Otto, Brayton and Rankine Cycles

Selected from Chapter 9 and 10

The content and the pictures are from the text book: Çengel, Y. A. and Boles, M. A., "Thermodynamics: An Engineering Approach," McGraw-Hill, New York, 6th Ed., 2008

Objective

- Basic considerations in the analysis of power cycles
- The Carnot cycle and its value in engineering
- Air-standard assumptions
- Otto cycle: The ideal cycle for spark-ignition engines
- Brayton cycle: The ideal cycle for gas-turbine engines
- Rankine cycle: The ideal cycle for vapor power cycles

Basic Considerations in Analysis of Power Cycles

The idealizations and simplifications in the analysis of power cycles:

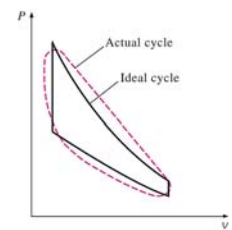
1. The cycle does not involve any *friction*. Therefore, the working fluid does not experience any pressure drop as it flows in pipes or devices such as heat exchangers.
2. All expansion and compression processes take place in a *quasi-equilibrium* manner.
3. The pipes connecting the various components of a system are well insulated, and *heat transfer* through them is negligible.

Basic Considerations in Analysis of Power Cycles

Most power-producing devices operate on cycles.

Ideal cycle: A cycle that resembles the actual cycle closely but is made up totally of **internally reversible** processes**.**

Reversible cycles such as **Carnot cycle** have the highest thermal efficiency of all heat engines operating between the same temperature levels. Unlike ideal cycles, they are **totally reversible**, and unsuitable as a realistic model.



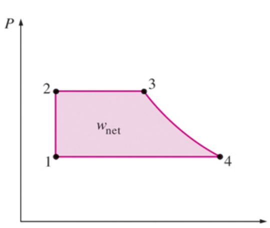
Thermal efficiency of heat engines:

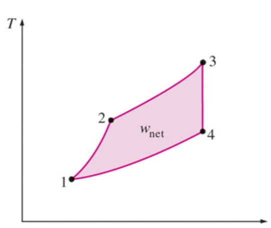
$$\eta_{th} = \frac{W_{net}}{Q_{in}} \quad \text{or} \quad \eta_{th} = \frac{w_{net}}{q_{in}}$$

The analysis of many complex processes can be reduced to a manageable level by utilizing some idealizations.

Basic Considerations in Analysis of Power Cycles

On a *T-s* diagram, the ratio of the area enclosed by the cyclic curve to the area under the heat-addition process curve represents the thermal efficiency of the cycle. Any modification that increases the ratio of these two areas will also increase the thermal efficiency of the cycle.



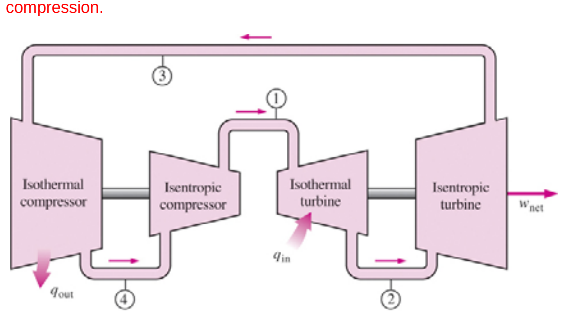


Care should be exercised in the interpretation of the results from ideal cycles.

On both *P-v* and *T-s* diagrams, the area enclosed by the process curve represents the net work of the cycle.

Carnot Cycle and Its Value in Engineering

The Carnot cycle is composed of four totally reversible processes: isothermal heat addition, isentropic expansion, isothermal heat rejection, and isentropic compression.

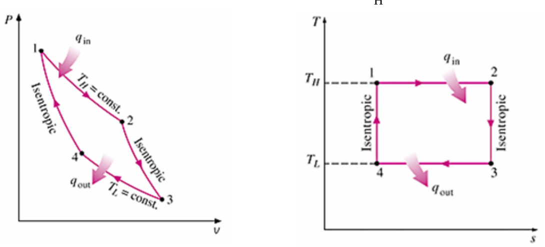


A steady-flow Carnot engine.

Carnot Cycle and Its Value in Engineering

For both ideal and actual cycles: Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system.

Thermal efficiency of Carnot cycle: $\eta_{th,rev} = 1 - \frac{T_L}{T_H}$

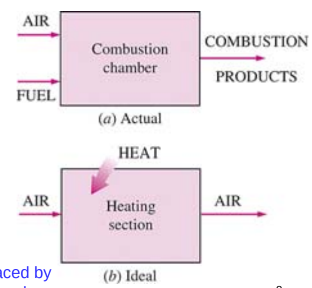


P-v and *T-s* diagrams of a Carnot cycle

Air-Standard Assumptions

Air-standard assumptions:

1. The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
2. All the processes that make up the cycle are internally reversible.
3. The combustion process is replaced by a heat-addition process from an external source.
4. The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

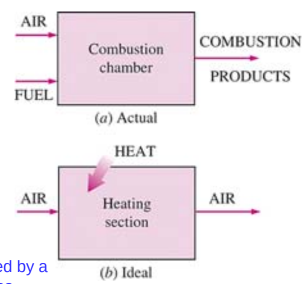


The combustion process is replaced by a heat-addition process in ideal cycles.

Air-Standard Assumptions

Cold-air-standard assumptions: When the working fluid is considered to be air with constant specific heats at room temperature (25°C).

Air-standard cycle: A cycle for which the air-standard assumptions are applicable.



The combustion process is replaced by a heat-addition process in ideal cycles.

Basic Considerations in Analysis of Power Cycles

Gas cycle: the working fluid remains a gas throughout the entire cycle.

- Otto cycle: The ideal cycle for spark-ignition engines
- Brayton cycle: The ideal cycle for gas-turbine engines

Vapor cycle: the working fluid remains is alternatively vaporized and condensed in a cycle.

- Rankine cycle: The ideal cycle for vapor power cycles

Reciprocating Engines

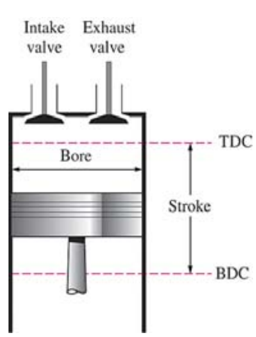
Nomenclature for reciprocating engines:

Bore: the diameter of the piston

TDC: top dead center, the position of the piston when it forms the smallest volume in the cylinder

BDC: bottom dead center, the position of the piston when it forms the largest volume in the cylinder

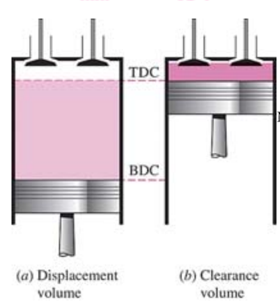
Stroke: the distance between TDC and BDC

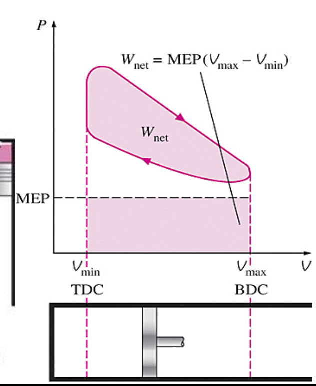


Reciprocating Engines

Compression ratio:

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$$



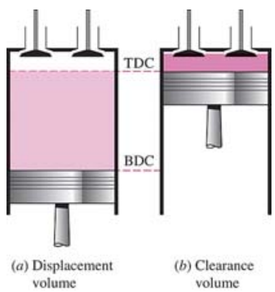


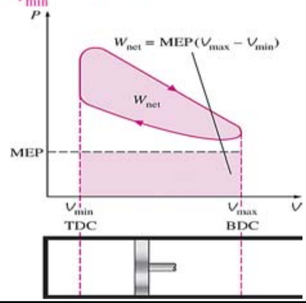
Reciprocating Engines

Mean effective pressure (MEP):

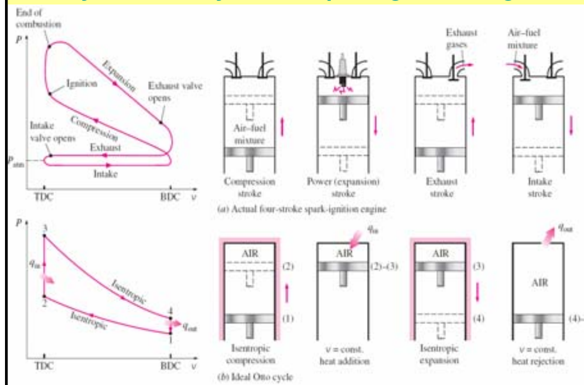
$$W_{net} = \text{MEP} \times \text{Piston area} \times \text{Stroke} = \text{MEP} \times \text{Displacement volume}$$

$$\text{MEP} = \frac{W_{net}}{V_{max} - V_{min}} = \frac{w_{net}}{v_{max} - v_{min}} \quad \text{(kPa)}$$



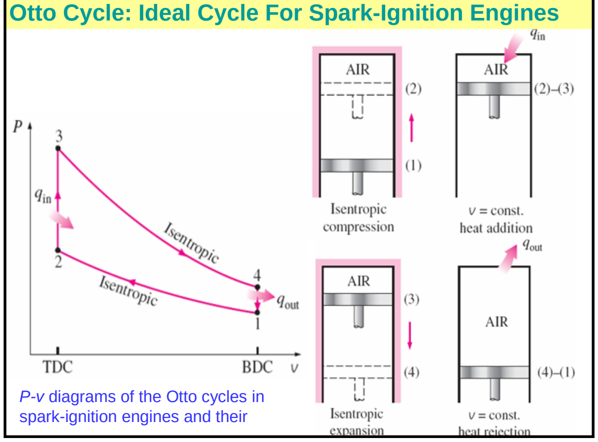


Otto Cycle: Ideal Cycle For Spark-Ignition Engines



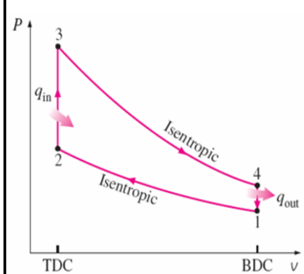
Actual and ideal cycles in spark-ignition engines and their *P-v* diagrams.

Otto Cycle: Ideal Cycle For Spark-Ignition Engines

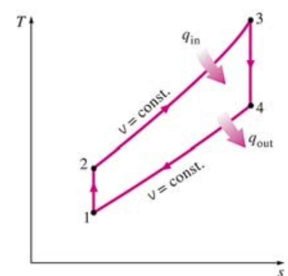


P-v diagrams of the Otto cycles in spark-ignition engines and their

Otto Cycle: Ideal Cycle For Spark-Ignition Engines



P-v diagrams of the Otto cycles in spark-ignition engines and their



T-s diagrams of the Otto cycles in spark-ignition engines and their

Otto Cycle: Energy Analysis

The Otto cycle is performed in a closed system. Neglecting the kinetic and potential energy, the energy balance on a unit mass basis:

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = \Delta u$$

Process 2-3 and 4-1 are volume-constant:

$$q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Otto} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

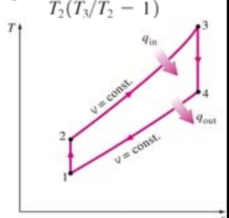
Process 1-2 and 3-4 are isentropic:

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$

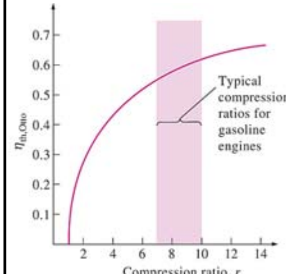
Combine the above equations:

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}}$$

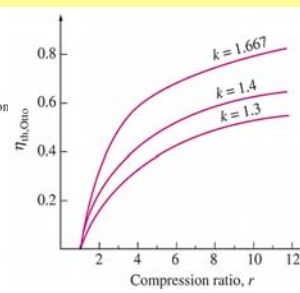
Compression ration: $r = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$



Otto Cycle: Energy Analysis



The thermal efficiency of the Otto cycle increases with the specific heat ratio *k* of the working fluid.



Thermal efficiency of the ideal Otto cycle as a function of compression ratio (*k* = 1.4).

Otto Cycle: Energy Analysis

EXAMPLE 9–2 The Ideal Otto Cycle

An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Accounting for the variation of specific heats of air with temperature, determine (*a*) the maximum temperature and pressure that occur during the cycle, (*b*) the net work output, (*c*) the thermal efficiency, and (*d*) the mean effective pressure for the cycle.

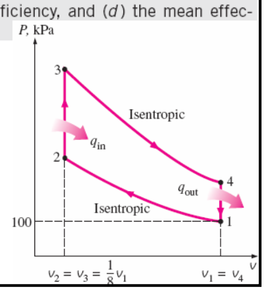


FIGURE 9–19

Otto Cycle: Energy Analysis

(*a*) The maximum temperature and pressure in an Otto cycle occur at the end of the constant-volume heat-addition process (state 3). But first we need to determine the temperature and pressure of air at the end of the isentropic compression process (state 2), using data from Table A–17:

$$T_1 = 290 \text{ K} \rightarrow u_1 = 206.91 \text{ kJ/kg}$$
$$v_{r1} = 676.1$$

Process 1-2 (isentropic compression of an ideal gas):

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} = \frac{1}{r} \rightarrow v_{r2} = \frac{v_{r1}}{r} = \frac{676.1}{8} = 84.51 \rightarrow T_2 = 652.4 \text{ K}$$
$$u_2 = 475.11 \text{ kJ/kg}$$

$$\frac{P_2v_2}{T_2} = \frac{P_1v_1}{T_1} \rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$$
$$= (100 \text{ kPa})\left(\frac{652.4 \text{ K}}{290 \text{ K}}\right)(8) = 1799.7 \text{ kPa}$$

Otto Cycle: Energy Analysis

Process 2-3 (constant-volume heat addition):

$$q_{in} = u_3 - u_2$$
$$800 \text{ kJ/kg} = u_3 - 475.11 \text{ kJ/kg}$$
$$u_3 = 1275.11 \text{ kJ/kg} \rightarrow T_3 = \mathbf{1575.1 \text{ K}}$$
$$v_{r3} = 6.108$$

$$\frac{P_3v_3}{T_3} = \frac{P_2v_2}{T_2} \rightarrow P_3 = P_2\left(\frac{T_3}{T_2}\right)\left(\frac{v_2}{v_3}\right)$$
$$= (1.7997 \text{ MPa})\left(\frac{1575.1 \text{ K}}{652.4 \text{ K}}\right)(1) = \mathbf{4.345 \text{ MPa}}$$

(*b*) The net work output for the cycle is determined either by finding the boundary ($P\,dV$) work involved in each process by integration and adding them or by finding the net heat transfer that is equivalent to the net work done during the cycle. We take the latter approach. However, first we need to find the internal energy of the air at state 4:

Process 3-4 (isentropic expansion of an ideal gas):

Otto Cycle: Energy Analysis

Process 3-4 (isentropic expansion of an ideal gas):

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = r \rightarrow v_{r4} = rv_{r3} = (8)(6.108) = 48.864 \rightarrow T_4 = 795.6 \text{ K}$$
$$u_4 = 588.74 \text{ kJ/kg}$$

Process 4-1 (constant-volume heat rejection):

$$-q_{out} = u_1 - u_4 \rightarrow q_{out} = u_4 - u_1$$
$$q_{out} = 588.74 - 206.91 = 381.83 \text{ kJ/kg}$$

Thus, $w_{net} = q_{net} = q_{in} - q_{out} = 800 - 381.83 = \mathbf{418.17 \text{ kJ/kg}}$

(*c*) The thermal efficiency of the cycle is determined from its definition:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{418.17 \text{ kJ/kg}}{800 \text{ kJ/kg}} = \mathbf{0.523 \text{ or } 52.3\%}$$

Under the cold-air-standard assumptions (constant specific heat values at room temperature), the thermal efficiency would be (Eq. 9–8)

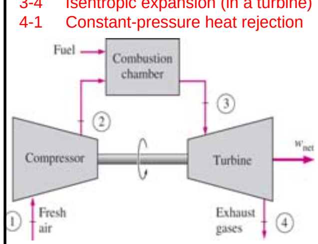
$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}} = 1 - r^{1-k} = 1 - (8)^{1-1.4} = 0.565 \text{ or } 56.5\%$$

which is considerably different from the value obtained above. Therefore care should be exercised in utilizing the cold-air-standard assumptions.

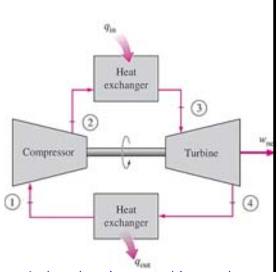
Brayton Cycle: Ideal Cycle for Gas-Turbine Engines

The combustion process is replaced by a constant-pressure heat-addition process from an external source, and the exhaust process is replaced by a constant-pressure heat-rejection process to the ambient air.

1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection

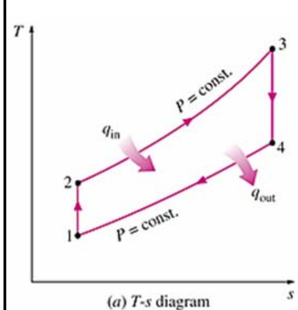


An open-cycle gas-turbine engine.

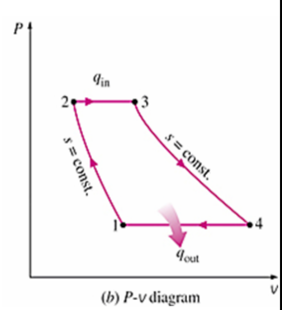


A closed-cycle gas-turbine engine.

Brayton Cycle: Ideal Cycle for Gas-Turbine Engines



T-s diagram of Brayton Cycle



P-v diagram of Brayton Cycle

Brayton Cycle: Energy analysis

The Brayton cycle is performed in a steady-flow control volume. Neglecting the kinetic and potential energy, the energy balance on a unit mass basis:

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

The heat transfer to and from the working fluid:

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

The thermal efficiency of the Brayton cycle:

$$\eta_{th,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

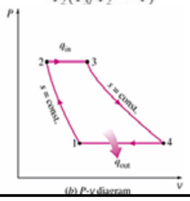
Process 1-2 and 3-4 are isentropic:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

Combine the above equations:

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

Pressure ratio: $r_p = \frac{P_2}{P_1}$

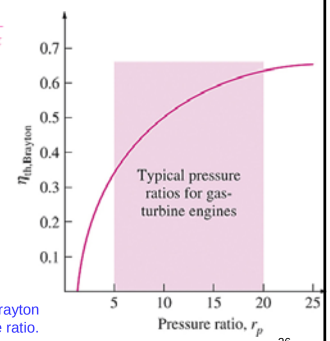


Brayton Cycle: Energy analysis

The thermal efficiency of the Brayton cycle:

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

Pressure ratio $r_p = \frac{P_2}{P_1}$



Thermal efficiency of the ideal Brayton cycle as a function of the pressure ratio.

Brayton Cycle: Energy analysis

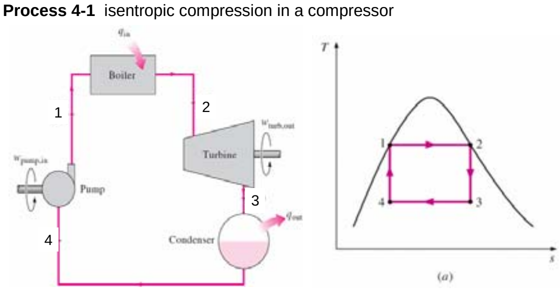
Example 9-5

In Page 521 in the textbook
Please study this example

The Carnot Vapor Cycle

Carnot vapor cycles:

Process 1-2 isothermal heat addition in a boiler
Process 2-3 isentropic expansion in a turbine
Process 3-4 isothermal heat rejection in a condenser
Process 4-1 isentropic compression in a compressor

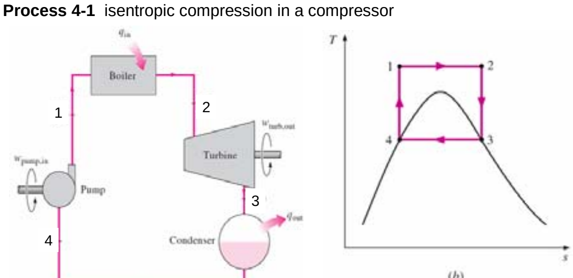


T-s diagram of two Carnot vapor cycles.

The Carnot Vapor Cycle

Carnot vapor cycles:

Process 1-2 isothermal heat addition in a boiler
Process 2-3 isentropic expansion in a turbine
Process 3-4 isothermal heat rejection in a condenser
Process 4-1 isentropic compression in a compressor



T-s diagram of two Carnot vapor cycles.

The Carnot Vapor Cycle

The Carnot cycle is the most efficient cycle operating between two specified temperature limits but it is not a suitable model for power cycles. Because:

Process 1-2 Limiting the heat transfer processes to two-phase systems severely limits the maximum temperature that can be used in the cycle (374°C for water)

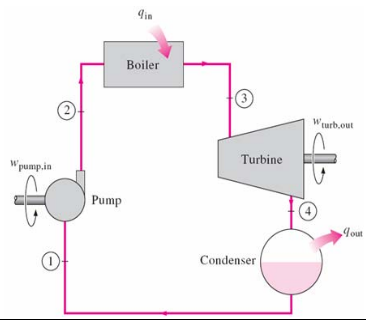
Process 2-3 The turbine cannot handle steam with a high moisture content because of the impingement of liquid droplets on the turbine blades causing erosion and wear.

Process 4-1 It is not practical to design a compressor that handles two phases.

The cycle in (b) is not suitable since it requires isentropic compression to extremely high pressures and isothermal heat transfer at variable pressures.

Rankine Cycle: Ideal Cycle For Vapor Power Cycles

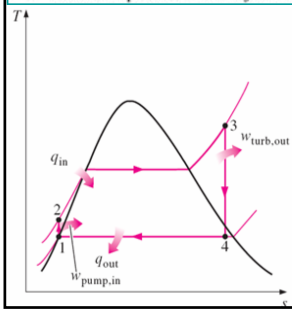
Many of the impracticalities associated with the Carnot cycle can be eliminated by superheating the steam in the boiler and condensing it completely in the condenser. The cycle that results is the **Rankine cycle**, the ideal cycle for vapor power plants, which does not involve any internal irreversibilities.

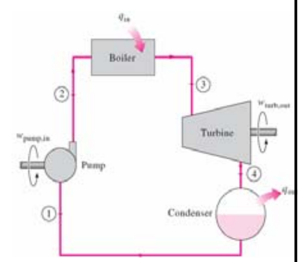


The simple ideal Rankine cycle.

Rankine Cycle: Ideal Cycle For Vapor Power Cycles

1-2	Isentropic compression in a pump
2-3	Constant pressure heat addition in a boiler
3-4	Isentropic expansion in a turbine
4-1	Constant pressure heat rejection in a condenser





The simple ideal Rankine cycle.

Rankine Cycle: Energy Analysis

Steady-flow energy equation:

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i \qquad \text{(kJ/kg)}$$

Pump $(q = 0)$: $\quad w_{pump,in} = h_2 - h_1$

$$w_{pump,in} = v(P_2 - P_1)$$

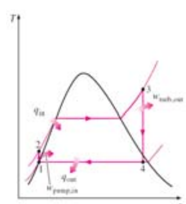
$$h_1 = h_{f\,@\,P_1} \quad \text{and} \quad v \cong v_1 = v_{f\,@\,P_1}$$

Boiler $(w = 0)$: $\quad q_{in} = h_3 - h_2$

Turbine $(q = 0)$: $\quad w_{turb,out} = h_3 - h_4$

Condenser $(w = 0)$: $\quad q_{out} = h_4 - h_1$

$$w_{net} = q_{in} - q_{out} = w_{turb,out} - w_{pump,in}$$



Rankine Cycle: Energy Analysis

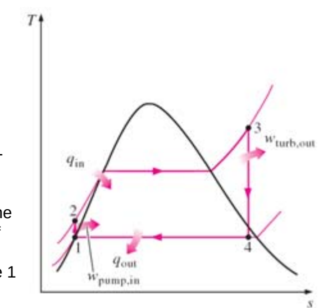
The thermal efficiency of Rankine Cycle:

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

The thermal efficiency can be interpreted as the ratio of the area enclosed by the cycle on a *T-s* diagram to the area under the heat-addition process.

The efficiency of power plants in the U.S. is often expressed in terms of **heat rate**, which is the amount of heat supplied, in Btu's, to generate 1 kWh of electricity.

$$\eta_{th} = \frac{3412 \text{ (Btu/kWh)}}{\text{Heat rate (Btu/kWh)}}$$



Rankine Cycle: Energy Analysis

EXAMPLE 10–1 **The Simple Ideal Rankine Cycle**

Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa. Determine the thermal efficiency of this cycle.

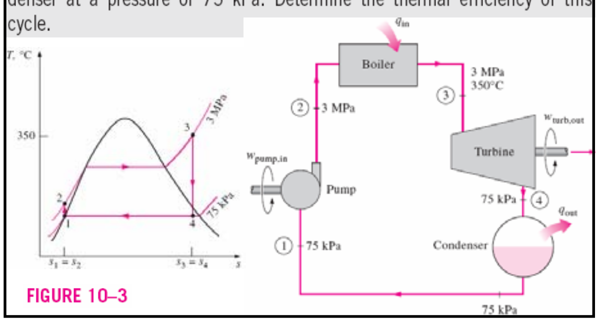


FIGURE 10–3

Rankine Cycle: Energy Analysis

Solution:

First we determine the enthalpies at various points in the cycle, using data from steam tables (Tables A–4, A–5, and A–6):

State 1: $\left.\begin{array}{l} P_1 = 75 \text{ kPa} \\ \text{Sat. liquid} \end{array}\right\}$ $\begin{array}{l} h_1 = h_{f\,@\,75\text{ kPa}} = 384.44 \text{ kJ/kg} \\ v_1 = v_{f\,@\,75\text{ kPa}} = 0.001037 \text{ m}^3\text{/kg} \end{array}$

State 2: $P_2 = 3$ MPa

$$s_2 = s_1$$

$$w_{pump,in} = v_1(P_2 - P_1) = (0.001037 \text{ m}^3\text{/kg})[(3000 - 75) \text{ kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa}\cdot\text{m}^3}\right)$$

$$= 3.03 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{pump,in} = (384.44 + 3.03) \text{ kJ/kg} = 387.47 \text{ kJ/kg}$$

State 3: $\left.\begin{array}{l} P_3 = 3 \text{ MPa} \\ T_3 = 350°\text{C} \end{array}\right\}$ $\begin{array}{l} h_3 = 3116.1 \text{ kJ/kg} \\ s_3 = 6.7450 \text{ kJ/kg}\cdot\text{K} \end{array}$

Rankine Cycle: Energy Analysis

State 4: $P_4 = 75 \text{ kPa}$ (sat. mixture)

$$s_4 = s_3$$

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7450 - 1.2132}{6.2426} = 0.8861$$

$$h_4 = h_f + x_4 h_{fg} = 384.44 + 0.8861(2278.0) = 2403.0 \text{ kJ/kg}$$

Thus,

$$q_{in} = h_3 - h_2 = (3116.1 - 387.47) \text{ kJ/kg} = 2728.6 \text{ kJ/kg}$$

$$q_{out} = h_4 - h_1 = (2403.0 - 384.44) \text{ kJ/kg} = 2018.6 \text{ kJ/kg}$$

and

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{2018.6 \text{ kJ/kg}}{2728.6 \text{ kJ/kg}} = \mathbf{0.260 \text{ or } 26.0\%}$$

Rankine Cycle: Energy Analysis

The thermal efficiency could also be determined from

$$w_{turb,out} = h_3 - h_4 = (3116.1 - 2403.0) \text{ kJ/kg} = 713.1 \text{ kJ/kg}$$

$$w_{net} = w_{turb,out} - w_{pump,in} = (713.1 - 3.03) \text{ kJ/kg} = 710.1 \text{ kJ/kg}$$

or

$$w_{net} = q_{in} - q_{out} = (2728.6 - 2018.6) \text{ kJ/kg} = 710.0 \text{ kJ/kg}$$

and

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{710.0 \text{ kJ/kg}}{2728.6 \text{ kJ/kg}} = \mathbf{0.260 \text{ or } 26.0\%}$$