Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c \tag{1}$$

$$\int \frac{1}{x} dx = \ln x + c \tag{2}$$

$$\int u dv = uv - \int v du \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + c \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} + c \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1} + c, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} + c \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c \tag{9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| + c \tag{10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} + c \tag{11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2| + c \tag{12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} + C \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x| + C \tag{15}$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} + C \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a} dx = \frac{2}{3} (x-a)^{3/2} + C \tag{17}$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a} + C \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} + C \tag{19}$$

$$\int x\sqrt{x-a} dx = \frac{2}{3} a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} + C \tag{20}$$

$$\int \sqrt{ax+b} dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b} + C \tag{21}$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a} (ax+b)^{5/2} + C \tag{22}$$

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3} (x \pm 2a) \sqrt{x \pm a} + C \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} + C \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln \left[\sqrt{x} + \sqrt{x+a} \right] + C \tag{25}$$

$$\int x\sqrt{ax+b} dx = \frac{2}{15a^2} (-2b^2 + abx + 3a^2x^2) \sqrt{ax+b} + C \tag{26}$$

$$\int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}} \left[(2ax+b)\sqrt{ax(ax+b)} - b^2 \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \right] + C \tag{27}$$

$$\int \sqrt{x^3(ax+b)} dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| + C \tag{28}$$

$$\int \sqrt{x^2 \pm a^2}dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \tag{29}$$

$$\int \sqrt{a^2 - x^2}dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1}\frac{x}{\sqrt{a^2 - x^2}} + C \tag{30}$$

$$\int x\sqrt{x^2 \pm a^2}dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2} + C \tag{31}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \tag{32}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a} + C \tag{33}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}}dx = \sqrt{x^2 \pm a^2} + C \tag{34}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}dx = -\sqrt{a^2 - x^2} + C \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \tag{36}$$

$$\int \sqrt{ax^2 + bx + c}dx = \frac{b + 2ax}{4a}\sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx^+c)}\right| + C \tag{37}$$

$$\int x\sqrt{ax^2 + bx + c} = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2 + bx + c}\right. - \left(3b^2 + 2abx + 8a(c + ax^2)\right) \left. + 3(b^3 - 4abc)\ln\left|b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + x}\right|\right) \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2 + bx + c}}dx = \frac{1}{\sqrt{a}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right| + C \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2 + bx + c}}dx = \frac{1}{a}\sqrt{ax^2 + bx + c} + \frac{b}{2a^{3/2}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right| + C \tag{40}$$

Integrals with Logarithms

$$\int \ln ax dx = x \ln ax - x + C \tag{41}$$

$$\int \frac{\ln ax}{x}dx = \frac{1}{2}\left(\ln ax\right)^2 + C \tag{42}$$

$$\int \ln(ax + b)dx = \left(x + \frac{b}{a}\right)\ln(ax + b) - x + C, a \neq 0 \tag{43}$$

$$\int \ln\left(a^2x^2 \pm b^2\right)dx = x\ln\left(a^2x^2 \pm b^2\right) + \frac{2b}{a}\tan^{-1}\frac{ax}{b} - 2x + C \tag{44}$$

$$\int \ln\left(a^2 - b^2x^2\right)dx = x\ln\left(ar - b^2x^2\right) + \frac{2a}{b}\tan^{-1}\frac{bx}{a} - 2x + C \tag{45}$$

$$\int \ln\left(ax^2 + bx + c\right)dx = \frac{1}{a}\sqrt{4ac - b^2}\tan^{-1}\frac{2ax + b}{\sqrt{4ac - b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2 + bx + c\right) + C \tag{46}$$

$$\int x\ln(ax + b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax + b) + C \tag{47}$$

$$\int x\ln\left(a^2 - b^2x^2\right)dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2 - b^2x^2\right) + C \tag{48}$$

Integrals with Exponentials

$$\int e^{ax}dx = \frac{1}{a}e^{ax} + C \tag{49}$$

$$\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\operatorname{erf}\left(i\sqrt{ax}\right) + C, \text{ where } \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dtet \tag{50}$$

$$\int xe^x dx = (x - 1)e^x + C \tag{51}$$

$$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} + C \tag{52}$$

$$\int x^2e^x dx = \left(x^2 - 2x + 2\right)e^x + C \tag{53}$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax} + C \tag{54}$$

$$\int x^3 e^x dx = \left(x^3 - 3x^2 + 6x - 6\right) e^x + C \tag{55}$$

$$\int x^n e^{ax} dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax], \quad \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} dt \tag{56}$$

$$\int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}\left(ix\sqrt{a}\right) \tag{57}$$

Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C \tag{58}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C \tag{59}$$

$$\int \sin^n ax dx = -\frac{1}{a} \cos ax \; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] + C \tag{60}$$

$$\int \sin^3 ax dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} + C \tag{61}$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + C \tag{62}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C \tag{63}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] + C \tag{64}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} + C \tag{65}$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + C, a \neq b \tag{66}$$

$$\int \sin^2 ax \cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} + C \tag{67}$$

$$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x + C \tag{68}$$

$$\int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} + C \tag{69}$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax + C \tag{70}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} + C \tag{71}$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} + C \tag{72}$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax + C \tag{73}$$

$$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax + C \tag{74}$$

$$\int \tan^n ax dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) + C \tag{75}$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax + C \tag{76}$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) + C \tag{77}$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + C \tag{78}$$

$$\int \sec^3 x \, \mathrm{dx} = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C \tag{79}$$

$$\int \sec x \tan x dx = \sec x + C \tag{80}$$

$$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x + C \tag{81}$$

$$\int \sec^n x \tan x dx = \frac{1}{n} \sec^n x + C, n \neq 0 \tag{82}$$

$$\int \csc x dx = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C \tag{83}$$

$$\int \csc^2 ax dx = -\frac{1}{a}\cot ax + C \quad (84)$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x| + C \quad (85)$$

$$\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x + C, n \neq 0 \quad (86)$$

$$\int \sec x \csc x dx = \ln|\tan x| + C \quad (87)$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x dx = \cos x + x \sin x + C \quad (88)$$

$$\int x \cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C \quad (89)$$

$$\int x^2 \cos x dx = 2x\cos x + \left(x^2 - 2\right)\sin x + C \quad (90)$$

$$\int x^2 \cos ax dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax + C \quad (91)$$

$$\int x^n cos x dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix) + (-1)^n\Gamma(n+1, ix)\right] + C \quad (92)$$

$$\int x^n cos ax dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right] + C \quad (93)$$

$$\int x \sin x dx = -x\cos x + \sin x + C \quad (94)$$

$$\int x \sin ax dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} + C \quad (95)$$

$$\int x^2 \sin x dx = \left(2 - x^2\right)\cos x + 2x\sin x + C \quad (96)$$

$$\int x^2 \sin ax dx = \frac{2 - a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} + C \quad (97)$$

$$\int x^n \sin x dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1, -ix) - (-1)^n\Gamma(n+1, -ix)\right] + C \quad (98)$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C \quad (99)$$

$$\int e^{bx}\sin ax dx = \frac{1}{a^2+b^2}e^{bx}(b\sin ax - a\cos ax) + C \quad (100)$$

$$\int e^x \cos x dx = \frac{1}{2}e^x(\sin x + \cos x) + C \quad (101)$$

$$\int e^{bx}\cos ax dx = \frac{1}{a^2+b^2}e^{bx}(a\sin ax + b\cos ax) + C \quad (102)$$

$$\int xe^x \sin x dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) + C \quad (103)$$

$$\int xe^x \cos x dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) + C \quad (104)$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a}\sinh ax + C \quad (105)$$

$$\int e^{ax}\cosh bx dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] + C & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} + C & a = b \end{cases} \quad (106)$$

$$\int \sinh ax dx = \frac{1}{a}\cosh ax + C \quad (107)$$

$$\int e^{ax}\sinh bx dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}[-b\cosh bx + a\sinh bx] + C & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} + C & a = b \end{cases} \quad (108)$$

$$\int e^{ax}\tanh bx dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)}{}_2F_1\left[1+\frac{a}{2b}, 1, 2+\frac{a}{2b}, -e^{2bx}\right] - \frac{1}{a}e^{ax}{}_2F_1\left[\frac{a}{2b}, 1, 1E, -e^{2bx}\right] + C & a \neq b \\ \frac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} + C & a = b \end{cases} \quad (109)$$

$$\int \tanh bx dx = \frac{1}{a}\ln\cosh ax + C \quad (110)$$

$$\int \cos ax \cosh bx dx = \frac{1}{a^2+b^2}\left[a\sin ax\cosh bx + b\cos ax\sinh bx\right] + C \quad (111)$$

$$\int \cos ax \sinh bx dx = \frac{1}{a^2+b^2}\left[b \cos ax \cosh bx + a \sin ax \sinh bx\right] + C \tag{112}$$

$$\int \sin ax \cosh bx dx = \frac{1}{a^2+b^2}\left[-a \cos ax \cosh bx + b \sin ax \sinh bx\right] + C \tag{113}$$

$$\int \sin ax \sinh bx dx = \frac{1}{a^2+b^2}\left[b \cosh bx \sin ax - a \cos ax \sinh bx\right] + C \tag{114}$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a}\left[-2ax + \sinh 2ax\right] + C \tag{115}$$

$$\int \sinh ax \cosh bx dx = \frac{1}{b^2-a^2}\left[b \cosh bx \sinh ax - a \cosh ax \sinh bx\right] + C \tag{116}$$