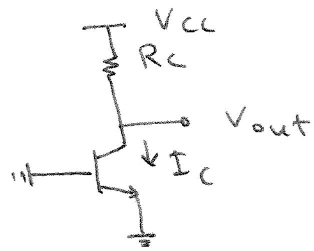
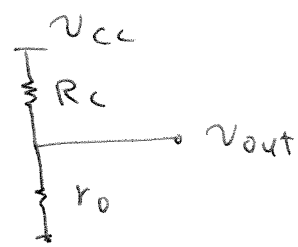
(1)

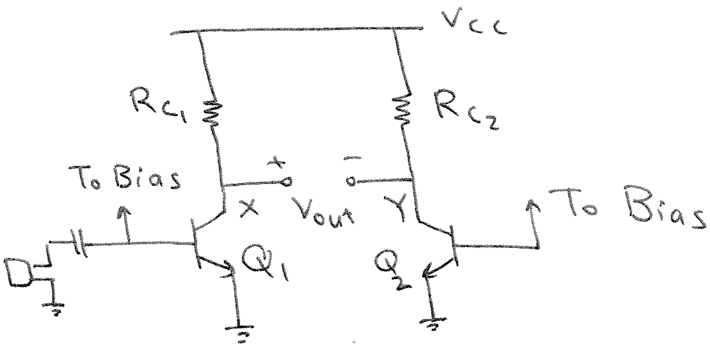


the small signal model is as follow:



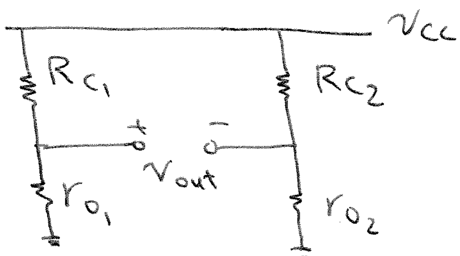
$$\frac{v_{out}}{v_{cc}} = \frac{r_o}{r_o + R_C} = \frac{\frac{V_A}{I_C}}{\frac{V_A}{I_C} + R_C} = \frac{V_A}{V_A + R_C I_C}$$

(2)



$R_{C_1} = R_{C_2}$

The small signal model is:

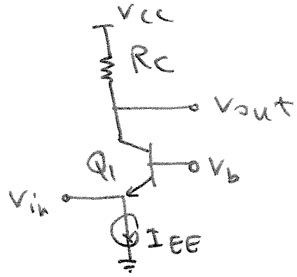


$r_{o_1} = r_{o_2}$
$R_{C_1} = R_{C_2}$

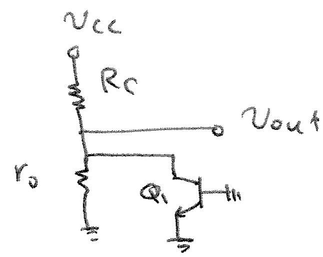
$$\frac{v_{out}}{v_{cc}} = \frac{1}{v_{cc}}\left(\frac{r_{o_1}}{R_{C_1}+r_{o_1}} - \frac{r_{o_2}}{R_{C_2}+r_{o_2}}\right)v_{cc} = 0$$

(3)

(a)

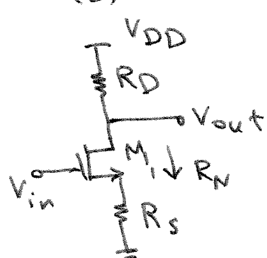


The small signal model is:

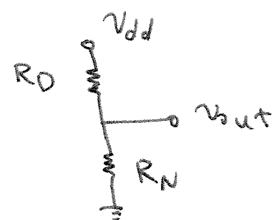


$$\frac{v_{out}}{v_{cc}} = \frac{r_o}{r_o + R_C} = \frac{V_A}{V_A + R_C I_{EE}}$$

(b)

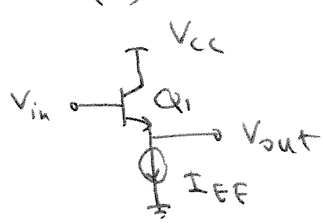


$\Rightarrow$

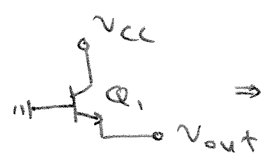


$$R_N = g_{m1} r_{o1} R_S + r_{o1} + R_S \quad , \quad \frac{v_{out}}{v_{dd}} = \frac{R_N}{R_N + R_D}$$

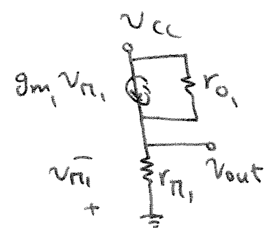
(c)



$\Rightarrow$



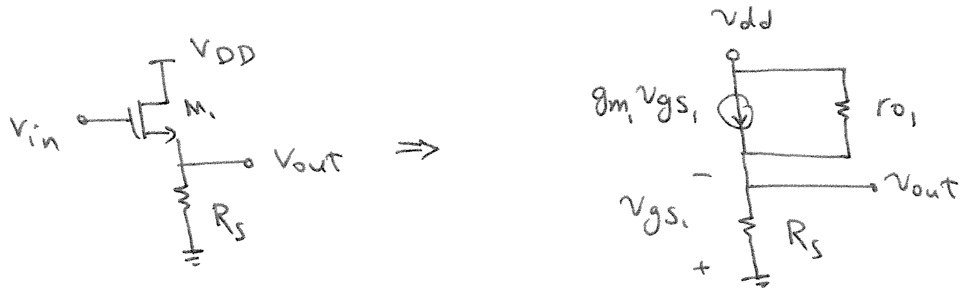
$\Rightarrow$



$$\frac{v_{out}}{r_{\pi 1}} + \frac{v_{out} - v_{cc}}{r_{o1}} + g_{m1} v_{out} = 0 \rightarrow (r_{\pi 1} + r_{o1} + g_{m1} r_{o1} r_{\pi 1}) v_{out} = r_{\pi 1} v_{cc}$$

$$\Rightarrow \frac{v_{out}}{v_{cc}} = \frac{r_{\pi 1}}{\beta r_{o1} + r_{o1} + r_{\pi 1}}$$

(3) (d)

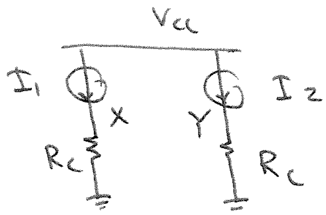


$$\frac{v_{out}}{R_S} + \frac{v_{out} - v_{dd}}{r_{o1}} + g_{m1} v_{out} = 0$$

$$\Rightarrow \left(r_{o1} + R_S + g_{m1} r_{o1} R_S \right) v_{out} = R_S \, v_{dd}$$

$$\Rightarrow \frac{v_{out}}{v_{dd}} = \frac{R_S}{g_{m1} r_{o1} R_S + R_S + r_{o1}}$$

(4)

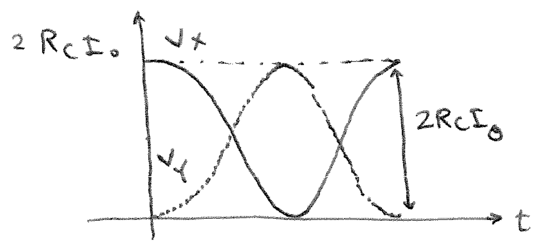


$I_1 = I_0 \cos\omega t + I_0$

$I_2 = -I_0 \cos\omega t + I_0$

$V_X = R_C I_1 = R_C I_0 (1 + \cos\omega t)$

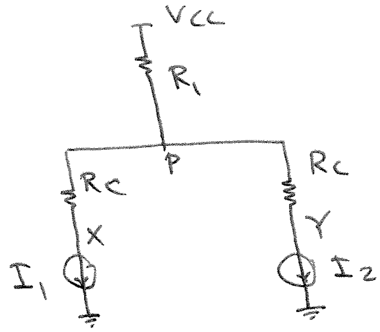
$V_Y = R_C I_2 = R_C I_0 (1 - \cos\omega t)$



$V_{X_{p\text{-}p}} = V_{Y_{p\text{-}p}} = 2 R_C I_0$

$V_{X,CM} = V_{Y,CM} = R_C I_0$

(5)



$I_1 = I_0 \cos\omega t + I_0$

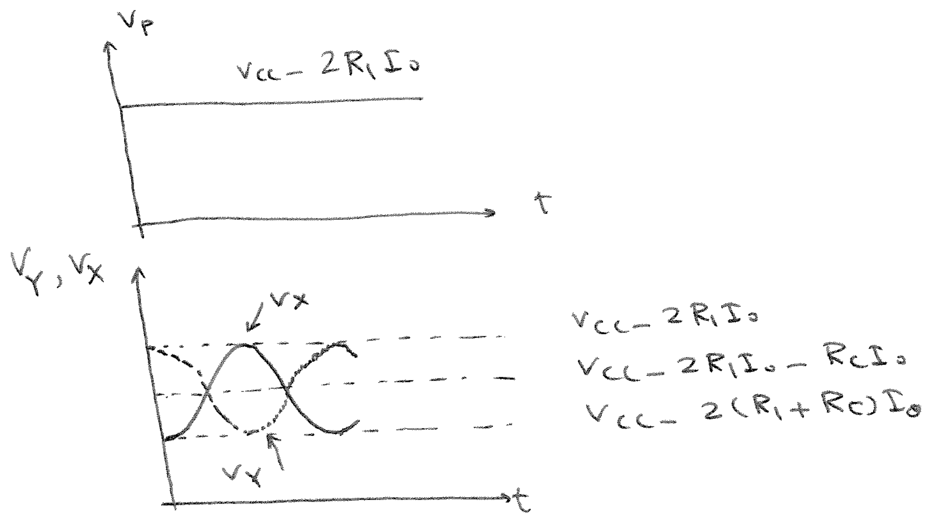
$I_2 = -I_0 \cos\omega t + I_0$

$V_P = V_{CC} - R_1(I_1 + I_2) = V_{CC} - 2R_1 I_0$

$V_X = V_P - R_C I_1 = V_{CC} - 2R_1 I_0 - R_C I_0 - R_C I_0 \cos\omega t$

$\Rightarrow V_X = V_{CC} - (2R_1 + R_C) I_0 - R_C I_0 \cos\omega t$

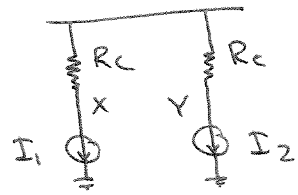
$V_Y = V_P - R_C I_2 = V_{CC} - (2R_1 + R_C) I_0 + R_C I_0 \cos\omega t$



$V_{X,CM} = V_{Y,CM} = V_{CC} - (2R_1 + R_C) I_0$

$V_{X,P\text{-}P} = V_{Y,P\text{-}P} = 2R_C I_0$

⑥

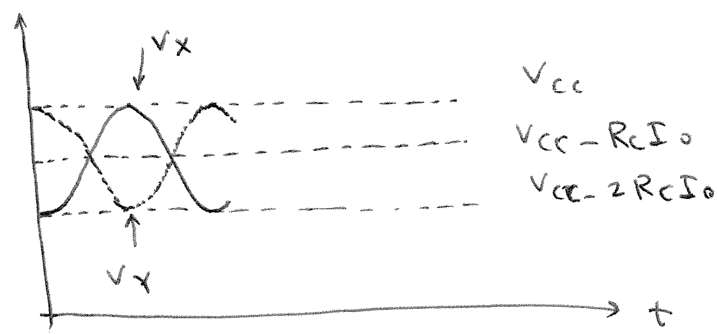


$I_1 = I_0 \cos\omega t + I_0$

$I_2 = -I_0 \cos\omega t + I_0$

$V_X = V_{CC} - R_C I_1 = V_{CC} - R_C I_0 (1 + \cos\omega t)$

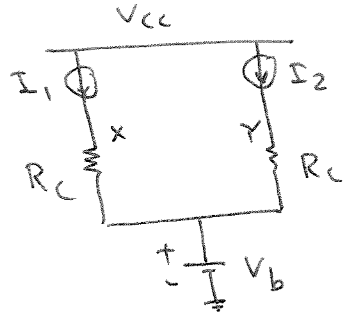
$V_Y = V_{CC} - R_C I_2 = V_{CC} - R_C I_0 (1 - \cos\omega t)$



$V_{X,CM} = V_{Y,CM} = V_{CC} - R_C I_0$

$V_{X,p\text{-}p} = V_{Y,p\text{-}p} = 2 R_C I_0$

⑦

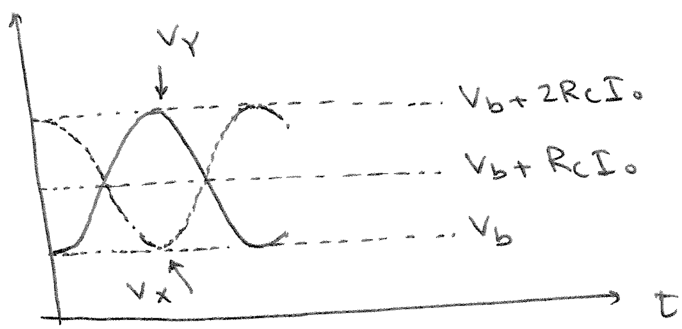


$I_1 = I_0 \cos\omega t + I_0$

$I_2 = -I_0 \cos\omega t + I_0$

$V_X = R_C I_1 + V_b = R_C I_0 (1 + \cos\omega t) + V_b$

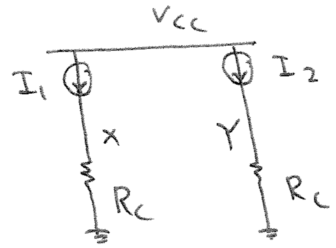
$V_Y = R_C I_2 + V_b = R_C I_0 (1 - \cos\omega t) + V_b$



$V_{X,CM} = V_{Y,CM} = V_b + R_C I_0$

$V_{X,p\text{-}p} = V_{Y,p\text{-}p} = 2 R_C I_0$

(8)

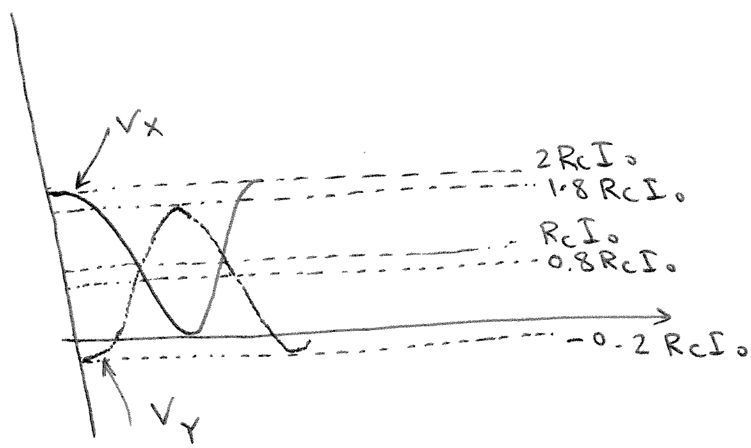


$$I_1 = I_0 \cos\omega t + I_0$$

$$I_2 = -I_0 \cos\omega t + 0.8 I_0$$

$$V_X = R_C I_1 = R_C I_0 (1 + \cos\omega t)$$

$$V_Y = R_C I_2 = R_C I_0 (0.8 - \cos\omega t)$$



$V_{X,CM} = R_C I_0$ $\quad$ $V_{Y,CM} = 0.8 R_C I_0$

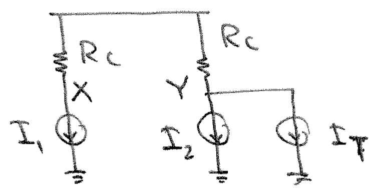
$V_{X,P-P} = 2 R_C I_0$ $\quad$ $V_{Y,P-P} = 2 R_C I_0$

X and Y are true differential signals.

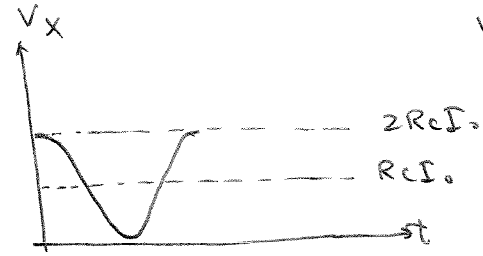
⑨

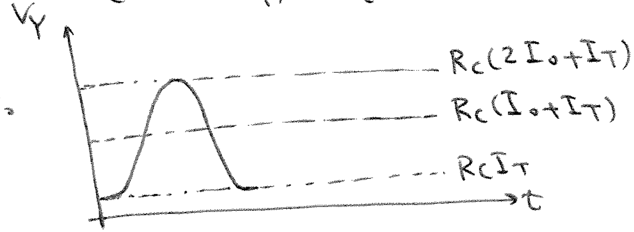
$$I_1 = I_0 \cos\omega t + I_0$$
$$I_2 = -I_0 \cos\omega t + I_0$$

(a)

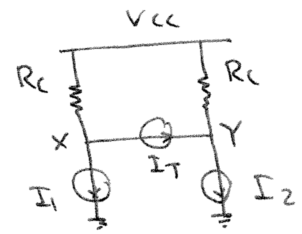


$$V_X = R_C I_1 = R_C I_0 (1 + \cos\omega t)$$
$$V_Y = R_C (I_2 + I_T) =$$
$$R_C (I_0 + I_T) - R_C I_0 \cos\omega t$$

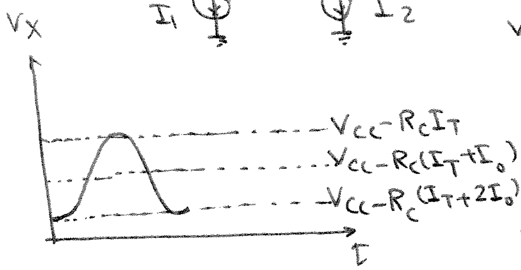


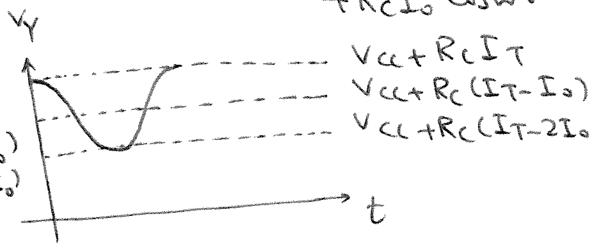


(b)

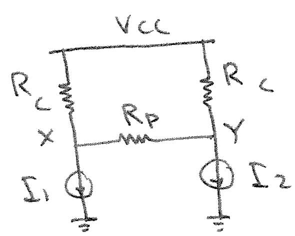


$$V_X = V_{CC} - R_C(I_1 + I_T) = V_{CC} - R_C(I_0 + I_T) - R_C I_0 \cos\omega t$$
$$V_Y = V_{CC} - R_C(I_2 - I_T) = V_{CC} - R_C(I_0 - I_T) + R_C I_0 \cos\omega t$$



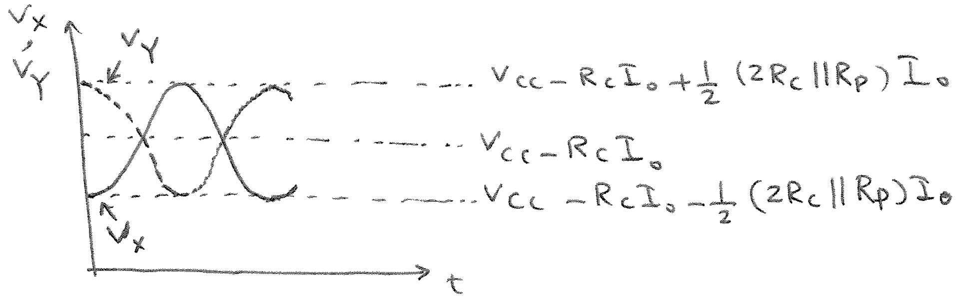


(c)

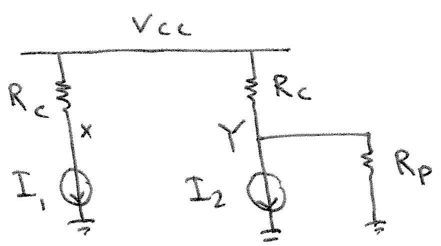


$$\begin{cases} \dfrac{V_X - V_{CC}}{R_C} + \dfrac{V_X - V_Y}{R_P} + I_1 = 0 \\[2ex] \dfrac{V_Y - V_{CC}}{R_C} + \dfrac{V_Y - V_X}{R_P} + I_2 = 0 \end{cases} \Rightarrow$$

$$V_X = V_{CC} - R_C I_0 - \frac{R_C R_P}{2R_C + R_P} I_0 \cos\omega t$$
$$V_Y = V_{CC} - R_C I_0 + \frac{R_C R_P}{2R_C + R_P} I_0 \cos\omega t$$



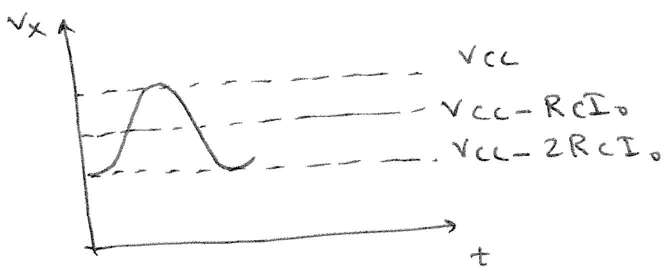
(d)

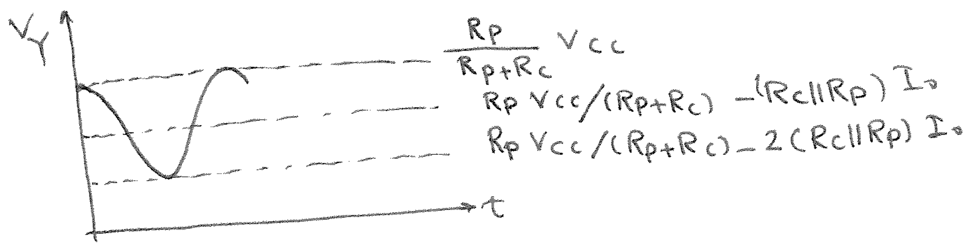


$$v_X = V_{CC} - R_C I_1 =$$
$$V_{CC} - R_C I_o (1 + \cos \omega t)$$

$$v_Y = \frac{R_P}{R_P + R_C} V_{CC} - (R_C \| R_P) I_2 =$$

$$\frac{R_P}{R_P + R_C} V_{CC} - (R_C \| R_P) I_o (1 - \cos \omega t)$$

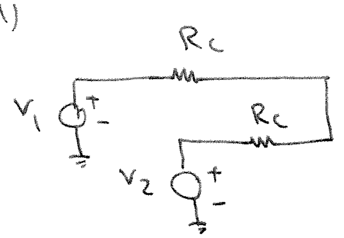




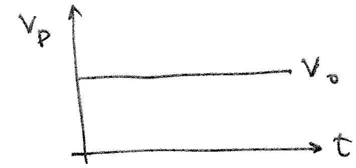
10)

$$V_1 = V_0 \cos\omega t + V_0$$
$$V_2 = -V_0 \cos\omega t + V_0$$

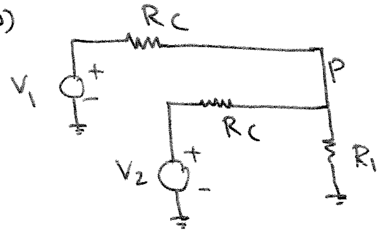
(a)



$$V_p = \frac{V_1 + V_2}{2} = V_0$$



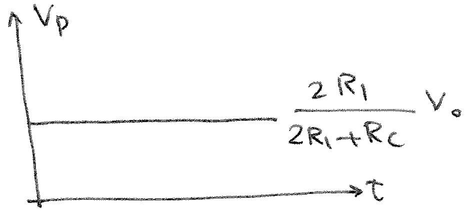
(b)



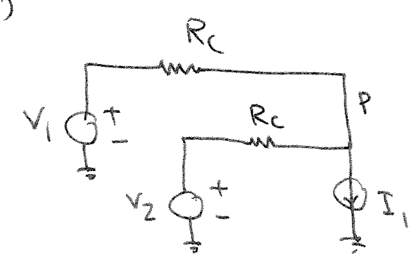
$$\frac{2V_p - V_1 - V_2}{R_c} + \frac{V_p}{R_1} = 0 \Rightarrow$$

$$\frac{2V_p - 2V_0}{R_c} + \frac{V_p}{R_1} = 0 \Rightarrow (2R_1 + R_c)V_p = 2V_0 R_1$$

$$\rightarrow V_p = \frac{2R_1}{2R_1 + R_c} V_0$$



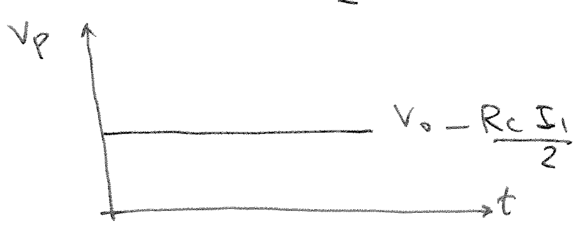
(c)



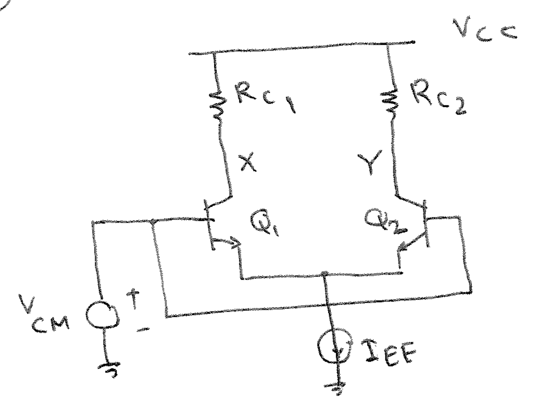
$$\frac{2V_p - V_1 - V_2}{R_c} + I_1 = 0 \Rightarrow$$

$$\frac{2V_p - 2V_0}{R_c} = -I_1 \Rightarrow$$

$$V_p = V_0 - \frac{R_c I_1}{2}$$



(11)



$R_{C1} = R_{C2} = R_C$

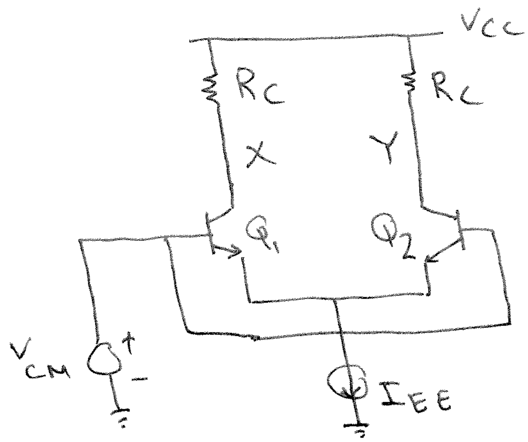
if Vcc changes by ΔV then:

$\Delta V_X = \Delta V$, $\Delta V_Y = \Delta V$ $\Rightarrow$

$\Delta (V_X - V_Y) = 0$

Since both V_X and V_Y response to the supply changes similarly, the circuit rejects supply noise.

12

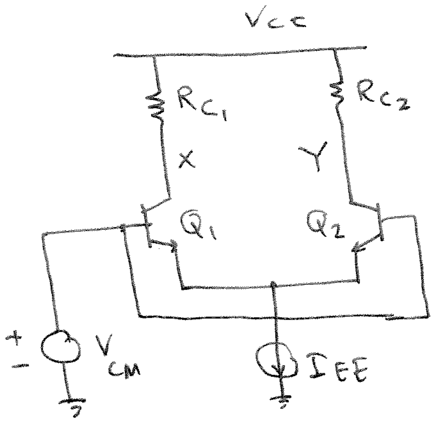


$$\Delta V_X = -\frac{R_C \Delta I_{EE}}{2} = -\frac{R_C \Delta I}{2}$$

$$\Delta V_Y = -\frac{R_C \Delta I_{EE}}{2} = -\frac{R_C \Delta I}{2}$$

$$\Rightarrow \Delta(V_X - V_Y) = 0$$

(13)



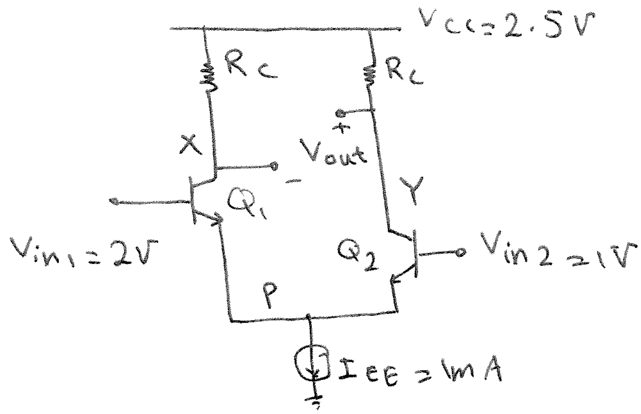
$$R_{C1} = R_{C2} + \Delta R$$

$$\Delta V_X = -\frac{R_{C1}\,\Delta I}{2} = -\frac{(R_{C2}+\Delta R)\,\Delta I}{2}$$

$$\Delta V_Y = -\frac{R_{C2}\,\Delta I}{2} \Rightarrow$$

$$\Delta(V_X - V_Y) = -\frac{\Delta R\,\Delta I}{2}$$

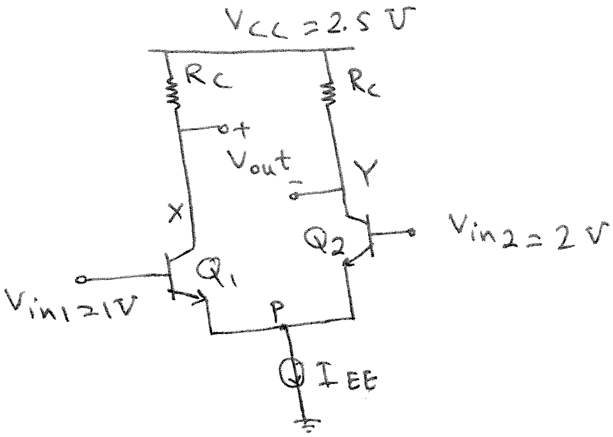
14



$$V_X \geq V_{in_1} \Rightarrow V_X \geq 2 \Rightarrow V_{CC} - R_C I_{EE} \geq 2$$

$$\Rightarrow 2.5 - \overset{(K\Omega)}{R_C} \geq 2 \Rightarrow R_C \leq 0.5\ K\Omega$$

15

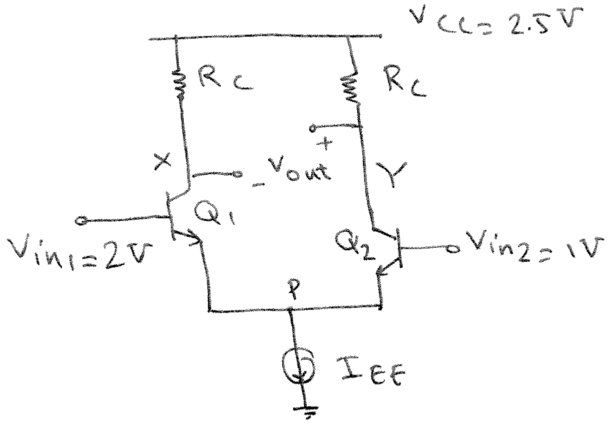


$R_C = 500\Omega$

$V_Y \geq V_{in2} \Rightarrow V_{CC} - R_C I_{EE} \geq 2 \Rightarrow$

$2.5 - 500 I_{EE} \geq 2 \Rightarrow I_{EE} \leq 1mA$

16



IEE= 1mA

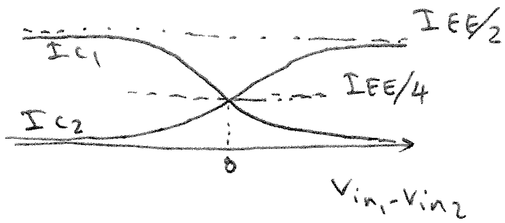
Rc= 800 Ω

$$V_X = V_{CC} - R_C I_{EE} = 2.5 - 0.8 = 1.7\ V$$

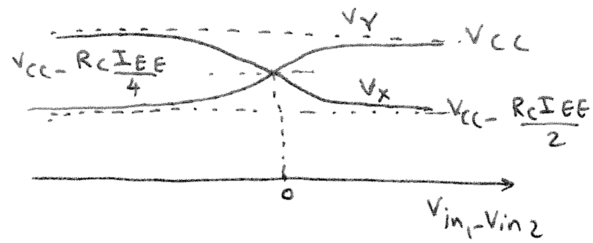
$\Rightarrow$ $V_X < V_{in1} \Rightarrow$ Q_1 is in saturation region.

(17)

(a)

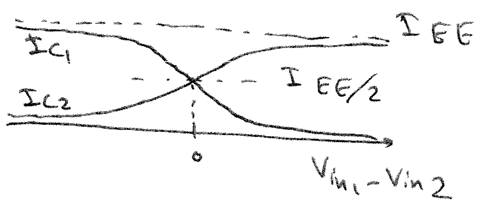


$I_{EE}/2$
I_{C1}
$I_{EE}/4$
I_{C2}
0
$V_{in1}-V_{in2}$

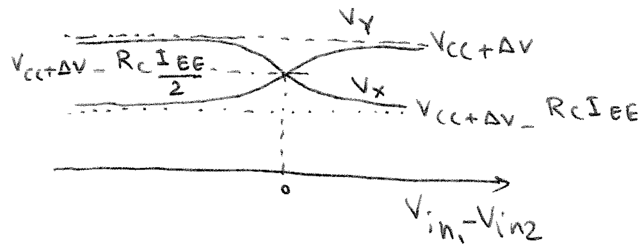


V_Y
V_{CC}
$V_{CC}-R_C\frac{I_{EE}}{4}$
V_X
$V_{CC}-\frac{R_CI_{EE}}{2}$
0
$V_{in1}-V_{in2}$

(b)

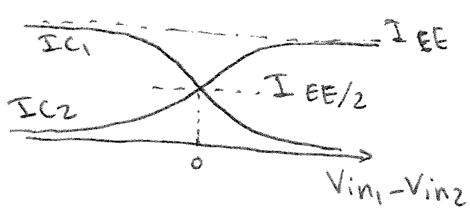


I_{EE}
I_{C1}
$I_{EE}/2$
I_{C2}
0
$V_{in1}-V_{in2}$

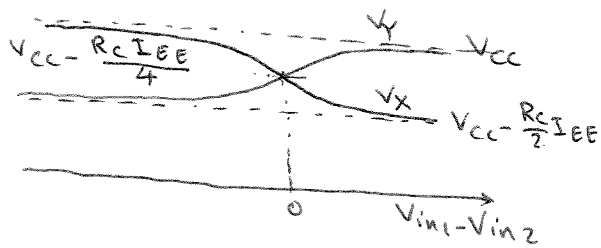


V_Y
$V_{CC}+\Delta V$
$V_{CC}+\Delta V-R_C\frac{I_{EE}}{2}$
V_X
$V_{CC}+\Delta V-R_CI_{EE}$
0
$V_{in1}-V_{in2}$

(c)

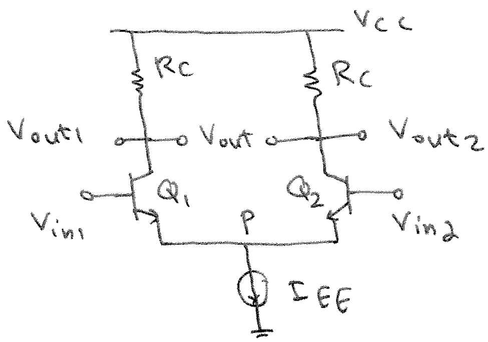


I_{EE}
I_{C1}
$I_{EE}/2$
I_{C2}
0
$V_{in1}-V_{in2}$



V_Y
V_{CC}
$V_{CC}-\frac{R_CI_{EE}}{4}$
V_X
$V_{CC}-\frac{R_C}{2}I_{EE}$
0
$V_{in1}-V_{in2}$

18



$$\frac{I_{C1}}{I_{C2}} = 5$$

$$V_{in1} - V_{in2} = V_T \ln \frac{I_{C1}}{I_{C2}} = 0.026 \ln 5 = 41.845\,mV$$

at $27°$, $V_T = 26mV$ $\Rightarrow$ at $100°C$,

$$V_T = \frac{(273+100)}{273+27}\,26^{mV} = 32.33\,mV$$

$$\Rightarrow \quad \underset{mV}{41.845} = \underset{mV}{32.33} \ln \frac{I_{C1}}{I_{C2}} \Rightarrow \frac{I_{C1}}{I_{C2}} = 3.65$$

19

$$I_{C2} = I_{C1} = \frac{I_{EE}}{2}$$

if I_{C2} changes by 10% then

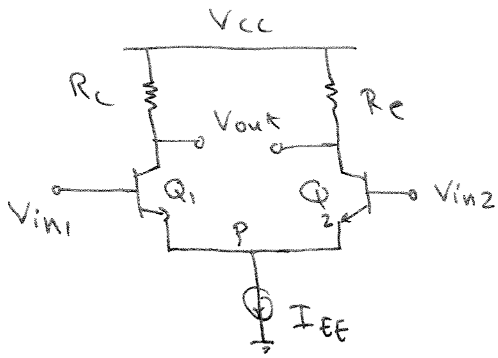
$$1.1 \times I_{C2\,bias} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$1.1 \times \frac{I_{EE}}{2} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}} \Rightarrow$$

$$V_{in1} - V_{in2} = V_T \ln \frac{0.9}{1.1} = -0.2\ V_T = -5.217\ mV$$

So the input differential voltage should change by no more than 5.2 mV.

(20)



$$I_{C2} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)}$$

$$I_{C2_{bias}} = \frac{I_{EE}}{2}$$

if the transconductance of Q_2 drops by a factor of 2, then $I_{C2} = \frac{I_{EE}}{4}$

$$\Rightarrow \quad \frac{I_{EE}}{4} = \frac{I_{EE}}{1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)} \Rightarrow$$

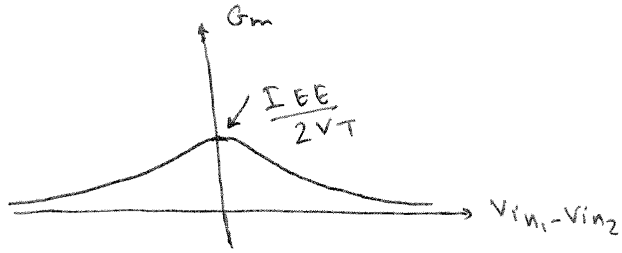
$$V_{in1} - V_{in2} = V_T \ln 3 = 1.0986 V_T = 28.564 \text{ mV}$$

(21) $V_{in1} - V_{in2} = \Delta V_{in}$

$$I_{C1} - I_{C2} = \frac{I_{EE} \exp \frac{\Delta V_{in}}{V_T}}{1 + \exp \frac{\Delta V_{in}}{V_T}} - \frac{I_{EE}}{1 + \exp \frac{\Delta V_{in}}{V_T}}$$

$$\Rightarrow \frac{\partial (I_{C1} - I_{C2})}{\partial (\Delta V_{in})} = I_{EE} \left[\frac{\frac{1}{V_T} \exp\left(\frac{\Delta V_{in}}{V_T}\right)\left(1 + \exp \frac{\Delta V_{in}}{V_T}\right) - \frac{\left(\exp \frac{\Delta V_{in}}{V_T}\right)^2}{V_T}}{\left(1 + \exp \frac{\Delta V_{in}}{V_T}\right)^2} + \frac{\frac{1}{V_T} \exp \frac{\Delta V_{in}}{V_T}}{\left(1 + \exp \frac{\Delta V_{in}}{V_T}\right)^2} \right]$$

$$= \frac{2 I_{EE}}{V_T} \frac{\exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)}{\left(1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)\right)^2}$$



$$\begin{cases} \max G_m = \frac{I_{EE}}{2V_T} \\ \text{At } V_{in1} - V_{in2} = 0 \end{cases}$$

if $G_m = \frac{1}{2} G_{m_{max}} = \frac{I_{EE}}{4V_T} \Rightarrow$

$$\frac{\exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)}{\left(1 + \exp\left(\frac{V_{in1} - V_{in2}}{V_T}\right)\right)^2} = \frac{1}{8} \Rightarrow V_{in1} - V_{in2} = \pm 1.763\, V_T$$

$$= \pm 45.838 \text{ mV}$$

(22)

$$V_{out1} - V_{out2} = -R_C I_{EE} \tanh \frac{V_{in1} - V_{in2}}{2V_T}$$

$$A_v = \frac{\partial (V_{out1} - V_{out2})}{\partial (V_{in1} - V_{in2})} = -\frac{2R_C I_{EE}}{V_T} \frac{\exp\left(\frac{V_{in1}-V_{in2}}{V_T}\right)}{\left[1+\exp\left(\frac{V_{in1}-V_{in2}}{V_T}\right)\right]^2}$$

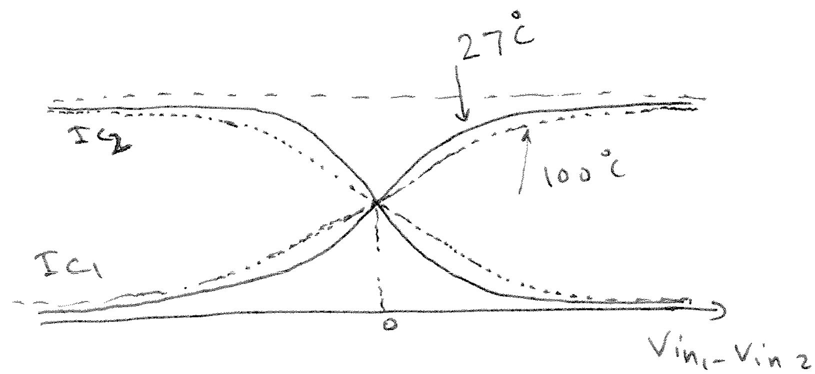
if $V_{in1} - V_{in2} = 30mV \Rightarrow$

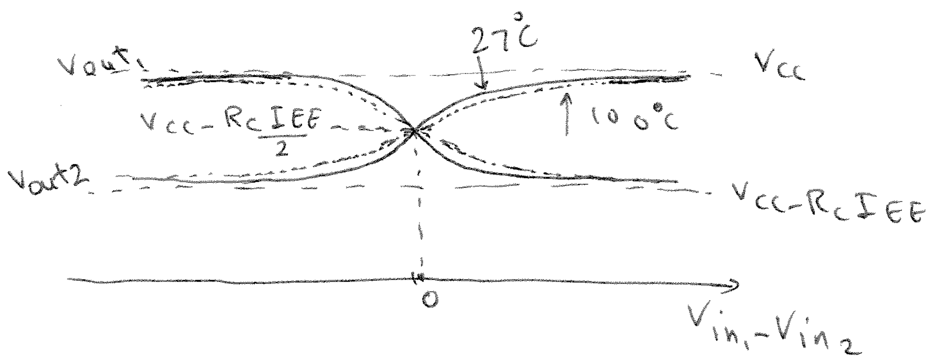
$$A_v = -14.02 \; R_C I_{EE}$$

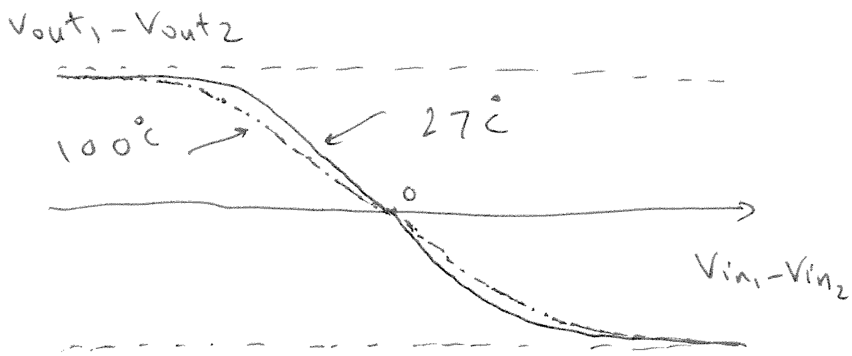
(23)

if the ambient temperature goes from 27°C to 100°C, V_T will change from 26mV to 32.33mV.

Therefor the curves stretch out as shown below to the sides:

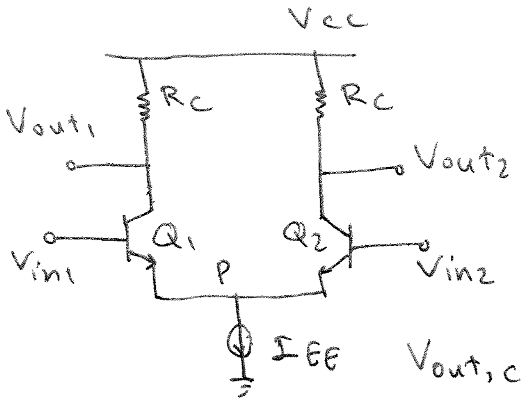






(24) $R_C = 500\Omega$, $I_{EE} = 1mA$, $V_{CC} = 2.5V$

$V_{in_1} = V_0 \sin \omega t + V_{CM}$ $\quad$ $V_{in_2} = -V_0 \sin\omega t + V_{CM}$, $V_{CM} = 1V$

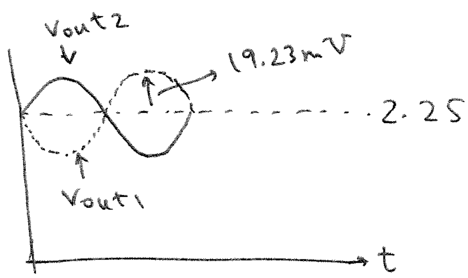


$$A_V = -g_m R_C = -\frac{I_{EE}}{2V_T} R_C = -\frac{10^{-3} \times 500}{2 \times 0.026} = -9.615$$

$$V_{out,CM} = V_{CC} - R_C \frac{I_{EE}}{2} = 2.5 - 0.5 \times 0.5 \Rightarrow$$

$$V_{out,CM} = 2.25$$

(a) $|V_{out}| = |A_V V_{in}| = 9.615 \times 2m = 19.23mV$

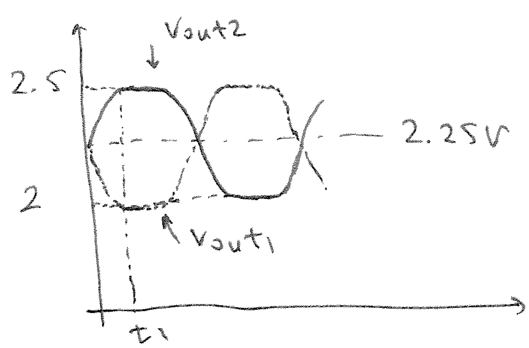


(b) $I_{C_1} = 0.95 I_{EE}$, $I_{C_2} = 0.05 I_{EE}$, $\frac{I_{C_1}}{I_{C_2}} = 19$

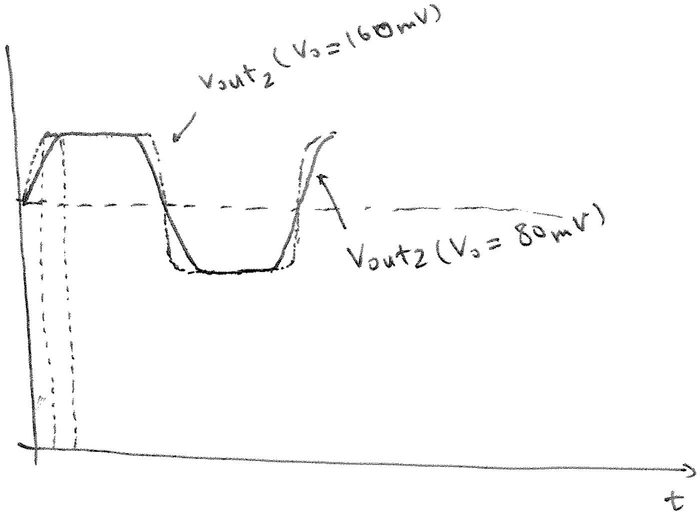
$$V_{in_1} - V_{in_2} = V_T \ln \frac{I_{C_1}}{I_{C_2}} = 76.555 mV$$

$$\frac{V_{in_1} - V_{in_2}}{2} = \underset{mV}{50} \sin\omega t_1 \Rightarrow \underset{mV}{38.278} = \underset{mV}{50} \sin\omega t_1 \Rightarrow$$

$$t_1 = \frac{0.872}{\omega}$$



(25)



The time at which one transistor takes 95% of the tail current source is achievable through:

$$\frac{I_{C_1}}{I_{C_2}} = 19 \Rightarrow V_{in_1} - V_{in_2} = V_T \ln \frac{I_{C_1}}{I_{C_2}} = 76.555\,mV$$

$$\frac{V_{in_1} - V_{in_2}}{2} = V_0 \sin \omega t_1 \Rightarrow t_1 = \frac{\text{Arc Sin } \frac{38.278}{V_0}}{\omega}$$

evidently as V_0 increases, t_1 decreases and the output waveform becomes sharper.

$$t_1 (V_0 = 80\,mV) = \frac{0.499}{\omega}$$

$$t_1 (V_0 = 160\,mV) = \frac{0.242}{\omega}$$

(26) $\omega = 2\pi \times (100MHz)$

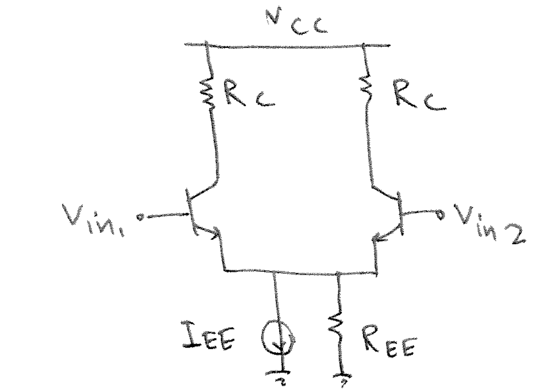
$$\text{Slope} \approx \frac{V_{cc} - V_{cm}}{t_1} = \frac{0.25\ \omega}{\text{ArcSin}\left(\frac{38.278}{V_0}\right)}$$

(mV)

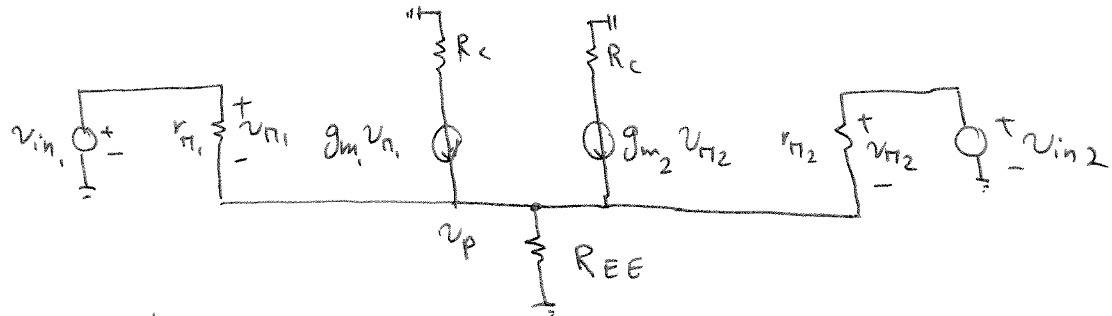
$\Rightarrow$ if $V_0 = 80mV \Rightarrow \text{Slope} = 3.148 \times 10^8\ V/s$

if $V_0 = 160mV \Rightarrow \text{slope} = 6.491 \times 10^8\ V/s$

(27)



The small signal model is:

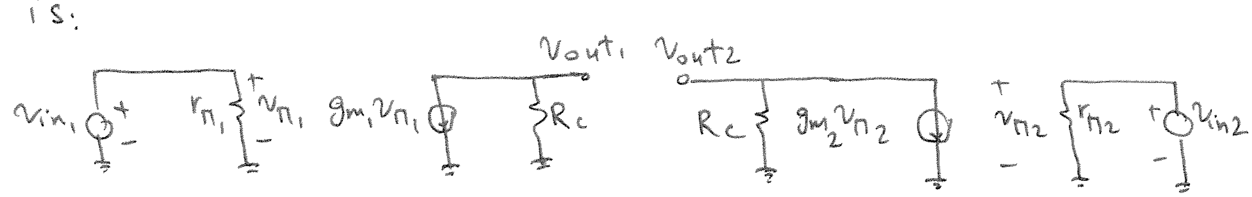


writing the node equation at P we have:

$$\frac{v_p}{R_{EE}} + \frac{v_p - v_{in_1}}{r_{\pi_1}} + g_{m_1}(v_p - v_{in_1}) + \frac{v_p - v_{in2}}{r_{\pi_2}} + g_{m_2}(v_p - v_{in_2}) = 0$$

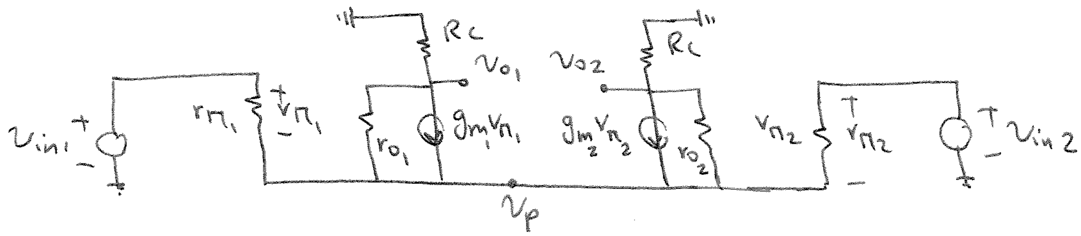
Since $v_{in1} = -v_{in2}$ and $\begin{cases} r_{\pi_1} = r_{\pi_2} \\ g_{m_1} = g_{m_2} \end{cases}$, the above equation simplifies to:

$$\frac{v_p}{R_{EE}} + \frac{2v_p}{r_{\pi_1}} + 2g_{m_1}v_p = 0 \Rightarrow v_p = 0 \Rightarrow$$ the small signal model is:



$$A_v = \frac{v_{out_1} - v_{out2}}{v_{in1} - v_{in2}} = \frac{-g_{m_1}v_{in_1}R_C + g_{m_2}v_{in_2}R_C}{v_{in_1} - v_{in_2}} = -g_{m_1}R_C$$

(28)



$v_{in_1} = -v_{in2} \rightarrow v_{in1} + v_{in2} = 0$

$g_{m_1} = g_{m2}, \ r_{\pi_1} = r_{\pi_2}, \ r_{o_1} = r_{o2}$

Writing the node equation at v_p:

$$\frac{v_p - v_{in1}}{r_{\pi_1}} + \frac{v_p - v_{o1}}{r_{o_1}} + g_{m_1}(v_p - v_{in_1}) + g_{m_2}(v_p - v_{in2}) +$$

$$\frac{v_p - v_{in2}}{r_{\pi_2}} + \frac{v_p - v_{o2}}{r_{o_2}} = 0 \Rightarrow 2 g_{m_1} v_p + \frac{2v_p - v_{o_1} - v_{o2}}{r_{o_1}} + \frac{v_p \times 2}{r_{\pi_1}} = 0 \quad (1)$$

Now the node equations at nodes v_{o_1} and v_{o2} leads to:

$$\begin{cases} \dfrac{v_{o1}}{R_C} + \dfrac{v_{o_1} - v_p}{r_{o_1}} + g_{m_1}(v_{in_1} - v_p) = 0 & (2) \\ \dfrac{v_{o2}}{R_C} + \dfrac{v_{o2} - v_p}{r_{o2}} + g_{m_2}(v_{in_2} - v_p) = 0 & (3) \end{cases} \Rightarrow (2) + (3) =$$

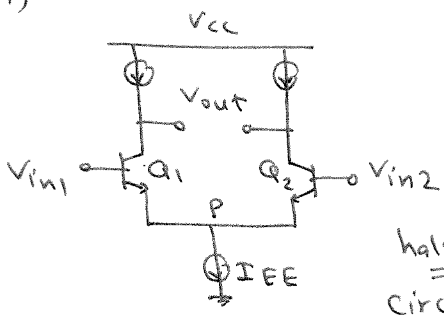
$$(v_{o_1} + v_{o2})\left(\frac{1}{R_C} + \frac{1}{r_{o_1}}\right) = \frac{2v_p}{r_{o_1}} + 2 g_{m_1} v_p \quad (4)$$

placing 4 in (1) $\Rightarrow$

$$2 g_{m_1} v_p + \frac{1}{r_{o_1}}\left(2v_p - \frac{1}{\frac{1}{R_C} + \frac{1}{r_{o_1}}}\left(\frac{2v_p}{r_{o_1}} + 2 g_{m_1} v_p\right)\right) + \frac{2v_p}{r_\pi} = 0$$

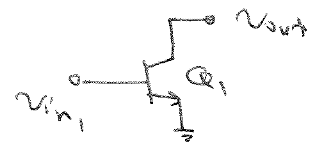
$$\Rightarrow \underline{v_p = 0}$$

(29)



$I_{EE} = 1mA$, $V_A = 5V$

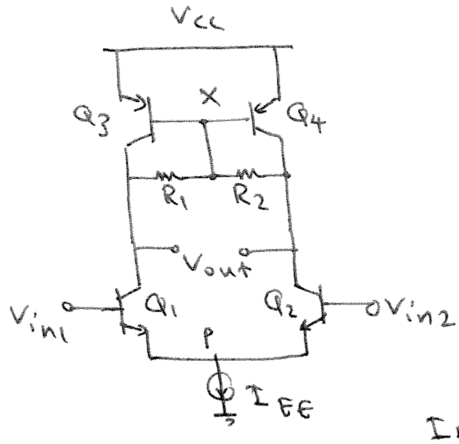
half $\Longrightarrow$ circuit



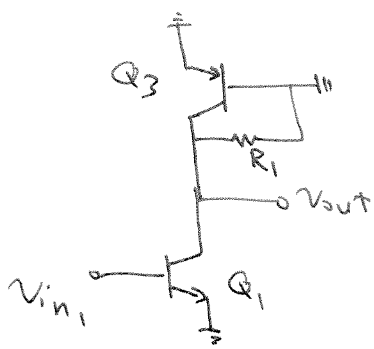
$$A_v = -g_{m_1} \cdot r_{o_1} = -\frac{I_{EE}}{2V_T} \frac{V_A}{\frac{I_{EE}}{2}} = -\frac{V_A}{V_T} = \frac{-5}{0.026}$$

$\rightarrow A_v = -192.31$

(30)



half $\Longrightarrow$ circuit



$I_{EE} = 2\,mA$, $V_{A,n} = 5\,V$, $V_{A,p} = 4\,V$

$$A_v = -g_{m_1}\,(r_{o_1} \parallel r_{o_3} \parallel R_1)$$

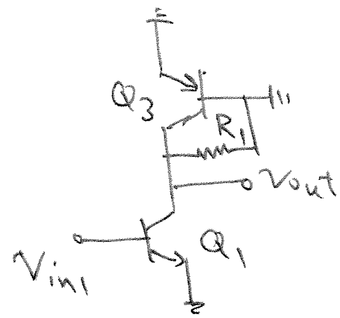
$$\Rightarrow 50 = \frac{I_{EE}}{2V_T}\left(\frac{V_{A,n}}{\frac{I_{EE}}{2}} \parallel \frac{V_{A,p}}{\frac{I_{EE}}{2}} \parallel R_1\right) \Rightarrow$$

$$50 = \frac{2}{2 \times 26}\left(\frac{5}{10^{-3}} \parallel \frac{4}{10^{-3}} \parallel R_1\right) \rightarrow$$

$$R_1 = 3132.53\,\Omega$$

(31)

The half circuit is:



$$A_v = -g_{m1}(r_{o1} \| r_{o3} \| R_1)$$

$\Rightarrow$

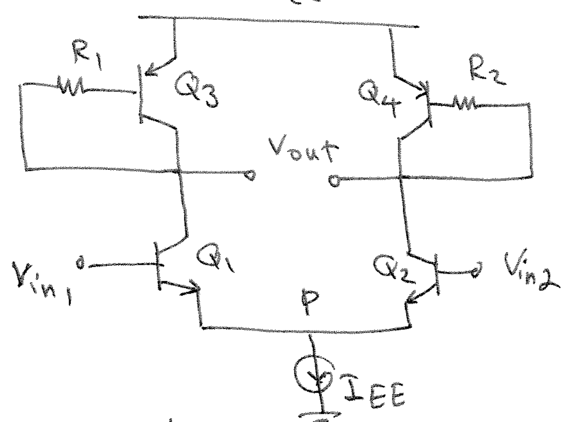
$$50 = \frac{I_{EE}}{2 \times 0.026}\left(\frac{5}{\frac{I_{EE}}{2}} \| \frac{4}{\frac{I_{EE}}{2}} \| 5^K\right) \Rightarrow$$

$$50 = \frac{1}{0.052}\left(10 \| 8 \| \underset{\mathrm{K}\Omega}{5}\underset{\mathrm{mA}}{I_{EE}}\right) \Rightarrow$$

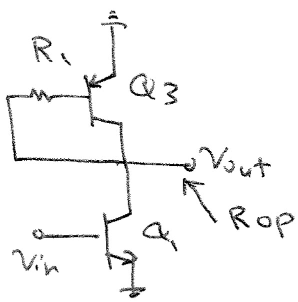
$$I_{EE} = 1.253 \text{ mA}$$

32

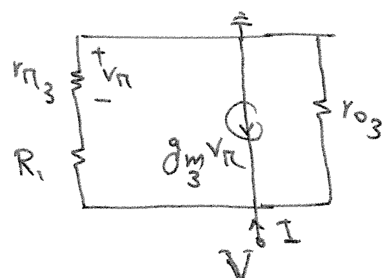
(a)



From half circuit concept we have: $A_v = -g_{m_1}(r_{o_1} \| R_{op})$



To calculate Rop, from small signal model we have:



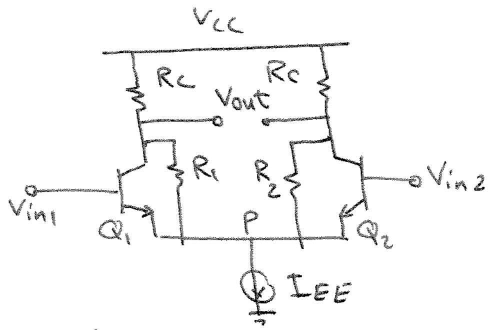
$$I = \frac{V}{r_{o_3}} - g_{m_3} V_\pi + \frac{V}{R_1 + r_{\pi 3}} = V\left[\frac{1}{r_{o3}} + \frac{1}{R_1 + r_{\pi 3}}\right] + g_{m3}\frac{r_{\pi 3}}{R_1 + r_{\pi 3}} V$$

$$\rightarrow R_{op} = \frac{V}{I} = r_{o3} \,\|\, (R_1 + r_{\pi 3}) \,\|\, \left(\left(1 + \frac{R_1}{r_{\pi 3}}\right)\frac{1}{g_{m_3}}\right)$$

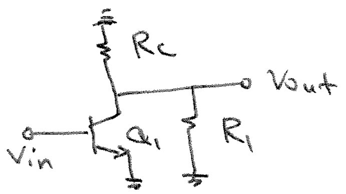
$$\rightarrow A_v = -g_{m_1}\left[r_{o_1} \,\|\, r_{o_3} \,\|\, (R_1 + r_{\pi 3}) \,\|\, \left(\left(1 + \frac{R_1}{r_{\pi 3}}\right)\frac{1}{g_{m_3}}\right)\right]$$

32

(b)

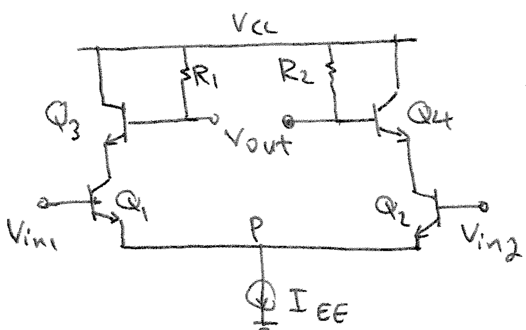


From half circuit concept:

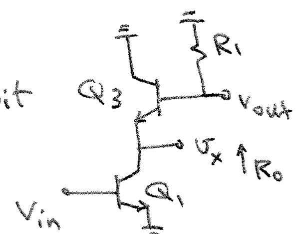


$\rightarrow A_v = -g_{m_1} (R_1 \| R_C \| r_{o_1})$

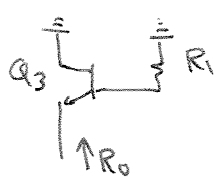
(C)

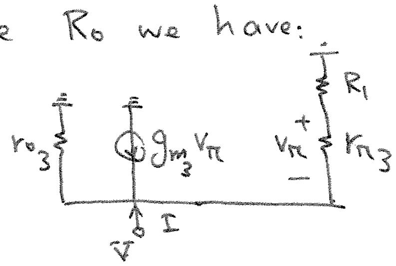


half circuit $\Longrightarrow$



To calculate R_o we have:





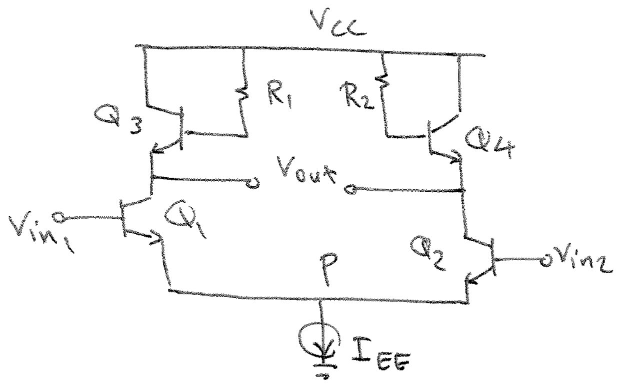
$R_o = \frac{V}{I}$

$I = \frac{V}{r_{o3}} + \frac{V}{R_1 + r_{\pi 3}} + g_{m_3} \frac{r_{\pi 3}}{r_{\pi 3} + R_1} V$

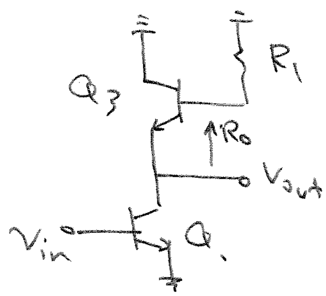
$\Longrightarrow R_o = r_{o3} \| (R_1 + r_{\pi 3}) \| \left(1 + \frac{R_1}{r_{\pi 3}}\right) \frac{1}{g_{m_3}}$

$A_v = \frac{v_{out}}{v_{in}} = \frac{v_x}{v_{in}} \frac{v_{out}}{v_x} = -g_{m_1} (r_{o_1} \| R_o) \frac{R_1}{R_1 + r_{\pi 3}}$

(32) (d)



From half circuit concept :



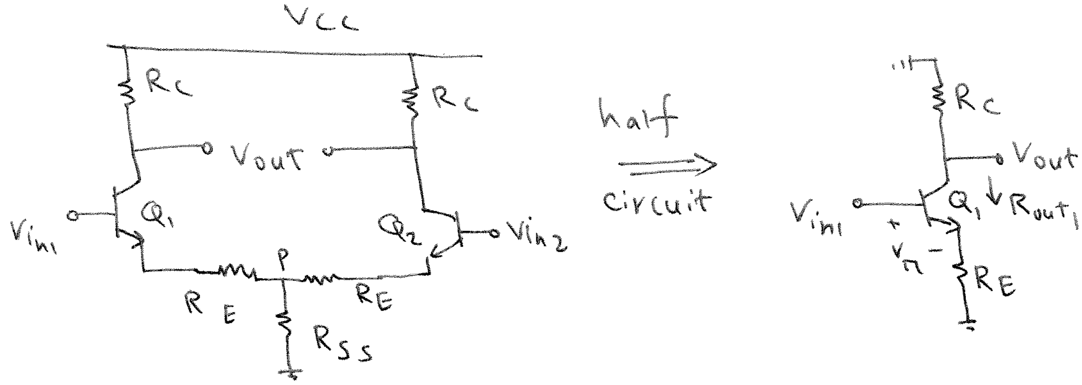
we already proved in part (c) that

$$R_o = r_{o3} \,||\, (R_1 + r_{\pi 3}) \,||\, \left(1 + \frac{R_1}{r_{\pi 3}}\right)\frac{1}{g_{m3}}$$

$$\rightarrow A_v = \frac{v_{out}}{v_{in}} = -g_{m1}(r_{o1} \,||\, R_o)$$

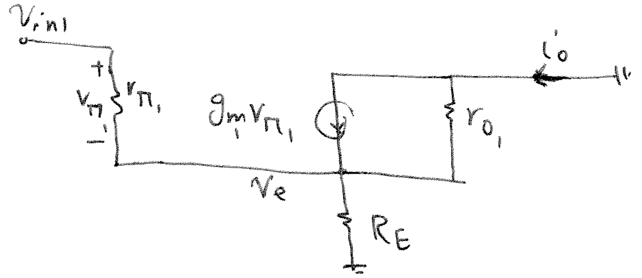
(33)

(a)



$$R_{out} = R_C \| R_{out_1} = R_C \| \left(g_{m_1} r_{o_1} (R_E \| r_{\pi_1}) + r_{o_1} + (R_E \| r_{\pi_1}) \right)$$

To calculate G_m, the small signal model is:



writing node equation at node v_e:

$$\frac{v_e}{R_E \| r_{o_1}} = \left(g_{m_1} + \frac{1}{r_{\pi_1}}\right) v_{\pi_1} = \underbrace{\left(g_{m_1} + \frac{1}{r_{\pi_1}}\right)}_{\approx g_{m_1}} (v_{in_1} - v_e)$$

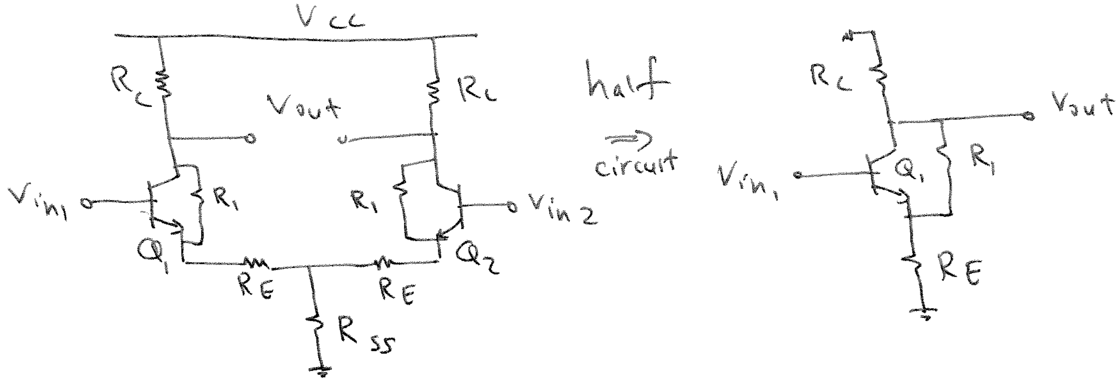
$$\Rightarrow v_e = \frac{g_{m_1}}{g_{m_1} + \frac{1}{R_E \| r_{o_1}}} v_{in_1} \Rightarrow i_o = \frac{v_e}{r_{o_1}} - g_{m_1} v_{\pi_1}$$

$$= \frac{v_e}{r_{o_1}} + g_{m_1} (v_e - v_{in_1}) \Rightarrow G_m = \frac{i_o}{v_{in_1}} = + \frac{g_{m_1} r_{o_1}}{g_{m_1} r_{o_1} R_E + r_{o_1} + R_E}$$

$$\Rightarrow A_v = -G_m R_{out}$$

(33)

(b)



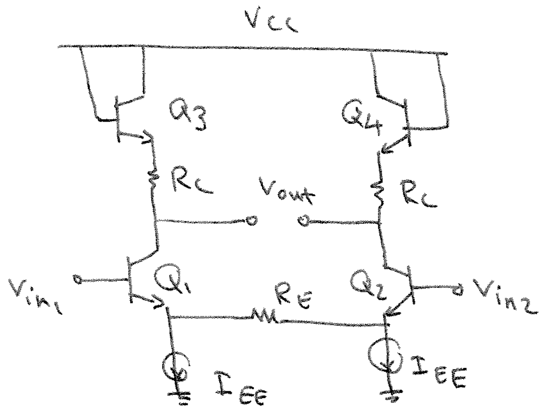
$$R_{out} = R_C \parallel \left(g_{m_1}(r_{o_1} \parallel R_1)(R_E \parallel r_{\pi_1}) + (r_{o_1} \parallel R_1) + (R_E \parallel r_{\pi_1}) \right)$$

Similar to the approach in part (a)

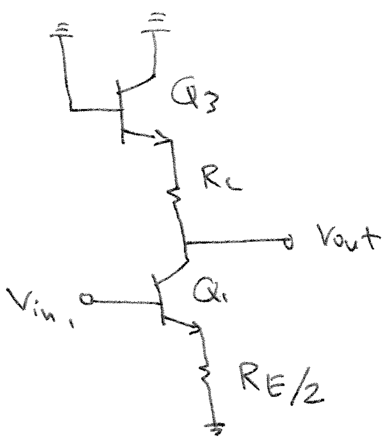
$$G_m = + \frac{g_{m_1}(r_{o_1} \parallel R_1)}{g_{m_1}(r_{o_1} \parallel R_1) R_E + (r_{o_1} \parallel R_1) + R_E}$$

$$\Rightarrow A_v = -G_m R_{out}$$

34



The half circuit is shown as:

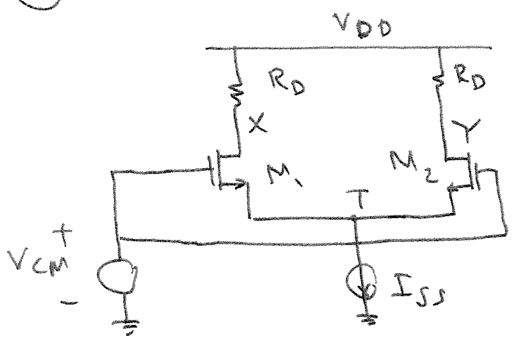


a) $$A_v = \frac{v_{out}}{v_{in1}} = -\frac{R_C + 1/g_{m3}}{R_E/2 + 1/g_{m1}}$$

b) if $\frac{R_C}{R_E/2} = A$, then if $\frac{1/g_{m3}}{1/g_{m1}} = A$

we conclude $A_v = -A$. So the circuit is very linear.

(35)



$$V_T = V_{CM} - V_{GS1} = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

(a) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{2W}{L}}}$$

The tail voltage increases

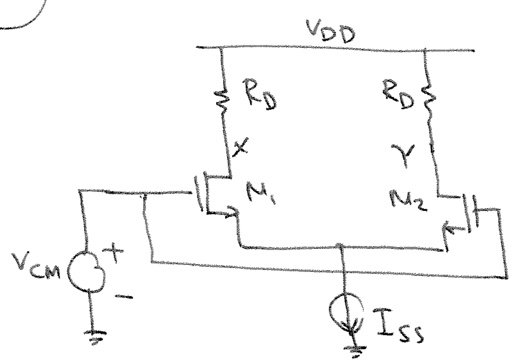
(b) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

The tail voltage decreases

(c) $$V_T = V_{CM} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_n \frac{C_{ox}}{2} \frac{W}{L}}}$$

The tail voltage decreases

(36)



$V_{CM} = 1V$
$I_{SS} = 1mA$
$R_D = 1k\Omega$

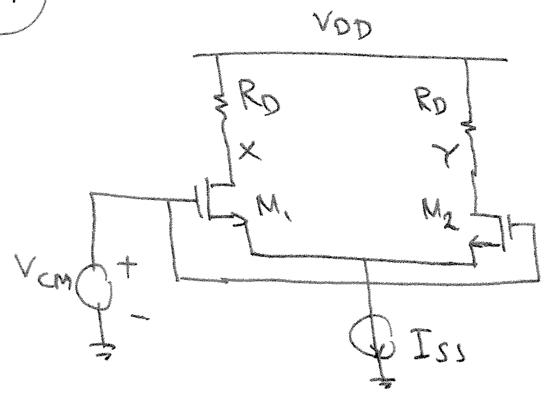
$$V_X = V_{DD} - R_D \frac{I_{SS}}{2} = V_{DD} - 0.5$$

To ensure that the devices work in saturation

$$V_{CM} \leqslant V_X - V_{TH} \rightarrow V_{DD} - 0.5 - V_{TH} \geq 1$$

if $V_{TH} = 0.5 \rightarrow V_{DD} > 2$

(37)



$V_{GS} - V_{TH} = 200\,mV$

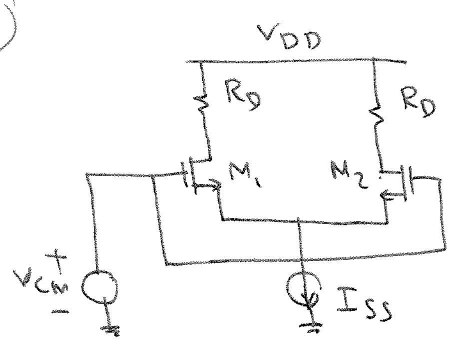
$\mu_n C_{ox} = 100\ \mu A/V^2$

$\frac{W}{L} = 20/0.18$

$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \rightarrow$$

$$0.2 = \sqrt{\frac{I_{SS}}{10^{-4} \times \frac{20}{0.18}}} \rightarrow I_{SS} = 0.44\,mA$$

38



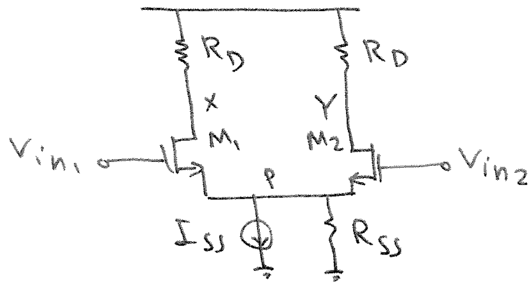
$$\frac{I_{D1}}{W} = \frac{I_{SS}}{2W} = J$$

current density

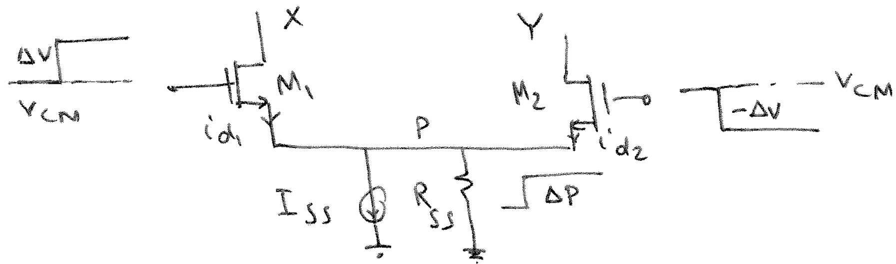
$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} =$$

$$\sqrt{\frac{2J}{\mu_n C_{ox} \frac{1}{L}}}$$

(39)



$g_{m1} = g_{m2} = g_m$



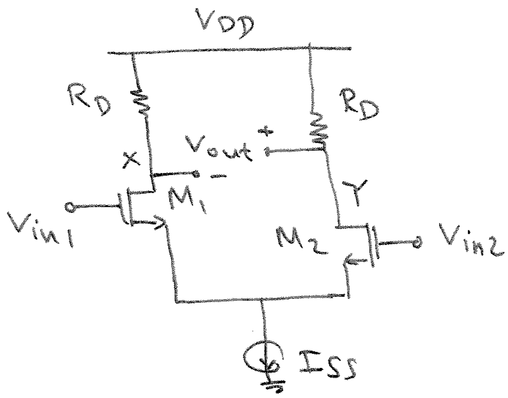
if V_{in1} and V_{in2} chang by ΔV and $-\Delta V$ we have:

node equation at P: $i_{d1} + i_{d2} + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow$

$$g_m(\Delta V - \Delta P) + g_m(-\Delta V - \Delta P) + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow$$

$$-g_m \Delta P - g_m \Delta P + \frac{\Delta P}{R_{SS}} = 0 \Rightarrow \Delta P = 0$$

40



$V_{in1} = 1.5$

$V_{in2} = 0.3$

$V_X - V_{TH} > V_{in1} \rightarrow V_{DD} - R_D I_{SS} - V_{TH} > 1.5$

41

$$P = 2\,mW$$

$$V_{DD} = 2V \rightarrow \quad I_{SS} = 1\,mA$$

$$V_{CM,out} = V_{DD} - R_D \frac{I_{SS}}{2} = 2 - 0.5 R_D \quad \rightarrow$$

$$R_D = \frac{2 - V_{CM,out}}{0.5} \quad (k\Omega)$$

$$g_m R_D = 5 \longrightarrow \quad \sqrt{2 \mu_n C_{ox} \frac{W}{L} \frac{I_{SS}}{2}} \quad \underbrace{\frac{2 - V_{CM,out}}{0.5}}^{k\Omega} = 5$$

$$\longrightarrow \quad \sqrt{2 \times 10^{-4} \frac{W}{L} \frac{10^{-3}}{2}} \quad 2 \times 10^{3} \left(2 - V_{CM,out} \right) = 5$$

if $V_{CM,out} = 1.6 \longrightarrow \quad \frac{W}{L} = 390.625$

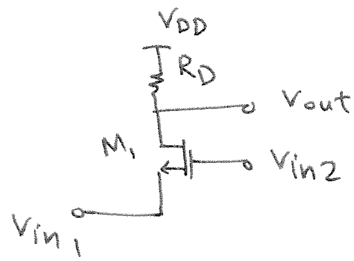
To formulate the trade off we have:

$$I_{SS} = \frac{2}{V_{DD}}\,mA \quad \Rightarrow \quad R_D = \frac{V_{DD} - V_{CM,out}}{1} \times V_{DD} \quad (k\Omega)$$

$$\Rightarrow |g_m R_D| = 5 \Rightarrow$$

$$\sqrt{2 \times 10^{-4} \left(\frac{W}{L}\right) \frac{1}{V_{DD}} \times 10^{-3}} \quad (V_{DD} - V_{CM,out})\, V_{DD} = 5$$

(42)



$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{in2} - V_{in1} - V_{TH}\right)^2$$

(1) The current is not an odd function of $(V_{in2} - V_{in1})$. Therefore it is not symmetric around $V_{in1} = V_{in2}$ $\left[(V_{in1} - V_{in2}) = 0\right]$.

(2) The input impedance seen at V_{in1} and V_{in2} are different

(3) The circuit cannot suppress the supply noise. because there is no differential output available.

(43)

$$(V_{in_1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - 2\sqrt{I_{D_1} I_{D2}})$$

(a)

$I_{D_1} = 0 \Rightarrow$

$$(V_{in_1} - V_{in_2})^2 = \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in_1} - V_{in_2} = -\sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum differential input voltage to turn M_1 off.

(b) $I_{D_1} = \frac{I_{SS}}{2} \Rightarrow I_{D_2} = \frac{I_{SS}}{2}$

$$(V_{in_1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{SS} - I_{SS}) = 0 \rightarrow V_{in_1} - V_{in_2} = 0$$

This is the equilibrium input case.

(c) $I_{D_1} = I_{SS} \rightarrow I_{D2} = 0$

$$(V_{in_1} - V_{in2})^2 = \frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}} \rightarrow V_{in_1} - V_{in2} = +\sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

This is the minimum input differential voltage to turn M_2 off.

(44)

$$I_{D_1} = \frac{I_{SS}}{2} - \frac{1}{4}\sqrt{4I_{SS}^2 - \left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right]}$$

The analyses which led to the above equation assume that the transistors work in saturation region.

So:

$$-(V_{in_1}-V_{in_2})_{max} \leq V_{in_1}-V_{in_2} \leq (V_{in_1}-V_{in_2})_{max}$$

$$(V_{in_1}-V_{in_2})_{max} = \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow$$

$$\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 \leq 2I_{SS} \Rightarrow$$

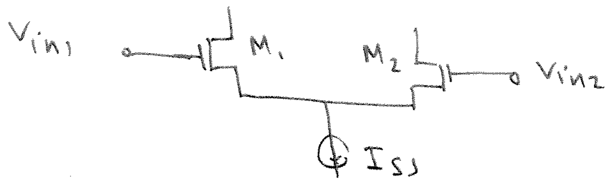
$$-\left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right] \geq 0 \Rightarrow$$

$$\frac{1}{4}\sqrt{4I_{SS}^2 - \left[\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})^2 - 2I_{SS}\right]} \geq \frac{1}{2}I_{SS}$$

$$\Rightarrow \quad I_{D_1} < 0$$

(45)

$$I_{D_1} - I_{D_2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in_1}-V_{in_2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in_1}-V_{in_2})^2}$$



The equilibrium overdrive voltage is:

$$(V_{GS_1}-V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} = V_{ov} \Rightarrow$$

$$\mu_n C_{ox}\frac{W}{L} = \frac{I_{SS}}{V_{ov}^2}$$ therefore

$$I_{D_1} - I_{D_2} = \frac{I_{SS}}{2}\frac{(V_{in_1}-V_{in_2})}{V_{ov}^2}\sqrt{\frac{4I_{SS}}{\frac{I_{SS}}{V_{ov}^2}} - (V_{in_1}-V_{in_2})^2} \Rightarrow$$

$$I_{D_1} - I_{D_2} = I_{SS} \Rightarrow$$

$$I_{\cancel{SS}} = \frac{I_{\cancel{SS}}}{2}\frac{(V_{in_1}-V_{in_2})}{V_{ov}^2}\sqrt{4V_{ov}^2 - (V_{in_1}-V_{in_2})^2}$$

$$\Rightarrow (V_{in_1}-V_{in_2})^4 - 4V_{ov}^2(V_{in_1}-V_{in_2})^2 + 4V_{ov}^4 = 0$$

$$\Rightarrow \left((V_{in_1}-V_{in_2})^2 - 2V_{ov}^2\right)^2 = 0 \Rightarrow$$

$$V_{in_1} - V_{in_2} = \sqrt{2}\,V_{ov} = \sqrt{2}\sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

(46)

$$I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

$$V_{ov} = (V_{GS} - V_{TH})_{equil} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow \mu_n C_{ox}\frac{W}{L} = \frac{I_{SS}}{V_{ov}^2}$$

$$\Rightarrow I_{D1} - I_{D2} = \frac{I_{SS}}{2}\,\frac{(V_{in1} - V_{in2})}{V_{ov}^2}\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}$$

$$\Rightarrow G_m = \frac{\partial(I_{D1} - I_{D2})}{\partial(V_{in1} - V_{in2})} =$$

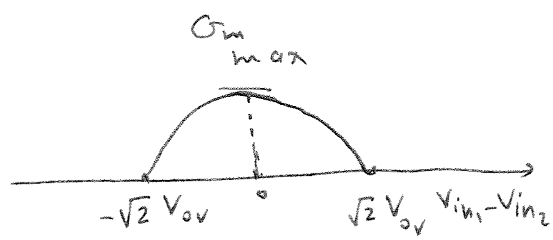
$$\frac{I_{SS}}{2V_{ov}^2}\left[\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2} - \frac{(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}}\right] =$$

$$\frac{I_{SS}}{2V_{ov}^2}\;\frac{4V_{ov}^2 - 2(V_{in1} - V_{in2})^2}{\sqrt{4V_{ov}^2 - (V_{in1} - V_{in2})^2}} =$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\;\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2(V_{in1} - V_{in2})^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2}}$$

$V_{in1} - V_{in2} = 0 \Rightarrow$

$$G_{m_{max}} = \sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}$$



(G_m plot)

47

From problem 46:

$$G_{m_{max}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = \sqrt{\frac{I_{SS}}{V_{ov}^2} I_{SS}} = \frac{I_{SS}}{V_{ov}}$$

$\Rightarrow$ if $G_m = \frac{1}{2} \frac{I_{SS}}{V_{ov}}$ we have

$$\frac{1}{2} \frac{I_{SS}}{V_{ov}} = \frac{I_{SS}}{2 V_{ov}^2} \frac{4V_{ov}^2 - 2(V_{in_1}-V_{in_2})^2}{\sqrt{4V_{ov}^2 - (V_{in_1}-V_{in_2})^2}} \Rightarrow$$

$$V_{ov} = \frac{4V_{ov}^2 - 2(V_{in_1}-V_{in_2})^2}{\sqrt{4V_{ov}^2 - (V_{in_1}-V_{in_2})^2}} \Rightarrow$$

$$\left(4V_{ov}^2 - (V_{in_1}-V_{in_2})^2\right) V_{ov}^2 = 16 V_{ov}^4 + 4(V_{in_1}-V_{in_2})^4 - 16 V_{ov}^2 (V_{in_1}-V_{in_2})^2$$

$$\Rightarrow 4(V_{in_1}-V_{in_2})^4 - 15 V_{ov}^2 (V_{in_1}-V_{in_2})^2 + 12 V_{ov}^4 = 0$$

$$\Rightarrow (V_{in_1}-V_{in_2})^2 = \frac{15 V_{ov}^2 \pm \sqrt{225 V_{ov}^4 - 192 V_{ov}^4}}{8} =$$

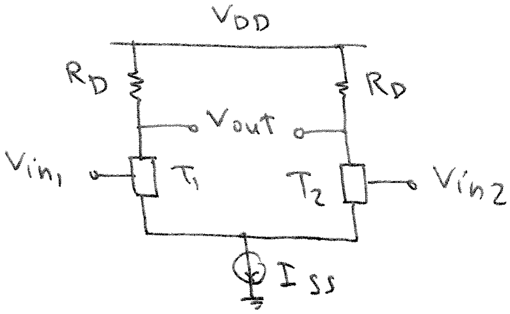
$$\frac{15 V_{ov}^2 \pm \sqrt{33}\, V_{ov}^2}{8}$$ positive sign is not accepted

because $(V_{in_1}-V_{in_2})^2 \le 2V_{ov}^2 \Rightarrow$

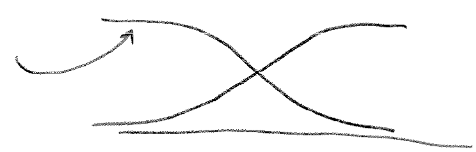
$$V_{in_1} - V_{in_2} = \pm \sqrt{\frac{15-\sqrt{33}}{8}}\, V_{ov} = \pm 1.0756\, V_{ov}$$

48-

$$I_D = \gamma (V_{GS} - V_{TH})^3$$



(a) The characteristic of $I_{D_1} - I_{D_2}$ v.s. $V_{in_1} - V_{in_2}$ is similar to the standard CMOS differential pair. because it has saturation part.



(b) $$I_D = \frac{I_{SS}}{2} = \gamma (V_{GS} - V_{TH})^3 \Rightarrow$$

$$(V_{GS} - V_{TH})_{equil} = \sqrt[3]{\frac{I_{SS}}{2\gamma}}$$

(c) $$I_{D_1} = I_{SS} = \gamma (V_{GS_1} - V_{TH})^3 \Rightarrow V_{GS_1} - V_{TH} = \sqrt[3]{\frac{I_{SS}}{\gamma}}$$

$$I_{D2} = 0 = \gamma (V_{GS2} - V_{TH})^3 \Rightarrow V_{GS2} - V_{TH} = 0$$

$$\Rightarrow V_{GS_1} - V_{GS_2} = V_{in_1} - V_{in_2} = \sqrt[3]{\frac{I_{SS}}{\gamma}} =$$

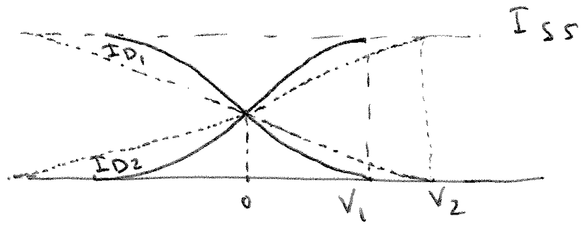
$$\sqrt[3]{2}\,(V_{GS} - V_{TH})_{equil}$$

(49)

(a)

gate oxide thickness is doubled $\Rightarrow$ Cox is halved $\Rightarrow$

$(V_{in_1} - V_{in_2})_{max}$ scales up by $\sqrt{2}$.



$V_2 = \sqrt{2}\, V_1$

so all the curves stretch out to the sides by $\sqrt{2}$ times.

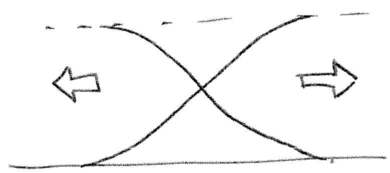
(b) if threshold voltage is halved, nothing will change in the curves. The reason is that the curves depend on $V_{in_1} - V_{in_2}$.

(C) In this case, $(V_{in_1} - V_{in_2})_{max}$ does not change so, all the curves scale half downward because I_{SS} is halved.

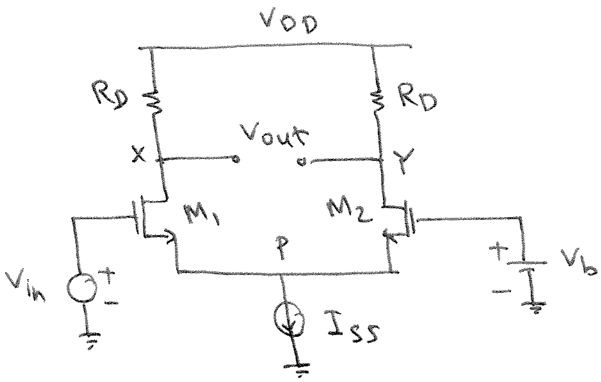
(50)

if mobility falls then $(V_{in1} - V_{in2})_{max}$ will increase because $(V_{in1} - V_{in2})_{max} = \sqrt{\dfrac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$

So the curves stretch out to the sides.

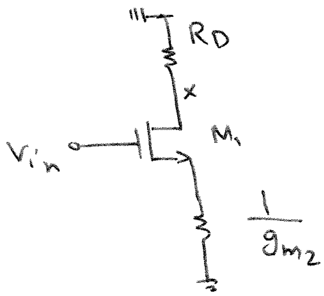


(51)



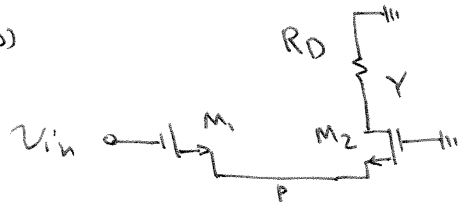
$g_{m_1} = g_{m_2} = g_m$

(a)



$$v_X = -g_{m_1} v_{gs_1} R_D = -g_{m_1} \frac{\frac{1}{g_{m_1}}}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}} v_{in} R_D = -\frac{g_{m_1} g_{m_2}}{g_{m_1} + g_{m_2}} R_D v_{in} = -\frac{g_m}{2} R_D v_{in}$$

(b)



$$v_P = \frac{\frac{1}{g_{m_2}}}{\frac{1}{g_{m_1}} + \frac{1}{g_{m_2}}} v_{in}$$

$$\Rightarrow v_P = \frac{g_{m_1}}{g_{m_1} + g_{m_2}} v_{in} \Rightarrow$$

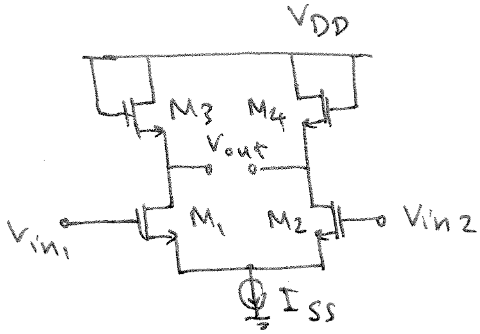
$$v_Y = -g_{m_2} v_{gs_2} R_D = g_{m_2} v_P R_D = \frac{g_{m_1} g_{m_2}}{g_{m_1} + g_{m_2}} R_D v_{in}$$

$$\rightarrow v_Y = \frac{g_m}{2} R_D v_{in}$$

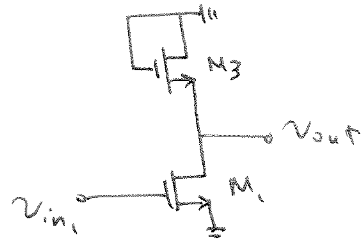
(c) $\frac{v_X - v_Y}{v_{in}} = -g_m R_D$ This value is equal to the gain of the differential amplifier.

(52)

(a)

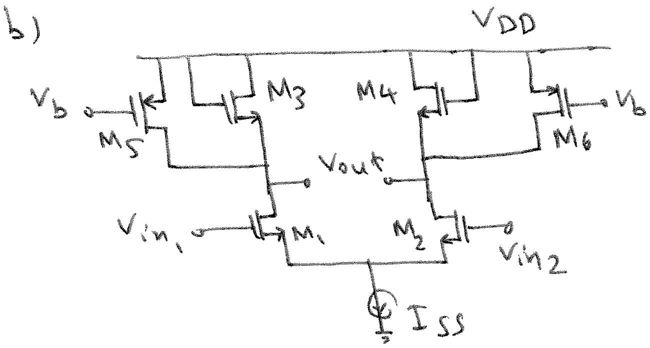


half $\Longrightarrow$ circuit

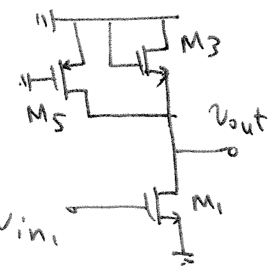


$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_3} \| \frac{1}{g_{m_3}}\right)$$

(b)

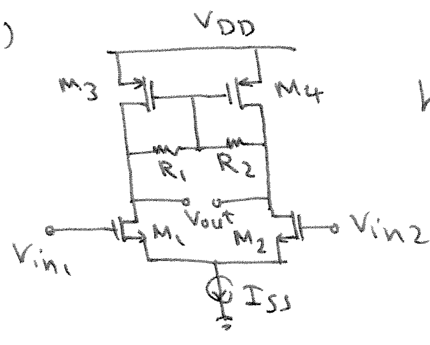


half $\Longrightarrow$ circuit

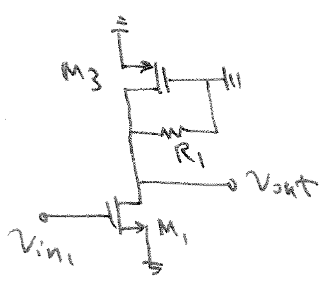


$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_5} \| \frac{1}{g_{m_3}} \| r_{o_3}\right)$$

(C)



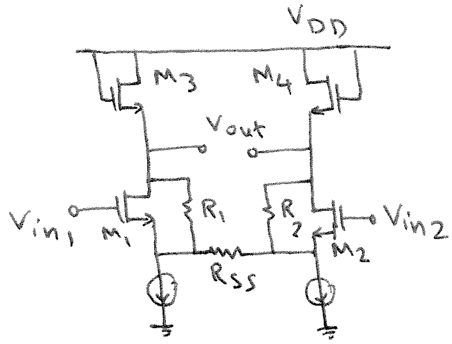
half $\Longrightarrow$ circuit



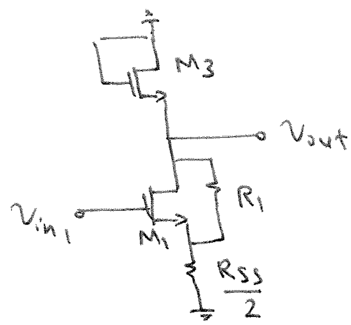
$$A_v = -g_{m_1}\left(r_{o_1} \| r_{o_3} \| R_1\right)$$

(53)

(a)

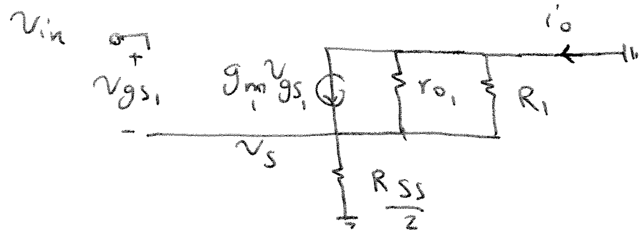


half $\Longrightarrow$ Circuit



$$R_{out} = \left(r_{o3} \,\|\, \frac{1}{g_{m3}}\right) \Big\| \left(g_{m1} (R_1 \| r_{o1}) \frac{R_{SS}}{2} + \frac{R_{SS}}{2} + R_1 \| r_{o1} \right)$$

To calculate G_m:



$v_{gs1} = v_{in} - v_s$

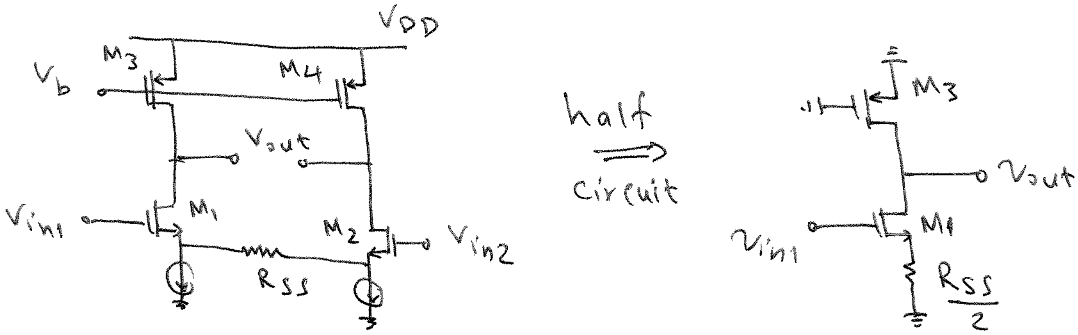
$$\frac{v_s}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}} + g_{m1} v_s = g_{m1} v_{in} \Longrightarrow v_s = \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}}$$

$$i_o = + \frac{v_s}{\frac{R_{SS}}{2}} = + \frac{1}{\frac{R_{SS}}{2}} \frac{g_{m1} v_{in}}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}} \Longrightarrow$$

$$G_m = \frac{i_o}{v_{in}} = + \frac{2 g_{m1}}{R_{SS}} \frac{1}{g_{m1} + \frac{1}{\frac{R_{SS}}{2} \| R_1 \| r_{o1}}}$$

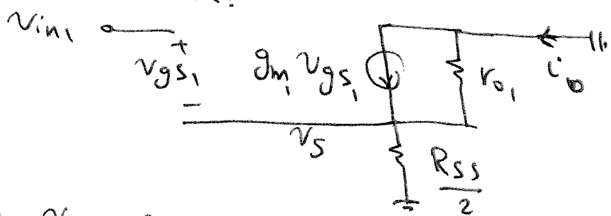
$A_v = -G_m R_{out}$

53 (b)



$$R_{out} = r_{o3} \,||\, \left(g_{m_1} r_{o_1} \frac{R_{SS}}{2} + r_{o_1} + \frac{R_{SS}}{2} \right)$$

To calculate G_m:

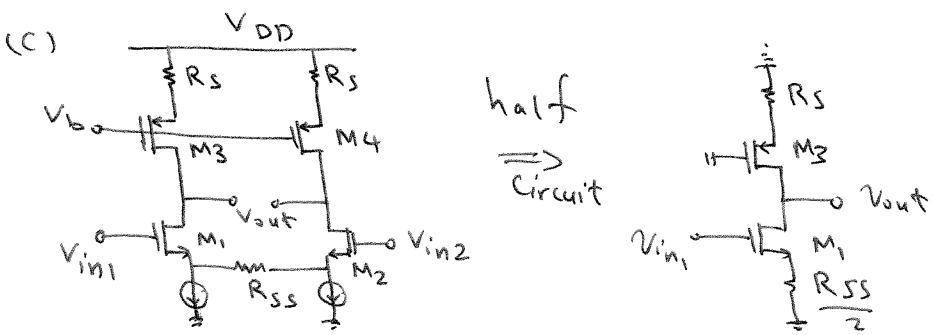


$$\frac{v_S}{r_{o_1} \,||\, \frac{R_{SS}}{2}} + g_{m_1} v_S = g_{m_1} v_{in} \implies v_S = \frac{g_{m_1} v_{in}}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \,||\, r_{o_1}}}$$

$$G_m = \frac{i_o}{v_{in}} = + \frac{v_S}{\frac{R_{SS}}{2}} \frac{1}{v_{in}} = + \frac{2 g_{m_1}}{R_{SS}} \frac{1}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \,||\, r_{o_1}}}$$

$\rightarrow A_v = -G_m R_{out}$

(c)

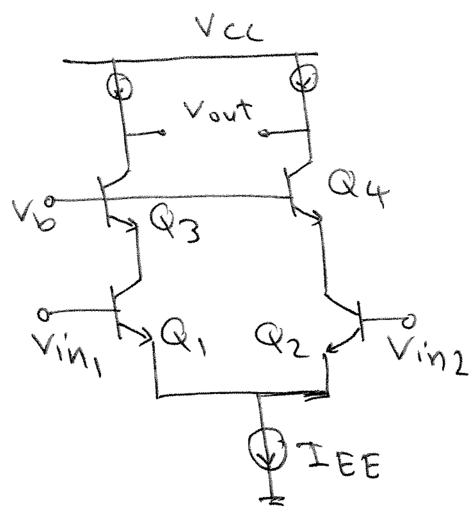


$$R_{out} = (g_{m_3} r_{o_3} R_S + r_{o_3} + R_S) \,||\, \left(g_{m_1} r_{o_1} \frac{R_{SS}}{2} + r_{o_1} + \frac{R_{SS}}{2} \right)$$

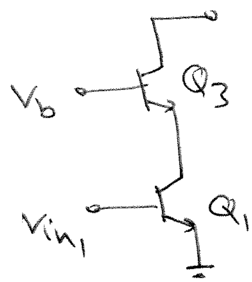
G_m for this circuit is equal to the one for part (b) so:

$$G_m = + \frac{2 g_{m_1}}{R_{SS}} \frac{1}{g_{m_1} + \frac{1}{\frac{R_{SS}}{2} \,||\, r_{o_1}}} \implies A_v = -G_m R_{out}$$

(54)



half $\Longrightarrow$ Circuit



$A_v = 4000$

$\beta = 100$

$$A_v = -g_{m_1}\left[g_{m_3}(r_{o_1} \| r_{\pi_3}) r_{o_3} + r_{o_3} + r_{o_1} \| r_{\pi_3}\right]$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} \qquad r_{o_{1-4}} = \frac{2V_A}{I_{EE}} \qquad r_{\pi_3} = \frac{2V_T\beta}{I_{EE}}$$

$$4000 = \frac{I_{EE}}{2V_T}\left[\frac{I_{EE}}{2V_T}\left(\frac{2V_A}{I_{EE}} \| \frac{2V_T\beta}{I_{EE}}\right)\frac{2V_A}{I_{EE}} + \frac{2V_A}{I_{EE}} + \left(\frac{2V_A}{I_{EE}} \| \frac{2V_T\beta}{I_{EE}}\right)\right]$$

$$\Rightarrow 4000 = \frac{1}{2V_T}\left[\frac{V_A}{V_T}(2V_A \| 2V_T\beta) + 2V_A + (2V_A \| 2V_T\beta)\right] \Rightarrow$$

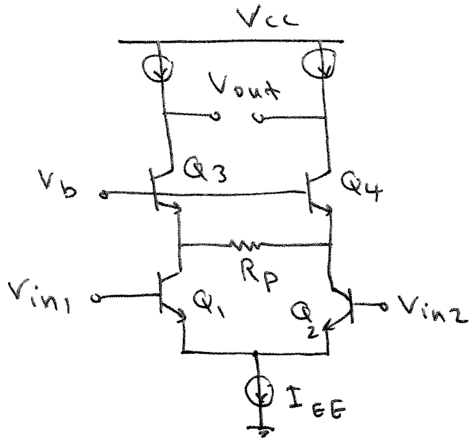
$$4000 = \frac{1}{V_T}\left[\frac{V_A}{V_T}(V_A \| \beta V_T) + V_A + (V_A \| \beta V_T)\right] \Rightarrow$$

$$4000 = \frac{1}{V_T}\left[\frac{\beta V_A^2}{\beta V_T + V_A} + V_A + \frac{\beta V_A V_T}{\beta V_T + V_A}\right] \Rightarrow$$

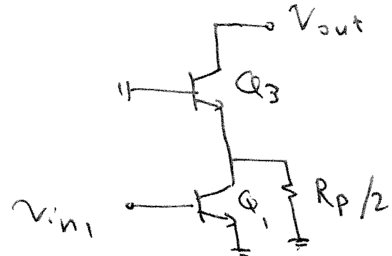
$$4000 = \frac{1}{0.026}\left[\frac{100 V_A^2}{2.6 + V_A} + V_A + \frac{2.6 V_A}{2.6 + V_A}\right] \Longrightarrow$$

$$V_A = 2.197$$

(55)



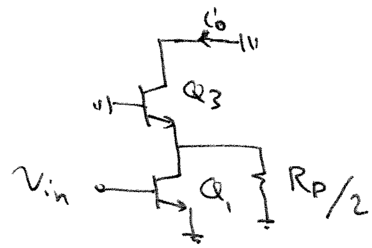
half $\Longrightarrow$ circuit



$$R_{out} = g_{m_3} r_{o3} \left(\frac{R_p}{2} \| r_{o1} \| r_{\pi 3}\right) + r_{o3} + \frac{R_p}{2} \| r_{o1} \| r_{\pi 3}$$

To calculate G_m

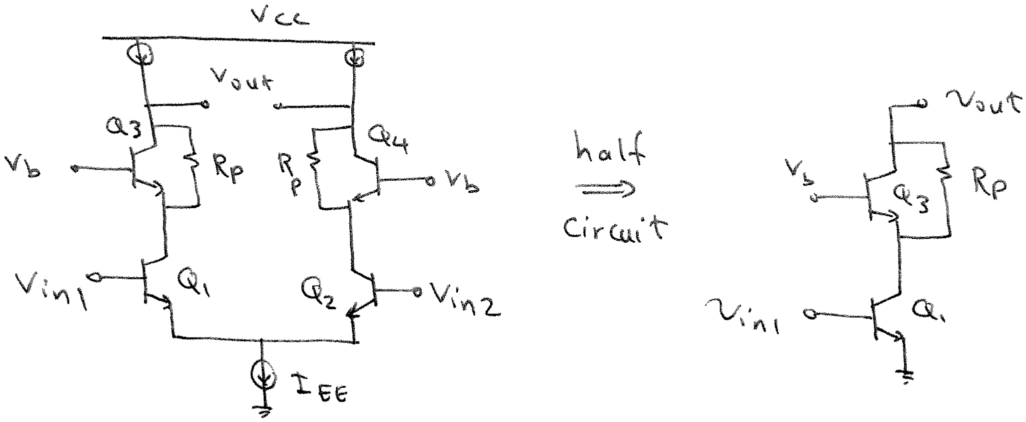
$$G_m = \frac{i_o'}{v_{in}}$$



$$\frac{i_o'}{v_{in}} = +g_{m_1} \frac{g_{m_3}\left(\frac{R_p}{2} \| r_{o1}\right)}{g_{m_3}\left(\frac{R_p}{2} \| r_{o1}\right) + 1}$$

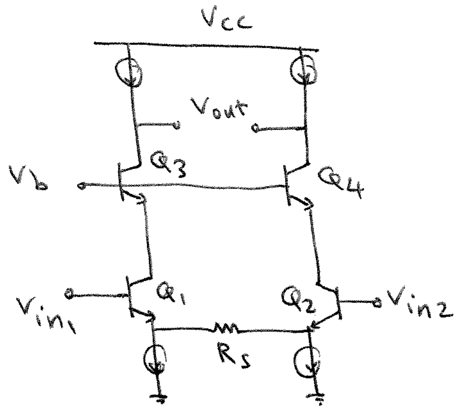
$$A_v = -G_m R_{out}$$

(56)

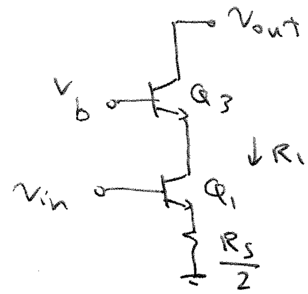


$$A_v = -g_{m_1}\left(g_{m_3}(r_{o_3} \| R_P)(r_{o_1} \| r_{\pi_3}) + (r_{o_3} \| R_P) + (r_{o_1} \| r_{\pi_3}) \right)$$

(57)



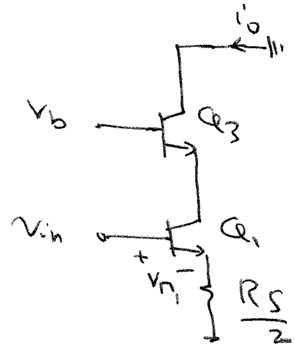
half ⟹ circuit



$$R_1 = g_{m_1} r_{o_1} \left(\frac{R_S}{2} \| r_{\pi_1}\right) + r_{o_1} + \frac{R_S}{2} \| r_{\pi_1}$$

$$R_{out} = g_{m_3} r_{o_3} (R_1 \| r_{\pi_3}) + r_{o_3} + (R_1 \| r_{\pi_3})$$

To calculate G_m:



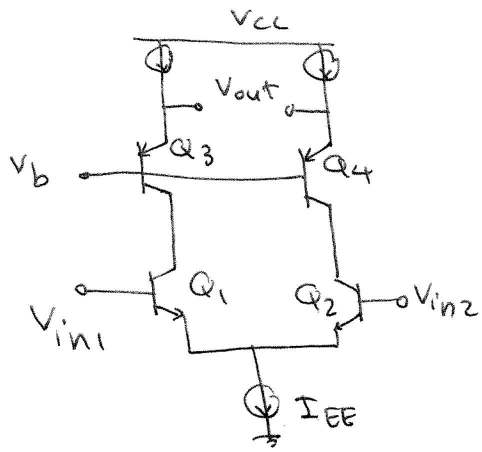
$$v_{\pi_1} \approx \frac{\frac{1}{g_{m_1}}}{\frac{1}{g_{m_1}} + \frac{R_S}{2}} v_{in}$$

$$= \frac{1}{1 + g_{m_1}\frac{R_S}{2}} v_{in}$$

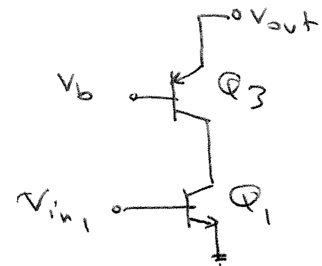
$$G_m = \frac{i_o}{v_{in}} = \frac{+g_{m_1} v_{\pi}}{v_{in}} = \frac{+g_{m_1}}{1 + g_{m_1}\frac{R_S}{2}}$$

$$A_v = -G_m R_{out}$$

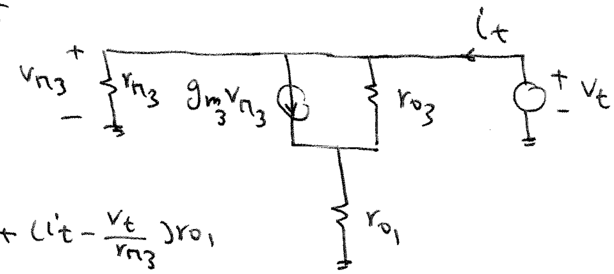
(58)



half $\Longrightarrow$ Circuit



To calculate Rout



$$v_{\pi 3} = v_t$$

$$v_t = \left(i_t - g_{m_3} v_t - \frac{v_t}{r_{\pi 3}}\right) r_{o3} + \left(i_t - \frac{v_t}{r_{\pi 3}}\right) r_{o1}$$

$$\rightarrow \quad v_t\left(1 + g_{m_3} r_{o3} + \frac{r_{o3}}{r_{\pi 3}} + \frac{r_{o1}}{r_{\pi 3}}\right) = i_t\left(r_{o1} + r_{o3}\right)$$

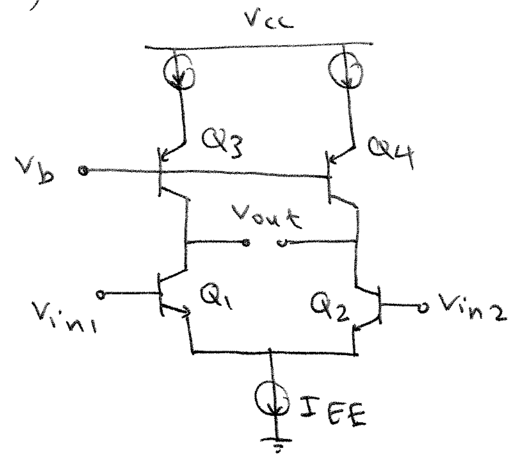
$$\rightarrow \frac{v_t}{i_t} = \frac{r_{o1} + r_{o3}}{1 + g_{m_3} r_{o3} + \frac{g_{m3} r_{o3}}{\beta_3} + \frac{g_{m_3} r_{o1}}{\beta_3}} = R_{out} \rightarrow$$

$$R_{out} \approx \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

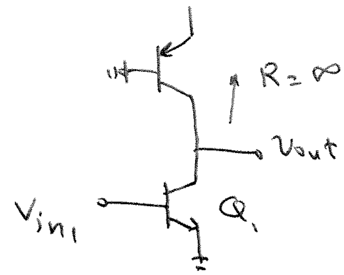
$$G_m = +g_{m_1} \quad \Rightarrow$$

$$A_v = -G_m R_{out} = -g_{m_1} \frac{r_{o1} + r_{o3}}{g_{m3} r_{o3}}$$

(59)

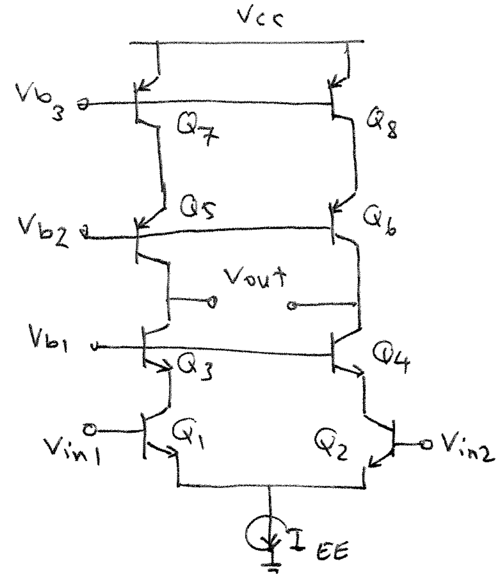


half ⟹ circuit

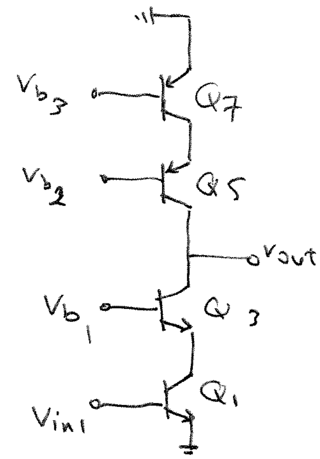


$$A_v = -g_{m1} r_{o1}$$

60



half circuit $\Longrightarrow$



$A_v = 800$, $\beta_n = 2\beta_p = 100$, $V_{A,n} = 2V_{A,p}$

$$A_v \approx -g_{m_1}\left[g_{m_3} r_{o_3}(r_{o_1} \| r_{\pi_3})\right] \| \left[g_{m_5} r_{o_5}(r_{o_7} \| r_{\pi_5})\right] =$$

$$-\frac{I_{EE}}{2V_T}\left[\frac{I_{EE}}{2V_T}\frac{V_{A,n}}{\frac{I_{EE}}{2}}\left(\frac{V_{A,n}}{\frac{I_{EE}}{2}} \| \frac{\beta_n V_T}{\frac{I_{EE}}{2}}\right)\right] \| \left[\frac{I_{EE}}{2V_T}\frac{V_{A,p}}{\frac{I_{EE}}{2}}\left(\frac{V_{A,p}}{\frac{I_{EE}}{2}} \| \frac{\beta_p V_T}{\frac{I_{EE}}{2}}\right)\right]$$

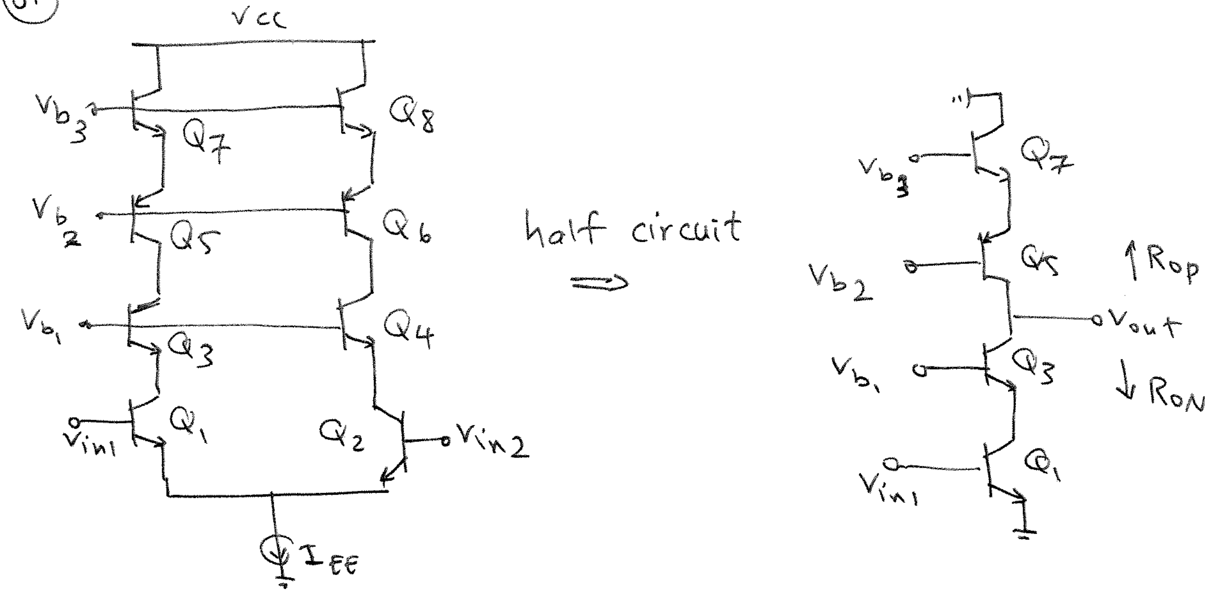
$$= -\frac{1}{V_T^2}\left[V_{A,n}(V_{A,n} \| \beta_n V_T)\right] \| \left[V_{A,p}(V_{A,p} \| \beta_p V_T)\right] =$$

$$-\frac{1}{V_T^2}\left[V_{A,n}(V_{A,n} \| \beta_n V_T)\right] \| \left[\frac{V_{A,n}}{2}\left(\frac{V_{A,n}}{2} \| \frac{\beta_n}{2} V_T\right)\right] \Rightarrow$$

$$800 = \frac{1}{5V_T^2}\left(V_{A,n}(V_{A,n} \| 2.6)\right) \Rightarrow V_{A,n} = 2.245 \text{ V}$$

$$\Rightarrow V_{A,p} = 1.225 \text{ V}$$

(65)



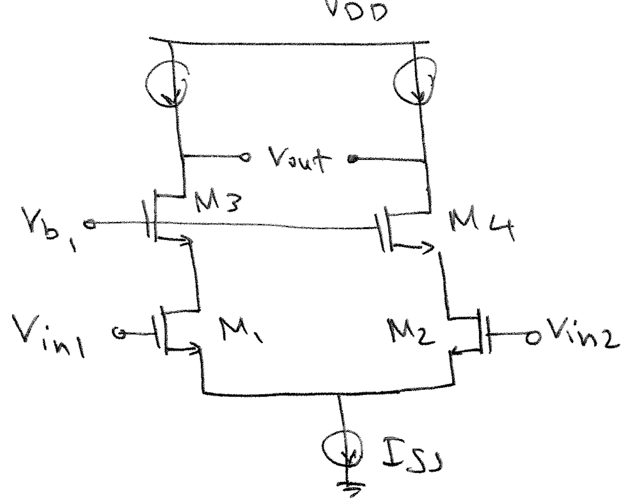
$$R_{oN} = (r_{o1} \| r_{\pi 3}) + r_{o3} + g_{m3} r_{o3} (r_{o1} \| r_{\pi 3})$$

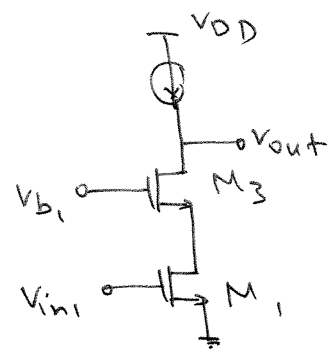
$$R_{op} = \frac{1}{g_{m7}} \| r_{o7} \| r_{\pi 5} + r_{o5} + g_{m5} r_{o5} \left(\frac{1}{g_{m7}} \| r_{o7} \| r_{\pi 5}\right) \approx$$

$$\frac{1}{g_{m7}} \| r_{\pi 5} + r_{o5} + g_{m5} r_{o5} \left(\frac{1}{g_{m7}} \| r_{\pi 5}\right)$$

$$A_v = -g_{m1} (R_{oN} \| R_{op})$$

(62)





half circuit

$A_v = 300$, $W/L = \frac{20}{0.18}$, $\mu_n C_{ox} = 100\ \mu A/V^2$

$\lambda = 0.1\ V^{-1}$

$$A_v \approx -g_{m_3} r_{o3}\, g_{m_1} r_{o1}$$

$$g_{m_1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \frac{I_{SS}}{2}} \qquad g_{m_3} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_3 \frac{I_{SS}}{2}}$$

$$\rightarrow g_{m_1} = g_{m_3} = \sqrt{10^{-4}\ \frac{20}{0.18}\ I_{SS}}$$

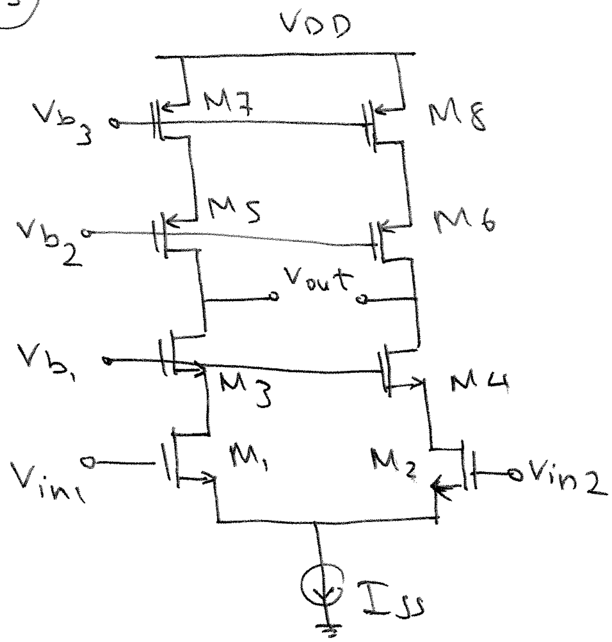
$$r_{o1} = \frac{1}{\lambda \frac{I_{SS}}{2}} \ , \quad r_{o3} = \frac{1}{\lambda \frac{I_{SS}}{2}} \rightarrow r_{o1} = r_{o3} = \frac{20}{I_{SS}}$$

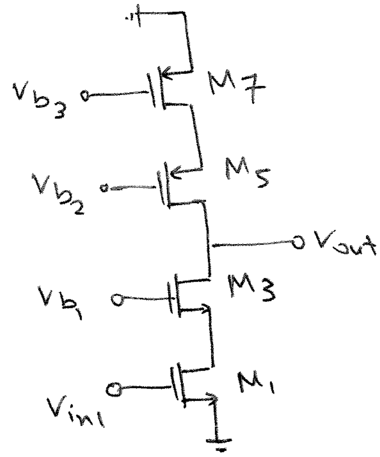
So:

$$300 = \left(10^{-4}\ \frac{20}{0.18}\ I_{SS}\right) \frac{\frac{400}{2}}{I_{SS}} \Rightarrow$$

$I_{SS} = 14.815\ mA$

(63)





$A_v = 200$, $I_{SS} = 1\text{mA}$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$\mu_p C_{ox} = 50\,\mu A/V^2$, $\lambda_n = 0.1\,V^{-1}$, $\lambda_p = 0.2\,V^{-1}$

$(\frac{W}{L})_1 = \cdots = (W/L)_8 = ?$

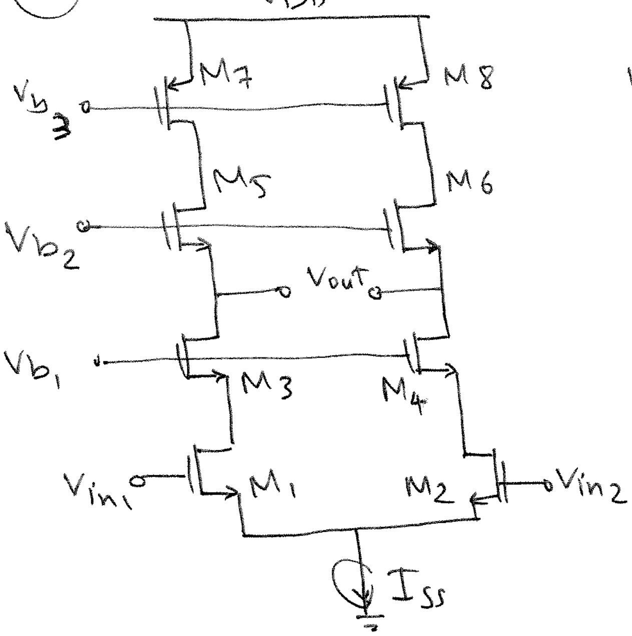
$$A_v \approx -g_{m1}\left[(g_{m3} r_{o3} r_{o1}) \| (g_{m5} r_{o5} r_{o7})\right] \Rightarrow$$

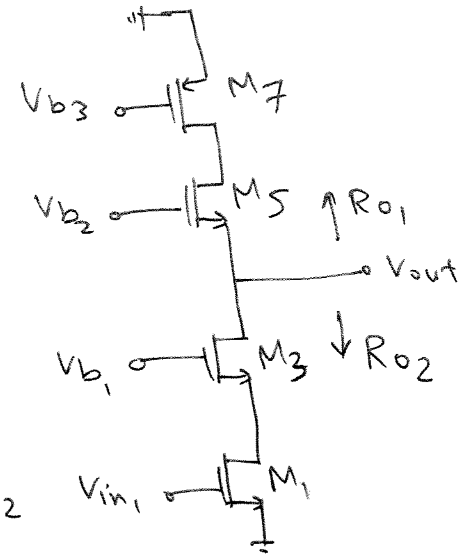
$$200 = \sqrt{\mu_n C_{ox} (\tfrac{W}{L})_1 I_{SS}} \left[\left(\sqrt{\mu_n C_{ox} (\tfrac{W}{L})_3 I_{SS}} \left(\frac{2}{\lambda_n I_{SS}}\right)^2\right) \| \left(\sqrt{\mu_p C_{ox} (\tfrac{W}{L})_5 I_{SS}} \left(\frac{2}{\lambda_p I_{SS}}\right)^2\right)\right]$$

$$\Rightarrow 200 = \sqrt{10^{-4} (\tfrac{W}{L})_1 10^{-3}} \left[\left(\sqrt{10^{-4} (\tfrac{W}{L}) 10^{-3}} \left(\frac{20}{10^{-3}}\right)^2\right) \| \left(\sqrt{0.5 \times 10^{-4} (\tfrac{W}{L}) 10^{-3}} \left(\frac{10}{10^{-3}}\right)^2\right)\right]$$

$$\Rightarrow \frac{W}{L} = 33.28$$

64

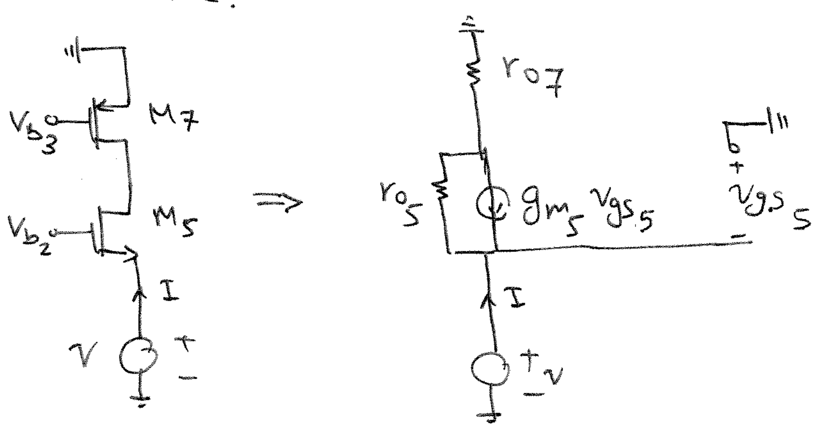




half circuit

$$R_{o2} = g_{m3} r_{o3} r_{o1} + r_{o1} + r_{o3}$$

To calculate R_{o1}, using the small signal model we have:

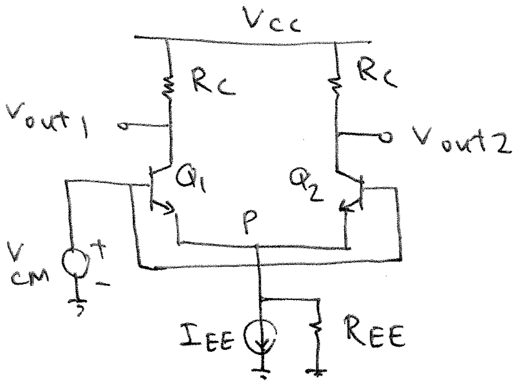


$$v_{gs_5} = -v \rightarrow g_{m_5} v_{gs_5} = -g_{m_5} v$$

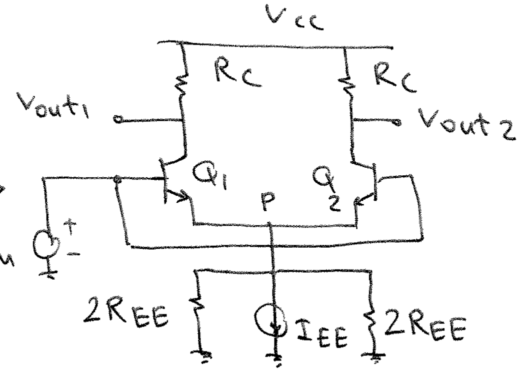
From KVL: $v = r_{o5}(I - g_{m_5} v) + r_{o7} I$

$$\rightarrow \frac{v}{I} = R_{o1} = \frac{r_{o5} + r_{o7}}{1 + g_{m_5} r_{o5}} \Rightarrow A_v = -g_{m_1}(R_{o1} \| R_{o2})$$

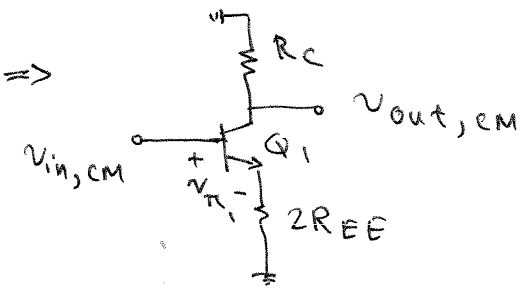
(65)



⇒



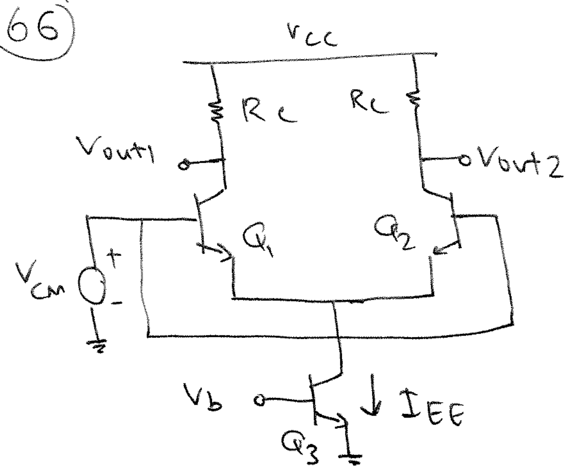
⇒



$$v_{out,cm} = -g_{m1} v_{\pi_1} R_C = -g_{m1} R_C \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + 2R_{EE}} v_{in,cm}$$

$$\Rightarrow \frac{v_{out,cM}}{v_{in,cM}} = -\frac{g_{m1} R_C}{1 + 2R_{EE} g_{m1}} = -\frac{\frac{R_C}{2}}{R_{EE} + \frac{1}{2g_{m1}}}$$

(66)

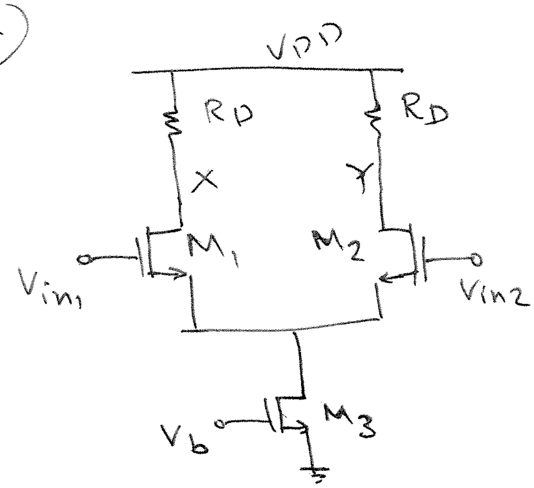


$$A_{CM} = \frac{\Delta V_{out,CM}}{\Delta V_{in,CM}} = -\frac{R_C/2}{\frac{1}{2g_m} + r_{o3}} \Rightarrow$$

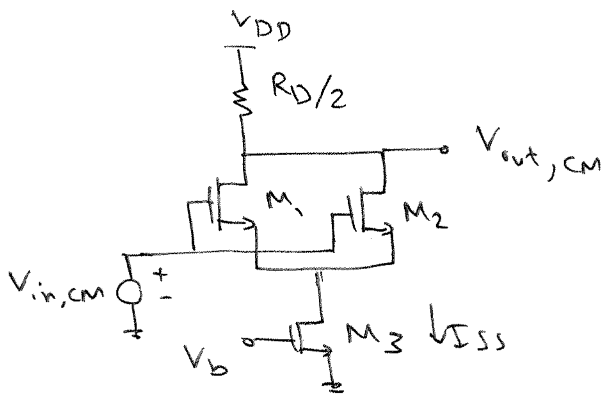
$$A_{CM} \leq 0.01 \Rightarrow \frac{R_C/2}{\frac{1}{2\frac{I_{EE}}{2V_T}} + \frac{V_A}{I_{EE}}} < 0.01 \Rightarrow$$

$$\frac{R_C I_{EE}}{2(V_A + V_T)} < 0.01 \Rightarrow R_C I_{EE} < 0.02(V_A + V_T)$$

(67)



The same value for the inputs common-mode leads to the following circuit:



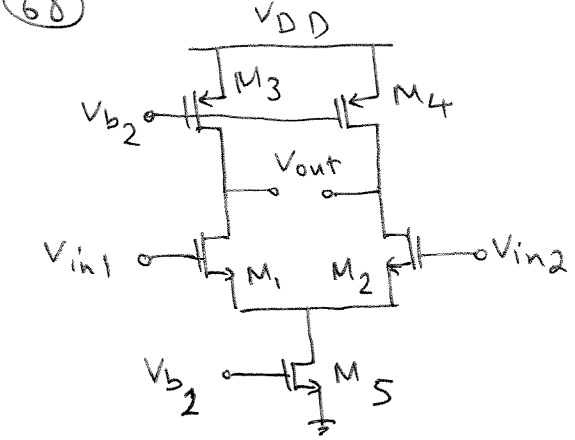
$$g_{m_1} = g_{m_2} = \frac{2\, I_{SS}/2}{(V_{GS}-V_{TH})_{eq}} \Rightarrow = \frac{I_{SS}}{(V_{GS}-V_{TH})_{eq}}$$

$$\frac{\Delta V_{out,\,CM}}{\Delta V_{in,\,CM}} = -\frac{R_D/2}{\dfrac{1}{2g_{m_1}} + r_{o3}}$$

$$= \frac{-R_D}{\dfrac{1}{g_{m_1}} + 2r_{o3}} = \frac{-R_D}{\dfrac{(V_{GS}-V_{TH})_{eq}}{I_{SS}} + \dfrac{2}{\lambda I_{SS}}} \Rightarrow$$

$$A_{CM} = -\frac{R_D\, I_{SS}}{\dfrac{2}{\lambda} + (V_{GS}-V_{TH})_{eq}}$$

68



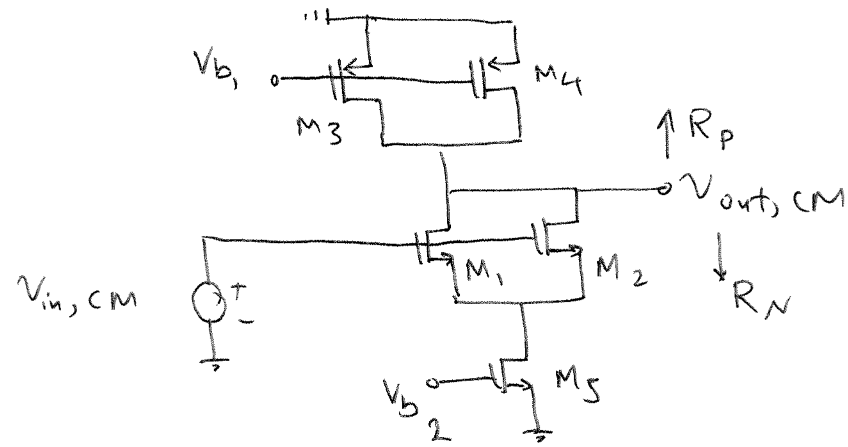
$\lambda > 0$, $g_m r_o \gg 1$

$r_{o3} = r_{o4}$

$r_{o1} = r_{o2}$

$g_{m1} = g_{m2}$

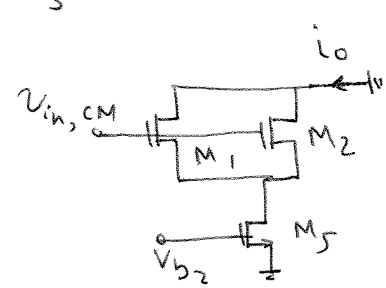
For the common mode input we have:



$$R_P = r_{o3} \| r_{o4} = \frac{r_{o3}}{2} = \frac{r_{o4}}{2}$$

$$R_N = r_{o5} + \frac{r_{o1}}{2} + 2g_{m1}\frac{r_{o1}}{2}r_{o5} =$$

$$g_{m1}r_{o1}r_{o5} + r_{o5} + \frac{r_{o1}}{2}$$



$$\frac{i_o}{v_{in,CM}} = G_m = \frac{2g_{m1}v_{gs_1}}{v_{in,CM}} = 2g_{m1}\frac{\frac{1}{2g_{m1}}}{\frac{1}{2g_{m1}} + r_{o5}}$$

$$\rightarrow G_m = \frac{2g_{m1}}{1 + 2g_{m1}r_{o5}} \approx \frac{1}{r_{o5}}$$

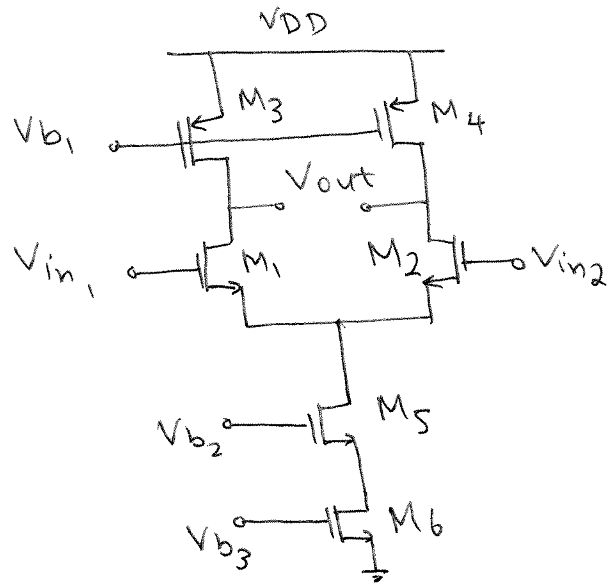
$$\Rightarrow A_{CM} = -G_m R_o$$

$$R_o = R_P \| R_N = \frac{r_{o4}}{2} \| \left(g_{m_1} r_{o_1} r_{o_5} + r_{o_5} + \frac{r_{o_1}}{2} \right)$$

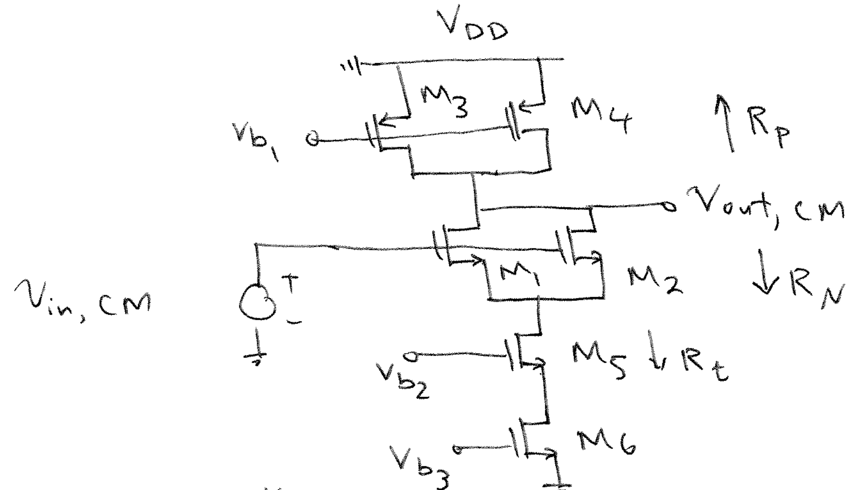
$$\approx \frac{r_{o4}}{2} \| g_{m_1} r_{o_1} r_{o_5} \approx \frac{r_{o4}}{2}$$

$$\rightarrow A_{CM} = -\frac{1}{r_{o_5}} \frac{r_{o4}}{2} = -\frac{r_{o4}}{2 r_{o_5}}$$

(69) (a)



For the common mode input:



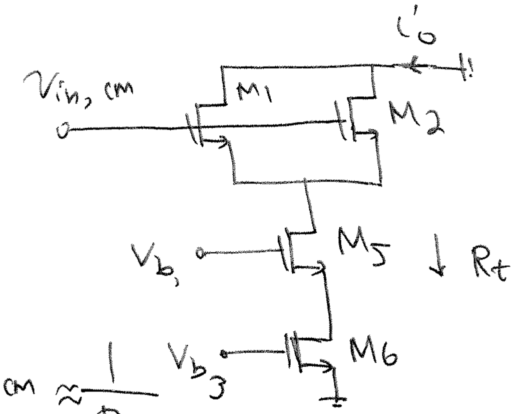
$$R_P = r_{o3} \| r_{o4} = \frac{r_{o3}}{2}$$

$$R_N = \frac{r_{o1}}{2} + R_t + 2g_{m1}\frac{r_{o1}}{2}R_t \simeq g_{m1}r_{o1}R_t \approx$$

$$g_{m1}r_{o1}\,g_{m5}r_{o5}\,r_{o6}$$

$$R_{out} = R_P \| R_N = \frac{r_{o3}}{2} \| g_{m1}g_{m5}r_{o1}r_{o5}r_{o6} \approx \frac{r_{o3}}{2}$$

To calculate G_m:



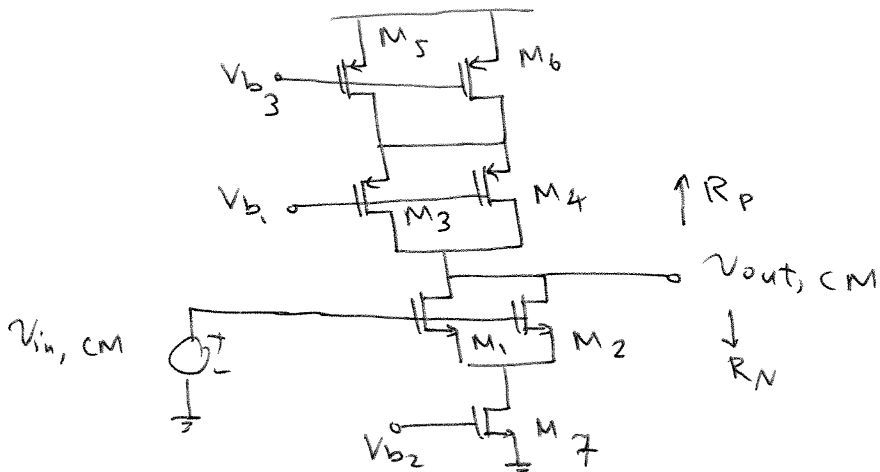
$$G_m = \frac{i_o'}{v_{in,CM}} = \frac{2g_{m_1} v_{gs_1}}{v_{in,CM}}$$

$$= \frac{2g_{m_1}}{v_{in,cm}} \frac{\frac{1}{2g_{m_1}}}{\frac{1}{2g_{m_1}} + R_t} v_{in,cm} \approx \frac{1}{R_t}$$

$$\rightarrow A_{CM} = -G_m R_{out} = -\frac{r_{o3}}{2R_t} = -\frac{r_{o3}}{2g_{m5} r_{o5} r_{o6}}$$

(69) (b)

For the common mode input, the circuit is:

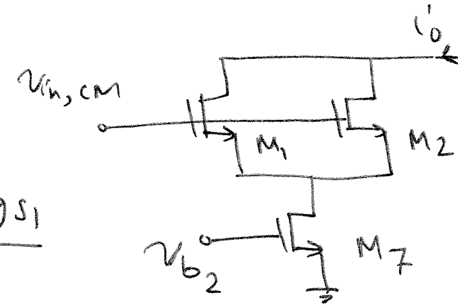


$$R_P = 2g_{m3}\frac{r_{o3}}{2}\frac{r_{o5}}{2} + \frac{r_{o3}}{2} + \frac{r_{o5}}{2} \approx \frac{g_{m3} r_{o3} r_{o5}}{2}$$

$$R_N = 2g_{m1}\frac{r_{o1}}{2} r_{o7} + \frac{r_{o1}}{2} + r_{o7} \approx g_{m1} r_{o1} r_{o7}$$

$$R_{out} = R_N \| R_P$$

To calculate G_m:



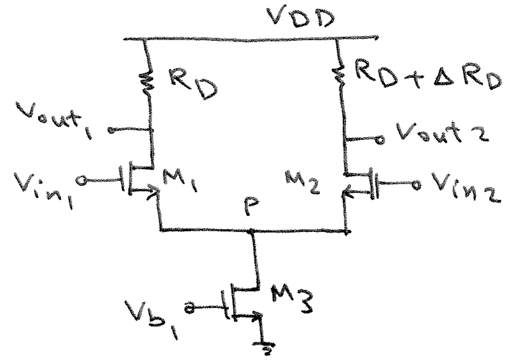
$$G_m = \frac{i_o'}{v_{in,CM}} = \frac{2g_{m1} v_{gs1}}{v_{in,CM}}$$

$$= \frac{2g_{m1}}{v_{in,CM}} \frac{\frac{1}{2g_{m1}}}{\frac{1}{2g_{m1}} + r_{o7}} v_{in,CM} \approx \frac{1}{r_{o7}}$$

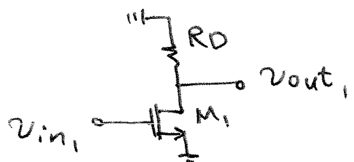
$\rightarrow A_{CM} = -G_m R_{out}$

(70)

(a)

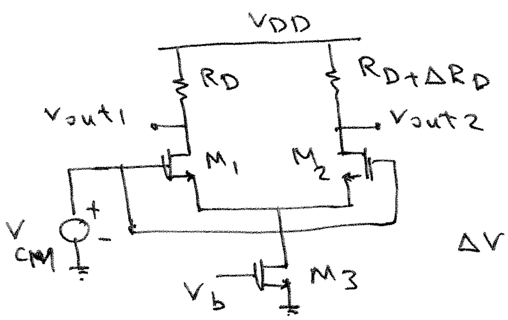


To calculate A_{DM}, using the half circuit:



$$A_{DM} = \frac{v_{out}}{v_{in_1}} = -g_{m_1} R_D$$

To calculate, A_{CM-DM} we have:



$$A_{CM-DM} = \frac{\Delta V_{out}}{\Delta V_{CM}}$$

$$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D r_{o3} \Rightarrow$$

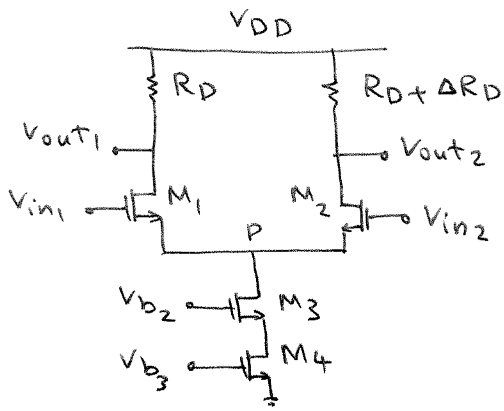
$$\Delta V_{CM} = \Delta I_D \left(\frac{1}{g_{m_1}} + 2r_{o3}\right)$$

$$\Delta V_{out} = \Delta V_{out_1} - \Delta V_{out2} = -\Delta R_D \, \Delta I_D \Rightarrow$$

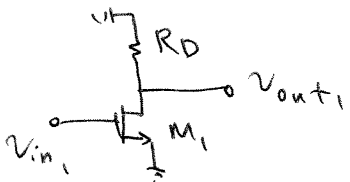
$$A_{CM-DM} = -\frac{\Delta R_D}{\frac{1}{g_{m_1}} + 2r_{o3}} \Rightarrow$$

$$CMRR = \frac{A_{DM}}{A_{CM-DM}} = \frac{g_{m_1} R_D}{\frac{\Delta R_D}{\frac{1}{g_{m_1}} + 2r_{o3}}} = (1 + 2g_{m_1} r_{o3}) \frac{R_D}{\Delta R_D}$$

(10) (b)



To calculate A_{DM}, using the half circuit, we have



$$A_{DM} = \frac{v_{out1}}{v_{in1}} = -g_{m1} R_D$$

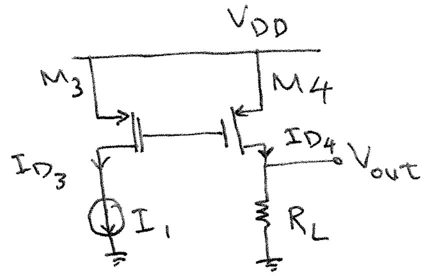
Similar to part (a) we have:

$$A_{CM-DM} = -\frac{\Delta R_D}{\frac{1}{g_{m1}} + 2\left[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}\right]}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM-DM}} = \left(1 + 2g_{m1}\left[g_{m3} r_{o3} r_{o4} + r_{o3} + r_{o4}\right]\right)\frac{R_D}{\Delta R_D}$$

Notice that CMMR of part (b) is much higher than the one for part (a).

71



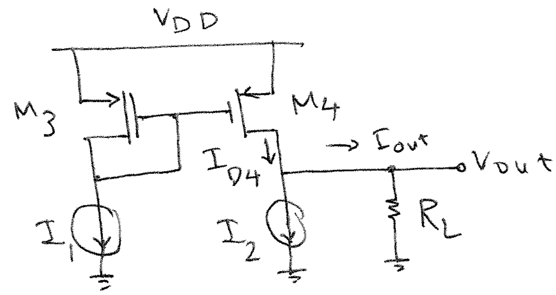
$$\left(\frac{W}{L}\right)_3 = N\left(\frac{W}{L}\right)_4$$

$$\left(\frac{W}{L}\right)_3 = N\left(\frac{W}{L}\right)_4 \Rightarrow I_{D3} = N I_{D4} \Rightarrow \underbrace{i_{d3} = N i_{d4}}_{\text{small signal}}$$

$$\begin{cases} i_{d3} = i_1 \\ v_{out} = R_L\, i_{d4} = \dfrac{R_L}{N} i_{d3} = \dfrac{R_L}{N} i_1 \Rightarrow \end{cases}$$

$$\frac{v_{out}}{i_1} = \frac{R_L}{N}$$

(72)



(a) $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4$

if $I_1 = I_2 = I_0$ $\Rightarrow \begin{cases} V_{out} = I_{out} \times R_L = 0 \quad \text{because} \\ I_{out} = I_{D_4} - I_2 = I_1 - I_2 = 0 \end{cases}$

if $I_1 = I_0 + \Delta I \Rightarrow I_{D4} = I_{D3} = I_1 = I_0 + \Delta I$

$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = 2\Delta I$

$V_{out} = I_{out} R_L = 2 R_L \Delta I$

(b) $\left(\frac{W}{L}\right)_3 = 2\left(\frac{W}{L}\right)_4$

$\Rightarrow I_{D3} = 2 I_{D4}$

if $I_1 = I_2 = I_0$ then $I_{D3} = I_1 = I_0 \Rightarrow I_{D4} = \frac{I_{D3}}{2}$

$\Rightarrow I_{D4} = \frac{I_0}{2} \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2}$

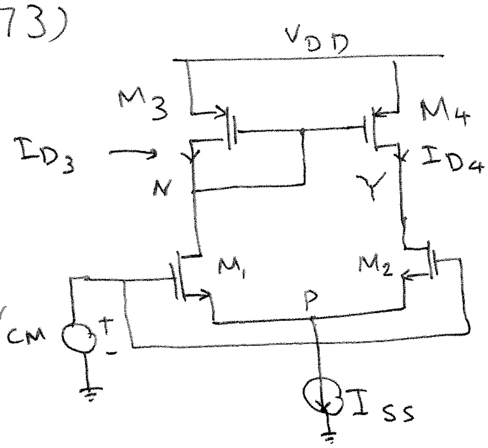
$\Rightarrow V_{out} = R_L I_{out} = -\frac{R_L I_0}{2}$

if $I_1 = I_0 + \Delta I \Rightarrow I_{D4} = \frac{I_{D3}}{2} = \frac{I_1}{2} = \frac{I_0 + \Delta I}{2}$

$I_2 = I_0 - \Delta I \Rightarrow I_{out} = I_{D4} - I_2 = -\frac{I_0}{2} + \frac{3\Delta I}{2}$

$\Rightarrow V_{out} = R_L I_{out} = + R_L\left(\frac{-I_0}{2} + \frac{3\Delta I}{2}\right)$

(73)



$V_{TH3} = |V_{TH,p}|$

(a) $I_{D3} = \frac{I_{SS}}{2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_3 (V_{SG_3} - V_{TH_3})^2 \Longrightarrow$

$$V_{SG_3} = V_{TH_3} + \sqrt{\frac{I_{SS}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}}$$

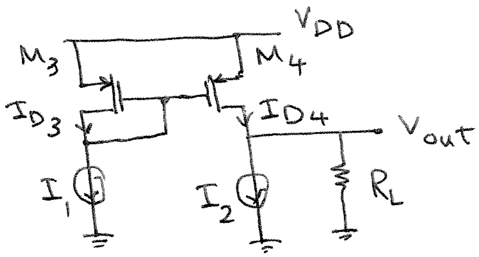
$$V_N = V_{DD} - V_{SG_3} = V_{DD} - V_{TH} - \sqrt{\frac{I_{SS}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_3}}$$

(b) Since the circuit is symmetric:

$$\begin{cases} I_{D3} = I_{D4} \\ V_{SG_3} = V_{SG_4} \end{cases} \Longrightarrow V_{SD_3} = V_{SD_4} \Longrightarrow V_Y = V_N$$

(c) if V_{DD} changes by a small amount ΔV, both V_N and V_Y will chang by ΔV.

(74)

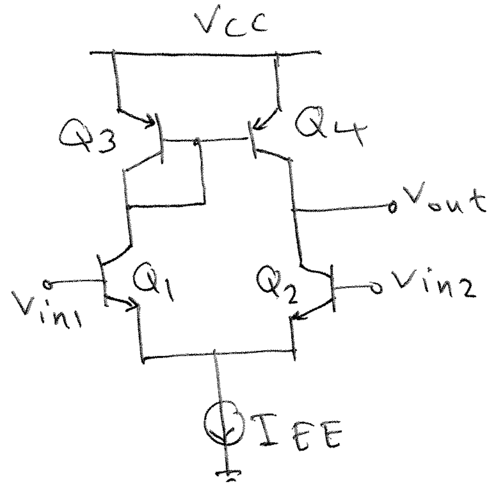


$$I_{D3} = I_{D4} = I_1$$

$$V_{out} = (I_{D4} - I_2) R_L = (I_1 - I_2) R_L$$

small $\Rightarrow$ signal $\quad \dfrac{v_{out}}{i_1} = R_L \quad , \quad \dfrac{v_{out}}{i_2} = -R_L$

(75)



$V_{A,n} = 5\,V$

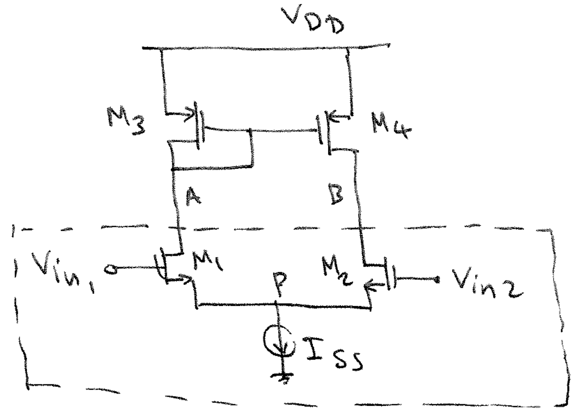
$A_v = 100$

$$\frac{v_{out}}{v_{in1} - v_{in2}} = g_{mN}(r_{oN} \| r_{op}) =$$

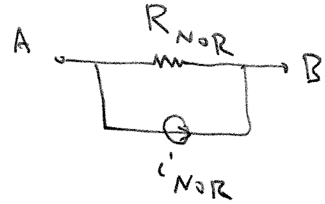
$$\frac{I_{EE}/2}{V_T}\left(\frac{V_{A,n}}{I_{EE}/2} \Big\| \frac{V_{A,p}}{I_{EE}/2}\right) = \frac{V_{A,n}\,V_{A,p}}{(V_{A,n}+V_{A,p})\,V_T}$$

$$\Rightarrow 100 = \frac{5\,V_{A,p}}{(5+V_{A,p})\,0.026} \Rightarrow V_{A,p} = 5.417\,V$$

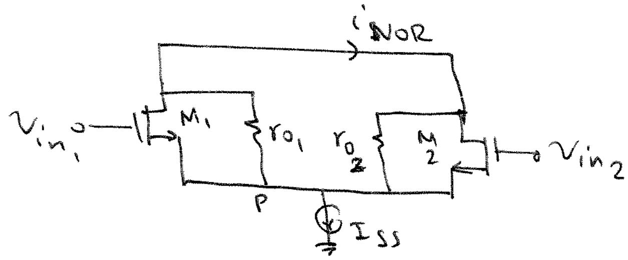
(76)



$\Rightarrow$



To calculate i'_{NOR} we have:

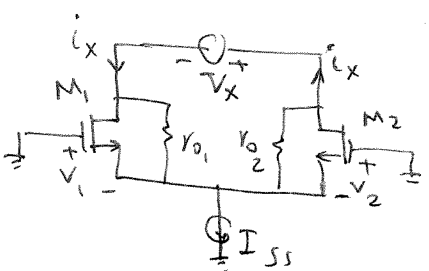


$$r_{o1}(i'_{NOR} + g_{m1} v_{in1}) + r_{o2}(i'_{NOR} - g_{m2} v_{in2}) = 0$$

$$\rightarrow 2 r_{oN} i'_{NOR} = -g_{m1} r_{oN}(v_{in1} - v_{in2}) \Rightarrow$$

$$i'_{NOR} = -\frac{g_{mN}}{2}(v_{in1} - v_{in2})$$

To calculate R_{NOR}:

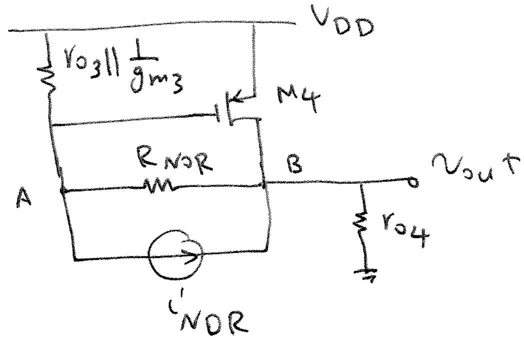


$v_1 = v_2$

$$(i_x - g_{m1} v_1) r_{o1} + (i_x + g_{m2} v_2) r_{o2} = v_x$$

$$\Rightarrow R_{NOR} = \frac{v_x}{i_x} = 2 r_{oN}$$

Therefore, utilizing the Norton model we have:



$$\begin{cases} \dfrac{v_A - v_B}{R_{NOR}} + \dfrac{v_A}{r_{o3} \| \frac{1}{g_{m3}}} + i_{NOR} = 0 \Rightarrow v_A = \dfrac{\frac{v_B}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \| \frac{1}{g_{m3}}}} \\ \dfrac{v_B - v_A}{R_{NOR}} + \dfrac{v_B}{r_{o4}} - i_{NOR} + g_{m4} v_A = 0 \end{cases}, \quad v_B = v_{out}$$

$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{o4}}\right) + \left(g_{m4} - \frac{1}{R_{NOR}}\right) \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{\frac{1}{R_{NOR}} + \frac{1}{r_{o3} \| \frac{1}{g_{m3}}}} = i_{NOR}$$

$$\frac{1}{g_{m3}} \ll r_{o3}, \quad \frac{1}{g_{m3}} \ll R_{NOR}, \quad g_{m3} = g_{m4} = g_m, \quad r_{o3} = r_{o4} = r_{op}$$

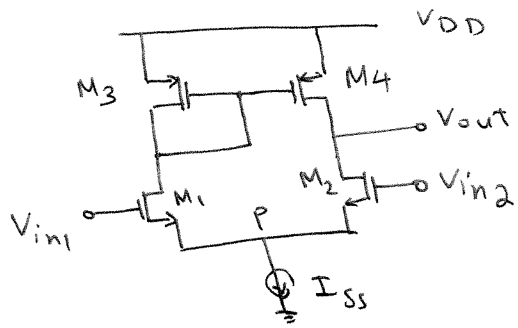
$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}}\right) + g_{m4} \frac{\frac{v_{out}}{R_{NOR}} - i_{NOR}}{g_{m3}} = i_{NOR}$$

$$\Rightarrow v_{out}\left(\frac{1}{R_{NOR}} + \frac{1}{r_{op}}\right) + \frac{v_{out}}{R_{NOR}} = 2 i_{NOR} \Rightarrow$$

$$\frac{2 v_{out}}{R_{NOR}} + \frac{v_{out}}{r_{op}} = 2 i_{NOR} \Rightarrow v_{out}\left(\frac{1}{r_{oN}} + \frac{1}{r_{op}}\right) = -g_{mN}(v_{in1} - v_{in2})$$

$$\Rightarrow \frac{v_{out}}{v_{in1} - v_{in2}} = -g_{mN}\,(r_{oN} \| r_{op})$$

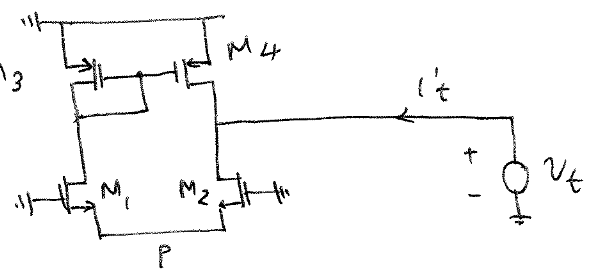
77



$g_m r_o \gg 1$

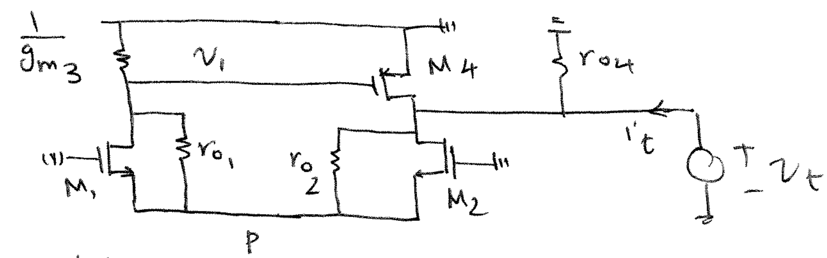
$g_{m1} = g_{m2}$

To calculate the output impedance we have the following circuit:



$R_{out} = \dfrac{v_t}{i_t'}$

$\Downarrow$ neglecting r_{o3} $\left(r_{o3} \gg \dfrac{1}{g_{m3}}\right)$

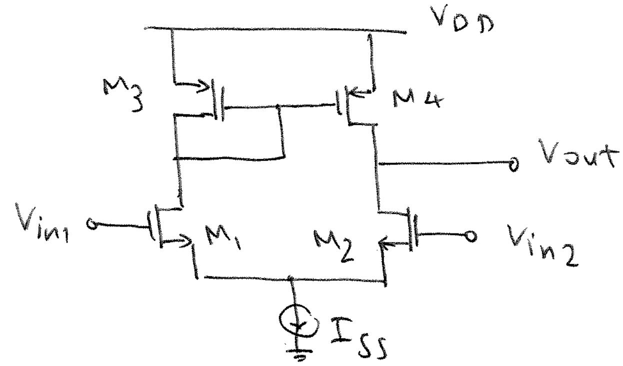


writing node equations of v_1 and v_p:

$$\begin{cases} g_{m3} v_1 + \dfrac{v_1 - v_p}{r_{o1}} - g_{m1} v_p = 0 \\ 2 g_{m1} v_p + \dfrac{v_p - v_1}{r_{o1}} + \dfrac{v_p - v_t}{r_{o2}} = 0 \end{cases} \xRightarrow{g_m r_o \gg 1} \begin{cases} g_{m3} v_1 \approx g_{m1} v_p \\ 2 g_{m1} v_p \approx 0 \end{cases}$$

$$\Rightarrow v_p \approx v_1 = 0 \Rightarrow R_{out} = \frac{v_t}{i_t'} = r_{o4} \| r_{o2} = r_{oN} \| r_{oP}$$

(78)

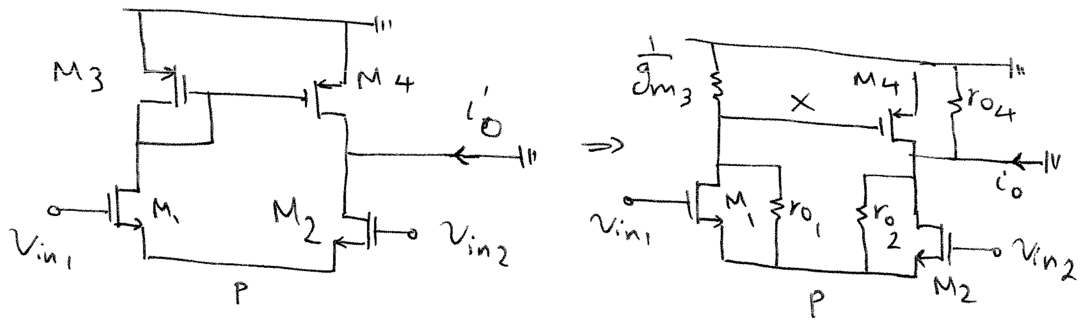


$g_m r_o >> 1$

$g_{m_1} = g_{m_2}$

$g_{m_3} = g_{m_4}$

To calculate G_m we have from small signal model:



writing node equations of nodes P and X we have:

$$\begin{cases} g_{m_1}(v_P - v_{in_1}) + g_{m_2}(v_P - v_{in_2}) + \dfrac{v_P - v_X}{r_{o_1}} + \dfrac{v_P}{r_{o_2}} = 0 \\ g_{m_3} v_X + g_{m_1}(v_{in_1} - v_P) + \dfrac{v_X - v_P}{r_{o_1}} = 0 \end{cases}$$

Since $g_m r_o >> 1$ we have

$$\begin{cases} g_{m_1}(v_P - v_{in_1}) + g_{m_2}(v_P - v_{in_2}) = 0 \Rightarrow v_P = \dfrac{v_{in_1} + v_{in_2}}{2} \\ v_X = -\dfrac{g_{m_1}}{g_{m_3}}(v_{in_1} - v_P) = -\dfrac{g_{m_1}}{g_{m_3}}\left(\dfrac{v_{in_1} - v_{in_2}}{2}\right) \end{cases}$$

$$i_o' = -\frac{v_P}{r_{o_2}} + g_{m_2}(v_{in_2} - v_P) - g_{m_4}(-v_X)$$

$$\Rightarrow \quad i_o \approx -g_{m4}(-v_x) + g_{m2}\left(v_{in_2} - \frac{v_{in_1}+v_{in_2}}{2}\right)$$

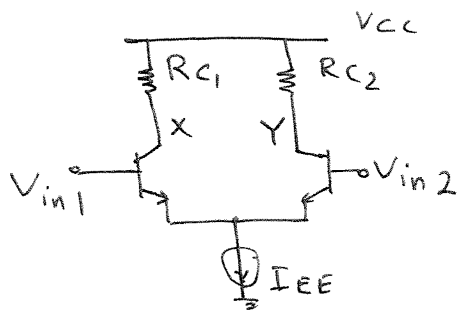
$$= -\left[g_{m4}\ \frac{g_{m1}}{g_{m3}}\left(\frac{v_{in_1}-v_{in_2}}{2}\right) + g_{m2}\left(\frac{v_{in_1}-v_{in_2}}{2}\right)\right]$$

$$= -g_{m1}\left(v_{in_1}-v_{in_2}\right)$$

$$G_m = \frac{i_o}{v_{in_1}-v_{in_2}} = -g_{m1} = -g_{mN}$$

$$\rightarrow A_v = -G_m R_{out} = g_{mN}\left(r_{oN} \| r_{oP}\right)$$

79



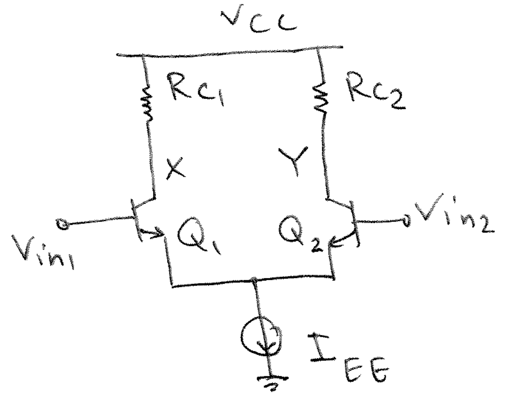
$A_v = 10,$
$P = 2\,mW$
$V_{cc} = 2.5\,V$
$V_A = \infty$

$$P = V_{cc} I_{EE} \Rightarrow 2 \times 10^{-3} = 2.5\, I_{EE} \Rightarrow I_{EE} = 0.8\,mA$$

$$A_v = \frac{v_{XY}}{v_{in_1} - v_{in_2}} = -g_m R_C = -\frac{I_{EE}/2}{V_T} R_C$$

$$\Rightarrow \quad 10 = \frac{0.4 \times 10^{-3}}{0.026} R_C \Rightarrow R_C = 650\,\Omega$$

(80)



$V_{in,CM} = 1.2\,V$

$P = 3\,mW$

$V_{CC} = 2.5\,V$

$$P = I_{EE} V_{CC} \Rightarrow 3\times10^{-3} = 2.5\, I_{EE} \Rightarrow I_{EE} = 1.2\,mA$$

$$A_v = -g_{m1} R_C = -\frac{I_{EE}/2}{V_T} R_C = -\frac{R_C I_{EE}}{2V_T}$$

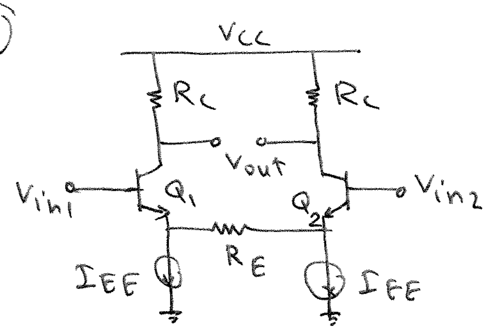
To maximize gain, $R_C I_{EE}$ and therefore R_C should be maximum. However, the upper bound of R_C value is limited by the voltage value of X. because:

$$V_{in,CM} \le V_X \Rightarrow 1.2 \le V_{CC} - R_C I_{EE}/2 \Rightarrow$$

$$R_C \le 2\frac{V_{CC}-1.2}{I_{EE}} \Rightarrow R_C \le 2\,\frac{2.5-1.2}{1.2\times10^{-3}} \Rightarrow$$

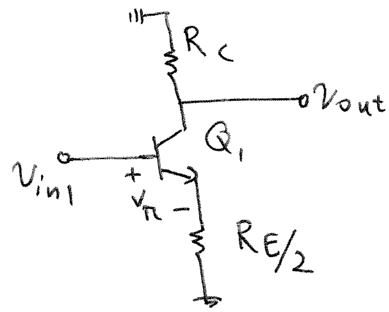
$$R_C \le 2.167\,k\Omega \Rightarrow R_C = 2.167\,k\Omega$$

(81)



$A_v = 5$
$P = 4\,mW$
$V_{CC} = 2.5\,V$
$V_A = \infty$

The half circuit is:



$$A_v = \frac{v_{out}}{v_{in1}} \simeq \frac{-g_{m1} v_\pi R_C}{v_{in1}} = -\frac{g_{m1} R_C}{v_{in1}} \frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}} + \frac{R_E}{2}} v_{in1}$$

$$= -\frac{R_C}{\frac{R_E}{2} + \frac{1}{g_{m1}}}$$

$P = 4mW = 2 I_{EE} V_{CC} = 5 I_{EE} \Rightarrow I_{EE} = 0.8mA$

$g_m = \frac{I_{EE}}{V_T} = 0.03077$

$$A_v = 5 \Rightarrow \frac{R_C}{\frac{R_E}{2} + 32.5} = 5 \quad (1)$$

if I_{EE} increases by 10%, the gain will be:

$$A_v = \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} \Rightarrow 5 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{1.1}} < 5 \times 1.02 \quad (2)$$

if I_{EE} decreases by 10% then:

$$5 \times 0.98 < \frac{R_C}{\frac{R_E}{2} + \frac{32.5}{0.9}} < 5 \quad (3)$$

The worse case is:

$$\begin{cases} \dfrac{R_C}{\dfrac{R_E}{2} + \dfrac{32.5}{1.1}} = 5 \times 1.02 & (4) \\[2ex] \dfrac{R_C}{\dfrac{R_E}{2} + \dfrac{32.5}{0.9}} = 5 \times 0.98 & (5) \end{cases}$$

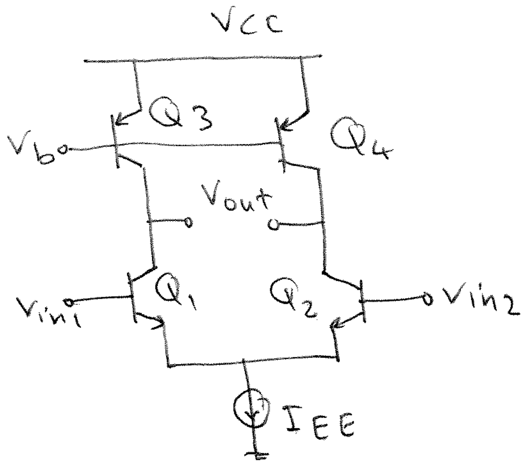
dividing (4) and (5) to (1) leads to:

$$\begin{cases} \dfrac{\dfrac{R_E}{2} + 32.5}{\dfrac{R_E}{2} + \dfrac{32.5}{1.1}} = 1.02 \Rightarrow R_E = 236.36\,\Omega \\[2ex] \dfrac{\dfrac{R_E}{2} + 32.5}{\dfrac{R_E}{2} + \dfrac{32.5}{0.9}} = 0.98 \Rightarrow R_E = 288.89\,\Omega \end{cases}$$

To ensure less than 2% gain variation for 10% current variation $R_E = 288.89\,\Omega$

From (1) $\quad R_C = 5\left(\frac{R_E}{2} + 32.5\right) = 884.72\,\Omega$

(82)



$A_v = 100$

$P = 1mW$

$V_{A,n} = 6$

$V_{cc} = 2.5\ V$

$$P = 1mW = I_{EE} V_{cc} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4mA$$

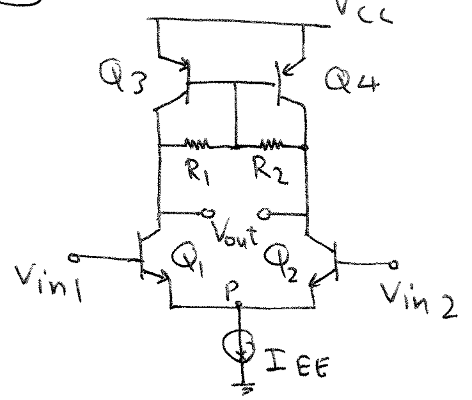
$$r_{o_N} = \frac{V_{A_n}}{I_{EE}/2} = \frac{6}{0.2 \times 10^{-3}} = 30k\Omega,\ g_{m_N} = \frac{I_{EE}/2}{V_T} = \frac{0.2}{26}\ S$$

$$A_v = -g_{m_N}(r_{o_N} \| r_{op}) \Rightarrow$$

$$100 = \frac{0.2}{26}(30 \times 10^3 \| r_{op}) \Rightarrow r_{op} = 22.94k\Omega$$

$$\Rightarrow V_{A,p} = r_{op}\frac{I_{EE}}{2} = 4.588\ V$$

83



$A_v = 50$

$P = 1mW$

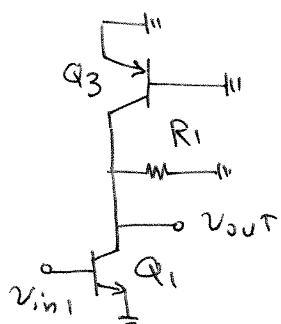
$V_{A,n} = 10\ V$

$V_{A,p} = 5\ V$

$V_{cc} = 2.5\ V$

$R_1 = R_2$

The half circuit is:



$$A_v = -g_{m_1}(R_1 \| r_{o_1} \| r_{o_3})$$

$$g_{m_1} = \frac{I_{EE}}{2V_T} \quad r_{o_1} = \frac{2V_{A,n}}{I_{EE}} \quad r_{o_3} = \frac{2V_{A,p}}{I_{EE}}$$

$$\Rightarrow 50 = \frac{1}{2V_T}\left((R_1 I_{EE}) \| \underbrace{(2V_{A,n})}_{20} \| \underbrace{(2V_{A,p})}_{10}\right) \Rightarrow$$

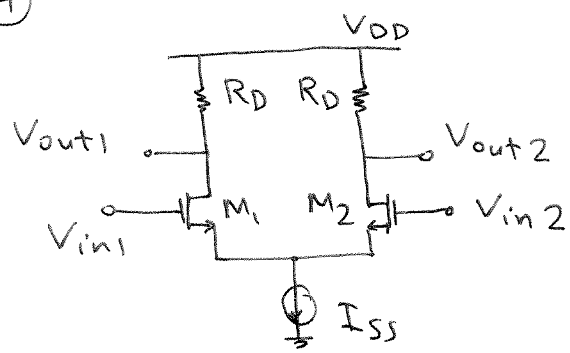
$$R_1 I_{EE} = 4.262$$

$$P = 1mw = I_{EE} V_{cc} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4\,mA$$

$$\rightarrow R_1 = \frac{4.262}{I_{EE}} = 10.655\ K\Omega$$

Notice that larger value of R_1, requires smaller I_{EE} and saves power.

(84)



$\Delta V_{in,max} = 0.3\,V$

$P = 3\,mW$

$R_D = 500\,\Omega$

$\lambda = 0$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$V_{DD} = 1.8\,V$

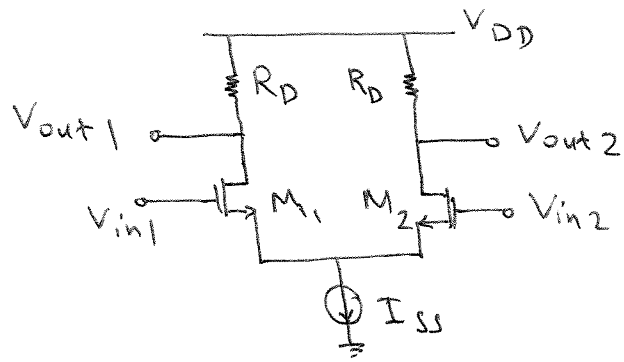
$P = V_{DD} I_{SS} \Rightarrow 3\times10^{-3} = 1.8\, I_{SS} \Rightarrow$

$I_{SS} = 1.67\ mA$

$$\Delta V_{in,max} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.3 = \sqrt{\frac{2\times1.67\times10^{-3}}{10^{-4}\times\frac{W}{L}}} \Rightarrow \frac{W}{L} = 370.37$$

(85)



$P = 2\,mW$

overdrive $= 100\,mV$

$V_{CM} = 1\,V$

$\lambda = 0$, $\mu_n C_{ox} = 100\,\mu A/V^2$

$V_{DD} = 1.8\,V$

$V_{TH,n} = 0.5$

$$P = I_{SS} V_{DD} \Rightarrow 2\times10^{-3} = 1.8\, I_{SS} \Rightarrow I_{SS} = 1.11\,mA$$

$$V_{GS_1} - V_{TH} = \sqrt{\frac{2 I_{D_1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \Rightarrow$$

$$0.1^2 = \frac{1.11\times10^{-3}}{10^{-4}\times\frac{W}{L}} \Rightarrow \frac{W}{L} = 1111.11$$

$$g_m = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} = \sqrt{10^{-4}\times1111.11\times1.11\times10^{-3}} = 0.011$$

To place the transistor at the edge of triode region:

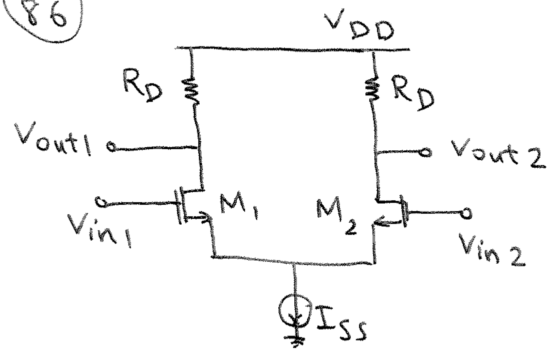
$$V_{in,CM} = V_{out1} + V_{TH,n}$$

$$1 = V_{DD} - R_D\frac{I_{SS}}{2} + 0.5 \Rightarrow$$

$$1 = 1.8 - R_D\frac{1.11\times10^{-3}}{2} + 0.5 \Rightarrow R_D = 2.34\,k\Omega$$

$$A_v = -g_m R_D = -25.74$$

86



$A_v = 5$

$P = 1mW$

$(V_{GS} - V_{TH})_{equil} = 150mV$

$\lambda = 0, \ \mu_n C_{ox} = 100 \mu A/V^2$

$V_{DD} = 1.8 \ V$

$$P = 1mw = V_{DD} I_{SS} = 1.8 I_{SS} \Rightarrow I_{SS} = 0.556mA$$

$$g_{m_1} = \frac{2 I_{D1}}{(V_{GS} - V_{TH})_{equil}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{equil}} = \frac{0.556 \times 10^{-3}}{0.15}$$

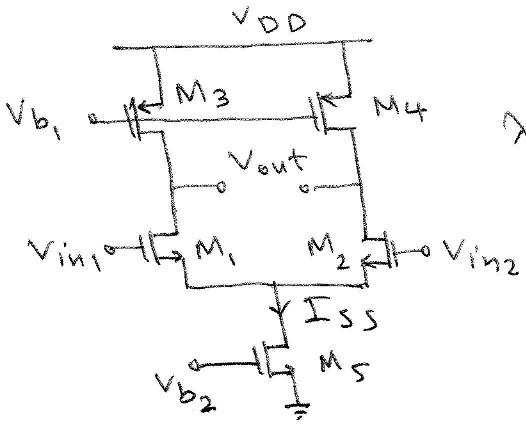
$$= 3.704 \ mS$$

$$A_v = -g_{m_1} R_D \Rightarrow 5 = 3.704 \times 10^{-3} \times R_D \Rightarrow R_D = 1.35 k\Omega$$

$$(V_{GS} - V_{TH})_{equil} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \Rightarrow$$

$$0.15 = \sqrt{\frac{0.556 \times 10^{-3}}{10^{-4} \times \frac{W}{L}}} \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 246.91$$

(87)



$A_V = 40$

$(V_{GS} - V_{TH})_{equil} = ?$

$\lambda_n = 0.1\,V^{-1} \quad \lambda_p = 0.2\,V^{-1}$

$\mu_n C_{ox} = 100\,\mu A/V^2$

$\mu_p C_{ox} = 50\,\mu A/V^2$

$V_{DD} = 1.8$

$P = 2\,mW$

$$A_V = -g_{m_N}(r_{o_P} \| r_{o_N}) = -\frac{I_{SS}}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\frac{I_{SS}}{2}\lambda_n} \Big\| \frac{1}{\frac{I_{SS}}{2}\lambda_p}\right)$$

$$= -\frac{2}{(V_{GS_1} - V_{TH})_{equil}}\left(\frac{1}{\lambda_n} \Big\| \frac{1}{\lambda_p}\right) \Rightarrow$$

$$\frac{2}{(V_{GS_1} - V_{TH})_{equil}}(10 \| 5) = 40 \Rightarrow (V_{GS_1} - V_{TH})_{equil} = 166.67\,mV$$

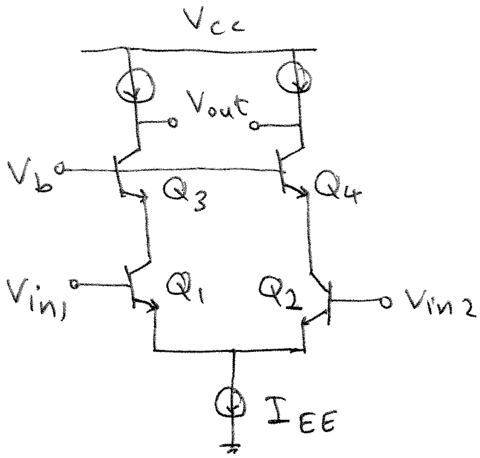
$$P = 2\times10^{-3} = V_{DD}\,I_{SS} \Rightarrow I_{SS} = \frac{2\times10^{-3}}{1.8} = 1.11\,mA$$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{I_{SS}}{\mu_n C_{ox}(V_{GS_1} - V_{TH})^2_{equil}} = \frac{1.11\times10^{-3}}{10^{-4}\times(0.16667)^2} = 400$$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{I_{SS}}{\mu_p C_{ox}(V_{GS} - V_{TH})_{equil}} = \frac{1.11\times10^{-3}}{0.5\times10^{-4}\times(0.16667)^2} = 800$$

$$\left(\frac{W}{L}\right)_5 = \frac{2\,I_{SS}}{\mu_n C_{ox}(V_{GS} - V_{TH})_{equil}} = \frac{2\times1.11\times10^{-3}}{10^{-4}\times(0.16667)^2} = 800$$

(88)



$A_v = 4000$

$\beta = 100$

$V_{CC} = 2.5\,V$

$P = 1\,mW$

$$P = I_{EE} V_{CC} = 10^{-3} \Rightarrow I_{EE} = \frac{10^{-3}}{2.5} = 0.4\,mA$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} = \frac{0.2}{26} = 7.692\,mS$$

$$r_{\pi_{1-4}} = \frac{\beta}{g_{m_1}} = 13\,K\Omega$$

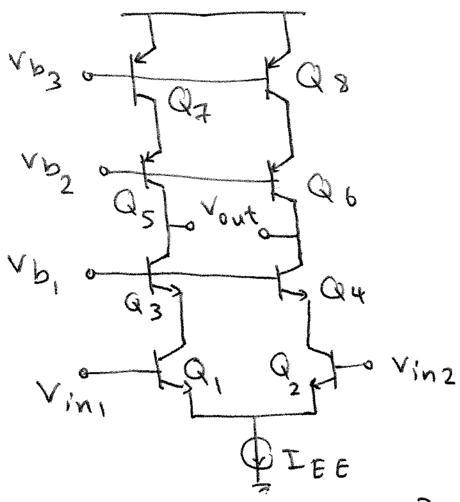
$$r_{o_{1-4}} = \frac{V_A}{\frac{I_{EE}}{2}} = 5\times10^3\,V_A$$

$$A_v = -g_{m_1}\left[g_{m_3}(r_{o_1} \| r_{\pi_3})r_{o_3} + (r_{o_1} \| r_{\pi_3}) + r_{o_3}\right] \Rightarrow$$

$$4000 = \frac{0.2}{26}\left[\frac{0.2}{26}(5\times10^3 V_A \| 13\times10^3)5\times10^3 V_A + (5\times10^3 V_A) \| 13\times10^3 + 5\times10^3 V_A\right]$$

$$\Rightarrow V_A = 2.197$$

(89)



$A_v = 2000$

$\beta_n = 100$

$\beta_p = 50$

$V_{A,n} = 5V$

$V_{CC} = 2.5\,V$

$P = 2\,mW$

$$P = I_{EE} V_{CC} = 2\times10^{-3} \Rightarrow I_{EE} = \frac{2\times10^{-3}}{2.5} = 0.8\,mA$$

$$g_{m_{1-8}} = \frac{I_{EE}}{2V_T} = \frac{0.4}{26} = 0.0154 \Rightarrow$$

$$r_{\pi_{1-4}} = \frac{\beta_n}{g_{m_1}} = 6.5\,K\Omega \qquad r_{\pi_{5-8}} = \frac{\beta_p}{g_{m_5}} = 3.25\,K\Omega$$

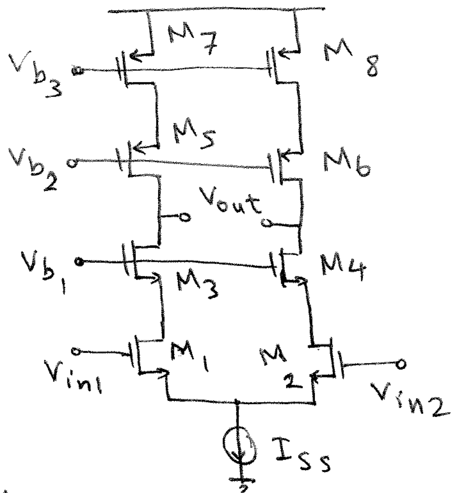
$$r_{o_{1-4}} = \frac{V_{A,n}}{I_{EE}/2} = 12.5\,K\Omega \qquad r_{o_{5-8}} = \frac{V_{A,p}}{I_{EE}/2}$$

$$A_v \approx -g_{m_1}\left[g_{m_3} r_{o_3} (r_{o_1} \| r_{\pi_3})\right] \| \left[g_{m_5} r_{o_5} (r_{o_7} \| r_{\pi_5})\right]$$

$$\Rightarrow \frac{0.4}{26}\left[\frac{0.4}{26}\times12.5\times10^{+3}(12.5\times10^{3} \| 6.5\times10^{3})\right] \| \left[\frac{0.4}{26}\,\frac{V_{A,P}}{I_{EE}/2}\left(\frac{V_{A,P}}{\frac{I_{EE}}{2}} \| 3250\right)\right] = 2000$$

$$\Rightarrow V_{A,P} = 2.027\ V$$

(90)



$A_V = 600$

$P = 4\,mW$

$(V_{GS} - V_{TH})_{NMOS} = 100\,mV$

$(V_{GS} - V_{TH})_{PMOS} = 150\,mV$

$\mu_n C_{ox} = 100\ \mu A/V^2$

$\mu_p C_{ox} = 50\ \mu A/V^2$

$\lambda_n = 0.1\ V^{-1}$

$$A_V \simeq -g_{m1}\left[(g_{m3} r_{o3} r_{o1}) \| (g_{m5} r_{o5} r_{o7})\right] = -600$$

$$P = 4\,mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{4\times10^{-3}}{1.8} = 2.22\,mA$$

$$g_{m_{1-4}} = \frac{2I_{D1}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{NMOS}} = \frac{2.22\times10^{-3}}{0.1} = 22.22\,mS$$

$$g_{m_{5-8}} = \frac{2I_{D5}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{I_{SS}}{(V_{GS}-V_{TH})_{PMOS}} = \frac{2.22\times10^{-3}}{0.15} = 14.815\,mS$$

$$r_{o_{1-4}} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1\times\frac{2.22}{2}\times10^{-3}} = 9K\Omega$$

$$r_{o_{5-8}} = \frac{1}{\lambda_p \frac{I_{SS}}{2}} = \frac{1}{\lambda_p \times \frac{2.22}{2}\times10^{-3}} = \frac{0.9\times10^{3}}{\lambda_p}$$

in A_V $\Rightarrow$ equation

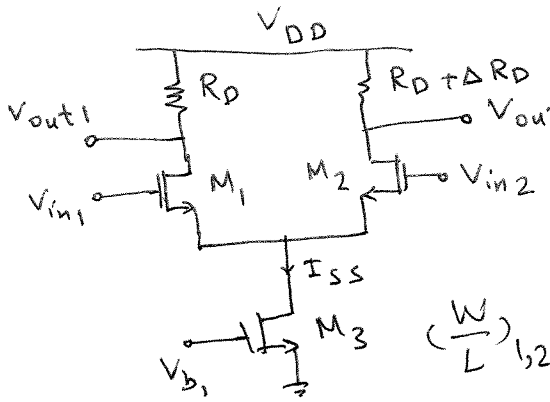
$$22.22\times10^{-3}\left[(22.22\times10^{-3}\times81\times10^{6}) \| (14.815\times10^{-3}\times\frac{0.81\times10^{6}}{\lambda_p^2})\right] = 600 \Rightarrow$$

$\lambda_p = 0.66\ V^{-1}$

$$\left(\frac{W}{L}\right)_{NMOS} = I_{SS}/(\mu_n C_{ox}(V_{GS}-V_{TH})^2_{NMOS}) = 2222.2$$

$$\left(\frac{W}{L}\right)_{PMOS} = I_{SS}/(\mu_p C_{ox}(V_{GS}-V_{TH})^2_{PMOS}) = 1975.31$$

(91)



CMMR = 60dB

P = 2mW

$A_{DM} = 5$

$\lambda(M_3) = ?$

$\left(\frac{W}{L}\right)_{1,2} = \frac{10}{0.18}$, $\mu_n C_{ox} = 100 \mu A/V^2$

$V_{DD} = 1.8$, $\frac{\Delta R_D}{R_D} = 2\%$

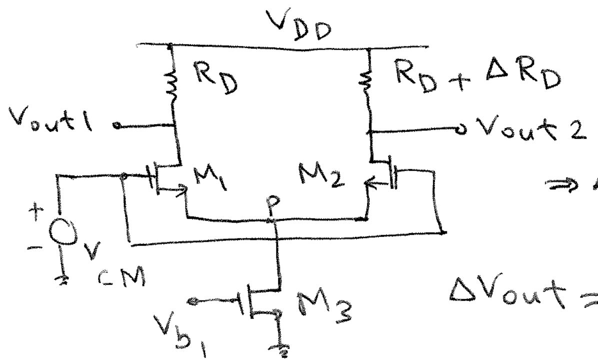
$$P = 2mW = I_{SS} V_{DD} \Rightarrow I_{SS} = \frac{2\times10^{-3}}{1.8} = 1.11\ mA$$

$$A_{DM} = -g_{m1} R_D$$

$$g_{m1} = \sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{SS}} = \sqrt{10^{-4} \times \frac{10}{0.18} \times 1.11\times10^{-3}} = 2.4845\ mS$$

$$\Rightarrow R_D = \frac{|A_{DM}|}{g_{m1}} = \frac{5}{2.4845\times10^{-3}} = 2.012\ k\Omega$$

To calculate $A_{CM,DM}$ we have:



$$\Delta V_{CM} = \Delta V_{GS} + 2\Delta I_D r_{o3}$$

$$\Rightarrow \Delta V_{CM} = \Delta I_D \left(\frac{1}{g_{m1}} + r_{o3}\right)$$

$$\Delta V_{out} = \Delta V_{out1} - \Delta V_{out2} \cong -\Delta R_D I_D$$

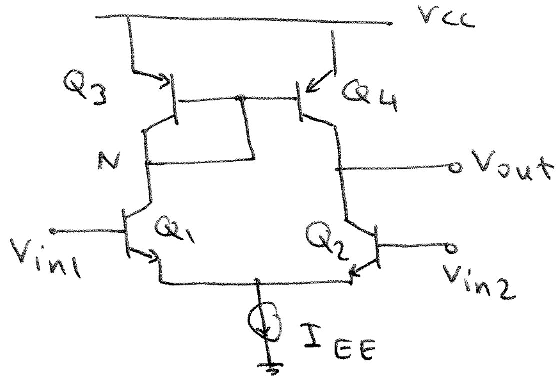
$$\Rightarrow A_{CM,DM} = \frac{\Delta V_{out}}{\Delta V_{CM}} = -\frac{\Delta R_D / 2}{\frac{1}{2g_{m1}} + r_{o3}}$$

$$\Rightarrow CMMR = \frac{A_{DM}}{A_{CM,DM}} = (1 + 2g_{m1} r_{o3}) \frac{R_D}{\Delta R_D}, \quad r_{o3} = \frac{1}{\lambda_3 I_{SS}}$$

$$\Rightarrow \quad CMMR = 60dB = 10^3 = \left(1 + 2 \times 2.4845 \times 10^{-3} \frac{1}{\lambda_3 \times 1.11 \times 10^{-3}}\right) 50$$

$$\Rightarrow \quad \lambda_3 = 0.2354$$

92



$A_v = 200$

$P = 3\,mW$

$V_{cc} = 2.5\,V$

$V_{A,n} = 2V_{A,p}$

$$P = V_{cc} I_{EE} \Rightarrow I_{EE} = \frac{3\times10^{-3}}{2.5} = 1.2\,mA$$

$$g_{m_{1-4}} = \frac{I_{EE}}{2V_T} = \frac{0.6}{26} = 23.077\ mS$$

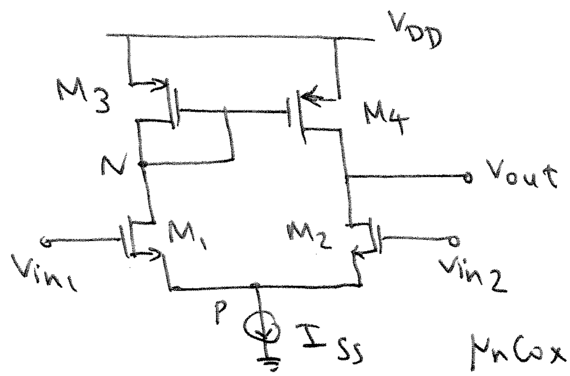
$$r_{o_1} = r_{o_2} = \frac{V_{A,n}}{\frac{I_{EE}}{2}} = \frac{V_{A,n}}{0.6\times10^{-3}}$$

$$r_{o_3} = r_{o_4} = \frac{V_{A,p}}{\frac{I_{EE}}{2}} = \frac{V_{A,n}}{1.2\times10^{-3}}$$

$$A_v = g_{m_1}(r_{o_2} \| r_{o_4}) \Rightarrow \frac{0.6}{26}\left(\frac{V_{A,n}}{0.6\times10^{-3}} \| \frac{V_{A,n}}{1.2\times10^{-3}}\right) = 200$$

$$\Rightarrow V_{A,n} = 15.6\ V$$

(93)



$A_V = 20$

$P = 1mW$

$V_{DD} = 1.8\,V$

$V_{in,CM} = 1\,V$

$\mu_n C_{ox} = 2\mu_p C_{ox} = 100\mu A/V^2$

$V_{TH,n} = 0.5V,\ V_{TH,p} = -0.4\,V$

$\lambda_n = \dfrac{\lambda_p}{2} = 0.1\,V^{-1}$

$$P = V_{DD} I_{SS} \Rightarrow I_{SS} = \frac{10^{-3}}{1.8} = 0.556\ mA$$

$$A_V = + g_{m_N} (r_{o_N} \| r_{o_P}) = 20$$

$$r_{o_N} = \frac{1}{\lambda_n \frac{I_{SS}}{2}} = \frac{1}{0.1 \ \frac{0.556\times 10^{-3}}{2}} = 36\,k\Omega$$

$$r_{o_P} = \frac{1}{\lambda_P \frac{I_{SS}}{2}} = 18\,k\Omega$$

$$g_{m_N} (36^k \| 18^k) = 20 \Rightarrow g_{m_N} = 1.667m\,S$$

$$\Rightarrow g_{m_N} = \frac{2 I_{D_N}}{(V_{GS} - V_{TH})_{NMOS}} = \frac{I_{SS}}{(V_{GS} - V_{TH})_{NMOS}} \Rightarrow$$

$$(V_{GS} - V_{TH})_{NMOS} = 0.333\ V$$

$$(V_{GS} - V_{TH})_{NMOS} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} (\frac{W}{L})_{NMOS}}} \Rightarrow \left(\frac{W}{L}\right)_{1,2} = 50$$

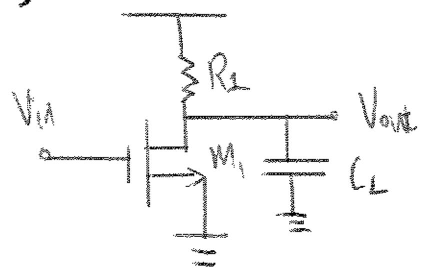
$$V_N = V_{in,CM} - V_{TH,n} = 1 - 0.5 = 0.5\ V$$

$$\rightarrow \quad |V_{GS_3}| - |V_{TH,p}| = 1.3 - 0.4 = 0.9\ V$$

$$\rightarrow \quad 0.9 = \sqrt{\frac{I_{SS}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_{PMOS}}} \quad \Rightarrow$$

$$\left(\frac{W}{L}\right)_{3,4} = 13.717$$

1)



$R_1 = 1\,K\Omega$

$C_L = 1pF$

$$V_{out} = -g_{m_1} R_1 // \frac{1}{C_L S} V_{in} \Rightarrow \frac{V_{out}}{V_{in}} = -g_m R_1 // \frac{1}{C_L S}$$

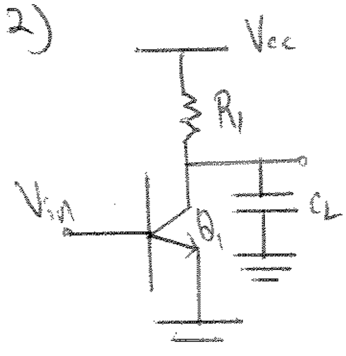
$$\frac{V_{out}}{V_{in}} = -g_{m_1} \left(\frac{R_1}{R_1 C_L S + 1} \right), \quad S \rightarrow j\omega, \quad \frac{V_{out}}{V_{in}}(j\omega) = -g_{m_1} \left(\frac{R_1}{R_1 C_L j\omega + 1} \right)$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_1}{\sqrt{1 + (\omega R_1 C_L)^2}}, \quad \text{Fall by } 10\% = \left| \frac{V_{out}}{V_{in}} \right| = g_m R_1\, 0.9$$

$$\Rightarrow \frac{g_m R_1}{\sqrt{1 + (\omega R_1 C_L)^2}} = g_m R_1\, 0.9 \Rightarrow \omega_{-10\%} = 4.84 \times 10^9 \text{ rad/s}$$

$$2\pi f = 4.84 \times 10^9 \Rightarrow f_{-10\%} = 7.708 \times 10^8 \text{ Hz}$$

2)



-3dB bandwidth = 1 GHz

$C_L = 2pF$

Power = 2mW

Low freq gain ?

Power = $2.5V\, I_c$, $I_c = 0.8\,mA$

Dominant Pole at the output = $\frac{1}{R_1 C_L} = 2\pi(1GHz)$

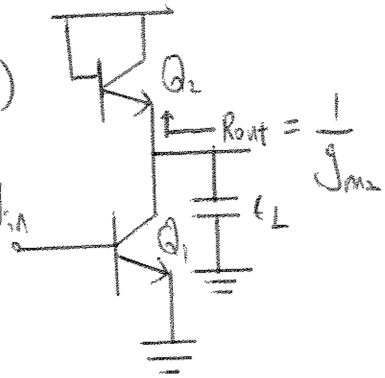
$R_1 = 79.58$ Ohm.

Low Freq gain : $-g_m R_1 = \frac{-I_c R_1}{V_T} = \frac{(79.58)(0.8)}{26}$

$A_v\Big|_{low\,freq} = -2.45$

3)

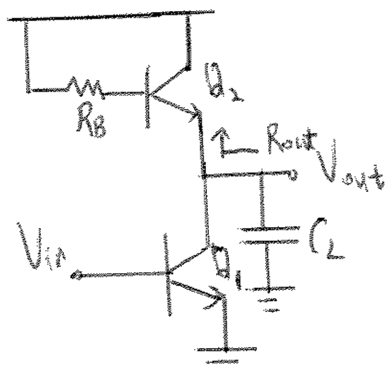
a)



Assume $\beta >> 1$

$$-3dB = \frac{g_{m2}}{C_L}$$

b)

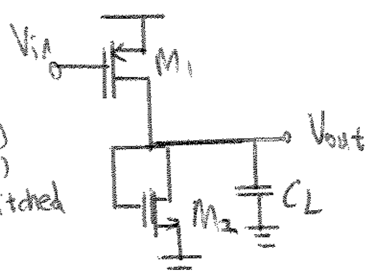


$$R_{out} = \frac{1}{g_{m2}} + \frac{R_B}{\beta+1}$$

$$-3dB = \frac{(\beta+1)\,g_{m2}}{C_L\,[\beta + R_B g_{m2}]}$$

d)

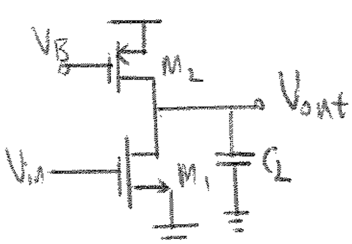
note, parts d) and c) are switched



$$R_{out} = \frac{1}{g_{m2}} \,//\, r_{o2} \,//\, r_{o1} \approx \frac{1}{g_{m2}}$$

$$-3dB = \frac{g_{m2}}{C_L}$$

c)

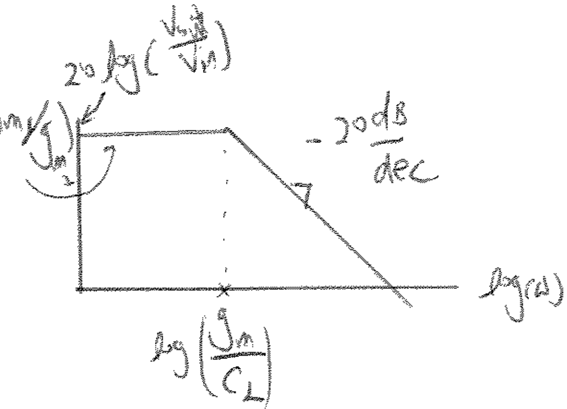


$$R_{out} = r_{o1} // r_{o2}$$

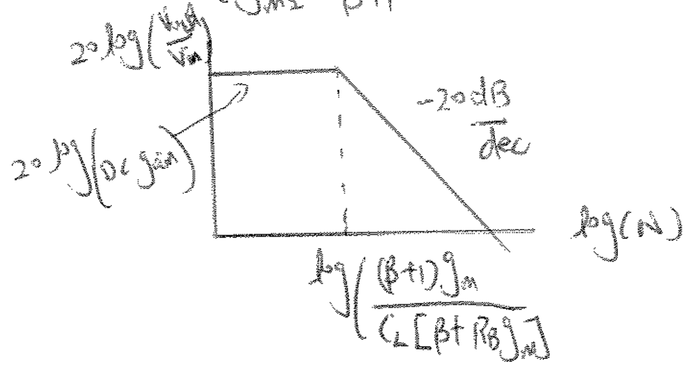
$$-3dB = \frac{1}{(r_{o1} // r_{o2})\,C_L}$$

4)

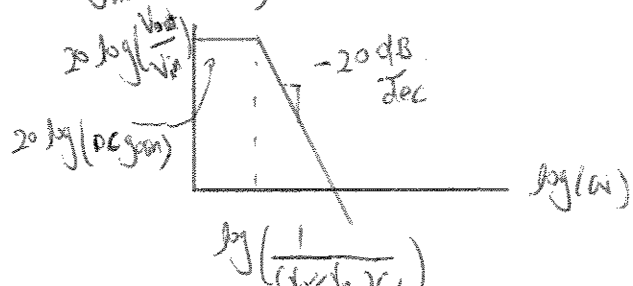
a) $|DC\ gain| = \dfrac{g_{m1}}{g_{m2}}$



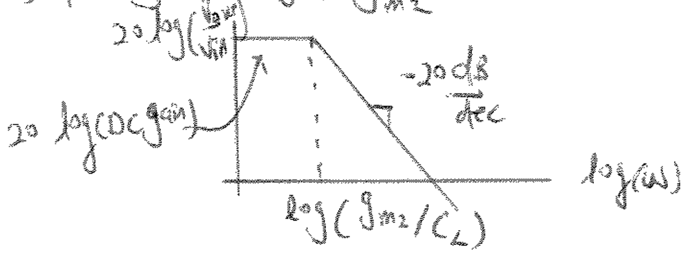
b) $|DC\ gain| = g_{m1}\left(\dfrac{1}{g_{m2}} + \dfrac{R_B}{\beta+1}\right)$



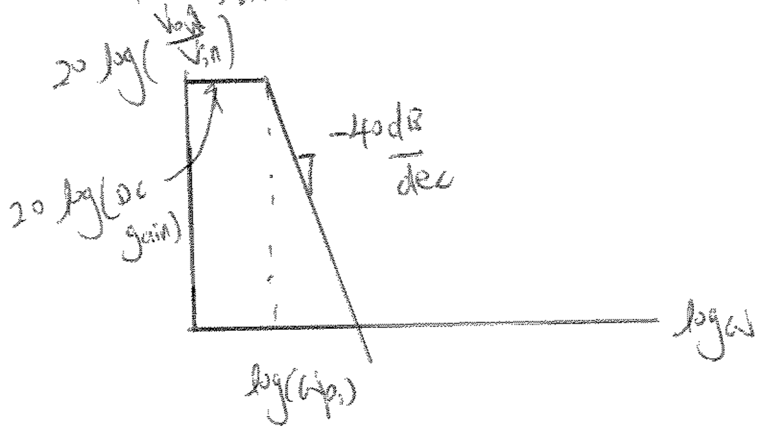
c) $|DC\ gain| = g_m (r_{o1} // r_{o2})$



d) $|DC\ gain| = g_{m1} / g_{m2}$



5) 2 poles at ω_{p1}, means slope is -40 dB/dec

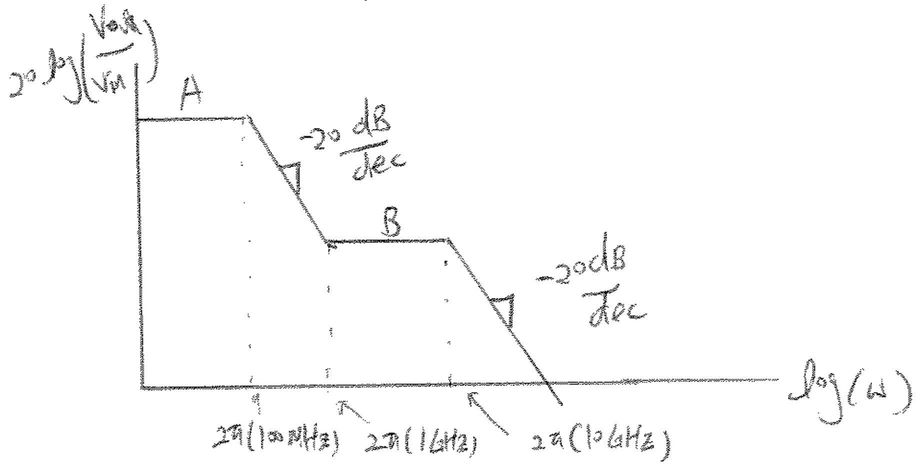


* Assuming transfer function is in the form of $\dfrac{A}{\left(\sqrt{\left(\frac{\omega}{\omega_{p_1}}\right)^2+1}\right)^2}$

6)

Poles at 100 MHz, 10 GHz

Zero at 1 GHz.

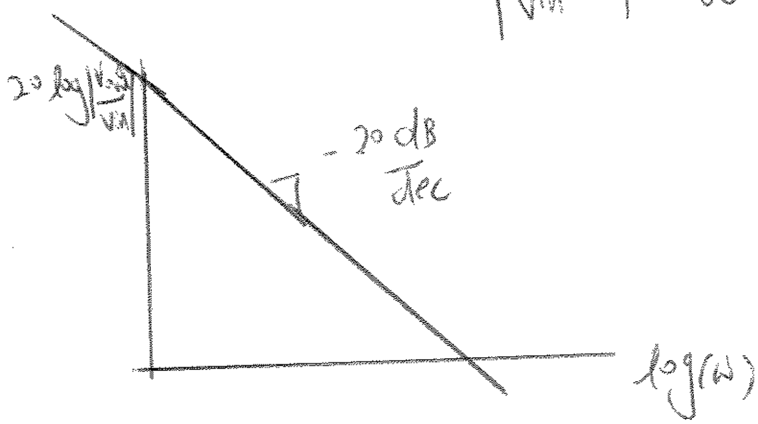


A (100 MHz) = B (1 GHz)

B = 0.1A

7) Ideal Integrator : $\frac{V_{out}}{V_{in}}(s) = \frac{1}{s}$

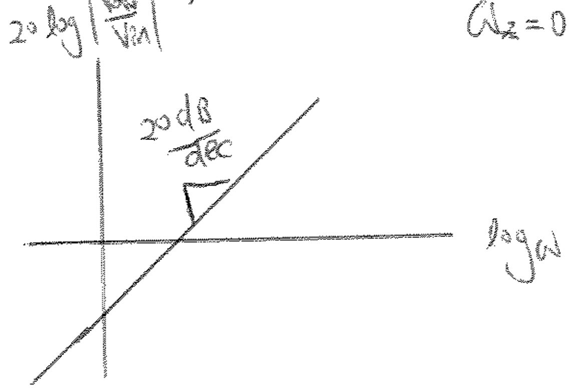
$$\left|\frac{V_{out}}{V_{in}}(j\omega)\right| = \frac{1}{\omega}$$



For an integrator, the gain at arbitrary low freq approaches infinity.

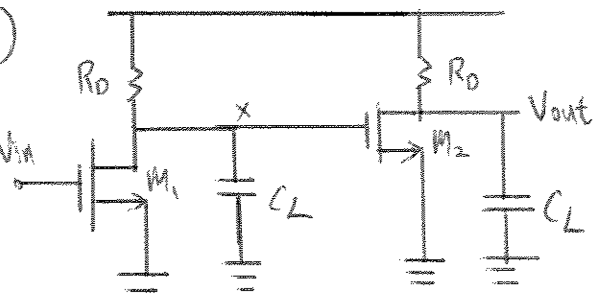
8) Ideal differentiator : $s = \frac{V_{out}}{V_{in}}$, $\left|\frac{V_{out}}{V_{in}}(j\omega)\right| = \omega$

$\omega_z = 0$



For an ideal differentiator, gain at arbitrary high freq approaches infinity.

9)



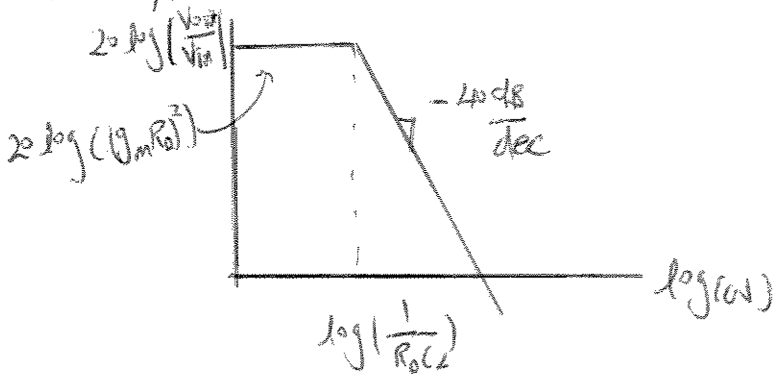
$\lambda = 0$, & neglect other caps.

DC gain: $\frac{V_x}{V_{in}} = -g_m R_D$, $\frac{V_{out}}{V_x} = -g_m R_D$

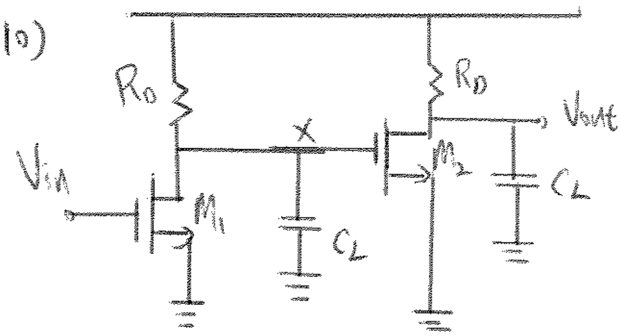
$$\frac{V_{out}}{V_{in}} = (g_m R_D)^2 \qquad (\text{At DC})$$

2 poles at $\frac{1}{R_D C_L}$

Bode plot:



10)



$$\frac{V_x}{V_{in}}(s) = -g_m\left(R_D \parallel \frac{1}{C_L s}\right), \quad \frac{V_{out}}{V_x}(s) = -g_m\left(\frac{R_D}{R_D C_L s + 1}\right)$$

$$= -g_m\left(\frac{R_D}{R_D C_L s + 1}\right)$$

$$H(s) = \frac{V_x}{V_{in}}(s)\,\frac{V_{out}}{V_x}(s) = \left(\frac{g_m R_D}{R_D C_L s + 1}\right)^2$$

$$s \to \omega j, \quad H(j\omega) = \left(\frac{g_m R_D}{1 + R_D C_L j\omega}\right)^2$$

$$|H(j\omega)| = \frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2}$$

-3dB Band Width:

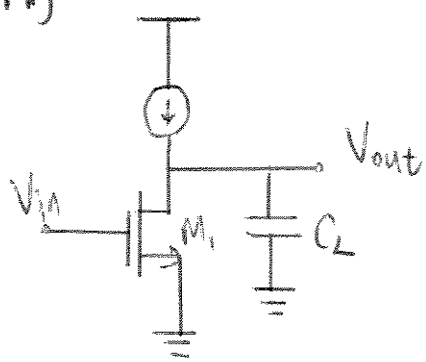
$$\frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2} = \frac{(g_m R_D)^2}{\sqrt{2}}$$

$$\Rightarrow (R_D C_L \omega)^2 + 1 = \sqrt{2}$$

$$\Rightarrow \omega = \frac{\sqrt{\sqrt{2} - 1}}{R_D C_L} = \frac{0.6436}{R_D C_L} \ (\text{rad}/s)$$

$$2\pi f = \frac{0.6436}{R_D C_L} \Rightarrow f = \frac{0.10243}{R_D C_L} \ (Hz)$$

1 a)



$\lambda > 0$

Since $\lambda > 0$, and we have an ideal current source, the impedance looking from out to ground is $r_o // \frac{1}{C_L s}$

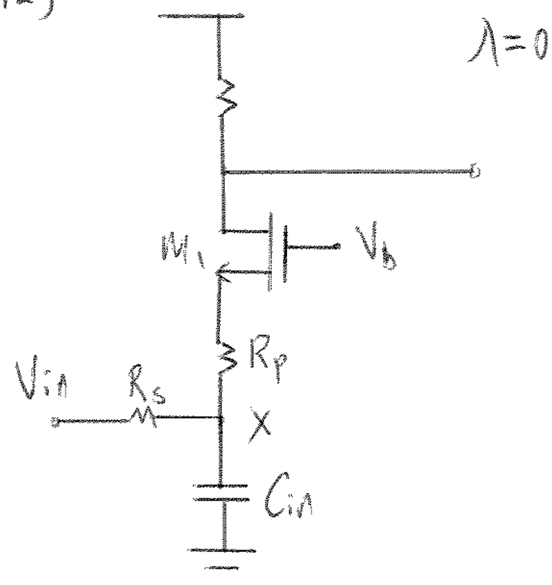
So, $V_{out} = -g_m V_{in} \left(r_o // \frac{1}{C_L s} \right)$

$$H(s) = -g_m \left(\frac{r_o}{r_o C_L s + 1} \right), \quad |H(j\omega)| = \frac{g_m r_o}{\sqrt{(r_o C_L \omega)^2 + 1}}$$

For $\lambda \to 0$, $r_o \to \infty \Rightarrow H(s) \to \frac{-g_m r_o}{r_o C_L s}$

$H(s) = \frac{-g_m}{C_L s}$. A pole at origin, thus operating as an ideal integrator.

12)



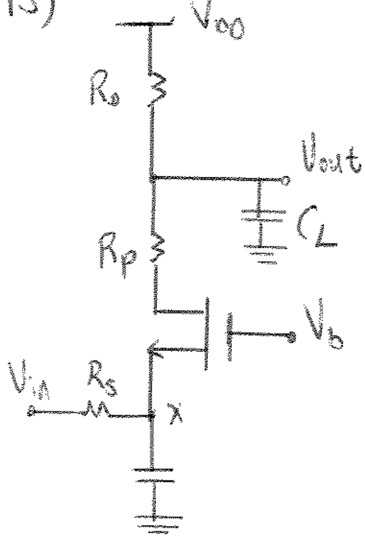
To find input pole, let $V_{in} = 0$ and find the equivalent resistance and capacitance from node X to ground.

$$R_x = R_s \,//\, \left(R_p + \frac{1}{g_{m_1}} \right), \qquad C_x = C_{in}$$

$$\omega_{p,in} = \frac{1}{C_{in}\left[R_s \,//\, \left(R_p + \frac{1}{g_m}\right)\right]}$$

$$\omega_{p,out} = \frac{1}{R_D C_L}$$

13)



$\lambda = 0$, neglect all other caps.

$$R_X = R_S // \frac{1}{g_m}$$

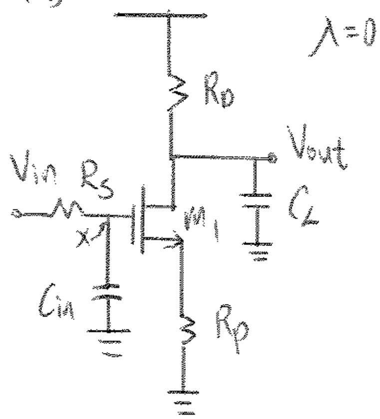
$$C_X = C_{in}$$

$$R_{out} = R_D \quad (\text{since } r_o = \infty)$$

$$C_{out} = C_L$$

$$\omega_{pin} = \frac{1}{(R_S // \frac{1}{g_m})\, C_{in}}, \qquad \omega_{pout} = \frac{1}{R_D C_L}$$

14)

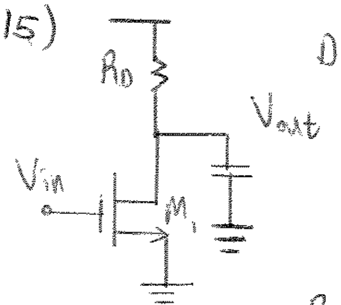


$R_X = R_S$, $R_{out} = R_D$

$C_X = C_{in}$, $C_{out} = C_L$

$$\omega_{pin} = \frac{1}{R_S C_{in}} \quad , \quad \omega_{pout} = \frac{1}{R_D C_L}$$

15)



DC Gain: $g_m R_D = \frac{2 I_D R_D}{V_{eff}}$

where $V_{eff} = V_{GS} - V_{th}$

Band Width: $\frac{1}{R_D C_L}$

Power Consumption: $V_{DD} I_D$

F.O.M. (11.5) $= \frac{\text{Gain} \times \text{Band Width}}{\text{Power Consumption}}$

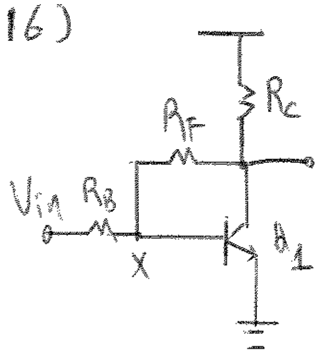
$$= \frac{\left(\frac{2 I_D R_D}{V_{eff}}\right)\left(\frac{1}{R_D C_L}\right)}{V_{DD} I_D}$$

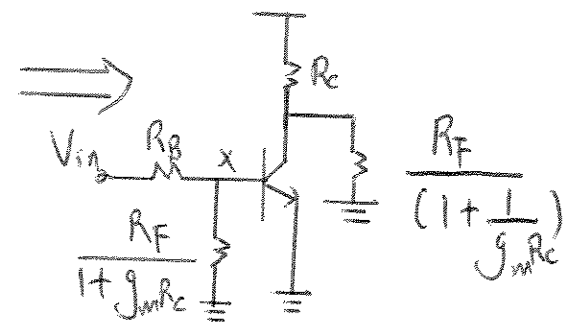
$$= \frac{2}{V_{eff} V_{DD} C_L}$$

For practical design, $V_{eff} > V_t$, thus bipolar has a larger F.O.M. than MOS.

16)

$V_A = \infty$, R_F is larg

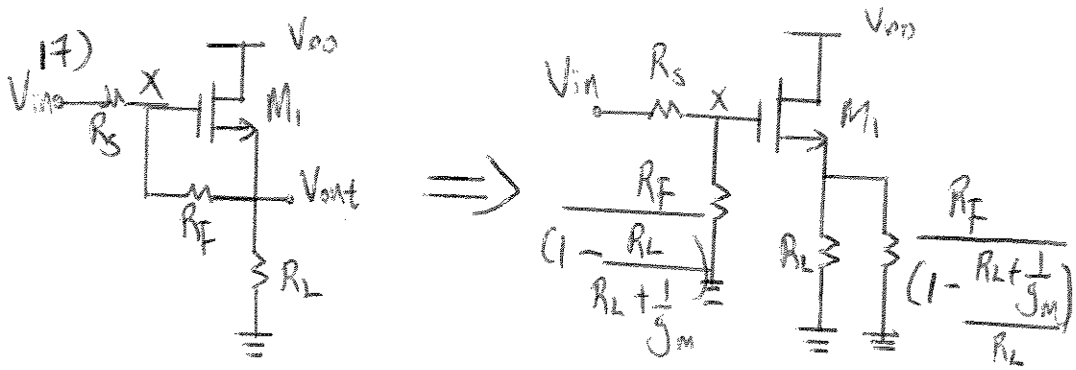




$$R_X = R_B \parallel \left(\frac{R_F}{1+g_m R_C}\right), \quad R_{out} = R_C \parallel \left(\frac{R_F}{1+\frac{1}{g_m R_C}}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-R_{out}}{\frac{1}{g_m} + \frac{R_X}{\beta+1}} = \frac{-R_C \parallel \left(\frac{R_F}{1+1/g_m R_C}\right)}{\frac{1}{g_m} + \frac{R_B \parallel (R_F/(1+g_m R_C))}{\beta+1}}$$

17)



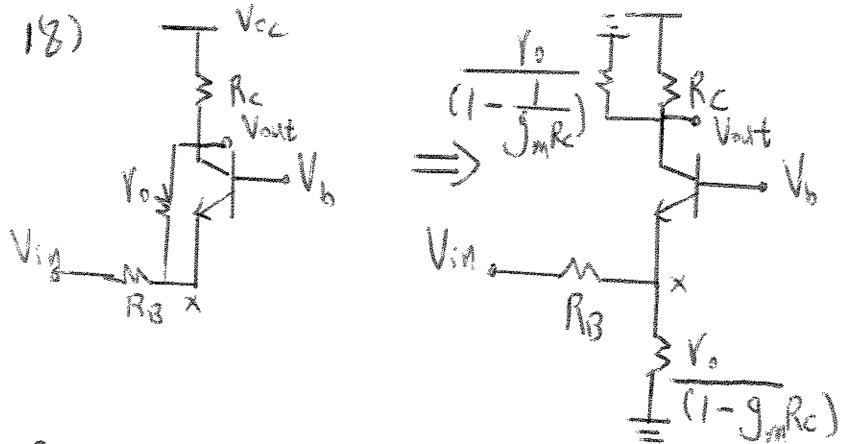
$$R_{out}: R_L \parallel \frac{R_F}{1-\frac{R_L+\frac{1}{g_m}}{R_L}} = R_L \parallel \frac{R_F}{-\frac{1}{g_m R_L}}$$

$$R_{out}: R_L \parallel -R_F g_m R_L$$

(note that R_{out} may be negative)

$$\frac{V_{out}}{V_{in}} = \frac{R_L \parallel -R_F g_m R_L}{R_L \parallel (-R_F R_L g_m) + \frac{1}{g_m}}$$

18)

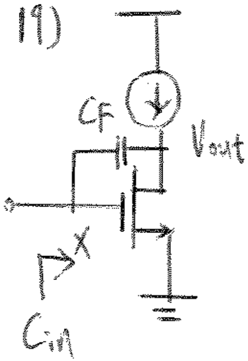


$$R_{out} = R_c \parallel \left(\frac{r_o}{1-\frac{1}{g_m R_c}} \right)$$

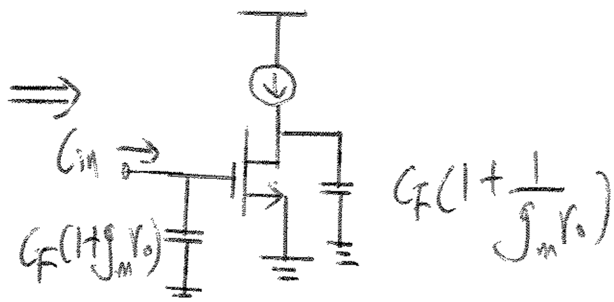
$$R_x = R_B \parallel \left(\frac{r_o}{1-g_m R_c} \right)$$

$$\frac{V_{out}}{V_{in}} = \frac{R_{out}}{R_x + \frac{1}{g_m}} = \frac{R_c \parallel \left(\frac{r_o}{1-\frac{1}{g_m R_c}} \right)}{R_B \parallel \left(\frac{r_o}{1-g_m R_c} \right) + \frac{1}{g_m}}$$

19)



$\lambda > 0$, DC gain $= -g_m r_o$

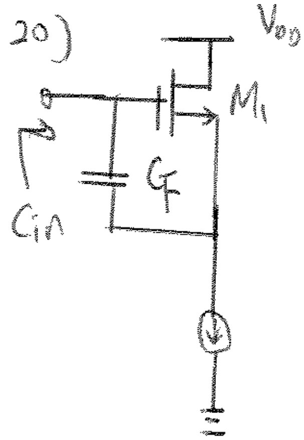


$C_{in} = C_F(1+g_m r_o)$, neglecting other caps.

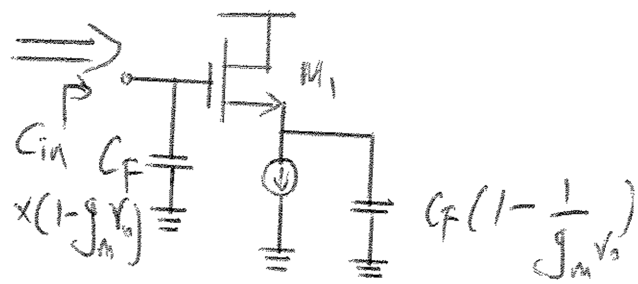
As $\lambda \to 0$, $r_o \to \infty$, DC gain $\to \infty$,

$C_{in} \to \infty$, this bandwidth will $\longrightarrow 0$.

20)



$\lambda > 0$, DC gain $= g_m r_o$

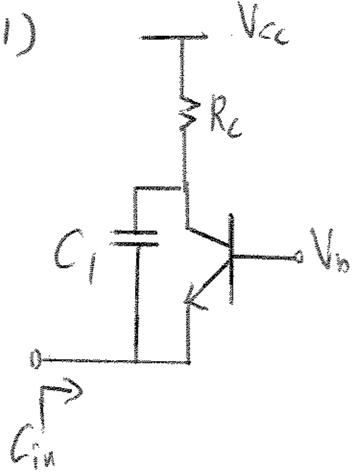


$$C_{in} = C_F(1 - g_m r_o)$$

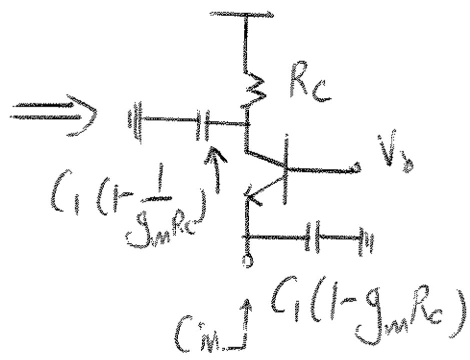
As $\lambda \to 0$, $r_o \to \infty$, $g_m r_o \to \infty$, $C_{in} = -\infty$

When $C \to$ negative in value, we have inductive activity. So right here, we have an effective infinite inductor.

21)



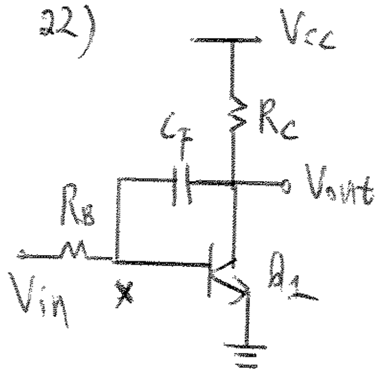
DC gain: $g_m R_c$



$$C_{in} = C_1(1 - g_m R_c).$$

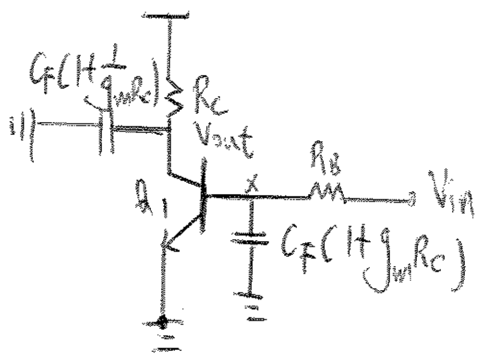
If $g_m R_c$ is designed to be larger than 1, as it normally would, we will have inductive action.

22)



DC gain (from x to out):

$-g_m R_c$



$C_{in} = C_F(1+g_m R_c)$

$R_{in} = R_B // r_\pi$

$C_{out} = C_F(1+\frac{1}{g_m R_c})$

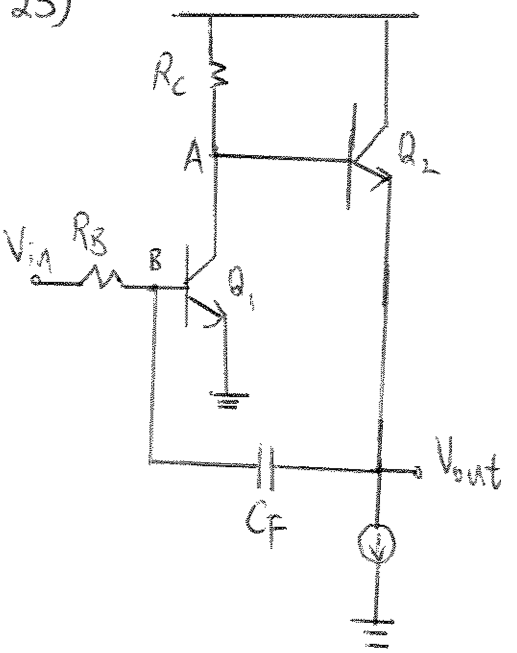
$R_{out} = R_c$

$$\omega_{pin} = \frac{1}{R_B // r_\pi \left[C_F(1+g_m R_c)\right]}$$

$$\omega_{pout} = \frac{1}{R_c C_F(1+\frac{1}{g_m R_c})} \approx \frac{1}{R_c C_F}$$

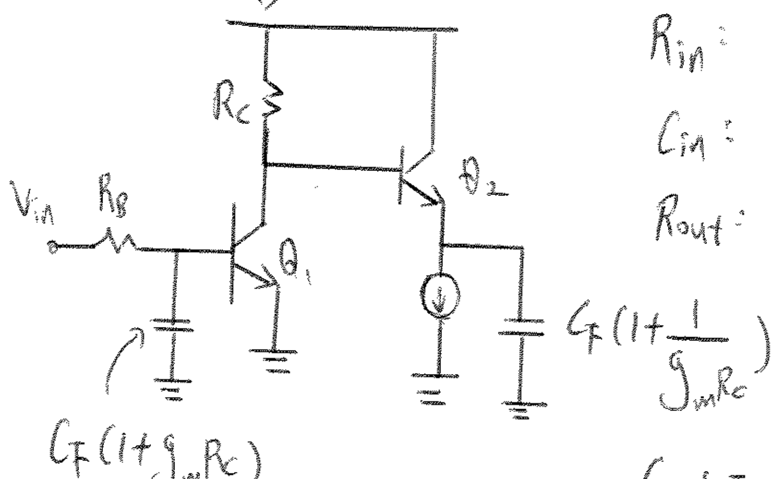
(If $g_m R_c \gg 1$)

23)



The gain from B to A is $-g_m R_c$, from A to out is 1 (since we have an ideal current source). So the gain from B to out is $-g_m R_c$.

$\Downarrow$



R_{in}: $R_B \,//\, r_\pi$

C_{in}: $C_F(1+g_m R_c)$

R_{out}: $\frac{1}{g_m} + \frac{R_c}{\beta+1}$

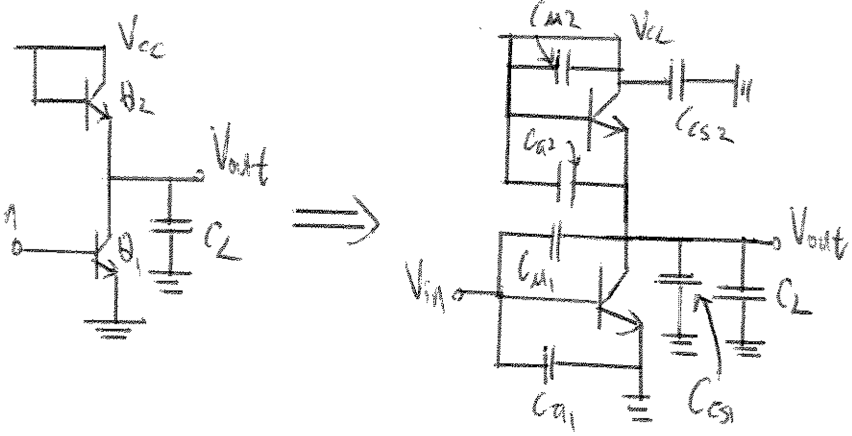
$C_{out} = C_F\left(1+\frac{1}{g_m R_c}\right)$

$$\omega_{pin} = \frac{1}{R_B // r_\pi \left[C_F(1+g_m R_c)\right]}$$

$$\omega_{pout} = \frac{1}{\left(\frac{1}{g_m}+\frac{R_c}{\beta+1}\right) C_F\left(1+\frac{1}{g_m R_c}\right)} \approx \frac{1}{\left(\frac{1}{g_m}+\frac{R_c}{\beta+1}\right) C_F}, \quad (g_m R_c \gg 1)$$

24)

a)

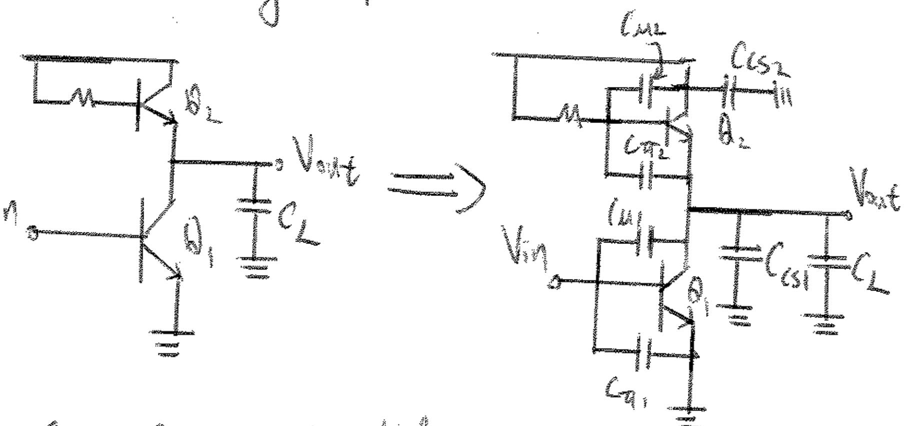


$C_{\pi 2}$, C_{CS1}, C_L are in parallel

$C_{\mu 2}$, C_{CS2} are grounded on both ends.

(and technically in parallel as well)

b)

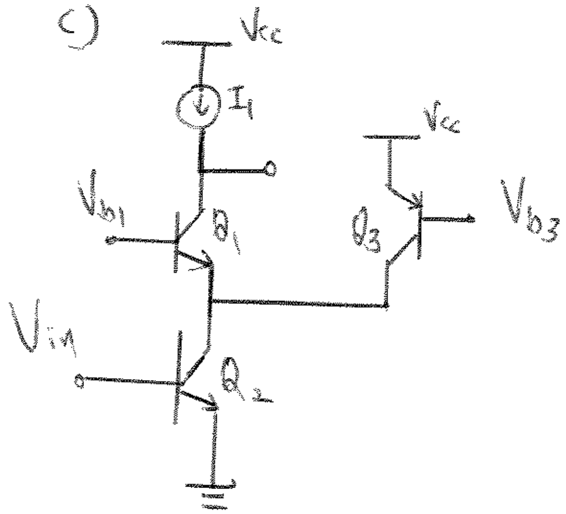


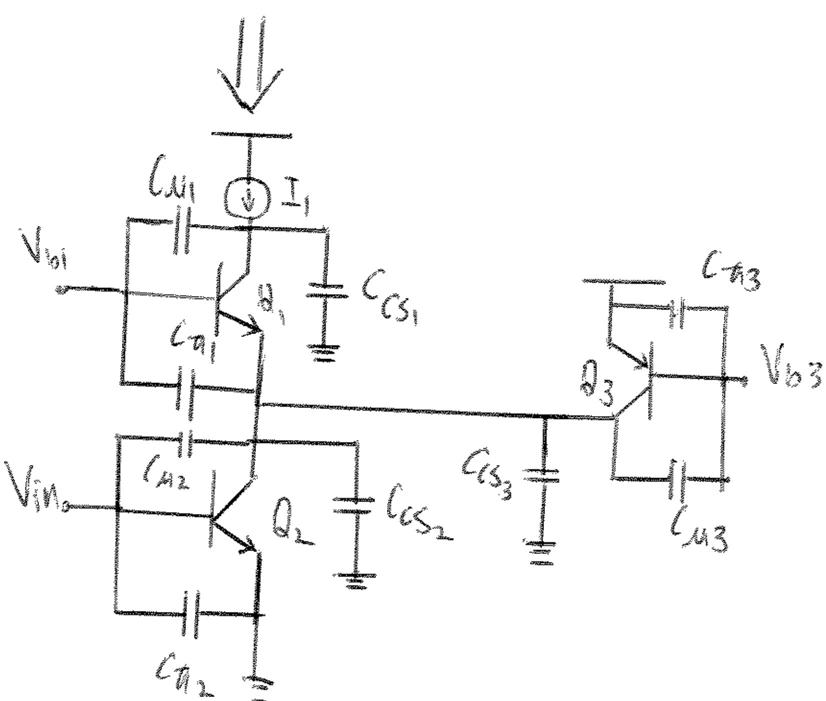
C_{CS1}, C_L are in parallel

C_{CS2} is grounded on both ends

24)

c)



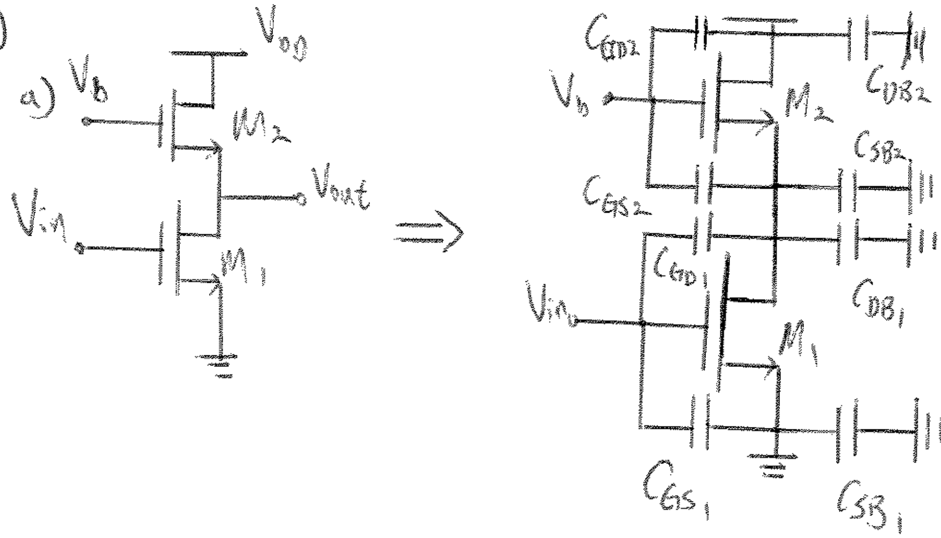


$C_{\pi 1}$, C_{CS_2}, C_{CS_3}, $C_{\mu 3}$ are in parallel
$C_{\mu 1}$, C_{CS_1} are also in parallel

$C_{\pi 3}$ is grounded on both ends

25)

a)

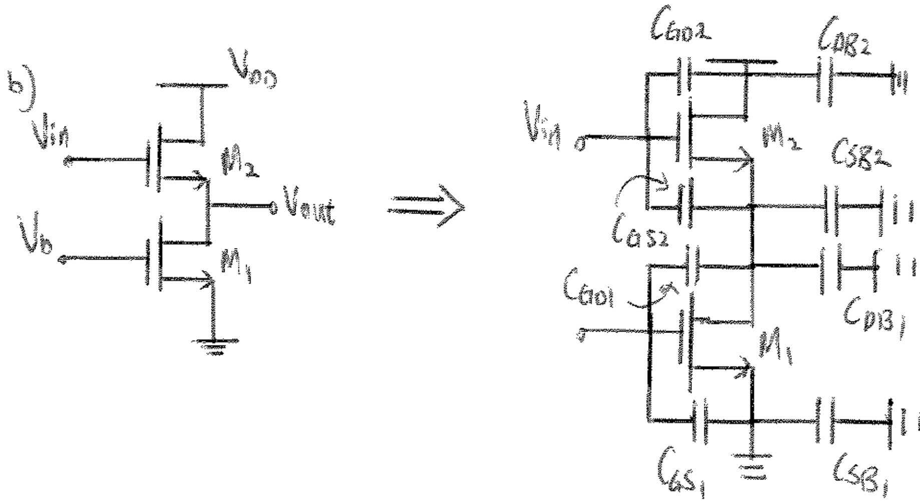


C_{GS2}, C_{SB2}, C_{DB1} are in parallel

C_{GD2}, C_{DB2} are in parallel and grounded on both ends

C_{SB1} is grounded on both ends.

b)



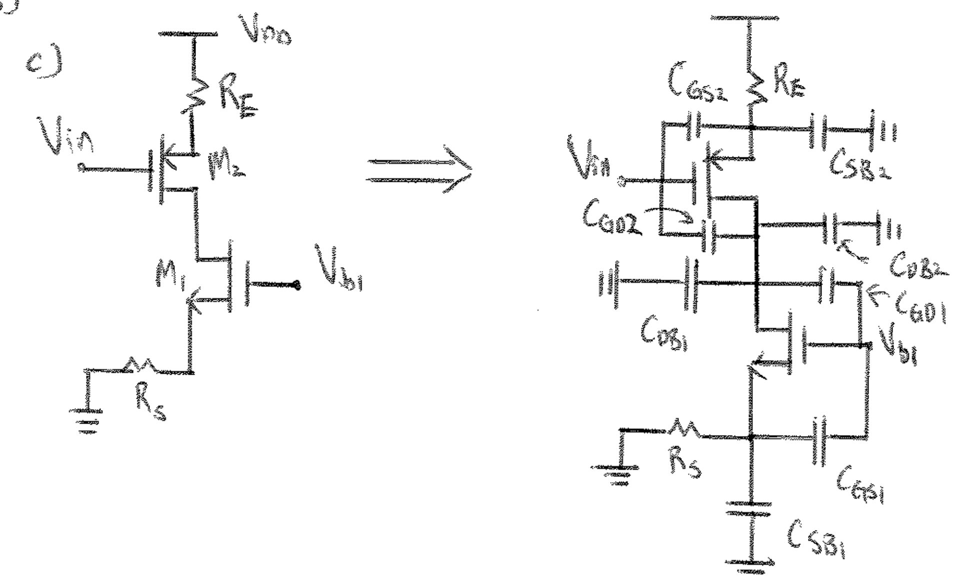
C_{GD1}, C_{DB1}, C_{SB2} are in parallel

C_{GS1}, C_{SB1} are in parallel and grounded on both ends

C_{DB2} is grounded on both ends.

25)

c)

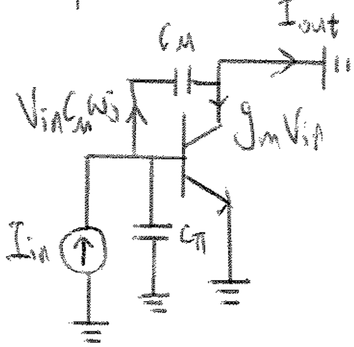


C_{DB2}, C_{GD1}, C_{DB1}, are in parallel

C_{SB1}, C_{GS1} are also in parallel.

2b)

Bipolar



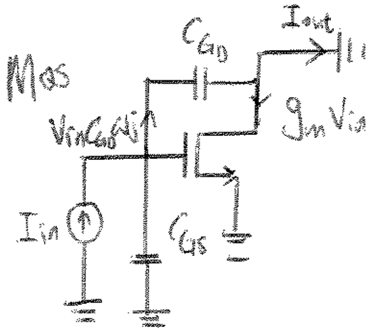
$$V_{in} = (I_{in})\left(\frac{1}{[C_\mu + C_\pi]\omega j}\right)$$ (Assuming we are at freq, and r_π can be neglected)

$$I_{out} = V_{in} C_\mu \omega j - g_m I_{in}\left(\frac{1}{[C_\mu + C_\pi]\omega j}\right)$$

$$\frac{I_{out}}{I_{in}} = \frac{C_\mu \omega j - g_m}{[C_\mu + C_\pi]\omega j} \Rightarrow \left|\frac{I_{out}}{I_{in}}\right| = \frac{\sqrt{(g_m)^2 + (C_\mu \omega_T)^2}}{[C_\mu + C_\pi]\omega_T} = 1$$

$$\omega_T^2 = \frac{g_m^2}{2C_\mu C_\pi + C_\pi^2} \Rightarrow \omega_T = 2\pi f_T = \frac{g_m}{\sqrt{2C_\mu C_\pi + C_\pi^2}}$$

MOS



Similarly for MOS, with C_μ and C_π replaced by C_{GD} and C_{GS} respectively.

$$\omega_T = 2\pi f_T = \frac{g_m}{\sqrt{2C_{GD}C_{GS} + C_{GS}^2}}$$

27)

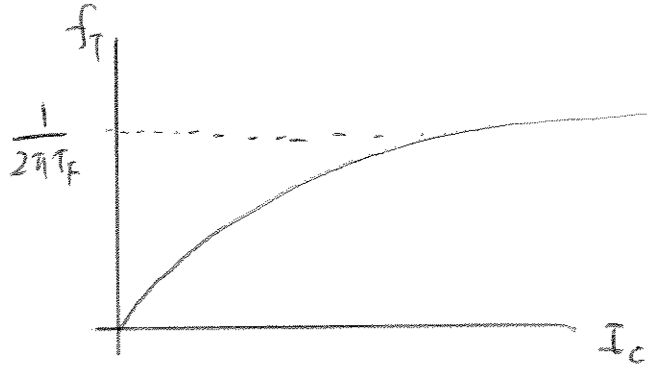
$$C_\pi = g_m \tau_F + C_{je}$$

$$2\pi f_T = \frac{g_m}{C_\pi} = \frac{g_m}{g_m \tau_F + C_{je}}$$

Assume C_{je} to be independent of I_C.

a) $$2\pi f_T = \frac{\frac{I_C}{V_T}}{\frac{I_C}{V_T}\tau_F + C_{je}} \Rightarrow f_T = \frac{I_C}{2\pi(I_C \tau_F + V_T C_{je})}$$

b)



As $I_C \to \infty$, $f_T \to \frac{1}{2\pi\tau_F}$

28)

$$C_{GS} \approx \left(\frac{2}{3}\right) W L C_{ox}$$

$$2\pi f_T = \frac{g_m}{C_{GS}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})}{\frac{2}{3} W L \; C_{ox}}$$

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(\, V_{GS} - V_{TH})$$

29)

$$2\pi f_T = \frac{3}{2}\frac{2I_D}{WLC_{ox}}\frac{1}{(V_{GS}-V_{TH})}$$

Apparently, f_T decreases with the overdrive. However, when we look closely, I_D is actually proportional to $(V_{GS}-V_{TH})^2$ (In Saturation), so f_T is proportional to $(V_{GS}-V_{TH})$.

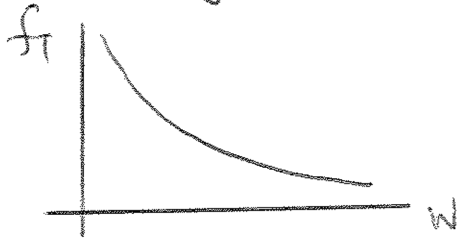
3c)

a) As $W \uparrow$, $(V_{GS} - V_{TH})$ has to $\downarrow$ by

$\frac{1}{\sqrt{W}}$ in order to maintain I_D constant

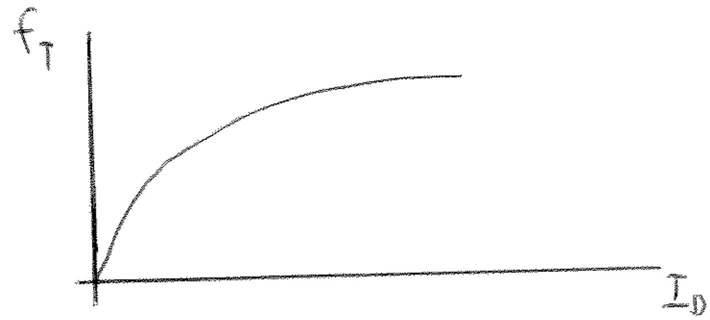
using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

$2\pi f_T \propto \frac{1}{\sqrt{W}}$



b) $I_D \uparrow$, W constant it means $V_{GS} - V_{TH} \uparrow$ with $\sqrt{I_D}$. Using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

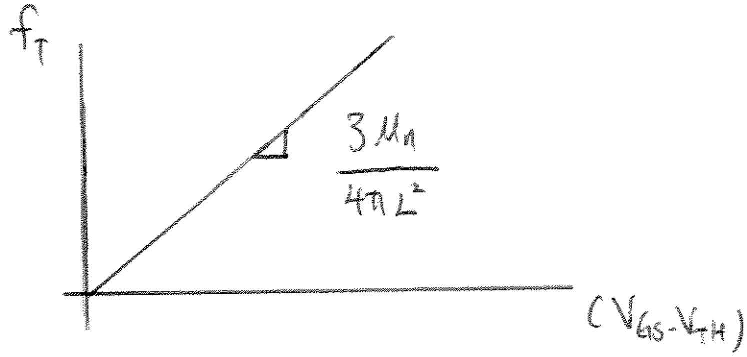
$2\pi f \propto \sqrt{I_D}$



31)

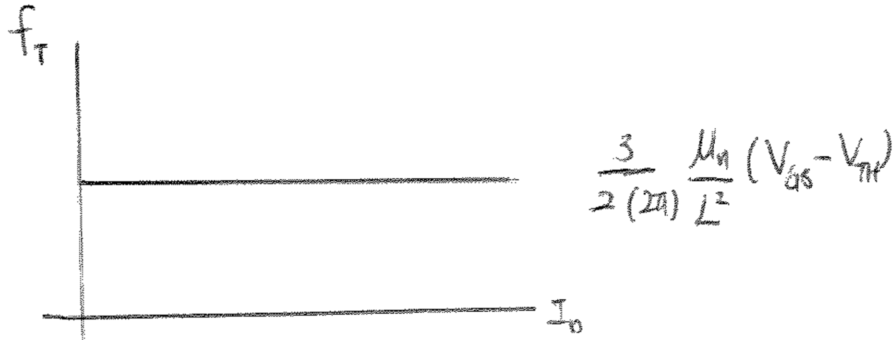
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

a) $2\pi f_T \propto (V_{GS}-V_{TH})$



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

$2\pi f$ = constant for all I_D.

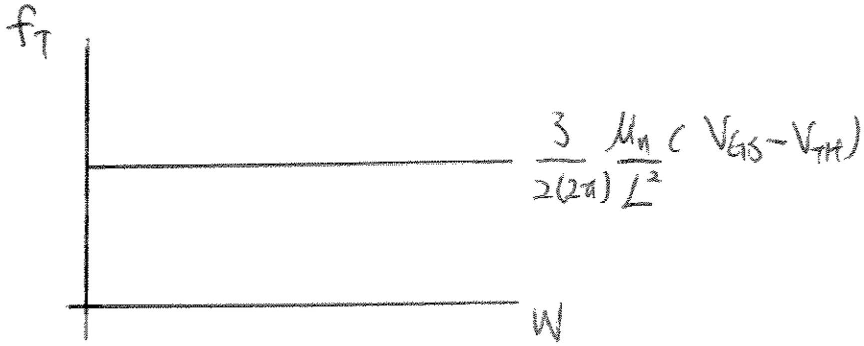


32)

a)

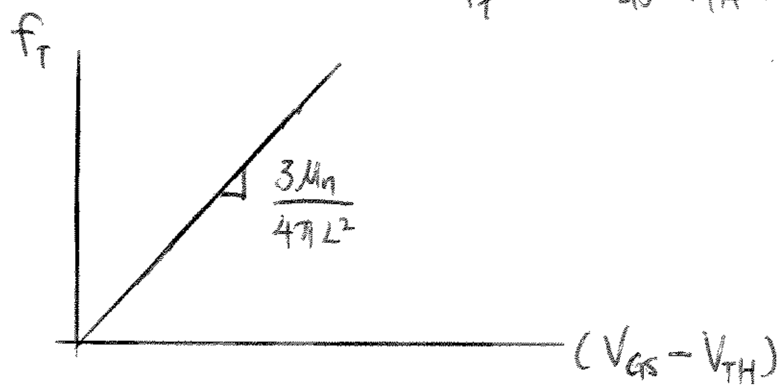
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$

we know that $2\pi f_T$ is constant for all W.



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$,

we know that $2\pi f_T \propto (V_{GS} - V_{TH})$.



33)

a) $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})^2$

As $L\uparrow$, to maintain the same current and overdrive voltage, $W\uparrow$ as well.

So W also 2X.

b) Since $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$, and

L 2X while $(V_{GS}-V_{TH})$ is constant,

$f_T \downarrow$ by $\frac{3}{4}$ or $f_{T_{new}} = \frac{1}{4} f_{T_{old}}$.

34)

a) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

Constant I_D and $W\uparrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

b) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

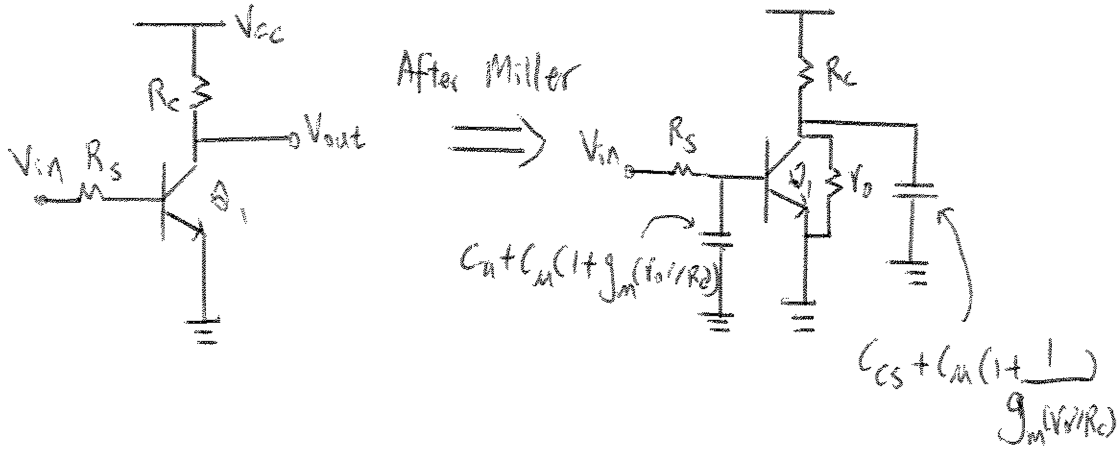
Constant W and $I_D \downarrow$ (L constant)

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

35)

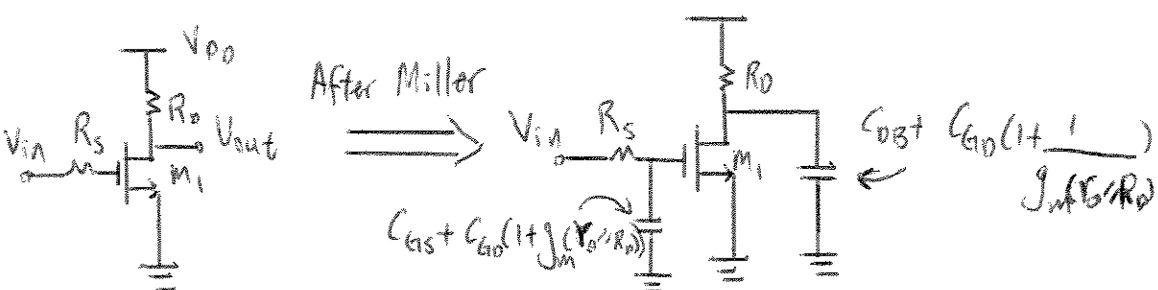
Bipolar CE Stage



$$\omega_{p_{in}}: \frac{1}{(R_S // r_\pi)[C_\pi + C_\mu(1+g_m(r_o // R_C))]}$$

$$\omega_{p_{out}}: \frac{1}{(R_C // r_o)[C_{CS} + C_\mu(1+1/g_m(r_o // R_C))]}$$

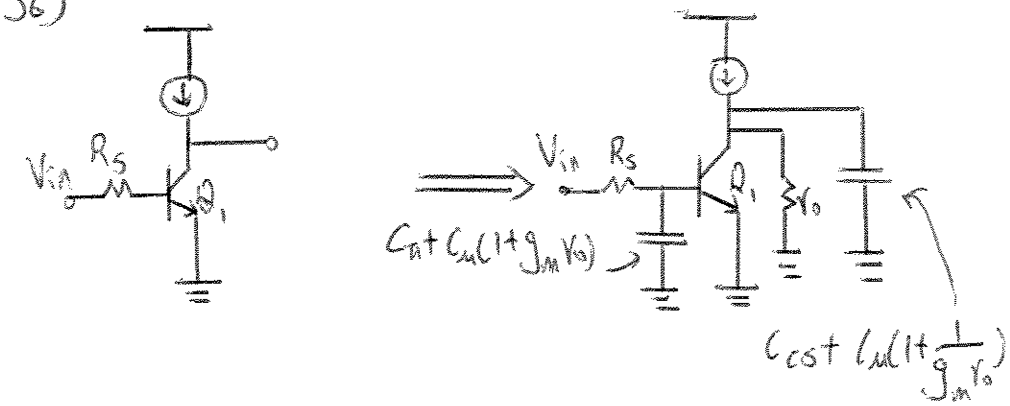
MOS CS Stage



$$\omega_{p_{in}}: \frac{1}{R_S[C_{GS} + C_{GD}(1+g_m(r_o // R_D))]}$$

$$\omega_{p_{out}}: \frac{1}{(R_D // r_o)[C_{DB} + C_{GD}(1+1/g_m(r_o // R_D))]}$$

36)



$$\omega_{p_{in}} = \frac{1}{(R_S \parallel r_\pi)[C_\pi + C_\mu(1+g_m r_o)]}$$

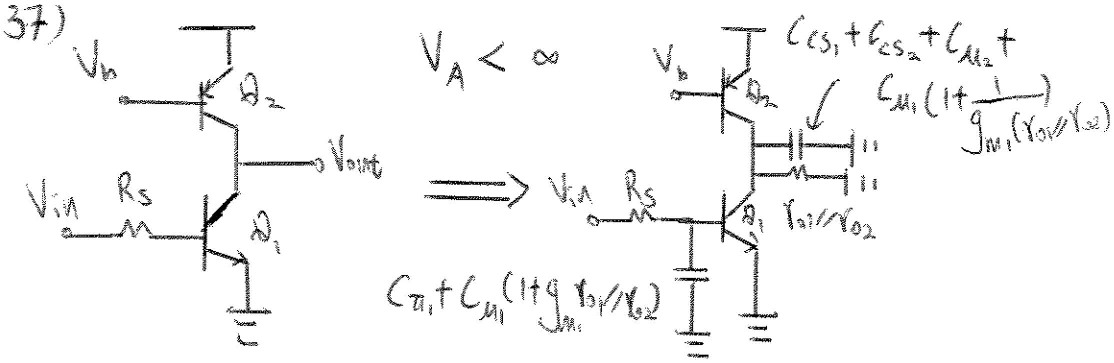
$$\omega_{p_{out}} = \frac{1}{r_o[C_{cs} + C_\mu(1+1/g_m r_o)]}$$

$$H(s) = \frac{\text{DC gain}}{\left(1+\frac{s}{\omega_{p_{in}}}\right)\left(1+\frac{s}{\omega_{p_{out}}}\right)}$$

$$H(s) = \frac{g_m r_o (r_\pi/(r_\pi+R_S))}{\left(1+\frac{s}{1/(R_S \parallel r_\pi)[C_\pi+C_\mu(1+g_m r_o)]}\right)\left(1+\frac{s}{1/(r_o[C_{cs}+C_\mu(1+\frac{1}{g_m r_o})])}\right)}$$

37)

$V_A < \infty$



$$\omega_{pin} = \frac{1}{(R_S // r_{\pi_1})\left[C_{\pi_1} + C_{\mu}(1 + g_{m_1}(r_{o1} // r_{o2}))\right]}$$

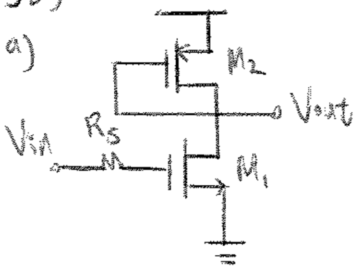
$$\omega_{pout} = \frac{1}{(r_{o1} // r_{o2})\left[C_{cs_1} + C_{cs_2} + C_{\mu_2} + C_{\mu_1}(1 + 1/(g_{m_1} r_{o1} // r_{o2}))\right]}$$

$$g_m r_o \gg 1$$

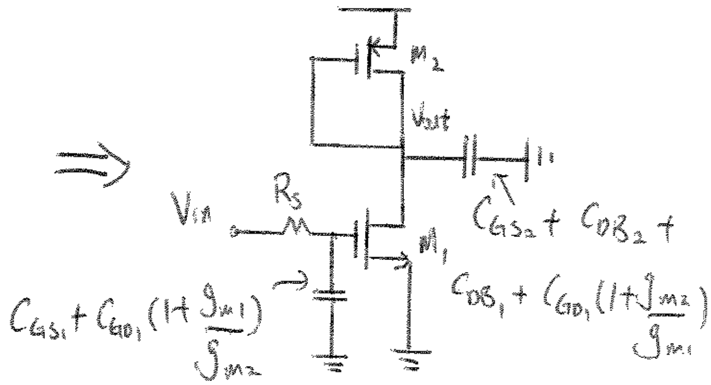
$$H(s) = \frac{g_m(r_{o1} // r_{o2})(r_\pi / r_\pi + R_S)}{(1 + R_S[C_{\pi_1} + C_\mu(g_m(r_{o1} // r_{o2}))]s)(1 + (r_{o1} // r_{o2})[C_{cs1} + C_{cs2} + C_{\mu 2} + C_{\mu 1}]s)}$$

38)

a)



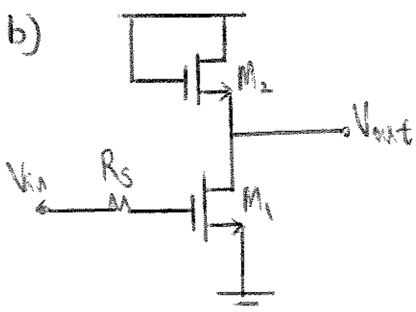
$\Longrightarrow$



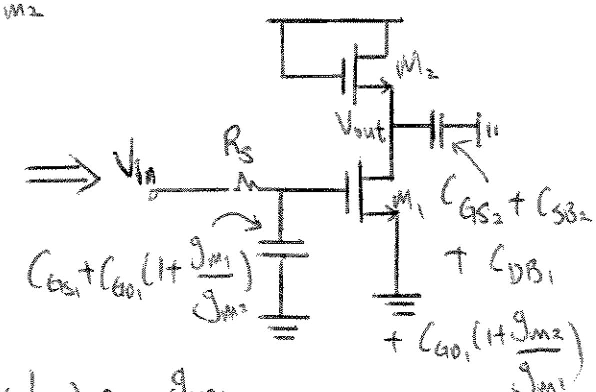
$$\text{DC gain} = -g_{m_1}\left(r_{o_1} \,//\, r_{o_2} \,//\, \frac{1}{g_{m_2}}\right) \approx -\frac{g_{m_1}}{g_{m_2}}$$

$$\omega_{p_{in}} = \frac{1}{R_S\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m_1}}{g_{m_2}}\right)\right)} \qquad \omega_{p_{out}} = \frac{g_{m_2}}{\left(C_{GS_2} + C_{DB_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m_2}}{g_{m_1}}\right)\right)}$$

b)



$\Longrightarrow$



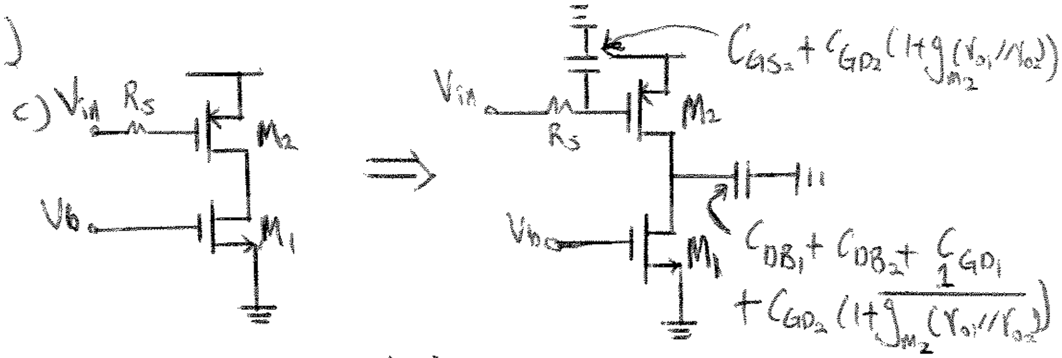
$$\text{DC gain} = -g_{m_1}\left(r_{o_1} \,//\, r_{o_2} \,//\, \frac{1}{g_{m_2}}\right) \approx -\frac{g_{m_1}}{g_{m_2}}$$

$$\omega_{p_{in}} = \frac{1}{R_S\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m_1}}{g_{m_2}}\right)\right)}$$

$$\omega_{p_{out}} = \frac{g_{m_2}}{C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m_2}}{g_{m_1}}\right)}$$

38)

c)



DC gain: $-g_{m_2}(r_{o1} // r_{o2})$

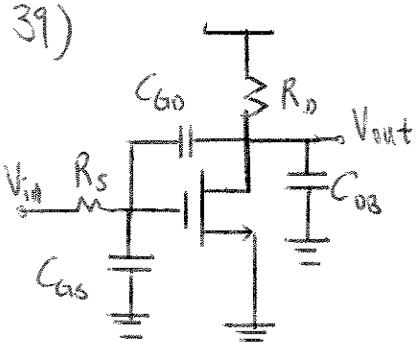
$$\omega_{pin} = \frac{1}{R_S\left(C_{GS_2} + C_{GD_2}(1 + g_{m_2}(r_{o1} // r_{o2}))\right)}$$

$$\omega_{pout} = \frac{1}{(r_{o1} // r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\left(1 + \frac{1}{g_{m_2}(r_{o1} // r_{o2})}\right)\right]}$$

$$\omega_{pout} \approx \frac{1}{(r_{o1} // r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\right]}$$

since $g_{m_2}(r_{o1} // r_{o2}) >> 1$

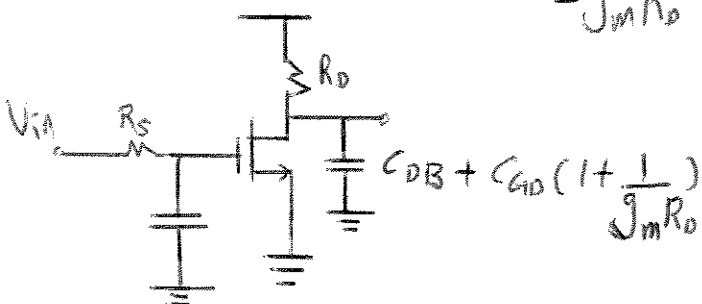
39)



$R_S = 200\Omega \quad C_{GS} = 50fF$
$R_D = 1K\Omega \quad C_{GD} = 10fF$
$I_D = 1mA \quad C_{DB} = 15fF$

a) Miller's approximation:

$$-g_m R_D = \frac{-(1)(2)(1)}{(0.2)} = -10$$



$C_{GS} + (1+g_m R_D)C_{GD}$

$$\omega_{pin} = \frac{1}{R_S(C_{GS} + (1+g_m R_D)C_{GD})} = \frac{1}{200(50fF + (11)(10fF))} = 31.25GHz$$

$$\omega_{pout} = \frac{1}{R_D(C_{DB} + (1+\frac{1}{10})C_{GD})} = \frac{1}{1000(15fF + (1+0.1)10fF)} = 38.46GHz$$

39)

b) Equation $\frac{V_{out}}{V_{thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$

$a = R_{thev} R_L (C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$

$R_{Thev} = R_S$, $C_{in} = C_{GS}$, $C_{out} = C_{DB}$, $R_L = R_D$, $C_{XY} = C_{GD}$

$a = (200)(1000)[(50)(10^{-15})(10)(10^{-15}) + (15)(10^{-15})(10)(10^{-15}) + (50)(10^{-15})(15)(10^{-15})] = 2.8 \times 10^{-22}$

$b = (1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})$

$b = (1+10)(10)(10^{-15})(200) + (200)(50 \times 10^{-15}) + (1000)(10)(10^{-15}) + 1000(15)(10^{-15})$

$b = 5.7 \times 10^{-11}$

So denominator $= (2.8 \times 10^{-22} s^2 + 5.7 \times 10^{-11} s + 1)$

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$S = -1.93909 \times 10^{10}, \; -1.84181 \times 10^{11}$$

$\omega_{p_1} = 19.39\,GHz$, $\omega_{p_2} = 184.2\,GHz$

Which one is ω_{pin}, and which one is ω_{pout}?

This can be seen from inspection, at output and high frequency C_{GD} starts to become a short and thus the output resistance collapses to $1/g_m$, and pushes the output pole out. Whereas at the input the pole location does not change too much because R_S is small and C_{GS} is large. Therefore, we conclude that when solving the transfer function directly, the ω_{pin} is 19.39 GHz (on the same order as

39)

b). that obtained from Miller's approximation), while ω_{pout} is pushed out significantly, 184.2 GHz (when compared to that obtained from Miller's approximation).

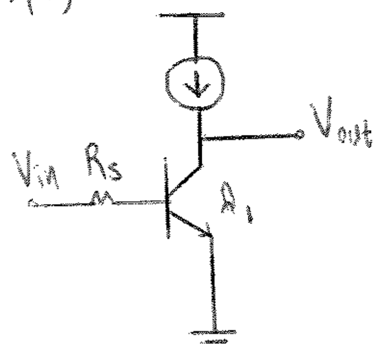
<u>Miller Approximation</u>

$\omega_{pin} = 31.25\,GHz$
$\omega_{pout} = 38.46\,GHz$

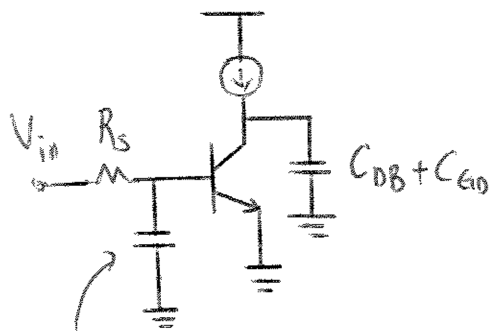
<u>Transfer Function</u>

$\omega_{pin} = 19.39\,GHz$
$\omega_{pout} = 184.2\,GHz$

40)



a) Miller's Approximation: DC gain: $-\infty$



$$\omega_{pin} = \frac{1}{R_S(\infty)} = 0, \quad \omega_{pout} = \frac{1}{\infty(C_{DB}+C_{GD})} = 0$$

b) Transfer Function:

$$\frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY}s - g_m)R_L}{as^2 + bs + 1}$$

$$a = R_{Thev}R_L(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})$$

$$b = (1+g_mR_L)C_{XY}R_{Thev} + R_{Thev}C_{in} + R_L(C_{XY} + C_{out})$$

40)

b) $R_L \to \infty$

$$\frac{V_{out}}{V_{thev}} = \frac{C_{XY}S - g_m}{S[R_{thev}(C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out})S + g_m C_{XY} R_{thev} + (C_{XY} + C_{out})]}$$

So $\omega_{p_1} = 0$

$$\omega_{p_2} = \frac{(g_m C_{XY} R_{thev} + (C_{XY} + C_{out}))}{R_{thev}[C_{in}C_{XY} + C_{out}C_{XY} + C_{in}C_{out}]}$$

$$\omega_{p_2} = \frac{g_m C_\mu R_S /\!/ R_\pi + C_\mu + C_{cs}}{R_S /\!/ r_\pi [C_\pi C_\mu + C_{cs}C_\mu + C_\pi C_{cs}]}$$

$\omega_{p_1} = \omega_{pin}$, $\omega_{p_2} = \omega_{pout}$.

<u>Miller:</u>

$\omega_{pin} = 0$, $\omega_{pout} = 0$

Again, the output pole predicted by the transfer function is pushed out, and the input poles are similar. (In fact, they are the same this time.)
This shows one of the short-comings of Miller's approximation.

4i) <u>Dominant-Pole approximation:</u>

$$\omega_{p_1} = \frac{1}{(1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}$$

$$\omega_{p_1} = 0 \qquad (\text{Since } R_L = \infty)$$

$$\omega_{p_2} = \frac{(1+g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}{R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

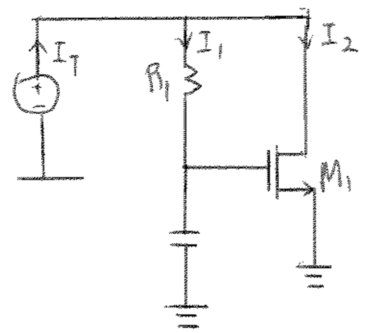
Since $R_L = \infty$

$$\omega_{p_2} = \frac{g_m C_{XY} R_{Thev} + C_{XY} + C_{out}}{R_{Thev} (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

$$\omega_{p_2} = \frac{g_m C_\mu R_S // R_\pi + C_\mu + C_{CS}}{R_S (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})}$$

Dominant-Pole approximation gives the same result as the transfer function method.

42)



$\lambda = 0$, and neglect other capacitances.

$$I_T = I_1 + I_2$$

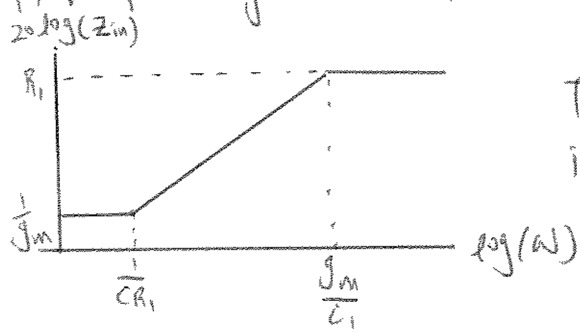
$$I_1 = \frac{V_T}{\left(R_1 + \frac{1}{C_1 s}\right)} , \quad I_2 = \frac{g_{m_1} V_T}{C_1 R_1 s + 1}$$

$$I_T = \frac{C_1 s V_T}{C_1 R_1 s + 1} + \frac{g_{m_1} V_T}{C_1 R_1 s + 1} \Rightarrow \frac{V_T}{I_T} = \frac{C_1 R_1 s + 1}{C_1 s + g_{m_1}}$$

$$s \to j\omega \Rightarrow \frac{C_1 R_1 (j\omega) + 1}{C_1 j\omega + g_{m_1}} = Z_T(j\omega)$$

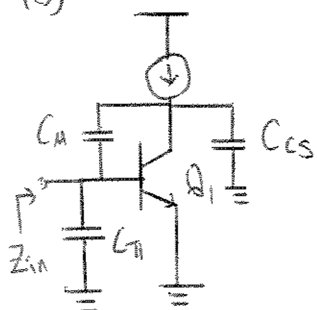
$$|Z_T| = |Z_{in}| = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{\sqrt{(C_1 \omega)^2 + g_{m_1}^2}} = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{g_m \sqrt{\left(\frac{C_1 \omega}{g_{m_1}}\right)^2 + 1}}$$

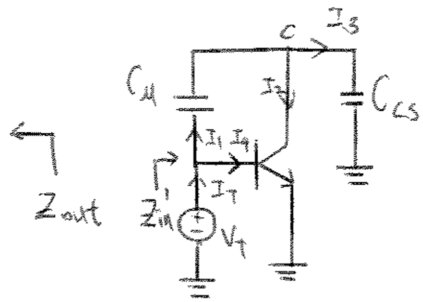
At $\omega = \frac{1}{C_1 R_1}$, we have a zero, at $\omega = \frac{g_{m_1}}{C_1}$, we have a pole. If $R_1 > \frac{1}{g_{m_1}}$, the zero is at a lower frequency than the pole, and the bode-plot for magnitude would look like the following.



The bode-plot shows an impedance that increases with frequency, an inductive behavior.

43)





$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s} \quad , \quad I_T = I_1 + I_4 = C_\mu s V_{bc} + \frac{g_m V_T}{\beta}$$

$$V_{bc} = V_T - V_c \quad , \quad V_c = (I_1 - g_m V_T)\frac{1}{C_{cs} s}$$

$$I_1 = \left[V_T - (I_1 - g_m V_T)\frac{1}{C_{cs} s} \right] C_\mu s$$

$$I_1 = V_T \left[C_\mu s + \frac{g_m C_\mu}{C_{cs}} \right] / \left(1 + \frac{C_\mu}{C_{cs}} \right)$$

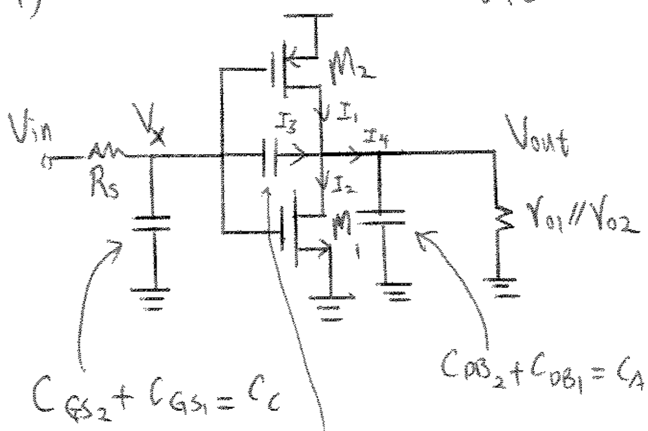
$$I_T = V_T \left[C_\mu s + \frac{g_m C_\mu}{C_{cs}} \right] / \left(1 + \frac{C_\mu}{C_{cs}} \right) + \frac{g_m V_T}{\beta}$$

$$Z_{in}' = \frac{V_T}{I_T} = \frac{1}{\frac{g_m}{\beta} + \frac{C_\mu s}{\left(1 + \frac{C_\mu}{C_{cs}}\right)} + \frac{g_m \frac{C_\mu}{C_{cs}}}{\left(1 + \frac{C_\mu}{C_{cs}}\right)}}$$

$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s} = r_\pi \,//\, \frac{1}{\frac{C_{cs} C_\mu s}{C_{cs} + C_\mu}} \,//\, \frac{1}{C_\pi s} \,//\, \frac{C_{cs} + C_\mu}{g_m C_\mu}$$

$$Z_{out} = \frac{1}{(C_\mu + C_{cs}) s}$$

44) $\lambda > 0$



$$V_{out} = I_4 \Big(\underbrace{r_{o1}//r_{o2} // \frac{1}{[C_{DB_2}+C_{DB_1}]s}}_{Z_{out}} \Big)$$

$$I_4 = I_1 + I_3 - I_2$$

$$I_1 = (0 - V_x) g_{m_2}$$

$$I_2 = V_x g_{m_1}$$

$$I_3 = (V_x - V_{out}) \big((C_{GD_1} + C_{GD_2}) s$$

$$I_4 = -V_x g_{m_2} + (V_x - V_{out}) C_B s - V_x g_{m_1}$$

$$V_{out} = Z_{out} \left[-V_x (g_{m_2} + g_{m_1}) + (V_x - V_{out}) C_B s \right]$$

Writing a node equation at X.

$$\frac{V_x - V_{in}}{R_s} + V_x C_c s + (V_x - V_{out}) C_B s = 0$$

$$V_x = \frac{V_{out} C_B s + V_{in}/R_s}{(1/R_s + C_c s + C_B s)}$$

44)

Subsitute everything and we get

$$V_{out} = Z_{out}\left[-(g_{m1}+g_{m2})\left(\frac{V_{out}C_BS + V_{in}/R_S}{1/R_S + C_cS + C_BS}\right) + \left(\frac{V_{out}C_BS + V_{in}/R_S}{1/R_S + C_cS + C_BS} - V_{out}\right)C_BS\right]$$

collect all the V_{out}'s on one-side and likewise for V_{in}'s, we will get

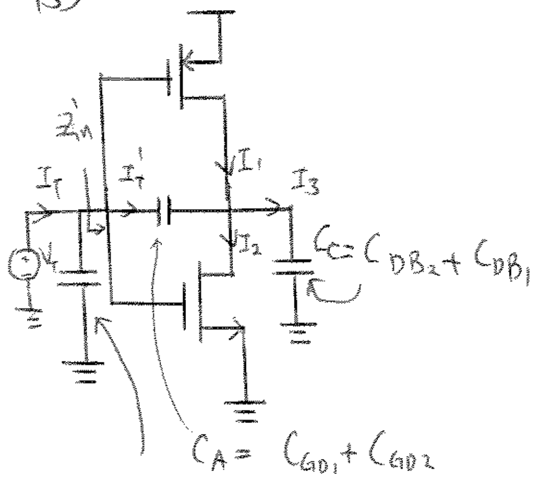
$$\frac{V_{out}}{V_{in}} = \frac{\frac{Z_{out}}{R_S}\left(C_BS - (g_{m1}+g_{m2})\right)}{1/R_S + (C_c+C_B)S + Z_{out}C_BS(g_{m1}+g_{m2}) + Z_{out}C_BS\left(\frac{1}{R_S}+(C_c+C_B)S\right) - Z_{out}C_B^2S^2}$$

where $Z_{out} = r_{o1} // r_{o2} // \dfrac{1}{[C_{DB1}+C_{DB2}]S}$

$$C_B = C_{GD1} + C_{GD2}$$

$$C_c = C_{GS1} + C_{GS2}$$

45)



$C_B = C_{GS_1} + C_{GS_2}$

$$Z_{in} = \frac{V_T}{I_T} = \frac{1}{C_B} \parallel Z_{in}'$$

$$Z_{in}' = \frac{V_T}{I_T'}$$

$$I_T' = \left[V_T - \left(I_3 \frac{1}{C_C s}\right)\right] C_A s$$

$$I_3 = I_T' - V_T g_{m2} - g_{m1} V_T$$

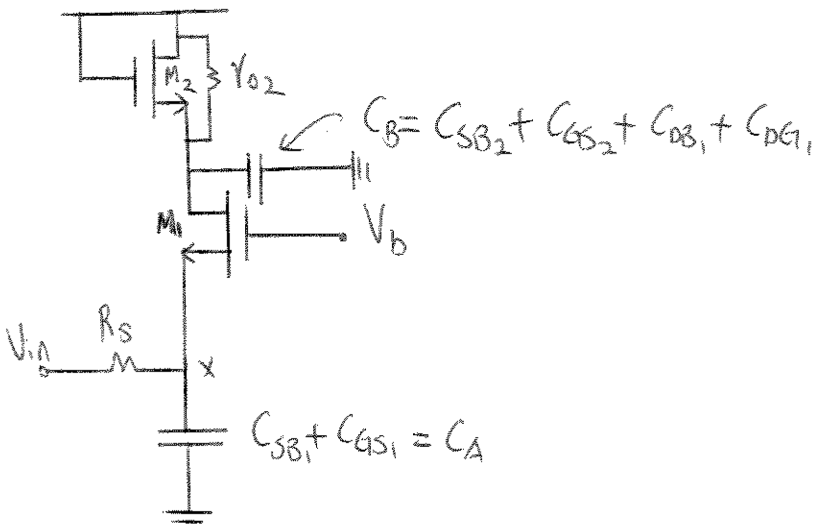
We get $\Rightarrow$ $I_T'\left(1 + \frac{C_A}{C_C}\right) = V_T\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]$

$$Z_{in}' = \frac{V_T}{I_T'} = \frac{\left(1 + \frac{C_A}{C_C}\right)}{\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]}$$

$$Z_{in} = \frac{1}{[C_{GS_1} + C_{GS_2}]s} \parallel \frac{\left(1 + \frac{C_{GD_1} + C_{GD_2}}{C_{DB_1} + C_{DB_2}}\right)}{\left[(C_{GD_1} + C_{GD_2})s + (g_{m1} + g_{m2})\frac{C_{GD_1} + C_{GD_2}}{C_{DB_2} + C_{DB_1}}\right]}$$

4b)

a)



$$V_{out} = -(0 - V_x) g_{m_1}\left[\frac{1}{g_{m_2}} \,//\, \frac{1}{C_B s}\right] = V_x g_{m_1}\left[\frac{1}{g_{m_2}} \,//\, \frac{1}{C_B s}\right]$$

Node equation at X, $\dfrac{V_x - V_{in}}{R_s} + V_x C_A s - g_m(0 - V_x) = 0$

$$V_x\left(\frac{1}{R_s} + C_A s + g_m\right) = \frac{V_{in}}{R_s} \Rightarrow V_x = \frac{V_{in}}{(1 + R_s C_A s + R_s g_m)}$$

subsitute in V_x and solving for V_{out}/V_{in} $\Rightarrow$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}\left[1/g_{m_2} \,//\, \frac{1}{C_B s}\right]}{(1 + R_s C_A s + R_s g_m)}$$

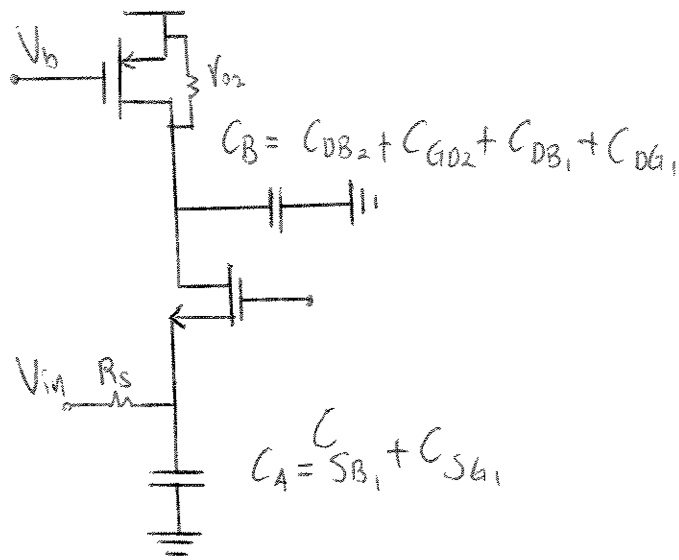
$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}(1/g_{m_2})}{(C_B(1/g_{m_2})s + 1)(1 + R_s C_A s + R_s g_{m_1})}$$

Where $C_B = C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{GS_1}$

4b)

b)



Similar to part a), with $\frac{1}{g_{m_2}}$ replaced by r_{o2},

and different C_B.

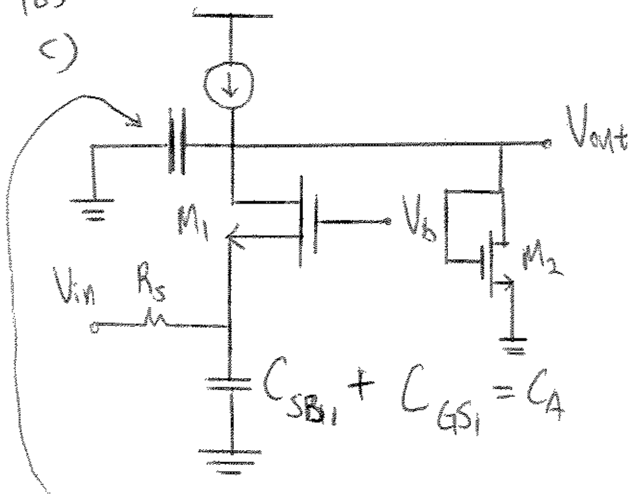
So $$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} r_{o2}}{(C_B r_o S + 1)(1 + R_s C_A S + R_s g_{m_1})}$$

Where $C_B = C_{DB_2} + C_{GD_2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{SG_1}$

4b)

c)



$C_B = C_{DB_1} + C_{GD_1} + C_{DB_2} + C_{GS_2}$

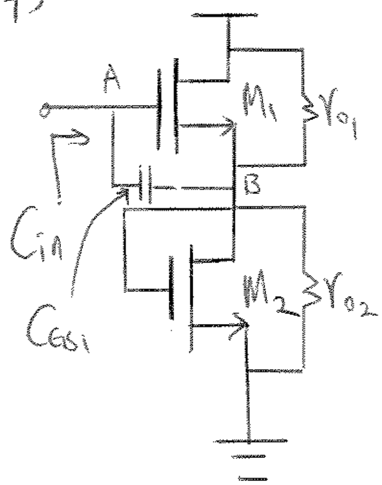
AC-wise, this circuit is very similar to part a). Its transfer function is the same as part a), except for C_B.

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}(1/g_{m_2})}{\left(C_B(1/g_{m_2})S + 1\right)\left(1 + R_S C_A S + R_S g_{m_1}\right)}$$

where $C_B = C_{DB_1} + C_{GD_1} + C_{DB_2} + C_{GS_2}$

$C_A = C_{SB_1} + C_{GS_1}$

47)



DC gain from A to B:

$$A_V = \frac{\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2}}{\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2} + \frac{1}{g_{m1}}}$$

$$A_V \approx \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + \frac{1}{g_{m1}}} = \frac{g_{m1}}{g_{m1} + g_{m2}}$$

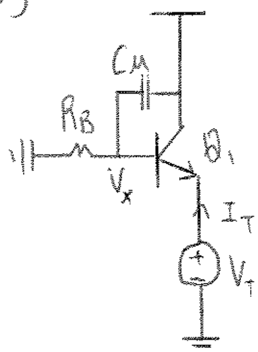
since $g_m r_o >> 1$

Using Miller's Capacitance:

$$C_{in} = C_{GS1}(1 - A_V) = C_{GS1}\left(1 - \frac{g_{m1}}{g_{m1} + g_{m2}}\right)$$

$$C_{in} = C_{GS1}\left(\frac{g_{m2}}{g_{m2} + g_{m1}}\right)$$

48)



$V_A = \infty$,

$$\frac{\beta}{\beta+1} \approx 1, \text{ if } \beta \gg 1$$

$$I_T = -(V_x - V_T)\, g_m \alpha \approx -(V_x - V_T)\, g_m$$

$$V_x = \frac{I_T}{\beta}\left(R_B // \frac{1}{C_\mu s}\right)$$

$$I_T = \left(V_T - \frac{I_T}{\beta}\left(R_B // \frac{1}{C_\mu s}\right)\right) g_m$$

$$I_T = g_m V_T - \frac{g_m}{\beta}\left(R_B // \frac{1}{C_\mu s}\right) I_T$$

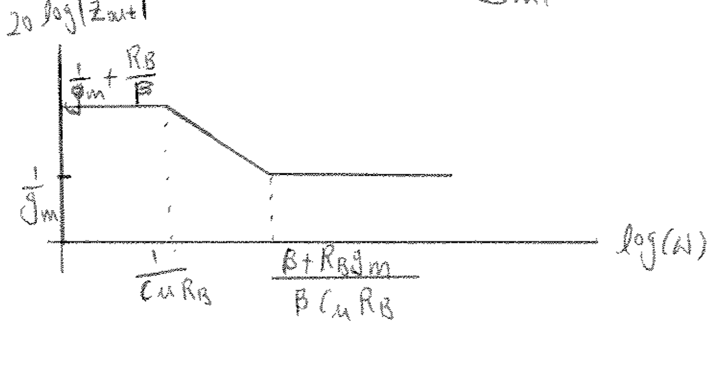
$$I_T\left(1 + \frac{g_m}{\beta}\left(R_B // \frac{1}{C_\mu s}\right)\right) = g_m V_T$$

$$\frac{V_T}{I_T} = \frac{1}{g_m} + \frac{R_B // \frac{1}{C_\mu s}}{\beta} = \frac{\beta C_\mu R_B \left(s + \frac{\beta + R_B g_m}{\beta C_\mu R_B}\right)}{g_m \beta (1 + C_\mu R_B s)}$$

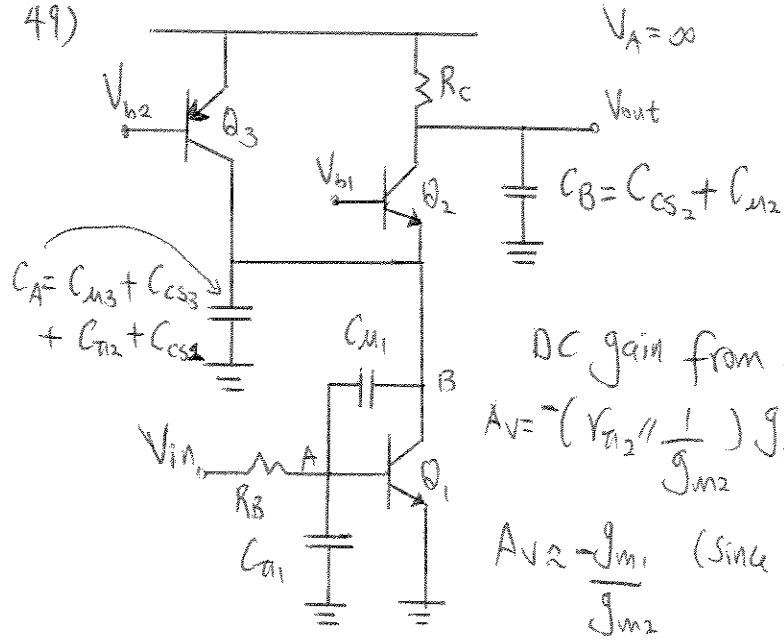
Zero: $\dfrac{\beta + R_B g_m}{\beta C_\mu R_B}$, Pole: $\dfrac{1}{C_\mu R_B}$

At DC, $|Z_{out}| = \dfrac{1}{g_{m1}} + \dfrac{R_B}{\beta}$

At very high freq: $|Z_{out}| = \dfrac{1}{g_{m1}}$



49)



DC gain from A to B:

$A_V = -\left(r_{\pi 2} // \frac{1}{g_{m2}}\right) g_{m1}$

$A_V \approx -\frac{g_{m1}}{g_{m2}}$ (Since $\beta \gg 1$)

We have $I_{C2} = 0.25 I_{C1}$, $g_m = \frac{I_C}{V_T} \Rightarrow g_{m2} = 0.25 g_{m1}$

$A_V = -\frac{g_{m1}}{g_{m2}} = -4$. (If $I_{C2} = I_{C1}$, $A_V = -1$)

Applying Miller's Theroem: $C_{in} = C_{\pi 1} + C_{\mu 1}(1+4) = C_{\pi 1} + 5C_{\mu 1}$

$C_B = C_A + C_\mu\left(1+\frac{1}{4}\right) = C_A + C_\mu\left(\frac{5}{4}\right)$

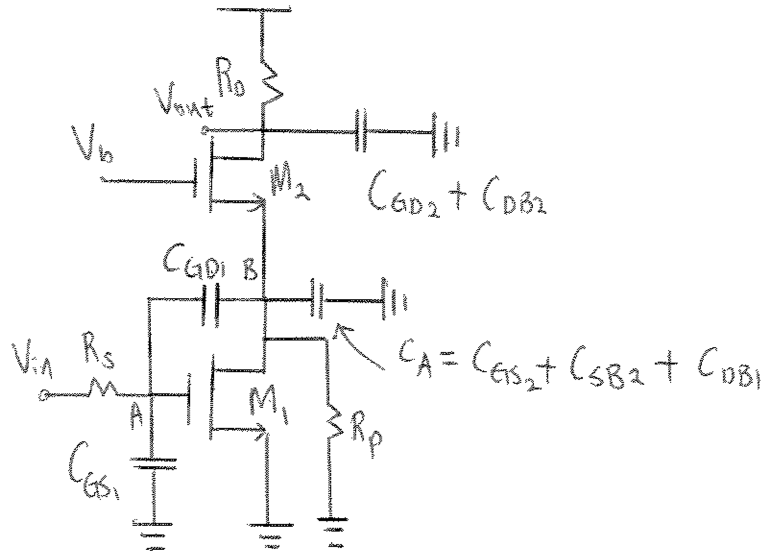
$\omega_{pin}\,(\omega_{pA}) = \frac{1}{(R_B // r_\pi)[C_{\pi 1} + 5C_\mu]}$, $\omega_{pB} = \frac{0.25 g_{m1}}{[C_A + \frac{5}{4}C_{\mu 1}]}$

$\omega_{pB} = \frac{g_{m2}}{[C_A + \frac{5}{4}C_{\mu 1}]}$, $\omega_{pout} = \frac{1}{R_C[C_{CS2} + C_{\mu 2}]}$

Where $C_A = C_{\mu 3} + C_{CS3} + C_{\pi 2} + C_{CS1}$.

Since the DC gain is increased (In magnitude), Miller effect is more significant.

50)



DC gain from A to B is $-g_{m_1}(R_P // \frac{1}{g_{m_2}})$

Applying Miller's Theorem:

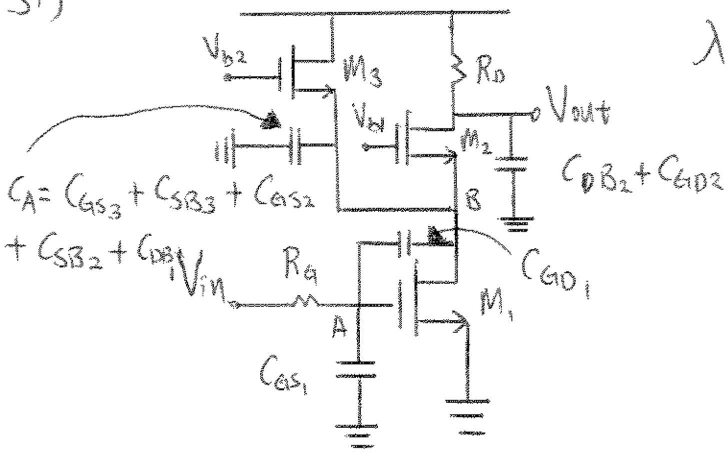
$$\omega_{p_{in}} (\omega_{p_A}) = \frac{1}{R_S(C_{GS_1} + C_{GD_1}(1 + g_{m_1}(R_P // \frac{1}{g_{m_2}})))}$$

$$\omega_{p_B} = \frac{1}{R_P // \frac{1}{g_{m_2}} [C_{GS_2} + C_{SB_2} + C_{DB_1} + C_{GD_1}(1 + 1/g_{m_1}(R_P // \frac{1}{g_{m_2}}))]}$$

$$\omega_{p_{out}} = \frac{1}{R_D(C_{GD_2} + C_{DB_2})}$$

5')

$\lambda = 0$



DC gain from A to B: $-g_{m1}\left(\frac{1}{g_{m3}} \parallel \frac{1}{g_{m2}}\right) = -g_{m1}\left(\frac{1}{g_{m2}+g_{m3}}\right)$

Applying Miller's Theorem:

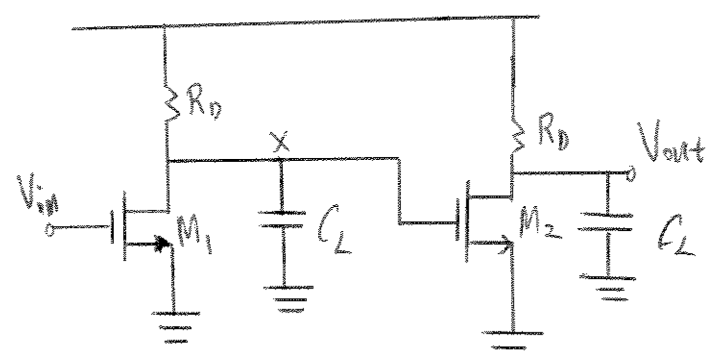
$$\omega_{pin}\ (\omega_{pA}) = \frac{1}{R_G\left(C_{GS1} + C_{GD1}\left(\frac{g_{m1}+g_{m2}+g_{m3}}{g_{m2}+g_{m3}}\right)\right)}$$

$$\omega_{pB} = \frac{g_{m3}+g_{m2}}{\left(C_A + C_{GD1}\left(\frac{g_{m1}+g_{m2}+g_{m3}}{g_{m1}}\right)\right)}$$

$$\omega_{pout} = \frac{1}{R_D\left(C_{DB2} + C_{GD2}\right)}$$

Where $C_A = C_{GS3} + C_{SB3} + C_{GS2} + C_{SB2} + C_{DB1}$

52)



Bias Current = 1mA (each stage)

$C_L = 50fF$

$\mu_n C_{ox} = 100\,\mu A/V^2$, $A_V = 20$, $-3dB$: $1GHz$

DC gain: $(g_m R_D)^2 = 20$

$-3dB$ bandwidth: $0.10243/(R_D C_L) = 1GHz$

Since $C_L = 50fF$, $R_D = 2048.6\,\Omega$

$(g_m R_D)^2 = 20 \Rightarrow g_m = 0.002183 = \frac{2I_D}{V_{eff}} \Rightarrow V_{eff} = 0.916V$

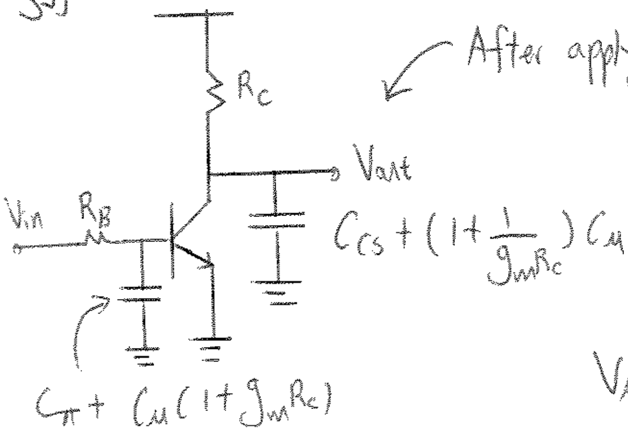
$V_{eff} = V_{GS} - V_{th} = 0.916V$

$g_m = \mu_n C_{ox} \frac{W}{L}(V_{eff}) \Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox}(V_{eff})} = 23.83$

So $R_D = 2.05K$, $C_L = 50fF$

$V_{GS} - V_{th} = 0.916V$, $W/L = 23.83$

53)



After apply Miller's theorem

$\omega_{pin} = (2\pi)(500MHz)$

$\omega_{pout} = (2\pi)(2G)$

$I_C = 1mA$, $C_\pi = 20fF$,

$C_\mu = 5fF$, $C_{CS} = 10fF$

$V_A = \infty$.

Low frequency Voltage gain: $\dfrac{V_{out}}{V_{in}} = \dfrac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pin} = \frac{1}{(R_B // r_\pi)(C_\pi + C_\mu(1 + g_m R_C))} = (2\pi)(500\,MHz)$$

$$\omega_{pout} = \frac{1}{R_C[C_{CS} + (1 + 1/(g_m R_C))C_\mu]} = (2\pi)(2G)$$

$$\Rightarrow g_m = 2\pi(2G)[g_m R_C C_{CS} + g_m R_C C_\mu + C_\mu]$$

$$\Rightarrow R_C = \left(\frac{g_m}{(2\pi)(2G)} - C_\mu\right) / (g_m(C_{CS} + C_\mu))$$

$$g_m = \frac{I_C}{V_T} = 0.0386\,\frac{1}{\Omega}, \quad R_C = 5296.53\,\Omega$$

53)

In order to maximize low frequency gain V_{out}/V_{in}, R_B should be as small as possible (restricted by the input pole location). So $R_B \parallel r_\pi \approx R_B$.

$$\omega_{pin} \approx \frac{1}{R_B(C_\pi + C_\mu(1+g_mR_C))} = (2\pi)(500\times10^6)$$

$$g_mR_c = 204.446$$

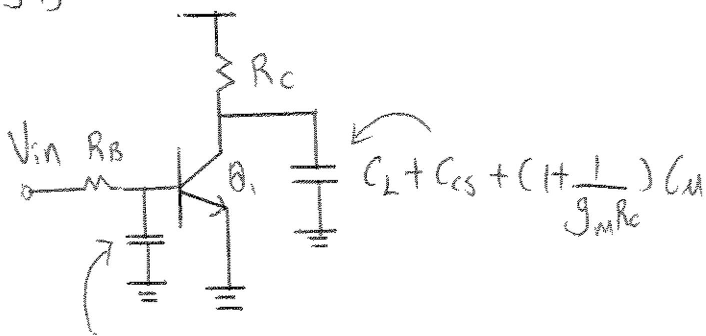
$$R_B = \frac{1}{\omega_{pin}(C_\pi + C_\mu(1+g_mR_c))} \approx 303.95\,\Omega$$

So

$$R_B = 303.95\,\Omega$$

$$R_c = 5296.53\,\Omega$$

54)



$C_\pi + (1+g_m R_c) C_\mu$

Low freq Voltage gain: $\dfrac{V_{out}}{V_{in}} = \dfrac{-R_c}{1/g_m + \dfrac{R_B}{\beta+1}}$

$$\omega_{pout} = \frac{1}{R_c\left[C_L + C_{cs} + \left(1+\frac{1}{g_m R_c}\right)C_\mu\right]} = (2\pi)(2\,GHz)$$

$g_m = \dfrac{I_c}{V_t} = 0.0386\ \dfrac{1}{\Omega}$

$g_m = (2\pi)(2\,GHz)\left[g_m R_c [C_L + C_{cs}] + g_m R_c C_\mu + C_\mu\right]$

$R_c = \left[\dfrac{g_m}{(2\pi)(2G)} - C_\mu\right] / \left(g_m [C_L + C_{cs} + C_\mu]\right)$

$R_c = 2269.94\,\Omega \approx 2.27\,K\Omega$

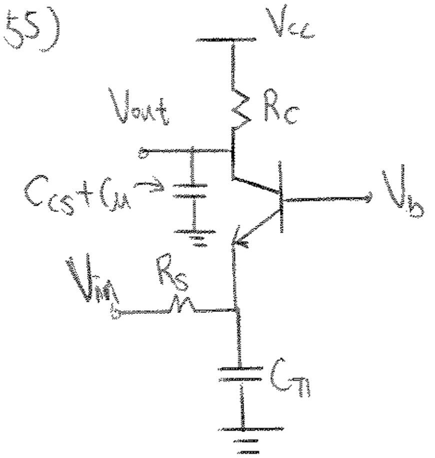
Again, to maximize low freq gain, R_B should be as small as possible, so $R_B // r_\pi \approx R_B$

$\omega_{pin} \approx \dfrac{1}{R_B(C_\pi + C_\mu(1+g_m R_c))} = (2\pi)(500\times10^6)$, $g_m R_c = 87.62$

$R_B = 687.35\,\Omega$

so, $R_c = 2.27\,K\Omega$, $R_B = 687.35\,\Omega$

55)



$V_A = \infty$, $I_C = 1mA$, $R_S = 50\Omega$,
$C_\pi = 20fF$, $C_{cs} = 20fF$, $C_\mu = 5fF$

$-3dB$ bandwidth: 10GHz

Since the output node sees a larger capacitance and resistance than the input, (R_C usually large for large gain), dominant pole and thus $-3dB$ bandwidth occurs at the output.

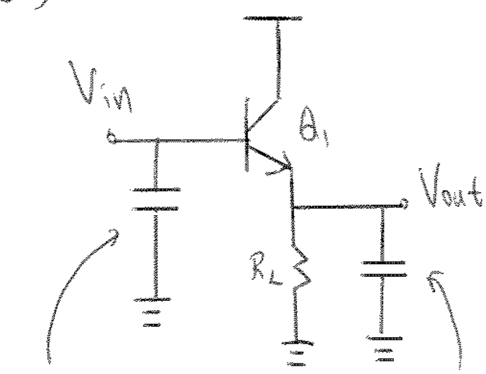
$$\omega_{pout} = \frac{1}{R_C\,[C_\mu + C_{cs}]} = (2\pi)(10GHz)$$

$$R_C = 636.62\,\Omega \quad , \quad \frac{1}{g_m} = \frac{25.9mV}{1mA}$$

Maximum achievable gain: $\dfrac{R_C}{R_S + \dfrac{1}{g_m}} = 8.4$

Here we have a tradeoff between gain and bandwidth.

56)



DC gain: $\dfrac{R_L}{R_L + 1/g_m}$

$V_A = \infty$

$C_\mu = 10fF$, $C_\pi = 100fF$

$C_\mu + C_\pi\left(1 - \dfrac{R_L}{R_L + 1/g_m}\right)$

$C_\pi\left(1 - \dfrac{R_L + 1/g_m}{R_L}\right)$

$$C_{in} < 50fF \Rightarrow C_\mu + C_\pi\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

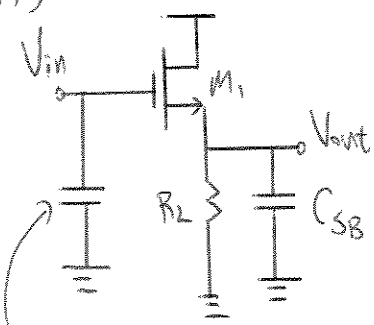
$$10fF + 100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

$$100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 40fF$$

$$\left(\frac{\frac{1}{g_m}}{R_L + 1/g_m}\right) < 0.4$$

$$R_L > \frac{3}{2g_m} = 38.85\Omega$$

57)



$C_{GD} + C_{GS}(1-0.8)$

$R_L = 100\Omega$, $I_D = 1mA$

$A_V = \frac{V_{out}}{V_{in}} = 0.8$ $\mu_n C_{ox} = 100 \mu A/V^2$

$L = 0.18\mu m$, $\lambda = 0$, $C_{GD} \approx 0$,

$C_{SB} \approx 0$, $C_{GS} = (2/3) WL C_{ox}$

$C_{ox} = 12 fF/(\mu m)^2$

$C_{in} = C_{GD} + C_{GS}(0.2)$, $C_{in} = C_{GS}(0.2) = C_{in(min)}$

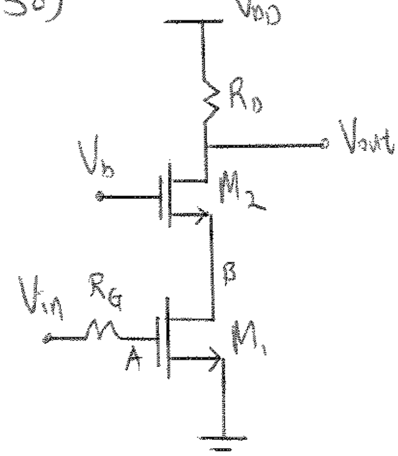
$A_V = \frac{R_L}{R_L + 1/g_m} = 0.8$, $\frac{1}{g_m} = 25 = \frac{V_{eff}}{2I_D}$

$V_{eff} = 50mV$, $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{eff})^2 \Rightarrow W = 1440$

$C_{in,min} = 0.2 C_{GS} = 0.2\left(\frac{2}{3}\right) WL C_{ox} = 414.72 fF$

or $C_{in,min} = 0.415 pF$

58)



$\omega_{p,in} = 5\text{ GHz}$, $\omega_{p,out} = 10\text{ GHz}$

$V_{eff} = 200\text{ mV } (V_{GS} - V_{th})$, $I_D = 0.5\text{mA}$

$\lambda = 0$, $C_{GS} = (2/3) WL\, C_{ox}$,

$L = 0.18\mu m$, $\mu_n C_{ox} = 100\mu A/V^2$

$C_{GD} = W C_o$, $C_o = 0.2 fF/\mu m$

$C_{ox} = 12 fF/\mu m^2$

DC gain from A to B: $\dfrac{-g_{m1}}{g_{m2}} = 1$

$C_{in} = C_{GS} + C_{GD}(1 + g_{m1}/g_{m2}) = C_{GS} + 2C_{GD}$

$I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{eff})^2 = 0.5\text{ mA} \Rightarrow \frac{W}{L} = 250$

$L = 0.18\mu$, $W = 45\mu$

$$\omega_{p,in} = (2\pi)(5\times10^9) = \frac{1}{R_G\left[\frac{2}{3}(45)(0.18)(12 fF/\mu m^2) + (0.2)(45)(2)\right]}$$

$R_G = 384.43\Omega$

$$\omega_{p,out} = \frac{1}{R_D[0.2W]} = (10\times10^9)(2\pi),\ W = 45\mu m$$

$\Rightarrow R_D = 1.8k\Omega$ (1768.4Ω) exact value

$$\text{Gain} = |g_m R_D| = \frac{2I_D}{V_{eff}} R_D = 8.842$$

59)

$W_2 = 4W_1$, $V_{eff_2} = \frac{V_{eff_1}}{2}$ (To maintain the current constant)

$V_{eff_1} = 200mV$, $V_{eff_2} = 100mV$ (Assume V_{eff_1} is not changed)

DC gain: $-\frac{g_{m_1}}{g_{m_2}} = \frac{-g_{m_1}}{2g_{m_1}} = -\frac{1}{2}$

$$\omega_{pin} = \frac{1}{R_G\left[\frac{2}{3}WLC_{ox} + (0.2)W\left(\frac{1}{2}\right)\right]} = (5\times10^9)(2\pi)$$

$W = 45\mu$

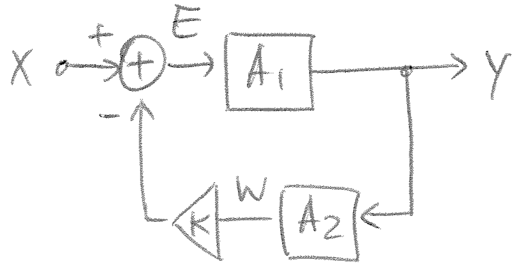
$\Rightarrow R_G = 459.32\,\Omega$

$$R_0 = \frac{1}{(10\times10^9)(2\pi)(0.2)(4)(45)} = 442.097\,\Omega$$

DC gain: $|g_{m_1}R_0| = \frac{2I_D}{V_{eff_1}}R_0 = 2.2105$

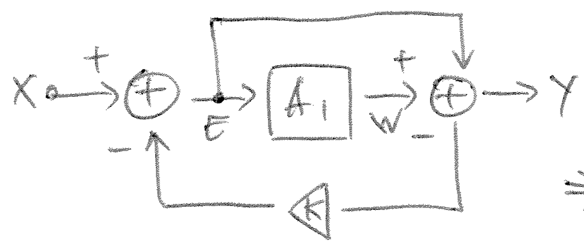
ch 12

1. (a)



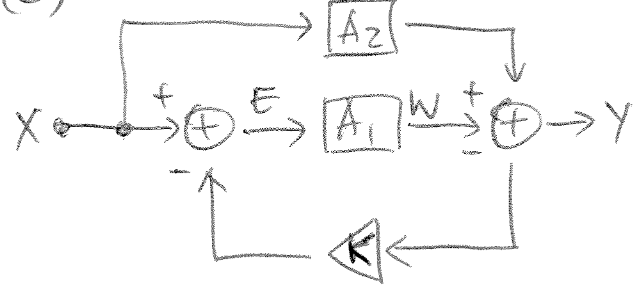
$$Y = A_1 E_1$$
$$= A_1[X - YA_2k]$$
$$\Rightarrow \frac{Y}{X} = \frac{A_1}{1 + kA_1A_2}$$

(b)



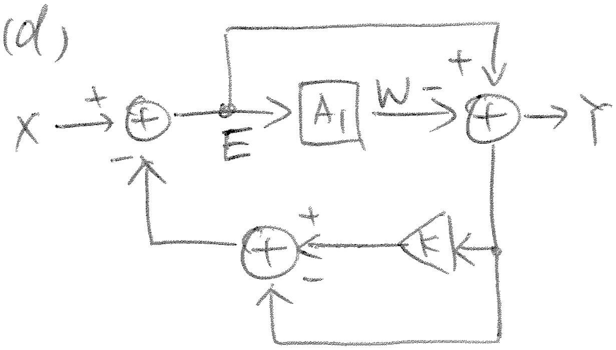
$$Y = E - A_1W$$
$$= (X - kY) - A_1(X - kY)$$
$$\Rightarrow \frac{Y}{X} = \frac{1 - A_1}{1 + (1 - A_1)k}$$

(c)



$$Y = XA_2 - W$$
$$= XA_2 - A_1(X - YK)$$
$$\Rightarrow \frac{Y}{X} = \frac{A_2 - A_1}{1 - A_1Y}$$

(d)



$$Y = E - W$$
$$= (X - (kY - Y)) -$$
$$A_1[X - (kY - Y)]$$
$$\Rightarrow \frac{Y}{X} = \frac{(1 - A_1)}{1 + (k - 1)(1 - A_1)}$$

2. (a) $W = A_2 Y = A_2\left[(X - KW)A_1\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1A_2}{1 + A_1A_2K}$$

(b) $W = A_1 \times E = A_1\left[X - K\left(\frac{W}{A_1} - W\right)\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1}{1 + K(1 - A_1)}$$

(c) $W = A_1 E = A_1\left[X - (A_2X - W)K\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1(1 - A_2K)}{(1 - A_1K)}$$

(d) $W = A_1 E = A_1\left[X - \left\{\left(\frac{W}{A_1} - W\right)K - \left(\frac{W}{A_1} - W\right)\right\}\right]$

$$\Rightarrow \frac{W}{X} = \frac{A_1}{1 + (K-1)(1 - A_1)}$$

3. (a) $E = X - kA_2A_1E$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + kA_2A_1}$$

(b) $E = X - k[E - A_1E]$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + k(1 - A_1)}$$

(c) $E = X - k[A_2X - A_1E]$

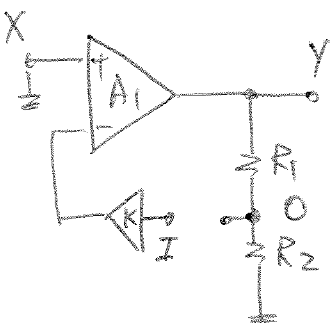
$$\Rightarrow \frac{E}{X} = \frac{1 - A_2k}{1 - A_1k}$$

(d) $E = X - \{k[E - A_1E] - [E - A_1E]\}$

$$\Rightarrow \frac{E}{X} = \frac{1}{1 + (k-1)(1 - A_1)}$$

4.

(a)

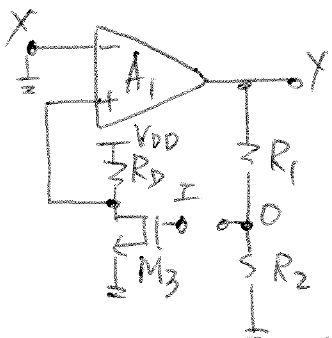


(X is grounded in loop-gain calculation)

$$O = Y\frac{R_2}{R_1+R_2} = (-Ik)A_1\left(\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +kA_1\left(\frac{R_2}{R_1+R_2}\right)$$

(b)

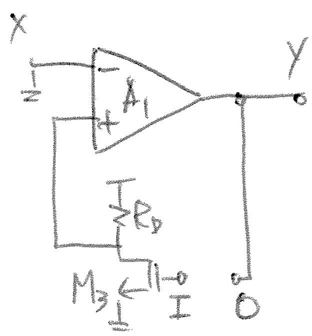


(X is grounded)

$$O = Y\left(\frac{R_2}{R_1+R_2}\right) = -Ig_{m3}R_D A_1\left(\frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +g_{m3}R_D A_1\left(\frac{R_2}{R_1+R_2}\right)$$

(c)

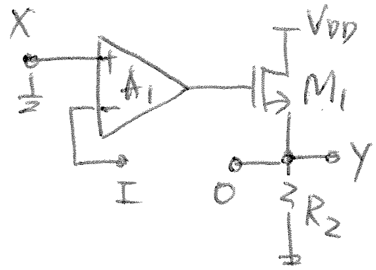


(X is grounded)

$$O = Y = -Ig_{m3}R_D A_1$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain} = +g_{m3}R_D A_1$$

(d)



(X is grounded)

$$O = Y = -I \times \frac{g_{m1}R_2}{1+g_{m1}R_2} \times A_1$$

$$\Rightarrow -\frac{O}{I} = \text{Loop Gain}$$

$$= +A_1 \frac{g_{m1}R_2}{1+g_{m1}R_2}$$

5\. (a) $$\frac{Y}{X} = \frac{A_{O.L.}}{1 + \text{Loop Gain}} = \frac{A_1}{1 + A_1 K\left(\frac{R_2}{R_1+R_2}\right)}$$

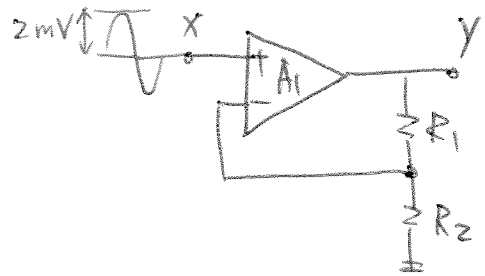
(b) $$\frac{Y}{X} = \frac{A_1}{1 + g_{m3} R_D A_1 \left(\frac{R_2}{R_1+R_2}\right)}$$

(c) $$\frac{Y}{X} = \frac{A_1}{1 + g_{m3} R_D A_1}$$

(d) $$\frac{Y}{X} = \frac{A_1}{1 + A_1 \left(\frac{g_{m1} R_2}{1 + g_{m1} R_2}\right)}$$

6. $A_1 = 500$

$R_1/R_2 = 7$



$$\frac{Y}{X} \approx 1 + \frac{R_1}{R_2} = 8$$

$$\Rightarrow \frac{R_2}{R_1+R_2} = \frac{1}{8} = K$$

$$E = \frac{X}{1+KA_1} = \frac{2\,mV}{1+500/8} \approx 0.031\ mV$$

$\therefore$ Amplitude of feedback waveform

$= X - E \approx 1.969\ mV$

Amplitude of output waveform

$$= X \frac{A_1}{1+KA_1} \simeq 15.75\ mV$$

7. $A_{CL} = \dfrac{A_1}{1+A_1K}$

$$\frac{dA_{CL}}{dA_1} = \frac{1}{(1+A_1K)^2} \Rightarrow dA_{CL} = \frac{dA_1}{(1+A_1K)^2}$$

$$\Rightarrow \frac{dA_{CL}}{A_{CL}} = \frac{dA_{CL}}{\left(\frac{A_1}{1+A_1K}\right)} = dA_1\left(\frac{1+A_1K}{A_1}\right)\left(\frac{1}{(1+A_1K)^2}\right)$$

$$= \frac{(dA_1/A_1)}{(1+A_1K)}$$

This equation implies that for a fractional change in A_{CL}, it is reduced by $(1+A_1K)$ compared to a fractional change in A_1.

$$\Rightarrow 0.01 > \frac{0.2}{1+A_1K} \Rightarrow A_1K > 19$$

8. $A_{OL} = -g_{m1} r_{o1}$ (max gain)

$$= -\frac{2}{\lambda V_{eff}}$$

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL}K}$$

$$\frac{dA_{CL}}{d\lambda} = \frac{d}{d\lambda}\left(\frac{A_{OL}}{1 + A_{OL}K}\right) = \frac{\frac{2}{\lambda^2 V_{eff}}}{(1 + A_{OL}K)^2} = \frac{-A_{OL} \times \frac{1}{\lambda}}{(1 + A_{OL}K)^2}$$

$$\Rightarrow \frac{dA_{CL}}{A_{CL}} = \frac{-A_{OL} \times \frac{d\lambda}{\lambda}}{(1 + A_{OL}K)^2} \times \frac{(1 + A_{OL}K)}{A_{OL}} = \frac{-\frac{d\lambda}{\lambda}}{(1 + A_{OL}K)}$$

$\Rightarrow$ This implies a change in λ $\left(\frac{d\lambda}{\lambda}\right)$ leads to a change in A_{CL} $\left(\frac{dA_{CL}}{A_{CL}} = \frac{d\lambda}{\lambda} \cdot \frac{1}{(1 + \text{Loop Gain})}\right)$ in the <u>opposite</u> direction (i.e. the sign)

$$5\% > \frac{20\%}{1 + A_{OL}K} \Rightarrow \text{Loop Gain} = A_{OL}K > 3$$

9. From the question,

$$(1-10\%)A_o = |A(j\omega')| \qquad \text{where } \omega' = \text{-1-dB bandwidth frequency}$$

$$0.9A_o = \frac{A_o}{\left|1+j\frac{\omega'}{\omega_o}\right|} = \frac{A_o}{\sqrt{1+\left(\frac{\omega'}{\omega_o}\right)^2}}$$

$\Rightarrow \omega' \cong 0.48\,\omega_o$

$\Rightarrow$ This is the open-loop -1dB bandwidth.

Similarly,

$$0.9\,\frac{A_o}{1+L.G.} = \left|\frac{Y}{X}(j\omega'')\right| \qquad \text{where } \omega'' = \text{-1dB bandwidth frequency}$$

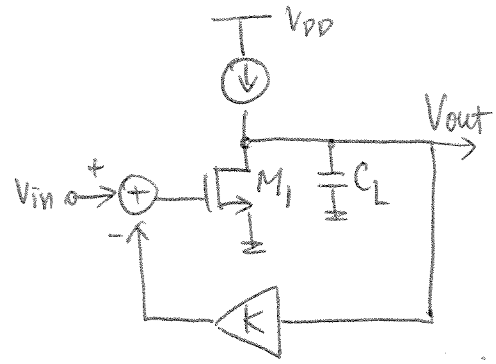
L.G = Loop Gain

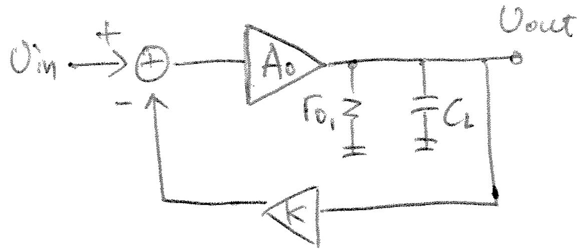
$$0.9\,\frac{A_o}{1+LG} = \frac{\frac{A_o}{1+L.G.}}{\left|1+j\frac{\omega''}{\omega_o(1+LG)}\right|} = \frac{\frac{A_o}{1+LG}}{\sqrt{1+\left[\frac{\omega''}{\omega_o(1+LG)}\right]^2}}$$

$\Rightarrow \omega'' \cong 0.48\,\omega_o\,(1+LG)$

$\therefore$ -1-dB bandwidth is boosted (expected) by $(1+LG)$ in closed-loop measurement.

10. An equivalent circuit is shown below:





Open-loop transfer function (without feedback):

$$A(s) = \frac{A_0}{1+\frac{s}{\omega_p}} = \frac{-g_{m1} r_{o1}}{1+\frac{s}{r_{o1} C_L}}$$

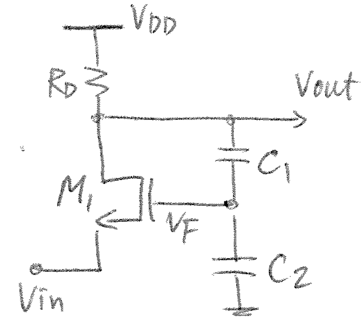
Loop Gain $= A_0 K = g_{m1} r_{o1} K$

$\Rightarrow$ Closed-loop $-3dB$ bandwidth

$= B = (r_{o1} C_L)(1 + A_0 K) = r_{o1} C_L (1 + g_{m1} r_{o1} K)$

$\therefore K = \left(\frac{B}{r_{o1} C_L} - 1\right) \times \frac{1}{g_{m1} r_{o1}}$

11.



Feedforward System

M_1 & R_D (common-gate stage)

Sense Mechanism (Feedback):

$C_1 + C_2$ (capacitive divider)

$\Rightarrow V_F = \frac{C_1}{C_2 + C_1} \times V_{out}$

Comparison Mechanism: M_1 itself.

Open-loop gain $= A_o = g_m R_D$

$$\text{Closed-loop gain} = \frac{V_{out}}{V_{in}} = \frac{A_o}{1 + KA_o} = \frac{g_m R_D}{1 + \frac{C_1}{C_1 + C_2} g_m R_D}$$

Open-loop $r_{in} = \frac{1}{g_m}$

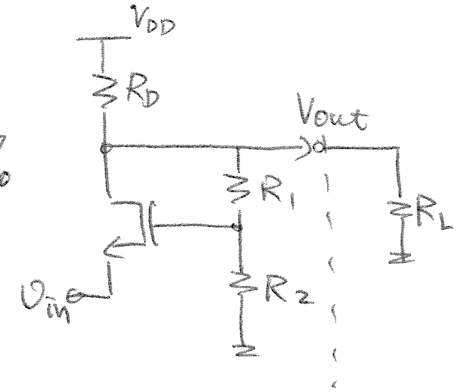
" $r_{out} = R_D$

$$\text{Closed-loop } r_{in} = r_{in,ol} \times (1 + KA_o) = \frac{1}{g_m}\left(1 + \frac{C_1}{C_1+C_2} g_m R_D\right)$$

$$\text{" } r_{out} = \frac{r_{out,ol}}{(1+KA_o)} = \frac{R_D}{1 + \frac{C_1}{C_1+C_2} g_m R_D}$$

12.

Given: $\dfrac{\text{Gain}_{loaded} - \text{Gain}_{loaded}}{\text{Gain}_{unloaded}} = 10\% = 0.1$



$$\text{Gain}_{unloaded} \cong \frac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D} \quad (\text{assume } R_1 + R_2 >> R_D)$$

$$\text{Gain}_{loaded} = \frac{g_m (R_D // R_L)}{1 + \frac{R_2}{R_1+R_2} g_m (R_D // R_L)}$$

$$\therefore\ 0.1 = \frac{\dfrac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D} - \dfrac{g_m (R_D // R_L)}{1 + \frac{R_2}{R_1+R_2} g_m (R_D // R_L)}}{\dfrac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}}$$

After solving for R_L:

$$R_L = \frac{9 R_D}{1 + \left(\frac{R_2}{R_1+R_2}\right) g_m R_D}$$

13. Gain at $x_1 = \dfrac{500}{1+500K}$

Gain at $x_2 = \dfrac{420}{1+420K}$

$$\Rightarrow \frac{\dfrac{500}{1+500K} - \dfrac{420}{1+420K}}{\dfrac{500}{1+500K}} < 0.05$$

$$\Rightarrow K > \frac{11}{2100}$$

$$A_{x_1} = \frac{500}{1+500(K)} = \frac{2625}{19} \approx 138.16$$

$$A_{x_2} = \frac{420}{1+420(K)} = \frac{525}{4} \cong 131.25$$

14. $y = \alpha_1 x - \alpha_3 x^3$

(a) $\dfrac{\partial y}{\partial x} = \alpha_1 - 3\alpha_3 x^2$

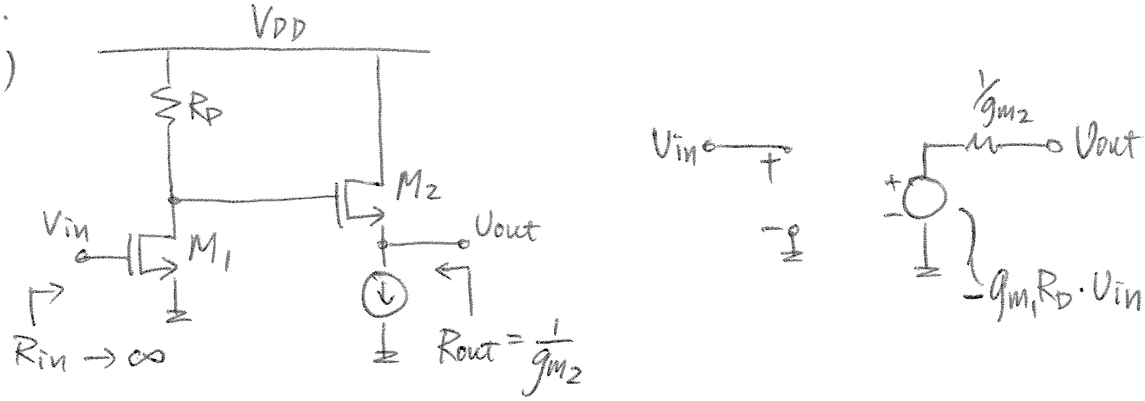
$\left.\dfrac{\partial y}{\partial x}\right|_{x=0} = \alpha_1$ $\qquad$ $\left.\dfrac{\partial y}{\partial x}\right|_{x=\Delta x} = \alpha_1$ (around $x=0$)

(b) $\text{Closed-loop}\,\Big|_{x=0} = \dfrac{\alpha_1}{1+\alpha_1 K}$

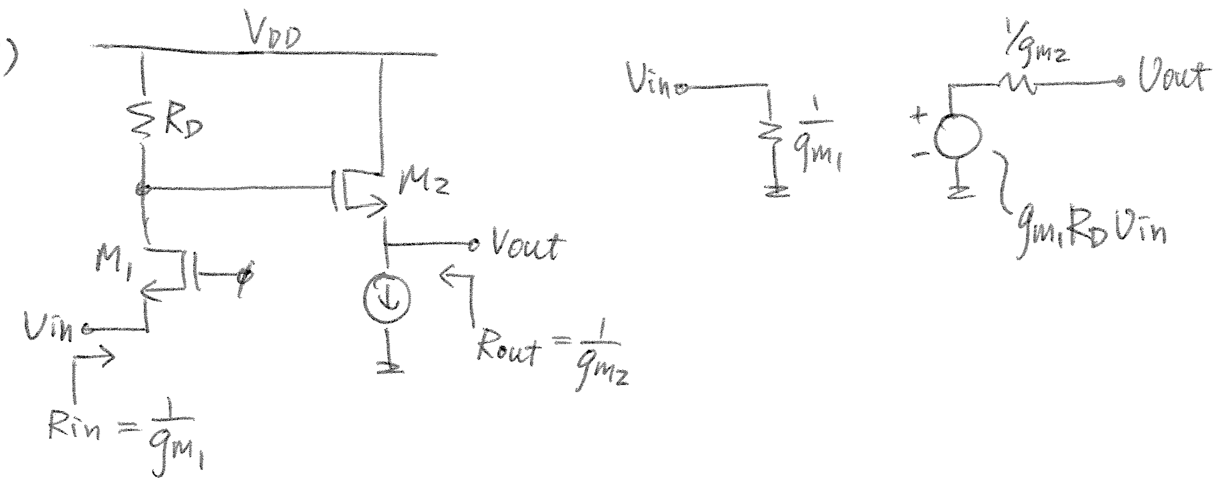
$\text{Closed-loop}\,\Big|_{x=\Delta x} = \dfrac{\alpha_1}{1+\alpha_1 K}$ (around $x=0$)

15.

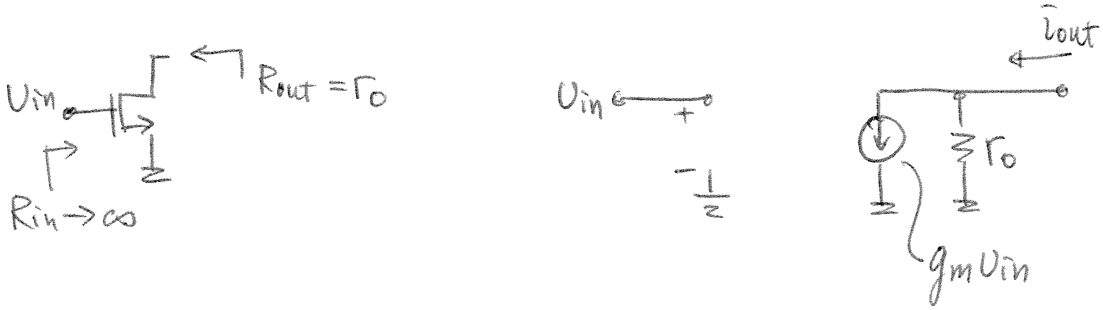
(a)


VDD
RD
M2
M1
Vin
Vout
Rin → ∞
Rout = 1/gm2
Vin
+
−
1/gm2
Vout
+
−
−gm1RD·Vin

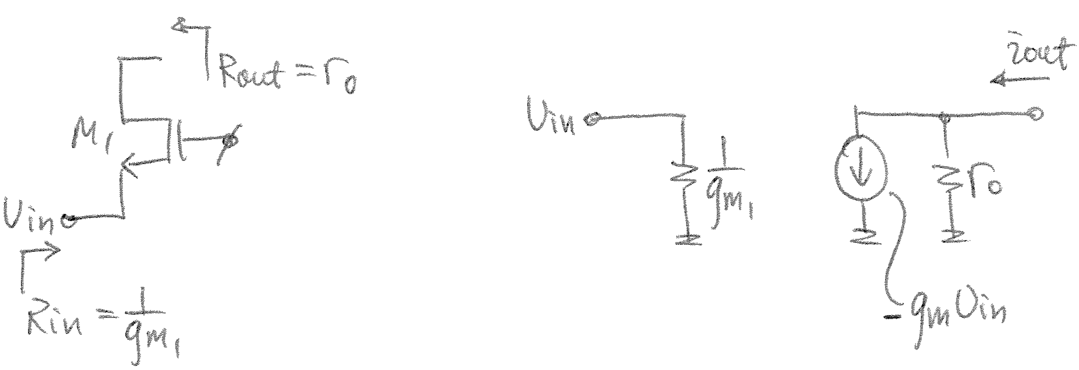
(b)


VDD
RD
M2
M1
Vin
Vout
Rout = 1/gm2
Rin = 1/gm1
Vin
1/gm1
1/gm2
Vout
+
−
gm1RDVin

(c)

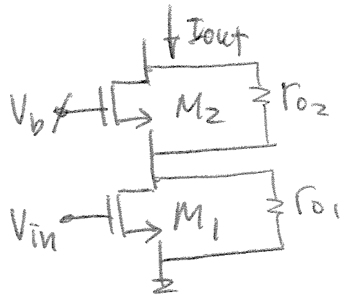

Rout = ro
Vin
Rin → ∞
Vin
+
−
iout
ro
gmVin

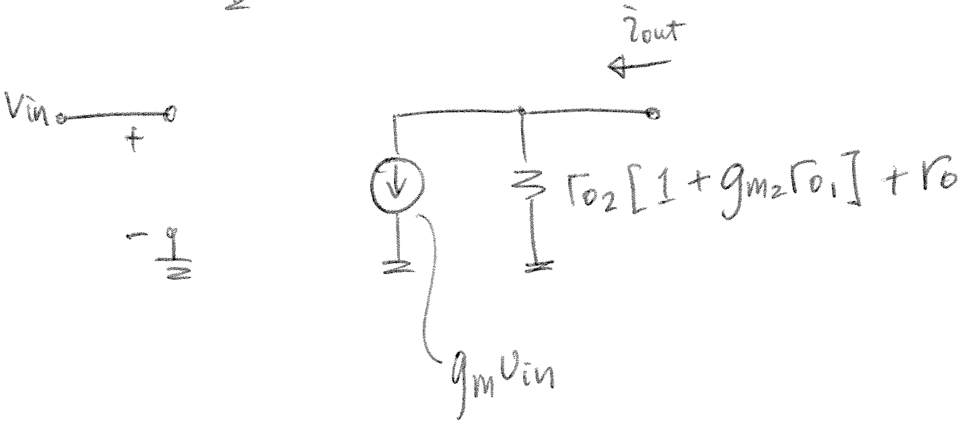
(d)


Rout = ro
M1
Vin
Rin = 1/gm1
Vin
1/gm1
iout
ro
−gmVin

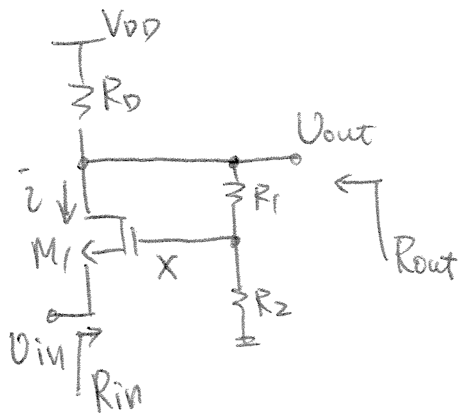
16.

$\lambda > 0$





17.

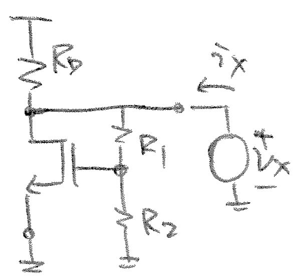


$$-v_{out} = i\,[R_D \,\|\, (R_1+R_2)]$$

$$i = g_{m1}(v_X - v_{in}) = g_{m1}\left(v_{out} \times \frac{R_2}{R_1+R_2} - v_{in}\right)$$

Combining the equations above yields:

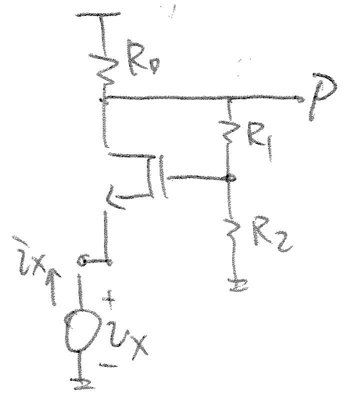
$$\frac{v_{out}}{v_{in}} = \frac{g_{m1}[R_D \| (R_1+R_2)]}{1 + \frac{R_2}{R_1+R_2} g_{m1}[R_D \| (R_1+R_2)]} \triangleq A_v$$



By KCL,

$$i_X = \frac{v_X}{R_1+R_2} + \frac{v_X}{R_D} + g_{m1}\left(v_X \frac{R_2}{R_1+R_2}\right)$$

$$\Rightarrow \frac{v_X}{i_X} = R_{out} = [(R_1+R_2) \| R_D]\left[1 + g_{m1}\frac{R_2}{R_1+R_2}(R_D \| (R_1+R_2))\right]$$



By KCL,

$$i_x = g_{m1}\left(v_x - v_P \frac{R_2}{R_1+R_2}\right)$$

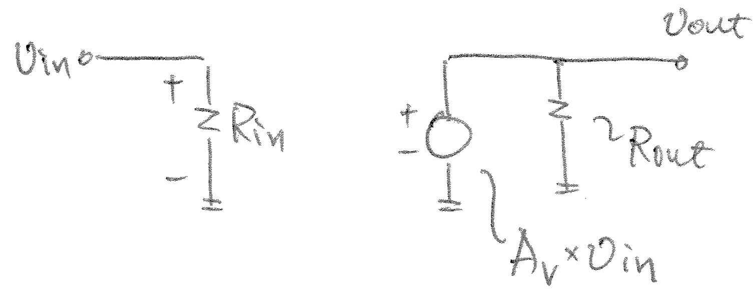
$$\Rightarrow v_P = \left(v_x - \frac{i_x}{g_{m1}}\right)\left(\frac{R_1+R_2}{R_2}\right) \quad \text{—— (1)}$$

$$i_x = \frac{v_P}{R_D \,\|\, (R_1+R_2)} \quad \text{—— (2)}$$

Substitute (1) into (2) & solve for $\frac{v_x}{i_x}$:

$$\frac{v_x}{i_x} = R_{in} = \frac{1}{g_{m1}}\left[1 + g_{m1}\left\{R_D \,\|\, (R_1+R_2)\right\}\frac{R_2}{R_1+R_2}\right]$$

Model:



18. (a) Sense Mechanism : Current from M_3
Return Mechanism : Voltage to Gate of M_2.

(b) Sense Mechanism: Voltage output (M_3)
Return Mechanism: Voltage to Gate of M_2.

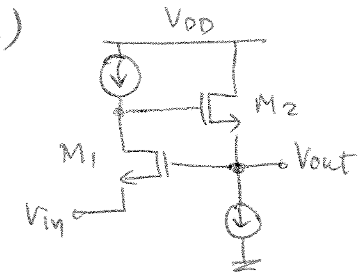
(c) Sense Mechanism: Current from M_3
Return Mechanism : Voltage to Gate of M_2.

(d) Sense Mechanism: Current from M_3
Return Mechanism: Voltage to Gate of M_2.

(e) Sense Mechanism: Voltage Divider ($\frac{R_2}{R_1+R_2}$)
Return Mechanism : Voltage to Gate of M_2.

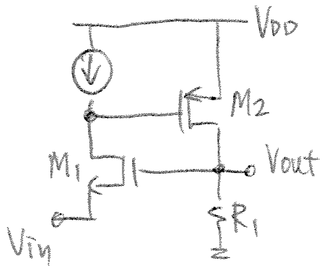
(f) Sense Mechanism: From Common Source of M_3
Return Mechanism : Voltage to Gate of M_2.

19. (a)



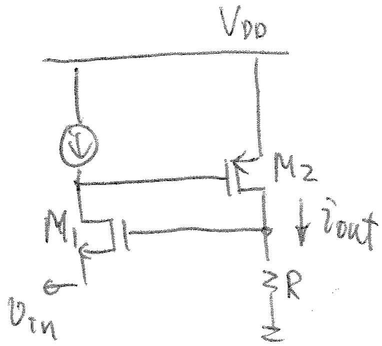
Sense Mechanism:
Voltage sensing at Vout.
Return Mechanism:
Voltage to Gate of M_1.

(b)



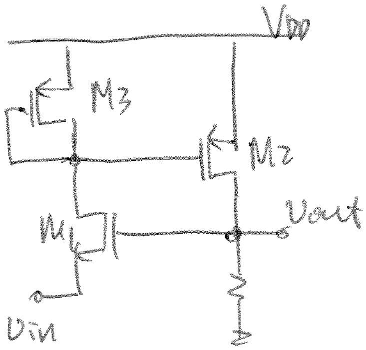
Sense Mechanism:
Voltage output from M_2.
Return Mechanism:
Voltage to Gate of M_1.

(c)



Sense Mechanism:
R_1
Return Mechanism:
Voltage to Gate of M_1.

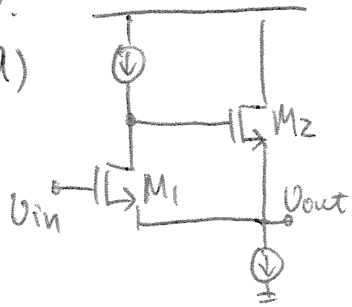
(d)



Sense Mechanism:
Voltage output of M_2
Return Mechanism:
Voltage to Gate of M_1

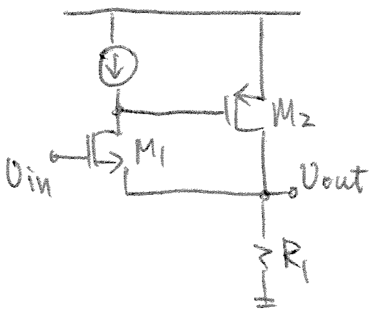
20.

(a)



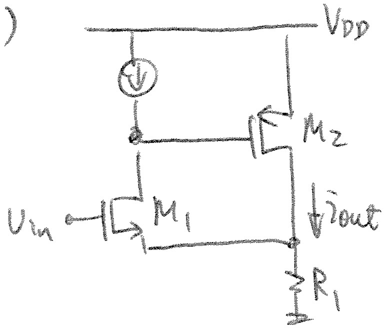
Sense Mechanism:
Voltage output of M_2
Return Mechanism:
Voltage to Source of M_1

(b)



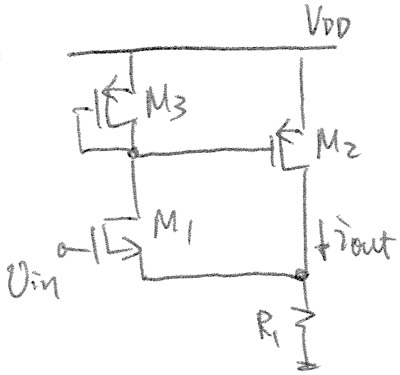
Sense Mechanism:
Voltage output of M_2
Return Mechanism:
Voltage to Source of M_1

(c)



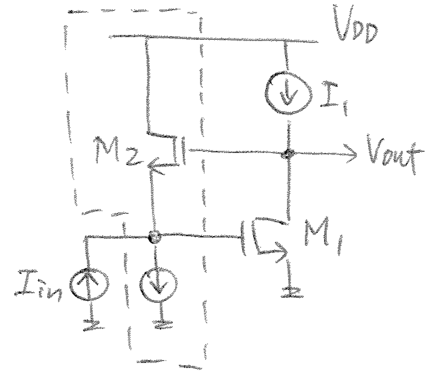
Sense Mechanism:
Current through R_1.
Return Mechanism:
$i_{out} \times R_1$ to Source of M_1.

(d)



Sense Mechanism:
Current through R_1.
Return Mechanism:
$i_{out} \times R_1$ to Source of M_1

21. (a)



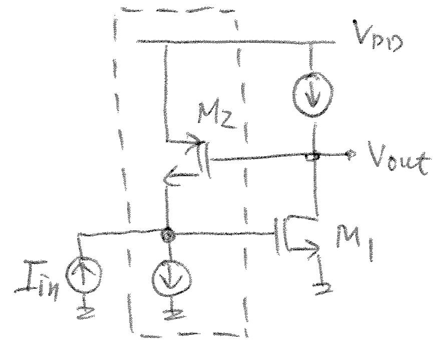
Sense Mechanism:

Gate of M_2 (Voltage)

Return Mechanism:

Current output of M_2.

(b)



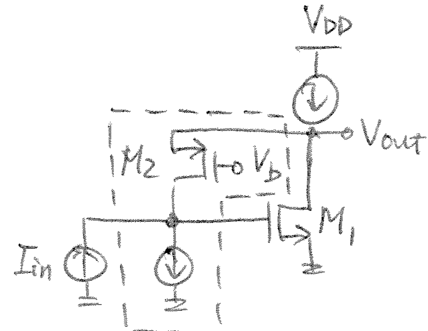
Sense Mechanism:

Gate of M_2 (Voltage)

Return Mechanism:

Current output of M_2.

(c)



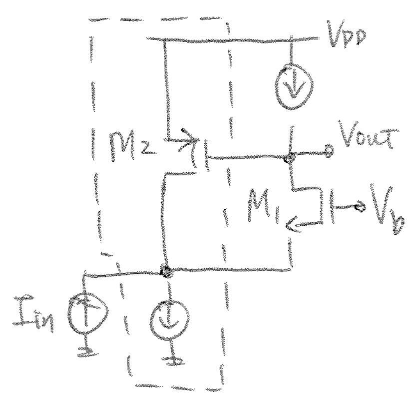
Sense Mechanism:

Source of M_2 (Voltage)

Return Mechanism:

Current output of M_2

(d) Sense Mechanism:



Gate of M_2. (Voltage)

Return Mechanism:

Current output of M_2

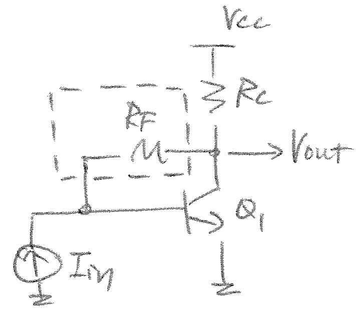
22.

(a) Sense Mechanism:

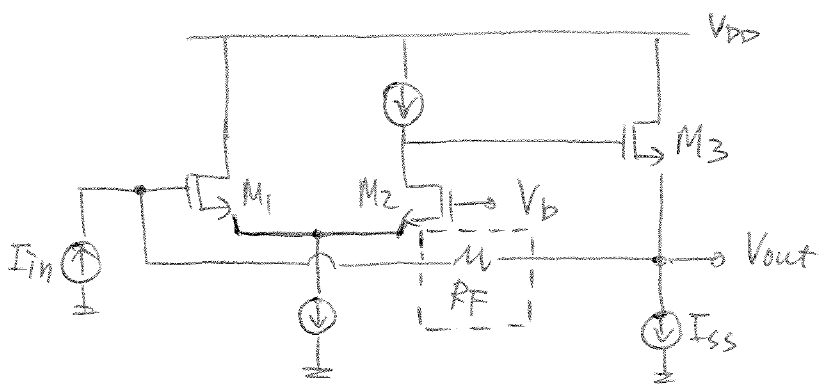
Resistor (R_F) — Voltage

Return Mechanism:

Current through R_F.



(b)



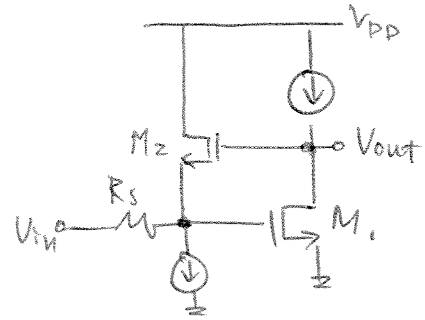
Sense Mechanism:

Resistor (R_F) — Voltage

Return Mechanism:

Current through R_F

23. First, recognize that
(a) both input & output are voltages.



* v_{in} primarily drives the Gate of M_1.

Sequence: Suppose v_{in} increases by Δv_{in}

$\Rightarrow$ v_{out} drops by $+g_{m_1} \Delta v_{in} \times r_{o_1}$ (Common-Source)

$\Rightarrow$ Source of M_2 decreases by same amount (Source follower)

$\therefore$ $v_{in} \uparrow \Rightarrow V_{M_1,D} \downarrow \Rightarrow V_{M_1,G} \downarrow$

$\Rightarrow$ effective v_{in} driving $M_{1,G} \downarrow$

$\Rightarrow$ negative feedback

(b) $v_{in} \uparrow \Rightarrow v_{out} \downarrow \Rightarrow v_{M_2,G} \uparrow$

$\Rightarrow$ effective v_{in} driving $M_{1,G} \uparrow$

$\Rightarrow$ positive feedback.

(c) $v_{in} \uparrow \Rightarrow v_{out} \downarrow \Rightarrow v_{M1,G} \downarrow$

$\Rightarrow$ effective v_{in} driving $M_{1,G} \downarrow$

$\Rightarrow$ negative feedback.

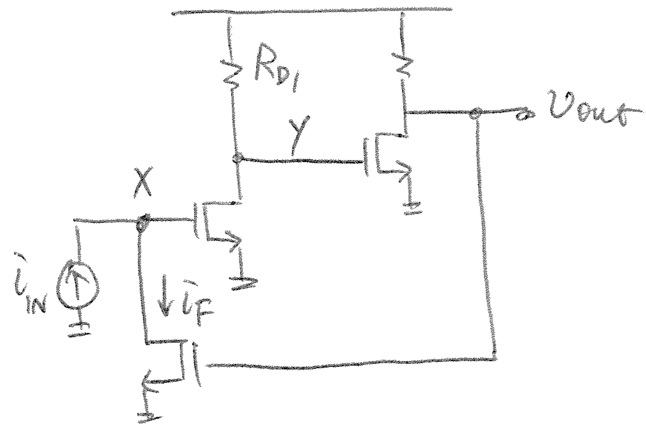
(d) $v_{in} \uparrow \Rightarrow v_{out} \uparrow$ (common-base, M_1)

$\Rightarrow v_{M1,S} \downarrow$

$\Rightarrow$ effective v_{in} driving $M_{1,S} \downarrow$

$\Rightarrow$ negative feedback.

24.



$i_{IN} \uparrow \Rightarrow v_X \uparrow \Rightarrow v_Y \downarrow \Rightarrow v_{out} \uparrow \Rightarrow i_F \uparrow$

$\Rightarrow$ effective $v_X \downarrow$

$\Rightarrow$ negative feedback.

25.

Figure 12.79

↑ = increases by
↓ = decreases by

(a) V_{in} ↑ $\Rightarrow V_{G,M3}$ ↑

$\Rightarrow V_{out}$ ↑ same amount (Emitter Follower)
$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2})$ ↓
$\Rightarrow$ Negative feedback.

(b) V_{in} ↑ $\Rightarrow V_{G,M_3}$ ↑

$\Rightarrow V_{out}$ ↑ by $\Delta V_{G,M_3} \times g_{m3}(r_{o3} \| R_1)$
$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2})$ ↓
$\Rightarrow$ Negative feedback.

(c) Same as (b)

(d) Same as (a)

(e) V_{in} ↑ $\Rightarrow V_{out}$ ↑

$\Rightarrow V_{G,M_2}$ ↑ by $\Delta V_{out} \times \dfrac{R_2}{R_1+R_2}$
$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2})$ ↓
$\Rightarrow$ Negative feedback.

(f) $V_{in} \uparrow \Rightarrow V_{G,M_3} \uparrow$

$\Rightarrow V_{out} \uparrow$ (Common-Source Stage)

$\Rightarrow$ effective V_{in} $(V_{G,M_1} - V_{G,M_2}) \downarrow$

$\Rightarrow$ Negative feedback.

Figure 12.80

(a) $V_{in} \uparrow \Rightarrow V_{G,M_2} \uparrow$ (Common-Gate)

$\Rightarrow V_{out} \uparrow$ (Source Follower)

$\Rightarrow$ effective V_{eff} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

(b) $V_{in} \uparrow \Rightarrow V_{G,M_2} \uparrow$ (Common-Gate)

$\Rightarrow V_{out} \uparrow$ (Common Source)

$\Rightarrow$ effective V_{eff} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

(c) Same as (b)

(d) $V_{in} \uparrow \Rightarrow V_{G,M_2} \uparrow$ (Common-Gate)

$\Rightarrow V_{out} \uparrow$ (Common Source)

$\Rightarrow$ effective V_{eff} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

Figure 12.81

(a) $V_{in} \uparrow \Rightarrow V_{G, M_2} \downarrow$ (Common Source)

$\Rightarrow V_{out} \uparrow$ (Source Follower)

$\Rightarrow$ effective V_{in} $(V_{in} - V_{out}) \uparrow$

$\Rightarrow$ Positive feedback.

(b) $V_{in} \uparrow \Rightarrow V_{G, M_2} \downarrow$ (Common Source)

$\Rightarrow V_{out} \uparrow$ (Common Source)

$\Rightarrow$ effective V_{in} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

(c) Same as (b) $(V_{out} = I_{out} \times R_1)$

(d) $V_{in} \uparrow \Rightarrow V_{G, M_2}$ (Common Source)

$\Rightarrow V_{out} \uparrow$ (Common Source)

$\Rightarrow$ effective V_{in} $(V_{in} - V_{out}) \downarrow$

$\Rightarrow$ Negative feedback.

Figure 12.82

(a) $i_{in} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Source, through current gain of M_1.)

$\Rightarrow i_{D,M2} \downarrow$ (Source Follower, M_2)

$\Rightarrow$ Counteracts the effect of i_{in}

$\Rightarrow$ Negative feedback.

(b) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Source, through current gain of M_1)

$\Rightarrow i_{D,M2} \uparrow$ (Common Source, M_2)

$\Rightarrow$ Enhances V_{out} through M_1

$\Rightarrow$ Positive feedback.

(c) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Source, through current gain of M_1)

$\Rightarrow i_{D,M2} \downarrow$ (Common Gate)

$\Rightarrow$ Counteracts the effect of i_{IN}

$\Rightarrow$ Negative feedback.

(d) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \uparrow$ (Common Gate)

$\Rightarrow i_{D,M2} \downarrow$ (Common Source)

$\Rightarrow$ Counteracts the effect of i_{IN}

$\Rightarrow$ Negative feedback.

Figure 12.83

(a) $i_{IN} \uparrow \Delta \Rightarrow V_{out} \downarrow$ (Common Emitter)

$\Rightarrow$ R_F demands more current momentarily

$\Rightarrow$ effective i_{IN} into $Q_1 \downarrow$

$\Rightarrow$ $V_{out} \uparrow$

$\Rightarrow$ Negative feedback.

(b) $i_{IN} \uparrow \Delta \Rightarrow V_{G, M_3} \uparrow$ (Diff Pair Action)

$\Rightarrow$ $V_{out} \uparrow$ (Source Follower)

$\Rightarrow$ V_{out} "tracks" behavior of i_{IN}.

(Current demanded by R_F from $i_{IN} \uparrow$)

$\Rightarrow$ Positive feedback.

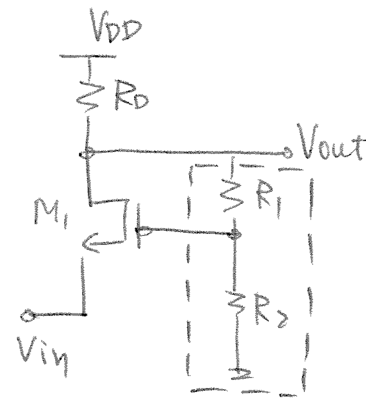
26.

(Without feedback)

$$\frac{v_{out}}{v_{in}} = A_{O.L.} = g_m R_D$$

Feedback factor, k:

$$k = \frac{R_2}{R_1 + R_2}$$



$R_1 + R_2 \gg R_D$

$\lambda = 0$

$$\Rightarrow A_{C.L.} = \left.\frac{v_{out}}{v_{in}}\right|_{C.L.} = \frac{A_{O.L}}{1 + A_{O.L} \cdot K} = \frac{g_m R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}$$

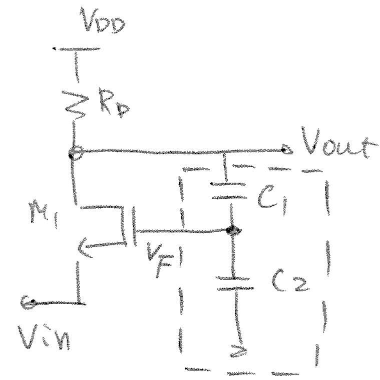
$$R_{in, CLOSED} = \frac{1}{g_{m1}}\left(1 + \frac{R_2}{R_1+R_2} g_m R_D\right)$$

$$R_{out, CLOSED} = \frac{R_D}{1 + \frac{R_2}{R_1+R_2} g_m R_D}$$

27.

(Without feedback)

$$A_{O.L.} = g_m R_D$$



$\lambda = 0$

C_1, C_2 small.

Feedback factor, k :

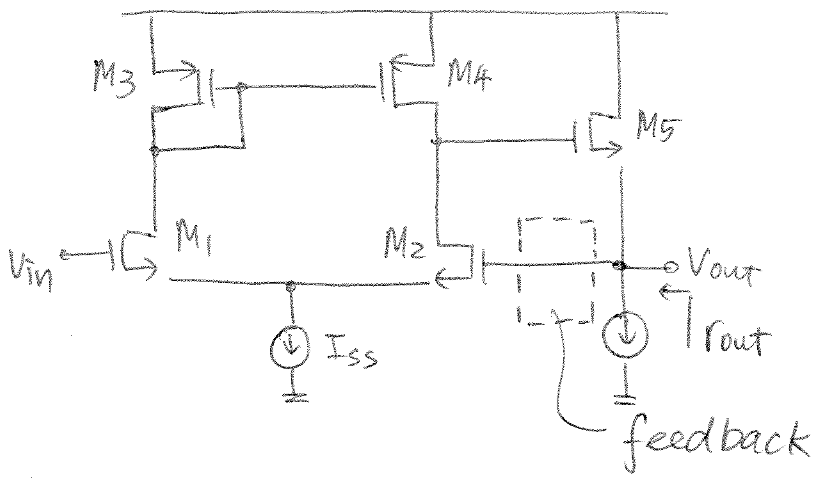
$$k = \frac{C_1}{C_1 + C_2}$$

$$\Rightarrow A_{C.L.} = \left.\frac{V_{out}}{V_{in}}\right|_{C.L.} = \frac{A_{O.L}}{1 + A_{O.L.} k} = \frac{g_m R_D}{1 + \frac{C_1}{C_1 + C_2} g_m R_D}$$

$$R_{in,CLOSED} = \frac{1}{g_m}\left[1 + \frac{C_1}{C_1 + C_2} g_m R_D\right]$$

$$R_{out,CLOSED} = \frac{R_D}{1 + \frac{C_1}{C_1 + C_2} g_m R_D}$$

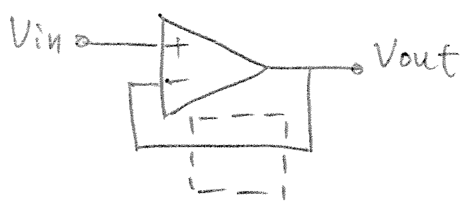
28.



$\lambda > 0$

r_{out} low.

Note that V_{out} is directly fed back to input:



$\therefore$ gain ≈ 1

(a buffer)

$\Rightarrow k = 1$.

$A_{O.L.}$ (i.e. without feedback)

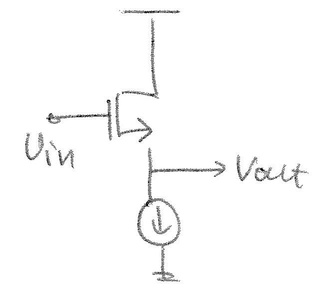
$$= g_{m1}(r_{o2} \parallel r_{o4}) \times \frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1} \quad \left(\approx g_{m1}(r_{o2} \parallel r_{o4}) \right)$$

$$\Rightarrow A_{C.L.} = \frac{A_{OL}}{1 + A_{OL} \cdot k} = \frac{g_{m1}(r_{o2} \parallel r_{o4}) \left(\frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1} \right)}{1 + g_{m1}(r_{o2} \parallel r_{o4}) \left(\frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1} \right)}$$

$$r_{out} = \frac{r_{out}\,(\text{no feedback})}{1 + A_{O.L.} \times k} = \frac{\left(\frac{1}{g_{m5}} \parallel r_{o5} \right)}{1 + g_{m1}(r_{o2} \parallel r_{o4}) \left(\frac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1} \right)}$$

$$\text{Gain} = A = \frac{g_m r_o}{g_m r_o + 1}$$

$$r_{out} = \frac{1}{g_m} \parallel r_o$$

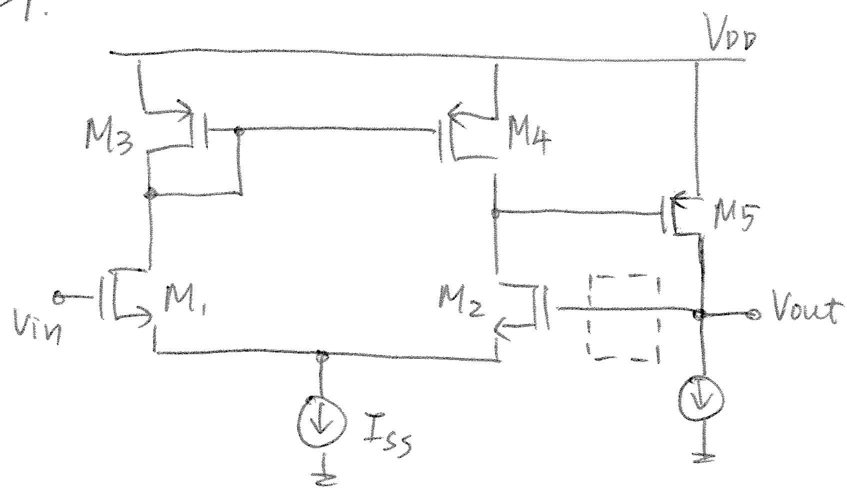


$\lambda > 0$

In comparison, the amplifier's gain is reduced by $\dfrac{g_{m1}(r_{o2} \parallel r_{o4})}{1 + g_{m1}(r_{o2} \parallel r_{o4})\left(\dfrac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1}\right)}$ times.

$\left(= \dfrac{A_{C.L.}}{A}\right)$. Output resistance of the amplifier is reduced by $\left[1 + g_{m1}(r_{o2} \parallel r_{o4})\left(\dfrac{g_{m5} r_{o5}}{g_{m5} r_{o5} + 1}\right)\right]$ times.

29.



(a) By inspection,

$V_{in} \uparrow \Rightarrow V_{G,M5} \uparrow$

$\Rightarrow$ $\underline{\underline{V_{out} \downarrow}}$ (Common Source)

$\Rightarrow$ effective V_{in} $(V_{in} - V_{out}) \uparrow \Rightarrow V_{G,M5} \uparrow$

$\Rightarrow$ Positive feedback.

(b) $A_{O.L.}$ (without feedback)

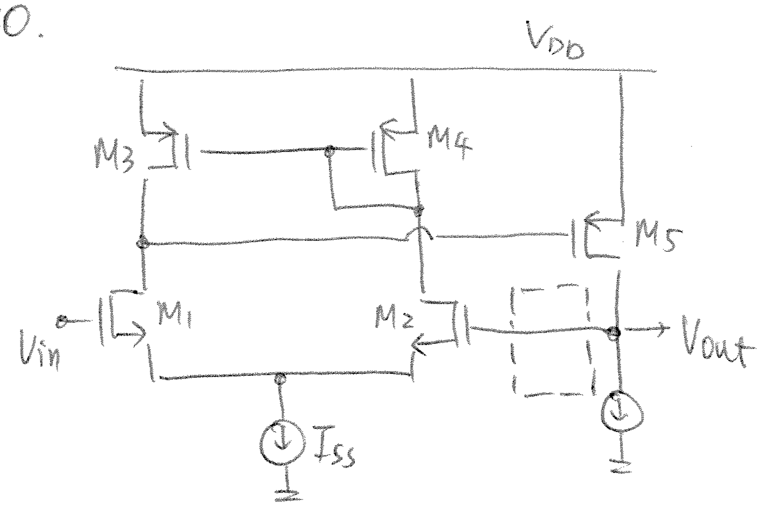
$= g_{m1}(r_{o2} \parallel r_{o4}) \times (-g_{m5} r_{o5})$

K (feedback factor) $= 1$

$\Rightarrow$ loop gain $= A_{O.L.} \times K = -g_{m5} r_{o5} \times g_{m1}(r_{o2} \parallel r_{o4})$

$\Rightarrow$ $A_{C.L.}$ becomes negative $\Rightarrow$ Positive feedback.

30.



$k = 1.$

$\lambda > 0$

$A_{O.L.}$ (without feedback)

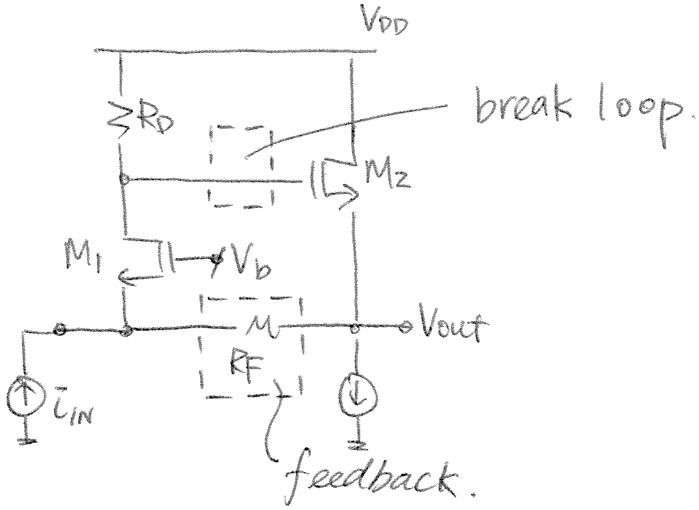
$$= -g_{m1}(r_{o1} \parallel r_{o3}) \times (-g_{m5} r_{o5}) = g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}$$

$$\Rightarrow A_{C.L.} = \frac{V_{out}}{V_{in}} = \frac{g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}{1 + g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}$$

$r_{in} \to \infty$

$$r_{out} = \frac{r_{out}\text{ (no feedback)}}{1 + A_{O.L.} \times K} = \frac{r_{o5}}{1 + g_{m1} g_{m5} (r_{o1} \parallel r_{o3}) r_{o5}}$$

31.



(a) $i_{IN} \uparrow \Rightarrow V_{G,M_2} \uparrow$ (Common Gate ; i_{IN} mostly flows to M_1 ∵ resistance $= \frac{1}{g_m}$)

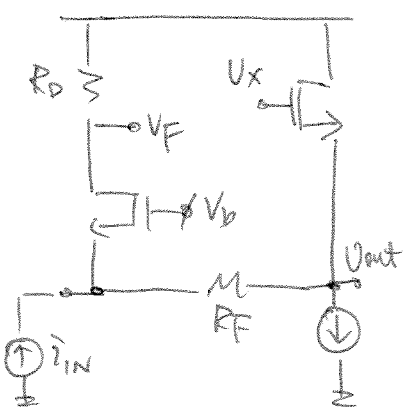
$\Rightarrow V_{out} \uparrow$ (Source Follower)

$\Rightarrow R_F$ momentarily provides <u>more</u> current to source of M_1)

$\Rightarrow V_{G,M_2} \uparrow$

$\Rightarrow$ Positive feedback.

(b)



$$v_F = v_X \frac{g_{m1} R_D}{1 + g_{m2}\left(R_F + \frac{1}{g_{m1}}\right)}$$

$$\Rightarrow -\frac{v_F}{v_X} = \text{Loop Gain} = -\frac{g_{m1} R_D}{1 + g_{m2}\left(R_F + \frac{1}{g_{m1}}\right)}$$

∴ feedback is positive.

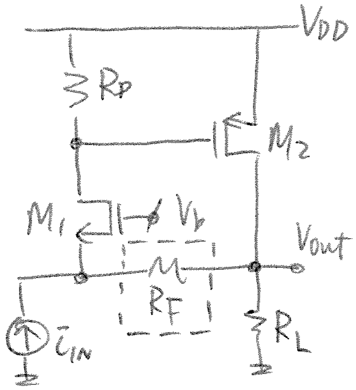
32.

(a) $i_{IN} \uparrow \Delta \Rightarrow \Delta i_{IN}$ mostly flows in $\frac{1}{g_{m1}} \Rightarrow V_{G,M2} \uparrow$ (Common Gate)

$\Rightarrow V_{out} \downarrow$ (Common Source)

$\Rightarrow R_F$ momentarily demands more current from i_{IN}

$\Rightarrow$ Negative feedback.



$\lambda = 0$

$R_F \gg 1.$

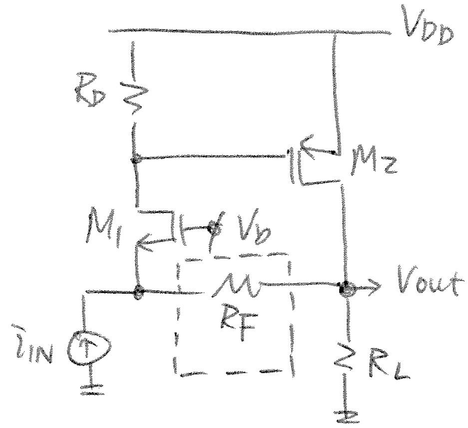
(b) $R_{O.L.} = \left.\frac{V_{out}}{i_{IN}}\right|_{O.L.} = -R_D \times g_{m2} R_L$

(c) K (feedback factor) $= \frac{-1}{R_F}$

$$\Rightarrow R_{C.L.} = \frac{R_{O.L}}{1 + R_{O.L.} \times K} = \frac{-R_D \times g_{m2} R_L}{1 + \frac{R_D}{R_F} g_{m2} R_L}$$

33.

$$R_{c.L.} = \frac{-g_{m2} R_D R_L}{1 + \frac{g_{m2} R_D R_L}{R_F}}$$

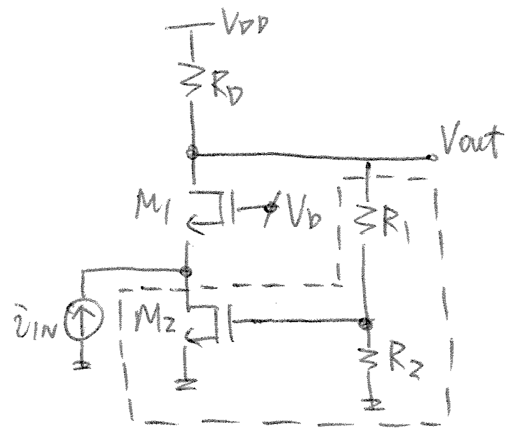


$$\text{loop gain} = \frac{g_{m2} R_D R_L}{R_F}$$

$$r_{in} \approx \frac{1}{g_{m1}} \qquad r_{out} \approx R_L \qquad (R_F \text{ large})$$

$$\Rightarrow r_{in}\Big|_{c.L.} = \frac{1/g_{m1}}{1 + \frac{g_{m2} R_D R_L}{R_F}} \qquad r_{out}\Big|_{c.L.} = \frac{R_L}{1 + \frac{g_{m2} R_D R_L}{R_F}}$$

34.



$\lambda = 0$

$R_1 + R_2 \gg 1.$

$$R_{O.L.} = \frac{v_{out}}{i_{IN}} \text{ (no feedback)}$$

$$= R_D$$

K (feedback factor)

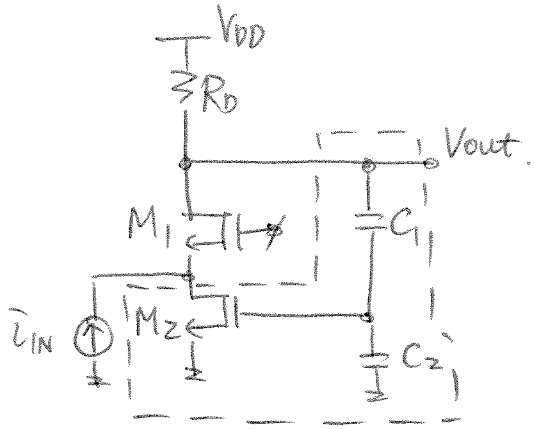
$$= g_{m2} \times \frac{R_2}{R_1 + R_2}$$

$$\Rightarrow R_{C.L.} = \frac{v_{out}}{i_{IN}} = \frac{R_D}{1 + R_D \times g_{m2} \frac{R_2}{R_1 + R_2}}$$

$$r_{in}\Big|_{C.L.} = \frac{1/g_{m1}}{1 + R_D \times g_{m2} \frac{R_2}{R_1 + R_2}}$$

$$r_{out}\Big|_{C.L.} = \frac{R_D}{1 + R_D \times g_{m2} \frac{R_2}{R_1 + R_2}}$$

35.



$$R_{O.L.} = \frac{V_{out}}{i_{IN}} \text{ (no feedback)}$$
$$= R_D$$

K (feedback factor)

$$= g_{m2} \times \frac{C_1}{C_1 + C_2}$$

$$\Rightarrow R_{C.L.} = \frac{V_{out}}{i_{IN}} = \frac{R_D}{1 + R_D \times g_{m2} \frac{C_1}{C_1 + C_2}}$$

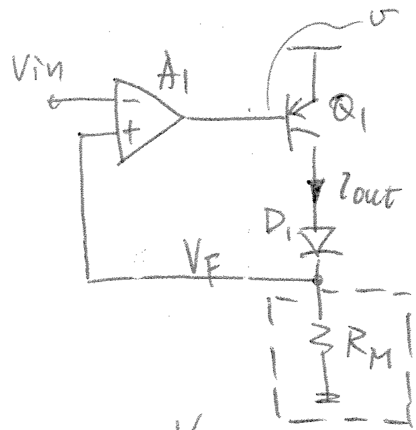
$$r_{in}|_{C.L.} = \frac{1/g_{m1}}{1 + R_D \times g_{m2} \frac{C_1}{C_1 + C_2}}$$

$$r_{out}|_{C.L.} = \frac{R_D}{1 + R_D \times g_{m2} \frac{C_1}{C_1 + C_2}}$$

3b.

(a) $G_{O.L.} = \dfrac{i_{out}}{v_{in}} = g_{m_1} A_1$

(common emitter)



$V_A \to \infty$

R_M small.

(b) K (feedback factor)

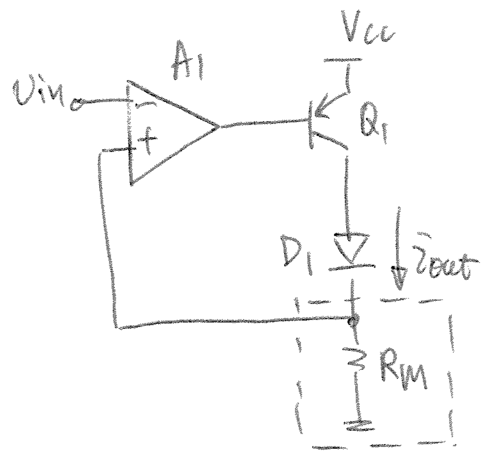
$\Rightarrow V_F = i_{out} \times R_M$

$\Rightarrow K = \dfrac{V_F}{i_{out}} = R_M$

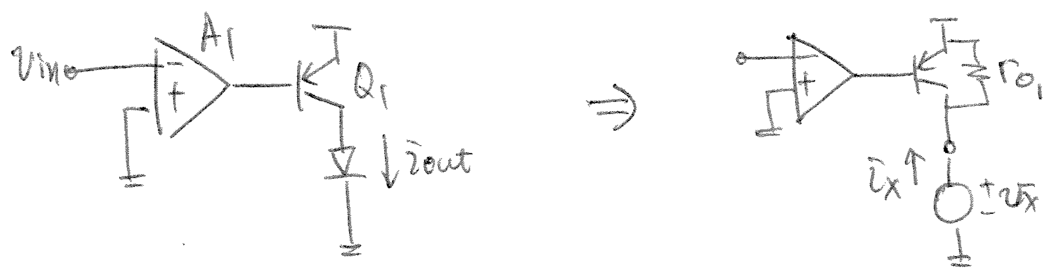
$\therefore$ Loop Gain $= G_{OL} K = g_{m_1} A_1 R_M$

$$G_{C.L.} = \frac{G_{OL}}{1 + G_{OL} K} = \frac{g_{m_1} A_1}{1 + g_{m_1} A_1 R_M}$$

37.



Since R_M is small, the following circuit results:

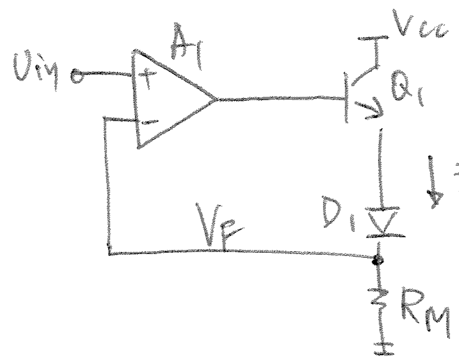


$\therefore \; R_{out,\,OPEN} = \dfrac{v_x}{i_x} = r_{o1}$

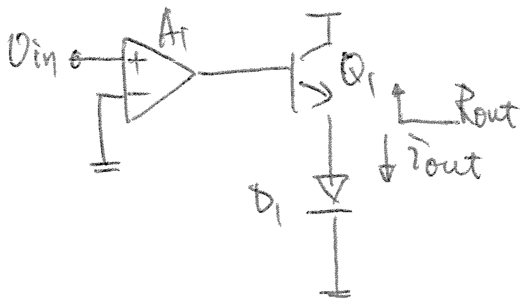
$G_{OL} = \dfrac{i_{out}}{v_{in}} = A_1 g_{m1}$ $\qquad K = R_M$

$$\therefore \; R_{out,\,CLOSED} = R_{out,\,OPEN}(1 + G_{OL}K)$$
$$= r_{o1}(1 + A_1 g_{m1} R_M)$$

38.



Since R_M is small, the open-loop equivalent becomes the following:



$$G_{OL} = \frac{i_{out}}{v_{in}} \approx A_1 g_{m1}$$

$$R_{out} = \frac{r_\pi}{\beta+1} \approx \frac{1}{g_{m1}}$$

$$K = \frac{V_F}{i_{out}} = R_M$$

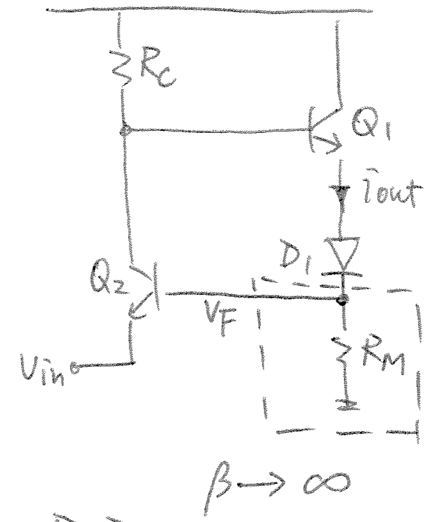
$$\Rightarrow G_{C.L.} = \frac{G_{OL}}{1+G_{OL}K} = \frac{A_1 g_{m1}}{1+g_{m1}A_1R_M}$$

$$\text{Loop Gain} = G_{OL}K = g_{m1}A_1R_M$$

$$R_{out,\,CLOSED} = \frac{1}{g_{m1}}(1+g_{m1}A_1R_M)$$

This circuit provides a much lower output resistance which in general is non-desirable (ideally any current source should have high impedance.)

39.



Using procedure in Ex 12.21

$$G_{O.L.} = \frac{i_{out}}{V_{in}} = g_{m2} R_C \times g_{m1}$$

K (feedback factor)

$$= \frac{V_F}{i_{out}} = R_M.$$

$\Rightarrow$ loop gain $= G_{O.L.} \times K = g_{m1} g_{m2} R_C R_M$

$\Rightarrow$ closed-loop gain $G_{CL} = \dfrac{g_{m1} g_{m2} R_C}{1 + g_{m1} g_{m2} R_C R_M}$

Using procedure in Ex. 12.22

$G_{OL} = g_{m1} g_{m2} R_C$ $\quad\quad$ $K = R_M$

$r_{in}|_{O.L.} = \dfrac{1}{g_{m1}}$ $\quad\quad$ $r_{out}|_{O.L.} \cong \dfrac{1}{g_{m2}}$

$$r_{in}|_{C.L.} = \frac{1}{g_{m1}}(1 + g_{m1} g_{m2} R_C R_M)$$

$$r_{out}|_{C.L.} = \frac{1}{g_{m2}}(1 + g_{m1} g_{m2} R_C R_M)$$

40.

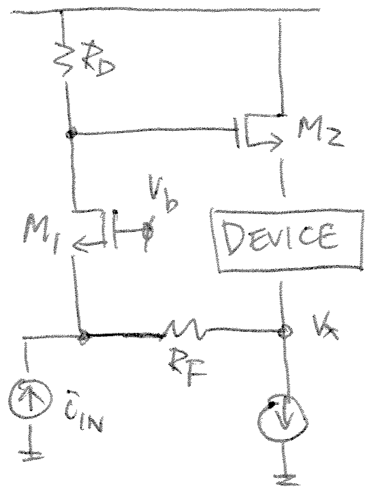
(a) $i_{IN} \uparrow \Delta \Rightarrow$ Most of i_{IN} flows into $1/g_{m1}$

$\Rightarrow V_{G,M2} \uparrow$ (Common Gate)

$\Rightarrow V_X \uparrow$ (Source Follower)

$\Rightarrow R_F$ momentarily provides more current to Source of M_1

$\Rightarrow V_{G,M2} \uparrow \Rightarrow$ Positive feedback.

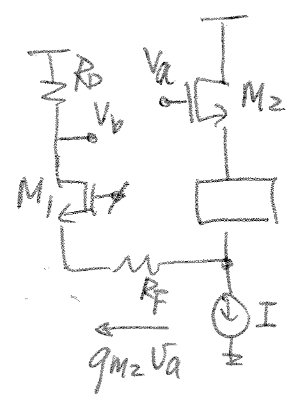


(b)

$v_a \times g_{m2} \times R_D = v_b$

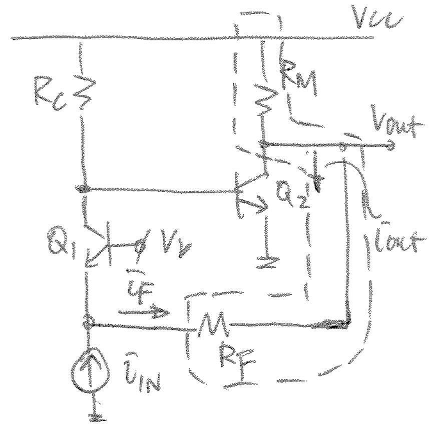
$\Rightarrow$ loop gain $= -\dfrac{v_b}{v_a} = -g_{m2} R_D$.

Since loop gain is negative, feedback is positive.



41.

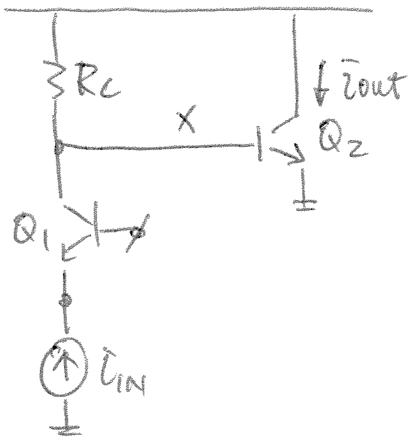
(a) The open-loop equivalent becomes:



$V_A \to \infty$

R_M small

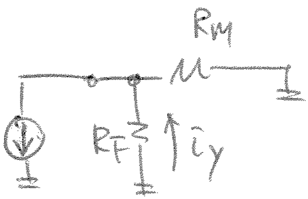
R_F large



$$A_I = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}} \cong g_{m2} \times R_C$$

$R_{in,OPEN} \cong \dfrac{1}{g_{m1}}$ $\qquad$ $R_{out,OPEN} \to \infty \quad (\because V_{A_2} \to \infty)$

$$k = \frac{i_Y}{i_X} = + \frac{(R_F \| R_M)}{R_F}$$

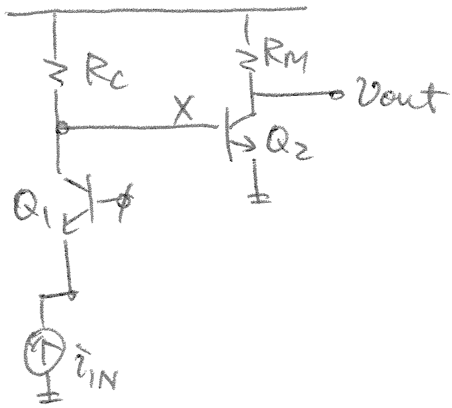


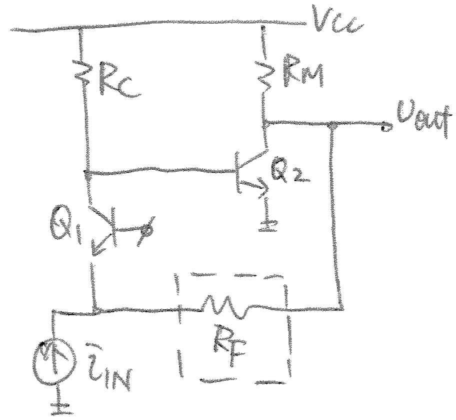
$$A_{CL} = \frac{A_I}{1 + A_I k} = \frac{g_{m2} R_C}{1 + g_{m2} R_C \dfrac{(R_F \| R_M)}{R_F}}$$

$$R_{in,CLOSED} = \frac{1/g_{m1}}{1 + g_{m2} R_C \dfrac{(R_F \| R_M)}{R_F}}$$

$R_{out,CLOSED} \to \infty$

(b) The open-loop equivalent becomes:





$V_A \to \infty$

R_F large

$$R_{OL} = \frac{v_{out}}{i_{IN}} = \frac{v_{out}}{v_X} \times \frac{v_X}{i_{IN}} \cong -g_{m2} R_M \times R_C$$

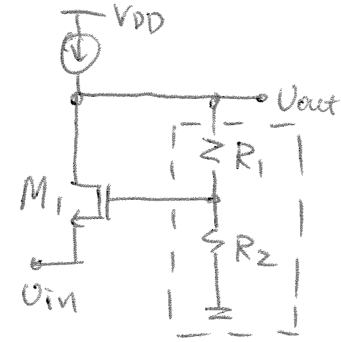
$$R_{in,OPEN} = \frac{1}{g_{m1}} \qquad R_{out,OPEN} = R_M$$

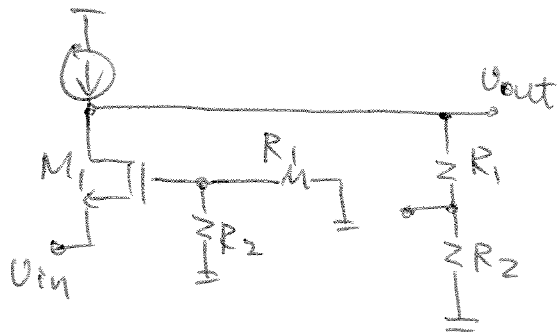
$$K = -\frac{1}{R_M}$$

$$R_{CL} = \frac{R_{OL}}{1 + R_{OL} K} = \frac{-g_{m2} R_M R_C}{1 + g_{m2} R_C}$$

$$R_{in,CLOSED} = \frac{1/g_{m1}}{1 + g_{m2} R_C} \qquad R_{out,CLOSED} = \frac{R_M}{1 + g_{m2} R_C}$$

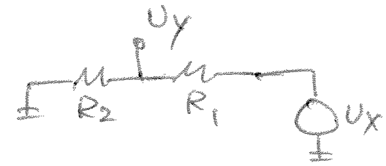
42. Breaking the feedback network results in the following circuit:





Feedback factor

$$= K = \frac{V_Y}{V_X} = \frac{R_2}{R_1+R_2}$$



$$A_{O.L.} = +g_{m1}(R_1+R_2)$$

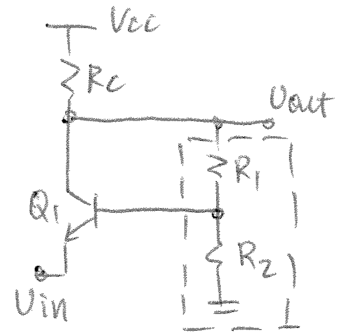
$$\text{Loop Gain} = A_{O.L.}K = g_{m1}R_2$$

$$\therefore A_{C.L.} = \frac{A_{OL}}{1+A_{OL}K} = \frac{g_{m1}(R_1+R_2)}{1+g_{m1}R_2}$$

$$R_{in,CLOSED} = \frac{1}{g_{m1}}(1+g_{m1}R_2)$$

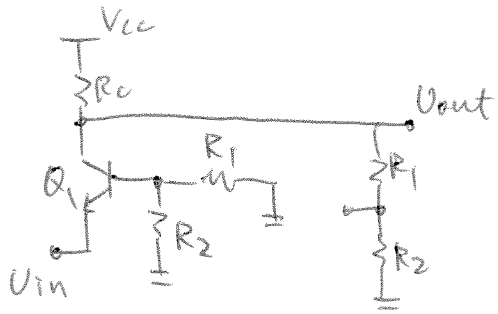
$$R_{out,CLOSED} = \frac{R_1+R_2}{1+g_{m1}R_2}$$

43. Breaking the feedback network results in the following circuit:



$V_A \to \infty$

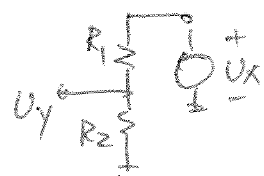
$1 \ll \beta < \infty$



$$A_{O.L.} \approx g_{m_1}\left[R_C \| (R_1 \| R_2)\right] \qquad \text{since } i_b \to 0 \ (\beta \gg 1)$$

Feedback factor

$$= K = \frac{v_y}{v_x} = \frac{R_2}{R_1 + R_2}$$



$$R_{in,OPEN} = \frac{r_{\pi_1} + (R_1 \| R_2)}{\beta_1 + 1} \approx \frac{1}{g_{m_1}} + \frac{R_1 \| R_2}{\beta_1 + 1}$$

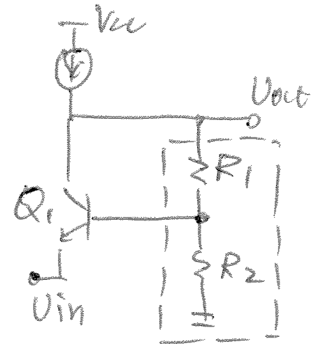
$$R_{out,OPEN} = R_C \| (R_1 + R_2)$$

$$\Rightarrow A_{C.L.} = \frac{A_{OL}}{1 + A_{OL}K} = \frac{g_{m_1}\left[R_C \| (R_1 + R_2)\right]}{1 + g_{m_1}\left[R_C \| (R_1 + R_2)\right] \times \frac{R_2}{R_1 + R_2}}$$

$$R_{in,CLOSED} = \left(\frac{1}{g_{m_1}} + \frac{R_1 \| R_2}{\beta_1 + 1}\right)(1 + A_{OL}K)$$

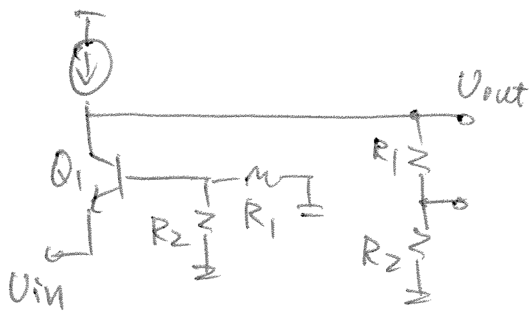
$$R_{out,CLOSED} = \frac{R_C \| (R_1 + R_2)}{1 + A_{OL}K}$$

44. Breaking the feedback network results in the following circuit:



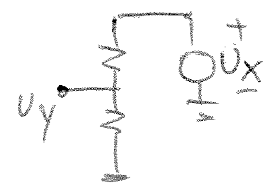
$V_A \to \infty$

$1 \ll \beta < \infty$



$A_{OL} \cong g_{m1}(R_1 + R_2)$ since $i_b \to 0$ $(\beta \gg 1)$

Feedback factor



$= K = \frac{V_Y}{V_X} = \frac{R_2}{R_1 + R_2}$

$R_{in,OPEN} \cong \frac{1}{g_{m1}} + \frac{R_1 \| R_2}{\beta_1 + 1}$

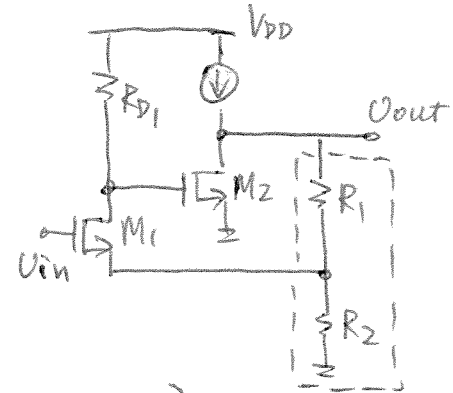
$R_{out,OPEN} = R_1 + R_2$

$\Rightarrow A_{CL} = \frac{A_{OL}}{1 + A_{OL}K} = \frac{g_{m1}(R_1 + R_2)}{1 + g_{m1}R_2}$

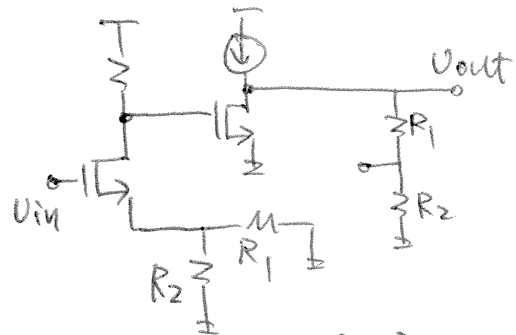
$R_{in,CLOSED} = \left(\frac{1}{g_{m1}} + \frac{R_1 \| R_2}{\beta_1 + 1}\right)(1 + g_{m1}R_2)$

$R_{out,CLOSED} = \frac{R_1 + R_2}{1 + g_{m1}R_2}$

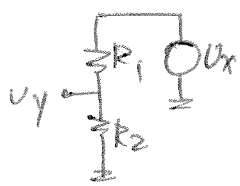
45. Breaking the feedback network results in the following circuit:



$\lambda = 0$



$$A_{OL} = \frac{-g_{m1} R_{D1}}{1 + g_{m1}(R_1 \| R_2)} \times -g_{m2}(R_1 + R_2)$$



$$K = \frac{v_Y}{v_X} = \frac{R_2}{R_1 + R_2}$$

$R_{in,OPEN} \to \infty$

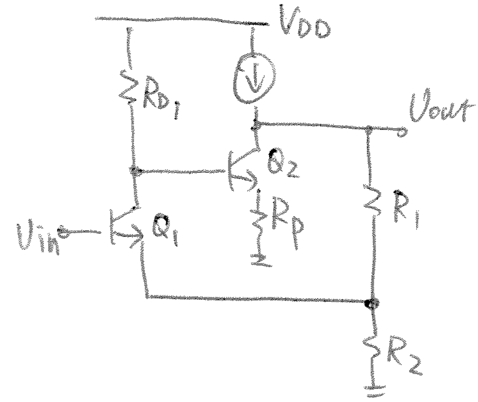
$R_{out,OPEN} = R_1 + R_2$

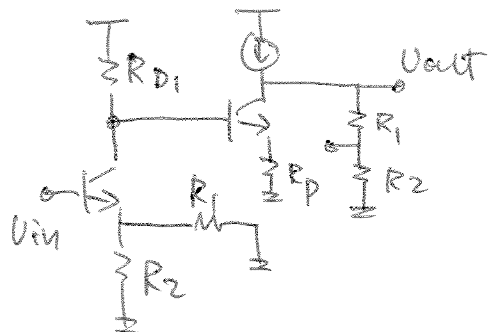
$$\therefore A_{CL} = \frac{A_{OL}}{1 + A_{OL}K} = \frac{\left[\dfrac{g_{m1} g_{m2} R_{D1}(R_1 + R_2)}{1 + g_{m1}(R_1 \| R_2)}\right]}{1 + \dfrac{g_{m1} g_{m2} R_{D1}}{1 + g_{m1}(R_1 \| R_2)}}$$

$R_{in,CLOSED} \to \infty$

$$R_{out,CLOSED} = \frac{R_1 + R_2}{1 + A_{OL}K}$$

46. Breaking the feedback network results in the following circuit:

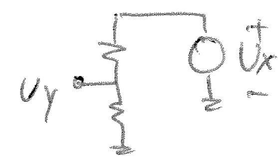




$$A_{OL} \cong \frac{-g_{m1} \times \{ R_{D1} \parallel [r_{\pi 2} + (\beta_2+1)R_p] \}}{1 + g_{m1}(R_1 \parallel R_2)} \times \frac{-g_{m2}(R_1+R_2)}{1+g_{m2}R_p}$$

$$R_{in,OPEN} = r_{\pi 1} + (R_1 \parallel R_2) \qquad R_{out,OPEN} = R_1 + R_2$$

$$K = \frac{v_Y}{v_X} = \frac{R_2}{R_1+R_2}$$

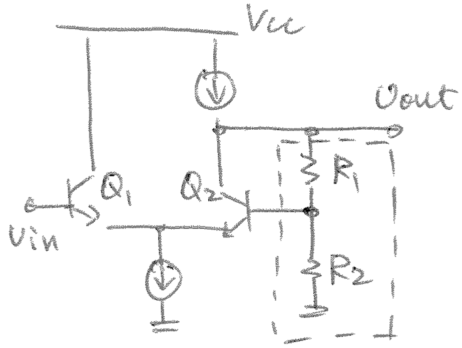


$$\Rightarrow A_{CL} = \frac{A_{OL}}{1 + A_{OL}K}$$

$$R_{in,CLOSED} = [r_{\pi 1} + (R_1 \parallel R_2)](1 + A_{OL}K)$$

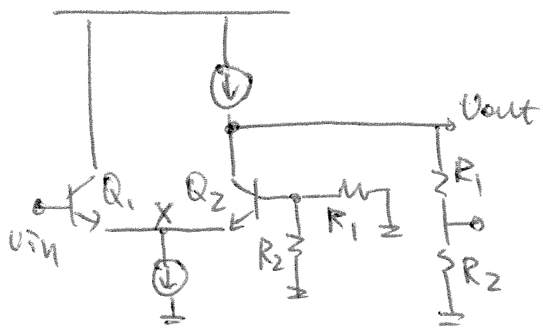
$$R_{out,CLOSED} = \frac{R_1+R_2}{1+A_{OL}K}$$

47. Breaking the feedback network results in the following circuit:



$(V_A \to \infty)$

$$K = \frac{R_2}{R_1 + R_2}$$



$$A_{OL} \cong \underbrace{\frac{g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)}{1 + g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)}}_{\text{emitter follower } (Q_1) = \frac{V_X}{V_{in}}} \times \underbrace{g_{m2}(R_1 + R_2)}_{\text{common-base stage } (Q_2) = \frac{V_{out}}{V_X}}$$

$$R_{in,OPEN} = r_{\pi 1} + (\beta_1 + 1)\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)$$

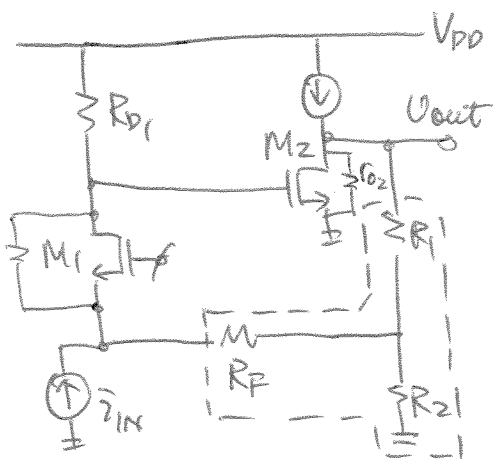
$$R_{out,OPEN} = R_1 + R_2$$

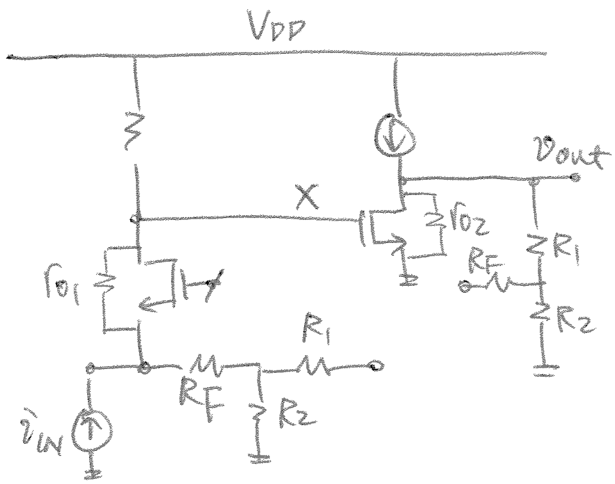
$$\therefore A_{CL} = \frac{A_{OL}}{1 + A_{OL}K}$$

$$R_{in,CLOSED} = \left[r_{\pi 1} + (\beta_1 + 1)\left(\frac{1}{g_{m2}} + \frac{R_1 \| R_2}{\beta_2 + 1}\right)\right] \times (1 + A_{OL}K)$$

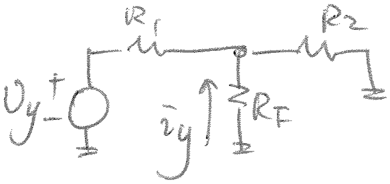
$$R_{out,CLOSED} = \frac{R_1 + R_2}{1 + A_{OL}K}$$

48. Breaking the feedback network results in the following circuit:





$$K = -\frac{(R_F \| R_2)/R_F}{R_F \| R_2 + R_1}$$



$$R_{OL} = \frac{v_{out}}{i_{IN}} = \frac{v_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

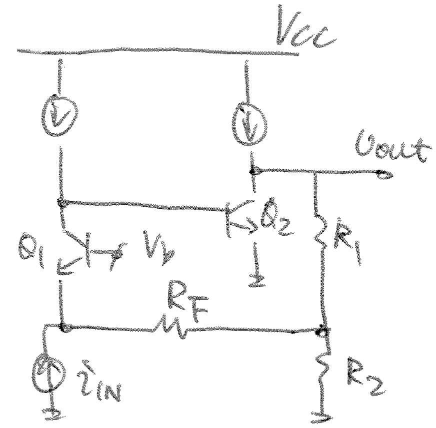
$$= -g_{m2}\left[r_{o2} \| (R_1+R_2)\right] \times \left\{ \frac{\frac{1}{r_{o1}} + \frac{1}{R_{D1}}}{\frac{1}{r_{o1}} + g_{m1}} \times \left[\frac{1}{(R_F+R_2) \| \frac{1}{g_{m1}}} + \frac{1}{r_{o1}} \right] - \frac{1}{r_{o1}} \right\}$$

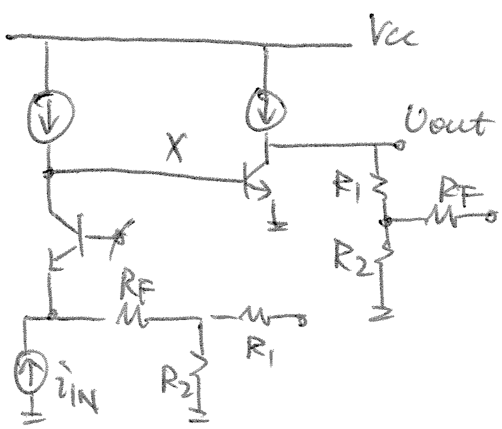
$$R_{in,OPEN} \cong \frac{1}{g_{m1}} \| (R_F+R_2) \| \frac{r_{o1}}{1-g_{m1}R_{D1}} \qquad R_{out,OPEN} = r_{o2} \| (R_1+R_2)$$

$$R_{CL} = \frac{R_{OL}}{1+R_{OL}K} \qquad R_{in,CLOSED} = \frac{R_{in,OPEN}}{1+R_{OL}K}$$

$$R_{out,CLOSED} = \frac{R_{out,OPEN}}{1+R_{OL}K}$$

49. Breaking the feedback network results in the following circuit:

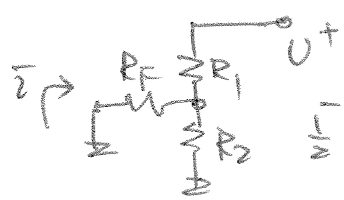




$$R_{OL} = \frac{v_{out}}{i_{IN}} = \frac{v_{out}}{v_X} \times \frac{v_X}{i_{IN}} = \left[-g_{m2}(R_1+R_2)\right] \times \left[g_{m1} r_{\pi 2} \left\{ \frac{1}{g_{m1}} \parallel (R_F+R_2) \right\} \right]$$

$$R_{in,OPEN} = \frac{1}{g_{m1}} \parallel (R_F+R_2) \qquad R_{out,OPEN} = R_1+R_2$$

$$K = \frac{\tilde{v}}{v} = -\frac{(R_2 \parallel R_F)/R_F}{R_1 + (R_2 \parallel R_F)}$$

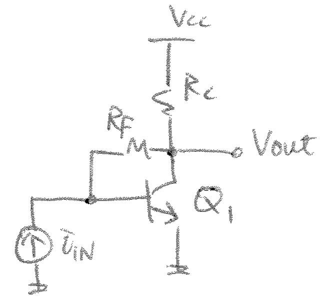


$$\therefore\ R_{CL} = \frac{R_{OL}}{1+R_{OL}K}$$

$$R_{in,CLOSED} = \frac{\frac{1}{g_{m1}} \parallel (R_F+R_2)}{1+R_{OL}K}$$

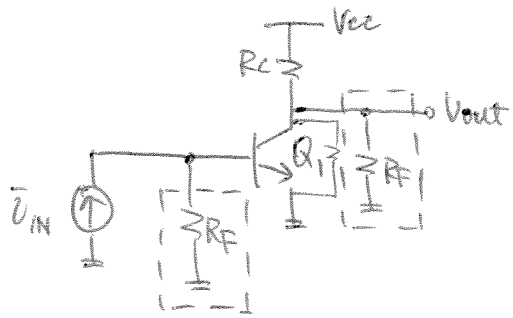
$$R_{out,CLOSED} = \frac{R_1+R_2}{1+R_{OL}K}$$

50. The feedback network consists of R_F.



Using the method discussed in lecture, break the circuit as follows:

$(V_A < \infty)$



This is the open-loop circuit with consideration of I/O loading.

- By inspection,

$$v_{out} = i_c \times (R_C \| R_F \| r_o)$$

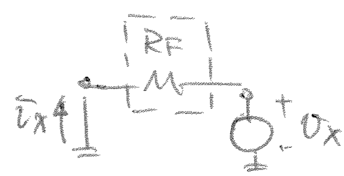
$$= -g_m(i_{IN} \times (R_F \| r_\pi)) \times (R_C \| R_F \| r_o)$$

$$\Rightarrow R_{O.L.} = \frac{v_{out}}{i_{IN}} = -g_m(R_F \| r_\pi)(R_C \| R_F \| r_o) \quad \text{— (1)}$$

$$R_{in,open} = (R_F \| r_\pi) \qquad R_{out,open} = (R_C \| R_F \| r_o)$$

- Feedback factor k:

$$k = \frac{v_x}{i_x} = -\frac{1}{R_F}$$

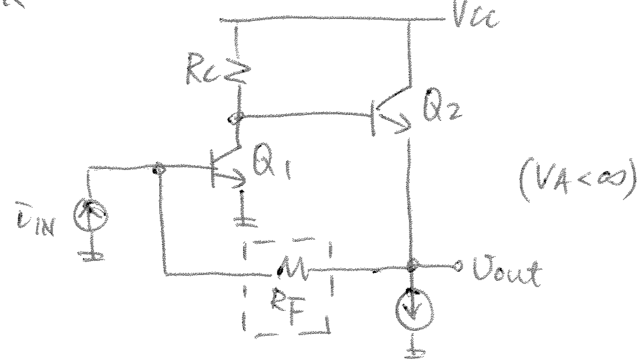


$$\therefore \quad R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K} = \frac{-g_m (R_F \| r_\pi)(R_C \| R_F \| r_o)}{1 + \frac{g_m (R_F \| r_\pi)(R_C \| R_F \| r_o)}{R_F}}$$

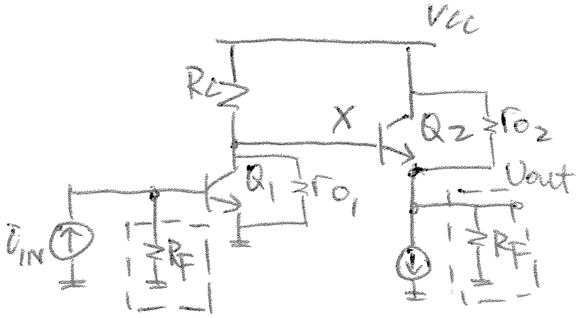
$$R_{in,CLOSED} = \frac{(R_F \| r_\pi)}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{(R_C \| R_F \| r_o)}{1 - \frac{R_{O.L.}}{R_F}}$$

where $R_{O.L.}$ is given by (1).

51. The feedback network consists of R_F.



Using the method discussed in lecture, break the circuit as follows:



This is the open-loop circuit with consideration of I/O loading.

- Gain of common-emitter stage:

$$\frac{v_X}{i_{IN}} = -g_{m_1}(R_F \| r_{\pi_1}) \times \left\{ R_C \| r_{o_1} \| \left[r_{\pi_2} + (\beta_2+1)(R_F \| r_{o_2}) \right] \right\}$$

- Gain of emitter-follower stage:

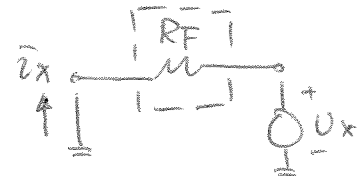
$$\frac{v_{out}}{v_X} = \frac{g_{m2}(R_F \| r_{o2})}{1 + g_{m2}(R_F \| r_{o2})}$$

$$\Rightarrow R_{O.L.} = \frac{v_X}{i_{IN}} \cdot \frac{v_{out}}{v_X} \qquad \text{—— (1)}$$

$$R_{in,OPEN} = R_F \| r_{\pi_1} \qquad R_{out,OPEN} \cong R_F \| r_{o_2} \| \frac{1}{g_{m_2}}$$

- Feedback factor k:

$$k = \frac{v_x}{i_x} = -\frac{1}{R_F}$$



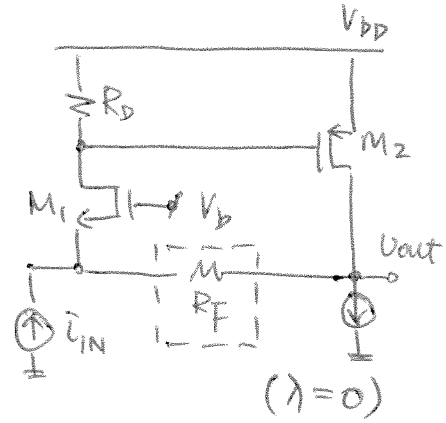
$$\therefore \; R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times k} = \frac{R_{O.L.}}{1 - R_{O.L.}/R_F}$$

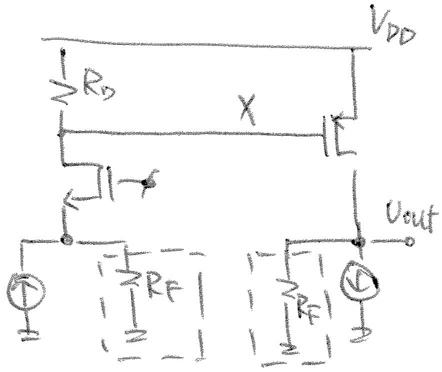
$$R_{in,CLOSED} = \frac{(R_F \parallel r_{\pi 1})}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,CLOSED} = \frac{R_F \parallel r_{o2} \parallel \frac{1}{g_{m2}}}{1 - \frac{R_{O.L.}}{R_F}}$$

where $R_{O.L.}$ is given by (1).

52.

(a) Breaking the feedback loop results in the following circuit:





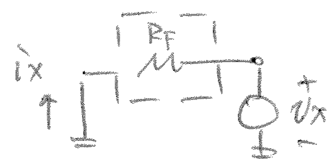
$$R_{O.L.} = \frac{v_X}{i_{IN}} \cdot \frac{v_{out}}{v_X}$$

$$= g_{m_1} R_D \left(\frac{1}{g_{m_1}} \| R_F\right) \times (-g_{m_2} R_F)$$

$$R_{in,OPEN} = \frac{1}{g_{m_1}} \| R_F \qquad R_{out,OPEN} = R_F$$

- Feedback factor K:

$$K = \frac{v_X}{i_X} = -\frac{1}{R_F}$$

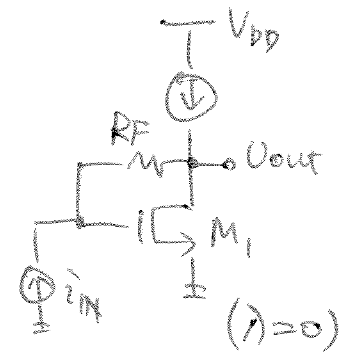


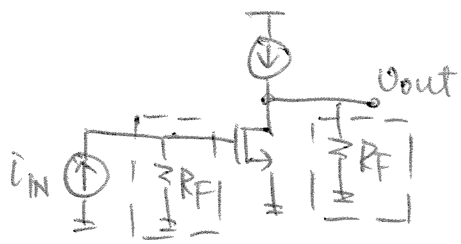
$$\Rightarrow R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K} = \frac{-g_{m_1} g_{m_2} R_D R_F \left(\frac{1}{g_{m_1}} \| R_F\right)}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

$$R_{in,CLOSED} = \frac{\left(\frac{1}{g_{m_1}} \| R_F\right)}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

$$R_{out,CLOSED} = \frac{R_F}{1 + g_{m_1} g_{m_2} R_D \left(\frac{1}{g_{m_1}} \| R_F\right)}$$

(b) Breaking the feedback loop results in the following circuit:

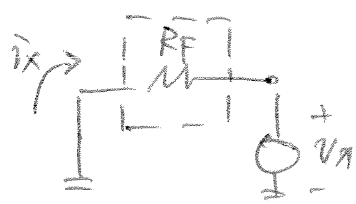




$$R_{O.L.} = \frac{v_{out}}{i_{IN}} = -g_m R_F R_F = -g_m R_F^2$$

$R_{in,\,OPEN} = R_F \qquad R_{out,\,OPEN} = R_F$

- Feedback factor K:



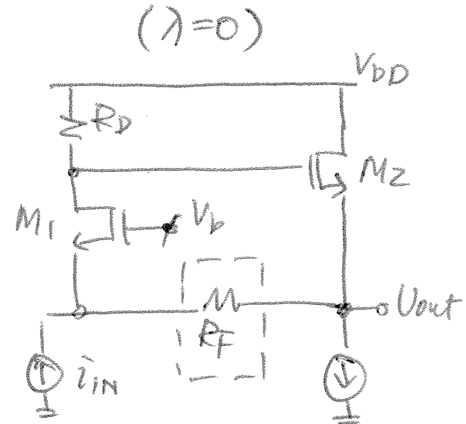
$$K = \frac{v_x}{i_x} = -\frac{1}{R_F}$$

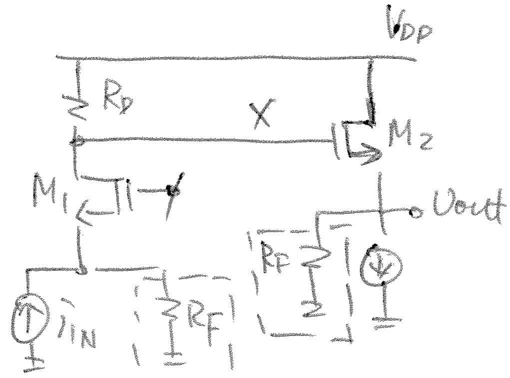
$$\Rightarrow R_{C.L.} = \frac{-g_m R_F^2}{1 + g_m R_F}$$

$$R_{in,\,CLOSED} = \frac{R_F}{1 + g_m R_F}$$

$$R_{out,\,CLOSED} = \frac{R_F}{1 + g_m R_F}$$

(c) Breaking the feedback loop results in the following circuit:





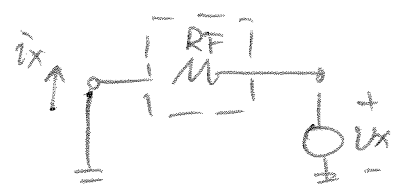
$$R_{O.L.} = \frac{v_{out}}{i_{IN}} = \frac{v_x}{i_{IN}} \cdot \frac{v_{out}}{v_x}$$

$$= g_{m1} R_D \left(\frac{1}{g_{m1}} \| R_F\right) \times g_{m2}\left(R_F \| \frac{1}{g_{m2}}\right)$$

$$R_{in,\,OPEN} = \left(\frac{1}{g_{m1}} \| R_F\right) \qquad R_{out,\,OPEN} = \left(R_F \| \frac{1}{g_{m2}}\right)$$

– Feedback factor K:

$$K = \frac{v_x}{i_x} = -\frac{1}{R_F}$$



(Note: Feedback is positive.)

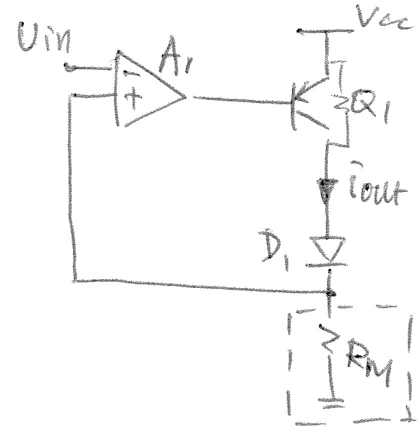
$$\Rightarrow R_{C.L.} = \frac{R_{O.L.}}{1 + R_{O.L.} \times K}$$

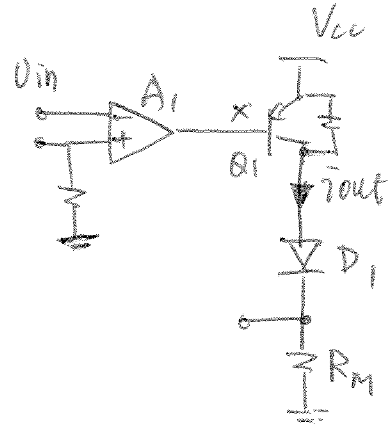
$$= \frac{g_{m1} g_{m2} R_D \left(1/g_{m1} \| R_F\right)\left(1/g_{m2} \| R_F\right)}{1 - g_{m1} g_{m2}\left(\frac{R_D}{R_F}\right)\left(\frac{1}{g_{m1}} \| R_F\right)\left(1/g_{m2} \| R_F\right)}$$

$$R_{IN,\,CLOSED} = \frac{\left(1/g_{m1} \| R_F\right)}{1 - \frac{R_{O.L.}}{R_F}} \qquad R_{out,\,CLOSED} = \frac{\left(1/g_{m2} \| R_F\right)}{1 - \frac{R_{O.L.}}{R_F}}$$

53. Breaking the feedback network (i.e. R_M) results in the following circuit:

($V_A < \infty$)

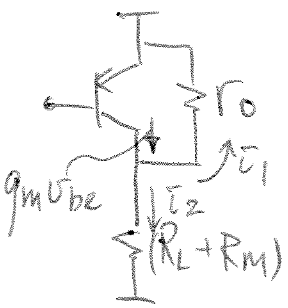




$$G_{O.L.} = \frac{i_{out}}{v_{in}} = \frac{i_{out}}{v_x} \times \frac{v_x}{v_{in}} \quad \text{— (1)}$$

$$= \underbrace{g_{m_1} \times \frac{[(R_L + R_M) \| r_{o_1}]}{(R_L + R_M)}}_{\text{(current division)}} \times (-A_1)$$

Note: current ($g_m v_{be}$) splits between r_o & [R_L (impedance of D_1) + R_M]



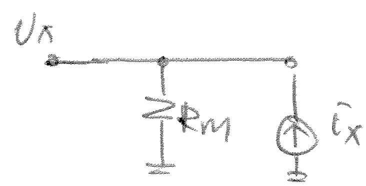
$g_m v_{be} = i_1 + i_2$

$R_{in, OPEN} \rightarrow \infty$

$R_{out, OPEN} = r_{o_1} + R_M$

- Feedback factor K:

$$K = \frac{v_x}{i_x} = R_M$$



$$\therefore \quad G_{C.L.} = \frac{G_{O.L.}}{1 + G_{O.L} \times K} = \frac{G_{O.L.}}{1 + G_{O.L.} \times R_M}$$

$$R_{in,\,CLOSED} \to \infty$$

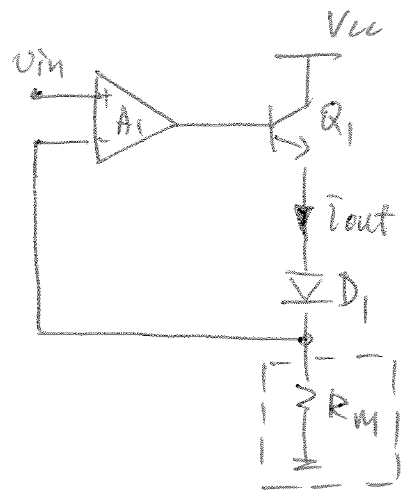
$$R_{out,\,CLOSED} = (r_{o1} + R_M)(1 + G_{O.L.} \times R_M)$$

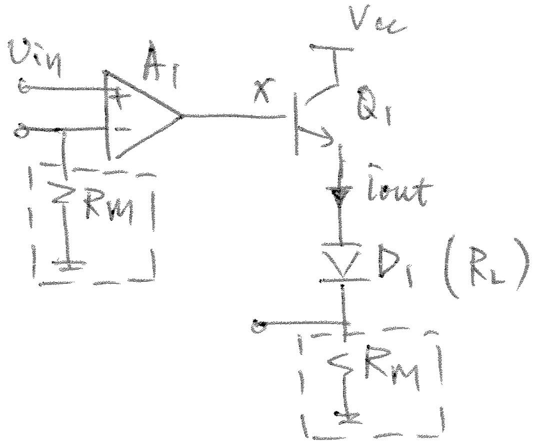
where $G_{O.L.}$ is given by (1)

54.

(a) Breaking the feedback loop results in the following circuit:

$(V_A \to \infty)$





$$G_{O.L.} = \frac{i_{out}}{v_{in}} = \frac{i_{out}}{v_X} \cdot \frac{v_X}{v_{in}} = \frac{g_{m1}(R_L + R_M)}{1 + g_{m1}(R_L + R_M)} \times A_1$$

$R_{in,OPEN} \to \infty$ $\qquad$ $R_{out,OPEN} \approx R_M + \frac{1}{g_{m1}}$

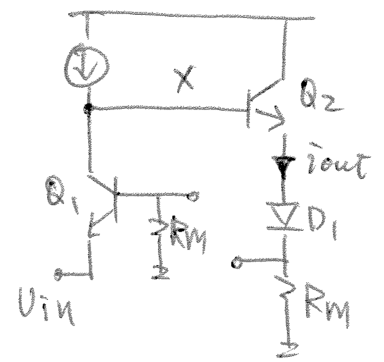
– Feedback factor K:

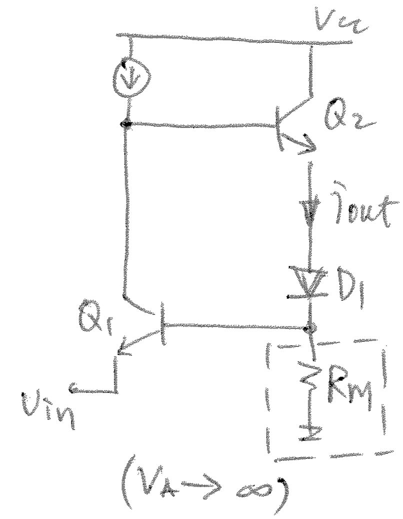
$$K = \frac{v_X}{i_X} = R_M$$

$$G_{C.L.} = \frac{G_{O.L.}}{1 + G_{O.L.} K} = \frac{G_{O.L.}}{1 + G_{O.L.} R_M}$$

$R_{in,CLOSED} \to \infty$ $\qquad$ $R_{out,CLOSED} = \left(R_M + \frac{1}{g_{m1}}\right) \times (1 + G_{O.L.} R_M)$

(b) Breaking the feedback loop results in the following circuit:



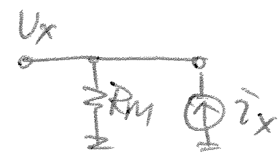


$$G_{O.L.} = \frac{i_{out}}{v_{in}} = \frac{i_{out}}{v_x} \times \frac{v_x}{v_{in}} \cong \frac{g_{m2}(R_L + R_M)}{1 + g_{m2}(R_L + R_M)} \times g_{m1}\left[r_{\pi 2} + (\beta_2 + 1)(R_L + R_M)\right]$$

$$R_{in,OPEN} = \frac{r_{\pi 1} + R_M}{\beta_1 + 1} \approx \frac{1}{g_{m1}} + \frac{R_M}{\beta_1 + 1}$$

$$R_{out,OPEN} = \frac{1}{g_{m2}} + R_M$$

- Feedback Factor K:



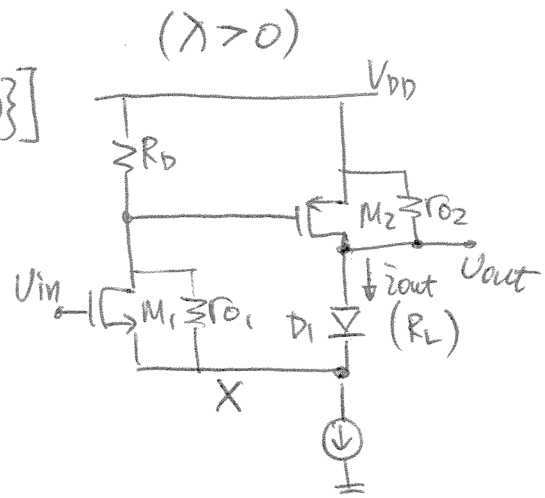
$$K = \frac{v_x}{i_x} = R_M$$

$$\Rightarrow G_{C.L.} = \frac{G_{O.L.}}{1 + G_{O.L.} \times K} = \frac{G_{O.L.}}{1 + G_{O.L.} R_M}$$

$$R_{in,CLOSED} = \left(\frac{1}{g_{m1}} + \frac{R_M}{\beta_1 + 1}\right)\left(1 + G_{O.L.} R_M\right)$$

$$R_{out,CLOSED} = \left(\frac{1}{g_{m2}} + R_M\right)\left(1 + G_{O.L.} R_M\right)$$

55.



$$v_{out} = \left[-g_{m1}(v_{in}-v_x)R_D\right]\cdot\left[-g_{m2}\left\{r_{o2} \parallel \left(\frac{1}{g_{m1}}+R_L\right)\right\}\right]$$

(ignore r_{o1} for now) — ①

$$\frac{v_{out}-v_x}{R_L} = i_{out}$$

$$\Rightarrow v_{out} = i_{out}R_L + v_x \quad \text{— ②}$$

$$i_{out} = -g_{m1}(v_{in}-v_x) \Rightarrow v_{in} + \frac{i_{out}}{g_{m1}} = v_x \quad \text{— ③}$$

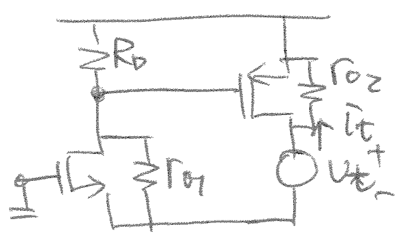
- Substitute ② & ③ into ① and solve for $\frac{i_{out}}{v_{in}}$:

$$\frac{i_{out}}{v_{in}} \cong -\frac{1}{R_L + \frac{1}{g_{m1}} + g_{m2}R_D\left\{r_{o2} \parallel \left(\frac{1}{g_{m1}}+R_L\right)\right\}} = G_{C.L.}$$

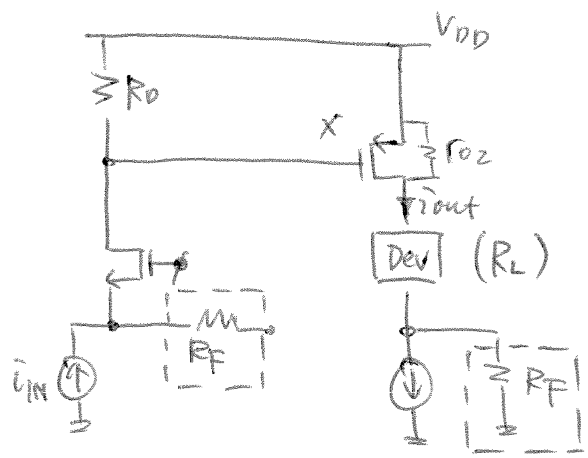
$R_{in,CLOSED} \to \infty$

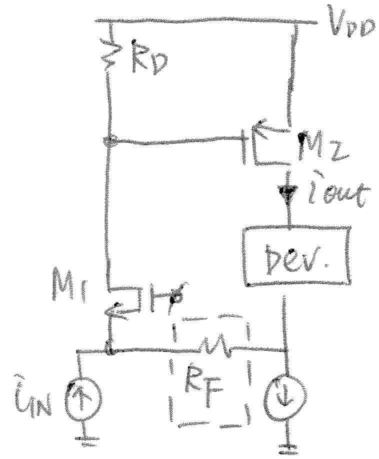
$R_{out,CLOSED}$

$$= r_{o2} + \left(\frac{r_{o1}+R_D}{1+g_{m1}r_{o1}}\right)$$



5b. Breaking the feedback loop results in the following circuit:





$(r_{o2} < \infty)$

$$A_{I,O.L.} = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

$$= -g_{m2} \times \frac{(R_L + R_F) \,||\, r_{o2}}{(R_L + R_F)} \times R_D$$

$$R_{in,OPEN} = \frac{1}{g_{m1}} \qquad R_{out,OPEN} = r_{o2} + R_F$$

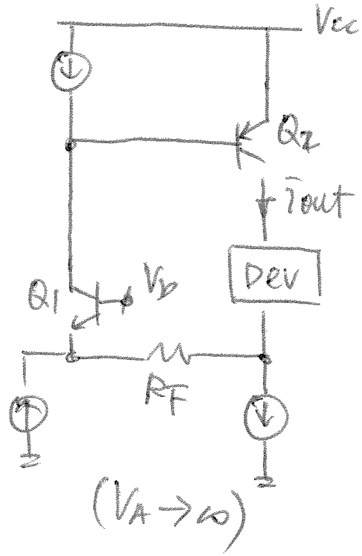
- Feedback factor K:

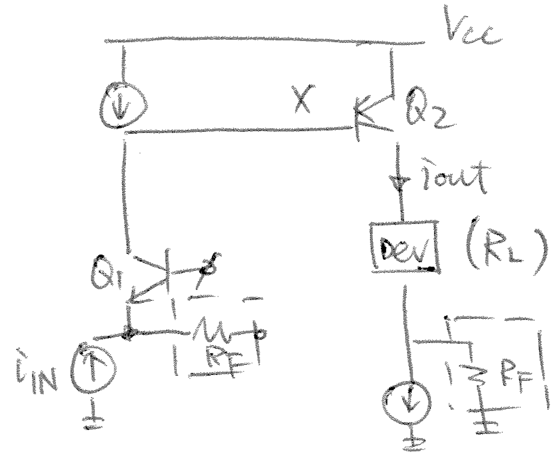
$$K = \frac{i_Y}{i_X} = -1$$

$$\Rightarrow A_{I,C.L.} = \frac{A_{I,O.L.}}{1 + A_{I,OL} \times K} = \frac{A_{I,OL}}{1 - A_{I,OL}}$$

$$R_{in,CLOSED} = \frac{1/g_{m1}}{1 - A_{I,OL}} \qquad R_{out,CLOSED} = (r_{o2} + R_F)(1 - A_{I,OL})$$

57. Breaking the feedback loop results in the following circuit:



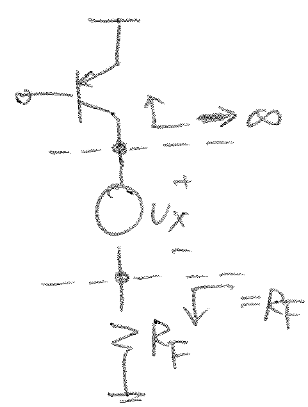


$$A_{I,o.L.} = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

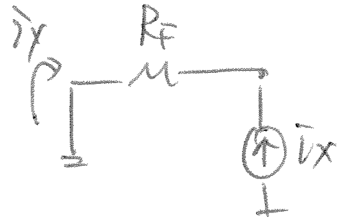
$$\cong -g_{m_2} \times r_{\pi_2} = \beta_2$$

$$R_{in,OPEN} = \frac{r_{\pi_1}}{\beta_1 + 1} \approx \frac{1}{g_{m_1}} \qquad R_{out,OPEN} \to \infty$$

(Note: $R_{out} \to \infty$ because there is no current path:)



- Feedback factor K:

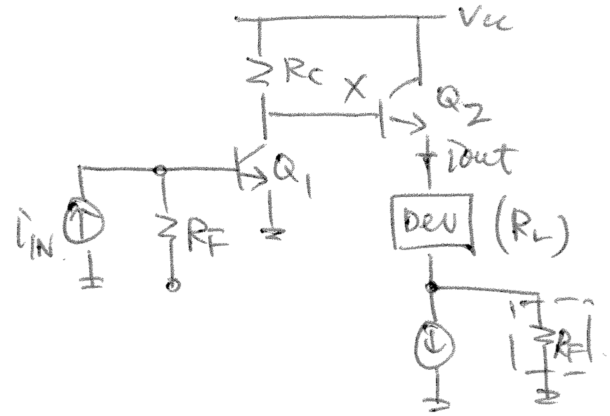


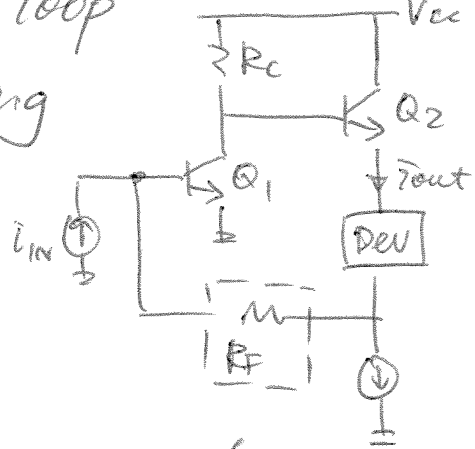
$$K = \frac{i_Y}{i_X} = -1.$$

$$\Rightarrow A_{I,C.L.} = \frac{A_{I,O.L.}}{1 + A_{I,OL} \times K} = \frac{\beta_2}{1 - \beta_2}$$

$$R_{in,\,CLOSED} = \frac{1/g_{m_1}}{1 - \beta_2} \qquad R_{out,\,CLOSED} \rightarrow \infty$$

58. Breaking the feedback loop results in the following circuit:





$(V_A \to \infty)$

$R_{in,open} = r_{\pi 1}$

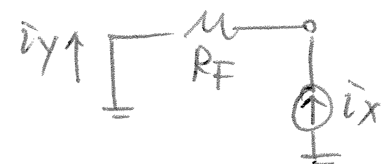
$R_{out,open} \cong \dfrac{1}{g_{m2}} + R_F$

$$A_{I,O.L.} = \frac{i_{out}}{i_{IN}} = \frac{i_{out}}{v_X} \times \frac{v_X}{i_{IN}}$$

$$= \frac{g_{m2}(R_L+R_F)}{1+g_{m2}(R_L+R_F)} \times \left[\beta_1 \times \left[R_C \parallel \left\{r_{\pi 2} + (\beta_2+1)(R_L+R_F)\right\}\right]\right]$$

- Feedback factor K:

$$K = \frac{i_Y}{i_X} = -1$$



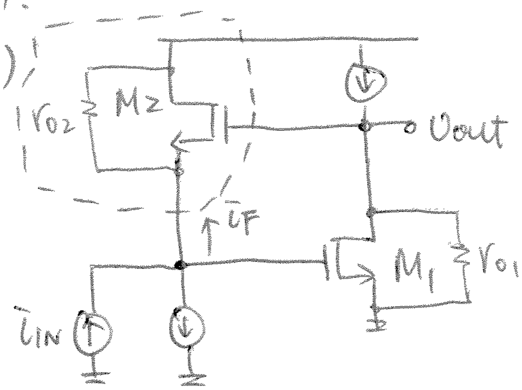
$$\Rightarrow A_{I,C.L.} = \frac{A_{I,OL}}{1 + A_{I,OL} \times K} = \frac{A_{I,OL}}{1 - A_{I,OL}}$$

$$R_{IN,CLOSED} = \frac{r_{\pi 1}}{1 - A_{I,OL}}$$

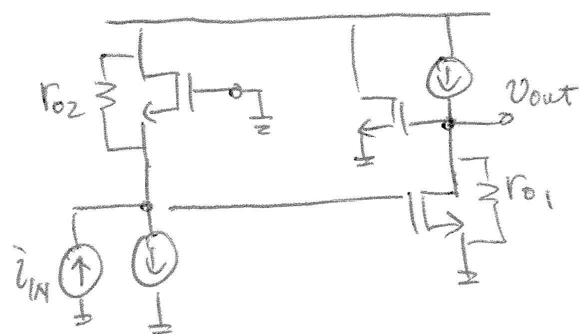
$$R_{OUT,CLOSED} = \left(\frac{1}{g_{m2}} + R_F\right)(1 - A_{I,OL})$$

59.

(a)



Breaking the feedback network results in the following:



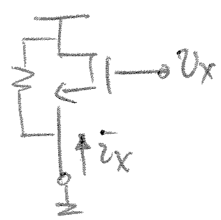
$$R_{OL} = \frac{v_{out}}{i_{IN}}$$

$$= -\left(r_{o2} \| \frac{1}{g_{m2}}\right) g_{m1} r_{o1}$$

$$R_{in,open} = r_{o2} \| \frac{1}{g_{m2}}$$

$$R_{out,open} = r_{o1}$$

K:



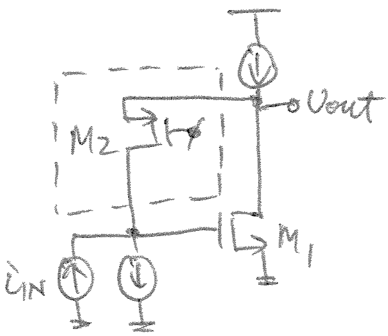
$$K = \frac{i_x}{v_x} = g_{m2}$$

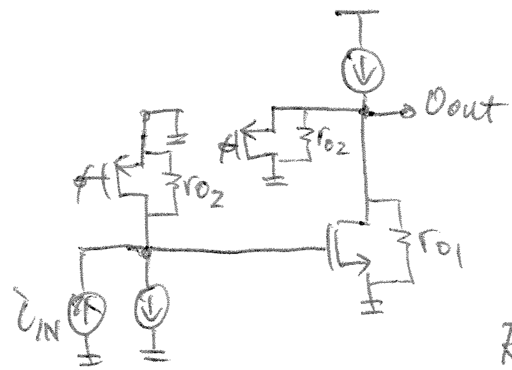
$$\therefore \quad R_{CL} = \frac{R_{OL}}{1 + R_{OL}K} = \frac{\left(r_{o2} \| \frac{1}{g_{m2}}\right) g_{m1} r_{o1}}{1 + g_{m1} g_{m2} r_{o1} \left(r_{o2} \| \frac{1}{g_{m2}}\right)}$$

$$R_{IN,closed} = \frac{\left(r_{o2} \| \frac{1}{g_{m2}}\right)}{1 + g_{m1} g_{m2} r_{o1} \left(r_{o2} \| 1/g_{m2}\right)}$$

$$R_{out,closed} = \frac{r_{o1}}{1 + g_{m1} g_{m2} r_{o1} \left(r_{o2} \| \frac{1}{g_{m2}}\right)}$$

(b) Breaking the feedback network results in the following:





$$R_{OL} = \frac{v_{out}}{i_{IN}}$$

$$= -r_{o2} \times g_{m1}\left[r_{o1} \,||\, r_{o2} \,||\, \frac{1}{g_{m2}}\right]$$

$$R_{IN,\,OPEN} = r_{o2}$$

$$R_{OUT,\,OPEN} = r_{o1} \,||\, r_{o2} \,||\, \frac{1}{g_{m2}}$$

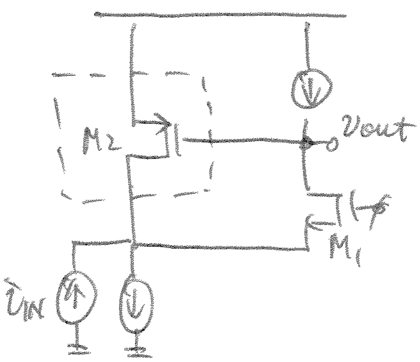
$$K = \frac{i_x}{v_x} = -g_{m2}$$

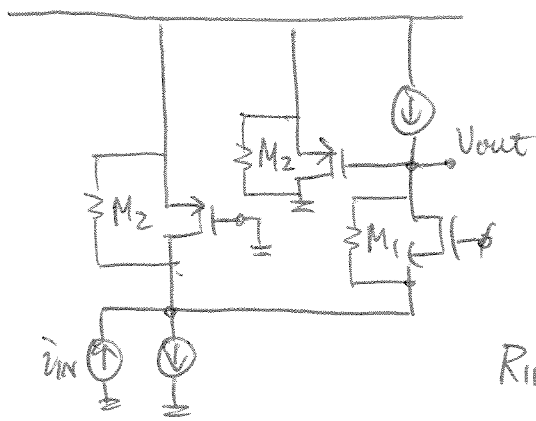
$$\therefore \; R_{CL} = \frac{R_{OL}}{1 + R_{OL}K} = \frac{-g_{m1} r_{o2}\left[r_{o1} \,||\, r_{o2} \,||\, \frac{1}{g_{m2}}\right]}{1 + g_{m1} g_{m2} r_{o2}\left[r_{o1} \,||\, r_{o2} \,||\, \frac{1}{g_{m2}}\right]}$$

$$R_{IN,\,CLOSED} = \frac{r_{o2}}{1 + R_{OL}K}$$

$$R_{OUT,\,CLOSED} = \frac{r_{o1} \,||\, r_{o2} \,||\, \frac{1}{g_{m2}}}{1 + R_{OL}K}.$$

(c) Breaking the feedback network results in the following:





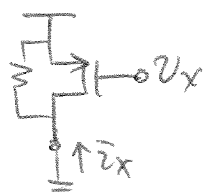
$$R_{OL} = \frac{v_{out}}{i_{IN}}$$

$$\approx \left(\frac{1}{g_{m1}} \parallel r_{o2}\right)(1+g_{m1}r_{o1})$$

$$R_{in,OPEN} \cong \frac{1}{g_{m1}} \parallel r_{o2}$$

$$R_{out,OPEN} \simeq r_{o2} + r_{o1}(1+g_{m1}r_{o2})$$

K:



$$K = \frac{i_x}{v_x} = g_{m2}$$

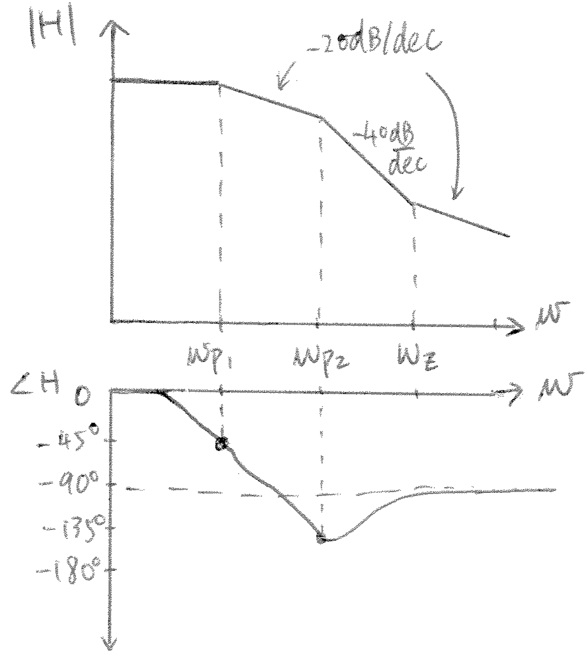
$$\therefore\ R_{CL} = \frac{R_{OL}}{1+R_{OL}K} \simeq \frac{\left(\frac{1}{g_{m2}} \parallel r_{o2}\right)(1+g_{m1}r_{o1})}{1+g_{m2}\left(\frac{1}{g_{m1}} \parallel r_{o2}\right)(1+g_{m1}r_{o1})}$$

$$R_{IN,CLOSED} = \frac{\frac{1}{g_{m1}} \parallel r_{o2}}{1+R_{OL}\cdot K}$$

$$R_{out,CLOSED} = \frac{r_{o2} + r_{o1}(1+g_{m1}r_{o2})}{1+R_{OL}\cdot K}$$

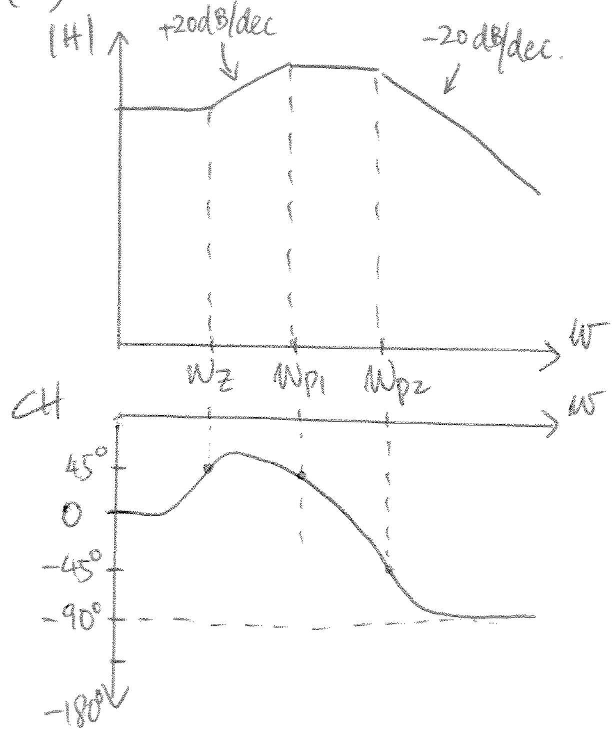
60.

(a)



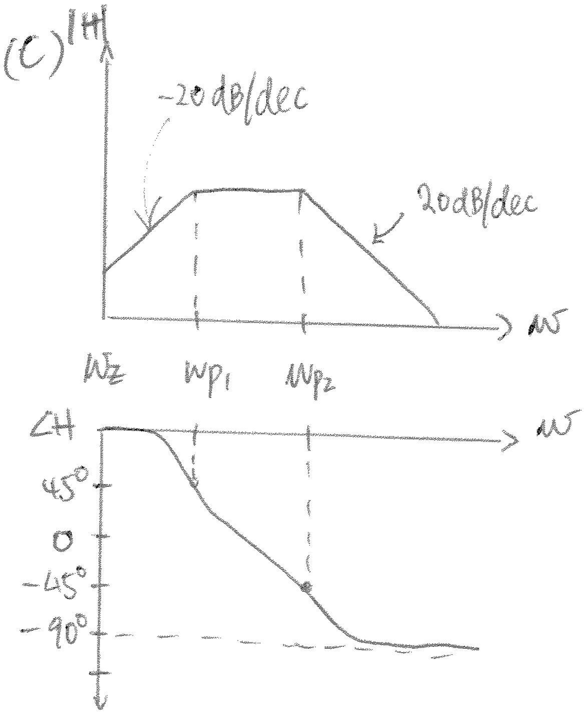
|H|
-20dB/dec
-40dB/dec
ω
ωp1
ωp2
ωz
∠H
0
-45°
-90°
-135°
-180°
ω

(b)



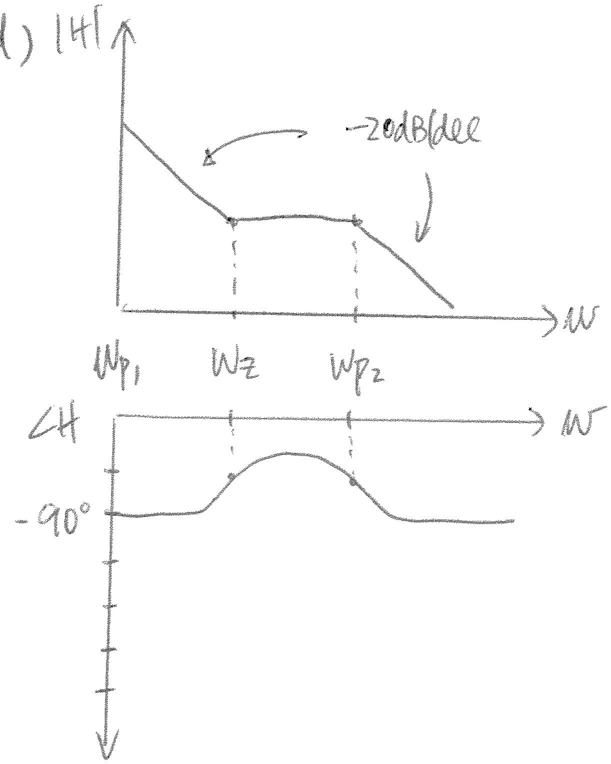
|H|
+20dB/dec
-20dB/dec.
ω
ωz
ωp1
ωp2
∠H
ω
45°
0
-45°
-90°
-180°

(c)



|H|
-20dB/dec
20dB/dec
ω
ωz
ωp1
ωp2
∠H
ω
45°
0
-45°
-90°

(d)

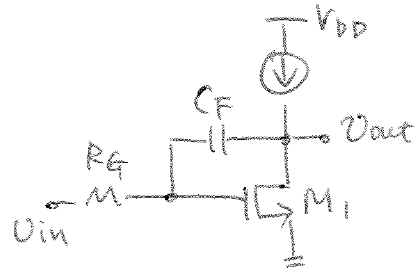


|H|
-20dB/dec
ω
ωp1
ωz
ωp2
∠H
ω
-90°

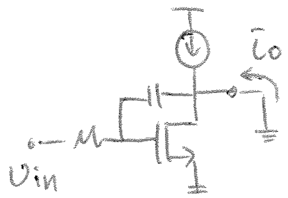
61. As w_z comes closer to w_{p_1} or w_{p_2}, it cancels out the effect (i.e. -20 dB/dec decrease) — pole-zero cancellation. It would appear as if nothing occured at that overlapping frequency.

62.

Calculate $G_m(s)$ of circuit:

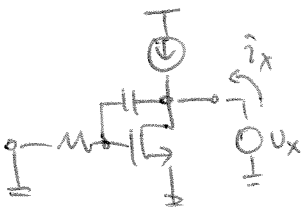


$(\lambda = 0)$



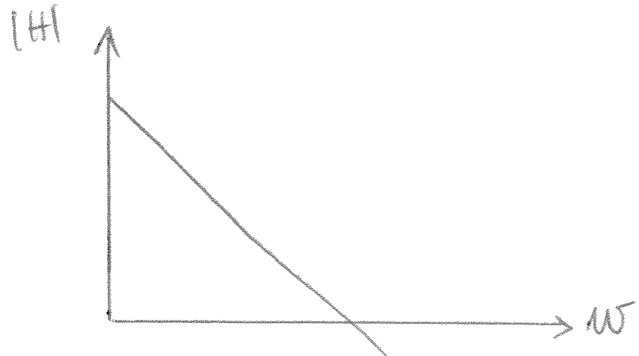
$$G_m = \frac{i_o}{V_{in}}$$

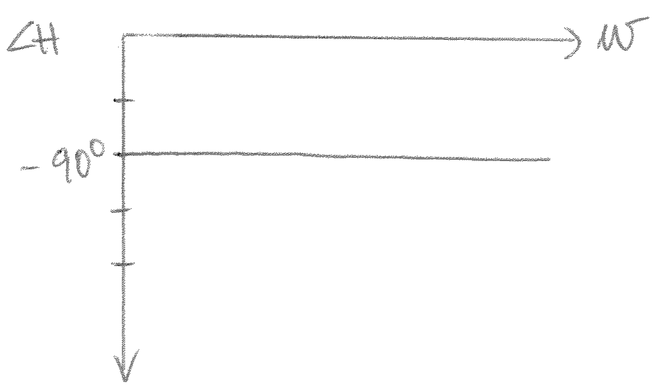
$$= g_{m1} \times \frac{1}{1 + sC_F R_G}$$



$$Z_{out} = \frac{V_x}{i_x} = \frac{1 + sC_F R_G}{sC_F \times (1 + g_{m1} R_G)}$$

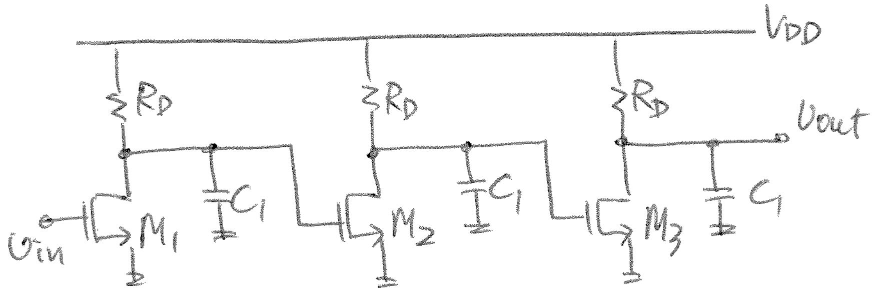
$$\Rightarrow H(s) = G_m \times Z_{out} = \frac{g_{m1}}{g_{m1} R_G + 1} \times \frac{1}{sC_F} = \frac{1}{(g_{m1} R_G + 1)\, s\left(\frac{C_F}{g_{m1}}\right)}$$





63. By Nyquist Criterion, decreasing k $(k \to 0)$ eventually leads to $|KH| < 1$ at $\angle H = -180°$, which implies stability.

64.



$(M_1 = M_2 = M_3)$

$$H(s) = \frac{v_{out}}{v_{in}}(s) = \frac{(-g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3} \quad \text{where } \omega_p = \frac{1}{R_D C_1}$$

$$\Rightarrow \angle H(j\omega) = \angle(-g_m R_D)^3 - \angle\left(1 + j\frac{\omega}{\omega_p}\right)^3$$

$$= 0 - 3\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$

$$\therefore \angle H\Big|_{\omega = 0.1\omega_p} = -3\tan^{-1}\left(\frac{0.1\,\omega_p}{\omega_p}\right) \cong -17.1^\circ$$

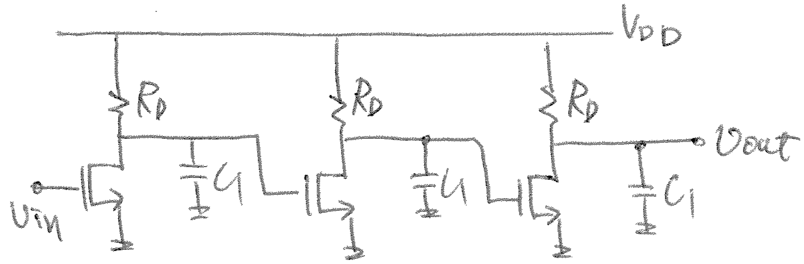
65. $$H(s) = \frac{(-g_m R_D)^3}{\left(1 + \frac{s}{\omega_p}\right)^3} \qquad (M_1 = M_2 = M_3)$$

$$\Rightarrow |H|\Big|_{\omega=\omega_p} = \frac{|g_m R_D{}^3|}{\left|\left(1 + j\frac{\omega_p}{\omega_p}\right)^3\right|} = \frac{(g_m R_D)^3}{(\sqrt{1+1})^3} = \frac{(g_m R_D)^3}{\sqrt{8}}$$

$$\Rightarrow 20 \log |H|\Big|_{\omega=\omega_p} = 20 \log (g_m R_D)^3 - 20 \log \sqrt{8}$$
$$\cong 20 \log (g_m R_D)^3 - (9 \text{ dB})$$

$\therefore$ $|H|$ falls by 9 dB due to the three coincident poles.

66.

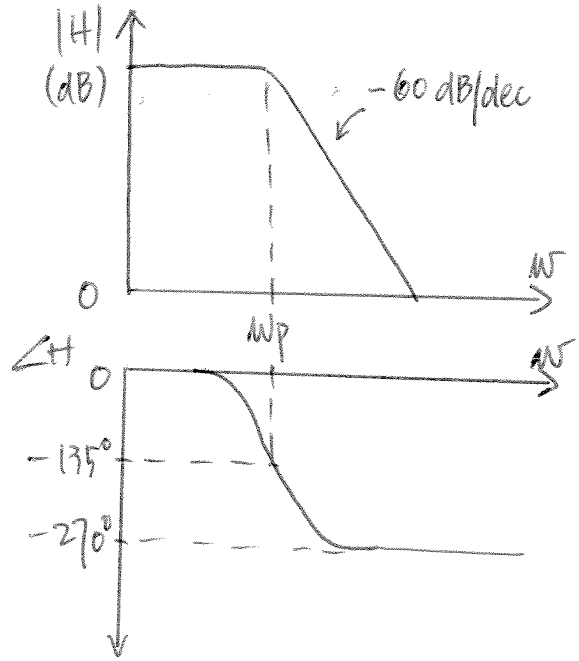


$\omega_p = \frac{1}{R_D C_1}$

$k = 0.1$

$$H(j\omega) = \frac{(-g_m R_D)^3}{\left(1 + j\frac{\omega}{\omega_p}\right)}$$

$$\angle H = -3\,\tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$



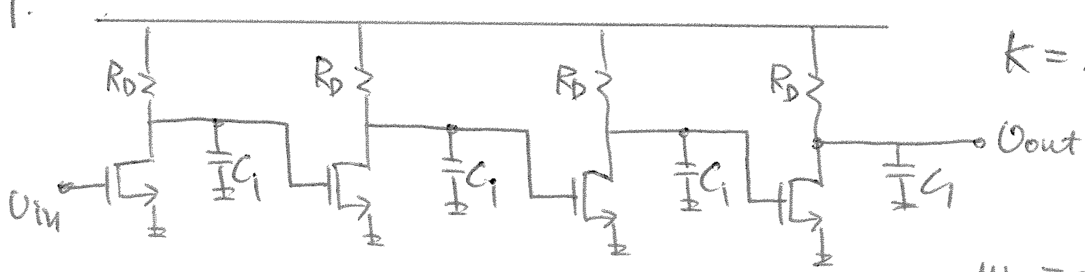
- Need to guarantee that $|kH| < 1$ when $\angle H = -180°$:

$$\angle H = -180° = -3\,\tan^{-1}\left(\frac{\omega}{\omega_p}\right) \Rightarrow \omega = \sqrt{3}\,\omega_p$$

$$|kH(j\omega)|_{\omega = \omega_p\sqrt{3}} = \frac{(g_m R_D)^3}{(\sqrt{1+3})^3} \times 0.1 < 1$$

$$\Rightarrow g_m R_D < \sqrt[3]{80} \approx 8.94$$

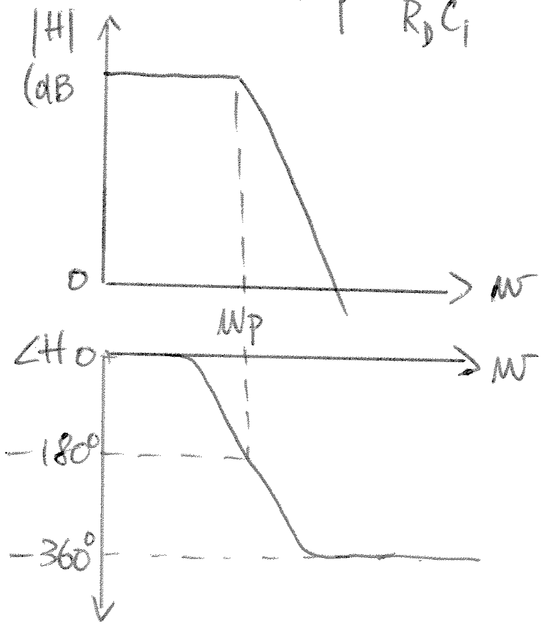
67.



$K = 1$

$\omega_p = \dfrac{1}{R_D C_1}$

$$H(j\omega) = \frac{(g_m R_D)^4}{\left(1 + j\frac{\omega}{\omega_p}\right)^4}$$



$$\angle H = -4\,\mathrm{TAN}^{-1}\left(\frac{\omega}{\omega_p}\right)$$

- To guarantee stability,

$|KH| < 1$ when $\angle H = -180°$.

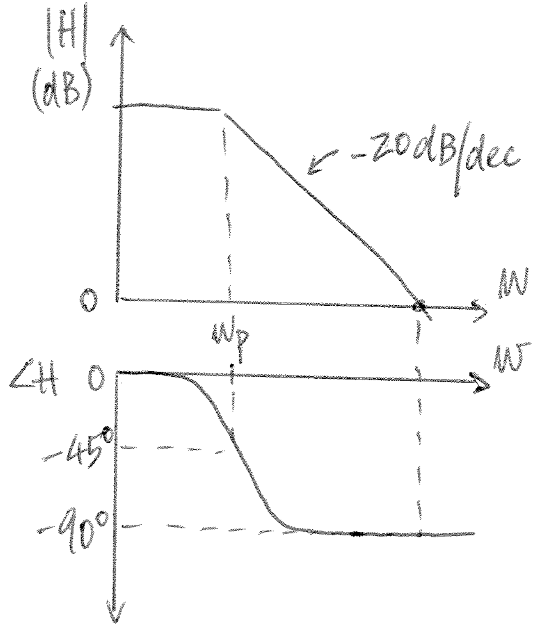
$$\angle H = -180° = -4\,\mathrm{TAN}^{-1}\left(\frac{\omega}{\omega_p}\right) \Rightarrow \omega = \omega_p$$

$$|KH|\Big|_{\omega=\omega_p} = \frac{(g_m R_D)^4}{(\sqrt{1+1})^4} < 1$$

$$\Rightarrow g_m R_D < \sqrt{2}$$

This four-pole system implies a lower upper-limit ($=\sqrt{2}$) on $g_m R_D$, which makes sense since $|H|$ drops faster here.

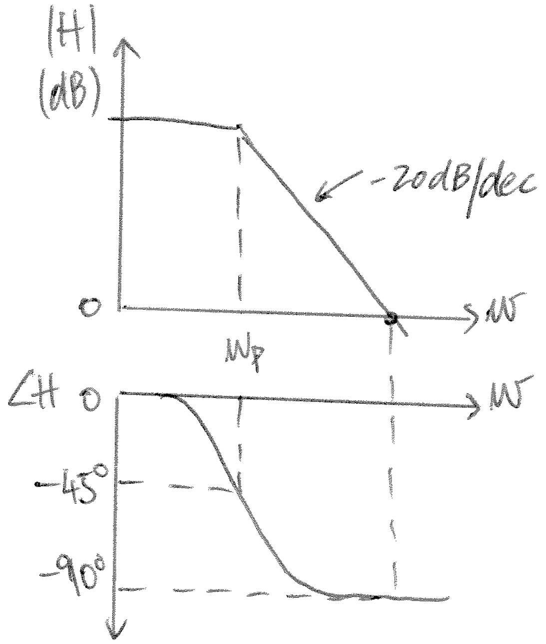
68. $H(s) = \dfrac{A_o}{1 + \dfrac{s}{\omega_p}}$ $\qquad K = 1.$



$\therefore$ Phase margin
$= 180° - 90° = 90°$

(i.e. system is stable)

69. $H(s) = \dfrac{A_0}{1 + \frac{s}{\omega_p}}$ $\qquad K = 0.5$



Phase margin = 90°
(independent of K for one-pole systems.)

70. All three scenarios will become stable eventually (depending on how far w_{Gx} is from w_{px}, & $w_{Gx} < w_{px}$.)

71. In the $20 \cdot \log|kH|$ vs. $\log|w|$ plot, the magnitude plot decreases at a rate of 20 dB/dec. between w_{p1} & w_{p2}.

$k = 1 \Rightarrow 20\log|H|$

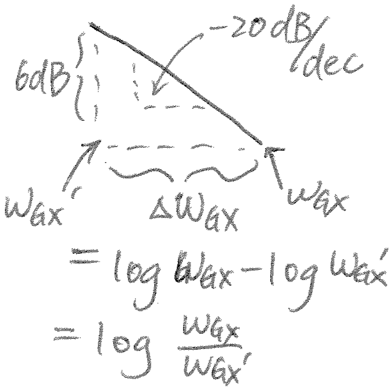
$k = 0.5 \Rightarrow 20\log\left|\frac{H}{2}\right| = 20\log|H| - 6\,dB$

$\therefore$ $k = 0.5$ induces a 6dB decrease on the magnitude plot at all frequencies.

$\Rightarrow \Delta w_{Gx}\,[\log] = \frac{6dB}{20dB} \approx -0.3$

$\frac{w_{Gx}}{w_{Gx}'} = 10^{\frac{6dB}{20dB}} \approx 0.5$

$\Rightarrow w_{Gx}' = \frac{w_{Gx}}{0.5} = 2w_{Gx}$



$\angle H = -\tan^{-1}\left[w\left(\frac{1}{w_{p1}} + \frac{1}{w_{p2}}\right)\right] \approx -\tan^{-1}\left(\frac{w}{w_{p1}}\right) \quad (w_{p2} >> w_{p1})$

$\Rightarrow$ Phase plot stays the same with $k = 0.5$.

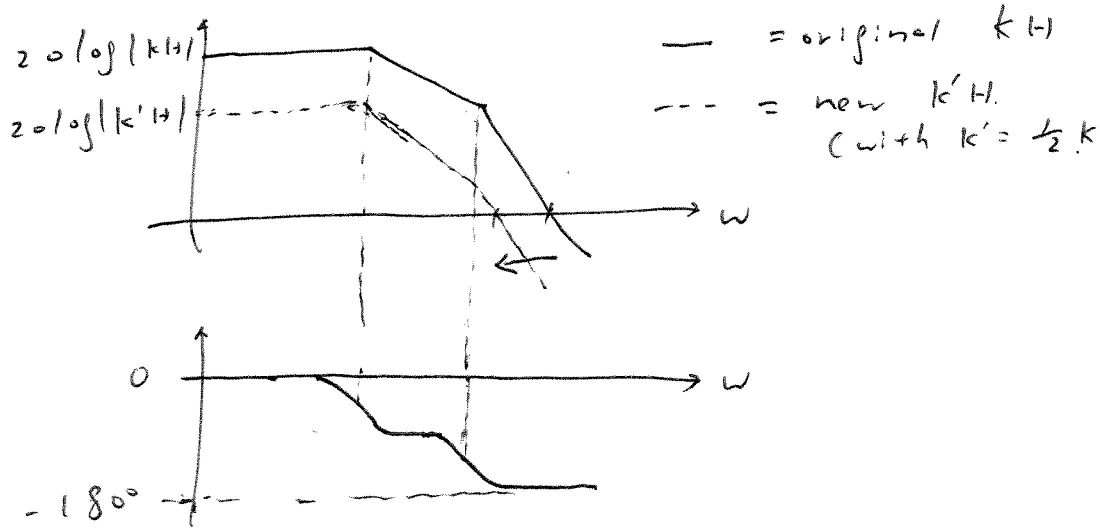
In original example $\angle H(w_{p2}) \cong -\tan^{-1}\left(\frac{w_{Gx}}{w_{p1}}\right) = -135°$

$\angle H(w_{Gx}') = \angle H(2w_{Gx}) \cong -\tan^{-1}\left(\frac{2w_{Gx}}{w_{p1}}\right)$

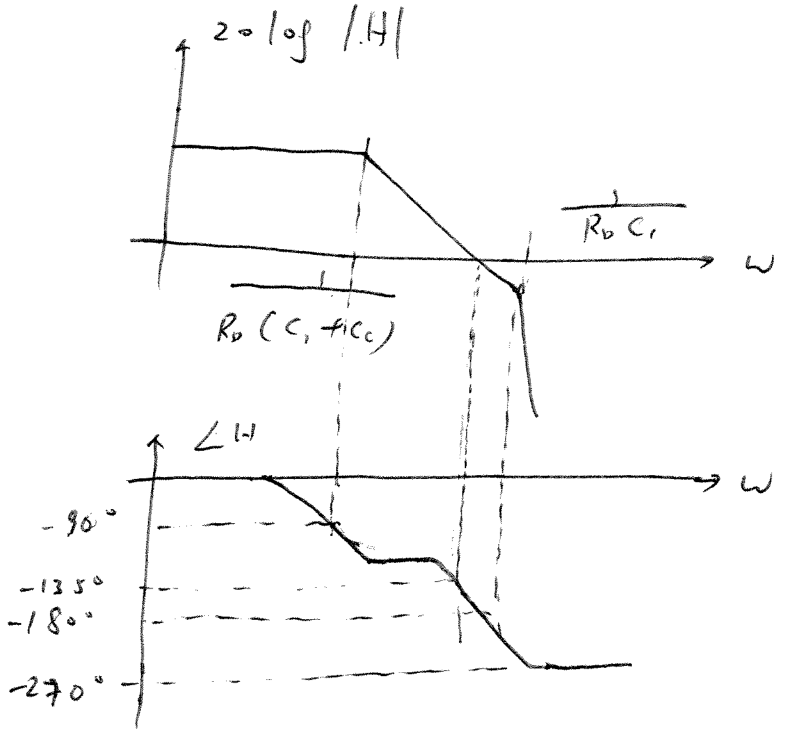
$\Rightarrow \angle H(w_{Gx}') = -\tan^{-1}\left[2\tan 135°\right] \cong -116°$

$\Rightarrow$ Phase Margin $= 180° + \angle H(w_{Gx}') \approx 63°$

(72) When k drops by a factor of 2, the phase margin improves. This is because a lower k corresponds to shifting the amplitude part of kH down by 6 dB. (The phase $\angle kH$ remains unchanged, since phase is only depends on pole location and is independent of amplitude of kH.) Thus, the gain $|kH|$ drops to 0 dB at a lower frequency. This results in a larger phase margin.



(73) With the compensation capacitor C_c in place, the pole associated with C_c becomes dominant. (i.e. at a lower frequency than the other 2 poles at $\frac{1}{R_D C_1}$).



(74) Open-loop gain, $A_o = g_{mN}(r_{oN} // r_{oP})$

$$= 50$$

Closed-loop gain, $\dfrac{V_{out}}{V_{in}} = \dfrac{g_{mN}\,(r_{oN} // r_{oP})}{1 + \dfrac{R_2}{R_1 + R_2}\, g_{mN}\,(r_{oN} // r_{oP})}$

$$= \frac{50}{1 + \dfrac{R_2}{R_1 + R_2} \times 50}$$

$$= 4$$

$$\therefore \quad 4 = \frac{50}{1 + \dfrac{R_2}{R_1 + R_2} \times 50}$$

$$1 + \frac{50 R_2}{R_1 + R_2} = 12.5$$

$$\frac{R_2}{R_1 + R_2} = 0.23$$

Choosing $r_{o2} // r_{o4} = 5\,k\Omega$.

i.e. $g_{mN} = 10\,mS$ //

and $R_1 + R_2 \approx 50\,k\Omega$.

$$\therefore \quad R_2 = 0.23 \times 50\,k\Omega$$

$$= 11.5\,k\Omega \;//$$

and $R_1 = 38.5\,k\Omega$ //

75

Open loop gain, $A_o = g_m R_D$

(assuming $R_1 + R_2$ is very large.)

i.e. $g_m R_D = 10$

$$\text{Closed-loop gain} = \frac{g_m R_D}{1 + \left(\frac{R_2}{R_1 + R_2}\right) g_m R_D}$$

$$= 2$$

$$\therefore \quad \frac{10}{1 + \left(\frac{R_2}{R_1 + R_2}\right) \times 10} = 2$$

$$\frac{R_2}{R_1 + R_2} = 0.4$$

$$\text{Closed-loop input impedance} = \frac{1}{g_m}\left[1 + \frac{R_2}{R_1 + R_2} \times 10\right]$$

$$= 50\,\Omega.$$

$$\therefore \quad \frac{1}{g_m} \times 5 = 50$$

$$g_m = 0.1\,S$$

$$\therefore \quad R_D = 100\,\Omega.$$

$$\therefore \quad R_1 + R_2 = 10 \times 100\,\Omega$$

$$= 1\,k\Omega.$$

$$\therefore \quad R_2 = 400\,\Omega$$

$$R_1 = 600\,\Omega$$

(76) Open-loop gain $= R_{D1}\,(g_{m2} R_{D2})$

$= 10\,k\Omega$

Closed-loop gain $= \dfrac{10\,k\Omega}{1 + \dfrac{10k\Omega}{R_F}}$

$= 1\,k\Omega.$

$$\frac{10}{1 + \frac{10k\Omega}{R_F}} = 1\,k\Omega$$

$\therefore R_F = 1.11\,k\Omega$ //

Closed-loop input impedance $= \dfrac{1}{g_{m1}}(1+9)^{-1}$

$= 50\Omega.$

$\therefore g_{m1} = 2mS$ //

Closed-loop output impedance $= \dfrac{R_{D2}}{10}$

$= 200\Omega.$

$\therefore R_{D2} = 2000\Omega$ //

$\because R_{D1} = 1\,k\Omega,$

$\therefore g_{m2} = 5\,mS$ //

(77) Assuming R_F is very large,

Open-loop gain $= R_D \, (g_{m2} \, R_L)$

$= 10 \, k\Omega$

Closed-loop gain $= \dfrac{10 \, k\Omega}{1 + \dfrac{10 k\Omega}{R_F}}$

$= 1 \, k\Omega.$

$\therefore \; R_F = 1.11 \, k\Omega$ //

Closed-loop input impedance $= \frac{1}{g_{m1}} \, (1 + \beta)^{-1} = 50 \Omega.$

$g_{m1} = 2 \, mS.$ //

Closed-loop output impedance $= \dfrac{R_L}{10}$

$\therefore \; R_L = 2000 \Omega.$ //

$R_D = 1 k\Omega.$

$\therefore \; g_{m2} = 5 \, mS$ //

(78) a) open-loop gain $= R_c\ (g_{m2}\ R_m)$

$= 20\ k\Omega.$

$g_{m2} = \frac{I}{V_T}$

$\therefore g_{m2} = \frac{1mA}{26mV} = 38.5\ mS$

$\therefore R_c\ R_m = \frac{20\ k\Omega}{38.5\ mS}$

open-loop output impandance $= R_m$ $(\because r_o = \infty)$

$\therefore R_m = 500\Omega$ //

$R_c = 1040\Omega$ //

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b) closed-loop gain $= \dfrac{20\ k\Omega}{1 + \frac{20k\Omega}{R_F}}$

$= 1\ k\Omega$

$\therefore R_F = 1053\ \Omega$ //

c) closed-loop input impedance $= \dfrac{\frac{1}{38.5mS}}{1 + \frac{20k}{1053}}$

$= 1.30\Omega$ //

closed-loop output impedance $\approx (500)(\frac{1}{20})$

$= 25\Omega$ //

(79) Open-loop voltage gain

$$= \frac{R_{D1}}{\frac{1}{g_{m1}} + R_1 // R_2} \cdot \left[g_{m2} (R_1 // R_2) \right]$$

$$= 20 \quad \text{———} \ (1)$$

$$\text{closed-loop gain} = \frac{20}{1 + 20\left(\frac{R_2}{R_1 + R_2}\right)} = 4$$

$$\therefore \frac{R_2}{R_1 + R_2} = 0.2. \quad \text{———} \ (2)$$

$$\text{Open-loop output impedance} = R_1 + R_2$$

$$= 2\ k\Omega.$$

$$\therefore R_2 = 0.4\ k\Omega. \ //$$

$$R_1 = 1.6\ k\Omega. \ //$$

From (1),

$$\frac{R_{D1}}{\frac{1}{g_{m1}} + 2\ k\Omega} \left[g_{m2} \times 320\ \Omega \right] = 20$$

$$\frac{g_{m1} R_{D1}}{1 + 2k\, g_{m1}} \times g_{m2} = 0.0625$$

$$\text{Setting } g_{m1} = g_{m2} = 20\,mS. \ //$$

$$R_{D1} \approx 6400\ \Omega. \ //$$

To minimize power consumption, g_{m1} and g_{m2} should be minimized. (and R_D maximized)

(80) Since $A_{OL} = \dfrac{g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 // R_2}{\beta_2 + 1}\right)}{1 + g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_1 // R_2}{\beta_2 + 1}\right)} \times g_{m2}(R_1 + R_2)$

$R_{out,\,closed} = \dfrac{R_1 + R_2}{1 + A_{OL} k}$

$\because A_{CL} = 2$

$\therefore 2 = \dfrac{A_{OL}}{1 + A_{OL} k}$

see $g_{m1} = g_{m2} = 50\,m\mho$, $(R_1 + R_2) = 1k$,

$A_{OL} = \dfrac{50\,m\mho\left(\frac{1}{50\,m\mho} + \frac{R_1 // R_2}{101}\right)}{1 + 50\,m\mho\left(\frac{1}{50\,m\mho} + \frac{R_1 // R_2}{101}\right)} \times 50\,m\mho\,(1\,k\Omega)$

$\approx \dfrac{1 + 10^{-5}(R_1 // R_2)}{2 + 10^{-5}(R_1 // R_2)} \times 50$

≈ 25

$\therefore 2 = \dfrac{25}{1 + 25k}$

$\therefore k = 0.46$ [$\therefore R_1 = 460\,\Omega$, $R_2 = 540\,\Omega$]

$R_{out,\,closed} = \dfrac{1\,k\Omega}{1 + 25 \times 0.46}$

$= 80\,\Omega$

(81) open-loop gain $(A_{OL}) = -g_{m1}(R_F \parallel r_{\pi 1}) \times [R_C \parallel [r_{\pi 2} + (\beta_2 + 1) R_F]]$

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL}\frac{1}{R_F}}$$

$\because$ $A_{CL} = 1000\Omega$.

$$1000 = \frac{A_{OL}}{1 + \frac{A_{OL}}{R_F}}$$

$$R_{in,\,closed} = \frac{R_F \parallel r_{\pi 1}}{1 + \frac{A_{OL}}{R_F}} = 50\Omega$$

$$R_{out,\,closed} = \frac{R_F}{1 + \frac{A_{OL}}{R_F}} = 50\Omega$$

setting $R_F = 1200\Omega$.

$\therefore$ $A_{OL} = 6000\ \Omega$ //

$\therefore$ $r_{\pi 1} = 400$ //

$g_{m2} = 2.5\text{m}\mho$ //

Assume $\beta_2 = 100$, and $g_{m1} = g_{m2}$.

$R_C = 8570\Omega$ //