$$Q_{net,\,23} = Q_{in} = mC_v(T_3 - T_2)$$

Apply First Law Closed System to Process 4-1,V = Constant

$$Q_{net,41} - W_{net,41} = \Delta U_{41}$$

$$W_{net,41} = W_{other,41} + W_{b,41} = 0 + \int_4^1 PdV = 0$$

$$Q_{net,41} = \Delta U_{41}$$

$$Q_{net,41} = -Q_{out} = mC_v(T_1 - T_4)$$

$$Q_{out} = -mC_v(T_1 - T_4) = mC_v(T_4 - T_1)$$

$$\eta_{th,\,Otto} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$= 1 - \frac{mC_v(T_4 - T_1)}{mC_v(T_3 - T_2)}$$

$\rightarrow$

$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1(T_4 / T_1 - 1)}{T_2(T_3 / T_2 - 1)}$$

Recall processes 1-2 and 3-4 are isentropic, so

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \quad \text{and} \quad \frac{T_3}{\mathrm{T}_4} = \left(\frac{v_4}{v_3}\right)^{k-1}$$

$$\mathrm{v}_3 = \mathrm{v}_2 \;\; \text{and} \;\; \mathrm{v}_4 = \mathrm{v}_1$$

$\rightarrow$

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

or

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\eta_{th,\,Otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1 (T_4 / T_1 - 1)}{T_2 (T_3 / T_2 - 1)}$$

1

$$\eta_{th,\,Otto} = 1 - \frac{T_1}{T_2}$$

Is this the same as the Carnot efficiency?

Efficiency of the Otto Cycle vs. Carnot Cycle

- There are only two temperatures in the Carnot cycle
  - Heat is added at T_H
  - Heat is rejected at T_L
- There are four temperatures in the Otto cycle!!
  - Heat is added over a range of temperatures
  - Heat is rejected over a range of temperatures

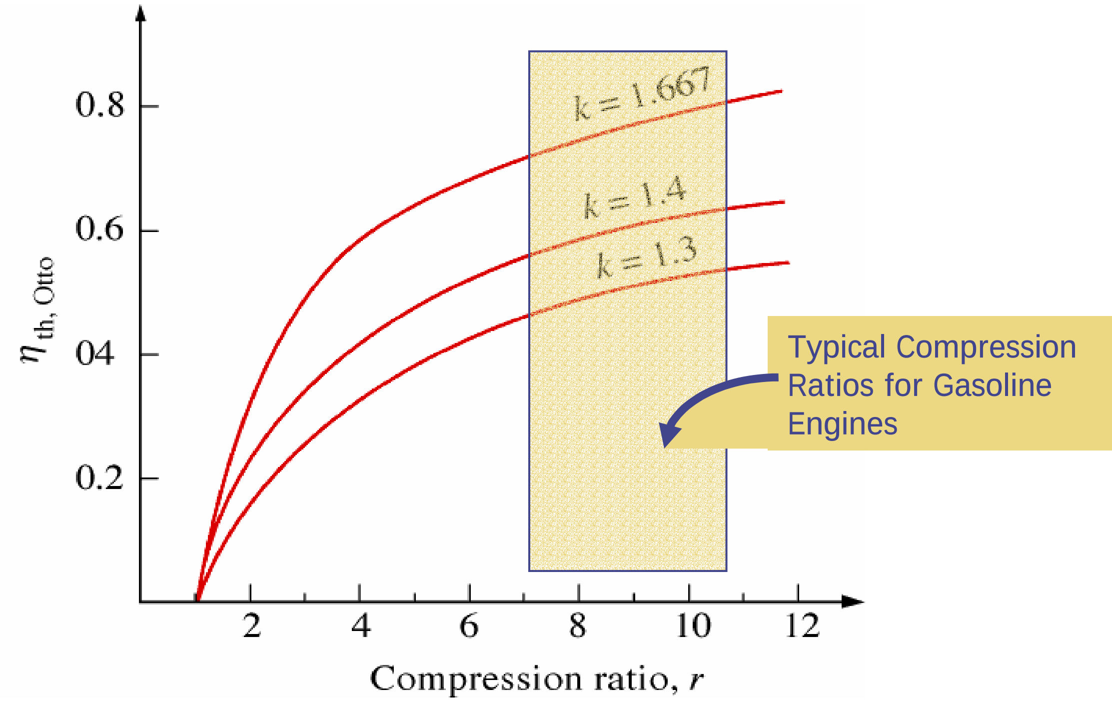
Since process 1-2 is isentropic,

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{k-1} = \frac{1}{r^{k-1}}$$

$$\eta_{th,\,Otto} = 1 - \frac{T_1}{T_2}$$

→

$$\eta_{th,\,Otto} = 1 - \frac{1}{r^{k-1}}$$



Increasing Compression Ratio Increases the Efficiency

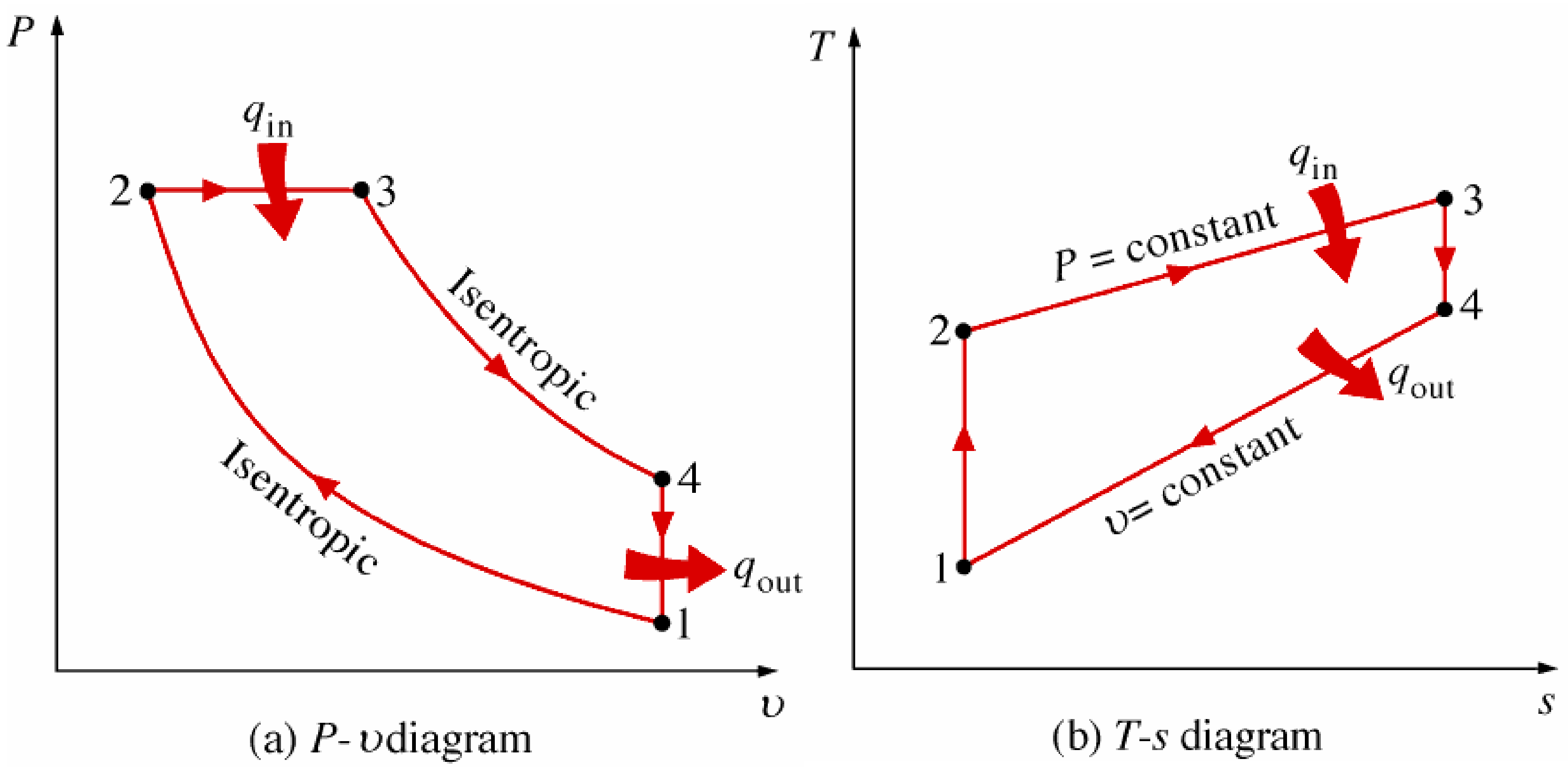
Why not use higher compression Ratios?

- Premature Ignition
- Causes “Knock”
- Reduces the Efficiency
- Mechanically need a better design

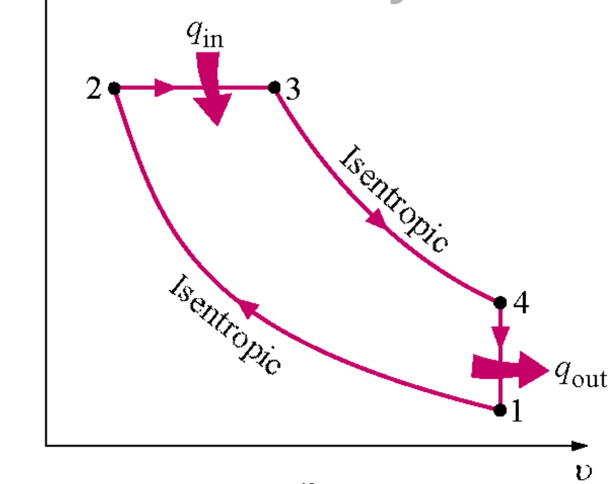
Diesel Engines

- No spark plug
- Fuel is sprayed into hot compressed air

State Diagrams for the Diesel Cycle

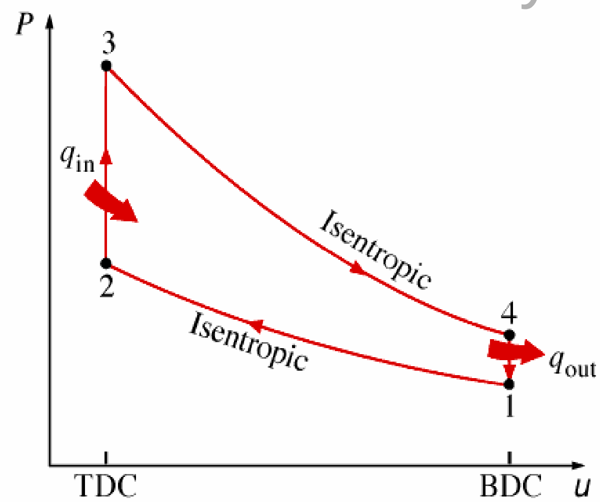


Diesel Cycle

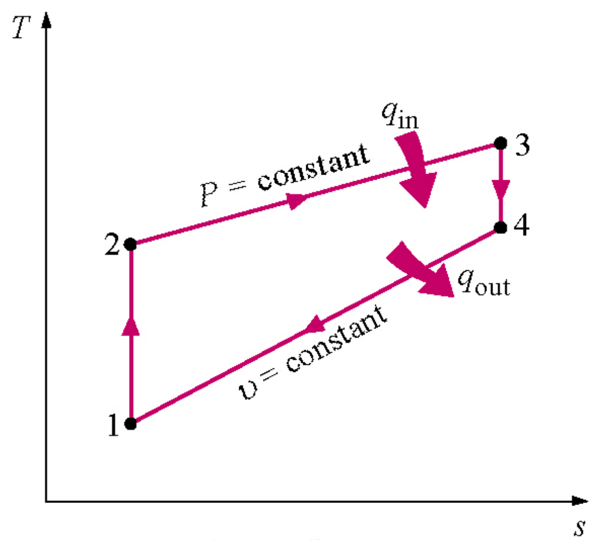


(a) P-υdiagram

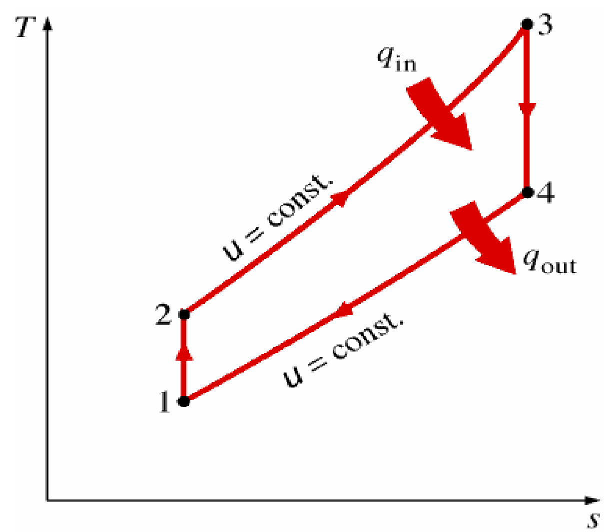
Otto Cycle



The only difference is in process 2-3



(b) T-s diagram



Consider Process 2-3

- This is the step where heat is transferred into the system
- We model it as constant pressure instead of constant volume

$$q_{in,23} - w_{b,out} = \Delta u = u_3 - u_2$$
$$q_{in,23} = \Delta u + P\Delta v = \Delta h = C_p\left(T_3 - T_2\right)$$

Consider Process 4-1

- This is where heat is rejected
- We model this as a constant v process
  - That means there is no boundary work

$$q_{41} - w_{41} = \Delta u$$

$$q_{41} = -q_{out} = \Delta u = C_v\left(T_1 - T_4\right)$$
$$q_{out} = C_v\left(T_4 - T_1\right)$$

As for any heat engine...

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$$q_{out} = C_v\left(T_4 - T_1\right) \text{ and } q_{in} = C_p\left(T_3 - T_2\right)$$

$$n_{th,diesel} = 1 - \frac{C_v\left(T_4 - T_1\right)}{C_p\left(T_3 - T_2\right)} = 1 - \frac{\left(T_4 - T_1\right)}{k\left(T_3 - T_2\right)}$$

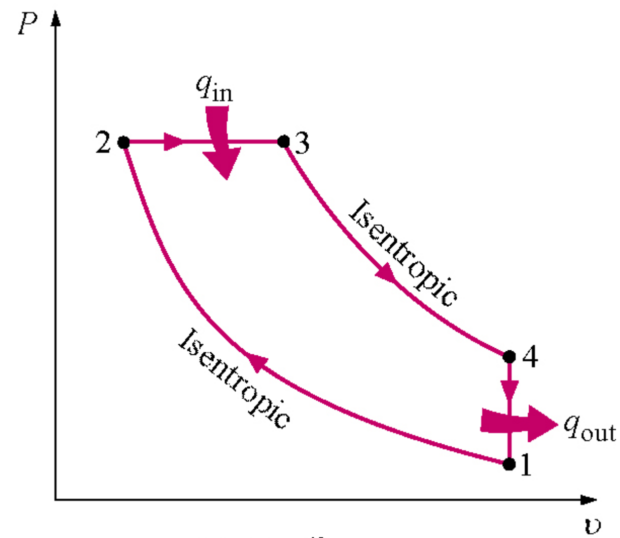
Rearrange

$$n_{th,diesel} = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{kT_2\left(T_3/T_2 - 1\right)}$$

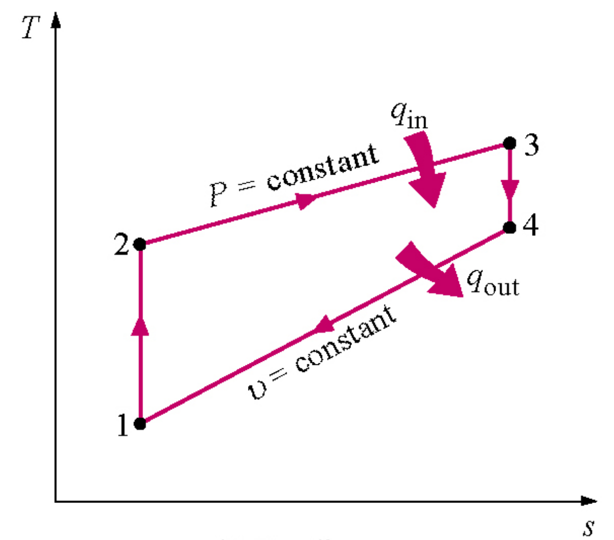
$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2} \quad \text{where } P_3 = P_2$$

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c$$

r_c is called the cutoff ratio – it's the ratio of the cylinder volume before and after the combustion process



(a) P-υ diagram

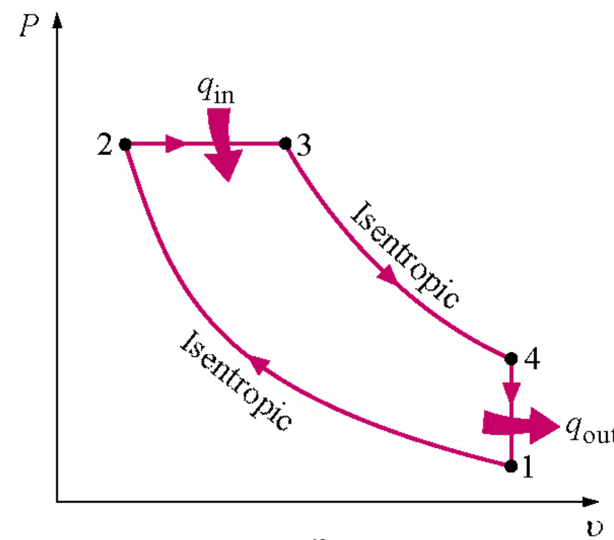


(b) T-s diagram

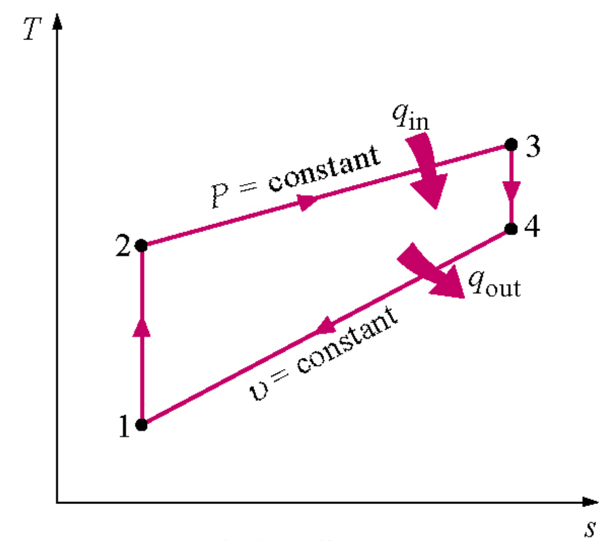
$$\eta_{th,diesel} = 1 - \frac{T_1\left(T_4/T_1 - 1\right)}{kT_2\left(r_c - 1\right)}$$

$$\frac{P_4V_4}{T_4} = \frac{P_1V_1}{T_1} \quad \text{where } V_4 = V_1$$

$$\frac{T_4}{T_1} = \frac{P_4}{P_1}$$



(a) P-υ diagram

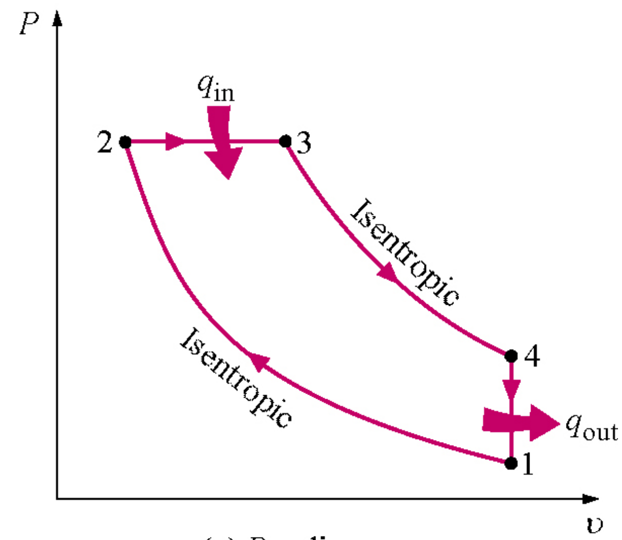


(b) T-s diagram

$$n_{th,diesel} = 1 - \frac{T_1\left(P_4/P_1 - 1\right)}{kT_2\left(r_c - 1\right)}$$

Since Process 1-2 and Process 3-4

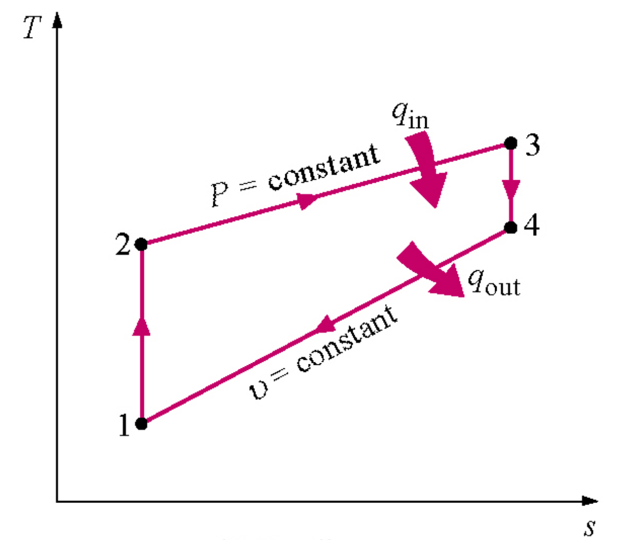
are both isentropic



(a) P-υ diagram

$$P_1V_1^k = P_2V_2^k \text{ and}$$

$$P_4V_4^k = P_3V_3^k$$



(b) T-s diagram

$$\frac{P_4}{P_1}\frac{V_4^k}{V_1^k} = \frac{P_3}{P_2}\frac{V_3^k}{V_2^k} = \left(\frac{V_3}{V_2}\right)^k = r_c^k$$

1 1

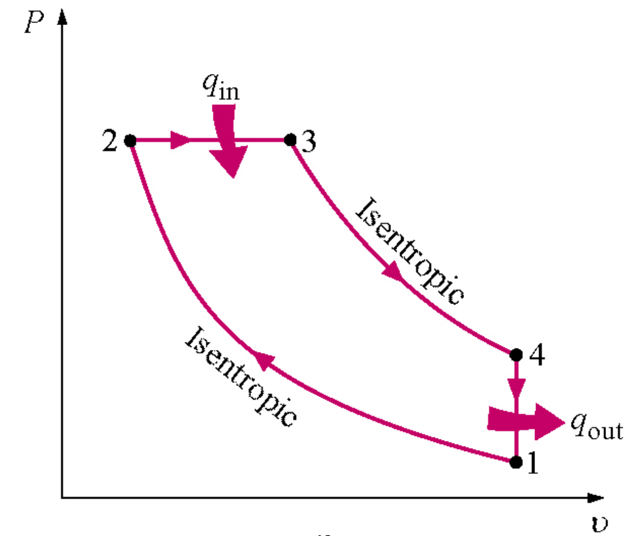
$$n_{th,diesel} = 1 - \frac{T_1\left(r_c^{\,k} - 1\right)}{kT_2\left(r_c - 1\right)}$$

Finally, Since process 1-2 is isentropic

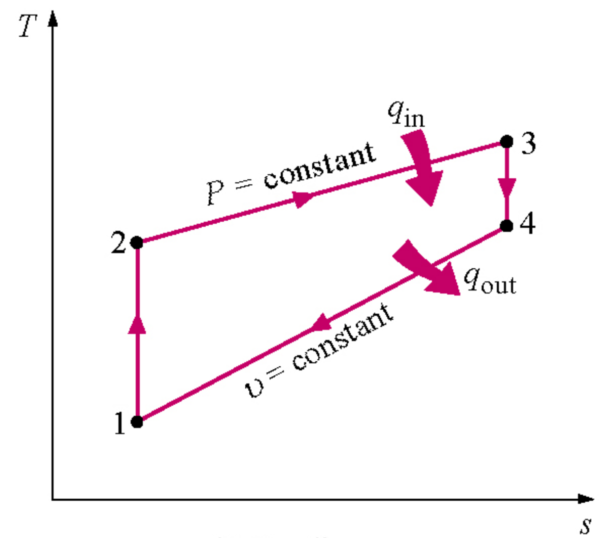
$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1}$$

The volume ratio from 1 to 2 is the compression ratio, r

$$\frac{T_1}{T_2} = \left(\frac{1}{r}\right)^{k-1}$$

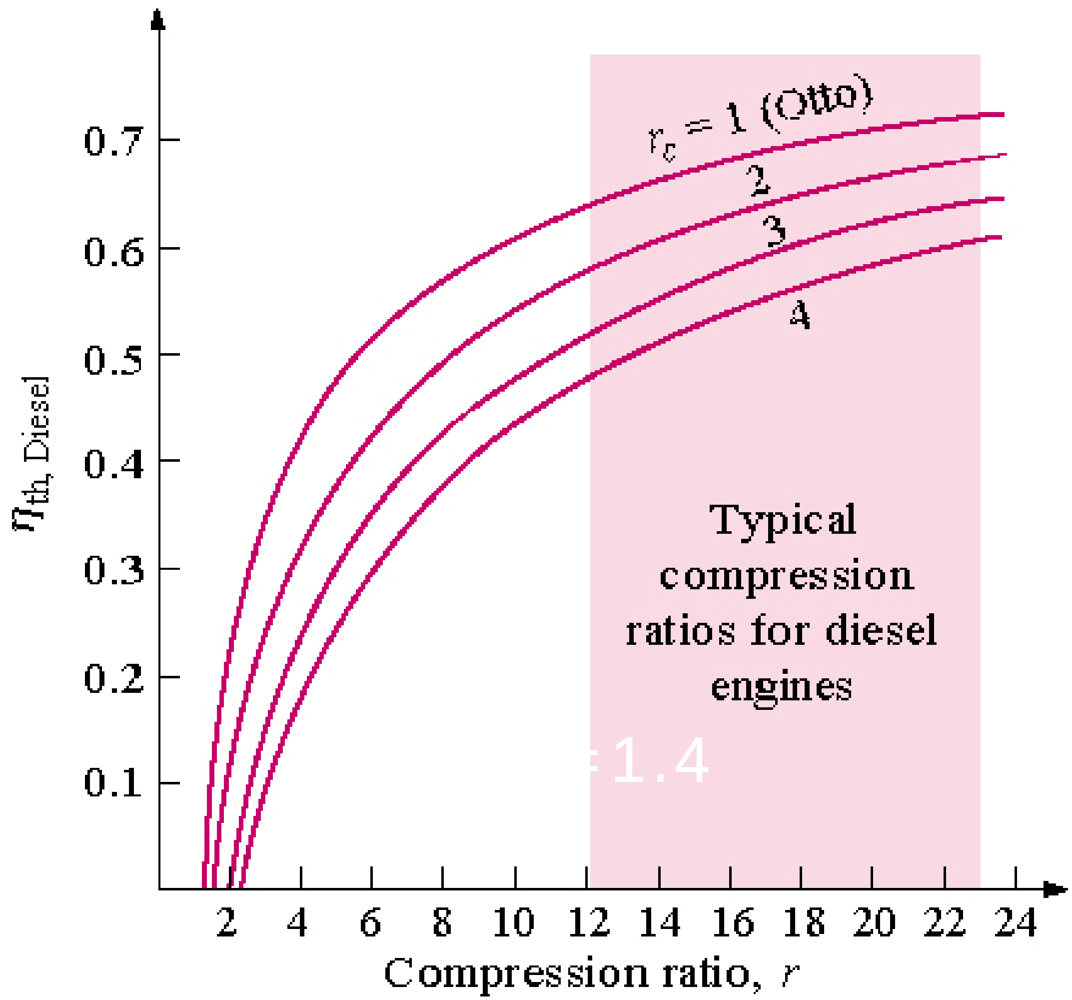


(a) P-υ diagram



(b) T-s diagram

$$\eta_{th,diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^{\,k} - 1}{k\left(r_c - 1\right)}\right]$$



The efficiency of the Otto cycle is always higher than the Diesel cycle

Why use the Diesel cycle?

Because you can use higher compression ratios