1. (a)

$$n_i(T=300K) = 1.66\cdot10^{15}(300)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot10^{-23}J/K)(300K)}\right]$$

$$= 2.5\cdot10^{13}\ cm^{-3}$$

$$n_i(T=600K) = 1.66\cdot10^{15}(600)^{3/2}\cdot \exp\left[\frac{-(0.66eV)}{2(1.38\cdot10^{-23}J/K)(600K)}\right]$$

$$= 4.15\cdot10^{16}\ cm^{-3}$$

Comparing these results with those in Example:

$$\frac{n_i(Ge\ @\ 300K)}{n_i(Si\ @\ 300K)} \approx 2315. \qquad \frac{n_i(Ge\ @\ 600K)}{n_i(Si\ @\ 600K)} \approx 27.$$

At higher temperature, the exponential terms approaches one, which implies that $n_i \sim T^{3/2}$, independent of bandgap energy, Eg.

(b) For any doped material, $n\cdot p = n_i^2$. Assuming at T = 300 K,

$$p = 5\cdot10^{16}cm^{-3}$$

$$n = [n_i(T=300K)]^2/p = \frac{(2.5\cdot10^{13}cm^{-3})^2}{5\cdot10^{16}cm^{-3}} = 1.25\cdot10^{10}cm^{-3}$$

2. (a) Mobility of electrons in Si = 1350 $cm^2/V\text{-}s$
Mobility of holes in Si = 480 $cm^2/V\text{-}s$

$\Rightarrow$ velocity of electrons $= \mu_n E = \left(1350 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 1.35 \cdot 10^4$ m/s

velocity of holes $= \mu_p E = \left(480 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$= 4.8 \cdot 10^3$ m/s

(b) Given $E = 0.1$ V/μm hole current negligible
$\mu_n = 1350$ $cm^2/V\text{-}s$ $\mu_p = 480$ $cm^2/V\text{-}s$

$$J_{tot} = 1\ mA/\mu m^2 = q[\mu_n n E + \mu_p p E] \approx q \mu_n n E$$

$$\therefore n = \frac{J_{tot}}{q \mu_n E} = \frac{1 mA/\mu m^2}{(1.6 \cdot 10^{-19} C)(1350\ cm^2/V\text{-}s)(0.1\ V/\mu m)}$$

$$= 4.6 \cdot 10^{17}\ cm^{-3}$$

3. Given $L = 0.1\,um$ $A = (0.05\,um)^2$ $V = 1V$

$\mu_n = 1350\ cm^2/V\text{-}s$ $\mu_p = 480\ cm^2/V\text{-}s$

$n = 10^{17}\ cm^{-3}$ (assuming n-type dopant)

(a) $$n_i(T=300K) = 5.2\cdot 10^{15}(300)^{3/2} \exp\left[\frac{-1.12\,eV}{2(1.38\cdot 10^{-23}\,J/K)(300K)}\right]$$

$$= 1.08\cdot 10^{10}\ cm^{-3}$$

$p = n_i^2/n = 1.17\cdot 10^3\ cm^{-3}$ $E = V/L = 10\ V/um$

$$\therefore I_{tot} = A\cdot J_{tot} = A\cdot q[\mu_n n + \mu_p p]E$$

$$= A\cdot q\,[\mu_n n + \mu_p (n_i^2/n)]E$$

$$= (0.05\,um)^2(1.6\cdot 10^{-19}C)\left[1350\frac{cm^2}{V\text{-}s}(10^{17}cm^{-3}) + 480\frac{cm^2}{V\text{-}s}(1.17\cdot 10^3\ cm^{-3})\right]\cdot(10\ V/um)$$

$\Rightarrow I_{tot} \approx 0.054\ mA$

(b) @ 400K : $n_i = 3.7 \cdot 10^{12}\ cm^{-3}$

$p = n_i^2/n = 1.4 \cdot 10^{8}\ cm^{-3}$

$E = 10\ V/um$

$\therefore I_{tot} = A \cdot q[\mu_n n + \mu_p(n_i^2/n)]E$

$= (0.05\ um)^2 (1.6 \cdot 10^{-19} C)\left[1350\ \frac{cm^2}{V\text{-}s}(10^{17} cm^{-3}) + 480\ \frac{cm^2}{V\text{-}s}(1.4 \cdot 10^{8} cm^{-3})\right]$

$\cdot (10\ V/um)$

$\Rightarrow I_{tot} \approx 0.054\ mA.$

4. Given $L = 0.1\ \mu m$ $A = (0.05\ \mu m)^2$ $V = 1V$

$\mu_n = 3900\ cm^2/V\text{-}s$ $\mu_p = 1900\ cm^2/V\text{-}s$

$n = 10^{17} cm^{-3}$ (assuming n-type dopant)

(a) From previous problem,

@ 300K: $n_i = 2.5 \cdot 10^{13}\ cm^{-3}$ $p = n_i^2/n = 6.3 \cdot 10^9 cm^{-3}$

$E = 10\ V/\mu m$

$$I_{tot} = A \cdot J_{tot} = A\, q \left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05\mu m)^2 (1.6 \cdot 10^{-19} C) \left[3900 \frac{cm^2}{V\text{-}s} (10^{17} cm^{-3}) + 1900 \frac{cm^2}{V\text{-}s} (6.3 \cdot 10^9 cm^{-3})\right] \cdot (10\ V/\mu m)$$

$\Rightarrow I_{tot} \approx 62.4\ mA$

(b) @ 400K: $n_i = 2.9 \cdot 10^{15} cm^{-3}$ $p = 8.5 \cdot 10^{13} cm^{-3}$

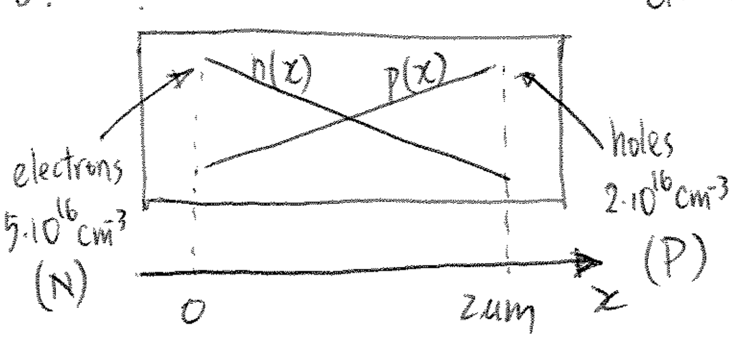
$E = 10\ V/\mu m$

$$I_{tot} = A\, q \left[\mu_n n + \mu_p (n_i^2/n)\right] E$$

$$= (0.05\mu m)^2 (1.6 \cdot 10^{-19} C) \left[3900 \frac{cm^2}{V\text{-}s} (10^{17} cm^{-3}) + 1900 \frac{cm^2}{V\text{-}s} (8.5 \cdot 10^{13} cm^{-3})\right] \cdot (10\ V/\mu m)$$

$\Rightarrow I_{tot} \approx 62.4\ mA$

5.



Given $D_n = 34\ cm^2/s$
$D_p = 12\ cm^2/s$
$L = 2\ \mu m$
$A = (1\ \mu m)^2$

The injected carriers diffuse from one end to the other.

$$I_{tot} = A \cdot J_{tot} = A \cdot q\left[\frac{dn}{dx}D_n - \frac{dp}{dx}D_p\right]$$

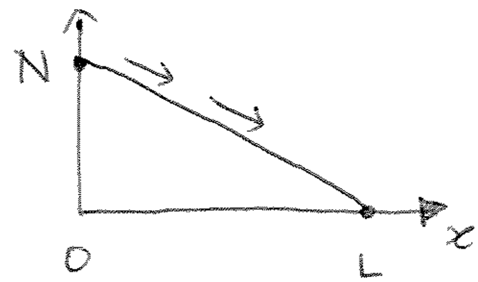
$$= A \cdot q\left[D_n\left(-\frac{N}{L}\right) - D_p\left(\frac{P}{L}\right)\right]$$

$$= (1\ \mu m)^2 (1.6 \cdot 10^{-19} C)\left[34\frac{cm^2}{s}\left(-\frac{5 \cdot 10^{16} cm^{-3}}{2\ \mu m}\right) - 12\frac{cm^2}{s}\left(\frac{2 \cdot 10^{16} cm^{-3}}{2\ \mu m}\right)\right]$$

$$= -15.5\ \mu A.$$

6. Given Area = a

find total e ectrons stored.



$n(x) = -\frac{N}{L}x + N$

∴ total electrons stored

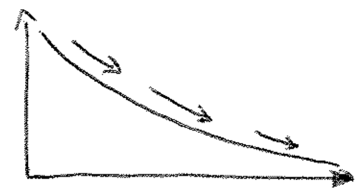
$$= \int a \cdot n(x)\, dx = \int_0^L a\left(-\frac{N}{L}x + N\right)dx$$

$$= aN\left(-\frac{x^2}{2L} + x\right)\Big|_0^L = \frac{aNL}{2}$$

7. Given Area = a

find total electrons stored.

$$n(x) = N \cdot \exp\left(\frac{-x}{L_d}\right)$$



∴ total electrons stored

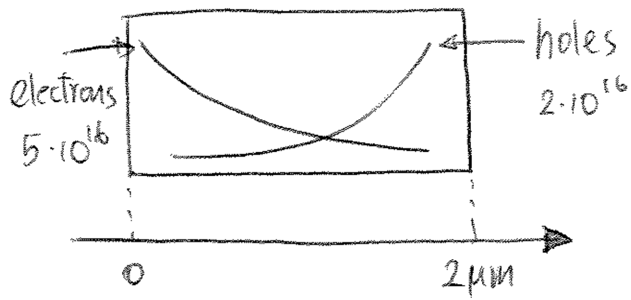
$$= \int a\, n(x)\, dx = \int_0^\infty a \cdot N \cdot \exp\left(\frac{-x}{L_d}\right) dx$$

$$= aN\left(-L_d \cdot \exp \frac{-x}{L_d}\right)\Bigg|_0^\infty = aNL_d.$$

For the linear profile, the result depends on the length, L.

For the exponential profile, the result is constant (since L_d is constant.)

8.



$$n(x) = N \exp(-x/L_d)$$

$$p(x) = P \exp\left(\frac{x-2}{L_d'}\right)$$

$N = 5\cdot 10^{16}\, cm^{-3}$ $\quad P = 2\cdot 10^{16}\, cm^{-3}$

total number of electrons $= \int a\cdot n \, dx$

$$= \int_0^2 a\cdot n(x)\, dx = aN \left(-L_d \cdot \exp(-x/L_d)\right|_0^2$$

$$= aNL_d \left[1 - \exp(-2/L_d)\right]$$

total number of holes $= \int a\cdot p\, dx$

$$= \int_0^2 a\cdot p(x)\, dx = aP\left(L_d' \cdot \exp\left(\frac{x-2}{L_d'}\right)\right|_0^2$$

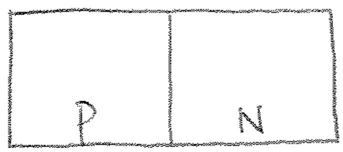
$$= aPL_d' \left[1 - \exp\left(-\frac{2}{L_d'}\right)\right]$$

9. Drift is analogous to water flow in a river.

Water flows from top of mountain to bottom because of gravitational field; electron flows from one terminal to the other because of electric field.

DRIFT		WATER FLOW
electrons	⟷	water
electric field	⟷	gravitational field.
drift / current	⟷	water flow

10. (a)



Assume Si.

$N_A = 4\cdot10^{16}\,cm^{-3}$ $\quad N_D = 5\cdot10^{17}\,cm^{-3}$

$$p_p \approx N_A = 4\cdot10^{16}\,cm^{-3}$$
$$n_p = \frac{n_i^2}{p_p} = \frac{(1.08\cdot10^{10}\,cm^{-3})^2}{4\cdot10^{16}\,cm^{-3}} \approx 2.9\cdot10^{3}\,cm^{-3}$$

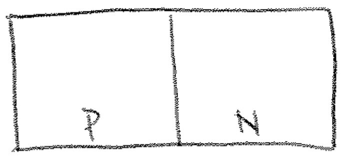
$$n_n \approx N_D = 5\cdot10^{17}\,cm^{-3}$$
$$p_n = \frac{n_i^2}{n_n} = \frac{(1.08\cdot10^{10}\,cm^{-3})^2}{5\cdot10^{17}\,cm^{-3}} \approx 2.3\cdot10^{2}\,cm^{-3}$$

(b) $$V_0 = \frac{kT}{q}\ln\left(\frac{N_A\cdot N_D}{n_i^2}\right)$$

@ 250K: $V_0 = 0.905\ V$
@ 300K: $V_0 = 0.848\ V$
@ 350K: $V_0 = 0.789\ V$

Towards higher temperatures, $V_0 \sim T\ln\left(\frac{1}{T^3}\right)$.
That is, overall, V_0 drops with higher T.

11. Given $N_D = 3 \cdot 10^{16}\ cm^{-3}$ $n_i = 1.08 \cdot 10^{10}\ cm^{-3}$



find V_0.

$$V_0 = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = \frac{kT}{q} \ln\left(\frac{N_D}{n_i}\right)$$

$$= \frac{(1.38 \cdot 10^{-23}\ J/K)(300K)}{1.6 \cdot 10^{-19} C} \ln\left(\frac{3 \cdot 10^{16}\ cm^{-3}}{1.08 \cdot 10^{10}\ cm^{-3}}\right)$$

$$= 0.384\ V$$

12. Given $N_D = 3\cdot 10^{16}\ cm^{-3}$ $N_A = 2\cdot 10^{15}\ cm^{-3}$

$V_R = 1.6\ V$ $\epsilon_{si} = 11.7\times 8.85\cdot 10^{-14}\ \frac{F}{cm^2}$

(a) $n_i = 1.08\cdot 10^{10}\ cm^{-3}$

$$V_0 = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \approx (26mV)\ln\left[\frac{3\cdot 10^{16}\times 2\cdot 10^{15}}{(1.08\cdot 10^{10})^2}\right]$$

$$= 0.698\ V$$

$$C_{j0} = \sqrt{\frac{\epsilon_{si}\cdot q}{2}\cdot\frac{N_A N_D}{N_A+N_D}\cdot\frac{1}{V_0}}$$

$$= \left[\frac{11.7\times 8.85\cdot 10^{-14}\times q}{2}\cdot\frac{3\times 10^{16}\times 2\times 10^{15}}{3\times 10^{16}+2\times 10^{15}}\cdot\frac{1}{V_0}\right]^{\frac{1}{2}}$$

$$= 0.149\ fF/\mu m^2$$

$$\therefore C_j(V_R) = [1 + 1.6/V_0]^{-\frac{1}{2}}\times C_{j0} = 0.082\ fF/\mu m^2$$

(b) Given $C_{j,new} = 2\cdot C_{j,old}$

$$\Rightarrow \sqrt{\frac{\frac{q\epsilon_{si}}{2}\cdot\frac{N_A N_D}{N_A+N_D}\cdot\frac{1}{V_0}}{1+\frac{V_R}{V_0}}} = \sqrt{\frac{\frac{q\epsilon_{si}}{2}\cdot\frac{N_A N_D'}{N_A+N_D'}\cdot\frac{1}{V_0'}}{1+\frac{V_R}{V_0'}}}\times 2$$

Squaring both sides & simplifying gives:

$$\frac{\left(\frac{N_D}{N_A+N_D}\right)}{V_0+V_R} = 4\cdot\frac{\left(\frac{N_D'}{N_A+N_D'}\right)}{V_0'+V_R}$$, where N_D' = old value.

Here, there is only one variable, N_D (new value). The solution can be found iteratively by solving this equation. But we can make an assumption that $V_0+V_R \approx V_0'+V_R$ since $V_R = 1.6\ V$, the dominant term. Then we verify V_0 & V_0' afterwards.

$$\Rightarrow \frac{N_D}{N_A+N_D} = 4\,\frac{N_D'}{N_A+N_D'}$$

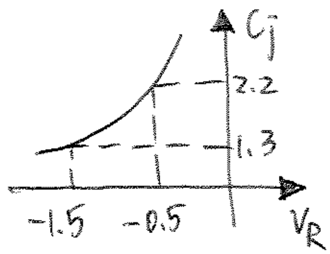
$$\Rightarrow N_D = \frac{4N_D'N_A}{N_A-3N_D'} = \frac{4(2\cdot10^{15})(3\cdot10^{16})}{(3\cdot10^{16})-3\cdot(2\cdot10^{15})} \approx 1.00\cdot10^{16}\,cm^{-3}$$

$$\Rightarrow \frac{N_D}{N_D'} = \frac{1\cdot10^{16}}{2\cdot10^{15}} \approx 5$$

Verify: $V_{0,old} = 0.698\,V \Rightarrow V_0+V_R \approx 2.3\,V$
$V_{0,new} = 0.740\,V \Rightarrow V_0+V_R \approx 2.3\,V$ (✓)

∴ Increase N_D by 5 times.

13.



$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{——} \ ①$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{——} \ ②$$

$$① \div ② : \quad \frac{1+1.5/V_0}{1+0.5/V_0} = \left(\frac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365 \ V$$

Substitute V_0 into ①:

$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43 \ fF/um^2$$

$$\Rightarrow \frac{N_A N_D}{N_A + N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$

$$= \left(8.43 \ \frac{fF}{um^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17} \ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A + N_D} \triangleq y$

$$N_A = 2 \cdot 10^{18}\,cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$$

$$= \frac{(3.13 \cdot 10^{17}\,cm^{-3})(2 \cdot 10^{18}\,cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\,cm^{-3}}$$

$$\approx 3.71 \cdot 10^{17}\,cm^{-3}$$

14 (a) In forward bias, $I_D = 1mA$, $V_D = 750mV$

$$\therefore I_S \approx I_D e^{-\frac{V_D}{V_T}} = (1mA)\exp[-750mV/26mV]$$

$$= 2.97 \cdot 10^{-16} A$$

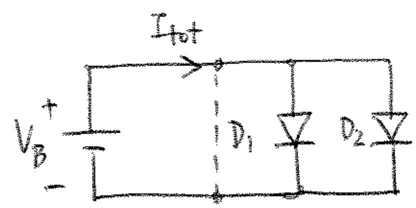
(b) Since $I_S \propto$ Area, doubling area implies doubling I_S. From (a),

$$I_D = 1\ mA = 2 \times I_S\, e^{V_D/V_T}$$

$$\therefore V_D = V_T \ln\left(\frac{I_D}{2I_S}\right) = (26mV)\ln\left(\frac{1mA}{2 \cdot 2.97 \times 10^{-16} A}\right)$$

$$= 0.732\ V$$

15 (a)



$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}(e^{V_B/V_T} - 1) + I_{S_2}(e^{V_B/V_T} - 1)$$

$$= (I_{S_1} + I_{S_2})(e^{V_B/V_T} - 1)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

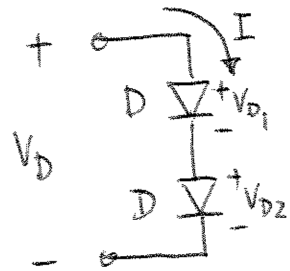
Also, $I_{tot} = I_{D_1} + I_{D_2} \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$

16. (a)



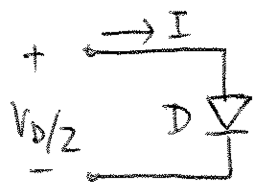
Suppose $I = I_s\left(e^{\frac{V_{D1}}{V_T}} - 1\right)$

$I_{D_2} = I_s\left(e^{\frac{V_{D2}}{V_T}} - 1\right)$

By KCL, $I_{D_1} = I_{D_2} = I$

$$\Rightarrow \left(e^{V_{D1}/V_T} - 1\right) = \left(e^{V_{D2}/V_T} - 1\right) \Rightarrow V_{D_1} = V_{D2} = \frac{V_D}{2}$$

$$\therefore \quad I = I_s\left(e^{(V_D/2)/V_T} - 1\right)$$



Therefore, a series combination can be viewed as a single two-terminal device with exponential characteristics.

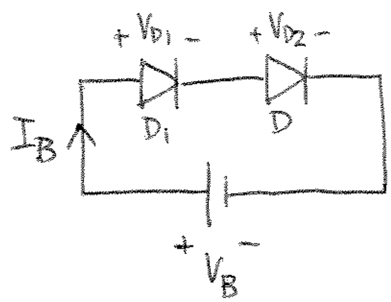
(b) Suppose V_i = initial V_D. Need 10x increase in I.
V_f = final V_D

$$\Rightarrow \quad 10 = \frac{I_s\left(e^{V_f/V_T} - 1\right)}{I_s\left(e^{V_i/V_T} - 1\right)} \approx e^{\frac{V_f - V_i}{V_T}}$$

$$\therefore \quad \Delta V = V_f - V_i = V_T \ln(10) = (26\,mV)\ln(10)$$

$$\approx 60.\ mV.$$

17.



Find I_B, V_{D_1}, V_{D2} in terms of V_B, $I_{,}$, I_{S2}

By KVL, $V_B = V_{D_1} + V_{D2} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) + V_T \ln\left(\frac{I_B}{I_{S2}}\right)$

$$\Rightarrow \quad V_B = V_T \ln\left(\frac{I_B^2}{I_{S_1} I_{S2}}\right)$$

$$\therefore \; I_B = \sqrt{I_{S_1} I_{S2} \cdot \exp V_B/V_T} = \sqrt{I_{S_1} I_{S2}} \cdot \exp\left(\frac{V_B}{2V_T}\right).$$

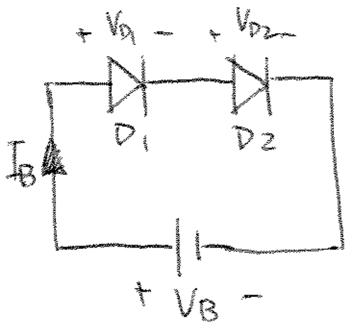
$$V_{D_1} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S_1}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S2}}{I_{S_1}}} + \frac{V_B}{2}$$

$$V_{D2} = V_T \ln\left(\frac{I_B}{I_{S2}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S2}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S1}}{I_{S2}}} + \frac{V_B}{2}$$

18.



$$V_B = V_T \ln \frac{I_B}{I_{S1}} + V_T \ln \frac{I_B}{I_{S2}} = V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right)$$

$$\Rightarrow I_B = \sqrt{I_{S1} I_{S2}} \cdot \exp \frac{V_B}{2V_T}$$

Increase I_B by 10 times:

$$I_{B,new} = 10 I_B$$

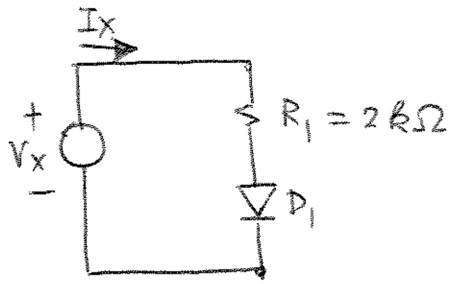
$$\Rightarrow V_{B,new} = V_T \ln\left(\frac{I_{B,new}^2}{I_{S1} I_{S2}}\right) = V_T \ln\left[\frac{(10 I_B)^2}{I_{S1} I_{S2}}\right]$$

$$= V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right) + V_T \ln 100$$

$$= V_B + V_T \ln 100 \approx V_B + 0.120\,V$$

$\therefore$ V_B increases by 0.120 V.

19.



$$I_{D_1} = I_S\left(e^{\frac{V_{D1}}{V_T}}-1\right)$$

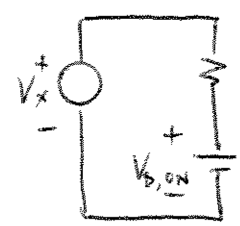
$$I_S = 2\cdot 10^{-15}A$$

By KVL, $V_X = I_X R_1 + V_{D1}$
$= I_X R_1 + V_T \ln(I_{D1}/I_S)$
$= I_X R_1 + V_T \ln(I_X/I_S)$

This can be solved directly with special programs or graphing calculators. But this can be solved iteratively, by hand.

$V_X = 0.5V$

We suppose that D_1 is on.
⇒ current flows through D_1.
Assume a $V_{D,ON}$:
⇒ $V_{D_1} = 0.4V$



⇒ $I_X = \frac{V_X - V_{D1}}{R_1} = \frac{(0.5-0.4)V}{2k\Omega} = 0.05\,mA$

$$V_{D_1} = V_T \ln\left(\frac{I_X}{I_S}\right) = (0.026V)\ln\left(\frac{0.05mA}{2\cdot 10^{-15}A}\right) \approx 0.62\,V$$

∴ Contradiction because V_{D_1} exceeds V_X !!
This means our assumption is incorrect
⇒ D_1 is OFF ⇒ $V_{D_1} = V_X = 0.5V$ $I_X = 0$

$\boxed{V_X = 0.8\ V}$ Suppose D_1 is on. (This is a reasonable assumption since most diodes turn on at around $V_D = 0.7\ V$.)

For startup, use $V_{D_1} = 0.7\ V$.

$$V_{D_1} = 0.7\ V \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = 0.05\ mA$$

$$\Rightarrow V_{D_1} = V_T \ln(I_X/I_{S_1}) \approx 0.622\ V$$

$$V_{D_1} = 0.622\ V \Rightarrow I_X = \frac{(0.8 - 0.622)V}{2\ k\Omega} = 0.089\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V)\ln\left(\frac{0.089\ mA}{2 \cdot 10^{-15} A}\right) \approx 0.637\ V$$

$$V_{D_1} = 0.637\ V \Rightarrow I_X = \frac{(0.8 - 0.637)V}{2\ k\Omega} = 0.082\ mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.082\ mA}{2 \cdot 10^{-15} A}\right) \approx 0.635V$$

$$V_{D_1} = 0.635V \Rightarrow I_X = \frac{(0.8 - 0.635)V}{2\ k\Omega} = 0.083\ mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.083\ mA}{2 \cdot 10^{-15} A}\right) \approx 0.635V$$

∴ With an accuracy of three decimal points,

$V_{D_1} \approx 0.635\ V$ (Of course, more iterations give a more accurate result.)

$I_X \approx 0.082\ mA$.

$V_X = 1\ V$ Suppose, again, that D_1 is on. Use V_{D_1} from previous calculations as starting point.

$$V_{D_1} = 0.635\ V \Rightarrow I_X = \frac{(1-0.635)V}{2\ k\Omega} = 0.18\ mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.18mA}{2\cdot10^{-15}A}\right) \approx 0.656\ V$$

$$V_{D_1} = 0.656V \Rightarrow I_X = \frac{(1-0.656)V}{2\ k\Omega} = 0.17\ mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.17mA}{2\cdot10^{-15}A}\right) \approx 0.655\ V$$

$$V_{D_1} = 0.655\ V \Rightarrow I_X = \frac{(1-0.655)V}{2\ k\Omega} = 0.17\ mA$$

$$\Rightarrow V_{D_1} = 0.655\ V$$

$\therefore$ $V_{D_1} \approx 0.655\ V$
$I_X \approx 0.17\ mA$.

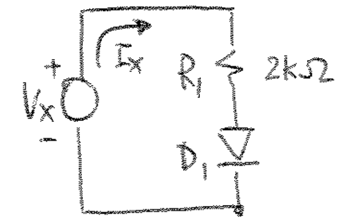
$V_X = 1.2\ V$ Using similar assumptions as those in previous calculations,

$V_{D_1} = 0.655\ V \Rightarrow I_X = 0.27\ mA \Rightarrow V_{D_1} \approx 0.667V$
$V_{D_1} = 0.667\ V \Rightarrow I_X = 0.27\ mA \Rightarrow V_{D_1} \approx 0.666V$
$V_{D_1} = 0.666V \Rightarrow I_X = 0.27\ mA \Rightarrow V_{D_1} \approx 0.666V$

$\therefore$ $I_X \approx 0.27\ mA$ $\quad V_{D_1} = 0.666\ V$

For more than 3x increase in I_x, V_{D_1} only increases by ~30mV, which is less than 10% of the turn-on voltage of the diode. In other words, once the diode conducts current, its voltage varies marginally (expected due to its exponential characteristic). This also implies that the diode, once on, can allow any amount of current to flow through (until $V_{D_1} \times I_{D_1}$ becomes so large that the diode simply "breaks down".)

20.



Since $I_{S_1} \propto$ Area, I_{D_1} becomes:

$$I_{D_1} = \underbrace{10 \times (2 \cdot 10^{-15} A)}_{I_{S_1}'} \left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

$\boxed{V_X = 0.8\ V}$ Suppose D_1 is on. Assume $V_{D_1} = 0.7V$

$$V_{D_1} = 0.7\ V \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = \frac{0.1\ V}{2\ k\Omega} = 0.05\ mA$$

$$\Rightarrow V_{D_1} = V_T \ln(I_X / I_{S_1}') = (0.026V) \ln\left(\frac{0.05\ mA}{20 \cdot 10^{-15} A}\right)$$

$$= 0.563\ V$$

$$V_{D_1} = 0.563V \Rightarrow I_X = \frac{(0.8 - 0.563)\ V}{2\ k\Omega} = 0.12\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.12\ mA}{20 \cdot 10^{-15} A}\right) \approx 0.585\ V$$

$$V_{D_1} = 0.585V \Rightarrow I_X = \frac{(0.8 - 0.585)V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.11\ mA}{20 \cdot 10^{-15} A}\right) \approx 0.583\ V$$

$$V_{D_1} = 0.583\ V \Rightarrow I_X = \frac{(0.8 - 0.583)V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = 0.583\ V$$

$\therefore$ $V_{D_1} \approx 0.583\ V$

$I_X \approx 0.11\ mA.$

$\boxed{V_X = 1.2\,V}$ Suppose D_1 is on. Use results from previous calculations as starting point.

$$V_{D_1} = 0.583\,V \Rightarrow I_X = \frac{(1.2-0.583)V}{2\,k\Omega} = 0.31\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.31mA}{20\times10^{-15}A}\right) \approx 0.610\,V$$

$$V_{D_1} = 0.610\,V \Rightarrow I_X = \frac{(1.2-0.610)V}{2\,k\Omega} = 0.30\,mA$$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.30\,mA}{20\cdot10^{-15}A}\right) \approx 0.609\,V$$

$$V_{D_1} = 0.609\,V \Rightarrow I_X = \frac{(1.2-0.609)V}{2\,k\Omega} = 0.30\,mA$$

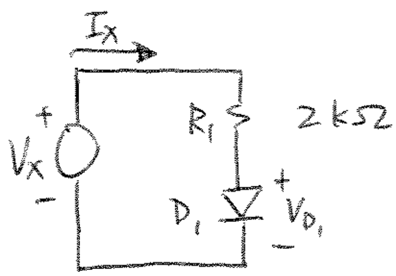
$$\Rightarrow V_{D_1} = 0.609\,V$$

$\therefore\ V_{D_1} \approx 0.609\,V$

$I_X \approx 0.30\,mA$.

By increasing the cross-section area of D_1, intuitively this means D_1 can conduct same amount of current with less V_{D_1}. The results have shown that in this problem, V_{D_1} is less and I_X is more.

21.



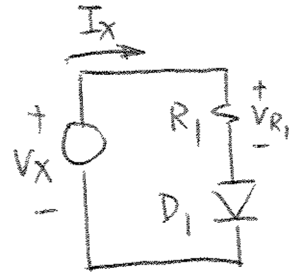
Given: @ $V_x = 2V$, $V_{D_1} = 850mV$

$$\Rightarrow I_x = \frac{V_x - V_{D_1}}{R_1} = 0.58\ mA$$

$$\therefore I_s = \frac{I_x}{(e^{V_D/V_T} - 1)} \approx I_x \exp[-V_{D}/V_T]$$

$$= (0.58\ mA) \exp[-0.85/0.026] \approx 3.64 \cdot 10^{-18} A$$

22.



Given $V_{R_1} = V_X/2$, find V_X.
$I_S = 2 \cdot 10^{-16} A$.

By KCL,

$$\frac{V_{R_1}}{R_1} = I_S\left(e^{V_{D1}/V_T} - 1\right)$$

Also, $V_{R_1} = V_{D_1} = V_X/2$ (KVL).

$$\therefore \frac{V_X/2}{R_1} = I_S \cdot \left(\exp\left[\frac{V_{D_1}/2}{V_T}\right] - 1\right)$$

This must be solved iteratively. From experience, suppose $V_X = 2V$.

$$V_X = 2V \Rightarrow I_X = \frac{V_X/2}{R_1} = \frac{1V}{2k\Omega} = 5mA$$

$$\Rightarrow V_X = 2 \cdot V_{D_1} = 2V_T \ln(I_X/I_S)$$

$$= 2(0.026V)\ln\left(\frac{5mA}{2\cdot 10^{-16}A}\right) \approx 1.48V$$

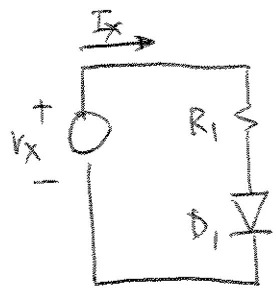
$$V_X = 1.48V \Rightarrow I_X = \frac{1.48/2\,V}{2k\Omega} = 0.37mA$$

$$\Rightarrow V_X = 2(0.026V)\ln\left(\frac{0.37mA}{2\cdot 10^{-16}A}\right) \approx 1.47V$$

$$V_X = 1.47V \Rightarrow I_X = \frac{(1.47)/2\,V}{2k\Omega} = 0.37mA$$

$$\Rightarrow V_X = 1.47V$$

23.



Given $V_X = 1V \Rightarrow I_X = 0.2mA$

$V_X = 2V \Rightarrow I_X = 0.5mA$

Find R_1 and I_S.

By KVL, $V_{D_1} = V_X - I_X R_1 = V_T \ln\left(\frac{I_X}{I_S}\right)$

$$\Rightarrow 1 - (0.2mA)R_1 = (0.026V)\ln\left(\frac{0.2mA}{I_S}\right) \quad \text{——— ①}$$

$$2 - (0.5mA)R_1 = (0.026V)\ln\left(\frac{0.5mA}{I_S}\right) \quad \text{——— ②}$$

② − ① : $1 - (0.3mA)R_1 = (0.026V)\ln\left(\frac{0.5}{0.2}\right)$

$$\Rightarrow R_1 = \frac{1-(0.026)\ V}{0.3m\ A} = 3.25\ k\Omega$$

Substitute R_1 into ①:

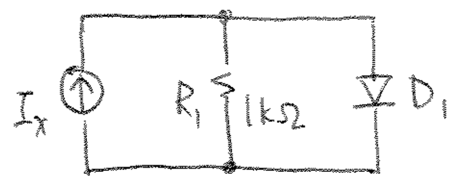
$$I_S = I_X \cdot \exp\left[-\frac{V_X - I_X R_1}{V_T}\right]$$

$$= (0.2mA)\exp\left[-\frac{1-(0.2m)(3.25k)}{0.026}\right] \approx 2.94 \cdot 10^{-10} A$$

$\therefore R_1 \approx 3.25\ k\Omega$

$I_S \approx 2.94 \cdot 10^{-10} A.$

24.



Given $I_S = 3 \cdot 10^{-16}$ A, find V_{D_1}.

By KCL, $I_X = \dfrac{V_{D_1}}{R_1} + I_{D_1} = \dfrac{V_T}{R_1} \ln\left(\dfrac{I_{D_1}}{I_S}\right) + I_{D_1}$

Since I_X, V_T, R_1 and I_S are known, this can be solved directly with special programs or graphing calculators. However, this can be also solved by iterations: Assume a V_{D_1}, calculate I_{D_1}, and re-iterate on V_{D_1}.

Assume $V_{D_1} = 0.7$ V as starting point.

$I_X = 1$ mA

$V_{D_1} = 0.7$ V $\Rightarrow I_{D_1} = I_X - V_{D_1}/R_1 = 1\text{mA} - \dfrac{0.7\text{V}}{1\text{k}\Omega} = 0.3\text{mA}$

$$\Rightarrow V_{D_1} = V_T \ln\left(I_X/I_S\right)$$
$$= (0.026\text{V}) \ln\left(\frac{0.3\text{mA}}{3 \cdot 10^{-16}\text{A}}\right) \approx 0.718\text{ V}$$

$V_{D_1} = 0.718$ V $\Rightarrow I_{D_1} = 1\text{ mA} - \dfrac{0.718\text{ V}}{1\text{ k}\Omega} = 0.28\text{ mA}$

$$\Rightarrow V_{D_1} = (0.026\text{V}) \ln\left(\frac{0.28\text{mA}}{3 \cdot 10^{-16}\text{A}}\right) \approx 0.717\text{ V}$$

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 1mA - \frac{0.717V}{1k\Omega} = 0.28mA$

$\Rightarrow V_{D_1} = 0.717\,V$

$\therefore V_{D_1} \approx 0.717\,V.$

$\boxed{I_X = 2mA}$ Assume $V_{D_1} = 0.717\,V$ from previous result.

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 2mA - \frac{0.717V}{1k\Omega} = 1.28\,mA$

$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{1.28mA}{3 \cdot 10^{-16}A}\right) \approx 0.756V$

$V_{D_1} = 0.756\,V \Rightarrow I_{D_1} = 2mA - \frac{0.756V}{1k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{1.24mA}{3 \cdot 10^{-16}A}\right) \approx 0.755V$

$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 2mA - \frac{0.755V}{1k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = 0.755V$

$\therefore V_{D_1} = 0.755V$

$\boxed{I_X = 4\ mA}$ Assume $V_{D_1} = 0.755\,V$ from previous result.

$$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 4mA - \frac{0.755V}{1k\Omega} = 3.25\ mA$$

$$\Rightarrow V_{D_1} = (0.026)V \ln\left(\frac{3.25mA}{3\cdot 10^{-16}A}\right) \approx 0.780\,V$$

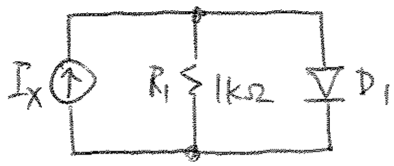
$$V_{D_1} = 0.780\,V \Rightarrow I_{D_1} = 4mA - \frac{0.780V}{1k\Omega} = 3.22\ mA$$

$$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\frac{3.22mA}{3\cdot 10^{-16}A}\right) \approx 0.780V$$

$\therefore V_{D_1} \approx 0.780\ V.$

Note: As I_X increases, I_{D_1} increases, while (V_{D1}/R_1) stays relatively the same. Because of the exponential characteristic, the diode, once on, will <u>absorb</u> as much current as necessary to satisfy KCL.

25.



Given $I_{D_1} = 0.5\,mA$ when $I_X = 1.3\,mA$, find I_S.

This means $V_{D_1} = (I_X - I_{D_1})R_1$

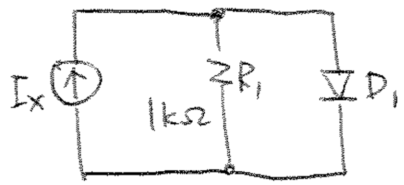
$= (0.8\,mA)\,1\,k\Omega = 0.8\,V$

$\Rightarrow I_S = I_{D_1} \cdot \exp\left[-V_{D_1}/V_T\right]$

$= (0.5\,mA)\,\exp\left[-0.8\,V/0.026\,V\right]$

$\approx 2.17 \cdot 10^{-17}\,A$

2b



Given $I_{R_1} = I_x/2$

$I_S = 3\cdot 10^{-16}$ A

find I_x.

$$V_{D_1} = \frac{I_x}{2}\cdot R_1 = V_T \ln\left(\frac{I_x/2}{I_S}\right)$$

This can be solved directly with special programs or graphing calculators. Alternatively, one can solve this iteratively by hand.

Assume $V_D = 0.8$ V.

$$V_D = 0.8\text{ V} \Rightarrow (I_x/2) = \frac{V_D}{R_1} = \frac{0.8\text{ V}}{1\text{ k}\Omega} = 0.8\text{ mA}$$

$$\Rightarrow V_D = V_T \ln\left(\frac{I_x/2}{I_S}\right) = (0.026\text{V})\ln\left(\frac{0.8\text{ mA}}{3\cdot 10^{-16}\text{A}}\right)$$

$$\approx 0.744\text{ V}$$

$$V_D = 0.744\text{ V} \Rightarrow I_x/2 = \frac{0.744\text{V}}{1\text{k}\Omega} = 0.744\text{ mA}$$

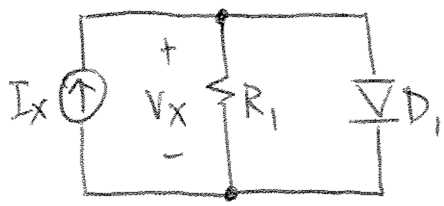
$$\Rightarrow V_D = (0.026\text{V})\ln\left(\frac{0.744\text{ mA}}{3\cdot 10^{-16}\text{A}}\right) \approx 0.742\text{ V}$$

$$V_D = 0.742\,V \Rightarrow I_x/2 = \frac{0.742\,V}{1\,k\Omega} = 0.742\ mA$$

$$\Rightarrow V_D = (0.026\,V)\ln\left(\frac{0.742\,mA}{3\cdot 10^{-16}A}\right) \approx 0.742\,V$$

$$\therefore I_x = 2(0.742\,mA) = 1.48\ mA$$

27.



Given $I_X = 1mA \rightarrow V_X = 1.2V$

$I_X = 2mA \rightarrow V_X = 1.8V$

find R_1 and I_S.

$$I_{D_1} = I_X - V_X/R_1 \quad \text{(KCL)}$$

By KVL, $V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X - V_X/R_1}{I_S}\right)$

$$\Rightarrow (1.2\,V) = (0.026\,V) \ln\left[\frac{(1mA) - (1.2V)/R_1}{I_S}\right] \text{ ——— ①}$$

$$(1.8\,V) = (0.026\,V) \ln\left[\frac{(2mA) - (1.8\,V)/R_1}{I_S}\right] \text{ ——— ②}$$

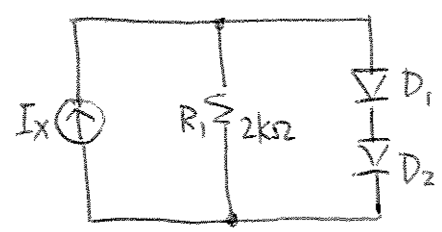
$$② - ①: \quad 0.6V = (0.026V) \ln\left(\frac{2mA - 1.8V/R_1}{1mA - 1.2V/R_1}\right)$$

$$\Rightarrow R_1 = \frac{1.2 \cdot \exp[0.6/0.026] - 1.8}{1\,mA \cdot \exp[0.6/0.026] - 2\,mA} \approx 1.2\,k\Omega$$

$$I_S = I_D \exp\left[-V_X/V_T\right] = \left(2mA - \frac{1.8V}{1.2k\Omega}\right) \exp\left[\frac{-1.8V}{0.026V}\right]$$

$$\approx 4.29 \cdot 10^{-34}\,A.$$

28.



Given $D_1 = D_2$ with $I_S = 5 \cdot 10^{-16} A$

Find V_{R_1} for $I_X = 2mA$.

Current through the diodes $= I_D$
$= I_X - \frac{V_{R_1}}{R_1}$ where V_{R_1} = voltage across R_1

$$\Rightarrow V_{R_1} = 2 \cdot V_T \ln\left(\frac{I_D}{I_S}\right) = 2\left[V_T \ln\left(\frac{I_X}{I_S} - \frac{V_{R_1}}{I_S R_1}\right)\right]$$

This can be solved directly with special programs or graphing calculators or by hand iteratively.

Assume a V_{R_1}, calculate I_D, and re-iterate on new $V_{R_1} = (2 \times V_{D_1})$. From experience, most diodes conduct at $V_D \approx 0.7V$. Assume $V_{R_1} = 1.4V$.

$$V_{R_1} = 1.4 V \Rightarrow I_D = I_X - \frac{V_{R_1}}{R_1} = 2mA - \frac{1.4V}{2k\Omega} = 1.3mA$$

$$\Rightarrow V_{R_1} = 2 V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 2(0.026V) \ln\left(\frac{1.3mA}{5 \times 10^{-16} A}\right) \approx 1.49 V$$

$V_{R_1} = 1.49V \Rightarrow I_D = 2mA - \frac{1.49}{2k\Omega} = 1.26\ mA$

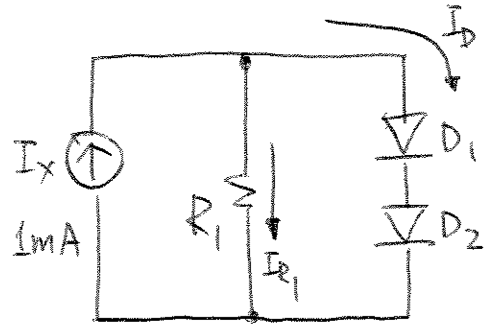
$\Rightarrow V_{R_1} = 2(0.026V)\ln\left(\frac{1.26\ mA}{5 \cdot 10^{-16}A}\right) \approx 1.48V$

$V_{R_1} = 1.48V \Rightarrow I_D = 2mA - \frac{1.48V}{2k\Omega} = 1.26\ mA$

$\Rightarrow V_{R_1} = 1.48V$

$\therefore$ voltage across $R_1 = 1.48\ V$

29.



Given $I_{R_1} = 0.5\,mA$,
$I_S = 5 \cdot 10^{-16} A$ for each diode.

Find R_1.

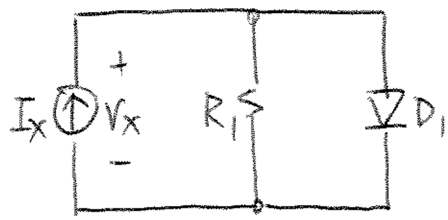
By KCL, $I_D = I_X - I_{R_1} = 0.5\,mA$

$$\Rightarrow V_{D_1} = V_{D_2} = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln\left(\frac{0.5mA}{5 \cdot 10^{-16}A}\right)$$

$$\approx 0.718\ V$$

$$\therefore R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{2V_{D_1}}{I_{R_1}} = \frac{2(0.718V)}{0.5mA} = 2.87\,k\Omega$$

30.

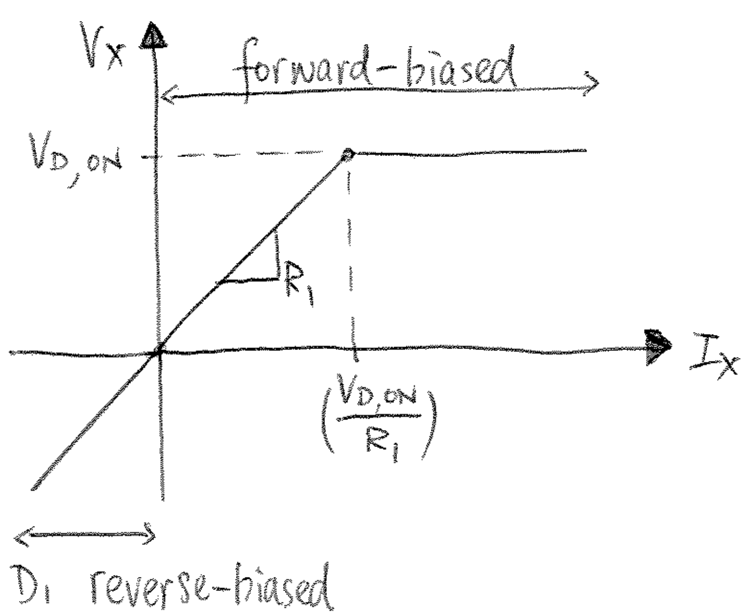


(a) Constant-voltage model:

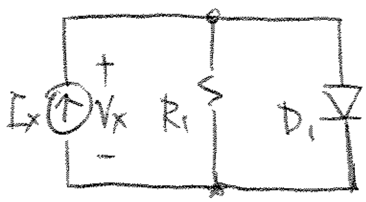
Consider, first, the extreme cases: when D_1 is off, we have the following:

This implies V_X is linearly proportional to I_X

When D_1 is on, V_X is fixed (by KVL) by D_1 ($= V_{D,ON}$). This implies that any additional current from I_X cannot flow through R_1, which means D_1 will absorb all the currents to satisfy KVL.



(b) exponential model:



Assume I_s negligible.

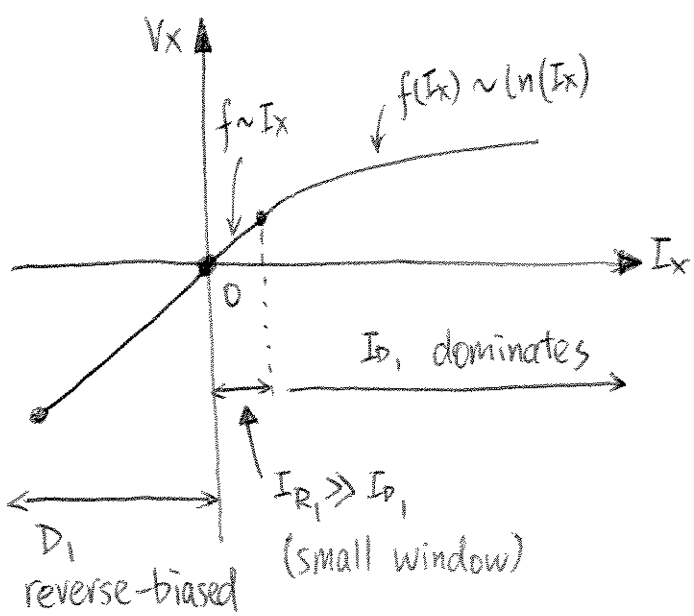
When D_1 is off, most of I_x flows through R_1. When D_1 is on, V_{D_1} (= V_x) follows this relationship:

$$V_{D_1} = V_x = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_x}{I_S} - \frac{V_x}{I_x R_1}\right)$$

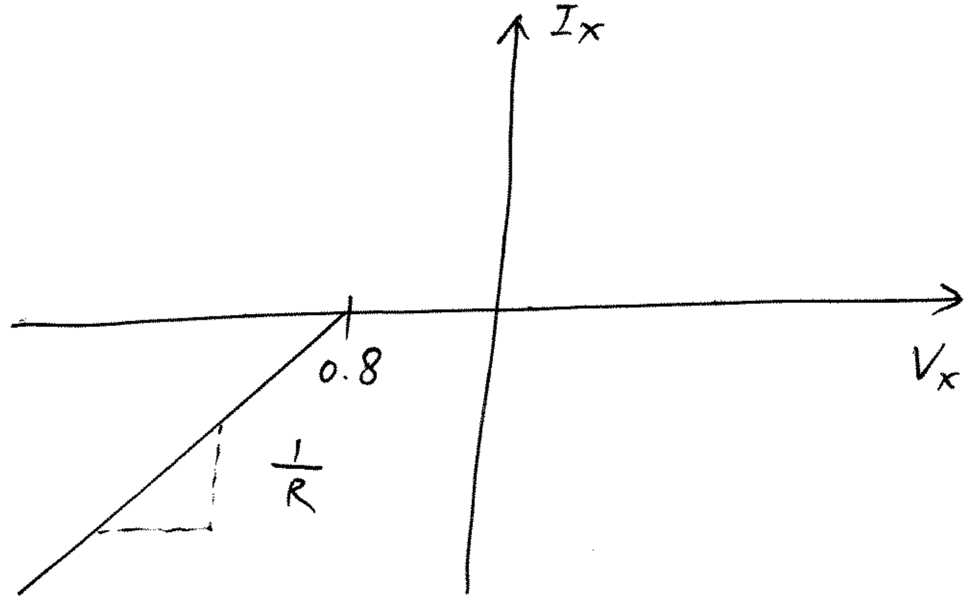
$$\Rightarrow I_x = I_S \exp(V_x/V_T) + V_x/R_1$$

$$\approx I_S \exp(V_x/V_T) \quad \text{when } D_1 \text{ is forward-biased } (V_x > V_T)$$

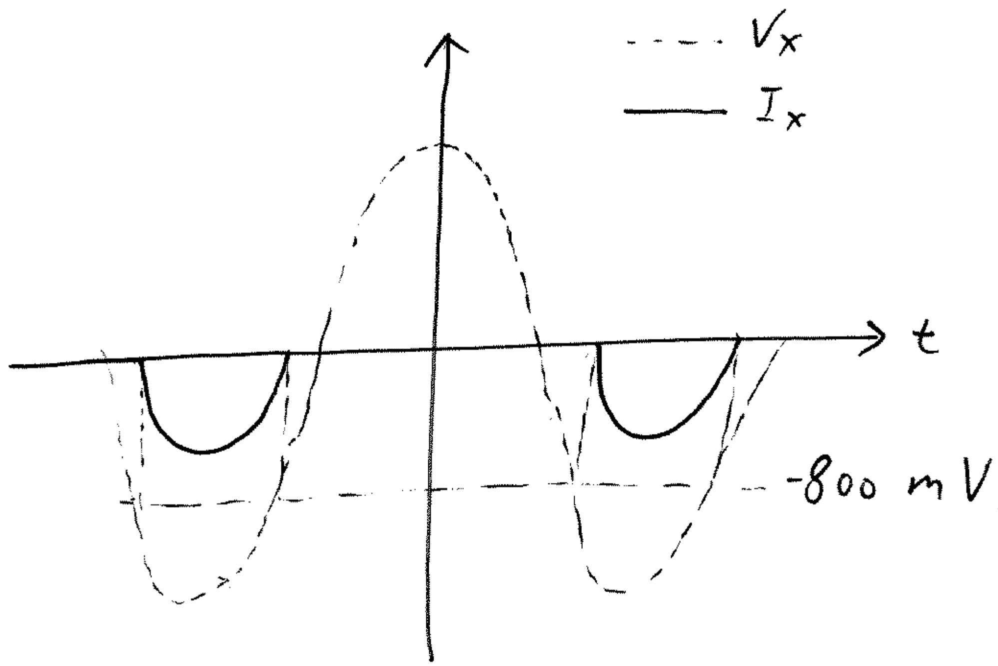
i.e. $V_x \approx V_T \ln(I_x/I_S)$



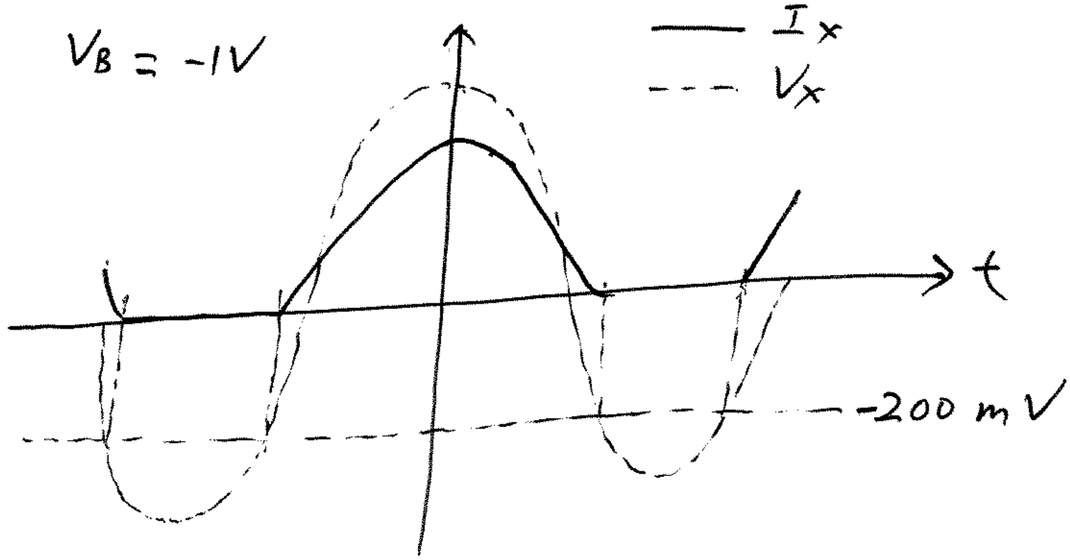
(1)

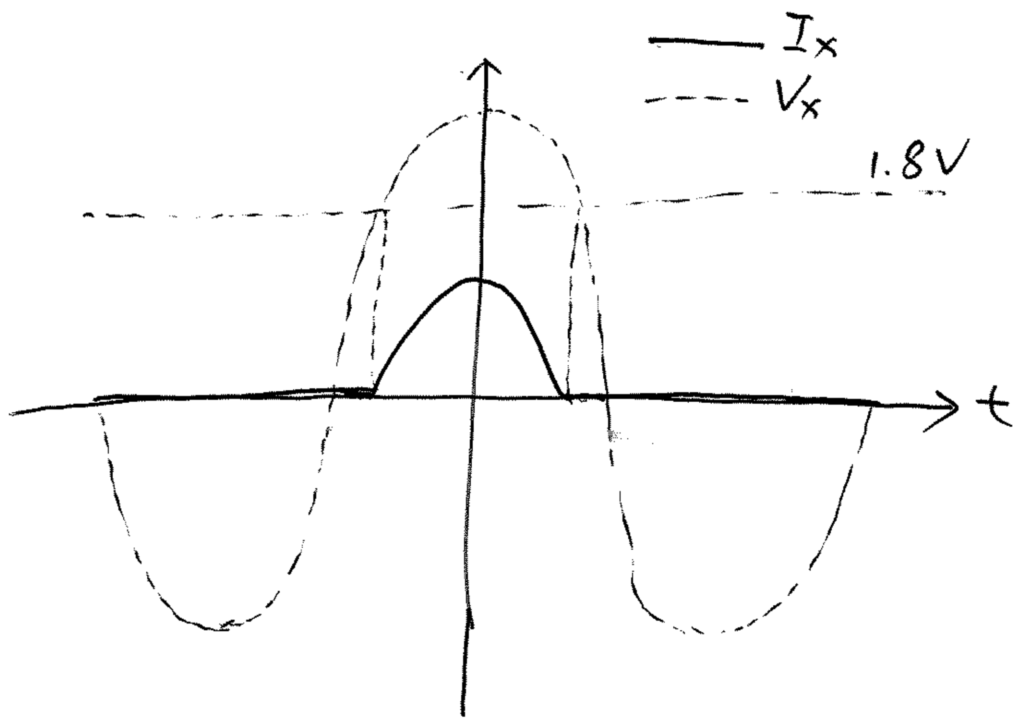


(2)

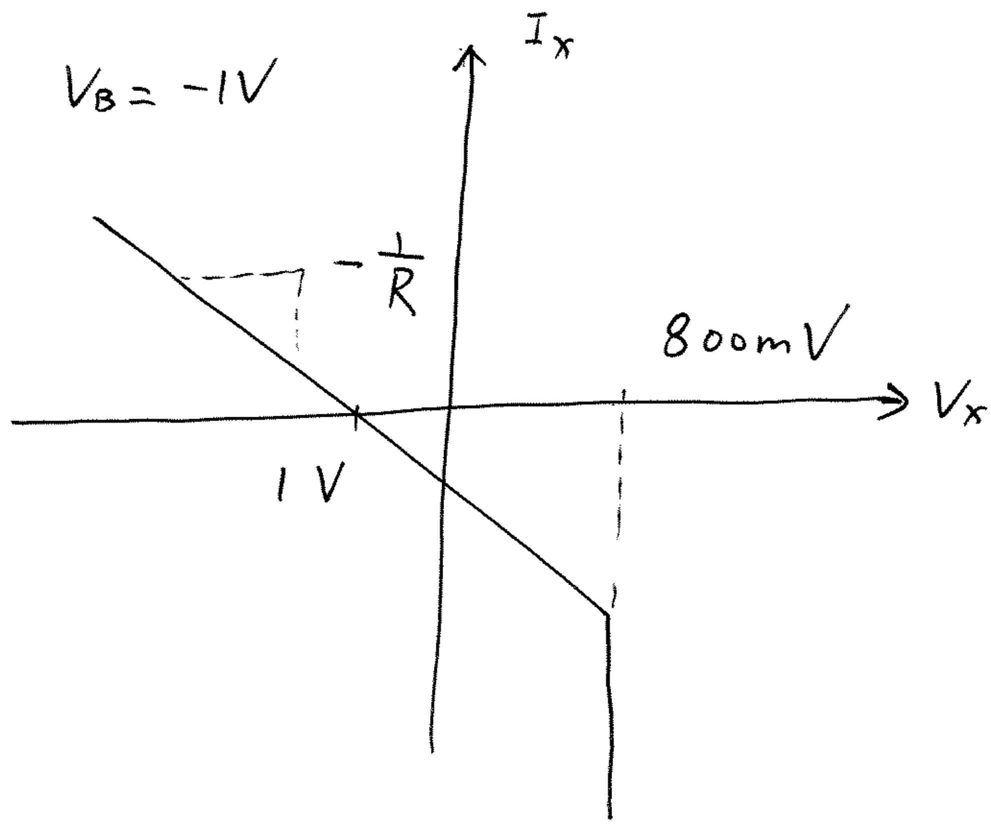


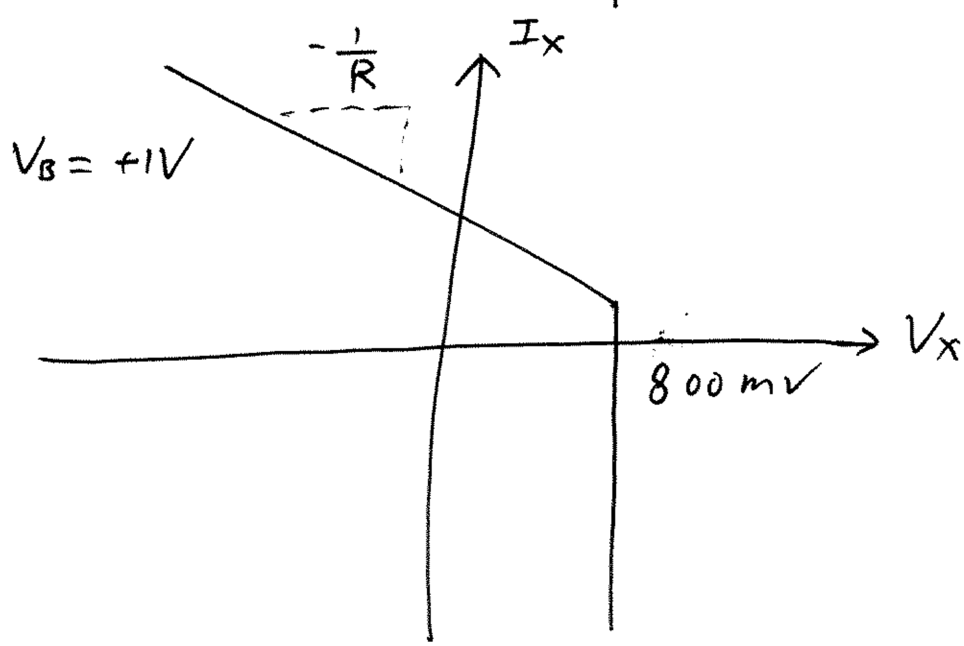
④



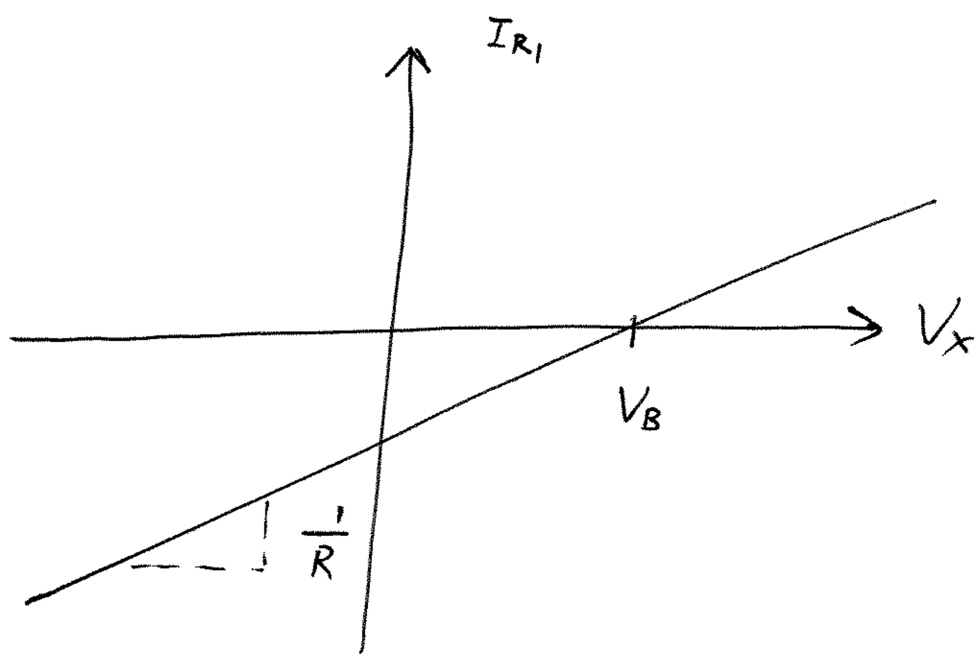


(5)





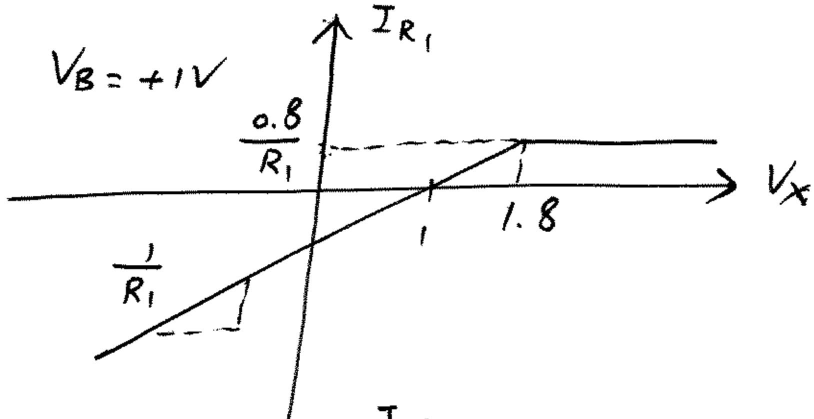
⑥

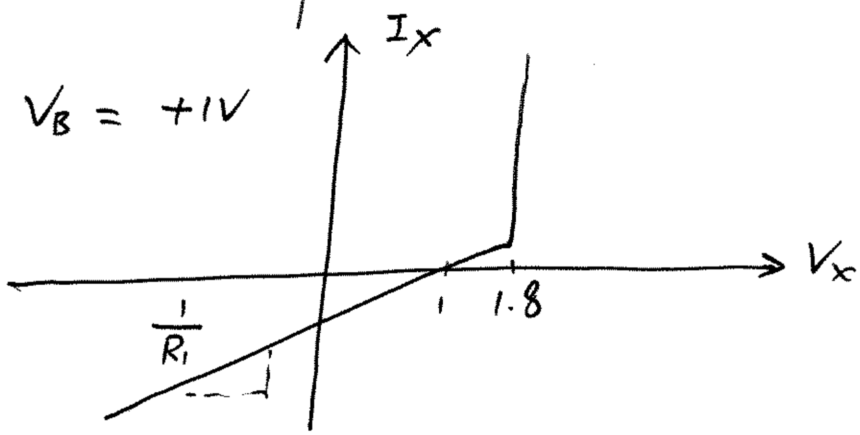


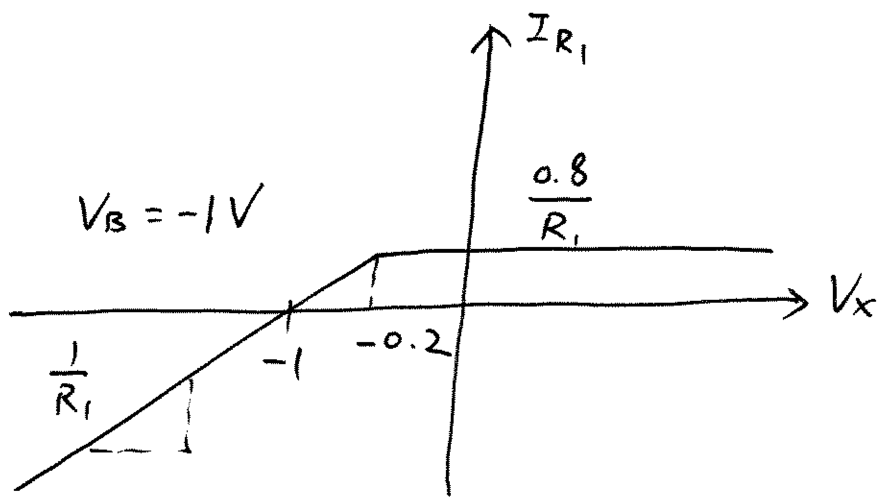
$I_{D1} = 0$ for all V_X

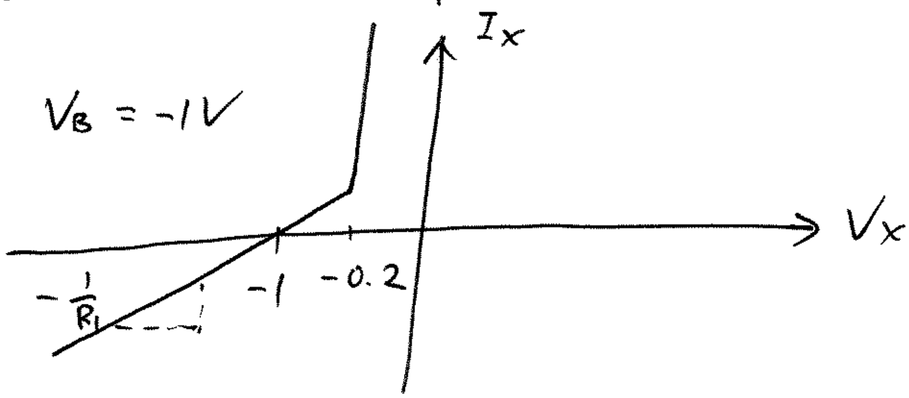
($\because$ $V_B > 0$, D_1 is reverse-biased)

(7)

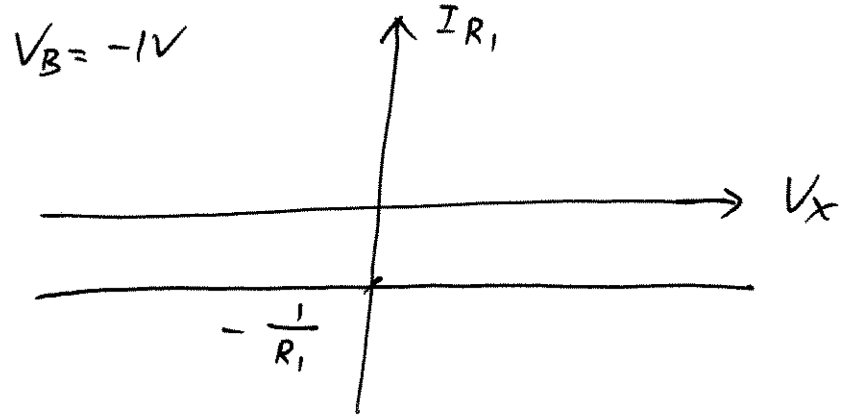

$V_B = +1V$
I_{R_1}
$\frac{0.8}{R_1}$
V_x
1
1.8
$\frac{1}{R_1}$


I_x
$V_B = +1V$
V_x
1
1.8
$\frac{1}{R_1}$

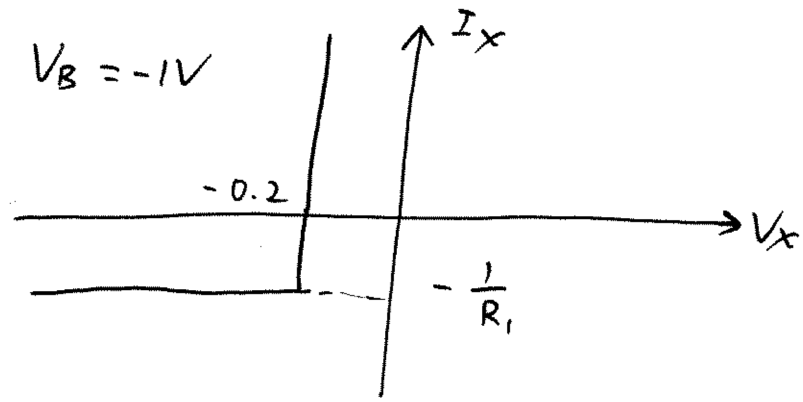

I_{R_1}
$\frac{0.8}{R_1}$
$V_B = -1V$
V_x
-1
-0.2
$\frac{1}{R_1}$


I_x
$V_B = -1V$
V_x
$-\frac{1}{R_1}$
-1
-0.2

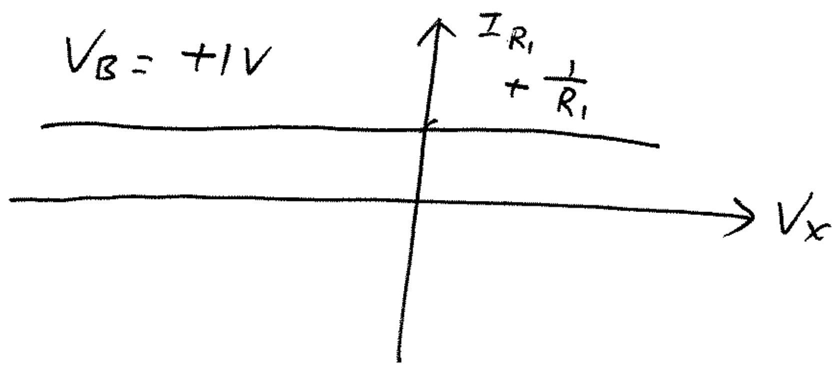
(8)



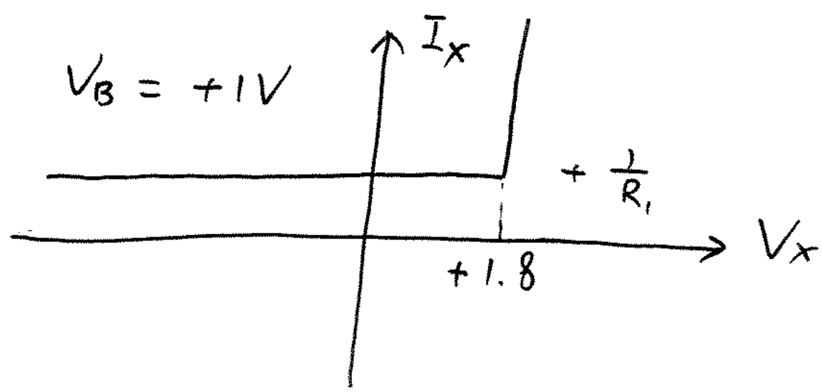
V_B = -1V
I_R1
V_x
-1/R_1



V_B = -1V
I_x
-0.2
V_x
-1/R_1



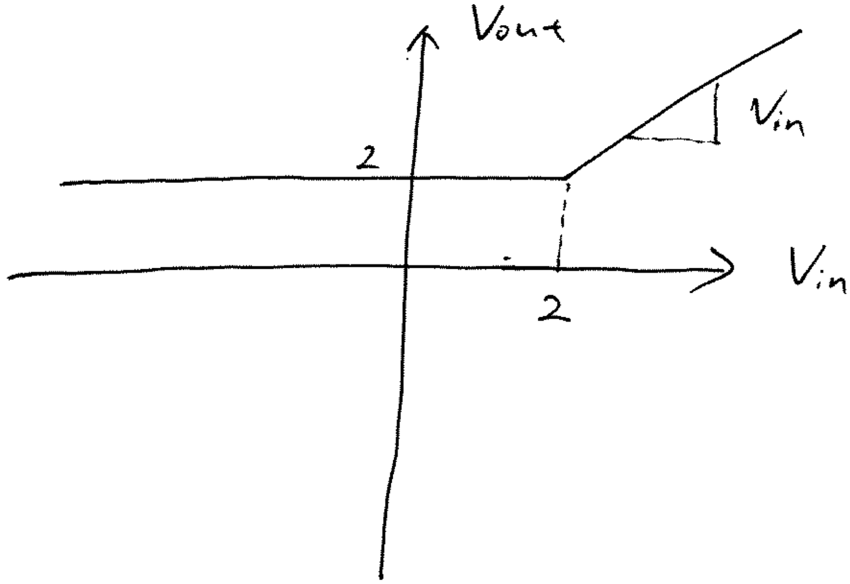
V_B = +1V
I_R1
+1/R_1
V_x



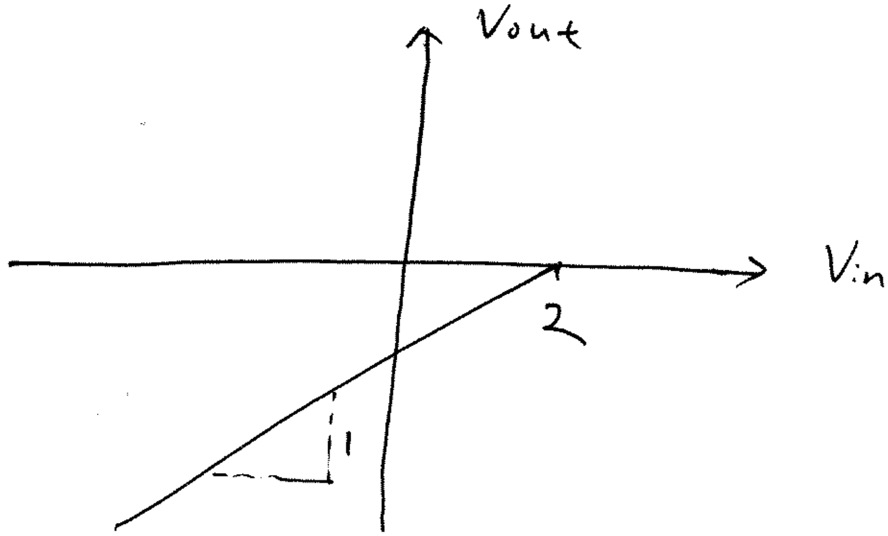
V_B = +1V
I_x
+1/R_1
V_x
+1.8

(9)

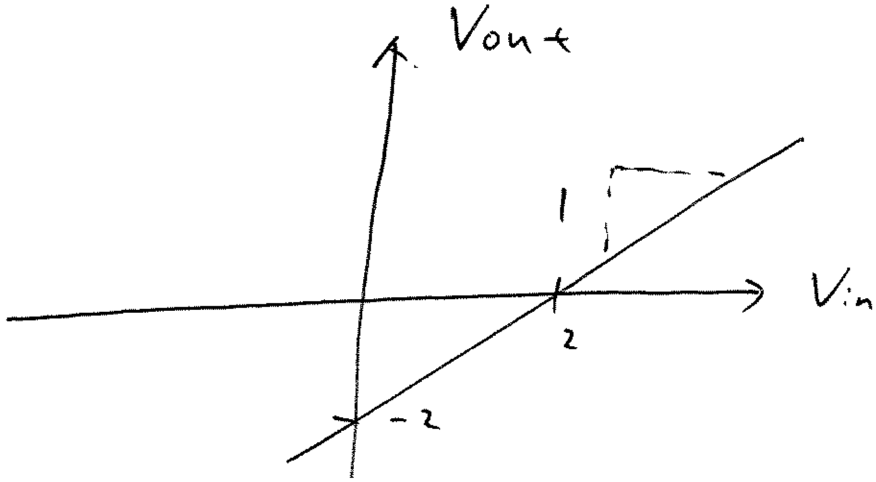
a)



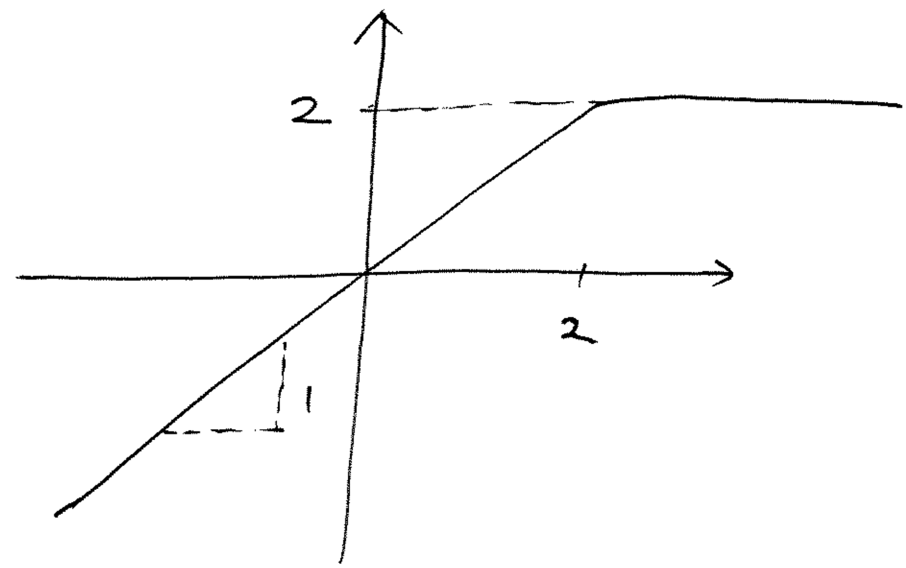
b)



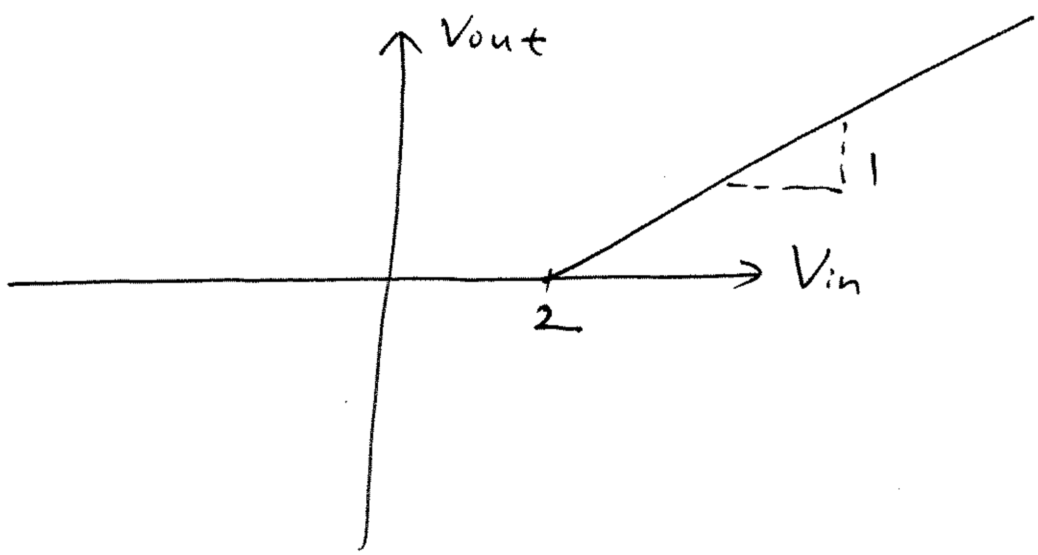
c)



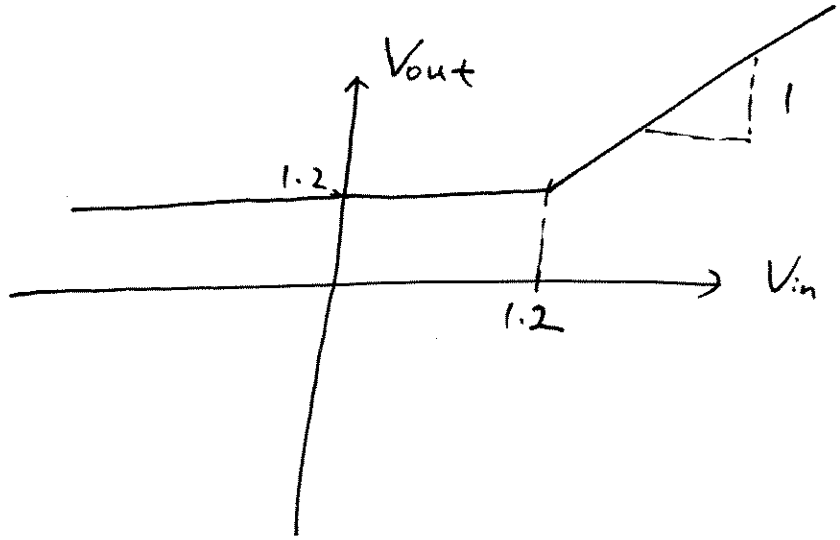
d)



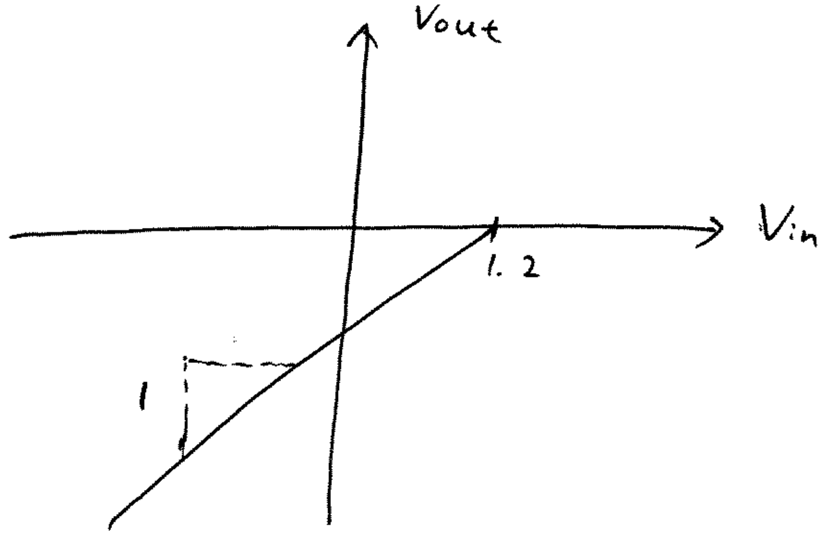
e)



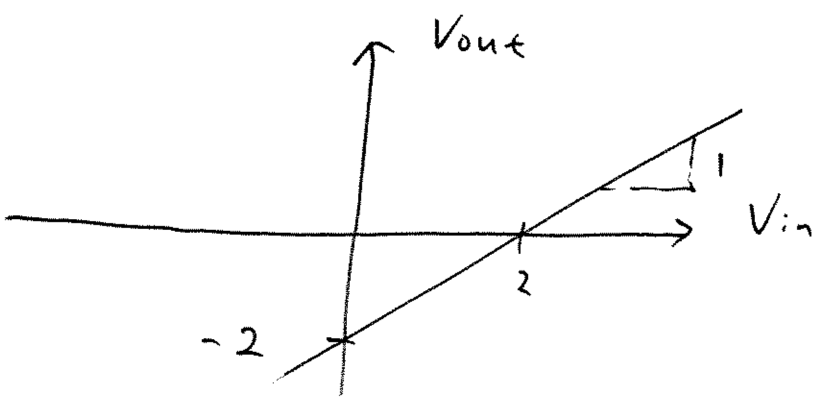
(10) a)



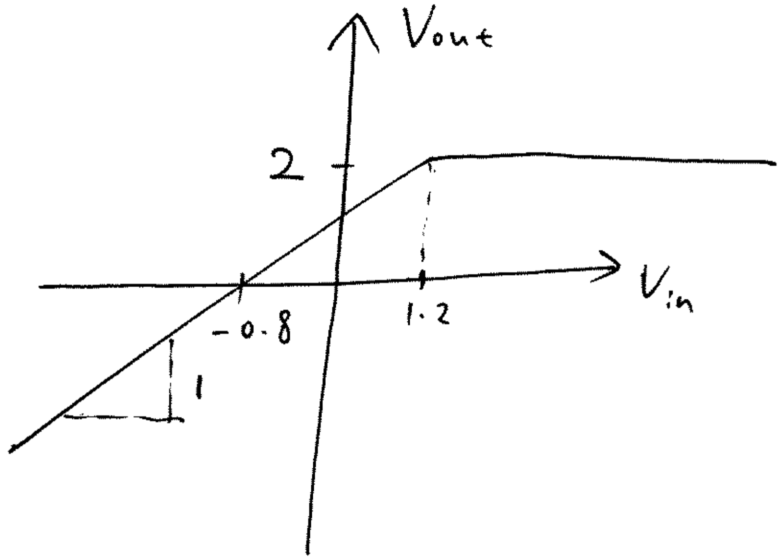
b)



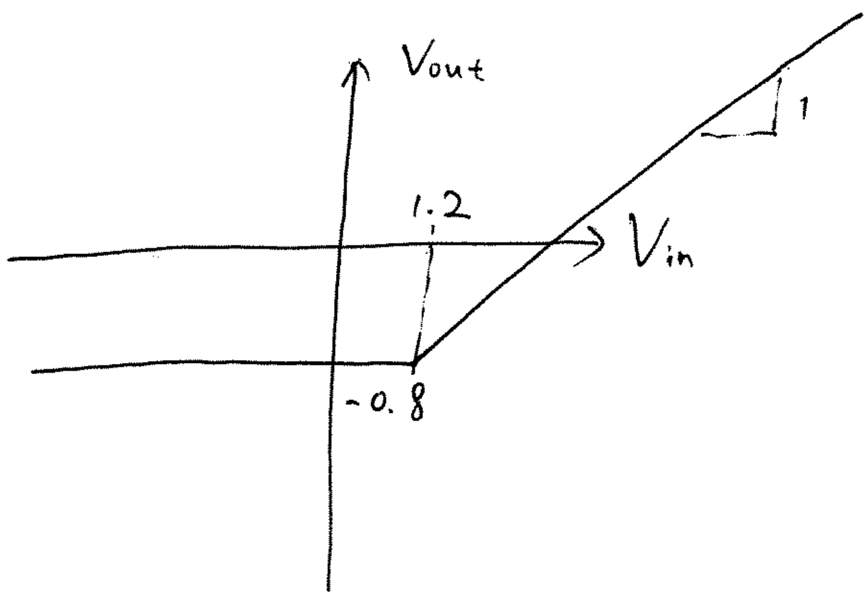
c)



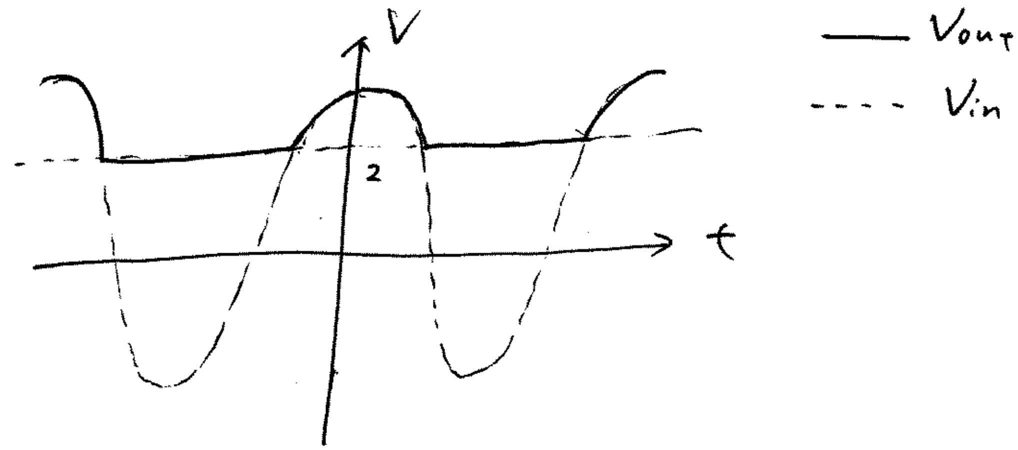
d)



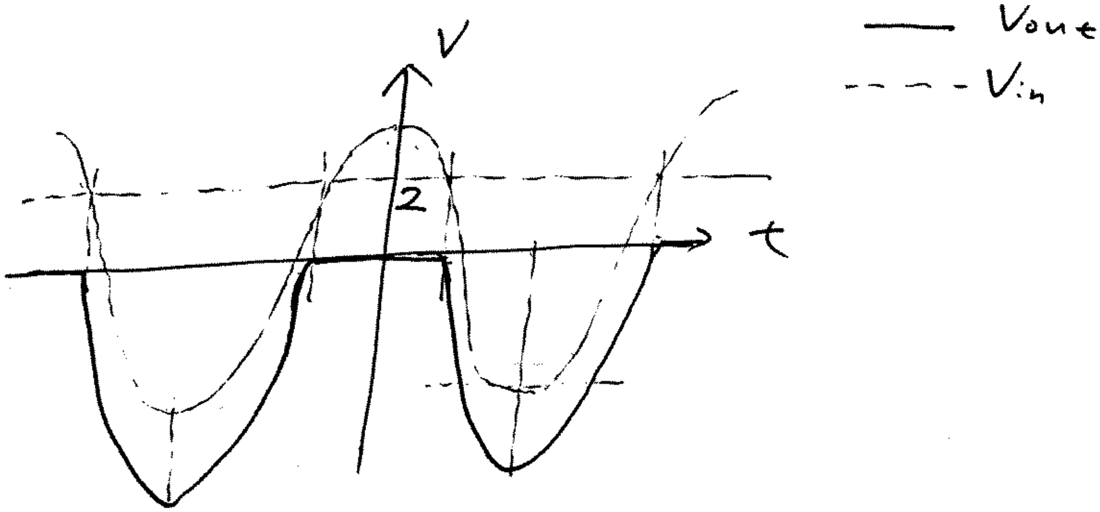
e)



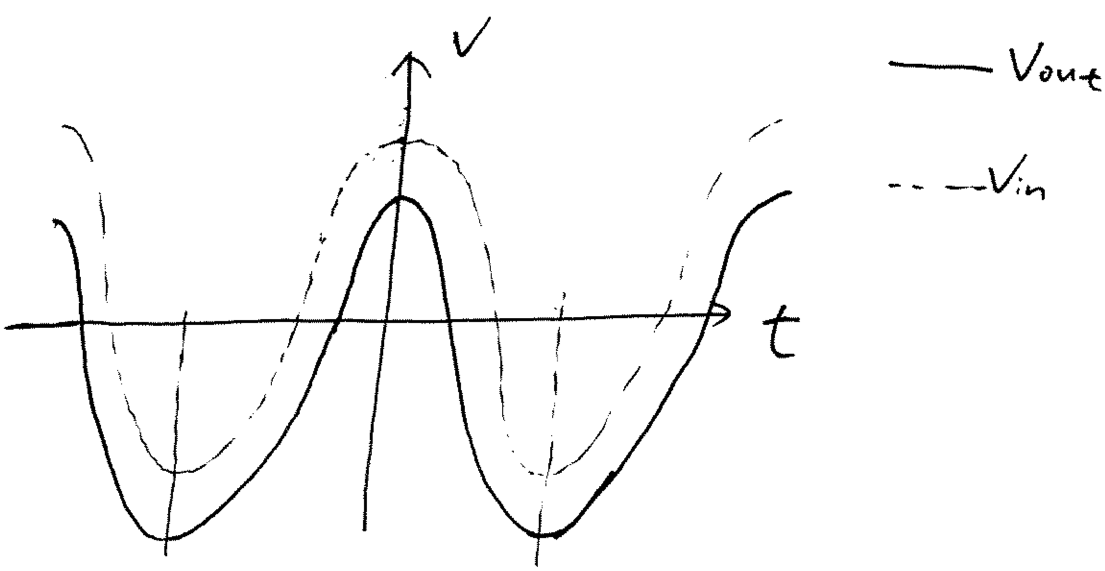
(11) a)



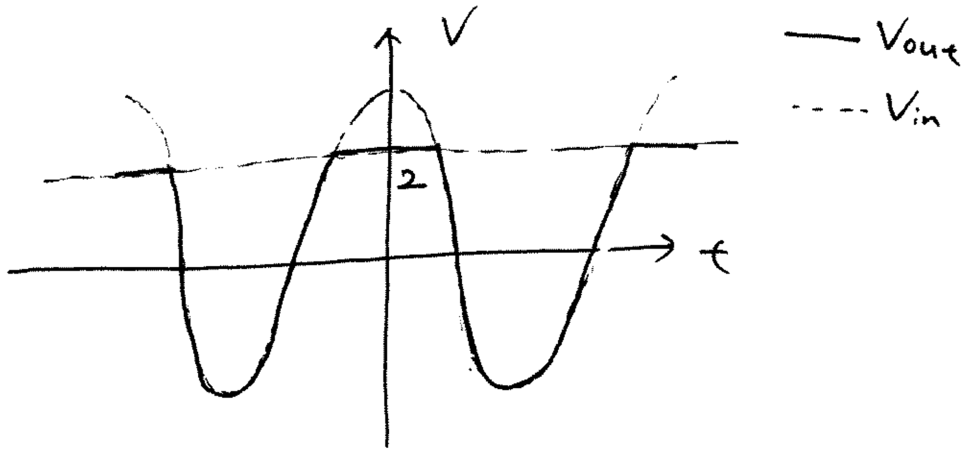
b)



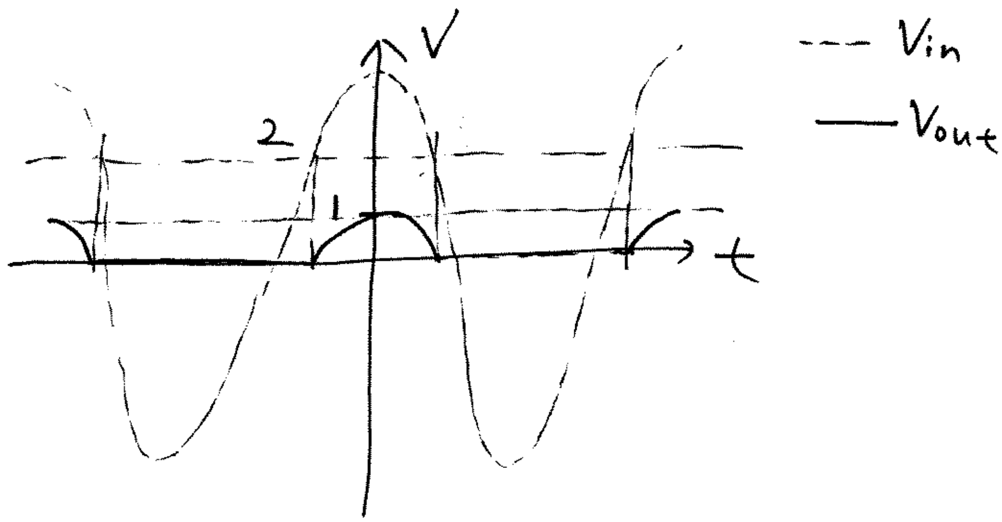
c)



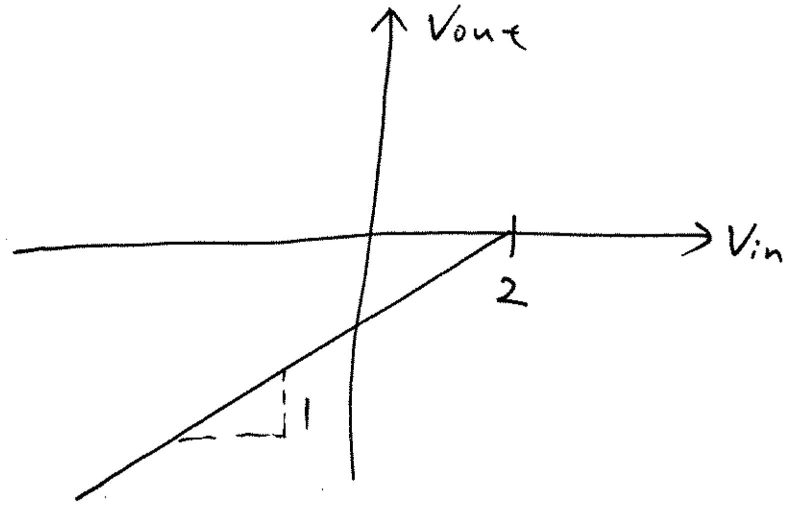
d)



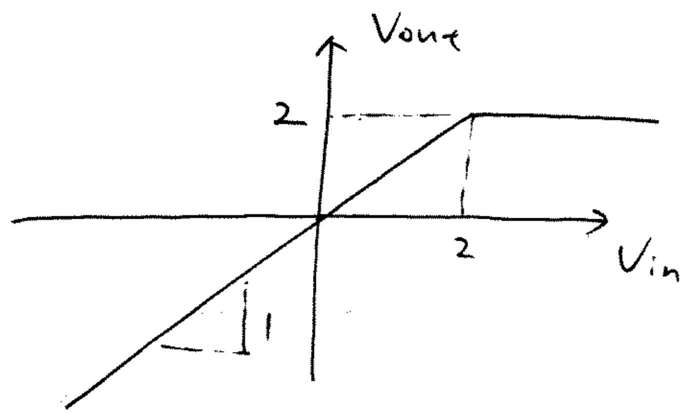
e)



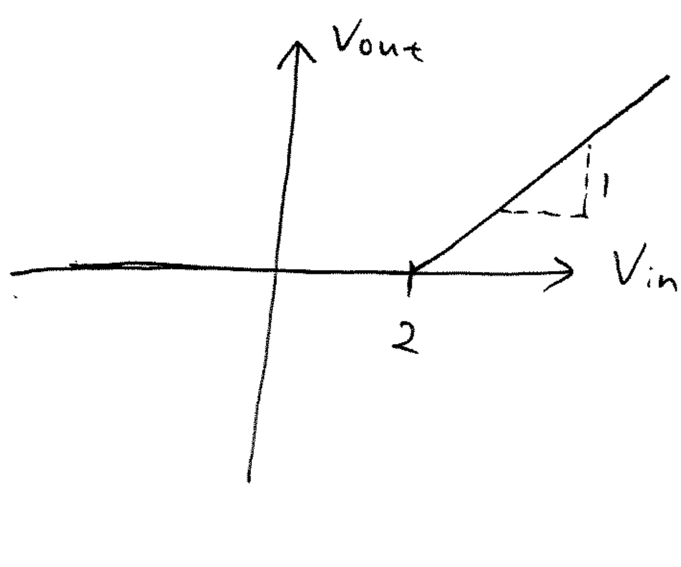
(12) a)



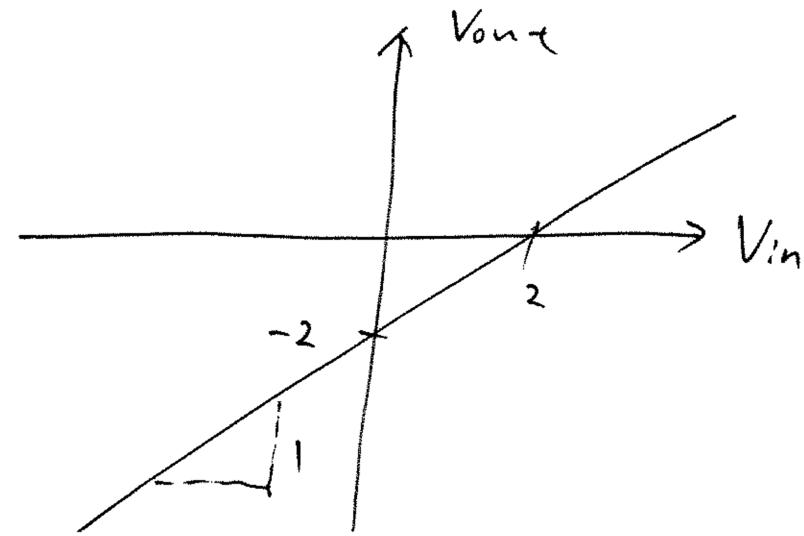
b)



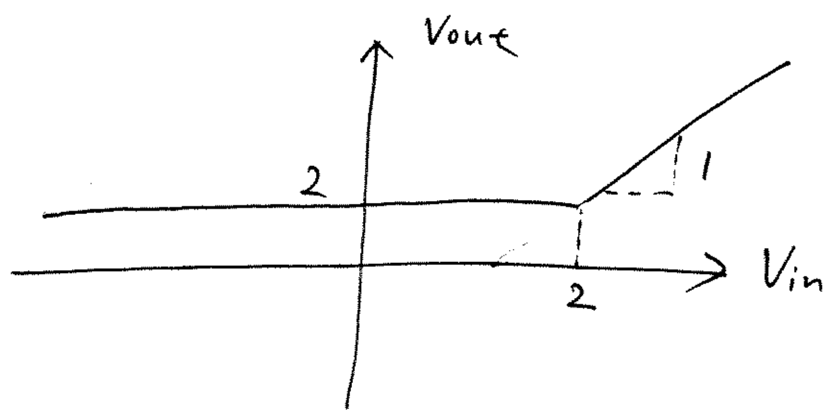
c)



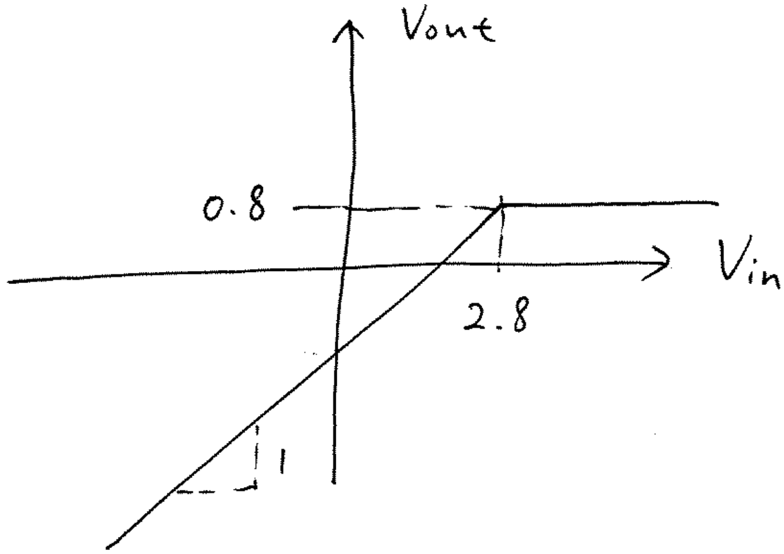
d)



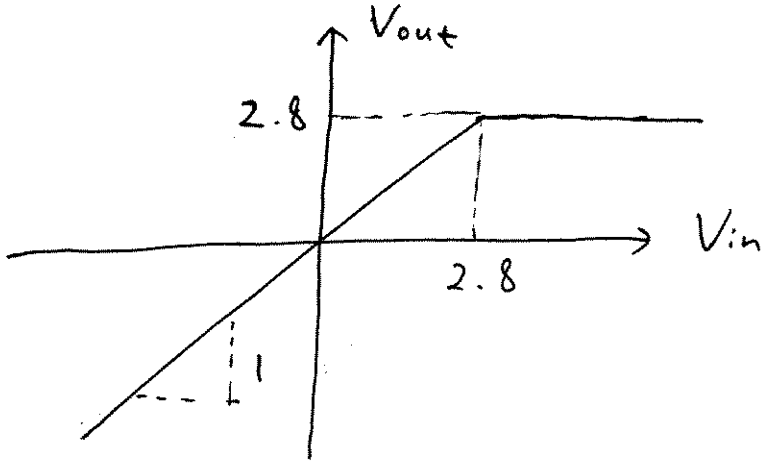
e)



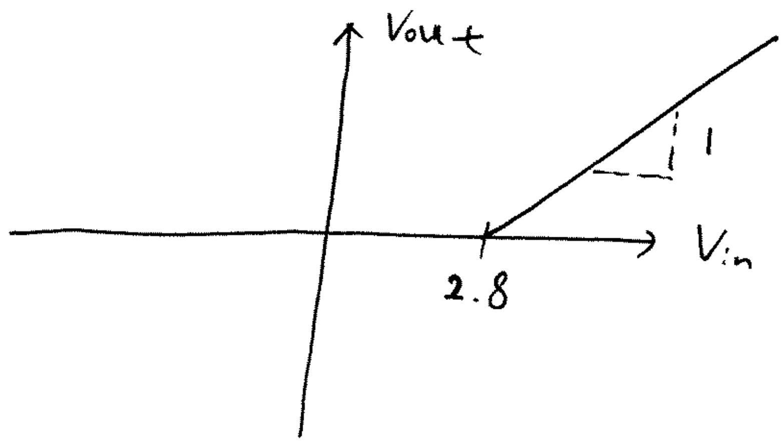
(13) a)



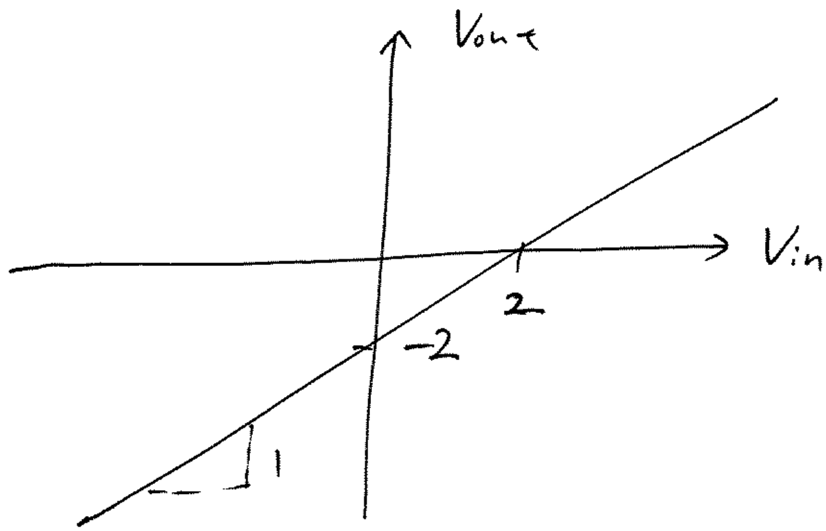
b)



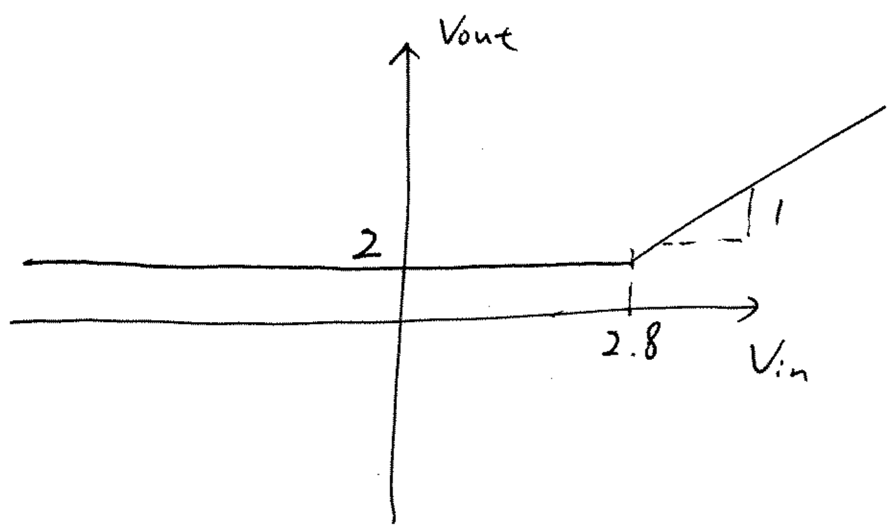
c)



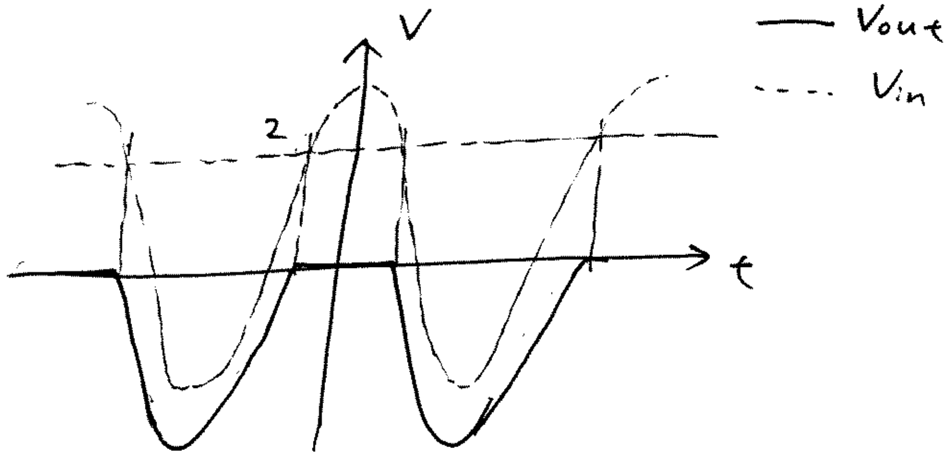
d)



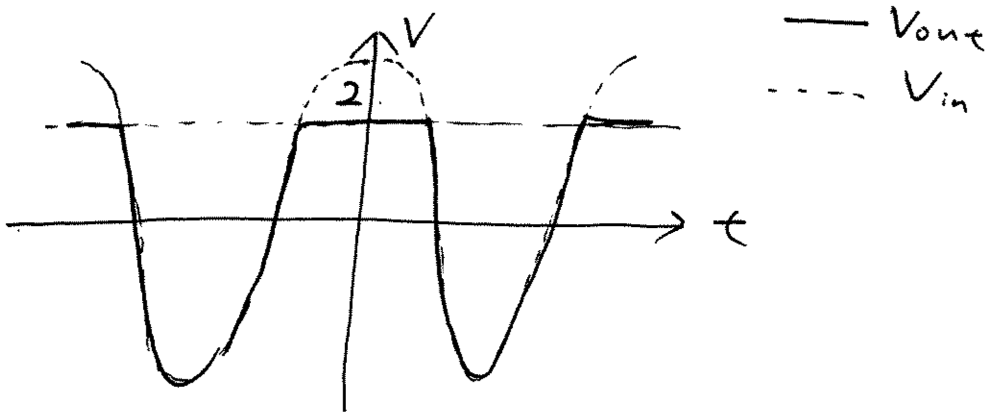
e)



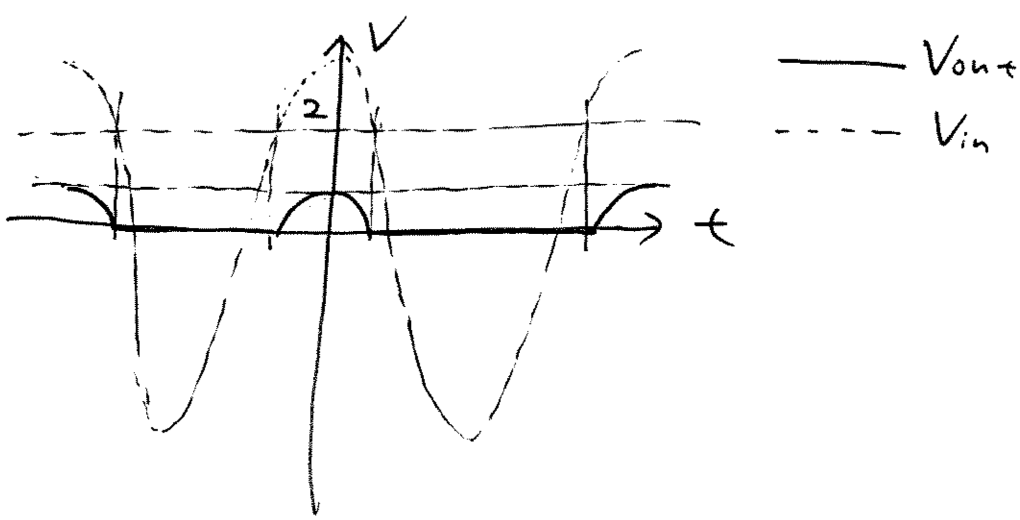
(14) a)



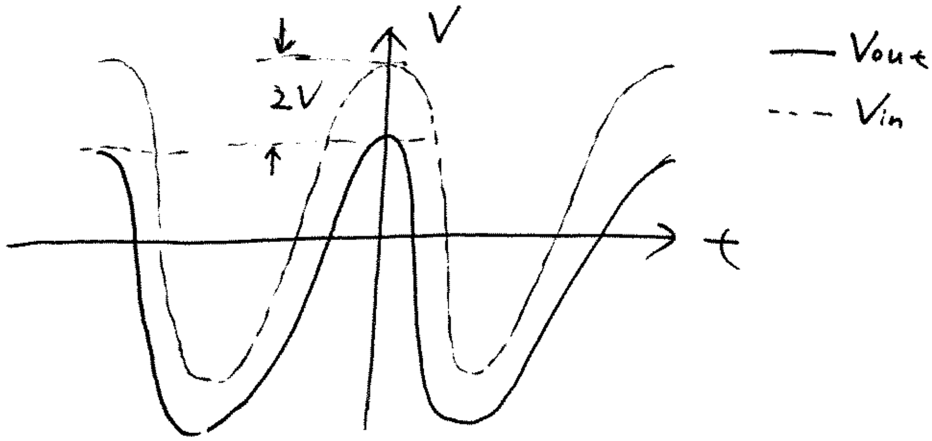
b)



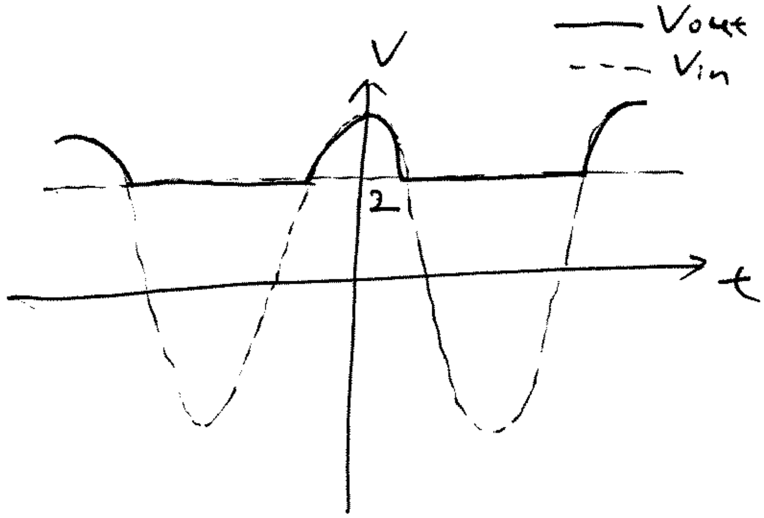
c)



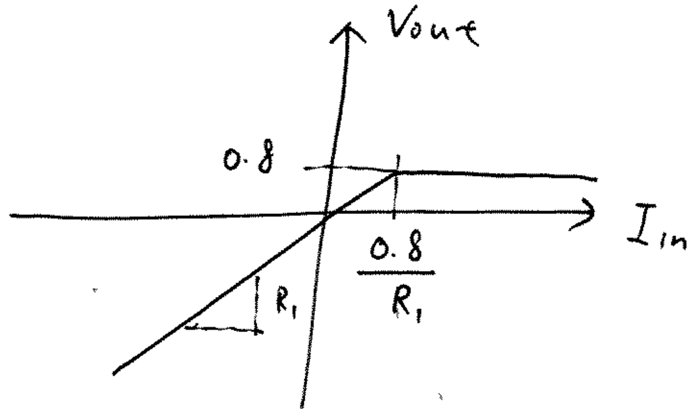
d)



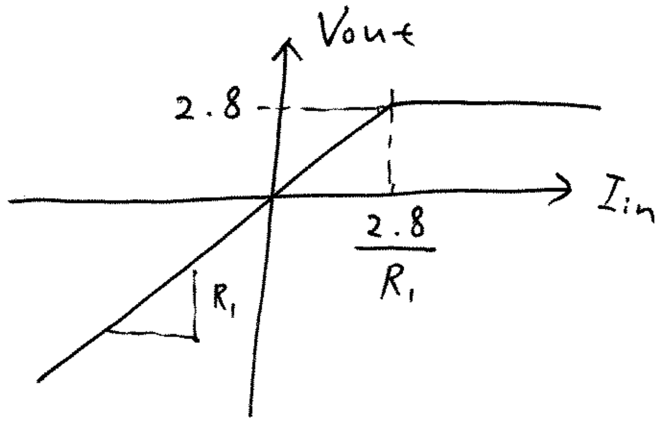
e).



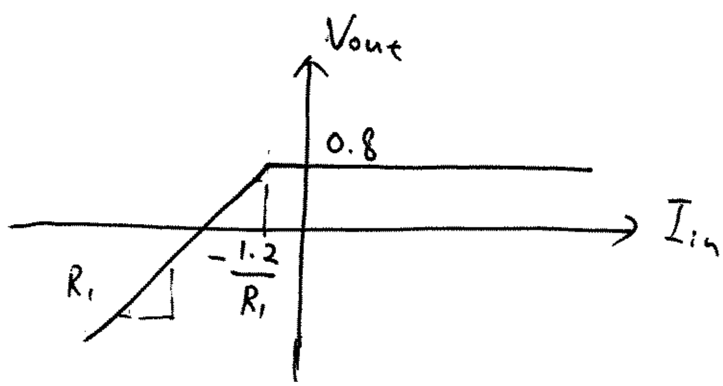
(15) a)


Vout
0.8
Iin
0.8
R1
R1

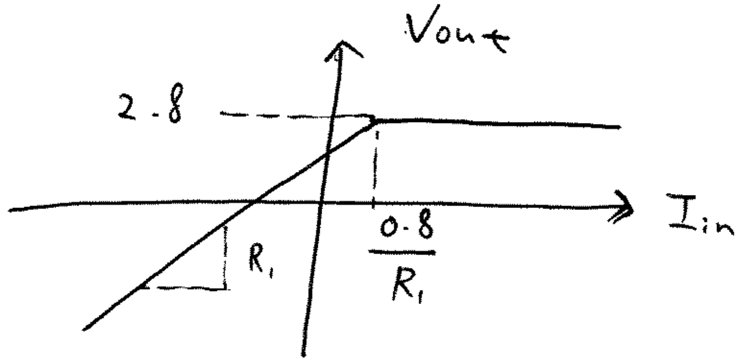
b)


Vout
2.8
Iin
2.8
R1
R1

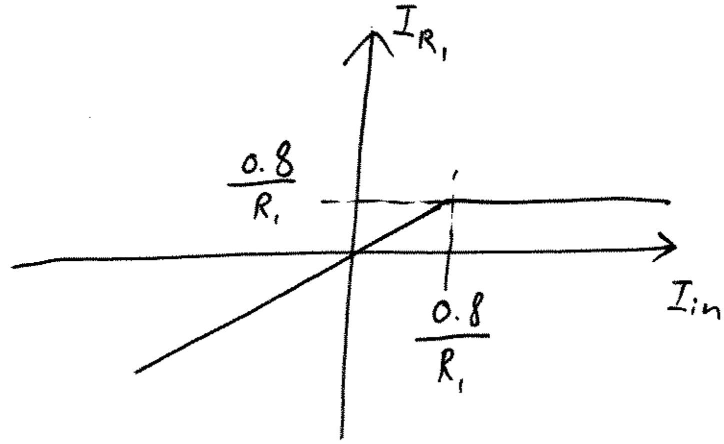
c)


Vout
0.8
Iin
-1.2
R1
R1

d)

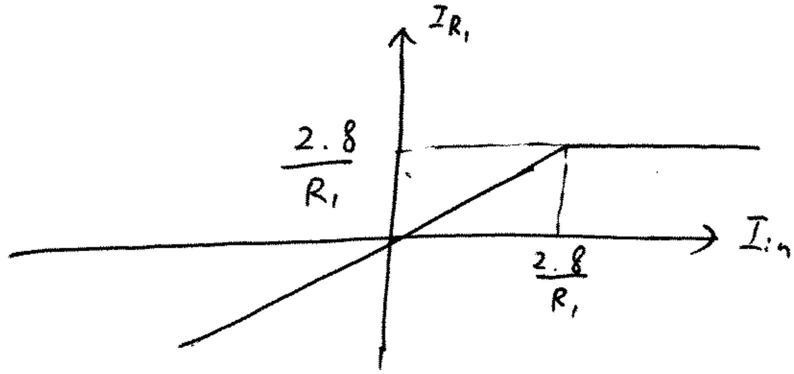

Vout
2.8
Iin
0.8
R1
R1

(16) a)



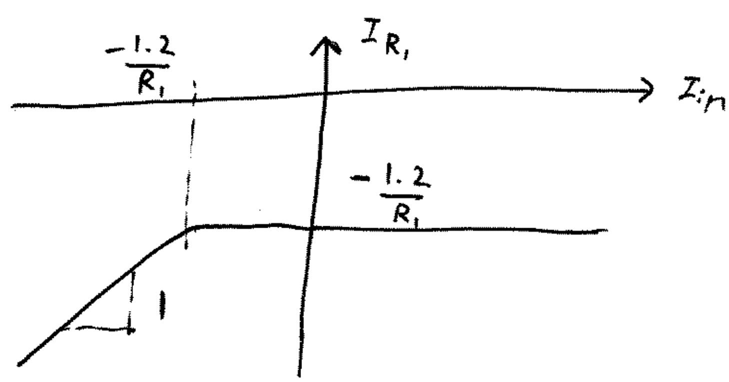
I_{R_1}
$\frac{0.8}{R_1}$
$\frac{0.8}{R_1}$
I_{in}

b)



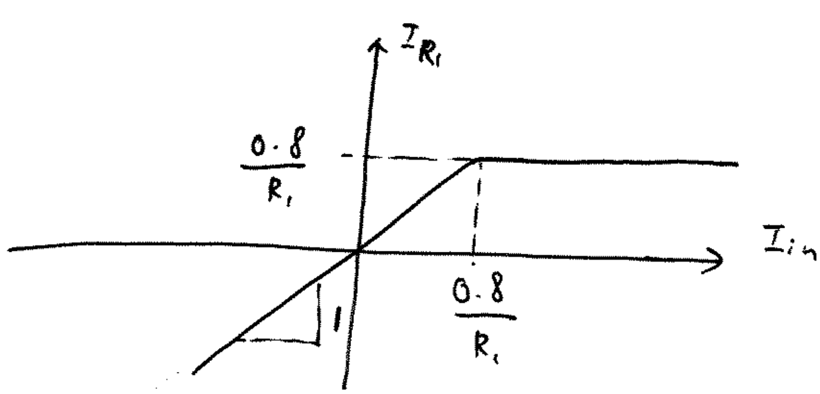
I_{R_1}
$\frac{2.8}{R_1}$
$\frac{2.8}{R_1}$
I_{in}

c)



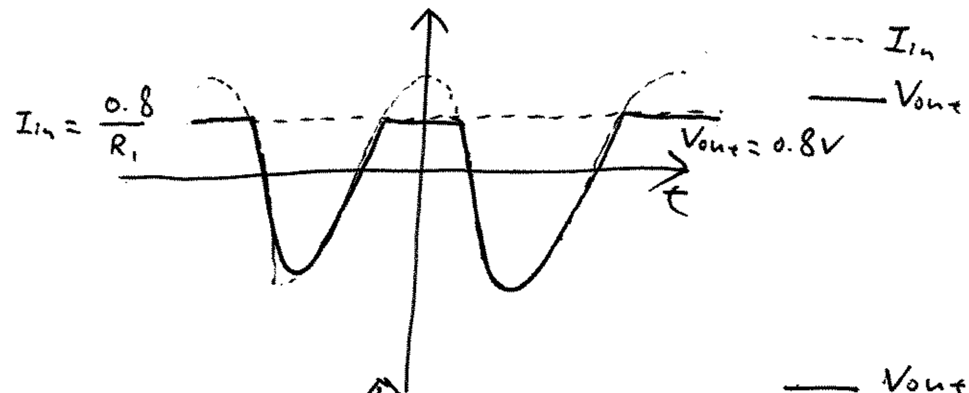
I_{R_1}
$-\frac{1.2}{R_1}$
I_{in}
$-\frac{1.2}{R_1}$
1

d)



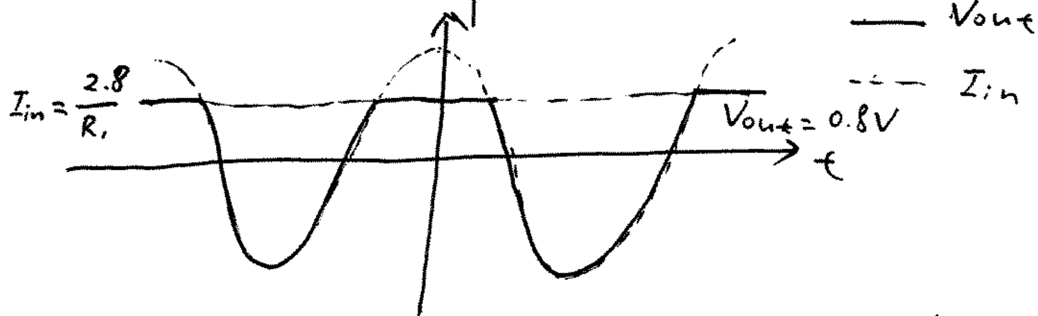
I_{R_1}
$\frac{0.8}{R_1}$
I_{in}
$\frac{0.8}{R_1}$
1

(17) a)



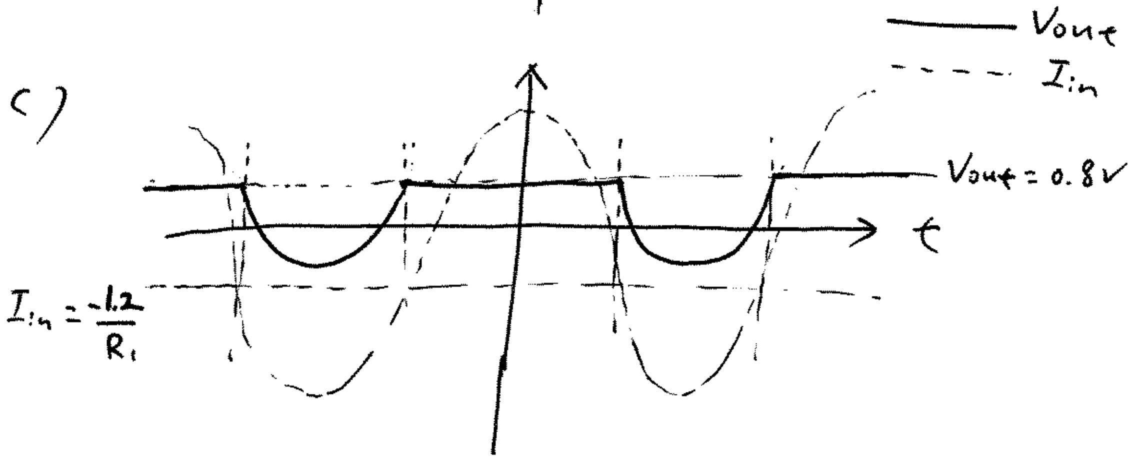
--- I_{in}
— V_{out}
$I_{in} = \frac{0.8}{R_1}$
$V_{out} = 0.8V$
t

b)



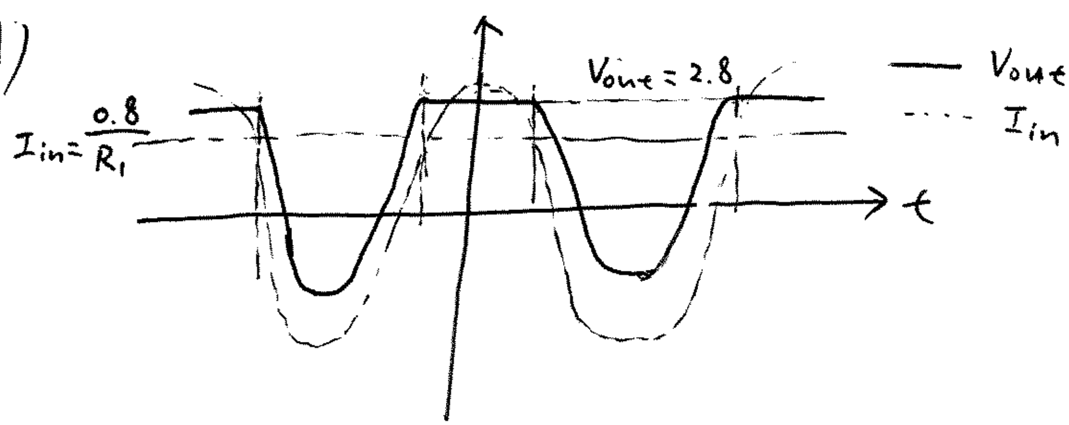
— V_{out}
--- I_{in}
$I_{in} = \frac{2.8}{R_1}$
$V_{out} = 0.8V$
t

c)



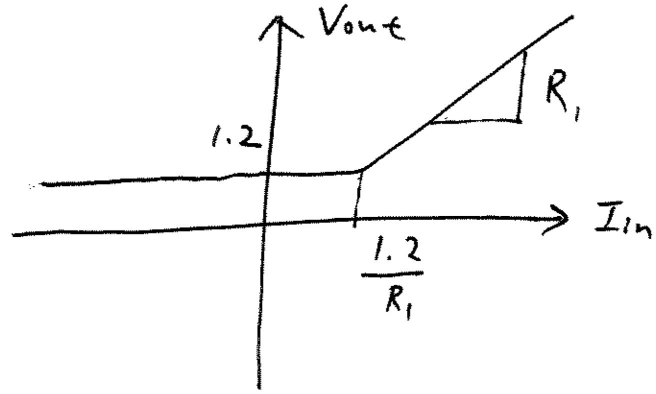
— V_{out}
--- I_{in}
$V_{out} = 0.8V$
t
$I_{in} = \frac{-1.2}{R_1}$

d)

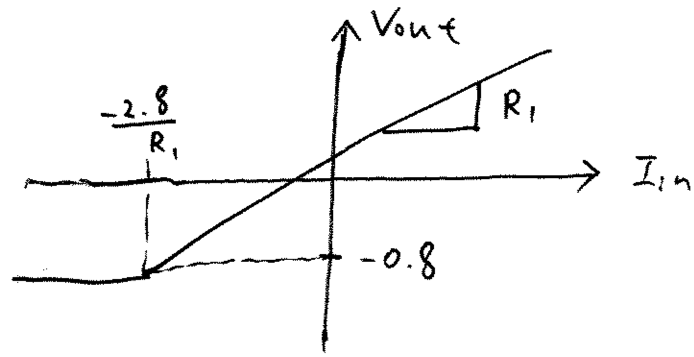


$V_{out} = 2.8$
— V_{out}
--- I_{in}
$I_{in} = \frac{0.8}{R_1}$
t

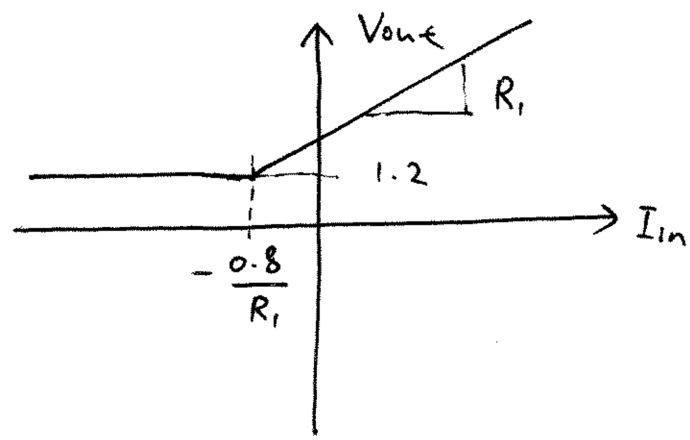
(18) a)



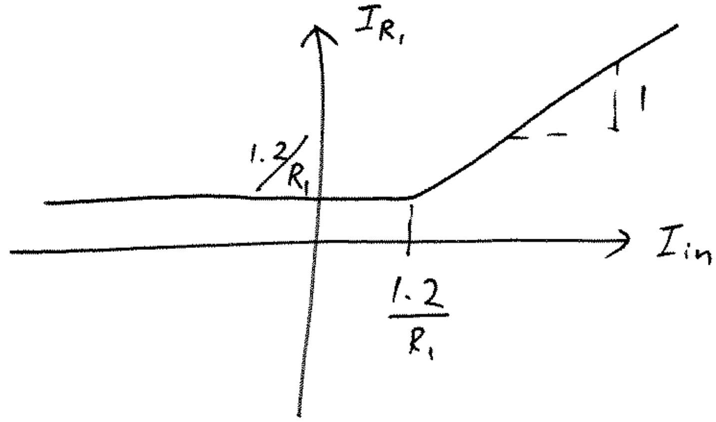
b)



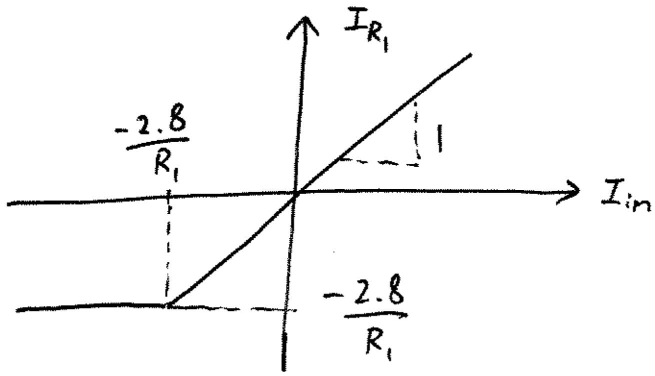
c)



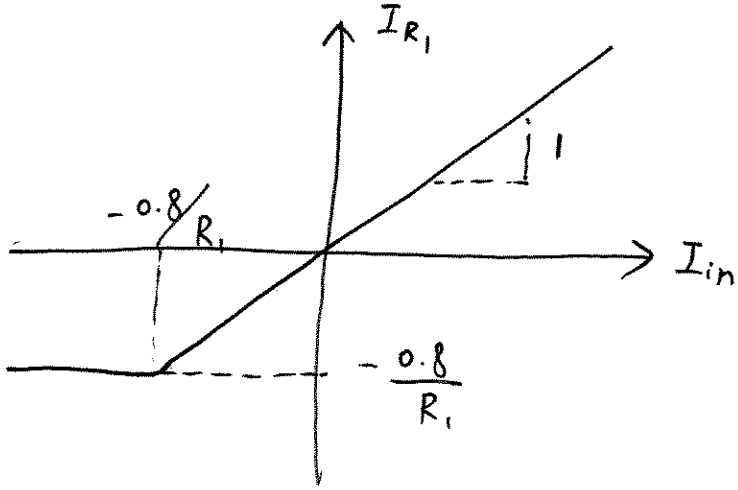
(19) a)


I_{R_1}
1.2/R_1
1
I_{in}
$\frac{1.2}{R_1}$

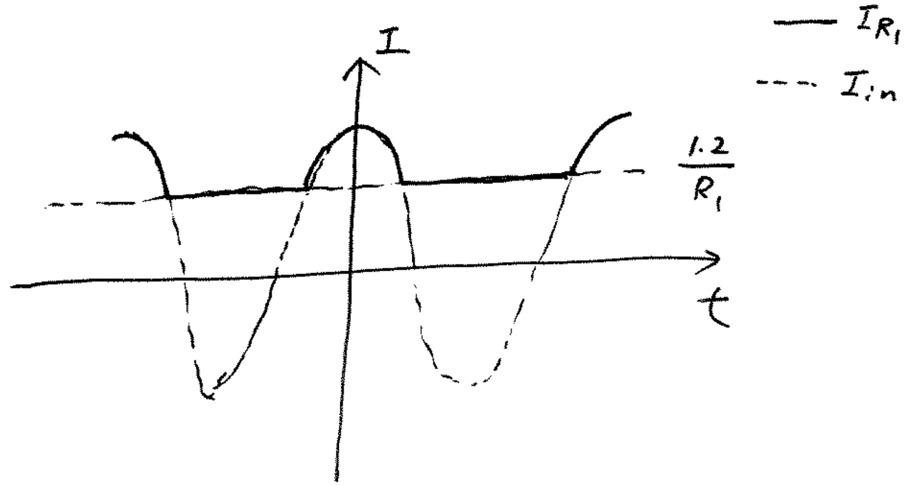
b)


I_{R_1}
$\frac{-2.8}{R_1}$
1
I_{in}
$-\frac{2.8}{R_1}$

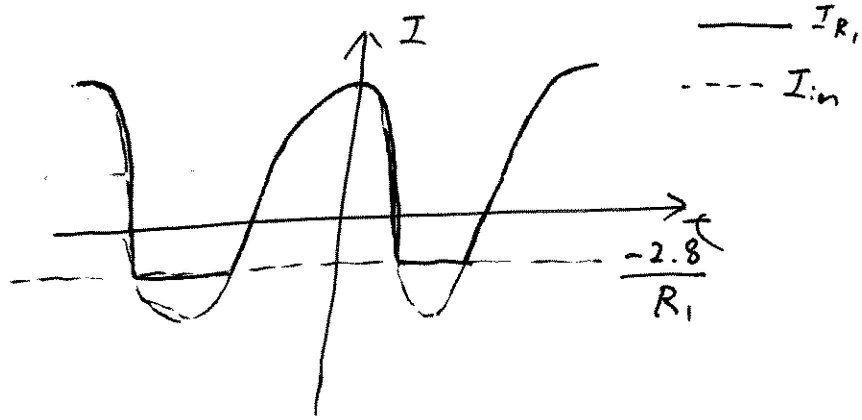
c)


I_{R_1}
-0.8/R_1
1
I_{in}
$-\frac{0.8}{R_1}$

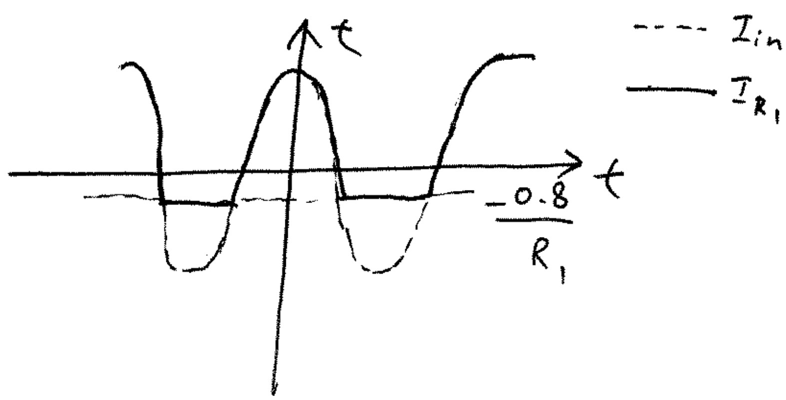
(20) a)



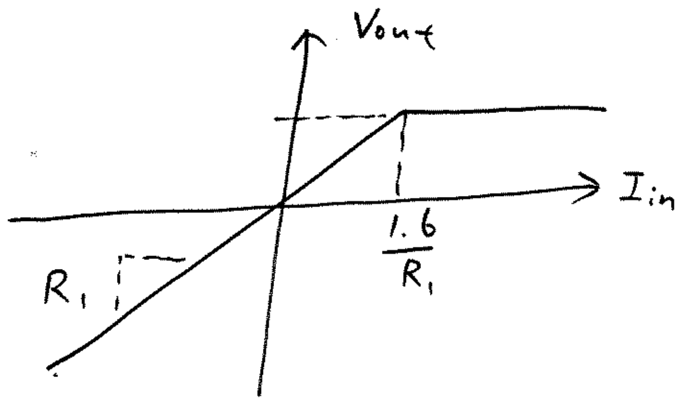
b)



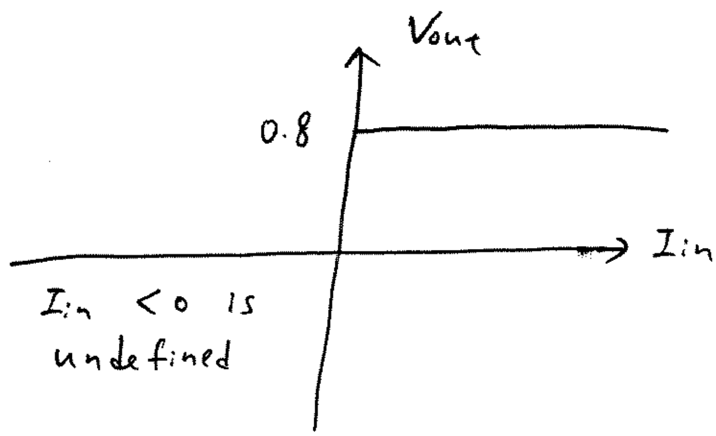
c)



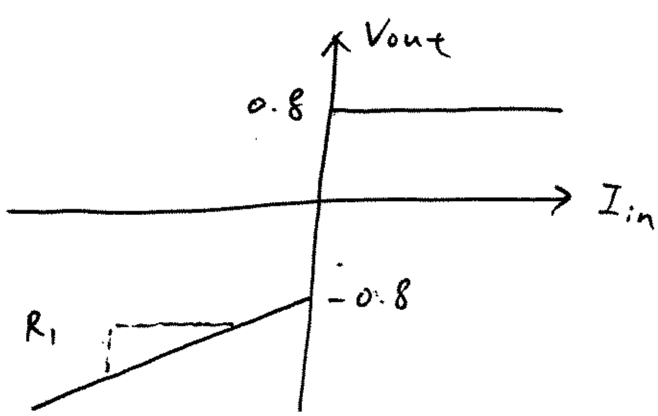
(21) a)


Vout
Iin
1.6
R1
R1

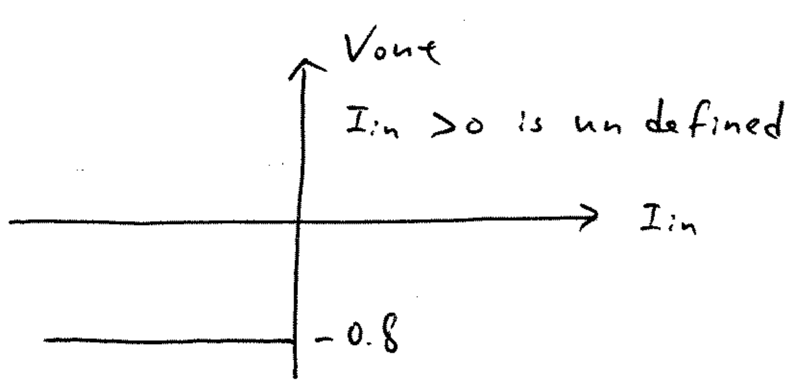
b)


Vout
0.8
Iin
Iin < 0 is
undefined

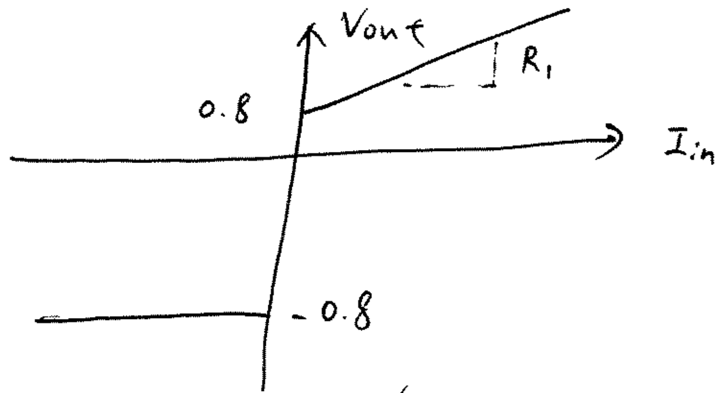
c)


Vout
0.8
Iin
-0.8
R1

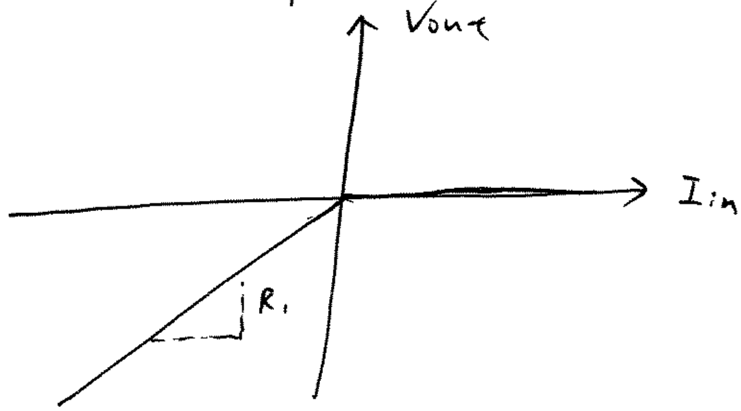
d)


Vout
Iin > 0 is undefined
Iin
-0.8

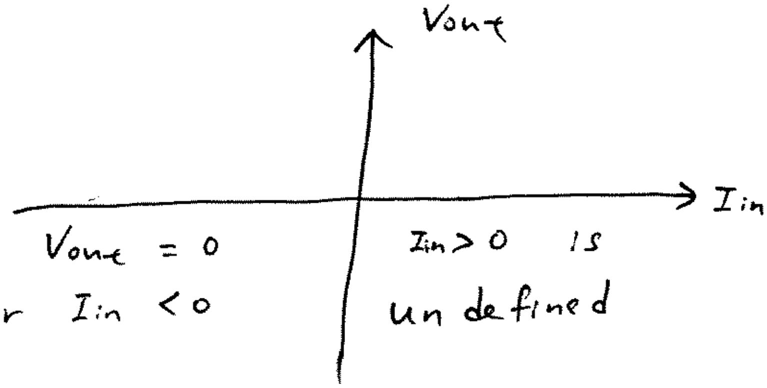
e)


Vout
R1
0.8
Iin
-0.8

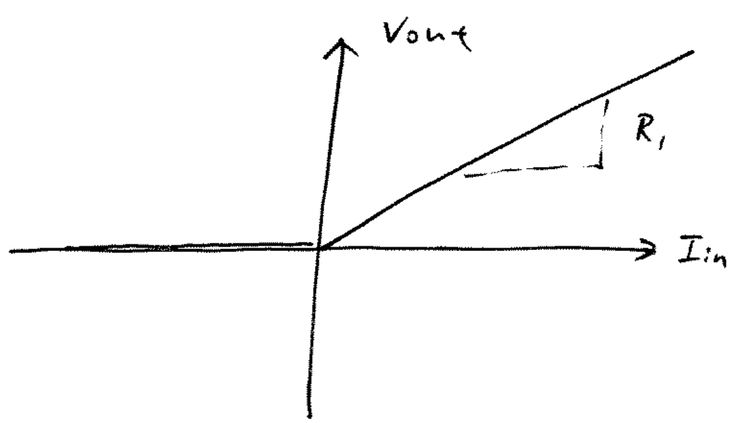
f)


Vout
Iin
R1

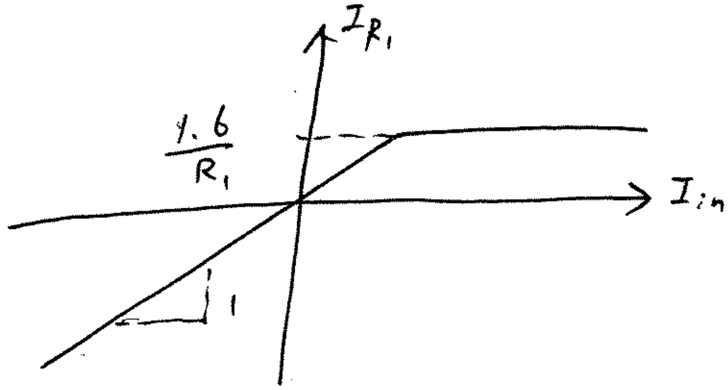
g)


Vout
Iin
Vout = 0
for Iin < 0
Iin > 0 is
undefined

h)

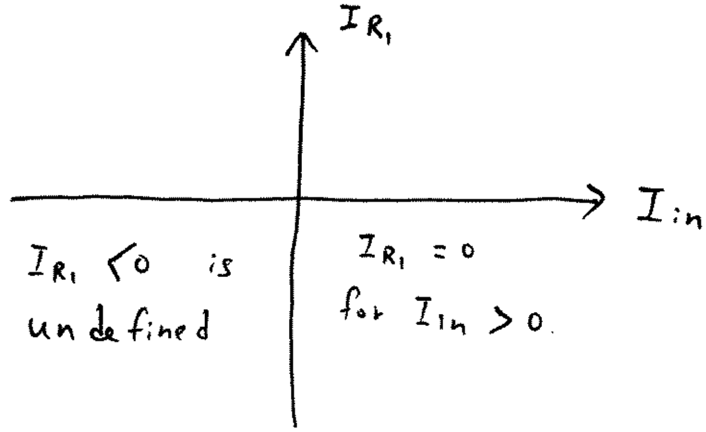

Vout
R1
Iin

(22) a)



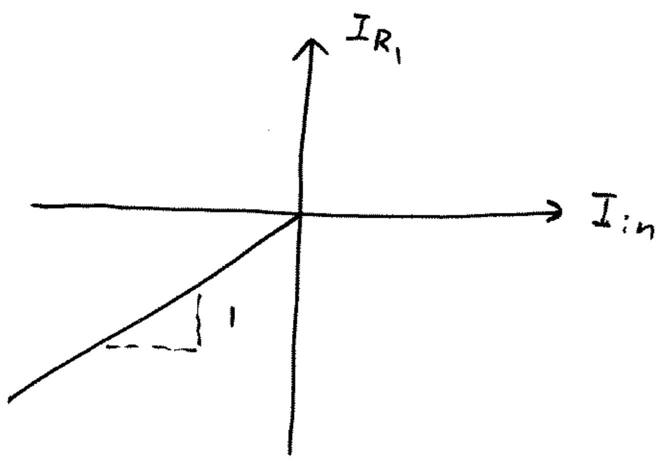
I_{R_1}
$\frac{4.6}{R_1}$
I_{in}
1

b)



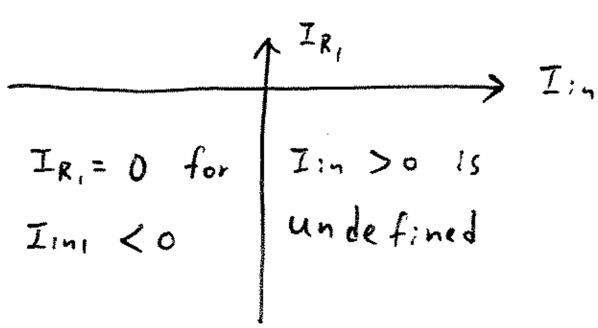
I_{R_1}
I_{in}
$I_{R_1} < 0$ is undefined
$I_{R_1} = 0$ for $I_{in} > 0$.

c)



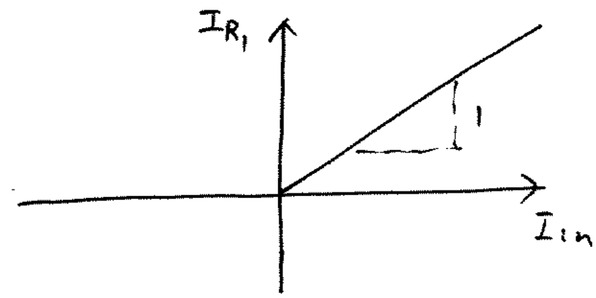
I_{R_1}
I_{in}
1

d)



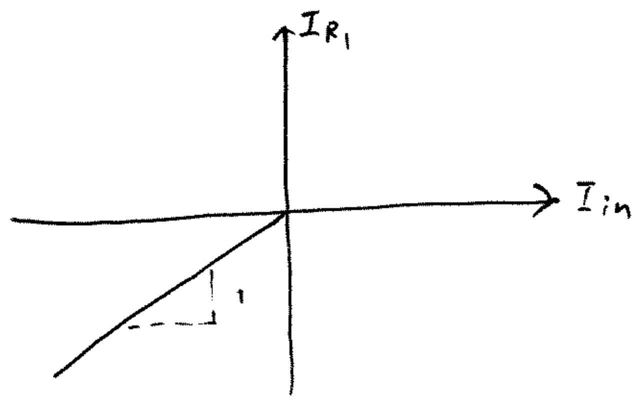
I_{R_1}
I_{in}
$I_{R_1} = 0$ for $I_{in1} < 0$
$I_{in} > 0$ is undefined

e)



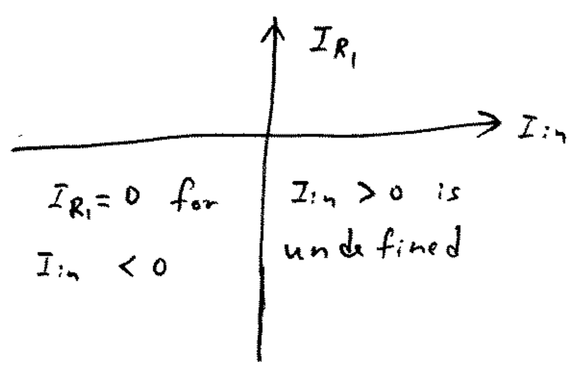
I_{R_1}
1
I_{in}

f)



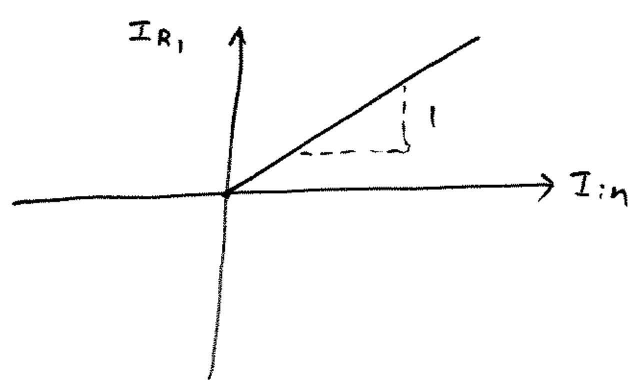
I_{R_1}
I_{in}
1

g)



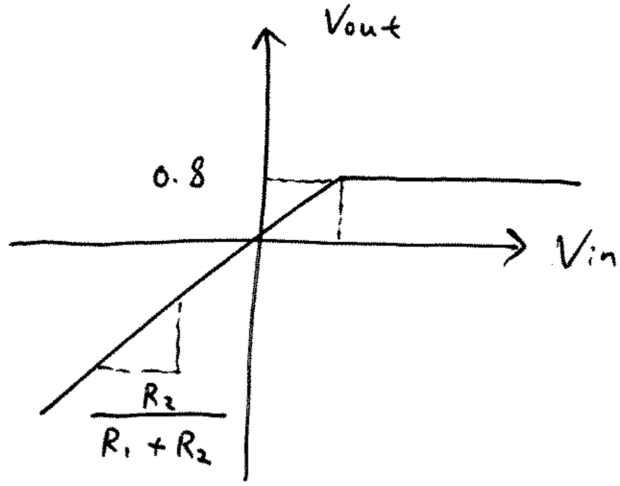
I_{R_1}
I_{in}
$I_{R_1} = 0$ for $I_{in} < 0$
$I_{in} > 0$ is undefined

h)

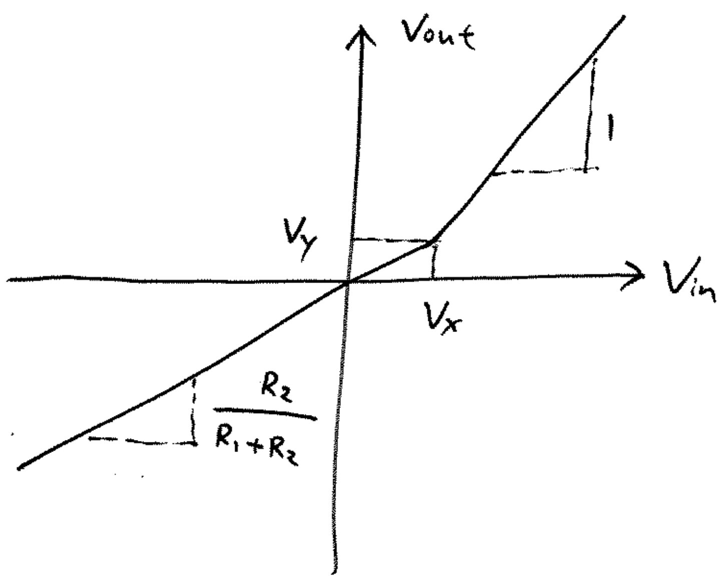


I_{R_1}
1
I_{in}

(23) a)



b)



Note: at the turning point when D_1 starts to conduct, V_x, V_y need to satisfy 2 conditions:

$$V_x - V_y = 0.8 \quad \text{—— (1)}$$

$$V_y = \frac{R_2}{R_1+R_2} V_x \quad \text{—— (2)}$$

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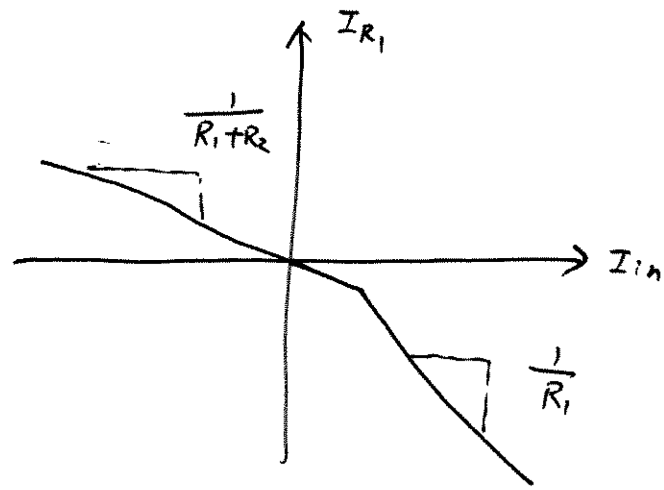
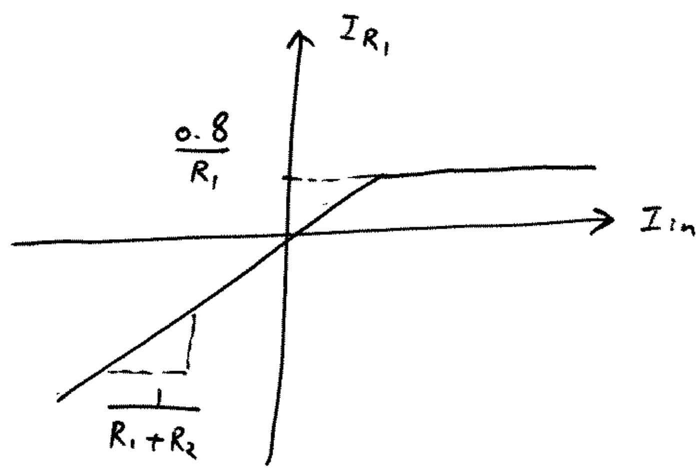
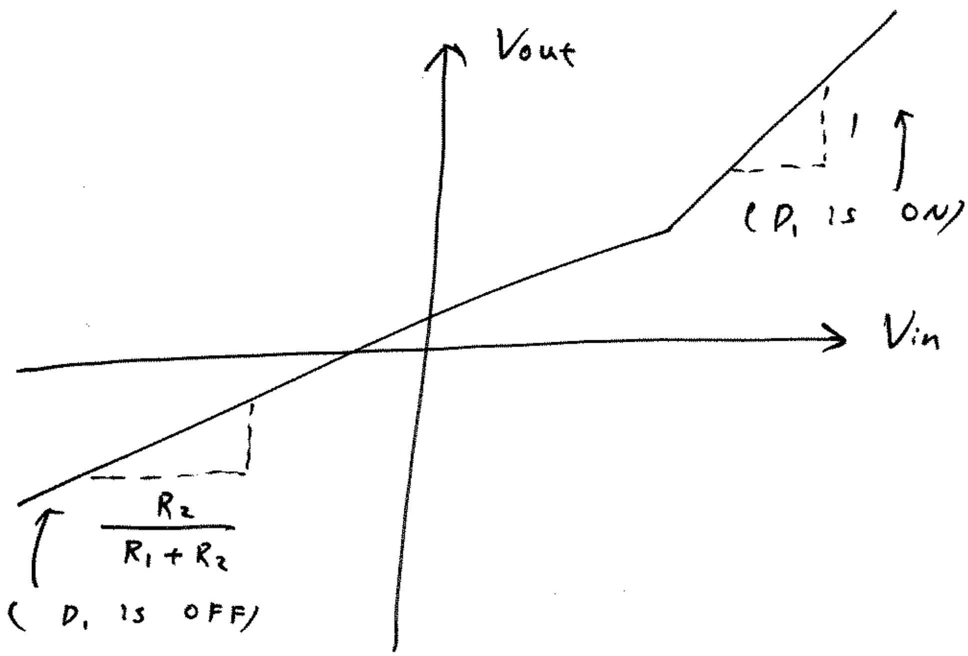
a)



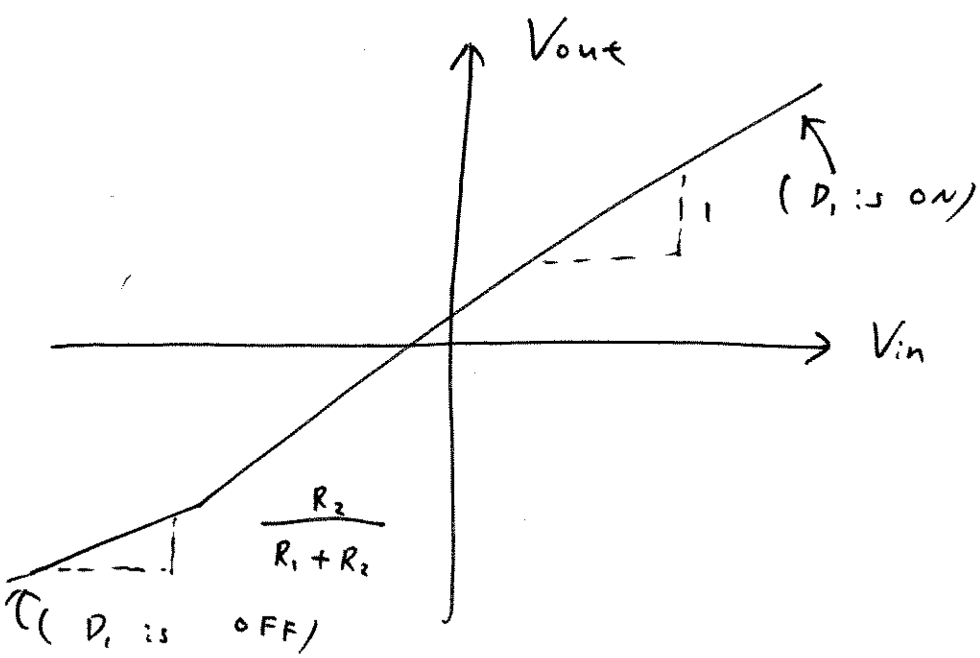
b)



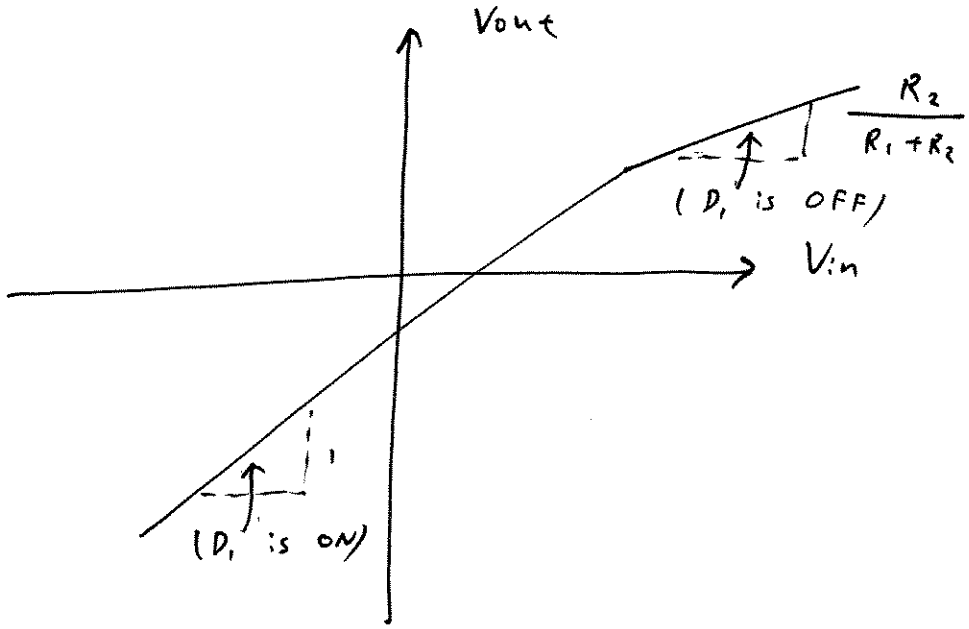
(25) a)


Vout
Vin
1
(D1 is ON)
R2
R1 + R2
(D1 is OFF)

b)

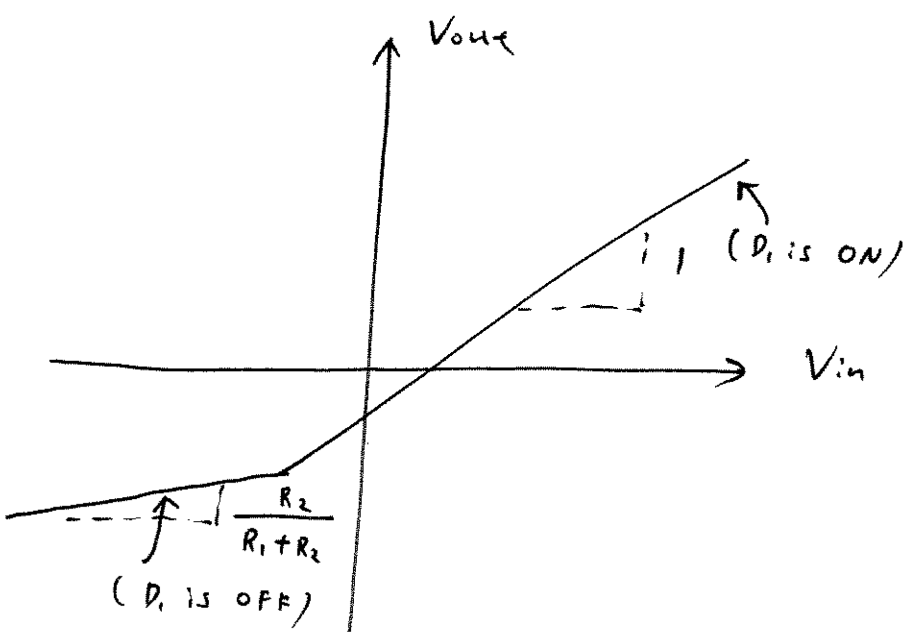

Vout
Vin
1
(D1 is ON)
R2
R1 + R2
(D1 is OFF)

c)



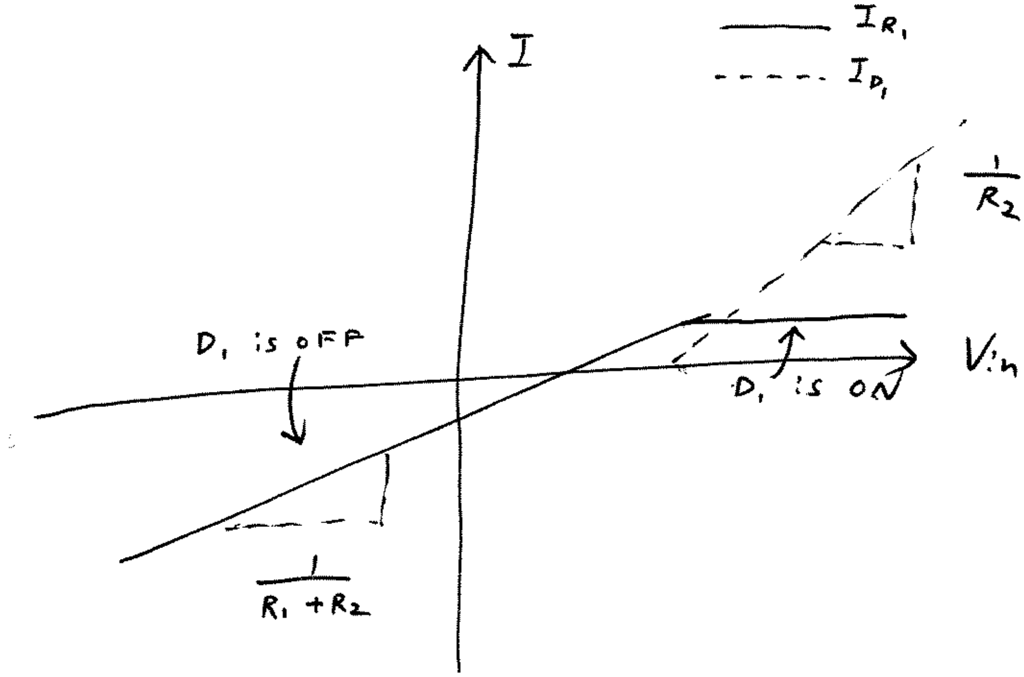
Vout
Vin
$\frac{R_2}{R_1+R_2}$
(D1 is OFF)
1
(D1 is ON)

d)

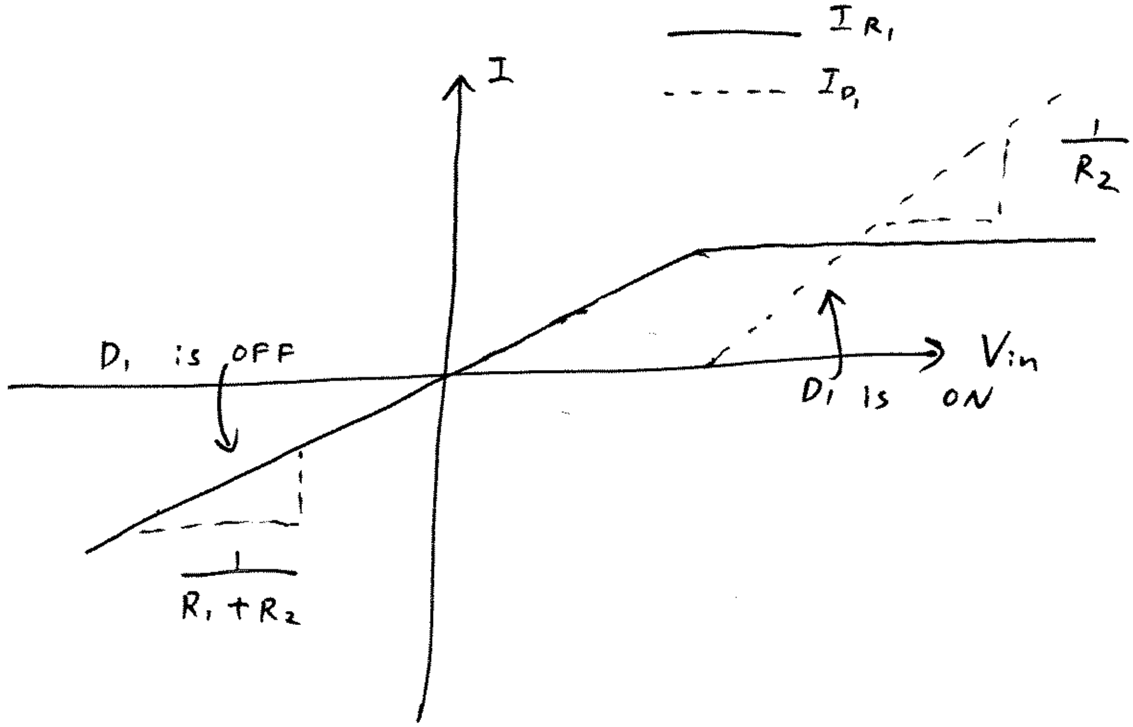


Vout
Vin
1
(D1 is ON)
$\frac{R_2}{R_1+R_2}$
(D1 is OFF)

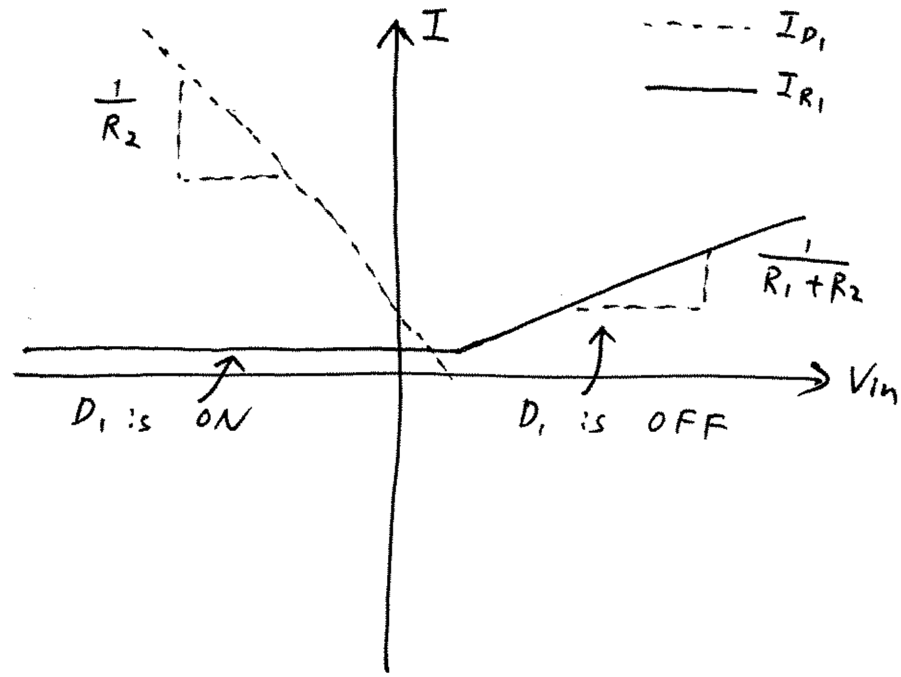
(26) a)


I
I_{R_1}
I_{D_1}
$\frac{1}{R_2}$
D_1 is OFF
D_1 is ON
V_{in}
$\frac{1}{R_1+R_2}$

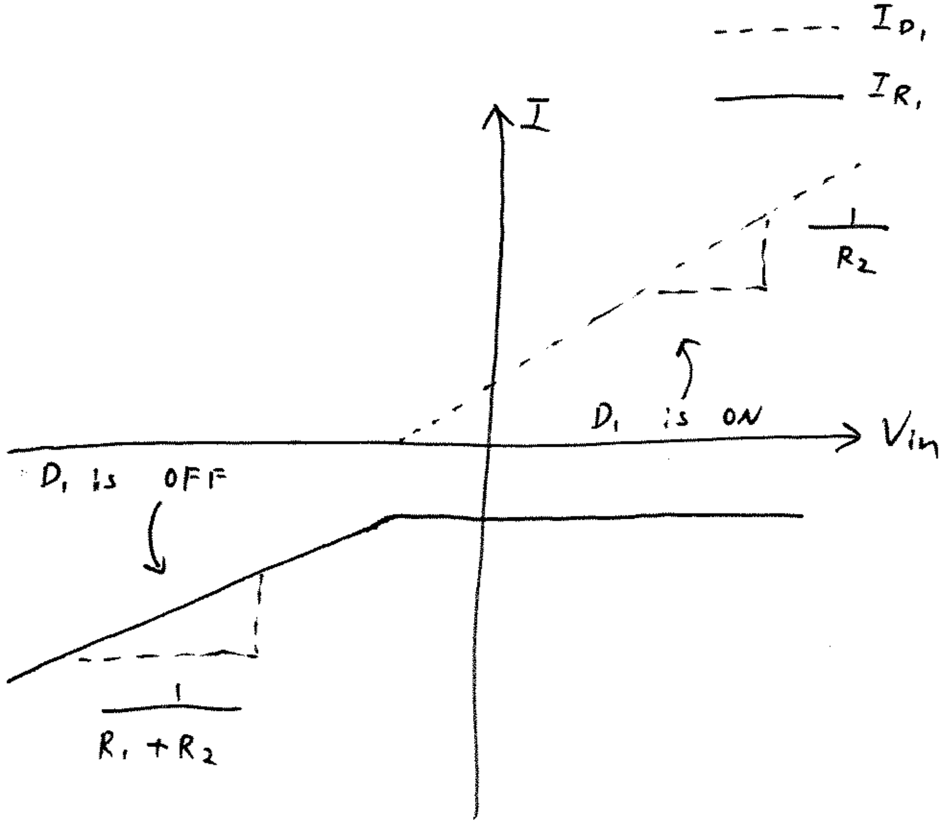
b)


I
I_{R_1}
I_{D_1}
$\frac{1}{R_2}$
D_1 is OFF
D_1 is ON
V_{in}
$\frac{1}{R_1+R_2}$

c/

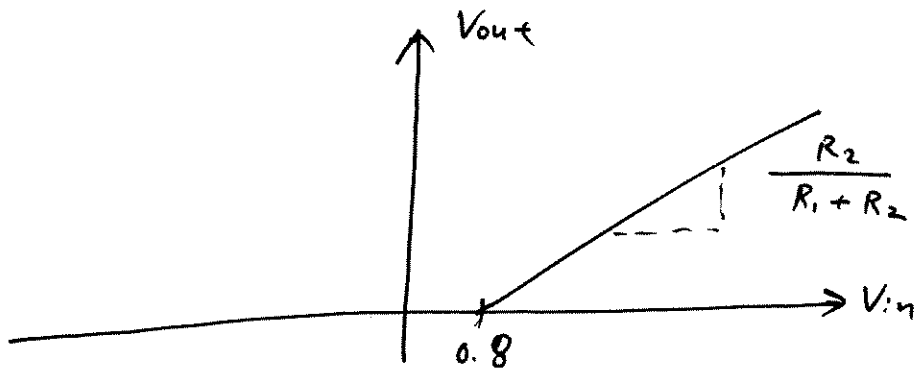

I
I_{D_1}
I_{R_1}
$\frac{1}{R_2}$
$\frac{1}{R_1+R_2}$
V_{in}
D_1 is ON
D_1 is OFF

d/


I_{D_1}
I_{R_1}
I
$\frac{1}{R_2}$
D_1 is ON
V_{in}
D_1 is OFF
$\frac{1}{R_1+R_2}$

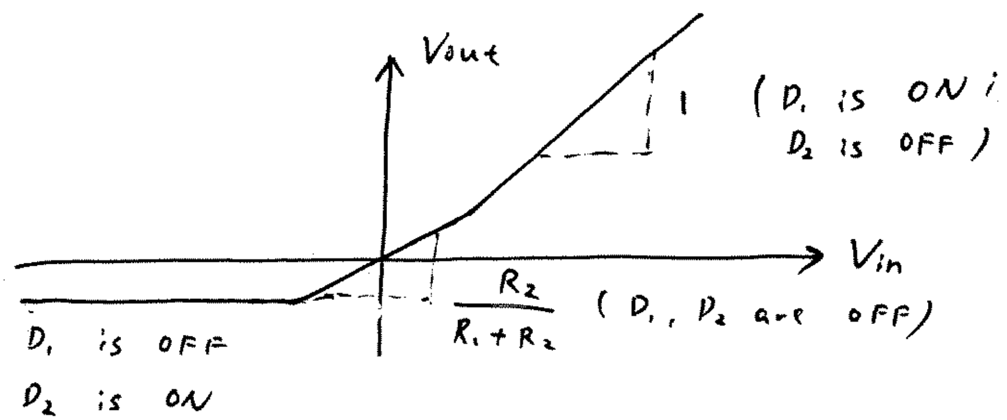
(27)

a)



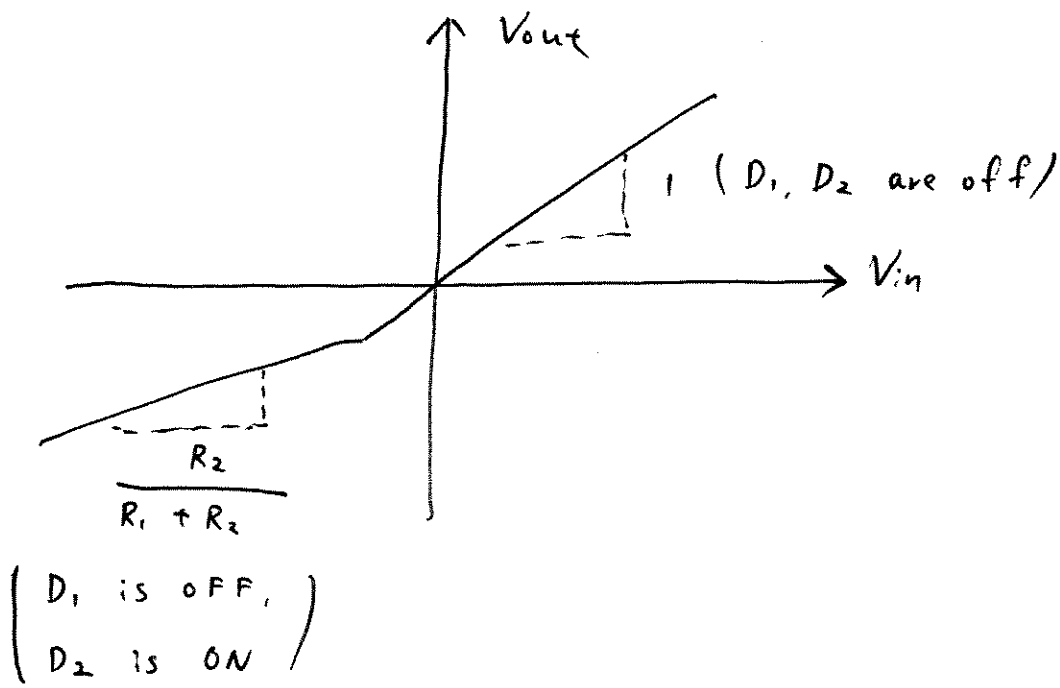
Vout
Vin
0.8
R2 / R1 + R2

b)



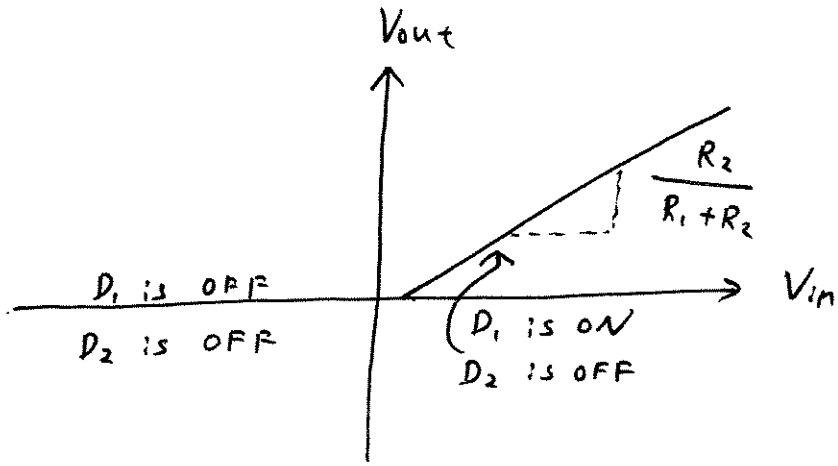
Vout
Vin
1 (D1 is ON, D2 is OFF)
R2 / R1 + R2 (D1, D2 are OFF)
D1 is OFF
D2 is ON

c)

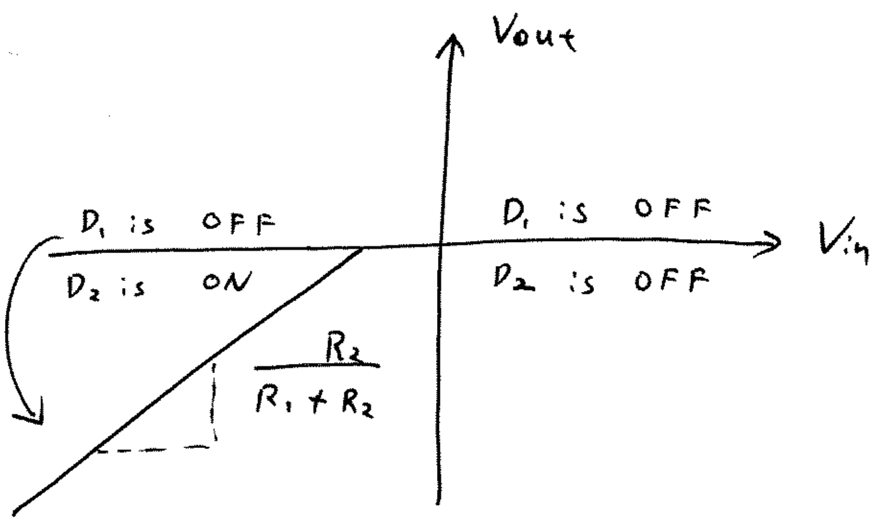


Vout
Vin
1 (D1, D2 are off)
R2 / R1 + R2
(D1 is OFF, D2 is ON)

d)

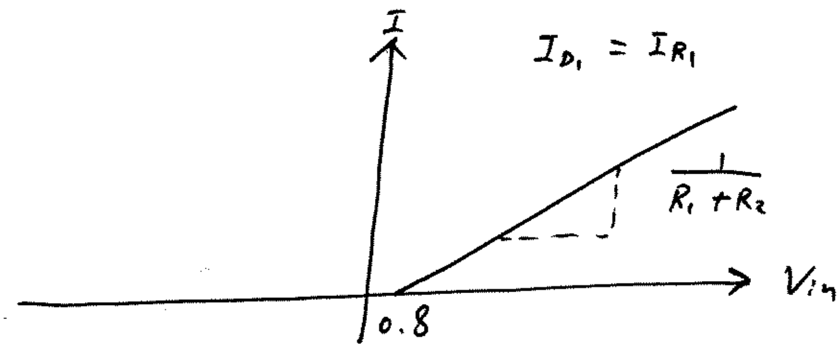

Vout
Vin
R2 / (R1 + R2)
D1 is OFF
D2 is OFF
D1 is ON
D2 is OFF

e)


Vout
Vin
D1 is OFF
D2 is ON
D1 is OFF
D2 is OFF
R2 / (R1 + R2)

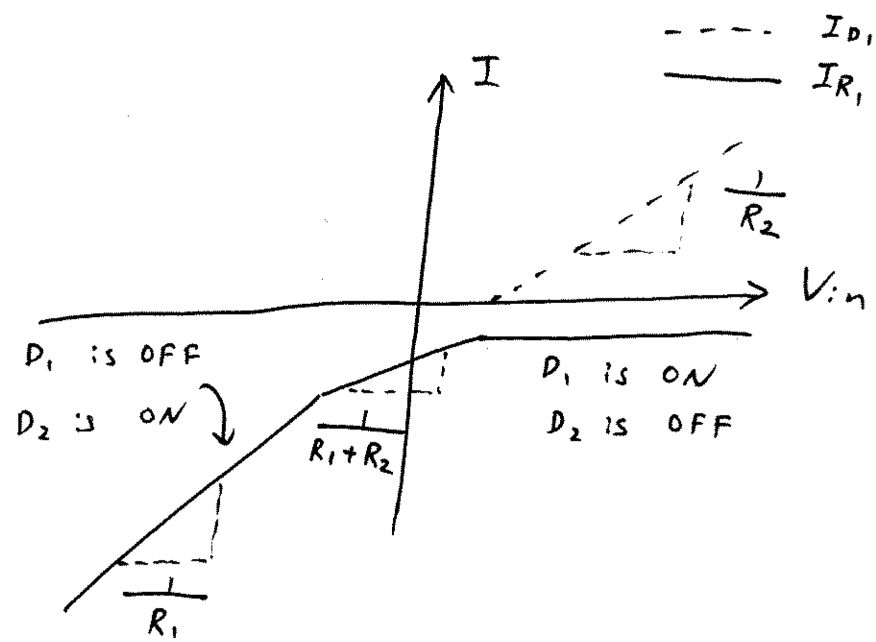
(28)

a)



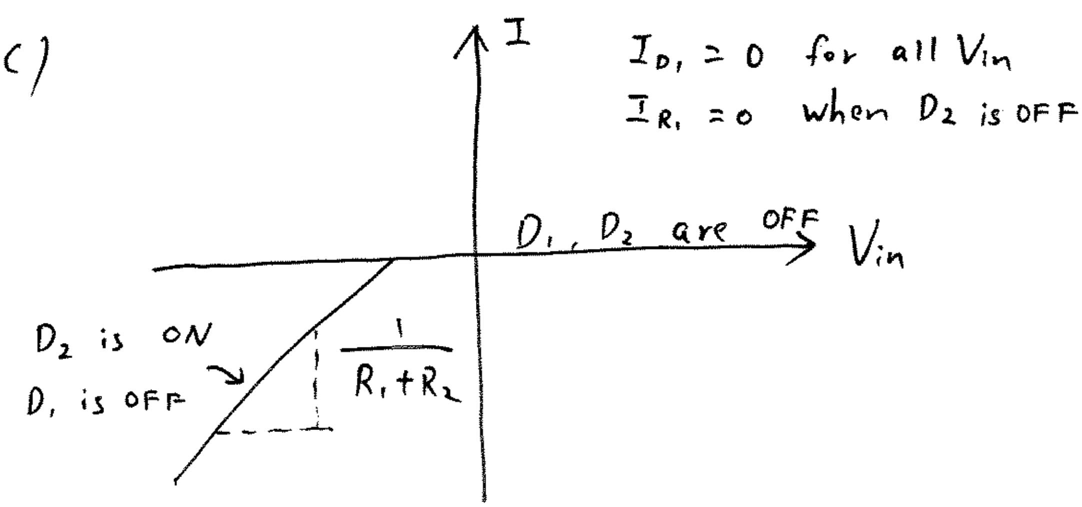
I
$I_{D_1} = I_{R_1}$
$\frac{1}{R_1 + R_2}$
V_{in}
0.8

b)



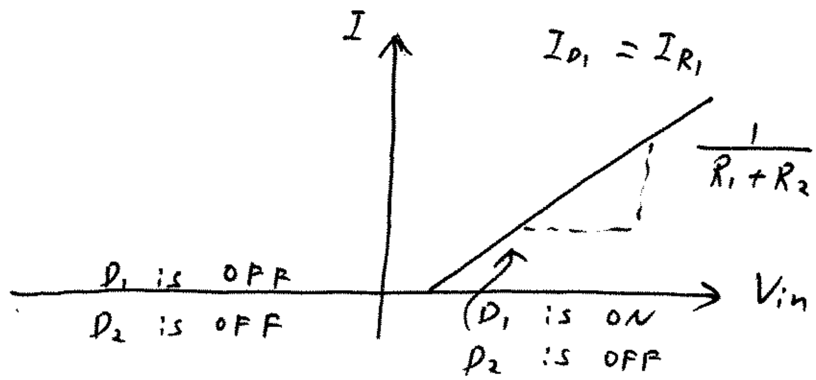
I
I_{D_1}
I_{R_1}
$\frac{1}{R_2}$
V_{in}
D_1 is OFF
D_2 is ON
D_1 is ON
D_2 is OFF
$\frac{1}{R_1 + R_2}$
$\frac{1}{R_1}$

c)

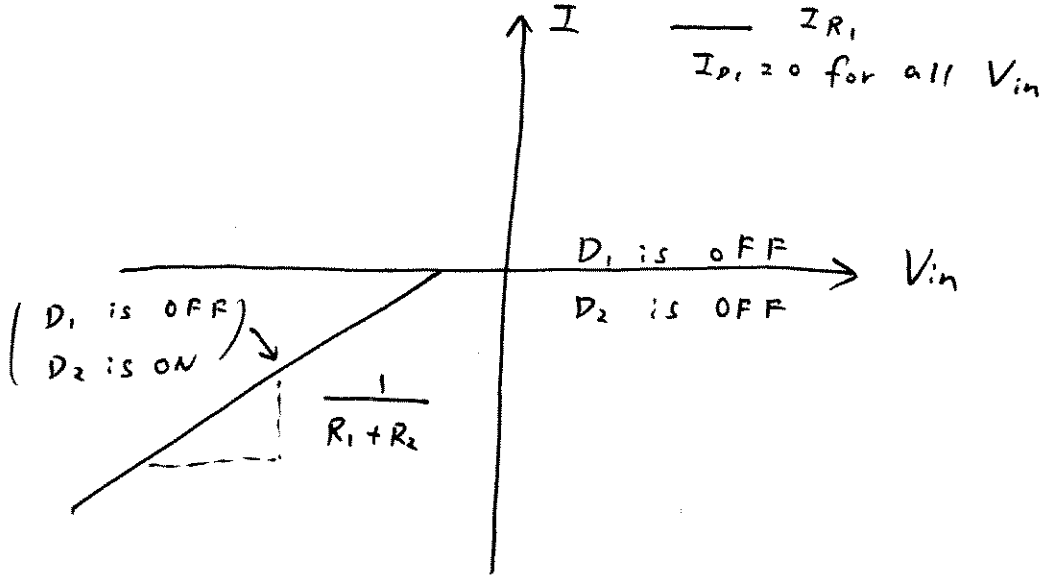


I
$I_{D_1} = 0$ for all V_{in}
$I_{R_1} = 0$ when D_2 is OFF
D_1, D_2 are OFF
V_{in}
D_2 is ON
D_1 is OFF
$\frac{1}{R_1 + R_2}$

d)

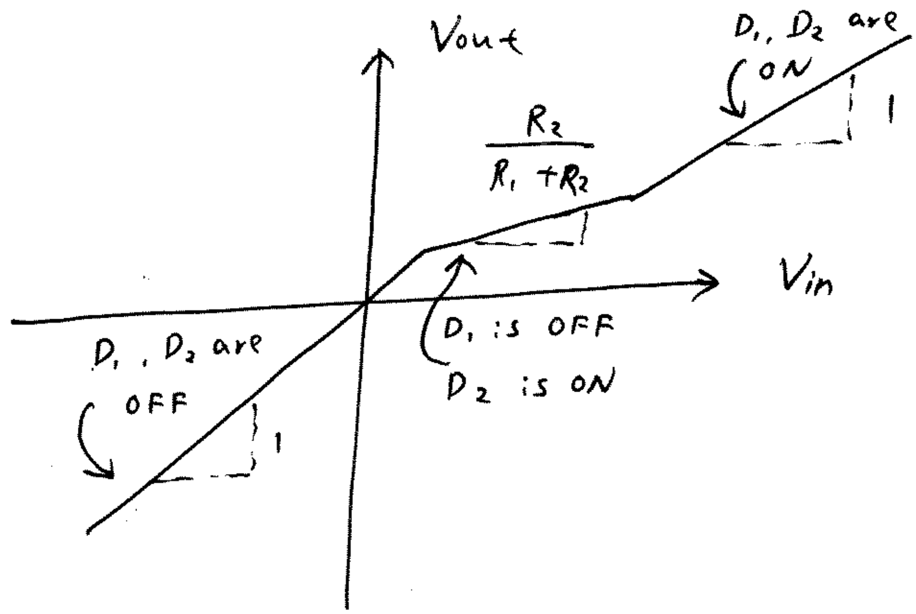

I
$I_{D_1} = I_{R_1}$
$\frac{1}{R_1 + R_2}$
V_{in}
D_1 is OFF
D_2 is OFF
D_1 is ON
D_2 is OFF

e)


I
I_{R_1}
$I_{D_1} = 0$ for all V_{in}
D_1 is OFF
D_2 is OFF
V_{in}
D_1 is OFF
D_2 is ON
$\frac{1}{R_1 + R_2}$

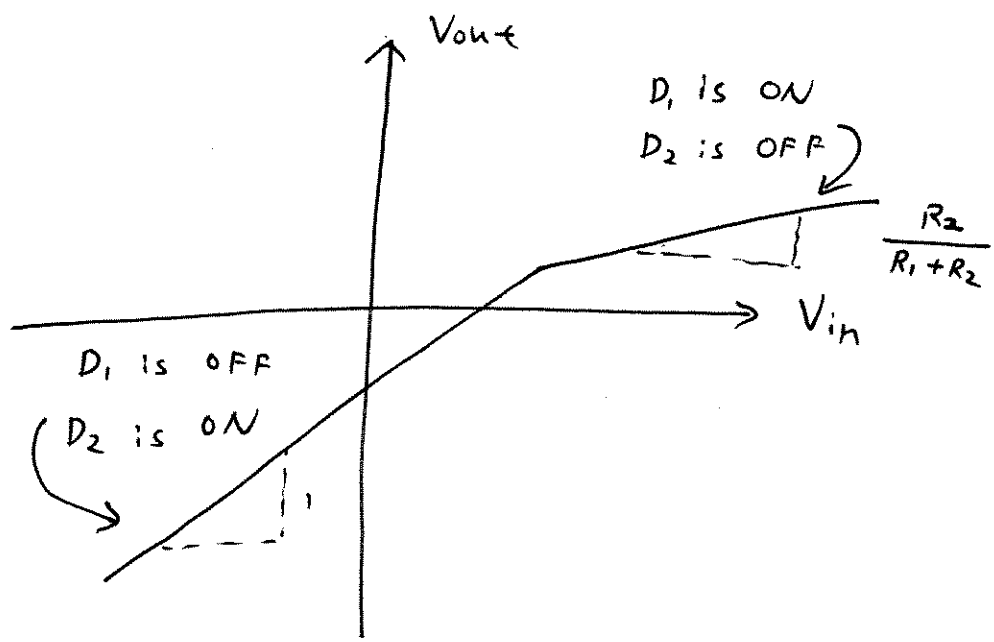
(29)

a)



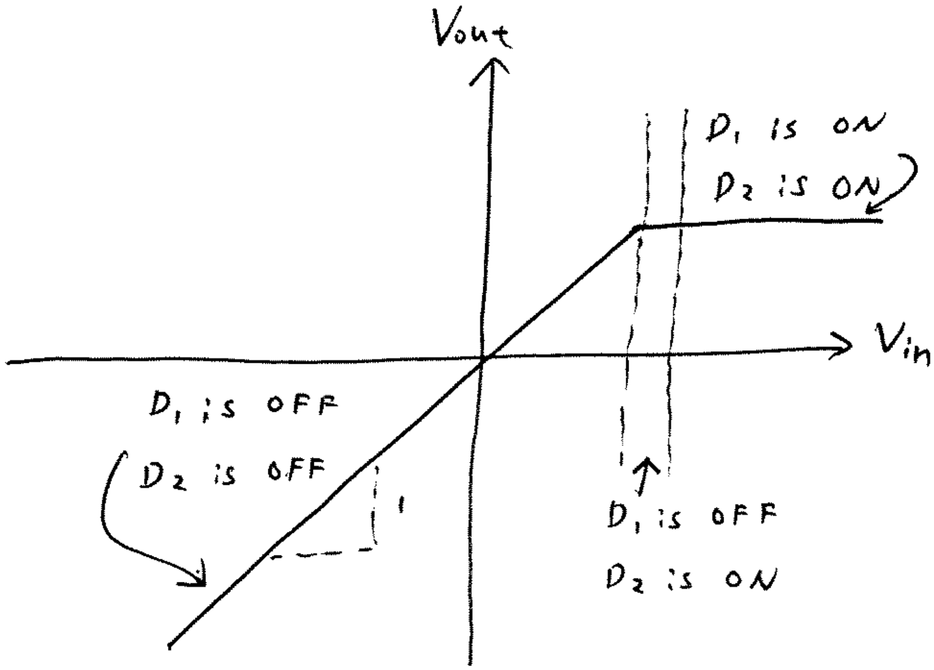
Vout
Vin
$\frac{R_2}{R_1+R_2}$
1
1
D_1, D_2 are ON
D_1 is OFF
D_2 is ON
D_1, D_2 are OFF

b)



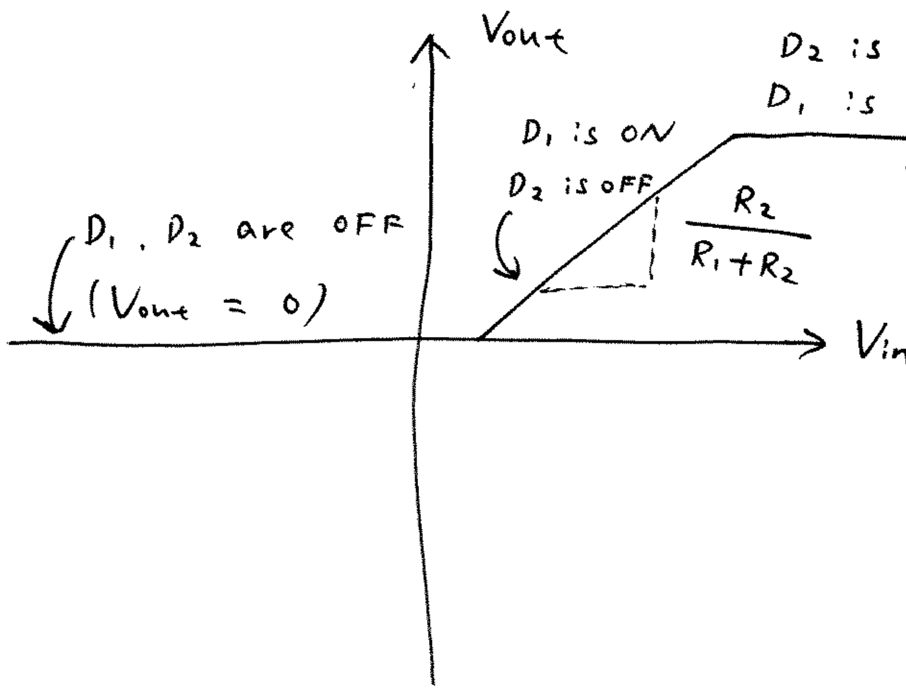
Vout
Vin
D_1 is ON
D_2 is OFF
$\frac{R_2}{R_1+R_2}$
1
D_1 is OFF
D_2 is ON

c)



Vout
Vin
D1 is ON
D2 is ON
D1 is OFF
D2 is OFF
1
D1 is OFF
D2 is ON

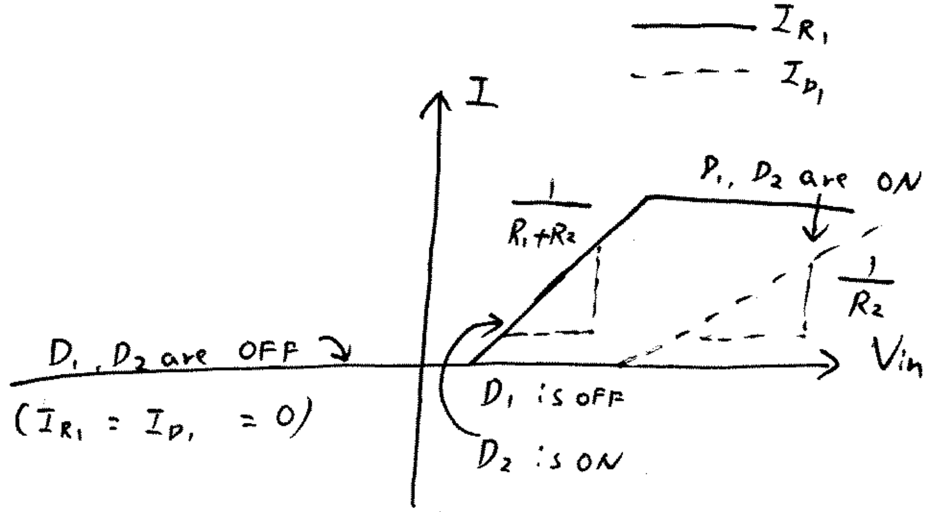
d)



Vout
Vin
D2 is ON
D1 is ON
D1 is ON
D2 is OFF
R2
R1+R2
D1, D2 are OFF
(Vout = 0)

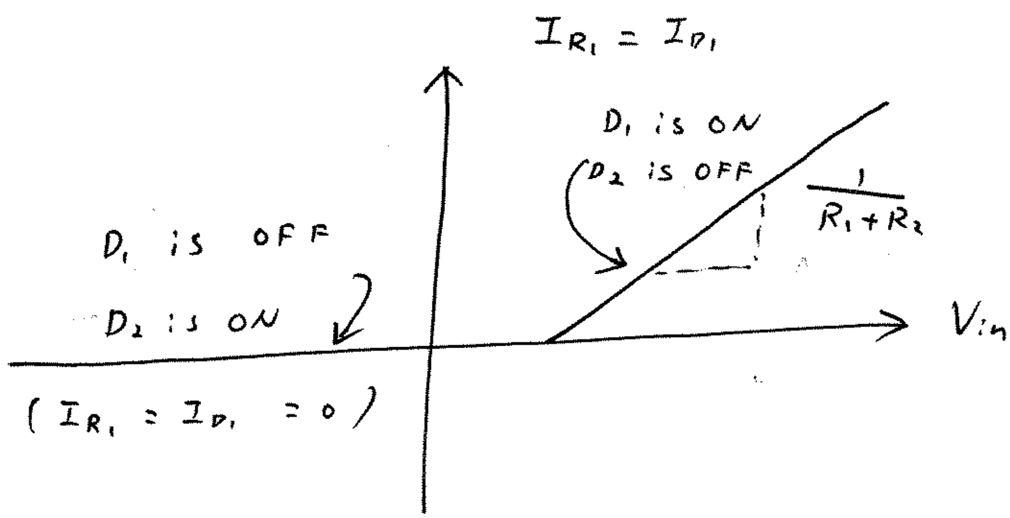
(30)

a)



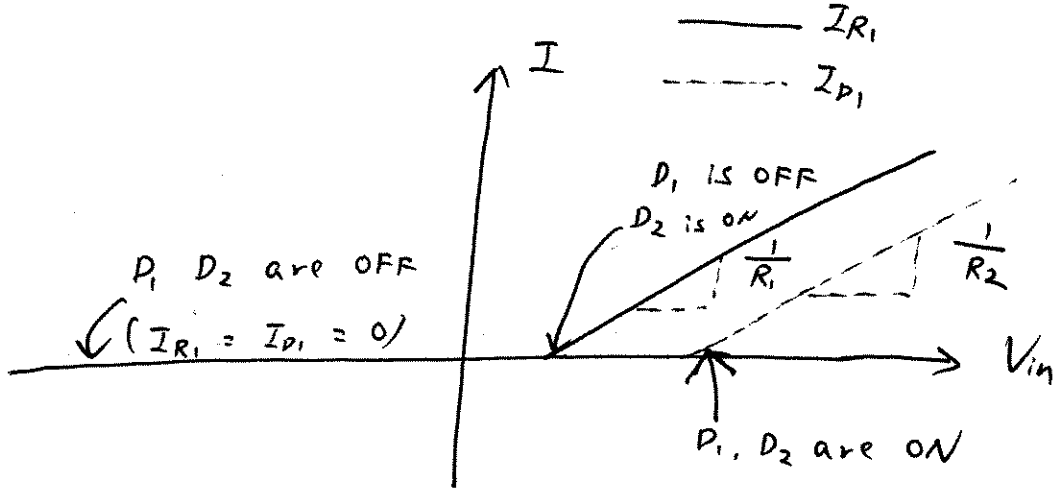
I_{R_1}
I_{D_1}
I
D_1, D_2 are ON
$\frac{1}{R_1+R_2}$
$\frac{1}{R_2}$
V_{in}
D_1, D_2 are OFF
$(I_{R_1} = I_{D_1} = 0)$
D_1 is OFF
D_2 is ON

b)

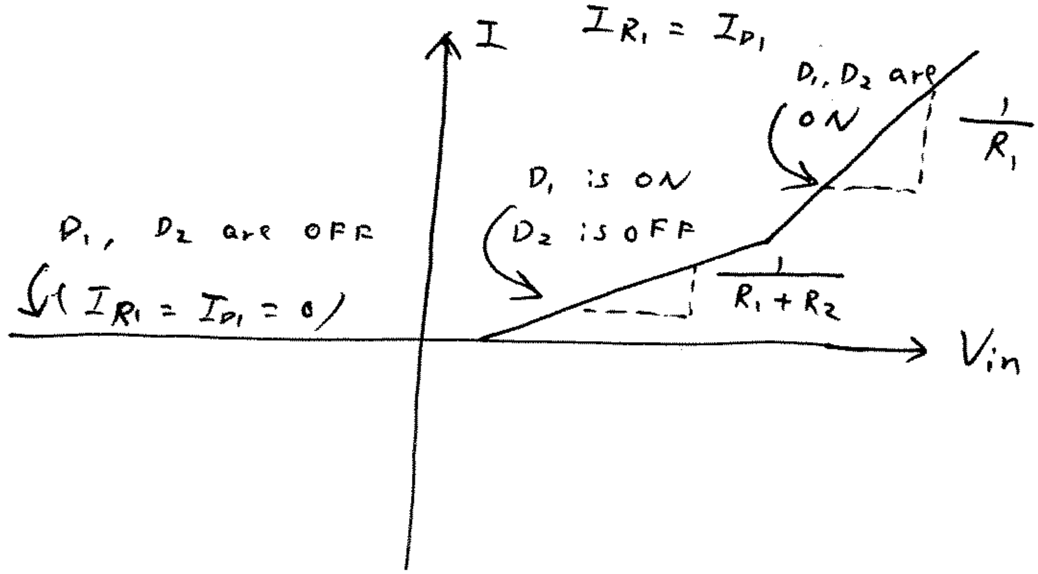


$I_{R_1} = I_{D_1}$
D_1 is ON
D_2 is OFF
$\frac{1}{R_1+R_2}$
D_1 is OFF
D_2 is ON
V_{in}
$(I_{R_1} = I_{D_1} = 0)$

c)


I
I_{R_1}
I_{D_1}
D_1 is OFF
D_2 is ON
$\frac{1}{R_1}$
$\frac{1}{R_2}$
D_1 D_2 are OFF
($I_{R_1} = I_{D_1} = 0$)
V_{in}
D_1, D_2 are ON

d)


I
$I_{R_1} = I_{D_1}$
D_1, D_2 are ON
$\frac{1}{R_1}$
D_1 is ON
D_2 is OFF
D_1, D_2 are OFF
($I_{R_1} = I_{D_1} = 0$)
$\frac{1}{R_1 + R_2}$
V_{in}

(31) a) when V_{in} changes from +2.4 V to +2.5 V,

D_1 is ON throughout the change.

$V_{out} \approx V_{in} - 0.8V$,

$\therefore$

i.e., V_{out} changes from +1.6V to +1.7V.

b) when V_{in} changes from +2.4V to +2.5V,

D_1 and D_2 are both ON.

$\therefore$ $V_{out} = V_{in} - V_{ON, D1}$,

i.e., V_{out} changes from +1.6 V to +1.7V.

c) when V_{in} changes from +2.4V to +2.5V,

D_1 and D_2 are both ON.

$V_{out} \approx V_{ON, D2}$,

i.e., V_{out} stays at +0.8V.

d) when V_{in} changes from +2.4V to +2.5V,

D_2 is ON.

$\therefore$ $V_{out} \approx V_{ON, D2}$,

i.e., V_{out} stays at +0.8V

(32) a) $V_{out} = i \times R_1$

$= 0.1\,mA \times 1\,k\Omega$

$= 0.1\,V$

b) $r_{d1} = r_{d2} = \dfrac{26\,mV}{3\,mA.}$ (Eq. 3.58)

$\simeq 8.67\,\Omega.$

$V_{out} = i \times (R_1 + r_{d2})$

$= 0.1\,mA\,(1.00867\,k\Omega)$

$\approx 1.009 \times 10^{-1}\,V$

c) $V_{out} = i \times r_{d2}$

$= 0.1\,mA \times 8.67$ (from (b))

$= 0.867\,mV$

d) $V_{out} = i \times (R_2 \,//\, r_{d2})$

$\approx i \times r_{d2}$ ($\because R_2 \gg r_{d2}$)

$= 0.867\,mV$

(33) a) $i_1 = i_{in}$

$= 0.1\ mA$

b) $i_{r_1} = i_{in}$

$= 0.1\ mA$

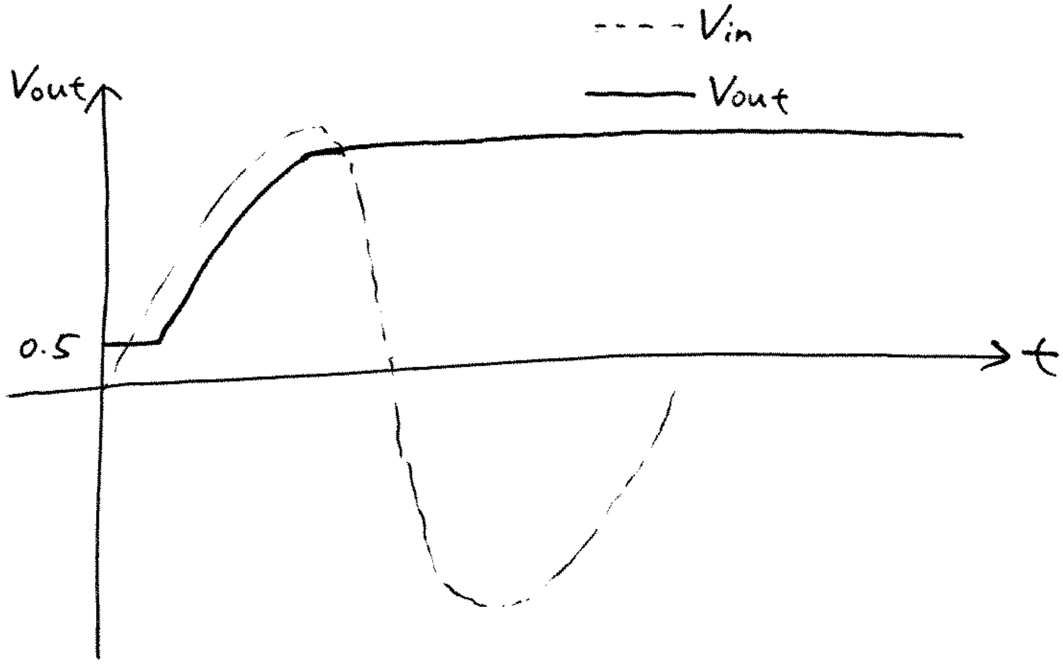
c) $i_{r_1} = i_{in}$

$= 0.1\ mA$

d) $i_{r_1} = i_{in}$

$= 0.1\ mA.$

(34).



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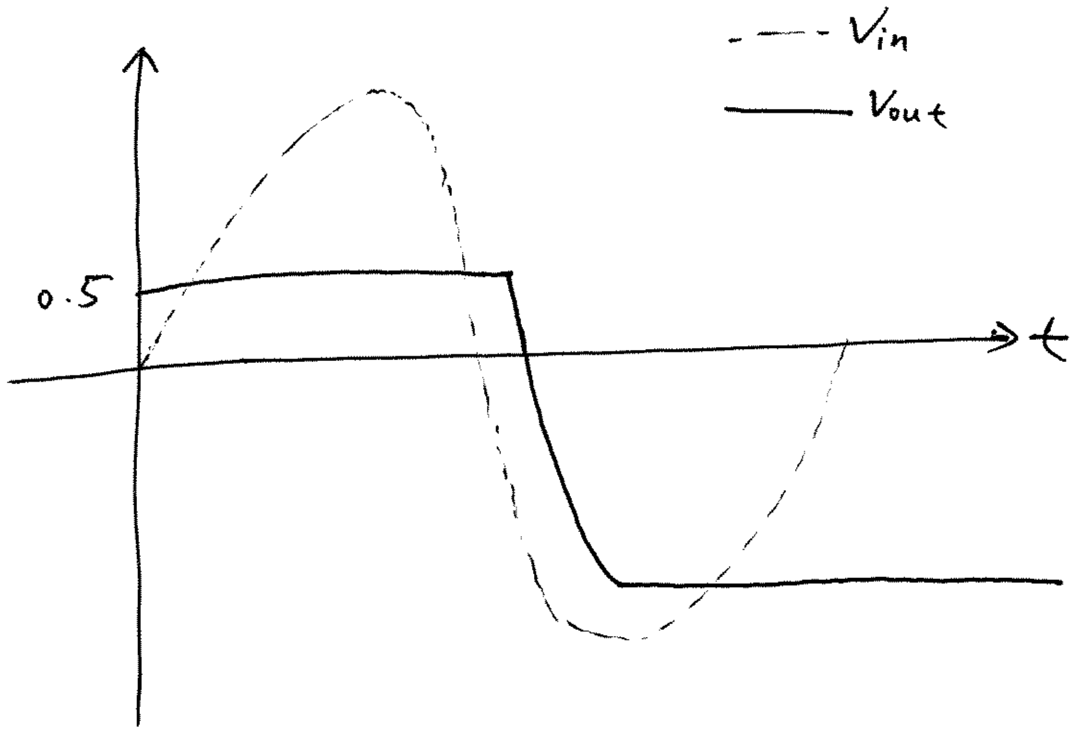


(36) From eq. (3.80),

$$\text{Ripple amplitude},\ V_R \approx \frac{V_P - V_{D,on}}{R_L C f_{in}}$$

$$= \frac{3.5 - 0.8}{10 \quad 1000 \times 10^{-6} \times 60}$$

$$= 0.45\,V$$

(37)

From Eq. (3.83),

$$V_R = \frac{I_L}{C f_{in}}$$

$$\because \quad V_R \lesssim 300\ mV$$

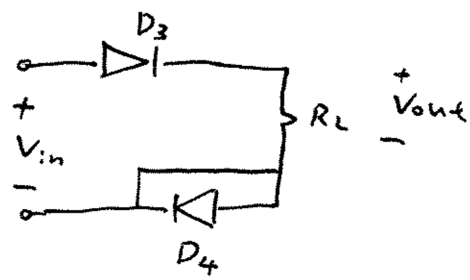
$$\frac{I_L}{C f_{in}} \lesssim 300\ mV$$

$$\therefore \quad C \geqslant \frac{I_L}{f_{in} \times 0.3}$$

$$C \geqslant \frac{0.5}{60 \times 0.3}$$

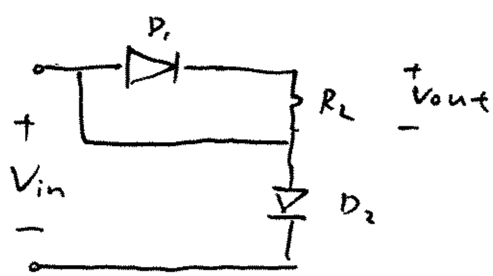
$$\text{i.e.} \quad C \geqslant 0.278\ F$$

(38) In the positive half of the cycle, when $V_{in+} > V_{in-}$, the circuit is operating as:



D_4 is shunted, and $D_3 - R_L$ forms a half-wave rectifier.

In the negative half of the cycle, when $V_{in-} > V_{in+}$, the circuit becomes:



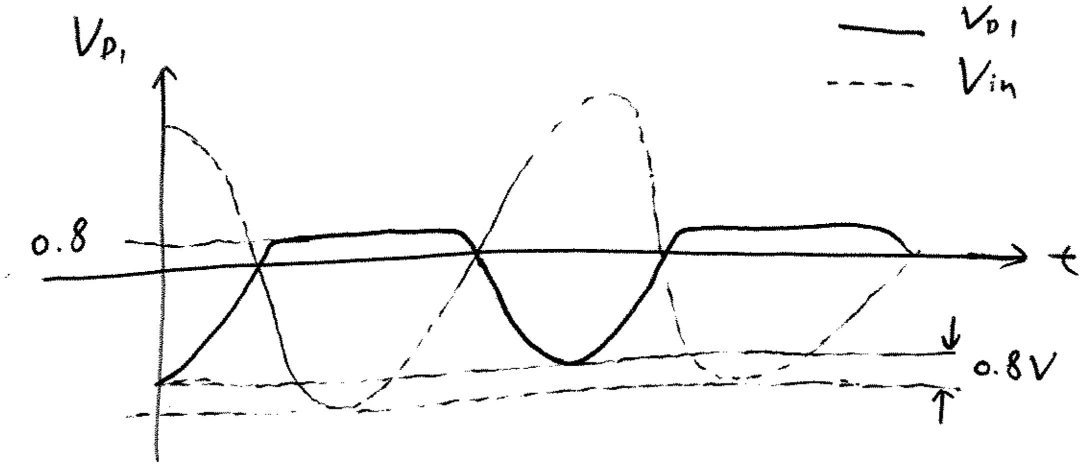
D_1 is shunted and is off.

Thus, $V_{out} = 0$.

Shunting the resistor load with a capacitor has no effect in the above two cases.

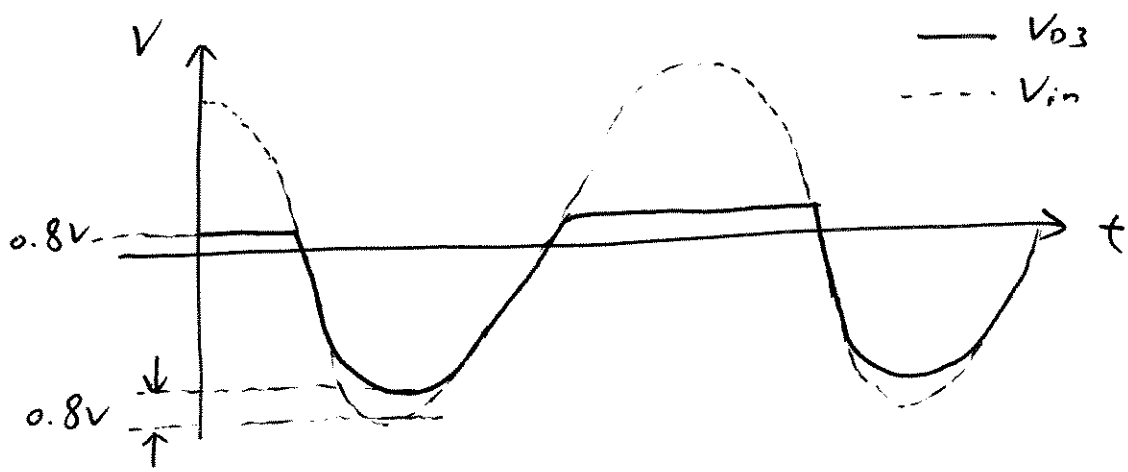
39

(i)

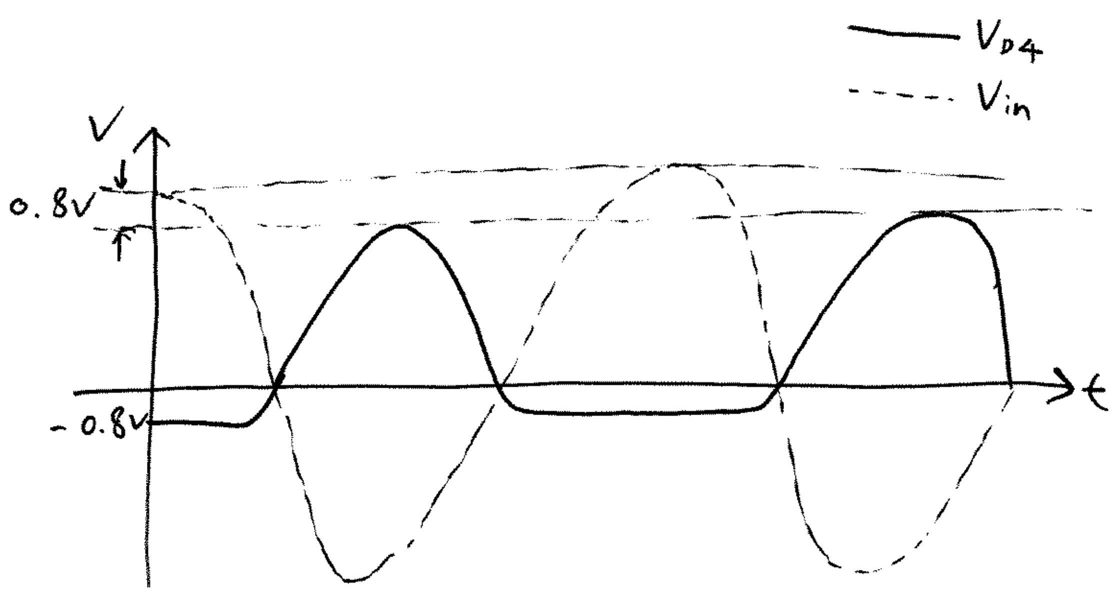


(ii) V_{D2} is same as V_{D1} (above)

(iii)



(iv)



(40) - This circuit would fail to function as a full-wave rectifier.

- It only rectifies for $V_{in-} > V_{in+}$ (Current flows through D_1 and D_2)
- But for $V_{in+} > V_{in-}$, there is no conduction path through the load.
- Thus, this circuit behave like a half-wave rectifier

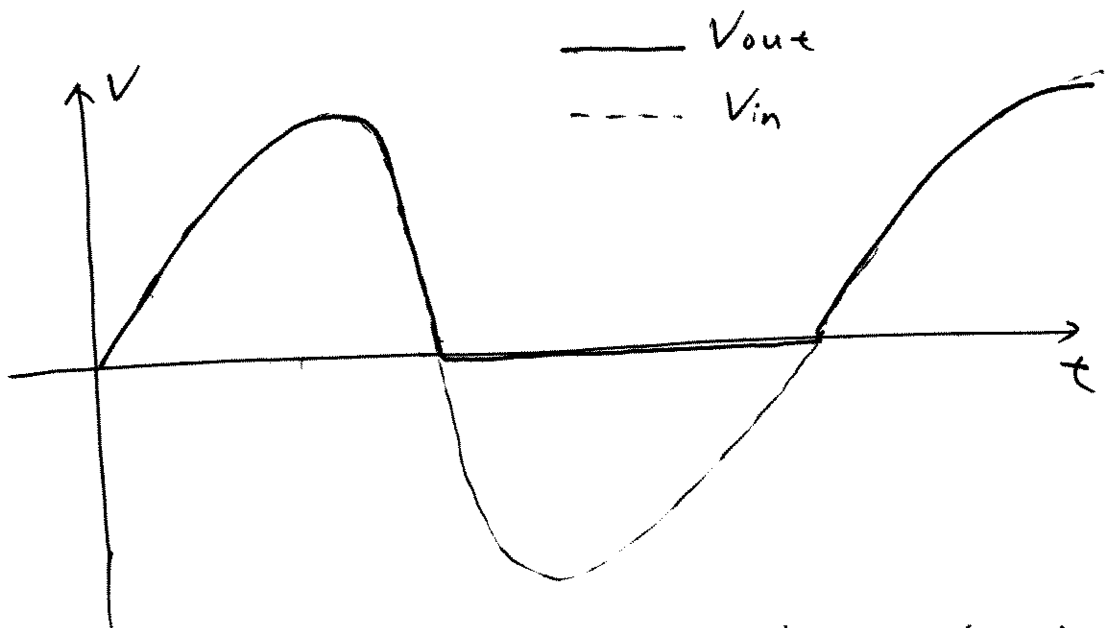
(41) Using Eq. (3.94),

$$V_R \approx \frac{1}{2} \cdot \frac{V_p - 2 V_{D,on}}{R_L C_1 f_{in}}$$

$$= \frac{1}{2} \cdot \frac{3 - 2 \times 0.8}{30 \times 1000 \times 10^{-6} \times 60}$$

$$= 0.389 V$$

(42)



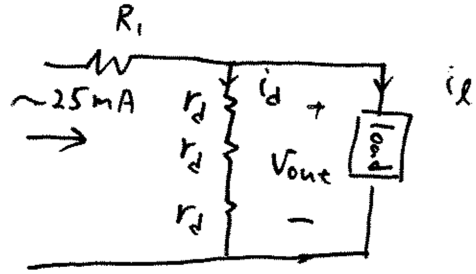
- With the two negative terminals shorted together, the circuit behaves like a half-wave rectifier.

- when $V_{in+} > V_{in-}$, D_3 and D_4 conduct as usual. There will be an additional path that bypasses D_4, since V_{in-} and V_{out-} are shorted. But this additional path causes no change to the Vout waveform.

- when $V_{in-} > V_{in+}$, both V_{out+} and V_{out-} track V_{in-}. V_{out+} connects to V_{in-} through D_1; V_{out-} connect to V_{in-} through the additional shorted path.

- Thus $(V_{out+}) - (V_{out-}) = 0$, i.e. $V_{out} = 0$

(43) The circuit can be simplified as:



First, find r_d:

$$r_d = \frac{V_T}{I_D} \quad \text{(from eq. 3.60)}$$

$$= \frac{26\,mV}{5\,mA}$$

$$= 5.2\,\Omega$$

Since $i_\ell = +1\,mA$.

$i_d = -1\,mA$.

∴ change in V_{out},

i.e. $V_{out} = (-1\,mA)(3 \times 5.2)$

$$= -15.6\,mV$$

(44) a) From Eq. (3.94),

the ripple amplitude, $V_R = \frac{1}{2} \cdot \frac{V_p - 2V_{D,on}}{R_L C_1 f_{in}}$

$= \frac{1}{2} \cdot \frac{5 - 2\times 0.8}{1000 \times 100\times 10^{-6} \times 60}$

$= 0.283\ V$

b) The ripple across the load,

$v_\ell = i \times 3r_d$,

where i is the change in current flowing through R_1, in series with the 3 diodes.

$\because r_d = \frac{V_T}{I_D}$

$\approx \frac{26\,mV}{5/R_1} = 5.2\,\Omega$

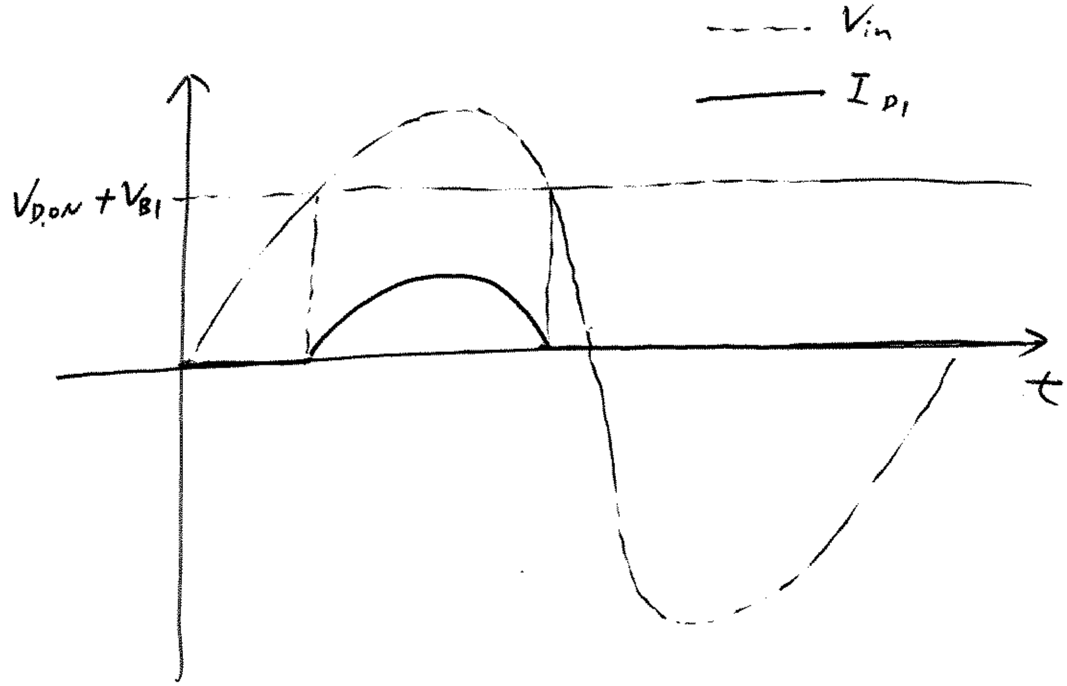
$i \approx \frac{V_R}{R_1 + 3r_d}$

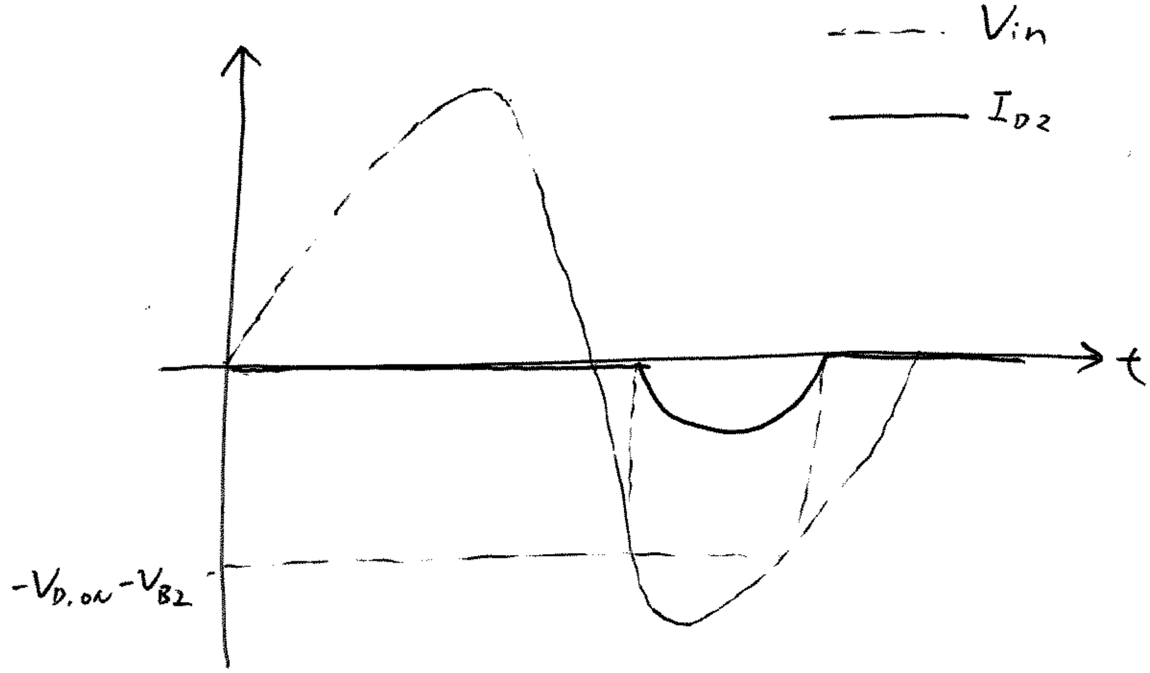
$= 0.279\ mA$

$\therefore v_\ell = 0.279\,mA \times 3 \times 5.2$

$= 4.35\ mV$

(45)





(46) With positive theshold = + 2.2V,

$$V_{B1} = 2.2 - 0.8$$

$$= +1.4V$$

with negative thres hold = −1.9V,

$$-V_{B2} = -1.9 + 0.8$$

$$= -1.1V.$$

$$V_{B2} = 1.1V$$

To meet the maximum current criterion,

Since $I_{R_1} = I_{D1}$ or I_{D2},

I_{D1} or I_{D2} is at max when

I_{R_1} is at max.

I_{R_1} is at max when $|V_R|$ is max,

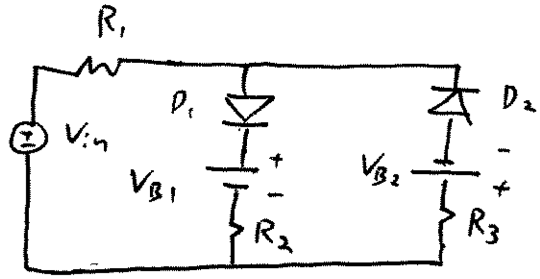
ie. $|V_R| = 5 - 1.9$

$$= 3.1V.$$

Since $I_{R_1} \leq 2$ mA.

$R_1 \geq \frac{3.1}{2mA}$, ie. $R_1 \geq 1550\Omega$

(47) The required circuit is:



Similar to Example 3.34,

$V_{B1} = V_{B2} = (2 - 0.8)V$

$= 1.2V$

To find R_2,

For $V_{in} > 2V$, $\frac{V_{out}}{V_{in}}$ has a slope of 0.5.

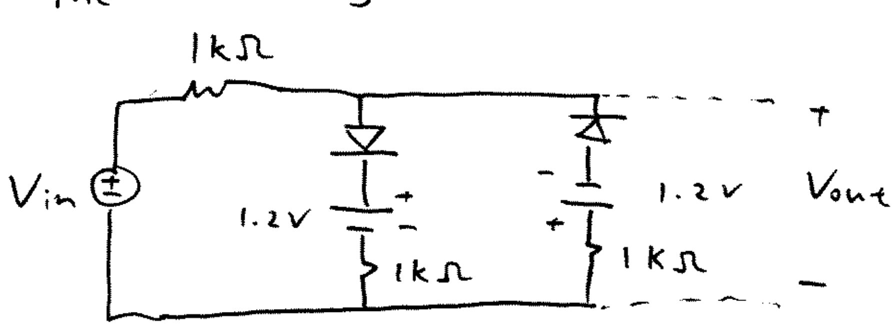
This implies $R_2 = R_1$

(R_1 and R_2 forms a volt. divider).

Similarly, $R_3 = R_1$

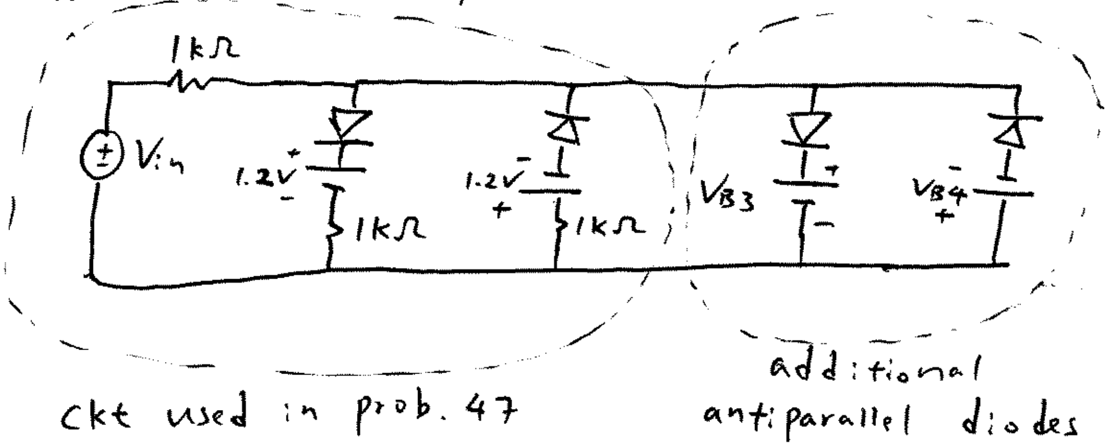
Thus, set $R_1 = R_2 = R_3 = 1\,k\Omega$.

The resulting circuit is:



(48) For $|V_{in}| < 4V$, the $V_{out} - V_{in}$ characteristic is similar to prob. (47).

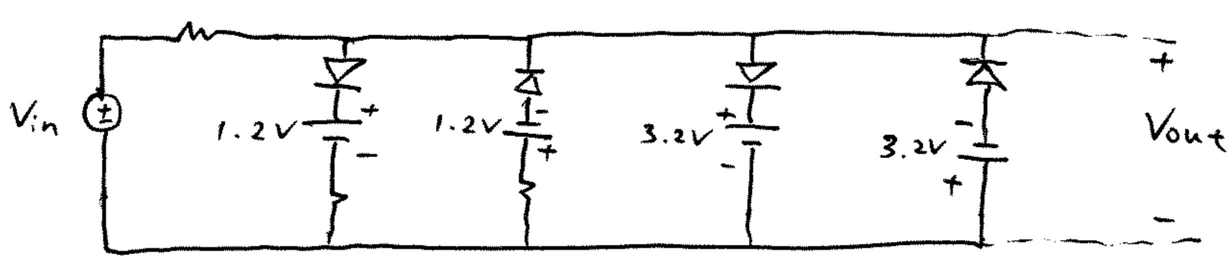
To get voltage limiting characteristic for $V_{in} > 4V$, and $V_{in} < -4V$, we can shunt the circuit used in prob(47) with two antiparallel diodes as below:



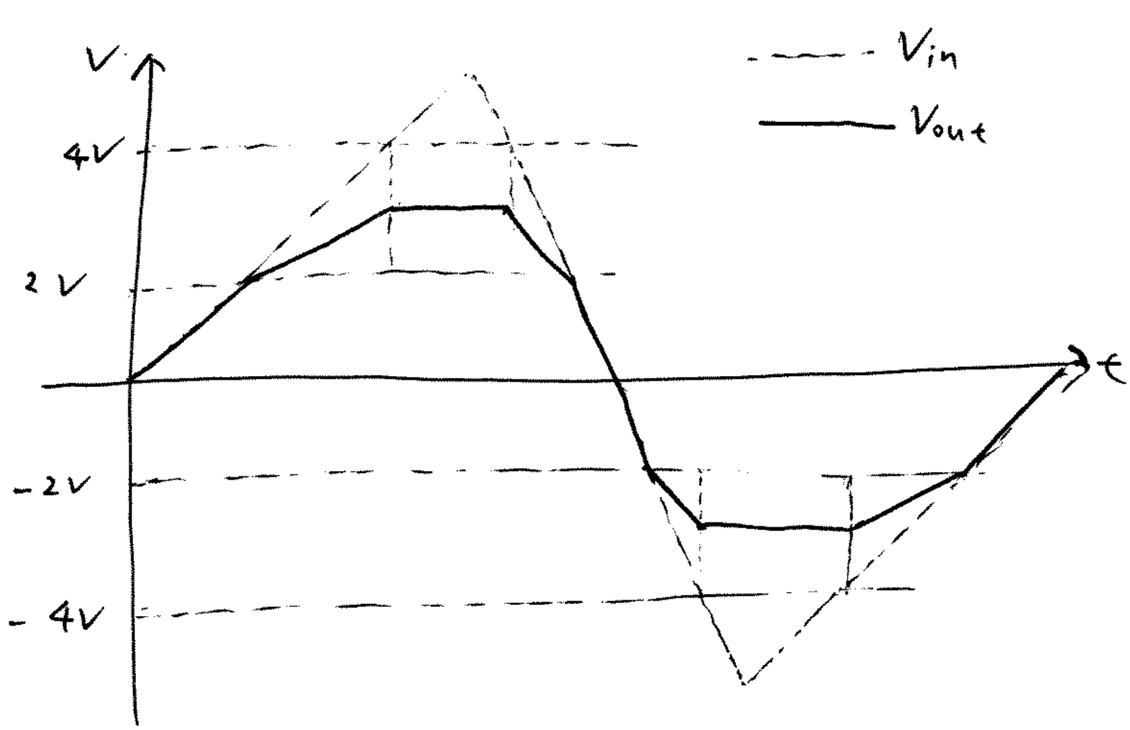
$$V_{B3} = V_{B4} = 4 - 0.8$$

$$= 3.2V$$

Resulting circuit is:

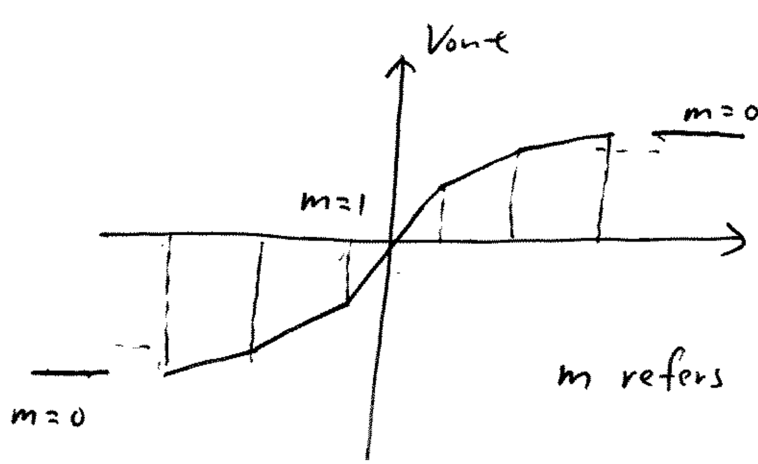


(49)



To get a better approximate of a sinusoid, the slope of the input-output characteristic should decrease <u>more gradually</u> from 1 to 0 through more sections.

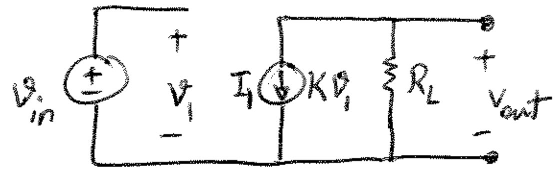
eg :



m refers to slope

Chapter 4

(4.1)



$K = 20\,mA/V$

$\left|\frac{V_{out}}{V_{in}}\right| = 15 \qquad V_{in} = V_1$

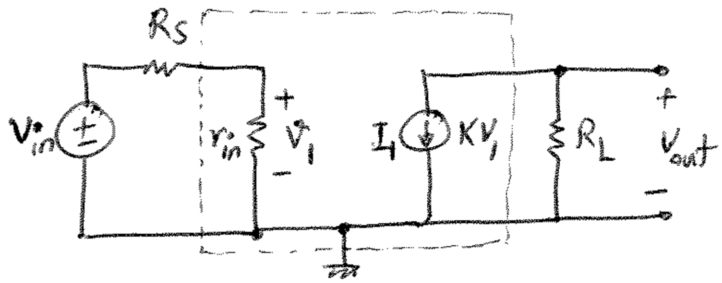
$$V_{out} = -I_1 R_L = -K R_L V_{in}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = -K R_L \Rightarrow \left|\frac{V_{out}}{V_{in}}\right| = K R_L$$

$$\Rightarrow K R_L = 15 \Rightarrow R_L = \frac{15}{20\,mA/V} = 750\,\Omega$$

$$\boxed{R_L = 750\,\Omega}$$

(4.2)



$$\frac{V_{out}}{V_{in}} = ?$$

$$V_1 = \frac{r_{in}}{r_{in} + R_S} V_{in}$$

$$I_1 = K V_1$$

$$V_{out} = -R_L I_1$$

$$\Rightarrow V_{out} = -K R_L V_1$$

$$\Rightarrow V_{out} = -K R_L \frac{r_{in}}{r_{in} + R_S} V_{in}$$

$$\Rightarrow \boxed{A_V = \frac{V_{out}}{V_{in}} = -K R_L \frac{r_{in}}{r_{in} + R_S}}$$

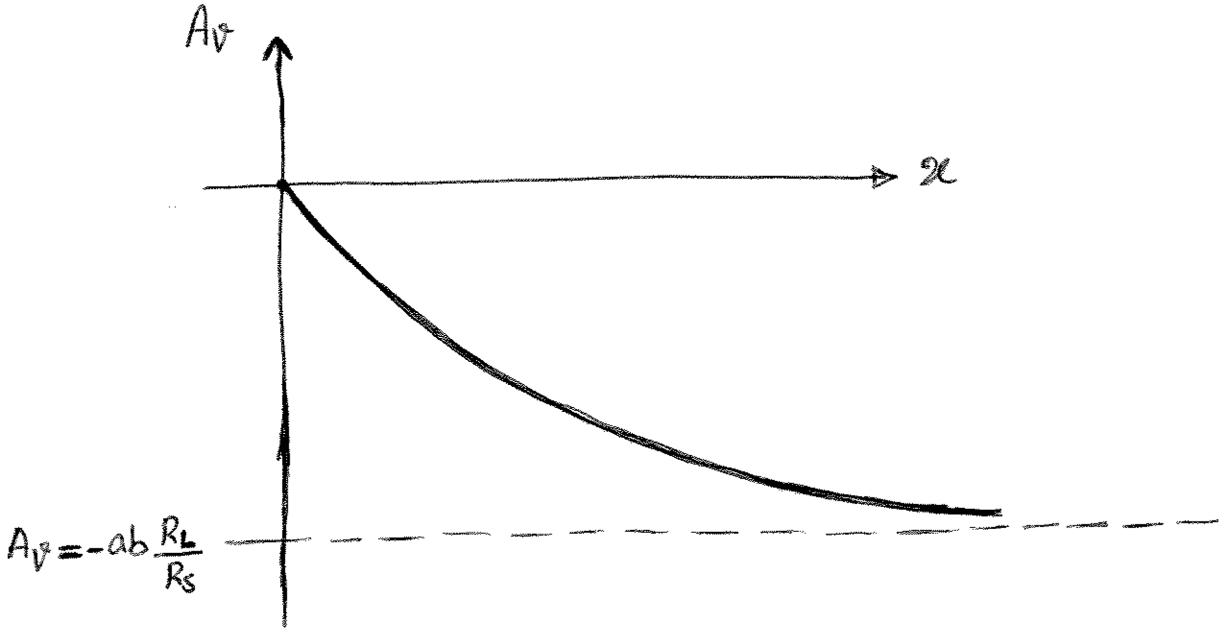
(4.3) From Solution for problem 4.2,

$a > 0$
$b > 0$
$x \geq 0$

$$A_v = -K R_L \frac{r_{in}}{r_{in} + R_S}$$

$$\xrightarrow[K = bx]{r_{in} = a/x} A_v = -bx R_L \frac{a/x}{a/x + R_S} = -b R_L \frac{a}{\frac{a}{x} + R_S}$$

$$\Rightarrow \boxed{A_v = -b R_L \left(\frac{x}{1 + \frac{R_S}{a} x} \right)}$$



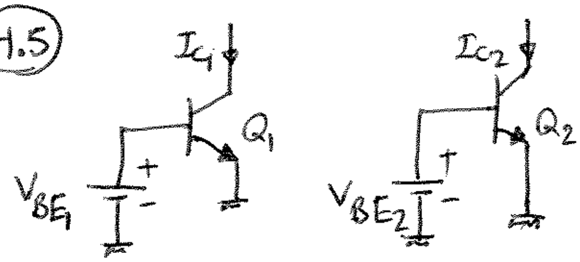
(4.4) From equation (4.8) page 136,

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}} \qquad W_B \equiv \text{width of the Base}$$

if $W_B \uparrow 2 \Rightarrow I_C \downarrow 2$

Collector current decreases by a factor of two

(4.5)



$V_T = 26\,mV$

$I_{C_1} = I_{C_2}$

$V_{BE_1} - V_{BE_2} = 20\,mV$

$$I_C = \frac{A_E q D_n n_i^2}{N_E W_B}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$$ equation (4.8) page 136

$$\Rightarrow I_C \simeq \frac{A_E q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE}}{V_T}}$$ $A_E \equiv$ Cross Section

if $I_{C_1} = I_{C_2}$

$$\Rightarrow A_{E_1} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_1}}{V_T}} = A_{E_2} \frac{q D_n n_i^2}{N_E W_B} e^{\frac{V_{BE_2}}{V_T}}$$

$$\Rightarrow \frac{A_{E2}}{A_{E_1}} = \frac{e^{V_{BE1}/V_T}}{e^{V_{BE2}/V_T}}$$

$$\Rightarrow \frac{A_{E2}}{A_{E_1}} = e^{(V_{BE_1} - V_{BE2})/V_T} = e^{20\,mV/26\,mV}$$

$$\Rightarrow \boxed{\frac{A_{E2}}{A_{E_1}} = e^{\frac{20}{26}} \simeq 2.16}$$

(6a) $I_X = 1^{mA} \Rightarrow I_{Q_1} = I_{Q_2} = 0.5^{mA}$

$$I_{Q_1} = I_{S_1} e^{\frac{V_{BE1}}{V_T}} \Rightarrow 5\times10^{-4} = 3\times10^{-16} e^{\frac{V_B}{26^{mV}}}$$

$$\Rightarrow V_B = 26^{mV} \ln\left(\tfrac{5}{3}\times10^{12}\right) \Rightarrow \boxed{V_B \simeq 731.7^{mV}}$$

(6b) $I_Y = I_{S_3} e^{\frac{V_B}{V_T}}$

$$\Rightarrow I_{S_3} = I_Y e^{-\frac{V_B}{V_T}} = 2.5\times10^{-3} \times e^{-\frac{V_B}{26^{mV}}} = 2.5\times10^{-3} \times \frac{1}{\frac{5}{3}\times10^{12}}$$

$$\Rightarrow \boxed{I_{S_3} = 1.5\times10^{-15} A}$$

(7a) $I_X = I_1 + I_2$

$$\Rightarrow I_X = I_{S_1} e^{\frac{V_B}{V_T}} + I_{S_2} e^{\frac{V_B}{V_T}} \Rightarrow I_X = (I_{S_1} + I_{S_2}) e^{\frac{V_B}{V_T}}$$

$$\Rightarrow V_B = V_T \ln\left(\frac{I_X}{I_{S_1} + I_{S_2}}\right) \xrightarrow{I_{S_1} = 2I_{S_2}} \boxed{V_B = V_T \ln\left(\frac{I_X}{\frac{3}{2} I_{S_1}}\right)}$$

$$V_B = 26\times10^{-3} \ln\left(\frac{1.2\times10^{-3}}{\frac{3}{2}\times5\times10^{-16}}\right) \Rightarrow \boxed{V_B \simeq 730.6\ mV}$$

(7b) Transistors at the edge of the active mode $\Rightarrow V_C = V_B$

applying KVL, we have:

$$V_{CC} = R_C I_X + V_B \Rightarrow \boxed{R_C = \frac{V_{CC} - V_B}{I_X}}$$

$$\Rightarrow R_C = \frac{2.5 - 0.73}{1.2\times10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 1475\ \Omega}$$

(8a) Same as 7a,

$$\boxed{V_B \simeq 730.6\,mV}$$

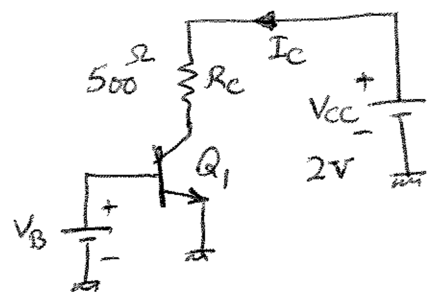
(8b) According to 7b,

$$R_C = \frac{V_{CC} - V_B}{I_X} = \frac{1.5 - 0.73}{1.2 \times 10^{-3}}$$

$$\Rightarrow \boxed{R_C \simeq 642\ \Omega}$$

(9) Q_1 is at the edge of the active region $\Rightarrow V_C = V_B$

applying KVL, we have:



$$V_{CC} = R_C I_C + V_C$$

$$\xrightarrow{V_C = V_B} V_{CC} = R_C I_C + V_B$$

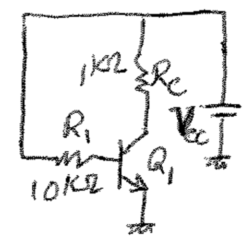
$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

$$\Rightarrow 500\Omega \times 5 \times 10^{-16} e^{\frac{V_B}{26\,mV}} + V_B = 2V$$

Using numerical methods or simply, trial & error:

$$\boxed{V_B \simeq 760\ mV}$$

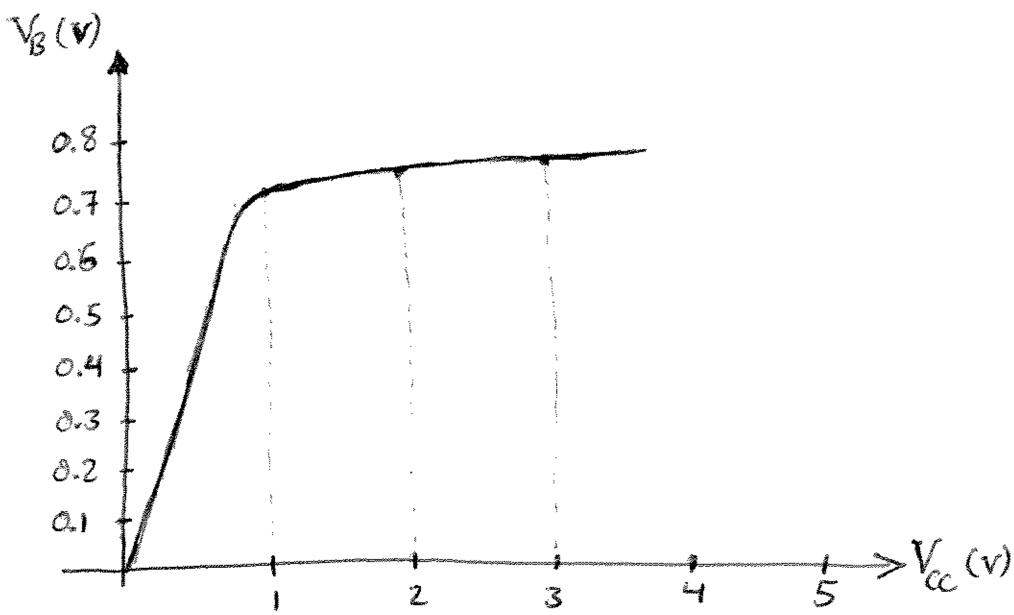
(10) Q_1 at the edge of saturation $\Rightarrow V_C = V_B$



Hence: $V_{CC} = R_C I_C + V_B$

$$\Rightarrow V_{CC} = R_C I_S e^{V_B/V_T} + V_B$$

$$\xrightarrow{I_S = 3\times10^{-16} A} V_{CC} = 3\times10^{-13} e^{V_B/V_T} + V_B \xrightarrow[V_{CC}=2^V]{\text{with}} \boxed{V_B \simeq 755 \text{ mV}}$$



(11) Assuming $I_E \simeq I_C$, we can write:

Applying KVL: $1.5^V = V_{BE} + V_X$ where $V_X = 1^{k\Omega} \times I_E$

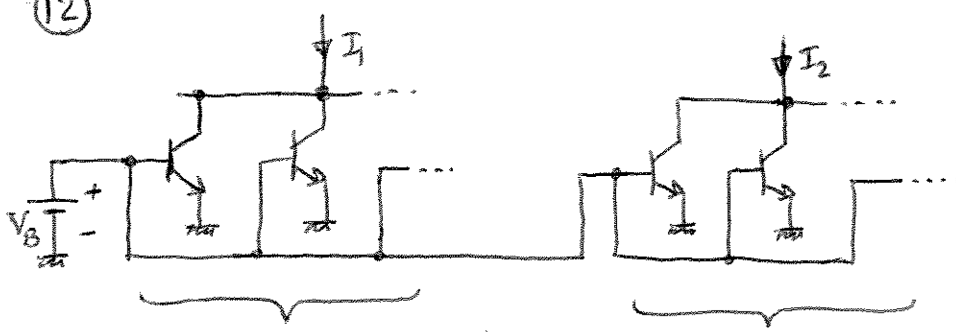
Hence: $1.5 = V_{BE} + 1^{k\Omega} \times I_C$

$$\Rightarrow 1.5 = V_{BE} + 1^{k\Omega} \times I_S e^{\frac{V_{BE}}{V_T}}$$

$$\xrightarrow[V_T = 26mV]{I_S = 6\times10^{-16} A} 1.5 = V_{BE} + 6\times10^{-13} e^{V_{BE}/26^{mV}} \Rightarrow \boxed{V_{BE} \simeq 724.5^{mV}}$$

$$V_X = 1.5 - V_{BE} \Rightarrow \boxed{V_X \simeq 775.5 \text{ mV}}$$

(12)



$$\left.\begin{aligned} I_1 &= n\, I_C = n\, I_S\, e^{V_B/V_T} \\ I_2 &= m\, I_C = m\, I_S\, e^{V_B/V_T} \end{aligned}\right\} \Rightarrow \frac{I_1}{I_2} = \frac{n}{m}$$

$$\Rightarrow \boxed{\frac{n}{m} = \frac{1\,mA}{1.5\,mA} = \frac{2}{3}} \xrightarrow{\text{choose}} \boxed{\begin{aligned} n &= 2 \\ m &= 3 \end{aligned}}$$

$$I_1 = n\, I_C = n\, I_S\, e^{\frac{V_B}{V_T}}$$

$$\Rightarrow I_1 = n \times 3\times 10^{-16}\, e^{\frac{V_B}{26m}} = 1^{mA} \overset{n=2}{\Longrightarrow} \boxed{V_B \simeq 750^{mV}}$$

(13) Using the same technique as in problem 12, we have:

$$\frac{n_1}{I_1} = \frac{n_2}{I_2} = \frac{n_3}{I_3}$$

$$\Rightarrow \frac{n_1}{0.2} = \frac{n_2}{0.3} = \frac{n_3}{0.45} \Rightarrow \boxed{\frac{n_1}{4} = \frac{n_2}{6} = \frac{n_3}{9}}$$

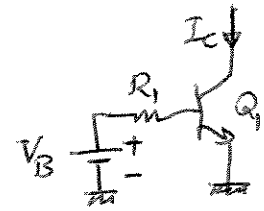
So lets choose $\boxed{\begin{cases} n_1 = 4 \\ n_2 = 6 \\ n_3 = 9 \end{cases}}$

Hence,

$$I_1 = n_1 I_S e^{V_B/V_T} \Rightarrow 0.2 \times 10^{-3} = 4 \times 3 \times 10^{-16} e^{\frac{V_B}{26mV}}$$

$$\Rightarrow \boxed{V_B \simeq 672 \text{ mV}}$$

(14) From KVL,



$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$I_B = \frac{I_C}{\beta} = \frac{1^{mA}}{100} \Rightarrow \boxed{I_B = 10^{-5} A}$$

$$V_{BE_{Q_1}} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26\times10^{-3} \ln\left(\frac{10^{-3}}{7\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE_{Q_1}} \simeq 727.7 \, mV}$$

Therefore,

$$V_B = R_1 I_B + V_{BE_{Q_1}}$$

$$\simeq 10^{K\Omega} \times 10^{-5} A + 728\times10^{-3}$$

$$\Rightarrow V_B \simeq 0.1 + 0.728 \Rightarrow \boxed{V_B \simeq 0.828 \text{ V}}$$

(15) According to the solution for problem 14, we have:

Applying KVL: $V_B = R_B I_B + V_{BE}$

$$\Rightarrow V_B = R_B \frac{I_C}{\beta} + V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$\Rightarrow 0.8 = 10^4 \times \frac{I_C}{100} + 26\times10^{-3} \ln\left(\frac{I_C}{7\times10^{-16}}\right)$$

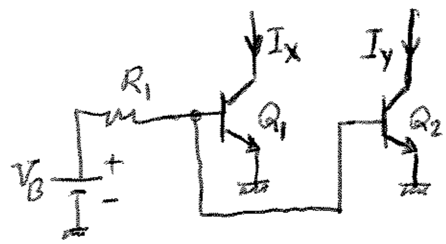
$$\Rightarrow 0.8 = 100 I_C + 26\times10^{-3} \ln\left(\frac{I_C}{7\times10^{-16}}\right)$$

using trial & error or numerical methods,

$$\boxed{I_C \simeq 7.85\times10^{-4} A = 785 \mu A}$$

(16)

$$\begin{cases} I_X = I_{S_1} \exp\left(\dfrac{V_{BE_1}}{V_T}\right) \\ I_Y = I_{S_2} \exp\left(\dfrac{V_{BE_2}}{V_T}\right) \\ V_{BE_1} = V_{BE_2} = V_{BE} \end{cases}$$



$$\Rightarrow \frac{I_X}{I_Y} = \frac{I_{S_1}}{I_{S_2}} = \frac{2 I_{S_2}}{I_{S_2}} \Rightarrow \boxed{\frac{I_X}{I_Y} = 2}$$

$$\begin{cases} I_X = \beta_1 I_{B_1} \\ I_Y = \beta_2 I_{B_2} \\ \beta_1 = \beta_2 \end{cases}$$

$$\Rightarrow \boxed{\frac{I_{B_1}}{I_{B_2}} = \frac{I_X}{I_Y} = 2}$$

Applying KVL:

$$V_B = R_1\left(I_{B_1} + I_{B_2}\right) + V_{BE}$$

$$V_{BE} = V_{BE_1} = V_T \ln\left(\frac{I_X}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{1^{mA}}{4\times10^{-16}}\right) \simeq 742\,mV$$

$$I_{B_1} = \frac{I_X}{\beta} \xrightarrow{\beta=100} I_{B_1} = \frac{1^{mA}}{100} = 10\mu A$$

$$\frac{I_{B_1}}{I_{B_2}} = 2 \longrightarrow I_{B_2} = \frac{I_{B_1}}{2} = \frac{10^{\mu A}}{2} \Rightarrow I_{B_2} = 5\mu A$$

Hence: $V_B = 5\times10^3\,\Omega \times (10\mu A + 5\mu A) + 0.742^V$

$$= 0.075 + 0.742 \Rightarrow \boxed{V_B \simeq 0.817^V}$$

(17) Applying KVL:

$$V_B = R_1(I_{B_1} + I_{B_2}) + V_{BE} \xlongequal{\beta_1=\beta_2=\beta} \frac{R_1}{\beta}(I_{C_1} + I_{C_2}) + V_{BE}$$

$$\Rightarrow V_B = \frac{R_1}{\beta}(I_{S_1} + I_{S_2})\exp\left(\frac{V_{BE}}{V_T}\right) + V_{BE}$$

$$\xRightarrow{\beta=100} 0.8^{V} = \frac{5000^{\Omega}}{100}(3\times10^{-16} + 5\times10^{-16})\exp\left(\frac{V_{BE}}{26^{mV}}\right) + V_{BE}$$

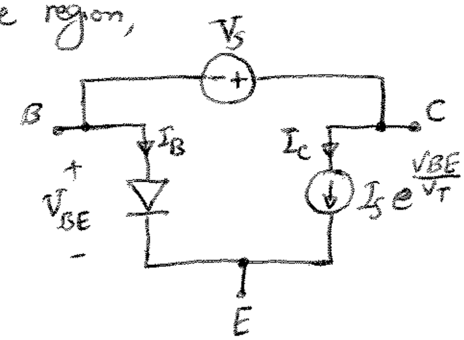
$$\Rightarrow 0.8^{V} = 4\times10^{-14}\cdot\exp\left(\frac{V_{BE}}{26^{mV}}\right) + V_{BE}$$

Numerical methods or Trial & error $\Rightarrow$ $\boxed{V_{BE} \simeq 732\,mV}$

$$I_X = I_{S_1}\exp\left(\frac{V_{BE}}{V_T}\right) = 3\times10^{-16}\left[\exp\left(\frac{732}{26}\right)\right] \Rightarrow \boxed{I_X \simeq 506\,\mu A}$$

$$I_Y = I_{S_2}\exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-16}\exp\left(\frac{732}{26}\right) \Rightarrow \boxed{I_Y \simeq 843\,\mu A}$$

(18) Since Transistor is in Forward active region,



No change across V_{BE}

$\Downarrow$

No change in I_B

$\Downarrow$

No change in I_C

(19) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow g_m = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T} \Rightarrow \boxed{V_{BE} = V_T \ln\left(\frac{g_m V_T}{I_S}\right)}$$

$$\xrightarrow[g_m = \frac{1}{13\Omega}]{I_S = 6\times10^{-16}A} V_{BE} = 26^{mV} \cdot \ln\left(\frac{\frac{1}{13\Omega}\times 26\times10^{-3}}{6\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 750\ mV}$$

(20) $g_m = \frac{I_C}{V_T}$

$$\Delta g_m = \frac{\Delta I_C}{V_T} = \frac{1}{V_T}\Delta\left(I_S e^{V_{BE}/V_T}\right) \simeq \frac{I_S}{V_T^2} e^{\frac{V_{BE}}{V_T}} \Delta V_{BE}$$

$$\Rightarrow \boxed{\Delta g_m \simeq \frac{I_C}{V_T^2} \Delta V_{BE}}$$

$$\Rightarrow \Delta g_m \simeq \frac{g_m}{V_T} \Delta V_{BE}$$

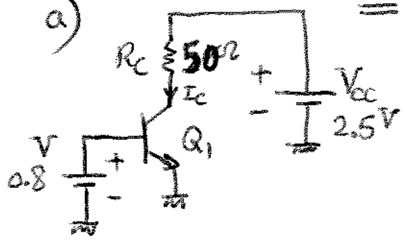
$$\Rightarrow \boxed{\frac{\Delta g_m}{g_m} \simeq \frac{1}{V_T} \Delta V_{BE}}$$

$$\left.\frac{\Delta g_m}{g_m}\right|_{I_C = 1mA} \overset{max}{=\!=} 0.1 \quad \Rightarrow \Delta V_{BE_{max}} = 0.1\, V_T$$

$$\Rightarrow \boxed{\Delta V_{BE} \leqslant 2.6 \text{ mV}}$$

(2) $V_A = \infty \Rightarrow r_o = \infty$, $I_S = 8\times10^{-16}$ A, $\beta = 100$

a)



If in Forward active region,

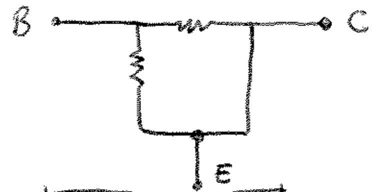
$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$= 8\times10^{-16} \exp\left(\frac{800}{26}\right)$$

$$\Rightarrow \boxed{I_C \simeq 18.5\,mA}$$

Hence Transistor should be in Forward Active

Small signal model

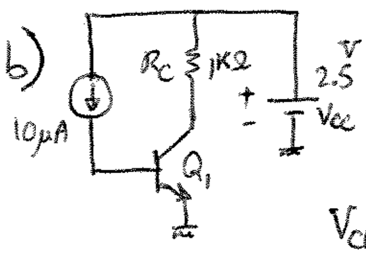


$$\Rightarrow \boxed{V_{CE} \simeq 1.575\ V}$$

$$I_{C_Q} = \frac{V_{CC} - V_{CE}}{R_C} = \frac{2.5 - 1.575}{50\Omega} = \frac{0.925}{50} \Rightarrow I_{C_Q} \simeq 18.5\,mA$$

which matches with I_C

b)



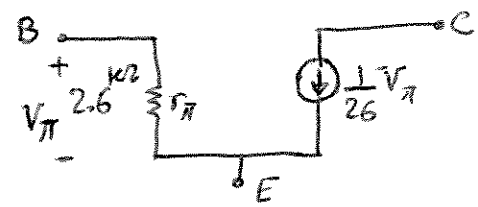
Assuming in Forward active region,

$$I_C = \beta I_B = 100 \times 10^{\mu A} \Rightarrow \boxed{I_C = 1^{mA}}$$

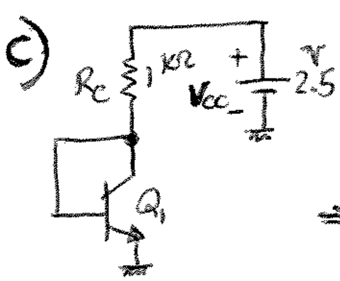
$$V_{CE} = V_{CC} - R_C I_C = 2.5 - 1^{K\Omega} \times 1^{mA} \Rightarrow \boxed{V_{CE} = 1.5^V}$$

$$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26}}\ \Omega^{-1}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi \simeq 2600^{\Omega}}$$



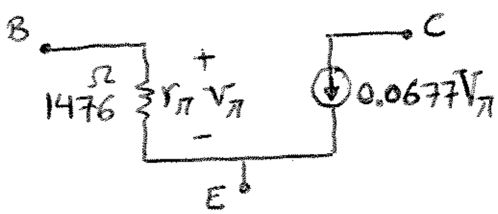
c)



Applying KVL,

$$V_{CC} \simeq R_C I_C + V_{BE}$$

$$\Rightarrow 2.5^V \simeq 1^{K\Omega} \times I_C + V_{BE}$$

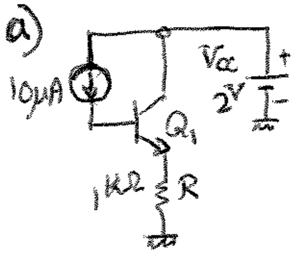


$$\Rightarrow 2.5^V \simeq 8\times10^{-13} \cdot \exp\left(\frac{V_{BE}}{V_T}\right) + V_{BE} \Rightarrow \boxed{V_{BE} \simeq 739^{mV}}$$

$$g_m = \frac{I_C}{V_T} = \frac{\frac{V_{CC} - V_{BE}}{R_C}}{V_T} \Rightarrow g_m = \frac{2.5 - 0.739}{1^{K\Omega} \times 0.026} \Rightarrow \boxed{g_m \simeq 67.7\ mS},\ \boxed{I_C = 1.76^{mA}}$$

(22) $V_A = \infty \Rightarrow r_o = \infty$, $I_S = 8\times10^{-16}A$, $\beta = 100$

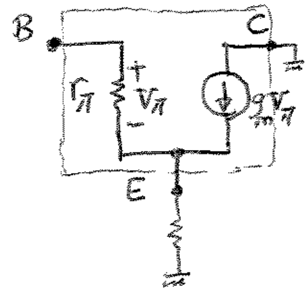
a)



$I_C = \beta I_B = 100\times 10^{\mu A} \Rightarrow \boxed{I_C = 1^{mA}}$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \times \ln\left(\frac{10^{-3}}{8\times10^{-16}}\right)$

$\Rightarrow \boxed{V_{BE} \simeq 724\ mV}$

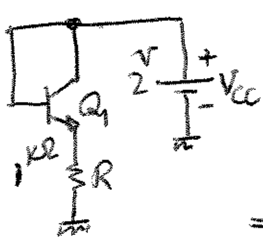


$V_{CE} = V_{CC} - RI_E \simeq V_{CC} - RI_C = 2 - 1^{k\Omega} \times 1^{mA} \Rightarrow \boxed{V_{CE} = 1^V}$

$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26\Omega}}$

$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi = 2.6^{k\Omega}}$

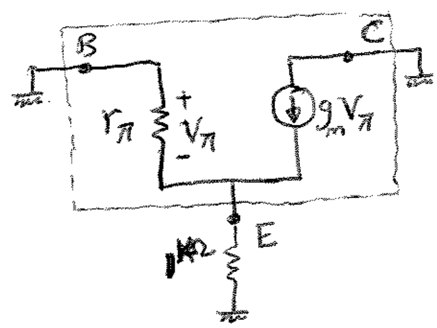
b)



Applying KVL,

$V_{CC} = V_{BE} + RI_E$

$\Rightarrow V_{CC} \simeq V_{BE} + RI_C$



$\Rightarrow V_{CC} \simeq V_{BE} + RI_S \exp\left(\frac{V_{BE}}{V_T}\right)$

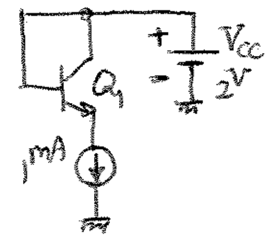
$\Rightarrow 2^V \simeq V_{BE} + 8\times10^{-13} \exp\left(\frac{V_{BE}}{26^{mV}}\right) \Rightarrow \boxed{V_{BE} \simeq 730\ mV}$

$\boxed{V_{CE} = V_{BE} = 730^{mV}}$ $I_C = 8\times10^{-16} \exp\left(\frac{730}{26}\right) \Rightarrow \boxed{I_C \simeq 1.2mA}$

$g_m = \frac{I_C}{V_T} = \frac{1.2^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 46\ mS}$ $r_\pi = \frac{\beta}{g_m} \xrightarrow{\beta=100} \boxed{r_\pi \simeq 2167^{\Omega}}$

(22) Continued ...

c)

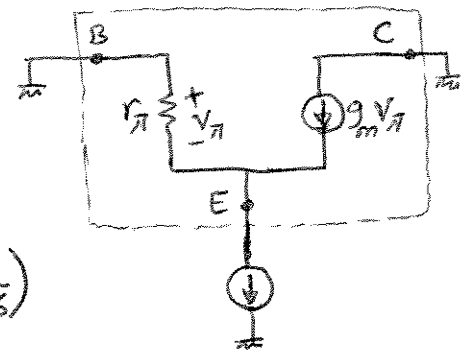


$$\boxed{I_C \simeq I_E = 1\,mA}$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$= 26^{mV} \ln\left(\frac{1\,mA}{8\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{BE} \simeq 724\,mV}$$

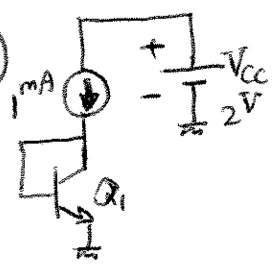


$$\boxed{V_{CE} = V_{BE} = 724\,mV}$$

$$g_m = \frac{I_C}{V_T} = \frac{1^{mA}}{26^{mV}} \Rightarrow \boxed{g_m = \frac{1}{26\Omega}}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{1/26} \Rightarrow \boxed{r_\pi \simeq 2.6\,k\Omega}$$

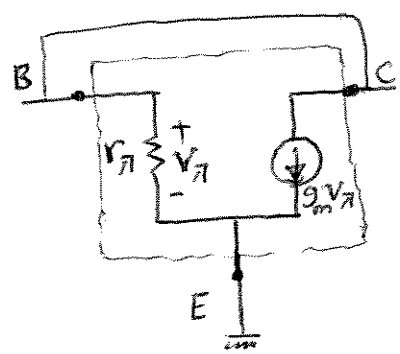
d)



As in part c, we have,

$$\boxed{I_C \simeq 1\,mA}$$

$$\boxed{V_{CE} \simeq 724\,mV}$$



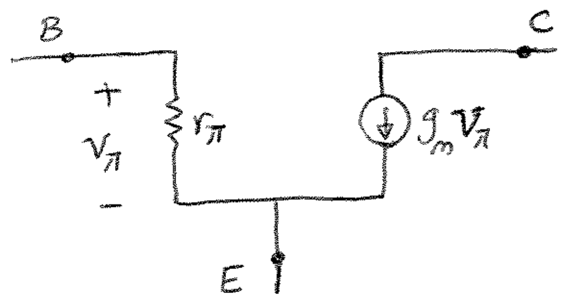
$$\boxed{g_m = \frac{1}{26\Omega}}$$

$$\boxed{r_\pi = 2.6\,k\Omega}$$

(23) $I_C = I_S \exp\left(\frac{V_{BE}}{nV_T}\right)$ $\qquad$ $I_C = \beta I_B$

$$g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{1}{nV_T} I_S \exp\left(\frac{V_{BE}}{nV_T}\right) = \frac{I_C}{nV_T}$$

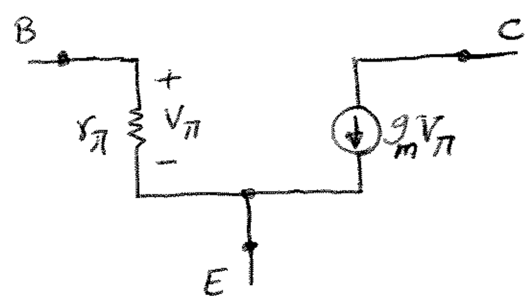
$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{\beta}\partial I_C} = \frac{\beta}{g_m} = \frac{n\beta V_T}{I_C}$$



(24) $I_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)$, $I_C = \alpha I_B^2 \Rightarrow \frac{\partial I_B}{\partial I_C} = \frac{1}{2\sqrt{\alpha I_C}}$

$$g_m = \frac{\partial I_C}{\partial v_{BE}} = \frac{I_S}{V_T}\exp\left(\frac{v_{BE}}{V_T}\right) = \boxed{\frac{I_C}{V_T}}$$

$$r_\pi = \frac{\partial V_{BE}}{\partial I_B} = \frac{\partial V_{BE}}{\frac{1}{2\sqrt{\alpha I_C}}\partial I_C} = \frac{2\sqrt{\alpha I_C}}{g_m} = \frac{2\sqrt{\alpha I_C}}{I_C/V_T} = \boxed{2V_T\sqrt{\frac{\alpha}{I_C}}}$$



(25) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$ $\quad V_{BE}$ is Constant

$$\Delta I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\,\Delta V_{CE}$$

$$\Rightarrow \frac{\Delta I_C}{I_C} = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A}\cdot \Delta V_{CE}}{I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]} = \frac{\Delta V_{CE}}{V_A + V_{CE}}$$

$$\frac{\Delta I_C}{I_{C_{min}}} < 0.05 \Rightarrow \frac{\Delta V_{CE}}{V_A + V_{CE_{min}}} < 0.05$$

$$\Rightarrow 20\,\Delta V_{CE} < V_A + V_{CE_{min}}$$

$$\left.\begin{array}{l}\Delta V_{CE} = 2^V \\ V_{CE_{min}} = 1^V\end{array}\right\} \Rightarrow 40 < V_A + 1 \Rightarrow \boxed{V_A > 39^V}$$

(26)

a) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) = 5\times10^{-17} \exp\left(\frac{800^{mV}}{26^{mV}}\right) \simeq 1.15\,mA$

$V_X = V_{CC} - R_C I_C = 2.5^{V} - 1^{k\Omega} \times 1.15^{mA}$

$V_X = 1.35\,V$ Transistor is in Forward Active Region

b) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$\Rightarrow I_C = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5^{V}}\right]$ equation 1

Also we know: $V_X = V_{CC} - R_C I_C \Rightarrow I_C = \frac{V_{CC} - V_X}{R_C}$ equation 2

equations 1, 2 $\Rightarrow \frac{V_{CC} - V_X}{R_C} = 5\times10^{-17} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right]$

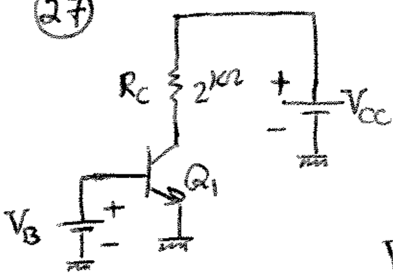
$\Rightarrow V_X + 5\times10^{-14} \exp\left(\frac{800}{26}\right)\left[1 + \frac{V_X}{5}\right] = 2.5$

$\Rightarrow 1.2306\, V_X \simeq 1.347$

$\Rightarrow V_X \simeq 1.095\ V \xrightarrow{\text{equation 1}} I_C \simeq 1.406\,mA$

Transistor is in Forward Active Region

(27)



$I_S = [illegible] \times 10^{-17} A \qquad V_A = 5^V$

Applying KVL:

$$V_{CC} = R_C I_C + V_{CE}$$

$$\Rightarrow V_{CC} = R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right] + V_{CE}$$

V_{BE} Constant

$$\Rightarrow \boxed{\Delta V_{CC} = \left[R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)\frac{1}{V_A} + 1\right] \cdot \Delta V_{CE}} \quad \text{equation 1}$$

$$I_C = I_S e^{V_{BE}/V_T}\left[1 + \frac{V_{CE}}{V_A}\right] \Rightarrow \Delta I_C = I_S e^{V_{BE}/V_T} \times \frac{1}{V_A} \Delta V_{CE}$$

$$\Rightarrow \boxed{\Delta V_{CE} = \frac{1}{I_S e^{V_{BE}/V_T} \times \frac{1}{V_A}} \cdot \Delta I_C} \quad \text{equation 2}$$

equations 1, 2

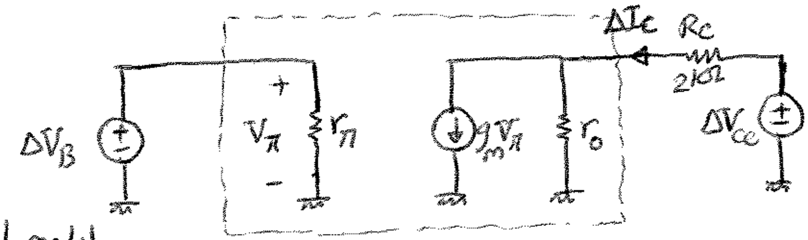
$$\Rightarrow \Delta I_C = \frac{I_S e^{V_{BE}/V_T} \times \frac{1}{V_A}}{1 + R_C I_S e^{V_{BE}/V_T} \times \frac{1}{V_A}} \cdot \Delta V_{CC}$$

$$\Rightarrow \boxed{\Delta I_C = \frac{I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_A + R_C I_S \exp\left(\frac{V_{BE}}{V_T}\right)} \cdot \Delta V_{CC}} = \frac{1}{r_o + R_C} \cdot \Delta V_{CC}$$

Could also be obtained using small signal model

$$\Rightarrow \Delta I_C = \frac{2.31 \times 10^{-4}}{5 + 0.4613} \times 0.5 \Rightarrow \Delta I_C \simeq 0.021 mA$$

(28)



We use small signal model,
Assuming that the required ΔV_B is small enough.

Applying superposition,

$$\Delta I_C = \left(\frac{1}{r_o + R_C}\right)\Delta V_{CC} + \left(\frac{g_m r_o}{r_o + R_C}\right)\Delta V_B$$

$$\Delta I_C = 0 \Rightarrow \boxed{\Delta V_B = -\frac{1}{g_m r_o}\Delta V_{CC}}$$

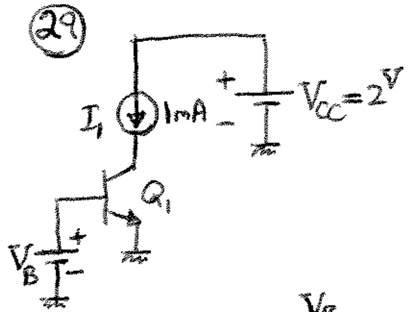
$$\Delta V_B = -\frac{1}{\frac{I_C}{V_T}\cdot\frac{V_A}{I_C}}\Delta V_{CC} \Rightarrow \Delta V_B = -\frac{V_T}{V_A}\Delta V_{CC}$$

$$\Rightarrow \Delta V_B = -\frac{26\times10^{-3}}{5}\times(3-2.5)$$

$$\Rightarrow \boxed{\Delta V_B = -2.6\ mV}$$

which is small enough for small signal model

(29)



$I_S = 3\times10^{-17}A$

a) $I_C = I_S e^{\frac{V_B}{V_T}} \implies V_B = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{10^{-3}}{3\times10^{-17}}\right)$

$\implies \boxed{V_B \simeq 809.6\ mV}$

b) $I_c = I_S\, e^{\frac{V_B}{V_T}}\left(1 + \frac{V_{CE}}{V_A}\right)$

$10^{-3} = 3\times10^{-17}\, e^{V_B/V_T}\left(1 + \frac{1.5}{5}\right) \implies e^{V_B/V_T} = \frac{10^{14}}{3.9}$

$\implies V_B = 26^{mV} \ln\left(\frac{10^{14}}{3.9}\right) \implies \boxed{V_B \simeq 802.8\ mV}$

(30) $I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1 + \frac{V_{CE}}{V_A}\right]$

$$r_o^{-1} = \frac{dI_C}{dV_{CE}} = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \cdot \frac{1}{V_A} \simeq \frac{I_C}{V_A} \Rightarrow r_o \simeq \frac{V_A}{I_C}$$

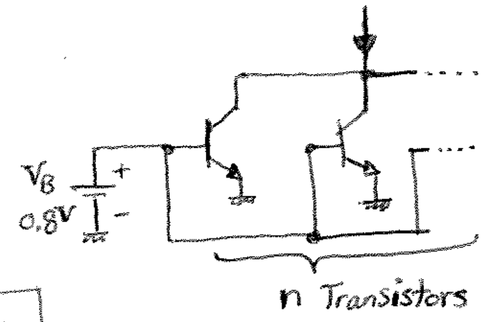
$$r_o > 10^{k\Omega} \Rightarrow \frac{V_A}{I_C} > 10^{k\Omega}$$

$$\Rightarrow V_A > 10^{k\Omega} \times 2^{mA}$$

$$\Rightarrow \boxed{V_A > 20\,V}$$

(31) $I_S = 5\times10^{-16}A$, $V_A = 8V$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1+\frac{V_{CE}}{V_A}\right]$$

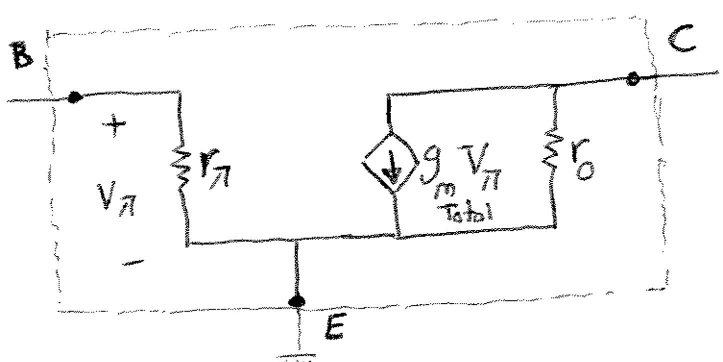


$$g_{m_{Total}} = \frac{I_{C_{Total}}}{V_T} \simeq \frac{n I_S \exp\left(\frac{V_{BE}}{V_T}\right)}{V_T}$$

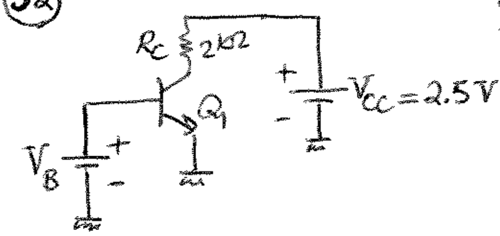
$$\Rightarrow g_{m_{Total}} \simeq \frac{n \times 5\times10^{-16} \exp\left(\frac{800}{26}\right)}{26\,mV} \Rightarrow \boxed{g_{m_{Total}} \simeq 0.4435\,n}$$

$$r_o^{-1} = \frac{\partial I_{C_{Total}}}{\partial V_{CE}} = \frac{\partial}{\partial V_{CE}}\left[n I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1+\frac{V_{CE}}{V_A}\right]\right]$$

$$\Rightarrow \boxed{r_o = \frac{V_A}{n I_S \exp\left(\frac{V_{BE}}{V_T}\right)}} \qquad \boxed{r_\pi = \frac{\beta}{g_{m_{Total}}} \overset{\beta=100}{\simeq} \frac{225.5}{n}}$$



(32)



$I_S = 6 \times 10^{-16} A$, $V_A = \infty$

a) Q_1 at the edge of the active region $\Rightarrow V_{CE} = V_{BE}$

applying KVL, $V_{CC} = R_C I_C + V_{CE}$

$$\xrightarrow[\text{edge}]{\text{at the}} V_{CC} = R_C I_C + V_{BE} \Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} = V_{CC}$$

$$\Rightarrow 2^{k\Omega} \times 6 \times 10^{-16} A \, e^{V_B / 26^{mV}} + V_B = 2.5 \Rightarrow \boxed{V_B \simeq 728.5^{mV}}$$

b) Applying KVL, $V_{CC} = R_C I_C + V_{CE}$

Soft Saturation $\Rightarrow V_{CE} = V_{BE} - 0.2^V$

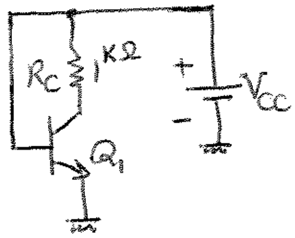
$$\Rightarrow V_{CC} = R_C I_C + V_{BE} - 0.2^V$$

$$\Rightarrow 2^{k\Omega} \times 6 \times 10^{-16} e^{V_B / 26^{mV}} + V_B = 2.7V$$

$$\Rightarrow \boxed{V_B \simeq 731.5 mV}$$

So V_B can increase by 3mV

(33)



$I_S = 7\times10^{-16}A$, $V_A = \infty \Downarrow r_o = \infty$

Applying KVL,

$$V_{CC} = R_C I_C + V_{CE} \xrightarrow{V_{CE}=V_{BE}-0.2^V} R_C I_C + V_{BE} - 0.2^V = V_{CC}$$

$$\Rightarrow R_C I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} - 0.2^V = V_{CC}$$

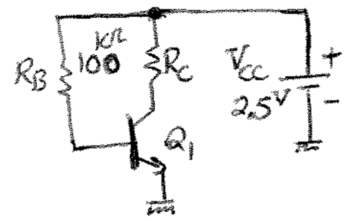
$$\xRightarrow{V_{BE}=V_{CC}} R_C I_S e^{\frac{V_{CC}}{V_T}} + \cancel{V_{CC}} - 0.2 = \cancel{V_{CC}}$$

$$\Rightarrow R_C I_S e^{V_{CC}/V_T} = 0.2^V$$

$$\Rightarrow 1^{K\Omega} \times 7\times10^{-16} e^{V_{CC}/26^{mV}} = 0.2^V$$

$$\Rightarrow \boxed{V_{CC} \simeq 686 \text{ mV}}$$

(34) $I_S = 2\times10^{-17}A$, $V_A = \infty$ $\beta = 100$



$$\begin{cases} V_{CC} = R_C I_C + V_{CE}, \quad V_{CE} = V_{BE} - 0.2^V \\ V_{CC} = R_B I_B + V_{BE} \Rightarrow V_{CC} = R_B \dfrac{I_C}{\beta} + V_{BE} \end{cases}$$

$$R_B \frac{I_C}{\beta} + V_{BE} = V_{CC} \Rightarrow \frac{R_B}{\beta} I_S e^{\frac{V_{BE}}{V_T}} + V_{BE} = V_{CC}$$

$$\Rightarrow \frac{100^{k\Omega}}{100} \times 2\times10^{-17} e^{\frac{V_{BE}}{26^{mV}}} + V_{BE} = 2.5^V$$

$$\Rightarrow \boxed{V_{BE} \simeq 833.5 \text{ mV}}$$

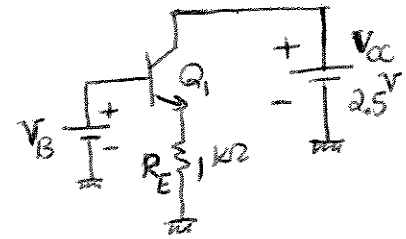
Soft Saturation $\Rightarrow V_{CE} = V_{BE} - 0.2^V \Rightarrow \boxed{V_{CE} = 692.5^{mV}}$

$$V_{CC} = R_C I_C + V_{CE} \Rightarrow R_C = \frac{V_{CC} - V_{CE}}{I_C}$$

$$\Rightarrow R_C = \frac{V_{CC} - V_{CE}}{I_S \exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{2.5 - 0.6925}{2\times10^{-17} \exp\left(\frac{892.5}{26}\right)}$$

$$\Rightarrow \boxed{R_C \simeq 112\ \Omega}$$

(35) $I_S = 5\times10^{-16}A$, $V_A = \infty \Rightarrow r_o = \infty$



Soft saturation $\Rightarrow V_{BC} = 200\,mV$

$\Rightarrow V_B = V_C + 0.2^V \Rightarrow \boxed{V_B = 2.7\,V}$

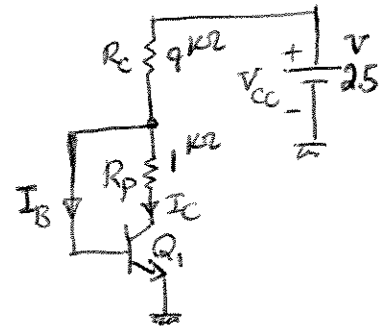
Applying KVL $\Rightarrow V_B = V_{BE} + R_E I_E \xrightarrow{I_E \simeq I_C} V_B = V_{BE} + R_E I_C$

$\Rightarrow V_{BE} + 1^{k\Omega} \times I_S e^{\frac{V_{BE}}{V_T}} = 2.7^V$

$\Rightarrow V_{BE} + 5\times10^{-13} e^{\frac{V_{BE}}{V_T}} = 2.7^V \Rightarrow \boxed{V_{BE} \simeq 754\,mV}$

$I_C = I_S e^{V_{BE}/V_T} = 5\times10^{-16} e^{\frac{0.754}{0.026}} \Rightarrow \boxed{I_C \simeq 2\,mA}$

(36) $\beta = 100, \quad V_A = \infty \Rightarrow r_o = \infty$



$$V_{BC} = 0.2\,V \Rightarrow R_P I_C = 0.2^V$$

$$\Rightarrow \boxed{I_C = \frac{0.2^V}{R_P}}$$

$$V_{BE} = V_{CC} - R_C(I_B + I_C)$$

$$\xrightarrow{\beta=100} V_{BE} = V_{CC} - \frac{\beta+1}{\beta} R_C I_C \Rightarrow \boxed{V_{BE} = V_{CC} - \frac{\beta+1}{\beta}\frac{R_C}{R_P} \times 0.2}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{BE}}{V_T}\right)$$

$$\Rightarrow \boxed{I_S = \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T} \cdot \frac{\beta+1}{\beta} \cdot \frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\xrightarrow{\beta=100} \boxed{I_S \simeq \frac{0.2}{R_P} \exp\left[\frac{0.2}{V_T}\frac{R_C}{R_P} - \frac{V_{CC}}{V_T}\right]}$$

$$\Rightarrow \boxed{I_S \simeq 4.06 \times 10^{-16}\,A}$$

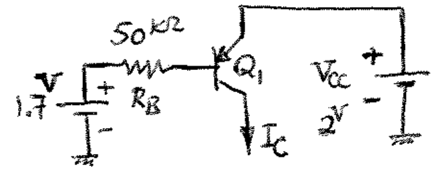
(37) $I_{S_1} = 3I_{S_2} = 6\times10^{-16}A$

$$I_1 = I_{S_1}\exp\left(\frac{V_{EB_1}}{V_T}\right) = 6\times10^{-16}\exp\left(\frac{300}{26}\right) \Rightarrow \boxed{I_1 \simeq 6.155\times10^{-11}\ A}$$

$$I_2 = I_{S_2}\exp\left(\frac{V_{EB_2}}{V_T}\right) = 2\times10^{-16}\exp\left(\frac{820}{26}\right) \Rightarrow \boxed{I_2 \simeq 10\ mA}$$

$$I_X = I_1 + I_2 \Rightarrow \boxed{I_X \simeq 10\ mA}$$

(38) $I_S = 2\times10^{-17}\,A \qquad \beta = 100$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B + 1.7^V$$

$$\Rightarrow 2^V = V_{EB} + R_B \frac{I_C}{\beta} + 1.7^V$$

$$\Rightarrow 0.3^V = V_{EB} + \frac{50^{k\Omega}}{100} I_C$$

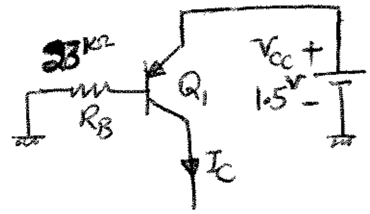
$$\Rightarrow 0.3^V = V_{EB} + 500 \times I_S\, e^{\frac{V_{EB}}{V_T}}$$

$$\Rightarrow 0.3^V = V_{EB} + 10^{-14}\, e^{V_{EB}/26mV} \Rightarrow \boxed{V_{EB} \simeq 0.3\ V}$$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow I_C = 2\times10^{-17}\, e^{\frac{300}{26}}$$

$$\Rightarrow \boxed{I_C \simeq 2.05\times10^{-12}\ A}$$

(39) $I_C = 3\,mA$, $\beta = 100$, $R_B = 23\,k\Omega$



Applying KVL,

$$V_{CC} = V_{EB} + R_B I_B \implies V_{CC} = V_{EB} + R_B \frac{I_C}{\beta}$$

$$\implies -I_C \frac{R_B}{\beta} + V_{CC} = V_{EB}$$

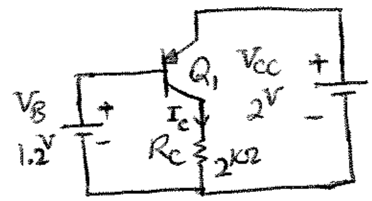
$$I_C = I_S e^{V_{EB}/V_T}$$

$$\implies I_S = I_C e^{-\frac{V_{EB}}{V_T}}$$

$$\implies \boxed{I_S = I_C e^{\frac{1}{V_T}\left(\frac{R_B I_C}{\beta} - V_{CC}\right)}}$$

$$\implies \boxed{I_S \simeq 8.85 \times 10^{-17}\,A}$$

(40) At the edge of active $\Rightarrow V_{BC} = 0$



$$I_C = \frac{V_B - V_{BC}}{R_C} = \frac{V_B}{R_C}$$

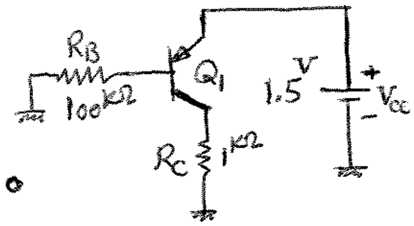
$$\Rightarrow I_C = \frac{1.2^V}{2^{k\Omega}} \Rightarrow \boxed{I_C \simeq 0.6\ mA}$$

$$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) \Rightarrow I_S = I_C \exp\left(-\frac{V_{EB}}{V_T}\right)$$

$$\Rightarrow I_S = 0.6 \times 10^{-3} \exp\left(-\frac{800}{26}\right)$$

$$\Rightarrow \boxed{I_S \simeq 2.6 \times 10^{-17}\ A}$$

(41) $I_S = 8 \times 10^{-16} A$



At the edge of the active mode $\Rightarrow V_{BC} = 0$

$\Rightarrow V_{EB} = V_{EC}$

Applying KVL,

$$V_{cc} = V_{EC} + R_C I_C \xrightarrow{V_{EB} = V_{EC}} V_{cc} = V_{EB} + R_C I_C$$

$$\Rightarrow V_{EB} + R_C I_S e^{\frac{V_{EB}}{V_T}} = V_{cc}$$

$$\Rightarrow V_{EB} + 8 \times 10^{-13} e^{\frac{V_{EB}}{26^{mV}}} = 1.5 \Rightarrow \boxed{V_{EB} \simeq 718 mV}$$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow \boxed{I_C \simeq 0.788 mA}$$

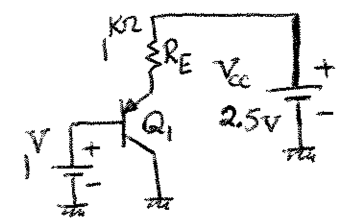
Applying KVL,

$$V_{BC} = 0 \Rightarrow V_B = V_C \Rightarrow R_B I_B = R_C I_C$$

$$\Rightarrow R_B \frac{I_C}{\beta} = R_C I_C \Rightarrow \boxed{\beta = \frac{R_B}{R_C}}$$

$$\Rightarrow \beta = \frac{100^{k\Omega}}{1^{k\Omega}} \Rightarrow \boxed{\beta = 100}$$

(42) $I_S = 3\times10^{-17}\,A$



Applying KVL,

$$V_{CC} = R_E I_E + V_{EB} + 1^V \xrightarrow{I_E \simeq I_C} V_{CC} = R_E I_C + V_{EB} + 1^V$$

$$\Rightarrow 2.5 = 1^{K\Omega} \times 3\times10^{-17} e^{V_{EB}/26^{mV}} + V_{EB} + 1^V$$

$$\Rightarrow V_{EB} + 3\times10^{-14} e^{V_{EB}/26^{mV}} = 1.5^V$$

$$\Rightarrow \boxed{V_{EB} \simeq 800.5\ mV}$$

$$I_C = I_S e^{\frac{V_{EB}}{V_T}} = 3\times10^{-17} e^{\frac{800.5}{26}} \Rightarrow \boxed{I_C \simeq 0.705\ mA}$$

(43) $I_S = 3\times10^{-17}A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

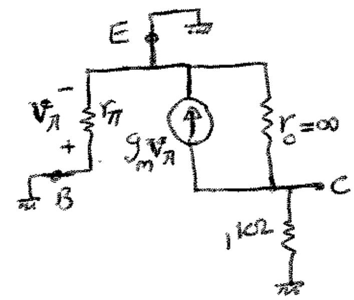
a) $V_{EB} = 2.5 - 1.7 = 0.8\ V$

$I_C = I_S \exp\left(\frac{V_{EB}}{V_T}\right) = 3\times10^{-17}\exp\left(\frac{800}{26}\right) \Rightarrow \boxed{I_C \simeq 0.692^{mA}}$

$V_{EC} = V_{CC} - R_C I_C = 2.5 - 1^{k\Omega}\times 0.692^{mA} \Rightarrow \boxed{V_{EC} \simeq 1.808^{V}}$

$g_m = \frac{I_C}{V_T} = \frac{0.692^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 26.6\,mS}$

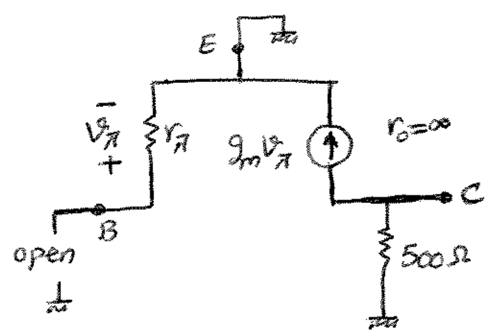
$r_\pi = \frac{\beta}{g_m} = \frac{100}{26.6\times10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3.76\ k\Omega}$



b) $V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) \Rightarrow V_{EB} = V_T \ln\left(\frac{\beta I_B}{I_S}\right)$

$\Rightarrow V_{EB} = 26^{mV}\times \ln\left(\frac{100\times20\times10^{-6}}{3\times10^{-17}}\right)$

$\Rightarrow \boxed{V_{EB} \simeq 827.6\,mV}$



$I_C = \beta I_B \Rightarrow \boxed{I_C = 2mA}$

$V_{EC} = V_{CC} - R_C I_C = 2.5 - 0.5^{k\Omega}\times 2^{mA} \Rightarrow \boxed{V_{EC} = 1.5\ V}$

$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_m \simeq 77\,mS}$ $\quad r_\pi = \frac{\beta}{g_m} \Rightarrow \boxed{r_\pi \simeq 1.3\,k\Omega}$

(43) Continued .....

c) Applying KVL,

$$V_{cc} = V_{EB} + (I_C + I_B)\times 2^{k\Omega} \simeq V_{EB} + 2^{k\Omega} \times I_C$$

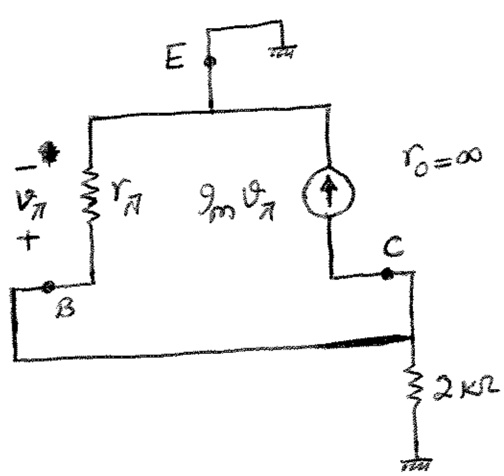
$$\Rightarrow V_{EB} + 2^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} = V_{cc}$$

$$\Rightarrow V_{EB} + 6\times 10^{-14} e^{V_{EB}/26^{mV}} = 2.5^{V} \Rightarrow \boxed{V_{EB} \simeq 805\,mV}$$

$$I_C \simeq \frac{V_{cc} - V_{EB}}{R} = \frac{2.5 - 0.805}{2^{k\Omega}} \Rightarrow \boxed{I_C \simeq 847.5\,\mu A}$$

$$g_m = \frac{I_C}{V_T} = \frac{0.8475\times 10^{-3}}{0.026} \Rightarrow \boxed{g_m \simeq 32.6\ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{32.6\times 10^{-3}} \Rightarrow \boxed{r_\pi \simeq 3068\ \Omega}$$

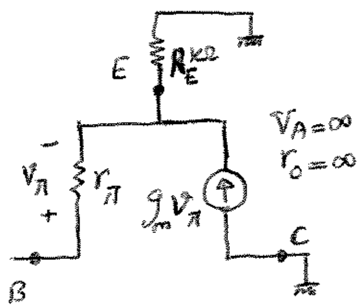


(44) $I_S = 3\times10^{-17} A$, $\beta = 100$, $V_A = \infty \Rightarrow \boxed{r_o = \infty}$

a) Applying KVL,

$$V_{CC} = R_E I_E + V_{EC} \xrightarrow{I_E \simeq I_C} V_{EC} = V_{CC} - R_E I_C \qquad \boxed{I_C = \beta I_B = 0.2^{mA}}$$

$$\Rightarrow \boxed{V_{EC} = V_{CC} - \beta R_E I_B} \xrightarrow[R_E = 2^{k\Omega},\ V_{CC} = 2.5^V]{I_B = 2^{\mu A}} \boxed{V_{EC} = 2.1\ V}$$



Transistor is in Forward Active Region

b) Applying KVL,

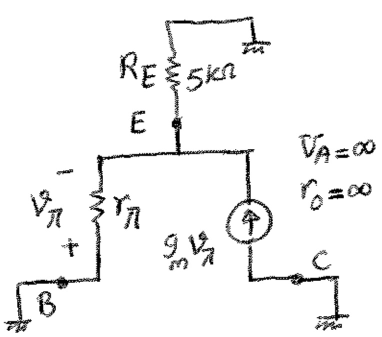
Forward Active Region

$$V_{CC} = R_E I_E + V_{EB} \Rightarrow V_{CC} \simeq R_E I_C + V_{EB}$$

$$\Rightarrow 2.5 = 5^{k\Omega} \times 3\times10^{-17} e^{V_{EB}/V_T} + V_{EB}$$

From Circuit $V_{EC} = V_{EB}$

$$\Rightarrow \boxed{V_{EB} \simeq 781.9\ mV}$$



$$g_m = \frac{I_C}{V_T} = \frac{3\times10^{-17} e^{\frac{781.9}{26}}}{0.026} \Rightarrow \boxed{g_m \simeq 0.0133\ S}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0133} \Rightarrow \boxed{r_\pi \simeq 7538\ \Omega}$$

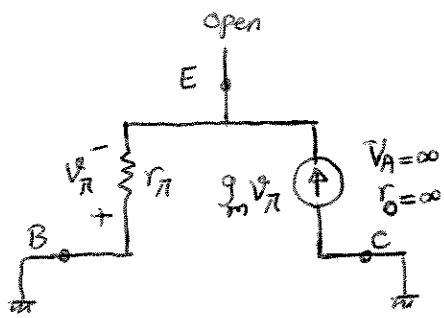
(44) Continued .....

c) $I_E = 0.5mA \Rightarrow \boxed{I_C \simeq 0.5mA}$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow 0.5mA = 3\times10^{-17} e^{\frac{V_{EB}}{26mV}} \Rightarrow \boxed{V_{EB} \simeq 791.6\,mV}$$

In the given Circuit : $V_{EC} = V_{EB}$

$$g_m = \frac{I_C}{V_T} = \frac{0.5\,mA}{26\,mV} \Rightarrow \boxed{g_m \simeq 19.2\,mS}$$

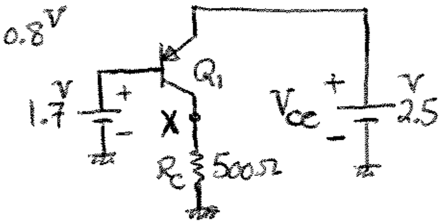


$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0192} \Rightarrow \boxed{r_\pi \simeq 5.2\,k\Omega}$$

Forward Active Region

(45) $I_S = 5\times10^{-17}\,A$

$V_{EB} = 0.8\,V$



a) $V_A = \infty \Rightarrow r_o = \infty$

$$I_C = I_S e^{V_{EB}/V_T} \Rightarrow I_C = 5\times10^{-17} e^{\frac{800}{26}} \Rightarrow \boxed{I_C \simeq 1.15\,mA}$$

$$V_X = R_C I_C = 0.5^{k\Omega} \times 1.15^{mA} \Rightarrow \boxed{V_X \simeq 0.58^{V}}$$

b) $V_A = 6\,V$

$$I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{EC}}{V_A}\right), \quad V_{EC} = V_{CC} - R_C I_C$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC} - R_C I_C}{V_A}\right)$$

$$\Rightarrow I_C = I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right) - \frac{I_S R_C}{V_A} e^{V_{EB}/V_T} I_C$$

$$\Rightarrow \boxed{I_C = \frac{I_S e^{V_{EB}/V_T}\left(1 + \frac{V_{CC}}{V_A}\right)}{1 + \frac{I_S R_C}{V_A} e^{V_{EB}/V_T}}} = \frac{5\times10^{-17} e^{\frac{800}{26}}\left(1 + \frac{2.5}{6}\right)}{1 + \frac{5\times10^{-17}\times 0.5^{k\Omega}}{6} e^{800/26}}$$

$$\Rightarrow \boxed{I_C = 1.49\,mA} \qquad V_X = R_C I_C = 500 \times 1.49 \times 10^{-3} \Rightarrow \boxed{V_X = .745^{V}}$$

0.745 V

(46) $r_o = 60\,k\Omega$, $I_C = 2\,mA$

$$r_o = \frac{V_A}{I_C} \Rightarrow 60\times10^{3}\,\Omega = \frac{V_A}{2\times10^{-3}\,A} \Rightarrow \boxed{V_A = 120\,V}$$

(47) $r_o = 60\ k\Omega$, $I_C = 1\ mA$

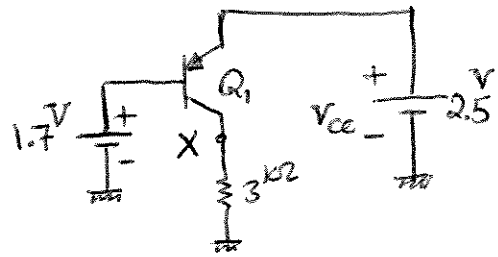
$$r_o = \frac{V_A}{I_C} \Rightarrow \boxed{V_A = r_o \cdot I_C} \Rightarrow V_A \propto I_C$$

$$\Rightarrow V_A = 60^{k\Omega} \times 1^{mA}$$

$$\Rightarrow \boxed{V_A = 60\ V}$$

V_A is half the value in problem 46 as V_A is proportional to I_C.

(48) $V_A = 5\,V$



a) At the edge of active mode

$$\Rightarrow \boxed{V_X = V_B = 1.7\,V}$$

$$I_C = \frac{V_X}{R_C} = \frac{1.7\,V}{3\,k\Omega} \Rightarrow \boxed{I_C \simeq 0.567\,mA}$$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}}\left(1 + \frac{V_{EC}}{V_A}\right) \Rightarrow \boxed{I_S = \frac{I_C\, e^{-\frac{V_{EB}}{V_T}}}{1 + \frac{V_{EC}}{V_A}}}$$

$$I_S = \frac{0.567 \times 10^{-3} \times e^{-\frac{800}{26}}}{1 + \frac{2.5 - 1.7}{5}} \Rightarrow \boxed{I_S \simeq 2.118 \times 10^{-17}\,A}$$

b) $V_A = \infty$

$$I_C = I_S\, e^{\frac{V_{EB}}{V_T}} \Rightarrow \boxed{I_S = I_C\, e^{-\frac{V_{EB}}{V_T}}}$$

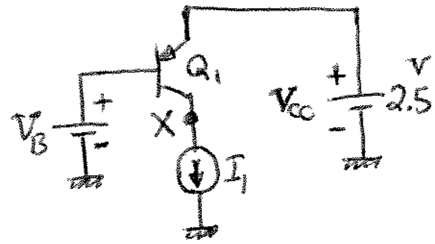
$$I_S = 0.567 \times 10^{-3}\, e^{-\frac{800}{26}} \Rightarrow \boxed{I_S \simeq 2.457 \times 10^{-17}\,A}$$

I_S increases

(49) The direction of Currents in large-signal model shows how Currents would flow when The pnp Transistor is properly DC biased.

The direction of currents in small-signal model shows how the ac Currents flow when ac Voltage across Base-Emitter increases.

(50) $I_S = 6\times10^{-16}A$, $V_A = 5V$, $I_1 = 2mA$



a) $$I_C = I_S e^{\frac{V_{EB}}{V_T}}\left(1+\frac{V_{EC}}{V_A}\right)$$

$$\Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{EC}}{V_A}\right)}\right)$$

$$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X} \boxed{V_B = V_{CC} - V_T \ln\left(\frac{I_C}{I_S\left(1+\frac{V_{CC}-V_X}{V_A}\right)}\right)}$$

$$\Rightarrow V_B = 2.5 - 0.026 \ln\left(\frac{2\times10^{-3}}{6\times10^{-16}\left(1+\frac{2.5-1}{5}\right)}\right) \Rightarrow \boxed{V_B \simeq 1.757V}$$

b) $$I_C = I_S e^{V_{EB}/V_T}\left(1+\frac{V_{EC}}{V_A}\right) \Rightarrow 1+\frac{V_{EC}}{V_A} = \frac{I_C}{I_S} e^{-V_{EB}/V_T}$$

$$\xrightarrow[V_{EB}=V_{CC}-V_B]{V_{EC}=V_{CC}-V_X} \boxed{V_X = V_{CC} - V_A\left(\frac{I_C}{I_S} e^{-V_{EB}/V_T} - 1\right)}$$

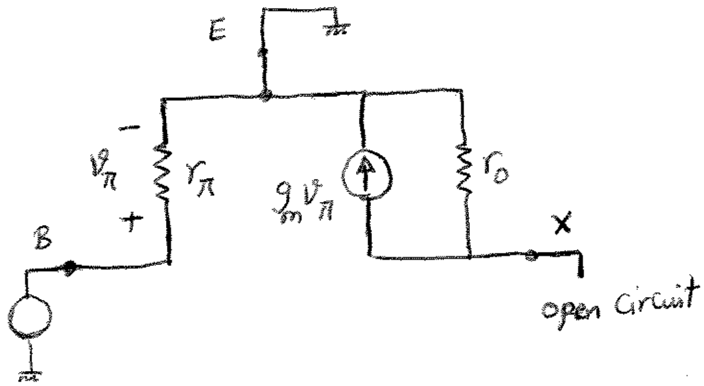
$$\Delta V_X \simeq \frac{dV_X}{dV_{EB}} \Delta V_{EB} \Rightarrow \Delta V_X \simeq \frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T} \Delta V_{EB}$$

$$\xrightarrow{\Delta V_{EB} = -\Delta V_B} \boxed{\Delta V_X \simeq -\frac{V_A}{V_T}\cdot\frac{I_C}{I_S} e^{-V_{EB}/V_T} \Delta V_B}$$

$$\Rightarrow \Delta V_X \simeq -\frac{5}{0.026}\times\frac{2\times10^{-3}}{6\times10^{-16}} \exp\left(-\frac{2.5-1.757}{0.026}\right)\times 0.1\times10^{-3} \Rightarrow \boxed{\Delta V_X \simeq -24.9\,mV}$$

(50) Continued .....

c)



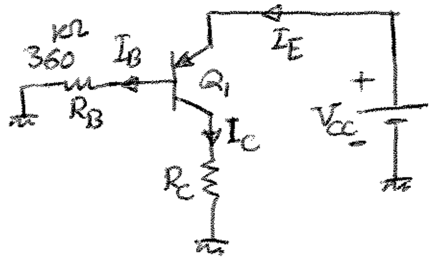
$$r_o = \frac{V_A}{I_C} = \frac{5^V}{2^{mA}} \Rightarrow \boxed{r_o \simeq 2.5\ k\Omega}$$

$$g_m = \frac{I_C}{V_T} = \frac{2^{mA}}{0.026\ V} \Rightarrow \boxed{g_m \simeq 76.9\ mS}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{\frac{2}{26}} \Rightarrow \boxed{r_\pi = 1.3\ k\Omega}$$

(51) $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

$R_B = 360\ k\Omega$



a) given: $V_C = V_B + 0.2^V$

$$\Rightarrow R_C I_C = R_B I_B + 0.2^V$$

$$\Rightarrow R_C I_C = R_B \frac{I_C}{\beta} + 0.2^V \Rightarrow \boxed{I_C = \frac{0.2^V}{R_C - \frac{R_B}{\beta}}} \Rightarrow \boxed{I_C = 0.5^{mA}}$$

$$I_C = I_S e^{+\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\frac{V_{EB}}{V_T}} \Rightarrow I_S = I_C e^{-\left(\frac{V_{CC} - R_B I_B}{V_T}\right)}$$

$$\Rightarrow \boxed{I_S = \left(\frac{0.2}{R_C - \frac{R_B}{\beta}}\right) \exp\left[-\frac{1}{V_T}\left(V_{CC} - R_B \times \frac{0.2^V}{\beta\left(R_C - \frac{R_B}{\beta}\right)}\right)\right]}$$

$$\Rightarrow \boxed{I_S \simeq 10^{-15} A = 1\ fA}$$

b) $g_m = \frac{I_C}{V_T}$

$$\Rightarrow \boxed{g_m = \frac{0.2^V}{V_T\left(R_C - \frac{R_B}{\beta}\right)}} \Rightarrow \boxed{g_m \simeq 19.23\ mS}$$

(52) $I_S = 5\times10^{-16}A$, $\beta = 100$, $V_A = \infty \Rightarrow r_o = \infty$

a) $V_{EB} = 0 \Rightarrow Q_1$ is off $\quad I_C \cong 0$

b) $I_B = 0 \Rightarrow Q_1$ is off

c) Applying KVL: $V_{CC} \cong V_{EB} + 1^{k\Omega} \times I_C$

$$\Rightarrow V_{EB} + 1^{k\Omega} \times I_S e^{\frac{V_{EB}}{V_T}} \simeq V_{CC} \Rightarrow V_{EB} + 5\times10^{-13} e^{\frac{V_{EB}}{26^{mV}}} \simeq 2.5^{V}$$

$$\Rightarrow \boxed{V_{EB} \simeq 751\, mV} \qquad I_C = 5\times10^{-16} e^{\frac{0.751}{0.026}} \Rightarrow \boxed{I_C \simeq 1.8mA}$$

with this current, Transistor is Saturated. Note $V_B < V_C$ Always

(d) $V_{BC} = 0 \Rightarrow$ Transistor is at the edge of Saturation

e) $I_C \cong 0.5mA \Rightarrow V_{EB} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26^{mV} \ln\left(\frac{0.5mA}{5\times10^{-16}}\right)$

$$\Rightarrow \boxed{V_{EB} \simeq 718\, mV}$$

$V_{Collector} = 500^{\Omega} \times I_C \Rightarrow \boxed{V_C = 0.25^{V}}$

As $V_B = 0$, $V_C = 0.25^{V} \Rightarrow$ Transistor is soft saturated

(53) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty \Rightarrow r_o = \infty$

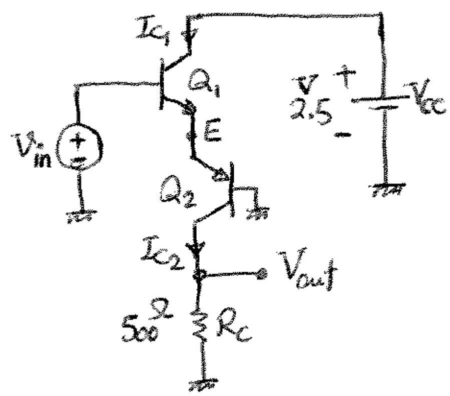
a) $V_{B_2} = \emptyset^V \xrightarrow[\text{Bias by } 200^{mV}_{max}]{BC\ Q_2\ \text{Forward}} V_{C_2 max} = 0.2\ V$

$\Rightarrow I_{C_2 max} = \dfrac{V_{C_2 max}}{R_C} = \dfrac{0.2^V}{500^\Omega}$

$\Rightarrow \boxed{I_{C_2 max} = 0.4\ mA}$

As shown

$I_{C_1} = I_{C_2}$



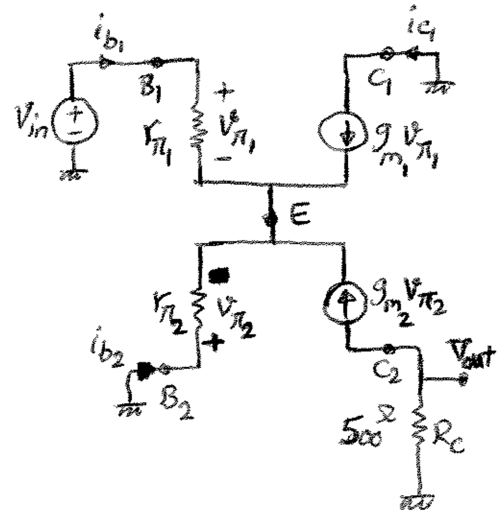
$V_{in_{max}} = V_{BE_{1 max}} + V_{EB_{2 max}} = V_T \ln \dfrac{I_{C_1 max}}{I_{S_1}} + V_T \ln \dfrac{I_{C_2 max}}{I_{S_2}}$

$\Rightarrow V_{in_{max}} = 26^{mV} \cdot \left[\ln \dfrac{0.4\times10^{-3}}{5\times10^{-16}} + \ln \dfrac{0.4\times10^{-3}}{5/3\times10^{-16}} \right] \Rightarrow \boxed{V_{in_{max}} \simeq 1.454^V}$

b) $g_{m_1} = \dfrac{I_{C_1}}{V_T} = \dfrac{0.4^{mA}}{26\ mV}$

$g_{m_2} = \dfrac{I_{C_2}}{V_T} = \dfrac{0.4^{mA}}{26^{mV}}$

$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\ mS}$

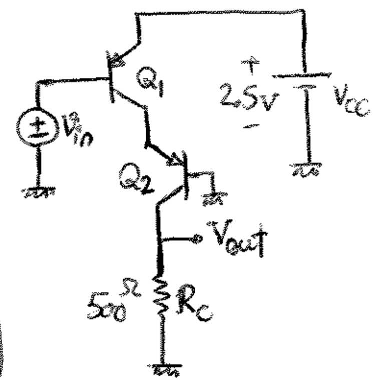


$r_{\pi_1} = \dfrac{\beta_1}{g_{m_1}} = \dfrac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5\ k\Omega}$

$r_{\pi_2} = \dfrac{\beta_2}{g_{m_2}} = \dfrac{50}{0.4/26} \Rightarrow \boxed{r_{\pi_2} = 3.25\ k\Omega}$

$V_A = \infty \Rightarrow \boxed{r_o = \infty}$

(54) $I_{S_1} = 3 I_{S_2} = 5\times10^{-16}\,A, \quad \beta_1 = 100, \quad \beta_2 = 50, \quad V_A = \infty$



a) $V_{B_2} = \phi \xrightarrow[\text{Forward biased by 200mV}]{Q_2 \text{ Base-Collector}} V_{C_2} = 0.2^V$

$$\Rightarrow I_{C_{2\,max}} = \frac{V_{C_2 max}}{R_C} = \frac{0.2^V}{500^\Omega} \Rightarrow \boxed{I_{C_{2\,max}} = 0.4\,mA}$$

As shown: $I_{C_1} \simeq I_{C_2}$ (Note: $I_{C_1} = I_{E_2} = \frac{\beta_2+1}{\beta_2} I_{C_2}$ precisely)

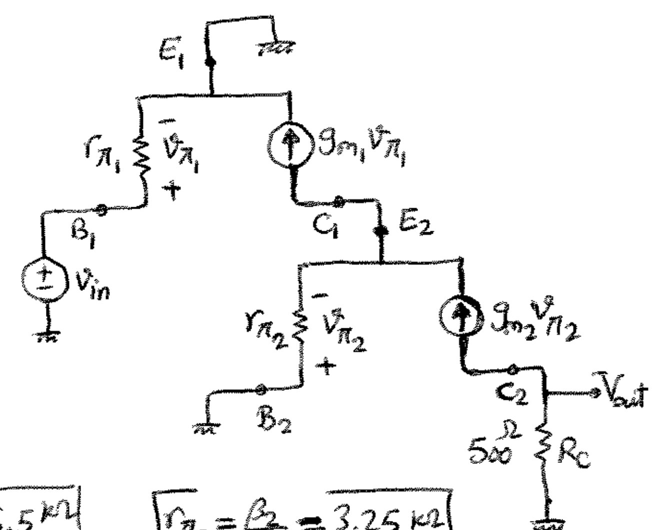
$$I_{C_1} \cong I_{S_1} e^{V_{EB_1}/V_T} \Rightarrow V_{EB_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \Rightarrow V_{CC} - V_{in} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)$$

$$\Rightarrow \boxed{V_{in} = V_{CC} - V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right)} \Rightarrow V_{in} = 2.5 - 0.026 \ln\left(\frac{4\times10^{-4}}{5\times10^{-16}}\right)$$

$$\Rightarrow \boxed{V_{in} \simeq 1.787\ V}$$ This is minimum acceptable V_{in}

b) $g_{m_1} = \frac{I_{C_1}}{V_T} \simeq \frac{0.4^{mA}}{26^{mV}}$

$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{0.4^{mA}}{26^{mV}}$



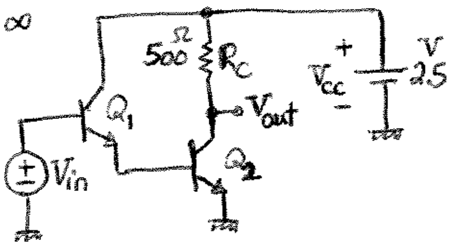
$$\Rightarrow \boxed{g_{m_1} = g_{m_2} \simeq 15.4\ mS}$$

$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{\frac{0.4}{26}} \Rightarrow \boxed{r_{\pi_1} = 6.5^{k\Omega}} \qquad \boxed{r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = 3.25\ k\Omega}$$

$$V_{EB_2} = V_T \ln\left(\frac{I_{C_2}}{I_{S_2}}\right) = 26^{mV} \ln\left(\frac{0.4\times10^{-3}}{\frac{5}{3}\times10^{-16}}\right) \Rightarrow \boxed{V_{EB_2} \simeq 741\ mV} \Rightarrow \boxed{V_{EC_1} \simeq 1.759\ V}$$

(Q_1 in active mode)

(55) $I_{S_1} = 3I_{S_2} = 5\times10^{-16}A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$



a) Q_2 is softly saturated $\Rightarrow V_{BC_2} = 0.2^V$

$$V_{BC_2} = 0.2^V \Rightarrow V_{B_2} - V_{C_2} = 0.2^V \Rightarrow V_{BE_2} - (V_{CC} - R_C I_{C_2}) = 0.2^V$$

$$\Rightarrow V_{BE_2} + R_C I_{S_2} e^{\frac{V_{BE_2}}{V_T}} = V_{CC} + 0.2$$

$$\Rightarrow V_{BE_2} + 500 \times \frac{5}{3}\times10^{-16} e^{\frac{V_{BE_2}}{V_T}} = 2.5 + 0.2 \Rightarrow \boxed{V_{BE_2} \simeq 800\,mV}$$

$$V_{BE_1} = V_T \ln\left(\frac{I_{C_1}}{I_{S_1}}\right) \qquad I_{C_2} = I_{S_2} e^{\frac{V_{BE_2}}{V_T}} = \frac{5}{3}\times10^{-16} e^{\frac{800}{26}} \Rightarrow \boxed{I_{C_2} \simeq 3.8^{mA}}$$

$$V_{BE_1} = V_T \ln\left(\frac{I_{C_2}/\beta_2}{I_{S_1}}\right) = 26^{mV} \ln\left(\frac{\frac{3.8\times10^{-3}}{50}}{5\times10^{-16}}\right) \Rightarrow \boxed{V_{BE_1} \simeq 669.4\,mV}$$

$$V_{in} = V_{BE_1} + V_{BE_2} \Rightarrow \boxed{V_{in} \simeq 1.469\ V}$$ Maximum allowable value for V_{in}

b)

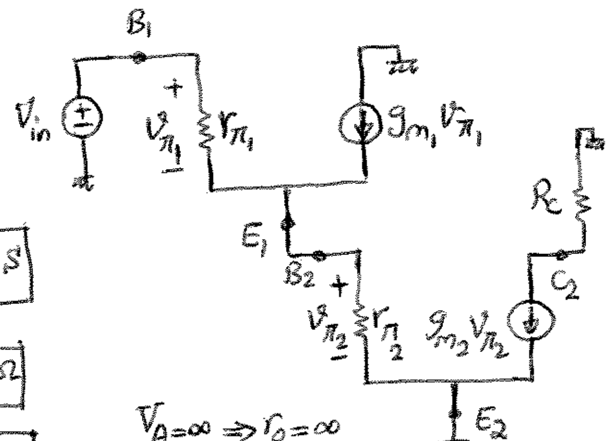
$$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{I_{C_2}/\beta_2}{V_T} = \frac{\frac{3.8^{mA}}{50}}{26^{mV}}$$

$$\Rightarrow \boxed{g_{m_1} \simeq 2.9\ mS}$$

$$g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{3.8^{mA}}{26^{mV}} \Rightarrow \boxed{g_{m_2} \simeq 146\ mS}$$

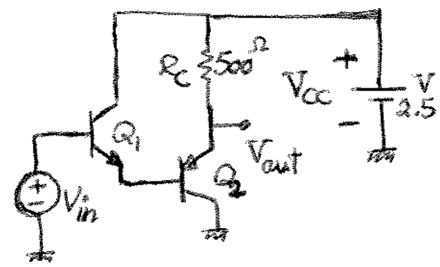
$$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{100}{2.9\times10^{-3}} \Rightarrow \boxed{r_{\pi_1} \simeq 34121\,\Omega}$$

$$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{50}{146\times10^{-3}} \Rightarrow \boxed{r_{\pi_2} \simeq 342\,\Omega}$$



$V_A = \infty \Rightarrow r_o = \infty$

(56) $I_{S_1} = 2 I_{S_2} = 6 \times 10^{-17} A$, $\beta_1 = 80$, $\beta_2 = 100$



a) $I_{C_2} = 2^{mA}$

$$V_{EB_2} = V_T \ln \frac{I_{C_2}}{I_{S_2}} = 26^{mV} \ln\left(\frac{2\times10^{-3}}{3\times10^{-17}}\right) \simeq 827.6^{mV}$$

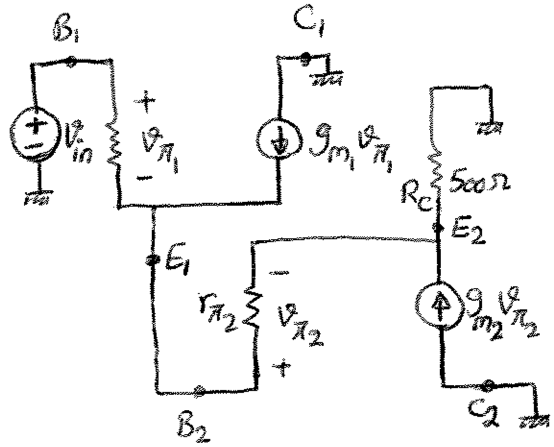
$$V_{BE_1} = V_T \ln \frac{I_{C_1}}{I_{S_1}} = 26^{mV} \ln\left(\frac{\frac{2\times10^{-3}}{100}}{6\times10^{-17}}\right) \simeq 689.9 \text{ mV}$$

$$\boxed{V_{in} = V_{CC} - R_C I_{C_2} - V_{EB_2} + V_{BE_1}} = 2.5 - 0.5^{k\Omega} \times 2^{mA} - 0.8276 + 0.6899$$

$$\Rightarrow \boxed{V_{in} \simeq 1.362 \text{ V}}$$

b) $g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{2^{mA}}{26^{mV}} \Rightarrow \boxed{g_{m_2} \simeq 76.9 \text{ mS}}$

$g_{m_1} = \frac{I_{C_1}}{V_T} = \frac{2^{mA}/100}{26^{mV}} \Rightarrow \boxed{g_{m_1} \simeq 769 \mu S}$



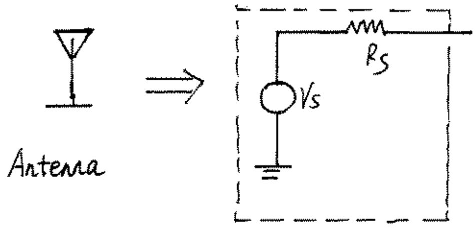
$r_{\pi_1} = \frac{\beta_1}{g_{m_1}} = \frac{80}{\frac{1}{1300}}$

$\Rightarrow \boxed{r_{\pi_1} = 104 \text{ k}\Omega}$

$r_{\pi_2} = \frac{\beta_2}{g_{m_2}} = \frac{100}{2/26} \Rightarrow \boxed{r_{\pi_2} = 1300 \Omega}$

$V_A = \infty \Rightarrow r_o = \infty$

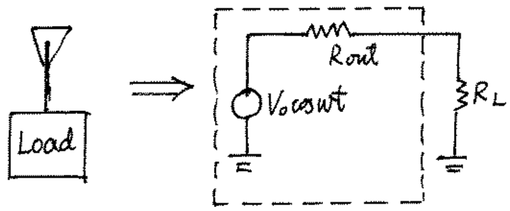
1)



Thevenin Equivalent:

$V_S = V_0 \cos\omega t$

$R_S = R_{out}$



Average power delivered to load $= (I_{RMS})^2 R_L$,

$$I_{RMS} = \frac{V_{RMS}}{R_{out} + R_L}, \quad V_{RMS} = \frac{V_0}{\sqrt{2}} \Rightarrow I_{RMS} = \frac{V_0}{\sqrt{2}(R_{out}+R_L)}$$

$$\text{Average power} = (I_{RMS})^2 R_L = \frac{V_0^2 R_L}{2(R_{out}+R_L)^2} \qquad \text{(Eq. 1)}$$

Plot of Average Power

When R_L is small, Eq. 1 is small.

When R_L is large, Eq. 1 is also small.

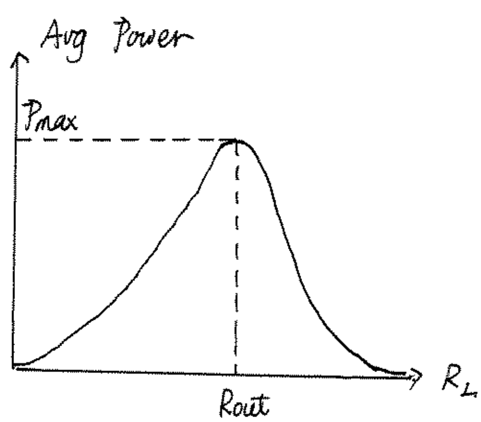
So for some R_L between zero and infinity, the average power will reach its peak. Lets take the derivative of Eq. 1 with respect to R_L to find the optimum R_L.

$$\frac{\partial}{\partial R_L}\left[\frac{V_0^2 R_L}{2(R_{out}+R_L)^2}\right] = \frac{V_0^2}{2(R_{out}+R_L)^2} - \frac{V_0^2 R_L}{(R_{out}+R_L)^3}$$

Setting it to zero and solve for R_L

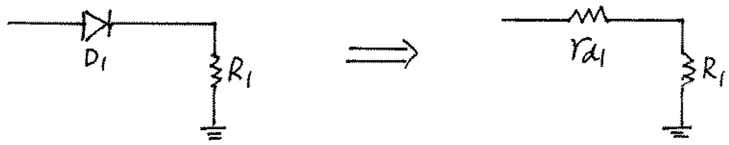
$$\frac{V_0^2}{2(R_{out}+R_L)^2} = \frac{V_0^2 R_L}{(R_{out}+R_L)^3} \Rightarrow \frac{(R_{out}+R_L)}{2} = R_L$$

$$\Rightarrow R_{out} + R_L = 2R_L \Rightarrow R_L = R_{out}$$



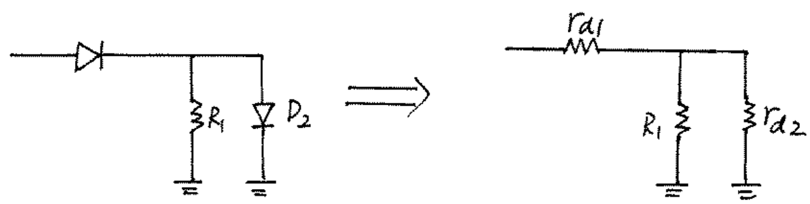
2) In small signal operation, a diode can be replaced by a linear resistor if changes are small.

a).



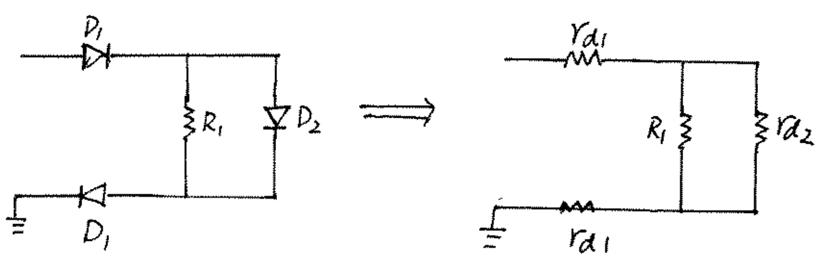
$R_{in} = r_{d1} + R_1$

b).



$R_{in} = r_{d1} + R_1 // r_{d2}$ (// means in parallel)

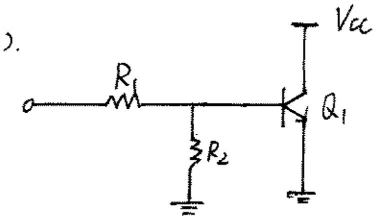
c).



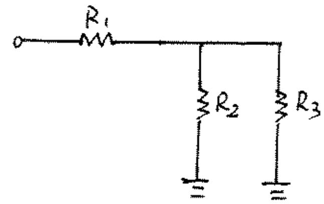
$R_{in} = 2r_{d1} + R_1 // r_{d2}$

3). When $V_A = \infty$, $V_o = \infty$.

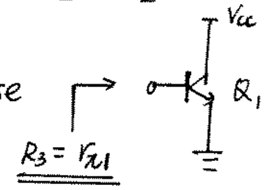
a).



$\Longrightarrow$

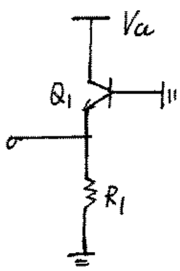


Replacing Q_1 by its equivalent resistance seen at base

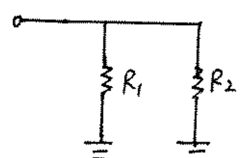


So $R_{in} = R_1 + R_2 /\!/ R_3 = R_1 + R_2 /\!/ r_{\pi 1}$

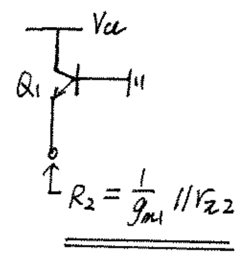
b).



$\Longrightarrow$

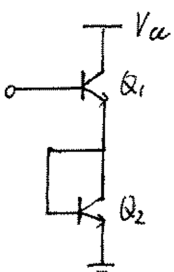


Replacing Q_1 by its equivalent resistance see at emitter

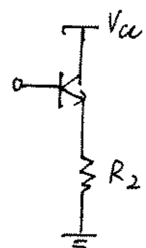


So $R_{in} = R_1 /\!/ R_2 = R_{in} /\!/ (\frac{1}{g_{m1}} /\!/ r_{\pi 1})$

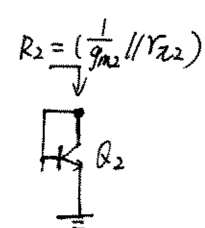
c).



$\Longrightarrow$



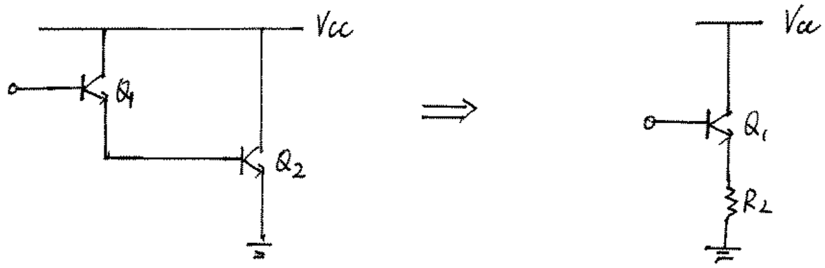
Replacing Q_2 by its equivalent diode-connected resistante



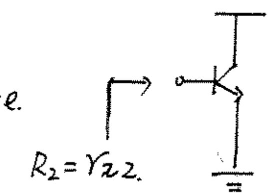
So $R_{in} = r_{\pi 1} + (1+\beta) R_2 = r_{\pi 1} + (1+\beta)(\frac{1}{g_{m2}} /\!/ r_{\pi 2})$

3).

d).



Replacing Q_2 by its equivalent resistance seen at Base.



So $R_{in} = r_{\pi 1} + (1+\beta) R_2 = r_{\pi 1} + (1+\beta) r_{\pi 2}$.

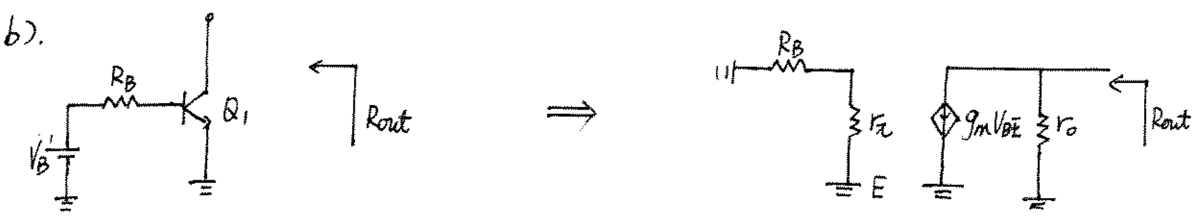
(Please refer to the textbook for all the equivalent resistances)

4). Since the problem doesn't say $V_A = \infty$, r_o must be considered in derivation.

a). Short V_B since it's a DC source, and replace Q_1 with an ideal transistor with its output resistance.

So $R_{out} = R_1 // r_o // \infty = R_1 // r_o$

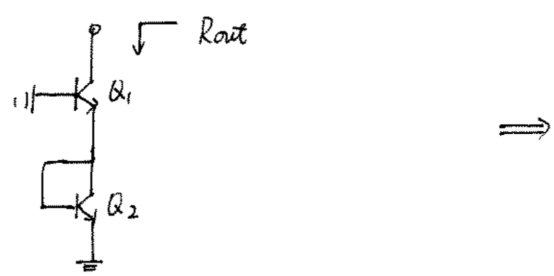
b).

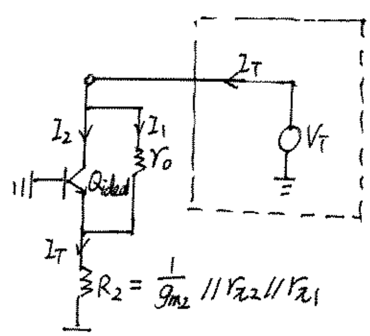


By drawing the small-signal model, it's easy to tell $V_{BE} = 0$ and $R_{out} = r_o$

c). Replace Q_1 with an ideal transistor and an output impedance r_{o1}.

Replace Q_2 with a resistor $\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1}$





Here, $r_{\pi 1}$ is included in R_2 because it is also connected from emitter to ground and it accounts for the base current of Q_1.

4) $I_1 = \frac{V_T - I_T R_2}{r_o}$, $I_2 = g_{m1}(0 - I_T R_2)$

$$I_T = I_1 + I_2 = \frac{V_T - I_T R_2}{r_o} - g_{m1} I_T R_2$$

$$\Rightarrow I_T + \frac{I_T R_2}{r_o} + g_m I_T R_2 = \frac{V_T}{r_o}$$

$$\Rightarrow \frac{V_T}{I_T} = r_o \left(1 + \frac{R_2}{r_o} + g_{m1} R_2\right)$$

$$\Rightarrow R_{out} = \frac{V_T}{I_T} = r_o(1 + g_{m1} R_2) + R_2$$

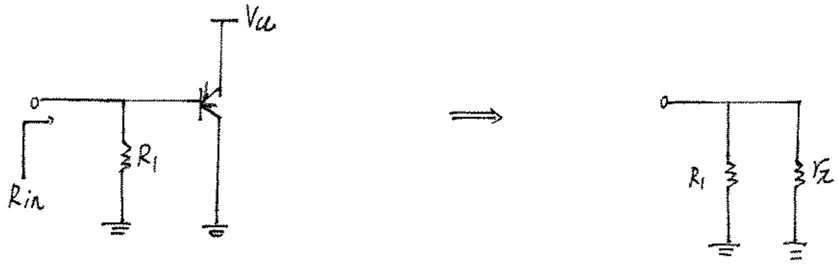
$$= r_o\left[1 + g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1}\right)\right] + \frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1}$$

Usually $\frac{1}{g_m} << r_\pi$, and if $Q_1 = Q_2$

$$R_{out} \approx \frac{1}{g_m} + 2r_o$$

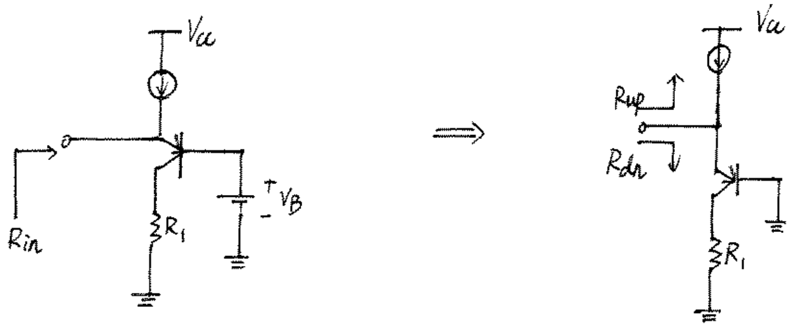
5). $V_A = \infty$, $r_o = \infty$

a).



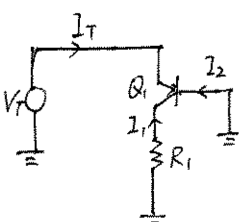
$R_{in} = R_1 // r_{\pi 1}$

b).



$R_{in} = R_{up} // R_{dn}$. $R_{up} = \infty$, since a DC current source is open.

Finding R_{dn}:



$$I_T = -(I_1 + I_2)$$

$$I_1 = g_m(0 - V_T) = -g_m V_T$$

$$I_2 = \frac{I_1}{\beta}.$$

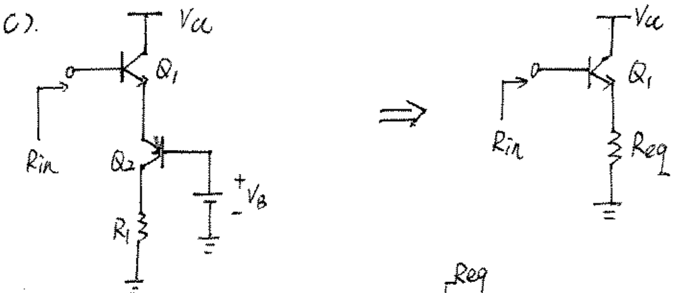
So $$I_T = -\left(-g_m V_T - \frac{g_m V_T}{\beta}\right) = \left(g_m + \frac{g_m}{\beta}\right) V_T$$

$$\frac{V_T}{I_T} = \frac{1}{\left(g_m + \frac{g_m}{\beta}\right)} = \frac{1}{g_m} // r_\pi$$

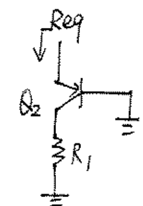
$$R_{dn} = \frac{V_T}{I_T} = \frac{1}{g_m} // r_\pi$$

So $R_{in} = R_{up} // R_{dn} = \infty // \frac{1}{g_m} // r_\pi = \frac{1}{g_m} // r_\pi$

5). c).



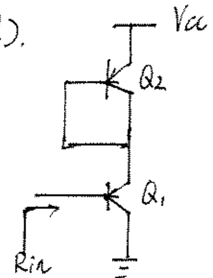
From b), we know that



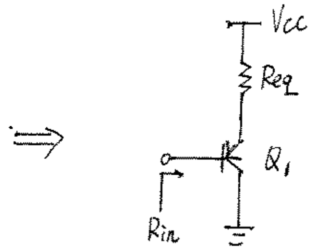
$R_{eq} = \frac{1}{g_{m2}} // r_{\pi 2}$.

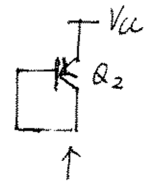
So $R_{in} = r_{\pi 1} + (1+\beta) R_{eq} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$.

d).



⇒

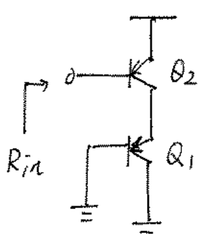




$R_{eq} = \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{in} = r_{\pi 1} + (1+\beta) R_{eq} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$

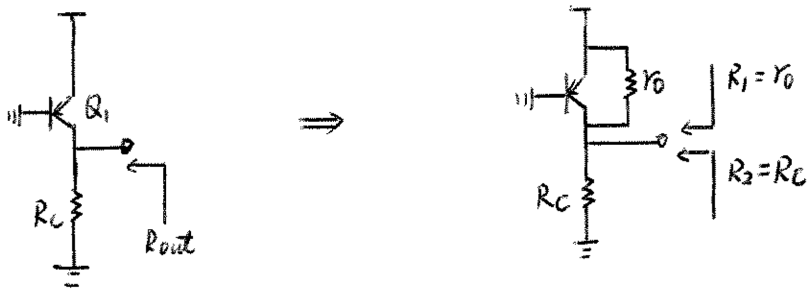
e)



$R_{in} = r_{\pi 2}$. Q_1 plays no role here since it's connected to the collector of Q_2. It can not be seen from the Base of Q_2.

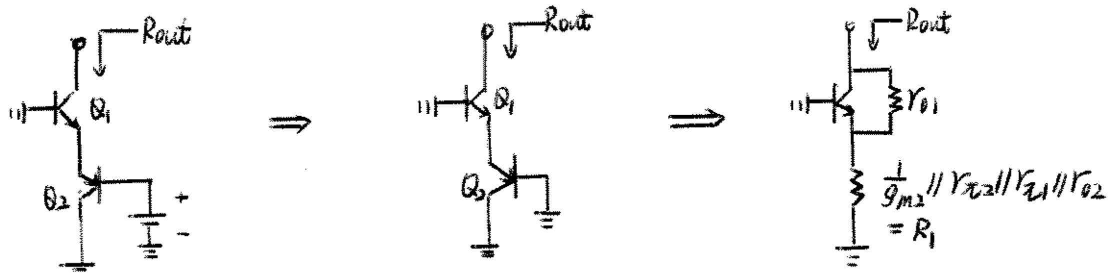
b) Since the problem doesn't state $V_A = \infty$, r_o is not ∞.

a)



$R_{out} = R_1 // R_2 = R_C // r_o$

b)

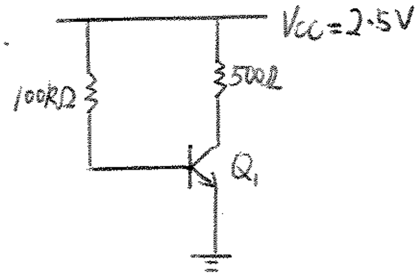


As shown is problem 4). c).

$R_{out} = R_1 + r_{o1} + g_{m2} r_{o1} R_1 = r_{o1} + (1 + g_{m1} r_{o1}) R_1$

$= r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} // r_{\pi 2} // r_{\pi 1} // r_{o2} \right)$

7) a).



$$I_B = \frac{V_{cc} - V_{BE}}{100\,K\Omega}, \quad I_c = \beta I_B$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.7V$,

$$I = \beta\left(\frac{V_{cc} - V_{BE}}{100K\Omega}\right) = 1.8\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747V, \text{ not } 0.7V, \text{ reiterate}$$

$V_{BE} = 0.747\,V$, $I_c = 1.753\,mA$

Verify V_{BE}, $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.746V$, converged

$$V_{CE} = 2.5 - (1.753)(0.5k) = 1.62\,V$$

$V_{CE} > V_{BE}$, Q_1 in forward active region.

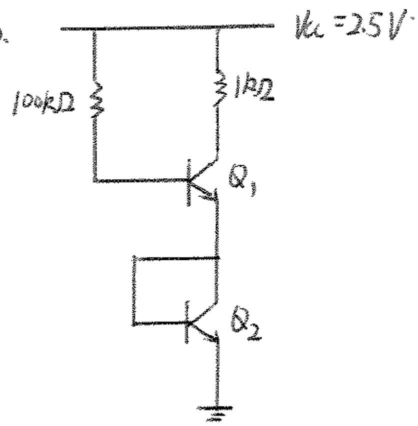
$I_c = 1.754\,mA$ $\quad$ $V_{CE} = 1.62V$

$I_B = 17.54\,\mu A$ $\quad$ $V_{BE} = 0.746V$

↑

operating point

7). b).



$$I_{B1} = \frac{2.5 - (V_{BE1} + V_{BE2})}{100K\Omega}$$

$$I_{C1} = \beta I_{B1}$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right), \quad V_{BE2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$

Assume $V_{BE1} = V_{BE2} = 0.8V$

$$I_{C1} = \beta\left(\frac{2.5 - 1.6}{100K}\right) = 0.9mA$$

$V_{BE1} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.728V$, not 0.8V, reiterate

$I_{C2} = 0.9mA$, since β's are the same

$V_{BE2} = 0.728V$

$$I_{C1} = \beta\left(\frac{2.5 - (2)(0.728)}{100K\Omega}\right) = 1.042mA = I_{C2}$$

$V_{BE1} = V_{BE2} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.733V$, iterate once more

$$I_{C1} = I_{C2} = \beta\left(\frac{2.5 - (2)(0.733)}{100K\Omega}\right) = 1.034mA$$

7) b)

$I_{c_1} = I_2 = 1.034\,mA$

$V_{BE_1} = V_{BE_2} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.733$, converges.

$V_{CE_1} = 2.5 - 0.733 - (1.034)(1K\Omega) = 0.733V$

$V_{CE} = V_{BE}$, Q_1 at the edge of active region.

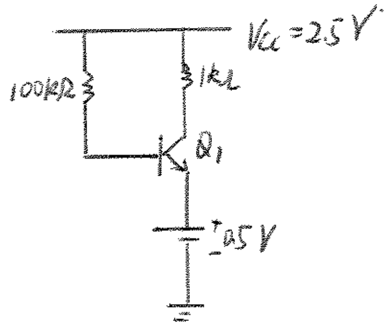
$V_{BE_2} = V_{CE_2} = 0.733V$

Operating point:

$I_{C_1} = 1.034mA$	$I_{C2} = 1.034mA$
$I_{B_1} = 0.01\,mA$	$I_{B2} = 0.01\,mA$
$V_{BE_1} = 0.733\,V$	$V_{BE_2} = 0.733\,V$
$V_{CE_1} = 0.733V$	$V_{CE_2} = 0.733V$

Although, for Q_2 $V_{BE} = V_{CE}$, it is at the edge of active region, the situation is not as severe as Q_1's. Since Q_2's configuration will always render $V_{BE} = V_{CE}$, whereas for Q_1, V_{CE} may drop below V_{BE}.

7) c).



$$I_B = \frac{V_{cc} - (V_{BE} + 0.5)}{100K}$$

$$I_c = \beta I_B$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.8V$,

$$I_c = \beta\left(\frac{2.5 - 1.3}{100K}\right) = 1.2\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$$, not 0.8, reiterate

$$V_{BE} = 0.736V, \quad I_c = \beta\left(\frac{2.5 - (0.736 + 0.5)}{100K\Omega}\right) = 1.26\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$$, converges.

$$V_{CE} = (2.5 - 0.5) - (1.26)(1K\Omega) = 0.74$$

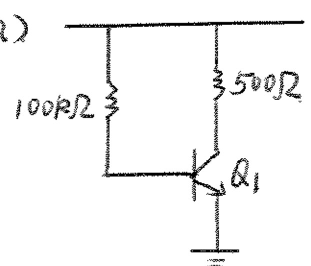
$V_{CE} > V_{BE}$, Q_1 in forward active region

Operating point

$I_c = 1.26\,mA$ $\quad V_{BE} = 0.738V$

$I_B = 0.0126\,mA$ $\quad V_{CE} = 0.74V$

8). a)



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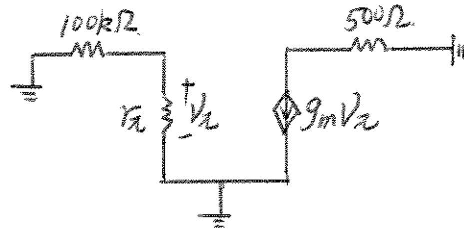
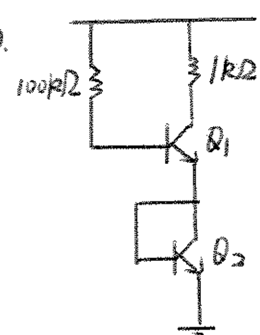


$$g_m = \frac{I_C}{V_T} = \frac{1.754mA}{26mV} = 0.0675\ S$$

$$r_{\pi 1} = \frac{\beta}{g_m} = \frac{100}{0.0675}\ \Omega = 1482.3\ \Omega$$

b).



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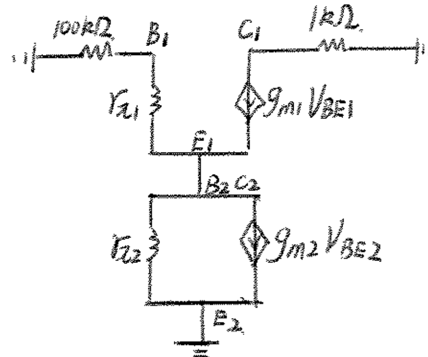


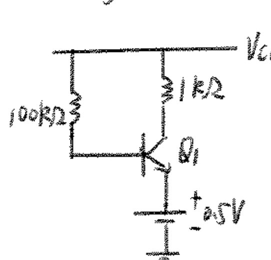
$$g_{m1} = \frac{I_C}{V_T} = \frac{1.034mA}{26mV} = 0.04\ S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.04}\ \Omega = 2500\ \Omega$$

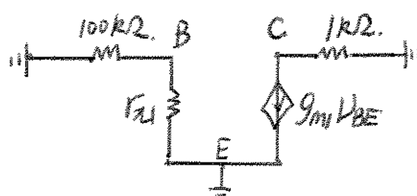
$$g_{m2} = \frac{I_C}{V_T} = \frac{1.034mA}{26mV} = 0.04\ S$$

$$r_{\pi 2} = \frac{\beta}{g_{m2}} = \frac{100}{0.04}\ \Omega = 2500\ \Omega.$$

c).



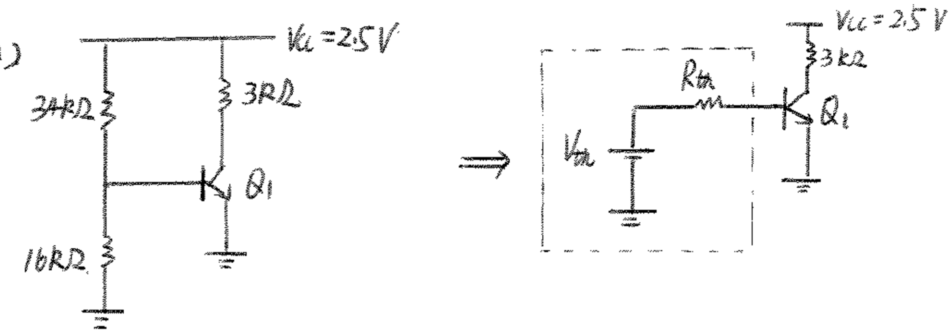
⟹



$$g_{m1} = \frac{I_C}{V_T} = \frac{1.26mA}{26mV} = 0.048\ S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.048}\ \Omega = 2083\ \Omega$$

9) a)



Thevenin Equivalent

$$R_{th} = \frac{34 \times 16}{34+16} k\Omega = 10.88\, k\Omega$$

$$V_{th} = \frac{2.5V \times 16}{34+16} = 0.8\, V.$$

$$I_c = \beta \left(\frac{0.8 - V_{BE}}{10.88K} \right), \quad V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Assume $V_{BE} = 0.7$,

$$I_c = \beta \left(\frac{0.8 - 0.7}{10.88K\Omega} \right) = 0.92\, mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$$

Iterate, $V_{BE} = 0.734V$

$$I_c = \beta \left(\frac{0.8 - 0.734}{10.88K\Omega} \right) = 0.61\, mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.724\, V$$

Iterate, $V_{BE} = 0.724\, V$

$$I_c = \beta \left(\frac{0.8 - 0.724}{10.88\, K\Omega} \right) = 0.699\, mA$$

9)

a) $I_c = 0.699 mA$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727 V$$

Iterate, $V_{BE} = 0.727V$

$$I_c = \beta\left(\frac{0.8 - 0.727}{10.88 K\Omega}\right) = 0.67 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.726 V, \text{ Converged !!}$$

$V_{CE} = 2.5 - (0.67)(3K\Omega) = 0.49$

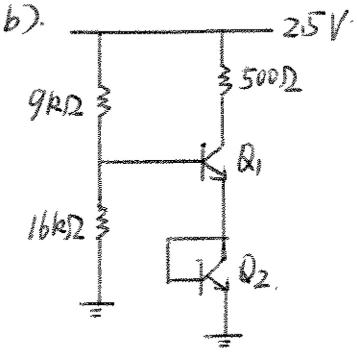
$V_{BE} - V_{CE} = 0.236V$, Soft-saturation, still ok.

Operating point:

$I_c = 0.67 mA$ $V_{BE} = 0.726 V$

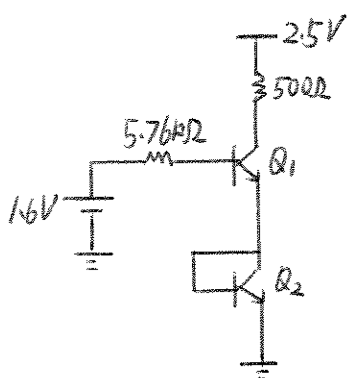
$I_B = 6.7 \mu A$ $V_{CE} = 0.49 V$

9) b).



$$R_{th} = \frac{9 \times 16}{9+16}\,k\Omega = 5.76\,k\Omega$$

$$V_{th} = 25V \times \frac{16}{9+16} = 1.6V$$



$$I_{C1} = \beta\left(\frac{1.6 - (V_{BE1} + V_{BE2})}{5.76\,k\Omega}\right)$$

$$V_{BE} = V_{BE1} = V_{BE2} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

$$I_{C1} = I_{C2} = I_c$$

Guess $V_{BE1} = V_{BE2} = 0.7V$

$$I_c = \beta\left(\frac{1.6 - 1.4}{5.76\,k\Omega}\right) = 3.47\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.769V$$

Iterate, $V_{BE} = 0.769V$

$$I_c = \beta\left(\frac{1.6 - (2)(0.769)}{5.76\,k\Omega}\right) = 1.08\,mA$$

9)

b)

$$I_c = 1.08 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738 V$$

Iterate, $V_{BE} = 0.738V$

$$I_c = \beta\left(\frac{1.6 - 2(0.738)}{5.76K\Omega}\right) = 2.15 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.756 V$$

Iterate, $V_{BE} = 0.756V$

$$I_c = \beta\left(\frac{1.6 - 2(0.756)}{5.76K\Omega}\right) = 1.53 mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747V$$

Iterate ... (For 3 more times)

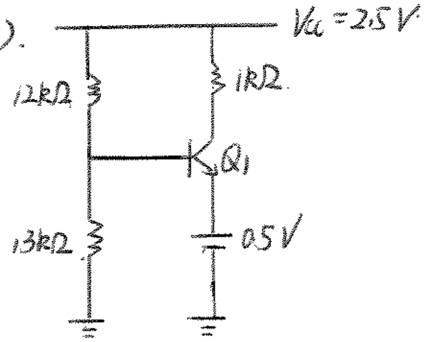
$V_{BE} = 0.75V$, $I_c = 1.74mA$ converged

$V_{CE} = 2.5 - 0.75 - (1.74)(0.5) = 0.88 V$

Operating Point

$I_{C_1} = 1.74 mA$	$I_{C_2} = 1.74mA$
$I_{B_1} = 17.4 \mu A$	$I_{B_2} = 17.4 \mu A$
$V_{BE} = 0.75V$ (Forward active)	$V_{BE_2} = 0.75V$ (Edge of forward active)
$V_{CE} = 0.88V$	$V_{CE_2} = 0.75V$

9). c).

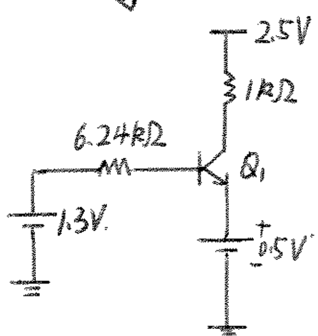


$\Rightarrow$

$$V_{th} = 2.5V \times \frac{13}{12+13} = 1.3V$$

$$R_{th} = \frac{12 \times 13}{12+13} k\Omega = 6.24k\Omega$$

$\Downarrow$



$$I_c = \beta\left(\frac{1.3-(V_{BE}+0.5)}{6.24K\Omega}\right)$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right)$$

Guess $V_{BE} = 0.743V$

$$I_c = \beta\left(\frac{1.3-(0.743+0.5)}{6.24K\Omega}\right) = 0.913\,mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$$

Iterate, $V_{BE} = 0.734V$

$$I_c = \beta\left(\frac{1.3-(0.734+0.5)}{6.24K\Omega}\right) = 1.06mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.738V$$

9)

c)

Iterate, $V_{BE} = 0.738V$

$$I_c = \beta\left(\frac{1.3-(0.738+0.5)}{6.24K\Omega}\right) = 0.99mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$$

$$V_{CE} = 2.5-0.5-(0.99)(1K\Omega) = 1.01V$$

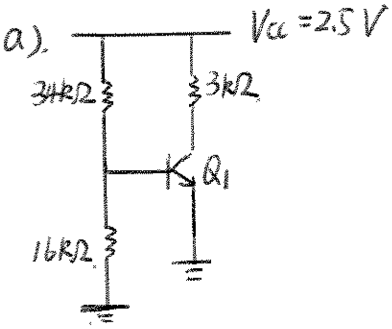
$V_{CE} > V_{BE} \Rightarrow$ Forward Active Region

Operating Point

$I_c = 0.99mA$ $\quad$ $V_{BE} = 0.736V$

$I_B = 9.9\mu A$ $\quad$ $V_{CE} = 1.01V$

10). a).



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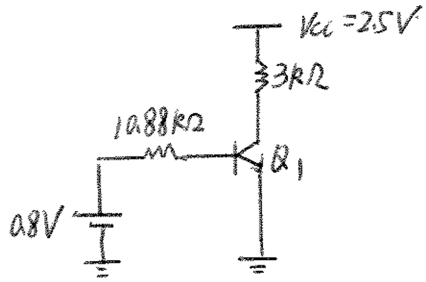

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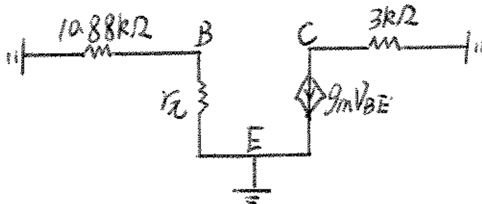
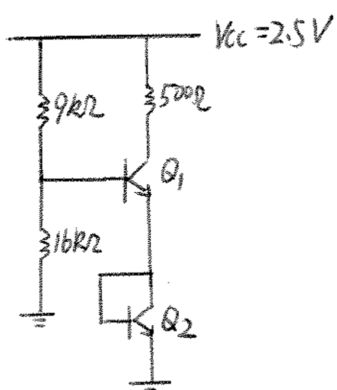


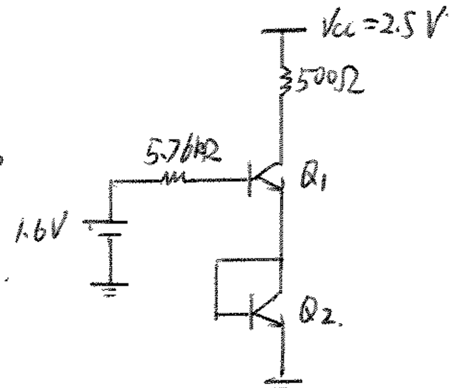
$$g_m = \frac{I_c}{V_T} = \frac{0.67mA}{26mV} = 0.026S$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.026}\Omega = 3846\Omega$$

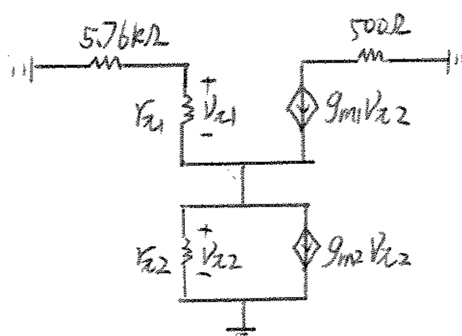
b).



⟹



⟹



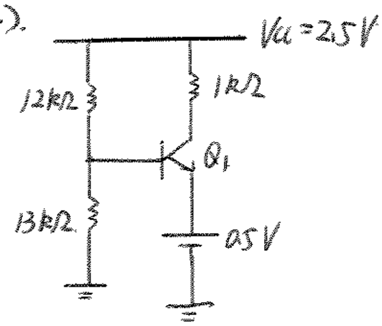
$$g_{m1} = \frac{1.74mA}{26mV} = 0.067S$$

$$r_{\pi 1} = \frac{\beta_1}{g_{m1}} = 1494.3\Omega$$

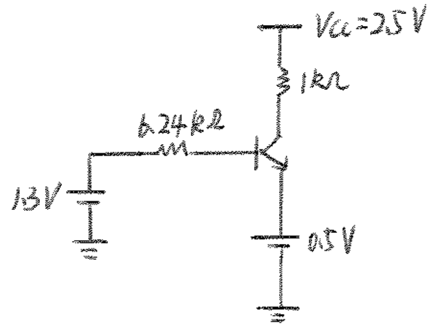
$$g_{m2} = 0.067S$$

$$r_{\pi 2} = 1494.3\Omega.$$

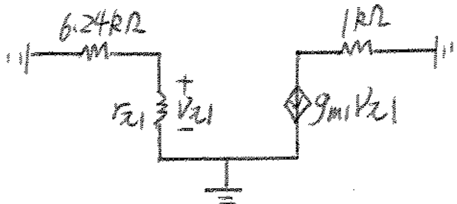
10) c).



⇒



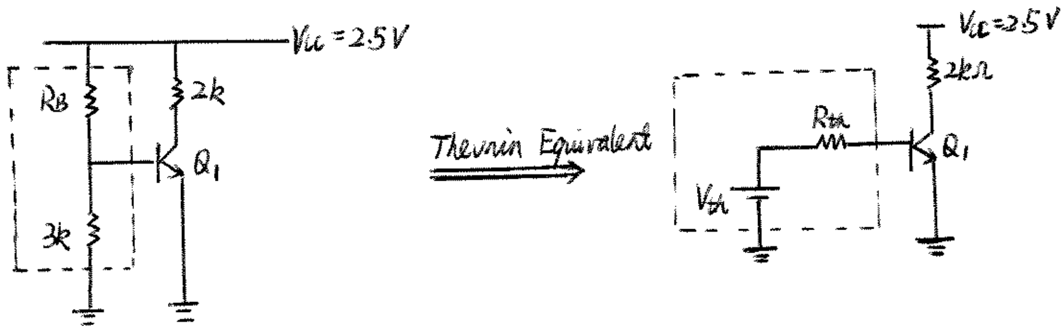
⇒



$$g_{m1} = \frac{I_C}{V_T} = \frac{0.99mA}{26mV} = 0.038\,S$$

$$r_{\pi 1} = \frac{\beta}{g_{m1}} = \frac{100}{0.038}\Omega = 2632\Omega.$$

11) a). Find the minimum R_B that guarantees forward active region.



$$R_{th} = \frac{R_B \times 3}{R_B + 3}, \qquad V_{th} = \frac{2.5 \times 3}{R_B + 3}$$

To maintain Q_1 in forward-active region, $V_{CE} \geq V_{BE}$ ⊛

$$V_{CE} = V_{cc} - I_C \cdot 2k, \quad I_C = \beta I_B, \quad I_B = \frac{V_{th} - V_{BE}}{R_{th}}$$

So $V_{CE} = V_{cc} - \beta \left(\frac{V_{th} - V_{BE}}{R_{th}} \right) \cdot 2k$

From ⊛

$$V_{cc} - \beta \left(\frac{V_{th} - V_{BE}}{R_{th}} \right) \cdot 2k \geq V_{BE} \qquad ①$$

And $V_{BE} = V_T \ln(I_C / I_S) = V_T \ln[\beta (V_{th} - V_{BE}) / R_{th} / I_S]$ ②

Find the minimum R_B by iteration. Guess $V_{BE} = 0.8$ as initial condition.

Use $V_{BE} = 0.8$, and subsitute R_{th} and V_{th} into ①, it can be calculated

$$R_B \geq 6.178\,k$$

Check the validity of V_{BE}. With $R_B = 6.178k$, from ②

$$V_{BE} = 0.727V.$$

So the initial guess of V_{BE} is not accurate.

Reiterate with $V_{BE} = 0.727$, it can be calculated from ①

$$R_B \geq 7.058\,k.$$

II) With $R_B \geqslant 7.06k$, from ②

$$V_{BE} = 0.728$$

It's very close to 0.727. So the results have converged. (Satisfy both ① & ②)

The final answer is

$$R_B \geqslant 7.06k$$

b). β changes from 100 to 200, so $\partial\beta$ is 100

$$V_{CB} = 2.5 - I_C(2k) - V_{BE} = 2.5 - \left(\frac{V_{th} - V_{BE}}{R_{th}}\right)\beta \cdot (2k) - V_{BE}$$

$$\frac{\partial V_{CB}}{\partial \beta} = -\left(\frac{V_{th} - V_{BE}}{R_{th}}\right) \cdot 2k$$

$$\partial V_{CB} = -\left(\frac{V_{th} - V_{BE}}{R_{th}}\right) \cdot 2k \cdot (\partial\beta) = -1.6627$$

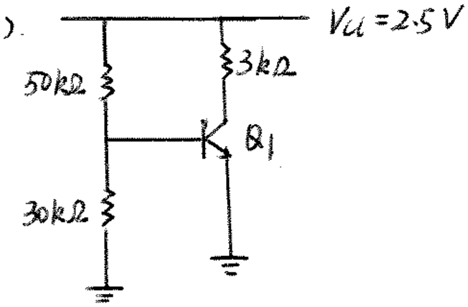
(Forward bias sustained during β's rising : 1.663 V)

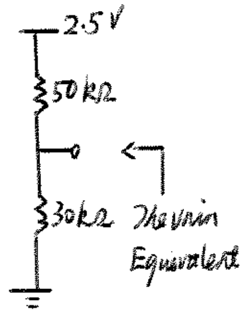
Original $V_{CB} = 0.01428$

Total net forward bias after β has rose to 200 :

$$1 - 1.6627 + 0.01428 = 1.648 \, (V)$$

12). a).





$V_{th} = 2.5 \times \frac{30}{50+30}$

$= 0.9375$

$R_{th} = \frac{30 \times 50}{30+50} k$

$= 18.75\ k$

Since $I_C = 0.5mA$, $I_B = \frac{I_C}{\beta} = 0.005mA$.

$I_B = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{BE} = V_{th} - I_B \cdot R_{th} = 0.84375$

$I_C = I_S\, e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = 4.03 \times 10^{-15} (mA)$

b). At the edge of saturation means $V_{BE} - V_{CE} = 0$.

(soft saturation not allowed)

$V_{CE} = 2.5 - I_C \cdot (3k)$, in which $I_C = \beta I_B = \beta \left(\frac{V_{th} - V_{BE}}{R_{th}}\right)$

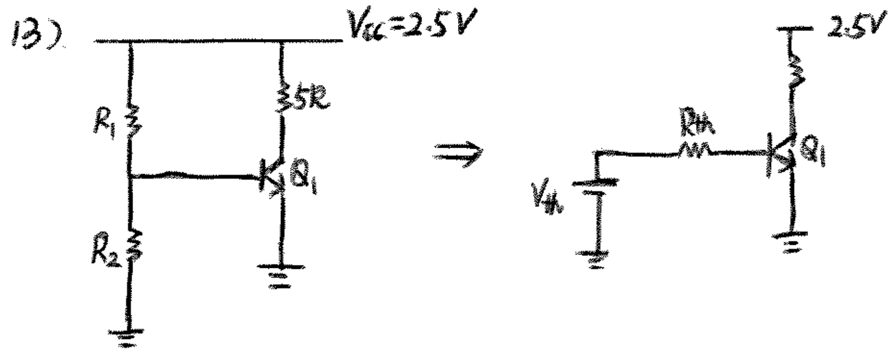
so $V_{BE} = 2.5 - \beta\left(\frac{V_{th} - V_{BE}}{R_{th}}\right) \cdot (3k)$

Solve this equation:

$V_{BE} = 0.83$.

$I_S = \frac{I_C}{e^{(V_{BE}/V_T)}} = \frac{\beta\left(\frac{V_{th} - V_{BE}}{R_{th}}\right)}{e^{(V_{BE}/V_T)}} = 7.84 \times 10^{-15} (mA)$

13)



$$V_{th} = 2.5 \times \frac{R_2}{R_1+R_2}$$

$$R_{th} = \frac{R_1 R_2}{R_1+R_2}$$

$$R_{in} = R_{th} \,//\, r_\pi = R_{th} \,//\, \frac{\beta}{g_m} = R_{th} \,//\, V_T\beta/I_C > 10k\Omega$$

$$g_m \geq \frac{1}{260\Omega} = 0.0038\,S$$

Let's choose g_m to be $0.0038\,S$

$$g_m = \frac{I_C}{V_T} \Rightarrow I_C = g_m V_T = 0.104\,(mA)$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.761\,(V)$$

Let $R_{in} = 10\,k\Omega$

$$R_{in} = R_{th} \,//\, V_T\beta/I_C \Rightarrow R_{th} = 16.13\,k\Omega \qquad ①$$

$$I_B = \frac{I_C}{\beta} = \frac{V_{th} - V_{BE}}{R_{th}} \Rightarrow V_{th} = V_{BE} + \frac{I_C \cdot (R_{th})}{\beta} = 0.78\,V \qquad ②$$

$$② \Rightarrow 2.5 \times \frac{R_2}{R_1+R_2} = 0.78\,V$$

$$① \Rightarrow \frac{R_1 R_2}{R_1+R_2} = 16.13\,k\Omega$$

It can be solved that $R_1 = 51.7\,k\Omega$; $R_2 = 23.44\,k\Omega$

This is only one possible solution set. The thought process is more important.

14). If g_m at least $\frac{1}{26} = 0.03848\ (\Omega^{-1})$

Let $g_m = 0.03848 = \frac{I_c}{V_T} \Rightarrow I_c = 0.99996\ (mA)$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.82\ V$

$V_{CE} = V_{CC} - I_C \cdot 5k = -2.5$

No solution exists because the transistor is in saturation mode

where g_m is essentially zero.

Whereas for problem 13),

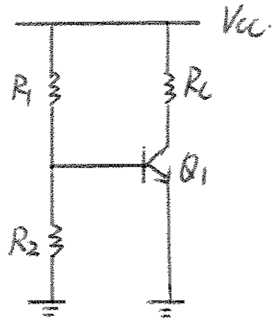
$V_{CE} = V_{CC} - I_C \cdot 5k = 2.5 - 0.104 \times 5 = 1.98\ V$

$V_{BE} = 0.761$

$V_{CE} > V_{BE}$

So Q_1 is still in forward-active region.

15).



Gain $= A_0$

$R_{out} = R_0 = R_C$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \parallel r_\pi \quad , \quad r_\pi = \frac{\beta}{g_m}$$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \parallel \frac{\beta}{g_m}$$

$$\text{Gain} = A_0 = g_m R_0 \Rightarrow g_m = \frac{A_0}{R_0} = \frac{I_c}{V_T} \Rightarrow I_c = \frac{A_0 V_T}{R_0}$$

(I_c is set)

Bias point analysis:

$$\frac{\dfrac{V_{cc} R_2}{R_1 + R_2} - V_{BE}}{\dfrac{R_1 R_2}{R_1 + R_2}} = \frac{A_0 V_T}{R_0}$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = V_T \ln\left(\frac{A_0 V_T}{R_0 I_s}\right)$$

15)

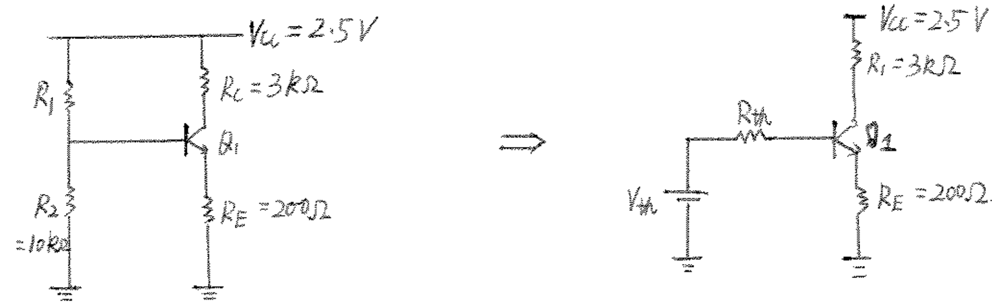
$$\frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_s}\right)}{\dfrac{R_1 R_2}{R_1+R_2}} = \frac{A_0}{R_0} V_T$$

$$\frac{R_1 R_2}{R_1+R_2} = \frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_s}\right)}{\dfrac{A_0}{R_0} V_T}$$

Max R_{in}:

$$\frac{\dfrac{V_{cc}R_2}{R_1+R_2} - V_T \ln\left(\dfrac{A_0 V_T}{R_0 I_s}\right)}{\dfrac{A_0}{R_0} V_T} \,//\, \beta \frac{R_0}{A_0}$$

1b).



a) $V_{th} = V_{cc} \cdot \frac{R_2}{R_1 + R_2} = 2.5 \times \frac{10k}{10k + R_1}$,

$R_{th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 \cdot (10k)}{R_1 + 10k}$

$V_{th} = I_B R_{th} + V_{BE} + I_E R_E$ (*)

Since $I_C = 0.25mA$, $I_B = 0.0025mA$, $I_E = \frac{0.25mA}{99} = 0.2525mA$.

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.696V$

(*) becomes

$$2.5 \times \frac{10k}{10k + R_1} = 0.0025m \times \frac{R_1 \cdot (10k)}{R_1 + 10k} + 0.696 + 0.2525 \times 0.2$$

So

$$R_1 = 22.73k.$$

b) If R_E deviates by 5%, changes in R_E is 10Ω.

$$I_B = \frac{V_{th} - (V_{BE} + I_E R_E)}{R_{th}} \Rightarrow \frac{I_C}{\beta} = \frac{V_{th} - \left(V_{BE} + \frac{I_C}{\alpha} R_E\right)}{R_{th}}$$

$$\Rightarrow I_C = \frac{\beta\alpha\,(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E}$$

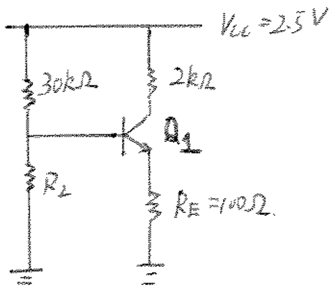
$$\Rightarrow \partial I_C = -\frac{\beta^2 \alpha\,(V_{th} - V_{BE})}{(\alpha R_{th} + \beta R_E)^2}\,\partial R_E$$

16) $\partial R_E = 10$, $V_{th} = 0.764$, $V_{BE} = 0.7465$, $R_{th} = 6.94k$, $\alpha = 0.99$, $\beta = 100$

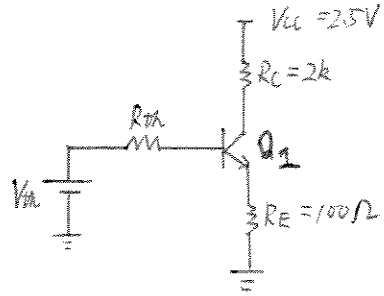
So $\partial I_C = -0.0024\ (mA)$

The error is $\frac{0.0024}{0.25} \times 100\% = 0.96\%$ in I_C.

17).



⟹



$$V_{th} = \frac{R_2 \times 2.5}{30k + R_2}, \quad R_{th} = \frac{30k \times R_2}{30k + R_2}$$

$V_{CE} \geq V_{BE}$ (To be guaranteed in active mode, soft saturation is not allowed.)

$$V_{CE} = V_{CC} - (I_C \cdot 2k + I_E \cdot 100)$$

$$I_C = \frac{\beta\alpha(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \quad \left(\because I_C = \frac{V_{th} - (V_{BE} + \frac{I_C}{\alpha} R_E)}{R_{th}} \right)$$

So $$V_{CE} = V_{CC} - \left[\frac{\beta\alpha(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \cdot 2k + \frac{\beta(V_{th} - V_{BE})}{\alpha R_{th} + \beta R_E} \times 100 \right]$$

$V_{CE} \geq V_{BE}$ means

$$2.5 - \left[\frac{99\left(\frac{R_2 \times 2.5}{30k + R_2} - V_{BE}\right)}{0.99 \times \frac{30k \times R_2}{30k + R_2} + 100 \times 100} \times 2k + \frac{100\left(\frac{R_2 \times 2.5}{30k + R_2} - V_{BE}\right)}{0.99 \times \frac{30k \times R_2}{30k + R_2} + 100 \times 100} \times 100 \right] \geq V_{BE} \quad ①$$

And

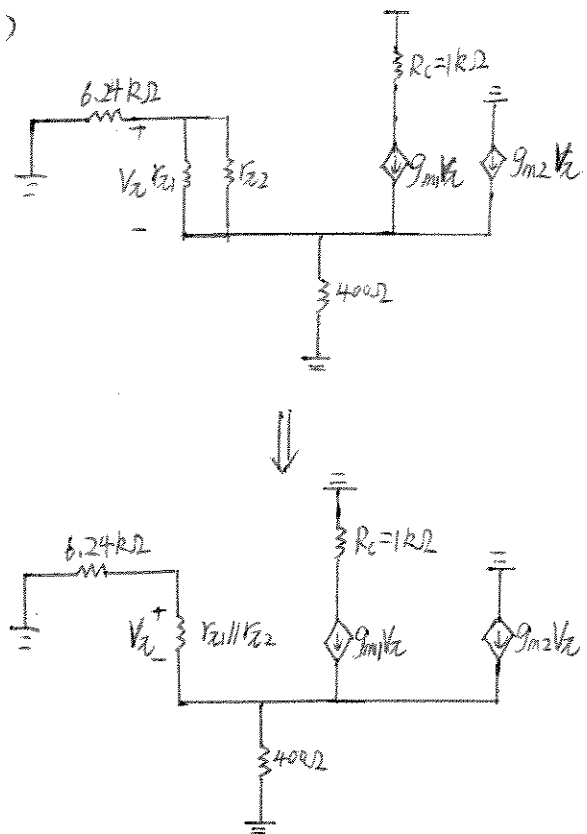
$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = V_T \ln\left[\frac{\beta\alpha(V_{th} - V_{BE})}{I_S(\alpha R_{th} + \beta R_E)}\right] \quad ②$$

There are two unknowns (R_2 and V_{BE}) and two equations (① and ②)

Since ② is a nonlinear equation, the problem can be solved by iteration.

Maximum $R_2 = 20.343k$.

18 b)



Small - Signal

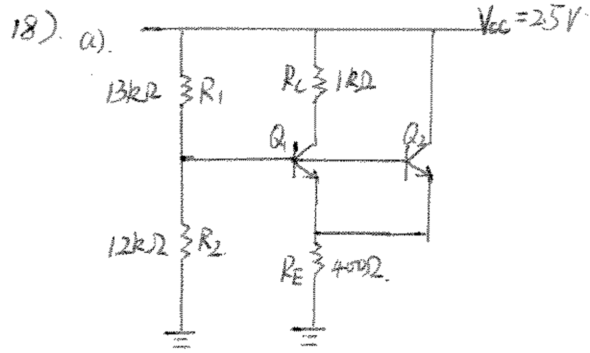
$g_{m1} = \frac{I_C}{V_T} = 0.02855\ (S)$

$g_{m2} = 0.0142\ (S)$

$r_{\pi 1} = 3571.4\ (\Omega)$

$r_{\pi 2} = 7042.3\ (\Omega)$

18). a).



$I_{S1} = 2 I_{S2} = 5\times10^{-16}$ A

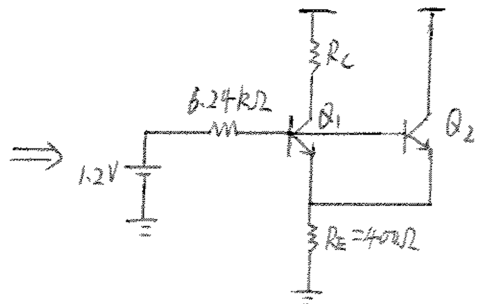
$\beta_1 = \beta_2 = 100$

$I_{S2} = 2.5\times10^{-16}$ A

$V_{th} = V_{cc} \times \frac{R_2}{R_1+R_2} = 1.2$ V

$R_{th} = R_1 // R_2 = 6.24$ kΩ

$\Rightarrow$



$I_{B2} = \frac{1.2 - (V_{BE} + 3I_{E2}\cdot R_E)}{6.24k}$, and $I_{B2} = \frac{I_{C2}}{\beta}$

so $\frac{I_{C2}}{\beta} = \frac{1.2 - (V_{BE} + 3I_{C2}/\alpha \cdot 0.4k)}{6.24k}$

$\beta = 100$, $\alpha = 0.99$

$\Rightarrow I_{C2} = \frac{(1.2 - V_{BE})\cdot(\beta\alpha)}{(\alpha\cdot 6.24k + 3\beta\cdot 0.4k)} = \frac{(1.2-V_{BE})(99)}{126.1776}$ (mA).

Guess $V_{BE} = 0.8$, $I_{C2} = 0.314$ mA

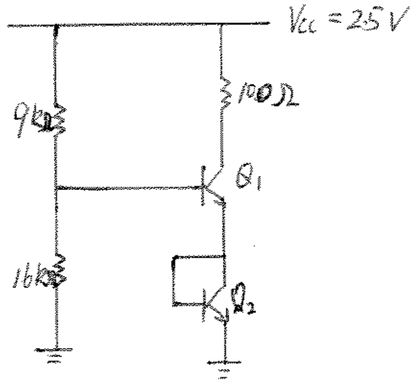
$V_{BE} = V_T \ln\left(\frac{I_C}{I_{S2}}\right) = 0.724$, not 0.8, so reiterate.

$I_{C2} = \frac{(1.2-0.724)(99)}{126.1776} = 0.3735$

$V_{BE} = 26 \ln\left(\frac{0.3735}{2.5\times10^{-13}}\right) = 0.728$, close. iterate again

$\Rightarrow V_{BE} \approx 0.729$ (V), $I_{C2} = 0.371$ (mA), $I_{C1} = 0.74$ (mA)

19).a)



$I_{S1} = I_{S2} = 4 \times 10^{-16} A$

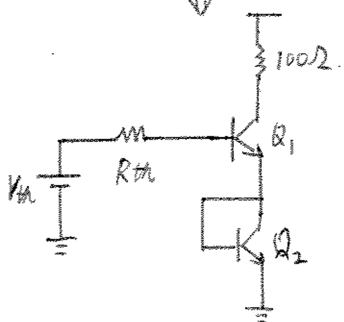
$\beta_1 = \beta_2 = 100$

$V_A = \infty$

$$V_{th} = \frac{(2.5)(16k)}{9k + 16k} = 1.6 (V)$$

$$R_{th} = 9k // 16k = 5.76k (\Omega)$$

⇓



$$I_{B1} = \frac{V_{th} - 2(V_{BE})}{R_{th}} \quad , \quad I_{C1} = \beta I_{B1} = \beta \frac{V_{th} - 2(V_{BE})}{R_{th}} \qquad ①$$

$$V_{BE} = V_T \ln\left(\frac{I_{C1}}{I_{S1}}\right). \qquad ②$$

Guess $V_{BE} = 0.7$,

① ⇒ $I_{C1} = 100 \times \frac{1.6 - 2 \times 0.7}{5.76} = 3.47 (mA)$

② ⇒ $V_{BE} = V_T \ln\left(\frac{3.47}{4 \times 10^{-13}}\right) = 0.7746$, not 0.7, reiterate

① ⇒ $I_{C1} = 0.8819$

② ⇒ $V_{BE} = 0.739$, not 0.7746, reiterate ...

After several iterations, V_{BE} converges to <u>0.755</u>

19) a)

$V_{BE} = 0.755\ (V)$

$I_{B1} = \frac{V_{th} - 2V_{BE}}{R_{th}} = 0.0156\ (mA)$

$I_{C2} = \beta I_{B1} = 1.56 (mA)$

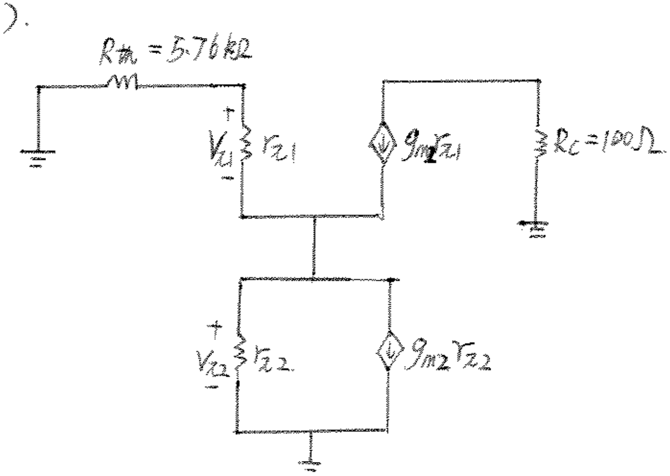
$V_{CE} = V_{CC} - [I_C \cdot (0.1) + V_{BE}] = 1.589\ (V)$

$I_{C2} = 1.56 (mA)$

$I_{B2} = (1/\beta) I_{C2} = 0.0156 (mA)$

$V_{CE2} = V_{BE} = 0.755 (\quad V)$

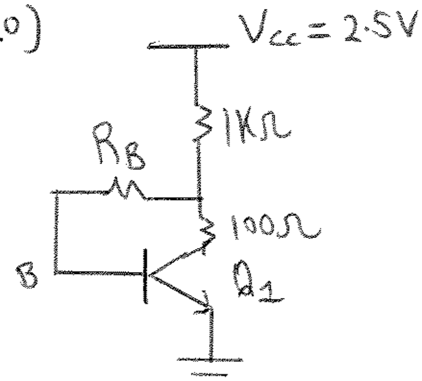
b).



$g_{m1} = g_{m2} = \frac{I_C}{V_T} = 0.065\ (S)$

$r_{\pi 1} = r_{\pi 2} = \frac{\beta}{g_m} = 1666.7\ (\Omega)$

20)



$I_c = 1mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750V$

$V_B = 2.5 - (I_E(1K\Omega) + I_B R_B) = 0.750V$

$I_E = 1.01mA$

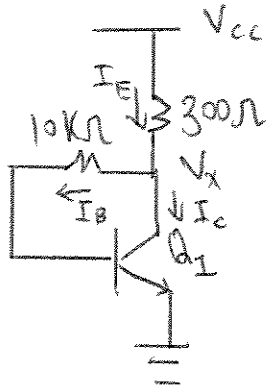
$I_B = 0.01mA$

$V_B = 2.5 - 1.01 - 0.01 R_B = 0.750$

$0.74 = 0.01 R_B$

$R_B = 74K\Omega$

21)



$V_x = 1.1V$

$\beta = 100$

$I_s = ?$

$$I_E = I_B + I_c$$

$$I_E = \frac{2.5 - 1.1}{300\Omega} = 4.67\,mA$$

$$I_B = \frac{I_c}{\beta}$$

$$I_E = \frac{I_c}{\beta} + I_c = 4.67\,mA$$

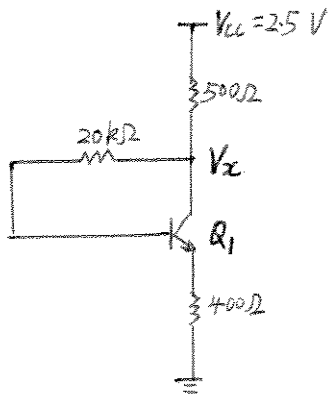
$$I_c = 4.624\,mA$$

$$I_s = \frac{I_c}{e^{\left(\frac{V_{BE}}{V_T}\right)}} \quad , \quad V_{BE} = 1.1 - \frac{4.624}{100}(10K) = 0.6376V$$

$$I_s = 1.035 \times 10^{-10}\,mA$$

$$I_s = 1.035 \times 10^{-13}\,A$$

22.



$I_S = 6 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$$\frac{V_{cc} - V_x}{0.5k} = I_C + I_B = I_C\left(1 + \frac{1}{\beta}\right) \Rightarrow V_x = 2.5 - 0.5k \cdot \frac{I_C}{\alpha} \qquad ①$$

$$\frac{V_x - (V_{BE} + I_E \cdot 0.4k)}{20k} = \frac{I_C}{\beta} \Rightarrow V_x = (20k)\left(\frac{I_C}{\beta}\right) + V_{BE} + \frac{I_C}{\alpha}\cdot(0.4k) \qquad ②$$

Equating V_x in ① and ②

$$2.5 - (0.5k)\left(\frac{I_C}{\alpha}\right) = (20k)\left(\frac{I_C}{\beta}\right) + V_{BE} + \frac{I_C}{\alpha}(0.4k)$$

$$I_C = \frac{2.5 - V_{BE}}{\frac{0.9k}{\alpha} + \frac{20k}{\beta}} = \frac{2.5 - V_{BE}}{1.11k} \qquad ③$$

First iteration $V_{BE} = 0.8$.

③ $\Rightarrow$ $I_C = 1.53\,(mA)$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.743$, not 0.8, reiterate

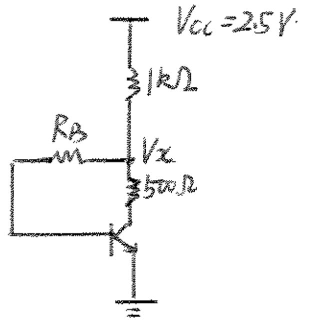
③ $\Rightarrow$ $I_C = \frac{2.5 - 0.743}{1.11} = 1.583\,(mA)$

$V_{BE} = V_T \ln\left(\frac{1.583}{I_S}\right) = 0.744$., converged.

So. $V_{BE} = 0.74(V)$ $\quad I_C = 1.58(mA)$, $I_B = I_C/\beta = 0.0158(mA)$

$V_C = 2.5 - \frac{1.583}{0.99} \times 0.5 = 1.7(V)$, $V_E = V_C - (I_B \cdot 20k + V_{BE}) = 0.644(V)$, $V_{CE} = V_C - V_E = 1.056(V)$

23).



$$I_c = \beta\left(\frac{2.5 - I_E(1K) - V_{BE}}{R_B}\right)$$

$$\frac{I_c R_B}{\beta} = 2.5 - I_E(1K) - V_{BE}$$

$$\left(I_E = \frac{I_c}{\alpha}\right)$$

$$I_c = \frac{2.5 - V_{BE}}{\frac{R_B}{\beta} + \frac{1K}{\alpha}} \quad (1)$$

$V_{BC} \leq 0.2V$

$$(V_x - I_B R_B) - (V_x - I_c 0.5) \leq 0.2V$$

$$I_c\left(0.5 - \frac{R_B}{\beta}\right) \leq 0.2V$$

$$\left(\frac{2.5 - V_{BE}}{\frac{R_B}{\beta} + \frac{1K}{\alpha}}\right)\left(0.5 - \frac{R_B}{\beta}\right) \leq 0.2V \quad (2)$$

Guess $V_{BE} = 0.75V \Rightarrow R_B \geq 34.513K\Omega$ (From (2))

Let $R_B = 34.513K\Omega$

$I_c = 1.291mA$, (From (1))

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7564V$, not 0.75, reiterate

$V_{BE} = 0.7564V \Rightarrow R_B \geq 34.461K\Omega$

23)

Let $R_B = 34.461\ K\Omega$

$I_c = 1.287\ mA$ (from (1))

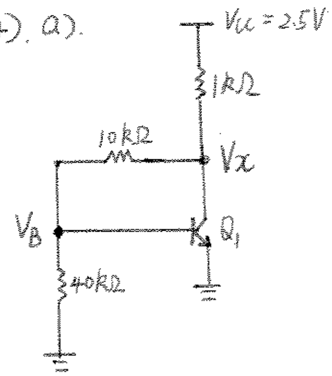
$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7563\ V$, converged !!

So $I_c = 1.287 mA$, $R_B = 34.46\ K\Omega$

Check V_{BC} : $V_{BC} = (1.287)(0.5) - \left(\frac{1.287}{100}\right)(34.46)$

$V_{BC} = 0.1999998$, less than $0.2V$

24). a).



$I_S = 8 \times 10^{-16} A$

$\beta = 100$

$V_A = \infty$

$V_x = 2.5 - \left(\frac{I_c}{\alpha} + \frac{V_B}{40k}\right) \cdot 1k$

$V_x = \left(\frac{V_B}{40k} + I_B\right) 10k + V_B = \left(\frac{V_B}{40k} + \frac{I_c}{\beta}\right) 10k + V_B$

Equating V_x $\Rightarrow$ $2.5 - \left(V_B + \frac{V_B \cdot 1k}{40k} + \frac{V_B \cdot 10k}{40k}\right) = \frac{I_c}{\alpha} \cdot 1k + \frac{I_c}{\beta} \cdot 10k$

$\Rightarrow I_c = \dfrac{2.5 - 1.275 V_B}{\frac{1k}{\alpha} + \frac{10k}{\beta}}$

Guess $V_B = 0.8$

$I_C = \dfrac{1.48}{\frac{1k}{0.99} + \frac{10k}{100}} = 1.33\, m(A)$

Then

$V_B = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.732$, not 0.8.

Reiterate

$I_C = \dfrac{1.3667}{1.11} = 1.4113 (mA)$

$V_B = V_T \ln\left(\frac{I_c}{I_S}\right) = 0.733$

So V_B converges to 0.73V

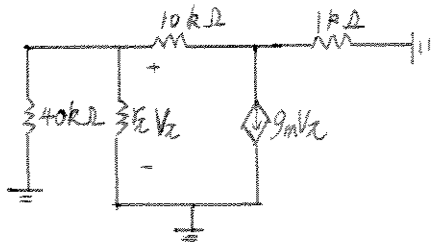
$I_C = 1.41\, mA$

$I_B = 14.1\, uA$

$V_{CE} = 2.5V - \left(\frac{1.41}{0.99} + \frac{0.73}{40}\right) \times 1V = 1.06V$

$V_{BE} = 0.73V$

24 b) Small Signal

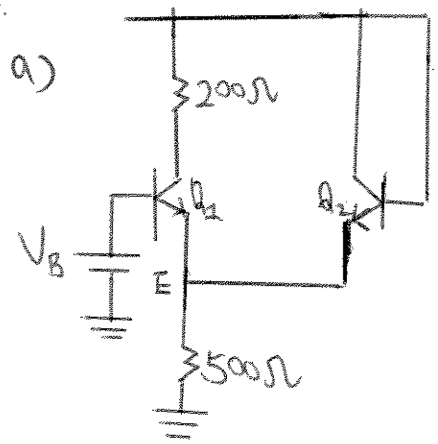


$$g_m = \frac{I_C}{V_T} = 0.054\ S$$

$$r_\pi = \frac{\beta}{g_m} = 1844\Omega$$

25).

a)



$I_{C_1} = 1mA$, $I_{E_1} = 1.01mA$

$I_{S_1} = I_{S_2} = 3\times10^{-16}A$

$V_A = \infty$

$\beta = 100$

$V_E = (I_{E_1} + I_{E_2})0.5K$, $V_{BE_1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) = 0.75V$

$V_E = 2.5 - V_{BE2}$

$V_B - (1.01 + I_{E_2})0.5 = 0.75V$

Guess $V_{BE_2} = 0.7V$

$V_E = 1.8 \Rightarrow I_{E_1} + I_{E_2} = 3.6mA \Rightarrow I_{E_2} = 2.59mA$

$I_{C_2} = 2.5641mA \Rightarrow V_{BE_2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right) = 0.774V$

Reiterate

$V_E = 1.726 \Rightarrow I_{E_2} = 2.442mA$, $I_{C_2} = 2.4176mA$

$V_{BE_2} = V_T \ln\left(\frac{I_{C_2}}{I_S}\right) = 0.773$, converged !!

$V_{BE} = 0.773V$, $I_{C_2} = 2.42mA$, $I_{E_2} = 2.44mA$

$V_B = 0.75 + (1.01 + 2.44)0.5 = 2.475V$

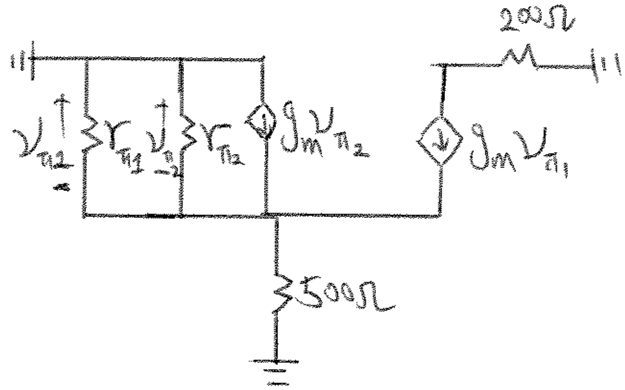
$V_C = 2.5 - (1)(0.2) = 2.3$

Q_1 in soft-saturation region.

25

b)

Small Signal Model



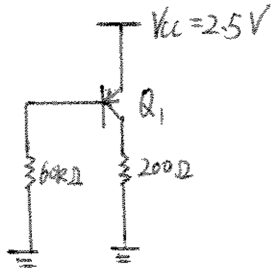
$$g_{m_1} = \frac{1mA}{26mV} = 0.0385 \left(\frac{1}{\Omega}\right) S$$

$$r_{\pi_1} = \frac{100}{0.0385} = 2.6 K\Omega$$

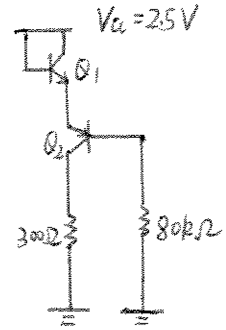
$$g_{m_2} = \frac{2.42mA}{26 mV} = 0.0931 \left(\frac{1}{\Omega}\right) S$$

$$r_{\pi_2} = \frac{100}{0.0931} = 1.07 K\Omega$$

26). $\beta_{npn} = 2\beta_{pnp} = 100$ $\quad I_S = 9\times10^{-16}$ A $\quad V_A = \infty$



(a)



(b)

a) $I_C = \dfrac{2.5 - |V_{BE}|}{60k} \beta_{pnp}$, $\quad V_{BE} = V_T \ln\left(\dfrac{I_C}{I_S}\right)$

Guess $|V_{BE}| = 0.8 \Rightarrow I_C = 1.42m(A)$

$|V_{BE}| = 26\times10^{-3} \ln\left(\dfrac{1.42}{9\times10^{-13}}\right) = 0.730(V)$, not 0.8

Reiterate, $I_C = \dfrac{2.5-0.73}{60k} \times 50 = 1.475m(A)$

$|V_{BE}| = 26\times10^{-3} \ln\left(\dfrac{1.42}{9\times10^{-13}}\right) = 0.731(V)$

Reiterate, $I_C = \dfrac{2.5-0.731}{60k} \times 50 = 1.474m(A)$

$|V_{BE}| = 26\times10^{-3} \ln\left(\dfrac{1.474}{9\times10^{-13}}\right) = 0.731(V)$, converged.

Q_1 : $|V_{BE}| = 0.731V$, $I_C = 1.47mA$, $I_B = 29.4\mu A$.

$|V_{CE}| = 2.206V$.

In forward active region.

2b)

b). $$I_{C2} = \frac{2.5 - (V_{BE1} + |V_{BE2}|)}{80k} \quad ①$$

$$I_{C2} \cdot \frac{\beta_{pnp}+1}{\beta_{pnp}} = \frac{I_{C1}(\beta_{npn}+1)}{\beta_{npn}}$$

$$\beta_{npn} = 2\beta_{pnp} = 100$$

$$\Rightarrow I_{C1} = \frac{2(\beta_{pnp}+1)}{2\beta_{pnp}+1} I_{C2} = 1.0099\, I_{C2} \quad ②$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) \quad ③$$

$$V_{BE2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right) \quad ④$$

Four unknowns: I_{C1}, I_{C2}, V_{BE1}, V_{BE2}, Four equations: ①. ②. ③. ④

Solve by iteration since ③ and ④ are exponential equations.

Guess $V_{BE2} = V_{BE1} = 0.8$

① $\Rightarrow I_{C2} = 50 \times \left(\frac{2.5-1.6}{80k}\right) A = 0.5625 mA$

② $\Rightarrow I_{C1} = 0.568\, mA$

③ $\Rightarrow V_{BE1} = V_T \ln\left(\frac{0.568}{9\times10^{-13}}\right) V = 0.706\, V$

④ $\Rightarrow V_{BE2} = V_T \ln\left(\frac{0.5625}{9\times10^{-13}}\right) V = 0.706\, V$

Reiterate,

$I_{C2} = 0.68 mA$, $I_{C1} = 0.6867 mA$, $V_{BE1} = 0.711 V$, $V_{BE2} = 0.711 V$.

Reiterate,

$I_{C2} = 0.674 mA$, $I_{C1} = 0.680 mA$, $V_{BE1} = 0.711 V$, $V_{BE2} = 0.711 V$

So.

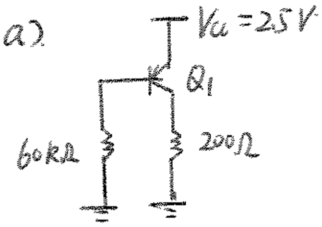
$I_{C1} = 0.680 mA$ $\quad$ $I_{C2} = 0.674 mA$

$I_{B1} = 6.8\, uA$ $\quad$ $I_{B2} = 13.48\, uA$

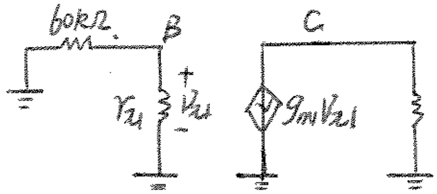
$V_{BE1} = 0.711 V$ $\quad$ $|V_{BE2}| = 0.711 V$

$V_{CE1} = 0.711 V$ $\quad$ $|V_{CE2}| = 2.5 V - 0.711 V - (0.674)(0.3) V = 1.5868 V$

27. a).



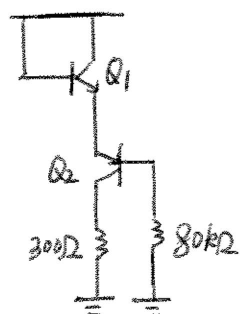
⟹



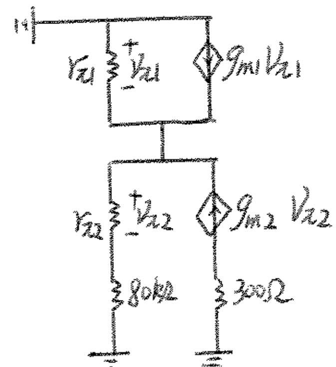
$$g_{m1} = \frac{I_C}{V_T} = \frac{1.47mA}{26mV} = 0.0565\,S$$

$$r_{\pi1} = \frac{\beta}{g_{m1}} = 884\Omega$$

b).



⟹



$$g_{m1} = \frac{I_{C1}}{V_T} = 0.02615\,S$$

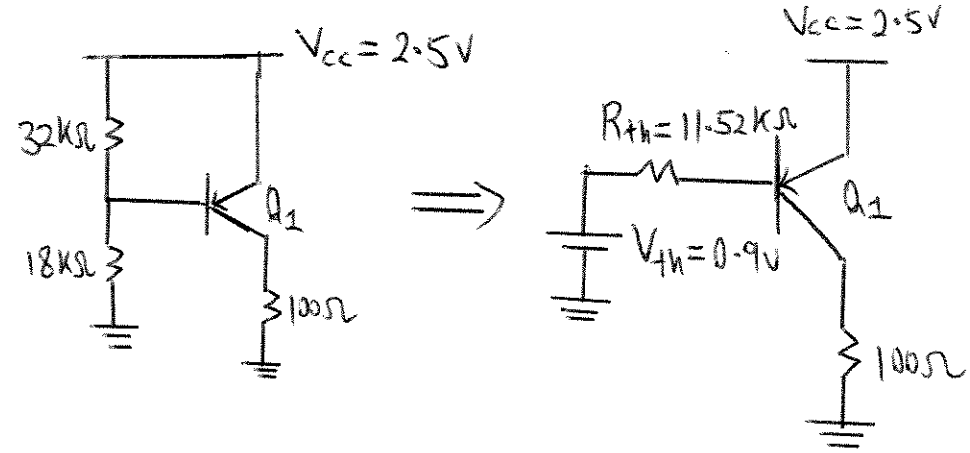
$$r_{\pi1} = 3823.5\Omega$$

$$g_{m2} = \frac{I_{C2}}{V_T} = 0.0259\,S$$

$$r_{\pi2} = 1928.8\Omega$$

28)

a)



$$I_c = \beta_{pnp}\left(\frac{2.5 - |V_{BE}| - V_{th}}{R_{th}}\right)$$

Guess $|V_{BE}| = 0.7\text{ V}$, $I_c = 3.91\text{mA}$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.757\text{ V}$

Reiterate, $|V_{BE}| = 0.757\text{V}$, $I_c = 3.66\text{mA}$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755\text{V}$

Reiterate, $|V_{BE}| = 0.755\text{ V}$, $I_c = 3.67\text{mA}$

$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.755\text{V}$, Converged!!

$V_c = (3.67\text{mA})(0.1\text{KΩ}) = 0.367\text{V}$, $V_B \approx 2.5 - 0.755 = 1.745\text{V}$

Q_1 in forward active.

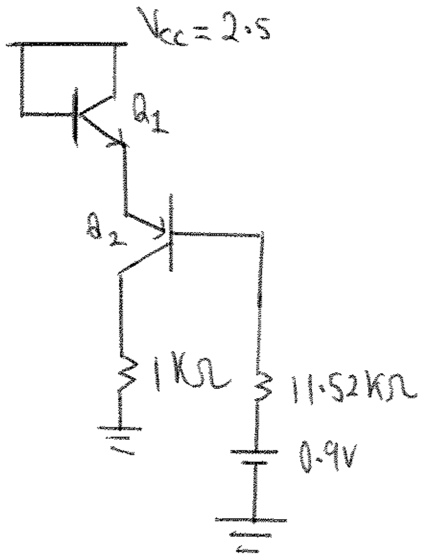
Bias point:

$I_c = 3.67\text{mA}$ $\quad |V_{BE}| = 0.755$

$I_B = 73.4\ \mu\text{A}$ $\quad |V_{CE}| = 2.5 - 0.367 = 2.133\text{V}$

28)

b)



$$I_{C2} = \left(\frac{2.5 - (V_{BE_1} + V_{BE_2}) - 0.9}{11.52\,K}\right)50$$

$$I_{C_1} = I_{C_2}(1.0099)$$

(From β relation)

$$V_{BE_1} = V_T \ln\left(\frac{I_{C_1}}{I_S}\right)$$

$$|V_{BE_2}| = V_T \ln\left(\frac{I_{C_2}}{I_S}\right)$$

Guess, $V_{BE_1} = V_{BE_2} = 0.7V$

$I_{C2} = 0.868\,mA$, $I_{C_1} = 0.877\,mA$

$V_{BE_1} = V_T \ln\left(\frac{I_{C_1}}{I_S}\right) = 0.718V$, $|V_{BE_2}| = V_T \ln\left(\frac{I_{C_2}}{I_S}\right) = 0.717$

Reiterate, $V_{BE_1} = 0.718V$, $|V_{BE_2}| = 0.717V$

$I_{C_2} = 0.716\,mA$, $I_{C_1} = 0.723\,mA$

$V_{BE_1} = V_T \ln\left(\frac{I_{C_1}}{I_S}\right) = 0.713V$, $|V_{BE_2}| = V_T \ln\left(\frac{I_{C_2}}{I_S}\right) = 0.712V$

Reiterate, $V_{BE_1} = 0.713V$, $|V_{BE_2}| = 0.712V$

$I_{C_2} = 0.760\,mA$, $I_{C_1} = 0.767\,mA$

$V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

28)

b)

Reiterate, $V_{BE_1} = 0.714V$, $|V_{BE_2}| = 0.714V$

$I_{C_2} = 0.747mA$, $I_{C_1} = 0.754mA$

$V_{BE_1} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.714V,$

$|V_{BE_2}| = 0.714\ V$

$V_{B_2} = \frac{(0.747mA)}{50}(11.52K\Omega) + 0.9 = 1.07\ V$

$V_{C_2} = (0.747mA)(1K\Omega) = 0.747\ V$

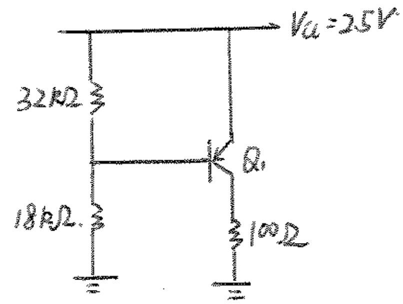
Q_2 is in forward-active region. Q_1 is always in forward-active region.

Bias point:

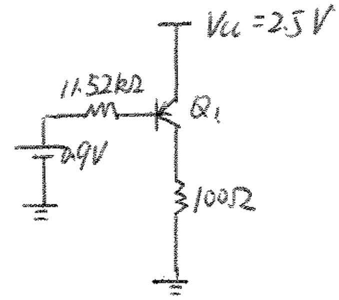
$V_{BE_1} = 0.714V$	$\lvert V_{BE_2}\rvert = 0.714V$
$I_{C_1} = 0.754\ mA$	$I_{C2} = 0.747mA$
$I_{B_1} = 7.54\mu A$	$I_{B2} = 14.94\ \mu A$
$V_{CE_1} = 0.714V$	$\lvert V_{CE_2}\rvert = 2.5 - 0.714 - 0.747 = 1.039V$

29).

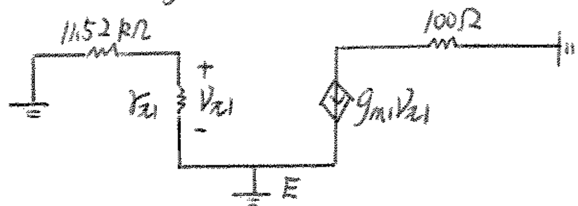
a).



⟹



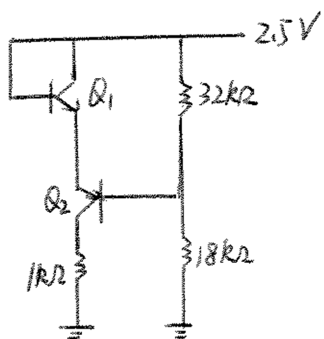
Small Signal:



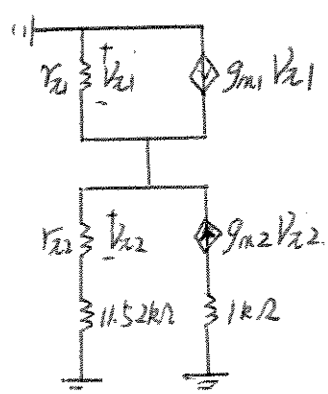
$$g_{m1} = \frac{3.67mA}{26mV} = 0.141\ S$$

$$r_\pi = \frac{50}{0.141}\ \Omega = 354.2\ \Omega$$

b).



⟹



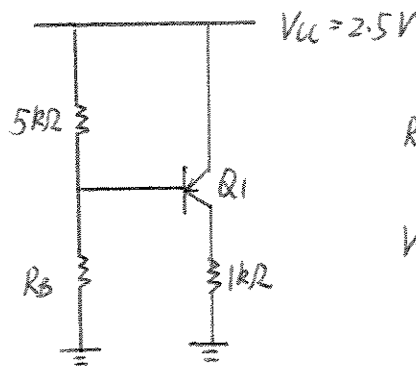
$g_{m1} = 0.029\ S$

$r_{\pi 1} = 3448.3\ \Omega$

$g_{m2} = 0.0287\ S$

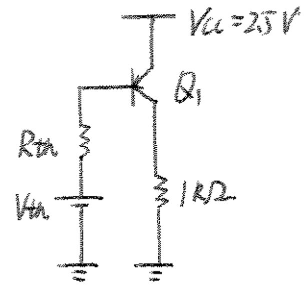
$r_{\pi 2} = 1740.3\ \Omega.$

30)



$$R_{th} = \frac{(R_B)(5k\Omega)}{R_B + 5k\Omega}$$

$$V_{th} = \frac{R_B}{R_B + 5k\Omega} \cdot 2.5V$$



$\beta = 50$, $I_s = 8 \times 10^{-16} A$, $V_A = \infty$

Edge of saturation: $|V_{BE}| = |V_{CE}|$

$$I_c = \frac{50(2.5 - |V_{BE}| - V_{th})}{R_{th}}, \quad |V_{CE}| = 2.5 - I_c \, 1K\Omega = |V_{BE}|$$

$$2.5 - \frac{50(2.5 - |V_{BE}| - V_{th})}{R_{th}}(1K\Omega) = |V_{BE}|$$

Subsitute in R_{th} and V_{th} and rearrange:

$$12.5R_B + 50|V_{BE}|R_B - |V_{BE}|(5K)R_B = 625 - |V_{BE}|250 \quad (1)$$

Guess $|V_{BE}| = 0.7V$, (1) $\Rightarrow$ $44R_B = 450 \Rightarrow R_B = 10.23K\Omega$

$V_{th} = 1.68V$ $\quad I_c = 1.7857\,mA$

$R_{th} = 3.36K\Omega$ $\quad |V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.739V$, not 0.7V, reiterate

$|V_{BE}| = 0.739V$, (1) $\Rightarrow 45.755R_B = 440.25$

$R_B = 9.62K\Omega$

$V_{th} = 1.645V$ $\quad I_c = 1.763mA$

$R_{th} = 3.29K\Omega$ $\quad |V_{BE}| = 0.739V$, converged.

3c)

So ±5% of 9.62KΩ.

+5% case:

$9.62K\Omega + 5\% = 10.101K\Omega$

$V_{th} = 1.67\,V, \quad R_{th} = 3.345\,K\Omega$

$I_c = \frac{(2.5 - 0.74 - 1.67)}{3.345} 50 = 1.3455\,mA$ (Assume $|V_{BE}| = 0.74V$)

Check for $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.732v$, iterate once

$I_c = 1.4651\,mA, \quad |V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.734V$, converged

$|V_{BE}| \approx 0.734, \quad |V_{CE}| = 2.5 - 1.4651(1k\Omega) = 1.0349V$

$V_{BC} = 0.3009\,V$ (Reverse bias)

-5% case:

$9.62k\Omega - 5\% = 9.139\,K\Omega$

$V_{th} = 1.616V, \quad R_{th} = 3.23184K\Omega$

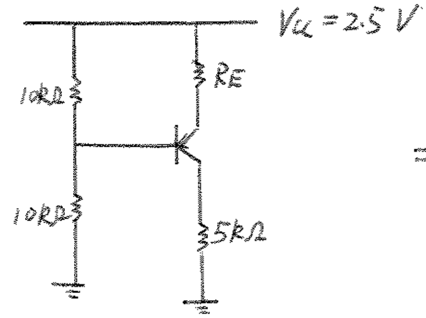
$I_c = \frac{(2.5 - 0.74 - 1.616)}{(3.23184)} 50 = 2.228mA$ ($|V_{BE}| = 0.7$), $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.745V$

reiterate: $|V_{BE}| = 0.745V, \quad I_c = 2.150\,mA$,

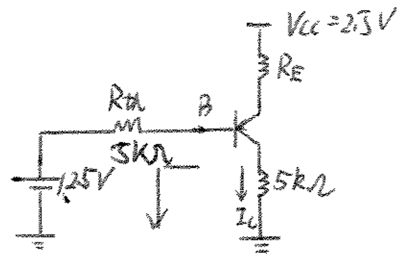
Verify V_{BE}, $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.744V$, Converged

$|V_{CE}| = 2.5 - 2.150(1K\Omega) = 0.35, \quad |V_{BE}| = 0.744V, \quad V_{BC} = -0.394V$ (Forward Bias)

31)



⟹



$$V_{BC} = 1.25 + I_B R_{th} - I_c 5K = 0.3$$

$$1.25 + \frac{I_c}{\beta}5 - I_c 5K = 0.3$$

$$\beta = 50 \Rightarrow I_c = 0.1939\,mA$$

$$|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.682\,V$$

$$I_B = \frac{\left(2.5 - \frac{I_c}{\alpha}R_E - |V_{BE}|\right) - 1.25}{5K}$$

, $\alpha = 0.9804$
$I_B = 0.003878\,mA$
$|V_{BE}| = 0.682\,V$
$I_c = 0.1939\,mA$

$$R_E = 2.89\,K\Omega$$

If R_E is halved ⟹ $R_E = 1.44\,K\Omega$

$$I_c = \beta\left(\frac{2.5 - |V_{BE}| - 1.25 - \alpha I_c R_E}{5k}\right)$$

$$I_c = \frac{62.5 - 50|V_{BE}|}{78.44}$$

, Guess $|V_{BE}| = 0.682V$

31)

$I_c = 0.3621\,mA$

Verify $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.698V$, not $0.682V$ reiterate

$$I_c = \frac{62.5 - 50(0.698)}{78.44} = 0.352\,mA$$

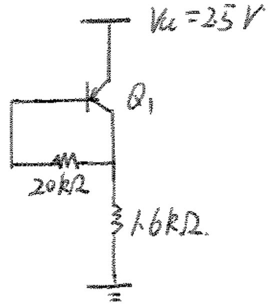
Verify $|V_{BE}| = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.697V$, converged

so $I_c = 0.352\,mA$, which is 1.82 times of $0.1939\,mA$

$$V_{BC} = 1.25 + \frac{(0.352)(5k\Omega)}{50} - (0.352)(5k\Omega) = -0.4748V$$

which drive Q_2 into saturation.

32).



$\beta = 80$

$V_A = \infty$

$V_B = (I_B)(20k\Omega) + I_E \cdot (1.6k\Omega)$

$I_C = 1mA$

$I_B = \frac{1}{80} mA, \quad I_E = \frac{1}{0.98765} mA = 1.0125 mA$

$V_B = (\frac{1}{80})(20)V + (1.0125)(1.6)V$

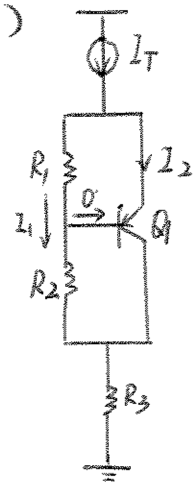
$= 1.87V$

$|V_{BE}| = 2.5V - 1.87V = 0.63V$

$I_C = I_S e^{(\frac{V_{BE}}{V_T})} \Rightarrow I_S = \frac{I_C}{e^{(\frac{V_{BE}}{V_T})}} = \frac{1mA}{e^{(\frac{0.63}{0.026})}}$

$I_S = 3 \times 10^{-11} mA$

33)



If Base current is neglected, $I_C = I_E$

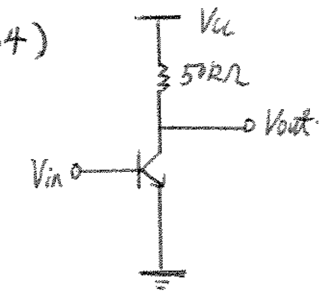
$$I_1 = \frac{V_E - V_C}{R_1 + R_2}$$

$$|V_{BE}| = I_1 R_1 = \frac{V_E - V_C}{R_1 + R_2} R_1 = \frac{|V_{CE}|}{R_1 + R_2} R_1$$

So $$\frac{|V_{CE}|}{|V_{BE}|} = \frac{R_1 + R_2}{R_1}$$

Let $A = \frac{R_1 + R_2}{R_1}$, $|V_{CE}| = A\,|V_{BE}|$, thus $|V_{BE}|$ is multiplied.

34)



$A_v = g_m R_C = 20$

$\frac{I_C R_C}{V_T} = 20 \Rightarrow I_C = \frac{20 V_T}{R_C}$

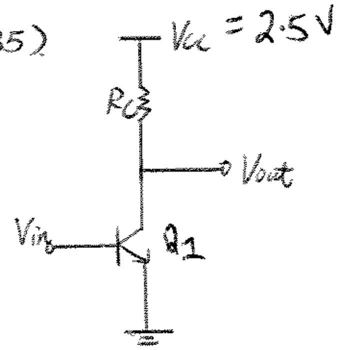
$I_C = 0.0104\,mA$

$V_{CC} - (50k\Omega)(0.0104\,mA) = V_{BE}$

$\Rightarrow V_{CC} - 50 \times 0.0104\,V = 0.8\,V$

$\Rightarrow V_{CC} = 1.32\,V$

35)



$$V_A = 10V,\quad r_o = \frac{V_A}{I_C},\quad g_m = \frac{I_c}{V_T}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m(R_C // r_o) = g_m\left(\frac{R_C r_o}{R_C + r_o}\right) = \frac{R_C V_A}{V_T\left(R_C + \frac{V_A}{I_C}\right)}$$

As the equation above shows, a large gain means a large I_c. However, a large I_c will drive Q_1 into saturation. So a tradeoff must be made. The maximum limit for I_c is when it drives Q_1 into the edge of saturation, namely, $V_{BE} = V_{CE}$.

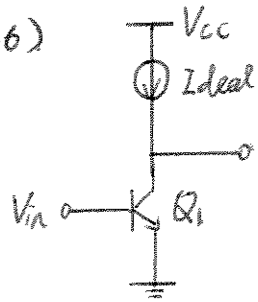
$$V_{CE} = V_{cc} - I_c(1K)$$

$$V_{BE} = 0.8V,\quad V_{cc} = 2.5V$$

$$0.8 = 2.5 - I_c 1K$$

$$I_c = 1.7mA$$

36)



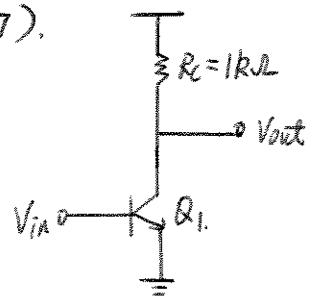
$A_v = 50$

$R_{out} = r_o = 10k\Omega$

$$A_v = g_m R_{out} = \frac{I_c}{V_T} R_{out} = 50$$

$$I_c = 50\left(\frac{V_T}{R_{out}}\right) = 0.13mA$$

37).



$$I_c = I_s \exp\left(\frac{V_{BE}}{2V_T}\right)$$

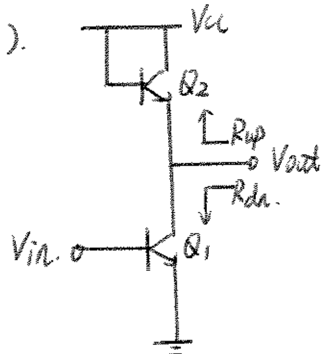
$$g_m = \frac{\partial I_c}{\partial V_{BE}} = \frac{I_c}{2V_T}$$

$$R_{out} = R_c$$

$$\left|\frac{V_{out}}{V_{in}}\right| = g_m R_{out} = \frac{I_c R_c}{2V_T} = \frac{(1mA)(1K\Omega)}{(2)(0.026V)} = 19.23$$

38). (Find A_v, R_{in}, R_{out})

a).



$V_A = \infty$

$R_{out} = R_{up} // R_{dn}$

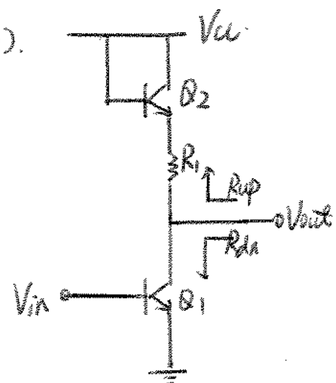
$R_{up} = \frac{1}{g_{m2}} // r_{\pi 2}$, $R_{dn} = \infty$

$R_{out} = \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{in} = r_{\pi 1}$

$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2}\right)$

b).



$V_A = \infty$

$R_{up} = R_1 + \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{dn} = \infty$

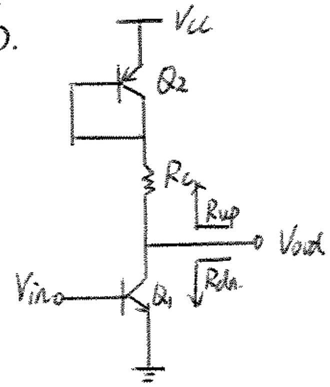
$R_{out} = R_{up} // R_{dn} = R_1 + \frac{1}{g_{m2}} // r_{\pi 2}$

$R_{in} = r_{\pi 1}$

$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(R_1 + \frac{1}{g_{m2}} // r_{\pi 2}\right)$

38

c).



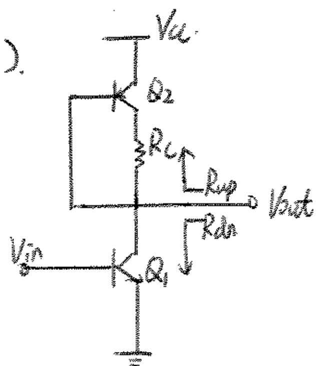
$V_A = \infty$

$R_{up} = R_c + \left(\frac{1}{g_{m_2}} // r_{\pi_2}\right)$, $R_{dn} = \infty$

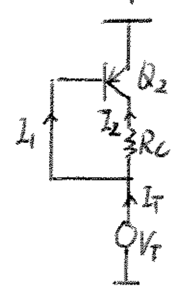
$R_{out} = R_c + \left(\frac{1}{g_{m_2}} // r_{\pi 2}\right)$, $R_{in} = r_{\pi 1}$

$A_v = g_{m1}\left(R_c + \left(\frac{1}{g_{m2}} // r_{\pi 2}\right)\right)$

d).



Find R_{up}:



$I_T = I_1 + I_2 = \frac{I_2}{\beta} + I_2$

$I_2 = g_m V_T$, $I_T = \frac{g_m V_T}{\beta} + g_m V_T$

$\frac{V_T}{I_T} = R_{up} = \frac{1}{\left(\frac{g_m}{\beta} + g_m\right)} = r_{\pi_2} // \frac{1}{g_{m_2}}$

$R_{dn} = \infty$

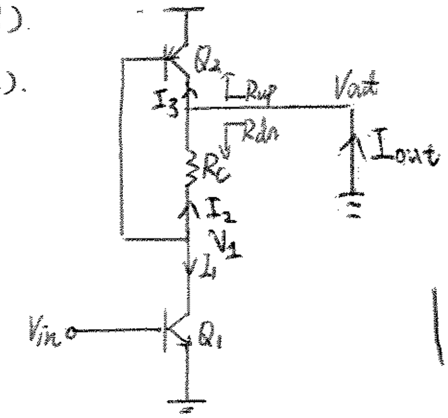
$R_{out} = R_{up} // R_{dn} = r_{\pi 2} // \frac{1}{g_{m2}}$

$R_{in} = r_{\pi 1}$

$A_v = g_{m1}\left(r_{\pi 2} // \frac{1}{g_{m2}}\right)$

38).

e).



$R_{up} = \infty$

$R_{dn} = R_c + r_{\pi 2}$

$R_{out} = R_c + r_{\pi 2}$

$|A_v| = G_m R_{out}$

where $G_m = \left| \frac{I_{out}}{V_{in}} \right|$

$I_{out} + I_2 = I_3$, $I_2 = \frac{V_1}{R_c}$

$I_3 = V_1 g_{m2}$

$V_1 = -I_1 (R_c // r_{\pi 2}) = -g_{m1} V_{in} (R_c // r_{\pi 2})$

$I_{out} = I_3 - I_2 = V_1 (g_{m2} - \frac{1}{R_c})$

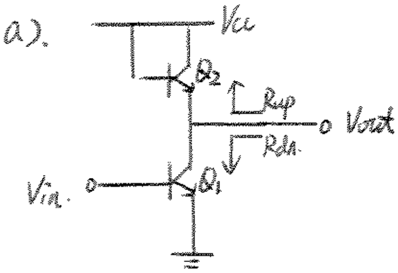
$I_{out} = -g_{m1} V_{in} (R_c // r_{\pi 2})(g_{m2} - \frac{1}{R_c})$

$G_m = \left| \frac{I_{out}}{V_{in}} \right| = g_{m1} (R_c // r_{\pi 2})(g_{m2} - \frac{1}{R_c})$

$|A_v| = g_{m1} (R_c // r_{\pi 2})(g_{m2} - \frac{1}{R_c})(R_c + r_{\pi 2})$

39). $V_A < \infty$, find A_v, R_{in}, R_{out}

a).



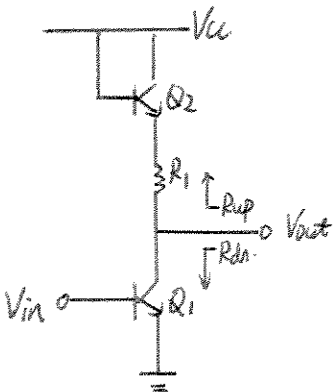
$$R_{up} = \frac{1}{g_{m2}} // r_{\pi 2} // r_{o2}, \quad R_{dn} = r_{o1}$$

$$R_{out} = \frac{1}{g_{m2}} // r_{\pi 2} // r_{o1} // r_{o2}$$

$$R_{in} = r_{\pi 1}$$

$$|A_v| = g_{m1} \left(\frac{1}{g_{m2}} // r_{\pi 1} // r_{o1} // r_{o2} \right)$$

b)



$$R_{up} = R_1 + \frac{1}{g_{m2}} // r_{\pi 2} // r_{o2}$$

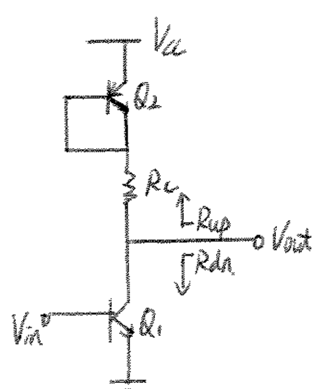
$$R_{dn} = r_{o1}$$

$$R_{out} = r_{o1} // \left(R_1 + \frac{1}{g_{m2}} // r_{\pi 2} // r_{o2} \right)$$

$$R_{in} = r_{\pi 1}$$

$$|A_v| = g_{m1} \left[r_{o1} // \left(R_1 + \frac{1}{g_{m2}} // r_{\pi 2} // r_{o2} \right) \right]$$

c).



$$R_{up} = R_C + \frac{1}{g_{m2}} // r_{\pi 2} // r_{o2}$$

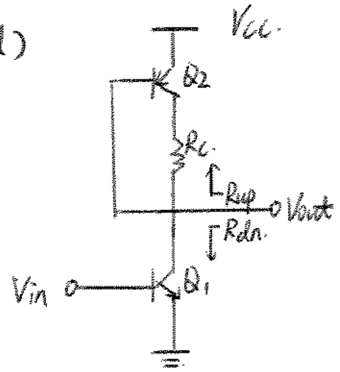
$$R_{dn} = r_{o1}$$

$$R_{in} = r_{\pi 1}$$

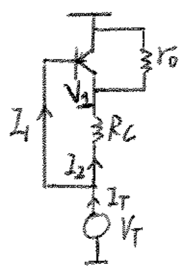
$$R_{out} = r_{o1} // \left(R_C + \frac{1}{g_{m2}} // r_{\pi 2} // r_{o2} \right)$$

$$|A_v| = g_{m1} \left[r_{o1} // \left(R_C + \frac{1}{g_{m2}} // r_{\pi 2} // r_{o2} \right) \right]$$

39 d)



Find R_{up}:



$$I_T = I_1 + I_2$$

$$I_2 = \frac{V_T - V_1}{R_C}, \quad I_1 = \frac{g_m V_T}{\beta}$$

$$V_1 = (I_2 - g_m V_T) r_o$$

$$I_2 = \frac{V_T - (I_2 - g_m V_T) r_o}{R_C}$$

$$I_2 = \frac{(1 + g_m r_o)}{R_c + r_o} V_T$$

$$I_T = \frac{g_m V_T}{\beta} + \frac{(1+g_m r_o)}{R_c + r_o} V_T$$

$$\frac{V_T}{I_T} = R_{up} = \frac{1}{\frac{g_m}{\beta} + \frac{(1+g_m r_o)}{R_c + r_o}}$$

$$R_{up} = r_\pi // \frac{R_c + r_o}{(1 + g_m r_o)}$$

$$R_{up} = r_{\pi_2} // \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}}$$

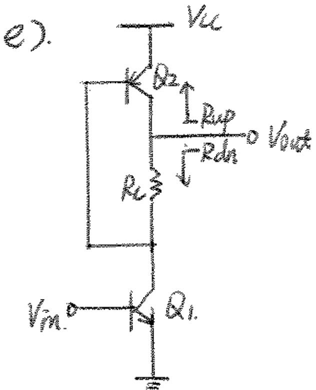
$$R_{dn} = r_{o1}$$

$$R_{out} = r_{\pi_2} // \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}} // r_{o1}$$

$$R_{in} = r_{\pi 1}$$

$$|A_v| = \left|\frac{V_{out}}{V_{in}}\right| = g_{m1}\left(r_{\pi_2} // \frac{(R_c + r_{o2})}{1 + g_{m2} r_{o2}} // r_{o1}\right)$$

39 e).



$R_{up} = r_{o2}$

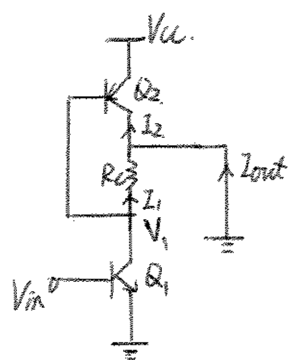
$R_{dn} = R_C + (r_{o1} \parallel r_{\pi 2})$

$R_{in} = r_{\pi 1}$

$R_{out} = r_{o2} \parallel [R_C + (r_{o1} \parallel r_{\pi 2})]$

$|A_v| = G_m R_{out}$

Finding G_m:



$G_m = \left| \frac{I_{out}}{V_{in}} \right|$

$I_{out} + I_1 = I_2$

$I_{out} = I_2 - I_1$

$I_1 = \frac{V_1}{R_C} \qquad I_2 = V_1 g_{m2}$

$V_1 = -(g_{m1} V_{in})(r_{o1} \parallel R_C \parallel r_{\pi 2})$

$I_{out} = V_1 (g_{m2} - \frac{1}{R_C}) = -g_{m1} V_{in} (r_{o1} \parallel R_C \parallel r_{\pi 2})(g_{m2} - \frac{1}{R_C})$

$G_m = \left| \frac{I_{out}}{V_{in}} \right| = g_{m1} (r_{o1} \parallel R_C \parallel r_{\pi 2})(g_{m2} - \frac{1}{R_C})$

$|A_v| = g_{m1} (r_{o1} \parallel R_C \parallel r_{\pi 2})(g_{m2} - \frac{1}{R_C}) \{ r_{o2} \parallel [R_C + (r_{o1} \parallel r_{\pi 2})] \}$

40)

Gain of a degenerated CE stage ($V_A = \infty$)

$$A_V = \frac{-R_C}{\frac{1}{g_m} + R_E} = \frac{-R_C g_m}{1 + R_E g_m}$$

$$\frac{\partial A_V}{\partial I_C} = R_C \left(\frac{g_m R_E}{(1+R_E g_m)^2} \frac{\partial g_m}{\partial I_C} - \frac{\partial g_m / \partial I_C}{1 + g_m R_E} \right)$$

$$\frac{\partial g_m}{\partial I_C} = \frac{1}{V_T} = \frac{1}{26mV} = 38.46 \left(\frac{1}{V}\right)$$

a) $g_m R_E = 3$

$$\frac{\partial A_V}{\partial I_C} = R_C(-2.404) \quad , \quad \partial I_C = 0.1 I_C$$

$$\partial A_V = -R_C I_C (0.24)$$

Relative change in gain $= \frac{\partial A_V}{A_V} = \frac{-0.24(R_C I_C)}{-\frac{R_C I_C}{V_T(1+R_E g_m)}} = 2.5\%$

40)

b) $g_m R_E = 7$

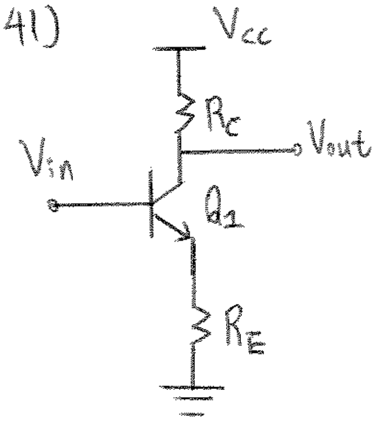
$$\frac{\partial A_v}{\partial I_c} = -R_c\, 0.6$$

$$\partial A_v = -R_c I_c (0.06)$$

Relative change in gain

$$\frac{\partial A_v}{A_v} = \frac{-0.06\,(R_c I_c)}{\dfrac{-R_c I_c}{V_T(1+R_E g_m)}} = 1.25\%$$

41)



$V_A = \infty$

$R_C I_C = 20 V_T$

$R_E I_C = 5 V_T$

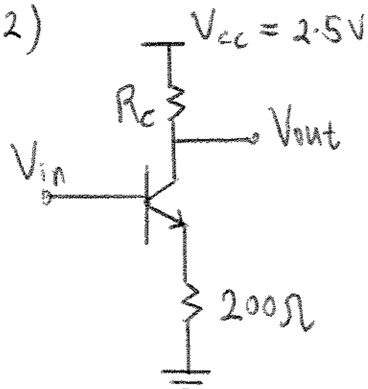
$$|A_v| = \frac{R_C}{R_E + \frac{1}{g_m}} = \frac{R_C}{R_E + \frac{V_T}{I_C}} = \frac{R_C I_C}{R_E I_C + V_T}$$

Assume β is large, so $I_C = I_E$.

$R_C I_C = 20 V_T$, $R_E I_C = 5 V_T$

$$|A_v| = \frac{20 V_T}{5 V_T + V_T} = \frac{20 V_T}{6 V_T} = 3.33$$

42)



$$|A_v| = \frac{R_C I_C}{R_E I_C + V_T} = 10$$

Edge of Saturation

$$V_{CE} = V_{BE} = 2.5 - I_C(R_C + R_E)$$

$V_{BE} = 0.8V \Rightarrow I_C R_C = 1.7 - I_C 0.2$ (operating point)

$|A_v| = 10 \Rightarrow R_C I_C = 10(R_E I_C + V_T)$ (Gain Equation)

Equating the two equations above $\Rightarrow$

$$1.7 - 0.2 I_C = 2 I_C + 0.26 \Rightarrow I_C = 0.655mA$$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.725$, not 0.8, Reiterate

$I_C R_C = 1.775 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

Equating the two equations $\Rightarrow I_C = 0.689mA$

Check for $V_{BE} \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.727V$, iterate 1 more time

$I_C R_C = 1.773 - I_C 0.2$ (operating point)

$I_C R_C = 2I_C + 0.26$ (Gain equation)

42)

Equating the two equations $\Rightarrow$ $I_c = 0.688\ mA$

Check for V_{BE} $\Rightarrow$ $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.727V$, converged

$I_c = 0.688 mA$

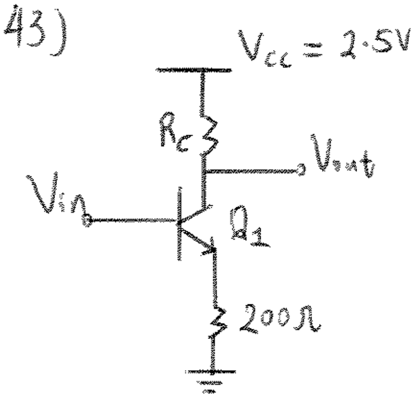
$$R_c = \frac{2I_c + 0.26}{I_c} = \frac{(2)(0.688) + 0.26}{0.688}$$

$R_c = 2.38 K\Omega$

$R_{in} = r_\pi + (1+\beta) R_E$

$$R_{in} = \frac{\beta}{g_m} + (101)(0.2) = 24.0 K\Omega$$

43)



$|A_v| = 100$

(Voltage gain)

$|A_v| = 100 \Rightarrow R_c I_c = 100(R_E I_c + V_T)$

$R_c I_c = 20 I_c + 2.6 \quad (1)$

$R_c I_c = 1.7 - I_c 0.2 \quad (2) \quad (\text{Assume } V_{BE} = 0.8)$

Equating (1) and (2) yield

$1.7 - I_c 0.2 = 20 I_c + 2.6 \Rightarrow I_c = -0.04455\,mA$

A negative I_c in forward active region is impossible, therefore, a solution does not exist. The reason is because $R_c I_c$ is too large to produce a gain of 100 that drive Q_1 into saturation region.

Maximum gain achievable:

$$\frac{R_c I_c}{R_E I_c + V_T} = |A_v| \quad (\text{Gain Equation})$$

$2.5 = R_c I_c + V_{CE} + R_E I_c$ (Operating Point Equation)

Let A = Maximum gain

43)

$$A I_c 0.2 + A 0.026 = 1.7 - I_c 0.2$$

$$\Rightarrow I_c = \frac{1.7 - A 0.026}{A 0.2 + 0.2}$$

Since I_c cannot be zero, set

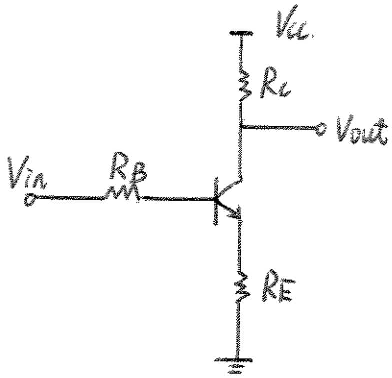
$$\frac{1.7 - A 0.026}{A 0.2 + 0.2} > 0$$

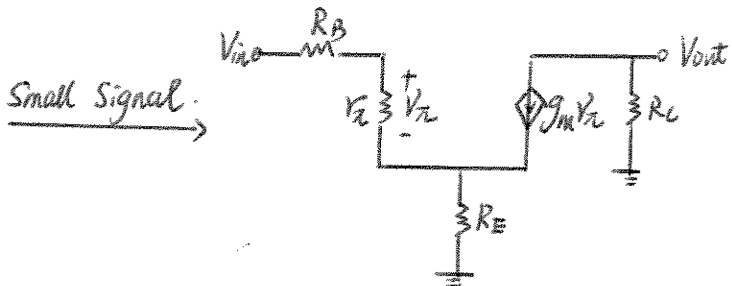
$$1.7 - A 0.026 > 0$$

$$1.7 > A 0.026$$

$$A < \frac{1.7}{0.026} = 65.4$$ (Maximum gain achievable)

44) $V_A = \infty$





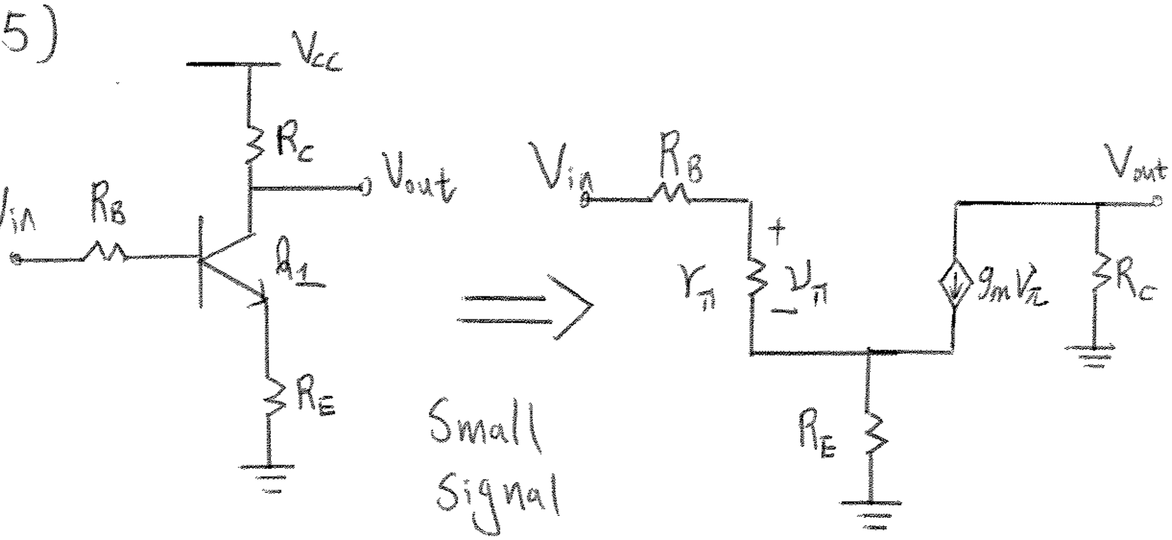
$$V_{out} = -g_m V_\pi R_C$$

$$V_\pi = \frac{V_{in} r_\pi}{R_B + r_\pi + (\beta+1) R_E}$$

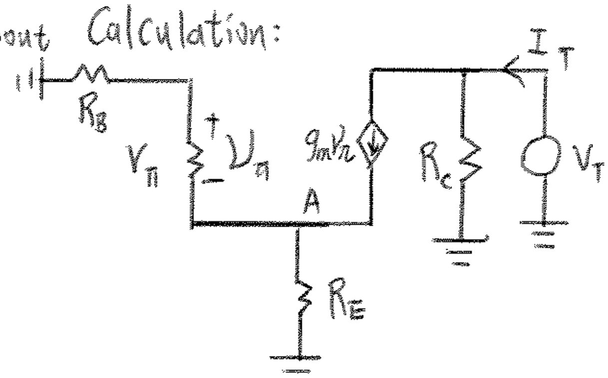
$$V_{out} = \frac{-g_m r_\pi R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-\beta R_C V_{in}}{R_B + r_\pi + (\beta+1) R_E} = \frac{-R_C V_{in}}{\frac{R_B}{\beta} + \frac{1}{g_m} + \frac{\beta+1}{\beta} R_E}$$

$$\frac{V_{out}}{V_{in}} \approx \frac{-R_C}{\frac{R_B}{\beta+1} + \frac{1}{g_m} + R_E}$$

45)



R_{out} Calculation:



$$V_A = g_m v_\pi (R_E \,//\, R_B + r_\pi) \qquad (1)$$

$$v_\pi = -\frac{V_A r_\pi}{r_\pi + R_B} \Rightarrow V_A = -\frac{v_\pi (r_\pi + R_B)}{r_\pi} \qquad (2)$$

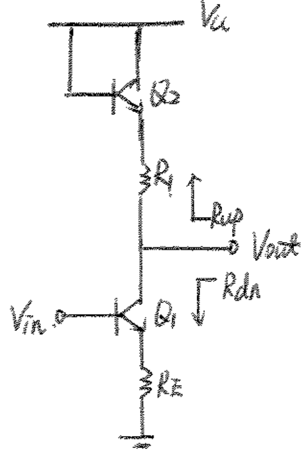
The only possible solution for 1) and 2) is $v_\pi = V_A = 0$, since 1) is positive and 2) is negative.

$$v_\pi = 0 \Rightarrow g_m v_\pi \Rightarrow 0 \Rightarrow \frac{V_T}{I_T} = R_C$$

Therefore, $R_{out} = R_C$

40) $V_A = \infty$

a)



$R_{up} = R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}$

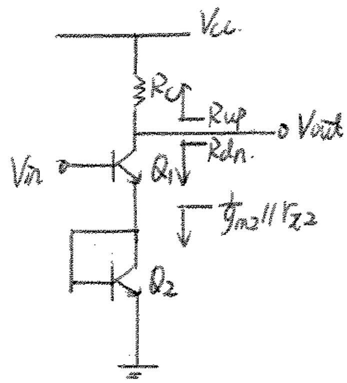
$R_{dn} = \infty$

$R_{out} = R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}$

$R_{in} = r_\pi + (1+\beta) R_E$

$$|A_v| = \frac{R_1 + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

b).



$R_{up} = R_C$

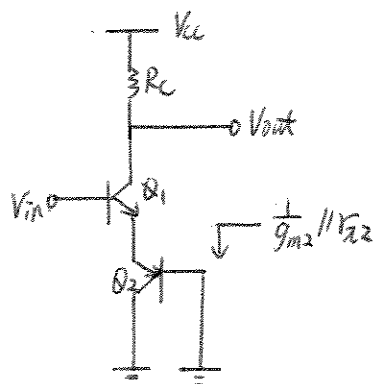
$R_{dn} = \infty$

$R_{out} = R_C$

$R_{in} = r_{\pi 1} + (\beta + 1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{1}{g_{m1}}}$$

c)

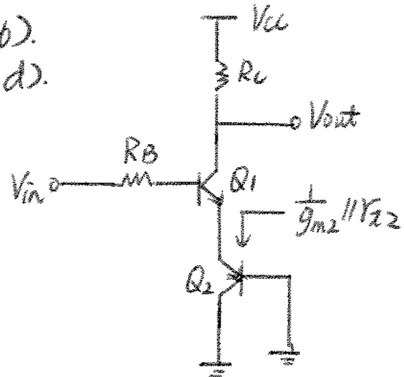


$R_{out} = R_C$

$R_{in} = r_{\pi 1} + (\beta + 1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{1}{g_{m1}}}$$

4b).
d).

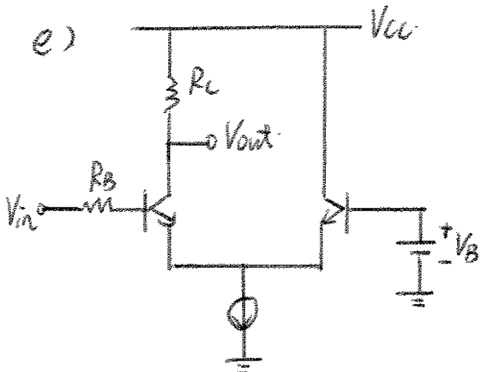


$$R_{out} = R_C$$

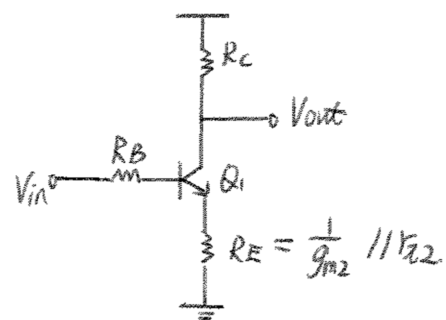
$$R_{in} = R_B + r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{1}{g_{m1}} + \frac{R_B}{\beta+1}}$$

e)



$\Rightarrow$



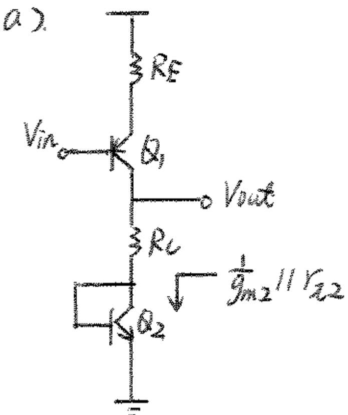
$$R_{out} = R_C$$

$$R_{in} = R_B + r_{\pi 1} + (\beta+1)\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

$$|A_v| = \frac{R_C}{\frac{1}{g_{m2}} \parallel r_{\pi 2} + \frac{1}{g_{m1}} + \frac{R_B}{\beta+1}}$$

47). $V_A = \infty$.

a).



$$R_{out} = R_C + \frac{1}{g_{m2}} \| r_{\pi 2}$$

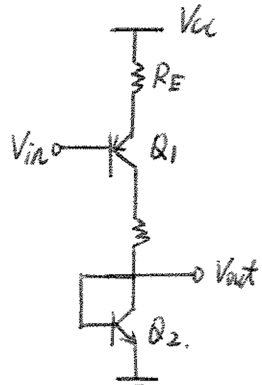
$$R_{in} = r_{\pi 1} + (1+\beta) R_E$$

$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \| r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$

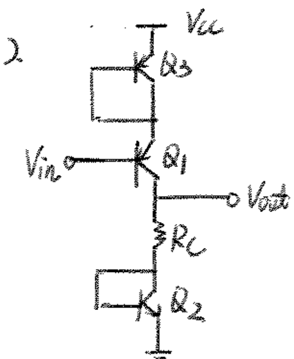
b). $R_{out} = \frac{1}{g_{m2}} \| r_{\pi 2}$

$$R_{in} = r_{\pi 1} + (1+\beta) R_E$$

$$|A_v| = \frac{\frac{1}{g_{m2}} \| r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$



c).



$$R_{out} = R_C + \frac{1}{g_{m2}} \| r_{\pi 2}$$

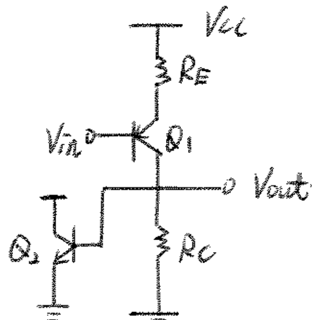
$$R_{in} = r_{\pi 1} + (1+\beta)\left(\frac{1}{g_{m3}} \| r_{\pi 3}\right)$$

$$|A_v| = \frac{R_C + \frac{1}{g_{m2}} \| r_{\pi 2}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}} \| r_{\pi 3}}$$

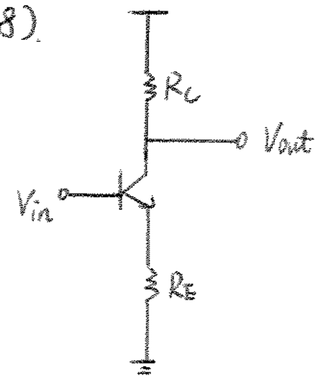
d) $R_{out} = R_C \| r_{\pi 2}$

$$R_{in} = r_{\pi 1} + (\beta+1) R_E$$

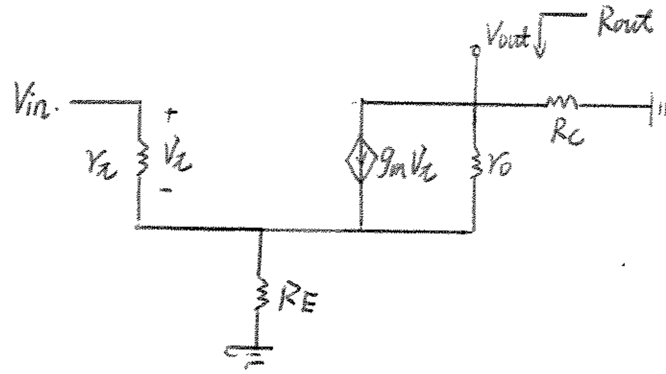
$$|A_v| = \frac{R_C \| r_{\pi 2}}{R_E + \frac{1}{g_{m1}}}$$



48).

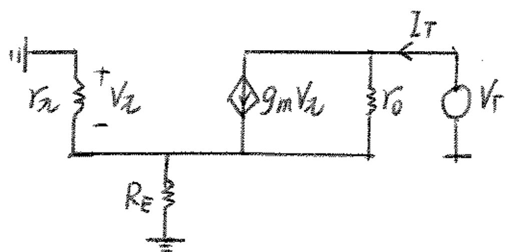


$\Longrightarrow$



$R_{out} = R_c \parallel R_{eq}$

Solve for Req.



$$I_T = g_m v_\pi + \frac{(V_T + v_\pi)}{r_o}$$

$$v_\pi = -I_T (r_\pi \parallel R_E)$$

$$I_T = -g_m I_T (r_\pi \parallel R_E) + \frac{(V_T - I_T (r_\pi \parallel R_E))}{r_o}$$

$$\frac{V_T}{I_T} = r_o \left(1 + \frac{(r_\pi \parallel R_E)}{r_o}\right) + g_m (r_\pi \parallel R_E))$$

$$\frac{V_T}{I_T} = r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

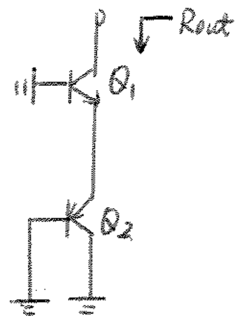
$$R_{eq} = r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

$$R_{out} = R_c \parallel r_o + (1 + g_m r_o)(r_\pi \parallel R_E)$$

$$R_{out} \approx R_c \parallel r_o (1 + g_m (r_\pi \parallel R_E)) \quad \text{since } g_m r_o \gg 1$$

49). $\beta >> 1$ and $V_A < \infty$ to have meaningful result.

a).

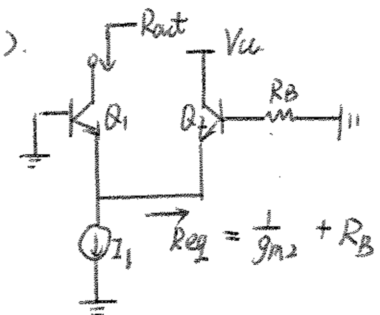


$\frac{1}{g_{m2}} \parallel r_{\pi 2} \approx \frac{1}{g_{m2}}$, since $\beta >> 1$

$$R_{out} = r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} \parallel r_{\pi 2} \right)$$

$$= r_{o1} (1 + g_{m1}/g_{m2})$$

b).

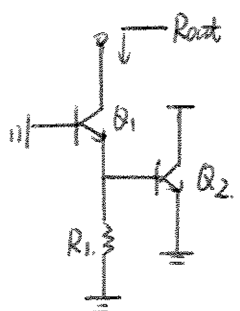


$$R_{out} = r_{o1} + (1 + g_{m1} r_{o1}) \left[\left(\frac{1}{g_{m2}} + \frac{R_B}{\beta + 1} \right) \parallel r_{\pi 1} \right]$$

$$\approx r_{o1} + (1 + g_{m1} r_{o1}) \left(\frac{1}{g_{m2}} + \frac{R_B}{\beta} \right)$$

$$\approx r_{o1} \left[1 + g_{m1} \left(\frac{1}{g_{m2}} + \frac{R_B}{\beta} \right) \right].$$

c).



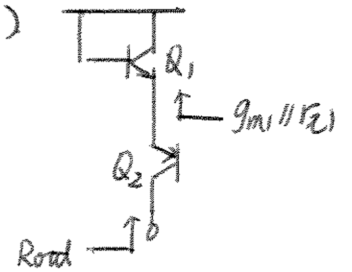
$$R_{out} = r_o + (1 + g_{m1} r_o)(R_1 \parallel r_{\pi 2} \parallel r_{\pi 1})$$

$R_1 \parallel r_{\pi 1} \parallel r_{\pi 2} \approx R_1$, since $\beta >> 1$.

$$R_{out} \approx r_o (1 + g_{m1} R_1)$$

50) $\beta >> 1$, $r_A \to \infty$, for meaningful results

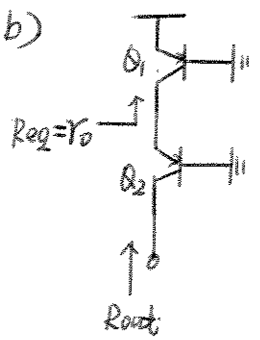
a)



$$R_{out} = r_{o2} + (1 + g_{m2} r_{o2})\left(\frac{1}{g_{m1}} \,//\, r_{\pi 2}\right)$$

$$\approx r_{o2}(1 + g_{m2}/g_{m1})$$

b)



$$R_{out} = r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} \,//\, r_{\pi 2})$$

$$\approx r_{o2}\left[1 + g_{m2}(r_{o1} \,//\, r_{\pi 2})\right]$$

The output impedance in b) is larger than a) because Q_2's connected for a high impedance load, whereas in a) it's connected to a low impedance load.

51). $r_\pi = \beta V_T / I_C$.

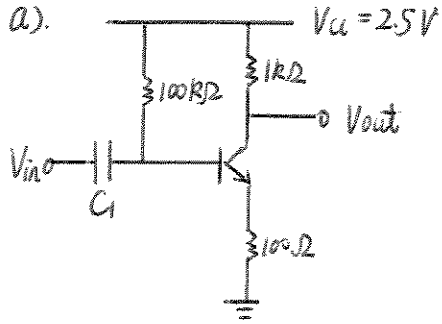
$$R_{in} = r_\pi // R_B = \frac{\frac{\beta V_T}{I_C} R_B}{\frac{\beta V_T}{I_C} + R_B} = \frac{V_T R_B}{V_T + \frac{I_C}{\beta} R_B} = \frac{V_T R_B}{V_T + I_B R_B}$$

Since $I_B R_B >> V_T \Rightarrow R_{in} \approx \frac{V_T R_B}{I_B R_B} = \frac{V_T}{I_B} = \frac{V_T}{\frac{I_C}{\beta}} = \frac{\beta V_T}{I_C} \approx r_\pi$

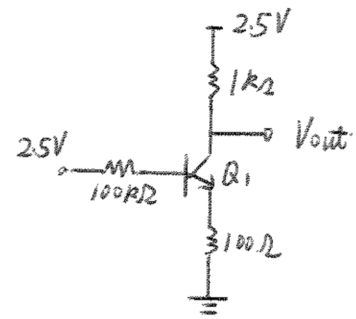
So $R_{in} = r_\pi // R_B \approx r_\pi$.

52). $I_S = 8\times10^{-6}A$, $\beta = 100$, $V_A = \infty$

a).



DC Analysis →



$$I_c = \frac{\beta\left(2.5 - \left(V_{BE} + \frac{I_c}{\alpha}0.1\right)\right)}{100K} \Rightarrow I_c = \frac{100\,(2.5 - V_{BE})}{100K + 10.1K}$$

Guess $V_{BE} = 0.75V$, $I_c = 1.59mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$, not $0.75V$, reiterate

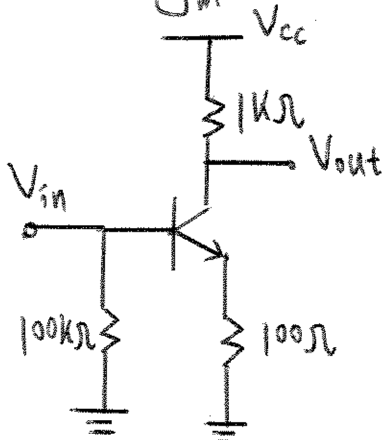
$V_{BE} = 0.736\,V$, $I_c = 1.60\,mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.736V$, converged !!

$I_c = 1.60mA$

$g_m = \frac{I_c}{V_T} = \frac{1.60mA}{26mV} = 0.0615\left(\frac{1}{\Omega}\right)s$

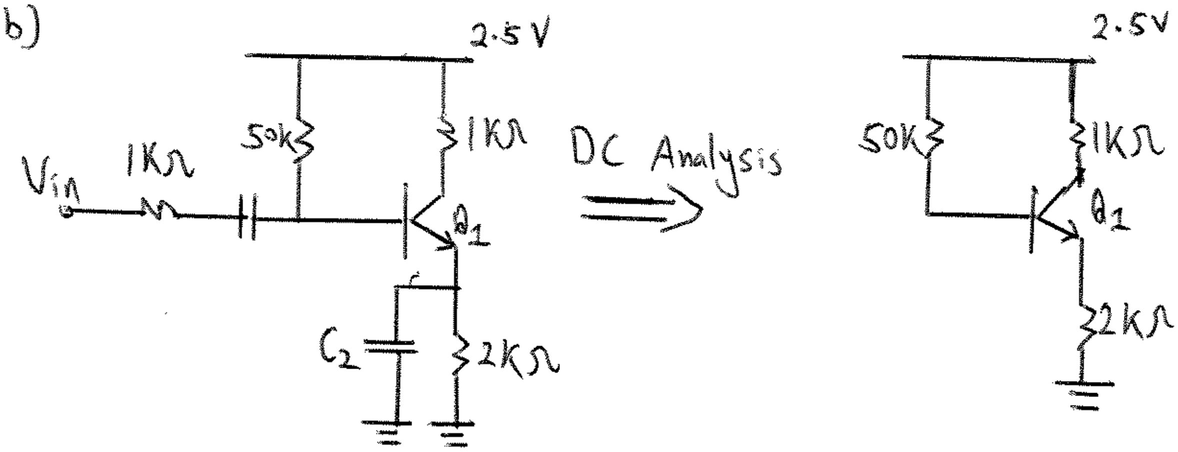
$r_\pi = \frac{\beta}{g_m} = 1.63\,K\Omega$



$$A_V = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1K}{0.1 + \frac{1}{g_m}} = 8.6$$

52)

b)



$$I_c = \beta\left(\frac{2.5-(V_{BE}+I_E 2K)}{50K}\right) \Longrightarrow I_c = \frac{100(2.5-V_{BE})}{50K + 202K}$$

Guess $V_{BE} = 0.7V$, $I_c = 0.714\ mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7155V$, reiterate

$V_{BE} = 0.7155V$, $I_c = 0.708\ mA$

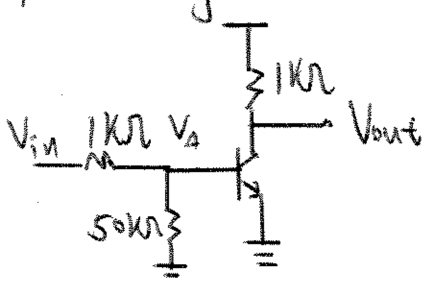
Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715\ V$, converged!!

$I_c = 0.708mA$

$g_m = \frac{I_c}{V_T} = 0.02723\left(\frac{1}{\Omega}\right) S$

$V_{BE} = 0.715V$

AC Analysis:



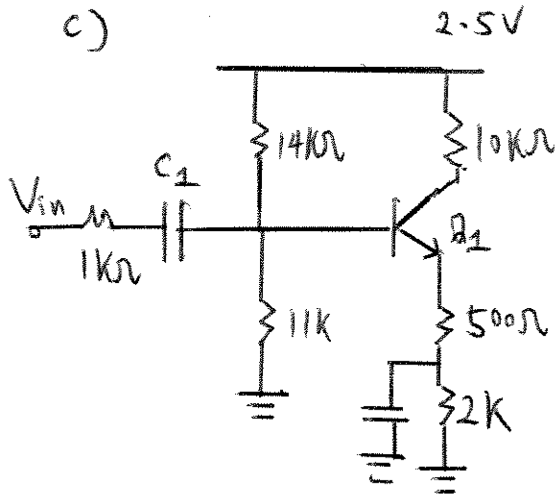
$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_{out}}{V_A}\right|\left|\frac{V_A}{V_{in}}\right| = 21.1$$

$$\left|\frac{V_{out}}{V_A}\right| = g_m\, 1K\Omega = 27.23$$

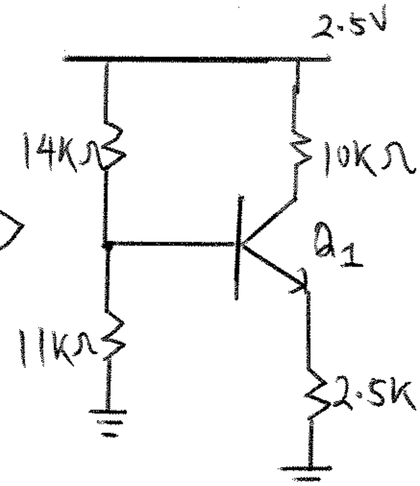
$$\left|\frac{V_A}{V_{in}}\right| = \frac{50K // V_\pi}{50K//V_\pi + 1K} = 0.77$$

52)

c)



⟹



$$I_c = \beta\left(\frac{1.1 - \left(V_{BE} + \frac{I_c}{\alpha}2.5\right)}{14K\Omega // 11K\Omega}\right) \Rightarrow I_c = \frac{100\,(1.1 - V_{BE})}{6.16 + 252.53}$$

Guess $V_{BE} = 0.7V$, $I_c = 0.1546\ mA$

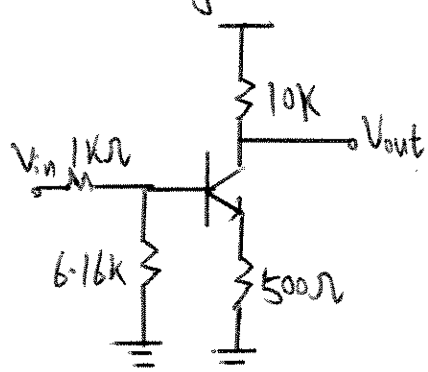
Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.676\ V$, not $0.7V$, reiterate

$V_{BE} = 0.676\ V$, $I_c = 0.164\ mA$

Verify $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.677\ V$, converged !!

$I_c = 0.164\ mA$, $V_{BE} = 0.677V$, $g_m = 0.00631\ \left(\frac{1}{\Omega}\right) S$, $r_\pi = 15.85k$

AC Analysis:

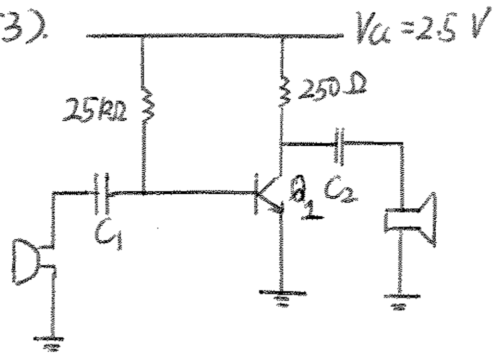


$$\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_{out}}{V_A}\right|\left|\frac{V_A}{V_{in}}\right| = 12.9$$

$$\left|\frac{V_{out}}{V_A}\right| = \frac{10K}{\frac{1}{g_m} + 0.5} = 15.2$$

$$\left|\frac{V_A}{V_{in}}\right| = \frac{6.16K // (15.85K + 101(0.5))}{6.16K // (15.85K + 101(0.5)) + 1K} = 0.85$$

53).



$R_B = 25\,K\Omega$

$R_C = 250\Omega$

$I_S = 5 \times 10^{-17} A$

$V_A = \infty$

DC Analysis: Assume collector bias voltage is still 1.5 V. So 1 V across $R_C \Rightarrow I_C = 4mA$.

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.832$$

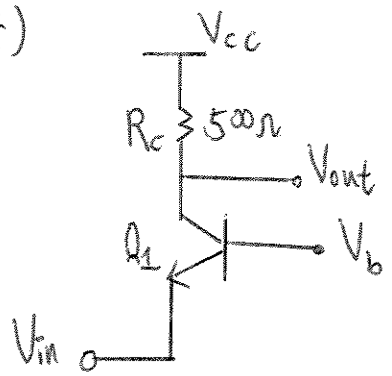
$$I_B = \frac{2.5 - V_{BE}}{25K} = 0.06673\ mA$$

$$\beta = \frac{I_C}{I_B} = \frac{0.832\ mA}{0.06673\ mA} = 60$$

$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_m(250\Omega // 8\Omega) = 1.2, \text{(Greater than unity)}$$

$$g_m = \frac{4mA}{26mV} = 0.1538 \left(\frac{1}{\Omega}\right) S$$

54)



$I_c = 2mA$

$V_A = \infty$

$g_m = \frac{I_c}{V_T} = 0.0769 \left(\frac{1}{\Omega}\right) S$, $\frac{1}{g_m} = 13\Omega$

a)

$|A_v| = g_m R_c = \frac{0.5}{0.013} = 38.5$

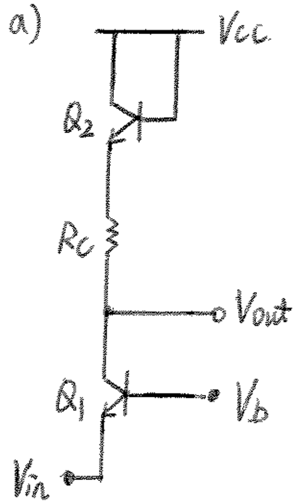
$R_{in} = \frac{1}{g_m} // r_\pi \approx \frac{1}{g_m} = 13\Omega$ (Since β is usually large)

$R_{out} = 500\,\Omega$

b) Since $|A_v| = g_m R_c$, and g_m is fixed by I_c. The only way to maximize $|A_v|$ is to maximize R_c. However a large R_c will push Q_1 into saturation, losing its gain altogether. Therefore, V_B has to be as small as possible to provide enough room for V_c to drop $\Rightarrow$ large R_c $\Rightarrow$ large gain.

55) $V_A = \infty$

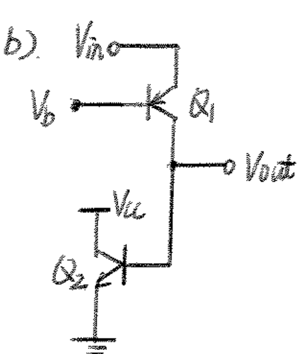
a)



$$|A_V| = \frac{R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}}{\frac{1}{g_{m1}}}$$

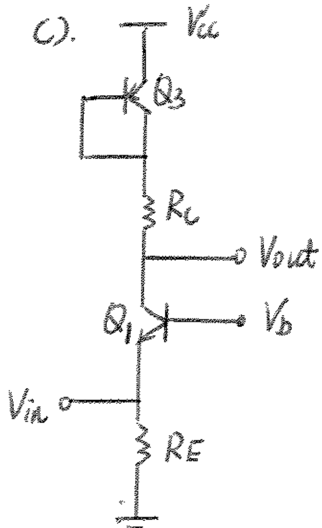
$$= g_{m1}\left(R_C + \frac{1}{g_{m2}} \parallel r_{\pi 2}\right)$$

b).



$$|A_V| = \frac{r_{\pi 2}}{\frac{1}{g_{m1}}} = g_{m1} r_{\pi 2}$$

c).

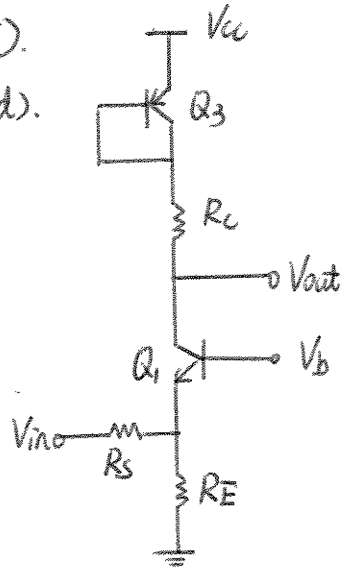


$$|A_V| = \frac{R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}}{\frac{1}{g_{m1}}}$$

$$= g_{m1}\left(R_C + \frac{1}{g_{m3}} \parallel r_{\pi 3}\right)$$

55).

d).

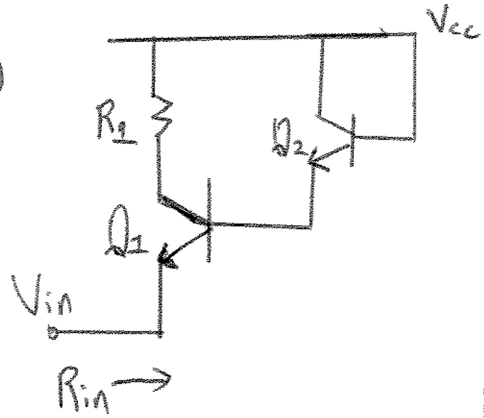


$$|A_v| = \left|\frac{V_{out}}{V_A}\right| \left|\frac{V_A}{V_{in}}\right|$$

$$= \left[g_{m1} \left(R_C + \frac{1}{g_{m3}} \,\|\, r_{\pi 3} \right) \right] \left(\frac{R_E \,\|\, \frac{1}{g_{m1}}}{R_E \,\|\, \frac{1}{g_{m1}} + R_S} \right)$$

56) $V_A = \infty$

a)

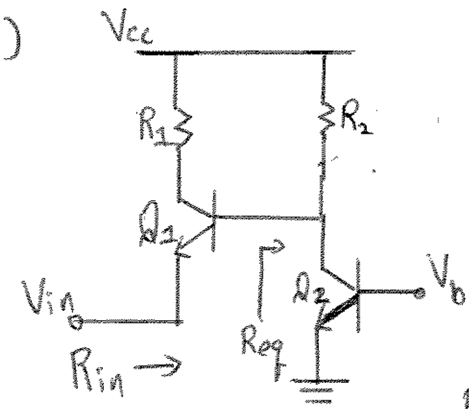


$$R_{in} = \frac{1}{g_{m1}} // r_{\pi 1} + \frac{\frac{1}{g_{m2}} // r_{\pi 2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{1}{g_{m2}(\beta_1 + 1)}$$

b)



$$R_{eq} = R_2 // \infty = R_2$$

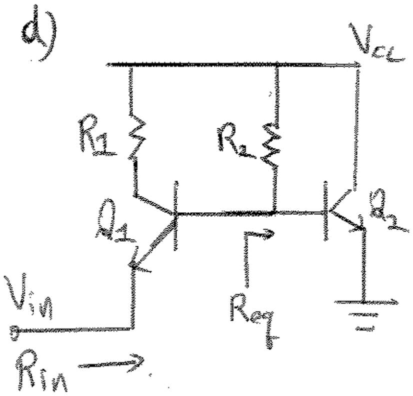
$$R_{in} = \frac{1}{g_{m1}} // r_{\pi 1} + \frac{R_2}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} = \frac{1}{g_{m1}} + \frac{R_2}{\beta_1 + 1}$$

* Note, part c) and d) have swapped places.

d)



$$R_{eq} = R_2 // r_{\pi 2}$$

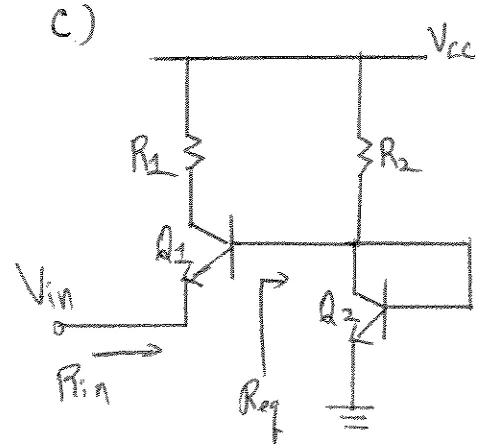
$$R_{in} = \frac{1}{g_{m1}} // r_{\pi 1} + \frac{R_2 // r_{\pi 2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m1}} + \frac{R_2 // r_{\pi 2}}{\beta_1 + 1}$$

56) * Note, part c) and d) have swapped places

c)



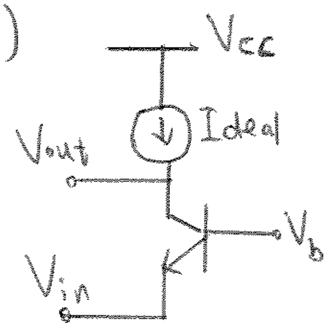
$$R_{eq} = R_2 \,//\, \frac{1}{g_{m_2}} \,//\, r_{\pi_2}$$

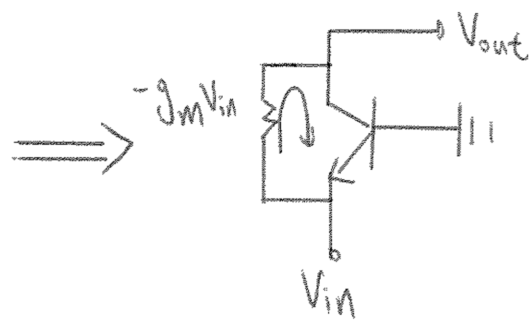
$$R_{in} = \frac{1}{g_{m_1}} \,//\, r_{\pi_1} + \frac{R_2 \,//\, \frac{1}{g_{m_2}} \,//\, r_{\pi_2}}{\beta_1 + 1}$$

Since β is usually very large

$$R_{in} \approx \frac{1}{g_{m_1}} + \frac{R_2 \,//\, \frac{1}{g_{m_2}}}{\beta_1 + 1}$$

57)





Since an ideal current source is an open circuit, the signal current produced by the transistor has no where to go but r_o.

So $V_{out} = -(g_m(0 - V_{in}))r_o + V_{in}$

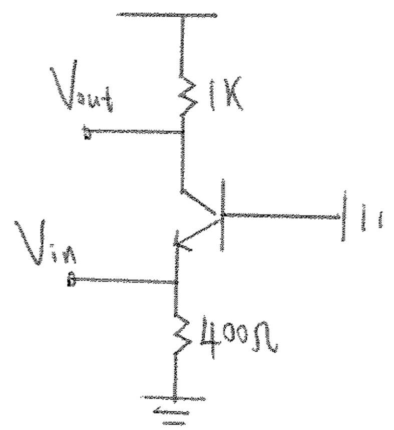
$V_{out} = g_m r_o V_{in} + V_{in}$

$V_{out} = V_{in}(g_m r_o + 1)$

$\frac{V_{out}}{V_{in}} = 1 + g_m r_o$

58)

b) AC Analysis



$$A_v = \left|\frac{V_{out}}{V_{in}}\right| = g_m 1K$$

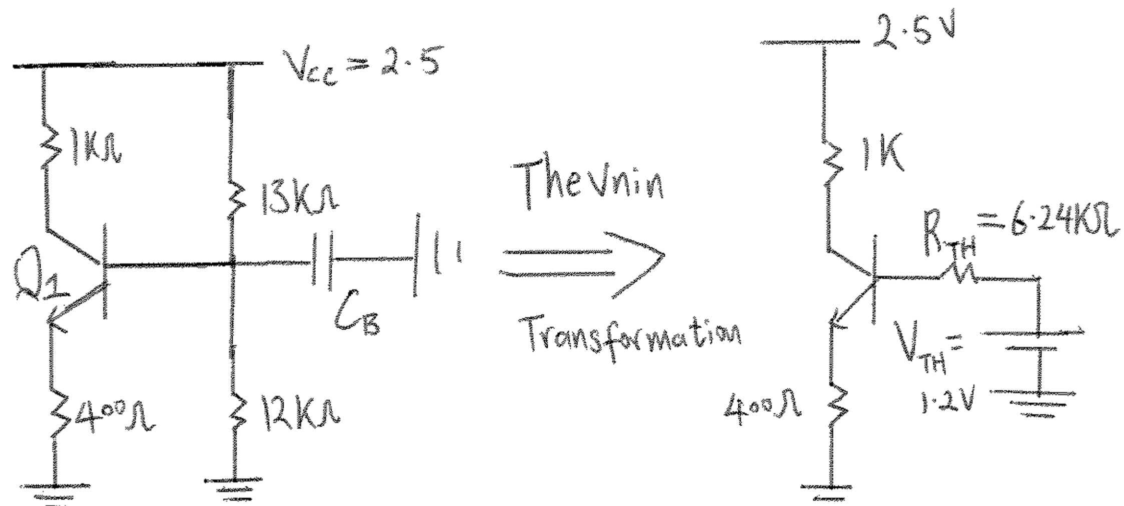
$$g_m = 0.0391 \left(\frac{1}{\Omega}\right) S$$

$$A_v = 39.1$$

$$R_{in} = 400\Omega \parallel \frac{1}{g_m} = 400\Omega \parallel 25.583\Omega = 24.0\Omega$$

$$R_{out} = 1K$$

58)



$\beta = 100, \quad I_s = 8\times10^{-16}A, \quad V_A = \infty, \quad C_B = \text{Very large}$

a) DC Analysis:

$$I_c = \beta\left(\frac{1.2 - (V_{BE} + I_E 0.4)}{6.24}\right) \Rightarrow \frac{\beta(1.2 - V_{BE})}{6.24 + \frac{0.4\beta}{\alpha}}$$

Guess $V_{BE} = 0.7 \Rightarrow I_c = 1.072\ mA$

Verify V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.726\ V$, not 0.7V, reiterate.

$V_{BE} = 0.726\ V$; $I_c = 1.0163\ mA$

Verify V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.725V$, Converged!!

$V_{BE} = 0.725$, $V_{CE} = 2.5 - \left[(1.0163)(1K) + 0.4\left(\frac{1.0163}{0.99}\right)\right]$

$V_{CE} = 1.07$

$I_c = 1.0163mA, \quad I_B = 10.163\ \mu A$

59)

$C_B = 0$

a) Since C_B was not considered during DC analysis, it has no effect on operating point analysis. So it is still the same as 58).

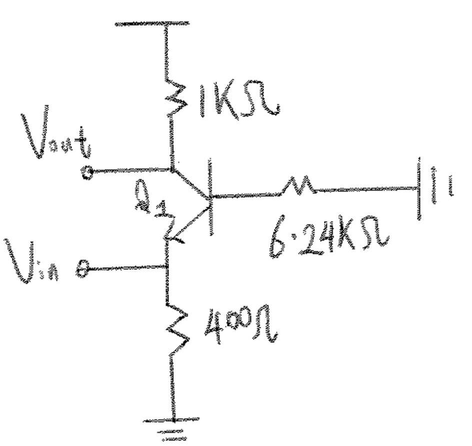
$V_{BE} = 0.725\,V$

$I_C = 1.0163\,mA$

$I_B = 10.163\,\mu A$

$V_{CE} = 1.07\,V$

b) Since capacitor is frequency dependent, the circuit's AC analysis will be different.



$$|A_v| = \frac{1k}{\frac{1}{g_m} + \frac{6.24k\Omega}{\beta+1}} = 11.4$$

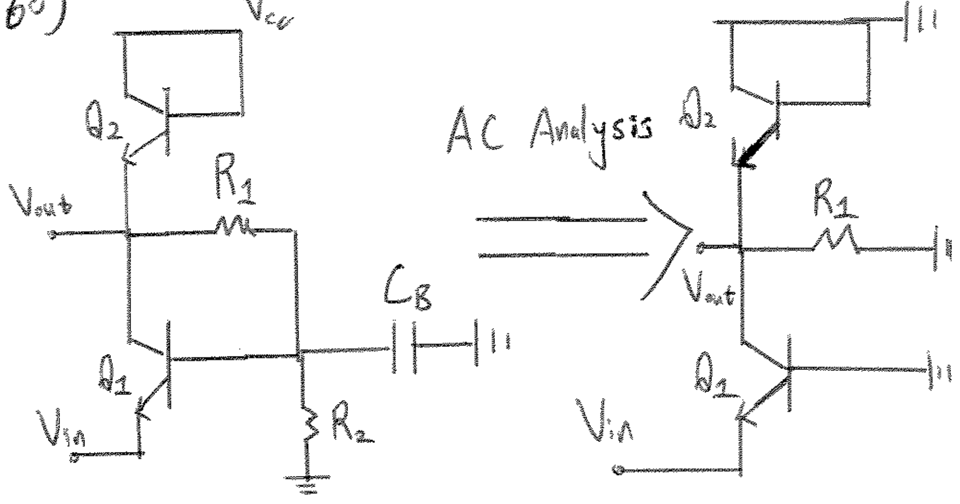
$$R_{in} = 400\Omega \,//\, \left(\frac{1}{g_m} + \frac{6.24k\Omega}{\beta+1}\right)$$

$$R_{in} = 71.7\Omega$$

Note: 6.24 KΩ is R_{THEV} of 13KΩ and 12KΩ combination.

$$R_{out} = 1K\Omega$$

60)

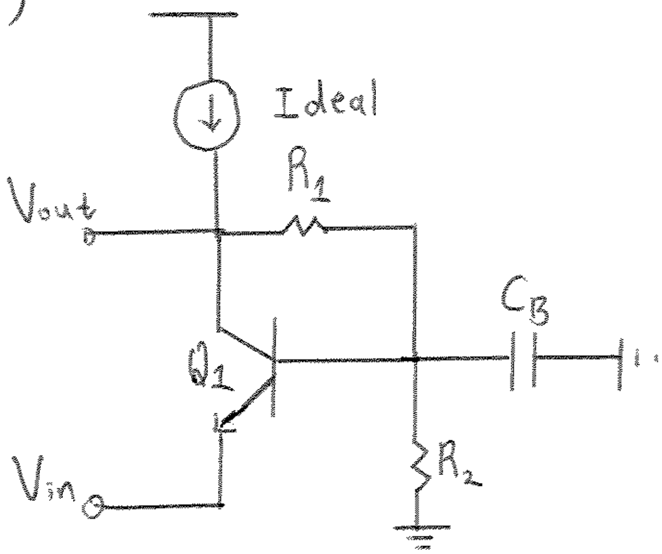


$$R_{out} = \frac{1}{g_{m2}} // r_{\pi 2} // R_1 \approx \frac{1}{g_{m2}} // R_1$$

$$A_v = \left| \frac{V_{out}}{V_{in}} \right| = g_{m1}\left(\frac{1}{g_{m2}} // r_{\pi 2} // R_1\right) \approx g_{m1}\left(\frac{1}{g_{m2}} // R_1\right)$$

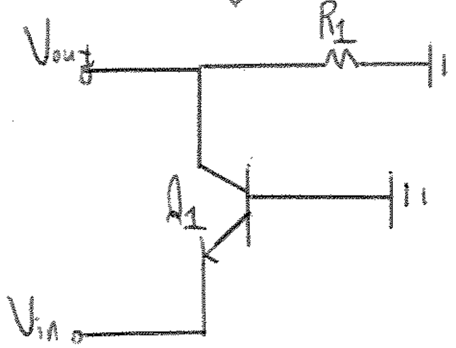
$$R_{in} = \frac{1}{g_m} // r_{\pi 1} \approx \frac{1}{g_{m1}}$$

61)



$V_A = \infty$
C_B large

⇓ AC Analysis



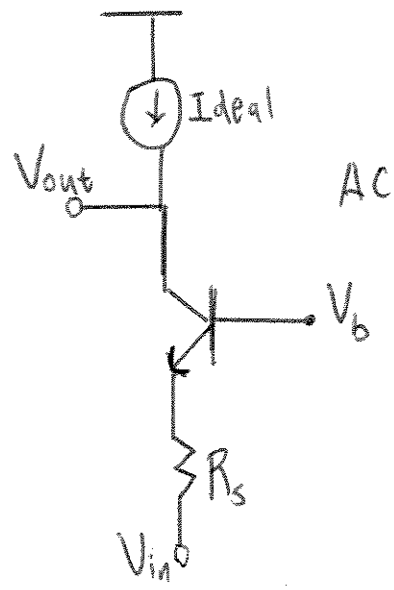
(R_2 shorted out)

$R_{out} = R_1$

$R_{in} = \frac{1}{g_{m1}} \parallel r_{\pi 1} \approx \frac{1}{g_{m1}}$

$|A_v| = g_{m1} R_1$

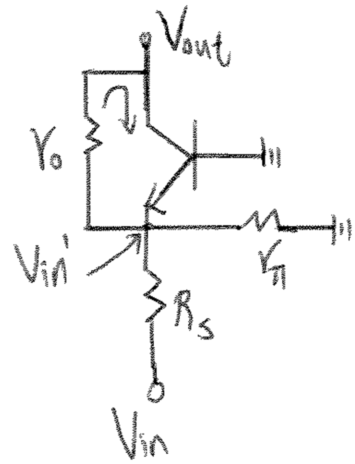
62)



$V_A < \infty$

AC Analysis

$\Longrightarrow$



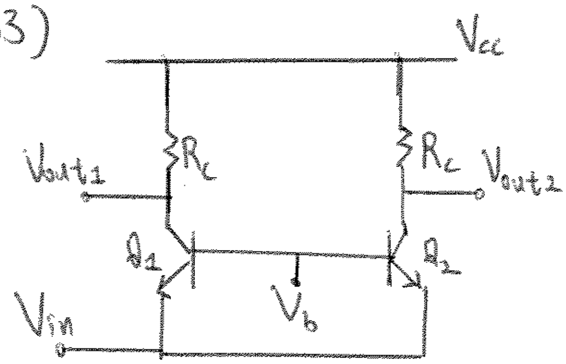
$$A_v = \frac{V_{out}}{V_{in}} = \left(\frac{V_{in}'}{V_{in}}\right)\left(\frac{V_{out}}{V_{in}'}\right), \quad \left(\frac{V_{in}'}{V_{in}}\right) = \frac{r_\pi}{r_\pi + R_s}$$

Since V_{out} is float, so looking at emitter and r_o, we will see an infinite impedance.

$$\frac{V_{out}}{V_{in}'} \Rightarrow \; -g_m(-V_{in}')r_o + V_{in}' = V_{out} \Rightarrow \frac{V_{out}}{V_{in}'} = (g_m r_o + 1)$$

$$A_v = (g_m r_o + 1)\left(\frac{r_\pi}{r_\pi + R_s}\right).$$

63)



$V_A = \infty$

$I_{S1} = 2I_{S2}$

$$\left|\frac{V_{out1}}{V_{in}}\right| = g_{m1} R_c \quad , \quad \left|\frac{V_{out2}}{V_{in}}\right| = g_{m2} R_c$$

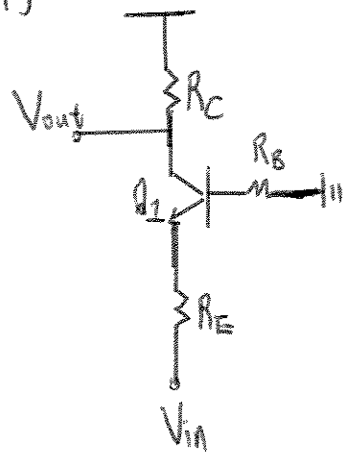
$$g_{m1} = \frac{V_T}{I_{c1}} = \frac{V_T}{2I_{s2} e^{(V_{BE}/V_T)}} \quad , \text{ Since } I_{s1} = 2I_{s2}$$

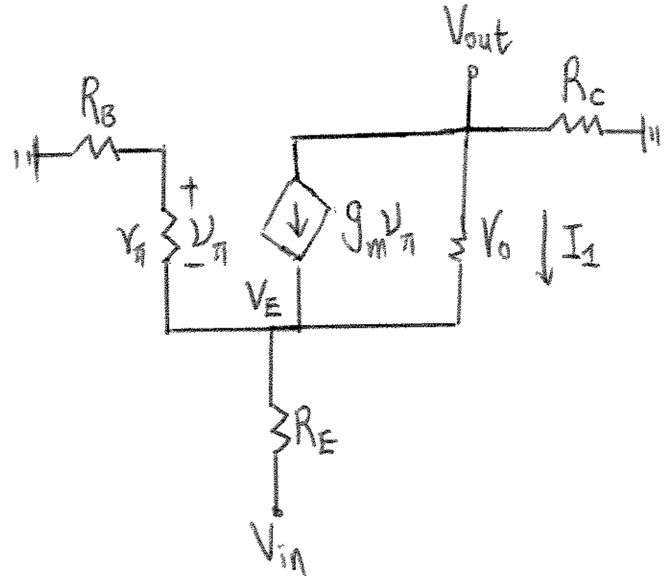
$$g_{m2} = \frac{V_T}{I_{c2}} = \frac{V_T}{I_{s2} e^{(V_{BE}/V_T)}}$$

$(V_{BE1} = V_{BE2} = V_{BE})$

$$\Rightarrow \quad g_{m1} = \frac{g_{m2}}{2} \Rightarrow \left|\frac{V_{out1}}{V_{in}}\right| = \frac{1}{2}\left|\frac{V_{out2}}{V_{in}}\right|$$

64)





$$V_{out} = -(I_1 + g_m v_\pi)R_c \,, \quad I_1 = \frac{V_{out} - V_E}{r_o}$$

$$V_{out} = -\left(\frac{V_{out} - V_E}{r_o} + g_m v_\pi\right)R_c \,, \quad V_E = \frac{-g_m v_\pi}{\beta}(r_\pi + R_B)$$

$$V_{out} = -\left(\frac{V_{out} + \frac{g_m v_\pi (r_\pi + R_B)}{\beta}}{r_o} + g_m v_\pi\right)R_c$$

Rearranging

$$v_\pi = \frac{-\left(1 + \frac{R_c}{r_o}\right)}{\frac{g_m(r_\pi + R_B)R_c}{\beta r_o} + g_m R_c} V_{out} = A V_{out}$$

Summing the voltage at node E.

$$V_E - \left(\left(1 + \frac{1}{\beta}\right)g_m v_\pi + \frac{(V_{out} - V_E)}{r_o}\right)R_E = V_{in} \quad (1)$$

64)

Writing V_E in terms of V_π, and V_π in terms of V_{out}

1) becomes

$$-\frac{g_m A V_{out}}{\beta}(r_\pi + R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A V_{out} R_E - \frac{V_{out}}{r_o} R_E = V_{in}$$

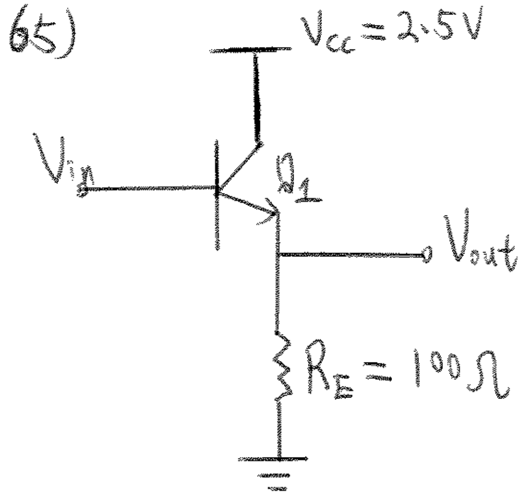
Solving V_{out}/V_{in} =>

$$\frac{V_{out}}{V_{in}} = \frac{1}{-\frac{g_m A}{\beta}(r_\pi+R_B)\left(1+\frac{R_E}{r_o}\right) - \left(1+\frac{1}{\beta}\right) g_m A R_E - \frac{R_E}{r_o}}$$

subsituting A into equation

$$\frac{V_{out}}{V_{in}} = \frac{\frac{g_m(r_\pi+R_B)R_c}{\beta r_o} + g_m R_c}{g_m\left(1+\frac{R_c}{r_o}\right)(r_\pi+R_B)\left(1+\frac{R_E}{r_o}\right) + \left(1+\frac{1}{\beta}\right) g_m\left(1+\frac{R_c}{r_o}\right) R_E - \frac{R_E}{r_o}\left(\frac{g_m(r_\pi+R_B)R_c}{\beta r_o} + g_m R_c\right)}$$

65)



$R_E = 100\Omega$
$V_A = \infty$
$|A_v| = 0.8$

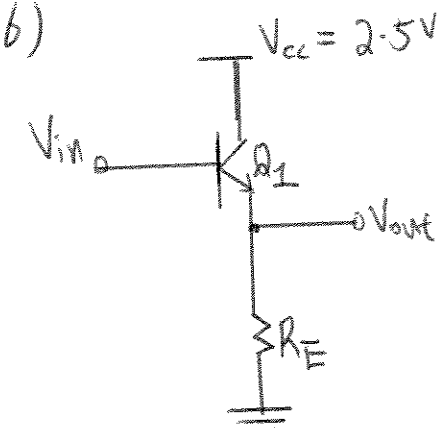
$$|A_v| = \frac{R_E}{R_E + \frac{1}{g_m}} = \frac{R_E I_C}{R_E I_C + V_T} = 0.8$$

$\Rightarrow R_E I_c = 0.8(R_E I_c + V_T)$, $R_E = 100\Omega$

$\Rightarrow 0.1 I_c = 0.08 I_c + 0.0208$ $\Rightarrow$ $0.02 I_c = 0.0208$

$\Rightarrow I_c = 1.04mA$

6b)



$|A_v| > 0.9$
$R_{in} > 10K\Omega$

$$|A_v| = \frac{R_E I_c}{R_E I_c + V_T} > 0.9 \Rightarrow R_E I_c > 0.9[R_E I_c + V_T]$$

$$\Rightarrow R_E I_c > 9V_T = 234\,mV, \text{ Let } R_E I_c = 240\,mV$$

$$R_{in} = r_\pi + (1+\beta)R_E > 10K \Rightarrow 100V_T + (101)R_E I_c > 10K\Omega I_c$$

Subsituting $R_E I_c = 240\,mV \Rightarrow I_c < 2.684\,mA$

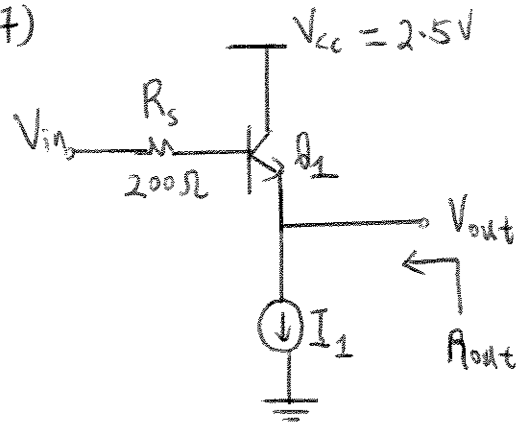
Choose I_c to be $2.5\,mA \Rightarrow R_E = 96\Omega$

To Verify:

$$R_{in} = 100\frac{(0.026)}{2.5} + (101)0.096 = 10.74K\Omega$$

$$|A_v| = \frac{(0.096)(2.5)}{(0.096)(2.5) + 0.026} = 0.902$$

67)



$\beta = 100$

$V_A = \infty$

$$R_{out} = \frac{1}{g_m} + \frac{R_s}{(\beta+1)} \leq 5\,\Omega \quad (\text{Assuming } r_\pi >> \frac{1}{g_m})$$

$$R_{out} = 0.026 + \frac{200\Omega I_c}{101} \leq 5\Omega I_c$$

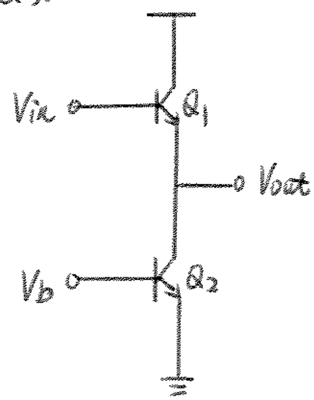
$$\Rightarrow I_c \geq 0.0086A$$

Pick $I_c = 0.009\,A$

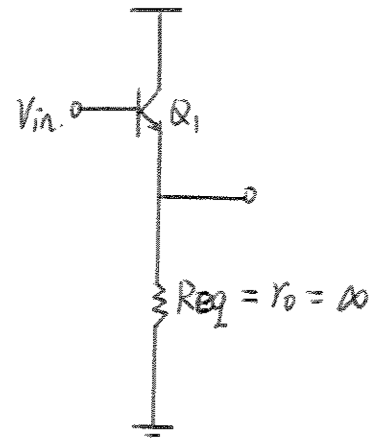
$$R_{out} = \frac{0.026V}{0.009A} + \frac{200}{101} = 4.87\Omega$$

68). a).

$V_A = \infty$



⇒



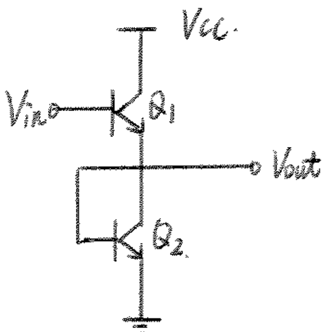
$$|A_v| = \frac{r_0}{r_0 + \frac{1}{g_m}}, \quad \text{since } r_0 = \infty$$

$$|A_v| = 1.$$

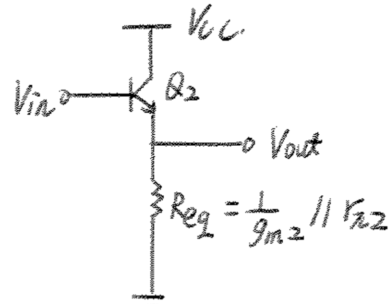
$$R_{in} = \infty \quad (\text{since } r_0 = \infty)$$

$$R_{out} = \infty \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1} = \frac{1}{g_{m1}} \,//\, r_{\pi 1}$$

b).



⇒



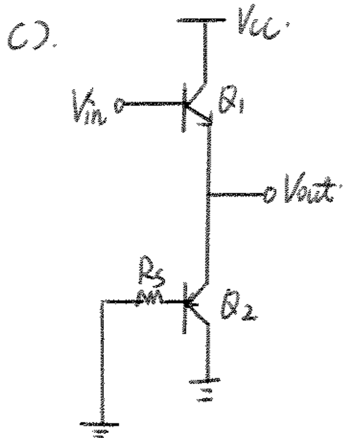
$$|A_v| = \frac{\frac{1}{g_{m2}} // r_{\pi 2}}{\frac{1}{g_{m2}} // r_{\pi 2} + \frac{1}{g_{m1}}}$$

$$R_{in} = r_{\pi 1} + (1+\beta)\frac{1}{g_{m2}} // r_{\pi 2}$$

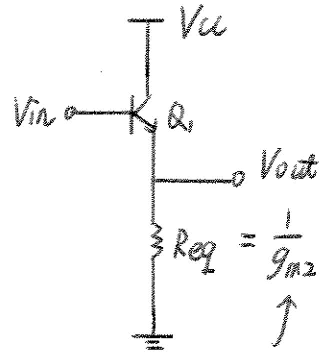
$$R_{out} = \frac{1}{g_{m2}} \,//\, r_{\pi 2} \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}$$

$$\left(\text{If } I_{S1} = I_{S2},\ g_{m1} = g_{m2} = g_m,\ r_{\pi 1} = r_{\pi 2} = r_\pi,\ R_{out} = \frac{1}{2g_m} // \frac{r_\pi}{2}\right)$$

c).



$\Longrightarrow$



$$R_{eq} = \frac{1}{g_{m2}} + \frac{R_S}{\beta+1}$$

(Assuming $r_\pi >> \frac{1}{g_m}$)

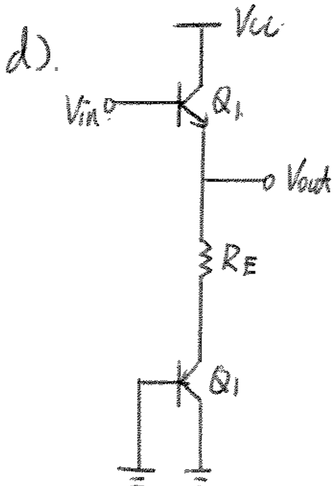
$$|A_v| = \frac{\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}}{\frac{1}{g_{m2}} + \frac{R_S}{\beta+1} + \frac{1}{g_{m1}}}$$

$$R_{in} = r_\pi + (1+\beta)\left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right)$$

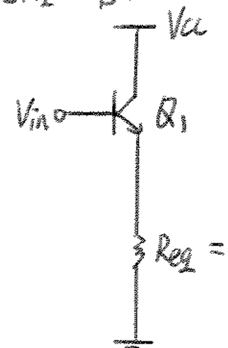
$$R_{out} = \left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right) // \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

$$R_{out} \approx \left(\frac{1}{g_{m2}} + \frac{R_S}{\beta+1}\right) // \frac{1}{g_{m1}}$$

d).



$\Longrightarrow$



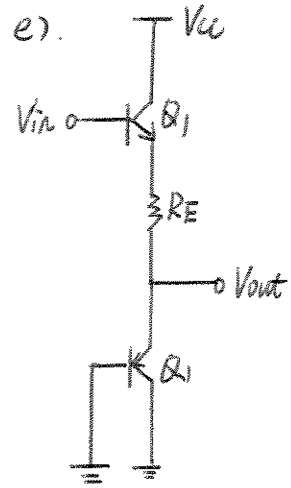
$$R_{eq} = R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

$$|A_v| = \frac{R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)}{R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right) + \frac{1}{g_{m1}}}$$

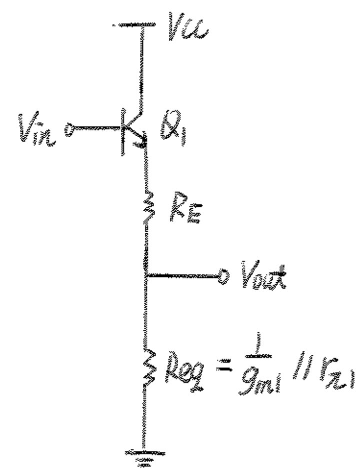
$$R_{in} = r_\pi + (1+\beta)\left[R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)\right]$$

$$R_{out} = \left[R_E + \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)\right] // \left(\frac{1}{g_{m1}} // r_{\pi 1}\right)$$

68). e).



$\Longrightarrow$

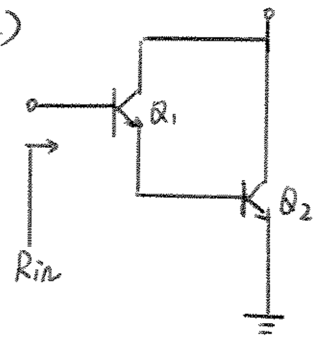


$$|A_v| = \frac{\frac{1}{g_{m1}} \,/\!/\, r_{\pi 1}}{\frac{1}{g_{m1}} \,/\!/\, r_{\pi 1} + R_E + \frac{1}{g_{m1}}}$$

$$R_{in} = r_\pi + (1+\beta)\left[R_E + \frac{1}{g_{m1}} \,/\!/\, r_{\pi 1}\right]$$

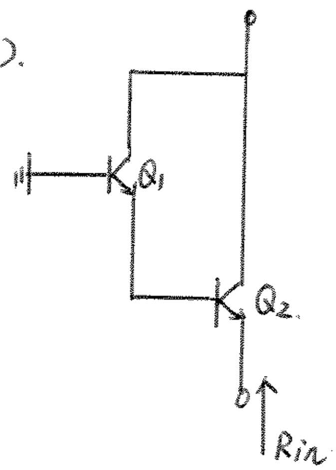
$$R_{out} = \left(\frac{1}{g_{m1}} \,/\!/\, r_{\pi 1}\right) /\!/ \left(R_E + \frac{1}{g_{m1}} \,/\!/\, r_{\pi 1}\right).$$

69 a)



$$R_{in} = r_{\pi 1} + (1+\beta) r_{\pi 2}$$

b).



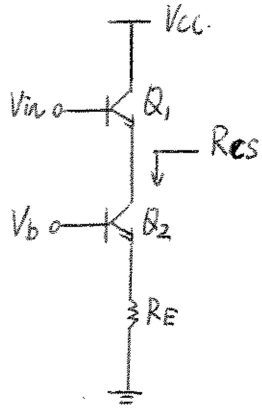
$$R_{in} = \frac{1}{g_{m2}} + \frac{1/g_{m1}}{(\beta+1)} \quad \left(\text{Assume } r_\pi \gg \frac{1}{g_m}\right)$$

c) $$\text{Current Gain} = \frac{(I_{C1} + I_{C2})}{I_{B1}} = \beta + \frac{I_{C2}}{I_{B1}} = \beta + \frac{\beta I_{B2}}{I_{B1}}$$

Since $I_{B2} = I_{C1} = \beta I_{B1}$

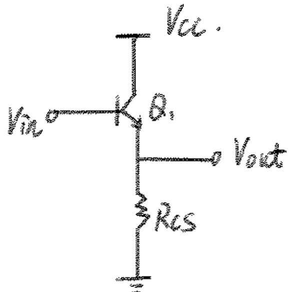
$$\text{Current Gain} = \beta + \beta^2 = \beta(\beta+1), \quad (\text{Assuming } \beta_1 = \beta_2 = \beta)$$

70).



a) $R_{cs} = r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})$

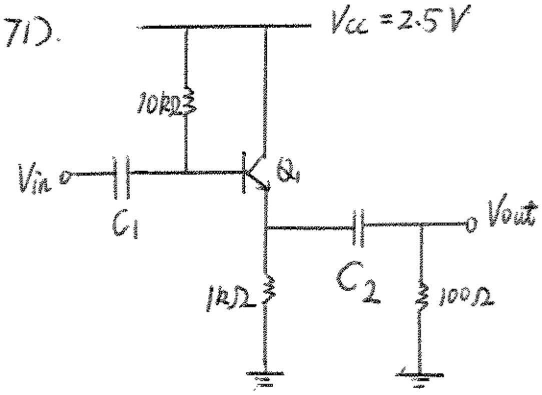
b).



$$A_v = \frac{R_{cs} // r_{o1}}{R_{cs} // r_{o1} + \frac{1}{g_{m1}}}$$

$$A_v = \frac{\left(r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})\right) // r_{o1}}{\left(r_{o2} + (1 + g_{m2} r_{o2})(R_E // r_{\pi 2})\right) // r_{o1} + \frac{1}{g_{m1}}}$$

7D.

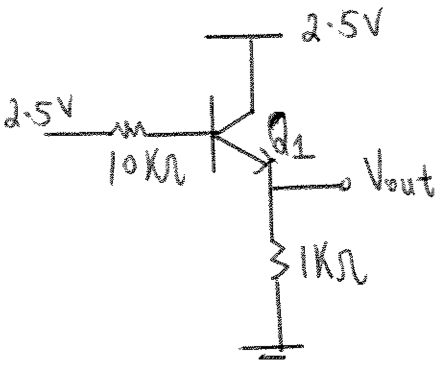


$I_s = 7 \times 10^{-16} A$

$\beta = 100$

$V_A = 5V$

DC Analysis: (Ignore r_o's effect)



$$I_c = \beta \left(\frac{2.5 - (V_{BE} + \frac{I_c}{\alpha} 1K\Omega)}{10K\Omega} \right)$$

rearrange

$$I_c = \frac{2.5 - V_{BE}}{\frac{10K\Omega}{\beta} + \frac{1K\Omega}{\alpha}}$$

Guess: $V_{BE} = 0.7V$, $I_c = 1.621mA$

check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, not 0.7, reiterate

$V_{BE} = 0.740V$, $I_c = 1.59mA$

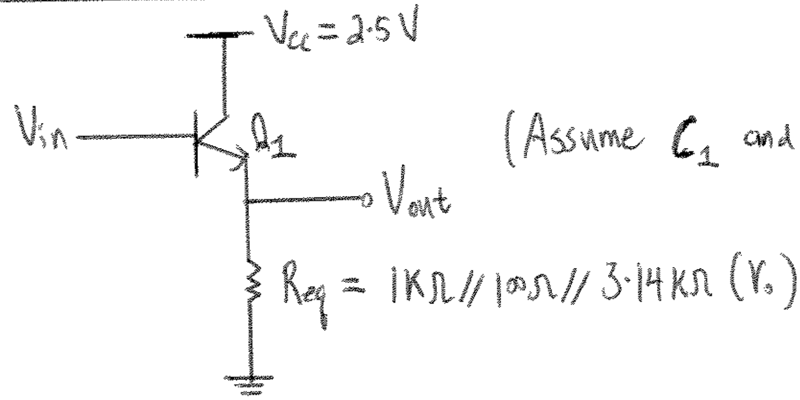
Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.740V$, converged.

So $I_c = 1.59mA$, $g_m = 0.0612 (\frac{1}{\Omega})S$, $\frac{1}{g_m} = 16.34\Omega$,

$r_o = 3.14K\Omega$

71)

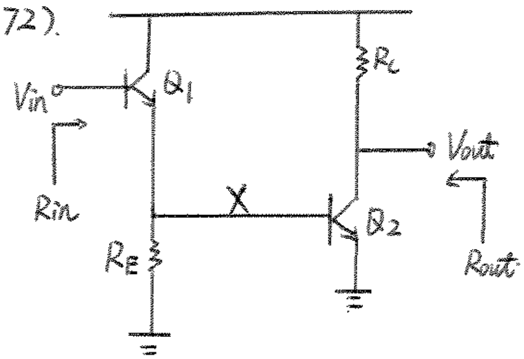
AC Analysis: (Include r_o)



(Assume C_1 and C_2 are large)

$$A_v = \frac{(1K\Omega // 100\Omega // 3.14K\Omega)}{16.34\Omega + (1K\Omega // 100\Omega / 3.14K\Omega)} = 0.84$$

72).



$V_A < \infty$

a) $R_{in} = r_{\pi 1} + (1+\beta)(R_E // r_{\pi 2} // r_{o1})$

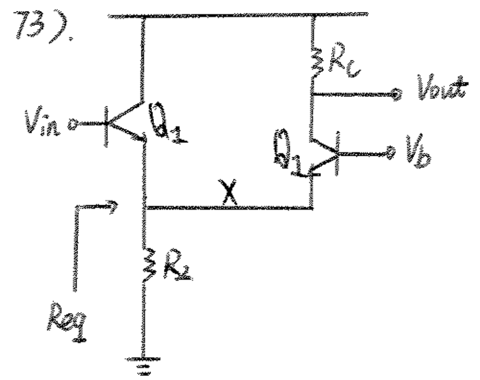
$R_{out} = R_C // r_{o2}$

b) $\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_X}{V_{in}}\right| \left|\frac{V_{out}}{V_X}\right|$

$\left|\frac{V_X}{V_{in}}\right| = \frac{(R_E // r_{\pi 2} // r_{o1})}{\frac{1}{g_{m1}} + R_E // r_{\pi 2} // r_{o1}}$, $\left|\frac{V_{out}}{V_X}\right| = g_{m2} R_C$

$\left|\frac{V_{out}}{V_{in}}\right| = (g_{m2} R_C)\left[\frac{R_E // r_{\pi 2} // r_{o1}}{\frac{1}{g_{m1}} + R_E // r_{\pi 2} // r_{o1}}\right]$

73).



$V_A = \infty$

a) $R_{eq} = R_E \,//\, r_{\pi 1} \,//\, \frac{1}{g_{m1}}$

$R_{in} = r_{\pi 1} + (1+\beta)\left[R_E \,//\, r_{\pi 1} \,//\, \frac{1}{g_{m1}}\right]$

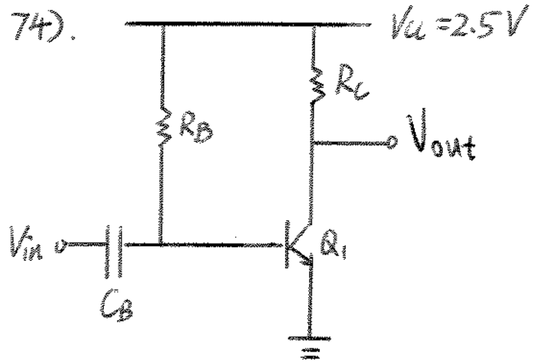
$R_{out} = R_C$

b) $\left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{V_X}{V_{in}}\right|\left|\frac{V_{out}}{V_X}\right|$

$\left|\frac{V_X}{V_{in}}\right| = \dfrac{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}{\frac{1}{g_{m1}} + R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}$, $\left|\frac{V_{out}}{V_X}\right| = g_{m1} R_C$

$$\left|\frac{V_{out}}{V_{in}}\right| = (g_{m1} R_C)\left(\frac{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1}}{R_E \,//\, \frac{1}{g_{m1}} \,//\, r_{\pi 1} + \frac{1}{g_{m1}}}\right)$$

74).



$A_V = 10$

$R_{in} > 5K\Omega$

$R_{out} = 1K$, $R_C = 1K\Omega$

$$A_V = \frac{R_C}{\frac{1}{g_m}} = 10 = \frac{I_C R_C}{V_T} \Rightarrow I_C = 0.26mA$$

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 0.697V.$$

$$I_C = 100\left(\frac{2.5 - 0.697}{R_B}\right) \Rightarrow R_B = 693K\Omega, \quad r_\pi = \frac{\beta V_T}{I_C} = 10K\Omega$$

$$R_{in} = 693K // 10K = 9.86K\Omega$$

$$\frac{1}{2\pi(200)C_B} = \frac{1}{10}\frac{1}{g_m} = 10\Omega \Rightarrow C_B = 80\mu f$$

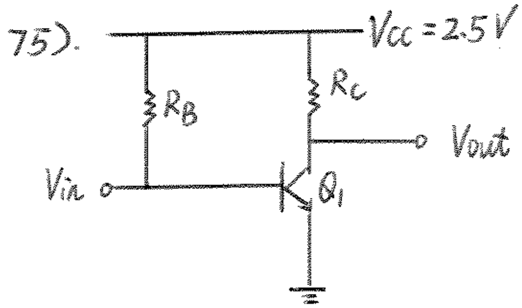
(To avoid gain degradation).

$R_C = 1K\Omega$
$R_B = 693K$
$C_B = 80\mu f$

$\Longrightarrow$

$A_V = 10$
$R_{out} = 1K\Omega$
$R_{in} = 9.86K\Omega$

75).



A_v = Maximum
$R_{out} \le 500\Omega$
$V_{BC} \le 400$ mV

$A_v = g_m R_c = \frac{I_c R_c}{V_T}$, gain is maximized by maximize $I_c R_c$

$R_{out} = R_c \le 500\Omega$, Choose $R_c = 450\Omega$, $R_{out} = 450\Omega$

$V_{BC} = V_{BE} - (2.5 - I_c R_c) \le 400$ mV

Guess $V_{BE} = 0.7$, and let $V_{BC} = 400$ mV to maximize $I_c R_c$

$0.7 - (2.5 - I_c\, 0.450) = 0.4$

$I_c = 4.89$ mA, $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.773$

Not 0.7, reiterate.

$0.773 - 2.5 + I_c\, 0.450 = 0.4$

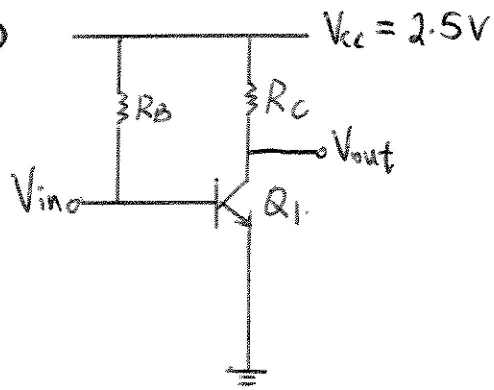
$I_c = 4.73$ mA, $V_{BE} = 0.772$ converged!!

$A_v = \left(\frac{I_c}{V_T}\right)(R_c) = \left(\frac{4.73}{26}\right)(450) = 81.9$

$R_B = 100\left(\frac{2.5 - 0.772}{4.73}\right) = 36.5K$

$R_B = 36.5K$
$R_c = 450\Omega$ $\Longrightarrow$ $A_v = 81.9$
$V_{BC} = 0.4V$
$R_{out} = 450\Omega$

76)



R_{in} : Maximum

$A_v \geq 20$

$R_{out} = R_c = 1K$

$$A_v = g_m R_c = \frac{I_c R_c}{V_T} \geq 20 \implies I_c \geq 0.52\,mA$$

$$R_{in} = R_B // r_\pi = \frac{\beta R_B V_T}{R_B I_c + V_T \beta} \quad 1), \quad I_c = \beta\left(\frac{2.5 - V_{BE}}{R_B}\right) \quad 2)$$

As we can see from 1), higher I_c means lower R_{in}.
So set I_c as low as possible, $I_c = 0.52\,mA$.

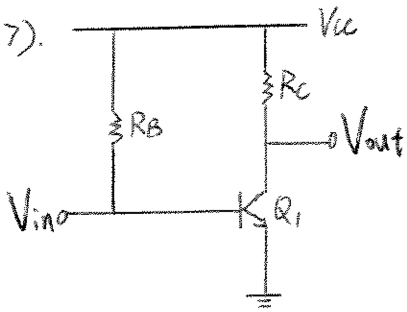
$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V$$

From 2), $R_B = \frac{100(2.5 - 0.715)}{0.52} = 343.3K\Omega$, $r_\pi = 5K\Omega$

$R_{in} = 4.93K\Omega$

$R_c = 1K\Omega$
$R_B = 343.3K\Omega$ $\implies$ $A_v = 20$
$R_{in} = 4.93K\Omega$
$R_{out} = 1K\Omega$

77).



Minimum Supply

$A_v = 15$

$R_{out} = 2K\Omega$, $R_c = 2K\Omega$

$V_{BC} \leq 0.4V$

$A_v = g_m R_c = \frac{I_c}{V_T} R_c = 15 \Rightarrow I_c = 0.195mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.689$

$V_{BC} = V_{BE} - (V_{cc} - I_c R_c) \leq 0.4V$, $I_c R_c = 0.39V$

$V_{cc} \geq 0.689 + 0.39 - 0.4 = 0.679V$

Since the problem is concerned with minimum power supply, let $V_{cc} = 0.69V$, Since $V_{BE} = 0.679V$ ($V_{cc} > V_{BE}$)

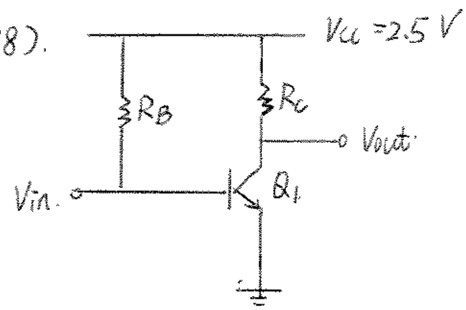
$I_c = \beta\left(\frac{V_{cc} - 0.689}{R_B}\right) \Rightarrow R_B = 100\left(\frac{0.69 - 0.689}{0.195}\right) = 512.8\Omega$

$R_c = 2K\Omega$

$R_B = 512.8\Omega \quad \Rightarrow \quad A_v = 15$

$V_{cc} = 0.69V \qquad R_{out} = 2K\Omega$

78).



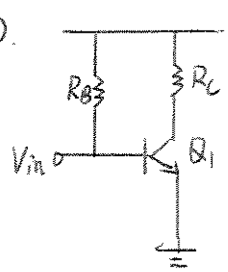
$$A_o = g_m R_c$$

$$A_o = \frac{I_c R_c}{V_T}, \quad \text{Power Dissipation} = I_c V_{cc}$$

$$R_{out} = R_c = \frac{A_o V_T}{I_c}$$

For large R_{out}, I_c has to be small, which decreases power.
So small power dissipation and small output impedance cannot be satisfied simultaneously.

79).



Power Budget = 1mW

$A_v = 20$

$A_v = g_m R_c = \frac{I_c R_c}{V_T} = 20$, $V_{cc} I_c = 1mW$

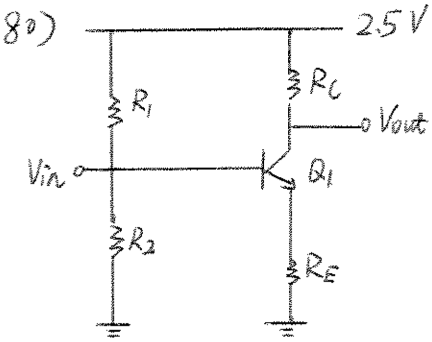
$I_c = 0.4mA$, $R_c = 1.3K$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.708V$, $I_c = \beta\left(\frac{V_{cc} - V_{BE}}{R_B}\right) = 100\left(\frac{2.5 - 0.708}{R_B}\right)$

$\Rightarrow R_B = 448K$, $R_{in} = 448 // (100)\left(\frac{26}{0.4}\right) = 6.4K$

$R_B = 448K$
$R_c = 1.3K$
$\Rightarrow$
$A_v = 20$
Power Budget = 1mW
$R_{out} = 1.3K\Omega$
$R_{in} = 6.4K$

80)



$A_V = 5$

$R_{out} = R_c = 500\Omega$

$R_E I_c \approx 300\,mV$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{300+26} \Rightarrow \begin{array}{l} R_c I_c = 1.63V \Rightarrow I_c = 3.26\,mA \\ R_E I_c \approx 300mV \Rightarrow R_E = 92\Omega \end{array}$$

$$R_1 = \frac{2.5 - (V_{BE} + 0.3)}{10\, I_B}, \quad \begin{array}{l} V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.7624 \\ 10 I_B = 0.326\,mA \end{array}$$

$$R_1 = \frac{2.5 - (0.7624 + 0.3)}{0.326} = 4.41K$$

$$R_2 = \frac{(0.7624 + 0.3)}{(9)(0.0326)} = 3.62K$$

$V_{CE} = 2.5 - 1.63 - 0.3 = 0.57$, $V_{BE} = 0.7624$.

Q_1 is in soft saturation region, so active region characteristics still apply.

$R_c = 500\Omega$

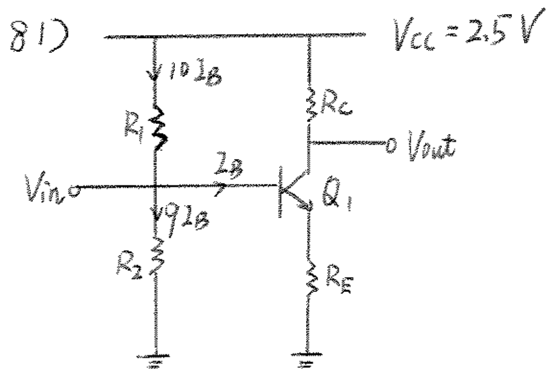
$R_1 = 4.41K\Omega$

$R_2 = 3.62K\Omega$

$R_E = 92\Omega$

$\Rightarrow A_V = 5$, $R_{out} = 500\Omega$

81)



A_V = Maximum

$R_{out} = R_c \le 1K\Omega$

$V_{BC} = 0.4V$

$R_E I_c \approx 200\,mV$

$$V_{BC} = (V_{BE} + 0.2) - (2.5 - I_c R_c) = 0.4 \quad \text{i)}$$

$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{R_c I_c}{0.226}$$

Rearrange, i) becomes $I_c R_c = 0.4 + 2.5 - (V_{BE} + 0.2)$

Guess $V_{BE} = 0.7 \Rightarrow I_c R_c = 2V$

Let $R_c = 1K \Rightarrow I_c = 2mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750$, not 0.7, reiterate

$V_{BE} = 0.75 \Rightarrow I_c R_c = 1.95\,V$

$R_c = 1K \Rightarrow I_c = 1.95mA$

Check for V_{BE}: $V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.750$, Converged!!

$I_c = 1.95mA$, $R_E I_c = 200mA \Rightarrow R_E = 103\Omega$

$I_B = 0.0195mA$

$$R_1 = \frac{2.5 - (0.750 + 0.2)}{(10)(0.0195)} = 7.95K$$

$$R_2 = \frac{(0.750 + 0.2)}{(9)(0.0195)} = 5.41K$$

81)

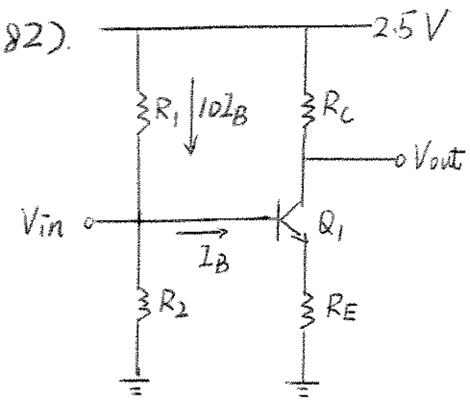
$$A_V = \frac{R_c I_c}{R_E I_c + V_T} = \frac{1.95}{0.226} = 8.63$$

This is the maximum gain we would get when R_{out} is $1K\Omega$ and V_{BC} is at 0.4 V. Since anything larger will violate either requirement.

$R_c = 1K\Omega$
$R_E = 103\Omega$
$R_1 = 7.95K\Omega$
$R_2 = 5.41K\Omega$

$\Longrightarrow$ $A_V = 8.63$
$R_{out} = 1K\Omega$

82).



Power budget = 5 mW

$A_V = 5$

$R_E I_C \approx 200\,mV$

$V_{cc}\left(I_c + \frac{I_c}{10}\right) = 5\,mW \Rightarrow I_C = 1.82\,mA, \quad I_B = 0.0182\,mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747$

$A_V = \frac{R_C I_C}{R_E I_C + V_T} = \frac{R_C I_C}{0.226} = 5 \Rightarrow R_C I_C = 1.13V \Rightarrow R_C = 621\,\Omega$

$R_1 = \frac{2.5 - (0.747 + 0.2)}{(10)(0.0182)} = 8.53\,K\Omega$

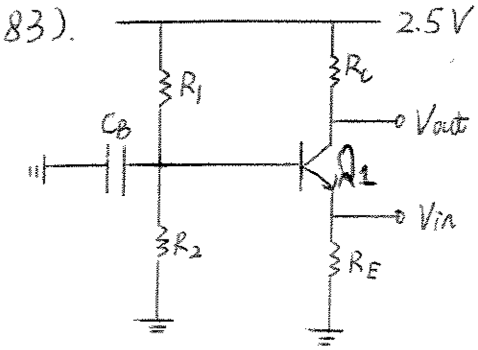
$R_2 = \frac{(0.747 + 0.2)}{(9)(0.0182)} = 5.78\,K\Omega$

$R_E I_C \approx 200\,mV \Rightarrow R_E = 110\,\Omega$

$R_C = 621\,\Omega$
$R_E = 110\,\Omega$
$R_1 = 8.53\,K\Omega$
$R_2 = 5.78\,K\Omega$

$\Longrightarrow$ $A_V = 5$, Power Budget = 5 mW

83).



$A_V = 20$

$R_{in} = 50\,\Omega$

$R_E I_C \approx 10 V_T = 260\text{ mV}$

$R_{in} = \frac{1}{g_m} = 50\Omega$, since R_E doesn't affect input impedance.

$\frac{V_T}{I_C} = 50\Omega \Rightarrow I_C = \frac{V_T}{50\Omega} = 0.52\text{ mA}, \quad I_B = 0.0052\text{ mA}$

$A_V = \frac{R_C}{1/g_m} = \frac{I_C R_C}{V_T} = 20 \Rightarrow R_C = 1K\Omega$

$R_1 = \frac{2.5 - (0.715 + 0.260)}{(10)(0.0052)} = 29.3K$

$R_2 = \frac{(0.715 + 0.260)}{(9)(0.0052)} = 20.83K$

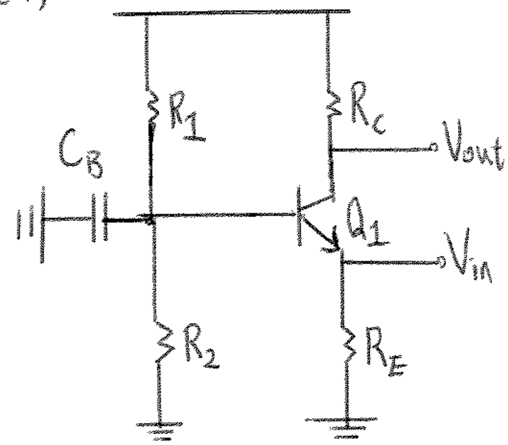
$R_E I_C \approx 260\text{ mV} \Rightarrow R_E \approx 500\Omega$

$\frac{1}{C_B(2\pi)(200)} = \frac{1}{10}\frac{1}{g_m} = 5\Omega \Rightarrow C_B = 159.1\,\mu f$

$R_C = 1K\Omega,\ R_E = 500\Omega,\ R_1 = 29.3K\Omega,\ R_2 = 20.83K,\ C_B = 159.1\,\mu f$

$\Rightarrow A_V = 20,\ R_{in} = 50\Omega$

84)



$A_v = 8$

$R_{out} = 500\Omega$

$R_{out} = R_c = 500\Omega$

$A_v = \frac{I_c}{V_T} R_c = 8 \Rightarrow I_c = 0.416\,mA, \quad I_B = 0.00416\,mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.709$

$R_E \approx \frac{260\,mV}{I_c} = 625\Omega, \quad R_1 = \frac{2.5 - (0.709 + 0.260)}{(10)(0.00416)} = 36.8K\Omega$

$R_2 = \frac{(0.709 + 0.260)}{(9)(0.00416)} = 25.9K\Omega$

$\frac{1}{g_m} = \frac{V_T}{I_c} = 62.5\Omega, \quad \frac{1}{C_B\, 200\,(2\pi)} = \frac{62.5}{10} \Rightarrow C_B = 127.3\,\mu f$

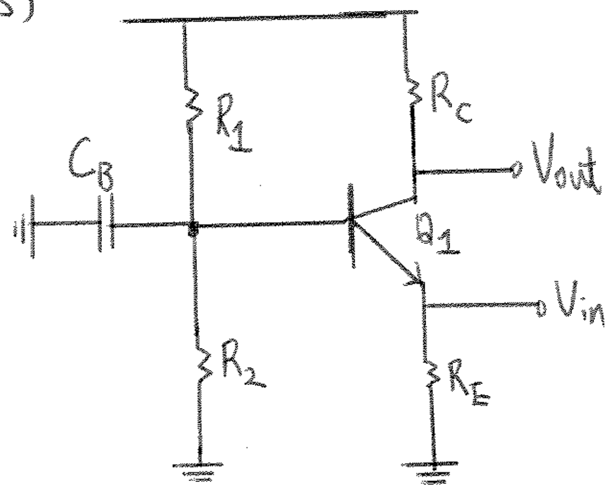
$C_B = 127.3\,\mu f$
$R_1 = 36.8K$
$R_2 = 25.9K$
$R_c = 500\Omega$
$R_E = 625\Omega$

$\Longrightarrow$

$A_v = 8$

$R_{out} = 500\Omega$

85)



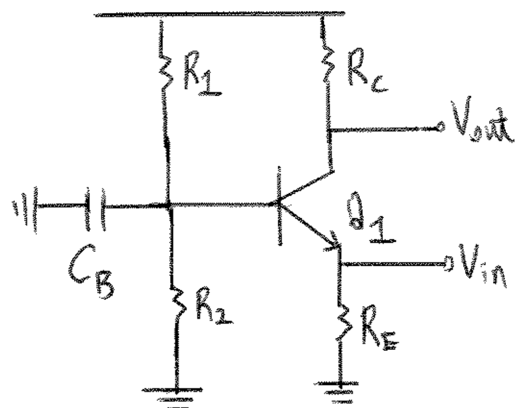
$A_V = 20$
$R_c = 200\Omega$
$(R_c = R_{out})$

$$A_V = \frac{I_c R_c}{V_T} = 20 \Rightarrow I_c = 2.6\,mA$$
$$I_B = 0.026\,mA, \quad 10I_B = 0.26\,mA$$

$$Power = V_{cc}(I_c + 10I_B) = 2.5(0.26\,mA + 2.6\,mA) = 7.15\,mW$$

This is the minimum power dissipation, since anything lower will lower the voltage gain.

86)



Power = 5mW

$A_V = 10$

$V_{cc} I_c + V_{cc} \frac{I_c}{10} = 5mW$, $V_{cc} I_c 1.1 = 5mW$, $I_c = 1.82mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.747$

$A_V = g_m R_c = \frac{I_c R_c}{V_T} = 10 \Rightarrow R_c = 0.143K\Omega$

$I_c R_E \approx 260mV$, $R_E \approx 142.9\Omega$

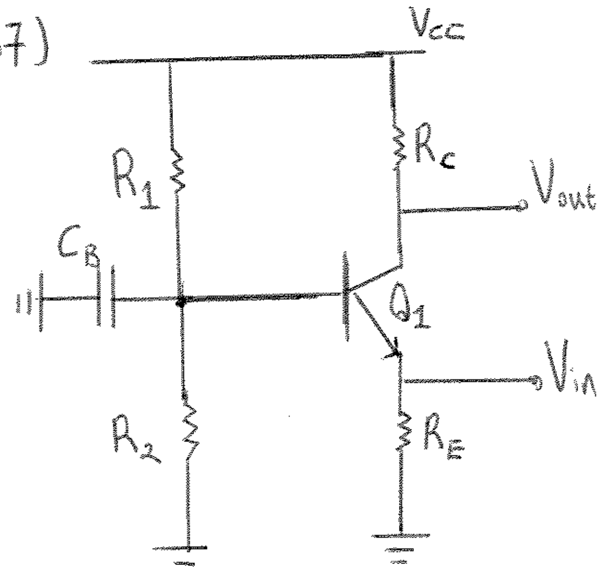
$R_1 = \frac{2.5 - (0.747 + 0.260)}{(10)(0.0182)} = 8.2K$, $R_2 = \frac{(0.747 + 0.260)}{(9)(0.0182)} = 6.15K$

$\frac{1}{g_m} = \frac{V_T}{I_c} = 14.3\Omega$, $\frac{1}{C_B 2\pi (200)} = \frac{14.3}{10} \Rightarrow C_B = 556.5\mu f$

$R_c = 143\Omega$, $R_E = 143\Omega$, $R_1 = 8.2K$, $R_2 = 6.15K$, $C_B = 556.5\mu f$.

$\Rightarrow$ $A_V = 10$, Power = 5mW

87)



$R_{in} = 50\Omega$
$A_v = 20$

Assume R_E doesn't affect R_{in} significantly,
$R_{in} \approx \frac{1}{g_m} = 50\Omega$

$A_v = \frac{R_c}{1/g_m} = 20 \Rightarrow R_c = 1K\Omega$, $\frac{1}{g_m} = \frac{V_T}{I_c} \Rightarrow I_c = \frac{26mV}{50\Omega} = 0.52\,mA$

$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.715V$, $R_E I_c = 260\,mV \Rightarrow R_E = 500\Omega$

$V_{cc} = I_c R_c + V_{CE} + I_c R_E = 0.52 + V_{CE} + 0.260$

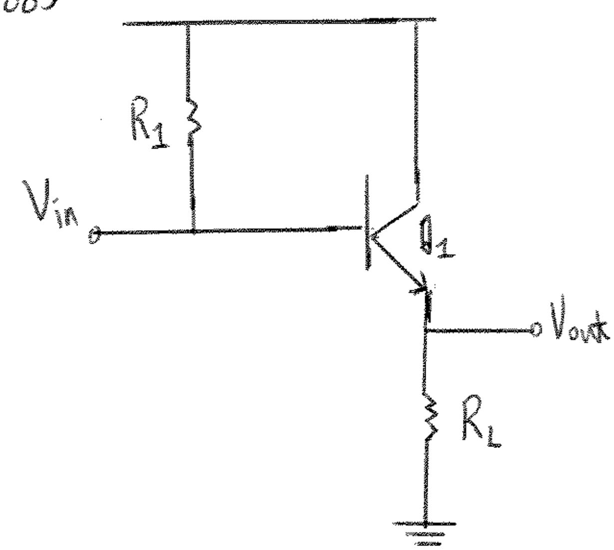
V_{BC} is forward biased to $0.4V$, $V_{CE} = V_{BE} - 0.4 = 0.315\,V$
$V_{cc} = 0.52 + 0.315 + 0.260 = 1.1V$. (Minimum supply Voltage)

$R_1 = \frac{1.1 - (0.715 + 0.260)}{0.052} = 2.4K$, $R_2 = \frac{(0.715 + 0.260)}{(9)(0.0052)} = 20.83K$

$\frac{1}{C_B 2\pi 200} = \frac{1}{10}\frac{1}{g_m} = 5 \Rightarrow C_B = 159.2\,\mu f$

$V_{cc} = 1.1V$, $R_1 = 2.4K$, $R_2 = 20.83K$, $R_c = 1K$, $R_E = 500\Omega$, $C_B = 159.2\mu f$
$\Rightarrow R_{in} = 50\Omega$, $A_v = 20$

88)



$A_v = 0.85$

$R_{in} > 10\text{k}\Omega$

$R_L = 200\Omega$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.85 \Rightarrow \frac{200}{200 + \frac{1}{g_m}} = 0.85$$

$$\Rightarrow 200 = 0.85\left(200 + \frac{1}{g_m}\right) \Rightarrow \frac{1}{g_m} = 35.294\Omega$$

$$\Rightarrow I_c = \frac{26\text{mV}}{35.294\Omega} = 0.737\text{mA}, \quad V_{BE} = V_T \ln\left(\frac{0.737}{6\times10^{-13}}\right) = 0.724\text{V}$$

$$R_{in} = R_1 \,//\, (r_\pi + (1+\beta)(200\Omega))$$

$$R_{in} = R_1 \,//\, 23.73\text{K}$$

$$R_{in} = \frac{R_1\, 23.73\text{K}}{R_1 + 23.73\text{K}} > 10\text{K} \Rightarrow R_1 > 17.28\text{K} \text{ (Input Impedance requirement)}$$

To support an I_c of 0.737, R_1 must be determined.

88)

$$R_1 = \frac{2.5 - (0.724 + (0.737)(0.2)/0.99)}{0.737/100}$$

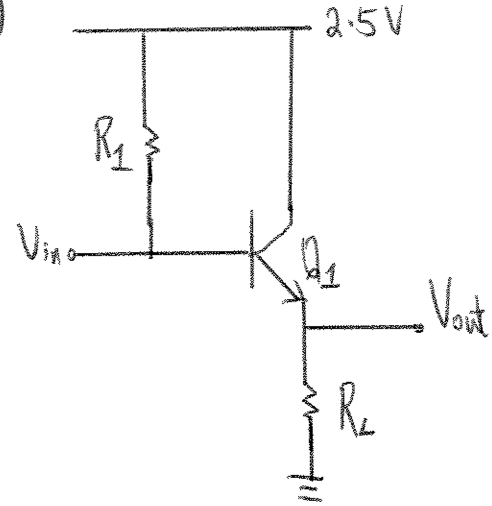
$$R_1 = 220.77K\Omega$$

$R_1 = 220.77K\Omega \Rightarrow R_{in} = 220.77K\Omega // 23.73K\Omega$

$R_{in} = 21.43K\Omega > 10K\Omega$

$R_1 = 220.77K\Omega$
$R_L = 200\Omega$ $\Rightarrow$ $A_v = 0.85$
$R_{in} = 21.43K\Omega$

89)



Power = 5 mW

$A_v = 0.9$

$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} = 0.9 \Rightarrow R_L = 0.9\left(R_L + \frac{1}{g_m}\right)$$

$$R_L = 9\frac{1}{g_m}$$

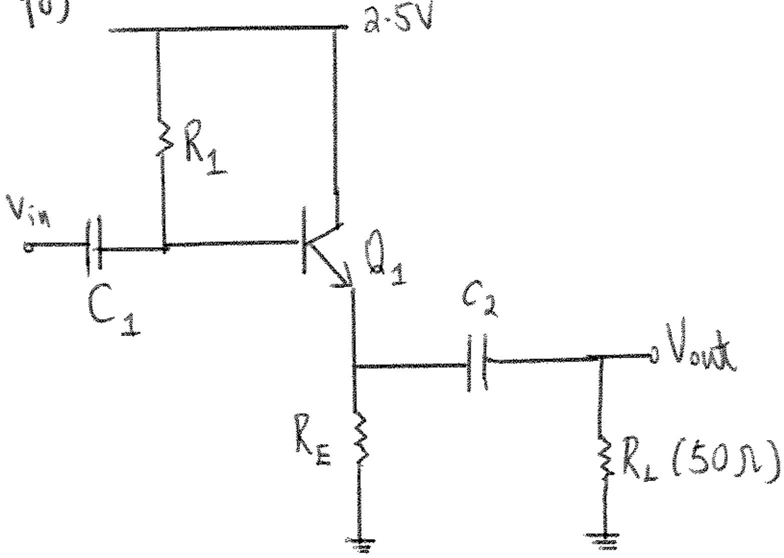
$$\text{Power} = 2.5\left(I_c + \frac{I_c}{\beta}\right) \Rightarrow I_c = 1.98\,mA$$

$$\frac{1}{g_m} = \frac{V_T}{I_c} = \frac{26mV}{1.98mA} = 13.13\,\Omega$$

$$R_L = (9)(13.13) = 118.17\,\Omega.$$

This is the minimum load resistance, since anything lower will lower the voltage gain.

40)



$A_v = 0.8$

Since R_E doesn't affect Voltage gain significantly.

$$A_v \approx \frac{R_L}{R_L + \frac{1}{g_m}} = 0.8$$

$$R_L = 0.8\left(R_L + \frac{1}{g_m}\right)$$

$$0.2 R_L = 0.8 \frac{1}{g_m}$$

$$R_L = 4\frac{1}{g_m} \Rightarrow \frac{R_L}{4} = \frac{1}{g_m} = 12.5 = \frac{V_T}{I_c}$$

$$I_c = \frac{26mV}{12.5\Omega} = 2.08mA$$

$$V_{BE} = V_T \ln\left(\frac{I_c}{I_s}\right) = 0.751V$$

Let $R_E I_c = 20V_T$ $(0.52V)$, $R_E = \frac{20}{g_m} = 250\Omega$

$$R_1 = \frac{2.5 - (0.751 + 0.52)}{0.0208mA} = 59.1K$$

9c)

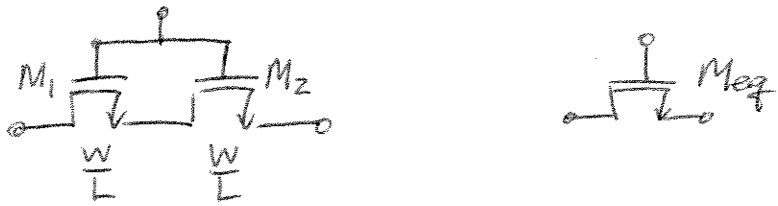
$$\frac{1}{(2\pi)(100\times10^6)C_1} = \frac{1}{10}\frac{1}{g_m} \Rightarrow C_1 = 1.27nf$$

$$\frac{1}{(2\pi)(100\times10^6)C_2} = \frac{1}{10}50 \Rightarrow C_2 = 0.32nf$$

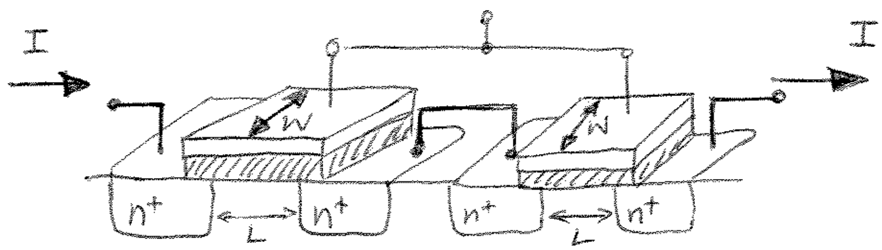
(So C_2 will not load Q_1).

$C_1 = 1.27$ nf
$C_2 = 0.32$ nf
$R_1 = 59.1$ KΩ $\Rightarrow$ $A_v = 0.8$
$R_E = 250$ Ω
$R_L = 50$ Ω

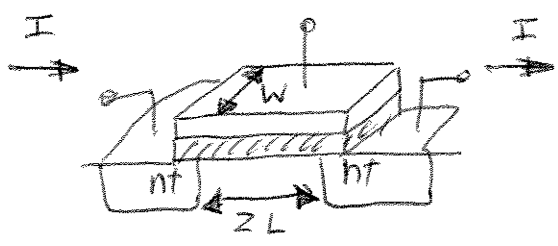
1.



Intuitively, this is similar to having twice of the original channel length:



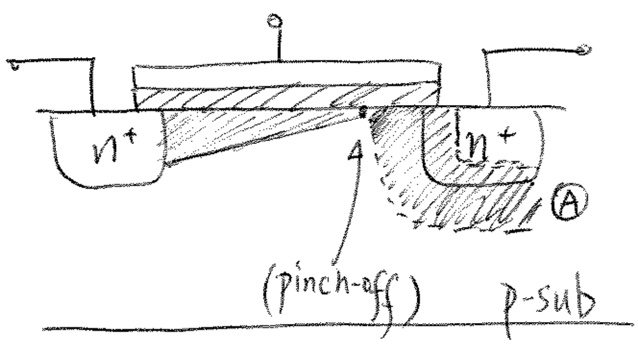
Since current flowing into either non-gate terminals must come out at the other terminal (KCL) and the intermediate node is equipotential, this is as if we have a Meq with width W & length 2L:



This approximation can simplify a lot of calculations.

2. A key point to remember: the charge density APPROACHES zero (not EQUALS) at pinch-off. In other words, Q is never exactly equal to zero (albeit very close.) Another way to view this phenomenon is by observing $I = Q \cdot v$: recognize that v is finite. Since we get some finite value of I at pinch-off, we expect $Q \neq 0$.

Consider the following:



The shaded region, Ⓐ, represents a reverse-biased pn junction. Just as a diode, there exist minority profiles on p & n sides, which $\neq 0$.

Pinch-off implies that the depletion region created no longer has <u>free</u> carriers. The depletion still sweeps all electrons from inversion channel to drain.

3. Given: $C_{ox} = 10\ fF/\mu m^2$ $W = 5\mu m$ $L = 0.1\mu m$

$V_{GS} - V_{TH} = 1V$ $V_{DS} = 0$

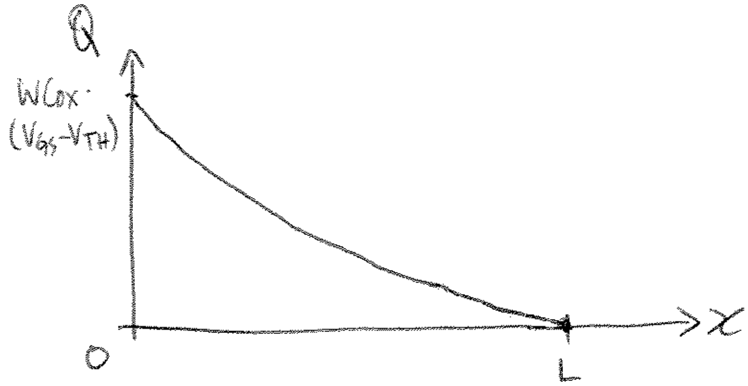
Find: total charge stored in channel, Q_{tot}

$$Q_{tot} = W C_{ox} (V_{GS} - V_{TH}) L$$

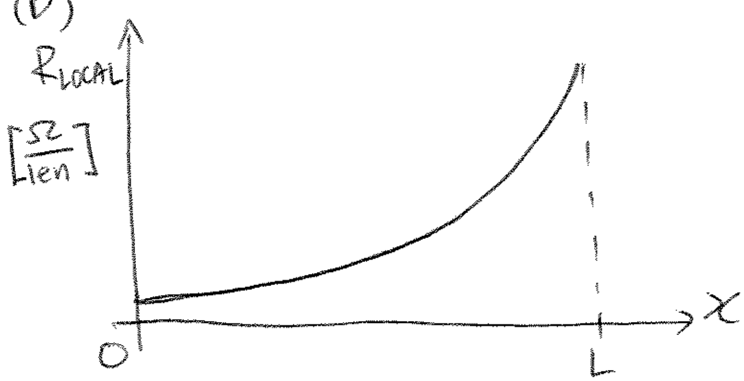
$$= (5\mu m)(10\ fF/\mu m^2)(1V)(0.1\mu m) = 5\ fC$$

4. (a) $Q = WC_{ox}(V_{GS} - V_{TH} - V(x))$

$= -WC_{ox} \cdot V(x) + WC_{ox}(V_{GS} - V_{TH})$



(b)



$R \propto \frac{1}{\mu Q}$

↑ mobility of charge.

5. $I_D = W C_{ox} [V_{GS} - V(x) - V_{TH}] \mu_n \dfrac{dV(x)}{dx}$

Define : $A = \dfrac{I_D}{W C_{ox} \mu_n}$, $B = V_{GS} - V_{TH}$

$$\Rightarrow \quad A = (B - V)\frac{dV}{dx} = \frac{d}{dx}\left(BV - \frac{V^2}{2}\right)$$

Integrating $A = \dfrac{d}{dx}(BV - V^2/2)$ gives:

$$Ax = BV - V^2/2 \Rightarrow V^2 - 2BV + 2Ax = 0$$

Using quadratic formula:

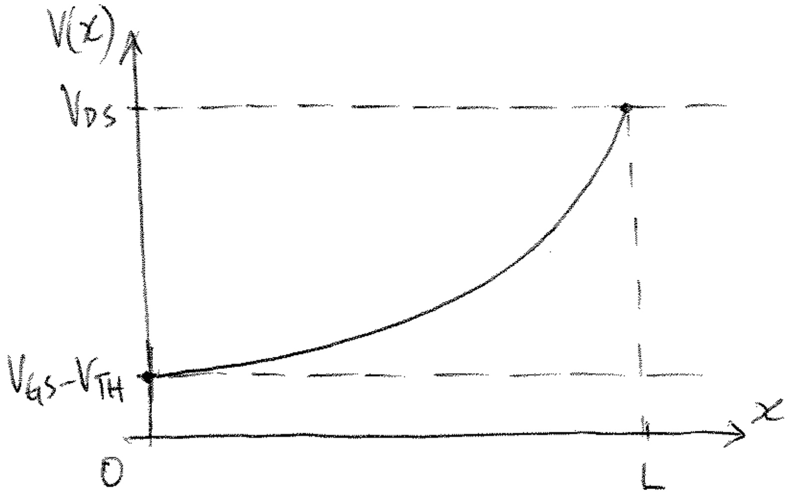
$$V_{+,-} = \frac{2B \pm \sqrt{4B^2 - 4 \cdot 2A}}{2} = B \pm \sqrt{B^2 - 2Ax}$$

$$= B\left(1 \pm \sqrt{1 - 2\left(\frac{A}{B^2}\right)x}\right)$$

$$= (V_{GS} - V_{TH})\left\{1 \pm \sqrt{1 - \left[2 \cdot \frac{I_D}{W C_{ox} \mu_n (V_{GS} - V_{TH})^2}\right] x}\right\}$$

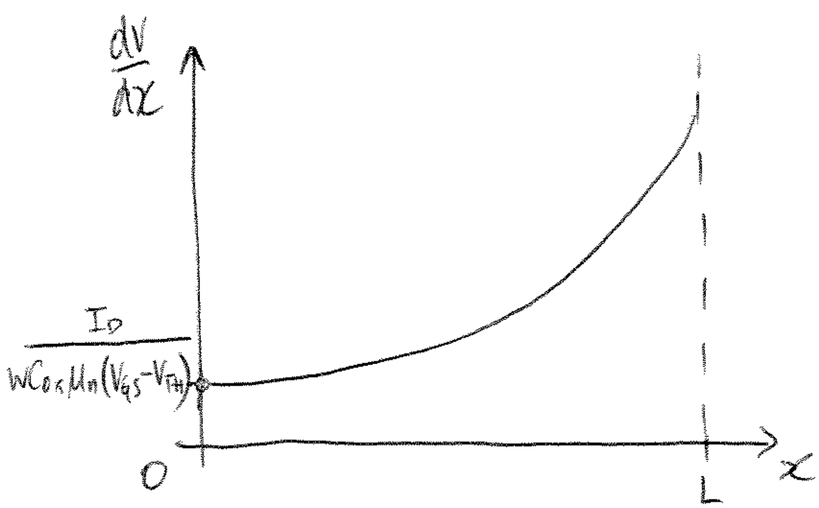
We know that $0 \le V(x) \le V_{GS} - V_{TH}$ (pinch-off), and the term inside the square root is > 0. Therefore, we take V_- as the solution.

i.e. $$V(x) = (V_{GS}-V_{TH})\left\{1-\sqrt{1-\left[\frac{2I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]x}\right\}$$



$\because I_D \propto W$
$\Rightarrow V(x)$ is independent of W.

$$\frac{dV}{dx} = \frac{I_D}{WC_{ox}\mu_n(V_{GS}-V_{TH})}\cdot\left[1-\frac{2I_D\cdot x}{WC_{ox}\mu_n(V_{GS}-V_{TH})^2}\right]^{-\frac{1}{2}}$$



6. No.

By varying $V_{GS} - V_{TH}$ & V_{DS}, we can only obtain $\mu_n C_{ox} \frac{W}{L}$, but not $\mu_n C_{ox}$ & $\frac{W}{L}$ individually.

7. Given: NMOS $\quad I_D = 1mA \quad V_{GS} - V_{TH} = 0.6V$

$I_D = 1.6mA \quad V_{GS} - V_{TH} = 0.8V$

(triode region) $\quad \mu_n C_{ox} = 200 \frac{\mu A}{V^2}$

Find V_{DS} & W/L.

$$1mA = \mu_n C_{ox} \frac{W}{L}\left[(0.6)V_{DS} - V_{DS}^2/2\right] \quad \text{——} \quad ①$$

$$1.6mA = \mu_n C_{ox} \frac{W}{L}\left[(0.8)V_{DS} - V_{DS}^2/2\right] \quad \text{——} \quad ②$$

$$② \div ① : \quad 1.6 = \frac{0.8 V_{DS} - V_{DS}^2/2}{0.6 V_{DS} - V_{DS}^2/2} = \frac{1.6 - V_{DS}}{1.2 - V_{DS}}$$

$$\Rightarrow V_{DS} = \frac{1.6(0.2)}{0.6} \approx 0.533 \ V$$

$$\Rightarrow \quad \frac{W}{L} = \frac{I_D}{\mu_n C_{ox}\left[(V_{GS} - V_{TH})V_{DS} - V_{DS}^2/2\right]}$$

$$= \frac{1mA}{200\frac{\mu A}{V^2}\left[(0.6V)(0.533V) - (0.533V)^2/2\right]}$$

$$\approx 28.$$

8. $$I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right]$$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} \mu C_{ox} \frac{W}{L} \cdot 2 V_{DS} = \mu C_{ox} \frac{W}{L} V_{DS}$$

$$g_m \big|_{V_{DS}=0} = 0 .$$

Intuitively, when $V_{GS} > V_{TH}$, mobile charges (channel) become available. This determines the on-resistance. But since there is no I_D ($\because V_{DS} = 0$), it does not matter if there is an incremental change in V_{GS} (i.e. ∂V_{gs}). Since varying V_{GS} gives no change in I_D, $g_m|_{V_{DS}=0} = 0$.

9. Given: $V_{DD} = 1.8\ V$ $\quad \frac{W}{L} = 20 \quad \mu_n C_{ox} = 200\ \frac{\mu A}{V^2}$

$V_{TH} = 0.4\ V$

Find minimum-on resistance.

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})}$$

$$= \frac{1}{\left(200\ \frac{\mu A}{V^2}\right)(20)\ (1.8 - 0.4)V} = 179.\ \Omega$$

10. $$500 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1 - V_{TH})}$$

$$400 = \frac{1}{\mu_n C_{ox} \frac{W}{L} (1.5 - V_{TH})}$$

For the same NMOS, $\mu_n C_{ox}$ & $\frac{W}{L}$ are fixed

$$\Rightarrow \quad 500(1 - V_{TH}) \overset{?}{=} 400(1.5 - V_{TH})$$

$$500(0.6) \neq 400(1.1)$$

$\therefore$ This is not possible.

11. $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]$

$$r_{DS,tri} \triangleq \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \left[\frac{\partial}{\partial V_{DS}}\left\{\frac{1}{2}\mu C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right]\right\}\right]^{-1}$$

$$= \left[\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) - \mu C_{ox}\frac{W}{L}V_{DS}\right]^{-1}$$

$$= \frac{1}{\mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH}-V_{DS})}$$

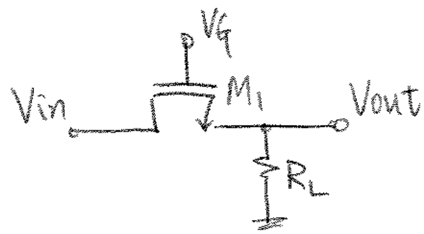
12. When MOS operates as a resistor,

$$R_{on} = \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

$$\Rightarrow \tau = R_{on} C_{GS} = \frac{W L C_{ox}}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} = \frac{L^2}{\mu (V_{GS} - V_{TH})}$$

To minimize the time constant,
1) use minimum channel length, and
2) maximize overdrive voltage.

13.



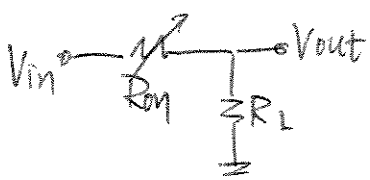
Given $V_{in} \approx 0$

$V_G = 1.8\ V$

$R_L = 100\Omega$

Find $\frac{W}{L}$ such that signal output attenuates by only 5%.

$V_{in} \approx 0$ implies that we can approximate M_1 as a linear resistance controlled by V_G. Therefore, the equivalent circuit becomes a resistive divider:



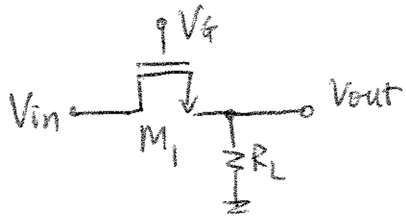
$$V_{out} = 0.95\, V_{in} = \frac{R_L}{R_{on} + R_L} V_{in}$$

$$\Rightarrow R_{on} \cong 5.3\,\Omega$$

$$\therefore \frac{W}{L} = \frac{1}{\mu C_{ox}(V_{GS} - V_{TH}) R_{on}} \approx \frac{1}{200\frac{\mu A}{V^2}(1.8 - 0.4)(5.3\Omega)}$$

$$= 674.$$

14.



$V_0 \sim$ few mV.

(a) $V_{in} = V_0 \cos\omega t \qquad V_{out} = 0.95(V_0 \cos\omega t)$

$$V_{out} = \frac{R_L}{R_{ON}+R_L} V_{in} \quad \Rightarrow \frac{R_L}{R_{ON}+R_L} = 0.95 V_0$$

$$R_{ON} = \frac{R_L}{\left(\frac{0.95V_0}{1-0.95V_0}\right)} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

$$\therefore \frac{W}{L} = \frac{0.95V_0/(1-0.95V_0)}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

(b) $V_{out} = 0.95\, V_{in} = 0.95(V_0 \cos\omega t + 0.5)$

$\approx 0.95 \times 0.5 = 0.475$

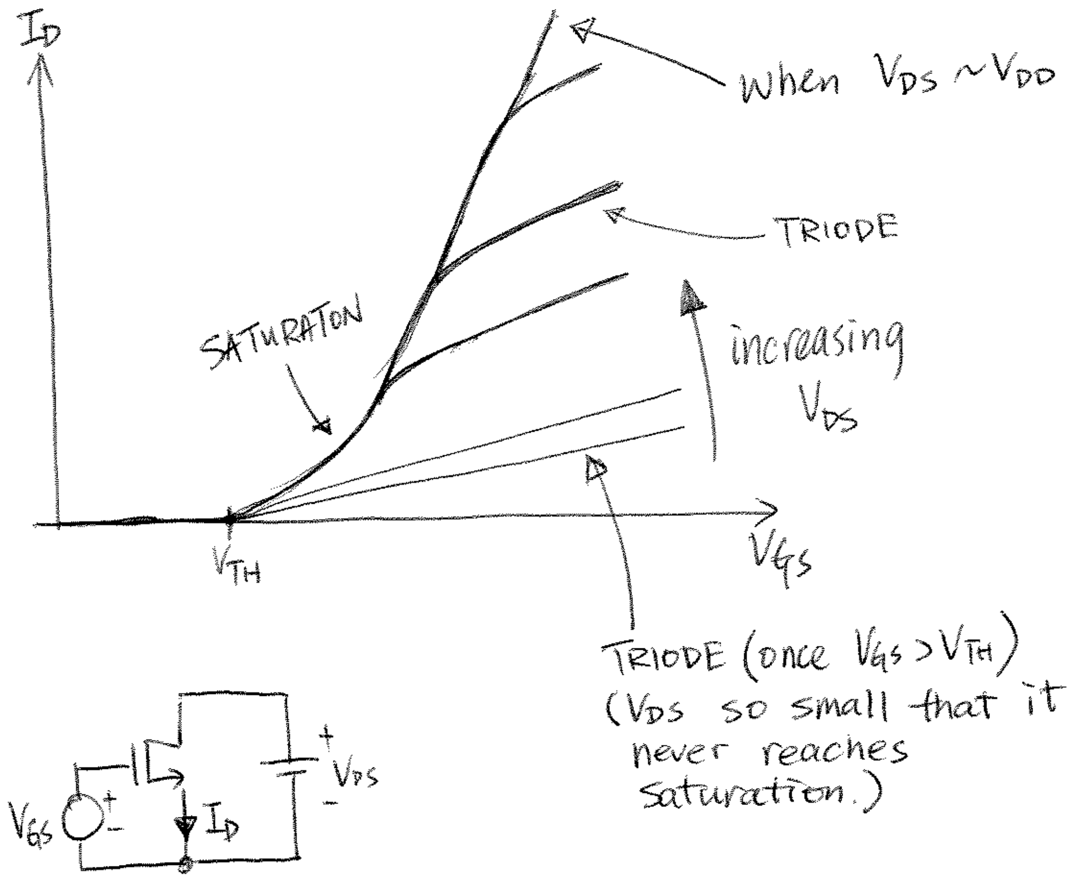
($\because V_0$ is relatively small)

$$\therefore R_{ON} = \frac{R_L}{0.9} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_G - V_{TH})}$$

$$\Rightarrow \frac{W}{L} = \frac{0.9}{\mu_n C_{ox} R_L (V_G - V_{TH})}$$

Results show that if there is no DC voltage as input, the R_{ON} varies with changing sinewave. With a DC bias voltage, R_{ON} becomes more stable (independent of V_0).

15.



I_D
when $V_{DS} \sim V_{DD}$
TRIODE
SATURATON
increasing V_{DS}
V_{TH}
V_{GS}
TRIODE (once $V_{GS} > V_{TH}$)
(V_{DS} so small that it never reaches saturation.)
V_{GS}
I_D
V_{DS}

16. The peak of the parabola signifies pinch-off (i.e. $V_{DS} = V_{GS} - V_{TH}$). This means that (with $\lambda = 0$) I_D cannot be increased further by increasing V_{DS}. Since this curve must be continuous, the peak I_D must originate from the peak of the parabola.

17. $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^{\alpha}$, $\alpha < 3$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot \alpha (V_{GS}-V_{TH})^{\alpha-1}$$

$$= \frac{\alpha}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^{\alpha-1}$$

$$= \frac{\alpha I_D}{(V_{GS}-V_{TH})}$$

18. $I_D = W C_{ox} (V_{GS} - V_{TH}) v_{SAT}$

$$g_m \triangleq \frac{\partial I_D}{\partial V_{GS}} = W C_{ox} v_{SAT}$$

19. (a) OFF $\because$ $V_{GS} = 0$

(b) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) TRIODE $\because$ $V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$
(REMEMBER: MOSFET is symmetric)

(e) TRIODE $\because$ $V_{GS} > V_{TH}$ & $V_{DS} < V_{GS} - V_{TH}$

(f) OFF $\because$ $V_{GS} = 0$

(g) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(h) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

(i) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

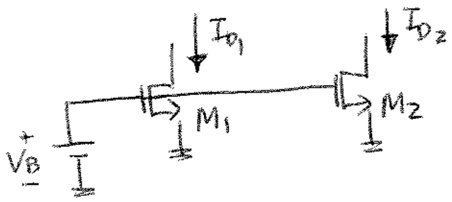
20. (a) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

(b) OFF $\because$ $V_{GS} = 0$ $(V_{GS} < V_{TH})$

(c) TRIODE (LINEAR) $\because$ $V_{GS} > V_{TH}$ &
$V_{DS} \ll 2(V_{GS} - V_{TH})$

(d) SATURATION $\because$ $V_{GS} > V_{TH}$ & $V_{DS} > V_{GS} - V_{TH}$

21.



$0.99 I_{D2} < I_{D1} < 1.01 I_{D2}$

Since M_1 & M_2 are treated as current sources, they are assumed to be in saturation.

Evaluate λ at boundaries:

$$I_{D1} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_B - V_{TH})^2 (1 + \lambda V_{DS1}) \quad \text{——— ①}$$

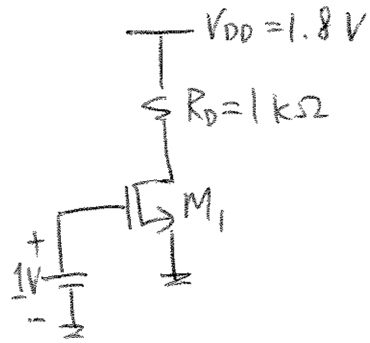
$$I_{D2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_B - V_{TH})^2 (1 + \lambda V_{DS2}) \quad \text{——— ②}$$

① ÷ ② : $\frac{I_{D1}}{I_{D2}} = \frac{0.99\, I_{D2}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}}$

$$\therefore \lambda = \frac{0.01}{0.99\, V_{DS2} - V_{DS1}} = \frac{0.01}{0.99(1V) - (0.5V)} = 0.02\ V^{-1}$$

Maximum tolerable $\lambda = 0.02 V^{-1}$

22.



$\lambda = 0$, $V_{TH} = 0.4\,V$

$\mu_n C_{ox} = 200\,\frac{\mu A}{V^2}$

M_1 sits at the edge of saturation when $V_{DS} = V_{GS} - V_{TH}$.

$\Rightarrow V_{DS,edge} = (1 - 0.4)\,V = 0.6\,V$

By KCL, $I_{D_1} = I_{R_D} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{1.2\,V}{1\,k\Omega} = 1.2\,mA$

$\therefore I_{D_1} = 1.2\,mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$

$$\Rightarrow \frac{W}{L} = \frac{2 I_{D_1}}{\mu_n C_{ox}(V_{GS} - V_{TH})^2} = \frac{2(1.2mA)}{\left(200\,\frac{\mu A}{V^2}\right)(1 - 0.4)^2 V^2}$$

$\cong 33.$

23. If gate oxide thickness, t_{ox}, doubles, the corresponding capacitance, $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$, is halved.

$\Rightarrow$ $\mu_n C_{ox}$ is also halved

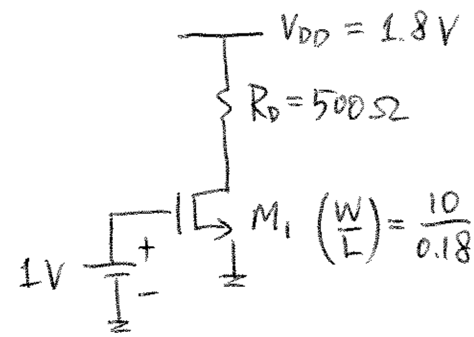
$\Rightarrow$ I_{D_1} is halved $\Rightarrow$ V_{DS} increases

$\Rightarrow$ M_1 stays in saturation ($V_{DS} > V_{GS} - V_{TH}$)

$$I_{D_1} = \frac{1.2mA}{2} = 0.6\,mA$$

$$\Rightarrow V_{DS} = (1.8\,V) - (0.6mA)(1\,k\Omega) = 1.2\,V$$

24.



$\lambda = 0$

To avoid triode region, $V_{DS} \geq V_{GS} - V_{TH}$.

$\Rightarrow V_{DS} \geq 1 - 0.4 = 0.6\,V$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

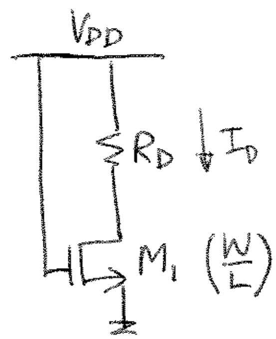
$$= \frac{1}{2}\left(200\,\frac{\mu A}{V^2}\right)\left(\frac{10}{0.18}\right)(0.6)^2 = 2\,mA$$

By KCL, $\frac{V_{DD} - V_{DS}}{R_D} = 2mA$

$\therefore V_{DD} = (2mA)(500\Omega) + 0.6V = 1.6\,V$

Minimum $V_{DD} = 1.6\,V$

25.



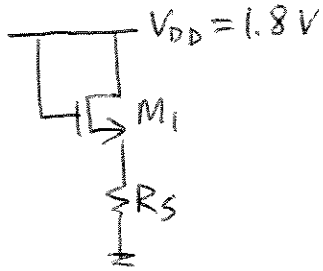
$\lambda = 0$

When M_1 operates at the edge of saturation, $V_{DS} = V_{GS} - V_{TH}$. Also, by KCL:

$$I_{R_D} = I_{D_1} \Rightarrow \frac{V_{DD} - (V_{DD} - V_{TH})}{R_D} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})^2$$

$$\therefore V_{TH} = R_D \cdot \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})^2}_{I_D}$$

26.



$\lambda = 0$

Find $\left(\frac{W}{L}\right)$ with bias current $= I_1$.

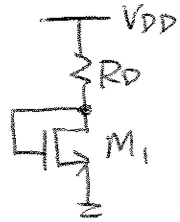
Since $V_{DS} = V_{GS}$ for M_1, this device always operates in saturation region (given $V_{GS} > V_{TH}$).

By KCL, $I_1 = I_{R_S}$; by Ohm's law, $V_S = I_1 R_S$

$$\Rightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{DD} - I_1 R_S - V_{TH})^2 = I_1$$

$$\therefore \frac{W}{L} = \frac{2 I_1}{\mu_n C_{ox} (V_{DD} - I_1 R_S - V_{TH})^2}$$

27.



Calculate I_1 if $\lambda = 0$.
Assume $V_{GS} > V_{TH}$

By KCL, $I_{R_D} = I_1$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}}_{\triangleq B}(V_{GS} - V_{TH})^2 = \frac{V_{DD} - V_{GS}}{R_D}$$

Re-arrange this to quadratic form:

$$V_{GS}^2(BR_D) - V_{GS}(2BR_DV_{TH} - 1) + (V_{TH}^2 \cdot BR_D - V_{DD}) = 0$$

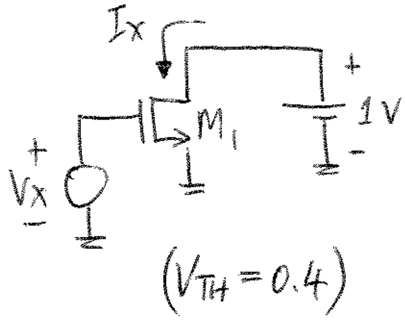
$$\Rightarrow V_{GS_{1,2}} = \frac{(BR_DV_{TH} - 1) \pm \sqrt{BR_D(V_{DD} - V_{TH}) + 1}}{BR_D}$$

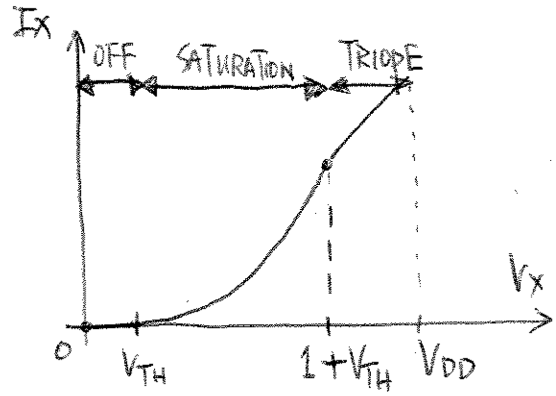
$$= \frac{\left(\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right) \cdot R_D \cdot V_{TH} - 1\right) \pm \sqrt{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)R_D(V_{DD} - V_{TH}) + 1}}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)}$$

Whether the answer is V_{GS_1} or V_{GS_2} depends on other parameters. Also note that since M_1 is diode-connected, it never goes into triode (i.e either OFF or SATURATION.) This helps in eliminating one of the solutions.

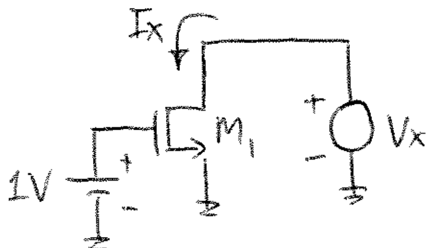
After solving V_{GS}, $I_D = I_1 = \frac{V_{DD} - V_{GS}}{R_D}$

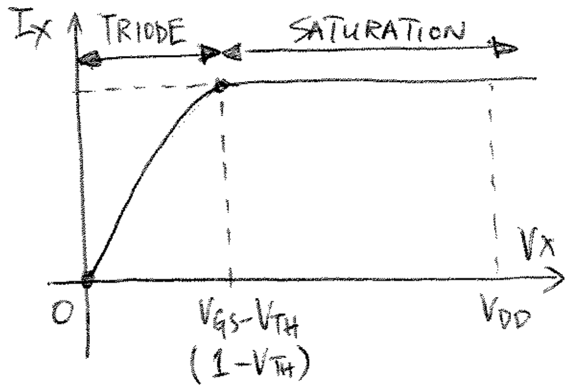
28. (a)


I_X
M_1
1V
V_X
$(V_{TH} = 0.4)$

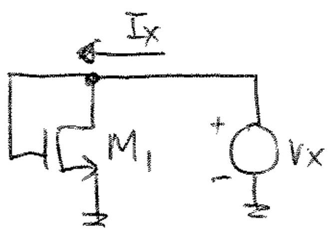

I_X
OFF
SATURATION
TRIODE
V_X
0
V_{TH}
$1+V_{TH}$
V_{DD}

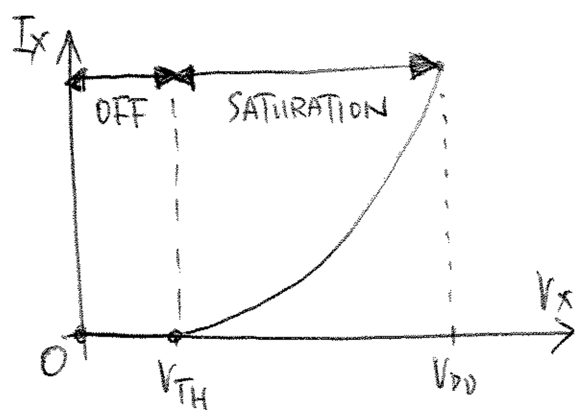
(b)


I_X
M_1
1V
V_X

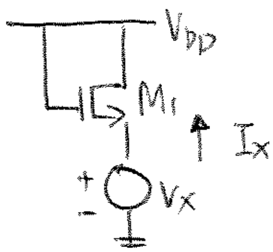

I_X
TRIODE
SATURATION
V_X
0
$V_{GS}-V_{TH}$
$(1-V_{TH})$
V_{DD}

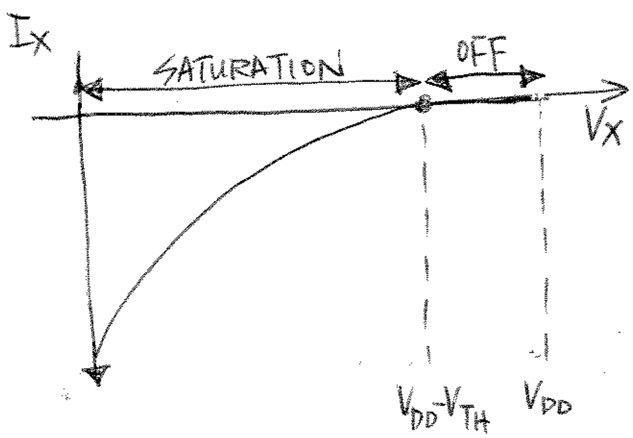
(c)


I_X
M_1
V_X

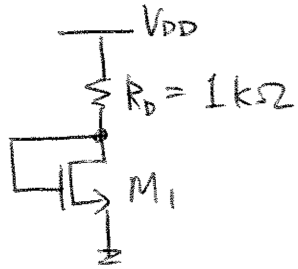

I_X
OFF
SATURATION
V_X
0
V_{TH}
V_{DD}

(d)


V_{DD}
M_1
I_X
V_X


I_X
SATURATION
OFF
V_X
$V_{DD}-V_{TH}$
V_{DD}

29.



$\left(\frac{W}{L}\right) = \frac{10}{0.18}$, $\lambda = 0.1\ V^{-1}$

Find I_{D1}

Since M_1 is diode-connected, it operates in saturation.

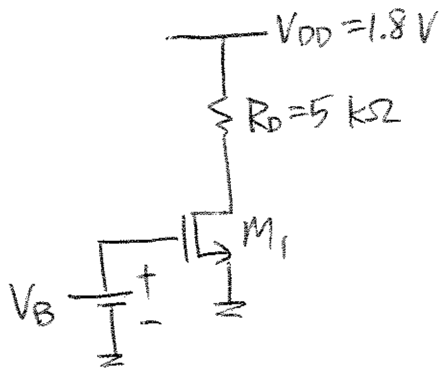
By KCL, $\frac{V_{DD} - V_G}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_G - V_{TH})^2(1 + \lambda V_G)$

One can solve this by (1) using a graphing calculator, (2) trial-and-error, (3) or iteratively finding V_G.

Using any method gives $V_G \approx 0.807\ V$

$\Rightarrow I_D = \frac{V_{DD} - V_G}{R_D} \approx 1\ mA$

30.



$\frac{W}{L} = \frac{20}{0.18}$, $\lambda = 0.1\ V^{-1}$

At the edge of saturation,

$$I_{D_1} = \frac{V_{DD} - (V_B - V_{TH})}{R_D} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_B - V_{TH})^2(1+\lambda(V_B - V_{TH}))$$

This equation can be solved by using a graphing calculator, special programs, or iteratively.

Using any method gives $V_B \approx 0.57\ V$
$(I_D \approx 0.33\ mA)$

31. An NMOS device with $\lambda = 0$ must provide a transconductance of $1/50 \ \frac{1}{\Omega}$.

(a) Given $I_D = 0.5 \text{ mA}$, find W/L.

$$g_m = \frac{1}{50} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\Rightarrow \frac{W}{L} = \frac{g_m^2}{2 \mu_n C_{ox} I_D} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)^2}{2\left(200 \frac{\mu A}{V^2}\right)(0.5 mA)} \approx 2000$$

(b) Given $V_{GS} - V_{TH} = 0.5V$, find W/L.

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{GS} - V_{TH})} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)}{\left(200 \frac{\mu A}{V^2}\right)(0.5 V)} \approx 200$$

(c) Given $V_{GS} - V_{TH} = 0.5V$, find I_D.

$$\Rightarrow I_D = \frac{g_m (V_{GS} - V_{TH})}{2} = \frac{\left(1/50 \ \frac{1}{\Omega}\right)(0.5V)}{2} \approx 5 mA$$

32. (a) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ (I_D constant)

Doubling (W/L) implies a $\sqrt{2}$ times increase in g_m: $g_{m_{NEW}} = \sqrt{2\mu_n C_{ox}\left(2\frac{W}{L}\right) I_D} = \sqrt{2}\, g_m$.

(b) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ (I_D constant)

Doubling $(V_{GS}-V_{TH})$ decreases g_m by half:
$g_{m_{NEW}} = \frac{2I_D}{2(V_{GS}-V_{TH})} = \frac{1}{2} g_m$

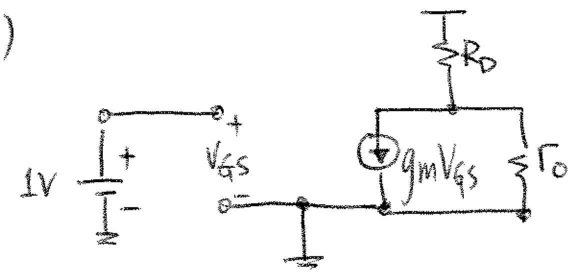
(c) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$ (W/L constant)

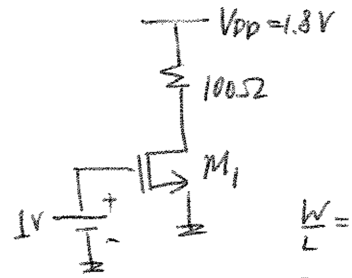
Doubling I_D increases g_m by $\sqrt{2}$ times.

(d) $g_m = \frac{2I_D}{V_{GS}-V_{TH}}$ ($V_{GS}-V_{TH}$ constant)

Doubling I_D increases g_m by 2 times.

33. (a)





$\frac{W}{L}=\frac{20}{0.18}$

$\lambda=0.1V^{-1}$

First, verify M_1 is in saturation:

$$V_{DS} = V_{DD} - I_D R_D = V_{DD} - R_D \cdot \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

$$= 1.8 - 100 \cdot \frac{1}{2}\, 200\frac{uA}{V^2}\left(\frac{20}{0.18}\right)(1-0.4)^2(1+\lambda V_{DS})$$

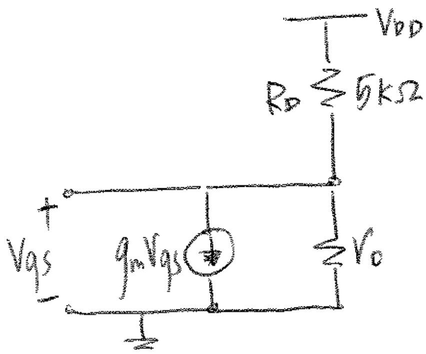
Solving this gives $V_{DS} \approx 1.35V > V_{GS}-V_{TH}$.

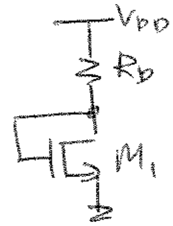
$$\therefore\ g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) = \left(200\frac{uA}{V^2}\right)\left(\frac{20}{0.18}\right)(1-0.4V)$$

$$\approx 0.013\ ^1/_\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda\left(\frac{V_{DD}-V_{DS}}{R_D}\right)} = \frac{1}{0.1V^{-1}\left(\frac{0.45V}{100\Omega}\right)} \approx 2222.\Omega$$

(b)





By KCL, $\dfrac{V_{DD}-V_{GS}}{R_D} = \dfrac{1}{2}\mu_n C_{ox}\dfrac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{GS})$

Solving this yields $V_{GS} \approx 0.546\,V > V_{TH}$

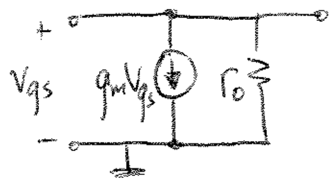
$$\Rightarrow g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH}) = \frac{200\mu A}{V^2}\left(\frac{20}{0.18}\right)(0.146V)$$

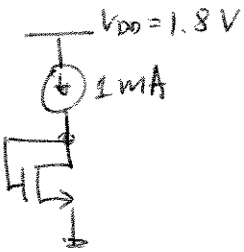
$$\approx 0.00324\ 1/\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda\left(\frac{V_{DD}-V_{GS}}{R_D}\right)} = \frac{1}{0.1V^{-1}\left(\frac{1.8-0.546}{5k}\right)}$$

$$\approx 40.\ k\Omega$$

(c)





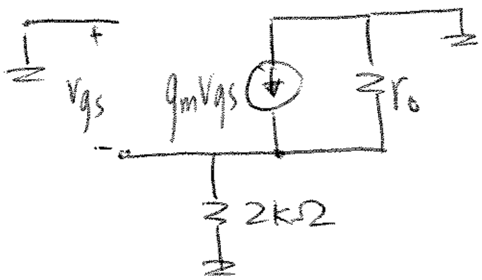
(Note: ideal current source is open in small signal)

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2\left(200 \frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(1\,mA)}$$

$$\approx 0.0067\ ^1/_{\Omega}$$

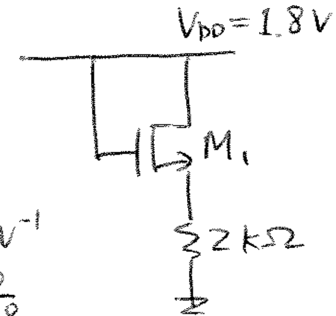
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\,V^{-1})(1\,mA)} = 10\,k\Omega$$

(d)



$\lambda = 0.1\ V^{-1}$

$\frac{W}{L} = \frac{20}{0.18}$



(Note: ideal voltage source is shorted; to GND in this problem because V_{DD} is si -ended.)

By KCL, $I_D = I_R$

$$\Rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD}-V_S)-V_{TH}\right]^2\left[1+\lambda(V_{DD}-V_S)\right] = V_S/2k\Omega.$$

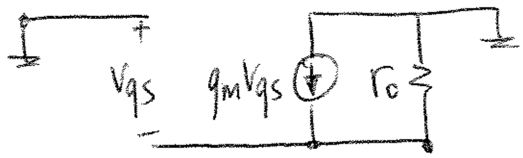
Solving this (analytically or numerically) gives $V_S \approx 1.18\ V$

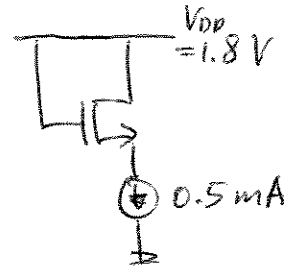
$$\Rightarrow I_D = V_S/2k\Omega \approx 0.59\ mA.$$

$$g_m = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2(0.59mA)}{(1.8-1.18-0.4)V} \approx 0.0054\ 1/\Omega$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.59mA)} \approx 16.9\ k\Omega$$

(e)





$$0.5\ mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD}-V_S)-V_{TH}\right]^2\left[1+\lambda(V_{DD}-V_S)\right]$$

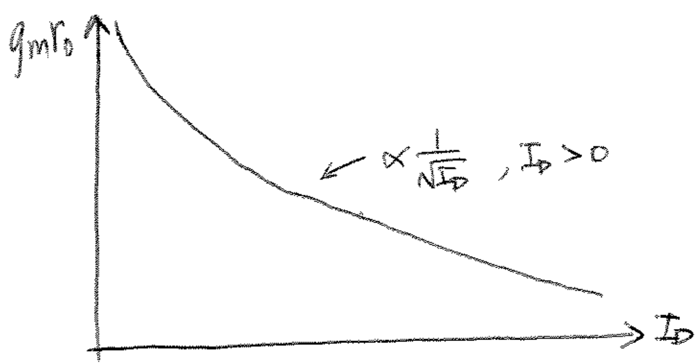
Solving this equation gives $V_S \approx 1.19\ V$

$$\Rightarrow g_m = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2(0.5mA)}{(1.8-1.19-0.4)V} \approx 0.0048\frac{1}{\Omega}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5mA)} = 20\ k\Omega.$$

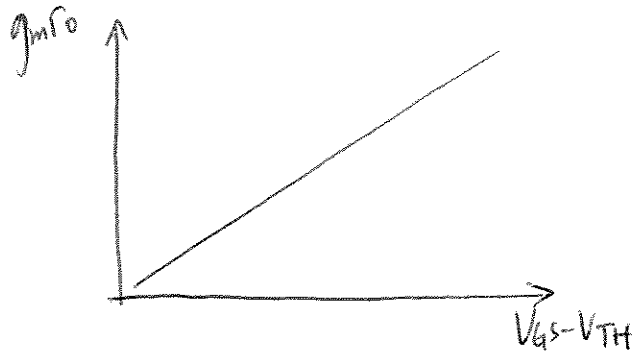
34. $g_m = \sqrt{2\mu C_{ox}\frac{W}{L} I_D}$ $\quad r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\sqrt{2\mu C_{ox}(W/L)\, I_D}}{\lambda I_D} = \frac{1}{\lambda}\sqrt{\frac{2\mu C_{ox}(W/L)}{I_D}}$$

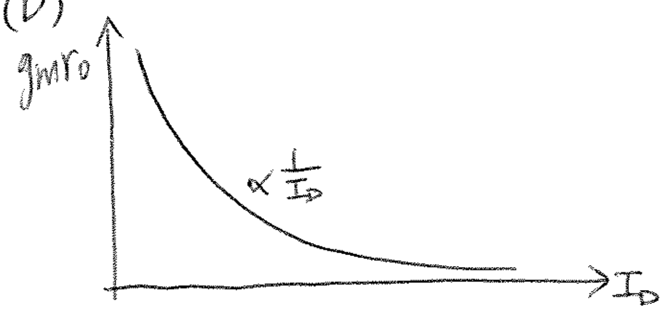


35 (a) $g_m = \mu C_{ox} \frac{W}{L}(V_{GS} - V_{TH})$ $r_o = \frac{1}{\lambda I_D}$

$$g_m r_o = \frac{\mu C_{ox}(W/L)(V_{GS} - V_{TH})}{\lambda I_D}$$



(b)



36. Given NMOS with $\lambda = 0.1\ V^{-1}$ $g_m r_o = 20$, $V_{DS} = 1.5\ V$

determine W/L if $I_D = 0.5\ mA$.

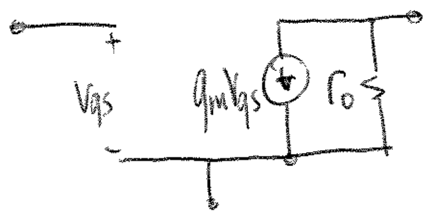
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.1\ V^{-1})(0.5\ mA)} = 20\ k\Omega$$

$$\Rightarrow g_m = \frac{20}{20\ k\Omega} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

$$\therefore \frac{W}{L} = \left(\frac{20}{20\ k\Omega}\right)^2 \frac{1}{2 \mu_n C_{ox} I_D}$$

$$= \left(\frac{1}{1\ k\Omega}\right)^2 \frac{1}{2\left(200 \frac{\mu A}{V^2}\right)(0.5\ mA)} \approx 5.$$

37.



Given $\lambda = 0.2\ V^{-1}$

$g_m r_o = 20$

$V_{DS} = 1.5\ V$

$I_D = 0.5\ mA$

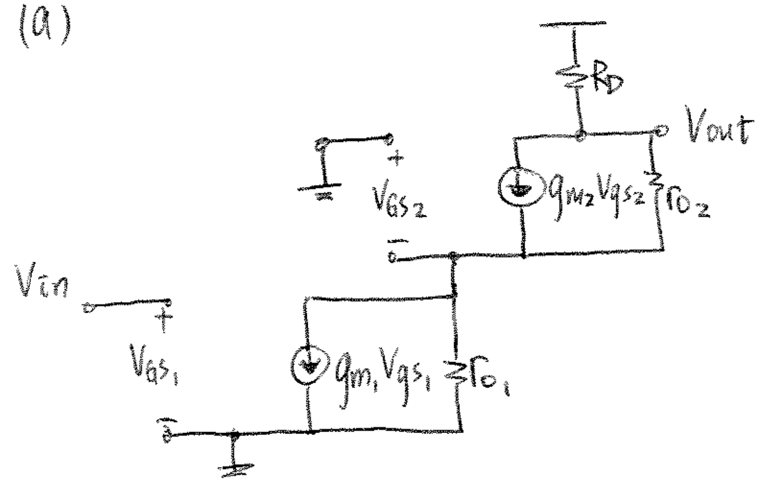
Calculate $\frac{W}{L}$.

$$g_m = \frac{20}{r_o} = 20 \cdot \lambda I_D = 20(0.2V^{-1})(0.5\,mA) = 0.002\ ^1/_\Omega$$

$$\Rightarrow g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}$$

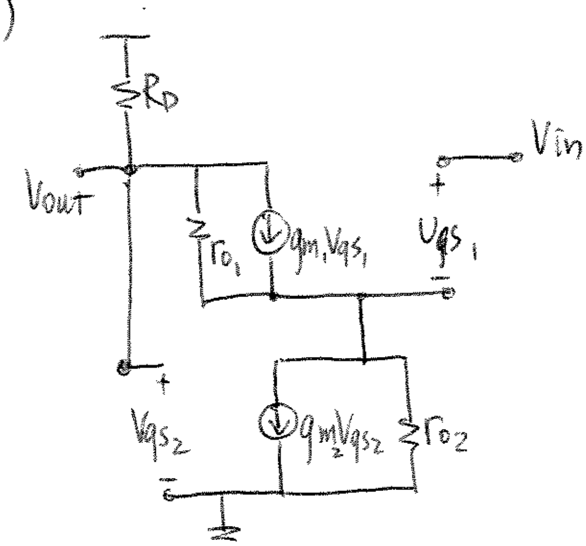
$$\therefore \frac{W}{L} = \frac{g_m^2}{2\mu_n C_{ox} I_D} = \frac{(0.0002\ ^1/_\Omega)^2}{2\left(200\frac{\mu A}{V^2}\right)(0.5\,mA)} = 20$$

38. (a)



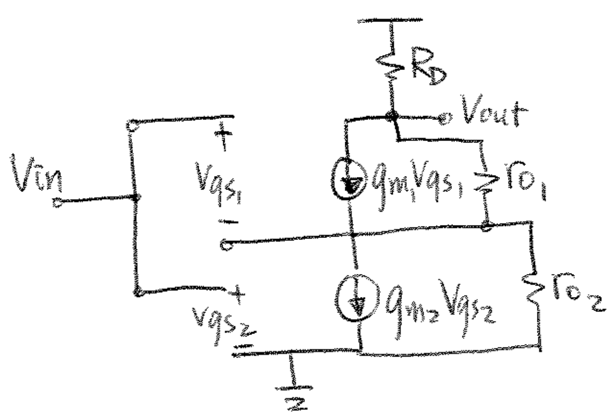
R_D
Vout
+
V_{GS_2}
−
$g_{m2}V_{gs2}$
r_{o2}
Vin
+
V_{GS_1}
−
$g_{m1}V_{gs1}$
r_{o1}

(b)



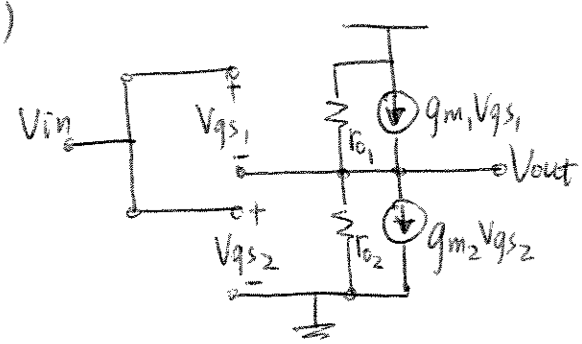
R_D
Vout
Vin
+
V_{gs_1}
−
r_{o1}
$g_{m1}V_{gs1}$
+
V_{gs_2}
−
$g_{m2}V_{gs2}$
r_{o2}

(c)



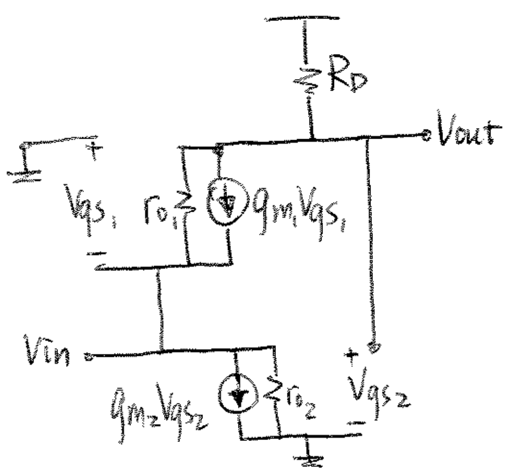
R_D
Vout
Vin
+
V_{gs_1}
−
$g_{m1}V_{gs1}$
r_{o1}
+
V_{gs2}
−
$g_{m2}V_{gs2}$
r_{o2}

(d)



Vin
Vgs1
gm1Vgs1
ro1
Vout
Vgs2
ro2
gm2Vgs2

(e)



RD
Vout
Vgs1
ro1
gm1Vgs1
Vin
gm2Vgs2
ro2
Vgs2

39. (a) OFF $\because$ $|V_{SG}| = 0$

(b) OFF $\because$ $|V_{SG}| < |V_{TH}| = 0.4V$

(c) SATURATION $\because$ $|V_{SD}| > |V_{SG}| - |V_{TH}|$

(d) OFF $\because$ $V_{SG} < |V_{TH}|$

40. (a) SATURATION $\because$ $V_{SD} > V_{SG} - |V_{TH}|$

(b) LINEAR (RESISTIVE) $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} \ll 2(V_{SG} - |V_{TH}|)$

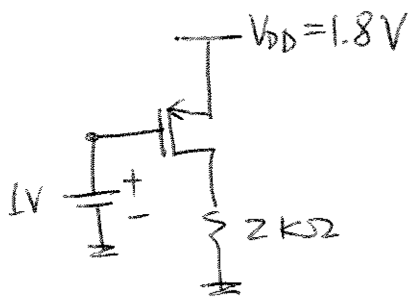
(c) (EDGE OF) SATURATION $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} = V_{SG} - |V_{TH}|$

(d) TRIODE $\because$ $V_{SG} > |V_{TH}|$

$V_{SD} < V_{SG} - |V_{TH}|$

41.



$\lambda = 0$

At the edge of saturation, $V_{SD} = V_{SG} - |V_{TH}|$
$\Rightarrow V_D = 1.4\ V.$

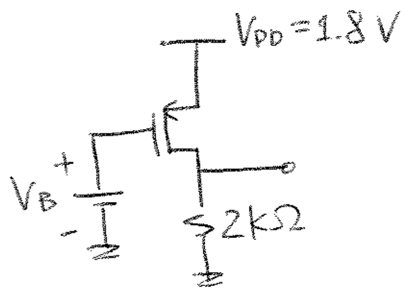
By KCL, $I_{D_1} = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2 = \frac{V_D}{2k\Omega}$$

$$\therefore \frac{W}{L} = \frac{V_D}{2k\Omega}\cdot\frac{2}{\mu_p C_{ox}(V_{SG} - |V_{TH}|)^2}$$

$$= \frac{1.4\ V}{2k\Omega}\cdot\frac{2}{100\ \frac{\mu A}{V^2}(0.8\ V - 0.4V)^2} \approx 87.5$$

42.



$\lambda = 0$

When $V_B = 1V$, $W/L = 87.5$

When $V_B = 0.8V$,

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{SG} - |V_{TH}|)^2$$

$$= \frac{1}{2}\left(100\frac{\mu A}{V^2}\right)(87.5)(1-0.4)^2 V^2 \approx 1.6\,mA$$

$\Rightarrow V_D = I_D(2k\Omega) \approx 3.2V$, which exceeds the supply voltage!

$\therefore$ PMOS goes into triode:
($\because I_D$ is too large)

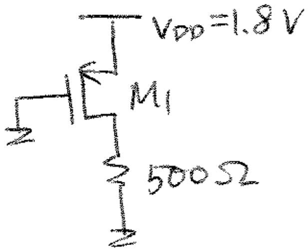
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG} - |V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = (V_{DD} - V_{SD})/2k\Omega$$

Solving this equation numerically (or trial-and-error) gives $V_{SD} \approx 0.18$ V

$$\Rightarrow I_D = \frac{V_{DD} - V_{SD}}{2k\Omega} = \frac{(1.8 - 0.18)V}{2k\Omega} \approx 0.81 \text{ mA}$$

43 (a)



Assume M_1 in triode (since V_{SG} is large). Note: if assumption is incorrect, results will show that.

By KCL, $I_D = I_R$

$$\Rightarrow \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{SG}-|V_{TH}|)\cdot 2V_{SD} - V_{SD}^2\right] = \frac{V_{DD}-V_{SD}}{500\Omega}$$

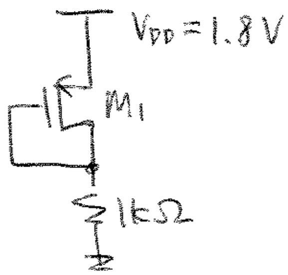
This is a quadratic relation on V_{SD}. Solving it yields $V_{SD} \approx 0.42\,V$

Verify assumption: $V_{SD} \overset{?}{<} V_{SG} - |V_{TH}|$

$0.42 < 1.8 - 0.4 = 1.4$ (✓)

$$I_D = \frac{V_{DD}-V_{SD}}{500\Omega} = \frac{(1.8-0.42)V}{500\Omega} \approx 2.8\,mA$$

(b)



$\frac{W}{L} = \frac{10}{0.18}$ $\lambda = 0$

$\mu_p C_{ox} = 100 \frac{\mu A}{V^2}$

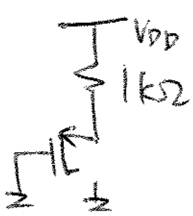
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_{TH}|\right)^2 = \frac{(V_{DD} - V_{SG})}{1k\Omega}$$

Solving this quadratic equation gives $V_{SG} \approx 0.61\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SG}}{1k\Omega} = \frac{(1.8 - 0.61)V}{1k\Omega} \approx 1.2\ mA$$

(c)



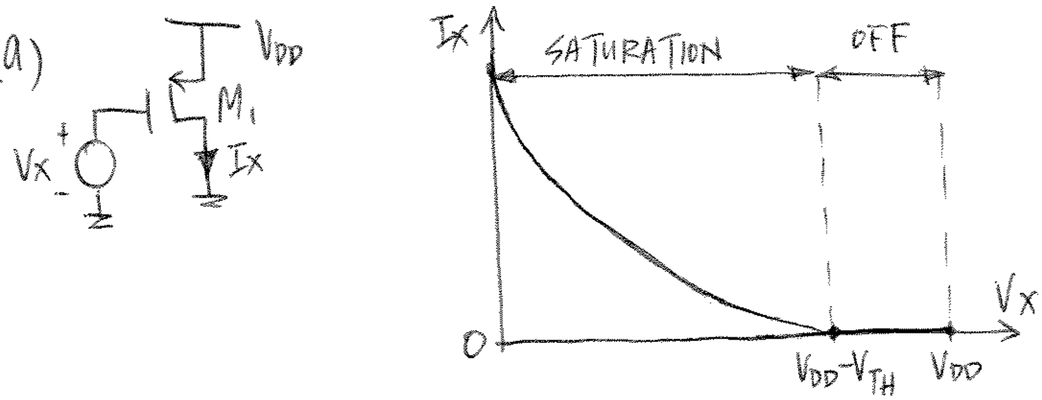
By KCL,

$$\frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left(V_{SG} - |V_{TH}|\right)^2 = \frac{V_{DD} - V_{SG}}{1k\Omega}$$

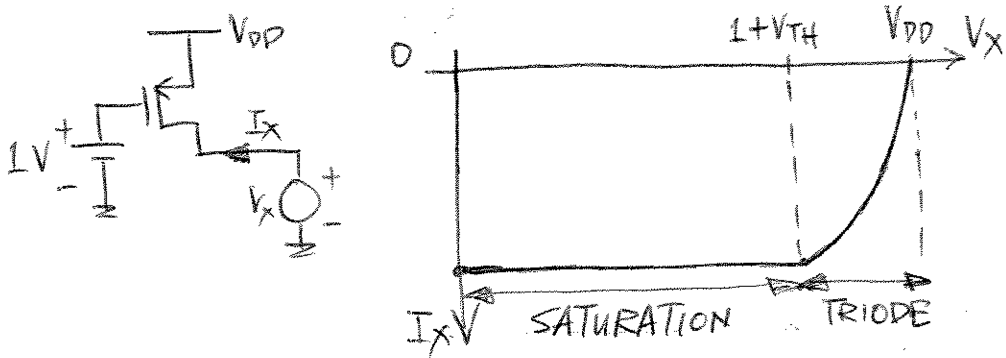
Solving this gives $V_{SG} \approx 0.61\ V$

$$\Rightarrow I_D = \frac{V_{DD} - V_{SG}}{1k\Omega} \approx 1.2\ mA$$

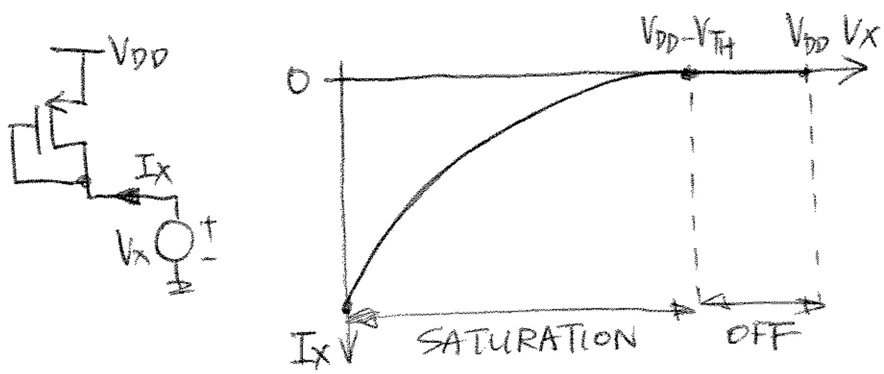
44. (a)


VDD
M1
Vx
Ix
SATURATION
OFF
0
VDD-VTH
VDD
Vx

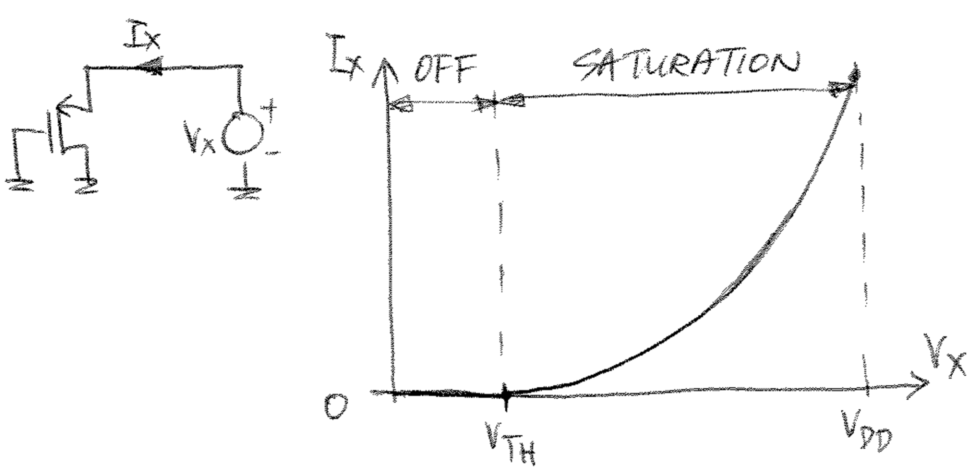
(b)


VDD
1V
Ix
Vx
0
1+VTH
VDD
Vx
SATURATION
TRIODE
Ix

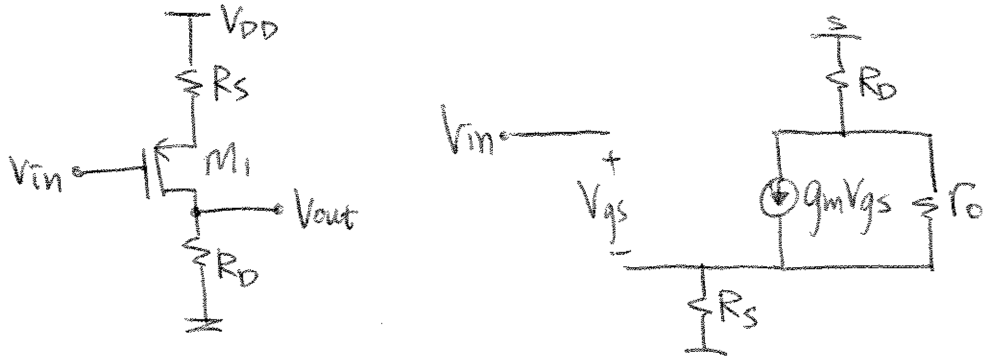
(c)


VDD
Ix
Vx
0
VDD-VTH
VDD
Vx
SATURATION
OFF
Ix

(d)

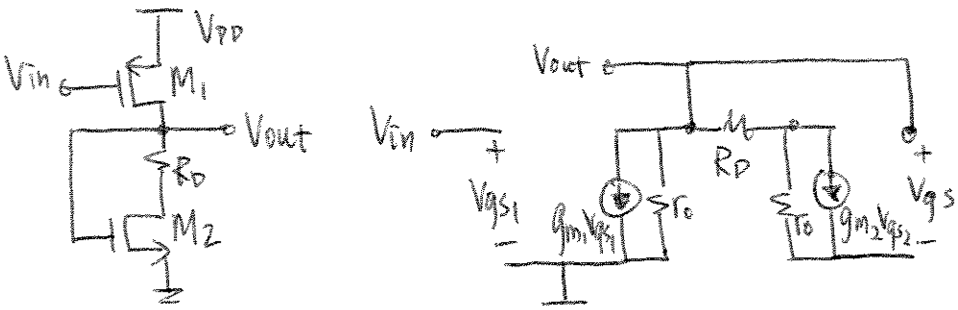

Ix
Vx
Ix
OFF
SATURATION
Vx
0
VTH
VDD

45. (a)



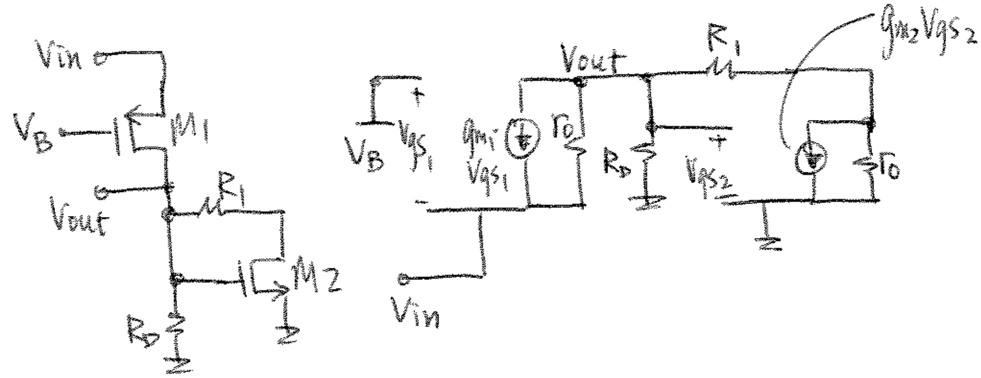
VDD
RS
M1
Vin
Vout
RD
Vin
+
Vgs
−
gmVgs
ro
RD
RS

(b)



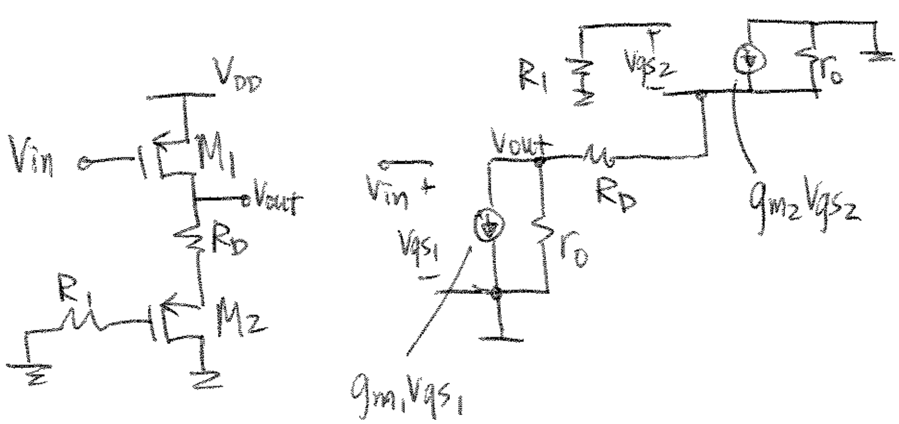
VDD
Vin
M1
Vout
RD
M2
Vout
Vin
+
Vgs1
−
gm1Vgs1
ro
RD
ro
gm2Vgs2
+
Vgs
−

(c)



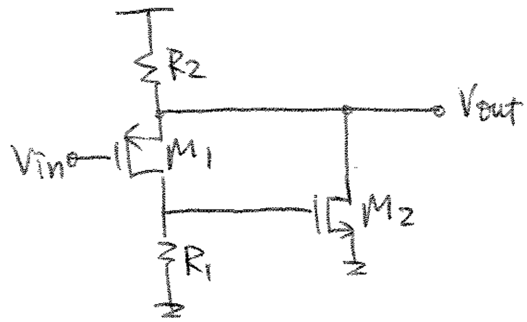
Vin
VB
M1
Vout
R1
M2
RD
Vout
R1
gm2Vgs2
VB
+
Vgs1
−
gm1
Vgs1
ro
RD
+
Vgs2
−
ro
Vin

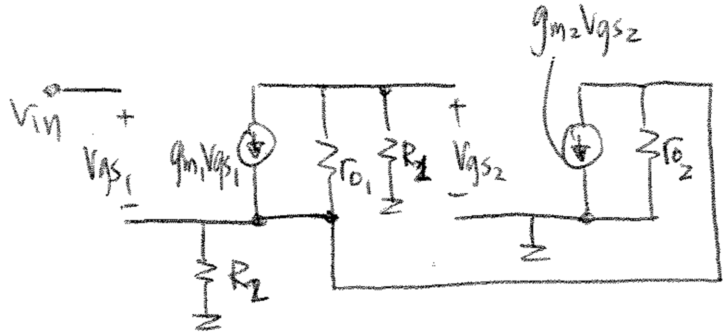
(d)



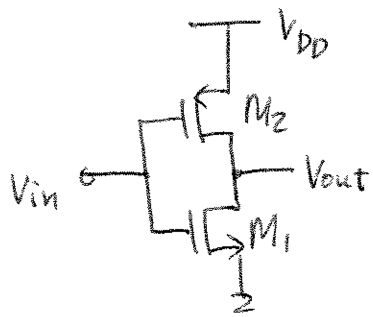
VDD
Vin
M1
Vout
RD
R1
M2
R1
+
Vgs2
−
ro
Vout
RD
Vin +
Vgs1
−
ro
gm2Vgs2
gm1Vgs1

(e)


R2
Vout
Vin
M1
M2
R1

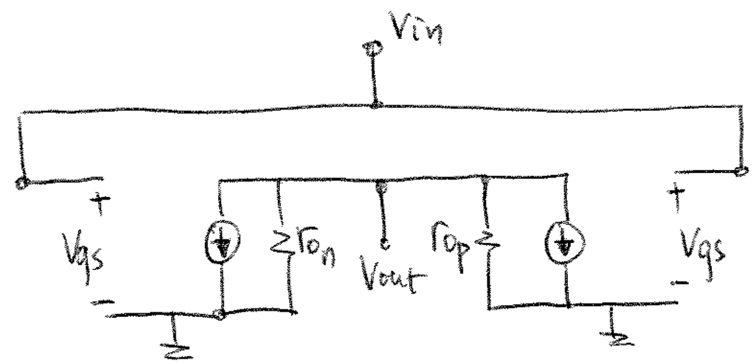

gm2Vgs2
Vin
+
Vgs1
gm1Vgs1
ro1
R1
+
Vgs2
ro2
R2

46.



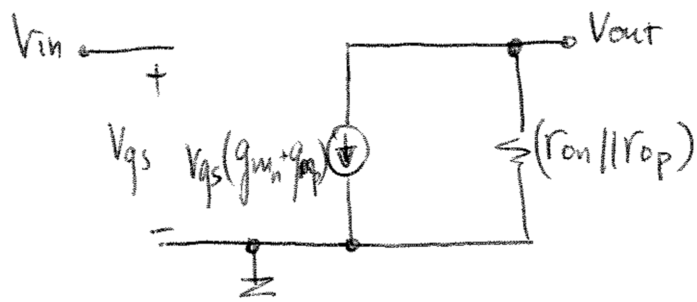
Assume λ_n & λ_p.

(a)



They are in "parallel" because from the small-signal model, both their respective SOURCE and DRAIN nodes are the same.

(b) Assuming both M_1 & M_2 are in saturation, we can combine r_o's & g_m's :



$$\therefore \frac{V_{out}}{V_{in}} = -(g_{mn} + g_{mp})(r_{on} // r_{op})$$

(1) For M_1 to stay in saturation,

$$V_{DS} > V_{GS} - V_{TH},$$

i.e. $$V_{DS} > V_{DD} - V_{TH}$$

$$V_{DS} > 1.4$$

$\because$ $$V_{DS} = V_{DD} - I_{DS}(R_L),$$

where $R_L = 1\,k\Omega$.

and $$I_{DS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2$$

$$= \frac{1}{2} \times 200 \times 10^{-6} \left(\frac{W}{L}\right)_1 (1.4)^2$$

$\therefore$ $$V_{DS} = V_{DD} - 10^{-4}\left(\frac{W}{L}\right)_1 (1.96) \times 1000$$

i.e. $$1.8 - 1.96 \times 10^{-4}\left(\frac{W}{L}\right)_1 > 1.4$$

$$\frac{0.4}{1.96 \times 10^{-1}} > \left(\frac{W}{L}\right)_1$$

Maximum allowable $\left(\frac{W}{L}\right)_1$ is 2

② To get $I_{DS} = 1\,mA$,

$$\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2 = 1 \times 10^{-3}\ A.$$

$$\frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)_1 (V_{GS} - V_{TH})^2 = 10^{-3}$$

$$(V_{GS} - V_{TH})^2 = 0.09$$

$$V_{GS} - V_{TH} = 0.3,$$

$$\text{ie. } V_{GS} = 0.7.,$$

$$\text{Since} \quad V_{GS} = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$0.7 = \frac{R_2}{R_1 + R_2.} \times 1.8$$

$$0.7\ R_1 = .\ R_2,$$

$$\therefore \frac{R_1}{R_2} = \frac{11}{7}. \quad \text{———— ①}$$

To get input impedance $\geqslant 20k$.

$$R_1 \parallel R_2 \geqslant 20\,k\Omega. \quad \text{———— ②}$$

By inspection, setting $R_1 = 55\,k\Omega$ and $R_2 = 35\,k\Omega$ Will satisfy both ① and ②.

(3) $V_G = 1.8V$

$V_S = I_{DS}(100)$

$V_D = 1.8 - 1000 I_{DS}$

For M_1 to be in saturation,

$$V_{DS} \geq V_{GS} - V_{TH}$$

$$\therefore V_D - V_S \geq V_G - V_S - V_{TH}$$

$$V_D \geq V_G - V_{TH}$$

$$V_D \geq 1.4V$$

$$\therefore I_{DS,max} = \frac{1.8 - 1.4}{1000} = 0.4mA.$$

and $\therefore V_S = (0.4 \times 10^{-3})(100)$

$$= 0.04V$$

$$V_{GS} = 1.76V$$

$$g_{m,max} = \frac{2 I_{DS,max}}{(1.76 - 0.4)}$$

$$= 0.588 mS$$

④ a). $\because \quad V_{RS} = 200\,mV,$

$\therefore \quad I_{DS}\,R_S = 200\,mV$

$$I_{DS} = \frac{0.2}{100}$$

$$I_{DS} = 2\,mA.$$

For M_1 to stay in saturation,

$$V_{DS} \geqslant V_{GS} - V_{TH}.$$

$\because \quad V_{DS} = V_D - V_S$

$$= [1.8 - (2\times10^{-3})\times500] - 0.2$$

$$= 0.6,$$

$\therefore \quad V_{GS} - V_{TH} \leqslant 0.6,$

Since $\quad I_{DS} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2,$

$\left(\frac{W}{L}\right)$ is min. when $(V_{GS} - V_{TH})$ is max,

$\therefore$ min $\left(\frac{W}{L}\right)_1$ is when $(V_{GS} - V_{TH}) = 0.6\,V,$

$$2\times10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)_1 (0.6)^2$$

$\therefore$ min. $\left(\frac{W}{L}\right)_1 \approx 56$

b) With $(V_{GS} - V_{TH}) = 0.6$,

$$V_{GS} = 1,$$

$$\therefore V_G = 1 + V_S$$

$$V_G = 1.2V,$$

i.e. $$1.8 \times \frac{R_2}{R_1 + R_2} = 1.2V,$$

$$\frac{R_2}{R_1} = 2 \qquad \text{—— ①}$$

Input impedance = $R_2 // R_1$,

i.e. $$R_2 // R_1 \geqslant 30k\Omega \qquad \text{—— ②}$$

Set $R_1 = 50k\Omega$ and $R_2 = 100k\Omega$ will satisfy both ① & ②.

(5)

$$V_S = V_{RS}$$
$$= I_{D1}\,(200) = 0.1V$$

$$I_{DS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2$$

$$0.5mA = \frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)(V_{GS} - V_{TH})^2$$

$$\therefore V_{GS} = 0.612\ V$$

$$\therefore V_G = 0.612 + 0.1$$
$$= 0.712.$$
$$\because V_G = V_{DD} - I_{R1} \times R_1$$

$$R_1 = \underline{\underline{21.76\ k\Omega}}.$$

and $V_{GS} = I_{R2} \times R_2.$

$$\therefore R_2 = \frac{0.712}{0.05 \times 10^{-3}}$$

$$= \underline{\underline{14.24\,k\Omega}}$$

⑥. $g_m = \sqrt{2\beta I_{DS}} = \frac{1}{100}$,

$\because I_{DS} = 1mA, \quad \beta = 0.05$,

and $I_{DS} = \frac{1}{2}\beta (V_{GS} - V_{TH})^2$,

where $\beta = \mu_n C_{ox} \left(\frac{W}{L}\right)$,

$\therefore 1mA = \frac{1}{2}(0.05)(V_{GS} - V_{TH})^2$.

$V_{GS} = 0.6$.

$\because V_{GS} = V_{DS} = V_{DD} - I_{DS} R_D$,

$0.6 = 1.8 - (0.5 \times 10^{-3}) R_D$,

$R_D = \underline{\underline{2.4 \ k\Omega}}$

⑦ $I_{DS} = \frac{1}{2} (\mu_n C_{ox})(\frac{W}{L}) (V_{GS} - V_{TH})^2$

$0.5 \times 10^{-3} = (100 \times 10^{-6})(\frac{50}{0.18}) (V_{GS} - V_{TH})^2$

$\therefore V_{GS} = 0.534V$

$\therefore R_2 = \frac{0.534}{0.05 \times 10^{-3}}$

$R_2 = \underline{\underline{10.68 k\Omega}}$

$\because V_{D1} = 1.8 - (1.1 \times I_{DS} \times 2k\Omega) = 0.1 I_{DS} (R_1 + R_2)$,

$\therefore 14k\Omega = R_1 + 10.68k\Omega$.

$\therefore R_1 = \underline{\underline{3320\Omega}}$

⑧ Without defect,

$$V_{GS} = V_{DS}, \quad (\text{i.e. } V_G = V_D)$$

$$\therefore \frac{20k}{10k+20k} \times 1.8 = 1.8 - I_{DS}(1k\Omega)$$

$$I_{DS} = 0.6\ mA.$$

$$\because \quad V_{GS} = V_G - V_S$$
$$= 1.2 - (0.6\times10^{-3})(200)$$
$$= 1.08.$$

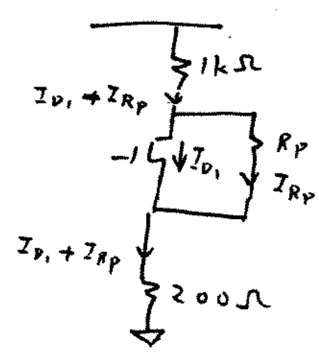
and. $$0.6\ mA = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)(V_{GS}-0.4)^2.$$

$$\frac{W}{L} \approx 13 //$$

with R_P

$$V_{GS} = V_{DS} + V_{TH},$$
$$1.2 = V_{DS} + 0.4,$$
$$\therefore V_{DS} = 0.8\ V.$$

$$\because V_{DS} = V_{DD} - V_{1k\Omega} - V_{200\Omega},$$



$$\therefore I_{D_1} + I_{R_P} = \frac{1\ V}{1k\Omega + 200\Omega}$$
$$= 0.833\ mA$$

$$\because I_{R_P} = \frac{V_{DS}}{R_P} = \frac{0.8}{R_P}$$

and $I_{D_1} = 0.6\ mA$ (from above)

$$\therefore \quad \frac{0.8V}{R_p} + 0.6\ mA = 0.833\ mA$$

$$R_p \approx 3430\,\Omega$$

(a) Without defects,

$$V_{GS} = 1.8V$$

i.e. $V_{DS} = (1.8 - 0.1)V$

$$= 1.7V$$

$$I_{DS} = \frac{(1.8-1.7)V}{2000\,\Omega} = 0.05\,mA.$$

$$\therefore\ 0.05\,mA = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1.8-0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) = 0.255$$

b) With defects,

$$V_{GS} = V_{DS} + 50\,mV$$

$$\therefore\ V_{Rp} = 50\,mV$$

$$I_{Rp} = \frac{50\,mV}{R_p}$$

$$V_{GS} = 1.8\,V - \frac{0.05V}{R_p} \times 30\,k\Omega \quad \text{—— (1)}$$

$$\because\ V_{DD} - \left(I_{DS} - \frac{50\,mV}{R_p}\right)2\,k\Omega = V_{DS}$$

$$V_{DD} - \left(I_{DS} - \frac{50\,mV}{R_p}\right)2\,k\Omega = V_{GS} - 50\,mV \quad \text{—— (2)}$$

$\because \quad I_{DS} = \frac{1}{2}\left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})^2$

$= \frac{1}{2}(0.255)(200 \times 10^{-6})(V_{GS} - 0.4)^2$

$= 2.55 \times 10^{-5}\ (V_{GS} - 0.4)^2$

$\therefore$ From ②,

$$1.8 - \left[2.55 \times 10^{-5}(V_{GS} - 0.4)^2 - \frac{0.2}{R_p}\right]2000$$

$$= V_{GS} - 0.05.$$

From ①, $\quad \frac{0.05}{R_p} = \frac{1.8 - V_{GS}}{30\ 000}$

$$\therefore\ 1.8 - \left[0.051(V_{GS} - 0.4)^2 - \frac{1.8 - V_{GS}}{15}\right] = V_{GS} - 0.3$$

$$1.85 - 0.051 V_{GS}^2 + 0.0408 V_{GS} - 0.00816 + \frac{1.8 - V_{GS}}{15} = V_{GS}$$

$$29.4276 - 15.388\ V_{GS} - 0.765\ V_{GS}^2 = 0$$

$$\therefore V_{GS} = 1.76\ V$$

$$R_p = \frac{0.05 \times 30\ 000}{1.8 - 1.76}$$

$$\approx 36.3\ k\Omega$$

(10) For M_1,

$$I_x = \frac{1}{2}(200 \times 100^{-6})\left(\frac{W_1}{0.25}\right)(0.8-0.4)^2 \times (1 + 0.1(0.8))$$

$$10^{-3} = 0.16 \times 10^{-4}\left(\frac{W_1}{0.25}\right)(1.08)$$

$$\therefore W_1 \approx 14.5\mu$$

For M_2,

$$0.5 \times 10^{-3} = 0.16 \times 10^{-4}\left(\frac{W_2}{0.25}\right)(1.08)$$

$$\therefore W_2 = 7.25\mu$$

$$\text{Output resistance} = r_o = \frac{1}{\lambda} \times \frac{1}{I_D}$$

$$\therefore r_{o1} = \left(\frac{1}{0.1}\right)\left(\frac{1}{10^{-3}}\right) = 10\,k\Omega$$

$$r_{o2} = \left(\frac{1}{0.1}\right)\left(\frac{1}{0.5 \times 10^{-3}}\right) = 20\,k\Omega$$

(11)

$$R_{out} = \frac{1}{\lambda}\left(\frac{1}{I_D}\right)$$

$$= \frac{1}{0.5\times10^{-3}\,\lambda} = 20k\Omega$$

$$\therefore \lambda = 0.1\ V^{-1}$$

(12) For M_1,

$$0.5 \times 10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W_1}{0.25}\right)(1-0.4)^2$$

$$W_1 \approx 3.47\ \mu m$$

For M_2,

$$0.5 \times 10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W_2}{0.25}\right)(1.2-0.4)^2$$

$$W_2 \approx 1.95\ \mu m$$

$$\frac{r_{o1}}{r_{o2}} = \frac{\frac{1}{\lambda I_x}}{\frac{1}{\lambda I_y}}$$

$$= 1 \quad (\because I_x = I_y)$$

$$\therefore r_{o1} = r_{o2}$$

(13) Impedance at source of M_1, $Z_S = \frac{1}{g_{mp}}$

$$g_{mp} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right) I_D (1 + \lambda V_{DS})}$$

$$= \sqrt{2 \times 100 \times 10^{-6} \left(\frac{10}{0.25}\right) (1 + 0.1 \times 1.2) I_D}$$

$$= \sqrt{0.0896 \, I_D}$$

$$I_D = \frac{1}{2} \times 100 \times 10^{-6} \times \left(\frac{10}{0.25}\right) (V_{B1} - V_x + 0.4)^2 \times (1 + 0.1 \times 1.2)$$

$$\approx 0.806 \text{ mA}$$

$$\therefore g_{mp} = \sqrt{0.0896 \times 0.806 \times 10^{-3}}$$

$$\approx 8.50 \text{ mS}$$

$$\therefore Z_S \approx 118 \, \Omega$$

(14)

$$I_X = \frac{1}{2}(100\times10^{-6})\left(\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 0.64\,mA$$

$$I_Y = \frac{1}{2}(100\times10^{-6})\left(2\times\frac{20}{0.25}\right)(1-1.8+0.4)^2$$

$$= 1.28\,mA$$

$\because r_o \propto \frac{1}{I}$

and $I_Y = 2I_X$

$\therefore r_{out,M_1} = 2\,r_{out,M_2}$

(15) $|I_{DS1}| = |I_{DS2}|$,

$$\frac{1}{2}(200\times10^{-6})\left(\frac{10}{0.18}\right)(V_B-0.4)^2(1+0.1\times0.9)$$

$$= \frac{1}{2}(100\times10^{-6})(1.8-V_B-0.4)^2(1+0.1\times0.9)\times\left(\frac{30}{0.18}\right)$$

$$2(V_B-0.4)^2 = 3(1.4-V_B)^2$$

$$\sqrt{\frac{2}{3}}(V_B-0.4) = (1.4-V_B)$$

$$1.816\,V_B = 1.7264$$

$$V_B = 0.95$$

(16) a) For M_1,

$$I_{D1} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{5}{0.18}\right)(V_B - 0.4)^2 (1 + 0.1 \times 0.9)$$

$\therefore V_B \approx 0.806\ V$

b) There are 3 regions of operation:

For $V_X < V_B - V_{TH,1}$, M_1 is in triode and $|I_{DS2}| > |I_{DS1}|$

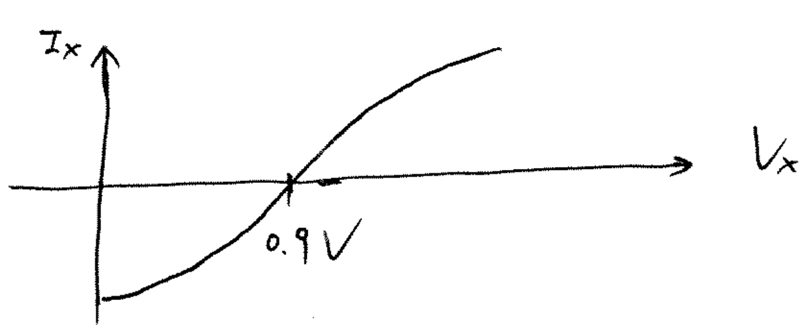
For $|V_X - V_{DD}| > |V_B - V_{DD} - V_{TH2}|$, M_2 is in triode and $I_{DS1} > |I_{DS2}|$

For $V_B - V_{TH} < V_X$ and $|V_X - V_{DD}| < |V_B - V \ - V_{TH2}|$

M_1 and M_2 are in saturation.

and $I_{DS1} = |I_{DS2}| = 0.5\ mA$ at $V_X = 0.9\ V$

In all cases, $I_X = I_{DS1} - |I_{DS2}|$



(17) a) $0.5\times10^{-3} = \frac{1}{2}(200\times10^{-6})(\frac{30}{0.18})(V_{GS} - V_{TH})^2$

$\therefore V_{GS} = 0.573\ V$

$V_{DS} = 1.8 - 0.5\times10^{-3}\times 2000$

$= 0.8.$

$\because V_{DS} > V_{GS} - V_{TH},$

M_1 is in saturation.

b) $\because \lambda = 0, \quad r_o = \infty.$

$\therefore A_v = g_m R_D$

$= \sqrt{2\times(200\times10^{-6})\times\frac{30}{0.18}\times 0.5mA} \times 2000$

$= 11.55$

(18)

a) $0.25\times10^{-3} = \frac{1}{2}\times(200\times10^{-6})\left(\frac{20}{0.18}\right)(V_{GS}-0.4)^2$

$\therefore V_{GS} = 0.55$

b) $V_{DS,min} = V_{GS} - V_{TH}$

$= 0.15V.$

with $V_{DS} = 0.15V,$

$I_{DS,max} = \frac{1.8-0.15}{2000}$

$= 0.825mA.$

$\therefore \frac{0.825\times10^{-3}}{0.25\times10^{-3}} = \frac{\left(\frac{W}{L}\right)'}{\left(\frac{W}{L}\right)},$

where $\left(\frac{W}{L}\right)'$ is the new $\left(\frac{W}{L}\right)$.

$\therefore \left(\frac{W}{L}\right)$ can be increased by 3.3 times.

$\because A_v \propto g_m$

$\propto \sqrt{\beta I}$

$\therefore A_v$ is also increased by 3.3 times.

(since both β & I increase by 3.3 times)

(19) Voltage gain (A_v) = 5,

i.e. $g_m R_D = 5$.

Power $(P) = I_{DS} \times V_{DD}$,

$\because P \leq 1\,mW$,

$I_{DS} \times 1.8 \leq 1\,mW$.

$I_{DS} \leq 0.556\,mA$.

$$g_m = \sqrt{2 \beta I_{DS}}$$

$$\therefore g_{m,max} = \sqrt{2 \times 200 \times 10^{-6} \times \left(\frac{20}{0.18}\right) \times 0.556\,mA}$$

$$= 0.00497\,\Omega^{-1}$$

$$\therefore R_D = \frac{5}{0.00497}$$

$$\approx 1006\,\Omega$$

$\therefore$ This is minimum value required for R_D.

(20) $|A_v| = g_{m1}\,(r_{o1} \,//\, r_{o2}) = 10.$

$$r_{o1} = \frac{1}{\lambda_1 I_1} = \frac{1}{0.1 \times 0.5 \times 10^{-3}}$$

$$= 20\,k\Omega.$$

$$r_{o2} = \frac{1}{\lambda_2 I_1} = \frac{1}{0.15 \times 0.5 \times 10^{-3}}$$

$$= 13.3\,k\Omega.$$

$$\therefore g_{m1} = \frac{10}{20k \,//\, 13.3k}$$

$$= 0.00138\,\Omega^{-1}$$

$$\because g_{m1} = \sqrt{2\beta_1 I_{DS}}$$

$$\beta_1 = 0.00192$$

$$200 \times 10^{-6} \left(\frac{W}{L}\right)_1 = 0.00192$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 9.6$$

b) $$0.5 \times 10^{-3} = \frac{1}{2}(100 \times 10^{-6})(1.8 - V_B - 0.4)^2 \left(\frac{20}{0.18}\right)$$

$$\therefore V_B \approx 1.1\,V$$

(21)

$$|A_v| = g_{m1}\,(r_{o1} \parallel r_{o2})$$

$$g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{20}{0.18}\right) \times (0.001)}$$

(Since V_{ds1} is not given, assume $(1 + \lambda_1 V_{DS1})$ has minimal effect on g_{m1})

$$= 6.67 \text{ mS} \qquad (S = \Omega^{-1})$$

$$r_{o1} = \frac{1}{\lambda_1 \times I_{D1}}$$

$$= \frac{1}{0.1 \times 1\text{mA}}$$

$$= 10\text{ k}\Omega.$$

$$r_{o2} \simeq \infty$$

$(\because \lambda_2 \ll \lambda_1)$

$$\therefore |A_v| = 6.67 \times 10^{-3} \times 10^{3} \times 10$$

$$= 66.7$$

(22) a) $A_v = g_{m1} \, (r_{o1} \parallel r_{o2})$

when length of M_1 and M_2 double,

r_o doubles ($\because r_o \propto \; \propto L$)

$$r_{o1} \parallel r_{o2} = \frac{r_{o1} \, r_{o2}}{r_{o1} + r_{o2}}$$

$$\therefore (r_{o1} \parallel r_{o2}) \propto \frac{L^2}{L},$$

i.e. $(r_{o1} \parallel r_{o2}) \propto L$.

g_{m1} is constant because both $\left(\frac{W}{L}\right)_{1,2}$ and I_{DS} are constant.

$\therefore$ Voltage gain is doubled.

b) When both length and bias current double, r_o remains the same.

$\because g_{m1} \propto \sqrt{I_{DS}}$.

$\therefore$ Voltage gain increased by $\sqrt{2}$.

(23). To get higher voltage gain,

(a) is preferred.

For the same dimensions of transistors and same bias current,

(a) has a high "g_m" than (b),

$\because g_{m1} > g_{m2}$.

(since $\mu_n C_{ox} > \mu_p C_{ox}$)

while $(r_{o1} \parallel r_{o2})$ is the same for both cases.

(24)

$$A_v = g_{m2} (r_{o1} \,//\, r_{o2})$$

$$r_{o1} = \frac{1}{0.15 \times 0.5mA}$$

$$= 13.3 \text{ k}\Omega.$$

$$r_{o2} = \frac{1}{0.05 \times 0.5mA}$$

$$= 40 \text{ k}\Omega.$$

$$\therefore r_{o1} // r_{o2} = 10 \text{ k}\Omega.$$

$$\therefore 15 = \left[\sqrt{2 \times (100 \times 10^{-6}) \left(\frac{W}{L}\right)_2 0.5mA}\right] \cdot (10 \text{ k}\Omega)$$

$$\left(\frac{W}{L}\right)_2 = 22.5$$

(25) From Eq (7.57),

$$3 = \sqrt{\frac{20/0.18}{(W/L)_2}}$$

$\therefore (W/L)_2 \approx 12.3$ //.

(26)

a) For M_1,

$$0.5 \times 10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{10}{0.18}\right)(V_{GS_1} - 0.4)^2$$

$$\therefore V_{GS1} = 0.7\,V$$

$$\therefore V_{DS1,\,min} = V_{GS1} - V_{TH}$$

$$= 0.3\,V$$

For M_2,

$$V_{S,\,min} = V_{DS1,\,min}$$

$$= 0.3\,V.$$

$$V_{GS,\,max} = 1.8 - 0.3$$

$$= 1.5\,V$$

$$\therefore I_{DS2} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)_2 (1.5 - 0.4)^2$$

$$= 0.5\,mA.$$

$$\left(\frac{W}{L}\right)_2 = 4.13$$

b) volt. gain $= -\sqrt{\dfrac{(W/L)_1}{(W/L)_2}}$

$$\approx \underline{3.67}$$

c) Because with M_1 at the edge of saturation, V_{GS} of M_2 is at maximum (V_S of M_2 is at minimum). Thus, a minimum (W/L) is required to set up the same bias current. With minimum (W/L), g_{m2} is at minimum. Since $A_v \propto \frac{1}{g_{m2}}$, A_v is at maximum.

(27) a) $$A_v = \sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$\therefore 5 = \sqrt{\frac{(W/L)_1}{(2/0.18)}}$$

$$\therefore (W/L)_1 \approx 277.8$$

b) For M_2,

$$I_{DS2} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{2}{0.18}\right)(1.8 - V_{S2} - 0.4)^2.$$

$$I_{DS2} = (0.00111)(1.4 - V_{S2})^2.$$

$$\because V_{S2} = \qquad I_{DS2} = (0.00111)(1.4 - V_{DS1})^2.$$

For M_1,

$$I_{DS1} = \frac{1}{2}(200 \times 10^{-6})(277.8)(V_{GS1} - 0.4)^2$$

$$= 0.02778\,(V_{GS1} - 0.4)^2$$

$$\because I_{DS1} = I_{DS2}$$

$$\therefore (0.02778)(V_{GS1} - 0.4)^2 = (0.00111)(1.4 - V_{DS1})^2$$

$$5\,(V_{GS1} - 0.4) = (1.4 - V_{DS1})$$

At edge of saturation,

$$V_{DS1} = V_{GS1} - 0.4,$$

Let $m = V_{DS1} = V_{GS1} - 0.4.$

$$\therefore 5m = 1.4 - m$$

$$m = 0.233$$

$$\because I_{DS1} = 0.02778 (V_{GS1} - 0.4)^2$$

$$= I_{Bias}$$

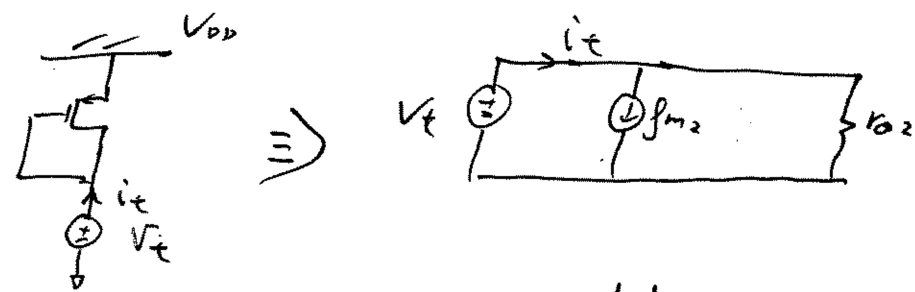
$$\therefore I_{Bias} = 0.02778 (0.233)^2$$

$$= 6.48 \, mA$$

(28) a/. $A_v = -g_{m1}\, r_{o1} \,//\, Z_2$,

where Z_2 is the impedance presented by M_2.

To find Z_2, apply a test voltage (V_T) at the drain of M_2:



From the small-signal model,

$$i_t = g_{m2} V_t + \frac{V_t}{r_{o2}},$$

$$Z_2 = \frac{V_t}{i_t} = r_{o2} \,//\, \frac{1}{g_{m2}}.$$

$$\therefore A_v = -g_{m1} \left(r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}} \right)$$

b/ $A_v = -g_{m1} (r_{o1} \,//\, Z_2 \,//\, Z_3)$

where Z_2 and Z_3 are impedances presented by M_2 and M_3 respectively.

From (a) $Z_3 = r_{o3} \,//\, \frac{1}{g_{m3}}$

By inspection, $Z_2 = r_{o2}$

$$\therefore A_v = -g_{m1} \left(r_{o1} \,//\, r_{o2} \,//\, r_{o3} \,//\, \frac{1}{g_{m3}} \right)$$

c) $A_v = -g_{m1} \, r_{o1} \parallel Z_2 \parallel Z_3$.

Similar to (b),

$Z_2 = r_{o2}$,

and $Z_3 = r_{o3} \parallel \frac{1}{g_{m3}}$

(the small signal model of M_3 in this case is equivalent to that of M_2 in (a))

$\therefore A_v = -g_{m1} \left(r_{o1} \parallel r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m3}} \right)$

d). M_2 is in CS arrangement. (similar to (c)).

$A_v = g_{m2} \, r_{o2} \parallel Z_1 \parallel Z_3$.

$Z_3 = \frac{1}{g_{m3}} \parallel r_{o3}$

$Z_1 = r_{o1}$

$\therefore A_v = g_{m2} \left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3} \right)$

e). $A_v = g_{m2} \left(r_{o2} \parallel Z_1 \parallel Z_3 \right)$

$Z_1 = r_{o1}$

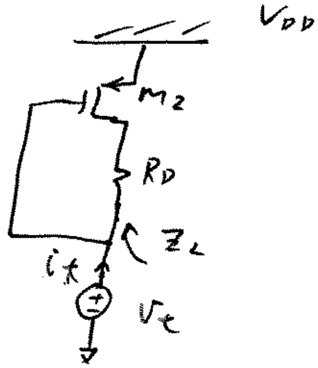
$Z_3 = \frac{1}{g_{m3}} \parallel r_{o3}$

(recall: impedance looking into source $= \frac{1}{g_m}$)

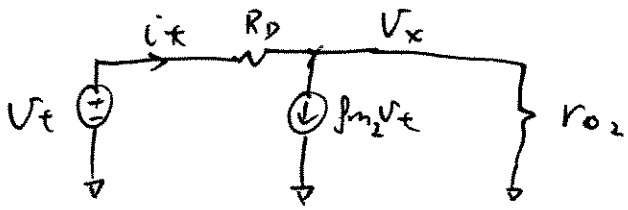
$\therefore A_v = g_{m2} \left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3} \right)$

(28) f) $A_v = -g_{m1} (r_{o1} \parallel Z_L)$,

where Z_L is the impedance depicted as follows:



The equivalent small-signal model is:



$$i_t = g_{m2} v_t + \frac{V_x}{r_{o2}},$$

$$V_x = V_t - R_D i_t$$

$$\therefore i_t = g_{m2} v_t + \frac{V_t}{r_{o2}} - \frac{R_D i_t}{r_{o2}}$$

$$i_T \left(1 + \frac{R_D}{r_{o2}}\right) = v_t \left(g_{m2} + \frac{1}{r_{o2}}\right)$$

$$\frac{V_t}{i_t} = \frac{r_{o2} + R_D}{g_{m2} r_{o2} + 1}$$

$$\therefore A_v = -g_{m1} \left(r_{o1} \parallel \frac{r_{o2} + R_D}{1 + g_{m2} r_{o2}} \right)$$

(30) a) From Eq. (7.67)

$$|A_v| = \frac{R_D}{\frac{1}{g_m} + R_S},$$

$$4 = \frac{1000}{\frac{1}{g_m} + \frac{0.2V}{1mA}}$$

$$\frac{4}{g_m} + 800 = 1000$$

$$\therefore \; g_m = 20 \text{ mS}.$$

$$\therefore \; 20 \times 10^{-3} = \sqrt{2 \times (200 \times 10^{-6})(\tfrac{W}{L})(1 \times 10^{-3})}$$

$$\therefore \; W/L = 1000$$

To check if M_1 is in saturation:

$$V_{DS} = V_D - V_S$$
$$= [1.8 - (10^{-3} \times 1k)] - 0.2$$
$$= 0.6 \text{ V}$$

and $10^{-3} = \frac{1}{2}(200 \times 10^{-6})(1000)(V_{GS} - 0.4)^2$

$$V_{GS} = 0.5$$

$$\therefore \; V_{DS} > V_{GS} - V_t,$$

ie. transistor is in operation.

b)

$$g_m = \sqrt{2 \times (200\times10^{-6}) \times \left(\frac{50}{0.18}\right) \times 10^{-3}}$$

$$\approx 10.5 \text{ mS}$$

$$|A_v| = \frac{R_D}{\frac{1}{g_m} + R_S},$$

$$4 = \frac{R_D}{\frac{1}{10.5\times10^{-3}} + 200}$$

$$\therefore R_D \approx 1179\,\Omega.$$

To check if M_1 is in saturation:

$$V_{DS} = [1.8 - (1179 \times 10^{-3})] - 0.2$$

$$= 0.421$$

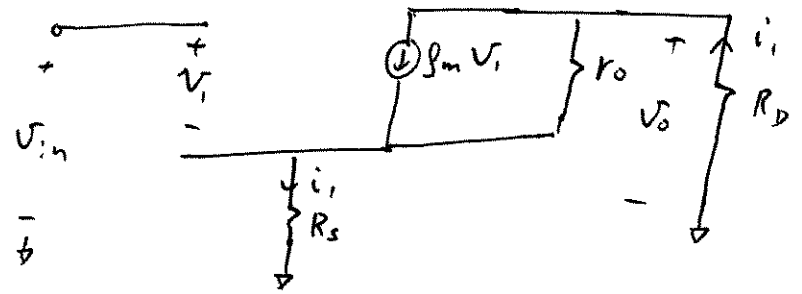
and $$10^{-3} = \frac{1}{2}(V_{GS} - 0.4)^2 (200\times10^{-6})\left(\frac{50}{0.18}\right)$$

$$V_{GS} \approx 0.590$$

$$\because \quad V_{DS} > V_{GS} - V_t,$$

Transistor is in saturation.

(31) The small signal model is:



$$v_o = -i_1 R_D \quad \text{——— (1)}$$

$$i_1 = g_m v_1 + \frac{v_o - v_1}{r_o}$$

$$= \frac{(g_m r_o - 1) v_1 + v_o}{r_o}$$

$$i_1 \approx g_m v_1 + \frac{v_o}{r_o}$$

$$\therefore -\frac{v_o}{R_D} = g_m v_1 + \frac{v_o}{r_o} \quad \text{——— (2)}$$

$$v_{in} = v_1 + i_1 R_s$$

$$\therefore v_1 = v_{in} + \frac{v_o}{R_D} R_s \quad \text{——— (3)}$$

(2) combines with (3):

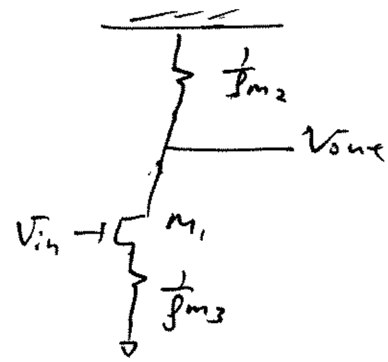
$$-\frac{v_o}{R_D} = g_m v_{in} + g_m v_o \frac{R_s}{R_D} + \frac{v_o}{r_o}$$

$$-v_o \left[\frac{1}{R_D} + g_m \frac{R_s}{R_D} + \frac{1}{r_o} \right] = g_m v_{in}$$

$$\therefore \text{Volt. gain} = \frac{v_o}{v_{in}} = -\left[\frac{g_m}{r_o + g_m R_s r_o + R_D}\right] (r_o R_D)$$

//

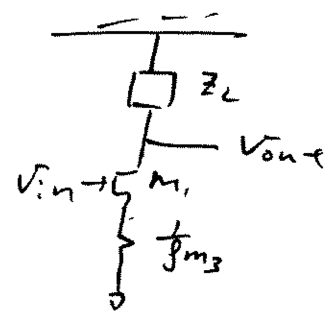
(32) a) Equivalent circuit is:



$$\therefore A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

b) Similar to Prob. 28(f),

Equivalent circuit is:

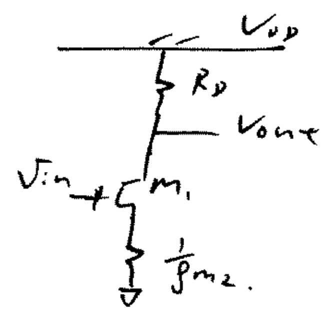


From Prob. 28(f),

$$Z_L = \frac{1}{g_{m2}} \quad (\text{as } r_{o2} \to \infty)$$

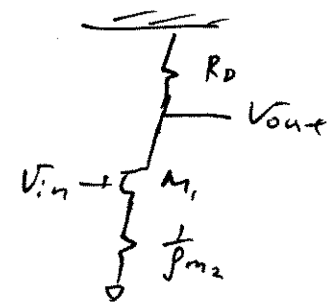
$$\therefore A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

c) Equivalent circuit is:



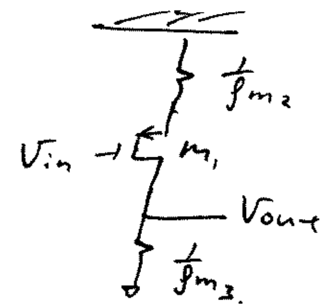
$$\therefore A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(d). Equivalent circuit is



$$A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(e) Equivalent circuit is



$$A_v = \frac{\frac{1}{g_{m3}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(33) a) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

b) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

c) From Eq. (7.71),

$$R_{out} = (1 + g_{m2} r_{o2}) \left(r_{o1} \parallel \frac{1}{g_{m3}} \right) + r_{o2}$$

d) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \left(r_{o2} \parallel \frac{1}{g_{m3}} \right) + r_{o1}$$

(34) To find $\left(\frac{W}{L}\right)$,

$$10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)(1-0.4)^2 \times (1+0.1\,V_{DS}),$$

where $V_{DS} = 1.8 - 1k\Omega \times 1mA$

$= 0.8V,$

$\therefore \left(\frac{W}{L}\right) \simeq 25.7$ //

Voltage gain, $(A_v) = -g_{m1}(r_{o1} // R_D)$

$$g_{m1} = \sqrt{2(200\times10^{-6})(25.7)\times10^{-3}\times(1+0.1\times0.8)}$$

$= 3.33\ mS.$

$$r_{o1} = \frac{1}{0.1\times10^{-3}}$$

$= 10k\Omega.$

$\therefore A_v = (-3.33\times10^{-3})(10k\Omega // 1k\Omega)$

$= -3.03$ //

(35) with $\lambda = 0$,

$$10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1-0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) \doteqdot 27.8$$

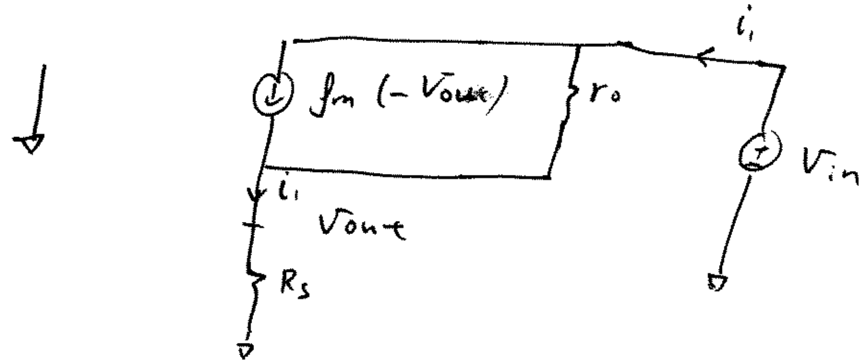
$$A_v = -g_{m1} R_D$$

$$= -\sqrt{2(200 \times 10^{-6})(27.8) \times 10^{-3}} \times 1000$$

$$= -3.33$$

Without r_o, gain increases due mainly to increase in load resistance.

(3b) The small-signal circuit is:



$$i_1 = \frac{V_{out}}{R_s} \quad \text{——— ①}$$

$$i_1 = g_m(-V_{out}) + \frac{V_{in} - V_{out}}{r_o} \quad \text{——— ②}$$

$$\therefore \frac{V_{out}}{R_s} = -g_m V_{out} + \frac{V_{in}}{r_o} - \frac{V_{out}}{r_o}$$

$$V_{out}\left(\frac{1}{R_s} + g_m + \frac{1}{r_o}\right) = \frac{V_{in}}{r_o}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{r_o}\left(\frac{R_s r_o}{r_o + g_m r_o R_s + R_s}\right)$$

$$= \frac{R_s}{g_m r_o R_s + r_o + R_s}$$

Since $(g_m r_o R_s + r_o) > 0$, the voltage gain < 1.

This is expected: Any variation in V_{in} causes minimal change in the bias current.

$\because V_{out}$ is determined largely by the amount of bias current ($\because V_{out}$ is set by V_{GS1})

$\therefore$ There is almost no variation in V_{out}. (ie. $\frac{V_{out}}{V_{in}} \ll 1$)

(37) a/ $|\text{Voltage gain}| = g_m R_D$

$$= 5$$

$$\therefore g_m = \frac{5}{500}$$

$$= 10\,mS.$$

$$= \sqrt{2(200\times10^{-6})\left(\frac{W}{L}\right)\times10^{-3}}$$

$$\therefore \frac{W}{L} = 250$$

b/ $V_D = 1.8 - 500\times10^{-3}$

$$= 1.3V$$

To obtain $V_{DS} \geq V_{GS} - V_{TH} + 0.2$,

$$V_D \geq V_G - 0.2$$

$$\therefore V_G \leq 1.5$$

Also, $I_{R_1+R_2} = 0.1\times10^{-3}A.$

$$\therefore R_1 + R_2 = \frac{1.8}{0.1\times10^{-3}}$$

$$= 18\,k\Omega.$$

choose $R_2 = 15\,k\Omega$ & $R_1 = 3\,k\Omega$

c) With twice of (W/L), M_1 will go further away from triode. As (W/L) doubles, & I_{bias} is fixed by the current source, V_{GS} is forced to decrease (so M_1 will have same I_{DS}). Thus, $(V_{GS} - V_{TH})$ decreases, and V_{DS} can be allowed to drop more before M_1 goes into triode.

Gain will be increased by $\sqrt{2}$, because gain $\propto g_m$, and $g_m \propto \sqrt{W/L}$.

(38) a) $V_G = 1.8V$.

$\therefore V_{D,min} = 1.8 - 0.4$ (for M_1 stays in saturation)

$= 1.4V$

$\therefore R_{D,max} = \frac{1.4V}{1mA}$

$= 1.4k\Omega$ //

b) $|\text{Voltage gain}| = g_m R_D$

$= 5$.

$\therefore g_m = \frac{5}{R_D}$

$= 3.57mS$.

$= \sqrt{2 \times (200 \times 10^{-6}) (\frac{W}{L}) \times 10^{-3}}$

$\therefore (\frac{W}{L}) = 31.9$ //

(39) To get $R_{in} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

volt gain $(A_v) = g_m R_D$

$$= 4,$$

$$\therefore R_D = \frac{4}{0.02}$$

$$R_D = 200\,\Omega$$

$$\because g_m = \sqrt{2\times(200\times10^{-6})\left(\frac{W}{L}\right)\times0.5\times10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) = 2000$$

(40) To get $R_{in} = 50\Omega$,

$$g_m = \frac{1}{50}$$

$$= 20\,mS.$$

Voltage gain $(A_v) = g_m R_D$

$$g_m = \sqrt{2\times(200\times10^{-6})\left(\frac{W}{L}\right)\times 0.5\times10^{-3}}$$

$$\therefore \frac{W}{L} = 2000.$$

$\because$ $V_G = V_B = 1V$,

$$V_{D,min} = V_G - V_{TH}$$

$$= 0.6V$$

$$\therefore R_{D,max} = \frac{1.8 - 0.6}{0.5\times10^{-3}}$$

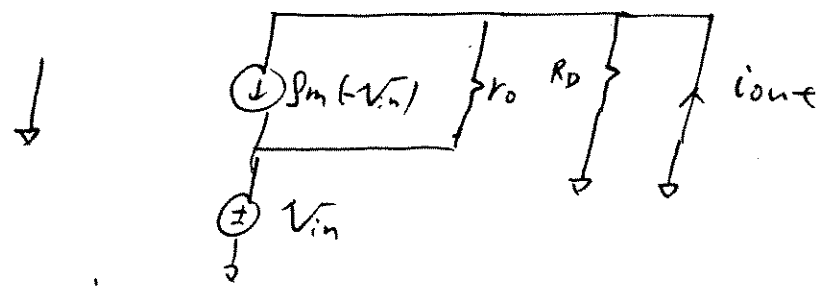
$$= 2400$$

$\therefore$ max. Voltage gain $= 0.02 \times 2400$

$$= 48$$

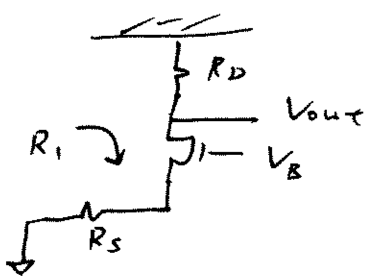
(41) Voltage gain $(A_v) = G_m R_{out}$,
where G_m and R_{out} are the transconductance and output resistance of the circuit respectively.

To find G_m:



$$G_m = \frac{i_{out}}{V_{in}} = g_m + \frac{1}{r_o}$$
$$\simeq g_m \quad (\because g_m r_o >> 1)$$

To find R_{out}:



$$R_{out} = R_D \,//\, R_1$$
$$= R_D \,//\, [(1+g_m r_o) R_s + r_o]$$
(from Eq. (7.110))
$$\simeq R_D \,//\, (g_m r_o R_s + r_o) \quad (\because g_m r_o >> 1)$$
$$= \frac{g_m r_o R_s R_D + r_o R_D}{R_D + g_m r_o R_s + r_o}$$

$$\therefore \text{Voltage gain} = g_m \left[\frac{g_m r_o R_D R_S + r_o R_D}{R_D + g_m r_o R_S + r_o} \right]$$

(42) a) To get $R_{in} = 50\,\Omega$,

$$g_m = \frac{1}{50} = 20\,mS.$$

To get $R_{out} = 500\,\Omega$,

$$R_D = 500\,\Omega. \quad (\because r_o = \infty)$$

$$\because \quad V_{D,min} = 1.8 - 0.4 = 1.4\,V$$

$$\therefore \quad I_{D,max} = \frac{1.8 - 1.4}{500} = 0.8\,mA$$

b) $$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) \times 0.8 \times 10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) = 1250$$

c) $$\text{Voltage gain} = 0.02 \times 500 = 10$$

(43) a) To place M_1 100mV away from triode,

$$V_{D,min} = V_G - V_{TH} + 0.1V$$
$$= 1.5V.$$

$$\therefore R_D = \frac{(1.8 - 1.5)V}{1mA}$$

$$= 300\,\Omega$$

b) Voltage gain $= g_m R_D$

$$\therefore g_m = \frac{5}{300}$$

$$\because \quad g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) 10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) \approx 694$$

(44) a) Voltage gain $(A_v) = \left[\dfrac{\frac{1}{g_{m1}}}{R_S + \frac{1}{g_{m1}}}\right] \dfrac{g_{m1}}{g_{m2}}$

$$= \frac{g_{m1}/g_{m2}}{1 + g_{m1}R_S}$$

b) Voltage gain $(A_v) = g_{m1} Z_2$

(similar to prob. 32(b))

$$= \frac{g_{m1}}{g_{m2}}$$

c) Voltage gain $= \left[\dfrac{\frac{1}{g_{m1}} // R_1}{R_S + \frac{1}{g_{m1}} // R_1}\right] \dfrac{g_{m1}}{g_{m2}}$

d) Voltage gain $= g_{m1}\left[R_D + r_{o3} // \frac{1}{g_{m2}}\right]$,

$\because r_{o3} = \infty$,

gain $= g_{m1}\left[R_D + \frac{1}{g_{m2}}\right]$

e) Voltage gain $= g_{m1}\left[R_D + \frac{1}{g_{m2}}\right]$

(45) a) $$\frac{V_x}{V_{in}} = -g_{m1}\left[R_{D1} \,//\, \frac{1}{g_{m2}}\right]$$

$$\frac{V_{out}}{V_x} = g_{m2} R_{D2}$$

$$\therefore \frac{V_{out}}{V_{in}} = -(g_{m2} R_{D2})\left[g_{m1}\left(R_{D1} \,//\, \frac{1}{g_{m2}}\right)\right]$$

b) if $R_{D1} \rightarrow \infty$,

$$\frac{V_{out}}{V_{in}} = (-g_{m2} R_{D2})\left(\frac{g_{m1}}{g_{m2}}\right)$$

$$= -g_{m1} R_{D2}$$

This is expected, because the circuit reduces to a cascode stage.

(∴ gain is the same as that of a cascode stage.)

(46) $$\frac{v_x}{v_{in}} = \left(R_{D1} \parallel \frac{1}{g_{m2}}\right) g_{m1}$$

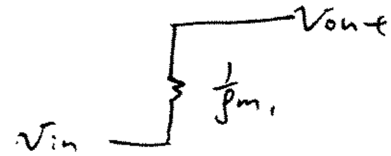
$$\frac{v_{out}}{v_x} = g_{m2} R_{D2}.$$

$$\therefore \frac{v_{out}}{v_{in}} = g_{m1} g_{m2} R_{D2} \left(R_{D1} \parallel \frac{1}{g_{m2}}\right)$$

Similar to prob. (45), voltage gain approaches that of cascode stage as R_{D1} approaches infinity. The gain is $g_{m1} R_{D2}$.

(47) with $\lambda = 0$, M_1 appears as a diode-connected device.

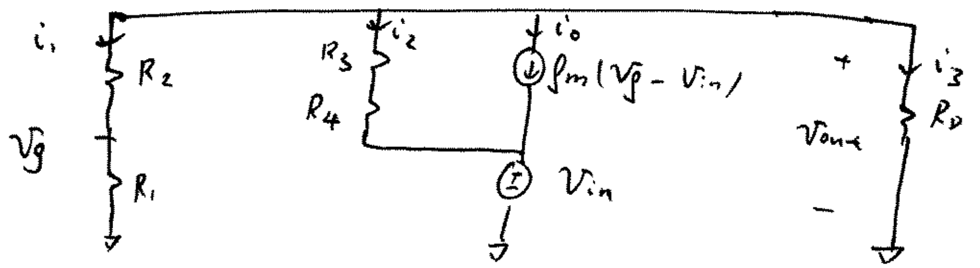
$\therefore$ the circuit becomes :



ie. $\frac{V_{out}}{V_{in}} = 1$ //

This is not a common-gate (CG) amplifier, because the gate is not fixed. (ie. gate is not at an "a.c. ground")

(48) The small-signal model is:



$$\because -i_0 = i_1 + i_2 + i_3$$

$$-g_m(V_g - V_{in}) = \frac{V_{out}}{R_2 + R_1} + \frac{V_{out} - V_{in}}{R_3 + R_4} + \frac{V_{out}}{R_D}$$

$$g_m\left(V_{in} - \frac{R_1}{R_1+R_2} V_{out}\right) = V_{out}\left(\frac{1}{R_1+R_2} + \frac{1}{R_3+R_4} + \frac{1}{R_D}\right) - \frac{V_{in}}{R_3+R_4}$$

$$V_{in}\left(g_m + \frac{1}{R_3+R_4}\right) = V_{out}\left(\frac{g_m R_1 + 1}{R_1+R_2} + \frac{1}{R_3+R_4} + \frac{1}{R_D}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{\left(g_m + \frac{1}{R_3+R_4}\right)}{\frac{1}{R_D} + \frac{1}{R_3+R_4} + \frac{g_m R_1 + 1}{R_1 + R_2}}$$

(49) Voltage gain $(A_v) = \dfrac{r_o // R_S}{\frac{1}{g_m} + r_o // R_S}$

To find I_{DS},

$$I_{DS} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)(1.8 - V_S - 0.4)^2$$

$$= 0.0111\ (1.4 - I_{DS} \times 1000)^2.$$

$$11100\, I_{DS}^2 - 32.08\, I_{DS} + 0.021756 = 0$$

$$\therefore I_{DS} = 1.80\,mA \text{ or } 1.08\,mA.$$

Reject $I_{DS} = 1.80\,mA$.

($\because V_S = 1.80\,V > V_{DD}$)

$$\therefore g_m = \sqrt{2 \times (200 \times 10^{-6}) \times 1.08 \times 10^{-3}}$$

(ignore channel-length modulation)

$$g_m = 0.659\,mS$$

$$r_o = \frac{1}{0.1 \times 1.08 \times 10^{-3}} \approx 9260\,\Omega$$

$$\therefore A_v = \frac{9260\,\Omega // 1000\,\Omega}{\frac{1}{0.659\,mS} + 9260\,\Omega // 1000\,\Omega}$$

$$\approx 0.372$$

(50)

$$A_v = \frac{R}{\frac{1}{g_m} + R}$$

$$= 0.8$$

$$\therefore 0.8 = \frac{500}{\frac{1}{g_m} + 500}$$

$$\frac{0.8}{g_m} + 400 = 500$$

$$\therefore g_m = 8\,mS.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6}) \times \left(\frac{30}{0.18}\right) I_{DS}}$$

$$\therefore I_{DS} = 0.96\,mA$$

$$\therefore V_S = 0.96 \times 10^{-3} \times 500$$

$$= 480\,mV$$

To find V_G:

$$0.96 \times 10^{-3} = \frac{1}{2} (200 \times 10^{-6}) \left(\frac{30}{0.18}\right) (V_G - 0.48 - 0.4)^2$$

$$\therefore V_G = 1.12\,V$$

(51)

$$A_v = \frac{R_s}{\frac{1}{g_m} + R_s}$$

$$= 0.8$$

$$0.8 = \frac{500}{\frac{1}{g_m} + 500}$$

$$\therefore g_m = 8\,mS.$$

$$\because I_{ds} = \frac{1}{2}\beta\,(V_{GS} - V_t)^2,$$

where $\beta = \left(\frac{W}{L}\right)\mu_n C_{ox}$

and $g_m = \beta\,(V_{GS} - V_t)$.

$$\therefore I_{ds} = \frac{1}{2} g_m\,(V_{GS} - V_t)$$

$$= \frac{1}{2} g_m\,(1.8 - I_{ds}(500) - 0.4)$$

$$I_{ds} = 4\times10^{-3}\,(1.4 - 500\,I_{ds})$$

$$\therefore I_{ds} = 1.87\,mA.$$

$$\because g_m = \sqrt{2\,(200\times10^{-6})\,\frac{W}{L}\times1.87\times10^{-3}}$$

$$\therefore \frac{W}{L} \approx 85.7$$

(52). To get $R_{out} = 100\,\Omega$,

$$\frac{1}{g_m} = 100$$

$$\therefore g_m = 10\ mS.$$

$$\because \quad I_{ds} = \frac{1}{2}\beta(V_{GS} - V_{TH})^2.$$

where $\beta = \mu_n C_{ox} \frac{W}{L}$.

and $g_m = \beta\,(V_{GS} - V_{TH})$

$$\therefore \quad I_{ds} = \frac{1}{2} g_m (V_{GS} - V_{TH})$$

$$= \frac{1}{2}(10 \times 10^{-3})(0.9 - 0.4)$$

$$\therefore I_{ds} = 2.5\ mA.$$

$$\because \quad g_m = \sqrt{2 \times (200 \times 10^{-6})\left(\frac{W}{L}\right)(2.5 \times 10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 100$$

(53). To get $R_{out} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 2 \times 10^{-3}\,W.$$

$$\therefore I_{DS} = 1.11\,mA.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6})\left(\frac{W}{L}\right)(1.11mA)}$$

$$\therefore \frac{W}{L} = 900$$

(54)

$$A_v = \frac{R_L}{\frac{1}{g_m} + R_L}$$

$$\therefore 0.8 = \frac{50}{\frac{1}{g_m} + 50}$$

$$g_m = 80 \text{ mS}$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 3 \text{ mW}$$

$$\therefore I_{DS} = 1.67 \text{ mA}$$

$$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) (1.67 \times 10^{-3})}$$

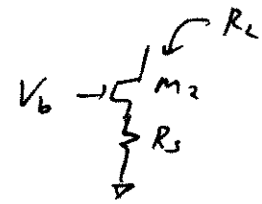
$$\therefore \left(\frac{W}{L}\right) = 9600$$

(55)

a) $$A_v = \frac{r_{o1} \,//\, (R_s + r_{o2})}{\frac{1}{g_{m1}} + r_{o1} \,//\, (R_s + r_{o2})}$$

b) $$A_v = \frac{r_{o1} \,//\, R_L}{\frac{1}{g_{m1}} + (r_{o1} \,//\, R_L)}$$

where R_L is :



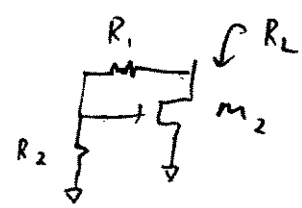
$$R_L = (1 + g_{m2} r_{o2}) R_s + r_{o2} \quad \text{Eq (7.110)}$$

$$\therefore A_v = \frac{r_{o1} \,//\, [(1 + g_{m2} r_{o2}) R_s + r_{o2}]}{\frac{1}{g_{m1}} + r_{o1} \,//\, [(1 + g_{m2} r_{o2}) R_s + r_{o2}]}$$

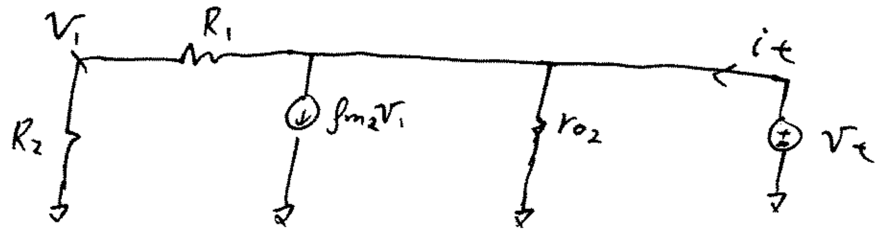
c) $$A_v = \frac{r_{o1} \,//\, \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + (r_{o1} \,//\, \frac{1}{g_{m2}})}$$

d) $$A_v = \frac{r_{o1} \,//\, R_L}{\frac{1}{g_{m1}} + (r_{o1} \,//\, R_L)}$$

where R_L is :



(c) Finding R_L with small-signal model:
(cont'd)



$$R_L = \frac{v_t}{i_t},$$

where
$$i_t = \frac{v_t}{r_{o2}} + g_{m2} v_1 + \frac{v_t}{R_1 + R_2}$$

$$= \frac{v_t}{r_{o2}} + \frac{g_{m2} R_2 v_t}{R_1 + R_2} + \frac{v_t}{R_1 + R_2}$$

$$\therefore R_L = \frac{r_{o2}(R_1 + R_2)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}$$

$$\therefore A_v = \frac{r_{o1} \mathbin{//} \dfrac{r_{o2}(R_1 + R_2)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}}{\dfrac{1}{g_{m1}} + r_{o1} \mathbin{//} \dfrac{r_{o2}(R_2 + R_1)}{R_2 + R_1 + r_{o2} + g_{m2} r_{o2} R_2}}$$

e) $$A_v = \frac{r_{o2} \mathbin{//} \left(\frac{1}{g_{m1}} \mathbin{//} r_{o3}\right)}{\frac{1}{g_{m2}} + r_{o2}\left(\frac{1}{g_{m1}} \mathbin{//} r_{o3}\right)}$$

f) $$A_v = \frac{r_{o1} \mathbin{//} [(1 + g_{m2} r_{o2}) r_{o3} + r_{o2}]}{\frac{1}{g_{m1}} + \{r_{o1} \mathbin{//} [(1 + g_{m2} r_{o2}) r_{o3} + r_{o2}]\}}$$

(5 6)

$$\frac{V_x}{V_{in}} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

$$\frac{V_{out}}{V_x} = g_{m2} R_D$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

b) if $g_{m1} = g_{m2}$,

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} R_D}{2}$$

(57) $\because R_{out} = 1k\Omega.$

$\therefore R_D = 1k\Omega.$

$\because A_v = 5$

$= g_{m1} R_D$

$\therefore g_{m1} (1000) = 5$

$g_{m1} = 5mS.$

$\because M_1$ is 00 mV away from triode,

$V_D = (V_G - V_{TH}) + 0.1.$

$V_D = (1.8 - 0.4) + 0.1$

$V_D = 1.5V$

$$\therefore I_{DS} = \frac{1.8 - 1.5}{R_D} = \frac{0.3}{R_D}$$

$= 0.3mA$

$$\because g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) I_{DS1}}$$

$\therefore \left(\frac{W}{L}\right) \approx 208$

$\therefore R_D = 1k\Omega$, $R_G = 10k\Omega$, $\left(\frac{W}{L}\right) = 208$

(58) $\because$ Power $(P) = 2mW$,

$$\therefore I_{DS} = \frac{2 \times 10^{-3}}{1.8}$$

$$= 1.11 \ mA.$$

$\because R_D I = 1$

$$\therefore R_D = 900 \Omega.$$

$\because |\text{gain } (A_v)| = 5$,

$$g_m R_D = 5$$

$$g_m = 5.56 \ mS.$$

$$\because g_m = \sqrt{2(200 \times 10^{-6})\left(\frac{W}{L}\right)(1.11 \times 10^{-3})}$$

$$\frac{W}{L} \approx 69.4$$

(58) $|A_v| = g_m R_L$.

$\therefore$ To achieve maximum gain, use maximum R_L

i.e. set $R_D = 500\,\Omega$.

For maximum g_m, use maximum I_{DS}

(while keeping M_1 in saturation),

i.e. $V_D \geq V_G - V_{TH}$

$1.8 - (I_{DS})(500) \geq 1.8 - 0.4$,

$\therefore$ $I_{DS} \leq \frac{0.4}{500}$

$I_{DS,max} = 0.8\,mA$.

<u>Note</u>: Setting a large R_D in this case would force $I_{DS,max}$ to be lower (in order to keep M_1 in saturation).

But since $A_v \propto R_D$, while $A_v \propto \sqrt{I_{DS}}$, sacrificing I_{DS} to get higher R_D would yield a higher gain.

(60) $\because$ Power $(P) = 2mW$,

$$\therefore I_{DS} = (0.95)\left(\frac{2\times10^{-3}}{1.8}\right)$$

(assuming $(R_1 + R_2)$ consumes 5% of total power)

$$I_{DS} = 1.06mA$$

$$\therefore R_S = \frac{0.2\,V}{1.06\,mA}$$

$$\approx 189\,\Omega$$

$$\because g_m = \beta V_{eff}$$

(where $\beta = \mu_n C_{ox}\left(\frac{W}{L}\right)$, $V_{eff} = V_{GS} - V_{TH}$)

and $I_{DS} = \frac{1}{2}\beta V_{eff}^2$

$$\therefore I_{DS} = \frac{1}{2} g_m V_{eff}.$$

Set $V_{eff} = 0.1\,V$ (< maximum allowable overdrive)

$$1.06\times10^{-3} = \frac{1}{2} g_m (0.1)$$

$$g_m = 21.2\ mS.$$

$$\because |A_V| = \frac{g_m R_D}{1 + g_m R_S} = 4.$$

$$\therefore \frac{21.2\times10^{-3}\times R_D}{1+(21.2\times10^{-3})\times189} = 4$$

$$R_D \approx 947\ \Omega.$$

With $V_{GS} - V_{TH} = 0.1V$,

$$V_{GS} = 0.1 + 0.4V$$
$$= 0.5V$$
$$= V_G - V_S$$

$\therefore V_G - 0.2V = 0.5V$

$\therefore V_G = 0.7V$

To find R_1 & R_2,

$\because I_{R_1+R_2} = (0.05)\left(\frac{2\times10^{-3}}{1.8}\right)$

$= 5.56\times10^{-5}$ A.

$\therefore R_1 + R_2 = \frac{1.8V}{5.56\times10^{-5}A}$

$= 32.4\,k\Omega$.

$$V_G = \frac{R_2}{R_1+R_2} \times 1.8 = 0.7V$$

$\therefore R_2 = 12.6\,k\Omega$,

$R_1 = (32.4 - 12.6)\,k\Omega = 19.8\,k\Omega$.

To find $\left(\frac{W}{L}\right)$,

$$g_m = \sqrt{2\times200\times10^{-6}\times\left(\frac{W}{L}\right)\times1.06\times10^{-3}} = 21.2\,mS$$

$\therefore \left(\frac{W}{L}\right) = 1060$

$\therefore R_1 = 19.8k$, $R_2 = 12.6k$, $R_S = 189\Omega$, $R_D = 947\Omega$

$\left(\frac{W}{L}\right) = 1060$, $I_{DS} = 1.06$ mA

(61)

Power $(P) = 6\,mW$

$\therefore I_{DS} = (0.95)\left(\frac{6\times10^{-3}}{1.8}\right) = 3.17\,mA$

Gain $(A_v) = 5,$

$\therefore \frac{g_m R_D}{1+g_m R_S} = 5$

$5 = (R_D - 5R_S)\,g_m$

for g_m to be positive,

$R_D > 5R_S$, i.e. $R_S < 50\,\Omega$

choose $R_S = 30\,\Omega$

$\because V_{ov}$ (over drive voltage) $= V_{RS}$

$\therefore V_{ov} = 3.17\times10^{-3} \times 30$

$= 95.1\,mV$.

From $A_v = \frac{g_m R_D}{1+g_m R_S} = 5,$

$g_m = 100\,mS$

$\because g_m = (\mu_n C_{ox})\left(\frac{W}{L}\right) V_{ov}$

$\therefore \left(\frac{W}{L}\right) \approx 5260$

To find R_1 and R_2,

$I_{R_1+R_2} = (0.05)\left(\frac{6\times10^{-3}}{1.8}\right) = 0.167\,mA$

$\therefore R_1 + R_2 = \frac{1.8}{0.167\times10^{-3}} = 10.8\,k\Omega$

$\because V_{GS} - V_{TH} = V_{OV} = 95.1mV,$

and $V_S = 95.1mV,$

$\therefore (V_G - 95.1mV) - 0.4 = 95.1mV$

$$V_G = 0.5902$$

$$\because V_G = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$\therefore R_2 = 3.54\,k\Omega$$

and $R_1 = 10.8k\Omega - 3.54\,k\Omega$

$= 7.26\,k\Omega$

$\therefore R_1 = 7.26k\Omega, \quad R_2 = 3.54k\Omega, \quad R_S = 30\Omega$

$\left(\frac{W}{L}\right) = 5260 \quad I_{DS} = 3.17\,mA.$

(62) Power $(P) = 2mW$.

$\therefore I_{DS} = \frac{2mW}{1.8V}$

$= 1.11 mA$.

$\because M_1$ operates 200 mV away from triode

$V_{DS} = (V_{GS} - V_{TH}) + 0.2$

$\therefore V_D = 1.6V$

$R_D = \frac{V_{RD}}{1.11 \times 10^{-3} A} = \frac{(1.8 - 1.6)V}{1.11 \times 10^{-3} A}$

$= 180 \Omega$

$\because$ gain $(A_v) = \frac{g_m R_D}{1 + g_m R_S} = 6$,

$\therefore 6 = (R_D - 6R_S) g_m$,

for $g_m > 0$, $R_D - R_S > 0$., i.e. $R_S < 30\Omega$.

Set $R_S = 20\Omega$,

$g_m = \frac{6}{180 - 6 \times 20} = 100 mS$.

$\because g_m = (\mu_n C_{ox})(\frac{W}{L})(V_{GS} - V_{TH})$

$0.1 = 200 \times 10^{-6} (\frac{W}{L})(1.8 - 1.11 \times 10^{-3} \times 20 - 0.4)$

$\therefore \frac{W}{L} \approx 363$

$$R_{in} = \frac{1}{sC_1} + R_1$$

$\because \frac{1}{sC_1}$ is negligible.

$$R_{in} = R_1 = 20\,k\Omega.$$

To make $\frac{1}{sC_1}$ negligible,

$$\frac{1}{sC_1} \ll R_1$$

$$\frac{1}{2\pi(10^6)C_1} \ll$$

$$\therefore C_1 \ll 7.96\,pF$$

Set $C_1 = 0.796\,pF$

To make $\frac{1}{sC_s}$ negligible,

$$\frac{1}{sC_s} \ll R_s \,//\, \frac{1}{g_{m_1}}$$

$$\frac{1}{2\pi(10^6)C_s} \ll 20 \,//\, \frac{1}{100\,ms}$$

$$C_s \ll 23.9\,nF$$

Set $C_s = 2.39\,nF$

$\therefore R_D = 180\,\Omega$, $R_s = 20\,\Omega$, $R_1 = 20\,k\Omega$, $\frac{W}{L} = 363$

$C_1 = 0.796\,pF$, $C_s = 2.39\,nF$

(63). Power $(P) = 2mW$.

$\therefore I_{DS1} = |I_{DS2}| = \dfrac{2mW}{1.8V} = 1.11mA$.

$r_{o1} = r_{o2} = \dfrac{1}{\lambda I_{DS}}$

$= \dfrac{1}{0.1 \times 1.11 \times 10^{-3}}$

$= 9000\,\Omega$.

Gain $(A_v) = g_{m1}(r_{o1} // r_{o2}) = 20$,

$g_{m1}\left(\dfrac{9000}{2}\right) = 20$.

$\therefore g_{m1} = 4.44\ mS$.

Set V_{DS1} (i.e. V_{out}) $= 1.2V$

(which is $< 1.5V$)

$\therefore V_{IN} = V_{GS1} \leqslant 1.2 + V_{TH}$

(for M_1 to stay in saturation)

Set $V_{GS1} = 1.2V$

$\because g_{m1} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH})$

$\left(\frac{W}{L}\right)_1 = 27.75$

For M_2, $\because M_2$ must be in saturation

for $V_{out} \leqslant 1.5V$.

$\therefore V_{DD} - V_B \leqslant V_{DD} - 1.5V + V_{TH}$.

$\therefore V_B \geqslant 1.1V$

Set $V_B = 1.2V$

$$|I_{DS2}| = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (|V_{GS2}| - V_{TH})^2 (1 + \lambda |V_{DS2}|)$$

$$1.11 \times 10^{-3} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_2 (0.6 - 0.2)^2 (1 + 0.1 \times (1.8 - 1.5))$$

(assuming $V_{out} = 1.5V$)

$$\therefore \left(\frac{W}{L}\right)_2 \approx 135$$

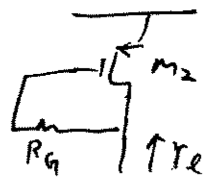
$\therefore \left(\frac{W}{L}\right)_1 = 27.75 \qquad \left(\frac{W}{L}\right)_2 = 135$

$V_{IN} = 1.2 \qquad V_b = 1.1$

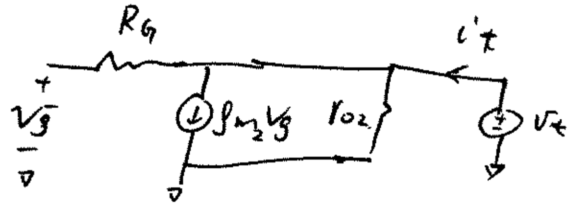
$I_{DS1} = I_{DS2} = 1.11mA$

(64) a) gain $(A_v) = -g_{m1}\, r_{o1} \,//\, r_e$,

where R_L is:



The small-signal model is:



$$r_e = \frac{v_t}{i_t} = r_{o2} \,//\, \frac{1}{g_{m2}}$$

$$\therefore \quad A_v = -g_{m1} \quad r_{o1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}$$

b) Power $= 3\,mW$

$$\therefore \; I_{DS1} = |I_{DS2}| = \frac{3\,mW}{1.8\,V}$$

$$= 1.67\,mA.$$

$$V_{OUT} = V_{G2} = \frac{V_{DD}}{2}$$

$$\therefore \; V_{GS2} = -0.9\,V$$

$$I_{DS2} = \frac{1}{2}(100\times 10^{-6})\left(\frac{W}{L}\right)_2 \times (|-0.9| - V_{TH})^2 \times \left(1 + 0.1 \times \frac{V_{DD}}{2}\right)$$

$$\therefore \left(\frac{W}{L}\right)_2 \approx 122.$$

$$g_{m2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH})$$

$$= 6.1\,mS.$$

From (a), $|A_v| = \frac{1}{g_{m1}} \times (r_{o1} // r_{o2} // \frac{1}{g_{m2}})$

$$\because r_{o1} = \frac{1}{0.1 \times 1.67 \times 10^{-3}} = 6000\,\Omega$$

$$r_{o2} = \frac{1}{0.2 \times 1.67 \times 10^{-3}} = 3000\,\Omega$$

$$\therefore 15 = g_{m1} (6000 // 3000 // \frac{1}{6.1mS})$$

$$g_{m1} = 99\ mS$$

$$g_{m1} = \sqrt{2\left(\frac{W}{L}\right)_1 \mu_n C_{ox} I_{DS1}}$$

$$\therefore \left(\frac{W}{L}\right)_1 = 14672$$

$\therefore \left(\frac{W}{L}\right)_1 = 14672$. $\left(\frac{W}{L}\right)_2 = 122$, $I_{DS1} = |I_{DS2}| = 1.67mA$.

(65) a) Impedance looking into drain of M_2

$$= (1 + g_{m2} r_{o2}) R_S + r_{o2}$$

$$= 10 r_{o1}$$

Assume $g_{m2} r_{o2} >> 1$,

$\therefore g_{m2} r_{o2} R_S + r_{o2} \approx 10 r_{o1}$

$\because r_{o1} = r_{o2}$ $(\lambda_1 = \lambda_2$ and $I_{DS1} = |I_{DS2}|)$

$\therefore g_{m2} R_S + 1 = 10$

$g_{m2} R_S = 9$ ——— (1)

Given $V_B = 1V$,

set $|V_{GS2}| = 0.6V$, (ie. $V_{GS2} - V_{TH} = 0.2V$)

$\therefore V_{S2} = 1.6V$,

$\therefore V_{R_S} = 1.8V - 1.6V = 0.2V$

$\because$ Power $= 2mW$

$I_{DS1} = |I_{DS2}| = \frac{2mW}{1.8V} = 1.11mA$.

$\therefore R_S = \frac{V_{R_S}}{1.11 \times 10^{-3}} \approx 180\Omega$ //

From (1), $g_{m2} = \frac{9}{180} = 50 mS$.

$\because g_{m2} = (\frac{W}{L})_2 (100 \times 10^{-6}) (V_{GS2} - V_{TH})$

$\therefore (\frac{W}{L})_2 = 2500$ //

b). $|\text{gain } (A_v)| = g_{m1} (r_{o1} \parallel 10 r_{o1})$

$$30 = g_{m1} (0.909 \, r_{o1})$$

$$r_{o1} = \frac{1}{0.1 \times 1.11 \times 10^{-3}}$$

$$= 9009 \, \Omega$$

$$\therefore g_{m1} = 3.66 \text{ mS.}$$

$$\because g_{m1} = \sqrt{2(\mu_n C_{ox})\left(\frac{W}{L}\right)_1 \times I_{DS1}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 30.2$$

(66) Power = 1mW

$\therefore I_{DS1} = I_{DS2} = \dfrac{1mW}{1.8V} = 0.556\,mA.$

Volt. gain $(A_v) = -\dfrac{g_{m1}}{g_{m2}} = -\sqrt{\dfrac{(W/L)_1}{(W/L)_2}}$

$= -4$

set $V_{GS1} = V_{GS2} = \dfrac{V_{DD}}{3}$.

$\because I_{DS1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH})^2$

$\therefore \left(\frac{W}{L}\right)_1 = 139$

$\therefore \left(\frac{W}{L}\right)_2 = \dfrac{139}{16}$

$\approx 8.69.$

and $V_{IN} = \dfrac{V_{DD}}{3} = 0.6V$

(67) $R_{in} = \frac{1}{g_{m1}} = 50\Omega$

$\therefore g_{m1} = 20\,mS$

Volt. gain $(A_v) = g_m R_D = 5$

$\therefore R_D = 250\Omega$

Power $= 3\,mW$

$\therefore I_{DS1} = \frac{3\,mW}{1.8\,V}$

$= 1.67\,mA.$

$\because g_{m1} = \sqrt{2 \times \mu_n C_{ox} \times (\frac{W}{L})_1 I_{DS1}}$

$(\frac{W}{L})_1 = 600$

$\therefore R_D = 250\Omega, \quad (\frac{W}{L})_1 = 600 \quad I_{DS} = 1.67\,mA.$

(68) Power (P) = 2 mW

$$I_{DS_1} = \frac{2\,mW}{1.8\,V} = 1.11\,mA.$$

$\because$ M_1 operates 100 mV away from triode,

$$V_{DS} = V_{GS} - V_{TH} + 0.1$$

$$V_D = 1.8 - 0.4 + 0.1 = 1.5\,V$$

$$\therefore R_D = \frac{1.8 - 1.5}{1.11 \times 10^{-3}} \approx 270\,\Omega$$

Volt. gain (A_v) $= g_{m_1} R_D = 4$

$$\therefore g_{m_1} = 14.8\,mS$$

$$\because I_{DS} = \frac{1}{2} g_{m_1} \times (V_{GS_1} - V_{TH})$$

$$V_{GS} \approx 0.550\,V$$

Set $V_G = 0.9\,V$, $\therefore V_S = (0.9 - 0.55)\,V = 0.35\,V$

$$R_S = \frac{0.35}{1.11 \times 10^{-3}} \approx 315\,\Omega$$

To find $\left(\frac{W}{L}\right)_1$: $g_{m_1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{TH})$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 135$$

$\therefore \left(\frac{W}{L}\right)_1 = 135$, $V_{IN} = 0.9\,V$, $R_S = 315\,\Omega$, $R_D = 270\,\Omega$

$I_{DS} = 1.11\,mA$

(69)

Power $= 5\,mW$

$\therefore I_{DS1} = \dfrac{5\times10^{-3}}{1.8} = 2.78\,mA$

Gain $(A_v) = g_m R_D = 5$

$V_{G1} = V_{OUT} = 1.8 - IR_D$

$V_{S1} = I R_S$

Let $R_S = \dfrac{10}{g_{m1}}$

$\therefore V_{S1} = \dfrac{10I}{g_{m1}}$

$\therefore V_{GS1} = 1.8 - IR_D - \dfrac{10I}{g_{m1}}$

$\because I_{DS} = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$2.78\times10^{-3} = \dfrac{g_m}{2}\left(1.8 - 2.78\times10^{-3} R_D - \dfrac{2.78\times10^{-2}}{g_m}\right)$

$= 0.9\,g_m - 1.39\times10^{-3} g_m R_D - 1.39\times10^{-2}$

$\because g_m R_D = A_v = 5$

$2.78\times10^{-3} = 0.9\,g_m - 6.95\times10^{-3} - 1.39\times10^{-2}$

$\therefore g_m \approx 26.3\,mS$

and $R_D = \dfrac{5}{26.3\times10^{-3}} \approx 190\,\Omega$

$R_S = \dfrac{10}{26.3\times10^{-3}} = 380\,\Omega$

$\because g_m = \sqrt{2\mu_n C_{ox}(\frac{W}{L}) I_{DS1}} \Rightarrow (\frac{W}{L}) \approx 622$

(70) $\because R_s \approx \frac{10}{g_m}$,

$\therefore R_{in} \approx \frac{1}{g_m} = 50\,\Omega$

i.e. $g_m = 20\,mS$ //

$$|\text{gain } (A_v)| = \frac{g_m R_D}{1 + g_m R_s} = 4$$

$$g_m R_D = 4 + 4 g_m R_s$$

$$R_D = \frac{4 + 0.08 R_s}{0.02} = 200 + 4R_s \quad \text{—(1)}$$

$\because R_s I_D + V_{GS} - V_{TH} + 0.25 = 1.8 - I_D R_D$ (given)

and $I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

i.e. $V_{GS} - V_{TH} = 100 I_D$

$\therefore R_s I_D + 100 I_D + 0.25 = 1.8 - I_D R_D$

From (1):

$$R_s I_D + 100 I_D + 0.25 = 1.8 - 200 I_D - 4 I_D R_s$$

$$5 R_s I_D + 300 I_D = 1.55$$

Set $R_s = \frac{10}{g_m} = 500\,\Omega$

$\therefore 2500 I_D + 300 I_D = 1.55$

$\therefore I_D = 0.554\,mA$ //

$\because I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$$0.554 \times 10^{-3} = \frac{1}{2} \times 20 \times 10^{-3} (V_{GS} - 0.4)$$

$\therefore V_{GS} = 0.455\,V$

To find $(\frac{W}{L})$:

$$g_m = \sqrt{2\,(\frac{W}{L})\,\mu_n C_{ox}\, I_{DS}}$$

$$\therefore (\frac{W}{L}) \approx 1805$$

To find R_D:

$$\because R_D = 200 + 4R_S \qquad \text{(from (1))}$$

$$R_D = 2200$$

To find R_1 and R_2,

$$\because R_1 + R_2 = 20\,k\Omega$$

and $V_{GS} = V_G - I_D R_S = 0.455V$

i.e. $V_G = 0.732\,V$

$$V_G = \frac{R_2}{R_1 + R_2} \times V_{DD}$$

$$\therefore R_1 = 8133\,\Omega$$

$$R_2 \approx 11.9\,k\Omega.$$

$\therefore R_1 = 8133\,\Omega$, $R_2 = 11.9\,k\Omega$, $R_D = 2200\,\Omega$ $R_S = 500\,\Omega$

$(\frac{W}{L}) = 1805$ $I_{DS} = 0.554\,mA$.

(7)

$R_{in} = R_G = 10\,k\Omega$

Power $= 2mW$

$\therefore I_{DS} = \dfrac{2mW}{1.8V} = 1.11mA$

$A_v = \dfrac{R_S}{\frac{1}{g_m} + R_S} = 0.8$

$\therefore R_S = \dfrac{4}{g_m}$ —— ①

$\because V_{OUT} = \dfrac{V_{DD}}{2} = I_{DS} R_S$

$I_{DS} R_S = 0.9$ —— ②

$\because V_G = 1.8V$ and $V_S = 0.9$

$\therefore V_{GS} = 0.9V$

From ②, $\because I_{DS} = 1.11mA$

$R_S = \dfrac{0.9V}{1.11mA} \approx 810\Omega$

From ①, $g_m = \dfrac{4}{810\Omega} \approx 4.94\,mS$

$\because g_m = \left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})$

$\frac{W}{L} \approx 49.4$

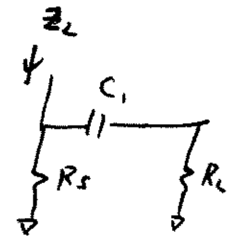
(72) $R_{in} = R_G = 20k\Omega$

$\because$ Power $= 3mW$

$\therefore I_{DS} = \dfrac{3mW}{1.8V} = 1.67mA$

$V_{x,\text{at DC}} = I_{DS} R_S = 0.9V$

$\therefore R_S = 540\Omega$



Load impedance, $Z_L = R_S // (\frac{1}{sC_1} + R_L)$
(at 100 MHz)

$$= 540 // \left(\frac{1}{2\pi \times 10^8 C_1} + 50\right)$$

Voltage gain $(A_v) = \dfrac{Z_L}{\frac{1}{g_m} + Z_L}$

$$g_m = \frac{2 I_{DS}}{V_{GS} - V_{TH}}$$

$$= \frac{2 \times 1.67 \times 10^{-3}}{(1.8 - 0.9) - 0.4}$$

$$= 6.67 mS.$$

$$\therefore A_v = \frac{Z_L}{+ Z_L} = 0.8$$

$$Z_L = 120 + Z_L(0.8)$$

$$\therefore Z_L = 150$$

$$\therefore 150 = 540 \parallel \left(\frac{1}{2\pi \times 10^8 C_1} + 50 \right)$$

$$= 540 \parallel \left[\frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1} \right]$$

$$= \frac{540 \times \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1}}{540 + \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1}}$$

$$\therefore \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1} \approx 208$$

$$\therefore C_1 \approx 10.1\,pF$$

To find $(\frac{W}{L})$:

$$\because g_m = \left(\frac{W}{L}\right) \mu_n C_{ox} (V_{GS} - V_{TH})$$

$$\frac{W}{L} = 66.7$$

$$\therefore \frac{W}{L} = 66.7, \quad C_1 = 10.1\,pF, \quad R_S = 540\,\Omega.$$

(73) Power = 3mW

$\therefore I_{DS_{1,2}} = \frac{3mW}{1.8V} = 1.67mA.$

$r_{o2} = \frac{1}{\lambda I_{DS2}}$

$= \frac{1}{0.1 \times 1.67 \times 10^{-3}} \approx 5990\Omega.$

$= r_{o1}$

$\because A_v = \frac{r_{o2} // r_{o1}}{\frac{1}{g_{m1}} + r_{o2} // r_{o1}} = 0.9$

$\therefore 0.9 = \frac{2995}{\frac{1}{g_{m1}} + 2995}$

$g_{m1} \approx 3mS$

$\because V_{DS2} \geq 0.3V$ (for M_2 to be in saturation).

Set V_{out} (i.e. $V_{DS2, nominal}$) = 0.3V

$\because g_{m1} = \frac{2 I_{DS1}}{V_{GS1} - V_{TH}}$

$3 \times 10^{-3} = \frac{2 \times 1.67 \times 10^{-3}}{V_G - 0.9 - 0.4}$

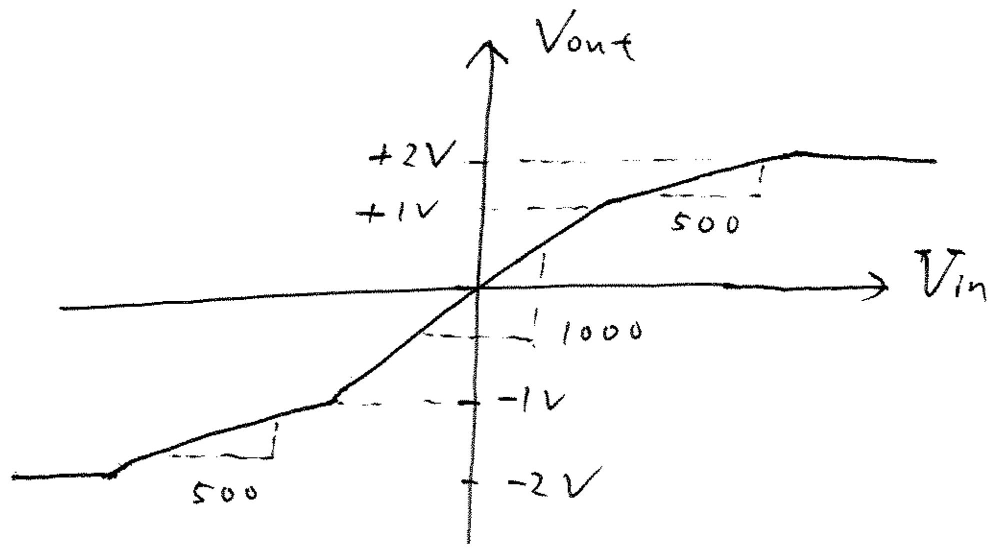
$\therefore V_{IN} = V_G \approx 1.81V$

$$g_m = \sqrt{2\left(\frac{W}{L}\right)\mu_n C_{ox} \quad I_{DS}}$$

$$\therefore \frac{W}{L} \approx 13.5$$

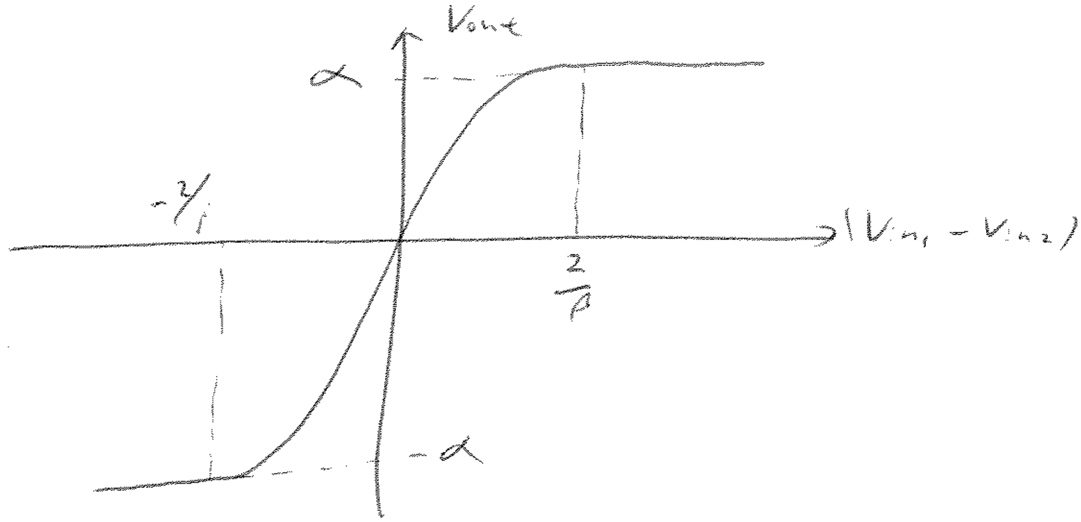
$$\therefore \frac{W}{L} = 13.5, \qquad V_{IN} = 1.81V, \qquad I_{DS} = 1.67mA.$$

① a)



b) The largest input swing is ± 1mV,
because gain is constant at 1000
over this range of input.

② $V_{out} = \alpha \tanh[\beta(V_{in1} - V_{in2})]$



To find small-signal gain,

$\because \tanh z = z - \frac{1}{3}z^3 + \frac{2}{15}z^5 + \cdots$

$\therefore$ for $\beta(V_{in1} - V_{in2}) \simeq 0$,

$$\frac{dV_{out}}{d(V_{in1} - V_{in2})} \simeq \frac{d}{d(V_{in1} - V_{in2})} \alpha\beta(V_{in1} - V_{in2})$$

$$= \underline{\underline{\alpha\beta}}$$

③ Closed-loop gain $= \left(1 + \frac{R_1}{R_2}\right)$

$$= 8$$

Gain error $= \left(1 + \frac{R_1}{R_2}\right)(A_0)^{-1}$

$$= \frac{8}{2000}$$

$$= \underline{\underline{0.4\%}}$$

④ Closed loop gain $= \left(1 + \frac{R_1}{R_2}\right)$

$= 4$

Gain error $= \left(1 + \frac{R_1}{R_2}\right)\left(\frac{1}{A_0}\right)$

$= 0.1\%$

$\therefore \quad 4/A_0 = 0.1\%$

$A_0 = \underline{\underline{4000}}$

⑤ Let $G_0 = \left(1 + \frac{R_1}{R_2}\right)$

Desired gain $= \alpha_1$

$$= \frac{A_0}{1 + \frac{R_2}{R_1 + R_2} A_0}$$

$$\therefore \quad \alpha_1 = \frac{A_0}{1 + \frac{A_0}{G_0}}$$

$$1 + \frac{A_0}{G_0} = \frac{A_0}{\alpha_1}$$

$$\frac{1}{G_0} = \frac{1}{\alpha_1} - \frac{1}{A_0}$$

$$G_0 = \frac{A_0 \alpha_1}{A_0 - \alpha_1}$$

$$\therefore \quad \frac{R_2}{R_1 + R_2} = \frac{1}{G_0} = \frac{1}{\alpha_1} - \frac{1}{A_0} \quad //$$

b) if A_0 drops to $0.6 A_0$,

$$\text{Actual gain} = \frac{0.6 A_0}{1 + \left(\frac{1}{\alpha_1} - \frac{1}{A_0}\right) 0.6 A_0}$$

$$= \frac{0.6 A_0}{0.4 + \frac{0.6 A_0}{\alpha_1}}$$

(5) b) (cont'd)

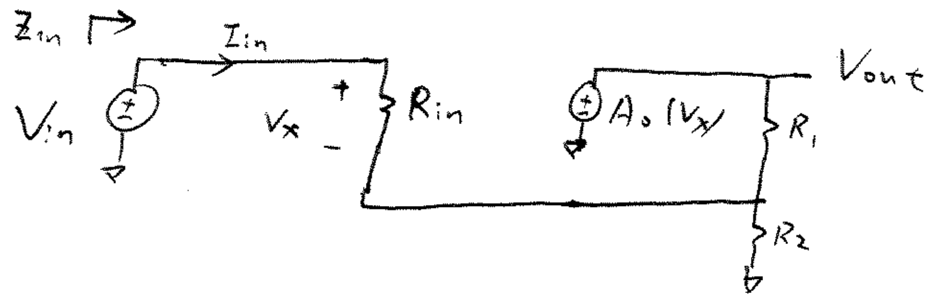
$$\text{Actual gain} = \frac{\alpha_1}{1 + \frac{0.4}{0.6}\frac{\alpha_1}{A_0}}$$

$$\simeq \alpha_1\left(1 - \frac{0.4}{0.6}\frac{\alpha_1}{A_0}\right)$$

$$\therefore \text{the gain error} \simeq \frac{0.4}{0.6}\frac{\alpha_1^2}{A_0}$$

$$= \frac{2}{3}\frac{\alpha_1^2}{A_0}$$

⑥ Using the model in Fig. 8.44,



$$V_x = V_{in} - V_{out} \frac{R_1}{R_1 + R_2}$$

$$V_{out} = A_o V_x$$

$$= A_o \left(V_{in} - V_{out} \frac{R_1}{R_1 + R_2} \right)$$

$$A_o V_{in} = V_{out} \left(1 + A_o \frac{R_1}{R_1 + R_2} \right)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_o}{1 + A_o \frac{R_1}{R_1 + R_2}} \quad \text{—— ①}$$

To find input impedance (Z_{in}),

$$I_{in} = \frac{V_x}{R_{in}}$$

$$= \frac{1}{R_{in}} \left(V_{in} - V_{out} \frac{R_1}{R_1 + R_2} \right)$$

$$= \frac{V_{in}}{R_{in}} \left(1 - \frac{V_{out}}{V_{in}} \frac{R_1}{R_1 + R_2} \right)$$

⑥ (cont'd)

$$I_{in} = \frac{V_{in}}{R_{in}}\left(1 - \frac{A_0}{1 + A_0 \frac{R_1}{R_1+R_2}} \frac{R_1}{R_1+R_2}\right)$$

$$= \frac{V_{in}}{R_{in}}\left(1 - \frac{1}{\frac{R_1+R_2}{A_0 R_1} + 1}\right)$$

$$= \frac{V_{in}}{R_{in}}\left(\frac{\frac{R_1+R_2}{A_0 R_1}}{\frac{R_1+R_2}{A_0 R_1} + 1}\right)$$

$$\therefore Z_{in} = \frac{V_{in}}{I_{in}} = R_{in}\left[\frac{1 + \frac{R_1+R_2}{A_0 R_1}}{\frac{R_1+R_2}{A_0 R_1}}\right] \quad \text{—— (2)}$$

As $A_0 \to \infty$,

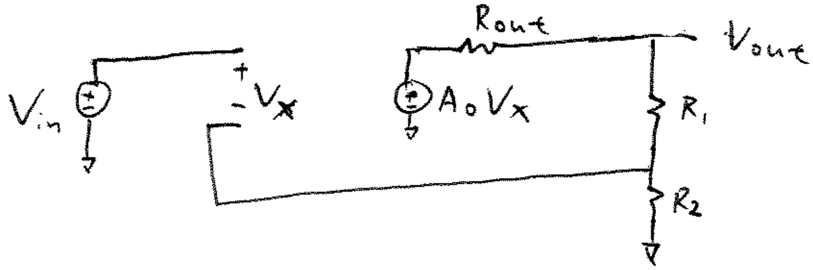
$$\text{gain} = \left.\frac{V_{out}}{V_{in}}\right|_{A_0 \to \infty} \quad [\text{From (1)}]$$

$$= 1 + \frac{R_2}{R_1}$$

$$Z_{in} = \left.\frac{V_{in}}{I_{in}}\right|_{A_0 \to \infty} \quad [\text{From (2)}]$$

$$= \infty$$

(7)



Similar to Prob. (6),

$$\text{gain} = \frac{V_{out}}{V_{in}}$$

$$V_x = V_{in} - V_{out}\frac{R_2}{R_1+R_2}$$

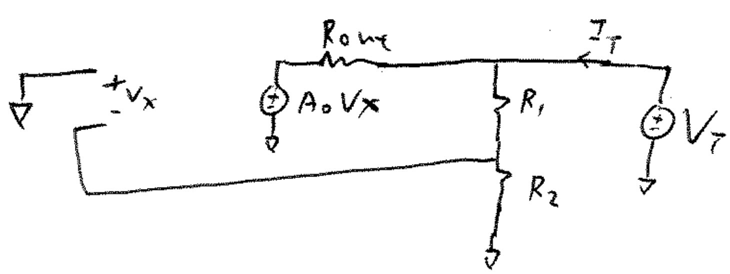
$$V_{out} = A_0 V_x \frac{R_1+R_2}{R_{out}+R_1+R_2}$$

$$= A_0\left(V_{in} - V_{out}\frac{R_2}{R_1+R_2}\right)\frac{R_1+R_2}{R_{out}+R_1+R_2}$$

$$V_{in} A_0 \frac{R_1+R_2}{R_{out}+R_1+R_2} = V_{out}\left(1+\frac{A_0 R_2}{R_{out}+R_1+R_2}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0\frac{R_1+R_2}{R_{out}+R_1+R_2}}{1+\frac{A_0 R_2}{R_{out}+R_1+R_2}}$$

To find output impedance (Z_{out})



(7) (cont'd) $V_x = \frac{R_2}{R_1 + R_2} V_T$

$$I_T = \frac{V_T}{R_1 + R_2} + \frac{V_T - A_0 V_x}{R_{out}}$$

$$= V_T \left[\frac{R_{out} + R_1 + R_2 - A_0 R_2}{(R_{out})(R_1 + R_2)} \right]$$

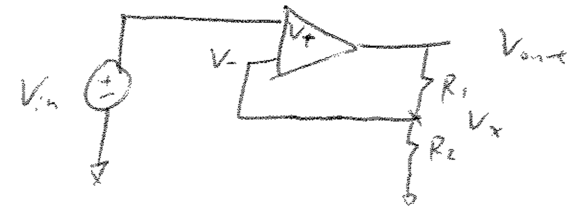
$$Z_{out} = \frac{V_T}{I_T} = \frac{(R_{out})(R_1 + R_2)}{R_{out} + R_1 + R_2 - A_0 R_2}$$

As $A_0 \to \infty$,

$$\text{gain} = 1 + \frac{R_1}{R_2}$$ //

$$Z_{out} = 0$$ //

(8)



ΔR for now.

$$V_{out} = A_0 (V_x)$$

$$V_x = V_{in} - \frac{R_2}{R_1 + R_2} V_{out}$$

$$\therefore \frac{-V_{out}}{A_0} = V_{in} - \frac{R_1}{R_1 + R_2} V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_0 (R_1 + R_2)}{A_0 R_1 - 1} = \text{nominal gain}$$

if $R_2' = \Delta R + R_2$

$$\left(\frac{V_{out}}{V_{in}}\right)' = \frac{A_0 (R_1 + \Delta R + R_2)}{A_0 R_1 - 1}$$

$$\therefore \text{gain error} = \frac{\left(\frac{V_{out}}{V_{in}}\right)' - \left(\frac{V_{out}}{V_{in}}\right)}{\frac{V_{out}}{V_{in}}}$$

$$= \frac{\Delta R}{A_0 R_1 - 1} \times \frac{A_0 R_1 - 1}{A_0 (R_1 + R_2)}$$

$$= \frac{\Delta R}{A_0 (R_1 + R_2)} \quad //$$

(9) Closed-loop gain $\approx \left(1+\frac{R_1}{R_2}\right)\left[1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right]$

$$= 5\left[1-\frac{5}{A_0}\right]$$

$\therefore$ As A_0 decreases to $0.8A_0$, closed-loop gain decreases along (deviating more from the norminal

A_0 drops to $0.8A_0$ when $|V_{in1} - V_{in2}| = 2mV$.

$$\because \quad V_{in2} = V_{out}\left(\frac{R_2}{R_1+R_2}\right)$$

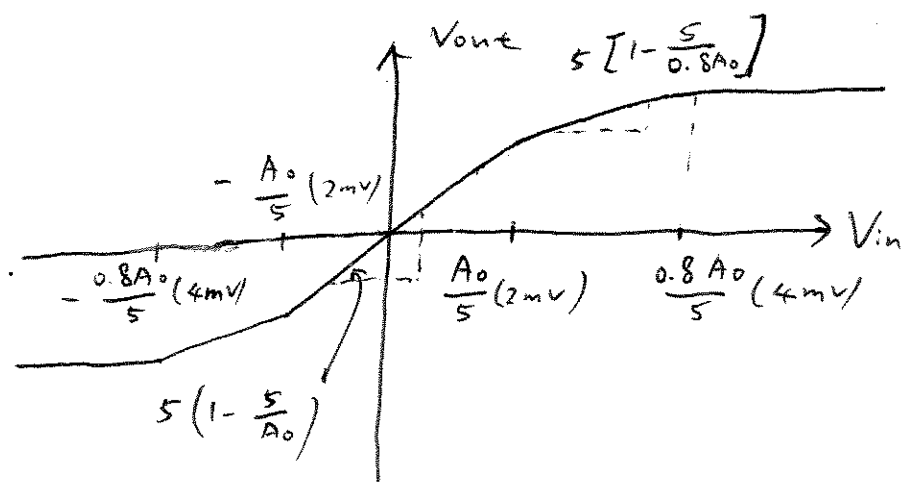
and $V_{out} = 5\left(1-\frac{5}{A_0}\right)V_{in1}$

$$\therefore \quad V_{in2} = 5\left(1-\frac{5}{A_0}\right)\left(\frac{1}{5}\right)V_{in1}$$

$$V_{in1} - V_{in2} = \frac{5}{A_0}V_{in1}$$

At $V_{in1} - V_{in2} = 2mV$,

$$V_{in1} = \frac{A_0}{5}(2mV)$$



(10) $\frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_S}$

$\because V_{in} = 1V,\quad V_{out} = 1 + \frac{R_1}{R_0 + \alpha W}$

$$\frac{dV_{out}}{dW} = -R_1 \alpha \left(R_0 + \alpha W \right)^{-2}$$

$$= \frac{-R_1 \alpha}{(R_0 + \alpha W)^2}$$

(11) if $A_0 = \infty$,

$$V_+ = V_- = V_{in}$$

$$V_- = \left(\frac{R_2}{R_2 + R_3}\right)\left[\frac{R_4 /\!/ (R_2 + R_3)}{R_1 + R_4 /\!/ (R_2 + R_3)}\right] V_{out}$$

$\therefore$ closed-loop gain $\dfrac{V_{out}}{V_{in}}$

$$= \frac{(R_2 + R_3)\,[R_1 + R_4 /\!/ (R_2 + R_3)]}{R_2\,[R_4 /\!/ (R_2 + R_3)]}$$

if $R_1 = 0$,

$$G\big|_{R_1 = 0} = 1 + \frac{R_3}{R_2}$$

if $R_3 = 0$,

$$G\big|_{R_3 = 0} = \frac{R_2\,[R_1 + R_4 /\!/ R_2]}{R_2\,[R_4 /\!/ R_2]}$$

$$= 1 + \frac{R_1}{R_4 /\!/ R_2}$$

(12) Gain Error $= \frac{1}{A_0}\left(1 + \frac{R_1}{R_2}\right)$

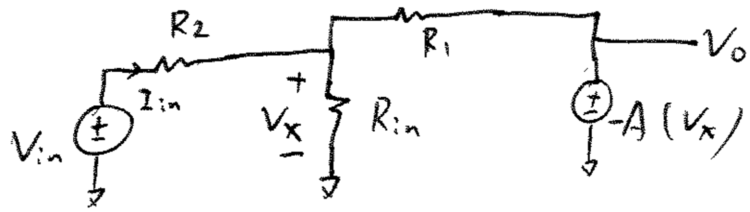
$= \frac{1}{A_0}(1+8)$

$= 0.2\%$

$\therefore \frac{1}{A_0}(9) = 0.2\%$

$A_0 = 4500$

(13)



$$V_o = -A V_x \quad \text{——— (1)}$$

$$\frac{V_{in} - V_x}{R_2} + \frac{V_o - V_x}{R_1} = \frac{V_x}{R_{in}} \quad \text{——— (2)}$$

Combining (1) and (2),

$$\frac{V_{in}}{R_2} = -\frac{V_o}{R_1} + \frac{V_o}{(-A)}\left(\frac{1}{R_{in}} + \frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{V_{in}}{R_2} = V_o\left[\frac{A R_{in} R_2 + R_1 R_2 + R_{in} R_2 + R_{in} R_1}{(-A) R_{in} R_1 R_2}\right]$$

$$\frac{V_o}{V_{in}} = -\frac{A R_{in} R_1}{R_1 R_2 + R_{in} R_2 + R_{in} R_1 + A R_{in} R_2}$$

Input impedance $(Z_{in}) = \dfrac{V_{in}}{I_{in}}$

$$I_{in} - \frac{V_x}{R_{in}} + \frac{(-A)V_x - V_x}{R_1} = 0$$

$$I_{in} = V_x\left[\frac{1}{R_{in}} + \frac{A+1}{R_1}\right]$$

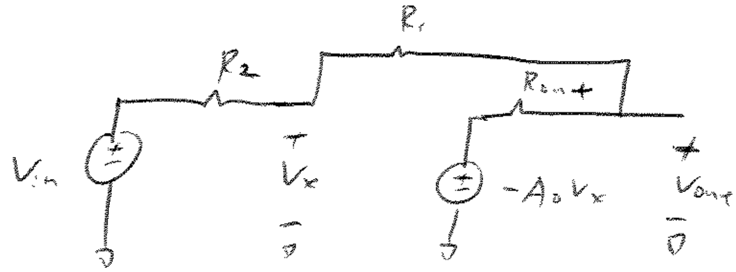
$$\because \quad V_x = V_{in} - I_{in} R_2$$

$$I_{in} = \left[V_{in} - I_{in} R_2 \right] \left[\frac{1}{R_{in}} + \frac{A+1}{R_1} \right]$$

$$I_{in} \left[1 + \frac{R_2}{R_{in}} + \frac{R_2}{R_1}(A+1) \right] = V_{in} \left(\frac{1}{R_{in}} + \frac{A+1}{R_1} \right)$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{1 + \frac{R_2}{R_{in}} + \frac{R_2}{R_1}(A+1)}{\frac{1}{R_{in}} + \frac{A+1}{R_1}}$$

(14)



By KCL,

$$\frac{V_{in} - V_x}{R_2} = - \frac{-A_0 V_x - V_x}{R_1 + R_{out}}$$

$$\therefore \quad V_x = -\frac{V_{out}}{A_0}$$

$$\frac{V_{in}}{R_2} = -\frac{V_{out}}{A_0 R_2} - \frac{A_0 + 1}{A_0} \frac{V_{out}}{R_1 + R_{out}}$$

$$\because \quad \frac{A_0 + 1}{A_0} \approx 1$$

$$\therefore \quad \frac{V_{in}}{R_2} \approx -\frac{V_{out}}{A_0 R_2} - \frac{V_{out}}{R_1 + R_{out}}$$

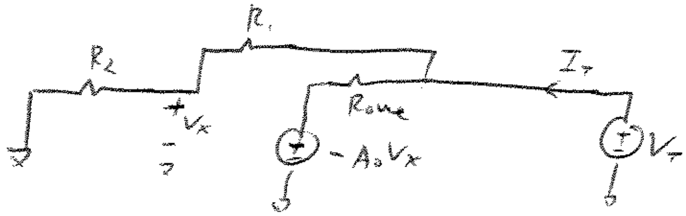
$$\frac{V_{in}}{R_2} \approx -V_{out} \left(\frac{R_1 + R_{out} + A_0 R_2}{A_0 R_2 (R_1 + R_{out})} \right)$$

$$\therefore \quad \frac{V_{out}}{V_{in}} \approx - \frac{A_0 (R_1 + R_{out})}{R_1 + R_{out} + A_0 R_2}$$

//

(14) cont'd:

To find output impedance (Z_{out})



$$Z_{out} = \frac{V_T}{I_T}$$

$$V_x = \frac{R_2}{R_1 + R_2} V_T \qquad \text{——— (1)}$$

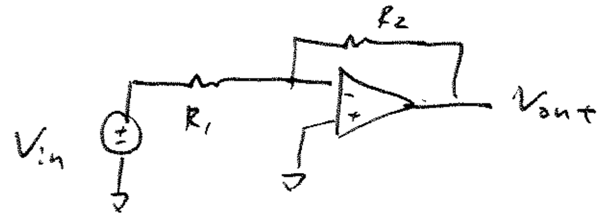
$$I_T = \frac{V_T}{R_1 + R_2} + \frac{V_T + A_0 V_x}{R_{out}} \qquad \text{——— (2)}$$

$$I_T = V_T \left[\frac{1}{R_1 + R_2} + \frac{1 + \frac{A_0 R_2}{R_1 + R_2}}{R_{out}} \right]$$

$$\frac{V_T}{I_T} = \frac{R_{out}(R_1 + R_2)}{R_{out} + R_1 + (A_0 + 1) R_2}$$

//

(15)



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_2}{R_1} = 4 \quad \text{——— ①}$$

$$\therefore R_2 = 4R_1$$

$$Z_{in} \approx R_1 = 10\,k\Omega \quad \text{—— ②}$$

$$\therefore R_2 = 40\,k\Omega$$

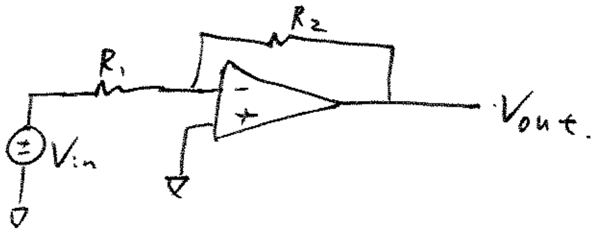
$$A_0 = 1000 \quad \text{——— ③}$$

$$\text{gain error} = \frac{1}{A_0}\left(1 + \frac{R_2}{R_1}\right)$$

$$= \frac{1}{1000}(1 + 4)$$

$$= 0.5\,\%$$

(16)



$$\text{Nominal gain} = \frac{R_2}{R_1} = 8 \quad \text{——} \ (1)$$

$$R_2 = 8R_1$$

$$\text{Input impedance} \approx R_1 = 1000\ \Omega \quad \text{——} \ (2)$$

$$\therefore R_2 = 8000\ \Omega.$$

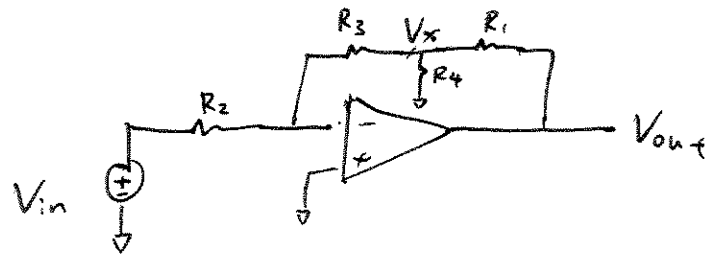
$$\text{gain error} = 0.1\% \quad \text{——} \ (3)$$

$$\therefore \frac{1}{A_0}\left(1 + \frac{R_2}{R_1}\right) = 0.1\%$$

$$\frac{1}{A_0}(9) = \frac{0.1}{100}$$

$$\therefore A_0 = 9000$$

(17)



$$V_- = V_+ = 0 \quad (\because A = \infty)$$

$$\frac{V_{in}}{R_2} = -\frac{V_x}{R_3} \quad \longrightarrow ①$$

$$V_x = \frac{R_3 // R_4}{R_1 + R_3 // R_4} V_{out} \quad \longrightarrow ②$$

Combining ① and ②,

$$\frac{V_{in}}{R_2} = -\frac{R_3 // R_4}{R_3 (R_1 + R_3 // R_4)} V_{out}$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_3}{R_2} \frac{(R_1 + R_3 // R_4)}{R_3 // R_4} \quad //$$

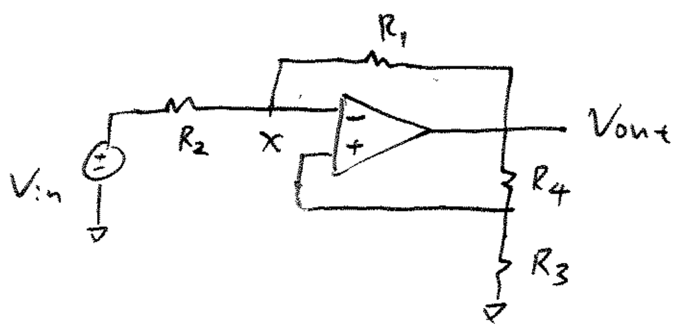
if $R_1 \longrightarrow 0$,

$$\frac{V_{out}}{V_{in}} = -\frac{R_3}{R_2} \quad // \quad \text{(typical inverting amplifier)}$$

if $R_3 \longrightarrow 0$,

$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{R_2} \quad // \quad \text{(typical inverting amplifier)}$$

(18)



$\because A = \infty$,

$$V_- = V_+$$

$$\therefore V_x = \frac{R_3}{R_3 + R_4} V_{out}$$

$$\frac{V_{in} - V_x}{R_2} = - \frac{V_{out} - V_x}{R_1}$$

$$\frac{V_{in}}{R_2} = V_x \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{V_{out}}{R_1}$$

$$= \left[\left(\frac{R_3}{R_3 + R_4} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{1}{R_1} \right] V_{out}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{\frac{1}{R_2}}{\left(\frac{R_3}{R_3 + R_4} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{1}{R_1}} \quad //$$

(19) From eq. (8.31),

$$V_{out} = -\frac{1}{R_1 C_1}\int V_{in}\,dt$$

$$= -\frac{1}{R_1 C_1}\int V_0 \sin wt\,dt$$

$$= \frac{V_0}{R_1 C_1 w}\cos wt$$

$\therefore$ Amplitude of output $= \dfrac{V_0}{R_1 C_1 w}$

(20) From prob. (19)

Amplification of the integrator $= \frac{1}{R_1 C_1 \omega}$

$$\therefore \quad \frac{1}{R_1 C_1 \omega} = 10$$

$$\frac{1}{\omega} = 10 \times 10\,ns$$

$$\therefore \omega = 10\,MHz.$$

∴ The frequency of the sinusoid is 10MHz.

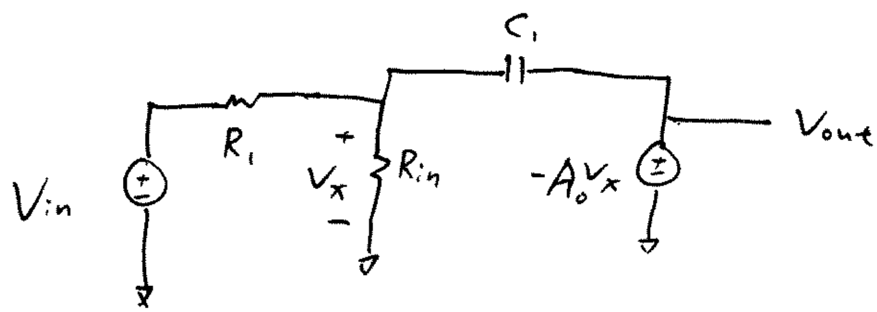
(21) From Eq. (8.37)

$$s_p = \frac{-1}{2\pi (A_0 + 1) R_1 C_1} \leq -1 \text{ Hz.}$$

$$\therefore \quad 2\pi (A_0 + 1)(10 \text{ k}\Omega)(1 \text{n F}) \geq 1$$

$$A_0 \geq 15915$$

(22)



$$\frac{V_{in} - V_x}{R_1} + \frac{V_{out} - V_x}{\frac{1}{sC_1}} = \frac{V_x}{R_{in}},$$

where $s = j\omega$

$\because \quad V_{out} = -A_o V_x$

$$\therefore \quad \frac{V_{in}}{R_1} = (sC_1)\left[-\frac{V_{out}}{A_o} - V_{out}\right] - \frac{V_{out}}{A_o}\left(\frac{1}{R_{in}} + \frac{1}{R_1}\right)$$

$$= -V_{out}\left[\frac{sC_1}{A_o} + sC_1 + \frac{1}{A_o R_{in}} + \frac{1}{A_o R_1}\right]$$

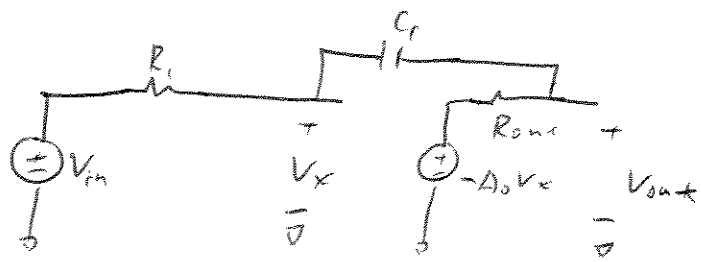
$$\frac{V_{out}}{V_{in}} = \frac{-1}{\left(\frac{1}{A_o} + \frac{R_1}{A_o R_{in}}\right) + \left(1 + \frac{1}{A_o}\right) s R_1 C_1}$$

To find the pole, equate denominator to zero,

$$S_p = \frac{-1}{(A_o + 1) R_1 C_1}\left(1 + \frac{R_1}{R_{in}}\right)$$

[$\therefore$ pole shifted out by $\left(1 + \frac{R_1}{R_{in}}\right)$]

(23)



By KCL,

$$\frac{V_{in} - V_x}{R_1} = -\frac{-A_0V_x - V_x}{R_{out} + \frac{1}{sC_1}}$$

$$\because \quad V_x = -\frac{V_{out}}{A_0}$$

$$\frac{V_{in}}{R_1} = -\frac{V_{out}}{A_0R_1} - \frac{(A_0+1)}{A_0}\frac{V_{out}}{R_{out} + \frac{1}{sC_1}}$$

$$\simeq -V_{out}\left[\frac{1}{A_0R_1} + \frac{1}{R_{out} + \frac{1}{sC_1}}\right]$$

$$\left(\because \frac{A_0+1}{A_0} \simeq 1\right)$$

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{A_0 \times \left(R_{out} + \frac{1}{sC_1}\right)}{\left(A_0R_1 + R_{out} + \frac{1}{sC_1}\right)}$$

$$\text{pole} = -\frac{1}{C_1\,(R_{out} + A_0R_1)}$$

(24) $\because A_0 = \infty$

$$|A_v| = \frac{R_1}{\frac{1}{\omega C_1}}$$

$$= \omega R_1 C_1$$

$$= 5$$

$$\therefore R_1 C_1 = \frac{5}{\omega}$$

$$= \frac{5}{2\pi \times 10^6}$$

$$= 7.958 \times 10^{-7}$$

(25) From eqn (8.55)

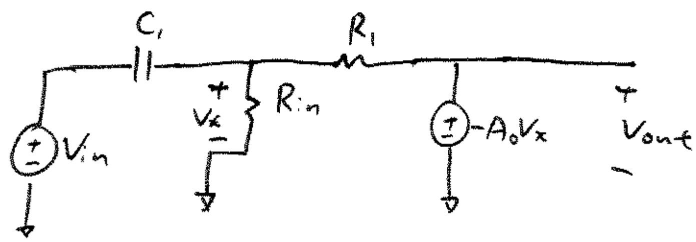
$$S_p = -\frac{A_o + 1}{R_1 C_1}$$

$$2\pi \times 100 \times 10^6 = \frac{A_o + 1}{1000 \times 10^{-9}}$$

(ie. R_1 and C_1 are chosen at minimum)

$$A_o \approx 627$$

(26)



By KCL,

$$(V_{in} - V_x) sC_1 = \frac{V_x}{R_{in}} + \frac{(V_x + A_0 V_x)}{R_1}$$

$$(V_{in}) sC_1 = V_x \left[sC_1 + \frac{1}{R_{in}} + (A_0 + 1)\frac{1}{R_1} \right]$$

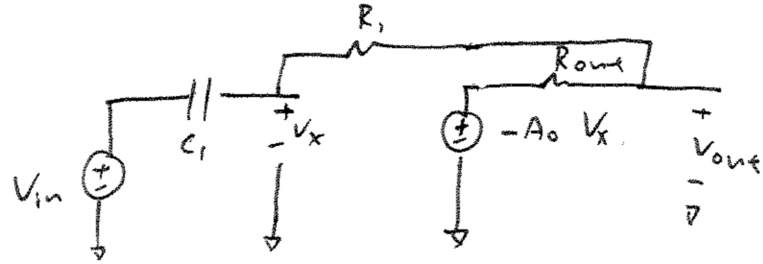
$$\frac{sC_1}{sC_1 + \frac{1}{R_{in}} + (A_0+1)\frac{1}{R_1}} = \frac{V_x}{V_{in}}$$

$\because$ $V_{out} = -A_0 V_x$,

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{A_0 \, sC_1}{sC_1 + \frac{1}{R_{in}} + (A_0+1)\frac{1}{R_1}}$$ //

As $A_0 \to \infty$, $\frac{V_{out}}{V_{in}} \to -R_1 C_1 s$. [8.42]

(27)



By KCL,

$$(V_{in} - V_x) sC_1 = (V_x + A_0 V_x) \frac{1}{R_1 + R_{out}}$$

$$(V_{in}) sC_1 = V_x \left[sC_1 + (A_0 + 1) \frac{1}{R_1 + R_{out}} \right]$$

$$\frac{V_x}{V_{in}} = \frac{sC_1}{sC_1 + (A_0 + 1) \frac{1}{R_1 + R_{out}}}$$

$$V_{out} = (-A_0 V_x - V_x) \frac{R_1}{R_1 + R_{out}}$$

(resistive divider)

$$= -V_x \frac{(A_0 + 1) R_1}{R_1 + R_{out}}$$

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{\frac{(A_0 + 1) R_1 sC_1}{R_1 + R_{out}}}{sC_1 + (A_0 + 1) \frac{1}{R_1 + R_{out}}}$$

$$\frac{V_{out}}{V_{in}} = -R_1 sC_1 \quad (\text{as } A_0 \to \infty)$$

[8.42]

(28) $\because$ $A_o = \infty$,

$V_+ = V_- = 0$

By KCL,

$$\frac{V_{in}}{R_1 \,//\, \frac{1}{sC_1}} = - \frac{V_{out}}{R_2 \,//\, \frac{1}{sC_2}}.$$

$$\therefore \frac{V_{out}}{V_{in}} = - \frac{R_2 \,//\, \frac{1}{sC_2}}{R_1 \,//\, \frac{1}{sC_1}}$$

$$= - \frac{R_2}{R_1} \times \frac{1 + R_1 C_1 s}{1 + R_2 C_2 s}.$$

For $\left| \frac{V_{out}}{V_{in}} \right| = 1$,

$$R_2 \,//\, \frac{1}{sC_2} = R_1 \,//\, \frac{1}{sC_1}$$

That is, choose the components such that the impedance of $R_2 // \frac{1}{sC_2}$ is equal to $R_1 // \frac{1}{sC_1}$ at the specific frequency.

(29) if $A_0 < \infty$,

Let V_- be the voltage at the negative input terminal of the opamp.

By KCL,

$$\frac{V_{in} - V_-}{R_1 // \frac{1}{sC_1}} = - \frac{V_{out} - V_-}{R_2 // \frac{1}{sC_2}}$$

$$V_{out} = -A_0 V_-,$$

$$\frac{V_{in} + \frac{V_{out}}{A_0}}{R_1 // \frac{1}{sC_1}} = - \frac{V_{out} + \frac{V_{out}}{A_0}}{R_2 // \frac{1}{sC_2}}$$

$$V_{in} = -\left[R_1 // \frac{1}{sC_1}\right] \left[\frac{\left(R_2 // \frac{1}{sC_2}\right)\frac{V_{out}}{A_0} + V_{out} + \frac{V_{out}}{A_0}}{R_2 // \frac{1}{sC_2}}\right.$$

$$\therefore \frac{V_{out}}{V_{in}} = - \frac{R_2 // \frac{1}{sC_2}}{R_1 // \frac{1}{sC_1}} \left[\frac{A_0}{(A_0+1) + \left(R_2 // \frac{1}{sC_2}\right)}\right]$$

To set $\left|\frac{V_{out}}{V_{in}}\right| = 1$,

Let $x = R_1 // \frac{1}{sC_1}$ and $y = R_2 // \frac{1}{sC_2}$.

$\therefore$ For $\left|\frac{V_{out}}{V_{in}}\right| = 1$, $\quad y A_0 = x\left[(A_0+1) + y\right]$

$$y(A_0 - 1) = x(A_0+1)$$

(29) Cont'd

$$\therefore \quad \frac{X}{Y} = \frac{A_o + 1}{A_o - 1},$$

ie. we need to set $\dfrac{R_1 \,//\, \frac{1}{sC_1}}{R_2 \,//\, \frac{1}{sC_1}} = \dfrac{A_o+1}{A_o-1}$.

Since A_o is generally rather large,

$\dfrac{A_o+1}{A_o-1}$ is a rational fraction,

in which the numerator and the denominator are large, and differ by a small amount.

(e.g. if $A_o = 1000$, $\dfrac{A_o+1}{A_o-1} = \dfrac{1001}{999}$)

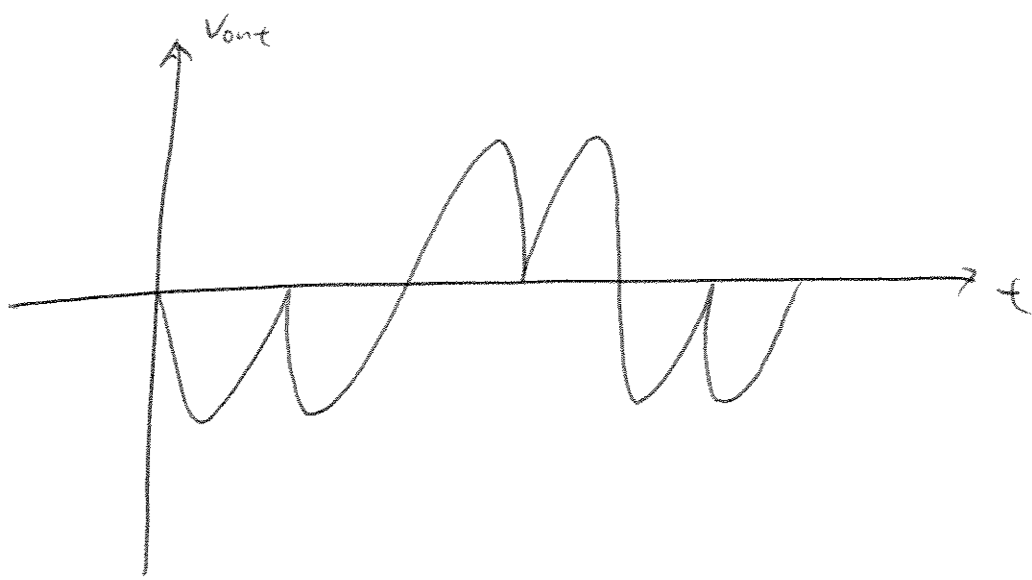
Hence, setting $\left|\dfrac{V_{out}}{V_{in}}\right|$ to unity is possible in principle, although it would be rather difficult to precisely control A_o.

(30) From eqn (8.63),

$$V_{out} = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

$\because R_1 = R_2$,

$$V_{out} = -\frac{R_F}{R_1}(V_1 + V_2)$$



(31) By KCL,

$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} = - \frac{V_{out} - V_x}{R_F}$$

$\because$ $V_{out} = -A_0 V_x$

$$V_x = -\frac{V_{out}}{A_0},$$

$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) + \frac{V_{out}}{A_0}\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{R_F}\right) = -\frac{V_{out}}{R_F}$$

$$-\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) = V_{out}\left[\frac{1}{R_F} + \frac{1}{A_0}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right)\right]$$

$$\therefore \quad V_{out} = -\left(\frac{1}{R_F} + \frac{1}{A_0}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right)\right)^{-1} \times \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

(32) For $A_o = \infty$,

$V_+ = V_- = 0$,

$\therefore$ No current flows through R_P,

i.e. No effect due to R_P

$$V_{out} = -R_F \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

For $A_o \neq \infty$,

By KCL

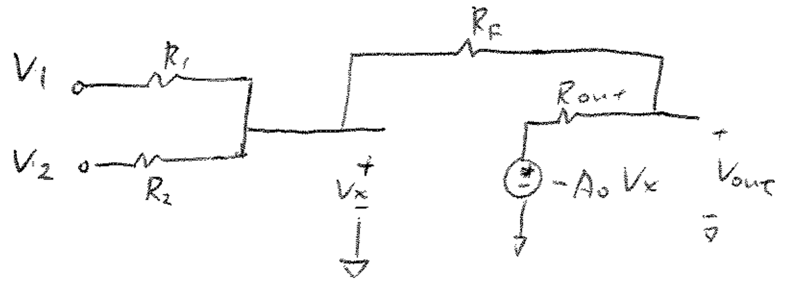
$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} - \frac{V_x}{R_P} = -\frac{V_{out} - V_x}{R_F}$$

$$\because \quad V_x = -\frac{V_{out}}{A_o},$$

$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right) + \frac{V_{out}}{A_o} \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_P} + \frac{1}{R_F} \right. = -\frac{V_{out}}{R_F}$$

$$V_{out} = -\left[\frac{1}{R_F} + \frac{1}{A_o} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_P} + \frac{1}{R_F} \right) \right]^{-1} \times \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

(33)



By KCL,

$$\frac{V_1 - V_x}{R_1} + \frac{V_2 - V_x}{R_2} = \frac{V_x (A_0 + 1)}{R_F + R_{out}}$$

$$\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right) = V_x \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{A_0 + 1}{R_F + R_{out}} \right]$$

$$\because V_{out} = (-A_0 V_x - V_x) \frac{R_F}{R_F + R_{out}}$$

$$= -V_x (1 + A_0) \frac{R_F}{R_F + R_{out}}$$

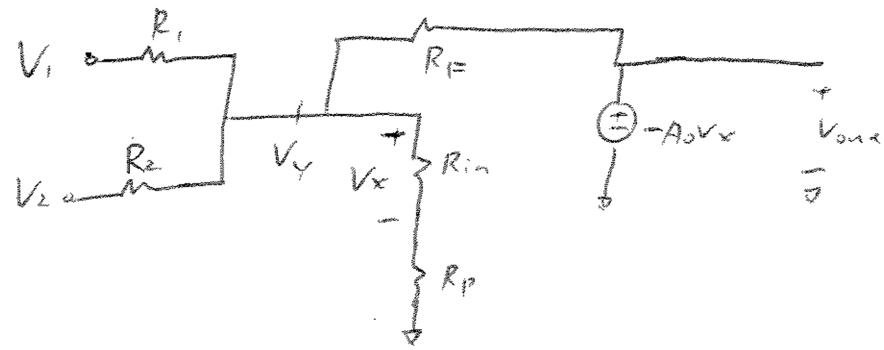
$$\therefore \frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{R_F + R_{out}}{R_F (A_0 + 1)} \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{A_0 + 1}{R_F + R_{out}} \right] V_{out}$$

$$V_{out} = -\frac{R_F (A_0 + 1)}{R_F + R_{out}} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{A_0 + 1}{R_F + R_{out}} \right)^{-1}$$

$$\times \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$$

//

(34)



By KCL,

$$\frac{V_1 - V_Y}{R_1} + \frac{V_2 - V_Y}{R_2} = \frac{V_Y + A_o V_x}{R_F} + \frac{V_Y}{R_{in} + R_p}$$

Using voltage divider,

$$V_x = V_Y \frac{R_{in}}{R_{in} + R_p}$$

$$V_Y = \frac{R_{in} + R_p}{R_p} V_x$$

$$\therefore \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) = V_Y\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p}\right) + \frac{A_o V_x}{R_F}$$

$$= \left[\left(\frac{R_{in} + R_p}{R_p}\right)\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p}\right) + \frac{A_o}{R_F}\right] \times V_x$$

$$\because V_{out} = -A_o V_x$$

$$\therefore \frac{V_1}{R_1} + \frac{V_2}{R_2} = -\left(\frac{V_{out}}{A_o}\right)\left[\left(\frac{R_{in}+R_p}{R_p}\right)\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p}\right) + \frac{A_o}{R_F}\right]$$

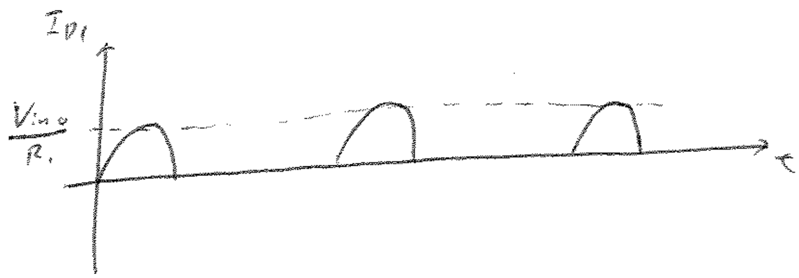
$$V_{out} = -A_o\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right) \times \left[\left(\frac{R_{in}+R_p}{R_p}\right)\left(\frac{1}{R_F} + \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{in}+R_p}\right) + \frac{A_o}{R_F}\right]^{-1}$$

(35) When D_1 is on, (i.e. when $V_{in} > 0$)

$$V_{out} = V_{in} = I_{D_1} R_1$$

$$\therefore I_{D_1} = \frac{V_{in}}{R_1}$$

when D_1 is off, $I_{D_1} = 0$.

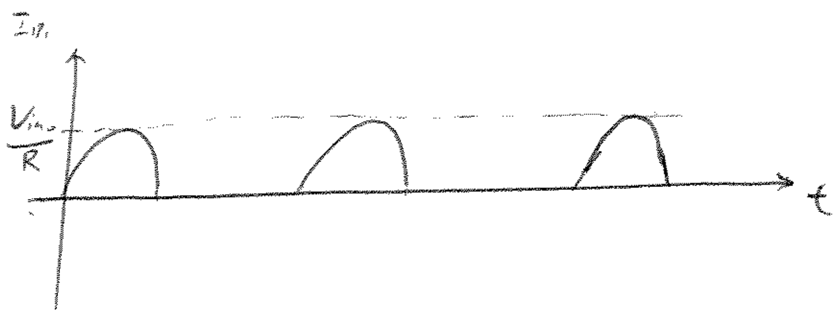


(36) D_1 is on when $V_{in} > 0$,

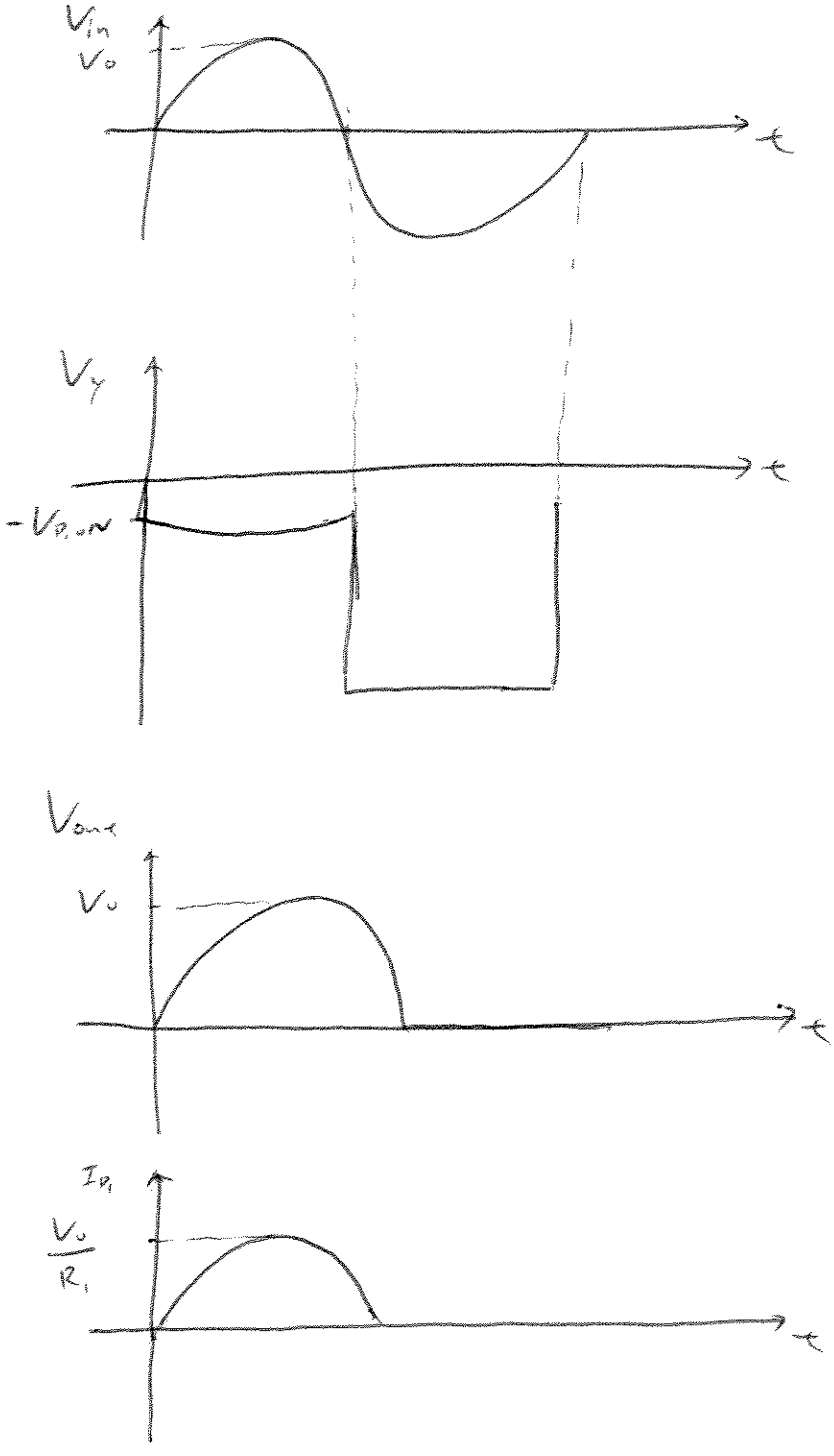
$V_x = 0$

By KCL;

$$I_{D1} = \frac{V_{in}}{R_1}$$



(37)



(38) $\because R_{D_1, on} << R_P$

$\therefore$ when diode is on, R_P has no effect.

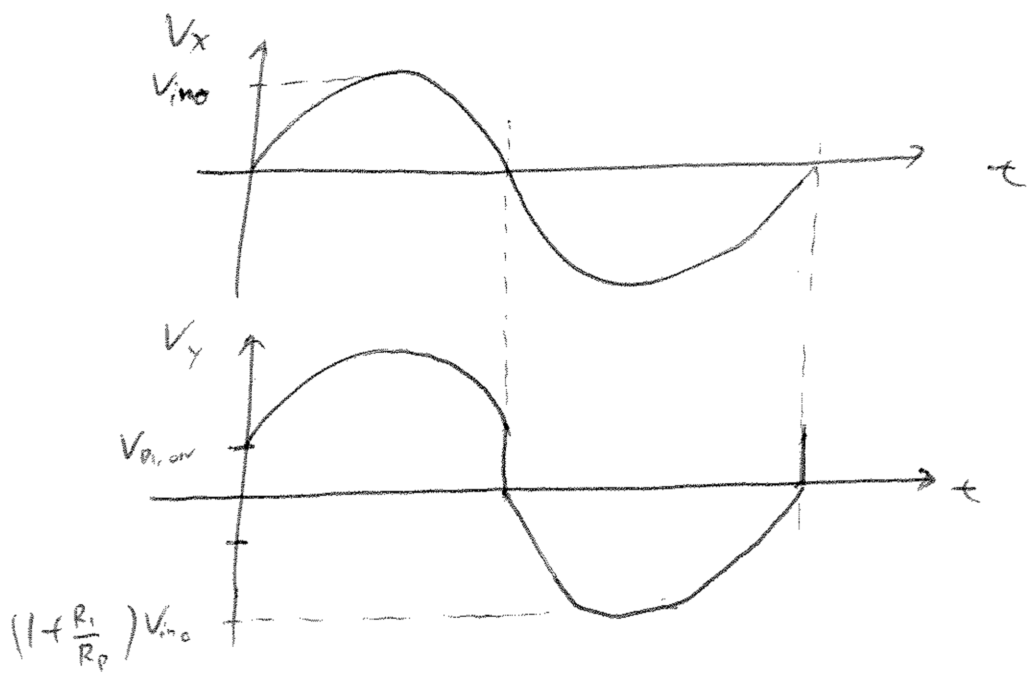
(diode "shorts" nodes X and Y)

when diode is off, R_P functions as a feedback resistor,

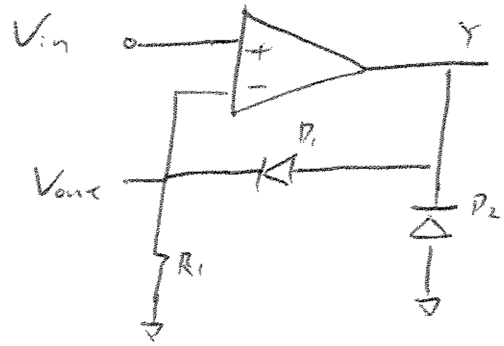
$$\therefore \frac{V_Y}{V_{in}} = 1 + \frac{R_1}{R_P}$$

and $V_{in} = V_{out}$.

$\therefore V_X = V_{in}$ for both D_1 is on and off.

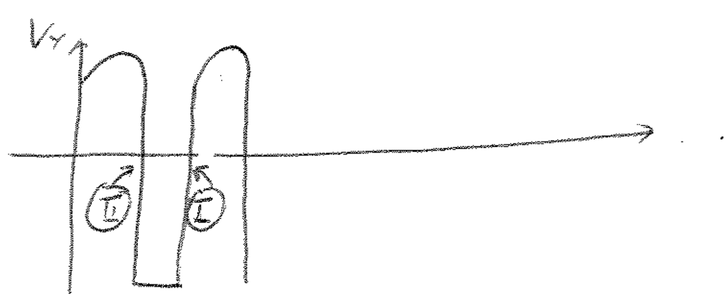


(39) Connecting a diode as below:



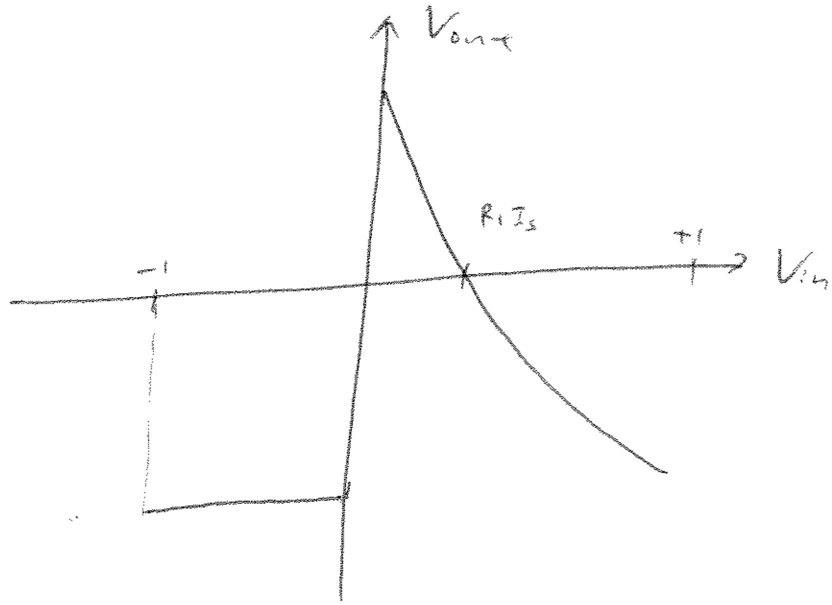
D2 allows the parasitic capacitance to charge up faster, right before D1 conducts.

This correspond to sharpening the transition (I) of VY, as shown below



But it will not speed up transition (II). (which is not critical)

(40)



(41) By KCL,

$$\frac{V_{in} - V_x}{R_1} = I_{R_1}$$

$$\therefore \quad V_{BE} = V_T \ln \frac{\frac{V_{in} - V_x}{R_1}}{I_s}$$

$$= -V_{out}$$

$$\because \; -A_0 V_x = V_{out}$$

$$V_x = -\frac{V_{out}}{A_0}$$

$$\therefore \quad V_{out} = -V_T \ln \frac{V_{in} + \frac{V_{out}}{A_0}}{R_1 I_s}$$

//

(42). This circuit will not function as a noninverting opamp:

assuming $A_o = \infty$,

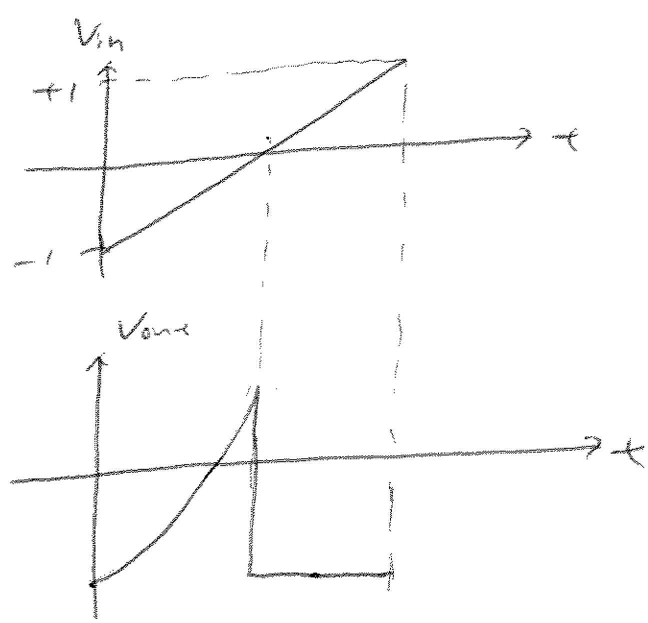
$$V_+ = V_- = V_{in}.$$

$$\therefore V_{BE} = V_T \ln \frac{\frac{-V_{in}}{R_1}}{I_s}$$

$$\therefore V_{out} = -V_{BE}$$

$$V_{out} = -V_T \ln \frac{-V_{in}}{R_1 I_s}$$

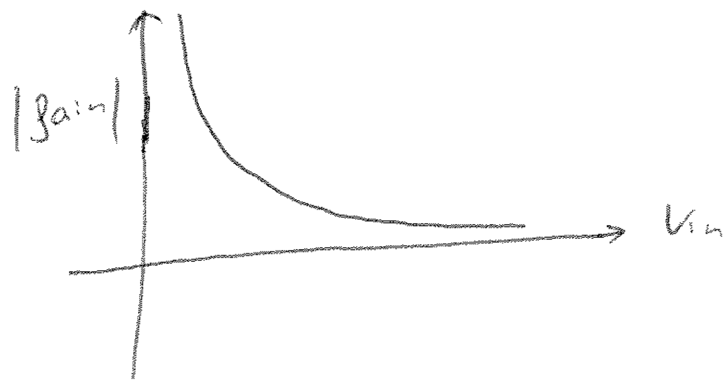
For example, as V_{in} varies from -1V to +1V:



(43)

$$V_{out} = -V_T \ln \frac{V_{in}}{R_1 I_s}$$

$$\frac{dV_{out}}{dV_{in}} = -\frac{V_T}{V_{in}}$$



the gain is compressive, because as V_{in} increases the magnitude of the gain decreases.

(44) Set $V_{out} = -0.5V$ when $V_{in} = 1V$

$$-0.5 = -V_T \ln \frac{1}{R_1 I_s}$$

$$\therefore R_1 I_s = 2.0612 \times 10^{-9}$$

when $V_{in} = 10V$,

$$V_{out} = -V_T \ln \frac{10}{2.0612 \times 10^{-9}}$$

$$= -0.558V \quad > -1V.$$

$\therefore$ setting $R_1 I_s = 2.0612 \times 10^{-9}$ meets the specification.

choose $I_s = 1 \times 10^{-16} A$.

$R_1 = 20.61 M\Omega$ //

(45) Assume $A_0 = \infty$,

$$I_{R_1} = \frac{V_{in} - V_{TH}}{R_1}$$

$$= \frac{1}{2} k' (V_{GS} - V_{TH})^2,$$

where $k' = \frac{W}{L} C_{ox}' \mu_n$

$\because V_{GS} = -V_{out}$,

$$\therefore \frac{1}{2} k' (-V_{out} - V_{TH})^2 = \frac{V_{in} - V_{TH}}{R_1}$$

$$(-V_{out} - V_{TH})^2 = \frac{2(V_{in} - V_{TH})}{k' R_1}$$

$$(-V_{out} - V_{TH}) = \sqrt{\frac{2(V_{in} - V_{TH})}{k' R_1}}$$

$$V_{out} = -\sqrt{\frac{2(V_{in} - V_{TH})}{k' R_1}} - V_{TH}$$

$$\text{small signal gain} = -\frac{d}{dV_{in}} \sqrt{\frac{2(V_{in} - V_{TH})}{k' R_1}}$$

$$= \frac{1}{k' R} \sqrt{\frac{k' R}{2(V_{in} - V_{TH})}}$$

$$= \sqrt{\frac{1}{2 k' R (V_{in} - V_{TH})}}$$

(46) By KCL,

$$\frac{V_x - V_{in}}{R_1} = I_{SD,M_1}$$

Assume $A_0 = \infty$, $\therefore V_x = V_+ = 0V$.

$$\therefore -\frac{V_{in}}{R_1} = \frac{1}{2} k' (V_{GS} - |V_{TH}|)^2,$$

where $K' = \mu_p \frac{W}{L} C_{ox}$.

$$\because V = -V_{out}$$

$$\therefore -\frac{V_{in}}{R_1} = \frac{1}{2} K' (- V_{out} - |V_{TH}|)^2$$

$$-\frac{2V_{in}}{R_1 k'} = (V_{out} + |V_{TH}|)^2$$

$$\therefore V_{out} = \sqrt{-\frac{2V_{in}}{R_1 k'}} - |V_{TH}|$$

//

(47) Assume $A_0 = \infty$,

$\therefore V_+ = V_- = V_{in}$

Using voltage divider:

$$V_{in} + V_{os} = V_{out} \frac{R_1}{R_1 + R_2}$$

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right)(V_{in} + V_{os})$$

(48) In Fig. (8.25),

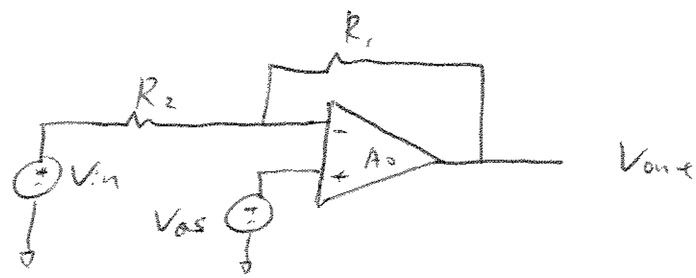
Assuming input is zero,

$$V_x = 10 \times V_{os,A_1}$$
$$= 30mV$$

$$\therefore V_{out} = 10 \times (V_{os,A_2} + V_x)$$
$$= 330mV$$

Thus, the maximum offset error is 330mV.

49



By KCL,

$$\frac{V_{in} - V_{os}}{R_2} = - \frac{V_{out} - V_{os}}{R_1}$$

$$V_{out} = - \frac{R_1}{R_2} (V_{in} - V_{os}) + V_{os}$$

(50) By eqn (8.72)

$$V_{out} = V_{os}\left(1 + \frac{R_2}{R_1}\right)$$

$$\therefore 20\,mV = 3\,mV\left(1 + \frac{R_2}{R_1}\right)$$

$$\frac{17}{3} = \frac{R_2}{R_1} \quad \text{———} (1)$$

$$\because \frac{1}{R_2 C_1} << 2\pi(1000)$$

and setting $C_1 = 100\,pF$,

$$\frac{1}{R_2 \times 100 \times 10^{-12}} << 2\pi(1000)$$

$$\frac{1}{R_2} << 6.283 \times 10^{-7}$$

$$\therefore R_2 >> 1.59\,M\Omega$$

choose $R_2 = 17\,M\Omega$.

$R_1 = 3\,M\Omega$. (From (1))

(51) From eqn (8.44),

$$V_{out} \propto \frac{dV_{in}}{dt}$$

(proportional)

Since offset is static (invariant with time)

i.e. $\frac{dV_{os}}{dt} = 0$.

$\therefore$ offset has no effect to V_{out}.

(5-2) From eqn (8.61),

with the presence of offset (V_{os}),

$$V_{out} = -V_T \ln \frac{V_{in} + V_{os}}{R_1 I_s}$$

The effect of offset to V_{out} is very small, because V_{out} is proportional to the log. of ($V_{in} + V_{os}$)

Thus, V_{out} is very insensitive to the magnitude of the offset.

(53). From eqn (8.76),

$$V_{out} = R_1 I_{B2}$$

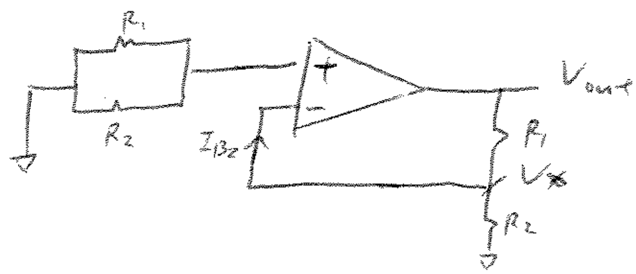
$\therefore V_{out}$ is independent of I_{B1}

Also I_{B1} will not affect $\frac{V_{out}}{V_{in}}$.

Thus, the small offset (ΔI) in the input bias currents has no effect on V_{out}.

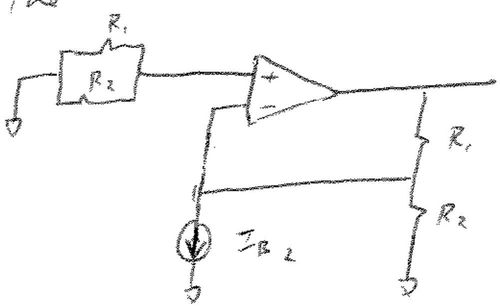
(54) Using superposition:

(I) turn off I_{B1}:



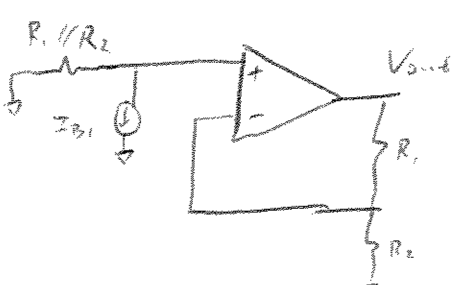
$\because V_+ = V_- = 0$, $\therefore V_x = 0$.

The circuit becomes:

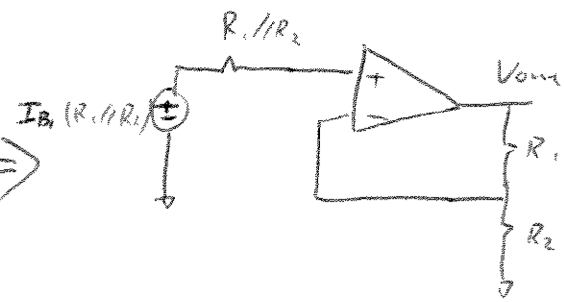


$\therefore$ From eqn (8.76), $V_{out,I_{B2}} = -R_1 I_{B2}$

(II) turn off I_{B2}:



$\Rightarrow$



$$\therefore V_{out,I_{B1}} = I_{B1}\left(\frac{R_1 R_2}{R_1+R_2}\right) \times \left(1+\frac{R_1}{R_2}\right)$$

$$= I_{B1} R_1$$

(54) Cont'd

$\therefore$ given $I_{B1} - I_{B2} = \Delta I$, and $V_{out} = I_{B1} + I_{B2}$,

$$I_{B1} R_1 - I_{B2} R_1 < \Delta V$$

$$\Delta I \; R_1 < \Delta V$$

$$\therefore R_1 < \frac{\Delta V}{\Delta I}$$

There is no dependence of output error on R_2.

(55) Using eqn. (8.84)

$$\text{Gain} = \frac{A_0}{1 + \frac{s}{\omega_1}}$$

For opamp (a): At 100 MHz:

$$\text{Gain}_{(a)} = \frac{1000}{1 + \frac{2\pi \times 100 \times 10^6}{2\pi \times 50}}$$

$$\approx 5 \times 10^{-4}$$

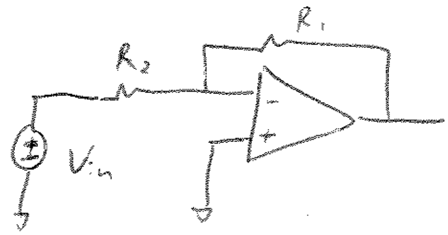
For opamp (b) at 100 MHz,

$$\text{Gain}_{(b)} = \frac{500}{1 + \frac{2\pi \times 100 \times 10^6}{2\pi \times 10}}$$

$$\approx 4.95 \quad > 4$$

$\therefore$ opamp (b) is a possible candidate

(5-6)



Using eqn (8.20),

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\frac{R_2}{R_1} + \frac{1}{A_0}\left(1+\frac{R_2}{R_1}\right)}$$

Here, A_0 becomes $\frac{A_0}{1+\frac{s}{\omega_1}}$.

$$\therefore \frac{V_{out}}{V_{in}} = \frac{-1}{\frac{R_2}{R_1} + \frac{A_0}{1+\frac{s}{\omega_1}}\left(1+\frac{R_2}{R_1}\right)}$$

$$= \frac{-\left(1+\frac{s}{\omega_1}\right)}{\left(1+\frac{s}{\omega_1}\right)\frac{R_2}{R_1} + A_0\left(1+\frac{R_2}{R_1}\right)}$$

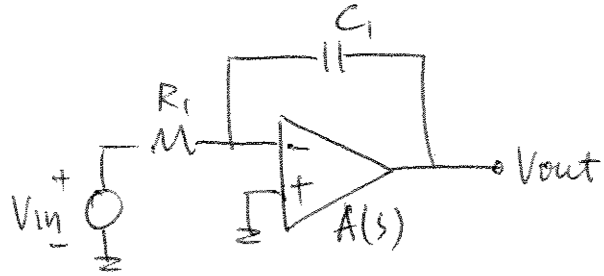
To find the pole, equate denominator to 0.

i.e. $\left(1+\frac{s}{\omega_1}\right)\frac{R_2}{R_1} + A_0\left(1+\frac{R_2}{R_1}\right) = 0$

$$\left(1+\frac{s}{\omega_1}\right) = -\frac{R_1}{R_2}A_0\left(1+\frac{R_2}{R_1}\right)$$

$$\therefore |\omega_{p,closed}| = \left(1+\frac{R_1}{R_2}A_0\left(1+\frac{R_2}{R_1}\right)\right)\omega_1$$

57.



$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

$$\omega_0 \gg \frac{1}{R_1 C_1}$$

$$\frac{V_{in} - V_{(-)}}{R_1} = (V_{(-)} - V_{out}) s C_1$$

$$-V_{(-)} \times A(s) = V_{out}$$

Substitute ② into ①:

$$\frac{V_{out}}{V_{in}}(s) = -\left[\frac{sC_1R_1 + 1}{A(s)} + sC_1R_1\right]^{-1}$$

$$= -\left[\frac{(sC_1R_1 + 1)(1 + s/\omega_0)}{A_0} + sC_1R_1\right]^{-1}$$

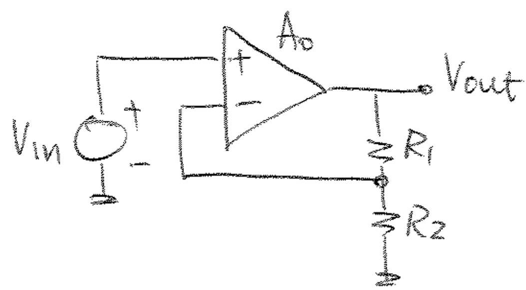
$$\simeq -\left\{\frac{1}{A_0\omega_0}\left[s\omega_0 C_1 R_1 + s^2 C_1 R_1 + s\right] + sCR\right\}^{-1}$$

$$\simeq -\left[s\left(C_1R_1 + \frac{1}{A_0\omega_0}\right) + s^2\left(\frac{C_1R_1}{A_0\omega_0}\right)\right]^{-1}$$

$$\approx -\left[sC_1R_1 + s^2\frac{C_1R_1}{A_0\omega_0}\right]^{-1}$$

$$= \frac{-1}{\left(1 + \frac{s}{A_0\omega_0}\right)\frac{s}{\left(\frac{1}{R_1C_1}\right)}}$$

58.



Nominal Gain = 4
Slew Rate = 1 V/ns
I/P = 0.5 V

$$V_{in}(t) = 0.5 \sin \omega t \Rightarrow V_{out} = 0.5 \times \underbrace{\left(1 + \frac{R_1}{R_2}\right)}_{=4} \sin \omega t.$$

$$\frac{dV_{out}}{dt} = 0.5\left(1 + \frac{R_1}{R_2}\right) \omega \cdot \cos \omega t.$$

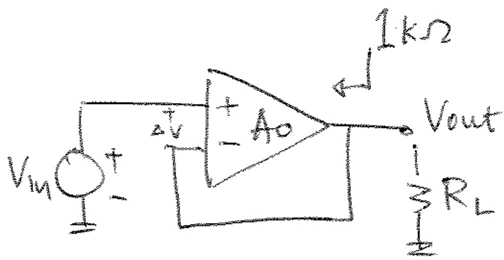
= maximum when $\cos \omega t = 1$

$$\Rightarrow \left.\frac{dV_{out}}{dt}\right|_{max} = 0.5\omega \left(1 + \frac{R_1}{R_2}\right) = 2\omega$$

∴ Highest frequency ⇒ $2\omega = 1$ V/ns

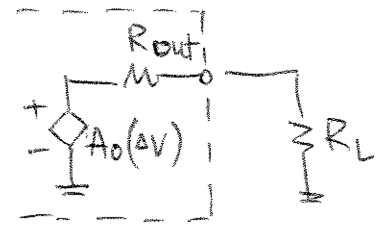
$\Rightarrow \omega = 0.5$ rad/ns $\Rightarrow f_{MAX} \simeq 79.6$ MHz

59.



$R_L = 100\,\Omega$

Gain Error $= 0.5\%$



$$(V_{in} - V_{out}) A_o \times \frac{R_L}{R_{out} + R_L} = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{R_{out} + R_L}{A_o R_L}} \simeq 1 - \underbrace{\frac{R_{out} + R_L}{A_o R_L}}_{=\epsilon}$$

$$\therefore \epsilon = \frac{R_{out} + R_L}{A_o R_L} \Rightarrow A_o = \frac{R_{out} + R_L}{\epsilon R_L} = \frac{1000 + 100}{0.5\% \times 100}$$

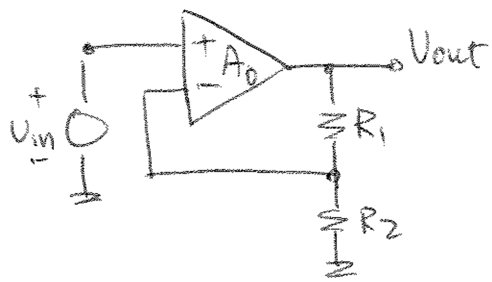
$$\approx 2200.$$

60.

Nominal Gain $= 4$

Gain Error $= 0.2\%$

$R_1 + R_2 = 20\,k\Omega$



$$\left[V_{in} - \frac{R_2}{R_1+R_2} \times V_{out}\right] A_0 = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \frac{R_2}{R_1+R_2} A_0} \simeq \left(1+\frac{R_1}{R_2}\right)\left[1-\left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0}\right]$$

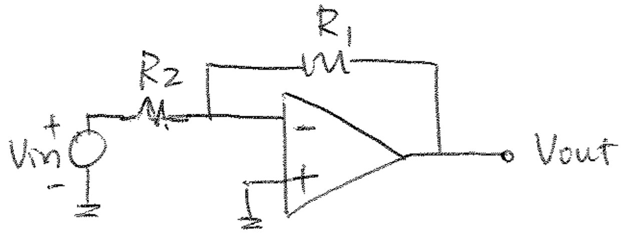
$$\left(1 + R_1/R_2\right) = 4 \quad \& \quad (R_1+R_2) = 20\,k\Omega$$

$$\Rightarrow R_1 = 15\,k\Omega \,, \; R_2 = 5\,k\Omega .$$

$$0.2\% = \left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0} \Rightarrow A_0 = \left(1+\frac{R_1}{R_2}\right) \times \frac{1}{0.2\%}$$

$$= 2000$$

61.



Nominal Gain = 8
Gain Error = 0.1%
R_{out} = 0.1%

$$V_x = V_{in} + (V_{out} - V_{in})\frac{R_2}{R_1+R_2} \qquad \text{— ①}$$

$$\frac{V_{out} - V_{in}}{R_1+R_2} = \frac{-A_0 V_x - V_{out}}{R_{out}} \qquad \text{— ②}$$

Substitute ② into ① gives:

$$\frac{V_{out}}{V_{in}} = \left(-\frac{R_1}{R_2}\right)\frac{A_0 - R_{out}/R_1}{\underbrace{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}}_{(1-\varepsilon)}}$$

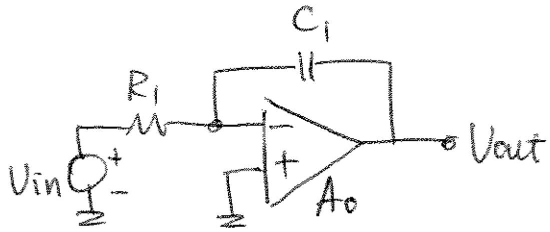
$\Rightarrow$ $8 = R_1/R_2$

$$0.1\% = 1 - \frac{A_0 - 100/R_1}{1 + \frac{100}{R_2} + A_0 + (8)}$$

$\Rightarrow$ Choose $R_1 = 8\,k\Omega$, $R_2 = 1\,k\Omega$

$\Rightarrow$ $A_0 \approx 9100$

62.



$= 100\ kHz$

pole $= 100\ Hz$

$C_{MAX} = 50\ pF$.

$$\frac{V_{in} - V_{(-)}}{R_1} = (V_{(-)} - V_{out})\, sC_1 \qquad —— \text{①}$$

$$V_{(-)} \cdot (-A_0) = V_{out} \qquad —— \text{②}$$

Substitute ② into ①:

$$\frac{V_{out}}{V_{in}} = \frac{-1}{\frac{1}{A_0} + \left(1 + \frac{1}{A_0}\right) R_1 C_1 s}$$

$$\Rightarrow S_P = \frac{-1}{(A_0+1)R_1 C_1} = -100\ Hz \qquad —— \text{①}$$

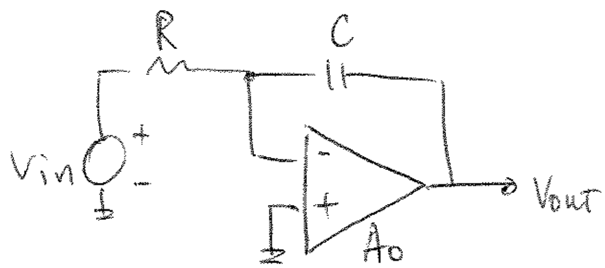
Attenuation above 100 kHz $\Rightarrow \left|\frac{V_{out}}{V_{in}}\right|_{100KHz} = 1$

$$\Rightarrow \left.\frac{A_0}{\sqrt{1 + \left[(A_0+1)R_1 C_1 \omega_z\right]^2}}\right|_{100KHz} = 1 \qquad —— \text{②}$$

Substitute ① into ②:

$\Rightarrow A_0 \simeq 1000.$ Choose $C = 50pF \Rightarrow R \simeq 200\ k\Omega$.

63.



$V(t) = \alpha t$

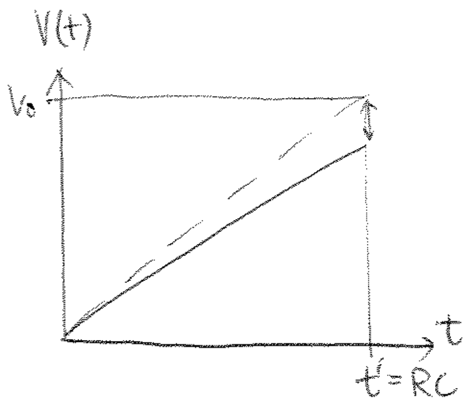
$0 < V(t) < V_o$

where $\alpha = 10\,V/\mu s$

$V_o = 1\,V$

$C_{max} = 20\,pF$

Error $< 0.1\%$



$V_{out}(t) = -\dfrac{V_o}{RC} t \quad , \quad t \in [0, RC]$

$V(t) = -\alpha t$

At $t = RC$, $\dfrac{\Delta V}{V_o} = \left.\dfrac{V_{out}(t) - V(t)}{V_o}\right|_{t=RC} < 0.1\%$

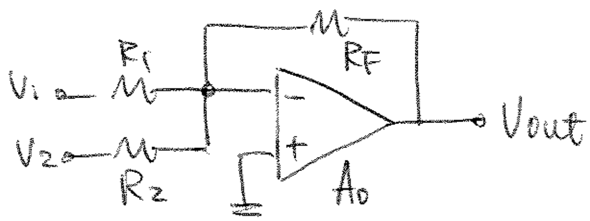
$\Rightarrow \Delta V = V_o \times 0.1\% = 0.001\,V$

$\Rightarrow \left. -\dfrac{V_o}{RC} \times t + \alpha t \right|_{t=RC} = 0.00 \quad V \quad (= \Delta V)$

Choose $C = 20\,pF$

$\therefore R = \dfrac{V_o - \Delta V}{\alpha C} = \dfrac{1V - 0.001\,V}{10\,V/\mu s \times 20pF} = 4995\,\Omega$

64.



$$V_{out} = \alpha_1 V_1 + \alpha_2 V_2$$

$\alpha_1 = 0.5$, $\alpha_2 = 1.5$

Error of $\alpha \le 0.5\%$

$r_{in} \ge 10\,k\Omega$.

$$\frac{V_1 - V_{(-)}}{R_1} + \frac{V_2 - V_{(-)}}{R_2} = \frac{V_{(-)} - V_{out}}{R_F} \quad \text{— (1)}$$

$$V(-A_0) = V_{out} \quad \text{— (2)}$$

Substitute (2) into (1) & solve for V_{out}:

$$V_{out} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right) \cdot \left[\frac{1}{A_0}\left(\frac{R_F}{R_1} + \frac{R_F}{R_2} + 1\right) + 1\right]^{-1}$$

$$\simeq -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2\right) \cdot \left[1 - \frac{1}{A_0}\left(\frac{R_F}{R_1} + \frac{R_F}{R_2} + 1\right)\right]$$

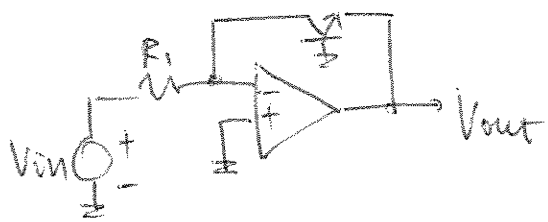
Choose $R_{in,v_2}\ (\simeq R_2) = 10\,k\Omega \Rightarrow R_F = \alpha_2 \times R_2 = 15\,k\Omega$

$\Rightarrow R_1 = R_F/\alpha_1 = 30\,k\Omega \simeq R_{in,v_1}$

$$\Rightarrow \epsilon = 0.5\% = \frac{1}{A_0}\left(\frac{R_F}{R_1} + \frac{R_F}{R_2} + 1\right)$$

$$\Rightarrow A_0 = \frac{1}{0.5\%}\left(0.5 + 1.5 + 1\right) = 600 \quad \text{(or larger)}$$

65.



$[0.1, 2]\,V \longmapsto [-0.5, -1]\,V$

$$V_{out} = -V_T \ln \frac{V_{in}}{I_S R_1}$$

$$-0.5\,V = -V_T \ln\left[\frac{(0.1)}{I_S R_1}\right] \Rightarrow I_S R_1 = 4.45 \cdot 10^{-10}\,V \quad —①$$

$$\Rightarrow -V_T \ln \frac{(2)}{I_S R_1} = -0.026\,V \ln\left(\frac{2}{4.45 \cdot 10^{-10}}\right) \approx -0.58\,V$$

∴ input range of $0.1 \leftrightarrow 2\,V$ corresponds to output range of $-0.5 \leftrightarrow -0.58\,V$

Choose $I_S = 1 \times 10^{-16}\,A \Rightarrow R_1 = 4.45\,M\Omega$.

(66.) No, this is not possible to ... requirements.

$$\frac{dV_{out}}{dV_{in}} = \frac{V_T}{V_{in}}$$

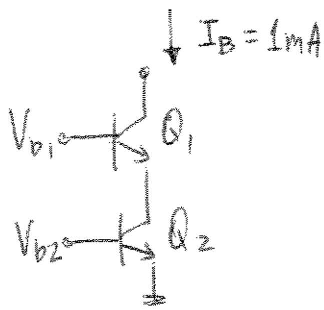
Assuming temperature is fixed, V_T is a fixed quantity that is both process and design independent.

At 25°C, $V_T \approx 25mV$.

$$\therefore \left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=1V} = 25\ mV/V$$

$$\left.\frac{dV_{out}}{dV_{in}}\right|_{V_{in}=2V} = 12.5\ mV/V.$$

1.



$I_S = 6 \cdot 10^{-17} A$ $\quad \beta = 100$

$V_A \to \infty$ $\quad V_T = \frac{kT}{q}$

$\alpha = \frac{\beta}{\beta + 1}$

(a) $V_{b2} = V_T \ln\left(\frac{I_B/\alpha^2}{I_S}\right) = (0.026V) \ln\left(\frac{1.02mA}{6 \cdot 10^{-17}A}\right)$

$\approx 0.792 \ V$

(b) From the configuration,

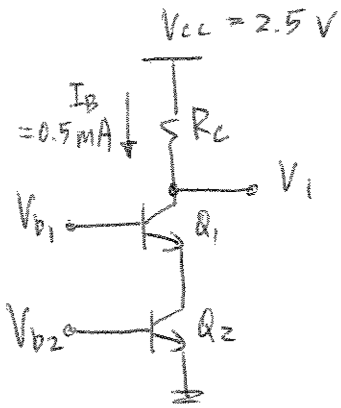
$V_{b1} = V_{CE_2} + V_{BE_1} = (V_{BE_2} - 300mV) + V_{BE_1}$

$V_{BE_1} = V_T \ln(I_B/I_S) = (0.026V) \ln\left(\frac{1mA}{6 \cdot 10^{-17}A}\right)$

$\approx 0.792 \ V$

$\therefore V_{b1} = (0.792 - 0.3) + 0.79 = 1.28 \ V$

2.



(a) $V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_B/\alpha^2}{I_S}\right) = (0.026V)\ln\left(\frac{0.51\,mA}{6\cdot 10^{-17}A}\right)$

$\approx 0.774\ V$

$V_{BE1} = V_{b1} - V_{C2} = V_{b1} - (V_{b2} - 300mV)$

$\Rightarrow V_{b1} = V_{BE1} + V_{b2} - 0.3V$

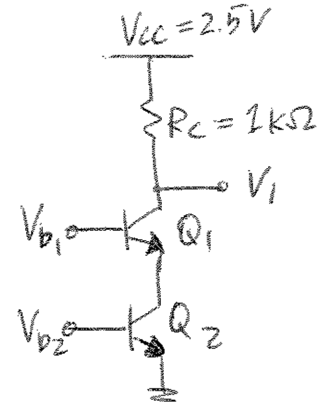
$= (0.026V)\ln\left(\frac{0.5mA}{6\cdot 10^{-17}A}\right) + (0.774V) - (0.3V)$

$\approx 1.25\ V$

(b) $V_1 = V_{b1} - 0.3V = 0.95V$

$\therefore\ R_C = \frac{V_{CC} - V_1}{I_B} = \frac{(2.5 - 0.95)V}{0.5mA} \approx 3.1\ k\Omega$

3. From previous experience, assume both V_{BE1} & V_{BE2} = 0.8 V



$$\Rightarrow V_1 = V_{CE_1} + V_{CE_2}$$

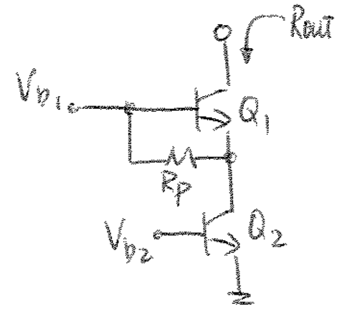
$$= (V_{BE_1} - 200mV) + (V_{b2} - 200mV)$$

$$= 1.2\ V$$

- By KCL, maximum bias current

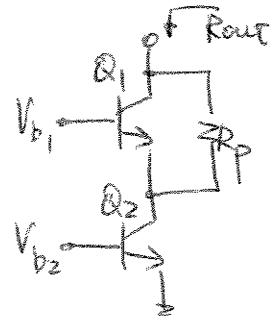
$$\approx \frac{V_{cc} - V_1}{R_C} = \frac{(2.5 - 1.2)V}{1k\Omega} = 1.3\ mA.$$

4. (a) R_p appears in parallel with r_{π_1}



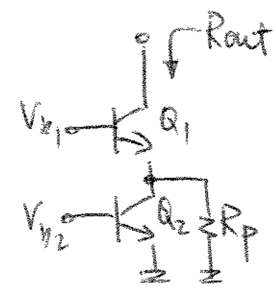
$$\therefore R_{out} = [1+g_{m_1}(r_{o2} \| r_{\pi_1} \| R_p)]r_{o1} + (r_{o2} \| r_{\pi_1} \| R_p)$$

(b) R_p appears in parallel with r_{o1}



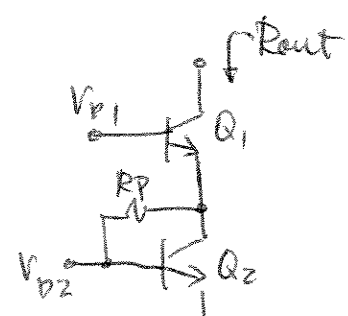
$$\therefore R_{out} = [1+g_{m_1}(r_{o2} \| r_{\pi_1})](r_{o1} \| R_p) + (r_{o2} \| r_{\pi_1})$$

(c) R_p appears in parallel with r_{o2}



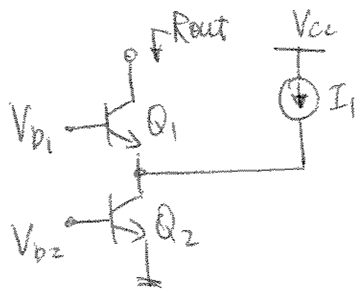
$$\therefore R_{out} = [1+g_{m_1}(r_{o2} \| r_{\pi_1} \| R_p)]r_{o1} + (r_{o2} \| r_{\pi_1} \| R_p)$$

(d) R_p appears in parallel with r_{o2} (in small-signal) $\because V_{b2}$ is AC GND.



$$\therefore R_{out} = [1+g_{m_1}(r_{o2} \| r_{\pi_1} \| R_p)]r_{o1} + (r_{o2} \| r_{\pi_1} \| R_p)$$

5.



$I_1 = 0.5\,mA$

$I_{C1} = 0.5\,mA \qquad I_{C2} = 1\,mA.$

$= 2I_{C1}$

$\beta = 100 \qquad V_A = 5V$

$$R_{out} = g_{m1} r_{o1} (r_{o2} \| r_{\pi 1})$$

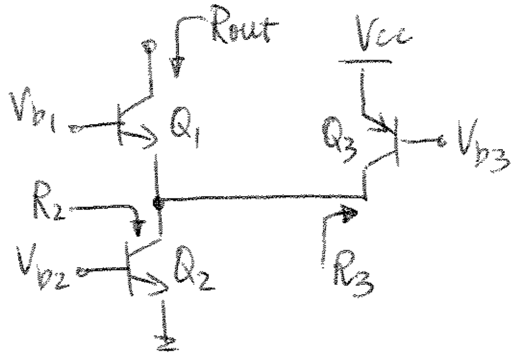
$$= \frac{I_{C1}}{V_T} \cdot \frac{V_A}{I_{C1}} \cdot \frac{V_{A2}/I_{C2} \cdot \beta V_T / I_{C1}}{V_{A2}/I_{C2} + \beta V_T / I_{C1}}$$

$$= \frac{V_A}{V_T} \cdot \frac{\frac{V_{A2}/2}{I_{C1}} \cdot \beta V_T / I_{C1}}{\frac{V_{A2}/2}{I_{C1}} + \beta V_T / I_{C1}} \approx \frac{1}{I_{C1}} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A + 2\beta V_T}$$

$$= \frac{1}{0.5mA} \cdot \frac{5V}{0.026V} \cdot \frac{100(5V)(0.026V)}{(5V) + 2(100)(0.026V)}$$

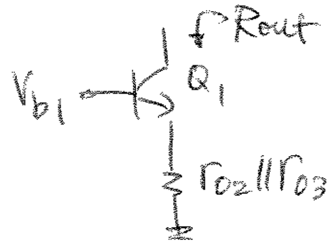
$\therefore\ R_{out} \approx 490.\ k\Omega.$

6.



$R_3 = r_{o3}$ (V_{cc} & V_{b3} are AC GND)

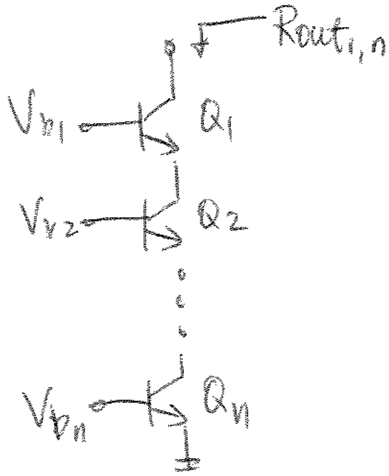
$R_2 = r_{o2}$ (V_{b2} is AC GND)



$$\therefore R_{out} = \left[(1 + g_{m1}(r_{o2} \| r_{o3} \| r_{\pi 1})\right] r_{o1} + (r_{o2} \| r_{o3} \| r_{\pi 1})$$

$$\approx g_{m1} r_{o1} (r_{o2} \| r_{o3} \| r_{\pi 1})$$

7.



Suppose $R_{out_{i,j}}$ is the output impedance of the cascode circuit with BJTs $Q_i, Q_{i+1}, Q_{i+2}, \ldots Q_{j-1}, Q_j$.

$$R_{out_{n-1,n}} = [1 + g_{m_{n-1}}(r_{o_n} \| r_{\pi_{n-1}})] r_{o_{n-1}} + (r_{o_n} \| r_{\pi_{n-1}})$$

$$\approx g_{m_{n-1}}(r_{o_n} \| r_{\pi_{n-1}}) r_{o_{n-1}} \approx g_{m_{n-1}} r_{\pi_{n-1}} r_{o_{n-1}}$$

$$= \beta r_o \qquad (\text{usually, } r_\pi \ll r_o)$$

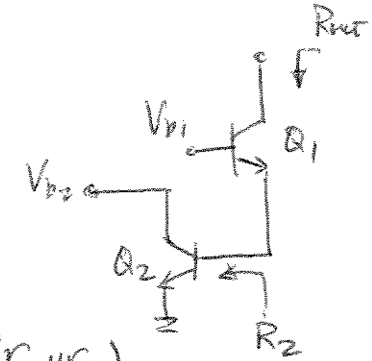
$$R_{out_{n-2,n}} = [1 + g_{m_{n-2}}(\beta r_o \| r_{\pi_{n-2}})] r_o + (\beta r_o \| r_{\pi_{n-2}})$$

$$\approx g_m r_\pi r_o + r_\pi \cong \beta r_o$$

This means that $R_{out} \approx \beta r_o$ even if an extra BJT is employed in the cascode configuration.

i.e. $R_{out_{MAX}} \approx \beta r_o$

8. (a) $R_2 = (r_{\pi_2} \| r_{\pi_1})$



$$\therefore R_{out} = [1 + g_{m_1} R_2] r_{o_1} + R_2$$

$$= [1 + g_{m_1}(r_{\pi_1} \| r_{\pi_2})] r_{o_1} + (r_{\pi_1} \| r_{\pi_2})$$

(b) In part (a), $I_{C2} = \beta I_{C1}$ $(= I_{B_2})$

$$\therefore R_{out(a)} = \left[1 + g_{m_1}\left(\frac{\beta V_T}{I_{C_1}} \| \frac{V_T}{I_{C_1}}\right)\right] r_{o_1} + (r_{\pi_1} \| r_{\pi_2})$$

$$\approx \left(1 + g_{m_1}\frac{V_T}{I_{C_1}}\right) r_{o_1} + \frac{V_T}{I_{C_1}}$$

$$= 2r_{o_1} + V_T/I_{C_1}$$

$$R_{out,cascode} = [1 + g_{m_1}(r_{o2} \| r_{\pi_1})] r_{o_1} + (r_{o2} \| r_{\pi_1})$$

$$\approx [1 + g_{m_1} r_{\pi_1}] r_{o_1} + r_{\pi_1}$$

$$\approx \beta r_{o_1} + r_{\pi_1} = \beta r_{o_1} + V_A/I_{C_1}$$

Compare term-by-term:

$$\left.\begin{array}{l} 2r_{o_1} \ll \beta r_{o_1} \\ V_T \ll V_A \end{array}\right\} \Rightarrow R_{out(a)} \ll R_{out,cascode}$$

i.e. using (a) reduces the effect of having a cascode configuration.

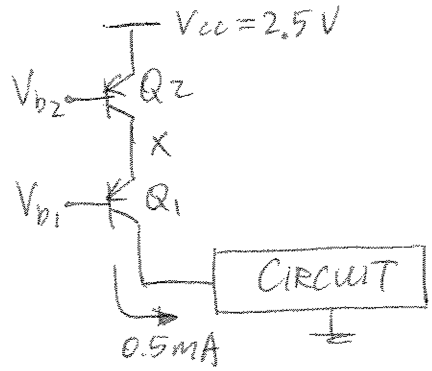
9. $$R_{out} = \frac{1}{I_C} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A + \beta V_T}$$

$$\approx \frac{1}{I_C} \cdot \frac{V_A}{V_T} \cdot \frac{\beta V_A V_T}{V_A} = \beta \frac{V_A}{I_C} = \beta r_o$$

$$= R_{out,max}$$

This means that $R_{out,max}$ is often achieved with 2-BJT cascode.

10.



$I_S = 10^{-16}$ A $\beta = 100$

$I_{BIAS} = 0.5$ mA

(a) $I_{BIAS} \approx I_{C2} = 0.5$ mA

$$\therefore V_{b2} = V_{CC} - |V_{be2}|$$
$$= V_{CC} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (2.5V) - (0.026V)\ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.74V$$

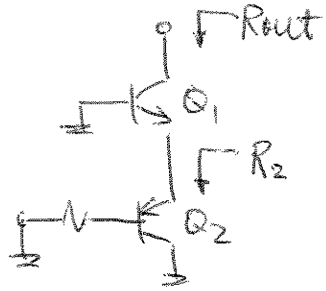
(b) $|V_{CB2}| = V_X - V_{b2} = 200$mV

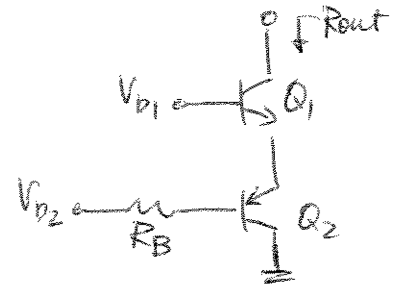
$\Rightarrow V_{C2} = V_{b2} + |V_{CB2}| = 1.94V$

$$\therefore V_{b1} = V_{C2} - |V_{BE1}| = V_{C2} - V_T \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)$$
$$= (1.94V) - (0.026V)\ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \approx 1.18V$$

$\Rightarrow$ Maximum allowable $V_{b1} = 1.18V$

11. (a)





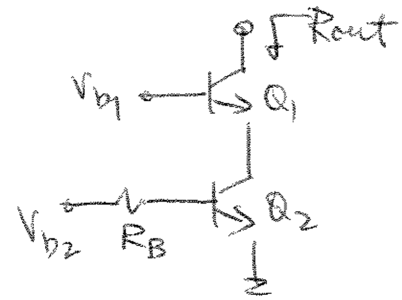
(AC-small signal)

Looking into emitter of Q2,

$$R_2 = \frac{1}{\left(\frac{\beta+1}{R_B + r_{\pi_2}} + \frac{1}{r_{o2}}\right)}$$

$$\Rightarrow R_{out} = \left[1 + g_{m_1}(R_2 \,\|\, r_{\pi_1})\right] r_{o_1} + (R_2 \,\|\, r_{\pi_1})$$

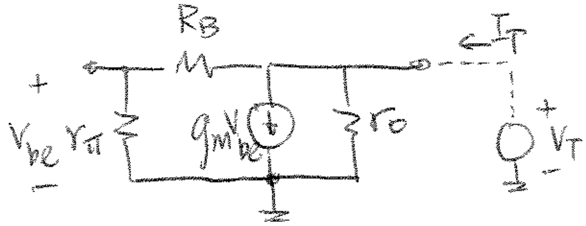
(b) R_B does not affect Q_2 in small-signal R_{out}:

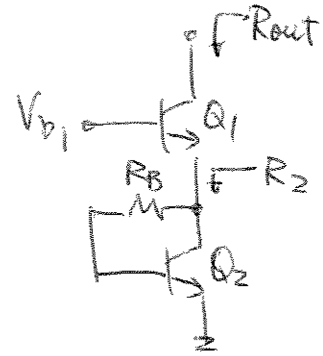


$$\therefore R_{out} = \left[1 + g_{m_1}(r_{o2} \,\|\, r_{\pi_1})\right] r_{o_1} + (r_{o2} \,\|\, r_{\pi_1})$$

This is a cascode stage.

(c) Use small-signal analysis:



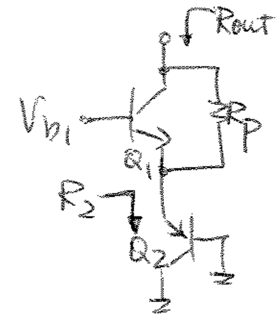


By KCL, $I_T = \dfrac{V_T}{R_B + r_{\pi_2}} + g_{m_2} \dfrac{V_T r_{\pi_2}}{r_{\pi_2} + R_B} + \dfrac{V_T}{r_{o_2}}$

$$\Rightarrow R_2 = \frac{V_T}{I_T} = \frac{1}{\left(\dfrac{\beta+1}{R_B + r_{\pi_2}} + \dfrac{1}{r_o}\right)} \approx \frac{1}{\beta/(R_B + r_{\pi_2}) + 1/r_{o_2}}$$

$$\therefore R_{out} = \left[1 + g_{m_1}(R_2 \| r_{\pi_1})\right] r_{o_1} + (R_2 \| r_{\pi_1})$$
$$\approx g_{m_1} r_{o_1} (R_2 \| r_{\pi_1})$$

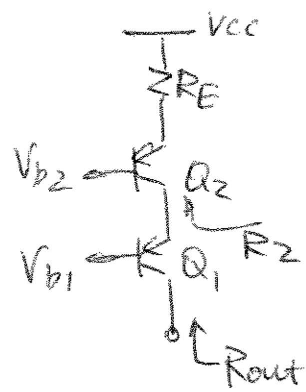
(d) R_p appears in parallel with r_{o1}.



$$R_2 = r_{o2} \,||\, \frac{1}{g_{m2}} \,||\, r_{\pi 2}$$
$$\approx r_{o2} \,||\, \frac{r_{\pi 2}}{\beta} \,||\, r_{\pi 2}$$
$$\approx r_{o2} \,||\, (r_{\pi 2}/\beta) \approx r_{\pi 2}/\beta$$

$$\therefore R_{out} = [1 + g_{m1}(R_2 \,||\, r_{\pi 1})](r_{o1} \,||\, R_p) + (R_2 \,||\, r_{\pi 1})$$
$$\approx g_{m1}(r_{o1} \,||\, R_p)(r_{\pi 1} \,||\, R_2)$$

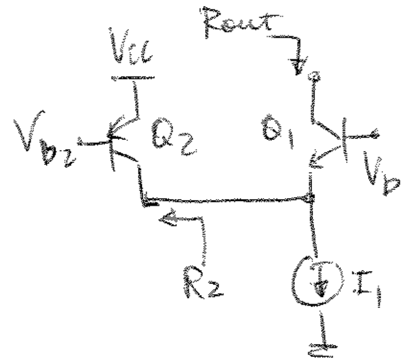
(e) $$R_2 = [1 + g_{m2}(R_E \,||\, r_{\pi 2})] r_{o2} + (R_E \,||\, r_{\pi 2})$$
$$\approx g_{m2}(R_E \,||\, r_{\pi 2}) r_{o2}$$



$$\therefore R_{out} = [1 + g_{m1}(R_2 \,||\, r_{\pi 1})] r_{o1} + (R_2 \,||\, r_{\pi 1})$$
$$\approx g_{m1}(R_2 \,||\, r_{\pi 1}) r_{o1}$$
$$= g_{m1} [g_{m2} r_{o2}(R_E \,||\, r_{\pi 2}) \,||\, r_{\pi 1}] r_{o1}$$

This is a cascode stage.

(f) $R_2 = r_{o2}$

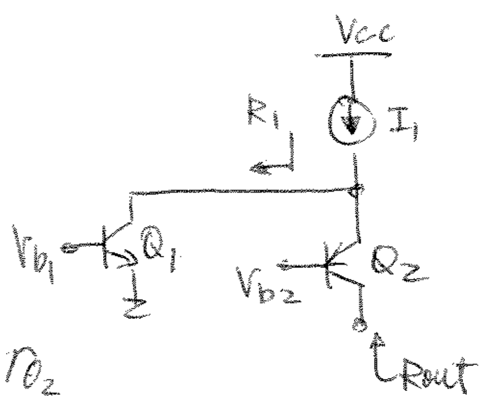


$$\therefore R_{out} = [1 + g_{m1}(R_2 \| r_{\pi 1})] r_{o1} + (R_2 \| r_{\pi 1})$$

$$\approx g_{m1} r_{o1} (R_2 \| r_{\pi 1})$$

$$= g_{m1} r_{o1} (r_{\pi 1} \| r_{o2})$$

(g) $R_1 = r_{o1}$
(output impedance of a common-emitter)

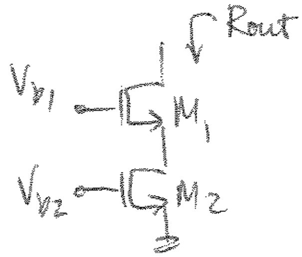


$$\therefore R_{out} = [1 + g_{m2}(R_1 \| r_{\pi 2})] r_{o2}$$

$$+ (R_1 \| r_{\pi 2})$$

$$\approx g_{m2} r_{o2} (r_{o1} \| r_{\pi 2})$$

12.



$I_D = 0.5\,mA$ $\quad R_{out} \geq 50k\Omega$

$\mu_n C_{ox} = 100 \frac{\mu A}{V^2}$ $\quad \frac{W}{L} = \frac{20}{0.18}$

Calculate max λ.

Assume M_1 & M_2 in saturation.

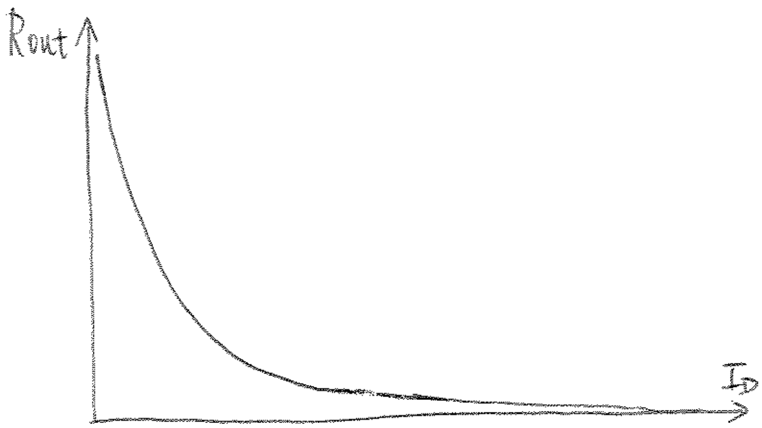
$$\Rightarrow R_{out} \approx g_{m1} r_{o1} r_{o2}$$
$$= \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \times \frac{1}{\lambda I_D} \times \frac{1}{\lambda I_D} \geq 50k\Omega.$$

(All quantities are known).
Solve for λ:

$$\lambda_{max} \approx 0.51\ V^{-1}$$

13. (a) $R_{out} = g_{m2} r_{o1} r_{o2} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$

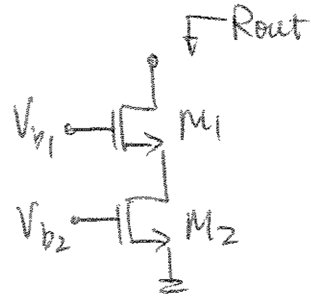
$= 2 \mu_n C_{ox} (W/L) \cdot (I_D)^{-3/2}$



(b) R_{out} (BJT) $\propto I_B^{-1}$
R_{out} (MOS) $\propto I_B^{-3/2}$

∴ MOS cascode is a stronger function of I in terms of R_{out}.

14.



$\left(\frac{W}{L}\right)_1 = 30/0.18 \qquad \left(\frac{W}{L}\right)_2 = 20/0.18$

$I_{BIAS} = 0.5\,mA$

$\mu_n C_{ox} = 100\,\frac{\mu A}{V^2} \qquad V_{TH} = 0.4\,V$

(a) $I_{D2} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + V_{TH}$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(20/0.18)}} + 0.4V \approx 0.7\,V$$

M_2 operates in saturation as long as $V_{GS2} - V_{TH} \le V_{DS2} \Rightarrow V_{DS2} \ge 0.3\,V$.

Observe that $V_{GS1} = V_{b1} - V_{DS2}$

$$I_{D1} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{DS2} - V_{TH})^2$$

$$\Rightarrow V_{b1} \ge \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + 0.4V + 0.3V$$

$$= \sqrt{\frac{2(0.5mA)}{(100\,\frac{\mu A}{V^2})(30/0.18)}} + 0.7V \approx 0.95\,V.$$

$\therefore$ Minimum $V_{b1} = 0.95V$.

(b) $R_{out} = (1 + g_{m1} r_{o2}) r_{o1} + r_{o2}$

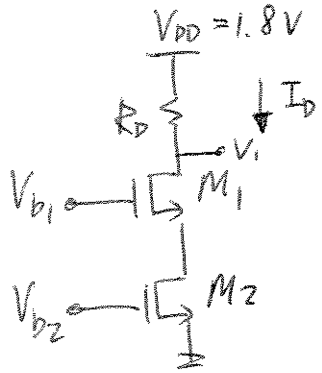
$$= \left(1 + \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{BIAS}} \cdot \frac{1}{\lambda I_{BIAS}}\right) \cdot \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$= \left[1 + \sqrt{2\left(100\frac{\mu A}{V^2}\right)\left(\frac{30}{0.18}\right)(0.5mA)} \cdot \frac{1}{(0.1)(0.5m)}\right] \cdot \frac{1}{(0.1)(0.5m)}$$

$$+ \frac{1}{(0.1)(0.5mA)}$$

$$\approx 1.67\ M\Omega$$

15.



$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\quad\left(\frac{W}{L}\right)_2 = \frac{40}{0.18}$

$\mu_n C_{ox} = 100\,\frac{\mu A}{V^2}$ $\quad V_{TH} = 0.4\,V.$

$I_D = 1\,mA$ $\quad R_D = 500\,\Omega$

(a) Both M_1 & M_2 must stay in saturation.

$$\Rightarrow V_1 = 1.8 - I_D R_D = 1.8 - (1mA)(500\Omega) = 1.3\,V.$$

Want this value equal to that which makes M_1 operates at the edge of saturation.

$$\therefore V_{b_1} = V_1 + V_{TH} = 1.3 + 0.4 = 1.7\,V.$$

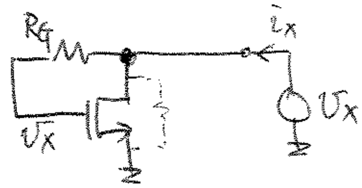
(b) $$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot \left[(V_{b_1} - V_X) - V_{TH}\right]^2 = 1mA.$$

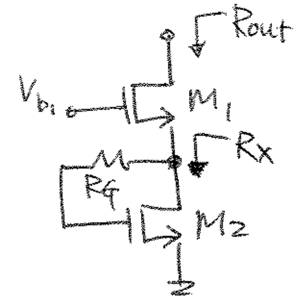
$$\Rightarrow V_X = V_{b_1} - V_{TH} - \sqrt{\frac{2 I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}}$$

$$= (1.7\,V) - (0.4\,V) - \sqrt{\frac{2(1mA)}{(100\,\mu A/V^2)(20/0.18)}}$$

$$\approx 1.276\,V.$$

1b. (a) First compute R_x:

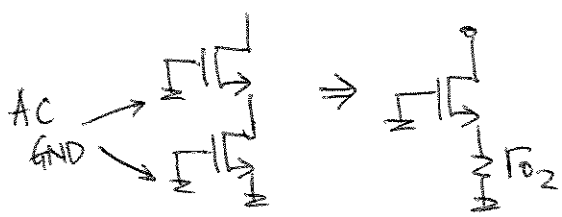


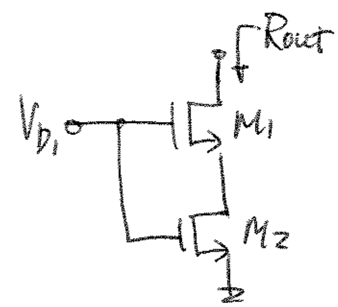


$$I_x = g_{m_2} v_x + v_x / r_{o_2} \Rightarrow R_x = v_x / i_x = \frac{1}{g_{m_2} + 1/r_{o_2}}$$

$$\therefore R_{out} = g_{m_1} r_{o_1} R_x = \frac{g_{m_1} r_{o_1}}{g_{m_2} + 1/r_{o_2}}$$

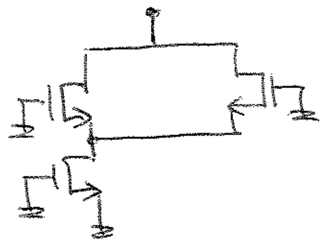
(b) Equivalent circuit:

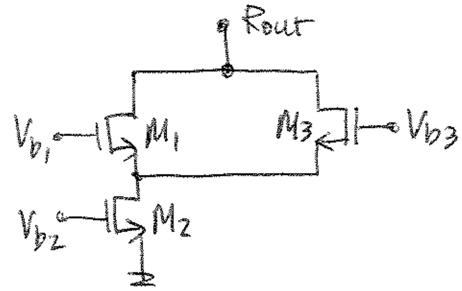


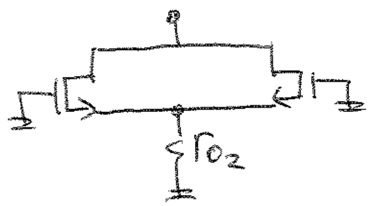


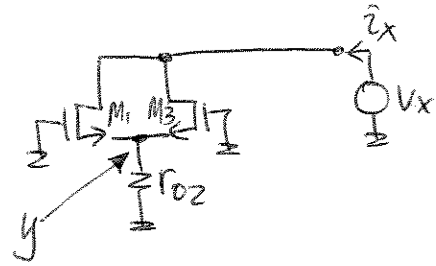
$$\therefore R_{out} = g_{m_1} r_{o_1} r_{o_2}$$

(c) Equivalent Circuit:









By KCL, $V_y = i_x \cdot r_{o2}$ ——— ①

$$i_x = g_{m1}(-V_y) + g_{m3}(-V_y) + (v_x - V_y)\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right) \quad ②$$

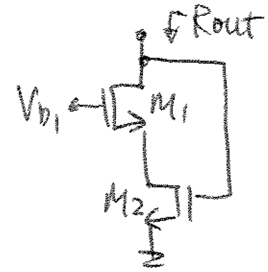
Substitute ① into ② and re-arrange:

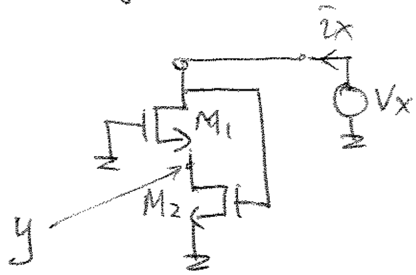
$$R_{out} = \frac{v_x}{i_x} = (r_{o1} \| r_{o3}) + r_{o2}(r_{o1} \| r_{o3})(g_{m1} + g_{m3}) + r_{o2}$$

$$\approx r_{o2}(r_{o1} \| r_{o3})(g_{m1} + g_{m3})$$

(Intuitively this makes sense because we have 2 NMOSs in parallel — (↓) $= g_m v_{gs}$ adds up, and r_o's are splitting total current, i_x. This is as if an equivalent NMOS replacing M_1 & M_3 with $g_m = (g_{m1} + g_{m3})$ & $r_o = (r_{o1} \| r_{o3})$.)

(d) Examine the equivalent circuit with a test voltage:





By observation, i_x must flow through both M_1 & M_2.

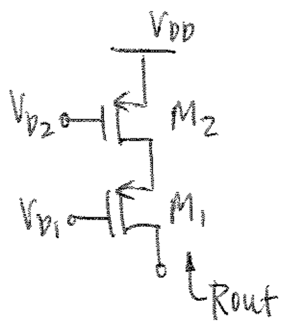
By KCL,
$$i_x = g_{m_2} v_x + v_y / r_{o_2}$$
$$i_x = g_{m_1}(-v_y) + (v_x - v_y)/r_{o_1}$$

Substitute ① into ② and re-arrange:

$$R_{out} = \frac{v_x}{i_x} = \frac{g_{m_1} r_{o_2} + \frac{r_{o_2}}{r_{o_1}} + 1}{g_{m_1} g_{m_2} r_{o_2} + (g_{m_2} r_{o_2} + 1)\left(\frac{1}{r_{o_1}}\right)}$$

$$\approx \frac{r_{o_2}\left(g_{m_1} + \frac{1}{r_{o_1}}\right)}{g_{m_2} r_{o_2}\left(g_{m_1} + \frac{1}{r_{o_1}}\right)} \approx \frac{1}{g_{m_2}}$$

17.



$I_{BIAS} = 0.5\ mA$

$R_{out} = 40\ k\Omega$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda = 0.2\ V^{-1}$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$

$$R_{out} = 40\ k\Omega = (g_{m1} r_{o2} + 1) r_{o1} + r_{o2}$$

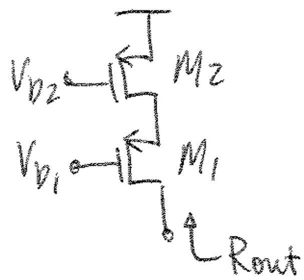
$$\Rightarrow g_{m1} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_1 I_{BIAS}} = \left(\frac{R_{out} - r_{o2}}{r_{o1}} - 1\right) \cdot \frac{1}{r_{o2}}$$

$$\therefore \left(\frac{W}{L}\right)_1 = \left[\left(\frac{R_{out} - r_{o2}}{r_{o1}} - 1\right) \frac{1}{r_{o2}}\right]^2 \cdot \frac{1}{2 \mu_p C_{ox} I_{BIAS}}$$

$$= \left\{\left[\frac{(40k\Omega) - [(0.2)(0.5m)]^{-1}}{[(0.2)(0.5m)]^{-1}} - 1\right] \cdot [0.2 \cdot 0.5m]\right\}^2 \cdot \frac{1}{2\left(50 \frac{\mu A}{V^2}\right)(0.5mA)}$$

$$\approx 0.8$$

18.



$$R_{out} = g_{m1} r_{o1} r_{o2} = \sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right)_1 I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

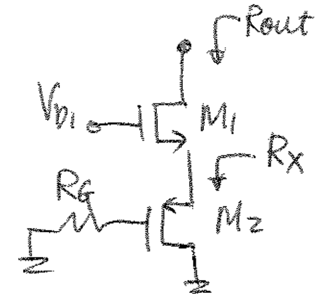
If W_1 & W_2 increase by N times and L_1, L_2, and I_D remain unchanged:

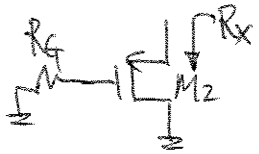
$$R_{out}(New) = \sqrt{2\mu_p C_{ox} \left(\frac{NW}{L}\right) I_D} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

$$= \sqrt{N} \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D} \left(\frac{1}{\lambda I_D}\right)^2 = \sqrt{N}\, R_{out}$$

∴ Rout is increased by $\sqrt{N}$ times.

19. (a) R_x is the input impedence of a common-gate configuration:



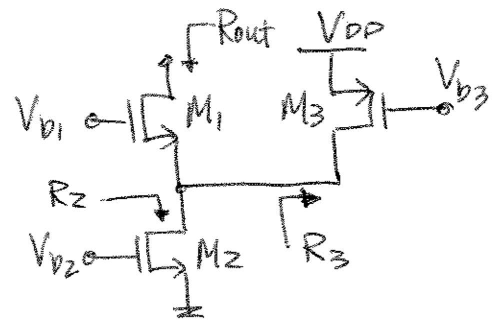


"Looking into" the source of M_2,

$$R_x = \frac{1}{g_{m2}} \parallel r_{o2}$$

$$\therefore R_{out} = g_{m1} r_{o1} R_x = g_{m1} r_{o1} \left(\frac{1}{g_{m2}} \parallel r_{o2}\right)$$

(b) From observation,



→ $R_3 = r_{o3}$ ($\because V_{SG} = 0$ in AC)

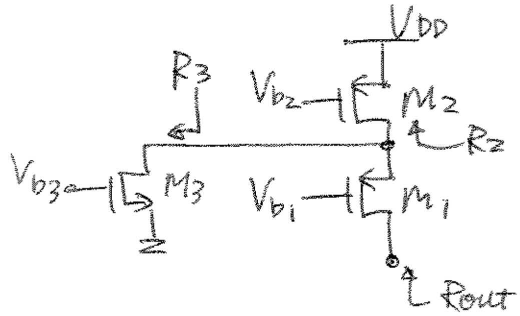
→ $R_2 = r_{o2}$ ($\because V_{SG} = 0$ in AC)

$$\therefore R_{out} = g_{m1} r_{o1} (R_2 \parallel R_3) = g_{m1} r_{o1} (r_{o2} \parallel r_{o3})$$

(c) By observation,

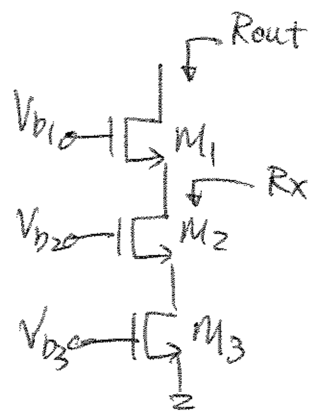
$R_2 = r_{o2}$ $(V_S = V_G = AC\ GND)$

$R_3 = r_{o3}$ $(V_S = V_G = AC\ GND)$



$\therefore R_{out} = g_{m1} r_{o1} (R_2 \| R_3) = g_{m1} r_{o1} (r_{o2} \| r_{o3})$

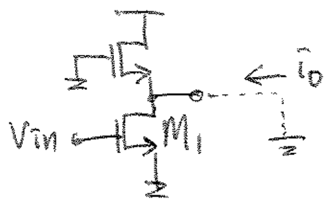
(d) $R_X = g_{m2} r_{o2} r_{o3}$

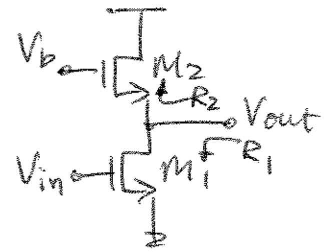


$\Rightarrow R_{out} = g_{m1} r_{o1} R_X$

$= g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}$

20.(a) Equivalent circuit:



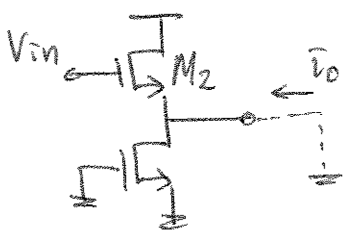


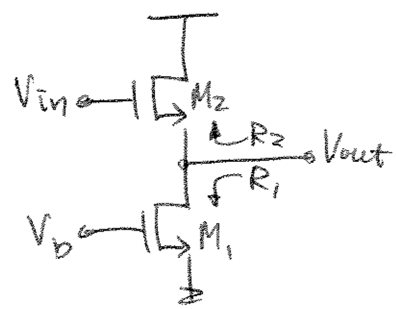
$$i_o = G_m v_{in} \Rightarrow G_m = \frac{i_o}{v_{in}} = g_{m1}$$

$$R_1 = r_{o1} \ ; \ R_2 = 1/g_{m2}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}(r_{o1} \parallel 1/g_{m2})$$

(b) Equivalent circuit:



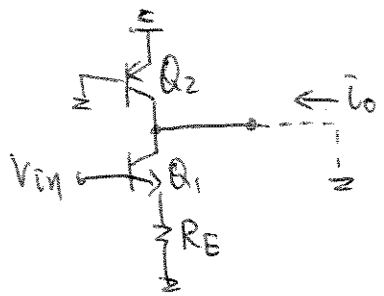


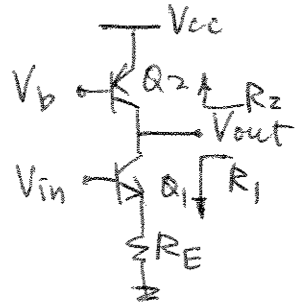
$$-i_o = g_{m2} V_{in} \Rightarrow G_m = \frac{i_o}{V_{in}} = -g_{m2}$$

$$R_1 = r_{o1} \ ; \ R_2 = 1/g_{m2}$$

$$\therefore A_v = -G_m R_{out} = g_{m2}(r_{o1} \parallel 1/g_{m2})$$

(c) Equivalent circuit:





With output node shorted, this is a common-emitter stage with degeneration.

$$\Rightarrow G_m = \frac{g_{m1}}{g_{m1}(R_E \| r_{o1}) + 1}$$

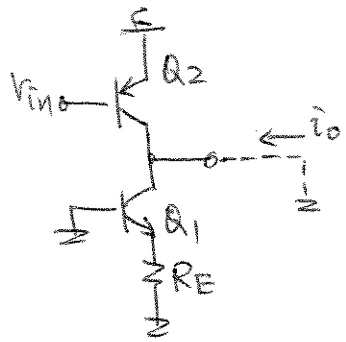
$$R_1 = [1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})$$

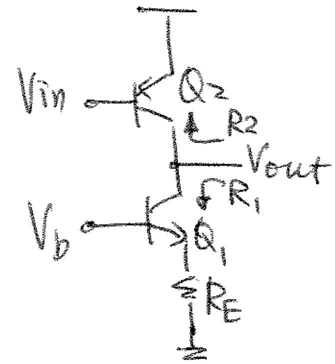
$$R_2 = r_{o2}$$

$$\Rightarrow R_{out} = R_1 \| R_2$$

$$\therefore A_v = -G_m R_{out} = -\frac{g_{m1}\left(\{[1 + g_{m1}(R_E \| r_{\pi 1})] r_{o1} + (R_E \| r_{\pi 1})\} \| r_{o2}\right)}{g_{m1}(R_E \| r_{o1}) + 1}$$

(d) Equivalent circuit:





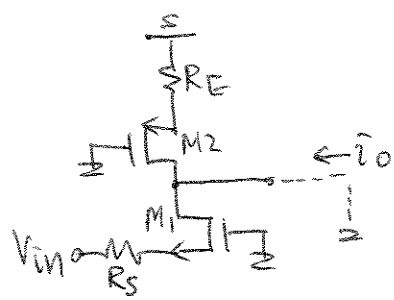
With output shorted to AC GND, circuit becomes a simple common-emitter stage:

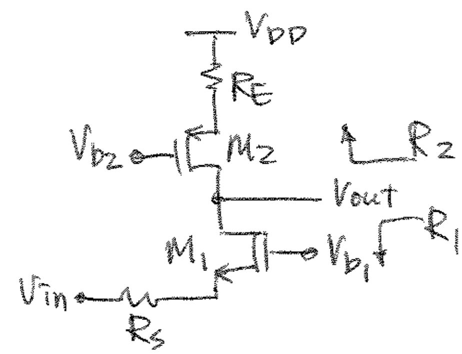
$$\Rightarrow G_m = g_{m2}$$

$$R_1 = [1 + g_{m1}(R_E \parallel r_{\pi 1})] r_{o1} + (R_E \parallel r_{\pi 1})$$
$$R_2 = r_{o2}$$
$$\Rightarrow R_{out} = R_1 \parallel R_2$$

$$\therefore A_v = -G_m R_{out} = -g_{m2}\left(\{[1+g_{m1}(R_E \parallel r_{\pi 1})] r_{o1} + (R_E \parallel r_{\pi 1})\} \parallel r_{o2}\right)$$

(e) Equivalent circuit:





Observing that i_o must flow through M_1 only:

$$i_o = g_{m1}(-\underbrace{(v_{in} + i_o R_S)}_{\text{gate voltage of } M_1})$$

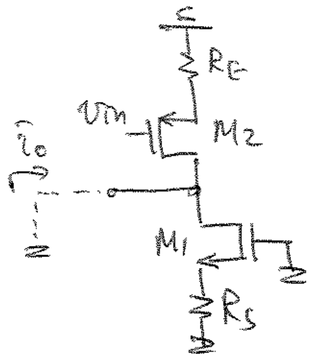
$$\Rightarrow G_m = \frac{i_o}{v_{in}} = \frac{-g_{m1}}{1+g_{m1}R_S}$$

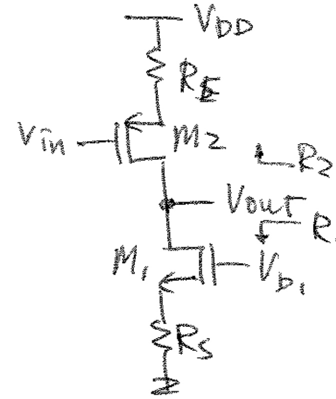
$$R_1 = (1+g_{m1}R_S)r_{o1} + R_S$$
$$R_2 = (1+g_{m2}R_E)r_{o2} + R_E$$
$$R_{out} = R_1 \parallel R_2$$

$$\therefore A_v = -G_m R_{out}$$

$$= \frac{g_{m1}}{1+g_{m1}R_S}\left\{[(1+g_{m1}R_S)r_{o1}+R_S] \parallel [(1+g_{m2}R_E)r_{o2}+R_E]\right\}$$

(f) Equivalent circuit:





This is a common-source stage with degeneration:

$$\Rightarrow G_m = \frac{g_{m2}}{1+g_{m2}R_E}$$

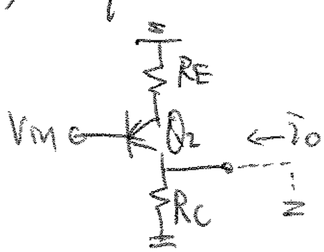
$$R_1 = (1+g_{m1}R_S)r_{o1} + R_S$$

$$R_2 = (1+g_{m2}R_E)r_{o2} + R_E$$

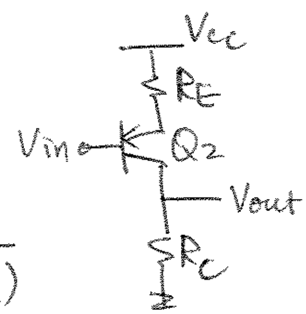
$$\therefore A_v = -G_m(R_1 \| R_2)$$

$$= \frac{g_{m2}}{1+g_{m2}R_E}\left\{\left[(1+g_{m1}R_S)r_{o1}+R_S\right] \| \left[(1+g_{m2}R_E)r_{o2}+R_E\right]\right\}$$

(g) Equivalent circuit:



$$\Rightarrow G_m = \frac{g_{m2}}{1+g_{m2}(R_E \| r_{\pi 2})}$$



$$R_{out} = \left\{\left[1+g_{m2}(R_E \| r_{\pi 2})\right]r_{o2} + (R_E \| r_{\pi 2})\right\} \| R_C$$

$$\Rightarrow A_v = -G_m R_{out} = \frac{g_{m2} R_{out}}{1+g_{m2}(R_E \| r_{\pi 2})}$$

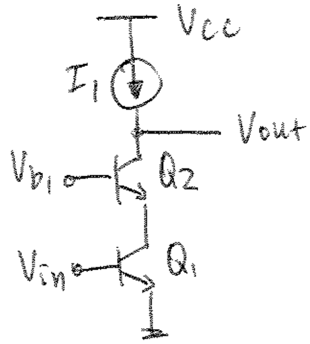
21. $A_v = -g_{m1} r_{o1} g_{m1} (r_{o1} \| r_{\pi 2})$

$$= -\frac{I_{C1}}{V_T} \cdot \frac{V_{A1}}{I_{C1}} \cdot \frac{I_{C1}}{V_T} \cdot \frac{1}{\frac{I_{C1}}{V_{A1}} + \frac{I_{C2}}{\beta V_T}}$$

Since $I_{C1} \approx I_{C2}$,

$$A_v \approx -\frac{V_{A1}/V_T^2}{\frac{1}{V_{A1}} + \frac{1}{\beta V_T}} = -\frac{\beta V_A^2}{V_T(V_A + \beta V_T)}$$

22.

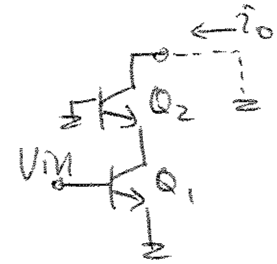


$A_v = 500$ $\quad I_1 = 1mA$

$\beta_1 = \beta_2 = 100$

Determine minimum $V_{A_1} = V_{A_2}$.

Using small-signal analysis,



$$G_M = \frac{i_o}{v_{in}} = g_{m_1}\left(\frac{\beta+1}{\beta}\right) = \frac{I_1}{V_T}\left(\frac{\beta+1}{\beta}\right)$$

$$R_{out} = \left[1 + g_{m_2}(r_{o_1} \| r_{\pi 2})\right] r_{o2} + (r_{o_1} \| r_{\pi 2})$$

$$\approx g_{m2}(r_{o_1} \| r_{\pi_2}) r_{o2} = \frac{\beta V_A^2}{I_C(V_A + \beta V_T)}$$

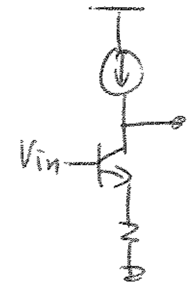
$$\Rightarrow A_v = -G_m R_{out}$$

$$= -\frac{I_1}{V_T}\left(\frac{\beta+1}{\beta}\right) \cdot \frac{\beta V_A^2}{I_C(V_A + \beta V_T)} = 500$$

$\Rightarrow$ All values are given. V_A is solved using the quadratic formula:

$\therefore V_A \approx 0.65$ V

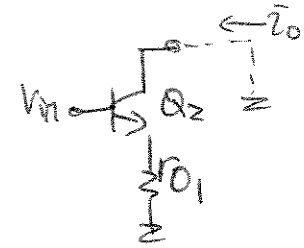
23. (a) Even though Rout is independent of where Vin is applied, Gm changes:



The circuit is a commom-emitter with degeneration, which always has Gm ≤ Gm of commom-emitter stage without degeneration.

Alternatively, this circuit has less gain because it only has one amplifier stage.

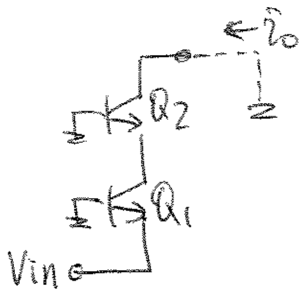
(b) $G_m = \frac{i_o}{v_{in}} = \frac{g_{m2}}{1 + g_{m2}(r_{o1} \| r_{o2})}$



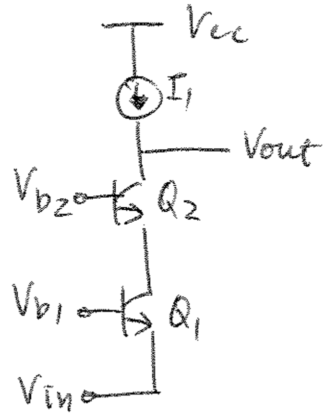
$R_{out} = [1 + g_{m2}(r_{o1} \| r_{\pi 2})] r_{o2} + r_{o1}$

$\Rightarrow A_v = -G_m R_{out} = \frac{g_{m2}\{[1 + g_{m2}(r_{o1} \| r_{\pi 2})] r_{o2} + r_{o1}\}}{1 + g_{m2}(r_{o1} \| r_{o2})}$

24. Equivalent circuit:



$\beta \to \infty.$

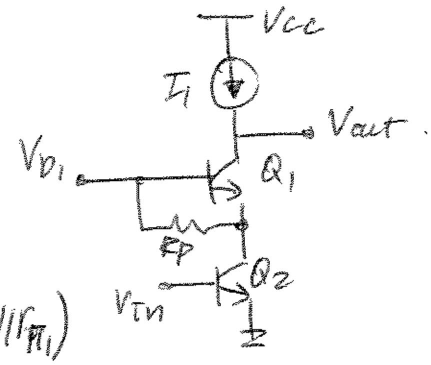


$$G_m = \frac{i_o}{v_{in}} \approx -g_{m1}$$

$$R_{out} = \left[1 + g_{m2}(r_{o1} \| r_{\pi 2})\right] r_o + (r_{o1} \| r_{\pi 2})$$

$$\Rightarrow A_v = -G_m R_{out} = g_{m1}\left[\left\{1 + g_{m2}(r_{o1} \| r_{\pi 2})\right\} r_{o2} + (r_{o1} \| r_{\pi 2})\right]$$

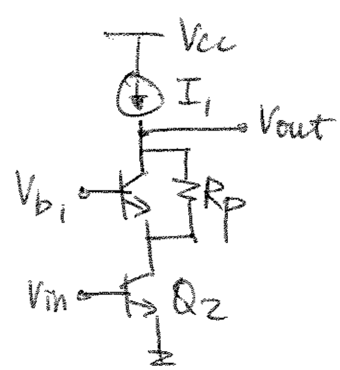
25. (a) From lecture, we know that the voltage gain of a BJT cascode circuit $\approx -g_{m2} r_{o2} g_{m2} (r_{o2} \| r_{\pi_1})$

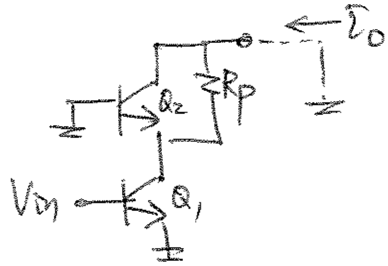


This circuit resembles such, and the only difference is that r_{π_1} now becomes $(r_{\pi_1} \| R_p)$

$$\therefore A_v \approx -g_{m2}^2 r_{o2} (r_{o2} \| r_{\pi_1} \| R_p)$$

(b) Equivalent circuit:



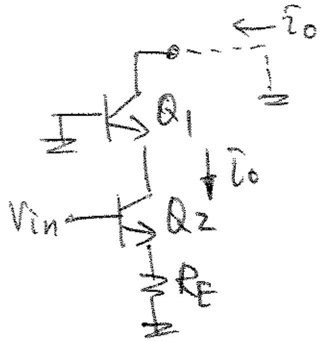


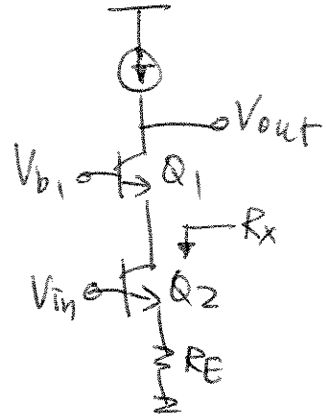
$$G_m = \frac{i_o}{v_{in}} = \frac{\beta+1}{\beta} g_{m1} \approx g_{m1}$$

$$R_{out} = [1 + g_{m2}(r_{o1} \| r_{\pi 2})](r_{o2} \| R_p) + (r_{o1} \| r_{\pi 2})$$

$$\therefore A_v = -G_m R_{out}$$
$$= -g_{m1} \{[1 + g_{m2}(r_{o1} \| r_{\pi 2})](r_{o2} \| R_p) + (r_{o1} \| r_{\pi 2})\}$$

(c) Equivalent circuit:





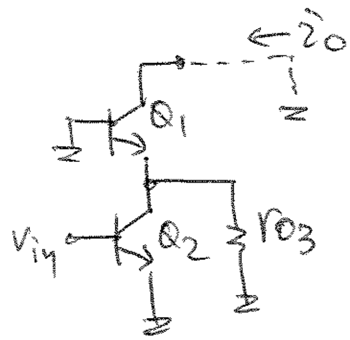
$$G_m = \frac{i_o}{v_{in}} = \frac{g_{m2}}{1 + g_{m2} R_E}$$ (small-signal analysis)

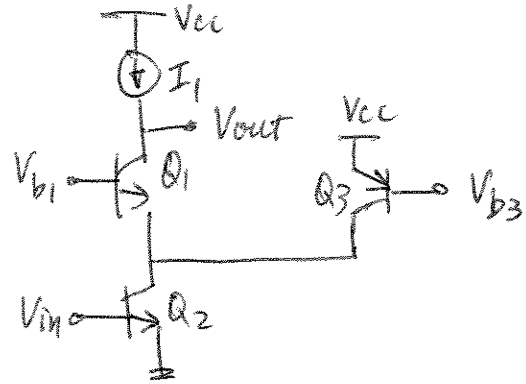
$$\begin{aligned} R_{out} &= (1 + g_{m1} R_X) r_{o1} + R_X \\ &= [1 + g_{m1} [(1 + g_{m2} R_E) r_{o2} + R_E]] r_{o1} \\ &\quad + [(1 + g_{m2} R_E) r_{o2} + R_E] \end{aligned}$$

$$\therefore A_v = -G_m R_{out}$$

$$= \frac{g_{m2} \{[1 + g_{m1}[(1+g_{m2}R_E)r_{o2} + R_E]] r_{o1} + [(1+g_{m2}R_E) r_{o2} + R_E]\}}{1 + g_{m2} R_E}$$

(d) Equivalent circuit:

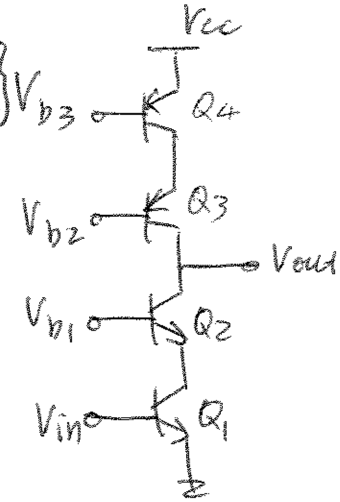




This resembles the BJT cascode topology, only now r_{o2} becomes $(r_{o2} \| r_{o3})$

$$\Rightarrow A_v \approx -g_{m2}^2 (r_{o2} \| r_{o3})(r_{o2} \| r_{o3} \| r_{\pi 1})$$

2b. $A_v = -g_{m1}\left\{\underbrace{\left[g_{m2} r_{o2}(r_{o1} \| r_{\pi 2})\right]}_{R_{on}} \| \underbrace{\left[g_{m3} r_{o3}(r_{o4} \| r_{\pi 3})\right]}_{R_{op}}\right\}$



$$R_{on} = \frac{(V_{An}/V_T)}{\left(\frac{1}{V_{An}} + \frac{1}{\beta_n V_T}\right) I_C}$$

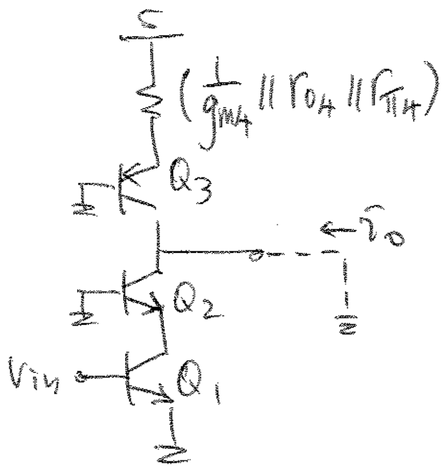
$$R_{op} = \frac{(V_{AP}/V_T)}{\left(\frac{1}{V_{Ap}} + \frac{1}{\beta_p V_T}\right) I_C} \qquad g_{m1} = \frac{I_C}{V_T}$$

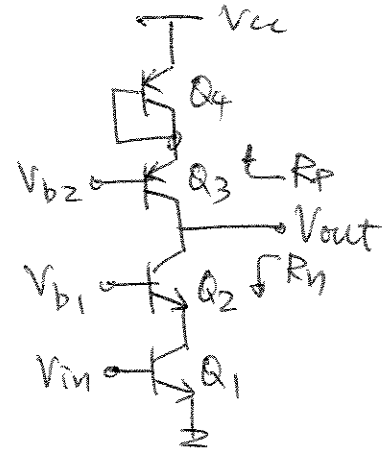
$$\therefore A_v = \frac{-\ (I_C/V_T)}{\frac{\left(\frac{1}{V_{An}} + \frac{1}{\beta_n V_T}\right) I_C}{V_{An}/V_T} + \frac{\left(\frac{1}{V_{Ap}} + \frac{1}{\beta_p V_T}\right) I_C}{V_{AP}/V_T}}$$

$$= \frac{V_{AN} \cdot V_{AP}}{V_T^2 \left(\frac{V_{AP}}{V_{AN}} + \frac{V_{AP}}{\beta_N V_T} + \frac{V_{AN}}{V_{AP}} + \frac{V_{AN}}{\beta_P V_T}\right)}$$

$\therefore$ A_v is independent of bias current, I_C.

27. Equivalent circuit.





$$G_m = g_{m1} = \frac{i_o}{v_{in}} = \frac{i_{c1}}{v_{in}}$$

$$R_{out} = R_p \| R_n$$

$$R_p = \left[1 + g_{m3}\left(\frac{1}{g_{m4}} \| r_{o4} \| r_{\pi 4} \| r_{\pi 3}\right)\right] r_{o3} + \left[\frac{1}{g_{m4}} \| r_{o4} \| r_{\pi 4} \| r_{\pi 3}\right]$$

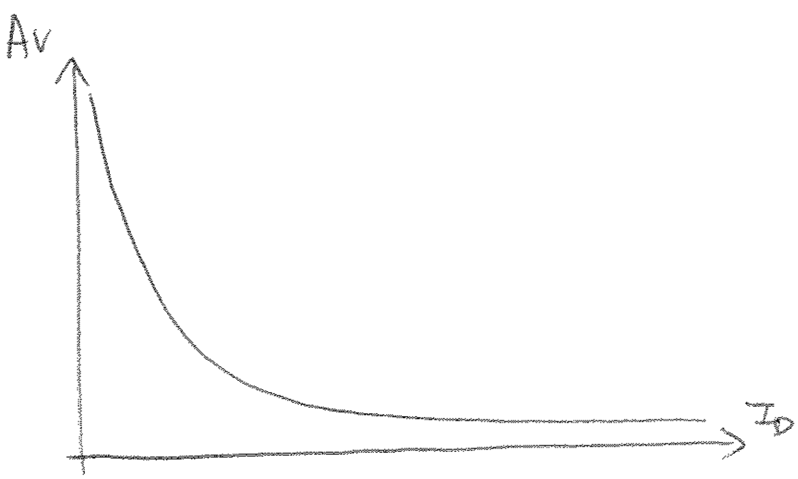
$$R_n = \left[1 + g_{m2}(r_{o1} \| r_{\pi 2})\right] r_{o2} + (r_{o1} \| r_{\pi 2})$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}(R_p \| R_n)$$

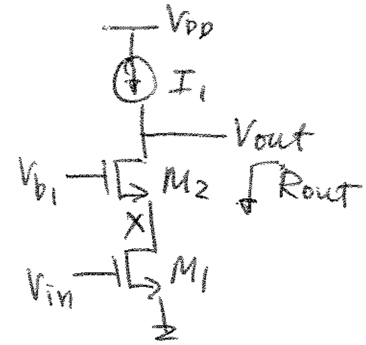
28. $|A_v| = g_{m1} r_{o1} g_{m2} r_{o2}$

$$= \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_D} \cdot \frac{1}{\lambda I_D} \cdot \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_D} \cdot \frac{1}{\lambda I_D}$$

$$= 2\mu_n C_{ox} \sqrt{\left(\frac{W}{L}\right)_1 \left(\frac{W}{L}\right)_2} \cdot \frac{1}{\lambda^2 I_D}$$

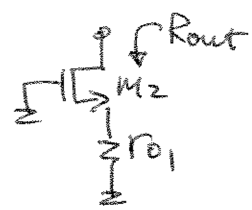


29. $|A_v| = 200$



$\mu_n C_{ox} = 100\ \frac{\mu A}{V^2}$ $\qquad \lambda = 0.1\ V^{-1}$

Determine $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$



$$R_{out} = (1 + g_{m2} r_{o1}) r_{o2} + r_{o1}$$

$G_m \cong g_{m1}$ (short-circuit current flows through both M_1 & M_2)

$$|A_v| = G_m R_{out} = g_{m1}\left[(1 + g_{m2} r_{o1}) r_{o2} + r_{o1}\right]$$

$$\approx g_{m1} g_{m2} r_{o1} r_{o2} = (g_m r_o)^2 = 200$$

$$\left(\because \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 \text{ and } I_{D1} = I_{D2}\right)$$

$$(g_m r_o)^2 = \left(\frac{2 I_D}{V_{GS} - V_{TH}} \cdot \frac{1}{\lambda I_D}\right)^2 = 200$$

$$\Rightarrow V_{GS} - V_{TH} = \left(\sqrt{200} \cdot \lambda/2\right)^{-1} = \left[\sqrt{200} \cdot (0.05\ V^{-1})\right]^{-1}$$

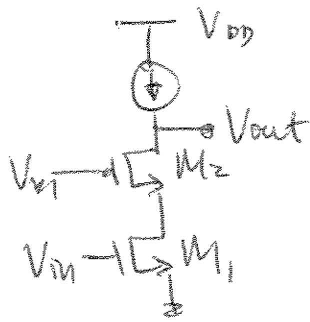
$$\approx 1.41\ V$$

$$\Rightarrow \quad I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2$$

$$\therefore \left(\frac{W}{L}\right) = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

$$= \frac{2(1\ mA)}{100\ \frac{uA}{V^2}\ (1.41V)^2} \approx 10$$

30.



$$\left(\frac{W}{L}\right)_{1,new} = N\left(\frac{W}{L}\right)_1$$

$$\left(\frac{W}{L}\right)_{2,new} = N\left(\frac{W}{L}\right)_2$$

$$\lambda_{n,1} = \lambda_{n,2}$$

$$A_{v_{new}} \approx -g_{m1} g_{m2} r_{o1} r_{o2}$$

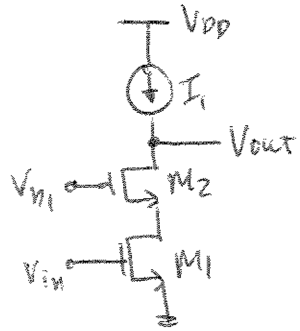
$$= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,new} I_D} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{2,new} I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

$$= -\sqrt{N}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} \cdot \sqrt{N}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_D} \cdot \frac{1}{\lambda I_D} \cdot \frac{1}{\lambda I_D}$$

$$= -N\left(g_{m1} g_{m2} r_{o1} r_{o2}\right) = -N \cdot A_{v,old}.$$

Gain is N times of original value.

31.



$$\left(\frac{W}{L}\right)_{1,new} = \frac{1}{N}\left(\frac{W}{L}\right)_1$$

$$\left(\frac{W}{L}\right)_{2,new} = \frac{1}{N}\left(\frac{W}{L}\right)_2$$

Assume $\lambda_{n,1} = \lambda_{n,2}$

$$A_{v_{new}} \approx -g_{m1}(g_{m2}(r_{o1}r_{o2}))$$

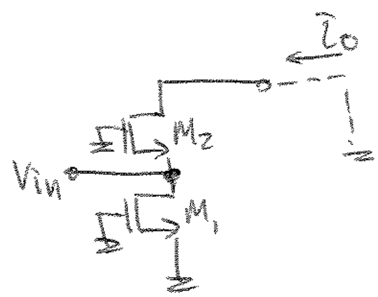
$$= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,new} \cdot I_{D1}} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_{2,new} I_{D1}} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

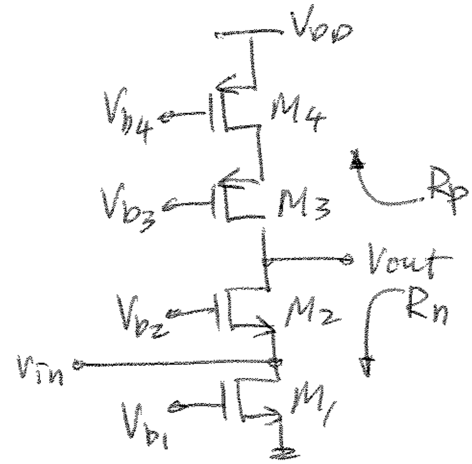
$$= -\sqrt{\frac{1}{N}}\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_D} \cdot \sqrt{\frac{1}{N}} \cdot \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_D} \cdot \left(\frac{1}{\lambda I_D}\right)^2$$

$$= -\frac{1}{N} g_{m1} g_{m2} r_{o1} r_{o2} = -\frac{1}{N}(A_{v,old})$$

Gain is $1/N$ of original value.

32.





By KCL,

$$\frac{i_o}{V_{in}} = -\left(g_{m2} + \frac{1}{r_{o1} \| r_{o2}}\right) = G_m$$

$$R_n = r_{o2} \qquad R_p \approx g_{m3} r_{o3} r_{o4}$$

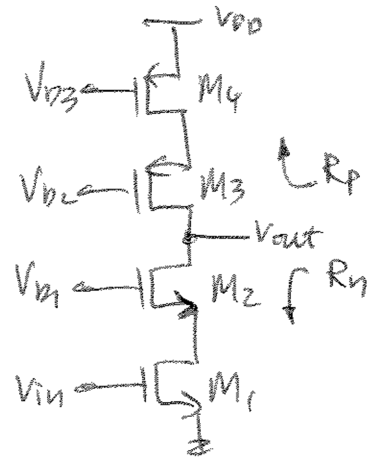
$$\therefore A_v = -G_m (R_n \| R_p) = \left(g_m + \frac{1}{r_{o1} \| r_{o2}}\right)(r_{o2} \| g_{m3} r_{o3} r_{o4})$$

33. $\left(\frac{W}{L}\right) = 20/0.18$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda_n = 0.1\ V^{-1} \quad \lambda_p = 0.15\ V^{-1}$



Calculate I_{BIAS} such as $A_v = 500$.

The cascode circuit has gain

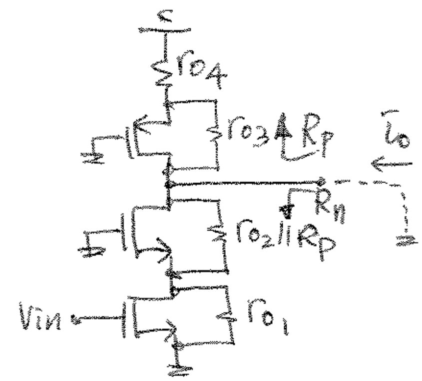
$$\approx -g_{m1} \cdot [g_{m2} r_{o1} r_{o2} \parallel g_{m3} r_{o3} r_{o4}]$$

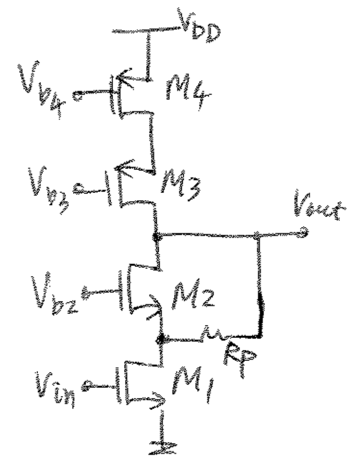
$$\Rightarrow 500 = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} \left(\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)}}{(\lambda_n)^2 I_D^{3/2}} \parallel \frac{\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)}}{(\lambda_p)^2 I_D^{3/2}} \right)$$

All quantities are known. Solving I_D gives:

$I_D = I_{BIAS} \approx 1.06$ mA.

34 (a) Equivalent circuit:



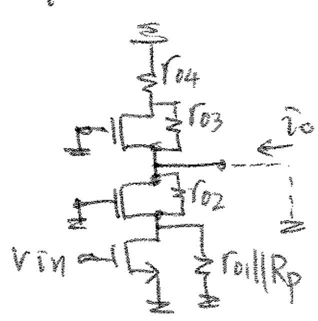


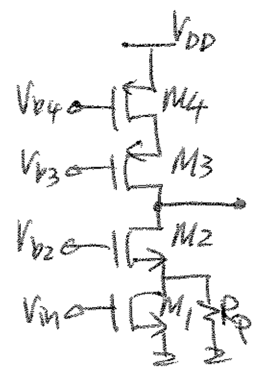
$$G_m = \frac{i_o}{v_{in}} \approx g_{m1} \quad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2}(r_{o2} \| R_p) r_{o1}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}\left[g_{m3} r_{o3} r_{o4} \,\|\, g_{m2}(r_{o2} \| R_p) r_{o1}\right]$$

(b) Equivalent circuit

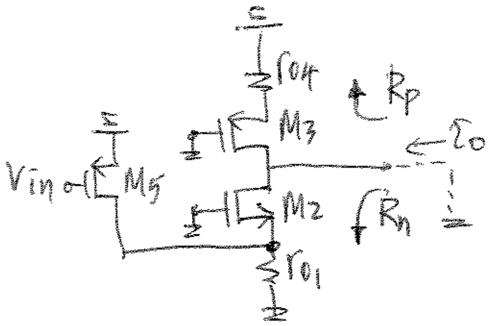


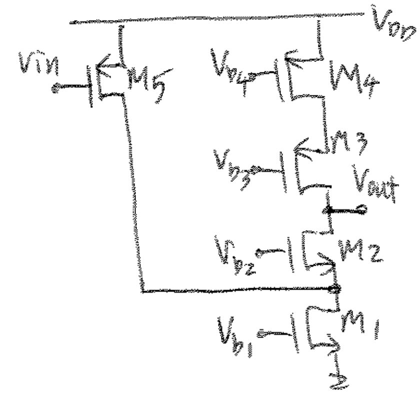


$$G_m = i_o / v_{in} \approx g_{m1} \quad (\because g_m r_o \gg 1)$$
$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2}(r_{o1} \| R_p) r_{o2}$$

$$\therefore A_v = -G_m R_{out} = -g_{m1}\left[g_{m3} r_{o3} r_{o4} \,\|\, g_{m2}(r_{o1} \| R_p) r_{o2}\right]$$

(c) Equivalent circuit:





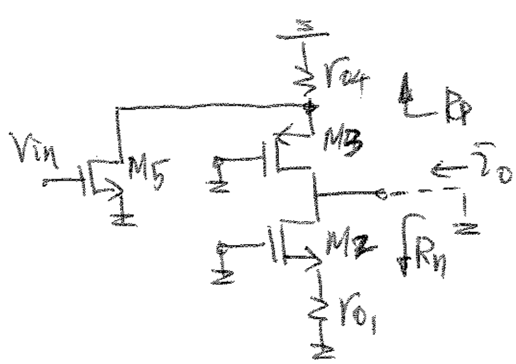
(Realize that r_{o1} & r_{o5} are in parallel.)

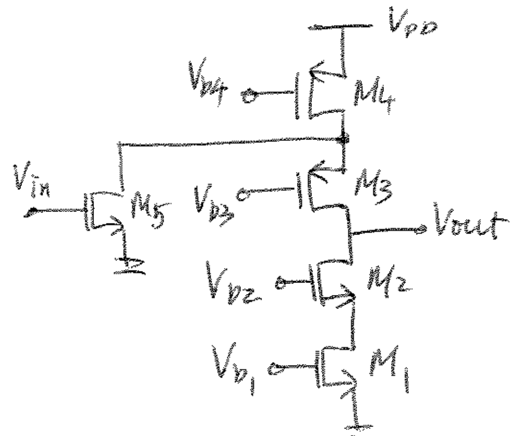
$$G_m = \frac{i_o}{v_{in}} \approx -g_{m5} \qquad (\because g_m r_o \gg 1)$$

$$R_p = g_{m3} r_{o3} r_{o4} \qquad R_n = g_{m2} r_{o2} (r_{o1} \| r_{o5})$$

$$\therefore A_v = -G_m R_{out} = g_{m5} \left[g_{m3} r_{o3} r_{o4} \| g_{m2} r_{o2} (r_{o1} \| r_{o5}) \right]$$

(d) Equivalent circuit:





$$G_m = \frac{i_o}{v_{in}} \approx g_{m5} \qquad R_p = g_{m3} r_{o3} (r_{o4} \| r_{o5})$$

$$R_n = g_{m2} r_{o2} r_{o1}$$

$$\therefore A_v = -G_m R_{out} = g_{m5} \left[g_{m3} r_{o3} (r_{o4} \| r_{o5}) \| g_{m2} r_{o2} r_{o1} \right]$$

35. $\frac{R_2}{R_1 + R_2} V_{CC} = V_T \ln\left(\frac{I_1}{I_S}\right)$

$$\Rightarrow I_1 = I_S \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$\frac{\partial I_1}{\partial V_{CC}} = \frac{I_S}{V_T} \cdot \frac{R_2}{R_1+R_2} \cdot \exp\left[\frac{V_{CC}}{V_T} \cdot \frac{R_2}{R_1+R_2}\right]$$

$$= \frac{I_1}{V_T} \cdot \frac{R_2}{R_1+R_2} = g_m\left(\frac{R_2}{R_1+R_2}\right)$$

Intuitively, we know that an exponential relationship exists between I_C & V_{BE}. Its transconductance is also a function (linear) of I_C. Since V_{BE} comes from a voltage divider (which is also linear), we expect a linear relationship between I_C & V_{CC}.

36. $I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)^2$

$$\frac{\partial I_1}{\partial V_{DD}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot 2 \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right) \cdot \frac{R_2}{R_1+R_2}$$

$$= \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} \right) \left(\frac{R_2 \cdot V_{DD}}{R_1+R_2} - V_{TH} \right).$$

$$= g_m \cdot \frac{R_2}{R_1+R_2}$$

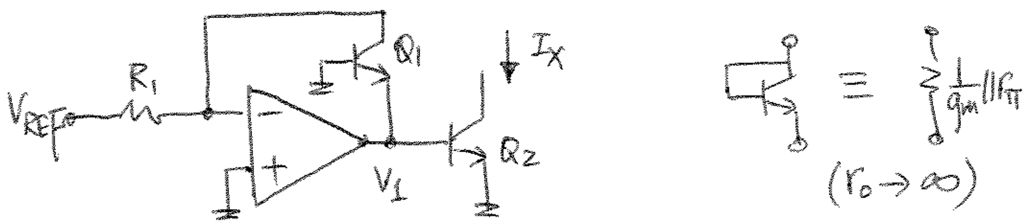
Intuitively, the voltage divider gives a linear relationship between V_{DD} & V_{GS1}. Since g_m of MOS is linearly proportional to $(V_{GS1} - V_{TH})$, we expect the same relationship between V_{DD} & $\frac{\partial I_1}{\partial V_{DD}}$

37. $I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)^2$

$$\frac{\partial I_1}{\partial V_{TH}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot 2 \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right) \cdot (-1)$$

$$= -\mu_n C_{ox} \frac{W}{L} \left(\frac{R_2}{R_1+R_2} V_{DD} - V_{TH} \right)$$

38.



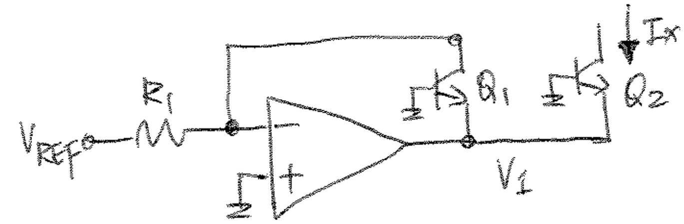
This is a negative feedback circuit.

The inverting input (−) of the op-amp is virtual ground. (∵ of feedback) in DC. ⇒ Q_1 becomes diode-connected.

$$\Rightarrow \frac{V_{REF}}{R_1} = \frac{0 - V_1}{\left(\frac{1}{g_{m1}} \parallel r_{\pi 1}\right)} \Rightarrow V_1 = -\frac{V_{REF}\left(\frac{1}{g_{m1}} \parallel r_{\pi 1}\right)}{R_1} < 0$$

This implies $V_{BE2} < 0 \Rightarrow I_X = 0$!

39.



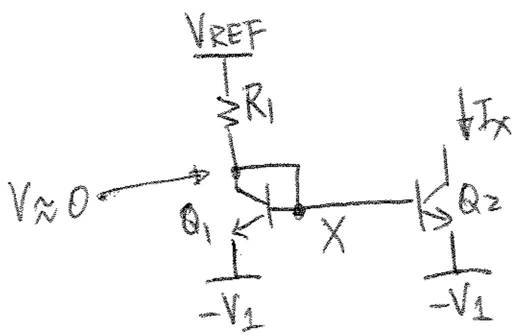
This is a negative feedback circuit.

The inverting input (−) is virtual ground, as a result. Q_1 then becomes diode-connected, and its resistance $= \left(\frac{1}{g_m} \| r_\pi\right)$, assuming $r_o \to \infty$.

$$\Rightarrow \frac{V_{REF}}{R_1} = \frac{-V_1}{\left(\frac{1}{g_{m1}} \| r_{\pi 1}\right)} \Rightarrow V_1 = -\frac{V_{REF}\left(\frac{1}{g_{m1}} \| r_{\pi 1}\right)}{R_1}$$

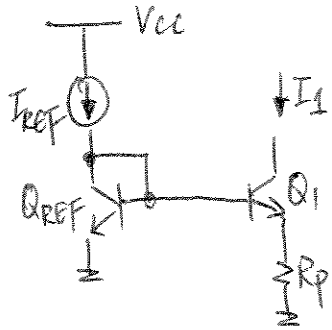
$$\Rightarrow V_{BE1} = V_{BE2} = -V_1$$

This circuit will work if the negative supply voltage of the op-amp allows value of $-V_1$ or lower.



- An equivalent circuit. (without op-amp). The op-amp guarantees a stable voltage at node X. (i.e. inverting input.)

40.



$Q_{REF} = Q_1$
$\beta \to \infty.$

$I_1 = \frac{I_{REF}}{2}$

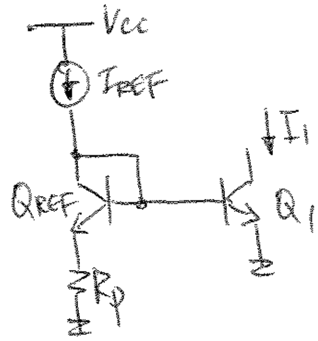
By KVL, $V_{BE,REF} = V_{BE_1} + I_1 R_p$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) = V_T \ln\left(\frac{I_{REF}/2}{I_{S,1}}\right) + \frac{I_{REF}}{2} R_p$$

$$V_T \ln(2) = \frac{I_{REF}}{2} R_p$$

$$R_p = 2 \cdot \ln(2) \cdot (V_T / I_{REF})$$

41.



$Q_{REF} = Q_1$

$\beta \to \infty$

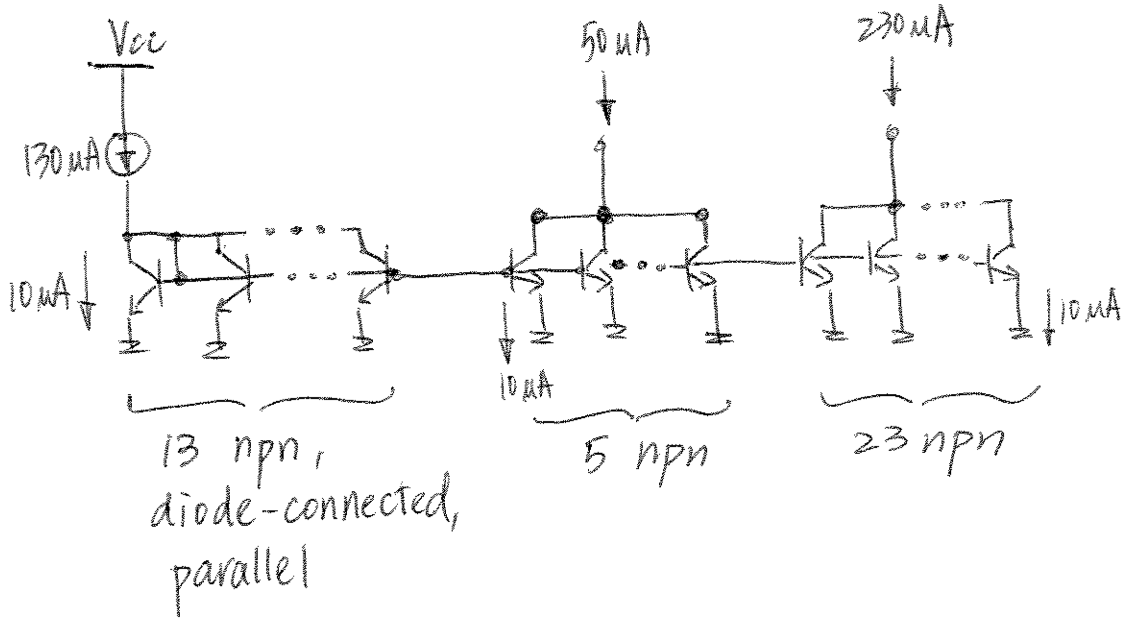
By KVL, $V_{BE,REF} + I_{REF} R_p = V_{BE,1}$

$$\Rightarrow V_T \ln\left(\frac{I_{REF}}{I_{S,REF}}\right) + I_{REF} R_p = V_T \ln\left(\frac{2 I_{REF}}{I_{S,1}}\right)$$

$$I_{REF} R_p = V_T \ln(2)$$

$$R_p = \frac{V_T}{I_{REF}} \ln(2)$$

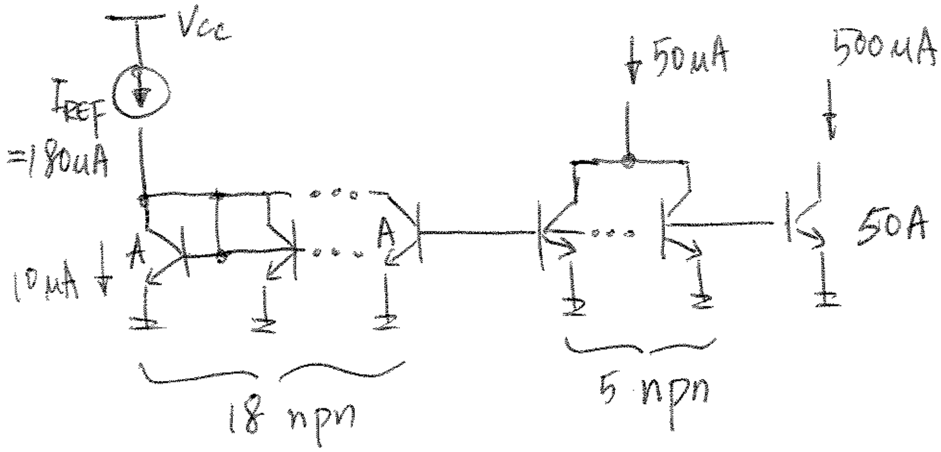
42.



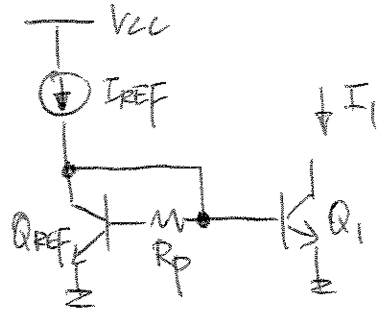
All the bases are the same node.

If the area of BJT is flexible, the 5-npn group can be replaced by one BJT that is 5 times as big in area. Similar concept applies to 23-npn grouping.

43.



44.



$Q_{REF} = Q_1$

I_1 10% larger. ($I_1 = 1.1\, I_{C,REF}$)
Solve for R_P.

By KVL,

$$V_{BE_{REF}} + \frac{I_{C,REF}}{\beta} \cdot R_P = V_{BE_1}$$

$$\Rightarrow V_T \ln\left(\frac{I_1}{I_S}\right) - V_T \ln\left(\frac{I_{C,REF}}{I_S}\right) = \frac{I_{C,REF}}{\beta} \cdot R_P$$

$$V_T \ln\left(\frac{I_1}{I_{C,REF}}\right) = \frac{I_{C,REF}}{\beta} \cdot R_P$$

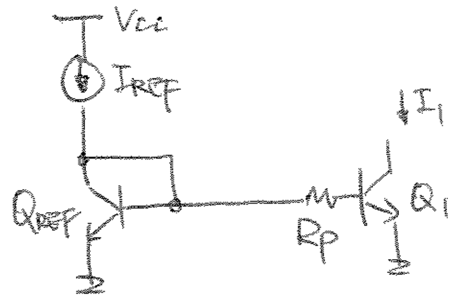
$$\Rightarrow V_T \ln(1.1) = \frac{I_{C,REF}}{\beta} R_P \quad \Rightarrow I_{C,REF} = \frac{\beta V_T \ln(1.1)}{R_P}$$

By KCL, $I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$

$$= \frac{\beta}{R_P} V_T \ln(1.1) \cdot \left(1 + 1/\beta\right) + I_1/\beta$$

$$\therefore R_P = \frac{(\beta + 1)\, V_T \ln(1.1)}{I_{REF} - I_1/\beta}$$

45.



$I_1 = 0.9 I_{C,REF}$

By KVL, $V_{BE_{REF}} = \frac{I_1}{\beta} R_p + V_{BE_1}$

$$\Rightarrow V_T \ln\left(\frac{I_{C,REF}}{I_1}\right) = \frac{I_1}{\beta} R_p$$

$$V_T \ln\left(\frac{1}{0.9}\right) = 0.9\, I_{C,REF} \frac{R_p}{\beta}$$

$$\Rightarrow I_{C,REF} = \frac{\beta}{0.9 R_p} V_T \ln\left(\frac{1}{0.9}\right)$$

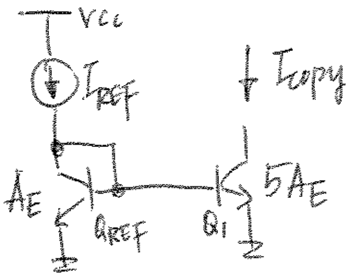
By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_1/\beta$$

$$\therefore I_{REF} - \frac{I_1}{\beta} = \frac{\beta}{0.9 R_p} V_T \ln\left(\frac{1}{0.9}\right)\left(1 + \frac{1}{\beta}\right)$$

$$\Rightarrow R_p = \frac{(\beta + 1) V_T \ln(10/9)}{0.9\,(I_{REF} - I_1/\beta)}$$

4b (a)



Q_1 has I_S 5 times as that of Q_{REF}

$$\Rightarrow I_{C_{REF}} = I_{copy} / 5$$

By KCL,

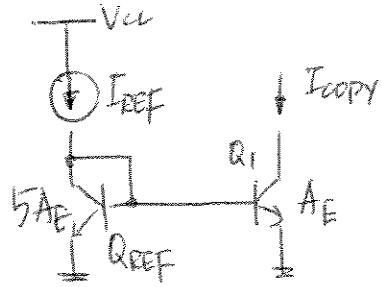
$$I_{REF} = I_{C_{REF}} + \frac{I_{C_{REF}}}{\beta} + \frac{I_{copy}}{\beta}$$

$$= \frac{I_{copy}}{5}\left(1 + \frac{1}{\beta}\right) + \frac{I_{copy}}{\beta}$$

$$\therefore I_{copy} = I_{REF}\left(\frac{5\beta}{\beta+6}\right)$$

$$\therefore \text{error} = \frac{I_{copy}}{I_{REF}} = \frac{5\beta}{\beta+6}$$

(b)



Q_1 & Q_{REF} have the same V_{BE}, but area of Q_{REF} is 5 times larger

$\Rightarrow I_{C,REF} = 5 \cdot I_{copy}$

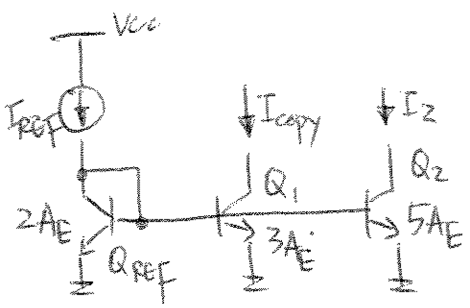
By KCL,

$$I_{REF} = I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta}$$

$$= I_{copy} \cdot 5 + \left(1+\frac{1}{\beta}\right) + I_{copy}\left(\frac{1}{\beta}\right)$$

$$\therefore I_{copy} = I_{REF}\left(\frac{\beta}{5\beta+6}\right)$$

(c)



Q_1 & Q_{REF} have identical V_{BE}, but area of Q_1 is 1.5 times larger.

$\Rightarrow 3 I_{C,REF} = 2 I_{copy}$

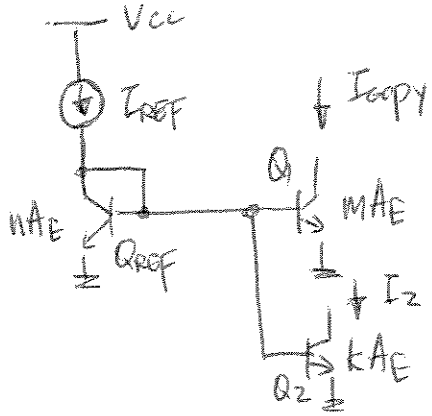
By KCL,

$$I_{REF} = I_{C,REF} + I_{C,REF}/\beta + I_{copy}/\beta + I_2/\beta$$

$$= I_{copy}\left(\frac{2}{3}\right)\left[\left(1+\frac{1}{\beta}\right) + \frac{1}{\beta} + \frac{5}{3}\left(\frac{1}{\beta}\right)\right]$$

$$\rightarrow I_{copy} = \frac{9\beta}{6\beta+22} I_{REF}$$

47.



By observing the areas of the BJTs,

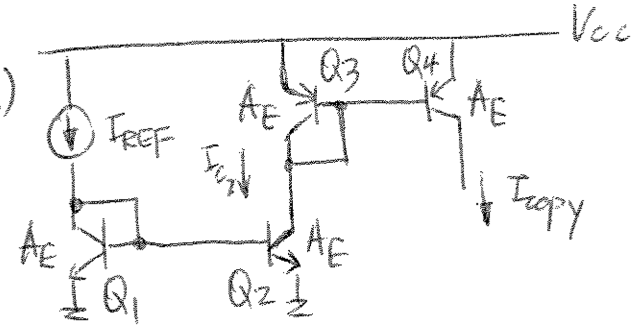
$$I_{C,REF} = \left(\frac{n}{m}\right) I_{copy} = \left(\frac{n}{k}\right) I_2$$

By KCL, $I_{C,REF} = I_{REF} - \frac{I_{C,REF}}{\beta} - \frac{I_{copy}}{\beta} - \frac{I_2}{\beta}$

$$\Rightarrow \frac{n}{m} I_{copy} = I_{REF} - \frac{\left(\frac{n}{m}\right) I_{copy}}{\beta} - \frac{I_{copy}}{\beta} - \frac{\left(\frac{k}{m}\right) I_{copy}}{\beta}$$

$$\therefore I_{copy} = I_{REF}\left[\frac{\beta m}{(\beta+1)n + k + m}\right]$$

48. (a)



$V_{BE_1} = V_{BE_2}$

$\Rightarrow I_{C_1} = I_{C2}$

$V_{BE_3} = V_{BE_4}$

$\Rightarrow I_{B_3} = I_{C_4}$

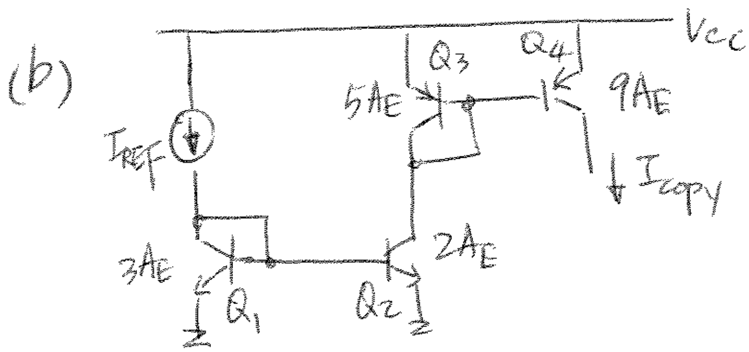
First compute $I_{C_1,2}$:

$$I_{C1} = I_{REF} - \frac{I_{C1}}{\beta} - \frac{I_{C2}}{\beta} \Rightarrow I_{C_2} = \frac{\beta}{\beta+2} \cdot I_{REF}.$$

View I_{C2} as the "I_{REF}" for the Q_3–Q_4 current mirror and apply the equation derived.

$$\Rightarrow I_{copy} = \frac{\beta}{\beta+2}\left[\frac{\beta}{\beta+2} \cdot I_{REF}\right] = I_{REF}\left(\frac{\beta}{\beta+2}\right)^2$$

(b)



$V_{BE_1} = V_{BE_2}$:

$\Rightarrow I_{C_1} = \frac{3}{2} I_{C2}$

$V_{BE_3} = V_{BE_4}$:

$\Rightarrow I_{copy} = \frac{9}{5} I_{C_3}$

- By KCL,

$$I_{REF} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_{C2}}{\beta}$$

$$\Rightarrow I_{C2} = \frac{2\beta}{3\beta+5} I_{REF} \qquad ①$$

- By KCL,

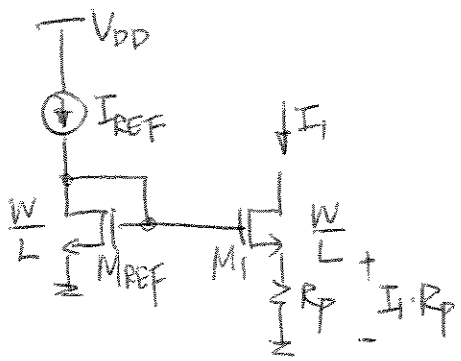
$$I_{C2} = I_{C3} + \frac{I_{C3}}{\beta} + \frac{I_{copy}}{\beta}$$

$$\Rightarrow I_{copy} = \frac{9\beta}{5\beta+14} I_{C2}$$

Substitute ① into Icopy :

$$\therefore I_{copy} = \frac{9\beta}{5\beta+14} \cdot \frac{2\beta}{3\beta+5} \cdot I_{REF}.$$

49.



Determine R_P such that $I_1 = \frac{I_{REF}}{2}$.

First calculate $V_{GS,REF}$:

$$V_{GS,REF} = \sqrt{\frac{2 I_{REF}}{\mu_n C_{ox} W/L}} + V_{TH} \quad \text{—— ①}$$

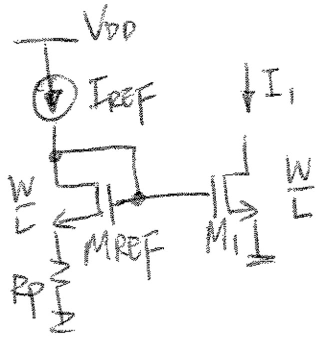
Assuming M_1 in saturation:

$$I_1 = \frac{I_{REF}}{2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[V_{GS_1} - V_{TH} \right]^2$$

$$\Rightarrow \quad \frac{I_{REF}}{2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[V_{GS,REF} - \frac{I_{REF}}{2}(R_P) - V_{TH} \right]^2$$

Rearrange, substitute ① into equation above and solve for R_P:

$$\therefore R_P = \frac{2(\sqrt{2}-1)}{\sqrt{I_{REF} \cdot \mu_n C_{ox} \frac{W}{L}}}$$

50. 

Determine R_P such that $I_1 = 2I_{REF}$.

First calculate V_{GS_1}:

$$V_{GS_1} = \sqrt{\frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{TH} = 2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} + V_{TH} \quad -① $$

Assuming I_1 is in saturation:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{GS,REF} - V_{TH}\right)^2$$

$$= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{GS_1} - I_{REF}R_P - V_{TH}\right]^2$$

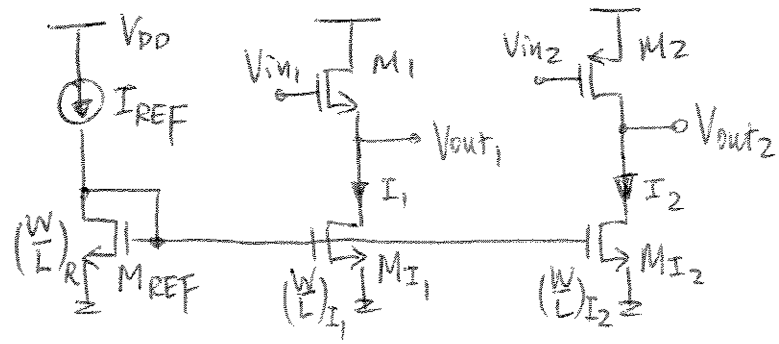
Substitute ① into I_{REF}:

$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left[2\sqrt{\frac{I_{REF}}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} - I_{REF}R_P\right]^2 \quad -②$$

Solve for R_P: $$R_P = \frac{(2-\sqrt{2})}{\sqrt{I_{REF}\cdot \mu_n C_{ox}\left(\frac{W}{L}\right)}}$$

From ②, we find that R_P is <u>independent</u> of any change in V_{TH}, ΔV!!

51.



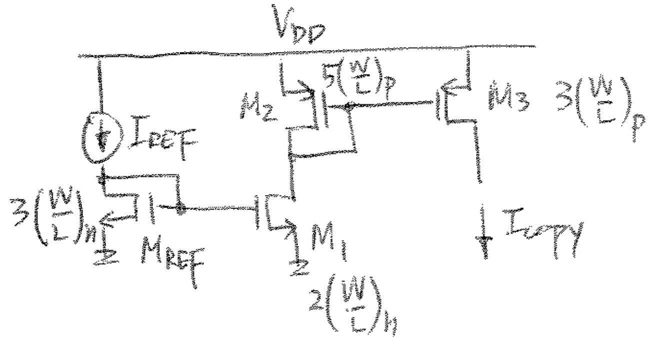
This figure implies that $V_{GS,REF} = V_{GS,I_1} = V_{GS,I_2}$.
Assuming all devices operate in saturation, with $(V_{GS}-V_{TH})$ fixed, $I_D \propto (\frac{W}{L})$

$\Rightarrow$ we have $(\frac{W}{L})_R = 7(\frac{W}{L})$

$$(\frac{W}{L})_{I_1} = 4(\frac{W}{L})$$

$$(\frac{W}{L})_{I_2} = 10(\frac{W}{L})$$

52. (a)

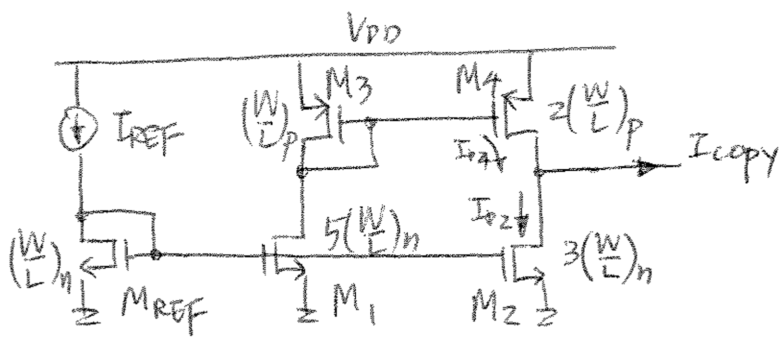


$V_{GS,REF} = V_{GS,1}$: $\Rightarrow I_{D,1} = \frac{2}{3} I_{REF}$

$V_{GS,2} = V_{GS,3}$: $\Rightarrow I_{copy} = \frac{3}{5} I_{D,2} = \frac{3}{5} I_{D,1}$

$= \frac{3}{5} \cdot \left(\frac{2}{3} I_{REF}\right) = \frac{2}{5} I_{REF}$

(b)



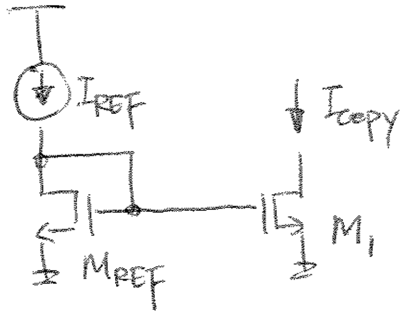
$V_{GS,REF} = V_{GS,1}$: $I_{D,1} = 5 I_{REF}$

$V_{GS,3} = V_{GS,4}$: $I_{D4} = 2 I_{D3} = 2 I_{D,1} = 10 I_{REF}$

$V_{GS,REF} = V_{GS,2}$: $I_{D2} = 3 I_{REF}$

$\therefore I_{copy} = I_{D4} - I_{D2} = 7 I_{REF}$.

53.



$V_{GS,REF} = V_{GS,1} = V_{GS}$

$\lambda \neq 0$

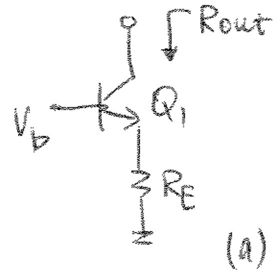
(a)
$$I_{REF} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2(1+\lambda V_{GS})$$
$$I_{copy} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2(1+\lambda V_{DS,1})$$

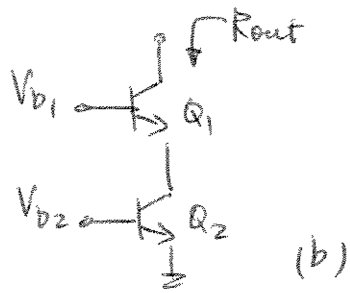
For $I_{REF} = I_{copy} \Rightarrow V_{DS,1} = V_{GS}$

(b)
$$\frac{I_{REF}}{I_{copy}} = \frac{1+\lambda V_{GS}}{1+\lambda(V_{GS}-V_{TH})}$$

$$\Rightarrow I_{copy} = I_{REF}\left(1-\frac{\lambda V_{TH}}{1+\lambda V_{GS}}\right)$$

54.





Given $I_{BIAS} = 1mA$, $V_{RE} \approx V_{CE,2} \approx 0.5V$, design the circuit.

R_E can be readily calculated:

$$R_E = \frac{V_{RE}}{I_{BIAS}/\alpha} = \frac{0.5V}{1mA/0.909} = 505\,\Omega$$

$$V_{be_1} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,1}}\right) = (0.026V)\ln\left(\frac{1mA}{6\cdot10^{-16}A}\right) \approx 0.732V$$

$$\Rightarrow V_b = V_{be_1} + V_{RE} = 0.732V + 0.5V = 1.232V$$

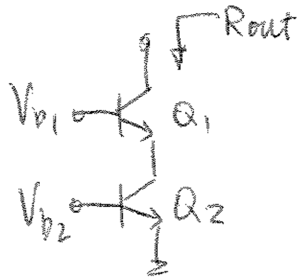
$$R_{out(a)} = [1 + g_{m_1}(R_E \parallel r_{\pi_1})]r_{o_1} + (R_E \parallel r_{\pi_1})$$

$$R_{out(b)} = [1 + g_{m_1}(r_{o2} \parallel r_{\pi_1})]r_{o_1} + (r_{o2} \parallel r_{\pi_1})$$

In most cases $r_o > r_\pi > R_E$

$\therefore$ $R_{out(b)}$ is relatively larger than $R_{out(a)}$

55.



$I_{BIAS} = 1mA$

$\beta = 100$

Given $R_{out} = 50\,k\Omega$, $V_{BC_2} = 100\,mV$, determine V_{b1}.

$$R_{out} = [1 + g_{m1}(r_{o2} \| r_{\pi 1})]r_{o1} + (r_{o2} \| r_{\pi 1})$$

$$\approx g_{m1}(r_{o2} \| r_{\pi 1})\, r_{o1}$$

$$= \frac{\beta V_A^2}{(V_A + \beta V_T) I_{BIAS}}$$

$$\Rightarrow I_{BIAS} = \left[\frac{R_{out}(V_A + \beta V_T)}{\beta V_A^2}\right]^{-1} = \left[\frac{(50k\Omega)(5V + 100 \cdot 0.026V)}{100(5V)^2}\right]^{-1}$$

$$\approx 6.6\,mA.$$

$$V_{b2} = V_{BE2} = V_T \ln\left(\frac{I_{BIAS}}{I_S}\right) = (0.026V)\ln\left(\frac{6.6mA}{6 \cdot 10^{-16}A}\right)$$

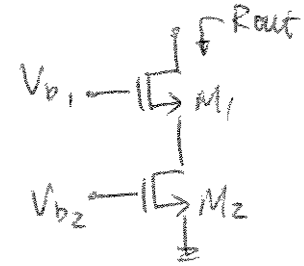
$$\approx 0.78\,V$$

$$\Rightarrow V_{C2} = V_{BE2} - 100mV = 0.68V$$

$$\therefore V_{b1} = V_{C2} + V_{BE1} = V_{C2} + V_T \ln\left(\frac{I_{BIAS}}{I_S}\right)$$

$$= 0.68V + (0.026V)\ln\left(\frac{6.6mA}{6 \cdot 10^{-16}A}\right) \approx 1.46\,V.$$

5b. Given $R_{out} = 200k\Omega$
$I_{BIAS} = 0.5 mA$



(a) Determine $(W/L)_1 = (W/L)_2$ with $\lambda = 0.1 V^{-1}$

$$R_{out} = (1+g_{m1} r_{o2}) r_{o1} + r_{o2}$$
$$= \left[1 + \sqrt{2 I_{BIAS} \mu_n C_{ox} \left(\frac{W}{L}\right)_1} \cdot \frac{1}{\lambda I_{BIAS}}\right] \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \cong \frac{\left[\left(R_{out} - \frac{1}{\lambda I_{BIAS}}\right)(\lambda I_{BIAS})^2\right]^2}{2 I_{BIAS} \mu_n C_{ox}}$$

$$= \frac{\left\{\left[(200k\Omega) - \frac{1}{(0.1V^{-1})(0.5mA)}\right](0.1V^{-1})^2(0.5mA)^2\right\}^2}{2(0.5mA)(100\ \frac{\mu A}{V^2})}$$

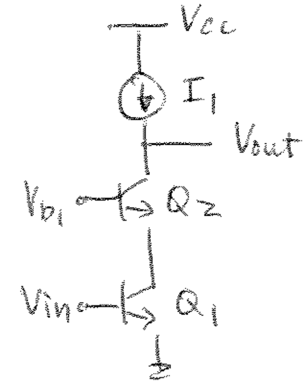
$$\approx 2.0$$

(b) $I_{BIAS} = 0.5\ mA = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{b2} - V_{TH,n})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox} (W/L)_1}} + V_{TH}$$
$$= \sqrt{\frac{2(0.5mA)}{(100\ \frac{\mu A}{V^2})(2.0)}} + (0.4V) \approx 2.62\ V$$

57. Given $|A_v| = 500$

$\beta = 100$



(a) $A_v = -g_{m1} r_{o1} g_{m2} (r_{o1} \| r_{\pi 2})$

$$= -\frac{V_A}{V_T} \times \frac{I_{C2}}{V_T}\left(\frac{V_A}{I_{C1}} \| \frac{\beta}{g_{m2}}\right)$$

Assume $I_{C1} \approx I_{C2}$. After expanding $(r_{o1} \| r_{\pi 2})$,

$$A_v \approx -\frac{V_A/V_T}{\frac{V_T}{V_A} + \frac{1}{\beta}} \Rightarrow V_A^2 + V_A\left(\frac{V_T A_v}{\beta}\right) + (A_v V_T^2) = 0$$

$\Rightarrow V_A \approx 0.65\ V$

(b) $V_{in} = V_T \ln\left(\frac{I_1}{I_S}\right) = (0.026\ V) \ln\left(\frac{0.5 mA}{6 \cdot 10^{-16} A}\right)$

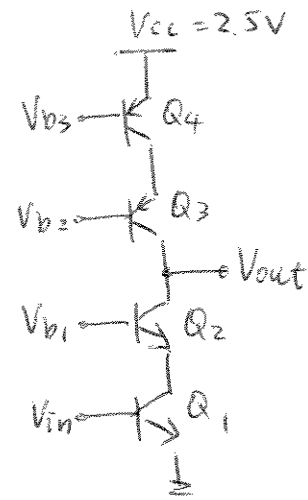
$\approx 0.71\ V$

(c) $V_{b1} = V_{BE2} + 500\ mV$

$= V_T \ln\left(\frac{I_1}{I_S}\right) + 0.5\ V$

$= 0.71\ V + 0.5\ V = 1.21\ V$

58. Given power budget = 2mW
$V_{BC_1} = V_{CB_4} = 200\ mV$,
calculate voltage gain.



$$\alpha_p = \frac{50}{50+1} \approx 0.98$$

$$\alpha_n = \frac{100}{100+1} \approx 0.99$$

∴ we assume $I_{c,p} \approx I_{e,p}$ & $I_{c,n} \approx I_{e,n}$

This implies that $I_{BIAS} = \frac{Power}{V_{cc}} = \frac{2mW}{2.5V}$

$$\approx 0.8mA$$

$$\Rightarrow V_{BE_1} = V_{in} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,1}}\right) = (0.026V)\cdot \ln\left(\frac{0.8mA}{6\cdot 10^{-16}A}\right) \approx 0.726V$$

$$V_{C,1} = V_{BE_1} - V_{BC_1} = 0.726V - 0.2V = 0.526V$$

$$\therefore V_{b_1} = V_{C_1} + V_{BE_2} = (0.526V) + (0.026V)\ln\left(\frac{0.8mA}{6\cdot 10^{-16}A}\right)$$

$$\approx 1.252V$$

$$\Rightarrow V_{EB_4} = V_{cc} - V_{b_3} = V_T \ln\left(\frac{I_{BIAS}}{I_{S,4}}\right) = 0.026V \cdot \ln\left(\frac{0.8mA}{6\cdot 10^{-16}A}\right)$$

$$\approx 0.726V$$

$V_{b3} = V_{cc} - 0.726V = 1.774\ V$

$V_{C4} = V_{D3} + V_{CB4} = 1.774V + 0.2V = 1.974\ V$

$\therefore V_{b2} = V_{C4} - V_{EB3} = (1.974V) - (0.026)\ln\left(\frac{0.8mA}{6 \cdot 10^{-16}A}\right)$

$\approx 1.248\ V$

$$A_V = -g_{m1}\left\{\left[g_{m2} r_{o2}(r_{o1} \| r_{\pi 2})\right] \| \left[g_{m3} r_{o3}(r_{o4} \| r_{\pi 3})\right]\right\}$$

After simplifying, A_v is independent of I_{BIAS}:

$$A_V \approx \frac{V_{AN} \cdot V_{AP}}{V_T^2\left(\frac{V_{AP}}{V_{AN}} + \frac{V_{AP}}{\beta_N V_T} + \frac{V_{AN}}{V_{AP}} + \frac{V_{AN}}{\beta_P V_T}\right)}$$

$$= \frac{5 \cdot 5}{(0.026V)^2\left(\frac{5}{5} + \frac{5}{100 \cdot 0.026} + \frac{5}{5} + \frac{5}{50 \cdot 0.026}\right)}$$

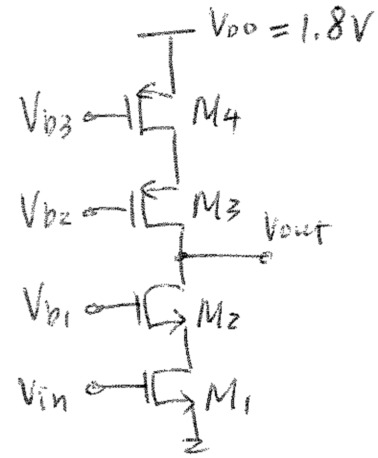
≈ 4760

59. Given $A_v = 200$

power budget = 2mW

all $\left(\frac{W}{L}\right) = \frac{20}{0.18}$

$V_{b1} = V_{b2} = 0.9\ V$



$\lambda_n = 0.1\ V^{-1}$

$\lambda_p = 0.2\ V^{-1}$

calculate V_{in} & V_{b3}

$$A_v \cong -g_{m1}(g_{m2}r_{o1}r_{o2} \parallel g_{m3}r_{o3}r_{o4}) = 200$$

$$power = V_{DD} \times I_{BIAS} \Rightarrow I_{BIAS} = \frac{power}{V_{DD}} = \frac{2mW}{1.8V} \approx 1.11\ mA$$

$$g_{m2}r_{o1}r_{o2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_{BIAS}} \cdot \left(\frac{1}{\lambda_n I_{BIAS}}\right)^2$$

$$= \sqrt{2 \cdot 100\ \mu A \cdot \frac{20}{0.18} \cdot 1.11\ mA} \cdot \left[\frac{1}{(0.1V^{-1})(1.11mA)}\right]^2$$

$$\approx 403\ k\Omega$$

$$g_{m3}r_{o3}r_{o4} \approx 71.\ k\Omega$$

We know that $\frac{|A_v|}{(g_{m2}r_{o1}r_{o2} \parallel g_{m3}r_{o3}r_{o4})} = g_{m1} = \frac{2I_D}{V_{GS1} - V_{TH}}$

$$\therefore V_{in} = V_{GS1} = V_{TH} + 2I_D \cdot \frac{g_{m2}r_{o1}r_{o2} \parallel g_{m3}r_{o3}r_{o4}}{A_v}$$

$$= (0.4\,V) + 2(1.11\,mA)\ \frac{(403\,k\Omega \parallel 71.\,k\Omega)}{200}$$

$$\approx 1.07\ V$$

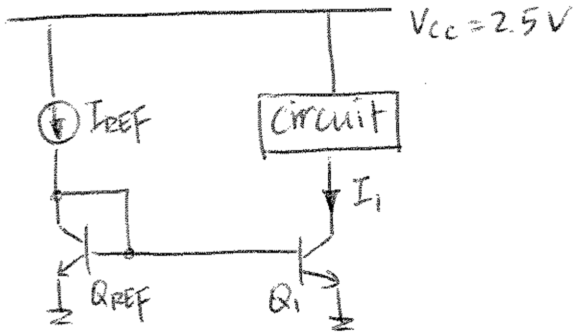
$$g_{m4} = \frac{2 I_D}{V_{DD} - V_{D3} - |V_{THp}|} = \sqrt{2 \mu_p C_{ox} \frac{W}{L} I_D}$$

$$\therefore V_{D3} = V_{DD} - |V_{THp}| - \frac{2 I_D}{\sqrt{2 \mu_p C_{ox} \frac{W}{L} I_D}}$$

$$= (1.8\,V) - (0.5\,V) - \frac{2(1.22\,mA)}{\sqrt{2 \cdot (50\,\frac{\mu A}{V^2})(\frac{20}{0.18})(1.22\,mA)}}$$

$$\approx 0.67\ V$$

60.



$I_1 = 0.5mA$
power = 2mW

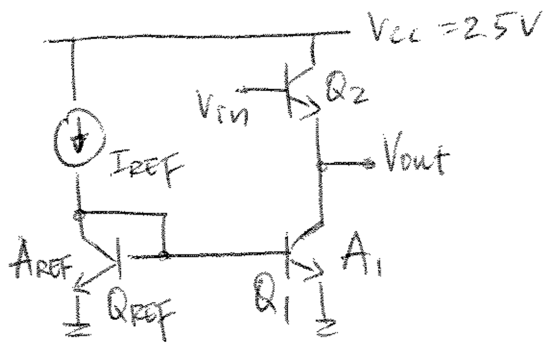
$$\text{power} = V_{cc}(I_{REF} + I_1)$$

$$\Rightarrow I_{REF} = \frac{\text{Power}}{V_{cc}} - I_1 = \frac{2mW}{2.5V} - 0.5mA = 0.3mA$$

Therefore, if Q_{REF} has area A_E, then Q_1 has area $\frac{5}{3}A_E$ for the currents specified.

i.e. $\frac{A_{REF}}{A_1} = \frac{3}{5}$

61.



power = 3mW
Rout = 50Ω

For an emitter follower, $R_{out} = r_{\pi 2} \parallel \frac{1}{g_{m2}}$

$$\Rightarrow R_{out} = 50\Omega = \frac{1}{\frac{I_{C2}}{V_T}\left(1+\frac{1}{\beta}\right)}$$

$$\therefore I_{C2} = \frac{V_T}{R_{out}} \cdot \frac{1}{1+1/\beta} = \frac{0.026}{50} \frac{1}{1+0.01} \approx 0.51 \text{ mA}$$

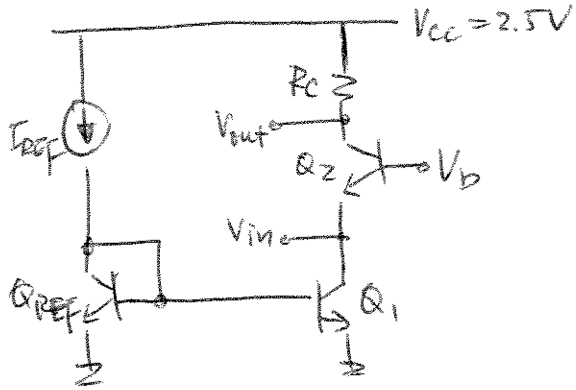
Realize that V_{CC} is providing current through I_{REF} & I_{C2}, and we are given

$$\text{power} = V_{CC}(I_{REF} + I_{C2}) = 3\text{mW}$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{CC}} - I_{C2} = \frac{3\text{mW}}{2.5\text{V}} - 0.51\text{mA} \approx 0.69\text{mA}$$

$$\Rightarrow \frac{I_{C2}}{I_{REF}} = \frac{A_1}{A_{REF}} = \frac{0.51}{0.69} \approx \frac{5}{7}$$

b2.



$R_{out} = 50\Omega$
$A_v = 20$
power = 1.5 mW
$\beta >> 1$, $V_A \to \infty$

$R_{out} = R_c \Rightarrow R_c = 50\Omega$

$A_v = g_m R_c = 20 \Rightarrow g_m = \frac{A_v}{R_c} = \frac{I_{c2}}{V_T}$

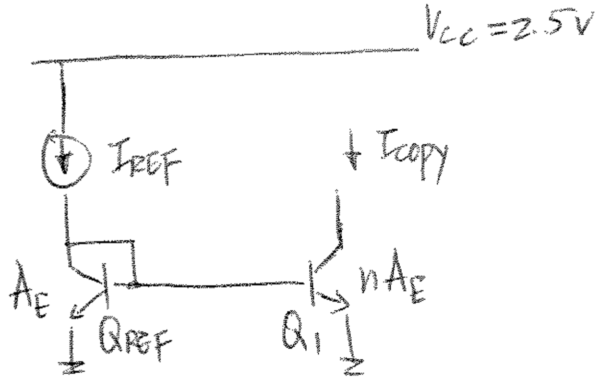
$\Rightarrow I_{c2} = \frac{A_v V_T}{R_c} = \frac{20(0.026V)}{50\Omega} \approx 10.4\ mA$

Realize that Vcc is providing current through I_{REF} & I_{c2}

power = $V_{cc}(I_{REF} + I_{c2})$

$\Rightarrow I_{REF} = \frac{\text{power}}{V_{cc}} - I_{c2} = \frac{1.5mW}{2.5V} - 10.4mA$

63.



Given $I_{copy} = 0.5mA$

By KCL, $I_{REF} = I_{C,REF} + \frac{I_{C,REF}}{\beta} + \frac{I_{copy}}{\beta}$

$$= \frac{I_{copy}}{n} + \frac{I_{copy}/n}{\beta} + \frac{I_{copy}}{\beta}$$

$$\Rightarrow I_{copy} = I_{REF} \cdot \frac{n}{1 + \frac{1}{\beta}(n+1)} = 0.5\ mA$$

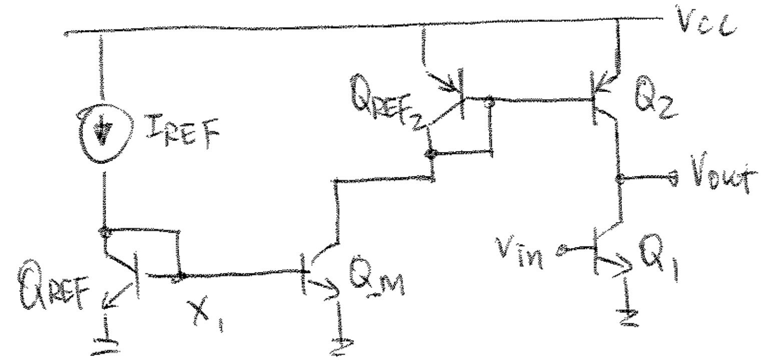
Within 1% implies that:

$$\Rightarrow I_{REF} \geq \frac{0.5mA}{0.99} \approx 0.505\ mA.$$

- For given n and β, $I_{copy} \leq n I_{REF}$. Since the error term causes $I_{copy} < n I_{REF}$ (strictly less than), one needs to increase I_{REF} in order to maintain the desired I_{copy}. This, however, means an increase of power (i.e. $\Delta p = V_{CC} \cdot \Delta I_{REF}$)

$\Rightarrow$ Trade off between accuracy & power dissipation.

64.



$$I_{C2} = I_{REF} \underbrace{\frac{(A_M/A_{REF})}{1+\frac{1}{\beta_n}\left(A_M/A_{REF}+1\right)}}_{X} \cdot \frac{(A_2/A_{REF_2})}{1+\frac{1}{\beta_p}\left(A_2/A_{REF_2}+1\right)}$$

Given $I_{C,M} \geq 0.98\, I_{REF}$ (less than 2% error)

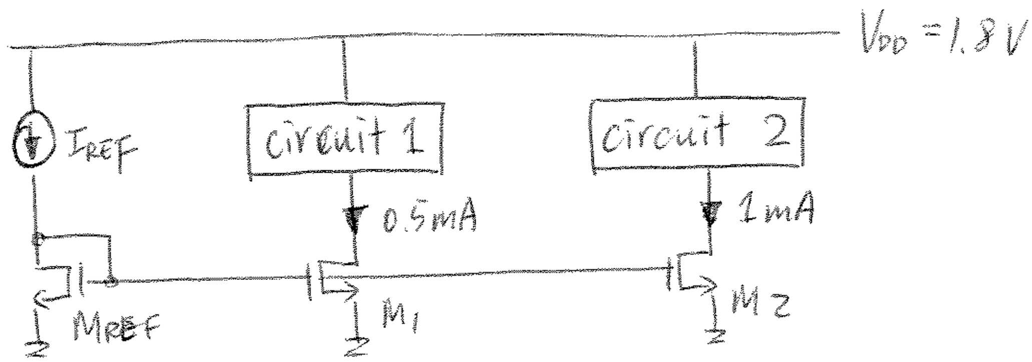
$$I_{C2} = 1\,mA = 0.98\, I_{REF} \cdot \frac{A_2/A_{REF_2}}{1+\frac{1}{50}\left(A_2/A_{REF_2}+1\right)}$$

Suppose $X = 0.98$ & $I_{REF} = 2mA$.

$$\Rightarrow \frac{A_2}{A_{REF_2}} \approx 0.5$$

Solution is not unique because no power constraint is present (i.e. I_{REF} is arbitrary.)

65.



power budget = 3 mW.

$$\text{power} = V_{DD}(I_{REF} + 0.5mA + 1mA)$$

$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{DD}} - 0.5mA - 1mA \approx 0.17mA$$

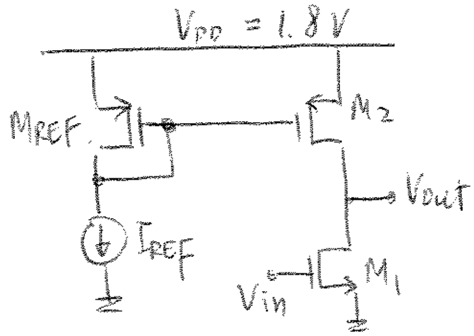
Assuming M_1 & M_2 operate in saturation,

If M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_1}{(W/L)_{REF}} = \frac{I_1}{I_{REF}} = \frac{50}{17}$$

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_2}{I_{REF}} = \frac{100}{17}$$

66.



$A_v = -20$

power = 2mW

$\left(\frac{W}{L}\right)_1 = \frac{20}{0.18}$ $\lambda_n = 0.1V^{-1}$

$\lambda_p = 0.2V^{-1}$

$$R_{out} = r_{o2} \parallel r_{o1} = \frac{1}{\lambda_n I_{D_1} + \lambda_p I_{D_1}}$$

$$\Rightarrow A_v = -G_m R_{out} = -\frac{g_{m_1}}{\lambda_n I_{D_1} + \lambda_p I_{D_1}} = -\frac{2I_{D_1}/(V_{GS_1}-V_{TH})}{I_{D_1}(\lambda_n+\lambda_p)}$$

$$\Rightarrow -20 = -\frac{2}{(V_{GS_1}-V_{TH})(\lambda_n+\lambda_p)}$$

$$\Rightarrow V_{GS_1} = \frac{1}{10(\lambda_n+\lambda_p)} + V_{TH_n}$$

$$= \frac{1}{10(0.1+0.2)V^{-1}} + 0.4V \approx 0.73\ V$$

$$\Rightarrow I_{D_1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{TH_n})^2$$

$$= \frac{1}{2}\left(100\ \frac{\mu A}{V^2}\right)\left(\frac{20}{0.18}\right)(0.33V)^2 \approx 0.61\ mA$$

$$\because \text{power} = V_{DD}(I_{REF} + I_{D_1})$$

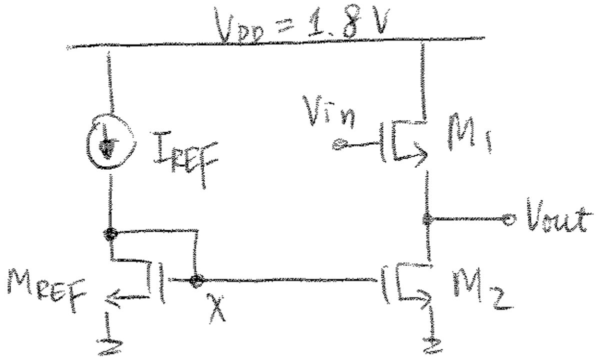
$$\Rightarrow I_{REF} = \frac{\text{power}}{V_{DD}} - I_{D_1} = \frac{2mW}{1.8V} - 0.61mA$$

$$\approx 0.5\ mA$$

$\therefore$ if M_{REF} has $\left(\frac{W}{L}\right)_{REF}$, then

$$\frac{(W/L)_2}{(W/L)_{REF}} = \frac{I_{D2}}{I_{REF}} = \frac{61}{50} \approx 1.2$$

67.



Given:

$A_v = 0.85$

$R_{out} = 100\,\Omega$

$(W/L)_2 = 10/0.18$

$\lambda_n = 0.1\,V^{-1}$, $\lambda_p = 0.2\,V^{-1}$

$$R_{out} = r_{o2} \,\|\, \left(\frac{1}{g_{m1}} \,\|\, r_{o1}\right) = \frac{1}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 100$$

For source follower,

$$A_v = \frac{g_{m1}}{g_{m1} + 1/r_{o2} + 1/r_{o1}} = 0.85$$

$$\Rightarrow g_{m1} = \frac{0.85}{100} = 8.5 \cdot 10^{-3}\,S$$

$$R_{out} = \frac{1}{g_{m1} + 2/r_o} = 100$$

$$\Rightarrow r_o = \frac{200}{1 - 100 g_{m1}} = \frac{200}{1 - 100(8.5 \cdot 10^{-3})}$$

$$\approx 1333\,\Omega$$

$$\Rightarrow I_{D1} = \frac{1}{\lambda_n r_{o1}} = 7.5\,mA.$$

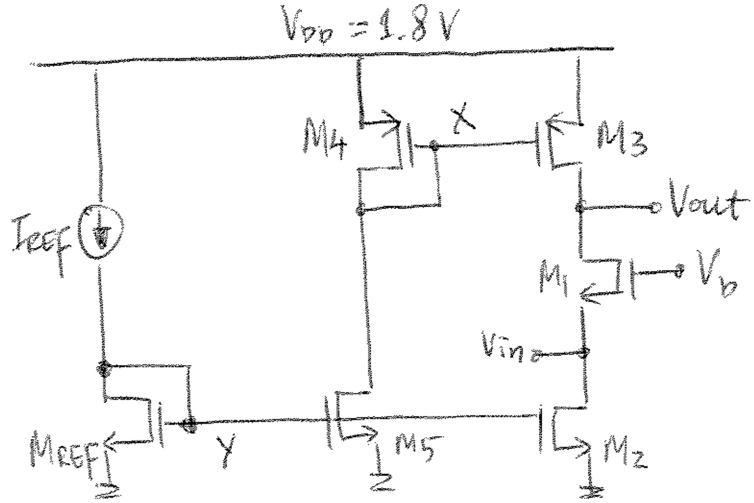
Assume $V_X \approx 1\,V$

$$\left(\frac{W}{L}\right)_2 = \frac{2 I_{D1}}{\mu_n C_{ox} (V_X - V_{TH})^2} \approx 416$$

Set $I_{REF} \approx 0.75$ mA.

$$\Rightarrow \left(\frac{W}{L}\right)_{REF} = \left(\frac{W}{L}\right)_{2} \frac{I_{REF}}{I_{D2}} \approx 42.$$

68.



$$\left(\frac{W}{L}\right)_3 = 20/0.18, \quad \lambda_n = 0.1\ V^{-1}, \quad \lambda_p = 0.2\ V^{-1}$$

$$A_v = 20, \quad R_{in} = 50\,\Omega$$

$$R_{in} = 50\,\Omega = r_{o2} \parallel 1/g_{m1} = \frac{1}{\lambda_n I_{D1} + g_{m1}} \quad \cdots \text{①}$$

$$R_{out} = r_{o3}$$

$$A_v = g_{m1} r_{o3} = \frac{g_{m1}}{\lambda_p I_{D1}} \quad \cdots \text{②}$$

Solve for g_{m1} in ② and substitute it into ①:

$$50 = \frac{1}{\lambda_n I_{D1} + A_v \lambda_p I_{D1}}$$

$$\Rightarrow I_{D1} = \frac{1}{(\lambda_n + A_v \lambda_p)(50\,\Omega)} = \frac{1}{(0.1 + 20(0.2))(50\,\Omega)} \approx 4.88\ mA$$

$$|V_{GS3}| = \sqrt{\frac{2 I_{D1}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}} + |V_{THp}| \approx 1.44 \text{ V}$$

$$g_{m1} = A_v \lambda_p I_{D1} \Rightarrow \left(\frac{W}{L}\right)_1 = \left[\frac{A_v \lambda_p I_{D1}}{\sqrt{2 \mu_n C_{ox} I_{D1}}}\right]^2$$

$$\approx 390.$$

Since $V_X \approx 0.4$ V, size up other transistors to allow them to operate in saturation.

Suppose $I_{D4} = 1.2$ mA $\Rightarrow \left(\frac{W}{L}\right)_4 = \dfrac{2 I_{D4}}{\mu_p C_{ox} \left(|V_{GS3}| - |V_{THp}|\right)^2}$

$$\approx 10/0.18$$

$$I_{D5} = I_{D4} \Rightarrow \left(\frac{W}{L}\right)_5 = \frac{2 I_{D5}}{\mu_n C_{ox} (V_y - V_{THn})^2} \approx \frac{100}{0.18}$$

(Assume $V_y = 0.6$; this is arbitrary, but must ensure M_5 in saturation.)

Set $I_{REF} = I_{D5} \Rightarrow \left(\frac{W}{L}\right)_{REF} \approx \dfrac{100}{0.18}$

$$I_{D2} \approx I_{D3} \Rightarrow \left(\frac{W}{L}\right)_2 = \frac{2 I_{D2}}{\mu_n C_{ox} (V_y - V_{THn})^2} \approx \frac{45}{0.18}$$

$$\text{Total power} = V_{DD}(I_{REF} + I_{D4} + I_{D3})$$
$$= 1.8(7.3) \text{ mW} \approx 13 \text{ mW}$$