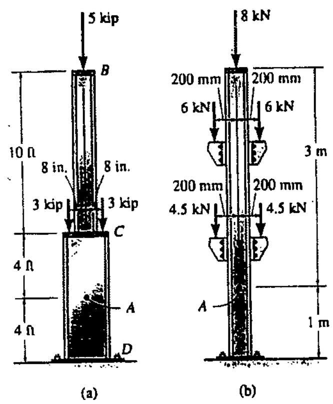
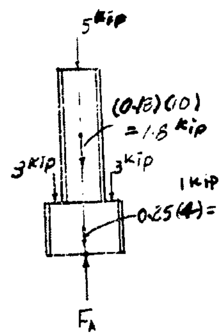
1–1 Determine the resultant internal normal force acting on the cross section through point A in each column. In (a), segment BC weighs 180 lb/ft and segment CD weighs 250 lb/ft. In (b), the column has a mass of 200 kg/m.



(a)

$$+\uparrow \Sigma F_y = 0; \qquad F_A - 1.0 - 3 - 3 - 1.8 - 5 = 0$$

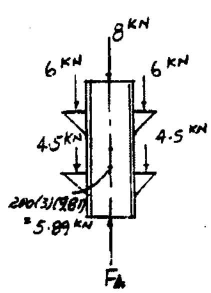
$$F_A = 13.8 \text{ kip} \qquad \textbf{Ans}$$



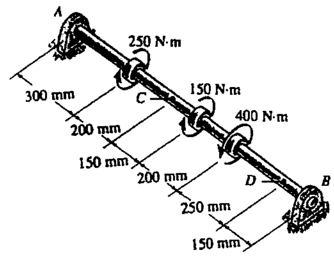
(b)

$$+\uparrow \Sigma F_y = 0; \qquad F_A - 4.5 - 4.5 - 5.89 - 6 - 6 - 8 = 0$$

$$F_A = 34.9 \text{ kN} \qquad \textbf{Ans}$$

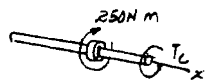


1–2 Determine the resultant internal torque acting on the cross sections through points *C* and *D*. The support bearings at *A* and *B* allow free turning of the shaft.



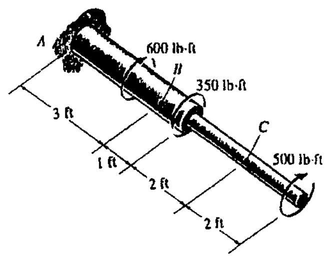
$\Sigma M_x = 0; \qquad T_C - 250 = 0$

$T_C = 250 \text{ N} \cdot \text{m}$ **Ans**



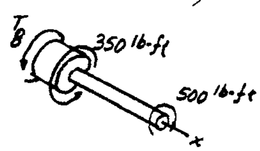
$\Sigma M_x = 0; \quad T_D = 0$ **Ans**

1–3 Determine the resultant internal torque acting on the cross sections through points *B* and *C*.



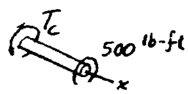
$\Sigma M_x = 0;\qquad T_B + 350 - 500 = 0$

$T_B = 150\ \text{lb}\cdot\text{ft}$ **Ans**

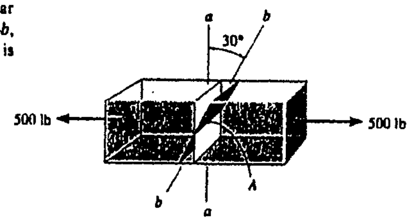


$\Sigma M_x = 0;\qquad T_C - 500 = 0$

$T_C = 500\ \text{lb}\cdot\text{ft}$ **Ans**



•1–4 Determine the resultant internal normal and shear force in the member at (*a*) section *a–a* and (*b*) section *b–b*, each of which passes through point *A*. The 500-lb load is applied along the centroidal axis of the member.

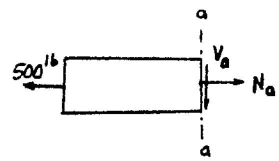


(a)

$\xrightarrow{+} \Sigma F_x = 0;\qquad N_a - 500 = 0$

$N_a = 500\text{ lb}$ **Ans**

$+\downarrow \Sigma F_y = 0;\qquad V_a = 0$ **Ans**



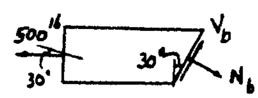
(b)

$\nwarrow^{+} \Sigma F_x = 0;\qquad N_b - 500\cos 30^\circ = 0$

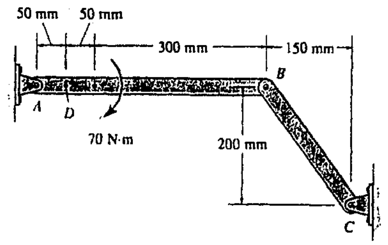
$N_b = 433\text{ lb}$ **Ans**

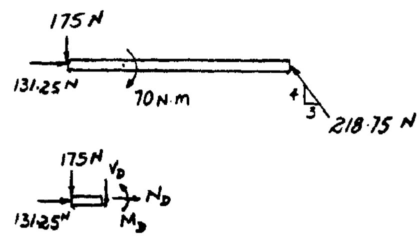
$+\nearrow \Sigma F_y = 0;\qquad V_b - 500\sin 30^\circ = 0$

$V_b = 250\text{ lb}$ **Ans**



1–5 Determine the resultant internal loadings acting on the cross section through point D of member AB.





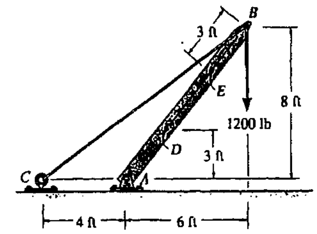
Segment AD **:**

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D + 131.25 = 0; \quad N_D = -131 \text{ N}$ **Ans**

$+\downarrow \Sigma F_y = 0; \quad V_D + 175 = 0; \quad V_D = -175 \text{ N}$ **Ans**

$\curvearrowleft + \Sigma M_D = 0; \quad M_D + 175(0.05) = 0; \quad M_D = -8.75 \text{ N} \cdot \text{m}$ **Ans**

1-6 The beam AB is pin supported at A and supported by a cable BC. Determine the resultant internal loadings acting on the cross section at point D.



$$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87°$$

$$\phi = \tan^{-1}\left(\frac{10}{8}\right) - 36.87° = 14.47°$$

Member AB :

$\circlearrowleft + \Sigma M_A = 0;\quad F_{BC}\sin 14.47°(10) - 1200(6) = 0$

$F_{BC} = 2881.46\text{ lb}$

Segment BD :

$+\nearrow \Sigma F_x = 0;\quad -N_D - 2881.46\cos 14.47° - 1200\cos 36.87° = 0$

$N_D = -3750\text{ lb} = -3.75\text{ kip}$ **Ans**

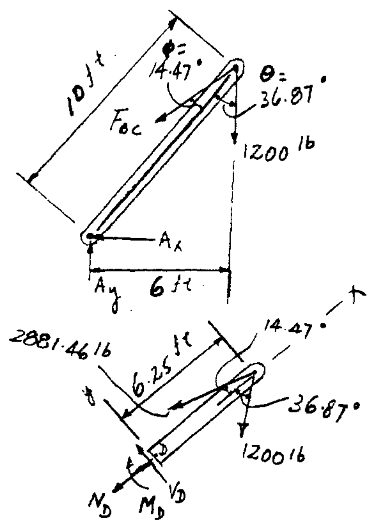
$+\nwarrow \Sigma F_y = 0;\quad V_D + 2881.46\sin 14.47° - 1200\sin 36.87° = 0$

$V_D = 0$ **Ans**

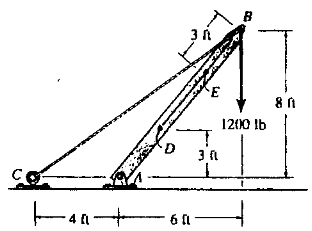
$\circlearrowleft + \Sigma M_D = 0;\quad 2881.46\sin 14.47°(6.25) - 1200\sin 36.87°(6.25) - M_D = 0$

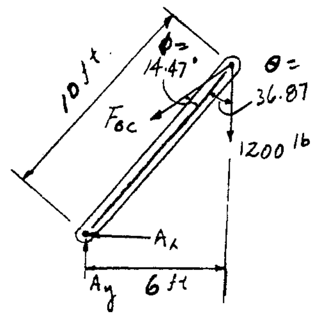
$M_D = 0$ **Ans**

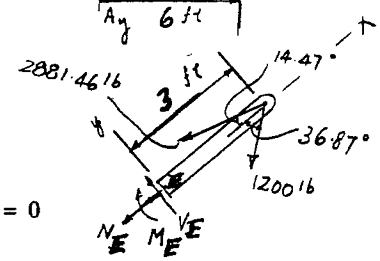
Notice that member AB is the two - force member ; therefore the shear force and moment are zero.



1–7 Solve Prob. 1–6 for the resultant internal loadings acting at point *E*.







$$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ$$

$$\phi = \tan^{-1}\left(\frac{10}{8}\right) - 36.87^\circ = 14.47^\circ$$

Member *AB* :

$\circlearrowleft + \Sigma M_A = 0;\quad F_{BC} \sin 14.47^\circ(10) - 1200(6) = 0$

$F_{BC} = 2881.46 \text{ lb}$

Segment *BE* :

$\xrightarrow{+} \Sigma F_x = 0;\quad -N_E - 2881.46 \cos 14.47^\circ - 1200 \cos 36.87^\circ = 0$

$N_E = -3750 \text{ lb} = -3.75 \text{ kip}$ **Ans**

$+ \Sigma F_y = 0;\quad V_E + 2881.46 \sin 14.47^\circ - 1200 \sin 36.87^\circ = 0$

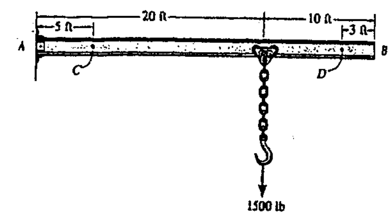
$V_E = 0$ **Ans**

$\circlearrowleft + \Sigma M_E = 0;\quad 2881.46 \sin 14.47^\circ(3) - 1200 \sin 36.87^\circ(3) - M_E = 0$

$M_E = 0$ **Ans**

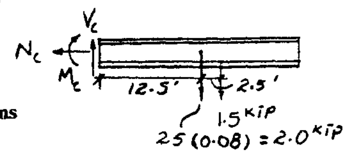
Notice that member *AB* is the two - force member ; therefore the shear force and moment are zero.

*1-8. The beam AB is fixed to the wall and has a uniform weight of 80 lb/ft. If the trolley supports a load of 1500 lb, determine the resultant internal loadings acting on the cross sections through points C and D.



Segment BC:

$\overset{+}{\leftarrow} \Sigma F_x = 0;$ $N_C = 0$ **Ans**



$+\uparrow \Sigma F_y = 0;$ $V_C - 2.0 - 1.5 = 0$

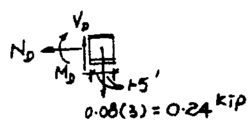
$V_C = 3.50$ kip **Ans**

$\circlearrowleft + \Sigma M_C = 0;$ $-M_C - 2(12.5) - 1.5(15) = 0$

$M_C = -47.5$ kip · ft **Ans**

Segment BD:

$\overset{+}{\leftarrow} \Sigma F_x = 0;$ $N_D = 0$ **Ans**



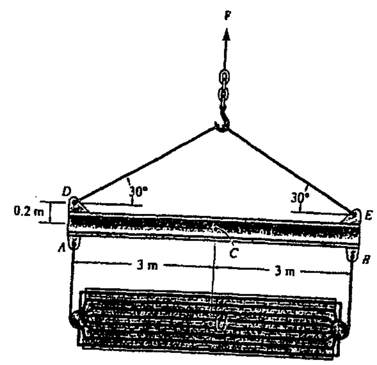
$+\uparrow \Sigma F_y = 0;$ $V_D - 0.24 = 0$

$V_D = 0.240$ kip **Ans**

$\circlearrowleft + \Sigma M_D = 0;$ $-M_D - 0.24(1.5) = 0$

$M_D = -0.360$ kip · ft **Ans**

1-9. Determine the resultant internal loadings acting on the cross section at point C. The cooling unit has a total weight of 52 kip and a center of gravity at G.



From FBD (a)

$\circlearrowleft + \Sigma M_A = 0; \quad T_B(6) - 52(3) = 0; \quad T_B = 26 \text{ kip}$

From FBD (b)

$\circlearrowleft + \Sigma M_D = 0; \quad T_E \sin 30°(6) - 26(6) = 0; \quad T_E = 52 \text{ kip}$

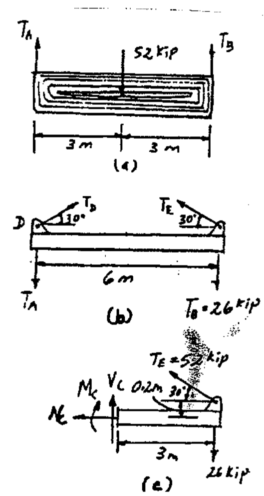
From FBD (c)

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_C - 52 \cos 30° = 0; \quad N_C = -45.0 \text{ kip}$ **Ans**

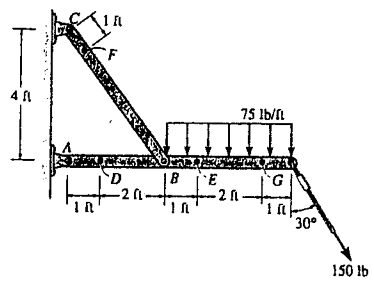
$+\uparrow \Sigma F_y = 0; \quad V_C + 52 \sin 30° - 26 = 0; \quad V_C = 0$ **Ans**

$\circlearrowleft + \Sigma M_C = 0; \quad 52 \cos 30°(0.2) + 52 \sin 30°(3) - 26(3) - M_C = 0$

$M_C = 9.00 \text{ kip} \cdot \text{ft}$ **Ans**



1-10 Determine the resultant internal loadings acting on the cross sections through points D and E of the frame.

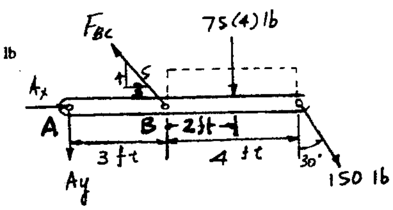


Member AG :

$\circlearrowleft + \Sigma M_A = 0;\quad \frac{4}{5}F_{BC}(3) - 75(4)(5) - 150\cos 30^\circ(7) = 0;\quad F_{BC} = 1003.89\text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;\quad A_y(3) - 75(4)(2) - 150\cos 30^\circ(4) = 0;\quad A_y = 373.20\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - \frac{3}{5}(1003.89) + 150\sin 30^\circ = 0;\quad A_x = 527.33\text{ lb}$



For point D :

$\xrightarrow{+} \Sigma F_x = 0;\quad N_D + 527.33 = 0$

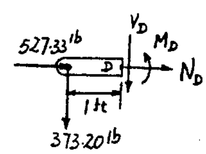
$N_D = -527\text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad -373.20 - V_D = 0$

$V_D = -373\text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_D = 0;\quad M_D + 373.20(1) = 0$

$M_D = -373\text{ lb}\cdot\text{ft}$ **Ans**



For point E :

$\xrightarrow{+} \Sigma F_x = 0;\quad 150\sin 30^\circ - N_E = 0$

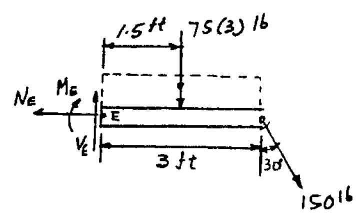
$N_E = 75.0\text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad V_E - 75(3) - 150\cos 30^\circ = 0$

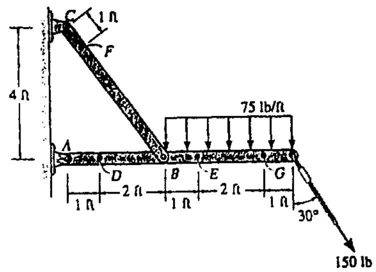
$V_E = 355\text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_E = 0;\quad -M_E - 75(3)(1.5) - 150\cos 30^\circ(3) = 0;$

$M_E = -727\text{ lb}\cdot\text{ft}$ **Ans**



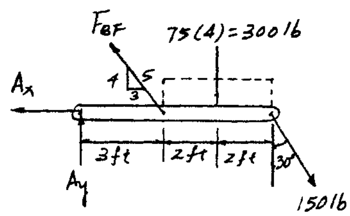
1-11 Determine the resultant internal loadings acting on the cross sections through points F and G of the frame.



Member AG :

$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5}F_{BF}(3) - 300(5) - 150\cos 30^\circ(7) = 0$

$F_{BF} = 1003.9 \text{ lb}$



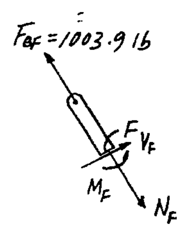
For point F :

$+\nearrow \Sigma F_{x'} = 0; \quad V_F = 0$ **Ans**

$+\nwarrow \Sigma F_{y'} = 0; \quad N_F - 1003.9 = 0$

$N_F = 1004 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_F = 0; \quad M_F = 0$ **Ans**



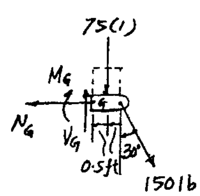
For point G :

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad N_G - 150\sin 30^\circ = 0$

$N_G = 75.0 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad V_G - 75(1) - 150\cos 30^\circ = 0$

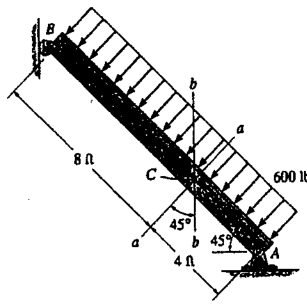
$V_G = 205 \text{ lb}$ **Ans**

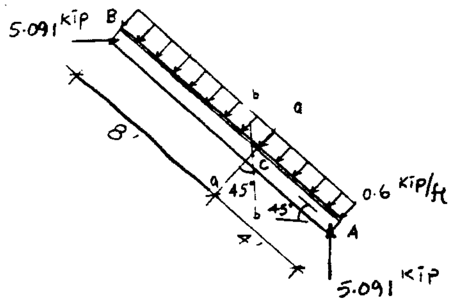


$\circlearrowleft + \Sigma M_G = 0; \quad -M_G - 75(1)(0.5) - 150\cos 30^\circ(1) = 0$

$M_G = -167 \text{ lb}\cdot\text{ft}$ **Ans**

•1–12 **Determine the resultant internal loadings acting on (*a*) section *a–a* and (*b*) section *b–b*. Each section is located through the centroid, point *C*.**





(a)

$\nwarrow\!+\ \Sigma F_x = 0;\qquad N_C + 5.091\sin 45° = 0$

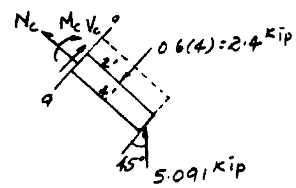
$N_C = -3.60\text{ kip}$ **Ans**

$\nearrow\!+\ \Sigma F_y = 0;\qquad V_C + 5.091\cos 45° - 2.4 = 0$

$V_C = -1.20\text{ kip}$ **Ans**

$\circlearrowleft\!+\ \Sigma M_C = 0;\qquad -M_C - 2.4(2) + 5.091\cos 45°(4) = 0$

$M_C = 9.60\text{ kip}\cdot\text{ft}$ **Ans**



(b)

$\overset{+}{\leftarrow}\Sigma F_x = 0;\qquad N_C + 2.4\cos 45° = 0$

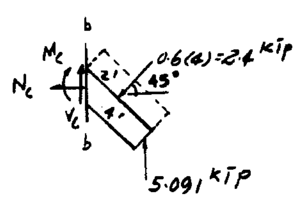
$N_C = -1.70\text{ kip}$ **Ans**

$+\uparrow\Sigma F_y = 0;\qquad V_C + 5.091 - 2.4\sin 45° = 0$

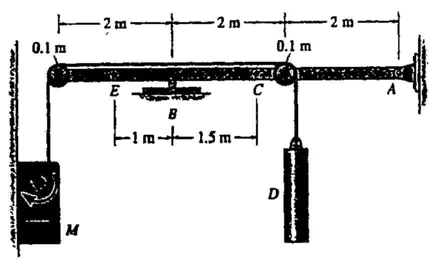
$V_C = -3.39\text{ kip}$ **Ans**

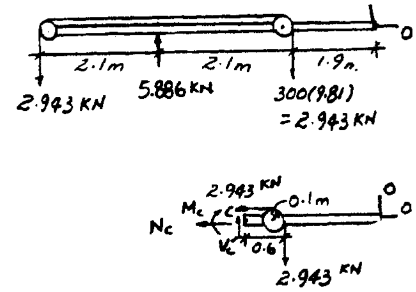
$\circlearrowleft\!+\Sigma M_C = 0;\qquad -M_C - 2.4(2) + 5.091\cos 45°(4) = 0$

$M_C = 9.60\text{ kip}\cdot\text{ft}$ **Ans**



1-13 Determine the resultant internal loadings acting on the cross section through point C in the beam. The load D has a mass of 300 kg and is being hoisted by the motor M with constant velocity.





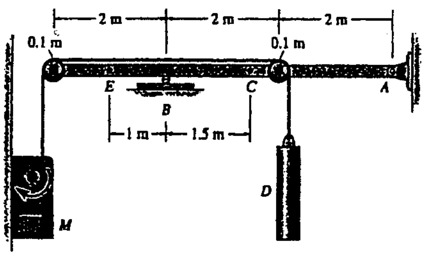
$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad N_C + 2.943 = 0;\quad N_C = -2.94\text{ kN}$ **Ans**

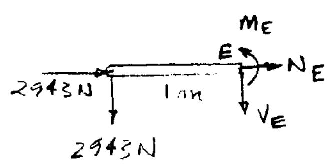
$+\uparrow \Sigma F_y = 0;\quad V_C - 2.943 = 0;\quad V_C = 2.94\text{ kN}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;\quad -M_C - 2.943(0.6) + 2.943(0.1) = 0$

$M_C = -1.47\text{ kN}\cdot\text{m}$ **Ans**

1-14 Determine the resultant internal loadings acting on the cross section through point E of the beam in Prob. 1-13.





$\xrightarrow{+} \Sigma F_x = 0;\quad N_E + 2943 = 0$

$N_E = -2.94\text{ kN}$ **Ans**

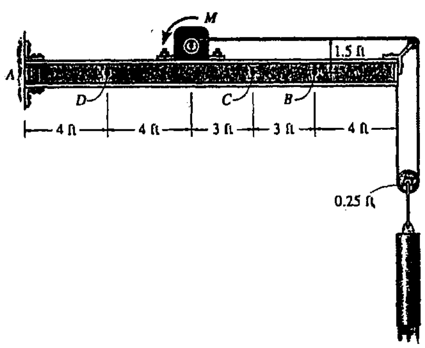
$+\uparrow \Sigma F_y = 0;\quad -2943 - V_E = 0$

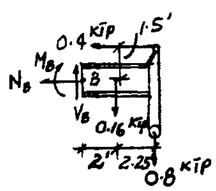
$V_E = -2.94\text{ kN}$ **Ans**

$\circlearrowleft + \Sigma M_E = 0;\quad M_E + 2943\,(1) = 0$

$M_E = -2.94\text{ kN}\cdot\text{m}$ **Ans**

1–15 The 800-lb load is being hoisted at a constant speed using the motor *M*, which has a weight of 90 lb. Determine the resultant internal loadings acting on the cross section through point *B* in the beam. The beam has a weight of 40 lb/ft and is fixed to the wall at *A*.





$\xrightarrow{+} \Sigma F_x = 0; \quad -N_B - 0.4 = 0$

$N_B = -0.4$ kip **Ans**

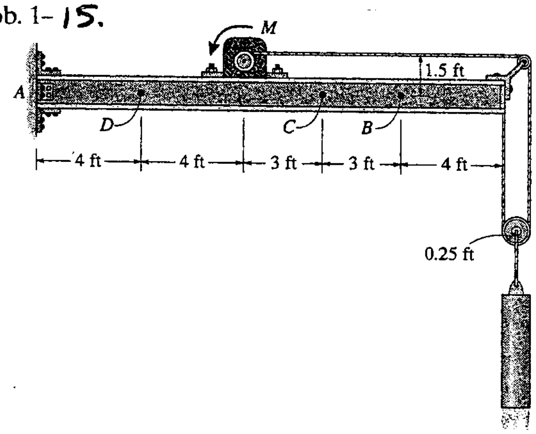
$+\uparrow \Sigma F_y = 0; \quad V_B - 0.8 - 0.16 = 0$

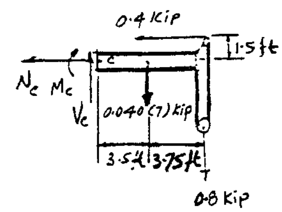
$V_B = 0.960$ kip **Ans**

$\circlearrowleft + \Sigma M_B = 0; \quad -M_B - 0.16(2) - 0.8(4.25) + 0.4(1.5) = 0$

$M_B = -3.12$ kip · ft **Ans**

***1-16.** Determine the resultant internal loadings acting on the cross section through points C and D of the beam in Prob. 1- 15.





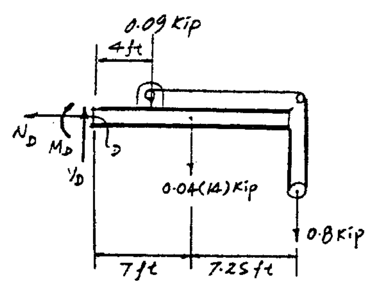
For point C :

$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad N_C + 0.4 = 0;\quad N_C = -0.4 \text{ kip}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad V_C - 0.8 - 0.04\,(7) = 0;\quad V_C = 1.08 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0;\quad -M_C - 0.8(7.25) - 0.04(7)(3.5) + 0.4(1.5) = 0$

$M_C = -6.18 \text{ kip}\cdot\text{ft}$ **Ans**



For point D :

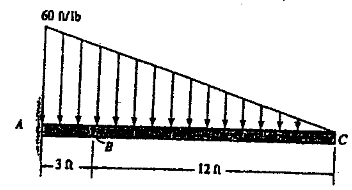
$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad N_D = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad V_D - 0.09 - 0.04(14) - 0.8 = 0;\quad V_D = 1.45 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_D = 0;\quad -M_D - 0.09(4) - 0.04(14)(7) - 0.8(14.25) = 0$

$M_D = -15.7 \text{ kip}\cdot\text{ft}$ **Ans**

1-17. Determine the resultant internal loadings acting on the cross section at point *B*.



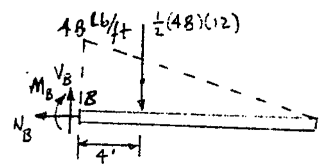
$\xrightarrow{+} \Sigma F_x = 0;\quad N_B = 0$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad V_B - \frac{1}{2}(48)(12) = 0$

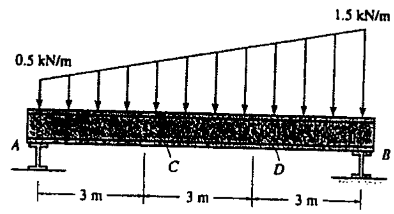
$V_B = 288 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_B = 0;\quad -M_B - \frac{1}{2}(48)(12)(4) = 0$

$M_B = -1152 \text{ lb}\cdot\text{ft} = -1.15 \text{ kip}\cdot\text{ft}$ **Ans**



1–18 The beam supports the distributed load shown. Determine the resultant internal loadings acting on the cross section through point C. Assume the reactions at the supports A and B are vertical.



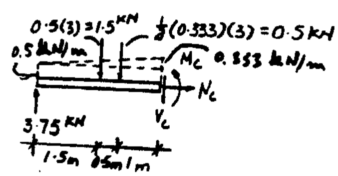
$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad N_C = 0$ **Ans**

$+\downarrow \Sigma F_y = 0; \qquad V_C + 0.5 + 1.5 - 3.75 = 0$

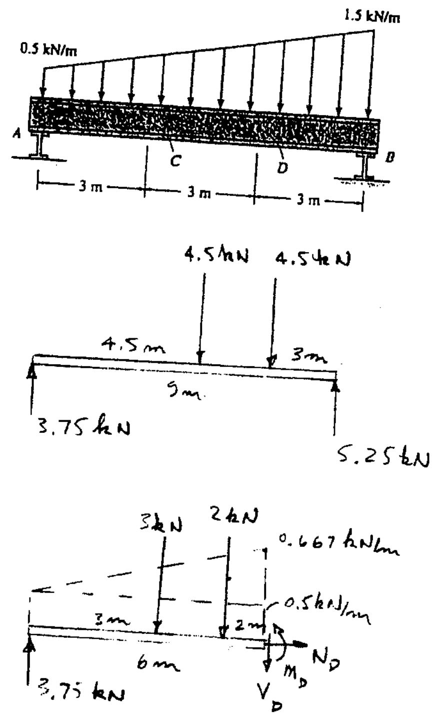
$V_C = 1.75 \text{ kN}$ **Ans**

$\circlearrowleft + \Sigma M_C = 0; \qquad M_C + 0.5(1) + 1.5(1.5) - 3.75(3) = 0$

$M_C = 8.50 \text{ kN} \cdot \text{m}$ **Ans**



1–19 Determine the resultant internal loadings acting on the cross section through point D in Prob. 1–18.



$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0 \quad$ **Ans**

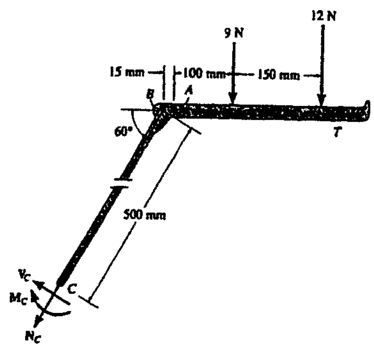
$+\uparrow \Sigma F_y = 0; \quad 3.75 - 3 - 2 - V_D = 0$

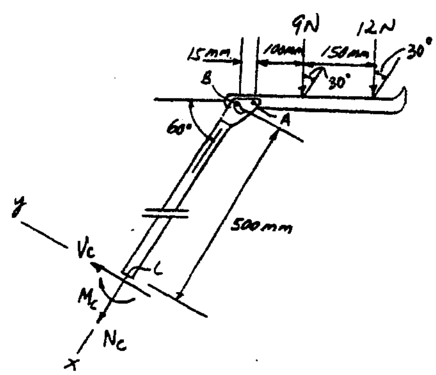
$V_D = -1.25 \text{ kN} \quad$ **Ans**

$\circlearrowleft + \Sigma M_D = 0; \quad M_D + 2(2) + 3(3) - 3.75(6) = 0$

$M_D = 9.50 \text{ kN}\cdot\text{m} \quad$ **Ans**

*1-20.** The serving tray T used on an airplane is supported on *each side* by an arm. The tray is pin connected to the arm at A, and at B there is a smooth pin. (The pin can move within the slot in the arms to permit folding the tray against the front passenger seat when not in use.) Determine the resultant internal loadings acting on the cross section of the arm through point C when the tray arm supports the loads shown.





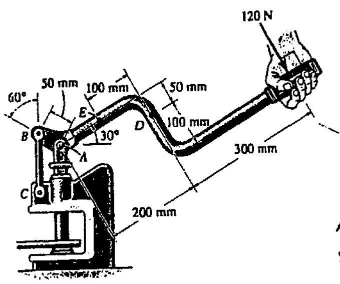
$\nearrow\!+ \Sigma F_x = 0;\quad N_C + 9\cos 30° + 12\cos 30° = 0;\quad N_C = -18.2\text{ N}$ **Ans**

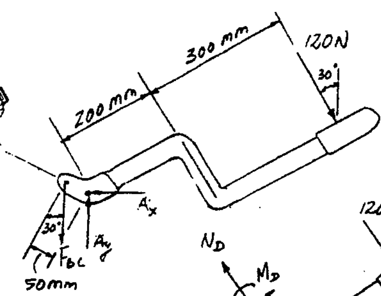
$\nwarrow\!+ \Sigma F_y = 0;\quad V_C - 9\sin 30° - 12\sin 30° = 0;\quad V_C = 10.5\text{ N}$ **Ans**

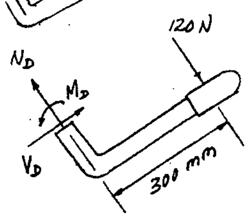
$\circlearrowleft\!+\Sigma M_C = 0;\quad -M_C - 9(0.5\cos 60° + 0.115) - 12(0.5\cos 60° + 0.265) = 0$

$M_C = -9.46\text{ N}\cdot\text{m}$ **Ans**

1-21. The metal stud punch is subjected to a force of 120 N on the handle. Determine the magnitude of the reactive force at the pin A and in the short link BC. Also, determine the internal resultant loadings acting on the cross section passing through the handle arm at D.







Member:

$\circlearrowleft + \Sigma M_A = 0;\quad F_{BC}\cos 30°(50) - 120(500) = 0$

$F_{BC} = 1385.6\text{ N} = 1.39\text{ kN}$ **Ans**

$+\uparrow \Sigma F_y = 0;\quad A_y - 1385.6 - 120\cos 30° = 0$

$A_y = 1489.56\text{ N}$

$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad A_x - 120\sin 30° = 0;\quad A_x = 60\text{ N}$

$F_A = \sqrt{1489.56^2 + 60^2}$

$= 1490\text{ N} = 1.49\text{ kN}$ **Ans**

Segment:

$\searrow + \Sigma F_{x'} = 0;\quad N_D - 120 = 0$

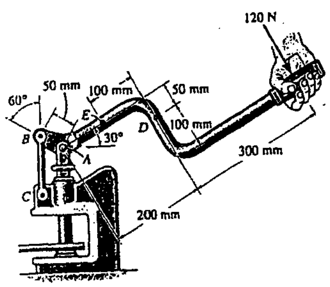
$N_D = 120\text{ N} = 0.12\text{ kN}$ **Ans**

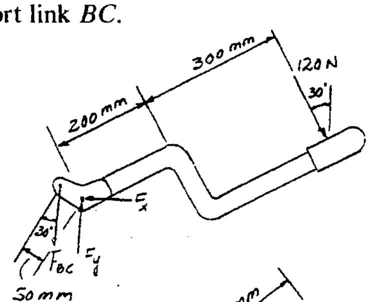
$+\nearrow \Sigma F_{y'} = 0;\quad V_D = 0$ **Ans**

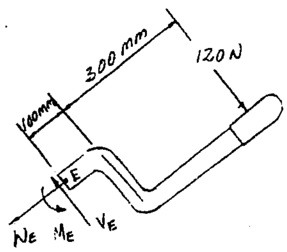
$\circlearrowleft + \Sigma M_D = 0;\quad M_D - 120(0.3) = 0$

$M_D = 36.0\text{ N}\cdot\text{m}$ **Ans**

1-22. Solve Prob. 1–21 for the resultant internal loadings acting on the cross section passing through the handle arm at E and at a cross section of the short link BC.







Member :

$\circlearrowleft + \Sigma M_A = 0; \quad F_{BC}\cos 30°(50) - 120(500) = 0$

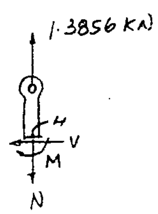
$F_{BC} = 1385.6\text{ N} = 1.3856\text{ kN}$

Segment :

$+\swarrow \Sigma F_{x'} = 0; \quad N_E = 0$ **Ans**

$+\nwarrow \Sigma F_{y'} = 0; \quad V_E - 120 = 0; \quad V_E = 120\text{ N}$ **Ans**

$\circlearrowleft + \Sigma M_E = 0; \quad M_E - 120(0.4) = 0; \quad M_E = 48.0\text{ N}\cdot\text{m}$ **Ans**



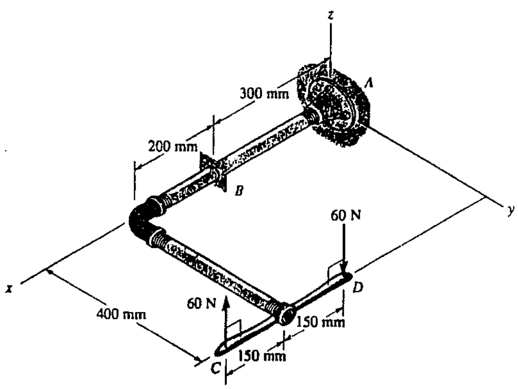
Short link :

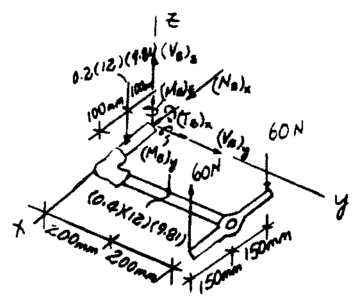
$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad V = 0$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad 1.3856 - N = 0; \quad N = 1.39\text{ kN}$ **Ans**

$\circlearrowleft + \Sigma M_H = 0; \quad M = 0$ **Ans**

1–23 The pipe has a mass of 12 kg/m. If it is fixed to the wall at *A*, determine the resultant internal loadings acting on the cross section at *B*. Neglect the weight of the wrench *CD*.





$\Sigma F_x = 0;\quad (N_B)_x = 0$ **Ans**

$\Sigma F_y = 0;\quad (V_B)_y = 0$ **Ans**

$\Sigma F_z = 0;\quad (V_B)_z - 60 + 60 - (0.2)(12)(9.81) - (0.4)(12)(9.81) = 0$

$(V_B)_z = 70.6\text{ N}$ **Ans**

$\Sigma M_x = 0;\quad (T_B)_x + 60(0.4) - 60(0.4) - (0.4)(12)(9.81)(0.2) = 0$

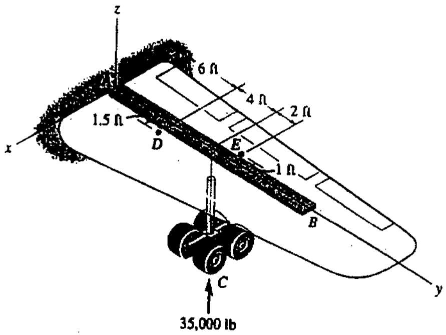
$(T_B)_x = 9.42\text{ N}\cdot\text{m}$ **Ans**

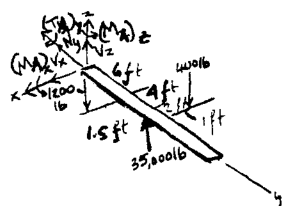
$\Sigma M_y = 0;\quad (M_B)_y + (0.2)(12)(9.81)(0.1) + (0.4)(12)(9.81)(0.2) - 60(0.3) = 0$

$(M_B)_y = 6.23\text{ N}\cdot\text{m}$ **Ans**

$\Sigma M_z = 0;\quad (M_B)_z = 0$ **Ans**

•1–24 The main beam AB supports the load on the wing of the airplane. The loads consist of the wheel reaction of 35 000 lb at C, the 1200-lb weight of fuel in the tank of the wing, having a center of gravity at D, and the 400-lb weight of the wing, having a center of gravity at E. If it is fixed to the fuselage at A, determine the resultant internal loadings on the beam at this point. Assume that the wing does not transfer any of the loads to the fuselage, except through the beam.





$\Sigma F_x = 0;\quad (V_A)_x = 0$ **Ans**

$\Sigma F_y = 0;\quad (N_A)_y = 0$ **Ans**

$\Sigma F_z = 0;\quad (V_A)_z - 1200 - 400 + 35000 = 0$

$(V_A)_z = -33.4 \text{ kip}$ **Ans**

$\Sigma M_x = 0;\quad (M_A)_x - 1200(6) + 35000(10) - 400(12) = 0$

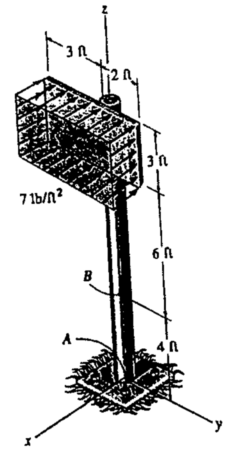
$(M_A)_x = 338 \text{ kip}\cdot\text{ft}$ **Ans**

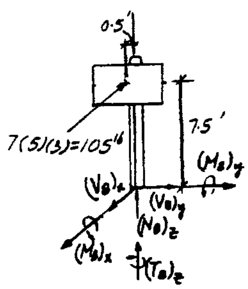
$\Sigma M_y = 0;\quad (T_A)_y + 1200(1.5) - 400(1) = 0$

$(T_A)_y = -1.40 \text{ kip}\cdot\text{ft}$ **Ans**

$\Sigma M_z = 0;\quad (M_A)_z = 0$ **Ans**

1–25 Determine the resultant internal loadings acting on the cross section through point B of the signpost. The post is fixed to the ground and a uniform pressure of 7 lb/ft² acts perpendicular to the face of the sign.





$\Sigma F_x = 0;$ $(V_B)_x - 105 = 0;$ $(V_B)_x = 105$ lb **Ans**

$\Sigma F_y = 0;$ $(V_B)_y = 0$ **Ans**

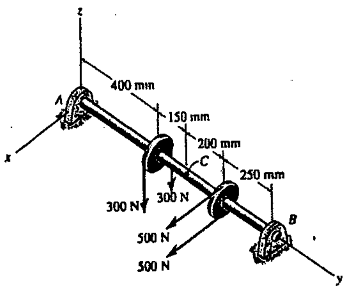
$\Sigma F_z = 0;$ $(N_B)_z = 0$ **Ans**

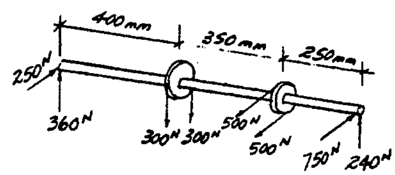
$\Sigma M_x = 0;$ $(M_B)_x = 0$ **Ans**

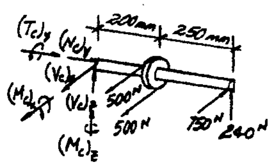
$\Sigma M_y = 0;$ $(M_B)_y - 105(7.5) = 0;$ $(M_B)_y = 788$ lb·ft **Ans**

$\Sigma M_z = 0;$ $(T_B)_z - 105(0.5) = 0;$ $(T_B)_z = 52.5$ lb·ft **Ans**

1–26. The shaft is supported at its ends by two bearings A and B and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section located at point C. The 300-N forces act in the $-z$ direction and the 500-N forces act in the $+x$ direction. The journal bearings at A and B exert only x and z components of force on the shaft.







$\Sigma F_x = 0;\quad (V_C)_x + 1000 - 750 = 0;\quad (V_C)_x = -250\text{ N}$ **Ans**

$\Sigma F_y = 0;\quad (N_C)_y = 0$ **Ans**

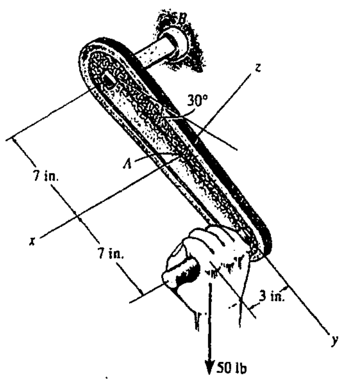
$\Sigma F_z = 0;\quad (V_C)_z + 240 = 0;\quad (V_C)_z = -240\text{ N}$ **Ans**

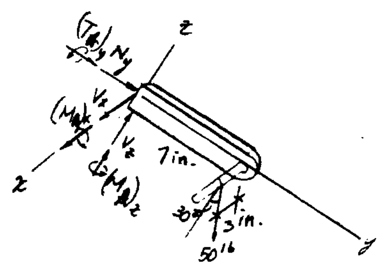
$\Sigma M_x = 0;\quad (M_C)_x + 240(0.45) = 0;\quad (M_C)_x = -108\text{ N}\cdot\text{m}$ **Ans**

$\Sigma M_y = 0;\quad (T_C)_y = 0$ **Ans**

$\Sigma M_z = 0;\quad (M_C)_z - 1000(0.2) + 750(0.45) = 0;\quad (M_C)_z = -138\text{ N}\cdot\text{m}$ **Ans**

1–27 A hand crank that is used in a press has the dimensions shown. Determine the resultant internal loadings acting on the cross section at *A* if a vertical force of 50 lb is applied to the handle as shown. Assume the crank is fixed to the shaft at *B*.





$\Sigma F_x = 0;\quad (V_A)_x = 0$ **Ans**

$\Sigma F_y = 0;\quad (N_A)_y + 50\sin 30° = 0;\quad (N_A)_y = -25\text{ lb}$ **Ans**

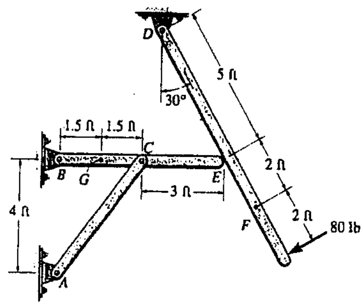
$\Sigma F_z = 0;\quad (V_A)_z - 50\cos 30° = 0;\quad (V_A)_z = 43.3\text{ lb}$ **Ans**

$\Sigma M_x = 0;\quad (M_A)_x - 50\cos 30°(7) = 0;\quad (M_A)_x = 303\text{ lb}\cdot\text{in.}$ **Ans**

$\Sigma M_y = 0;\quad (T_A)_y + 50\cos 30°(3) = 0;\quad (T_A)_y = -130\text{ lb}\cdot\text{in.}$ **Ans**

$\Sigma M_z = 0;\quad (M_A)_z + 50\sin 30°(3) = 0;\quad (M_A)_z = -75\text{ lb}\cdot\text{in.}$ **Ans**

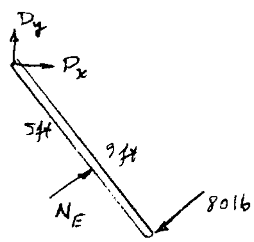
•1–28 Determine the resultant internal loadings acting on the cross section of the frame at points F and G. The contact at E is smooth.



Member DEF :

$\curvearrowleft + \Sigma M_D = 0; \quad N_E\,(5) - 80\,(9) = 0$

$N_E = 144\text{ lb}$



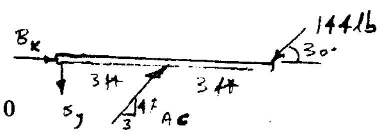
Member BCE :

$\curvearrowleft + \Sigma M_B = 0; \quad F_{AC}\,(\frac{4}{5})(3) - 144\sin 30°\,(6) = 0$

$F_{AC} = 180\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 180\,(\frac{3}{5}) - 144\cos 30° = 0$

$B_x = 16.708\text{ lb}$

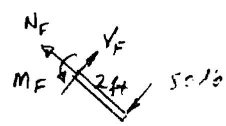


$+\uparrow \Sigma F_y = 0; \quad -B_y + 180(\frac{4}{5}) - 144\sin 30° = 0$

$B_y = 72.0\text{ lb}$

For point F :

$\nwarrow + \Sigma F_x = 0; \quad N_F = 0$ **Ans**

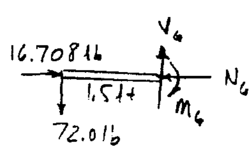


$\nearrow + \Sigma F_y = 0; \quad V_F - 80 = 0; \quad V_F = 80\text{ lb}$ **Ans**

$\curvearrowleft + \Sigma M_F = 0; \quad M_F - 80\,(2) = 0; \quad M_F = 160\text{ lb}\cdot\text{ft}$ **Ans**

For point G :

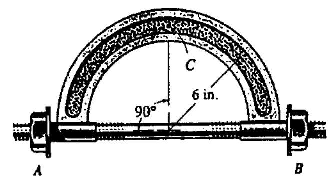
$\xrightarrow{+} \Sigma F_x = 0; \quad 16.708 - N_G = 0; \quad N_G = 16.7\text{ lb}$ **Ans**



$+\uparrow \Sigma F_y = 0; \quad V_G - 72.0 = 0; \quad V_G = 72.0\text{ lb}$ **Ans**

$\curvearrowleft + \Sigma M_G = 0; \quad 72\,(1.5) - M_G = 0; \quad M_G = 108\text{ lb}\cdot\text{ft}$ **Ans**

1–29 The bolt shank is subjected to a tension of 80 lb. Determine the resultant internal loadings acting on the cross section at point *C*.

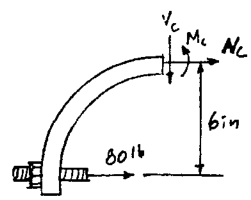


Segment AC:

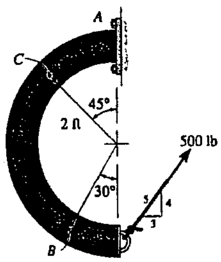
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C + 80 = 0; \quad N_C = -80 \text{ lb}$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad V_C = 0$ **Ans**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 80(6) = 0; \quad M_C = -480 \text{ lb} \cdot \text{in.}$ **Ans**



1-30 Determine the resultant internal loadings acting on the cross section at points *B* and *C* of the curved member.



From FBD (a)

$\nearrow + \Sigma F_{x'} = 0;\qquad 400\cos 30° + 300\cos 60° - V_B = 0$

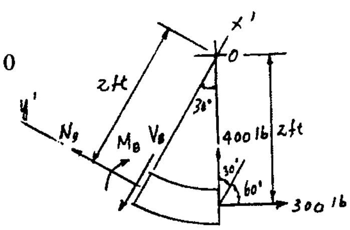
$V_B = 496\text{ lb}$ **Ans**

$\searrow + \Sigma F_{y'} = 0;\qquad N_B + 400\sin 30° - 300\sin 60° = 0$

$N_B = 59.80 = 59.8\text{ lb}$ **Ans**

$\curvearrowleft + \Sigma M_O = 0;\qquad 300(2) - 59.80(2) - M_B = 0$

$M_B = 480\text{ lb}\cdot\text{ft}$ **Ans**



(a)

From FBD (b)

$\nearrow + \Sigma F_{x'} = 0;\qquad 400\cos 45° + 300\cos 45° - N_C = 0$

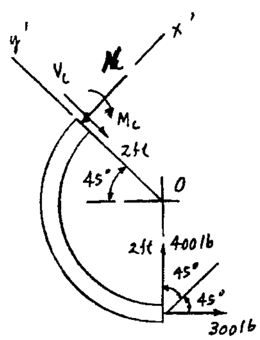
$N_C = 495\text{ lb}$ **Ans**

$\nwarrow + \Sigma F_{y'} = 0;\qquad -V_C + 400\sin 45° - 300\sin 45° = 0$

$V_C = 70.7\text{ lb}$ **Ans**

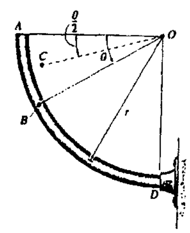
$\curvearrowleft + \Sigma M_O = 0;\qquad 300(2) + 495(2) - M_C = 0$

$M_C = 1590\text{ lb}\cdot\text{ft} = 1.59\text{ kip}\cdot\text{ft}$ **Ans**



(b)

1–31 The curved rod AD of radius r has a weight per length of w. If it lies in the vertical plane, determine the resultant internal loadings acting on the cross section through point B. *Hint:* The distance from the centroid C of segment AB to point O is $OC = [2r \sin(\theta/2)]/\theta$.



$\searrow + \Sigma F_x = 0;$ $\quad N_B + wr\theta\cos\theta = 0$

$N_B = -wr\theta\cos\theta$ **Ans**

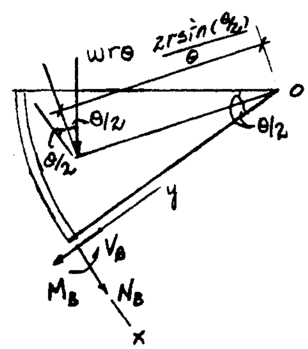
$\nearrow + \Sigma F_y = 0;$ $\quad -V_B - wr\theta\sin\theta = 0$

$V_B = -wr\theta\sin\theta$ **Ans**

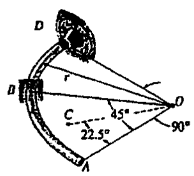
$\circlearrowleft + \Sigma M_O = 0;$ $\quad wr\theta\left(\cos\frac{\theta}{2}\right)\left(\frac{2r\sin(\theta/2)}{\theta}\right) + (N_B)r + M_B = 0$

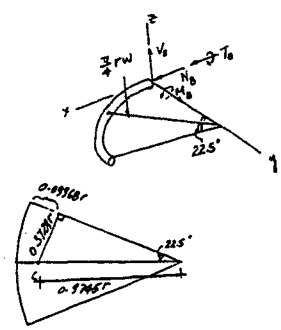
$M_B = -N_B r - wr^2\, 2\sin(\theta/2)\cos(\theta/2)$

$M_B = wr^2(\theta\cos\theta - \sin\theta)$ **Ans**



*1-32. The curved rod AD of radius r has a weight per length of w. If it lies in the horizontal plane, determine the resultant internal loadings acting on the cross section through point B. *Hint:* The distance from the centroid C of segment AB to point O is $CO = 0.9745r$.





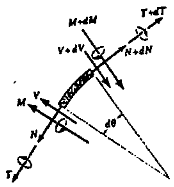
$\Sigma F_z = 0; \quad V_B - \frac{\pi}{4} rw = 0; \quad V_B = 0.785\, w\, r$ **Ans**

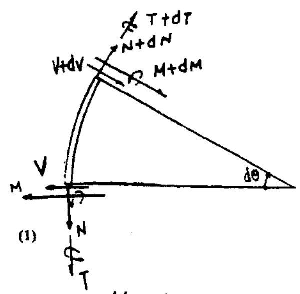
$\Sigma F_x = 0; \quad N_B = 0$ **Ans**

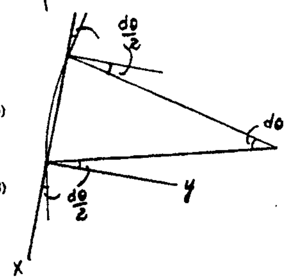
$\Sigma M_x = 0; \quad T_B - \frac{\pi}{4} rw(0.09968r) = 0; \quad T_B = 0.0783\, w\, r^2$ **Ans**

$\Sigma M_y = 0; \quad M_B + \frac{\pi}{4} rw(0.3729\, r) = 0; \quad M_B = -\,0.293\, w\, r^2$ **Ans**

1-33. A differential element taken from a curved bar is shown in the figure. Show that $dN/d\theta = V$, $dV/d\theta = -N$, $dM/d\theta = -T$, and $dT/d\theta = M$.







$\Sigma F_x = 0;$

$$N\cos\frac{d\theta}{2} + V\sin\frac{d\theta}{2} - (N + dN)\cos\frac{d\theta}{2} + (V + dV)\sin\frac{d\theta}{2} = 0 \qquad (1)$$

$\Sigma F_y = 0;$

$$N\sin\frac{d\theta}{2} - V\cos\frac{d\theta}{2} + (N + dN)\sin\frac{d\theta}{2} + (V + dV)\cos\frac{d\theta}{2} = 0 \qquad (2)$$

$\Sigma M_x = 0;$

$$T\cos\frac{d\theta}{2} + M\sin\frac{d\theta}{2} - (T + dT)\cos\frac{d\theta}{2} + (M + dM)\sin\frac{d\theta}{2} = 0 \qquad (3)$$

$\Sigma M_y = 0;$

$$T\sin\frac{d\theta}{2} - M\cos\frac{d\theta}{2} + (T + dT)\sin\frac{d\theta}{2} + (M + dM)\cos\frac{d\theta}{2} = 0 \qquad (4)$$

Since $\frac{d\theta}{2}$ is small, then $\sin\frac{d\theta}{2} = \frac{d\theta}{2}, \cos\frac{d\theta}{2} = 1$

Eq. (1) becomes $V d\theta - dN + \frac{dV d\theta}{2} = 0$

Neglecting the second order term, $V d\theta - dN = 0$

$\frac{dN}{d\theta} = V$ **QED**

Eq.(2) becomes $N d\theta + dV + \frac{dN d\theta}{2} = 0$

Neglecting the second order term, $N d\theta + dV = 0$

$\frac{dV}{d\theta} = -N$ **QED**

Eq.(3) becomes $M d\theta - dT + \frac{dM d\theta}{2} = 0$

Neglecting the second order term, $M d\theta - dT = 0$

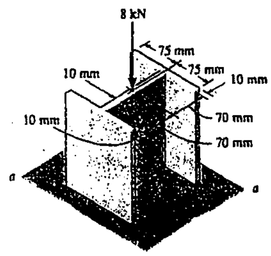
$\frac{dT}{d\theta} = M$ **QED**

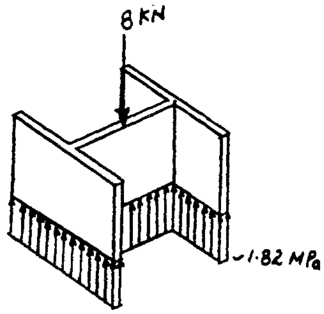
Eq. (4) becomes $T d\theta + dM + \frac{dT d\theta}{2} = 0$

Neglecting the second order term, $T d\theta + dM = 0$

$\frac{dM}{d\theta} = -T$ **QED**

1-34. The column is subjected to an axial force of 8 kN at its top. If the cross-sectional area has the dimensions shown in the figure, determine the average normal stress acting at section *a–a*. Show this distribution of stress acting over the area's cross section.

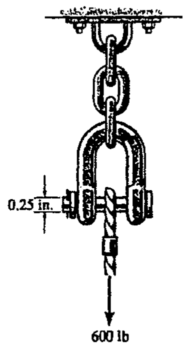


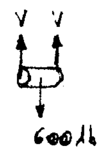


$$A = (2)(150)(10) + (140)(10)$$
$$= 4400\ \text{mm}^2 = 4.4\,(10^{-3})\ \text{m}^2$$

$$\sigma = \frac{P}{A} = \frac{8\,(10^3)}{4.4\,(10^{-3})} = 1.82\ \text{MPa} \qquad \textbf{Ans}$$

1–35 The anchor shackle supports a cable force of 600 lb. If the pin has a diameter of 0.25 in., determine the average shear stress in the pin.



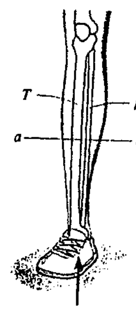


$$+\uparrow \Sigma F_y = 0; \qquad 2V - 600 = 0$$

$$V = 300 \text{ lb}$$

$$\tau_{avg} = \frac{V}{A} = \frac{300}{\frac{\pi}{4}(0.25)^2} = 6.11 \text{ ksi} \qquad \textbf{Ans}$$

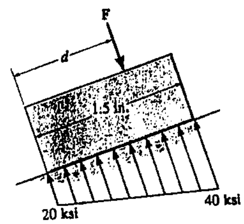
•1–36 While running the foot of a 150-lb man is momentarily subjected to a force which is 5 times his weight. Determine the average normal stress developed in the tibia T of his leg at the mid section a–a. The cross section can be assumed circular, having an outer diameter of 1.75 in. and an inner diameter of 1 in. Assume the fibula F does not support a load.



$P = 5(150 \text{ lb}) = 750 \text{ lb}$

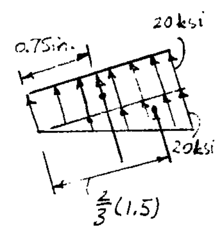
$$\sigma = \frac{P}{A} = \frac{750}{\frac{\pi}{4}((1.75)^2 - (1)^2)} = 463 \text{ psi} \qquad \textbf{Ans}$$

1–37 The small block has a thickness of 0.5 in. If the stress distribution at the support developed by the load varies as shown, determine the force **F** applied to the block, and the distance *d* to where it is applied.



$F = \int \sigma \, dA$ = volume under load diagram

$F = 20(1.5)(0.5) + \frac{1}{2}(20)(1.5)(0.5) = 22.5$ kip **Ans**



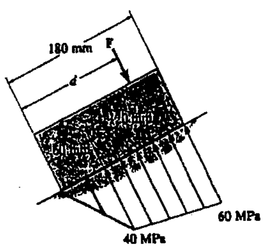
$Fd = \int x(\sigma \, dA)$

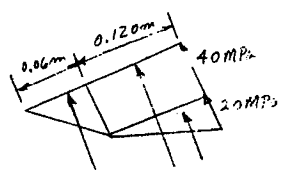
$(22.5)\,d = (0.75)(20)(1.5)(0.5) + \frac{2}{3}(1.5)(\frac{1}{2})(20)(1.5)(0.5)$

$(22.5)\,d = 18.75$

$d = 0.833$ in. **Ans**

1-38. The small block has a thickness of 5 mm. If the stress distribution at the support developed by the load varies as shown, determine the force **F** applied to the block, and the distance d to where it is applied.





$F = \int \sigma\, dA$ = volume under stress diagram

$$F = \frac{1}{2}(0.06)(40)(10^6)(0.005) + (0.120)(40)(10^6)(0.005) + \frac{1}{2}(0.120)(20)(10^6)(0.005)$$

$F = 36$ kN **Ans**

Require

$F\,d = \int x(\sigma\, dA)$

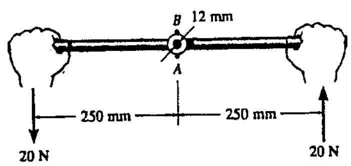
$$36.0(10^3)d = \frac{2}{3}(0.06)(\frac{1}{2})(0.06)(40)(10^6)(0.005) + (0.06 + \frac{1}{2}(0.120))(0.120)(40)(10^6)(0.005) +$$

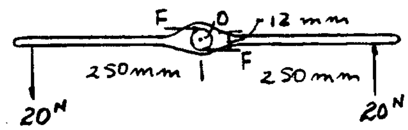
$$(0.06 + \frac{2}{3}(0.120))(\frac{1}{2})(0.120)(20)(10^6)(0.005)$$

$36.0(10^3)d = 3960$

$d = 0.110 = 110$ mm **Ans**

1-39 The lever is held to the fixed shaft using a tapered pin *AB*, which has a mean diameter of 6 mm. If a couple is applied to the lever, determine the average shear stress in the pin between the pin and lever.

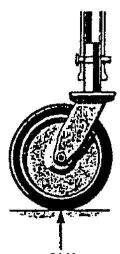




$\circlearrowleft+\Sigma M_O = 0; \quad F(12) - 20(500) = 0; \quad F = 833.33\text{ N}$

$\tau_{avg} = \frac{V}{A} = \frac{833.33}{\frac{\pi}{4}(\frac{6}{1000})^2} = 29.5\text{ MPa}$ **Ans**

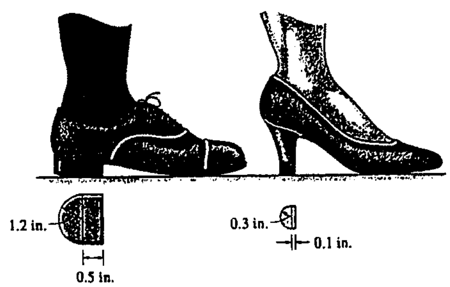
***1-40.** The supporting wheel on a scaffold is held in place on the leg using a 4-mm-diameter pin as shown. If the wheel is subjected to a normal force of 3 kN, determine the average shear stress developed in the pin. Neglect friction between the inner scaffold puller leg and the tube used on the wheel.



$+\uparrow \Sigma F_y = 0; \quad 3\text{ kN} - 2V = 0; \quad V = 1.5\text{ kN}$

$$\tau_{avg} = \frac{V}{A} = \frac{1.5(10^3)}{\frac{\pi}{4}(0.004)^2} = 119\text{ MPa} \quad \textbf{Ans}$$

1–41 A 175-lb woman stands on a vinyl floor wearing stiletto high-heel shoes. If the heel has the dimensions shown, determine the average normal stress she exerts on the floor and compare it with the average normal stress developed when a man having the same weight is wearing flat-heeled shoes. Assume the load is applied slowly, so that dynamic effects can be ignored. Also, assume the entire weight is supported only by the heel of one shoe.



Stiletto shoes :

$$A = \frac{1}{2}(\pi)(0.3)^2 + (0.6)(0.1) = 0.2014 \text{ in}^2$$

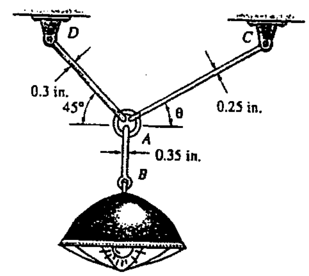
$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{0.2014 \text{ in}^2} = 869 \text{ psi} \qquad \textbf{Ans}$$

Flat - heeled shoes :

$$A = \frac{1}{2}(\pi)(1.2)^2 + 2.4(0.5) = 3.462 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{3.462 \text{ in}^2} = 50.5 \text{ psi} \qquad \textbf{Ans}$$

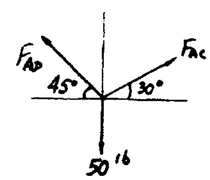
1–42 The 50-lb lamp is supported by three steel rods connected by a ring at A. Determine which rod is subjected to the greater average normal stress and compute its value. Take $\theta = 30°$. The diameter of each rod is given in the figure.



$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\cos 30° - F_{AD}\cos 45° = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\sin 30° + F_{AD}\sin 45° - 50 = 0$$

$$F_{AC} = 36.60 \text{ lb}, \qquad F_{AD} = 44.83 \text{ lb}$$



Rod AB :

$$\sigma_{AB} = \frac{50}{\frac{\pi}{4}(0.35)^2} = 520 \text{ psi}$$

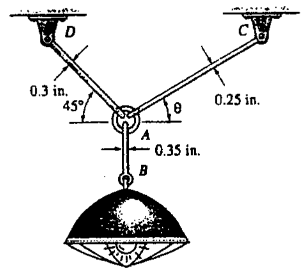
Rod AD :

$$\sigma_{AD} = \frac{44.83}{\frac{\pi}{4}(0.3)^2} = 634 \text{ psi}$$

Rod AC :

$$\sigma_{AC} = \frac{36.60}{\frac{\pi}{4}(0.25)^2} = 746 \text{ psi} \qquad \textbf{Ans}$$

1–43 Solve Prob. 1–42 for $\theta = 45°$.



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AC} \cos 45° - F_{AD} \cos 45° = 0$

$+\uparrow \Sigma F_y = 0; \qquad F_{AC} \sin 45° + F_{AD} \sin 45° - 50 = 0$

$$F_{AC} = F_{AD} = 35.36 \text{ lb}$$

Rod *AB* :

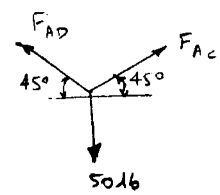
$$\sigma_{AB} = \frac{50}{\frac{\pi}{4}(0.35)^2} = 520 \text{ psi}$$

Rod *AC* :

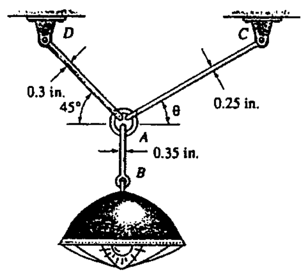
$$\sigma_{AC} = \frac{35.36}{\frac{\pi}{4}(0.25)^2} = 720 \text{ psi} \qquad \textbf{Ans}$$

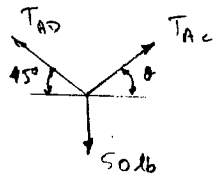
Rod *AD* :

$$\sigma_{AD} = \frac{35.36}{\frac{\pi}{4}(0.3)^2} = 500 \text{ psi}$$



•1–44 The 50-lb lamp is supported by three steel rods connected by a ring at A. Determine the angle of orientation θ of AC such that the average normal stress in rod AC is twice the average normal stress in rod AD. What is the magnitude of stress in each rod? The diameter of each rod is given in the figure.





$$\sigma_{AD} = \frac{T_{AD}}{\frac{\pi}{4}(0.3)^2}; \qquad T_{AD} = (0.070686)\sigma_{AD}$$

$$\sigma_{AC} = 2\sigma_{AD} = \frac{T_{AC}}{\frac{\pi}{4}(0.25)^2}; \qquad T_{AC} = (0.098175)\sigma_{AD}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -T_{AD}\cos 45° + T_{AC}\cos\theta = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{AC}\sin\theta + T_{AD}\sin 45° - 50 = 0 \qquad (2)$$

Thus

$$-(0.070686)\sigma_{AD}(\cos 45°) + (0.098175)\sigma_{AD}(\cos\theta) = 0$$

$$\theta = 59.39° = 59.4° \qquad \textbf{Ans}$$

From Eq. (2) :

$$(0.098175)\sigma_{AD}\sin 59.39° + (0.070686)\sigma_{AD}\sin 45° - 50 = 0$$

$$\sigma_{AD} = 371.8 \text{ psi} = 372 \text{ psi} \qquad \textbf{Ans}$$

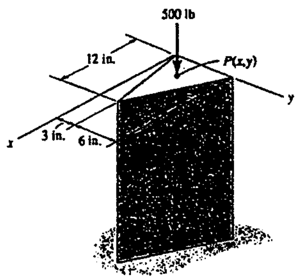
Hence,

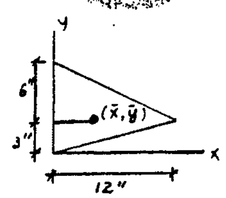
$$\sigma_{AC} = 2(371.8) = 744 \text{ psi} \qquad \textbf{Ans}$$

And,

$$\sigma_{AB} = \frac{T_{AB}}{\frac{\pi}{4}(0.35)^2} = \frac{50}{\frac{\pi}{4}(0.35)^2} = 520 \text{ psi} \qquad \textbf{Ans}$$

1-45. The pedestal has a triangular cross section as shown. If it is subjected to a compressive force of 500 lb, specify the x and y coordinates for the location of point $P(x, y)$, where the load must be applied on the cross section, so that the average normal stress is uniform. Compute the stress and sketch its distribution acting on the cross section at a location removed from the point of load application.

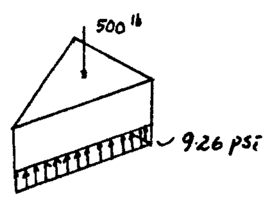




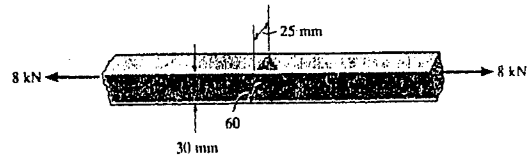
$$\bar{x} = \frac{\frac{1}{2}(3)(12)(\frac{12}{3}) + \frac{1}{2}(6)(12)(\frac{12}{3})}{\frac{1}{2}(9)(12)} = 4 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\frac{1}{2}(3)(12)(3)(\frac{2}{3}) + \frac{1}{2}(6)(12)(3 + \frac{6}{3})}{\frac{1}{2}(9)(12)} = 4 \text{ in.} \qquad \textbf{Ans}$$

$$\sigma = \frac{P}{A} = \frac{500}{\frac{1}{2}(9)(12)} = 9.26 \text{ psi} \qquad \textbf{Ans}$$



1-46 The two steel members are joined together using a 60° scarf weld. Determine the average normal and average shear stress resisted in the plane of the weld.

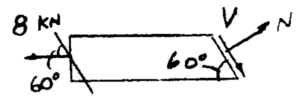


$+\nearrow \Sigma F_x = 0;\quad N - 8 \sin 60° = 0$

$N = 6.928 \text{ kN}$

$\nwarrow + \Sigma F_y = 0;\quad V - 8 \cos 60° = 0$

$V = 4 \text{ kN}$

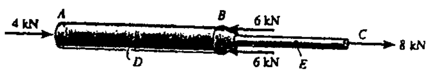


$$A = (25)\left(\frac{30}{\sin 60°}\right) = 866.03 \text{ mm}^2$$

$$\sigma = \frac{N}{A} = \frac{6.928\,(10^3)}{0.8660\,(10^{-3})} = 8 \text{ MPa} \qquad \textbf{Ans}$$

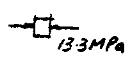
$$\tau_{avg} = \frac{V}{A} = \frac{4\,(10^3)}{0.8660\,(10^{-3})} = 4.62 \text{ MPa} \qquad \textbf{Ans}$$

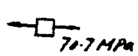
1-47. The built-up shaft consists of a pipe AB and solid rod BC. The pipe has an inner diameter of 20 mm and outer diameter of 28 mm. The rod has a diameter of 12 mm. Determine the average normal stress at points D and E and represent the stress on a volume element located at each of these points.



At D:

$$\sigma_D = \frac{P}{A} = \frac{4(10^3)}{\frac{\pi}{4}(0.028^2 - 0.02^2)} = 13.3 \text{ MPa (C)} \quad \textbf{Ans}$$

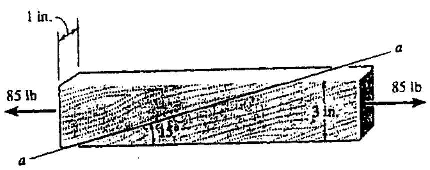


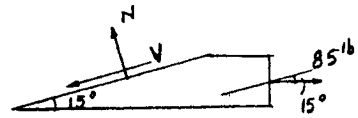


At E:

$$\sigma_E = \frac{P}{A} = \frac{8(10^3)}{\frac{\pi}{4}(0.012^2)} = 70.7 \text{ MPa (T)} \quad \textbf{Ans}$$

•1–48 The board is subjected to a tensile force of 85 lb. Determine the average normal and average shear stress developed in the wood fibers that are oriented along section a–a at 15° with the axis of the board.





$$+\swarrow \Sigma F_x = 0; \quad V - 85\cos 15^\circ = 0$$

$$V = 82.10 \text{ lb}$$

$$+\nwarrow \Sigma F_y = 0; \quad N - 85\sin 15^\circ = 0$$

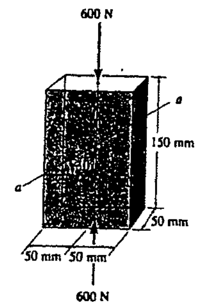
$$N = 22.00 \text{ lb}$$

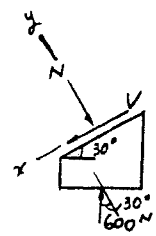
$$A = (1)\left(\frac{3}{\sin 15^\circ}\right) = 11.591 \text{ in}^2$$

$$\sigma = \frac{N}{A} = \frac{22.0}{11.591} = 1.90 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{avg} = \frac{V}{A} = \frac{82.10}{11.591} = 7.08 \text{ psi} \qquad \textbf{Ans}$$

1-49. The plastic block is subjected to an axial compressive force of 600 N. Assuming that the caps at the top and bottom distribute the load uniformly throughout the block, determine the average normal and average shear stress acting along section $a-a$.





Along a-a:

$+\nearrow \Sigma F_x = 0;\quad V - 600\sin 30^\circ = 0$

$V = 300\text{ N}$

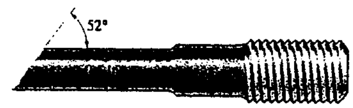
$+\nwarrow \Sigma F_y = 0;\quad -N + 600\cos 30^\circ = 0$

$N = 519.6\text{ N}$

$$\sigma_{a\text{-}a} = \frac{519.6}{(0.05)(\frac{0.1}{\cos 30^\circ})} = 90.0\text{ kPa} \quad \textbf{Ans}$$

$$\tau_{a\text{-}a} = \frac{300}{(0.05)(\frac{0.1}{\cos 30^\circ})} = 52.0\text{ kPa} \quad \textbf{Ans}$$

1-50 The specimen failed in a tension test at an angle of 52° when the axial load was 19.80 kip. If the diameter of the specimen is 0.5 in., determine the average normal and average shear stress acting on the area of the inclined failure plane. Also, what is the average normal stress acting on the *cross section* when failure occurs?

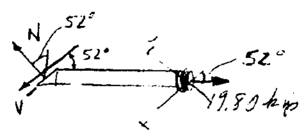


$+\nearrow \Sigma F_x = 0; \quad V - 19.80 \cos 52° = 0$

$V = 12.19$ kip

$+\nwarrow \Sigma F_y = 0; \quad N - 19.80 \sin 52° = 0$

$N = 15.603$ kip



Inclined plane :

$\sigma' = \frac{P}{A}; \quad \sigma' = \frac{15.603}{\frac{\pi(0.25)^2}{\sin 52°}} = 62.6$ ksi **Ans**

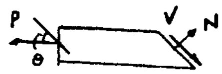
$\tau'_{avg} = \frac{V}{A}; \quad \tau'_{avg} = \frac{12.19}{\frac{\pi(0.25)^2}{\sin 52°}} = 48.9$ ksi **Ans**

Cross section :

$\sigma = \frac{P}{A}; \quad \sigma = \frac{19.80}{\pi(0.25)^2} = 101$ ksi **Ans**

$\tau_{avg} = \frac{V}{A}; \quad \tau_{avg} = 0$ **Ans**

1–51 A tension specimen having a cross-sectional area A is subjected to an axial force **P**. Determine the maximum average shear stress in the specimen and indicate the orientation θ of a section on which it occurs.



$+\nwarrow \Sigma F_y = 0; \quad V - P\cos\theta = 0; \quad V = P\cos\theta$

$$\tau = \frac{P\cos\theta}{A/\sin\theta} = \frac{P\cos\theta\sin\theta}{A} = \frac{P\sin 2\theta}{2A}$$

$$\frac{d\tau}{d\theta} = \frac{P\cos 2\theta}{A} = 0$$

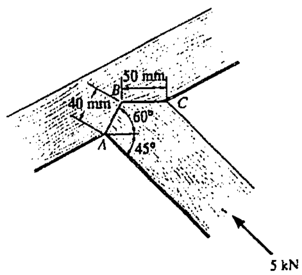
$$\cos 2\theta = 0$$

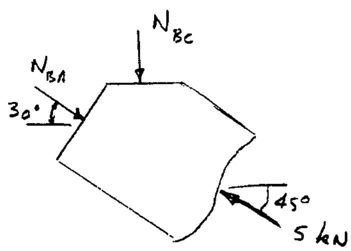
$$2\theta = 90^\circ$$

$$\theta = 45^\circ \qquad \textbf{Ans}$$

$$\tau_{max} = \frac{P}{2A}\sin 90^\circ = \frac{P}{2A} \qquad \textbf{Ans}$$

•1–52 The joint is subjected to the axial member force of 5 kN. Determine the average normal stress acting on sections *AB* and *BC*. Assume the member is smooth and is 50 mm thick.





$\xrightarrow{+} \Sigma F_x = 0; \qquad N_{BA} \cos 30° - 5 \cos 45° = 0$

$N_{BA} = 4.082 \text{ kN}$

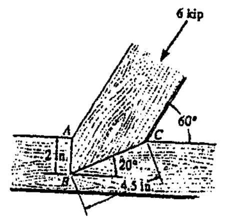
$+\uparrow \Sigma F_y = 0; \qquad -N_{BC} - 4.082 \sin 30° + 5 \sin 45° = 0$

$N_{BC} = 1.494 \text{ kN}$

$$\sigma_{BA} = \frac{N_{BA}}{A_{BA}} = \frac{4.082(10^3)}{(0.04)(0.05)} = 2.04 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{1.494(10^3)}{(0.05)(0.05)} = 0.598 \text{ MPa} \qquad \textbf{Ans}$$

1-53. The joint in subjected to the axial member force of 6 kip. Determine the average normal stress acting on sections AB and BC. Assume the member is smooth and is 1.5 in. thick.

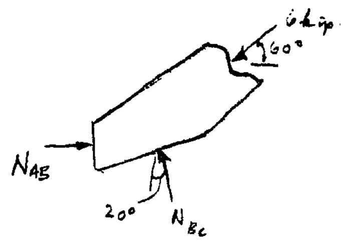


$+\uparrow \Sigma F_y = 0;\qquad -6 \sin 60° + N_{BC} \cos 20° = 0$

$N_{BC} = 5.530$ kip

$\xrightarrow{+} \Sigma F_x = 0;\qquad N_{AB} - 6 \cos 60° - 5.530 \sin 20° = 0$

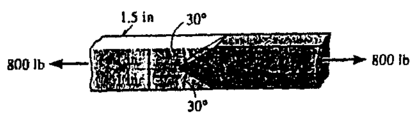
$N_{AB} = 4.891$ kip

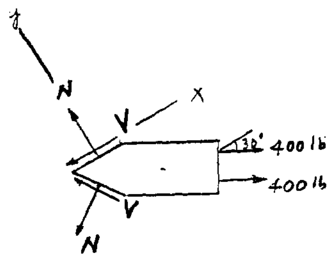


$$\sigma_{AB} = \frac{N_{AB}}{A_{AB}} = \frac{4.891}{(1.5)(2)} = 1.63 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{N_{BC}}{A_{BC}} = \frac{5.530}{(1.5)(4.5)} = 0.819 \text{ ksi} \qquad \textbf{Ans}$$

1-54 The two members used in the construction of an aircraft fuselage are joined together using a 30° fish-mouth weld. Determine the average normal and average shear stress on the plane of each weld. Assume each inclined plane supports a horizontal force of 400 lb.





$$N - 400\sin 30° = 0; \qquad N = 200 \text{ lb}$$

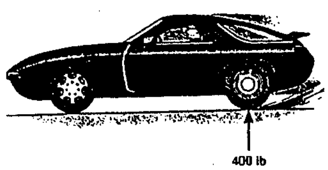
$$400\cos 30° - V = 0; \qquad V = 346.41 \text{ lb}$$

$$A' = \frac{1.5(1)}{\sin 30°} = 3 \text{ in}^2$$

$$\sigma = \frac{N}{A'} = \frac{200}{3} = 66.7 \text{ psi} \qquad \textbf{Ans}$$

$$\tau = \frac{V}{A'} = \frac{346.41}{3} = 115 \text{ psi} \qquad \textbf{Ans}$$

1-55. The driver of the sports car applies his rear brakes and causes the tires to slip. If the normal force on each rear tire is 400 lb and the coefficient of kinetic friction between the tires and the pavement is $\mu_k = 0.5$, determine the average shear stress developed by the friction force on the tires. Assume the rubber of the tires is flexible and each tire is filled with an air pressure of 32 psi.

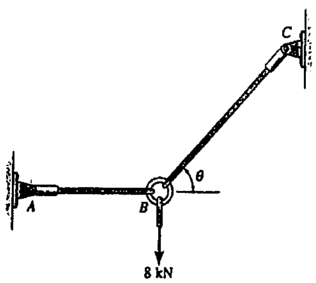


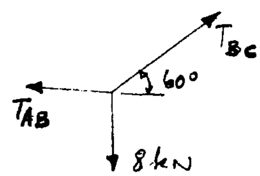
$$F = \mu_k N = 0.5\,(400) = 200 \text{ lb}$$

$$p = \frac{N}{A}; \qquad A = \frac{400}{32} = 12.5 \text{ in}^2$$

$$\tau_{avg} = \frac{F}{A} = \frac{200}{12.5} = 16 \text{ psi} \qquad \textbf{Ans}$$

•1–56 Rods AB and BC have diameters of 4 mm and 6 mm, respectively. If the load of 8 kN is applied to the ring at B, determine the average normal stress in each rod if $\theta = 60°$.





$+\uparrow \Sigma F_y = 0; \quad T_{BC} \sin 60° - 8 = 0$

$T_{BC} = 9.2376$ kN

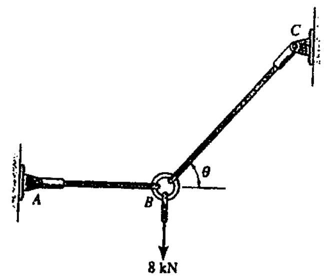
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 9.2376 \cos 60° - T_{AB} = 0$

$T_{AB} = 4.6188$ kN

$$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{4.6188(10^3)}{\frac{\pi}{4}(0.004)^2} = 368 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{T_{BC}}{A_{BC}} = \frac{9.2376(10^3)}{\frac{\pi}{4}(0.006)^2} = 327 \text{ MPa} \qquad \textbf{Ans}$$

1–57 Rods *AB* and *BC* have diameters of 4 mm and 6 mm, respectively. If the vertical load of 8 kN is applied to the ring at *B*, determine the angle θ of rod *BC* so that the average normal stress in each rod is equivalent. What is this stress?

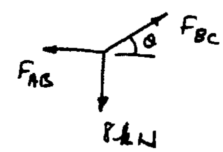


$F_{AB} = \sigma A_{AB} = \sigma(\pi)(0.002)^2$

$F_{BC} = \sigma A_{BC} = \sigma(\pi)(0.003)^2$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \sigma(\pi)(0.003^2)\cos\theta - \sigma\pi(0.002^2) = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0; \quad \sigma\pi(0.003^2)\sin\theta - 8(10^3) = 0 \qquad (2)$



From Eq. (1) :

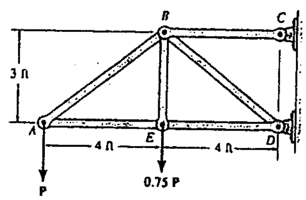
$$\cos\theta = \left(\frac{0.002}{0.003}\right)^2$$

$\theta = 63.6°$ **Ans**

From Eq. (2) :

$$\sigma = \frac{8(10^3)}{\pi(0.003)^2 \sin 63.6°} = 316 \text{ MPa}$$ **Ans**

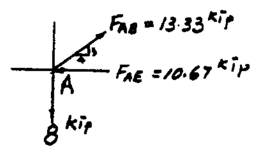
1-58. The bars of the truss each have a cross-sectional area of 1.25 in^2. Determine the average normal stress in each member due to the loading $P = 8$ kip. State whether the stress is tensile or compressive.



Joint A :

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{13.33}{1.25} = 10.7 \text{ ksi} \quad \text{(T)} \quad \textbf{Ans}$$

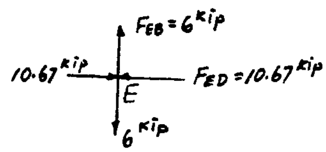
$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}} = \frac{10.67}{1.25} = 8.53 \text{ ksi} \quad \text{(C)} \quad \textbf{Ans}$$



Joint E :

$$\sigma_{ED} = \frac{F_{ED}}{A_{ED}} = \frac{10.67}{1.25} = 8.53 \text{ ksi} \quad \text{(C)} \quad \textbf{Ans}$$

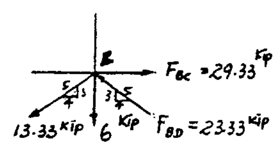
$$\sigma_{EB} = \frac{F_{EB}}{A_{EB}} = \frac{6.0}{1.25} = 4.80 \text{ ksi} \quad \text{(T)} \quad \textbf{Ans}$$



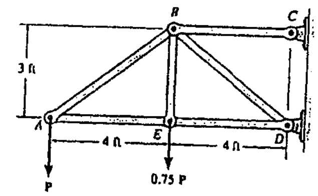
Joint B :

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{29.33}{1.25} = 23.5 \text{ ksi} \quad \text{(T)} \quad \textbf{Ans}$$

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{23.33}{1.25} = 18.7 \text{ ksi} \quad \text{(C)} \quad \textbf{Ans}$$



1-59. The bars of the truss each have a cross-sectional area of 1.25 in^2. If the maximum average normal stress in any bar is not to exceed 20 ksi, determine the maximum magnitude P of the loads that can be applied to the truss.



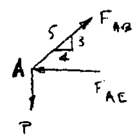
Joint A:

$+\uparrow \Sigma F_y = 0;\quad -P + (\frac{3}{5})F_{AB} = 0$

$F_{AB} = (1.667)P$

$\xrightarrow{+} \Sigma F_x = 0;\quad -F_{AE} + (1.667)P(\frac{4}{5}) = 0$

$F_{AE} = (1.333)P$



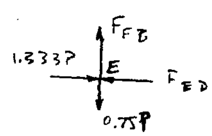
Joint E:

$+\uparrow \Sigma F_y = 0;\quad F_{EB} - (0.75)P = 0$

$F_{EB} = (0.75)P$

$\xrightarrow{+} \Sigma F_x = 0;\quad (1.333)P - F_{ED} = 0$

$F_{ED} = (1.333)P$



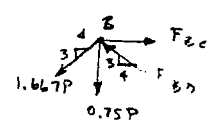
Joint B:

$+\uparrow \Sigma F_y = 0;\quad (\frac{3}{5})F_{BD} - (0.75)P - (1.667)P(\frac{3}{5}) = 0$

$F_{BD} = (2.9167)P$

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{BC} - (2.9167)P(\frac{4}{5}) - (1.667)P(\frac{4}{5}) = 0$

$F_{BC} = (3.67)P$

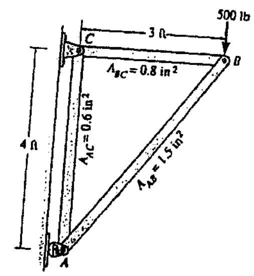


The highest stressed member is BC:

$$\sigma_{BC} = \frac{(3.67)P}{1.25} = 20$$

$P = 6.82$ kip **Ans**

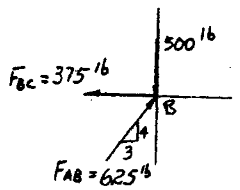
***1-60.** The truss is made from three pin-connected members having the cross-sectional areas shown in the figure. Determine the average normal stress developed in each member when the truss is subjected to the load shown. State whether the stress is tensile or compressive.



Joint B :

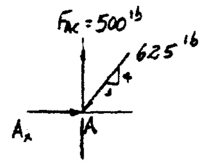
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{625}{1.5} = 417 \text{ psi} \quad \text{(C)} \quad \textbf{Ans}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{375}{0.8} = 469 \text{ psi} \quad \text{(T)} \quad \textbf{Ans}$$

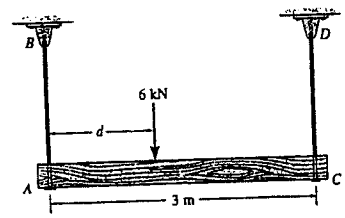


Joint A :

$$\sigma_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{500}{0.6} = 833 \text{ psi} \quad \text{(T)} \quad \textbf{Ans}$$



1–61 The uniform beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2, respectively. If d = 1 m, determine the average normal stress in each rod.

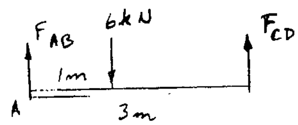


$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(3) - 6(1) = 0$

$F_{CD} = 2 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - 6 + 2 = 0$

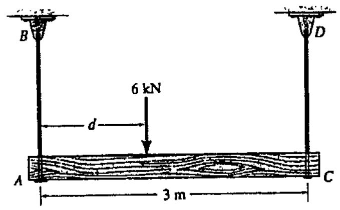
$F_{AB} = 4 \text{ kN}$

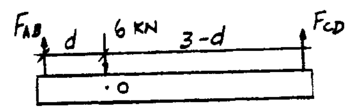


$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{4(10^3)}{12(10^{-6})} = 333 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{2(10^3)}{8(10^{-6})} = 250 \text{ MPa} \qquad \textbf{Ans}$$

1–62 The uniform beam is supported by two rods AB and CD that have cross-sectional areas of 12 mm^2 and 8 mm^2, respectively. Determine the position d of the 6-kN load so that the average normal stress in each rod is the same.





$$\curvearrowleft + \Sigma M_O = 0; \qquad F_{CD}\,(3 - d) - F_{AB}\,(d) = 0 \qquad (1)$$

$$\sigma = \frac{F_{AB}}{12} = \frac{F_{CD}}{8}$$

$$F_{AB} = 1.5\,F_{CD} \qquad (2)$$

From Eqs. (1) and (2),

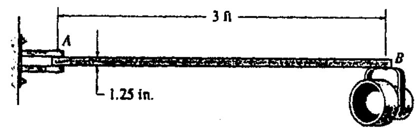
$$F_{CD}\,(3 - d) - 1.5\,F_{CD}(d) = 0$$

$$F_{CD}\,(3 - d - 1.5\,d) = 0$$

$$3 - 2.5\,d = 0$$

$$d = 1.20\text{ m} \qquad \textbf{Ans}$$

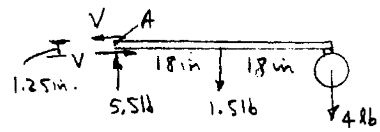
1–63 The railcar docklight is supported by the $\frac{1}{8}$-in.-diameter pin at A. If the lamp weighs 4 lb, and the extension arm AB has a weight of 0.5 lb/ft, determine the average shear stress in the pin needed to support the lamp. *Hint:* The shear force in the pin is caused by the couple moment required for equilibrium at A.



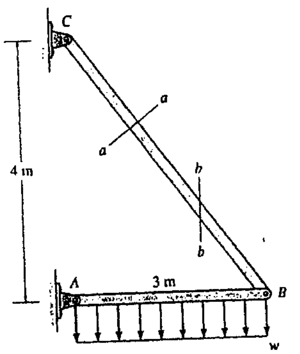
$$\curvearrowleft + \Sigma M_A = 0; \qquad V(1.25) - 1.5(18) - 4(36) = 0$$

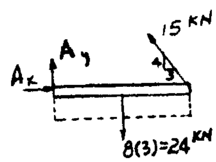
$$V = 136.8 \text{ lb}$$

$$\tau_{avg} = \frac{V}{A} = \frac{136.8}{\frac{\pi}{4}(\frac{1}{8})^2} = 11.1 \text{ ksi} \qquad \textbf{Ans}$$



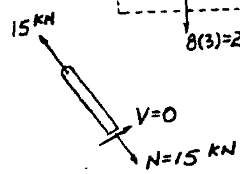
•1–64 The two-member frame is subjected to the distributed loading shown. Determine the average normal stress and average shear stress acting at sections a–a and b–b. Member CB has a square cross section of 35 mm on each side. Take w = 8 kN/m.





At setion a - a :

$$\sigma_{a\text{-}a} = \frac{15(10^3)}{(0.035)^2} = 12.2\text{ MPa} \qquad \textbf{Ans}$$

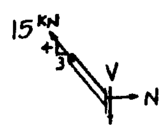


$$\tau_{a\text{-}a} = 0 \qquad \textbf{Ans}$$

At section b - b :

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N - 15(3/5) = 0; \qquad N = 9\text{ kN}$$

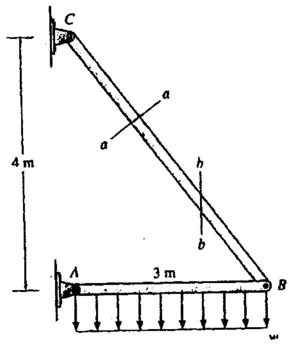
$$+\downarrow \Sigma F_y = 0; \qquad V - 15(4/5) = 0; \qquad V = 12\text{ kN}$$



$$\sigma_{b\text{-}b} = \frac{9(10^3)}{(0.035)(0.035/0.6)} = 4.41\text{ MPa} \qquad \textbf{Ans}$$

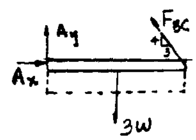
$$\tau_{b\text{-}b} = \frac{12(10^3)}{(0.035)(0.035/0.6)} = 5.88\text{ MPa} \qquad \textbf{Ans}$$

1–65 The two-member frame is subjected to the distributed loading shown. Determine the intensity w of the largest uniform loading that can be applied to the frame without causing either the average normal stress or the average shear stress at section b–b to exceed $\sigma = 15$ MPa and $\tau = 16$ MPa, respectively. Member CB has a square cross section of 35 mm on each side.



$\curvearrowleft + \Sigma M_A = 0; \quad (4/5)F_{BC}(3) - 3w(1.5) = 0$

$2.4\,F_{BC} - 4.5w = 0 \qquad (1)$

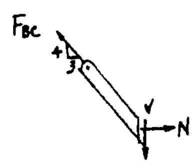


$\xrightarrow{+} \Sigma F_x = 0; \quad N - (3/5)F_{BC} = 0; \quad N = 0.6F_{BC}$

$+\downarrow \Sigma F_y = 0; \quad V - (4/5)F_{BC} = 0; \quad V = 0.8F_{BC}$

$$\sigma = 15(10^6) = \frac{0.6F_{BC}}{(0.035)(0.035/0.6)}$$

$F_{BC} = 51.04$ kN



$$\tau = 16(10^6) = \frac{0.8F_{BC}}{(0.035)(0.035/0.6)}$$

$F_{BC} = 40.83$ kN (controls)

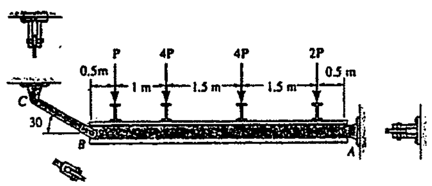
From Eq. (1),

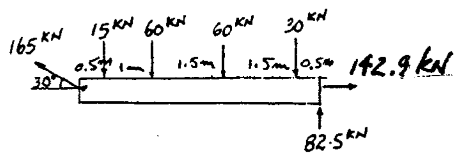
$2.4(40.83) - 4.5w = 0$

$w = 21.8$ kN/m **Ans**

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

1-67. The beam is supported by a pin at A and a short link BC. If $P = 15$ kN, determine the average shear stress developed in the pins at A, B, and C. All pins are in double shear as shown, and each has a diameter of 18 mm.





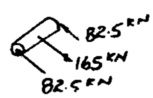
For pins B and C :

$$\tau_B = \tau_C = \frac{V}{A} = \frac{82.5\,(10^3)}{\frac{\pi}{4}\,(\frac{18}{1000})^2} = 324 \text{ MPa} \qquad \textbf{Ans}$$

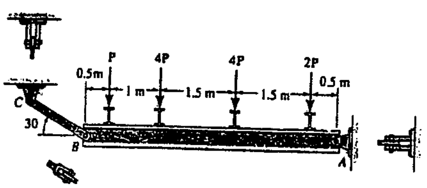
For pin A :

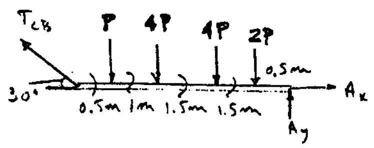
$$F_A = \sqrt{(82.5)^2 + (142.9)^2} = 165 \text{ kN}$$

$$\tau_A = \frac{V}{A} = \frac{82.5\,(10^3)}{\frac{\pi}{4}\,(\frac{18}{1000})^2} = 324 \text{ MPa} \qquad \textbf{Ans}$$



***1-68.** The beam is supported by a pin at A and a short link BC. Determine the maximum magnitude P of the loads the beam will support if the average shear stress in each pin is not to exceed 80 MPa. All pins are in double shear as shown, and each has a diameter of 18 mm.





$$\circlearrowleft + \Sigma M_A = 0; \quad 2P(0.5) + 4P(2) + 4P(3.5) + P(4.5) - (T_{CB} \sin 30°)(5) = 0$$

$$T_{CB} = 11P$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 11P \cos 30° = 0$$

$$A_x = 9.5263P$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 11P + 11P \sin 30° = 0$$

$$A_y = 5.5P$$

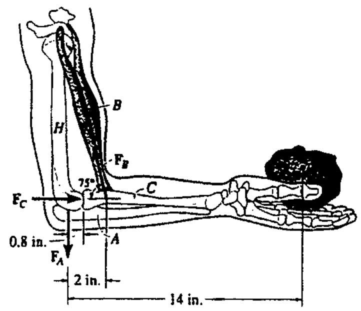
$$F_A = \sqrt{(9.5263P)^2 + (5.5P)^2} = 11P$$

Require;

$$\tau = \frac{V}{A}; \quad 80(10^6) = \frac{11P/2}{\frac{\pi}{4}(0.018)^2}$$

$$P = 3.70 \text{ kN} \quad \textbf{Ans}$$

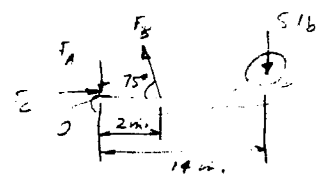
1–69 When the hand is holding the 5-lb stone, the humerus H, assumed to be smooth, exerts normal forces F_C and F_A on the radius C and ulna A, respectively, as shown. If the smallest cross-sectional area of the ligament at B is 0.30 in^2, determine the greatest average tensile stress to which it is subjected.



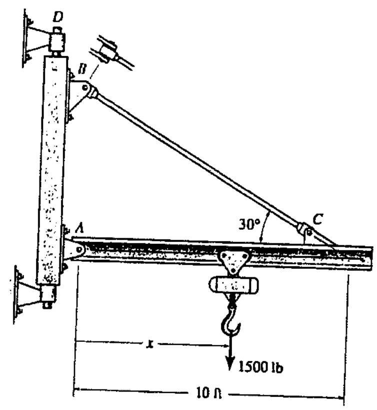
$$\curvearrowleft + \Sigma M_O = 0; \qquad F_B \sin 75°(2) - 5(14) = 0$$

$$F_B = 36.235 \text{ lb}$$

$$\sigma = \frac{P}{A} = \frac{36.235}{0.30} = 121 \text{ psi} \qquad \textbf{Ans}$$



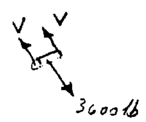
1–70 The jib crane is pinned at A and supports a chain hoist that can travel along the bottom flange of the beam, $1\text{ ft} \le x \le 12\text{ ft}$. If the hoist is rated to support a maximum of 1500 lb, determine the maximum average normal stress in the $\frac{3}{4}$-in.-diameter tie rod BC and the maximum average shear stress in the $\frac{5}{8}$-in.-diameter pin at B.



$$\circlearrowleft + \Sigma M_A = 0; \qquad T_{BC}\sin 30°(10) - 1500(x) = 0$$

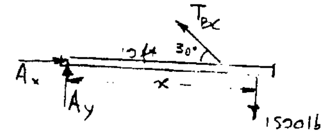
Maximum T_{BC} occurs when $x = 12$ ft

$$T_{BC} = 3600\text{ lb}$$

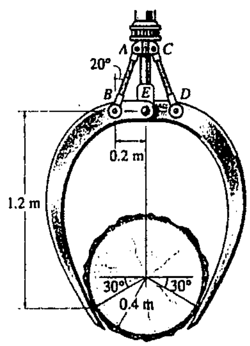


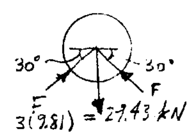
$$\sigma = \frac{P}{A} = \frac{3600}{\frac{\pi}{4}(0.75)^2} = 8.15\text{ ksi} \qquad \textbf{Ans}$$

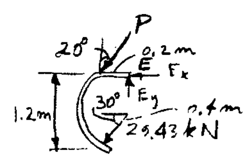
$$\tau = \frac{V}{A} = \frac{3600/2}{\frac{\pi}{4}(5/8)^2} = 5.87\text{ ksi} \qquad \textbf{Ans}$$



1–71 Determine the average normal stress developed in links *AB* and *CD* of the two-tine grapple that supports the log having a mass of 3 Mg. The cross-sectional area of each link is 400 mm².







$+\uparrow \Sigma F_y = 0; \qquad 2(F \sin 30°) - 29.43 = 0$

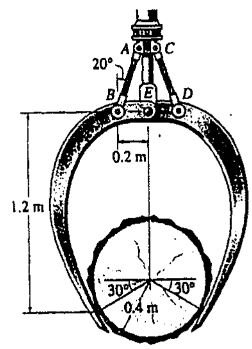
$F = 29.43 \text{ kN}$

$\circlearrowleft + \Sigma M_E = 0; \qquad P \cos 20°(0.2) - (29.43 \cos 30°)(1.2) + (29.43 \sin 30°)(0.4 \cos 30°) = 0$

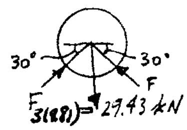
$P = 135.61 \text{ kN}$

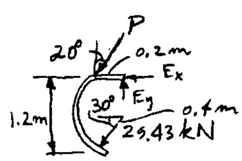
$$\sigma = \frac{P}{A} = \frac{135.61(10^3)}{400(10^{-6})} = 339 \text{ MPa} \qquad \textbf{Ans}$$

***1–72** Determine the average shear stress developed in pins A and B of the two-tine grapple that supports the log having a mass of 3 Mg. Each pin has a diameter of 25 mm and is subjected to double shear.



Prob. 1–72





$$+\uparrow \Sigma F_y = 0; \qquad 2(F \sin 30°) - 29.43 = 0$$

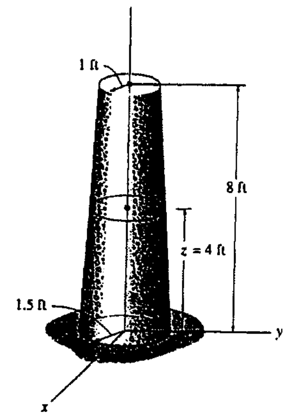
$$F = 29.43 \text{ kN}$$

$$\circlearrowleft + \Sigma M_E = 0; \qquad P \cos 20°(0.2) - (29.43 \cos 30°)(1.2) + (29.43 \sin 30°)(0.4 \cos 30°) = 0$$

$$P = 135.61 \text{ kN}$$

$$\tau_A = \tau_B = \frac{V}{A} = \frac{\frac{135.61(10^3)}{2}}{\frac{\pi}{4}(0.025)^2} = 138 \text{ MPa} \qquad \textbf{Ans}$$

1–73 The pedestal in the shape of a frustum of a cone is made of concrete having a specific weight of 150 lb/ft³. Determine the average normal stress acting in the pedestal at its base. *Hint:* The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.

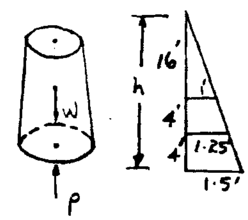


$$\frac{h}{1.5} = \frac{h-8}{1}, \qquad h = 24\text{ ft}$$

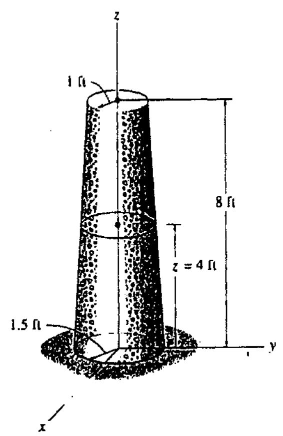
$$V = \frac{1}{3}\pi(1.5)^2(24) - \frac{1}{3}\pi(1)^2(16); \qquad V = 39.794\text{ ft}^3$$

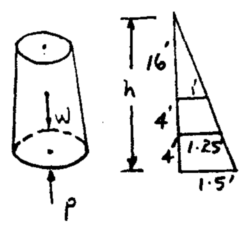
$$W = 150(39.794) = 5.969\text{ kip}$$

$$\sigma = \frac{P}{A} = \frac{5.969}{\pi(1.5)^2} = 844\text{ psf} = 5.86\text{ psi} \qquad \textbf{Ans}$$



1–74 The pedestal in the shape of a frustum of a cone is made of concrete having a specific weight of 150 lb/ft³. Determine the average normal stress acting in the pedestal at its midheight, $z = 4$ ft. *Hint:* The volume of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.





$$W = \left[\frac{1}{3}\pi\,(1.25)^2\,20 - \frac{1}{3}(\pi)\,(1^2)(16)\right](150) = 2395.5\text{ lb}$$

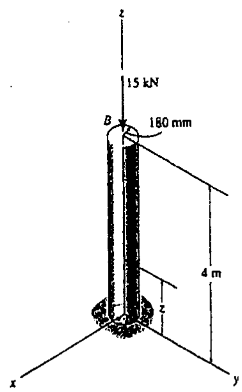
$$\frac{h}{1.5} = \frac{h-8}{1}, \qquad h = 24\text{ ft}$$

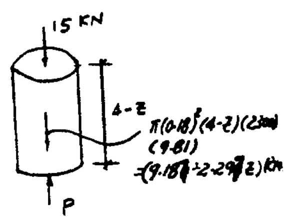
$$+\uparrow \Sigma F_y = 0; \quad P - 2395.5 = 0$$

$$P = 2395.5\text{ lb}$$

$$\sigma = \frac{P}{A} = \frac{2395.5}{\pi\,(1.25)^2} = 488\text{ psf} = 3.39\text{ psi} \qquad \textbf{Ans}$$

1–75. The column is made of concrete having a density of 2.30 Mg/m^3. At its top B it is subjected to an axial compressive force of 15 kN. Determine the average normal stress in the column as a function of the distance z measured from its base. *Note:* The result will be useful only for finding the average normal stress at a section removed from the ends of the column, because of localized deformation at the ends.



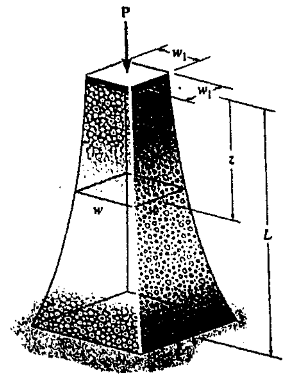


$$+\uparrow \Sigma F_y = 0 \qquad P - 15 - 9.187 + 2.297z = 0$$

$$P = 24.187 - 2.297z$$

$$\sigma = \frac{P}{A} = \frac{24.187 - 2.297z}{\pi(0.18)^2} = (238 - 22.6z)\ \text{kPa} \qquad \textbf{Ans}$$

•1–76 The pier is made of material having a specific weight γ. If it has a square cross section, determine its width w as a function of z so that the average normal stress in the pier remains constant. The pier supports a constant load **P** at its top where its width is w_1.



Assume constant stress σ_1, then at the top,

$$\sigma_1 = \frac{P}{w_1{}^2} \qquad (1)$$

For an increase in z the area must increase,

$$dA = \frac{dW}{\sigma_1} = \frac{\gamma A\, dz}{\sigma_1} \qquad \text{or} \qquad \frac{dA}{A} = \frac{\gamma}{\sigma_1} dz$$

For the top section :

$$\int_{A_1}^{A} \frac{dA}{A} = \frac{\gamma}{\sigma_1}\int_0^z dz$$

$$\ln\frac{A}{A_1} = \frac{\gamma}{\sigma_1} z$$

$$A = A_1 e^{\left(\frac{\gamma}{\sigma_1}\right) z}$$

$$A = w^2$$

$$A_1 = w_1^2$$

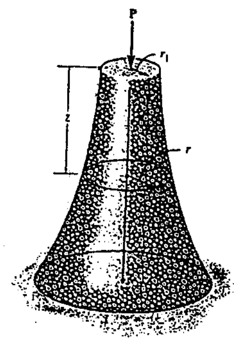
$$w = w_1\, e^{\left(\frac{\gamma}{2\sigma_1}\right) z}$$

From Eq. (1),

$$w = w_1\, e^{\left[\frac{w_1^2 \gamma}{2P}\right] z} \qquad \textbf{Ans}$$

Also see the method used in Prob. 1 - 77.

1-77. The pedestal supports a load **P** at its center. If the material has a mass density ρ, determine the radial dimension r as a function of z so that the average normal stress in the pedestal remains constant. The cross section is circular.

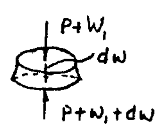


Require :

$$\sigma = \frac{P + W_1}{A} = \frac{P + W_1 + dW}{A + dA}$$

$$P\,dA + W_1\,dA = A\,dW$$

$$\frac{dW}{dA} = \frac{P + W_1}{A} = \sigma \qquad (1)$$



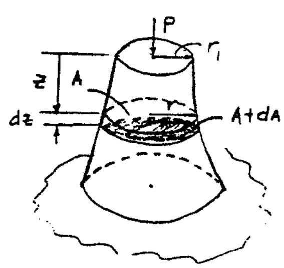
$$dA = \pi(r + dr)^2 - \pi r^2 = 2\pi r\,dr$$

$$dW = \pi r^2(\rho g)\,dz$$

From Eq. (1),

$$\frac{\pi r^2(\rho g)\,dz}{2\pi r\,dr} = \sigma$$

$$\frac{r\rho g\,dz}{2\,dr} = \sigma$$



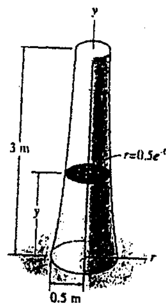
$$\frac{\rho g}{2\sigma}\int_0^z dz = \int_{r_1}^{r}\frac{dr}{r}$$

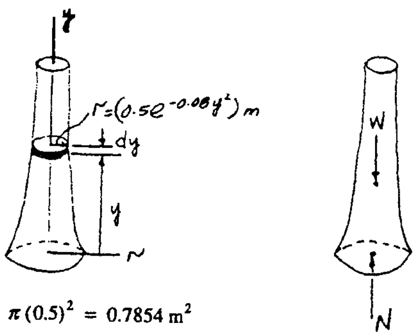
$$\frac{\rho g\,z}{2\sigma} = \ln\frac{r}{r_1}; \qquad r = r_1 e^{\left(\frac{\rho g}{2\sigma}\right)z}$$

However, $\sigma = \dfrac{P}{\pi r_1^2}$

$$r = r_1\, e^{\left(\frac{\pi r_1^2 \rho g}{2P}\right)z} \qquad \textbf{Ans}$$

1-78. The radius of the pedestal is defined by $r = (0.5e^{-0.08y^2})$ m, where y is given in meters. If the material has a density of 2.5 Mg/m^3, determine the average normal stress at the support.





$$A = \pi(0.5)^2 = 0.7854\ \text{m}^2$$

$$dV = \pi(r^2)\,dy = \pi(0.5)^2\,(e^{-0.08y^2})^2$$

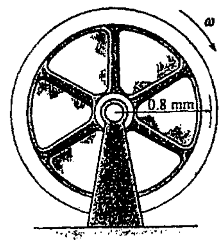
$$V = \int_0^3 \pi(0.5)^2\,(e^{-0.08y^2})^2 dy = 0.7854\int_0^3 (e^{-0.08y^2})^2\,dy$$

$$W = \rho g\,V = (2500)(9.81)(0.7854)\int_0^3 (e^{-0.08y^2})^2\,dy$$

$$W = 19.262(10^3)\int_0^3 (e^{-0.08y^2})^2\,dy = 38.849\ \text{kN}$$

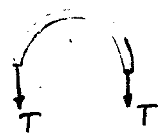
$$\sigma = \frac{W}{A} = \frac{38.849}{0.7854} = 49.5\ \text{kPa} \qquad \textbf{Ans}$$

1–79 Determine the greatest constant angular velocity ω of the flywheel so that the average normal stress in its rim does not exceed σ = 15 MPa. Assume the rim is a thin ring having a thickness of 3 mm, width of 20 mm, and a mass of 30 kg/m. Rotation occurs in the horizontal plane. Neglect the effect of the spokes in the analysis. *Hint:* Consider a free-body diagram of a semicircular portion of the ring. The center of mass for a semicircular segment is located at $\bar{r} = 2r/\pi$ from the diameter.



$$+\downarrow \Sigma F_n = m(a_G)_n; \qquad 2T = m(\bar{r})\omega^2$$

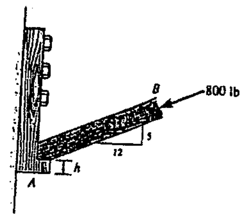
$$2\sigma A = m\left(\frac{2r}{\pi}\right)\omega^2$$



$$2(15(10^6))(0.003)(0.020) = \pi(0.8)(30)\left(\frac{2(0.8)}{\pi}\right)\omega^2$$

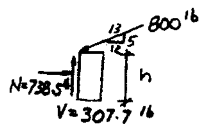
$$\omega = 6.85 \text{ rad/s} \qquad \textbf{Ans}$$

*__1-80.__ Member B is subjected to a compressive force of 800 lb. If A and B are both made of wood and are $\frac{3}{8}$ in. thick, determine to the nearest $\frac{1}{4}$ in. the smallest dimension h of the support so that the average shear stress does not exceed $\tau_{allow} = 300$ psi.



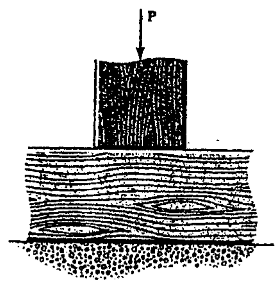
$$\tau_{allow} = 300 = \frac{307.7}{(\frac{3}{8})h}$$

$$h = 2.74 \text{ in.}$$



Use $h = 2\frac{3}{4}$ in. **Ans**

1-81 The 60 mm × 60 mm oak post is supported on the pine block. If the allowable bearing stresses for these materials are $\sigma_{oak} = 43$ MPa and $\sigma_{pine} = 25$ MPa, determine the greatest load P that can be supported. If a rigid bearing plate is used between these materials, determine its required area so that the maximum load P can be supported. What is this load?



For failure of pine block :

$$\sigma = \frac{P}{A}; \qquad 25(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 90\text{ kN} \qquad \textbf{Ans}$$

For failure of oak post :

$$\sigma = \frac{P}{A}; \qquad 43(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 154.8\text{ kN}$$

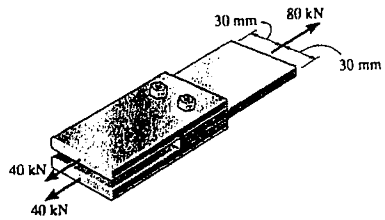
Area of plate based on strength of pine block :

$$\sigma = \frac{P}{A}; \qquad 25(10^6) = \frac{154.8(10)^3}{A}$$

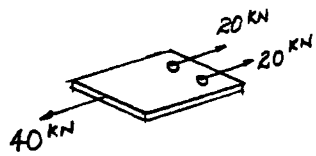
$$A = 6.19(10^{-3})\text{ m}^2 \qquad \textbf{Ans}$$

$$P_{max} = 155\text{ kN} \qquad \textbf{Ans}$$

1–82 The joint is fastened together using two bolts. Determine the required diameter of the bolts if the allowable shear stress for the bolts is $\tau_{allow} = 110$ MPa. Assume each bolt supports an equal portion of the load.

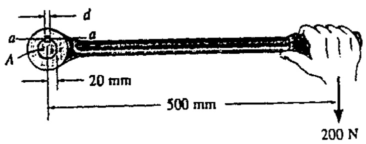


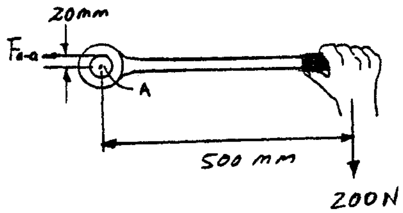
$$\tau_{allow} = 110\,(10^6) = \frac{20\,(10^3)}{\frac{\pi}{4}\,d^2}$$



$$d = 0.0152 \text{ m} = 15.2 \text{ mm} \qquad \textbf{Ans}$$

1–83 The lever is attached to the shaft A using a key that has a width d and length of 25 mm. If the shaft is fixed and a vertical force of 200 N is applied perpendicular to the handle, determine the dimension d if the allowable shear stress for the key is $\tau_{allow} = 35$ MPa.





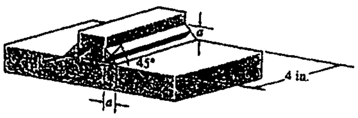
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{a\text{-}a}(20) - 200(500) = 0$$

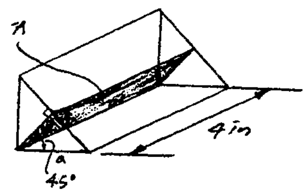
$$F_{a\text{-}a} = 5000 \text{ N}$$

$$\tau_{allow} = \frac{F_{a\text{-}a}}{A_{a\text{-}a}}; \qquad 35(10^6) = \frac{5000}{d(0.025)}$$

$$d = 0.00571 \text{ m} = 5.71 \text{ mm}$$ **Ans**

***1-84.** The fillet weld size a is determined by computing the average shear stress along the shaded plane, which has the smallest cross section. Determine the smallest size a of the two welds if the force applied to the plate is $P = 20$ kip. The allowable shear stress for the weld material is $\tau_{allow} = 14$ ksi.



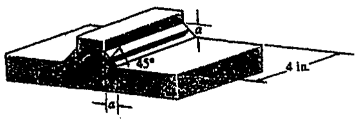


Shear plane $A = a \sin 45°(4) = 2.8284\,a$

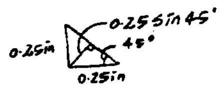
$$\tau_{allow} = \frac{V}{A}; \quad 14(10^3) = \frac{\frac{20(10^3)}{2}}{2.8284\,a}$$

$a = 0.253$ in. **Ans**

1-85. The fillet weld size $a = 0.25$ in. If the joint is assumed to fail by shear on both sides of the block along the shaded plane, which is the smallest cross section, determine the largest force P that can be applied to the plate. The allowable shear stress for the weld material is $\tau_{allow} = 14$ ksi.



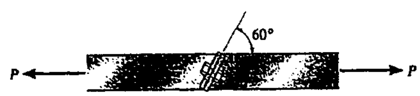
$$\text{Area} = (2)[(4)(0.707)(0.25)] = 1.414 \text{ in}^2$$

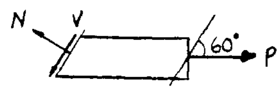


$$\tau_{allow} = \frac{V}{A}; \quad 14 = \frac{P}{1.414}$$

$$P = 19.8 \text{ kip} \qquad \textbf{Ans}$$

1–86 The tension member is fastened together using *two* bolts, one on each side of the member as shown. Each bolt has a diameter of 0.3 in. Determine the maximum load P that can be applied to the member if the allowable shear stress for the bolts is $\tau_{allow} = 12$ ksi and the allowable average normal stress is $\sigma_{allow} = 20$ ksi.





$$\nwarrow + \Sigma F_y = 0; \quad N - P\sin 60^\circ = 0$$

$$P = 1.1547\,N \qquad (1)$$

$$\nearrow + \Sigma F_x = 0; \quad V - P\cos 60^\circ = 0$$

$$P = 2\,V \qquad (2)$$

Assume failure due to shear :

$$\tau_{allow} = 12 = \frac{V}{(2)\,\frac{\pi}{4}\,(0.3)^2}$$

$V = 1.696$ kip

From Eq. (2) ,

$P = 3.39$ kip

Assume failure due to normal force :

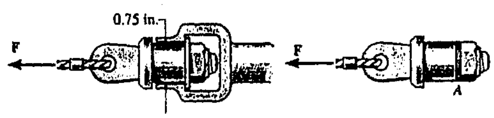
$$\sigma_{allow} = 20 = \frac{N}{(2)\,\frac{\pi}{4}\,(0.3)^2}$$

$N = 2.827$ kip

From Eq. (1),

$P = 3.26$ kip (controls) **Ans**

1–87 The steel swivel bushing in the elevator control of an airplane is held in place using a nut and washer as shown in Fig. (*a*). Failure of the washer *A* can cause the push rod to separate as shown in Fig. (*b*). If the maximum average normal stress for the washer is $\sigma_{max} = 60$ ksi and the maximum average shear stress is $\tau_{max} = 21$ ksi, determine the force **F** that must be applied to the bushing that will cause this to happen. The washer is $\frac{1}{16}$ in. thick.



(a) (b)

$$\tau_{avg} = \frac{V}{A}; \qquad 21(10^3) = \frac{F}{2\pi(0.375)(\frac{1}{16})}$$

$$F = 3{,}092.5\ \text{lb} = 3.09\ \text{kip} \qquad \textbf{Ans}$$

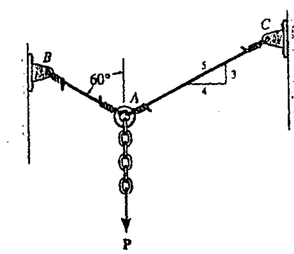
•1–88 The two steel wires AB and AC are used to support the load. If both wires have an allowable tensile stress of $\sigma_{allow} = 200$ MPa, determine the required diameter of each wire if the applied load is $P = 5$ kN.

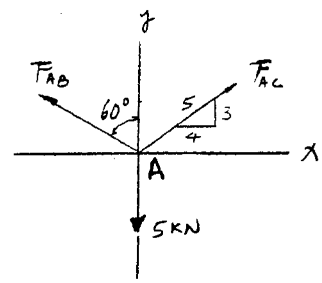
$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{4}{5}F_{AC} - F_{AB}\sin 60^\circ = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{AC} + F_{AB}\cos 60^\circ - 5 = 0 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$F_{AB} = 4.3496$ kN; $F_{AC} = 4.7086$ kN





Applying $\sigma_{allow} = \frac{P}{A}$

For wire AB,

$$200(10^6) = \frac{4.3496(10^3)}{\frac{\pi}{4}(d_{AB})^2}$$

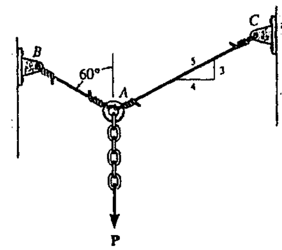
$d_{AB} = 0.00526$ m $= 5.26$ mm **Ans**

For wire AC,

$$200(10^6) = \frac{4.7086(10^3)}{\frac{\pi}{4}(d_{AC})^2}$$

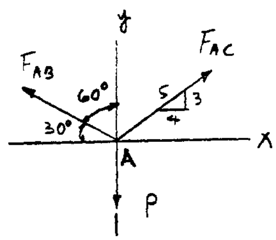
$d_{AC} = 0.00548$ m $= 5.48$ mm **Ans**

1-89 The two steel wires *AB* and *AC* are used to support the load. If both wires have an allowable tensile stress of σ_{allow} = 180 MPa, and wire *AB* has a diameter of 6 mm and *AC* has a diameter of 4 mm, determine the greatest force *P* that can be applied to the chain before one of the wires fails.



$$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{4}{5}F_{AC} - F_{AB}\sin 60° = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{AC} + F_{AB}\cos 60° - P = 0 \qquad (2)$$



Assume failure of *AB* :

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \quad 180(10^6) = \frac{F_{AB}}{\frac{\pi}{4}(0.006)^2}$$

$$F_{AB} = 5089.38 \text{ N} = 5.089 \text{ kN}$$

Solving Eqs.(1) and (2) yields :

F_{AC} = 5.509 kN ; P = 5.85 kN

Assume failure of *AC* :

$$\sigma_{allow} = \frac{F_{AC}}{A_{AC}}; \quad 180(10^6) = \frac{F_{AC}}{\frac{\pi}{4}(0.004)^2}$$

$$F_{AC} = 2261.94 \text{ N} = 2.262 \text{ kN}$$

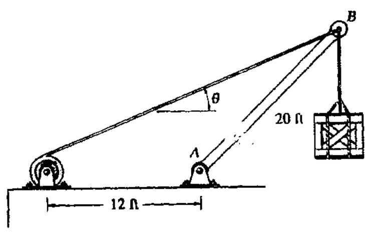
Solving Eqs. (1) and (2) yields :

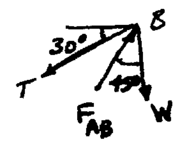
F_{AB} = 2.089 kN ; P = 2.40 kN

Choose the smallest value

P = 2.40 kN **Ans**

1–90 The boom is supported by the winch cable that has a diameter of 0.25 in. and an allowable normal stress of σ_{allow} = 24 ksi. Determine the greatest load that can be supported without causing the cable to fail when θ = 30°. Neglect the size of the winch.





$$\sigma = \frac{P}{A}; \qquad 24(10^3) = \frac{T}{\frac{\pi}{4}(0.25)^2};$$

$$T = 1178.10 \text{ lb}$$

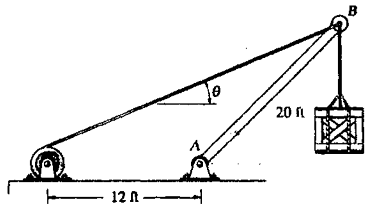
$$\xrightarrow{+} \Sigma F_x = 0; \qquad -1178.10 \cos 30° + F_{AB} \sin 45° = 0$$

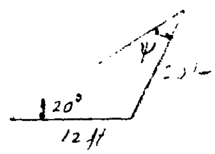
$$+\uparrow \Sigma F_y = 0; \qquad -W + F_{AB} \cos 45° - 1178.10 \sin 30° = 0$$

$$W = 431 \text{ lb}$$ **Ans**

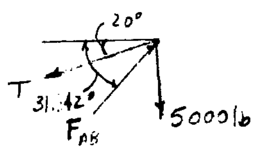
$$F_{AB} = 1442.9 \text{ lb}$$

1–91 The boom is supported by the winch cable that has an allowable normal stress of $\sigma_{allow} = 24$ ksi. If it is required that it be able to slowly lift 5000 lb, from $\theta = 20°$ to $\theta = 50°$, determine the smallest diameter of the cable to the nearest $\frac{1}{16}$ in. The boom AB has a length of 20 ft. Neglect the size of the winch.





Maximum tension in cable occurs when $\theta = 20°$,



$$\frac{\sin 20°}{20} = \frac{\sin \psi}{12}$$

$$\psi = 11.842°$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -T\cos 20° + F_{AB}\cos 31.842° = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\sin 31.842° - T\sin 20° - 5000 = 0$$

$$T = 20\,698.3 \text{ lb}$$

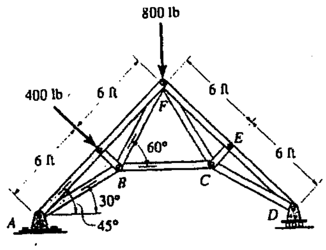
$$F_{AB} = 22\,896 \text{ lb}$$

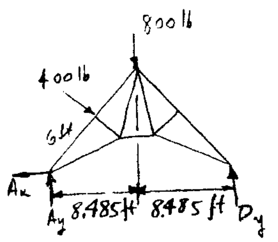
$$\sigma = \frac{P}{A}; \qquad 24(10^3) = \frac{20\,698.3}{\frac{\pi}{4}(d)^2}$$

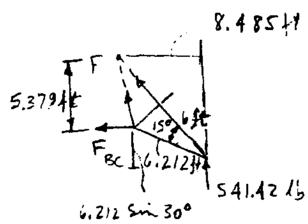
$$d = 1.048 \text{ in.}$$

Use $\quad d = 1\frac{1}{16}$ in. $\quad$ **Ans**

•1-92 The truss is used to support the loading shown. Determine the required cross-sectional area of member *BC* if the allowable normal stress is $\sigma_{allow} = 24$ ksi.







$$+\Sigma M_A = 0; \qquad -400(6) - 800(8.485) + 2(8.485)(D_y) = 0$$

$$D_y = 541.42 \text{ lb}$$

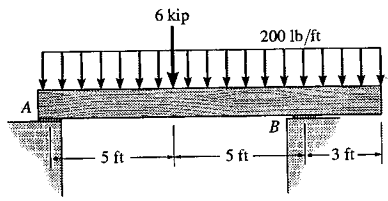
$$+\Sigma M_F = 0; \qquad 541.42(8.485) - F_{BC}(5.379 \sin 30°) = 0$$

$$F_{BC} = 853.98 \text{ lb}$$

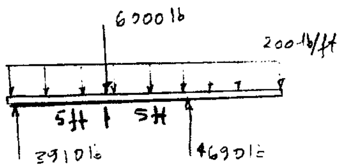
$$\sigma = \frac{P}{A}; \qquad 24000 = \frac{853.98}{A}$$

$$A = 0.0356 \text{ in}^2$$ **Ans**

1–93 The beam is made from southern pine and is supported at its ends by base plates resting on brick work. If the allowable bearing stresses for the materials are $(\sigma_{pine})_{allow} = 2.81$ ksi $(\sigma_{brick})_{allow} = 6.70$ ksi, determine the required length of the base plates at A and B to the nearest $\frac{1}{4}$ inch in order to support the load shown. The plates are 3 in. wide.



Prob. 1–93



The design must be based on strength of the pine.

At A:

$$\sigma = \frac{P}{A}; \qquad 2810 = \frac{3910}{l_A(3)}$$

$$l_A = 0.464 \text{ in.}$$

Use $l_A = \frac{1}{2}$ in. **Ans**

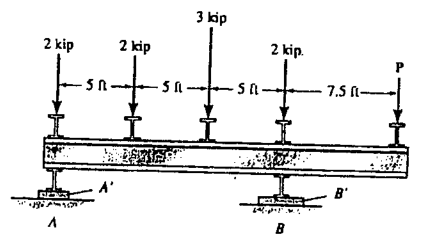
At B:

$$\sigma = \frac{P}{A}; \qquad 2810 = \frac{4690}{l(3)}$$

$$l_B = 0.556 \text{ in.}$$

Use $l_B = \frac{3}{4}$ in. **Ans**

1–94 If the allowable bearing stress for the material under the supports at *A* and *B* is $(\sigma_b)_{allow}$ = 400 psi, determine the size of *square* bearing plates *A'* and *B'* required to support the loading. Take P = 1.5 kip. Dimension the plates to the nearest $\frac{1}{2}$ in. The reactions at the supports are vertical.



For Plate *A* :

$$\sigma_{allow} = 400 = \frac{3.583\,(10^3)}{a_{A'}^2}$$

$a_{A'}$ = 2.99 in.

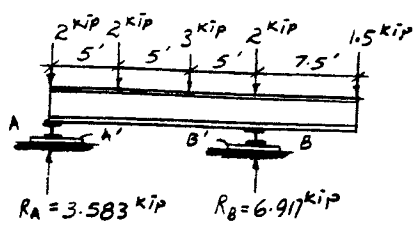
Use a 3 in. x 3 in. plate **Ans**

For Plate *B'* :

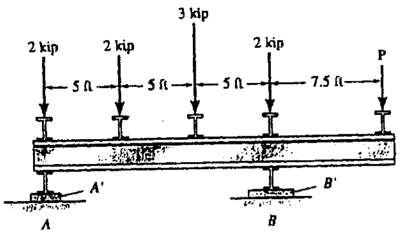
$$\sigma_{allow} = 400 = \frac{6.917\,(10^3)}{a_{B'}^2}$$

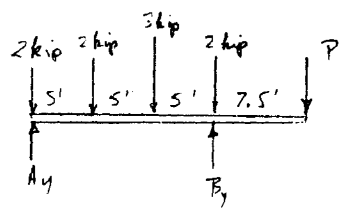
$a_{B'}$ = 4.16 in.

Use a $4\frac{1}{2}$ in. x $4\frac{1}{2}$ in. plate **Ans**



1–95 If the allowable bearing stress for the material under the supports at A and B is $(\sigma_b)_{allow} = 400$ psi, determine the maximum load **P** that can be applied to the beam. The bearing plates A' and B' have square cross sections of 2 in. × 2 in. and 4 in. × 4 in., respectively.





$$\circlearrowleft + \Sigma M_A = 0; \quad B_y(15) - 2(5) - 3(10) - 2(15) - P(22.5) = 0$$

$$B_y = 1.5P + 4.667$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + 1.5P + 4.667 - 9 - P = 0$$

$$A_y = 4.333 - 0.5P$$

At A:

$$0.400 = \frac{4.333 - 0.5P}{2(2)}$$

$$P = 5.47 \text{ kip}$$

At B:

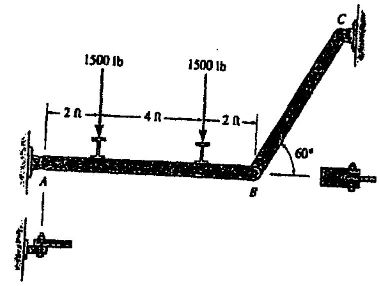
$$0.400 = \frac{1.5P + 4.667}{4(4)}$$

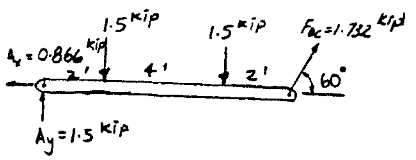
$$P = 1.16 \text{ kip}$$

Thus,

$$P_{allow} = 1.16 \text{ kip} \qquad \textbf{Ans}$$

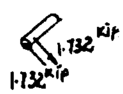
*1-96.** Determine the required cross-sectional area of member BC and the diameter of the pins at A and B if the allowable normal stress is $\sigma_{\text{allow}} = 3\text{ ksi}$ and the allowable shear stress is $\tau_{\text{allow}} = 4\text{ ksi}$.





Member BC:

$$\sigma_{\text{allow}} = 3\,(10^3) = \frac{1.732\,(10^3)}{A_{BC}}$$

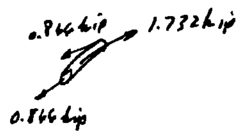


$A_{BC} = 0.577\text{ in}^2$ **Ans**

Pin A:

$$F_A = \sqrt{(0.866)^2 + (1.5)^2} = 1.732\text{ kip}$$

$$\tau_{\text{allow}} = 4\,(10^3) = \frac{1.732\,(10^3)}{\frac{\pi}{4}(d_A)^2}$$



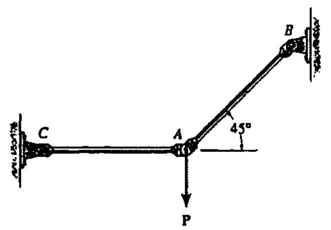
$d_A = 0.743\text{ in.}$ **Ans**

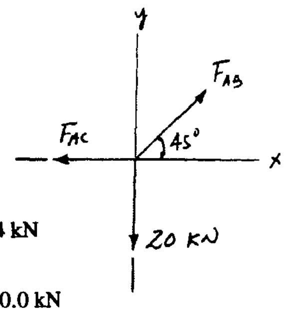
Pin B:

$$\tau_{\text{allow}} = 4\,(10^3) = \frac{0.866\,(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$d_B = 0.525\text{ in.}$ **Ans**

1-97 The two aluminum rods support the vertical force of $P = 20$ kN. Determine their required diameters if the allowable tensile stress for the aluminum is $\sigma_{allow} = 150$ MPa.





$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 45° - 20 = 0; \qquad F_{AB} = 28.284 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 28.284 \cos 45° - F_{AC} = 0; \qquad F_{AC} = 20.0 \text{ kN}$$

For rod AB :

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \qquad 150(10^6) = \frac{28.284(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

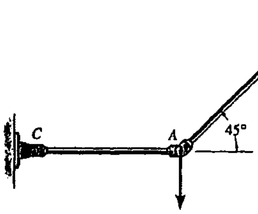
$d_{AB} = 0.0155 \text{ m} = 15.5 \text{ mm}$ **Ans**

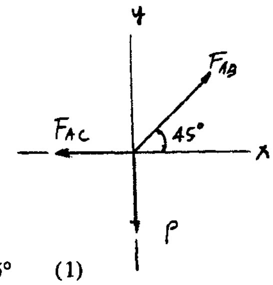
For rod AC :

$$\sigma_{allow} = \frac{F_{AC}}{A_{AC}}; \qquad 150(10^6) = \frac{20.0(10^3)}{\frac{\pi}{4} d_{AC}^2}$$

$d_{AC} = 0.0130 \text{ m} = 13.0 \text{ mm}$ **Ans**

1–98 The two aluminum rods *AB* and *AC* have diameters of 10 mm and 8 mm, respectively. Determine the largest vertical force **P** that can be supported. The allowable tensile stress for the aluminum is $\sigma_{allow} = 150$ MPa.





$+\uparrow\Sigma F_y = 0; \quad F_{AB}\sin 45° - P = 0; \quad P = F_{AB}\sin 45° \quad (1)$

$\xrightarrow{+}\Sigma F_x = 0; \quad F_{AB}\cos 45° - F_{AC} = 0 \quad (2)$

Assume failure of rod *AB* :

$$\sigma_{allow} = \frac{F_{AB}}{A_{AB}}; \quad 150(10^6) = \frac{F_{AB}}{\frac{\pi}{4}(0.01)^2}$$

$F_{AB} = 11.78$ kN

From Eq. (1),

$P = 8.33$ kN

Assume failure of rod *AC* :

$$\sigma_{allow} = \frac{F_{AC}}{A_{AC}}; \quad 150(10^6) = \frac{F_{AC}}{\frac{\pi}{4}(0.008)^2}$$

$F_{AC} = 7.540$ kN

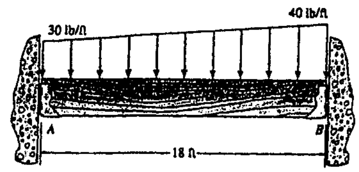
Solving Eqs. (1) and (2) yields :

$F_{AB} = 10.66$ kN ; $\quad P = 7.54$ kN

Choose the smallest value

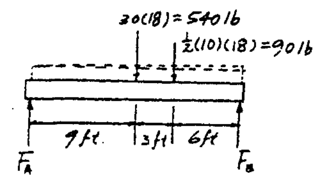
$P = 7.54$ kN **Ans**

1-99. The hangers support the joist uniformly, so that it is assumed the four nails on each hanger carry an equal portion of the load. If the joist is subjected to the loading shown, determine the average shear stress in each nail of the hanger at ends *A* and *B*. Each nail has a diameter of 0.25 in. The hangers only support vertical loads.



$$\circlearrowleft + \Sigma M_A = 0; \quad F_B(18) - 540(9) - 90(12) = 0; \quad F_B = 330\ \text{lb}$$

$$+\uparrow \Sigma F_y = 0; \quad F_A + 330 - 540 - 90 = 0; \quad F_A = 300\ \text{lb}$$



For nails at *A*,

$$\tau_{avg} = \frac{F_A}{A_A} = \frac{300}{4(\frac{\pi}{4})(0.25)^2}$$

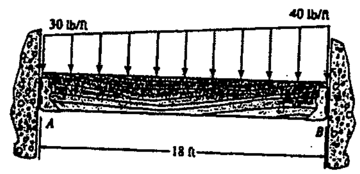
$$= 1528\ \text{psi} = 1.53\ \text{ksi} \qquad \textbf{Ans}$$

For nails at *B*,

$$\tau_{avg} = \frac{F_B}{A_B} = \frac{330}{4(\frac{\pi}{4})(0.25)^2}$$

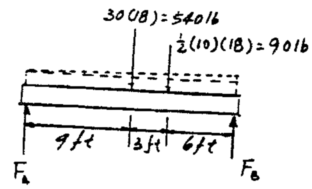
$$= 1681\ \text{psi} = 1.68\ \text{ksi} \qquad \textbf{Ans}$$

***1-100.** The hangers support the joists uniformly, so that it is assumed the four nails on each hanger carry an equal portion of the load. Determine the smallest diameter of the nails at A and at B if the allowable shear stress for the nails is $\tau_{allow} = 4$ ksi. The hangers only support vertical loads.



$\curvearrowleft + \Sigma M_A = 0; \quad F_B(18) - 540(9) - 90(12) = 0; \quad F_B = 330 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad F_A + 330 - 540 - 90 = 0; \quad F_A = 300 \text{ lb}$



For nails at A,

$\tau_{allow} = \frac{F_A}{A_A}; \quad 4(10^3) = \frac{300}{4(\frac{\pi}{4})d_A^2}$

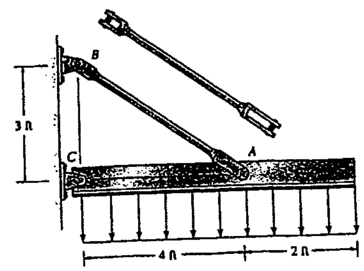
$d_A = 0.155$ in. **Ans**

For nails at B,

$\tau_{allow} = \frac{F_B}{A_B}; \quad 4(10^3) = \frac{330}{4(\frac{\pi}{4})d_B^2}$

$d_B = 0.162$ in. **Ans**

1-101. The hanger assembly is used to support a distributed loading of $w = 0.8$ kip/ft. Determine the average shear stress in the 0.40-in.-diameter bolt at A and the average tensile stress in rod AB, which has a diameter of 0.5 in. If the yield shear stress for the bolt is $\tau_y = 25$ ksi, and the yield tensile stress for the rod is $\sigma_y = 38$ ksi, determine the factor of safety with respect to yielding in each case.



For bolt A :

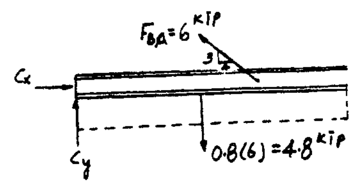
$$\tau = \frac{V}{A} = \frac{3}{\frac{\pi}{4}(0.4^2)} = 23.9 \text{ ksi} \quad \textbf{Ans}$$

$$\text{F. S.} = \frac{\tau_y}{\tau} = \frac{25}{23.9} = 1.05 \quad \textbf{Ans}$$

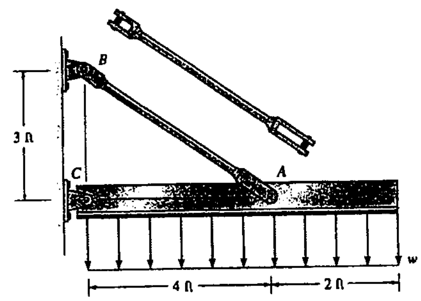
For rod AB :

$$\sigma = \frac{P}{A} = \frac{6}{\frac{\pi}{4}(0.5^2)} = 30.6 \text{ ksi} \quad \textbf{Ans}$$

$$\text{F. S.} = \frac{\sigma_y}{\sigma} = \frac{38}{30.6} = 1.24 \quad \textbf{Ans}$$



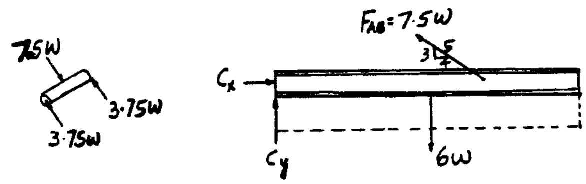
1-102 Determine the intensity w of the maximum distributed load that can be supported by the hanger assembly so that an allowable shear stress of $\tau_{allow} = 13.5$ ksi is not exceeded in the 0.40-in.-diameter bolts at A and B, and an allowable tensile stress of $\sigma_{allow} = 22$ ksi is not exceeded in the 0.5-in.-diameter rod AB.



Assume failure of pin A or B :

$$\tau_{allow} = 13.5 = \frac{3.75w}{\frac{\pi}{4}(0.4^2)}$$

$$w = 0.452 \text{ kip/ft (controls)}$$ **Ans**

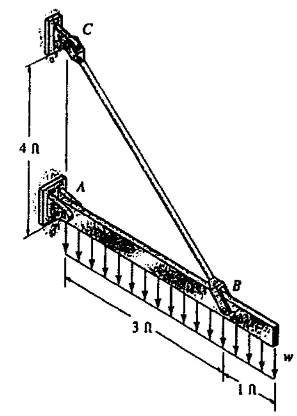


Assuming failure of rod AB :

$$\sigma_{allow} = 22 = \frac{7.5w}{\frac{\pi}{4}(0.5^2)}$$

$$w = 0.576 \text{ kip/ft}$$

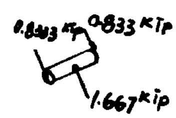
1-103. The assembly is used to support the distributed loading of $w = 500$ lb/ft. Determine the factor of safety with respect to yielding for the steel rod BC and the pins at B and C if the yield stress for the steel in tension is $\sigma_y = 36$ ksi and in shear $\tau_y = 18$ ksi. The rod has a diameter of 0.4 in., and the pins each have a diameter of 0.30 in.



For rod BC:

$$\sigma = \frac{P}{A} = \frac{1.667}{\frac{\pi}{4}(0.4^2)} = 13.26 \text{ ksi}$$

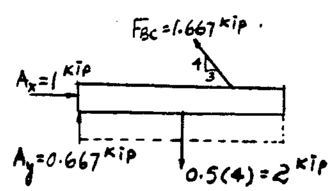
$$\text{F. S.} = \frac{\sigma_y}{\sigma} = \frac{36}{13.26} = 2.71 \qquad \textbf{Ans}$$



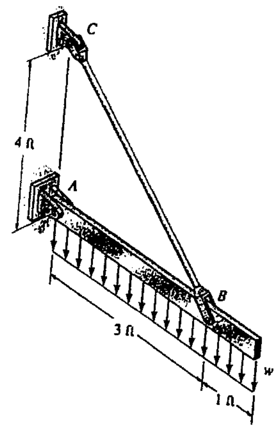
For pins B and C:

$$\tau = \frac{V}{A} = \frac{0.8333}{\frac{\pi}{4}(0.3^2)} = 11.79 \text{ ksi}$$

$$\text{F. S.} = \frac{\tau_y}{\tau} = \frac{18}{11.79} = 1.53 \qquad \textbf{Ans}$$



•1–104 If the allowable shear stress for each of the 0.3-in.-diameter steel pins at A, B, and C is $\tau_{allow} = 12.5$ ksi, and the allowable normal stress for the 0.40-in.-diameter rod is $\sigma_{allow} = 22$ ksi, determine the largest intensity w of the uniform distributed load that can be suspended from the beam.



Assume failure of pins B and C:

$$\tau_{allow} = 12.5 = \frac{1.667w}{\frac{\pi}{4}(0.3^2)}$$

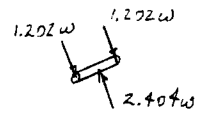
$$w = 0.530 \text{ kip/ft} \qquad \text{(controls)} \qquad \textbf{Ans}$$

Assume failure of pin A:

$$F_A = \sqrt{(2w)^2 + (1.333w)^2} = 2.404\,w$$

$$\tau_{allow} = 12.5 = \frac{1.202w}{\frac{\pi}{4}(0.3^2)}$$

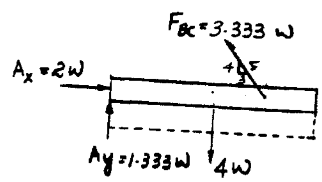
$$w = 0.735 \text{ kip/ft}$$



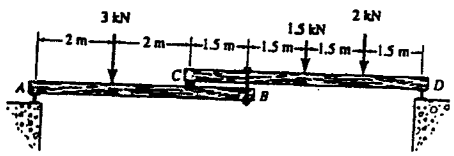
Assume failure of rod BC:

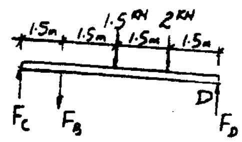
$$\sigma_{allow} = 22 = \frac{3.333w}{\frac{\pi}{4}(0.4^2)}$$

$$w = 0.829 \text{ kip/ft}$$

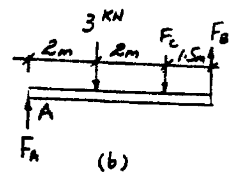


1-105. The compound wooden beam is connected together by a bolt at B. Assuming that the connections at A, B, C, and D exert only vertical forces on the beam, determine the required diameter of the bolt at B and the required outer diameter of its washers if the all wable tensile stress for the bolt is $(\sigma_t)_{\text{allow}} = 150$ MPa and the allowable bearing stress for the wood is $(\sigma_b)_{\text{allow}} = 28$ MPa. Assume that the hole in the washers has the same diameter as the bolt.





(a)



(b)

From FBD (a) :

$\curvearrowleft + \Sigma M_D = 0; \quad F_B(4.5) + 1.5(3) + 2(1.5) - F_C(6) = 0$

$$4.5\,F_B - 6\,F_C = -7.5 \qquad (1)$$

From FBD (b) :

$\curvearrowleft + \Sigma M_A = 0; \quad F_B(5.5) - F_C(4) - 3(2) = 0$

$$5.5\,F_B - 4\,F_C = 6 \qquad (2)$$

Solving Eqs. (1) and (2) yields

$$F_B = 4.40 \text{ kN}; \qquad F_C = 4.55 \text{ kN}$$

For bolt :

$$\sigma_{\text{allow}} = 150\,(10^6) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$$d_B = 0.00611 \text{ m}$$

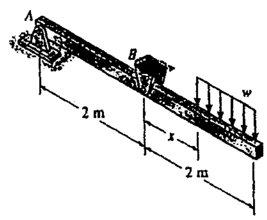
$$= 6.11 \text{ mm} \qquad \textbf{Ans}$$

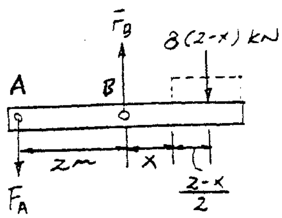
For washer :

$$\sigma_{\text{allow}} = 28\,(10^6) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_w^2 - 0.00611^2)}$$

$$d_w = 0.0154 \text{ m} = 15.4 \text{ mm} \qquad \textbf{Ans}$$

1–106 The bar is held in equilibrium by the pin supports at A and B. Note that the support at A has a single leaf and therefore it involves single shear in the pin, and the support at B has a double leaf and therefore it involves double shear. The allowable shear stress for both pins is $\tau_{allow} = 150$ MPa. If a uniform distributed load of $w = 8$ kN/m is placed on the bar, determine its minimum allowable position x from B. Pins A and B each have a diameter of 8 mm. Neglect any axial force in the bar.





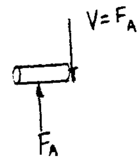
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(2) - 8(2 - x)(3 + \frac{x}{2}) = 0$$

$$2F_B - 48 + 16x + 4x^2 = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad F_A(2) - 8(2 - x)(\frac{x}{2} + 1) = 0$$

$$2F_A - 16 + 4x^2 = 0 \qquad (2)$$

Assume failure of pin A

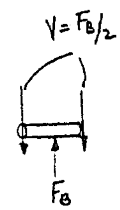


$$\tau_{allow} = \frac{F_A}{A_A}; \qquad 150(10^6) = \frac{F_A}{\frac{\pi}{4}(0.008)^2}$$

$$F_A = 7539.8 \text{ N} = 7.5398 \text{ kN}$$

Substitute $F_A = 7.5398$ kN into Eq. (2), $x = 0.480$ m

Assume failure of pin B



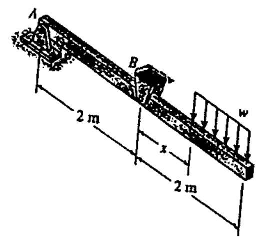
$$\tau_{allow} = \frac{\frac{F_B}{2}}{A_B}; \qquad 150(10^6) = \frac{\frac{F_B}{2}}{\frac{\pi}{4}(0.008)^2}$$

$$F_B = 15079.6 \text{ N} = 15.0796 \text{ kN}$$

Substitute $F_B = 15.0796$ kN into Eq. (1), $x = 0.909$ m

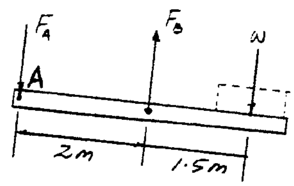
Choose the larger $x = 0.909$ m **Ans**

1–107 The bar is held in equilibrium by the pin supports at A and B. Note that the support at A has a single leaf and therefore it involves single shear in the pin, and the support at B has a double leaf and therefore it involves double shear. The allowable shear stress for both pins is $\tau_{allow} = 125$ MPa. If $x = 1$ m, determine the maximum distributed load w the bar will support. Pins A and B each have a diameter of 8 mm. Neglect any axial force in the bar.



$\curvearrowleft + \Sigma M_A = 0; \qquad F_B(2) - w(3.5) = 0 ; \quad F_B = 1.75w$

$+\uparrow \Sigma F_y = 0; \qquad 1.75w - w - F_A = 0 ; \quad F_A = 0.75w$



For pin A,

$$\tau_{allow} = \frac{F_A}{A_A}; \qquad 125(10^6) = \frac{0.75w}{\frac{\pi}{4}(0.008)^2}$$

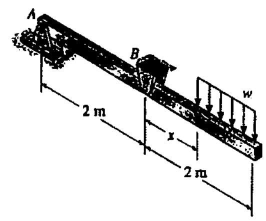
$w = 8377 \text{ N/m} = 8.38 \text{ kN/m}$

For pin B,

$$\tau_{allow} = \frac{\frac{F_B}{2}}{A_B}; \qquad 125(10^6) = \frac{\frac{1.75w}{2}}{\frac{\pi}{4}(0.008)^2}$$

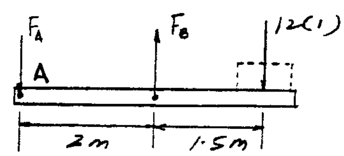
$w = 7181 \text{ N/m} = 7.18 \text{ kN/m}$ (controls) **Ans**

•1–108 The bar is held in equilibrium by the pin supports at A and B. Note that the support at A has a single leaf and therefore it involves single shear in the pin, and the support at B has a double leaf and therefore it involves double shear. The allowable shear stress for both pins is $\tau_{allow} = 125$ MPa. If $x = 1$ m and $w = 12$ kN/m, determine the smallest required diameter of pins A and B. Neglect any axial force in the bar.



$$\circlearrowleft + \Sigma M_A = 0; \quad F_B(2) - 12(3.5) = 0; \quad F_B = 21 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad 21 - 12 - F_A = 0; \quad F_A = 9 \text{ kN}$$



For pin A,

$$\tau_{allow} = \frac{F_A}{A_A}; \quad 125(10^6) = \frac{9(10^3)}{\frac{\pi}{4}(d_A)^2}$$

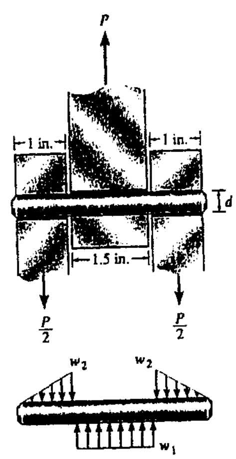
$d_A = 0.00957$ m $= 9.57$ mm **Ans**

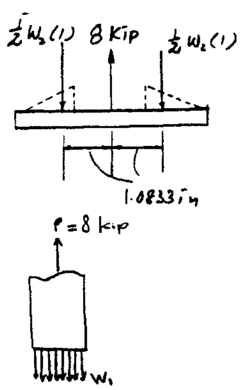
For pin B,

$$\tau_{allow} = \frac{\frac{F_B}{2}}{A_B}; \quad 125(10^6) = \frac{\frac{21(10^3)}{2}}{\frac{\pi}{4}(d_B)^2}$$

$d_B = 0.0103$ m $= 10.3$ mm **Ans**

1–109 The pin is subjected to double shear since it is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. Determine the diameter d of the pin if the allowable shear stress is $\tau_{allow} = 10$ ksi and the load $P = 8$ kip. Also, determine the load intensities w_1 and w_2.





Pin :

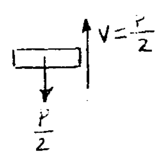
$+\uparrow \Sigma F_y = 0; \qquad 8 - 1.5\,w_1 = 0$

$w_1 = 5.33 \text{ kip / in.}$ **Ans**

Link :

$+\uparrow \Sigma F_y = 0; \qquad -2(\frac{1}{2}w_2)(1) + 8 = 0$

$w_2 = 8 \text{ kip / in.}$ **Ans**

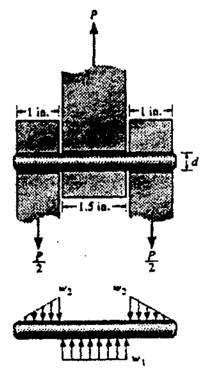


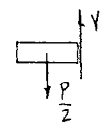
Shear stress

$$\tau_{allow} = \frac{\frac{P}{2}}{\frac{\pi}{4}(d)^2}; \qquad 10 = \frac{\frac{8}{2}}{\frac{\pi}{4}(d)^2}$$

$d = 0.714$ in. **Ans**

1-110. The pin is subjected to double shear since it is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. Determine the maximum load P the connection can support if the allowable shear stress for the material is $\tau_{\text{allow}} = 8$ ksi and the diameter of the pin is 0.5 in. Also, determine the load intensities w_1 and w_2.

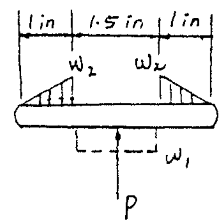




$$V = \frac{P}{2}$$

$$\tau_{\text{allow}} = \frac{\frac{P}{2}}{A}; \qquad 8\,(10^3) = \frac{\frac{P}{2}}{\frac{\pi}{4}(0.5)^2}$$

$$P = 3.1416 = 3.14 \text{ kip} \qquad \textbf{Ans}$$



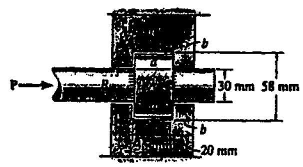
$$3.1416 \text{ kip} = w_1\,(1.5)$$

$$w_1 = 2.09 \text{ kip/in.} \qquad \textbf{Ans}$$

$$\frac{3.1416}{2} = \frac{1}{2}\,w_2\,(1)$$

$$w_2 = 3.14 \text{ kip/in.} \qquad \textbf{Ans}$$

1-111. The thrust bearing consists of a circular collar A fixed to the shaft B. Determine the maximum axial force P that can be applied to the shaft so that it does not cause the shear stress along a cylindrical surface a or b to exceed an allowable shear stress of $\tau_{\text{allow}} = 170$ MPa.



Assume failure along a:

$$\tau_{\text{allow}} = 170(10^6) = \frac{P}{\pi(0.03)(0.035)}$$

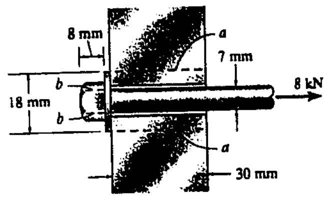
$P = 561$ kN (controls) **Ans**

Assume failure along b:

$$\tau_{\text{allow}} = 170(10^6) = \frac{P}{\pi(0.058)(0.02)}$$

$P = 620$ kN

•1–112 The long bolt passes through the 30-mm-thick plate. If the force in the bolt shank is 8 kN, determine the average normal stress in the shank, the average shear stress along the cylindrical area of the plate defined by the section lines $a - a$, and the average shear stress in the bolt head along the cylindrical area defined by the section lines $b - b$.

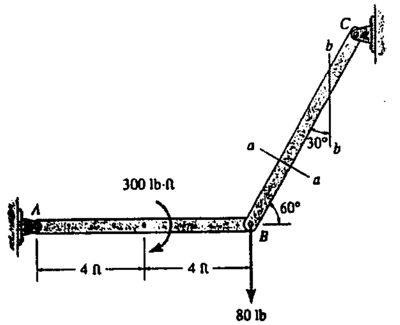


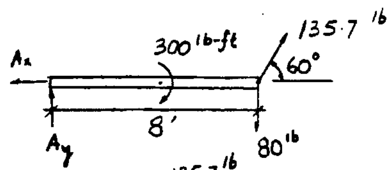
$$\sigma_s = \frac{P}{A} = \frac{8\,(10^3)}{\frac{\pi}{4}\,(0.007)^2} = 208\text{ MPa}$$ **Ans**

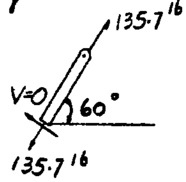
$$(\tau_{avg})_a = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.018)(0.030)} = 4.72\text{ MPa}$$ **Ans**

$$(\tau_{avg})_b = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.007)(0.008)} = 45.5\text{ MPa}$$ **Ans**

1–113 The two-member frame is subjected to the loading shown. Determine the average normal stress and the average shear stress acting at sections *a–a* and *b–b*. Member *CB* has a square cross section of 2 in. on each side.





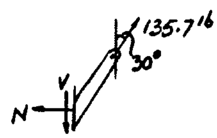


For *a - a* :

$$\sigma_{a-a} = \frac{P}{A} = \frac{135.7}{2\,(2)} = 33.9 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{a-a} = 0 \qquad \textbf{Ans}$$

For *b - b* :



$$\overset{+}{\leftarrow} \Sigma F_x = 0 \qquad N - 135.7 \sin 30^\circ = 0$$

$$N = 67.84 \text{ lb}$$

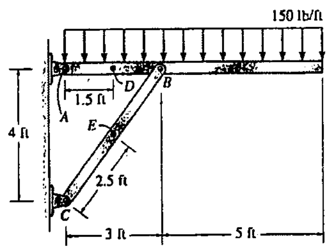
$$+\downarrow \Sigma F_y = 0 \qquad V - 135.7 \cos 30^\circ = 0$$

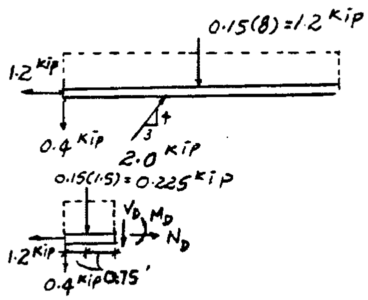
$$V = 117.5 \text{ lb}$$

$$\sigma_{b-b} = \frac{67.84}{(2)(\frac{2}{\sin 30^\circ})} = 8.48 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{b-b} = \frac{117.5}{(2)(\frac{2}{\sin 30^\circ})} = 14.7 \text{ psi} \qquad \textbf{Ans}$$

1–114 Determine the resultant internal loadings acting on the cross sections located through points D and E of the frame.





Segment AD :

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D - 1.2 = 0; \quad N_D = 1.20 \text{ kip}$ **Ans**

$+\downarrow \Sigma F_y = 0; \quad V_D + 0.225 + 0.4 = 0; \quad V_D = -0.625 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_D = 0; \quad M_D + 0.225(0.75) + 0.4(1.5) = 0$

$M_D = -0.769 \text{ kip} \cdot \text{ft}$ **Ans**

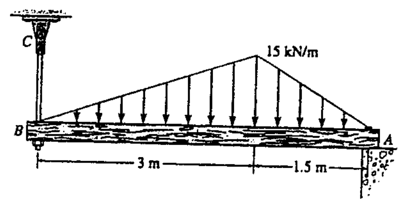
Segment CE :

$\nearrow + \Sigma F_x = 0; \quad N_E + 2.0 = 0; \quad N_E = -2.00 \text{ kip}$ **Ans**

$\nwarrow + \Sigma F_y = 0; \quad V_E = 0$ **Ans**

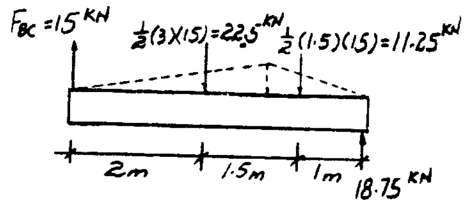
$\circlearrowleft + \Sigma M_E = 0; \quad M_E = 0$ **Ans**

1–115 The rod BC is made of steel having an allowable tensile stress of $\sigma_{allow} = 155$ MPa. Determine its smallest diameter so that it can support the load shown. The beam is assumed to be pin-connected at A.

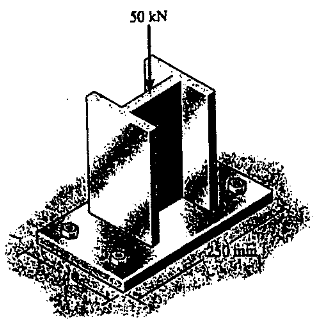


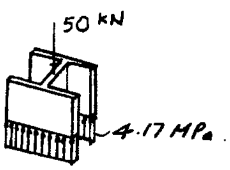
$$\sigma_{allow} = 155\,(10^6) = \frac{15\,(10^3)}{\frac{\pi}{4}(d_{BC})^2}$$

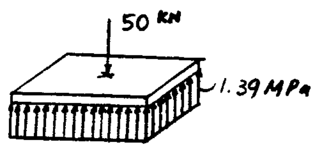
$d_{BC} = 0.0111$ m $= 11.1$ mm **Ans**



•1–116 The column has a cross-sectional area of $12(10^3)$ mm². It is subjected to an axial force of 50 kN. If the base plate to which the column is attached has a length of 250 mm, determine its width d so that the average bearing stress under the plate at the ground is one-third of the average compressive stress in the column. Sketch the stress distributions acting over the column's cross-sectional area and at the bottom of the base plate.







$$\sigma_c = \frac{P}{A} = \frac{50\,(10^3)}{\frac{12\,(10^3)}{(1000)^2}} = 4.167\text{ MPa}$$

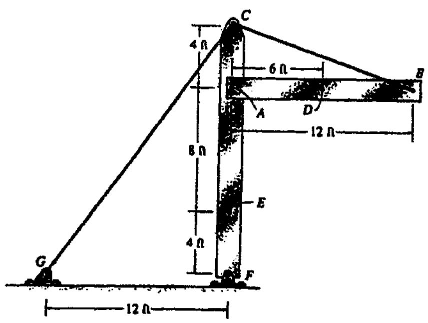
$$\frac{1}{3}\sigma_c = \sigma_b$$

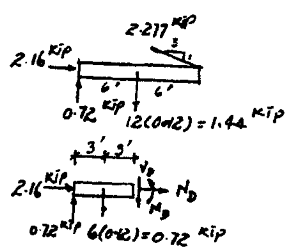
$$\frac{4.167\,(10^6)}{3} = \frac{50\,(10^3)}{(0.25)\,d}$$

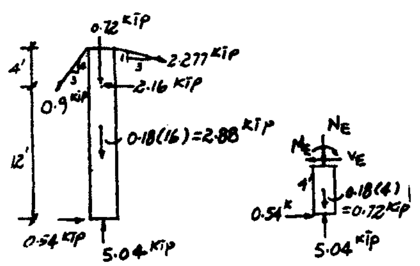
$$d = 0.144\text{ m} = 144\text{ mm} \qquad \textbf{Ans}$$

$$\sigma_b = \frac{1}{3}(4.167) = 1.39\text{ Mpa}$$

1-117. The beam AB is pin supported at A and supported by a cable BC. A *separate* cable CG is used to hold up the frame. If AB weighs 120 lb/ft and the column FC has a weight of 180 lb/ft, determine the resultant internal loadings acting on cross sections located at points D and E. Neglect the thickness of both the beam and column in the calculation.







Segment BD :

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D + 2.16 = 0; \quad N_D = -2.16 \text{ kip}$ **Ans**

$+\downarrow \Sigma F_y = 0; \quad V_D + 0.72 - 0.72 = 0; \quad V_D = 0$ **Ans**

$\circlearrowleft + \Sigma M_D = 0; \quad M_D - 0.72(3) = 0; \quad M_D = 2.16 \text{ kip}\cdot\text{ft}$ **Ans**

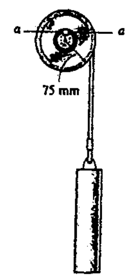
Segment FE :

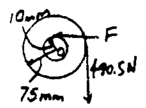
$\xleftarrow{+} \Sigma F_x = 0; \quad V_E - 0.54 = 0; \quad V_E = 0.540 \text{ kip}$ **Ans**

$+\downarrow \Sigma F_y = 0; \quad N_E + 0.72 - 5.04 = 0; \quad N_E = 4.32 \text{ kip}$ **Ans**

$\circlearrowleft + \Sigma M_E = 0; \quad -M_E + 0.54(4) = 0; \quad M_E = 2.16 \text{ kip}\cdot\text{ft}$ **Ans**

1-118. The pulley is held fixed to the 20-mm-diameter shaft using a key that fits within a groove cut into the pulley and shaft. If the suspended load has a mass of 50 kg, determine the average shear stress in the key along section *a–a*. The key is 5 mm by 5 mm square and 12 mm long.

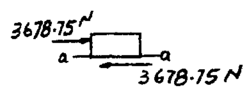




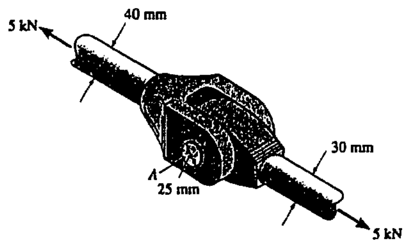
$\curvearrowleft + \Sigma M_O = 0; \quad F(10) - 490.5(75) = 0$

$F = 3678.75 \text{ N}$

$$\tau_{avg} = \frac{V}{A} = \frac{3678.75}{(0.005)(0.012)} = 61.3 \text{ MPa} \quad \textbf{Ans}$$



1–119 The yoke-and-rod connection is subjected to a tensile force of 5 kN. Determine the average normal stress in each rod and the average shear stress in the pin A between the members.

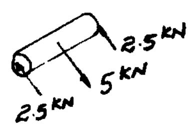


For the 40 – mm – dia. rod :

$$\sigma_{40} = \frac{P}{A} = \frac{5\,(10^3)}{\frac{\pi}{4}\,(0.04)^2} = 3.98\ \text{MPa} \qquad \textbf{Ans}$$

For the 30 – mm – dia. rod :

$$\sigma_{30} = \frac{V}{A} = \frac{5\,(10^3)}{\frac{\pi}{4}\,(0.03)^2} = 7.07\ \text{MPa} \qquad \textbf{Ans}$$



Average shear stress for pin A :

$$\tau_{avg} = \frac{P}{A} = \frac{2.5\,(10^3)}{\frac{\pi}{4}\,(0.025)^2} = 5.09\ \text{MPa} \qquad \textbf{Ans}$$

2–1 An air filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball's diameter becomes 7 in., determine the average normal strain in the rubber.

$d_0 = 6 \text{ in.}$

$d = 7 \text{ in.}$

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.} \qquad \textbf{Ans}$$

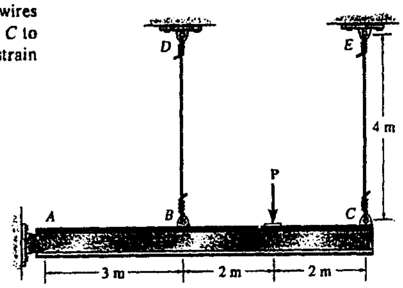
2-2 A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

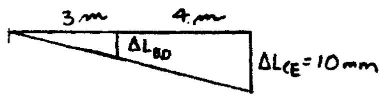
$$L_0 = 15 \text{ in.}$$

$$L = \pi(5 \text{ in.})$$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.} \qquad \textbf{Ans}$$

2–3 The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.





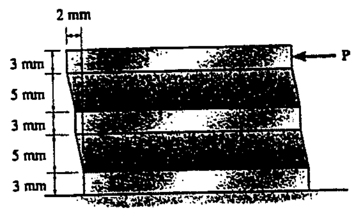
$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286\text{ mm}$$

$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250\text{ mm/mm}$$ **Ans**

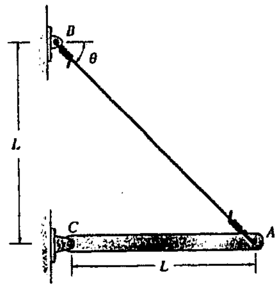
$$\varepsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107\text{ mm/mm}$$ **Ans**

•2–4 Nylon strips are fused to glass plates. When moderately heated the nylon will become soft while the glass stays approximately rigid. Determine the average shear strain in the nylon due to the load **P** when the assembly deforms as indicated.



$$\gamma = \tan^{-1}\left(\frac{2}{10}\right) = 11.31° = 0.197 \text{ rad} \qquad \textbf{Ans}$$

2-5 The wire AB is unstretched when $\theta = 45°$. If a load is applied to the bar AC, which causes $\theta = 47°$, determine the normal strain in the wire.



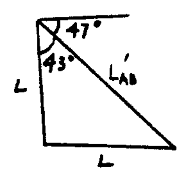
$$L^2 = L^2 + L'^2_{AB} - 2LL'_{AB}\cos 43°$$

$$L'_{AB} = 2L\cos 43°$$

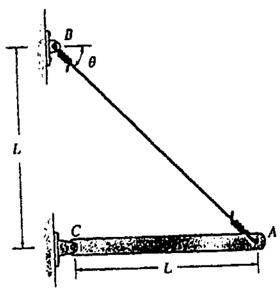
$$\varepsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{2L\cos 43° - \sqrt{2}\,L}{\sqrt{2}\,L}$$

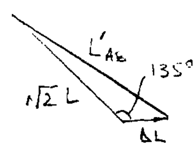
$$= 0.0343 \quad \textbf{Ans}$$



2–6 If a load applied to bar AC causes point A to be displaced to the right by an amount ΔL, determine the normal strain in wire AB. Originally, $\theta = 45°$.



$$L'_{AB} = \sqrt{\left(\sqrt{2}L\right)^2 + \Delta L^2 - 2(\sqrt{2}\,L)(\Delta L)\cos 135°}$$



$$= \sqrt{2L^2 + \Delta L^2 + 2L\Delta L}$$

$$\varepsilon_{AB} = \frac{L'_{AB} - L_{AB}}{L_{AB}}$$

$$= \frac{\sqrt{2L^2 + \Delta L^2 + 2L\Delta L} - \sqrt{2}\,L}{\sqrt{2}\,L}$$

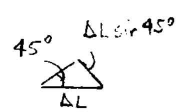
$$= \sqrt{1 + \frac{\Delta L^2}{2L^2} + \frac{\Delta L}{L}} - 1$$

Neglecting the higher – order terms,

$$\varepsilon_{AB} = (1 + \frac{\Delta L}{L})^{\frac{1}{2}} - 1$$

$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots\ldots - 1 \qquad \text{(binomial theorem)}$$

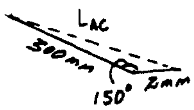
$$= \frac{0.5\Delta L}{L} \qquad \textbf{Ans}$$

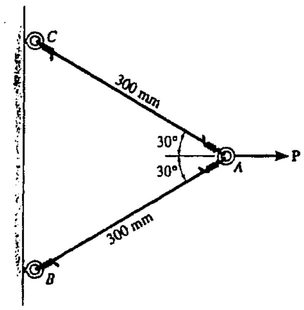


Also,

$$\varepsilon_{AB} = \frac{\Delta L \sin 45°}{\sqrt{2}\,L} = \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

2–7 The two wires are connected together at A. If the force **P** causes point A to be displaced horizontally 2 mm, determine the normal strain developed in each wire.

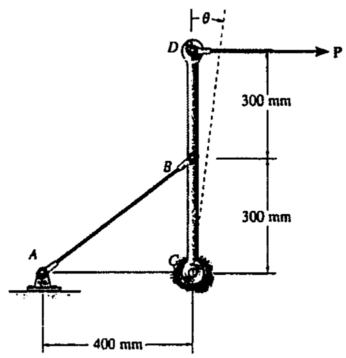




$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150^\circ} = 301.734\text{ mm}$$

$$\varepsilon_{AC} = \varepsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578\text{ mm/mm} \qquad \textbf{Ans}$$

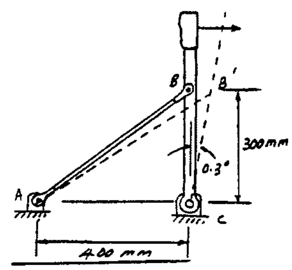
•2–8 Part of a control linkage for an airplane consists of a rigid member CBD and a flexible cable AB. If a force is applied to the end D of the member and causes it to rotate by $\theta = 0.3°$, determine the normal strain in the cable. Originally the cable is unstretched.



$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3°}$$

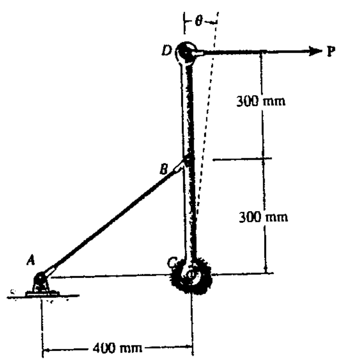
$$= 501.255 \text{ mm}$$

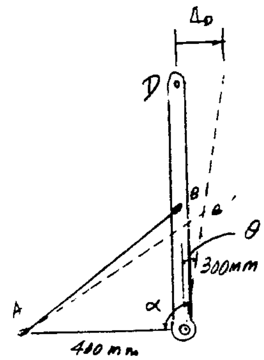


$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm} \qquad \textbf{Ans}$$

2–9 Part of a control linkage for an airplane consists of a rigid member *CBD* and a flexible cable *AB*. If a force is applied to the end *D* of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point *D*. Originally the cable is unstretched.





$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$AB' = AB + \varepsilon_{AB}AB$$
$$= 500 + 0.0035(500) = 501.75 \text{ mm}$$

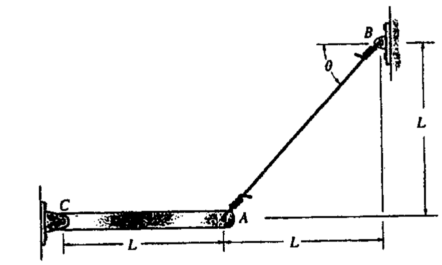
$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

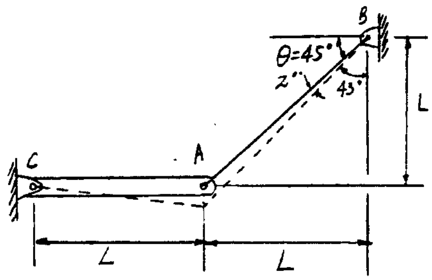
$$\alpha = 90.4185°$$

$$\theta = 90.4185° - 90° = 0.4185° = \frac{\pi}{180°}(0.4185) \text{ rad}$$

$$\Delta_D = 600(\theta) = 600(\frac{\pi}{180°})(0.4185) = 4.38 \text{ mm}$$ **Ans**

2–10 The wire AB is unstretched when $\theta = 45°$. If a vertical load is applied to bar AC, which causes $\theta = 47°$, determine the normal strain in the wire.





$$AB = \sqrt{L^2 + L^2} = \sqrt{2}\,L$$

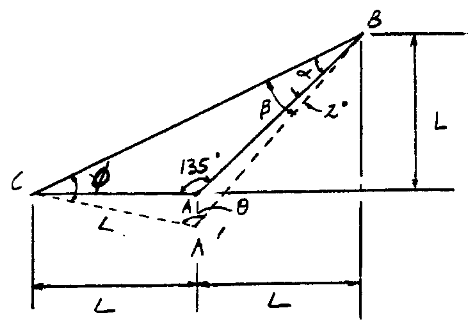
$$CB = \sqrt{(2L)^2 + L^2} = \sqrt{5}\,L$$

From triangle ABC,

$$\frac{\sin\alpha}{L} = \frac{\sin 135°}{\sqrt{5}\,L}$$

$$\alpha = 18.435°$$

$$\beta = 18.435° + 2° = 20.435°$$



From triangle $A'BC$,

$$\frac{\sin\theta}{\sqrt{5}\,L} = \frac{\sin 20.435°}{L}$$

$$\theta = 128.674°$$

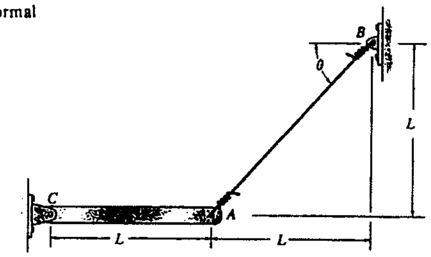
$$\phi = 180° - 128.674° - 20.435° = 30.891°$$

$$\frac{A'B}{\sin 30.891°} = \frac{L}{\sin 20.435°}$$

$$A'B = 1.47047L$$

$$\varepsilon_{AB} = \frac{A'B - AB}{AB} = \frac{1.47047L - \sqrt{2}\,L}{\sqrt{2}\,L} = 0.0398 \qquad \textbf{Ans}$$

2–11 If a load applied to bar AC causes point A to be displaced to the left by an amount ΔL, determine the normal strain in wire AB. Originally, $\theta = 45°$.

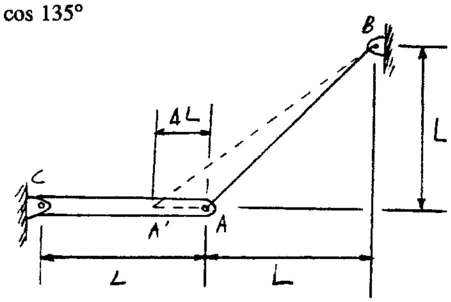


$$AB = \sqrt{L^2 + L^2} = \sqrt{2}\,L$$

From triangle $A'AB$,

$$A'B = \sqrt{\Delta L^2 + (\sqrt{2}L)^2 - 2(\Delta L)\sqrt{2}L \cos 135°}$$

$$= \sqrt{\Delta L^2 + 2L^2 + 2L\,\Delta L}$$



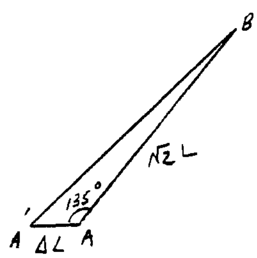
$$\varepsilon_{AB} = \frac{A'B - AB}{AB}$$

$$= \frac{\sqrt{\Delta L^2 + 2L^2 + 2L\,\Delta L} - \sqrt{2}\,L}{\sqrt{2}\,L}$$

$$= \sqrt{\frac{\Delta L^2}{2L^2} + 1 + \frac{\Delta L}{L}} - 1$$

Neglecting the higher order terms,

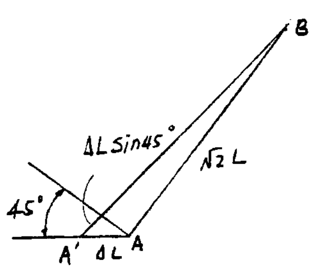
$$\varepsilon_{AB} = (1 + \frac{\Delta L}{L})^{\frac{1}{2}} - 1$$



$$= 1 + \frac{1}{2}\frac{\Delta L}{L} + \ldots - 1 \quad \text{(Binomial theorem)}$$

$$= \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

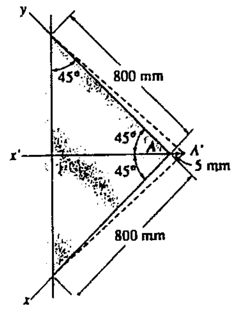
Also,



$$\varepsilon_{AB} = \frac{\Delta L \ \sin\ 45°}{\sqrt{2}\,L}$$

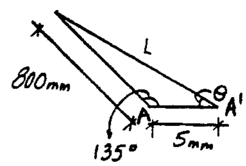
$$= \frac{0.5\,\Delta L}{L} \qquad \textbf{Ans}$$

*2–12 The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the shear strain γ_{xy} at A.



$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54\text{ mm}$$

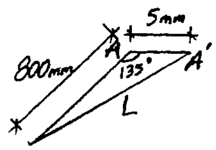
$$\frac{\sin 135^\circ}{803.54} = \frac{\sin \theta}{800}; \qquad \theta = 44.75^\circ = 0.7810\text{ rad}$$

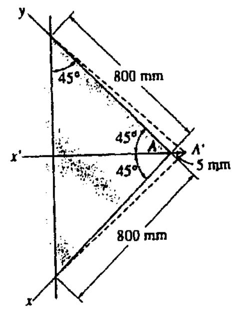


$$\gamma_{xy} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2(0.7810)$$

$$= 0.00880\text{ rad} \qquad \textbf{Ans}$$

2–13 The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain ϵ_x along the x axis.

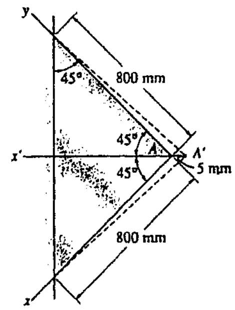


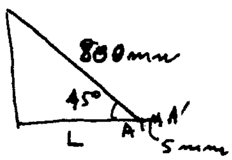


$$L = \sqrt{800^2 + 5^2 - 2(800)(5)\cos 135^\circ} = 803.54 \text{ mm}$$

$$\varepsilon_x = \frac{803.54 - 800}{800} = 0.00443 \text{ mm/mm} \qquad \textbf{Ans}$$

2–14 The triangular plate is fixed at its base, and its apex A is given a horizontal displacement of 5 mm. Determine the average normal strain $\epsilon_{x'}$ along the x' axis.

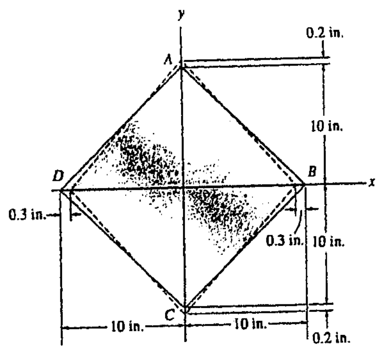




$$L = 800 \cos 45° = 565.69 \text{ mm}$$

$$\varepsilon_{x'} = \frac{5}{565.69} = 0.00884 \text{ mm/mm} \qquad \textbf{Ans}$$

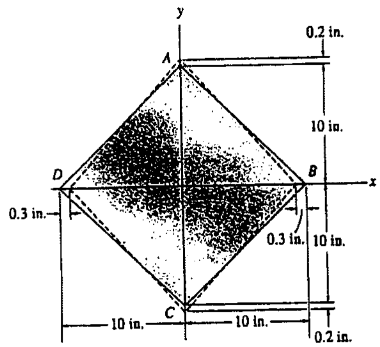
2–15 The corners of the square plate are given the displacements indicated. Determine the average normal strains ϵ_x and ϵ_y along the x and y axes.



$$\varepsilon_x = \frac{-0.3}{10} = -0.03 \text{ in./in.} \qquad \textbf{Ans}$$

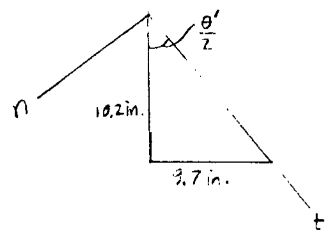
$$\varepsilon_y = \frac{0.2}{10} = 0.02 \text{ in./in.} \qquad \textbf{Ans}$$

•2–16 The corners of the square plate are given the displacements indicated. Determine the shear strain along the edges of the plate at *A* and *B*.



At *A* :

$$\frac{\theta'}{2} = \tan^{-1}\left(\frac{9.7}{10.2}\right) = 43.561°$$



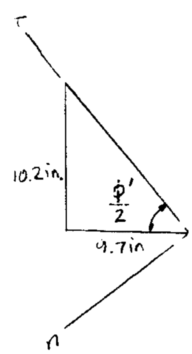
$$\theta' = 1.52056 \text{ rad}$$

$$(\gamma_A)_{nt} = \frac{\pi}{2} - 1.52056$$

$$= 0.0502 \text{ rad} \qquad \textbf{Ans}$$

At *B* :

$$\frac{\phi'}{2} = \tan^{-1}\left(\frac{10.2}{9.7}\right) = 46.439°$$

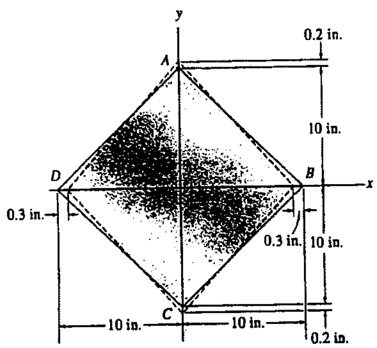


$$\phi' = 1.62104 \text{ rad}$$

$$(\gamma_B)_{nt} = \frac{\pi}{2} - 1.62104$$

$$= -0.0502 \text{ rad} \qquad \textbf{Ans}$$

2-17 The corners of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonals AC and DB.

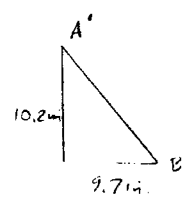


For AB :

$$A'B' = \sqrt{(10.2)^2 + (9.7)^2} = 14.0759 \text{ in.}$$

$$AB = \sqrt{(10)^2 + (10)^2} = 14.14214 \text{ in.}$$

$$\varepsilon_{AB} = \frac{14.0759 - 14.14214}{14.14214} = -0.00469 \text{ in./in.} \qquad \textbf{Ans}$$



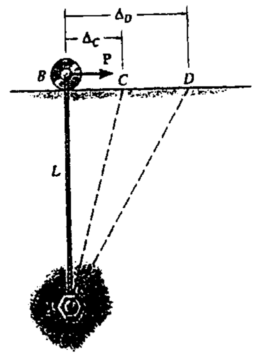
For AC :

$$\varepsilon_{AC} = \frac{20.4 - 20}{20} = 0.0200 \text{ in./in.} \qquad \textbf{Ans}$$

For DB :

$$\varepsilon_{DB} = \frac{19.4 - 20}{20} = -0.0300 \text{ in./in.} \qquad \textbf{Ans}$$

2-18 The nylon cord has an original length L and is tied to a fixed bolt at A and a roller at B. If a force **P** is applied to the roller, determine the normal strain in the cord when the roller is at C, ϵ_C, and D, ϵ_D. If the cord was originally unstrained when it was at C, determine the normal strain ϵ_{CD} when the roller moves to D. Show that if the displacements Δ_C and Δ_D are small, then $\epsilon_{CD} = \epsilon_D - \epsilon_C$.



$$L_C = \sqrt{L^2 + \Delta_C^2}$$

$$\varepsilon_C = \frac{\sqrt{L^2 + \Delta_C^2} - L}{L}$$

$$= \frac{L\sqrt{1 + (\frac{\Delta_C^2}{L^2})} - L}{L} = \sqrt{1 + (\frac{\Delta_C^2}{L^2})} - 1$$

For small Δ_C,

$$\varepsilon_C = 1 + \frac{1}{2}(\frac{\Delta_C^2}{L^2}) - 1 = \frac{1}{2}\frac{\Delta_C^2}{L^2} \qquad \textbf{Ans}$$

In the same manner,

$$\varepsilon_D = \frac{1}{2}\frac{\Delta_D^2}{L^2} \qquad \textbf{Ans}$$

$$\varepsilon_{CD} = \frac{\sqrt{L^2 + \Delta_D^2} - \sqrt{L^2 + \Delta_C^2}}{\sqrt{L^2 + \Delta_C^2}} = \frac{\sqrt{1 + \frac{\Delta_D^2}{L^2}} - \sqrt{1 + \frac{\Delta_C^2}{L^2}}}{\sqrt{1 + \frac{\Delta_C^2}{L^2}}}$$

For small Δ_C and Δ_D,

$$\varepsilon_{CD} = \frac{(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2}) - (1 + \frac{1}{2}\frac{\Delta_D^2}{L^2})}{(1 + \frac{1}{2}\frac{\Delta_C^2}{L^2})} = \frac{\frac{1}{2L^2}(\Delta_C^2 - \Delta_D^2)}{\frac{1}{2L^2}(2L^2 + \Delta_C^2)}$$

$$\varepsilon_{CD} = \frac{\Delta_C^2 - \Delta_D^2}{2L^2 - \Delta_C^2} = \frac{1}{2L^2}(\Delta_C^2 - \Delta_D^2) = \varepsilon_C - \varepsilon_D \qquad \textbf{QED}$$

Also this problem can be solved as follows :

$$A_C = L \sec\theta_C ; \quad A_D = L \sec\theta_D$$

$$\varepsilon_C = \frac{L\sec\theta_C - L}{L} = \sec\theta_C - 1$$

$$\varepsilon_D = \frac{L\sec\theta_D - L}{L} = \sec\theta_D - 1$$

Expanding $\sec\theta$

$$\sec\theta = 1 + \frac{\theta^2}{2!} + \frac{5\theta^4}{4!} \ldots\ldots$$

For small θ neglect the higher order terms

$$\sec\theta = 1 + \frac{\theta^2}{2}$$

Hence,

$$\varepsilon_C = 1 + \frac{\theta_C^2}{2} - 1 = \frac{\theta_C^2}{2}$$

$$\varepsilon_D = 1 + \frac{\theta_D^2}{2} - 1 = \frac{\theta_D^2}{2}$$

$$\varepsilon_{CD} = \frac{L\sec\theta_D - L\sec\theta_C}{L\sec\theta_C} = \frac{\sec\theta_D}{\sec\theta_C} - 1 = \sec\theta_D\cos\theta_C - 1$$

Since $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \ldots\ldots$

$$\sec\theta_D\cos\theta_C = (1 + \frac{\theta_D^2}{2}\ldots\ldots)(1 - \frac{\theta_C^2}{2}\ldots\ldots)$$

$$= 1 - \frac{\theta_C^2}{2} + \frac{\theta_D^2}{2} - \frac{\theta_C^2\theta_D^2}{4}$$

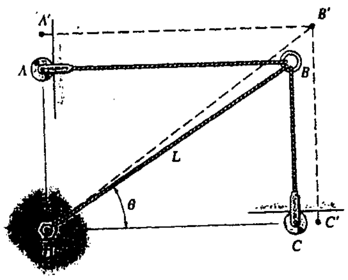
Neglecting the higher order terms

$$\sec\theta_D\cos\theta_C = 1 + \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$\varepsilon_{CD} = [1 + \frac{\theta_2^2}{2} - \frac{\theta_1^2}{2}] - 1 = \frac{\theta_D^2}{2} - \frac{\theta_C^2}{2}$$

$$= \varepsilon_D - \varepsilon_C \qquad \textbf{QED}$$

2–19 The three cords are attached to the ring at B. When a force is applied to the ring it moves it to point B', such that the normal strain in AB is ϵ_{AB} and the normal strain in CB is ϵ_{CB}. Provided these strains are small, determine the normal strain in DB. Note that AB and CB remain horizontal and vertical, respectively, due to the roller guides at A and C.



Coordinates of B $(L\cos\theta, L\sin\theta)$

Coordinates of B' $(L\cos\theta + \varepsilon_{AB} L\cos\theta, L\sin\theta + \varepsilon_{CB} L\sin\theta)$

$$L_{DB'} = \sqrt{(L\cos\theta + \varepsilon_{AB} L\cos\theta)^2 + (L\sin\theta + \varepsilon_{CB} L\sin\theta)^2}$$

$$L_{DB'} = L\sqrt{\cos^2\theta(1 + 2\varepsilon_{AB} + \varepsilon_{AB}^2) + \sin^2\theta(1 + 2\varepsilon_{CB} + \varepsilon_{CB}^2)}$$

Since ε_{AB} and ε_{CB} are small,

$$L_{DB'} = L\sqrt{1 + (2\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)}$$

Use the binomial theorem,

$$L_{DB'} = L\left(1 + \frac{1}{2}(2\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)\right)$$

$$= L(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta)$$

Thus, $$\varepsilon_{DB} = \frac{L(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta) - L}{L}$$

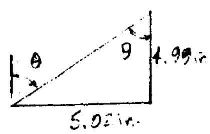
$$\varepsilon_{DB} = \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta$$ **Ans**

*2–20 The rectangular plate is subjected to the deformation shown by the dashed lines. Determine the shear strains γ_{xy} and $\gamma_{x'y'}$ developed at point A.

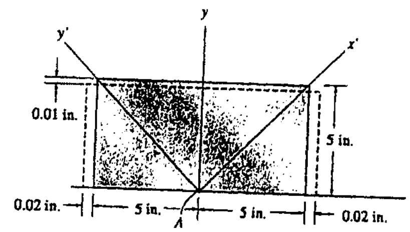
Since the right angle of an element along the x, y axes does not distort, then

$\gamma_{xy} = 0$ **Ans**

$$\tan\theta = \frac{5.02}{4.99}$$



$\theta = 45.17^\circ = 0.7884$ rad

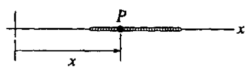


$$\gamma_{x'y'} = \frac{\pi}{2} - 2\theta$$

$$= \frac{\pi}{2} - 2\,(0.7884)$$

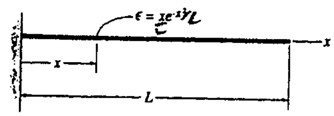
$$= -\ 0.00599 \text{ rad}$$ **Ans**

2-21 A thin wire, lying along the x axis, is strained such that each point on the wire is displaced $\Delta x = kx^2$ along the x axis. If k is constant, what is the normal strain at any point P along the wire?



$$\varepsilon = \frac{d(\Delta x)}{dx} = 2kx \qquad \textbf{Ans}$$

2–22 The wire is subjected to a normal strain that is defined by $\epsilon = xe^{-x^2/L}$, where x is in millimeters. If the wire has an initial length L, determine the increase in its length.

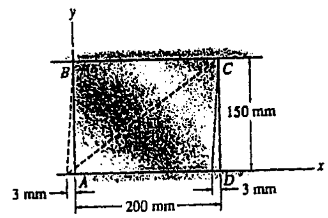


$$\Delta L = \frac{1}{L}\int_0^L xe^{-(x/L)^2}\,dx$$

$$= -L\left[\frac{e^{-(x/L)^2}}{2}\right]_0^L = \frac{L}{2}\left[1-(1/e)\right]$$

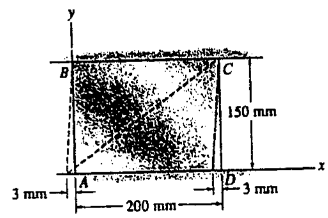
$$= \frac{L}{2e}\left[e-1\right] \qquad \textbf{Ans.}$$

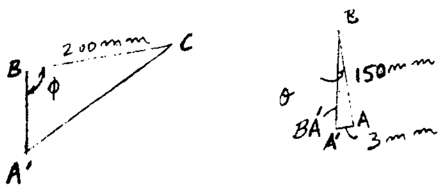
2–23 The rectangular plate is subjected to the deformation shown by the dashed lines. Determine the average shear strain γ_{xy} of the plate.



$$\gamma_{xy} \approx \tan \gamma_{xy} = \frac{3}{150} = 0.02 \text{ rad} \qquad \textbf{Ans}$$

•2–24 The rectangular plate is subjected to the deformation shown by the dashed lines. Determine the average normal strains along the diagonal AC and side AB.





For AC :

$$\theta = \tan^{-1}\left(\frac{3}{150}\right)$$

$$\theta = 1.1458°$$

$$\phi = 90° + 1.1458° = 91.1458°$$

$$BA' = \sqrt{(150)^2 + (3)^2} = 150.0300\text{ mm}$$

$$A'C' = \sqrt{(150.0300)^2 + (200)^2 - 2(150.0300)(200)\cos 91.1458°}$$

$$A'C' = 252.4064\text{ mm}$$

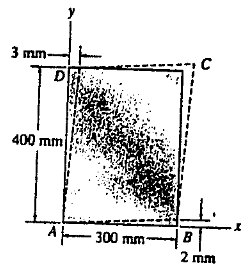
$$AC = \sqrt{(200)^2 + (150)^2} = 250\text{ mm}$$

$$\varepsilon_{AC} = \frac{252.4064 - 250}{250} = 0.00963\text{ mm/mm} \qquad \textbf{Ans}$$

For AB :

$$\varepsilon_{AB} = \frac{150.0300 - 150}{150} = 0.000200\text{ mm/mm} \qquad \textbf{Ans}$$

2–25 The piece of rubber is originally rectangular. Determine the average shear strain γ_{xy} if the corners B and D are subjected to the displacements that cause the rubber to distort as shown by the dashed lines.

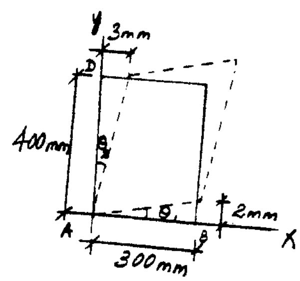


$$\theta_1 = \tan\theta_1 = \frac{2}{300} = 0.006667 \text{ rad}$$

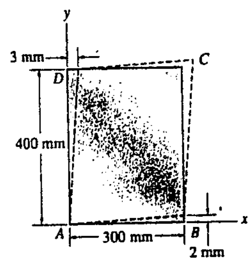
$$\theta_2 = \tan\theta_2 = \frac{3}{400} = 0.0075 \text{ rad}$$

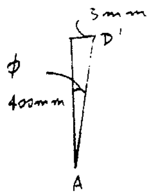
$$\gamma_{xy} = \theta_1 + \theta_2$$

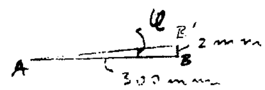
$$= 0.006667 + 0.0075 = 0.0142 \text{ rad} \qquad \textbf{Ans}$$



2–26 The piece of rubber is originally rectangular and subjected to the deformation shown by the dashed lines. Determine the average normal strain along the diagonal *DB* and side *AD*.

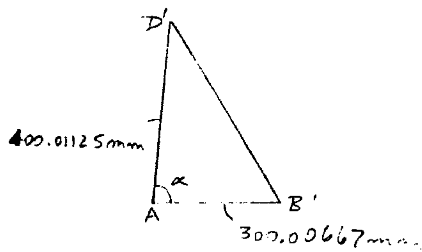






$$AD' = \sqrt{(400)^2 + (3)^2} = 400.01125 \text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{3}{400}\right) = 0.42971^\circ$$



$$AB' = \sqrt{(300)^2 + (2)^2} = 300.00667$$

$$\varphi = \tan^{-1}\left(\frac{2}{300}\right) = 0.381966^\circ$$

$$\alpha = 90^\circ - 0.42971^\circ - 0.381966^\circ = 89.18832^\circ$$

$$D'B' = \sqrt{(400.01125)^2 + (300.00667)^2 - 2(400.01125)(300.00667)\cos(89.18832^\circ)}$$

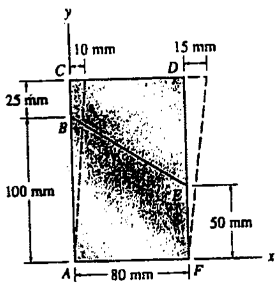
$$D'B' = 496.6014 \text{ mm}$$

$$DB = \sqrt{(300)^2 + (400)^2} = 500 \text{ mm}$$

$$\varepsilon_{DB} = \frac{496.6014 - 500}{500} = -0.00680 \text{ mm/mm}$$ **Ans**

$$\varepsilon_{AD} = \frac{400.01125 - 400}{400} = 0.0281(10^{-3}) \text{ mm/mm}$$ **Ans**

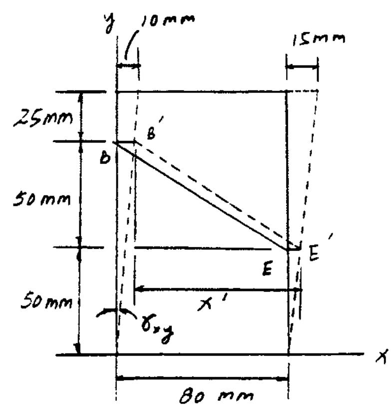
2-27 The material distorts into the dashed position shown. Determine (a) the average normal strains ϵ_x, ϵ_y and the shear strain γ_{xy}, and (b) the average normal strain along line BE.



Since there is no deformation occuring along the y and x axes,

$\varepsilon_x = 0$ **Ans.**

$\varepsilon_y = \dfrac{\sqrt{(125)^2 + (10)^2} - 125}{125} = 0.00319$ **Ans.**



$\tan \gamma_{xy} = \dfrac{10}{125}$

$\gamma_{xy} = 0.0798$ rad **Ans**

From geometry :

$\dfrac{BB'}{100} = \dfrac{10}{125}; \quad BB' = 8$ mm

$\dfrac{EE'}{50} = \dfrac{15}{125}; \quad EE' = 6$ mm

$BE = \sqrt{50^2 + 80^2} = \sqrt{8900}$ mm

$x' = 80 + EE' - BB' = 80 + 6 - 8 = 78$ mm

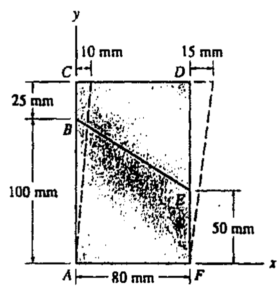
$B'E' = \sqrt{50^2 + 78^2} = \sqrt{8584}$ mm

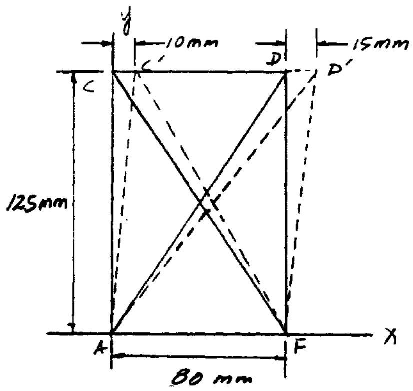
$\varepsilon_{BE} = \dfrac{B'E' - BE}{BE} = \dfrac{\sqrt{8584} - \sqrt{8900}}{\sqrt{8900}}$

$= -0.0179$ mm/mm **Ans**

Negative sign indicates shortening of BE.

*2–28 The material distorts into the dashed position shown. Determine the average normal strain that occurs along the diagonals *AD* and *CF*.





$AD = CF = \sqrt{(80)^2 + (125)^2} = \sqrt{22025}$ mm

$C'F = \sqrt{(70)^2 + (125)^2} = \sqrt{20525}$ mm

$AD' = \sqrt{(95)^2 + (125)^2} = \sqrt{24650}$ mm

$$\varepsilon_{AD} = \frac{AD' - AD}{AD}$$

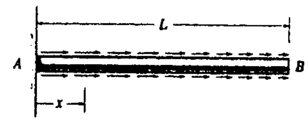
$$= \frac{\sqrt{24650} - \sqrt{22025}}{\sqrt{22025}}$$

$= 0.0579$ mm/mm **Ans**

$$\varepsilon_{CF} = \frac{C'F - CF}{CF} = \frac{\sqrt{20525} - \sqrt{22025}}{\sqrt{22025}}$$

$= -\ 0.0347$ mm/mm **Ans**

2–29 The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = kx^2$, where k is a constant. Determine the displacement of the end B. Also, what is the average normal strain in the rod?

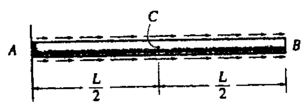


$$\frac{d(\Delta x)}{dx} = \varepsilon_x = k x^2$$

$$(\Delta x)_B = \int_0^L k x^2 = \frac{k L^3}{3}$$ **Ans**

$$(\varepsilon_x)_{avg} = \frac{(\Delta x)_B}{L} = \frac{\frac{kL^3}{3}}{L} = \frac{k L^2}{3}$$ **Ans**

2–30 The nonuniform loading causes a normal strain in the shaft that can be expressed as $\epsilon_x = k \sin\left(\frac{\pi}{L}x\right)$, where k is a constant. Determine the displacement of the center C and the average normal strain in the entire rod.



$$\varepsilon_x = k\sin\left(\frac{\pi}{L}x\right)$$

$$(\Delta x)_C = \int_0^{L/2} \varepsilon_x\,dx = \int_0^{L/2} k\sin\left(\frac{\pi}{L}x\right)dx$$

$$= -k\left(\frac{L}{\pi}\right)\cos\left(\frac{\pi}{L}x\right)\Big|_0^{L/2} = -k\left(\frac{L}{\pi}\right)\left(\cos\frac{\pi}{2} - \cos 0\right)$$

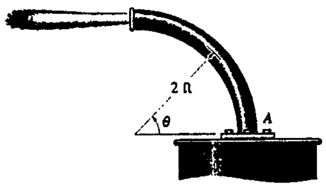
$$= \frac{kL}{\pi} \qquad \textbf{Ans}$$

$$(\Delta x)_B = \int_0^{L} k\sin\left(\frac{\pi}{L}x\right)dx$$

$$= -k\left(\frac{L}{\pi}\right)\cos\left(\frac{\pi}{L}x\right)\Big|_0^{L} = -k\left(\frac{L}{\pi}\right)(\cos\pi - \cos 0) = \frac{2kL}{\pi}$$

$$\varepsilon_{avg} = \frac{(\Delta x)_B}{L} = \frac{2k}{\pi} \qquad \textbf{Ans}$$

2–31 The curved pipe has an original radius of 2 ft. If it is heated nonuniformly, so that the normal strain along its length is $\epsilon = 0.05 \cos \theta$, determine the increase in length of the pipe.



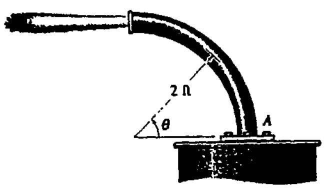
$$\varepsilon = 0.05 \cos\theta$$

$$\Delta L = \int \varepsilon \, dL$$

$$= \int_0^{90^\circ} (0.05 \cos \theta)(2 \, d\theta)$$

$$= 0.1 \int_0^{90^\circ} \cos \theta \, d\theta = 0.1[\sin \theta] \Big|_0^{90^\circ}] = 0.10 \text{ ft} \qquad \textbf{Ans}$$

•2–32 Solve Prob. 2–31 if $\epsilon = 0.08 \sin \theta$.



$$dL = 2\,d\theta$$

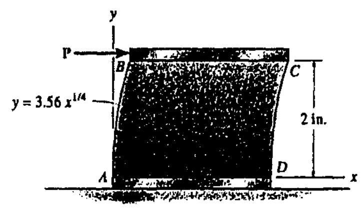
$$\varepsilon = 0.08 \sin \theta$$

$$\Delta L = \int \varepsilon\, dL$$

$$= \int_0^{90^\circ} (0.08 \sin \theta)(2\,d\theta)$$

$$= 0.16 \int_0^{90^\circ} \sin \theta\, d\theta = 0.16[-\cos \theta] \Big|_0^{90^\circ} = 0.16 \text{ ft} \qquad \textbf{Ans}$$

2–33 The Polysulfone block is glued at its top and bottom to the rigid plates. If a tangential force, applied to the top plate, causes the material to deform so that its sides are described by the equation $y = 3.56x^{1/4}$, determine the shear strain in the material at its corners *A* and *B*.



Prob. 2–33

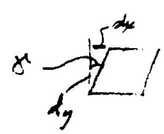
$$y = 3.56\,x^{1/4}$$

$$\frac{dy}{dx} = 0.890\,x^{-3/4}$$

$$\frac{dx}{dy} = 1.123\,x^{3/4}$$

At *A*, $x = 0$

$$\gamma_A = \frac{dx}{dy} = 0 \qquad \textbf{Ans}$$



At *B*,

$$2 = 3.56\,x^{1/4}$$

$$x = 0.0996 \text{ in.}$$

$$\gamma_B = \frac{dx}{dy} = 1.123(0.0996)^{3/4} = 0.199 \text{ rad} \qquad \textbf{Ans}$$

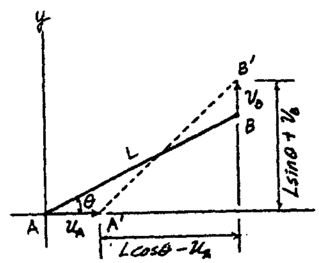
2-34. The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B, respectively, determine the normal strain in the fiber when it is in position $A'B'$.

Geometry :

$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$
$$= \sqrt{L^2 + u_A^2 + v_B^2 + 2L(v_B\sin\theta - u_A\cos\theta)}$$

Average Normal strain :

$$\varepsilon_{AB} = \frac{L_{A'B'} - L}{L}$$
$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}} - 1$$



Neglecting higher terms u_A^2 and v_B^2

$$\varepsilon_{AB} = \left[1 + \frac{2(v_B\sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem :

$$\varepsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B\sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$
$$= \frac{v_B\sin\theta}{L} - \frac{u_A\cos\theta}{L} \qquad \textbf{Ans}$$

2-35. If the normal strain is defined in reference to the final length, that is,

$$\epsilon_n' = \lim_{p \to p'} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon_n - \epsilon_n' = \epsilon_n \epsilon_n'$.

$$\varepsilon_n = \frac{\Delta S' - \Delta S}{\Delta S}$$

$$\varepsilon_n - \varepsilon_n' = \frac{\Delta S' - \Delta S}{\Delta S} - \frac{\Delta S' - \Delta S}{\Delta S'}$$

$$= \frac{\Delta S'^2 - \Delta S \Delta S' - \Delta S' \Delta S + \Delta S^2}{\Delta S \Delta S'}$$

$$= \frac{\Delta S'^2 + \Delta S^2 - 2\Delta S' \Delta S}{\Delta S \Delta S'}$$

$$= \frac{(\Delta S' - \Delta S)^2}{\Delta S \Delta S'} = \left(\frac{\Delta S' - \Delta S}{\Delta S} \right) \left(\frac{\Delta S' - \Delta S}{\Delta S'} \right)$$

$$= \varepsilon_n \varepsilon_n' \quad (Q.E.D)$$

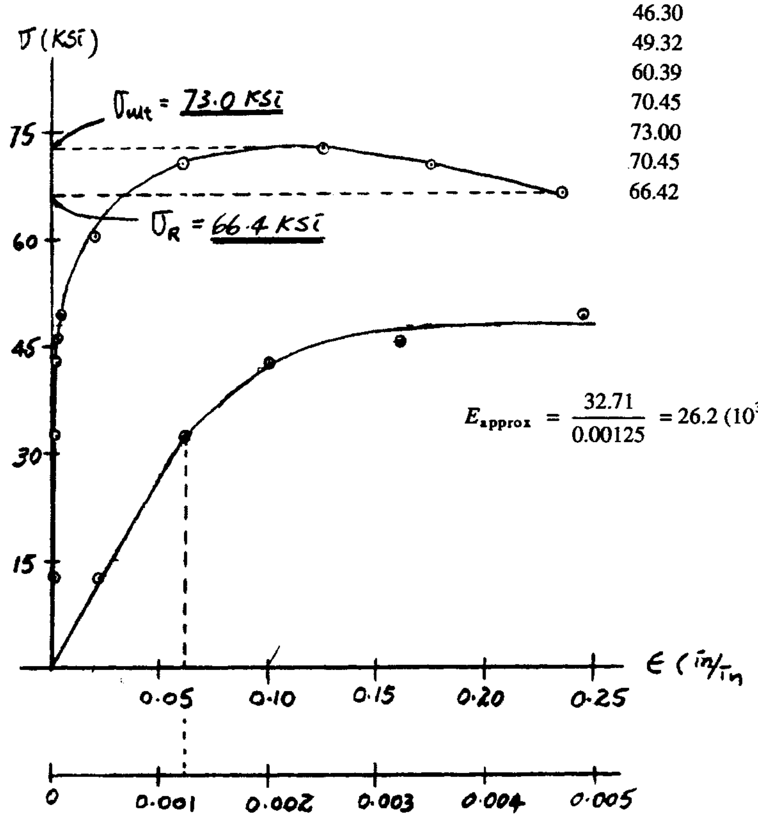
3–1 A tension test was performed on a steel specimen having an original diameter of 0.503 in. and a gauge length of 2.00 in. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the ultimate stress, and the rupture stress. Use a scale of 1 in. = 15 ksi and 1 in. = 0.05 in./in. Redraw the linear-elastic region, using the same stress scale but a strain scale of 1 in. = 0.001 in.

Load (kip)	Elongation (in.)
0	0
2.50	0.0009
6.50	0.0025
8.50	0.0040
9.20	0.0065
9.80	0.0098
12.0	0.0400
14.0	0.1200
14.5	0.2500
14.0	0.3500
13.2	0.4700

$A = \frac{1}{4}\pi(0.503)^2 = 0.19871 \text{ in}^2$

$L = 2.00$ in.

$\sigma = \frac{P}{A}$ (ksi)	$\varepsilon = \frac{\Delta L}{L}$ (in./in.)
0	0
12.58	0.00045
32.71	0.00125
42.78	0.0020
46.30	0.00325
49.32	0.0049
60.39	0.02
70.45	0.06
73.00	0.125
70.45	0.175
66.42	0.235



$$E_{approx} = \frac{32.71}{0.00125} = 26.2\,(10^3)\text{ ksi} \qquad \textbf{Ans}$$

3-2 A tension test was performed on a steel specimen having an original diameter of 0.503 in. and gauge length of 2.00 in. Using the data listed in the table, plot the stress–strain diagram and determine approximately the modulus of toughness.

Load (kip)	Elongation (in.)
0	0
2.50	0.0009
6.50	0.0025
8.50	0.0040
9.20	0.0065
9.80	0.0098
12.0	0.0400
14.0	0.1200
14.5	0.2500
14.0	0.3500
13.2	0.4700

Modulus of toughness (approx)

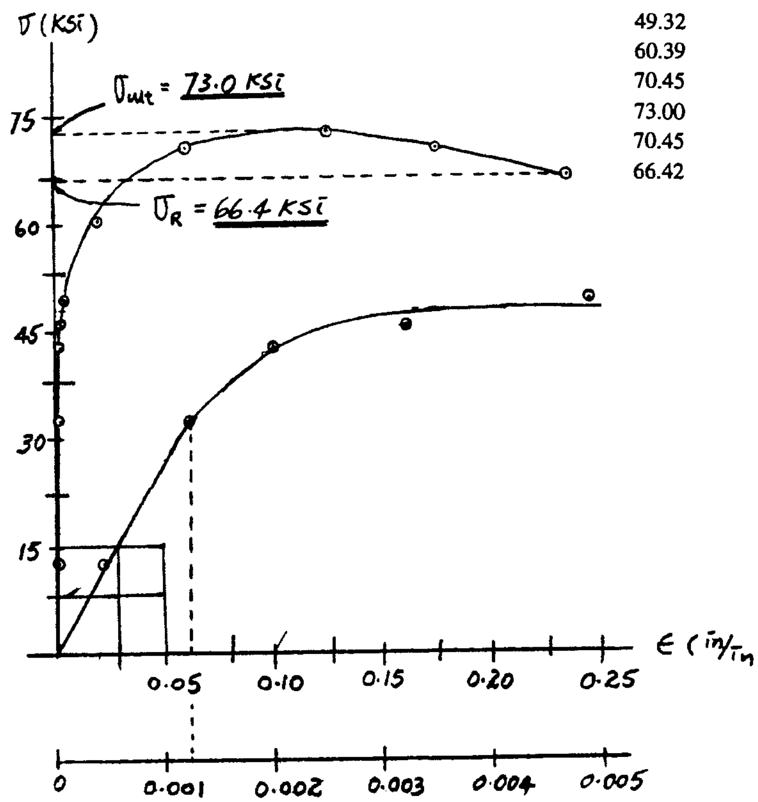
$u_t =$ total area under the curve

$= 87\,(7.5)\,(0.025) \qquad (1)$

$= 16.3\ \dfrac{\text{in.}\cdot\text{kip}}{\text{in}^3}$ **Ans**

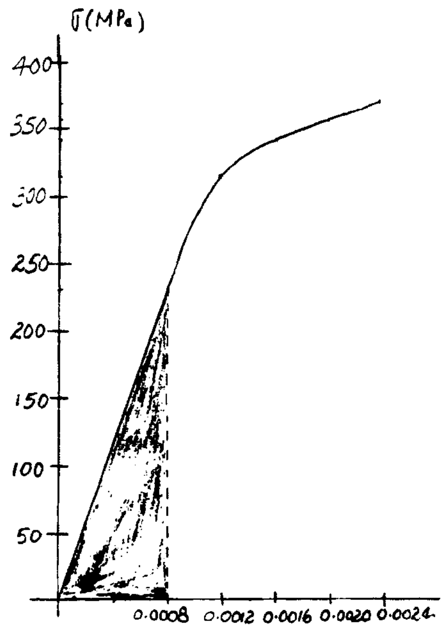
In Eq.(1), 87 is the number of squares under the curve.

$\sigma = \frac{P}{A}$ (ksi)	$\varepsilon = \frac{\Delta L}{L}$ (in./in.)
0	0
12.58	0.00045
32.71	0.00125
42.78	0.0020
46.30	0.00325
49.32	0.0049
60.39	0.02
70.45	0.06
73.00	0.125
70.45	0.175
66.42	0.235



3-3 Data taken from a stress–strain test for a ceramic is given in the table. The curve is linear between the origin and the first point. Plot the curve, and determine the modulus of elasticity and the modulus of resilience.

σ (MPa)	ϵ (mm/mm)
0	0
229	0.0008
314	0.0012
341	0.0016
355	0.0020
368	0.0024



$$E = \frac{229(10^6)}{0.0008} = 286\text{ GPa} \qquad \textbf{Ans}$$

$$u_r = \frac{1}{2}(229)(10^6)\text{ N/m}^2(0.0008)\text{ mm/mm}$$

$$= 91.6\text{ kJ/m}^3 \qquad \textbf{Ans}$$

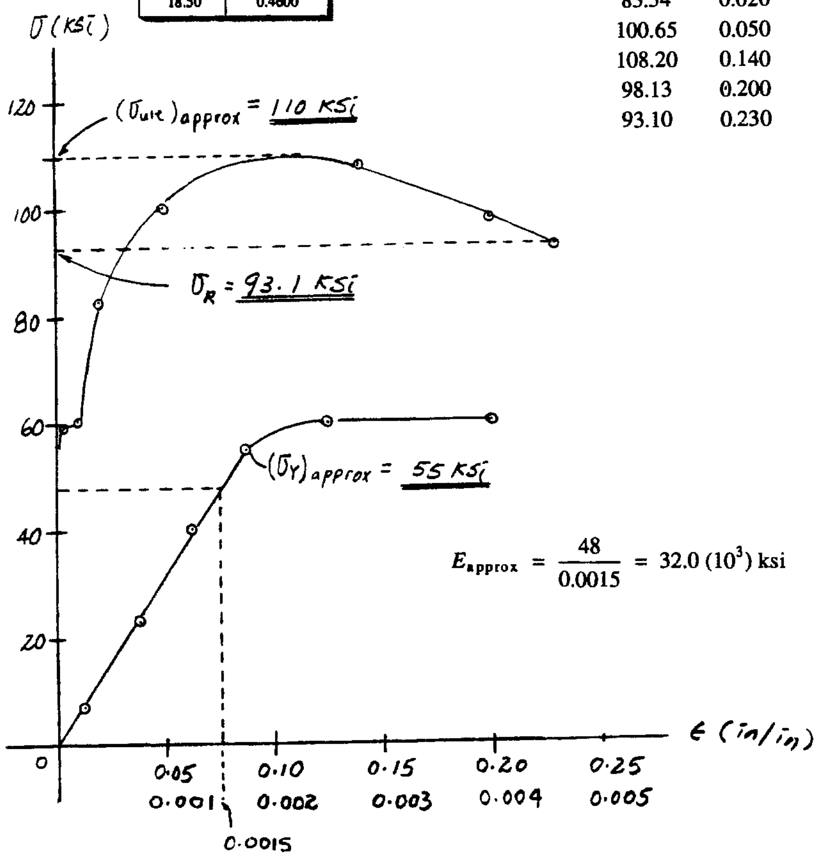
•3–4 A tension test was performed on a steel specimen having an original diameter of 0.503 in. and gauge length of 2.00 in. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the yield stress, the ultimate stress, and the rupture stress. Use a scale of 1 in. = 20 ksi and 1 in. = 0.05 in./in. Redraw the elastic region, using the same stress scale but a strain scale of 1 in. = 0.001 in./in.

Load (kip)	Elongation (in.)
0	0
1.50	0.0005
4.60	0.0015
8.00	0.0025
11.00	0.0035
11.80	0.0050
11.80	0.0080
12.00	0.0200
16.60	0.0400
20.00	0.1000
21.50	0.2800
19.50	0.4000
18.50	0.4600

$$A = \frac{1}{4}\pi(0.503)^2 = 0.1987 \text{ in}^2$$

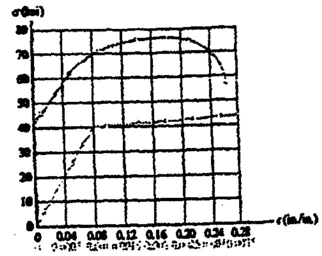
$L = 2.00$ in.

σ(ksi)	ε(in./in.)
0	0
7.55	0.00025
23.15	0.00075
40.26	0.00125
55.36	0.00175
59.38	0.0025
59.38	0.0040
60.39	0.010
83.54	0.020
100.65	0.050
108.20	0.140
98.13	0.200
93.10	0.230



$$E_{approx} = \frac{48}{0.0015} = 32.0\,(10^3)\text{ ksi} \qquad \textbf{Ans}$$

3-5. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



Modulus of Elasticity : From the stress – strain diagram, $\sigma = 40$ ksi when $\varepsilon = 0.001$ in./in.

$$E_{approx} = \frac{40 - 0}{0.001 - 0} = 40.0\left(10^3\right)\ \text{ksi} \qquad \textbf{Ans}$$

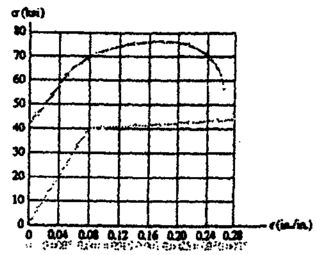
Yield Load : From the stress – strain diagram, $\sigma_Y = 40.0$ ksi.

$$P_Y = \sigma_Y A = 40.0\left[\left(\frac{\pi}{4}\right)\left(0.5^2\right)\right] = 7.85\ \text{kip} \qquad \textbf{Ans}$$

Ultimate Load : From the stress – strain diagram, $\sigma_u = 76.25$ ksi.

$$P_u = \sigma_u A = 76.25\left[\left(\frac{\pi}{4}\right)\left(0.5^2\right)\right] = 15.0\ \text{kip} \qquad \textbf{Ans}$$

3-6. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. If the specimen is loaded until it is stressed to 70 ksi, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.



Modulus of Elasticity : From the stress – strain diagram, $\sigma = 40$ ksi when $\varepsilon = 0.001$ in./in.

$$E = \frac{40-0}{0.001-0} = 40.0(10^3)\ \text{ksi}$$

Elastic Recovery :

$$\text{Elastic recovery} = \frac{\sigma}{E} = \frac{70}{40.0(10^3)} = 0.00175\ \text{in./in.}$$

Thus,

The amount of Elastic Recovery = 0.00175(2) = 0.00350 in. **Ans**

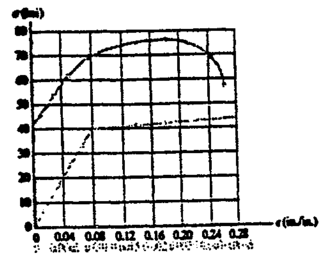
Permanent Set :

Permanent set = 0.08 – 0.00175 = 0.07825 in./in.

Thus,

Permanent elongation = 0.07825(2) = 0.1565 in. **Ans**

3-7. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.



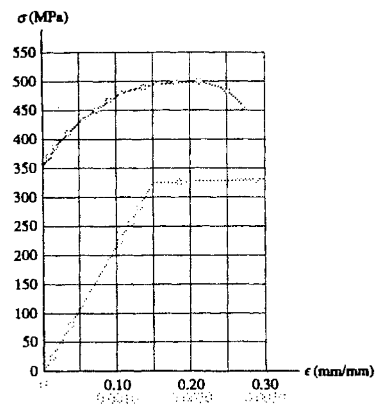
Modulus of Resilience : The modulus of resilience is equal to the area under the *linear portion* of the stress – strain diagram.

$$(u_r)_{approx} = \frac{1}{2}(40.0)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.001\ \frac{\text{in.}}{\text{in.}}\right) = 20.0\ \frac{\text{in}\cdot\text{lb}}{\text{in}^3}$$ **Ans.**

Modulus of Toughness : The modulus of toughness is equal to the total area under the stress – strain diagram and can be approximated by counting the number of squares. The total number of squares is 45.

$$(u_t)_{approx} = 45\left(10\ \frac{\text{kip}}{\text{in}^2}\right)\left(0.04\ \frac{\text{in.}}{\text{in.}}\right) = 18.0\ \frac{\text{in}\cdot\text{kip}}{\text{in}^3}$$ **Ans**

•3–8 The stress–strain diagram for a steel bar is shown in the figure. Determine approximately the modulus of elasticity, the proportional limit, the ultimate stress, and the modulus of resilience. If the bar is loaded until it is stressed to 450 MPa, determine the amount of elastic strain recovery and the permanent set or strain in the bar when it is unloaded.



Prob. 3–8

$\sigma_{pl} = 325 \text{ MPa}$ **Ans**

$\sigma_{ult} = 500 \text{ MPa}$ **Ans**

Modulus of elasticity :

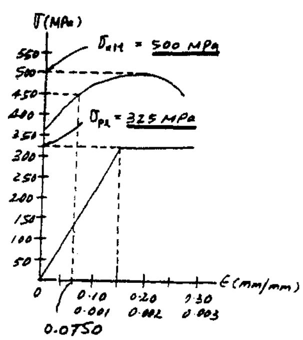
$$E = \frac{325(10^6)}{0.0015} = 217 \text{ GPa}$$ **Ans**

Modulus of resilience

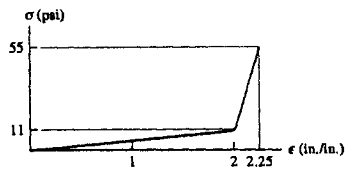
$$u_r = \frac{1}{2}(0.0015 \text{ mm/mm})(325)(10^6) \text{ N/m}^2$$
$$= 244 \text{ kJ/m}^3$$ **Ans**

$$\text{Elastic recovery} = \frac{450(10^6)}{E} = \frac{450(10^6)}{217(10^9)}$$
$$= 0.00207 \text{ mm/mm}$$ **Ans**

$$\text{Permanent set} = 0.0750 - 0.00207$$
$$= 0.0729 \text{ mm/mm}$$ **Ans**



3–9 The σ–ϵ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.

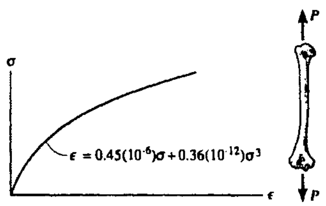


$$E = \frac{11}{2} = 5.5 \text{ psi} \qquad \textbf{Ans}$$

$$u_t = \frac{1}{2}(2)(11) + \frac{1}{2}(55 + 11)(2.25 - 2) = 19.25 \text{ psi} \qquad \textbf{Ans}$$

$$u_r = \frac{1}{2}(2)(11) = 11 \text{ psi} \qquad \textbf{Ans}$$

3–10 The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.



$$\varepsilon = 0.45(10^{-6})\sigma + 0.36(10^{-12})\sigma^3$$

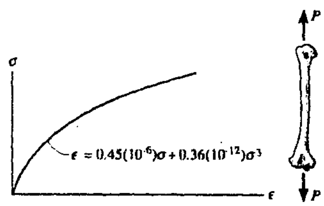
For 0.3% = 0.003 mm/mm offset

$$3000 = 0.45\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root yields

$\sigma = 1.82$ MPa **Ans**

•3–11 The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.

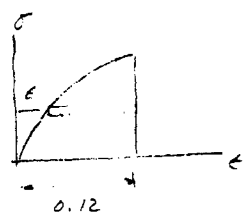


When $\varepsilon = 0.12$

$$120(10^3) = 0.45\,\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root :

$$\sigma = 6873.52 \text{ kPa}$$



$$u_t = \int_A dA = \int_0^{6873.52} (0.12 - \varepsilon)d\sigma$$

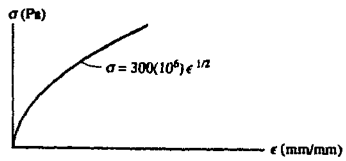
$$u_t = \int_0^{6873.52} (0.12 - 0.45(10^{-6})\sigma - 0.36(10^{-12})\sigma^3)d\sigma$$

$$= 0.12\sigma - 0.225(10^{-6})\sigma^2 - 0.09(10^{-12})\sigma^4|_0^{6873.52}$$

$$= 613 \text{ kPa} \qquad \textbf{Ans}$$

$$\delta = \varepsilon L = 0.12(200) = 24 \text{ mm} \qquad \textbf{Ans}$$

•3–12 Fiberglass has a stress–strain diagram as shown. If a 50-mm-diameter bar of length 2 m made from this material is subjected to an axial tensile load of 60 kN, determine its elongation.



$$\sigma = \frac{P}{A} = \frac{60(10^3)}{\pi(0.025)^2} = 30.558 \text{ MPa}$$

$$\sigma = 300(10^6)\varepsilon^{1/2}$$

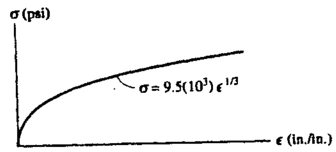
$$30.558(10^6) = 300(10^6)\varepsilon^{1/2}$$

$$\varepsilon = 0.010375 \text{ mm/mm}$$

$$\delta = L\varepsilon = 2(0.010375) = 0.0208 \text{ m}$$

$\delta = 20.8$ mm **Ans**

3–13 Acetal plastic has a stress–strain diagram as shown. If a bar of this material has a length of 3 ft and cross-sectional area of 0.875 in², and is subjected to an axial load of 2.5 kip, determine its elongation.



$$\sigma = \frac{P}{A} = \frac{2.5}{0.875} = 2.857 \text{ ksi}$$

$$\sigma = 9.5(10^3)\varepsilon^{1/3}$$

$$2.857(10^3) = 9.5(10^3)\varepsilon^{1/3}$$

$$\varepsilon = 0.0272 \text{ in./in.}$$

$$\delta = L\varepsilon = 3(12)(0.0272) = 0.979 \text{ in.}$$ **Ans**

3-14 A specimen is originally 1 ft long, has a diameter of 0.5 in., and is subjected to a force of 500 lb. When the force is increased to 1800 lb, the specimen elongates 0.9 in. Determine the modulus of elasticity for the material if it remains elastic.

$$\sigma_1 = \frac{P}{A} = \frac{500}{\frac{\pi}{4}(0.5)^2} = 2.546 \text{ ksi}$$

$$\sigma_2 = \frac{P}{A} = \frac{1800}{\frac{\pi}{4}(0.5)^2} = 9.167 \text{ ksi}$$

$$\Delta\varepsilon = \frac{0.9}{12} = 0.075 \text{ in./in.}$$

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{9.167 - 2.546}{0.075} = 88.3 \text{ ksi}$$

3–15 A structural member in a nuclear reactor is made from a zirconium alloy. If an axial load of 4 kip is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 with respect to yielding. What is the load on the member if it is 3 ft long and its elongation is 0.02 in.? $E_{zr} = 14(10^3)$ ksi, $\sigma_Y = 57.5$ ksi. The material has elastic behavior.

$$\text{F.S.} = 3 = \frac{\sigma_y}{\sigma_{allow}}$$

$$\sigma_{allow} = \frac{57.5}{3} = 19.17 \text{ ksi}$$

$$\sigma_{allow} = 19.17 = \frac{4}{A}$$

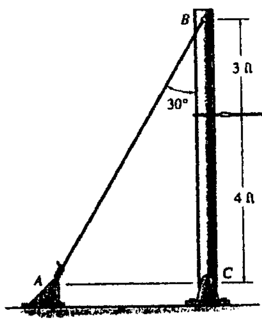
$$A = 0.209 \text{ in}^2 \qquad \textbf{Ans}$$

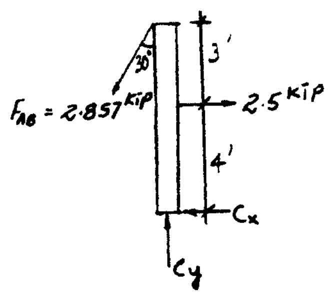
$$\varepsilon = \frac{\delta}{L} = \frac{0.02}{3\,(12)} = 0.000555$$

$$\sigma = E\varepsilon = 14\,(10^3)(0.000555) = 7.78 \text{ ksi}$$

$$P = \sigma A = 7.78\,(0.209) = 1.62 \text{ kip} \qquad \textbf{Ans}$$

•3–16 The pole is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when a horizontal force of 2.5 kip acts on the pole.





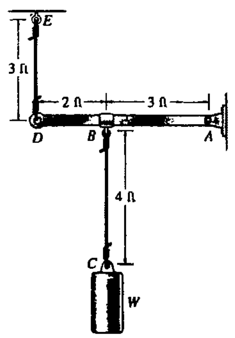
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{2.857}{\frac{\pi}{4}(0.2^2)} = 90.94\text{ ksi}$$

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{90.94}{29(10^3)} = 0.003136$$

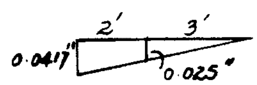
$$\delta_{AB} = \varepsilon_{AB}\, L_{AB} = 0.003136\left(\frac{7(12)}{\cos 30^\circ}\right)$$

$$= 0.304\text{ in.} \qquad \textbf{Ans}$$

3–17 The bar *DA* is rigid and is originally held in the horizontal position when the weight *W* is supported from *C*. If the weight causes *B* to be displaced downward 0.025 in., determine the strain in wires *DE* and *BC*. Also, if the wires are made of A-36 steel and have a cross-sectional area of 0.002 in^2, determine the weight *W*.



$$\frac{3}{0.025} = \frac{7}{\delta}$$

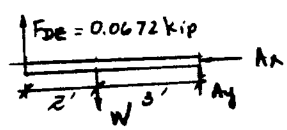


$\delta = 0.0417$ in.

$$\varepsilon_{DE} = \frac{\delta}{L} = \frac{0.0417}{3\,(12)} = 0.00116 \text{ in./in.} \qquad \textbf{Ans}$$

$$\sigma_{DE} = E\varepsilon_{DE} = 29(10^3)(0.00116) = 33.56 \text{ ksi}$$

$$F_{DE} = \sigma_{DE}A_{DE} = 33.56\,(0.002) = 0.0672 \text{ kip}$$



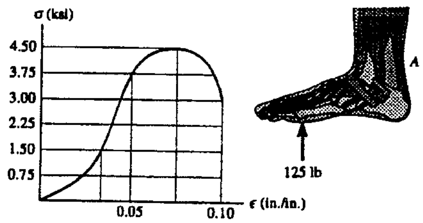
$$\circlearrowleft + \Sigma M_A = 0; \qquad -(0.0672)\,(5) + 3(W) = 0$$

$W = 0.112$ kip $= 112$ lb **Ans**

$$\sigma_{BC} = \frac{W}{A_{BC}} = \frac{0.112}{0.002} = 55.94 \text{ ksi}$$

$$\varepsilon_{BC} = \frac{\sigma_{BC}}{E} = \frac{55.94}{29\,(10^3)} = 0.00193 \text{ in./in.} \qquad \textbf{Ans}$$

3-18 The σ–ϵ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 6.5 in. and an approximate cross-sectional area of 0.229 in^2, determine its elongation if the foot supports a load of 125 lb, which causes a tension in the tendon of 343.75 lb.

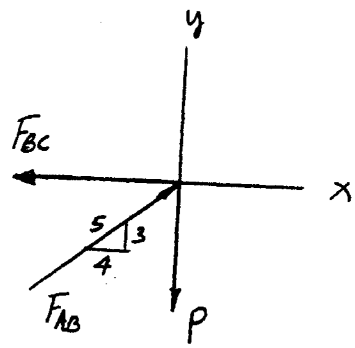


$$\sigma = \frac{P}{A} = \frac{343.75}{0.229} = 1.50 \text{ ksi}$$

From the graph $\varepsilon \approx 0.025$ in./in.

$\delta = \varepsilon L = 0.025(6.5) = 0.162$ in. **Ans**

3-19. The two bars are made of polystyrene, which has the stress–strain diagram shown. If the cross-sectional area of bar AB is 1.5 in^2 and BC is 4 in^2, determine the largest force P that can be supported before any member ruptures. Assume that buckling does not occur.



$+\uparrow\Sigma F_y = 0;\quad \frac{3}{5}F_{AB} - P = 0;\quad F_{AB} = 1.6667\,P \qquad [1]$

$\overset{+}{\leftarrow}\Sigma F_x = 0;\quad F_{BC} - \frac{4}{5}(1.6667P) = 0;\quad F_{BC} = 1.333\,P \qquad [2]$

Assuming failure of bar BC :

From the stress - strain diagram $(\sigma_R)_t = 5$ ksi

$\sigma = \frac{F_{BC}}{A_{BC}};\quad 5 = \frac{F_{BC}}{4};\quad F_{BC} = 20.0$ kip

From Eq. [2], $P = 15.0$ kip

Assuming failure of bar AB :

From stress - strain diagram $(\sigma_R)_c = 25.0$ ksi

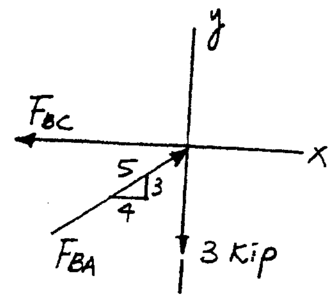
$\sigma = \frac{F_{AB}}{A_{AB}};\quad 25.0 = \frac{F_{AB}}{1.5};\quad F_{AB} = 37.5$ kip

From Eq. [1], $P = 22.5$ kip

Choose the smallest value

$P = 15.0$ kip **Ans**

***3-20.** The two bars are made of polystyrene, which has the stress–strain diagram shown. Determine the cross-sectional area of each bar so that the bars rupture simultaneously when the load $P = 3$ kip. Assume that buckling does not occur.



$$+\uparrow \Sigma F_y = 0; \quad F_{BA}\left(\frac{3}{5}\right) - 3 = 0; \quad F_{BA} = 5\text{ kip}$$

$$\underset{\rightarrow}{+} \Sigma F_x = 0; \quad -F_{BC} + 5\left(\frac{4}{5}\right) = 0; \quad F_{BC} = 4\text{ kip}$$

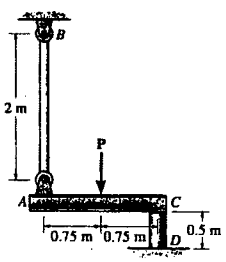
For member BC :

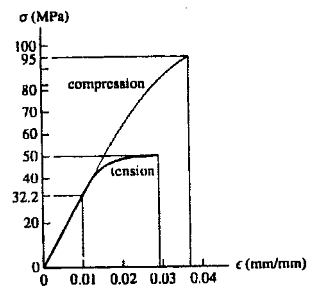
$$(\sigma_{\max})_t = \frac{F_{BC}}{A_{BC}}; \quad A_{BC} = \frac{4\text{ kip}}{5\text{ ksi}} = 0.8\text{ in}^2 \qquad \textbf{Ans}$$

For member BA :

$$(\sigma_{\max})_c = \frac{F_{BA}}{A_{BA}}; \quad A_{BA} = \frac{5\text{ kip}}{25\text{ ksi}} = 0.2\text{ in}^2 \qquad \textbf{Ans}$$

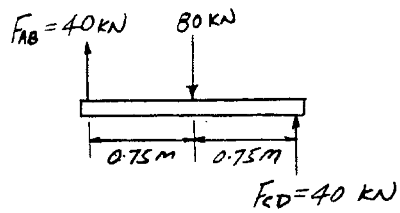
3–21 The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD, both made from this material, and subjected to a load of $P = 80$ kN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.





From the stress - strain diagram,

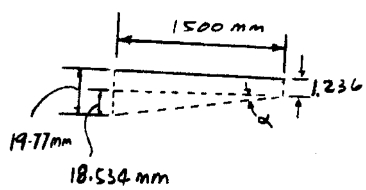
$$E = \frac{32.2(10)^6}{0.01} = 3.22(10^9)\text{ Pa}$$



Thus,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40(10^3)}{\frac{\pi}{4}(0.04)^2} = 31.83\text{ MPa}$$

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83(10^6)}{3.22(10^9)} = 0.009885\text{ mm/mm}$$



$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40(10^3)}{\frac{\pi}{4}(0.08)^2} = 7.958\text{ MPa}$$

$$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958(10^6)}{3.22(10^9)} = 0.002471\text{ mm/mm}$$

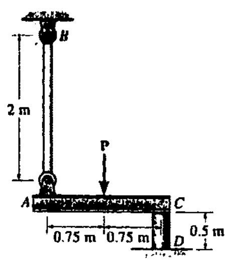
$$\delta_{AB} = \varepsilon_{AB}L_{AB} = 0.009885(2000) = 19.77\text{ mm}$$

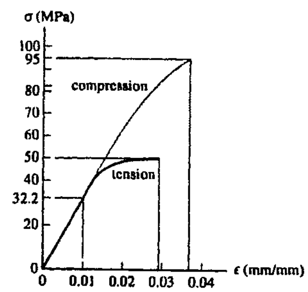
$$\delta_{CD} = \varepsilon_{CD}L_{CD} = 0.002471(500) = 1.236\text{ mm}$$

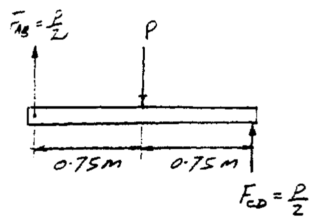
Angle of tilt α :

$$\tan\alpha = \frac{18.534}{1500}; \qquad \alpha = 0.708^\circ \qquad \textbf{Ans}$$

3-22 The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD made from this material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.







Rupture of strut AB :

$$\sigma_R = \frac{F_{AB}}{A_{AB}}; \qquad 50(10^6) = \frac{P/2}{\frac{\pi}{4}(0.012)^2};$$

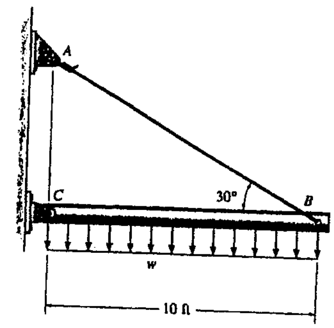
$P = 11.3\text{ kN}$ (controls) **Ans**

Rupture of post CD :

$$\sigma_R = \frac{F_{CD}}{A_{CD}}; \qquad 95(10^6) = \frac{P/2}{\frac{\pi}{4}(0.04)^2}$$

$$P = 239\text{ kN}$$

3–23 The pipe is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when a distributed load of w = 100 lb/ft acts on the pipe. The material remains elastic.



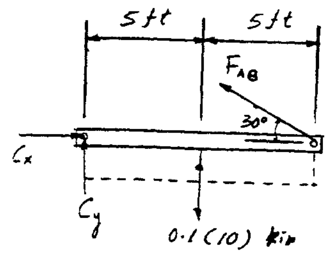
$$+\Sigma M_C = 0; \qquad F_{AB}\sin 30°(10) - 0.1(10)(5) = 0;$$

$$F_{AB} = 1.0 \text{ kip}$$

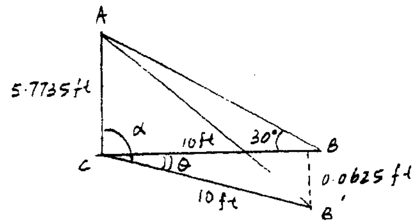
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{1.0}{\frac{\pi}{4}(0.2)^2} = 31.83 \text{ ksi}$$

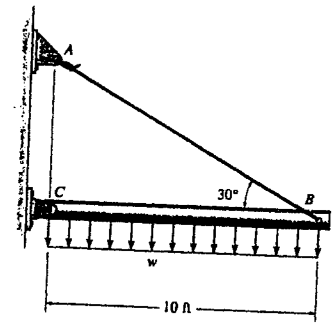
$$\sigma = E\,\varepsilon; \qquad 31.83 = 29(10^3)\,\varepsilon_{AB}; \qquad \varepsilon_{AB} = 0.0010981 \text{ in./in.}$$

$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0010981(\frac{120}{\cos 30°}) = 0.152 \text{ in.}$$ **Ans**



•3–24 The pipe is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine the distributed load w if the end B is displaced 0.75 in. downward.





$$\sin\theta = \frac{0.0625}{10}; \qquad \theta = 0.3581°$$

$$\alpha = 90 + 0.3581° = 90.3581°$$

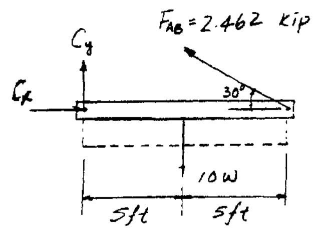
$$AB = \frac{10}{\cos 30°} = 11.5470 \text{ ft}$$

$$AB' = \sqrt{10^2 + 5.7735^2 - 2(10)(5.7735)\cos 90.3581°}$$
$$= 11.5782 \text{ ft}$$

$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{11.5782 - 11.5470}{11.5470} = 0.002703 \text{ in./in.}$$

$$\sigma_{AB} = E\,\varepsilon_{AB} = 29(10^3)(0.002703) = 78.38 \text{ ksi}$$

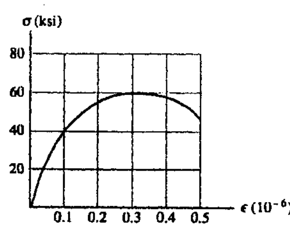
$$F_{AB} = \sigma_{AB} A_{AB} = 78.38\left(\frac{\pi}{4}\right)(0.2)^2 = 2.462 \text{ kip}$$



$$+\Sigma M_C = 0; \qquad 2.462 \sin 30°(10) - 10w(5) = 0;$$

$w = 0.246$ kip/ft **Ans**

3–25 The stress–strain diagram for many metal alloys can be described analytically using the Ramberg-Osgood three parameter equation $\epsilon = \sigma/E + k\sigma^n$, where E, k, and n are determined from measurements taken from the diagram. Using the stress–strain diagram shown in the figure, take $E = 30(10^3)$ ksi and determine the other two parameters k and n and thereby obtain an analytical expression for the curve.



Choose,

$\sigma = 40$ ksi, $\varepsilon = 0.1$

$\sigma = 60$ ksi, $\varepsilon = 0.3$

$$0.1 = \frac{40}{30(10^3)} + k(40)^n$$

$$0.3 = \frac{60}{30(10^3)} + k(60)^n$$

$$0.098667 = k(40)^n$$

$$0.29800 = k(60)^n$$

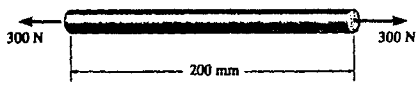
$$0.3310962 = (0.6667)^n$$

$$\ln(0.3310962) = n \ln(0.6667)$$

$n = 2.73$ **Ans**

$k = 4.23(10^{-6})$ **Ans**

3–26 The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. E_p = 2.70 GPa, ν_p = 0.4.



$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.697 \text{ MPa}$$

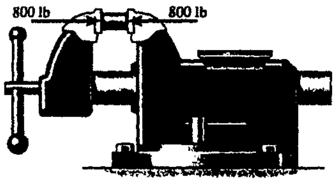
$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{1.697(10^6)}{2.70\,(10^9)} = 0.0006288$$

$$\delta = \varepsilon_{\text{long}}\, L = 0.0006288\,(200) = 0.126 \text{ mm}$$ **Ans**

$$\varepsilon_{\text{lat}} = -\nu\, \varepsilon_{\text{long}} = -0.4\,(0.0006288) = -0.0002515$$

$$\Delta d = \varepsilon_{\text{lat}} d = -0.0002515\,(15) = -0.00377 \text{ mm}$$ **Ans**

3-27 The short cylindrical block of 2014-T6 aluminum, having an original diameter of 0.5 in. and a length of 1.5 in., is placed in the smooth jaws of a vise and squeezed until the axial load applied is 800 lb. Determine (*a*) the decrease in its length and (*b*) its new diameter.



a)

$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}(0.5)^2} = 4074.37 \text{ psi}$$

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{-4074.37}{10.6(10^6)} = -0.0003844$$

$$\delta = \varepsilon_{long} L = -0.0003844\,(1.5) = -0.577\,(10^{-3}) \text{ in.}$$ **Ans**

b)

$$\nu = \frac{-\varepsilon_{lat}}{\varepsilon_{long}} = 0.35$$

$$\varepsilon_{lat} = -0.35\,(-0.0003844) = 0.00013453$$

$$\Delta d = \varepsilon_{lat} d = 0.00013453\,(0.5) = 0.00006727$$

$$d' = d + \Delta d = 0.5000673 \text{ in.}$$ **Ans**

•3–28 A short cylindrical block of bronze C86100, having an original diameter of 1.5 in. and a length of 3 in., is placed in a compression machine and squeezed until its length becomes 2.98 in. Determine the new diameter of the block.

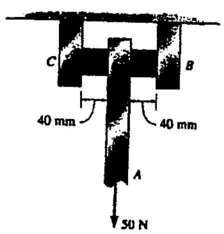
$$\varepsilon_{\text{long}} = \frac{-0.02}{3} = -0.0066667 \text{ in./in.}$$

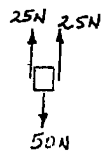
$$\varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -0.34(-0.0066667) = 0.0022667 \text{ in./in.}$$

$$\Delta d = \varepsilon_{\text{lat}} d = 0.0022667(1.5) = 0.0034 \text{ in.}$$

$$d' = d + \Delta d = 1.5 + 0.0034 = 1.5034 \text{ in.}$$ **Ans**

3-29. The support consists of three rigid plates, which are connected together using two symmetrically placed rubber pads. If a vertical force of 50 N is applied to plate A, determine the approximate vertical displacement of this plate due to shear strains in the rubber. Each pad has cross-sectional dimensions of 30 mm and 20 mm. $G_r = 0.20$ MPa.



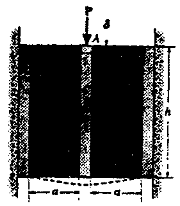


$$\tau_{avg} = \frac{V}{A} = \frac{25}{(0.03)(0.02)} = 41666.67 \text{ Pa}$$

$$\gamma = \frac{\tau}{G} = \frac{41666.67}{0.2\,(10^6)} = 0.2083 \text{ rad}$$

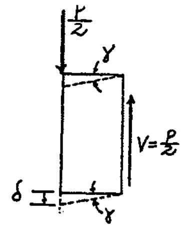
$$\delta = 40\,(0.2083) = 8.33 \text{ mm} \qquad \textbf{Ans}$$

3-30. A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A if a vertical load $\mathbf{P}$ is applied to this plate. Assume that the displacement is small so that $\delta = a \tan \gamma \approx a\gamma$.



Average Shear Stress : The rubber block is subjected to a shear force of $V = \frac{P}{2}$.

$$\tau = \frac{V}{A} = \frac{\frac{P}{2}}{bh} = \frac{P}{2bh}$$



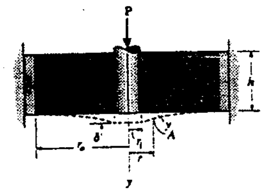
Shear Strain : Applying Hooke's law for shear

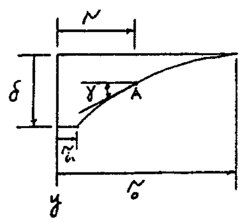
$$\gamma = \frac{\tau}{G} = \frac{\frac{P}{2bh}}{G} = \frac{P}{2bhG}$$

Thus,

$$\delta = a\gamma = \frac{Pa}{2bhG} \qquad \textbf{Ans}$$

3-31. A shear spring is made by bonding the rubber annulus to a rigid fixed ring and a plug. When an axial load **P** is placed on the plug, show that the slope at point y in the rubber is $dy/dr = -\tan\gamma = -tan(P/(2\pi hGr))$. For small angles we can write $dy/dr = -P/(2\pi hGr)$. Integrate this expression and evaluate the constant of integration using the condition that $y = 0$ at $r = r_o$. From the result compute the deflection $y = \delta$ of the plug.





Shear Stress – Strain Relationship : Applying Hooke's law with $\tau_A = \frac{P}{2\pi r h}$.

$$\gamma = \frac{\tau_A}{G} = \frac{P}{2\pi h G r}$$

$$\frac{dy}{dr} = -\tan\gamma = -\tan\left(\frac{P}{2\pi h G r}\right) \quad (Q.E.D)$$

If γ is small, then $\tan\gamma \approx \gamma$. Therefore,

$$\frac{dy}{dr} = -\frac{P}{2\pi h G r}$$

$$y = -\frac{P}{2\pi h G}\int\frac{dr}{r}$$

$$y = -\frac{P}{2\pi h G}\ln r + C$$

At $r = r_o$, $y = 0$

$$0 = -\frac{P}{2\pi h G}\ln r_o + C$$

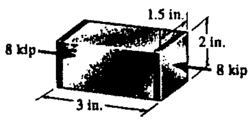
$$C = \frac{P}{2\pi h G}\ln r_o$$

Then, $y = \frac{P}{2\pi h G}\ln\frac{r_o}{r}$

At $r = r_i$, $y = \delta$

$$\delta = \frac{P}{2\pi h G}\ln\frac{r_o}{r_i} \qquad \textbf{Ans}$$

•3–32 The aluminum block has a rectangular cross section and is subjected to an axial compressive force of 8 kip. If the 1.5-in. side changed its length to 1.500132 in., determine Poisson's ratio and the new length of the 2-in. side. $E_{al} = 10(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{8}{(2)(1.5)} = 2.667 \text{ ksi}$$

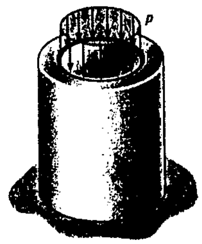
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{-2.667}{10(10^3)} = -0.0002667$$

$$\varepsilon_{lat} = \frac{1.500132 - 1.5}{1.5} = 0.0000880$$

$$\nu = \frac{-0.0000880}{-0.0002667} = 0.330 \qquad \textbf{Ans}$$

$$h' = 2 + 0.0000880(2) = 2.000176 \text{ in.} \qquad \textbf{Ans}$$

3–33 The plug has a diameter of 30 mm and fits within a rigid sleeve having an inner diameter of 32 mm. Both the plug and the sleeve are 50 mm long. Determine the axial pressure p that must be applied to the top of the plug to cause it to contact the sides of the sleeve. Also, how far must the plug be compressed downward in order to do this? The plug is made from a material for which $E = 5$ MPa, $\nu = 0.45$.



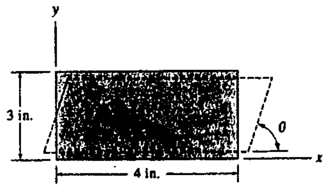
$$\varepsilon_{lat} = \frac{d' - d}{d} = \frac{32 - 30}{30} = 0.06667 \text{ mm/mm}$$

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}; \quad \varepsilon_{long} = -\frac{\varepsilon_{lat}}{\nu} = -\frac{0.06667}{0.45} = -0.1481 \text{ mm/mm}$$

$$p = \sigma = E\,\varepsilon_{long} = 5(10^6)(0.1481) = 741 \text{ kPa} \qquad \textbf{Ans}$$

$$\delta = |\varepsilon_{long} L| = |-0.1481(50)| = 7.41 \text{ mm} \qquad \textbf{Ans}$$

3-34 The rubber block is subjected to an elongation of 0.03 in. along the x axis, and its vertical faces are given a tilt so that $\theta = 89.3°$. Determine the strains ϵ_x, ϵ_y and γ_{xy}. Take $\nu_r = 0.5$.

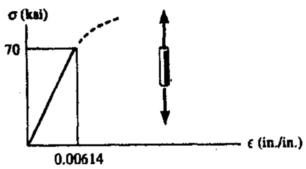


$$\varepsilon_x = \frac{\delta L}{L} = \frac{0.03}{4} = 0.0075 \text{ in./in.} \qquad \textbf{Ans}$$

$$\varepsilon_y = -\nu\,\varepsilon_x = -0.5(0.0075) = -0.00375 \text{ in./in.} \qquad \textbf{Ans}$$

$$\gamma_{xy} = \frac{\pi}{2} - \theta = \frac{\pi}{2} - 89.3°\left(\frac{\pi}{180°}\right) = 0.0122 \text{ rad} \qquad \textbf{Ans}$$

3-35 The elastic portion of the tension stress-strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. When the applied load is 9 kip, the new diameter of the specimen is 0.49935 in. Compute the shear modulus G_{al} for the aluminum.



From the stress-strain diagram,

$$E_{al} = \frac{\sigma}{\varepsilon} = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

When specimen is loaded with a 9-kip load,

$$\sigma = \frac{P}{A} = \frac{9}{\frac{\pi}{4}(0.5)^2} = 45.84 \text{ ksi}$$

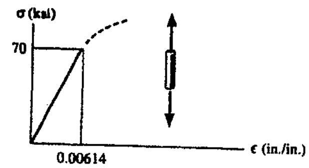
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{45.84}{11400.65} = 0.0040208 \text{ in./in.}$$

$$\varepsilon_{lat} = \frac{d' - d}{d} = \frac{0.49935 - 0.5}{0.5} = -0.0013 \text{ in./in.}$$

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} = -\frac{-0.0013}{0.0040208} = 0.32332$$

$$G_{al} = \frac{E_{al}}{2(1+\nu)} = \frac{11.4(10^3)}{2(1+0.32332)} = 4.31(10^3) \text{ ksi}$$ **Ans**

•3-36 The elastic portion of the tension stress-strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.5)^2} = 50.9296 \text{ ksi}$$

From the stress - strain diagram

$$E = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{50.9296}{11400.65} = 0.0044673 \text{ in./in.}$$

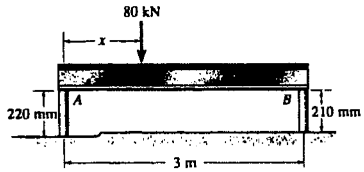
$$G = \frac{E}{2(1+\nu)}; \qquad 3.8(10^3) = \frac{11400.65}{2(1+\nu)}; \qquad \nu = 0.500$$

$$\varepsilon_{lat} = -\nu\varepsilon_{long} = -0.500(0.0044673) = -0.002234 \text{ in./in.}$$

$$\Delta d = \varepsilon_{lat} d = -0.002234(0.5) = -0.001117 \text{ in.}$$

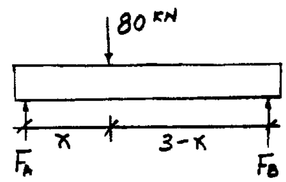
$$d' = d + \Delta d = 0.5 - 0.001117 = 0.4989 \text{ in.}$$ **Ans**

3–37 The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.



$\curvearrowleft + \Sigma M_A = 0; \quad F_B(3) - 80(x) = 0; \quad F_B = \frac{80x}{3}$ (1)

$\curvearrowleft + \Sigma M_B = 0; \quad -F_A(3) + 80(3-x) = 0; \quad F_A = \frac{80(3-x)}{3}$ (2)



Since the beam is held horizontally, $\delta_A = \delta_B$

$\sigma = \frac{P}{A}; \quad \varepsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$

$\delta = \varepsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$

$\delta_A = \delta_B; \quad \frac{\frac{80(3-x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$

$80(3-x)(220) = 80x(210)$

$x = 1.53 \text{ m}$ **Ans**

From Eq. (2),

$F_A = 39.07 \text{ kN}$

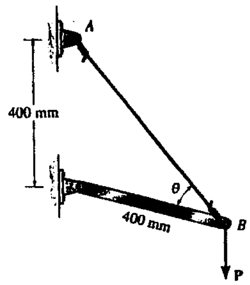
$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$

$\varepsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$

$\varepsilon_{lat} = -\nu\varepsilon_{long} = -0.35(-0.000756) = 0.0002646$

$d'_A = d_A + d\,\varepsilon_{lat} = 30 + 30(0.0002646) = 30.008 \text{ mm}$ **Ans**

3-38 A short cylindrical block of 6061-T6 aluminum, having an original diameter of 20 mm and a length of 75 mm, is placed in a compression machine and squeezed until the axial load applied is 5 kN. Determine (a) the decrease in its length and (b) its new diameter.



a) $\sigma = \frac{P}{A} = \frac{-5(10^3)}{\frac{\pi}{4}(0.02)^2} = -15.915 \text{ MPa}$

$\sigma = E\,\varepsilon_{long}; \qquad -15.915(10^6) = 68.9(10^9)\varepsilon_{long}$

$$\varepsilon_{long} = -0.0002310 \text{ mm/mm}$$

$\delta = \varepsilon_{long} L = -0.0002310(75) = -0.0173 \text{ mm}$ **Ans**

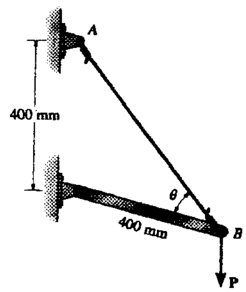
b) $\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}; \qquad 0.35 = -\frac{\varepsilon_{lat}}{-0.0002310}$

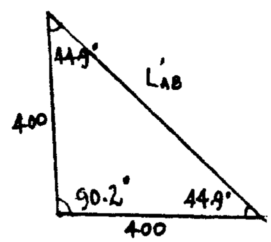
$$\varepsilon_{lat} = 0.00008085 \text{ mm/mm}$$

$\Delta d = \varepsilon_{lat} d = 0.00008085(20) = 0.0016 \text{ mm}$

$d' = d + \Delta d = 20 + 0.0016 = 20.0016 \text{ mm}$ **Ans**

3-39 The A-36 steel wire AB has a cross-sectional area of $10\ mm^2$ and is unstretched when $\theta = 45.0°$. Determine the applied load P needed to cause $\theta = 44.9°$.





$$\frac{L'_{AB}}{\sin 90.2^\circ} = \frac{400}{\sin 44.9^\circ}$$

$$L'_{AB} = 566.67\ \text{mm}$$

$$L_{AB} = \frac{400}{\sin 45^\circ} = 565.69$$

$$\varepsilon = \frac{L'_{AB} - L_{AB}}{L_{AB}} = \frac{566.67 - 565.69}{565.69} = 0.001744$$

$$\sigma = E\varepsilon = 200(10^9)\,(0.001744) = 348.76\ \text{MPa}$$

$\curvearrowleft + \Sigma M_A = 0$

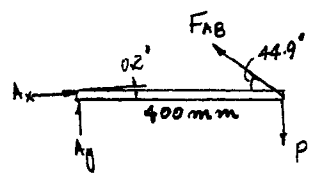
$$P(400\cos 0.2^\circ) - F_{AB}\sin 44.9^\circ\,(400) = 0 \qquad (1)$$

However,

$$F_{AB} = \sigma A = 348.76\,(10^6)(10)(10^{-6}) = 3.488\ \text{kN}$$

From Eq. (1),

$P = 2.46\ \text{kN}$ **Ans**



•3–40 While undergoing a tension test, a copper-alloy specimen having a gauge length of 2 in. is subjected to a strain of 0.40 in./in. when the stress is 70 ksi. If $\sigma_Y = 45$ ksi when $\epsilon_Y = 0.0025$ in./in., determine the distance between the gauge points when the load is released.

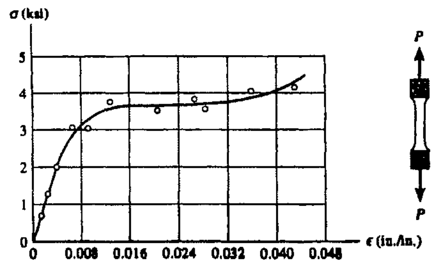
$$\text{Elastic recovery} = 70\frac{(0.0025)}{45} = 0.0038889 \text{ in./in.}$$

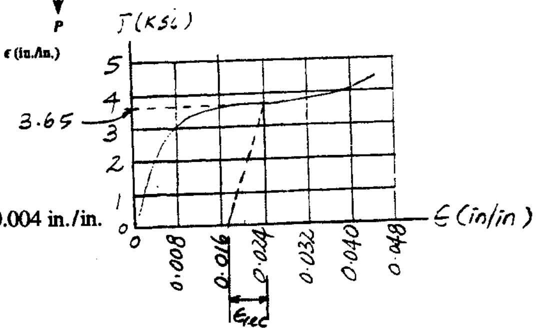
$$\text{Permanent set} = 0.4 - 0.0038889 = 0.3961 \text{ in./in.}$$

$$\delta = 0.3961(2) = 0.792 \text{ in.}$$

$$L = 2 + 0.792 = 2.792 \text{ in.}$$ **Ans**

3–41 The stress–strain diagram for polyethylene, which is used to sheath coaxial cables, is determined from testing a specimen that has a gauge length of 10 in. If a load P on the specimen develops a strain of $\epsilon = 0.024$ in./in., determine the approximate length of the specimen, measured between the gauge points, when the load is removed. Assume the specimen recovers elastically.





From the graph $\sigma = 2$ ksi, $\varepsilon = 0.004$ in./in.

$$E = \frac{\sigma}{\varepsilon} = \frac{2(10^3)}{0.004} = 500 \text{ ksi}$$

$\varepsilon = 0.024$ in./in. , $\sigma \approx 3.65$ ksi

$L' = 10 \text{ in.} + 0.024(10) = 10.24 \text{ in.}$

Elastic strain recovery :

$$\varepsilon_{rec} = \frac{\sigma}{E} = \frac{3.65 \text{ ksi}}{500 \text{ ksi}} = 0.0073 \text{ in./in.}$$

$\delta = L\,\varepsilon_{rec} = 10(0.0073) = 0.073$ in.

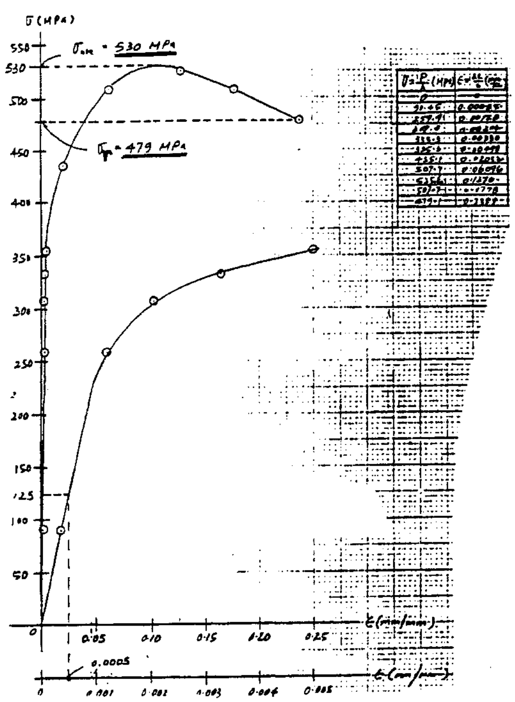
$L = L' - \delta = 10.24 \text{ in.} - 0.073 \text{ in.} = 10.17 \text{ in.}$ **Ans**

3–42 A tension test was performed on a steel specimen having an original diameter of 12.5 mm and a gauge length of 50 mm. The data is listed in the table. Plot the stress–strain diagram and determine approximately the modulus of elasticity, the ultimate stress, and the rupture stress. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm. Redraw the linear-elastic region, using the same stress scale but a strain scale of 20 mm = 0.001 mm/mm.

Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380

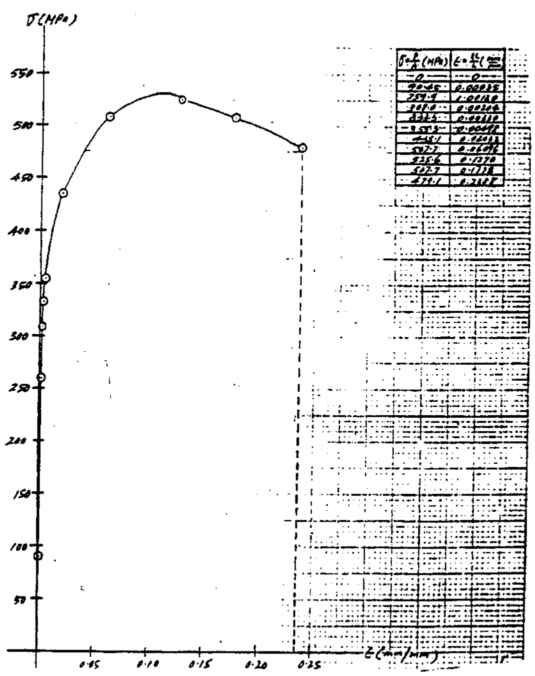
$$A = \frac{1}{4}\pi(0.0125)^2 = 0.12272(10)^{-3}\,\text{m}^2$$

$$E_{\text{approx}} = \frac{125(10^6)}{0.0005} = 250\text{ GPa} \qquad \textbf{Ans}$$



3-43 A tension test was performed on a steel specimen having an original diameter of 12.5 mm and a gauge length of 50 mm. Using the data listed in the table, plot the stress–strain diagram and determine approximately the modulus of toughness. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm.

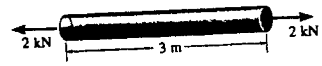
Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380



The modulus of toughness = Total area under the curve. By counting squares we have (approximately)

$$u_t = (188.5 \text{ squares})\left(25x10^6\,\frac{N}{m^2}\right)\left(0.025\,\frac{m}{m}\right) = 118(10^6)\frac{N}{m^2}$$

•3-44 An 8-mm-diameter brass rod has a modulus of elasticity of E_{br} = 100 GPa. If it is 3 m long and subjected to an axial load of 2 kN, determine its elongation. What is its elongation under the same load if its diameter is 6 mm?



$$\sigma = \frac{P}{A} = \frac{2(10^3)}{\frac{\pi}{4}(0.008^2)} = 39.789 \text{ MPa}$$

$$\varepsilon = \frac{\sigma}{E} = \frac{39.789(10^6)}{100(10^9)} = 0.00039789$$

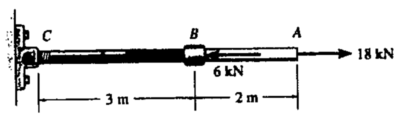
$$\delta = \varepsilon L = 0.00039789(3000) = 1.19 \text{ mm}$$ **Ans**

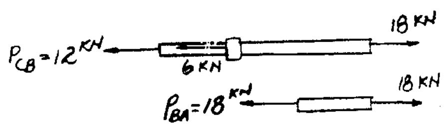
$$\sigma' = \frac{P}{A} = \frac{2(10^3)}{\frac{\pi}{4}(0.006^2)} = 70.735 \text{ MPa}$$

$$\varepsilon' = \frac{\sigma}{E} = \frac{70.735(10^6)}{100(10^9)} = 0.00070735$$

$$\delta' = \varepsilon' L = 0.00070735(3000) = 2.12 \text{ mm}$$ **Ans**

4-1 The assembly consists of a steel rod CB and an aluminum rod BA, each having a diameter of 12 mm. If the rod is subjected to the axial loadings at A and at the coupling B, determine the displacement of the coupling B and the end A. The unstretched length of each segment is shown in the figure. Neglect the size of the connections at B and C, and assume that they are rigid. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



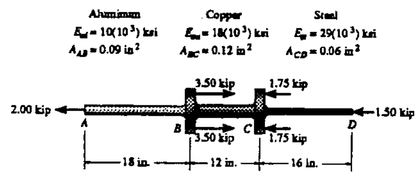


$$\delta_B = \frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} = 0.00159 \text{ m} = 1.59 \text{ mm} \qquad \textbf{Ans}$$

$$\delta_A = \Sigma\frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} + \frac{18(10^3)(2)}{\frac{\pi}{4}(0.012)^2(70)(10^9)}$$

$$= 0.00614 \text{ m} = 6.14 \text{ mm} \qquad \textbf{Ans}$$

4–2 The composite shaft, consisting of aluminum, copper, and steel sections, is subjected to the loading shown. Determine the displacement of end A with respect to end D and the normal stress in each section. The cross-sectional area and modulus of elasticity for each section are shown in the figure. Neglect the size of the collars at B and C.



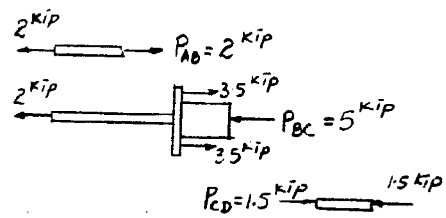
$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{2}{0.09} = 22.2 \text{ ksi} \quad \text{(T)} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{5}{0.12} = 41.7 \text{ ksi} \quad \text{(C)} \qquad \textbf{Ans}$$

$$\sigma_{CD} = \frac{P_{BC}}{A_{BC}} = \frac{1.5}{0.06} = 25.0 \text{ ksi} \quad \text{(C)} \qquad \textbf{Ans}$$

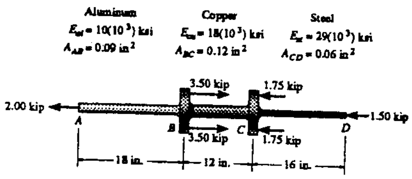
$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{2\,(18)}{(0.09)(10)(10^3)} + \frac{(-5)(12)}{(0.12)(18)(10^3)} + \frac{(-1.5)(16)}{(0.06)(29)(10^3)}$$

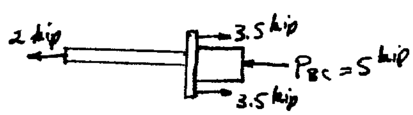
$$= -0.00157 \text{ in.} \qquad \textbf{Ans}$$



The negative sign indicates end A moves towards end D.

4–3 Determine the displacement of *B* with respect to *C* of the composite shaft in Prob. 4–2.

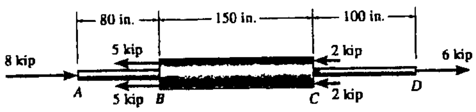


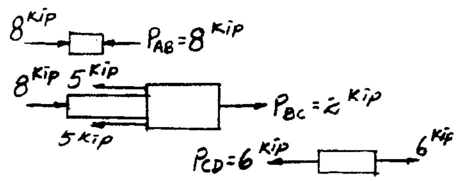


$$\delta_{B/C} = \frac{PL}{AE} = \frac{(-5)(12)}{(0.12)(18)(10^3)} = -0.0278 \text{ in.} \qquad \textbf{Ans}$$

The negative sign indicates end *B* moves towards end *C*.

•4–4 The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D if the diameters of each segment are $d_{AB} = 0.75$ in., $d_{BC} = 1$ in., and $d_{CD} = 0.5$ in. Take $E_{cu} = 18(10^3)$ ksi.



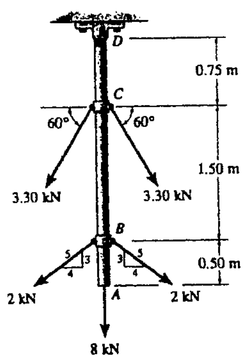


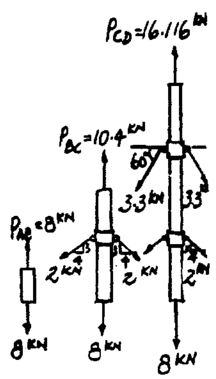
$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{-8(80)}{\frac{\pi}{4}(0.75)^2(18)(10^3)} + \frac{2(150)}{\frac{\pi}{4}(1)^2(18)(10^3)} + \frac{6(100)}{\frac{\pi}{4}(0.5)^2(18)(10^3)}$$

$= 0.111$ in. **Ans**

The positive sign indicates that end A moves away from end D.

4–5 The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 60 mm², determine the displacement of B and A. Neglect the size of the couplings at B, C, and D.



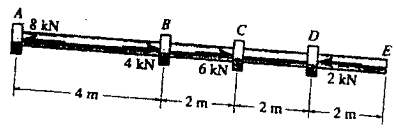


$$\delta_B = \Sigma\frac{PL}{AE} = \frac{16.116\,(10^3)(0.75)}{60\,(10^{-6})(200)(10^9)} + \frac{10.4\,(10^3)(1.50)}{60(10^{-6})(200)(10^9)}$$

$$= 0.00231 \text{ m} = 2.31 \text{ mm} \qquad \textbf{Ans}$$

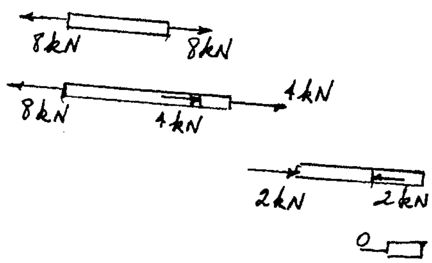
$$\delta_A = \delta_B + \frac{8\,(10^3)(0.5)}{60(10^{-6})(200)(10^9)} = 0.00264 \text{ m} = 2.64 \text{ mm} \qquad \textbf{Ans}$$

4–6 The 2014-T6 aluminum rod has a diameter of 30 mm and supports the load shown. Determine the displacement of A with respect to E. Neglect the size of the couplings.

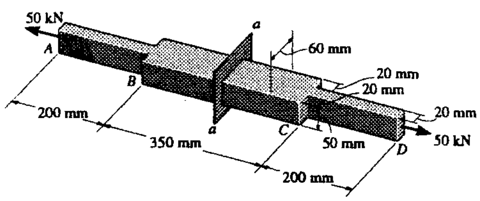


$$\delta_{A/E} = \Sigma\frac{PL}{AE} = \frac{1}{AE}[8(4) + 4(2) - 2(2) + 0(2)](10^3)$$

$$= \frac{36(10^3)}{\frac{\pi}{4}(0.03)^2(73.1)(10^9)} = 0.697\ (10^{-3}) = 0.697\ \text{mm} \qquad \textbf{Ans}$$



4–7 The steel bar has the original dimensions shown in the figure. If it is subjected to an axial load of 50 kN, determine the change in its length and its new cross-sectional dimensions at section *a–a*. E_{st} = 200 GPa, ν_{st} = 0.29.



$$\delta_{A/D} = \Sigma\frac{PL}{AE} = \frac{2(50)(10^3)(200)}{(0.02)(0.05)(200)(10^9)} + \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)}$$

$$= 0.129 \text{ mm} \qquad \textbf{Ans}$$

$$\delta_{B/C} = \frac{PL}{AE} = \frac{50(10^3)(350)}{(0.06)(0.05)(200)(10^9)} = 0.02917 \text{ mm}$$

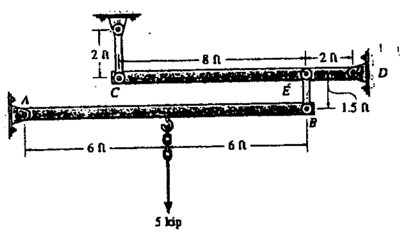
$$\varepsilon_{BC} = \frac{\delta_{B/C}}{L_{BC}} \frac{0.02917}{350} = 0.00008333$$

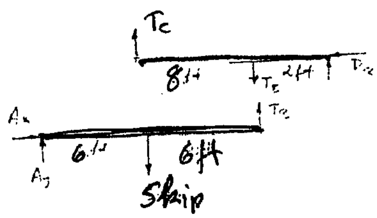
$$\varepsilon_{lat} = -\nu\,\varepsilon_{long} = -(0.29)(0.00008333) = -0.00002417$$

$$h' = 50 - 50(0.00002417) = 49.9988 \text{ mm} \qquad \textbf{Ans}$$

$$w' = 60 - 60(0.00002417) = 59.9986 \text{ mm} \qquad \textbf{Ans}$$

***4-8.** The assembly consists of two rigid bars that are originally horizontal. They are supported by pins and 0.25-in.-diameter A-36 steel rods. If the vertical load of 5 kip is applied to the bottom bar AB, determine the displacement at C, B, and E.



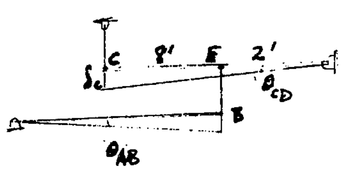


$\circlearrowleft + \Sigma M_A = 0;\quad T_B(12) - 5(6) = 0$

$T_B = 2.5 \text{ kip}$

$\circlearrowleft + \Sigma M_D = 0;\quad 2.5(2) - T_C(10) = 0$

$T_C = 0.5 \text{ kip}$



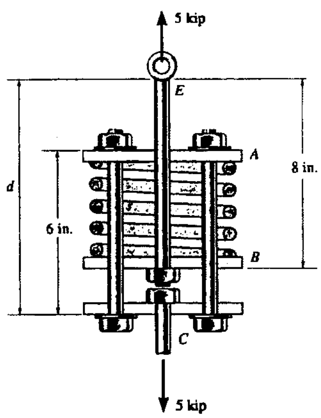
$$\delta_{B/E} = \frac{PL}{AE} = \frac{2.5(1.5)(12)}{\frac{\pi}{4}(0.25)^2(29)(10^3)} = 0.0316 \text{ in.}$$

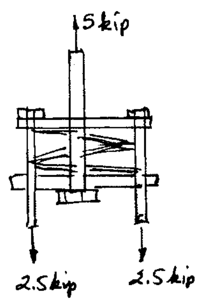
$$\delta_C = \frac{PL}{AE} = \frac{0.5(2)(12)}{\frac{\pi}{4}(0.25)^2(29)(10^3)} = 0.0084297 \text{ in.} = 0.00843 \text{ in.} \qquad \textbf{Ans}$$

$$\delta_E = \left(\frac{2}{10}\right)\delta_C = \frac{2}{10}(0.0084297) = 0.00169 \text{ in.} \qquad \textbf{Ans}$$

$$\delta_B = \delta_E + \delta_{B/E} = 0.00169 + 0.0316 = 0.0333 \text{ in.} \qquad \textbf{Ans}$$

4–9 The coupling is subjected to a force of 5 kip. Determine the distance d' between C and E accounting for the compression of the spring and the deformation of the vertical segments of the bolts. When no load is applied the spring is unstretched and d = 10 in. The material is A-36 steel and each bolt has a diameter of 0.25 in. The plates at A, B, and C are rigid and the spring has a stiffness of k = 12 kip/in.





$$\delta_{\text{center bolt}} = \frac{PL}{AE} = \frac{5(10^3)(8)}{\frac{\pi}{4}(0.25)^2(29)(10^6)} = 0.028099 \text{ in.} \uparrow$$

$$\delta_{\text{side bolts}} = \frac{PL}{AE} = \frac{2.5(10^3)(6)}{\frac{\pi}{4}(0.25)^2(29)(10^6)} = 0.010537 \text{ in.} \downarrow$$

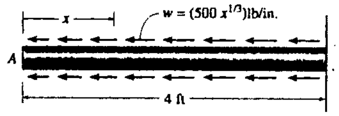
$$\delta_{sp} = \frac{P}{k} = \frac{5}{12} = 0.41667 \text{ in.} \uparrow$$

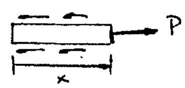
$$\delta d = 0.41667 + 0.028099 + 0.010537$$

$$\delta d = 0.455 \text{ in.}$$

$d = 10 + 0.455 = 10.455$ in. **Ans**

4–10 The bar has a cross-sectional area of $A = 3\ \text{in}^2$, and $E = 35(10^3)$ksi. Determine the displacement of its end A when it is subjected to the distributed loading.



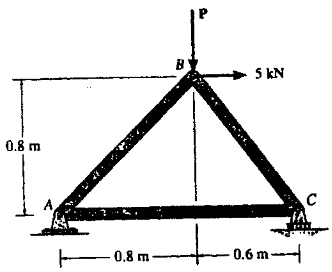


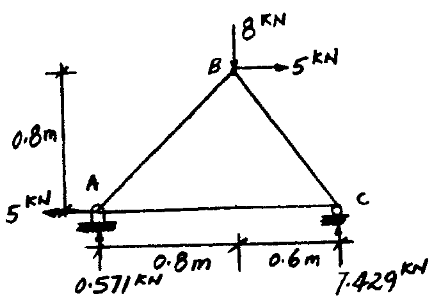
$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \frac{1500}{4}x^{\frac{4}{3}}$$

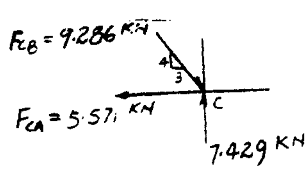
$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \frac{1}{(3)(35)(10^6)}\int_0^{4(12)} \frac{1500}{4}x^{\frac{4}{3}}\,dx = \left(\frac{1500}{(3)(35)(10^6)(4)}\right)\left(\frac{3}{7}\right)(48)^{\frac{7}{3}}$$

$\delta_A = 0.0128$ in. **Ans**

4-11 The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm^2. Determine the horizontal displacement of the roller at C when $P = 8$ kN.





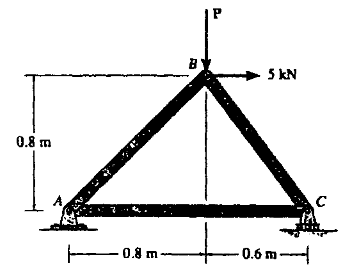


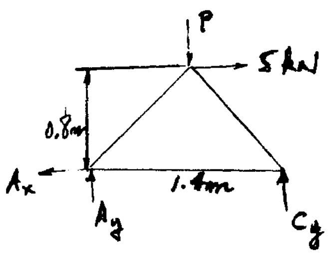
By observation the horizontal displacement of roller C is equal to the displacement of point C obtained from member AC.

$F_{CA} = 5.571$ kN

$$\delta_{C_h} = \frac{F_{CA}L}{AE} = \frac{5.571(10^3)(1.40)}{(400)(10^{-6})(200)(10^6)} = 0.0975 \text{ mm} \qquad \textbf{Ans}$$

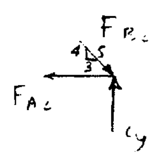
***4–12** The truss is made of three A-36 steel members, each having a cross-sectional area of 400 mm². Determine the magnitude P required to displace the roller to the right 0.2 mm.





$$\curvearrowleft + M_A = 0; \qquad -P(0.8) - 5(0.8) + C_y(1.4) = 0$$

$$C_y = 0.5714\,P + 2.857$$



$$+\uparrow \Sigma F_y = 0; \qquad C_y - F_{BC}\left(\frac{4}{5}\right) = 0$$

$$F_{BC} = 1.25\,C_y$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC} + 1.25\,C_y(0.6) = 0$$

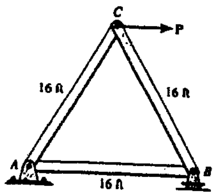
$$F_{AC} = 0.75\,C_y = 0.4286\,P + 2.14286$$

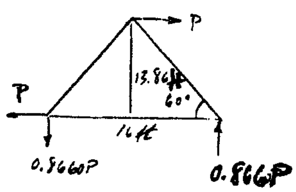
Require,

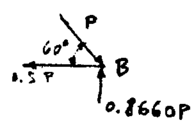
$$\delta_{C_h} = 0.0002 = \frac{(0.4286\,P + 2.14286)(10^3)(1.4)}{(400)(10^{-6})(200)(10^9)}$$

$$P = 21.7\text{ kN} \qquad \textbf{Ans}$$

4-13. The truss consists of three members, each made from A-36 steel and having a cross-sectional area of $0.75\ in^2$. Determine the greatest load P that can be applied so that the roller support at B is not displaced more than 0.03 in.



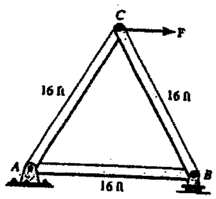


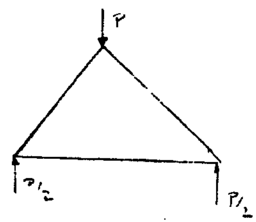


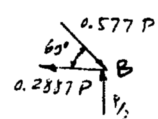
$$\delta_{B_h} = 0.03 \text{ in.} = \frac{(0.5)P(16)(12)}{(0.75)(29)(10^6)}$$

$P = 6.80$ kip **Ans**

4-14. Solve Prob. 4–13 when the load **P** acts vertically downward at *C*.





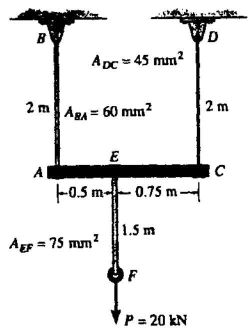


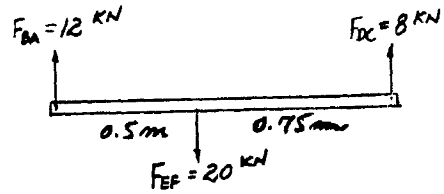
Require,

$$\delta_{B_A} = 0.03 \text{ in.} = \frac{0.2887\,P(16)(12)}{(0.75)(29)(10^6)}$$

$P = 11.8$ kip **Ans**

4–15 The assembly consists of three titanium rods and a rigid bar AC. The cross-sectional area of each rod is given in the figure. If a vertical force of $P = 20$ kN is applied to the ring F, determine the vertical displacement of point F. $E_{ti} = 350$ GPa.





$$\delta_A = \frac{PL}{AE} = \frac{12(10^3)(2000)}{(60)(10^{-6})(350)(10^9)} = 1.1429 \text{ mm}$$

$$\delta_C = \frac{PL}{AE} = \frac{8(10^3)(2000)}{45(10^{-6})(350)(10^9)} = 1.0159 \text{ mm}$$

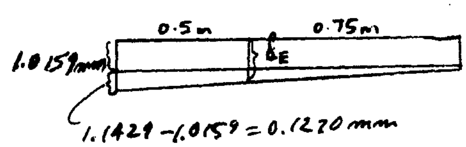
$$\delta_{F/E} = \frac{PL}{AE} = \frac{20(10^3)(1500)}{75(10^{-6})(350)(10^9)} = 1.1429 \text{ mm}$$

$$\delta_E = 1.0159 + \frac{0.75}{1.25}(0.1270) = 1.092 \text{ mm}$$

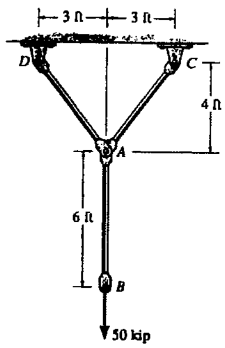
$\delta_F = \delta_E + \delta_{F/E}$

$= 1.092 + 1.1429$

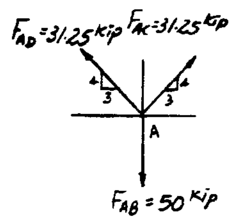
$= 2.23$ mm **Ans**



•4-16 The linkage is made of three pin-connected A-36 steel members, each having a cross-sectional area of 0.730 in². If a vertical force of $P = 50$ kip is applied to the end B of member AB, determine the vertical displacement of point B.



$$\delta_{A/D} = \delta_{A/C} = \frac{PL}{AE} = \frac{31.25(5)(12)}{(0.730)(29)(10^3)} = 0.08857 \text{ in.}$$



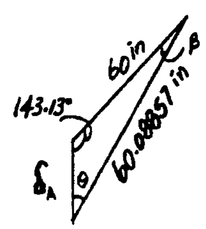
$$\delta_{B/A} = \frac{PL}{AE} = \frac{50(6)(12)}{(0.730)(29)(10^3)} = 0.17005 \text{ in.}$$

$$\phi = 90° + \tan^{-1}\left(\frac{4}{3}\right) = 143.13°$$

$$\frac{\sin\theta}{60} = \frac{\sin 143.13°}{60.08857}\ ;\ \theta = 36.806584°$$

$$\beta = 180° - 36.806584° - 143.130102° = 0.06331297°$$

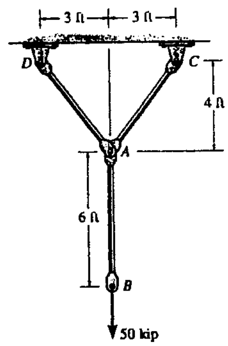
$$\frac{\delta_A}{\sin 0.06331297°} = \frac{60}{\sin 36.806584°}$$

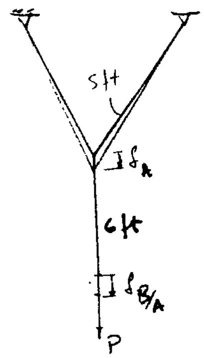


$$\delta_A = 0.11066 \text{ in.}$$

$$\delta_B = \delta_A + \delta_{B/A} = 0.11066 + 0.17005 = 0.281 \text{ in.} \qquad \textbf{Ans}$$

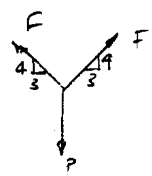
4–17 The linkage is made of three pin-connected 304 stainless steel members, each having a cross-sectional area of 0.75 in². Determine the magnitude of the force **P** needed to displace point *B* 0.10 in. downward.





$$\delta_B = \delta_A + \delta_{B/A} = 0.10 \text{ in.} \qquad (1)$$

$$\delta_{B/A} = \frac{PL}{AE} = \frac{P(6)(12)}{(0.75)(29)(10^3)} = 0.0033103P$$



$$+\uparrow \Sigma F_y = 0; \qquad 2F\left(\frac{4}{5}\right) - P = 0$$

$$F = 0.625\,P$$

$$\delta_{A/C} = \delta_{A/D} = \frac{0.625P(5)(12)}{(0.75)(29)(10^3)} = 0.0017241P$$

$$\delta_A = \delta_{A/C}\left(\frac{5}{4}\right) = 0.0021552P$$

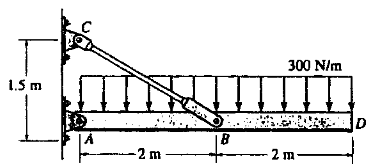
From Eq. (1),

$$0.0033103P + 0.0021552P = 0.10$$

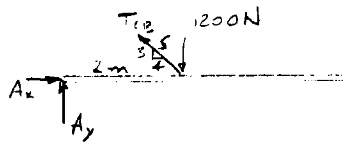
$P = 18.3$ kip **Ans.**

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

4–19 The rigid bar is supported by the pin-connected rod *CB* that has a cross-sectional area of 14 mm² and is made from 6061-T6 aluminum. Determine the vertical deflection of the bar at *D* when the distributed load is applied.



$\curvearrowleft + \Sigma M_A = 0; \qquad 1200(2) - T_{CB}(0.6)(2) = 0$



$$T_{CB} = 2000 \text{ N}$$

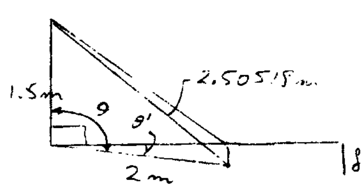
$$\delta_{B/C} = \frac{PL}{AE} = \frac{(2000)(2.5)}{14(10^{-6})(68.9)(10^9)} = 0.0051835$$

$$(2.5051835)^2 = (1.5)^2 + (2)^2 - 2(1.5)(2)\cos\theta$$

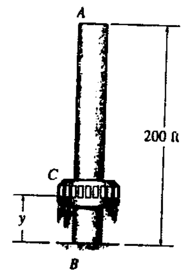
$$\theta = 90.248°$$

$$\theta' = 90.248° - 90° = 0.2478° = 0.004324 \text{ rad}$$

$$\delta_D = \theta r = 0.004324(4000) = 17.3 \text{ mm}$$ **Ans**



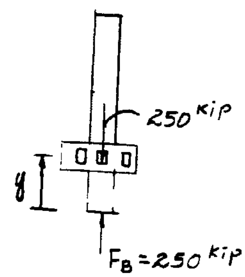
•4-20 The observation cage C has a weight of 250 kip and through a system of gears, travels upward at constant velocity along the A-36 steel column, which has a height of 200 ft. The column has an outer diameter of 3 ft and is made from steel plate having a thickness of 0.25 in. Neglect the weight of the column, and determine the average normal stress in the column at its base, B, as a function of the cage's position y. Also, determine the displacement of end A as a function of y.



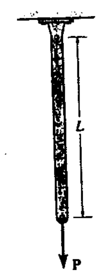
$$\sigma_B = \frac{P}{A} = \frac{250}{\frac{\pi}{4}(36^2 - 35.5^2)} = 8.90 \text{ ksi}$$

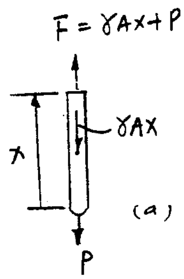
σ_B is independent of y.

$$\delta_A = \frac{PL}{AE} = \frac{250y}{\frac{\pi}{4}(36^2 - 35.5^2)(29)(10^3)} = [\,0.307(10^{-3})y]\ \text{ft} \qquad \textbf{Ans}$$



4–21 The bar has a length L and cross-sectional area A. Determine its elongation due to both the force **P** and its own weight. The material has a specific weight γ (weight/volume) and a modulus of elasticity E.

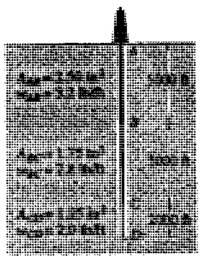


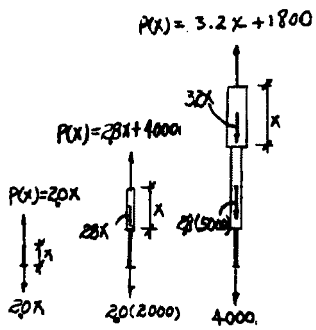


$$\delta = \int \frac{P(x)\,dx}{A(x)\,E} = \frac{1}{AE}\int_0^L (\gamma Ax + P)\,dx$$

$$= \frac{1}{AE}\left(\frac{\gamma AL^2}{2} + PL\right) = \frac{\gamma L^2}{2E} + \frac{PL}{AE} \qquad \textbf{Ans}$$

4–22 The A-36 steel drill shaft of an oil well extends 12 000 ft into the ground. Assuming that the pipe used to drill the well is suspended freely from the derrick at A, determine the maximum average normal stress in each pipe segment and the elongation of its end D with respect to the fixed end at A. The shaft consists of three different sizes of pipe, AB, BC, and CD, each having the length, weight per unit length, and cross-sectional area indicated. *Hint:* Use the results of Prob. 4–21.





$$\sigma_A = \frac{P}{A} = \frac{3.2(5000) + 18000}{2.5} = 13.6 \text{ ksi} \qquad \textbf{Ans}$$

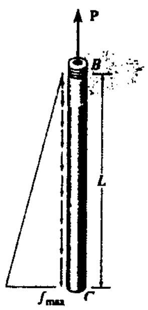
$$\sigma_B = \frac{P}{A} = \frac{2.8(5000) + 4000}{1.75} = 10.3 \text{ ksi} \qquad \textbf{Ans}$$

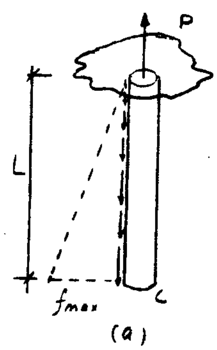
$$\sigma_C = \frac{P}{A} = \frac{2(2000)}{1.25} = 3.2 \text{ ksi} \qquad \textbf{Ans}$$

$$\delta_D = \Sigma \int \frac{P(x)\,dx}{A(x)\,E} = \int_0^{2000} \frac{2x\,dx}{(1.25)(29)(10^6)} + \int_0^{5000} \frac{(2.8x + 4000)dx}{(1.75)(29)(10^6)} + \int_0^{5000} \frac{(3.2x + 18000)dx}{(2.5)(29)(10^6)}$$

$$= 2.99 \text{ ft} \qquad \textbf{Ans}$$

4–23 The pipe is stuck in the ground so that when it is pulled upward the frictional force along its length varies linearly from zero at *B* to f_{max} (force/length) at *C*. Determine the initial force *P* required to pull the pipe out and the pipe's associated elongation just before it starts to slip. The pipe has a length *L*, cross-sectional area *A*, and the material from which it is made has a modulus of elasticity *E*.

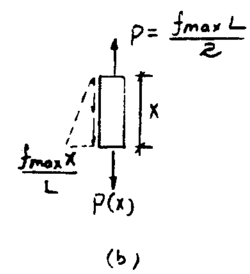




From FBD (a)

$$+\uparrow \Sigma F_y = 0; \qquad P - \frac{1}{2}(F_{max}\, L) = 0$$

$$P = \frac{F_{max}\, L}{2} \qquad \textbf{Ans}$$



From FBD (b)

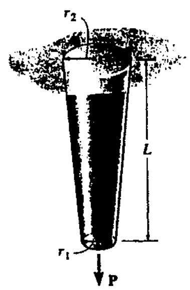
$$+\downarrow \Sigma F_y = 0; \qquad P(x) + \frac{1}{2}\left(\frac{F_{max}\, x}{L}\right)x - \frac{F_{max}\, L}{2} = 0$$

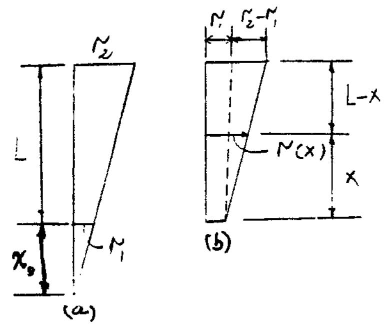
$$P(x) = \frac{F_{max} L}{2} - \frac{F_{max}\, x^2}{2L}$$

$$\delta = \int_0^L \frac{P(x)\, dx}{A(x)E} = \int_0^L \frac{F_{max} L}{2AE} dx - \int_0^L \frac{F_{max}\, x^2}{2AEL} dx$$

$$= \frac{F_{max} L^2}{3AE} \qquad \textbf{Ans}$$

•4-24 The rod has a slight taper and length L. It is suspended from the ceiling and supports a load **P** at its end. Show that the displacement of its end due to this load is $\delta = PL/(\pi E r_2 r_1)$. Neglect the weight of the material. The modulus of elasticity is E.





$$\frac{L + x_0}{r_2} = \frac{x_0}{r_1}; \qquad x_0 = \frac{L r_1}{r_2 - r_1}$$

Thus, $r(x) = r_1 + \dfrac{r_2 - r_1}{L}x = \dfrac{r_1 L + (r_2 - r_1)x}{L}$

$$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$$

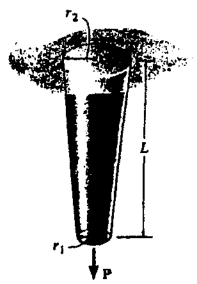
$$\delta = \int \frac{P\,dx}{A(x)E} = \frac{PL^2}{\pi E}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$$

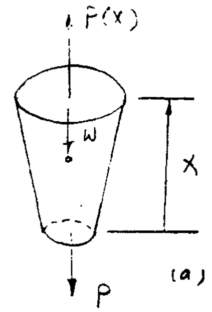
$$= -\frac{PL^2}{\pi E}\left[\frac{1}{(r_2 - r_2)(r_1 L + (r_2 - r_1)x)}\right]\Bigg|_0^L = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_1 L + (r_2 - r_1)L} - \frac{1}{r_1 L}\right]$$

$$= -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_2 L} - \frac{1}{r_1 L}\right] = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_1 - r_2}{r_2 r_1 L}\right]$$

$$= \frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_2 - r_1}{r_2 r_1 L}\right] = \frac{PL}{\pi E r_2 r_1} \qquad \textbf{QED}$$

4–25 Solve Prob. 4-24 by including the weight of the material, considering its specific weight to be γ (weight/volume).

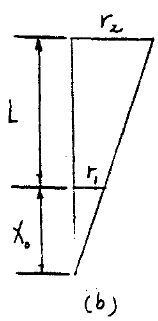




$$+\uparrow \Sigma F_x = 0; \quad P(x) - P - W = 0; \quad P(x) = P + W$$

From diagram (b)

$$\frac{L + x_0}{r_2} = \frac{x_0}{r_1}; \quad x_0 = \frac{L\,r_1}{r_2 - r_1}$$



From diagram (c)

$$r(x) = r_1 + \frac{r_2 - r_1}{L}x = \frac{r_1 L + (r_2 - r_1)x}{L}$$

$$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$$

$$W = \frac{\gamma\pi}{3L^2}[r_1 L + (r_2 - r_1)x]^2[x + \frac{L\,r_1}{r_2 - r_1}] - \frac{\gamma\pi}{3}(r_1^2)(\frac{L\,r_1}{r_2 - r_1})$$

$$= \frac{\gamma\pi}{3L^2(r_2 - r_1)}\{[r_1 L + (r_2 - r_1)x]^3 - r_1^3 L^3\}$$

$$\delta = \int\frac{W\,dx}{A(x)E} = \frac{\gamma}{3E(r_2 - r_1)}\int_0^L \frac{[r_1 L + (r_2 - r_1)x]^3 - r_1^3 L^3}{[r_1 L + (r_2 - r_1)x]^2}dx$$

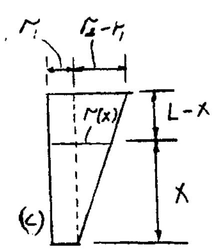
$$= \frac{\gamma}{3E(r_2 - r_1)}\int_0^L [r_1 L + (r_2 - r_1)x]\,dx - \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$$

$$= \frac{\gamma}{3E(r_2 - r_1)}[r_1 L x + \frac{(r_2 - r_1)x^2}{2}]\Big|_0^L + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}[\frac{1}{r_1 L + (r_2 - r_1)x}]\Big|_0^L$$

$$= \frac{\gamma}{3E(r_2 - r_1)}[r_1 L^2 + \frac{(r_2 - r_1)L^2}{2}] + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}[\frac{1}{r_2 L} - \frac{1}{r_1 L}]$$

$$= \frac{\gamma}{6E(r_2 - r_1)}[2r_1 L^2 + r_2 L^2 - r_1 L^2] + \frac{\gamma r_1^3 L^3}{3E(r_2 - r_1)^2}[\frac{-(r_2 - r_1)}{r_2 r_1 L}]$$

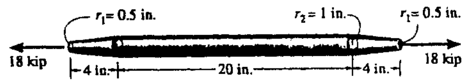
$$\delta = \frac{\gamma L^2(r_2 + r_1)}{6E(r_2 - r_1)} - \frac{\gamma L^2 r_1^2}{3E\,r_2(r_2 - r_1)}$$



Therefore, adding the result of Prob. (4 - 24) we have

$$\delta = \frac{PL}{\pi E\,r_2 r_1} + \frac{\gamma L^2(r_2 + r_1)}{6E(r_2 - r_1)} - \frac{\gamma L^2 r_1^2}{3E\,r_2(r_2 - r_1)} \qquad \textbf{Ans}$$

4–26 Determine the elongation of the tapered A-36 steel shaft when it is subjected to an axial force of 18 kip. *Hint:* Use the result of Prob. 4–24.

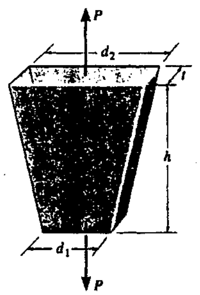


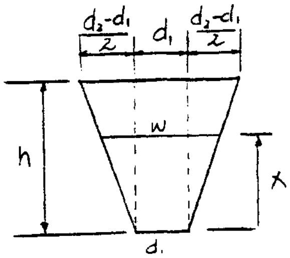
$$\delta = (2)\frac{PL_1}{\pi E\, r_2 r_1} + \frac{PL_2}{AE}$$

$$= \frac{(2)(18)(4)}{\pi(29)(10^3)(1)(0.5)} + \frac{18(20)}{\pi(1)^2(29)(10^3)}$$

$$= 0.00711 \text{ in.}$$ **Ans**

4–27 Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.





$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

$$\delta = \int \frac{P(x)\,dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\frac{[d_1 h + (d_2 - d_1)x]t}{h}}$$

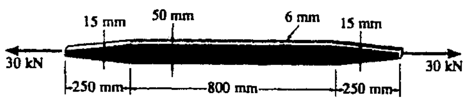
$$= \frac{Ph}{Et}\int_0^h \frac{dx}{d_1 h + (d_2 - d_1)x}$$

$$= \frac{Ph}{Etd_1 h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1 h}x} = \frac{Ph}{Etd_1 h}\left(\frac{d_1 h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1 h}x\right)\right]\Big|_0^h$$

$$= \frac{Ph}{Et(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{Et(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

$$= \frac{Ph}{Et(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right]$$ **Ans**

•4–28 Determine the elongation of the aluminum strap when it is subjected to an axial force of 30 kN. E_{al} = 70 GPa. *Hint:* Use the result of Prob. 4–27.

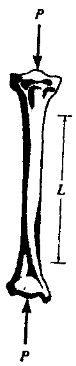


$$\delta = (2)\frac{Ph}{Et(d_2 - d_1)}\ln\frac{d_2}{d_1} + \frac{PL}{AE}$$

$$= \frac{2(30)(10^3)(250)}{(70)(10^9)(0.006)(0.05 - 0.015)}(\ln\frac{50}{15}) + \frac{30(10^3)(800)}{(0.006)(0.05)(70)(10^9)}$$

$= 2.37$ mm **Ans**

4-29. Bone material has a stress–strain diagram that can be defined by the relation $\sigma = E[\epsilon/(1 + kE\epsilon)]$, where k and E are constants. Determine the compression within the length L of the bone, where it is assumed the cross-sectional area A of the bone is constant.



$$\sigma = \frac{P}{A}; \qquad \varepsilon = \frac{\delta x}{dx}$$

$$\sigma = E\left(\frac{\varepsilon}{1 + kE\varepsilon}\right); \qquad \frac{P}{A} = \frac{E\left(\frac{\delta x}{dx}\right)}{1 + kE\left(\frac{\delta x}{dx}\right)}$$

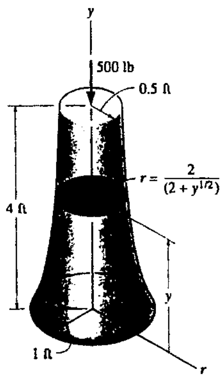
$$\frac{P}{A} + \frac{PkE}{A}\left(\frac{\delta x}{dx}\right) = E\left(\frac{\delta x}{dx}\right)$$

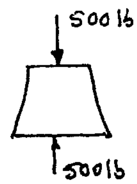
$$\frac{P}{A} = \left(E - \frac{PkE}{A}\right)\left(\frac{\delta x}{dx}\right)$$

$$\int_0^{\delta} \delta x = \int_0^{L} \frac{P\,dx}{A\left(kE - \frac{Pk}{A}\right)}$$

$$\delta = \frac{\frac{PL}{AE}}{\left(1 - \frac{Pk}{A}\right)} = \frac{PL}{E(A - Pk)}$$ **Ans.**

4-30 The pedestal is made in a shape that has a radius defined by the function $r = 2/(2 + y^{1/2})$ ft, where y is in feet. If the modulus of elasticity for the material is $E = 14(10^3)$ psi, determine the displacement of its top when it supports the 500-lb load.





$$\delta = \int \frac{P(y)\,dy}{A(y)E}$$

$$= \frac{500}{14(10^3)(144)} \int_0^4 \frac{dy}{\pi\left(\frac{2}{2+y^{\frac{1}{2}}}\right)^2}$$

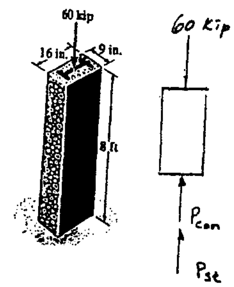
$$= 0.01974(10^{-3}) \int_0^4 (4 + 4y^{\frac{1}{2}} + y)\,dy$$

$$= 0.01974(10^{-3})\left[4y + 4\left(\frac{2}{3}y^{\frac{3}{2}}\right) + \frac{1}{2}y^2\right]_0^4$$

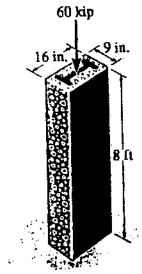
$$= 0.01974(10^{-3})(45.33)$$

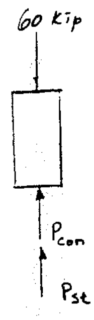
$$= 0.8947(10^{-3})\text{ ft} = 0.0107\text{ in.} \qquad \textbf{Ans}$$

4-31. The A-36 steel column, having a cross-sectional area of 18 in^2, is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the average compressive stress in the concrete and in the steel. How far does the column shorten? It has an original length of 8 ft.



•4-32 The A-36 steel column is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the required area of the steel so that the force is shared equally between the steel and concrete. How far does the column shorten? It has an original length of 8 ft.





The force of 60 kip is shared equally by the concrete and steel. Hence

$$P_{st} = P_{con} = P = 30\text{ kip}$$

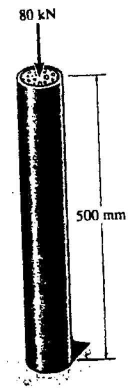
$$\delta_{con} = \delta_{st}; \qquad \frac{PL}{A_{con}E_{con}} = \frac{PL}{A_{st}E_{st}}$$

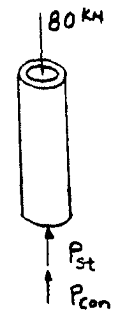
$$A_{st} = \frac{A_{con}E_{con}}{E_{st}} = \frac{[9(16) - A_{st}]\,4.20(10^3)}{29(10^3)}$$

$$= 18.2\text{ in}^2 \qquad \textbf{Ans}$$

$$\delta = \frac{P_{st}L}{A_{st}E_{st}} = \frac{30(8)(12)}{18.2(29)(10^3)} = 0.00545\text{ in.} \qquad \textbf{Ans}$$

4–33 The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm. $E_{st} = 200$ GPa, $E_c = 24$ GPa.





$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{con} - 80 = 0 \qquad (1)$$

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}\,L}{\frac{\pi}{4}(0.08^2 - 0.07^2)(200)(10^9)} = \frac{P_{con}\,L}{\frac{\pi}{4}(0.07^2)(24)(10^9)}$$

$$P_{st} = 2.5510\,P_{con} \qquad (2)$$

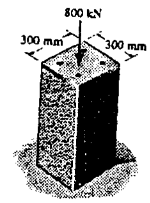
Solving Eqs. (1) and (2) yields

$$P_{st} = 57.47 \text{ kN} \qquad P_{con} = 22.53 \text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{57.47\,(10^3)}{\frac{\pi}{4}(0.08^2 - 0.07^2)} = 48.8 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{22.53\,(10^3)}{\frac{\pi}{4}(0.07^2)} = 5.85 \text{ MPa} \qquad \textbf{Ans}$$

4-34. The concrete column is reinforced using four steel reinforcing rods, each having a diameter of 18 mm. Determine the stress in the concrete and the steel if the column is subjected to an axial load of 800 kN. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



Equilibrium :

$$+\uparrow \Sigma F_y = 0; \quad P_{st} + P_{con} - 800 = 0 \qquad [1]$$

Compatibility :

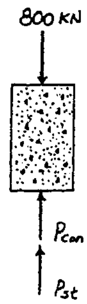
$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}(L)}{4\left(\frac{\pi}{4}\right)(0.018^2)(200)(10^9)} = \frac{P_{con}(L)}{\left[0.3^2 - 4\left(\frac{\pi}{4}\right)(0.018^2)\right](25)(10^9)}$$

$$P_{st} = 0.091513\, P_{con} \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$$P_{st} = 67.072 \text{ kN} \qquad P_{con} = 732.928 \text{ kN}$$

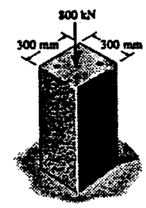


Average Normal Sress :

$$\sigma_{st} = \frac{67.072(10^3)}{4\left(\frac{\pi}{4}\right)(0.018^2)} = 65.9 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{con} = \frac{732.928(10^3)}{\left[0.3^2 - 4\left(\frac{\pi}{4}\right)(0.018^2)\right]} = 8.24 \text{ MPa} \qquad \textbf{Ans}$$

4-35. The column is constructed from high-strength concrete and four A-36 steel reinforcing rods. If it is subjected to an axial force of 800 kN, determine the required diameter of each rod so that one-fourth of the load is carried by the steel and three-fourths by the concrete. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



Equilibrium : Require $P_{st} = \frac{1}{4}(800) = 200$ kN and

$P_{con} = \frac{3}{4}(800) = 600$ kN.

Compatibility :

$$\delta_{con} = \delta_{st}$$

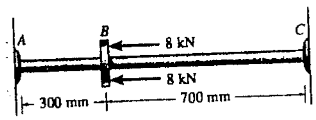
$$\frac{P_{con} L}{(0..3^2 - A_{st})(25.0)(10^9)} = \frac{P_{st} L}{A_{st}(200)(10^9)}$$

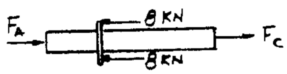
$$A_{st} = \frac{0.09 P_{st}}{8 P_{con} + P_{st}}$$

$$4\left[\left(\frac{\pi}{4}\right)d^2\right] = \frac{0.09(200)}{8\,(600) + 200}$$

$$d = 0.03385\text{m} = 33.9\text{mm}$$ **Ans**

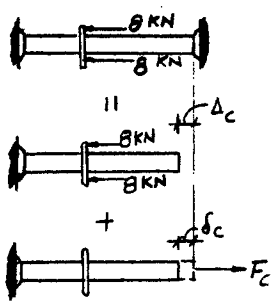
•4-36 The A-36 steel pipe has an outer radius of 20 mm and an inner radius of 15 mm. If it fits snugly between the fixed walls before it is loaded, determine the reaction at the walls when it is subjected to the load shown.





$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_C - 16 = 0 \qquad (1)$

By superposition :



$(\xrightarrow{+}) \qquad 0 = -\Delta_C + \delta_C$

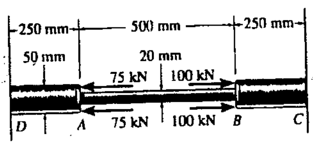
$$0 = \frac{-16\,(300)}{AE} + \frac{F_C\,(1000)}{AE}$$

$F_C = 4.80\text{ kN}$ **Ans**

From Eq. (1),

$F_A = 11.2\text{ kN}$ **Ans**

4-37 The composite bar consists of a 20-mm-diameter A-36 steel segment *AB* and 50-mm-diameter red brass C83400 end segments *DA* and *CB*. Determine the average normal stress in each segment due to the applied load.



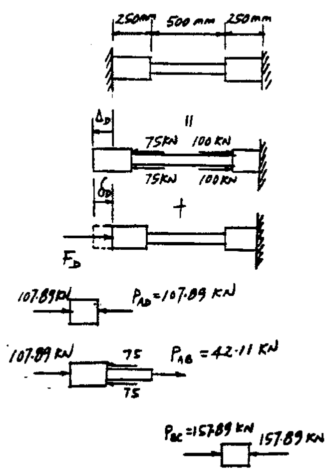
$$\overset{+}{\leftarrow}\Sigma F_x = 0; \qquad F_C - F_D + 75 + 75 - 100 - 100 = 0$$

$$F_C - F_D - 50 = 0 \qquad (1)$$

$$\overset{+}{\leftarrow} \qquad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(0.5)}{\frac{\pi}{4}(0.02)^2(200)(10^9)} - \frac{50(0.25)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.02)^2(200)(10^9)}$$

$$F_D = 107.89 \text{ kN}$$



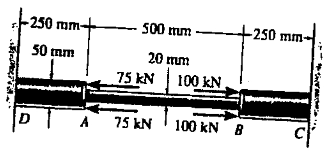
From Eq. (1), $F_C = 157.89$ kN

$$\sigma_{AD} = \frac{P_{AD}}{A_{AD}} = \frac{107.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 55.0 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{42.11(10^3)}{\frac{\pi}{4}(0.02^2)} = 134 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{157.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 80.4 \text{ MPa} \qquad \textbf{Ans}$$

4-38 The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the displacement of A with respect to B due to the applied load.

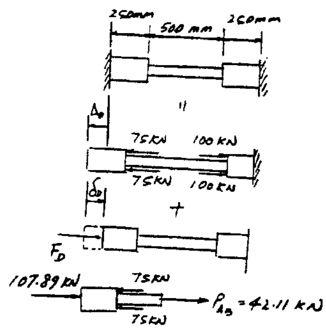


$$\overset{+}{\leftarrow} \qquad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(10^3)(500)}{\frac{\pi}{4}(0.02)^2(200)(10^9)} - \frac{50(10^3)(250)}{\frac{\pi}{4}(0.05^2)(101)(10^9)}$$

$$- \frac{F_D(500)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(500)}{\frac{\pi}{4}(0.02)^2(200)(10^9)}$$

$$F_D = 107.89 \text{ kN}$$

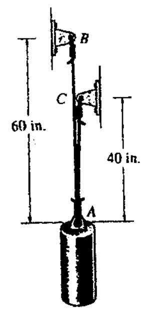


Displacement :

$$\delta_{A/B} = \frac{P_{AB}L_{AB}}{A_{AB}E_{st}} = \frac{42.11(10^3)(500)}{\frac{\pi}{4}(0.02^2)200(10^9)}$$

$$= 0.335 \text{ mm}$$ **Ans**

4-39 The load of 2800 lb is to be supported by the two essentially vertical A-36 steel wires. If originally wire AB is 60 in. long and wire AC is 40 in. long, determine the force developed in each wire after the load is suspended. Each wire has a cross-sectional area of 0.02 in².

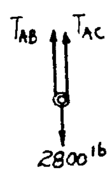


$$+\uparrow \Sigma F_y = 0; \quad T_{AB} + T_{AC} - 2800 = 0$$

$$\delta_{AB} = \delta_{AC}$$

$$\frac{T_{AB}\,(60)}{AE} = \frac{T_{AC}\,(40)}{AE}$$

$$1.5T_{AB} = T_{AC}$$

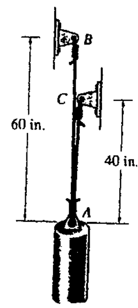


Solving,

$T_{AB} = 1.12$ kip **Ans**

$T_{AC} = 1.68$ kip **Ans**

•4–40 The load of 2800 lb is to be supported by the two essentially vertical A-36 steel wires. If originally wire AB is 60 in. long and wire AC is 40 in. long, determine the cross-sectional area of AB if the load is to be shared equally between both wires. Wire AC has a cross-sectional area of 0.02 in².



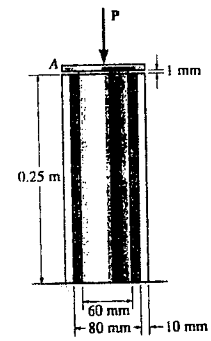
$$T_{AC} = T_{AB} = \frac{2800}{2} = 1400 \text{ lb}$$

$$\delta_{AC} = \delta_{AB}$$

$$\frac{1400(40)}{(0.02)(29)(10^6)} = \frac{1400(60)}{A_{AB}(29)(10^6)}$$

$$A_{AB} = 0.03 \text{ in}^2 \qquad \textbf{Ans}$$

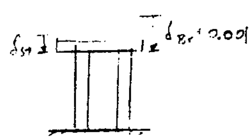
4–41 The support consists of a solid red brass C83400 post surrounded by a 304 stainless steel tube. Before the load is applied the gap between these two parts is 1 mm. Given the dimensions shown, determine the greatest axial load that can be applied to the rigid cap A without causing yielding of any one of the materials.



Require,

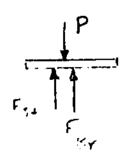
$$\delta_{st} = \delta_{br} + 0.001$$

$$\frac{F_{st}(0.25)}{\pi\left[(0.05)^2 - (0.04)^2\right]193(10^9)} = \frac{F_{br}(0.25)}{\pi(0.03)^2(101)(10^9)} + 0.001$$



$$0.45813\, F_{st} = 0.87544\, F_{br} + 10^6 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{st} + F_{br} - P = 0 \qquad (2)$$



Assume brass yields, then

$$(F_{br})_{max} = \sigma_Y A_{br} = 70(10^6)(\pi)(0.03)^2 = 197\,920.3 \text{ N}$$

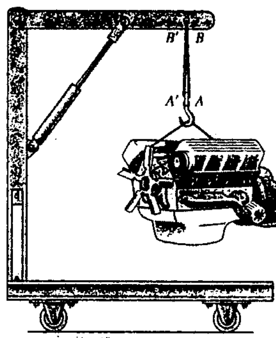
$$(\varepsilon_Y)_{br} = \sigma_Y/E = \frac{70.0(10^6)}{101(10^9)} = 0.6931(10^{-3}) \text{ mm/mm}$$

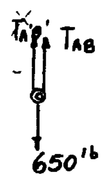
$$\delta_{br} = (\varepsilon_Y)_{br} L = 0.6931(10^{-3})(0.25) = 0.1733 \text{ mm} < 1 \text{ mm}$$

Thus only the brass is loaded.

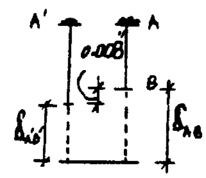
$$P = F_{br} = 198 \text{ kN} \qquad \textbf{Ans}$$

4–42 Two A-36 steel wires are used to support the 650-lb engine. Originally, AB is 32 in. long and $A'B'$ is 32.008 in. long. Determine the force supported by each wire when the engine is suspended from them. Each wire has a cross-sectional area of 0.01 in^2.





$$+\uparrow \Sigma F_y = 0; \qquad T_{A'B'} + T_{AB} - 650 = 0 \qquad (1)$$



$$\delta_{AB} = \delta_{A'B'} + 0.008$$

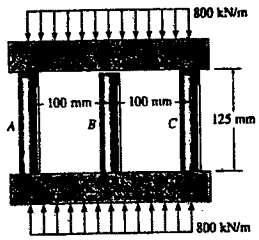
$$\frac{T_{AB}(32)}{(0.01)(29)(10^6)} = \frac{T_{A'B'}(32.008)}{(0.01)(29)(10^6)} + 0.008$$

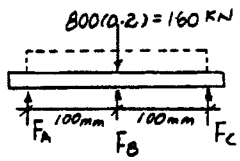
$$32T_{AB} - 32.008T_{A'B'} = 2320$$

$T_{AB} = 361$ lb **Ans**

$T_{A'B'} = 289$ lb **Ans**

4-43. The center post B of the assembly has an original length of 124.7 mm, whereas posts A and C have a length of 125 mm. If the caps on the top and bottom can be considered rigid, determine the average normal stress in each post. The posts are made of aluminum and have a cross-sectional area of 400 mm². E_{al} = 70 GPa.



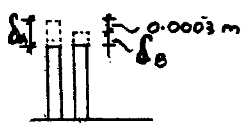


$\circlearrowleft + \Sigma M_B = 0; \quad -F_A(100) + F_C(100) = 0$

$F_A = F_C = F \qquad (1)$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_B - 160 = 0 \qquad (2)$

$\delta_A = \delta_B + 0.0003$



$$\frac{F\,(0.125)}{400\,(10^{-6})(70)(10^{6})} = \frac{F_B\,(0.1247)}{400\,(10^{-6})(70)(10^{6})} + 0.0003$$

$0.125\,F - 0.1247F_B = 8.4 \qquad (3)$

Solving Eqs. (2) and (3)

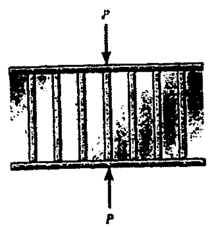
$F = 75.726$ kN

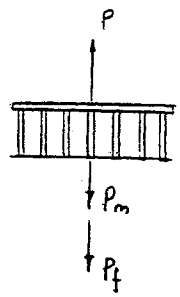
$F_B = 8.547$ kN

$$\sigma_A = \sigma_C = \frac{75.726\,(10^3)}{400(10^{-6})} = 189 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{8.547\,(10^3)}{400\,(10^{-6})} = 21.4 \text{ MPa} \qquad \textbf{Ans}$$

*4-44. The specimen represents a filament-reinforced matrix system made from plastic (matrix) and glass (fiber). If there are n fibers, each having a cross-sectional area of A_f and modulus of E_f, embedded in a matrix having a cross-sectional area of A_m and modulus of E_m, determine the stress in the matrix and each fiber when the force P is imposed on the specimen.





$$+\uparrow\Sigma F_y = 0; \quad P - P_m - P_f = 0 \qquad (1)$$

$$\delta_m = \delta_f$$

$$\frac{P_m L}{A_m E_m} = \frac{P_f L}{n A_f E_f}; \quad P_m = \frac{A_m E_m}{n A_f E_f} P_f \qquad (2)$$

Solving Eqs. (1) and (2) yields

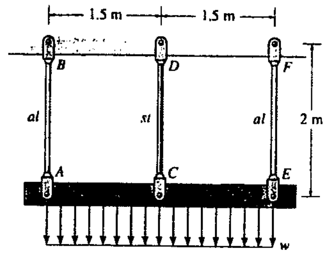
$$P_m = \frac{A_m E_m}{n A_f E_f + A_m E_m} P; \quad P_f = \frac{n A_f E_f}{n A_f E_f + A_m E_m} P$$

Normal stress :

$$\sigma_m = \frac{P_m}{A_m} = \frac{\left(\frac{A_m E_m}{n A_f E_f + A_m E_m} P\right)}{A_m} = \frac{E_m}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans}$$

$$\sigma_f = \frac{P_f}{n A_f} = \frac{\left(\frac{n A_f E_f}{n A_f E_f + A_m E_m} P\right)}{n A_f} = \frac{E_f}{n A_f E_f + A_m E_m} P \qquad \textbf{Ans}$$

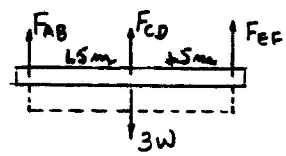
4–45 The distributed loading is supported by the three suspender bars. AB and EF are made from aluminum and CD is made from steel. If each bar has a cross-sectional area of 450 mm^2, determine the maximum intensity w of the distributed loading so that an allowable stress of $(\sigma_{allow})_{st} = 180$ MPa in the steel, and $(\sigma_{allow})_{al} = 94$ MPa in the aluminum is not exceeded. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



$\circlearrowleft + \Sigma M_C = 0; \quad F_{EF}(1.5) - F_{AB}(1.5) = 0$

$F_{EF} = F_{AB} = F$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_{CD} - 3w = 0 \qquad (1)$



Compatibility condition :

$\delta_A = \delta_C$

$$\frac{FL}{A(70)(10^9)} = \frac{F_{CD}L}{A(200)(10^9)}; \qquad F = 0.35\,F_{CD} \qquad (2)$$

Assume failure of AB and EF :

$F = (\sigma_{allow})_{al}A$

$= 94(10^6)(450)(10^{-6})$

$= 42300$ N

From Eq. (2) $\quad F_{CD} = 120857.14$ N

From Eq. (1) $\quad w = 68.5$ kN/m

Assume failure of CD :

$F_{CD} = (\sigma_{allow})_{st}A$

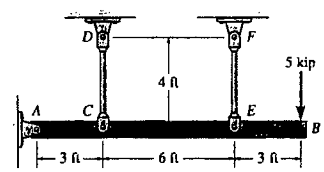
$= 180(10^6)(450)(10^{-6})$

$= 81000$ N

From Eq. (2) $\quad F = 28350$ N

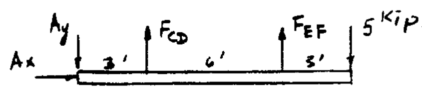
From Eq. (1) $\quad w = 45.9$ kN/m $\quad$ (controls) $\quad$ **Ans**

4–46 The beam is pinned at A and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the beam is assumed to be rigid and initially horizontal, determine the displacement of the end B when the force of 5 kip is applied.



$\curvearrowleft + \Sigma M_A = 0; \quad F_{CD}(3) + F_{EF}(9) - 5(12) = 0$

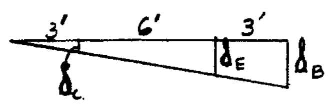
$$3F_{CD} + 9F_{EF} = 60 \qquad (1)$$



$$\frac{\delta_C}{3} = \frac{\delta_E}{9}$$

$$\frac{F_{CD}(L)}{3AE} = \frac{F_{EF}(L)}{9AE}$$

$$F_{EF} = 3F_{CD} \qquad (2)$$



Solving Eqs. (1) and (2) yields

$F_{CD} = 2$ kip

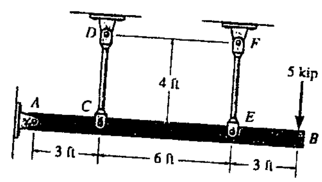
$F_{EF} = 6$ kip

$$\delta_E = \frac{F_{EF}L}{AE} = \frac{6(4)(12)}{\frac{\pi}{4}(1)^2(10)(10^3)} = 0.03667 \text{ in.}$$

$$\frac{\delta_B}{12} = \frac{\delta_E}{9}$$

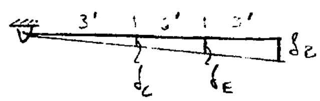
$$\delta_B = \left(\frac{12}{9}\right)(0.03667) = 0.0489 \text{ in.} \qquad \textbf{Ans}$$

4–47 The bar is pinned at A and supported by two aluminum rods, each having a diameter of 1 in. and a modulus of elasticity $E_{al} = 10(10^3)$ ksi. If the bar is assumed to be rigid and initially horizontal, determine the force in each rod when the 5-kip load is applied.



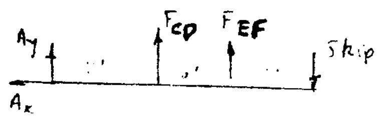
$\curvearrowleft + \Sigma M_A = 0; \quad F_{CD}(3) + F_{EF}(9) - 5(12) = 0 \quad (1)$

$\frac{\delta_C}{3} = \frac{\delta_E}{9}; \quad \delta_E = 3\,\delta_C$



$\frac{F_{EF}L}{AE} = \frac{3F_{CD}L}{AE}$

$F_{EF} = 3F_{CD}$

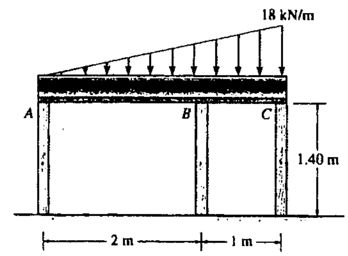


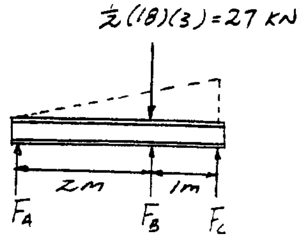
From Eq. (1),

$F_{CD} = 2$ kip **Ans**

$F_{EF} = 6$ kip **Ans**

*4–48 The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the vertical reactions at the supports. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.



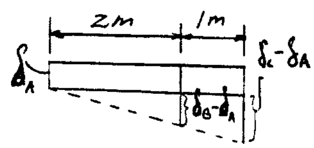


$\curvearrowleft + \Sigma M_B = 0; \quad F_C(1) - F_A(2) = 0 \qquad (1)$

$+\uparrow \Sigma F_y = 0; \quad F_A + F_B + F_C - 27 = 0 \qquad (2)$

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \quad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_BL}{AE} - \frac{F_AL}{AE} = \frac{2F_CL}{AE}; \quad 3F_B - F_A = 2F_C \qquad (3)$$



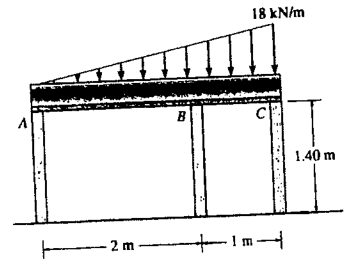
Solving Eqs. (1) – (3) yields :

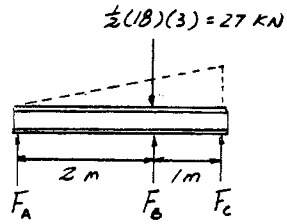
$F_A = 5.79$ kN **Ans**

$F_B = 9.64$ kN **Ans**

$F_C = 11.6$ kN **Ans**

4-49 The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the angle of tilt of the beam after the load is applied. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.



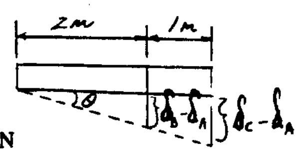


$$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(1) - F_A(2) = 0 \qquad (1)$$

$$\uparrow + \Sigma F_y = 0; \qquad F_A + F_B + F_C - 27 = 0 \qquad (2)$$

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \qquad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_BL}{AE} - \frac{F_AL}{AE} = \frac{2F_CL}{AE}; \quad 3F_B - F_A = 2F_C \qquad (3)$$



Solving Eqs. (1) – (3) yields :

$F_A = 5.7857$ kN; $\quad F_B = 9.6428$ kN; $\quad F_C = 11.5714$ kN

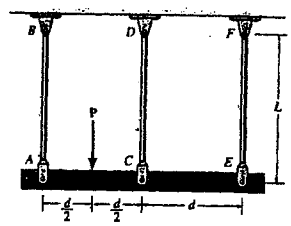
$$\delta_A = \frac{F_AL}{AE} = \frac{5.7857(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.0597(10^{-3})\text{ m}$$

$$\delta_C = \frac{F_CL}{AE} = \frac{11.5714(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.1194(10^{-3})\text{ m}$$

$$\tan\theta = \frac{0.1194 - 0.0597}{3}(10^{-3})$$

$$\theta = 0.0199(10^{-3})\text{ rad} = 1.14(10^{-3})^\circ \qquad \textbf{Ans}$$

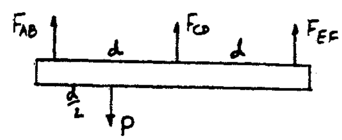
4-50. The three suspender bars are made of the same material and have equal cross-sectional areas A. Determine the average normal stress in each bar if the rigid beam ACE is subjected to the force **P**.



$$\circlearrowleft + \Sigma M_A = 0; \quad F_{CD}(d) + F_{EF}(2d) - P(\frac{d}{2}) = 0$$

$$F_{CD} + 2F_{EF} = \frac{P}{2} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad F_{AB} + F_{CD} + F_{EF} - P = 0 \qquad (2)$$

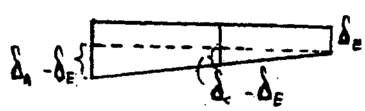


$$\frac{\delta_C - \delta_E}{d} = \frac{\delta_A - \delta_E}{2d}$$

$$2\delta_C = \delta_A + \delta_E$$

$$\frac{2F_{CD}L}{AE} = \frac{F_{AB}L}{AE} + \frac{F_{EF}L}{AE}$$

$$2F_{CD} - F_{AB} - F_{EF} = 0 \qquad (3)$$



Solving Eqs. (1), (2) and (3) yields

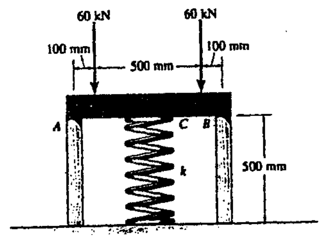
$$F_{AB} = \frac{7P}{12} \quad F_{CD} = \frac{P}{3} \quad F_{EF} = \frac{P}{12}$$

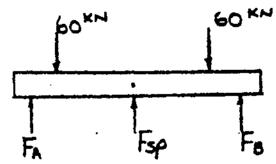
$$\sigma_{AB} = \frac{7P}{12A} \quad \textbf{Ans}$$

$$\sigma_{CD} = \frac{P}{3A} \quad \textbf{Ans}$$

$$\sigma_{EF} = \frac{P}{12A} \quad \textbf{Ans}$$

4-51. The rigid bar is supported by the two short wooden posts and a spring. If each of the posts has an unloaded length of 500 mm and a cross-sectional area of 800 mm^2, and the spring has a stiffness of $k = 1.8$ MN/m and an unstretched length of 520 mm, determine the force in each post after the load is applied to the bar. $E_w = 11$ GPa.





Due to symmetrical system and loading

$$F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \quad F_{sp} + 2F - 120(10^3) = 0 \quad (1)$$

Spring equation :

$$F_{sp} = k(\delta_A + 0.02)$$

$$= 1.8(10^6)\left(\frac{F(0.5)}{800(10^{-6})(11)(10^9)} + 0.02\right)$$

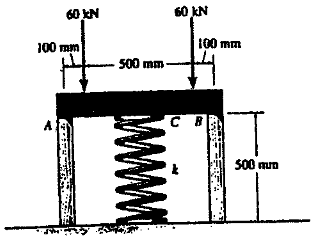
$$= 0.10227F + 36000 \quad (2)$$

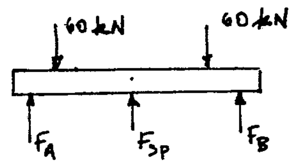
Solving Eqs. (1) and (2) yields

$$F_{sp} = 40.1 \text{ kN}$$

$$F = 40.0 \text{ kN}$$ **Ans**

***4-52.** The rigid bar is supported by the two short white spruce wooden posts and a spring. If each of the posts has an unloaded length of 500 mm and a cross-sectional area of 800 mm^2, and the spring has a stiffness of $k = 1.8\ \text{MN/m}$ and an unstretched length of 520 mm, determine the vertical displacement of A and B after the load is applied to the bar.





Due to symmetrical system and loading

$$F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{sp} + 2F - 120\,(10^3) = 0 \qquad (1)$$

Spring equation :

$$F_{sp} = k\,(\delta_A + 0.02)$$

$$= 1.8\,(10^6)\,\left(\frac{F\,(0.5)}{800\,(10^{-6})(11)(10^9)} + 0.02\right)$$

$$= 0.10227\,F + 36000 \qquad (2)$$

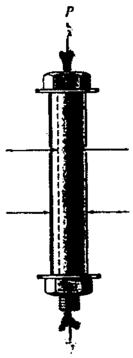
Solving Eqs. (1) and (2) yields

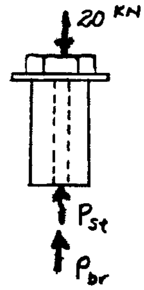
$$F_{sp} = 40.1\ \text{kN}$$

$$F = 40.0\ \text{kN}$$

$$\delta_A = \delta_B = \frac{FL}{AE} = \frac{40.0(10^3)(0.5)}{800(10^{-6})(11)(10^9)} = 0.00227\ \text{m} = 2.27\ \text{mm} \qquad \textbf{Ans}$$

4–53 The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the bolt is subjected to a compressive force of $P = 20$ kN, determine the average normal stress in the steel and the bronze. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.





$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 20 = 0 \qquad (1)$$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st} L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br} L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br} \qquad (2)$$

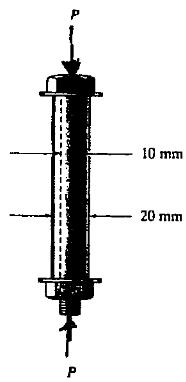
Solving Eqs (1) and (2) yields

$$P_{st} = 8 \text{ kN} \qquad P_{br} = 12 \text{ kN}$$

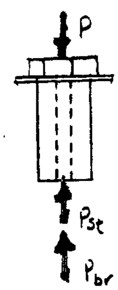
$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{8(10^3)}{\frac{\pi}{4}(0.01^2)} = 102 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{12(10^3)}{\frac{\pi}{4}(0.02^2 - 0.01^2)} = 50.9 \text{ MPa} \qquad \textbf{Ans}$$

4–54 The 10-mm-diameter steel bolt is surrounded by a bronze sleeve. The outer diameter of this sleeve is 20 mm, and its inner diameter is 10 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load P that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.



$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - P = 0 \qquad (1)$$



Assume failure of bolt :

$$P_{st} = (\sigma_Y)_{st}(A) = 640(10^6)(\frac{\pi}{4})(0.01^2)$$
$$= 50265.5 \text{ N}$$

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}L}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_{br}L}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_{st} = 0.6667\, P_{br}$$

$$50\,265.5 = 0.6667 P_{br}$$

$$P_{br} = 75398.2 \text{ N}$$

From Eq. (1)

$$P = 50265.5 + 75398.2$$
$$= 125663.7 \text{ N} = 126 \text{ kN} \qquad \text{(controls)} \qquad \textbf{Ans}$$

Assume failure of sleeve :

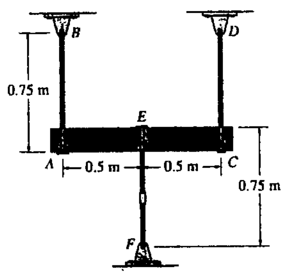
$$P_{br} = (\sigma_Y)_{br}(A) = 520(10^6)(\frac{\pi}{4})(0.02^2 - 0.01^2) = \mathbf{122\,522.11\ N}$$

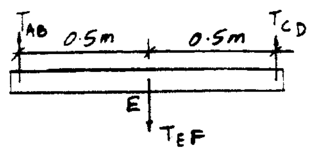
$$P_{st} = 0.6667\, P_{br}$$
$$= 0.6667(122\,522.11)$$
$$= 81\,681.4 \text{ N}$$

From Eq. (1),

$$P = 122\,522.11 + 81\,681.4$$
$$= 204\,203.52 \text{ N}$$
$$= 204 \text{ kN}$$

4–55 The rigid member is held in the position shown by three A-36 steel tie rods. Each rod has an unstretched length of 0.75 m and a cross-sectional area of 125 mm². Determine the forces in the rods if a turnbuckle on rod *EF* undergoes one full turn. The lead of the screw is 1.5 mm. Neglect the size of the turnbuckle and assume that it is rigid. *Note:* The lead would cause the rod, when *unloaded*, to shorten 1.5 mm when the turnbuckle is rotated one revolution.



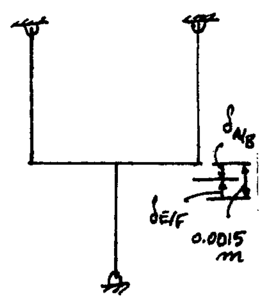


$\circlearrowleft + \Sigma M_E = 0; \qquad -T_{AB}(0.5) + T_{CD}(0.5) = 0$

$T_{AB} = T_{CD} = T \qquad (1)$

$+\downarrow \Sigma F_y = 0; \qquad T_{EF} - 2T = 0$

$T_{EF} = 2T \qquad (2)$



Rod *EF* shortens 1.5mm causing *AB* (and *DC*) to elongate. Thus;

$$0.0015 = \delta_{A/B} + \delta_{E/F}$$

$$0.0015 = \frac{T(0.75)}{(125)(10^{-6})(200)(10^9)} + \frac{2T(0.75)}{(125)(10^{-6})(200)(10^9)}$$

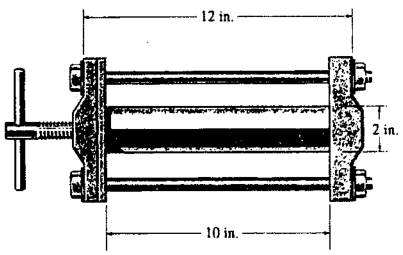
$$2.25T = 37500$$

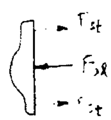
$$T = 16666.67 \text{ N}$$

$T_{AB} = T_{CD} = 16.7$ kN **Ans**

$T_{EF} = 33.3$ kN **Ans**

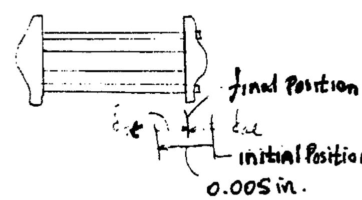
*4–56 The press consists of two rigid heads that are held together by the two A-36 steel $\frac{1}{2}$-in.-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. If it is then tightened one-half turn, determine the average normal stress in the rods and in the cylinder. The single-threaded screw on the bolt has a lead of 0.01 in. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.





$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

$$\delta_{st} = 0.005 - \delta_{al}$$



$$\frac{F_{st}(12)}{(\frac{\pi}{4})(0.5)^2(29)(10^3)} = 0.005 - \frac{F_{al}(10)}{\pi(1)^2(10)(10^3)}$$

Solving,

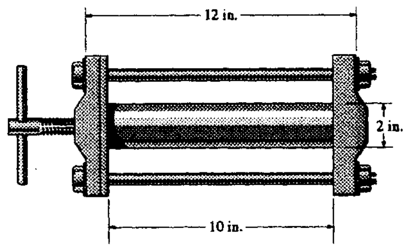
$F_{st} = 1.822$ kip

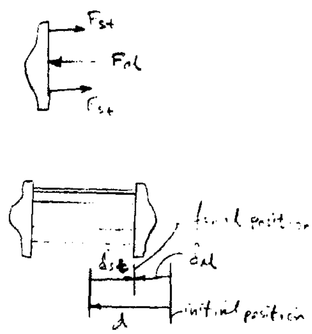
$F_{al} = 3.644$ kip

$$\sigma_{rod} = \frac{F_{st}}{A_{st}} = \frac{1.822}{(\frac{\pi}{4})(0.5)^2} = 9.28 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{cyl} = \frac{F_{al}}{A_{al}} = \frac{3.644}{\pi(1)^2} = 1.16 \text{ ksi} \qquad \textbf{Ans}$$

4-57 The press consists of two rigid heads that are held together by the two A-36 steel $\frac{1}{2}$-in.-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. Determine the angle through which the screw can be turned before the rods or the specimen begin to yield. The single-threaded screw on the bolt has a lead of 0.01 in. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.





$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

$$\delta_{st} = d - \delta_{al}$$

$$\frac{F_{st}(12)}{(\frac{\pi}{4})(0.5)^2(29)(10^3)} = d - \frac{F_{al}(10)}{\pi(1)^2(10)(10^3)} \qquad (1)$$

Assume steel yields first,

$$\sigma_Y = 36 = \frac{F_{st}}{(\frac{\pi}{4})(0.5)^2}; \qquad F_{st} = 7.068 \text{ kip}$$

Then $F_{al} = 14.137$ kip;

$$\sigma_{al} = \frac{14.137}{\pi(1)^2} = 4.50 \text{ ksi}$$

4.50 ksi < 37 ksi steel yields first as assumed. From Eq. (1),

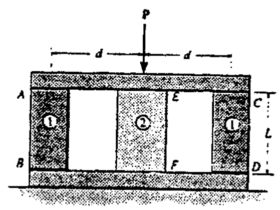
$$d = 0.01940 \text{ in.}$$

Thus,

$$\frac{\theta}{360°} = \frac{0.01940}{0.01}$$

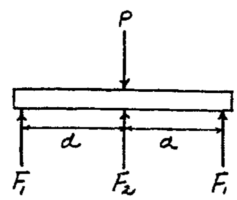
$\theta = 698°$ **Ans**

4-58. The assembly consists of two posts made from material 1 having a modulus of elasticity of E_1 and each a cross-sectional area A_1, and a material 2 having a modulus of elasticity E_2 and cross-sectional area A_2. If a central load **P** is applied to the rigid cap, determine the force in each material.



Equilibrium :

$$+\uparrow \Sigma F_y = 0; \quad 2F_1 + F_2 - P = 0 \qquad [1]$$



Compatibility :

$$\delta = \delta_1 = \delta_2$$

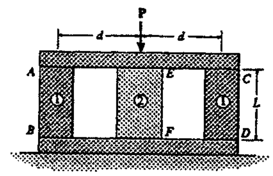
$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad [2]$$

Solving Eq. [1] and [2] yields :

$$F_1 = \left(\frac{A_1 E_1}{2 A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans}$$

$$F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P \qquad \textbf{Ans}$$

4-59. The assembly consists of two posts AB and CD made from material 1 having a modulus of elasticity of E_1 and each a cross-sectional area A_1, and a central post EF made from material 2 having a modulus of elasticity E_2 and a cross sectional area A_2. If posts AB and CD are to be replaced by those having a material 2, determine the required cross-sectional area of these new posts so that both assemblies deform the same amount when loaded.



$$+\uparrow \Sigma F_y = 0; \quad 2F_1 + F_2 - P = 0 \qquad [1]$$

Compatibility :

$$\delta_{in} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad [2]$$

Solving Eq.[1] and [2] yields :

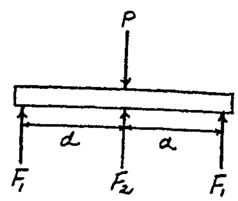
$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{in} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) PL}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

Compatibility : When material 1 has been replaced by material 2 for two side posts, then

$$\delta_{final} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1' E_2} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1'}{A_2}\right) F_2 \qquad [3]$$



Solving for F_2 from Eq.[1] and [3]

$$F_2 = \left(\frac{A_2}{2A_1' + A_2}\right) P$$

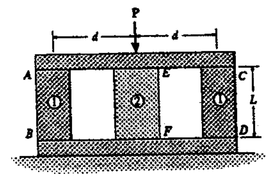
$$\delta_{final} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2}{2A_1' + A_2}\right) PL}{A_2 E_2} = \frac{PL}{E_2 (2A_1' + A_2)}$$

Requires, $\quad \delta_{in} = \delta_{final}$

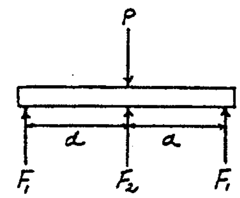
$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_2 (2A_1' + A_2)}$$

$$A_1' = \left(\frac{E_1}{E_2}\right) A_1 \qquad \textbf{Ans}$$

***4-60.** The assembly consists of two posts AB and CD made from material 1 having a modulus of elasticity of E_1 and each a cross-sectional area A_1, and a central post EF made from material 2 having a modulus of elasticity E_2 and a cross-sectional area A_2. If post EF is to be replaced by one having a material 1, determine the required cross-sectional area of this new post so that both assemblies deform the same amount when loaded.



$$+\uparrow \Sigma F_y = 0; \quad 2F_1 + F_2 - P = 0 \qquad [1]$$



Compatibility:

$$\delta_{in} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2 E_2} \qquad F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \qquad [2]$$

Solving Eq.[1] and [2] yields :

$$F_1 = \left(\frac{A_1 E_1}{2A_1 E_1 + A_2 E_2}\right) P \qquad F_2 = \left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P$$

$$\delta_{in} = \frac{F_2 L}{A_2 E_2} = \frac{\left(\frac{A_2 E_2}{2A_1 E_1 + A_2 E_2}\right) P}{A_2 E_2} = \frac{PL}{2A_1 E_1 + A_2 E_2}$$

Compatibility: When material 2 has been replaced by material 1 for central posts, then

$$\delta_{final} = \delta_1 = \delta_2$$

$$\frac{F_1 L}{A_1 E_1} = \frac{F_2 L}{A_2' E_1} \qquad F_2 = \left(\frac{A_2'}{A_1}\right) F_1 \qquad [3]$$

Solving for F_1 from Eq.[1] and [3]

$$F_1 = \left(\frac{A_1}{2A_1 + A_2'}\right) P$$

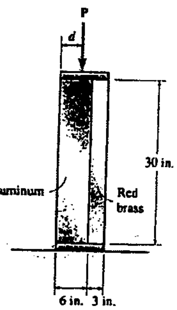
$$\delta_{final} = \frac{F_1 L}{A_1 E_1} = \frac{\left(\frac{A_1}{2A_1 + A_2'}\right) PL}{A_1 E_1} = \frac{PL}{E_1 (2A_1 + A_2')}$$

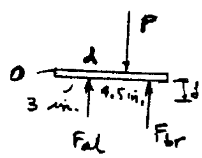
Requires, $\delta_{in} = \delta_{final}$

$$\frac{PL}{2A_1 E_1 + A_2 E_2} = \frac{PL}{E_1 (2A_1 + A_2')}$$

$$A_2' = \left(\frac{E_2}{E_1}\right) A_2 \qquad \textbf{Ans}$$

4-61. The assembly consists of a 6061-T6-aluminum member and a C83400-red-brass member that rest on the rigid plates. Determine the distance d where the vertical load **P** should be placed on the plates so that the plates remain horizontal when the materials deform. Each member has a width of 8 in. and they are not bonded together.





$+\uparrow \Sigma F_y = 0; \quad -P + F_{al} + F_{br} = 0$

$+\Sigma M_O = 0; \quad 3\,F_{al} + 7.5\,F_{br} - Pd = 0$

$\delta = \delta_{br} = \delta_{al}$

$$\frac{F_{br}L}{A_{br}E_{br}} = \frac{F_{al}L}{A_{al}E_{al}}$$

$$F_{br} = F_{al}\left(\frac{A_{br}E_{br}}{A_{al}E_{al}}\right) = F_{al}\left(\frac{(3)(8)(14.6)(10^3)}{6(8)(10)(10^3)}\right) = 0.730\,F_{al}$$

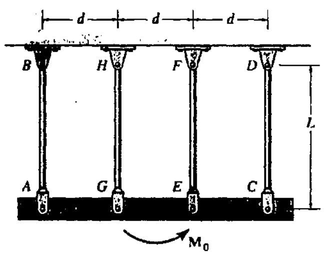
Thus,

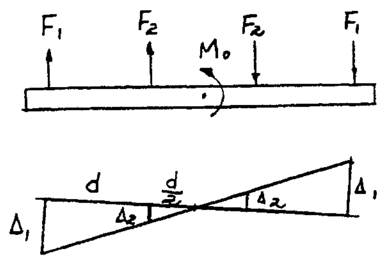
$P = 1.730\,F_{al}$

$3\,F_{al} + 7.5(0.730\,F_{al}) = (1.730\,F_{al})d$

$d = 4.90$ in. **Ans**

4–62 The rigid beam is supported by a symmetrical arrangement of bars of equal area A and length L. Bars AB and CD have a modulus of elasticity E_1 and bars EF and GH have a modulus of elasticity E_2. Determine the average normal stress in each bar if a couple moment M_0 is applied to the beam.





$$F_{AB} = F_{CD} = F_1; \qquad F_{GH} = F_{EF} = F_2$$

$$+\Sigma M_O = 0; \qquad -F_1(3d) - F_2(d) + M_0 = 0 \qquad (1)$$

$$\frac{\delta_1}{1.5d} = \frac{\delta_2}{0.5d}; \qquad 0.5\delta_1 = 1.5\delta_2$$

$$\frac{0.5F_1(L)}{AE_1} = \frac{1.5F_2(L)}{AE_2}$$

$$F_1 = 3\left(\frac{E_1}{E_2}\right)F_2 \qquad (2)$$

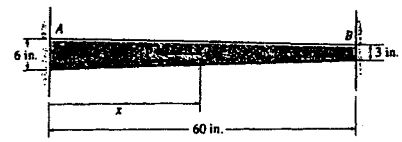
Solving Eqs. (1) and (2),

$$F_2 = \frac{M_0 E_2}{d[9E_1 + E_2]}; \qquad F_1 = \frac{3E_1 M_0}{d[9E_1 + E_2]}$$

$$\sigma_{AB} = \sigma_{CD} = \frac{3E_1 M_0}{Ad[9E_1 + E_2]} \qquad \textbf{Ans}$$

$$\sigma_{GH} = \sigma_{EF} = \frac{M_0 E_2}{Ad[9E_1 + E_2]} \qquad \textbf{Ans}$$

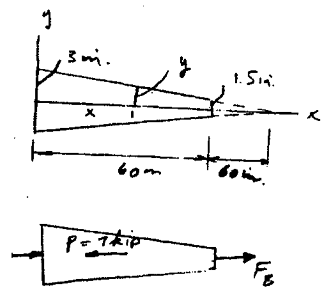
4-63. The tapered member is fixed connected at its ends A and B and is subjected to a load $P = 7$ kip at $x = 30$ in. Determine the reactions at the supports. The material is 2 in. thick and is made from 2014-T6 aluminum.



$$\frac{y}{120-x} = \frac{1.5}{60}$$

$$y = 3 - 0.025\,x$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - 7 = 0 \qquad (1)$$



$$\delta_{A/B} = 0$$

$$-\int_0^{30} \frac{F_A\,dx}{2(3-0.025\,x)(2)(E)} + \int_{30}^{60} \frac{F_B\,dx}{2(3-0.025\,x)(2)(E)} = 0$$

$$-F_A \int_0^{30} \frac{dx}{(3-0.025\,x)} + F_B \int_{30}^{60} \frac{dx}{(3-0.025\,x)} = 0$$

$$40\,F_A \ln(3-0.025\,x)\big|_0^{30} - 40\,F_B \ln(3-0.025\,x)\big|_{30}^{60} = 0$$

$$-F_A(0.2876) + 0.40547\,F_B = 0$$

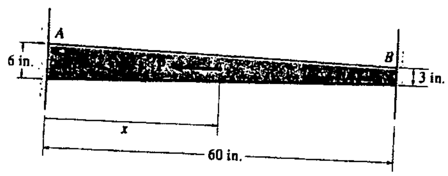
$$F_A = 1.40942\,F_B$$

Thus, from Eq. (1),

$F_A = 4.09$ kip **Ans**

$F_B = 2.91$ kip **Ans**

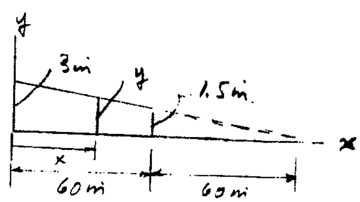
*4–64 The tapered member is fixed connected at its ends A and B and is subjected to a load **P**. Determine the location x of the load and its greatest magnitude if the allowable normal stress for the material is $\sigma_{allow} = 4$ ksi. The member is 2 in. thick.



$$\frac{y}{120-x} = \frac{1.5}{60}$$

$$y = 3 - 0.025\,x$$

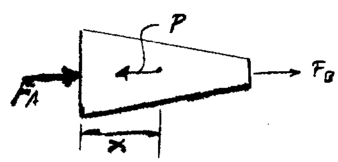
$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - P = 0$$



$$\delta_{A/B} = 0$$

$$-\int_0^x \frac{F_A\,dx}{2(3-0.025\,x)(2)(E)} + \int_x^{60} \frac{F_B\,dx}{2(3-0.025\,x)(2)(E)} = 0$$

$$-F_A\int_0^x \frac{dx}{(3-0.025\,x)} + F_B\int_x^{60} \frac{dx}{(3-0.025\,x)} = 0$$



$$F_A(40)\ln(3-0.025\,x)|_0^x - F_B(40)\ln(3-0.025\,x)|_x^{60} = 0$$

$$F_A \ln\left(1-\frac{0.025\,x}{3}\right) = -F_B\ln\left(2-\frac{0.025\,x}{1.5}\right)$$

For greatest magnitude of P require,

$$4 = \frac{F_A}{2(3-0.025\,x)(2)}; \qquad F_A = 48 - 0.4\,x$$

$$4 = \frac{F_B}{2(3)}; \qquad F_B = 24 \text{ kip}$$

Thus,

$$(48-0.4\,x)\ln\left(1-\frac{0.025\,x}{3}\right) = -24\ln\left(2-\frac{0.025\,x}{1.5}\right)$$

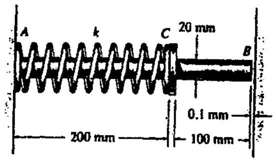
Solving by trial and error,

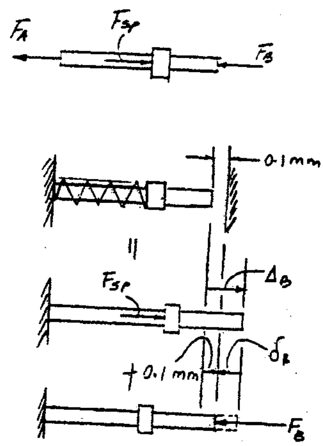
$x = 28.9$ in. **Ans**

Therefore,

$F_A = 36.4$ kip
$P = 60.4$ kip **Ans**

4-65. The spring has an unstretched length of 250 mm and stiffness $k = 400$ kN/m. If it is compressed and placed over the 200-mm-long portion AC of the aluminum bar AB and released, determine the force that the bar exerts on the wall at A. Before loading there is a gap of 0.1 mm between the bar and the wall at B. The bar is fixed to the wall at A. Neglect the thickness of the rigid plate at C. $E_{al} = 70$ GPa.





$\xrightarrow{+} \Sigma F_x = 0; \quad F_{sp} - F_A - F_B = 0 \qquad (1)$

$\xrightarrow{+} \quad 0.1 = \Delta_B - \delta_B$

$$0.1 = \frac{F_{sp}(200)}{\frac{\pi}{4}(0.02^2)(70)(10^9)} - \frac{F_B(300)}{\frac{\pi}{4}(0.02^2)(70)(10^9)}$$

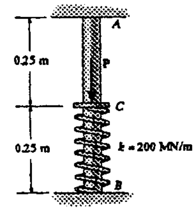
$2F_{sp} - 3F_B = 21991.15 \qquad (2)$

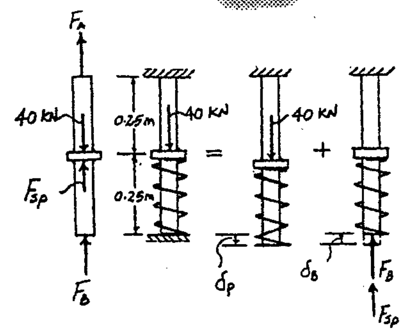
$F_{sp} = k\Delta x = 400(10^3)(0.25 - 0.2001) = 19960$ N

From Eq. (2),
$F_B = 5976.28$ N

From Eq. (1),
$F_A = 13983.7$ N $= 14.0$ kN **Ans**

4-66. The post is made from 6061-T6 aluminum and has a diameter of 50 mm. It is fixed supported at A and B and at its center C there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the reactions at A and B when the force $P = 40$ kN is applied to the collar.





Equations of Equilibrium :

$$+\uparrow \Sigma F_y = 0; \qquad F_A + F_B + F_{sp} - 40(10^3) = 0 \qquad [1]$$

Compatibility :

$$0 = \delta_P - \delta_B$$

$$0 = \frac{40(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 23119.45 \qquad [2]$$

Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057 F_{sp} \qquad [3]$$

Solving Eq. [2] and [3] yields

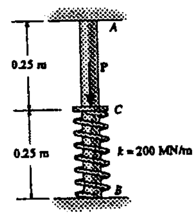
$F_{sp} = 6238.9$ N

$F_B = 16880.6$ N $= 16.9$ kN **Ans**

Substitute the results into Eq. [1]

$F_A = 16880.6$ N $= 16.9$ kN **Ans**

4-67. The post is made from 6061-T6 aluminum and has a diameter of 50 mm. It is fixed supported at A and B and at its center C there is a coiled spring attached to the rigid collar. If the spring is originally uncompressed, determine the compression in the spring when the load of $P = 50$ kN is applied to the collar.

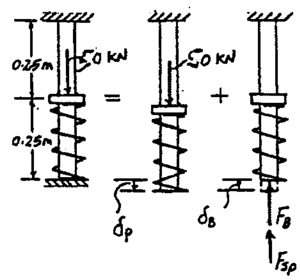


Compatibility :

$$0 = \delta_P - \delta_B$$

$$0 = \frac{50(10^3)(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} - \left[\frac{(F_B + F_{sp})(0.25)}{\frac{\pi}{4}(0.05^2)68.9(10^9)} + \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}\right]$$

$$F_B + F_{sp} = 28899.31 \qquad [1]$$



Also,

$$\delta_{sp} = \delta_{BC}$$

$$\frac{F_{sp}}{200(10^6)} = \frac{F_B + F_{sp}}{\frac{\frac{\pi}{4}(0.05^2)68.9(10^9)}{0.25} + 200(10^6)}$$

$$F_B = 2.7057 F_{sp} \qquad [2]$$

Solving Eqs. [1] and [2] yield

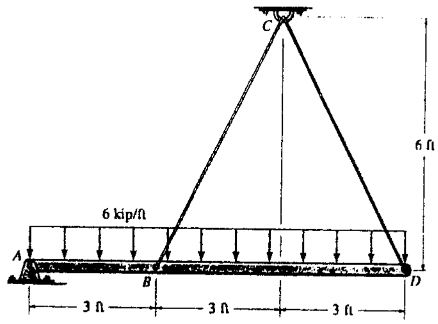
$$F_{sp} = 7798.6 \text{ N} \qquad F_B = 21100.7 \text{ N}$$

Thus,

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{7798.6}{200(10^6)}$$

$$= 0.0390(10^{-3}) \text{ m} = 0.0390 \text{ mm} \qquad \textbf{Ans}$$

*4–68 The rigid bar supports the uniform distributed load of 6 kip/ft. Determine the force in each cable if each cable has a cross-sectional area of 0.05 in², and $E = 31(10^3)$ ksi.



$$\circlearrowleft + \Sigma M_A = 0; \quad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(3) - 54(4.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)9 = 0 \qquad (1)$$

$$\theta = \tan^{-1}\frac{6}{6} = 45^\circ$$

$$L_{B'C}^2 = (3)^2 + (8.4853)^2 - 2(3)(8.4853)\cos\theta'$$

Also,

$$L_{D'C}^2 = (9)^2 + (8.4853)^2 - 2(9)(8.4853)\cos\theta \qquad (2)$$

Thus, eliminating $\cos\theta'$,

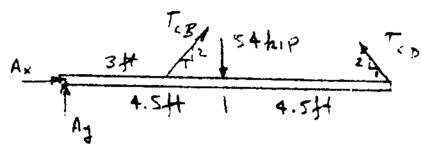
$$-L_{B'C}^2(0.019642) + 1.5910 = -L_{DC'}^2(0.0065473) + 1.001735$$

$$L_{B'C}^2(0.019642) = 0.0065473\,L_{DC'}^2 + 0.589256$$

$$L_{B'C}^2 = 0.333\,L_{DC'}^2 + 30$$

But,

$$L_{B'C} = \sqrt{45} + \delta_{BC}\,, \qquad L_{D'C} = \sqrt{45} + \delta_{DC}$$



Neglect squares or δ's since small strain occurs.

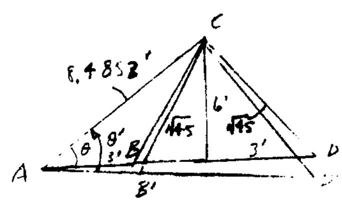
$$L_{B'C}^2 = (\sqrt{45} + \delta_{BC})^2 = 45 + 2\sqrt{45}\,\delta_{BC}$$

$$L_{D'C}^2 = (\sqrt{45} + \delta_{DC})^2 = 45 + 2\sqrt{45}\,\delta_{DC}$$

$$45 + 2\sqrt{45}\,\delta_{BC} = 0.333(45 + 2\sqrt{45}\,\delta_{DC}) + 30$$

$$2\sqrt{45}\,\delta_{BC} = 0.333(2\sqrt{45})\,\delta_{DC}$$

$$\delta_{DC} = 3\delta_{BC}$$



Thus,

$$\frac{T_{CD}\sqrt{45}}{AE} = 3\frac{T_{CB}\sqrt{45}}{AE}$$

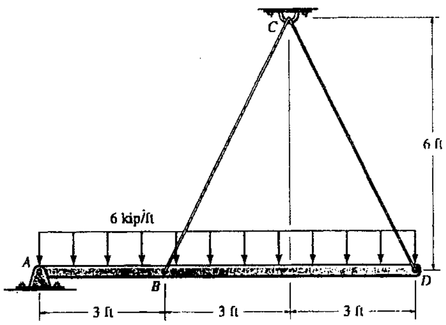
$$T_{CD} = 3\,T_{CB}$$

From Eq. (1),

$T_{CD} = 27.1682$ kip $= 27.2$ kip **Ans**

$T_{CB} = 9.06$ kip **Ans**

4–69 The rigid bar is originally horizontal and is supported by two cables each having a cross-sectional area of 0.05 in², and $E = 31(10^3)$ ksi. Determine the slight rotation of the bar when the uniform load is applied.



See solution of Prob. 4 - 68,

$$T_{CD} = 27.1682 \text{ kip}$$

$$\delta_{DC} = \frac{T_{CD}\sqrt{45}}{0.05(31)(10^3)} = \frac{27.1682\sqrt{45}}{0.05(31)(10^3)} = 0.1175806 \text{ ft}$$

Using Eq. (2) of Prob. 4 - 68,

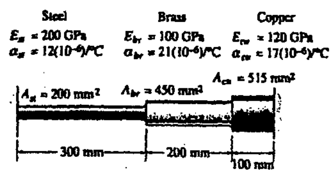
$$(\sqrt{45} + 0.1175806)^2 = (9)^2 + (8.4852)^2 - 2(9)(8.4852)\cos\theta'$$

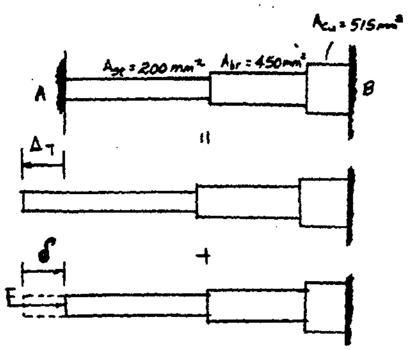
$$\theta' = 45.838°$$

Thus,

$$\Delta\theta = 45.838° - 45° = 0.838° \qquad \textbf{Ans}$$

4-70. Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°C$. The material properties and cross-sectional area of each bar are given in the figure.





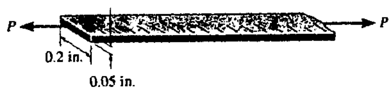
$(\overset{+}{\leftarrow})\quad 0 = \Delta_T - \delta$

$$0 = 12(10^{-6})(6)(0.3) + 21(10^{-6})(6)(0.2) + 17(10^{-6})(6)(0.1)$$

$$-\frac{F(0.3)}{200(10^{-6})(200)(10^9)} - \frac{F(0.2)}{450(10^{-6})(100)(10^9)} - \frac{F(0.1)}{515(10^{-6})(120)(10^9)}$$

$F = 4202\text{ N} = 4.20\text{ kN}$ **Ans**

4-71 A steel surveyor's tape is to be used to measure the length of a line. The tape has a rectangular cross section of 0.05 in. by 0.2 in. and a length of 100 ft when $T_1 = 60°F$ and the tension or pull on the tape is 20 lb. Determine the true length of the line if the tape shows the reading to be 463.25 ft when used with a pull of 35 lb at $T_2 = 90°F$. The ground on which it is placed is flat. $\alpha_{st} = 9.60(10^{-6})/°F$, $E_{st} = 29(10^3)$ ksi.

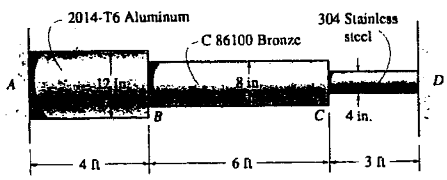


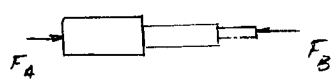
$$\delta_T = \alpha \Delta T L = 9.6(10^{-6})(90 - 60)(463.25) = 0.133416 \text{ ft}$$

$$\delta = \frac{PL}{AE} = \frac{(35 - 20)(463.25)}{(0.2)(0.05)(29)(10^6)} = 0.023961 \text{ ft}$$

$$L = 463.25 + 0.133416 + 0.023961 = 463.41 \text{ ft}$$ **Ans**

•4–72 The assembly has the diameters and material make-up indicated. If it fits securely between its fixed supports when the temperature is $T_1 = 70°F$, determine the average normal stress in each material when the temperature reaches $T_2 = 110°F$.





$\Sigma F_x = 0; \qquad F_A = F_B = F$

$$\delta_{A/D} = 0; \qquad -\frac{F(4)(12)}{\pi(6)^2(10.6)(10^6)} + 12.8(10^{-6})(110-70)(4)(12)$$

$$-\frac{F(6)(12)}{\pi(4)^2(15)(10^6)} + 9.60(10^{-6})(110-70)(6)(12)$$

$$-\frac{F(3)(12)}{\pi(2)^2(28)(10^6)} + 9.60(10^{-6})(110-70)(3)(12) = 0$$

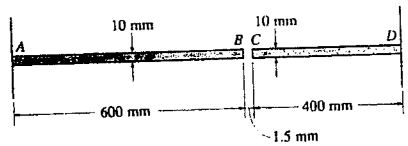
$F = 277.69$ kip

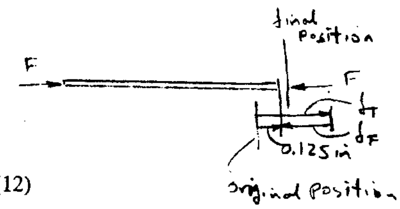
$$\sigma_{al} = \frac{277.69}{\pi(6)^2} = 2.46 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{br} = \frac{277.69}{\pi(4)^2} = 5.52 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{st} = \frac{277.69}{\pi(2)^2} = 22.1 \text{ ksi} \qquad \textbf{Ans}$$

4–73 A high-strength concrete driveway slab has a length of 20 ft when its temperature is 20°F. If there is a gap of 0.125 in. on one side before it touches its fixed abutment, determine the temperature required to close the gap. What is the compressive stress in the concrete if the temperature becomes 110°F?





Require,

$$\delta_T = \alpha \Delta T L$$

$$0.125 = 6(10^{-6})(T - 20°)(20)(12)$$

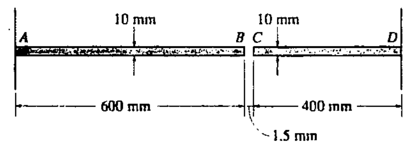
$T = 107°$ F **Ans**

$$0.125 = \delta_T - \delta_F$$

$$0.125 = 6(10^{-6})(110° - 20°)(20)(12) - \frac{F(20)(12)}{A(4.20(10^6))}$$

$\sigma = \frac{F}{A} = 80.5$ psi **Ans**

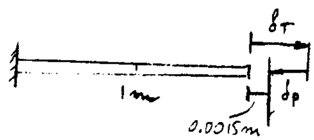
4-74. A thermo gate consists of two 6061-T6-aluminum plates that have a width of 15 mm and are fixed supported at their ends. If the gap between them is 1.5 mm when the temperature is $T_1 = 25°C$, determine the temperature required to just close the gap. Also, what is the axial force in each plate if the temperature becomes $T_2 = 100°C$? Assume bending or buckling will not occur.



$\delta_T = \alpha \Delta T L$

Require,

$\delta_T = 0.0015 \text{ m} = \delta_{B/A} + \delta_{C/D}$



$0.0015 = 24(10^{-6})(T_2 - 25)(1)$

$T_2 = 87.5° \text{ C}$ **Ans**

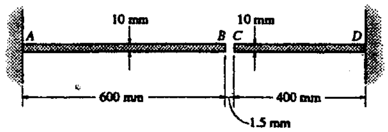
The problem is equivalent to that shown in diagram.

$(\overset{+}{\rightarrow})\ \delta_T - \delta_P = 0.0015$

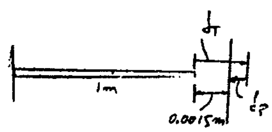
$$\alpha \Delta TL - \frac{FL}{AE} = 0.0015$$

$$24(10^{-6})(100-25)(1) - \frac{F(1)}{(0.015)(0.010)(68.9)(10^9)} = 0.0015$$

4-75. A thermo gate consists of a 6061-T6-aluminum plate *AB* and an Am-1004-T61-magnesium plate *CD*, each having a width of 15 mm and fixed supported at their ends. If the gap between them is 1.5 mm when the temperature is $T_1 = 25°C$, determine the temperature required to just close the gap. Also, what is the axial force in each plate if the temperature becomes $T_2 = 100°C$? Assume bending or buckling will not occur.



$\delta_T = \alpha \Delta T L$



Require,

$\delta_T = 0.0015 \text{ m} = \delta_{B/A} + \delta_{C/D}$

$$0.0015 = 24(10^{-6})(T_2 - 25)(0.6) + 26(10^{-6})(T_2 - 25)(0.4)$$

$$T_2 = 85.5° \text{ C} \quad \textbf{Ans}$$

The problem is equivalent to that shown in the diagram. **Require,**

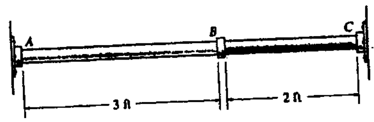
$$(\overset{+}{\rightarrow}) \quad \delta_T - \delta_P = 0.0015$$

$$\Sigma(\alpha \Delta T L) - \Sigma(\frac{FL}{AE}) = 0.0015$$

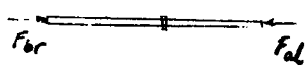
$$24(10^{-6})(100 - 25)(0.6) + 26(10^{-6})(100 - 25)(0.4) - \frac{F(0.6)}{(0.01)(0.015)(68.9)(10^9)} - \frac{F(0.4)}{(0.01)(0.015)(44.7)(10^9)} = 0.0015$$

$$F = 3.06 \text{ kN} \quad \textbf{Ans}$$

***4-76.** The C83400-red-brass rod *AB* and 2014-T6-aluminum rod *BC* are joined at the collar *B* and fixed connected at their ends. If there is no load in the members when $T_1 = 50°F$, determine the average normal stress in each member when $T_2 = 120°F$. Also, how far will the collar be displaced? The cross-sectional area of each member is $1.75\ in^2$.



$\Sigma F_x = 0;\quad F_{br} = F_{al} = F$



$$\delta_{A/C} = 0$$

$$-\frac{F_{br}L_{AB}}{A_{AB}E_{br}} + \alpha_B \Delta T\, L_{AB} - \frac{F_{al}L_{BC}}{A_{BC}E_{al}} + \alpha_{al}\Delta T\, L_{BC} = 0$$

$$-\frac{F(3)(12)}{(1.75)(14.6)(10^6)} + 9.80(10^{-6})(120-50)(3)(12)$$

$$-\frac{F(2)(12)}{1.75(10.6)(10^6)} + 12.8(10^{-6})(120-50)(2)(12) = 0$$

$$F = 17\ 093.4\ \text{lb}$$

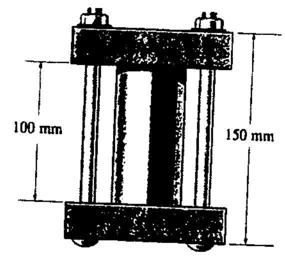
$$\sigma_{br} = \sigma_{al} = \frac{17\ 093.4}{1.75} = 9.77\ \text{ksi} \qquad \textbf{Ans}$$

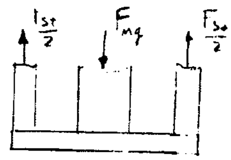
$$9.77\ \text{ksi} < (\sigma_Y)_{al}\ \text{ and } (\sigma_Y)_{br} \qquad \textbf{OK}$$

$$\delta_B = -\frac{17\ 093.4(3)(12)}{1.75(14.6)(10^6)} + 9.80(10^{-6})(120-50)(3)(12)$$

$$\delta_B = 0.611(10^{-3})\ \text{in.} \rightarrow \qquad \textbf{Ans}$$

4–77 The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 20°C$. If the 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the force in the cylinder when the temperature rises to $T_2 = 130°C$.





$$+\uparrow \Sigma F_y = 0; \qquad F_{st} = F_{mg} = F$$

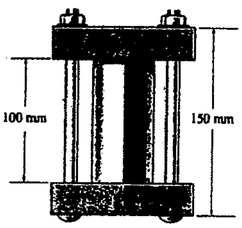
$$\delta_{mg} = \delta_{st}$$

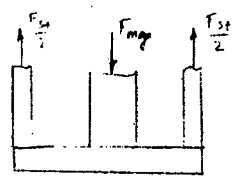
$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$

$$26(10^{-6})(0.1)(110) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(110) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

$$F = 904 \text{ N} \qquad \textbf{Ans}$$

4-78. The 50-mm-diameter cylinder is made from Am 1004-T61 magnesium and is placed in the clamp when the temperature is $T_1 = 15°C$. If the two 304-stainless-steel carriage bolts of the clamp each have a diameter of 10 mm, and they hold the cylinder snug with negligible force against the rigid jaws, determine the temperature at which the average normal stress in either the magnesium or steel becomes 12 MPa.





$+\uparrow \Sigma F_y = 0; \quad F_{st} = F_{mg} = F$

$$\delta_{mg} = \delta_{st}$$

$$\alpha_{mg} L_{mg} \Delta T - \frac{F_{mg} L_{mg}}{E_{mg} A_{mg}} = \alpha_{st} L_{st} \Delta T + \frac{F_{st} L_{st}}{E_{st} A_{st}}$$

$$26(10^{-6})(0.1)(\Delta T) - \frac{F(0.1)}{44.7(10^9)\frac{\pi}{4}(0.05)^2} = 17(10^{-6})(0.150)(\Delta T) + \frac{F(0.150)}{193(10^9)(2)\frac{\pi}{4}(0.01)^2}$$

The steel has the smallest cross - sectional area.

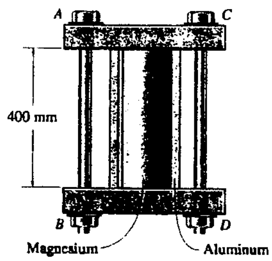
$$F = \sigma A = 12(10^6)(2)(\frac{\pi}{4})(0.01)^2 = 1885.0 \text{ N}$$

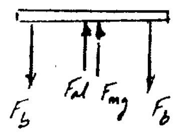
Thus,

$\Delta T = 229°$

$T_2 = 229° + 15° = 244°$ **Ans**

4-79 The assembly consists of a 2014-T6-aluminum cylinder having an outer diameter of 200 mm and inner diameter of 150 mm, together with a concentric solid inner cylinder of Am 1004-T61 magnesium, having a diameter of 125 mm. If the clamping force in the bolts AB and CD is 4 kN when the temperature is $T_1 = 16°C$, determine the force in the bolts when the temperature becomes $T_2 = 48°C$. Assume the bolts and the restraining bars are rigid.





For aluminum :

$$-\delta_F + \delta_T = 0$$

$$-\frac{F_{al}(0.4)}{\frac{\pi}{4}((0.2)^2 - (0.15)^2)73.1(10^9)} + 23(10^{-6})(48 - 16)(0.4) = 0$$

$$F_{al} = 739.47 \text{ kN}$$

For magnesium :

$$-\delta_F + \delta_T = 0$$

$$-\frac{F_{mg}(0.4)}{\frac{\pi}{4}(0.125)^2(44.7)(10^9)} + 26(10^{-6})(48 - 16)(0.4) = 0$$

$$F_{mg} = 456.39 \text{ kN}$$

$$+\uparrow \Sigma F_z = 0; \qquad 739.47 + 456.39 - 2F_b = 0$$

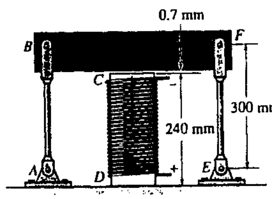
$$F_b = 598 \text{ kN} \qquad \textbf{Ans}$$

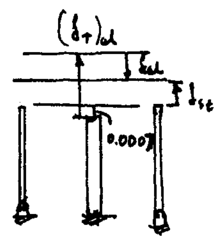
Note :

$$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{739.47(10)^3}{\frac{\pi}{4}\left[(0.2)^2 - (0.15)^2\right]} = 53.8 \text{ MPa} < 414 \text{ MPa} = (\sigma_Y)_{al}$$

$$\sigma_{mg} = \frac{F_{mg}}{A_{mg}} = \frac{456.39(10)^3}{\frac{\pi}{4}(0.125)^2} = 37.2 \text{ MPa} < 152 \text{ MPa} = (\sigma_Y)_{mg}$$

•4–80 The center rod CD of the assembly is heated from $T_1 = 30°C$ to $T_2 = 180°C$ using electrical resistance heating. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm^2. CD is made of aluminum and has a cross-sectional area of 375 mm^2. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, and $\alpha_{al} = 23(10^{-6})/°C$.



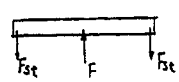


$$\delta_{st} = (\delta_T)_{al} - \delta_{al} - 0.0007$$

$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} = 23(10^{-6})(150)(0.24) - \frac{F(0.24)}{(375)(10^{-6})(70)(10^9)} - 0.0007$$

$$12F_{st} = 128\,000 - 9.1428F \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F - 2F_{st} = 0 \qquad (2)$$

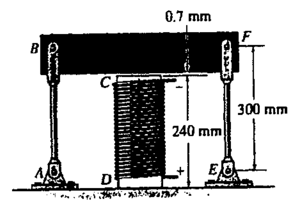


Solving Eqs. (1) and (2) yields,

$F_{AB} = F_{EF} = F_{st} = 4.23$ kN **Ans**

$F_{CD} = F = 8.45$ kN

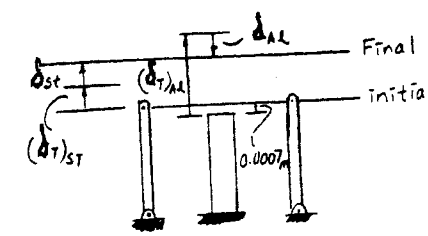
4–81 The center rod CD of the assembly is heated from $T_1 = 30°C$ to $T_2 = 180°C$ using electrical resistance heating. Also, the two end rods AB and EF are heated from $T_1 = 30°C$ to $T_2 = 50°C$. At the lower temperature T_1 the gap between C and the rigid bar is 0.7 mm. Determine the force in rods AB and EF caused by the increase in temperature. Rods AB and EF are made of steel, and each has a cross-sectional area of 125 mm². CD is made of aluminum and has a cross-sectional area of 375 mm². $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 12(10^{-6})/°C$, and $\alpha_{al} = 23(10^{-6})/°C$.



$$\delta_{st} + (\delta_T)_{st} = (\delta_T)_{al} - \delta_{al} - 0.0007$$

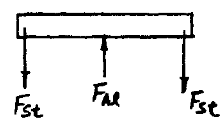
$$\frac{F_{st}(0.3)}{(125)(10^{-6})(200)(10^9)} + 12(10^{-6})(50 - 30)(0.3)$$

$$= 23(10^{-6})(180 - 30)(0.24) - \frac{F_{al}(0.24)}{375(10^{-6})(70)(10^9)} - 0.0007$$



$$12.0F_{st} + 9.14286F_{al} = 56000 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{al} - 2F_{st} = 0 \qquad (2)$$

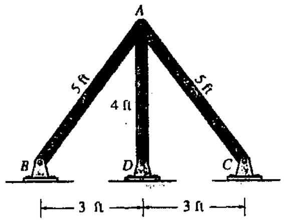


Solving Eqs. (1) and (2) yields :

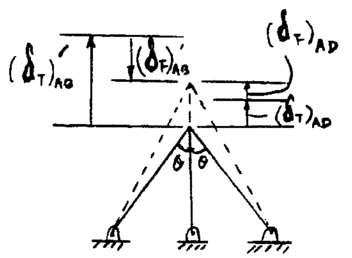
$F_{AB} = F_{EF} = F_{st} = 1.85$ kN **Ans**

$F_{CD} = F_{al} = 3.70$ kN

4–82 The three bars are made of A-36 steel and form a pin-connected truss. If the truss is constructed when $T_1 = 50°F$, determine the force in each bar when $T_2 = 110°F$. Each bar has a cross-sectional area of 2 in^2.



$$(\delta_T')_{AB} - (\delta_F')_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD} \qquad (1)$$

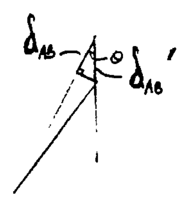


However, $\delta_{AB} = \delta'_{AB}\cos\theta$;

$$\delta'_{AB} = \frac{\delta_{AB}}{\cos\theta} = \frac{5}{4}\delta_{AB}$$

Substitute into Eq. (1)

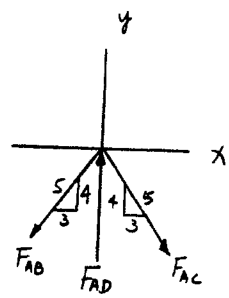
$$\frac{5}{4}(\delta_T)_{AB} - \frac{5}{4}(\delta_F)_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD}$$



$$\frac{5}{4}[6.60(10^{-6})(110° - 50°)(5)(12) - \frac{F_{AB}(5)(12)}{2(29)(10^3)}]$$

$$= 6.60(10^{-6})(110° - 50°)(4)(12) + \frac{F_{AD}(4)(12)}{2(29)(10^3)}$$

$$620.136 = 75F_{AB} + 48F_{AD} \qquad (2)$$



$$\xrightarrow{+}\Sigma F_x = 0; \qquad \frac{3}{5}F_{AC} - \frac{3}{5}F_{AB} = 0; \qquad F_{AC} = F_{AB}$$

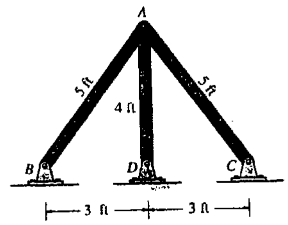
$$+\uparrow\Sigma F_y = 0; \qquad F_{AD} - 2(\frac{4}{5}F_{AB}) = 0 \qquad (3)$$

Solving Eqs. (2) and (3) yields :

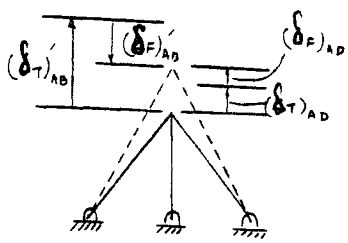
$F_{AD} = 6.54$ kip **Ans**

$F_{AC} = F_{AB} = 4.09$ kip **Ans**

4-83 The three bars are made of A-36 steel and form a pin-connected truss. If the truss is constructed when $T_1 = 50°F$, determine the vertical displacement of joint A when $T_2 = 150°F$. Each bar has a cross-sectional area of 2 in^2.



$$(\delta_T')_{AB} - (\delta_F')_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD} \qquad (1)$$



However, $\delta_{AB} = \delta'_{AB}\cos\theta$;

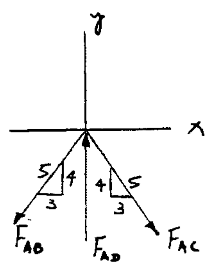
$$\delta'_{AB} = \frac{\delta_{AB}}{\cos\theta} = \frac{5}{4}\delta_{AB}$$

Substitute into Eq. (1)

$$\frac{5}{4}(\delta_T)_{AB} - \frac{5}{4}(\delta_F)_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD}$$

$$\frac{5}{4}[6.60(10^{-6})(150° - 50°)(5)(12) - \frac{F_{AB}(5)(12)}{2(29)(10^3)}]$$
$$= 6.60(10^{-6})(150° - 50°)(4)(12) + \frac{F_{AD}(4)(12)}{2(29)(10^3)}$$

$$239.25 - 6.25F_{AB} = 153.12 + 4F_{AD}$$
$$4F_{AD} + 6.25F_{AB} = 86.13 \qquad (2)$$



$\xrightarrow{+}\Sigma F_x = 0;\quad \frac{3}{5}F_{AC} - \frac{3}{5}F_{AB} = 0;\qquad F_{AC} = F_{AB}$

$+\uparrow\Sigma F_y = 0;\quad F_{AD} - 2(\frac{4}{5}F_{AB}) = 0;$

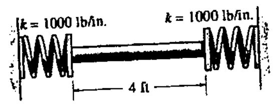
$$F_{AD} = 1.6F_{AB} \qquad (3)$$

Solving Eqs. (2) and (3) yields :

$F_{AB} = 6.8086$ kip; $\quad F_{AD} = 10.8939$ **kip**

$$(\delta_A)_v = (\delta_T)_{AD} + (\delta_F)_{AD}$$
$$= 6.60(10^{-6})(150° - 50°)(4)(12) + \frac{10.8939(4)(12)}{2(29)(10^3)}$$
$$= 0.0407 \text{ in. } \uparrow \qquad \textbf{Ans}$$

•4-84 The rod is made of A-36 steel and has a diameter of 0.25 in. If the springs are compressed 0.5 in. when the temperature of the rod is $T = 40°F$, determine the force in the rod when its temperature is $T = 160°F$.

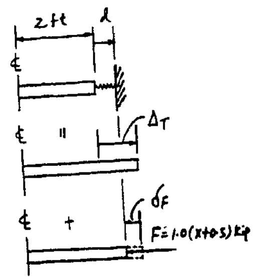


$$x = \Delta_T - \delta_F$$

$$x = 6.60(10^{-6})(160° - 40°)(2)(12) - \frac{1.0(x + 0.5)(2)(12)}{\frac{\pi}{4}(0.25^2)(29)(10^3)}$$

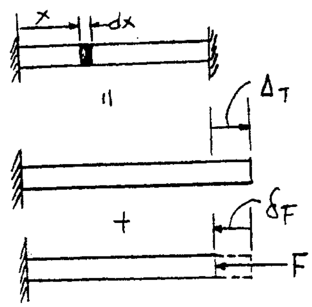
$$x = 0.0104 \text{ in.}$$

$$F = 1.0(0.0104 + 0.5) = 0.510 \text{ kip} = 510 \text{ lb}$$ **Ans**



4–85 The bar has a cross-sectional area A, length L, modulus of elasticity E, and coefficient of thermal expansion α. The temperature of the bar changes uniformly from an original temperature of T_A to T_B so that at any point x along the bar $T = T_A + x(T_B - T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar.

$$\xrightarrow{+} \quad 0 = \Delta_T - \delta_F \qquad (1)$$



However,

$$d\Delta_T = \alpha\,\Delta_T\,dx = \alpha(T_A + \frac{T_B - T_A}{L}x - T_A)dx$$

$$\Delta_T = \alpha\int_0^L \frac{T_B - T_A}{L}x\,dx = \alpha[\frac{T_B - T_A}{2L}x^2]\Big|_0^L$$

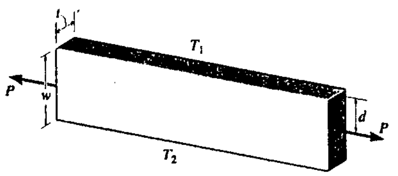
$$= \alpha[\frac{T_B - T_A}{2}L] = \frac{\alpha L}{2}(T_B - T_A)$$

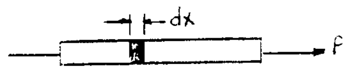
From Eq.(1).

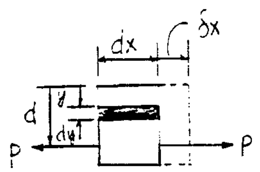
$$0 = \frac{\alpha L}{2}(T_B - T_A) - \frac{FL}{AE}$$

$$F = \frac{\alpha AE}{2}(T_B - T_A) \qquad \textbf{Ans}$$

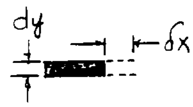
4-86 The metal strap has a thickness t and width w and is subjected to a temperature gradient T_1 to T_2 ($T_1 < T_2$). This causes the modulus of elasticity for the material to vary linearly from E_1 at the top to a smaller amount E_2 at the bottom. As a result, for any vertical position y, $E = [(E_2 - E_1)/w]\,y + E_1$. Determine the position d where the axial force P must be applied so that the bar stretches uniformly over its cross section.



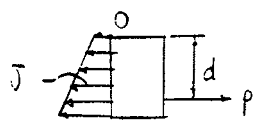




$\epsilon = \frac{\delta x}{dx}$



$\epsilon = \frac{\sigma}{E}$



$\varepsilon = \text{constant} = \varepsilon_0$

$$\varepsilon_0 = \frac{\sigma}{E} = \frac{\sigma}{\left(\left(\frac{E_2 - E_1}{w}\right)y + E_1\right)}$$

$$\sigma = \varepsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right)$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P - \int_A \sigma\, dA = 0$$

$$P = \int_0^w \sigma\, t\, dy = \int_0^w \varepsilon_0\left(\frac{E_2 - E_1}{w}y + E_1\right) t\, dy$$

$$P = \varepsilon_0\, t\left(\frac{(E_2 - E_1)w}{2} + E_1 w\right) = \varepsilon_0 t\left(\frac{E_2 + E_1}{2}\right)w$$

$$\curvearrowleft + \Sigma M_0 = 0; \qquad P(d) - \int_A y\,\sigma dA = 0$$

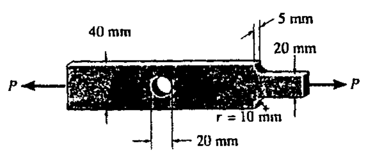
$$\varepsilon_0 t\left(\frac{E_2 + E_1}{2}\right)wd = \int_0^w \varepsilon_0\left(\left(\frac{E_2 - E_1}{w}\right)y^2 + E_1 y\right) t\, dy$$

$$\varepsilon_0\, t\left(\frac{E_2 + E_1}{2}\right)wd = \varepsilon_0 t\left(\frac{E_2 - E_1}{3}w^2 + \frac{E_1}{2}w^2\right)$$

$$\left(\frac{E_2 + E_1}{2}\right) d = \frac{1}{6}(2E_2 + E_1)w$$

$$d = \left(\frac{2E_2 + E_1}{3(E_2 + E_1)}\right)w \qquad \textbf{Ans}$$

4-87 Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



For the fillet :

$$\frac{w}{h} = \frac{40}{20} = 2 \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 10-23. $K = 1.4$

$$\sigma_{max} = K\sigma_{avg}$$
$$= 1.4\left(\frac{8\,(10^3)}{0.02\,(0.005)}\right)$$
$$= 112 \text{ MPa}$$

For the hole :

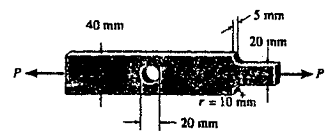
$$\frac{r}{w} = \frac{10}{40} = 0.25$$

From Fig. 4-24. $K = 2.375$

$$\sigma_{max} = K\sigma_{avg}$$
$$= 2.375\left(\frac{8\,(10^3)}{(0.04 - 0.02)(0.005)}\right)$$

$$= 190 \text{ MPa}$$ **Ans**

***4-88.** If the allowable normal stress for the bar is $\sigma_{allow} = 120$ MPa, determine the maximum axial force P that can be applied to the bar.



Assume failure of the fillet.

$$\frac{w}{h} = \frac{40}{20} = 2; \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4-23, $K = 1.4$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120\,(10^6) = 1.4\left(\frac{P}{0.02\,(0.005)}\right)$$

$$P = 8.57 \text{ kN}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{10}{40} = 0.25$$

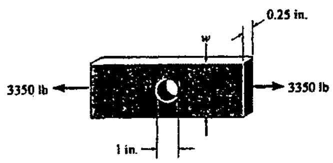
From Fig. 4-24, $K = 2.375$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$120\,(10^6) = 2.375\left(\frac{P}{(0.04 - 0.02)\,(0.005)}\right)$$

$$P = 5.05 \text{ kN} \qquad \text{(controls)} \qquad \textbf{Ans}$$

4-89. The member is to be made from a steel plate that is 0.25 in. thick. If a 1-in. hole is drilled through its center, determine the approximate width w of the plate so that it can support an axial force of 3350 lb. The allowable stress is $\sigma_{allow} = 22$ ksi.



$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$22 = K[\frac{3.35}{(w - 1)(0.25)}]$$

$$w = \frac{3.35K + 5.5}{5.5}$$

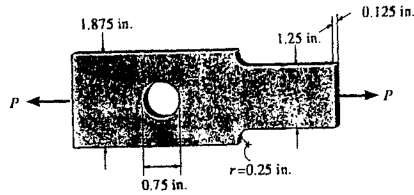
By trial and error, from Fig. 4-24,

choose $\frac{r}{w} = 0.2$: $\quad K = 2.45$

$$w = \frac{3.35(2.45) + 5.5}{5.5} = 2.49 \text{ in.}$$ **Ans**

Since $\frac{r}{w} = \frac{0.5}{2.49} = 0.2$ OK

4-90 Determine the maximum axial force P that can be applied to the bar. The bar is made from steel and has an allowable stress of $\sigma_{allow} = 21$ ksi.



Assume failure of the fillet.

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4-23. $K = 1.75$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 1.75\left(\frac{P}{1.25\,(0.125)}\right)$$

$$P = 1.875 \text{ kip}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

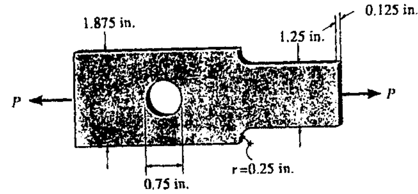
From Fig. 4-24. $K = 2.45$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 2.45\left(\frac{P}{(1.875 - 0.75)(0.125)}\right)$$

$P = 1.21$ kip (controls) **Ans**

4–91 Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 2$ kip.



At fillet:

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4-23, $K = 1.73$

$$\sigma_{max} = K\left(\frac{P}{A}\right) = 1.73\left[\frac{2}{1.25(0.125)}\right] = 22.1\text{ ksi}$$

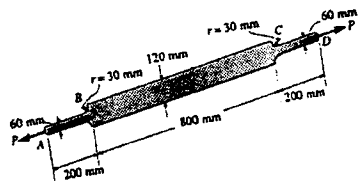
At hole:

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

From Fig. 4-24, $K = 2.45$

$$\sigma_{max} = 2.45\left[\frac{2}{(1.875 - 0.75)(0.125)}\right] = 34.8\text{ ksi} \quad \textbf{(Controls)} \quad \textbf{Ans}$$

***4-92.** The A-36 steel plate has a thickness of 12 mm. If there are shoulder fillets at B and C, and $\sigma_{allow} = 150$ MPa, determine the maximum axial load P that it can support. Compute its elongation neglecting the effect of the fillets.



Maximum Normal Stress at fillet :

$$\frac{r}{h} = \frac{30}{60} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{120}{60} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.06(0.012)}\right]$$

$$P = 77142.86 \text{ N} = 77.1 \text{ kN} \qquad \textbf{Ans}$$

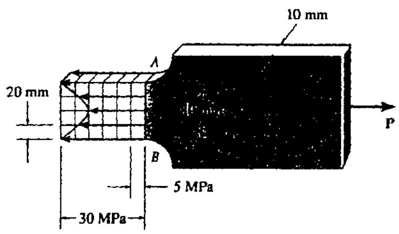
Displacement :

$$\delta = \Sigma\frac{PL}{AE}$$

$$= \frac{77142.86(400)}{(0.06)(0.012)(200)(10^9)} + \frac{77142.86(800)}{(0.12)(0.012)(200)(10^9)}$$

$$= 0.429 \text{ mm} \qquad \textbf{Ans}$$

4-93 The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



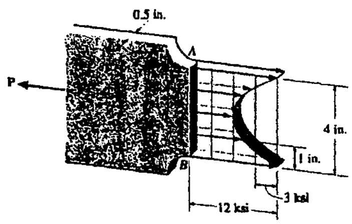
Number of squares ≈ 19

$$P = 19(5)(10^6)(0.02)(0.01) = 19\text{ kN} \qquad \textbf{Ans}$$

$$\sigma_{avg} = \frac{P}{A} = \frac{19(10^3)}{0.08(0.01)} = 23.75\text{ MPa}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{30\text{ MPa}}{23.75\text{ MPa}} = 1.26 \qquad \textbf{Ans}$$

4-94. The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



$P = \int \sigma dA$ = Volume under curve

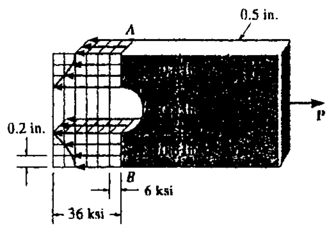
Number of squares = 10

$P = 10(3)(1)(0.5) = 15$ kip **Ans**

$$\sigma_{avg} = \frac{P}{A} = \frac{15 \text{ kip}}{(4 \text{ in.})(0.5 \text{ in.})} = 7.5 \text{ ksi}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{12 \text{ ksi}}{7.5 \text{ ksi}} = 1.60$$ **Ans**

4-95 The resulting stress distribution along section *AB* for the bar is shown. From this distribution, determine the approximate resultant axial force *P* applied to the bar. Also, what is the stress-concentration factor for this geometry?



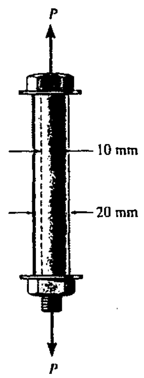
Number of squares ≈ 28

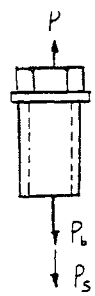
$P = 28(6)(0.2)(0.5) = 16.8$ kip **Ans**

$$\sigma_{avg} = \frac{P}{A} = \frac{16.8}{2(0.6)(0.5)} = 28 \text{ ksi}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{36}{28} = 1.29 \qquad \textbf{Ans}$$

•4–96 The 10-mm-diameter shank of the steel bolt has a bronze sleeve bonded to it. The outer diameter of this sleeve is 20 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the largest possible value of P that can be applied to the bolt. Assume the materials to be elastic perfectly plastic. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.





$+\uparrow \Sigma F_y = 0; \quad P - P_b - P_s = 0 \quad (1)$

The largest possible P that can be applied is when P causes both bolt and sleeve to yield. Hence,

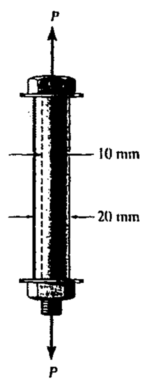
$P_b = (\sigma_{st})_Y A_b = 640(10^6)(\frac{\pi}{4})(0.01^2) = 50.265$ kN

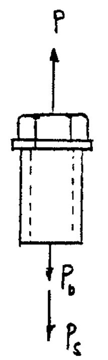
$P_s = (\sigma_{br})_Y A_s = 520(10^6)(\frac{\pi}{4})(0.02^2 - 0.01^2)$
$= 122.52$ kN

From Eq. (1),

$P = 50.265 + 122.52 = 173$ kN **Ans**

4-97 The 10-mm-diameter shank of the steel bolt has a bronze sleeve bonded to it. The outer diameter of this sleeve is 20 mm. If the yield stress for the steel is $(\sigma_Y)_{st} = 640$ MPa, and for the bronze $(\sigma_Y)_{br} = 520$ MPa, determine the magnitude of the largest elastic load P that can be applied to the assembly. $E_{st} = 200$ GPa, $E_{br} = 100$ GPa.





$$+\uparrow \Sigma F_y = 0; \quad P - P_b - P_s = 0 \qquad (1)$$

$$\Delta_b = \Delta_s; \quad \frac{P_b(L)}{\frac{\pi}{4}(0.01^2)(200)(10^9)} = \frac{P_s(L)}{\frac{\pi}{4}(0.02^2 - 0.01^2)(100)(10^9)}$$

$$P_b = 0.6667\,P_s \qquad (2)$$

Assume yielding of the bolt :

$P_b = (\sigma_{st})_Y A_b = 640\,(10^6)(\frac{\pi}{4})(0.01^2) = 50.265$ **kN**

Using $P_b = 50.265$ kN and solving Eqs. (1) and (2) :

$P_s = 75.40$ kN; $\quad P = 125.66$ kN

Assume yielding of the sleeve :

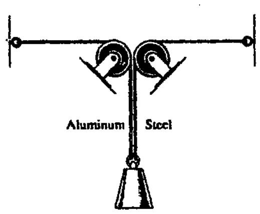
$P_s = (\sigma_Y)_{br} A_s = 520\,(10^6)(\frac{\pi}{4})(0.02^2 - 0.01^2) = 122.52$ **kN**

Use $P_s = 122.52$ kN, and solving Eqs. (1) and (2) :

$P_b = 81.68$ kN $\quad P = 204.20$ kN

$P = 126$ kN (controls) **Ans**

4-98. The weight is suspended from steel and aluminum wires, each having the same initial length of 3 m and cross-sectional area of 4 mm^2. If the materials can be assumed to be elastic perfectly plastic, with $(\sigma_Y)_{st} = 120$ MPa and $(\sigma_Y)_{al} = 70$ MPa, determine the force in each wire if the weight is (a) 600 N and (b) 720 N. $E_{al} = 70$ GPa, $E_{st} = 200$ GPa.

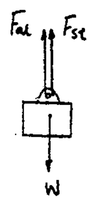


$+\uparrow \Sigma F_y = 0; \quad F_{al} + F_{st} - W = 0 \qquad (1)$

Assume both wires behave elastically.

$\delta_{al} = \delta_{st}; \quad \frac{F_{al} L}{A(70)} = \frac{F_{st} L}{A(200)}$

$F_{al} = 0.35\, F_{st} \qquad (2)$



a) When $W = 600$ N, solving Eqs. (1) and (2) yields :

$F_{st} = 444.44 \text{ N} = 444 \text{ N}$ **Ans**

$F_{al} = 155.55 \text{ N} = 156 \text{ N}$ **Ans**

$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{155.55}{4(10^{-6})} = 38.88 \text{ MPa} < (\sigma_Y)_{al} = 70 \text{ MPa}$ **OK**

$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{444.44}{4(10^{-6})} = 111.11 \text{ MPa} < (\sigma_Y)_{st} = 120 \text{ MPa}$ **OK**

The elastic analysis is valid for both wires

b) When $W = 720$ N, solving Eqs. (1) and (2) yields :

$F_{st} = 533.33 \text{ N}; \quad F_{al} = 186.67 \text{ N}$

$\sigma_{al} = \frac{F_{al}}{A_{al}} = \frac{186.67}{4(10^{-6})} = 46.67 \text{ MPa} < (\sigma_Y)_{al} = 70 \text{ MPa}$ **OK**

$\sigma_{st} = \frac{F_{st}}{A_{st}} = \frac{533.33}{4(10^{-6})} = 133.33 \text{ MPa} > (\sigma_Y)_{st} = 120 \text{ MPa}$

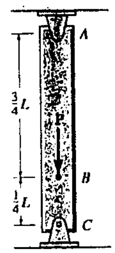
Therefore, the steel wire yields. Hence,

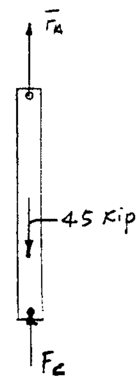
$F_{st} = (\sigma_Y)_{st} A_{st} = 120(10^6)(4)(10^{-6}) = 480 \text{ N}$ **Ans**

From Eq. (1), $F_{al} = 240$ N **Ans**

$\sigma_{al} = \frac{240}{4(10^{-6})} = 60 \text{ MPa} < (\sigma_Y)_{al}$ **OK**

4-99 The bar has a cross-sectional area of 1 in^2. If a force of $P = 45$ kip is applied at B and then removed, determine the residual stress in sections AB and BC. $\sigma_Y = 30$ ksi.

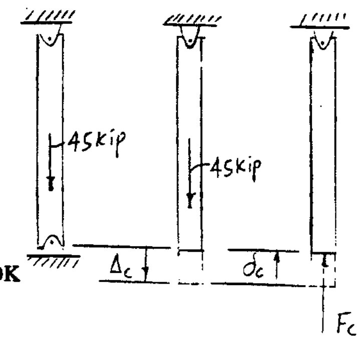




$+\uparrow \Sigma F_y = 0: \quad F_A + F_C - 45 = 0 \qquad (1)$

Assume both segment AB and BC behave elastically.

$+\downarrow \quad 0 = \Delta_C - \delta_C : \quad \dfrac{45(\frac{3}{4}L)}{AE} = \dfrac{F_C L}{AE}$



$F_C = 33.75$ kip

From Eq. (1), $F_A = 11.25$ kip

$\sigma_{AB} = \dfrac{F_A}{A} = \dfrac{11.25}{1} = 11.25 \text{ ksi} < \sigma_Y = 30 \text{ ksi}$ **OK**

$\sigma_{BC} = \dfrac{F_C}{A} = \dfrac{33.75}{1} = 33.75 \text{ ksi} > \sigma_Y = 30 \text{ ksi}$

Therefore segment BC yields and the elastic analysis is invalid.

Plastic analysis : Assume segment BC yields and AB behaves elastically.

$F_C = \sigma_Y(A) = 30(1) = 30.0$ kip

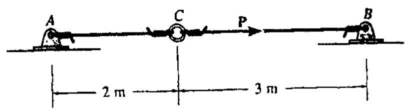
From Eq. (1), $F_A = 15.0$ kip and $\sigma_{AB} = \dfrac{15}{1} = 15.0 \text{ ksi} < \sigma_Y = 30 \text{ ksi}$ **OK**

A reversed force of 45 kip applied results in a reversed $F_C = 33.75$ kip and $F_A = 11.25$ kip which produces $\sigma_{BC} = 33.75$ ksi (T) and $\sigma_{AB} = 11.25$ ksi (C). Hence,

$(\sigma_{AB})_r = 15 - 11.25 = 3.75$ ksi (T) **Ans**

$(\sigma_{BC})_r = -30 + 33.75 = 3.75$ ksi (T) **Ans**

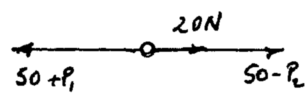
*4–100 Two steel wires, each having a cross-sectional area of 2 mm^2, are tied to a ring at C, and then stretched and tied between the two pins A and B. The initial tension in the wires is 50 N. If a horizontal force **P** is applied to the ring, determine the force in each wire if $P = 20$ N. What is the smallest force P that must be applied to the ring to reduce the force in wire CB to zero? Take $\sigma_Y = 300$ MPa. $E_{st} = 200$ GPa.



Equilibrium :

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 20 + (50 - P_2) - (50 + P_1) = 0$

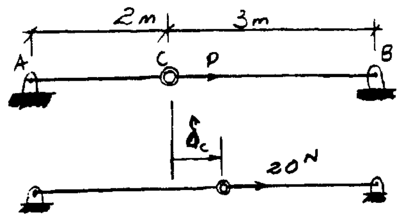
$P_1 + P_2 = 20 \qquad (1)$



Compatibility condition :

$$\delta_C = \frac{P_1(2)}{AE} = \frac{P_2(3)}{AE}$$

$P_1 = 1.5\,P_2 \qquad (2)$



Solving Eqs. (1) and (2) yields:

$P_1 = 12$ N, $\quad P_2 = 8$ N

$F_{AC} = 50 + 12 = 62$ N **Ans**

$F_{BC} = 50 - 8 = 42$ N **Ans**

For $F_{CB} = 0$; $\quad 50 - P_2 = 0$

$P_2 = 50$ N

$P_1 = 1.5(50) = 75$ N

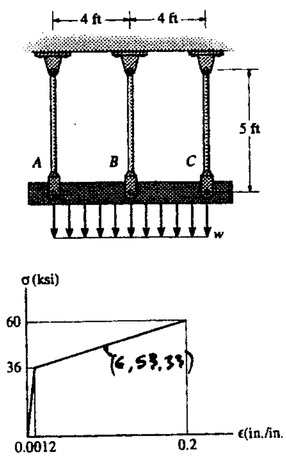
$P = 75 + 50 = 125$ N **Ans**

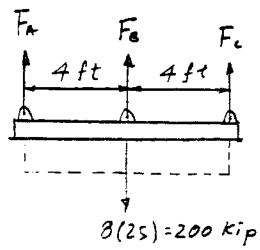
$F_{AC} = 50 + 75 = 125$ N

$$\sigma_{AC} = \frac{125}{2(10^{-6})} = 62.5 \text{ MPa}$$

$62.5 \text{ MPa} < \sigma_Y \qquad$ OK

4–101 The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 1.25 in^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. If a load of $w = 25$ kip/ft is applied to the beam, determine the stress in each bar and the vertical displacement of the beam.





$\circlearrowleft + \Sigma M_B = 0$: $\quad F_C(4) - F_A(4) = 0$;

$F_A = F_C = F$

$+\uparrow \Sigma F_y = 0$: $\quad 2F + F_B - 200 = 0 \qquad (1)$

Since the loading and geometry are symmetrical, the bar will remain horizontal. Therefore, the displacement of the bars is the same and hence, the force in each bar is the same. From Eq. (1).

$F = F_B = 66.67$ kip

Thus,

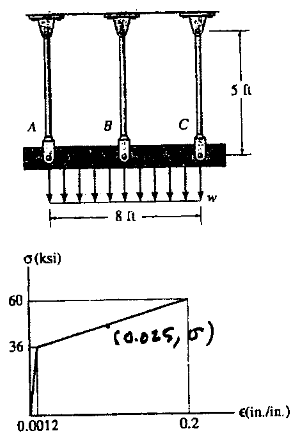
$$\sigma_A = \sigma_B = \sigma_C = \frac{66.67}{1.25} = 53.33 \text{ ksi} \qquad \textbf{Ans}$$

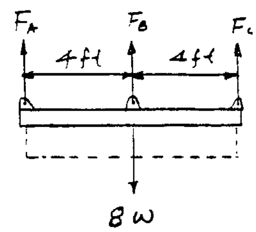
From the stress - strain diagram :

$$\frac{53.33 - 36}{\varepsilon - 0.0012} = \frac{60 - 36}{0.2 - 0.0012}; \qquad \varepsilon = 0.14477 \text{ in./in.}$$

$$\delta = \varepsilon L = 0.14477(5)(12) = 8.69 \text{ in.} \qquad \textbf{Ans}$$

4–102 The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 0.75 in^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. Determine the intensity of the distributed loading w needed to cause the beam to be displaced downward 1.5 in.





$\circlearrowleft + \Sigma M_B = 0; \quad F_C(4) - F_A(4) = 0; \quad F_A = F_C = F$

$+\uparrow \Sigma F_y = 0; \quad 2F + F_B - 8\,w = 0 \qquad (1)$

Since the system and the loading are symmetrical, the bar will remain horizontal. Hence the displacement of the bars is the same and the force supported by each bar is the same.

From Eq. (1),

$F_B = F = 2.6667\,w \qquad (2)$

From the stress – strain diagram :

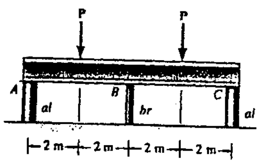
$$\varepsilon = \frac{1.5}{5\,(12)} = 0.025 \text{ in./in.}$$

$$\frac{\sigma - 36}{0.025 - 0.0012} = \frac{60 - 36}{0.2 - 0.0012}; \qquad \sigma = 38.87 \text{ ksi}$$

Hence $F = \sigma A = 38.87\,(0.75) = 29.15$ kip

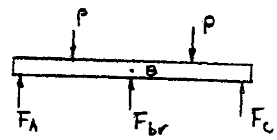
From Eq.(2), $\quad w = 10.9$ kip/ft $\qquad$ **Ans**

4-103. The rigid beam is supported by the three posts A, B, and C of equal length. Posts A and C have a diameter of 75 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post B has a diameter of 20 mm and is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. Determine the smallest magnitude of **P** so that (a) only rods A and C yield and (b) all the posts yield.



$\Sigma M_B = 0;\quad F_A = F_C = F_{al}$

$+\uparrow \Sigma F_y = 0;\quad F_{br} + 2F_{al} - 2P = 0 \qquad (1)$



(a)

Post A and C will yield,

$$F_{al} = (\sigma_y)_{al}A = 20(10^6)(\tfrac{\pi}{4})(0.075)^2 = 88.36 \text{ kN}$$

$$(\varepsilon_{al})_Y = \frac{(\sigma_Y)_{al}}{E_{al}} = \frac{20(10^6)}{70(10^9)} = 0.0002857$$

Compatibility condition :

$$\delta_{br} = \delta_{al} = 0.0002857(L)$$

$$\frac{F_{br}(L)}{\frac{\pi}{4}(0.02)^2(100)(10^9)} = 0.0002857\,L$$

$$F_{br} = 8.976 \text{ kN}$$

$$\sigma_{br} = \frac{8.976(10^3)}{\frac{\pi}{4}(0.02^2)} = 28.6 \text{ MPa} < \sigma_Y \qquad \text{OK}$$

From Eq. (1),

$$8.976 + 2(88.36) - 2P = 0$$

$P = 92.8$ kN **Ans**

(b) All the posts yield :

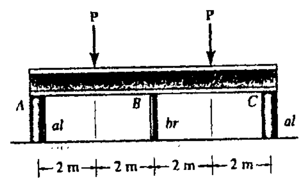
$$F_{br} = (\sigma_Y)_{br}A = (590)(10^6)(\tfrac{\pi}{4})(0.02^2) = 185.35 \text{ kN}$$

$$F_{al} = 88.36 \text{ kN}$$

From Eq. (1) : $185.35 + 2(88.36) - 2P = 0$

$P = 181$ kN **Ans**

•4–104 The rigid beam is supported by the three posts *A*, *B*, and *C*. Posts *A* and *C* have a diameter of 60 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post *B* is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. If $P = 130$ kN, determine the largest diameter of post *B* so that all the posts yield at the same time.



$$+\uparrow \Sigma F_y = 0; \qquad 2(F_Y)_{al} + F_{br} - 260 = 0 \qquad (1)$$

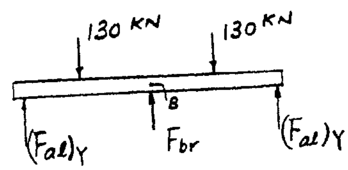
$$(F_{al})_Y = (\sigma_Y)_{al} A$$

$$= 20(10^6)(\tfrac{\pi}{4})(0.06)^2 = 56.55 \text{ kN}$$

From Eq. (1),

$$2(56.55) + F_{br} - 260 = 0$$

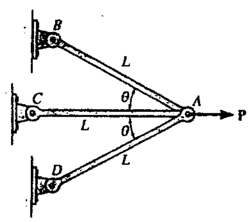
$$F_{br} = 146.9 \text{ kN}$$



$$(\sigma_Y)_{br} = 590(10^6) = \frac{146.9(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$$d_B = 0.01779 \text{ m} = 17.8 \text{ mm} \qquad \textbf{Ans}$$

4–105 The three bars are pinned together and subjected to the load **P**. If each bar has a cross-sectional area A, length L, and is made from an elastic perfectly plastic material, for which the yield stress is σ_Y, determine the largest load (ultimate load) that can be supported by the bars, i.e., the load P that causes all the bars to yield. Also, what is the horizontal displacement of point A when the load reaches its ultimate value? The modulus of elasticity is E.

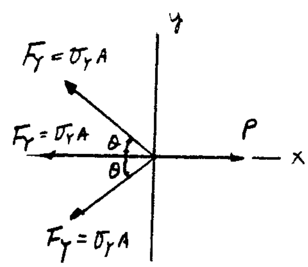


When all bars yield, the force in each bar is,

$F_Y = \sigma_Y A$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad P - 2\sigma_Y A\cos\theta - \sigma_Y A = 0$

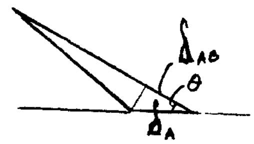
$P = \sigma_Y A(2\cos\theta + 1)$ **Ans**



Bar AC will yield first followed by bars AB and AD.

$$\delta_{AB} = \delta_{AD} = \frac{F_Y(L)}{AE} = \frac{\sigma_Y AL}{AE} = \frac{\sigma_Y L}{E}$$

$$\delta_A = \frac{\delta_{AB}}{\cos\theta} = \frac{\sigma_Y L}{E\cos\theta}$$ **Ans**



4-106. A material has a stress–strain diagram that can be described by the curve $\sigma = c\epsilon^{1/2}$. Determine the deflection δ of the end of a rod made from this material if it has a length L, cross-sectional area A, and a specific weight γ.

$\sigma = c\,\varepsilon^{\frac{1}{2}}; \qquad \sigma^2 = c^2\varepsilon$

$$\sigma^2(x) = c^2\varepsilon(x) \qquad (1)$$

However $\sigma(x) = \dfrac{P(x)}{A}; \qquad \varepsilon(x) = \dfrac{d\delta}{dx}$

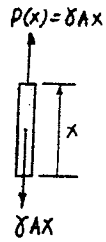
From Eq. (1),

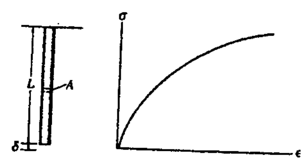
$$\frac{P^2(x)}{A^2} = c^2\frac{d\delta}{dx}; \qquad \frac{d\delta}{dx} = \frac{P^2(x)}{A^2c^2}$$

$$\delta = \frac{1}{A^2c^2}\int P^2(x)\,dx = \frac{1}{A^2c^2}\int_0^L(\gamma Ax)^2dx$$

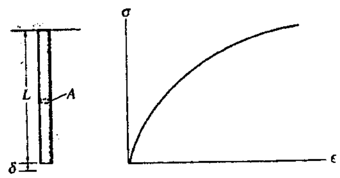
$$= \frac{\gamma^2}{c^2}\int_0^L x^2dx = \frac{\gamma^2}{c^2}\frac{x^3}{3}\Big|_0^L$$

$$\delta = \frac{\gamma^2L^3}{3c^2} \qquad \textbf{Ans}$$

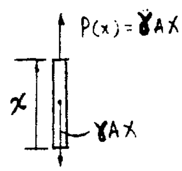




4–107 Solve Prob. 4-106 if the stress–strain diagram is defined by $\sigma = c\epsilon^{3/2}$.



Prob. 4–107



$$\sigma = c\varepsilon^{\frac{3}{2}} \ ; \qquad \varepsilon = \frac{\sigma^{\frac{2}{3}}}{c^{\frac{2}{3}}} \qquad (1)$$

However $\sigma(x) = \dfrac{P(x)}{A}$; $\quad \varepsilon(x) = \dfrac{d\delta}{dx}$

From Eq. (1),

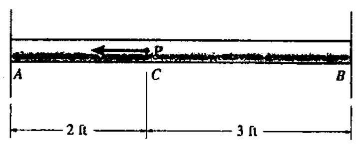
$$\frac{d\delta}{dx} = \frac{1}{c^{\frac{2}{3}}}\frac{P^{\frac{2}{3}}}{A^{\frac{2}{3}}}$$

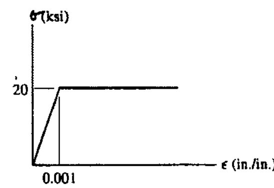
$$\delta = \frac{1}{c^{\frac{2}{3}}A^{\frac{2}{3}}}\int P^{\frac{2}{3}}\,dx = \frac{1}{(cA)^{\frac{2}{3}}}\int_0^L(\gamma Ax)^{\frac{2}{3}}dx$$

$$= \frac{1}{(cA)^{\frac{2}{3}}}(\gamma A)^{\frac{2}{3}}\int_0^L x^{\frac{2}{3}}dx = \left(\frac{\gamma}{c}\right)^{\frac{2}{3}}\left(\frac{3}{5}\right)x^{\frac{5}{3}}\Big|_0^L$$

$$\delta = \frac{3}{5}\left(\frac{\gamma}{c}\right)^{\frac{2}{3}}L^{\frac{5}{3}} \qquad \textbf{Ans}$$

•4–108 The bar having a diameter of 2 in. is fixed connected at its ends and supports the axial load **P**. If the material is elastic perfectly plastic as shown by the stress–strain diagram, determine the smallest load P needed to cause both segments AC and CB to yield. If this load is released, determine the permanent displacement of point C.





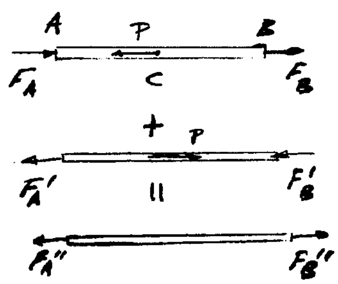
When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad (1)$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip} \qquad \textbf{Ans}$$



The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\, F_B'$$

So that from Eq. (1)

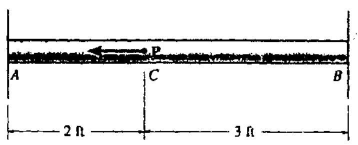
$$F_B' = 0.4P$$

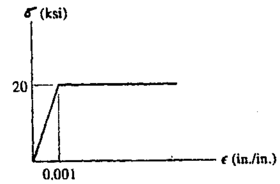
$$F_A' = 0.6P$$

$$\delta_C' = \frac{F_B' L}{AE} = \frac{0.4(P)(3)(12)}{AE} = \frac{0.4(125.66)(3)(12)}{\pi(1)^2(20/0.001)} = 0.02880 \text{ in.} \rightarrow$$

$$\Delta\delta = 0.036 - 0.0288 = 0.00720 \text{ in.} \leftarrow \qquad \textbf{Ans}$$

4-109 Determine the elongation of the bar in Prob. 4-108 when both the load **P** and the supports are removed.





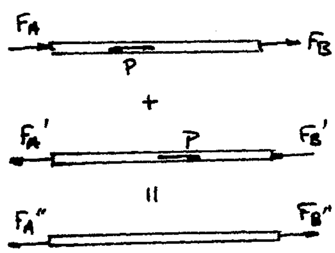
When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_A + F_B - P = 0 \qquad (1)$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip} \qquad \textbf{Ans}$$



The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\, F_B'$$

So that from Eq. (1)

$$F_B' = 0.4P$$

$$F_A' = 0.6P$$

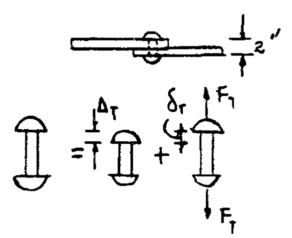
The resultant reactions are

$$F_A'' = F_B'' = -62.832 + 0.4(125.66) = 62.832 - 0.4(125.66) = 12.568 \text{ kip}$$

When the supports are removed the elongation will be,

$$\delta = \frac{PL}{AE} = \frac{12.568(5)(12)}{\pi(1)^2(20/0.001)} = 0.0120 \text{ in.} \qquad \textbf{Ans}$$

4-110. A 0.25-in.-diameter steel rivet having a temperature of 1500°F is secured between two plates such that at this temperature it is 2 in. long and exerts a clamping force of 250 lb between the plates. Determine the approximate clamping force between the plates when the rivet cools to 70°F. For the calculation, assume that the heads of the rivet and the plates are rigid. Take $\alpha_{st} = 8(10^{-6})/°F$, $E_{st} = 29(10^3)$ ksi. Is the result a conservative estimate of the actual answer? Why or why not?



By superposition;

$(+\downarrow) \qquad 0 = \Delta_T - \delta_T$

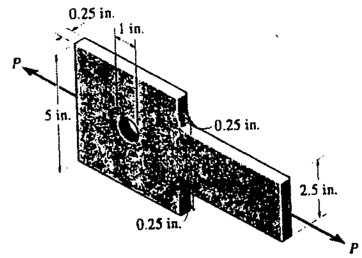
$$0 = 8(10^{-6})(1500-70)(2) - \frac{F_T(2)}{\frac{\pi}{4}(0.25^2)(29)(10^3)}$$

$F_T = 16.285$ kip

$F = 0.25 + 16.285 = 16.5$ kip **Ans**

Yes, because as the rivet cools, the plates and rivet head will also deform. Consequently, the force F_T on the rivet will not be as great.

4–111 Determine the maximum axial force P that can be applied to the steel plate. The allowable stress is $\sigma_{allow} = 21$ ksi.



Assume failure at fillet

$\frac{r}{h} = \frac{0.25}{2.5} = 0.1\ ; \qquad \frac{w}{h} = \frac{5}{2.5} = 2$

From Fig. 4 - 23, $K = 2.4$

$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$

$21 = 2.4[\frac{P}{2.5(0.25)}]\ ; \qquad P = 5.47$ kip

Assume failure at hole

$\frac{r}{w} = \frac{0.5}{5} = 0.1;$ From Fig. 4 - 24, $K = 2.65$

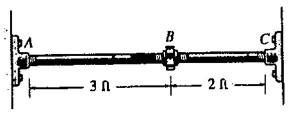
$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$

$21 = 2.65[\frac{P}{(5 - 1)(0.25)}]$

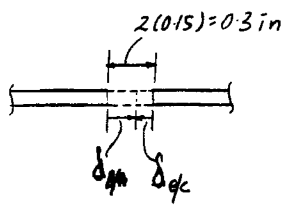
$P = 7.92$ kip

$P = 5.47$ kip (controls) **Ans**

•4–112 Two A-36 steel pipes, each having a cross-sectional area of 0.32 in², are screwed together using a union at *B* as shown. Originally the assembly is adjusted so that no load is on the pipe. If the union is then tightened so that its screw, having a lead of 0.15 in., undergoes two full turns, determine the average normal stress developed in the pipe. Assume that the union at *B* and couplings at *A* and *C* are rigid. Neglect the size of the union. *Note:* The lead would cause the pipe, when *unloaded*, to shorten 0.15 in. when the union is rotated one revolution.



The loads acting on both segments *AB* and *BC* are the same since no external load acts on the system.



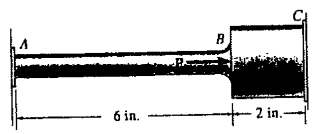
$$0.3 = \delta_{B/A} + \delta_{B/C}$$

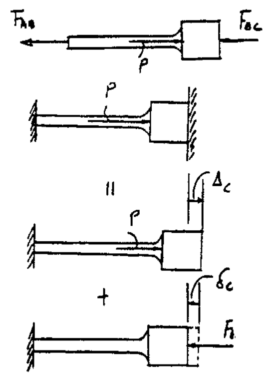
$$0.3 = \frac{P(3)(12)}{0.32(29)(10^3)} + \frac{P(2)(12)}{0.32(29)(10^3)}$$

$$P = 46.4 \text{ kip}$$

$$\sigma_{AB} = \sigma_{BC} = \frac{P}{A} = \frac{46.4}{0.32} = 145 \text{ ksi} \qquad \textbf{Ans}$$

4–113 The force **P** is applied to the bar, which is composed of an elastic perfectly plastic material. Construct a graph to show how the force in each section AB and BC (ordinate) varies as P (abscissa) is increased. The bar has cross-sectional areas of 1 in^2 in region AB and 4 in^2 in region BC, and σ_Y = 30 ksi.





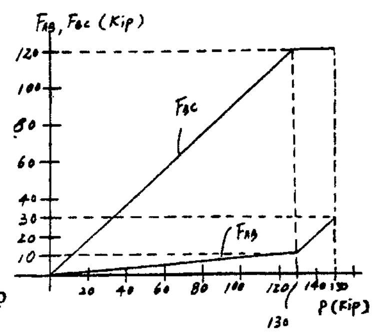
$\underset{\rightarrow}{+}\Sigma F_x = 0; \qquad P - F_{AB} - F_{BC} = 0 \qquad (1)$

Elastic behavior : $\underset{\rightarrow}{+}\; 0 = \Delta_C - \delta_C$;

$$0 = \frac{P(6)}{(1)E} - [\frac{F_{BC}(2)}{(4)E} + \frac{F_{BC}(6)}{(1)E}]$$

$F_{BC} = 0.9231\,P \qquad (2)$

Substituting Eq. (2) into (1) :
$F_{AB} = 0.07692\,P \qquad (3)$



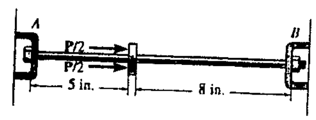
By comparison, segment BC will yield first. Hence,
$(F_{BC})_Y = \sigma_Y A = 30(4) = 120$ kip

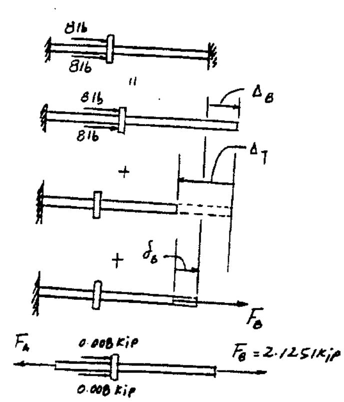
From Eq. (1) and (3) using $F_{BC} = (F_{BC})_Y = 120$ kip
$P = 130$ kip: $\quad F_{AB} = 10$ kip

When segment AB yields,
$(F_{AB})_Y = \sigma_Y A = 30(1) = 30$ kip ; $\qquad (F_{BC})_Y = 120$ kip

From Eq. (1),
$P = 150$ kip

4-114 The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at A and B when $T_1 = 70°F$. If the temperature becomes $T_2 = -10°F$, and an axial force of $P = 16$ lb is applied to the rigid collar as shown, determine the reactions at A and B.





$$\xrightarrow{+} \quad 0 = \Delta_B - \Delta_T + \delta_B$$

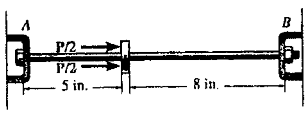
$$0 = \frac{0.016(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[70° - (-10°)](13) + \frac{F_B(13)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)}$$

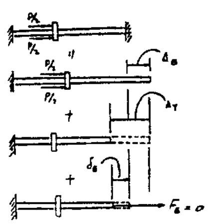
$F_B = 2.1251 \text{ kip} = 2.13 \text{ kip}$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0; \quad 2(0.008) + 2.1251 - F_A = 0$

$F_A = 2.14 \text{ kip}$ **Ans**

4–115 The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at *A* and *B* when $T_1 = 70°F$. Determine the force *P* that must be applied to the collar so that, when $T = 0°F$, the reaction at *B* is zero.



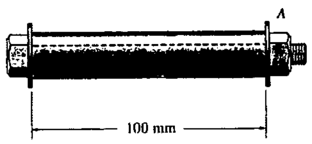


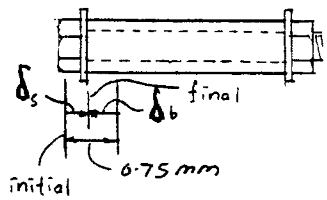
$$\xrightarrow{+} \quad 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{P(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[(70)(13)] + 0$$

$$P = 4.85 \text{ kip} \qquad \textbf{Ans}$$

*4–116 The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. If it is then tightened one-half turn, determine the force in the bolt and the sleeve. The single-threaded screw on the bolt has a lead of 1.5 mm. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa. *Note:* The lead represents the distance the nut advances along the bolt for one complete turn of the nut.



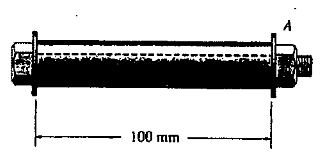


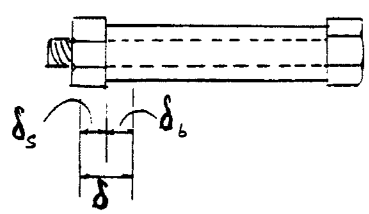
$$0.75 = \delta_s + \delta_b$$

$$0.75 = \frac{F(100)}{\frac{\pi}{4}(0.01^2 - 0.008^2)(70)(10^9)} + \frac{F(100)}{\frac{\pi}{4}(0.007^2)(200)(10^9)}$$

$$F = 11808 \text{ N} = 11.8 \text{ kN} \qquad \textbf{Ans}$$

4-117 The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. Determine the amount of turn the nut at A is to be tightened so that the force in the bolt and sleeve will be 12 kN. The single-threaded screw on the bolt has a lead of 1.5 mm. E_{st} = 200 GPa, E_{al} = 70 GPa. *Note:* The lead represents the distance the nut advances along the bolt for one complete turn of the nut.





Since no external load was applied, the force acting on the sleeve must be equal to that acting on bolt i.e. 12 kN.

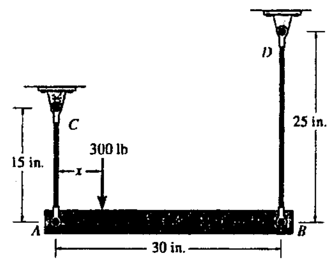
$\delta = \delta_s + \delta_b$

$$= \frac{12(10^3)(100)}{\frac{\pi}{4}(0.01^2 - 0.008^2)(70)(10^9)} + \frac{12(10^3)(100)}{\frac{\pi}{4}(0.007^2)(200)(10^9)}$$

$= 0.7622$ mm

The number of turns $= \dfrac{0.7622}{1.5} = 0.508$ of a turn **Ans**

4-118 The assembly consists of two A-36 steel suspender rods AC and BD attached to the 100-lb uniform rigid beam AB. Determine the position x for the 300-lb loading so that the beam remains in a horizontal position both before and after the load is applied. Each rod has a diameter of 0.5 in.



$$+\Sigma M_A = 0; \qquad F_{BD}(30) - 300x - 100(15) = 0$$

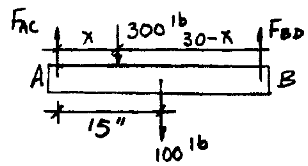
$$F_{BD} = 10x + 50 \qquad (1)$$

$$+\Sigma M_B = 0; \qquad -F_{AC}(30) + 300(30-x) + 100(15) = 0$$

$$F_{AC} = 350 - 10x \qquad (2)$$

$$\delta_A = \delta_B$$

From Eq. (1) and (2),

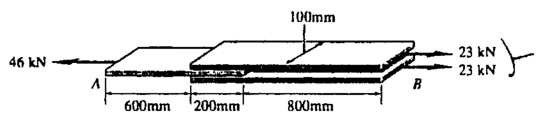


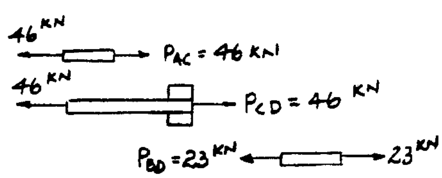
$$\frac{(350 - 10x)(15)}{AE} = \frac{(10x + 50)(25)}{AE}$$

$$5250 - 150x = 250x + 1250$$

$$x = 10 \text{ in.} \qquad \textbf{Ans}$$

4–119 The joint is made from three A-36 steel plates that are bonded together at their seams. Determine the displacement of end A with respect to end B when the joint is subjected to the axial loads shown. Each plate has a thickness of 5 mm.

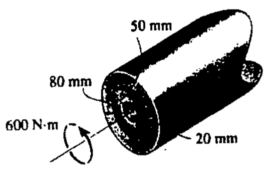




$$\delta_{A/B} = \Sigma\frac{PL}{AE} = \frac{46(10^3)(600)}{(0.005)(0.1)(200)(10^9)} + \frac{46(10^3)(200)}{3(0.005)(0.1)(200)(10^9)} + \frac{23(10^3)(800)}{(0.005)(0.1)(200)(10^9)}$$

$= 0.491$ mm **Ans**

5–1 The tube is subjected to a torque of 600 N · m. Determine the amount of this torque that is resisted by the shaded section. Solve the problem two ways: (*a*) by using the torsion formula; (*b*) by finding the resultant of the shear-stress distribution.



a)

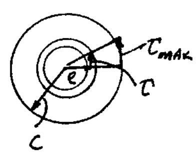
$$\tau_{max} = \frac{Tc}{J} = \frac{600\,(0.08)}{\frac{\pi}{2}(0.08^4 - 0.02^4)} = 748964 \text{ Pa}$$

$$\tau_{max} = \frac{T'c}{J'}$$

$$748\,964 = \frac{T'\,(0.08)}{\frac{\pi}{2}(0.08^4 - 0.05^4)}$$

$T' = 510$ N · m **Ans**

b)



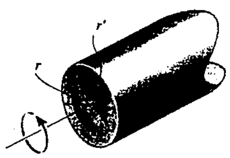
$$\tau = \tau_{max}\left(\frac{\rho}{c}\right) \qquad dA = 2\pi\rho\, d\rho$$

$$dT' = \rho\tau\, dA = \rho\tau_{max}\left(\frac{\rho}{c}\right)2\pi\rho\, d\rho$$

$$T' = \frac{2\pi\,\tau_{max}}{c}\int \rho^3\, d\rho = \frac{2\pi\,(748964)}{0.08}\,\frac{\rho^4}{4}\Bigg|_{0.05}^{0.08}$$

$= 510$ N · m **Ans**

5–2 The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque ($T/2$). Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



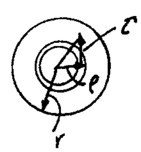
a) $\tau_{max} = \dfrac{Tc}{J} = \dfrac{Tr}{\frac{\pi}{2}r^4} = \dfrac{2T}{\pi r^3}$

$\tau = \dfrac{(\frac{T}{2})r'}{\frac{\pi}{2}(r')^4} = \dfrac{T}{\pi(r')^3}$

Since $\tau = \dfrac{r'}{r}\tau_{max}$; $\quad \dfrac{T}{\pi(r')^3} = \dfrac{r'}{r}(\dfrac{2T}{\pi r^3})$

$r' = \dfrac{r}{2^{\frac{1}{4}}} = 0.841\,r$ **Ans**

b) $\int_0^{\frac{T}{2}} dT = 2\pi\int_0^{r'} \tau\rho^2\, d\rho$



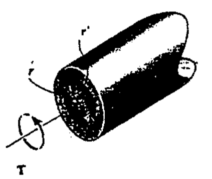
$\int_0^{\frac{T}{2}} dT = 2\pi\int_0^{r'} \frac{\rho}{r}\tau_{max}\rho^2\, d\rho$

$\int_0^{\frac{T}{2}} dT = 2\pi\int_0^{r'} \frac{\rho}{r}(\frac{2T}{\pi r^3})\rho^2\, d\rho$

$\dfrac{T}{2} = \dfrac{4T}{r^4}\int_0^{r'} \rho^3\, d\rho$

$r' = \dfrac{r}{2^{\frac{1}{4}}} = 0.841r$ **Ans**

5-3. The solid shaft of radius r is subjected to a torque **T**. Determine the radius r' of the inner core of the shaft that resists one-quarter of the applied torque ($T/4$). Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



a) $\tau_{max} = \frac{Tc}{J} = \frac{T(r)}{\frac{\pi}{2}(r^4)} = \frac{2T}{\pi r^3}$

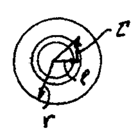
Since $\tau = \frac{r'}{r}\tau_{max} = \frac{2Tr'}{\pi r^4}$

$\tau' = \frac{T'c'}{J'}; \quad \frac{2Tr'}{\pi r^4} = \frac{(\frac{T}{4})r'}{\frac{\pi}{2}(r')^4}$

$r' = \frac{r}{4^{\frac{1}{4}}} = 0.707\, r$ **Ans**

b) $\tau = \frac{\rho}{c}\tau_{max} = \frac{\rho}{r}(\frac{2T}{\pi r^3}) = \frac{2T}{\pi r^4}\rho; \qquad dA = 2\pi\rho\, d\rho$

$dT = \rho\tau\, dA = \rho[\frac{2T}{\pi r^4}\rho](2\pi\rho\, d\rho) = \frac{4T}{r^4}\rho^3 d\rho$

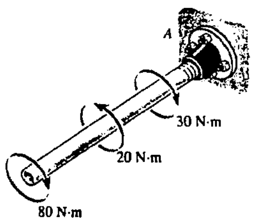


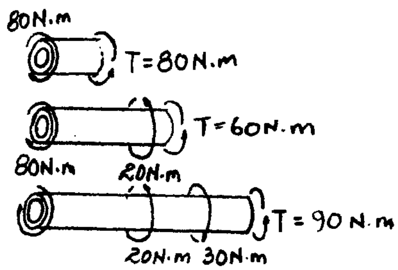
$\int_0^{\frac{T}{4}} dT = \frac{4T}{r^4}\int^{r'} \rho^3 d\rho$

$\frac{T}{4} = \frac{4T}{r^4}\frac{\rho^4}{4}\Big|_0^{r'}; \quad \frac{1}{4} = \frac{(r')^4}{r^4}$

$r' = 0.707\, r$ **Ans**

•5–4 The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall at A and three torques are applied to it as shown, determine the absolute maximum shear stress developed in the pipe.

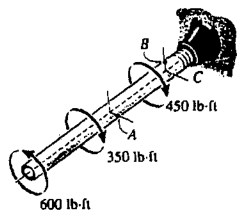


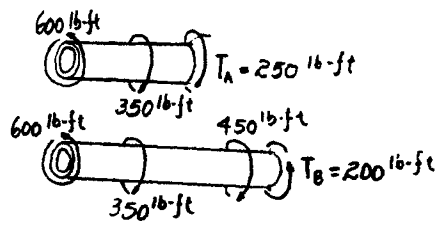


$$\tau_{max} = \frac{T_{max}\,c}{J} = \frac{90(0.02)}{\frac{\pi}{2}(0.02^4 - 0.0185^4)}$$

$$= 26.7 \text{ MPa} \qquad \textbf{Ans}$$

5–5 The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and three torques are applied to it as shown, determine the shear stress developed at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.

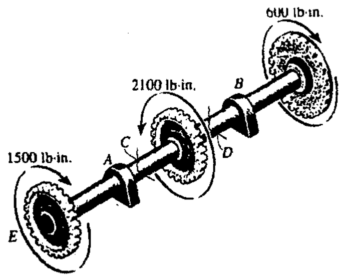


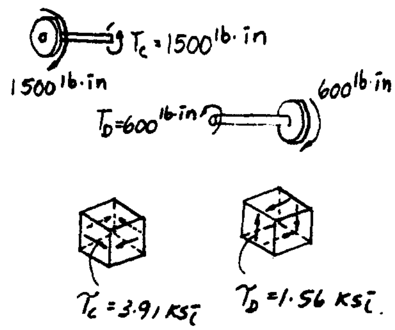


$$\tau_A = \frac{Tc}{J} = \frac{250(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.45 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_B = \frac{Tc}{J} = \frac{200(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 2.76 \text{ ksi} \qquad \textbf{Ans}$$

5–6 The solid 1.25-in.-diameter shaft is used to transmit the torques applied to the gears. If it is supported by smooth bearings at A and B, which do not resist torque, determine the shear stress developed in the shaft at points C and D. Indicate the shear stress on volume elements located at these points.

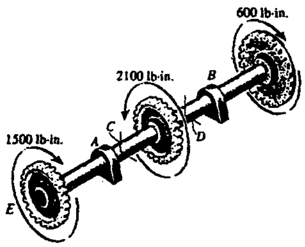




$$\tau_C = \frac{Tc}{J} = \frac{1500(0.625)}{\frac{\pi}{2}(0.625^4)} = 3.91 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_D = \frac{Tc}{J} = \frac{600(0.625)}{\frac{\pi}{2}(0.625^4)} = 1.56 \text{ ksi} \qquad \textbf{Ans}$$

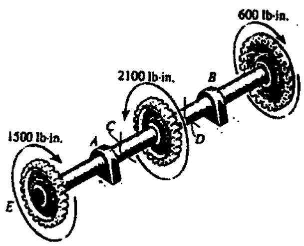
5-7. The shaft has an outer diameter of 1.25 in. and an inner diameter of 1 in. If it is subjected to the applied torques as shown, determine the absolute maximum shear stress developed in the shaft. The smooth bearings at *A* and *B* do not resist torque.



$T_{max} = 1500 \text{ lb} \cdot \text{in.}$

$$\tau_{max} = \frac{Tc}{J} = \frac{1500(0.625)}{\frac{\pi}{2}\left[(0.625)^4 - (0.5)^4\right]} = 6.62 \text{ ksi} \qquad \textbf{Ans}$$

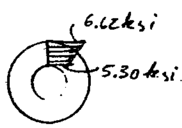
***5-8.** The shaft has an outer diameter of 1.25 in. and an inner diameter of 1 in. If it is subjected to the applied torques as shown, plot the shear-stress distribution acting along a radial line lying within region *EA* of the shaft. The smooth bearings at *A* and *B* do not resist torque.



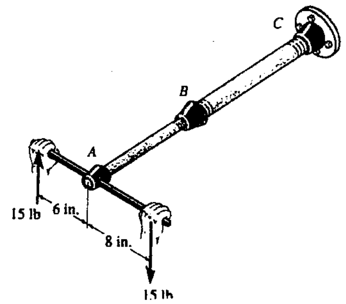
$T = 1500\text{ lb}\cdot\text{in.}$

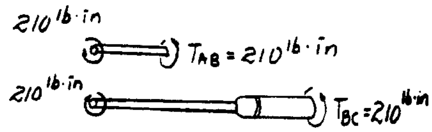
$$\tau_{max} = \frac{Tc}{J} = \frac{1500(0.625)}{\frac{\pi}{2}\left[(0.625)^4 - (0.5)^4\right]} = 6.62\text{ ksi}$$

$$\tau_2 = \frac{T\rho}{J} = \frac{1500(0.5)}{\frac{\pi}{2}\left[(0.625)^4 - (0.5)^4\right]} = 5.30\text{ ksi}$$



5–9 The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at *B*. The smaller pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in., whereas the larger pipe has an outer diameter of 1 in. and an inner diameter of 0.86 in. If the pipe is tightly secured to the wall at *C*, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.

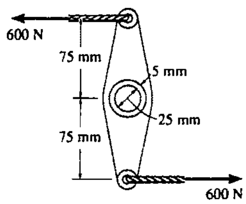




$$\tau_{AB} = \frac{Tc}{J} = \frac{210(0.375)}{\frac{\pi}{2}(0.375^4 - 0.34^4)} = 7.82 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_{BC} = \frac{Tc}{J} = \frac{210(0.5)}{\frac{\pi}{2}(0.5^4 - 0.43^4)} = 2.36 \text{ ksi} \qquad \textbf{Ans}$$

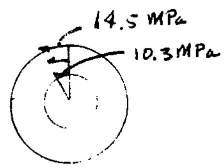
5–10 The link acts as part of the elevator control for a small airplane. If the attached aluminum tube has an inner diameter of 25 mm and a wall thickness of 5 mm, determine the maximum shear stress in the tube when the cable force of 600 N is applied to the cables. Also, sketch the shear-stress distribution over the cross section.



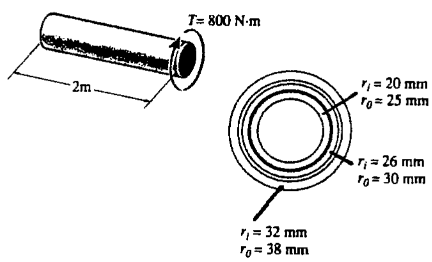
$T = 600(0.15) = 90\ \text{N}\cdot\text{m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{90(0.0175)}{\frac{\pi}{2}[(0.0175)^4 - (0.0125)^4]} = 14.5\ \text{MPa} \qquad \textbf{Ans}$$

$$\tau_i = \frac{T\rho}{J} = \frac{90(0.0125)}{\frac{\pi}{2}[(0.0175)^4 - (0.0125)^4]} = 10.3\ \text{MPa}$$



5–11 The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of $T = 800$ N · m is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.

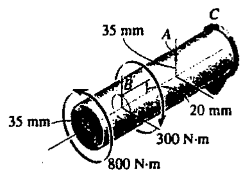


$$J = \frac{\pi}{2}((0.038)^4 - (0.032)^4) + \frac{\pi}{2}((0.030)^4 - (0.026)^4) + \frac{\pi}{2}((0.025)^4 - (0.020)^4)$$

$$J = 2.545(10^{-6})\text{m}^4$$

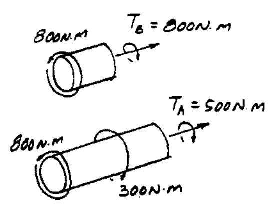
$$\tau_{max} = \frac{Tc}{J} = \frac{800(0.038)}{2.545(10^{-6})} = 11.9 \text{ MPa} \qquad \textbf{Ans}$$

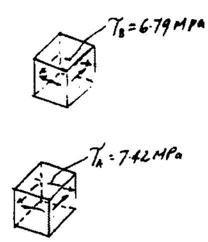
•5–12 The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume elements located at these points.



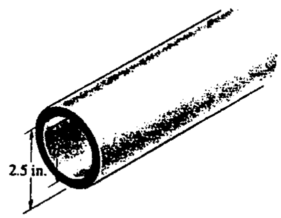
$$\tau_B = \frac{T_B \rho}{J} = \frac{800(0.02)}{\frac{\pi}{2}(0.035^4)} = 6.79 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_A = \frac{T_A\, c}{J} = \frac{500(0.035)}{\frac{\pi}{2}(0.035^4)} = 7.42 \text{ MPa} \qquad \textbf{Ans}$$





5-13. A steel tube having an outer diameter of 2.5 in. is used to transmit 350 hp when turning at 27 rev/min. Determine the inner diameter d of the tube to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{allow} = 10$ ksi.



$$\omega = \frac{27(2\pi)}{60} = 2.8274 \text{ rad/s}$$

$$P = T\omega$$

$$350(550) = T(2.8274)$$

$$T = 68\ 082.9 \text{ lb}\cdot\text{ft}$$

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

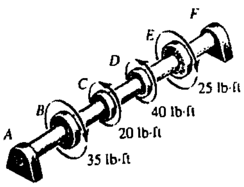
$$10(10^3) = \frac{68\ 082.9\ (12)(1.25)}{\frac{\pi}{2}(1.25^4 - c_i^4)}$$

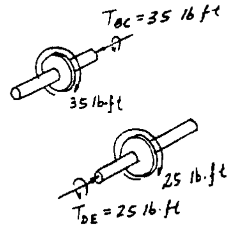
$$c_i = 1.2416 \text{ in.}$$

$$d = 2.48 \text{ in.}$$

Use $d = 2\frac{3}{8}$ in. **Ans**

5-14 The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions *BC* and *DE* of the shaft. The bearings at *A* and *F* allow free rotation of the shaft.

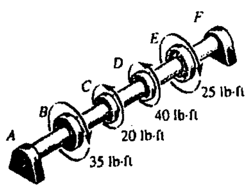


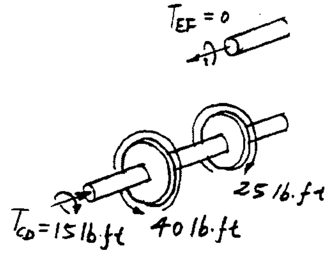


$$(\tau_{BC})_{\max} = \frac{T_{BC}\,c}{J} = \frac{35(12)(0.375)}{\frac{\pi}{2}(0.375)^4} = 5070 \text{ psi} = 5.07 \text{ ksi}$$ **Ans**

$$(\tau_{DE})_{\max} = \frac{T_{DE}\,c}{J} = \frac{25(12)(0.375)}{\frac{\pi}{2}(0.375)^4} = 3621 \text{ psi} = 3.62 \text{ ksi}$$ **Ans**

5–15 The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions *CD* and *EF* of the shaft. The bearings at *A* and *F* allow free rotation of the shaft.



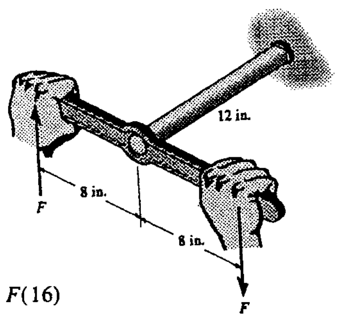


$$(\tau_{EF})_{max} = \frac{T_{EF}\, c}{J} = 0 \qquad \textbf{Ans}$$

$$(\tau_{CD})_{max} = \frac{T_{CD}\, c}{J} = \frac{15(12)(0.375)}{\frac{\pi}{2}(0.375)^4}$$

$$= 2173 \text{ psi} = 2.17 \text{ ksi} \qquad \textbf{Ans}$$

***5–16** The steel shaft has a diameter of 1 in. and is screwed into the wall using a wrench. Determine the largest couple forces F that can be applied to the shaft without causing the steel to yield. $\tau_Y = 8$ ksi.

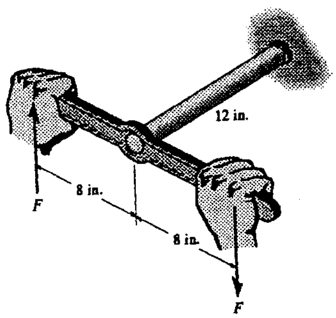


$$T = F(16)$$

$$\tau_{max} = \frac{Tc}{J}; \qquad 8(10^3) = \frac{F(16)(0.5)}{\frac{\pi}{2}(0.5)^4}$$

$$F = 98.2 \text{ lb}$$ **Ans**

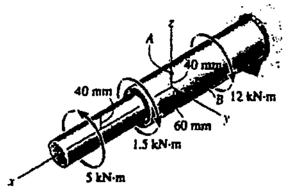
5–17 The steel shaft has a diameter of 1 in. and is screwed into the wall using a wrench. Determine the maximum shear stress in the shaft if the couple forces have a magnitude of $F = 30$ lb.



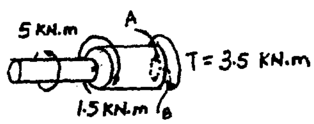
$$T = 30(16) = 480 \text{ lb} \cdot \text{in.}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{480(0.5)}{\frac{\pi}{2}(0.5)^4} = 2.44 \text{ ksi} \qquad \textbf{Ans}$$

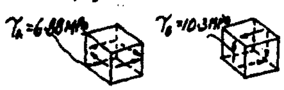
5–18. The steel shaft is subjected to the torsional loading shown. Determine the shear stress developed at points A and B and sketch the shear stress on volume elements located at these points. The shaft where A and B are located has an outer radius of 60 mm.



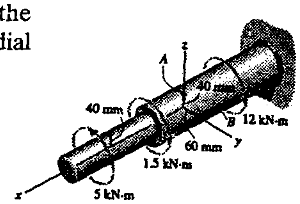
$$\tau_A = \frac{T\rho}{J} = \frac{3.5(10^3)(0.04)}{\frac{\pi}{2}(0.06)^4} = 6.88 \text{ MPa} \qquad \textbf{Ans}$$



$$\tau_B = \frac{Tc}{J} = \frac{3.5(10^3)(0.06)}{\frac{\pi}{2}(0.06)^4} = 10.3 \text{ MPa} \qquad \textbf{Ans}$$



5–19. The steel shaft is subjected to the torsional loading shown. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line where it is maximum.

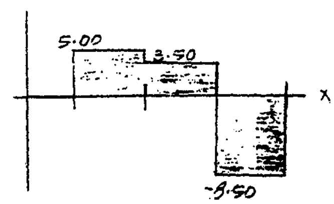


Maximum torque is 8.5 kN · m; however, two sections of the shaft should be considered since J is different.

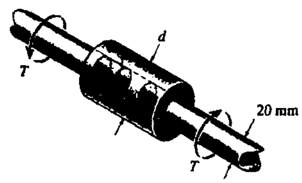
$$\tau_{max} = \frac{Tc}{J} = \frac{5(10^3)(0.04)}{\frac{\pi}{2}(0.04)^4} = 49.7\text{ MPa}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{8.5(10^3)(0.06)}{\frac{\pi}{2}(0.06)^4} = 25.1\text{ MPa}$$

$$\tau_{max\,abs} = 49.7\text{ MPa}$$ **Ans**



***5–20** The 20-mm-diameter steel shafts are connected using a brass coupling. If the yield point for the steel is $(\tau_Y)_{st} = 100$ MPa and for the brass $(\tau_Y)_{br} = 250$ MPa, determine the required outer diameter d of the coupling so that the steel and brass begin to yield at the same time when the assembly is subjected to a torque **T**. Assume that the coupling has an inner diameter of 20 mm.



For the steel shaft :

$$\tau_{max} = \frac{Tc}{J}; \qquad 100(10^6) = \frac{T(0.01)}{\frac{\pi}{2}(0.01)^4}; \qquad T = 157.08\ \text{N}\cdot\text{m}$$

For the brass coupling :

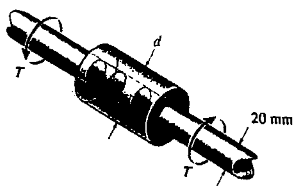
$$\tau_{max} = \frac{Tc}{J}; \qquad 250(10^6) = \frac{157.08(\frac{d}{2})}{\frac{\pi}{2}[(\frac{d}{2})^4 - (0.01)^4]}$$

$$24.5437(10^6)(d^4) - 78.54d - 3.9270 = 0$$

Solving,

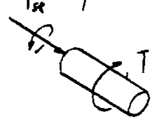
$d = 0.0219\ \text{m} = 21.9\ \text{mm}$ **Ans**

5–21 The 20-mm-diameter steel shafts are connected using a brass coupling. If the yield point for the steel is $(\tau_Y)_{st}$ = 100 MPa, determine the applied torque **T** necessary to cause the steel to yield. If d = 40 mm, determine the maximum shear stress in the brass. The coupling has an inner diameter of 20 mm.



For the steel shaft :

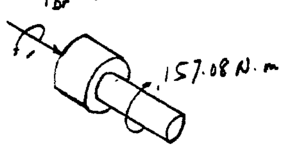
$$(\tau_Y)_{st} = \frac{Tc}{J}; \qquad 100(10^6) = \frac{T(0.01)}{\frac{\pi}{2}(0.01)^4}$$



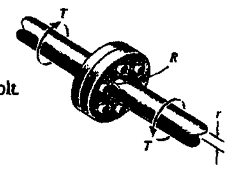
$T = 157.08\ \mathrm{N \cdot m} = 157\ \mathrm{N \cdot m}$ **Ans**

For the brass shaft :

$$(\tau_{max})_{br} = \frac{Tc}{J} = \frac{157.08(0.02)}{\frac{\pi}{2}[0.02^4 - 0.01^4]} = 13.3\ \mathrm{MPa}$$ **Ans**



5-22. The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is *uniform*, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shear stress in the bolts. Each bolt has a diameter d.



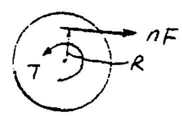
n is the number of bolts and F is the shear force in each bolt.

$$T - nFR = 0; \qquad F = \frac{T}{nR}$$

$$\tau_{avg} = \frac{F}{A} = \frac{\frac{T}{nR}}{(\frac{\pi}{4})d^2} = \frac{4T}{nR\pi d^2}$$

Maximum shear stress for the shaft :

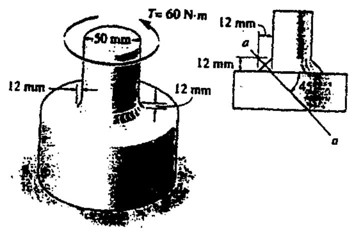
$$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$



$$\tau_{avg} = \tau_{max}; \qquad \frac{4T}{nR\pi d^2} = \frac{2T}{\pi r^3}$$

$$n = \frac{2r^3}{Rd^2} \qquad \textbf{Ans}$$

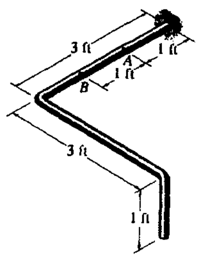
5-23. The steel shafts are connected together using a fillet weld as shown. Determine the average shear stress in the weld along section *a–a* if the torque applied to the shafts is $T = 60\ \text{N}\cdot\text{m}$. *Note:* The critical section where the weld fails is along section *a–a*.

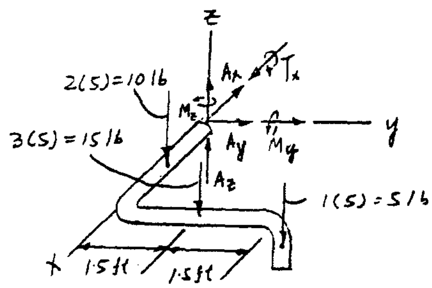


$$\tau_{avg} = \frac{V}{A} = \frac{(60/(0.025+0.006))}{2\pi(0.025+0.006)(0.012\sin45°)}$$

$\tau_{avg} = 1.17\ \text{MPa}$ **Ans**

*5–24 The rod has a diameter of 0.5 in. and a weight of 5 lb/ft. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.





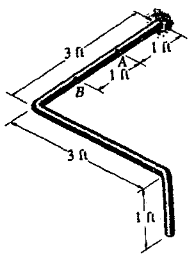
$\Sigma M_x = 0; \qquad T_x - 15(1.5) - 5(3) = 0;$

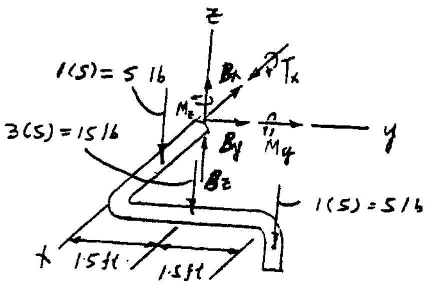
$T_x = 37.5\ \text{lb} \cdot \text{ft}$

$$(\tau_A)_{max} = \frac{T\,c}{J} = \frac{37.5(12)(0.25)}{\frac{\pi}{2}(0.25)^4}$$

$= 18.3\ \text{ksi}$ **Ans**

5–25 Solve Prob. 5–24 for the maximum torsional stress at *B*.





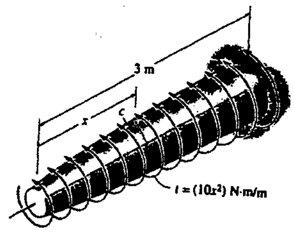
$\Sigma M_x = 0; \qquad -15(1.5) - 5(3) + T_x = 0;$

$T_x = 37.5 \text{ lb} \cdot \text{ft} = 450 \text{ lb} \cdot \text{in.}$

$$(\tau_B)_{max} = \frac{Tc}{J} = \frac{450(0.25)}{\frac{\pi}{2}(0.25)^4} = 18.3 \text{ ksi} \qquad \textbf{Ans}$$

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

5-27. The shaft is subjected to a distributed torque along its length of $t = (10x^2)$ N · m/m, where x is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius c of the shaft for $0 \le x \le 3$ m.

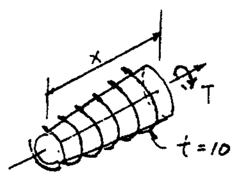


$$T = \int t\,dx = \int_0^x 10x^2 dx = \frac{10}{3}x^3$$

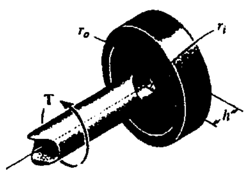
$$\tau = \frac{Tc}{J}; \qquad 80(10^6) = \frac{(\frac{10}{3})x^3 c}{\frac{\pi}{2}c^4}$$

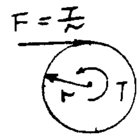
$$c^3 = 26.526\,(10^{-9})\,x^3$$

$$c = (2.98\,x)\text{ mm} \qquad \textbf{Ans}$$



***5-28** A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque **T** is applied to the shaft, determine the maximum shear stress in the rubber.



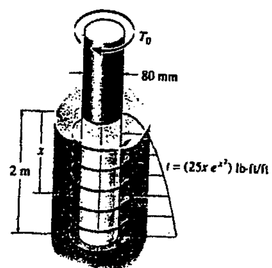


$$\tau = \frac{F}{A} = \frac{\frac{T}{r}}{2\pi r h} = \frac{T}{2\pi r^2 h}$$

Shear stress is maximum when r is the smallest, i.e. $r = r_i$. Hence,

$$\tau_{max} = \frac{T}{2\pi r_i^2 h}$$ **Ans**

■5-29. The shaft has a diameter of 80 mm and due to friction at its surface within the hole, it is subjected to a variable torque described by the function $t = (25xe^{x^2})\ \mathrm{N \cdot m/m}$, where x is in meters. Determine the minimum torque T_0 needed to overcome friction and cause it to twist. Also, determine the absolute maximum stress in the shaft.

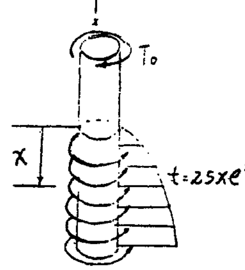


$$t = 25(x\,e^{x^2})\,; \qquad T_0 = \int_0^2 25(x\,e^{x^2})\,dx$$

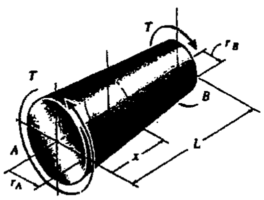
Integrating using Simpson's rule, we get

$T_0 = 669.98 = 670\ \mathrm{N \cdot m}$ **Ans**

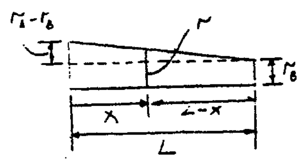
$$\tau_{\substack{abs\\max}} = \frac{T_0\,c}{J} = \frac{(669.98)(0.04)}{\frac{\pi}{2}(0.04)^4} = 6.66\ \mathrm{MPa}$$ **Ans**



5-30. The solid shaft has a linear taper from r_A at one end to r_B at the other. Derive an equation that gives the maximum shear stress in the shaft at a location x along the shaft's axis.



$$r = r_B + \frac{r_A - r_B}{L}(L - x) = \frac{r_B L + (r_A - r_B)(L - x)}{L}$$

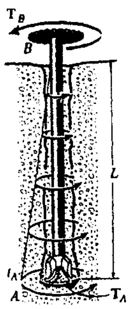


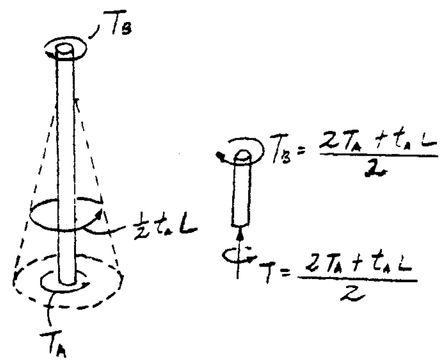
$$= \frac{r_A(L - x) + r_B x}{L}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$= \frac{2T}{\pi[\frac{r_A(L - x) + r_B x}{L}]^3} = \frac{2TL^3}{\pi[r_A(L - x) + r_B x]^3} \qquad \textbf{Ans}$$

5–31 When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and compute the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.





$$T_A + \frac{1}{2}t_A L - T_B = 0$$

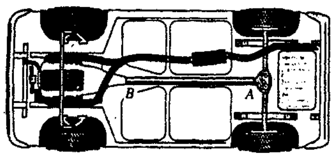
$$T_B = \frac{2T_A + t_A L}{2} \qquad \textbf{Ans}$$

Maximum shear stress : The maximum torque is within the region above the distributed torque.

$$\tau_{max} = \frac{T\,c}{J}$$

$$\tau_{max} = \frac{\left[\frac{(2T_A + t_A L)}{2}\right](r_o)}{\frac{\pi}{2}(r_o^4 - r_i^4)} = \frac{(2T_A + t_A L)r_o}{\pi(r_o^4 - r_i^4)} \qquad \textbf{Ans}$$

*5–32 The drive shaft AB of an automobile is made of a steel having an allowable shear stress of $\tau_{allow} = 8$ ksi. If the outer diameter of the shaft is 2.5 in. and the engine delivers 200 hp to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.



$$\omega = \frac{1140(2\pi)}{60} = 119.38 \text{ rad/s}$$

$$P = T\omega$$

$$200(550) = T(119.38)$$

$$T = 921.42 \text{ lb}\cdot\text{ft}$$

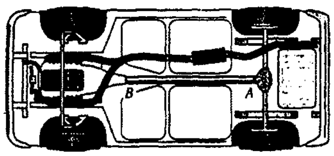
$$\tau_{allow} = \frac{Tc}{J}$$

$$8(10^3) = \frac{921.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.0762 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.0762$$

$t = 0.174$ in. **Ans**

5-33 The drive shaft AB of an automobile is to be designed as a thin-walled tube. The engine delivers 150 hp when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 2.5 in. The material has an allowable shear stress of τ_{allow} = 7 ksi.



$$\omega = \frac{1500(2\pi)}{60} = 157.08 \text{ rad/s}$$

$$P = T\omega$$

$$150(550) = T(157.08)$$

$$T = 525.21 \text{ lb}\cdot\text{ft}$$

$$\tau_{allow} = \frac{Tc}{J}$$

$$7(10^3) = \frac{525.21(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.1460 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.1460$$

$t = 0.104$ in. **Ans**

5–34 The drive shaft of a tractor is to be designed as a thin-walled tube. The engine delivers 200 hp when the shaft is turning at 1200 rev/min. Determine the minimum thickness of the wall of the shaft if the shaft's outer diameter is 3 in. The material has an allowable shear stress of $\tau_{allow} = 7$ ksi.

$$\omega = 1200\,\frac{\text{rev}}{\text{min}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right]\frac{1\ \text{min}}{60\ \text{s}} = 40\,\pi\ \text{rad/s}$$

$$P = 200\ \text{hp}\left[\frac{550\ \text{ft}\cdot\text{lb/s}}{1\ \text{hp}}\right] = 110{,}000\ \text{ft}\cdot\text{lb/s}$$

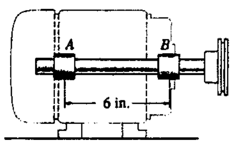
$$T = \frac{P}{\omega} = \frac{110{,}000}{40\pi} = 875.35\ \text{lb}\cdot\text{ft}$$

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$7(10^3) = \frac{875.35(12)(1.5)}{\frac{\pi}{2}(1.5^4 - r_i^4)}\ ; \qquad r_i = 1.380\ \text{in.}$$

$$t = r_o - r_i = 1.5 - 1.380 = 0.120\ \text{in.}$$ **Ans**

5–35 A motor delivers 500 hp to the steel shaft AB, which is tubular and has an outer diameter of 2 in. If it is rotating at 200 rad/s, determine its largest inner diameter to the nearest $\frac{1}{8}$ in. if the allowable shear stress for the material is $\tau_{allow} = 25$ ksi.



$$P = 500 \text{ hp} \left[\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right] = 275000 \text{ ft} \cdot \text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{275000}{200} = 1375 \text{ lb} \cdot \text{ft}$$

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$25(10^3) = \frac{1375(12)(1)}{\frac{\pi}{2}[1^4 - (\frac{d_i}{2})^4]}$$

Use $d = 1\frac{5}{8}$ in. **Ans**

•5–36 The drive shaft of a tractor is made of a steel tube having an allowable shear stress of $\tau_{allow} = 6$ ksi. If the outer diameter is 3 in. and the engine delivers 175 hp to the shaft when it is turning at 1250 rev/min, determine the minimum required thickness of the shaft's wall.

$$\omega = 1250 \frac{\text{rev}}{\text{min}} \left[\frac{2\pi \text{ rad}}{\text{rev}} \right] \frac{1 \text{ min}}{60 \text{ s}} = 130.90 \text{ rad/s}$$

$$P = 175 \text{ hp} \left[\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}} \right] = 96{,}250 \text{ ft} \cdot \text{lb/s}$$

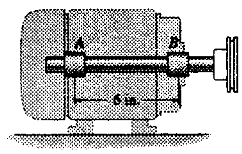
$$T = \frac{P}{\omega} = \frac{96{,}250}{130.90} = 735.30 \text{ lb} \cdot \text{ft}$$

$$\tau_{max} = \tau_{allow} = \frac{T c}{J}$$

$$6(10^3) = \frac{735.30(12)(1.5)}{\frac{\pi}{2}(1.5^4 - r_i^4)} ; \qquad r_i = 1.383 \text{ in.}$$

$$t = r_o - r_i = 1.5 - 1.383 = 0.117 \text{ in.} \qquad \textbf{Ans}$$

5–37 A motor delivers 500 hp to the steel shaft AB, which is tubular and has an outer diameter of 2 in. and an inner diameter of 1.84 in. Determine the *smallest* angular velocity at which it can rotate if the allowable shear stress for the material is $\tau_{allow} = 25$ ksi.



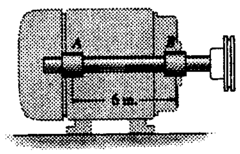
$$P = 500\text{ hp}\left[\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right] = 275000\text{ ft}\cdot\text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{275000}{\omega}$$

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$25(10^3) = \frac{\left(\frac{275000}{\omega}\right)(12)(1)}{\frac{\pi}{2}(1^4 - 0.92^4)}$$

5–38 The 0.75-in.-diameter shaft for the electric motor develops 0.5 hp and runs at 1740 rev/min. Determine the torque produced and compute the maximum shear stress in the shaft. The shaft is supported by ball bearings at A and B.



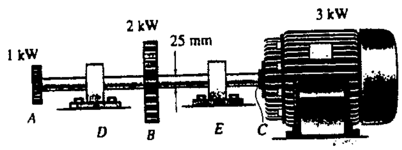
$$\omega = 1740 \frac{\text{rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \frac{1 \text{ min}}{60 \text{ s}} = 58\,\pi \text{ rad/s}$$

$$P = 0.5 \text{ hp} \left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}} \right) = 275 \text{ ft} \cdot \text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{275}{58\,\pi} = 1.5092 \text{ lb} \cdot \text{ft} = 1.51 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{1.5092(12)(0.375)}{\frac{\pi}{2}(0.375^4)} = 219 \text{ psi} \qquad \textbf{Ans}$$

5–39 The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



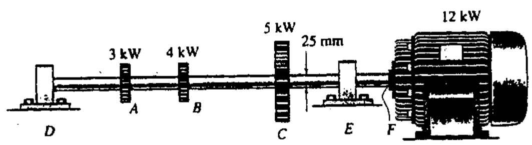
$$T_C = \frac{P}{\omega} = \frac{3(10^3)}{50(2\pi)} = 9.549\ \text{N}\cdot\text{m}$$

$$T_A = \frac{1}{3}T_C = 3.183\ \text{N}\cdot\text{m}$$

$$(\tau_{AB})_{max} = \frac{Tc}{J} = \frac{3.183\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 1.04\ \text{MPa} \qquad \textbf{Ans}$$

$$(\tau_{BC})_{max} = \frac{Tc}{J} = \frac{9.549\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 3.11\ \text{MPa} \qquad \textbf{Ans}$$

•5–40 The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C, remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.

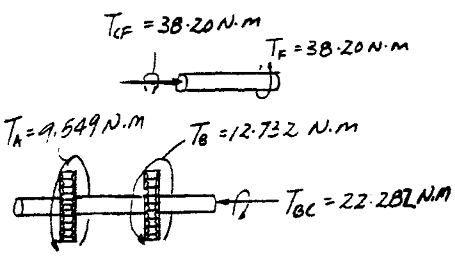


$$\omega = 50\ \frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\,\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\,\pi} = 9.549\ \text{N}\cdot\text{m}$$

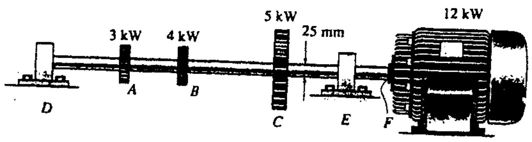
$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\,\pi} = 12.73\ \text{N}\cdot\text{m}$$



$$(\tau_{max})_{CF} = \frac{T_{CF}\,c}{J} = \frac{38.20(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans**

$$(\tau_{max})_{BC} = \frac{T_{BC}\,c}{J} = \frac{22.282(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 7.26\ \text{MPa}$$ **Ans**

5-41. Determine the absolute maximum shear stress developed in the shaft in Prob. 5–40.

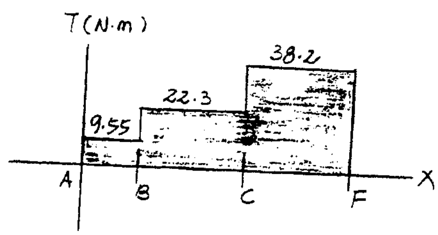


$$\omega = 50\ \frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\pi} = 9.549\ \text{N}\cdot\text{m}$$

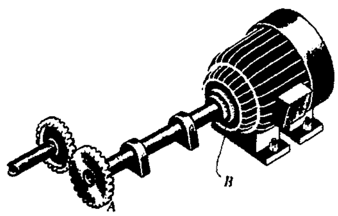
$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\pi} = 12.73\ \text{N}\cdot\text{m}$$



From the torque diagram,
$T_{max} = 38.2\ \text{N}\cdot\text{m}$

$$\tau_{\substack{abs\\max}} = \frac{Tc}{J} = \frac{38.2(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa} \qquad \textbf{Ans}$$

5–42 The motor delivers 500 hp to the steel shaft *AB*, which is tubular and has an outer diameter of 2 in. and an inner diameter of 1.84 in. Determine the *smallest* angular velocity at which it can rotate if the allowable shear stress for the material is $\tau_{allow} = 25$ ksi.



$$\tau_{allow} = \frac{Tc}{J}$$

$$25(10^3) = \frac{T(1)}{\frac{\pi}{2}(1^4 - 0.92^4)}$$

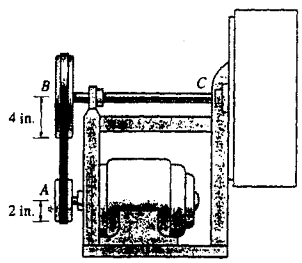
$$T = 11137.22 \text{ lb} \cdot \text{in.}$$

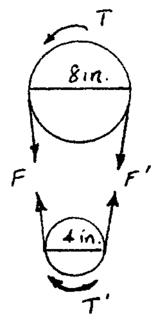
$$P = T\omega$$

$$500(550) = \frac{11137.22}{12}(\omega)$$

$$\omega = 296 \text{ rad/s} \qquad \textbf{Ans}$$

5–43 The motor delivers 50 hp while turning at a constant rate of 1350 rpm at A. Using the belt and pulley system this loading is delivered to the steel blower shaft BC. Determine to the nearest $\frac{1}{8}$ in. the smallest diameter of this shaft if the allowable shear stress for the steel is $\tau_{allow} = 12$ ksi.





$$P = T\omega$$

$$50(550) = T'(1350\ \text{rev/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$T' = 194.52$ lb · ft

$4(F' - F) = T'$

$4(F' - F) = (194.52)(12)$

$(F' - F) = 583.57$ lb

$T = 8(F' - F)$

$= 8(583.57) = 4668.5$ lb · in

$$\tau_{allow} = \frac{Tc}{J}; \qquad 12(10^3) = \frac{4668.5c}{\frac{\pi}{2}(c)^4}$$

$$c = 0.628\ \text{in.}$$

Use $1\frac{3}{8}$ in. – diameter shaft. **Ans**

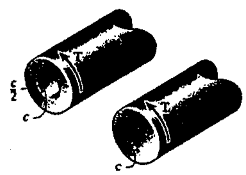
***5-44.** The propellers of a ship are connected to a A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

$$T = \frac{P}{\omega} = \frac{4.5(10^6)}{20} = 225(10^3)\ \text{N} \cdot \text{m}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{225(10^3)(0.170)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4)} = 44.3\ \text{MPa}$$ **Ans**

$$\phi = \frac{TL}{JG} = \frac{225(10^3)(60)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4)75(10^9)} = 0.2085\ \text{rad} = 11.9°$$ Ans.

5-45. A shaft is subjected to a torque **T**. Compare the effectiveness of using the tube shown in the figure with that of a solid section of radius c. To do this, compute the percent increase in torsional stress and angle of twist per unit length for the tube versus the solid section.



Shear stress :
For the tube,

$$(\tau_t)_{max} = \frac{Tc}{J_t}$$

For the solid shaft,

$$(\tau_s)_{max} = \frac{Tc}{J_s}$$

$$\% \text{ increase in shear stress} = \frac{(\tau_s)_{max} - (\tau_t)_{max}}{(\tau_t)_{max}}(100) = \frac{\frac{Tc}{J_t} - \frac{Tc}{J_s}}{\frac{Tc}{J_s}}(100)$$

$$= \frac{J_s - J_t}{J_t}(100) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]}(100)$$

$= 6.67\ \%$ **Ans**

Angle of twist :
For the tube,

$$\phi_t = \frac{TL}{J_t(G)}$$

For the shaft,

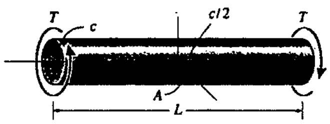
$$\phi_s = \frac{TL}{J_s(G)}$$

$$\% \text{ increase in } \phi = \frac{\phi_t - \phi_s}{\phi_s}(100\%) = \frac{\frac{TL}{J_t(G)} - \frac{TL}{J_s(G)}}{\frac{TL}{J_s(G)}}(100\%)$$

$$= \frac{J_s - J_t}{J_t}(100\%) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{c}{2})^4]}(100\%)$$

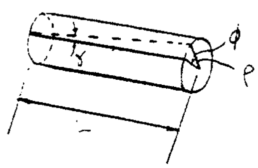
$= 6.67\ \%$ **Ans**

5–46 The solid shaft of radius c is subjected to a torque T at its ends. Show that the maximum shear strain developed in the shaft is $\gamma_{max} = Tc/JG$. What is the shear strain on an element located at point A, $c/2$ from the center of the shaft? Sketch the strain distortion of this element.



From the geometry :

$$\gamma L = \rho\,\phi\,; \qquad \gamma = \frac{\rho\,\phi}{L}$$

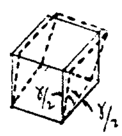


Since $\phi = \dfrac{TL}{JG}$, then

$$\gamma = \frac{T\rho}{JG} \qquad (1)$$

However the maximum shear strain occurs when $\rho = c$

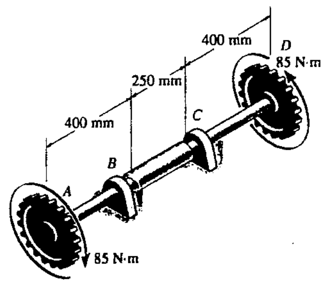
$$\gamma_{max} = \frac{T\,c}{JG} \qquad \textbf{QED}$$

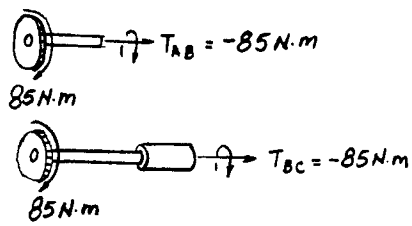


Shear strain when $\rho = \dfrac{c}{2}$ is from Eq. (1),

$$\gamma = \frac{T\,(c/2)}{JG} = \frac{T\,c}{2\,JG} \qquad \textbf{Ans}$$

5–47 The A-36 steel axle is made from tubes *AB* and *CD* and a solid section *BC*. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N · m torques, determine the angle of twist of gear *A* relative to gear *D*. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm.



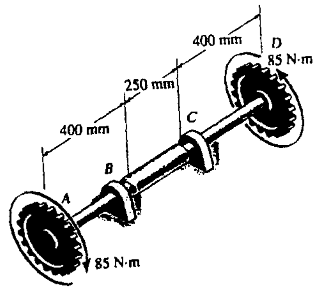


$$\phi_{A/D} = \Sigma\frac{TL}{JG}$$

$$= \frac{2(85)(0.4)}{\frac{\pi}{2}(0.015^4 - 0.01^4)(75)(10^9)} + \frac{(85)(0.25)}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

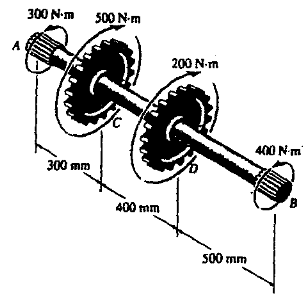
$$= 0.01534 \text{ rad} = 0.879^\circ \qquad \textbf{Ans}$$

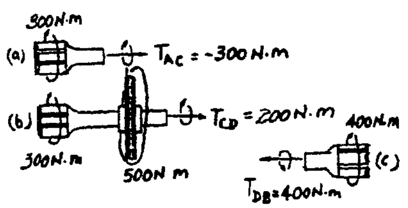
•5–48 The A-36 steel axle is made from tubes AB and CD and a solid section BC. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N · m torques, determine the angle of twist of the end B of the solid section relative to end C. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm.



$$\phi_{B/C} = \frac{TL}{JG} = \frac{85(0.250)}{\frac{\pi}{2}(0.020)^4(75)(10^9)} = 0.00113 \text{ rad} = 0.0646° \qquad \textbf{Ans}$$

5-49. The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of 40 mm.



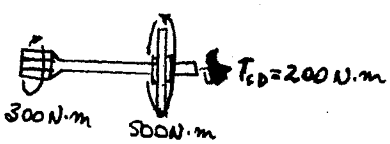


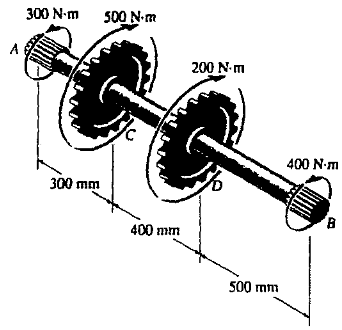
$$\phi_{B/A} = \Sigma\frac{TL}{JG} = \frac{-300(0.3)}{JG} + \frac{200(0.4)}{JG} + \frac{400(0.5)}{JG}$$

$$= \frac{190}{JG} = \frac{190}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

$$= 0.01008 \text{ rad} = 0.578^\circ \quad \textbf{Ans}$$

5–50 The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of gear C with respect to gear D. The shaft has a diameter of 40 mm.

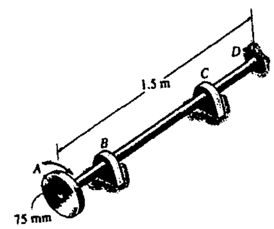




$$\phi_{C/D} = \frac{200(0.4)}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

$$= 0.004244 \text{ rad} = 0.243^\circ \qquad \textbf{Ans}$$

5-51. The rotating flywheel-and-shaft, when brought to a sudden stop at D, begins to oscillate clockwise-counterclockwise such that a point A on the outer edge of the flywheel is displaced through a 6-mm arc. Determine the maximum shear stress developed in the tubular A-36 steel shaft due to this oscillation. The shaft has an inner diameter of 24 mm an outer diameter of 32 mm. The bearings at B and C allow the shaft to rotate freely, whereas the support at D holds the shaft fixed.



$s = r\theta$

$6 = 75\,\phi \qquad \phi = 0.08 \text{ rad}$

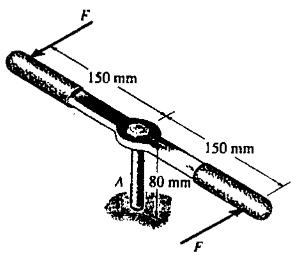
$$\phi = \frac{TL}{JG}$$

$$0.08 = \frac{T(1.5)}{J(75)(10^9)}$$

$$T = 4(10^9)\,J$$

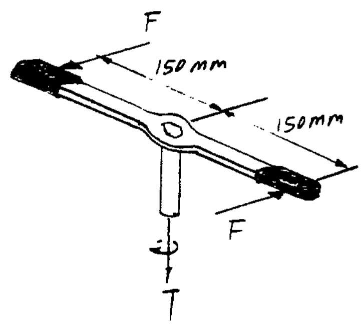
$$\tau_{max} = \frac{Tc}{J} = \frac{4(10^9)(J)(0.016)}{J} = 64.0 \text{ MPa} \qquad \textbf{Ans}$$

*5–52 The 8-mm-diameter A-36 bolt is screwed tightly into a block at A. Determine the couple forces F that should be applied to the wrench so that the maximum shear stress in the bolt becomes 18 MPa. Also, compute the corresponding displacement of each force F needed to cause this stress. Assume that the wrench is rigid.



$$T - F(0.3) = 0 \qquad (1)$$

$$\tau_{max} = \frac{Tc}{J}; \qquad 18(10^6) = \frac{T(0.004)}{\frac{\pi}{2}(0.004^4)}$$



$T = 1.8096 \text{ N}\cdot\text{m}$

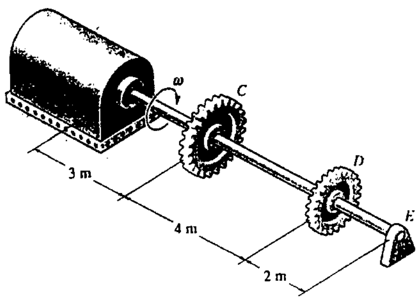
From Eq. (1),

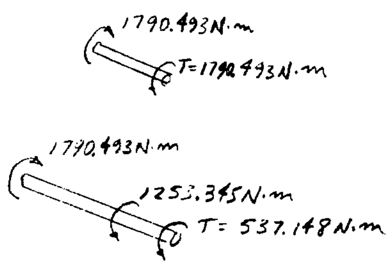
$F = 6.03 \text{ N}$ **Ans**

$$\phi = \frac{TL}{JG} = \frac{1.8096(0.08)}{\frac{\pi}{2}[(0.0040)^4]75(10^9)} = 0.00480 \text{ rad}$$

$s = r\phi = 0.15(0.00480) = 0.00072 \text{ m} = 0.720 \text{ mm}$ Ans.

5–53 The turbine develops 150 kW of power, which is transmitted to the gears such that C receives 70% and D receives 30%. If the rotation of the 100-mm-diameter A-36 steel shaft is ω = 800 rev/min., determine the absolute maximum shear stress in the shaft and the angle of twist of end E of the shaft relative to B. The journal bearing at E allows the shaft to turn freely about its axis.





$$P = T\omega; \qquad 150(10^3)W = T(800\frac{\text{rev}}{\text{min}})(\frac{1\text{ min}}{60\text{ sec}})(\frac{2\pi\text{ rad}}{1\text{ rev}})$$

$$T = 1790.493\text{ N}\cdot\text{m}$$

$$T_C = 1790.493(0.7) = 1253.345\text{ N}\cdot\text{m}$$
$$T_D = 1790.493(0.3) = 537.148\text{ N}\cdot\text{m}$$

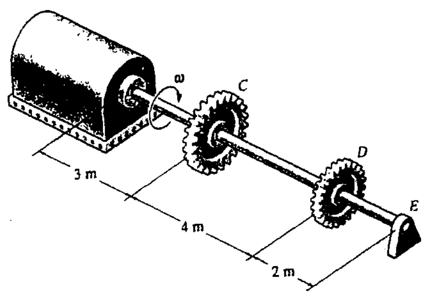
Maximum torque is in region BC.

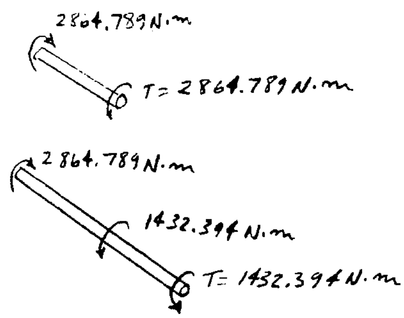
$$\tau_{max} = \frac{Tc}{J} = \frac{1790.493(0.05)}{\frac{\pi}{2}(0.05)^4} = 9.12\text{ MPa} \qquad \textbf{Ans}$$

$$\phi_{E/B} = \Sigma(\frac{TL}{JG}) = \frac{1}{JG}[1790.493(3) + 537.148(4) + 0]$$

$$= \frac{7520.171}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0102\text{ rad} = 0.585^\circ \qquad \textbf{Ans}$$

5-54 The turbine develops 150 kW of power, which is transmitted to the gears such that both C and D receive an equal amount. If the rotation of the 100-mm-diameter A-36 steel shaft is $\omega = 500$ rev/min., determine the absolute maximum shear stress in the shaft and the rotation of end B of the shaft relative to E. The journal bearing at C allows the shaft to turn freely about its axis.





$$P = T\omega; \qquad 150(10^3)W = T(500\,\frac{\text{rev}}{\text{min}})(\frac{1\text{ min}}{60\text{ sec}})(\frac{2\pi\text{ rad}}{1\text{ rev}})$$

$$T = 2864.789\ \text{N}\cdot\text{m}$$

$$T_C = T_D = \frac{T}{2} = 1432.394\ \text{N}\cdot\text{m}$$

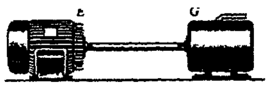
Maximum torque is in region BC.

$$\tau_{max} = \frac{Tc}{J} = \frac{2864.789(0.05)}{\frac{\pi}{2}(0.05)^4} = 14.6\ \text{MPa} \qquad \textbf{Ans}$$

$$\phi_{E/B} = \Sigma(\frac{TL}{JG}) = \frac{1}{JG}[2864.789(3) + 1432.394(4) + 0]$$

$$= \frac{14323.945}{\frac{\pi}{2}(0.05)^4(75)(10^9)} = 0.0195\ \text{rad} = 1.11^\circ \qquad \textbf{Ans}$$

5-55. The A-36 hollow steel shaft is 2 m long and has an outer diameter of 40 mm. When it is rotating at 80 rad/s, it transmits 32 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{allow} = 140$ MPa and the shaft is restricted not to twist more than 0.05 rad.



$P = T\omega$

$32(10^3) = T(80)$

$T = 400\ \text{N}\cdot\text{m}$

Shear stress failure

$$\tau = \frac{Tc}{J}$$

$$\tau_{allow} = 140(10^6) = \frac{400(0.02)}{\frac{\pi}{2}(0.02^4 - r_i^{\ 4})}$$

$r_i = 0.01875$ m

Angle of twist limitation :

$$\phi = \frac{TL}{JG}$$

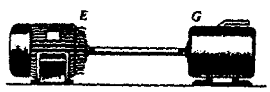
$$0.05 = \frac{400(2)}{\frac{\pi}{2}(0.02^4 - r_i^{\ 4})(75)(10^9)}$$

$r_i = 0.01247$ m (controls)

$t = r_o - r_i = 0.02 - 0.01247$

$= 0.00753$ m

***5-56.** The A-36 solid steel shaft is 3 m long and has a diameter of 50 mm. It is required to transmit 35 kW of power from the engine E to the generator G. Determine the smallest angular velocity the shaft can have if it is restricted not to twist more than 1°.



$$\phi = \frac{TL}{JG}$$

$$\frac{1°(\pi)}{180°} = \frac{T(3)}{\frac{\pi}{2}(0.025^4)(75)(10^9)}$$

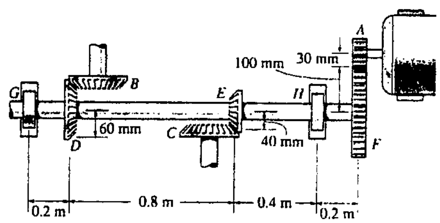
$$T = 267.73\ \text{N}\cdot\text{m}$$

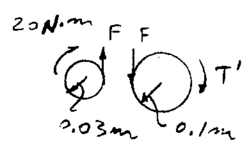
$$P = T\omega$$

$$35(10^3) = 267.73(\omega)$$

$$\omega = 131\ \text{rad/s} \qquad \textbf{Ans}$$

5–57 The motor produces a torque of $T = 20\ \text{N}\cdot\text{m}$ on gear A. If gear C is suddenly locked so it does not turn, yet B can freely turn, determine the angle of twist of F with respect to E and F with respect to D of the L2-steel shaft, which has an inner diameter of 30 mm and an outer diameter of 50 mm. Also, calculate the absolute maximum shear stress in the shaft. The shaft is supported on journal bearings at G and H.





$F(0.03) = 20$

$F = 666.67\ \text{N}$

$T' = (666.67)(0.1) = 66.67\ \text{N}\cdot\text{m}$

Since shaft is held fixed at C, the torque is only in region EF of the shaft.

$$\phi_{F/E} = \frac{TL}{JG} = \frac{66.67(0.6)}{\frac{\pi}{2}[(0.025)^4 - (0.015)^4]75(10^9)} = 0.999(10)^{-3}\ \text{rad} \quad \textbf{Ans.}$$

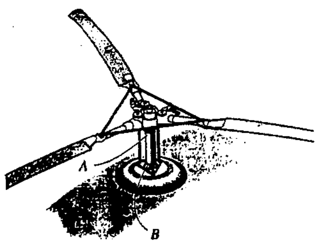
Since the torque in region ED is zero,

$$\phi_{F/D} = 0.999(10)^{-3}\ \text{rad} \quad \textbf{Ans}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{66.67(0.025)}{\frac{\pi}{2}((0.025)^4 - (0.015)^4)}$$

$= 3.12\ \text{MPa}$ **Ans**

5-58 The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is τ_{allow} = 8 ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{allow} = \frac{Tc}{J}$$

$$8(10^3) = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.3586 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4(11.0)(10^6)}$$

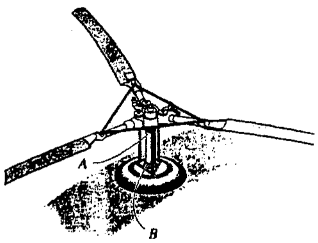
$$c = 0.967 \text{ in.}$$

Shear - stress failure controls the design.

$$d = 2c = 2\,(1.3586) = 2.72 \text{ in.}$$

Use d = 2.75 in. **Ans**

5–59 The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 10.5$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{allow} = 10.5(10)^3 = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.2408 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4(11.0)(10^6)}$$

$$c = 0.967 \text{ in.}$$

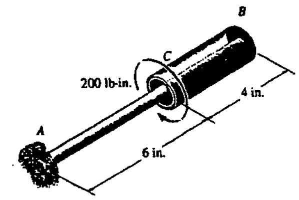
Shear stress failure controls the design

$$d = 2c = 2\,(1.2408) = 2.48 \text{ in.}$$

Use $d = 2.50$ in. **Ans**

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

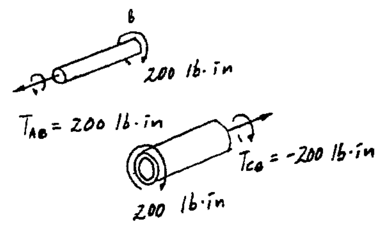
5-61. The A-36 steel assembly consists of a tube having an outer radius of 1 in. and a wall thickness of 0.125 in. Using a rigid plate at B, it is connected to the solid 1-in-diameter shaft AB. Determine the rotation of the tube's end C if a torque of 200 lb · in. is applied to the tube at this end. The end A of the shaft is fixed-supported.



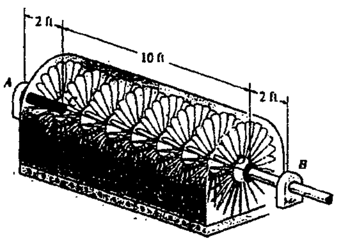
$$\phi_B = \frac{T_{AB}L}{JG} = \frac{200(10)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} = 0.001852 \text{ rad}$$

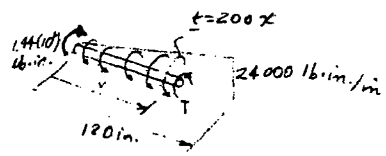
$$\phi_{C/B} = \frac{T_{CB}L}{JG} = \frac{-200(4)}{\frac{\pi}{2}(1^4 - 0.875^4)(11.0)(10^6)} = -0.0001119 \text{ rad}$$

$$\phi_C = \phi_B + \phi_{C/B}$$
$$= 0.001852 + 0.0001119$$
$$= 0.001964 \text{ rad} = 0.113^\circ \qquad \textbf{Ans}$$



5-62. The 6-in.-diameter L-2 steel shaft on the turbine is supported on journal bearings at A and B. If C is held fixed and the turbine blades create a torque on the shaft that increases linearly from zero at C to 2000 lb · ft at D, determine the angle of twist of the shaft of end D relative to end C. Also, compute the absolute maximum shear stress in the shaft. Neglect the size of the blades.





$$T_{max} = \frac{1}{2}(120)(200(120)) = 1.44(10^6)$$
$$T = 1.44(10^6) - \frac{1}{2}(x)(200x) = 1.44(10^6) - 100x^2$$

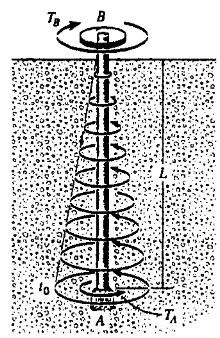
$$\phi_{D/C} = \int \frac{T\,dx}{JG} = \frac{1}{JG}\int_0^{120} 1.44(10^6)dx - 100x^2 dx$$

$$= \frac{1.44(10^6)(120)}{\pi\frac{(3)^4}{2}(11.0(10^6))} - \frac{100(120)^3}{3(\pi\frac{(3)^4}{2})(11.0(10^6))} = 0.0823 \text{ rad} \quad \textbf{Ans}$$

Maximum torque occurs at $x = 0$

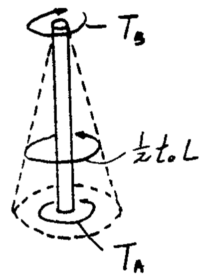
$$\tau_{abs\,max} = \frac{Tc}{J} = \frac{1.44(10^6)(3)}{\pi\frac{(3)^4}{2}} = 34.0 \text{ ksi} \quad \textbf{Ans}$$

5–63 When drilling a well, the deep end of the drill pipe is assumed to encounter a torsional resistance T_A. Furthermore, soil friction along the sides of the pipe creates a linear distribution of torque per unit length, varying from zero at the surface B to t_0 at A. Determine the necessary torque T_B that must be supplied by the drive unit to turn the pipe. Also, what is the relative angle of twist of one end of the pipe with respect to the other end at the instant the pipe is about to turn? The pipe has an outer radius r_o and an inner radius r_i. The shear modulus is G.



$$\frac{1}{2}t_0 L + T_A - T_B = 0$$

$$T_B = \frac{t_0 L + 2T_A}{2} \qquad \textbf{Ans}$$



$$T(x) + \frac{t_0}{2L}x^2 - \frac{t_0 L + 2\,T_A}{2} = 0$$

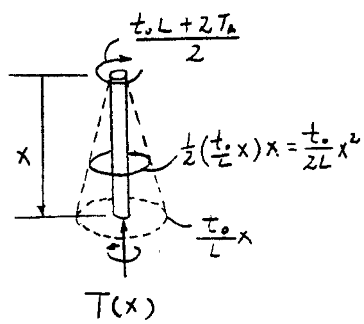
$$T(x) = \frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L}x^2$$

$$\phi = \int \frac{T(x)\,dx}{JG}$$

$$= \frac{1}{JG}\int_0^L \left(\frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L}x^2\right) dx$$

$$= \frac{1}{JG}\left[\frac{t_0 L + 2T_A}{2}x - \frac{t_0}{6L}x^3\right]\Big|_0^L$$

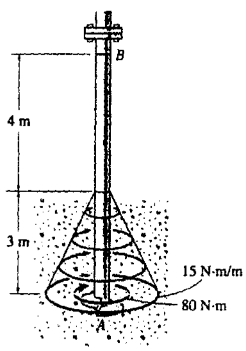
$$= \frac{t_0 L^2 + 3T_A L}{3JG}$$

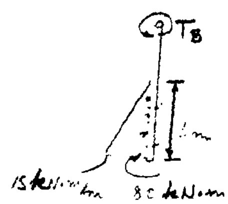


However, $J = \frac{\pi}{2}(r_o^{\,4} - r_i^{\,4})$

$$\phi = \frac{2L(t_0 L + 3T_A)}{3\pi\,(r_o^{\,4} - r_i^{\,4})G} \qquad \textbf{Ans}$$

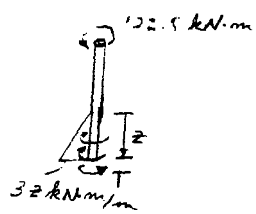
•5–64 The A-36 steel posts are "drilled" at constant angular speed into the soil using the rotary installer. If the post has an inner diameter of 200 mm and an outer diameter of 225 mm, determine the relative angle of twist of end A of the post with respect to end B when the post reaches the depth indicated. Due to soil friction, assume the torque along the post varies linearly as shown, and a concentrated torque of 80 kN · m acts at the bit.





$$\Sigma M_z = 0; \qquad T_B - 80 - \frac{1}{2}(15)(3) = 0$$

$$T_B = 102.5 \text{ kN} \cdot \text{m}$$



$$\Sigma M_z = 0; \qquad 102.5 - \frac{1}{2}(5\,z)(z) - T = 0$$

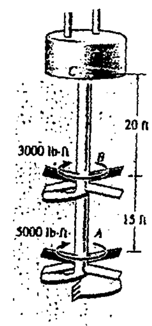
$$T = (102.5 - 2.5z^2) \text{ kN} \cdot \text{m}$$

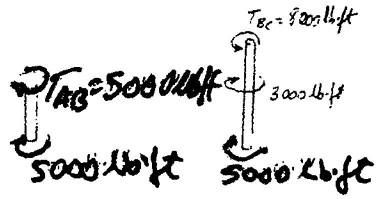
$$\phi_{A/B} = \frac{TL}{JG} + \int \frac{T\,dz}{JG}$$

$$= \frac{102.5(10^3)(4)}{\frac{\pi}{2}((0.1125)^4 - (0.1)^4)(75)(10^9)} + \int_0^3 \frac{(102.5 - 2.5z^2)(10^3)dz}{\frac{\pi}{2}((0.1125)^4 - (0.1)^4)(75)(10^9)}$$

$= 0.0980$ rad **Ans**

5-65. The device shown is used to mix soils in order to provide in-situ stabilization. If the mixer is connected to an A-36 steel tubular shaft that has an inner diameter of 3 in. and an outer diameter of 4.5 in, determine the angle of twist of the shaft of A relative to B and the absolute maximum shear stress in the shaft if each mixing blade is subjected to the torques shown.

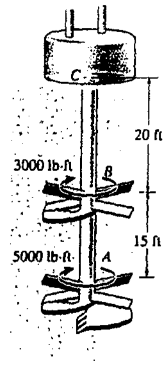


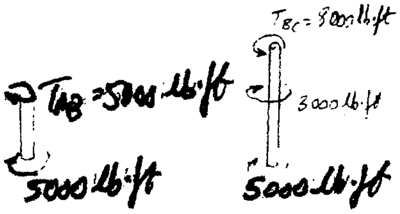


$$\phi_{A/B} = \frac{TL}{JG} = \frac{5000(12)(15)(12)}{\frac{\pi}{2}[(2.25)^4 - (1.5)^4]11(10^6)} = 0.03039 \text{ rad} = 1.74° \quad \textbf{Ans.}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{8000(12)(2.25)}{\frac{\pi}{2}[(2.25)^4 - (1.50)^4)]} = 6.69 \text{ ksi} \qquad \textbf{Ans.}$$

5–66 The device shown is used to mix soils in order to provide in-situ stabilization. If the mixer is connected to an A-36 steel tubular shaft that has an inner diameter of 3 in. and an outer diameter of 4.5 in., determine the angle of twist of the shaft of A relative to C if each mixing blade is subjected to the torques shown.

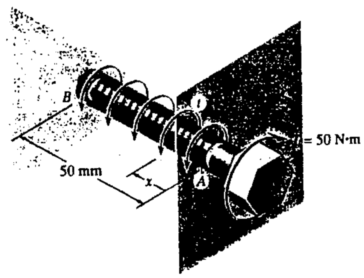


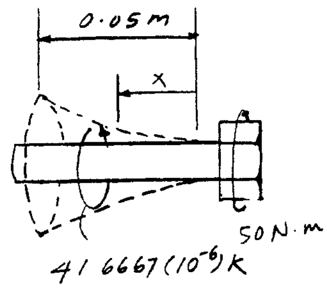


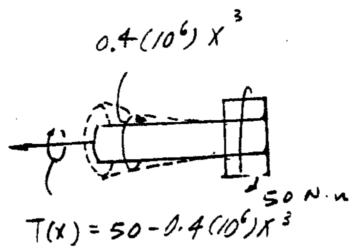
$$\phi_{A/C} = \Sigma\left(\frac{TL}{JG}\right) = \frac{5000(12)(15)(12)}{\frac{\pi}{2}((2.25)^4 - (1.5)^4)(11)(10^6)} + \frac{8000(12)(20)(12)}{\frac{\pi}{2}((2.25)^4 - (1.5)^4)(11)(10^6)}$$

$= 0.0952 \text{ rad} = 5.45°$ **Ans**

***5–68** The A-36 bolt is tightened within a hole so that the reactive torque on the shank AB can be expressed by the equation $t = (kx^2)$ N · m/m, where x is in meters. If a torque of $T = 50$ N · m is applied to the bolt head, determine the constant k and the amount of twist in the 50-mm length of the shank. Assume the shank has a constant radius of 4 mm.







$$dT = t\,dx$$

$$T = \int_0^{0.05\text{ m}} kx^2\,dx = k\frac{x^3}{3}\Big|_0^{0.05} = 41.667(10^{-6})\,k$$

$$50 - 41.6667(10^{-6})\,k = 0$$

$$k = 1.20(10^6)\text{ N/m}^2 \qquad \textbf{Ans}$$

In the general position, $T = \int_0^x 1.20(10^6)x^2\,dx = 0.4(10^6)x^3$

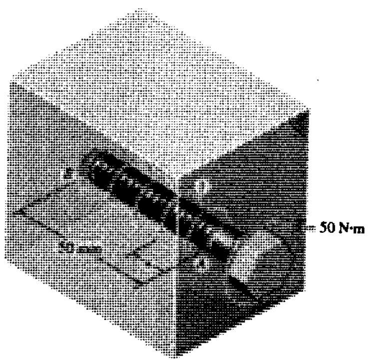
$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{0.05\text{ m}} [50 - 0.4(10^6)x^3]\,dx$$

$$= \frac{1}{JG}\left[50x - \frac{0.4(10^6)x^4}{4}\right]\Big|_0^{0.05\text{ m}}$$

$$= \frac{1.875}{JG} = \frac{1.875}{\frac{\pi}{2}(0.004^4)(75)(10^9)}$$

$$= 0.06217\text{ rad} = 3.56^\circ \qquad \textbf{Ans}$$

5–69 Solve Prob. 5–68 if the distributed torque is $t = (kx^{2/3})$ N · m/m.



$$dT = t\,dx$$

$$T = \int_0^{0.05} kx^{\frac{2}{3}}dx = k\frac{3}{5}x^{\frac{5}{3}}\Big|_0^{0.05} = (4.0716)(10^{-3})\,k$$

$$50 - 4.0716(10^{-3})\,k = 0$$

$$k = 12.28(10^3) \qquad \textbf{Ans}$$

In the general position,

$$T = \int_0^x 12.28(10^3)x^{\frac{2}{3}}dx = 7.368(10^3)\,x^{\frac{5}{3}}$$

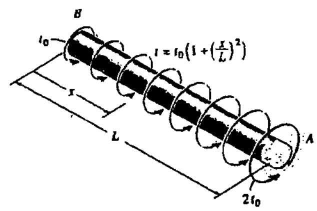
Angle of twist :

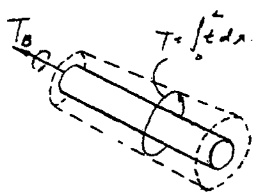
$$\phi = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{0.05\text{ m}}[50 - 7.3681(10^3)x^{\frac{5}{3}}]dx$$

$$= \frac{1}{JG}[50x - 7.3681(10^3)(\frac{3}{8})x^{\frac{8}{3}}]\Big|_0^{0.05\text{ m}}$$

$$= \frac{1.5625}{\frac{\pi}{2}(0.004^4)(75)(10^9)} = 0.0518\text{ rad} = 2.97° \qquad \textbf{Ans}$$

5-70. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the angle of twist at end A. The shear modulus is G.

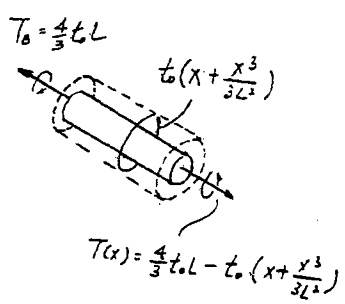




$$T_B - \int t\,dx = 0$$

$$T_B = \int t\,dx = t_0 \int (1 + \frac{x^2}{L^2})\,dx$$

$$= t_0[x + \frac{x^3}{3L^2}]\Big|_0^L = t_0(L + \frac{L}{3}) = \frac{4}{3}t_0 L$$



$$\phi = \int \frac{T(x)\,dx}{JG}$$

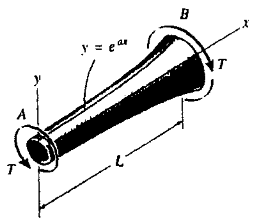
$$= \frac{1}{JG}\int_0^L [\frac{4}{3}t_0 L - t_0(x + \frac{x^3}{3L^2})]\,dx$$

$$= \frac{t_0}{JG}[\frac{4}{3}Lx - (\frac{x^2}{2} + \frac{x^4}{12L^2})]\Big|_0^L = \frac{7\,t_0 L^2}{12\,JG}$$

However $J = \frac{\pi}{2}c^4$,

$$\phi = \frac{7\,t_0 L^2}{6\,\pi c^4 G} \qquad \textbf{Ans}$$

5–71 The contour of the surface of the shaft is defined by the equation $y = e^{ax}$, where a is a constant. If the shaft is subjected to a torque T at its ends, determine the angle of twist of end A with respect to end B. The shear modulus is G.



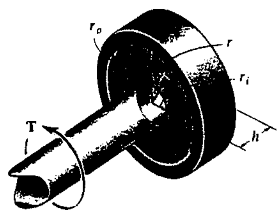
$$\phi = \int \frac{T\,dx}{J(x)G} \qquad \text{where, } J(x) = \frac{\pi}{2}(e^{ax})^4$$

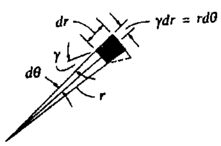
$$= \frac{2T}{\pi G}\int_0^L \frac{dx}{e^{4ax}} = \frac{2T}{\pi G}\left[-\frac{1}{4\,a\,e^{4ax}}\right]\Big|_0^L$$

$$= \frac{2T}{\pi G}\left[-\frac{1}{4\,a\,e^{4aL}} + \frac{1}{4a}\right] = \frac{T}{2\,a\,\pi\,G}\left[\frac{e^{4aL} - 1}{e^{4aL}}\right]$$

$$= \frac{T}{2\,a\,\pi\,G}\left[1 - e^{-4aL}\right] \qquad \textbf{Ans}$$

*5-72 A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque T is applied to the rigid shaft, determine the angle of twist of the shaft. The shear modulus of the rubber is G. *Hint:* As shown in the figure, the deformation of the element at radius r can be determined from $r\,d\theta = dr\,\gamma$. Use this expression along with $\tau = T/(2\pi r^2 h)$, from Prob. 5-28, to obtain the result.





$$r\,d\theta = \gamma\,dr$$

$$d\theta = \frac{\gamma dr}{r} \qquad (1)$$

From Prob. 5-28,

$$\tau = \frac{T}{2\pi r^2 h} \qquad \text{and} \qquad \gamma = \frac{\tau}{G}$$

$$\gamma = \frac{T}{2\pi r^2 hG}$$

From (1),

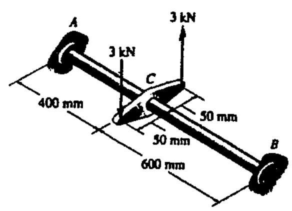
$$d\theta = \frac{T}{2\pi hG}\frac{dr}{r^3}$$

$$\theta = \frac{T}{2\pi hG}\int_{r_i}^{r_o}\frac{dr}{r^3} = \frac{T}{2\pi hG}\left[-\frac{1}{2r^2}\right]\Big|_{r_i}^{r_o}$$

$$= \frac{T}{2\pi hG}\left[-\frac{1}{2r_o^2} + \frac{1}{2r_i^2}\right]$$

$$= \frac{T}{4\pi hG}\left[\frac{1}{r_i^2} - \frac{1}{r_o^2}\right] \qquad \textbf{Ans}$$

5-73. The steel shaft has a diameter of 40 mm and is fixed at its ends A and B. If it is subjected to the couple, determine the maximum shear stress in regions AC and CB of the shaft. $G_{st} = 10.8(10^3)$ ksi.

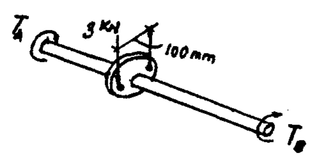


Equilibrium :

$$T_A + T_B - 3000(0.1) = 0 \qquad (1)$$

Compatibility condition :

$$\phi_{C/A} = \phi_{C/B}$$



$$\frac{T_A(400)}{JG} = \frac{T_B(600)}{JG}$$

$$T_A = 1.5\,T_B \qquad (2)$$

Solving Eqs (1) and (2) yields :

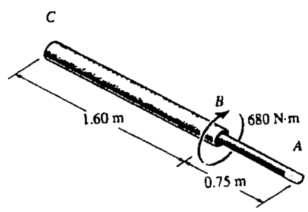
$$T_B = 120\ \text{N}\cdot\text{m}$$

$$T_A = 180\ \text{N}\cdot\text{m}$$

$$(\tau_{AC})_{max} = \frac{Tc}{J} = \frac{180(0.02)}{\frac{\pi}{2}(0.02^4)} = 14.3\ \text{MPa} \qquad \textbf{Ans}$$

$$(\tau_{CB})_{max} = \frac{Tc}{J} = \frac{120(0.02)}{\frac{\pi}{2}(0.02^4)} = 9.55\ \text{MPa} \qquad \textbf{Ans}$$

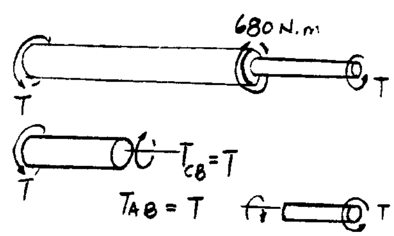
5–74 A rod is made from two segments: AB is steel and BC is brass. It is fixed at its ends and subjected to a torque of $T = 680$ N · m. If the steel portion has a diameter of 30 mm, determine the required diameter of the brass portion so the reactions at the walls will be the same. $G_{st} = 75$ GPa, $G_{br} = 39$ GPa.



Compatibility condition :

$$\phi_{B/C} = \phi_{B/A}$$

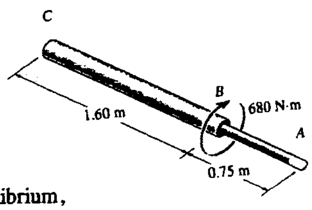
$$\frac{T(1.60)}{\frac{\pi}{2}(c^4)(39)(10^9)} = \frac{T(0.75)}{\frac{\pi}{2}(0.015^4)(75)(10^9)}$$

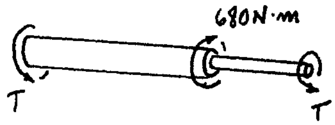


$c = 0.02134$ m

$d = 2c = 0.04269$ m $= 42.7$ mm **Ans**

5–75 Determine the absolute maximum shear stress in the shaft of Prob. 5–74.





Equilibrium,

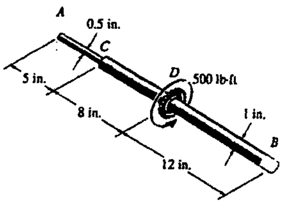
$$2T = 680$$
$$T = 340 \text{ N} \cdot \text{m}$$

$\tau_{\text{abs max}}$ occurs in the steel. See solution to Prob. 5 - 74.

$$\tau_{\text{abs max}} = \frac{Tc}{J} = \frac{340(0.015)}{\frac{\pi}{2}(0.015)^4}$$

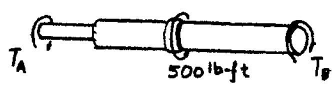
$$= 64.1 \text{ MPa} \quad \textbf{Ans}$$

***5–76.** The steel shaft is made from two segments: *AC* has a diameter of 0.5 in, and *CB* has a diameter of 1 in. If it is fixed at its ends *A* and *B* and subjected to a torque of 500 lb · ft, determine the maximum shear stress in the shaft. $G_{st} = 10.8(10^3)$ ksi.



Equilibrium :

$T_A + T_B - 500 = 0 \qquad (1)$



Compatibility condition :

$\phi_{D/A} = \phi_{D/B}$

$$\frac{T_A(5)}{\frac{\pi}{2}(0.25^4)G} + \frac{T_A(8)}{\frac{\pi}{2}(0.5^4)G} = \frac{T_B(12)}{\frac{\pi}{2}(0.5^4)G}$$

$1408\, T_A = 192\, T_B \qquad (2)$

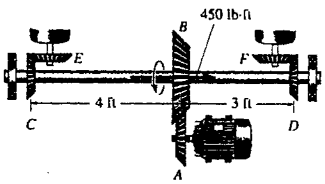
Solving Eqs. (1) and (2) yields

$T_A = 60 \text{ lb}\cdot\text{ft} \qquad T_B = 440 \text{ lb}\cdot\text{ft}$

$$\tau_{AC} = \frac{Tc}{J} = \frac{60(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 29.3 \text{ ksi} \quad \text{(max)} \qquad \textbf{Ans}$$

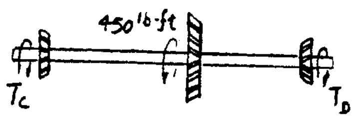
$$\tau_{DB} = \frac{Tc}{J} = \frac{440(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 26.9 \text{ ksi}$$

5–77 The motor A develops a torque at gear B of 450 lb · ft, which is applied along the axis of the 2-in.-diameter steel shaft CD. This torque is to be transmitted to the pinion gears at E and F. If these gears are temporarily fixed, determine the maximum shear stress in segments CB and BD of the shaft. Also, what is the angle of twist of each of these segments? The bearings at C and D only exert force reactions on the shaft and do not resist torque. $G_{st} = 12(10^3)$ ksi.



Equilibrium :

$$T_C + T_D - 450 = 0 \qquad (1)$$



Compatibility condition :

$$\phi_{B/C} = \phi_{B/D}$$

$$\frac{T_C(4)}{JG} = \frac{T_D(3)}{JG}$$

$$T_C = 0.75\, T_D \qquad (2)$$

Solving Eqs. (1) and (2), yields

$$T_D = 257.14 \text{ lb} \cdot \text{ft}$$

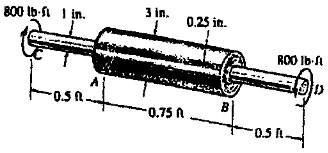
$$T_C = 192.86 \text{ lb} \cdot \text{ft}$$

$$(\tau_{BC})_{max} = \frac{192.86(12)(1)}{\frac{\pi}{2}(1^4)} = 1.47 \text{ ksi} \qquad \textbf{Ans}$$

$$(\tau_{BD})_{max} = \frac{257.14(12)(1)}{\frac{\pi}{2}(1^4)} = 1.96 \text{ ksi} \qquad \textbf{Ans}$$

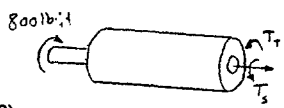
$$\phi = \frac{192.86(12)(4)(12)}{\frac{\pi}{2}(1^4)(12)(10^6)} = 0.00589 \text{ rad} = 0.338^\circ \qquad \textbf{Ans}$$

5-78. The composite shaft consists of a mid-section that includes the 1-in.-diameter solid shaft and a tube that is welded to the rigid flanges at A and B. Neglect the thickness of the flanges and determine the angle of twist of end C of the shaft relative to end D. The shaft is subjected to a torque of 800 lb · ft. The material is A-36 steel.



Equilibrium :

$800(12) - T_T - T_S = 0$



Compatibility condition :

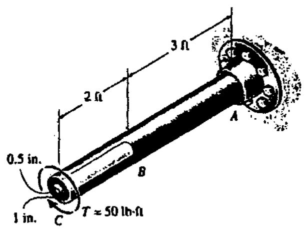
$$\phi_T = \phi_S; \qquad \frac{T_T(0.75)(12)}{\frac{\pi}{2}((1.5)^4 - (1.25)^4)G} = \frac{T_S(0.75)(12)}{\frac{\pi}{2}(0.5)^4 G}$$

$T_T = 9376.42$ lb · in.

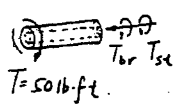
$T_S = 223.58$ lb · in.

$$\phi_{C/D} = \Sigma\frac{TL}{JG} = \frac{800(12)(1)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} + \frac{223.58(0.75)(12)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} = 0.1085 \text{ rad} = 6.22° \qquad \textbf{Ans}$$

5-79. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50\ \text{lb}\cdot\text{ft}$ is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear and maximum shear strain in the brass and steel. Take $G_{st} = 11.5(10^3)$ ksi, $G_{br} = 5.6(10^3)$ ksi.



Equilibrium :



$$T_{br} + T_{st} - 50 = 0 \qquad (1)$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$

$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{br}(2)(12)}{\frac{\pi}{2}(0.5^4)(5.6)(10^6)} = \frac{T_{st}(2)(12)}{\frac{\pi}{2}(1^4 - 0.5^4)(11.5)(10^6)}$$

$$T_{br} = 0.032464\, T_{st} \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$T_{st} = 48.428$ lb · ft; $\quad T_{br} = 1.572$ lb · ft

$$\phi_C = \Sigma\frac{TL}{JG} = \frac{1.572(12)(2)(12)}{\frac{\pi}{2}(0.5^4)(5.6)(10^6)} + \frac{50(12)(3)(12)}{\frac{\pi}{2}(1^4)(11.5)(10^6)}$$

$$= 0.002019 \text{ rad} = 0.116^\circ \qquad \textbf{Ans}$$

$$(\tau_{st})_{\max\,AB} = \frac{T_{AB}c}{J} = \frac{50(12)(1)}{\frac{\pi}{2}(1^4)} = 382 \text{ psi}$$

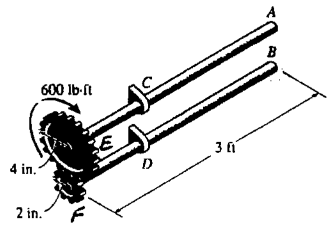
$$(\tau_{st})_{\max\,BC} = \frac{T_{st}c}{J} = \frac{48.428(12)(1)}{\frac{\pi}{2}(1^4 - 0.5^4)} = 394.63 \text{ psi} = 395 \text{ psi (Max)} \qquad \textbf{Ans}$$

$$(\gamma_{st})_{\max} = \frac{(\tau_{st})_{\max}}{G} = \frac{394.63}{11.5(10^6)} = 34.3(10^{-6}) \text{ rad} \qquad \textbf{Ans}$$

$$(\tau_{br})_{\max} = \frac{T_{br}c}{J} = \frac{1.572(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 96.07 \text{ psi} = 96.1 \text{ psi (Max)} \qquad \textbf{Ans}$$

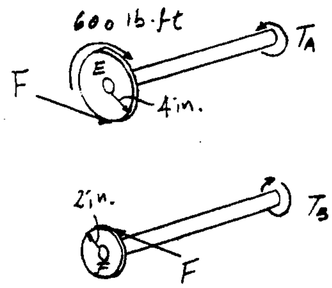
$$(\gamma_{br})_{\max} = \frac{(\tau_{br})_{\max}}{G} = \frac{96.07}{5.6(10^6)} = 17.2(10^{-6}) \text{ rad} \qquad \textbf{Ans}$$

•5–80 The two 3-ft-long shafts are made of 2014-T6 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 600 lb · ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.



$$T_A + F\left(\frac{4}{12}\right) - 600 = 0 \qquad (1)$$

$$T_B - F\left(\frac{2}{12}\right) = 0 \qquad (2)$$



From Eqs. (1) and (2)

$$T_A + 2T_B - 600 = 0 \qquad (3)$$

$$4(\phi_E) = 2(\phi_F); \qquad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \qquad T_A = 0.5T_B \qquad (4)$$

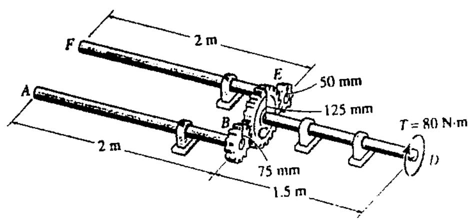
Solving Eqs. (3) and (4) yields :

$T_B = 240$ lb · ft ; $\quad T_A = 120$ lb · ft

$$(\tau_{BD})_{\max} = \frac{T_B\, c}{J} = \frac{240(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 4.35 \text{ ksi} \qquad \textbf{Ans}$$

$$(\tau_{AC})_{\max} = \frac{T_A\, c}{J} = \frac{120(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 2.17 \text{ ksi} \qquad \textbf{Ans}$$

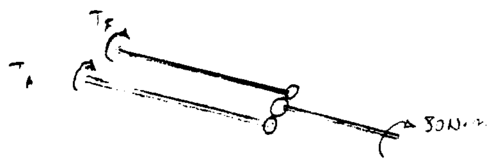
5–81 The two shafts AB and EF are fixed at their ends and fixed connected to gears that are in mesh with a common gear at C, which is fixed connected to shaft CD. If a torque of $T = 80$ N · m is applied to end D, determine the angle of twist of end D. Each shaft has a diameter of 20 mm and is made from A-36 steel.



If gear C rotates ϕ_C then,

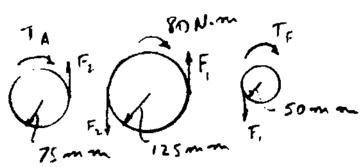
$$50\,\phi_E = 125\,\phi_C$$
$$\phi_E = 2.5\,\phi_C$$

$$75\,\phi_B = 125\,\phi_C$$
$$\phi_B = 1.667\,\phi_C$$



$$\phi_E = \frac{T_F(2)}{JG} \qquad (1)$$

$$\phi_B = \frac{T_A(2)}{JG} \qquad (2)$$



$\Sigma M_{CD} = 0;\quad F_2(125) + F_1(125) - 80 = 0$

$\Sigma M_{BA} = 0;\quad -T_A + F_2(75) = 0$

$\Sigma M_{EF} = 0;\quad F_1(50) - T_F = 0$

or

$$1.667\,T_A + 2.5\,T_F = 80$$

From Eqs. (1) and (2)

$$\phi_C = \frac{T_F(2)}{2.5\,JG} \qquad (3)$$

$$\phi_C = \frac{T_A(2)}{1.667\,JG} \qquad (4)$$

Thus,

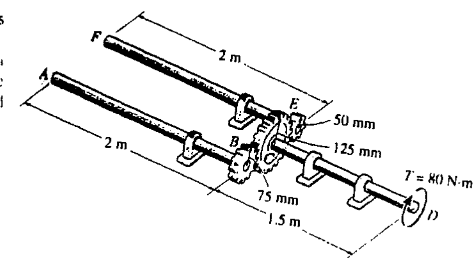
$$\phi_C\left[\frac{1.667(1.667JG)}{2} + \frac{2.5(2.5JG)}{2}\right] = 80$$

$$\phi_C\,JG = 17.723$$

$$\phi_C = \frac{17.723}{(\pi\frac{(0.01)^4}{2})(75)(10^9)} = 0.01504 \text{ rad}$$

$$\phi_D = \phi_C + \frac{80(1.5)}{(\pi\frac{(0.01)^4}{2})(75)(10^9)} = 0.1169 \text{ rad} = 6.70° \qquad \textbf{Ans}$$

5–82 The two shafts AB and EF are fixed at their ends and fixed connected to gears that are in mesh with a common gear at C, which is fixed connected to shaft CD. If a torque of $T = 80\ \text{N}\cdot\text{m}$ is applied to end D, determine the torque at A and F. Each shaft has a diameter of 20 mm and is made from A-36 steel.



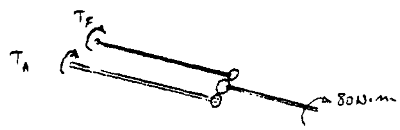
If gear C rotates ϕ_C then,

$$50\,\phi_E = 125\,\phi_C$$

$$\phi_E = 2.5\,\phi_C$$

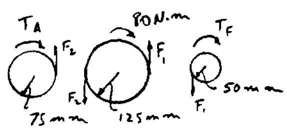
$$75\,\phi_B = 125\,\phi_C$$

$$\phi_B = 1.667\,\phi_C$$



$$\phi_E = \frac{T_F(2)}{JG} \qquad (1)$$

$$\phi_B = \frac{T_A(2)}{JG} \qquad (2)$$



$\Sigma M_{CD} = 0;\qquad F_2(125) + F_1(125) - 80 = 0$

$\Sigma M_{BA} = 0;\qquad -T_A + F_2(75) = 0$

$\Sigma M_{EF} = 0;\qquad F_1(50) - T_F = 0$

or

$$1.667\,T_A + 2.5\,T_F = 80$$

From Eqs. (1) and (2)

$$\phi_C = \frac{T_F(2)}{2.5\,JG} \qquad (3)$$

$$\phi_C = \frac{T_A(2)}{1.667\,JG} \qquad (4)$$

Thus,

$$\phi_C\left[\frac{1.667(1.667JG)}{2} + \frac{2.5(2.5JG)}{2}\right] = 80$$

$$\phi_C\,JG = 17.723$$

$$\phi_C = \frac{17.723}{(\pi\frac{(0.01)^4}{2})(75)(10^9)} = 0.01504\ \text{rad}$$

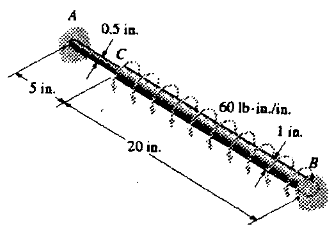
$$T_F = \frac{2.5\,JG\,\phi_C}{2} = \frac{2.5(\pi\frac{(0.01)^4}{2})(75)(10^9)(0.01504)}{2}$$

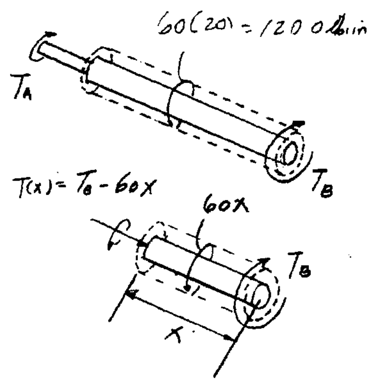
$T_F = 22.1\ \text{N}\cdot\text{m}$ **Ans**

$$T_A = \frac{1.667\,JG\,\phi_C}{2} = \frac{1.667(\pi\frac{(0.01)^4}{2})(75)(10^9)(0.01504)}{2}$$

$T_A = 14.8\ \text{N}\cdot\text{m}$ **Ans**

5–83 The A-36 steel shaft is made from two segments: *AC* has a diameter of 0.5 in. and *CB* has a diameter of 1 in. If the shaft is fixed at its ends *A* and *B* and subjected to a uniform distributed torque of 60 lb · in./in. along segment *CB*, determine the absolute maximum shear stress in the shaft.





Equilibrium :

$$T_A + T_B - 60(20) = 0 \qquad (1)$$

Compatibility condition :

$$\phi_{C/B} = \phi_{C/A}$$

$$\phi_{C/B} = \int \frac{T(x)\,dx}{JG} = \int_0^{20} \frac{(T_B - 60x)\,dx}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

$$= 18.52(10^{-6})T_B - 0.011112$$

$$18.52(10^{-6})T_B - 0.011112 = \frac{T_A(5)}{\frac{\pi}{2}(0.25^4)(11.0)(10^6)}$$

$$18.52(10^{-6})T_B - 74.08(10^{-6})T_A = 0.011112$$

$$18.52T_B - 74.08T_A = 11112 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

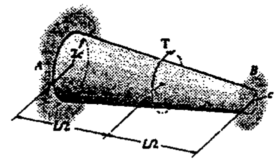
$T_A = 120.0\ \text{lb}\cdot\text{in.}$; $\quad T_B = 1080\ \text{lb}\cdot\text{in.}$

$$(\tau_{max})_{BC} = \frac{T_B\,c}{J} = \frac{1080(0.5)}{\frac{\pi}{2}(0.5^4)} = 5.50\ \text{ksi}$$

$$(\tau_{max})_{AC} = \frac{T_A\,c}{J} = \frac{120.0(0.25)}{\frac{\pi}{2}(0.25^4)} = 4.89\ \text{ksi}$$

$\tau_{abs\,max} = 5.50\ \text{ksi}$ **Ans**

***5-84.** The tapered shaft is confined by the fixed supports at A and B. If a torque $\mathbf{T}$ is applied at its mid-point, determine the reactions at the supports.



Equilibrium :

$$T_A + T_B - T = 0 \qquad [1]$$

Section Properties :

$$r(x) = c + \frac{c}{L}x = \frac{c}{L}(L+x)$$

$$J(x) = \frac{\pi}{2}\left[\frac{c}{L}(L+x)\right]^4 = \frac{\pi c^4}{2L^4}(L+x)^4$$

Angle of Twist :

$$\phi_T = \int \frac{T dx}{J(x) G} = \int_{\frac{L}{2}}^{L} \frac{T dx}{\frac{\pi c^4}{2L^4}(L+x)^4 G}$$

$$= \frac{2TL^4}{\pi c^4 G}\int_{\frac{L}{2}}^{L} \frac{dx}{(L+x)^4}$$

$$= -\frac{2TL^4}{3\pi c^4 G}\left[\frac{1}{(L+x)^3}\right]\Bigg|_{\frac{L}{2}}^{L}$$

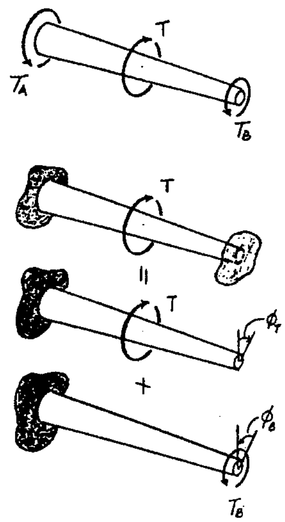
$$= \frac{37TL}{324\pi c^4 G}$$

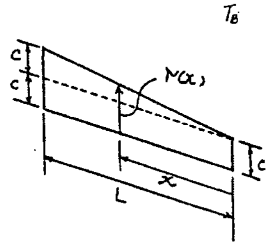
$$\phi_B = \int \frac{T dx}{J(x) G} = \int_0^L \frac{T_B dx}{\frac{\pi c^4}{2L^4}(L+x)^4 G}$$

$$= \frac{2T_B L^4}{\pi c^4 G}\int_0^L \frac{dx}{(L+x)^4}$$

$$= -\frac{2T_B L^4}{3\pi c^4 G}\left[\frac{1}{(L+x)^3}\right]\Bigg|_0^L$$

$$= \frac{7T_B L}{12\pi c^4 G}$$





Compatibility :

$$0 = \phi_T - \phi_B$$

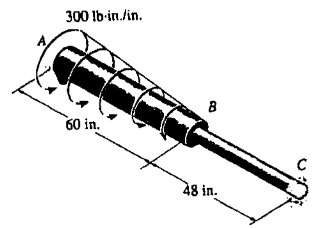
$$0 = \frac{37TL}{324\pi c^4 G} - \frac{7T_B L}{12\pi c^4 G}$$

$$T_B = \frac{37}{189}T \qquad \textbf{Ans}$$

Substituting the result into Eq.[1] yields :

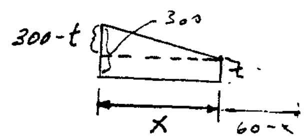
$$T_A = \frac{152}{189}T \qquad \textbf{Ans}$$

5-85 A portion of the A-36 steel shaft is subjected to a linearly distributed torsional loading. If the shaft has the dimensions shown, determine the reactions at the fixed supports A and C. Segment AB has a diameter of 1.5 in. and segment BC has a diameter of 0.75 in.



Equilibrium :

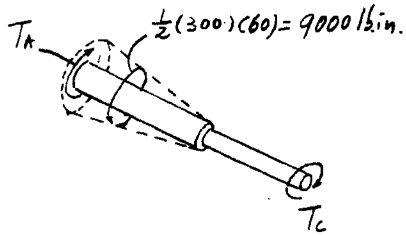
$$T_A + T_C - 9000 = 0 \qquad (1)$$



$$T_R = tx + \frac{1}{2}(300 - t)x = 150x + \frac{tx}{2}$$

But $\dfrac{t}{60 - x} = \dfrac{300}{60}$; $\quad t = 5(60 - x)$

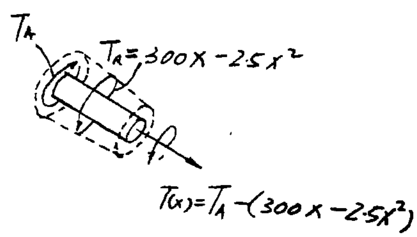
$$T_R = 150x + \frac{1}{2}[5(60 - x)]x$$
$$= (300x - 2.5x^2)\ \text{lb}\cdot\text{in.}$$



Compatibility condition :

$$\phi_{B/A} = \phi_{B/C}$$

$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60} [T_A - (300x - 2.5x^2)]\,dx$$
$$= \frac{1}{JG}[T_A x - 150x^2 + 0.8333x^3]\Big|_0^{60}$$
$$= \frac{60T_A - 360\,000}{JG}$$



$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

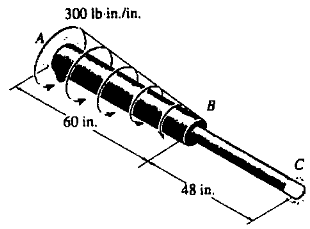
$$60T_A - 768T_C = 360\,000 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$T_C = 217.4$ lb·in. $= 18.1$ lb·ft **Ans**

$T_A = 8782.6$ lb·in. $= 732$ lb·ft **Ans**

5–86 Determine the rotation of joint B and the absolute maximum shear stress in the shaft in Prob. 5–85.

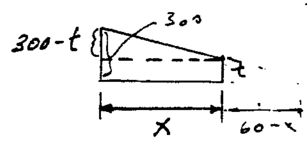


Equilibrium :

$$T_A + T_C - 9000 = 0 \qquad (1)$$

$$T_R = tx + \frac{1}{2}(300 - t)x = 150x + \frac{tx}{2}$$

But $\dfrac{t}{60 - x} = \dfrac{300}{60}; \quad t = 5(60 - x)$

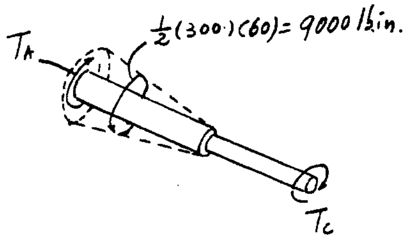


$$T_R = 150x + \frac{1}{2}[5(60 - x)]x$$
$$= (300x - 2.5x^2)\ \text{lb}\cdot\text{in.}$$

Compatibility condition :

$$\phi_{B/A} = \phi_{B/C}$$

$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60}[T_A - (300x - 2.5x^2)]\,dx$$
$$= \frac{1}{JG}[T_Ax - 150x^2 + 0.8333x^3]\Big|_0^{60}$$
$$= \frac{60T_A - 360\,000}{JG}$$



$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

$$60T_A - 768T_C = 360\,000 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

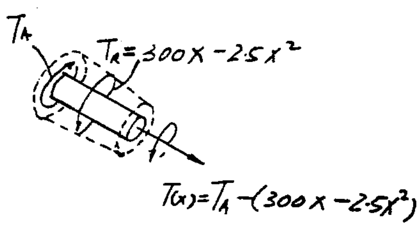
$T_C = 217.4$ lb·in. $= 18.1$ lb·ft

$T_A = 8782.6$ lb·in. $= 732$ lb·ft

For segment BC :

$$\phi_B = \phi_{B/C} = \frac{T_CL}{JG} = \frac{217.4(48)}{\frac{\pi}{2}(0.375)^4(11.0)(10^6)} = 0.030540\ \text{rad}$$

$\phi_B = 1.75°$ **Ans**



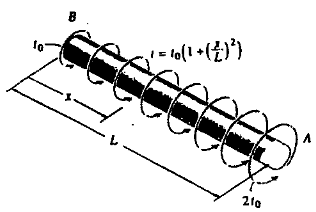
$$\tau_{max} = \frac{Tc}{J} = \frac{217.4(0.375)}{\frac{\pi}{2}(0.375)^4} = 2.62\ \text{ksi}$$

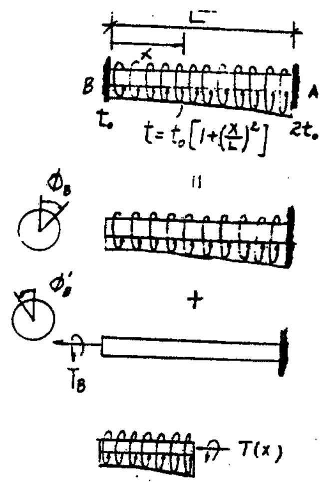
For segment AB,

$$\tau_{max} = \frac{Tc}{J} = \frac{8782.6(0.75)}{\frac{\pi}{2}(0.75)^4} = 13.3\ \text{ksi}$$

$\tau_{abs\,max} = 13.3$ ksi **Ans**

5-87. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.





$$T(x) = \int_0^x t_0\left(1 + \frac{x^2}{L^2}\right)dx = t_0\left(x + \frac{x^3}{3L^2}\right)$$ (1)

By superposition :

$$0 = \phi_B - \phi'_B$$

$$0 = \int_0^L \frac{t_0\left(x + \frac{x^3}{3L^2}\right)dx}{JG} - \frac{T_B(L)}{JG} = \frac{7t_0L^2}{12} - T_B(L)$$

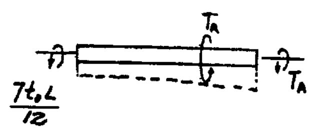
$$T_B = \frac{7t_0L}{12}$$ **Ans**

From Eq. (1),

$$T_R = t_0\left(L + \frac{L^3}{3L^2}\right) = \frac{4t_0L}{3}$$

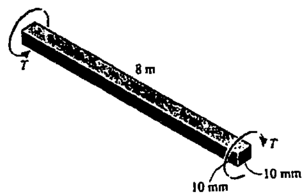
$$T_A + \frac{7t_0L}{12} - \frac{4t_0L}{3} = 0$$

$$T_A = \frac{3t_0L}{4}$$ **Ans**



***5-88.** The aluminum rod has a square cross section of 10 mm by 10 mm. If it is 8 m long, determine the torque T that is required to rotate one end relative to the other end by 90°. $G_{al} = 28$ GPa, $(\tau_Y)_{al} = 240$ MPa.

$$\phi = \frac{7.10\,T\,L}{a^4 G}$$



$$\frac{\pi}{2} = \frac{7.10T(8)}{(0.01)^4(28)(10^9)}$$

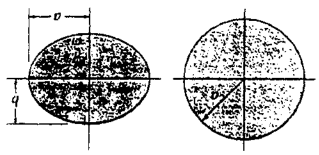
$$T = 7.74\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

$$\tau_{max} = \frac{4.81T}{a^3}$$

$$= \frac{4.81(7.74)}{0.01^3}$$

$$= 37.2\ \text{MPa} < \tau_Y \qquad \text{OK}$$

5–89 Determine the amount the maximum shear stress in the shaft having an elliptical cross section is increased compared to the shaft having a circular cross section if both shafts withstand the same torque.



For the circular shaft :

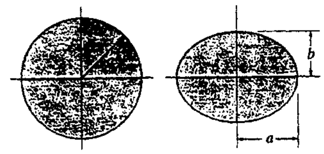
$$(\tau_{max})_c = \frac{Tc}{J} = \frac{2T}{\pi a^3}$$

For the elliptical shaft :

$$(\tau_{max})_e = \frac{2T}{\pi a b^2}$$

$$\text{Fraction of increase} = \frac{(\tau_{max})_e}{(\tau_{max})_c} = \frac{\frac{2T}{\pi a b^2}}{\frac{2T}{\pi a^3}} = (\frac{a}{b})^2 \qquad \textbf{Ans}$$

5–90 If $a = 25$ mm and $b = 15$ mm, determine the maximum shear stress in the circular and elliptical shafts when the applied torque is $T = 80$ N · m. By what percentage is the shaft of circular cross section more efficient at withstanding the torque than the shaft of elliptical cross section?



For the circular shaft :

$$(\tau_{max})_c = \frac{Tc}{J} = \frac{80(0.025)}{\frac{\pi}{2}(0.025^4)} = 3.26 \text{ MPa}$$ **Ans**

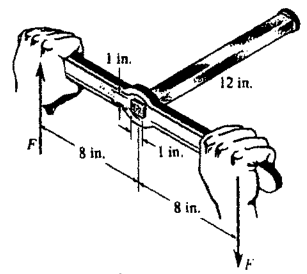
For the elliptical shaft :

$$(\tau_{max})_e = \frac{2T}{\pi a b^2} = \frac{2(80)}{\pi(0.025)(0.015^2)} = 9.05 \text{ MPa}$$ **Ans**

$$\% \text{ more efficient} = \frac{(\tau_{max})_e - (\tau_{max})_c}{(\tau_{max})_c}(100\%)$$

$$= \frac{9.05 - 3.26}{3.26}(100\%) = 178\ \%$$ **Ans**

5-91 The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the largest couple forces F that can be applied to the shaft without causing the steel to yield. $\tau_Y = 8$ ksi.

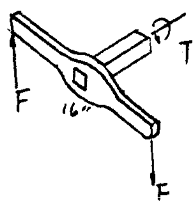


$$F(16) - T = 0 \qquad (1)$$

$$\tau_{max} = \tau_Y = \frac{4.81T}{a^3}$$

$$8(10^3) = \frac{4.81T}{(1)^3}$$

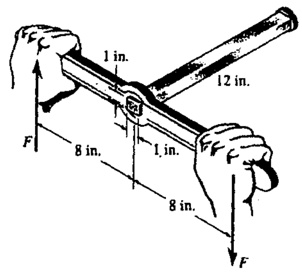
$$T = 1663.2 \text{ lb} \cdot \text{in.}$$



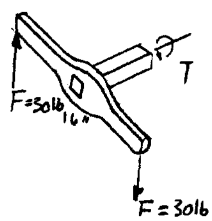
From Eq. (1),

$F = 104$ lb **Ans**

•5-92 The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the maximum shear stress in the shaft and the amount of displacement that each couple force undergoes if the couple forces have a magnitude of $F = 30$ lb. $G_{st} = 10.8(10^3)$ ksi.



$$T - 30(16) = 0$$
$$T = 480 \text{ lb}\cdot\text{in.}$$

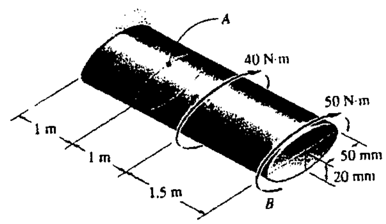


$$\tau_{max} = \frac{4.81T}{a^3} = \frac{4.81(480)}{(1)^3}$$
$$= 2.31 \text{ ksi} \qquad \textbf{Ans}$$

$$\phi = \frac{7.10TL}{a^4 G} = \frac{7.10(480)(12)}{(1)^4(10.8)(10^6)} = 0.00379 \text{ rad}$$

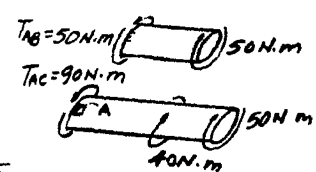
$$\delta_F = 8(0.00379) = 0.0303 \text{ in.} \qquad \textbf{Ans}$$

5–93 The shaft is made of plastic and has an elliptical cross-section. If it is subjected to the torsional loading shown, determine the shear stress at point A and show the shear stress on a volume element located at this point. Also, determine the angle of twist ϕ at the end B. $G_p = 15$ GPa.



$$\tau_A = \frac{2(T_{AC})}{\pi a b^2}$$

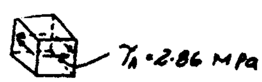
$$= \frac{2(90)}{\pi(0.05)(0.02)^2} = 2.86 \text{ MPa} \qquad \textbf{Ans}$$



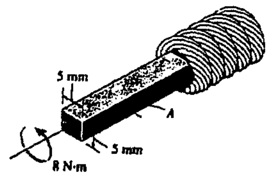
$$\phi = \sum \frac{(a^2 + b^2)TL}{\pi a^3 b^3 G}$$

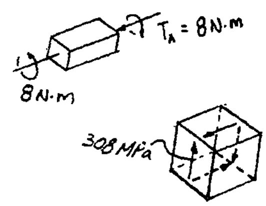
$$= \frac{(0.05^2 + 0.02^2)(50)(1.5)}{\pi(0.05^3)(0.02^3)(15)(10^9)} + \frac{(0.05^2 + 0.02^2)(90)(2)}{\pi(0.05^3)(0.02^3)(15)(10^9)}$$

$$= 0.0157 \text{ rad} = 0.899^\circ \qquad \textbf{Ans}$$



5–94. The square shaft is used at the end of a drive cable in order to register the rotation of the cable on a gauge. If it has the dimensions shown and is subjected to a torque of 8 N · m, determine the shear stress in the shaft at point A. Sketch the shear stress on a volume element located at this point.

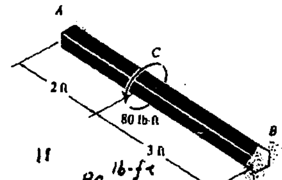




Maximum shear stress :

$$(\tau_{max})_A = \frac{4.81T}{a^3} = \frac{4.81(8)}{(0.005)^3} = 308 \text{ MPa} \qquad \textbf{Ans}$$

5-95. The aluminum strut is fixed between the two walls at A and B. If it has a 2 in. by 2 in. square cross section, and it is subjected to the torque of 80 lb·ft at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 3.8(10^3)$ ksi.



By superposition :

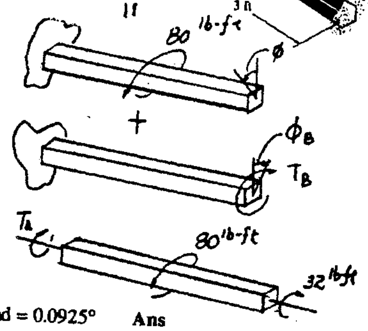
$$0 = \phi - \phi_B$$

$$0 = \frac{7.10(80)(2)}{a^4 G} - \frac{7.10(T_B)(5)}{a^4 G}$$

$T_B = 32$ lb·ft **Ans**

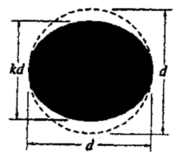
$$T_A + 32 - 80 = 0$$

$T_A = 48$ lb·ft **Ans**



$$\phi_C = \frac{7.10(32)(12)(3)(12)}{(2^4)(3.8)(10^6)} = 0.00161 \text{ rad} = 0.0925°$$ **Ans**

*5–96 It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



For the circular shaft :

$$(\tau_{max})_c = \frac{Tc}{J} = \frac{T(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4} = \frac{16T}{\pi d^3}$$

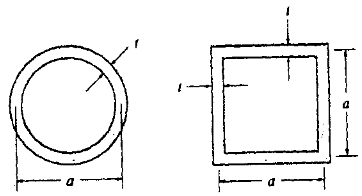
For the elliptical shaft :

$$(\tau_{max})_e = \frac{2T}{\pi a b^2} = \frac{2T}{\pi(\frac{d}{2})(\frac{kd}{2})^2} = \frac{16T}{\pi k^2 d^3}$$

$$\text{Factor of increase in shear stress} = \frac{(\tau_{max})_e}{(\tau_{max})_c} = \frac{\frac{16T}{\pi k^2 d^3}}{\frac{16T}{\pi d^3}}$$

$$= \frac{1}{k^2} \qquad \textbf{Ans}$$

5–97 A torque **T** is applied to two tubes having the cross-sections shown. Compare the shear flow developed in each tube.



Circular tube :

$$q_{ct} = \frac{T}{2A_m} = \frac{T}{2\pi\,(a/2)^2} = \frac{2T}{\pi\,a^2}$$

Square tube :

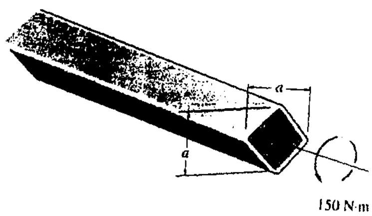
$$q_{st} = \frac{T}{2A_m} = \frac{T}{2a^2}$$

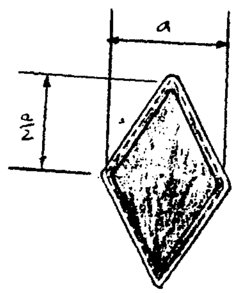
$$\frac{q_{st}}{q_{ct}} = \frac{T/(2a^2)}{2T/(\pi\,a^2)} = \frac{\pi}{4}$$

Thus;

$$q_{st} = \frac{\pi}{4}\,q_{ct}$$ **Ans**

5–98 The plastic tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress if $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm. Neglect stress concentrations at the corners.



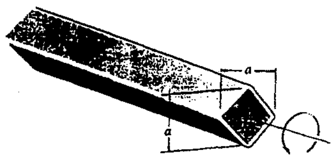


$$A_m = 4\left[\frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\right] = \frac{a^2}{2}$$

$$\tau_{avg} = \frac{T}{2\,t A_m}; \qquad 60(10^6) = \frac{150}{2(0.003)\,\frac{1}{2}\,a^2}$$

$$a = 0.0289 \text{ m} = 28.9 \text{ mm}$$ **Ans**

5-99. The plastic tube is subjected to a torque of 150 N · m. Determine the average shear stress in the tube if the mean dimension $a = 200$ mm. Each side has a thickness of $t = 3$ mm. Neglect stress concentrations at the corners.

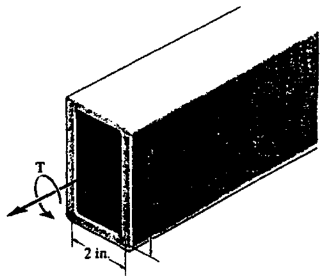


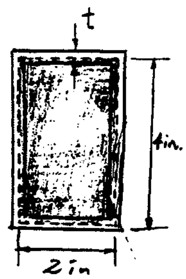
$$A_m = 4\left[\frac{1}{2}\left(\frac{0.2}{2}\right)\left(\frac{0.2}{2}\right)\right] = 0.02\ \text{m}^2$$

$$\tau_{avg} = \frac{T}{2\,t\,A_m} = \frac{150}{2(0.003)(0.02)}$$

$$= 1.25\ \text{MPa} \qquad \textbf{Ans}$$

*5–100 Determine the constant thickness of the rectangular tube if the average shear stress is not to exceed 12 ksi when a torque of $T = 20$ kip · in. is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.





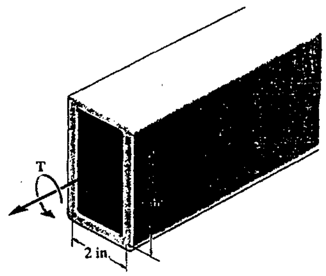
$$A_m = 2(4) = 8 \text{ in}^2$$

$$\tau_{avg} = \frac{T}{2\, t A_m}$$

$$12 = \frac{20}{2\, t(8)}$$

$$t = 0.104 \text{ in.} \qquad \textbf{Ans}$$

5–101 Determine the torque T that can be applied to the rectangular tube if the average shear stress is not to exceed 12 ksi. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 0.125 in.

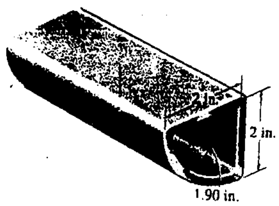


$$A_m = 2(4) = 8\ \text{in}^2$$

$$\tau_{avg} = \frac{T}{2\,t\,A_m}; \qquad 12 = \frac{T}{2(0.125)(8)}$$

$$T = 24\ \text{kip}\cdot\text{in.} = 2\ \text{kip}\cdot\text{ft}$$ **Ans**

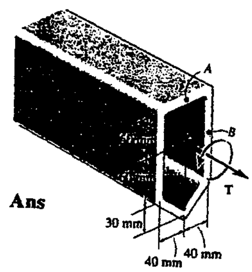
5–102 A torque of 2 kip · in. is applied to the tube. If the wall thickness is 0.1 in., determine the average shear stress in the tube.



$$A_m = \frac{\pi(1.95^2)}{4} = 2.9865\ \text{in}^2$$

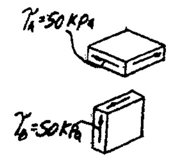
$$\tau_{avg} = \frac{T}{2tA_m} = \frac{2(10^3)}{2(0.1)(2.9865)} = 3.35\ \text{ksi} \qquad \textbf{Ans}$$

5-103. The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if it is subjected to the torque of $T = 5\ \text{N}\cdot\text{m}$. Show the shear stress on volume elements located at these points.

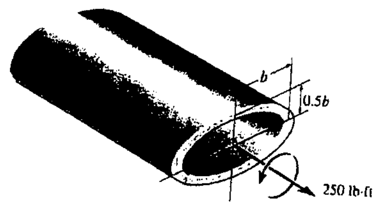


$$A_m = (0.11)(0.08) + \frac{1}{2}(0.08)(0.03) = 0.01\ \text{m}^2$$

$$\tau_A = \tau_B = \tau_{avg} = \frac{T}{2tA_m} = \frac{5}{2(0.005)(0.01)} = 50\ \text{kPa} \qquad \textbf{Ans}$$



•5–104 The steel tube has an elliptical cross section of mean dimensions shown and a constant thickness of $t = 0.2$ in. If the allowable shear stress is $\tau_{allow} = 8$ ksi, and the tube is to resist a torque of $T = 250$ lb · ft, determine the necessary dimension b. The mean area for the ellipse is $A_m = \pi b(0.5b)$.

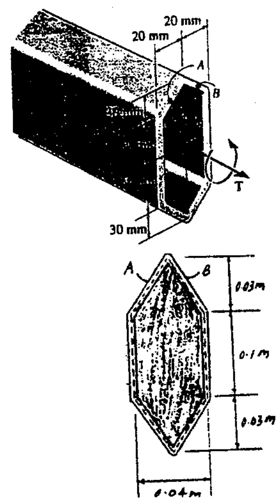


$$\tau_{avg} = \tau_{allow} = \frac{T}{2\,t\,A_m}$$

$$8(10^3) = \frac{250(12)}{2(0.2)(\pi)(b)(0.5b)}$$

$$b = 0.773 \text{ in.} \qquad \textbf{Ans}$$

5-105. The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if the tube is subjected to the torque of $T = 500\ \text{N}\cdot\text{m}$. Show the shear stress on volume elements located at these points. Neglect stress concentrations at the corners.

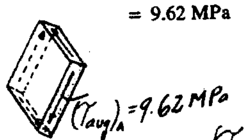


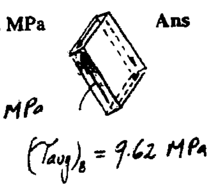
$$A_m = 2[\frac{1}{2}(0.04)(0.03)] + 0.1(0.04) = 0.0052\ \text{m}^2$$

$$(\tau_{avg})_A = (\tau_{avg})_B = \frac{T}{2\,t A_m}$$

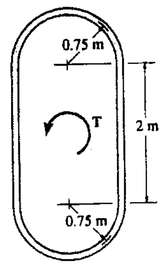
$$= \frac{500}{2(0.005)(0.0052)}$$

$$= 9.62\ \text{MPa} \qquad \textbf{Ans}$$





5-106. A portion of an airplane fuselage can be approximated by the cross section shown. If the thickness of its 2014-T6-aluminum skin is 10 mm, determine the maximum wing torque **T** that can be applied if $\tau_{allow} = 4$ MPa. Also, in a 4-m long section, determine the angle of twist.



$$\tau_{avg} = \frac{T}{2tA_m}$$

$$4(10^6) = \frac{T}{2(0.01)\left[(\pi)(0.75)^2 + 2(1.5)\right]}$$

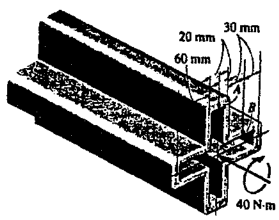
$$T = 381.37(10^3) = 381 \text{ kN} \cdot \text{m} \qquad \textbf{Ans}$$

$$\phi = \frac{TL}{4A_m^2 G} \int \frac{ds}{t}$$

$$\phi = \frac{381.37(10^3)(4)}{4\left[(\pi(0.75)^2 + 2(1.5))^2 27(10^9)\right]} \left[\frac{4 + 2\pi(0.75)}{0.010}\right]$$

$$\phi = 0.542(10^{-3}) \text{ rad} \qquad \textbf{Ans}$$

5-107. The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of $T = 40\ \text{N}\cdot\text{m}$, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.



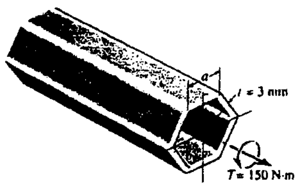
$$A_m = 4(0.04)(0.06) + (0.04)^2 = 0.0112\ \text{m}^2$$

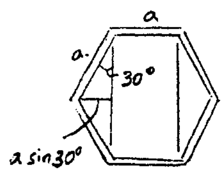
$$\tau_{avg} = \frac{T}{2\,t\,A_m}$$

$$(\tau_{avg})_A = (\tau_{avg})_B = \frac{40}{2(0.005)(0.0112)} = 357\ \text{kPa} \qquad \textbf{Ans}$$

$\tau_A = \tau_B = 357$ kPa

***5-108.** The plastic hexagonal tube is subjected to a torque of 150 N · m. Determine the mean dimension a of its sides if the allowable shear stress is $\tau_{allow} = 60$ MPa. Each side has a thickness of $t = 3$ mm.





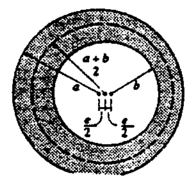
$$A_m = 4\left[\frac{1}{2}(a\cos 30^\circ)(a\sin 30^\circ)\right] + (a)(2a)\cos 30^\circ = 2.5981\,a^2$$

$$\tau_{avg} = \tau_{allow} = \frac{T}{2\,t A_m}$$

$$60(10^6) = \frac{150}{(2)(0.003)(2.5981\,a^2)}$$

$a = 0.01266$ m $= 12.7$ mm **Ans**

5-109. Due to fabrication, the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity e is one-fourth of the difference in the radii?



Average Shear Stress :

For the aligned tube

$$\tau_{avg} = \frac{T}{2\,t A_m} = \frac{T}{2(a-b)(\pi)\left(\frac{a+b}{2}\right)^2}$$

$$T = \tau_{avg}(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2$$

For the eccentric tube

$$\tau_{avg} = \frac{T'}{2\,t A_m}$$

$$t = a - \frac{e}{2} - \left(\frac{e}{2} + b\right) = a - e - b$$

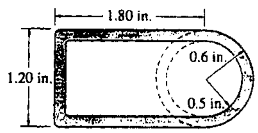
$$= a - \frac{1}{4}(a-b) - b = \frac{3}{4}(a-b)$$

$$T' = \tau_{avg}(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2$$

$$\text{Factor} = \frac{T'}{T} = \frac{\tau_{avg}(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2}{\tau_{avg}(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2} = \frac{3}{4}$$

$$\text{Percent reduction in strength} = \left(1 - \frac{3}{4}\right) \times 100\,\% = 25\,\% \qquad \textbf{Ans}$$

5–110 For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 0.1 in. thick.



$$A_m = (1.10)(1.75) - \frac{\pi(0.55^2)}{2} = 1.4498 \text{ in}^2$$

$$A_m' = (1.10)(1.75) + \frac{\pi(0.55^2)}{2} = 2.4002 \text{ in}^2$$

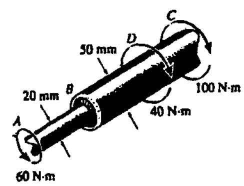
$$\tau_{max} = \frac{T}{2t A_m}$$

$$T = 2\, t A_m \tau_{max}$$

$$\text{Factor} = \frac{2t A_m' \tau_{max}}{2t A_m \tau_{max}}$$

$$= \frac{A_m'}{A_m} = \frac{2.4002}{1.4498} = 1.66 \qquad \textbf{Ans}$$

5–111. The steel shaft is made from two segments AB and BC, which are connected using a fillet weld having a radius of 2.8 mm. Determine the maximum shear stress developed in the shaft.

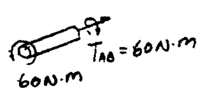


$$(\tau_{max})_{CD} = \frac{T_{CD}c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)}$$

$$= 4.07 \text{ MPa}$$

For the fillet :

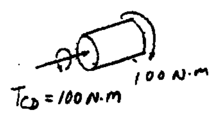
$$\frac{D}{d} = \frac{50}{20} = 2.5; \qquad \frac{r}{d} = \frac{2.8}{20} = 0.14$$



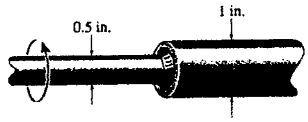
From Fig. 5–36, $K = 1.325$

$$(\tau_{max})_f = K\frac{T_{AB}c}{J} = 1.325\left[\frac{60(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$= 50.6 \text{ MPa (max)} \qquad \textbf{Ans}$$



•5–112 The shaft is used to transmit 0.8 hp while turning at 450 rpm. Determine the maximum shear stress in the shaft. The segments are connected together using a fillet weld having a radius of 0.075 in.



$$\frac{D}{d} = \frac{1}{0.5} = 2 \qquad \frac{r}{d} = \frac{0.075}{0.5} = 0.15$$

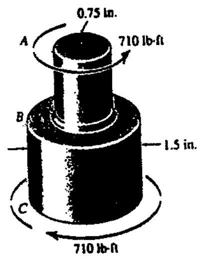
From Fig. 5-36, $K = 1.30$.

$$\omega = \frac{450(2\pi)}{60} = 47.124 \text{ rad/s}$$

$$P = T\omega$$
$$0.8(550) = T(47.124)$$
$$T = 9.337 \text{ lb} \cdot \text{ft}$$

$$\tau_{max} = K\frac{Tc}{J} = \frac{1.30(9.337)(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 5.93 \text{ ksi} \qquad \textbf{Ans}$$

5-113. The assembly is subjected to a torque of 710 lb · in. If the allowable shear stress for the material is τ_{allow} = 12 ksi, determine the radius of the smallest size fillet that can be used to transmit the torque.



$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$12(10^3) = \frac{K(710)(0.375)}{\frac{\pi}{2}(0.375^4)}$$

$$K = 1.40$$

$$\frac{D}{d} = \frac{1.5}{0.75} = 2$$

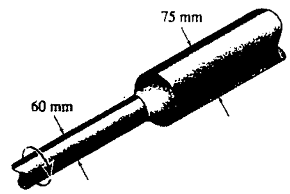
From Fig. 5-36,

$$\frac{r}{d} = 0.1; \qquad r = 0.1(0.75) = 0.075 \text{ in.} \qquad \textbf{Ans}$$

Check :

$$\frac{D-d}{2} = \frac{1.5-0.75}{2} = 0.375 > 0.075 \text{ in.} \qquad \textbf{OK}$$

5–114 The built-up shaft is to be designed to rotate at 720 rpm while transmitting 30 kW of power. Is this possible? The allowable shear stress is $\tau_{allow} = 12$ MPa.



$$\omega = 720\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 24\,\pi\ \text{rad/s}$$

$$T = \frac{P}{\omega} = \frac{30(10^3)}{24\,\pi} = 397.89\ \text{N}\cdot\text{m}$$

$$\tau_{max} = K\frac{Tc}{J}\ ; \qquad 12(10^6) = K\left[\frac{397.89(0.03)}{\frac{\pi}{2}(0.03^4)}\right]; \qquad K = 1.28$$

$$\frac{D}{d} = \frac{75}{60} = 1.25$$

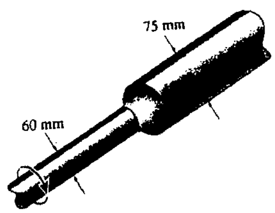
From Fig. 5–36, $\frac{r}{d} = 0.133$

$$\frac{r}{60} = 0.133\ ; \qquad r = 7.98\ \text{mm}$$

Check : $\frac{D-d}{2} = \frac{75-60}{2} = \frac{15}{2} = 7.5\ \text{mm} < 7.98\ \text{mm}$

No, it is not possible. **Ans**

5–115 The built-up shaft is designed to rotate at 540 rpm. If the radius of the fillet weld connecting the shafts is r 7.20 mm, and the allowable shear stress for the material is τ_{allow} = 55 MPa, determine the maximum power the shaft can transmit.



$$\frac{D}{d} = \frac{75}{60} = 1.25; \qquad \frac{r}{d} = \frac{7.2}{60} = 0.12$$

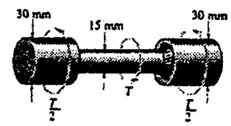
From Fig. 5 – 36, $K = 1.30$

$$\tau_{max} = K\frac{Tc}{J}; \qquad 55(10^6) = 1.30\left[\frac{T(0.03)}{\frac{\pi}{2}(0.03^4)}\right]; \qquad T = 1794.33\ \text{N}\cdot\text{m}$$

$$\omega = 540\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\frac{1\ \text{min}}{60\ \text{s}} = 18\,\pi\ \text{rad/s}$$

$$P = T\omega = 1794.33(18\pi) = 101466\ \text{W} = 101\ \text{kW}$$ **Ans**

***5-116.** The steel used for the shaft has an allowable shear stress of $\tau_{allow} = 8$ MPa. If the members are connected together with a fillet weld of radius $r = 2.25$ mm, determine the maximum torque T that can be applied.



Allowable Shear Stress :

$$\frac{D}{d} = \frac{30}{15} = 2 \quad \text{and} \quad \frac{r}{d} = \frac{2.25}{15} = 0.15$$

From the text, $K = 1.30$

$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$8(10^6) = 1.3\left[\frac{\left(\frac{T}{2}\right)(0.0075)}{\frac{\pi}{2}(0.0075^4)}\right]$$

$$T = 8.16 \text{ N}\cdot\text{m}$$ **Ans**

5–117 A solid shaft is subjected to the torque T, which causes the material to yield. If the material is elastic plastic, show that the torque can be expressed in terms of the angle of twist ϕ of the shaft as $T = \frac{4}{3} T_Y(1 - \phi^3_Y/4\phi^3)$, where T_Y and ϕ_Y are the torque and angle of twist when the material begins to yield.

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y}{\rho_Y} L$$

$$\rho_Y = \frac{\gamma_Y L}{\phi} \qquad (1)$$

When $\rho_Y = c, \qquad \phi = \phi_Y$

From Eq. (1),

$$c = \frac{\gamma_Y L}{\phi_Y} \qquad (2)$$

Dividing Eq. (1) by Eq. (2) yields :

$$\frac{\rho_Y}{c} = \frac{\phi_Y}{\phi} \qquad (3)$$

Use Eq. 5 – 26 from the text.

$$T = \frac{\pi \tau_Y}{6}(4c^3 - \rho_Y^3) = \frac{2\pi \tau_Y c^3}{3}\left(1 - \frac{\rho_Y^3}{4c^3}\right)$$

Use Eq. 5 - 24, $T_Y = \frac{\pi}{2}\tau_Y c^3$ from the text and Eq. (3)

$$T = \frac{4}{3}T_Y\left(1 - \frac{\phi_Y^3}{4\phi^3}\right) \qquad \textbf{QED}$$

5–118 A solid shaft having a diameter of 2 in. is made of elastic-plastic material having a yield stress of $\tau_Y = 16$ ksi and shear modulus of $G = 12\ (10^3)$ ksi. Determine the torque required to develop an elastic core in the shaft having a diameter of 1 in. Also, what is the plastic torque?

Use Eq. 5 – 26 from the text :

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y{}^3) = \frac{\pi\,(16)}{6}[4(1^3) - 0.5^3]$$

$$= 32.46\ \text{kip}\cdot\text{in.} = 2.71\ \text{kip}\cdot\text{ft}$$ **Ans**

Use Eq. 5 – 27 from the text :

$$T_p = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(16)(1^3)$$

$$= 33.51\ \text{kip}\cdot\text{in.} = 2.79\ \text{kip}\cdot\text{ft}$$ **Ans**

5–119 Determine the torque needed to twist a short 3-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic plastic and having a yield stress of $\tau_Y = 80$ MPa. Assume that the material becomes fully plastic.

When the material becomes fully plastic then, from Eq. 5 – 2 in the text,

$$T_p = \frac{2\,\pi\,\tau_Y}{3}c^3 = \frac{2\,\pi\,(80)(10^6)}{3}(0.0015^3) = 0.565\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

*5–120 A solid shaft has a diameter of 40 mm and length of 1 m. It is made from an elastic-plastic material having a yield stress of $\tau_Y = 100$ MPa. Determine the maximum elastic torque T_Y and the corresponding angle of twist. What is the angle of twist if the torque is increased to $T = 1.2T_Y$? $G = 80$ GPa.

Maximum elastic torque T_Y,

$$\tau_Y = \frac{T_Y\, c}{J}$$

$$T_Y = \frac{\tau_Y J}{c} = \frac{100(10^6)(\frac{\pi}{2})(0.02^4)}{0.02} = 1256.64\ \text{N}\cdot\text{m} = 1.26\ \text{kN}\cdot\text{m} \qquad \textbf{Ans}$$

Angle of twist :

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{100(10^6)}{80(10^9)} = 0.00125\ \text{rad}$$

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{0.00125}{0.02}(1) = 0.0625\ \text{rad} = 3.58^\circ \qquad \textbf{Ans}$$

Also,

$$\phi = \frac{T_Y L}{JG} = \frac{1256.64(1)}{\frac{\pi}{2}(0.02^4)(80)(10^9)} = 0.0625\ \text{rad} = 3.58^\circ$$

From Eq. 5 – 26 of the text,

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y^3)\,; \qquad 1.2(1256.64) = \frac{\pi\,(100)(10^6)}{6}[4(0.02^3) - \rho_Y^3]$$

$$\rho_Y = 0.01474\ \text{m}$$

$$\phi' = \frac{\gamma_Y}{\rho_Y}L = \frac{0.00125}{0.01474}(1) = 0.0848\ \text{rad} = 4.86^\circ \qquad \textbf{Ans}$$

5–121 Determine the torque needed to twist a short 2-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic-plastic and having a yield stress of $\tau_Y = 50$ MPa. Assume that the material becomes fully plastic.

Fully plastic torque is applied. From Eq. 5 – 27,

$$T_p = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(50)(10^6)(0.001^3) = 0.105 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$

5–122 A bar having a circular cross section of 3 in. diameter is subjected to a torque of 100 in. · kip. If the material is elastic plastic, with $\tau_Y = 16$ ksi, determine the radius of the elastic core.

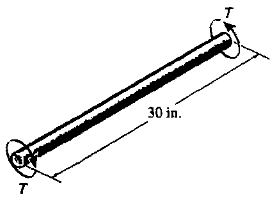
Using Eq. 5 – 26 of the text,

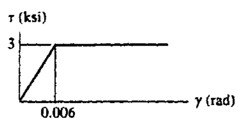
$$T = \frac{\pi\,\tau_Y}{6}\,(4c^3 - \rho_Y^3)$$

$$100(10^3) = \frac{\pi\,(16)(10^3)}{6}\,(4\,(1.5^3 - \rho_Y{}^3)$$

$\rho_Y = 1.16$ in. **Ans**

5–123 A shaft of radius $c = 0.75$ in. is made from an elastic-plastic material as shown. Determine the torque T that must be applied to its ends so that it has an elastic core of radius $\rho = 0.6$ in. If the shaft is 30 in. long, determine the angle of twist.





Use Eq. 5 - 26 of the text.

$$T = \frac{\pi \tau_Y}{6}(4c^3 - \rho_Y^3)$$

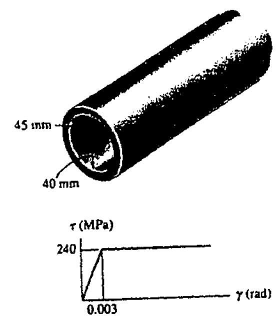
$$= \frac{\pi(3)(10^3)}{6}(4(0.75^3) - 0.6^3)$$

$$= 2311 \text{ lb}\cdot\text{in.} = 193 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

See Example 5 - 20 of the text.

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y L}{\rho_Y} = \frac{0.006(30)}{0.6} = 0.300 \text{ rad} = 17.2^\circ \qquad \textbf{Ans}$$

***5-124.** The 2-m-long tube is made from an elastic-plastic material as shown. Determine the applied torque T, which subjects the material of the tube's outer edge to a shearing strain of $\gamma_{max} = 0.008$ rad. What would be the permanent angle of twist of the tube when the torque is removed? Sketch the residual stress distribution of the tube.



$$\phi = \frac{\gamma_{max} L}{c} = \frac{0.008\,(2)}{0.045}$$

$$\phi = 0.3556 \text{ rad}$$

However,

$$\phi = \frac{\gamma_Y}{\rho_Y} L$$

$$0.3556 = \frac{0.003}{\rho_Y}(2)$$

$$\rho_Y = 0.016875 \text{ m} < 0.04 \text{ m}$$

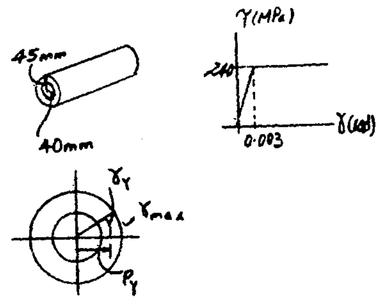
Therefore the tube is fully plastic.

Also,

$$\frac{0.008}{45} = \frac{r}{40}$$

$$r = 0.00711 > 0.003$$

Again, the tube is fully plastic.



$$T_p = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2 \, d\rho$$

$$= \frac{2\pi \tau_Y}{3}(c_o^3 - c_i^3)$$

$$= \frac{2\pi (240)(10^6)}{3}(0.045^3 - 0.04^3)$$

$$= 13634.5 \text{ N}\cdot\text{m} = 13.6 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

The torque is removed and the opposite torque of $T_p = 13634.5$ N·m is applied.

$$\phi' = \frac{T_p L}{JG} \qquad G = \frac{240(10^6)}{0.003} = 80 \text{ GPa}$$

$$= \frac{13634.5\,(2)}{\frac{\pi}{2}(0.045^4 - 0.04^4)(80)(10^9)}$$

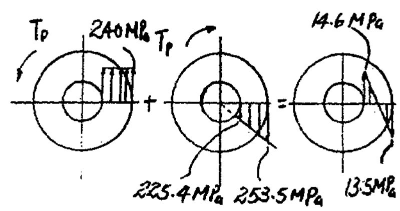
$$= 0.14085 \text{ rad}$$

$$\phi_r = \phi - \phi' = 0.35555 - 0.14085$$

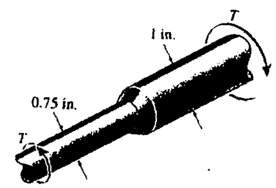
$$= 0.215 \text{ rad} = 12.3^\circ \qquad \textbf{Ans}$$

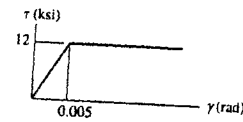
$$\tau'_{p_o} = \frac{T_p c}{J} = \frac{13634.5\,(0.045)}{\frac{\pi}{2}(0.045^4 - 0.04^4)} = 253.5 \text{ MPa}$$

$$\tau'_{p_i} = \frac{0.04}{0.045}(253.5) = 225.4 \text{ MPa}$$



5–125 The shaft consists of two sections that are rigidly connected. If the material is elastic plastic as shown, determine the largest torque T that can be applied to the shaft. Also, draw the shear-stress distribution over a radial line for each section. Neglect the effect of stress concentration.



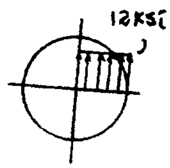


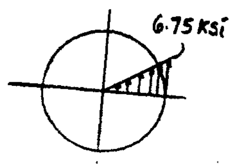
0.75 in. diameter segment will be fully plastic.
From Eq. 5 – 27 of the text :

$$T = T_p = \frac{2\pi\,\tau_Y}{3}(c^3)$$

$$= \frac{2\pi\,(12)(10^3)}{3}(0.375^3)$$

$$= 1325.36\ \text{lb}\cdot\text{in.} = 110\ \text{lb}\cdot\text{ft} \qquad \textbf{Ans}$$



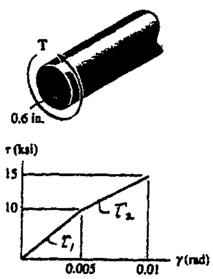


For 1 – in. diameter segment :

$$\tau_{max} = \frac{Tc}{J} = \frac{1325.36(0.5)}{\frac{\pi}{2}(0.5)^4}$$

$$= 6.75\ \text{ksi} < \tau_Y$$

5-126. The shaft is made from a strain-hardening material having a $\tau - \gamma$ diagram as shown. Determine the torque T that must be applied to the shaft in order to create an elastic core in the shaft having a radius of $\rho_c = 0.5$ in.

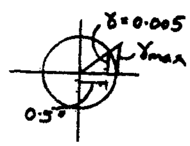


$$\frac{\tau_1}{\gamma} = \frac{10(10^3)}{0.005}$$

$$\tau_1 = 2\,(10^6)\,\gamma \qquad (1)$$

$$\frac{\tau_2 - 10(10^3)}{\gamma - 0.005} = \frac{15\,(10^3) - 10(10^3)}{0.01 - 0.005}$$

$$\tau_2 = 1(10^6)\,\gamma + 5\,(10^3) \qquad (2)$$



$$\gamma_{max} = \frac{0.6}{0.5}(0.005) = 0.006$$

$$\gamma = \frac{\rho}{c}\gamma_{max} = \frac{\rho}{0.6}(0.006) = 0.01\,\rho$$

Substituting γ into Eqs. (1) and (2) yields :

$$\tau_1 = 20(10^3)\,\rho$$

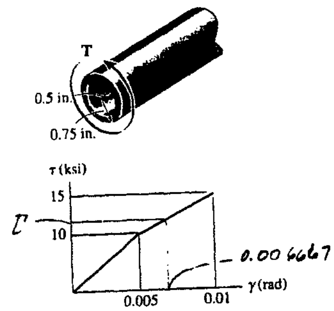
$$\tau_2 = 10(10^3)\,\rho + 5(10^3)$$

$$T = 2\pi \int_0^c \tau \rho^2\, d\rho$$

$$= 2\pi \int_0^{0.5} 20(10^3)\,\rho^3\, d\rho + 2\pi \int_{0.5}^{0.6} [10(10^3)\,\rho + 5(10^3)]\,\rho^2\, d\rho$$

$$= 3970 \text{ lb}\cdot\text{in.} = 331 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

5–127 The tubular shaft is made from a strain-hardening material having a τ-γ diagram as shown. Determine the torque T that must be applied to the shaft so that the maximum shear strain is 0.01 rad.



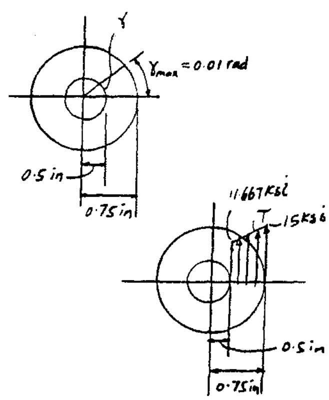
From the shear – strain diagram,

$$\frac{\gamma}{0.5} = \frac{0.01}{0.75}; \qquad \gamma = 0.006667 \text{ rad}$$

From the shear stress - strain diagram,

$$\frac{\tau - 10}{0.006667 - 0.005} = \frac{15 - 10}{0.01 - 0.005}; \quad \tau = 11.667 \text{ ksi}$$

$$\frac{\tau - 11.667}{\rho - 0.5} = \frac{15 - 11.667}{0.75 - 0.50}; \qquad \tau = 13.333\,\rho + 5$$



$$T = 2\pi \int_{c_i}^{c_o} \tau \rho^2 \, d\rho$$

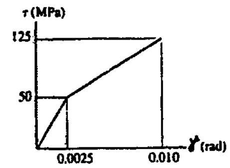
$$= 2\pi \int_{0.5}^{0.75} (13.333\rho + 5)\, \rho^2 \, d\rho$$

$$= 2\pi \int_{0.5}^{0.75} (13.333\rho^3 + 5\rho^2)\, d\rho$$

$$= 2\pi \left[\frac{13.333\rho^4}{4} + \frac{5\rho^3}{3}\right]\Big|_{0.5}^{0.75}$$

$$= 8.426 \text{ kip} \cdot \text{in.} = 702 \text{ lb} \cdot \text{ft}$$ **Ans**

***5-128.** The shear stress–strain diagram for a solid 50-mm diameter shaft can be approximated as shown in the figure. Determine the torque required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 3 m long, what is the corresponding angle of twist?.

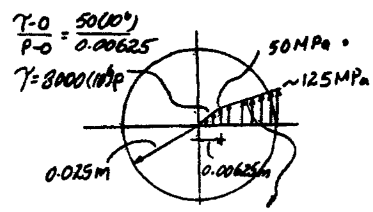


$$\gamma = \frac{\rho}{c}\gamma_{max}$$

$$\gamma_{max} = 0.01$$

When $\gamma = 0.0025$

$$\rho = \frac{c\gamma}{\gamma_{max}} = \frac{0.025(0.0025)}{0.010} = 0.00625$$



$$\frac{\tau - 0}{\rho - 0} = \frac{50(10^6)}{0.00625}$$

$$\tau = 8000\,(10^6)(\rho)$$

$$\frac{\tau - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau = 4000\,(10^6)(\rho) + 25(10^6)$$

$$T = 2\pi\int_0^c \tau\rho^2\,d\rho$$

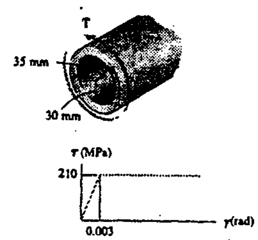
$$= 2\pi\int_0^{0.00625} 8000(10^6)\,\rho^3\,d\rho + 2\pi\int_{0.00625}^{0.025} [4000(10^6)\rho + 25(10^6)]\rho^2\,d\rho$$

$T = 3269\ \text{N}\cdot\text{m} = 3.27\ \text{kN}\cdot\text{m}$ **Ans**

$$\phi = \frac{\gamma_{max}}{c}L = \frac{0.01}{0.025}(3)$$

$= 1.20\ \text{rad} = 68.8^\circ$ **Ans**

5-129. The 2-m-long tube is made from an elastic-plastic material as shown. Determine the applied torque T, which subjects the material at the tube's outer edge to a shear strain of $\gamma_{max} = 0.006$ rad. What would be the permanent angle of twist of the tube when this torque is removed? Sketch the residual stress distribution in the tube.

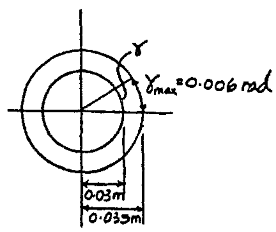


Plastic Torque : The tube is fully plastic if $\gamma_i \geq \gamma_Y = 0.003$ rad.

$$\frac{\gamma}{0.03} = \frac{0.006}{0.035}; \quad \gamma = 0.005143 \text{ rad}$$

Therefore the tube is fully plastic.

$$T_P = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2 \, d\rho$$
$$= \frac{2\pi \tau_Y}{3}(c_o^3 - c_i^3)$$
$$= \frac{2\pi(210)(10^6)}{3}(0.035^3 - 0.03^3)$$
$$= 6982.19 \text{ N}\cdot\text{m} = 6.98 \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$



Angle of Twist :

$$\phi_P = \frac{\gamma_{max}}{c_o}L = \left(\frac{0.006}{0.035}\right)(2) = 0.34286 \text{ rad}$$

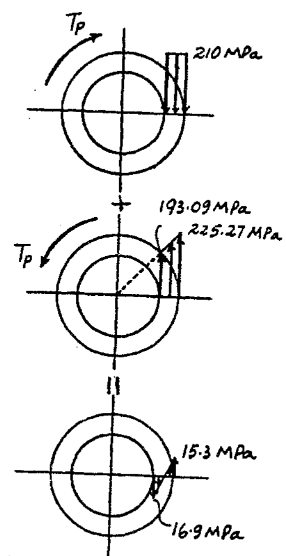
When a reverse torque of $T_P = 6982.19$ N · m is applied,

$$G = \frac{\tau_Y}{\gamma_Y} = \frac{210(10^6)}{0.003} = 70 \text{ GPa}$$

$$\phi_P' = \frac{T_P L}{JG} = \frac{6982.19(2)}{\frac{\pi}{2}(0.035^4 - 0.03^4)(70)(10^9)} = 0.18389 \text{ rad}$$

Permanent angle of twist,

$$\phi_r = \phi_P - \phi_P'$$
$$= 0.34286 - 0.18389 = 0.1590 \text{ rad} = 9.11° \qquad \textbf{Ans}$$



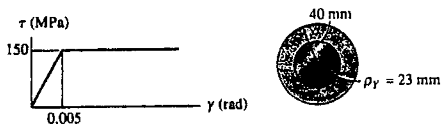
Residual Shear Stress :

$$\tau_{P_o}' = \frac{T_P c}{J} = \frac{6982.19(0.035)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 225.27 \text{ MPa}$$

$$\tau_{P_i}' = \frac{T_P \rho}{J} = \frac{6982.19(0.03)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 193.09 \text{ MPa}$$

$$(\tau_r)_o = -\tau_Y + \tau_{P_o}' = -210 + 225.27 = 15.3 \text{ MPa}$$
$$(\tau_r)_i = -\tau_Y + \tau_{P_i}' = -210 + 193.09 = -16.9 \text{ Mpa}$$

5–130 The solid shaft is made from an elastic-plastic material as shown. Determine the torque T needed to form an elastic core in the shaft having a radius of $\rho_Y = 23$ mm. If the shaft is 2 m long, through what angle does one end of the shaft twist with respect to the other end? When the torque is removed, determine the residual stress distribution in the shaft and the permanent angle of twist.



Use Eq. 5 – 26 of the text.

$$T = \frac{\pi\,\tau_Y}{6}(4c^3 - \rho_Y^3) = \frac{\pi(150)(10^6)}{6}(4(0.04^3) - 0.023^3)$$

$$= 19\,151\text{ N}\cdot\text{m} = 19.2\text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y L}{\rho_Y} = \frac{0.005(2)(1000)}{23} = 0.4348\text{ rad} = 24.9^\circ \qquad \textbf{Ans}$$

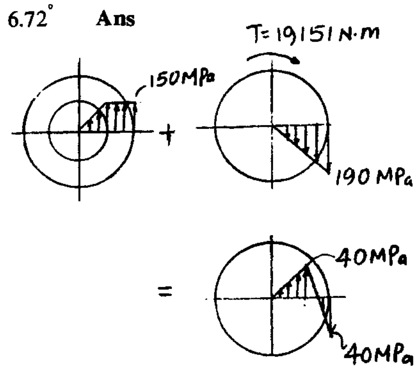
An opposite torque $T = 19\,151$ N · m is applied :

$$\tau_r = \frac{TC}{J} = \frac{19\,151(0.04)}{\frac{\pi}{2}(0.04^4)} = 190\text{ MPa}$$

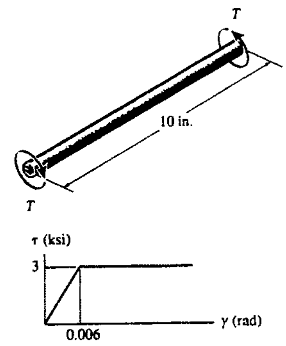
$$G = \frac{150(10^6)}{0.005} = 30\text{ GPa}$$

$$\phi_p' = \frac{TL}{JG} = \frac{19151(2)}{\frac{\pi}{2}(0.04^4)(30)(10^9)} = 0.3175\text{ rad}$$

$$\phi_r = 0.4348 - 0.3175 = 0.117\text{ rad} = 6.72^\circ \qquad \textbf{Ans}$$



5–131 A 1.5-in.-diameter shaft is made from an elastic-plastic material as shown. Determine the radius of its elastic core if it is subjected to a torque of $T = 200$ lb · ft. If the shaft is 10 in. long, determine the angle of twist.



Use Eq. 5 – 26 from the text :

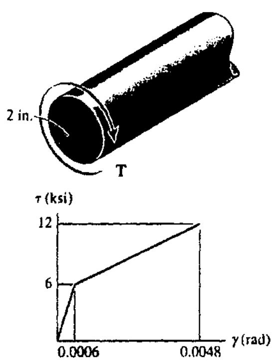
$$T = \frac{\pi\, \tau_Y}{6}(4\,c^3 - \rho_Y^3)$$

$$200(12) = \frac{\pi\,(3)(10^3)}{6}[4(0.75^3) - \rho_Y^3]$$

$\rho_Y = 0.542$ in. **Ans**

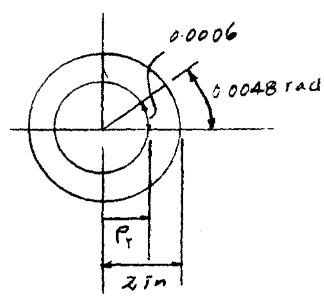
$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{0.006}{0.542}(10) = 0.111 \text{ rad} = 6.34° \qquad \textbf{Ans}$$

*5–132 The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain-hardening as shown by the shear stress–strain diagram.



From the shear – strain diagram,

$$\frac{\rho_Y}{0.0006} = \frac{2}{0.0048}; \qquad \rho_Y = 0.25 \text{ in.}$$



From the shear stress - strain diagram,

$$\tau_1 = \frac{6}{0.25}\rho = 24\rho$$

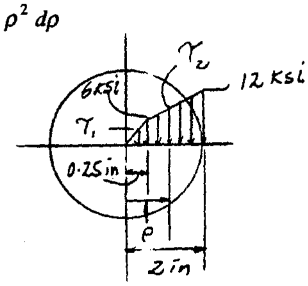
$$\frac{\tau_2 - 6}{\rho - 0.25} = \frac{12 - 6}{2 - 0.25}; \qquad \tau_2 = 3.4286\,\rho + 5.1429$$

$$T = 2\pi \int_0^c \tau \rho^2 \, d\rho$$

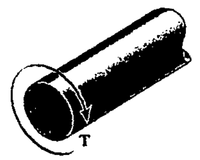
$$= 2\pi \int_0^{0.25} 24\rho^3 d\rho + 2\pi \int_{0.25}^{2} (3.4286\rho + 5.1429)\, \rho^2 \, d\rho$$

$$= 2\pi \left[6\rho^4\right] \Big|_0^{0.25} + 2\pi \left[\frac{3.4286\rho^4}{4} + \frac{5.1429\rho^3}{3}\right] \Big|_{0.25}^{2}$$

$$= 172.30 \text{ kip} \cdot \text{in.} = 14.4 \text{ kip} \cdot \text{ft}$$ **Ans**



5–133 A torque is applied to the shaft of radius r. If the material has a shear stress–strain relation of $\tau = k\gamma^{1/6}$, where k is a constant, determine the maximum shear stress in the shaft.



$$\gamma = \frac{\rho}{c}\gamma_{max} = \frac{\rho}{r}\gamma_{max}$$

$$\tau = k\gamma^{\frac{1}{6}} = k\left(\frac{\gamma_{max}}{r}\right)^{\frac{1}{6}}\rho^{\frac{1}{6}}$$

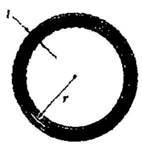
$$T = 2\pi\int_0^r \tau\rho^2\, d\rho$$

$$= 2\pi\int_0^r k\left(\frac{\gamma_{max}}{r}\right)^{\frac{1}{6}}\rho^{\frac{13}{6}}\, d\rho = 2\pi k\left(\frac{\gamma_{max}}{r}\right)^{\frac{1}{6}}\left(\frac{6}{19}\right) r^{\frac{19}{6}} = \frac{12\pi k\gamma_{max}^{\frac{1}{6}} r^3}{19}$$

$$\gamma_{max} = \left(\frac{19T}{12\pi k r^3}\right)^6$$

$$\tau_{max} = k\gamma_{max}^{\frac{1}{6}} = \frac{19T}{12\pi r^3} \qquad \textbf{Ans}$$

5–134 Consider a thin-walled tube of mean radius r and thickness t. Show that the maximum shear stress in the tube due to an applied torque T approaches the average shear stress computed from Eq. 5–18 as $r/t \rightarrow \infty$.



$$r_o = r + \frac{t}{2} = \frac{2r + t}{2}; \qquad r_i = r - \frac{t}{2} = \frac{2r - t}{2}$$

$$J = \frac{\pi}{2}\left[\left(\frac{2r + t}{2}\right)^4 - \left(\frac{2r - t}{2}\right)^4\right]$$

$$= \frac{\pi}{32}[(2r + t)^4 - (2r - t)^4] = \frac{\pi}{32}[64\, r^3 t + 16\, r\, t^3]$$

$$\tau_{max} = \frac{Tc}{J}; \qquad c = r_o = \frac{2\, r + t}{2}$$

$$= \frac{T\left(\frac{2\, r + t}{2}\right)}{\frac{\pi}{32}[64\, r^3 t + 16\, r\, t^3]} = \frac{T\left(\frac{2\, r + t}{2}\right)}{2\pi\, r\, t[r^2 + \frac{1}{4}t^2]}$$

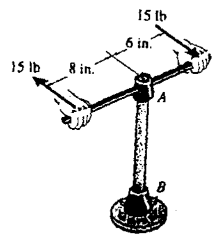
$$= \frac{T\left(\frac{2r}{2r^2} + \frac{t}{2r^2}\right)}{2\pi\, r\, t\left[\frac{r^2}{r^2} + \frac{1}{4}\frac{t^2}{r^2}\right]}$$

As $\frac{r}{t} \rightarrow \infty$, then $\frac{t}{r} \rightarrow 0$

$$\tau_{max} = \frac{T\left(\frac{1}{r} + 0\right)}{2\pi\, r\, t\,(1 + 0)} = \frac{T}{2\pi\, r^2 t}$$

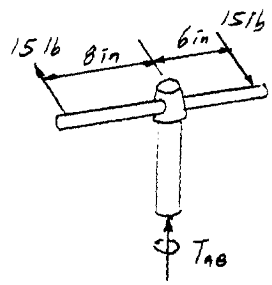
$$= \frac{T}{2\, t A_m}$$ **QED**

5–135 The pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in. If it is tightly secured to the flange, determine the maximum shear stress developed in the pipe when the couple shown is applied to the handles of the wrench.



$T - 15(6) - 15(8) = 0$

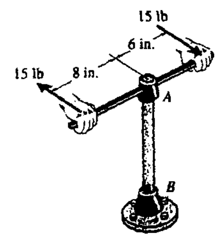
$T = 210 \text{ lb} \cdot \text{in.}$



$$\tau_{max} = \frac{Tc}{J} = \frac{210(0.375)}{\frac{\pi}{2}(0.375^4 - 0.34^4)}$$

$= 7.82 \text{ ksi}$ **Ans**

•5–136 The pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in. If it is tightly secured to the flange at B, determine the shear-stress distribution acting along a radial line lying on the midsection of the pipe when the couple shown is applied to the handles of the wrench.

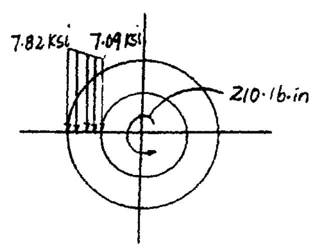


See Prob. 5 - 135.

$\tau_{max} = 7.82$ ksi

At $\rho = 0.34$ in.

$$\tau = \frac{T\rho}{J} = \frac{210(0.34)}{\frac{\pi}{2}((0.375)^4 - (0.34)^4)} = 7.09 \text{ ksi} \qquad \textbf{Ans}$$



5–137 The drilling pipe on an oil rig is made from steel pipe having an outside diameter of 4.5 in. and a thickness of 0.25 in. If the pipe is turning at 650 rev/min while being powered by a 15-hp motor, determine the maximum shear stress in the pipe.

$$\omega = \frac{650(2\pi)}{60} = 68.068 \text{ rad/s}$$

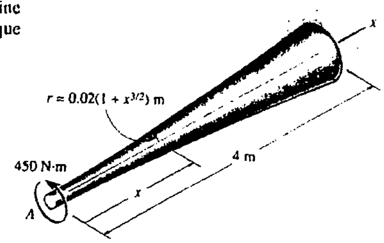
$$P = T\omega$$

$$15(550) = T(68.068)$$

$$T = 121.20 \text{ lb} \cdot \text{ft}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{121.20(12)(2.25)}{\frac{\pi}{2}(2.25^4 - 2^4)} = 216 \text{ psi} \qquad \textbf{Ans}$$

5-138 The tapered shaft is made from 2014-T6 aluminum alloy, and has a radius which can be described by the function $r = 0.02(1 + x^{3/2})$ m, where x is in meters. Determine the angle of twist of its end A if it is subjected to a torque of 450 N · m.



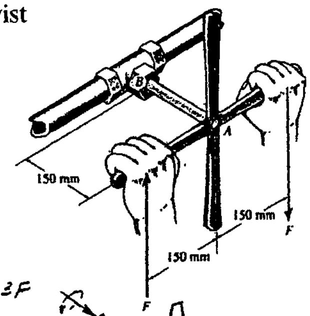
$T = 450 \text{ N} \cdot \text{m}$

$$\phi_A = \int \frac{T dx}{JG} = \int_0^4 \frac{450\, dx}{\frac{\pi}{2}(0.02)^4(1 + x^{\frac{3}{2}})^4 (27)(10^9)} = 0.066315 \int_0^4 \frac{dx}{(1 + x^{\frac{3}{2}})^4}$$

Evaluating the integral using Simpson's rule, we have

$$\phi_A = 0.066315[0.4179] \text{ rad}$$
$$= 0.0277 \text{ rad} = 1.59^\circ \qquad \textbf{Ans}$$

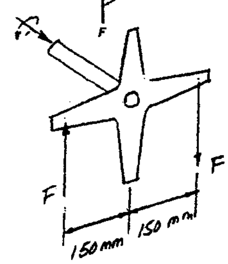
5-139. If the solid shaft AB to which the valve handle is attached is made of C83400 red brass and has a diameter of 10 mm, determine the maximum couple forces F that can be applied to the handle just before the material starts to fail. Take $\tau_{allow} = 40$ MPa. What is the angle of twist of the handle? The shaft is fixed at A.



$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$40(10^6) = \frac{0.3F(0.005)}{\frac{\pi}{2}(0.005)^4}$$

$T = 0.3F$



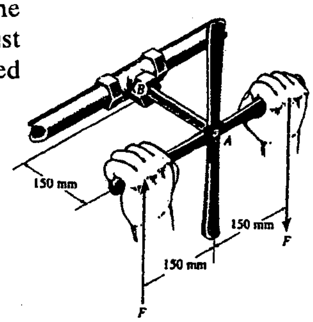
$F = 26.18\text{ N} = 26.2\text{ N}$ **Ans**

$T = 0.3F = 7.85\text{ N}\cdot\text{m}$

$$\phi = \frac{TL}{JG} = \frac{7.85(0.15)}{\frac{\pi}{2}(0.005)^4(37)(10^9)}$$

$= 0.03243\text{ rad} = 1.86°$ **Ans**

***5-140.** If the solid shaft AB to which the valve handle is attached is made of C83400 red brass, determine the smallest diameter of the handle so that the angle of twist does not exceed 0.5° and the shear stress does not exceed 40 MPa when $F = 25$ N.



Failure by shear :

$$T = 25(0.3) = 7.5\ \text{N}\cdot\text{m}$$

$$\tau_{allow} = \frac{Tc}{J}; \quad 40(10^6) = \frac{7.5c}{\frac{\pi}{2}c^4}$$

$$c = 4.92\ \text{mm}$$
$$d = 9.85\ \text{mm}$$

Failure by twist :

$$\phi = \frac{TL}{JG}; \quad 0.5\left(\frac{\pi}{180°}\right) = \frac{7.5(0.15)}{\frac{\pi}{2}(c)^4(37)(10^9)}$$

$$c = 6.86\ \text{mm}$$
$$d = 13.7\ \text{mm (controls)} \qquad \textbf{Ans}$$

5–141 The material of which each of three shafts is made has a yield stress of τ_Y and a shear modulus of G. Determine which shaft geometry will resist the largest torque without yielding. What percentage of this torque can be carried by the other two shafts? Assume that each shaft is made of the same amount of material and that it has the same cross-sectional area A.

For circular shaft :

$A = \pi c^2; \qquad c = \left(\frac{A}{\pi}\right)^{\frac{1}{2}}$

$\tau_{max} = \frac{Tc}{J}; \qquad \tau_Y = \frac{Tc}{\frac{\pi}{2}c^4}$

$T = \frac{\pi c^3}{2}\tau_Y = \frac{\pi\left(\frac{A}{\pi}\right)^{\frac{3}{2}}}{2}\tau_Y$

$T_{cir} = 0.282\,A^{\frac{3}{2}}\tau_Y$ (controls) **Ans**

For the square shaft :

$A = a^2; \qquad a = A^{\frac{1}{2}}$

$\tau_{max} = \frac{4.81T}{a^3}; \qquad \tau_Y = \frac{4.81T}{A^{\frac{3}{2}}}$

$T = 0.2079\,A^{\frac{3}{2}}\tau_Y$

For the triangular shaft :

$A = \frac{1}{2}(a)(a\sin 60^\circ); \qquad a = 1.5197A^{\frac{1}{2}}$

$\tau_{max} = \frac{20T}{a^3}; \qquad \tau_Y = \frac{20T}{(1.5197)^3A^{\frac{3}{2}}}$

$T = 0.1755A^{\frac{3}{2}}\tau_Y$

The circular shaft will carry the largest torque. **Ans.**

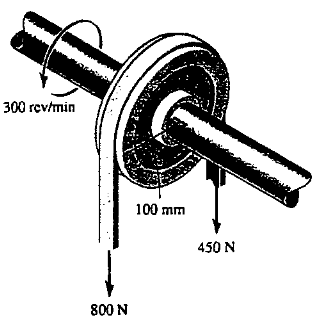
For the square shaft

$\% = \frac{0.2079}{0.2821}(100\%) = 73.7\,\%$ **Ans**

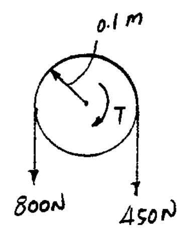
For the triangular shaft ,

$\% = \frac{0.1755}{0.2821}(100\%) = 62.2\,\%$ **Ans**

5–142 The 60-mm-diameter shaft rotates at 300 rev/min. This motion is caused by the unequal belt tensions on the pulley of 800 N and 450 N. Determine the power transmitted and the maximum shear stress developed in the shaft.



$$\omega = 300\,\frac{\text{rev}}{\text{min}}\left[\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right]\frac{1\ \text{min}}{60\ \text{s}} = 10\,\pi\ \text{rad/s}$$



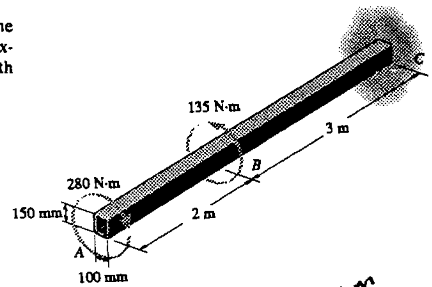
$$T + 450(0.1) - 800(0.1) = 0$$

$$T = 35.0\ \text{N}\cdot\text{m}$$

$$P = T\omega = 35.0(10\pi) = 1100\ \text{W} = 1.10\ \text{kW} \qquad \textbf{Ans}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{35.0(0.03)}{\frac{\pi}{2}(0.03^4)} = 825\ \text{kPa} \qquad \textbf{Ans}$$

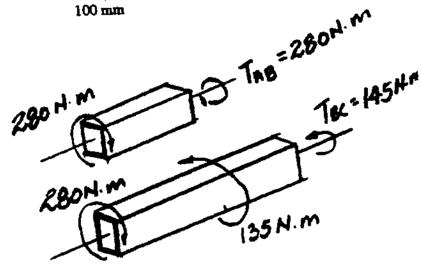
5–143 The aluminum tube has a thickness of 5 mm and the outer cross-sectional dimensions shown. Determine the maximum average shear stress in the tube. If the tube has a length of 5 m, determine the angle of twist. G_{al} = 28 GPa.



$$A_m = (0.145)(0.095) = 0.013775\ \text{m}^2$$

$$(\tau_{avg})_{max} = \frac{T_{AB}}{2A_m t} = \frac{280}{2(0.013775)(0.005)}$$

$$= 2.03\ \text{MPa} \qquad \textbf{Ans}$$

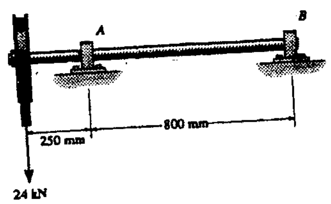


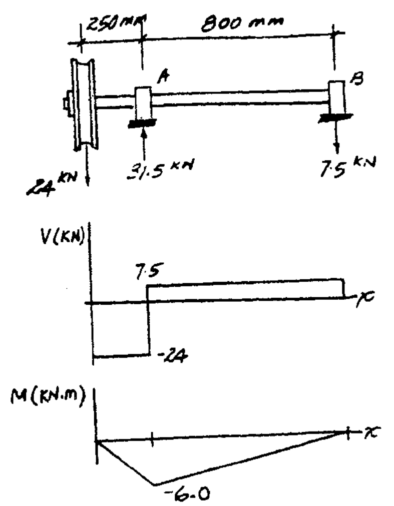
$$\phi = \frac{TL}{4A_m^2 G}\int \frac{ds}{t}$$

$$\int \frac{ds}{t} = \frac{2(0.145)+2(0.095)}{0.005} = 96$$

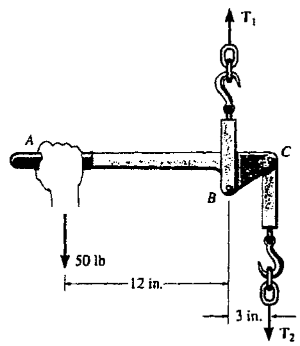
$$\phi = \frac{96}{4(0.013775)^2(28)(10^9)}[280(2)+145(3)] = 0.00449\ \text{rad} = 0.258^\circ \qquad \textbf{Ans}$$

6–1. Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft.





6–2 The load binder is used to support a load. If the force applied to the handle is 50 lb, determine the tensions T_1 and T_2 in each end of the chain and then draw the shear and moment diagrams for the arm ABC.

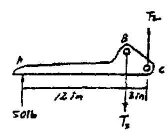


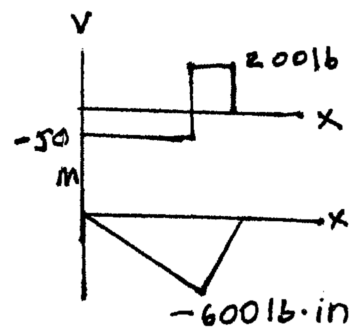
$$\circlearrowleft + \Sigma M_C = 0; \qquad -50(15) + T_1(3) = 0$$

$$T_1 = 250 \text{ lb} \qquad \textbf{Ans}$$

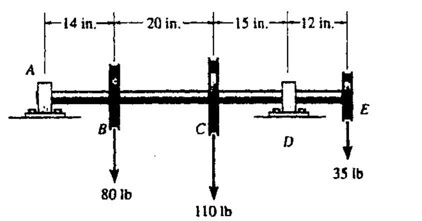
$$+\uparrow \Sigma F_y = 0; \qquad 50 - 250 + T_2 = 0$$

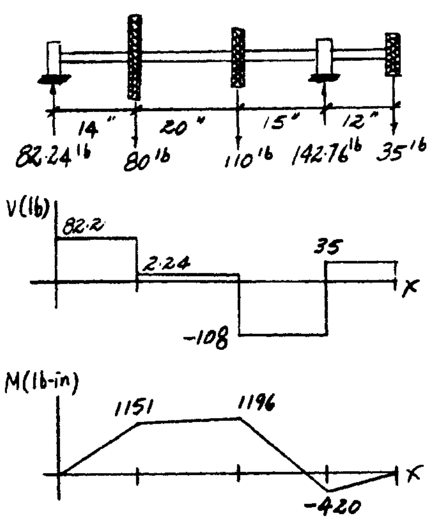
$$T_2 = 200 \text{ lb} \qquad \textbf{Ans}$$



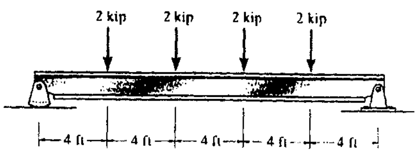


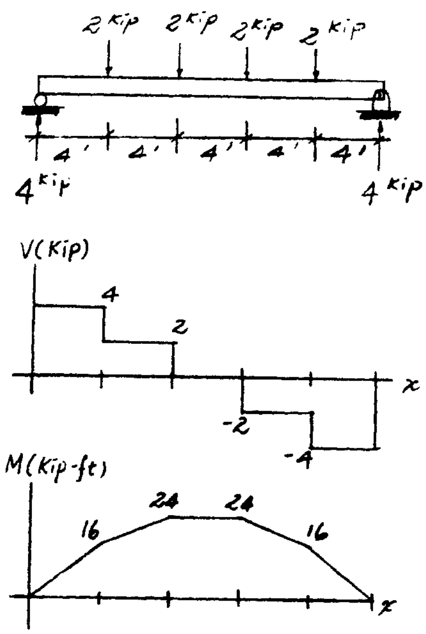
6–3 Draw the shear and moment diagrams for the shaft. The bearings at A and D exert only vertical reactions on the shaft. The loading is applied to the pulleys at B and C and E.



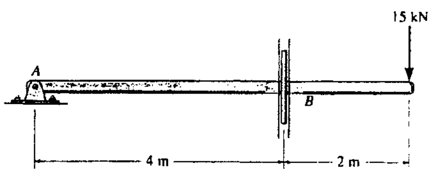


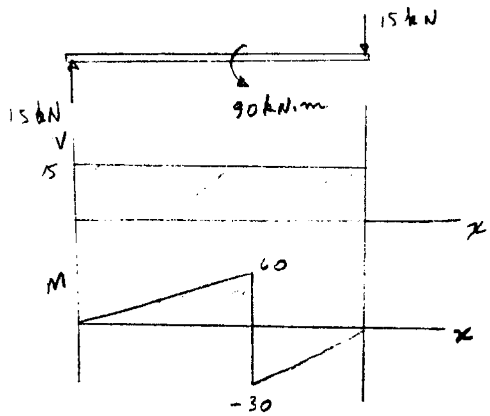
*6–4 Draw the shear and moment diagrams for the beam.



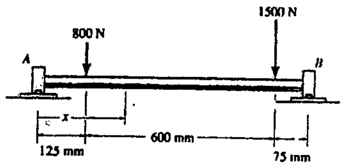


6–5 Draw the shear and moment diagrams for the rod. It is supported by a pin at A and a smooth plate at B. The plate slides within the groove and so it cannot support a vertical force, although it can support a moment.





6–6. Draw the shear and moment diagrams for the shaft. The bearings at A and B exert only vertical reactions on the shaft. Also, express the shear and moment in the shaft as a function of x within the region 125 mm $< x <$ 725 mm.

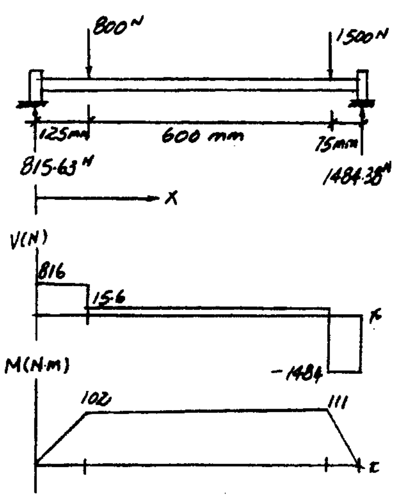


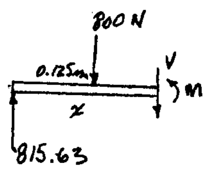
$+\uparrow \Sigma F_y = 0;\quad 815.63 - 800 - V = 0$

$V = 15.6\ \text{N}$ **Ans**

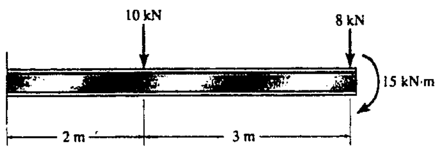
$\circlearrowleft + \Sigma M = 0;\quad M + 800(x - 0.125) - 815.63\,x = 0$

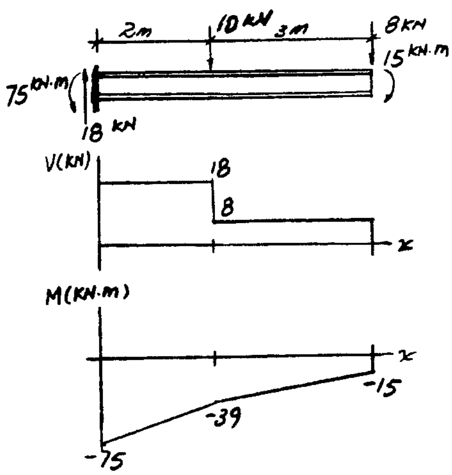
$M = (15.6x + 100)\ \text{N}\cdot\text{m}$ **Ans**



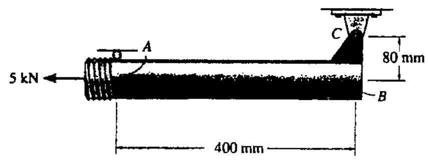


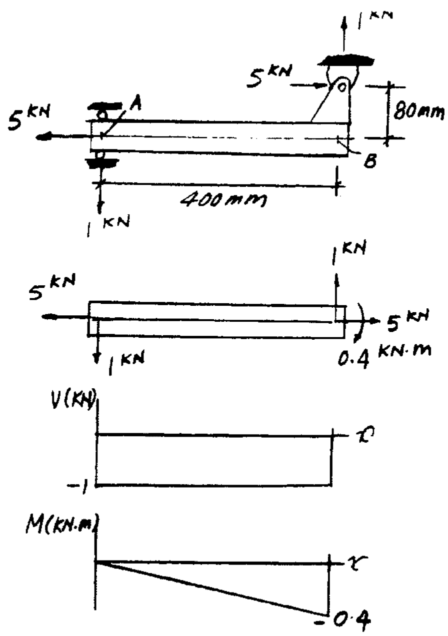
6–7 Draw the shear and moment diagrams for the beam.



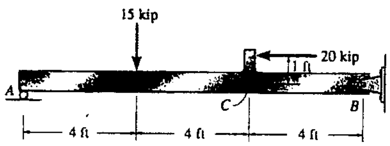


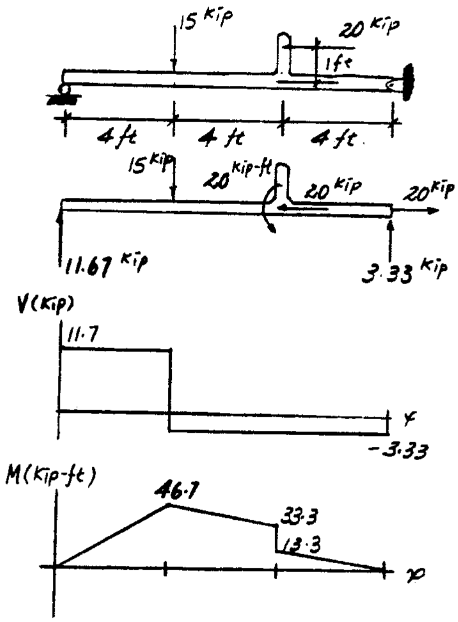
•6–8 Draw the shear and moment diagrams for the pipe. The end screw is subjected to a horizontal force of 5 kN. *Hint:* The reactions at the pin *C* must be replaced by equivalent loadings at point *B* on the axis of the pipe.



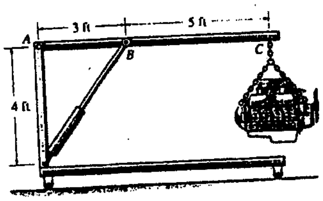


6–9 Draw the shear and moment diagrams for the beam. *Hint:* The 20-kip load must be replaced by equivalent loadings at point *C* on the axis of the beam.





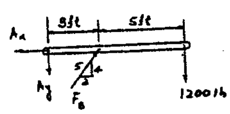
6-10. The engine crane is used to support the engine, which has a weight of 1200 lb. Draw the shear and moment diagrams of the boom ABC when it is in the horizont position shown.

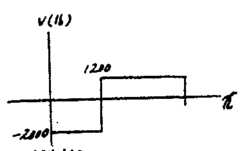


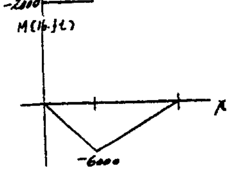
$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5}F_B(3) - 1200(8) = 0; \quad F_B = 4000\text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(4000) - 1200 = 0; \quad A_y = 2000\text{ lb}$

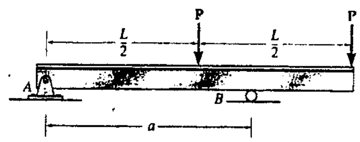
$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x - \frac{3}{5}(4000) = 0; \quad A_x = 2400\text{ lb}$

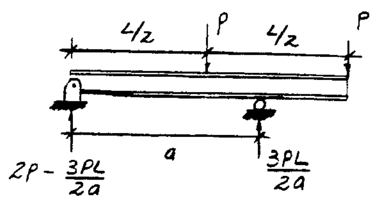






6–11 Determine the placement distance a of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.



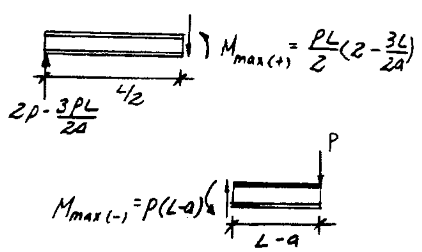


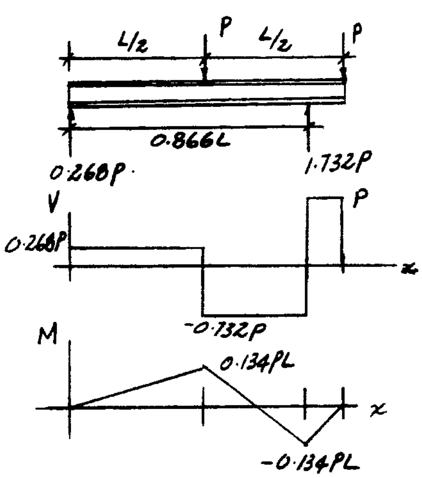
To get absolute minimum moment;

$$M_{max\,(+)} = M_{max\,(-)}$$

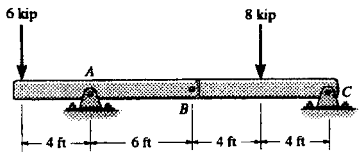
$$\frac{PL}{2}(2 - \frac{3L}{2a}) = P(L - a)$$

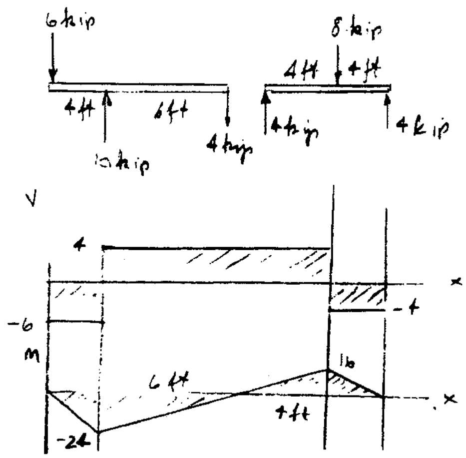
$$a = \frac{\sqrt{3}}{2}L = 0.866\,L \qquad \textbf{Ans}$$



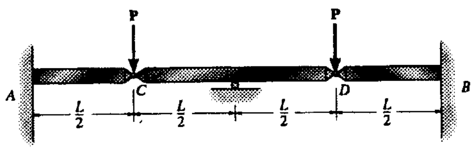


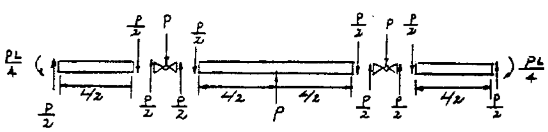
***6–12** Draw the shear and moment diagrams for the compound beam which is pin connected at *B*.

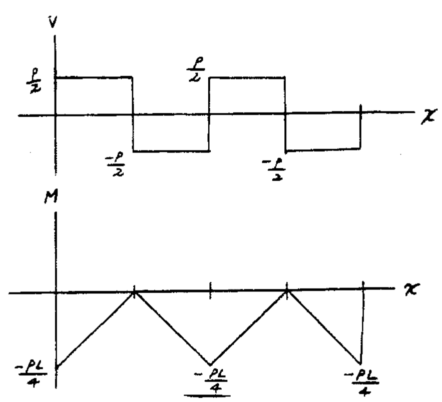




6–13 The bars are connected by pins at *C* and *D*. Draw the shear and moment diagrams for the assembly. Neglect the effect of axial load.

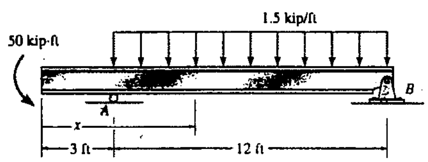


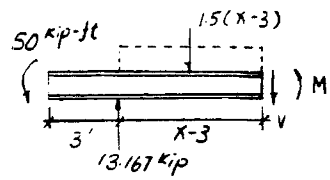




Because of the number and variety of potential correct solutions to this problem, no solution is being given.

6–15 Draw the shear and moment diagrams for the beam. Also, determine the shear and moment in the beam as functions of x, where $3 \text{ ft} < x \leq 15 \text{ ft}$.





$+\uparrow \Sigma F_y = 0; \quad -V - 1.5(x-3) + 13.167 = 0$

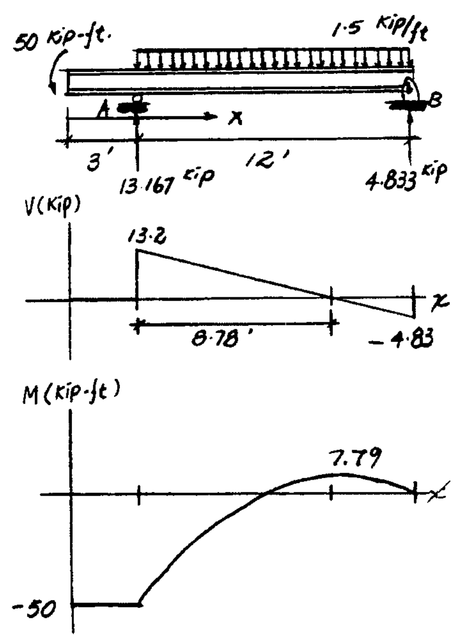
$$V = 17.7 - 1.5x \qquad \textbf{Ans}$$

$\curvearrowleft + \Sigma M = 0; \quad M + 1.5(x-3)\dfrac{(x-3)}{2} + 50 - 13.167(x-3) = 0$

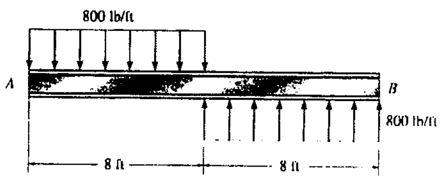
$$M = -0.75x^2 + 17.7x - 96.25 \qquad \textbf{Ans}$$

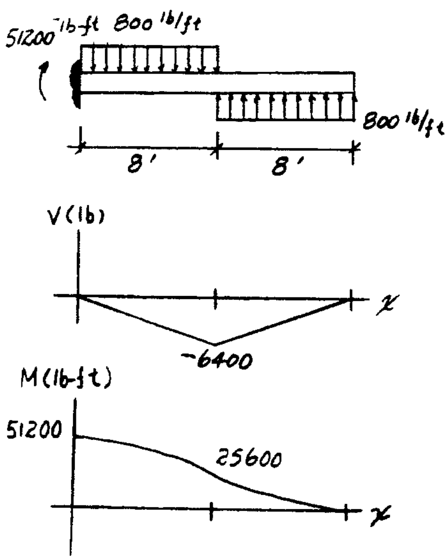
$V = 0$ at $x = \dfrac{17.7}{1.5} = 11.778$ ft

$M_{max} = -0.75(11.778)^2 + 17.7(11.778) - 96.25 = 7.79$ ft **Ans**

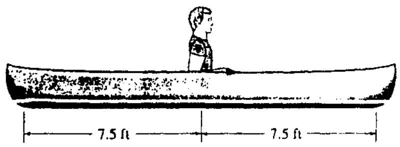


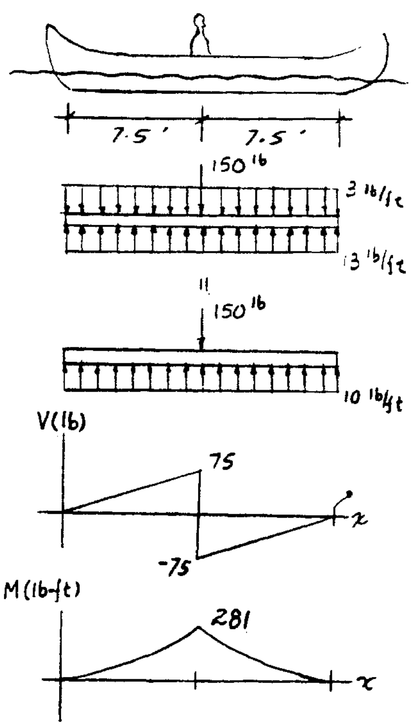
*6–16 Draw the shear and moment diagrams for the beam.





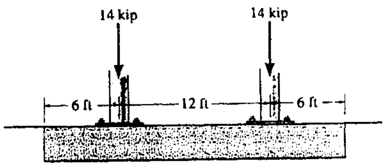
6–17 The 150-lb man sits in the center of the boat, which has a uniform width and a weight per linear foot of 3 lb/ft. Determine the maximum bending moment exerted on the boat. Assume that the water exerts a uniform distributed load upward on the bottom of the boat.

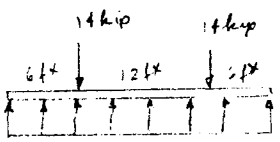




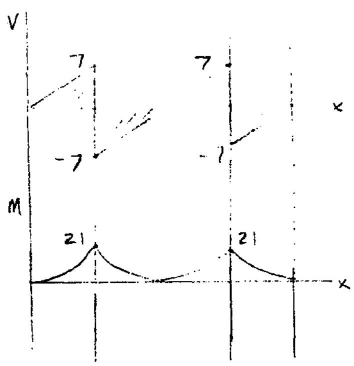
$M_{max} = 281 \text{ lb} \cdot \text{ft}$ **Ans**

6–18 The footing supports the load transmitted by the two columns. Draw the shear and moment diagrams for the footing if the reaction of soil pressure on the footing is assumed to be uniform.

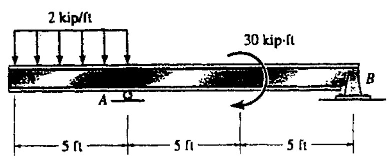


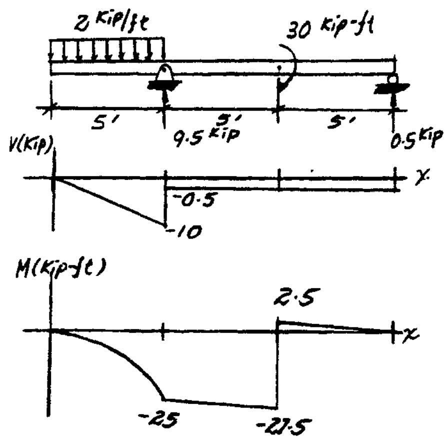


$w = \frac{28}{24} = 1.167$ kip/ft

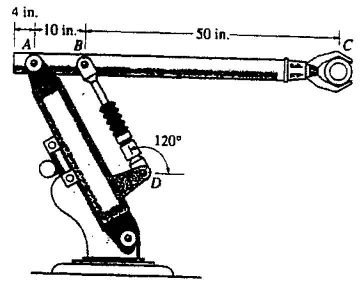


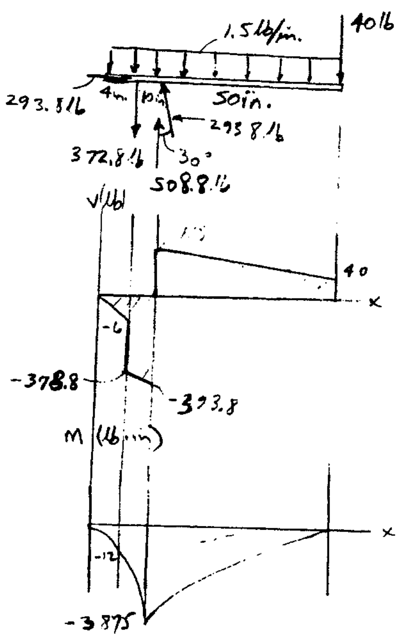
6–19 Draw the shear and moment diagrams for the beam.



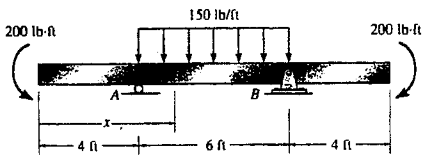


***6-20.** The industrial robot is held in the stationary position shown. Draw the shear and moment diagrams of the arm *ABC* if it is pin connected at *A* and connected to a hydraulic cylinder (two-force member) *BD*. Assume the arm and grip have a uniform weight of 1.5 lb/in. and support the load of 40 lb at *C*.





6–21 Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where 4 ft < x < 10 ft.

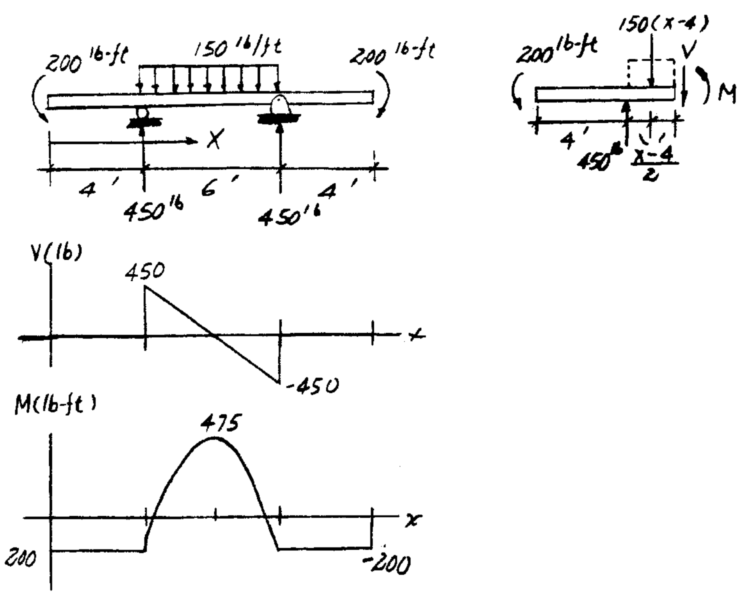


$+\uparrow \Sigma F_y = 0;\qquad -150(x-4) - V + 450 = 0$

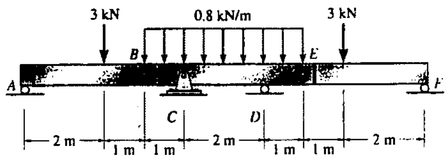
$V = 1050 - 150x$ **Ans**

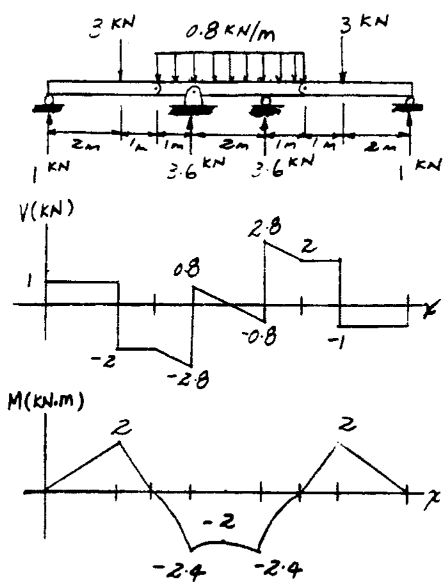
$\circlearrowleft + \ \Sigma M = 0;\qquad -200 - 150(x-4)\frac{(x-4)}{2} - M + 450(x-4) = 0$

$M = -75x^2 + 1050x - 3200$ **Ans**

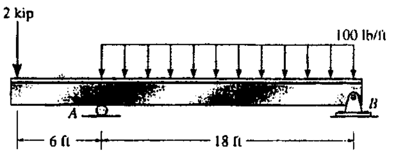


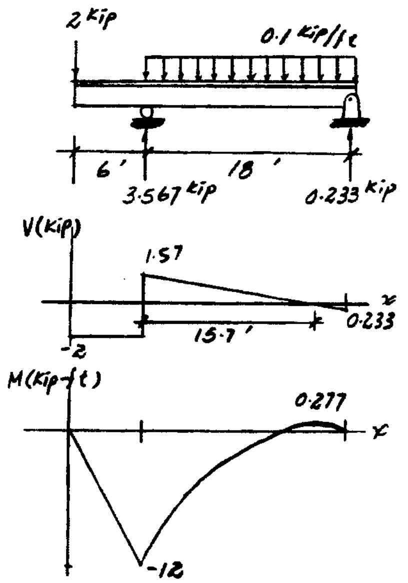
6–22 Draw the shear and moment diagrams for the compound beam. The three segments are connected by pins at *B* and *E*.



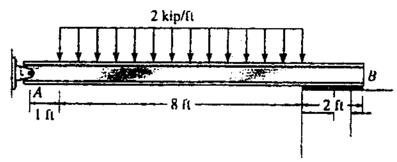


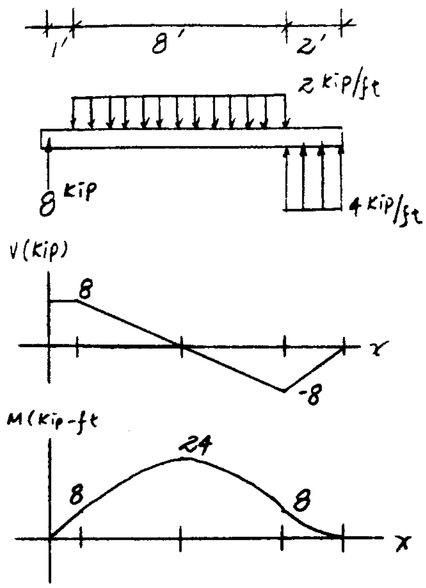
6–23 The T-beam is subjected to the loading shown. Draw the shear and moment diagrams for the beam.



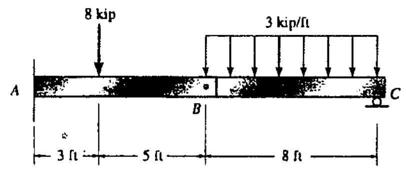


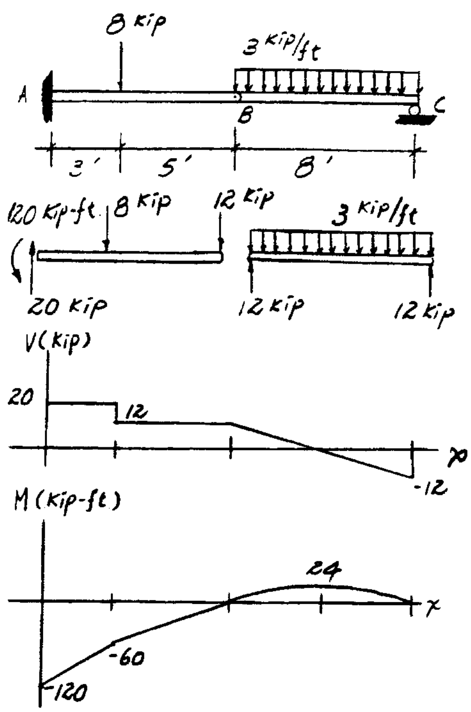
*6–24 The beam is bolted or pinned at A and rests on a bearing pad at B that exerts a uniform distributed loading on the beam over its 2-ft length. Draw the shear and moment diagrams for the beam if it supports a uniform loading of 2 kip/ft.





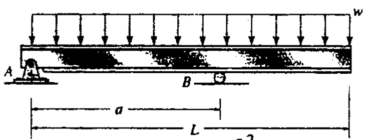
6–25 Draw the shear and moment diagrams for the beam. The two segments are joined together at *B*.

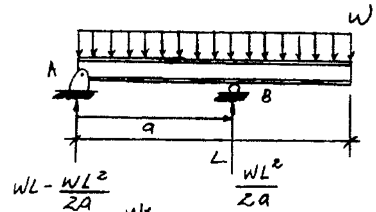




Because of the number and variety of potential correct solutions to this problem, no solution is being given.

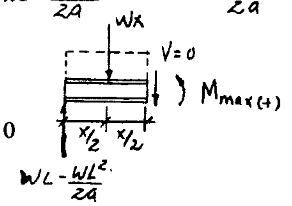
6–27 Determine the placement distance *a* of the roller support so that the largest absolute value of the moment is a minimum. Draw the shear and moment diagrams for this condition.





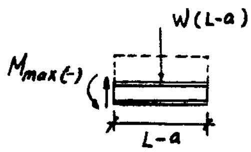
$+\uparrow \Sigma F_y = 0; \qquad wL - \dfrac{wL^2}{2a} - wx = 0$

$$x = L - \frac{L^2}{2a}$$



$\circlearrowleft + \Sigma M = 0; \qquad M_{\max\,(+)} + wx\left(\dfrac{x}{2}\right) - \left(wL - \dfrac{wL^2}{2a}\right)x = 0$

Substitute $x = L - \dfrac{L^2}{2a}$;

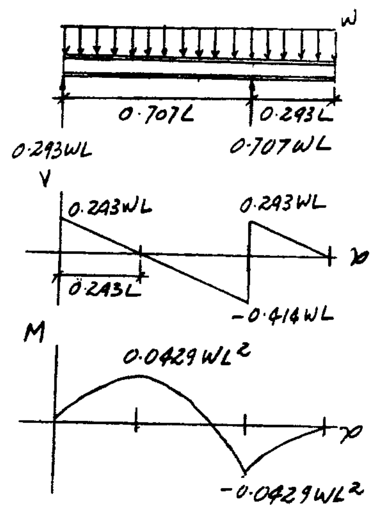


$$M_{\max\,(+)} = \left(wL - \frac{wL^2}{2a}\right)\left(L - \frac{L^2}{2a}\right) - \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

$$= \frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2$$

$\Sigma M = 0; \qquad M_{\max\,(-)} - w(L-a)\dfrac{(L-a)}{2} = 0$

$$M_{\max\,(-)} = \frac{w(L-a)^2}{2}$$



To get absolute minimum moment,

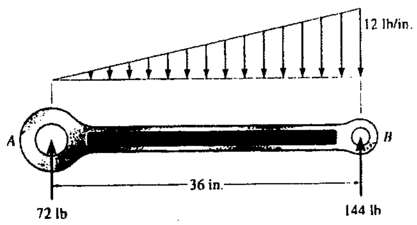
$$M_{\max\,(+)} = M_{\max\,(-)}$$

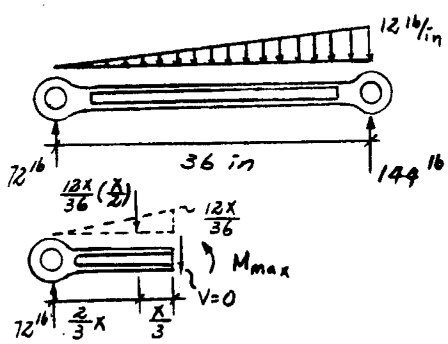
$$\frac{w}{2}\left(L - \frac{L^2}{2a}\right)^2 = \frac{w}{2}(L-a)^2$$

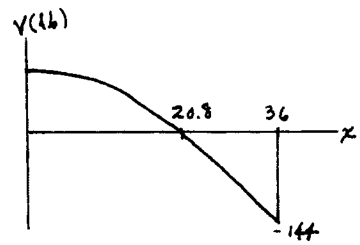
$$L - \frac{L^2}{2a} = L - a$$

$$a = \frac{L}{\sqrt{2}} \qquad \textbf{Ans}$$

*6–28 Draw the shear and moment diagrams for the connecting rod. Only vertical reactions occur at its ends A and B.





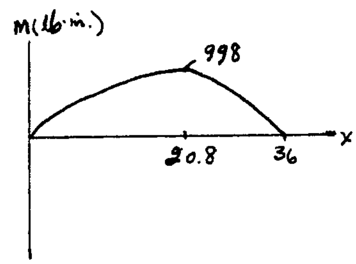


$$+\uparrow \Sigma F_y = 0; \qquad 72 - \frac{12x}{36}\left(\frac{x}{2}\right) = 0$$

$$x = 20.784 \text{ in.}$$

$$\curvearrowleft + \Sigma M = 0; \qquad M_{max} + \frac{12x}{36}\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) - 72x = 0$$

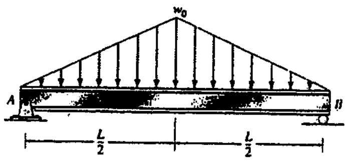
$$M_{max} = -\frac{x^3}{18} + 72x$$

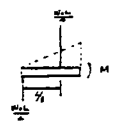


Substitute $x = 20.784$ in.,

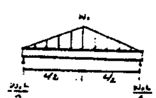
$$M_{max} = 997.66 \text{ lb} \cdot \text{in.}$$

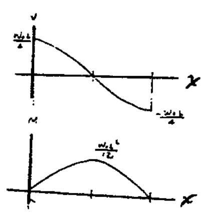
6-29. Draw the shear and moment diagrams for the beam.



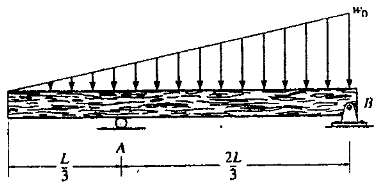


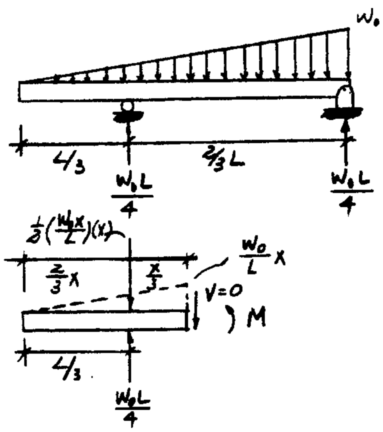
$$\circlearrowleft + \Sigma M = 0; \quad M - \frac{w_0 L}{4}\left(\frac{L}{3}\right) = 0; \quad M = \frac{w_0 L^2}{12}$$

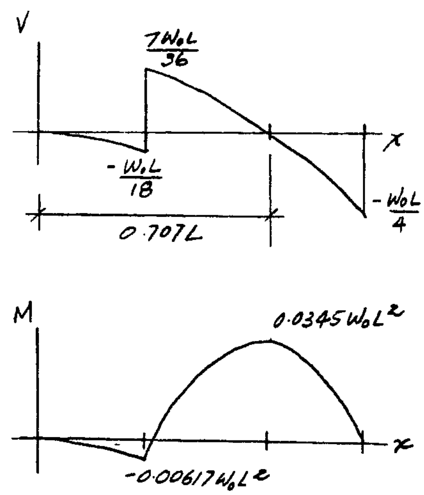




6–30 Draw the shear and moment diagrams for the beam.







$+\uparrow \Sigma F_y = 0; \quad \frac{w_0 L}{4} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x) = 0$

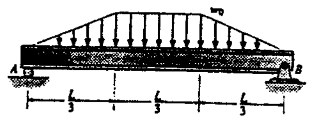
$x = 0.7071\,L$

$\circlearrowleft + \Sigma M_{NA} = 0; \quad M + \frac{1}{2}\left(\frac{w_0 x}{L}\right)(x)\left(\frac{x}{3}\right) - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right) = 0$

Substitute $x = 0.7071L$,

$M = 0.0345\, w_0 L^2$

6-31. Draw the shear and moment diagrams for the beam.

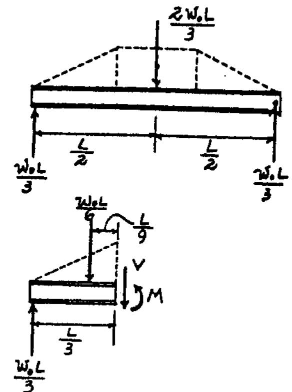


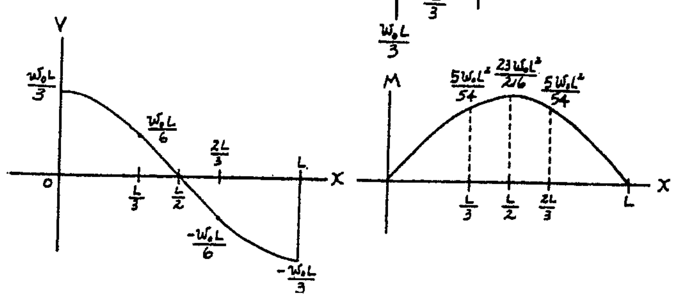
Support Reactions : As shown on FBD.
Shear and Moment Diagram : Shear and moment at $x = L/3$ can be determined using the method of sections.

$$+\uparrow \Sigma F_y = 0; \quad \frac{w_0 L}{3} - \frac{w_0 L}{6} - V = 0 \quad V = \frac{w_0 L}{6}$$

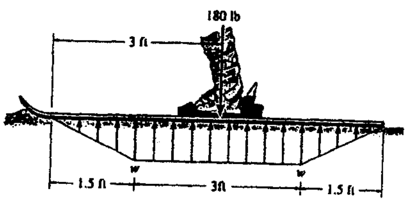
$$\curvearrowleft + \Sigma M_{NA} = 0; \quad M + \frac{w_0 L}{6}\left(\frac{L}{9}\right) - \frac{w_0 L}{3}\left(\frac{L}{3}\right) = 0$$

$$M = \frac{5 w_0 L^2}{54}$$

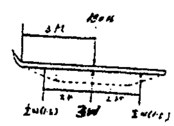




*6-32. The ski supports the 180-lb weight of the man. If the snow loading on its bottom surface is trapezoidal as shown, determine the intensity w, and then draw the shear and moment diagrams for the ski.



Ski:



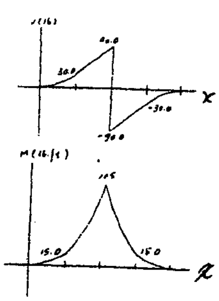
$$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2}w(1.5) + 3w + \frac{1}{2}w(1.5) - 180 = 0$$

$$w = 40.0 \text{ lb/ft} \qquad \textbf{Ans}$$

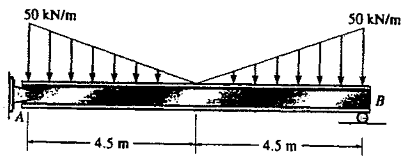
Segment:

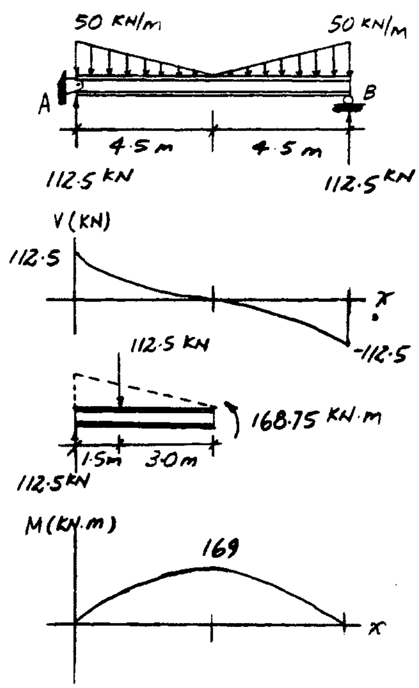
$$+\uparrow \Sigma F_y = 0; \quad 30 - V = 0; \quad V = 30.0 \text{ lb}$$

$$\circlearrowleft + \Sigma M = 0; \quad M - 30(0.5) = 0; \quad M = 15.0 \text{ lb}\cdot\text{ft}$$

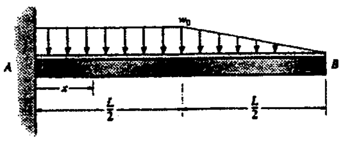


6–33 Draw the shear and moment diagrams for the beam.





6-34. Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x.



Support Reactions : As shown on FBD.
Shear and Moment Functions :

For $0 \le x < L/2$

$$+\uparrow \Sigma F_y = 0; \quad \frac{3w_0L}{4} - w_0x - V = 0$$

$$V = \frac{w_0}{4}(3L - 4x) \qquad \text{Ans}$$

$$\curvearrowleft + \Sigma M_{NA} = 0; \quad \frac{7w_0L^2}{24} - \frac{3w_0L}{4}x + w_0x\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w_0}{24}\left(-12x^2 + 18Lx - 7L^2\right) \qquad \text{Ans}$$

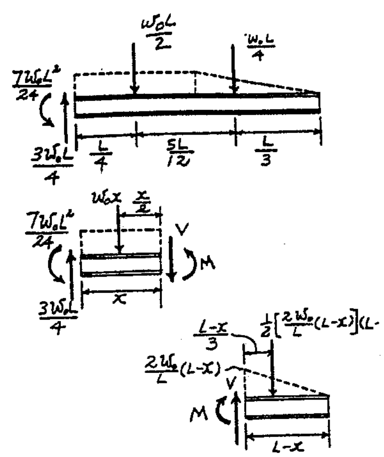
For $L/2 < x \le L$

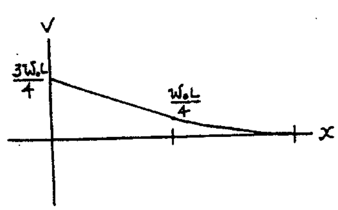
$$+\uparrow \Sigma F_y = 0; \quad V - \frac{1}{2}\left[\frac{2w_0}{L}(L-x)\right](L-x) = 0$$

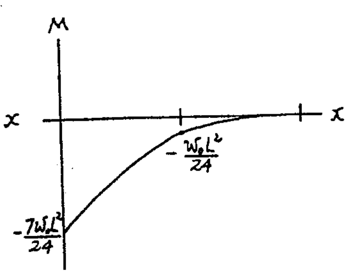
$$V = \frac{w_0}{L}(L-x)^2 \qquad \text{Ans}$$

$$\curvearrowleft + \Sigma M_{NA} = 0; \quad -M - \frac{1}{2}\left[\frac{2w_0}{L}(L-x)\right](L-x)\left(\frac{L-x}{3}\right) = 0$$

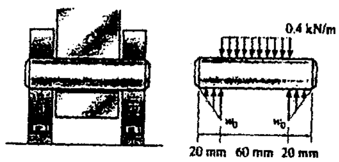
$$M = -\frac{w_0}{3L}(L-x)^3 \qquad \text{Ans}$$





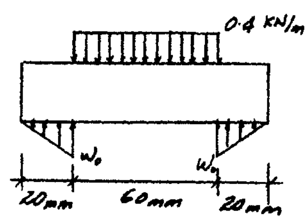


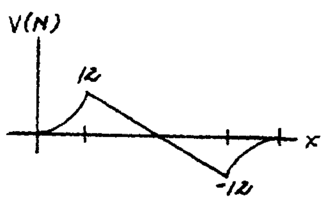
6-35. The smooth pin is supported by two leaves A and B and subjected to a compressive load of 0.4 kN/m caused by bar C. Determine the intensity of the distributed load w_0 of the leaves on the pin and draw the shear and moment diagrams for the pin.

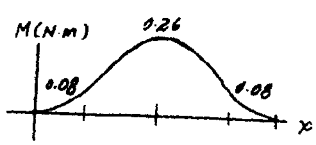


$$+\uparrow \Sigma F_y = 0; \qquad 2(w_0)(20)\left(\frac{1}{2}\right) - 60(0.4) = 0$$

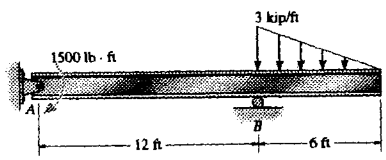
$$w_0 = 1.2\ \text{kN/m} \qquad \textbf{Ans}$$

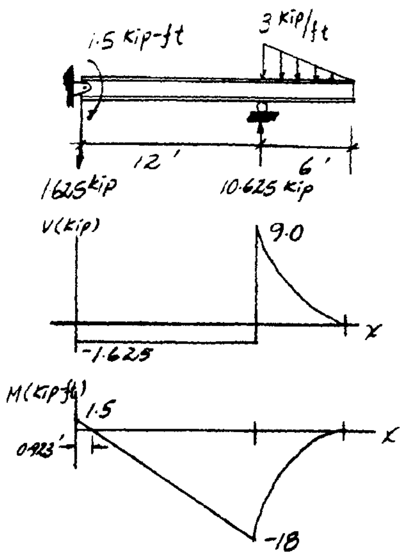




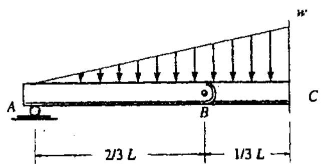


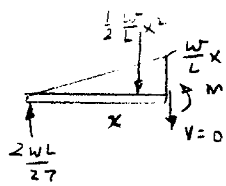
•6–36 Draw the shear and moment diagrams for the beam.





6–37 The compound beam consists of two segments that are pinned together at B. Draw the shear and moment diagrams if it supports the distributed loading shown.



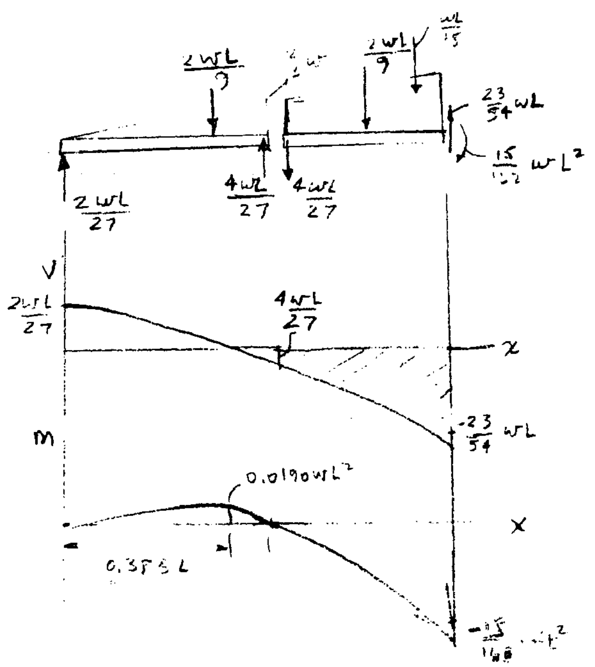


$$+\uparrow \Sigma F_y = 0; \quad \frac{2wL}{27} - \frac{1}{2}\frac{w}{L}x^2 = 0$$

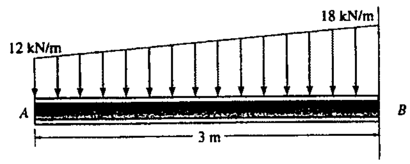
$$x = \sqrt{\frac{4}{27}}\, L = 0.385\, L$$

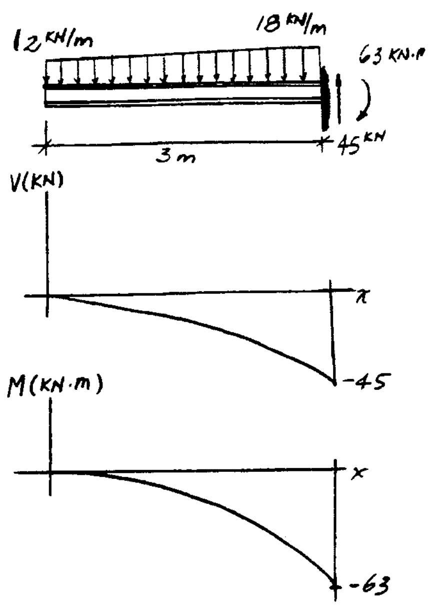
$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}\frac{w}{L}(0.385L)^2\left(\frac{1}{3}\right)(0.385L) - \frac{2wL}{27}(0.385L) = 0$$

$$M = 0.0190\, wL^2$$

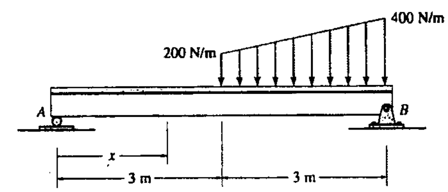


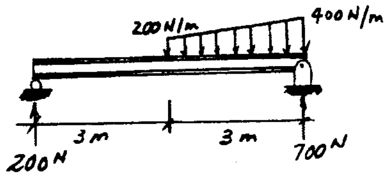
6–38 Draw the shear and moment diagrams for the beam.





6–39 Draw the shear and moment diagrams for the beam and determine the shear and moment as functions of x.





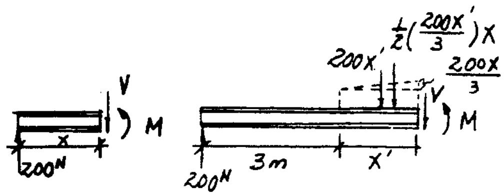
For $0 \le x \le 3$ m :

$+\uparrow \Sigma F_y = 0; \quad 200 - V = 0$

$$V = 200$$

$\circlearrowleft + \Sigma M = 0; \quad M - 200x = 0$

$$M = 200x$$



For $3\text{m} \le x \le 6\text{m}$:

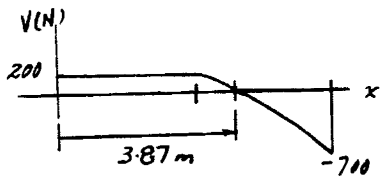
$+\uparrow \Sigma F_y = 0; \quad 200 - 200x' - \dfrac{200x'^2}{6} - V = 0$

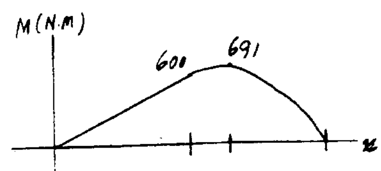
$$V = 200 - 200x' - \frac{100}{3}x'^2$$

$$x' = x - 3$$

$$V = 200 - 200(x-3) - \frac{100}{3}(x-3)^2$$

$$V = 500 - \frac{100}{3}x^2 \qquad \textbf{Ans}$$





Set $V = 0; \quad x = 3.873$ m

$\circlearrowleft + \Sigma M = 0; \quad 200(x'+3) - 200x'(\dfrac{x'}{2}) - \dfrac{100}{3}x'^2(\dfrac{x'}{3}) - M = 0$

$$M = 200(x'+3) - 100x'^2 - \frac{100}{9}x'^3$$

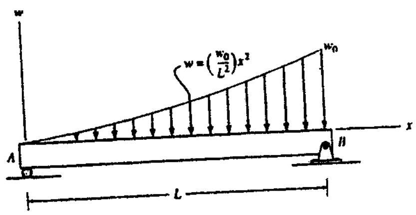
Substitute $x' = x - 3$,

$$M = -\frac{100}{9}x^3 + 500x - 600 \qquad \textbf{Ans}$$

Substitute $x = 3.873$ m,

$$M = 691\ \text{N}\cdot\text{m}$$

6-40. Draw the shear and moment diagrams for the beam.



$$F_R = \int_A dA = \int_0^L w\,dx = \frac{w_0}{L^2}\int_0^L x^2 dx = \frac{w_0 L}{3}$$

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\frac{w_0}{L^2}\int_0^L x^3 dx}{\frac{w_0 L}{3}} = \frac{3L}{4}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{12} - \frac{w_0 x^3}{3L^2} = 0$$

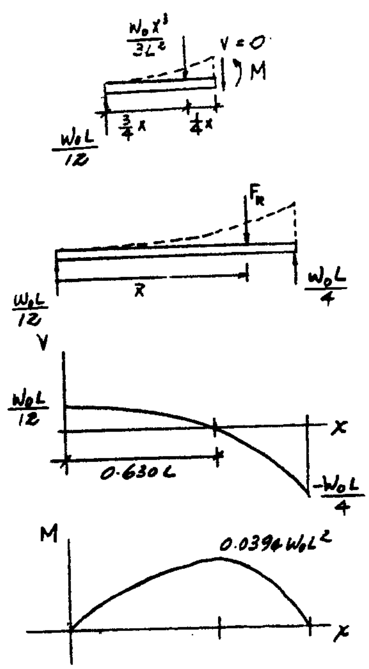
$$x = \left(\frac{1}{4}\right)^{1/3} L = 0.630\,L$$

$$\circlearrowleft + \Sigma M = 0; \qquad \frac{w_0 L}{12}(x) - \frac{w_0 x^3}{3L^2}\left(\frac{1}{4}x\right) - M = 0$$

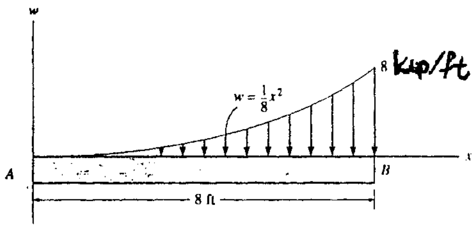
$$M = \frac{w_0 L x}{12} - \frac{w_0 x^4}{12L^2}$$

Substitute $x = 0.630L$

$M = 0.0394\, w_0 L^2$

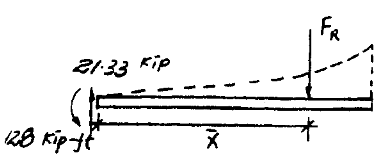


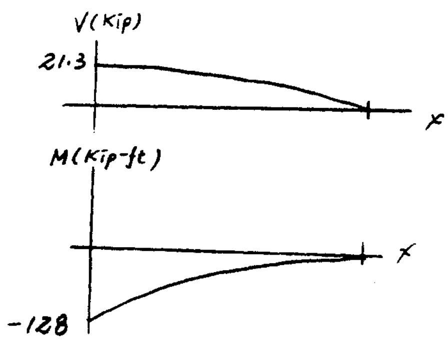
6–41 Draw the shear and moment diagrams for the beam.



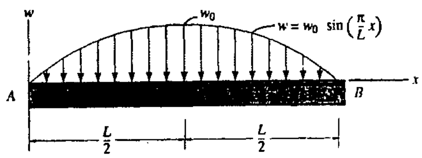
$$F_R = \frac{1}{8}\int_0^8 x^2 dx = 21.33 \text{ kip}$$

$$\bar{x} = \frac{\frac{1}{8}\int_0^8 x^3 dx}{21.33} = 6.0 \text{ ft}$$

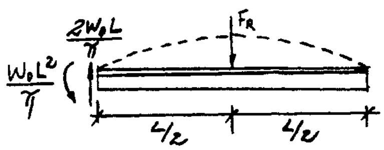


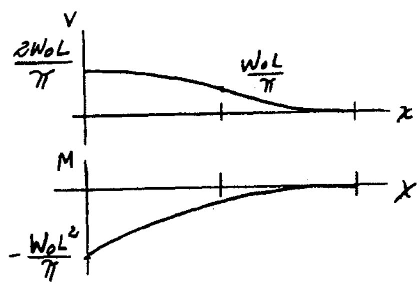


6–42 Draw the shear and moment diagrams for the beam.

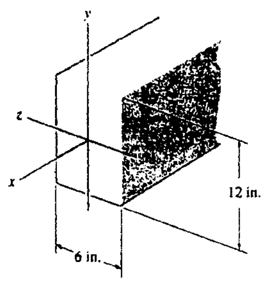


$$F_R = \int_A dA = w_0 \int_0^L \sin\left(\frac{\pi}{L}x\right)dx = \frac{2w_0 L}{\pi}$$





6-43 A member having the dimensions shown is to be used to resist an internal bending moment of $M = 2$ kip · ft. Determine the maximum stress in the member if the moment is applied (*a*) about the *z* axis, (*b*) about the *y* axis. Sketch the stress distribution for each case.



$$I_z = \frac{1}{12}(6)(12^3) = 864 \text{ in}^4$$

$$I_y = \frac{1}{12}(12)(6^3) = 216 \text{ in}^4$$

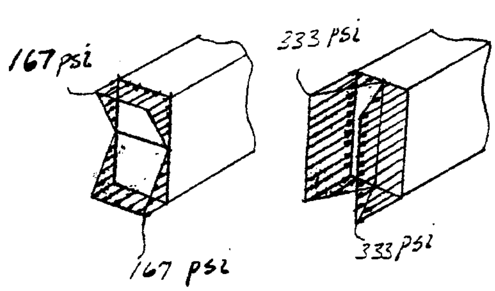
a) Maximum stress :

For z-z axis :

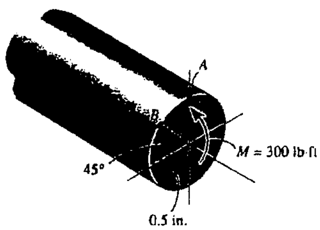
$$\sigma_{max} = \frac{Mc}{I_z} = \frac{2(10^3)(12)(6)}{864} = 167 \text{ psi} \qquad \textbf{Ans}$$

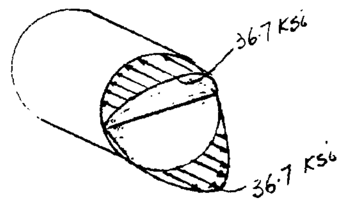
b) For y-y axis :

$$\sigma_{max} = \frac{Mc}{I_y} = \frac{2(10^3)(12)(3)}{216} = 333 \text{ psi} \qquad \textbf{Ans}$$



•6–44 The steel rod having a diameter of 1 in. is subjected to an internal moment of $M = 300$ lb · ft. Determine the stress created at points A and B. Also, sketch a three-dimensional view of the stress distribution acting over the cross section.



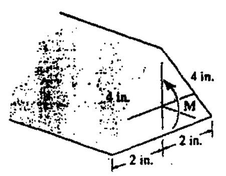


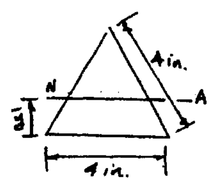
$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.5^4) = 0.0490874 \text{ in}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{300(12)(0.5)}{0.0490874} = 36.7 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{300(12)(0.5 \sin 45^\circ)}{0.0490874} = 25.9 \text{ ksi} \qquad \textbf{Ans}$$

6-45. A member has the triangular cross section shown. Determine the largest internal moment M that can be applied to the cross section without exceeding allowable tensile and compressive stresses of $(\sigma_{allow})_t = 22$ ksi and $(\sigma_{allow})_c = 15$ ksi, respectively.





$$\bar{y}\,(\text{From base}) = \frac{1}{3}\sqrt{4^2 - 2^2} = 1.1547 \text{ in.}$$

$$I = \frac{1}{36}(4)(\sqrt{4^2 - 2^2})^3 = 4.6188 \text{ in}^4$$

Assume failure due to tensile stress :

$$\sigma_{max} = \frac{My}{I}; \qquad 22 = \frac{M(1.1547)}{4.6188}$$

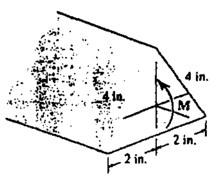
$$M = 88.0 \text{ kip}\cdot\text{in.} = 7.33 \text{ kip}\cdot\text{ft}$$

Assume failure due to compressive stress:

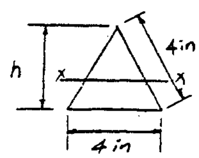
$$\sigma_{max} = \frac{Mc}{I}; \qquad 15 = \frac{M(3.4641 - 1.1547)}{4.6188}$$

$$M = 30.0 \text{ kip}\cdot\text{in.} = 2.50 \text{ kip}\cdot\text{ft} \quad \text{(controls)}$$ **Ans**

6-46. A member has the triangular cross section shown. If a moment of $M = 800$ lb · ft is applied to the cross section, determine the maximum tensile and compressive bending stresses in the member. Also, sketch a three-dimensional view of the stress distribution acting over the cross section.



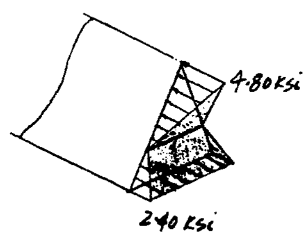
$$h = \sqrt{4^2 - 2^2} = 3.4641 \text{ in.}$$



$$I_x = \frac{1}{36}(4)(3.4641)^3 = 4.6188 \text{ in}^4$$

$$c = \frac{2}{3}(3.4641) = 2.3094 \text{ in.}$$

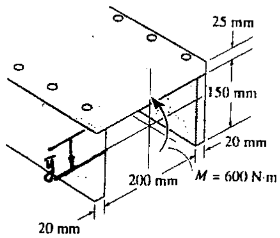
$$y = \frac{1}{3}(3.4641) = 1.1547 \text{ in.}$$



$$(\sigma_{max})_t = \frac{My}{I} = \frac{800(12)(1.1547)}{4.6188} = 2.40 \text{ ksi} \qquad \textbf{Ans}$$

$$(\sigma_{max})_c = \frac{Mc}{I} = \frac{800(12)(2.3094)}{4.6188} = 4.80 \text{ ksi} \qquad \textbf{Ans}$$

6–47 The beam is made from three boards nailed together as shown. If the moment acting on the cross section is M 600 N · m, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section.



$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2\,(0.1)(0.15)(0.02)}{0.24\,(0.025) + 2\,(0.15)(0.02)} = 0.05625\text{ m}$$

$$I = \frac{1}{12}(0.24)(0.025^3) + (0.24)(0.025)(0.04375^2)$$

$$+ 2\,(\frac{1}{12})(0.02)(0.15^3) + 2\,(0.15)(0.02)(0.04375^2)$$

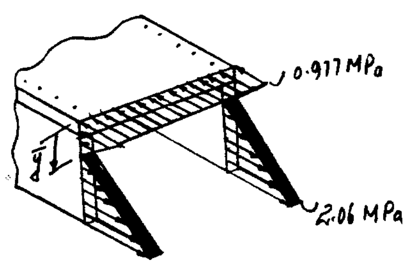
$$= 34.53125\,(10^{-6})\text{ m}^4$$

$$\sigma_{max} = \sigma_B = \frac{Mc}{I}$$

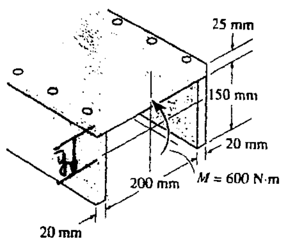
$$= \frac{600\,(0.175 - 0.05625)}{34.53125\,(10^{-6})}$$

$$= 2.06\text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_C = \frac{My}{I} = \frac{600\,(0.05625)}{34.53125\,(10^{-6})} = 0.977\text{ MPa}$$

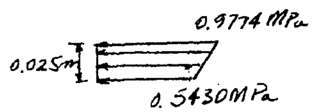


•6–48 The beam is made from three boards nailed together as shown. If the moment acting on the cross section is M = 600 N · m, determine the resultant force the bending stress produces on the top board.



$$\bar{y} = \frac{(0.0125)(0.24)(0.025) + 2\,(0.15)(0.1)(0.02)}{0.24\,(0.025) + 2\,(0.15)(0.02)} = 0.05625 \text{ m}$$

$$I = \frac{1}{12}(0.24)(0.025^3) + (0.24)(0.025)(0.04375^2) + 2\,(\frac{1}{12})(0.02)(0.15^3) + 2\,(0.15)(0.02)(0.04375^2) = 34.53125\,(10^{-6})\text{ m}^4$$

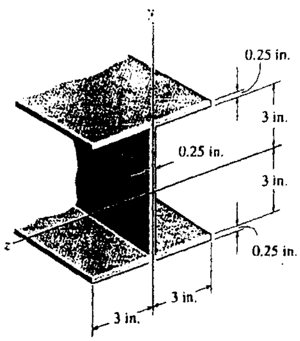


$$\sigma_t = \frac{My}{I} = \frac{600(0.05625)}{34.53125(10^{-6})} = 0.9774 \text{ MPa}$$

$$\sigma_b = \frac{My}{I} = \frac{600(0.05625 - 0.025)}{34.53125(10^{-6})} = 0.5430 \text{ MPa}$$

$$F = \frac{1}{2}(0.025)(0.9774 + 0.5430)(10^6)(0.240) = 4.56 \text{ kN} \qquad \textbf{Ans}$$

6–49 A beam has the cross section shown. If it is made of steel that has an allowable stress of $\sigma_{allow} = 24$ ksi, determine the largest internal moment the beam can resist if the moment is applied (*a*) about the *z* axis, (*b*) about the *y* axis.



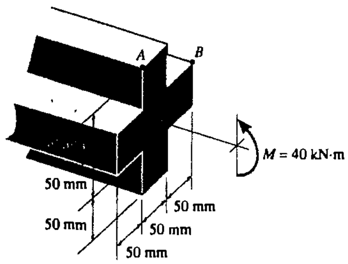
$$I_z = \frac{1}{12}(6)(6.5^3) - \frac{1}{12}(5.75)(6^3) = 33.8125 \text{ in}^4$$

$$I_y = 2[\frac{1}{12}(0.25)(6^3)] + \frac{1}{12}(6)(0.25^3) = 9.0078 \text{ in}^4$$

a) $$(M_{allow})_z = \frac{\sigma_{allow} I_z}{c} = \frac{24(33.8125)}{3.25}$$
$$= 249.7 \text{ kip} \cdot \text{in.} = 20.8 \text{ kip} \cdot \text{ft}$$ **Ans**

b) $$(M_{allow})_y = \frac{\sigma_{allow} I_y}{c} = \frac{24(9.0078)}{3}$$
$$= 72.0625 \text{ kip} \cdot \text{in.} = 6.00 \text{ kip} \cdot \text{ft}$$ **Ans**

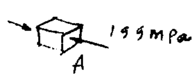
6–50 The beam is subjected to a moment of $M = 40$ kN · m. Determine the bending stress acting at points A and B. Sketch the results on a volume element acting at each of these points.

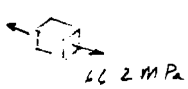


$$I = \frac{1}{12}(0.150)(0.05)^3 + 2[\frac{1}{12}(0.05)(0.05)^3 + (0.05)(0.05)(0.05)^2] = 15.1042(10^{-6})\text{ m}^4$$

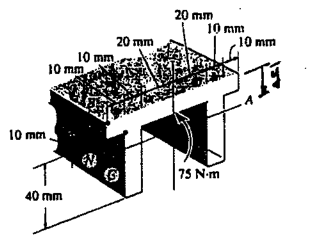
$$\sigma_A = \frac{Mc}{I} = \frac{40(10^3)(0.075)}{15.1042(10^{-6})} = 199\text{ MPa} \qquad \textbf{Ans}$$

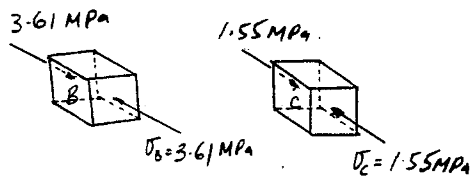
$$\sigma_B = \frac{My}{I} = \frac{40(10^3)(0.025)}{15.1042(10^{-6})} = 66.2\text{ MPa} \qquad \textbf{Ans}$$





6-51. The aluminum machine part is subjected to a moment of $M = 75\ \text{N}\cdot\text{m}$. Determine the bending stress created at points B and C on the cross section. Sketch the results on a volume element located at each of these points.





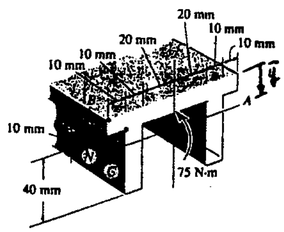
$$\bar{y} = \frac{0.005(0.08)(0.01) + 2[0.03(0.04)(0.01)]}{0.08(0.01) + 2(0.04)(0.01)} = 0.0175\ \text{m}$$

$$I = \frac{1}{12}(0.08)(0.01^3) + 0.08(0.01)(0.0125^2) + 2[\frac{1}{12}(0.01)(0.04^3) + 0.01(0.04)(0.0125^2)] = 0.3633(10^{-6})\ \text{m}^4$$

$$\sigma_B = \frac{Mc}{I} = \frac{75(0.0175)}{0.3633(10^{-6})} = 3.61\ \text{MPa} \qquad \textbf{Ans}$$

$$\sigma_C = \frac{My}{I} = \frac{75(0.0175 - 0.01)}{0.3633(10^{-6})} = 1.55\ \text{MPa} \qquad \textbf{Ans}$$

***6-52.** The aluminum machine part is subjected to a moment of $M = 75\ \text{N}\cdot\text{m}$. Determine the maximum tensile and compressive bending stresses in the part.



$$\bar{y} = \frac{0.005(0.08)(0.01) + 2[0.03(0.04)(0.01)]}{0.08(0.01) + 2(0.04)(0.01)} = 0.0175\ \text{m}$$

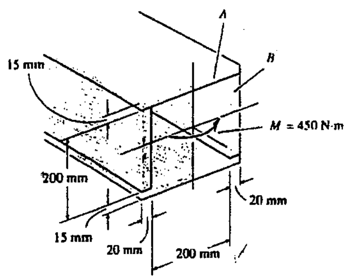
$$I = \frac{1}{12}(0.08)(0.01^3) + 0.08(0.01)(0.0125^2)$$

$$+ 2[\frac{1}{12}(0.01)(0.04^3) + 0.01(0.04)(0.0125^2)] = 0.3633(10^{-6})\ \text{m}^4$$

$$(\sigma_{max})_t = \frac{Mc}{I} = \frac{75(0.050-0.0175)}{0.3633(10^{-6})} = 6.71\ \text{MPa} \qquad \textbf{Ans}$$

$$(\sigma_{max})_c = \frac{My}{I} = \frac{75(0.0175)}{0.3633(10^{-6})} = 3.61\ \text{MPa} \qquad \textbf{Ans}$$

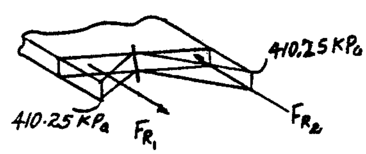
6–53. A beam is constructed from four pieces of wood, glued together as shown. If the moment acting on the cross section is $M = 450$ N · m, determine the resultant force the bending stress produces on the top board A and on the side board B.



$$I_y = \frac{1}{12}(0.23)(0.24^3) - \frac{1}{12}(0.2)(0.2^3) = 1.31626\,(10^{-4})\text{ m}^4$$

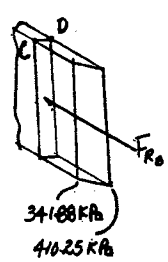
$$\sigma_D = \frac{Mx}{I_y} = \frac{450\,(0.12)}{1.31626\,(10^{-4})} = 410.25\text{ kPa}$$

$$\sigma_C = \frac{Mx}{I_y} = \frac{450\,(0.1)}{1.31626\,(10^{-4})} = 341.88\text{ kPa}$$

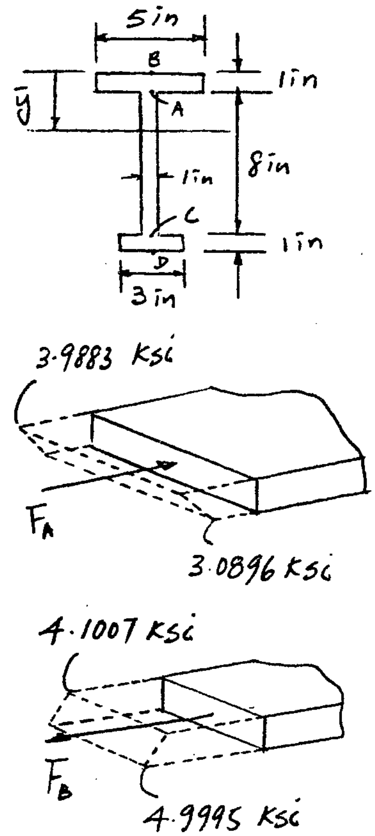


$$F_{R_A} = F_{R_1} - F_{R_2}$$
$$= \frac{1}{2}(410.25)(10^3)(0.12)(0.015) - \frac{1}{2}(410.25)(10^3)(0.12)(0.015)$$
$$= 0 \quad \textbf{Ans}$$

$$F_{R_B} = 341.88\,(10^3)(0.2)(0.02) + \frac{1}{2}(410.25 - 341.88)(10^3)(0.2)(0.02)$$
$$= 1.50\text{ kN} \quad \textbf{Ans}$$



6-54. The beam is subjected to a moment of 15 kip · ft. Determine the resultant force the bending stress produces on the top flange A and bottom flange B. Also compute the maximum bending stress developed in the beam.



$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.5(1)(5) + 5(8)(1) + 9.5(3)(1)}{1(5) + 8(1) + 3(1)} = 4.4375 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5(1)(4.4375 - 0.5)^2 + \frac{1}{12}(1)(8^3) + 8(1)(5 - 4.4375)^2$$
$$+ \frac{1}{12}(3)(1^3) + 3(1)(9.5 - 4.4375)^2$$
$$= 200.27 \text{ in}^4$$

Using flexure formula $\sigma = \dfrac{M\,y}{I}$

$$\sigma_A = \frac{15(12)(4.4375 - 1)}{200.27} = 3.0896 \text{ ksi}$$

$$\sigma_B = \frac{15(12)(4.4375)}{200.27} = 3.9883 \text{ ksi}$$

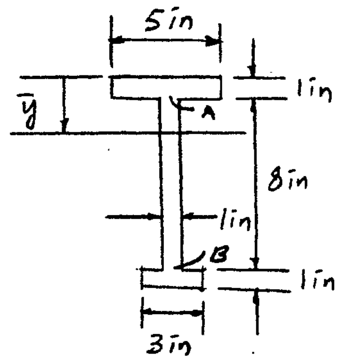
$$\sigma_C = \frac{15(12)(9 - 4.4375)}{200.27} = 4.1007 \text{ ksi}$$

$$\sigma_{max} = \frac{15(12)(10 - 4.4375)}{200.27} = 4.9995 \text{ ksi} = 5.00 \text{ ksi (Max)}$$ **Ans**

$$F_A = \frac{1}{2}(3.0896 + 3.9883)(1)(5) = 17.7 \text{ kip}$$ **Ans**

$$F_B = \frac{1}{2}(4.9995 + 4.1007)(1)(3) = 13.7 \text{ kip}$$ **Ans**

6-55. The beam is subjected to a moment of 15 kip · ft. Determine the percentage of this moment that is resisted by the web D of the beam.



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.5(1)(5) + 5(8)(1) + 9.5(3)(1)}{1(5) + 8(1) + 3(1)} = 4.4375 \text{ in.}$$

$$I = \frac{1}{12}(5)(1^3) + 5(1)(4.4375 - 0.5)^2 + \frac{1}{12}(1)(8^3) + 8(1)(5 - 4.4375)^2$$
$$+ \frac{1}{12}(3)(1^3) + 3(1)(9.5 - 4.4375)^2$$
$$= 200.27 \text{ in}^4$$

Using flexure formula $\sigma = \frac{My}{I}$

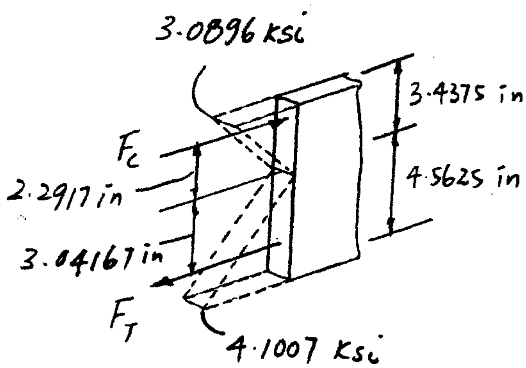
$$\sigma_A = \frac{15(12)(4.4375 - 1)}{200.27} = 3.0896 \text{ ksi}$$

$$\sigma_B = \frac{15(12)(9 - 4.4375)}{200.27} = 4.1007 \text{ ksi}$$

$$F_C = \frac{1}{2}(3.0896)(3.4375)(1) = 5.3102 \text{ kip}$$

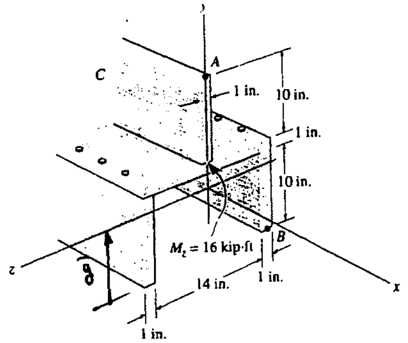
$$F_T = \frac{1}{2}(4.1007)(4.5625)(1) = 9.3547 \text{ kip}$$

$$M = 5.3102(2.2917) + 9.3547(3.0417)$$
$$= 40.623 \text{ kip . in.} = 3.3852 \text{ kip . ft}$$



$$\% \text{ of moment carried by web} = \frac{3.3852}{15} \times 100 = 22.6\ \% \qquad \textbf{Ans}$$

*6–56 The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 16$ kip · ft, determine the stress at points A and B. Sketch a three-dimensional view of the stress distribution.



$$\bar{y} = \frac{2[5(10)(1)] + 10.5(16)(1) + 16(10)(1)}{2(10)(1) + 16(1) + 10(1)}$$

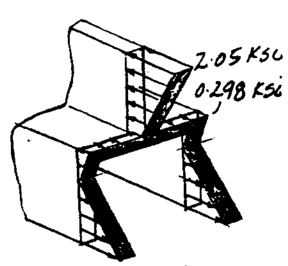
$$= 9.3043 \text{ in.}$$

$$I = 2[\frac{1}{12}(1)(10^3) + 1(10)(9.3043-5)^2] + \frac{1}{12}(16)(1^3) + 16(1)(10.5-9.3043)^2$$

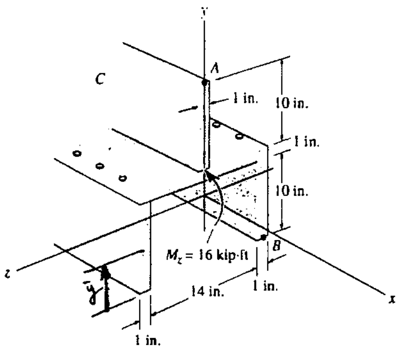
$$+ \frac{1}{12}(1)(10^3) + 1(10)(16-9.3043) = 1093.07 \text{ in}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{16(12)(21 - 9.3043)}{1093.07} = 2.05 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{16(12)(9.3043)}{1093.07} = 1.63 \text{ ksi} \qquad \textbf{Ans}$$



6-57 The beam is constructed from four boards as shown. If it is subjected to a moment of $M_z = 16$ kip · ft, determine the resultant force the stress produces on the top board C.



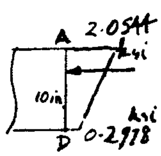
$$\bar{y} = \frac{2[5(10)(1)] + 10.5(16)(1) + 16(10)(1)}{2(10)(1) + 16(1) + 10(1)} = 9.3043 \text{ in.}$$

$$I = 2[\frac{1}{12}(1)(10^3) + (10)(9.3043-5)^2] + \frac{1}{12}(16)(1^3) + 16(1)(10.5-9.3043)^2 + \frac{1}{12}(1)(10^3) + 1(10)(16-9.3043)^2 = 1093.07 \text{ in}^4$$

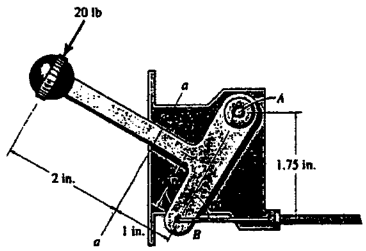
$$\sigma_A = \frac{Mc}{I} = \frac{16(12)(21 - 9.3043)}{1093.07} = 2.0544 \text{ ksi}$$

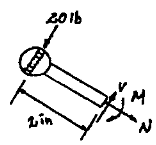
$$\sigma_D = \frac{My}{I} = \frac{16(12)(11 - 9.3043)}{1093.07} = 0.2978 \text{ ksi}$$

$$(F_R)_C = \frac{1}{2}(2.0544 + 0.2978)(10)(1) = 11.8 \text{ kip}$$ **Ans**



6-58. The control lever is used on a riding lawn mower. Determine the maximum bending stress in the lever at section $a - a$ if a force of 20 lb is applied to the handle. The lever is supported by a pin at A and a wire at B. Section $a - a$ is square, 0.25 in. by 0.25 in.

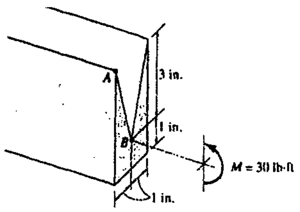




$\circlearrowleft + \Sigma M = 0; \quad 20(2) - M = 0; \quad M = 40 \text{ lb} \cdot \text{in.}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{40(0.125)}{\frac{1}{12}(0.25)(0.25^3)} = 15.4 \text{ ksi} \qquad \textbf{Ans}$$

6-59. The beam is subjected to a moment of $M = 30\ \text{lb}\cdot\text{ft}$. Determine the bending stress acting at points A and B. Also, sketch a three-dimensional view of the stress distribution acting over the entire cross-sectional area.



$$\bar{y} = \frac{2(4)(2) - 3(\frac{1}{2})(2)(3)}{4(2) - \frac{1}{2}(2)(3)} = 1.40 \text{ in.}$$

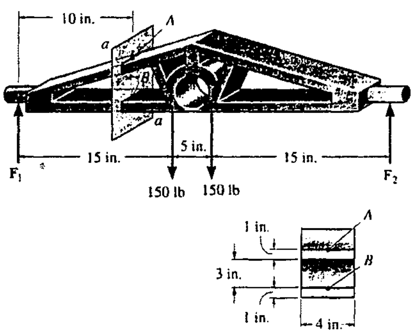
$$I = \frac{1}{12}(2)(4)^3 + (4)(2)(2 - 1.40)^2 - (\frac{1}{36}(2)(3)^3 + \frac{1}{2}(2)(3)(3 - 1.40)^2) = 4.367 \text{ in}^4$$

$$\sigma_A = \frac{My}{I} = \frac{(30)(12)(4 - 1.40)}{4.367} = 214 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{30(12)(1.40 - 1)}{4.367} = 33.0 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_C = \frac{My}{I} = \frac{30(12)(1.40)}{4.367} = 115 \text{ psi}$$

*6–60 The tapered casting supports the loading shown. Determine the bending stress at points A and B. The cross section at section a–a is given in the figure.

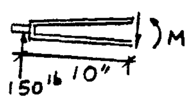


Casting :

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_1(35) - 150(20) - 150(15) = 0$$

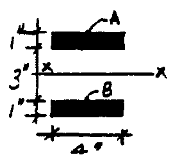
$$F_1 = 150 \text{ lb}$$

Section :



$$\circlearrowleft + \Sigma M = 0; \qquad M - 150(10) = 0$$

$$M = 1500 \text{ lb} \cdot \text{in.}$$

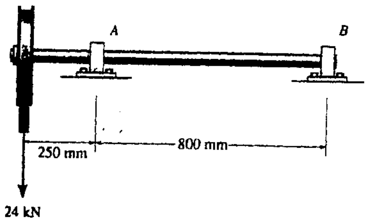


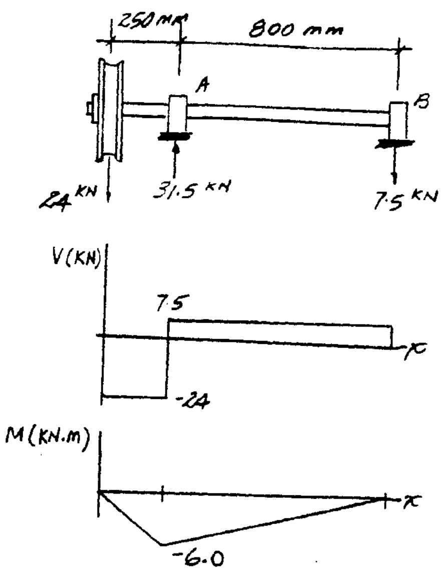
$$I_x = \frac{1}{12}(4)(5^3) - \frac{1}{12}(4)(3)^3 = 32.67 \text{ in}^4$$

$$\sigma_A = \frac{Mc}{I} = \frac{1500(2.5)}{32.67} = 115 \text{ psi (C)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{1500(1.5)}{32.67} = 68.9 \text{ psi (T)} \qquad \textbf{Ans}$$

6–61 If the shaft in Prob. 6–1 has a diameter of 100 mm, determine the absolute maximum bending stress in the shaft.

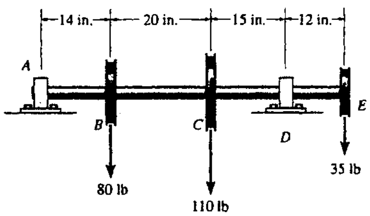


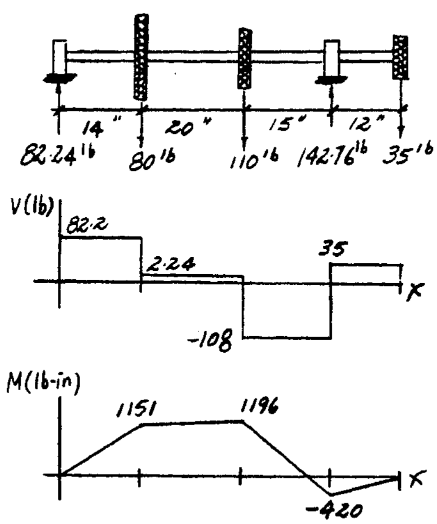


$M_{max} = 6000\ \mathrm{N \cdot m}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{(6000)(0.05)}{\frac{1}{4}\pi(0.05)^4} = 61.1\ \mathrm{MPa} \qquad \textbf{Ans}$$

6–62 If the shaft in Prob. 6–3 has a diameter of 1.5 in., determine the absolute maximum bending stress in the shaft.

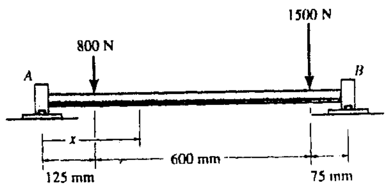


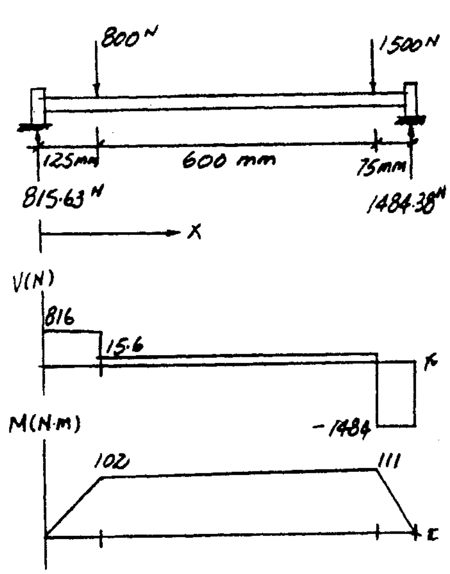


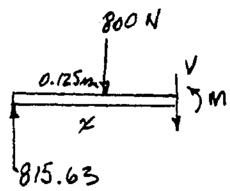
$M_{max} = 1196 \text{ lb} \cdot \text{in.}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{1196(0.75)}{\frac{1}{4}\pi(0.75)^4} = 3.61 \text{ ksi} \qquad \textbf{Ans}$$

6–63 If the shaft in Prob. 6–6 has a diameter of 50 mm, determine the absolute maximum bending stress in the shaft.



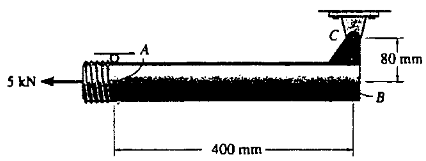


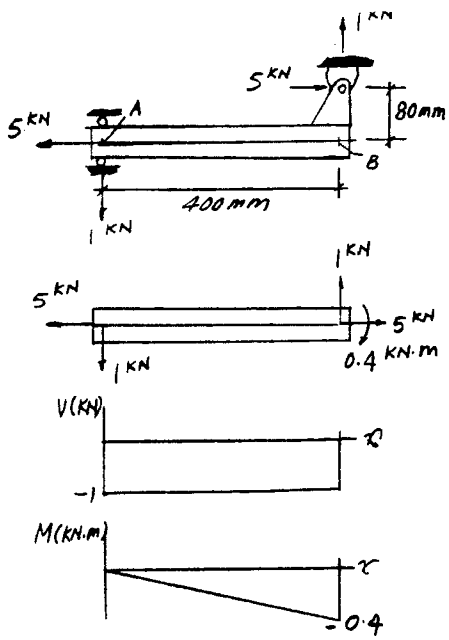


$$M_{max} = 111\ \text{N}\cdot\text{m}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{111(0.025)}{\frac{1}{4}\pi(0.025)^4} = 9.05\ \text{MPa} \qquad \textbf{Ans}$$

*6–64 If the pipe in Prob. 6–8 has an outer diameter of 30 mm and thickness of 10 mm, determine the absolute maximum bending stress in the shaft.

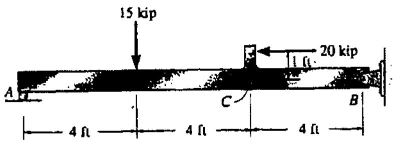


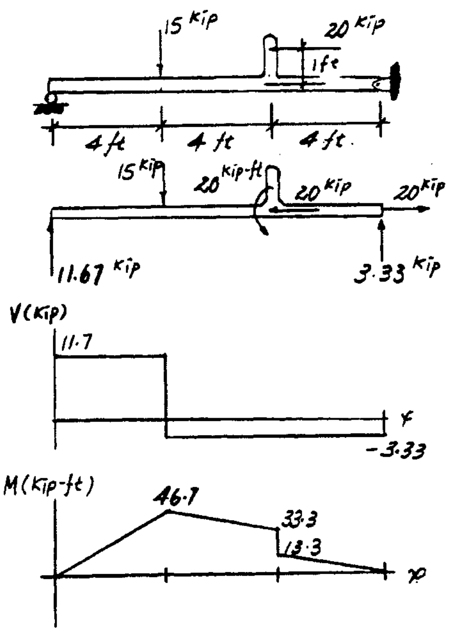


$M_{max} = 0.4 \text{ N} \cdot \text{m}$

$$\sigma_{max} = \frac{400(0.015)}{\frac{1}{4}\pi((0.015)^4 - (0.005)^4)} = 153 \text{ MPa} \qquad \textbf{Ans}$$

6–65 If the beam ACB in Prob. 6–9 has a square cross section, 6 in. by 6 in., determine the absolute maximum bending stress in the beam.

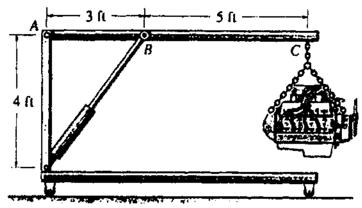




$M_{max} = 46.7 \text{ kip} \cdot \text{ft}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{46.7(10^3)(12)(3)}{\frac{1}{12}(6)(6^3)} = 15.6 \text{ ksi} \qquad \textbf{Ans}$$

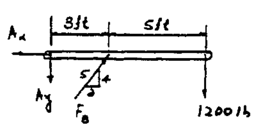
6–66 If the crane boom *ABC* in Prob. 6–10 has a rectangular cross section with a base of 2.5 in., determine its required height *h* to the nearest 1/4 in. if the allowable bending stress is σ_{allow} = 24ksi.

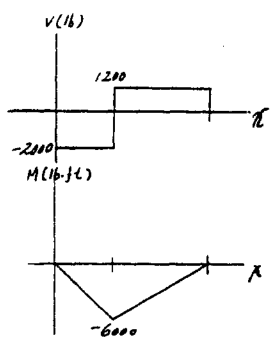


$$\circlearrowleft + \Sigma M_A = 0; \quad \frac{4}{5}F_B(3) - 1200(8) = 0; \quad F_B = 4000\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad -A_y + \frac{4}{5}(4000) - 1200 = 0; \quad A_y = 2000\text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad A_x - \frac{3}{5}(4000) = 0; \quad A_x = 2400\text{ lb}$$



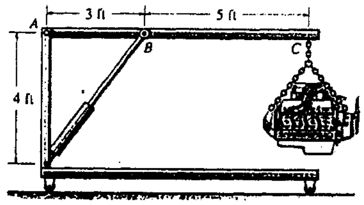


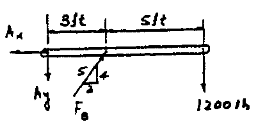
$$\sigma_{max} = \frac{Mc}{I} = \frac{6000(12)\left(\frac{h}{2}\right)}{\frac{1}{12}(2.5)(h^3)} = 24(10)^3$$

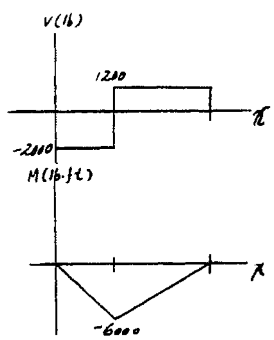
$h = 2.68$ in.

Use $h = 2.75$ in. **Ans**

6–67 If the crane boom ABC in Prob. 6–10 has a rectangular cross section with a base of 2 in. and a height of 3 in., determine the absolute maximum bending stress in the boom.



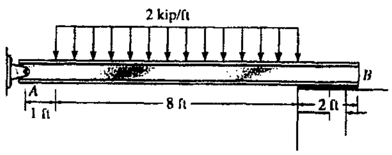


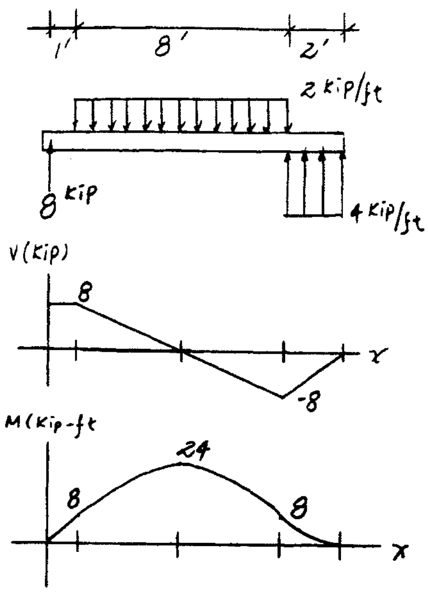


$M_{max} = 6000\ \text{lb}\cdot\text{ft}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{6000(12)(1.5)}{\frac{1}{12}(2)(3^3)} = 24\ \text{ksi} \qquad \textbf{Ans}$$

•6–68 Determine the absolute maximum bending stress in the beam in Prob. 6–24. The cross section is rectangular with a base of 3 in. and height of 4 in.

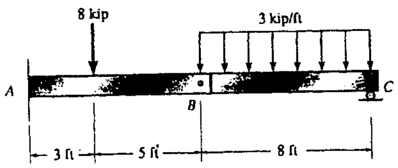


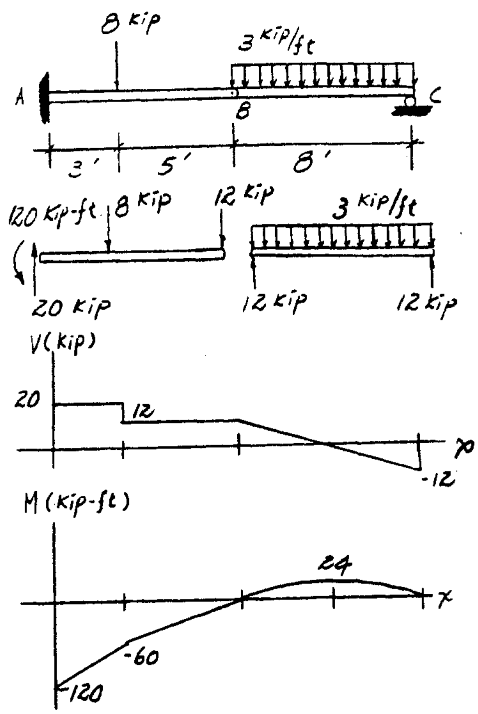


$M_{max} = 24 \text{ kip} \cdot \text{ft}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{24(12)(10^3)(2)}{\frac{1}{12}(3)(4)^3} = 36 \text{ ksi} \qquad \textbf{Ans}$$

6–69 Determine the absolute maximum bending stress in the beam in Prob. 6–25. Each segment has a rectangular cross section with a base of 4 in. and height of 8 in.

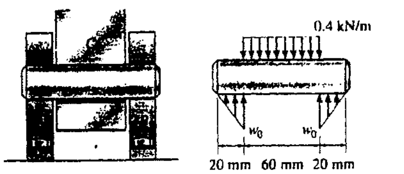




$$M_{max} = 120 \text{ kip} \cdot \text{ft}$$

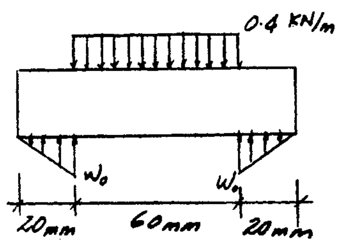
$$\sigma_{max} = \frac{Mc}{I} = \frac{120(12)(10^3)(4)}{\frac{1}{12}(4)(8)^3} = 33.8 \text{ ksi} \qquad \textbf{Ans}$$

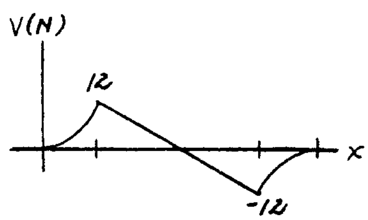
6–70 Determine the absolute maximum bending stress in the 20-mm-diameter pin in Prob. 6–35.

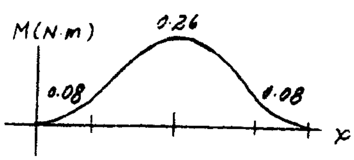


$$+\uparrow \Sigma F_y = 0; \qquad 2(w_0)(20)(\frac{1}{2}) - 60(0.4) = 0$$

$$w_0 = 1.2\text{ kN/m}$$

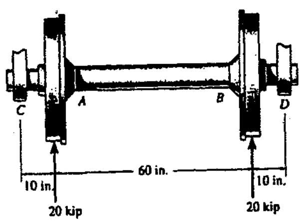


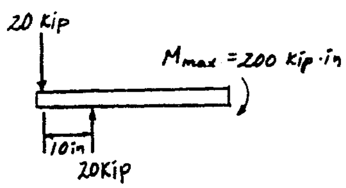




$$\sigma_{max} = \frac{Mc}{I} = \frac{0.26(0.01)}{\frac{1}{4}\pi(0.01)^4} = 331\text{ kPa} \qquad \textbf{Ans}$$

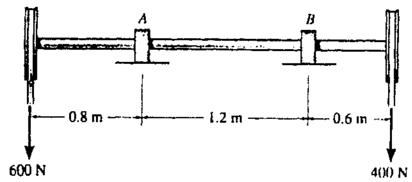
6-71. The axle of the freight car is subjected to wheel loadings of 20 kip. If it is supported by two journal bearings at C and D, determine the maximum bending stress developed at the center of the axle, where the diameter is 5.5 in.

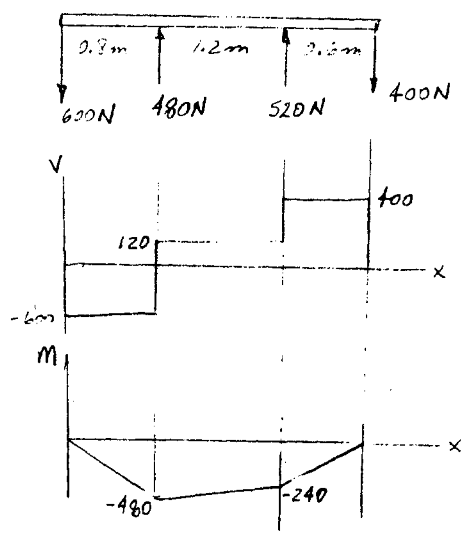




$$\sigma_{max} = \frac{Mc}{I} = \frac{200(2.75)}{\frac{1}{4}\pi(2.75)^4} = 12.2\text{ ksi} \qquad \textbf{Ans}$$

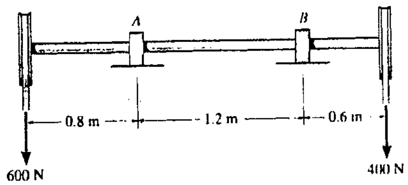
•6–72 Determine the absolute maximum bending stress in the 30-mm-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.

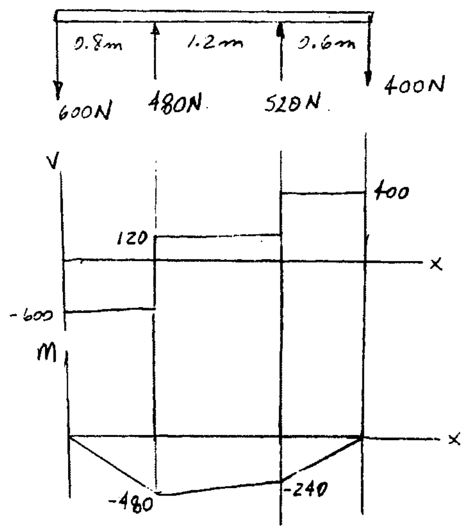




$$\sigma_{max} = \frac{Mc}{I} = \frac{480(0.015)}{\frac{1}{4}\pi(0.015)^4} = 181 \text{ MPa} \qquad \textbf{Ans}$$

6–73 Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{allow} = 160$ MPa.





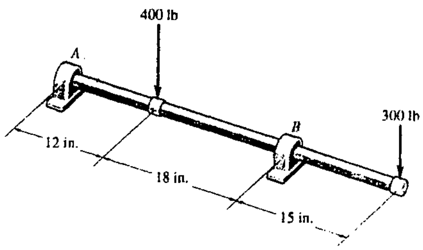
$M_{max} = 480\ \text{N}\cdot\text{m}$

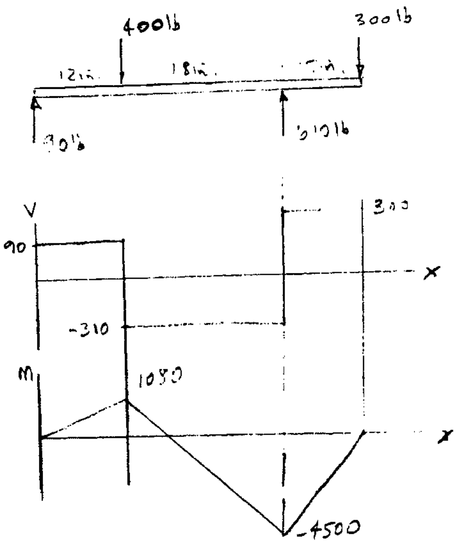
$$\sigma_{allow} = \frac{Mc}{I}; \qquad 160(10^6) = \frac{480c}{\frac{1}{4}\pi c^4}$$

$c = 0.01563$ m

$d = 31.3$ mm **Ans**

6–74 Determine the absolute maximum bending stress in the 1.5-in.-diameter shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces.

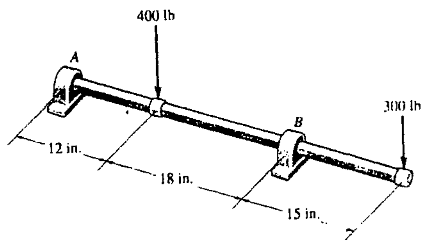


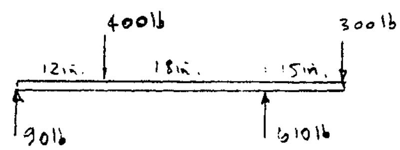


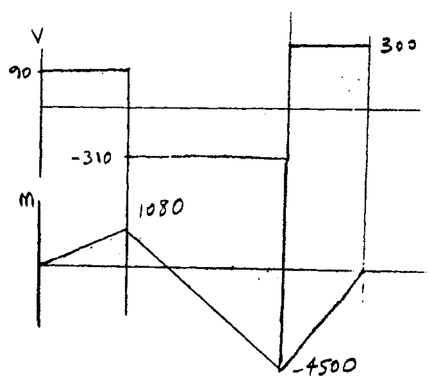
$M_{max} = 4500 \text{ lb} \cdot \text{in.}$

$$\sigma = \frac{Mc}{I} = \frac{4500(0.75)}{\frac{1}{4}\pi(0.75)^4} = 13.6 \text{ ksi} \qquad \textbf{Ans}$$

6–75 Determine the smallest allowable diameter of the shaft which is subjected to the concentrated forces. The sleeve bearings at A and B support only vertical forces, and the allowable bending stress is $\sigma_{allow} = 22$ ksi.







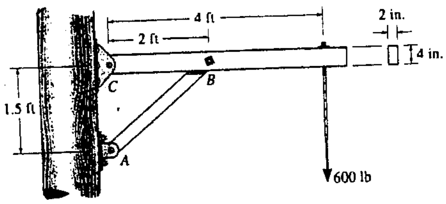
$M_{max} = 4500$ lb · in.

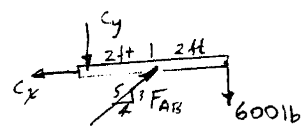
$$\sigma = \frac{Mc}{I}; \qquad 22(10^3) = \frac{4500c}{\frac{1}{4}\pi c^4}$$

$c = 0.639$ in.

$d = 1.28$ in. **Ans**

***6-76.** The strut on the utility pole supports the cable having a weight of 600 lb. Determine the absolute maximum bending stress in the strut if *A*, *B*, and *C* are assumed to be pinned.





$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(\frac{3}{5})(2) - 600(4) = 0$

$F_{AB} = 2000 \text{ lb}$

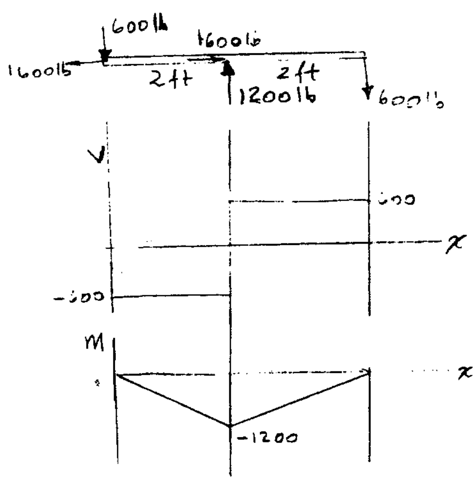
$+\uparrow \Sigma F_y = 0; \qquad -C_y + 2000(\frac{3}{5}) - 600 = 0$

$C_y = 600 \text{ lb}$

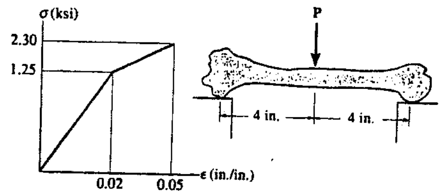
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2000(\frac{4}{5}) - C_x = 0$

$C_x = 1600 \text{ lb}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{1200(12)(2)}{\frac{1}{12}(2)(4)^3} = 2.70 \text{ ksi} \qquad \textbf{Ans}$$

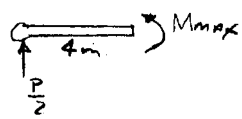


6–77 A portion of the femur can be modeled as a tube having an inner diameter of 0.375 in. and an outer diameter of 1.25 in. Determine the maximum elastic static force P that can be applied to its center without causing failure. Assume the bone to be roller supported at its ends. The σ–ϵ diagram for the bone mass is shown and is the same in tension as in compression.



$$I = \frac{1}{4}\pi\left[\left(\frac{1.25}{2}\right)^4 - \left(\frac{0.375}{2}\right)^4\right] = 0.11887 \text{ in}^4$$

$$M_{max} = \frac{P}{2}(4) = 2P$$



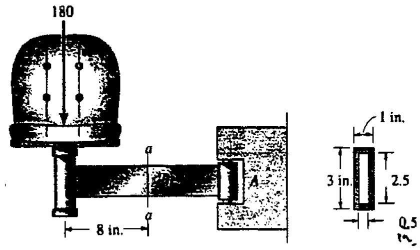
Require $\sigma_{max} = 1.25$ ksi

$$\sigma_{max} = \frac{Mc}{I}$$

$$1.25 = \frac{2P(1.25/2)}{0.11887}$$

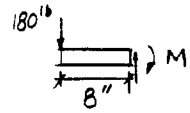
$P = 0.119$ kip $= 119$ lb **Ans**

6–78 The chair is supported by an arm that is hinged so it rotates about the vertical axis at A. If the load on the chair is 180 lb and the arm is a hollow tube section having the dimensions shown, determine the maximum bending stress at section a–a.



$\curvearrowleft + \Sigma M = 0; \quad M - 180(8) = 0$

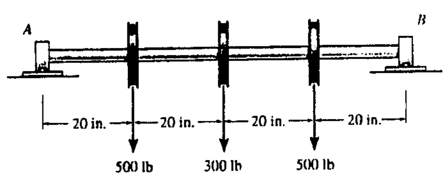
$M = 1440 \text{ lb} \cdot \text{in.}$

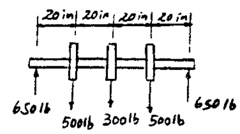


$$I_x = \frac{1}{12}(1)(3^3) - \frac{1}{12}(0.5)(2.5^3) = 1.59896 \text{ in}^4$$

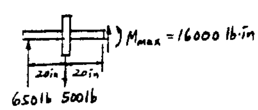
$$\sigma_{max} = \frac{Mc}{I} = \frac{1440(1.5)}{1.59896} = 1.35 \text{ ksi} \qquad \textbf{Ans}$$

6-79 The steel shaft has a circular cross section with a diameter of 2 in. It is supported on smooth journal bearings *A* and *B*, which exert only vertical reactions on the shaft. Determine the absolute maximum bending stress in the shaft if it is subjected to the pulley loadings shown.



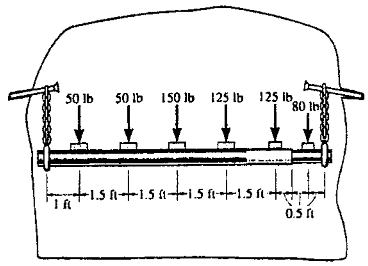


$$I = \frac{1}{4}\pi(1^4) = 0.7854\ \text{in}^4$$



$$\sigma_{max} = \frac{Mc}{I} = \frac{16000\,(1)}{0.7854} = 20.4\ \text{ksi} \qquad \textbf{Ans}$$

*6-80. The end supports of a drillers' scaffold used in coal mining consist of a suspended 4-in.-outside-diameter pipe and telescoping 3-in.-outside-diameter pipe having a length of 1.5 ft. Each pipe has a thickness of 0.25 in. If the end reactions of the supported planks are given, determine the absolute maximum bending stress in each pipe. Neglect the size of the planks in the calculation.



4 in. diameter pipe:

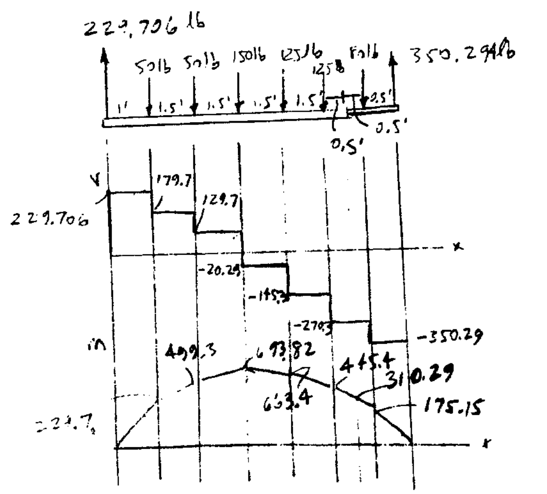
$$M_{max} = 693.82 \text{ lb} \cdot \text{ft}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{693.82(12)(2)}{\frac{1}{4}\pi((2)^4 - (1.75)^4)} = 3.20 \text{ ksi} \qquad \textbf{Ans}$$

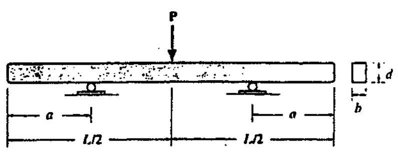
3 in. diameter pipe:

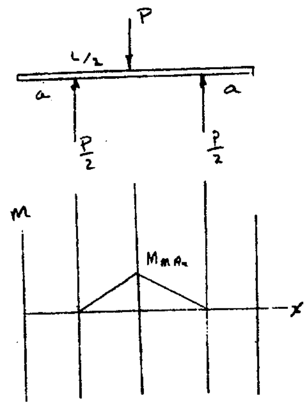
$$M_{max} = 310.29 \text{ lb} \cdot \text{ft}$$

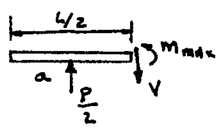
$$\sigma_{max} = \frac{Mc}{I} = \frac{310.29(12)(1.5)}{\frac{1}{4}\pi((1.5)^4 - (1.25)^4)} = 2.71 \text{ ksi} \qquad \textbf{Ans}$$



6-81. The beam is subjected to the load **P** at its center. Determine the placement *a* of the supports so that the absolute maximum bending stress in the beam is as large as possible. What is this stress?







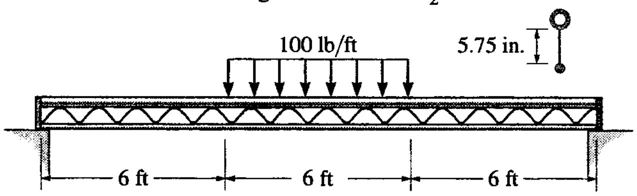
$$M_{max} = \frac{P}{2}\left(\frac{L}{2} - a\right)$$

For the largest M_{max} require,

$a = 0$ **Ans**

$$\sigma_{max} = \frac{Mc}{I} = \frac{(P/2)(\frac{L}{2})(\frac{d}{2})}{\frac{1}{12} b\, d^3} = \frac{3\, PL}{2\, bd^2} \quad \textbf{Ans}$$

6-82. The simply supported truss is subjected to the central distributed load. Neglect the effect of the diagonal lacing and determine the absolute maximum bending stress in the truss. The top member is a pipe having an outer diameter of 1 in. and thickness of $\frac{3}{16}$ in., and the bottom member is a solid rod having a diameter of $\frac{1}{2}$ in.



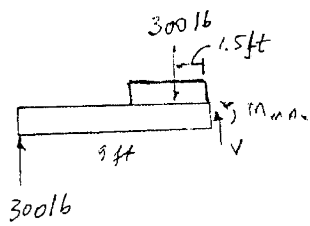
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0 + (6.50)(0.4786)}{0.4786 + 0.19635} = 4.6091 \text{ in.}$$

$$I = [\frac{1}{4}\pi(0.5)^4 - \frac{1}{4}\pi(0.3125)^4] + 0.4786(6.50 - 4.6091)^2 + \frac{1}{4}\pi(0.25)^4 + 0.19635(4.6091)^2 = 5.9271 \text{ in}^4$$

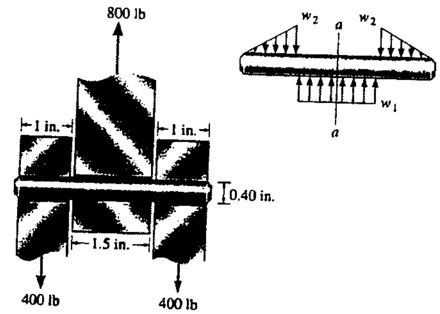
$$M_{max} = 300(9 - 1.5)(12) = 27\,000 \text{ lb} \cdot \text{in.}$$

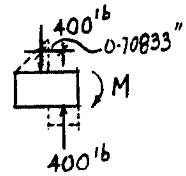
$$\sigma_{max} = \frac{Mc}{I} = \frac{27\,000(4.6091 + 0.25)}{5.9271}$$

$$= 22.1 \text{ ksi} \qquad \textbf{Ans}$$



6–83 The pin is used to connect the three links together. Due to wear, the load is distributed over the top and bottom of the pin as shown on the free-body diagram. If the diameter of the pin is 0.40 in., determine the maximum bending stress on the cross-sectional area at the center section a–a. For the solution it is first necessary to determine the load intensities w_1 and w_2.





$$\frac{1}{2} w_2 (1) = 400; \qquad w_2 = 800 \text{ lb/in.}$$

$$w_1 (1.5) = 800; \qquad w_1 = 533 \text{ lb/in.}$$

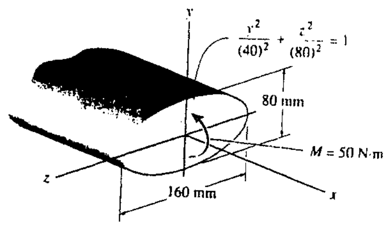
$$M = 400 (0.70833) = 283.33 \text{ lb} \cdot \text{in}$$

$$I = \frac{1}{4} \pi (0.2^4) = 0.0012566 \text{ in}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{283.33 (0.2)}{0.0012566}$$

$$= 45.1 \text{ ksi} \qquad \textbf{Ans}$$

***6–84** A shaft is made of a polymer having an elliptical cross-section. If it resists an internal moment of $M = 50\ \text{N}\cdot\text{m}$, determine the maximum bending stress developed in the material (*a*) using the flexure formula, where $I_z = \frac{1}{4}\pi(0.08\ \text{m})(0.04\ \text{m})^3$, (*b*) using integration. Sketch a three-dimensional view of the stress distribution acting over the cross-sectional area.



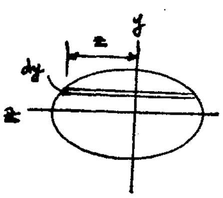
a) $$I = \frac{1}{4}\pi ab^3 = \frac{1}{4}\pi(0.08)(0.04^3) = 4.021238(10^{-6})\text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa} \qquad \textbf{Ans}$$

b)

$$M = \frac{\sigma_{\max}}{c}\int_A y^2 dA$$

$$= \frac{\sigma_{\max}}{c}\int y^2 2z dy$$



$$z = \sqrt{0.0064 - 4y^2} = 2\sqrt{(0.04)^2 - y^2}$$

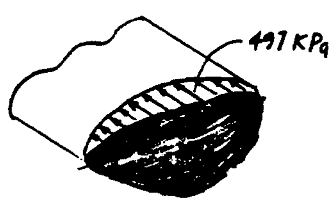
$$2\int_{-0.04}^{0.04} y^2 z dy = 4\int_{-0.04}^{0.04} y^2\sqrt{(0.04)^2 - y^2}\, dy$$

$$= 4\left[\frac{(0.04)^4}{8}\sin^{-1}\left(\frac{y}{0.04}\right) - \frac{1}{8}y\sqrt{0.04^2 - y^2}(0.04^2 - 2y^2)\right]\Bigg|_{-0.04}^{0.04}$$

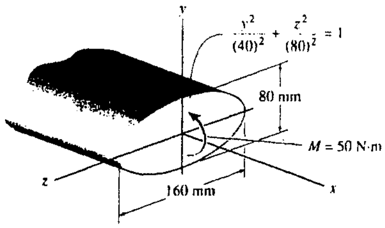
$$= \frac{(0.04)^4}{2}\sin^{-1}\left(\frac{y}{0.04}\right)\Bigg|_{-0.04}^{0.04}$$

$$= 4.021238(10^{-6})\text{m}^4$$

$$\sigma_{\max} = \frac{50(0.04)}{4.021238(10^{-6})} = 497\ \text{kPa} \qquad \textbf{Ans}$$



6–85 Solve Prob. 6–84 if the moment $M = 50$ N · m is applied about the y axis instead of the x axis. Here $I_y = \frac{1}{4}\pi\ (0.04\ \text{m})(0.08\ \text{m})^3$.



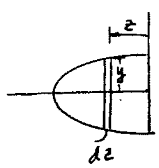
a)

$$I = \frac{1}{4}\pi\, ab^3 = \frac{1}{4}\pi(0.04)(0.08)^3 = 16.085(10^{-6})\ \text{m}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{50(0.08)}{16.085(10^{-6})} = 249\ \text{kPa} \qquad \textbf{Ans}$$

b)

$$M = \int_A z(\sigma\, dA) = \int_A z\left(\frac{\sigma_{max}}{0.08}\right)(z)(2y)dz$$

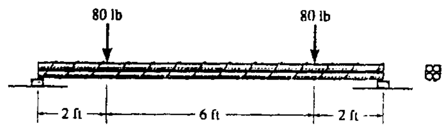


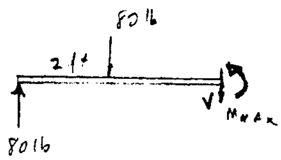
$$50 = 2\left(\frac{\sigma_{max}}{0.04}\right)\int_0^{0.08} z^2\left(1 - \frac{z^2}{(0.08)^2}\right)^{1/2}(0.04)dz$$

$$50 = 201.06(10^{-6})\sigma_{max}$$

$$\sigma_{max} = 249\ \text{kPa} \qquad \textbf{Ans}$$

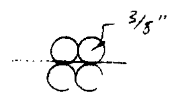
6–86 The simply supported beam is made from four 3/4-in.-diameter rods, which are bundled as shown. Determine the maximum bending stress in the beam due to the loading shown.





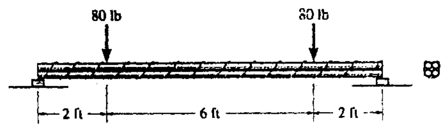
$$M_{max} = 80(2) = 160 \text{ lb} \cdot \text{ft}$$

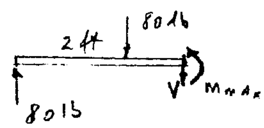
$$I = 4[(\frac{1}{4}\pi)(3/8)^4 + \pi(3/8)^2(3/8)^2] = 0.31063 \text{ in}^4$$



$$\sigma_{max} = \frac{Mc}{I} = \frac{160(12)(3/4)}{0.31063} = 4.64 \text{ ksi}$$ **Ans.**

6–87 Solve Prob. 6–86 if the bundle is rotated 45° and set on the supports.

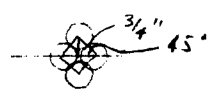




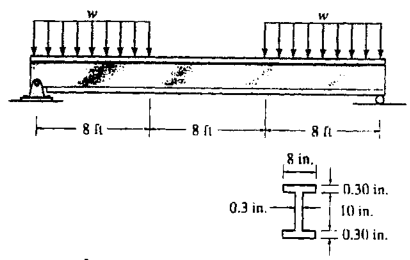
$$M_{max} = 80(2) = 160 \text{ lb} \cdot \text{ft}$$

$$I = 2[\frac{1}{4}\pi(3/8)^4] + 2[\frac{1}{4}\pi(3/8)^4 + \pi\,(3/8)^2((3/4)\sin 45°)^2] = 0.31063 \text{ in}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{160(12)(\frac{3}{4}\sin 45° + \frac{3}{8})}{0.31063} = 5.60 \text{ ksi}$$ **Ans**



*6-88 The steel beam has the cross-sectional area shown. Determine the largest intensity of distributed load w that it can support so that the bending stress does not exceed σ_{max} = 22 ksi.

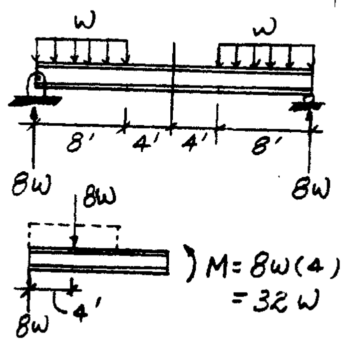


$$I = \frac{1}{12}(8)(10.6)^3 - \frac{1}{12}(7.7)(10^3) = 152.344\ \text{in}^4$$

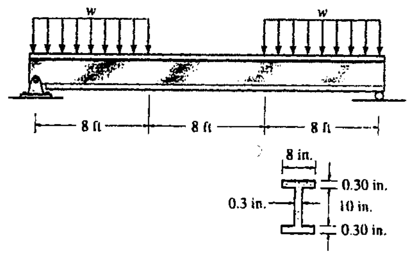
$$\sigma_{max} = \frac{Mc}{I}$$

$$22 = \frac{32w(12)(5.3)}{152.344}$$

w = 1.65 kip/ft **Ans**



6–89 The steel beam has the cross-sectional area shown. If $w = 5$ kip/ft, determine the absolute maximum bending stress in the beam.



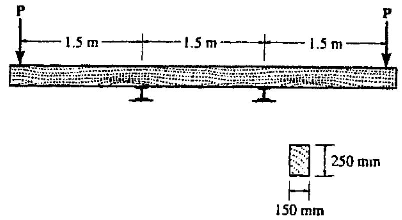
From Prob. 6 - 88;

$$M = 32w = 32(5)(12) = 1920 \text{ kip} \cdot \text{in.}$$

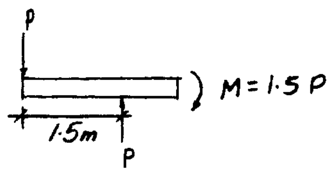
$$I = 152.344 \text{ in}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{1920(5.3)}{152.344} = 66.8 \text{ ksi} \qquad \textbf{Ans}$$

6–90 The beam has a rectangular cross section as shown. Determine the largest load P that can be supported on its overhanging ends so that the bending stress does not exceed $\sigma_{max} = 10$ MPa.



$$I = \frac{1}{12}(0.15)(0.25^3) = 1.953125(10^{-4})\text{m}^4$$

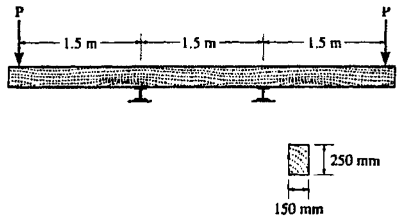


$$\sigma_{max} = \frac{Mc}{I}$$

$$10(10^6) = \frac{1.5P(0.125)}{1.953125(10^{-4})}$$

$$P = 10.4 \text{ kN} \qquad \textbf{Ans}$$

6–91 The beam has the rectangular cross section shown. If $P = 12$ kN, determine the absolute maximum bending stress in the beam. Sketch the stress distribution acting over the cross section.

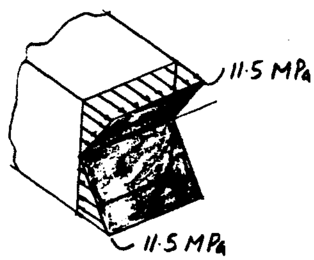


From Prob. 6-90 :

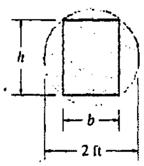
$$M = 1.5P = 1.5(12)(10^3) = 18000\ \text{N}\cdot\text{m}$$

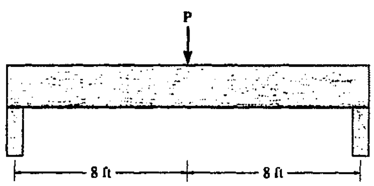
$$I = 1.953125(10^{-4})\text{m}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{18000(0.125)}{1.953125(10^{-4})} = 11.5\ \text{MPa} \qquad \textbf{Ans}$$

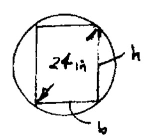


*6–92 A log that is 2 ft in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress for the wood is $\sigma_{allow} = 8$ ksi, determine the required width b and height h of the beam that will support the largest load possible. What is this load?





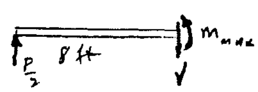
$$(24)^2 = b^2 + h^2$$



$$M_{max} = \frac{P}{2}(8)(12) = 48P$$

$$\sigma_{allow} = \frac{Mc}{I} = \frac{M_{max}(\frac{h}{2})}{\frac{1}{12}(b)(h)^3}$$

$$\sigma_{allow} = \frac{6\,M_{max}}{bh^2}$$



$$bh^2 = \frac{6}{8000}(48\,P)$$

$$b(24)^2 - b^3 = 0.036\,P$$

$$(24)^2 - 3\,b^2 = 0.036\,\frac{dP}{db} = 0$$

$b = 13.856$ in.

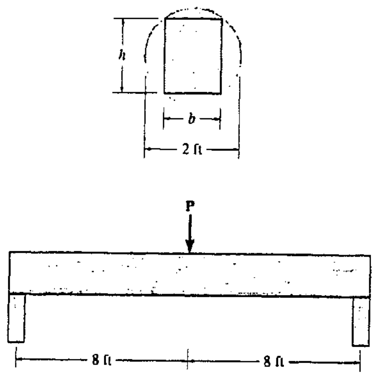
Thus, from the above equations,

$b = 13.9$ in. **Ans**

$h = 19.6$ in. **Ans**

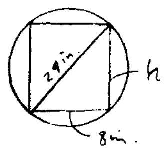
$P = 148$ kip **Ans**

6–93 A log that is 2 ft in diameter is to be cut into a rectangular section for use as a simply supported beam. If the allowable bending stress for the wood is $\sigma_{allow} = 8$ ksi, determine the largest load P that can be supported if the width of the beam is $b = 8$ in.



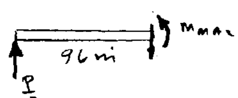
$$24^2 = h^2 + 8^2$$

$$h = 22.63 \text{ in.}$$



$$M_{max} = \frac{P}{2}(96) = 48\,P$$

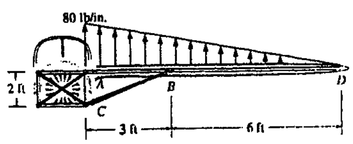
$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

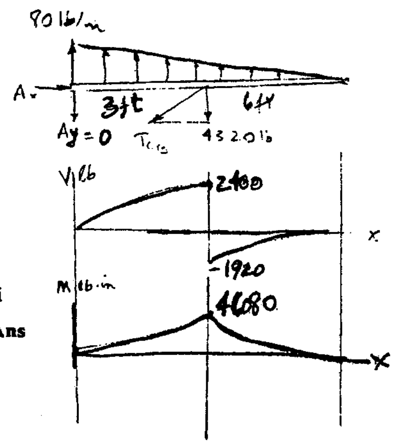


$$8(10^3) = \frac{48P(\frac{22.63}{2})}{\frac{1}{12}(8)(22.63)^3}$$

$P = 114$ kip **Ans**

6-94. The wing spar *ABD* of a light plane is made from 2014–T6 aluminum and has a cross-sectional area of 1.27 in., a depth of 3 in., and a moment of inertia about its neutral axis of 2.68 in^4. Determine the absolute maximum bending stress in the spar if the anticipated loading is to be as shown. Assume *A*, *B*, and *C* are pins. Connection is made along the central longitudinal axis of the spar.



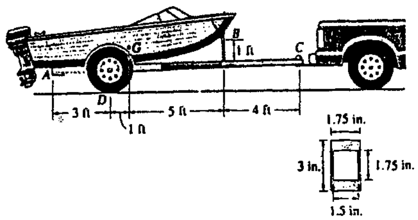


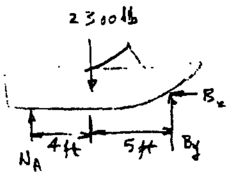
$$\sigma_{max} = \frac{Mc}{I}; \qquad \sigma_{max} = \frac{46080(1.5)}{2.68} = 25.8 \text{ ksi}$$

Ans

Note that 25.8 ksi < σ_Y = 60 ksi OK

6-95. The boat has a weight of 2300 lb and a center of gravity at G. If it rests on the trailer at the smooth contact A and can be considered pinned at B, determine the absolute maximum bending stress developed in the main strut of the trailer. Consider the strut to be a box-beam having the dimensions shown and pinned at C.





Boat :

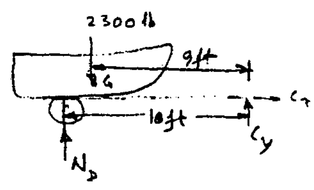
$\xrightarrow{+} \Sigma F_x = 0; \quad B_x = 0$

$\curvearrowleft + \Sigma M_B = 0; \quad -N_A(9) + 2300(5) = 0$

$N_A = 1277.78 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad 1277.78 - 2300 + B_y = 0$

$B_y = 1022.22 \text{ lb}$



Assembly :

$\curvearrowleft + \Sigma M_C = 0; \quad -N_D(10) + 2300(9) = 0$

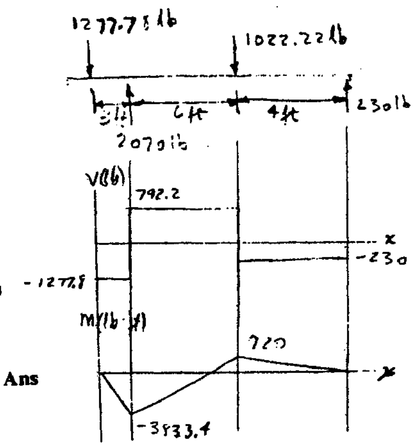
$N_D = 2070 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad C_y + 2070 - 2300 = 0$

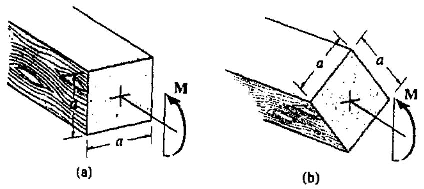
$C_y = 230 \text{ lb}$

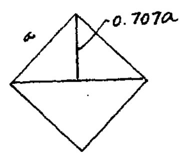
$$I = \frac{1}{12}(1.75)(3)^3 - \frac{1}{12}(1.5)(1.75)^3 = 3.2676 \text{ in}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{3833.4(12)(1.5)}{3.2676} = 21.1 \text{ ksi} \qquad \textbf{Ans}$$



***6–96** A wooden beam has a square cross section as shown. Determine which orientation of the beam provides the greatest strength at resisting the moment **M**. What is the difference in the resulting maximum stress in both cases?





Case (a) :

$$\sigma_{max} = \frac{Mc}{I} = \frac{M(a/2)}{\frac{1}{12}(a)^4} = \frac{6M}{a^3}$$

Case (b) :

$$I = 2[\frac{1}{36}(\frac{2}{\sqrt{2}}a)(\frac{1}{\sqrt{2}}a)^3 + \frac{1}{2}(\frac{2}{\sqrt{2}}a)(\frac{1}{\sqrt{2}}a)[(\frac{1}{\sqrt{2}}a)(\frac{1}{3})]^2] = 0.08333\,a^4$$

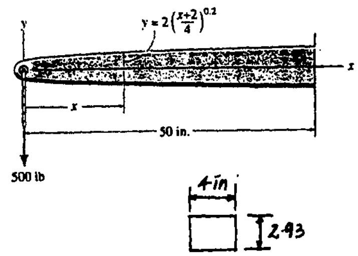
$$\sigma_{max} = \frac{Mc}{I} = \frac{M(\frac{1}{\sqrt{2}}a)}{0.08333\,a^4} = \frac{8.4853\,M}{a^3}$$

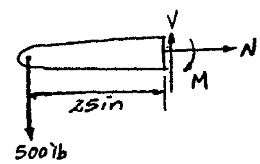
Case (a) provides higher strength since the resulting maximum stress is less for a given M and a.

Case (a) **Ans**

$$\Delta\sigma_{max} = \frac{8.4853\,M}{a^3} - \frac{6M}{a^3} = 2.49\,(\frac{M}{a^3}) \qquad \textbf{Ans}$$

6-97. The cantilevered beam has a thickness of 4 in. and a variable depth that can be described by the function $y = 2[(x + 2)/4]^{0.2}$, where x is in inches. Determine the maximum bending stress in the beam at its center.





At the same mid point :

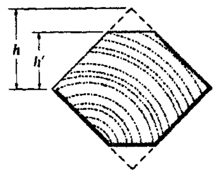
$$y = 2\left(\frac{25 + 2}{4}\right)^{0.2} = 2.93 \text{ in.}$$

$$\curvearrowleft + \Sigma M = 0; \qquad 500(25) - M = 0$$

$$M = 500(25) = 12{,}500 \text{ lb}\cdot\text{in.}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{12500(\frac{2.93}{2})}{\frac{1}{12}(4)(2.93)^3} = 2.18 \text{ ksi} \qquad \textbf{Ans}$$

6–98 A timber beam has a cross section which is originally square. If it is oriented as shown, determine the height h' so that it can resist the maximum moment possible. By what factor is this moment greater than that of the beam without its top or bottom flattened?



$$\frac{x}{h - h'} = \frac{2h}{h}; \qquad x = 2(h - h')$$

$$y = h' + \frac{h - h'}{3} = \frac{2h' + h}{3}$$

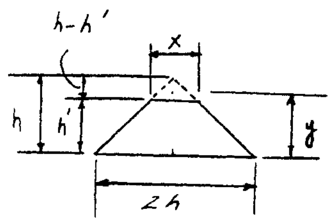
$$I = 2\{\frac{1}{12}(2h)(h^3) - [\frac{1}{36}(2)(h - h')(h - h')^3 + \frac{1}{2}(2)(h - h')(h - h')(\frac{2h' + h}{3})^2]\}$$

$$= \frac{1}{3}h^4 - \frac{1}{9}(h - h')^4 - \frac{2}{9}(h - h')^2(2h' + h)^2$$

$$= \frac{1}{3}h^4 - \frac{1}{9}(h - h')^2[3h^2 + 9h'^2 + 6hh']$$

$$= \frac{1}{3}h^4 - \frac{1}{9}(3h^4 + 9h'^4 - 12hh'^3)$$

$$= \frac{4}{3}hh'^3 - h'^4$$



$$\sigma_{max} = \frac{Mc}{I}$$

$$M = \frac{I}{c}\sigma_{max} \qquad (1)$$

$$= \frac{\frac{4}{3}hh'^3 - h'^4}{h'}\sigma_{max} = (\frac{4}{3}hh'^2 - h'^3)\sigma_{max}$$

$$\frac{dM}{dh'} = (\frac{8}{3}hh' - 3h'^2)\sigma_{max}$$

In - order to have maximum moment,

$$\frac{dM}{dh'} = 0 = \frac{8}{3}hh' - 3h'^2$$

$$h' = \frac{8}{9}h \qquad \textbf{Ans}$$

For the square beam,

$$I = \bar{I} + Ad^2$$

$$I = 2[\frac{1}{36}(2h)(h)^3 + \frac{1}{12}(2h)(h)(\frac{h}{3})^2] = \frac{h^4}{3}$$

From Eq. (1)

$$M = \frac{\frac{h^4}{3}}{h}\sigma_{max} = \frac{h^3}{3}\sigma_{max} = 0.3333h^3\sigma_{max}$$

For the flattened beam :

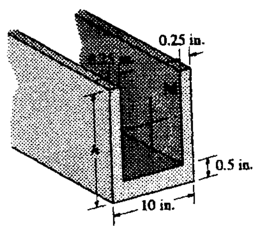
$$I = \frac{4}{3}h(\frac{8}{9}h)^3 - (\frac{8}{9}h)^4 = 0.312147\,h^4$$

From Eq. (1)

$$M' = \frac{0.312147\,h^4}{\frac{8}{9}h}\sigma_{max} = 0.35117\,h^3\sigma_{max}$$

$$\text{Factor} = \frac{M'}{M'} = \frac{0.35117\,h^3\sigma_{max}}{0.3333h^3\sigma_{max}} = 1.05 \qquad \textbf{Ans}$$

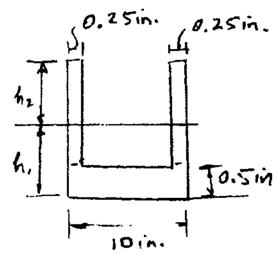
6–99 A beam is to be molded from polyethylene plastic and have the cross section shown. Determine its largest required height so that it supports the greatest moment M. What is this moment? The allowable tensile and compressive stress for the material is $(\sigma_{allow})_t$ = 10 ksi and $(\sigma_{allow})_c$ = 30 ksi, respectively.



Require,

$$\sigma_c = \frac{Mh_2}{I}, \qquad \sigma_t = \frac{Mh_1}{I}$$

Since $\sigma_c = 3\,\sigma_t$



$$\frac{Mh_2}{I} = \frac{3\,Mh_1}{I}$$

$$h_2 = 3\,h_1$$
$$h_1 + h_2 = h$$
$$h_1 + 3h_1 = h$$
$$h_1 = 0.25\,h$$
$$h_2 = 0.75h$$

Also,

$\Sigma \bar{y}A = 0;$ $\quad 2[(0.75h)(0.25)(0.375h)] - 2[(0.25h - 0.5)(0.25)(0.25h - 0.5)/2] - (0.5)(10)(0.25h - 0.25) = 0$

$$0.140625\,h^2 - 0.015625\,h^2 + 0.0625\,h - 0.0625 - 1.25\,h + 1.25 = 0$$

$$0.125\,h^2 - 1.1875\,h + 1.1875 = 0$$

Roots are

h = 8.364 in. and 1.136 in.

Choosing the largest root,

h = 8.364 in. = 8.36 in. **Ans**

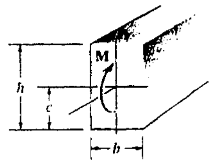
$h_1 = 0.25(8.364) = 2.091$ in.

$h_2 = 0.75(8.364) = 6.273$ in.

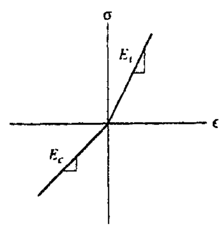
$$I = \left[\frac{1}{12}(10)(8.364)^3 + (10)(8.364)\left(\frac{8.364}{2} - 6.2731\right)^2\right]$$
$$-\left[\frac{1}{12}(9.5)(8.364 - 0.5)^3 + 9.5(8.364 - 0.5)\left(\frac{(8.364 - 0.5)}{2} - 6.273\right)^2\right] = 58.863\ \text{in}^4$$

$$\sigma_{max} = \frac{Mc}{I}; \qquad 10 = \frac{M(2.091)}{58.863}; \qquad M = 281.5\ \text{kip}\cdot\text{in.} = 23.5\ \text{kip}\cdot\text{ft} \quad \textbf{Ans}$$

*6–100 A beam is made of a material that has a modulus of elasticity in compression different from that given for tension. Determine the location c of the neutral axis, and derive an expression for the maximum tensile stress in the beam having the dimensions shown if it is subjected to the bending moment M.



$$(\varepsilon_{max})_c = \frac{(\varepsilon_{max})_t(h-c)}{c}$$



$$(\sigma_{max})_c = E_c(\varepsilon_{max})_c = \frac{E_c(\varepsilon_{max})_t(h-c)}{c}$$

Location of neutral axis :

$$\xrightarrow{+}\Sigma F = 0; \qquad -\frac{1}{2}(h-c)(\sigma_{max})_c(b) + \frac{1}{2}(c)(\sigma_{max})_t(b) = 0$$

$$(h-c)(\sigma_{max})_c = c(\sigma_{max})_t \qquad [1]$$

$$(h-c)E_c(\varepsilon_{max})_t\frac{(h-c)}{c} = cE_t(\varepsilon_{max})_t\,; \qquad E_c(h-c)^2 = E_t c^2$$

Taking positive root :

$$\frac{c}{h-c} = \sqrt{\frac{E_c}{E_t}}$$

$$c = \frac{h\sqrt{\frac{E_c}{E_t}}}{1+\sqrt{\frac{E_c}{E_t}}} = \frac{h\sqrt{E_c}}{\sqrt{E_t}+\sqrt{E_c}} \qquad [2] \qquad \textbf{Ans}$$

$$\Sigma M_{NA} = 0;$$

$$M = [\frac{1}{2}(h-c)(\sigma_{max})_c(b)](\frac{2}{3})(h-c) + [\frac{1}{2}(c)(\sigma_{max})_t(b)](\frac{2}{3})(c)$$

$$M = \frac{1}{3}(h-c)^2(b)(\sigma_{max})_c + \frac{1}{3}c^2 b(\sigma_{max})_t$$

From Eq. [1], $(\sigma_{max})_c = \frac{c}{h-c}(\sigma_{max})_t$

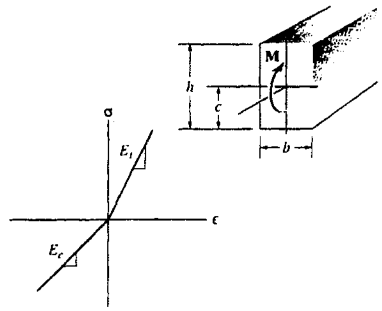
$$M = \frac{1}{3}(h-c)^2(b)(\frac{c}{h-c})(\sigma_{max})_t + \frac{1}{3}c^2 b(\sigma_{max})_t$$

$$M = \frac{1}{3}bc(\sigma_{max})_t(h-c+c)\,; \qquad (\sigma_{max})_t = \frac{3M}{b\,h\,c}$$

From Eq. [2]

$$(\sigma_{max})_t = \frac{3M}{b\,h^2}(\frac{\sqrt{E_t}+\sqrt{E_c}}{\sqrt{E_c}}) \qquad \textbf{Ans}$$

6–101 The beam has a rectangular cross section and is subjected to a bending moment M. If the material from which it is made has a different modulus of elasticity for tension and compression as shown, determine the location c of the neutral axis and the maximum compressive stress in the beam.



See the solution to Prob. 6 – 100

$$c = \frac{h\sqrt{E_c}}{\sqrt{E_t} + \sqrt{E_c}} \qquad \textbf{Ans}$$

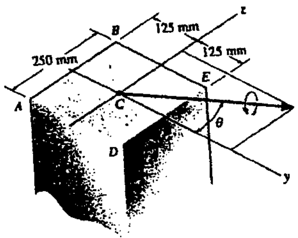
Since $(\sigma_{max})_c = \frac{c}{h - c}(\sigma_{max})_t = \frac{h\sqrt{E_c}}{(\sqrt{E_t} + \sqrt{E_c})[h - (\frac{h\sqrt{E_c}}{\sqrt{E_t} + \sqrt{E_c}})]}(\sigma_{max})_t$

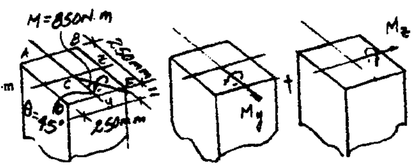
$$(\sigma_{max})_c = \frac{\sqrt{E_c}}{\sqrt{E_t}}(\sigma_{max})_t$$

$$(\sigma_{max})_c = \frac{\sqrt{E_c}}{\sqrt{E_t}}(\frac{3M}{bh^2})(\frac{\sqrt{E_t} + \sqrt{E_c}}{\sqrt{E_c}})$$

$$(\sigma_{max})_c = \frac{3M}{bh^2}(\frac{\sqrt{E_t} + \sqrt{E_c}}{\sqrt{E_t}}) \qquad \textbf{Ans}$$

6-102. The member has a square cross section and is subjected to a resultant internal bending moment of $M = 850\ \text{N}\cdot\text{m}$ as shown. Determine the stress at each corner and sketch the stress distribution produced by **M**. Set $\theta = 45°$.





$$M_y = 850 \cos 45° = 601.04\ \text{N}\cdot\text{m}$$

$$M_z = 850 \sin 45° = 601.04\ \text{N}\cdot\text{m}$$

$$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\ \text{m}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{601.04\,(-0.125)}{0.3255208(10^{-3})} + \frac{601.04\,(-0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans}$$

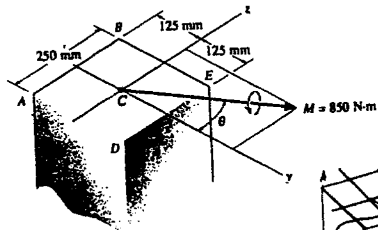
$$\sigma_B = -\frac{601.04\,(-0.125)}{0.3255208(10^{-3})} + \frac{601.04\,(0.125)}{0.3255208(10^{-3})} = 462\ \text{kPa} \qquad \textbf{Ans}$$

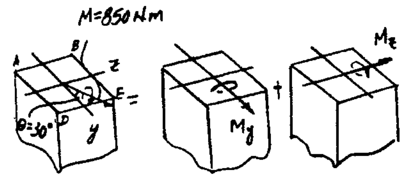
$$\sigma_D = -\frac{601.04\,(0.125)}{0.3255208(10^{-3})} + \frac{601.04\,(-0.125)}{0.3255208(10^{-3})} = -462\ \text{kPa} \qquad \textbf{Ans}$$

$$\sigma_E = -\frac{601.04\,(0.125)}{0.3255208(10^{-3})} + \frac{601.04\,(0.125)}{0.3255208(10^{-3})} = 0 \qquad \textbf{Ans}$$

The negative sign indicates compressive stress.

6-103. The member has a square cross section and is subjected to a resultant internal bending moment of $M = 850\ \mathrm{N \cdot m}$ as shown. Determine the stress at each corner and sketch the stress distribution produced by **M**. Set $\theta = 30°$.





$M_y = 850 \cos 30° = 736.12\ \mathrm{N \cdot m}$

$M_z = 850 \sin 30° = $ [illegible]$5\ \mathrm{N \cdot m}$

$I_z = I_y = \frac{1}{12}(0.25)(0.25)^3 = 0.3255208(10^{-3})\ \mathrm{m}^4$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

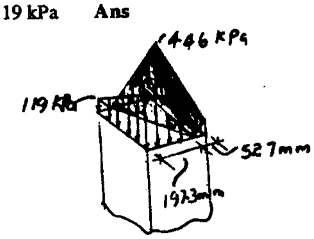
$$\sigma_A = -\frac{425\,(-0.125)}{0.3255208(10^{-3})} + \frac{736.12\,(-0.125)}{0.3255208(10^{-3})} = -119\ \mathrm{kPa} \quad \textbf{Ans}$$

$$\sigma_B = -\frac{425\,(-0.125)}{0.3255208(10^{-3})} + \frac{736.12\,(0.125)}{0.3255208(10^{-3})} = 446\ \mathrm{kPa} \quad \textbf{Ans}$$

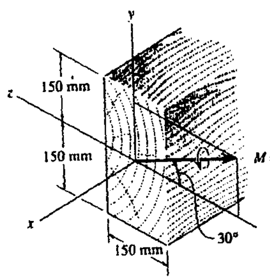
$$\sigma_D = -\frac{425\,(0.125)}{0.3255208(10^{-3})} + \frac{736.12\,(-0.125)}{0.3255208(10^{-3})} = -446\ \mathrm{kPa} \quad \textbf{Ans}$$

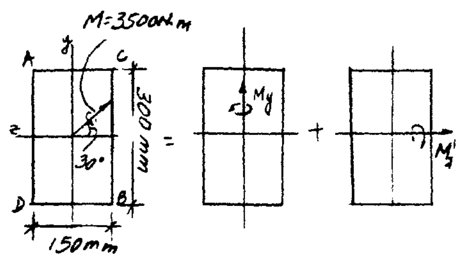
$$\sigma_E = -\frac{425\,(0.125)}{0.3255208(10^{-3})} + \frac{736.12\,(0.125)}{0.3255208(10^{-3})} = 119\ \mathrm{kPa} \quad \textbf{Ans}$$

The negative signs indicate compressive stress.



•6–104 The beam has a rectangular cross section. If it is subjected to a moment of $M = 3500\ \text{N}\cdot\text{m}$ directed as shown, determine the maximum bending stress in the beam and the orientation of the neutral axis.





$$M_y = 3500 \sin 30^\circ = 1750\ \text{N}\cdot\text{m}$$

$$M_z = 3500 \cos 30^\circ = -3031.09\ \text{N}\cdot\text{m}$$

$$I_y = \frac{1}{12}(0.3)(0.15^3) = 84.375(10^{-6})\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.15)(0.3^3) = 0.3375(10^{-3})\ \text{m}^4$$

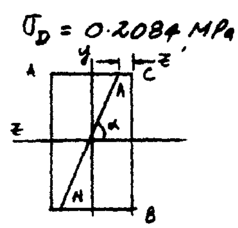
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-3031.09\,(0.15)}{0.3375(10^{-3})} + \frac{1750\,(0.075)}{84.375(10^{-6})} = 2.90\ \text{MPa (max)} \qquad \textbf{Ans}$$

$$\sigma_B = -\frac{-3031.09\,(-0.15)}{0.3375(10^{-3})} + \frac{1750\,(-0.075)}{84.375(10^{-6})} = -2.90\ \text{MPa (max)} \qquad \textbf{Ans}$$

$$\sigma_C = -\frac{-3031.09\,(0.15)}{0.3375(10^{-3})} + \frac{1750\,(-0.075)}{84.375(10^{-6})} = -0.2084\ \text{MPa}$$

$$\sigma_D = 0.2084\ \text{MPa}$$



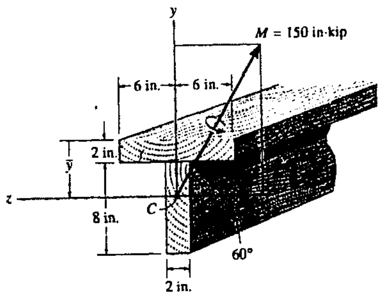
$$\frac{z'}{0.2084} = \frac{150 - z'}{2.90}$$

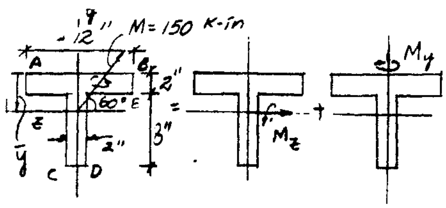
$$z' = 10.0\ \text{mm}$$

$$\tan \alpha = \frac{I_z}{I_y}\tan\theta = \frac{3.375\,(10^{-4})}{8.4375(10^{-5})}\tan(-30^\circ)$$

$$\alpha = -66.6^\circ \qquad \textbf{Ans}$$

6–105 The T-beam is subjected to a moment of $M = 150$ kip · in. directed as shown. Determine the maximum bending stress in the beam and the orientation of the neutral axis. The location $\bar{y}$ of the centroid, C, must be determined.





$$M_y = 150 \sin 60^\circ = 129.9 \text{ kip} \cdot \text{in.}$$

$$M_z = -150 \cos 60^\circ = -75 \text{ kip} \cdot \text{in.}$$

$$\bar{y} = \frac{(1)(12)(2) + (6)(8)(2)}{12(2) + 8(2)} = 3 \text{ in.}$$

$$I_y = \frac{1}{12}(2)(12^3) + \frac{1}{12}(8)(2^3) = 293.33 \text{ in}^4$$

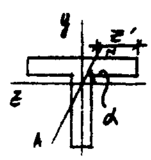
$$I_z = \frac{1}{12}(12)(2^3) + 12(2)(2^2) + \frac{1}{12}(2)(8^3) + 2(8)(3^2) = 333.33 \text{ in}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-(-75)(3)}{333.33} + \frac{129.9(6)}{293.33} = 3.33 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_D = \frac{-(-75)(-7)}{333.33} + \frac{129.9(-1)}{293.33} = -2.02 \text{ ksi}$$

$$\sigma_B = \frac{-(-75)(3)}{333.33} + \frac{129.9(-6)}{293.33} = -1.982 \text{ ksi}$$



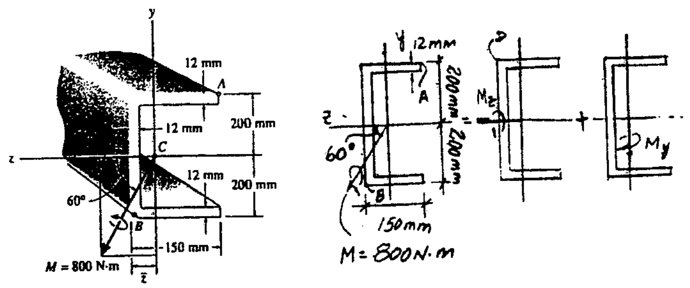
$$\frac{z'}{1.982} = \frac{12 - z'}{3.333}$$

$$z' = 4.47 \text{ in.}$$

$$\tan \alpha = \frac{I_z}{I_y} \tan \theta = \frac{333.33}{293.33} \tan(-60^\circ)$$

$$\alpha = -63.1^\circ \qquad \textbf{Ans}$$

6-106. If the internal moment acting on the cross section of the strut has a magnitude of $M = 800\ \text{N}\cdot\text{m}$ and is directed as shown, determine the bending stress at points A and B. The location $\bar{z}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.



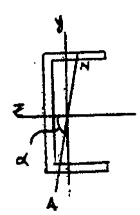
$$M_z = 800\cos 60^\circ = 400\ \text{N}\cdot\text{m}$$
$$M_y = -800\sin 60^\circ = -692.82\ \text{N}\cdot\text{m}$$

$$\bar{z} = \frac{400\,(12)(6) + 2\,(138)(12)(81)}{400\,(12) + 2\,(138)(12)} = 36.6\ \text{mm}$$

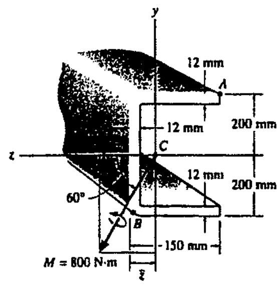
$$I_z = \frac{1}{12}(0.15)(0.4^3) - \frac{1}{12}(0.138)(0.376^3) = 0.18869\,(10^{-3})\ \text{m}^4$$
$$I_y = \frac{1}{12}(0.4)(0.012^3) + (0.4)(0.012)(0.03062^2)$$
$$+2\left[\frac{1}{12}(0.012)(0.138^3) + (0.138)(0.012)(0.04438^2)\right] = 16.3374\,(10^{-6})\ \text{m}^4$$

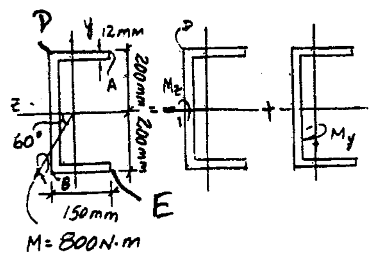
$$\sigma = -\frac{M_z\,y}{I_z} + \frac{M_y\,z}{I_y}$$
$$\sigma_A = \frac{-(400)(0.2)}{0.18869\,(10^{-3})} + \frac{(-692.82)(-0.11338)}{16.3374\,(10^{-6})} = 4.38\ \text{MPa} \qquad \textbf{Ans}$$
$$\sigma_B = \frac{-(400)(-0.2)}{0.18869\,(10^{-3})} + \frac{(-692.82)(0.036621)}{16.3374\,(10^{-6})} = -1.13\ \text{MPa} \qquad \textbf{Ans}$$

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$
$$\tan\alpha = \frac{1.8869\,(10^{-4})}{1.63374(10^{-5})}\tan(-60)$$
$$\alpha = -87.1^\circ \qquad \textbf{Ans}$$



6-107. The resultant moment acting on the cross section of the aluminum strut has a magnitude of $M = 800\ \text{N}\cdot\text{m}$ and is directed as shown. Determine the maximum bending stress in the strut. The location $\bar{y}$ of the centroid C of the strut's cross-sectional area must be determined. Also, specify the orientation of the neutral axis.





$M_z = 800 \cos 60° = 400\ \text{N}\cdot\text{m}$

$M_y = -800 \sin 60° = -692.82\ \text{N}\cdot\text{m}$

$\bar{z} = \dfrac{400\,(12)(6) + 2\,(138)(12)(81)}{400\,(12) + 2\,(138)(12)} = 36.6\ \text{mm}$ **Ans**

$I_z = \dfrac{1}{12}(0.15)(0.4^3) - \dfrac{1}{12}(0.138)(0.376^3) = 0.18869\,(10^{-3})\ \text{m}^4$

$I_y = \dfrac{1}{12}(0.4)(0.012^3) + (0.4)(0.012)(0.03062^2)$

$+ 2\left[\dfrac{1}{12}(0.012)(0.138^3) + (0.138)(0.012)(0.04438^2)\right] = 16.3374\,(10^{-6})\ \text{m}^4$

$\sigma = -\dfrac{M_z\,y}{I_z} + \dfrac{M_y\,z}{I_y}$

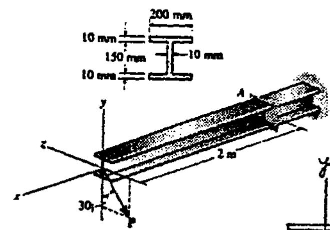
$\sigma_A = \dfrac{-(400)(0.2)}{0.18869\,(10^{-3})} + \dfrac{(-692.82)(-0.11338)}{16.3374\,(10^{-6})} = 4.38\ \text{MPa}$

$\sigma_B = \dfrac{-(400)(-0.2)}{0.18869\,(10^{-3})} + \dfrac{(-692.82)(0.036621)}{16.3374\,(10^{-6})} = -1.13\ \text{MPa}$

$\sigma_D = \dfrac{-(400)(0.2)}{0.18869\,(10^{-3})} + \dfrac{(-692.82)(0.036621)}{16.3374\,(10^{-6})} = -1.977\ \text{MPa}$

$\sigma_E = -\dfrac{(400)(-0.2)}{0.18869(10^{-3})} + \dfrac{(-692.82)(-0.11338)}{16.3374(10^{-6})} = 5.23\ \text{MPa}$ **Ans**

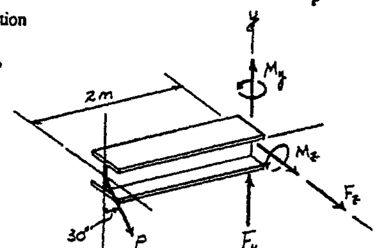
***6-108.** The cantilevered wide-flange steel beam is subjected to the concentrated force **P** at its end. Determine the largest magnitude of this force so that the bending stress developed at A does not exceed $\sigma_{allow} = 180$ MPa.

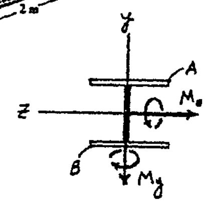


Internal Moment Components : Using method of section

$\Sigma M_z = 0;\quad M_z + P\cos 30°(2) = 0\quad M_z = -1.732P$

$\Sigma M_y = 0;\quad M_y + P\sin 30°(2) = 0;\quad M_y = -1.00P$





Section Properties :

$$I_z = \frac{1}{12}(0.2)(0.17^3) - \frac{1}{12}(0.19)(0.15^3) = 28.44583(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)(0.2^3)\right] + \frac{1}{12}(0.15)(0.01^3) = 13.34583(10^{-6})\ \text{m}^4$$

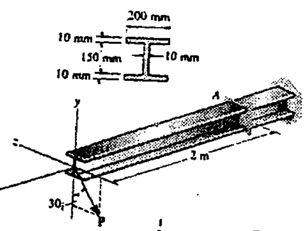
Allowable Bending Stress : By inspection, maximum bending stress occurs at points A and B. Applying the flexure formula for biaxial bending at point A,

$$\sigma_A = \sigma_{allow} = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$180(10^6) = -\frac{(-1.732P)(0.085)}{28.44583(10^{-6})} + \frac{-1.00P(-0.1)}{13.34583(10^{-6})}$$

$P = 14208\ \text{N} = 14.2\ \text{kN}$ **Ans**

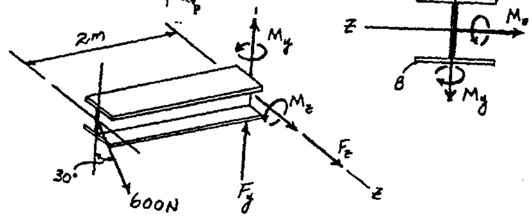
6-109. The cantilevered wide-flange steel beam is subjected to the concentrated force of $P = 600$ N at its end. Determine the maximum bending stress developed in the beam at section A.



Internal Moment Components : Using method of sections

$\Sigma M_z = 0;\quad M_z + 600\cos 30°(2) = 0 \quad M_z = -1039.23\ \text{N}\cdot\text{m}$

$\Sigma M_y = 0;\quad M_y + 600\sin 30°(2) = 0; \quad M_y = -600.0\ \text{N}\cdot\text{m}$



Section Properties :

$$I_z = \frac{1}{12}(0.2)(0.17^3) - \frac{1}{12}(0.19)(0.15^3) = 28.44583(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{1}{12}(0.01)(0.2^3)\right] + \frac{1}{12}(0.15)(0.01^3) = 13.34583(10^{-6})\ \text{m}^4$$

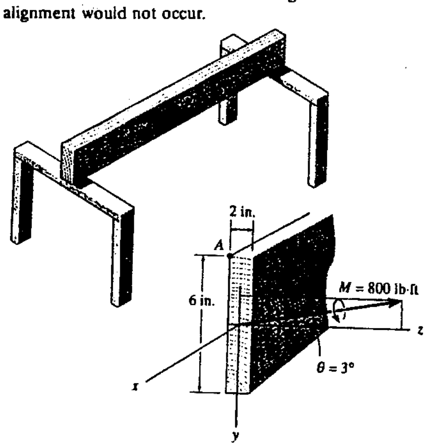
Maximum Bending Stress : By inspection, maximum bending stress occurs at A and B. Applying the flexure formula for biaxial bending at points A and B

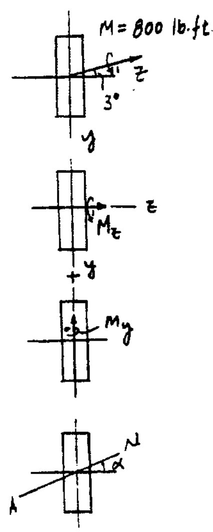
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{-1039.32(0.085)}{28.44583(10^{-6})} + \frac{-600.0(-0.1)}{13.34583(10^{-6})}$$
$= 7.60$ MPa (T) **(Max)** **Ans**

$$\sigma_B = -\frac{(-1039.32)(-0.085)}{28.44583(10^{-6})} + \frac{-600.0(0.1)}{13.34583(10^{-6})}$$
$= -7.60$ MPa $= 7.60$ MPa (C) **(Max)** **Ans**

6-110 The board is used as a simply supported floor joist. If a bending moment of $M = 800$ lb · ft is applied 3° from the z axis, determine the bending stress developed in the board at the corner A. Compare this stress with that developed by the same moment applied along the z axis ($\theta = 0°$). What is the angle α for the neutral axis when $\theta = 3°$? *Comment:* Normally, floor boards would be nailed to the top of the beam so that $\theta \approx 0°$ and the high stress due to misalignment would not occur.





$$M_z = 800 \cos 3° = 798.904 \text{ lb} \cdot \text{ft}$$

$$M_y = -800 \sin 3° = -41.869 \text{ lb} \cdot \text{ft}$$

$$I_z = \frac{1}{12}(2)(6^3) = 36 \text{ in}^4 ; \qquad I_y = \frac{1}{12}(6)(2^3) = 4 \text{ in}^4$$

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = -\frac{798.904(12)(-3)}{36} + \frac{-41.869(12)(-1)}{4} = 924 \text{ psi}$$ **Ans**

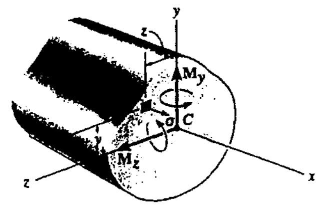
$$\tan \alpha = \frac{I_z}{I_y} \tan \theta ; \qquad \tan \alpha = \frac{36}{4} \tan(-3°)$$

$$\alpha = -25.3°$$ **Ans**

When $\theta = 0°$

$$\sigma_A = \frac{Mc}{I} = \frac{800(12)(3)}{36} = 800 \text{ psi}$$ **Ans**

6–111 Consider the general case of a prismatic beam subjected to bending-moment components $\mathbf{M}_y$ and $\mathbf{M}_z$, as shown, when the x, y, z axes pass through the centroid of the cross section. If the material is linear-elastic, the normal stress in the beam is a linear function of position such that $\sigma = a + by + cz$. Using the equilibrium conditions $0 = \int_A \sigma \, dA$, $M_y = \int_A z\sigma \, dA$, $M_z = \int_A -y\sigma \, dA$, determine the constants a, b, and c, and show that the normal stress can be determined from the equation $\sigma = [-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z]/(I_y I_z - I_{yz}^2)$, where the moments and products of inertia are defined in Appendix A.



$$\sigma_x = a + by + cz$$

$$0 = \int_A \sigma_x \, dA = \int_A (a + by + cz)\, dA$$

$$= a\int_A dA + b\int_A y\, dA + c\int_A z\, dA$$

$$M_y = \int_A z\,\sigma_x \, dA = \int_A z(a + by + cz)\, dA$$

$$= a\int_A z\, dA + b\int_A yz\, dA + c\int_A z^2\, dA$$

$$M_z = \int_A -y\,\sigma_x \, dA = \int_A -y(a + by + cz)\, dA$$

$$= -a\int_A y dA - b\int_A y^2\, dA - c\int_A yz\, dA$$

The integrals are defined in Appendix A. Note that $\int_A y\, dA = \int_A z\, dA = 0.$

Thus, $0 = aA$

$$M_y = bI_{yz} + cI_y \; ; \qquad M_z = -bI_z - cI_{yz}$$

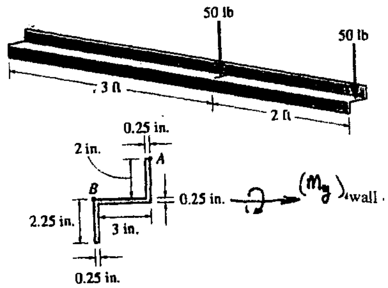
Solving for a, b, c :

$a = 0$ (Since $A \neq 0$)

$$b = -\left(\frac{I_y M_z + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right); \qquad c = \frac{I_z M_y + M_z I_{yz}}{I_y I_z - I_{yz}^2}$$

Thus, $\quad \sigma_x = -\left(\dfrac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\dfrac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z \qquad$ **QED**

***6-112.** The cantilevered beam is made from the Z section having the cross-section shown. If it supports the two loadings, determine the bending stress at the wall in the beam at point A. Use the result of Prob. 6–111.



$$(M_y)_{wall} = 50(3) + 50(5) = 400\ \text{lb}\cdot\text{ft} = 4.80(10^3)\text{lb}\cdot\text{in.}$$

$$I_y = \frac{1}{12}(3.25)(0.25)^3 + 2[\frac{1}{12}(0.25)(2)^3 + (0.25)(2)(1.125)^2] = 1.60319\ \text{in}^4$$

$$I_z = \frac{1}{12}(0.25)(3.25)^3 + 2[\frac{1}{12}(2)(0.25)^3 + (0.25)(2)(1.5)^2] = 2.970378\ \text{in}^4$$

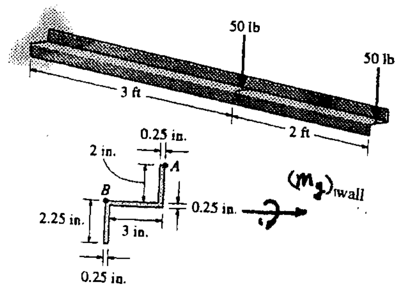
$$I_{yz} = 2[1.5(1.125)(2)(0.25)] = 1.6875\ \text{in}^4$$

Using the equation developed in Prob. 6 - 111,

$$\sigma = -(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2})y + (\frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2})z$$

$$\sigma_A = \frac{\{-[0 + (4.80)(10^3)(1.6875)](1.625) + [(4.80)(10^3)(2.970378) + 0](2.125)\}}{[1.60319(2.970378) - (1.6875)^2]} = 8.95\ \text{ksi} \qquad \textbf{Ans}$$

6-113. The cantilevered beam is made from the Z section having the cross-section shown. If it supports the two loadings, determine the bending stress at the wall in the beam at point B. Use the result of Prob. 6–111.



$(M_y)_{wall} = 50(3) + 50(5) = 400\ \text{lb}\cdot\text{ft} = 4.80(10^3)\text{lb}\cdot\text{in.}$

$I_y = \frac{1}{12}(3.25)(0.25)^3 + 2[\frac{1}{12}(0.25)(2)^3 + (0.25)(2)(1.125)^2] = 1.60319\ \text{in}^4$

$I_z = \frac{1}{12}(0.25)(3.25)^3 + 2[\frac{1}{12}(2)(0.25)^3 + (0.25)(2)(1.5)^2] = 2.970378\ \text{in}^4$

$I_{yz} = 2[1.5(1.125)(2)(0.25)] = 1.6875\ \text{in}^4$

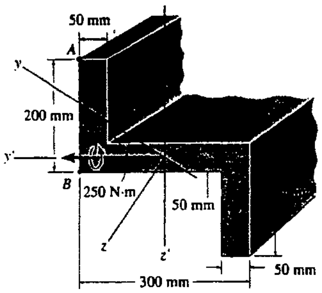
Using the equation developed in Prob. 6-111,

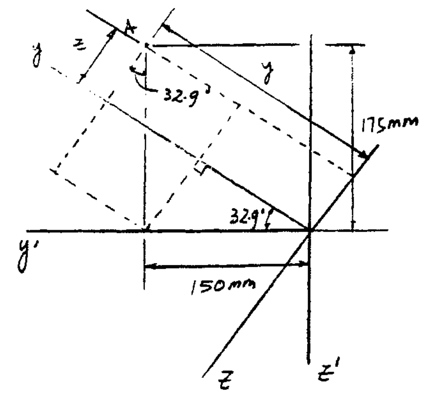
$$\sigma = -(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2})y + (\frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2})z$$

$$\sigma_B = \frac{-[0 + (4.80)(10^3)(1.6875)](-1.625) + [(4.80)(10^3)(2.976378) + 0](0.125)}{[(1.60319)(2.970378) - (1.6875)^2]}$$

$= 7.81$ ksi **Ans**

6-114. Using the techniques outlined in Appendix A, Example A.5 or A.6, the Z section has principal moments of inertia of $I_y = 0.060(10^{-3})\ \text{m}^4$ and $I_z = 0.471(10^{-3})\ \text{m}^4$, computed about the principal axes of inertia y and z, respectively. If the section is subjected to an internal moment of $M = 250\ \text{N}\cdot\text{m}$ directed horizontally as shown, determine the stress produced at point A. Solve the problem using Eq. 6–17.





$$M_y = 250\cos 32.9^\circ = 209.9\ \text{N}\cdot\text{m}$$

$$M_z = 250\sin 32.9^\circ = 135.8\ \text{N}\cdot\text{m}$$

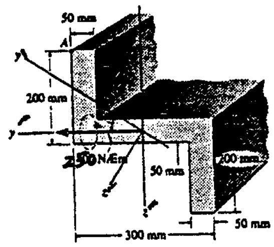
$$y = 0.15\cos 32.9^\circ + 0.175\sin 32.9^\circ = 0.2210\ \text{m}$$

$$z = -(0.175\cos 32.9^\circ - 0.15\sin 32.9^\circ) = -0.06546\ \text{m}$$

$$\sigma_A = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} = \frac{-135.8(0.2210)}{0.471(10^{-3})} + \frac{209.9(-0.06546)}{60.0(10^{-6})}$$

$$= -293\ \text{kPa} = 293\ \text{kPa (C)} \qquad \textbf{Ans}$$

* 6-115. Solve Prob. 6–114 using the equation developed in Prob. 6–111.



Internal Moment Components :

$$M_y = 250\ \text{N}\cdot\text{m} \qquad M_z = 0$$

Section Properties :

$$I_y = \frac{1}{12}(0.3)\left(0.05^3\right) + 2\left[\frac{1}{12}(0.05)\left(0.15^3\right) + 0.05(0.15)\left(0.1^2\right)\right]$$
$$= 0.18125\left(10^{-3}\right)\ \text{m}^4$$

$$I_z = \frac{1}{12}(0.05)\left(0.3^3\right) + 2\left[\frac{1}{12}(0.15)\left(0.05^3\right) + 0.15(0.05)\left(0.125^2\right)\right]$$
$$= 0.350\left(10^{-3}\right)\ \text{m}^4$$

$$I_{yz} = 0.15(0.05)(0.125)(-0.1) + 0.15(0.05)(-0.125)(0.1)$$
$$= -0.1875\left(10^{-3}\right)\ \text{m}^4$$

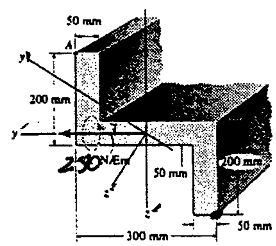
Bending Stress : Using formula developed in Prob. 6 - 110

$$\sigma = \frac{-(M_z I_y + M_y I_{yz})y + (M_y I_z + M_z I_{yz})z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{-[0 + 250(-0.1875)(10^{-3})](0.15) + [250(0.350)(10^{-3}) + 0](-0.175)}{0.18125(10^{-3})(0.350)(10^{-3}) - [0.1875(10^{-3})]^2}$$

$$= -293\ \text{kPa} = 293\ \text{kPa (C)} \qquad \textbf{Ans}$$

***6-116.** Using the techniques outlined in Appendix *A*, Example *A*.5 or *A*.6, the *Z* section has principal moments of inertia of $I_y = 0.060(10^{-3})\ \mathrm{m^4}$ and $I_z = 0.471(10^{-3})\ \mathrm{m^4}$, computed about the principal axes of inertia *y* and *z*, respectively. If the section is subjected to an internal moment of $M = 250\ \mathrm{N \cdot m}$ directed horizontally as shown, determine the stress produced at point *B*. Solve the problem using Eq. 6–17.

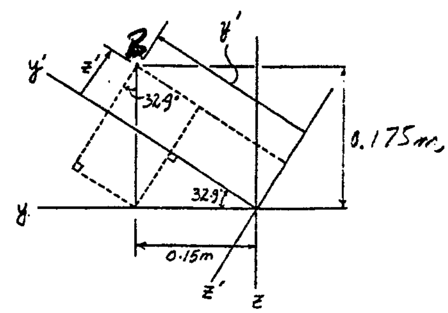


Internal Moment Components :

$$M_{y'} = 250 \cos 32.9° = 209.9\ \mathrm{N \cdot m}$$
$$M_{z'} = 250 \sin 32.9° = 135.8\ \mathrm{N \cdot m}$$

Section Property :

$$y' = 0.15 \cos 32.9° + 0.175 \sin 32.9° = 0.2210\ \mathrm{m}$$
$$z' = 0.15 \sin 32.9° - 0.175 \cos 32.9° = -0.06546\ \mathrm{m}$$

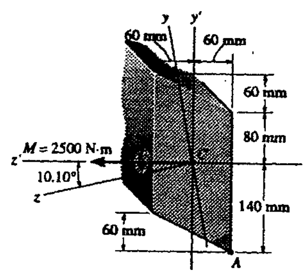


Bending Stress : Applying the flexure formula for biaxial bending

$$\sigma = -\frac{M_{z'} y'}{I_{z'}} + \frac{M_{y'} z'}{I_{y'}}$$

$$\sigma_B = -\frac{135.8(0.2210)}{0.471(10^{-3})} + \frac{209.9(-0.06546)}{0.060(10^{-3})}$$
$$= -293\ \mathrm{kPa} = 293\ \mathrm{kPa\ (C)} \qquad \textbf{Ans}$$

6-117. For the section, $I_{y'} = 31.7(10^{-6})\ \text{m}^4$, $I_{z'} = 114(10^{-6})\ \text{m}^4$, $I_{y'z'} = 15.1(10^{-6})\ \text{m}^4$. Using the techniques outlined in Appendix *A*, the member's cross-sectional area has principal moments of inertia of $I_y = 29.0(10^{-6})\ \text{m}^4$ and $I_z = 117(10^{-6})\ \text{m}^4$, computed about the principal axes of inertia *y* and *z*, respectively. If the section is subjected to a moment of $M = 2500\ \text{N}\cdot\text{m}$ directed as shown, determine the stress produced at point *A*, using Eq. 6–17.



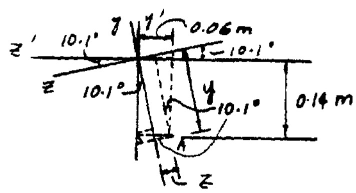
$I_z = 117(10^{-6})\text{m}^4 \qquad I_y = 29.0(10^{-6})\text{m}^4$

$M_y = 2500 \sin 10.1° = 438.42\ \text{N}\cdot\text{m}$

$M_z = 2500 \cos 10.1° = 2461.26\ \text{N}\cdot\text{m}$

$y = -0.06 \sin 10.1° - 0.14 \cos 10.1° = -0.14835\ \text{m}$

$z = 0.14 \sin 10.1° - 0.06 \cos 10.1° = -0.034519\ \text{m}$



$$\sigma_A = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$= \frac{-2461.26(-0.14835)}{117(10^{-6})} + \frac{438.42(-0.034519)}{29.0(10^{-6})} = 2.60\ \text{MPa (T)} \qquad \textbf{Ans}$$

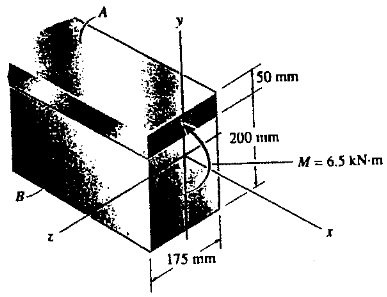
6-118. Solve Prob. 6–117 using the equation developed in Prob. 6–111.

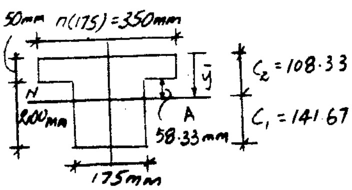
$$\sigma_A = \frac{-(M_{z'} I_{y'} + M_{y'} I_{y'z'})y' + (M_{y'} I_{z'} + M_{z'} I_{y'z'})z'}{I_{y'} I_{z'} - I_{y'z'}^{2}}$$

$$= \frac{-[2500(31.7)(10^{-6}) + 0](-0.14) + [0 + 2500(15.1)(10^{-6})](-0.06)}{31.7(10^{-6})(114)(10^{-6}) - [(15.1)(10^{-6})]^2} = 2.60 \text{ MPa (T)}$$

Ans

6–119 The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If it is subjected to a moment of $M = 6.5$ kN · m, determine the maximum bending stress in the brass and steel. Also, what is the stress in each material at the seam where they are bonded together? $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.





$$n = \frac{E_{st}}{E_{br}} = \frac{200(10^9)}{100(10^9)} = 2$$

$$\bar{y} = \frac{(350)(50)(25) + (175)(200)(150)}{350(50) + 175(200)} = 108.33 \text{ mm}$$

$$I = \frac{1}{12}(0.35)(0.05^3) + (0.35)(0.05)(0.08333^2) + \frac{1}{12}(0.175)(0.2^3) + (0.175)(0.2)(0.04167^2) = 0.3026042(10^{-3})\text{m}^4$$

Maximum stress in brass :

$$(\sigma_{br})_{max} = \frac{Mc_1}{I} = \frac{6.5(10^3)(0.14167)}{0.3026042(10^{-3})} = 3.04 \text{ MPa} \qquad \textbf{Ans}$$

Maximum stress in steel :

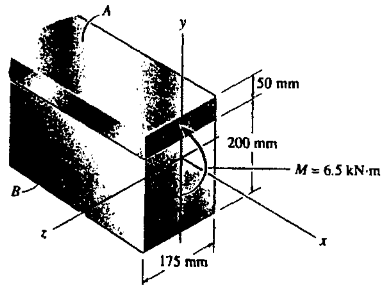
$$(\sigma_{st})_{max} = \frac{nMc_2}{I} = \frac{(2)(6.5)(10^3)(0.10833)}{0.3026042(10^{-3})} = 4.65 \text{ MPa} \qquad \textbf{Ans}$$

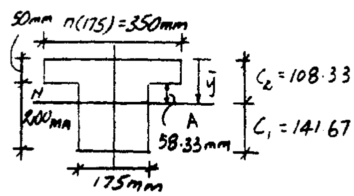
Stress at the junction :

$$\sigma_{br} = \frac{M\rho}{I} = \frac{6.5(10^3)(0.05833)}{0.3026042(10^{-3})} = 1.25 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_{st} = n\sigma_{br} = 2(1.25) = 2.51 \text{ MPa} \qquad \textbf{Ans}$$

*6–120 The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{allow})_{st} = 180$ MPa, and for the brass $(\sigma_{allow})_{br} = 60$ MPa, determine the maximum moment M that can be applied to the beam. $E_{br} = 100$ GPa, $E_{st} = 200$ GPa.





$$n = \frac{E_{st}}{E_{br}} = \frac{200(10^9)}{100(10^9)} = 2$$

$$\bar{y} = \frac{(350)(50)(25) + (175)(200)(150)}{350(50) + 175(200)} = 108.33 \text{ mm}$$

$$I = \frac{1}{12}(0.35)(0.05^3) + (0.35)(0.05)(0.08333^2) + \frac{1}{12}(0.175)(0.2^3) + (0.175)(0.2)(0.04167^2) = 0.3026042(10^{-3})\text{m}^4$$

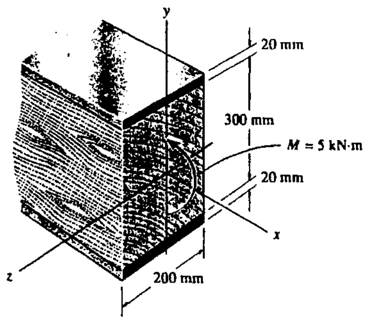
$$(\sigma_{st})_{allow} = \frac{nMc_2}{I}; \qquad 180(10^6) = \frac{(2)M(0.10833)}{0.3026042(10^{-3})}$$

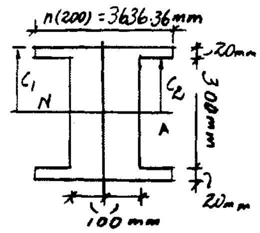
$$M = 251 \text{ kN}\cdot\text{m}$$

$$(\sigma_{br})_{allow} = \frac{Mc_1}{I}; \qquad 60(10^6) = \frac{M(0.14167)}{0.3026042(10^{-3})}$$

$$M = 128 \text{ kN}\cdot\text{m (controls)} \qquad \textbf{Ans}$$

6–121 A wood beam is reinforced with steel straps at its top and bottom as shown. Determine the maximum bending stress developed in the wood and steel if the beam is subjected to a bending moment of $M = 5$ kN · m. Sketch the stress distribution acting over the cross section. Take $E_w = 11$ GPa, $E_{st} = 200$ GPa.





$$I = \frac{1}{12}(3.63636)(0.34)^3 - \frac{1}{12}(3.43636)(0.3)^3 = 4.17848(10^{-3})\text{m}^4$$

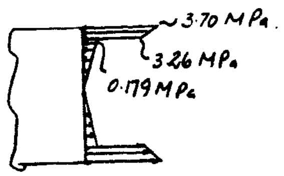
Maximum stress in steel :

$$(\sigma_{st})_{max} = \frac{nMc_1}{I} = \frac{18.182(5)(10^3)(0.17)}{4.17848(10^{-3})} = 3.70 \text{ MPa} \qquad \textbf{Ans}$$

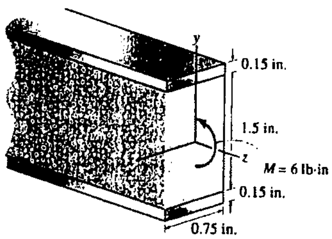
Maximum stress in wood :

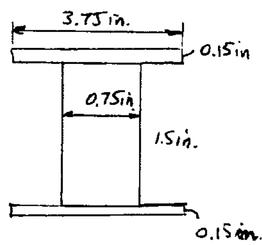
$$(\sigma_w)_{max} = \frac{Mc_2}{I} = \frac{5(10^3)(0.15)}{4.17848(10^{-3})} = 0.179 \text{ MPa} \qquad \textbf{Ans}$$

$$(\sigma_{st}) = n(\sigma_w)_{max} = 18.182(0.179) = 3.26 \text{ MPa}$$



6–122 The sandwich beam is used as a strut in a surfboard. It consists of top and bottom face segments that are made from thin strips of aluminum and an inner core of plastic resin. Determine the maximum bending stress in the aluminum and plastic if the beam is subjected to a moment of $M = 6$ lb · in. $E_{al} = 10(10^3)$ ksi, $E_{pl} = 2(10^3)$ ksi.





$$n = \frac{10(10^3)}{2(10^3)} = 5$$

$$I = \frac{1}{12}(3.75)(1.8)^3 - \frac{1}{12}(3)(1.5)^3 = 0.97875 \text{ in}^4$$

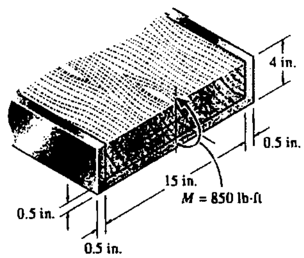
Maximum stress in the aluminum :

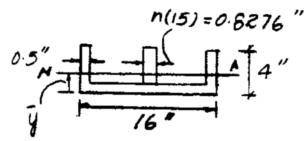
$$(\sigma_{al})_{max} = n\frac{Mc_1}{I} = \frac{5(6)(0.75+0.15)}{0.97875} = 27.6 \text{ psi} \qquad \textbf{Ans}$$

Maximum stress in the plastic :

$$(\sigma_{pl})_{max} = \frac{Mc_2}{I} = \frac{6(0.75)}{0.97875} = 4.60 \text{ psi} \qquad \textbf{Ans}$$

6–123 The steel channel is used to reinforce the wood beam. Determine the maximum bending stress in the steel and in the wood if the beam is subjected to a moment of $M = 850$ lb · ft. $E_{st} = 29(10^3)$ ksi, $E_w = 1600$ ksi.





$$\bar{y} = \frac{(0.5)(16)(0.25) + 2(3.5)(0.5)(2.25) + (0.8276)(3.5)(2.25)}{0.5(16) + 2(3.5)(0.5) + (0.8276)(3.5)} = 1.1386 \text{ in.}$$

$$I = \frac{1}{12}(16)(0.5^3) + (16)(0.5)(0.8886^2) + 2(\frac{1}{12})(0.5)(3.5^3) + 2(0.5)(3.5)(1.1114^2)$$
$$+ \frac{1}{12}(0.8276)(3.5^3) + (0.8276)(3.5)(1.1114^2) = 20.914 \text{ in}^4$$

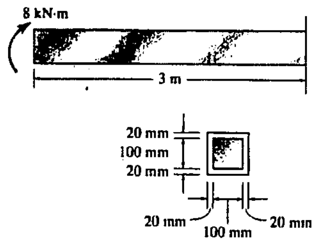
Maximum stress in steel :

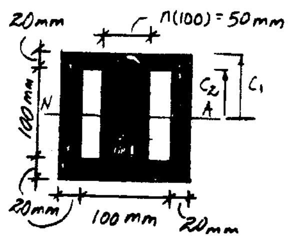
$$(\sigma_{st}) = \frac{Mc}{I} = \frac{850(12)(4 - 1.1386)}{20.914} = 1395 \text{ psi} = 1.40 \text{ ksi} \qquad \textbf{Ans}$$

Maximum stress in wood :

$$(\sigma_w) = n(\sigma_{st})_{max}$$
$$= 0.05517(1395) = 77.0 \text{ psi} \qquad \textbf{Ans}$$

•6–124 The member has a brass core bonded to a steel casing. If a couple moment of 8 kN · m is applied at its end, determine the maximum bending stress in the member. E_{br} = 100 GPa, E_{st} = 200 GPa.





$$n = \frac{E_{br}}{E_{st}} = \frac{100}{200} = 0.5$$

$$I = \frac{1}{12}(0.14)(0.14)^3 - \frac{1}{12}(0.05)(0.1)^3 = 27.84667(10^{-6})\text{m}^4$$

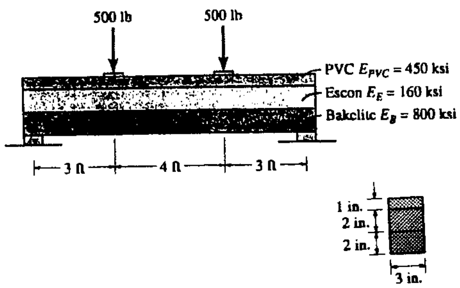
Maximum stress in steel :

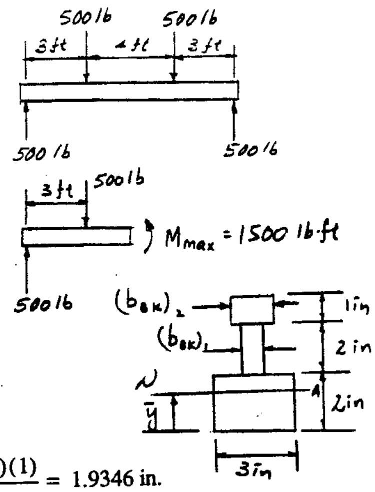
$$(\sigma_{st})_{\max} = \frac{Mc_1}{I} = \frac{8(10^3)(0.07)}{27.84667(10^{-6})} = 20.1 \text{ MPa} \quad \text{(max)} \quad \textbf{Ans}$$

Maximum stress in brass :

$$(\sigma_{br})_{\max} = \frac{nMc_2}{I} = \frac{0.5(8)(10^3)(0.05)}{27.84667(10^{-6})} = 7.18 \text{ MPa}$$

6–125 The beam is made from three types of plastic that are identified and have the moduli of elasticity shown in the figure. Determine the maximum bending stress in the PVC.





$$(b_{bk})_1 = n_1 b_{Es} = \frac{160}{800}(3) = 0.6 \text{ in.}$$

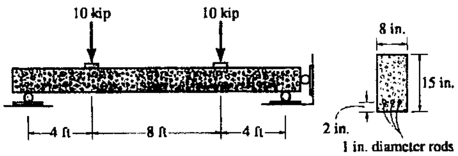
$$(b_{bk})_2 = n_2 b_{pvc} = \frac{450}{800}(3) = 1.6875 \text{ in.}$$

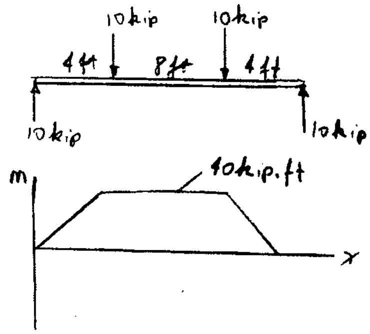
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{(1)(3)(2) + 3(0.6)(2) + 4.5(1.6875)(1)}{3(2) + 0.6(2) + 1.6875(1)} = 1.9346 \text{ in.}$$

$$I = \frac{1}{12}(3)(2^3) + 3(2)(0.9346^2) + \frac{1}{12}(0.6)(2^3) + 0.6(2)(1.0654^2)$$
$$+ \frac{1}{12}(1.6875)(1^3) + 1.6875(1)(2.5654^2) = 20.2495 \text{ in}^4$$

$$(\sigma_{max})_{pvc} = n_2 \frac{Mc}{I} = \left(\frac{450}{800}\right)\frac{1500(12)(3.0654)}{20.2495}$$
$$= 1.53 \text{ ksi}$$ **Ans**

6-126 The reinforced concrete beam is used to support the loading shown. Determine the absolute maximum normal stress in each of the A-36 steel reinforcing rods and the absolute maximum compressive stress in the concrete. Assume the concrete has a high strength in compression and neglect its strength in supporting tension.

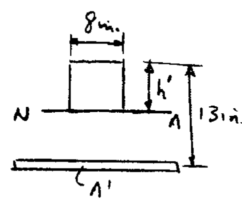




$$M_{max} = (10\text{ kip})(4\text{ ft}) = 40\text{ kip}\cdot\text{ft}$$

$$A_{st} = 3(\pi)(0.5)^2 = 2.3562\text{ in}^2$$

$$E_{st} = 29.0(10^3)\text{ ksi}$$
$$E_{con} = 4.20(10^3)\text{ ksi}$$



$$A' = nA_{st} = \frac{29.0(10^3)}{4.20(10^3)}(2.3562) = 16.2690\text{ in}^2$$

$$\Sigma\tilde{y}A = 0; \qquad 8(h')(\frac{h'}{2}) - 16.2690(13 - h') = 0$$

$$h'^2 + 4.06724h - 52.8741 = 0$$

Solving for the postive root :

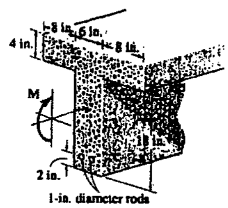
$$h' = 5.517\text{ in.}$$

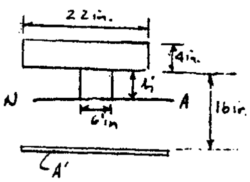
$$I = [\frac{1}{12}(8)(5.517)^3 + 8(5.517)(5.517/2)^2] + 16.2690(13 - 5.517)^2 = 1358.781\text{ in}^4$$

$$(\sigma_{con})_{max} = \frac{My}{I} = \frac{40(12)(5.517)}{1358.781} = 1.95\text{ ksi} \qquad \textbf{Ans}$$

$$(\sigma_{st})_{max} = n(\frac{My}{I}) = (\frac{29.0(10^3)}{4.20(10^3)})(\frac{40(12)(13 - 5.517)}{1358.781}) = 18.3\text{ ksi} \qquad \textbf{Ans}$$

6-127. The reinforced concrete beam is made using two steel reinforcing rods. If the allowable tensile stress, for the steel is $(\sigma_{st})_{\text{allow}} = 40$ ksi, and the allowable compressive stress for the concrete is $(\sigma_{conc})_{\text{allow}} = 3$ ksi, determine the maximum moment M that can be applied to the section. Assume the concrete cannot support a tensile stress. $E_{st} = 29(10^3)$ ksi, $E_{conc} = 3.8(10^3)$ ksi.





$$A_{st} = 2(\pi)(0.5)^2 = 1.5708 \text{ in}^2$$

$$A' = nA_{st} = \frac{29(10^3)}{3.8(10^3)}(1.5708) = 11.9877 \text{ in}^2$$

$$\Sigma \tilde{y}A = 0; \qquad 22(4)(h'+2) + h'(6)(h'/2) - 11.9877(16-h') = 0$$

$$3h'^2 + 99.9877h' - 15.8032 = 0$$

Solving for the positive root :

$$h' = 0.15731 \text{ in.}$$

$$I = [\frac{1}{12}(22)(4)^3 + 22(4)(2.15731)^2] + [\frac{1}{12}(6)(0.15731)^3 + 6(0.15731)(0.15731/2)^2]$$
$$+ 11.9877(16 - 0.15731)^2 = 3535.69 \text{ in}^4$$

Assume concrete fails :

$$(\sigma_{con})_{\text{allow}} = \frac{My}{I}; \qquad 3 = \frac{M(4.15731)}{3535.69}$$

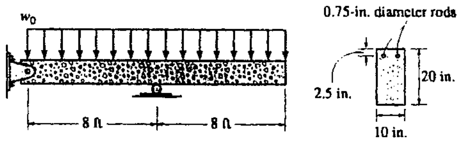
$$M = 2551 \text{ kip} \cdot \text{in.}$$

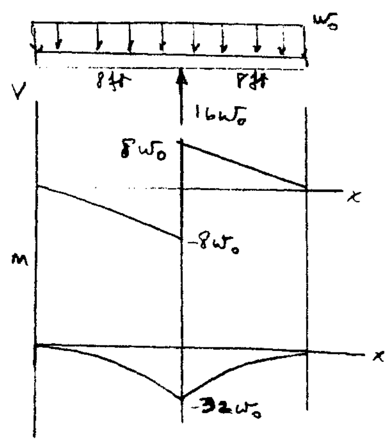
Assume steel fails :

$$(\sigma_{st})_{\text{allow}} = n(\frac{My}{I}); \qquad 40 = (\frac{29(10^3)}{3.8(10^3)})(\frac{M(16-0.15731)}{3535.69})$$

$$M = 1169.7 \text{ kip} \cdot \text{in.} = 97.5 \text{ kip} \cdot \text{ft (controls)} \qquad \textbf{Ans}$$

•6–128 Determine the maximum uniform distributed load w_0 that can be supported by the reinforced concrete beam if the allowable tensile stress for the steel is $(\sigma_{st})_{allow} = 28$ ksi, and the allowable compressive stress for the concrete is $(\sigma_{conc})_{allow} = 3$ ksi. Assume the concrete cannot support a tensile stress. Take $E_{st} = 29(10^3)$ ksi, $E_{conc} = 3.6(10^3)$ ksi.





$M_{max} = -32w_0$

$A_{st} = 2\pi(0.375)^2 = 0.883573 \text{ in}^2$

$A' = nA_{st} = \frac{29(10^3)}{3.6(10^3)}(0.883573) = 7.11767 \text{ in}^2$

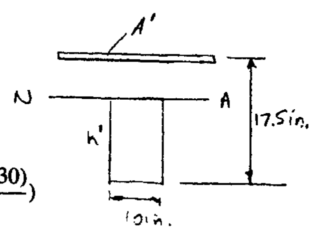
$\Sigma \tilde{y}A = 0; \quad -10(h')(h'/2) + 7.11767(17.5 - h') = 0$

$5h'^2 + 7.11767h' - 124.559 = 0$

Solving for the positive root:

$h' = 4.330$ in.

$I = [\frac{1}{12}(10)(4.330)^3 + (10)(4.330)(4.330/2)^2] + 7.11767(17.5 - 4.330)^2] = 1505.161 \text{ in}^4$



Assume concrete fails:

$(\sigma_{con})_{allow} = \frac{My}{I}; \quad 3 = \frac{M(4.330)}{1505.161}$

$M = 1043.9$ kip · in.

Assume steel fails:

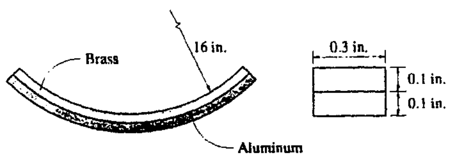
$(\sigma_{st})_{allow} = n(\frac{My}{I}); \quad 28 = (\frac{29(10^3)}{3.6(10^3)})(\frac{M(17.5 - 4.330)}{1505.161})$

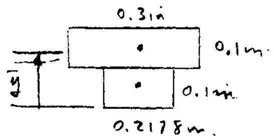
$M = 397.2$ kip · in.

Thus, steel fails first:

$\frac{397.2}{12} = 32w_0; \quad w_0 = 1.03$ kip/ft **Ans**

6–129 A bimetallic strip is made from pieces of 2014-T6 aluminum and C83400 red brass, having the cross section shown. A temperature increase causes its neutral surface to be bent into a circular arc having a radius of 16 in. Determine the moment that must be acting on its cross section due to the thermal stress.





Transform the section to brass.

$$n = \frac{E_{al}}{E_{br}} = \frac{10.6}{14.6} = 0.7260$$

Thus,

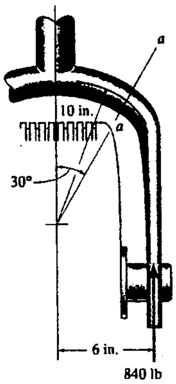
$$\bar{y} = \frac{0.05(0.1)(0.2178) + (0.15)(0.1)(0.3)}{(0.1)(0.2178) + (0.1)(0.3)} = 0.10794 \text{ in.}$$

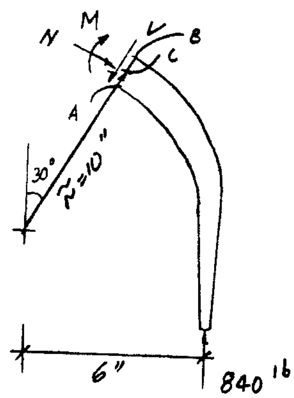
$$I = \frac{1}{12}(0.2178)(0.1)^3 + (0.2178)(0.1)(0.10794 - 0.05)^2 + \frac{1}{12}(0.3)(0.1)^3 + (0.1)(0.3)(0.15 - 0.10794)^2 = 169.34(10^{-6}) \text{ in}^4$$

$$\frac{1}{\rho} = \frac{M}{EI}$$

$$M = \frac{14.6(10^6)(169.34)(10^{-6})}{16.092} = 154 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

6–130 The fork is used as part of a nosewheel assembly for an airplane. If the maximum wheel reaction at the end of the fork is 840 lb, determine the maximum bending stress in the curved portion of the fork at section *a-a*. There the cross-sectional area is circular, having a diameter of 2 in.





$\curvearrowleft + \Sigma M_C = 0; \qquad M - 840(6 - 10\sin 30°) = 0$

$$M = 840 \text{ lb}\cdot\text{in.}$$

$$\int_A \frac{dA}{r} = 2\pi\left(\bar{r} - \sqrt{\bar{r}^2 - c^2}\right)$$

$$= 2\pi\left(10 - \sqrt{10^2 - (1)^2}\right)$$

$$= 0.314948615 \text{ in.}$$

$$A = \pi c^2 = \pi (1)^2 = \pi \text{ in}^2$$

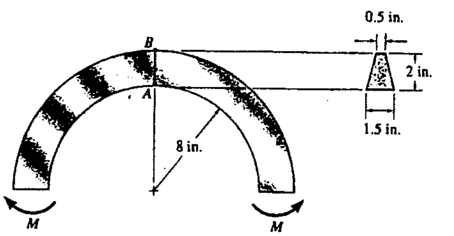
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{\pi}{0.314948615} = 9.974937173 \text{ in.}$$

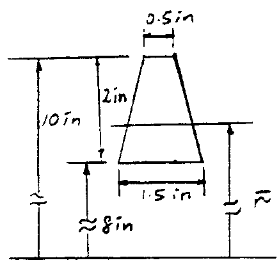
$$\bar{r} - R = 10 - 9.974937173 = 0.025062827 \text{ in.}$$

$$\sigma_A = \frac{M(R - r_A)}{A r_A(\bar{r} - R)} = \frac{840(9.974937173 - 9)}{\pi(9)(0.025062827)} = 1.16 \text{ ksi (T) (max)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B(\bar{r} - R)} = \frac{840(9.974937173 - 11)}{\pi(11)(0.025062827)} = -0.994 \text{ ksi (C)}$$

6–131 The curved member is symmetric and is subjected to a moment of $M = 600$ lb · ft. Determine the bending stress in the member at points A and B. Show the stress acting on volume elements located at these points.





$$A = 0.5(2) + \frac{1}{2}(1)(2) = 2\text{ in}^2$$

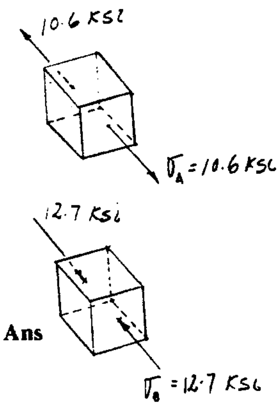
$$\bar{r} = \frac{\Sigma \tilde{r}A}{\Sigma A} = \frac{9(0.5)(2) + 8.6667(\frac{1}{2})(1)(2)}{2} = 8.83333\text{ in.}$$

$$\int_A \frac{dA}{r} = 0.5 \ln\frac{10}{8} + \left[\frac{1(10)}{(10-8)}\left[\ln\frac{10}{8}\right] - 1\right] = 0.22729\text{ in.}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2}{0.22729} = 8.7993\text{ in.}$$

$$\bar{r} - R = 8.83333 - 8.7993 = 0.03398\text{ in.}$$

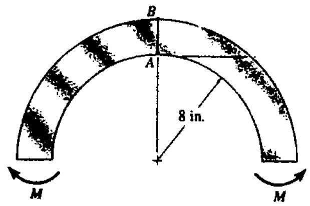
$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$



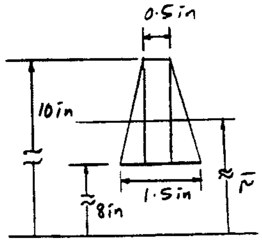
$$\sigma_A = \frac{600(12)(8.7993 - 8)}{2(8)(0.03398)} = 10.6\text{ ksi (T)}$$ **Ans**

$$\sigma_B = \frac{600(12)(8.7993 - 10)}{2(10)(0.03398)} = -12.7\text{ ksi} = 12.7\text{ ksi (C)}$$ **Ans**

***6–132** The curved member is symmetric and is subjected to a moment of $M = 400$ lb · ft. Determine the maximum tensile and compressive stress in the member. Compare these values with those for a straight member having the same cross section and loaded with the same moment.



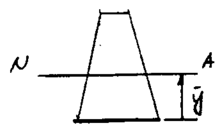
$$A = 0.5(2) + \frac{1}{2}(1)(2) = 2\text{ in}^2$$



$$\bar{r} = \frac{\Sigma \tilde{r}A}{\Sigma A} = \frac{9(0.5)(2) + 8.6667(\frac{1}{2})(1)(2)}{2} = 8.8333\text{ in.}$$

$$\int_A \frac{dA}{r} = 0.5\ln\frac{10}{8} + [\frac{1(10)}{(10-8)}[\ln\frac{10}{8}] - 1] = 0.22729\text{ in.}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2}{0.22729} = 8.7993\text{ in.}$$



$$\bar{r} - R = 8.8333 - 8.7993 = 0.03398\text{ in.}$$

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

$$(\sigma_{max})_t = \frac{400(12)(8.7993 - 8)}{2(8)(0.3398)} = 7.06\text{ ksi}$$ **Ans**

$$(\sigma_{max})_c = \frac{400(12)(8.7993 - 10)}{2(10)(0.3398)} = -8.48\text{ ksi} = 8.48\text{ ksi (C)}$$ **Ans**

For straight beam : $\bar{y} = \bar{r} - 8 = 0.83333$ in.

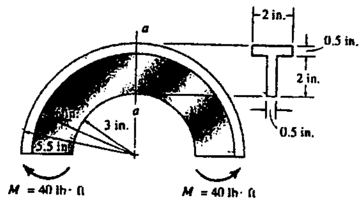
$$I = \frac{1}{12}(0.5)(2^3) + 0.5(2)(0.1666^2) + \frac{1}{36}(1)(2^3) + \frac{1}{2}(1)(2)(0.1666^2) = 0.61111\text{ in}^4$$

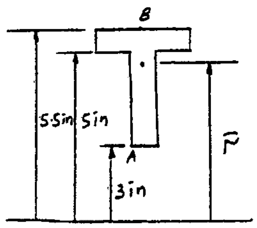
Bending stress :

$$(\sigma_{max})_c = \frac{Mc}{I} = \frac{400(12)(1.1666)}{0.61111} = 9.16\text{ ksi}$$ **Ans**

$$(\sigma_{max})_t = \frac{My}{I} = \frac{400(12)(0.8333)}{0.61111} = 6.54\text{ ksi}$$ **Ans**

6-133. The curved beam is subjected to a bending moment of $M = 40 \text{ lb}\cdot\text{ft}$. Determine the maximum bending stress in the beam. Also, sketch a two-dimensional view of the stress distribution acting on section *a–a*.





Section properties :

$$\bar{r} = \frac{4(2)(0.5) + 5.25(2)(0.5)}{2(0.5) + 2(0.5)} = 4.625 \text{ in.}$$

$$\Sigma\int_A \frac{dA}{r} = 0.5 \ln\frac{5}{3} + 2\ln\frac{5.5}{5} = 0.446033 \text{ in.}$$

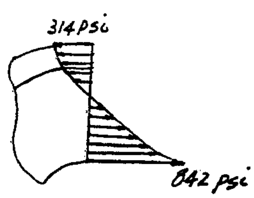
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2}{0.446033} = 4.4840 \text{ in.}$$

$$\bar{r} - R = 4.625 - 4.4840 = 0.1410 \text{ in.}$$

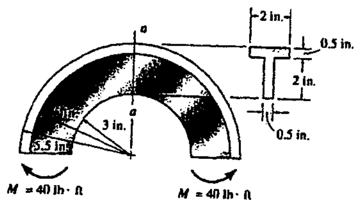
$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

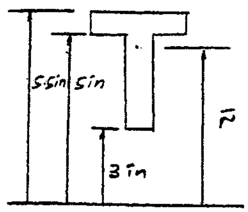
$$\sigma_A = \frac{40(12)(4.4840 - 3)}{2(3)(0.1410)} = 842 \text{ psi (T) (Max)}$$ **Ans**

$$\sigma_B = \frac{40(12)(4.4840 - 5.5)}{2(5.5)(0.1410)} = -314 \text{ psi} = 314 \text{ psi (C)}$$



6-134. The curved beam is made from a material having an allowable bending stress of $\sigma_{allow} = 24$ ksi. Determine the maximum moment M that can be applied to the beam.





$$\bar{r} = \frac{4(2)(0.5) + 5.25(2)(0.5)}{2(0.5) + 2(0.5)} = 4.625 \text{ in.}$$

$$\Sigma\int_A \frac{dA}{r} = 0.5 \ln\frac{5}{3} + 2 \ln\frac{5.5}{5} = 0.4460 \text{ in.}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2}{0.4460} = 4.4840 \text{ in.}$$

$$\bar{r} - R = 4.625 - 4.4840 = 0.1410 \text{ in.}$$

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

Assume tension failure :

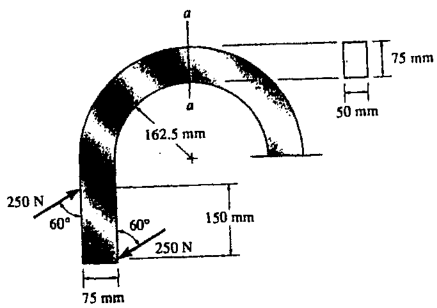
$$24 = \frac{M(4.484 - 3)}{2(3)(0.1410)}$$

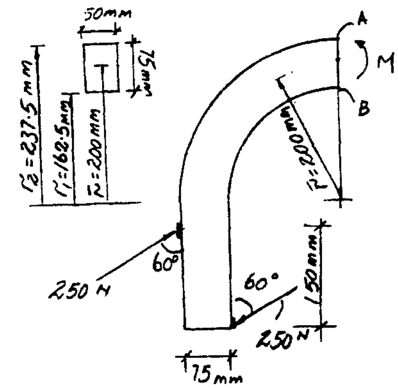
$M = 13.68$ kip · in. $= 1.14$ kip · ft (controls) **Ans**

Assume compression failure :

$$-24 = \frac{M(4.484 - 5.5)}{2(5.5)(0.1410)}; \quad M = 36.64 \text{ kip} \cdot \text{in.} = 3.05 \text{ kip} \cdot \text{ft}$$

6–135 The curved bar used on a machine has a rectangular cross section. If the bar is subjected to a couple as shown, determine the maximum tensile and compressive stress acting at section *a–a*. Sketch the stress distribution on the section in three dimensions.





$\curvearrowleft + \Sigma M_O = 0;\qquad M - 250\cos 60°\,(0.075) - 250\sin 60°\,(0.15) = 0$

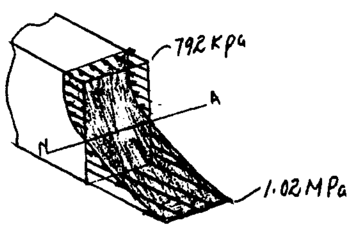
$$M = 41.851\ \text{N}\cdot\text{m}$$

$$\int_A \frac{dA}{r} = b\ln\frac{r_2}{r_1} = 0.05\ln\frac{0.2375}{0.1625} = 0.018974481\ \text{m}$$

$$A = (0.075)(0.05) = 3.75(10^{-3})\ \text{m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{3.75(10^{-3})}{0.018974481} = 0.197633863\ \text{m}$$

$$\bar{r} - R = 0.2 - 0.197633863 = 0.002366137$$

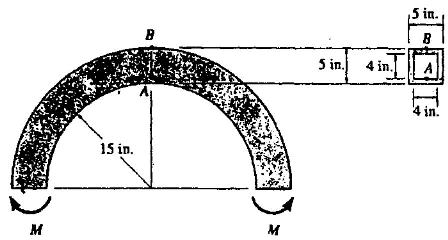


$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{41.851(0.197633863 - 0.2375)}{3.75(10^{-3})(0.2375)(0.002366137)} = -791.72\ \text{kPa}$$

$= 792$ kPa (C) **Ans**

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{41.851(0.197633863 - 0.1625)}{3.75(10^{-3})(0.1625)(0.002366137)} = 1.02\ \text{MPa (T)}$$ **Ans**

•6–136 The curved box member is symmetric and is subjected to a moment of $M = 500$ lb · ft. Determine the bending stress in the member at points A and B. Show the stress acting on volume elements located at these points.



$$\int \frac{dA}{r} = \Sigma\, b \ln \frac{r_2}{r_1} = 5 \ln \frac{20}{15} - 4 \ln \frac{19.5}{15.5} = 0.520112595 \text{ in.}$$

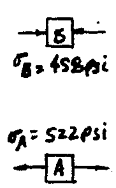
$$A = (5)(5) - (4)(4) = 9 \text{ in}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{9}{0.520112595} = 17.30394549 \text{ in.}$$

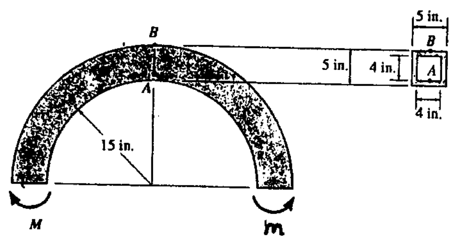
$$\bar{r} - R = 17.5 - 17.30394549 = 0.196054513 \text{ in.}$$

$$\sigma_A = \frac{M(R - r_A)}{A r_A(\bar{r} - R)} = \frac{500(12)(17.30394549 - 15.5)}{9(15.5)(0.196054513)} = 396 \text{ psi (T)} \qquad \textbf{Ans.}$$

$$\sigma_B = \frac{M(R - r_B)}{A r_B(\bar{r} - R)} = \frac{500(12)(17.30394549 - 20)}{9(20)(0.19605413)} = -458 \text{ psi (C)} \qquad \textbf{Ans}$$



6–137 The curved box member is symmetric and is subjected to a moment of $M = 350$ lb · ft. Determine the maximum tensile and compressive stress in the member. Compare these values with those for a straight member having the same cross section and loaded with the same moment.



$$\int \frac{dA}{r} = \Sigma b \ln \frac{r_2}{r_1} = 5 \ln \frac{20}{15} - 4 \ln \frac{19.5}{15.5} = 0.520112595 \text{ in.}$$

$$A = (5)(5) - (4)(4) = 9 \text{ in}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{9}{0.520112595} = 17.30394549 \text{ in.}$$

$$\bar{r} - R = 17.5 - 17.30394549 = 0.196054513 \text{ in.}$$

Tensile stress :

$$(\sigma_t)_{max} = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{350(12)(17.30394549 - 15)}{(9)(15)(0.196054513)} = 366 \text{ psi} \qquad \textbf{Ans}$$

Compressive stress :

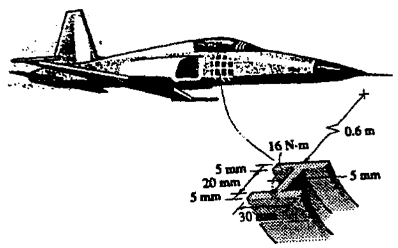
$$(\sigma_c)_{max} = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{350(12)(17.30394549 - 20)}{9(20)(0.196054513)} = -321 \text{ psi} \qquad \textbf{Ans}$$

Straight beam analysis :

$$I = \frac{1}{12}(5)(5^3) - \frac{1}{12}(4)(4)^3 = 30.75 \text{ in}^4$$

$$(\sigma_t)_{max} = (\sigma_c)_{max} = \frac{Mc}{I} = \frac{350(12)(2.5)}{30.75} = 341 \text{ psi} \qquad \textbf{Ans}$$

6-138. While in flight, the curved rib on the jet plane is subjected to an anticipated moment of $M = 16\ \text{N}\cdot\text{m}$ at the section. Determine the maximum bending stress in the rib at this section, and sketch a two-dimensional view of the stress distribution.



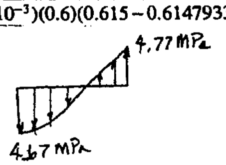
$$\int_A dA/r = (0.03)\ln\frac{0.605}{0.6} + (0.005)\ln\frac{0.625}{0.605} + (0.03)\ln\frac{0.630}{0.625} = 0.650625(10^{-3})\text{ in.}$$

$$A = 2(0.005)(0.03) + (0.02)(0.005) = 0.4(10^{-3})\text{ in}^2$$

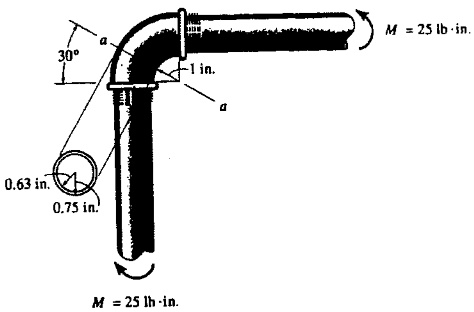
$$R = \frac{A}{\int_A dA/r} = \frac{0.4(10^{-3})}{0.650625(10^{-3})} = 0.6147933$$

$$(\sigma_c)_{max} = \frac{M(R - r_c)}{Ar_A(\bar{r} - R)} = \frac{16(0.6147933 - 0.630)}{0.4(10^{-3})(0.630)(0.615 - 0.6147933)} = -4.67\text{ MPa}$$

$$(\sigma_t)_{max} = \frac{M(R - r_t)}{Ar_A(\bar{r} - R)} = \frac{16(0.6147933 - 0.6)}{0.4(10^{-3})(0.6)(0.615 - 0.6147933)} = 4.77\text{ MPa} \qquad \textbf{Ans}$$



6–139 The elbow of the pipe has an outer radius of 0.75 in. and an inner radius of 0.63 in. If the assembly is subjected to the moments of $M = 25$ lb · in., determine the maximum bending stress developed at section a–a.



$$\int_A \frac{dA}{r} = \Sigma\, 2\pi\,(\bar{r} - \sqrt{\bar{r}^2 - c^2}\,)$$

$$= 2\pi\,(1.75 - \sqrt{1.75^2 - 0.75^2}\,) - 2\pi\,(1.75 - \sqrt{1.75^2 - 0.63^2}\,)$$

$$= 0.32375809 \text{ in.}$$

$$A = \pi\,(0.75^2) - \pi\,(0.63^2) = 0.1656\,\pi$$

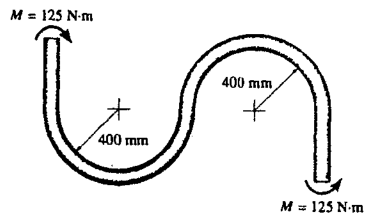
$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.1656\,\pi}{0.32375809} = 1.606902679 \text{ in.}$$

$$\bar{r} - R = 1.75 - 1.606902679 = 0.14309732 \text{ in.}$$

$$(\sigma_{max})_t = \frac{M(R - r_A)}{A r_A(\bar{r} - R)} = \frac{25(1.606902679 - 1)}{0.1656\,\pi\,(1)(0.14309732)} = 204 \text{ psi (T)} \qquad \textbf{Ans}$$

$$(\sigma_{max})_c = \frac{M(R - r_B)}{A r_B(\bar{r} - R)} = \frac{25(1.606902679 - 2.5)}{0.1656\pi(2.5)(0.14309732)} = 120 \text{ psi (C)} \qquad \textbf{Ans}$$

*6–140 A 100-mm-diameter circular rod is bent into an S shape. If it is subjected to the applied moments $M = 125\ \text{N}\cdot\text{m}$ at its ends, determine the maximum tensile and compressive stress developed in the rod.



$$\int_A \frac{dA}{r} = 2\pi(\bar{r} - \sqrt{\bar{r}^2 - c^2})$$

$$= 2\pi(0.45 - \sqrt{0.45^2 - 0.05^2}) = 0.01750707495 \text{ m}$$

$$A = \pi c^2 = \pi(0.05^2) = 2.5(10^{-3})\pi \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2.5(10^{-3})\pi}{0.017507495} = 0.448606818$$

$$\bar{r} - R = 0.45 - 0.448606818 = 1.39318138(10^{-3})\text{m}$$

Compressive stress at A and D :

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{-125(0.448606818 - 0.4)}{2.5(10^{-3})\pi(0.4)(1.39318138)(10^{-3})} = -1.39 \text{ MPa (max)} \qquad \textbf{Ans}$$

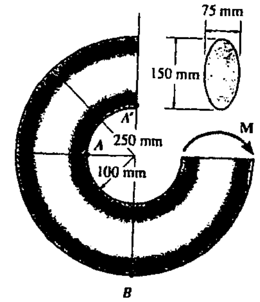
$$\sigma_D = \frac{M(R - r_D)}{Ar_D(\bar{r} - R)} = \frac{125(0.448606818 - 0.5)}{2.5(10^{-3})\pi(0.5)(1.39318138)(10^{-3})} = -1.17 \text{ MPa}$$

Tensile Stress at B and C :

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{-125(0.448606818 - 0.5)}{2.5(10^{-3})\pi(0.5)(1.39318138)(10^{-3})} = 1.17 \text{ MPa}$$

$$\sigma_C = \frac{M(R - r_C)}{Ar_C(\bar{r} - R)} = \frac{125(0.448606818 - 0.4)}{2.5(10^{-3})\pi(0.4)(1.39318138)(10^{-3})} = 1.39 \text{ MPa} \qquad \textbf{Ans}$$

6-141 The member has an elliptical cross section. If it is subjected to a moment of $M = 50\text{ N}\cdot\text{m}$, determine the bending stress at points A and B. Is the stress at point A', which is located on the member near the wall, the same as that at A? Explain.



$$\int_A \frac{dA}{r} = \frac{2\pi b}{a}\left(\bar{r} - \sqrt{\bar{r}^2 - a^2}\right)$$

$$= \frac{2\pi(0.0375)}{0.075}\left(0.175 - \sqrt{0.175^2 - 0.075^2}\right) = 0.053049301 \text{ m}$$

$$A = \pi ab = \pi(0.075)(0.0375) = 2.8125(10^{-3})\pi$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{2.8125(10^{-3})\pi}{0.053049301} = 0.166556941$$

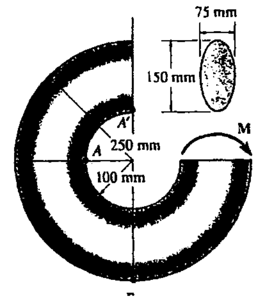
$$\bar{r} - R = 0.175 - 0.166556941 = 0.0084430586$$

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{50(0.166556941 - 0.1)}{2.8125(10^{-3})\pi(0.1)(0.0084430586)} = 446 \text{ kPa (T)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{50(0.166556941 - 0.25)}{2.8125(10^{-3})\pi(0.25)(0.0084430586)} = 224 \text{ kPa (C)} \qquad \textbf{Ans}$$

No, because of localized stress concentration at the wall. **Ans**

6–142 The member has an elliptical cross section. If the allowable bending stress is $\sigma_{allow} = 125$ MPa, determine the maximum moment M that can be applied to the member.



$a = 0.075$ m; $\quad b = 0.0375$ m

$$A = \pi(0.075)(0.0375) = 0.002825\pi \text{ m}^2$$

$$\int_A \frac{dA}{r} = \frac{2\pi b}{a}(\bar{r} - \sqrt{\bar{r}^2 - a^2}) = \frac{2\pi(0.0375)}{0.075}(0.175 - \sqrt{0.175^2 - 0.075^2}) = 0.053049301 \text{ m}$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{0.0028125\pi}{0.053049301} = 0.166556941 \text{ m}$$

$$\bar{r} - R = 0.175 - 0.166556941 = 8.4430586(10^{-3}) \text{ m}$$

$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)}$$

Assume tension failure.

$$125(10^6) = \frac{M(0.166556941 - 0.1)}{0.0028125\pi(0.1)(8.4430586)(10^{-3})}$$

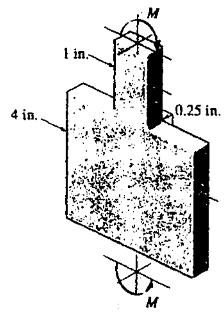
$M = 14.0$ kN . m (controls) **Ans**

Assume compression failure :

$$-125(10^6) = \frac{M(0.166556941 - 0.25)}{0.0028125\pi(0.25)(8.4430586)(10^{-3})}$$

$M = 27.9$ kN·m

6–143 The bar has a thickness of 0.25 in. and is made of a material having an allowable bending stress of $\sigma_{allow} = 18$ ksi. Determine the maximum moment M that can be applied.



$$\frac{w}{h} = \frac{4}{1} = 4 \qquad \frac{r}{h} = \frac{0.25}{1} = 0.25$$

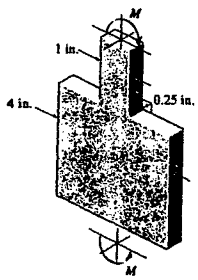
From Fig. 6-48, $K = 1.45$

$$\sigma_{max} = K\frac{Mc}{I}$$

$$18(10^3) = \frac{1.45(M)(0.5)}{\frac{1}{12}(0.25)(1^3)}$$

$M = 517\ \text{lb}\cdot\text{in.} = 43.1\ \text{lb}\cdot\text{ft}$ **Ans**

***6-144.** The bar has a thickness of 0.5 in. and is subjected to a moment of 60 lb · ft. Determine the maximum bending stress in the bar.

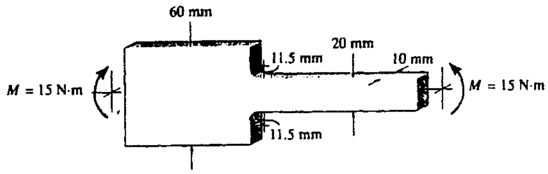


$\frac{w}{h} = \frac{4}{1} = 4; \quad \frac{r}{h} = \frac{0.25}{1} = 0.25$

From Fig. 6-48, $K = 1.45$

$\sigma_{max} = K\frac{Mc}{I} = 1.45\left[\frac{60(12)(0.5)}{\frac{1}{12}(0.5)(1)^3}\right] = 12.5\text{ ksi}$ **Ans**

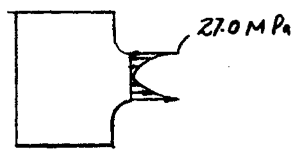
6–145 The bar is subjected to a moment of $M = 15\ N \cdot m$. Determine the maximum bending stress in the bar and sketch, approximately, how the stress varies over the critical section.



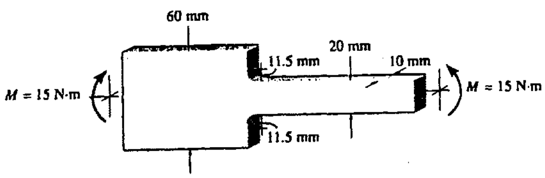
$$\frac{w}{h} = \frac{60}{20} \approx 3 \qquad \frac{r}{h} = \frac{11.5}{20} = 0.575$$

From Fig. 6 - 48, $K = 1.2$

$$\sigma_{max} = K\frac{Mc}{I} = 1.2[\frac{(15)(0.01)}{\frac{1}{12}(0.01)(0.02^3)}] = 27.0\ \text{MPa} \qquad \textbf{Ans}$$



6-146 The allowable bending stress for the bar is σ_{allow} = 175 MPa. Determine the maximum moment M that can be applied to the bar.



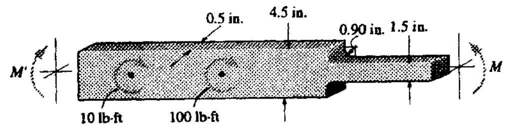
$$\frac{w}{h} = \frac{60}{20} = 3; \qquad \frac{r}{h} = \frac{11.5}{20} = 0.575$$

From Fig. 6-48, $K = 1.2$

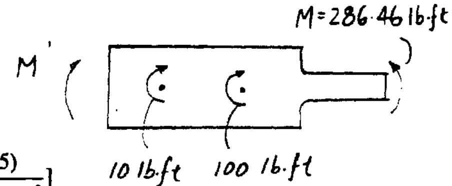
$$\sigma_{allow} = K\frac{Mc}{I}; \qquad 175(10^6) = 1.2\left[\frac{M(0.01)}{\frac{1}{12}(0.01)(0.02)^3}\right]$$

$M = 97.2$ N·m **Ans**

6–147 The bar is subjected to four couple moments. If it is in equilibrium, determine the magnitudes of the largest moments **M** and **M′** that can be applied without exceeding an allowable bending stress of $\sigma_{allow} = 22$ ksi.



$$\frac{w}{h} = \frac{4.5}{1.5} = 3; \qquad \frac{r}{h} = \frac{0.9}{1.5} = 0.6$$



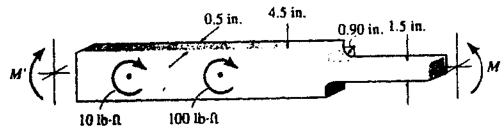
From Fig. 6-48, $K = 1.2$

$$\sigma_{allow} = K\frac{Mc}{I}; \qquad 22(10^3) = 1.2\left[\frac{M(0.75)}{\frac{1}{12}(0.5)(1.5)^3}\right]$$

$M = 3437.5\ \text{lb}\cdot\text{in.} = 286.46\ \text{lb}\cdot\text{ft} = 286\ \text{lb}\cdot\text{ft}$ **Ans**

$\curvearrowleft + \Sigma M = 0;\quad 286.46 - 10 - 100 - M' = 0;\quad M' = 176\ \text{lb}\cdot\text{ft}$ **Ans**

•6–148 The bar is subjected to four couple moments. If $M = 180$ lb · ft and $M' = 70$ lb · ft, determine the maximum bending stress developed in the bar.

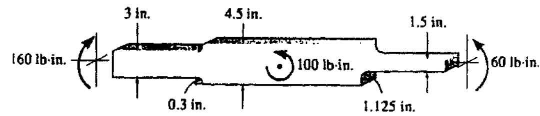


$$\frac{w}{h} = \frac{4.5}{1.5} = 3; \qquad \frac{r}{h} = \frac{0.9}{1.5} = 0.6$$

From Fig. 6 - 48, $K = 1.2$

$$\sigma_{max} = K\frac{Mc}{I} = 1.2\left[\frac{180(12)(0.75)}{\frac{1}{12}(0.5)(1.5)^3}\right] = 13.8 \text{ ksi} \qquad \textbf{Ans}$$

6–149 Determine the maximum bending stress developed in the bar if it is subjected to the couples shown. The bar has a thickness of 0.25 in.



For the larger section :

$$\frac{w}{h} = \frac{4.5}{3} = 1.5; \quad \frac{r}{h} = \frac{0.3}{3} = 0.1$$

From Fig. 6-48, $K = 1.755$

$$\sigma_{max} = K\frac{Mc}{I} = 1.755[\frac{160(1.5)}{\frac{1}{12}(0.25)(3)^3}] = 749 \text{ psi (controls)} \qquad \textbf{Ans}$$

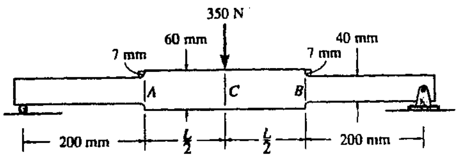
For the smaller section :

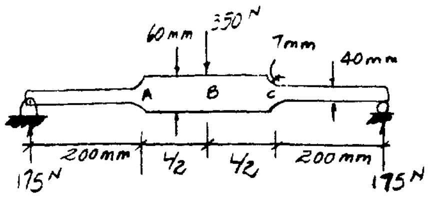
$$\frac{w}{h} = \frac{4.5}{1.5} = 3; \quad \frac{r}{h} = \frac{1.125}{1.5} = 0.75$$

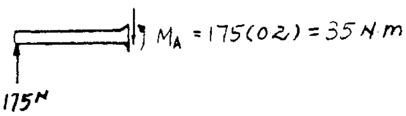
From Fig. 6-48, $K = 1.15$

$$\sigma_{max} = K\frac{Mc}{I} = 1.15[\frac{60(0.75)}{\frac{1}{12}(0.25)(1.5)^3}] = 736 \text{ psi}$$

6–150 Determine the length L of the center portion of the bar so that the maximum bending stress at A, B, and C is the same. The bar has a thickness of 10 mm.







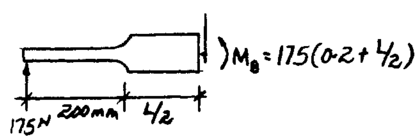
$$\frac{w}{h} = \frac{60}{40} = 1.5 \qquad \frac{r}{h} = \frac{7}{40} = 0.175$$

From Fig. 6-48, $K = 1.5$

$$(\sigma_A)_{max} = K\frac{M_A c}{I} = 1.5\left[\frac{(35)(0.02)}{\frac{1}{12}(0.01)(0.04^3)}\right] = 19.6875\text{ MPa}$$

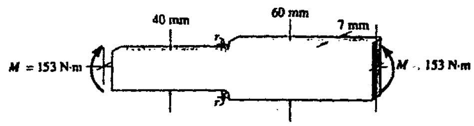
$$(\sigma_B)_{max} = (\sigma_A)_{max} = \frac{M_B c}{I}$$

$$19.6875(10^6) = \frac{175(0.2 + \frac{L}{2})(0.03)}{\frac{1}{12}(0.01)(0.06^3)}$$



$L = 0.95\text{ m} = 950\text{ mm}$ **Ans**

6-151. The bar is subjected to a moment of $M = 153\ \text{N}\cdot\text{m}$. Determine the smallest radius r of the fillets so that an allowable bending stress of $\sigma_{allow} = 120\ \text{MPa}$ is not exceeded.



Probs. 6-151/6-152

$$\sigma_{max} = K\frac{Mc}{I}$$

$$120(10^6) = K\left[\frac{(153)(0.02)}{\frac{1}{12}(0.007)(0.04)^3}\right]$$

$$K = 1.46$$

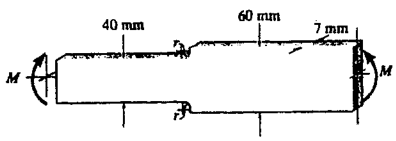
$$\frac{w}{h} = \frac{60}{40} = 1.5$$

From Fig. 6-48,

$$\frac{r}{h} = 0.2$$

$$r = 0.2(40) = 8.0\ \text{mm} \qquad \textbf{Ans}$$

*6–152 The bar is subjected to a moment of $M = 17.5\ \text{N}\cdot\text{m}$. If $r = 6$ mm determine the maximum bending stress in the material.



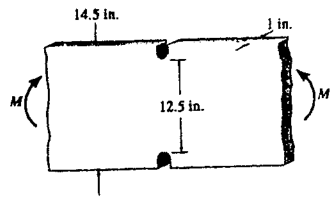
$$\frac{w}{h} = \frac{60}{40} = 1.5; \qquad \frac{r}{h} = \frac{6}{40} = 0.15$$

From Fig. 6-48,

$K = 1.555$

$$\sigma_{max} = K\frac{Mc}{I} = 1.555\left[\frac{17.5(0.02)}{\frac{1}{12}(0.007)(0.04)^3}\right] = 14.6\ \text{MPa} \qquad \textbf{Ans}$$

6–153 If the radius of each notch on the plate is $r = 0.5$ in. determine the largest moment that can be applied. The allowable bending stress for the material is $\sigma_{allow} = 18$ ksi.



$$b = \frac{14.5 - 12.5}{2} = 1.0 \text{ in.}$$

$$\frac{b}{r} = \frac{1}{0.5} = 2.0 \qquad \frac{r}{h} = \frac{0.5}{12.5} = 0.04$$

From Fig. 6 - 50 :

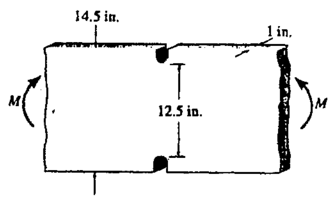
$K = 2.60$

$$\sigma_{max} = K\frac{Mc}{I}$$

$$18(10^3) = 2.60[\frac{(M)(6.25)}{\frac{1}{12}(1)(12.5)^3}]$$

$M = 180\,288$ lb · in. = 15.0 kip · ft **Ans**

6-154 The symmetric notched plate is subjected to bending. If the radius of each notch is $r = 0.5$ in. and the applied moment is $M = 10$ kip · ft, determine the maximum bending stress in the plate.



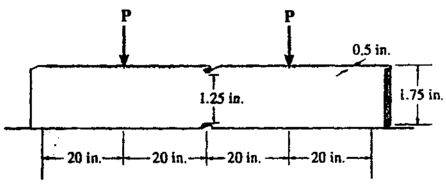
$$\frac{b}{r} = \frac{1}{0.5} = 2.0 \qquad \frac{r}{h} = \frac{0.5}{12.5} = 0.04$$

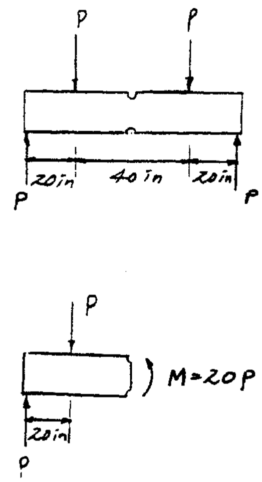
From Fig. 6-50 :

$K = 2.60$

$$\sigma_{max} = K\frac{Mc}{I} = 2.60\left[\frac{(10)(12)(6.25)}{\frac{1}{12}(1)(12.5)^3}\right] = 12.0\ \text{ksi} \qquad \textbf{Ans}$$

6–155 The simply supported notched bar is subjected to two forces **P**. Determine the largest magnitude of **P** that can be applied without causing the material to yield. The material is A-36 steel. Each notch has a radius of $r = 0.125$ in.





$$b = \frac{1.75 - 1.25}{2} = 0.25$$

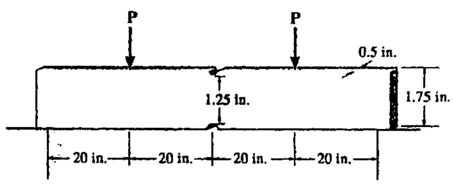
$$\frac{b}{r} = \frac{0.25}{0.125} = 2; \qquad \frac{r}{h} = \frac{0.125}{1.25} = 0.1$$

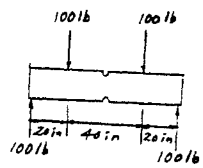
From Fig. 6-50, $K = 1.92$

$$\sigma_Y = K\frac{Mc}{I}; \qquad 36 = 1.92\left[\frac{20P(0.625)}{\frac{1}{12}(0.5)(1.25)^3}\right]$$

$P = 122$ lb **Ans**

*6-156 The simply supported notched bar is subjected to the two loads, each having a magnitude of $P = 100$ lb. Determine the maximum bending stress developed in the bar, and sketch the bending-stress distribution acting over the cross section at the center of the bar. Each notch has a radius of $r = 0.125$ in.



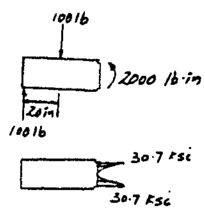


$$b = \frac{1.75 - 1.25}{2} = 0.25$$

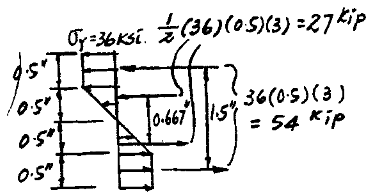
$$\frac{b}{r} = \frac{0.25}{0.125} = 2; \quad \frac{r}{h} = \frac{0.125}{1.25} = 0.1$$

From Fig. 6-50, $K = 1.92$

$$\sigma_{max} = K\frac{Mc}{I} = 1.92\left[\frac{2000(0.625)}{\frac{1}{12}(0.5)(1.25)^3}\right] = 29.5 \text{ ksi} \qquad \textbf{Ans}$$



6–157 A bar having a width of 3 in. and height of 2 in. is made of an elastic plastic material for which $\sigma_Y = 36$ ksi. Determine the moment applied about the horizontal axis that will cause half the bar to yield.



$M = 54(1.5) + 27(0.667) = 99.0$ kip · in. $= 8.25$ kip · ft **Ans**

Also;

$$\sigma_Y = \frac{M_Y c}{I}$$

$$36(10^3) = \frac{M_Y(1)}{\frac{1}{12}(3)(2^3)}$$

$M_Y = 72\,000$ lb · in.

$$M = \frac{3}{2}M_Y\left[1 - \frac{4}{3}\left(\frac{y_Y^2}{h^2}\right)\right]$$

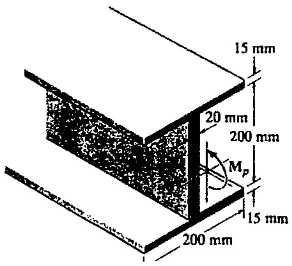
Half of the bar will yield, $y_Y = h/4$

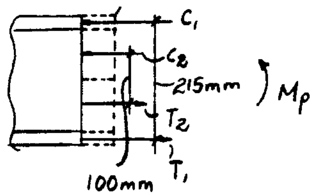
$$M = \frac{11}{8}M_Y$$

$= \frac{11}{8}(72000) = 99000$ lb · in. $= 8.25$ kip · ft **Ans**

Note : The above equation is valid only for rectangular sections.

6–158 Determine the plastic section modulus and the shape factor for the wide-flange beam.





$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\sigma_Y$$

$$M_p = 0.003\sigma_Y(0.215) + 0.002\sigma_Y(0.1) = 0.000845\ \sigma_Y$$

$$\sigma_Y = \frac{M_p}{Z}$$

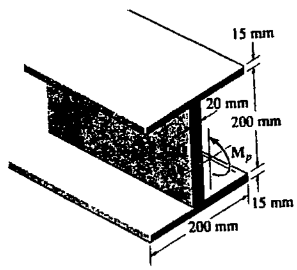
$$Z = \frac{0.000845\sigma_Y}{\sigma_Y} = 845(10^{-6})\text{m}^3 \qquad \textbf{Ans}$$

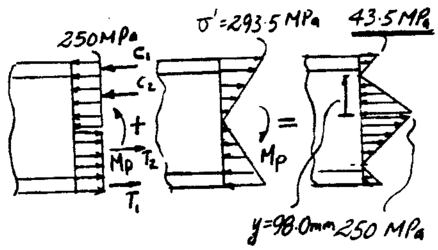
$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y(82.78333)10^{-6})}{0.115} = 0.000719855\ \sigma_Y$$

$$K = \frac{M_p}{M_Y} = \frac{0.000845\sigma_Y}{0.000719855\sigma_Y} = 1.17 \qquad \textbf{Ans}$$

6-159 The beam is made of an elastic plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.





$$I_x = \frac{1}{12}(0.2)(0.23)^3 - \frac{1}{12}(0.18)(0.2)^3 = 82.78333(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(0.2)(0.015) = 0.003\sigma_Y$$

$$C_2 = T_2 = \sigma_Y(0.1)(0.02) = 0.002\sigma_Y$$

$$M_p = 0.003\sigma_Y(0.215) + 0.002\sigma_Y(0.1) = 0.000845\ \sigma_Y$$

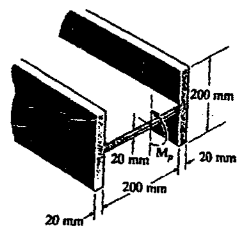
$$= 0.000845(250)(10^6) = 211.25\ \text{kN}\cdot\text{m}$$

$$\sigma' = \frac{M_p c}{I} = \frac{211.25(10^3)(0.115)}{82.78333(10^{-6})} = 293.5\ \text{MPa}$$

$$\frac{y}{250} = \frac{0.115}{293.5}; \qquad y = 0.09796\ \text{m} = 98.0\ \text{mm}$$

$$\sigma_{top} = \sigma_{bottom} = 293.5 - 250 = 43.5\ \text{MPa} \qquad \textbf{Ans}$$

***6-160.** Determine the shape factor for the cross section of the H-beam.

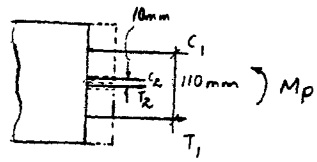


$$I_x = \frac{1}{12}(0.2)(0.02^3) + 2(\frac{1}{12})(0.02)(0.2^3) = 26.8(10^{-6})\text{m}^4$$

$$C_1 = T_1 = \sigma_Y(2)(0.09)(0.02) = 0.0036\sigma_y$$

$$C_2 = T_2 = \sigma_Y(0.01)(0.24) = 0.0024\sigma_y$$

$$M_p = 0.0036\sigma_Y(0.11) + 0.0024\sigma_Y(0.01) = 0.00042\sigma_Y$$

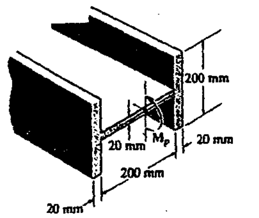


$$\sigma_Y = \frac{M_Y c}{I}$$

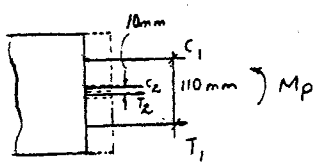
$$M_Y = \frac{\sigma_Y(26.8)(10^{-6})}{0.1} = 0.000268\sigma_Y$$

$$K = \frac{M_p}{M_Y} = \frac{0.00042\sigma_Y}{0.000268\sigma_Y} = 1.57 \quad \textbf{Ans}$$

6-161. The H-beam is made of an elastic-plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the top and bottom of the beam after the plastic moment $\mathbf{M}_p$ is applied and then released.



$$I_x = \frac{1}{12}(0.2)(0.02^3) + 2(\frac{1}{12})(0.02)(0.2^3) = 26.8(10^{-6})\text{m}^4$$



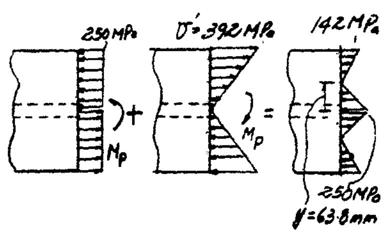
$$C_1 = T_1 = \sigma_Y(2)(0.09)(0.02) = 0.0036\sigma_y$$

$$C_2 = T_2 = \sigma_Y(0.01)(0.24) = 0.0024\sigma_y$$

$$M_p = 0.0036\sigma_Y(0.11) + 0.0024\sigma_Y(0.01) = 0.00042\sigma_Y$$

$$M_p = 0.00042(250)(10^6) = 105 \text{ kN}\cdot\text{m}$$

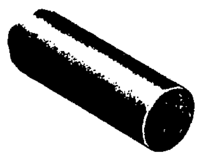
$$\sigma' = \frac{M_p c}{I} = \frac{105(10^3)(0.1)}{26.8(10^{-6})} = 392 \text{ MPa}$$



$$\frac{y}{250} = \frac{0.1}{392}; \qquad y = 0.0638 = 63.8 \text{ mm}$$

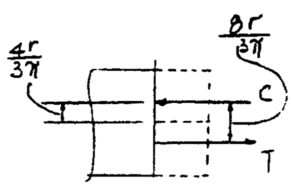
$$\sigma_T = \sigma_B = 392 - 250 = 142 \text{ MPa} \qquad \textbf{Ans}$$

6–162 The rod has a circular cross section. If it is made of an elastic plastic material, determine the shape factor and the plastic section modulus Z.



Plastic moment :

$$C = T = \sigma_y\left(\frac{\pi r^2}{2}\right) = \frac{\pi r^2}{2}\sigma_y$$



$$M_p = \frac{\pi r^2}{2}\sigma_y\left(\frac{8r}{3\pi}\right) = \frac{4r^3}{3}\sigma_y$$

Elastic moment :

$$I = \frac{1}{4}\pi r^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(\frac{1}{4}\pi r^4)}{r} = \frac{\pi r^3}{4}\sigma_Y$$

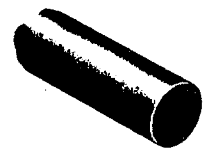
Shape factor :

$$K = \frac{M_p}{M_Y} = \frac{\frac{4r^3}{3}\sigma_Y}{\frac{\pi r^3}{4}\sigma_Y} = \frac{16}{3\pi} = 1.70 \qquad \textbf{Ans}$$

Plastic section modulus :

$$Z = \frac{M_p}{\sigma_Y} = \frac{\frac{4r^3}{3}\sigma_Y}{\sigma_Y} = \frac{4r^3}{3} \qquad \textbf{Ans}$$

6–163 The rod has a circular cross section. If it is made of an elastic plastic material, determine the maximum elastic moment and plastic moment that can be applied to the cross section. Take $r = 3$ in., $\sigma_Y = 36$ ksi.



Elastic moment :

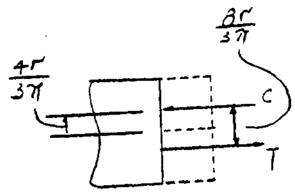
$$I = \frac{1}{4}\pi r^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(\frac{1}{4}\pi r^4)}{r} = \frac{\pi r^3}{4}\sigma_Y$$

$$= \frac{\pi(3^3)}{4}(36) = 763.4 \text{ kip} \cdot \text{in.}$$

$$= 63.6 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

Plastic moment :

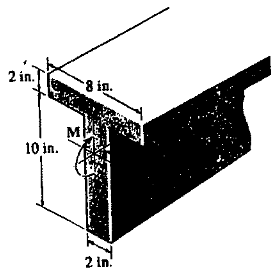


$$C = T = \sigma_Y(\frac{\pi r^2}{2}) = \frac{\pi r^2}{2}\sigma_Y$$

$$M_p = \frac{\pi r^2}{2}\sigma_Y(\frac{8r}{3\pi}) = \frac{4r^3}{3}\sigma_Y = \frac{4}{3}(3^3)(36)$$

$$= 1296 \text{ kip} \cdot \text{in.} = 108 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

•6–164 The T-beam is made of an elastic-plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. $\sigma_Y = 36$ ksi.



Elastic Analysis :

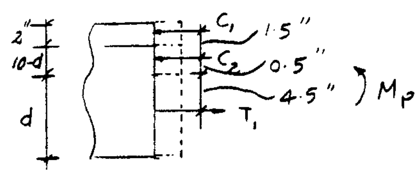
$$\bar{y} = \frac{8(2)(1) + 10(2)(7)}{8(2) + 10(2)} = 4.333 \text{ in.}$$

$$I = \frac{1}{12}(8)(2^3) + 8(2)(3.3333^2) + \frac{1}{12}(2)(10^3) + 10(2)(2.6667^2) = 492 \text{ in}^4$$

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{492(36)}{7.667} = 2310 \text{ kip} \cdot \text{in.} = 193 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

Plastic analysis :



$$\int \sigma\, dA = 0; \qquad C_1 + C_2 - T_1 = 0$$

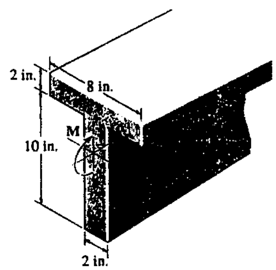
$$36(8)(2) + 36(2)(10-d) - 36(2)(d) = 0$$

$$d = 9 \text{ in.} < 10 \text{ in.} \qquad \text{OK}$$

$$M_p = 36(8)(2)(2) + 36(2)(1)(0.5) + 36(2)(9)(4.5)$$

$$= 4104 \text{ kip} \cdot \text{in.} = 342 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

6–165 Determine the plastic section modulus and the shape factor for the beam.



Elastic Analysis :

$$\bar{y} = \frac{8\,(2)(1) + 10(2)(7)}{8(2) + 10(2)} = 4.333 \text{ in.}$$

$$I = \frac{1}{12}(8)(2^3) + 8(2)(3.3333^2) + \frac{1}{12}(2)(10^3) + 10(2)(2.6667^2) = 492 \text{ in}^4$$

$$\sigma_Y = \frac{M_Y c}{I}$$

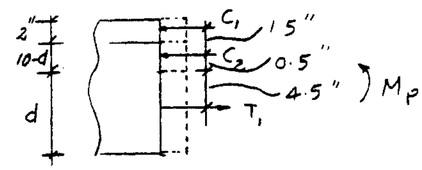
$$M_Y = \frac{492(36)}{7.667} = 2310 \text{ kip}\cdot\text{in.}$$

Plastic analysis :

$$\int \sigma\, dA = 0; \qquad C_1 + C_2 - T_1 = 0$$

$$36(8)(2) + 36(2)(10 - d) - 36(2)(d) = 0$$

$$d = 9 \text{ in.} < 10 \text{ in.} \qquad \text{OK}$$



$$M_p = 36(8)(2)(2) + 36(2)(1)(0.5) + 36(2)(9)(4.5)$$
$$= 4104 \text{ kip}\cdot\text{in.}$$

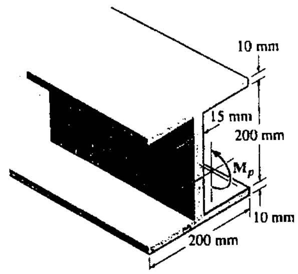
$$\sigma_Y = \frac{M_p}{Z}$$

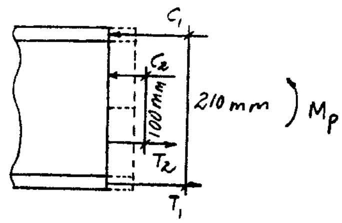
$$Z = \frac{M_p}{\sigma_Y} = \frac{4104}{36} = 114 \text{ in}^3 \qquad \textbf{Ans}$$

$$K = \frac{M_p}{M_Y}$$

$$K = \frac{4104}{2310} = 1.78 \qquad \textbf{Ans}$$

6–166 Determine the plastic section modulus and the shape factor for the cross section of the beam.





$$I = \frac{1}{12}(0.2)(0.22)^3 - \frac{1}{12}(0.185)(0.2)^3 = 54.133(10^{-6})\ \text{m}^4$$

$$C_1 = \sigma_Y(0.01)(0.2) = (0.002)\sigma_Y$$

$$C_2 = \sigma_Y(0.1)(0.015) = (0.0015)\sigma_Y$$

$$M_p = 0.002\sigma_Y(0.21) + 0.0015\sigma_Y(0.1) = 0.00057\sigma_Y$$

$$\sigma_Y = \frac{M_p}{Z}$$

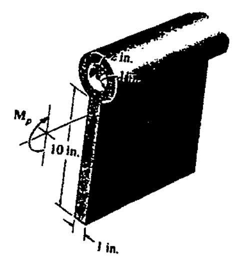
$$Z = \frac{0.00057\sigma_Y}{\sigma_Y} = 570(10^{-6})\ \text{m}^3$$ **Ans**

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{\sigma_Y(54.133)(10^{-6})}{0.11} = 0.0005\sigma_y$$

$$K = \frac{M_p}{M_Y} = \frac{0.0006\sigma_Y}{0.0005\sigma_Y} = 1.16$$ **Ans**

6-167. Determine the plastic moment $\mathbf{M}_p$ that can be supported by a beam having the cross section shown. $\sigma_Y = 30$ ksi.



$\int \sigma \, dA = 0$

$C_1 + C_2 - T_1 = 0$

$\pi(2^2 - 1^2)(30) + (10 - d)(1)(30) - d(1)(30) = 0$

$3\pi + 10 - 2d = 0$

$d = 9.7124 \text{ in.} < 10 \text{ in.}$ OK

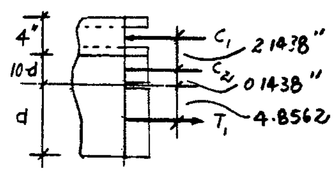
$M_p = \pi(2^2 - 1^2)(30)(2.2876)$

$+ (0.2876)(1)(30)(0.1438)$

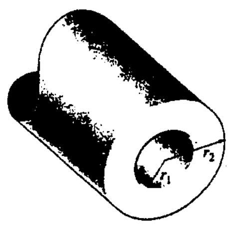
$+ (9.7124)(1)(30)(4.8562)$

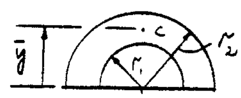
$= 2063 \text{ kip} \cdot \text{in.}$

$= 172 \text{ kip} \cdot \text{ft}$ **Ans**



***6–168** The thick-walled tube is made from an elastic-plastic material. Determine the shape factor and the plastic section modulus Z.

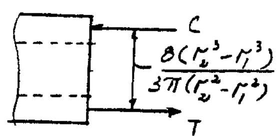




Plastic analysis :

Location of centroid C,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{\frac{4r_2}{3\pi}(\frac{\pi}{2}r_2^2) - \frac{4r_1}{3\pi}(\frac{\pi}{2}r_1^2)}{\frac{\pi}{2}(r_2^2 - r_1^2)} = \frac{4(r_2^3 - r_1^3)}{3\pi(r_2^2 - r_1^2)}$$



$$T = C = \frac{\pi}{2}(r_2^2 - r_1^2)\sigma_Y$$

$$M_p = \frac{\pi}{2}(r_2^2 - r_1^2)\sigma_Y[\frac{8(r_2^3 - r_1^3)}{3\pi(r_2^2 - r_1^2)}] = \frac{4}{3}(r_2^3 - r_1^3)\sigma_Y$$

Elastic analysis :

$$I = \frac{\pi}{4}(r_2^4 - r_1^4)$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\frac{\pi}{4}(r_2^4 - r_1^4)}{r_2}\sigma_Y = \frac{\pi(r_2^4 - r_1^4)}{4r_2}\sigma_Y$$

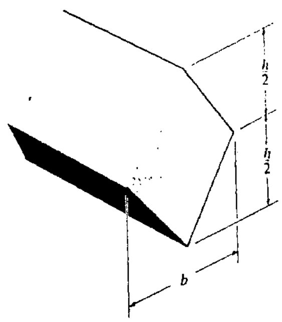
Shape factor :

$$K = \frac{M_p}{M_Y} = \frac{\frac{4}{3}(r_2^3 - r_1^3)\sigma_Y}{\frac{\pi(r_2^4 - r_1^4)}{4r_2}\sigma_Y} = \frac{16r_2(r_2^3 - r_1^3)}{3\pi(r_2^4 - r_1^4)}$$ **Ans**

Plastic section modulus :

$$Z = \frac{M_p}{\sigma_Y} = \frac{\frac{4}{3}(r_2^3 - r_1^3)\sigma_Y}{\sigma_Y} = \frac{4}{3}(r_2^3 - r_1^3)$$ **Ans**

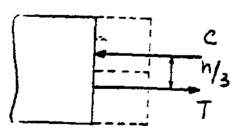
6-169 Determine the shape factor and the plastic section modulus for the member.



Plastic analysis :

$$T = C = \frac{1}{2}(b)(\frac{h}{2})\sigma_Y = \frac{b\,h}{4}\sigma_Y$$

$$M_p = \frac{b\,h}{4}\sigma_Y(\frac{h}{3}) = \frac{b\,h^2}{12}\sigma_Y$$



Elastic analysis :

$$I = 2[\frac{1}{12}(b)(\frac{h}{2})^3] = \frac{b\,h^3}{48}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(\frac{bh^3}{48})}{\frac{h}{2}} = \frac{b\,h^2}{24}\sigma_Y$$

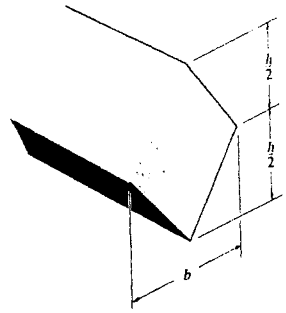
Shape factor :

$$K = \frac{M_p}{M_Y} = \frac{\frac{bh^2}{12}\sigma_Y}{\frac{bh^2}{24}\sigma_Y} = 2 \qquad \textbf{Ans}$$

Plastic section modulus :

$$Z = \frac{M_p}{\sigma_Y} = \frac{\frac{bh^2}{12}\sigma_Y}{\sigma_Y} = \frac{b\,h^2}{12} \qquad \textbf{Ans}$$

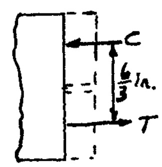
6–170 The member is made from an elastic-plastic material. Determine the maximum elastic moment and the plastic moment that can be applied to the cross section. Take b = 4 in., h = 6 in., σ_Y = 36 ksi.



Elastic analysis :

$$I = 2[\frac{1}{12}(4)(3)^3] = 18\text{ in}^4$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{36(18)}{3} = 216\text{ kip}\cdot\text{in.} = 18\text{ kip}\cdot\text{ft}$$ **Ans**

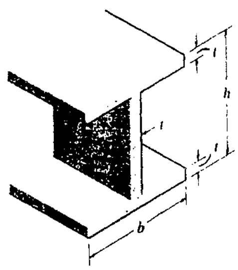


Plastic analysis :

$$T = C = \frac{1}{2}(4)(3)(36) = 216\text{ kip}$$

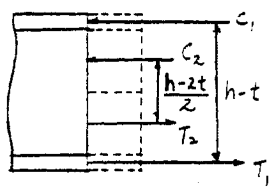
$$M_p = 216(\frac{6}{3}) = 432\text{ kip}\cdot\text{in.} = 36\text{ kip}\cdot\text{ft}$$ **Ans**

6–171 The wide-flange member is made from an elastic-plastic material. Determine the shape factor and the plastic section modulus Z.



Plastic analysis :

$$T_1 = C_1 = \sigma_Y b\, t\,; \qquad T_2 = C_2 = \sigma_Y\left(\frac{h-2t}{2}\right)t$$



$$M_p = \sigma_Y b\, t(h-t) + \sigma_Y\left(\frac{h-2t}{2}\right)(t)\left(\frac{h-2t}{2}\right)$$

$$= \sigma_Y\left[b\, t(h-t) + \frac{t}{4}(h-2\,t)^2\right]$$

Elastic analysis :

$$I = \frac{1}{12}b\,h^3 - \frac{1}{12}(b-t)(h-2t)^3$$

$$= \frac{1}{12}\left[b\,h^3 - (b-t)(h-2\,t)^3\right]$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y\left(\frac{1}{12}\right)\left[bh^3 - (b-t)(h-2t)^3\right]}{\frac{h}{2}}$$

$$= \frac{bh^3 - (b-t)(h-2t)^3}{6\,h}\sigma_Y$$

Shape factor :

$$K = \frac{M_p}{M_Y} = \frac{\left[b\, t(h-t) + \frac{t}{4}(h-2t)^2\right]\sigma_Y}{\frac{bh^3 - (b-t)(h-2t)^3}{6h}\sigma_Y}$$

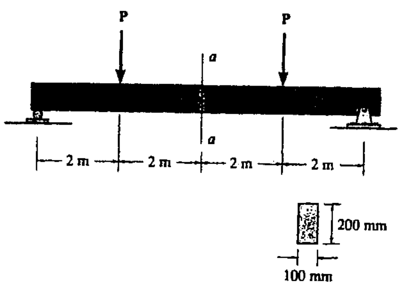
$$= \frac{3h}{2}\left[\frac{4b\, t(h-t) + t\,(h-2t)^2}{b\,h^3 - (b-t)(h-2t)^3}\right]$$ **Ans**

Plastic section modulus :

$$Z = \frac{M_p}{\sigma_Y} = \frac{\sigma_Y\left[b\, t(h-t) + \frac{t}{4}(h-2t)^2\right]}{\sigma_Y}$$

$$= b\, t(h-t) + \frac{t}{4}(h-2t)^2$$ **Ans**

•6–172 The beam is made of an elastic-plastic material for which $\sigma_Y = 200$ MPa. If the largest moment in the beam occurs within the center section a–a, determine the magnitude of each force **P** that causes this moment to be (a) the largest elastic moment and (b) the largest plastic moment.



$$M = 2P \qquad (1)$$

a) Elastic moment

$$I = \frac{1}{12}(0.1)(0.2^3) = 66.667(10^{-6})\ \text{m}^4$$

$$\sigma_y = \frac{M_Y c}{I}$$

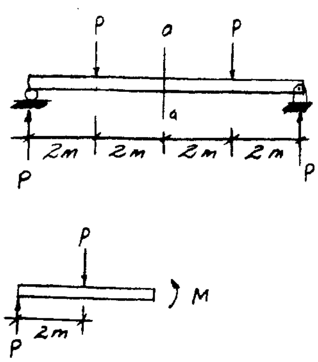
$$M_Y = \frac{200(10^6)(66.667)(10^{-6})}{0.1}$$

$$= 133.33\ \text{kN}\cdot\text{m}$$

From Eq. (1)

$$133.33 = 2P$$

$$P = 66.7\ \text{kN} \qquad \textbf{Ans}$$



b) Plastic moment

$$M_p = \frac{b h^2}{4}\sigma_Y$$

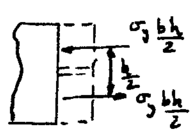
$$= \frac{0.1(0.2^2)}{4}(200)(10^6)$$

$$= 200\ \text{kN}\cdot\text{m}$$

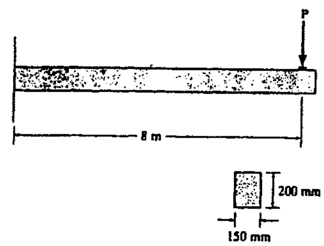
From Eq. (1)

$$200 = 2P$$

$$P = 100\ \text{kN} \qquad \textbf{Ans}$$

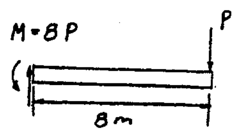


6-173. The beam is made from an elastic-plastic material for which $\sigma_Y = 200$ MPa. Determine the magnitude of force **P** that causes this moment to be (a) the largest elastic moment and (b) the largest plastic moment.



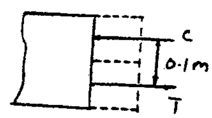
a) **Elastic analysis :**

$$M_Y = \frac{\sigma_Y I}{c} = \frac{200(10^6)(\frac{1}{12})(0.15)(0.2^3)}{0.1} = 200 \text{ kN} \cdot \text{m}$$



$M = 8P = 200;\quad P = 25.0$ kN **Ans**

b) **Plastic analysis :**

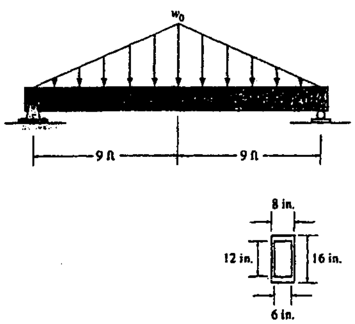


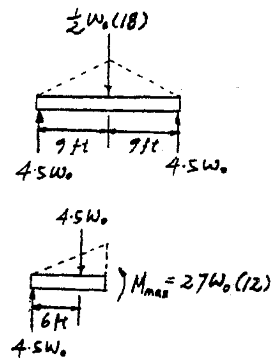
$T = C = 200(10^6)(0.15)(0.1) = 3000$ kN

$M_p = 3000(0.1) = 300$ kN · m

$M = 8P = 300;\quad P = 37.5$ kN **Ans**

6-174. The box beam is made from an elastic-plastic material for which $\sigma_Y = 25$ ksi. Determine the intensity of the distributed load w_0 that will cause the moment to be (a) the largest elastic moment and (b) the largest plastic moment.





Elastic analysis :

$$I = \frac{1}{12}(8)(16^3) - \frac{1}{12}(6)(12^3) = 1866.67 \text{ in}^4$$

$$M_{max} = \frac{\sigma_Y I}{c}; \qquad 27w_0(12) = \frac{25(1866.67)}{8}$$

$w_0 = 18.0$ kip/ft **Ans**

Plastic analysis :

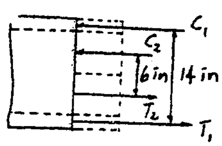
$$C_1 = T_1 = 25(8)(2) = 400 \text{ kip}$$

$$C_2 = T_2 = 25(6)(2) = 300 \text{ kip}$$

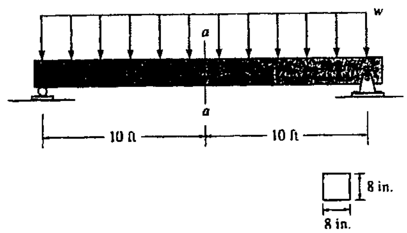
$$M_p = 400(14) + 300(6) = 7400 \text{ kip} \cdot \text{in.}$$

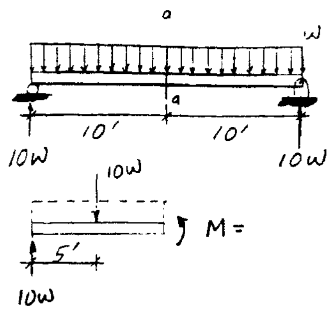
$$27w_0(12) = 7400$$

$w_0 = 22.8$ kip/ft **Ans**



6–175 The beam is made of an elastic plastic material for which $\sigma_Y = 30$ ksi. If the largest moment in the beam occurs at the center section *a–a*, determine the intensity of the distributed load *w* that causes this moment to be (*a*) the largest elastic moment and (*b*) the largest plastic moment.





$$M = 50\,w \qquad (1)$$

a) Elastic moment

$$I = \frac{1}{12}(8)(8^3) = 341.33 \text{ in}^4$$

$$\sigma_Y = \frac{M_Y c}{I}$$

$$M_Y = \frac{30\,(341.33)}{4}$$

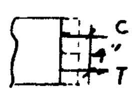
$$= 2560 \text{ kip}\cdot\text{in.} = 213.33 \text{ kip}\cdot\text{ft}$$

From Eq. (1),

$$213.33 = 50\,w$$

$$w = 4.27 \text{ kip/ft} \qquad \textbf{Ans}$$

b) Plastic moment



$$C = T = 30\,(8)(4) = 960 \text{ kip}$$

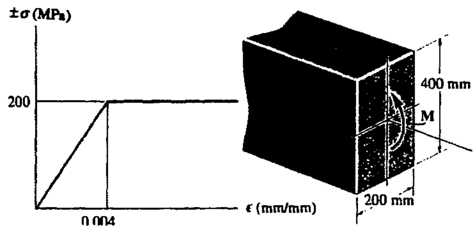
$$M_p = 960\,(4) = 3840 \text{ kip}\cdot\text{in.} = 320 \text{ kip}\cdot\text{ft}$$

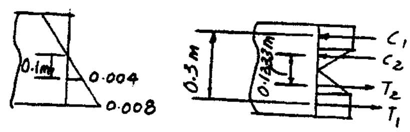
From Eq. (1)

$$320 = 50\,w$$

$$w = 6.40 \text{ kip/ft} \qquad \textbf{Ans}$$

*6–176 The beam has a rectangular cross section and is made of an elastic-plastic material having a stress strain diagram as shown. Determine the magnitude of the moment **M** that must be applied to the beam in order to create a maximum strain in its outer fibers of $\epsilon_{max} = 0.008$.



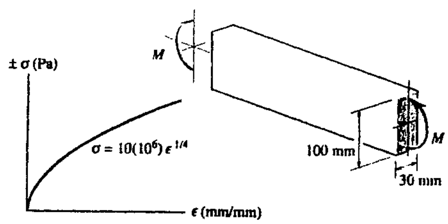


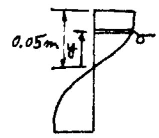
$$C_1 = T_1 = 200(10^6)(0.1)(0.2) = 4000 \text{ kN}$$

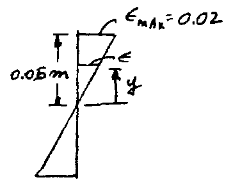
$$C_2 = T_2 = \frac{1}{2}(200)(10^6)(0.1)(0.2) = 2000 \text{ kN}$$

$$M = 4000(0.3) + 2000(0.1333) = 1467 \text{ kN}\cdot\text{m} = 14.7 \text{ MN}\cdot\text{m} \quad \textbf{Ans}$$

6–177 A beam is made from polypropylene plastic and has a stress–strain diagram that can be approximated by the curve shown. If the beam is subjected to a maximum tensile and compressive strain of $\epsilon = 0.02$ mm/mm, determine the maximum moment M.







$\varepsilon_{max} = 0.02$

$\sigma = 10(10^6)(0.02)^{1/4} = 3.761$ MPa

$$\frac{0.02}{0.05} = \frac{\varepsilon}{y}$$

$\varepsilon = 0.4\,y$

$\sigma = 10(10^6)(0.4)^{1/4} y^{1/4}$

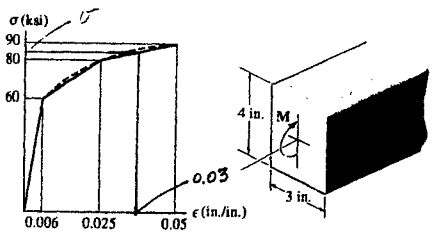
$\sigma = 7.9527(10^6) y^{1/4}$

$$M = \int_A y\,\sigma\,dA = 2\int_0^{0.05} y(7.9527)(10^6)y^{1/4}(0.03)dy$$

$$M = 0.47716(10^6)\int_0^{0.05} y^{1/4}dy = 0.47716(10^6)(\frac{4}{5})(0.05)^{5/4}$$

$M = 9.03$ kN · m **Ans**

6–178 The bar is made of an aluminum alloy having a stress-strain diagram that can be approximated by the straight line segments shown. Assuming that this diagram is the same for both tension and compression, determine the moment the bar will support if the maximum strain at the top and bottom fibers of the beam is $\epsilon_{max} = 0.03$.

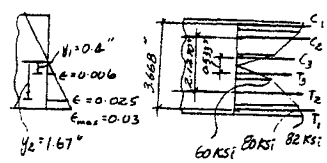


$$\frac{\sigma - 80}{0.03 - 0.025} = \frac{90 - 80}{0.05 - 0.025}; \qquad \sigma = 82 \text{ ksi}$$

$$C_1 = T_1 = \frac{1}{2}(0.3333)(80 + 82)(3) = 81 \text{ kip}$$

$$C_2 = T_2 = \frac{1}{2}(1.2666)(60 + 80)(3) = 266 \text{ kip}$$

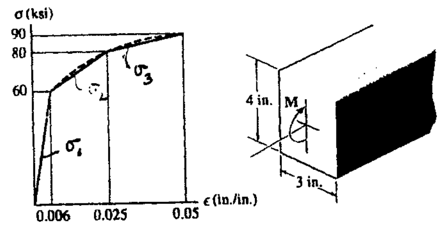
$$C_3 = T_3 = \frac{1}{2}(0.4)(60)(3) = 36 \text{ kip}$$



$$M = 81(3.6680) + 266(2.1270) + 36(0.5333)$$
$$= 882.09 \text{ kip} \cdot \text{in.} = 73.5 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans}$$

Note : The centroid of a trapezodial area was used in calculation of moment areas.

6–179 The bar is made of an aluminum alloy having a stress–strain diagram that can be approximated by the straight line segments shown. Assuming that this diagram is the same for both tension and compression, determine the moment the bar will support if the maximum strain at the top and bottom fibers of the beam is $\epsilon_{max} = 0.05$.

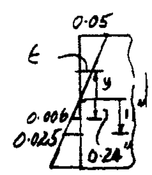


$$\sigma_1 = \frac{60}{0.006}\varepsilon = 10(10^3)\varepsilon$$

$$\frac{\sigma_2 - 60}{\varepsilon - 0.006} = \frac{80 - 60}{0.025 - 0.006}$$

$$\sigma_2 = 1052.63\varepsilon + 53.684$$

$$\frac{\sigma_3 - 80}{\varepsilon - 0.025} = \frac{90 - 80}{0.05 - 0.025}; \qquad \sigma_3 = 400\varepsilon + 70$$



$$\varepsilon = \frac{0.05}{2}(y) = 0.025y$$

Substitute ε into σ expression :

$\sigma_1 = 250y \qquad 0 \le y < 0.24$ in.

$\sigma_2 = 26.315y + 53.684 \qquad 0.24 < y < 1$ in.

$\sigma_3 = 10y + 70 \qquad 1$ in. $< y \le 2$ in.

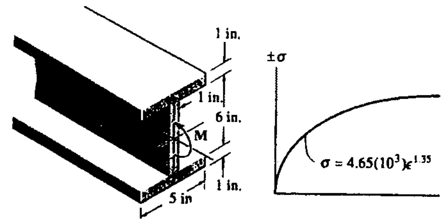
$$dM = y\sigma\, dA = y\sigma(3\, dy)$$

$$M = 2[3\int_0^{0.24} 250y^2\, dy + 3\int_{0.24}^{1}(26.315y^2 + 53.684y)\, dy + 3\int_1^2 (10y^2 + 70y)\, dy]$$

$= 980.588$ kip · in. $= 81.7$ kip · ft **Ans**

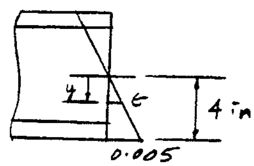
Also, the solution can be obtained from stress blocks as in Prob . 6 - 178.

*6–180 A member is made of a polymer having the stress–strain diagram shown. If the curve can be represented by the equation $\sigma = 4.65(10)^3\,\epsilon^{1.35}$ ksi, determine the magnitude of the moment **M** that can be applied without causing the maximum strain in the member to exceed $\epsilon_{max} = 0.005$ in./in.



From the strain diagram :

$$\frac{\varepsilon}{y} = \frac{0.005}{4}; \qquad \varepsilon = 0.00125\,y$$



Substitute $\varepsilon = 0.00125\,y$ into σ expression

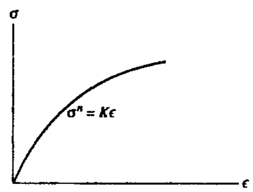
$$\sigma = 4.65(10^3)(0.00125\,y)^{1.35} = 0.56012\,y^{1.35}$$

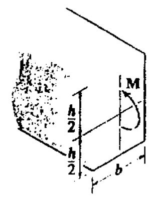
$$dM = (\sigma\,dA)y = \sigma by\,dy = 0.56012 b\,y^{2.35}\,dy$$

$$M = 2[0.56012(1)\int_0^3 y^{2.35}\,dy + 0.56012(5)\int_3^4 y^{2.35}\,dy]$$

$$= 120.79 \text{ kip}\cdot\text{in.} = 10.1 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans}$$

6–181 A material has a stress–strain diagram such that within the elastic range the tensile or compressive stress can be related to the tensile or compressive strain by the equation $\sigma^n = K\epsilon$, where K and n are constants. If the material is subjected to a bending moment M, derive an expression between the maximum stress in the material and the moment. The cross section has a moment of inertia of I about its neutral axis.





$$\sigma^n = k\varepsilon$$

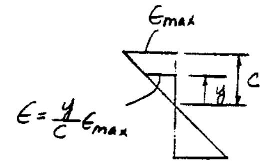
Due to symmetry, $T = C$

$$\varepsilon = \frac{y}{C}\varepsilon_{max}$$

$$\frac{\sigma^n}{k} = \frac{2y}{h}\frac{\sigma^n_{max}}{k}$$

$$\sigma^n = \frac{2y}{h}\sigma^n_{max}$$

$$\sigma = (\frac{2y}{h})^{\frac{1}{n}}\sigma_{max}$$

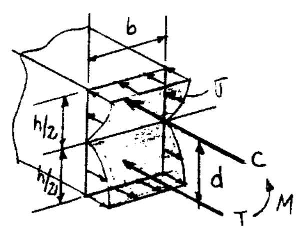


$$M = \int_A y\sigma\, dA = 2\int_0^{\frac{h}{2}} y(\frac{2y}{h})^{\frac{1}{n}}\sigma_{max}\, b\, dy$$

$$M = \frac{2(2)^{\frac{1}{n}}b}{h^{\frac{1}{n}}}\sigma_{max}\int_0^{\frac{h}{2}} y^{\frac{(n+1)}{n}}\, dy$$

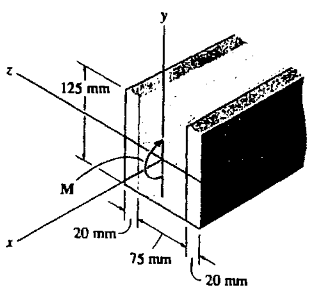
$$M = \frac{2(2)^{\frac{1}{n}}b}{h^{\frac{1}{n}}}\sigma_{max}[(\frac{n}{2n+1})\, y^{\frac{(2n+1)}{n}}]_0^{\frac{h}{2}}$$

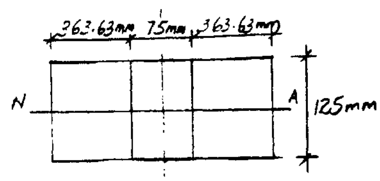
$$M = \frac{n}{(2n+1)}(\frac{2(2)^{\frac{1}{n}}b}{h^{\frac{1}{n}}})\,\sigma_{max}[\frac{h}{2}]^{\frac{2n+1}{n}}$$



$$M = \frac{nbh^2}{2(2n+1)}\sigma_{max}$$ **Ans**

6–182 The composite beam consists of a wood core and two plates of steel. If the allowable bending stress for the wood is $(\sigma_{allow})_w = 20$ MPa, and for the steel $(\sigma_{allow})_{st} = 130$ MPa, determine the maximum moment that can be applied to the beam. $E_w = 11$ GPa, $E_{st} = 200$ GPa.





$$n = \frac{E_{st}}{E_w} = \frac{200(10^9)}{11(10^9)} = 18.182$$

$$I = \frac{1}{12}(0.80227)(0.125^3) = 0.130578(10^{-3})\text{m}^4$$

Failure of wood :

$$(\sigma_w)_{max} = \frac{Mc}{I}$$

$$20(10^6) = \frac{M(0.0625)}{0.130578(10^{-3})}; \qquad M = 41.8 \text{ kN} \cdot \text{m}$$

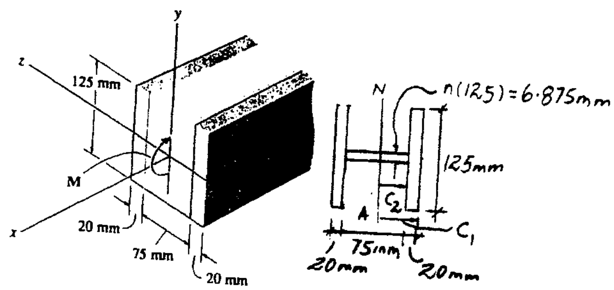
Failure of steel :

$$(\sigma_{st})_{max} = \frac{nMc}{I}$$

$$130(10^6) = \frac{18.182(M)(0.0625)}{0.130578(10^{-3})}$$

$M = 14.9$ kN · m (controls) **Ans**

6-183. Solve Prob. 6–182 if the moment is applied about the y axis instead of the z axis as shown.



$$n = \frac{11(10^9)}{200(10^4)} = 0.055$$

$$I = \frac{1}{12}(0.125)(0.115^3) - \frac{1}{12}(0.118125)(0.075^3) = 11.689616(10^{-6})$$

Failure of wood :

$$(\sigma_w)_{max} = \frac{nMc_2}{I}$$

$$20(10^6) = \frac{0.055(M)(0.0375)}{11.689616(10^{-6})}; \qquad M = 113 \text{ kN} \cdot \text{m}$$

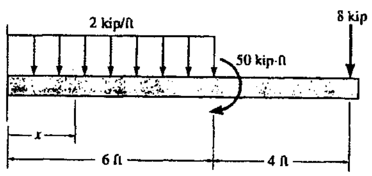
Failure of steel :

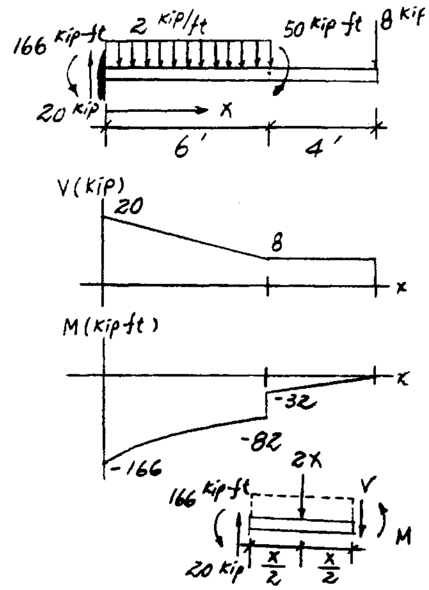
$$(\sigma_{st})_{max} = \frac{Mc_1}{I}$$

$$130(10^6) = \frac{M(0.0575)}{11.689616(10^{-6})}$$

$M = 26.4$ kN · m (controls) **Ans**

•6–184 Draw the shear and moment diagrams for the beam and determine the shear and moment in the beam as functions of x, where $0 \le x < 6$ ft.





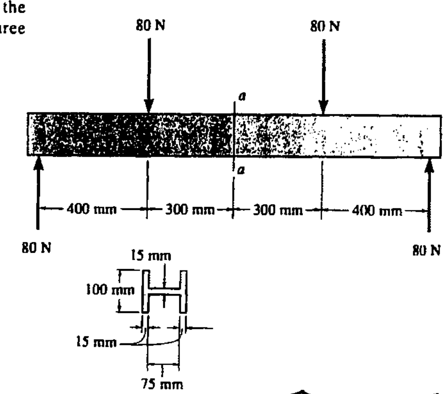
$$+\uparrow \Sigma F_y = 0; \qquad 20 - 2x - V = 0$$

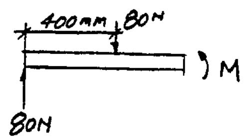
$$V = 20 - 2x \qquad \textbf{Ans}$$

$$\circlearrowleft + \Sigma M_{NA} = 0; \qquad 20x - 166 - 2x\left(\frac{x}{2}\right) - M = 0$$

$$M = -x^2 + 20x - 166 \qquad \textbf{Ans}$$

6–185 Determine the bending stress distribution in the beam at section *a–a*. Sketch the distribution in three dimensions acting over the cross section.



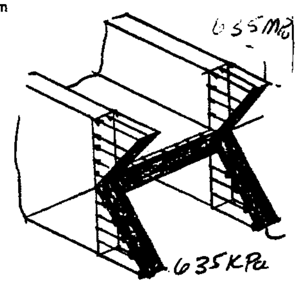


$\curvearrowleft + \Sigma M = 0; \qquad M - 80(0.4) = 0$

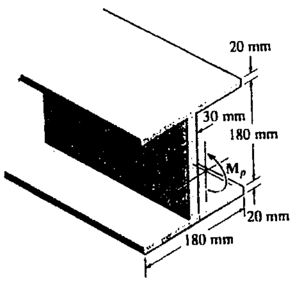
$M = 32 \text{ N}\cdot\text{m}$

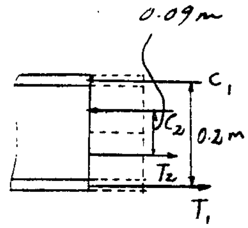
$$I_z = \frac{1}{12}(0.075)(0.015^3) + 2(\frac{1}{12})(0.015)(0.1^3) = 2.52109(10^{-6})\text{m}^4$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{32(0.05)}{2.52109(10^{-6})} = 635 \text{ kPa} \qquad \textbf{Ans}$$



6–186 Determine the plastic section modulus and the shape factor for the wide-flange beam.





$$I = \frac{1}{12}(0.18)(0.22^3) - \frac{1}{12}(0.15)(0.18^3)$$
$$= 86.82(10^{-6})\ \text{m}^4$$

Plastic moment :

$$M_p = \sigma_Y(0.18)(0.02)(0.2) + \sigma_Y(0.09)(0.03)(0.09)$$
$$= 0.963(10^{-3})\sigma_Y$$

Plastic section modulus :

$$Z = \frac{M_p}{\sigma_Y} = \frac{0.963(10^{-3})\sigma_Y}{\sigma_Y}$$
$$= 0.963(10^{-3})\ \text{m}^3$$ **Ans**

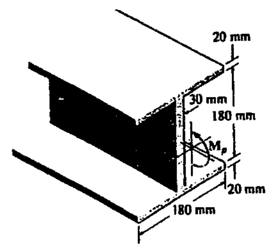
Shape factor :

$$M_Y = \frac{\sigma_Y I}{c} = \frac{\sigma_Y(86.82)(10^{-6})}{0.11} = 0.789273(10^{-3})\sigma_Y$$

$$K = \frac{M_p}{M_Y} = \frac{0.963(10^{-3})\sigma_Y}{0.789273(10^{-3})\sigma_Y} = 1.22$$ **Ans**

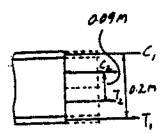
6-187. The beam is made of an elastic plastic material for which $\sigma_Y = 250$ MPa. Determine the residual stress in the beam at its top and bottom after the plastic moment $\mathbf{M}_p$ is applied and then released.

$$I = \frac{1}{12}(0.18)(0.22^3) - \frac{1}{12}(0.15)(0.18^3)$$
$$= 86.82(10^{-6})\ \text{m}^4$$



Plastic moment :

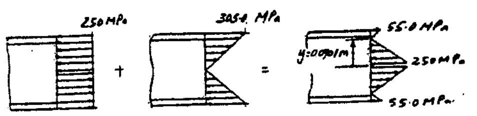
$$M_p = 250(10^6)(0.18)(0.02)(0.2)$$
$$+ 250(10^6)(0.09)(0.03)(0.09)$$
$$= 240750\ \text{N}\cdot\text{m}$$



Applying a reverse $M_p = 240750\ \text{N}\cdot\text{m}$

$$\sigma_p = \frac{M_p c}{I} = \frac{240750(0.11)}{86.82(10^{-6})} = 305.03\ \text{MPa}$$

$$\sigma'_{top} = \sigma'_{bottom} = 305 - 250 = 55.0\ \text{MPa} \qquad \textbf{Ans}$$



•6–188 For the curved beam in Fig. 6–44*a*, show that when the radius of curvature approaches infinity, the curved-beam formula, Eq. 6–24, reduces to the flexure formula, Eq. 6–13.

$\sigma = \frac{M(R-r)}{Ar(\bar{r}-R)}$, $\quad R = \frac{A}{\int_A \frac{dA}{r}} = \frac{A}{A'}$, $\quad A' = \int_A \frac{dA}{r}$

$$\sigma = \frac{M(A - rA')}{Ar\,(\bar{r}A' - A)} \qquad (1)$$

But,

$$r = \bar{r} + y, \qquad (2)$$

$$\bar{r}A' = \bar{r}\int_A \frac{dA}{r} = \int_A \left(\frac{\bar{r}}{\bar{r}+y} - 1 + 1\right)dA = \int_A \left(\frac{\bar{r}-\bar{r}-y}{\bar{r}+y} + 1\right)dA$$

$$= A - \int_A \frac{y}{\bar{r}+y}\,dA \qquad (3)$$

Denominator of Eq (1) becomes :

$$Ar\,(\bar{r}A' - A) = Ar\left(A - \int_A \frac{y}{\bar{r}+y}\,dA - A\right) = -Ar\int_A \frac{y}{\bar{r}+y}\,dA$$

Using Eq. (2) :

$$Ar\,(\bar{r}A' - A) = -A\int_A \left(\frac{\bar{r}y}{\bar{r}+y} + y - y\right)dA - Ay\int_A \frac{y}{\bar{r}+y}\,dA$$

$$Ar\,(\bar{r}A' - A) = A\int_A \frac{y^2}{\bar{r}+y}dA - A\int_A y\,dA - Ay\int_A \frac{y}{\bar{r}+y}dA$$

$$Ar\,(\bar{r}A' - A) = \frac{A}{\bar{r}}\int_A \left(\frac{y^2}{1+y/\bar{r}}\right)dA - A\int_A y\,dA - \frac{Ay}{\bar{r}}\int_A \left(\frac{y}{1+y/\bar{r}}\right)dA$$

But,

$$\int_A y\,dA = 0; \qquad \text{as } \frac{y}{\bar{r}} \to 0$$

Then,

$$Ar\,(\bar{r}A' - A) \to \frac{A}{\bar{r}}I$$

Equation (1) becomes :

$$\sigma = \frac{M\bar{r}}{AI}(A - rA')$$

Using Eq. (2) :

$$\sigma = \frac{M\bar{r}}{AI}(A - \bar{r}A' - yA')$$

Using Eq. (3)

$$\sigma = \frac{M\bar{r}}{AI}\left[A - \left(A - \int_A \frac{y}{\bar{r}+y}\,dA\right) - y\int_A \frac{dA}{\bar{r}+y}\right] = \frac{M\bar{r}}{AI}\left[\int_A \frac{y}{\bar{r}+y}\,dA - y\int_A \frac{dA}{\bar{r}+y}\right]$$

$$= \frac{M\bar{r}}{AI}\left[\int_A \frac{y/\bar{r}}{1+y/\bar{r}}dA - \frac{y}{\bar{r}}\int_A \frac{dA}{1+y/\bar{r}}\right]$$

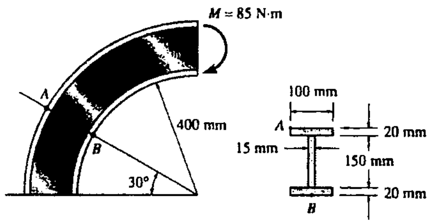
As $\frac{y}{\bar{r}} \to 0$

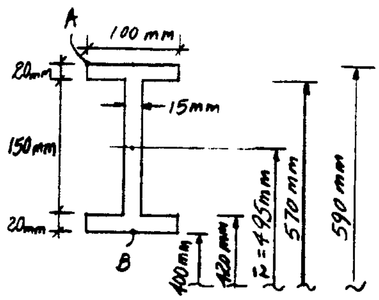
$$\int_A \frac{y/\bar{r}}{1+y/\bar{r}}dA = 0$$

$$\frac{y}{\bar{r}}\int_A \frac{dA}{1+y/\bar{r}} = \frac{y}{\bar{r}}\int_A dA = \frac{yA}{\bar{r}}$$

$$\sigma = \frac{M\bar{r}}{AI}\left(-\frac{yA}{\bar{r}}\right) = -\frac{My}{I} \qquad \textbf{QED}$$

6–189 The curved beam is subjected to a bending moment of $M = 85$ N · m as shown. Determine the stress at points *A* and *B* and show the stress on a volume element located at these points.





$$\int_A \frac{dA}{r} = b \ln \frac{r_2}{r_1} = 0.1 \ln \frac{0.42}{0.40} + 0.015 \ln \frac{0.57}{0.42} + 0.1 \ln \frac{0.59}{0.57}$$
$$= 0.012908358 \text{ m}$$

$$A = 2(0.1)(0.02) + (0.15)(0.015) = 6.25(10^{-3}) \text{ m}^2$$

$$R = \frac{A}{\int_A \frac{dA}{r}} = \frac{6.25(10^{-3})}{0.012908358} = 0.484182418 \text{ m}$$

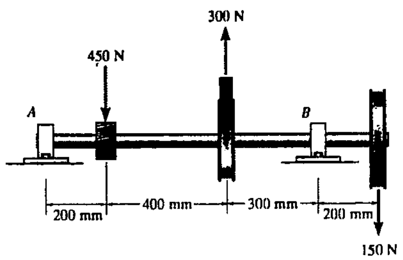
$$\bar{r} - R = 0.495 - 0.484182418 = 0.010817581 \text{ m}$$

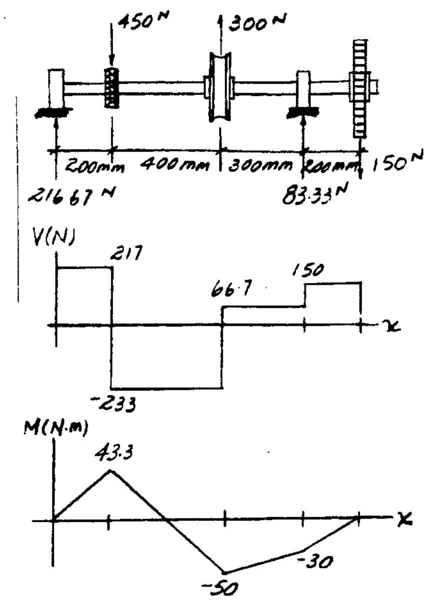
$$\sigma_A = \frac{M(R - r_A)}{A r_A (\bar{r} - R)} = \frac{85(0.484182418 - 0.59)}{6.25(10^{-3})(0.59)(0.010817581)} = -225.48 \text{ kPa}$$

$\sigma_A = 225$ kPa (C) **Ans**

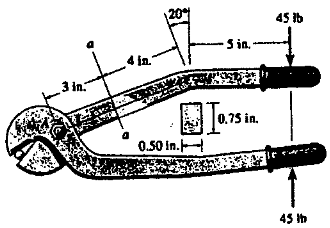
$$\sigma_B = \frac{M(R - r_B)}{A r_B (\bar{r} - R)} = \frac{85(0.484182418 - 0.40)}{6.25(10^{-3})(0.40)(0.010817581)} = 265 \text{ kPa (T)}$$ **Ans**

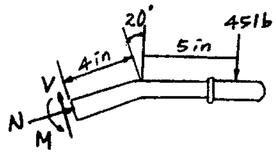
6–190 Draw the shear and moment diagrams for the shaft if it is subjected to the vertical loadings of the belt, gear, and flywheel. The bearings at *A* and *B* exert only vertical reactions on the shaft.





6-191. Determine the maximum bending stress in the handle of the cable cutter at section *a–a*. A force of 45 lb is applied to the handles. The cross-sectional area is shown in the figure.



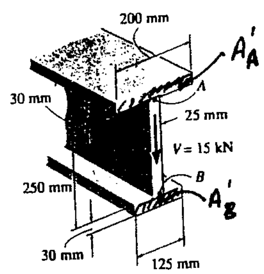


$\curvearrowleft + \Sigma M = 0; \quad M - 45(5 + 4\cos 20^\circ) = 0$

$M = 394.14 \text{ lb} \cdot \text{in.}$

$$\sigma_{max} = \frac{Mc}{I} = \frac{394.14(0.375)}{\frac{1}{12}(0.5)(0.75^3)} = 8.41 \text{ ksi} \qquad \textbf{Ans}$$

7–1 If the beam is subjected to a shear of $V = 15$ kN, determine the web's shear stress at A and B. Indicate the shear-stress components on a volume element located at these points. Set $w = 125$ mm. Show that the neutral axis is located at $\bar{y} = 0.1747$ m from the bottom and $I_{NA} = 0.2182(10^{-3})$ m^4.



$$\bar{y} = \frac{(0.015)(0.125)(0.03) + (0.155)(0.025)(0.25) + (0.295)(0.2)(0.03)}{0.125(0.03) + (0.025)(0.25) + (0.2)(0.03)} = 0.1747 \text{ m}$$

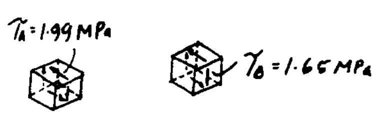
$$I = \frac{1}{12}(0.125)(0.03^3) + 0.125(0.03)(0.1747-0.015)^2$$
$$+\frac{1}{12}(0.025)(0.25^3) + 0.25(0.025)(0.1747-0.155)^2$$
$$+\frac{1}{12}(0.2)(0.03^3) + 0.2(0.03)(0.295-0.1747)^2 = 0.218182\,(10^{-3})\text{ m}^4$$

$$Q_A = \bar{y}'A'_A = (0.310-0.015-0.1747)(0.2)(0.03) = 0.7219\,(10^{-3})\text{ m}^3$$

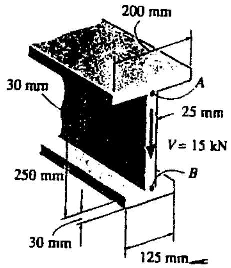
$$Q_B = \bar{y}'A'_B = (0.1747-0.015)(0.125)(0.03) = 0.59883\,(10^{-3})\text{ m}^3$$

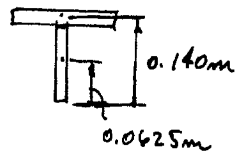
$$\tau_A = \frac{VQ_A}{I\,t} = \frac{15(10^3)(0.7219)(10^{-3})}{0.218182(10^{-3})0.025)} = 1.99 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_B = \frac{VQ_B}{I\,t} = \frac{15(10^3)(0.59883)(10^{-3})}{0.218182(10^{-3})0.025)} = 1.65 \text{ MPa} \qquad \textbf{Ans}$$



7–2 If the wide-flange beam is subjected to a shear of $V = 30$ kN, determine the maximum shear stress in the beam. Set $w = 200$ mm.





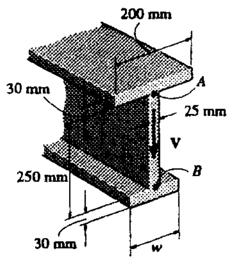
Section Properties :

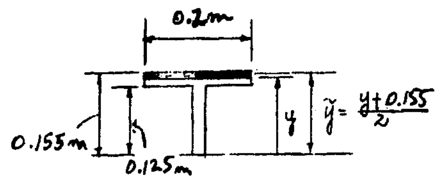
$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \mathrm{m}^4$$

$$Q_{max} = \Sigma \bar{y}A = 0.0625(0.125)(0.025) + 0.140(0.2)(0.030) = 1.0353(10)^{-3}\ \mathrm{m}^3$$

$$\tau_{max} = \frac{VQ}{It} = \frac{30(10)^3(1.0353)(10)^{-3}}{268.652(10)^{-6}(0.025)} = 4.62\ \mathrm{MPa} \qquad \textbf{Ans}$$

7–3 If the wide-flange beam is subjected to a shear of V = 30 kN, determine the shear force resisted by the web of the beam. Set w = 200 mm.





$$I = \frac{1}{12}(0.2)(0.310)^3 - \frac{1}{12}(0.175)(0.250)^3 = 268.652(10)^{-6}\ \text{m}^4$$

$$Q = \left(\frac{0.155 + y}{2}\right)(0.155 - y)(0.2) = 0.1(0.024025 - y^2)$$

$$\tau_f = \frac{30(10)^3(0.1)(0.024025 - y^2)}{268.652(10)^{-6}(0.2)}$$

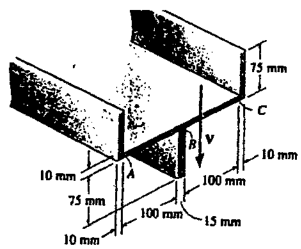
$$V_f = \int \tau_f\, dA = 55.8343(10)^6 \int_{0.125}^{0.155} (0.024025 - y^2)(0.2\, dy)$$

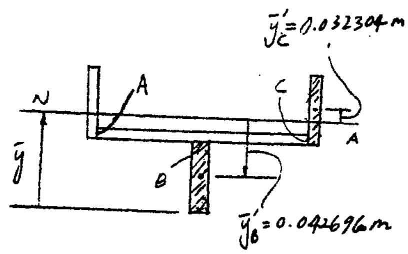
$$= 11.1669(10)^6 \left[0.024025y - \frac{1}{3}y^3 \right]_{0.125}^{0.155}$$

$$V_f = 1.457 \text{ kN}$$

$$V_w = 30 - 2(1.457) = 27.1 \text{ kN}$$ **Ans**

***7-4.** The beam is fabricated from three steel plates, and it is subjected to a shear force of $V = 150$ kN. Determine the shear stress at points A and C where the plates are joined. Show $\bar{y} = 0.080196$ m from the bottom and $I_{NA} = 4.8646(10^{-6})$ m^4.





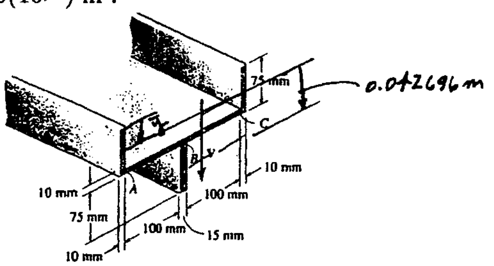
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.0375(0.075)(0.015) + 0.08(0.215)(0.01) + 2[0.1125(0.075)(0.01)]}{0.075(0.015) + 0.215(0.01) + 2(0.075)(0.01)}$$
$$= 0.080196 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.075^3) + (0.015)(0.075)(0.080196 - 0.0375)^2$$
$$+ \frac{1}{12}(0.215)(0.01^3) + 0.215(0.01)(0.080196 - 0.08)^2$$
$$+ 2[\frac{1}{12}(0.01)(0.075^3) + 0.01(0.075)(0.1125 - 0.080196)^2] = 4.8646(10^{-6}) \text{ m}^4$$

$$Q_A = Q_C = \bar{y}'_C A = 0.032304(0.075)(0.01) = 24.2277(10^{-6}) \text{ m}^3$$

$$\tau_A = \tau_C = \frac{VQ}{It} = \frac{150(10^3)(24.2277)(10^{-6})}{4.8646(10^{-6})(0.01)} = 74.7 \text{ MPa} \quad \textbf{Ans}$$

7-5. The beam is fabricated from three steel plates, and it is subjected to a shear force of $V = 150$ kN. Determine the shear stress at point B where the plates are joined. Show $\bar{y} = 0.080196$ m from the bottom and $I_{NA} = 4.8646(10^{-6})$ m^4.



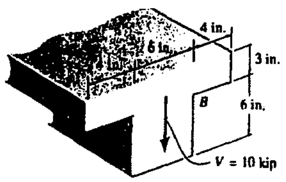
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.0375(0.075)(0.015) + 0.08(0.215)(0.01) + 2[0.1125(0.075)(0.01)]}{0.075(0.015) + 0.215(0.01) + 2(0.075)(0.01)}$$
$$= 0.080196 \text{ m}$$

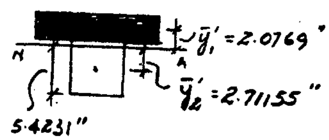
$$I = \frac{1}{12}(0.015)(0.075^3) + (0.015)(0.075)(0.080196 - 0.0375)^2$$
$$+ \frac{1}{12}(0.215)(0.01^3) + 0.215(0.01)(0.080196 - 0.08)^2$$
$$+ 2[\frac{1}{12}(0.01)(0.075^3) + 0.01(0.075)(0.1125 - 0.080196)^2] = 4.8646(10^{-6}) \text{ m}^4$$

$$Q_B = \bar{y}'_B A = 0.042696(0.075)(0.015) = 48.0333(10^{-6}) \text{ m}^3$$

$$\tau_B = \frac{VQ_B}{I\,t} = \frac{150(10^3)(48.0333)(10^{-6})}{4.8646(10^{-6})(0.015)} = 98.7 \text{ MPa} \qquad \textbf{Ans}$$

7-6. If the T-beam is subjected to a vertical shear of $V = 10$ kip, determine the maximum shear stress in the beam. Also, compute the shear-stress jump at the flange–web junction AB. Sketch the variation of the shear-stress intensity over the entire cross section. Show that $I_{NA} = 532.04\ \text{in}^4$.





$$\bar{y} = \frac{(1.5)(3)(14) + 6(6)(6)}{3(14) + 6(6)} = 3.5769\ \text{in.}$$

$$I = \frac{1}{12}(14)(3^3) + 3(14)(3.5769 - 1.5)^2 + \frac{1}{12}(6)(6^3) + 6(6)(6 - 3.5769)^2 = 532.04\ \text{in}^4$$

$$Q_{max} = \bar{y}_2 A' = 2.71155(5.4231)(6) = 88.23\ \text{in}^4$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{10(10^3)(88.23)}{532.04\,(6)} = 276\ \text{psi} \qquad \textbf{Ans}$$

$$Q_{AB} = \bar{y}_1 A' = 2.0769(3)(14) = 87.23\ \text{in}^3$$

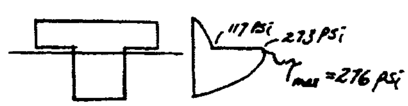
$$(\tau_{AB})_f = \frac{VQ_{AB}}{I\,t_f} = \frac{10(10^3)(87.23)}{532.04\,(14)} = 117.1\ \text{psi}$$

$$(\tau_{AB})_w = \frac{VQ_{AB}}{I\,t_w} = \frac{10(10^3)(87.23)}{532.04\,(6)} = 273.3\ \text{psi}$$

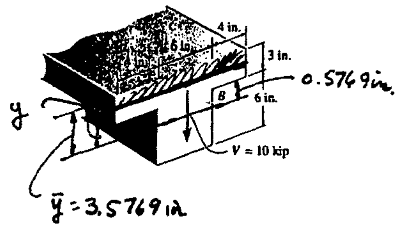
Shear stress jump $= (\tau_{AB})_w - (\tau_{AB})_f$

$= 273.3 - 117.1$

$= 156$ psi **Ans**



7-7. If the T-beam is subjected to a vertical shear of $V = 10$ kip, determine the vertical shear force resisted by the flange. Show that $I_{NA} = 532.04\ \text{in}^4$.



$$\bar{y} = \frac{(1.5)(3)(14) + 6(6)(6)}{3(14) + 6(6)} = 3.5769\ \text{in.}$$

$$I = \frac{1}{12}(14)(3^3) + 3(14)(3.5769 - 1.5)^2$$

$$+ \frac{1}{12}(6)(6^3) + 6(6)(6 - 3.5769)^2 = 532.04\ \text{in}^4$$

$$Q = (3.5769 - y)(14)\left(\frac{3.5769 + y}{2}\right) = 7(3.5769^2 - y^2)$$

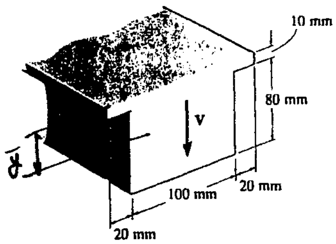
$$\tau = \frac{10(7)(3.5769^2 - y^2)}{(532.04)(14)} = 0.009398(3.5769^2 - y^2)$$

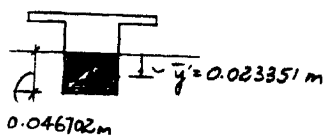
$$V_f = \int \tau\, dA = \int_{0.5769}^{3.5769} 0.009398(3.5769^2 - y^2)(14\, dy)$$

$$= 0.13157\left(3.5769^2 y - \frac{1}{3}y^3\right)\Big|_{0.5769}^{3.5769}$$

$$= 3.05\ \text{kip} \qquad \textbf{Ans}$$

•7–8 Determine the maximum shear stress in the strut if it is subjected to a shear force of $V = 15$ kN. Show that $I_{NA} = 6.691(10^{-6})\ \text{m}^4$.





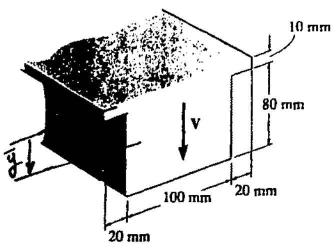
$$\bar{y} = \frac{(0.005)(0.01)(0.14) + (0.05)(0.1)(0.08)}{(0.01)(0.14) + (0.1)(0.08)} = 0.043298 \text{ m}$$

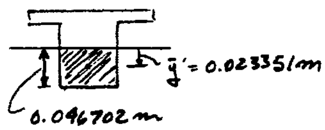
$$I = \frac{1}{12}(0.14)(0.01^3) + (0.14)(0.01)(0.043298 - 0.005)^2$$
$$+ \frac{1}{12}(0.1)(0.08^3) + (0.1)(0.08)(0.05 - 0.43298)^2 = 6.6911(10^{-6}) \text{ m}^4$$

$$Q_{max} = \bar{y}'A' = (0.023351)(0.046702)(0.1) = 0.1090544\,(10^{-3}) \text{ m}^3$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{15(10^3)(0.1090544)(10^{-3})}{6.6911(10^{-6})(0.1)} = 2.44 \text{ MPa}$$

7–9 Determine the maximum shear force V that the strut can support if the allowable shear stress for the material is $\tau_{allow} = 50$ MPa. Show that $I_{NA} = 6.691(10^{-6})$ m^4.





$$\bar{y} = \frac{(0.005)(0.01)(0.14) + (0.05)(0.1)(0.08)}{(0.01)(0.14) + (0.1)(0.08)} = 0.043298 \text{ m}$$

$$I = \frac{1}{12}(0.14)(0.01^3) + (0.14)(0.01)(0.043298 - 0.005)^2 + \frac{1}{12}(0.1)(0.08^3) + (0.1)(0.08)(0.043298 - 0.05)^2 = 6.6911(10^{-6}) \text{ m}^4$$

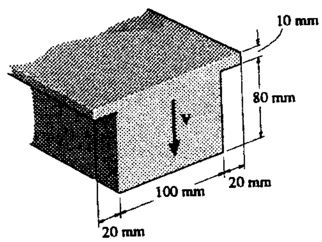
$$Q_{max} = \bar{y}'A' = (0.023351)(0.046702)(0.1) = 0.1090544\,(10^{-3}) \text{ m}^3$$

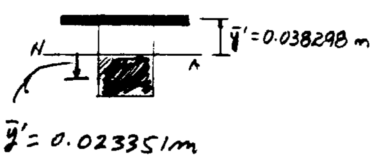
$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$50(10^6) = \frac{V(0.1090544)\,(10^{-3})}{6.6911\,(10^{-6})\,(0.1)}$$

$V = 307$ kN **Ans**

7–10 Determine the intensity of the shear stress distributed over the cross section of the strut if it is subjected to a shear force of $V = 12$ kN. Show that $I_{NA} = 6.691(10^{-6})$ m^4.





$$\bar{y} = \frac{(0.005)(0.01)(0.14) + (0.05)(0.1)(0.08)}{(0.01)(0.14) + (0.1)(0.08)} = 0.043298 \text{ m}$$

$$I = \frac{1}{12}(0.14)(0.01^3) + (0.14)(0.01)(0.043298 - 0.005)^2$$

$$+ \frac{1}{12}(0.1)(0.08^3) + (0.1)(0.08)(0.05 - 0.043298)^2 = 6.6911(10^{-6}) \text{ m}^4$$

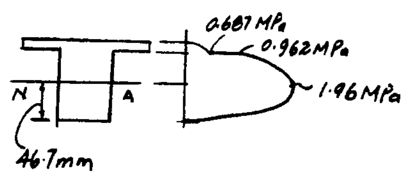
$$Q_{max} = \bar{y}'A' = (0.023351)(0.046702)(0.1) = 0.1090544\,(10^{-3}) \text{ m}^3$$

$$Q = \bar{y}'A' = (0.038298)(0.14)(0.01) = 53.6172\,(10^{-6}) \text{ m}^3$$

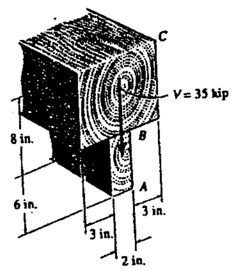
$$\tau_f = \frac{VQ}{It} = \frac{12(10^3)(53.6172)(10^{-6})}{6.6911\,(10^{-6})(0.14)} = 0.687 \text{ MPa}$$

$$\tau_w = \frac{12(10^3)(53.6172)(10^{-6})}{6.6911\,(10^{-6})(0.1)} = 0.962 \text{ MPa}$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{12(10^3)(0.1090544)\,(10^{-3})}{6.6911\,(10^{-6})(0.1)} = 1.96 \text{ MPa} \qquad \textbf{Ans}$$

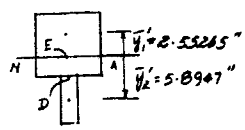


7-11. Sketch the intensity of the shear-stress distribution acting over the beam's cross-sectional area, and determine the resultant shear force acting on the segment AB. The shear acting at the section is $V = 35$ kip. Show that $I_{NA} = 872.49\ \text{in}^4$.



$$\bar{y} = \frac{(4)(8)(8) + (11)(6)(2)}{8(8) + 6(2)} = 5.1053\ \text{in.}$$

$$I = \frac{1}{12}(8)(8^3) + 8(8)(5.1053 - 4)^2 + \frac{1}{12}(2)(6^3) + 2(6)(11 - 5.1053)^2 = 872.49\ \text{in}^4$$



$$Q_E = \bar{y}'_1 A' = (2.55265)(5.1053)(8) = 104.26\ \text{in}^3$$

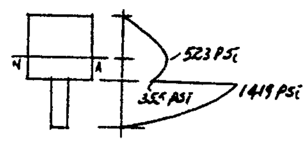
$$Q_D = \bar{y}'A' = (5.8947)(6)(2) = 70.74\ \text{in}^3$$

$$\tau = \frac{VQ}{It}$$

$$\tau_E = \frac{35(10^3)(104.26)}{872.49\ (8)} = 523\ \text{psi}$$

$$(\tau_D)_{t = 2\ \text{in.}} = \frac{35(10^3)(70.74)}{872.49\ (2)} = 1419\ \text{psi}$$

$$(\tau_D)_{t = 8\ \text{in.}} = \frac{35(10^3)(70.74)}{872.49\ (8)} = 355\ \text{psi}$$

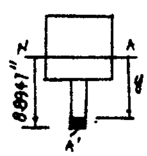


$$A' = 2(8.8947 - y)$$

$$\bar{y}' = y + \frac{(8.8947 - y)}{2} = \frac{(8.8947 + y)}{2}$$

$$Q = \bar{y}A' = 79.1157 - y^2$$

$$\tau = \frac{VQ}{It} = \frac{35\ (79.1157 - y^2)}{872.49\ (2)} = 1.586866 - 0.0200575\, y^2$$



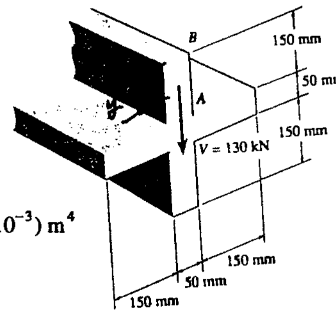
$$V = \int \tau\, dA \qquad dA = 2\, dy$$

$$V = \int (1.586866 - 0.0200575\, y^2)\, 2\, dy$$

$$= \int_{2.8947}^{8.8947} (3.173732 - 0.040115\, y^2)\, dy$$

$$= 9.96\ \text{kip} \qquad \textbf{Ans}$$

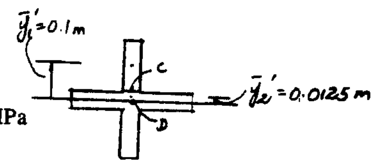
•7–12 The strut is subjected to a vertical shear of $V = 130$ kN. Plot the intensity of the shear-stress distribution acting over the cross-sectional area, and compute the resultant shear force developed in the vertical segment AB.



$$I = \frac{1}{12}(0.05)(0.35^3) + \frac{1}{12}(0.3)(0.05^3) = 0.18177083(10^{-3})\ \text{m}^4$$

$$Q_C = \bar{y}'A' = (0.1)(0.05)(0.15) = 0.75(10^{-3})\ \text{m}^3$$

$$Q_D = \Sigma\bar{y}A' = (0.1)(0.05)(0.15) + (0.0125)(0.35)(0.025)$$
$$= 0.859375(10^{-3})\ \text{m}^3$$

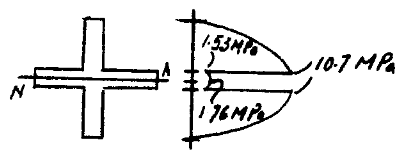


$$\tau = \frac{VQ}{It}$$

$$(\tau_C)_{t = 0.05\text{m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.05)} = 10.7\ \text{MPa}$$

$$(\tau_C)_{t = 0.35\text{m}} = \frac{130(10^3)(0.75)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.53\ \text{MPa}$$

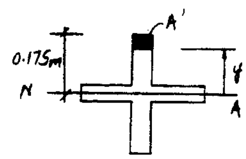
$$\tau_D = \frac{130(10^3)(0.859375)(10^{-3})}{0.18177083(10^{-3})(0.35)} = 1.76\ \text{MPa}$$



$$A' = (0.05)(0.175 - y)$$

$$\bar{y}' = y + \frac{(0.175 - y)}{2} = \frac{1}{2}(0.175 + y)$$

$$Q = \bar{y}'A' = 0.025\,(0.030625 - y^2)$$



$$\tau = \frac{VQ}{It}$$
$$= \frac{130(0.025)(0.030625 - y^2)}{0.18177083(10^{-3})(0.05)}$$
$$= 10951.3 - 357593.1\,y^2$$

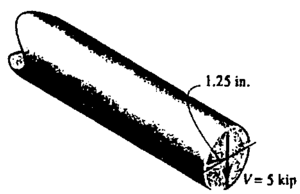
$$V = \int \tau\, dA \qquad dA = 0.05\, dy$$

$$= \int_{0.025}^{0.175} (10951.3 - 357593.1y^2)(0.05\, dy)$$

$$= \int_{0.025}^{0.175} (547.565 - 17879.66y^2)\, dy$$

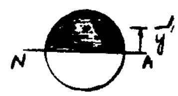
$= 50.3$ kN **Ans**

7–13 The steel rod has a radius of 1.25 in. If it is subjected to a shear of $V = 5$ kip, determine the maximum shear stress.



$$\bar{y}' = \frac{4r}{3\pi} = \frac{4(1.25)}{3\pi} = \frac{5}{3\pi}$$

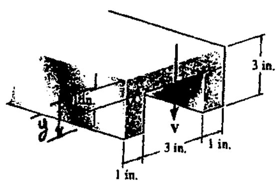
$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi (1.25)^4 = 0.610351\,\pi$$



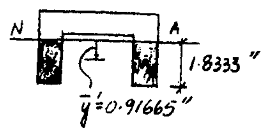
$$Q = \bar{y}'A' = \frac{5}{3\pi}\,\frac{\pi(1.25^2)}{2} = 1.3020833 \text{ in}^3$$

$$\tau_{max} = \frac{VQ}{It} = \frac{5(10^3)(1.3020833)}{0.610351(\pi)(2.50)} = 1358 \text{ psi} = 1.36 \text{ ksi} \qquad \textbf{Ans}$$

7-14. Determine the largest shear force V that the member can sustain if the allowable shear stress is $\tau_{allow} = 8$ ksi.



$$\bar{y} = \frac{(0.5)(1)(5) + 2\,[(2)(1)(2)]}{1\,(5) + 2\,(1)(2)} = 1.1667 \text{ in.}$$



$$I = \frac{1}{12}(5)(1^3) + 5\,(1)(1.1667 - 0.5)^2$$

$$+ 2\,(\frac{1}{12})(1)(2^3) + 2\,(1)(2)(2 - 1.1667)^2 = 6.75 \text{ in}^4$$

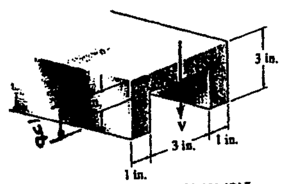
$$Q_{max} = \Sigma\bar{y}'A' = 2\,(0.91665)(1.8333)(1) = 3.3611 \text{ in}^3$$

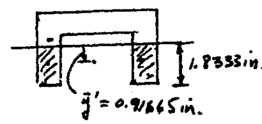
$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$8\,(10^3) = \frac{V(3.3611)}{6.75\,(2)(1)}$$

$V = 32132$ lb $= 32.1$ kip **Ans**

7-15. If the applied shear force $V = 18$ kip, determine the maximum shear stress in the member.





$$\bar{y} = \frac{(0.5)(1)(5) + 2\,[(2)(1)(2)]}{1\,(5) + 2\,(1)(2)} = 1.1667 \text{ in.}$$

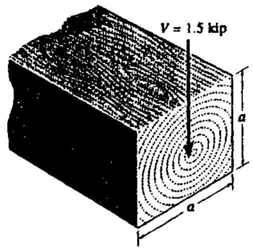
$$I = \frac{1}{12}(5)(1^3) + 5\,(1)(1.1667 - 0.5)^2$$

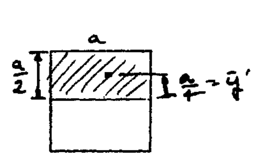
$$+ 2\left(\frac{1}{12}\right)(1)(2^3) + 2\,(1)(2)(2 - 1.1667) = 6.75 \text{ in}^4$$

$$Q_{max} = \Sigma\bar{y}'A' = 2\,(0.91665)(1.8333)(1) = 3.3611 \text{ in}^3$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{18(3.3611)}{6.75\,(2)(1)} = 4.48 \text{ ksi} \qquad \textbf{Ans}$$

•7–16 The beam has a square cross section and is made of wood having an allowable shear stress of $\tau_{allow} = 1.4$ ksi. If it is subjected to a shear of $V = 1.5$ kip, determine the smallest dimension a of its sides.





$$I = \frac{1}{12}a^4$$

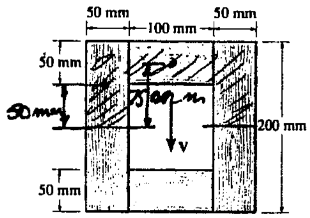
$$Q_{max} = \bar{y}'A' = (\frac{a}{4})(\frac{a}{2})a = \frac{a^3}{8}$$

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$1.4 = \frac{1.5\,(\frac{a^3}{8})}{\frac{1}{12}(a^4)(a)}$$

$a = 1.27$ in. **Ans**

7–17 The wood beam has an allowable shear stress of τ_{allow} = 7 MPa. Determine the maximum shear force V that can be applied to the cross section.



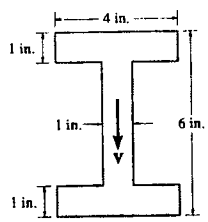
$$I = \frac{1}{12}(0.2)(0.2)^3 - \frac{1}{12}(0.1)(0.1)^3 = 125(10^{-6})\ \text{m}^4$$

$$\tau_{allow} = \frac{VQ_{max}}{It}$$

$$7(10^6) = \frac{V[(0.075)(0.1)(0.05) + 2(0.05)(0.1)(0.05)]}{125(10^{-6})(0.1)}$$

$V = 100$ kN **Ans**

7–18 The beam is made from a polymer and is subjected to a shear of $V = 7$ kip. Determine the maximum shear stress in the beam and plot the shear-stress distribution over the cross section. Report the values of the shear stress every 0.5 in. of beam depth.



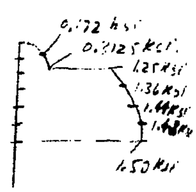
$$I = \frac{1}{12}(1)(4)^3 + 2\left[\frac{1}{12}(4)(1)^3 + 4(1)(2.5)^2\right] = 56\ \text{in}^4$$

$$\tau_1 = \frac{VQ}{It} = \frac{7(2.75)(4)(0.5)}{56(4)} = 0.172\ \text{ksi}$$

$$\tau_{2^+} = \frac{VQ}{It} = \frac{7(2.5)(4)(1)}{56(4)} = 0.3125\ \text{ksi}$$

$$\tau_{2^-} = \frac{VQ}{It} = \frac{7(2.5)(4)(1)}{56(1)} = 1.25\ \text{ksi}$$

$$\tau_3 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.75)(1)(0.5)]}{56(1)} = 1.36\ \text{ksi}$$

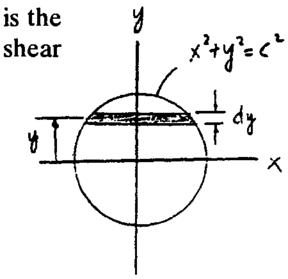


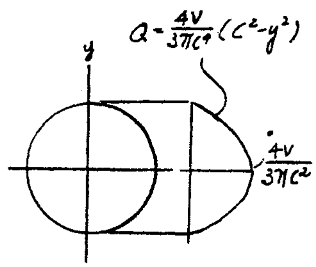
$$\tau_4 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.5)(1)(1)]}{56(1)} = 1.44\ \text{ksi}$$

$$\tau_4 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1.25)(1)(1.5)]}{56(1)} = 1.48\ \text{ksi}$$

$$\tau_{max} = \tau_5 = \frac{VQ}{It} = \frac{7[(2.5)(4)(1) + (1)(1)(2)]}{56(1)} = 1.50\ \text{ksi} \qquad \textbf{Ans}$$

7-19. Plot the shear-stress distribution over the cross section of a rod that has a radius c. By what factor is the maximum shear stress greater than the average shear stress acting over the cross section?





$x = \sqrt{c^2 - y^2}\,; \quad I = \frac{\pi}{4}c^4$

$t = 2x = 2\sqrt{c^2 - y^2}$

$dA = 2x\,dy = 2\sqrt{c^2 - y^2}\,dy$

$dQ = y\,dA = 2y\sqrt{c^2 - y^2}\,dy$

$$Q = \int_y^c 2y\sqrt{c^2 - y^2}\,dy = -\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}\Big|_y^c = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$

$$\tau = \frac{VQ}{It} = \frac{V[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}]}{(\frac{\pi}{4}c^4)(2\sqrt{c^2 - y^2})} = \frac{4V}{3\pi c^4}(c^2 - y^2)$$

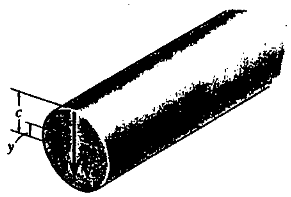
The maximum shear stress occur when $y = 0$

$$\tau_{max} = \frac{4V}{3\pi c^2}$$

$$\tau_{avg} = \frac{V}{A} = \frac{V}{\pi c^2}$$

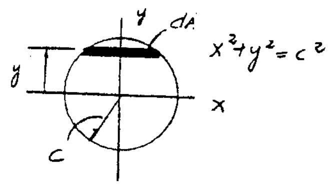
The factor $= \dfrac{\tau_{max}}{\tau_{avg}} = \dfrac{\frac{4V}{3\pi c^2}}{\frac{V}{\pi c^2}} = \dfrac{4}{3}$ **Ans**

•7-20 Develop an expression for the average vertical component of shear stress acting on the horizontal plane through the shaft, located a distance y from the neutral axis.



$$dA = 2x\,dy = 2\sqrt{c^2 - y^2}\,dy$$

$$Q = \int y\,dA = \int_y^c 2y\sqrt{c^2 - y^2}\,dy = \frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}$$

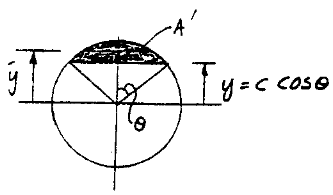


$$I = \frac{\pi}{4}c^4; \qquad t = 2(c^2 - y^2)^{\frac{1}{2}}$$

$$\tau = \frac{VQ}{It} = \frac{V[\frac{2}{3}(c^2 - y^2)^{\frac{3}{2}}]}{\frac{\pi}{4}c^4(2)(c^2 - y^2)^{\frac{1}{2}}}$$

$$= \frac{4V(c^2 - y^2)}{3\pi c^4} \qquad \textbf{Ans}$$

Also,



$$\bar{y} = \frac{\frac{2c\sin\theta}{3\theta}\theta\,c^2 - \frac{2}{3}\,c\cos\theta\,(\frac{1}{2})(2c\sin\theta)(c\cos\theta)}{A'}$$

$$= \frac{2c^3\sin\theta - 2c^3\sin\theta\cos^2\theta}{3A'}$$

$$Q = \bar{y}'A' = \frac{2}{3}c^3\sin\theta\,(1 - \cos^2\theta) = \frac{2}{3}c^3\sin^3\theta$$

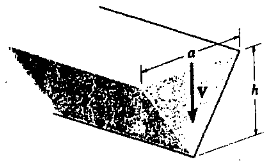
$$I = \frac{1}{4}\pi c^4; \qquad t = 2c\sin\theta$$

$$\tau = \frac{VQ}{It} = \frac{V(\frac{2}{3}c^3\sin^3\theta)}{\frac{1}{4}\pi c^4(2c\sin\theta)} = \frac{4V\sin^2\theta}{3\pi c^2}$$

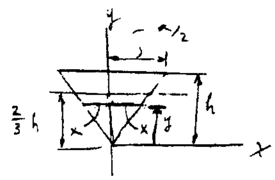
$$\sin\theta = \frac{\sqrt{c^2 - y^2}}{c}$$

Therefore, $\tau = \dfrac{4V}{3\pi c^2}\,\dfrac{c^2 - y^2}{c^2} = \dfrac{4V(c^2 - y^2)}{3\pi c^4}$ **Ans**

7–21 A member has a cross section in the form of an equilateral triangle. If it is subjected to a shear force **V**, determine the maximum average shear stress in the member. Can the shear formula be used to predict this value? Explain.



$$I = \frac{1}{36}(a)(h)^3$$



$$\frac{y}{x} = \frac{h}{a/2}; \qquad y = \frac{2h}{a}x$$

$$Q = \int_{A'} y\, dA = 2[(\frac{1}{2})(x)(y)(\frac{2}{3}h - \frac{2}{3}y)]$$

$$Q = (\frac{4h^2}{3a})(x^2)(1 - \frac{2x}{a})$$

$$t = 2x$$

$$\tau = \frac{VQ}{It} = \frac{V(4h^2/3a)(x^2)(1 - \frac{2x}{a})}{((1/36)(a)(h^3))(2x)}$$

$$\tau = \frac{24V(x - \frac{2}{a}x^2)}{a^2 h}$$

$$\frac{d\tau}{dx} = \frac{24V}{a^2h^2}(1 - \frac{4}{a}x) = 0$$

$$\text{At } x = \frac{a}{4}$$

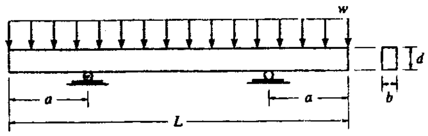
$$y = \frac{2h}{a}(\frac{a}{4}) = \frac{h}{2}$$

$$\tau_{max} = \frac{24V}{a^2h}(\frac{a}{4})(1 - \frac{2}{a}(\frac{a}{4}))$$

$$\tau_{max} = \frac{3V}{ah} \qquad \textbf{Ans}$$

No, because the shear stress is not perpendicular to the boundary. See Sec. 7 - 3.

7–22 The beam is subjected to a uniform load w. Determine the placement a of the supports so that the shear stress in the beam is as small as possible. What is this stress?



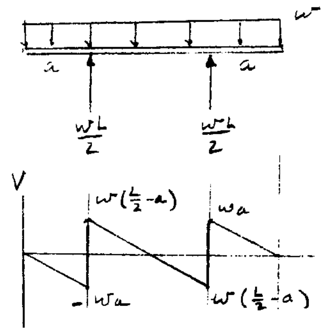
Require,

$$w\left(\frac{L}{2}-a\right) = wa$$

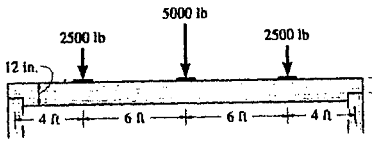
$$a = \frac{L}{4} \qquad \textbf{Ans}$$

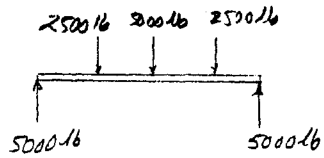
$$V = wa$$

$$\tau_{max} = \frac{VQ}{It} = \frac{w(L/4)(d/4)(b)(d/2)}{\left[\frac{1}{12}(b)(d)^3\right](b)} = \frac{3wL}{8bd} \qquad \textbf{Ans}$$



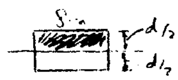
7–23 The timber beam is to be notched at its ends as shown. If it is to support the loading shown, determine the smallest depth d of the beam at the notch if the allowable shear stress is $\tau_{allow} = 450$ psi. The beam has a width of 8 in.





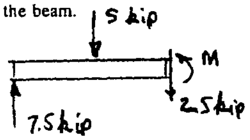
$V = 5000$ lb

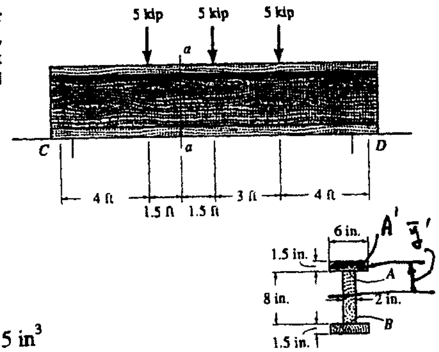
$$\tau = \frac{VQ}{It}; \qquad 450 = \frac{5000(d/4)(d/2)(8)}{\frac{1}{12}(8)(d)^3(8)}$$



$d = 2.08$ in. **Ans**

•7–24 The beam is made from three boards glued together at the seams *A* and *B*. If it is subjected to the loading shown, determine the shear stress developed in the glued joints at section *a–a*. The supports at *C* and *D* exert only vertical reactions on the beam.





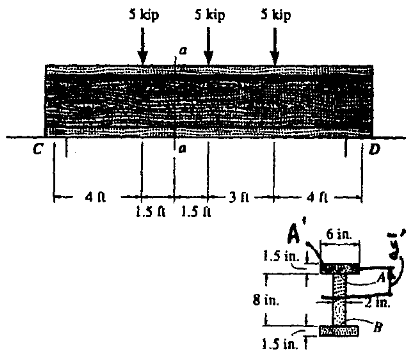
$$I = \frac{1}{12}(6)(11^3) - \frac{1}{12}(4)(8^3) = 494.83 \text{ in}^4$$

$$Q_A = Q_B = \bar{y}A' = (4 + \frac{1.5}{2})(6)(1.5) = 42.75 \text{ in}^3$$

$$\tau = \frac{VQ}{It}$$

$$\tau_A = \tau_B = \frac{2.5(10^3)(42.75)}{494.83\,(2)} = 108 \text{ psi} \qquad \textbf{Ans}$$

7–25 The beam is made from three boards glued together at the seams A and B. If it is subjected to the loading shown, determine the maximum shear stress developed in the glued joints. The supports at C and D exert only vertical reactions on the beam.



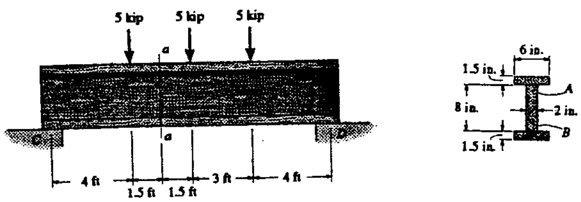
$V_{max} = 7.5$ kip (at C or D)

$$I = \frac{1}{12}(6)(11)^3 - \frac{1}{12}(4)(8)^3 = 494.83 \text{ in}^4$$

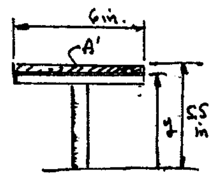
$$Q_A = Q_B = \bar{y}'A' = (4 + \frac{1.5}{2})(6)(1.5) = 42.75 \text{ in}^3$$

$$\tau_A = \tau_B = \frac{VQ}{I} = \frac{7.5(10^3)(42.75)}{494.83(2)} = 324 \text{ psi} \qquad \textbf{Ans}$$

7-26. The beam is made from three boards glued together at the seams A and B. If it is subjected to the loading shown, determine the maximum vertical shear force resisted by the top flange of the beam. The supports at C and D exert only vertical reactions on the beam.



$V_{max} = 7.5$ kip (at C or D)



$$I = \frac{1}{12}(6)(11)^3 - \frac{1}{12}(4)(8)^3 = 494.83 \text{ in}^4$$

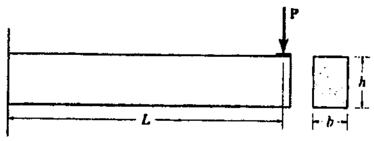
$$F_f = \int_{A_f} \tau dA$$

$$\tau = \frac{VQ}{It} = \frac{7.5(10^3)(5.5-y)(6)[(5.5+y)/2]}{494.83(6)} = 7.57836(30.25-y^2)$$

$$F_f = \int_4^{5.5} 7.57836(30.25-y^2)(6dy)$$

$$= 45.4702(30.25y - \frac{1}{3}y^3)\Big|_4^{5.5} = 512 \text{ lb} \quad \textbf{Ans}$$

7–27 Determine the length of the cantilevered beam so that the maximum bending stress in the beam is equivalent to the maximum shear stress. Comment on the validity of your results.



$$V_{max} = P$$

$$M_{max} = PL$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{PL(h/2)}{I} = \frac{PLh}{2I}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{P(h/2)(b)(h/4)}{Ib} = \frac{Ph^2}{8I}$$

Require,

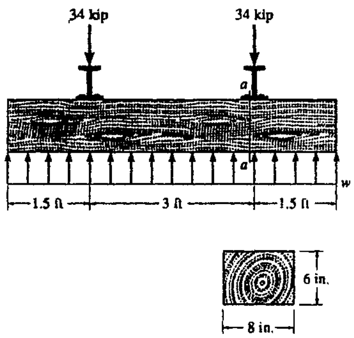
$$\sigma_{max} = \tau_{max}$$

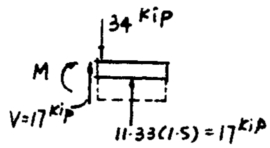
$$\frac{PLh}{2I} = \frac{Ph^2}{8I}$$

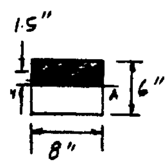
$$L = \frac{h}{4} \quad \textbf{Ans}$$

Shear stress is important only for very short beams. Note also, that this result is not all that accurate since Saint-Venant's Principle must be considered.

•7–28 Railroad ties must be designed to resist large shear loadings. If the tie is subjected to the 34-kip rail loadings and an assumed uniformly distributed ground reaction, determine the intensity w for equilibrium, and compute the maximum shear stress in the tie at section a–a, which is located just to the left of the rail.







$+\uparrow \Sigma F_y = 0; \qquad 6\,w - 2\,(34) = 0$

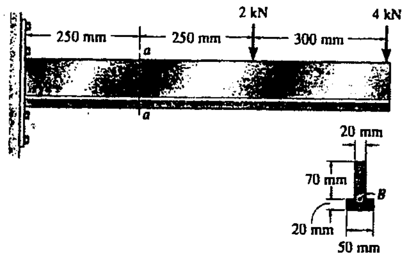
$w = 11.3 \text{ kip/ft}$ **Ans**

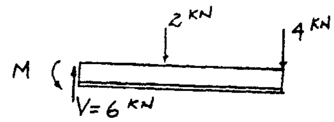
$I = \frac{1}{12}(8)(6^3) = 144 \text{ in}^4$

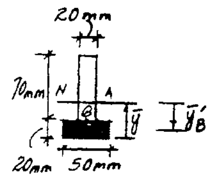
$Q_{max} = \bar{y}'A' = 1.5\,(3)(8) = 36 \text{ in}^3$

$\tau_{max} = \frac{VQ_{max}}{I\,t} = \frac{17(10^3)(36)}{144\,(8)} = 531 \text{ psi}$ **Ans**

7-29 Determine the shear stress at point *B* on the web of the cantilevered strut at section *a–a*.







$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625\ \text{m}$$

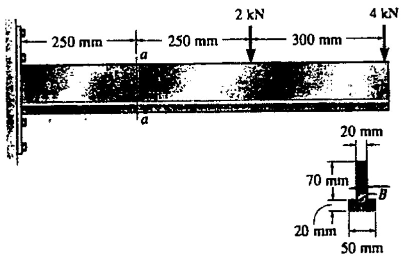
$$I = \frac{1}{12}(0.05)(0.02^3) + (0.05)(0.02)(0.03625 - 0.01)^2$$
$$+ \frac{1}{12}(0.02)(0.07^3) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625(10^{-6})\ \text{m}^4$$

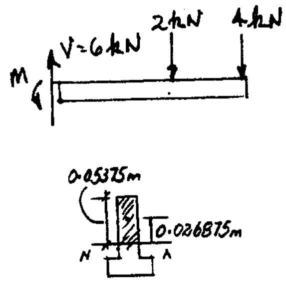
$$\bar{y}'_B = 0.03625 - 0.01 = 0.02625\ \text{m}$$

$$Q_B = (0.02)(0.05)(0.02625) = 26.25(10^{-6})\ \text{m}^3$$

$$\tau_B = \frac{VQ_B}{It} = \frac{6(10^3)(26.25)(10^{-6})}{1.78622(10^{-6})(0.02)}$$
$$= 4.41\ \text{MPa} \qquad \textbf{Ans}$$

7–30 Determine the maximum shear stress acting at section a–a of the cantilevered strut.





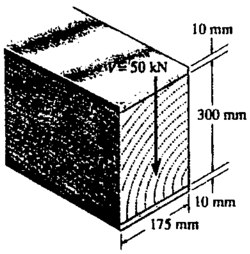
$$\bar{y} = \frac{(0.01)(0.05)(0.02) + (0.055)(0.07)(0.02)}{(0.05)(0.02) + (0.07)(0.02)} = 0.03625 \text{ m}$$

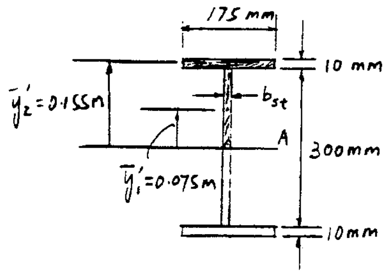
$$I = \frac{1}{12}(0.05)(0.02^3) + (0.05)(0.02)(0.03625 - 0.01)^2$$
$$+ \frac{1}{12}(0.02)(0.07^3) + (0.02)(0.07)(0.055 - 0.03625)^2 = 1.78625(10^{-6}) \text{ m}^4$$

$$Q_{max} = \bar{y}'A' = (0.026875)(0.05375)(0.02) = 28.8906(10^{-6}) \text{ m}^3$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{6(10^3)(28.8906)(10^{-6})}{1.78625(10^{-6})(0.02)}$$
$$= 4.85 \text{ MPa} \qquad \textbf{Ans}$$

7-31 The composite beam is constructed from wood and reinforced with a steel strap. Use the method of Sec. 6.6 and compute the maximum shear stress in the beam when it is subjected to a vertical shear of $V = 50$ kN. Take $E_{st} = 200$ GPa, $E_w = 15$ GPa.





$$b_{st} = nb_w = \frac{15}{200}(0.175) = 0.013125 \text{ m}$$

$$I = \frac{1}{12}(0.175)(0.32^3) - \frac{1}{12}(0.175-0.013125)(0.3^3) = 0.113648(10^{-3}) \text{ m}^4$$

$$Q_{max} = \Sigma\bar{y}'A' = 0.075(0.013125)(0.15) + 0.155(0.175)(0.01) = 0.4189(10^{-3}) \text{ m}^3$$

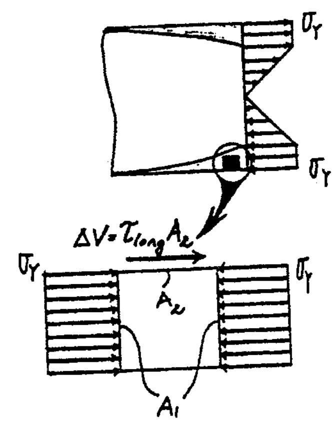
$$\tau_{max} = n\frac{VQ_{max}}{It} = \left(\frac{15}{200}\right)\frac{50(10^3)(0.4189)(10^{-3})}{0.113648(10^{-3})(0.013125)}$$

$$= 1.05 \text{ MPa} \qquad \textbf{Ans}$$

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

7-34. The beam has a rectangular cross section and is subjected to a load P that is just large enough to develop a fully plastic moment $M_p = PL$ at the fixed support. If the material is elastic-plastic, then at a distance $x < L$ the moment $M = Px$ creates a region of plastic yielding with an associated elastic core having a height $2y'$. This situation has been described by Eq. 6–30 and the moment **M** is distributed over the cross section as shown in Fig. 6–54e. Prove that the maximum shear stress developed in the beam is given by $\tau_{max} = \frac{3}{2}(P/A')$, where $A' = 2y'b$, the cross-sectional area of the elastic core.



Force Equilibrium : The shaded area indicates the plastic zone. Isolate an element in the plastic zone and write the equation of equilibrium.

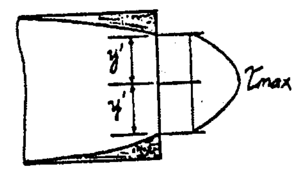
$$\xrightarrow{+} \Sigma F_x = 0; \qquad \tau_{long} A_2 + \sigma_Y A_1 - \sigma_Y A_1 = 0$$
$$\tau_{long} = 0$$

This proves that the longitudinal shear stress, τ_{long}, is equal to zero. Hence the corresponding transverse stress, τ_{trans}, is also equal to zero in the plastic zone. Therefore, the shear force $V = P$ is carried by the material only in the elastic zone.

Section Properties :

$$I_{NA} = \frac{1}{12}(b)(2y')^3 = \frac{2}{3}b\,y'^3$$

$$Q_{max} = \bar{y}'A' = \frac{y'}{2}(y')(b) = \frac{y'^2 b}{2}$$



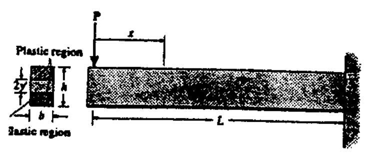
Maximum Shear Stress : Applying the shear formula

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{V\left(\frac{y'^2 b}{2}\right)}{\left(\frac{2}{3}by'^3\right)(b)} = \frac{3P}{4by'}$$

However, $\quad A' = 2by' \quad$ hence

$$\tau_{max} = \frac{3P}{2A'} \qquad (Q.E.D.)$$

7-35. The beam in Fig. 6–54*f* is subjected to a fully plastic moment $\mathbf{M}_p$. Prove that the longitudinal and transverse shear stresses in the beam are zero. *Hint:* Consider an element of the beam as shown in Fig. 7–4*d*.

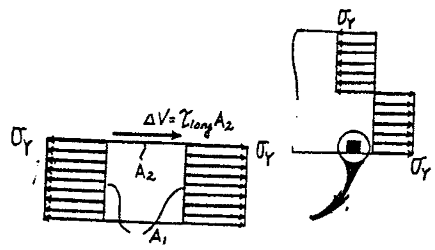


Force Equilibrium : If a fully plastic moment acts on the cross section, then an element of the material taken from the top or bottom of the cross section is subjected to the loading shown. For equilibrium

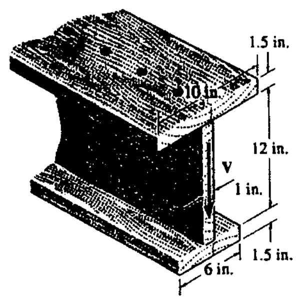
$$\xrightarrow{+} \Sigma F_x = 0; \qquad \sigma_Y A_1 + \tau_{\text{long}} A_2 - \sigma_Y A_1 = 0$$

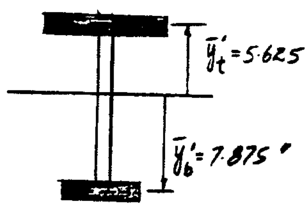
$$\tau_{\text{long}} = 0$$

Thus no shear stress is developed on the longitudinal or transverse plane of the element. *(Q. E. D.)*



***7–36** The beam is constructed from three boards. If it is subjected to a shear of $V = 5$ kip, determine the spacing s of the nails used to hold the top and bottom flanges to the web. Each nail can support a shear force of 500 lb.





$$\bar{y} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ in.}$$

$$I = \frac{1}{12}(10)(1.5^3) + 10(1.5)(6.375-0.75)^2 + \frac{1}{12}(1)(12^3) + (1)(12)(7.5-6.375)^2$$
$$+\frac{1}{12}(6)(1.5^3) + (1.5)(6)(14.25-6.375)^2 = 1196.4375 \text{ in}^4$$

$$Q_t = \bar{y}_t'A' = 5.625(10)(1.5) = 84.375 \text{ in}^3$$

$$Q_b = \bar{y}_b'A' = 7.875(6)(1.5) = 70.875 \text{ in}^3$$

$$q_t = \frac{VQ_t}{I} = \frac{5\,(10^3)(84.375)}{1196.4375} = 352.61 \text{ lb/in.}$$

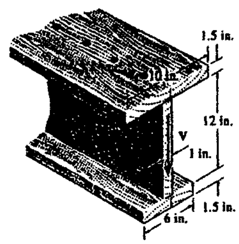
$$q_b = \frac{VQ_b}{I} = \frac{5\,(10^3)(70.875)}{1196.4375} = 296.19 \text{ lb/in.}$$

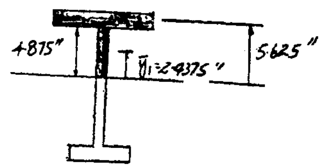
$$F = q\,s; \qquad s = \frac{F}{q}$$

$$s_t = \frac{500}{352.61} = 1.42 \text{ in.} \qquad \textbf{Ans}$$

$$s_b = \frac{500}{296.19} = 1.69 \text{ in.} \qquad \textbf{Ans}$$

7-37. The beam is constructed from three boards. Determine the maximum shear V that it can sustain if the allowable shear stress for the wood is $\tau_{\text{allow}} = 400$ psi. What is the required spacing s of the nails if each nail can resist a shear force of 400 lb?





$$\bar{y} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ in.}$$

$$I = \frac{1}{12}(10)(1.5^3) + 10(1.5)(6.375-0.75)^2 + \frac{1}{12}(1)(12^3) + (1)(12)(7.5-6.375)^2$$
$$+ \frac{1}{12}(6)(1.5^3) + (1.5)(6)(14.25-6.375)^2 = 1196.4375 \text{ in}^4$$

$$Q_{max} = \Sigma\bar{y}'A' = 5.625(10)(1.5) + 2.4375(4.875)(1) = 96.258 \text{ in}^3$$

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$0.4 = \frac{V(96.258)}{1196.4375(1)}$$

$V = 4.97$ kip **Ans**

$$Q_t = \bar{y}_t'A' = 5.625(10)(1.5) = 84.375 \text{ in}^3$$

$$Q_b = \bar{y}_b'A' = 7.875(6)(1.5) = 70.875 \text{ in}^3$$

$$q_t = \frac{4.9718(10^3)(84.375)}{1196.4375} = 350.62 \text{ lb/in.}$$

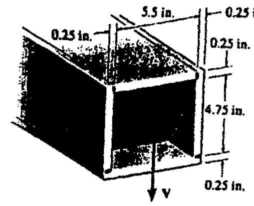
$$q_b = \frac{4.9718(10^3)(70.875)}{1196.4375} = 294.52 \text{ lb/in.}$$

$$s = \frac{F}{q}$$

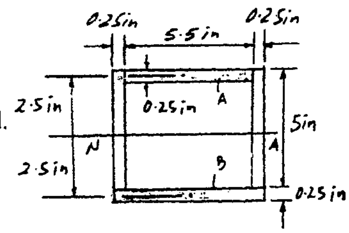
$s_t = \dfrac{400}{350.62} = 1.14$ in. **Ans**

$s_b = \dfrac{400}{294.52} = 1.36$ in. **Ans**

7-38. The box beam is made from four pieces of plastic that are glued together as shown. If the glue has an allowable strength of 400 lb/in^2, determine the maximum shear the beam will support.



$$I = \frac{1}{12}(6)(5.25^3) - \frac{1}{12}(5.5)(4.75^3) = 23.231 \text{ in}^4$$



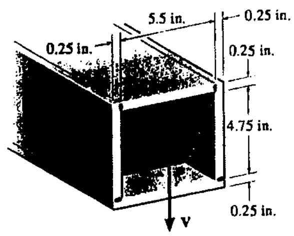
$$Q_B = \bar{y}'A' = 2.5(6)(0.25) = 3.75 \text{ in}^3$$

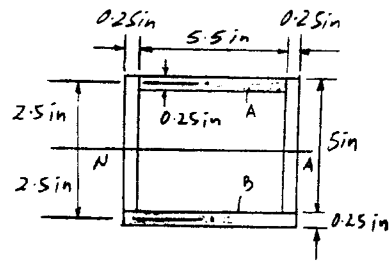
The beam will fail at the glue joint for board B since Q is a maximum for this board.

$$\tau_{allow} = \frac{VQ_B}{It}; \qquad 400 = \frac{V(3.75)}{23.231(2)(0.25)}$$

$$V = 1239 \text{ lb} = 1.24 \text{ kip} \qquad \textbf{Ans}$$

7–39 The box beam is made from four pieces of plastic that are glued together as shown. If the shear $V = 2$ kip, determine the shear stress resisted by the seam at each of the glued joints.





$$I = \frac{1}{12}(6)(5.25^3) - \frac{1}{12}(5.5)(4.75^3) = 23.231 \text{ in}^4$$

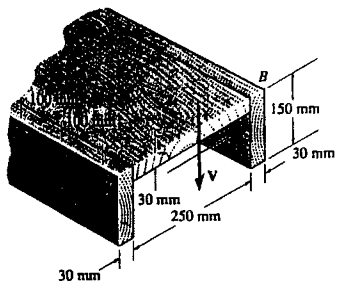
$$Q_B = \bar{y}'A' = 2.5(6)(0.25) = 3.75 \text{ in}^3$$

$$Q_A = 2.5(5.5)(0.25) = 3.4375$$

$$\tau_B = \frac{VQ_B}{It} = \frac{2(10^3)(3.75)}{23.231(2)(0.25)} = 646 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{2(10^3)(3.4375)}{23.231(2)(0.25)} = 592 \text{ psi} \qquad \textbf{Ans}$$

•7–40 The beam is subjected to a shear of $V = 800$ N. Determine the average shear stress developed in the nails along the sides A and B if the nails are spaced $s = 100$ mm apart. Each nail has a diameter of 2 mm.



$$\bar{y} = \frac{0.015\,(0.03)(0.25) + 2\,(0.075)(0.15)(0.03)}{0.03(0.25) + 2(0.15)(0.03)} = 0.04773 \text{ m}$$

$$I = \frac{1}{12}(0.25)(0.03^3) + (0.25)(0.03)(0.04773 - 0.015)^2$$

$$+ (2)(\frac{1}{12})\,(0.03)(0.15^3) + 2(0.03)(0.15)(0.075 - 0.04773)^2$$

$$= 32.164773(10^{-6})\text{ m}^4$$

$$Q = \bar{y}'A' = 0.03273(0.25)(0.03) = 0.245475(10^{-3})\text{ m}^3$$

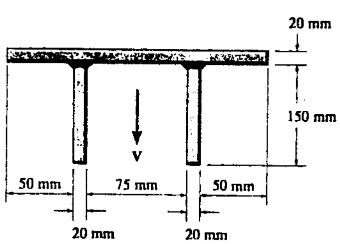
$$q = \frac{VQ}{I} = \frac{800\,(0.245475)(10^{-3})}{32.164773(10^{-6})} = 6105.44 \text{ N/m}$$

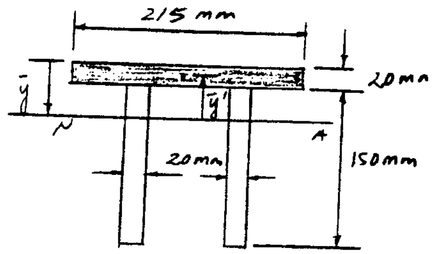
$$F = q\,s = 6105.44\,(0.1) = 610.544 \text{ N}$$

Since each side of the beam resists this shear force then

$$\tau_{avg} = \frac{F}{2A} = \frac{610.544}{2(\frac{\pi}{4})(0.002^2)} = 97.2 \text{ MPa} \qquad \textbf{Ans}$$

7–41 The double T-beam is fabricated by welding the three plates together as shown. Determine the shear stress in the weld necessary to support the shear force of $V = 80$ kN.





$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$

$$I = \frac{1}{12}(0.215)(0.02^3) + 0.215(0.02)(0.059515 - 0.01)^2$$

$$+ 2[\frac{1}{12}(0.02)(0.15^3) + 0.02(0.15)(0.095 - 0.059515)^2] = 29.4909(10^{-6}) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

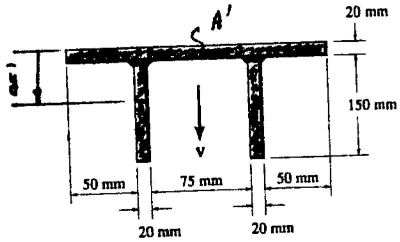
$$Q = \bar{y}'A' = 0.049515(0.215)(0.02) = 0.2129(10^{-3}) \text{ m}^3$$

Shear stress :

$$\tau = \frac{VQ}{It} = \frac{80(10^3)(0.2129)(10^{-3})}{29.4909(10^{-6})(2)(0.02)}$$

$$= 14.4 \text{ MPa} \qquad \textbf{Ans}$$

7–42 The double T-beam is fabricated by welding the three plates together as shown. If the weld can resist a shear stress τ_{allow} = 90 MPa, determine the maximum shear V that can be applied to the beam.



$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.01(0.215)(0.02) + 2[0.095(0.15)(0.02)]}{0.215(0.02) + 2(0.15)(0.02)} = 0.059515 \text{ m}$$

$$I = \frac{1}{12}(0.215)(0.02^3) + 0.215(0.02)(0.059515 - 0.01)^2$$
$$+ 2[\frac{1}{12}(0.02)(0.15^3) + 0.02(0.15)(0.095 - 0.059515)^2] = 29.4909(10^{-6}) \text{ m}^4$$

$$\bar{y}' = 0.059515 - 0.01 = 0.049515 \text{ m}$$

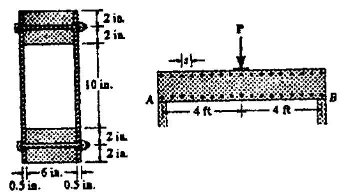
$$Q = \bar{y}'A' = 0.049515(0.215)(0.02) = 0.2129(10^{-3}) \text{ m}^3$$

$$\tau = \frac{VQ}{It}$$

$$90(10^6) = \frac{V(0.2129)(10^{-3})}{29.491(10^{-6})(2)(0.02)}$$

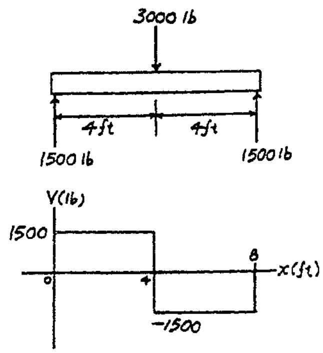
V = 499 kN **Ans**

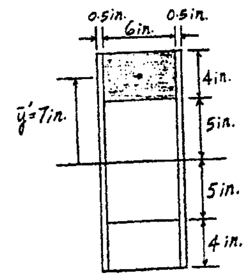
7-43. The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. If each fastener can support 600 lb in single shear, determine the required spacing s of the fasteners needed to support the loading $P = 3000$ lb. Assume A is pinned and B is a roller.



Support Reactions : As shown on FBD.

Internal Shear Force : As shown on shear diagram, $V_{max} = 1500$ lb.





Section Properties :

$$I_{NA} = \frac{1}{12}(7)(18^3) - \frac{1}{12}(6)(10^3) = 2902 \text{ in}^4$$

$$Q = \bar{y}'A' = 7(4)(6) = 168 \text{ in}^3$$

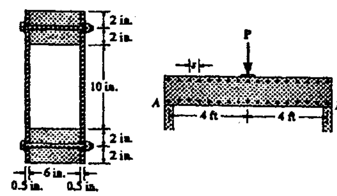
Shear Flow : Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(600)}{s} = \frac{1200}{s}$.

$$q = \frac{VQ}{I}$$

$$\frac{1200}{s} = \frac{1500(168)}{2902}$$

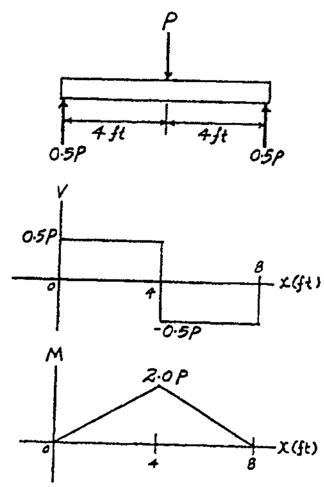
$$s = 13.8 \text{ in.} \qquad \textbf{Ans}$$

***7-44.** The double-web girder is constructed from two plywood sheets that are secured to wood members at its top and bottom. The allowable bending stress for the wood is $\sigma_{allow} = 8$ ksi and the allowable shear stress is $\tau_{allow} = 3$ ksi. If the fasteners are spaced $s = 6$ in. and each fastener can support 600 lb in single shear, determine the maximum load P that can be applied to the beam.



Support Reactions : As shown on FBD.

Internal Shear Force and Moment : As shown on shear and moment diagram, $V_{max} = 0.500P$ and $M_{max} = 2.00P$.

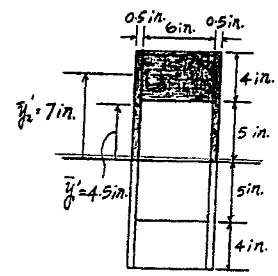


Section Properties :

$$I_{NA} = \frac{1}{12}(7)(18^3) - \frac{1}{12}(6)(10^3) = 2902 \text{ in}^4$$

$$Q = \bar{y}_2' A' = 7(4)(6) = 168 \text{ in}^3$$

$$Q_{max} = \Sigma \bar{y}' A' = 7(4)(6) + 4.5(9)(1) = 208.5 \text{ in}^3$$



Shear Flow : Assume bolt failure. Since there are two shear planes on the bolt, the allowable shear flow is $q = \frac{2(600)}{6} = 200$ lb/in.

$$q = \frac{VQ}{I}$$

$$200 = \frac{0.500P(168)}{2902}$$

$P = 6910 \text{ lb} = 6.91$ kip (*Controls!*) **Ans**

Shear Stress : Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{VQ_{max}}{It}$$

$$3000 = \frac{0.500P(208.5)}{2902(1)}$$

$$P = 22270 \text{ lb} = 83.5 \text{ kip}$$

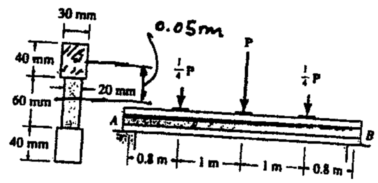
Bending Stress : Assume failure due to bending stress.

$$\sigma_{max} = \sigma_{allow} = \frac{Mc}{I}$$

$$8(10^3) = \frac{2.00P(12)(9)}{2902}$$

$$P = 107 \text{ ksi}$$

7-45. The beam is made from three polystyrene strips that are glued together as shown. If the glue has a shear strength of 80 kPa, determine the maximum load P that can be applied without causing the glue to lose its bond.



Maximum shear is at the supports.

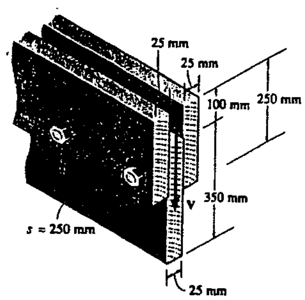
$$V_{max} = \frac{3P}{4}$$

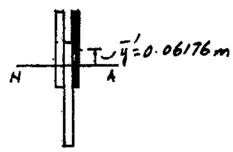
$$I = \frac{1}{12}(0.02)(0.06)^3 + 2[\frac{1}{12}(0.03)(0.04)^3 + (0.03)(0.04)(0.05)^2] = 6.68(10^{-6})\text{m}^4$$

$$\tau = \frac{VQ}{It}; \qquad 80(10^3) = \frac{(3P/4)(0.05)(0.04)(0.03)}{6.68(10^{-6})(0.02)}$$

$$P = 238 \text{ N} \qquad \textbf{Ans}$$

7-46. A beam is constructed from three boards bolted together as shown. Determine the shear force developed in each bolt if the bolts are spaced $s = 250$ mm apart and the applied shear is $V = 35$ kN.





$$\bar{y} = \frac{2\,(0.125)(0.25)(0.025) + 0.275\,(0.35)(0.025)}{2\,(0.25)(0.025) + 0.35\,(0.025)} = 0.18676\text{ m}$$

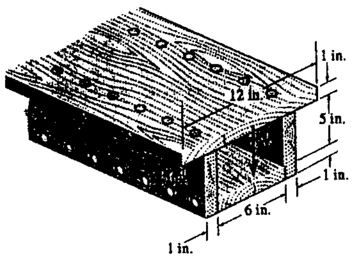
$$I = (2)(\frac{1}{12})(0.025)(0.25^3) + 2\,(0.025)(0.25)(0.18676 - 0.125)^2$$
$$+ \frac{1}{12}(0.025)(0.35)^3 + (0.025)(0.35)(0.275 - 0.18676)^2$$
$$= 0.270236\,(10^{-3})\text{ m}^4$$

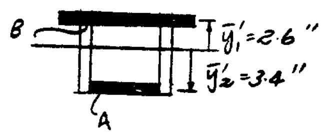
$$Q = \bar{y}'A' = 0.06176(0.025)(0.25) = 0.386(10^{-3})\text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{35\,(0.386)(10^{-3})}{0.270236\,(10^{-3})} = 49.993\text{ kN/m}$$

$$F = q\,(s) = 49.993\,(0.25) = 12.5\text{ kN} \qquad \textbf{Ans}$$

7–47 The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 in. If each nail can resist a shear of 50 lb, determine the greatest shear V that can be applied to the beam without causing failure of the nails.





$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5\,(12)(1) + 2\,(4)(6)(1) + (6.5)(6)(1)}{12(1) + 2(6)(1) + (6)(1)} = 3.1 \text{ in.}$$

$$I = \frac{1}{12}(12)(1^3) + 12(1)(3.1 - 0.5)^2$$

$$+ 2(\frac{1}{12})(1)(6^3) + 2(1)(6)(4 - 3.1)^2$$

$$+ \frac{1}{12}(6)(1^3) + 6(1)(6.5 - 3.1)^2 = 197.7 \text{ in}^4$$

$$Q_B = \bar{y}_1' A' = 2.6(12)(1) = 31.2 \text{ in}^3$$

$$q_B = \frac{1}{2}(\frac{VQ_B}{I}) = \frac{V(31.2)}{2(197.7)} = 0.0789\,V$$

$$q_B\,s = 0.0789V(2) = 50$$

$V = 317$ lb (controls) **Ans**

$$Q_A = \bar{y}_2' A' = 3.4(6)(1) = 20.4 \text{ in}^3$$

$$q_A = \frac{1}{2}(\frac{VQ_A}{I}) = \frac{V(20.4)}{2(197.7)} = 0.0516\,V$$

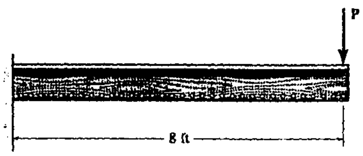
$$q_A\,s = 0.0516V(2) = 50$$

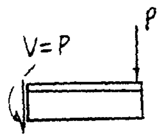
$V = 485$ lb

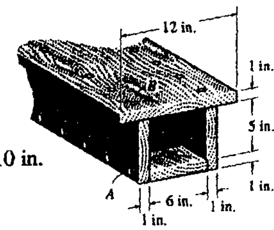
Because of the number and variety of potential correct solutions to this problem, no solution is being given.

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

7-50 The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 in. If each nail can resist a shear force of 50 lb, determine the largest force P that can be applied to the beam without causing failure of the nails.



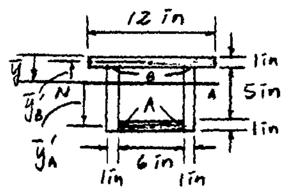




Shear force : $V = P$

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.5(12)(1) + 2[4(6)(1)] + 6.5(6)(1)}{12(1) + 2(6)(1) + 6(1)} = 3.10 \text{ in.}$$

$$I = \frac{1}{12}(12)(1^3) + 12(1)(3.10-0.5)^2 + 2[\frac{1}{12}(1)(6^3) + (1)(6)(4-3.10)^2]$$
$$+ \frac{1}{12}(6)(1^3) + 6(1)(6.5-3.10)^2 = 197.7 \text{ in}^4$$



$$Q_B = \bar{y}_B'A' = (3.10 - 0.5)(12)(1) = 31.2 \text{ in}^3$$

$$Q_A = \bar{y}_A'A' = (6.5 - 3.10)(6)(1) = 20.4 \text{ in}^3$$

For B :

$$q_B = \frac{VQ_B}{I} = \frac{P(31.2)}{197.7} = 0.1578\, P, \qquad \text{however } q = \frac{2(50)}{2} = 50 \text{ lb/in.}$$

$50 = 0.1578\, P$

$P = 317$ lb (controls) **Ans**

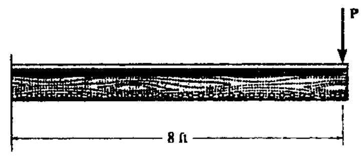
For A :

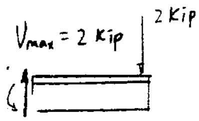
$$q_A = \frac{VQ_A}{I} = \frac{P(20.4)}{197.7} = 0.1032\, P \qquad \text{However } q = \frac{2(50)}{2} = 50 \text{ lb/in.}$$

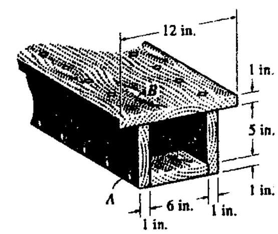
$50 = 0.1032\, P$

$P = 485$ lb

7-51 The box beam is constructed from four boards that are fastened together using nails spaced along the beam every 2 in. If a force $P = 2$ kip is applied to the beam, determine the shear force resisted by each nail at A and B.





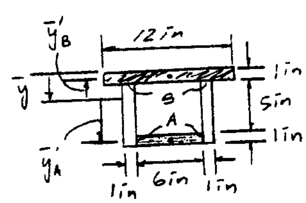


As shown on FBD, $V_{max} = 2$ kip

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.5(12)(1) + 2[4(6)(1)] + 6.5(6)(1)}{12(1) + 2(6)(1) + 6(1)} = 3.10 \text{ in.}$$

$$I = \frac{1}{12}(12)(1^3) + 12(1)(3.10-0.5)^2 + 2[\frac{1}{12}(1)(6^3) + (1)(6)(4-3.10)^2]$$
$$+ \frac{1}{12}(6)(1^3) + 6(1)(6.5-3.10)^2 = 197.7 \text{ in}^4$$

$Q_B = \bar{y}_B'A' = (3.10 - 0.5)(12)(1) = 31.2 \text{ in}^3$



$Q_A = \bar{y}_A'A' = (6.5 - 3.10)(6)(1) = 20.4 \text{ in}^3$

$V = P = 2$ kip

$$q_B = \frac{1}{2}(\frac{VQ_B}{I}) = \frac{1}{2}[\frac{2(31.2)}{197.7}] = 157.81 \text{ lb/in.}$$

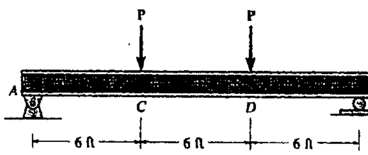
$$q_A = \frac{1}{2}(\frac{VQ_A}{I}) = \frac{1}{2}[\frac{2(20.4)}{197.7}] = 103.19 \text{ lb/in.}$$

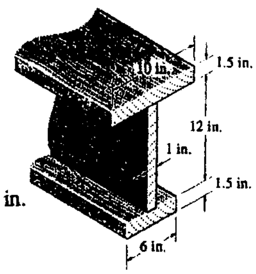
Shear force in nail :

$F_B = q_B s = 157.81(2) = 316$ lb **Ans**

$F_A = q_A s = 103.19(2) = 206$ lb **Ans**

•7–52 The beam is constructed from three boards. If it is subjected to loads of $P = 5$ kip, determine the spacing s of the nails within regions AC, CD, and DB used to hold the top and bottom flanges to the web. Each nail can support a shear force of 500 lb.



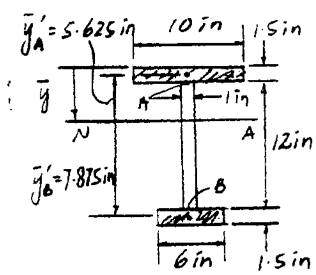


Shear force : Shear force in region BD is equal to shear force in region AC due to the symetrical loading and geometry

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ in.}$$

$$I = \frac{1}{12}(10)(1.5^3) + 10(1.5)(6.375-0.75)^2 + \frac{1}{12}(1)(12^3)$$
$$+ 1(12)(7.5-6.375)^2 + \frac{1}{12}(6)(1.5^3) + 6(1.5)(14.25-6.375)^2 = 1196.4375 \text{ in}^4$$

$$Q_A = \bar{y}'_A A' = 5.625(10)(1.5) = 84.375 \text{ in}^3$$

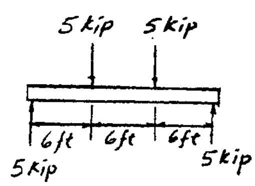


$$Q_B = \bar{y}'_B A' = 7.875(6)(1.5) = 70.875 \text{ in}^3$$

For region AC and BD, $V = 5$ kip

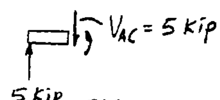
$$q_A = \frac{VQ_A}{I} = \frac{5(10^3)(84.375)}{1196.4375} = 352.61 \text{ lb/in.}$$

$$q_B = \frac{VQ_B}{I} = \frac{5(10^3)(70.875)}{1196.4375} = 296.19 \text{ lb/in.}$$



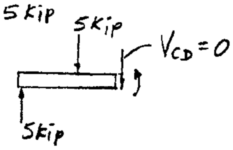
Spacing of nails at top flange

$$s_{top} = \frac{F}{q_A} = \frac{500}{352.61} = 1.42 \text{ in.} \quad \text{(Regions } AC \text{ and } BD\text{)} \quad \textbf{Ans}$$



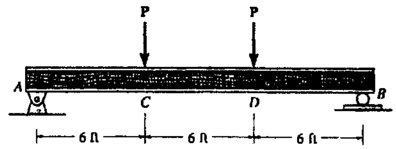
Spacing of nails at bottom flange,

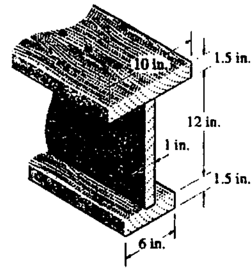
$$s_{bottom} = \frac{F}{q_B} = \frac{500}{296.19} = 1.69 \text{ in.} \quad \text{(Regions } AC \text{ and } BD\text{)} \quad \textbf{Ans}$$



For region CD, theoretically no nails are required to hold the flange and web together since $V_{CD} = 0$. However, it is advisible to provide some nails within this region. **Ans**

7–53 The beam is constructed from three boards. Determine the maximum loads P that it can support if the allowable shear stress for the wood is $\tau_{allow} = 400$ psi. What is the required spacing s of the nails used to hold the top and bottom flanges to the web if each nail can resist a shear force of 400 lb?





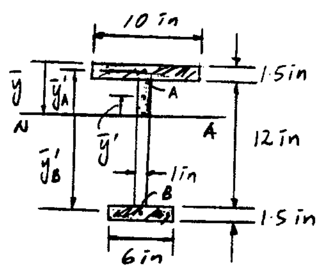
As shown on FBD

$V_{max} = P,\ V_{AC} = V_{DB} = P,\ V_{CD} = 0$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.75(10)(1.5) + 7.5(12)(1) + 14.25(6)(1.5)}{10(1.5) + 12(1) + 6(1.5)} = 6.375 \text{ in.}$$

$$I = \frac{1}{12}(10)(1.5^3) + 10(1.5)(6.375-0.75)^2 + \frac{1}{12}(1)(12^3) + 1(12)(7.5-6.375)^2 + \frac{1}{12}(6)(1.5^3) + 6(1.5)(7.875^2)$$

$$= 1196.4375 \text{ in}^4$$



$Q_A = \bar{y}_A' A' = 5.625(10)(1.5) = 84.375 \text{ in}^3$

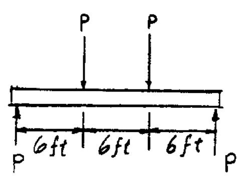
$Q_B = \bar{y}_B' A' = 7.875(6)(1.5) = 70.875 \text{ in}^3$

$Q_{max} = \Sigma \bar{y}' A' = 5.625(10)(1.5) + \frac{4.875}{2}(4.875)(1) = 96.2578 \text{ in}^3$

Maximum shear stress :

$$\tau_{allow} = \frac{VQ_{max}}{It}; \quad 400 = \frac{P(96.2578)}{1196.4375(1)}$$

$P = 4971.8 \text{ lb} = 4.97 \text{ kip}$ **Ans**



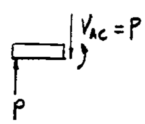
For region AC and BD

$$q_A = \frac{VQ_A}{I} = \frac{4971.8(84.375)}{1196.4375} = 350.62 \text{ lb/in.}$$

$$q_B = \frac{VQ_B}{I} = \frac{4971.8(70.875)}{1196.4375} = 294.52 \text{ lb/in.}$$

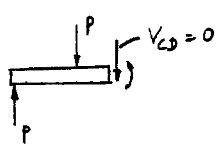
Nail spacing at top and bottom flange

$$s_{top} = \frac{F}{q_A} = \frac{400}{350.62} = 1.14 \text{ in.}$$ **Ans** (Regions AC and BD)

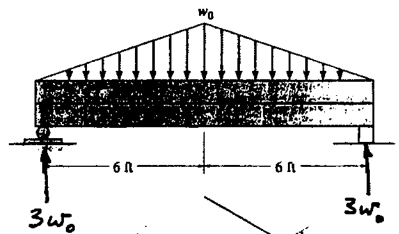


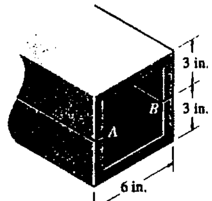
$$s_{bottom} = \frac{F}{q_B} = \frac{400}{294.52} = 1.36 \text{ in.}$$ **Ans** (Regions AC and BD)

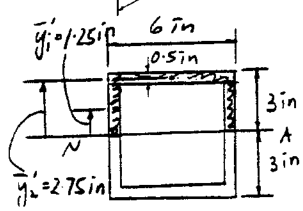
For region CD, theoretically no nails are required to hold the flange and web together since $V_{CD} = 0$. However, it is advisible to provide some nails within this region. **Ans**



7–54 The member consists of two plastic channel strips 0.5 in. thick, bonded together at A and B. If the glue can support an allowable shear stress of $\tau_{allow} = 600$ psi, determine the maximum intensity w_0 of the triangular distributed loading that can be applied to the member based on the strength of the glue.







Maximum shear force : $V_{max} = 3w_0$

$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5)(5^3) = 55.916\text{ in}^4$$

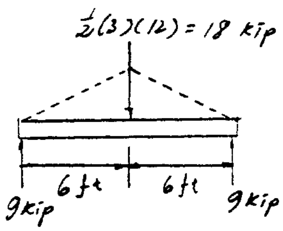
$$Q = \Sigma\tilde{y}'A' = 2[1.25(2.5)(0.5)] + 2.75(6)(0.5) = 11.375\text{ in}^3$$

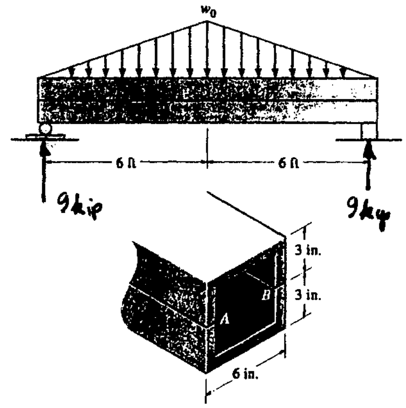
$$q = \tau_{allow}\, t = \frac{VQ}{I}$$

$$600(2)(0.5) = \frac{3w_0(11.375)}{55.916}$$

$w_0 = 983$ lb/ft **Ans**

7–55 The member consists of two plastic channel strips, 0.5 in. thick, glued together at A and B. If the distributed load has a maximum intensity of w_0 = 3 kip/ft, determine the maximum shear stress resisted by the glue.



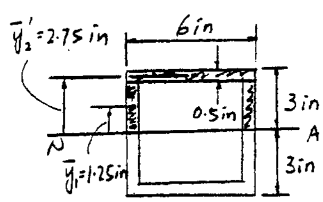


V_{max} = 9 kip

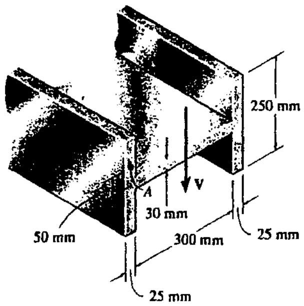
$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(5)(5^3) = 55.916 \text{ in}^4$$

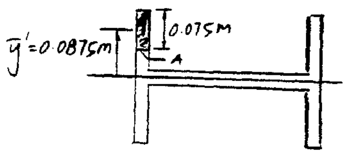
$$Q = \Sigma \bar{y}'A' = 2[1.25(2.5)(0.5)] + 2.75(6)(0.5) = 11.375 \text{ in}^3$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{9(11.375)}{55.916(1)} = 1.83 \text{ ksi} \qquad \textbf{Ans}$$



•7–56 The H-beam is subjected to a shear of $V = 80$ kN. Determine the shear flow at point A.



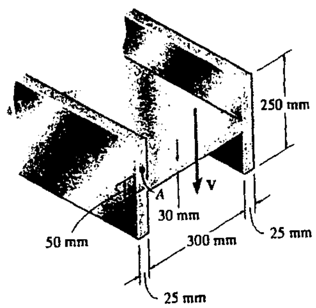


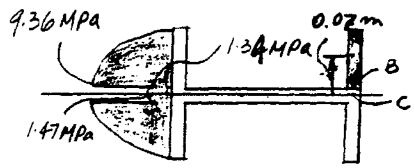
$$I = 2\left[\frac{1}{12}(0.025)(0.25^3) + \frac{1}{12}(0.3)(0.03^3)\right] = 65.7792(10^{-6})\ \text{m}^4$$

$$Q_A = \bar{y}'A' = 0.0875(0.075)(0.025) = 0.1641(10^{-3})\ \text{m}^3$$

$$q_A = \frac{VQ_A}{I} = \frac{80(10^3)(0.1641)(10^{-3})}{65.7792(10^{-6})} = 200\ \text{kN/m} \qquad \textbf{Ans}$$

7-57 The H-beam is subjected to a shear of $V = 80$ kN. Sketch the shear stress distribution acting along one of its side segments. Indicate all peak values.





$$I = 2[\frac{1}{12}(0.025)(0.25^3) + \frac{1}{12}(0.3)(0.03^3) = 65.7792(10^{-6})\ \text{m}^4$$

$$Q_B = (0.070)(0.025)(0.110) = 0.1925(10^{-3})\ \text{m}^3$$

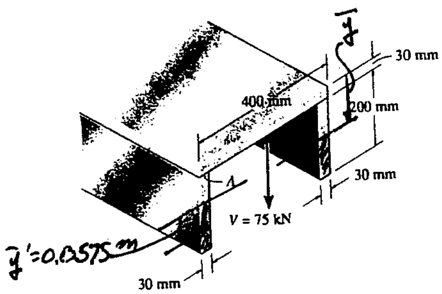
$$\tau_{B'} = \frac{VQ}{It} = \frac{80(10^3)(0.1925)(10^{-3})}{65.7792(10^{-6})(0.025)} = 9.36\ \text{MPa}$$

$$\tau_B = \frac{VQ}{It} = \frac{80(10^3)[2(0.1925)(10^{-3})]}{65.7792(10^{-6})(0.35)} = 1.3378\ \text{MPa}$$

$$Q_{max} = 2(0.07)(0.025)(0.110) + (0.0075)(0.35)(0.015) = 0.4244(10^{-3})\ \text{m}^3$$

$$\tau_C = \frac{VQ}{It} = \frac{80(10^3)(0.4244)(10^{-3}))}{65.7792(10^{-6})(0.35)} = 1.47\ \text{MPa}$$

7–58 The channel is subjected to a shear of $V = 75$ kN. Determine the shear flow developed at point A.



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015(0.4)(0.03) + 2[0.13(0.2)(0.03)]}{0.4(0.03) + 2(0.2)(0.03)} = 0.0725 \text{ m}$$

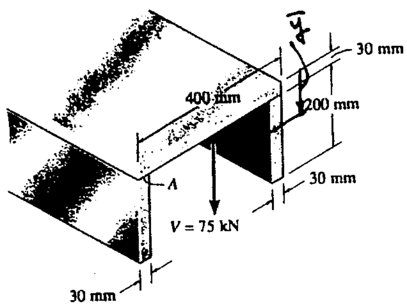
$$I = \frac{1}{12}(0.4)(0.03^3) + 0.4(0.03)(0.0725 - 0.015)^2 + 2[\frac{1}{12}(0.03)(0.2^3) + 0.03(0.2)(0.13 - 0.0725)^2] = 0.12025(10^{-3}) \text{ m}^4$$

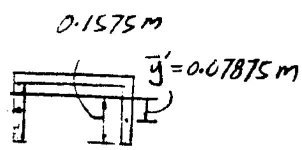
$$Q_A = \bar{y}'_A A' = 0.0575(0.2)(0.03) = 0.3450(10^{-3}) \text{ m}^3$$

$$q = \frac{VQ}{I}$$

$$q_A = \frac{75(10^3)(0.3450)(10^{-3})}{0.12025(10^{-3})} = 215 \text{ kN/m} \qquad \textbf{Ans}$$

7-59 The channel is subjected to a shear of $V = 75$ kN. Determine the maximum shear flow in the channel.





$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.015(0.4)(0.03) + 2[0.13(0.2)(0.03)]}{0.4(0.03) + 2(0.2)(0.03)}$$

$$= 0.0725 \text{ m}$$

$$I = \frac{1}{12}(0.4)(0.03^3) + 0.4(0.03)(0.0725 - 0.015)^2$$

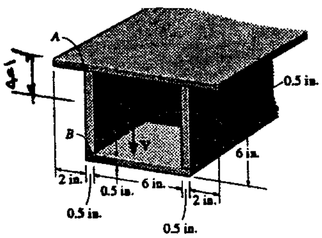
$$+ 2[\frac{1}{12}(0.03)(0.2^3) + 0.03(0.2)(0.13 - 0.0725)^2]$$

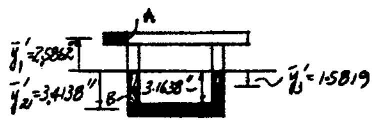
$$= 0.1202(10^{-3}) \text{ m}^4$$

$$Q_{max} = \bar{y}'A' = 0.07875(0.1575)(0.03) = 0.37209(10^{-3}) \text{ m}^3$$

$$q_{max} = \frac{75(10^3)(0.37209)(10^{-3})}{0.12025(10^{-3})} = 232 \text{ kN/m} \qquad \textbf{Ans}$$

***7-60.** The assembly is subjected to a vertical shear of V = 7 kip. Determine the shear flow at points A and B and the maximum shear flow in the cross section.





$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(0.25)(11)(0.5) + 2(3.25)(5.5)(0.5) + 6.25(7)(0.5)}{0.5(11) + 2(0.5)(5.5) + 7(0.5)} = 2.8362 \text{ in.}$$

$$I = \frac{1}{12}(11)(0.5^3) + 11(0.5)(2.8362 - 0.25)^2 + 2(\frac{1}{12})(0.5)(5.5^3) + 2(0.5)(5.5)(3.25 - 2.8362)^2 +$$
$$\frac{1}{12}(7)(0.5^3) + (0.5)(7)(6.25 - 2.8362)^2 = 92.569 \text{ in}^4$$

$Q_A = \bar{y}_1{}'A_1{}' = (2.5862)(2)(0.5) = 2.5862 \text{ in}^3$

$Q_B = \bar{y}_2{}'A_2{}' = (3.4138)(7)(0.5) = 11.9483 \text{ in}^3$

$Q_{max} = \Sigma \bar{y}'A' = (3.4138)(7)(0.5) + 2(1.5819)(3.1638)(0.5) = 16.9531 \text{ in}^3$

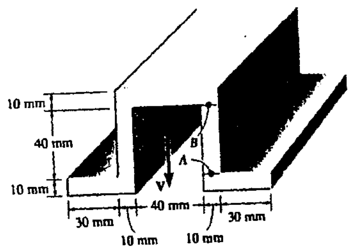
$$q = \frac{VQ}{I}$$

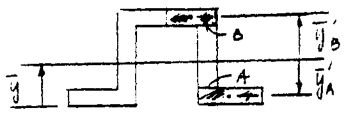
$$q_A = \frac{7(10^3)(2.5862)}{92.569} = 196 \text{ lb/in.} \qquad \textbf{Ans}$$

$$q_B = \frac{1}{2}(\frac{7(10^3)(11.9483)}{92.569}) = 452 \text{ lb/in.} \qquad \textbf{Ans}$$

$$q_{max} = \frac{1}{2}(\frac{7(10^3)(16.9531)}{92.569}) = 641 \text{ lb/in.} \qquad \textbf{Ans}$$

7–61 The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the shear flow at points *A* and *B*.





$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)} = 0.027727 \text{ m}$$

$$I = 2[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727 - 0.005)^2]$$
$$+ 2[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03 - 0.027727)^2]$$
$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055 - 0.027727)^2 = 0.98197(10^{-6}) \text{ m}^4$$

$$\bar{y}_B' = 0.055 - 0.027727 = 0.027272 \text{ m}$$

$$\bar{y}_A' = 0.027727 - 0.005 = 0.022727 \text{ m}$$

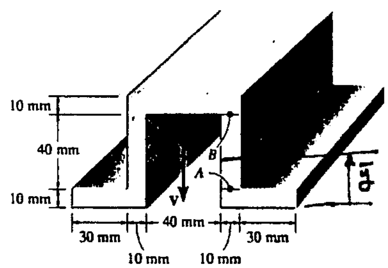
$$Q_A = \bar{y}_A'A' = 0.022727(0.04)(0.01) = 9.0909(10^{-6}) \text{ m}^3$$

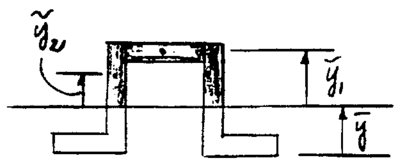
$$Q_B = \bar{y}_B'A' = 0.027272(0.03)(0.01) = 8.1818(10^{-6}) \text{ m}^3$$

$$q_A = \frac{VQ_A}{I} = \frac{150(9.0909)(10^{-6})}{0.98197(10^{-6})} = 1.39 \text{ kN/m}$$ **Ans**

$$q_B = \frac{VQ_B}{I} = \frac{150(8.1818)(10^{-6})}{0.98197(10^{-6})} = 1.25 \text{ kN/m}$$ **Ans**

7–62 The aluminum strut is 10 mm thick and has the cross section shown. If it is subjected to a shear of $V = 150$ N, determine the maximum shear flow in the strut.





$$\bar{y} = \frac{2[0.005(0.03)(0.01)] + 2[0.03(0.06)(0.01)] + 0.055(0.04)(0.01)}{2(0.03)(0.01) + 2(0.06)(0.01) + 0.04(0.01)}$$
$$= 0.027727 \text{ m}$$

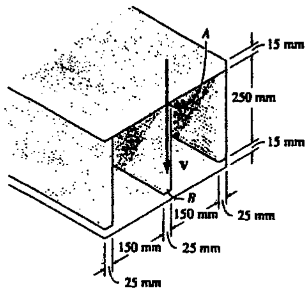
$$I = 2[\frac{1}{12}(0.03)(0.01)^3 + 0.03(0.01)(0.027727-0.005)^2]$$
$$+ 2[\frac{1}{12}(0.01)(0.06)^3 + 0.01(0.06)(0.03-0.027727)^2]$$
$$+ \frac{1}{12}(0.04)(0.01)^3 + 0.04(0.01)(0.055-0.027727)^2$$
$$= 0.98197(10^{-6}) \text{ m}^4$$

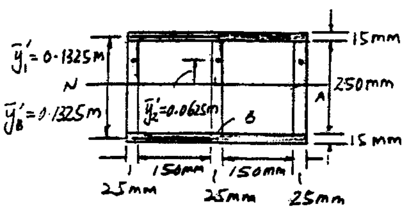
$$Q_{max} = (0.055 - 0.027727)(0.04)(0.01) + 2[(0.06 - 0.027727)(0.01)](\frac{0.06 - 0.0277}{2})$$

$$= 21.3(10^{-6}) \text{ m}^3$$

$$q_{max} = \frac{1}{2}(\frac{VQ_{max}}{I}) = \frac{1}{2}\left(\frac{150(21.3(10^{-6}))}{0.98197(10^{-6})}\right) = 1.63 \text{ kN/m} \qquad \textbf{Ans.}$$

7-63. The box girder is subjected to a shear of $V = 15$ kN. Determine (a) the shear flow developed at point B and (b) the maximum shear flow in the girder's web AB.





$$I = \frac{1}{12}(0.375)(0.28^3) - \frac{1}{12}(0.3)(0.25^3) = 0.295375(10^{-3})\text{ m}^4$$

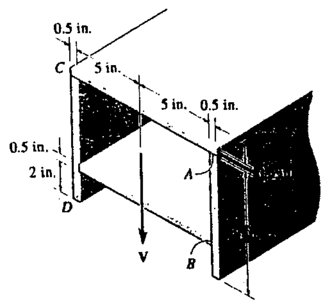
$$Q_B = \bar{y}'_B A' = 0.1325(0.375)(0.015) = 0.7453125(10^{-3})\text{ m}^3$$

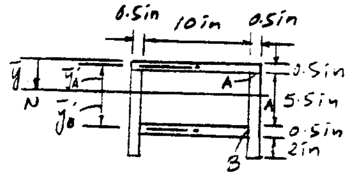
$$Q_{max} = \Sigma \bar{y}' A' = 0.1325(0.375)(0.015) + 3[(0.0625)(0.125)(0.025)] = 1.33125(10^{-3})\text{ m}^3$$

a) $$q_B = \frac{1}{3}\left[\frac{VQ_B}{I}\right] = \frac{1}{3}\left[\frac{15(10^3)(0.7453125)(10^{-3})}{0.295375(10^{-3})}\right] = 12.6\text{ kN/m}$$ **Ans**

b) $$q_{max} = \frac{1}{3}\left[\frac{VQ_{max}}{I}\right] = \frac{1}{3}\left[\frac{15(10^3)(1.33125)(10^{-3})}{0.295375(10^{-3})}\right] = 22.5\text{ kN/m}$$ **Ans**

•7-64 The beam is subjected to a shear force of $V = 5$ kip. Determine the shear flow at points A and B.





$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.25(11)(0.5) + 2[4.5(8)(0.5)] + 6.25(10)(0.5)}{11(0.5) + 2(8)(0.5) + 10(0.5)} = 3.70946 \text{ in.}$$

$$I = \frac{1}{12}(11)(0.5^3) + 11(0.5)(3.70946-0.25)^2 + 2[\frac{1}{12}(0.5)(8^3) + 0.5(8)(4.5-3.70946)^2]$$
$$+ \frac{1}{12}(10)(0.5^3) + 10(0.5)(6.25-3.70946)^2$$
$$= 145.98 \text{ in}^4$$

$$\bar{y}'_A = 3.70946 - 0.25 = 3.45946 \text{ in.}$$

$$\bar{y}'_B = 6.25 - 3.70946 = 2.54054 \text{ in.}$$

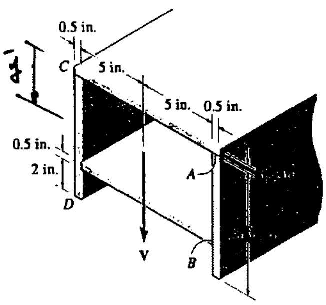
$$Q_A = \bar{y}'_A A' = 3.45946(11)(0.5) = 19.02703 \text{ in}^3$$

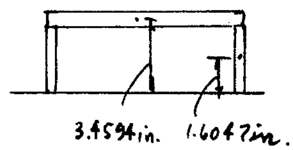
$$Q_B = \bar{y}'_B A' = 2.54054(10)(0.5) = 12.7027 \text{ in}^3$$

$$q_A = \frac{1}{2}(\frac{VQ_A}{I}) = \frac{1}{2}(\frac{5(10^3)(19.02703)}{145.98}) = 326 \text{ lb/in.}$$ **Ans**

$$q_B = \frac{1}{2}(\frac{VQ_B}{I}) = \frac{1}{2}(\frac{5(10^3)(12.7027)}{145.98}) = 218 \text{ lb/in.}$$ **Ans**

7–65 The beam is constructed from four plates and is subjected to a shear force of $V = 5$ kip. Determine the maximum shear flow in the cross section.





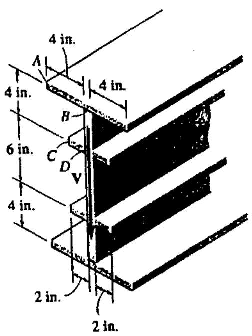
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.25(11)(0.5) + 2[4.5(8)(0.5)] + 6.25(10)(0.5)}{11(0.5) + 2(8)(0.5) + 10(0.5)} = 3.70946 \text{ in.}$$

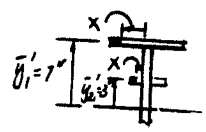
$$I = \frac{1}{12}(11)(0.5^3) + 11(0.5)(3.4595^2) + 2[\frac{1}{12}(0.5)(8^3) + 0.5(8)(0.7905^2)]$$
$$+ \frac{1}{12}(10)(0.5^3) + 10(0.5)(2.5405^2)$$
$$= 145.98 \text{ in}^4$$

$$Q_{max} = 3.4594\,(11)(0.5) + 2[(1.6047)(0.5)(3.7094 - 0.5)]$$
$$= 24.177 \text{ in}^3$$

$$q_{max} = \frac{1}{2}(\frac{VQ_{max}}{I}) = \frac{1}{2}(\frac{5(10^3)(24.177)}{145.98})$$
$$= 414 \text{ lb/in.} \qquad \textbf{Ans}$$

7–66 The stiffened beam is constructed from plates having a thickness of 0.25 in. If it is subjected to a shear of $V = 8$ kip, determine the shear-flow distribution in segments AB and CD of the beam. What is the resultant shear supported by these segments? Also, sketch how the shear flow passes through the cross section. The vertical dimensions are measured to the centerline of each horizontal segment.

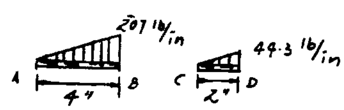




As an approximation :

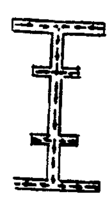
$$I = \frac{1}{12}(0.25)(14^3) + 2(8)(0.25)(7^2) + 2(0.25)(4)(3^2) = 271.167 \text{ in}^4$$

$$Q_{AB} = 7(4 - x)(0.25) = 7 - 1.75x$$
$$Q_{CD} = 3(2 - x)(0.25) = 1.5 - 0.75x$$



$$q_{AB} = \frac{VQ_{AB}}{I} = \frac{8(10^3)(7 - 1.75x)}{271.167} = 207 - 51.6x \qquad \textbf{Ans}$$

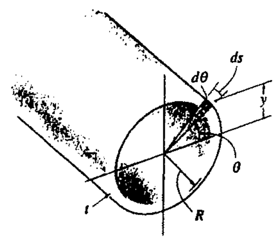
$$q_{CD} = \frac{VQ_{CD}}{I} = \frac{8(10^3)(1.5 - 0.75x)}{271.167} = 44.3 - 22.1x \qquad \textbf{Ans}$$



$$F_{AB} = \int q_{AB}dx = \int_0^4 (207 - 51.6x)dx = 413 \text{ lb} \qquad \textbf{Ans}$$

$$F_{CD} = \int q_{CD}dx = \int_0^2 (44.3 - 22.1x)dx = 44.3 \text{ lb} \qquad \textbf{Ans}$$

7-67 Determine the shear-stress variation over the cross section of the thin-walled tube as a function of elevation y and show that $\tau_{max} = 2V/A$, where $A = 2\pi rt$. *Hint:* Choose a differential area element $dA = Rt\,d\theta$. Using $dQ = y\,dA$, formulate Q for a circular section from θ to $(\pi - \theta)$ and show that $Q = 2R^2 t \cos\theta$, where $\cos\theta = \sqrt{R^2 - y^2}/R$.



$$dA = R\,t\,d\theta$$

$$dQ = y\,dA = yR\,t\,d\theta$$

Here $y = R\sin\theta$

Therefore $dQ = R^2 t \sin\theta\,d\theta$

$$Q = \int_{\theta}^{\pi-\theta} R^2 t \sin\theta\,d\theta = R^2 t(-\cos\theta)\Big|_{\theta}^{\pi-\theta}$$

$$= R^2 t[-\cos(\pi-\theta) - (-\cos\theta)] = 2R^2 t\cos\theta$$

$$dI = y^2 dA = y^2 R\,t\,d\theta = R^3 t \sin^2\theta\,d\theta$$

$$I = \int_0^{2\pi} R^3 t \sin^2\theta\,d\theta = R^3 t \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2}\,d\theta$$

$$= \frac{R^3 t}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]\Big|_0^{2\pi} = \frac{R^3 t}{2}[2\pi - 0] = \pi R^3 t$$

$$\tau = \frac{VQ}{It} = \frac{V(2R^2 t\cos\theta)}{\pi R^3 t(2t)} = \frac{V\cos\theta}{\pi R\,t}$$

Here $\cos\theta = \dfrac{\sqrt{R^2 - y^2}}{R}$

$$\tau = \frac{V}{\pi R^2 t}\sqrt{R^2 - y^2} \qquad \textbf{Ans}$$

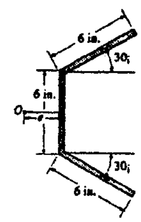
τ_{max} occurs at $y = 0$; therefore

$$\tau_{max} = \frac{V}{\pi R\,t}$$

$A = 2\pi Rt$; therefore

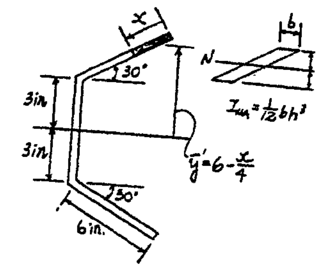
$$\tau_{max} = \frac{2V}{A} \qquad \textbf{QED}$$

***7-68.** Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown where $b_2 > b_1$. The member segments have the same thickness t.



Section Properties :

$$I = \frac{1}{12}t(6^3) + 2\left[\frac{1}{12}\left(\frac{t}{\sin 30^\circ}\right)(6\sin 30^\circ)^3 + (6t)(3+3\sin 30^\circ)^2\right]$$
$$= 270t$$



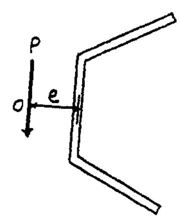
$$\bar{y}' = 3 + 6\sin 30^\circ - \frac{x}{2}\sin 30^\circ = 6 - \frac{x}{4}$$

$$Q = \bar{y}'A' = \left(6 - \frac{x}{4}\right)(x)(t) = t\left(6x - \frac{x^2}{4}\right)$$

Shear Flow Resultant :

$$q = \frac{VQ}{I} = \frac{P\,t\left(6x - \frac{x^2}{4}\right)}{270\,t} = \frac{P\left(6x - \frac{x^2}{4}\right)}{270}$$

$$F_1 = \int_0^{6\,\text{in}} q\,dx = \frac{P}{270}\int_0^{6\,\text{in}}\left(6x - \frac{x^2}{4}\right)dx = \frac{1}{3}P$$



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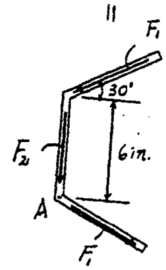
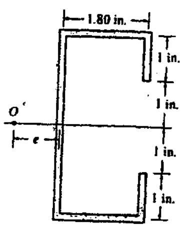


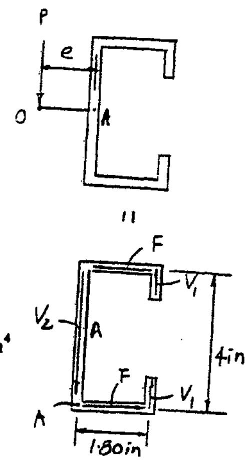
Shear Center : Summing moments about point A,

$$Pe = F_1\cos 30^\circ(6)$$
$$Pe = \frac{1}{3}P\cos 30^\circ(6)$$

$$e = 1.73\text{ in.}$$ **Ans**

7-69. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.





Summing moments about A,

$$Pe = F(4) + 2V_1(1.8) \qquad (1)$$

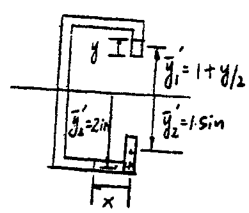
$$I = 2[\frac{1}{12}t(4^3)] - \frac{1}{12}t(2^3) + 2[(1.80)(t)(2^2)] = 24.4\,t\,\text{in}^4$$

$$Q_1 = \bar{y}_1'A' = (1 + \frac{y}{2})(y\,t) = t(y + \frac{y^2}{2})$$

$$Q_2 = \Sigma\bar{y}'A' = 1.5(1)(t) + 2(x)(t) = t(1.5 + 2x)$$

$$q_1 = \frac{VQ_1}{I} = \frac{P\,t(y + \frac{y^2}{2})}{24.4\,t} = \frac{P(y + \frac{y^2}{2})}{24.4}$$

$$q_2 = \frac{VQ_2}{I} = \frac{P\,t(1.5 + 2x)}{24.4\,t} = \frac{P(1.5 + 2x)}{24.4}$$



$$V_1 = \int q_1\,dy = \frac{P}{24.4}\int_0^1 (y + \frac{y^2}{2})\,dy = 0.0273\,P$$

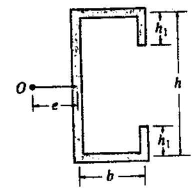
$$F = \int q_2\,dy = \frac{P}{24.4}\int_0^{1.80} (1.5 + 2x)dx = 0.2434\,P$$

From Eq. (1),

$$Pe = 0.2434\,P(4) + 2(0.0273)P(1.8)$$

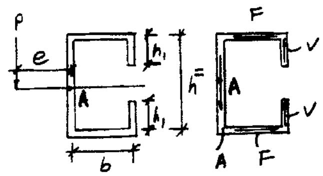
$$e = 1.07\text{ in.}$$ **Ans**

7–70 Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Summing moments about A,

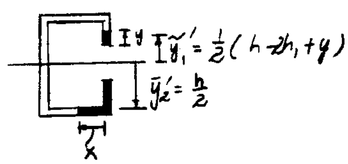
$$Pe = F(h) + 2V(b) \qquad (1)$$



$$I = \frac{1}{12}(t)(h^3) + 2b(t)(\frac{h}{2})^2 + \frac{1}{12}(t)[h^3 - (h-2h_1)^3]$$
$$= \frac{th^3}{6} + \frac{bth^2}{2} - \frac{t(h-2h_1)^3}{12}$$

$$Q_1 = \bar{y}'A' = \frac{1}{2}(h - 2h_1 + y)yt = \frac{t(hy - 2h_1 y + y^2)}{2}$$

$$q_1 = \frac{VQ}{I} = \frac{Pt(hy - 2h_1 y + y^2)}{2I}$$



$$V = \int q_1\, dy = \frac{Pt}{2I}\int_0^{h_1}(hy - 2h_1 y + y^2)dy = \frac{Pt}{2I}[\frac{hh_1^2}{2} - \frac{2}{3}h_1^3]$$

$$Q_2 = \Sigma\bar{y}'A' = \frac{1}{2}(h - h_1)h_1 t + \frac{h}{2}(x)(t) = \frac{1}{2}t[h_1(h - h_1) + hx]$$

$$q_2 = \frac{VQ_2}{I} = \frac{Pt}{2I}(h_1(h - h_1) + hx)$$

$$F = \int q_2\, dx = \frac{Pt}{2I}\int_0^b [h_1(h - h_1) + hx]dx = \frac{Pt}{2I}(h_1 hb - h_1^2 b + \frac{hb^2}{2})$$

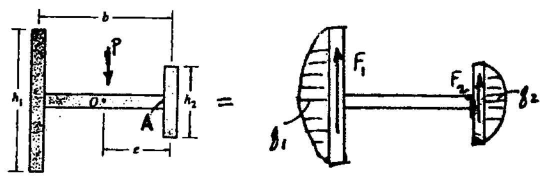
From Eq. (1),

$$Pe = \frac{Pt}{2I}[h_1 h^2 b - h_1^2 hb + \frac{h^2 b^2}{2} + hh_1^2 b - \frac{4}{3}h_1^3 b]$$

$$I = \frac{t}{12}(2h^3 + 6bh^2 - (h - 2h_1)^3)$$

$$e = \frac{t}{12I}(6h_1 h^2 b + 3h^2 b^2 - 8h_1^3 b) = \frac{b(6h_1 h^2 + 3h^2 b - 8h_1^3)}{2h^3 + 6bh^2 - (h - 2h_1)^3} \qquad \textbf{Ans}$$

7-71. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Summing moments about A,

$$eP = bF_1 \qquad (1)$$

$$I = \frac{1}{12}(t)(h_1)^3 + \frac{1}{12}(t)(h_2)^3 = \frac{1}{12}t\,(h_1^3 + h_2^3)$$

$$q_1 = \frac{P(h_1/2)(t)(h_1/4)}{I} = \frac{Ph_1^2 t}{8I}$$

$$F_1 = \frac{2}{3}q_1(h_1) = \frac{Ph_1^3 t}{12\,I}$$

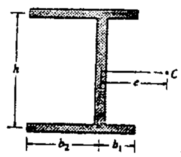
From Eq. (1),

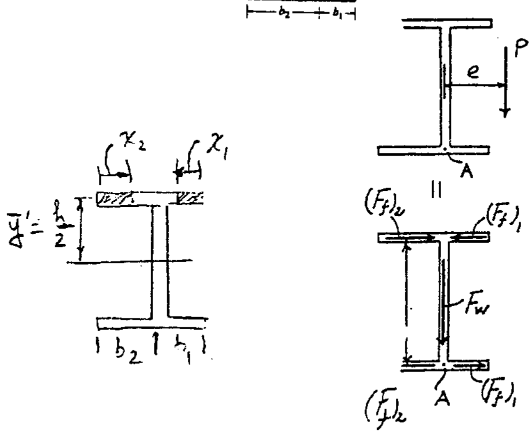
$$e = \frac{b}{P}\left(\frac{Ph_1^3 t}{12\,I}\right)$$

$$= \frac{h_1^3 b}{(h_1^3 + h_2^3)}$$

$$= \frac{b}{1 + (h_2/h_1)^3} \qquad \textbf{Ans}$$

***7-72.** Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown, where $b_2 > b_1$. The member segments have the same thickness t.





Section Properties :

$$I = \frac{1}{12}th^3 + 2\left[(b_1 + b_2)t\left(\frac{h}{2}\right)^2\right] = \frac{th^2}{12}\left[h + 6(b_1 + b_2)\right]$$

$$Q_1 = \bar{y}'A' = \frac{h}{2}(x_1)t = \frac{ht}{2}x_1$$

$$Q_2 = \bar{y}'A' = \frac{h}{2}(x_2)t = \frac{ht}{2}x_2$$

Shear Flow Resultant :

$$q_1 = \frac{VQ_1}{I} = \frac{P\left(\frac{ht}{2}x_1\right)}{\frac{th^2}{12}\left[h + 6(b_1 + b_2)\right]} = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}x_1$$

$$q_2 = \frac{VQ_2}{I} = \frac{P\left(\frac{ht}{2}x_2\right)}{\frac{th^2}{12}\left[h + 6(b_1 + b_2)\right]} = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}x_2$$

$$(F_f)_1 = \int_0^{b_1} q_1\,dx_1 = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}\int_0^{b_1} x_1\,dx_1 = \frac{3Pb_1^2}{h\left[h + 6(b_1 + b_2)\right]}$$

$$(F_f)_2 = \int_0^{b_2} q_2\,dx_2 = \frac{6P}{h\left[h + 6(b_1 + b_2)\right]}\int_0^{b_2} x_2\,dx_2 = \frac{3Pb_2^2}{h\left[h + 6(b_1 + b_2)\right]}$$

Shear Center : Summing moment about point A.

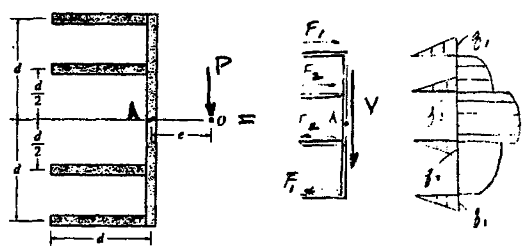
$$Pe = (F_f)_2 h - (F_f)_1 h$$

$$Pe = \frac{3Pb_2^2}{h\left[h + 6(b_1 + b_2)\right]}(h) - \frac{3Pb_1^2}{h\left[h + 6(b_1 + b_2)\right]}(h)$$

$$e = \frac{3(b_2^2 - b_1^2)}{h + 6(b_1 + b_2)} \qquad \textbf{Ans}$$

Note that if $b_2 = b_1$, $e = 0$ (I shape).

7-73. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



Summing moments about point A :

$$Pe = F_2 d + F_1(2d) \qquad (1)$$

$$I = 2[dt(d)^2] + 2[dt(d/2)^2] = \frac{1}{12}t(2d)^3 = \frac{19}{6}t\,d^3$$

$$q_1 = \frac{P(dt)(d)}{\frac{19}{6}t\,d^3} = \frac{6P}{19\,d}$$

$$F_1 = \frac{1}{2}\left(\frac{6P}{19d}\right)(d) = \frac{3}{19}P$$

$$q_2 = \frac{P(dt)(d/2)}{\frac{19}{6}t\,d^3} = \frac{3P}{19d}$$

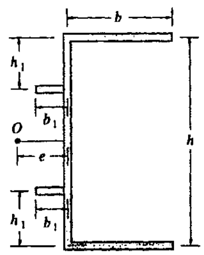
$$F_2 = \frac{1}{2}\left(\frac{3P}{19d}\right)d = \frac{1.5P}{19}$$

From Eq. (1) :

$$Pe = 2d\left(\frac{3}{19}P\right) + d\left(\frac{1.5P}{19}\right)$$

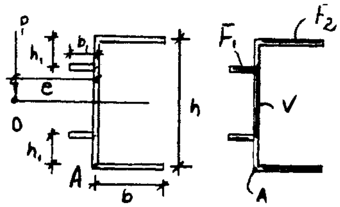
$$e = \frac{7.5}{19}d = \frac{15}{38}d \qquad \textbf{Ans}$$

7–74 Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.



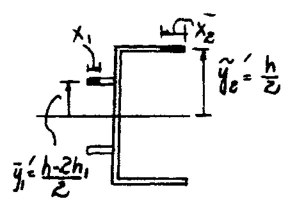
Summing moments about A,

$$Pe = F_2(h) - F_1(h - 2h_1) \qquad (1)$$



$$I = \frac{1}{12}(t)h^3 + 2(b)(t)(\frac{h}{2})^2 + 2(b_1)(t)(\frac{h-2h_1}{2})^2$$

$$= \frac{1}{12}th^3 + \frac{bth^2}{2} + \frac{b_1 t(h-2h_1)^2}{2}$$



$$Q_1 = \bar{y}_1' A_1' = \frac{h-2h_1}{2}(x_1)t$$

$$q_1 = \frac{VQ_1}{I} = \frac{Pt(\frac{h-2h_1}{2})x_1}{I} = \frac{Pt(h-2h_1)}{2I}x_1$$

$$F_1 = \int_0^{b_1} q_1\,dx_1 = \frac{Pt(h-2h_1)}{2I}\int_0^{b_1} x_1\,dx = \frac{Ptb_1^2(h-2h_1)}{4I}$$

$$Q_2 = \bar{y}_2' A_2' = \frac{h}{2}(x_2)t$$

$$q_2 = \frac{VQ_2}{I} = \frac{P(\frac{h}{2})(x_2)t}{I}$$

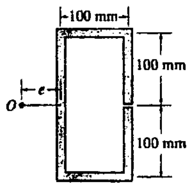
$$F_2 = \int q_2\,dx_2 = \frac{Pht}{2I}\int_0^{b} x_2\,dx_2 = \frac{Phb^2t}{4I}$$

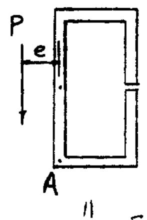
From Eq. (1),

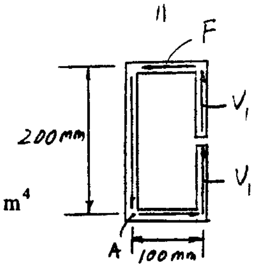
$$Pe = \frac{Ph^2b^2t}{4I} - \frac{Ptb_1^2(h-2h_1)^2}{4I}$$

$$e = \frac{h^2b^2t - tb_1^2(h-2h_1)^2}{4\left(\frac{1}{12}th^3 + \frac{bth^2}{2} + \frac{b_1t(h-2h_1)^2}{2}\right)} = \frac{3[h^2b^2 - (h-2h_1)^2b_1^2]}{h^3 + 6bh^2 + 6b_1(h-2h_1)^2} \qquad \textbf{Ans}$$

7–75 Determine the location e of the shear center, point O, for the thin-walled member having a slit along its side.



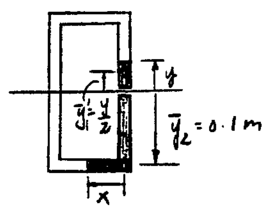




Suming moments about A,

$$Pe = 2V_1(100) + F(200) \qquad (1)$$

$$I = 2\left[\frac{1}{12}t(0.2^3)\right] + 2[(0.1)(t)(0.1^2)] = 3.3333(10^{-3})\,t \ \text{m}^4$$



$$Q_1 = \bar{y}_1'A' = \frac{y}{2}(y)\,t = 0.5y^2\,t$$

$$Q_2 = \Sigma\bar{y}A = 0.05(0.1)(t) + 0.1(x)(t) = 0.005\,t + 0.1x\,t$$

$$q_1 = \frac{VQ_1}{I} = \frac{P(0.5y^2t)}{3.3333(10^{-3})\,t} = 150P\,y^2$$

$$q_2 = \frac{VQ_2}{I} = \frac{P\,(0.005\,t + 0.1x\,t)}{3.3333(10^{-3})\,t} = 300P(0.005 + 0.1x)$$

$$V_1 = \int_0^{0.1} q_1\,dy = 150P\int_0^{0.1} y^2dy = 150P\left[\frac{y^3}{3}\right]_0^{0.1} = 0.05P$$

$$F = \int_0^{0.1} q_2\,dx = 300P\int_0^{0.1}(0.005 + 0.1x)dx$$

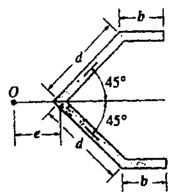
$$= 300P\left[0.005x + \frac{0.1x^2}{2}\right]\Big|_0^{0.1} = 0.3P$$

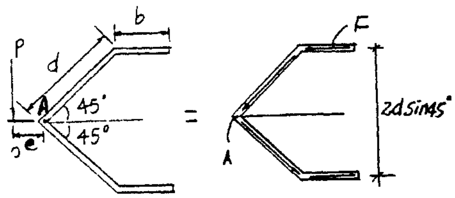
From Eq. (1) ;

$$Pe = 2(0.05P)(100) + 0.3P(200)$$

$e = 70$ mm **Ans**

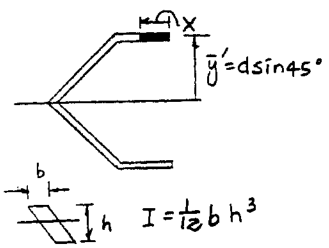
•7-76 Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown. The member segments have the same thickness t.





Summing moments about A,

$$Pe = F(\,2d\sin 45°) \qquad (1)$$



$$I = \frac{1}{12}b\,h^3$$

Here $b = \dfrac{t}{\cos 45°}$

$$h = 2d\sin 45°$$

$$I = \frac{1}{12}\left(\frac{t}{\cos 45°}\right)(2d\sin 45°)^3 + 2(t)b(d\sin 45°)^2 = \frac{1}{3}t\,d^3 + t\,b\,d^2$$

$$Q = \bar{y}A' = (d\sin 45°)(x)(t)$$

$$q = \frac{VQ}{I} = \frac{P(0.7071\,d\,t\,x)}{\frac{1}{3}t\,d^3 + t\,b\,d^2} = \frac{2.1213\,P}{d\,(d + 3b)}\,x$$

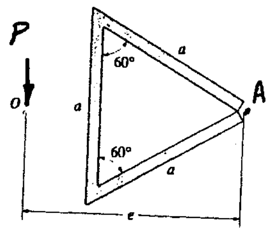
$$F = \int_0^b q\,dx = \frac{2.1213\,P}{d\,(d + 3b)}\int_0^b x\,dx = \frac{1.0607\,Pb^2}{d\,(d + 3b)}$$

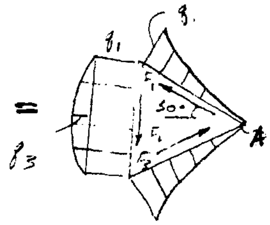
From Eq.(1),

$$Pe = \frac{1.0607\,Pb^2}{d\,(d + 3b)}(\,2d\sin 45°)$$

$$e = \frac{1.5\,b^2}{d + 3b} \qquad \textbf{Ans}$$

7-77 Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown.





Summing moments about A :

$$Pe = F_2\left(\frac{\sqrt{3}}{2}a\right)$$

$$I = \frac{1}{12}(t)(a)^3 + \frac{1}{12}\left(\frac{t}{\sin 30^\circ}\right)(a)^3 = \frac{1}{4}t\,a^3$$

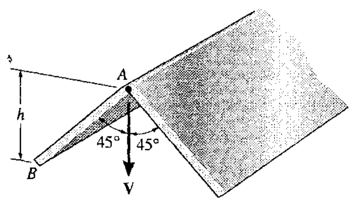
$$q_1 = \frac{V(a)(t)(a/4)}{\frac{1}{4}t\,a^3} = \frac{V}{a}$$

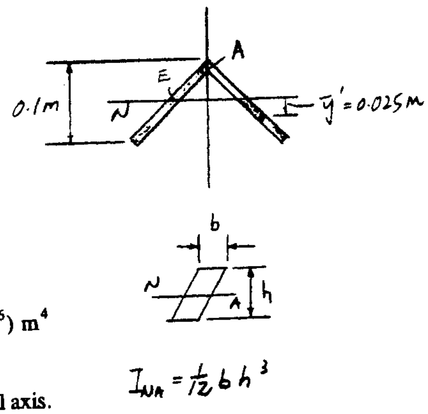
$$q_2 = q_1 + \frac{V(a/2)(t)(a/4)}{\frac{1}{4}t\,a^3} = q_1 + \frac{V}{2a}$$

$$F_2 = \frac{V}{a}(a) + \frac{2}{3}\left(\frac{V}{2a}\right)(a) = \frac{4V}{3}$$

$$e = \frac{2\sqrt{3}}{3}a \qquad \textbf{Ans}$$

7-78. If the angle has a thickness of 3 mm, a height $h = 100$ mm, and it is subjected to a shear of $V = 50$ N, determine the shear flow at point A and the maximum shear flow in the angle.





$$b = \frac{0.003}{\cos 45^\circ} = 0.00424264 \text{ m}$$

$$h = 0.1 \text{ m}$$

$$I = 2[\frac{1}{12}(0.00424264)(0.1^3)] = 0.7071(10^{-6}) \text{ m}^4$$

$$I_{NA} = \frac{1}{12} b h^3$$

Centroid E of the shaded area lies on the neutral axis.

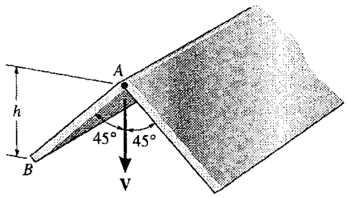
Therefore, $Q_A = 0$

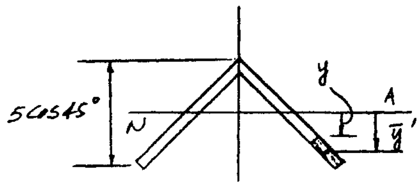
$$Q_{max} = \bar{y}'A' = 0.025(\frac{0.05}{\cos 45^\circ})(0.003) = 5.3033(10^{-6}) \text{ m}^3$$

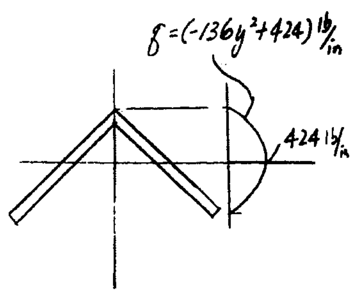
$$q_A = \frac{VQ_A}{I} = 0 \quad \textbf{Ans}$$

$$q_{max} = \frac{VQ_{max}}{I} = \frac{50(5.3033)(10^{-6})}{0.7071(10^{-6})}] = 375 \text{ N/m} \quad \textbf{Ans}$$

7-79. The angle is subjected to a shear of $V = 2$ kip. Sketch the distribution of shear flow along the leg AB. Indicate numerical values at all peaks. The thickness is 0.25 in. and the legs (AB) are 5 in.





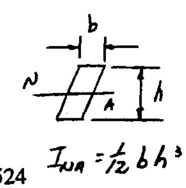


$$b = \frac{0.25}{\cos 45^\circ} = 0.3536 \text{ in.}$$

$$h = 5\cos 45^\circ = 3.5355 \text{ in.}$$

$$I = 2[\frac{1}{12}(0.3536)(3.5355^3)] = 2.6042 \text{ in}^4$$

$$\bar{y}' = y + \frac{2.5\cos 45^\circ - y}{2} = 0.5y + 0.8839$$



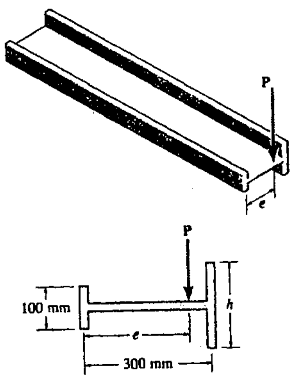
$$A' = (2.5\cos 45^\circ - y)(\frac{0.25}{\cos 45^\circ}) = 0.625 - 0.3536y$$

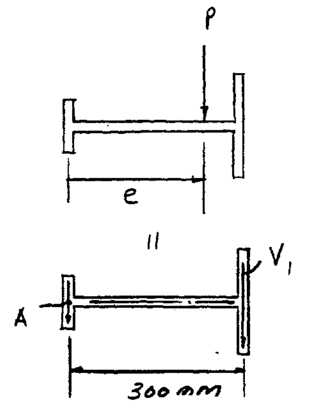
$$Q = \bar{y}A' = (0.5y + 0.8839)(0.625 - 0.3536y) = -0.1768y^2 + 0.5524$$

$$q = \frac{VQ}{I} = \frac{2(10^3)(-0.1768y^2 + 0.5524)}{2.6042}$$
$$= (-136y^2 + 424) \text{ lb/in.} \qquad \textbf{Ans}$$

$$q_{max} = 424 \text{ lb/in.} \qquad \textbf{Ans}$$

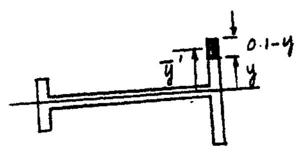
•7–80 Determine the placement e for the force $\mathbf{P}$ so that the beam bends downward without twisting. Take $h = 200$ mm.





Summing moments about A,

$$Pe = 300V \qquad (1)$$



$$I = \frac{1}{12}t(0.1^3) + \frac{1}{12}(t)(0.2^3) = 0.75(10^{-3})\,t\ \text{m}^4$$

$$\bar{y}' = y + \frac{0.1 - y}{2} = \frac{1}{2}(y + 0.1)$$

$$Q = \bar{y}'A' = \frac{1}{2}(y + 0.1)(0.1 - y)\,t = \frac{t}{2}(0.01 - y^2)$$

$$q = \frac{VQ}{I} = \frac{P(\frac{t}{2})(0.01 - y^2)}{0.75(10^{-3})\,t} = 666.67P(0.01 - y^2)$$

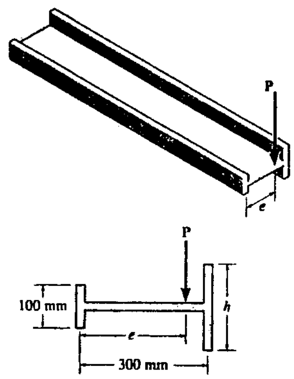
$$V_1 = \int_{-0.1}^{0.1} q\,dy = 666.67P\int_{-0.1}^{0.1}(0.01 - y^2)$$

$$= 666.67P\left[0.01y - \frac{y^3}{3}\right]_{-0.1}^{0.1} = 0.8889P$$

From Eq. (1) ; $\quad Pe = 300(0.8889P)$

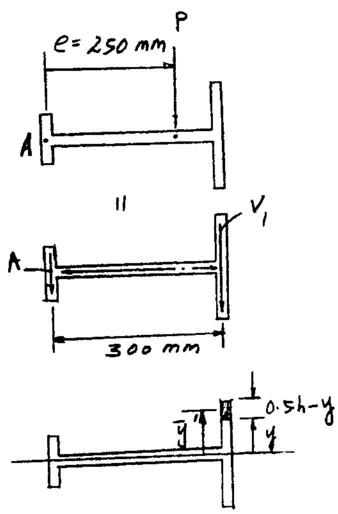
$e = 267$ mm $\qquad$ **Ans**

7–81 A force **P** is applied to the web of the beam as shown. If e = 250 mm, determine the height h of the right flange so that the beam will deflect downward without twisting. The member segments have the same thickness t.



Summing moments about A,

$$P(250) = V_1(300)\,; \qquad V_1 = 0.8333P \qquad (1)$$



$$I = \frac{1}{12}t(0.1^3) + \frac{1}{12}(t)(h^3) = \frac{t}{12}(0.001 + h^3)$$

$$\bar{y}' = y + \frac{0.5h - y}{2} = \frac{1}{2}(y + 0.5h)$$

$$Q = \bar{y}'A' = \frac{1}{2}(y + 0.5h)(0.5h - y)\,t = \frac{t}{2}(0.25h^2 - y^2)$$

$$q = \frac{VQ}{I} = \frac{P(\frac{t}{2})(0.25h^2 - y^2)}{\frac{t}{12}(0.001 + h^3)} = \frac{6P(0.25h^2 - y^2)}{(0.001 + h^3)}$$

$$V_1 = \int_{-\frac{h}{2}}^{h} q\,dy = \frac{6P}{0.001 + h^3}\int_{-\frac{h}{2}}^{\frac{h}{2}}(0.25h^2 - y^2)\,dy$$

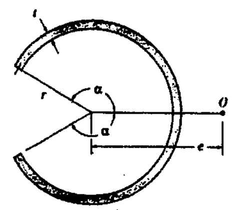
$$= \frac{6P}{0.001 + h^3}\left[0.25h^2y - \frac{y^3}{3}\right]_{-\frac{h}{2}}^{\frac{h}{2}} = \frac{P\,h^3}{0.001 + h^3}$$

From Eq. (1)

$$0.8333P = \frac{P\,h^3}{0.001 + h^3}$$

h = 0.171 m = 171 mm **Ans**

7-82. Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown.



Summing moments about A,

$$Pe = r\int dF \qquad (1)$$

$$dA = t\,ds = t\,r\,d\theta$$

$$y = r\sin\theta$$

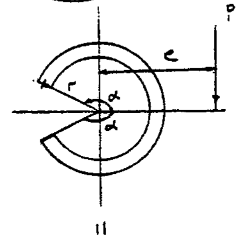
$$dI = y^2 dA = r^2\sin^2\theta(t\,r\,d\theta) = r^3 t\sin^2\theta\,d\theta$$

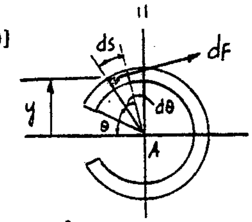
$$I = r^3 t\int \sin^2\theta\,d\theta = r^3 t\int_{\pi-\alpha}^{\pi+\alpha}\frac{1-\cos 2\theta}{2}d\theta$$

$$= \frac{r^3 t}{2}\left(\theta - \frac{\sin 2\theta}{2}\right)\Big|_{\pi-\alpha}^{\pi+\alpha}$$

$$= \frac{r^3 t}{2}\left[\left(\pi+\alpha - \frac{\sin 2(\pi+\alpha)}{2}\right) - \left(\pi - \alpha - \frac{\sin 2(\pi-\alpha)}{2}\right)\right]$$

$$= \frac{r^3 t}{2}2\sin\alpha\cos\alpha = \frac{r^3 t}{2}(2\alpha - \sin 2\alpha)$$





$$dQ = y\,dA = r\sin\theta\,(t\,r\,d\theta) = r^2 t\sin\theta\,d\theta$$

$$Q = r^2 t\int_{\pi-\alpha}^{\theta}\sin\theta\,d\theta = r^2 t(-\cos\theta)\Big|_{\pi-\alpha}^{\theta} = r^2 t(-\cos\theta - \cos\alpha) = -r^2 t(\cos\theta + \cos\alpha)$$

$$q = \frac{VQ}{I} = \frac{P(-r^2 t)(\cos\theta + \cos\alpha)}{\frac{r^3 t}{2}(2\alpha - \sin 2\alpha)} = \frac{-2P(\cos\theta + \cos\alpha)}{r(2\alpha - \sin 2\alpha)}$$

$$\int dF = \int q\,ds = \int q\,r\,d\theta$$

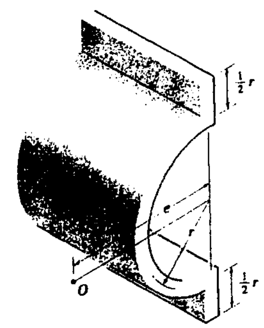
$$\int dF = \frac{-2Pr}{r(2\alpha - \sin 2\alpha)}\int_{\pi-\alpha}^{\pi+\alpha}(\cos\theta + \cos\alpha)\,d\theta = \frac{-2P}{2\alpha - \sin 2\alpha}(2\alpha\cos\alpha - 2\sin\alpha)$$

$$= \frac{4P}{2\alpha - \sin 2\alpha}(\sin\alpha - \alpha\cos\alpha)$$

From Eq.(1); $\quad Pe = r\left[\frac{4P}{2\alpha - \sin 2\alpha}(\sin\alpha - \alpha\cos\alpha)\right]$

$$e = \frac{4r(\sin\alpha - \alpha\cos\alpha)}{2\alpha - \sin 2\alpha}$$ **Ans**

7–83 Determine the location e of the shear center, point O, for the beam having the cross section shown. The thickness is t.

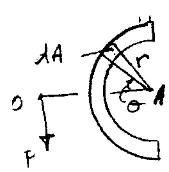


$$I = (2)[\frac{1}{12}(t)(r/2)^3 + (r/2)(t)(r + \frac{r}{4})^2] + I_{semi\text{-}circle}$$

$$= 1.583333 t\,r^3 + I_{semi\text{-}circle}$$

$$I_{semi\text{-}circle} = \int_{-\pi/2}^{\pi/2} (r \sin\theta)^2\, t\,r\,d\theta = t\,r^3 \int_{-\pi/2}^{\pi/2} \sin^2\theta\, d\theta$$

$$I_{semi\text{-}circle} = t\,r^3(\frac{\pi}{2})$$

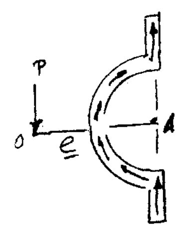


Thus,

$$I = 1.583333 t\,r^3 + t\,r^3(\frac{\pi}{2}) = 3.15413 t\,r^3$$

$$Q = (\frac{r}{2})t(\frac{r}{4} + r) + \int_{\theta}^{\pi/2} r \sin\theta\,(t\,r\,d\theta)$$

$$Q = 0.625 + r^2 + t\,r^2 \cos\theta$$



$$q = \frac{VQ}{I} = \frac{P(0.625 + \cos\theta)t\,r^2}{3.15413\,t\,r^3}$$

Summing moments about A:

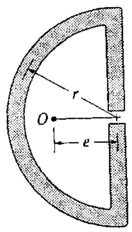
$$Pe = \int_{-\pi/2}^{\pi/2} (q\,r\,d\theta)r$$

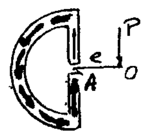
$$Pe = \frac{Pr}{3.15413}\int_{-\pi/2}^{\pi/2} (0.625 + \cos\theta)d\theta$$

$$e = \frac{r\,(1.9634 + 2)}{3.15413}$$

$$e = 1.26\,r \qquad \textbf{Ans}$$

***7-84.** Determine the location e of the shear center, point O, for the beam having the cross section shown. The thickness is t.





$$I = 2(\frac{1}{12} t\, r^3 + t\, r\, (r/2)^2) + \int_{-\pi/2}^{\pi/2} (r \sin\theta)^2\, t\, r\, d\theta$$

$$= 0.6667\, t r^3 + t\, r^3 (\frac{\pi}{2}) = 2.2375\, t\, r^3$$

$$Q = (\frac{r}{2})(r\, t) + \int_{\theta}^{\pi/2} r \sin\theta\, (t\, r\, d\theta)$$

$$= 0.5\, t\, r^2 + t\, r^2 \cos\theta$$

$$q = \frac{VQ}{I} = \frac{P(0.5 + \cos\theta) t\, r^2}{2.2375\, t\, r^3}$$

Summing moments about A,

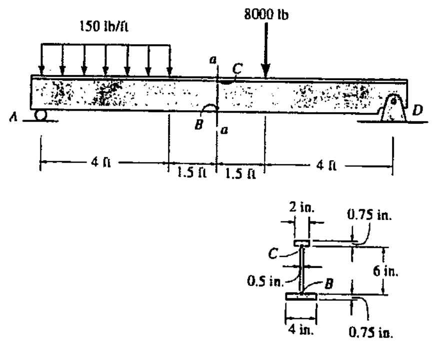
$$Pe = \int_{-\pi/2}^{\pi/2} (q\, r\, d\theta) r$$

$$Pe = \frac{P r}{2.2375} \int_{-\pi/2}^{\pi/2} (0.5 + \cos\theta) d\theta$$

$$e = \frac{r\, (1.5708 + 2)}{2.2375}$$

$e = 1.60\, r$ **Ans**

7–85 Determine the shear stress at points B and C on the web of the beam located at section a–a.

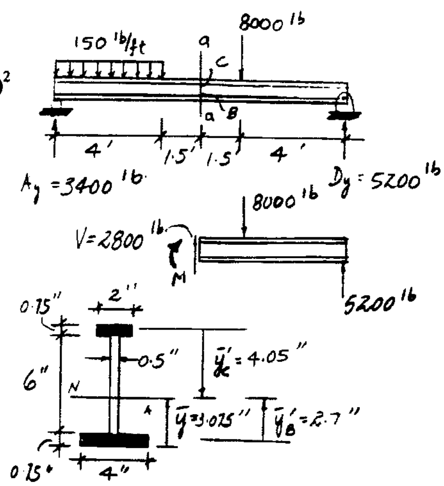


$$\bar{y} = \frac{(0.375)(4)(0.75) + (3.75)(6)(0.5) + (7.125)(2)(0.75)}{4(0.75) + 6(0.5) + 2(0.75)} = 3.075 \text{ in.}$$

$$I = \frac{1}{12}(4)(0.75^3) + 4(0.75)(3.075-0.375)^2$$
$$+ \frac{1}{12}(0.5)(6^3) + 0.5(6)(3.75-3.075)^2$$
$$+ \frac{1}{12}(2)(0.75^3) + 2(0.75)(7.125-3.075)^2$$
$$= 57.05 \text{ in}^4$$

$$Q_B = \bar{y}'_B A' = 2.7(4)(0.75) = 8.1 \text{ in}^3$$

$$Q_C = \bar{y}'_C A' = 4.05(2)(0.75) = 6.075 \text{ in}^3$$

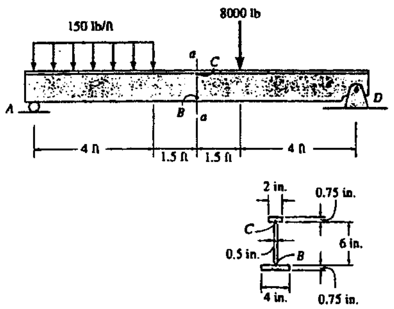


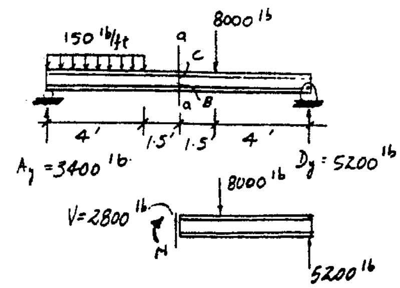
$$\tau = \frac{VQ}{It}$$

$$\tau_B = \frac{2800(8.1)}{57.05(0.5)} = 795 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_C = \frac{2800(6.075)}{57.05(0.5)} = 596 \text{ psi} \qquad \textbf{Ans}$$

7-86. Determine the maximum shear stress acting at section *a–a* in the beam.





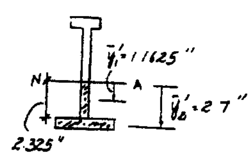
$$\bar{y} = \frac{(0.375)(4)(0.75) + (3.75)(6)(0.5) + (7.125)(2)(0.75)}{4(0.75) + 6(0.5) + 2(0.75)} = 3.075 \text{ in.}$$

$$I = \frac{1}{12}(4)(0.75^3) + 4(0.75)(3.075 - 0.375)^2$$

$$+ \frac{1}{12}(0.5)(6^3) + 0.5(6)(3.75 - 3.075)^2$$

$$+ \frac{1}{12}(2)(0.75^3) + 2(0.75)(7.125 - 3.075)^2$$

$$= 57.05 \text{ in}^4$$



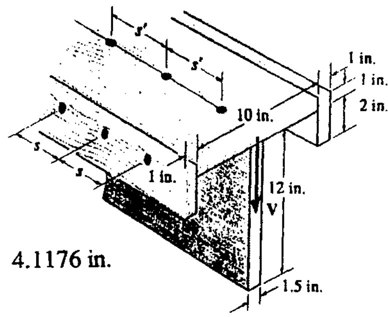
$$Q_{max} = \Sigma \bar{y}'A'$$

$$= 2.7(4)(0.75) + 2.325(0.5)(1.1625)$$

$$= 9.4514 \text{ in}^3$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{2800(9.4514)}{57.05(0.5)} = 928 \text{ psi} \qquad \textbf{Ans}$$

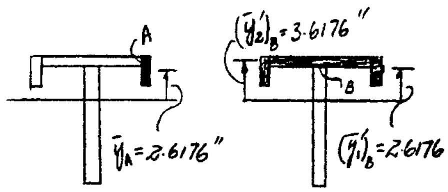
7-87 The beam is made from four boards nailed together as shown. If the nails can each support a shear force of 100 lb, determine their required spacings s' and s if the beam is subjected to a shear of V = 700 lb.



$$\bar{y} = \frac{(0.5)(10)(1) + (2)(1.5)(3)(1) + (7)(12)(1.5)}{(10)(1) + (2)(3)(1) + (12)(1.5)} = 4.1176 \text{ in.}$$

$$I = \frac{1}{12}(10)(1^3) + 10(1)(4.1176-0.5)^2$$
$$+ 2[(\frac{1}{12})(1)(3^3) + (1)(3)(4.1176-1.5)^2]$$
$$+ \frac{1}{12}(1.5)(12^3) + (12)(1.5)(7-4.1176)^2 = 542.86 \text{ in}^4$$

$$Q_A = \bar{y}'_A A' = 2.6176(3)(1) = 7.8528 \text{ in}^3$$



$$Q_B = \Sigma \bar{y}'_B A' = 2(2.6176)(3)(1) + 3.6176(10)(1)$$
$$= 51.8816 \text{ in}^3$$

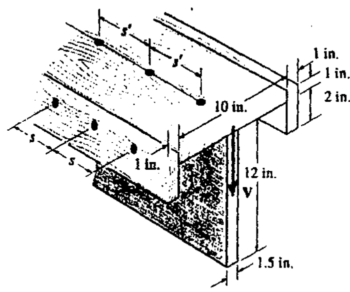
$$q_A = \frac{VQ_A}{I} = \frac{700(7.8528)}{542.86} = 10.126 \text{ lb / in.}$$

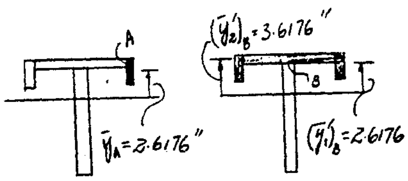
$$q_B = \frac{VQ_B}{I} = \frac{700(51.8816)}{542.86} = 66.90 \text{ lb / in.}$$

$$s' = \frac{100}{q_B} = \frac{100}{66.90} = 1.49 \text{ in.}$$ **Ans**

$$s = \frac{100}{q_A} = \frac{100}{10.126} = 9.88 \text{ in.}$$ **Ans**

*7–88 The beam is made from four boards nailed together as shown. If the beam is subjected to a shear of $V = 1200$ lb, determine the shear force in each nail. The spacing along the side is $s = 3$ in. and at the top, $s' = 4.5$ in.





$$\bar{y} = \frac{(0.5)(10)(1) + (2)(1.5)(3)(1) + (7)(12)(1.5)}{(10)(1) + (2)(3)(1) + (12)(1.5)} = 4.1176 \text{ in.}$$

$$I = \frac{1}{12}(10)(1^3) + 10(1)(4.1176-0.5)^2$$
$$+ 2[(\frac{1}{12})(1)(3^3) + (1)(3)(4.1176-1.5)^2]$$
$$+ \frac{1}{12}(1.5)(12^3) + (12)(1.5)(7-4.1176)^2 = 542.86 \text{ in}^4$$

$$Q_A = \bar{y}_A A' = 2.6176(3)(1) = 7.8528 \text{ in}^3$$

$$Q_B = \Sigma \bar{y}_B A' = 2(2.6176)(3)(1) + 3.6176(10)(1) = 51.8816 \text{ in}^3$$

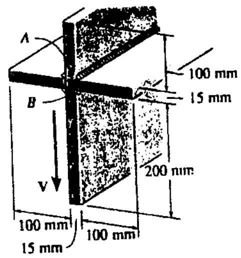
$$q_A = \frac{VQ_A}{I} = \frac{1200(7.8528)}{542.86} = 17.359 \text{ lb / in.}$$

$$q_B = \frac{VQ_B}{I} = \frac{1200(51.8816)}{542.86} = 114.685 \text{ lb / in.}$$

$$s' = \frac{F'}{q_B}; \quad 4.5 = \frac{F'}{114.685}; \quad F' = 516 \text{ lb}$$ **Ans**

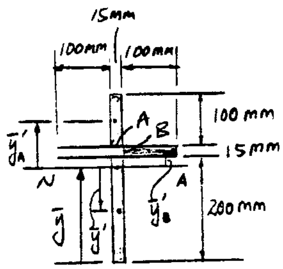
$$s = \frac{F}{q_A}; \quad 3 = \frac{F}{17.359}; \quad F = 52.1 \text{ lb}$$ **Ans**

7–89 The beam is made from three thin plates welded together as shown. If it is subjected to a shear of $V = 48$ kN, determine the shear flow at points A and B. Also, calculate the maximum shear stress in the beam.



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.1575(0.315)(0.015) + 2[0.2075(0.1)(0.015)]}{0.315(0.015) + 2(0.1)(0.015)} = 0.17692 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.315^3) + (0.015)(0.315)(0.17692-0.1575)^2$$
$$+ 2[\frac{1}{12}(0.1)(0.015^3) + 0.1(0.015)(0.2075-0.17692)^2] = 43.71347(10^{-6}) \text{ m}^4$$



$$\bar{y}'_A = 0.315 - 0.17692 - 0.05 = 0.08808 \text{ m}$$

$$\bar{y}'_B = 0.315 - 0.17692 - 0.1075 = 0.03058 \text{ m}$$

$$\bar{y}' = \frac{0.17692}{2} = 0.08846 \text{ m}$$

$$Q_A = \bar{y}'_A A' = 0.08808(0.1)(0.015) = 0.13212(10^{-3}) \text{ m}^3$$

$$Q_B = \bar{y}'_B A' = 0.03058(0.1)(0.015) = 45.87(10^{-6}) \text{ m}^3$$

$$Q_{max} = \bar{y}' A' = 0.08846(0.17692)(0.015) = 0.234755(10^{-3}) \text{ m}^3$$

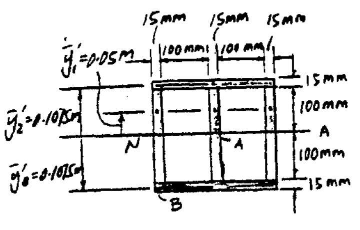
$$q = \frac{VQ}{I}$$

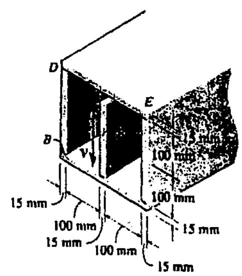
$$q_A = \frac{48(10^3)(0.13212)(10^{-3})}{43.71347(10^{-6})} = 145 \text{ kN/m}$$ **Ans**

$$q_B = \frac{48(10^3)(45.87)(10^{-6})}{43.71347(10^{-6})} = 50.4 \text{ kN/m}$$ **Ans**

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{48(10^3)(0.234755)(10^{-3})}{43.71347(10^{-6})(0.015)}$$
$$= 17.2 \text{ MPa}$$ **Ans**

7-90. The beam is subjected to a shear of $V = 25$ kN. Determine the shear stress at points A and B and compute the maximum shear stress in the beam. There is a very small gap at C.





$$I = \frac{1}{12}(0.245)(0.23^3) - \frac{1}{12}(0.2)(0.2^3) = 0.1151(10^{-3})\ \text{m}^4$$

$$Q_A = \Sigma y^1 A^1 = (0.100)(0.015)(0.05)$$
$$= 75 \times 10^{-6}\ \text{m}^3$$

$$Q_B = \bar{y}_B' A' = 0.1075(0.245)(0.015) = 0.3951(10^{-3})\ \text{m}^3$$

$$\tau = \frac{VQ}{It}$$

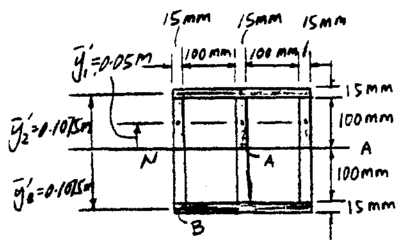
$$\tau_A = \frac{25(10^3)(75)(10^{-6})}{0.1151(10^{-3})(0.015)} = 1.09\text{MPa} \qquad \text{Ans.}$$

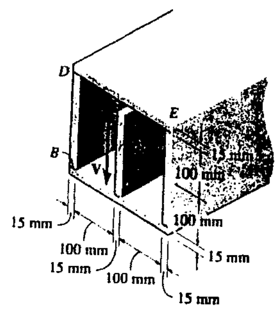
$$\tau_B = \frac{25(10^3)(0.3951)(10^{-3})}{0.1151(10^{-3})(2)(0.015)} = 2.86\text{MPa} \qquad \text{Ans.}$$

$$Q_{max} = \Sigma y^1 A^1 = (0.1075)(0.245)(0.015) + 2[(0.100)(0.015)(0.05)] + (0)(0.1)(0.015)$$
$$= 0.5451(10)^{-3}$$

$$\tau_{max} = \frac{25(10^3)(0.5451)(10^{-3})}{0.1151(10^{-3})(2)(0.015)} = 3.95\ \text{MPa} \qquad \text{Ans.}$$

7–91 The beam is subjected to a shear of $V = 25$ kN. Determine the shear stress at points A and B and compute the maximum shear stress in the beam. Assume the gap at C is closed so that the center plate is fixed to the top plate.





$$I = \frac{1}{12}(0.245)(0.23^3) - \frac{1}{12}(0.2)(0.2^3) = 0.1151(10^{-3})\ \text{m}^4$$

$$Q_A = \Sigma\bar{y}A' = 3[0.05(0.1)(0.015)] + 0.1075(0.245)(0.015) = 0.6201(10^{-3})\ \text{m}^3$$

$$Q_B = \bar{y}_B'A' = 0.1075(0.245)(0.015) = 0.3951(10^{-3})\ \text{m}^3$$

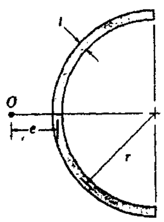
$$\tau = \frac{VQ}{It}$$

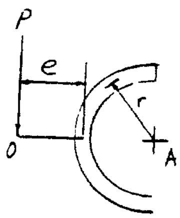
$$\tau_A = \frac{25(10^3)(0.6201)(10^{-3})}{0.1151(10^{-3})(3)(0.015)} = 2.99\ \text{MPa} \qquad \textbf{Ans}$$

$$\tau_B = \frac{25(10^3)(0.3951)(10^{-3})}{0.1151(10^{-3})(3)(0.015)} = 1.91\ \text{MPa} \qquad \textbf{Ans}$$

$$\tau_{max} = \tau_A = 2.99\ \text{MPa} \qquad \textbf{Ans}$$

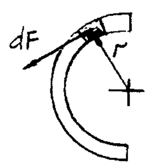
*7–92 **Determine the location e of the shear center, point O, for the thin-walled member having the cross section shown.**





Summing moments about A,

$$P(e + r) = r\int dF \qquad (1)$$

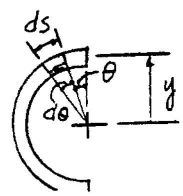


$$y = r\cos\theta; \qquad dA = t\,ds$$

$dI = y^2 dA = r^2\cos^2\theta(t)\,ds;$ however $ds = r\,d\theta$, then,

$$I = r^3 t\int_0^{\pi}\cos^2\theta\,d\theta = r^3 t\int_0^{\pi}\left(\frac{\cos 2\theta + 1}{2}\right)d\theta$$

$$= \frac{r^3 t}{2}(\pi) = \frac{\pi r^3 t}{2}$$



$$dQ = y\,dA = r\cos\theta(t\,r\,d\theta) = r^2 t\cos\theta\,d\theta$$

$$Q = r^2 t\int_0^{\theta}\cos\theta\,d\theta = r^2 t\sin\theta$$

$$q = \frac{VQ}{I} = \frac{p(r^2 t\sin\theta)}{\frac{1}{2}\pi r^3 t} = \frac{2P\sin\theta}{\pi r}$$

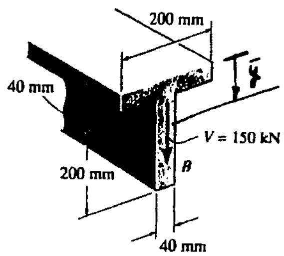
$$F = \int dF = \int q\,ds = \int q\,r\,d\theta$$

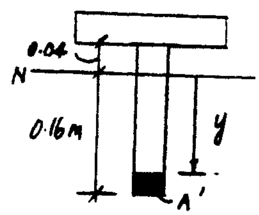
From Eq. (1)

$$P(e + r) = r\int_0^{\pi}\frac{2P\sin\theta}{\pi r}(r)\,d\theta; \qquad P(e + r) = \frac{2Pr}{\pi}\int_0^{\pi}\sin\theta\,d\theta$$

$$e = \frac{2r}{\pi}\int_0^{\pi}\sin\theta\,d\theta - r = \frac{4r}{\pi} - r = 0.273r \qquad \textbf{Ans}$$

7-93 The T-beam is subjected to a shear of $V = 150$ kN. Determine the amount of this force that is supported by the web *B*.





$$\bar{y} = \frac{(0.02)(0.2)(0.04) + (0.14)(0.2)(0.04)}{0.2(0.04) + 0.2(0.04)} = 0.08 \text{ m}$$

$$I = \frac{1}{12}(0.2)(0.04^3) + 0.2(0.04)(0.08 - 0.02)^2$$

$$+ \frac{1}{12}(0.04)(0.2^3) + 0.2(0.04)(0.14 - 0.08)^2 = 85.3333(10^{-6}) \text{ m}^4$$

$$A' = 0.04(0.16 - y)$$

$$\bar{y}' = y + \frac{(0.16 - y)}{2} = \frac{(0.16 + y)}{2}$$

$$Q = \bar{y}'A' = 0.02(0.0256 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{150(10^3)(0.02)(0.0256 - y^2)}{85.3333(10^{-6})(0.04)} = 22.5(10^6) - 878.9(10^6)y^2$$

$$V = \int \tau\, dA, \qquad dA = 0.04\, dy$$

$$V = \int_{-0.04}^{0.16} (22.5(10^6) - 878.9(10^6)y^2)\, 0.04\, dy$$

$$= \int_{-0.04}^{0.16} (900(10^3) - 35.156(10^6)y^2)\, dy$$

$$= 131\,250 \text{ N} = 131 \text{ kN} \qquad \textbf{Ans}$$

8–1 A spherical gas tank has an inner radius of r = 1.5 m. If it is subjected to an internal pressure of p = 300 kPa, determine its required thickness if the maximum normal stress is not to exceed 12 MPa.

$$\sigma_{allow} = \frac{p\,r}{2\,t}; \qquad 12(10^6) = \frac{300(10^3)(1.5)}{2\,t}$$

t = 0.0188 m = 18.8 mm **Ans**

8–2 A pressurized spherical tank is to be made of 0.5-in.-thick steel. If it is subjected to an internal pressure of p = 200 psi, determine its outer radius if the maximum normal stress is not to exceed 15 ksi.

$$\sigma_{\text{allow}} = \frac{p\,r}{2\,t}; \qquad 15(10^3) = \frac{200\,r_i}{2(0.5)}$$

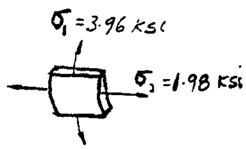
$$r_i = 75 \text{ in.}$$

$$r_o = 75 \text{ in.} + 0.5 \text{ in.} = 75.5 \text{ in.} \qquad \textbf{Ans}$$

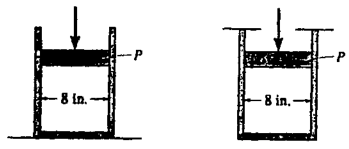
8-3. The tank of a cylindrical air compressor is subjected to an internal pressure of 90 psi. If the internal diameter of the tank is 22 in., and the wall thickness is 0.25 in., determine the stress components acting at a point. Draw a volume element of the material at this point, and show the results on the element.

$$\sigma_1 = \frac{p\,r}{t} = \frac{90\,(11)}{0.25} = 3960\text{ psi} = 3.96\text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{p\,r}{2\,t} = \frac{90(11)}{2(0.25)} = 1980\text{ psi} = 1.98\text{ ksi} \qquad \textbf{Ans}$$



•8–4 The thin-walled cylinder can be supported in one or two ways as shown. Determine the state of stress in the wall of the cylinder for both cases if the piston P causes the internal pressure to be 65 psi. The wall has a thickness of 0.25 in. and the inner diameter of the cylinder is 8 in.



Case (a) :

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04 \text{ ksi} \qquad \textbf{Ans}$$

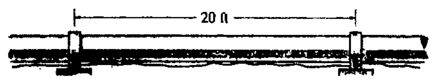
$$\sigma_2 = 0 \qquad \textbf{Ans}$$

Case (b) :

$$\sigma_1 = \frac{pr}{t}; \qquad \sigma_1 = \frac{65(4)}{0.25} = 1.04 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{pr}{2t}; \qquad \sigma_2 = \frac{65(4)}{2(0.25)} = 520 \text{ psi} \qquad \textbf{Ans}$$

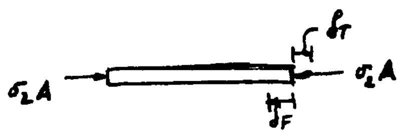
8–5 The gas pipe line is supported every 20 ft by concrete piers and also lays on the ground. If there are rigid retainers at the piers that hold the pipe fixed, determine the longitudinal and hoop stress in the pipe if the temperature rises 60°F from the temperature at which it was installed. The gas within the pipe is at a pressure of 600 lb/in². The pipe has an inner diameter of 20 in. and thickness of 0.25 in. The material is A-36 steel.



Require,

$$\delta_F = \delta_T; \qquad \delta_F = \frac{PL}{AE} = \frac{\sigma L}{E}, \qquad \delta_T = \alpha \Delta T L$$

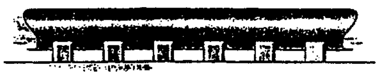
$$\frac{\sigma_2(20)(12)}{29(10^6)} = (6.60)(10^{-6})(60)(20)(12)$$



$\sigma_2 = 11.5$ ksi **Ans**

$$\sigma_1 = \frac{pr}{t} = \frac{600(10)}{0.25} = 24 \text{ ksi}$$ **Ans**

8-6. The open-ended polyvinyl chloride pipe has an inner diameter of 4 in. and thickness of 0.2 in. If it carries flowing water at 60 psi pressure, determine the state of stress in the walls of the pipe.

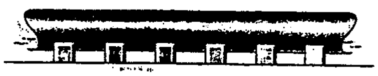


$$\sigma_1 = \frac{p\,r}{t} = \frac{60(2)}{0.2} = 600 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

There is no stress component in the longitudinal direction since the pipe has open ends.

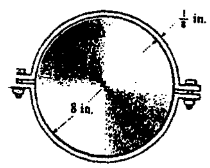
8-7. If the flow of water within the pipe in Prob. 8–6 is stopped due to the closing of a valve, determine the state of stress in the walls of the pipe. Neglect the weight of the water. Assume the supports only exert vertical forces on the pipe.



$$\sigma_1 = \frac{p\,r}{t} = \frac{60(2)}{0.2} = 600 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{p\,r}{2\,t} = \frac{60(2)}{2(0.2)} = 300 \text{ psi} \qquad \textbf{Ans}$$

***8-8.** The A-36-steel band is 2 in. wide and is secured around the smooth rigid cylinder. If the bolts are tightened so that the tension in them is 400 lb, determine the normal stress in the band, the pressure exerted on the cylinder, and the distance half the band stretches.



$$\sigma_1 = \frac{400}{2(1/8)(1)} = 1600 \text{ psi}$$

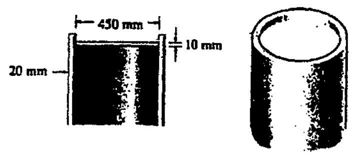
$$\sigma_1 = \frac{pr}{t}; \qquad 1600 = \frac{p(8)}{(1/8)}$$

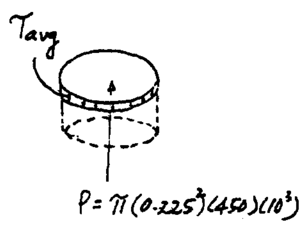
$$p = 25 \text{ psi} \qquad \textbf{Ans}$$

$$\varepsilon_1 = \frac{\sigma_1}{E} = \frac{1600}{29(10^6)} = 55.1724(10^{-6})$$

$$\delta = \varepsilon_1 L = 55.1724(10^{-6})(\pi)(8 + \frac{1}{16}) = 0.00140 \text{ in.} \qquad \textbf{Ans}$$

8-9. A pressure-vessel head is fabricated by gluing the circular plate to the end of the vessel as shown. If the vessel sustains an internal pressure of 450 kPa, determine the average shear stress in the glue and the state of stress in the wall of the vessel.





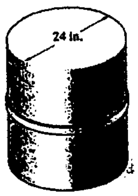
$+\uparrow \Sigma F_y = 0; \quad \pi(0.225)^2 450(10^3) - \tau_{avg}(2\pi)(0.225)(0.01) = 0;$

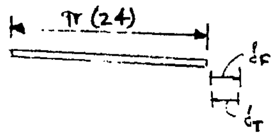
$\tau_{avg} = 5.06 \text{ MPa}$ **Ans**

$$\sigma_1 = \frac{p\,r}{t} = \frac{450(10^3)(0.225)}{0.02} = 5.06 \text{ MPa}$$ **Ans**

$$\sigma_2 = \frac{p\,r}{2\,t} = \frac{450(10^3)(0.225)}{2(0.02)} \ 2.53 \text{ MPa}$$ **Ans**

8–10. An A-36-steel hoop has an inner diameter of 23.99 in., thickness of 0.25 in., and width of 1 in. If it and the 24-in.-diameter rigid cylinder have a temperature of 65° F, determine the temperature to which the hoop should be heated in order for it to just slip over the cylinder. What is the pressure the hoop exerts on the cylinder, and the tensile stress in the ring when it cools back down to 65° F?





$\delta_T = \alpha \Delta T L$

$\pi(24) - \pi(23.99) = 6.60(10^{-6})(T_1 - 65)(\pi)(23.99)$

$T_1 = 128.16° F = 128°$ **Ans**

Cool down :

$\delta_F = \delta_T$

$$\frac{FL}{AE} = \alpha \Delta T L$$

$$\frac{F(\pi)(24)}{(1)(0.25)(29)(10^6)} = 6.60(10^{-6})(128.16 - 65)(\pi)(24)$$

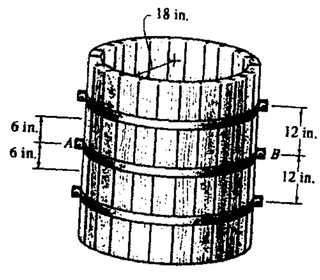
$F = 3022.21$ lb

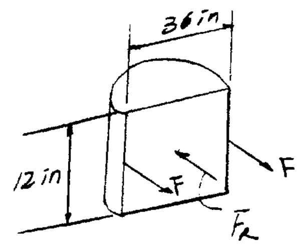
$\sigma_1 = \frac{F}{A};\qquad \sigma_1 = \frac{3022.21}{(1)(0.25)} = 12\,088 \text{ psi} = 12.1 \text{ ksi}$ **Ans**

$\sigma_1 = \frac{pr}{t};\qquad 12\,088 = \frac{p(12)}{(0.25)}$

$p = 252$ psi **Ans**

8–11 The staves or vertical members of the wooden tank are held together using semicircular hoops having a thickness of 0.5 in. and a width of 2 in. Determine the normal stress in hoop AB if the tank is subjected to an internal gauge pressure of 2 psi and this loading is transmitted directly to the hoops. Also, if 0.25-in.-diameter bolts are used to connect each hoop together, determine the tensile stress in each bolt at A and B. Assume hoop AB supports the pressure loading within a 12-in. length of the tank as shown.





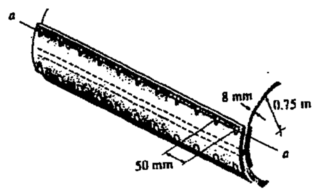
$$F_R = 2(36)(12) = 864 \text{ lb}$$

$$\Sigma F = 0; \quad 864 - 2F = 0; \quad F = 432 \text{ lb}$$

$$\sigma_h = \frac{F}{A_h} = \frac{432}{0.5(2)} = 432 \text{ psi} \qquad \textbf{Ans.}$$

$$\sigma_b = \frac{F}{A_b} = \frac{432}{\frac{\pi}{4}(0.25)} = 8801 \text{ psi} = 8.80 \text{ ksi} \qquad \textbf{Ans.}$$

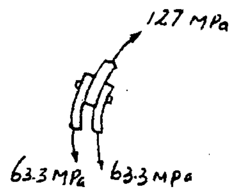
***8-12.** A boiler is constructed of 8-mm steel plates that are fastened together at their ends using a butt joint consisting of two 8-mm cover plates and rivets having a diameter of 10 mm and spaced 50 mm apart as shown. If the steam pressure in the boiler is 1.35 MPa, determine (a) the circumferential stress in the boiler's plate apart from the seam, (b) the circumferential stress in the outer cover plate along the rivet line *a–a*, and (c) the shear stress in the rivets.



a) $\sigma_1 = \frac{p\,r}{t} = \frac{1.35(10^6)(0.75)}{0.008} = 126.56\,(10^6) = 127\text{ MPa}$ **Ans**

b) $126.56\,(10^6)(0.05)(0.008) = \sigma_1'(2)(0.04)(0.008)$

$\sigma_1' = 79.1\text{MPa}$ **Ans**

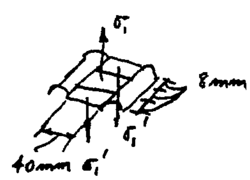


c) From FBD (a)

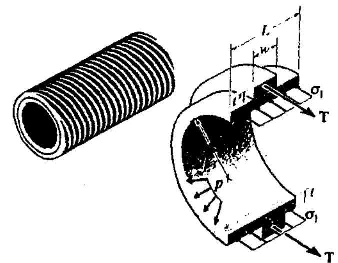
$+\uparrow\Sigma F_y = 0;\quad F_b - 79.1(10^6)[(0.008)(0.04)] = 0$

$F_b = 25.3\text{ kN}$

$(\tau_{avg})_b = \frac{F_b}{A} = \frac{25312.5}{\frac{\pi}{4}(0.01)^2} = 322\text{ MPa}$ **Ans**



8–13 In order to increase the strength of the pressure vessel, filament winding of the same material is wrapped around the circumference of the vessel as shown. If the pretension in the filament is T, and the vessel is subjected to an internal pressure p, determine the hoop stresses in the filament and in the wall of the vessel. Use the free-body diagram shown, and assume the filament winding has a thickness t' and width w for every length L of the vessel.



$$\sigma_{fil} = \frac{T}{t'w}$$

Equilibrium over entire length of the cylinder without internal pressure p.

$$-2\sigma_1'(L)(t) + 2T(\frac{L}{w}) = 0$$

$$\sigma_1' = \frac{T}{wt}$$

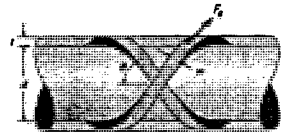
After applying the internal pressure p, the stress in the filament is

$$\sigma_{fil} = \frac{pr}{(t+t')} + \frac{T}{wt} \qquad \textbf{Ans}$$

And for the cylinder,

$$\sigma_1 = \frac{pr}{(t+t')} - \frac{T}{wt} \qquad \textbf{Ans}$$

8–14. A closed-ended pressure vessel is fabricated by cross winding glass filaments over a mandrel, so that the wall thickness t of the vessel is composed entirely of filament and an epoxy binder as shown. Consider a segment of the vessel of width w and wrapped at an angle θ. If the vessel is subjected to an internal pressure p, show that the force in the segment is $F_\theta = \sigma_0 wt$, where σ_0 is the stress in the filaments. Also, show that the stresses in the hoop and longitudinal directions are $\sigma_h = \sigma_0 \sin^2\theta$ and $\sigma_l = \sigma_0 \cos^2\theta$, respectively. At what angle θ (optimum winding angle) would the filaments have to be wound so that the hoop and longitudinal stresses are equivalent?



The Hoop and Longitudinal Stresses : Applying Eq. 8 – 1 and Eq. 8 – 2

$$\sigma_1 = \frac{pr}{t} = \frac{p\left(\frac{d}{2}\right)}{t} = \frac{pd}{2t}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{p\left(\frac{d}{2}\right)}{2t} = \frac{pd}{4t}$$

The Hoop and Longitudinal Force for Filament :

$$F_h = \sigma_1 A = \frac{pd}{2t}\left(\frac{w}{\sin\theta}t\right) = \frac{pdw}{2\sin\theta}$$

$$F_l = \sigma_2 A = \frac{pd}{4t}\left(\frac{w}{\cos\theta}t\right) = \frac{pdw}{4\cos\theta}$$

Hence,

$$F_\theta = \sqrt{F_h^2 + F_l^2}$$
$$= \sqrt{\left(\frac{pdw}{2\sin\theta}\right)^2 + \left(\frac{pdw}{4\cos\theta}\right)^2}$$
$$= \frac{pdw}{4}\sqrt{\frac{4}{\sin^2\theta} + \frac{1}{\cos^2\theta}}$$
$$= \frac{pdw}{4}\sqrt{\frac{4\cos^2\theta + \sin^2\theta}{\sin^2\theta\cos^2\theta}}$$
$$= \frac{pdw}{2\sqrt{2}\sin 2\theta}\sqrt{3\cos 2\theta + 5}$$

$$\sigma_\theta = \frac{F_\theta}{A} = \frac{\frac{pdw}{2\sqrt{2}\sin 2\theta}\sqrt{3\cos 2\theta + 5}}{wt}$$
$$= \frac{pd}{2\sqrt{2}t}\left(\frac{\sqrt{3\cos 2\theta + 5}}{\sin 2\theta}\right) \qquad (Q.E.D.)$$

$\frac{d\sigma_\theta}{d\theta} = 0$ when σ_θ is minimum.

$$\frac{d\sigma_\theta}{d\theta} = \frac{pd}{2\sqrt{2}t}\left[-\frac{2\cos 2\theta}{\sin^2 2\theta}\left(\sqrt{3\cos 2\theta + 5}\right) - \frac{3}{\sqrt{3\cos 2\theta + 5}}\right] = 0$$

$$\frac{2\cos 2\theta}{\sin^2 2\theta}\left(\sqrt{3\cos 2\theta + 5}\right) + \frac{3}{\sqrt{3\cos 2\theta + 5}} = 0$$

$$\left(\sqrt{3\cos 2\theta + 5}\right)\left(\frac{2\cos\theta}{\sin^2 2\theta} + \frac{3}{3\cos 2\theta + 5}\right) = 0$$

$$\left(\sqrt{3\cos 2\theta + 5}\right)\left[\frac{3\cos^2 2\theta + 10\cos 2\theta + 3}{\sin^2 2\theta(3\cos 2\theta + 5)}\right] = 0$$

However, $\sqrt{3\cos 2\theta + 5} \neq 0$. Therefore,

$$\frac{3\cos^2 2\theta + 10\cos 2\theta + 3}{\sin^2 2\theta(3\cos 2\theta + 5)} = 0$$

$$3\cos^2 2\theta + 10\cos 2\theta + 3 = 0$$

$$\cos 2\theta = \frac{-10 \pm \sqrt{10^2 - 4(3)(3)}}{2(3)}$$

$$\cos 2\theta = -0.3333$$

$$\theta = 54.7^\circ \qquad \textbf{Ans}$$

Force in θ Direction : Consider a portion of the cylinder. For a filament wire the cross-sectional area is $A = wt$, then

$$F_\theta = \sigma_0 w t \qquad (Q.E.D.)$$

Hoop Stress : The force in hoop direction is $F_h = F_\theta \sin\theta = \sigma_0 w t \sin\theta$ and the area is $A = \frac{wt}{\sin\theta}$. Then due to the internal pressure p,

$$\sigma_h = \frac{F_h}{A} = \frac{\sigma_0 wt\sin\theta}{wt/\sin\theta}$$

$$= \sigma_0 \sin^2\theta \qquad (Q.E.D.)$$

Longitudinal Stress : The force in the longitudinal direction is $F_l = F_\theta\cos\theta = \sigma_0 w t\cos\theta$ and the area is $A = \frac{wt}{\cos\theta}$. Then due to the internal pressure p,

$$\sigma_l = \frac{F_h}{A} = \frac{\sigma_0 wt\cos\theta}{wt/\cos\theta}$$

$$= \sigma_0 \cos^2\theta \qquad (Q.E.D.)$$

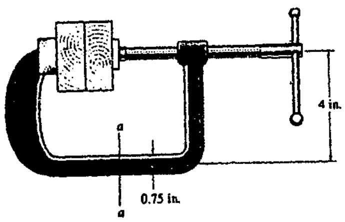
Optimum Wrap Angle : This require $\frac{\sigma_h}{\sigma_l} = \frac{pd/2t}{pd/4t} = 2$. Then

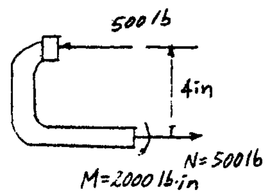
$$\frac{\sigma_h}{\sigma_l} = \frac{\sigma_0\sin^2\theta}{\sigma_0\cos^2\theta} = 2$$

$$\tan^2\theta = 2$$

$$\theta = 54.7^\circ \qquad \textbf{Ans}$$

8–15. The screw of the clamp exerts a compressive force of 500 lb on the wood blocks. Determine the maximum normal stress developed along section *a–a*. The cross section there is rectangular, 0.75 in. by 0.50 in.





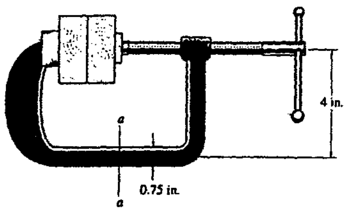
$A = 0.75(0.5) = 0.375 \text{ in}^2$

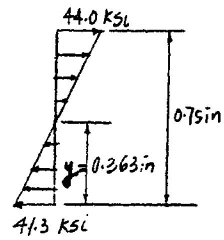
$I = \frac{1}{12}(0.5)(0.75^3) = 0.017578 \text{ in}^4$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$= \frac{500}{0.375} + \frac{2000(0.375)}{0.017578} = 44.0 \text{ ksi (T)}$$ **Ans**

***8-16.** The screw of the clamp exerts a compressive force of 500 lb on the wood blocks. Sketch the stress distribution along section *a–a* of the clamp. The cross section there is rectangular, 0.75 in. by 0.50 in.





$$A = 0.75(0.5) = 0.375 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)(0.75^3) = 0.017578 \text{ in}^4$$

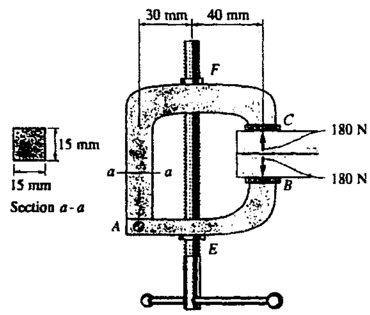
$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{500}{0.375} + \frac{2000(0.375)}{0.017578} = 44.0 \text{ ksi (T)}$$

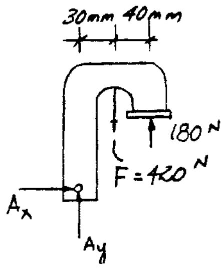
$$\sigma_{min} = \frac{P}{A} - \frac{Mc}{I} = \frac{500}{0.375} - \frac{2000(0.375)}{0.017578} = -41.3 \text{ ksi (C)}$$

$$\frac{y}{41.33} = \frac{(0.75 - y)}{44.0}$$

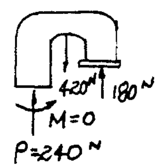
$$y = 0.363 \text{ in.}$$

8–17 The clamp is made from members AB and AC, which are pin connected at A. If it exerts a compressive force at C and B of 180 N, determine the maximum compressive stress in the clamp at section a–a. The screw EF is subjected only to a tensile force along its axis.



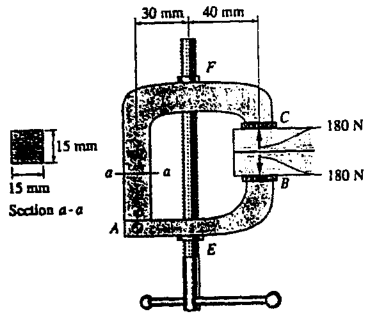


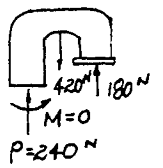
There is no moment in this problem .Therefore, the compressive stress is produced by axial force only.



$$\sigma_{max} = \frac{P}{A} = \frac{240}{(0.015)(0.015)} = 1.07 \text{ MPa} \qquad \textbf{Ans}$$

8–18 The clamp is made from members AB and AC, which are pin connected at A. If it exerts a compressive force at C and B of 180 N, sketch the stress distribution acting over section a–a. The screw EF is subjected only to a tensile force along its axis.

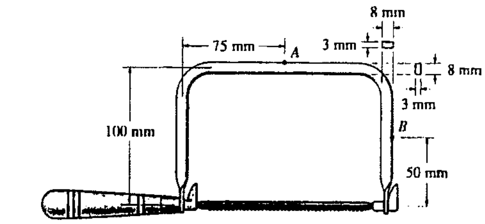


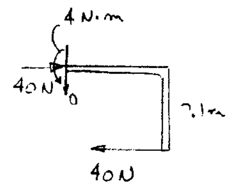


There is no moment in this problem .Therefore, the compressive stress is produced by axial force only.

$$\sigma_{max} = \frac{P}{A} = \frac{240}{(0.015)(0.015)} = 1.07 \text{ MPa}$$

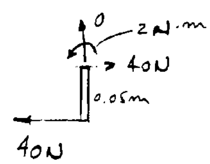
8–19 The coping saw has an adjustable blade that is tightened with a tension of 40 N. Determine the state of stress in the frame at points *A* and *B*.

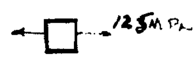


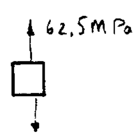


$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = -\frac{40}{(0.008)(0.003)} + \frac{4(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 123 \text{ MPa}$$ **Ans.**

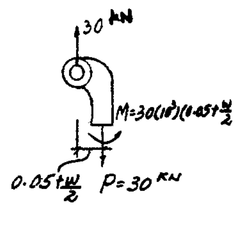
$$\sigma_B = \frac{Mc}{I} = \frac{2(0.004)}{\frac{1}{12}(0.003)(0.008)^3} = 62.5 \text{ MPa}$$ **Ans**

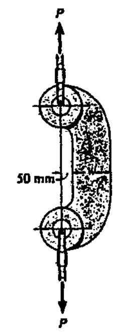






***8-20.** The offset link supports the loading of $P = 30$ kN. Determine its required width w if the allowable normal stress is $\sigma_{allow} = 73$ MPa. The link has a thickness of 40 mm.





σ due to axial force:

$$\sigma_a = \frac{P}{A} = \frac{30\,(10^3)}{(w)(0.04)} = \frac{750\,(10^3)}{w}$$

σ due to bending:

$$\sigma_b = \frac{Mc}{I} = \frac{30\,(10^3)(0.05 + \frac{w}{2})(\frac{w}{2})}{\frac{1}{12}(0.04)(w)^3}$$

$$= \frac{4500\,(10^3)(0.05 + \frac{w}{2})}{w^2}$$

$$\sigma_{max} = \sigma_{allow} = \sigma_a + \sigma_b$$

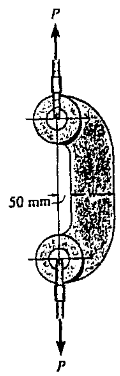
$$73\,(10^6) = \frac{750\,(10^3)}{w} + \frac{4500\,(10^3)(0.05 + \frac{w}{2})}{w^2}$$

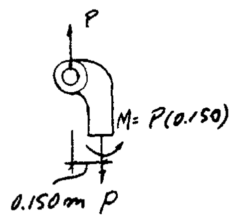
$$73\,w^2 = 0.75\,w + 0.225 + 2.25\,w$$

$$73\,w^2 - 3\,w - 0.225 = 0$$

$$w = 0.0797 \text{ m} = 79.7 \text{ mm} \qquad \textbf{Ans}$$

8–21 The offset link has a width of w = 200 mm and a thickness of 40 mm. If the allowable normal stress is σ_{allow} = 75 MPa, determine the maximum load P that can be applied to the cables.





$$A = 0.2(0.04) = 0.008 \text{ m}^2$$

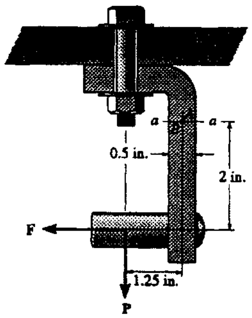
$$I = \frac{1}{12}(0.04)(0.2)^3 = 26.6667(10^{-6}) \text{ m}^4$$

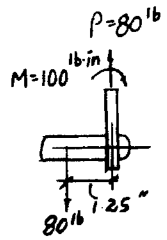
$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$75(10^6) = \frac{P}{0.008} + \frac{0.150\,P(0.1)}{26.6667(10^{-6})}$$

$$P = 109 \text{ kN} \qquad \textbf{Ans}$$

8–22 The joint is subjected to a force of $P = 80$ lb and $F = 0$. Sketch the normal-stress distribution acting over section a–a if the member has a rectangular cross-sectional area of width 2 in. and thickness 0.5 in.





σ due to axial force :

$$\sigma = \frac{P}{A} = \frac{80}{(0.5)(2)} = 80 \text{ psi}$$

σ due to bending :

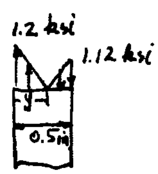
$$\sigma = \frac{M\,c}{I} = \frac{100\,(0.25)}{\frac{1}{12}(2)(0.5)^3} = 1200 \text{ psi}$$

$(\sigma_{max})_t = 80 + 1200 = 1280\text{psi} = 1.28 \text{ ksi}$ **Ans**

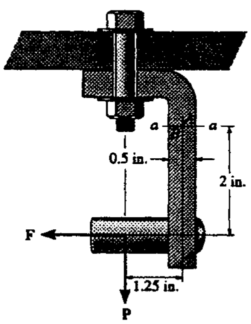
$(\sigma_{max})_c = 1200 - 80 = 1120 \text{ psi} = 1.12 \text{ ksi}$ **Ans**

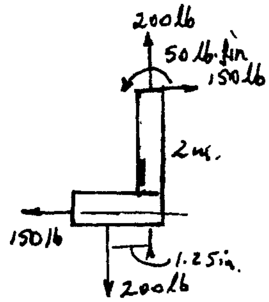
$$\frac{y}{1.25} = \frac{(0.5 - y)}{1.12}$$

$y = 0.264$ in.



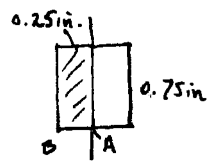
8–23 The joint is subjected to a force of P = 200 lb and F = 150 lb. Determine the state of stress at points A and B and sketch the results on differential elements located at these points. The member has a rectangular cross-sectional area of width 0.75 in. and thickness 0.5 in.





$$A = 0.5(0.75) = 0.375\ \text{in}^2$$

$$Q_A = \bar{y}'_A A' = 0.125(0.75)(0.25) = 0.0234375\ \text{in}^3\ ; \qquad Q_B = 0$$

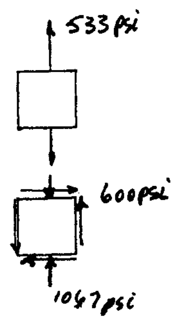


$$I = \frac{1}{12}(0.75)(0.5^3) = 0.0078125\ \text{in}^4$$

Normal Stress :

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{200}{0.375} + 0 = 533\ \text{psi (T)} \qquad \textbf{Ans}$$



$$\sigma_B = \frac{200}{0.375} - \frac{50(0.25)}{0.0078125} = -1067\ \text{psi} = 1067\ \text{psi (C)}$$

Ans

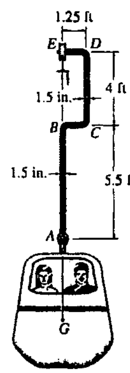
Shear stress :

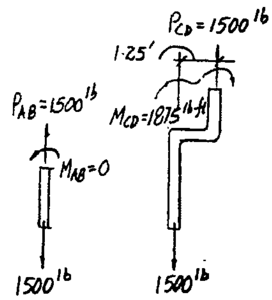
$$\tau = \frac{VQ}{It},$$

$$\tau_A = \frac{150(0.0234375)}{(0.0078125)(0.75)} = 600\ \text{psi} \qquad \textbf{Ans}$$

$$\tau_B = 0 \qquad \textbf{Ans}$$

•8–24 The gondola and passengers have a weight of 1500 lb and center of gravity at G. The suspender arm AE has a square cross-sectional area of 1.5 in. by 1.5 in., and is pin connected at its ends A and E. Determine the largest tensile stress developed in regions AB and DC of the arm.





Segment AB :

$$(\sigma_{max})_{AB} = \frac{P_{AB}}{A} = \frac{1500}{(1.5)(1.5)} = 667 \text{ psi} \qquad \textbf{Ans}$$

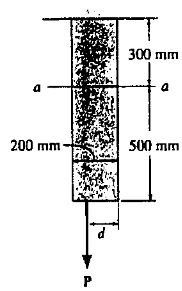
Segment CD :

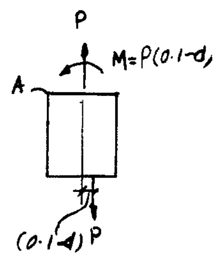
$$\sigma_a = \frac{P_{CD}}{A} = \frac{1500}{(1.5)(1.5)} = 666.67 \text{ psi}$$

$$\sigma_b = \frac{Mc}{I} = \frac{1875(12)(0.75)}{\frac{1}{12}(1.5)(1.5^3)} = 40\,000 \text{ psi}$$

$$(\sigma_{max})_{CD} = \sigma_a + \sigma_b = 666.67 + 40\,000$$
$$= 40\,666.67 \text{ psi} = 40.7 \text{ ksi} \qquad \textbf{Ans}$$

8-25 The vertical force **P** acts on the bottom of the plate having a negligible weight. Determine the shortest distance *d* to the edge of the plate at which it can be applied so that it produces no compressive stresses on the plate at section *a–a*. The plate has a thickness of 10 mm and **P** acts along the center line of this thickness.





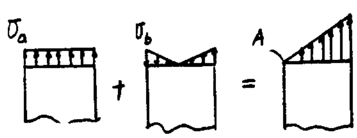
$$\sigma_A = 0 = \sigma_a - \sigma_b$$

$$0 = \frac{P}{A} - \frac{Mc}{I}$$

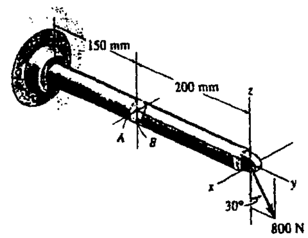
$$0 = \frac{P}{(0.2)(0.01)} - \frac{P\,(0.1 - d)(0.1)}{\frac{1}{12}(0.01)(0.2^3)}$$

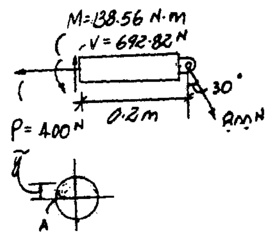
$$P\,(-1000 + 15000\,d) = 0$$

$$d = 0.0667\text{m} = 66.7\text{ mm} \qquad \textbf{Ans}$$



8-26. The bar has a diameter of 40 mm. If it is subjected to a force of 800 N as shown, determine the stress components that act at point A and show the results on a volume element located at this point.





$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\text{ m}^4$$

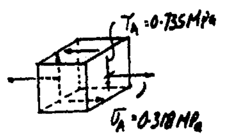
$$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\text{ m}^2$$

$$Q_A = \bar{y}'A' = \left(\frac{4\,(0.02)}{3\pi}\right)\left(\frac{\pi\,(0.02)^2}{2}\right) = 5.3333\,(10^{-6})\text{ m}^3$$

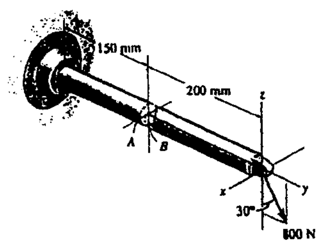
$$\sigma_A = \frac{P}{A} + \frac{Mz}{I}$$

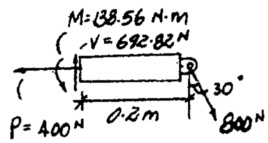
$$= \frac{400}{1.256637\,(10^{-3})} + 0 = 0.318\text{ MPa} \qquad \textbf{Ans}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{692.82\,(5.3333)\,(10^{-6})}{0.1256637\,(10^{-6})(0.04)} = 0.735\text{ MPa} \qquad \textbf{Ans}$$



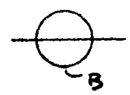
8-27. Solve Prob. 8–26 for point B.





$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\ \text{m}^4$

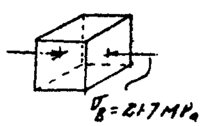
$A = \pi r^2 = \pi(0.02^2) = 1.256637\,(10^{-3})\ \text{m}^2$



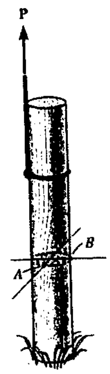
$Q_B = 0$

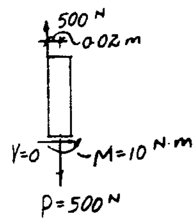
$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{400}{1.256637\,(10^{-3})} - \frac{138.56\,(0.02)}{0.1256637\,(10^{-6})} = -21.7\ \text{MPa}$ **Ans**

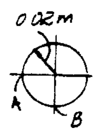
$\tau_B = 0$ **Ans**



***8–28** The cylindrical post, having a diameter of 40 mm, is being pulled from the ground using a sling of negligible thickness. If the rope is subjected to a vertical force of $P = 500$ N, determine the stress at points A and B. Show the results on a volume element located at each of these points.







$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}(\pi)(0.02^4) = 0.1256637\,(10^{-6})\text{ m}^4$$

$$A = \pi r^2 = \pi\,(0.02^2) = 1.256637\,(10^{-3})\text{ m}^2$$

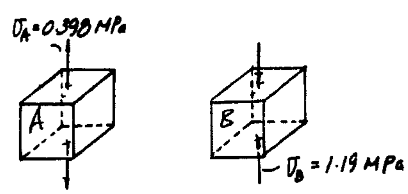
$$\sigma_A = \frac{P}{A} + \frac{Mx}{I}$$

$$= \frac{500}{1.256637\,(10^{-3})} + 0 = 0.398\text{ MPa} \qquad \textbf{Ans}$$

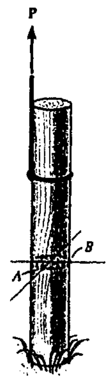
$$\sigma_B = \frac{P}{A} - \frac{Mc}{I}$$

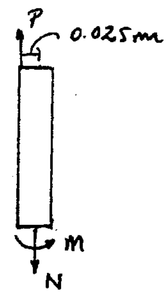
$$= \frac{500}{1.256637\,(10^{-3})} - \frac{10\,(0.02)}{0.1256637\,(10^{-6})}$$

$$= -\,1.19\text{ MPa} \qquad \textbf{Ans}$$



8–29 Determine the maximum load P that can be applied to the sling having a negligible thickness so that the normal stress in the post does not exceed $\sigma_{allow} = 30$ MPa. The post has a diameter of 50 mm.





$+\downarrow \Sigma F = 0; \quad N - P = 0; \quad N = P$

$\circlearrowleft + \Sigma M = 0; \quad M - P(0.025) = 0; \quad M = 0.025P$

$$A = \frac{\pi}{4} d^2 = \pi (0.025^2) = 0.625\,(10^{-3})\pi \text{ m}^2$$

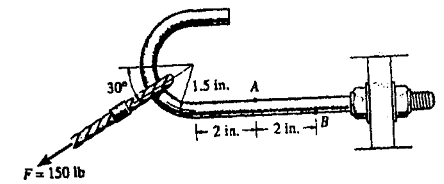
$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.025^4) = 97.65625\,(10^{-9})\pi \text{ m}^4$$

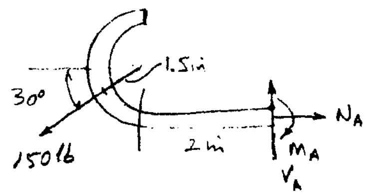
$$\sigma = \frac{N}{A} + \frac{My}{I}$$

$$\sigma = 30(10^6) = \frac{P}{0.625\,(10^{-3})\pi} + \frac{P(0.025)(0.025)}{97.65625\,(10^{-9})\pi}$$

$P = 11.8$ kN **Ans**

8–30 The $\frac{1}{2}$-in.-diameter bolt hook is subjected to the load of $F = 150$ lb. Determine the stress components at point A on the shank. Show the results on a volume element located at this point.





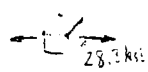
$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A - 150 \cos 30° = 0$

$$N_A = 129.9038 \text{ lb}$$

$+\uparrow \Sigma F_y = 0; \qquad V_A - 150 \sin 30° = 0$

$$V_A = 75 \text{ lb}$$

$\circlearrowleft + \Sigma M_A = 0; \qquad 150 \cos 30°(1.5) + 150 \sin 30°(2) - M_A = 0$

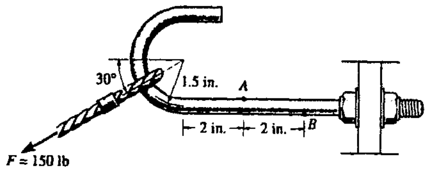


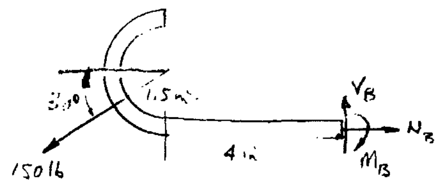
$$M_A = 344.8557 \text{ lb} \cdot \text{in.}$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{129.9038}{\pi(\frac{1}{4})^2} + \frac{344.8557(\frac{1}{4})}{\frac{\pi}{4}(\frac{1}{4})^4} = 28.8 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_A = 0 \;\; (\text{ since } Q_A = 0) \qquad \textbf{Ans}$$

8–31 The $\frac{1}{2}$-in.-diameter bolt hook is subjected to the load of $F = 150$ lb. Determine the stress components at point B on the shank. Show the results on a volume element located at this point.





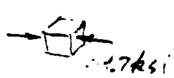
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_B - 150 \cos 30° = 0; \quad N_B = 129.9038$

$+\uparrow \Sigma F_y = 0; \quad V_B - 150 \sin 30° = 0; \quad V_B = 75 \text{ lb}$

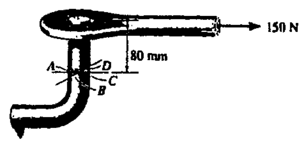
$\circlearrowleft + \Sigma M_B = 0; \quad 150 \cos 30°(1.5) + 150 \sin 30°(4) - M_B = 0$

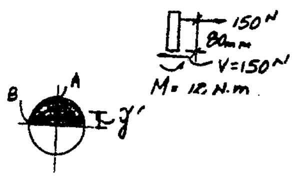
$$M_B = 494.8557 \text{ lb} \cdot \text{in.}$$

$$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{129.9038}{\pi(\frac{1}{4})^2} - \frac{494.8557(\frac{1}{4})}{\frac{\pi}{4}(\frac{1}{4})^4} = -39.7 \text{ ksi} \qquad \textbf{Ans}$$



*8-32.** The pin support is made from a steel rod and has a diameter of 20 mm. Determine the stress components at points A and B and represent the results on a volume element located at each of these points.





$$I = \frac{1}{4}(\pi)(0.01^4) = 7.85398\,(10^{-9})\ \text{m}^4$$

$$Q_B = \bar{y}'A' = \frac{4\,(0.01)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.01^2) = 0.66667\,(10^{-6})\ \text{m}^3$$

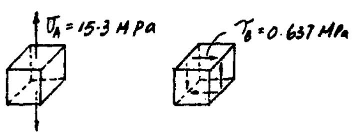
$$Q_A = 0$$

$$\sigma_A = \frac{Mc}{I} = \frac{12(0.01)}{7.85398\,(10^{-9})} = 15.3\ \text{MPa} \qquad \textbf{Ans}$$

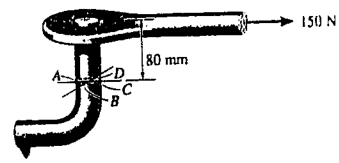
$\tau_A = 0$ **Ans**

$\sigma_B = 0$ **Ans**

$$\tau_B = \frac{VQ_B}{It} = \frac{150\,(0.6667)(10^{-6})}{7.85398\,(10^{-9})(0.02)} = 0.637\ \text{MPa} \qquad \textbf{Ans}$$



8–33 Solve Prob. 8-32 for points C and D.

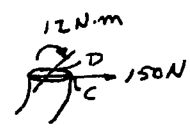


$$I = \frac{1}{4}(\pi)(0.01^4) = 7.85398\,(10^{-9})\ \text{m}^4$$

$$Q_D = \bar{y}'A' = \frac{4\,(0.01)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.01^2) = 0.66667\,(10^{-6})\ \text{m}^3$$

$$Q_C = 0$$

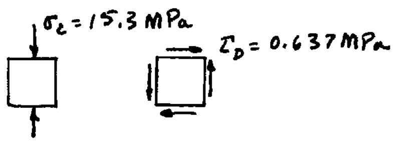
$$\sigma_C = \frac{Mc}{I} = \frac{12(0.01)}{7.85398(10^{-9})} = 15.3\ \text{MPa} \qquad \textbf{Ans}$$



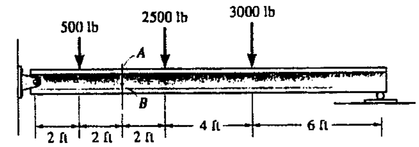
$\tau_C = 0$ **Ans**

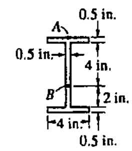
$\sigma_D = 0$ **Ans**

$$\tau_D = \frac{VQ_D}{It} = \frac{150(0.6667)(10^{-6})}{7.8539(10^{-9})(0.02)} = 0.637\ \text{MPa} \qquad \textbf{Ans}$$

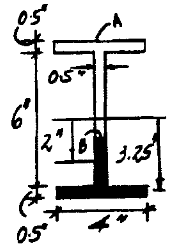


8–34 The wide-flange beam is subjected to the loading shown. Determine the stress components at points A and B and show the results on a volume element at each of these points. Use the shear formula to compute the shear stress.





$$I = \frac{1}{12}(4)(7^3) - \frac{1}{12}(3.5)(6^3) = 51.33\ \text{in}^4$$

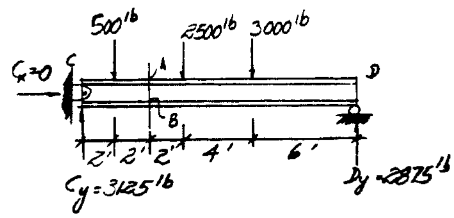


$$A = 2(0.5)(4) + 6(0.5) = 7\ \text{in}^2$$

$$Q_B = \Sigma \bar{y}A' = 3.25(4)(0.5) + 2(2)(0.5) = 8.5\ \text{in}^3$$

$$Q_A = 0$$

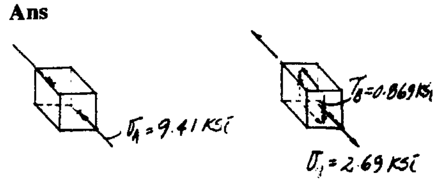
$$\sigma_A = \frac{-Mc}{I} = \frac{-11500\,(12)(3.5)}{51.33} = -9.41\ \text{ksi} \qquad \textbf{Ans}$$

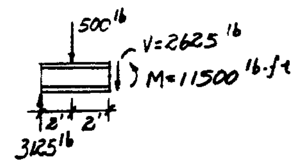


$$\tau_A = 0 \qquad \textbf{Ans}$$

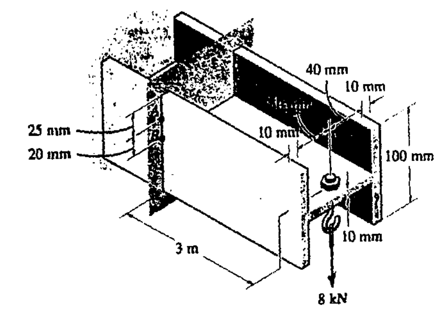
$$\sigma_B = \frac{My}{I} = \frac{11500(12)(1)}{51.33} = 2.69\ \text{ksi} \qquad \textbf{Ans}$$

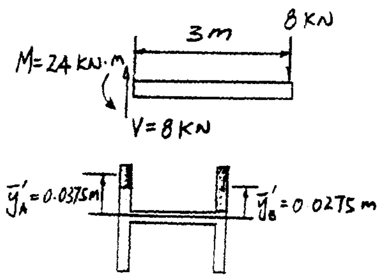
$$\tau_B = \frac{VQ_B}{It} = \frac{2625(8.5)}{51.33(0.5)} = 0.869\ \text{ksi} \qquad \textbf{Ans}$$





8-35 The cantilevered beam is used to support the load of 8 kN. Determine the state of stress at points A and B, and sketch the results on differential elements located at each of these points.





$$I = 2[\frac{1}{12}(0.01)(0.1^3)] + \frac{1}{12}(0.08)(0.01^3) = 1.6733(10^{-6})\ \text{m}^4$$

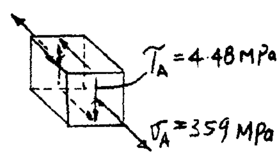
$$A = 2[0.01(0.1)] + 0.08(0.01) = 0.0028\ \text{m}^2$$

$$Q_A = \bar{y}'_A A = 0.0375(0.025)(0.01) = 9.375(10^{-6})\ \text{m}^3$$

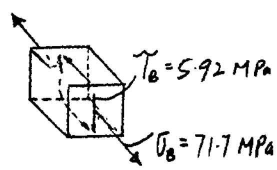
$$Q_B = \bar{y}'_B A = 0.0275(0.045)(0.01) = 12.375(10^{-6})\ \text{m}^3$$

$$\sigma = \frac{My}{I}$$

$$\sigma_A = \frac{24(10^3)(0.025)}{1.6733(10^{-6})} = 359\ \text{MPa (T)} \qquad \textbf{Ans}$$



$$\sigma_B = \frac{24(10^3)(0.005)}{1.6733(10^{-6})} = 71.7\ \text{MPa(T)} \qquad \textbf{Ans}$$

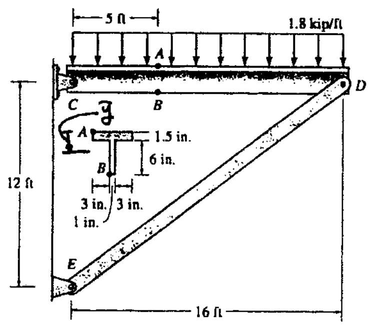


$$\tau = \frac{VQ}{It}$$

$$\tau_A = \frac{8(10^3)(9.375)(10^{-6})}{1.6733(10^{-6})(0.01)} = 4.48\ \text{MPa} \qquad \textbf{Ans}$$

$$\tau_B = \frac{8(10^3)(12.375)(10^{-6})}{1.6733(10^{-6})(0.01)} = 5.92\ \text{MPa} \qquad \textbf{Ans}$$

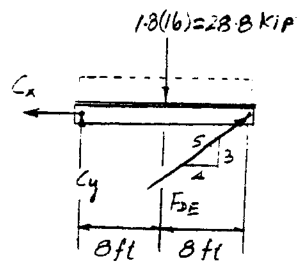
•8–36 The frame supports a centrally applied distributed load of 1.8 kip/ft. Determine the state of stress at points A and B on member CD and indicate the results on a volume element located at each of these points. The pins at C and D are at the same location as the neutral axis for the cross section.



Member CD :

$$\curvearrowleft + \Sigma M_C = 0; \quad \frac{3}{5}F_{DE}(16) - 28.8(8) = 0;$$

$$F_{DE} = 24.0 \text{ kip}$$



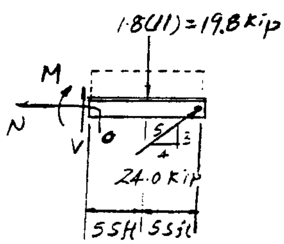
Segment :

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad N - \frac{4}{5}(24.0) = 0; \quad N = 19.2 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \quad V + \frac{3}{5}(24.0) - 19.8 = 0; \quad V = 5.40 \text{ kip}$$

$$\curvearrowleft + \Sigma M_O = 0; \quad -M - 19.8(5.5) + \frac{3}{5}(24.0)(11) = 0;$$

$$M = 49.5 \text{ kip}\cdot\text{ft}$$



$$A = 7(1.5) + 6(1) = 16.5 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.75(1.5)(7) + 4.5(6)(1)}{16.5} = 2.1136 \text{ in.}$$

$$I = \frac{1}{12}(7)(1.5^3) + 7(1.5)(2.1136 - 0.75)^2 + \frac{1}{12}(1)(6^3) + 1(6)(4.5 - 2.1136)^2 = 73.662 \text{ in}^4$$

$$Q_A = Q_B = 0$$

Normal Stress :

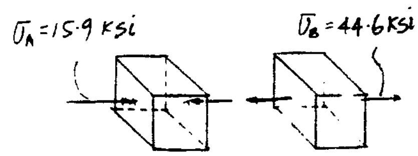
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{19.2}{16.5} - \frac{49.5(12)(7.5 - 2.1136)}{73.662} = -15.9 \text{ ksi} = 15.9 \text{ ksi(C)} \qquad \textbf{Ans}$$

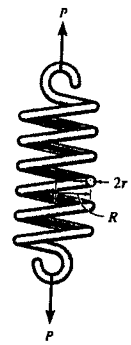
$$\sigma_B = \frac{19.2}{16.5} + \frac{49.5(12)(5.3864)}{73.662} = 44.6 \text{ ksi(T)} \qquad \textbf{Ans}$$

Shear Stress : Since $Q_A = Q_B = 0$,

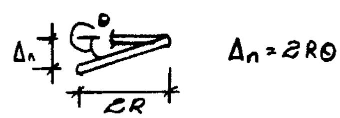
$$\tau_A = \tau_B = 0 \qquad \textbf{Ans}$$



8–37 The coiled spring is subjected to a force P. If we assume the shear stress caused by the shear force at any vertical section of the coil wire to be uniform, show that the maximum shear stress in the coil is $\tau_{max} = P/A + PRr/J$, where J is the polar moment of inertia of the coil wire and A is its cross-sectional area.



$$\tau_{max} = \frac{V}{A} + \frac{Tc}{J} = \frac{P}{A} + \frac{PRr}{J} \qquad \textbf{QED}$$

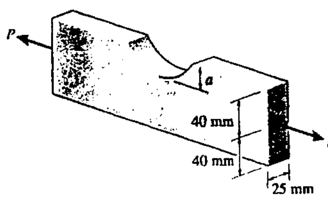


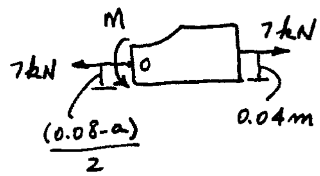
$$\tau_{max} = \frac{Vq_{max}}{It} + \frac{Tc}{J}$$

$$\frac{VQ}{It} = \frac{4}{3}\frac{V}{A}$$

$$\frac{Tc}{J} = \text{max on perimeter} = \frac{PRr}{J}$$

8-38 The metal link is subjected to the axial force of P = 7 kN. Its original cross section is to be altered by cutting a circular groove into one side. Determine the distance a the groove can penetrate into the cross section so that the tensile stress does not exceed σ_{allow} = 175 MPa. Offer a better way to remove this depth of material from the cross section and calculate the tensile stress for this case. Neglect the effects of stress concentration.





$$\circlearrowleft + \Sigma M_O = 0; \qquad M - 7(10^3)(0.04 - (\frac{0.08-a}{2})) = 0$$

$$M = 3.5(10^3)a$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$175(10^6) = \frac{7(10^3)}{(0.025)(0.08-a)} + \frac{3.5\,(10^3)a(0.08-a)/2}{\frac{1}{12}(0.025)(0.08-a)^3}$$

Set $x = 0.08 - a$

$$4375 = \frac{7}{x} + \frac{21(0.08-x)}{x^2}$$

$$4375x^2 + 14x - 1.68 = 0$$

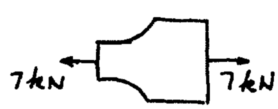
Choose positive root :

$x = 0.01806$

$a = 0.08 - 0.01806 = 0.0619$ m

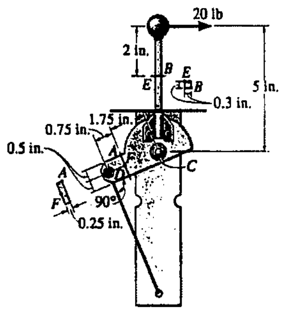
$a = 61.9$ mm **Ans**

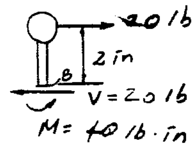
Remove material equally from both sides.

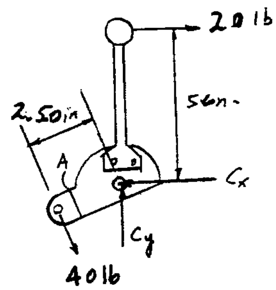


$$\sigma = \frac{7(10^3)}{(0.025)(0.01806)} = 15.5 \text{ MPa} \qquad \textbf{Ans}$$

8–39 The control lever is subjected to a horizontal force of 20 lb on the handle. Determine the state of stress at points *A* and *B*. Sketch the results on differential elements located at each of these points. The assembly is pin-connected at *C* and attached to a cable at *D*.



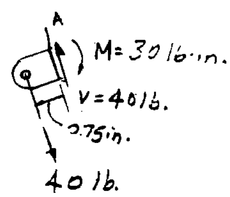




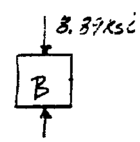
For point B :

$$I = \frac{1}{12}(0.3)(0.3^3) = 0.675(10^{-3}) \text{ in}^4$$

$$\sigma_B = \frac{Mc}{I} = \frac{40(0.15)}{0.675(10^{-3})} = 8.89 \text{ ksi (C)} \qquad \textbf{Ans}$$



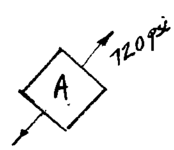
$\tau_B = 0 \quad (\text{since } Q_B = 0)$ **Ans**



For point A :

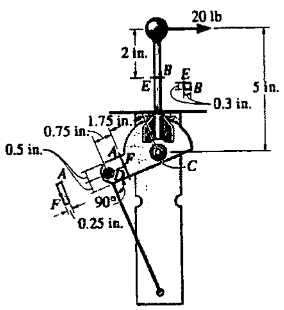
$$I = \frac{1}{12}(0.25)(1^3) = 0.020833 \text{ in}^4$$

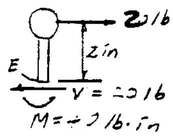
$$\sigma_A = \frac{Mc}{I} = \frac{30(0.5)}{0.020833} = 720 \text{ psi (T)} \qquad \textbf{Ans}$$

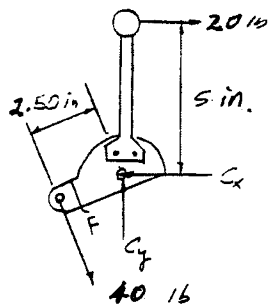


$\tau_A = 0 \quad (\text{since } Q_A = 0)$ **Ans**

***8–40** The control lever is subjected to a horizontal force of 20 lb on the handle. Determine the state of stress at points *E* and *F*. Sketch the results on differential elements located at each of these points. The assembly is pin-connected at *C* and attached to a cable at *D*.



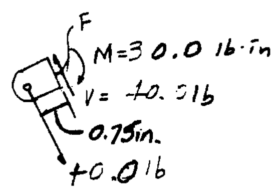




For point *E* :

$$I = \frac{1}{12}(0.3)(0.3^3) = 0.675(10^{-3})\ \text{in}^4$$

$$\sigma_E = \frac{Mc}{I} = \frac{40(0.15)}{0.675(10^{-3})} = 8.89\ \text{ksi (T)} \qquad \textbf{Ans}$$



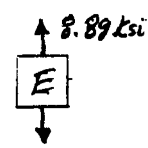
$$\tau_E = 0 \qquad (\text{since } Q_E = 0) \qquad \textbf{Ans}$$

For point *F* :

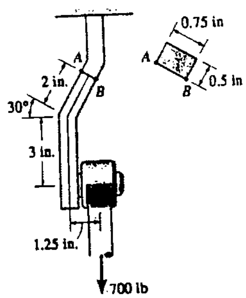
$$I = \frac{1}{12}(0.25)(1^3) = 0.020833\ \text{in}^4$$

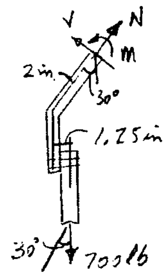
$$\sigma_F = 0 \qquad \textbf{Ans}$$

$$\tau_F = \frac{VQ}{It} = \frac{40(0.25)(0.5)(0.25)}{\frac{1}{12}(0.25)(1)^3(0.25)} = 240\ \text{psi} \qquad \textbf{Ans}$$



8–41 The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point *A*. The support is 0.5 in. thick.





$\Sigma F_x = 0; \qquad N - 700\cos 30° = 0; \qquad N = 606.218 \text{ lb}$

$\Sigma F_y = 0; \qquad V - 700\sin 30° = 0; \qquad V = 350 \text{ lb}$

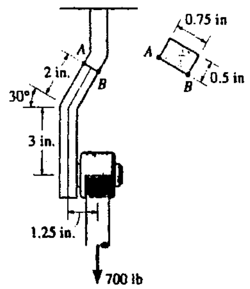
$\circlearrowleft + \Sigma M = 0; \qquad M - 700(1.25 - 2\sin 30°) = 0; \qquad M = 175 \text{ lb}\cdot\text{in.}$

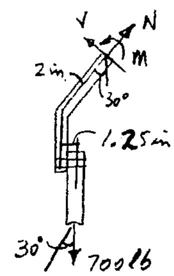
$$\sigma_A = \frac{N}{A} - \frac{Mc}{I} = \frac{606.218}{(0.75)(0.5)} - \frac{(175)(0.375)}{\frac{1}{12}(0.5)(0.75)^3}$$

$\sigma_A = -2.12 \text{ ksi}$ **Ans**

$\tau_A = 0 \quad (\text{since } Q_A = 0)$ **Ans**

8–42 The bearing pin supports the load of 700 lb. Determine the stress components in the support member at point *B*. The support is 0.5 in. thick.





$\Sigma F_x = 0; \qquad N - 700\cos 30° = 0; \qquad N = 606.218 \text{ lb}$

$\Sigma F_y = 0; \qquad V - 700\sin 30° = 0; \qquad V = 350 \text{ lb}$

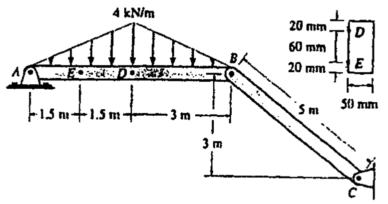
$\curvearrowleft + \Sigma M = 0; \qquad M - 700(1.25 - 2\sin 30°) = 0; \qquad M = 175 \text{ lb}\cdot\text{in.}$

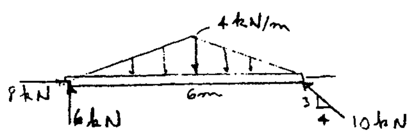
$$\sigma_B = \frac{N}{A} + \frac{Mc}{I} = \frac{606.218}{(0.75)(0.5)} + \frac{175(0.375)}{\frac{1}{12}(0.5)(0.75)^3}$$

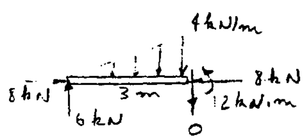
$\sigma_B = 5.35 \text{ ksi}$ **Ans**

$\tau_B = 0 \qquad (\text{since } Q_B = 0)$ **Ans**

8-43. The frame supports the distributed load shown. Determine the state of stress acting at point D. Show the results on a differential element located at this point.





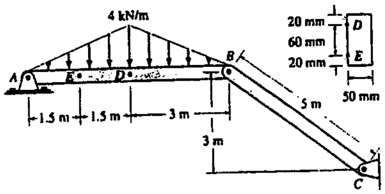


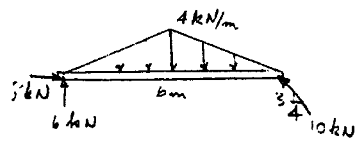
$$\sigma_D = -\frac{P}{A} - \frac{My}{I} = -\frac{8(10^3)}{(0.1)(0.05)} - \frac{12(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3}$$

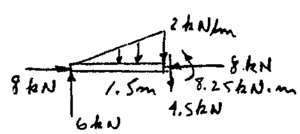
$\sigma_D = -88.0\text{ MPa}$ **Ans**

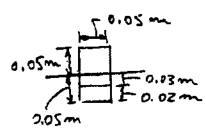
$\tau_D = 0$ **Ans**

***8-44.** The frame supports the distributed load shown. Determine the state of stress acting at point E. Show the results on a differential element located at this point.



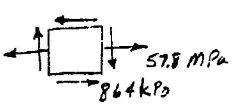




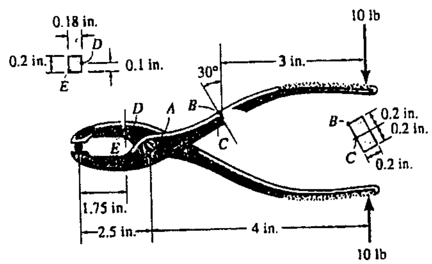


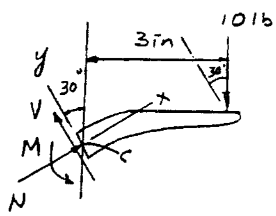
$$\sigma_E = -\frac{P}{A} - \frac{My}{I} = -\frac{8(10^3)}{(0.1)(0.05)} + \frac{8.25(10^3)(0.03)}{\frac{1}{12}(0.05)(0.1)^3} = 57.8\text{ MPa} \qquad \textbf{Ans}$$

$$\tau_E = \frac{VQ}{It} = \frac{4.5(10^3)(0.04)(0.02)(0.05)}{\frac{1}{12}(0.05)(0.1)^3(0.05)} = 864\text{ kPa} \qquad \textbf{Ans}$$



8–45 The pliers are made from two steel parts pinned together at A. If a smooth bolt is held in the jaws and a gripping force of 10 lb is applied at the handles, determine the state of stress developed in the pliers at points B and C. Here the cross section is rectangular, having the dimensions shown in the figure.

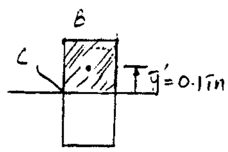




$$+\Sigma F_x = 0; \quad N - 10\sin 30° = 0; \quad N = 5.0 \text{ lb}$$

$$+\Sigma F_y = 0; \quad V - 10\cos 30° = 0; \quad V = 8.660 \text{ lb}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad M - 10(3) = 0; \quad M = 30 \text{ lb}\cdot\text{in.}$$



$$A = 0.2(0.4) = 0.08 \text{ in}^2$$

$$I = \frac{1}{12}(0.2)(0.4^3) = 1.0667(10^{-3}) \text{ in}^4$$

$$Q_B = 0$$
$$Q_C = \bar{y}A' = 0.1(0.2)(0.2) = 4(10^{-3}) \text{ in}^3$$

Point B :

$$\sigma_B = \frac{N}{A} + \frac{My}{I} = \frac{-5.0}{0.08} + \frac{30(0.2)}{1.0667(10^{-3})} = 5.56 \text{ ksi(T)}$$ **Ans**

$$\tau_B = \frac{VQ}{It} = 0$$ **Ans**

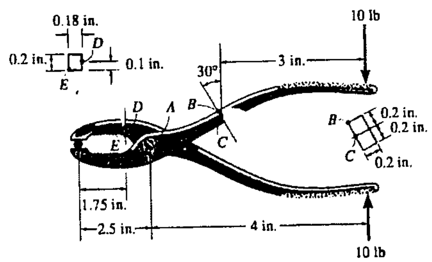
Point C :

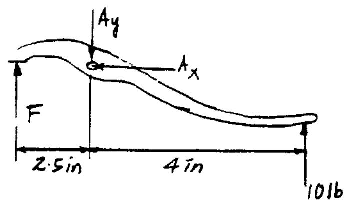
$$\sigma_C = \frac{N}{A} + \frac{My}{I} = \frac{-5.0}{0.08} + 0 = -62.5 \text{ psi} = 62.5 \text{ psi(C)}$$ **Ans**

Shear Stress :

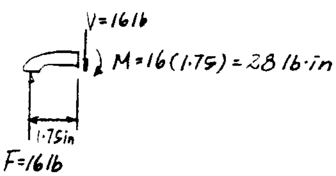
$$\tau_C = \frac{VQ}{It} = \frac{8.660(4)(10^{-3})}{1.0667(10^{-3})(0.2)} = 162 \text{ psi}$$ **Ans**

8–46 Solve Prob. 8–45 for points D and E.





$\curvearrowleft + \Sigma M_A = 0; \qquad -F(2.5) + 4(10) = 0; \qquad F = 16\ \text{lb}$



Point D :

$\sigma_D = 0$ **Ans**

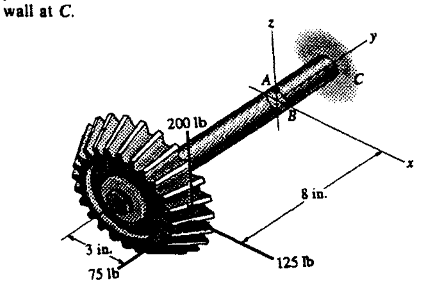
$$\tau_D = \frac{VQ}{It} = \frac{16(0.05)(0.1)(0.18)}{[\frac{1}{12}(0.18)(0.2)^3](0.18)} = 667\ \text{psi} \qquad \textbf{Ans}$$

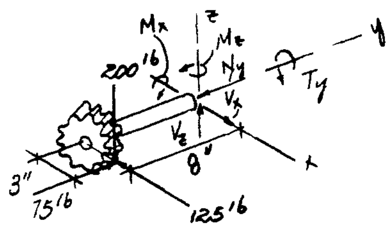
Point E :

$$\sigma_E = \frac{My}{I} = \frac{28(0.1)}{\frac{1}{12}(0.18)(0.2)^3} = 23.3\ \text{ksi (C)} \qquad \textbf{Ans}$$

$\tau_E = 0$ **Ans**

8-47 The beveled gear is subjected to the loads shown. Determine the stress components acting on the shaft at point A, and show the results on a volume element located at this point. The shaft has a diameter of 1 in. and is fixed to the wall at C.





$\Sigma F_x = 0; \quad V_x - 125 = 0; \quad V_x = 125 \text{ lb}$

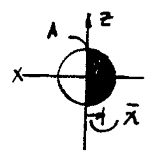
$\Sigma F_y = 0; \quad 75 - N_y = 0; \quad N_y = 75 \text{ lb}$

$\Sigma F_z = 0; \quad V_z - 200 = 0; \quad V_z = 200 \text{ lb}$

$\Sigma M_x = 0; \quad 200(8) - M_x = 0; \quad M_x = 1600 \text{ lb} \cdot \text{in.}$

$\Sigma M_y = 0; \quad 200(3) - T_y = 0; \quad T_y = 600 \text{ lb} \cdot \text{in.}$

$\Sigma M_z = 0; \quad M_z + 75(3) - 125(8) = 0; \quad M_z = 775 \text{ lb} \cdot \text{in.}$



$A = \pi(0.5^2) = 0.7854 \text{ in}^2$

$J = \frac{\pi}{2}(0.5^4) = 0.098175 \text{ in}^4$

$I = \frac{\pi}{4}(0.5^4) = 0.049087 \text{ in}^4$

$(Q_A)_x = 0$

$(Q_A)_z = \frac{4(0.5)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.5^2) = 0.08333 \text{ in}^3$

$$(\sigma_A)_y = -\frac{N_y}{A} + \frac{M_x c}{I}$$

$$= -\frac{75}{0.7854} + \frac{1600(0.5)}{0.049087}$$

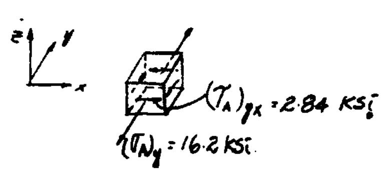
$$= 16202 \text{ psi} = 16.2 \text{ ksi (T)} \qquad \textbf{Ans}$$

$$(\tau_A)_{yx} = (\tau_A)_V - (\tau_A)_{\text{twist}}$$

$$= \frac{V_x (Q_A)_z}{I\,t} - \frac{T_y c}{J}$$

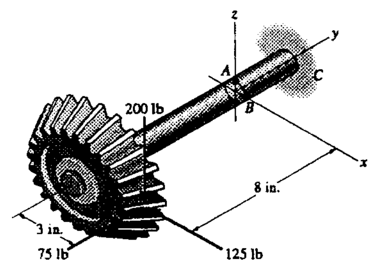
$$= \frac{125(0.08333)}{0.049087\,(1)} - \frac{600(0.5)}{0.098175}$$

$$= -2843 \text{ psi} = -2.84 \text{ ksi} \qquad \textbf{Ans}$$



$$(\tau_A)_{yz} = \frac{V_z (Q_A)_x}{I\,t} = 0 \qquad \textbf{Ans}$$

***8–48** The beveled gear is subjected to the loads shown. Determine the stress components acting on the shaft at point *B*, and show the results on a volume element located at this point. The shaft has a diameter of 1 in. and is fixed to the wall at *C*.



$\Sigma F_x = 0; \quad V_x - 125 = 0; \quad V_x = 125\ \text{lb}$

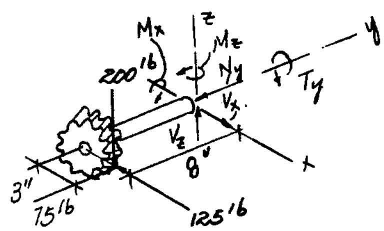
$\Sigma F_y = 0; \quad 75 - N_y = 0; \quad N_y = 75\ \text{lb}$

$\Sigma F_z = 0; \quad V_z - 200 = 0; \quad V_z = 200\ \text{lb}$

$\Sigma M_x = 0; \quad 200(8) - M_x = 0; \quad M_x = 1600\ \text{lb}\cdot\text{in.}$

$\Sigma M_y = 0; \quad 200(3) - T_y = 0; \quad T_y = 600\ \text{lb}\cdot\text{in.}$

$\Sigma M_z = 0; \quad M_z + 75(3) - 125(8) = 0; \quad M_z = 775\ \text{lb}\cdot\text{in.}$



$A = \pi(0.5^2) = 0.7854\ \text{in}^2$

$J = \dfrac{\pi}{2}(0.5^4) = 0.098175\ \text{in}^4$

$I = \dfrac{\pi}{4}(0.5^4) = 0.049087\ \text{in}^4$

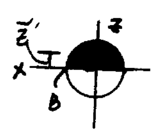
$(Q_B)_z = 0$

$(Q_B)_x = \dfrac{4(0.5)}{3\pi}(\dfrac{1}{2})(\pi)(0.5^2) = 0.08333\ \text{in}^3$

$$(\sigma_B)_y = -\frac{P_y}{A} + \frac{M_z\, c}{I}$$

$$= -\frac{75}{0.7854} + \frac{775(0.5)}{0.049087}$$

$= 7.80\ \text{ksi (T)}$ **Ans**



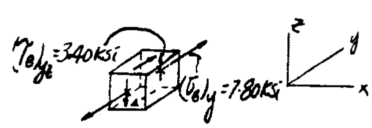
$$(\tau_B)_{yz} = (\tau_B)_V + (\tau_B)_{\text{twist}}$$

$$= \frac{V_z (Q_B)_x}{I\,t} + \frac{T_y\, c}{J}$$

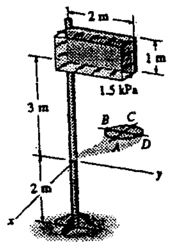
$$= \frac{200(0.08333)}{0.049087\,(1)} + \frac{600(0.5)}{0.098175}$$

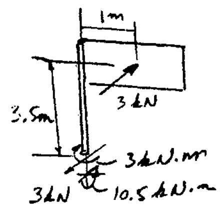
$= 3395\text{psi} = 3.40\ \text{ksi}$ **Ans**

$$(\tau_B)_{yx} = \frac{V_x\,(Q_B)_z}{I\,t} = 0$$ **Ans**



8-49. The sign is subjected to the uniform wind loading. Determine the stress components at points A and B on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.

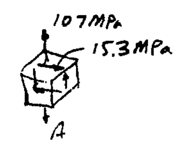




Point A :

$$\sigma_A = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa} \quad \textbf{Ans}$$

$$\tau_A = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa} \quad \textbf{Ans}$$

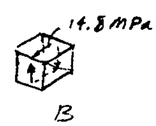


Point B :

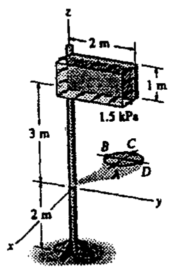
$\sigma_B = 0$ **Ans**

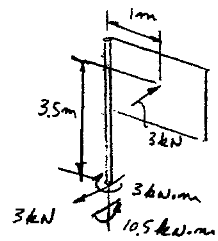
$$\tau_B = \frac{Tc}{J} - \frac{VQ}{It} = 15.279(10^6) - \frac{3000(4(0.05)/3\pi))(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)}$$

$\tau_B = 14.8$ MPa **Ans**



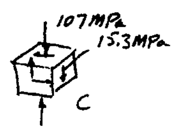
8-50. The sign is subjected to the uniform wind loading. Determine the stress components at points C and D on the 100-mm-diameter supporting post. Show the results on a volume element located at each of these points.





Point C:

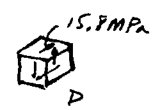
$$\sigma_C = \frac{Mc}{I} = \frac{10.5(10^3)(0.05)}{\frac{\pi}{4}(0.05)^4} = 107 \text{ MPa (C)} \qquad \textbf{Ans}$$



$$\tau_C = \frac{Tc}{J} = \frac{3(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 15.279(10^6) = 15.3 \text{ MPa} \qquad \textbf{Ans}$$

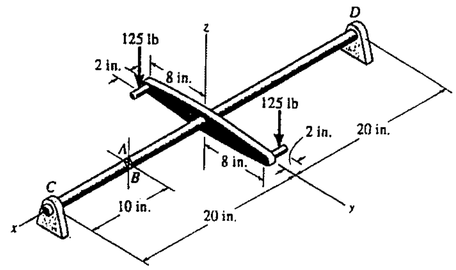
Point D:

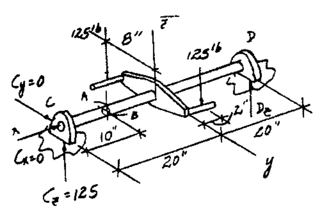
$$\sigma_D = 0 \qquad \textbf{Ans}$$



$$\tau_D = \frac{Tc}{J} + \frac{VQ}{It} = 15.279(10^6) + \frac{3(10^3)(4(0.05)/3\pi)(\frac{1}{2})(\pi)(0.05)^2}{\frac{\pi}{4}(0.05)^4(0.1)} = 15.8 \text{ MPa} \qquad \textbf{Ans}$$

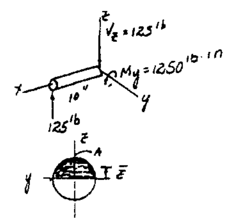
8–51 The $\frac{3}{4}$-in.-diameter shaft is subjected to the loading shown. Determine the stress components at point A. Sketch the results on a volume element located at this point. The journal bearing at C can exert only force components $\mathbf{C}_y$ and $\mathbf{C}_z$ on the shaft, and the thrust bearing at D can exert force components $\mathbf{D}_x$, $\mathbf{D}_y$, and $\mathbf{D}_z$ on the shaft.





$$A = \frac{\pi}{4}(0.75^2) = 0.44179\ \text{in}^2$$

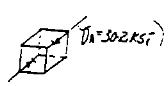
$$I = \frac{\pi}{4}(0.375^4) = 0.015531\ \text{in}^4$$



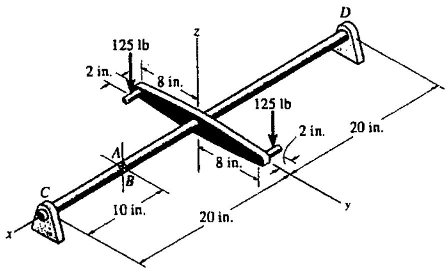
$$Q_A = 0$$

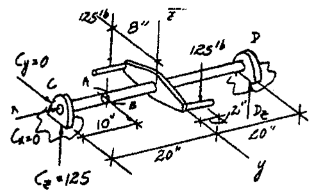
$$\tau_A = 0$$ **Ans**

$$\sigma_A = \frac{M_y c}{I} = \frac{-1250(0.375)}{0.015531} = -30.2\ \text{ksi} = 30.2\ \text{ksi (C)}$$ **Ans**



*8-52 Solve Prob. 8-51 for the stress components at point B.

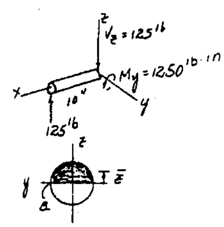




$$A = \frac{\pi}{4}(0.75^2) = 0.44179 \text{ in}^2$$

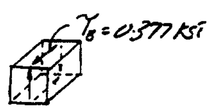
$$I = \frac{\pi}{4}(0.375^4) = 0.015531 \text{ in}^4$$

$$Q_B = \bar{y}'A' = \frac{4(0.375)}{3\pi}(\frac{1}{2})(\pi)(0.375^2) = 0.035156 \text{ in}^3$$

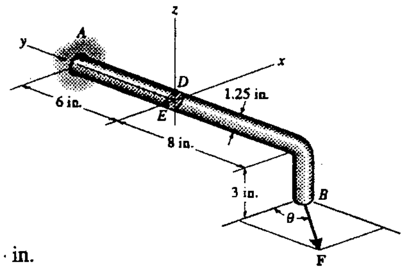


$$\sigma_B = 0$$ **Ans**

$$\tau_B = \frac{V_z Q_B}{I t} = \frac{125(0.035156)}{0.015531(0.75)} = 0.377 \text{ ksi}$$ **Ans**



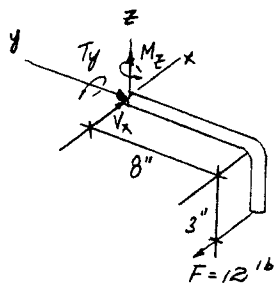
8–53 The bent shaft is fixed in the wall at A. If a force $\mathbf{F}$ is applied at B, determine the stress components at points D and E. Show the results on a differential element located at each of these points. Take $F = 12$ lb and $\theta = 0°$.



$\Sigma F_x = 0; \quad V_x - 12 = 0; \quad V_x = 12 \text{ lb}$

$\Sigma M_y = 0; \quad -T_y + 12(3) = 0; \quad T_y = 36 \text{ lb}\cdot\text{in.}$

$\Sigma M_z = 0; \quad M_z - 12(8) = 0; \quad M_z = 96 \text{ lb}\cdot\text{in.}$



$A = \pi(0.625^2) = 1.2272 \text{ in}^2$

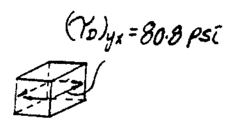
$I = \dfrac{1}{4}\pi(0.625^4) = 0.1198 \text{ in}^4$

$J = \dfrac{1}{2}\pi(0.625^4) = 0.2397 \text{ in}^4$

Point D :

$(Q_D)_z = \dfrac{4(0.625)}{3\pi}\,\dfrac{1}{2}(\pi)(0.625^2) = 0.1628 \text{ in}^3$

$\sigma_D = \dfrac{M_z\, x}{I} = 0$ **Ans**



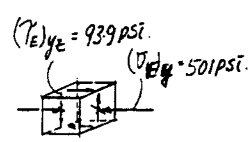
$(\tau_D)_{yx} = (\tau_D)_V - (\tau_D)_{\text{twist}}$

$= \dfrac{V_x (Q_D)_z}{I\,t} - \dfrac{T_y\, c}{J}$

$= \dfrac{12(0.1628)}{0.1198(1.25)} - \dfrac{36(0.625)}{0.2397} = -\,80.8 \text{ psi}$ **Ans**

Point E :

$(\sigma_E)_y = \dfrac{M_z x}{I} = \dfrac{-96(0.625)}{0.1198} = -501 \text{ psi}$ **Ans**

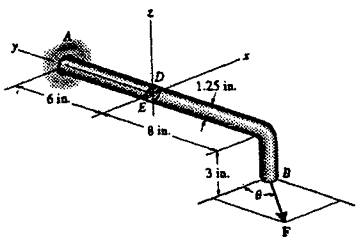


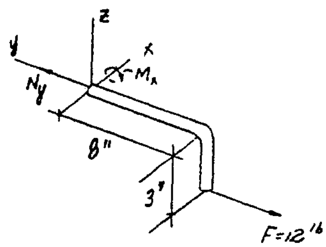
$(\tau_E)_{yz} = (\tau_E)_V - (\tau_E)_{\text{twist}}$

$= 0 - \dfrac{T_y\, c}{J} = \dfrac{-36(0.625)}{0.2397}$

$= -\,93.9 \text{ psi}$ **Ans**

8-54. The bent shaft is fixed in the wall at A. If a force $\mathbf{F}$ is applied at B, determine the stress components at points D and E. Show the results on a differential element located at each of these points. Take $F = 12$ lb and $\theta = 90°$.





$\Sigma F_y = 0; \quad N_y - 12 = 0; \quad N_y = 12 \text{ lb}$

$\Sigma M_x = 0; \quad M_x - 12(3) = 0; \quad M_x = 36 \text{ lb} \cdot \text{in.}$

$A = \pi(0.625^2) = 1.2272 \text{ in}^2$

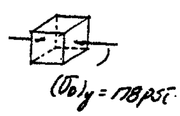
$I = \frac{1}{4}\pi(0.625^4) = 0.1198 \text{ in}^4$

Point D :

$(\sigma_D)_y = \frac{N_y}{A} - \frac{M_x z}{I} = \frac{12}{1.2272} - \frac{36(0.625)}{0.1198}$

$= -178 \text{ psi}$ **Ans**

$(\tau_D)_{yx} = (\tau_D)_{yz} = 0$ **Ans**

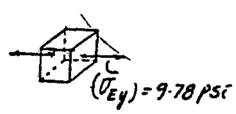


Point E :

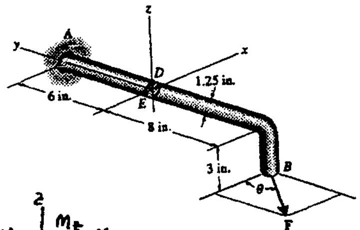
$(\sigma_E)_y = \frac{N_y}{A} + \frac{M_x z}{I} = \frac{12}{1.2272}$

$= 9.78 \text{ psi}$ **Ans**

$(\tau_E)_{yx} = (\tau_E)_{yz} = 0$ **Ans**



8-55. The bent shaft is fixed in the wall at A. If a force $\mathbf{F}$ is applied at B, determine the stress components at points D and E. Show the results on a volume element located at each of these points. Take $F = 12$ lb and $\theta = 45°$.



$\Sigma F_x = 0; \quad V_x - 12\cos 45° = 0; \quad V_x = 8.485 \text{ lb}$

$\Sigma F_y = 0; \quad N_y - 12\sin 45° = 0; \quad N_y = 8.485 \text{ lb}$

$\Sigma M_x = 0; \quad M_x - 12\sin 45°(3) = 0;$

$M_x = 25.456 \text{ lb}\cdot\text{in.}$

$\Sigma M_y = 0; \quad -T_y + 12\cos 45°(3) = 0; \quad T_y = 25.456 \text{ lb}\cdot\text{in.}$

$\Sigma M_z = 0; \quad M_z - 12\cos 45°(8) = 0; \quad M_z = 67.882 \text{ lb}\cdot\text{in.}$

$A = \pi(0.625^2) = 1.2272 \text{ in}^2$

$I = \frac{1}{4}\pi(0.625^4) = 0.1198 \text{ in}^4$

$J = \frac{1}{2}\pi(0.625^4) = 0.2397 \text{ in}^4$

Point D :

$(Q_D)_z = \frac{4(0.625)}{3\pi}\,\frac{1}{2}(\pi)(0.625^2) = 0.1628 \text{ in}^3$

$(\sigma_D)_y = \frac{N_y}{A} - \frac{M_x z}{I} = \frac{8.485}{1.2272} - \frac{25.456(0.625)}{0.1198}$

$= -126 \text{ psi}$ **Ans**

$(\tau_D)_{yx} = \frac{V_x (Q_D)_z}{I\,t} - \frac{T_y\,c}{J}$

$= \frac{8.485(0.1628)}{0.1198(1.25)} - \frac{(25.456)(0.625)}{0.2397}$

$= -57.2 \text{ psi}$ **Ans**

Point E :

$(\sigma_E)_z = 0$

$(\sigma_E)_y = \frac{N_y}{A} - \frac{M_z x}{I} = \frac{8.485}{1.2272} - \frac{(67.882)(0.625)}{0.1198}$

$= -347 \text{ psi}$ **Ans**

$(\tau_E)_{yz} = \frac{V_x Q_E}{I\,t} - \frac{T\,c}{J}$

$= 0 - \frac{(25.456)(0.625)}{0.2397}$

$= -66.4 \text{ psi}$ **Ans**