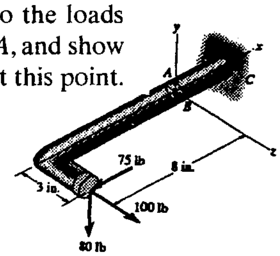
***8-56.** The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point A, and show the results on a differential element located at this point.



$\Sigma F_z = 0; \quad V_z + 100 = 0; \quad V_z = -100 \text{ lb}$

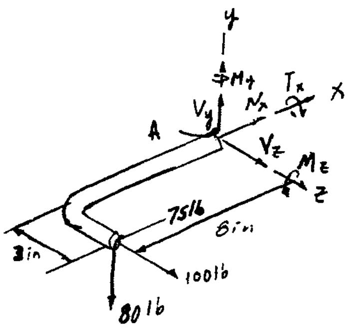
$\Sigma F_x = 0; \quad N_x - 75 = 0; \quad N_x = 75 \text{ lb}$

$\Sigma F_y = 0; \quad V_y - 80 = 0; \quad V_y = 80 \text{ lb}$

$\Sigma M_z = 0; \quad M_z + 80(8) = 0; \quad M_z = -640 \text{ lb}\cdot\text{in.}$

$\Sigma M_x = 0; \quad T_x + 80(3) = 0; \quad T_x = -240 \text{ lb}\cdot\text{in.}$

$\Sigma M_y = 0; \quad M_y + 100(8) - 75(3) = 0; \quad M_y = -575 \text{ lb}\cdot\text{in.}$



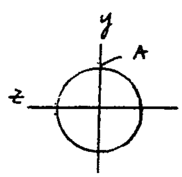
$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1^2) = \frac{1}{4}\pi \text{ in}^2$

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$

$(Q_y)_A = 0$

$(Q_z)_A = \bar{y}'A = \frac{4(0.5)}{3\pi}\,\frac{1}{2}(\pi)(0.5^2) = 0.08333 \text{ in}^3$

$I_y = I_z = \frac{\pi}{4} r^4 = \frac{\pi}{4}(0.5^4) = 0.015625\pi \text{ in}^4$



Normal stress : $\sigma = \frac{P}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$

$\sigma_A = \frac{75}{\frac{1}{4}\pi} + \frac{640\,(0.5)}{0.0156\pi} + 0 = 6.61 \text{ ksi (T)}$ **Ans**

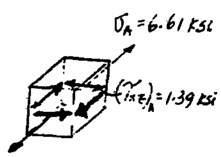
Shear stress :

$\tau = \frac{VQ}{It} + \frac{Tc}{J}$

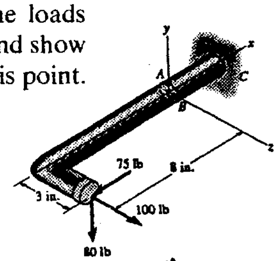
$(\tau_{xz})_A = \frac{100(0.08333)}{0.0156\pi(1)} + \frac{240(0.5)}{0.0312\pi}$

$= 1.39 \text{ ksi}$ **Ans**

$(\tau_{xy})_A = 0$ **Ans**



8-57. The 1-in.-diameter rod is subjected to the loads shown. Determine the state of stress at point B, and show the results on a differential element located at this point.



$\Sigma F_z = 0;\quad V_z + 100 = 0;\quad V_z = -100\text{ lb}$

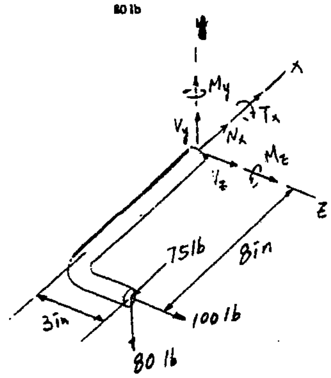
$\Sigma F_x = 0;\quad N_x - 75 = 0;\quad N_x = 75.0\text{ lb}$

$\Sigma F_y = 0;\quad V_y - 80 = 0;\quad V_y = 80\text{ lb}$

$\Sigma M_z = 0;\quad M_z + 80(8) = 0;\quad M_z = -640\text{ lb}\cdot\text{in.}$

$\Sigma M_x = 0;\quad T_x + 80(3) = 0;\quad T_x = -240\text{ lb}\cdot\text{in.}$

$\Sigma M_y = 0;\quad M_y + 100(8) - 75(3) = 0;\quad M_y = -575\text{ lb}\cdot\text{in.}$



$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1^2) = \frac{\pi}{4}\text{ in}^2$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.5^4) = 0.03125\pi\text{ in}^4$

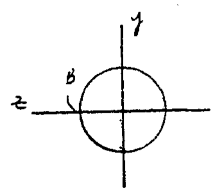
$(Q_y)_B = \frac{4(0.5)}{3\pi}\,\frac{1}{2}\left(\frac{\pi}{4}\right)(1^2) = 0.08333\text{ in}^3$

$I_y = I_z = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.5^4) = 0.015625\pi\text{ in}^4$

Normal stress :

$\sigma = \frac{P}{A} + \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$

$\sigma_B = \frac{75}{\frac{\pi}{4}} + 0 - \frac{575(0.5)}{0.015625\pi} = -5.76\text{ ksi} = 5.76\text{ ksi (C)}$ **Ans**

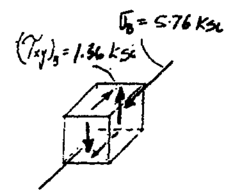


Shear stress :

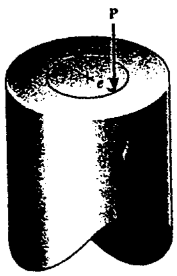
$\tau = \frac{VQ}{It}$ and $\tau = \frac{Tc}{J}$

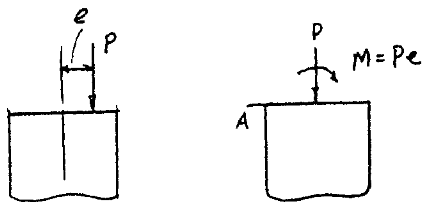
$(\tau_{xy})_B = \frac{Tc}{J} - \frac{VQ}{It} = \frac{240(0.5)}{0.03125\pi} + \frac{80(0.0833)}{0.015625\pi(1)} = 1.36\text{ ksi}$ **Ans**

$(\tau_{xz})_B = 0$ **Ans**



8-58 The post has a circular cross section of radius c. Determine the maximum radius e at which the load can be applied so that no part of the post experiences a tensile stress. Neglect the weight of the post.



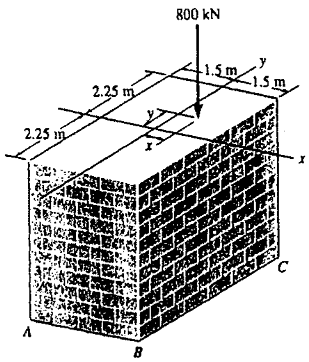


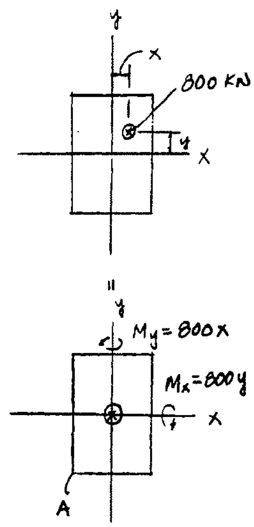
Require $\sigma_A = 0$

$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}; \qquad 0 = \frac{-P}{\pi c^2} + \frac{(Pe)c}{\frac{\pi}{4}c^4}$$

$$e = \frac{c}{4}$$ **Ans**

8–59 The masonry pier is subjected to the 800-kN load. For the range y>0, x>0, determine the equation of the line $y = f(x)$ along which the load can be placed without causing a tensile stress in the pier. Neglect the weight of the pier.





$$A = 3(4.5) = 13.5\ \text{m}^2$$

$$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125\ \text{m}^4$$

$$I_y = \frac{1}{12}(4.5)(3^3) = 10.125\ \text{m}^4$$

Normal stress : Require $\sigma_A = 0$

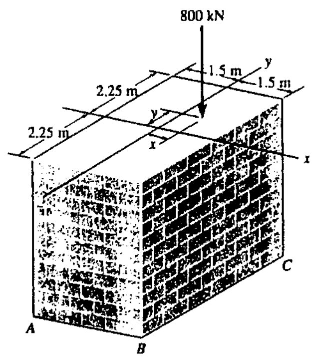
$$\sigma_A = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

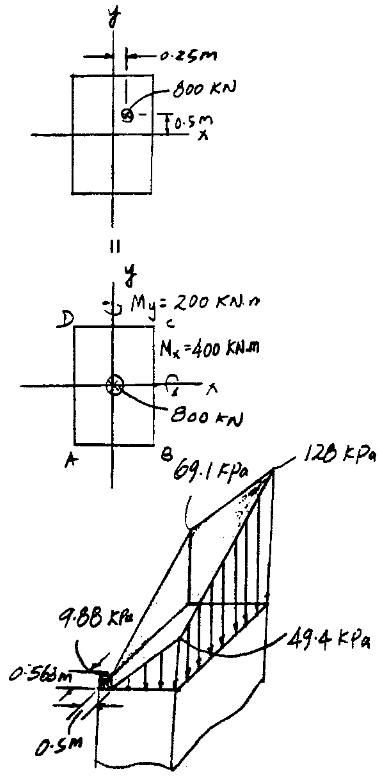
$$0 = \frac{-800(10^3)}{13.5} + \frac{800(10^3)y(2.25)}{22.78125} + \frac{800(10^3)x(1.5)}{10.125}$$

$$0 = 0.148x + 0.0988y - 0.0741$$

$$y = 0.75 - 1.5x \qquad \textbf{Ans}$$

*8-60 The masonry pier is subjected to the 800-kN load. If $x = 0.25$ m and $y = 0.5$ m, determine the normal stress at each corner A, B, C, D (not shown) and plot the stress distribution over the cross section. Neglect the weight of the pier.





$$A = 3(4.5) = 13.5\ \text{m}^2$$

$$I_x = \frac{1}{12}(3)(4.5^3) = 22.78125\ \text{m}^4$$

$$I_y = \frac{1}{12}(4.5)(3^3) = 10.125\ \text{m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

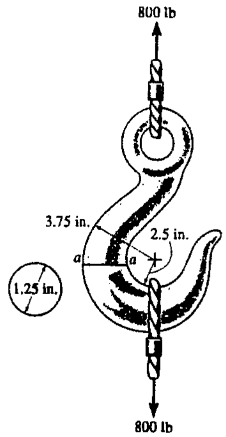
$$\sigma_A = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$
$= 9.88$ kPa (T) **Ans**

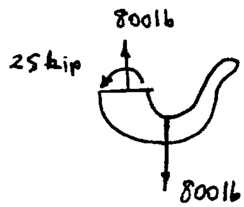
$$\sigma_B = \frac{-800(10^3)}{13.5} + \frac{400(10^3)(2.25)}{22.78125} - \frac{200(10^3)(1.5)}{10.125}$$
$= -49.4$ kPa $= 49.4$ kPa (C) **Ans**

$$\sigma_C = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} - \frac{200(10^3)(1.5)}{10.125}$$
$= -128$ kPa $= 128$ kPa (C) **Ans**

$$\sigma_D = \frac{-800(10^3)}{13.5} - \frac{400(10^3)(2.25)}{22.78125} + \frac{200(10^3)(1.5)}{10.125}$$
$= -69.1$ kPa $= 69.1$ kPa (C) **Ans**

8–61 The eye hook has the dimensions shown. If it supports a cable loading of 80 kN, determine the maximum normal stress at section *a–a* and sketch the stress distribution acting over the cross section.

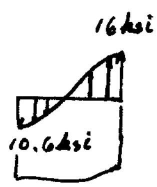




$$\int \frac{dA}{r} = 2\pi\,(3.125 - \sqrt{(3.125)^2 - (0.625)^2}\,) = 0.395707$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi\,(0.625)^2}{0.396707} = 3.09343 \text{ in.}$$

$$M = 800(3.125) = 2.5(10^3)$$

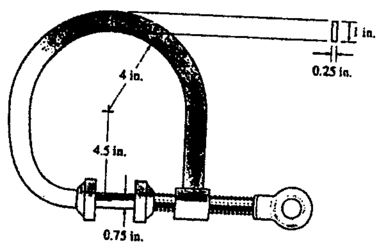


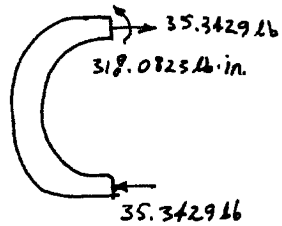
$$\sigma = \frac{M(R - r)}{Ar(\bar{r} - R)} + \frac{P}{A}$$

$$(\sigma_t)_{max} = \frac{2.5(10^3)(3.09343 - 2.5)}{\pi\,(0.625)^2(2.5)(3.125 - 3.09343)} + \frac{800}{\pi\,(0.625)^2} = 16.0 \text{ ksi} \qquad \textbf{Ans}$$

$$(\sigma_c)_{max} = \frac{2.5(10^3)(3.09343 - 3.75)}{\pi\,(0.625)^2(3.75)(3.125 - 3.09343)} + \frac{800}{\pi\,(0.625)^2} = -10.6 \text{ ksi} \qquad \textbf{Ans}$$

8-62. The C-clamp applies a compressive stress on the cylindrical block of 80 psi. Determine the maximum normal stress developed in the clamp.





$$\int_A \frac{dA}{r} = 0.25 \ln \frac{5}{4} = 0.055786$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{1(0.25)}{0.055786} = 4.48142$$

$$P = \sigma_b A = 80\pi(0.375)^2 = 35.3429 \text{ lb}$$

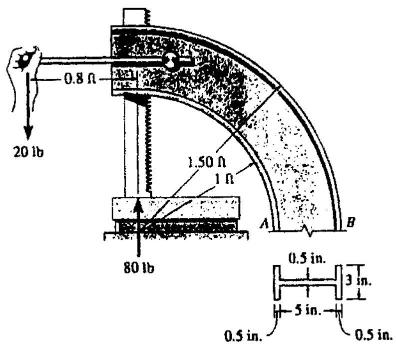
$$M = 35.3429(9) = 318.0863 \text{ kip} \cdot \text{in.}$$

$$\sigma = \frac{M(R-r)}{Ar(\bar{r}-R)} + \frac{P}{A}$$

$$(\sigma_t)_{max} = \frac{318.0863(4.48142-4)}{(1)(0.25)(4)(4.5-4.48142)} + \frac{35.3429}{(1)(0.25)} = 8.38 \text{ ksi} \qquad \textbf{Ans}$$

$$(\sigma_c)_{max} = \frac{318.0863(4.48142-5)}{1(0.25)(5)(4.5-4.48142)} + \frac{35.3429}{(1)(0.25)} = -6.96 \text{ ksi}$$

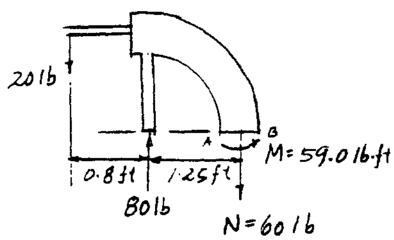
8-63 The handle of the press is subjected to a force of 20 lb. Due to internal gearing, this causes the block to be subjected to a compressive force of 80 lb. Determine the normal stress acting in the frame at points along the outside flanges A and B. Use the curved-beam formula to compute the bending stress.



Normal stress due to axial force :

$A = 2[0.5(3)] + 5(0.5) = 5.5 \text{ in}^2$

$\sigma_A = \dfrac{P}{A} = \dfrac{60}{5.5} = 10.9090 \text{ psi} \qquad \text{(T)}$



Normal stress due to bending :

$\bar{r} = 15 \text{ in.} \qquad r_A = 12 \text{ in.} \qquad r_B = 18 \text{ in.}$

$$\Sigma\int \frac{dA}{r} = \Sigma\, b \ln\frac{r_2}{r_1} = 3\ln\frac{12.5}{12} + 0.5\ln\frac{17.5}{12.5} + 3\ln\frac{18}{17.5} = 0.3752 \text{ in.}$$

$$R = \frac{A}{\int\frac{dA}{r}} = \frac{5.5}{0.3752} = 14.6583 \text{ in.}$$

$\bar{r} - R = 0.3417 \text{ in.}$

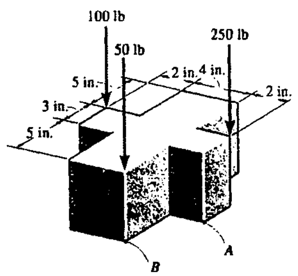
$$(\sigma_A)_b = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{59.0(12)(14.6583 - 12)}{5.5(12)(0.3417)} = 83.4468 \text{ psi (T)}$$

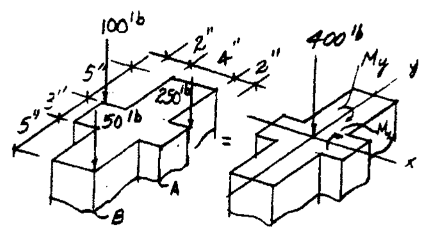
$$(\sigma_B)_b = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{59.0(12)(14.6583 - 18)}{5.5(18)(0.3417)} = -69.9342 \text{ psi} = 69.9342 \text{ psi (C)}$$

$\sigma_A = 83.4468 + 10.9090 = 94.4 \text{ psi (T)}$ **Ans**

$\sigma_B = 69.9342 - 10.9090 = 59.0 \text{ psi (C)}$ **Ans**

•8-64 The block is subjected to the three axial loads shown. Determine the normal stress developed at points A and B. Neglect the weight of the block.





$$M_x = -250(1.5) - 100(1.5) + 50(6.5) = -200 \text{ lb}\cdot\text{in.}$$

$$M_y = 250(4) + 50(2) - 100(4) = 700 \text{ lb}\cdot\text{in.}$$

$$I_x = \frac{1}{12}(4)(13^3) + 2(\frac{1}{12})(2)(3^3) = 741.33 \text{ in}^4$$

$$I_y = \frac{1}{12}(3)(8^3) + 2(\frac{1}{12})(5)(4^3) = 181.33 \text{ in}^4$$

$$A = 4(13) + 2(2)(3) = 64 \text{ in}^2$$

$$\sigma = \frac{P}{A} - \frac{M_y\, x}{I_y} + \frac{M_x\, y}{I_x}$$

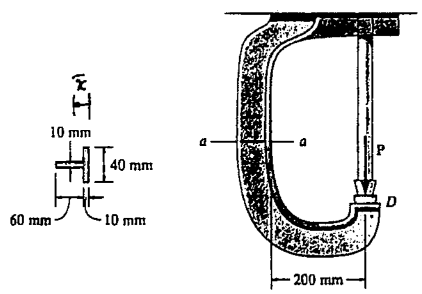
$$\sigma_A = -\frac{400}{64} - \frac{700(4)}{181.33} + \frac{-200\ (-1.5)}{741.33}$$

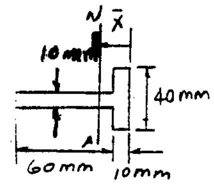
$$= -21.3 \text{ psi} \qquad \textbf{Ans}$$

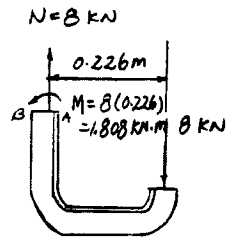
$$\sigma_B = -\frac{400}{64} - \frac{700(2)}{181.33} + \frac{-200\ (-6.5)}{741.33}$$

$$= -12.2 \text{ psi} \qquad \textbf{Ans}$$

8-65 The C-frame is used in a riveting machine. If the force at the ram on the clamp at D is $P = 8$ kN, sketch the stress distribution acting over the section a–a.







$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

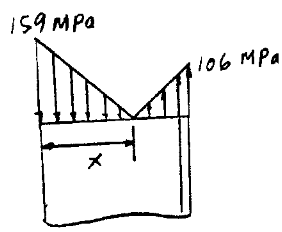
$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$

$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026-0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040-0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

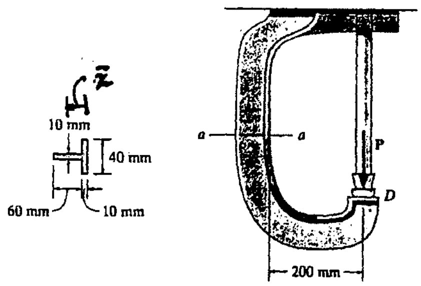
$$(\sigma_{max})_t = \frac{P}{A} + \frac{Mx}{I} = \frac{8(10^3)}{0.001} + \frac{1.808(10^3)(0.07-0.26)}{0.4773(10^{-6})} = 106.48 \text{ MPa} = 106 \text{ MPa}$$

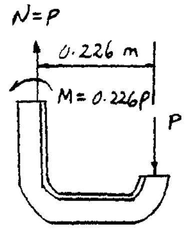
$$(\sigma_{max})_c = \frac{P}{A} - \frac{Mc}{I} = \frac{8(10^3)}{0.001} - \frac{1.808(10^3)(0.070-0.026)}{0.4773(10^{-6})} = -158.66 \text{ MPa} = -159 \text{ MPa}$$

$$\frac{x}{158.66} = \frac{70-x}{106.48}; \quad x = 41.9 \text{ mm}$$



8-66 Determine the maximum ram force P that can be applied to the clamp at D if the allowable normal stress for the material is $\sigma_{allow} = 180$ MPa.





$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(0.005)(0.04)(0.01) + 0.04(0.06)(0.01)}{0.04(0.01) + 0.06(0.01)} = 0.026 \text{ m}$$

$$A = 0.04(0.01) + 0.06(0.01) = 0.001 \text{ m}^2$$

$$I = \frac{1}{12}(0.04)(0.01^3) + (0.04)(0.01)(0.026 - 0.005)^2 + \frac{1}{12}(0.01)(0.06^3) + 0.01(0.06)(0.040 - 0.026)^2 = 0.4773(10^{-6}) \text{ m}^4$$

$$\sigma = \frac{P}{A} \pm \frac{Mx}{I}$$

Assume tension failure,

$$180(10^6) = \frac{P}{0.001} + \frac{0.226\,P(0.026)}{0.4773(10^{-6})}$$

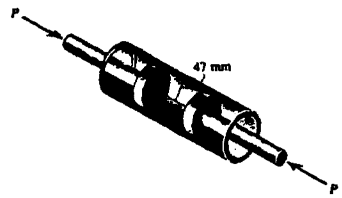
$$P = 13524 \text{ N} = 13.5 \text{ kN}$$

Assume compression failure,

$$-180(10^6) = \frac{P}{0.001} - \frac{0.226\,P(0.070 - 0.026)}{0.4773(10^{-6})}$$

$P = 9076 \text{ N} = 9.08 \text{ kN}$ (controls) **Ans**

8-67. Air pressure in the cylinder is increased by exerting forces $P = 2$ kN on the two pistons, each having a radius of 45 mm. If the cylinder has a wall thickness of 2 mm, determine the state of stress in the wall of the cylinder.



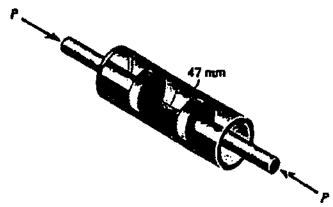
$$p = \frac{P}{A} = \frac{2(10^3)}{\pi(0.045^2)} = 314\,380.13 \text{ Pa}$$

$$\sigma_1 = \frac{p\,r}{t} = \frac{314\,380.13(0.045)}{0.002} = 7.07 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

The pressure p is supported by the surface of the pistons in the longitudinal direction.

*8-68.** Determine the maximum force P that can be exerted on each of the two pistons so that the circumferential stress component in the cylinder does not exceed 3 MPa. Each piston has a radius of 45 mm and the cylinder has a wall thickness of 2 mm.

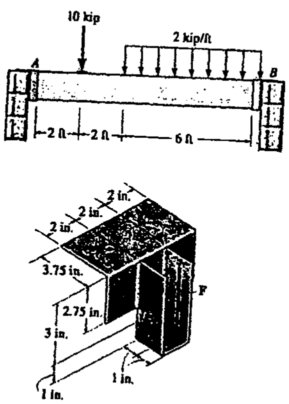


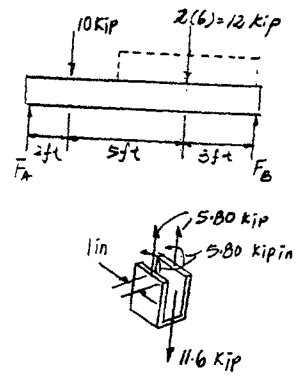
$$\sigma = \frac{p\,r}{t}; \qquad 3(10^6) = \frac{p(0.045)}{0.002}$$

$$p = 133.3 \text{ kPa} \qquad \textbf{Ans}$$

$$P = pA = 133.3(10^3)(\pi)(0.045)^2 = 848 \text{ N} \qquad \textbf{Ans}$$

8-69. The wall hanger has a thickness of 0.25 in. and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points C and D on the strap at A. Assume the vertical reaction **F** at this end acts in the center and on the edge of the bracket as shown.





$\curvearrowleft + \Sigma M_B = 0; \qquad 12(3) + 10(8) - F_A(10) = 0$

$F_A = 11.60 \text{ kip}$

$I = 2[\frac{1}{12}(0.25)(2)^3] = 0.333 \text{ in}^4$

$A = 2(0.25)(2) = 1 \text{ in}^2$

At point C,

$\sigma_C = \frac{P}{A} = \frac{2(5.80)}{1} = 11.6 \text{ ksi}$ **Ans**

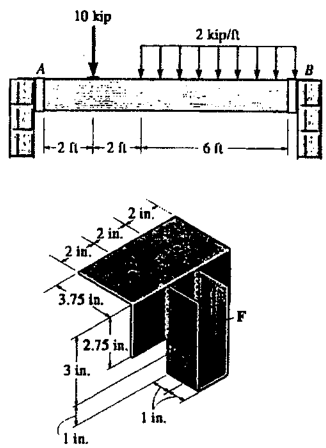
$\tau_C = 0$ **Ans**

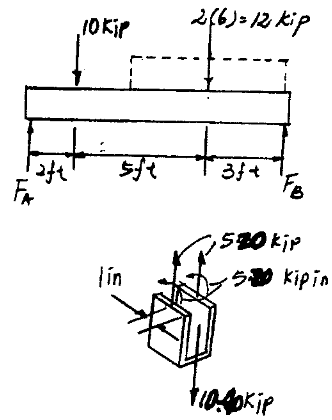
At point D,

$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2(5.80)}{1} - \frac{[2(5.80)](1)}{0.333} = -23.2 \text{ ksi}$ **Ans**

$\tau_D = 0$ **Ans**

8-70 The wall hanger has a thickness of 0.25 in. and is used to support the vertical reactions of the beam that is loaded as shown. If the load is transferred uniformly to each strap of the hanger, determine the state of stress at points C and D of the strap at B. Assume the vertical reaction F at this end acts in the center and on the edge of the bracket as shown.





$\circlearrowleft + \Sigma M_A = 0; \quad F_B(10) - 10(2) - 12(7) = 0; \quad F_B = 10.40 \text{ kip}$

$I = 2[\frac{1}{12}(0.25)(2)^3] = 0.333 \text{ in}^4; \quad A = 2(0.25)(2) = 1 \text{ in}^2$

At point C:

$\sigma_C = \frac{P}{A} = \frac{2(5.20)}{1} = 10.4 \text{ ksi}$ **Ans**

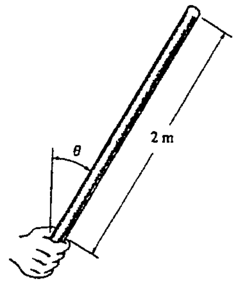
$\tau_C = 0$ **Ans**

At point D:

$\sigma_D = \frac{P}{A} - \frac{Mc}{I} = \frac{2(5.20)}{1} - \frac{[2(5.20)](1)}{0.333} = -20.8 \text{ ksi}$ **Ans**

$\tau_D = 0$ **Ans**

8–71 A bar having a square cross section of 30 mm by 30 mm is 2 m long and is held upward. If it has a mass of 5 kg/m, determine the largest angle θ, measured from the vertical, at which it can be supported before it is subjected to a tensile stress along its axis near the grip.



$$A = 0.03(0.03) = 0.9(10^{-3})\ \text{m}^2$$

$$I = \frac{1}{12}(0.03)(0.03^3) = 67.5(10^{-9})\ \text{m}^4$$

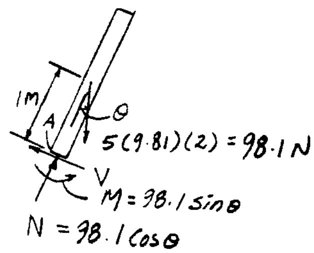
Require $\sigma_A = 0$

$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}$$

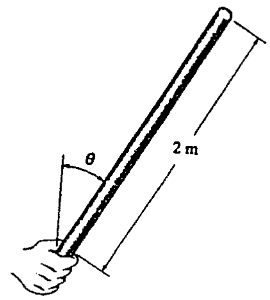
$$0 = \frac{-98.1\cos\theta}{0.9(10^{-3})} + \frac{98.1\sin\theta(0.015)}{67.5(10^{-9})}$$

$$0 = -1111.11\cos\theta + 222222.22\sin\theta$$

$\tan\theta = 0.005; \qquad \theta = 0.286°$ **Ans**



*8–72 Solve Prob. 8–71 if the bar has a circular cross section of 30-mm diameter.

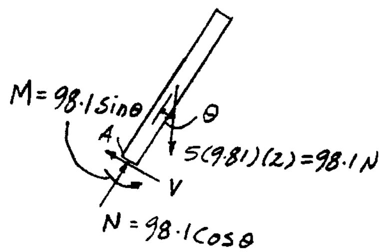


$$A = \frac{\pi}{4}(0.03^2) = 0.225\pi(10^{-3})\ \text{m}^2$$

$$I = \frac{\pi}{4}(0.015^4) = 12.65625\pi\,(10^{-9})\ \text{m}^4$$

Require $\sigma_A = 0$

$$\sigma_A = 0 = \frac{P}{A} + \frac{Mc}{I}$$



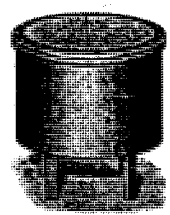
$$0 = \frac{-98.1\cos\theta}{0.225\pi(10^{-3})} + \frac{98.1\sin\theta(0.015)}{12.65625\,\pi(10^{-9})}$$

$$0 = -\,4444.44\cos\theta + 1185185.185\sin\theta$$

$$\tan\theta = 0.00375$$

$\theta = 0.215°$ **Ans**

8-73. The cap on the cylindrical tank is bolted to the tank along the flanges. The tank has an inner diameter of 1.5 m and a wall thickness of 18 mm. If the largest normal stress is not to exceed 150 MPa, determine the maximum pressure the tank can sustain. Also, compute the number of bolts required to attach the cap to the tank if each bolt has a diameter of 20 mm. The allowable stress for the bolts is $(\sigma_{\text{allow}})_b = 180$ MPa.

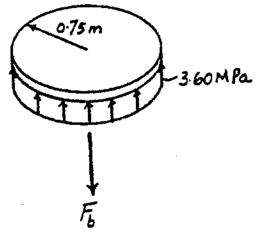


Hoop Stress for Cylindrical Tank : Since $\frac{r}{t} = \frac{750}{18}$ $= 41.6 > 10$, then *thin wall* analysis can be used. Applying Eq. 8 – 1

$$\sigma_1 = \sigma_{\text{allow}} = \frac{pr}{t}$$

$$150(10^6) = \frac{p(750)}{18}$$

$$p = 3.60 \text{ MPa} \qquad \textbf{Ans}$$



Force Equilibrium for the Cap :

$$+\uparrow \Sigma F_y = 0; \quad 3.60(10^6)\left[\pi(0.75^2)\right] - F_b = 0$$

$$F_b = 6.3617(10^6) \text{ N}$$

Allowable Normal Stress for Bolts :

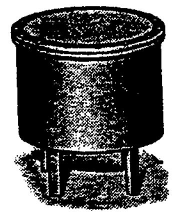
$$(\sigma_{\text{allow}})_b = \frac{P}{A}$$

$$180(10^6) = \frac{6.3617(10^6)}{n\left[\frac{\pi}{4}(0.02^2)\right]}$$

$$n = 112.5$$

Use $n = 113$ bolts $\qquad$ **Ans**

8-74. The cap on the cylindrical tank is bolted to the tank along the flanges. The tank has an inner diameter of 1.5 m and a wall thickness of 18 mm. If the pressure in the tank is $p = 1.20$ MPa, determine the force in the 16 bolts that are used to attach the cap to the tank. Also, specify the state of stress in the wall of the tank.



Hoop Stress for Cylindrical Tank : Since $\frac{r}{t} = \frac{750}{18}$ $= 41.6 > 10$, then *thin wall* analysis can be used. Applying Eq. 8 – 1

$$\sigma_1 = \frac{pr}{t} = \frac{1.20(10^6)(750)}{18} = 50.0 \text{ MPa} \qquad \textbf{Ans}$$

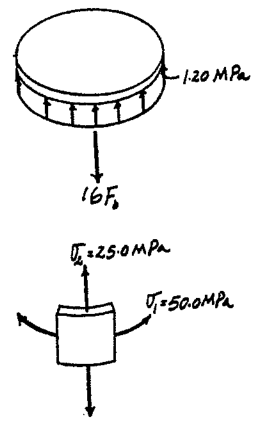
Longitudinal Stress for Cylindrical Tank :

$$\sigma_2 = \frac{pr}{2t} = \frac{1.20(10^6)(750)}{2(18)} = 25.0 \text{ MPa} \qquad \textbf{Ans}$$

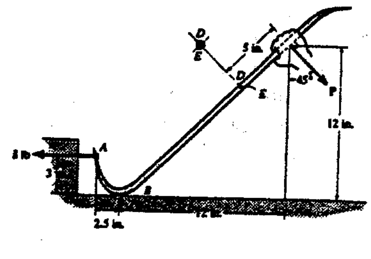
Force Equilibrium for the Cap :

$$+\uparrow \Sigma F_y = 0; \quad 1.20(10^6)\left[\pi(0.75^2)\right] - 16F_b = 0$$

$$F_b = 132536 \text{ N} = 133 \text{ kN} \qquad \textbf{Ans}$$



8-75. The crowbar is used to pull out the nail at A. If a force of 8 lb is required, determine the stress components in the bar at points D and E. Show the results on a differential volume element located at each of these points. The bar has a circular cross section with a diameter of 0.5 in. No slipping occurs at B.



Support Reactions :

$$\circlearrowleft + \Sigma M_B = 0; \quad 8(3) - P(16.97) = 0 \quad P = 1.414 \text{ lb}$$

Internal Forces and Moment :

$$+\Sigma F_{x'} = 0; \quad N = 0$$
$$+\Sigma F_{y'} = 0; \quad V - 1.414 = 0 \quad V = 1.414 \text{ lb}$$
$$\circlearrowleft + \Sigma M_O = 0; \quad M - 1.414(5) = 0 \quad M = 7.071 \text{ lb} \cdot \text{in.}$$

Section Properties :

$$A = \pi(0.25^2) = 0.0625\pi \text{ in}^2$$
$$I = \frac{\pi}{4}(0.25^4) = 0.9765625\pi(10^{-3}) \text{ in}^4$$
$$Q_D = 0$$
$$Q_E = \bar{y}'A' = \frac{4(0.25)}{3\pi}\left[\frac{1}{2}(\pi)(0.25^2)\right] = 0.0104167 \text{ m}^3$$

Normal Stress : Since $N = 0$, the normal stress is caused by bending stress only.

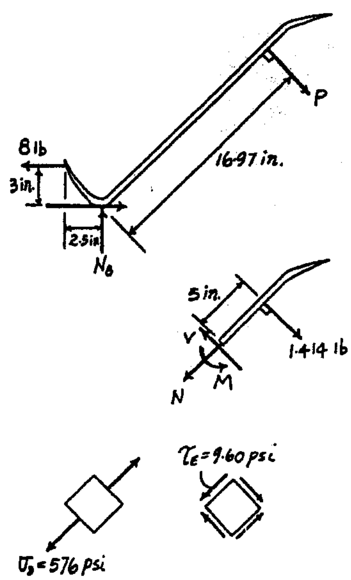
$$\sigma_D = \frac{Mc}{I} = \frac{7.071(0.25)}{0.9765625\pi(10^{-3})} = 576 \text{ psi (T)} \qquad \textbf{Ans}$$

$$\sigma_E = \frac{My}{I} = \frac{7.071(0)}{0.9765625\pi(10^{-3})} = 0 \qquad \textbf{Ans}$$

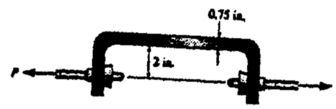
Shear Stress : Applying the shear formul,.

$$\tau_D = \frac{VQ_D}{It} = 0 \qquad \textbf{Ans}$$

$$\tau_E = \frac{VQ_E}{It} = \frac{1.414(0.0104167)}{0.9765625\pi(10^{-3})(0.5)} = 9.60 \text{ psi} \qquad \textbf{Ans}$$



*8-76.** The steel bracket is used to connect the ends of two cables. If the applied force $P = 500$ lb, determine the maximum normal stress in the bracket. The bracket has a thickness of 0.5 in. and a width of 0.75 in.

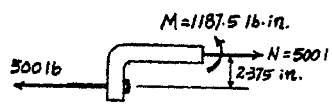


Internal Force and Moment : As shown on FBD.

Section Properties :

$$A = 0.5(0.75) = 0.375 \text{ in}^2$$
$$I = \frac{1}{12}(0.5)\left(0.75^3\right) = 0.01758 \text{ in}^4$$

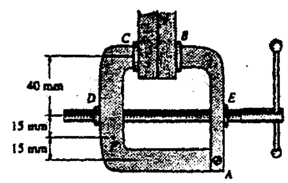
Maximum Normal Stress : The maximum normal stress occurs at the bottom of the steel bracket.

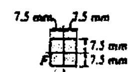


$$\sigma_{max} = \frac{N}{A} + \frac{Mc}{I}$$
$$= \frac{500}{0.375} + \frac{1187.5(0.375)}{0.01758}$$
$$= 26.7 \text{ ksi}$$

Ans

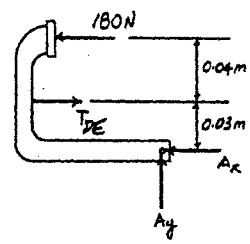
8-77. The clamp is made from members AB and AC, which are pin connected at A. If the compressive force at C and B is 180 N, determine the state of stress at point F, and indicate the results on a differential volume element. The screw DE is subjected only to a tensile force along its axis.





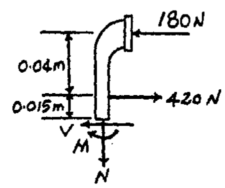
Support Reactions :

$$\curvearrowleft + \Sigma M_A = 0; \quad 180(0.07) - T_{DE}(0.03) = 0$$
$$T_{DE} = 420 \text{ N}$$



Internal Forces and Moment :

$$\xrightarrow{+} \Sigma F_x = 0; \quad 420 - 180 - V = 0 \quad V = 240 \text{ N}$$
$$+\uparrow \Sigma F_y = 0; \quad N = 0$$
$$+\Sigma M_O = 0; \quad 180(0.055) - 420(0.015) - M = 0$$
$$M = 3.60 \text{ N} \cdot \text{m}$$

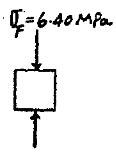


Section Properties :

$$A = 0.015(0.015) = 0.225(10^{-3}) \text{ m}^2$$
$$I = \frac{1}{12}(0.015)(0.015^3) = 4.21875(10^{-9}) \text{ m}^4$$
$$Q_F = 0$$

Normal Stress : Since $N = 0$, the normal stress is caused by bending stress only.

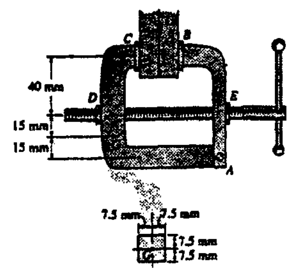
$$\sigma_F = \frac{Mc}{I} = \frac{3.60(0.0075)}{4.21875(10^{-9})} = 6.40 \text{ MPa (C)} \qquad \textbf{Ans}$$



Shear Stress : Applying shear formula, we have

$$\tau_F = \frac{VQ_F}{It} = 0 \qquad \textbf{Ans}$$

8-78. The clamp is made from members AB and AC, which are pin-connected at A. If the compressive force at C and B is 180 N, determine the state of stress at point G, and indicate the results on a differential volume element. The screw DE is subjected only to a tensile force along its axis.

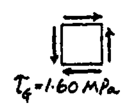


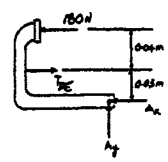
Support Reactions :

$$\circlearrowleft + \Sigma M_A = 0; \quad 180(0.07) - T_{DE}(0.03) = 0$$
$$T_{DE} = 420 \text{ N}$$

Internal Forces and Moment :

$$\xrightarrow{+} \Sigma F_x = 0; \quad 420 - 180 - V = 0 \quad V = 240 \text{ N}$$
$$+\uparrow \Sigma F_y = 0; \quad N = 0$$
$$\circlearrowleft + \Sigma M_O = 0; \quad 180(0.055) - 420(0.015) - M = 0$$
$$M = 3.60 \text{ N}\cdot\text{m}$$





Section Properties :

$$A = 0.015(0.015) = 0.225\left(10^{-3}\right) \text{ m}^2$$
$$I = \frac{1}{12}(0.015)\left(0.015^3\right) = 4.21875\left(10^{-9}\right) \text{ m}^4$$
$$Q_G = \bar{y}'A' = 0.00375(0.0075)(0.015) = 0.421875\left(10^{-6}\right) \text{ m}^3$$

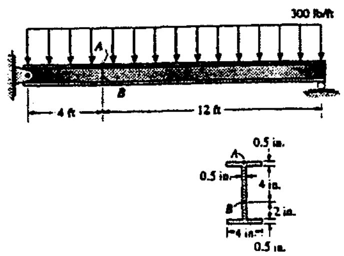
Normal Stress : Since $N = 0$, the normal stress is caused by bending stress only.

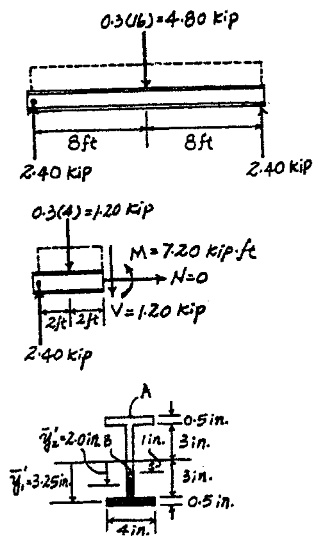
$$\sigma_G = \frac{My}{I} = \frac{3.60(0)}{4.21875(10^{-9})} = 0 \qquad \textbf{Ans}$$

Shear Stress : Applying shear formula, we have

$$\tau_G = \frac{VQ_G}{It} = \frac{240\left[0.421875(10^{-6})\right]}{4.21875(10^{-9})(0.015)} = 1.60 \text{ MPa} \qquad \textbf{Ans}$$

8-79. The wide-flange beam is subjected to the loading shown. Determine the state of stress at points A and B, and show the results on a differential volume element located at each of these points.





Support Reactions : As shown on FBD.

Internal Forces and Moment : As shown on FBD.

Section Properties :

$$A = 4(7) - 3.5(6) = 7.00 \text{ in}^2$$

$$I = \frac{1}{12}(4)(7^3) - \frac{1}{12}(3.5)(6^3) = 51.333 \text{ in}^4$$

$$Q_A = 0$$

$$Q_B = \Sigma \bar{y}'A' = 3.25(0.5)(4) + 2.00(0.5)(2) = 8.50 \text{ in}^3$$

Normal Stress : Since $N = 0$, the normal stress is contributed by bending stress only.

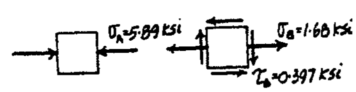
$$\sigma_A = \frac{Mc}{I} = \frac{7.20(12)(3.5)}{51.333} = 5.89 \text{ ksi (C)} \qquad \textbf{Ans}$$

$$\sigma_B = \frac{My}{I} = \frac{7.20(12)(1)}{51.333} = 1.68 \text{ ksi (T)} \qquad \textbf{Ans}$$

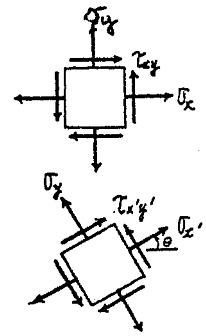
Shear Stress : Applying the shear formula.

$$\tau_A = \frac{VQ_A}{It} = 0 \qquad \textbf{Ans}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{1.20(8.50)}{51.333(0.5)} = 0.397 \text{ ksi} \qquad \textbf{Ans}$$



9–1. Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant.

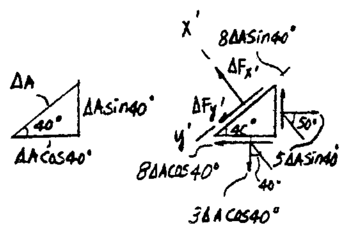


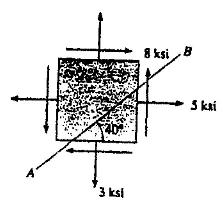
Stress Transformation Equations : Applying Eqs. 9-1 and 9-3 of the text.

$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$+ \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad (Q.E.D.)$$

9–2. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\nwarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} + (8\Delta A \sin 40°)\cos 40° - (5\Delta A \sin 40°)\cos 50° - (3\Delta A \cos 40°)\cos 40° + (8\Delta A \cos 40°)\cos 50° = 0$

$$\Delta F_{x'} = -4.052\Delta A$$

$\swarrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - (8\Delta A \sin 40°)\sin 40° - (5\Delta A \sin 40°)\sin 50° + (3\Delta A \cos 40°)\sin 40° + (8\Delta A \cos 40°)\sin 50° = 0$

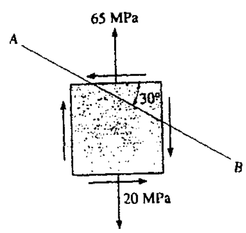
$$\Delta F_{y'} = -0.4044\Delta A$$

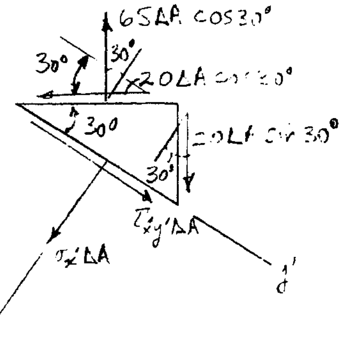
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -4.05 \text{ ksi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -0.404 \text{ ksi} \qquad \textbf{Ans}$$

The negative signs indicate that the sense of $\sigma_{x'}$ and $\tau_{x'y'}$ are opposite to that shown on FBD

9–3 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





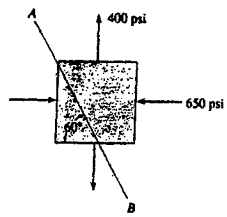
$\nearrow + \Sigma F_{x'} = 0;\qquad \sigma_{x'}\Delta A + 20\Delta A \sin 30^\circ \cos 30^\circ + 20\Delta A \cos 30^\circ \cos 60^\circ - 65\Delta A \cos 30^\circ \cos 30^\circ = 0$

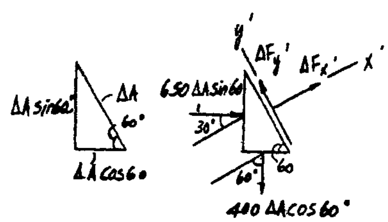
$\sigma_{x'} = 31.4\text{ MPa}$ **Ans**

$\searrow + \Sigma F_{y'} = 0;\qquad \tau_{x'y'}\Delta A + 20\Delta A \sin 30^\circ \sin 30^\circ - 20\Delta A \cos 30^\circ \sin 60^\circ - 65\,\Delta A \cos 30^\circ \sin 30^\circ = 0$

$\tau_{x'y'} = 38.1\text{ MPa}$ **Ans**

***9-4.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane *AB*. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\nearrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 400(\Delta A \cos 60°)\cos 60° + 650(\Delta A \sin 60°)\cos 30° = 0$

$$\Delta F_{x'} = -387.5 \Delta A$$

$\nwarrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 650(\Delta A \sin 60°)\sin 30° - 400(\Delta A \cos 60°)\sin 60° = 0$

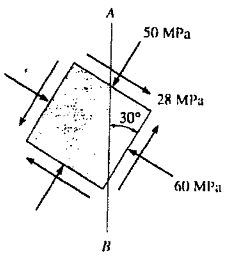
$$\Delta F_{y'} = 455\ \Delta A$$

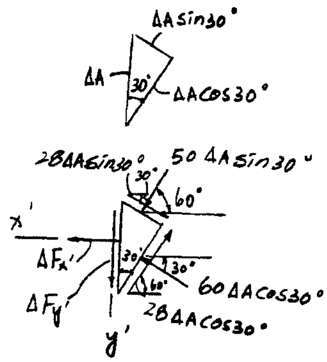
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -388 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 455 \text{ psi} \qquad \textbf{Ans}$$

The negative sign indicates that the sense of $\sigma_{x'}$ is opposite to that shown on FBD.

9–5 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\overset{+}{\leftarrow} \Sigma F_{x'} = 0; \quad \Delta F_{x'} + 60\,\Delta A \cos 30^\circ \cos 30^\circ - 28\Delta A \cos 30^\circ \cos 60^\circ$

$+ 50\Delta A \sin 30^\circ \cos 60^\circ - 28\,\Delta A \sin 30^\circ \cos 30^\circ = 0$

$\Delta F_{x'} = -33.251\,\Delta A$

$+\downarrow \Sigma F_{y'} = 0; \quad \Delta F_{y'} - 28\Delta A \cos 30^\circ \sin 60^\circ - 60\Delta A \cos 30^\circ \sin 30^\circ$

$+ 50\Delta A \sin 30^\circ \sin 60^\circ + 28\Delta A \sin 30^\circ \sin 30^\circ = 0$

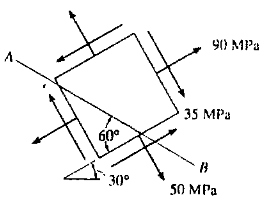
$\Delta F_{y'} = 18.33\,\Delta A$

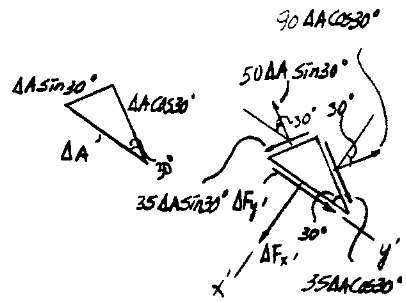
$\sigma_{x'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{x'}}{\Delta A} = -33.3\text{ MPa}$ **Ans**

$\tau_{x'y'} = \lim_{\Delta A \to 0} \dfrac{\Delta F_{y'}}{\Delta A} = 18.3\text{ MPa}$ **Ans**

The negative sign indicates that the sense of $\sigma_{x'}$ is opposite to that shown on FBD.

9–6 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\searrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 50\Delta A \sin 30^\circ \cos 30^\circ - 35\Delta A \sin 30^\circ \text{ oos } 60^\circ +$
$90\Delta A \cos 30^\circ \sin 30^\circ + 35\Delta A \cos 30^\circ \sin 60^\circ = 0$
$\Delta F_{y'} = -34.82\Delta A$

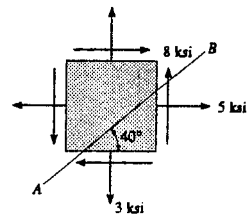
$\swarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 50\Delta A \sin 30^\circ \sin 30^\circ + 35\Delta A \sin 30^\circ \sin 60^\circ -$
$- 90\Delta A \cos 30^\circ \cos 30^\circ + 35\Delta A \cos 30^\circ \cos 60^\circ = 0$
$\Delta F_{x'} = 49.69\ \Delta A$

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = 49.7 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -34.8 \text{ MPa} \qquad \textbf{Ans}$$

The negative signs indicate that the sense of $\sigma_{x'}$ and $\tau_{x'y'}$ are opposite to that shown on FBD.

9–7 Solve Prob. 9–2 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = 5\text{ ksi} \qquad \sigma_y = 3\text{ ksi} \qquad \tau_{xy} = 8\text{ ksi} \qquad \theta = 130°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{5+3}{2} + \frac{5-3}{2}\cos 260° + 8\sin 260° = -4.05\text{ ksi} \qquad \textbf{Ans}$$

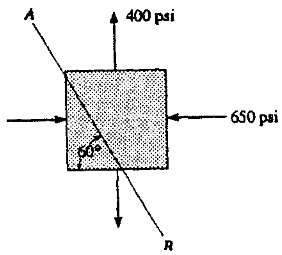
The negative sign indicates $\sigma_{x'}$ is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{5-3}{2}\right)\sin 260° + 8\cos 260° = -0.404\text{ ksi} \qquad \textbf{Ans}$$

The negative sign indicates $\tau_{x'y'}$ is in the $-y'$ direction.

*9–8 Solve Prob. 9–4 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = -650$ psi $\quad \sigma_y = 400$ psi $\quad \tau_{xy} = 0 \quad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

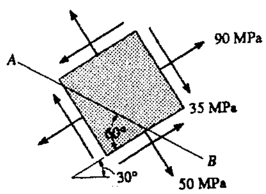
$$= \frac{-650+400}{2} + \frac{-650-400}{2}\cos 60° + 0 = -388 \text{ psi} \qquad \textbf{Ans}$$

The negative sign indicates $\sigma_{x'}$ is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-650-400}{2}\right)\sin 60° = 455 \text{ psi} \qquad \textbf{Ans}$$

9–9 Solve Prob. 9–6 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = 90\text{ MPa} \qquad \sigma_y = 50\text{ MPa} \qquad \tau_{xy} = -35\text{ MPa} \qquad \theta = -150°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{90+50}{2} + \frac{90-50}{2}\cos(-300°) + (-35)\sin(-300°)$$

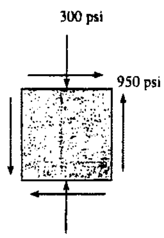
$$= 49.7\text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -(\frac{90-50}{2})\sin(-300°) + (-35)\cos(-300°) = -34.8\text{ MPa} \qquad \textbf{Ans}$$

The negative sign indicates $\tau_{x'y'}$ acts in $-y'$ direction.

9–10 Determine the equivalent state of stress on an element if the element is oriented 30° clockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = 0 \qquad \sigma_y = -300 \text{ psi} \qquad \tau_{xy} = 950 \text{ psi} \qquad \theta = -30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

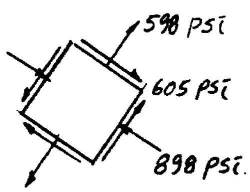
$$= \frac{0-300}{2} + \frac{0-(-300)}{2}\cos(-60°) + 950\sin(-60) = -898 \text{ psi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

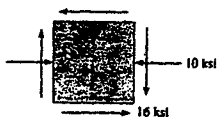
$$= -\left(\frac{0-(-300)}{2}\right)\sin(-60°) + 950\cos(-60°) = 605 \text{ psi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{0-300}{2} - \left(\frac{0-(-300)}{2}\right)\cos(-60°) - 950\sin(-60°) = 598 \text{ psi} \qquad \textbf{Ans}$$



9–11. Determine the equivalent state of stress on an element if it is oriented 50° counterclockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = -10\text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16\text{ ksi}$

$\theta = +50°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

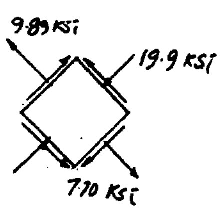
$$= \frac{-10+0}{2} + \frac{-10-0}{2}\cos 100° + (-16)\sin 100° = -19.9\text{ ksi} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

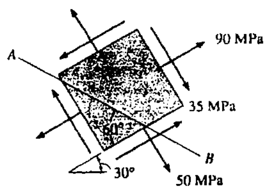
$$= -\left(\frac{-10-0}{2}\right)\sin 100° + (-16)\cos 100° = 7.70\text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-10+0}{2} - \left(\frac{-10-0}{2}\right)\cos 100° - (-16)\sin 100° = 9.89\text{ ksi} \qquad \textbf{Ans}$$



•9–12 Solve Prob. 9–6 using the stress-transformation equations.



$\theta = 120^\circ \qquad \sigma_x = 50 \text{ MPa} \qquad \sigma_y = 90 \text{ MPa} \qquad \tau_{xy} = 35 \text{ MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{50 + 90}{2} + \frac{50 - 90}{2}\cos 240^\circ + (35)\sin 240^\circ$$

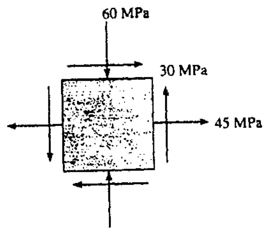
$$= 49.7 \text{ MPa}$$ **Ans**

The negative sign indicates $\sigma_{x'}$ is a compressive stress

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{50 - 90}{2}\sin 240^\circ + (35)\cos 240^\circ = -34.8 \text{ MPa}$$ **Ans**

9–13 The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



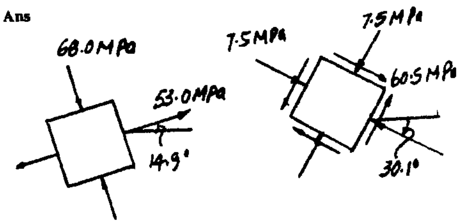
$\sigma_x = 45$ MPa $\quad \sigma_y = -60$ MPa $\quad \tau_{xy} = 30$ MPa

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

$\sigma_1 = 53.0$ MPa **Ans** $\qquad \sigma_2 = -68.0$ MPa **Ans**

Orientation of principal stress :



$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$\theta_p = 14.87, \quad -75.13$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2 :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \quad \text{where } \theta = 14.87^\circ$$

$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74^\circ + 30 \sin 29.74^\circ = 53.0 \text{ MPa}$$

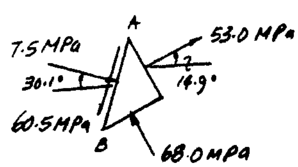
Therefore $\theta_{p1} = 14.9^\circ$ **Ans** and $\theta_{p2} = -75.1^\circ$ **Ans**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5 \text{ MPa} \quad \textbf{Ans}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa} \quad \textbf{Ans}$$

Orientation of maximum in - plane shear stress :

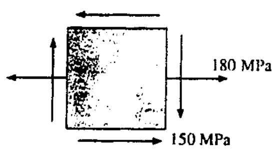


$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

$\theta_s = -30.1^\circ$ **Ans** and $\theta_s = 59.9^\circ$ **Ans**

By observation, in order to preserve equilibrium along AB, $\tau_{\max}$ has to act in the direction shown.

9–14 The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

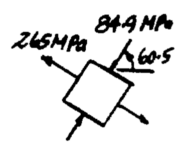


$\sigma_x = 180\text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -150\text{ MPa}$

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{180+0}{2} \pm \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2}$$

$\sigma_1 = 265\text{ MPa}$ **Ans** $\qquad \sigma_2 = -84.9\text{ MPa}$ **Ans**



Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-150}{(180-0)/2} = -1.6667$$

$\theta_p = 60.482°$ and $-29.518°$

Use Eq. 9 - 1 to determine the pricipal plane of σ_1 and σ_2 :

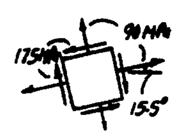
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 60.482°$$

$$= \frac{180+0}{2} + \frac{180-0}{2}\cos 2(60.482°) + (-150)\sin 2(60.482°) = -84.9\text{ MPa}$$

Therefore $\theta_{p1} = 60.5°$ **Ans** and $\theta_{p2} = -29.5°$ **Ans**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{180-0}{2}\right)^2 + (-150)^2} = 175\text{ MPa} \quad \textbf{Ans}$$



$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{180+0}{2} = 90.0\text{ MPa} \quad \textbf{Ans}$$

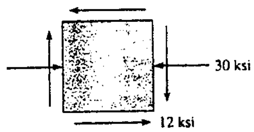
Orientation of maximum in - plane shear stress :

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(180-0)/2}{-150} = 0.6$$

$\theta_s = 15.5°$ **Ans** and $\theta = -74.5°$ **Ans**

By observation, in order to preserve equilibrium along *AB*, $\tau_{\max}$ has to act in the direction shown.

9-15 The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.



$\sigma_x = -30$ ksi $\qquad \sigma_y = 0 \qquad \tau_{xy} = -12$ ksi

a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{-30+0}{2} \pm \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2}$$

$\sigma_1 = 4.21$ ksi **Ans** $\qquad \sigma_2 = -34.2$ ksi **Ans**

Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$\theta_p = 19.33°$ and $-70.67°$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = 19.33°$

$$\sigma_{x'} = \frac{-30+0}{2} + \frac{-30-0}{2}\cos 2(19.33°) + (-12)\sin 2(19.33°) = -34.2 \text{ ksi}$$

Therefore $\theta_{p_2} = 19.3°$ **Ans** and $\theta_{p_1} = -70.7°$ **Ans**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-30-0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi} \quad \textbf{Ans}$$

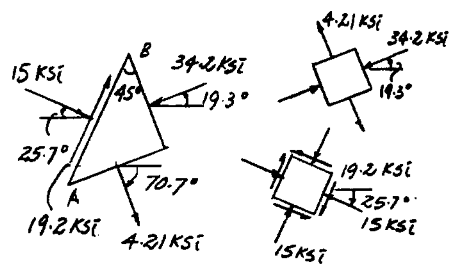
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30+0}{2} = -15 \text{ ksi} \quad \textbf{Ans}$$

Orientation of max. in - plane shear stress :

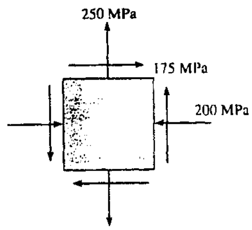
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30-0)/2}{-12} = -1.25$$

$\theta_s = -25.7°$ and $64.3°$ **Ans**

By observation, in order to preserve equilibrium along *AB*, τ_{max} has to act in the direction shown in the figure.



*9–16 The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

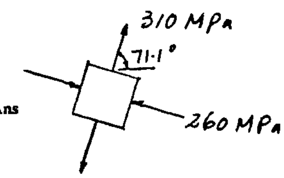


a) $\sigma_x = -200\text{ MPa}$ $\quad\sigma_y = 250\text{ MPa}$ $\quad\tau_{xy} = 175\text{ MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-200 + 250}{2} \pm \sqrt{\left(\frac{-200 - 250}{2}\right)^2 + 175^2}$$

$\sigma_1 = 310\text{ MPa}$ $\qquad\sigma_2 = -260\text{ MPa}$ $\qquad$ **Ans**



Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{175}{\frac{-200-250}{2}} = -0.7777$$

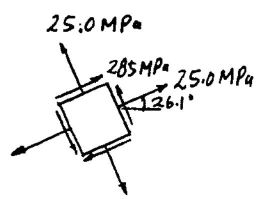
$\theta_p = -18.94°$ and $71.06°$

Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = -18.94°$

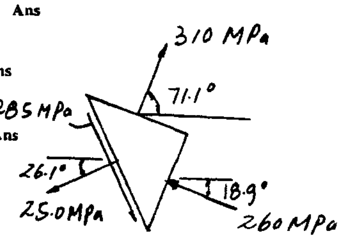
$$\sigma_{x'} = \frac{-200 + 250}{2} + \frac{-200 - 250}{2}\cos(-37.88°) + 175\sin(-37.88°) = -260\text{ MPa} = \sigma_2$$



Therefore $\theta_{p_1} = 71.1°$ $\qquad\theta_{p_2} = -18.9°$ $\qquad$ **Ans**

b) $\tau_{\substack{max \\ in\ plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-200 - 250}{2}\right)^2 + 175^2} = 285\text{ MPa}$ $\quad$ **Ans**

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-200 + 250}{2} = 25.0\text{ MPa}$ $\quad$ **Ans**



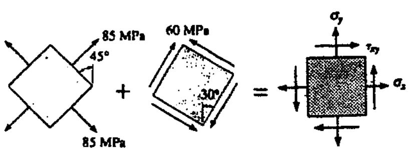
Orientation of maximum in - plane shear stress :

$$\tan 2\theta_s = -\frac{\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}} = -\frac{\frac{-200-250}{2}}{175} = 1.2857$$

$\theta_s = 26.1°$ **Ans** and $-63.9°$ $\qquad$ **Ans**

By observation, in order to preserve equilibrium, $\tau_{max} = 285\text{ MPa}$ has to act in the direction shown in the figure.

9-17. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.



For element *a* :

$\sigma_x = \sigma_y = 85\text{ MPa} \quad \tau_{xy} = 0 \quad \theta = -45°$

$$(\sigma_{x'})_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= \frac{85 + 85}{2} + \frac{85 - 85}{2}\cos(-90°) + 0 = 85\text{ MPa}$$

$$(\sigma_{y'})_a = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$= \frac{85 + 85}{2} - \frac{85 - 85}{2}\cos(-90°) - 0 = 85\text{ MPa}$$

$$(\tau_{x'y'})_a = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= -\frac{85 - 85}{2}\sin(-90°) + 0 = 0$$

For element *b* :

$\sigma_x = \sigma_y = 0 \quad \tau_{xy} = 60\text{ MPa} \quad \theta = -60°$

$$(\sigma_{x'})_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$= 0 + 0 + 60\sin(-120°) = -51.96\text{ MPa}$$

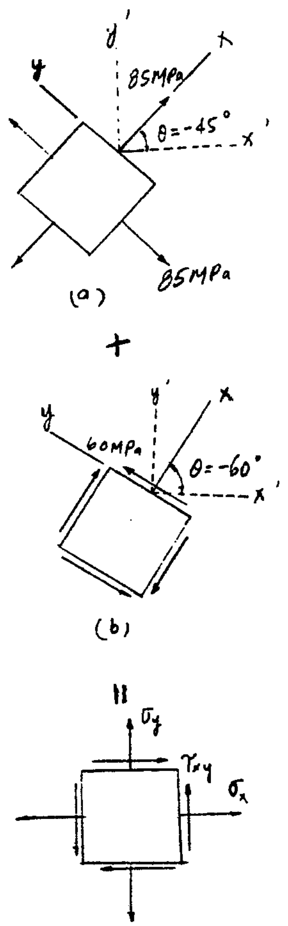
$$(\sigma_{y'})_b = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$= 0 - 0 - 60\sin(-120°) = 51.96\text{ MPa}$$

$$(\tau_{x'y'})_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= -\frac{85 - 85}{2}\sin(-120°) + 60\cos(-120°) = -30\text{ MPa}$$

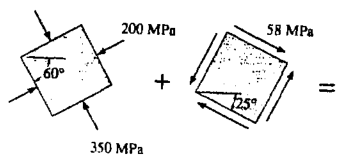
$\sigma_x = (\sigma_{x'})_a + (\sigma_{x'})_b = 85 + (-51.96) = 33.0\text{ MPa}$ **Ans**

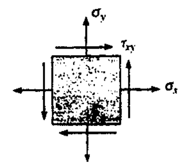
$\sigma_y = (\sigma_{y'})_a + (\sigma_{y'})_b = 85 + 51.96 = 137\text{ MPa}$ **Ans**

$\tau_{xy} = (\tau_{x'y'})_a + (\tau_{x'y'})_b = 0 + (-30) = -30\text{ MPa}$ **Ans**



9-18 A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.





For element *a* :

$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$-200 = 0.5(\sigma_x)_a + 0.5(\sigma_y)_a + 0.25(\sigma_x)_a - 0.25(\sigma_y)_a + 0.8660(\tau_{xy})_a$

$-200 = 0.75(\sigma_x)_a + 0.25(\sigma_y)_a + 0.866(\tau_{xy})_a$ (1)

$-350 = 0.5(\sigma_x)_a + 0.5(\sigma_y)_a + 0.25(\sigma_y)_a - 0.25(\sigma_x)_a - 0.8660(\tau_{xy})_a$

$-350 = 0.25(\sigma_x)_a + 0.75(\sigma_y)_a - 0.866(\tau_{xy})_a$ (2)

$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

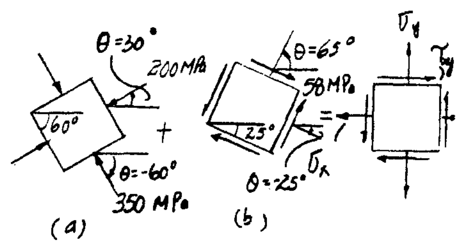
$0 = -0.4330(\sigma_x)_a + 0.4330(\sigma_y)_a + 0.5(\tau_{xy})_a$ (3)

Solving Eqs. (1), (2), and (3) yields :

$(\sigma_x)_a = -237.5$ MPa

$(\sigma_y)_a = -312.5$ MPa

$(\tau_{xy})_a = 64.95$ MPa



For element *b* :

$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$\sigma_{x'} = 0 \qquad \theta = -25°$

$0 = 0.5(\sigma_x)_b + 0.5(\sigma_y)_b + 0.3214(\sigma_x)_b - 0.3214(\sigma_y)_b - 0.7660(\tau_{xy})_b$

$0 = 0.8214(\sigma_x)_b + 0.1768(\sigma_y)_b - 0.7660(\tau_{xy})_b$ (4)

$\sigma_{x'} = 0 \qquad \theta = 65°$

$0 = 0.5(\sigma_x)_b + 0.5(\sigma_y)_b - 0.3214(\sigma_x)_b + 0.3214(\sigma_y)_b + 0.766(\tau_{xy})_b$

$0 = 0.1786(\sigma_x)_b + 0.8214(\sigma_y)_b + 0.766(\tau_{xy})_b$ (5)

$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$\theta = -25° \qquad \tau_{x'y'} = 58$ MPa

$58 = 0.3830(\sigma_x)_b - 0.3830(\sigma_y)_b + 0.6428(\tau_{xy})_b$ (6)

Solving Eqs. (4), (5), and (6) yields :

$(\sigma_x)_b = 44.43$ MPa

$(\sigma_y)_b = -44.43$ MPa

$(\tau_{xy})_b = 37.28$ MPa

$\sigma_x = (\sigma_x)_a + (\sigma_x)_b$

$= -237.5 + 44.43 = -193$ MPa **Ans**

$\sigma_y = (\sigma_y)_a + (\sigma_y)_b$

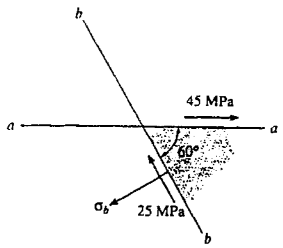
$= -312.5 - 44.43 = -357$ MPa **Ans**

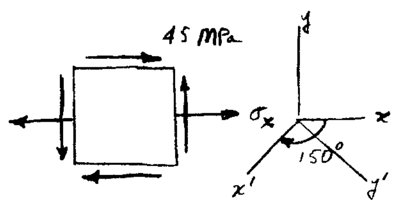
$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b$

$= 64.95 + 37.28 = 102$ MPa **Ans**

Note : This problem can also be solved by using $\sigma_x = -200$ MPa
$\sigma_y = -350$ MPa, $\tau_{xy} = 0$, and $\theta = -30°$
for element *a*, and $\sigma_x = 0$, $\sigma_y = 0$, $\tau_{xy} = 58$ MPa and
$\theta = 25°$ for element *b*.

9–19 The stress along two planes at a point is indicated. Determine the normal stresses on plane *b-b* and the principal stresses.





$$\tau_b = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-25 = -\frac{(\sigma_x - 0)}{2}\sin(-300^\circ) + 45\cos(-300^\circ)$$

$$\sigma_x = 109.70\ \text{MPa}$$

$$\sigma_b = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.70 + 0}{2} + \frac{109.70 - 0}{2}\cos(-300^\circ) + 45\sin(-300^\circ)$$

$\sigma_b = 121$ MPa **Ans**

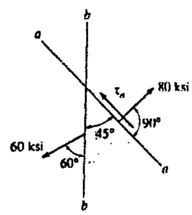
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

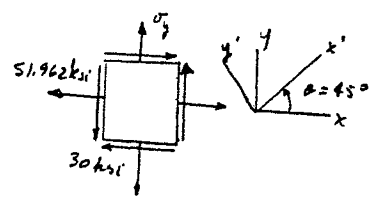
$$= \frac{109.70 + 0}{2} \pm \sqrt{\left(\frac{109.70 - 0}{2}\right)^2 + (45)^2}$$

$\sigma_1 = 126$ MPa **Ans**

$\sigma_2 = -16.1$ MPa **Ans**

***9-20.** The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a-a* and the principal stresses at the point.





$$\sigma_x = 60 \sin 60° = 51.962 \text{ ksi}$$

$$\tau_{xy} = 60 \cos 60° = 30 \text{ ksi}$$

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2} \cos(90°) + 30 \sin(90°)$$

$$\sigma_y = 48.038 \text{ ksi}$$

$$\tau_a = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos \theta$$

$$= -\left(\frac{51.962 - 48.038}{2}\right) \sin(90°) + 30 \cos(90°)$$

$\tau_a = -1.96$ ksi **Ans**

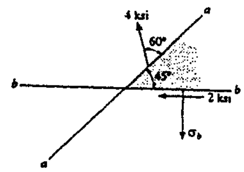
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{51.962 + 48.038}{2} \pm \sqrt{\left(\frac{51.962 - 48.038}{2}\right)^2 + (30)^2}$$

$\sigma_1 = 80.1$ ksi **Ans**

$\sigma_2 = 19.9$ ksi **Ans**

9-21. The stress acting on two planes at a point is indicated. Determine the normal stress σ_b and the principal stresses at the point.



Stress Transformation Equations : Applying Eqs. 9 – 3 and 9 – 1 with $\theta = -135°$, $\sigma_y = 3.464$ ksi, $\tau_{xy} = 2.00$ ksi, $\tau_{x'y'} = -2$ ksi, and $\sigma_{x'} = \sigma_b$.,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

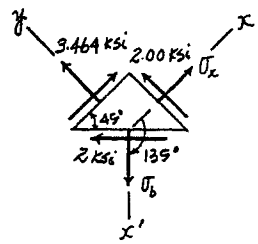
$$-2 = -\frac{\sigma_x - 3.464}{2}\sin(-270°) + 2\cos(-270°)$$

$$\sigma_x = 7.464 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_b = \frac{7.464 + 3.464}{2} + \frac{7.464 - 3.464}{2}\cos(-270°) + 2\sin(-270°)$$

$$= 7.46 \text{ ksi} \qquad \textbf{Ans}$$



In - Plane Principal Stress : Applying Eq.9 – 5,

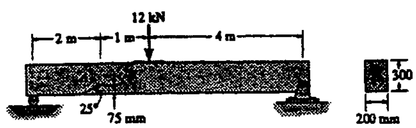
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

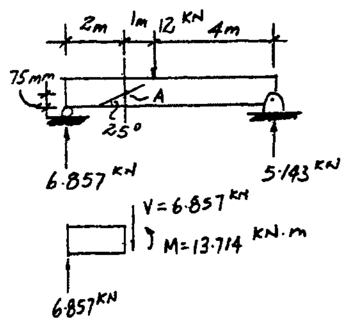
$$= \frac{7.464 + 3.464}{2} \pm \sqrt{\left(\frac{7.464 - 3.464}{2}\right)^2 + 2^2}$$

$$= 5.464 \pm 2.828$$

$$\sigma_1 = 8.29 \text{ ksi} \qquad \sigma_2 = 2.64 \text{ ksi} \qquad \textbf{Ans}$$

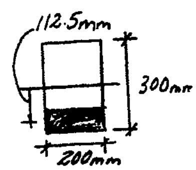
9-22. The wood beam is subjected to a load of 12 kN. If grains of wood in the beam at point A make an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular to the grains due to the loading.





$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$



$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa (T)}$$

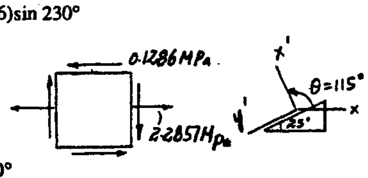
$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$

$\sigma_x = 2.2857$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -0.1286$ MPa $\quad \theta = 115°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} = \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos 230° + (-0.1286)\sin 230°$$

$= 0.507$ MPa **Ans**

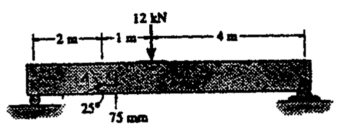


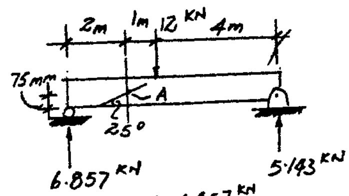
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

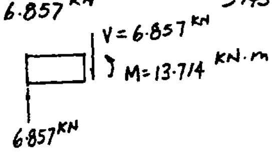
$$= -\left(\frac{2.2857 - 0}{2}\right)\sin 230° + (-0.1286)\cos 230°$$

$= 0.958$ MPa **Ans**

9-23. The wood beam is subjected to a distributed loading. Determine the principal stresses at point A and specify the orientation of the element.



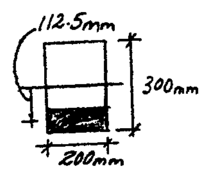




$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa (T)}$$



$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$

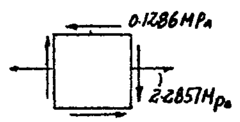
$\sigma_x = 2.2857$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2.2857 + 0}{2} \pm \sqrt{\left(\frac{2.2857 - 0}{2}\right)^2 + (-0.1286)^2}$$

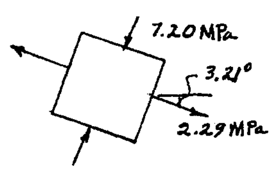
$\sigma_1 = 2.29$ MPa **Ans**

$\sigma_2 = -7.20$ kPa **Ans**



$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.1286}{(2.2857 - 0)/2}$$

$\theta_p = -3.21°$



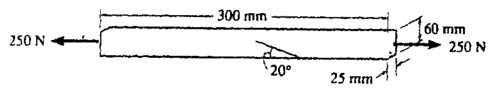
Check direction of principal stress :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos(-6.42°) - 0.1285\sin(-6.42)$$

$$= 2.29\ \text{MPa}$$

*9-24. The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stress that act perpendicular to the grains if the board is subjected to an axial load of 250 N.



$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$

$\sigma_y = 0 \qquad \tau_{xy} = 0$

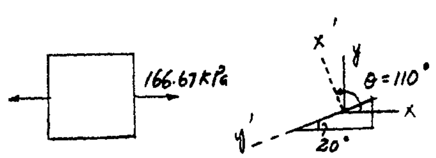
$\theta = 110°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

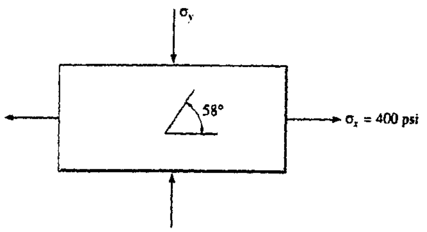
$$= \frac{166.67 + 0}{2} + \frac{166.67 - 0}{2}\cos 220° + 0 = 19.5 \text{ kPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{166.67 - 0}{2}\right)\sin 220° + 0 = 53.6 \text{ kPa} \qquad \textbf{Ans}$$



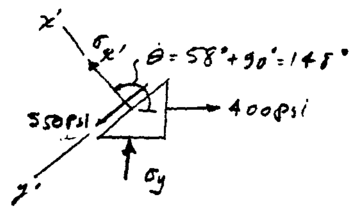
9–25 The wooden block will fail if the shear stress acting along the grain is 550 psi. If the normal stress $\sigma_x = 400$ psi, determine the necessary compressive stress σ_y that will cause failure.



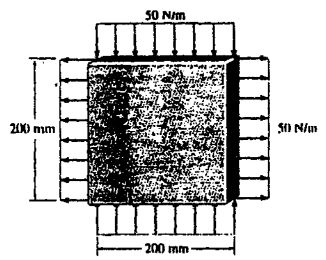
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$550 = -\left(\frac{400 - \sigma_y}{2}\right)\sin 296° + 0$$

$\sigma_y = -824$ psi **Ans**



9-26. The square steel plate has a thickness of 10 mm and is subjected to the edge loading shown. Determine the maximum in-plane shear stress and the average normal stress developed in the steel.



$\sigma_x = 5\text{ kPa} \qquad \sigma_y = -5\text{ kPa} \qquad \tau_{xy} = 0$

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{5+5}{2}\right)^2 + 0} = 5\text{ kPa} \qquad \textbf{Ans}$$

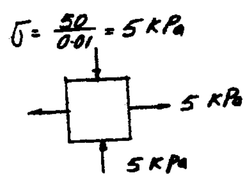
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{5-5}{2} = 0 \qquad \textbf{Ans}$$

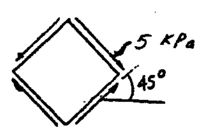
Note:

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

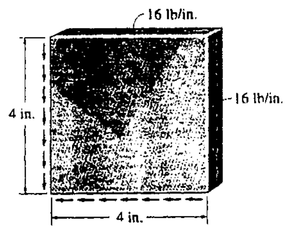
$$\tan 2\theta_s = \frac{-(5+5)/2}{0} = \infty$$

$$\theta_s = 45^0$$

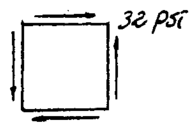




9-27 The square steel plate has a thickness of 0.5 in. and is subjected to the edge loading shown. Determine the principal stresses developed in the steel.



$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 32 \text{ psi}$

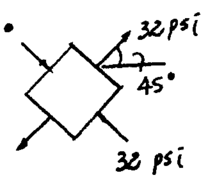


$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + 32^2}$$

$\sigma_1 = 32 \text{ psi}$ **Ans**

$\sigma_2 = -32 \text{ psi}$ **Ans**

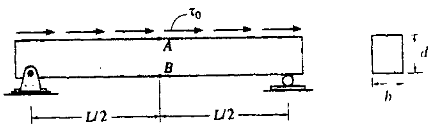


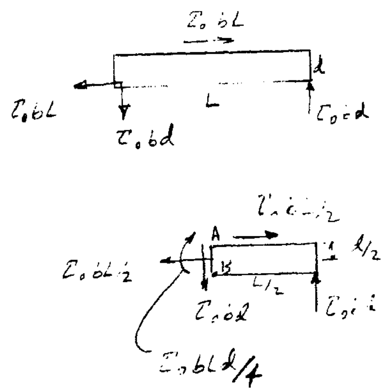
Note :

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{32}{0} = \infty$$

$$\theta_p = 45^0$$

•9–28 The simply supported beam is subjected to the traction stress τ_0 on its top surface. Determine the principal stresses at points A and B.

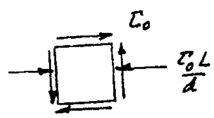




Point A:

$$\sigma_A = -\frac{Mc}{I} + \frac{P}{A} = -\frac{(\tau_0 bLd/4)(d/2)}{\frac{1}{12}(b)(d)^3} + \frac{\tau_0 bL/2}{bd} = -\frac{\tau_0 L}{d}$$

$$\tau_A = \tau_0$$



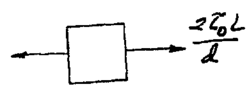
Thus,

$$\sigma_{1,2} = \frac{-\tau_0 L}{2d} \pm \sqrt{(\frac{\tau_0 L}{2d})^2 + \tau_0^2}$$

$$\sigma_{1,2} = \frac{-\tau_0 L}{2d} \pm \tau_0\sqrt{(\frac{L}{2d})^2 + 1} \qquad \textbf{Ans}$$

Point B:

$$\sigma_B = \frac{Mc}{I} + \frac{P}{A} = \frac{(\tau_0 bLd/4)(d/2)}{\frac{1}{12}bd^3} + \frac{\tau_0 bL/2}{bd} = \frac{2\tau_0 L}{d}$$

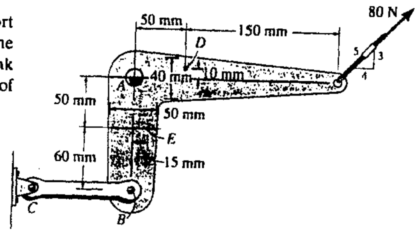


$$\tau_B = 0$$

$$\sigma_1 = \frac{2\tau_0 L}{d} \qquad \textbf{Ans}$$

$$\sigma_2 = 0 \qquad \textbf{Ans}$$

9–29 The bell crank is pinned at A and supported by a short link BC. If it is subjected to the force of 80 N, determine the principal stresses at (*a*) point D and (*b*) point E. The crank is constructed from an aluminum plate having a thickness of 20 mm.

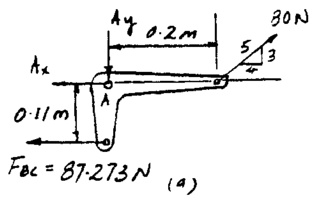


Point D :

$A = 0.04(0.02) = 0.8(10^{-3})\ \text{m}^2$

$I = \frac{1}{12}(0.02)(0.04^3) = 0.1067(10^{-6})\ \text{m}^4$

$Q_D = \bar{y}A' = 0.015(0.02)(0.01) = 3(10^{-6})\ \text{m}^3$

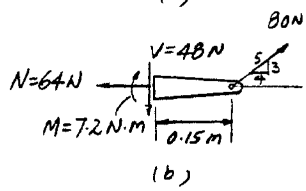


Normal stress :

$$\sigma_D = \frac{P}{A} + \frac{My}{I} = \frac{64}{0.8(10^{-3})} - \frac{7.2(0.01)}{0.1067(10^{-6})} = -0.595\ \text{MPa}$$

Shear stress :

$$\tau_D = \frac{VQ}{It} = \frac{48(3)(10^{-6})}{0.1067(10^{-6})(0.02)} = 0.0675\ \text{MPa}$$



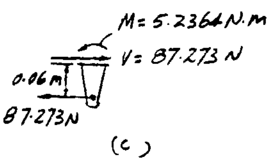
Principal stress : $\sigma_x = -0.595$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 0.0675$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-0.595 + 0}{2} \pm \sqrt{\left(\frac{-0.595 - 0}{2}\right)^2 + 0.0675^2}$$

$\sigma_1 = 7.56$ kPa **Ans**

$\sigma_2 = -603$ kPa **Ans**



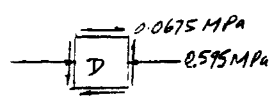
Point E :

$I = \frac{1}{12}(0.02)(0.05^3) = 0.2083(10^{-6})\ \text{m}^4$

$Q_E = \bar{y}A' = 0.02(0.01)(0.02) = 4.0(10^{-6})\ \text{m}^3$

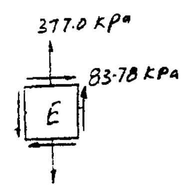
Normal stress :

$$\sigma_E = \frac{My}{I} = \frac{5.2364(0.015)}{0.2083(10^{-6})} = 377.0\ \text{kPa}$$



Shear stress :

$$\tau_E = \frac{VQ}{It} = \frac{87.273(4.0)(10^{-6})}{0.2083(10^{-6})(0.02)} = 83.78\ \text{kPa}$$



Principal stress : $\sigma_x = 0 \quad \sigma_y = 377.0$ kPa $\quad \tau_{xy} = 83.78$ kPa

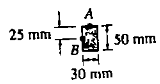
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

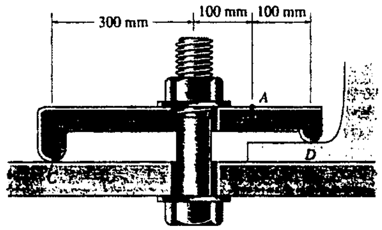
$$= \frac{0 + 377.0}{2} \pm \sqrt{\left(\frac{0 - 377.0}{2}\right)^2 + 83.78^2}$$

$\sigma_1 = 395$ kPa **Ans**

$\sigma_2 = -17.8$ kPa **Ans**

9–30 The clamp bears down on the smooth surfaces at C and D when the bolt is tightened. If the tensile force in the bolt is 40 kN, determine the principal stresses at points A and B and show the results on elements located at each of these points. The cross-sectional area at A and B is shown in the adjacent figure.

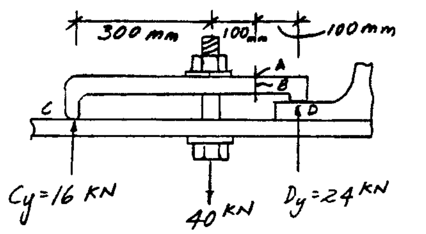




$$I = \frac{1}{12}(0.03)(0.05^3) = 0.3125(10^{-6})\ \text{m}^4$$

$$Q_A = 0$$

$$Q_B = (0.0125)(0.025)(0.03) = 9.375(10^{-6})\ \text{m}^3$$



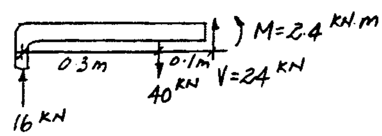
Point A:

$$\sigma_A = \frac{-Mc}{I} = \frac{-2.4(10^3)(0.025)}{0.3125(10^{-6})} = -192\ \text{MPa}$$

Here,

$\sigma_1 = 0$ **Ans**

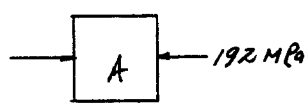
$\sigma_2 = -192$ MPa **Ans**



Point B:

$$\sigma_B = \frac{My}{I} = 0$$

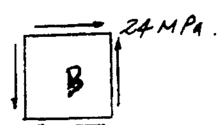
$$\tau_B = \frac{VQ_B}{It} = \frac{24(10^3)(9.375)(10^{-6})}{0.3125(10^{-6})(0.03)} = 24.0\ \text{MPa}$$



$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -24$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + (24)^2}$$

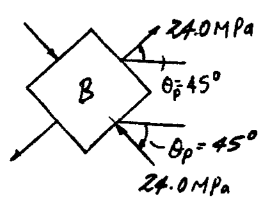


$\sigma_1 = 24.0$ MPa **Ans**

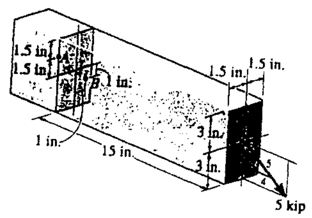
$\sigma_2 = -24.0$ MPa **Ans**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{24}{0} = \infty$$

$\theta_p = 45°$ and $\theta_p = -45°$

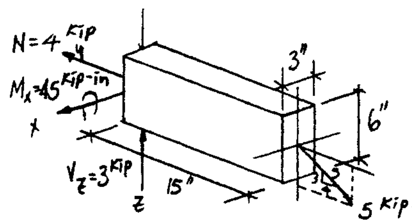


9–31 The cantilevered rectangular bar is subjected to the force of 5 kip. Determine the principal stresses at points *A* and *B*.



$I = \frac{1}{12}(3)(6^3) = 54\ \text{in}^4 \qquad A = (6)(3) = 18\ \text{in}^2$

$Q_A = 2.25(1.5)(3) = 10.125\ \text{in}^3 \qquad Q_B = 2(2)(3) = 12\ \text{in}^3$



Point *A* :

$$\sigma_A = \frac{P}{A} + \frac{M_x z}{I} = \frac{4}{18} + \frac{45(1.5)}{54} = 1.472\ \text{ksi}$$

$$\tau_A = \frac{V_z Q_A}{It} = \frac{3(10.125)}{54(3)} = 0.1875\ \text{ksi}$$

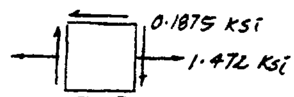
$\sigma_x = 1.472\ \text{ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.1875\ \text{ksi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{1.472 + 0}{2} \pm \sqrt{\left(\frac{1.472 - 0}{2}\right)^2 + 0.1875^2}$$

$\sigma_1 = 1.50\ \text{ksi}$ **Ans**

$\sigma_2 = -0.0235\ \text{ksi}$ **Ans**



Point *B* :

$$\sigma_B = \frac{P}{A} - \frac{M_x z}{I} = \frac{4}{18} - \frac{45(1)}{54} = -0.6111\ \text{ksi}$$

$$\tau_B = \frac{V_z Q_B}{It} = \frac{3(12)}{54(3)} = 0.2222\ \text{ksi}$$

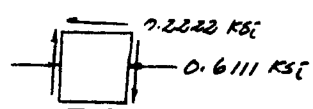
$\sigma_x = -0.6111\ \text{ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.2222\ \text{ksi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

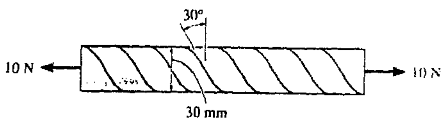
$$= \frac{-0.611 + 0}{2} \pm \sqrt{\left(\frac{-0.6111 - 0}{2}\right)^2 + 0.222^2}$$

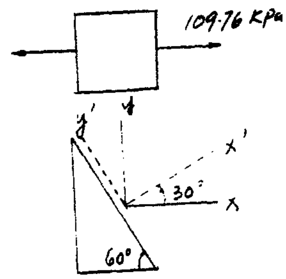
$\sigma_1 = 0.0723\ \text{ksi}$ **Ans**

$\sigma_2 = -0.683\ \text{ksi}$ **Ans**



•9–32 A paper tube is formed by rolling a paper strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 30° from the vertical, when the tube is subjected to an axial force of 10 N. The paper is 1 mm thick and the tube has an outer diameter of 30 mm.





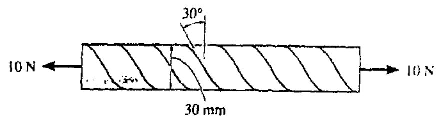
$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$\sigma_x = 109.76 \text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0 \qquad \theta = 30°$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{109.76 - 0}{2}\sin 60° + 0 = -47.5 \text{ kPa}$$ **Ans**

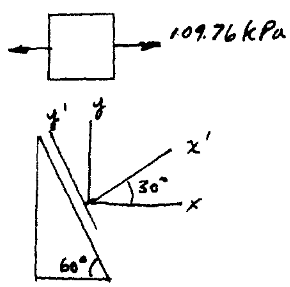
9–33 Solve Prob. 9–32 for the normal stress acting perpendicular to the seam.



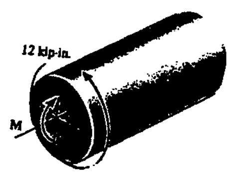
$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{109.76 + 0}{2} + \frac{109.76 - 0}{2}\cos(60°) + 0 = 82.3 \text{ kPa}$$ **Ans**



9-34. A rod has a circular cross section with a diameter of 2 in. It is subjected to a torque of 12 kip · in. and a bending moment **M**. The greater principal stress at the point of maximum flexural stress is 15 ksi. Determine the magnitude of the bending moment.



$$J = \frac{\pi}{2}(1)^4 = 1.5708 \text{ in}^4$$

$$I = \frac{\pi}{4}(1)^2 = 0.7854 \text{ in}^4$$

$$\tau = \frac{Tc}{J} = \frac{12(1)}{1.5708} = 7.639 \text{ ksi}$$

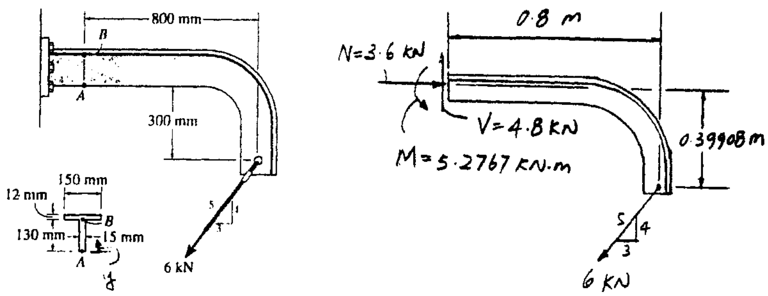
$$\sigma_x = \frac{Mc}{I} = \frac{M(1)}{0.7854} = 1.2732M$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2}$$

$$15 = \frac{1.2732M}{2} + \sqrt{(\frac{-1.2732M}{2})^2 + 7.639^2}$$

$M = 8.73$ kip · in. **Ans**

9-35 Determine the principal stresses acting at point A of the supporting frame. Show the results on a properly oriented element located at this point.



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$

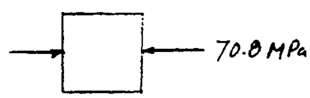
$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2 + \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$

$Q_A = 0$

$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$

Normal stress :

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$



$$\sigma_A = \frac{-3.6(10^3)}{3.75(10^{-3})} - \frac{5.2767(10^3)(0.0991)}{7.4862(10^{-6})} = -70.80 \text{ MPa}$$

Shear stress :

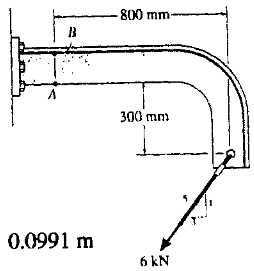
$\tau_A = 0$

Principal stress :

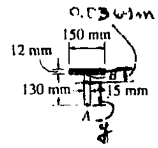
$\sigma_1 = 0$ **Ans**

$\sigma_2 = -70.8 \text{ MPa}$ **Ans**

•9–36 Determine the principal stresses acting at point B, which is located just on the web, below the horizontal segment on the cross section. Show the results on a properly oriented element located at this point. Although it is not very accurate, use the shear formula to calculate the shear stress.

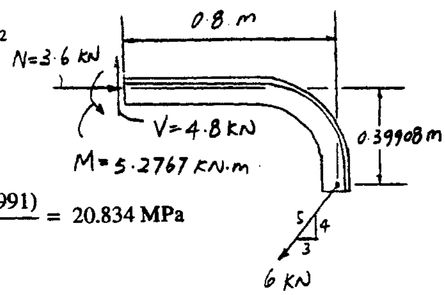


$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$



$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2 + \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$

$$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$$



Normal stress :

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_B = -\frac{3.6(10^3)}{3.75(10^{-3})} + \frac{5.2767(10^3)(0.130 - 0.0991)}{7.4862(10^{-6})} = 20.834 \text{ MPa}$$

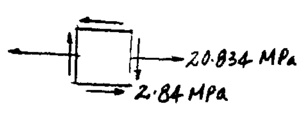
Shear stress :

$$\tau_B = \frac{VQ}{It} = \frac{-4.8(10^3)(0.0369)(0.15)(0.012)(0.0369)}{7.4862(10^{-6})(0.015)} = -2.84 \text{ MPa}$$

Principal stress :

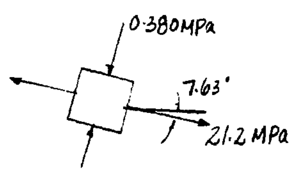
$$\sigma_{1,2} = \left(\frac{20.834 + 0}{2}\right) \pm \sqrt{\left(\frac{20.834 - 0}{2}\right)^2 + (-2.84)^2}$$

$\sigma_1 = 21.2$ MPa **Ans**



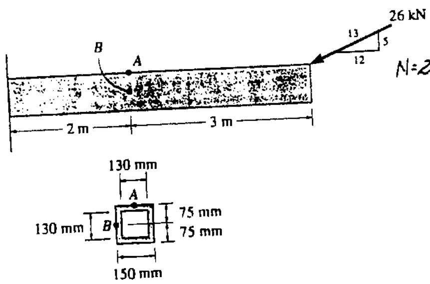
$\sigma_2 = -0.380$ MPa **Ans**

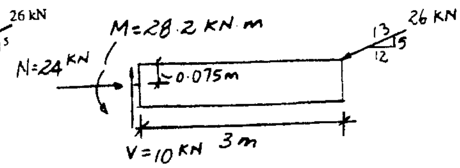
$$\tan 2\theta_p = \frac{-2.84}{\left(\frac{20.834 - 0}{2}\right)}$$



$\theta_p = -7.63°$ **Ans**

9–37 The box beam is subjected to the 26-kN force that is applied at the center of its width, 75 mm from each side. Determine the principal stresses at point A and show the results on an element located at this point. Use the shear formula to compute the shear stress.





$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.13)(0.13^3) = 18.38667(10^{-6})\ \text{m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6(10^{-3})\ \text{m}^2$$

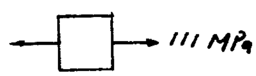
$$Q_A = 0$$

$$\tau_A = 0$$

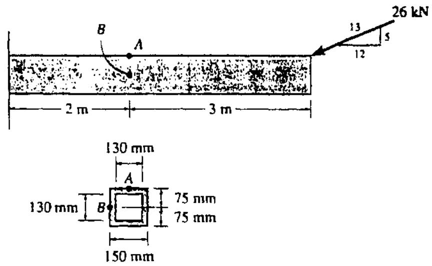
$$\sigma_A = -\frac{P}{A} + \frac{Mc}{I} = \frac{-24(10^3)}{5.6(10^{-3})} + \frac{28.2(10^3)(0.075)}{18.38667(10^{-6})} = 111\ \text{MPa}$$

$\sigma_1 = 111$ MPa **Ans**

$\sigma_2 = 0$ **Ans**



9–38 Solve Prob. 9-37 for point *B*.

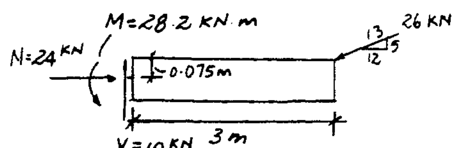


$$I = \frac{1}{12}(0.15)(0.15^3) - \frac{1}{12}(0.13)(0.13^3) = 18.38667(10^{-6})\ \text{m}^4$$

$$A = 0.15^2 - 0.13^2 = 5.6(10^{-3})\ \text{m}^2$$

$$Q_B = (0.07)(0.15)(0.01) + 2(0.0325)(0.065)(0.01) = 0.14725(10^{-3})\ \text{m}^3$$

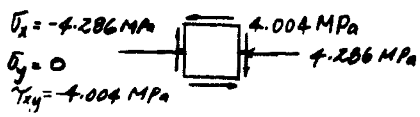
$$\sigma_B = -\frac{P}{A} = -\frac{24(10^3)}{5.6(10^{-3})} = -4.286\ \text{MPa}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{10(10^3)(0.14725)(10^{-3})}{18.38667(10^{-6})(0.02)} = 4.004\ \text{MPa}$$

$\sigma_x = -4.286$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -4.004$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

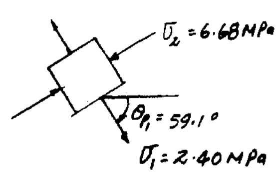


$$= \frac{-4.286 + 0}{2} \pm \sqrt{\left(\frac{-4.286 - 0}{2}\right)^2 + (-4.004^2)}$$

$\sigma_1 = 2.40$ MPa **Ans** $\qquad \sigma_2 = -6.68$ MPa **Ans**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-4.004}{(-4.286 - 0)/2}$$

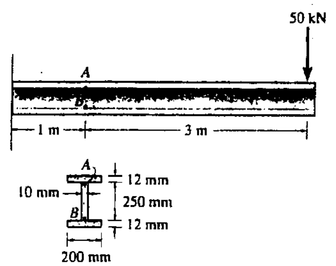
$\theta_p = 30.9°$ or $-59.1°$

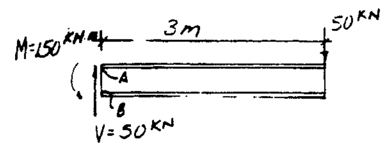


Use Eq 9 - 1,

$\theta_{p1} = -59.1° \qquad \theta_{p2} = 30.9°$

9–39 The wide-flange beam is subjected to the 50-kN force. Determine the principal stresses in the beam at point *A* located on the *web* at the bottom of the upper flange. Although not very accurate, use the shear formula to compute the shear stress.

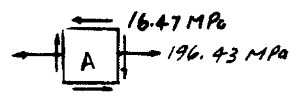




$$I = \frac{1}{12}(0.2)(0.274)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_A = (0.131)(0.012)(0.2) = 0.3144(10^{-3})\text{m}^3$$

$$\sigma_A = \frac{My}{I} = \frac{150(10^3)(0.125)}{95.451233(10^{-6})} = 196.43\ \text{MPa}$$



$$\tau_A = \frac{VQ_A}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

$\sigma_x = 196.43$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -16.47$ MPa

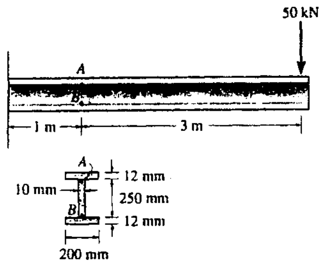
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

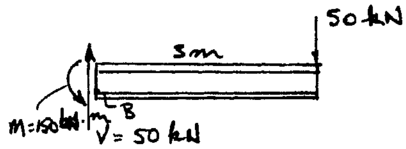
$$= \frac{196.43 + 0}{2} \pm \sqrt{\left(\frac{196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 198$ MPa **Ans**

$\sigma_2 = -1.37$ MPa **Ans**

•9–40 Solve Prob. 9-39 for point *B* located on the *web* at the top of the bottom flange.

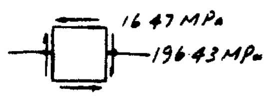




$$I = \frac{1}{12}(0.2)(0.247)^3 - \frac{1}{12}(0.19)(0.25)^3 = 95.451233(10^{-6})\ \text{m}^4$$

$$Q_B = (0.131)(0.012)(0.2) = 0.3144(10^{-3})$$

$$\sigma_B = -\frac{My}{I} = -\frac{150(10^3)(0.125)}{95.451233(10^{-6})} = -196.43\ \text{MPa}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{50(10^3)(0.3144)(10^{-3})}{95.451233(10^{-6})(0.01)} = 16.47\ \text{MPa}$$

$\sigma_x = -196.43$ MPa $\sigma_y = 0$ $\tau_{xy} = -16.47$ MPa

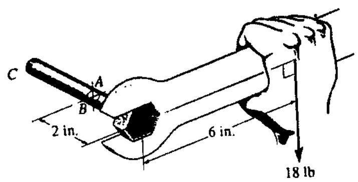
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-196.43 + 0}{2} \pm \sqrt{\left(\frac{-196.43 - 0}{2}\right)^2 + (-16.47)^2}$$

$\sigma_1 = 1.37$ MPa **Ans**

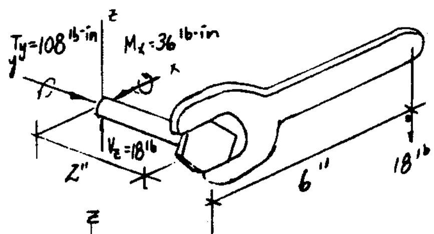
$\sigma_2 = -198$ MPa **Ans**

9–41 The bolt is fixed to its support at C. If a force of 18 lb is applied to the wrench to tighten it, determine the principal stresses developed in the bolt shank at point A. Represent the results on an element located at this point. The shank has a diameter of 0.25 in.



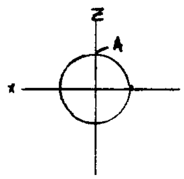
$$I_x = I_z = \frac{\pi}{4}(0.125^4) = 0.1917476(10^{-3})\text{ in}^4$$

$$J = \frac{\pi}{2}(0.125)^4 = 0.383495(10^{-3})\text{ in}^4$$



$$\sigma_A = \frac{M_x c}{I} = \frac{36(0.125)}{0.1917476(10^{-3})} = 23.47\text{ ksi}$$

$$\tau_A = \frac{T_y c}{J} = \frac{108(0.125)}{0.383495(10^{-3})} = 35.20\text{ ksi}$$



$\sigma_x = 23.47$ ksi, $\sigma_y = 0$, $\tau_{xy} = 35.20$ ksi

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{23.47 + 0}{2} \pm \sqrt{\left(\frac{23.47 - 0}{2}\right)^2 + 35.2^2}$$

$\sigma_1 = 48.8$ ksi **Ans**

$\sigma_2 = -25.4$ ksi **Ans**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{35.20}{(23.47 - 0)/2}$$

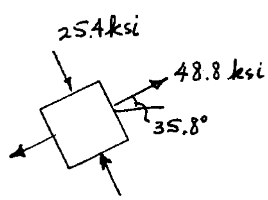
$\theta_p = 35.78°$ and $\theta_p = -54.22°$

Use Eq. 9 - 1 to determine the principal plane for σ_1 and σ_2 :

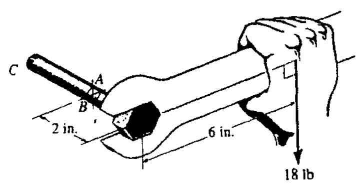
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

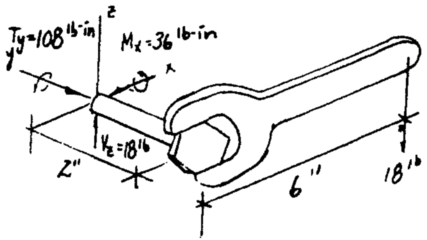
$$= \frac{23.47 + 0}{2} + \frac{23.47 - 0}{2}\cos 71.56° + 35.20 \sin 71.56° = 48.8\text{ ksi}$$

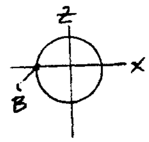
$\theta_{p1} = 35.78°$ $\theta_{p2} = -54.22°$



9–42 Solve Prob. 9-41 for point *B*.







$$I_x = I_z = \frac{\pi}{4}(0.125^4) = 0.1917476(10^{-3})\ \text{in}^4$$

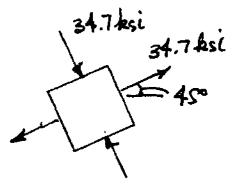
$$J = \frac{\pi}{2}(0.125)^4 = 0.383495(10^{-3})\ \text{in}^4$$

$$\sigma_B = 0$$

$$Q_B = \bar{y}'A' = \frac{4(0.125)}{3\pi}\left(\frac{1}{2}\right)(\pi)(0.125^2) = 1.3020833(10^{-3})\ \text{in}^3$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{18(1.3020833)(10^{-3})}{0.1917476(10^{-3})(0.25)} - \frac{108(0.125)}{0.383495(10^{-3})} = -34.71\ \text{ksi}$$

$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 34.71$ ksi



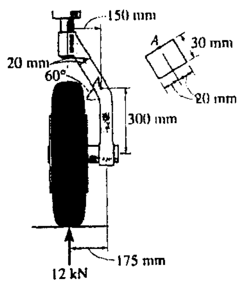
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

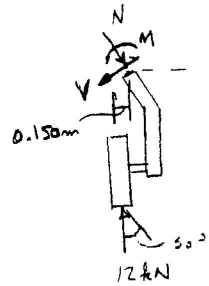
$$= 0 \pm \sqrt{(0)^2 + (34.71)^2}$$

$\sigma_1 = 34.7$ ksi **Ans**

$\sigma_2 = -34.7$ ksi **Ans**

9–43 The nose wheel of the plane is subjected to a design load of 12 kN. Determine the principal stresses acting on the aluminum wheel support at point A.





$\nwarrow + \Sigma F_y = 0; \quad 12\cos 30° - N = 0; \quad N = 10.392 \text{ kN}$

$\nearrow + \Sigma F_x = 0; \quad -12\sin 30° + V = 0; \quad V = 6 \text{ kN}$

$\curvearrowleft + \Sigma M_A = 0; \quad M - (12)(0.150) = 0; \quad M = 1.80 \text{ kN}\cdot\text{m}$

$$\sigma = \frac{P}{A} = \frac{10.392(10^3)}{(0.03)(0.04)} = 8.66 \text{ MPa} \ (C)$$

$$\tau = \frac{VQ}{It} = \frac{6(10^3)(0.01)(0.03)(0.02)}{\frac{1}{12}(0.03)(0.04)^3(0.03)} = 7.50 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

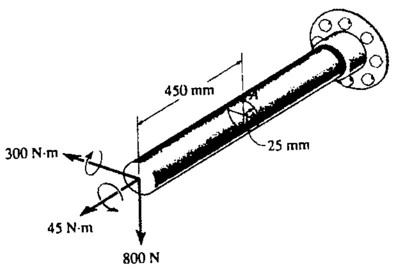
$$= \frac{-8.66 + 0}{2} \pm \sqrt{\left(\frac{8.66 - 0}{2}\right)^2 + (7.50)^2}$$

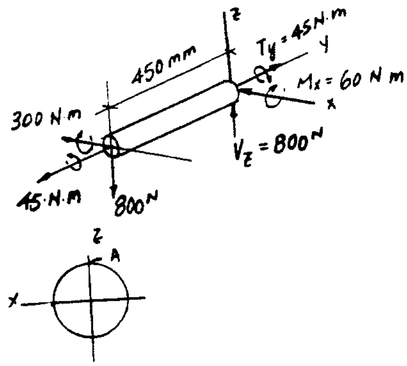
$$= -4.33 \pm 8.66025$$

$\sigma_1 = -12.990 = -13.0 \text{ MPa}$ **Ans**

$\sigma_2 = 4.33 \text{ MPa}$ **Ans**

•9–44 The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at point A.





$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

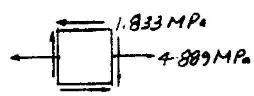
$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{M_x c}{I} = \frac{60(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\tau_A = \frac{T_y c}{J} = \frac{45(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$

$\sigma_x = 4.889$ MPa $\qquad \sigma_y = 0 \qquad \tau_{xy} = -1.833$ MPa



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

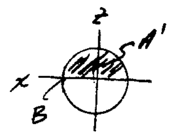
$$= \frac{4.889 + 0}{2} \pm \sqrt{\left(\frac{4.889 - 0}{2}\right)^2 + (-1.833)^2}$$

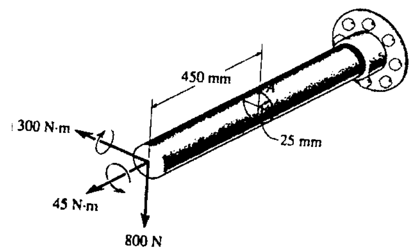
$\sigma_1 = 5.50$ MPa **Ans**

$\sigma_2 = -0.611$ MPa **Ans**

9–45 Solve prob. 9-44 for point *B*.

$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$





$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$

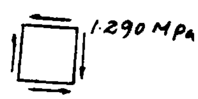
$Q_B = \bar{y}A' = \frac{4(0.025)}{3\pi}(\frac{1}{2})\pi\,(0.025^2) = 10.4167(10^{-6})\ \text{m}$

$\sigma_B = 0$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{800(10.4167)(10^{-6})}{0.306796(10^{-6})(0.05)} - \frac{45(0.025)}{0.61359(10^{-6})} = -1.290\ \text{MPa}$$

$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.290\ \text{MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}{}^2}$$

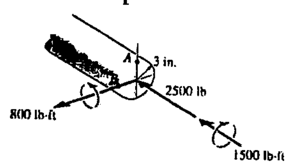


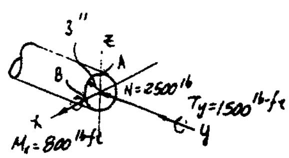
$$= 0 \pm \sqrt{(0)^2 + (-1.290)^2}$$

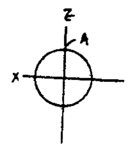
$\sigma_1 = 1.29\ \text{MPa}$ **Ans**

$\sigma_2 = -1.29\ \text{MPa}$ **Ans**

9-46. The internal loadings at a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb · ft, and a torsional moment of 1500 lb · ft. Determine the principal stresses at point A. Also calculate the maximum in-plane shear stress at this point.







$$A = \pi(3)^2 = 28.274\ \text{in}^2$$

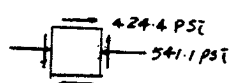
$$J = \frac{\pi}{2}(3^4) = 127.23\ \text{in}^4$$

$$I = \frac{\pi}{4}(3^4) = 63.62\ \text{in}^4$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x c}{I} = -\frac{2500}{28.274} - \frac{800(12)(3)}{63.62} = -541.1\ \text{psi}$$

$$\tau_A = \frac{T_y c}{J} = \frac{1500(12)(3)}{127.23} = 424.4\ \text{psi}$$

$$\sigma_x = -541.1\ \text{psi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 424.4\ \text{psi}$$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-541.1 + 0}{2} \pm \sqrt{\left(\frac{-541.1 - 0}{2}\right)^2 + (424.4)^2}$$

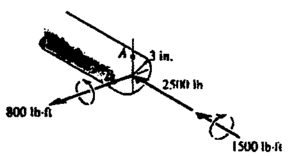
$\sigma_1 = 233$ psi **Ans**

$\sigma_2 = -774$ psi **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-541.1 - 0}{2}\right)^2 + (424.4)^2} = 503\ \text{psi}$$ **Ans**

9-47. The internal loadings at a cross section through the 6-in.-diameter drive shaft of a turbine consist of an axial force of 2500 lb, a bending moment of 800 lb · ft, and a torsional moment of 1500 lb · ft. Determine the principal stresses at point B. Also calculate the maximum in-plane shear stress at this point.



$$A = \pi(3)^2 = 28.274 \text{ in}^2$$

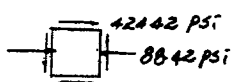
$$J = \frac{\pi}{2}(3^4) = 127.23 \text{ in}^4$$

$$I = \frac{\pi}{4}(3^4) = 63.62 \text{ in}^4$$

$$\sigma_B = -\frac{P}{A} = -\frac{2500}{28.274} = -88.42 \text{ psi}$$

$$\tau_B = \frac{T_y c}{J} = \frac{1500(12)(3)}{127.23} = 424.42$$

$\sigma_x = -88.42$ psi $\qquad \sigma_y = 0 \qquad \tau_{xy} = 424.4$ psi



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-88.42 + 0}{2} \pm \sqrt{\left(\frac{-88.42 - 0}{2}\right)^2 + (424.4)^2}$$

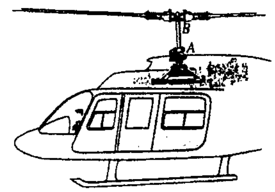
$\sigma_1 = 382$ psi **Ans**

$\sigma_2 = -471$ psi **Ans**

$$\tau_{\substack{max \\ in\text{-}plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

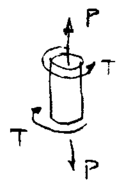
$$= \sqrt{\left(\frac{-88.42 - 0}{2}\right)^2 + (424.42)^2} = 427 \text{ psi} \quad \textbf{Ans}$$

•9–48 The 2-in.-diameter drive shaft AB on the helicopter is subjected to an axial tension of 10 000 lb and a torque of 300 lb · ft. Determine the principal stresses and the maximum in-plane shear stress that act at a point on the surface of the shaft.



$$\sigma = \frac{P}{A} = \frac{10\,000}{\pi(1)^2} = 3.183 \text{ ksi}$$

$$\tau = \frac{Tc}{J} = \frac{300(12)(1)}{\frac{\pi}{2}(1)^4} = 2.292 \text{ ksi}$$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3.183 + 0}{2} \pm \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

$\sigma_1 = 4.38$ ksi **Ans**

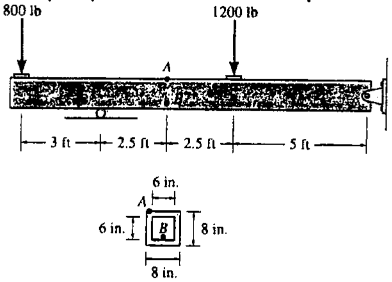
$\sigma_2 = -1.20$ ksi **Ans**

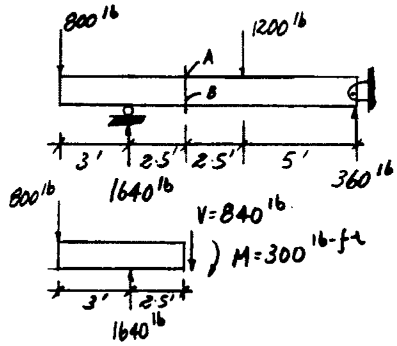
$$\tau_{\substack{max \\ in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

$= 2.79$ ksi **Ans**

9–49 The box beam is subjected to the loading shown. Determine the principal stresses in the beam at points A and B.





$$I = \frac{1}{12}(8)(8)^3 - \frac{1}{12}(6)(6)^3 = 233.33 \text{ in}^4$$

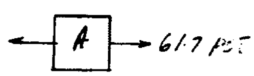
$Q_A = 0$

$Q_B = 0$

For point A :

$\tau_A = 0$

$$\sigma_A = \frac{Mc}{I} = \frac{300(12)(4)}{233.33} = 61.7 \text{ psi}$$



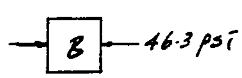
$\sigma_1 = 61.7$ psi **Ans**

$\sigma_2 = 0$ **Ans**

For point B :

$\tau_B = 0$

$$\sigma_B = -\frac{My}{I} = \frac{-300(12)(3)}{233.33} = -46.3 \text{ psi}$$



$\sigma_1 = 0$ **Ans**

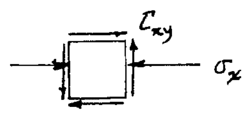
$\sigma_2 = -46.3$ psi **Ans**

9–50 A bar has a circular cross section with a diameter of 1 in. It is subjected to a torque and a bending moment. At the point of maximum bending stress the principal stresses are 20 ksi and –10 ksi. Determine the torque and the bending moment.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

In this problem $\sigma_y = 0$

$$20 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$



$$\left(20 - \frac{\sigma_x}{2}\right)^2 = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$400 + \frac{\sigma_x^2}{4} - 20\sigma_x = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$400 - 20\sigma_x = \tau_{xy}^2 \qquad (1)$$

$$-10 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(-10 - \frac{\sigma_x}{2}\right)^2 = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$100 + \frac{\sigma_x^2}{4} + 10\sigma_x = \frac{\sigma_x^2}{4} + \tau_{xy}^2$$

$$100 + 10\sigma_x = \tau_{xy}^2 \qquad (2)$$

Solving Eqs. (1) and (2) :

$\sigma_x = 10\text{ ksi} \qquad \tau_{xy} = 14.14\text{ ksi}$

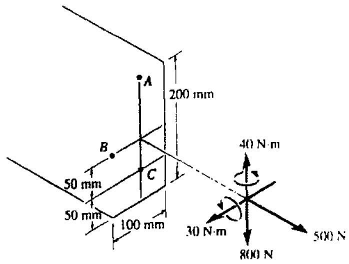
$$\tau_{xy} = \frac{Tc}{J}; \qquad 14.14 = \frac{T(0.5)}{\frac{\pi}{2}(0.5^4)}$$

$T = 2.776\text{ kip}\cdot\text{in.} = 231\text{ lb}\cdot\text{ft}$ **Ans**

$$\sigma = \frac{Mc}{I}; \qquad 10 = \frac{M(0.5)}{\frac{\pi}{4}(0.5^4)}$$

$M = 0.981\text{ kip}\cdot\text{in.} = 81.8\text{ lb}\cdot\text{ft}$ **Ans**

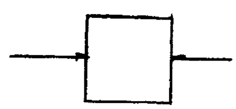
9–51 The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N · m and 40 N · m. Determine the principal stresses at point A. Also compute the maximum in-plane shear stress at this point.



$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\ \text{in}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{P}{A} - \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} - \frac{30(0.1)}{66.67(10^{-6})} = -20\ \text{kPa}$$



$$\tau_A = 0$$

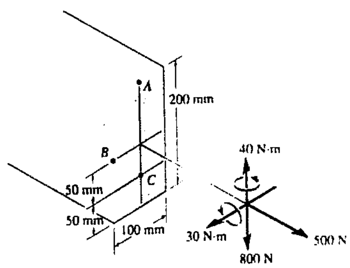
Here, the principal stresses are

$\sigma_1 = \sigma_y = 0$ **Ans** $\sigma_2 = \sigma_x = -20\ \text{kPa}$ **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

$$= \sqrt{\left(\frac{-20-0}{2}\right)^2 + 0} = 10\ \text{kPa} \quad \textbf{Ans}$$

•9–52 The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N · m and 40 N · m. Determine the principal stresses at point B. Also compute the maximum in-plane shear stress at this point.

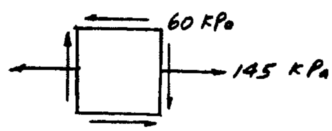


$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67\left(10^{-6}\right)\text{m}^4$$

$$I_z = \frac{1}{12}(0.2)(0.1)^3 = 16.67(10^{-6})\text{m}^4$$

$$Q_B = \bar{z}'A' = (0.05)(0.1)(0.1) = 0.5(10^{-3})\text{m}^3$$

$$\sigma_B = \frac{P}{A} + \frac{M_z x}{I} = \frac{500}{(0.1)(0.2)} + \frac{40(0.05)}{16.67(10^{-6})} = 145\text{ kPa}$$



$$\tau_B = \frac{V_z Q_B}{I_x t} = \frac{800(0.5)(10^{-3})}{66.67(10^{-6})(0.1)} = 60\text{ kPa}$$

$\sigma_x = 145\text{ kPa}$ $\quad\sigma_y = 0$ $\quad\tau_{xy} = -60\text{ kPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{145+0}{2} \pm \sqrt{\left(\frac{145-0}{2}\right)^2 + (-60)^2}$$

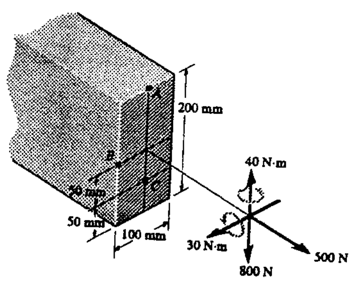
$\sigma_1 = 167\text{ kPa}$ **Ans**

$\sigma_2 = -21.6\text{ kPa}$ **Ans**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{145-0}{2}\right)^2 + (-60)^2} = 94.1\text{ kPa}$$ **Ans**

9–53 The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N · m and 40 N · m. Determine the principal stresses at point *C*. Also compute the maximum in-plane shear stress at this point.



$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\text{m}^4$$

$$I_z = \frac{1}{12}(0.2)(0.1)^3 = 16.67(10^{-6})\text{m}^4$$

$$Q_C = (0.075)(0.05)(0.1) = 0.375(10^{-3})\text{m}^3$$

$$\sigma_C = \frac{P}{A} + \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} + \frac{30(0.05)}{66.67(10^{-6})} = 47.5\text{ kPa}$$

$$\tau_C = \frac{V_x Q_C}{I_x t} = \frac{800(0.375)(10^{-3})}{66.67(10^{-6})(0.1)} = 45\text{ kPa}$$

$\sigma_x = 47.5$ kPa $\quad \sigma_y = 0 \quad \tau_{xy} = -45$ kPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{47.5 + 0}{2} \pm \sqrt{\left(\frac{47.5 - 0}{2}\right)^2 + (-45)^2}$$

$\sigma_1 = 74.6$ kPa **Ans**

$\sigma_2 = -27.1$ kPa **Ans**

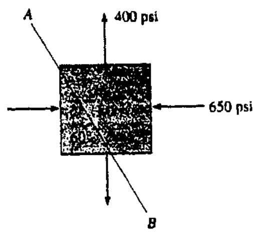
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{47.5 - 0}{2}\right)^2 + (-45)^2} = 50.9\text{ kPa}$$ **Ans**

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

•9–56 Solve Prob. 9–4 using Mohr's circle.



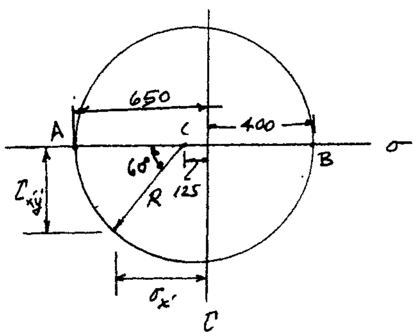
$$\frac{\sigma_x + \sigma_y}{2} = \frac{-650 + 400}{2} = -125$$

$A(-650, 0)$ $\qquad$ $B(400, 0)$ $\qquad$ $C(-125, 0)$

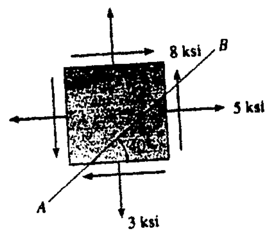
$R = CA = 650 - 125 = 525$

$\sigma_{x'} = -125 - 525\cos 60° = -388$ psi $\quad$ **Ans**

$\tau_{x'y'} = 525\sin 60° = 455$ psi $\quad$ **Ans**



9–57 Solve Prob. 9–2 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{5+3}{2} = 4\text{ ksi}$$

$$R = \sqrt{(5-4)^2 + 8^2} = 8.0623$$

$$\phi = \tan^{-1}\frac{8}{(5-4)} = 82.875°$$

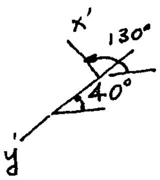
$$2\theta = 2(130°) = 260°$$

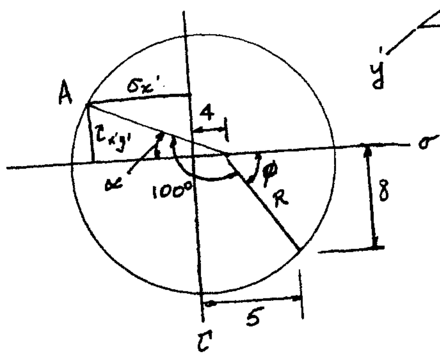
$$360° - 260° = 100°$$

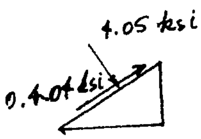
$$\alpha = 100° + 82.875° - 180° = 2.875°$$

$$\sigma_{x'} = 8.0623\cos 2.875° - 4 = -4.05\text{ ksi}$$ **Ans**

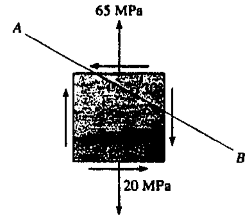
$$\tau_{x'y'} = -8.0623\sin 2.875° = -0.404\text{ ksi}$$ **Ans**







9–58 Solve Prob. 9–3 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{0 + 65}{2} = 32.5 \text{ MPa}$$

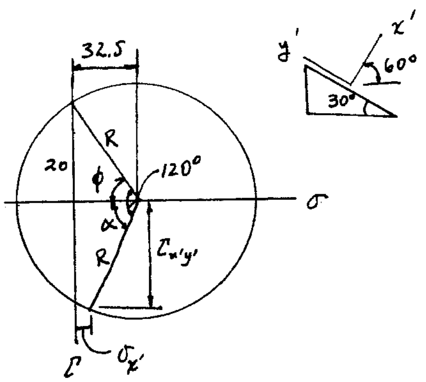
$$R = \sqrt{(32.5)^2 + (20)^2} = 38.1608$$

$$\phi = \tan^{-1} \frac{20}{32.5} = 31.6075°$$

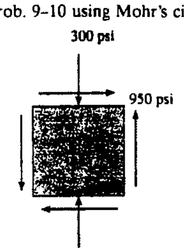
$\alpha = 120° - 31.6075° = 88.392°$

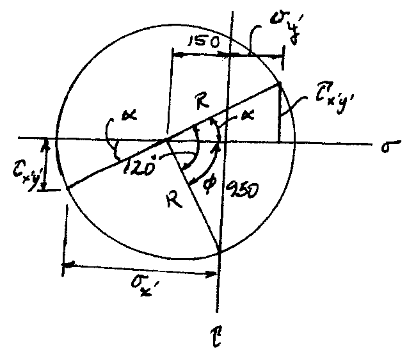
$\sigma_{x'} = 32.5 - 38.1608 \cos 88.392° = 31.4$ MPa **Ans**

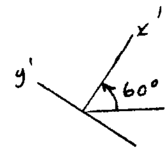
$\tau_{x'y'} = 38.1608 \sin 88.392° = 38.1$ MPa **Ans**

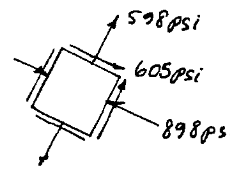


9–59 Solve Prob. 9–10 using Mohr's circle.









$$\frac{\sigma_x + \sigma_y}{2} = \frac{0 - 300}{2} = -150 \text{ psi}$$

$$R = \sqrt{(150)^2 + (950)^2} = 961.769 \text{ psi}$$

$$\phi = \tan^{-1}\frac{950}{150} = 81.0274°$$

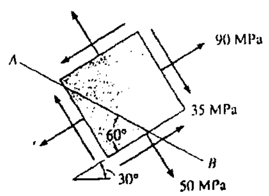
$\alpha = 180° - 60° - 81.0274° = 38.973°$

$\sigma_{x'} = -961.769 \cos 38.973° - 150 = -898 \text{ psi}$ **Ans**

$\tau_{x'y'} = 961.769 \sin 38.973° = 605 \text{ psi}$ **Ans**

$\sigma_{y'} = 961.769 \cos 38.973 - 150 = 598 \text{ psi}$ **Ans**

*9–60 Solve Prob. 9–6 using Mohr's circle.



$\sigma_x = 90\text{ MPa} \qquad \sigma_y = 50\text{ MPa} \qquad \tau_{xy} = -35\text{ MPa} \qquad A(90,-35)$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{90 + 50}{2} = 70$$

$$R = \sqrt{(90 - 70)^2 + (35)^2} = 40.311$$

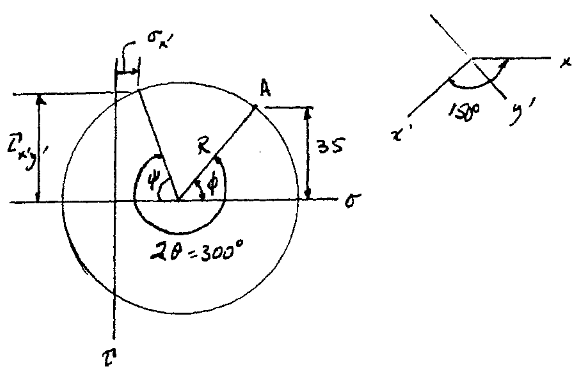
Coordinates of point B :

$$\phi = \tan^{-1}\left(\frac{35}{20}\right) = 60.255°$$

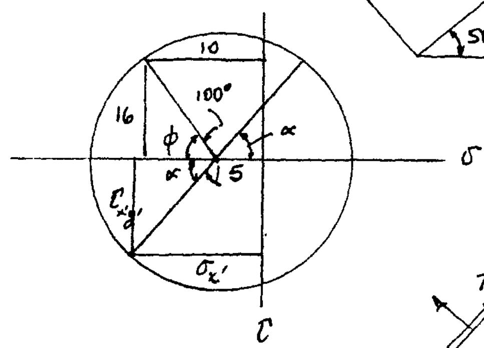
$$\psi = 300° - 180° - 60.255° = 59.745°$$

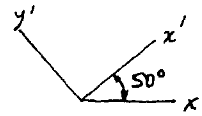
$\sigma_{x'} = 70 - 40.311 \cos 59.745° = 49.7\text{ MPa}$ **Ans**

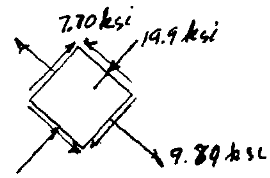
$\tau_{x'} = -40.311 \sin 59.745° = -34.8\text{ MPa}$ **Ans**



9–61 Solve Prob. 9–11 using Mohr's circle.







$$\frac{\sigma_x + \sigma_y}{2} = \frac{-10+0}{2} = -5 \text{ ksi}$$

$$R = \sqrt{(10-5)^2 + (16)^2} = 16.763 \text{ ksi}$$

$$\phi = \tan^{-1} \frac{16}{(10)(5)} = 72.646^\circ$$

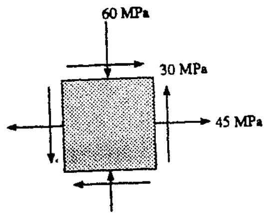
$$\alpha = 100 - 72.646 = 27.354^\circ$$

$\sigma_{x'} = -5 - 16.763 \cos 27.354^\circ = -19.9$ ksi **Ans**

$\tau_{x'y'} = 16.763 \sin 27.354^\circ = 7.70$ ksi **Ans**

$\sigma_{y'} = 16.763 \cos 27.354^\circ - 5 = 9.89$ ksi

9–62 Solve Prob. 9–13 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$

$\sigma_1 = 60.467 - 7.5 = 53.0 \text{ MPa}$ **Ans**

$\sigma_2 = -60.467 - 7.5 = -68.0 \text{ MPa}$ **Ans**

$$2\theta_{p1} = \tan^{-1}\frac{30}{(45 + 7.5)}$$

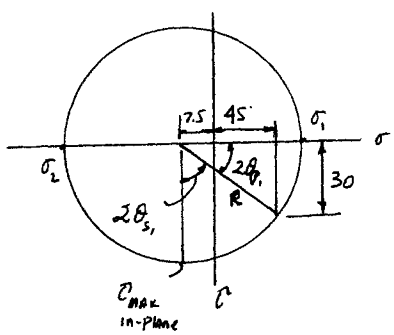
$\theta_{p1} = 14.9°$ counterclockwise **Ans**

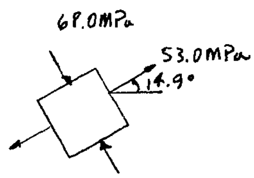
$\tau_{\substack{max \\ in-plane}} = 60.5 \text{ MPa}$ **Ans**

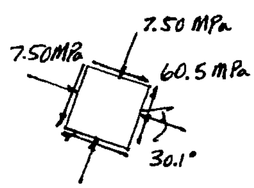
$\sigma_{avg} = -7.50 \text{ MPa}$ **Ans**

$$2\theta_{s1} = 90° - \tan^{-1}\frac{30}{(45 + 7.5)}$$

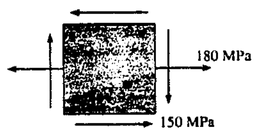
$\theta_{s1} = 30.1°$ clockwise **Ans**



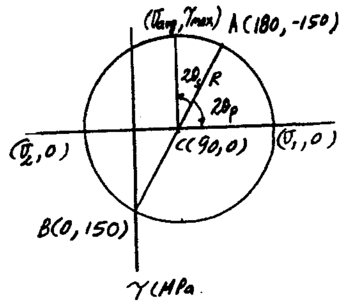




9–63 Solve Prob. 9–14 using Mohr's circle.



$A(180,-150) \qquad B(0,150) \qquad C(90,0)$



$R = CA = \sqrt{90^2 + 150^2} = 174.93$

$\sigma_1 = 90 + 174.93 = 265 \text{ MPa}$ **Ans**

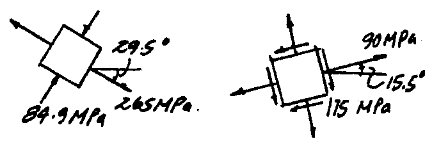
$\sigma_2 = 90 - 174.93 = -84.9 \text{ MPa}$ **Ans**

$\tan 2\theta_p = \dfrac{150}{90}; \qquad 2\theta_p = 59.04^\circ$

$\theta_p = 29.5^\circ$ clockwise **Ans**

$\tau_{\substack{max \\ \text{in-plane}}} = R = 174.93 = 175 \text{ MPa}$ **Ans**

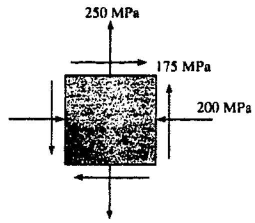
$\sigma_{avg} = 90 \text{ MPa}$ **Ans**



$2\theta_s = 90 - 59.04$

$\theta_s = 15.5^\circ$ counterclockwise **Ans**

*9–64 Solve Prob. 9–16 using Mohr's circle.

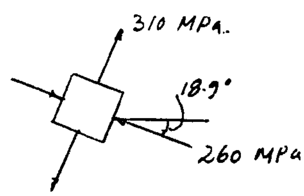


$A(-200, 175) \qquad B(250, -175) \qquad C(25, 0)$

$$R = CA = \sqrt{(200 + 25)^2 + 175^2} = 285.04$$

$$\tan 2\theta_p = \frac{175}{(200 + 25)} = 0.7777$$

$\theta_p = 18.9°$ **Ans**



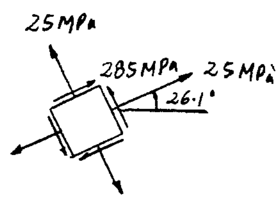
$\sigma_1 = 25 + 285.04 = 310\ \text{MPa}$ **Ans**

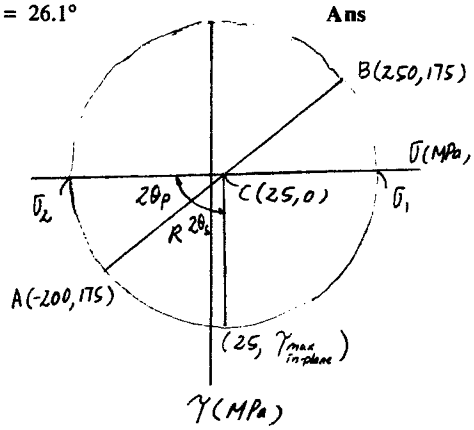
$\sigma_2 = 25 - 285.04 = -260\ \text{MPa}$ **Ans**

$\tau_{\substack{\max \\ \text{in-plane}}} = R = 285\ \text{MPa}$ **Ans**

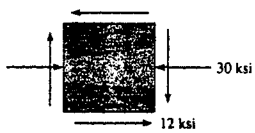
$$\tan 2\theta_s = \frac{200 + 25}{175} = 1.2857$$

$\theta_s = 26.1°$ **Ans**





9–65 Solve Prob. 9–15 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{-30+0}{2} = -15$$

$$R = \sqrt{(30-15)^2 + (12)^2} = 19.21 \text{ ksi}$$

$\sigma_1 = 19.21 - 15 = 4.21$ ksi **Ans**

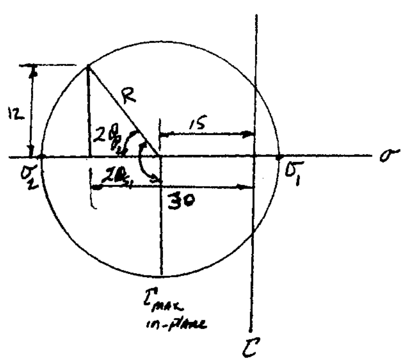
$\sigma_2 = -19.21 - 15 = -34.2$ ksi **Ans**

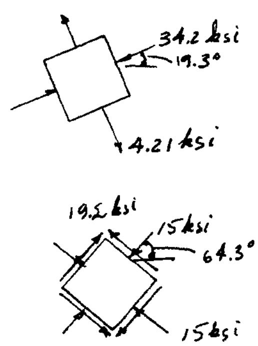
$2\theta_{p2} = \tan^{-1}\dfrac{12}{(30-15)}$; $\theta_{p2} = 19.3°$

$\tau_{\substack{max \\ in-plane}} = R = 19.2$ ksi **Ans**

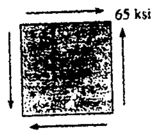
$\sigma_{avg} = -15$ ksi **Ans**

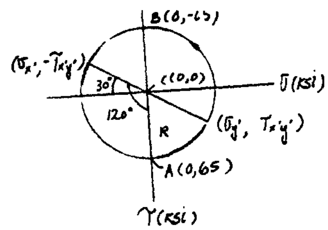
$2\theta_{s2} = \tan^{-1}\dfrac{12}{(30-15)} + 90°$; $\theta_{s2} = 64.3°$





9–66 Determine the equivalent state of stress if an element is oriented 60° clockwise from the element shown.





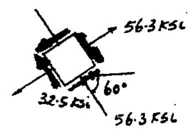
$A(0,65) \qquad B(0,\,-65) \qquad C(0,0)$

$R = 65$

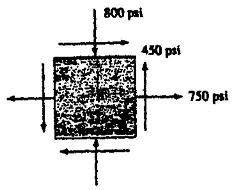
$\sigma_{x'} = 0 - 65\cos 30° = -56.3$ ksi **Ans**

$\sigma_{y'} = 0 + 65\cos 30° = 56.3$ ksi **Ans**

$\tau_{x'y'} = -65\sin 30° = -32.5$ ksi **Ans**



9-67. Determine the equivalent state of stress if an element is oriented 60° counterclockwise from the element shown.



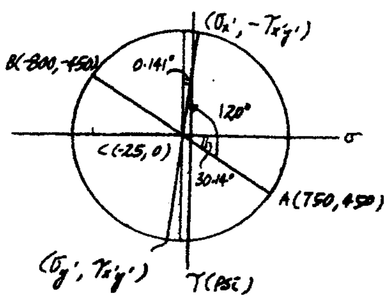
$A(750, 450)$ $\quad B(-800, -450)$ $\quad C(-25, 0)$

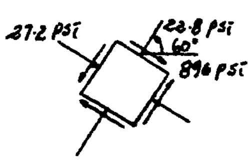
$R = CA = CB = \sqrt{775^2 + 450^2} = 896.17$

$\sigma_{x'} = 25 + 896.17 \sin 0.141° = -22.8$ psi **Ans**

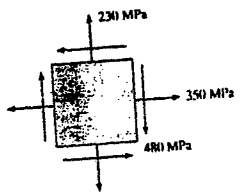
$\tau_{x'y'} = -896.17 \cos 0.141° = -896$ psi **Ans**

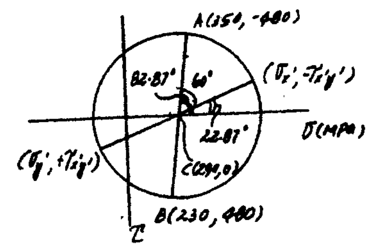
$\sigma_{y'} = -25 - 896.17 \sin 0.141° = -27.2$ psi **Ans**





***9-68.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown.





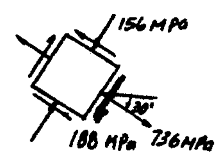
$A(350, -480)$ $\quad B(230, 480)$ $\quad C(290, 0)$

$R = \sqrt{60^2 + 480^2} = 483.73$

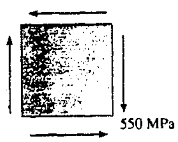
$\sigma_{x'} = 290 + 483.73 \cos 22.87° = 736$ MPa **Ans**

$\sigma_{y'} = 290 - 483.73 \cos 22.87° = -156$ MPa **Ans**

$\tau_{x'y'} = -483.73 \sin 22.87° = -188$ MPa **Ans**



9–69 Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.



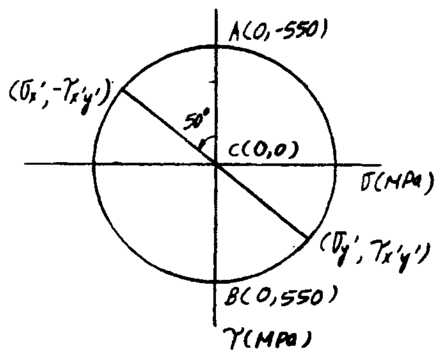
$A(0,-550) \quad B(0,550) \quad C(0,0)$

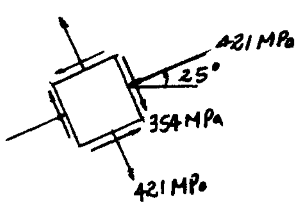
$R = CA = CB = 550$

$\sigma_{x'} = -550 \sin 50° = -421$ MPa **Ans**

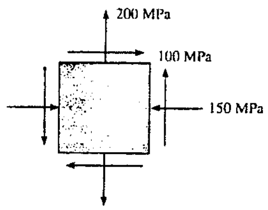
$\tau_{x'y'} = -550 \cos 50° = -354$ MPa **Ans**

$\sigma_{y'} = 550 \sin 50° = 421$ MPa **Ans**





9–70 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-150, 100) \qquad B(200, -100) \qquad C(25, 0)$

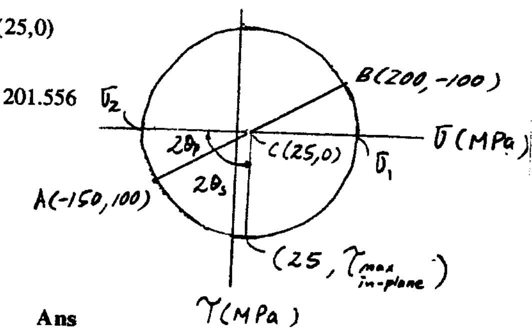
$$R = CA = \sqrt{(150+25)^2 + 100^2} = 201.556$$

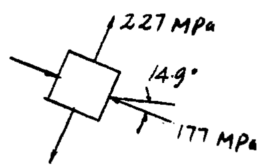
$$\tan 2\theta_p = \frac{100}{150+25} = 0.5714$$

$\theta_p = -14.9°$ **Ans**

$\sigma_1 = 25 + 201.556 = 227\text{ MPa}$ **Ans**

$\sigma_2 = 25 - 201.556 = -177\text{MPa}$ **Ans**



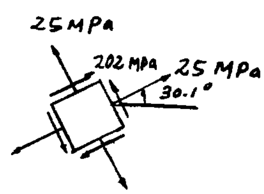


$\tau_{\substack{max \\ in-plane}} = R = 202\text{ MPa}$ **Ans**

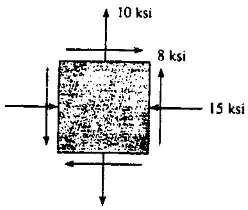
$\sigma_{avg} = 25\text{ MPa}$ **Ans**

$$\tan 2\theta_s = \frac{150+25}{100} = 1.75$$

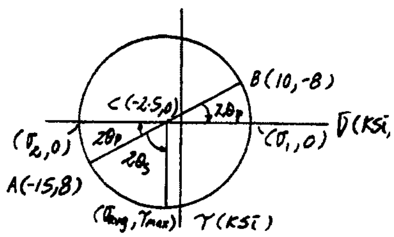
$\theta_s = 30.1°$ **Ans**



9–71 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-15,8)$, $B(10,-8)$, $C(-2.5,0)$



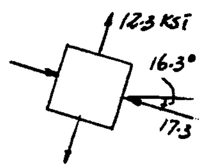
$$R = CA = CB = \sqrt{12.5^2 + 8^2} = 14.84$$

a)

$\sigma_1 = -2.5 + 14.84 = 12.3$ ksi **Ans**

$\sigma_2 = -2.5 - 14.84 = -17.3$ ksi **Ans**

$\tan 2\theta_p = \dfrac{8}{12.5}$ $2\theta_p = 32.62°$ $\theta_p = 16.3°$ **Ans**



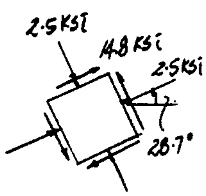
b)

$\tau_{\substack{max \\ in-plane}} = R = 14.8$ ksi **Ans**

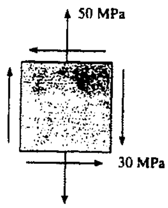
$\sigma_{avg} = -2.5$ ksi **Ans**

$2\theta_s = 90 - 2\theta_p$

$\theta_s = 28.7°$



•9–72 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



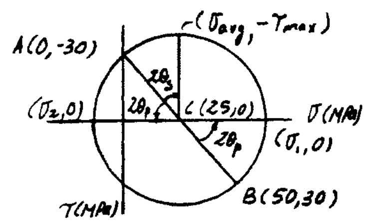
$A(0,-30) \quad B(50,30) \quad C(25,0)$

$R = CA = CB = \sqrt{25^2 + 30^2} = 39.05$

a)

$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa}$ **Ans**

$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa}$ **Ans**



$\tan 2\theta_p = \frac{30}{25} \qquad 2\theta_p = 50.19° \qquad \theta_p = 25.1°$

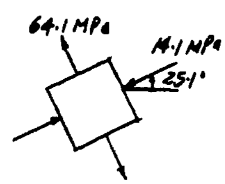
b)

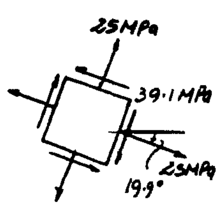
$\tau_{\substack{max \\ in-plane}} = R = 39.1 \text{ MPa}$ **Ans**

$\sigma_{avg} = 25 \text{ MPa}$ **Ans**

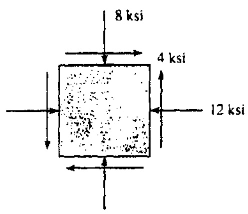
$2\theta_s = 90 - 2\theta_p$

$\theta_s = -19.9°$

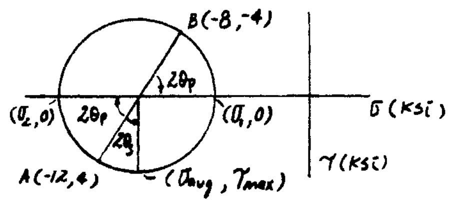




9-73 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(-12,4)$ $B(-8,-4)$ $C(-10,0)$



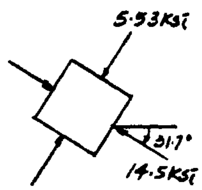
$R = CA = CB = \sqrt{2^2 + 4^2} = 4.472$

a)

$\sigma_1 = -10 + 4.472 = -5.53 \text{ ksi}$ **Ans**

$\sigma_2 = -10 - 4.472 = -14.5 \text{ ksi}$ **Ans**

$\tan 2\theta_p = \frac{4}{2}$ $2\theta_p = 63.43°$ $\theta_p = 31.7°$



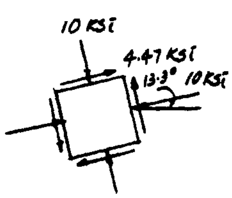
b)

$\tau_{\substack{max \\ in-plane}} = R = 4.47 \text{ ksi}$ **Ans**

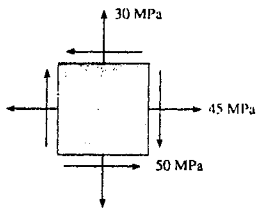
$\sigma_{avg} = -10 \text{ ksi}$ **Ans**

$2\theta_s = 90 - 2\theta_p$

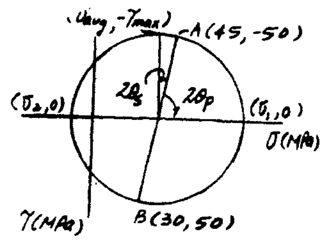
$\theta_s = 13.3°$



9–74 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(45,-50)$ $\quad B(30,50)$ $\quad C(37.5,0)$



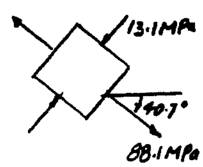
$R = CA = CB = \sqrt{7.5^2 + 50^2} = 50.56$

a)

$\sigma_1 = 37.5 + 50.56 = 88.1$ MPa **Ans**

$\sigma_2 = 37.5 - 50.56 = -13.1$ MPa **Ans**

$\tan 2\theta_p = \dfrac{50}{7.5} \qquad 2\theta_p = 81.47^\circ \qquad \theta_p = -40.7^\circ$



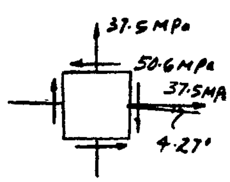
b)

$\tau_{\substack{max \\ \text{in-plane}}} = R = 50.6$ MPa **Ans**

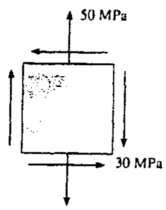
$\sigma_{avg} = 37.5$ MPa **Ans**

$2\theta_s = 90 - 2\theta_p$

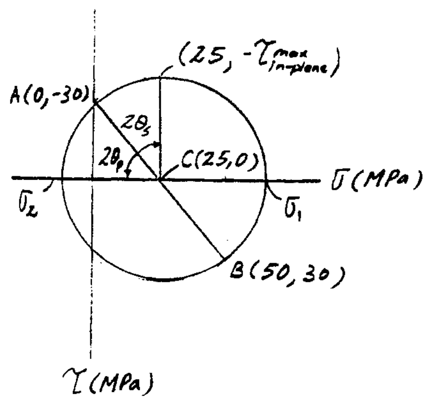
$\theta_s = 4.27^\circ$



9–75 Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(0,-30)$ $\quad B(50,30)$ $\quad C(25,0)$



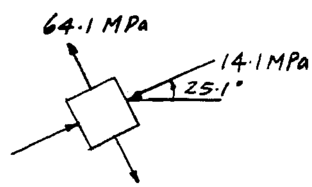
$$R = CA = \sqrt{(25-0)^2 + 30^2} = 39.05$$

$$\sigma_1 = 25 + 39.05 = 64.1 \text{ MPa} \qquad \textbf{Ans}$$

$$\sigma_2 = 25 - 39.05 = -14.1 \text{ MPa} \qquad \textbf{Ans}$$

$$\tan 2\theta_p = \frac{30}{25-0} = 1.2$$

$$\theta_p = 25.1° \qquad \textbf{Ans}$$

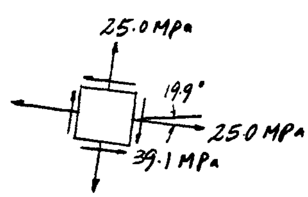


$$\sigma_{avg} = 25.0 \text{ MPa} \qquad \textbf{Ans}$$

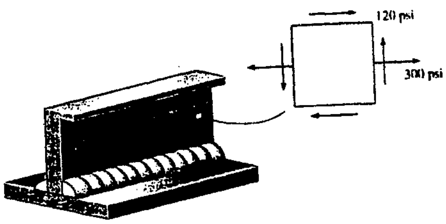
$$\tau_{\substack{max \\ in-plane}} = R = 39.1 \text{ MPa} \qquad \textbf{Ans}$$

$$\tan 2\theta_s = \frac{25-0}{30} = 0.8333$$

$$\theta_s = -19.9° \qquad \textbf{Ans}$$



*9-76. Determine (a) the principal stresses and (b) the maximum in-plane shear stress and average normal stress. Specify the orientation of the element in each case.



$A(300, 120) \quad B(0, -120) \quad C(150, 0)$

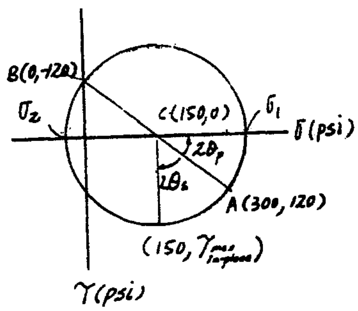
$R = \sqrt{(300 - 150)^2 + 120^2} = 192.094$

$\sigma_1 = 150 + 192.094 = 342 \text{ psi}$ **Ans**

$\sigma_2 = 150 - 192.094 = -42.1 \text{ psi}$ **Ans**

$\tan 2\theta_p = \dfrac{120}{300 - 150} = 0.8$

$\theta_p = 19.3°$ **Ans**

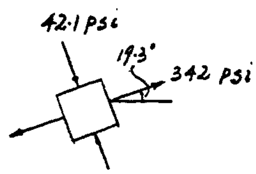


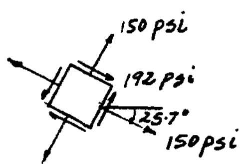
$\sigma_{avg} = 150 \text{ psi}$ **Ans**

$\tau_{\substack{max \\ in\text{-}plane}} = 192 \text{ psi}$ **Ans**

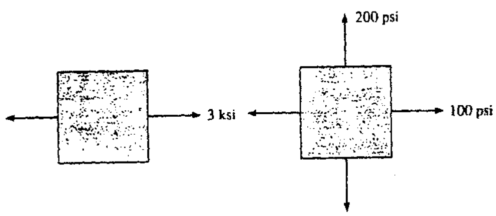
$\tan 2\theta_s = \dfrac{300 - 150}{120} = 1.25$

$\theta_s = -25.7°$ **Ans**

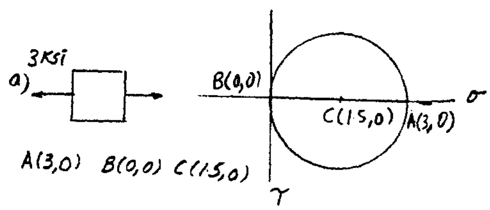


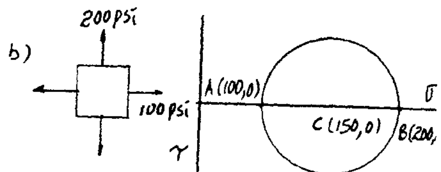


9–77 Draw Mohr's circle that describes each of the following states of stress.



(a) (b)

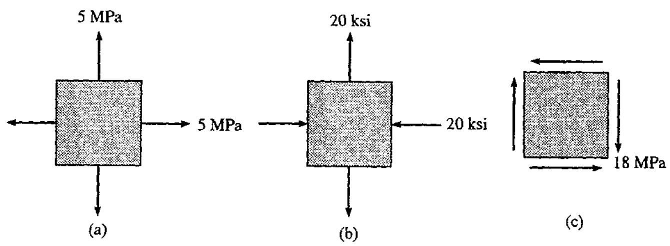


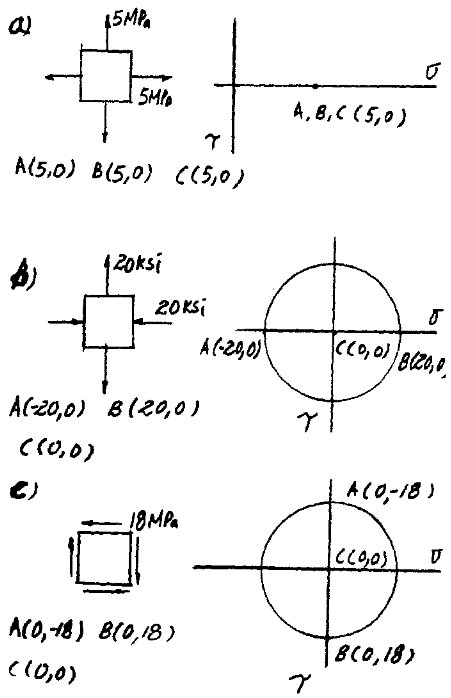


A(100,0) B(200,0)

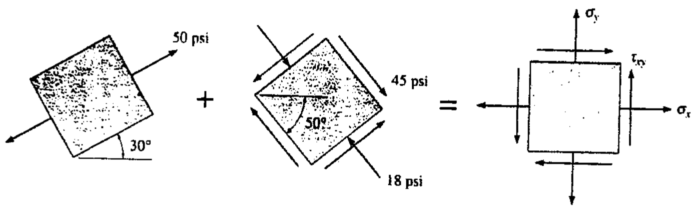
C(150,0)

9–78 Draw Mohr's circle that describes each of the following states of stress.





9–79 A point on a thin plate is subjected to two successive states of stress as shown. Determine the resulting state of stress with reference to an element oriented as shown on the right.



For element a :

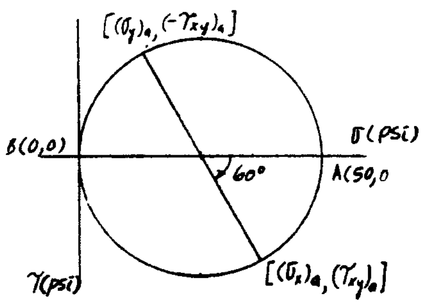
$A(50,0)$ $\quad B(0,0)$ $\quad C(25,0)$

$R = 50 - 25 = 25$

$(\sigma_x)_a = 25 + 25\cos 60° = 37.5$ psi

$(\sigma_y)_a = 25 - 25\cos 60° = 12.5$ psi

$(\tau_{xy})_a = 25\sin 60° = 21.65$ psi



For element b :

$A(0,-45)$ $\quad B(-18,45)$ $\quad C(-9,0)$

$R = \sqrt{9^2 + 45^2} = 45.89$

$(\sigma_x)_b = -9 + 45.89\cos 1.31° = 36.88$ psi

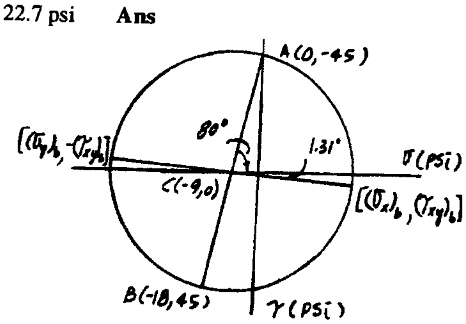
$(\sigma_y)_b = -9 - 45.89\cos 1.31° = -54.88$ psi

$(\tau_{xy})_b = 45.89\sin 1.31° = 1.049$ psi

$\sigma_x = (\sigma_x)_a + (\sigma_x)_b = 37.5 + 36.88 = 74.4$ psi **Ans**

$\sigma_y = (\sigma_y)_a + (\sigma_y)_b = 12.5 - 54.88 = -42.4$ psi **Ans**

$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 21.65 + 1.049 = 22.7$ psi **Ans**



•9–80 Mohr's circle for the state of stress in Fig. 9-15*a* is shown in Fig. 9-15*b*. Show that finding the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress-transformation Eqs. 9-1 and 9-2.

$$A(\sigma_x, \tau_{xy}) \qquad B(\sigma_y, -\tau_{xy}) \qquad C((\frac{\sigma_x + \sigma_y}{2}), 0)$$

$$R = \sqrt{[\sigma_x - (\frac{\sigma_x + \sigma_y}{2})\ ]^2 + \tau_{xy}^2} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}\cos\theta' \qquad (1)$$

$$\theta' = 2\theta_p - 2\theta$$

$$\cos(2\theta_p - 2\theta) = \cos 2\theta_p \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad (2)$$

From the circle :

$$\cos 2\theta_p = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}} \qquad (3)$$

$$\sin 2\theta_p = \frac{\tau_{xy}}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}} \qquad (4)$$

Substitute Eq. (2), (3) and (4) into Eq. (1)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

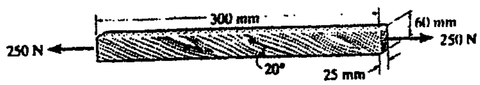
$$\tau_{x'y'} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}\sin\theta' \qquad (5)$$

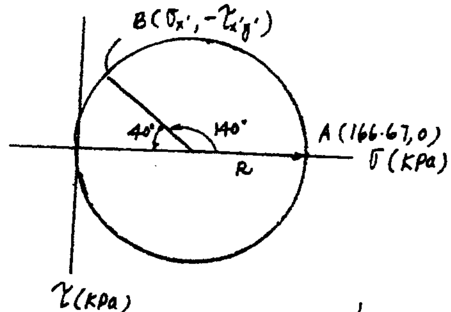
$$\sin\theta' = \sin(2\theta_p - 2\theta)$$
$$= \sin 2\theta_p \cos 2\theta - \sin 2\theta \cos 2\theta_p \qquad (6)$$

Substitute Eq. (3), (4), (6) into Eq. (5),

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

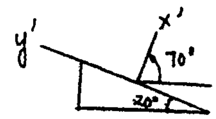
9-81. The grains of wood in the board make an angle of 20° with the horizontal as shown. Determine the normal and shear stresses that act perpendicular and parallel to the grains if the board is subjected to an axial load of 250 N.





$$\sigma_x = \frac{P}{A} = \frac{250}{(0.06)(0.025)} = 166.67 \text{ kPa}$$

$R = 83.33$



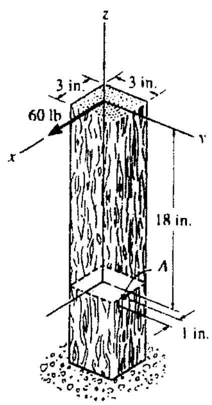
Coordinates of point B :

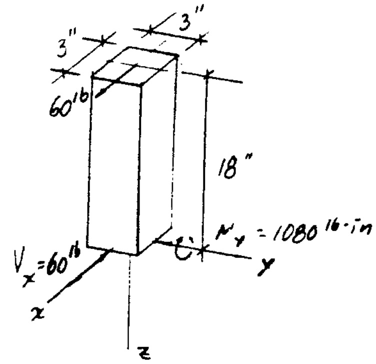
$\sigma_{x'} = 83.33 - 83.33 \cos 40°$

$\sigma_{x'} = 19.5 \text{ kPa}$ **Ans**

$\tau_{x'y'} = -83.33 \sin 40° = -53.6 \text{ kPa}$ **Ans**

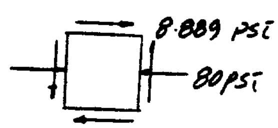
9–82 The post has a square cross-sectional area. If it is fixed-supported at its base and a horizontal force is applied at its end as shown, determine (*a*) the maximum in-plane shear stress developed at *A* and (*b*) the principal stresses at *A*.





$$I = \frac{1}{12}(3)(3^3) = 6.75 \text{ in}^4 \qquad Q_A = (1)(1)(3) = 3 \text{ in}^3$$

$$\sigma_A = -\frac{M_y x}{I} = -\frac{1080(0.5)}{6.75} = -80 \text{ psi}$$



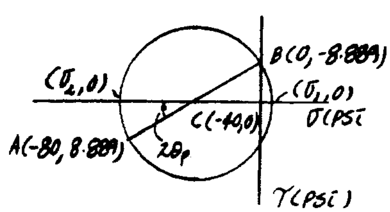
$$\tau_A = \frac{V_y Q_A}{It} = \frac{60(3)}{6.75(3)} = 8.889 \text{ psi}$$

$A(-80, 8.889) \qquad B(0, -8.889) \qquad C(-40, 0)$

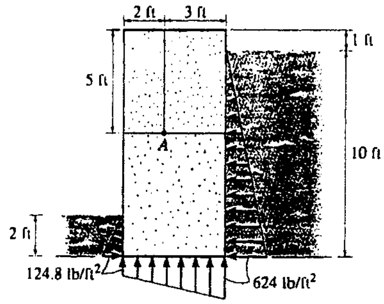
$\tau_{\substack{max \\ in-plane}} = R = \sqrt{40^2 + 8.889^2} = 41.0$ psi **Ans**

$\sigma_1 = -40 + 40.9757 = 0.976$ psi **Ans**

$\sigma_2 = -40 - 40.9757 = -81.0$ psi **Ans**

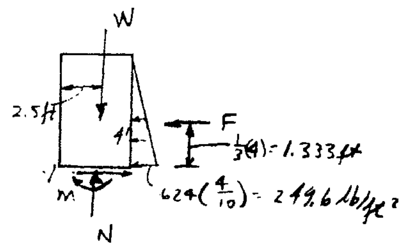


9-83 The concrete dam rests on a pervious foundation and is subjected to the hydrostatic pressures shown. If it has a width of 6 ft, determine the principal stresses acting in the concrete at point A. Show the results on a properly oriented element at the point. The specific weight of the concrete is $\gamma = 150\ \text{lb/ft}^3$.



$$W = 150(6)(5)(5) = 22\,500\ \text{lb}$$

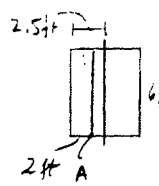
$$F = \frac{1}{2}(249.6)(4)(6) = 2995.2\ \text{lb}$$



$$\xrightarrow{+} \Sigma F_x = 0; \qquad V - 2995.2 = 0$$
$$V = 2995.2\ \text{lb}$$

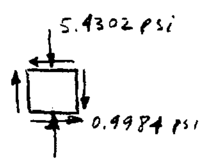
$$+\uparrow \Sigma F_y = 0; \qquad N - 22\,500 = 0$$
$$N = 22\,500\ \text{lb}$$

$$\circlearrowleft + \Sigma M = 0; \qquad -M + 2995.2(1.3333) = 0$$
$$M = 3993.6\ \text{lb}\cdot\text{ft}$$



$$\sigma_A = \frac{P}{A} + \frac{My}{I} = \frac{22500}{(5)(6)} + \frac{3993.6(0.5)}{\frac{1}{12}(6)(5)^3} = 781.95\ \text{psf} = 5.4302\ \text{psi}$$

$$\tau_A = \frac{VQ}{It} = \frac{2995.2(1.5)(2)(6)}{\frac{1}{12}(6)(5)^3(6)} = 143.77\ \text{psf} = 0.9984\ \text{psi}$$

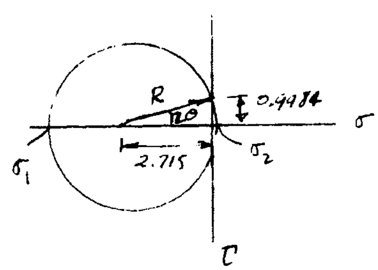


$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 - 5.4302}{2} = -2.715\ \text{psi}$$

$$R = \sqrt{(0.9984)^2 + (2.715)^2} = 2.8929\ \text{psi}$$

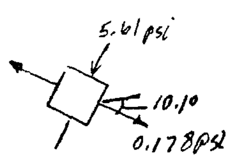
$\sigma_1 = 2.8929 - 2.715 = 0.178$ psi **Ans**

$\sigma_2 = -(2.715 + 2.8929) = -5.61$ psi **Ans**

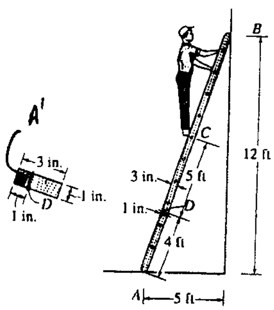


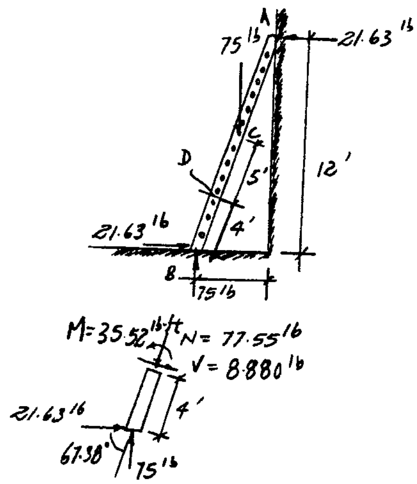
$$2\theta = \tan^{-1}\left(\frac{0.9984}{2.715}\right) = 20.2°$$

$\theta = 10.1°$



•9–84 The ladder is supported on the rough surface at A and by a smooth wall at B. If a man weighing 150 lb stands upright at C, determine the principal stresses in one of the legs at point D. Each leg is made from a 1-in.-thick board having a rectangular cross section. Assume that the total weight of the man is exerted vertically on the rung at C and is shared equally by each of the ladder's two legs. Neglect the weight of the ladder and the forces developed by the man's arms.



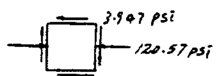


$$A = 3(1) = 3 \text{ in}^2 \qquad I = \frac{1}{12}(1)(3^3) = 2.25 \text{ in}^4$$

$$Q_D = y'A' = (1)(1)(1) = 1 \text{ in}^3$$

$$\sigma_D = \frac{-P}{A} - \frac{My}{I} = \frac{-77.55}{3} - \frac{35.52(12)(0.5)}{2.25} = -120.570 \text{ psi}$$

$$\tau_D = \frac{VQ_D}{It} = \frac{8.88(1)}{2.25(1)} = 3.947 \text{ psi}$$

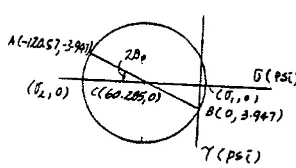


$A(-120.57, -3.947)$ $\quad B(0, 3.947)$ $\quad C(-60.285, 0)$

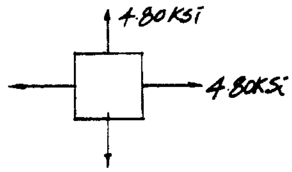
$$R = \sqrt{(60.285)^2 + (3.947)^2} = 60.412$$

$\sigma_1 = -60.285 + 60.4125 = 0.129$ psi **Ans**

$\sigma_2 = -60.285 - 60.4125 = -121$ psi **Ans**

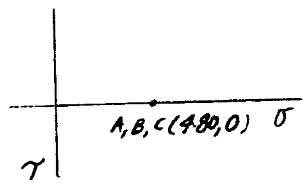


9–85 A spherical pressure vessel has an inner radius of 5 ft and a wall thickness of 0.5 in. Draw Mohr's circle for the state of stress at a point on the vessel and explain the significance of the result. The vessel is subjected to an internal pressure of 80 psi.



Normal stress :

$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t} = \frac{80(5)(12)}{2(0.5)} = 4.80 \text{ ksi}$$

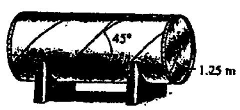


Mohr's circle :

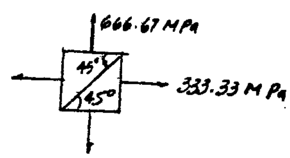
$A(4.80, 0)$ $B(4.80, 0)$ $C(0, 0)$

Regardless of the orientation of the element, the shear stress is zero and the state of stress is represented by the same two normal stress components.

9-86. The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.



$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

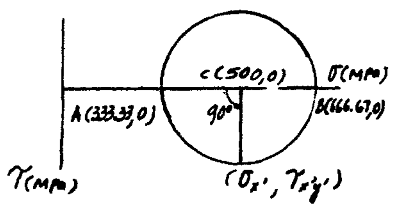


$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$

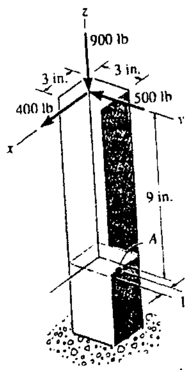
$A(333.33,0)$ $\quad B(666.67,0)$ $\quad C(500,0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa} \qquad \textbf{Ans}$$

$$\tau_{x'y'} = R = 666.67 - 500 = 167 \text{ MPa} \qquad \textbf{Ans}$$



9–87 The post has a square cross-sectional area. If it is fixed-supported at its base and the loadings are applied at its end as shown, determine (*a*) the maximum in-plane shear stress developed at *A* and (*b*) the principal stresses at *A*.



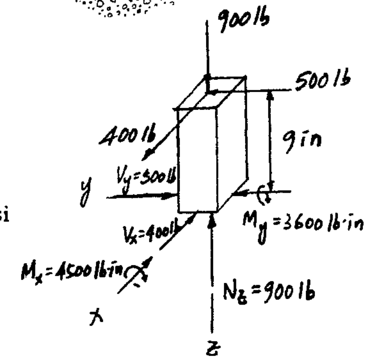
Section properties :

$$I_x = I_y = \frac{1}{12}(3)(3^3) = 6.75\ \text{in}^4$$

$$A = 3(3) = 9\ \text{in}^2$$

$$(Q_A)_x = \bar{y}'A' = (1)(1)(3) = 3\ \text{in}^3$$

$$(Q_A)_y = 0$$

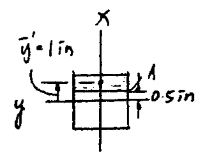


Normal stress : Applying $\sigma = \frac{P}{A} + \frac{M_x y}{I_x} + \frac{M_y x}{I_y}$

$$\sigma_A = -\frac{900}{9} + \frac{4500(1.5)}{6.75} - \frac{3600(0.5)}{6.75} = 633.33\ \text{psi}$$

Shear stress : Applying $\tau = \frac{VQ}{It}$

$$\tau_{zx} = \frac{400(3)}{6.75(3)} = 59.259\ \text{psi}$$

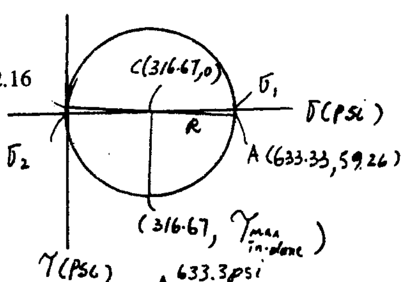


$$\tau_{zy} = 0$$

Mohr's circle :

$A(633.33, 59.26)$ $\quad C(316.67, 0)$

$$R = CA = \sqrt{(633.33 - 316.67)^2 + 59.26^2} = 322.16$$



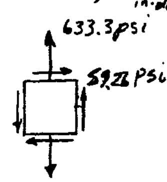
a)

$$\tau_{\substack{max \\ in-plane}} = R = 322\ \text{psi} \qquad \textbf{Ans}$$

b)

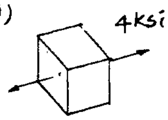
$$\sigma_1 = 316.67 + 322.16 = 639\ \text{psi} \qquad \textbf{Ans}$$

$$\sigma_2 = 316.67 - 322.16 = -5.50\ \text{psi} \qquad \textbf{Ans}$$



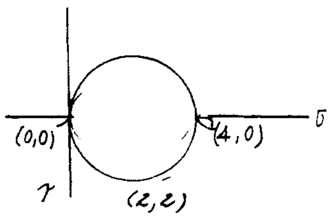
•9–88 Draw the three Mohr's circles that describe each of the following states of stress.

a)

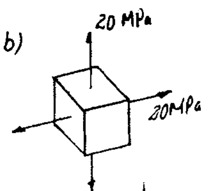


$\sigma_1 = 4$ ksi

$\sigma_2 = 0 \quad \sigma_3 = 0$

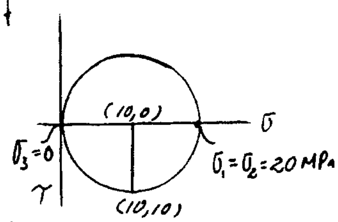


b)

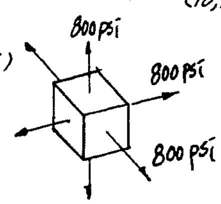


$\sigma_1 = \sigma_2 = 20$ MPa

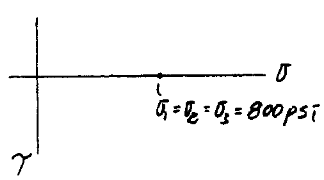
$\sigma_3 = 0$

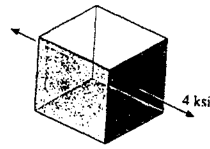


c)

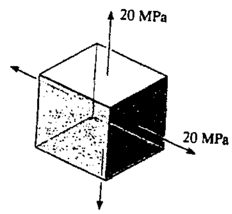


$\sigma_1 = \sigma_2 = \sigma_3 = 800$ psi

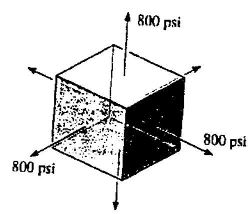




(a)

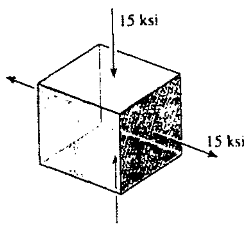


(b)

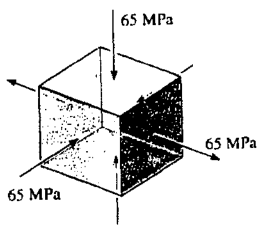


(c)

9–89 Draw the three Mohr's circles that describe each of the following states of stress.

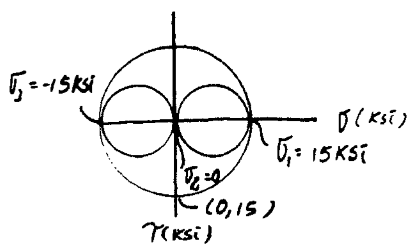


(a)

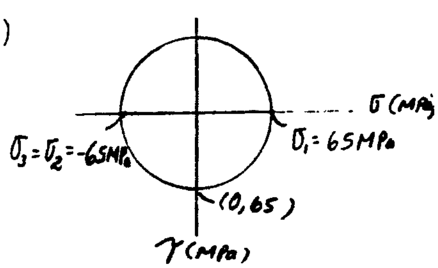


(b)

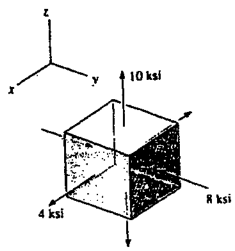
a)

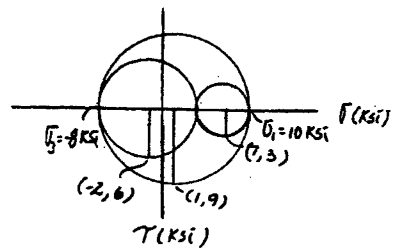


b)



9-90. The principal stresses acting at a point in a body are shown. Draw the three Mohr's circles that describe this state of stress and find the maximum in-plane shear stresses and associated average normal stresses for the x–y, y–z, and x–z planes. For each case, show the results on the element oriented in the appropriate direction.





From Mohr's circle :

For x-y plane :

$\tau_{max} = 6.0$ ksi   **Ans**

$\sigma_{avg} = -2.0$ ksi   **Ans**

For x-z plane :

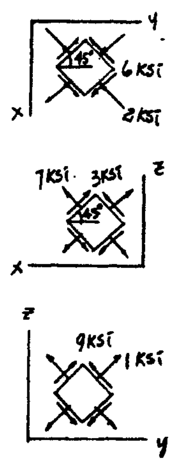
$\tau_{max} = 3.0$ ksi   **Ans**

$\sigma_{avg} = 7.0$ ksi   **Ans**

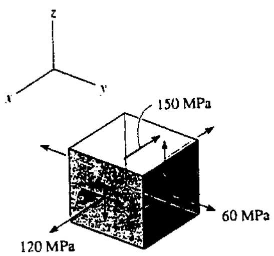
For y-z plane :

$\tau_{max} = 9.0$ ksi   **Ans**

$\sigma_{avg} = 1.0$ ksi   **Ans**



9–91 The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.

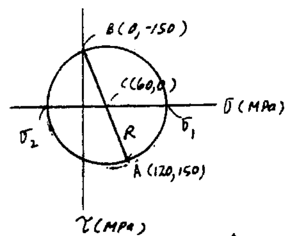


For $x-y$ plane :

$$R = CA = \sqrt{(120-60)^2 + 150^2} = 161.55$$

$$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$$

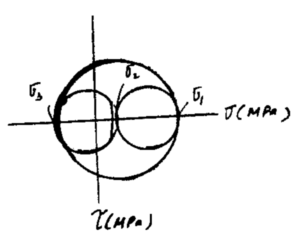
$$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$$



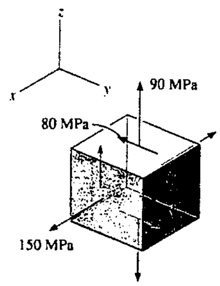
Three Mohr's circles

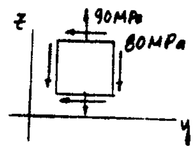
$\sigma_1 = 222$ MPa $\quad \sigma_2 = 60.0$ MPa $\quad \sigma_3 = -102$ MPa **Ans**

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{221.55 - (-101.55)}{2} = 162 \text{ MPa}$$ **Ans**



•9–92 The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.





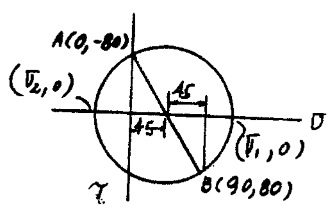
For y-z plane :

$A(0,-80) \qquad B(90,80) \qquad C(45,0)$

$R = \sqrt{45^2 + 80^2} = 91.79$

$\sigma_1 = 45 + 91.79 = 136.79$ MPa

$\sigma_2 = 45 - 91.79 = -46.79$ MPa



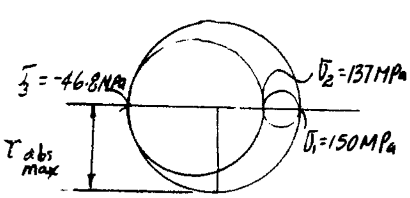
Thus,

$\sigma_1 = 150$ MPa **Ans**

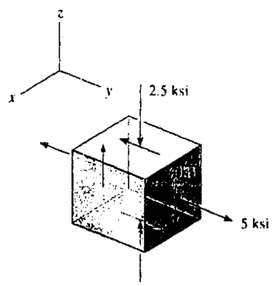
$\sigma_2 = 137$ MPa **Ans**

$\sigma_3 = -46.8$ MPa **Ans**

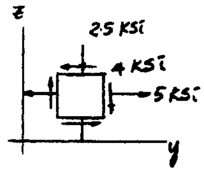
$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{150 - (-46.8)}{2} = 98.4 \text{ MPa} \qquad \textbf{Ans}$$



9–93 The state of stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



For y - z plane :



$A(5,-4)$ $B(-2.5,4)$ $C(1.25,0)$

$R = \sqrt{3.75^2 + 4^2} = 5.483$

$\sigma_1 = 1.25 + 5.483 = 6.733$ ksi

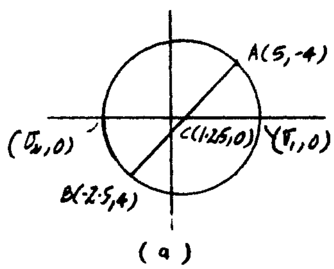
$\sigma_2 = 1.25 - 5.483 = -4.233$ ksi

Thus,

$\sigma_1 = 6.73$ ksi **Ans**

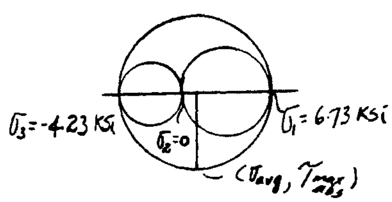
$\sigma_2 = 0$ **Ans**

$\sigma_3 = -4.23$ ksi **Ans**



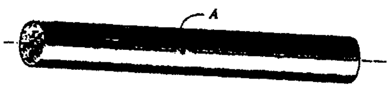
$$\sigma_{avg} = \frac{6.73 + (-4.23)}{2} = 1.25 \text{ ksi}$$

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.73 - (-4.23)}{2} = 5.48 \text{ ksi} \quad \textbf{Ans}$$



Because of the number and variety of potential correct solutions to this problem, no solution is being given.

9-95. The solid cylinder having a radius r is placed in a sealed container and subjected to a pressure p. Determine the stress components acting at point A located on the center line of the cylinder. Draw Mohr's circles for the element at this point.

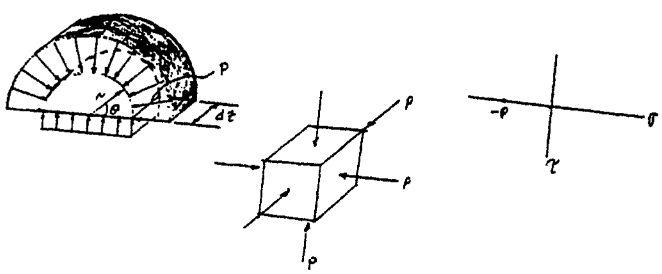


$$-\sigma(dz)(2r) = \int_0^{\pi} p(r\,d\theta)\,dz \sin\theta$$

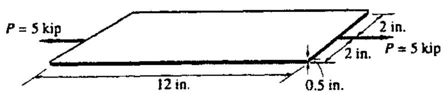
$$-\sigma = p\int_0^{\theta} \sin\theta\,d\theta = p(-\cos\theta)\Big|_0^{\pi}$$

$$\sigma = -p$$

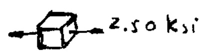
The stress in every direction is $\sigma_1 = \sigma_2 = \sigma_3 = -p$ **Ans**



*9–96 The plate is subjected to a tensile force $P = 5$ kip. If it has the dimensions shown, determine the principal stresses and the absolute maximum shear stress. If the material is ductile it will fail in shear. Make a sketch of the plate showing how this failure would appear. If the material is brittle the plate will fail due to the principal stresses. Show how this failure occurs.



$$\sigma = \frac{P}{A} = \frac{5000}{(4)(0.5)} = 2500 \text{ psi} = 2.50 \text{ ksi}$$



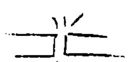
$\sigma_1 = 2.50$ ksi **Ans**

$\sigma_2 = \sigma_3 = 0$ **Ans**

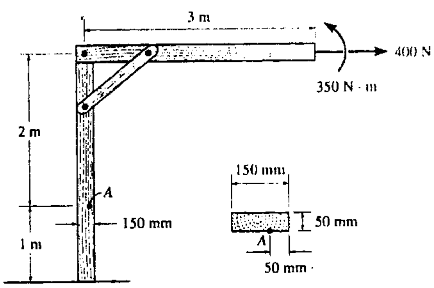
$$\tau_{\substack{abs \\ max}} = \frac{\sigma_1}{2} = 1.25 \text{ ksi}$$ **Ans**

Failure by shear :

Failure by principal stress :



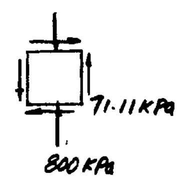
9–97 The frame is subjected to a horizontal force and couple moment at its end. Determine the principal stresses and the absolute maximum shear stress at point A. The cross-sectional area at this point is shown.



$$I = \frac{1}{12}(0.05)(0.15^3) = 14.0625(10^{-6})\ \text{m}^4$$

$$Q_A = 0.05(0.05)(0.05) = 0.125(10^{-3})\ \text{m}^3$$

$$\sigma_A = -\frac{Mx}{I} = -\frac{450(0.025)}{14.0625(10^{-6})} = -800\ \text{kPa}$$



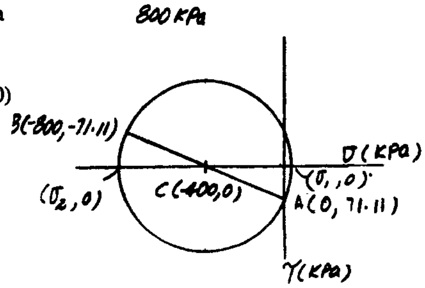
$$\tau_A = \frac{VQ_A}{It} = \frac{400(0.125)(10^{-3})}{14.0625(10^{-6})(0.05)} = 71.11\ \text{kPa}$$

$A(0,71.11)$ $B(-800,-71.11)$ $C(-400,0)$

$R = \sqrt{400^2 + 71.11^2} = 406.272$

$\sigma_1 = -400 + 406.272 = 6.72\ \text{kPa}$

$\sigma_2 = -400 - 406.272 = -806\ \text{kPa}$



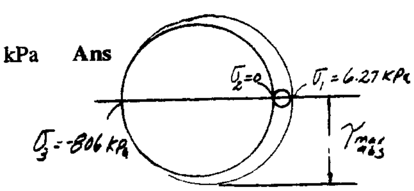
Thus,

$\sigma_1 = 6.27\ \text{kPa}$ **Ans**

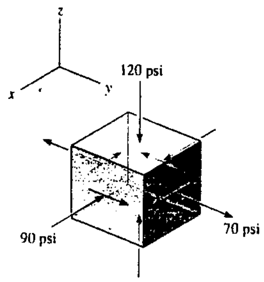
$\sigma_2 = 0$ **Ans**

$\sigma_3 = -806\ \text{kPa}$ **Ans**

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.27 - (-806.27)}{2} = 406\ \text{kPa} \quad \textbf{Ans}$$



9–98 The stress at a point is shown on the element. Determine the principal stresses and the absolute maximum shear stress.



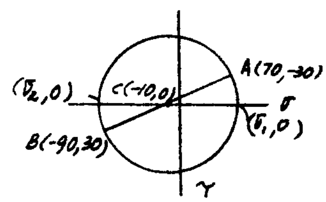
For x - y plane :

$A(70,-30)$ $B(-90,30)$ $C(-10,0)$

$$R = \sqrt{80^2 + 30^2} = 85.44$$

$$\sigma_1 = -10 + 85.44 = 75.44 \text{ psi}$$

$$\sigma_2 = -10 - 85.44 = -95.44 \text{ psi}$$



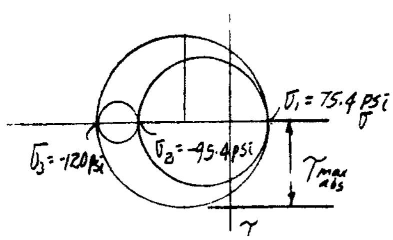
Here

$\sigma_1 = 75.4$ psi **Ans**

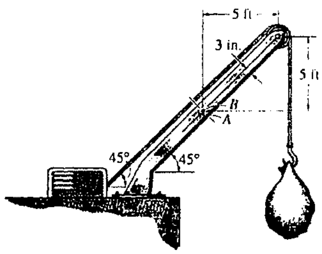
$\sigma_2 = -95.4$ psi **Ans**

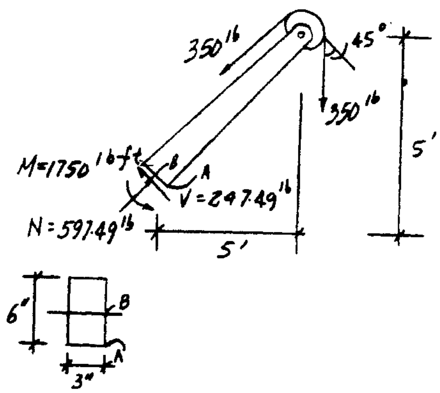
$\sigma_3 = -120$ psi **Ans**

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{75.44 - (-120)}{2} = 97.7 \text{ psi} \qquad \textbf{Ans}$$



9–99 The crane is used to support the 350-lb load. Determine the principal stresses acting in the boom at points A and B. The cross section is rectangular and has a width of 6 in. and a thickness of 3 in. Use Mohr's circle.





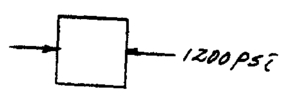
$A = 6(3) = 18\ \text{in}^2 \qquad I = \frac{1}{12}(3)(6^3) = 54\ \text{in}^4$

$Q_B = (1.5)(3)(3) = 13.5\ \text{in}^3$

$Q_A = 0$

For point A:

$$\sigma_A = -\frac{P}{A} - \frac{My}{I} = -\frac{597.49}{18} - \frac{1750(12)(3)}{54} = -1200\ \text{psi}$$

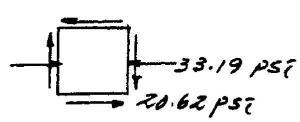


$\tau_A = 0$

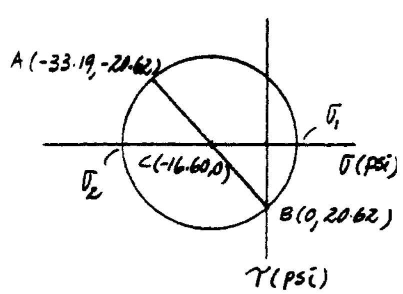
$\sigma_1 = 0$ **Ans** $\qquad \sigma_2 = -1200\ \text{psi} = -1.20\ \text{ksi}$ **Ans**

For point B:

$$\sigma_B = -\frac{P}{A} = -\frac{597.49(13.5)}{18} = -33.19\ \text{psi}$$



$$\tau_B = \frac{VQ_B}{It} = \frac{247.49}{54(3)} = 20.62\ \text{psi}$$



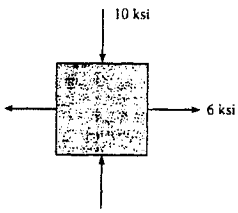
$A(-33.19, -20.62) \qquad B(0, 20.62) \qquad C(-16.60, 0)$

$R = \sqrt{16.60^2 + 20.62^2} = 26.47$

$\sigma_1 = -16.60 + 26.47 = 9.88\ \text{psi}$ **Ans**

$\sigma_2 = -16.60 - 26.47 = -43.1\ \text{psi}$ **Ans**

*9–100 Determine the equivalent state of stress if an element is oriented 40° clockwise from the element shown. Use Mohr's circle.



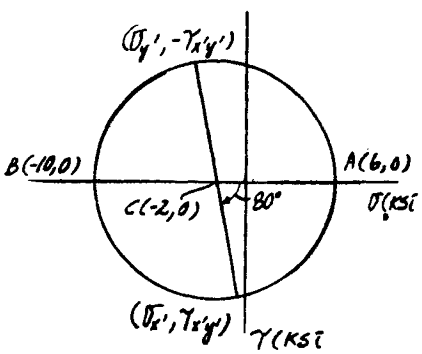
$A(6,0)$ $B(-10,0)$ $C(-2,0)$

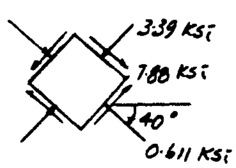
$R = CA = CB = 8$

$\sigma_{x'} = -2 + 8 \cos 80° = -0.611$ ksi **Ans**

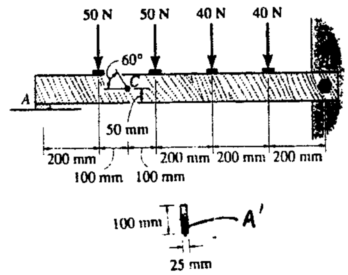
$\tau_{x'y'} = 8 \sin 80° = 7.88$ ksi **Ans**

$\sigma_{y'} = -2 - 8 \cos 80° = -3.39$ ksi **Ans**



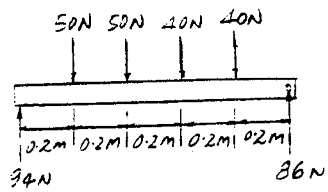


9-101. The wooden strut is subjected to the loading shown. Determine the principal stresses that act at point C and specify the orientation of the element at this point. The strut is supported by a bolt (pin) at B and smooth support at A.



$$Q_C = \bar{y}'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

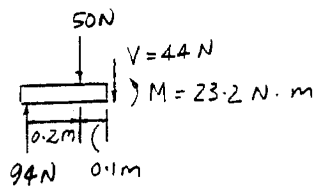
$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$



Normal stress : $\sigma_C = 0$

Shear stress :

$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$



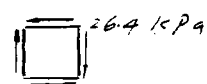
Principal stress :

$$\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\ \text{kPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$= 0 \pm \sqrt{0 + (26.4)^2}$$

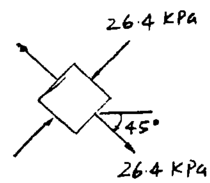
$$\sigma_1 = 26.4\ \text{kPa} \quad ; \quad \sigma_2 = -26.4\ \text{kPa}$$



Orientation of principal stress :

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{(\sigma_x - \sigma_y)}{2}} = -\infty$$

$$\theta_p = +45° \text{ and } -45°$$



Use Eq. 9 - 1 to determine the principal plane of σ_1 and σ_2

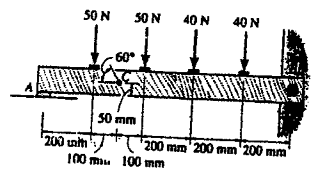
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

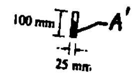
$$\theta = \theta_p = -45°$$

$$\sigma_{x'} = 0 + 0 + (-26.4)\sin -90° = 26.4\ \text{kPa}$$

Therefore, $\theta_{p_1} = -45°; \quad \theta_{p_2} = 45°$ **Ans**

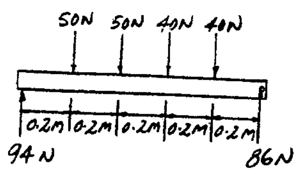
9-102. The wooden strut is subjected to the loading shown. If grains of wood in the strut at point C make an angle of 60° with the horizontal as shown, determine the normal and shear stresses that act perpendicular and parallel to the grains, respectively, due to the loading. The strut is supported by a bolt (pin) at B and smooth support at A.

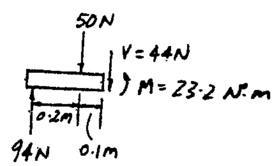




$$Q_C = \bar{y}'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$

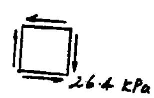




Normal stress : $\sigma_C = 0$

Shear stress :

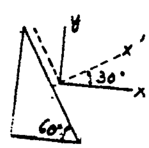
$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$



Stress transformation : $\sigma_x = \sigma_y = 0$; $\tau_{xy} = -26.4$ kPa; $\theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

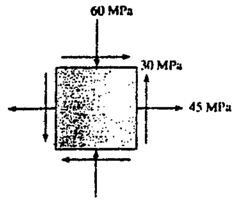
$$= 0 + 0 + (-26.4)\sin 60° = -22.9\ \text{kPa} \qquad \textbf{Ans}$$



$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -0 + (-26.4)\cos 60° = -13.2\ \text{kPa} \qquad \textbf{Ans}$$

9-103. The state of stress at a point is shown on the element. Determine (*a*) the principal stresses and (*b*) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case.

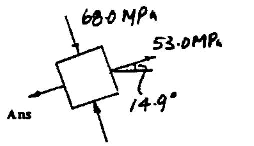


a) $\sigma_x = 45$ MPa $\quad \sigma_y = -60$ MPa $\quad \tau_{xy} = 30$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2}$$

$\sigma_1 = 53.0$ MPa $\qquad \sigma_2 = -68.0$ MPa



$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{30}{\frac{45 - (-60)}{2}} = 0.5714$$

$\theta_p = 14.87°$ and $-75.13°$

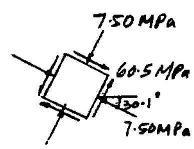
Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = \theta_p = 14.87°$

$$\sigma_{x'} = \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30\sin 29.74° = 53.0 \text{ MPa}$$

Therefore, $\theta_{p_1} = 14.9°$; $\qquad \theta_{p_2} = -75.1°$ $\qquad$ Ans



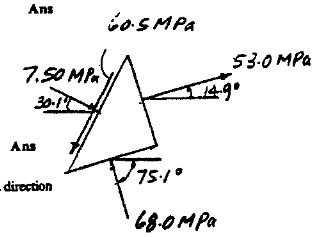
b) $$\tau_{\max\text{ in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5 \text{ MPa}$$ Ans

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa}$$ Ans

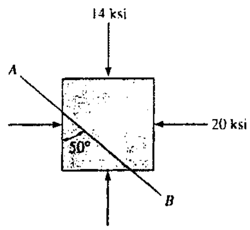
$$\tan 2\theta_s = -\frac{\frac{(\sigma_x - \sigma_y)}{2}}{\tau_{xy}} = -\frac{\frac{(45 - (-60))}{2}}{30} = -1.75$$

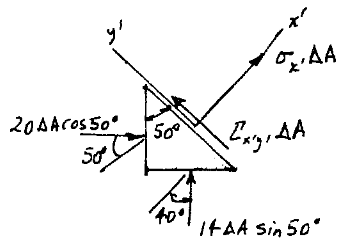
$\theta_s = -30.1°$ Ans and $59.9°$ $\qquad$ Ans

By observation, in order to preserve equilibrium, $\tau_{max} = 60.5$ MPa has to act in the direction shown in the figure.



•9–104 The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.





$\nearrow + \Sigma F_{x'} = 0;\qquad \sigma_{x'}\Delta A + 14\,\Delta A \sin 50^\circ \cos 40^\circ + 20\,\Delta A \cos 50^\circ \cos 50^\circ = 0$

$\sigma_{x'} = -16.5\text{ ksi}$ **Ans**

$\nwarrow + \Sigma F_{y'} = 0;\qquad \tau_{x'y'}\,\Delta A + 14\,\Delta A \sin 50^\circ \sin 40^\circ - 20\,\Delta A \cos 50^\circ \sin 50^\circ = 0$

$\tau_{x'y'} = 2.95\text{ ksi}$ **Ans**

10–1 Prove that the sum of the normal strains in perpendicular directions is constant.

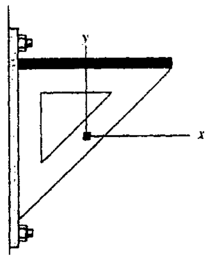
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (2)$$

Adding Eq. (1) and Eq. (2) yields :

$\varepsilon_{x'} + \varepsilon_{y'} = \varepsilon_x + \varepsilon_y = \text{constant}$ **QED**

10–2 The state of strain at the point on the bracket has components $\epsilon_x = -200(10^{-6})$, $\epsilon_y = -650(10^{-6})$, $\gamma_{xy} = -175(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 20°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



$$\varepsilon_x = -200(10^{-6}) \qquad \varepsilon_y = -650(10^{-6}) \qquad \gamma_{xy} = -175(10^{-6}) \qquad \theta = 20°$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

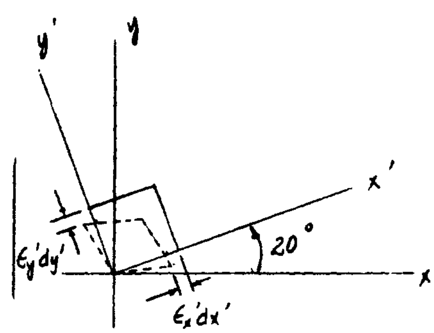
$$= [\frac{-200 + (-650)}{2} + \frac{(-200) - (-650)}{2}\cos(40°) + \frac{(-175)}{2}\sin(40°)](10^{-6}) = -309(10^{-6})$$ **Ans**

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

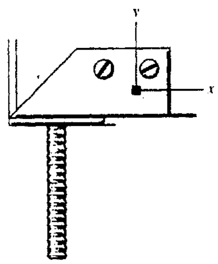
$$= [\frac{-200 + (-650)}{2} - \frac{-200 - (-650)}{2}\cos(40°) - \frac{(-175)}{2}\sin(40°)](10^{-6}) = -541(10^{-6})$$ **Ans**

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-200 - (-650))\sin(40°) + (-175)\cos(40°)](10^{-6}) = -423(10^{-6})$$ **Ans**



10–3 A differential element on the bracket is subjected to plane strain that has the following components: $\epsilon_x = 150(10^{-6})$, $\epsilon_y = 200(10^{-6})$, $\gamma_{xy} = -700(10^{-6})$. Use the strain-transformation equations and determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 60°$ counterclockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



$\varepsilon_x = 150\,(10^{-6}) \qquad \varepsilon_y = 200\,(10^{-6}) \qquad \gamma_{xy} = -700\,(10^{-6}) \qquad \theta = 60°$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

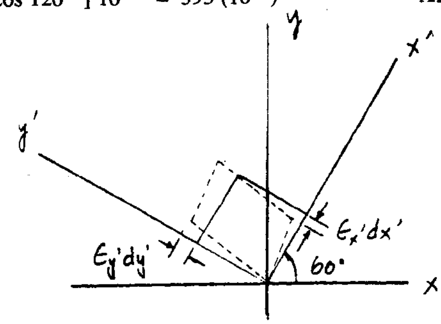
$$= [\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos 120° + (\frac{-700}{2})\sin 120°\,]\,10^{-6} = -116\,(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

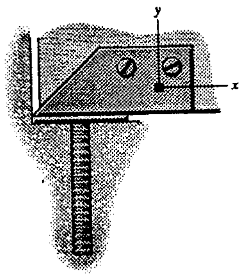
$$= [\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos 120° - (\frac{-700}{2})\sin 120°\,]\,10^{-6} = 466\,(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\,[-\frac{150 - 200}{2}\sin 120° + (\frac{-700}{2})\cos 120°\,]\,10^{-6} = 393\,(10^{-6}) \qquad \textbf{Ans}$$



*10–4 Solve Prob. 10-3 for an element oriented $\theta = 30°$ clockwise.



$\varepsilon_x = 150(10^{-6}) \qquad \varepsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \theta = -30°$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

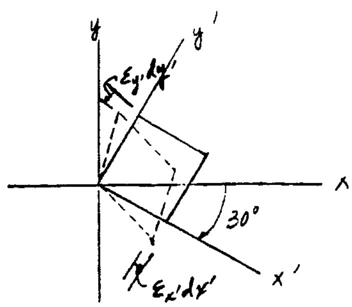
$$= \left[\frac{150 + 200}{2} + \frac{150 - 200}{2}\cos(-60°) + \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6} = 466(10^{-6})$$ **Ans**

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

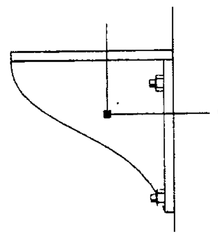
$$= \left[\frac{150 + 200}{2} - \frac{150 - 200}{2}\cos(-60°) - \left(\frac{-700}{2}\right)\sin(-60°)\right]10^{-6} = -116(10^{-6})$$ **Ans**

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2\left[-\frac{150 - 200}{2}\sin(-60°) + \frac{-700}{2}\cos(-60°)\right]10^{-6} = -393(10^{-6})$$ **Ans**



10–5 The state of strain at the point on the bracket has components $\epsilon_x = 400(10^{-6})$, $\epsilon_y = -250(10^{-6})$, $\gamma_{xy} = 310(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ clockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



$\varepsilon_x = 400(10^{-6}) \qquad \varepsilon_y = -250(10^{-6}) \qquad \gamma_{xy} = 310(10^{-6}) \qquad \theta = -30°$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

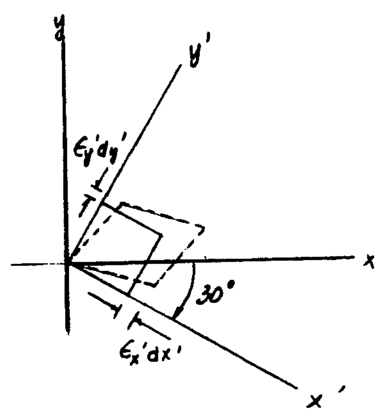
$$= [\frac{400 + (-250)}{2} + \frac{400 - (-250)}{2}\cos(-60°) + (\frac{310}{2})\sin(-60°)](10^{-6}) = 103(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= [\frac{400 + (-250)}{2} - \frac{400 - (-250)}{2}\cos(-60°) - \frac{310}{2}\sin(-60°)](10^{-6}) = 46.7(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(400 - (-250))\sin(-60°) + 310\cos(-60°)](10^{-6}) = 718(10^{-6}) \qquad \textbf{Ans}$$



10-6 The state of strain at the point on the wrench has components $\epsilon_x = 120(10^{-6})$, $\epsilon_y = -180(10^{-6})$, $\gamma_{xy} = 150(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x-y plane.

$\varepsilon_x = 120(10^{-6}) \qquad \varepsilon_y = -180(10^{-6}) \qquad \gamma_{xy} = 150(10^{-6})$

a) $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{120 + (-180)}{2} \pm \sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right] 10^{-6}$$

$\varepsilon_1 = 138\,(10^{-6}); \qquad \varepsilon_2 = -198(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{150}{[120 - (-180)]} = 0.5$$

$\theta_p = 13.28°$ and $-76.72°$

Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2

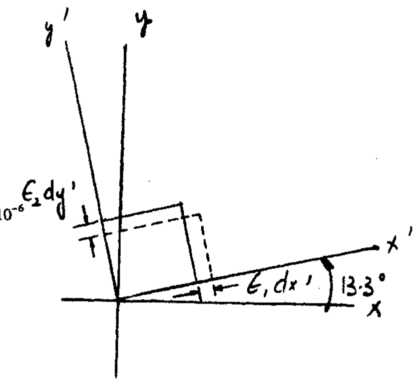
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = 13.28°$

$$\varepsilon_{x'} = \left[\frac{120 + (-180)}{2} + \frac{120 - (-180)}{2}\cos(26.56°) + \frac{150}{2}\sin 26.56°\right] 10^{-6}$$

$$= 138\,(10^{-6}) = \varepsilon_1$$

Therefore $\theta_{p_1} = 13.3°; \qquad \theta_{p_2} = -76.7°$ **Ans**



b) $$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]10^{-6} = 335\,(10^{-6})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{120 + (-180)}{2}\right]10^{-6} = -30.0\,(10^{-6})$$ **Ans**

Orientation of γ_{max}

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-[120 - (-180)]}{150} = -2.0$$

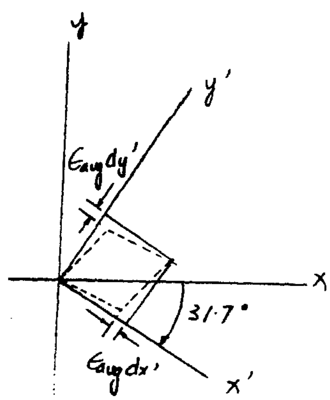
$\theta_s = -31.7°$ and $58.3°$ **Ans**

Use Eq. 10 - 11 to determine the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$

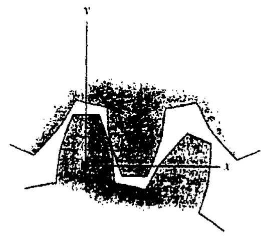
$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$\theta = \theta_s = -31.7°$

$$\gamma_{x'y'} = 2\left[-\frac{120 - (-180)}{2}\sin(-63.4°) + \frac{150}{2}\cos(-63.4°)\right]10^{-6} = 335\,(10^{-6})$$



10–7 The state of strain at the point on the gear tooth has components $\epsilon_x = 850(10^{-6})$, $\epsilon_y = 480(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



$\varepsilon_x = 850(10^{-6}) \qquad \varepsilon_y = 480(10^{-6}) \qquad \gamma_{xy} = 650(10^{-6})$

a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

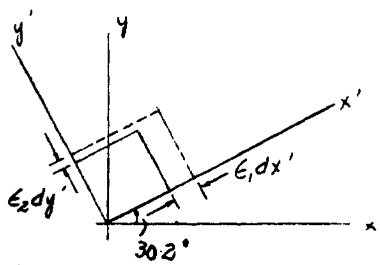
$$= \left[\frac{850+480}{2} \pm \sqrt{\left(\frac{850-480}{2}\right)^2 + \left(\frac{650}{2}\right)^2}\right](10^{-6})$$

$\varepsilon_1 = 1039(10^{-6})$ **Ans** $\qquad \varepsilon_2 = 291(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{650}{850-480}$$

$\theta_p = 30.18° \quad$ and $\quad 120.18°$



Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2 :

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = 30.18°$

$$\varepsilon_{x'} = \left[\frac{850+480}{2} + \frac{850-480}{2}\cos(60.35°) + \frac{650}{2}\sin(60.35°)\right](10^{-6}) = 1039(10^{-6})$$

Therefore, $\theta_{p1} = 30.2°$ **Ans** $\qquad \theta_{p2} = 120°$ **Ans**

b)

$$\frac{\gamma_{\max\ \text{in-plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

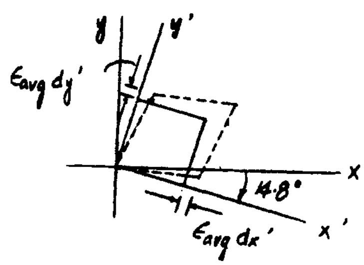
$$\gamma_{\max\ \text{in-plane}} = 2\left[\sqrt{\left(\frac{850-480}{2}\right)^2 + \left(\frac{650}{2}\right)^2}\right](10^{-6}) = 748(10^{-6}) \quad \textbf{Ans}$$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{850+480}{2}\right)(10^{-6}) = 665(10^{-6}) \quad \textbf{Ans}$$

Orientation of $\gamma_{\max}$:

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-(850-480)}{650}$$

$\theta_s = -14.8°$ and $75.2°$ **Ans**

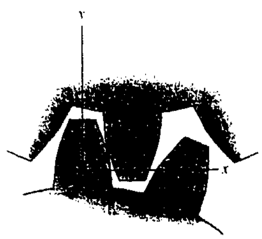


Use Eq 10 - 6 to determine the sign of $\gamma_{\max\ \text{in-plane}}$:

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta; \qquad \theta = \theta_s = -14.8°$$

$$\gamma_{x'y'} = [-(850-480)\sin(-29.65°) + 650\cos(-29.65°)](10^{-6}) = 748(10^{-6})$$

*10–8 The state of strain at the point on the gear tooth has the components $\epsilon_x = 520(10^{-6})$, $\epsilon_y = -760(10^{-6})$, $\gamma_{xy} = -750(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the *x–y* plane.



$\varepsilon_x = 520(10^{-6}) \qquad \varepsilon_y = -760(10^{-6}) \qquad \gamma_{xy} = -750(10^{-6})$

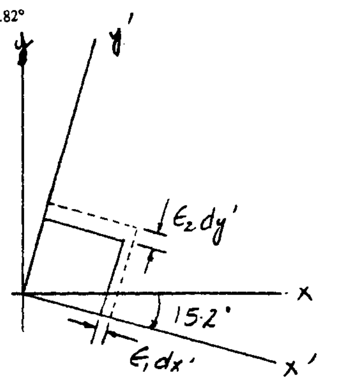
a) $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{520 + (-760)}{2} \pm \sqrt{\left(\frac{520 - (-760)}{2}\right)^2 + \left(\frac{-750}{2}\right)^2}\right] 10^{-6}$$

$\varepsilon_1 = 622\,(10^{-6})\,; \qquad \varepsilon_2 = -862(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-750}{[520 - (-760)]} = -0.5859\,; \qquad \theta_p = -15.18^\circ \text{ and } \theta_p = 74.82^\circ$$



Use Eq. 10-5 to determine the direction of ε_1 and ε_2.

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -15.18^\circ$

$$\varepsilon_{x'} = \left[\frac{520 + (-760)}{2} + \frac{520 - (-760)}{2}\cos(-30.36^\circ) + \frac{-750}{2}\sin(-30.36^\circ)\right] 10^{-6}$$

$$= 622\,(10^{-6}) = \varepsilon_1$$

Therefore $\theta_{p_1} = -15.2^\circ$ and $\theta_{p_2} = 74.8^\circ$ **Ans**

b) $$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

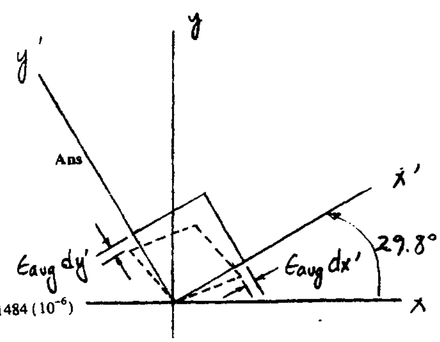
$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{520 - (-760)}{2}\right)^2 + \left(\frac{-750}{2}\right)^2}\right]10^{-6} = -1484\,(10^{-6})$$ **Ans**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{520 + (-760)}{2}\right]10^{-6} = -120\,(10^{-6})$$ **Ans**

Orientation of $\gamma_{\substack{max \\ in\text{-}plane}}$:

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-[520 - (-760)]}{-750} = 1.7067$$

$\theta_s = 29.8^\circ$ and $\theta_s = -60.2^\circ$ **Ans**

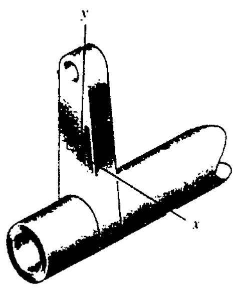


Use Eq. 10-6 to check the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta\,; \qquad \theta = \theta_s = 29.8^\circ$$

$$\gamma_{x'y'} = 2\left[-\frac{520 - (-760)}{2}\sin(59.6^\circ) + \frac{-750}{2}\cos(59.6^\circ)\right]10^{-6} = -1484\,(10^{-6})$$

10–9 The state of strain at the point on the arm has components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = -450(10^{-6})$ $\gamma_{xy} = -825(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the $x-y$ plane.



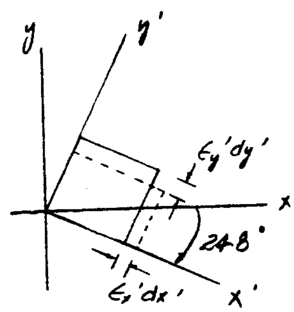
$\epsilon_x = 250(10^{-6})$ $\quad \epsilon_y = -450(10^{-6})$ $\quad \gamma_{xy} = -825(10^{-6})$

a)

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250-450}{2} \pm \sqrt{\left(\frac{250-(-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6})$$

$\epsilon_1 = 441(10^{-6})$ **Ans** $\quad \epsilon_2 = -641(10^{-6})$ **Ans**



Orientation of ϵ_1 and ϵ_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-825}{250-(-450)}$$

$\theta_p = -24.84°$ and $\theta_p = 65.16°$

Use Eq. 10-5 to determine the direction of ϵ_1 and ϵ_2 :

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -24.84°$

$$\epsilon_{x'} = \left[\frac{250-450}{2} + \frac{250-(-450)}{2}\cos(-49.69°) + \frac{-825}{2}\sin(-49.69°)\right](10^{-6}) = 441(10^{-6})$$

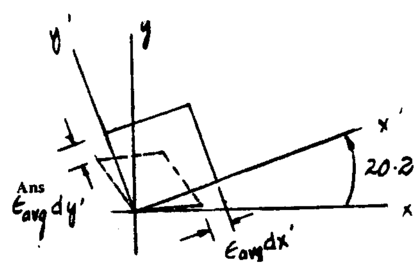
Therefore, $\theta_{p1} = -24.8°$ **Ans** $\quad \theta_{p2} = 65.2°$ **Ans**

b)

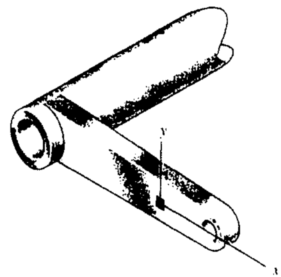
$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{250-(-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6}) = 1.08(10^{-3})$$ **Ans**

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{250-450}{2}\right)(10^{-6}) = -100(10^{-6})$$ **Ans**



10-10 The state of strain at the point on the bracket has components $\epsilon_x = -130(10^{-6})$, $\epsilon_y = 280(10^{-6})$, $\gamma_{xy} = 75(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the *x*–*y* plane.



$\epsilon_x = -130(10^{-6}) \qquad \epsilon_y = 280(10^{-6}) \qquad \gamma_{xy} = 75(10^{-6})$

a)

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

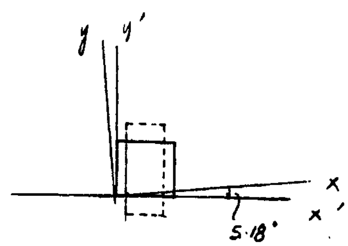
$$= \left[\frac{-130+280}{2} \pm \sqrt{\left(\frac{-130-280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right](10^{-6})$$

$\epsilon_1 = 283(10^{-6})$ **Ans** $\qquad \epsilon_2 = -133(10^{-6})$ **Ans**

Orientation of ϵ_1 and ϵ_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{75}{-130-280}$$

$\theta_p = -5.18° \quad$ and $\quad 84.82°$



Use Eq. 10-5 to determine the direction of ϵ_1 and ϵ_2 :

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -5.18°$

$$\epsilon_{x'} = \left[\frac{-130+280}{2} + \frac{-130-280}{2}\cos(-10.37°) + \frac{75}{2}\sin(-10.37°)\right](10^{-6}) = -133(10^{-6})$$

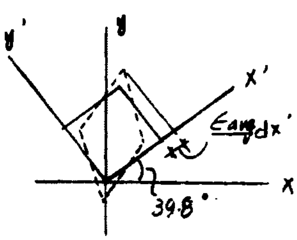
Therefore $\theta_{p1} = 84.8°$ **Ans** $\qquad \theta_{p2} = -5.18°$ **Ans**

b)

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{-130-280}{2}\right)^2 + \left(\frac{75}{2}\right)^2}\right](10^{-6}) = 417(10^{-6}) \qquad \textbf{Ans}$$

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left(\frac{-130+280}{2}\right)(10^{-6}) = 75.0(10^{-6}) \qquad \textbf{Ans}$$



Orientation of γ_{max} :

$$\tan 2\theta_s = \frac{-(\epsilon_x - \epsilon_y)}{\gamma_{xy}} = \frac{-(250-(-450))}{-825}$$

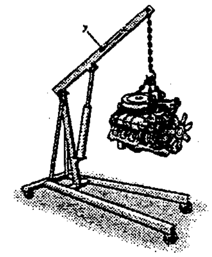
$\theta_s = 20.2°$ and $\theta_s = 110°$ **Ans**

Use Eq. 10-16 to determine the sign of $\gamma_{\substack{max \\ in\text{-}plane}}$:

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta; \qquad \theta = \theta_s = 20.2°$$

$$\gamma_{x'y'} = [-(250-(-450))\sin 40.4° + (-825)\cos 40.4°](10^{-6}) = -1082(10^{-6})$$

10–[illegible]. The state of strain at the point on the boom of the hydraulic engine crane has components of $\epsilon_x = 250(10^{-6})$, $\epsilon_y = 300(10^{-6})$, and $\gamma_{xy} = -180(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



Normal Strain and Shear strain : In accordance with the sign convention,

$$\epsilon_x = 250(10^{-6}) \qquad \epsilon_y = 300(10^{-6}) \qquad \gamma_{xy} = -180(10^{-6})$$

a)

In-Plane Principal Strain : Applying Eq. 10–9,

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250+300}{2} \pm \sqrt{\left(\frac{250-300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$$= 275 \pm 93.41$$

$$\epsilon_1 = 368(10^{-6}) \qquad \epsilon_2 = 182(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Principal Strain : Applying Eq. 10–8,

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{-180(10^{-6})}{(250-300)(10^{-6})} = 3.600$$

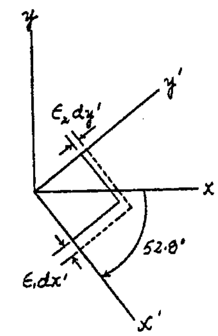
$$\theta_p = 37.24^\circ \quad \text{and} \quad -52.76^\circ$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = 37.24^\circ$.

$$\epsilon_{x'} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{250+300}{2} + \frac{250-300}{2}\cos 74.48^\circ + \frac{-180}{2}\sin 74.48^\circ\right](10^{-6})$$

$$= 182(10^{-6}) = \epsilon_2$$



Hence, $\qquad \theta_{p_1} = -52.8^\circ$ and $\theta_{p_2} = 37.2^\circ \qquad$ **Ans**

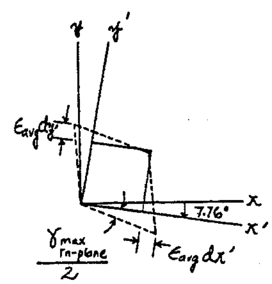
b)

Maximum In-Plane Shear Strain : Applying Eq. 10–11,

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{250-300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$$= 187(10^{-6}) \qquad \textbf{Ans}$$



Orientation of Maximum In-Plane Shear Strain : Applying Eq. 10–10,

$$\tan 2\theta_s = -\frac{\epsilon_x - \epsilon_y}{\gamma_{xy}} = -\frac{250-300}{-180} = -0.2778$$

$$\theta_s = -7.76^\circ \quad \text{and} \quad 82.2^\circ \qquad \textbf{Ans}$$

The proper sign of $\gamma_{\substack{max \\ in\text{-}plane}}$ can be determined by substituting $\theta = -7.76^\circ$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\epsilon_x - \epsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

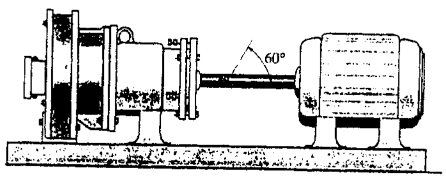
$$\gamma_{x'y'} = \{-[250-300]\sin(-15.52^\circ) + (-180)\cos(-15.52^\circ)\}(10^{-6})$$

$$= -187(10^{-6})$$

Average Normal Strain : Applying Eq. 10–12,

$$\epsilon_{avg} = \frac{\epsilon_x + \epsilon_y}{2} = \left[\frac{250+300}{2}\right](10^{-6}) = 275(10^{-6}) \qquad \textbf{Ans}$$

•10–12 A strain gauge is mounted on the 1-in.-diameter A-36 steel shaft in the manner shown. When the shaft is rotating with an angular velocity of $\omega = 1760$ rev/min, using a slip ring the reading on the strain gauge is $\epsilon = 800(10^{-6})$. Determine the power output of the motor. Assume the shaft is only subjected to a torque.



$$\omega = (1760 \text{ rev/min})(\frac{1 \text{ min}}{60 \text{ sec}})(\frac{2\pi \text{ rad}}{1 \text{ rev}}) = 184.307 \text{ rad/s}$$

$$\varepsilon_x = \varepsilon_y = 0$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$800(10^{-6}) = 0 + 0 + \frac{\gamma_{xy}}{2}\sin 120°$$

$$\gamma_{xy} = 1.848(10^{-3}) \text{ rad}$$

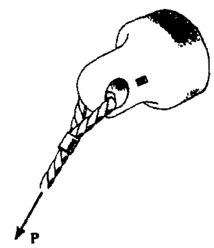
$$\tau = G\,\gamma_{xy} = 11(10^3)(1.848)(10^{-3}) = 20.323 \text{ ksi}$$

$$\tau = \frac{Tc}{J}; \qquad 20.323 = \frac{T(0.5)}{\frac{\pi}{2}(0.5)^4};$$

$T = 3.99$ lb · in = 332.5 lb · ft

$P = T\omega = 0.332.5\ (184.307) = 61.3$ lb · ft / s = 111 hp **Ans.**

10–13 The state of strain at the point on the support has components of $\epsilon_x = 350(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = -675(10^{-6})$. Use the strain-transformation equations to determine (*a*) the in-plane principal strains and (*b*) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.

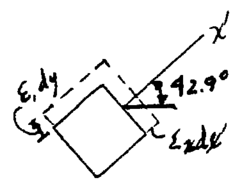


a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{350 + 400}{2} \pm \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$\varepsilon_1 = 713(10^{-6})$ **Ans** $\varepsilon_2 = 36.6(10^{-6})$ **Ans**



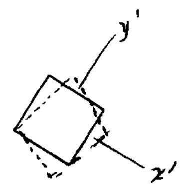
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-675}{(350 - 400)}$$

$\theta_p = 42.9°$ **Ans**

b)

$$\frac{(\gamma_{x'y'})_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{(\gamma_{x'y'})_{max}}{2} = \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$



$(\gamma_{x'y'})_{max} = 677(10^{-6})$ **Ans**

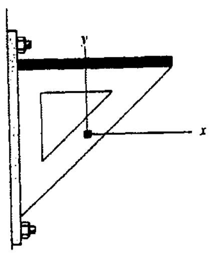
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \frac{350 + 400}{2} = 375(10^{-6})$$ **Ans**

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{350 - 400}{675}$$

$\theta_s = -2.12°$ **Ans**

Because of the number and variety of potential correct solutions to this problem, no solution is being given.

10-15 Solve Prob. 10-2 using Mohr's circle.



$\varepsilon_x = -200(10^{-6}) \qquad \varepsilon_y = -650(10^{-6}) \qquad \gamma_{xy} = -175(10^{-6}) \quad \frac{\gamma_{xy}}{2} = -87.5(10^{-6})$

$\theta = 20°, \quad 2\theta = 40°$

$A(-200,-87.5)(10^{-6}) \qquad C(-425,0)(10^{-6})$

$R = [\sqrt{(-200-(-425))^2 + 87.5^2}\,](10^{-6}) = 241.41(10^{-6})$

$\tan\alpha = \frac{87.5}{-200-(-425)}; \qquad \alpha = 21.25°$

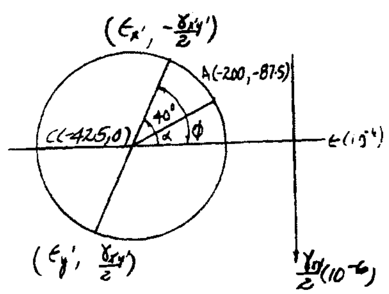
$\phi = 40 + 21.25 = 61.25°$

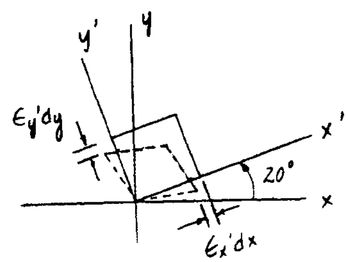
$\varepsilon_{x'} = (-425 + 241.41\cos 61.25°)(10^{-6}) = -309(10^{-6})$ **Ans**

$\varepsilon_{y'} = (-425 - 241.41\cos 61.25°)(10^{-6}) = -541(10^{-6})$ **Ans**

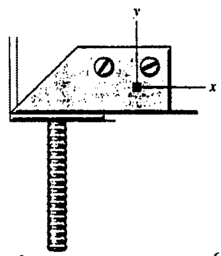
$\frac{-\gamma_{x'y'}}{2} = 241.41(10^{-6})\sin 61.25°$

$\gamma_{x'y'} = -423(10^{-6})$ **Ans**





*10–16 Solve Prob. 10–4 using Mohr's circle.



$\varepsilon_x = 150(10^{-6})$ $\varepsilon_y = 200(10^{-6})$ $\gamma_{xy} = -700(10^{-6})$ $\frac{\gamma_{xy}}{2} = -350(10^{-6})$

$\theta = -30^\circ$ $2\theta = -60^\circ$

$A(150, -350)$; $C(175, 0)$

$R = \sqrt{(175 - 150)^2 + (-350)^2} = 350.89$

Coordinates of point B :

$\varepsilon_{x'} = 350.89 \cos 34.09^\circ + 175$

$= 466(10^{-6})$ **Ans**

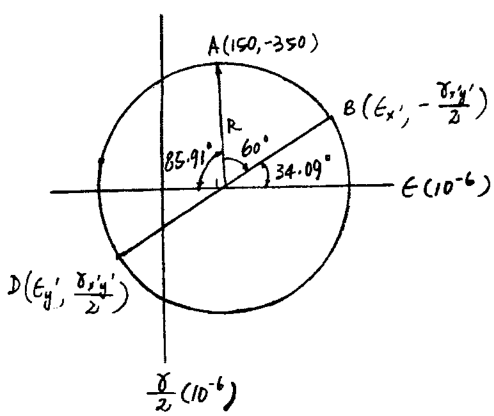
$\frac{\gamma_{x'y'}}{2} = -350.89 \sin 34.09^\circ$

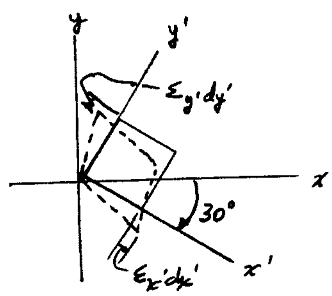
$\gamma_{x'y'} = -393(10^{-6})$ **Ans**

Coordinates of point D :

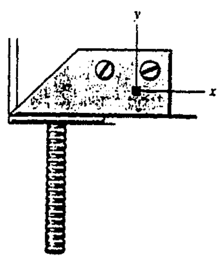
$\varepsilon_{y'} = 175 - 350.89 \cos 34.09^\circ$

$= -116(10^{-6})$ **Ans**





10–17 Solve Prob. 10-3 using Mohr's circle.



$\varepsilon_x = 150(10^{-6}) \qquad \varepsilon_y = 200(10^{-6}) \qquad \gamma_{xy} = -700(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = -350\,(10^{-6})$

$\theta = 60^\circ \quad 2\theta = 120^\circ$ (Mohr's circle)

$A\,(150,\ -350)\,10^{-6} \qquad C\,(175,\ 0)\,10^{-6}$

$R = CA = [\sqrt{(175-150)^2 + 350^2}]\,10^{-6} = 350.89\,(10^{-6})$

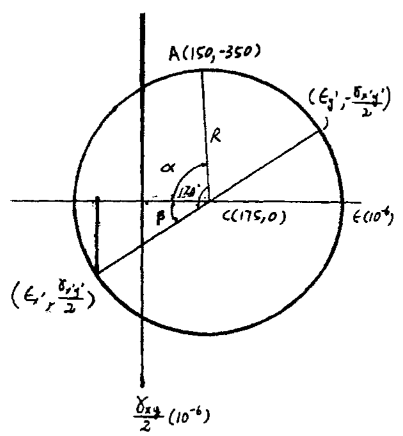
$\tan\alpha = \dfrac{350}{175-150} \qquad \alpha = 85.91^\circ$

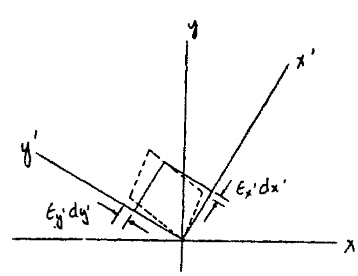
$\beta = 120 - 85.91 = 34.09^\circ$

$\varepsilon_{x'} = (175 - 350.89\cos 34.09^\circ)\,10^{-6} = -116\,(10^{-6})$ **Ans**

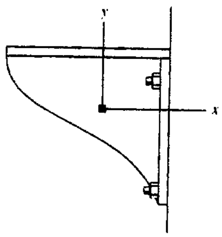
$\varepsilon_{y'} = (175 + 350.89\cos 34.09^\circ)\,10^{-6} = 466\,(10^{-6})$ **Ans**

$\gamma_{x'y'} = 2[350.89\sin 34.09^\circ]\,10^{-6} = 393\,(10^{-6})$ **Ans**





10–18 Solve Prob. 10-5 using Mohr's circle.



$\varepsilon_x = 400(10^{-6})$ $\varepsilon_y = -250(10^{-6})$ $\gamma_{xy} = 310(10^{-6})$ $\frac{\gamma_{xy}}{2} = 155(10^{-6})$ $\theta = 30°$

$A(400, 155)(10^{-6})$ $C(75, 0)(10^{-6})$

$R = [\sqrt{(400-75)^2 + 155^2}\,](10^{-6}) = 360.1(10^{-6})$

$\tan\alpha = \frac{155}{400-75};\quad \alpha = 25.50°$

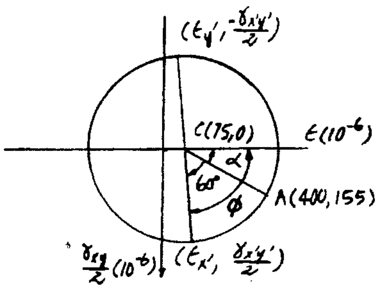
$\phi = 60 + 25.50 = 85.5°$

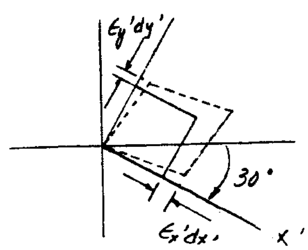
$\varepsilon_{x'} = (75 + 360.1\cos 85.5°)(10^{-6}) = 103(10^{-6})$ **Ans**

$\varepsilon_{y'} = (75 - 360.1\cos 85.5°)(10^{-6}) = 46.7(10^{-6})$ **Ans**

$\frac{\gamma_{x'y'}}{2} = (360.1\sin 85.5°(10^{-6})$

$\gamma_{x'y'} = 718(10^{-6})$ **Ans**





10–19 Solve Prob. 10-11 using Mohr's circle.

Construction of the Circle : In accordance with the sign convention, $\varepsilon_x = 250(10^{-6})$, $\varepsilon_y = 300(10^{-6})$, and $\frac{\gamma_{xy}}{2} = -90(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{250+300}{2}\right)(10^{-6}) = 275(10^{-6}) \qquad \textbf{Ans}$$

The coordinates for reference points A and C are

$$A(250,\ -90)(10^{-6}) \qquad C(275,\ 0)(10^{-6})$$

The radius of the circle is

$$R = \left(\sqrt{(275-250)^2 + 90^2}\right)(10^{-6}) = 93.408$$

In - Plane Principal Strain : The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (275 + 93.408)(10^{-6}) = 368(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_2 = (275 - 93.408)(10^{-6}) = 182(10^{-6}) \qquad \textbf{Ans}$$

Orientation of Principal Strain : From the circle,

$$\tan 2\theta_{p_2} = \frac{90}{275-250} = 3.600 \qquad 2\theta_{p_2} = 74.48°$$

$$2\theta_{p_1} = 180° - 2\theta_{p_2}$$

$$\theta_{p_1} = \frac{180° - 74.78°}{2} = 52.8° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

Maximum In - Plane Shear Strain : Represented by the coordinates of point E on the circle.

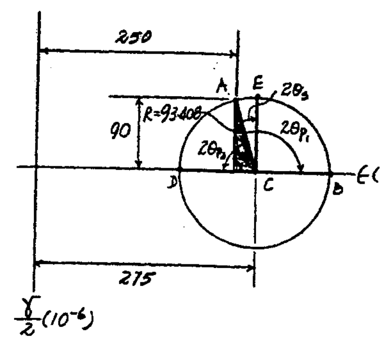
$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = -R = -93.408(10^{-6})$$

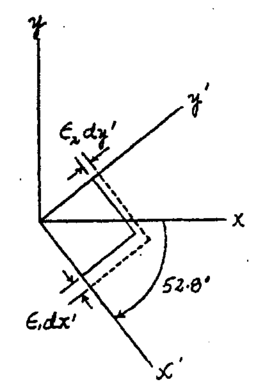
$$\gamma_{\substack{max \\ in\text{-}plane}} = -187(10^{-6}) \qquad \textbf{Ans}$$

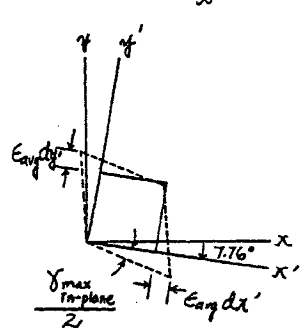
Orientation of Maximum In - Plane Shear Strain : From the circle,

$$\tan 2\theta_s = \frac{275-250}{90} = 0.2778$$

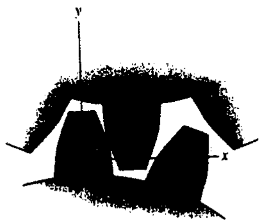
$$\theta_s = 7.76° \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$



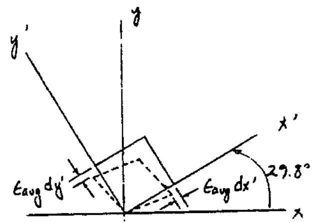




•10–20 Solve Prob. 10–8 using Mohr's circle.



a) $\varepsilon_x = 520(10^{-6})$ $\quad \varepsilon_y = -760(10^{-6})$ $\quad \gamma_{xy} = -750(10^{-6})$ $\quad \frac{\gamma_{xy}}{2} = -375(10^{-6})$

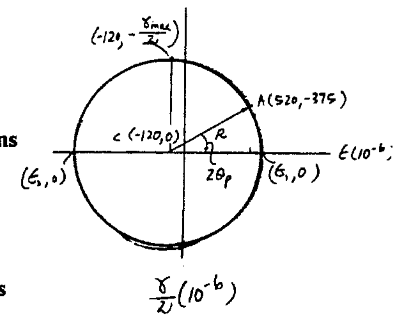


$A(520, -375)$; $\quad C(-120, 0)$

$R = \sqrt{(520 + 120)^2 + 375^2} = 741.77$

$\varepsilon_1 = 741.77 - 120 = 622(10^{-6})$ **Ans**

$\varepsilon_2 = -120 - 741.77 = -862(10^{-6})$ **Ans**



$\tan 2\theta_{p_1} = \frac{375}{(120 + 520)} = 0.5859$

$\theta_{p_1} = 15.2°$ **Ans**

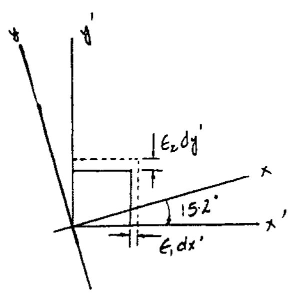
b) $\gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(741.77)$

$\gamma_{\substack{max \\ in\text{-}plane}} = -1484(10^{-6})$ **Ans**

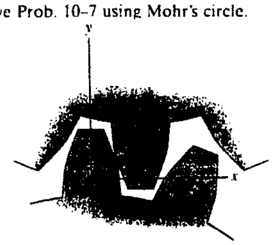
$\varepsilon_{avg} = -120(10^{-6})$ **Ans**

$\tan 2\theta_s = \frac{(120 + 520)}{375} = 1.7067$

$\theta_s = 29.8°$ **Ans**



10-21 Solve Prob. 10-7 using Mohr's circle.



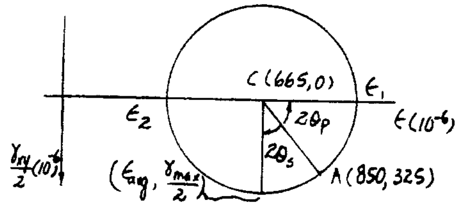
$\varepsilon_x = 850(10^{-6}) \qquad \varepsilon_y = 480(10^{-6}) \qquad \gamma_{xy} = 650(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = 325(10^{-6})$

$A(850, 325)(10^{-6}) \qquad C(665, 0)(10^{-6})$

$R = [\sqrt{(850-665)^2 + 325^2}](10^{-6}) = 373.97(10^{-6})$

$\varepsilon_1 = (665 + 373.97)(10^{-6}) = 1039(10^{-6})$ **Ans**

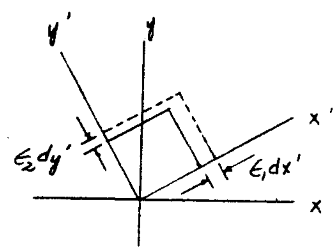
$\varepsilon_2 = (665 - 373.97)(10^{-6}) = 291(10^{-6})$ **Ans**



$\tan 2\theta_p = \dfrac{325}{850 - 665}$

$2\theta_p = 60.35°$ (Mohr's circle)

$\theta_p = 30.2°$ (element)



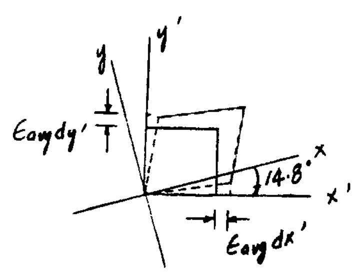
$\dfrac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = R$

$\gamma_{\substack{max \\ in\text{-}plane}} = 2(373.97)(10^{-6}) = 748(10^{-6})$ **Ans**

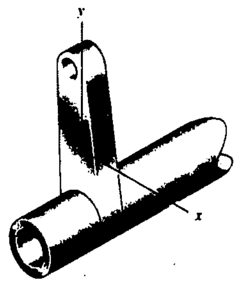
$\varepsilon_{avg} = 665(10^{-6})$ **Ans**

$2\theta_s = 90° - 2\theta_p = 29.65°$ (Mohr's circle)

$\theta_s = -14.8°$ (element)



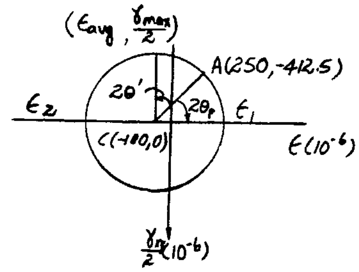
10–22 Solve Prob. 10–9 using Mohr's circle.



$\varepsilon_x = 250(10^{-6}) \qquad \varepsilon_y = -450(10^{-6}) \qquad \gamma_{xy} = -825(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = -412.5(10^{-6})$

$A(250, -412.5)(10^{-6}) \qquad C(-100, 0)(10^{-6})$

$R = [\sqrt{(250-(-100))^2 + (-412.5)^2}](10^{-6}) = 540.98(10^{-6})$



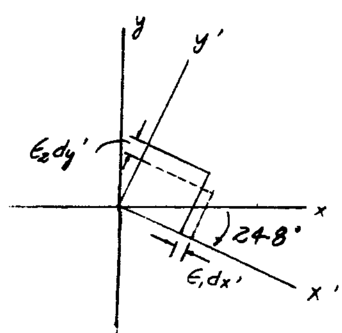
$\varepsilon_1 = (-100 + 540.98)(10^{-6}) = 441(10^{-6})$ **Ans**

$\varepsilon_2 = (-100 - 540.98)(10^{-6}) = -641(10^{-6})$ **Ans**

$\tan 2\theta_p = \frac{412.5}{250-(-100)}$

$2\theta_p = 49.68°$ (Mohr's circle)

$\theta_p = -24.8°$ (element)



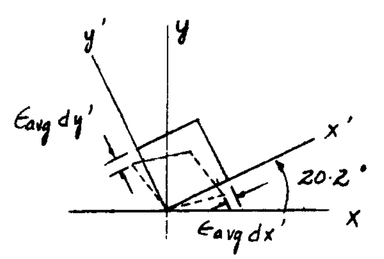
$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = R$

$\gamma_{\substack{max \\ in\text{-}plane}} = 2(540.98)(10^{-6}) = 1.08(10^{-3})$ **Ans**

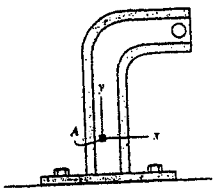
$\varepsilon_{avg} = -100(10^{-6})$ **Ans**

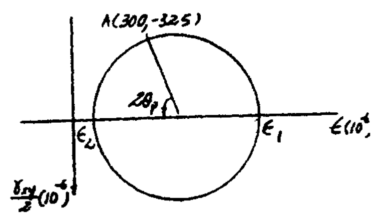
$2\theta_s - 90° - 2\theta_p = 40.32$ (Mohr's circle)

$\theta_s = 20.2°$ (element) **Ans**



10-23. The strain at point A on the bracket has components $\epsilon_x = 300(10^{-6})$, $\epsilon_y = 550(10^{-6})$, $\gamma_{xy} = -650(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.

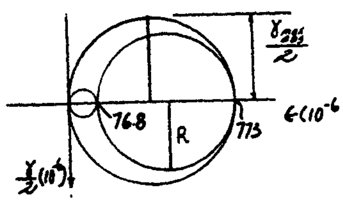




$\epsilon_x = 300(10^{-6})$ $\quad \epsilon_y = 550(10^{-6})$ $\quad \gamma_{xy} = -650(10^{-6})$ $\quad \frac{\gamma_{xy}}{2} = -325(10^{-6})$

$A(300, -325)10^{-6}$ $\quad C(425, 0)10^{-6}$

$R = [\sqrt{(425-300)^2 + (-325)^2}]10^{-6} = 348.2(10^{-6})$



a)

$\epsilon_1 = (425 + 348.2)(10^{-6}) = 773(10^{-6})$ **Ans**

$\epsilon_2 = (425 - 348.2)(10^{-6}) = 76.8(10^{-6})$ **Ans**

b)

$\gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(348.2)(10^{-6}) = 696(10^{-6})$ **Ans**

c)

$\frac{\gamma_{\substack{abs \\ max}}}{2} = \frac{773(10^{-6})}{2}$; $\quad \gamma_{\substack{abs \\ max}} = 773(10^{-6})$ **Ans**

•10–24 The strain at point A on the beam has components $\epsilon_x = 450(10^{-6})$, $\epsilon_y = 825(10^{-6})$, $\gamma_{xy} = 275(10^{-6})$, $\epsilon_z = 0$. Determine (*a*) the principal strains at A, (*b*) the maximum shear strain in the $x-y$ plane, and (*c*) the absolute maximum shear strain.

$\varepsilon_x = 450(10^{-6}) \qquad \varepsilon_y = 825(10^{-6}) \qquad \gamma_{xy} = 275(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = 137.5(10^{-6})$

$A(450, 137.5)10^{-6} \qquad C(637.5, 0)10^{-6}$

$$R = [\sqrt{(637.5-450)^2 + 137.5^2}]\,10^{-6} = 232.51(10^{-6})$$

a)

$\varepsilon_1 = (637.5 + 232.51)(10^{-6}) = 870(10^{-6})$ **Ans**

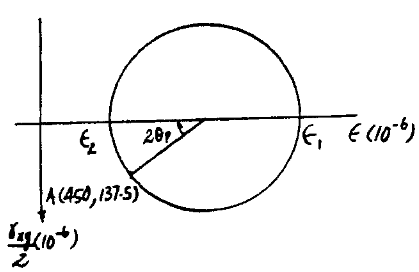
$\varepsilon_2 = (637.5 - 232.51)(10^{-6}) = 405(10^{-6})$ **Ans**

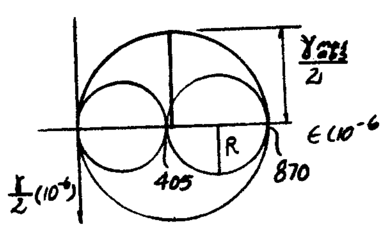
b)

$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(232.51)(10^{-6}) = 465(10^{-6})$ **Ans**

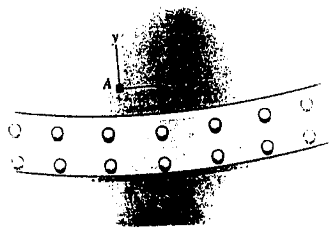
c)

$\frac{\gamma_{\substack{\text{abs} \\ \max}}}{2} = \frac{870(10^{-6})}{2}; \qquad \gamma_{\substack{\text{abs} \\ \max}} = 870(10^{-6})$ **Ans**





10–25 The strain at point A on the pressure-vessel wall has components $\epsilon_x = 480(10^{-6})$, $\epsilon_y = 720(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$, $\epsilon_z = 0$. Determine (*a*) the principal strains at A, (*b*) the maximum shear strain in the x–y plane, and (*c*) the absolute maximum shear strain.



$\varepsilon_x = 480(10^{-6}) \qquad \varepsilon_y = 720(10^{-6}) \qquad \gamma_{xy} = 650(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = 325(10^{-6})$

$A(480, 325)10^{-6} \qquad C(600, 0)10^{-6}$

$$R = \left(\sqrt{(600-480)^2 + 325^2}\right)10^{-6} = 346.44(10^{-6})$$

a)

$\varepsilon_1 = (600 + 346.44)10^{-6} = 946(10^{-6})$ **Ans**

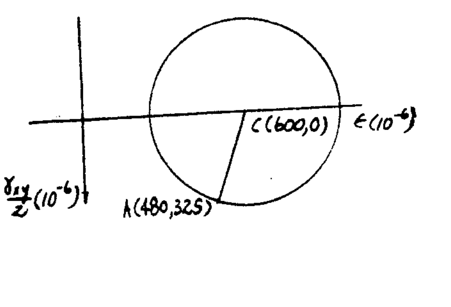
$\varepsilon_2 = (600 - 346.44)10^{-6} = 254(10^{-6})$ **Ans**

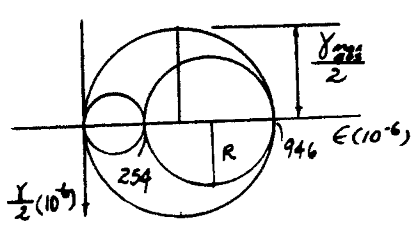
b)

$\gamma_{\substack{\text{max} \\ \text{in-plane}}} = 2R = 2(346.44)10^{-6} = 693(10^{-6})$ **Ans**

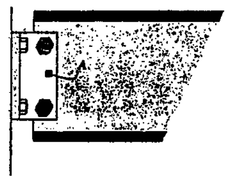
c)

$\frac{\gamma_{\substack{\text{abs} \\ \text{max}}}}{2} = \frac{946(10^{-6})}{2}; \qquad \gamma_{\substack{\text{abs} \\ \text{max}}} = 946(10^{-6})$ **Ans**





10-26. The strain at point A on the leg of the angle has components $\epsilon_x = -140(10^{-6})$, $\epsilon_y = 180(10^{-6})$, $\gamma_{xy} = -125(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



$\varepsilon_x = -140(10^{-6}) \qquad \varepsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -125(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = -62.5(10^{-6})$

$A(-140, -62.5)10^{-6} \qquad C(20, 0)10^{-6}$

$$R = \left(\sqrt{(20-(-140))^2 + (-62.5)^2}\right)10^{-6} = 171.77(10^{-6})$$

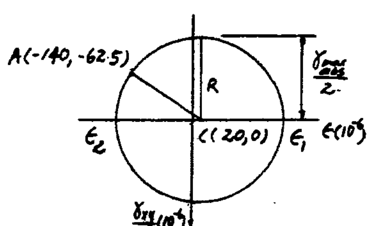
a)

$\varepsilon_1 = (20 + 171.77)(10^{-6}) = 192(10^{-6})$ **Ans**

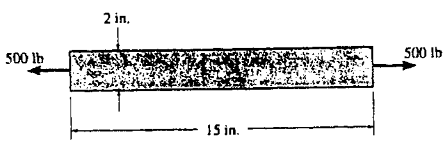
$\varepsilon_2 = (20 - 171.77)(10^{-6}) = -152(10^{-6})$ **Ans**

b,c)

$\gamma_{\substack{abs \\ max}} = \gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(171.77)(10^{-6}) = 344(10^{-6})$ **Ans**



10–27 The steel bar is subjected to the tensile load of 500 lb. If it is 0.5 in. thick determine the absolute maximum shear strain. $E = 29(10^3)$ ksi, $\nu = 0.3$.



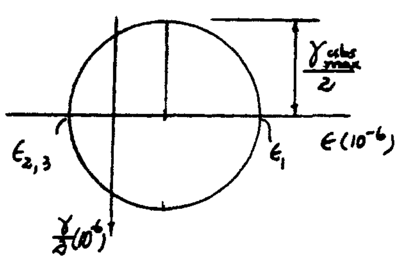
$$\sigma_x = \frac{500}{2(0.5)} = 500 \text{ psi} \qquad \sigma_y = 0 \qquad \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x) = \frac{1}{29(10^6)}(500) = 17.2414(10^{-6})$$

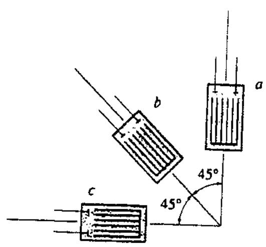
$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -0.3(17.2414)\left(10^{-6}\right) = -5.1724\left(10^{-6}\right)$$

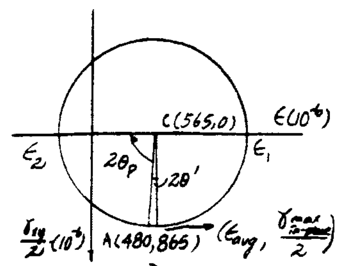
$$\varepsilon_1 = 17.2414(10^{-6}) \qquad \varepsilon_{2,3} = -5.1724(10^{-6})$$

$$\gamma_{\substack{abs \\ max}} = \varepsilon_1 - \varepsilon_2 = (17.2414 - (-5.1724))(10^{-6}) = 22.4(10^{-6}) \qquad \textbf{Ans}$$



*10–28 The 45° strain rosette is mounted on a machine element. The following readings are obtained from each gauge: $\epsilon_a = 650(10^{-6})$, $\epsilon_b = -300(10^{-6})$, $\epsilon_c = 480(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and associated average normal strain. In each case show the deformed element due to these strains.





$\varepsilon_a = 650(10^{-6})$ $\quad \varepsilon_b = -300(10^{-6})$ $\quad \varepsilon_c = 480(10^{-6})$

$\theta_a = 90°$ $\quad \theta_b = 135°$ $\quad \theta_c = 180°$

$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$

$650(10^{-6}) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy}\sin 90° \cos 90°$

$\varepsilon_y = 650(10^{-6})$

$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$

$480(10^{-6}) = \varepsilon_x \cos^2 180° + \varepsilon_y \sin^2 180° + \gamma_{xy}\sin 180° \cos 180°$

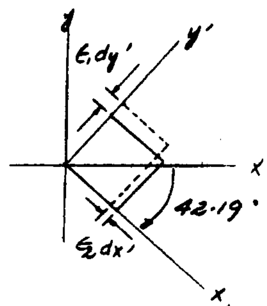
$\varepsilon_x = 480(10^{-6})$

$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$

$-300(10^{-6}) = 480(10^{-6})\cos^2 135° + 650(10^{-6})\sin^2 135° + \gamma_{xy}\sin 135° \cos 135°$

$\gamma_{xy} = 1730(10^{-6})$

$\frac{\gamma_{xy}}{2} = 865(10^{-6})$



$A(480, 865)10^{-6}$ $\quad C(565, 0)10^{-6}$

$R = [\sqrt{(565-480)^2 + 865^2}]10^{-6} = 869.17(10^{-6})$

a)

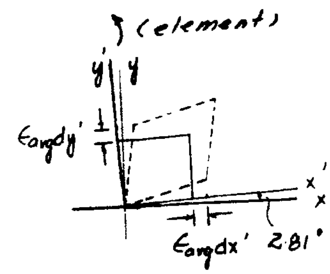
$\varepsilon_1 = (565 + 869.17)10^{-6} = 1434(10^{-6})$ **Ans**

$\varepsilon_2 = (565 - 869.17)10^{-6} = -304(10^{-6})$ **Ans**

$\tan 2\theta_p = \frac{865}{565-480}$

$2\theta_p = 84.39°$ (Mohr's circle)

$\theta_p = 42.19°$ (element)



b)

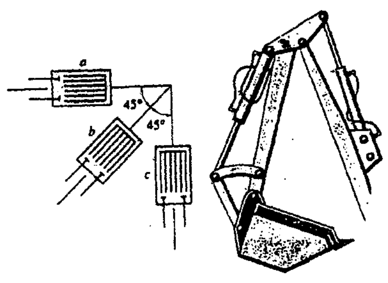
$\gamma_{\max \atop \text{in-plane}} = 2R = 2(869.17)(10^{-6}) = 1738(10^{-6})$ **Ans**

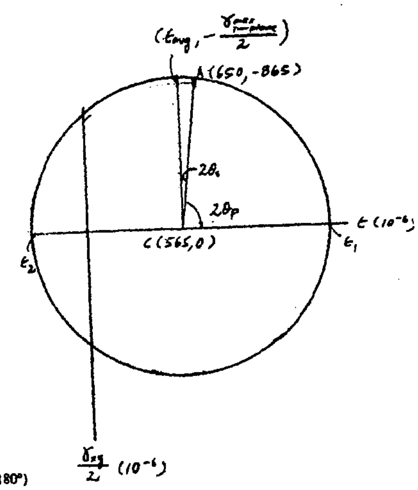
$\varepsilon_{avg} = 565(10^{-6})$ **Ans**

$2\theta_s = 90° - 2\theta_p = 5.61°$ (Mohr's circle)

$\theta_s = 2.81°$ (element)

10-29. The 45° strain rosette is mounted on the link of the backhoe. The following readings are obtained from each gauge: $\epsilon_a = 650(10^{-6})$, $\epsilon_b = -300(10^{-6})$, $\epsilon_c = 480(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and associated average normal strain.





$\epsilon_a = 650(10^{-6})$; $\epsilon_b = -300(10^{-6})$; $\epsilon_c = 480(10^{-6})$

$\theta_a = 180°$; $\theta_b = 225°$ $\theta_c = 270°$

Applying Eq. 10-15, $\epsilon = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$

$650(10^{-6}) = \epsilon_x \cos^2(180°) + \epsilon_y \sin^2(180°) + \gamma_{xy}\sin(180°)\cos(180°)$

$\epsilon_x = 650(10^{-6})$

$480(10^{-6}) = \epsilon_x \cos^2(270°) + \epsilon_y \sin^2(270°) + \gamma_{xy}\sin(270°)\cos(270°)$

$\epsilon_y = 480(10^{-6})$

$-300(10^{-6}) = 650(10^{-6})\cos^2(225°) + 480(10^{-6})\sin^2(225°) + \gamma_{xy}\sin(225°)\cos(225°)$

$\gamma_{xy} = -1730(10^{-6})$

Therefore, $\epsilon_x = 650(10^{-6})$ $\epsilon_y = 480(10^{-6})$ $\gamma_{xy} = -1730(10^{-6})$

$\frac{\gamma_{xy}}{2} = -865(10^{-6})$

Mohr's circle :

$A(650, -865)10^{-6}$ $C(565, 0)10^{-6}$

$R = CA = [\sqrt{(650-565)^2 + 865^2}]10^{-6} = 869.17(10^{-6})$

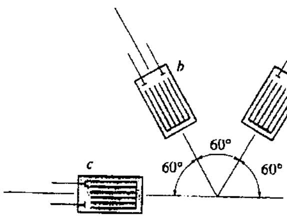
a) $\epsilon_1 = [565 + 869.17]10^{-6} = 1434(10^{-6})$ **Ans**

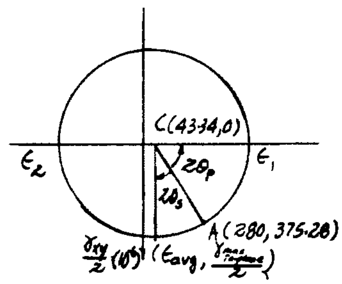
$\epsilon_2 = [565 - 869.17]10^{-6} = -304(10^{-6})$ **Ans**

b) $\gamma_{\max \text{ in-plane}} = 2R = 2(869.17)(10^{-6}) = 1738(10^{-6})$ **Ans**

$\epsilon_{avg} = 565(10^{-6})$ **Ans**

10–30 The 60° strain rosette is mounted on a beam. The following readings are obtained from each gauge: $\epsilon_a = 250(10^{-6})$, $\epsilon_b = -400(10^{-6})$, $\epsilon_c = 280(10^{-6})$. Determine (*a*) the in-plane principal strains and their orientation, and (*b*) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.





$\varepsilon_a = 250(10^{-6})$ $\quad \varepsilon_b = -400(10^{-6})$ $\quad \varepsilon_c = 280(10^{-6})$

$\theta_a = 60°$ $\quad \theta_b = 120°$ $\quad \theta_c = 180°$

$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$

$280(10^{-6}) = \varepsilon_x \cos^2 180° + \varepsilon_y \sin^2 180° + \gamma_{xy} \sin 180° \cos 180°$

$\varepsilon_x = 280(10^{-6})$

$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$

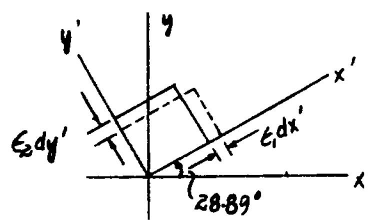
$250(10^{-6}) = \varepsilon_x \cos^2 60° + \varepsilon_y \sin^2 60° + \gamma_{xy} \sin 60° \cos 60°$

$250(10^{-6}) = 0.25\varepsilon_x + 0.75\varepsilon_y + 0.433\gamma_{xy}$ (1)

$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$

$-400(10^{-6}) = \varepsilon_x \cos^2 120° + \varepsilon_y \sin^2 120° + \gamma_{xy} \sin 120° \cos 120°$

$-400(10^{-6}) = 0.25\varepsilon_x + 0.75\varepsilon_y - 0.433\gamma_{xy}$ (2)



Subtract Eq. (2) from Eq. (1)

$650(10^{-6}) = 0.866\gamma_{xy}$

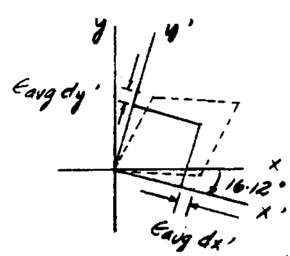
$\gamma_{xy} = 750.56(10^{-6})$

$\varepsilon_y = -193.33(10^{-6})$

$\frac{\gamma_{xy}}{2} = 375.28(10^{-6})$

$A(280, 375.28)10^{-6}$ $\quad C(43.34, 0)10^{-6}$

$R = [\sqrt{(280-43.34)^2 + 375.28^2}]10^{-6} = 443.67(10^{-6})$



a)

$\varepsilon_1 = (43.34 + 443.67)10^{-6} = 487(10^{-6})$ **Ans**

$\varepsilon_2 = (43.34 - 443.67)10^{-6} = -400(10^{-6})$ **Ans**

$\tan 2\theta_p = \frac{375.28}{280 - 43.34}$

$2\theta_p = 57.76°$ (Mohr's circle)

$\theta_p = 28.89°$ (element)

b)

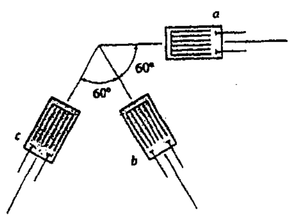
$\gamma_{\substack{max \\ in\text{-}plane}} = 2R = 2(443.67)(10^{-6}) = 887(10^{-6})$ **Ans**

$\varepsilon_{avg} = 43.3(10^{-6})$ **Ans**

$2\theta_s = 90° - 2\theta_p = 32.24°$ (Mohr's circle)

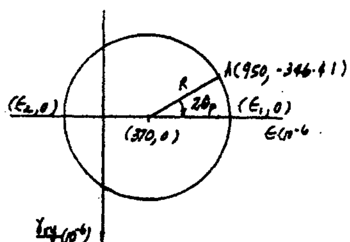
$\theta_s = 16.12°$ (element)

10–31. The 60° strain rosette is mounted on the surface of an aluminum plate. The following readings are obtained from each gauge: $\epsilon_a = 950(10^{-6})$, $\epsilon_b = 380(10^{-6})$, $\epsilon_c = -220(10^{-6})$. Determine the in-plane principal strains and their orientation.



$\varepsilon_a = 950(10^{-6}) \qquad \varepsilon_b = 380(10^{-6}) \qquad \varepsilon_c = -220(10^{-6})$

$\theta_a = 0° \qquad \theta_b = -60° \qquad \theta_c = -120°$



$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$$

$$950(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + \gamma_{xy} \sin 0° \cos 0°$$

$$\varepsilon_x = 950(10^{-6})$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy} \sin\theta_c \cos\theta_c$$

$$380(10^{-6}) = \varepsilon_x \cos^2(-60°) + \varepsilon_y \sin^2(-60°) + \gamma_{xy} \sin(-60°) \cos(-60°)$$

$$380(10^{-6}) = 0.25\varepsilon_x + 0.75\varepsilon_y - 0.433\gamma_{xy} \qquad (1)$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy} \sin\theta_b \cos\theta_b$$

$$-220(10^{-6}) = \varepsilon_x \cos^2(-120°) + \varepsilon_y \sin^2(-120°) + \gamma_{xy} \sin(-120°) \cos(-120°)$$

$$-220(10^{-6}) = 0.25\varepsilon_x + 0.75\varepsilon_y + 0.433\gamma_{xy} \qquad (2)$$

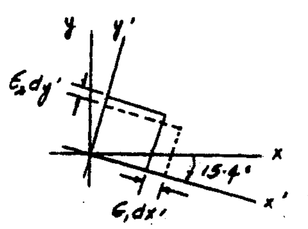
Subtract Eq. (2) from Eq. (1)

$$600(10^{-6}) = -0.866\gamma_{xy}$$

$$\gamma_{xy} = -692.82(10^{-6})$$

$$\varepsilon_y = -210(10^{-6})$$

$$\frac{\gamma_{xy}}{2} = -346.41(10^{-6})$$



$A(950, -346.41)10^{-6} \qquad C(370, 0)10^{-6}$

$$R = \{\sqrt{(950-370)^2 + 346.41^2}\}10^{-6} = 675.57(10^{-6})$$

$$\varepsilon_1 = (370 + 675.57)10^{-6} = 1046(10^{-6}) \qquad \textbf{Ans}$$

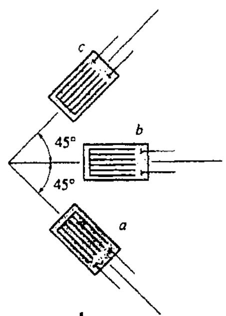
$$\varepsilon_2 = (370 - 675.57)10^{-6} = -306(10^{-6}) \qquad \textbf{Ans}$$

$$\tan 2\theta_p = \frac{346.41}{950-370}$$

$2\theta_p = 30.8°$ (Mohr's circle)

$\theta_p = -15.4°$ (element) **Ans**

•10–32 The 45° strain rosette is mounted on a steel shaft. The following readings are obtained for each gauge: $\epsilon_a = 800(10^{-6})$, $\epsilon_b = 520(10^{-6})$, $\epsilon_c = -450(10^{-6})$. Determine the in-plane principal strains and their orientation.



$\varepsilon_a = 800(10^{-6}) \qquad \varepsilon_b = 520(10^{-6}) \qquad \varepsilon_c = -450(10^{-6})$

$\theta_a = -45° \qquad \theta_b = 0° \qquad \theta_c = 45°$

$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$

$520(10^{-6}) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + \gamma_{xy}\sin 0° \cos 0°$

$\varepsilon_x = 520(10^{-6})$

$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$

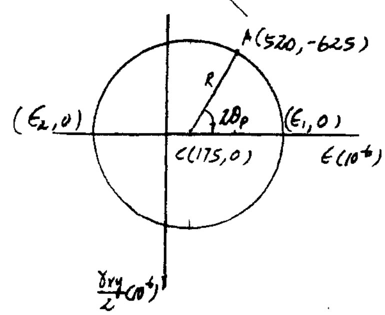
$800(10^{-6}) = \varepsilon_x \cos^2(-45°) + \varepsilon_y \sin^2(-45°) + \gamma_{xy}\sin(-45°)\cos(-45°)$

$800(10^{-6}) = 0.5\varepsilon_x + 0.5\varepsilon_y - 0.5\gamma_{xy} \qquad (1)$

$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$

$-450(10^{-6}) = \varepsilon_x \cos^2 45° + \varepsilon_y \sin^2 45° + \gamma_{xy}\sin 45° \cos 45°$

$-450(10^{-6}) = 0.5\varepsilon_x + 0.5\varepsilon_y + 0.5\gamma_{xy} \qquad (2)$



Subtract Eq. (2) from Eq. (1)

$1250(10^{-6}) = -\gamma_{xy}$

$\gamma_{xy} = -1250(10^{-6})$

$\varepsilon_y = -170(10^{-6})$

$\frac{\gamma_{xy}}{2} = -625(10^{-6})$

$A(520, -625)10^{-6} \qquad C(175, 0)10^{-6}$

$R = [\sqrt{(520-175)^2 + 625^2}]10^{-6} = 713.90(10^{-6})$

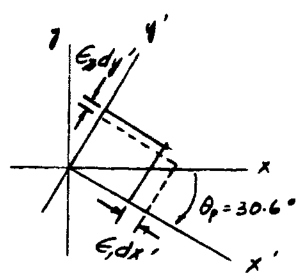
$\varepsilon_1 = (175 + 713.9)10^{-6} = 889(10^{-6})$ **Ans**

$\varepsilon_2 = (175 - 713.9)10^{-6} = -539(10^{-6})$ **Ans**

$\tan 2\theta_p = \frac{625}{520-175}$

$2\theta_p = 61.1°$ (Mohr's circle)

$\theta_p = 30.6°$ (element) **Ans**



Because of the number and variety of potential correct solutions to this problem, no solution is being given.

10-34. For the case of plane stress, show that Hooke's law can be written as

$$\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y), \qquad \sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x)$$

Generalized Hooke's Law : For plane stress, $\sigma_z = 0$.
Applying Eq. 10-18,

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\nu E\varepsilon_x = (\sigma_x - \nu\sigma_y)\nu$$

$$\nu E\varepsilon_x = \nu\sigma_x - \nu^2\sigma_y \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$E\varepsilon_y = -\nu\sigma_x + \sigma_y \qquad [2]$$

Adding Eq. [1] and Eq. [2] yields,

$$\nu E\varepsilon_x + E\varepsilon_y = \sigma_y - \nu^2\sigma_y$$

$$\sigma_y = \frac{E}{1-\nu^2}(\nu\varepsilon_x + \varepsilon_y) \qquad (Q.E.D.)$$

Substituting σ_y into Eq. [2]

$$E\varepsilon_y = -\nu\sigma_x + \frac{E}{1-\nu^2}(\nu\varepsilon_x + \varepsilon_y)$$

$$\sigma_x = \frac{E(\nu\varepsilon_x + \varepsilon_y)}{\nu(1-\nu^2)} - \frac{E\varepsilon_y}{\nu}$$

$$= \frac{E\nu\varepsilon_x + E\varepsilon_y - E\varepsilon_y + E\varepsilon_y\nu^2}{\nu(1-\nu^2)}$$

$$= \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \qquad (Q.E.D.)$$

10–35 Use Hooke's law, Eq. 10–18, to develop the strain-transformation equations, Eqs. 10–5 and 10–6, from the stress-transformation equations, Eqs. 9–1 and 9–2.

Stress transformation equations :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (1)$$

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (2)$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad (3)$$

Hooke's law :

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\,\sigma_y}{E} \qquad (4)$$

$$\varepsilon_y = \frac{-\nu\,\sigma_x}{E} + \frac{\sigma_y}{E} \qquad (5)$$

$$\tau_{xy} = G\,\gamma_{xy} \qquad (6)$$

$$G = \frac{E}{2(1+\nu)} \qquad (7)$$

From Eqs. (4) and (5)

$$\varepsilon_x + \varepsilon_y = \frac{(1-\nu)(\sigma_x + \sigma_y)}{E} \qquad (8)$$

$$\varepsilon_x - \varepsilon_y = \frac{(1+\nu)(\sigma_x - \sigma_y)}{E} \qquad (9)$$

From Eqs. (6) and (7)

$$\tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} \qquad (10)$$

From Eq. (4)

$$\varepsilon_{x'} = \frac{\sigma_{x'}}{E} - \frac{\nu\,\sigma_{y'}}{E} \qquad (11)$$

Substitute Eqs. (1) and (3) into Eq. (11)

$$\varepsilon_{x'} = \frac{(1-\nu)(\sigma_x + \sigma_y)}{2E} + \frac{(1+\nu)(\sigma_x - \sigma_y)}{2E}\cos 2\theta + \frac{(1+\nu)\tau_{xy}\sin 2\theta}{E} \qquad (12)$$

By using Eqs. (8), (9) and (10) and substitute into Eq. (12),

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{QED}$$

From Eq. (6),

$$\tau_{x'y'} = G\,\gamma_{x'y'} = \frac{E}{2(1+\nu)}\gamma_{x'y'} \qquad (13)$$

Substitute Eqs. (13), (6) and (9) into Eq.(2),

$$\frac{E}{2(1+\nu)}\gamma_{x'y'} = -\frac{E(\varepsilon_x - \varepsilon_y)}{2(1+\nu)}\sin 2\theta + \frac{E}{2(1+\nu)}\gamma_{xy}\cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\varepsilon_x - \varepsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \qquad \textbf{QED}$$

*10–36 A bar of copper alloy is loaded in a tension machine and it is determined that $\epsilon_x = 940(10^{-6})$ and $\sigma_x = 14$ ksi, $\sigma_y = 0$, $\sigma_z = 0$. Determine the modulus of elasticity, E_{cu}, and the dilatation, e_{cu}, of the copper. $\nu_{cu} = 0.35$.

$$\varepsilon_x = \frac{1}{E}[\sigma_x - v(\sigma_y + \sigma_z)]$$

$$940(10^{-6}) = \frac{1}{E_{cu}}[14(10^3) - 0.35(0+0)]$$

$$E_{cu} = 14.9(10^3)\text{ ksi}$$ **Ans**

$$e_{cu} = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{1-2(0.35)}{14.9(10^3)}(14+0+0) = 0.282(10^{-3})$$ **Ans**

10–37 The principal plane stresses and associated strains in a plane at a point are $\sigma_1 = 36$ ksi, $\sigma_2 = 16$ ksi, $\epsilon_1 = 1.02(10^{-3})$, $\epsilon_2 = 0.180(10^{-3})$. Determine the modulus of elasticity and Poisson's ratio.

$$\sigma_3 = 0$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

$$1.02(10^{-3}) = \frac{1}{E}[36 - \nu\,(16)]$$

$$1.02(10^{-3})E = 36 - 16\nu \qquad (1)$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu\,(\sigma_1 + \sigma_3)]$$

$$0.180(10^{-3}) = \frac{1}{E}[16 - \nu\,(36)]$$

$$0.180(10^{-3})E = 16 - 36\nu \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$E = 30.7(10^3)$ ksi **Ans**

$\nu = 0.291$ **Ans**

10–38 Determine the bulk modulus for each of the following materials: (*a*) rubber, $E_r = 0.4$ ksi, $\nu_r = 0.48$, and (*b*) glass, $E_g = 8(10^3)$ ksi, $\nu_g = 0.24$.

a) For rubber :

$$K_r = \frac{E_r}{3\,(1 - 2\,\nu_r)} = \frac{0.4}{3[1 - 2(0.48)]} = 3.33 \text{ ksi} \qquad \textbf{Ans}$$

b) For glass :

$$K_g = \frac{E_g}{3\,(1 - 2\,\nu_g)} = \frac{8(10^3)}{3[1 - 2(0.24)]} = 5.13\,(10^3) \text{ ksi} \qquad \textbf{Ans}$$

10-39 The principal strains at a point on the aluminum fuselage of a jet aircraft are $\epsilon_1 = 780(10^{-6})$ and $\epsilon_2 = 400(10^{-6})$. Determine the associated principal stresses at the point in the same plane. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.33$. *Hint:* See Prob. 10-34.

Plane stress, $\sigma_3 = 0$

Use the formula developed in Prob. 10 - 34.

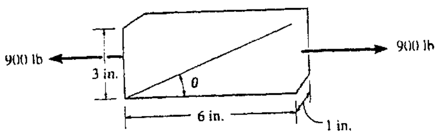
$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

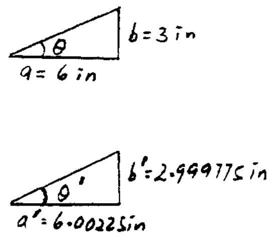
$$= \frac{10(10^3)}{1-0.33^2}(780(10^{-6}) + 0.33(400)(10^{-6})) = 10.2 \text{ ksi} \qquad \textbf{Ans}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

$$= \frac{10(10^3)}{1-0.33^2}(400(10^{-6}) + 0.33(780)(10^{-6})) = 7.38 \text{ ksi} \qquad \textbf{Ans}$$

•10–40 The polyvinyl chloride bar is subjected to an axial force of 900 lb. If it has the original dimensions shown, determine the *change* in the angle θ after the load is applied. $E_{pvc} = 800(10^3)$ psi, $\nu_{pvc} = 0.20$.





$$\sigma_x = \frac{900}{3(1)} = 300 \text{ psi}$$

$$\sigma_y = 0 \qquad \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$= \frac{1}{800(10^3)}[300 - 0] = 0.375\,(10^{-3})$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$
$$= \frac{1}{800(10^3)}[0 - 0.2(300 + 0)] = -75\,(10^{-6})$$

$$a' = 6 + 6\,(0.375)(10^{-3}) = 6.00225 \text{ in.}$$

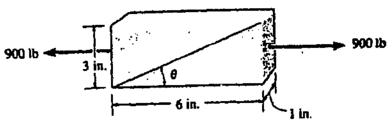
$$b' = 3 + 3\,(-75)(10^{-6}) = 2.999775 \text{ in.}$$

$$\theta = \tan^{-1}(\frac{3}{6}) = 26.56505118°$$

$$\theta' = \tan^{-1}(\frac{2.999775}{6.00225}) = 26.55474088°$$

$$\Delta\theta = \theta - \theta' = 26.56505118° - 26.55474088° = 0.0103°$$ **Ans**

10-41. The polyvinyl chloride bar is subjected to an axial force of 900 lb. If it has the original dimensions shown, determine the value of Poisson's ratio if the angle θ decreases by $\Delta\theta = 0.01°$ after the load is applied. $E_{pvc} = 800(10^3)$ psi.



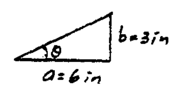
$$\sigma_x = \frac{900}{3(1)} = 300 \text{ psi} \qquad \sigma_y = 0 \qquad \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu_{pvc}(\sigma_y + \sigma_z)]$$

$$= \frac{1}{800(10^3)}[300 - 0] = 0.375(10^{-3})$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu_{pvc}(\sigma_x + \sigma_z)]$$

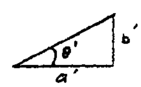
$$= \frac{1}{800(10^3)}[0 - \nu_{pvc}(300 + 0)] = -0.375(10^{-3})\nu_{pvc}$$



$$a' = 6 + 6(0.375)(10^{-3}) = 6.00225 \text{ in.}$$

$$b' = 3 + 3(-0.375)(10^{-3})\nu_{pvc} = 3 - 1.125(10^{-3})\nu_{pvc}$$

$$\theta = \tan^{-1}\left(\frac{3}{6}\right) = 26.56505118°$$



$$\theta' = 26.56505118° - 0.01° = 26.55505118°$$

$$\tan\theta' = 0.49978185 = \frac{3 - 1.125(10^{-3})\nu_{pvc}}{6.00225}$$

$$\nu_{pvc} = 0.164 \qquad \textbf{Ans}$$

10–42 A rod has a radius of 10 mm. If it is subjected to an axial load of 15 N such that the axial strain in the rod is $\epsilon_x = 2.75(10^{-6})$, determine the modulus of elasticity E and the change in its diameter. $\nu = 0.23$.

$$\sigma_x = \frac{15}{\pi(0.01)^2} = 47.746 \text{ kPa}, \qquad \sigma_y = 0, \qquad \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$2.75(10^{-6}) = \frac{1}{E}[47.746(10^3) - 0.23(0 + 0)]$$

$E = 17.4$ GPa **Ans**

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -0.23(2.75)(10^{-6}) = -0.632(10^{-6})$$

$\Delta d = 20(-0.632(10^{-6})) = -12.6(10^{-6})$ mm **Ans**

10–43 The principal strains at a point on the aluminum surface of a tank are $\epsilon_1 = 630(10^{-6})$ and $\epsilon_2 = 350(10^{-6})$. If this is a case of plane stress, determine the associated principal stresses at the point in the same plane. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.33$. *Hint:* See Prob. 10–34.

For plane stress $\sigma_3 = 0$.

Use the formula developed in Prob. 10 - 34.

$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$= \frac{10\,(10^3)}{1 - 0.33^2}[630\,(10^{-6}) + 0.33\,(350)(10^{-6})]$$

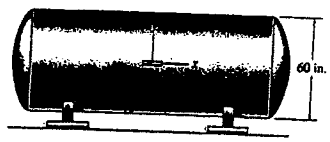
$$= 8.37 \text{ ksi}$$ **Ans**

$$\sigma_2 = \frac{E}{1 - \nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

$$= \frac{10\,(10^3)}{1 - 0.33^2}[350\,(10^{-6}) + 0.33\,(630)(10^{-6})]$$

$$= 6.26 \text{ ksi}$$ **Ans**

***10-44.** The strain gauge is placed on the surface of a thin-walled steel boiler as shown. If it is 0.5 in. long, determine the pressure in the boiler when the gauge elongates $0.2(10^{-3})$ in. The boiler has a thickness of 0.5 in. and inner diameter of 60 in. Also, determine the maximum x, y in-plane shear strain in the material. $E_{st} = 29(10^3)$ ksi, $\nu_{st} = 0.3$.



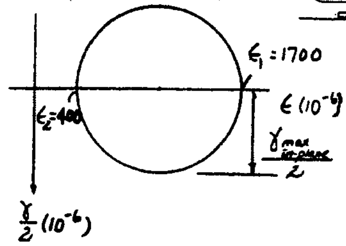
$$\varepsilon_2 = \frac{0.2(10^{-3})}{0.5} = 400(10^{-6})$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)]$$

where, $\sigma_2 = \frac{1}{2}\sigma_1 \qquad \sigma_3 = 0$

$$400(10^{-6}) = \frac{1}{29(10^3)}[\frac{1}{2}\sigma_1 - 0.3\sigma_1]$$

$$\sigma_1 = 58 \text{ ksi}$$



Thus,

$$p = \frac{\sigma_1 t}{r} = \frac{58(0.5)}{30} = 0.967 \text{ ksi} \qquad \textbf{Ans}$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$$

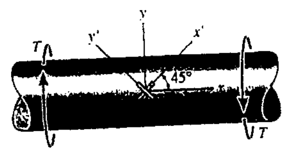
where, $\sigma_3 = 0$ and $\sigma_2 = \frac{58}{2} = 29$ ksi

$$\varepsilon_1 = \frac{1}{29(10^3)}[58 - 0.3(29 + 0)] = 1700(10^{-6})$$

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = \frac{\varepsilon_1 - \varepsilon_2}{2}$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = (1700 - 400)(10^{-6}) = 1.30(10^{-3}) \qquad \textbf{Ans}$$

10–45 The steel shaft has a radius of 15 mm. Determine the torque T in the shaft if the two strain gauges, attached to the surface of the shaft, report strains of $\epsilon_{x'} = -80(10^{-6})$ and $\epsilon_{y'} = 80(10^{-6})$. Also, compute the strains acting in the x and y directions. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



$\varepsilon_{x'} = -80(10^{-6}) \qquad \varepsilon_{y'} = 80(10^{-6})$

Pure shear $\quad \varepsilon_x = \varepsilon_y = 0 \qquad$ **Ans**

$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$$
$$\theta = 45°$$

$$-80(10^{-6}) = 0 + 0 + \gamma_{xy} \sin 45° \cos 45°$$
$$\gamma_{xy} = -160(10^{-6}) \qquad \textbf{Ans}$$

Also, $\theta = 135°$

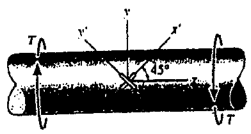
$$80(10^{-6}) = 0 + 0 + \gamma \sin 135° \cos 135°$$
$$\gamma_{xy} = -160(10^{-6})$$

$$G = \frac{E}{2(1+\nu)} = \frac{200(10^9)}{2(1+0.3)} = 76.923(10^9)$$

$$\tau = G\gamma = 76.923(10^9)(160)(10^{-6}) = 12.308(10^6)\ \text{Pa}$$

$$T = \frac{\tau J}{c} = \frac{12.308(10^6)(\frac{\pi}{2})(0.015)^4}{0.015} = 65.2\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

10-46. The shaft has a radius of 15 mm and is made of L2 tool steel. Determine the strains in the x' and y' directions if a torque $T = 2\ \text{kN}\cdot\text{m}$ is applied to the shaft.



$$\tau = \frac{Tc}{J} = \frac{2(10^3)(0.015)}{\frac{\pi}{2}(0.015^4)} = 377.26\ \text{MPa}$$

Stress - strain relationship :

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{377.26(10^6)}{75.0(10^9)} = 5.030(10^{-3})\ \text{rad}$$

This is a pure shear case, therefore,

$$\varepsilon_x = \varepsilon_y = 0$$

Applying Eq. 10-15,

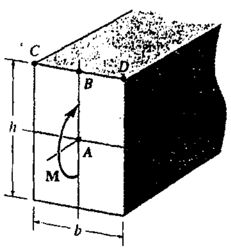
$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

Here $\theta_a = 45°$

$$\varepsilon_{x'} = 0 + 0 + 5.030(10^{-3})\sin 45°\cos 45° = 2.52(10^{-3})$$

$$\varepsilon_{x'} = \varepsilon_{y'} = 2.52(10^{-3})$$ **Ans.**

10–47 The cross section of the rectangular beam is subjected to the bending moment **M**. Determine an expression for the increase in length of lines *AB* and *CD*. The material has a modulus of elasticity *E* and Poisson's ratio is ν.



For line *AB*,

$$\sigma_z = -\frac{M y}{I} = -\frac{M y}{\frac{1}{12} b h^3} = -\frac{12 M y}{b h^3}$$

$$\varepsilon_y = -\frac{\nu \sigma_z}{E} = \frac{12 \nu M y}{E b h^3}$$

$$\Delta L_{AB} = \int_0^{\frac{h}{2}} \varepsilon_y \, dy = \frac{12 \nu M}{E b h^3} \int_0^{\frac{h}{2}} y \, dy$$

$$= \frac{3 \nu M}{2 E b h} \qquad \textbf{Ans}$$

For line *CD*,

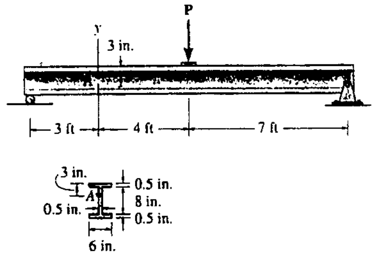
$$\sigma_z = -\frac{M c}{I} = -\frac{M \frac{h}{2}}{\frac{1}{12} b h^3} = -\frac{6 M}{b h^2}$$

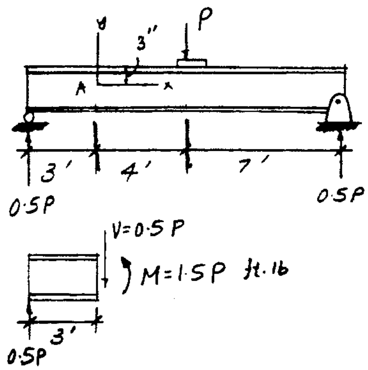
$$\varepsilon_x = -\frac{\nu \sigma_z}{E} = \frac{6 \nu M}{E b h^2}$$

$$\Delta L_{CD} = \varepsilon_x L_{CD} = \frac{6 \nu M}{E b h^2} (b)$$

$$= \frac{6 \nu M}{E h^2} \qquad \textbf{Ans}$$

***10–48** The strain in the x direction at point A on the steel beam is measured and found to be $\epsilon_x = -100(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A? $E_{st} = 29(10^3)$ ksi, $\nu_{st} = 0.3$.





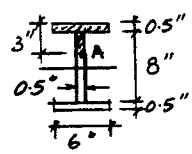
$$I_x = \frac{1}{12}(6)(9)^3 - \frac{1}{12}(5.5)(8^3) = 129.833 \text{ in}^4$$

$$Q_A = (4.25)(0.5)(6) + (2.75)(0.5)(2.5) = 16.1875 \text{ in}^3$$

$$\sigma = E\varepsilon_x = 29(10^3)(100)(10^{-6}) = 2.90 \text{ ksi}$$

$$\sigma = \frac{My}{I}, \qquad 2.90 = \frac{1.5P(12)(1.5)}{129.833}$$

$$P = 13.945 = 13.9 \text{ kip} \qquad \textbf{Ans}$$

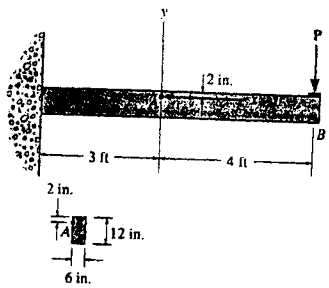


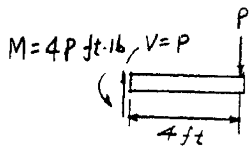
$$\tau_A = \frac{VQ}{It} = \frac{0.5(13.945)(16.1875)}{129.833(0.5)} = 1.739 \text{ ksi}$$

$$G = \frac{E}{2(1+\nu)} = \frac{29(10^3)}{2(1+0.3)} = 11.154(10^3) \text{ ksi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{1.739}{11.154(10^3)} = 0.156(10^{-3}) \text{ rad} \qquad \textbf{Ans}$$

10–49 The strain in the x direction at point A on the A-36 structural-steel beam is measured and found to be $\epsilon_x = 100(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A?





Section properties :

$$I = \frac{1}{12}(6)(12^3) = 864 \text{ in}^4$$

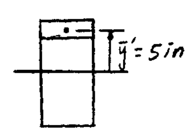
$$Q_A = \bar{y}' A' = 5\,(6)(2) = 60 \text{ in}^3$$

Normal stress :

$$\sigma = E\,\varepsilon_x = 29\,(10^3)(100)(10^{-6}) = 2.90 \text{ ksi}$$

$$\sigma = \frac{M\,y}{I}; \qquad 2.90(10^3) = \frac{4\,P\,(12)(4)}{864}$$

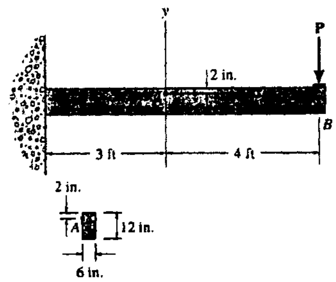
$$P = 13050 \text{ lb} = 13.0 \text{ kip} \qquad \textbf{Ans}$$



$$\tau_{xy} = \frac{V\,Q}{I\,t} = \frac{13.05\,(10^3)(60)}{864\,(6)} = 151.04 \text{ psi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{151.04}{11.0\,(10^6)} = 13.7\,(10^{-6}) \qquad \textbf{Ans}$$

10–50 The strain in the x direction at point A on the A-36 structural-steel beam is measured and found to be $\epsilon_x = 200(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A?



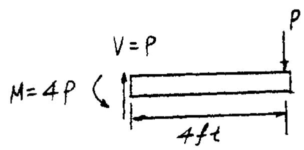
Section properties :

$$Q_A = \bar{y}' A' = 5\,(6)(2) = 60 \text{ in}^3$$

$$I = \frac{1}{12}(6)(12^3) = 864 \text{ in}^4$$

Normal stress :

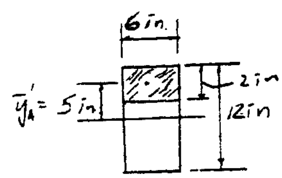
$$\sigma = E\,\varepsilon_x = 29\,(10^3)(200)(10^{-6}) = 5.80 \text{ ksi}$$



$$\sigma = \frac{M\,y}{I}; \qquad 5.80(10^3) = \frac{4\,P\,(12)(4)}{864}$$

$P = 26.1$ kip **Ans**

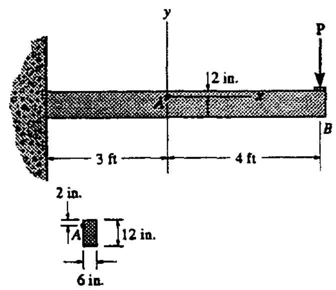
Shear stress and shear strain :

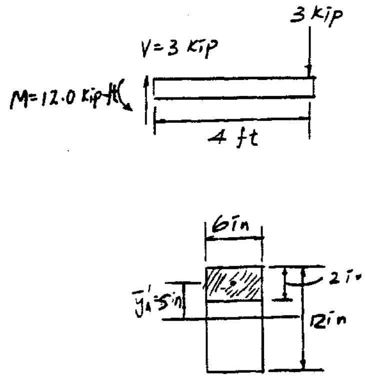


$$\tau_A = \frac{V\,Q}{I\,t} = \frac{26.1\,(60)}{864\,(6)} = 0.302 \text{ ksi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{0.302}{11.0\,(10^3)} = 27.5\,(10^{-6}) \text{ rad}$$ **Ans**

10–51 If a load of $P = 3$ kip is applied to the A-36 structural-steel beam, determine the strain ϵ_x and γ_{xy} at point A.





Section properties :

$$Q_A = \bar{y}' A' = 2\,(6)(5) = 60 \text{ in}^3$$

$$I = \frac{1}{12}(6)(12^3) = 864 \text{ in}^4$$

Normal stress and strain :

$$\sigma_A = \frac{My}{I} = \frac{12.0\,(10^3)(12)(4)}{864} = 666.7 \text{ psi}$$

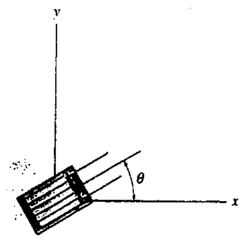
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{666.7}{29\,(10^6)} = 23.0\,(10^{-6}) \qquad \textbf{Ans}$$

Shear stress and shear strain :

$$\tau_A = \frac{VQ}{It} = \frac{3(10^3)\,(60)}{864\,(6)} = 34.72 \text{ psi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{34.72}{11.0\,(10^6)} = 3.16\,(10^{-6}) \qquad \textbf{Ans}$$

*10–52 A material is subjected to principal stresses σ_x and σ_y. Determine the orientation θ of a strain gauge placed at the point so that its reading of normal strain responds only to σ_y and not σ_x. The material constants are E and ν.



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

Since $\tau_{xy} = 0$,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + (\sigma_x - \sigma_y)\cos^2\theta - \frac{\sigma_x}{2} + \frac{\sigma_y}{2}$$

$$= \sigma_y(1 - \cos^2\theta) + \sigma_x \cos^2\theta$$

$$= \sigma_x \cos^2\theta + \sigma_y \sin^2\theta$$

$$\sigma_{n+90^\circ} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)(2\cos^2\theta - 1)$$

$$= \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - (\sigma_x - \sigma_y)\cos^2\theta + \frac{\sigma_x}{2} - \frac{\sigma_y}{2}$$

$$= \sigma_x(1 - \cos^2\theta) + \sigma_y \cos^2\theta$$

$$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta$$

$$\varepsilon_n = \frac{1}{E}(\sigma_n - \nu\, \sigma_{n+90^\circ})$$

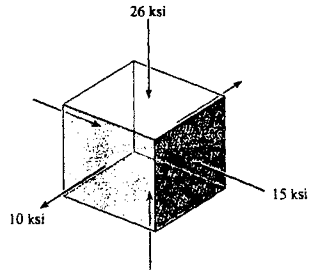
$$= \frac{1}{E}(\sigma_x \cos^2\theta + \sigma_y \sin^2\theta - \nu\, \sigma_x \sin^2\theta - \nu\, \sigma_y \cos^2\theta)$$

If ε_n is to be independent of σ_x, then

$\cos^2\theta - \nu \sin^2\theta = 0$ or $\tan^2\theta = 1/\nu$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{\nu}}\right)$$ **Ans**

10–53 The principal stresses at a point are shown in the figure. If the material is aluminum for which $E_{al} = 10(10^3)$ ksi and $\nu_{al} = 0.33$, determine the principal strains.



$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{10(10^3)}(10 - 0.33(-15 - 26)) = 2.35(10^{-3})$$ **Ans**

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = \frac{1}{10(10^3)}(-15 - 0.33)(10 - 26)) = -0.972(10^{-3})$$ **Ans**

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = \frac{1}{10(10^3)}(-26 - 0.33(10 - 15)) = -2.44(10^{-3})$$ **Ans**

10–54 A thin-walled cylindrical pressure vessel has an inner radius r, thickness t, and length L. If it is subjected to an internal pressure p, show that the increase in its inner radius is $dr = r\epsilon_1 = pr^2(1 - \frac{1}{2}\nu)/Et$ and the increase in its length is $\Delta L = pLr(\frac{1}{2} - \nu)/Et$. Using these results, show that the change in internal volume becomes $dV = \pi r^2(1 + \epsilon_1)^2(1 + \epsilon_2)L - \pi r^2 L$. Since ϵ_1 and ϵ_2 are small quantities, show further that the change in volume per unit volume, called *volumetric strain*, can be written as $dV/V = Pr(2.5 - 2\nu)/Et$.

Normal stress :

$$\sigma_1 = \frac{p\,r}{t}; \qquad \sigma_2 = \frac{p\,r}{2\,t}$$

Normal strain : Applying Hooke's law

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}(\frac{p\,r}{t} - \frac{\nu\,p\,r}{2\,t}) = \frac{p\,r}{E\,t}(1 - \frac{1}{2}\nu)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{E\,t}(1 - \frac{1}{2}\nu) \qquad \textbf{QED}$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}(\frac{p\,r}{2\,t} - \frac{\nu\,p\,r}{t}) = \frac{p\,r}{E\,t}(\frac{1}{2} - \nu)$$

$$\Delta L = \varepsilon_2\,L = \frac{p\,L\,r}{E\,t}(\frac{1}{2} - \nu) \qquad \textbf{QED}$$

$$V' = \pi(r + \varepsilon_1\,r)^2(L + \varepsilon_2 L); \qquad V = \pi r^2 L$$

$$dV = V' - V = \pi r^2(1 + \varepsilon_1)^2(1 + \varepsilon_2)L - \pi r^2 L \qquad \textbf{QED}$$

$$(1 + \varepsilon_1)^2 = 1 + 2\,\varepsilon_1 \quad \text{neglect } \varepsilon_1^2 \text{ term}$$

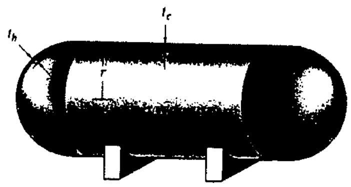
$$(1 + \varepsilon_1)^2(1 + \varepsilon_2) = (1 + 2\,\varepsilon_1)(1 + \varepsilon_2) = 1 + \varepsilon_2 + 2\,\varepsilon_1 \text{ neglect } \varepsilon_1\varepsilon_2 \text{ term}$$

$$\frac{dV}{V} = 1 + \varepsilon_2 + 2\,\varepsilon_1 - 1 = \varepsilon_2 + 2\,\varepsilon_1$$

$$= \frac{p\,r}{E\,t}(\frac{1}{2} - \nu) + \frac{2\,p\,r}{E\,t}(1 - \frac{1}{2}\nu)$$

$$= \frac{p\,r}{E\,t}(2.5 - 2\,\nu) \qquad \textbf{QED}$$

10-55 The cylindrical pressure vessel is fabricated using hemispherical end caps in order to reduce the bending stress that would occur if flat ends were used. The bending stresses at the seam where the caps are attached can be eliminated by proper choice of the thickness t_h and t_c of the caps and cylinder, respectively. This requires the radial expansion to be the same for both the hemispheres and cylinder. Show that this ratio is $t_c/t_h = (2 - \nu)/(1 - \nu)$. Assume that the vessel is made of the same material and both the cylinder and hemispheres have the same inner radius. If the cylinder is to have a thickness of 0.5 in., what is the required thickness of the hemispheres? Take $\nu = 0.3$.



For cylindrical vessel :

$$\sigma_1 = \frac{p\,r}{t_c}; \qquad \sigma_2 = \frac{p\,r}{2\,t_c}$$

$$\varepsilon_1 = \frac{1}{E}[\,\sigma_1 - \nu(\sigma_2 + \sigma_3)\,] \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t_c} - \frac{\nu\,p\,r}{2\,t_c}\right) = \frac{p\,r}{E\,t_c}\left(1 - \frac{1}{2}\nu\right)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) \qquad (1)$$

For hemispherical end caps :

$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t_h}$$

$$\varepsilon_1 = \frac{1}{E}[\,\sigma_1 - \nu(\sigma_2 + \sigma_3)\,]; \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t_h} - \frac{\nu\,p\,r}{2\,t_h}\right) = \frac{p\,r}{2\,E\,t_h}(1 - \nu)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu) \qquad (2)$$

Equate Eqs. (1) and (2) :

$$\frac{p\,r^2}{E\,t_c}\left(1 - \frac{1}{2}\nu\right) = \frac{p\,r^2}{2\,E\,t_h}(1 - \nu)$$

$$\frac{t_c}{t_h} = \frac{2\left(1 - \frac{1}{2}\nu\right)}{1 - \nu} = \frac{2 - \nu}{1 - \nu} \qquad \textbf{QED}$$

$$t_h = \frac{(1 - \nu)\,t_c}{2 - \nu} = \frac{(1 - 0.3)(0.5)}{2 - 0.3} = 0.206 \text{ in.} \qquad \textbf{Ans}$$

•10–56 A thin-walled spherical pressure vessel has an inner radius r, thickness t, and is subjected to an internal pressure p. If the material constants are E and ν, determine the strain in the circumferential direction in terms of the stated parameters.

Principal stresses :

$$\sigma_1 = \sigma_2 = \sigma = \frac{p\,r}{2\,t}; \qquad \sigma_3 = 0$$

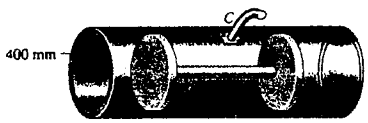
Applying Hooke's law :

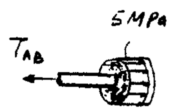
$$\varepsilon_1 = \varepsilon_2 = \varepsilon = \frac{1}{E}[\sigma_1 - v(\sigma_2 + \sigma_3)]$$

$$\varepsilon = \frac{1}{E}[\sigma - v\,\sigma] = \frac{1-v}{E}\,\sigma$$

$$= \frac{1-v}{E}\left(\frac{p\,r}{2\,t}\right) = \frac{p\,r}{2\,t\,E}(1 - v) \qquad \textbf{Ans}$$

10–57 Air is pumped into the steel thin-walled pressure vessel at C. If the ends of the vessel are closed using two pistons connected by a rod AB, determine the increase in the diameter of the pressure vessel when the internal gauge pressure is 5 MPa. Also, what is the tensile stress in rod AB if it has a diameter of 100 mm? The inner radius of the vessel is 400 mm, and its thickness is 10 mm. E_{st} = 200 GPa and ν_{st} = 0.3.





Circumferential stress :

$$\sigma = \frac{p\,r}{t} = \frac{5\,(400)}{10} = 200\text{ MPa}$$

Note : longitudinal and radial stresses are zero.

Circumferential strain :

$$\varepsilon = \frac{\sigma}{E} = \frac{200\,(10^6)}{200\,(10^9)} = 1.0\,(10^{-3})$$

$$\Delta d = \varepsilon\, d = 1.0\,(10^{-3})\,(800) = 0.800\text{ mm} \qquad \textbf{Ans}$$

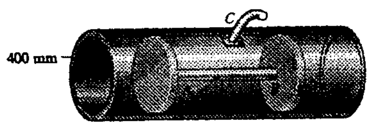
For rod AB :

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad T_{AB} - 5(10^6)(\frac{\pi}{4})(0.8^2 - 0.1^2) = 0$$

$$T_{AB} = 2474\text{ kN}$$

$$\sigma_{AB} = \frac{T_{AB}}{A_{AB}} = \frac{2474\,(10^3)}{\frac{\pi}{4}\,(0.1^2)} = 315\text{ MPa} \qquad \textbf{Ans}$$

10–58 Determine the increase in the diameter of the pressure vessel in Prob. 10–57 if the pistons are replaced by walls connected to the ends of the vessel.



Principal stress :

$$\sigma_1 = \frac{p\,r}{t} = \frac{5\,(400)}{10} = 200\text{ MPa}; \qquad \sigma_3 = 0$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 100\text{ MPa}$$

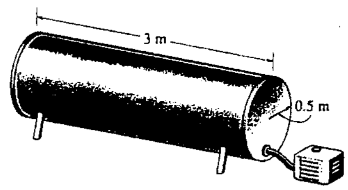
Circumferential strain :

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)] = \frac{1}{200\,(10^9)}[200\,(10^6) - 0.3\{\,100\,(10^6) + 0\,\}]$$

$$= 0.85\,(10^{-3})$$

$$\Delta d = \varepsilon_1\, d = 0.85\,(10^{-3})\,(800) = 0.680\text{ mm}$$ **Ans**

10–59 The thin-walled cylindrical pressure vessel of inner radius r and thickness t is subjected to an internal pressure p. If the material constants are E and ν, determine the strains in the circumferential and longitudinal directions. Using these results, compute the increase in both the diameter and the length of a steel pressure vessel filled with air and having an internal gauge pressure of 15 MPa. The vessel is 3 m long, and has an inner radius of 0.5 m and a thickness of 10 mm. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



Normal stress :

$$\sigma_1 = \frac{p\,r}{t} \qquad \sigma_2 = \frac{p\,r}{2\,t} \qquad \sigma_3 = 0$$

Normal strain :

$$\varepsilon_{cir} = \frac{1}{E}[\,\sigma_1 - \nu(\sigma_2 + \sigma_3)\,]$$

$$= \frac{1}{E}\left(\frac{p\,r}{t} - \frac{\nu\,p\,r}{2\,t}\right) = \frac{p\,r}{2\,E\,t}(2 - \nu) \qquad \textbf{Ans}$$

$$\varepsilon_{long} = \frac{1}{E}[\,\sigma_2 - \nu(\sigma_1 + \sigma_3)\,]$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t} - \frac{\nu\,p\,r}{t}\right) = \frac{p\,r}{2\,E\,t}(1 - 2\nu) \qquad \textbf{Ans}$$

Numerical substitution :

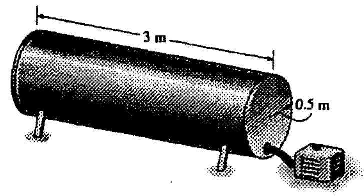
$$\varepsilon_{cir} = \frac{15\,(10^6)(0.5)}{2\,(200)(10^9)(0.01)}(2 - 0.3) = 3.1875\,(10^{-3})$$

$$\Delta d = \varepsilon_{cir}\,d = 3.1875\,(10^{-3})\,(1000) = 3.19\text{ mm} \qquad \textbf{Ans}$$

$$\varepsilon_{long} = \frac{15\,(10^6)(0.5)}{2\,(200)(10^9)(0.01)}(1 - 2(0.3)) = 0.75\,(10^{-3})$$

$$\Delta L = \varepsilon_{long}\,L = 0.75\,(10^{-3})\,(3000) = 2.25\text{ mm} \qquad \textbf{Ans}$$

*10–60 Estimate the increase in volume of the tank in Prob. 10–59. *Suggestion:* Use the results of Prob. 10–54 as a check.



By basic principles,

$$\Delta V = \pi (r + \Delta r)^2 (L + \Delta L) - \pi r^2 L = \pi (r^2 + \Delta r^2 + 2\, r\, \Delta r)(L + \Delta L) - \pi r^2 L$$

$$= \pi (r^2 L + r^2 \Delta L + \Delta r^2 L + \Delta r^2 \Delta L + 2\, r\, \Delta r\, L + 2\, r\, \Delta r\, \Delta L - r^2 L)$$

$$= \pi (r^2 \Delta L + \Delta r^2 L + \Delta r^2 \Delta L + 2\, r\, \Delta r\, L + 2\, r\, \Delta r\, \Delta L)$$

Neglecting the second order terms,

$$\Delta V = \pi (r^2 \Delta L + 2\, r\, \Delta r\, L)$$

From Prob. 10-59,

$$\Delta L = 0.00225 \text{ m} \qquad \Delta r = \frac{\Delta d}{2} = 0.00159375 \text{ m}$$

$$\Delta V = \pi[(0.5^2)\,(0.00225) + 2\,(0.5)(0.00159375)(3)] = 0.0168 \text{ m}^3$$ **Ans**

Or use the result of Prob. 10-54

$$\frac{dV}{V} = \frac{p\,r}{E\,t}(2.5 - 2\,\nu)$$

$$\Delta V = \frac{p\,r}{E\,t}(2.5 - 2\,\nu)\,V = \frac{15\,(10^6)(0.5)}{200\,(10^9)(0.01)}[2.5 - 2(0.3)]\,\pi\,(0.5^2)(3)$$

$$= \mathbf{0.0168\ m^3}$$ **Ans**

10-61. A soft material is placed within the confines of a rigid cylinder which rests on a rigid support. Assuming that $\epsilon_x = 0$ and $\epsilon_y = 0$, determine the factor by which the modulus of elasticity will be increased when a load is applied if $v = 0.3$ for the material.

Normal Strain : Since the material is confined in a rigid cylinder, $\varepsilon_x = \varepsilon_y = 0$. Applying the generalized Hooke's Law,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right]$$

$$0 = \sigma_x - v\left(\sigma_y + \sigma_z\right) \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - v(\sigma_x + \sigma_z)\right]$$

$$0 = \sigma_y - v(\sigma_x + \sigma_z) \qquad [2]$$

Solving Eqs. [1] and [2] yields :

$$\sigma_x = \sigma_y = \frac{v}{1-v}\sigma_z$$

Thus,

$$\begin{aligned}\varepsilon_z &= \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right] \\ &= \frac{1}{E}\left[\sigma_z - v\left(\frac{v}{1-v}\sigma_z + \frac{v}{1-v}\sigma_z\right)\right] \\ &= \frac{\sigma_z}{E}\left[1 - \frac{2v^2}{1-v}\right] \\ &= \frac{\sigma_z}{E}\left[\frac{1-v-2v^2}{1-v}\right] \\ &= \frac{\sigma_z}{E}\left[\frac{(1+v)(1-2v)}{1-v}\right]\end{aligned}$$

Thus, when the material is not being confined and undergoes the same normal strain of ε_z, then the required modulus of elasticity is

$$E' = \frac{\sigma_z}{\varepsilon_z} = \frac{1-v}{(1-2v)(1+v)}E$$

The increased factor is
$$k = \frac{E'}{E} = \frac{1-v}{(1-2v)(1+v)} = \frac{1-0.3}{[1-2(0.3)](1+0.3)} = 1.35 \qquad \textbf{Ans}$$

10–62 A thin-walled spherical pressure vessel having an inner radius r and thickness t is subjected to an internal pressure p. Show that the increase in the volume within the vessel is $\Delta V = (2p\pi r^4/Et)(1 - \nu)$. Use a small-strain analysis.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\sigma_3 = 0$$

$$\varepsilon_1 = \varepsilon_2 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

$$\varepsilon_1 = \varepsilon_2 = \frac{pr}{2t\,E}(1-\nu)$$

$$\varepsilon_3 = \frac{1}{E}(-\nu(\sigma_1 + \sigma_2))$$

$$\varepsilon_3 = \frac{\nu\,pr}{t\,E}$$

$$V = \frac{4\pi\,r^3}{3}$$

$$V+\Delta V = \frac{4\pi}{3}(r+\Delta r)^3 = \frac{4\pi\,r^3}{3}(1+\frac{\Delta r}{r})^3$$

where $\Delta V << V$, $\Delta r << r$
Using Eq. 2 - 5,

$$V+\Delta V \approx \frac{4\pi\,r^3}{3}(1+3\frac{\Delta r}{r})$$

$$e_{\text{Vol}} = \frac{\Delta V}{V} = 3(\frac{\Delta r}{r})$$

Since $\varepsilon_1 = \varepsilon_2 = \dfrac{2\pi(r+\Delta r) - 2\pi\,r}{2\pi\,r} = \dfrac{\Delta r}{r}$

$$e_{\text{Vol}} = 3\varepsilon_1 = \frac{3pr}{2t\,E}(1-\nu)$$

$$\Delta V = Ve_{\text{Vol}} = \frac{2p\pi\,r^4}{E\,t}(1-\nu) \qquad \textbf{QED}$$

10–63 A material is subjected to plane stress. Express the distortion-energy theory of failure in terms of σ_x, σ_y, and τ_{xy}.

Maximum distortion energy theory :

$$(\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2) = \sigma_Y^2 \qquad (1)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Let $a = \frac{\sigma_x + \sigma_y}{2}$ and $b = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_1 = a + b; \qquad \sigma_2 = a - b$

$\sigma_1^2 = a^2 + b^2 + 2\,a\,b; \qquad \sigma_2^2 = a^2 + b^2 - 2\,a\,b$

$\sigma_1 \sigma_2 = a^2 - b^2$

From Eq. (1)

$$(a^2 + b^2 + 2\,a\,b - a^2 + b^2 + a^2 + b^2 - 2\,a\,b) = \sigma_Y^2$$

$$(a^2 + 3\,b^2) = \sigma_Y^2$$

$$\frac{(\sigma_x + \sigma_y)^2}{4} + 3\,\frac{(\sigma_x - \sigma_y)^2}{4} + 3\,\tau_{xy}^2 = \sigma_Y^2$$

$$\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\,\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans}$$

*10–64 A material is subjected to plane stress. Express the maximum-shear-stress theory of failure in terms of σ_x, σ_y, and τ_{xy}. Assume that the principal stresses are of different algebraic signs.

Maximum shear stress theory :

$$|\sigma_1 - \sigma_2| = \sigma_Y \qquad (1)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

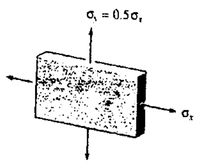
$$\left|\sigma_1 - \sigma_2\right| = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

From Eq. (1)

$$4\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] = \sigma_Y^2$$

$$(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2 = \sigma_Y^2 \qquad \textbf{Ans}$$

10–65 The plate is made of hard copper, which yields at $\sigma_Y = 105$ ksi. Using the maximum-shear-stress theory, determine the tensile stress σ_x that can be applied to the plate if a tensile stress $\sigma_y = 0.5\sigma_x$ is also applied.

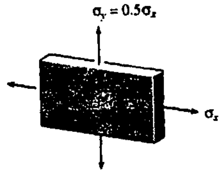


$$\sigma_1 = \sigma_x \qquad \sigma_2 = \frac{1}{2}\sigma_x$$

$$|\sigma_1| = \sigma_Y$$

$$\sigma_x = 105 \text{ ksi}$$ **Ans**

10–66 Solve Prob. 10–65 using the maximum-distortion-energy theory.



$\sigma_1 = \sigma_x$

$\sigma_2 = \dfrac{\sigma_x}{2}$

$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$

$\sigma_x^2 - \dfrac{\sigma_x^2}{2} + \dfrac{\sigma_x^2}{4} = (105)^2$

$\sigma_x = 121$ ksi **Ans**

10–67 The yield stress for a zirconium-magnesium alloy is $\sigma_Y = 15.3$ ksi. If a machine part is made of this material and a critical point in the material is subjected to in-plane principal stresses σ_1 and $\sigma_2 = -0.5\sigma_1$, determine the magnitude of σ_1 that will cause yielding according to the maximum-shear-stress theory.

$\sigma_Y = 15.3$ ksi

$\sigma_1 - \sigma_2 = 15.3$

$\sigma_1 - (-0.5\,\sigma_1) = 15.3$

$\sigma_1 = 10.2$ ksi **Ans**

***10–68** Solve Prob. 10–67 using the maximum-distortion-energy theory.

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\sigma_1^2 - \sigma_1(-0.5\sigma_1) + (-0.5\sigma_1)^2 = \sigma_Y^2$$

$$1.75\,\sigma_1^2 = (15.3)^2$$

$\sigma_1 = 11.6\text{ ksi}$ **Ans**

10–69 If a shaft is made of a material for which $\sigma_Y = 50$ ksi, determine the maximum torsional shear stress required to cause yielding using the maximum-distortion-energy theory.

$$\sigma_1 = \tau, \qquad \sigma_2 = -\tau$$

$$\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$3\tau^2 = 50^2$$

$$\tau = 28.9 \text{ ksi} \qquad \textbf{Ans}$$

10–70 Solve Prob. 10–69 using the maximum-shear-stress theory.

$\sigma_1 = \tau \qquad \sigma_2 = -\tau$

$|\sigma_1 - \sigma_2| = \sigma_Y$

$\tau - (-\tau) = 50$

$\tau = 25\text{ ksi}$ **Ans**

10–71 The yield stress for a plastic material is $\sigma_Y = 110$ MPa. If this material is subjected to plane stress and elastic failure occurs when one principal stress is 120 MPa, what is the smallest magnitude of the other principal stress? Use the maximum distortion-energy theory.

Using the distortion - energy theory :

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$120^2 - 120\,\sigma_2 + \sigma_2^2 = 110^2$$

$$\sigma_2^2 - 120\,\sigma_2 + 2300 = 0$$

Solving for the positive root :

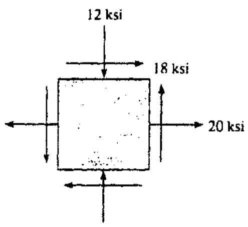
$\sigma_2 = 23.9$ MPa **Ans**

*10–72 Solve Prob. 10–71 using the maximum-shear-stress theory. Both principal stresses have the same sign.

The material will fail for any σ_2 since

120 MPa > 110 MPa **Ans.**

10–73 The state of plane stress at a critical point in a steel machine bracket is shown. If the yield stress for steel is $\sigma_Y = 36$ ksi, determine if yielding occurs using the maximum-distortion-energy theory.



$\sigma_x = 20$ ksi $\quad \sigma_y = -12$ ksi $\quad \tau_{xy} = 18$ ksi

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{20 - 12}{2} \pm \sqrt{\left(\frac{20 - (-12)}{2}\right)^2 + 18^2}$$

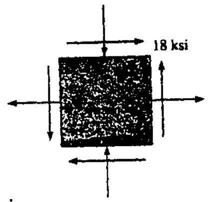
$\sigma_1 = 28.08$ ksi $\qquad \sigma_2 = -20.08$ ksi

$(\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2) = (28.08^2 - 28.08(-20.08) + (-20.08)^2)$

$1756 > \sigma_Y^2 = 1296$

Yes. **Ans**

10–74 Solve Prob. 10–73 using the maximum-shear-stress theory.



$\sigma_x = 20\text{ ksi} \qquad \sigma_y = -12\text{ ksi} \qquad \tau_{xy} = 18\text{ ksi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{20 - 12}{2} \pm \sqrt{\left(\frac{20 - (-12)}{2}\right)^2 + 18^2}$$

$\sigma_1 = 28.08\text{ ksi} \qquad \sigma_2 = -20.08\text{ ksi}$

$|\sigma_1 - \sigma_2| = 28.08 - (-20.08) = 48.16\text{ ksi} > \sigma_Y = 36\text{ ksi}$

Yes. **Ans**

10–75 An aluminum alloy 6061-T6 is to be used for a solid drive shaft such that it transmits 40 hp at 2400 rev/min. Using a factor of safety of 2 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-shear-stress theory.

$$\omega = (2400\,\frac{\text{rev}}{\text{min}})(\frac{2\pi\ \text{rad}}{\text{rev}})(\frac{1\ \text{min}}{60\ \text{s}}) = 80\,\pi\ \text{rad/s}$$

$$T = \frac{P}{\omega} = \frac{40\,(550)\,(12)}{80\,\pi} = \frac{3300}{\pi}\ \text{lb}\cdot\text{in.}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{(\frac{3300}{\pi})\,c}{\frac{\pi}{2}\,c^4} = \frac{6600}{\pi^2\,c^3}$$

The principal stresses :

$$\sigma_1 = \tau = \frac{6600}{\pi^2\,c^3}; \qquad \sigma_2 = -\tau = -\frac{6600}{\pi^2\,c^3}$$

Maximum shear stress theory : Both principal stresses have opposite sign, hence,

$$\left|\sigma_1 - \sigma_2\right| = \frac{\sigma_Y}{\text{F.S.}}; \qquad 2\,(\frac{6600}{\pi^2\,c^3}) = [\frac{37\,(10^3)}{2}]$$

$c = 0.4166$ in.

$d = 0.833$ in. **Ans**

•10–76 Solve Prob. 10–75 using the maximum-distortion-energy theory.

$$\omega = (2400\ \frac{\text{rev}}{\text{min}})(\frac{2p\ \text{rad}}{\text{rev}})(\frac{1\ \text{min}}{60\ \text{s}}) = 80\,\pi\ \text{rad/s}$$

$$T = \frac{P}{\omega} = \frac{40\,(550)\,(12)}{80\,\pi} = \frac{3300}{\pi}\ \text{lb}\cdot\text{in.}$$

Applying $\tau = \frac{T\,c}{J}$

$$\tau = \frac{(\frac{3300}{\pi})\,c}{\frac{\pi}{2}\,c^4} = \frac{6600}{\pi^2\,c^3}$$

The principal stresses :

$$\sigma_1 = \tau = \frac{6600}{\pi^2\,c^3};\qquad \sigma_2 = -\tau = -\frac{6600}{\pi^2\,c^3}$$

The maximum distortion - energy theory :

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = (\frac{\sigma_Y}{\text{F.S.}})^2$$

$$3\,[\frac{6600}{\pi^2 c^3}]^2 = (\frac{37(10^3)}{2})^2$$

$c = 0.3971$ in.

$d = 0.794$ in. **Ans.**

10–77 An aluminum alloy is to be used for a drive shaft such that it transmits 25 hp at 1500 rev/min. Using a factor of safety of 2.5 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-distortion-energy theory. $\sigma_Y = 3.5$ ksi.

$$T = \frac{P}{\omega} \qquad \omega = \frac{1500(2\pi)}{60} = 50\pi$$

$$T = \frac{25(550)(12)}{50\pi} = \frac{3300}{\pi}$$

$$\tau = \frac{Tc}{J}, \qquad J = \frac{\pi}{2}c^4$$

$$\tau = \frac{\frac{3300}{\pi}c}{\frac{\pi}{2}c^4} = \frac{6600}{\pi^2 c^3}$$

$$\sigma_1 = \frac{6600}{\pi^2 c^3} \qquad \sigma_2 = \frac{-6600}{\pi^2 c^3}$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = (\frac{\sigma_Y}{\text{F.S.}})^2$$

$$3(\frac{6600}{\pi^2 c^3})^2 = (\frac{3.5(10^3)}{2.5})^2$$

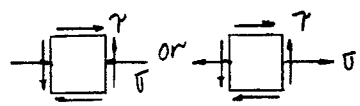
$c = 0.9388$ in.

$d = 1.88$ in. **Ans**

10–78 A bar with a circular cross-sectional area is made of A-36 steel. If the bar is subjected to a torque of 16 kip · in. and a bending moment of 20 kip · in., determine the required diameter of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 2 with respect to yielding.

$$I = \frac{\pi}{4}c^4 \qquad J = \frac{\pi}{2}c^4$$

$$\sigma = \frac{Mc}{I} = \frac{20c}{\frac{\pi}{4}c^4} = \frac{80}{\pi c^3}$$

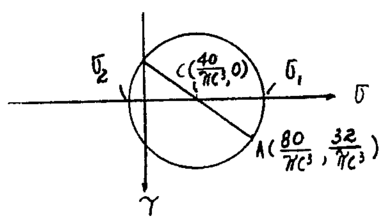


$$\tau = \frac{Tc}{J} = \frac{16\,c}{\frac{\pi}{2}c^4} = \frac{32}{\pi c^3}$$

Critical state of stress :

$$A(\frac{80}{\pi c^3}, \frac{32}{\pi c^3}) \qquad B(0, -\frac{32}{\pi c^3}) \qquad C(\frac{40}{\pi c^3}, 0)$$

$$R = \sqrt{(\frac{40}{\pi c^3})^2 + (\frac{32}{\pi c^3})^2} = \frac{51.225}{\pi c^3}$$



$$\sigma_1 = \frac{40}{\pi c^3} + \frac{51.225}{\pi c^3} = \frac{91.225}{\pi c^3}$$

$$\sigma_2 = \frac{40}{\pi c^3} - \frac{51.225}{\pi c^3} = \frac{-11.225}{\pi c^3}$$

$$\sigma_{allow} = \frac{36}{\text{F.S.}} = \frac{36}{2} = 18 \text{ ksi}$$

$$(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2) = \sigma_{allow}^2$$

$$(\frac{91.225}{\pi c^3})^2 - (\frac{91.225}{\pi c^3})(-\frac{11.225}{\pi c^3}) + (-\frac{11.225}{\pi c^3})^2 = (18)^2$$

$c = 1.20$ in.

$d = 2c = 2.40$ in. **Ans**

10–79 Solve Prob. 10–78 using the maximum-shear-stress theory.

$$I = \frac{\pi}{4}c^4 \qquad J = \frac{\pi}{2}c^4$$

$$\sigma = \frac{Mc}{I} = \frac{20c}{\frac{\pi}{4}c^4} = \frac{80}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{16\,c}{\frac{\pi}{2}c^4} = \frac{32}{\pi c^3}$$

Critical state of stress :

$$A(\frac{80}{\pi c^3}, \frac{32}{\pi c^3}) \qquad B(0,-\frac{32}{\pi c^3}) \qquad C(\frac{40}{\pi c^3}, 0)$$

$$R = \sqrt{(\frac{40}{\pi c^3})^2 + (\frac{32}{\pi c^3})^2} = \frac{51.225}{\pi c^3}$$

$$\sigma_1 = \frac{40}{\pi c^3} + \frac{51.225}{\pi c^3} = \frac{91.225}{\pi c^3}$$

$$\sigma_2 = \frac{40}{\pi c^3} - \frac{51.225}{\pi c^3} = \frac{-11.225}{\pi c^3}$$

$$\sigma_{allow} = \frac{36}{\text{F.S.}} = \frac{36}{2} = 18 \text{ ksi}$$

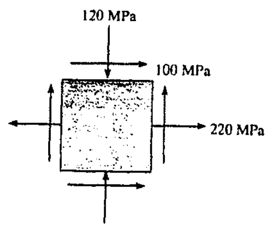
$$|\sigma_1 - \sigma_2| = \sigma_{allow}$$

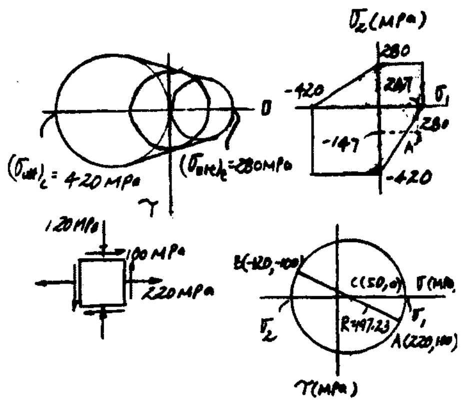
$$\left|\frac{91.225}{\pi c^3} - (\frac{-11.225}{\pi c^3})\right| = 18$$

$c = 1.219$ in.

$d = 2c = 2.44$ in. **Ans**

*10–80 Cast iron when tested in tension and compression has an ultimate strength of $(\sigma_{ult})_t$ = 280 MPa and $(\sigma_{ult})_c$ = 420 MPa, respectively. Also, when subjected to pure torsion it can sustain an ultimate shear stress of τ_{ult} = 168 MPa. Plot the Mohr's circles for each case and establish the failure envelope. If a part made of this material is subjected to the state of plane stress shown, determine if it fails according to Mohr's failure criterion.



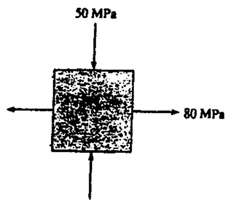


$$\sigma_1 = 50 + 197.23 = 247 \text{ MPa}$$
$$\sigma_2 = 50 - 197.23 = -147 \text{ MPa}$$

The principal stress coordinate is located at point A which is outside the shaded region. Therefore the material fails according to Mohr's failure criterion.

Yes. **Ans**

10-81. The principal plane stresses acting on a differential element are shown. If the material is machine steel having a yield stress of $\sigma_Y = 700$ MPa, determine the factor of safety with respect to yielding if the maximum-shear-stress theory is considered.



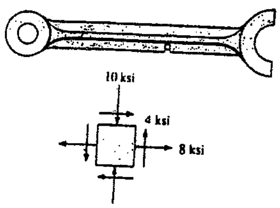
$\sigma_{max} = 80$ MPa $\quad \sigma_{min} = -50$ MPa

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{80-(-50)}{2} = 65 \text{ MPa}$$

$$\tau_{max} = \frac{\sigma_Y}{2} = \frac{700}{2} = 350 \text{ MPa}$$

$$\text{F.S.} = \frac{\tau_{max}}{\tau_{\substack{abs \\ max}}} = \frac{350}{65} = 5.38 \qquad \textbf{Ans}$$

10-82. The state of stress acting at a critical point on a machine element is shown in the figure. Determine the smallest yield stress for a steel that might be selected for the part, based on the maximum-shear-stress theory.



The principal stresses :

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{8 - 10}{2} \pm \sqrt{\left(\frac{8 - (-10)}{2}\right)^2 + 4^2}$$

$\sigma_1 = 8.8489$ ksi $\quad \sigma_2 = -10.8489$ ksi

Maximum shear stress theory : Both principal stresses have opposite sign, hence,

$|\sigma_1 - \sigma_2| = \sigma_Y \qquad 8.8489 - (-10.8489) = \sigma_Y$

$\sigma_Y = 19.7$ ksi **Ans**

10–83 The yield stress for a uranium alloy is $\sigma_Y = 160$ MPa. If a machine part is made of this material and a critical point in the material is subjected to plane stress, such that the principal stresses are σ_1 and $\sigma_2 = 0.25\sigma_1$, determine the magnitude of σ_1 that will cause yielding according to the maximum-distortion-energy theory.

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\sigma_1^2 - (\sigma_1)(0.25\sigma_1) + (0.25\sigma_1)^2 = \sigma_Y^2$$

$$0.8125\sigma_1^2 = \sigma_Y^2$$

$$0.8125\sigma_1^2 = (160)^2$$

$\sigma_1 = 178$ MPa **Ans**

•10–84 Solve Prob. 10–83 using the maximum-shear-stress theory.

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_1}{2} \qquad \tau_{allow} = \frac{\sigma_Y}{2} = \frac{160}{2} = 80 \text{ MPa}$$

$$\tau_{\substack{abs \\ max}} = \tau_{allow}$$

$$\left|\frac{\sigma_1}{2}\right| = 80 ; \qquad \sigma_1 = 160 \text{ MPa} \qquad \textbf{Ans}$$

10–85 An aluminum alloy is to be used for a solid drive shaft such that it transmits 30 hp at 1200 rev/min. Using a factor of safety of 2.5 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-shear-stress theory. $\sigma_Y = 10$ ksi.

$$T = \frac{P}{\omega} \qquad \omega = \frac{2\pi\,(1200)}{60} = 40\,\pi$$

$$T = \frac{30(550)(12)}{40\,\pi} = \frac{4950}{\pi}$$

$$\tau = \frac{Tc}{J} = \frac{\frac{4950}{\pi}c}{\frac{\pi}{2}c^4} = \frac{9900}{\pi^2 c^3}$$

$$\sigma_1 = \frac{9900}{\pi^2 c^3} \qquad \sigma_2 = \frac{-9900}{\pi^2 c^3}$$

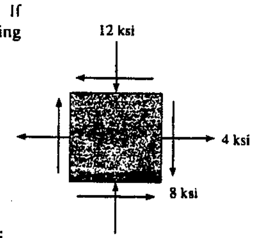
$$|\sigma_1 - \sigma_2| = \frac{\sigma_Y}{F.S.}$$

$$2\left(\frac{9900}{\pi^2 c^3}\right) = \frac{10(10^3)}{2.5}$$

$c = 0.7945$ in.

$d = 2c = 1.59$ in. **Ans**

10–86 The element is subjected to the stresses shown. If $\sigma_Y = 36$ ksi, determine the factor of safety for the loading based on the maximum-shear-stress theory.



$\sigma_x = 4\text{ ksi} \qquad \sigma_y = -12\text{ ksi} \qquad \tau_{xy} = -8\text{ ksi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4-12}{2} \pm \sqrt{\left(\frac{4-(-12)}{2}\right)^2 + (-8)^2}$$

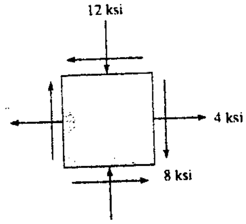
$\sigma_1 = 7.314\text{ ksi} \qquad \sigma_2 = -15.314\text{ ksi}$

$$\tau_{abs_{max}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{7.314-(-15.314)}{2} = 11.314\text{ ksi}$$

$$\tau_{allow} = \frac{\sigma_Y}{2} = \frac{36}{2} = 18\text{ ksi}$$

$$\text{F.S.} = \frac{\tau_{allow}}{\tau_{abs_{max}}} = \frac{18}{11.314} = 1.59 \qquad \textbf{Ans}$$

10–87 Solve Prob. 10–86 using the maximum-distortion-energy theory.



$\sigma_x = 4\text{ ksi} \qquad \sigma_y = -12\text{ ksi} \qquad \tau_{xy} = -8\text{ ksi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4-12}{2} \pm \sqrt{\left(\frac{4-(-12)}{2}\right)^2 + (-8)^2}$$

$\sigma_1 = 7.314\text{ ksi} \qquad \sigma_2 = -15.314\text{ ksi}$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$\text{F.S.} = \sqrt{\frac{36^2}{(7.134)^2 - (7.314)(-15.314) + (-15.314)^2}} = 1.80 \qquad \textbf{Ans}$$

•10–88 If a solid shaft having a diameter d is subjected to a torque **T** and moment **M**, show that by the maximum-shear-stress theory the maximum allowable shear stress is $\tau_{allow} = (16/\pi d^3)\sqrt{M^2 + T^2}$. Assume the principal stresses to be of opposite algebraic signs.

Section properties :

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}; \qquad J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

Thus,

$$\sigma = \frac{Mc}{I} = \frac{M(\frac{d}{2})}{\frac{\pi d^4}{64}} = \frac{32M}{\pi d^3}$$

$$\tau = \frac{Tc}{J} = \frac{T(\frac{d}{2})}{\frac{\pi d^4}{32}} = \frac{16T}{\pi d^3}$$

The principal stresses :

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16M}{\pi d^3} \pm \sqrt{\left(\frac{16M}{\pi d^3}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} = \frac{16M}{\pi d^3} \pm \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

Assume σ_1 and σ_2 have opposite sign, hence,

$$\tau_{allow} = \frac{\sigma_1 - \sigma_2}{2} = \frac{2[\frac{16}{\pi d^3}\sqrt{M^2 + T^2}]}{2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \qquad \textbf{QED}$$

10–89 Derive an expression for an equivalent torque T_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

$$\tau = \frac{T_e\, c}{J}$$

Principal stress :

$$\sigma_1 = \tau; \quad \sigma_2 = -\tau$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1+\nu}{3E}(3\,\tau^2) = \frac{1+\nu}{3E}\left(\frac{3\,T_e^2\,c^2}{J^2}\right)$$

Bending moment and torsion :

$$\sigma = \frac{M\,c}{I}; \qquad \tau = \frac{T\,c}{J}$$

Principal stress :

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Let $a = \frac{\sigma}{2}$ $\quad b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3\,b^2 + a^2$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_2 = \frac{1+\nu}{3E}(3\,b^2 + a^2) = \frac{1+\nu}{3E}\left(\frac{3\,\sigma^2}{4} + 3\,\tau^2 + \frac{\sigma^2}{4}\right)$$

$$= \frac{1+\nu}{3E}(\sigma^2 + 3\,\tau^2) = \frac{c^2(1+\nu)}{3E}\left(\frac{M^2}{I^2} + \frac{3\,T^2}{J^2}\right)$$

$$(u_d)_1 = (u_d)^2$$

$$\frac{c^2(1+\nu)}{3E}\frac{3\,T_e^2}{J^2} = \frac{c^2(1+\nu)}{3E}\left(\frac{M^2}{I^2} + \frac{3\,T^2}{J^2}\right)$$

$$T_e = \sqrt{\frac{J^2}{I^2}\frac{M^2}{3} + T^2}$$

For circular shaft

$$\frac{J}{I} = \frac{\frac{\pi}{2}c^4}{\frac{\pi}{4}c^4} = 2$$

10–90 Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same maximum shear stress as the combination of an applied moment M and torque T. Assume that the principal stresses are of opposite algebraic signs.

Bending and Torsion :

$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}; \qquad \tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

The principal stresses :

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\frac{4M}{\pi c^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{4M}{\pi c^3} - 0}{2}\right)^2 + \left(\frac{2T}{\pi c^3}\right)^2}$$

$$= \frac{2M}{\pi c^3} \pm \frac{2}{\pi c^3}\sqrt{M^2 + T^2}$$

$$\tau_{\substack{abs \\ max}} = \sigma_1 - \sigma_2 = 2\left[\frac{2}{\pi c^3}\sqrt{M^2 + T^2}\right] \qquad (1)$$

Pure bending :

$$\sigma_1 = \frac{Mc}{I} = \frac{M_e c}{\frac{\pi}{4}c^4} = \frac{4M_e}{\pi c^3}; \qquad \sigma_2 = 0$$

$$\tau_{\substack{abs \\ max}} = \sigma_1 - \sigma_2 = \frac{4M_e}{\pi c^3} \qquad (2)$$

Equating Eq. (1) and (2) yields :

$$\frac{4}{\pi c^3}\sqrt{M^2 + T^2} = \frac{4M_e}{\pi c^3}$$

$$M_e = \sqrt{M^2 + T^2} \qquad \textbf{Ans}$$

10–91 Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

Principal stresses :

$$\sigma_1 = \frac{M_e\, c}{I}; \qquad \sigma_2 = 0$$

$$u_d = \frac{1+\nu}{3E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1+\nu}{3E}\left(\frac{M_e^2\, c^2}{I^2}\right) \qquad (1)$$

Principal stress :

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Distortion Energy :

Let $a = \dfrac{\sigma}{2}$, $\quad b = \sqrt{\dfrac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2ab$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2ab$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3b^2 + a^2$$

Apply $\sigma = \dfrac{Mc}{I}; \quad \tau = \dfrac{Tc}{J}$

$$(u_d)_2 = \frac{1+\nu}{3E}(3b^2 + a^2) = \frac{1+\nu}{3E}\left(\frac{\sigma^2}{4} + \frac{3\sigma^2}{4} + 3\tau^2\right)$$

$$= \frac{1+\nu}{3E}(\sigma^2 + 3\tau^2) = \frac{1+\nu}{3E}\left(\frac{M^2c^2}{I^2} + \frac{3T^2c^2}{J^2}\right) \qquad (2)$$

Equating Eq. (1) and (2) yields :

$$\frac{(1+\nu)}{3E}\left(\frac{M_e\, c^2}{I^2}\right) = \frac{1+\nu}{3E}\left(\frac{M^2c^2}{I^2} + \frac{3T^2c^2}{J^2}\right)$$

$$\frac{M_e^2}{I^2} = \frac{M^2}{I^2} + \frac{3T^2}{J^2}$$

$$M_e^2 = M^2 + 3T^2\left(\frac{I}{J}\right)^2$$

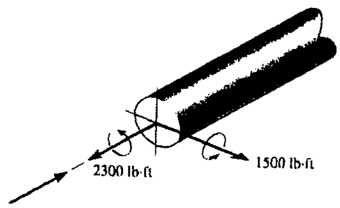
For circular shaft

$$\frac{I}{J} = \frac{\frac{\pi}{4}c^4}{\frac{\pi}{2}c^4} = \frac{1}{2}$$

Hence, $M_e^2 = M^2 + 3T^2\left(\dfrac{1}{2}\right)^2$

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2} \qquad \textbf{Ans}$$

*10–92 The internal loadings at a critical section along the steel drive shaft of a ship are calculated to be a torque of 2300 lb · ft, a bending moment of 1500 lb · ft, and an axial thrust of 2500 lb. If the yield points for tension and shear are $\sigma_Y = 100$ ksi and $\tau_Y = 50$ ksi, respectively, determine the required diameter of the shaft using the maximum-shear-stress theory.



$$A = \pi c^2 \qquad I = \frac{\pi}{4}c^4 \qquad J = \frac{\pi}{2}c^4$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = -(\frac{2500}{\pi c^2} + \frac{1500(12)(c)}{\frac{\pi c^4}{4}}) = -(\frac{2500}{\pi c^2} + \frac{72\,000}{\pi c^3})$$

$$\tau_A = \frac{Tc}{J} = \frac{2300(12)(c)}{\frac{\pi c^4}{2}} = \frac{55\,200}{\pi c^3}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

$$= -(\frac{2500\,c + 72\,000}{2\pi c^3}) \pm \sqrt{(\frac{2500c + 72\,000}{2\pi c^3})^2 + (\frac{55\,200}{\pi c^3})^2} \qquad (1)$$

Assume σ_1 and σ_2 have opposite signs :

$|\sigma_1 - \sigma_2| = \sigma_Y$

$$2\sqrt{(\frac{2500c + 72\,000}{2\pi c^3})^2 + (\frac{55\,200}{\pi c^3})^2} = 100(10^3)$$

$$(2500c + 72000)^2 + 110400^2 = 10\,000(10^6)\pi^2 c^6$$

$$6.25c^2 + 360c + 17372.16 - 10\,000\pi^2 c^6 = 0$$

By trial and error :

$c = 0.750\,55$ in.

Substitute c into Eq. (1) :

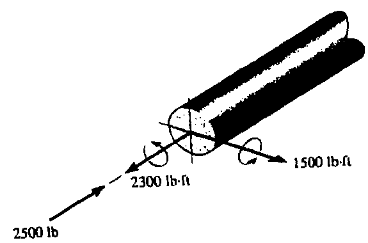
$\sigma_1 = 22\,191$ psi $\qquad \sigma_2 = -77\,809$ psi

σ_1 and σ_2 are of opposite signs OK

Therefore,

$d = 1.50$ in. **Ans**

10–93 The internal loadings at a critical section along the steel drive shaft of a ship are calculated to be a torque of 2300 lb · ft, a bending moment of 1500 lb · ft, and an axial thrust of 2500 lb. If the yield points for tension and shear are $\sigma_Y = 100$ ksi and $\tau_Y = 50$ ksi, respectively, determine the required diameter of the shaft using the maximum-distortion-energy theory.



$$A = \pi c^2 \qquad I = \frac{\pi}{4}c^4 \qquad J = \frac{\pi}{2}c^4$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = -\left(\frac{2500}{\pi c^2} + \frac{1500(12)(c)}{\frac{\pi c^4}{4}}\right) = -\left(\frac{2500}{\pi c^2} + \frac{72\,000}{\pi c^3}\right)$$

$$\tau_A = \frac{Tc}{J} = \frac{2300(12)(c)}{\frac{\pi c^4}{2}} = \frac{55\,200}{\pi c^3}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -\left(\frac{2500\,c + 72\,000}{2\pi c^3}\right) \pm \sqrt{\left(\frac{2500c + 72\,000}{2\pi c^3}\right)^2 + \left(\frac{55\,200}{\pi c^3}\right)^2}$$

Let $\quad a = \dfrac{2500c + 72\,000}{2\pi c^3}, \qquad b = \sqrt{\left(\dfrac{2500c + 72\,000}{2\pi c^3}\right)^2 + \left(\dfrac{55\,200}{\pi c^3}\right)^2}$

$$\sigma_1^2 = (-a+b)^2 = a^2 + b^2 - 2ab$$
$$\sigma_1\sigma_2 = (-a+b)(-a-b) = a^2 - b^2$$
$$\sigma_2^2 = (-a-b)^2 = (a^2 + b^2 - 2ab)$$
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = a^2 + 3b^2$$
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$
$$a^2 + 3b^2 = \sigma_Y^2$$

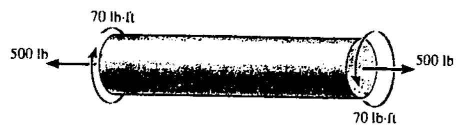
$$\left(\frac{2500c + 72\,000}{2\pi c^3}\right)^2 + 3\left[\left(\frac{2500c + 72\,000}{2\pi c^3}\right)^2 + \left(\frac{55\,200}{\pi c^3}\right)^2\right] = (100(10^3))^2$$

$$25c^2 + 1440c + 57300 - 394784c^6 = 0$$

By trial and error :

$c = 0.72715$ in. $\qquad d = 2c = 1.45$ in. $\qquad$ **Ans**

10–94 The 304-stainless-steel cylinder has an inner diameter of 4 in. and a wall thickness of 0.1 in. If it is subjected to an internal pressure of $p = 80$ psi, axial load of 500 lb and a torque of 70 lb · ft, determine if yielding occurs according to the maximum-distortion energy theory.



$$\sigma_x = \frac{P}{A} + \frac{pr}{2t} = \frac{500}{\pi(2.1)^2 - \pi(2)^2} + \frac{80(2)}{2(0.1)} = 1188.18 \text{ psi}$$

$$\sigma_y = \frac{pr}{t} = \frac{80(2)}{(0.1)} = 1600 \text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{70(12)(2.1)}{\frac{\pi}{2}(2.1)^4 - \frac{\pi}{2}(2)^4} = 325.686 \text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{1188.18 + 1600}{2} \pm \sqrt{\left(\frac{1188.18 - 1600}{2}\right)^2 + (325.686)^2}$$

$$\sigma_1 = 1779.408 \text{ psi}$$

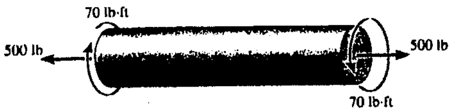
$$\sigma_2 = 1008.77 \text{ psi}$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = (1779.408)^2 - (1779.408)(1008.77) + (1008.77)^2 \leq [(30)(10^3)]^2$$

$$= 2.389(10^6) \leq 900(10^6)$$

Yielding will not occur. **Ans**

10–95 The 304-stainless-steel cylinder has an inner diameter of 4 in. and a wall thickness of 0.1 in. If it is subjected to an internal pressure of p = 80 psi, axial load of 500 lb and a torque of 70 lb · ft, determine if yielding occurs according to the maximum-shear-stress theory.



$$\sigma_x = \frac{P}{A} + \frac{pr}{2t} = \frac{500}{\pi(2.1)^2 - \pi(2)^2} + \frac{80(2)}{2(0.1)} = 1188.18 \text{ psi}$$

$$\sigma_y = \frac{pr}{t} = \frac{80(2)}{(0.1)} = 1600 \text{ psi}$$

$$\tau = \frac{Tc}{J} = \frac{70(12)(2.1)}{\frac{\pi}{2}(2.1)^2 - \frac{\pi}{2}(2)^2} = 325.686 \text{ psi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{1188.18 + 1600}{2} \pm \sqrt{\left(\frac{1188.18 - 1600}{2}\right)^2 + (325.686)^2}$$

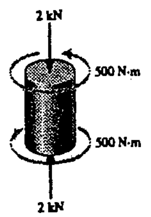
$$\sigma_1 = 1779.408 \text{ psi}$$

$$\sigma_2 = 1008.77 \text{ psi}$$

Since

$\sigma_1 = 1.78$ **ksi < 30 ksi** yielding will not occur. **Ans**

***10-96** The short concrete cylinder having a diameter of 50 mm is subjected to a torque of 500 N · m and an axial compressive force of 2 kN. Determine if it fails according to the maximum-normal-stress theory. The ultimate stress of the concrete is $\sigma_{ult} = 28$ MPa.



$$A = \frac{\pi}{4}(0.05)^2 = 1.9635(10^{-3})\ \text{m}^2$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.61359(10^{-6})\ \text{m}^4$$

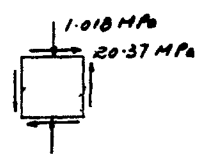
$$\sigma = \frac{P}{A} = \frac{2(10^3)}{1.9635(10^{-3})} = 1.019\ \text{MPa}$$

$$\tau = \frac{Tc}{J} = \frac{500(0.025)}{0.61359(10^{-6})} = 20.372\ \text{MPa}$$

$$\sigma_x = 0 \qquad \sigma_y = -1.019\ \text{MPa} \qquad \tau_{xy} = 20.372\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{0 - 1.018}{2} \pm \sqrt{\left(\frac{0-(-1.019)}{2}\right)^2 + 20.372^2}$$



$$\sigma_1 = 19.87\ \text{MPa} \qquad \sigma_2 = -20.89\ \text{MPa}$$

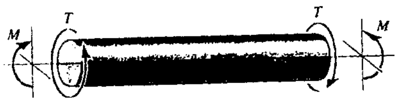
Failure criteria:

$|\sigma_1| < \sigma_{ult} = 28$ MPa OK

$|\sigma_2| < \sigma_{ult} = 28$ MPa OK

No. **Ans**

*10–97 If a solid shaft having a diameter d is subjected to a torque **T** and moment **M**, show that by the maximum-normal-stress theory the maximum allowable principal stress is $\sigma_{allow} = (16/\pi d^3)(M + \sqrt{M^2 + T^2})$.



Section properties :

$$I = \frac{\pi d^4}{64}; \qquad J = \frac{\pi d^4}{32}$$

Stress components :

$$\sigma = \frac{M c}{I} = \frac{M(\frac{d}{2})}{\frac{\pi}{64} d^4} = \frac{32 M}{\pi d^3}; \qquad \tau = \frac{T c}{J} = \frac{T(\frac{d}{2})}{\frac{\pi}{32} d^4} = \frac{16 T}{\pi d^3}$$

The principal stresses :

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \frac{\frac{32 M}{\pi d^3} + 0}{2} \pm \sqrt{(\frac{\frac{32 M}{\pi d^3} - 0}{2})^2 + (\frac{16 T}{\pi d^3})^2}$$

$$= \frac{16 M}{\pi d^3} \pm \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

Maximum normal stress theory. Assume $\sigma_1 > \sigma_2$

$$\sigma_{allow} = \sigma_1 = \frac{16 M}{\pi d^3} + \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

$$= \frac{16}{\pi d^3}[M + \sqrt{M^2 + T^2}] \qquad \textbf{QED}$$

10–98 Determine the bulk modulus for hard rubber if $E_r = 0.68(10^3)$ ksi, $\nu_r = 0.43$.

$$K_r = \frac{E_r}{3(1 - 2\nu_r)} = \frac{0.68(10^3)}{3[1 - 2(0.43)]} = 1.62(10^3)\text{ ksi} \qquad \textbf{Ans}$$

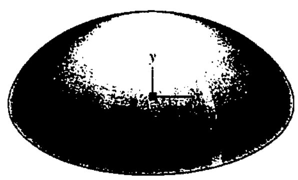
10–99 A thin-walled spherical pressure vessel has an inner radius r, thickness t, and is subjected to an internal pressure p. If the material constants are E and ν, determine the strain in the circumferential direction in terms of the stated parameters.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon = \frac{1}{E}(\sigma - \nu\sigma)$$

$$\varepsilon = \frac{1-\nu}{E}\sigma = \frac{1-\nu}{E}\left(\frac{pr}{2t}\right) = \frac{pr}{2Et}(1-\nu) \qquad \textbf{Ans}$$

•10–100 The strain at point A on the shell has components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = 275(10^{-6})$, $\epsilon_z = 0$. Determine (*a*) the principal strains at A, (*b*) the maximum shear strain in the $x-y$ plane, and (*c*) the absolute maximum shear strain.



$\varepsilon_x = 250(10^{-6}) \qquad \varepsilon_y = 400(10^{-6}) \qquad \gamma_{xy} = 275(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = 137.5(10^{-6})$

$A(250, 137.5)10^{-6} \qquad C(325, 0)10^{-6}$

$$R = \left(\sqrt{(325-250)^2 + (137.5)^2}\right)10^{-6} = 156.62(10^{-6})$$

a)

$\varepsilon_1 = (325 + 156.62)10^{-6} = 482(10^{-6})$ **Ans**

$\varepsilon_2 = (325 - 156.62)10^{-6} = 168(10^{-6})$ **Ans**

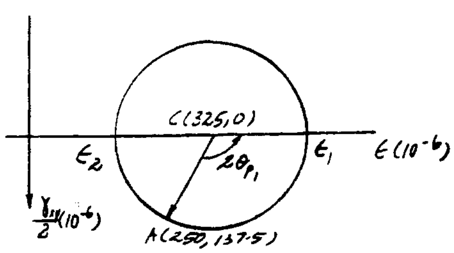
b)

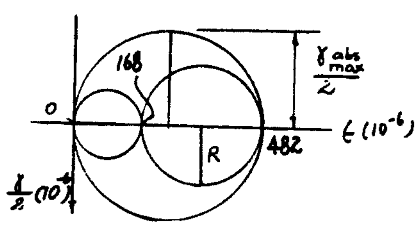
$\gamma_{\substack{max \\ in-plane}} = 2R = 2(156.62)(10^{-6}) = 313(10^{-6})$ **Ans**

c)

$\frac{\gamma_{\substack{abs \\ max}}}{2} = \frac{482(10^{-6})}{2}$

$\gamma_{\substack{abs \\ max}} = 482(10^{-6})$ **Ans**





10–101 A differential element is subjected to plane strain that has the following components: $\epsilon_x = 950(10^{-6})$, $\epsilon_y = 420(10^{-6})$, $\gamma_{xy} = -325(10^{-6})$. Use the strain-transformation equations and determine (*a*) the principal strains and (*b*) the maximum in-plane shear strain and the associated average strain. In each case specify the orientation of the element and show how the strains deform the element.

$$\varepsilon_{1,2} = \frac{\varepsilon_x+\varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \gamma_{xy}^2}$$

$$= \left[\frac{950+420}{2} \pm \sqrt{\left(\frac{950-420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6})$$

$\varepsilon_1 = 996(10^{-6})$ **Ans** $\varepsilon_2 = 374(10^{-6})$ **Ans**

Orientation of ε_1 and ε_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-325}{950-420}$$

$$\theta_p = -15.76°,\ 74.24°$$

Use Eq. 10 - 5 to determine the direction of ε_1 and ε_2.

$$\varepsilon_x' = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\theta = \theta_p = -15.76°$$

$$\varepsilon_x' = \left\{\frac{950+420}{2} + \frac{950-420}{2}\cos(-31.52°) + \frac{(-325)}{2}\sin(-31.52°)\right\}(10^{-6}) = 996(10^{-6})$$

$\theta_{p_1} = -15.8°$ **Ans** $\theta_{p_2} = 74.2°$ **Ans**

b)

$$\frac{\gamma_{\substack{max\\ in\text{-}plane}}}{2} = \sqrt{\left(\frac{\varepsilon_x-\varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max\\ in\text{-}plane}} = 2\left[\sqrt{\left(\frac{950-420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6}) = 622(10^{-6}) \quad \textbf{Ans}$$

$$\varepsilon_{avg} = \frac{\varepsilon_x+\varepsilon_y}{2} = \left(\frac{950+420}{2}\right)(10^{-6}) = 685(10^{-6}) \quad \textbf{Ans}$$

Orientation of γ_{max} :

$$\tan 2\theta_s = \frac{-(\varepsilon_x-\varepsilon_y)}{\gamma_{xy}} = \frac{-(950-420)}{-325}$$

$\theta_s = 29.2°$ and $\theta_s = 119°$ **Ans**

Use Eq. 10 - 6 to determine the sign of $\gamma_{\substack{max\\ in\text{-}plane}}$:

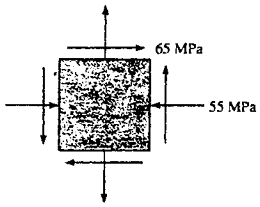
$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x-\varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\theta = \theta_s = 29.2°$$

$$\gamma_{x'y'} = 2\left[\frac{-(950-420)}{2}\sin(58.4°) + \frac{-325}{2}\cos(58.4°)\right](10^{-6})$$

$$\gamma_{x'y'} = -622(10^{-6})$$

10–102 The components of plane stress at a critical point on a thin steel,shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-distortion-energy theory. The yield stress for the steel is $\sigma_Y = 650$ MPa.



$\sigma_x = -55$ MPa $\sigma_y = 340$ MPa $\tau_{xy} = 65$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \gamma_{xy}^2}$$

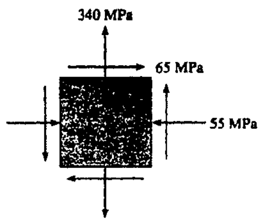
$$= \frac{-55 + 340}{2} \pm \sqrt{\left(\frac{-55 - 340}{2}\right)^2 + 65^2}$$

$\sigma_1 = 350.42$ MPa $\sigma_2 = -65.42$ MPa

$$(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2) = [350.42^2 - 350.42(-65.42) + (-65.42)^2]$$
$$= 150\,000 < \sigma_Y^2 = 422\,500 \quad \text{OK}$$

No. **Ans**

10–103 Solve Prob. 10–102 using the maximum-shear-stress theory.



$\sigma_x = -55 \text{ MPa} \qquad \sigma_y = 340 \text{ MPa} \qquad \tau_{xy} = 65 \text{ MPa}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \gamma_{xy}^2}$$

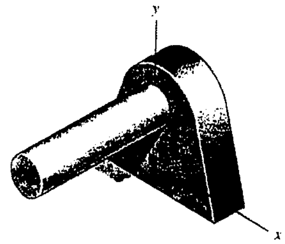
$$= \frac{-55+340}{2} \pm \sqrt{(\frac{-55-340}{2})^2 + 65^2}$$

$\sigma_1 = 350.42 \text{ MPa} \qquad \sigma_2 = -65.42 \text{ MPa}$

$|\sigma_1 - \sigma_2| = 350.42 - (-65.42) = 415.84 \text{ MPa} < \sigma_Y = 650 \text{ MPa}$ OK

No. **Ans**

10–104 The state of strain at the point on the bracket has components $\epsilon_x = 350(10^{-6})$, $\epsilon_y = -860(10^{-6})$, $\gamma_{xy} = 250(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 45°$ clockwise from the original position. Sketch the deformed element within the x–y plane due to these strains.



$$\varepsilon_x = 350(10^{-6}) \qquad \varepsilon_y = -860(10^{-6}) \qquad \gamma_{xy} = 250(10^{-6}) \qquad \theta = -45°$$

$$\varepsilon_x' = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

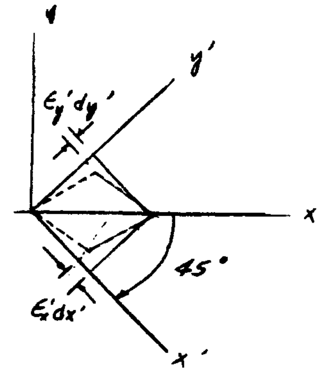
$$= [\frac{350-860}{2} + \frac{350-(-860)}{2}\cos(-90°) + \frac{250}{2}\sin(-90°)](10^{-6}) = -380(10^{-6}) \qquad \textbf{Ans}$$

$$\varepsilon_y' = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

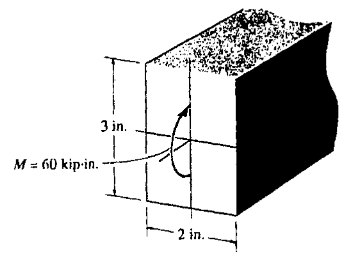
$$= [\frac{350-860}{2} - \frac{350-(-860)}{2}\cos(-90°) - \frac{250}{2}\sin(-90°)](10^{-6}) = -130(10^{-6}) \qquad \textbf{Ans}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma}{2}\cos 2\theta$$

$$\gamma_{x'y'} = 2[-(\frac{350-(-860)}{2})\sin(-90°) + \frac{250}{2}\cos(-90°)](10^{-6}) = 1.21(10^{-3}) \qquad \textbf{Ans}$$



10–105 The aluminum beam has the rectangular cross section shown. If it is subjected to a bending moment of M = 60 kip · in., determine the increase in the 2-in. dimension at the top of the beam and the decrease in this dimension at the bottom. $E_{al} = 10(10^3)$ ksi, $\nu_{al} = 0.3$.



In general for the top or bottom of the beam :

$$\sigma_z = -\frac{Mc}{I} = -\frac{M\frac{h}{2}}{\frac{1}{12}bh^3} = -\frac{6M}{bh^2}$$

$$\varepsilon_x = -\frac{\nu\sigma_z}{E} = \frac{6\nu M}{Ebh^2}$$

$$\Delta b = \varepsilon_x b = \frac{6\nu M}{Ebh^2}(b)$$

$$= \frac{6\nu M}{Eh^2}$$

At the top :

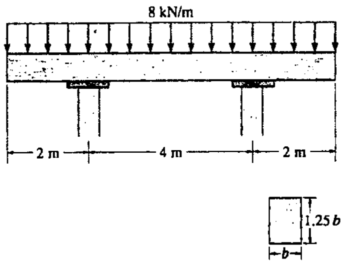
$$\Delta b = \frac{6(0.3)(60)}{10(10^3)(3^2)} = 1.2(10^{-3}) \text{ in.}$$ **Ans**

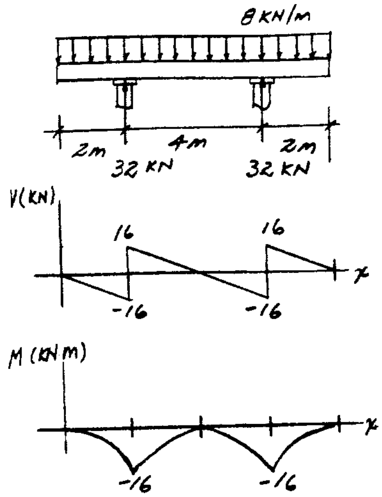
At the bottom :

$$\Delta b = -1.2(10^{-3}) \text{ in.}$$ **Ans**

The negative sign indicates shortening.

11–1 The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 6.5$ MPa and an allowable shear stress of $\tau_{allow} = 500$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25.





$$I_x = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$Q_{max} = \bar{y}'A' = (0.3125b)(0.625b)(b) = 0.1953125b^3$$

Assume bending moment controls :

$$M_{max} = 16\ \text{kN}\cdot\text{m}$$

$$\sigma_{allow} = \frac{M_{max}c}{I}$$

$$6.5(10^6) = \frac{16(10^3)(0.625b)}{0.16276b^4}$$

$b = 0.21143\text{m} = 211$ mm **Ans**

$h = 1.25b = 264\text{mm}$ Ans

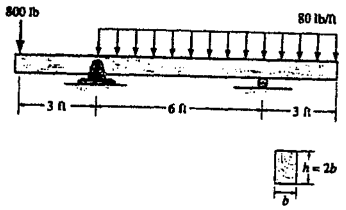
Check shear :

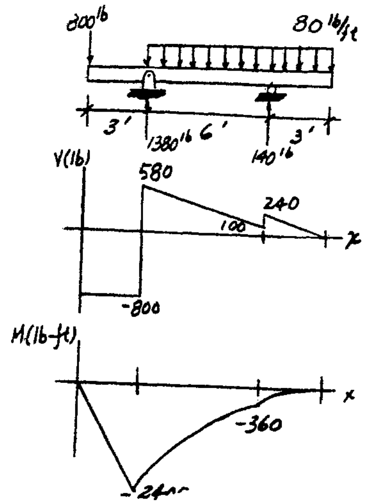
$$Q_{max} = 1.846159(10^{-3})\ \text{m}^3$$

$$I = 0.325248(10^{-3})\text{m}^4$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{16(10^3)(1.846159)(10^{-3})}{0.325248(10^{-3})(0.21143)} = 429\ \text{kPa} < 500\ \text{kPa} \qquad \text{OK}$$

11-2. The beam is made of Douglas fir having an allowable bending stress of $\sigma_{allow} = 1.1$ ksi and an allowable shear stress of $\tau_{allow} = 0.70$ ksi. Determine the width b of the beam if the height $h = 2b$.





$$I_x = \frac{1}{12}(b)(2b)^3 = 0.6667\,b^4$$

$$Q_{max} = \bar{y}'A' = (0.5b)(b)(b) = 0.5b^3$$

Assume bending moment controls :

$$M_{max} = 2400 \text{ lb} \cdot \text{ft}$$

$$\sigma_{allow} = \frac{M_{max}c}{I}$$

$$1100 = \frac{2400(12)(b)}{0.6667b^4}$$

$b = 3.40$ in. **Ans**

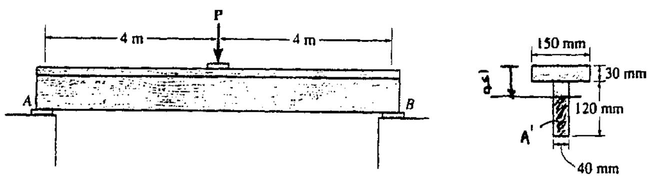
Check shear :

$$Q_{max} = 19.65 \text{ in}^3$$

$$I = 89.09 \text{ in}^4$$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{800(19.65)}{89.09(3.40)} = 51.9 \text{ psi} < 700 \text{ psi} \quad \text{OK}$$

11-3 The timber beam is to be loaded as shown. If the ends support only vertical forces, determine the greatest magnitude of **P** that can be applied. $\sigma_{allow} = 25$ MPa, $\tau_{allow} = 700$ kPa.



$$\bar{y} = \frac{(0.015)(0.150)(0.03) + (0.09)(0.04)(0.120)}{(0.150)(0.03) + (0.04)(0.120)} = 0.05371 \text{ m}$$

$$I = \frac{1}{12}(0.150)(0.03)^3 + (0.15)(0.03)(0.05371 - 0.015)^2 + \frac{1}{12}(0.04)(0.120)^3 + (0.04)(0.120)(0.09 - 0.05371)^2 = 19.162(10^{-6}) \text{ m}^4$$

Maximum moment at center of beam :

$$M_{max} = \frac{P}{2}(4) = 2P$$

$$\sigma = \frac{Mc}{I}; \qquad 25(10^6) = \frac{(2P)(0.15 - 0.05371)}{19.162(10^{-6})}$$

$$P = 2.49 \text{ kN}$$

Maximum shear at end of beam :

$$V_{max} = \frac{P}{2}$$

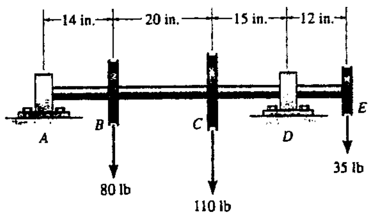
$$\tau = \frac{VQ}{It}; \qquad 700(10^3) = \frac{\left[\frac{1}{2}(0.15 - 0.05371)(0.04)(0.15 - 0.05371)\right]}{19.162(10^{-6})}$$

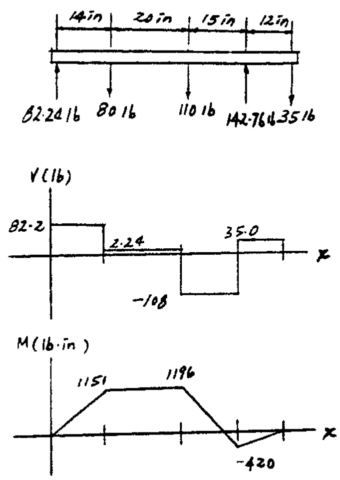
$$P = 145 \text{ kN}$$

Thus,

$P = 2.49$ kN **Ans**

•11–4 Draw the shear and moment diagrams for the shaft, and determine its required diameter to the nearest $\frac{1}{4}$ in. if $\sigma_{allow} = 7$ ksi and $\tau_{allow} = 3$ ksi. The bearings at A and D exert only vertical reactions on the shaft. The loading is applied to the pulleys at B, C, and E.





$$\sigma_{allow} = \frac{M_{max}\, c}{I}$$

$$7(10^3) = \frac{1196\, c}{\frac{\pi}{4} c^4}; \qquad c = 0.601 \text{ in.}$$

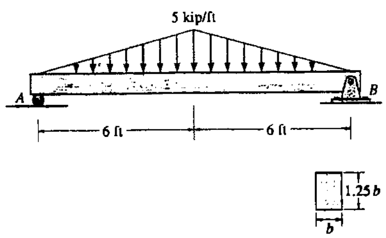
$$d = 2c = 1.20 \text{ in.}$$

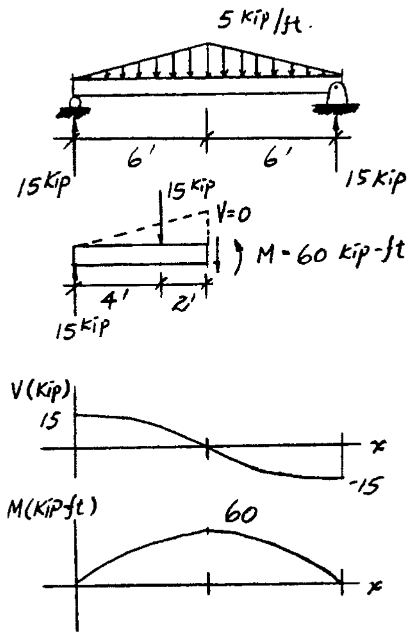
Use $d = 1.25$ in. **Ans**

Check shear :

$$\tau_{max} = \frac{V_{max} Q}{I t} = \frac{108\left(\frac{4(0.625)}{3\pi}\right)(\pi)\left(\frac{0.625^2}{2}\right)}{\frac{\pi}{4}(0.625)^4(1.25)} = 117 \text{ psi} < 3 \text{ ksi} \qquad \textbf{OK}$$

11-5 The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 960$ psi and an allowable shear stress of $\tau_{allow} = 75$ psi. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of 1.25.





$$I = \frac{1}{12}(b)(1.25b)^3 = 0.16276b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.16276b^4}{0.625b} = 0.26042b^3$$

Assume bending moment controls :

$$M_{max} = 60 \text{ kip} \cdot \text{ft}$$

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}$$

$$960 = \frac{60(10^3)(12)}{0.26042\,b^3}$$

$b = 14.2$ in.

Check shear :

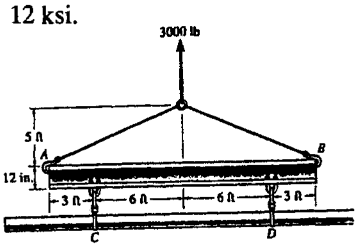
$$\tau_{max} = \frac{1.5V}{A} = \frac{1.5(15)(10^3)}{(14.2)(1.25)(14.2)} = 88.9 \text{ psi} > 75 \text{ psi} \qquad \text{NG}$$

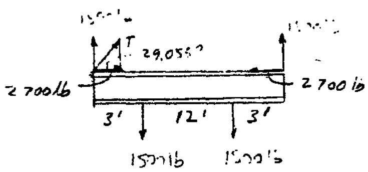
Shear controls :

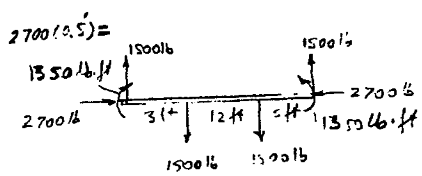
$$\tau_{allow} = \frac{1.5V}{A} = \frac{1.5(15)(10^3)}{(b)(1.25b)}$$

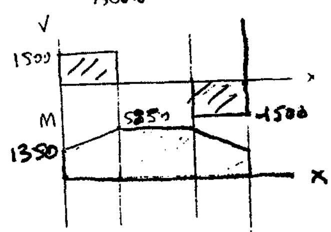
$b = 15.5$ in. **Ans**

11-6. The spreader beam AB is used to lift slowly the 3000-lb pipe that is centrally located on the straps at C and D. If the beam is a $W\ 12 \times 45$, determine if it can safely support the load. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.









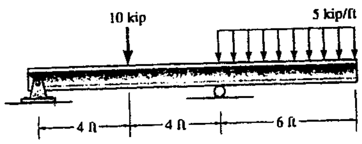
$$h = \frac{1500}{\tan 29.055^\circ} = 2700 \text{ lb}$$

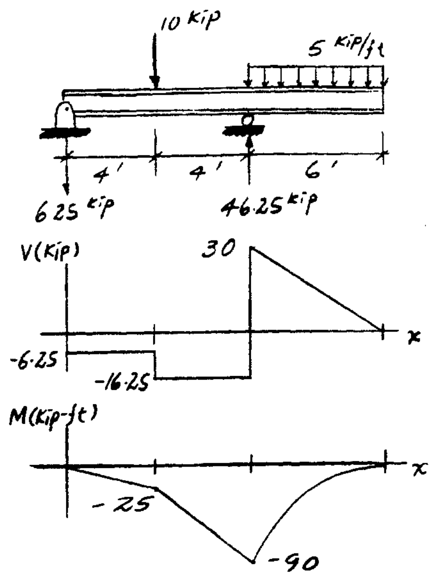
$$\sigma = \frac{M}{S}; \qquad \sigma = \frac{5850(12)}{58.1} = 1.21 \text{ ksi} < 22 \text{ ksi} \qquad \text{OK}$$

$$\tau = \frac{V}{A_{web}}; \qquad \tau = \frac{1500}{(12.06)(0.335)} = 371 \text{ psi} < 12 \text{ ksi} \qquad \text{OK}$$

Yes. **Ans**

11–7 Select the lightest-weight steel wide-flange beam from Appendix *B* that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.





Assume bending moment controls.

$M_{max} = 90$ kip · ft

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{90(12)}{24} = 45 \text{ in}^3$$

Select a W 16 x 31

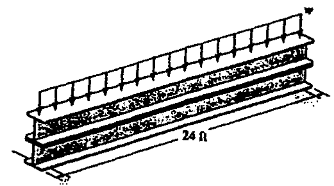
$S_x = 47.5 \text{ in}^3$ $\quad d = 15.88$ in. $\quad t_w = 0.275$ in.

Check shear :

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{30}{(15.88)(0.275)} = 6.87 \text{ ksi} < 14 \text{ ksi} \qquad \text{OK}$$

Use W 16 x 31 **Ans**

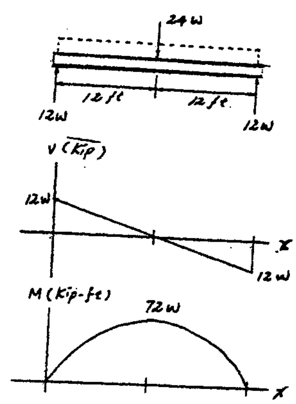
***11-8.** The simply supported beam is composed of two W 12 × 22 sections built up as shown. Determine the maximum uniform loading w the beam will support if the allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.



Section properties : For W 12 x 22 (d = 12.31 in. $I_x = 156\text{ in}^4$ t_w = 0.260 in. $A = 6.48\text{ in}^2$)

$$I = 2[156 + 6.48(\frac{12.31}{2})^2] = 802.98\text{ in}^4$$

$$S = \frac{I}{c} = \frac{802.98}{12.31} = 65.23\text{ in}^3$$



Maximum Loading : Assume moment controls.

$$M = \sigma_{allow} S$$

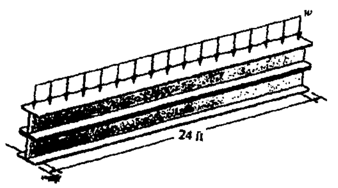
$$(72\,w)(12) = 22(65.23)$$

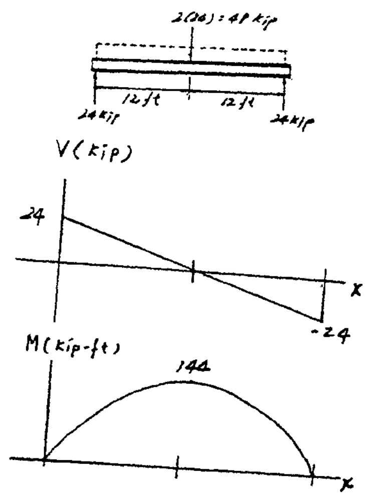
$$w = 1.66\text{ kip / ft} \qquad \textbf{Ans}$$

Check Shear : (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{12(1.66)}{2(12.31)(0.26)} = 3.11\text{ ksi} < \tau_{allow} = 14\text{ ksi} \quad \textbf{OK}$$

11-9. The simply supported beam is composed of two $W\ 12 \times 22$ sections built up as shown. Determine if the beam will safely support a loading of $w = 2$ kip/ft. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.





Section properties :

For $W\ 12 \times 22$ ($d = 12.31$ in. $I_x = 156\ \text{in}^4$ $t_w = 0.260$ in. $A = 6.48\ \text{in}^2$)

$$I = 2[156 + 6.48(6.155^2)] = 802.98\ \text{in}^4$$

$$S = \frac{I}{c} = \frac{802.98}{12.31} = 65.23\ \text{in}^3$$

Bending stress :

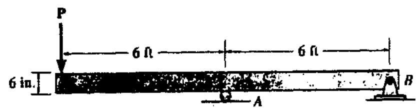
$$\sigma_{max} = \frac{M_{allow}}{S} = \frac{144\,(12)}{65.23} = 26.5\ \text{ksi} > \sigma_{allow} = 22\ \text{ksi}$$

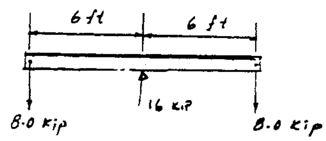
No, the beam fails due to bending stress criteria. **Ans**

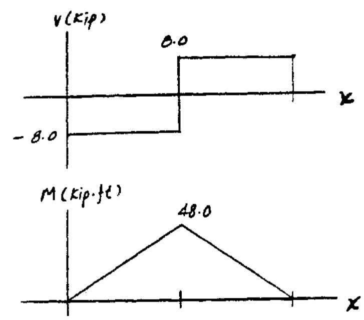
Check shear : (Neglect area of flanges.)

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{24}{2(12.31)(0.26)} = 3.75\ \text{ksi} < \tau_{allow} = 14\ \text{ksi} \quad \text{OK}$$

11–10 Determine the minimum width of the beam to the nearest $\frac{1}{4}$ in. that will safely support the loading of $P = 8$ kip. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 15$ ksi.







Beam design : Assume moment controls.

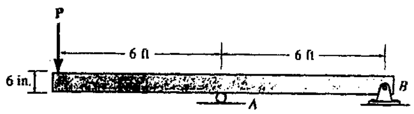
$$\sigma_{allow} = \frac{Mc}{I}; \qquad 24 = \frac{48.0(12)(3)}{\frac{1}{12}(b)(6^3)}$$

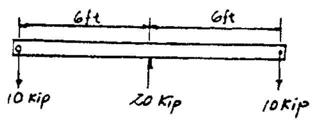
$b = 4$ in. **Ans**

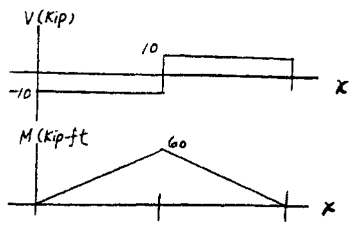
Check shear :

$$\tau_{max} = \frac{VQ}{It} = \frac{8(1.5)(3)(4)}{\frac{1}{12}(4)(6)^3(4)} = 0.5 \text{ ksi} < 15 \text{ ksi} \qquad \text{OK}$$

11–11 Solve Prob. 11–10 if $P = 10$ kip.







Beam design : Assume moment controls.

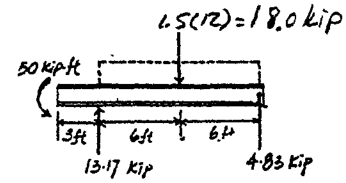
$$\sigma_{allow} = \frac{Mc}{I}; \qquad 24 = \frac{60(12)(3)}{\frac{1}{12}(b)(6^3)}$$

$b = 5$ in. **Ans**

Check shear :

$$\tau_{max} = \frac{VQ}{It} = \frac{10(1.5)(3)(5)}{\frac{1}{12}(5)(6)^3(5)} = 0.5\text{ ksi} < 15\text{ ksi} \quad \text{OK}$$

***11-12.** Draw the shear and moment diagrams for the W 12 × 14 beam and check if the beam will safely support the loading. Take $\sigma_{allow} = 22$ ksi and $\tau_{allow} = 12$ ksi.



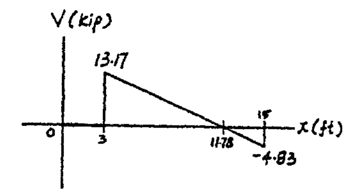
Bending Stress : From the moment diagram, $M_{max} = 50.0$ kip · ft. Applying the flexure formula with $S = 14.9$ in^3 for a wide-flange section W12 × 14,

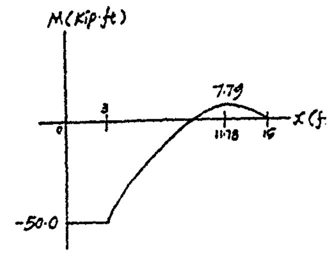
$$\sigma_{max} = \frac{M_{max}}{S}$$
$$= \frac{50.0(12)}{14.9} = 40.27 \text{ ksi} > \sigma_{allow} = 22 \text{ ksi } (\textit{No Good!})$$

Shear Stress : From the shear diagram, $V_{max} = 13.17$ kip. Using $\tau = \dfrac{V}{t_w d}$ where $d = 11.91$ in. and $t_w = 0.20$ in. for W12 × 14 wide flange section,

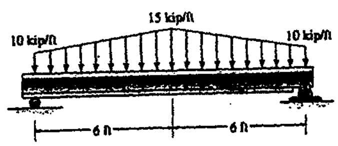
$$\tau_{max} = \frac{V_{max}}{t_w d}$$
$$= \frac{13.17}{0.20(11.91)}$$
$$= 5.53 \text{ ksi} < \tau_{allow} = 12 \text{ ksi } (\textit{O.K!})$$

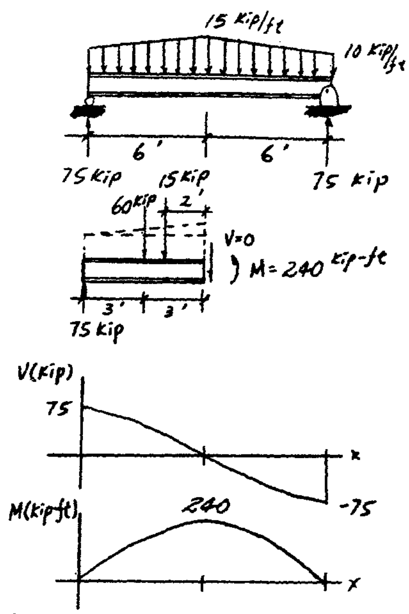
Hence, the **wide flange section** W12 × 14 **fails due to the bending stress and will not safely support the loading.** **Ans**





11-13. Select the lightest-weight steel wide-flange beam from Appendix *B* that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi.





Assume bending controls.

$M_{max} = 240 \text{ kip} \cdot \text{ft}$

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{240(12)}{24} = 120 \text{ in}^3$$

Select a $W\,24 \times 62$,

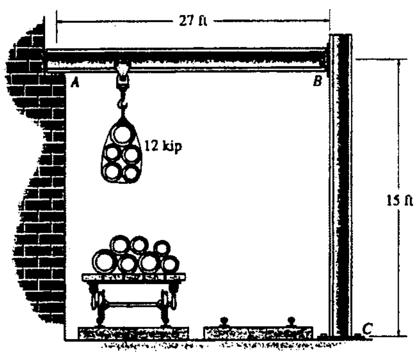
$S_x = 131 \text{ in}^3$ $\quad d = 23.74$ in. $\quad t_w = 0.430$ in.

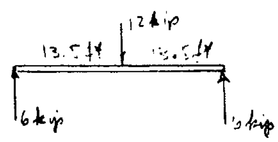
Check shear :

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{75}{(23.74)(0.430)} = 7.35 \text{ ksi} < 14 \text{ ksi}$$

Use $W\,24 \times 62$ **Ans**

11–14 The beam is used in a railroad yard for loading and unloading cars. If the maximum anticipated hoist load is 12 kip, select the lightest-weight steel wide-flange section from Appendix *B* that will safely support the loading. The hoist travels along the bottom flange of the beam, 1 ft $\le x \le$ 25 ft, and has negligible size. Assume the beam is pinned to the column at *B* and roller supported at *A*. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.





Maximum moment occurs when load is in the center of beam.

$M_{max} = (6\text{ kip})(13.5\text{ ft}) = 81\text{ lb}\cdot\text{ft}$

$\sigma_{allow} = \dfrac{M}{S}; \qquad 24 = \dfrac{81(12)}{S_{req'd}}$

$S_{req'd} = 40.5\text{ in}^3$

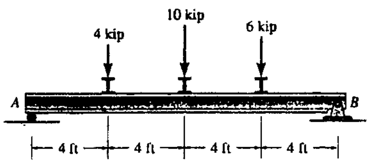
Select a W 14 x30, $S_x = 42.0\text{ in}^3$, $d = 13.84$ in, $t_w = 0.270$ in.

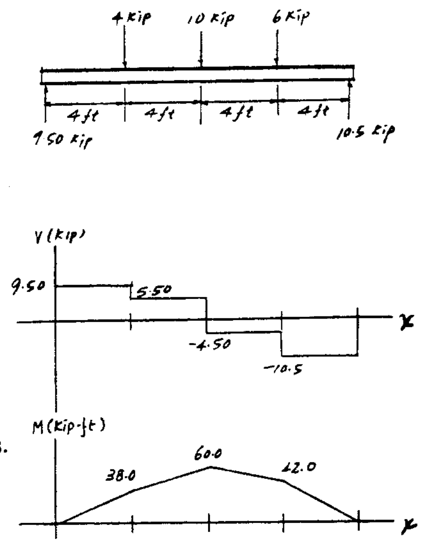
At $x = 1$ ft, $V = 11.56$ kip

$\tau = \dfrac{V}{A_{web}} = \dfrac{11.36}{(13.84)(0.270)} = 3.09\text{ksi} < 12\text{ksi}$

Use W14 × 30 **Ans.**

11–15 Select the shortest and lightest-weight steel wide-flange beam from Appendix *B* that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.





Beam design : Assume bending moment controls.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{60.0(12)}{22} = 32.73 \text{ in}^3$$

Select a W 12 x 26

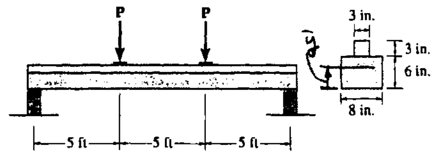
$S_x = 33.4 \text{ in}^3$, $d = 12.22$ in., $t_w = 0.230$ in.

Check shear :

$$\tau_{avg} = \frac{V}{A_{web}} = \frac{10.5}{(12.22)(0.230)} = 3.74 \text{ ksi} < 12 \text{ ksi}$$

Use W 12 x 26 **Ans**

•11-16 Two acetyl plastic members are to be glued together and used to support the loading shown. If the allowable bending stress for the plastic is $\sigma_{allow} = 13$ ksi and the allowable shear stress is $\tau_{allow} = 4$ ksi, determine the greatest load P that can be supported and specify the required shear stress capacity of the glue.



$M_{max} = P(5)(12) = 60\,P$

$V_{max} = P$

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{3(6)(8) + 7.5(3)(3)}{(6)(8) + (3)(3)} = 3.7105 \text{ in.}$$

$$I = \frac{1}{12}(8)(6)^3 + 8(6)(3.7105 - 3)^2 + \frac{1}{12}(3)(3)^3 + 3(3)(7.5 - 3.7105)^2 = 304.22 \text{ in}^4$$

Bending :

$$\sigma = \frac{Mc}{I}; \qquad 13 = \frac{60\,P(9 - 3.7105)}{304.22}$$

$$P = 12.462 = 12.5 \text{ kip}$$

Shear :

$$\tau = \frac{VQ}{It};$$

At neutral axis :

$$4 = \frac{P(3.7105/2)(8)(3.7105)}{304.22(8)}, \qquad P = 177 \text{ kip}$$

Also check just above glue seam.

$$4 = \frac{P(7.5 - 3.7105)(3)(3)}{304.22(3)}, \qquad P = 107 \text{ kip}$$

Bending governs, thus

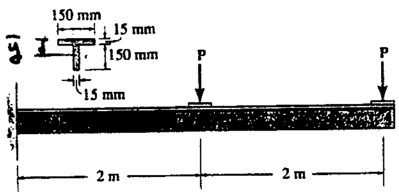
$P = 12.5$ kip **Ans**

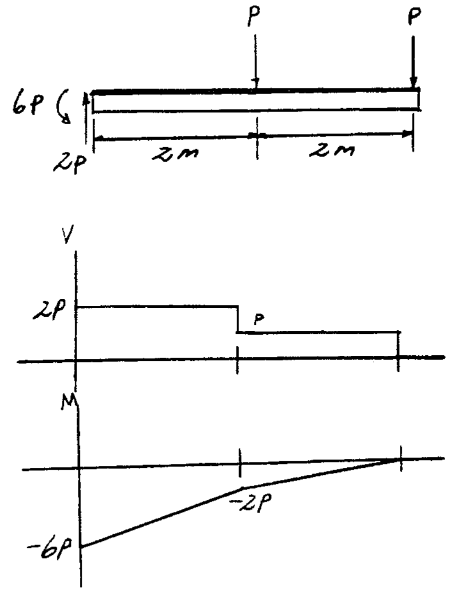
Glue strength :

$$\tau = \frac{VQ}{It}; \qquad \tau_{req'd} = \frac{12.462(7.5 - 3.7105)(3)(3)}{304.22(3)}$$

$\tau_{req'd} = 466$ psi **Ans**

11–17 The steel cantilevered T-beam is made from two plates welded together as shown. Determine the maximum loads P that can be safely supported on the beam if the allowable bending stress is $\sigma_{allow} = 170$ MPa and the allowable shear stress is $\tau_{allow} = 95$ MPa.





Section properties :

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.0075(0.15)(0.015) + 0.09(0.15)(0.015)}{0.15(0.015) + 0.15(0.015)} = 0.04875 \text{ m}$$

$$I = \frac{1}{12}(0.15)(0.015)^3 + 0.15(0.015)(0.04875 - 0.0075)^2 + \frac{1}{12}(0.015)(0.15)^3 + 0.015(0.15)(0.09 - 0.04875)^2 = 11.9180(10^{-6}) \text{ m}^4$$

$$S = \frac{I}{c} = \frac{11.9180(10^{-6})}{(0.165 - 0.04875)} = 0.10252(10^{-3}) \text{ m}^3$$

$$Q_{max} = \bar{y}'A' = (\frac{(0.165 - 0.04875)}{2})(0.165 - 0.04875)(0.015) = 0.101355(10^{-3}) \text{ m}^3$$

Maximum load : Assume failure due to bending moment.

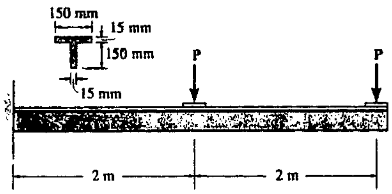
$$M_{max} = \sigma_{allow} S; \quad 6P = 170(10^6)(0.10252)(10^{-3})$$

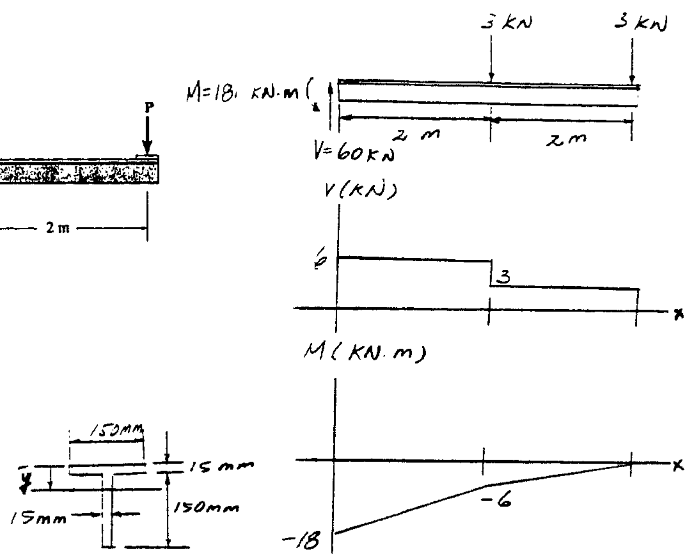
$$P = 2904.7 \text{ N} = 2.90 \text{ kN}$$ **Ans**

Check shear :

$$\tau_{max} = \frac{V_{max} Q_{max}}{I t} = \frac{2(2904.7)(0.101353)(10^{-3})}{11.9180(10^{-6})(0.015)} = 3.29 \text{ MPa} < \tau_{allow} = 95 \text{ MPa}$$

11–18 Determine if the steel cantilevered T-beam can safely support the two loads of $P = 3$ kN if the allowable bending stress is $\sigma_{allow} = 170$ MPa and the allowable shear stress is $\tau_{allow} = 95$ MPa.





Maximum shear and moment are at support.

$V_{max} = 3 + 3 = 6\text{ kN} \qquad M_{max} = 2(3) + 4(3) = 18\text{ kN}\cdot\text{m}$

Section properties :

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.0075(0.15)(0.015) + 0.09(0.15)(0.015)}{0.15(0.015) + 0.15(0.015)} = 0.04875\text{ m}$$

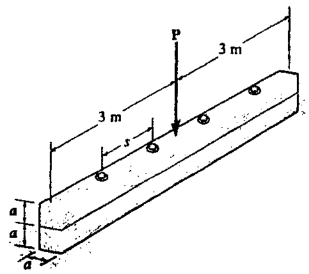
$$I = \frac{1}{12}(0.15)(0.015)^3 + 0.15(0.015)(0.04875 - 0.0075)^2 + \frac{1}{12}(0.015)(0.15)^3 + 0.015(0.15)(0.09 - 0.04875)^2 = 11.9180(10^{-6})\text{ m}^4$$

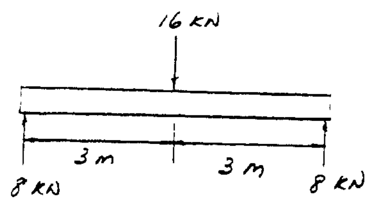
$$S = \frac{I}{c} = \frac{11.9180(10^{-6})}{(0.165 - 0.04875)} = 0.10252(10^{-3})\text{ m}^3$$

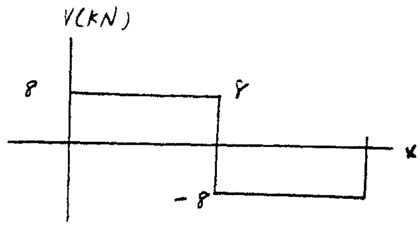
$$\sigma_{max} = \frac{M_{max}}{S} = \frac{18(10^3)}{0.10252(10^{-3})} = 176\text{ MPa} > 170\text{ MPa}$$

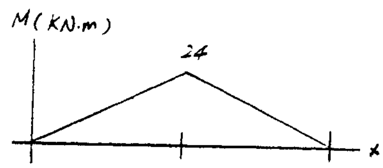
No. **Ans**

11-19. The simply supported beam supports a load of $P = 16$ kN. Determine the smallest dimension a of each timber if the allowable bending stress for the wood is $\sigma_{allow} = 30$ MPa and the allowable shear stress is $\tau_{allow} = 800$ kPa. Also, if each bolt can sustain a shear of 2.5









Section properties :

$$I = \frac{1}{12}(a)(2a)^3 = 0.66667\,a^4$$

$$Q_{max} = \bar{y}'A' = \frac{a}{2}(a)(a) = 0.5\,a^3$$

Assume bending controls.

$$\sigma_{allow} = \frac{M_{max}c}{I}; \qquad 30(10^6) = \frac{24(10^3)a}{0.66667\,a^4}$$

$a = 0.106266$ m $= 106$ mm **Ans**

Check shear :

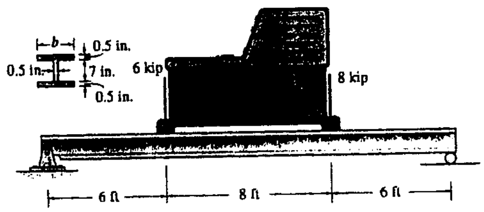
$$\tau_{max} = \frac{VQ}{It} = \frac{8(10^3)(0.106266/2)(0.106266)^2}{0.66667(0.106266^4)(0.106266)} = 531 \text{ kPa} < \tau_{allow} = 800 \text{ kPa} \quad \text{OK}$$

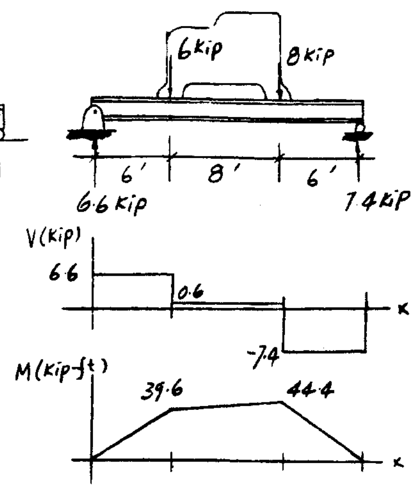
Bolt spacing :

$$q = \frac{VQ}{I} = \frac{8(10^3)(0.106266/2)(0.106266^2)}{0.66667\,(0.106266^4)} = 56462.16 \text{ N/m}$$

$$s = \frac{2.5(10^3)}{56462.16} = 0.04427 \text{ m} = 44.3 \text{ mm}$$ **Ans**

*11–20 The beam is to be used to support the machine, which exerts the forces of 6 kip and 8 kip as shown. If the maximum bending stress is not to exceed $\sigma_{allow} = 22$ ksi, determine the required width b of the flanges.





Section Properties :

$$I = \frac{1}{12}(b)(8^3) - \frac{1}{12}(b - 0.5)(7^3) = 14.083b + 14.292$$

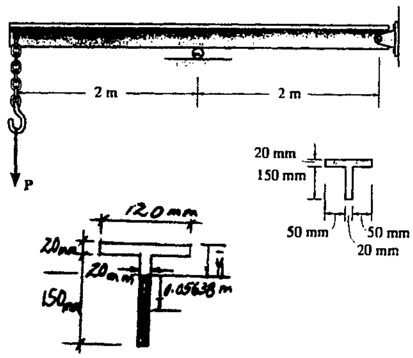
$$S = \frac{I}{c} = \frac{14.083b + 14.292}{4} = 3.5208b + 3.5729$$

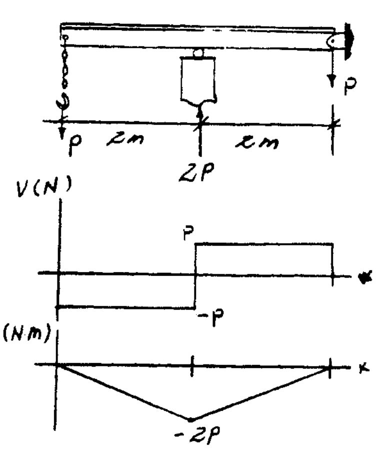
$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}}$$

$$3.5208b + 3.5729 = \frac{44.4(12)}{22}$$

$b = 5.86$ in. **Ans**

11–21 The steel beam has an allowable bending stress σ_{allow} = 140 MPa and an allowable shear stress of τ_{allow} = 90 MPa. Determine the maximum load that can safely be supported.





Section properties :

$$\bar{y} = \frac{(10)(120)(20) + (95)(150)(20)}{120(20) + 150(20)} = 57.22 \text{ mm}$$

$$Q_{max} = \bar{y}'A' = (0.05638)(0.02)(0.170 - 0.05722) = 0.127168(10^{-3})\text{m}^3$$

$$I = \frac{1}{12}(0.12)(0.02^3) + 0.12(0.02)(0.05722 - 0.01)^2 + \frac{1}{12}(0.02)(0.15^3) + 0.15(0.02)(0.095 - 0.05722)^2 = 15.3383(10^{-6})\text{ m}^4$$

$$S = \frac{I}{c} = \frac{15.3383(10^{-6})}{(0.170 - 0.05722)} = 0.136005(10^{-3})\text{ m}^3$$

For moment :

$$M = \sigma_{allow} S$$

$$2P = 140(10^6)(0.136005)(10^{-3})$$

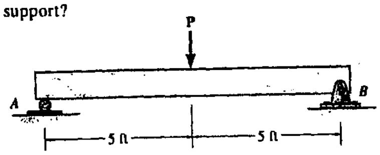
$P = 9520 \text{ N} = 9.52 \text{ kN}$ (Controls) **Ans**

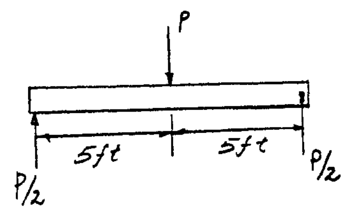
For shear :

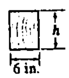
$$V \doteq \tau_{allow}\left(\frac{It}{Q_{max}}\right)$$

$$P = 90(10^6)\left(\frac{15.3383(10^{-6})(0.02)}{0.127168(10^{-3})}\right) = 217106 = 217 \text{ kN}$$

11-22 The timber beam has a rectangular cross section. If the width of the beam is 6 in., determine its height h so that it simultaneously reaches its allowable bending stress of $\sigma_{allow} = 1.50$ ksi and an allowable shear stress of $\tau_{allow} = 50$ psi. Also, what is the maximum load P that the beam can then support?





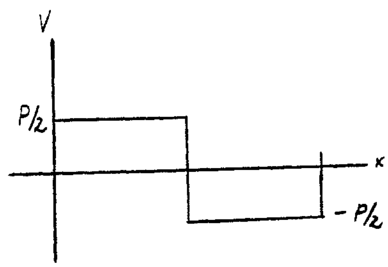


Section properties :

$$I = \frac{1}{12}(6)(h^3) = 0.5\,h^3$$

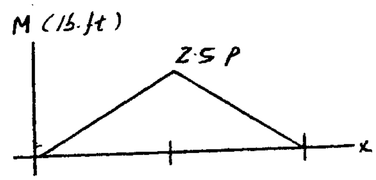
$$S = \frac{I}{c} = \frac{0.5h^3}{0.5h} = h^2$$

$$Q_{max} = 0.25h(0.5h)(6) = 0.75\,h^2$$



If shear controls :

$$\tau_{allow} = \frac{V_{max}Q_{max}}{It}; \qquad 50 = \frac{(\frac{P}{2})(0.75h^2)}{0.5h^3(6)}$$



$$150h = 0.375\,P \qquad (1)$$

If bending controls :

$$\sigma_{allow} = \frac{M_{max}}{S}$$

$$S = \frac{I}{c} = \frac{0.5(h^3)}{\frac{h}{2}} = h^2$$

$$1.50(10^3) = \frac{2.5P(12)}{h^2}$$

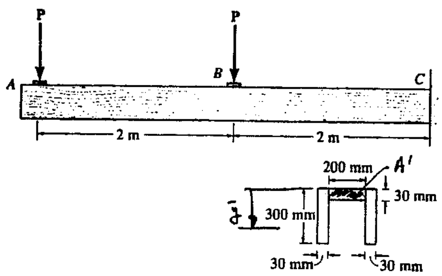
$$1.50(10^3)h^2 = 30P \qquad (2)$$

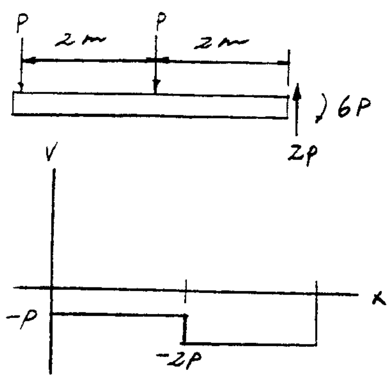
Solving Eqs. (1) and (2) yields :

$h = 8.0$ in. **Ans**

$P = 3200$ lb **Ans**

11–23 The beam is constructed from three plastic strips. If the glue can support a shear stress of $\tau_{allow} = 8$ kPa, determine the largest magnitude of the loads **P** that can be applied to the beam.





Section properties :

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.015(0.2)(0.03) + 2[0.15(0.3)(0.03)]}{0.2(0.03) + 2(0.3)(0.03)} = 0.11625 \text{ m}$$

$$I = \frac{1}{12}(0.2)(0.03)^3 + 0.2(0.03)(0.11625-0.015)^2 + 2[\frac{1}{12}(0.03)(0.3)^3 + 0.03(0.3)(0.15-0.11625)^2]$$
$$= 0.2174625(10^{-3}) \text{ m}^4$$

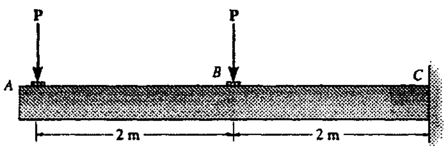
$$Q_A = \bar{y}'A' = (0.11625-0.015)(0.2)(0.03) = 0.6075(10^{-3}) \text{ m}^3$$

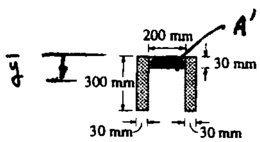
Maximum load :

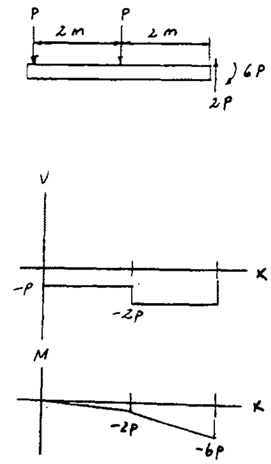
$$\tau_{allow} = \frac{V_{max}Q_A}{It}; \qquad 8000 = \frac{2P(0.6075)(10^{-3})}{0.2174625(10^{-3})(2)(0.03)}$$

$P = 85.9$ N **Ans**

•11–24 The beam is constructed from three wood boards. If the allowable bending stress is $\sigma_{allow} = 6$ MPa, and the glue can support a shear stress of $\tau_{allow} = 8$ kPa, determine the largest magnitude of the loads **P** that can be applied to the beam.







Section properties :

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{0.015(0.2)(0.03) + 2[0.15(0.3)(0.03)]}{0.2(0.03) + 2(0.3)(0.03)} = 0.1162 \text{ m}$$

$$I = \frac{1}{12}(0.2)(0.03)^3 + 0.2(0.03)(0.11625 - 0.015)^2$$

$$+ 2[\frac{1}{12}(0.03)(0.3)^3 + 0.03(0.3)(0.15 - 0.11625)^2] = 0.2174625(10^{-3}) \text{ m}^4$$

Maximum load :

$$\sigma_{allow} = \frac{M_{max}c}{I}; \qquad 6(10^6) = \frac{6P(0.3 - 0.11625)}{0.2174625(10^{-3})}$$

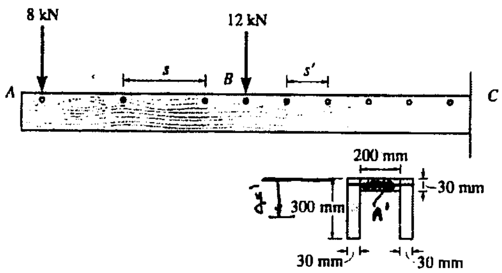
$P = 1183$ N $= 1.18$ kN

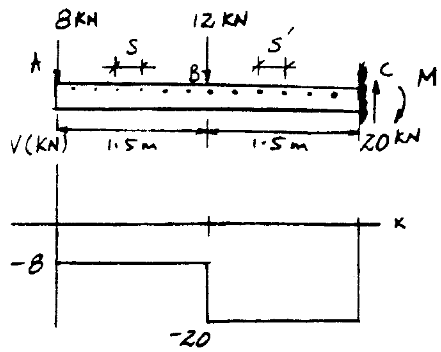
Check glue : $Q_A = \bar{y}'A' = (0.11625 - 0.015)(0.2)(0.03) = 0.6075(10^{-3})$ m^3

$$\tau_{allow} = \frac{V_{max}Q_A}{It}; \qquad 8000 = \frac{2P(0.6075)(10^{-3})}{0.2174625(10^{-3})(2)(0.03)}$$

$P = 85.9$ N **Ans**

11-25 The beam is constructed from three boards as shown. If each nail can support a shear force of 300 N, determine the maximum spacing of the nails, s and s', within regions AB and BC.





$$\bar{y} = \frac{(0.015)(0.2)(0.03) + 2[(0.150)(0.3)(0.03)]}{0.2(0.03) + 2(0.3)(0.03)} = 0.11625 \text{ m}$$

$$I = \frac{1}{12}(0.2)(0.03^3) + (0.2)(0.03)(0.11625 - 0.015)^2 +$$
$$2[\frac{1}{12}(0.03)(0.3^3) + (0.03)(0.3)(0.150 - 0.11625)^2] = 0.2174625(10^{-3})\text{m}^4$$

$$Q = \bar{y}'A' = (0.11625 - 0.015)(0.2)(0.03) = 0.6075(10^{-3}) \text{ m}^3$$

Region AB :

$V = 8$ kN

$$q = \frac{VQ}{I} = \frac{8(10^3)(0.6075)(10^{-3})}{0.2174625(10^{-3})} = 22348.68 \text{ N/m}$$

$$s = \frac{300}{22348.68/2} = 0.0268 \text{ m} = 26.8 \text{ mm} \qquad \textbf{Ans}$$

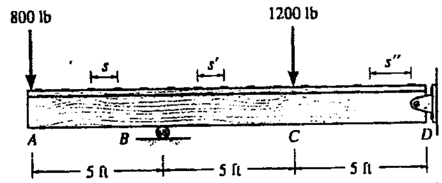
Region BC :

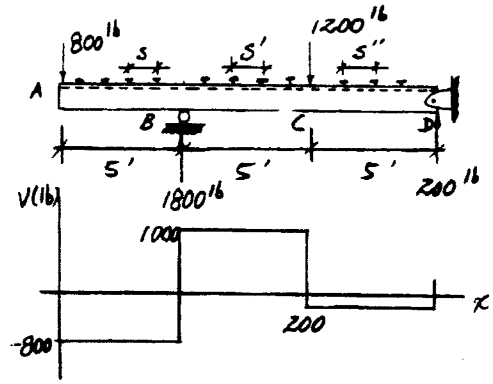
$V = 20$ kN

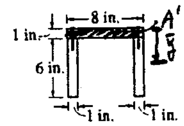
$$q = \frac{VQ}{I} = \frac{20(10^3)(0.6075)(10^{-3})}{0.2174625(10^{-3})} = 55871.70 \text{ N}$$

$$s' = \frac{300}{55871.70/2} = 0.0107 \text{ m} = 10.7 \text{ mm} \qquad \textbf{Ans}$$

11–26 The beam is constructed from three boards as shown. If each nail can support a shear force of 50 lb, determine the maximum spacing of the nails, *s*, *s'*, and *s''*, for regions *AB*, *BC*, and *CD*, respectively.







Section properties :

$$\bar{y} = \frac{(0.5)8(1) + 2[(4)(6)(1)]}{8(1) + 2[(6)(1)]} = 2.6 \text{ in.}$$

$$I = \frac{1}{12}(8)(1^3) + 8(1)(2.6 - 0.5)^2 + 2(\frac{1}{12})(1)(6^3) + 2(1)(6)(4 - 2.6)^2 = 95.47 \text{ in}^4$$

$$Q = (2.6 - 0.5)(8)(1) = 16.8 \text{ in}^3$$

Region *AB* :

$$V = 800 \text{ lb} \qquad q = \frac{VQ}{I} = \frac{800(16.8)}{95.47} = 140.8 \text{ lb/in.}$$

$$s = \frac{50}{140.8/2} = 0.710 \text{ in.}$$ **Ans**

Region *BC* :

$$V = 1000 \text{ lb}, \qquad q = \frac{VQ}{I} = \frac{1000(16.8)}{95.47} = 176.0 \text{ lb/in.}$$

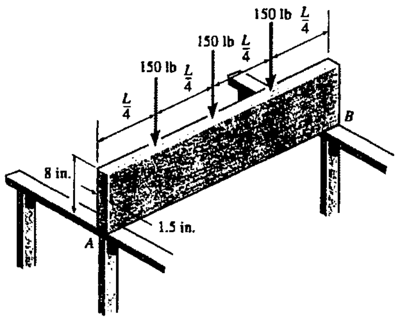
$$s' = \frac{50}{176.0/2} = 0.568 \text{ in.}$$ **Ans**

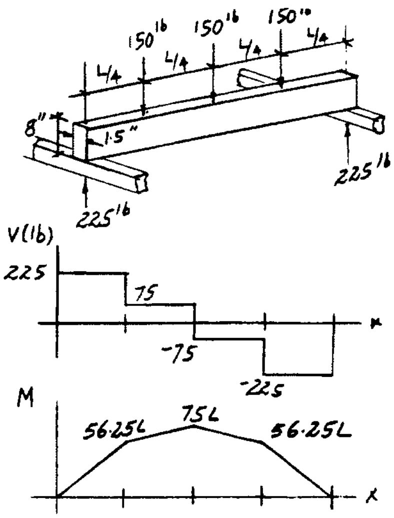
Region *CD* :

$$V = 200 \text{ lb} \qquad q = \frac{VQ}{I} = \frac{200(16.8)}{95.47} = 35.2 \text{ lb/in.}$$

$$s'' = \frac{50}{35.2/2} = 2.84 \text{ in.}$$ **Ans**

11–27 The joist AB used in housing construction is to be made from 8-in. by 1.5-in. Southern-pine boards. If the design loading on each board is placed as shown, determine the largest room width L that the boards can span. The allowable bending stress for the wood is $\sigma_{allow} = 2$ ksi and the allowable shear stress is $\tau_{allow} = 180$ psi. Assume that the beam is simply supported from the walls at A and B.





Check shear :

$$\tau_{max} = \frac{1.5V}{A} = \frac{1.5(225)}{(1.5)(8)} = 28.1 \text{ psi}$$

28.1 psi < 180 psi OK

For bending moment :

$$M_{max} = 75\,L$$

$$I = \frac{1}{12}(1.5)(8^3) = 64 \text{ in}^4$$

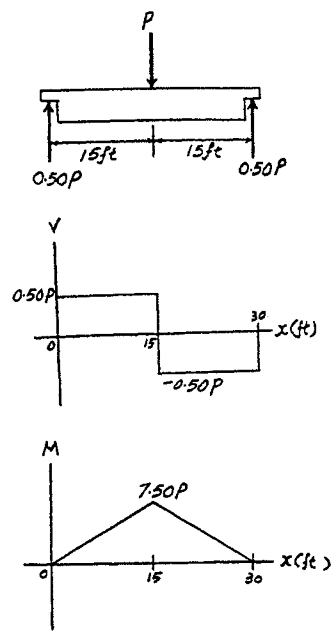
$$S = \frac{I}{c} = \frac{64}{4} = 16 \text{ in}^3$$

$$M_{max} = \sigma_{allow} S$$

$$75L(12) = 2000(16)$$

$$L = 35.6 \text{ ft} \qquad \textbf{Ans}$$

*11-28. The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{allow} = 350$ psi and the allowable bending stress is $\sigma_{allow} = 1500$ psi, determine the height h that will cause the beam to reach both allowable stresses at the same time. Also, what load P causes this to happen? Neglect the stress concentration at the notch.



Bending Stress : From the moment diagram. $M_{max} = 7.50P$.
Applying the flexure formula,

$$\sigma_{allow} = \frac{M_{max}\,c}{I}$$

$$1500 = \frac{7.50P(12)(5)}{\frac{1}{12}(2)(10^3)}$$

$$P = 555.56 \text{ lb} = 556 \text{ lb} \qquad \textbf{Ans}$$

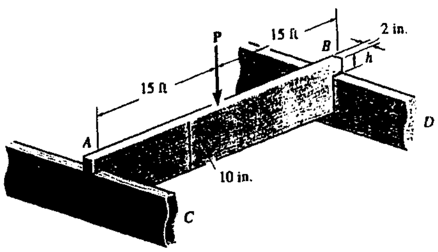
Shear Stress : From the shear diagram, $V_{max} = 0.500P = 277.78$ lb.
The notch is the critical section. Using the shear formula for a rectangular section,

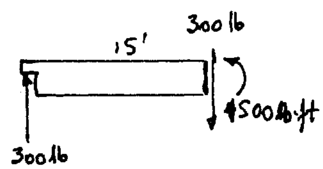
$$\tau_{allow} = \frac{3V_{max}}{2A}$$

$$350 = \frac{3(277.78)}{2(2)h}$$

$$h = 0.595 \text{ in.} \qquad \textbf{Ans}$$

11-29 The simply supported joist is used in the construction of a floor for a building. In order to keep the floor low with respect to the sill beams C and D, the ends of the joists are notched as shown. If the allowable shear stress for the wood is $\tau_{allow} = 350$ psi and the allowable bending stress is $\sigma_{allow} = 1700$ psi, determine the smallest height h so that the beam will support a load of $P = 600$ lb. Also, will the entire joist safely support the load? Neglect the stress concentration at the notch.





The reaction at the support is $\frac{600}{2} = 300$ lb

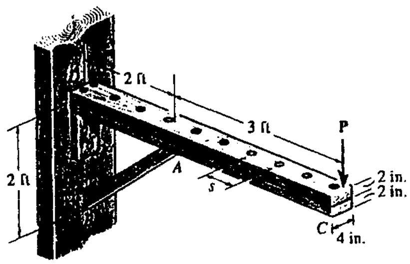
$$\tau_{allow} = \frac{1.5V}{A}; \qquad 350 = \frac{1.5(300)}{(2)(h)}$$

$h = 0.643$ in. **Ans**

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{4500(12)(5)}{\frac{1}{12}(2)(10)^3} = 1620 \text{ psi} < 1700 \text{ psi} \quad \text{OK}$$

Yes, the joist will safely support the load. **Ans**

11-30 The overhang beam is constructed using two 2-in. by 4-in. pieces of wood braced as shown. If the allowable bending stress is $\sigma_{allow} = 600$ psi, determine the largest load P that can be applied. Also, determine the associated maximum spacing of nails, s, along the beam section AC if each nail can resist a shear force of 800 lb. Assume the beam is pin-connected at A, B, and D. Neglect the axial force developed in the beam along DA.



$M_A = M_{max} = 3P$

Section properties :

$$I = \frac{1}{12}(4)(4)^3 = 21.33 \text{ in}^4$$

$$S = \frac{I}{c} = \frac{21.33}{2} = 10.67 \text{ in}^3$$

$M_{max} = \sigma_{allow} S$

$3P(12) = 600(10.67)$

$P = 177.78 = 178$ lb **Ans**

Nail Spacing :

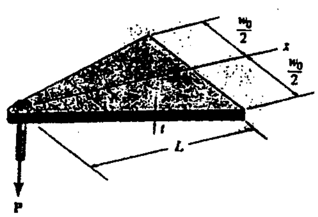
$V = P = 177.78$ lb

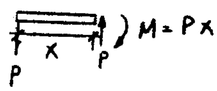
$Q = (4)(2)(1) = 8 \text{ in}^3$

$$q = \frac{VQ}{I} = \frac{177.78(8)}{21.33} = 66.67 \text{ lb/in.}$$

$$S = \frac{800 \text{ lb}}{66.67 \text{ lb/in.}} = 12.0 \text{ in.}$$ **Ans**

11-31. Determine the variation in the width w as a function of x for the cantilevered beam that supports a concentrated force **P** at its end so that it has a maximum bending stress σ_{allow} throughout its length. The beam has a constant thickness t.





Section properties :

$$I = \frac{1}{12}(w)(t^3) \qquad S = \frac{I}{c} = \frac{\frac{1}{12}(w)(t^3)}{t/2} = \frac{wt^2}{6}$$

$$\sigma_{allow} = \frac{M}{S} = \frac{Px}{w\,t^2/6} \qquad (1)$$

At $x = L$,

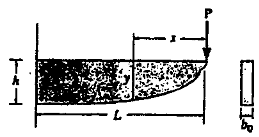
$$\sigma_{allow} = \frac{PL}{w_0\,t^2/6} \qquad (2)$$

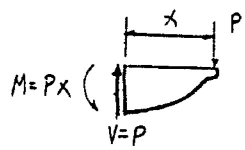
Equate Eqs (1) and (2),

$$\frac{Px}{w\,t^2/6} = \frac{PL}{w_0\,t^2/6}$$

$$w = \frac{w_0}{L}x \qquad \textbf{Ans}$$

***11-32.** Determine the variation in the depth of a cantilevered beam that supports a concentrated force **P** at its end so that it has a constant maximum bending stress σ_{allow} throughout its length. The beam has a constant width b_0.





Section properties :

$$I = \frac{1}{12} b_0 d^3; \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b_0 d^3}{\frac{d}{2}} = \frac{b_0 d^2}{6}$$

Maximum bending stress :

$$\sigma_{allow} = \frac{M}{S} = \frac{Px}{b_0 \frac{d^2}{6}} = \frac{6Px}{b_0 d^2} \qquad (1)$$

At $x = L, \quad d = h$

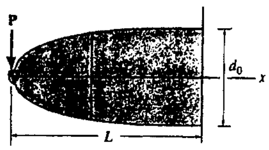
$$\sigma_{allow} = \frac{6PL}{b_0 h^2} \qquad (2)$$

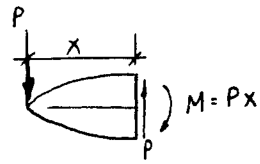
Equating Eqs. (1) and (2),

$$\frac{6Px}{b_0 d^2} = \frac{6PL}{b_0 h^2}$$

$$d = h\sqrt{\frac{x}{L}} \qquad \textbf{Ans}$$

11–33 Determine the variation in the depth d of a cantilevered beam that supports a concentrated force **P** at its end so that it has a constant maximum bending stress σ_{allow} throughout its length. The beam has a constant width b_0.





Section properties :

$$I = \frac{1}{12}(b_0)(d^3) \qquad S = \frac{I}{c} = \frac{\frac{1}{12}(b_0)(d^3)}{d/2} = \frac{b_0 d^2}{6}$$

$$\sigma_{allow} = \frac{M}{S} = \frac{Px}{b_0 d^2/6} \qquad (1)$$

At $x = L$

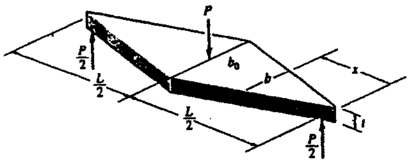
$$\sigma_{allow} = \frac{PL}{b_0 d_0^2/6} \qquad (2)$$

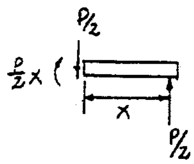
Equate Eqs. (1) and (2) :

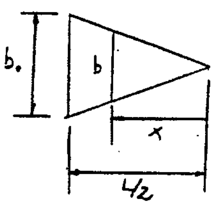
$$\frac{Px}{d_0{}^2/6} = \frac{PL}{b_0 d_0{}^2/6}$$

$$d^2 = (\frac{d_0{}^2}{L})x\,; \qquad d = d_0\sqrt{\frac{x}{L}} \qquad \textbf{Ans}$$

11-34. The beam is made from a plate having a constant thickness t and a width that varies as shown. If it supports a concentrated force **P** at its center, determine the absolute maximum bending stress in the beam and specify its location x, $0 < x < L/2$.







Section properties :

$$\frac{b}{b_0} = \frac{x}{\frac{L}{2}}; \qquad b = \frac{2b_0}{L}x$$

$$I = \frac{1}{12}\left(\frac{2b_0}{L}x\right)t^3 = \frac{b_0 t^3}{6L}x$$

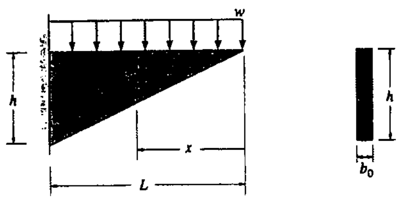
$$S = \frac{I}{c} = \frac{\frac{b_0 t^3}{6L}x}{\frac{t}{2}} = \frac{b_0 t^2}{3L}x$$

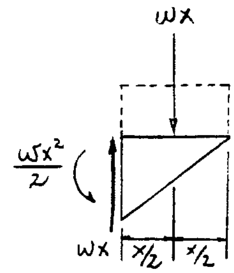
Bending stress :

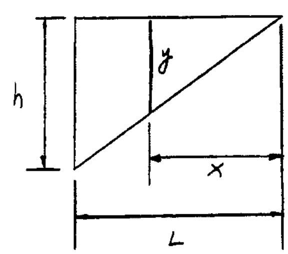
$$\sigma = \frac{M}{S} = \frac{\frac{P}{2}x}{\frac{b_0 t^2}{3L}x} = \frac{3PL}{2b_0 t^2} \qquad \textbf{Ans}$$

The bending stress is independent of x. Therefore, the stress is constant throughout the span. **Ans**

11–35 The tapered beam supports a uniform distributed load w. If it is made from a plate that has a constant width b_0, determine the absolute maximum bending stress in the beam.







Section properties :

$$\frac{y}{h} = \frac{x}{L}; \qquad y = \frac{h}{L}x$$

$$I = \frac{1}{12}(b_0)(\frac{h}{L}x)^3 = \frac{b_0 h^3}{12L^3}x^3$$

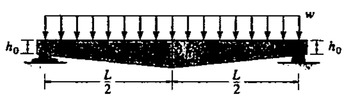
$$S = \frac{I}{c} = \frac{\frac{b_0 h^3}{12L^3}x^3}{\frac{h}{2L}x} = \frac{b_0 h^2}{6L^2}x^2$$

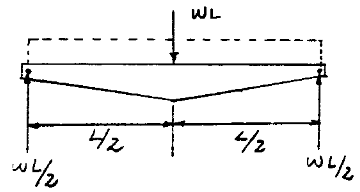
Bending stress :

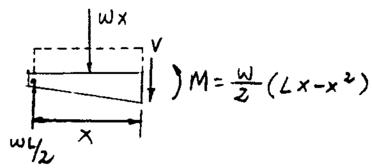
$$\sigma_{max} = \frac{M}{S} = \frac{\frac{w}{2}x^2}{\frac{b_0 h^2}{6L^2}x^2} = \frac{3wL^2}{b_0 h^2}$$ **Ans**

The bending stress is independent of x. Therefore, the stress is constant throughout the span. **Ans**

•11-36 The tapered beam supports a uniform distributed load w. If it is made from a plate and has a constant width b, determine the absolute maximum bending stress in the beam.

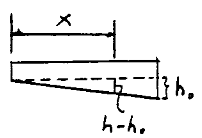






Section properties :

$$\frac{h - h_0}{x} = \frac{h_0}{\frac{L}{2}}; \qquad h = h_0\left(\frac{2}{L}x + 1\right)$$



$$I = \frac{1}{12}bh_0^3\left(\frac{2}{L}x + 1\right)^3$$

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh_0^3\left(\frac{2}{L}x + 1\right)^3}{\frac{h_0}{2}\left(\frac{2}{L}x + 1\right)} = \frac{1}{6}bh_0^2\left(\frac{2}{L}x + 1\right)^2$$

Bending stress :

$$\sigma = \frac{M}{S} = \frac{\frac{w}{2}(Lx - x^2)}{\frac{1}{6}bh_0^2\left(\frac{2}{L}x + 1\right)^2} = \frac{3w}{bh_0^2}\left[\frac{Lx - x^2}{\left(\frac{2}{L}x + 1\right)^2}\right] \qquad (1)$$

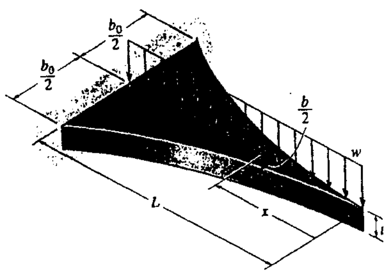
$$\frac{d\sigma}{dx} = \frac{3w}{bh_0^2}\left[\frac{\left(\frac{2}{L}x + 1\right)^2(L - 2x) - (Lx - x^2)(2)\left(\frac{2}{L}x + 1\right)\left(\frac{2}{L}\right)}{\left(\frac{2}{L}x + 1\right)^4}\right] = 0$$

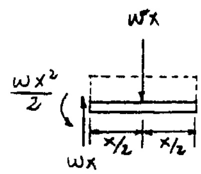
$$\left(\frac{2}{L}x + 1\right)(L - 2x) - \frac{4}{L}(Lx - x^2) = 0; \qquad x = \frac{L}{4}$$

Hence, from Eq. (1),

$$\sigma_{max} = \frac{3w}{bh_0^2}\left[\frac{L\left(\frac{L}{4}\right) - \left(\frac{L}{4}\right)^2}{\left(\frac{2}{L}\left(\frac{L}{4}\right) + 1\right)^2}\right] = \frac{wL^2}{4bh_0^2} \qquad \textbf{Ans}$$

11–37 Determine the variation in the width b as a function of x for the cantilevered beam that supports a uniform distributed load along its centerline so that it has the same maximum bending stress σ_{allow} throughout its length. The beam has a constant depth t.





Section properties :

$$I = \frac{1}{12} b\, t^3 \qquad S = \frac{I}{c} = \frac{\frac{1}{12} b\, t^3}{\frac{t}{2}} = \frac{t^2}{6} b$$

Bending stress :

$$\sigma_{allow} = \frac{M}{S} = \frac{\frac{w\,x^2}{2}}{\frac{t^2}{6} b} = \frac{3wx^2}{t^2 b} \qquad (1)$$

At $x = L, b = b_0$

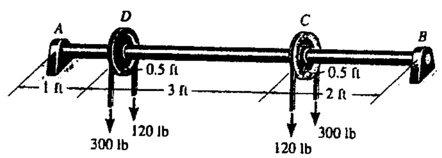
$$\sigma_{allow} = \frac{3wL^2}{t^2 b_0} \qquad (2)$$

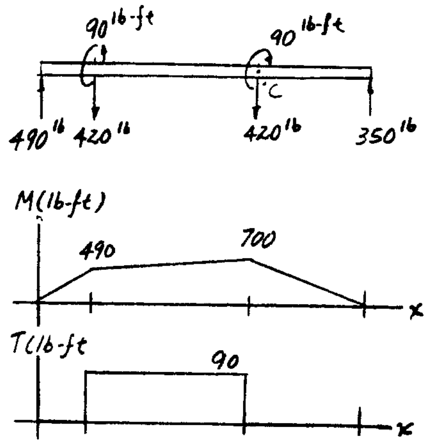
Equating Eqs. (1) and (2) yields :

$$\frac{3wx^2}{t^2 b} = \frac{3wL^2}{t^2 b_0}$$

$$b = \frac{b_0}{L^2} x^2 \qquad \textbf{Ans}$$

11–38 The two pulleys attached to the shaft are loaded as shown. If the bearings at A and B exert only vertical forces on the shaft, determine the required diameter of the shaft to the nearest $\frac{1}{8}$ in. using the maximum-shear-stress theory. $\tau_{allow} = 12$ ksi.





Section just to the left of point C is the most critical.

$$c = \left(\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right)^{1/3} = \left(\frac{2}{\pi\,(12)(10^3)}\sqrt{[700(12)]^2 + [90(12)]^2}\right)^{1/3}$$

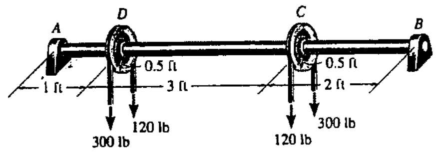
$c = 0.766$ in.

$d = 2c = 1.53$ in.

Use $d = 1\frac{5}{8}$ in. **Ans**

11–39 Solve Prob. 11–38 using the maximum-distortion-energy theory. $\sigma_{allow} = 67$ ksi.

Section just to the left of point C is the most critical.
Both states of stress will yield the same result.



$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = B$

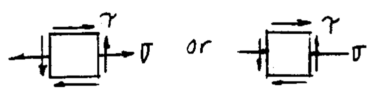
$$\sigma_a^2 = (A+B)^2 \qquad \sigma_b^2 = (A-B)^2$$

$$\sigma_a \sigma_b = (A+B)(A-B)$$

$$\begin{aligned}\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 &= A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB \\ &= A^2 + 3B^2 \\ &= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right) \\ &= \sigma^2 + 3\tau^2\end{aligned}$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad (1)$$



$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq (1)

$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{allow}^2$$

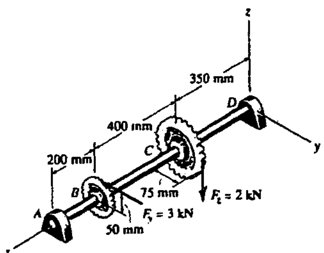
$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right)^{1/6} = \left(\frac{16((700)(12))^2 + 12((90)(12))^2}{\pi^2((67)(10^3))^2}\right)^{1/6}$$

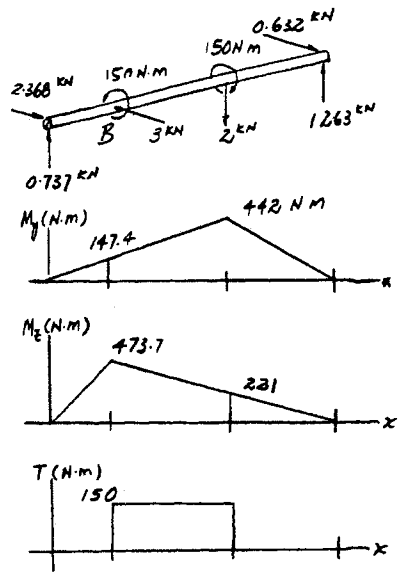
$c = 0.544$ in.

$d = 2c = 1.087$ in.

Use $d = 1\frac{1}{8}$ in. **Ans**

*11-40. The bearings at A and D exert only y and z components of force on the shaft. If $\tau_{allow} = 60$ MPa, determine to the nearest millimeter the smallest-diameter shaft that will support the loading. Use the maximum-shear-stress theory of failure.





Critical moment is at point B :

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1 \text{ N}\cdot\text{m}$$

$$T = 150 \text{ N}\cdot\text{m}$$

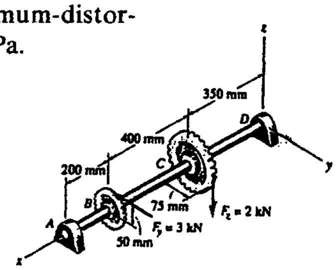
$$c = \left(\frac{2}{\pi\, \tau_{allow}}\sqrt{M^2 + T^2}\right)^{1/3} = \left(\frac{2}{\pi(60)(10^6)}\sqrt{496.1^2 + 150^2}\right)^{1/3} = 0.0176 \text{ m}$$

$c = 0.0176$ m $= 17.6$ mm

$d = 2c = 35.3$ mm

Use $d = 36$ mm **Ans**

11-41. Solve Prob. 11–40 using the maximum-distortionenergy theory of failure. $\sigma_{\text{allow}} = 130$ MPa.



The critical moment is at B.

$$M = \sqrt{(473.7)^2 + (147.4)^2} = 496.1\ \text{N}\cdot\text{m}$$

$$T = 150\ \text{N}\cdot\text{m}$$

Since,

$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = B$

$$\sigma_a^2 = (A+B)^2 \qquad \sigma_b^2 = (A-B)^2$$

$$\sigma_a \sigma_b = (A+B)(A-B)$$

$$\begin{aligned} \sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 &= A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB \\ &= A^2 + 3B^2 \\ &= \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right) \\ &= \sigma^2 + 3\tau^2 \end{aligned}$$

$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 = \sigma_{\text{allow}}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{\text{allow}}^2 \qquad (1)$$

$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

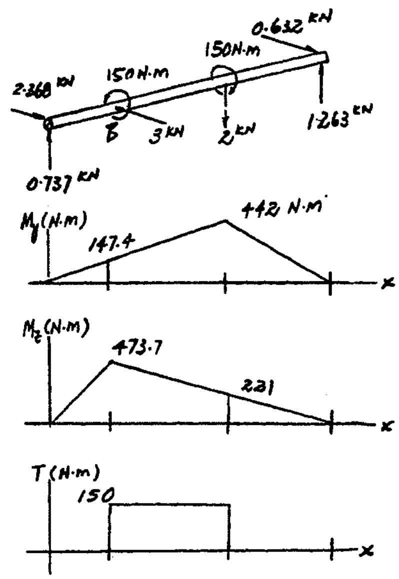
From Eq (1)

$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{\text{allow}}^2$$

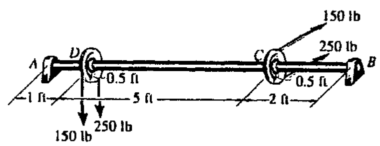
$$c = \left(\frac{16M^2 + 12T^2}{\pi^2 \sigma_{\text{allow}}^2}\right)^{1/6}$$

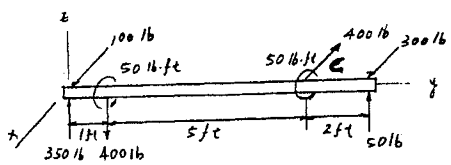
$$= \left(\frac{16(496.1)^2 + 12(150)^2}{\pi^2((130)(10^6))^2}\right)^{1/6} = 0.01712\ \text{m}$$

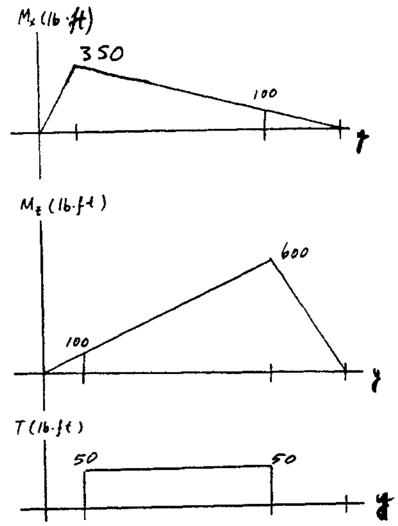
$d = 2c = 34.3$ mm **Ans**



11–42 The pulleys attached to the shaft are loaded as shown. If the bearings at A and B exert only horizontal and vertical forces on the shaft, determine the required diameter of the shaft to the nearest $\frac{1}{8}$ in. using the maximum-shear-stress theory of failure. $\tau_{allow} = 12$ ksi.







Section just to the left of point C is the most critical.

$$M = \sqrt{600^2 + 100^2} = 608.28\ \text{lb}\cdot\text{ft}$$

$$T = 50\ \text{lb}\cdot\text{ft}$$

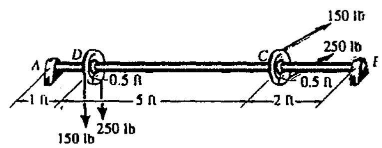
$$c = \left[\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(12)(10^3)}\sqrt{[(608.28)(12)]^2 + [50(12)]^2}\right]^{\frac{1}{3}}$$

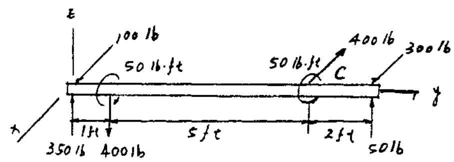
$$c = 0.7297\ \text{in.}$$

$$d = 2c = 1.46\ \text{in.}$$

Use $d = 1\frac{1}{2}$ in. **Ans**

11–43 Solve Prob. 11–42 using the maximum-distortion-energy theory of failure, $\sigma_{allow} = 20$ ksi.





Section just to the left of point C is the most critical.

$M = \sqrt{600^2 + 100^2} = 608.28$ lb · ft

$T = 50$ lb · ft

Both states of stress will yield the same result.

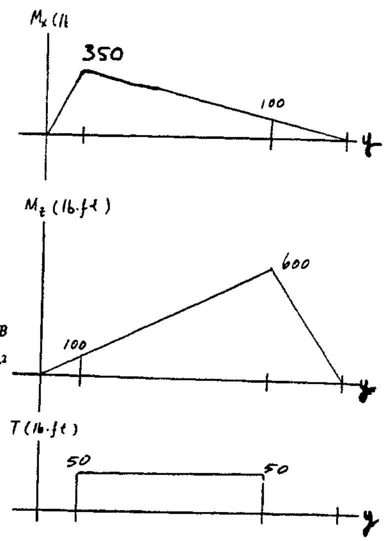
$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Let $\frac{\sigma}{2} = A$ and $\sqrt{\frac{\sigma^2}{4} + \tau^2} = B$

$\sigma_a^2 = (A + B)^2, \quad \sigma_b^2 = (A - B)^2$

$\sigma_a\,\sigma_b = (A + B)(A - B) = A^2 - B^2$

$$\sigma_a^2 - \sigma_a\,\sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$
$$= A^2 + 3B^2 = \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right) = \sigma^2 + 3\tau^2$$



$\sigma_a^2 - \sigma_a\,\sigma_b + \sigma_b^2 = \sigma_{allow}^2$

$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad (1)$

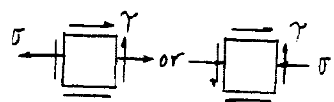
$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$

From Eq. (1)

$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{allow}^2$$

$$c = \left[\frac{16M^2 + 12T^2}{\pi^2 \sigma_{allow}^2}\right]^{\frac{1}{6}}$$

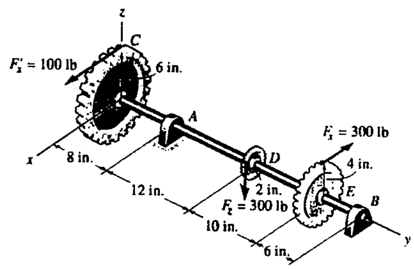


$$c = \left(\frac{16[(608.28)(12)]^2 + 12[50(12)]^2}{\pi^2[20(10^3)]^2}\right)^{\frac{1}{6}}$$

$c = 0.7752$ in.; $\quad d = 2c = 1.55$ in.

Use $d = 1\frac{5}{8}$ in. **Ans**

•11–44 **The shaft is supported by bearings at A and B that exert force components only in the x and z directions on the shaft. If the allowable normal stress for the shaft is σ_{allow} = 15 ksi, determine to the nearest $\frac{1}{8}$ in. the smallest diameter of the shaft that will support the loading. Use the maximum distortion-energy theory of failure.**

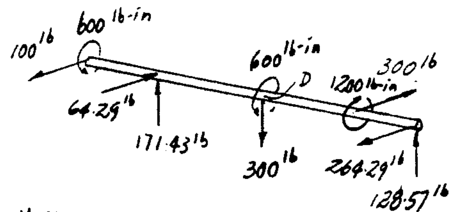


Critical moment is just to the right of D.

$M = \sqrt{2057^2 + 1229^2} = 2396\ \text{lb}\cdot\text{in.}$

$T = 1200\ \text{lb}\cdot\text{in.}$

Both states of stress will yield the same result.



$$\sigma_{a,b} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

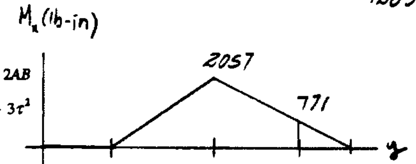
Let $\frac{\sigma}{2} = A$ and $\sqrt{\frac{\sigma^2}{4} + \tau^2} = B$

$$\sigma_a^2 = (A + B)^2, \quad \sigma_b^2 = (A - B)^2$$

$$\sigma_a\,\sigma_b = (A + B)(A - B) = A^2 - B^2$$

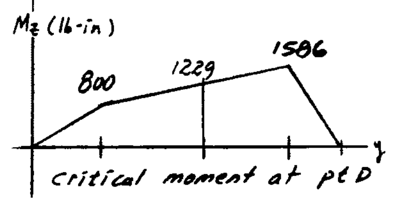
$$\sigma_a^2 - \sigma_a\,\sigma_b + \sigma_b^2 = A^2 + B^2 + 2AB - A^2 + B^2 + A^2 + B^2 - 2AB$$

$$= A^2 + 3B^2 = \frac{\sigma^2}{4} + 3\left(\frac{\sigma^2}{4} + \tau^2\right) = \sigma^2 + 3\tau^2$$



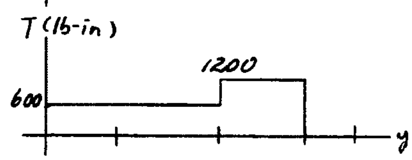
$$\sigma_a^2 - \sigma_a\,\sigma_b + \sigma_b^2 = \sigma_{allow}^2$$

$$\sigma^2 + 3\tau^2 = \sigma_{allow}^2 \qquad (1)$$



$$\sigma = \frac{Mc}{I} = \frac{Mc}{\frac{\pi}{4}c^4} = \frac{4M}{\pi c^3}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3}$$



From Eq. (1)

$$\frac{16M^2}{\pi^2 c^6} + \frac{12T^2}{\pi^2 c^6} = \sigma_{allow}^2$$

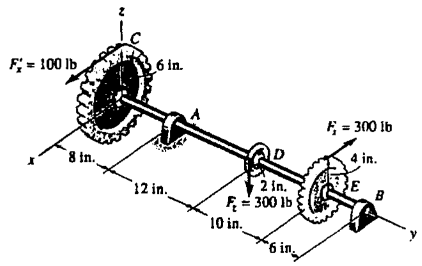
$$c = \left[\frac{16M^2 + 12T^2}{\pi^2\sigma_{allow}^2}\right]^{\frac{1}{6}}$$

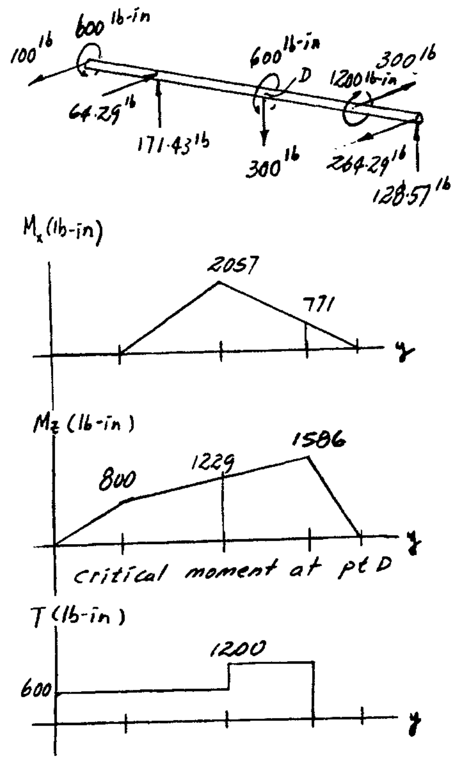
$$c = \left(\frac{16M^2 + 12T^2}{\pi^2\sigma_{allow}^2}\right)^{1/6} = \left[\frac{16(2396)^2 + 12(1200^2)}{\pi^2((15)(10^3))^2}\right]^{1/6} = 0.605\ \text{in.}$$

$d = 2c = 1.210$ in.

Use $d = 1\frac{1}{4}$ in. **Ans**

11–45 Solve Prob. 11–44 using the maximum-shear-stress theory of failure. Take $\tau_{allow} = 6$ ksi.





Critical moment is just to the right of D.

$$M = \sqrt{(2057)^2 + (1229)^2} = 2396 \text{ lb} \cdot \text{in.}$$

$$T = 1200 \text{ lb} \cdot \text{in.}$$

Use Eq. 11-2,

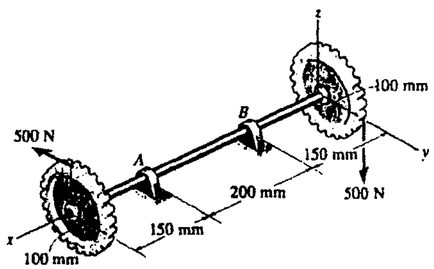
$$c = \left(\frac{2}{\pi\, \tau_{allow}}\sqrt{M^2 + T^2}\right)^{1/3}$$

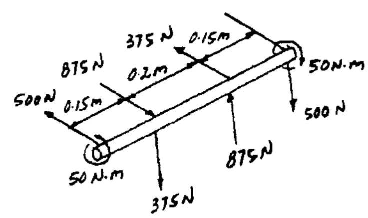
$$c = \left(\frac{2}{\pi\,(6)(10^3)}\sqrt{(2396)^2 + (1200)^2}\right)^{1/3} = 0.6576 \text{ in.}$$

$$d_{req'd} = 2c = 1.315 \text{ in.}$$

Use $d = 1\frac{3}{8}$ in. **Ans**

•11–46 The tubular shaft has an inner diameter of 15 mm. Determine to the nearest millimeter its outer diameter if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Use an allowable shear stress of $\tau_{allow} = 70$ MPa, and base the design on the maximum-shear-stress theory of failure.





$$I = \frac{\pi}{4}(c^4 - 0.0075^4) \text{ and } J = \frac{\pi}{2}(c^4 - 0.0075^4)$$

$$\tau_{allow} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

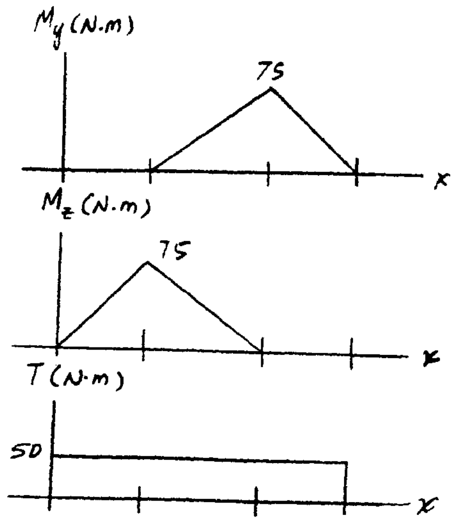
$$\tau_{allow} = \sqrt{\left(\frac{Mc}{2I}\right)^2 + \left(\frac{Tc}{J}\right)^2}$$

$$\tau_{allow}^2 = \frac{M^2c^2}{4I^2} + \frac{T^2c^2}{J^2}$$

$$\left(\frac{c^4 - 0.0075^4}{c}\right)^2 = \frac{4M^2}{\pi^2} + \frac{4T^2}{\pi^2}$$

$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}$$

$$\frac{c^4 - 0.0075^4}{c} = \frac{2}{\pi(70)(10^6)}\sqrt{75^2 + 50^2}$$



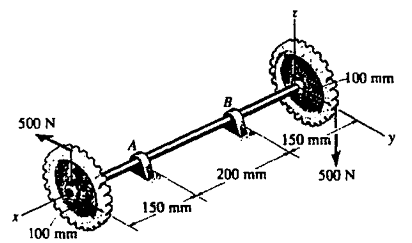
$$c^4 - 0.0075^4 = 0.8198(10^{-6})c$$

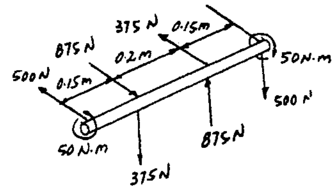
Solving, $c = 0.0103976$ m

$d = 2c = 0.0207952 \text{ m} = 20.8 \text{ mm}$

Use $d = 21$ mm **Ans**

11–47 Determine to the nearest millimeter the diameter of the solid shaft if it is subjected to the gear loading. The bearings at A and B exert force components only in the y and z directions on the shaft. Base the design on the maximum-distortion-energy theory of failure with $\sigma_{allow} = 150$ MPa.





$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \frac{\sigma_x}{2}$, $b = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$

$\sigma_1 = a + b$, $\sigma_2 = a - b$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{allow}$$

$$a^2 + 3b^2 = \sigma_{allow}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{allow}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{allow}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{allow}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{allow}^2$$

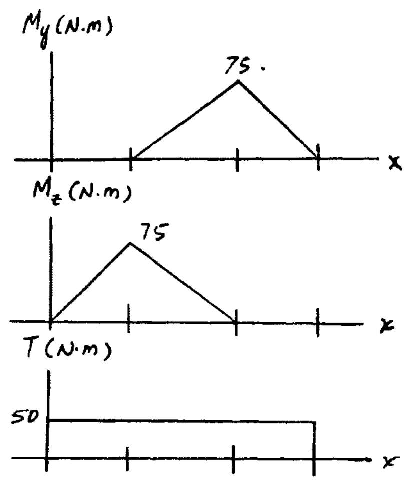
$$c^6 = \frac{16}{\sigma_{allow}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{allow}^2\pi^2}$$

$$c = \left(\frac{4}{\sigma_{allow}^2\pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{6}}$$

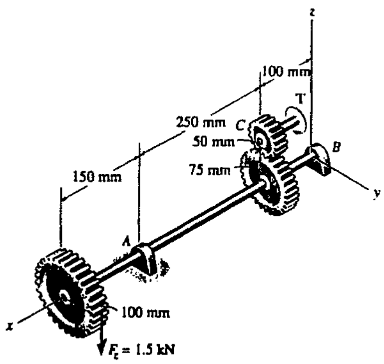
$$= \left[\frac{4}{(150(10^6))^2(\pi)^2}(4(75)^2 + 3(50)^2)\right]^{\frac{1}{6}} = 0.009025 \text{ m}$$

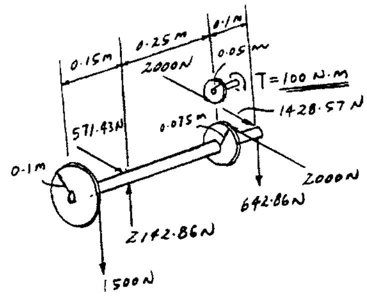
$d = 2c = 0.0181$ m

Use $d = 19$ mm **Ans**



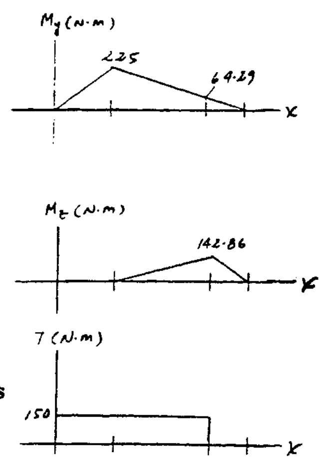
•11–48 The end gear connected to the shaft is subjected to the loading shown. If the bearings at A and B exert only y and z components of force on the shaft, determine the equilibrium torque T at gear C and then determine the smallest diameter of the shaft to the nearest millimeter that will support the loading. Use the maximum-shear-stress theory of failure with $\tau_{allow} = 60$ MPa.





From the free-body diagrams :

$T = 100\ \text{N} \cdot \text{m}$ **Ans**



Critical section is at support A.

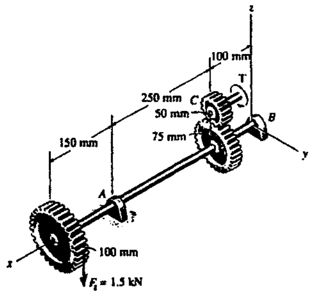
$$c = \left[\frac{2}{\pi\,\tau_{allow}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(60)(10^6)}\sqrt{225^2 + 150^2}\right]^{\frac{1}{3}}$$

$$= 0.01421\ \text{m}$$

$d = 2c = 0.0284\ \text{m} = 28.4\ \text{mm}$

Use $d = 29\ \text{mm}$ **Ans**

11-49. Solve Prob. 11–48 using the maximum-distortion energy theory of failure with $\sigma_{\text{allow}} = 80$ MPa.



From the free-body diagrams.

$T = 100\ \text{N}\cdot\text{m}$ **Ans**

Critical section is at support A.

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\text{Let } a = \frac{\sigma_x}{2},\ b = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

$$\sigma_1 = a + b,\ \sigma_2 = a - b$$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{\text{allow}}^2$$

$$a^2 + 3b^2 = \sigma_{\text{allow}}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{\text{allow}}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{\text{allow}}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{\text{allow}}^2$$

$$c^6 = \frac{16}{\sigma_{\text{allow}}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{\text{allow}}^2\pi^2}$$

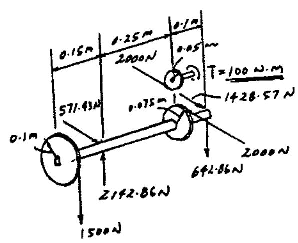
$$c = \left(\frac{4}{\sigma_{\text{allow}}^2\pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{6}}$$

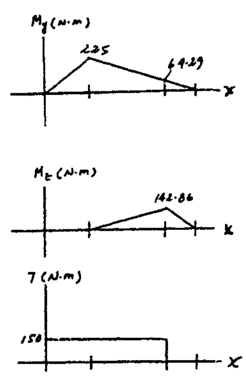
$$= \left[\frac{4}{(80(10^6))^2(\pi)^2}(4(225)^2 + 3(150)^2)\right]^{\frac{1}{6}}$$

$$= 0.01605 \text{ m}$$

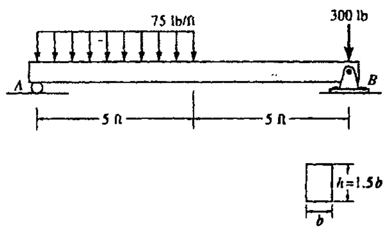
$$d = 2c = 0.0321 \text{ m} = 32.1 \text{ mm}$$

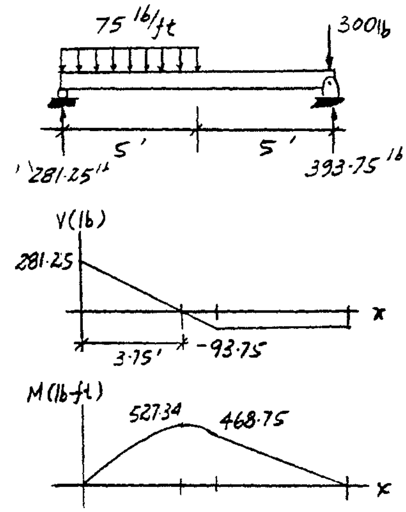
Use $d = 33$ mm **Ans**





11–50 The beam is made of cypress having an allowable bending stress of $\sigma_{allow} = 850$ psi and an allowable shear stress of $\tau_{allow} = 80$ psi. Determine the width b of the beam if the height $h = 1.5b$.





$$I_x = \frac{1}{12}(b)(1.5b)^3 = 0.28125\, b^4$$

$$Q_{max} = \bar{y}'A' = (0.375b)\ (0.75b)(b) = 0.28125\, b^3$$

Assume bending controls.

$M_{max} = 527.34 \text{ lb} \cdot \text{ft}$

$$\sigma_{allow} = \frac{M_{max}c}{I}; \qquad 850 = \frac{527.34(12)(0.75\, b)}{0.28125\, b^4}$$

$b = 2.71$ in. **Ans**

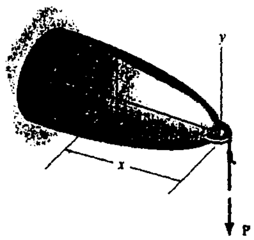
Check shear :

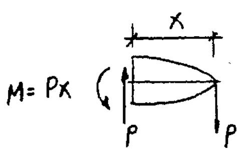
$I = 15.12 \text{ in}^4 \qquad Q_{max} = 5.584 \text{ in}^3$

$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{281.25(5.584)}{15.12(2.71)}$$

$= 38.36 \text{ psi} < 80 \text{ psi}$ **OK**

11–51 **The cantilevered beam has** a circular cross section. If it **supports a force P at its end,** determine its radius y as a **function of** x **so that it is subjected** to a constant maximum bending stress σ_{allow} throughout its length.





Section properties :

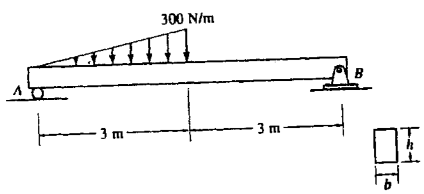
$$I = \frac{\pi}{4} y^4$$

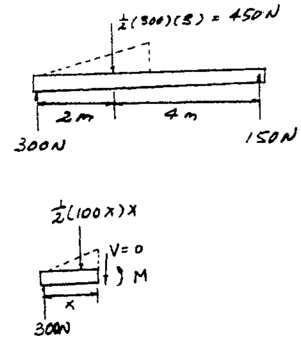
$$S = \frac{I}{c} = \frac{\frac{\pi}{4} y^4}{y} = \frac{\pi}{4} y^3$$

$$\sigma_{allow} = \frac{M}{S} = \frac{Px}{\frac{\pi}{4} y^3}$$

$$y = [\frac{4P}{\pi \sigma_{allow}} x]^{\frac{1}{3}}$$ **Ans**

•11–52 The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 8$ MPa and an allowable shear stress of $\tau_{allow} = 750$ kPa. Determine its dimensions if it is to be rectangular and have a height-to-width ratio of $h/b = 1.25$.





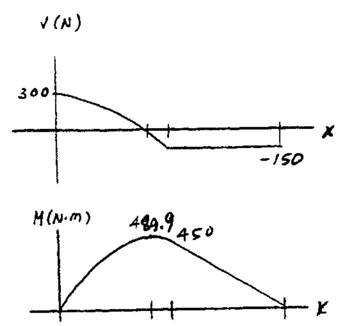
From the free - body diagram of the segment :

$$+\uparrow \Sigma F_y = 0; \qquad 300 - \frac{1}{2}(100x)x = 0; \qquad x = 2.449 \text{ m}$$

$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}[100(2.449)](2.449)(\frac{2.449}{3}) - 300(2.449) = 0; \quad M = 489.9 \text{ N} \cdot \text{m}$$

$$I = \frac{1}{12}(b)(1.25b)^3 = 0.16276\, b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.16276\, b^4}{0.625\, b} = 0.26042\, b^3$$



Assume bending controls.

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}; \qquad 8(10^6) = \frac{489.9}{0.26042\, b^3}$$

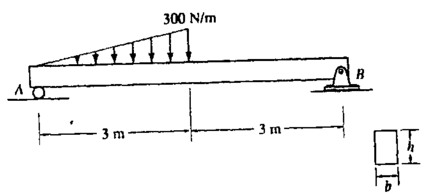
$b = 0.06172 \text{ m} = 61.7 \text{ mm}$ **Ans**

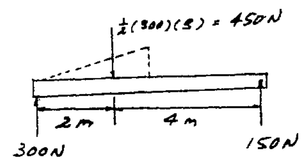
$h = 1.25(0.06172) = 77.2 \text{ mm}$ **Ans**

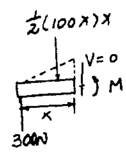
Check shear :

$$\tau_{max} = \frac{1.5V_{max}}{A} = \frac{1.5(300)}{(0.06172)(1.25)(0.0617)} = 94.5 \text{ kPa} < \tau_{allow} = 750 \text{ kPa} \quad \text{OK}$$

11-53 Solve Prob. 11-52 if the height-to-width ratio of the beam is to be $h/b = 1.5$.







From the free-body diagram of the segment :

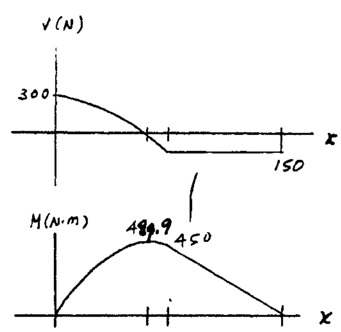
$$+\uparrow \Sigma F_y = 0; \quad 300 - \frac{1}{2}(100x)x = 0; \quad x = 2.449 \text{ m}$$

$$\circlearrowleft + \Sigma M = 0; \quad M + \frac{1}{2}[100(2.449)](2.449)(\frac{2.449}{3}) - 300(2.449) = 0; \quad M = 489.9 \text{ N} \cdot \text{m}$$

Section properties :

$$I = \frac{1}{12}(b)(1.5b)^3 = 0.28125\, b^4$$

$$S_{req'd} = \frac{I}{c} = \frac{0.28125\, b^4}{0.75\, b} = 0.375\, b^3$$



Beam design : Assume bending moment controls.

$$\sigma_{allow} = \frac{M_{max}}{S_{req'd}}; \quad 8(10^6) = \frac{489.9}{0.375\, b^3}$$

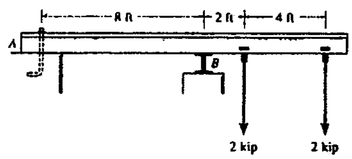
$b = 0.0547$ m $= 54.7$ mm **Ans**

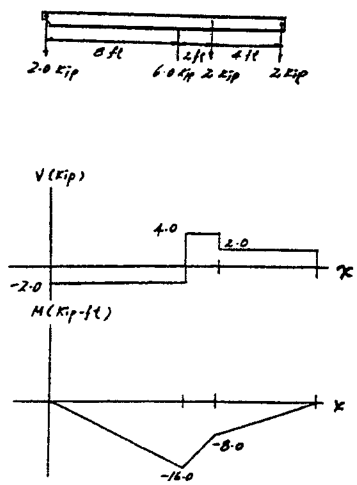
$h = 1.5(0.0547) = 82.0$ mm **Ans**

Check shear :

$$\tau_{max} = \frac{1.5V_{max}}{A} = \frac{1.5(300)}{(0.0547)(1.5)(0.0547)} = 100 \text{ kPa} < \tau_{allow} = 750 \text{ kPa} \quad \text{OK}$$

11–54. Select the lightest-weight steel wide-flange overhanging beam from Appendix B that will safely support the loading. Assume the support at A is a pin and the support at B is a roller. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\sigma_{allow} = 14$ ksi.





Assume bending controls.

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{16.0(12)}{24} = 8.0\ \text{in}^3$$

Select a $W\,10 \times 12$

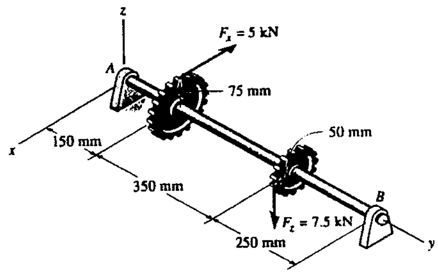
$S_x = 10.9\ \text{in}^3$, $d = 9.87$ in., $t_w = 0.190$ in.

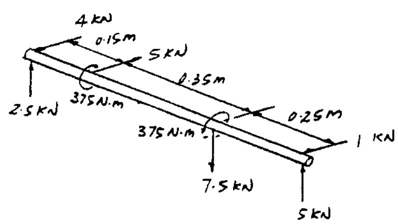
Check shear :

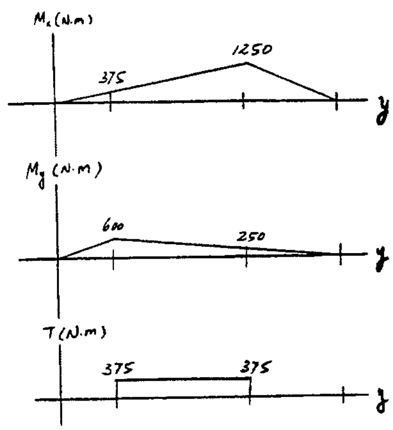
$$\tau_{avg} = \frac{V_{max}}{A_{web}} = \frac{4}{9.87(0.190)} = 2.13\ \text{ksi} < 14\ \text{ksi} \quad \text{OK}$$

Use $W\,10 \times 12$ **Ans**

11–55 The bearings at A and B exert only x and z components of force on the steel shaft. Determine the shaft's diameter to the nearest millimeter so that it can resist the loadings of the gears without exceeding an allowable shear stress of $\tau_{allow} = 80$ MPa. Use the maximum-shear-stress theory of failure.







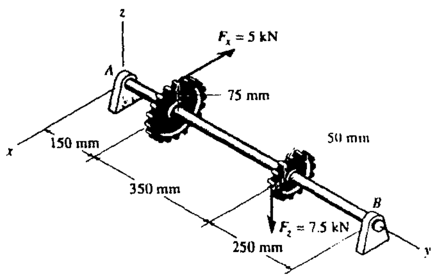
Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75$ N · m

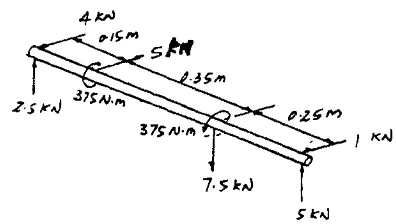
$$c = \left[\frac{2}{\pi\, \tau_{allow}}\sqrt{M^2 + T^2}\right]^{\frac{1}{3}} = \left[\frac{2}{\pi(80)(10^6)}\sqrt{1274.75^2 + 375^2}\right]^{\frac{1}{3}} = 0.0219 \text{ m}$$

$d = 2c = 0.0439 \text{ m} = 43.9 \text{ mm}$

Use $d = 44$ mm **Ans**

•11–56 Solve Prob. 11–55 using the maximum-distortion-energy theory of failure with $\sigma_{allow} = 200$ MPa.





Maximum resultant moment $M = \sqrt{1250^2 + 250^2} = 1274.75\ \text{N}\cdot\text{m}$

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$$

Let $a = \frac{\sigma_x}{2}$, $b = \sqrt{\frac{\sigma_x^2}{4} + \tau_{xy}^2}$

$\sigma_1 = a + b, \qquad \sigma_2 = a - b$

Require,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{allow}^2$$

$$a^2 + 2ab + b^2 - [a^2 - b^2] + a^2 - 2ab + b^2 = \sigma_{allow}^2$$

$$a^2 + 3b^2 = \sigma_{allow}^2$$

$$\frac{\sigma_x^2}{4} + 3\left(\frac{\sigma_x^2}{4} + \tau_{xy}^2\right) = \sigma_{allow}^2$$

$$\sigma_x^2 + 3\tau_{xy}^2 = \sigma_{allow}^2$$

$$\left(\frac{Mc}{\frac{\pi}{4}c^4}\right)^2 + 3\left(\frac{Tc}{\frac{\pi}{2}c^4}\right)^2 = \sigma_{allow}^2$$

$$\frac{1}{c^6}\left[\left(\frac{4M}{\pi}\right)^2 + 3\left(\frac{2T}{\pi}\right)^2\right] = \sigma_{allow}^2$$

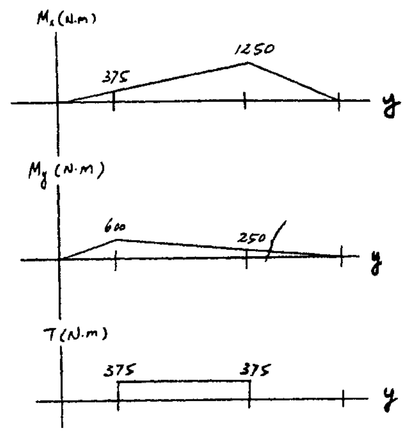
$$c^6 = \frac{16}{\sigma_{allow}^2\pi^2}M^2 + \frac{12T^2}{\sigma_{allow}^2\pi^2}$$

$$c = \left(\frac{4}{\sigma_{allow}^2\pi^2}(4M^2 + 3T^2)\right)^{\frac{1}{6}}$$

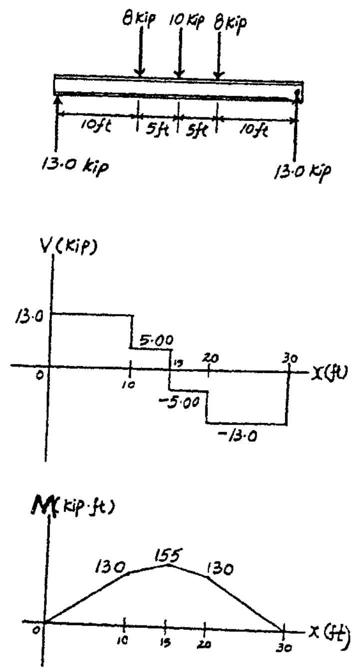
$$= \left[\frac{4}{(200(10^6))^2(\pi)^2}\left(4(1274.75)^2 + 3(375)^2\right)\right]^{\frac{1}{6}}$$

$$= 0.0203\ \text{m} = 20.3\ \text{mm}$$

$d = 40.6$ mm **Ans**



11-57. Select the lightest-weight steel wide-flange beam from Appendix *B* that will safely support the loading shown. The allowable bending stress is $\sigma_{allow} = 22$ ksi and the allowable shear stress is $\tau_{allow} = 12$ ksi.



Bending Stress : From the moment diagram, $M_{max} = 155$ kip·ft. Assume bending controls the design. Applying the flexure formula,

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{155(12)}{22} = 84.55 \text{ in}^3$$

Select W18 × 50 $\left(S_x = 88.9 \text{ in}^3,\ d = 17.99 \text{ in.},\ t_w = 0.355 \text{ in.}\right)$

Shear Stress : Provide a shear stress check using $\tau = \frac{V}{t_w d}$ for a W18 × 50 wide-flange section. From the shear diagram, $V_{max} = 13.0$ kip.

$$\tau_{max} = \frac{V_{max}}{t_w d} = \frac{13.0}{0.355(17.99)} = 2.04 \text{ ksi} < \tau_{allow} = 12 \text{ ksi } (O.K!)$$

Hence, **Use** W18×50 **Ans**

12–1 An L2 steel strap having a thickness of 0.125 in. and a width of 2 in. is bent into a circular arc of radius 600 in. Determine the maximum bending stress in the strap.

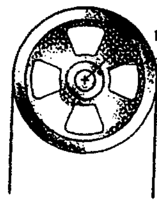
$$\frac{1}{\rho} = \frac{M}{EI} \qquad M = \frac{EI}{\rho}$$

However,

$$\sigma = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = \left(\frac{c}{\rho}\right)E$$

$$\sigma = \frac{0.0625}{600}(29)(10^3) = 3.02 \text{ ksi} \qquad \textbf{Ans}$$

12–2 The L2 steel blade of the band saw wraps around the pulley having a radius of 12 in. Determine the maximum normal stress in the blade. The blade is made of steel having a width of 0.75 in and a thickness of 0.0625 in.



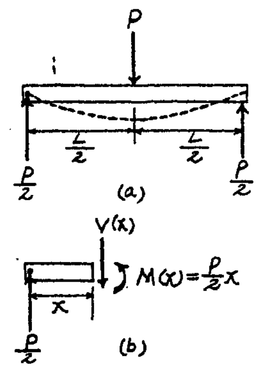
$$\frac{1}{\rho} = \frac{M}{EI}; \qquad M = \frac{EI}{\rho}$$

However,

$$\sigma = \frac{Mc}{I} = \frac{(EI/\rho)c}{I} = (\frac{c}{\rho})E$$

$$\sigma = (\frac{0.03125}{12})(29)(10^3) = 75.5 \text{ ksi} \qquad \textbf{Ans}$$

12-3. Determine the equation of the elastic curve for the beam using the x coordinate that is valid for $0 \le x < L/2$. Specify the slope at A and the beam's maximum deflection. EI is constant.



Support Reactions and Elastic Curve : As shown on FBD(a).

Moment Function : As shown on FBD(b).

Slope and Elastic Curve :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI\frac{dv}{dx} = \frac{P}{4}x^2 + C_1 \qquad [1]$$

$$EI\,v = \frac{P}{12}x^3 + C_1x + C_2 \qquad [2]$$

Boundary Conditions : Due to symmetry, $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$.

Also, $v = 0$ at $x = 0$.

From Eq. [1] $\quad 0 = \frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 \qquad C_1 = -\frac{PL^2}{16}$

From Eq. [2] $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

The Slope : Substitute the value of C_1 into Eq. [1],

$$\frac{dv}{dx} = \frac{P}{16EI}\left(4x^2 - L^2\right)$$

$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{PL^2}{16EI} \qquad \textbf{Ans}$$

The negative sign indicates clockwise rotation.

The Elastic Curve : Substitute the values of C_1 and C_2 into Eq. [2],

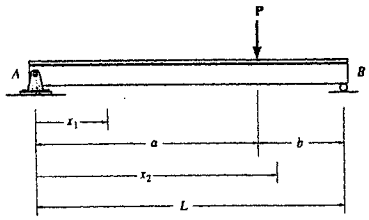
$$v = \frac{Px}{48EI}\left(4x^2 - 3L^2\right) \qquad \textbf{Ans}$$

v_{max} occurs at $x = \frac{L}{2}$,

$$v_{max} = -\frac{PL^3}{48EI} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

•12–4 Determine the equations of the elastic curve using the x_1, and x_2 coordinates. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$M_1 = \frac{Pb}{L}x_1$$

$$EI\frac{d^2v_1}{dx_1{}^2} = \frac{Pb}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Pb}{2L}x_1{}^2 + C_1 \qquad (1)$$

$$EI\,v_1 = \frac{Pb}{6L}x_1{}^3 + C_1x_1 + C_2 \qquad (2)$$

$$M_2 = \frac{Pb}{L}x_2 - P(x_2 - a)$$

But $b = L - a$. Thus

$$M_2 = Pa\,(1 - \frac{x_2}{L})$$

$$EI\frac{d^2v_2}{dx_2{}^2} = Pa\,(1 - \frac{x_2}{L})$$

$$EI\frac{dv_2}{dx_2} = Pa\,(x_2 - \frac{x_2^2}{2L}) + C_3 \qquad (3)$$

$$EI\,v_2 = Pa\,(\frac{x_2^2}{2} - \frac{x_2^3}{6L}) + C_3x_2 + C_4 \qquad (4)$$

Applying the boundary conditions :

$v_1 = 0$ at $x_1 = 0$

Therefore, $C_2 = 0$,

$v_2 = 0$ at $x_2 = L$

$$0 = \frac{Pa\,L^2}{3} + C_3L + C_4 \qquad (5)$$

Applying the continuity conditions :

$$v_1|_{x_1=a} = v_2|_{x_2=a}$$

$$\frac{Pb}{6L}a^3 + C_1a = Pa\,(\frac{a^2}{2} - \frac{a^3}{6L}) + C_3a + C_4 \qquad (6)$$

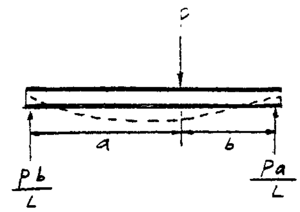
$$\frac{dv_1}{dx_1}\Big|_{x_1=a} = \frac{dv_2}{dx_2}\Big|_{x_2=a}$$

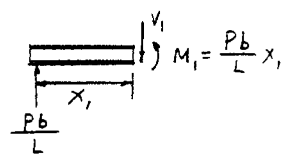
$$\frac{Pb}{2L}a^2 + C_1 = Pa\,(a - \frac{a^2}{2L}) + C_3 \qquad (7)$$

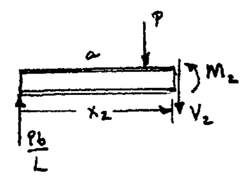
Solving Eqs. (5), (6) and (7) simultaneously yields,

$$C_1 = -\frac{Pb}{6L}(L^2 - b^2)\,; \qquad C_3 = -\frac{Pa}{6L}(2L^2 + a^2)$$

$$C_4 = \frac{Pa^3}{6}$$







Thus,

$$EIv_1 = \frac{Pb}{6L}x_1^3 - \frac{Pb}{6L}(L^2 - b^2)x_1$$

or

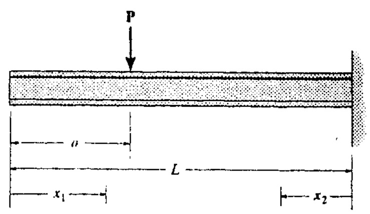
$$v_1 = \frac{Pb}{6EIL}(x_1^3 - (L^2 - b^2)x_1) \qquad \textbf{Ans}$$

and

$$EIv_2 = Pa\,(\frac{x_2^2}{2} - \frac{x_2^3}{6L}) - \frac{Pa}{6L}(2L^2 + a^2)x_2 + \frac{Pa^3}{6}$$

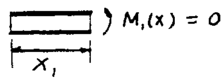
$$v_2 = \frac{Pa}{6EIL}[3x_2^2L - x_2^3 - (2L^2 + a^2)x_2 + a^2L] \qquad \textbf{Ans}$$

12–5 Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



$$EI\frac{d^2v_1}{dx_1^2} = M_1(x)$$

$$M_1(x) = 0; \quad EI\frac{d^2v_1}{dx_1^2} = 0$$



$$EI\frac{dv_1}{dx_1} = C_1 \qquad (1)$$

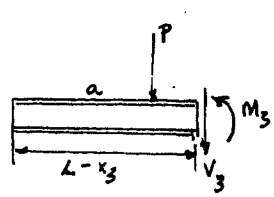
$$EI\,v_1 = C_1x_1 + C_2 \qquad (2)$$

$$M_2(x) = Px_2 - P(L-a)$$

$$EI\frac{d^2v_2}{dx_2^2} = Px_2 - P(L-a)$$

$$EI\frac{dv_2}{dx_2} = \frac{P}{2}x_2^2 - P(L-a)x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = \frac{P}{6}x_2^3 - \frac{P(L-a)x_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$



Boundary conditions :

At $x_2 = 0$, $\frac{dv_2}{dx_2} = 0$

From Eq.(3), $0 = C_3$

At $x_2 = 0$, $v_2 = 0$

$0 = C_4$

Continuity condition :

At $x_1 = a$, $x_2 = L - a$; $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$C_1 = -\left[\frac{P(L-a)^2}{2} - P(L-a)^2\right]; \qquad C_1 = \frac{P(L-a)^2}{2}$$

At $x_1 = a$, $x_2 = L - a$, $v_1 = v_2$

From Eqs. (2) and (4),

$$\left(\frac{P(L-a)^2}{2}\right)a + C_2 = \frac{P(L-a)^3}{6} - \frac{P(L-a)^3}{2}$$

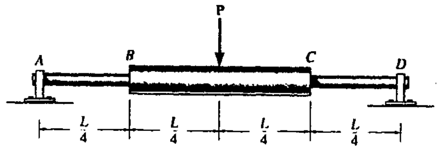
$$C_2 = -\frac{Pa(L-a)^2}{2} - \frac{P(L-a)^3}{3}$$

From Eq. (2),

$$v_1 = \frac{P}{6EI}[-3(L-a)^2x_1 - 3a(L-a)^2 - 2(L-a)^3] \qquad \textbf{Ans}$$

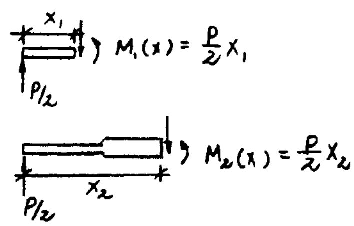
$$v_2 = \frac{P}{6EI}[x_2^3 - 3(L-a)x_2^2] \qquad \textbf{Ans}$$

12–6 The simply-supported shaft has a moment of inertia of $2I$ for region BC and a moment of inertia I for regions AB and CD. Determine the maximum deflection of the beam due to the load **P**.



$$M_1(x) = \frac{P}{2}x_1$$

$$M_2(x) = \frac{P}{2}x_2$$



Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{P}{2}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{4} + C_1 \qquad (1)$$

$$EIv_1 = \frac{Px_1^3}{12} + C_1x_1 + C_2 \qquad (2)$$

$$2EI\frac{d^2v_2}{dx_2^2} = \frac{P}{2}x_2$$

$$2EI\frac{dv_2}{dx_2} = \frac{Px_2^2}{4} + C_3 \qquad (3)$$

$$2EIv_2 = \frac{Px_2^3}{12} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_1 = 0$ at $x_1 = 0$

From Eq. (2), $C_2 = 0$

$\frac{dv_2}{dx_2} = 0$ at $x_2 = \frac{L}{2}$

From Eq. (3),

$$0 = \frac{PL^2}{16} + C_3$$

$$C_3 = -\frac{PL^2}{16}$$

Continuity conditions :

$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ at $x_1 = x_2 = \frac{L}{4}$

From Eqs. (1) and (3),

$$\frac{PL^2}{64} + C_1 = \frac{PL^2}{128} - \frac{1}{2}\left(\frac{PL^2}{16}\right)$$

$$C_1 = \frac{-PL^2}{128}$$

$v_1 = v_2$ at $x_1 = x_2 = \frac{L}{4}$

From Eqs. (2) and (4)

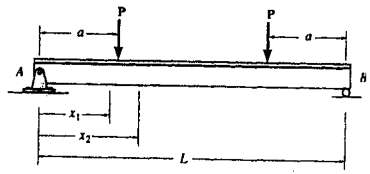
$$\frac{PL^3}{768} - \frac{5PL^2}{128}\left(\frac{L}{4}\right) = \frac{PL^3}{1536} - \frac{1}{2}\left(\frac{PL^2}{16}\right)\left(\frac{L}{4}\right) + \frac{1}{2}C_4$$

$$C_4 = \frac{-PL^3}{384}$$

$$v_2 = \frac{P}{768EI}(32x_2^3 - 24L^2x_2 - L^3)$$

$$v_{max} = v_2\Big|_{x_2 = \frac{L}{2}} = \frac{-3PL^3}{256EI} = \frac{3PL^3}{256EI}\downarrow \qquad \textbf{Ans}$$

12–7 Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. EI is constant.



Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = Px_1$

$$EI\frac{d^2v_1}{dx_1^2} = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1 \qquad (1)$$

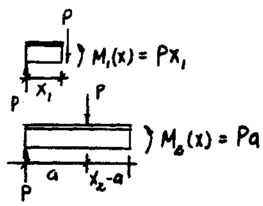
$$EIv_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = Pa$

$$EI\frac{d^2v_2}{dx_2^2} = Pa$$

$$EI\frac{dv_2}{dx_2} = Pax_2 + C_3 \qquad (3)$$

$$EIv_2 = \frac{Pax_2^2}{2} + C_3x_2 + C_4 \qquad (4)$$



Boundary Conditions:

$v_1 = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

Due to symmetry:

$\frac{dv_2}{dx_2} = 0$ at $x_2 = \frac{L}{2}$

From Eq. (3)

$$0 = Pa\frac{L}{2} + C_3$$

$$C_3 = -\frac{PaL}{2}$$

Continuity conditions:

$v_1 = v_2$ at $x_1 = x_2 = a$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} - \frac{Pa^2L}{2} + C_4$$

$$C_1a - C_4 = \frac{Pa^3}{3} - \frac{Pa^2L}{2} \qquad (5)$$

$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ at $x_1 = x_2 = a$

$$\frac{Pa^2}{2} + C_1 = Pa^2 - \frac{PaL}{2}$$

$$C_1 = \frac{Pa^2}{2} - \frac{PaL}{2}$$

Substitute C_1 into Eq. (5)

$$C_4 = \frac{Pa^3}{6}$$

$$\frac{dv_1}{dx_1} = \frac{P}{2EI}(x_1^2 + a^2 - aL)$$

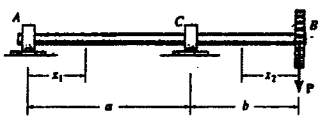
$$\theta_A = \frac{dv_1}{dx_1}\bigg|_{x_1=0} = \frac{Pa(a-L)}{2EI} \qquad \textbf{Ans}$$

$$v_1 = \frac{Px_1}{6EI}[x_1^2 + 3a(a-L)] \qquad \textbf{Ans}$$

$$v_2 = \frac{Pa}{6EI}[3x(x-L) + a^2] \qquad \textbf{Ans}$$

$$v_{max} = v_2\Big|_{x=\frac{L}{2}} = \frac{Pa}{24EI}(4a^2 - 3L^2) \qquad \textbf{Ans}$$

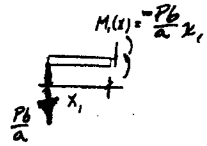
***12-8.** The shaft is supported at A by a journal bearing that exerts only vertical reactions on the shaft, and at C by a thrust bearing that exerts horizontal and vertical reactions on the shaft. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.



Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

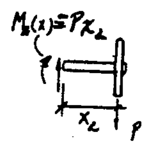
For $M_1(x) = -\frac{Pb}{a}x_1$



$$EI\frac{d^2v_1}{dx_1^2} = -\frac{Pb}{a}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{Pb}{2a}x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = -\frac{Pb}{6a}x_1^3 + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = -Px_2$

$$EI\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI\frac{dv_2}{dx_2} = \frac{-Px_2^2}{2} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-Px_2^3}{6} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_1 = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v_1 = 0$ at $x_1 = a$

From Eq. (2),

$$0 = -\frac{Pb}{6a}a^3 + C_1a$$

$$C_1 = \frac{Pab}{6}$$

$v_2 = 0$ at $x_2 = b$

From Eq. (4),

$$0 = -\frac{Pb^3}{6} + C_3b + C_4$$

$$C_3b + C_4 = \frac{Pb^3}{6} \qquad (5)$$

Continuity conditions :

$$\frac{dv_1}{dx_1} = \frac{-dv_2}{dx_2} \quad \text{at} \quad x_1 = a \quad x_2 = b$$

From Eqs. (1) and (3)

$$-\frac{Pb}{2a}(a^2) + \frac{Pab}{6} = \frac{Pb^2}{2} - C_3$$

$$C_3 = \frac{Pab}{3} + \frac{Pb^2}{2}$$

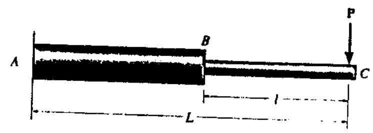
Substitute C_3 into Eq. (5)

$$C_4 = -\frac{Pb^3}{3} - \frac{Pab^2}{3}$$

$$v_1 = \frac{-Pb}{6aEI}[x_1^3 - a^2x_1] \qquad \textbf{Ans}$$

$$v_2 = \frac{P}{6EI}\left(-x_2^3 + b(2a+3b)x_2 - 2b^2(a+b)\right) \qquad \textbf{Ans}$$

12-9 The beam is made of two rods and is subjected to the concentrated load **P**. Determine the maximum deflection of the beam if the moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is E.



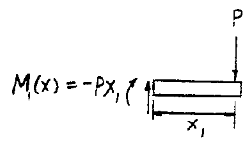
$$EI\frac{d^2v}{dx^2} = M(x)$$

$$M_1(x) = -Px_1$$

$$EI_{BC}\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI_{BC}\frac{dv_1}{dx_1} = -\frac{Px_1^2}{2} + C_1 \qquad (1)$$

$$EI_{BC}\,v_1 = -\frac{Px_1^3}{6} + C_1x_1 + C_2 \qquad (2)$$

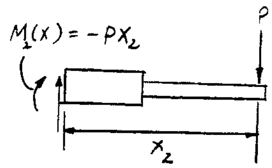


$$M_2(x) = -Px_2$$

$$EI_{AB}\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI_{AB}\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \qquad (3)$$

$$EI_{AB}\,v_2 = -\frac{P}{6}x_2^3 + C_3x_2 + C_4 \qquad (4)$$



Boundary conditions :

At $x_2 = L$, $\frac{dv_2}{dx_2} = 0$

$$0 = -\frac{PL^2}{2} + C_3; \quad C_3 = \frac{PL^2}{2}$$

At $x_2 = L$, $v = 0$

$$0 = -\frac{PL^3}{6} + \frac{PL^3}{2} + C_4; \quad C_4 = -\frac{PL^3}{3}$$

Continuity conditions :

At $x_1 = x_2 = l$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$\frac{1}{EI_{BC}}[-\frac{Pl^2}{2} + C_1] = \frac{1}{EI_{AB}}[-\frac{Pl^2}{2} + \frac{PL^2}{2}]$$

$$C_1 = \frac{I_{BC}}{I_{AB}}[-\frac{Pl^2}{2} + \frac{PL^2}{2}] + \frac{Pl^2}{2}$$

At $x_1 = x_2 = l$, $v_1 = v_2$

From Eqs. (2) and (4),

$$\frac{1}{EI_{BC}}\{-\frac{Pl^3}{6} + [\frac{I_{BC}}{I_{AB}}(-\frac{Pl^2}{2} + \frac{PL^2}{2}) + \frac{Pl^2}{2}]\,l + C_2\} = \frac{1}{EI_{AB}}[-\frac{Pl^3}{6} + \frac{PL^2l}{2} - \frac{PL^3}{3}]$$

$$C_2 = \frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}$$

Therefore,

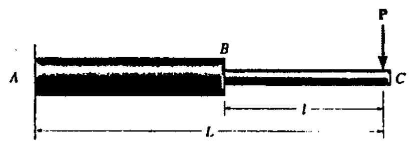
$$v_1 = \frac{1}{EI_{BC}}\{-\frac{Px_1^3}{6} + [\frac{I_{BC}}{I_{AB}}(-\frac{Pl^2}{2} + \frac{PL^2}{2}) + \frac{Pl^2}{2}]\,x_1 + \frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}\}$$

At $x_1 = 0$, $v_1|_{x=0} = v_{max}$

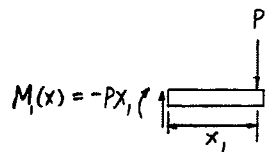
$$v_{max} = \frac{1}{EI_{BC}}\{\frac{I_{BC}}{I_{AB}}\frac{Pl^3}{3} - \frac{I_{BC}}{I_{AB}}\frac{PL^3}{3} - \frac{Pl^3}{3}\} = \frac{P}{3EI_{AB}}\{l^3 - L^3 - (\frac{I_{AB}}{I_{BC}})l^3\}$$

$$= \frac{P}{3EI_{AB}}\{(1 - \frac{I_{AB}}{I_{BC}})l^3 - L^3\} \qquad \textbf{Ans}$$

12–10 The beam is made of two rods and is subjected to the concentrated load **P**. Determine the slope at *C*. The moments of inertia of the rods are I_{AB} and I_{BC}, and the modulus of elasticity is *E*.



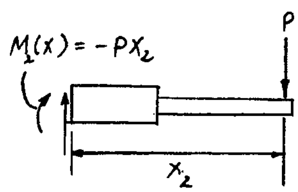
$$EI\frac{d^2v}{dx^2} = M(x)$$



$$M_1(x) = -Px_1$$

$$EI_{BC}\frac{d^2v_1}{dx_1^2} = -Px_1$$

$$EI_{BC}\frac{dv_1}{dx_1} = -\frac{Px_1^2}{2} + C_1 \qquad (1)$$



$$M_2(x) = -Px_2$$

$$EI_{AB}\frac{d^2v_2}{dx_2^2} = -Px_2$$

$$EI_{AB}\frac{dv_2}{dx_2} = -\frac{P}{2}x_2^2 + C_3 \qquad (2)$$

Boundary conditions :

At $x_2 = L$, $\frac{dv_2}{dx_2} = 0$

$$0 = -\frac{PL^2}{2} + C_3; \qquad C_3 = \frac{PL^2}{2}$$

Continuity conditions :

At $x_1 = x_2 = l$, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (2),

$$\frac{1}{EI_{BC}}[-\frac{Pl^2}{2} + C_1] = \frac{1}{EI_{AB}}[-\frac{Pl^2}{2} + \frac{PL^2}{2}]$$

$$C_1 = \frac{I_{BC}}{I_{AB}}[-\frac{Pl^2}{2} + \frac{PL^2}{2}] + \frac{Pl^2}{2}$$

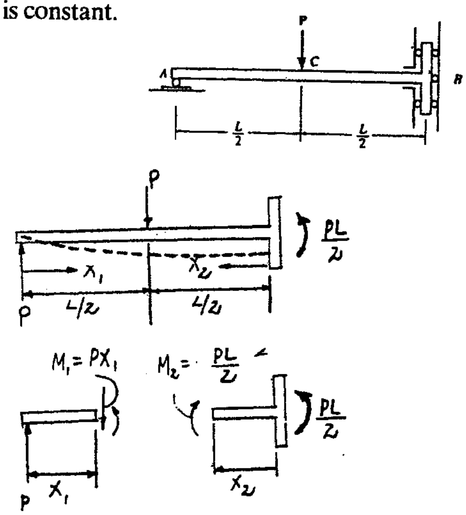
At $x_1 = 0$, $E\,I_{BC}\frac{dv_1}{dx_1} = C_1$

Thus,

$$\frac{dv_1}{dx_1} = \theta_C = \frac{1}{E\,I_{AB}}[-\frac{Pl^2}{2} + \frac{PL^2}{2}] + \frac{Pl^2}{2I_{BC}}$$

$$\theta_C = \frac{-P}{2E}[\frac{1}{I_{AB}}(L^2 - l^2) + \frac{l^2}{I_{BC}}] \qquad \textbf{Ans}$$

12-11. The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2^2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary conditions :

At $x_1 = 0$, $v_1 = 0$

$$0 = 0 + 0 + C_2; \quad C_2 = 0$$

At $x_2 = 0$, $\frac{dv_2}{dx_2} = 0$

$$0 + C_3 = 0; \quad C_3 = 0$$

At $x_1 = \frac{L}{2}$, $x_2 = \frac{L}{2}$, $v_1 = v_2$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

$$\frac{P(\frac{L}{2})^3}{6} + C_1(\frac{L}{2}) = \frac{PL(\frac{L}{2})^2}{4} + C_4$$

$$\frac{P(\frac{L}{2})^2}{2} + C_1 = -\frac{PL(\frac{L}{2})}{2}; \quad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

At $x_1 = 0$

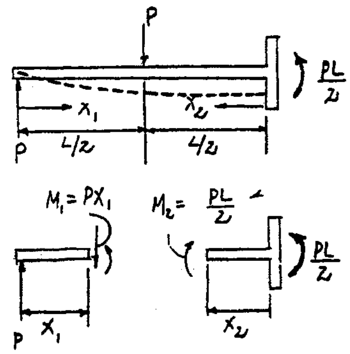
$$\frac{dv_1}{dx_1} = \theta_A = -\frac{3PL^2}{8EI}$$ **Ans**

At $x_1 = \frac{L}{2}$

$$v_C = \frac{P(\frac{L}{2})^3}{6EI} - (\frac{3}{8}PL^2)(\frac{L}{2}) + 0$$

$$v_C = \frac{-PL^3}{6EI}$$ **Ans**

***12-12.** Determine the deflection at B of the bar in Prob.



$$EI\frac{d^2v_1}{dx_1^2} = M_1 = Px_1$$

$$EI\frac{dv_1}{dx_1} = \frac{Px_1^2}{2} + C_1$$

$$EI\,v_1 = \frac{Px_1^3}{6} + C_1x_1 + C_2$$

$$EI\frac{d^2v_2}{dx_2} = M_2 = \frac{PL}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{PL}{2}x_2 + C_3$$

$$EI\,v_2 = \frac{PL}{4}x_2^2 + C_3x_2 + C_4$$

Boundary conditions :

At $x_1 = 0$, $v_1 = 0$

$$0 = 0 + 0 + C_2; \qquad C_2 = 0$$

At $x_2 = 0$, $\quad \frac{dv_2}{dx_2} = 0$

$$0 + C_3 = 0; \qquad C_3 = 0$$

At $x_1 = \frac{L}{2}$, $\quad x_2 = \frac{L}{2}$, $v_1 = v_2$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

$$\frac{P(\frac{L}{2})^3}{6} + C_1(\frac{L}{2}) = \frac{PL(\frac{L}{2})^2}{4} + C_4$$

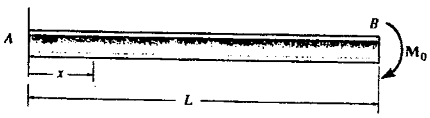
$$\frac{P(\frac{L}{2})^2}{2} + C_1 = -\frac{PL(\frac{L}{2})}{2}; \qquad C_1 = -\frac{3}{8}PL^2$$

$$C_4 = -\frac{11}{48}PL^3$$

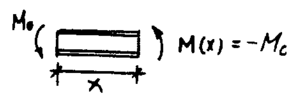
At $x_2 = 0$,

$$v_B = -\frac{11PL^3}{48EI} \qquad \text{Ans}$$

12–13 Determine the elastic curve for the cantilevered beam, which is subjected to the couple moment M_0. Also compute the maximum slope and maximum deflection of the beam. EI is constant.



Elastic curve and slope :



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -M_0$$

$$EI\frac{dv}{dx} = -M_0x + C_1 \qquad (1)$$

$$EIv = \frac{-M_0x^2}{2} + C_1x + C_2 \qquad (2)$$

Boundary Conditions :

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = 0$$

From Eq. (1), $C_1 = 0$

$$v = 0 \quad \text{at} \quad x = 0$$

From Eq. (2), $C_2 = 0$

$$\frac{dv}{dx} = \frac{-M_0x}{EI}$$

$$\theta_{max} = \frac{dv}{dx}\bigg|_{x=L} = \frac{-M_0L}{EI} \qquad \textbf{Ans}$$

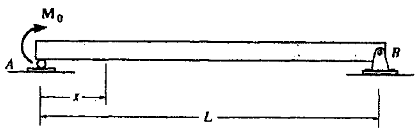
The negative sign indicates clockwise rotation.

$$v = \frac{-M_0x^2}{2EI} \qquad \textbf{Ans}$$

$$v_{max} = v\bigg|_{x=L} = -\frac{M_0L^2}{2EI} \qquad \textbf{Ans}$$

Negative sign indicates downward displacement.

12–14 Determine the equation of the elastic curve for the beam using the x coordinate. Specify the slope at A and the maximum deflection. EI is constant.

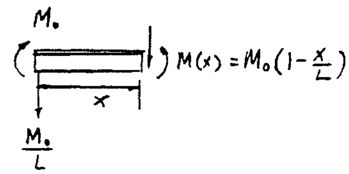


$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_0(1-\frac{x}{L})$$

$$EI\frac{dv}{dx} = M_0(x-\frac{x^2}{2L}) + C_1 \qquad (1)$$

$$EI\,v = M_0(\frac{x^2}{2}-\frac{x^3}{6L}) + C_1x + C_2 \qquad (2)$$



Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = M_0(\frac{L^2}{2}-\frac{L^2}{6}) + C_1L; \qquad C_1 = -\frac{M_0L}{3}$$

$$\frac{dv}{dx} = \frac{M_0}{EI}(x-\frac{x^2}{2L}-\frac{L}{3}) \qquad (3)$$

$$\theta_A = \frac{dv}{dx}\bigg|_{x=0} = -\frac{M_0L}{3EI} \qquad \textbf{Ans}$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{EI}(x-\frac{x^2}{2L}-\frac{L}{3})$$

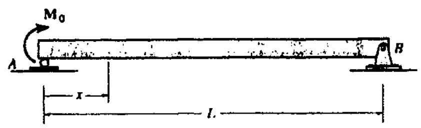
$$3x^2 - 6Lx + 2L^2 = 0; \qquad x = 0.42265\,L$$

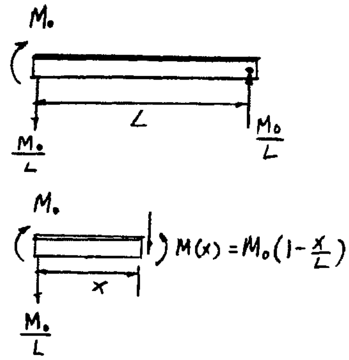
$$v = \frac{M_0}{6EIL}(3Lx^2 - x^3 - 2L^2x) \qquad (4) \qquad \textbf{Ans}$$

Substitute x into v,

$$v_{max} = \frac{-0.0642M_0L^2}{EI} \qquad \textbf{Ans}$$

12–15 Determine the deflection at the center of the beam and the slope at B. EI is constant.





$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_0(1 - \frac{x}{L})$$

$$EI\frac{dv}{dx} = M_0(x - \frac{x^2}{2L}) + C_1 \qquad (1)$$

$$EI\,v = M_0(\frac{x^2}{2} - \frac{x^3}{6L}) + C_1 x + C_2 \qquad (2)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (2), $\quad C_2 = 0$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = M_0(\frac{L^2}{2} - \frac{L^2}{6}) + C_1 L; \qquad C_1 = -\frac{M_0 L}{3}$$

$$\frac{dv}{dx} = \frac{M_0}{EI}(x - \frac{x^2}{2L} - \frac{L}{3}) \qquad (3)$$

$$\theta_A = \frac{dv}{dx}\Big|_{x=0} = -\frac{M_0 L}{3EI} \qquad \textbf{Ans}$$

$$\frac{dv}{dx} = 0 = \frac{M_0}{EI}(x - \frac{x^2}{2L} - \frac{L}{3})$$

$$3x^2 - 6Lx + 2L^2 = 0; \qquad x = 0.42265\,L$$

$$v = \frac{M_0}{6EIL}(3Lx^2 - x^3 - 2L^2x) \qquad (4) \qquad \textbf{Ans}$$

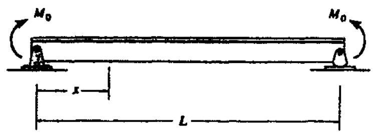
From Eq. (1) at $x = L$,

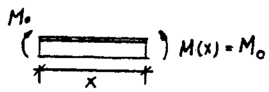
$$\theta_B = \frac{dv}{dx}\Big|_{x=L} = \frac{M_0 L}{6EI} \qquad \textbf{Ans}$$

From Eq. (2),

$$v\Big|_{x=\frac{L}{2}} = \frac{-M_0 L^2}{16EI} \qquad \textbf{Ans}$$

***12-16.** Determine the elastic curve for the simply supported beam, which is subjected to the couple moments $\mathbf{M}_0$. Also, compute the maximum slope and the maximum deflection of the beam. EI is constant.





Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M_0$$

$$EI\frac{dv}{dx} = M_0x + C_1 \qquad (1)$$

$$EIv = \frac{M_0x^2}{2} + C_1x + C_2 \qquad (2)$$

Boundary Conditions:

$v = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = \frac{M_0L^2}{2} + C_1L$$

$$C_1 = \frac{-M_0L}{2}$$

$$\frac{dv}{dx} = \frac{M_0}{2EI}(2x - L)$$

$$|\theta_{max}| = |\theta_A| = |\theta_B| = \frac{M_0L}{2EI} \qquad \textbf{Ans}$$

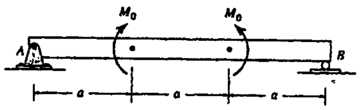
$$v = \frac{M_0x}{2EI}(x - L) \qquad \textbf{Ans}$$

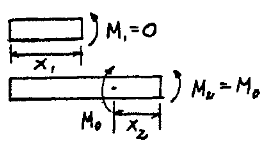
Due to symmetry, v_{max} occurs at $x = \frac{L}{2}$

$$v_{max} = -\frac{M_0L^2}{8EI} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

12-17. Determine the maximum deflection of the beam and the slope at A. EI is constant.





$M_1 = 0$

$EI\frac{d^2v_1}{dx_1^2} = 0; \quad EI\frac{dv_1}{dx_1} = C_1$

$EIv_1 = C_1x_1 + C_2$

At $x_1 = 0$, $v_1 = 0$; $\quad C_2 = 0$

$M_2 = M_0; \quad EI\frac{d^2v_2}{dx_2^2} = M_0$

$EI\frac{dv_2}{dx_2} = M_0x_2 + C_3$

$EI\,v_2 = \frac{1}{2}M_0x_2^2 + C_3x_2 + C_4$

At $x_2 = \frac{a}{2}$, $\frac{dv_2}{dx_2} = 0$; $\quad C_3 = \frac{-M_0a}{2}$

At $x_1 = a$, $x_2 = 0$, $\quad v_1 = v_2$, $\quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

$C_1a = C_4$

$C_1 = \frac{-M_0a}{2}, \quad C_4 = \frac{-M_0a^2}{2}$

At $x_1 = 0$,

$EI\frac{dv_1}{dx_1} = -\frac{M_0a}{2}$

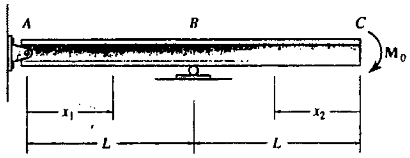
$\theta_A = \frac{M_0a}{2EI}$ **Ans**

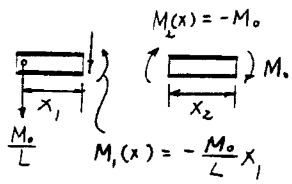
At $x_2 = \frac{a}{2}$,

$EI\,v_{max} = \frac{1}{2}M_0\left(\frac{a^2}{4}\right) - \frac{M_0a}{2}\left(\frac{a}{2}\right) - \frac{M_0a^2}{2}$

$v_{max} = -\frac{5M_0a^2}{8EI}$ **Ans**

12–18 Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the deflection and slope at C. EI is constant.





$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -M_0$; $\quad EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions :

At $x_1 = 0$, $v_1 = 0$

From Eq. (2),

$0 = 0 + 0 + C_2$; $\quad C_2 = 0$

At $x_1 = x_2 = L$, $v_1 = v_2 = 0$

From Eq. (2),

$0 = -\frac{M_0L^2}{6} + C_1L$; $\quad C_1 = \frac{M_0L}{6}$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad (5)$$

Continuity condition :

At $x_1 = x_2 = L$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3)$; $\quad C_3 = \frac{4M_0L}{3}$

Substituting C_3 into Eq. (5) yields,

$$C_4 = -\frac{5M_0L^2}{6}$$

The slope :

$$\frac{dv_2}{dx_2} = \frac{1}{EI}[-M_0x_2 + \frac{4M_0L}{3}]$$

$$\theta_C = \left.\frac{dv_2}{dx_2}\right|_{x_2=0} = \frac{4M_0L}{3EI} \qquad \textbf{Ans}$$

The elastic curve :

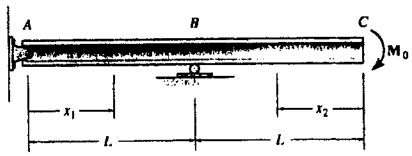
$$v_1 = \frac{M_0}{6EIL}[-x_1^3 + L^2x_1] \qquad \textbf{Ans}$$

$$v_2 = \frac{M_0}{6EIL}[-3Lx_2^3 + 8L^2x_2 - 5L^3] \qquad \textbf{Ans}$$

$$v_C = v_2\Big|_{x_2=0} = -\frac{5M_0L^2}{6EI} \qquad \textbf{Ans}$$

The negative sign indicates downward deflection.

12–19 Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope at A. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x_1) = -\frac{M_0}{L}x_1$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{M_0}{L}x_1$$

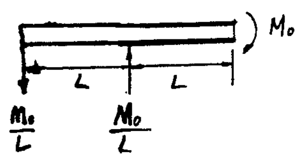
$$EI\frac{dv_1}{dx_1} = -\frac{M_0}{2L}x_1^2 + C_1 \qquad (1)$$

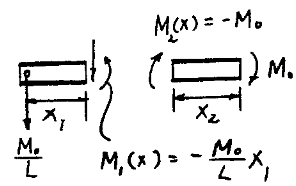
$$EI\,v_1 = -\frac{M_0}{6L}x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -M_0$; $\quad EI\frac{d^2v_2}{dx_2^2} = -M_0$

$$EI\frac{dv_2}{dx_2} = -M_0x_2 + C_3 \qquad (3)$$

$$EI\,v_2 = -\frac{M_0}{2}x_2^2 + C_3x_2 + C_4 \qquad (4)$$





Boundary conditions :

At $x_1 = 0$, $v_1 = 0$

From Eq. (2),

$0 = 0 + 0 + C_2; \quad C_2 = 0$

At $x_1 = x_2 = L$, $v_1 = v_2 = 0$

From Eq. (2),

$$0 = -\frac{M_0L^2}{6} + C_1L; \quad C_1 = \frac{M_0L}{6}$$

From Eq. (4),

$$0 = -\frac{M_0L^2}{2} + C_3L + C_4 \qquad (5)$$

Continuity condition :

At $x_1 = x_2 = L$, $\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{M_0L}{2} + \frac{M_0L}{6} = -(-M_0L + C_3); \quad C_3 = \frac{4M_0L}{3}$$

Substituting C_3 into Eq. (5) yields,

$$C_4 = -\frac{5M_0L^2}{6}$$

The elastic curve :

$$v_1 = \frac{M_0}{6EIL}[-x_1^3 + L^2x_1] \qquad \textbf{Ans}$$

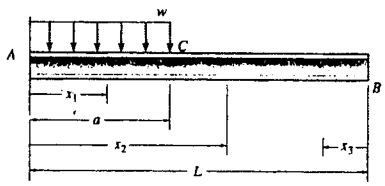
$$v_2 = \frac{M_0}{6EIL}[-3Lx_2^2 + 8L^2x_2 - 5L^3] \qquad \textbf{Ans}$$

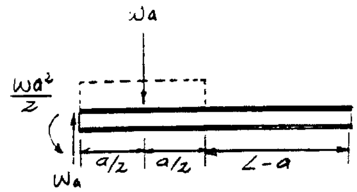
From Eq. (1),

$$EI\frac{dv_1}{dx_1} = 0 + C_1 = \frac{M_0L}{6}$$

$$\theta_A = \frac{dv_1}{dx_1} = \frac{M_0L}{6EI} \qquad \textbf{Ans}$$

•12–20 Determine the equations of the elastic curve using the coordinates x_1 and x_2, and specify the slope and deflection at B. EI is constant.





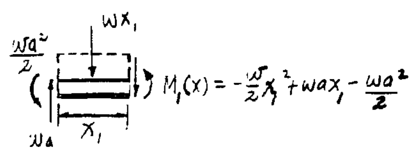
$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

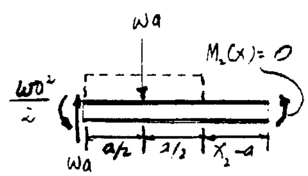
$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$



For $M_2(x) = 0$; $\quad EI\frac{d^2v_2}{dx_2^2} = 0$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad (3)$$

$$EI\,v_2 = C_3x_2 + C_4 \qquad (4)$$



Boundary conditions :

At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$

From Eq. (1), $\quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

From Eq. (2) ; $\quad C_2 = 0$

Continuity conditions :

At $x_1 = a$, $\quad x_2 = a$; $\quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = C_3; \qquad C_3 = -\frac{wa^3}{6}$$

From Eqs. (2) and (4),

At $x_1 = a$, $x_2 = a$ $\quad v_1 = v_2$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = -\frac{wa^4}{6} + C_4; \qquad C_4 = \frac{wa^4}{24}$$

The slope, from Eq.(3),

$$\theta_B = \frac{dv_2}{dx_2} = -\frac{wa^3}{6EI} \qquad \textbf{Ans}$$

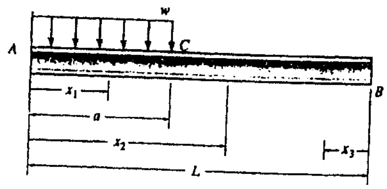
The elastic curve :

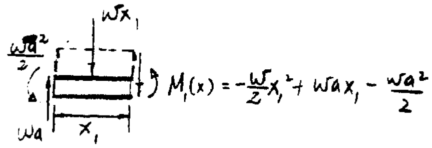
$$v_1 = \frac{w}{24EI}[-x_1^4 + 4ax_1^3 - 6a^2x_1^2] \qquad \textbf{Ans}$$

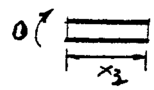
$$v_2 = \frac{wa^3}{24EI}[-4x_2 + a] \qquad \textbf{Ans}$$

$$v_B = v_2\Big|_{x_2 = L} = \frac{wa^3}{24EI}(-4L + a) \qquad \textbf{Ans}$$

12–21 Determine the equations of the elastic curve using the coordinates x_1 and x_3, and specify the slope and deflection at point B. EI is constant.







$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = -\frac{w}{2}x_1^2 + wa\,x_1 - \frac{wa^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = -\frac{w}{2}x_1^2 + wax_1 - \frac{wa^2}{2}$$

$$EI\frac{dv_1}{dx_1} = -\frac{w}{6}x_1^3 + \frac{wa}{2}x_1^2 - \frac{wa^2}{2}x_1 + C_1 \qquad (1)$$

$$EI\,v_1 = -\frac{w}{24}x_1^4 + \frac{wa}{6}x_1^3 - \frac{wa^2}{4}x_1^2 + C_1x_1 + C_2 \qquad (2)$$

For $M_3(x) = 0$; $\quad EI\frac{d^2v_3}{dx_3^2} = 0$

$$EI\frac{dv_3}{dx_3} = C_3 \qquad (3)$$

$$EI\,v_3 = C_3x_3 + C_4 \qquad (4)$$

Boundary conditions :

At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$

From Eq. (1),

$0 = -0 + 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$

From Eq. (2),

$0 = -0 - 0 - 0 + 0 + C_2; \quad C_2 = 0$

Continuity conditions :

At $x_1 = a$, $\quad x_3 = L - a$; $\quad \frac{dv_1}{dx_1} = -\frac{dv_3}{dx_3}$

$$-\frac{wa^3}{6} + \frac{wa^3}{2} - \frac{wa^3}{2} = -C_3; \quad C_3 = +\frac{wa^3}{6}$$

At $x_1 = a$, $x_3 = L - a$ $\quad v_1 = v_3$

$$-\frac{wa^4}{24} + \frac{wa^4}{6} - \frac{wa^4}{4} = \frac{wa^3}{6}(L-a) + C_4; \quad C_4 = \frac{wa^4}{24} - \frac{wa^3L}{6}$$

The slope

$$\frac{dv_3}{dx_3} = \frac{wa^3}{6EI}$$

$$\theta_B = \left.\frac{dv_3}{dx_3}\right|_{x_3=0} = \frac{wa^3}{6EI} \qquad \textbf{Ans}$$

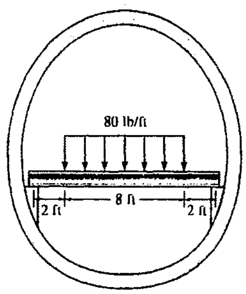
The elastic curve

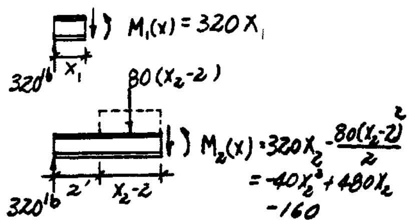
$$v_1 = \frac{wx_1^2}{24EI}[-x_1^2 + 4ax_1 - 6a^2] \qquad \textbf{Ans}$$

$$v_3 = \frac{wa^3}{24EI}[4x_3 + a - 4L] \qquad \textbf{Ans}$$

$$v_B = v_3\Big|_{x_3=0} = \frac{wa^3}{24EI}(a - 4L) \qquad \textbf{Ans}$$

12–22 The floor beam of the airplane is subjected to the loading shown. Assuming that the fuselage exerts only vertical reactions on the ends of the beam, determine the maximum deflection of the beam. EI is constant.





Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = 320x_1$

$$EI\frac{d^2v_1}{dx_1^2} = 320x_1$$

$$EI\frac{dv_1}{dx_1} = 160x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = 53.33x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -40x_2^2 + 480x_2 - 160$

$$EI\frac{d^2v_2}{dx_2^2} = -40x_2^2 + 480x_2 - 160$$

$$EI\frac{dv_2}{dx_2} = -13.33x_2^3 + 240x_2^2 - 160x_2 + C_3 \qquad (3)$$

$$EIv_2 = -3.33x_2^4 + 80x_2^3 - 80x_2^2 + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_1 = 0$ at $x_1 = 0$

From Eq. (2), $C_2 = 0$

Due to symmetry,

$\frac{dv_2}{dx_2} = 0$ at $x_2 = 6$ ft

From Eq. (3),

$$-2880 + 8640 - 960 + C_3 = 0$$

$$C_3 = -4800$$

Continuity conditions :

$\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ at $x_1 = x_2 = 2$ ft

From Eqs. (1) and (3),

$$640 + C_1 = -106.67 + 960 - 320 - 4800$$

$$C_1 = -4906.67$$

$v_1 = v_2$ at $x_1 = x_2 = 2$ ft

From Eqs. (2) and (4),

$$426.67 - 9813.33 = -53.33 + 640 - 320 - 9600 + C_4$$

$$C_4 = -53.33$$

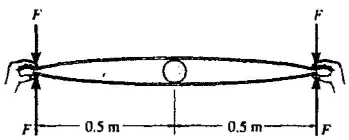
$$v_2 = \frac{1}{EI}(-3.33x_2^4 + 80x_2^3 - 80x_2^2 - 4800x_2 - 53.33)$$

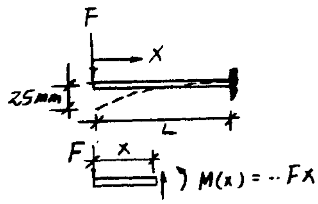
v_{max} occurs at $x_2 = 6$ ft.

$$v_{max} = v_2\Big|_{x_2=6} = \frac{-18.8 \text{ kip}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

12–23 The two wooden meter sticks are separated at their centers by a smooth rigid cylinder having a diameter of 50 mm. Determine the force F that must be applied at each end in order to just make their ends touch. Each stick has a width of 20 mm and a thickness of 5 mm. $E_w = 11$ GPa.





Slope at mid - span is zero, therefore we can model the problem as follows :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -Fx$$

$$EI\frac{dv}{dx} = \frac{-Fx^2}{2} + C_1 \qquad (1)$$

$$EIv = \frac{-Fx^3}{6} + C_1x + C_2 \qquad (2)$$

Boundary conditions :

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L$$

From Eq. (1),

$$0 = \frac{-FL^2}{2} + C_1$$

$$C_1 = \frac{FL^2}{2}$$

$$v = 0 \quad \text{at} \quad x = L$$

From Eq. (2),

$$0 = \frac{-FL^3}{6} + \frac{FL^3}{2} + C_2$$

$$C_2 = -\frac{FL^3}{3}$$

$$v = \frac{F}{6EI}(-x^3 + 3L^2x - 2L^3)$$

Require :

$$v = -0.025 \text{ m} \quad \text{at} \quad x = 0$$

$$-0.025 = \frac{F}{6EI}(0 + 0 - 2L^3)$$

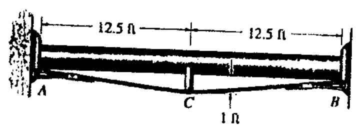
$$F = \frac{0.075EI}{L^3}$$

where

$$I = \frac{1}{12}(0.02)(0.005^3) = 0.20833(10^{-9})\text{m}^4$$

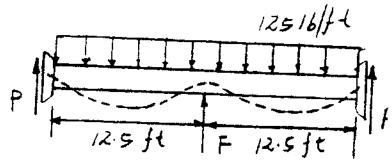
$$F = \frac{0.075(11)(10^9)(0.20833)(10^{-9})}{(0.5^3)} = 1.375 \text{ N} \qquad \textbf{Ans}$$

*12–24 The pipe can be assumed roller supported at its ends and by a rigid saddle C at its center. The saddle rests on a cable that is connected to the supports. Determine the force that should be developed in the cable if the saddle keeps the pipe from sagging or deflecting at its center. The pipe and fluid within it have a combined weight of 125 lb/ft. EI is constant.



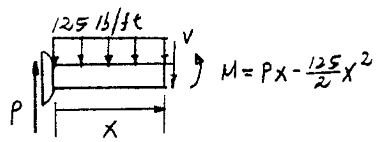
$$2P + F - 125(25) = 0$$

$$2P + F = 3125$$



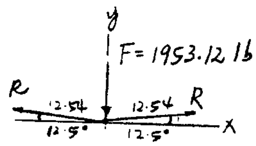
$$M = Px - \frac{125}{2}x^2$$

$$EI\frac{d^2v}{dx^2} = Px - \frac{125}{2}x^2$$



$$EI\frac{dv}{dx} = \frac{Px^2}{2} - 20.833x^3 + C_1$$

$$EIv = \frac{Px^3}{6} - 5.2083x^4 + C_1x + C_2$$



At $x = 0$, $v = 0$. Therefore $C_2 = 0$

At $x = 12.5$ ft, $v = 0$.

$$0 = \frac{P(12.5)^3}{6} - 5.2083(12.5)^4 + C_1(12.5) \qquad (1)$$

At $x = 12.5$ ft, $\frac{dv}{dx} = 0$.

$$0 = \frac{P(12.5)^2}{2} - 20.833(12.5)^3 + C_1 \qquad (2)$$

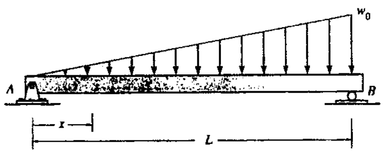
Solving Eqs. (1) and (2) for P,

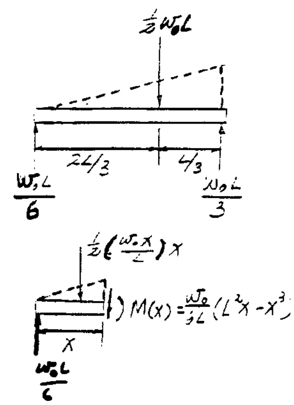
$P = 585.94$ $F = 3125 - 2(585.94) = 1953.12$ lb

$+\uparrow \Sigma F_y = 0;$ $2R(\frac{1}{12.54}) - 1953.12 = 0$

$R = 12\,246$ lb $= 12.2$ kip **Ans**

12–25 The beam is subjected to the linearly varying distributed load. Determine the maximum slope of the beam. EI is constant.





$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0}{6L}(L^2x - x^3)$$

$$EI\frac{dv}{dx} = \frac{w_0}{6L}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) + C_1 \qquad (1)$$

$$EI\,v = \frac{w_0}{6L}\left(\frac{L^2x^3}{6} - \frac{x^5}{20}\right) + C_1x + C_2 \qquad (2)$$

Boundary conditions :

At $x = 0,\ v = 0.$

From Eq. (2), $\quad C_2 = 0$

At $x = L,\ v = 0$

From Eq. (2),

$$0 = \frac{w_0}{6L}\left(\frac{L^5}{6} - \frac{L^5}{20}\right) + C_1L; \qquad C_1 = -\frac{7w_0L^3}{360}$$

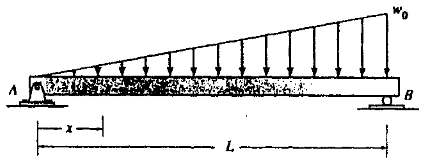
The slope :

From Eq.(1),

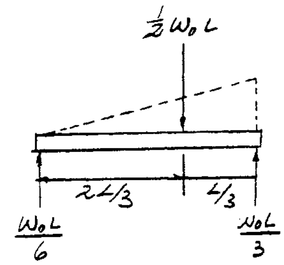
$$\frac{dv}{dx} = \frac{w_0}{6EIL}\left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

$$\theta_{max} = \left.\frac{dv}{dx}\right|_{x=L} = \frac{w_0L^3}{45EI} \qquad \textbf{Ans}$$

12-26 The beam is subjected to the linearly varying distributed load. Determine the maximum deflection of the beam. EI is constant.



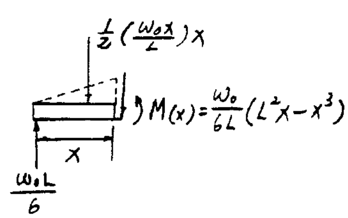
$$EI\frac{d^2v}{dx^2} = M(x)$$



$$EI\frac{d^2v}{dx^2} = \frac{w_0}{6L}(L^2x - x^3)$$

$$EI\frac{dv}{dx} = \frac{w_0}{6L}\left(\frac{L^2x^2}{2} - \frac{x^4}{4}\right) + C_1 \qquad (1)$$

$$EI\,v = \frac{w_0}{6L}\left(\frac{L^2x^3}{6} - \frac{x^5}{20}\right) + C_1x + C_2 \qquad (2)$$



Boundary conditions :

$v = 0$ at $x = 0$.

From Eq. (2), $\quad C_2 = 0$

$v = 0$ at $x = L$.

From Eq. (2),

$$0 = \frac{w_0}{6L}\left(\frac{L^5}{6} - \frac{L^5}{20}\right) + C_1L; \qquad C_1 = -\frac{7w_0L^3}{360}$$

$$\frac{dv}{dx} = \frac{w_0}{6EIL}\left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

$$\frac{dv}{dx} = 0 = \left(\frac{L^2x^2}{2} - \frac{x^4}{4} - \frac{7L^4}{60}\right)$$

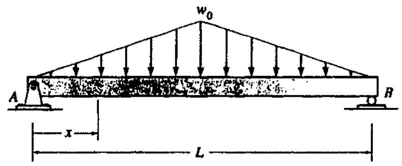
$$15x^4 - 30L^2x^2 + 7L^4 = 0; \qquad x = 0.5193L$$

$$v = \frac{w_0x}{360EIL}(10L^2x^2 - 3x^4 - 7L^4)$$

Substitute $x = 0.5193L$ into v,

$$v_{max} = -\frac{0.00652w_0L^4}{EI} \qquad \textbf{Ans}$$

12–27 Determine the elastic curve for the simply supported beam using the x coordinate $0 \le x \le L/2$. Also, compute the slope at A and the maximum deflection of the beam. EI is constant.



$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w_0L}{4}x - \frac{w_0}{3L}x^3$$

$$EI\frac{dv}{dx} = \frac{w_0L}{8}x^2 - \frac{w_0}{12L}x^4 + C_1 \qquad (1)$$

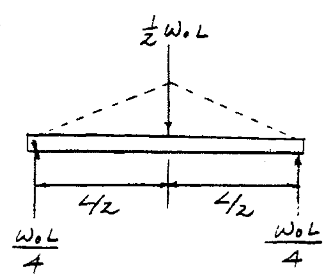
$$EIv = \frac{w_0L}{24}x^3 - \frac{w_0}{60L}x^5 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

Due to symmetry, at $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$

From Eq. (1),

$$0 = \frac{w_0L}{8}\left(\frac{L^2}{4}\right) - \frac{w_0}{12L}\left(\frac{L^4}{16}\right) + C_1; \qquad C_1 = -\frac{5w_0L^3}{192}$$



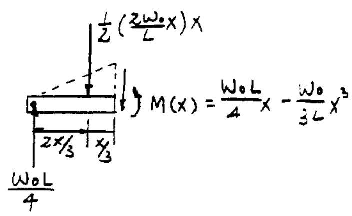
At $x = 0$, $v = 0$

From Eq. (2),

$$0 = 0 - 0 + 0 + C_2; \qquad C_2 = 0$$

From Eq. (1),

$$\frac{dv}{dx} = \frac{w_0}{192EIL}[24L^2x^2 - 16x^4 - 5L^4]$$



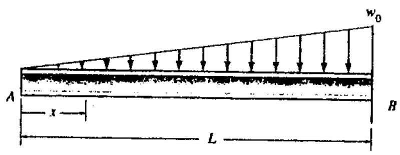
$$\theta_A = \left.\frac{dv}{dx}\right|_{x=0} = -\frac{5w_0L^3}{192EI} = \frac{5w_0L^3}{192EI} \qquad \textbf{Ans}$$

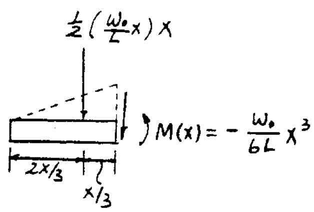
From Eq. (2),

$$v = \frac{w_0x}{960EIL}[40L^2x^2 - 16x^4 - 25L^4] \qquad \textbf{Ans}$$

$$v_{max} = v\Big|_{x=\frac{L}{2}} = -\frac{w_0L^4}{120EI} = \frac{w_0L^4}{120EI} \qquad \textbf{Ans}$$

•12–28 Determine the elastic curve for the cantilevered beam using the x coordinate. Also compute the maximum slope and maximum deflection. EI is constant.





$$EI\frac{d^2v}{dx^2} = M(x)\,; \qquad EI\frac{d^2v}{dx^2} = -\frac{w_0x^3}{6L}$$

$$EI\frac{dv}{dx} = -\frac{w_0x^4}{24L} + C_1 \qquad (1)$$

$$EI\,v = -\frac{w_0x^5}{120L} + C_1x + C_2 \qquad (2)$$

Boundary conditions :

$\frac{dv}{dx} = 0$ at $x = L$

From Eq. (1),

$$0 = -\frac{w_0}{24L}(L^4) + C_1\,; \qquad C_1 = \frac{w_0L^3}{24}$$

$v = 0$ at $x = L$

From Eq. (2),

$$0 = -\frac{w_0}{120\,L}(L^5) + \frac{w_0L^3}{24}(L) + C_2\,; \qquad C_2 = -\frac{w_0L^4}{30}$$

The slope :

From Eq.(1),

$$\frac{dv}{dx} = \frac{w_0}{24EIL}(-x^4 + L^4)$$

$$\theta_{max} = \frac{dv}{dx}\bigg|_{x=0} = \frac{w_0L^3}{24EI} \qquad \textbf{Ans}$$

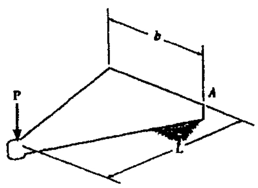
The elastic curve :

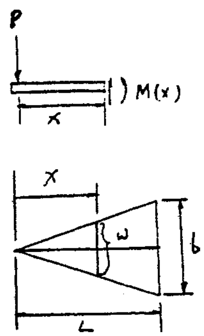
From Eq. (2),

$$v = \frac{w_0}{120EIL}(-x^5 + 5L^4x - 4L^5) \qquad \textbf{Ans}$$

$$v_{max} = v\bigg|_{x=0} = \frac{w_0L^4}{30EI} \qquad \textbf{Ans}$$

12-29. The tapered beam has a rectangular cross section. Determine the deflection of its end in terms of the load P, length L, modulus of elasticity E, and the moment of inertia I_0 of its end.





Moment function :

$$M(x) = -Px$$

Moment of inertia :

$$w = \frac{b}{L}x; \qquad I = \frac{1}{12}\left(\frac{b}{L}x\right)t^3 = \frac{1}{12}bt^3\left(\frac{x}{L}\right) = \frac{I_0}{L}x$$

Slope and elastic curve :

$$EI(x)\frac{d^2\upsilon}{dx^2} = M(x)$$

$$E\left(\frac{I_0}{L}\right)x\frac{d^2\upsilon}{dx^2} = -Px; \qquad EI_0\frac{d^2\upsilon}{dx^2} = -PL$$

$$EI_0\frac{d\upsilon}{dx} = -PLx + C_1 \qquad (1)$$

$$EI_0\upsilon = \frac{-PL}{2}x^2 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

$$\frac{d\upsilon}{dx} = 0, \text{ at } x = L$$

From Eq. (1),

$$0 = -PL^2 + C_1; \qquad C_1 = PL^2$$

$$\upsilon = 0, \text{ at } x = L$$

From Eq. (2),

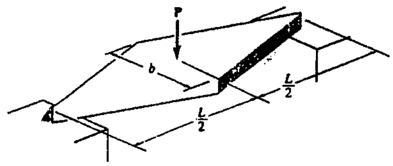
$$0 = -\frac{PL^3}{2} + PL^3 + C_2; \qquad C_2 = -\frac{PL^3}{2}$$

$$\upsilon = \frac{PL}{2EI_0}(-x^2 + Lx - L^2)$$

$$\text{at } x = 0, \qquad \upsilon_{max} = \upsilon\Big|_{x=0} = -\frac{PL^3}{2EI_0} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

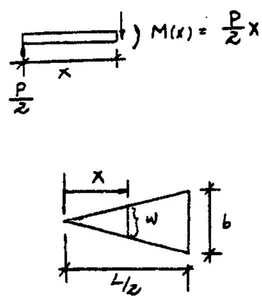
12-30. The tapered beam has a rectangular cross section. Determine the deflection of its center in terms of the load P, length L, modulus of elasticity E, and the moment of inertia I_c of its center.



Moment of inertia :

$$w = \frac{2b}{L}x$$

$$I = \frac{1}{12}\left(\frac{2b}{L}x\right)(t^3) = \frac{1}{12}(b)(t^3)\left(\frac{2x}{L}\right) = \left(\frac{2I_C}{L}\right)x$$



Elastic curve and slope :

$$EI(x)\frac{d^2v}{dx^2} = M(x)$$

$$E\left(\frac{2I_C}{L}\right)(x)\frac{d^2v}{dx^2} = \frac{P}{2}x$$

$$EI_C\frac{dv}{dx} = \frac{PL}{4}x + C_1 \qquad (1)$$

$$EI_Cv_1 = \frac{PL}{8}x^2 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

Due to symmetry :

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = \frac{L}{2}$$

From Eq. (1),

$$0 = \frac{PL^2}{8} + C_1$$

$$C_1 = -\frac{PL^2}{8}$$

$$v = 0 \quad \text{at} \quad x = 0$$

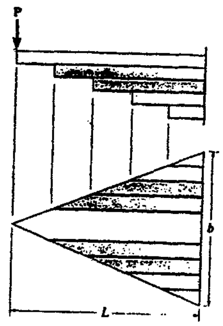
$$C_2 = 0$$

$$v = \frac{PLx}{8EI_C}(x - L)$$

$$v_C = v\Big|_{x=\frac{L}{2}} = -\frac{PL^3}{32EI_C} \qquad \textbf{Ans}$$

The negative sign indicates downward displacement.

12-31. The beam is made from a plate that has a constant thickness t and a width that varies linearly. The plate is cut into strips to form a series of leaves that are stacked to make a leaf spring consisting of n leaves. Determine the deflection at its end when loaded. Neglect friction between the leaves.



Use the triangular plate for the calculation.

$$M = Px$$

$$I = \frac{1}{12}\left(\frac{b}{L}x\right)(t)^3$$

$$\frac{d^2v}{dx^2} = \frac{M}{EI} = \frac{Px}{E(\frac{1}{12})(\frac{b}{L})x(t)^3}$$

$$\frac{d^2v}{dx^2} = \frac{12PL}{Ebt^3}$$

$$\frac{dv}{dx} = \frac{12PL}{Ebt^3}x + C_1$$

$$v = \frac{6PL}{Ebt^3}x^2 + C_1x + C_2$$

$\frac{dv}{dx} = 0$ at $x = L$

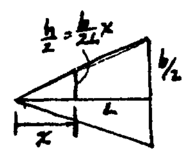
$$C_1 = \frac{-12PL^2}{Ebt^3}$$

$v = 0$ at $x = L$

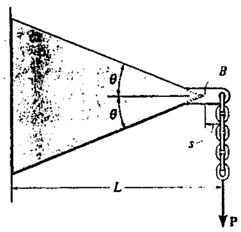
$$C_2 = \frac{6PL^3}{Ebt^3}$$

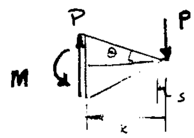
When $x = 0$

$$v_{max} = \frac{6PL^3}{Ebt^3} \quad \textbf{Ans}$$



•12–32 The beam has a constant width b and is tapered as shown. If it supports a load **P** at its end, determine the deflection at B. The load **P** is applied a short distance s from the tapered end B, where $s \ll L$. EI is constant.





$M = Px$

$I = \frac{1}{12}(b)(2x\tan\theta)^3 = \frac{2}{3}b\tan^3\theta\, x^3$

$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{P(x)}{E(\frac{2}{3})b\tan^3\theta\, x^3} = \frac{3P}{2Eb\tan^3\theta}\ \frac{x}{x^3} = \frac{k}{x^2}$

where $k = \frac{3P}{2Eb\tan^3\theta}$

$\frac{dy}{dx} = -k(\frac{1}{x}) + C_1$

At $x = L$, $\frac{dy}{dx} = 0$,

$C_1 = k(\frac{1}{L})$

$y = -k(\ln x) + \frac{k}{L}x + C_2$

When $x = L$, $y = 0$,

$C_2 = k(\ln L - 1)$

$y = -k\ln x + \frac{k}{L}x + k(\ln L - 1)$

At $x = s$,

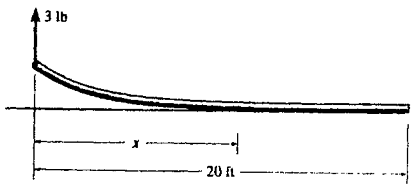
$y = -k(\ln s) + \frac{ks}{L} + k(\ln L - 1)$

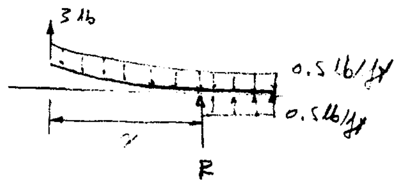
Since $L >> s$,

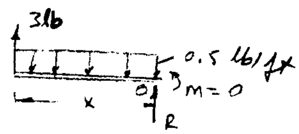
$y = k\ln(\frac{L}{s}) - k$

$y = \frac{3P}{2Eb\tan^3\theta}(\ln\frac{L}{s} - 1)$ **Ans**

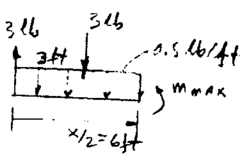
12–33 A thin flexible 20-ft-long rod having a weight of 0.5 lb/ft rests on the smooth surface. If a force of 3 lb is applied at its end to lift it, determine the suspended length x and the maximum moment developed in the rod.







Since the horizontal section has no curvature the the moment in the rod is zero. Hence, **R** acts at the end of the suspended portion and this portion acts like a simply - supported beam. Thus,



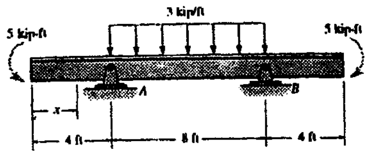
$$\circlearrowleft + \Sigma M_0 = 0; \qquad -3(x) + (0.5)(x)\left(\frac{x}{2}\right) = 0$$

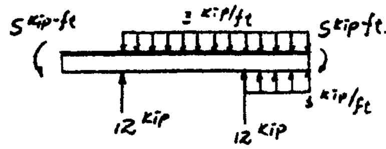
$$x = 12 \text{ ft} \qquad \textbf{Ans}$$

Maximum moment occurs at center.

$$M_{max} = 3(3) = 9 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans}$$

12-34. The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.





$$M = -5 < x-0 >^0 - (-12) < x-4 > - \frac{3}{2}(x-4>^2 - (-12) < x-12 > - (\frac{-3}{2}) < x-12 >^2$$

$$M = -5 + 12 < x-4 > - \frac{3}{2} < x-4 >^2 + 12 < x-12 > + \frac{3}{2} < x-12 >^2$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -5 + 12 < x-4 > - \frac{3}{2} < x-4 >^2 + 12 < x-12 > + \frac{3}{2} < x-12 >^2$$

$$EI\frac{dv}{dx} = -5x + 6 < x-4 >^2 - \frac{1}{2} < x-4 >^3 + 6 < x-12 >^2 + \frac{1}{2} < x-12 >^3 + C_1 \qquad (1)$$

$$EIv = \frac{-5}{2}x^2 + 2 < x-4 >^3 - \frac{1}{8} < x-4 >^4 + 2 < x-12 >^3 + \frac{1}{8} < x-12 >^4 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

$v = 0$ at $x = 4$ ft

From Eq. (2)

$0 = -40 + 0 - 0 + 0 + 0 + 4C_1 + C_2$

$4C_1 + C_2 = 40 \qquad (3)$

$v = 0$ at $x = 12$ ft.

$0 = -360 + 1024 - 512 + 0 + 0 + 12C_1 + C_2$

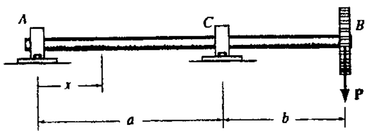
$12C_1 + C_2 = -152 \qquad (4)$

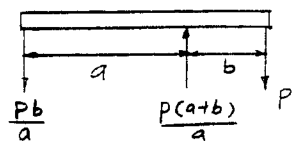
Solving Eqs. (3) and (4) yields :

$C_1 = -24 \qquad C_2 = 136$

$$v = \frac{1}{EI}[-2.5x^2 + 2 < x-4 >^3 - \frac{1}{8} < x-4 >^4 + 2 < x-12 >^3 + \frac{1}{8} < x-12 >^4 - 24x + 136] \text{kip} \cdot \text{ft}^3 \qquad \textbf{Ans}$$

12–35 The shaft is supported at A by a journal bearing that exerts only vertical reactions on the shaft, and at C by a thrust bearing that exerts horizontal and vertical reactions on the shaft. Determine the equation of the elastic curve. EI is constant.





$$M = -\frac{Pb}{a}\langle x-0\rangle - \left(-\frac{P(a+b)}{a}\langle x-a\rangle\right) = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x-a\rangle$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{Pb}{a}x + \frac{P(a+b)}{a}\langle x-a\rangle$$

$$EI\frac{dv}{dx} = -\frac{Pb}{2a}x^2 + \frac{P(a+b)}{2a}\langle x-a\rangle^2 + C_1 \qquad (1)$$

$$EIv = -\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x-a\rangle^3 + C_1x + C_2 \qquad (2)$$

Boundary condition :

At $x = 0$, $v = 0$

From Eq. (2)

$0 = -0 + 0 + 0 + C_2; \qquad C_2 = 0$

At $x = a$, $v = 0$

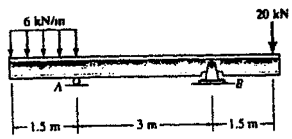
From Eq. (2)

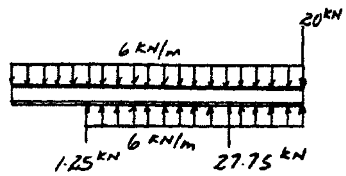
$0 = -\frac{Pb}{6a}(a^3) + 0 + C_1a + 0; \qquad C_1 = \frac{Pab}{6}$

From Eq. (2)

$$v = \frac{1}{EI}\left[-\frac{Pb}{6a}x^3 + \frac{P(a+b)}{6a}\langle x-a\rangle^3 + \frac{Pab}{6}x\right] \qquad \textbf{Ans}$$

***12-36.** The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.





$$M = -\frac{6}{2} < x-0 >^2 -(-1.25) < x-1.5 > -(-\frac{6}{2}) < x-1.5 >^2 -(-27.75) < x-4.5 >$$

$$M = -3x^2 + 1.25 < x-1.5 > +3 < x-1.5 >^2 +27.75 < x-4.5 >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -3x^2 + 1.25 < x-1.5 > +3 < x-1.5 >^2 +27.75 < x-4.5 >$$

$$EI\frac{dv}{dx} = -x^3 + 0.625 < x-1.5 >^2 + < x-1.5 >^3 +13.875 < x-4.5 >^2 + C_1$$

$$EIv = -0.25x^4 + 0.208 < x-1.5 >^3 + 0.25 < x-1.5 >^4 + 4.625 < x-4.5 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 1.5$ m

From Eq. (1)

$0 = -1.266 + 1.5C_1 + C_2$

$1.5C_1 + C_2 = 1.266 \qquad (2)$

$v = 0$ at $x = 4.5$ m

From Eq. (1)

$0 = -102.516 + 5.625 + 20.25 + 4.5C_1 + C_2$

$4.5C_1 + C_2 = 76.641 \qquad (3)$

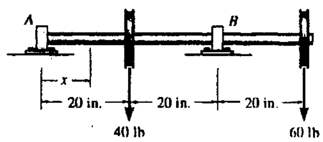
Solving Eqs. (2) and (3) yields:

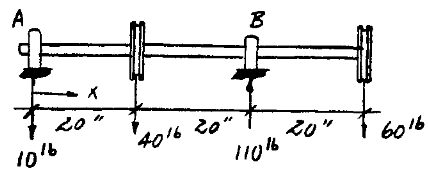
$C_1 = 25.12$

$C_2 = -36.42$

$$v = \frac{1}{EI}[-0.25x^4 + 0.208 < x-1.5 >^3 + 0.25 < x-1.5 >^4 + 4.625 < x-4.5 >^3 + 25.1x - 36.4] \text{kN} \cdot \text{m}^3 \qquad \textbf{Ans}$$

12–37 The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.





$$M = -10 < x-0 > -40 < x-20 > -(-110) < x-40 >$$
$$M = -10x - 40 < x-20 > +110 < x-40 >$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M$$
$$EI\frac{d^2v}{dx^2} = -10x - 40 < x-20 > +110 < x-40 >$$
$$EI\frac{dv}{dx} = -5x^2 - 20 < x-20 >^2 +55 < x-40 >^2 + C_1$$
$$EIv = -1.667x^3 - 6.667 < x-20 >^3 +18.33 < x-40 >^3 + C_1 x + C_2 \qquad (1)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (1) :

$C_2 = 0$

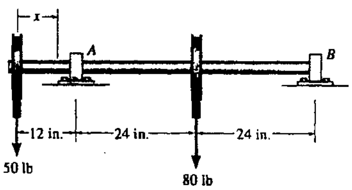
$v = 0$ at $x = 40$ in.

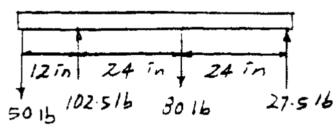
$0 = -106,666.67 - 53,333.33 + 0 + 40C_1$

$C_1 = 4000$

$$v = \frac{1}{EI}[-1.67x^3 - 6.67 < x-20 >^3 +18.3 < x-40 >^3 +4000x] \text{lb} \cdot \text{in}^3$$ **Ans**

12–38 The shaft supports the two pulley loads shown. Determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.





$$M = -50\langle x-0\rangle - (-102.5)\langle x-12\rangle - 80\langle x-36\rangle$$
$$= -50x + 102.5\langle x-12\rangle - 80\langle x-36\rangle$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -50x + 102.5\langle x-12\rangle - 80\langle x-36\rangle$$

$$EI\frac{dv}{dx} = -25x^2 + \frac{102.5}{2}\langle x-12\rangle^2 - 40\langle x-36\rangle^2 + C_1 \qquad (1)$$

$$EIv = -\frac{25}{3}x^3 + \frac{102.5}{6}\langle x-12\rangle^3 - \frac{40}{3}\langle x-36\rangle^3 + C_1x + C_2 \qquad (2)$$

Boundary conditions :
At $x = 12$ in., $v = 0$

From Eq. (2),
$$0 = -\frac{25}{3}(12)^3 + 0 - 0 + 12C_1 + C_2$$

$$12C_1 + C_2 = 14400 \qquad (3)$$

At $x = 60$ in., $v = 0$

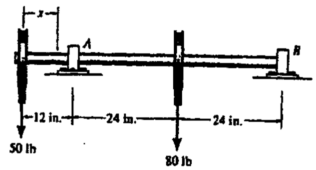
$$0 = -\frac{25}{3}(60)^3 + \frac{102.5}{6}(60-12)^3 - \frac{40}{3}(60-36)^3 + 60C_1 + C_2$$

$$60C_1 + C_2 = 95040 \qquad (4)$$

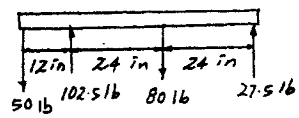
Solving Eqs. (3) and (4) yields :
$C_1 = 1680 \qquad C_2 = -5760$

The elastic curve. From Eq. (2),
$$v = \frac{1}{EI}[-8.33x^3 + 17.1\langle x-12\rangle^3 - 13.3\langle x-36\rangle^3 + 1680x - 5760]\ \text{lb}\cdot\text{in}^3 \qquad \textbf{Ans}$$

12-39. The shaft supports the two pulley loads shown. Determine the slope of the shaft at the bearings A and B. The bearings exert only vertical reactions on the shaft. EI is constant.



$$M = -50\langle x-0\rangle - (-102.5)\langle x-12\rangle - 80\langle x-36\rangle$$
$$= -50x + 102.5\langle x-12\rangle - 80\langle x-36\rangle$$



$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -50x + 102.5\langle x-12\rangle - 80\langle x-36\rangle$$

$$EI\frac{dv}{dx} = -25x^2 + \frac{102.5}{2}\langle x-12\rangle^2 - 40\langle x-36\rangle^2 + C_1 \qquad (1)$$

$$EIv = -\frac{25}{3}x^3 + \frac{102.5}{6}\langle x-12\rangle^3 - \frac{40}{3}\langle x-36\rangle^3 + C_1x + C_2 \qquad (2)$$

Boundary conditions :
At $x = 12$ in., $v = 0$

From Eq. (2),
$$0 = -\frac{25}{3}(12)^3 + 0 - 0 + 12C_1 + C_2$$

$$12C_1 + C_2 = 14400 \qquad (3)$$

At $x = 60$ in., $v = 0$

$$0 = -\frac{25}{3}(60)^3 + \frac{102.5}{6}(60-12)^3 - \frac{40}{3}(60-36)^3 + 60C_1 + C_2$$

$$60C_1 + C_2 = 95040 \qquad (4)$$

Solving Eqs. (3) and (4) yields :
$C_1 = 1680 \qquad C_2 = -5760$

$$EI\frac{dv}{dx} = -25x^2 + 51.25\langle x-12\rangle^2 - 40\langle x-36\rangle^2 + 1680$$

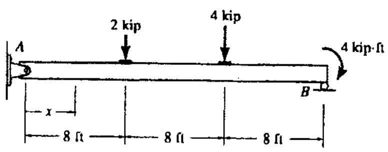
At $x = 12$ in.
$$\theta_A = \frac{1}{EI}(-25(12^2) + 0 - 0 + 1680)$$

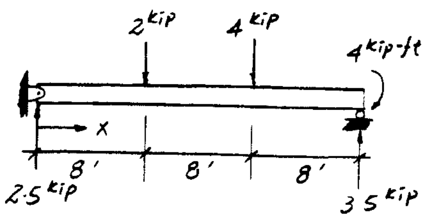
$$\theta_A = \frac{-1920}{EI} \qquad \textbf{Ans}$$

At $x = 60$ in.,
$$\theta_B = \frac{1}{EI}(-25(60^2) + 51.25(60-12)^2 - 40(60-36)^2 + 1680)$$

$$\theta_B = \frac{6720\ \text{lb}\cdot\text{in}^2}{EI} \qquad \textbf{Ans}$$

•12–40 The beam is subjected to the loads shown. Determine the equation of the elastic curve. EI is constant.





$M = -(-2.5) < x-0 > -2 < x-8 > -4 < x-16 >$

$M = 2.5x - 2 < x-8 > -4 < x-16 >$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = 2.5x - 2 < x-8 > -4 < x-16 >$$

$$EI\frac{dv}{dx} = 1.25x^2 - < x-8 >^2 - 2 < x-16 >^2 + C_1$$

$$EIv = 0.417x^3 - 0.333 < x-8 >^3 - 0.667 < x-16 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (1), $C_2 = 0$

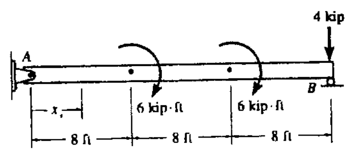
$v = 0$ at $x = 24$ ft

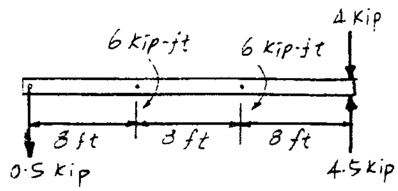
$0 = 5760 - 1365.33 - 341.33 + 24C_1$

$C_1 = -169$

$$v = \frac{1}{EI}[0.417x^3 - 0.333 < x-8 >^3 - 0.667 < x-16 >^3 - 169x] \text{ kip}\cdot\text{ft}^3$$ **Ans**

12–41 Determine the equation of the elastic curve. *EI* is constant.





$$M = -0.5\langle x-0\rangle - (-6)\langle x-8\rangle^0 - (-6)\langle x-16\rangle^0$$
$$= -0.5x + 6\langle x-8\rangle^0 + 6\langle x-16\rangle^0$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -0.5x + 6\langle x-8\rangle^0 + 6\langle x-16\rangle^0$$

$$EI\frac{dv}{dx} = -0.25x^2 + 6\langle x-8\rangle + 6\langle x-16\rangle + C_1 \qquad (1)$$

$$EI\,v = -\frac{0.25}{3}x^3 + 3\langle x-8\rangle^2 + 3\langle x-16\rangle^2 + C_1x + C_2 \qquad (2)$$

Boundary conditions :
At $x = 0$, $v = 0$

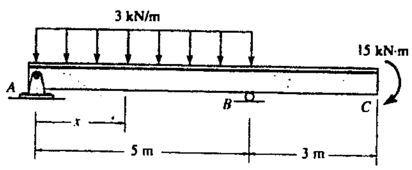
From Eq. (2),
$0 = -0 + 0 + 0 + 0 + C_2; \qquad C_2 = 0$

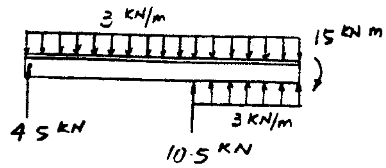
At $x = 24$ ft, $v = 0$

$$0 = -\frac{0.25}{3}(24)^3 + 3(24-8)^2 + 3(24-16)^2 + 24C_1; \qquad C_1 = 8.0$$

The elastic curve :
$$v = \frac{1}{EI}[-0.0833x^3 + 3\langle x-8\rangle^2 + 3\langle x-16\rangle^2 + 8.00x] \qquad \textbf{Ans}$$

12–42 The beam is subjected to the load shown. Determine the equations of the slope and elastic curve. EI is constant.





$$M = -(-4.5) < x-0 > -\frac{3}{2} < x-0 >^2 -(-10.5) < x-5) - (\frac{-3}{2}) < x-5 >^2$$

$$M = 4.5x - 1.5x^2 + 10.5 < x-5 > +1.5 < x-5 >^2$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = 4.5x - 1.5x^2 + 10.5 < x-5 > +1.5 < x-5 >^2$$

$$EI\frac{dv}{dx} = 2.25x^2 - 0.5x^3 + 5.25 < x-5 >^2 +0.5 < x-5 >^3 + C_1 \quad (1)$$

$$EIv = 0.75x^3 - 0.125x^4 + 1.75 < x-5 >^3 +0.125 < x-5 >^4 + C_1 x + C_2 \quad (2)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$v = 0$ at $x = 5$

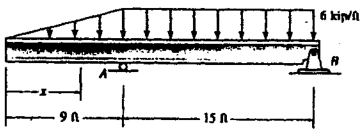
$0 = 93.75 - 78.125 + 5C_1$

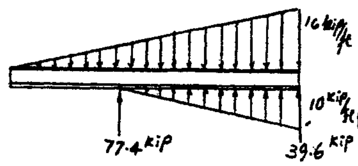
$C_1 = -3.125$

$$\frac{dv}{dx} = \frac{1}{EI}[2.25x^2 - 0.5x^3 + 5.25 < x-5 >^2 +0.5 < x-5 >^3 -3.125]\text{kN}\cdot\text{m}^2$$ **Ans**

$$v = \frac{1}{EI}[0.75x^3 - 0.125x^4 + 1.75 < x-5 >^3 +0.125 < x-5 >^4 -3.125x]\text{kN}\cdot\text{m}^3$$ **Ans**

12-43. The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.





$$M = -\frac{1}{6}\left(\frac{16}{24}\right) < x-0 >^3 - (-77.4) < x-9 > - \left(-\frac{1}{6}\right)\left(\frac{10}{15}\right) < x-9 >^3$$

$$M = -0.1111x^3 + 77.4 < x-9 > + 0.1111 < x-9 >^3$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = -0.1111x^3 + 77.4 < x-9 > + 0.1111 < x-9 >^3$$

$$EI\frac{dv}{dx} = -0.02778x^4 + 38.7 < x-9 >^2 + 0.02778 < x-9 >^4 + C_1$$

$$EIv = -0.005556x^5 + 12.9 < x-9 >^3 + 0.005556 < x-9 >^5 + C_1 x + C_2 \qquad (1)$$

Boundary conditions :

$v = 0$ at $x = 9$ ft

From Eq. (1)

$$0 = -328.05 + 0 + 0 + 9C_1 + C_2$$

$$9C_1 + C_2 = 328.05 \qquad (2)$$

$v = 0$ at $x = 24$ ft

$$0 = -44236.8 + 43537.5 + 4218.75 + 24C_1 + C_2$$

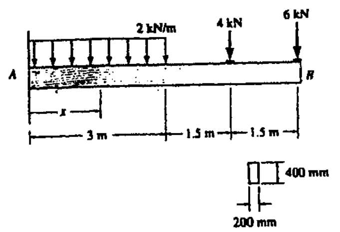
$$24C_1 + C_2 = -3519.45 \qquad (3)$$

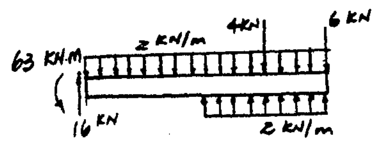
Solving Eqs. (2) and (3) yields,

$C_1 = -256 \qquad C_2 = 2637$

$$v = \frac{1}{EI}[-0.00556x^5 + 12.9 < x-9 >^3 + 0.00556 < x-9 >^5 - 256x + 2637] \text{ kip} \cdot \text{ft}^3 \qquad \textbf{Ans}$$

***12-44.** The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. If $E_w = 12$ GPa, determine the deflection and the slope at end B.





$$M = -63 < x-0 >^0 - (-16) < x-0 > -\frac{2}{2} < x-0 >^2 - (-\frac{2}{2}) < x-3 >^2 - 4 < x-4.5 >$$

$$M = -63 + 16x - x^2 + < x-3 >^2 - 4 < x-4.5 >$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = 63 + 16x - x^2 + < x-3 >^2 - 4 < x-4.5 >$$

$$EI\frac{dv}{dx} = -63x + 8x^2 - \frac{x^3}{3} + \frac{1}{3} < x-3 >^3 - 2 < x-4.5 >^2 + C_1 \qquad (1)$$

$$EIv = -31.5x^2 + \frac{8}{3}x^3 - \frac{x^4}{12} + \frac{1}{12} < x-3 >^4 - \frac{2}{3} < x-4.5 >^3 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

$\frac{dv}{dx} = 0$ at $x = 0$

From Eq. (1), $C_1 = 0$

$v = 0$ at $x = 0$

From Eq. (2), $C_2 = 0$

$$\frac{dv}{dx} = \frac{1}{EI}[-63x + 8x^2 - \frac{x^3}{3} + \frac{1}{3} < x-3 >^3 - 2 < x-4.5 >^2] \qquad (3)$$

$$v = \frac{1}{EI}[-31.5x^2 + \frac{8}{3}x^3 - \frac{x^4}{12} + \frac{1}{12} < x-3 >^4 - \frac{2}{3} < x-4.5 >^3] \text{kN}\cdot\text{m}^3 \qquad (4) \qquad \text{Ans}$$

$$I = \frac{1}{12}(0.20)(0.40)^3 = 1.067(10^{-3})\text{m}^4$$

At point B, $x = 6$m

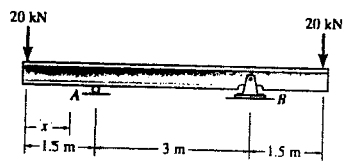
$$\theta_B = \frac{dv}{dx}\bigg|_{x=6\text{m}} = \frac{-157.5}{EI} = \frac{-157.5(10^3)}{12(10^9)(1.067)(10^{-3})} = -0.0123 \text{ rad} = -0.705° \qquad \text{Ans}$$

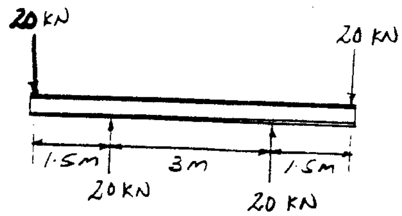
The negative sign indicates clockwise rotation.

$$v_B = \frac{-661.5}{EI} = \frac{-661.5(10^3)}{12(10^9)(1.067)(10^{-3})} = -0.0517\text{m} = -51.7 \text{ mm} \qquad \text{Ans}$$

The negative sign indicates downward displacement.

12–45 The beam is subjected to the load shown. Determine the equation of the elastic curve. EI is constant.





$$M = -20 < x-0 > - (-20) < x-1.5 > - (-20) < x-4.5 >$$
$$= -20x + 20 < x-1.5 > + 20 < x-4.5 >$$

$$EI\frac{d^2 v}{dx^2} = M$$

$$EI\frac{d^2 v}{dx^2} = -20x + 20 < x-1.5 > + 20 < x-4.5 >$$

$$EI\frac{dv}{dx} = -10x^2 + 10 < x-1.5 >^2 + 10 < x-4.5 >^2 + C_1 \qquad (1)$$

$$EIv = -\frac{10}{3}x^3 + \frac{10}{3} < x-1.5 >^3 + \frac{10}{3} < x-4.5 >^3 + C_1 x + C_2 \qquad (2)$$

Boundary conditions :

Due to symmetry, at $x = 3$ m, $\frac{dv}{dx} = 0$

From Eq. (1),

$0 = -10(3^2) + 10(1.5)^2 + 0 + C_1; \qquad C_1 = 67.5$

At $x = 1.5$ m, $v = 0$

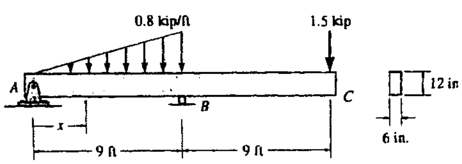
From Eq. (2),

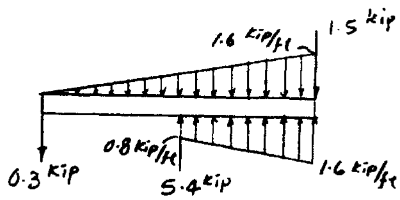
$0 = -\frac{10}{3}(1.5)^3 + 0 + 0 + 67.5(1.5) + C_2; \qquad C_2 = -90.0$

Hence,

$$v = \frac{1}{EI}[-\frac{10}{3}x^3 + \frac{10}{3} < x-1.5 >^3 + \frac{10}{3} < x-4.5 >^3 + 67.5x - 90] \text{ kN} \cdot \text{m}^3 \qquad \textbf{Ans}$$

12–46 The wooden beam is subjected to the load shown. Determine the equation of the elastic curve. Specify the deflection at the end C. $E_w = 1.6(10^3)$ ksi.





$$M = -0.3 < x-0 > -\frac{1}{6}\left(\frac{1.6}{18}\right) < x-0 >^3 -(-5.4) < x-9 > -\left(-\frac{0.8}{2}\right) < x-9 >^2 -\frac{1}{6}\left(-\frac{0.8}{9}\right) < x-9 >^3$$

$$M = -0.3x - 0.0148x^3 + 5.4 < x-9 > +0.4 < x-9 >^2 +0.0148 < x-9 >^3$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = -0.3x - 0.0148x^3 + 5.4 < x-9 > +0.4 < x-9 >^2 +0.0148 < x-9 >^3$$

$$EI\frac{dv}{dx} = -0.15x^2 - 0.003704x^4 + 2.7 < x-9 >^2 +0.1333 < x-9 >^3 + 0.003704 < x-9 >^4 + C_1$$

$$EIv = -0.05x^3 - 0.0007407x^5 + 0.9 < x-9 >^3 +0.03333 < x-9 >^4 + 0.0007407 < x-9 >^5 + C_1x + C_2 \quad (1)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (1)

$C_2 = 0$

$v = 0$ at $x = 9$ ft

From Eq. (1)

$0 = -36.45 - 43.74 + 0 + 0 + 0 + 9C_1$

$C_1 = 8.91$

$$v = \frac{1}{EI}[-0.05x^3 - 0.000741x^5 + 0.9 < x-9 >^3 +0.0333 < x-9 >^4 + 0.000741 < x-9 >^5 + 8.91x] \text{ kip} \cdot \text{ft}^3$$

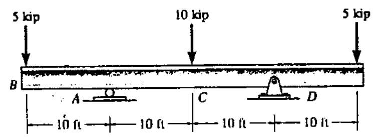
Ans

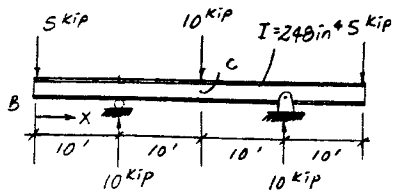
At point C, $x = 18$ ft

$$v_C = \frac{-612.29 \text{ kip} \cdot \text{ft}^3}{EI} = \frac{-612.29(12^3)}{1.6(10^3)(\frac{1}{12})(6)(12^3)} = -0.765 \text{ in.}$$ **Ans**

The negative sign indicates downward displacement.

12–47 Determine the slope at B and the deflection at C for the $W10 \times 45$ beam. $E_{st} = 29(10^3)$ ksi.





$M = -5 < x-0 > -(-10) < x-10 > -10 < x-20 > -(-10) < x-30 >$

$M = -5x + 10 < x-10 > -10 < x-20 > +10 < x-30 >$

Elastic curve and slope :

$EI\frac{d^2v}{dx^2} = M = -5x + 10 < x-10 > -10 < x-20 > +10 < x-30 >$

$EI\frac{dv}{dx} = -2.5x^2 + 5 < x-10 >^2 -5 < x-20 >^2 +5 < x-30 >^2 + C_1$

$EIv = -0.833x^3 + 1.67 < x-10 >^3 -1.67 < x-20 >^3 +1.67 < x-30 >^3 + C_1x + C_2$ (1)

Boundary conditions :

$v = 0$ at $x = 10$ ft

From Eq. (1)

$0 = -833.33 + 0 = 0 = 0 = 10C_1 + C_2$

$10C_1 + C_2 = 833.33$ (2)

$v = 0$ at $x = 30$ ft

From Eq. (1)

$0 = -22500 + 13333.33 - 1666.67 + 30C_1 + C_2$

$30C_1 + C_2 = 10833.33$ (3)

Solving Eqs. (2) and (3) yields :

$C_1 = 500 \qquad C_2 = -4167$

$\frac{dv}{dx} = \frac{1}{EI}[-2.5x^2 + 5 < x-10 >^2 -5 < x-20 >^2 +5 < x-30 >^2 +500]$

$\theta_B = \left.\frac{dv}{dx}\right|_{x=0} = \frac{500}{EI}$, where $I = 248$ in^4 for a W 10x45

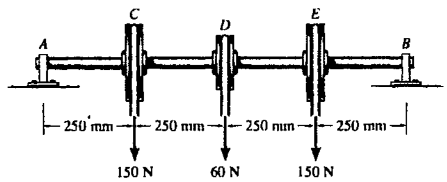
$= \frac{500(144)}{29(10^3)(248)} = 0.0100 \text{ rad} = 0.574°$ **Ans**

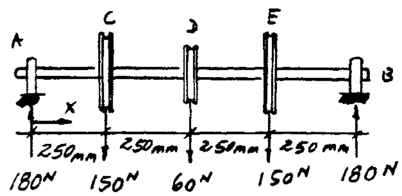
$v = \frac{1}{EI}[-0.833x^3 + 1.67 < x-10 >^3 -1.67 < x-20 >^3 +1.67 < x-30 >^3 +500x - 4167]$

At point C, $x = 20$ ft

$v_C = \frac{833.33}{EI} = \frac{833.33(1728)}{29(10^3)(248)} = 0.200$ in. **Ans**

•12-48 Determine the deflection at each of the pulleys C, D, and E. The shaft is made of steel and has a diameter of 30 mm. The bearings at A and B exert only vertical reactions on the shaft. $E_{st} = 200$ GPa.





$M = -(-180) < x - 0 > -150 < x - 0.25 > -60 < x - 0.5 > -150 < x - 0.75 >$

$M = 180x - 150 < x - 0.25 > -60 < x - 0.5 > -150 < x - 0.75 >$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = 180x - 150 < x - 0.25 > -60 < x - 0.5 > -150 < x - 0.75 >$$

$$EI\frac{dv}{dx} = 90x^2 - 75 < x - 0.25 >^2 -30 < x - 0.50 >^2 -75 < x - 0.75 >^2 + C_1 \quad (1)$$

$$EIv = 30x^3 - 25 < x - 0.25 >^3 -10 < x - 0.50 >^3 -25 < x - 0.75 >^3 + C_1 x + x_2 \quad (2)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

$v = 0$ at $x = 1.0$ m

$0 = 30 - 10.55 - 1.25 - 0.39 + C_1$

$C_1 = -17.8125$

$$\frac{dv}{dx} = \frac{1}{EI}[90x^2 - 75 < x - 0.25 >^2 -30 < x - 0.5 >^2 -75 < x - 0.75 >^2 -17.8125] \quad (3)$$

$$v = \frac{1}{EI}[30x^3 - 25 < x - 0.25 >^3 -10 < x - 0.5 >^3 -25 < x - 0.75 >^3 -17.8125x]$$

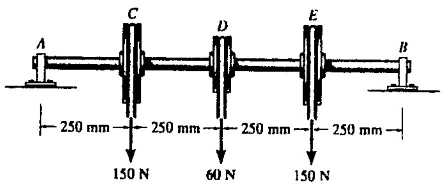
$$v_C = v\Big|_{x=0.25\text{ m}} = \frac{-3.984}{EI} = \frac{-3.984}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.000501\text{ m} = -0.501\text{ mm}$$ **Ans**

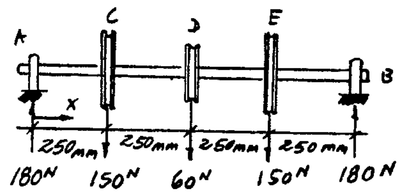
$$v_D = v\Big|_{x=0.5\text{ m}} = \frac{-5.547}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.000698\text{ m} = -0.698\text{ mm}$$ **Ans**

$$v_E = v\Big|_{x=0.75\text{ m}} = \frac{-3.984}{EI} = -0.501\text{ mm}$$ **Ans** (symmetry check !)

The negative signs indicate downward displacement.

12–49 Determine the slope of the shaft at the bearings at A and B. The shaft is made of steel and has a diameter of 30 mm. The bearings at A and B exert only vertical reactions on the shaft. $E_{st} = 200$ GPa.





$$M = -(-180) < x-0 > -150 < x-0.25 > -60 < x-0.5 > -150 < x-0.75 >$$
$$M = 180x - 150 < x-0.25 > -60 < x-0.5 > -150 < x-0.75 >$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = 180x - 150 < x-0.25 > -60 < x-0.5 > -150 < x-0.75 >$$

$$EI\frac{dv}{dx} = 90x^2 - 75 < x-0.25 >^2 -30 < x-0.50 >^2 -75 < x-0.75 >^2 + C_1 \qquad (1)$$

$$EIv = 30x^3 - 25 < x-0.25 >^3 -10 < x-0.50 >^3 -25 < x-0.75 >^3 + C_1 x + x_2 \qquad (2)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

$v = 0$ at $x = 1.0$ m

$0 = 30 - 10.55 - 1.25 - 0.39 + C_1$

$C_1 = -17.8125$

$$\frac{dv}{dx} = \frac{1}{EI}[90x^2 - 75 < x-0.25 >^2 -30 < x-0.5 >^2 -75 < x-0.75 >^2 -17.8125] \qquad (3)$$

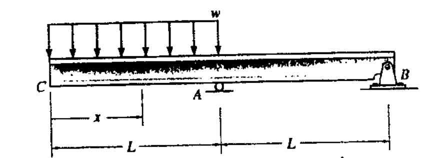
$$\theta_A = \frac{dv}{dx}\bigg|_{x=0} = \frac{-17.8125}{EI} = \frac{-17.8125}{200(10^9)\frac{\pi}{4}(0.015)^4} = -0.00224 \text{ rad} = -0.128° \qquad \textbf{Ans}$$

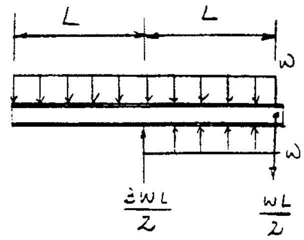
The negative sign indicates clockwise rotation.

$$\theta_B = \frac{dv}{dx}\bigg|_{x=1 \text{ m}} = \frac{17.8125}{EI} = 0.128° \qquad \textbf{Ans}$$

The positive result indicates counterclockwise rotation.

12-50 Determine the equation of the elastic curve. Specify the slope at A. EI is constant.





$$M = -\frac{1}{2}w < x-0 >^2 - (-\frac{3wL}{2}) < x-L > - (-\frac{1}{2}w) < x-L >^2$$

$$= -\frac{1}{2}wx^2 + \frac{3wL}{2} < x-L > + \frac{1}{2}w < x-L >^2$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{1}{2}wx^2 + \frac{3wL}{2} < x-L > + \frac{1}{2}w < x-L >^2$$

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{4} < x-L >^2 + \frac{w}{6} < x-L >^3 + C_1 \qquad (1)$$

$$EI\,v = -\frac{w}{24}x^4 + \frac{wL}{4} < x-L >^3 + \frac{w}{24} < x-L >^4 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

At $x = L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}L^4 + C_1L + C_2 \qquad (3)$$

At $x = 2L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}(2L)^4 + \frac{wL}{4}(2L - L)^3 + \frac{w}{24}(2L - L)^4 + C_1(2L) + C_2$$

$$0 = -\frac{3wL^4}{8} + 2LC_1 + C_2 \qquad (4)$$

Solving Eqs. (3) and (4) yields :

$$C_1 = \frac{wL^3}{3}, \qquad C_2 = -\frac{7wL^4}{24}$$

The elastic curve :

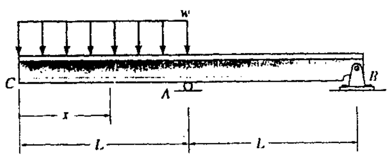
$$v = \frac{1}{EI}[-\frac{w}{24}x^4 + \frac{wL}{4} < x-L >^3 + \frac{w}{24} < x-L >^4 + \frac{wL^3}{3}x - \frac{7wL^4}{24}] \qquad \textbf{Ans}$$

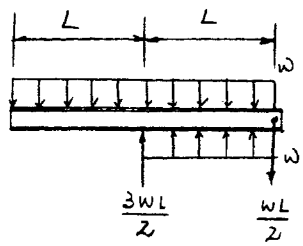
At $x = L$, from Eq. (1),

$$EI\frac{dv}{dx} = -\frac{w}{6}L^3 + 0 + 0 + \frac{wL^3}{3}$$

$$\theta_A = \frac{wL^3}{6EI} \qquad \textbf{Ans}$$

12–51 Determine the equation of the elastic curve. Specify the deflection at *C*. *EI* is constant.





$$M = -\frac{1}{2}w<x-0>^2 - (-\frac{3wL}{2})<x-L> - (-\frac{1}{2}w)<x-L>^2$$

$$= -\frac{1}{2}wx^2 + \frac{3wL}{2}<x-L> + \frac{1}{2}w<x-L>^2$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{1}{2}wx^2 + \frac{3wL}{2}<x-L> + \frac{1}{2}w<x-L>^2$$

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{4}<x-L>^2 + \frac{w}{6}<x-L>^3 + C_1 \qquad (1)$$

$$EI\,v = -\frac{w}{24}x^4 + \frac{wL}{4}<x-L>^3 + \frac{w}{24}<x-L>^4 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

At $x = L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}L^4 + C_1L + C_2 \qquad (3)$$

At $x = 2L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}(2L)^4 + \frac{wL}{4}(2L - L)^3 + \frac{w}{24}(2L - L)^4 + C_1(2L) + C_2$$

$$0 = -\frac{3wL^4}{8} + 2LC_1 + C_2 \qquad (4)$$

Solving Eqs. (3) and (4) yields :

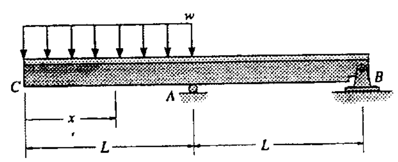
$$C_1 = \frac{wL^3}{3}, \qquad C_2 = -\frac{7wL^4}{24}$$

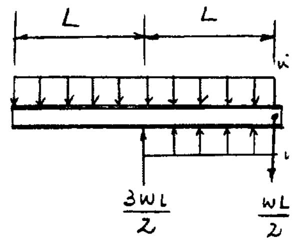
$$v = \frac{1}{EI}[-\frac{w}{24}x^4 + \frac{wL}{4}<x-L>^3 + \frac{w}{24}<x-L>^4 + \frac{wL^3}{3}x - \frac{7wL^4}{24}]$$

At $x = 0$,

$$v_C = -\frac{7wL^4}{24EI} \qquad \textbf{Ans}$$

•12–52 Determine the equation of the elastic curve. Specify the slope at B. EI is constant.





$$M = -\frac{1}{2}w\langle x-0\rangle^2 - (-\frac{3wL}{2})\langle x-L\rangle - (-\frac{1}{2}w)\langle x-L\rangle^2$$

$$= -\frac{1}{2}wx^2 + \frac{3wL}{2}\langle x-L\rangle + \frac{1}{2}w\langle x-L\rangle^2$$

$$EI\frac{d^2v}{dx^2} = M$$

$$EI\frac{d^2v}{dx^2} = -\frac{1}{2}wx^2 + \frac{3wL}{2}\langle x-L\rangle + \frac{1}{2}w\langle x-L\rangle^2$$

$$EI\frac{dv}{dx} = -\frac{w}{6}x^3 + \frac{3wL}{4}\langle x-L\rangle^2 + \frac{w}{6}\langle x-L\rangle^3 + C_1 \qquad (1)$$

$$EI\,v = -\frac{w}{24}x^4 + \frac{wL}{4}\langle x-L\rangle^3 + \frac{w}{24}\langle x-L\rangle^4 + C_1x + C_2 \qquad (2)$$

Boundary conditions :

At $x = L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}L^4 + C_1L + C_2 \qquad (3)$$

At $x = 2L$, $v = 0$

From Eq. (2),

$$0 = -\frac{w}{24}(2L)^4 + \frac{wL}{4}(2L - L)^3 + \frac{w}{24}(2L - L)^4 + C_1(2L) + C_2$$

$$0 = -\frac{3wL^4}{8} + 2LC_1 + C_2 \qquad (4)$$

Solving Eqs. (3) and (4) yields :

$$C_1 = \frac{wL^3}{3}, \qquad C_2 = -\frac{7wL^4}{24}$$

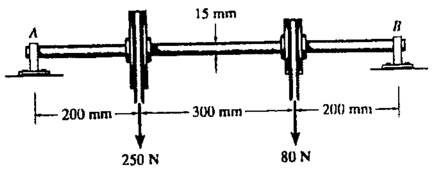
$$v = \frac{1}{EI}[-\frac{w}{24}x^4 + \frac{wL}{4}\langle x-L\rangle^3 + \frac{w}{24}\langle x-L\rangle^4 + \frac{wL^3}{3}x - \frac{7wL^4}{24}]$$

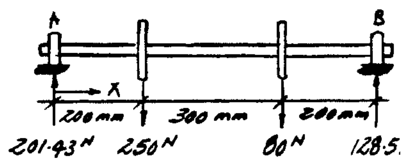
From Eq. (1), at $x = 2L$,

$$EI\frac{dv}{dx} = -\frac{w}{6}(2L)^3 + \frac{3wL}{4}(L)^2 + \frac{w}{6}(L)^3 + \frac{w}{3}(L^3)$$

$$\theta_B = -\frac{wL^3}{12EI} \qquad \textbf{Ans}$$

12-53 The shaft is made of steel and has a diameter of 15 mm. Determine its maximum deflection. The bearings at A and B exert only vertical reactions on the shaft. $E_{st} = 200$ GPa.





$M = -(-201.43) < x-0 > -250 < x-0.2 > -80 < x-0.5 >$
$M = 201.43x - 250 < x-0.2 > -80 < x-0.5 >$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M = 201.43x - 250 < x-0.2 > -80 < x-0.5 >$$
$$EI\frac{dv}{dx} = 100.72x^2 - 125 < x-0.2 >^2 -40 < x-0.5 >^2 + C_1$$
$$EIv = 33.72x^3 - 41.67 < x-0.2 >^3 -13.33 < x-0.5 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions :

$v = 0$ at $x = 0$

From Eq. (1)

$C_2 = 0$

$v = 0$ at $x = 0.7$ m

$0 = 11.515 - 5.2083 - 0.1067 + 0.7C_1$

$C_1 = -8.857$

$$\frac{dv}{dx} = \frac{1}{EI}[100.72x^2 - 125 < x-0.2 >^2 -40 < x-0.5 >^2 -8.857]$$

Assume v_{max} occurs at $0.2 \text{ m} < x < 0.5 \text{ m}$

$$\frac{dv}{dx} = 0 = \frac{1}{EI}[100.72x^2 - 125(x-0.2)^2 - 8.857]$$

$24.28x^2 - 50x + 13.857 = 0$

$x = 0.3300$ m OK

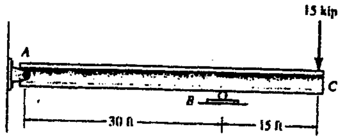
$$v = \frac{1}{EI}[33.57x^3 - 41.67 < x-0.2 >^3 -13.33 < x-0.5 >^3 -8.857x]$$

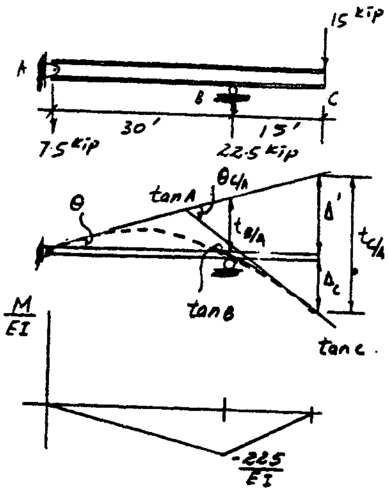
Substitute $x = 0.3300$ m into the elastic curve :

$$v_{max} = -\frac{1.808 \text{ N}\cdot\text{m}^3}{EI} = -\frac{1.808}{200(10^9)\frac{\pi}{4}(0.0075)^4} = -0.00364 = -3.64 \text{ mm} \qquad \text{Ans}$$

The negative sign indicates downward displacement.

12-54. Determine the slope and deflection at C. EI is constant.





$$\theta_A = \frac{|t_{B/A}|}{30}$$

$$t_{B/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30)(10) = \frac{-33\,750}{EI}$$

$$\theta_A = \frac{1125}{EI}$$

$$\theta_{C/A} = \frac{1}{2}\left(\frac{-225}{EI}\right)(30) + \frac{1}{2}\left(\frac{-225}{EI}\right)(15) = \frac{-5062.5}{EI} = \frac{5062.5}{EI}$$

$$\theta_C = \theta_{C/A} + \theta_A$$

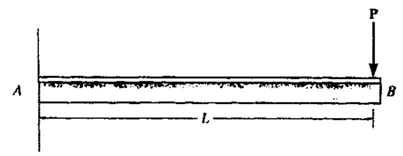
$$\theta_C = \frac{5062.5}{EI} - \frac{1125}{EI} = \frac{3937.5}{EI} \qquad \textbf{Ans}$$

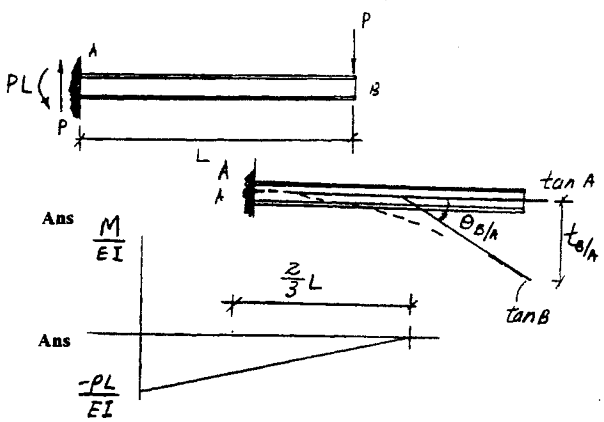
$$\Delta_C = |t_{C/A}| - \frac{45}{30}|t_{B/A}|$$

$$t_{C/A} = \frac{1}{2}\left(-\frac{225}{EI}\right)(30)(25) + \frac{1}{2}\left(-\frac{225}{EI}\right)(15)(10) = -\frac{101\,250}{EI}$$

$$\Delta_C = \frac{101\,250}{EI} - \frac{45}{30}\left(\frac{33\,750}{EI}\right) = \frac{50\,625}{EI} \qquad \textbf{Ans}$$

12–55 Determine the slope and deflection at B. EI is constant.





$$\theta_{B/A} = \frac{1}{2}\left(\frac{-PL}{EI}\right)(L) = \frac{-PL^2}{2EI} = \frac{PL^2}{2EI}$$

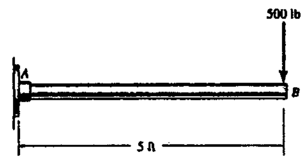
$$\theta_B = \theta_{B/A} = \theta_A$$

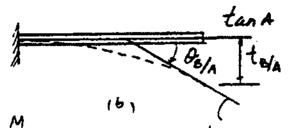
$$\theta_B = \frac{PL^2}{2EI} + 0 = \frac{PL^2}{2EI}$$ **Ans**

$$\Delta_B = |t_{B/A}| = \frac{1}{2}\left(\frac{-P\,L}{EI}\right)(L)\left(\frac{2}{3}L\right)$$

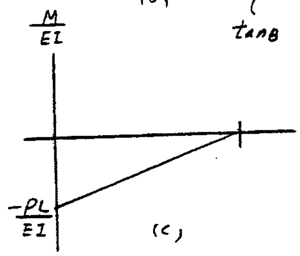
$$= \frac{PL^3}{3EI}$$ **Ans**

***12-56.** Determine the slope and deflection at B if the A-36 steel beam is (a) a solid rod having a diameter of 3 in., (b) a tube having an outer diameter of 3 in. and thickness of 0.25 in.

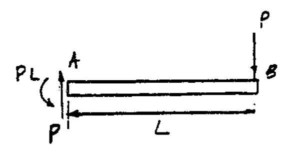




(b)



(c)



$$\theta_{B/A} = \frac{1}{2}(\frac{-PL}{EI})(L) = \frac{-PL^2}{2EI} = \frac{PL^2}{2EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

$$\theta_B = \frac{PL^2}{2EI} + 0 = \frac{PL^2}{2EI}$$

$$\Delta_B = |t_{B/A}| = \frac{1}{2}(\frac{-PL}{EI})(L)(\frac{2L}{3}) = \frac{PL^3}{3EI}$$

a) Numerical substitution :

$P = 500$ lb $\qquad L = 5(12) = 60$ in.

$I = \frac{\pi}{4}(1.5^4) = 3.9761$ in^4

$$\theta_B = \frac{500(60)^2}{2(29)(10^6)(3.9761)} = 0.00781 \text{ rad} \qquad \textbf{Ans}$$

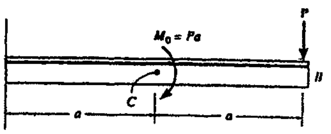
$$\Delta_B = \frac{500(60^3)}{3(29)(10^6)(3.9761)} = 0.312 \text{ in.} \qquad \textbf{Ans}$$

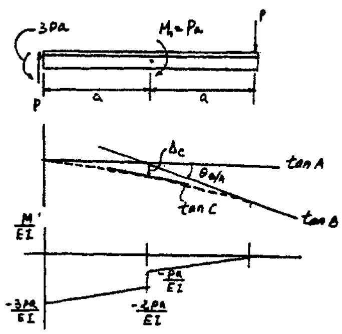
b) $I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586$ in^4

$$\theta_B = \frac{500(60)^2}{2(29)(10^6)(2.0586)} = 0.0151 \text{ rad} \qquad \textbf{Ans}$$

$$\Delta_B = \frac{500(60^3)}{3(29)(10^6)(2.0586)} = 0.603 \text{ in.} \qquad \textbf{Ans}$$

12-57. Determine the slope at B and the deflection at C. EI is constant.





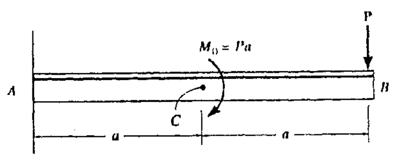
$$\theta_B = \theta_{B/A} = \frac{1}{2}(\frac{-Pa}{EI})(a) + \frac{1}{2}[-\frac{3Pa}{EI} - \frac{2Pa}{EI}](a)$$

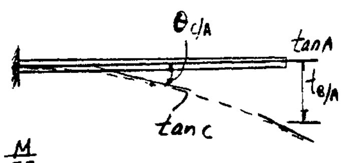
$$= \frac{3Pa^2}{EI} \qquad \textbf{Ans}$$

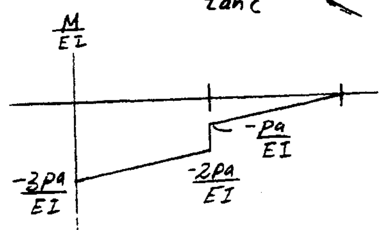
$$\Delta_C = \frac{1}{2}(a)(\frac{-2Pa}{EI})(a) + \frac{2}{3}(a)[(\frac{1}{2})\frac{-Pa}{EI}](a)$$

$$= \frac{4Pa^3}{3EI} \qquad \textbf{Ans}$$

12-58 Determine the slope at C and the deflection at B. EI is constant.







$$\theta_{C/A} = \left(-\frac{2Pa}{EI}\right)a + \frac{1}{2}\left(-\frac{Pa}{EI}\right)a = -\frac{5Pa^2}{2EI} = \frac{5Pa^2}{2EI}$$

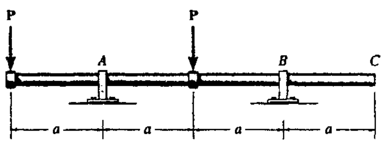
$$\theta_C = \theta_{C/A}$$

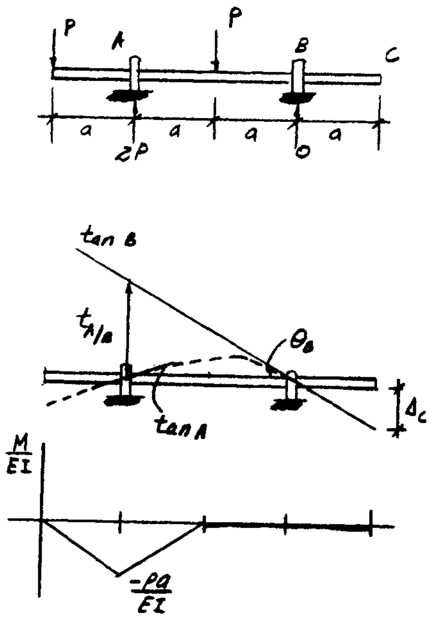
$$\theta_C = \frac{5Pa^2}{2EI} \qquad \textbf{Ans}$$

$$\Delta_B = |t_{B/A}| = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right) a \left(a + \frac{2a}{3}\right) + \left(-\frac{2Pa}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

$$= \frac{25Pa^3}{6EI} \qquad \textbf{Ans}$$

12–59 If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at B and the deflection at C. EI is constant.



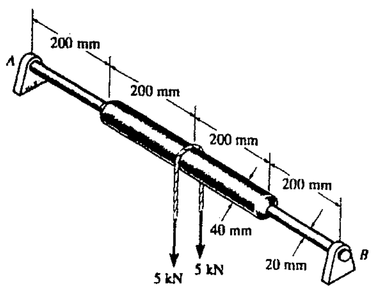


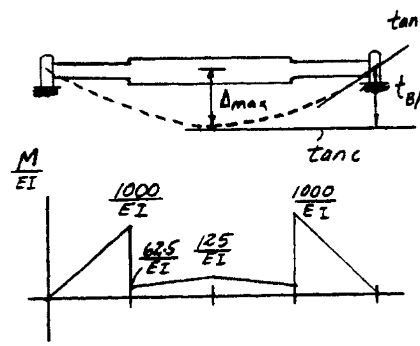
$$t_{A/B} = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{a}{3}\right) = -\frac{Pa^3}{6EI}$$

$$\theta_B = \frac{|t_{A/B}|}{2a} = \frac{Pa^3/6EI}{2a} = \frac{Pa^2}{12EI} \qquad \textbf{Ans}$$

$$\Delta_C = \theta_B a = \frac{Pa^2}{12EI}(a) = \frac{Pa^3}{12EI} \qquad \textbf{Ans}$$

*12–60 The composite simply-supported steel shaft is subjected to a force of 10 kN at its center. Determine its maximum deflection. $E_{st} = 200$ GPa.

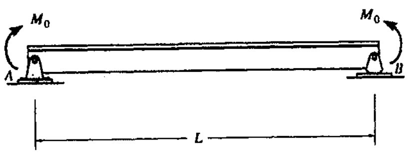


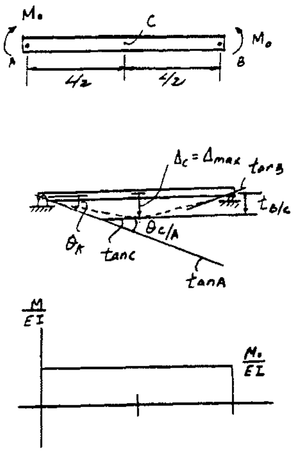


$$\Delta_{max} = |t_{B/C}| = \frac{62.5}{EI}(0.2)(0.3) + \frac{1}{2}(\frac{62.5}{EI})(0.2)(0.3333) + \frac{1}{2}(\frac{1000}{EI})(0.2)(0.1333)$$

$$= \frac{19.167}{EI} = \frac{19.167}{200(10^9)(7.8540)(10^{-9})} = 0.0122 \text{ m} = 12.2 \text{ mm} \qquad \textbf{Ans}$$

12-61 Determine the maximum slope and the maximum deflection of the beam. EI is constant.





$$\theta_{C/A} = \frac{M_0}{EI}\left(\frac{L}{2}\right) = \frac{M_0 L}{2EI}$$

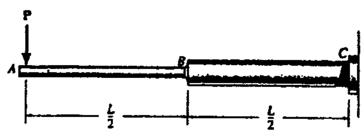
$$\theta_C = \theta_{C/A} + \theta_A$$

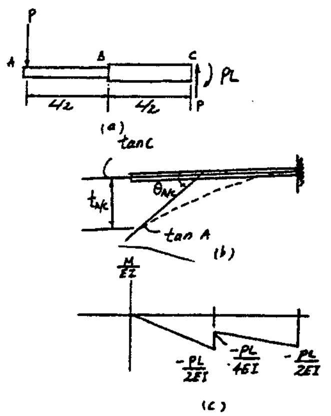
$$0 = \frac{M_0 L}{2EI} + \theta_A$$

$$\theta_{max} = \theta_A = \frac{-M_0 L}{2EI} = \frac{M_0 L}{2EI} \qquad \textbf{Ans}$$

$$\Delta_{max} = |t_{B/C}| = \frac{M_0}{EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{M_0 L^2}{8EI} \qquad \textbf{Ans}$$

12-62. The rod is constructed from two shafts for which the moment of inertia of AB is I and of BC is $2I$. Determine the maximum slope and deflection of the rod due to the loading. The modulus of elasticity is E.





$$\theta_{A/C} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{-5PL^2}{16EI} = \frac{5PL^2}{16EI}$$

$$\theta_A = \theta_{A/C} + \theta_C$$

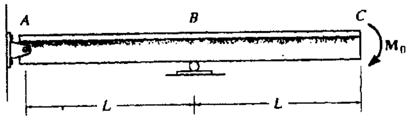
$$\theta_{max} = \theta_A = \frac{5PL^2}{16EI} + 0 = \frac{5PL^2}{16EI} \qquad \textbf{Ans}$$

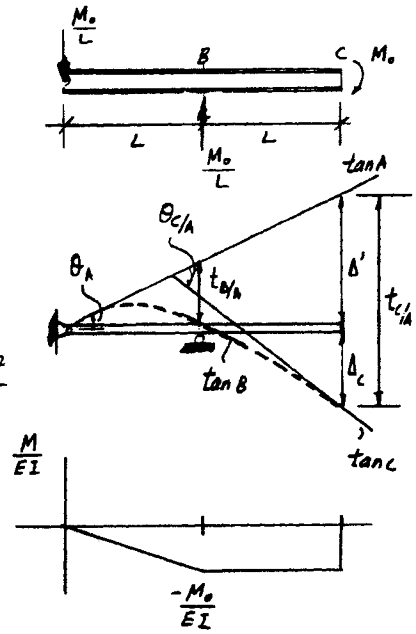
$$\Delta_{max} = \Delta_A = |t_{A/C}|$$

$$= \left|\frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{3}\right) + \left(\frac{-PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2} + \frac{L}{4}\right)\right|$$

$$= \frac{3PL^3}{16EI} \qquad \textbf{Ans}$$

12–63 Determine the deflection and slope at C. EI is constant.





$$t_{B/A} = \frac{1}{2}\left(\frac{-M_0}{EI}\right)(L)\left(\frac{1}{3}\right)(L) = -\frac{M_0 L^2}{6EI}$$

$$\Delta_C = |t_{C/A}| - 2|t_{B/A}|$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-M_0}{EI}\right)(L)\left(L + \frac{L}{3}\right) + \left(\frac{-M_0}{EI}\right)(L)\left(\frac{L}{2}\right) = -\frac{7M_0 L^2}{6EI}$$

$$\Delta_C = \frac{7M_0 L^2}{6EI} - (2)\left(\frac{M_0 L^2}{6EI}\right) = \frac{5M_0 L^2}{6EI} \qquad \textbf{Ans}$$

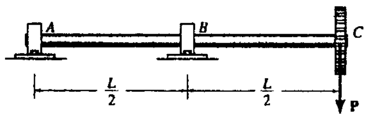
$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{M_0 L}{6EI}$$

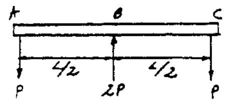
$$\theta_{C/A} = \frac{1}{2}\left(-\frac{M_0}{EI}\right)(L) + \left(-\frac{M_0}{EI}\right)(L) = -\frac{3M_0 L}{2EI} = \frac{3M_0 L}{2EI}$$

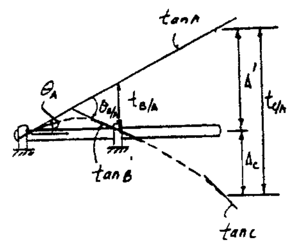
$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = \frac{3M_0 L}{2EI} - \frac{M_0 L}{6EI} = \frac{4M_0 L}{3EI} \qquad \textbf{Ans}$$

*12–64 The shaft supports the pulley at its end C. Determine the deflection at C and the slopes at the bearings A and B. EI is constant.





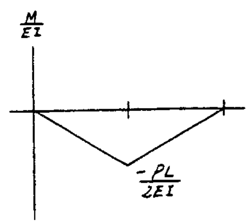


$$t_{B/A} = \frac{1}{2}(\frac{-PL}{2EI})(\frac{L}{2})(\frac{L}{6}) = \frac{-PL^3}{48EI}$$

$$t_{C/A} = \frac{1}{2}(\frac{-PL}{2EI})(L)(\frac{L}{2}) = \frac{-PL^3}{8EI}$$

$$\Delta_C = |t_{C/A}| - (\frac{L}{\frac{L}{2}})|t_{B/A}|$$

$$= \frac{PL^3}{8EI} - 2(\frac{PL^3}{48EI}) = \frac{PL^3}{12EI} \qquad \textbf{Ans}$$



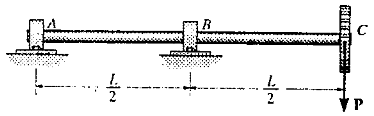
$$\theta_A = \frac{|t_{B/A}|}{\frac{L}{2}} = \frac{\frac{PL^3}{48EI}}{\frac{L}{2}} = \frac{PL^2}{24EI} \qquad \textbf{Ans}$$

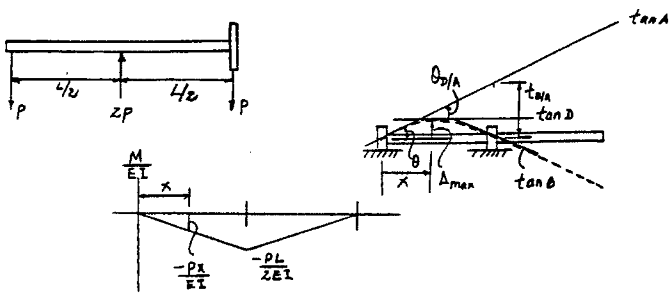
$$\theta_{B/A} = \frac{1}{2}(\frac{-PL}{2EI})(\frac{L}{2}) = \frac{-PL^2}{8EI} = \frac{PL^2}{8EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

$$\theta_B = \frac{PL^2}{8EI} - \frac{PL^2}{24EI} = \frac{PL^2}{12EI} \qquad \textbf{Ans}$$

12–65 The shaft supports the pulley at its end C. Determine its maximum deflection within region AB. EI is constant. The bearings exert only vertical reactions on the shaft.





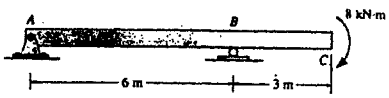
$$\theta_{D/A} = \frac{t_{B/A}}{(\frac{L}{2})}$$

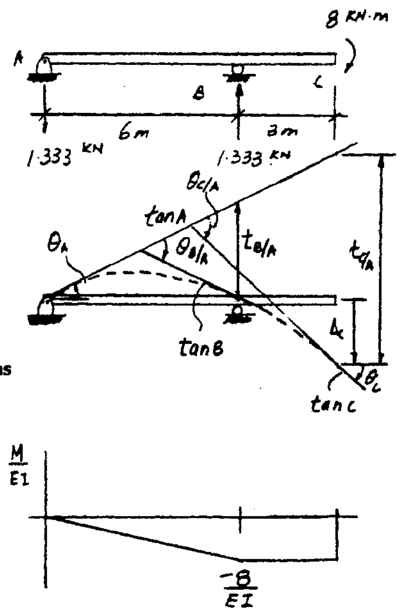
$$\frac{1}{2}(\frac{Px}{EI})x = \frac{\frac{1}{2}(\frac{L}{2})(\frac{PL}{2EI})(\frac{1}{3})(\frac{L}{2})}{(\frac{L}{2})}; \qquad x = 0.288675\,L$$

$$\Delta_{max} = \frac{1}{2}(\frac{P(0.288675\,L)}{EI})(0.288675\,L)(\frac{2}{3})(0.288675\,L)$$

$$\Delta_{max} = \frac{0.00802PL^3}{EI}$$ **Ans**

12-66. Determine the deflection at C and the slope of the beam at A, B, and C. EI is constant.





$$t_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(2) = \frac{-48}{EI}$$

$$t_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6)(3+2) + \left(\frac{-8}{EI}\right)(3)(1.5) = \frac{-156}{EI}$$

$$\Delta_C = |t_{C/A}| - \frac{9}{6}|t_{B/A}| = \frac{156}{EI} - \frac{9(48)}{6(EI)} = \frac{84}{EI}$$ **Ans**

$$\theta_A = \frac{|t_{B/A}|}{6} = \frac{8}{EI}$$ **Ans**

$$\theta_{B/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) = \frac{-24}{EI} = \frac{24}{EI}$$

$$\theta_B = \theta_{B/A} + \theta_A$$

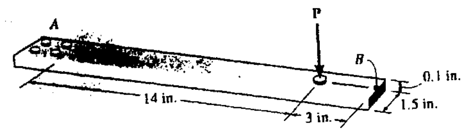
$$\theta_B = \frac{24}{EI} - \frac{8}{EI} = \frac{16}{EI}$$ **Ans**

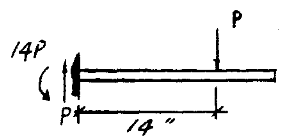
$$\theta_{C/A} = \frac{1}{2}\left(\frac{-8}{EI}\right)(6) + \left(\frac{-8}{EI}\right)(3) = \frac{-48}{EI} = \frac{48}{EI}$$

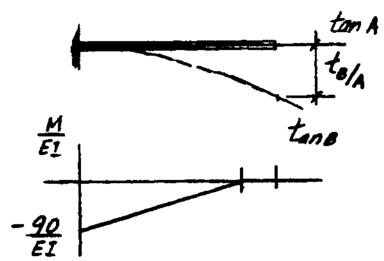
$$\theta_C = \theta_{C/A} + \theta_A$$

$$\theta_C = \frac{48}{EI} - \frac{8}{EI} = \frac{40}{EI}$$ **Ans**

12–67 The flat spring is made of A-36 steel and has a rectangular cross section as shown. Determine the maximum elastic load P that can be applied. What is the deflection at B when P reaches its maximum value? Assume that the spring is fixed supported at A.







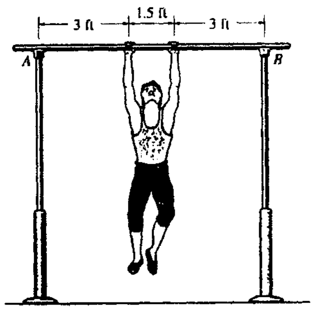
$$I = \frac{1}{12}(1.5)(0.1)^3 = 0.125(10^{-3})\text{ in}^4$$

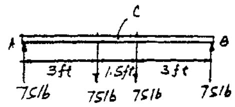
$$\sigma_y = \frac{Mc}{I}; \qquad 36(10^3) = \frac{14P(0.05)}{0.125(10^{-3})} \qquad P = 6.43\text{ lb} \qquad \textbf{Ans}$$

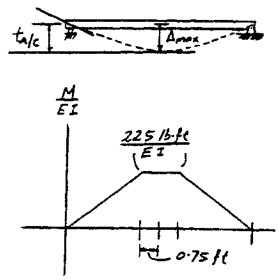
$$\Delta_B = t_{B/A} = \frac{1}{2}\left(\frac{-90}{EI}\right)(14)(9.333+3)$$

$$= \frac{-7770}{EI} = \frac{-7770}{29(10^6)(0.125)(10^{-3})} = 2.14\text{ in.} \qquad \textbf{Ans}$$

*12–68 The acrobat has a weight of 150 lb, and suspends himself uniformly from the center of the high bar. Determine the maximum bending stress in the pipe (bar) and its maximum deflection. The pipe is made of L2 steel and has an outer diameter of 1 in. and a wall thickness of 0.125 in.







$$M_{max} = 75(3) = 225 \text{ lb} \cdot \text{ft}$$

$$I = \frac{\pi}{4}(0.5^4 - 0.375^4) = 0.033556 \text{ in}^4$$

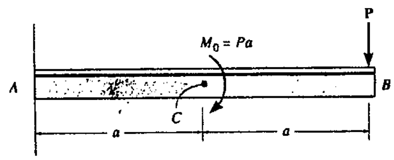
$$\sigma_{max} = \frac{Mc}{I} = \frac{225(12)(0.5)}{0.033556} = 40.2 \text{ ksi} \qquad \textbf{Ans}$$

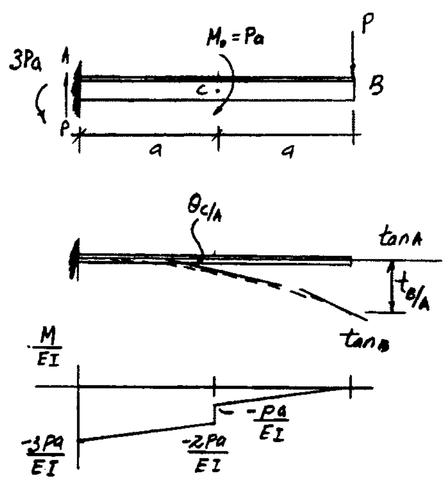
$$40.2 \text{ ksi} < \sigma_Y = 102 \text{ ksi} \qquad \text{OK}$$

$$\Delta_{max} = t_{A/C} = \left(\frac{225}{EI}\right)(0.75)(3.375) + \frac{1}{2}\left(\frac{225}{EI}\right)(3)(2) = \frac{1244.53 \text{ lb} \cdot \text{ft}^3}{EI}$$

$$\Delta_{max} = \frac{1244.53(12^3)}{29(10^6)(0.033556)} = 2.21 \text{ in.} \qquad \textbf{Ans}$$

12–69 Determine the slope at C and the deflection at B. EI is constant.





$$\theta_{C/A} = \left(-\frac{2Pa}{EI}\right)(a) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)$$

$$= -\frac{5Pa^2}{2EI} = \frac{5Pa^2}{2EI} \circlearrowright$$

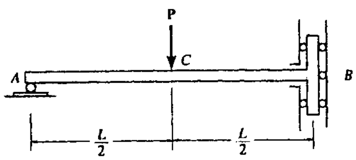
$$\theta_C = \theta_{C/A}$$

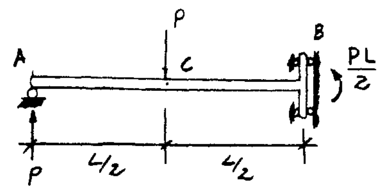
$$\circlearrowleft + \theta_C = +\frac{5Pa^2}{2EI} \triangleleft$$

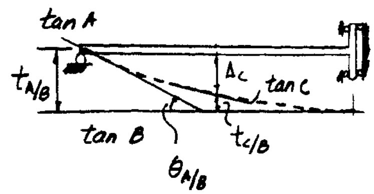
$$\Delta_B = |t_{B/A}| = \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(-\frac{Pa}{EI}\right)(a)\left(a + \frac{2a}{3}\right) + \left(-\frac{2Pa}{EI}\right)(a)\left(a + \frac{a}{2}\right)$$

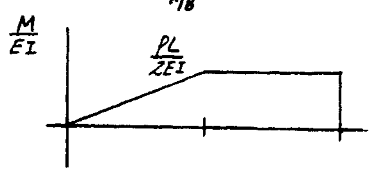
$$= \frac{25Pa^3}{6EI} \downarrow$$ **Ans**

12-70 The bar is supported by a roller constraint at B, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at A and the deflection at C. EI is constant.









$$\theta_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right) = \frac{3PL^2}{8EI}$$

$$\theta_A = \theta_{A/B}$$

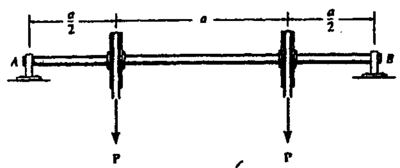
$$\theta_A = \frac{3PL^2}{8EI} \qquad \textbf{Ans}$$

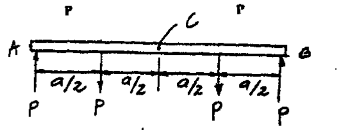
$$t_{A/B} = \frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right) + \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{4}\right) = \frac{11PL^3}{48EI}$$

$$t_{C/B} = \frac{PL}{2EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) = \frac{PL^3}{16EI}$$

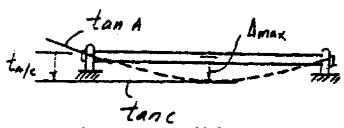
$$\Delta_C = t_{A/B} - t_{C/B} = \frac{11PL^3}{48EI} - \frac{PL^3}{16EI} = \frac{PL^3}{6EI} \qquad \textbf{Ans}$$

12-71. Determine the maximum deflection of the shaft. *EI* is constant. The bearings exert only vertical reactions on the shaft.

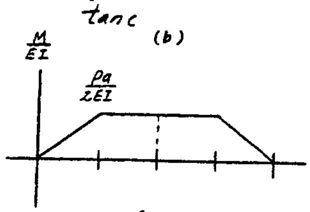




(a)



(b)



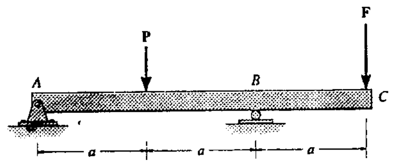
(c)

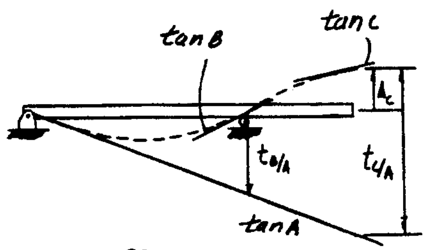
$$\Delta_{max} = t_{A/C}$$

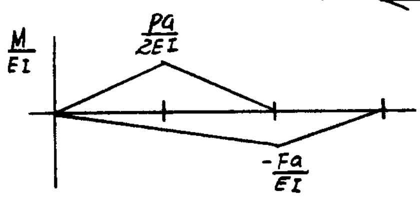
$$= \left(\frac{Pa}{2EI}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2} + \frac{a}{4}\right) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)\left(\frac{a}{2}\right)\left(\frac{a}{3}\right)$$

$$= \frac{11Pa^3}{48EI} \qquad \textbf{Ans}$$

•12–72 The beam is subjected to the load **P** as shown. Determine the magnitude of force **F** that must be applied at the end of the overhang C so that the deflection at C is zero. EI is constant.







$$t_{B/A} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)(a) + \frac{1}{2}\left(-\frac{Fa}{EI}\right)(2a)\left(\frac{2}{3}a\right) = \frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}$$

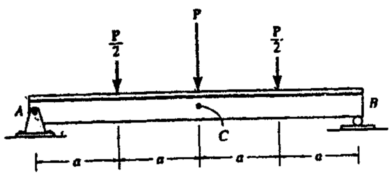
$$t_{C/A} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(2a)(2a) + \frac{1}{2}\left(\frac{-Fa}{EI}\right)(2a)\left(a + \frac{2a}{3}\right) + \frac{1}{2}\left(\frac{-Fa}{EI}\right)(a)\left(\frac{2a}{3}\right) = \frac{Pa^3}{EI} - \frac{2Fa^3}{EI}$$

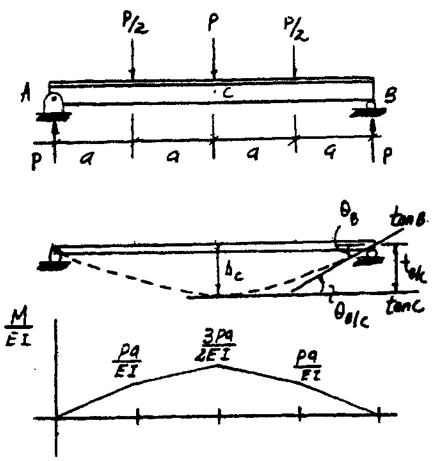
$$\Delta_C = t_{C/A} - \frac{3}{2}t_{B/A} = 0$$

$$\frac{Pa^3}{EI} - \frac{2Fa^3}{EI} - \frac{3}{2}\left(\frac{Pa^3}{2EI} - \frac{2Fa^3}{3EI}\right) = 0$$

$$F = \frac{P}{4} \quad \textbf{Ans}$$

12-73. Determine the slope at B and deflection at C. EI is constant.



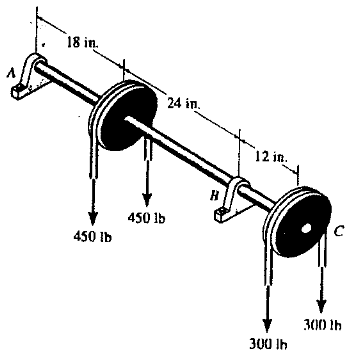


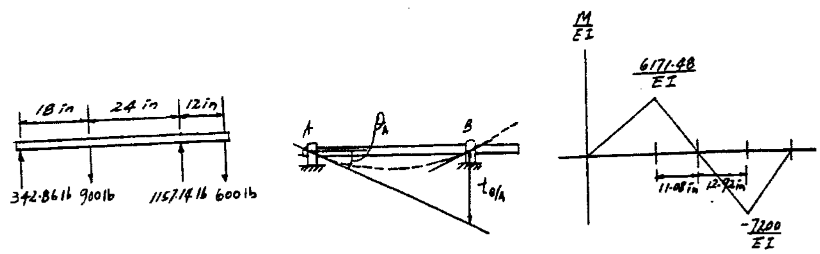
$$\theta_{B/C} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) + \left(\frac{Pa}{EI}\right)(a) = \frac{7Pa^2}{4EI}$$

$$\theta_B = \theta_{B/C} = \frac{7Pa^2}{4EI} \qquad \textbf{Ans}$$

$$\Delta_C = |t_{B/C}| = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(\frac{2a}{3}\right) + \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a)\left(a + \frac{2a}{3}\right) + \left(\frac{Pa}{EI}\right)(a)\left(a + \frac{a}{2}\right) = \frac{9Pa^3}{4EI} \qquad \textbf{Ans}$$

12-74 The A-36 steel shaft is subjected to the loadings developed in the belts passing over the two pulleys. If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at A. The shaft has a diameter of 0.75 in.





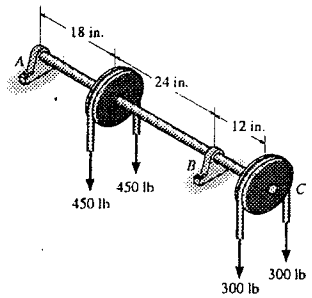
$$t_{B/A} = \frac{1}{2}\left(\frac{6171.48}{EI}\right)(18)(30) + \frac{1}{2}\left(\frac{6171.48}{EI}\right)(11.08)(20.31)$$

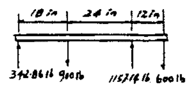
$$+ \frac{1}{2}\left(\frac{-7200}{EI}\right)(12.92)(4.31) = \frac{2160231.8}{EI}$$

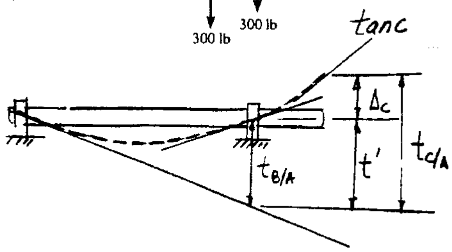
$$\theta_A = \frac{|t_{B/A}|}{42} = \frac{51434.1}{EI} = \frac{51434.1}{29(10^6)(\frac{\pi}{4})(0.375)^4}$$

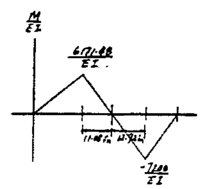
$$= 0.114 \text{ rad} = 6.54° \qquad \textbf{Ans}$$

12–75 The A-36 steel shaft is subjected to the loadings developed in the belts passing over the two pulleys. If the bearings at A and B exert only vertical reactions on the shaft, determine the deflection at C. The shaft has a diameter of 0.75 in.









$$t_{B/A} = \frac{1}{2}\left(\frac{6171.48}{EI}\right)(18)(30) + \frac{1}{2}\left(\frac{6171.48}{EI}\right)(11.08)(20.31)$$

$$+ \frac{1}{2}\left(\frac{-7200}{EI}\right)(12.92)(4.31) = \frac{2160231.8}{EI}$$

$$t_B' = (t_{B/A})\left(\frac{54}{42}\right) = \frac{2777441}{EI}$$

$$t_{C/A} = \frac{1}{2}\left(\frac{6171.48}{EI}\right)(18)(42) + \frac{1}{2}\left(\frac{6171.48}{EI}\right)(11.08)(32.31)$$

$$+ \frac{1}{2}\left(\frac{-7200}{EI}\right)(12.92)(16.31) + \frac{1}{2}\left(\frac{-7200}{EI}\right)(12)(8) = \frac{2333287.6}{EI}$$

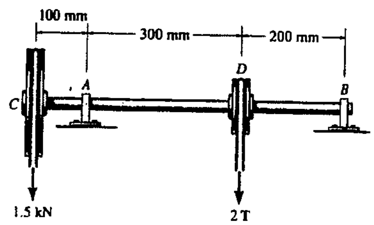
$$\Delta_C = t_{C/A} - t_B' = \frac{-444,153}{29(10^6)(\frac{\pi}{4})(0.375)^4}$$

$$\Delta_C = 0.987 \text{ in.} \qquad \textbf{Ans}$$

•12–76 The 25-mm-diameter A-36 steel shaft is supported at A and B by bearings. If the tension in the belt on the pulley at C is 0.75 kN, determine the largest belt tension T on the pulley at D so that the slope of the shaft at A or B does not exceed 0.02 rad. The bearings exert only vertical reactions on the shaft.

$$t_{B/A} = \frac{1}{2}\left(\frac{0.24T}{EI}\right)(0.2)(0.1333) + \frac{1}{2}\left(\frac{0.24T}{EI}\right)(0.3)(0.2 + 0.1)$$

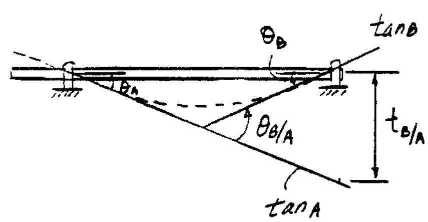
$$+ \frac{1}{2}\left(\frac{-150}{EI}\right)(0.5)(0.3333) = \frac{0.014\,T}{EI} - \frac{12.5}{EI}$$



$$\theta_A = \frac{|t_{B/A}|}{0.5} = \frac{0.028\,T}{EI} - \frac{25}{EI} \qquad (1)$$

$$\theta_{B/A} = \frac{1}{2}\left(\frac{0.24\,T}{EI}\right)(0.2) + \frac{1}{2}\left(\frac{0.24T}{EI}\right)(0.3)$$

$$+ \frac{1}{2}\left(\frac{-150}{EI}\right)(0.5) = \frac{0.06\,T}{EI} - \frac{37.5}{EI}$$

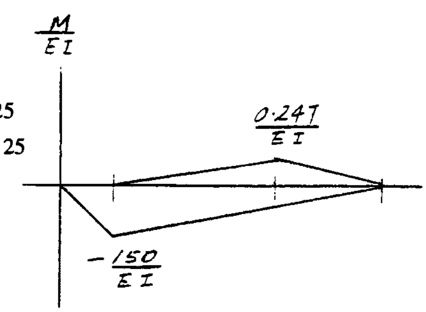


$$\theta_B = \theta_{B/A} + \theta_A$$

$$\theta_B = \frac{0.06\,T}{EI} - \frac{37.5}{EI} - \left(\frac{0.028\,T}{EI} - \frac{25}{EI}\right)$$

$$= \frac{0.032\,T}{EI} - \frac{12.5}{EI} \qquad (2)$$

For $\theta_A = 0.02$ rad



From Eq. (1) :

$$0.02(EI) = 0.028\,T - 25$$

$$0.02(200)(10^9)(19.175)(10^{-9}) = 0.028\,T - 25$$

$T = 3632$ N

For $\theta_B = 0.02$ rad

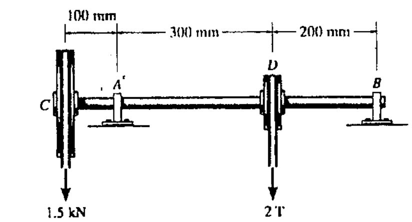
From Eq. (2) :

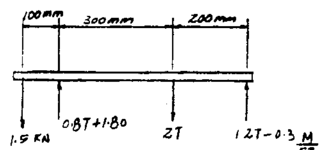
$$0.02(E\,I) = 0.032\,T - 12.5$$

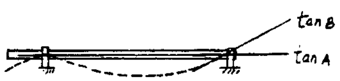
$$0.02(200)(10^9)(19.175)(10^{-9}) = 0.032\,T - 12.5;$$

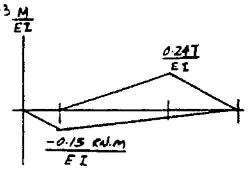
$T = 2787\text{ N} = 2.79\text{ kN}$ controls **Ans**

12-77 The 25-mm-diameter A-36 steel shaft is supported at A and B by bearings. If the tension in the belt on the pulley at C is 0.75 kN, determine the largest belt tension T on the pulley at D so that the slope of the shaft at A is zero. The bearings exert only vertical reactions on the shaft.







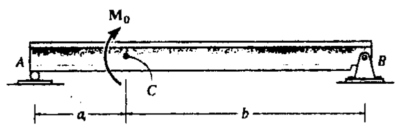


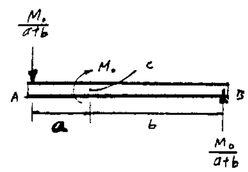
Require $t_{B/A} = 0$.

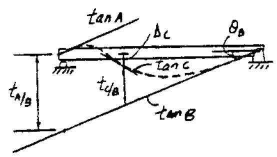
$$t_{B/A} = \frac{1}{2}\left(\frac{-1500}{EI}\right)(0.5)\left(\frac{2}{3}\right)(0.5) + \frac{1}{2}\left(\frac{0.24\,T}{EI}\right)(0.3)(0.3) + \frac{1}{2}\left(\frac{0.24\,T}{EI}\right)(0.2)\left(\frac{2}{3}\right)(0.2) = 0$$

$T = 8928.6\text{ N} = 8.93\text{ kN}$ **Ans**

12–78 The beam is subjected to the loading shown. Determine the slope at B and deflection at C. EI is constant.





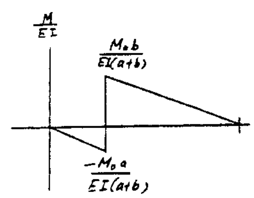


The slope :

$$t_{A/B} = \frac{1}{2}[\frac{-M_0 a}{EI(a+b)})(a)(\frac{2}{3}a)$$

$$+ \frac{1}{2}[\frac{M_0 b}{EI(a+b)})(b)(a + \frac{b}{3})$$

$$= \frac{M_0(b^3 + 3ab^2 - 2a^3)}{6EI(a + b)}$$



$$\theta_B = \frac{t_{A/B}}{a + b} = \frac{M_0(b^3 + 3ab^2 - 2a^3)}{6EI(a + b)^2} \qquad \textbf{Ans}$$

The deflection :

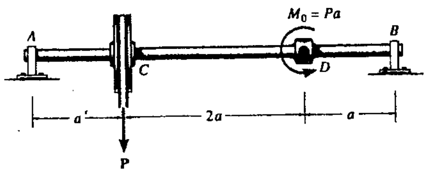
$$t_{C/B} = \frac{1}{2}[\frac{M_0 b}{EI(a+b)})(b)(\frac{b}{3}) = \frac{M_0 b^3}{6EI(a + b)}$$

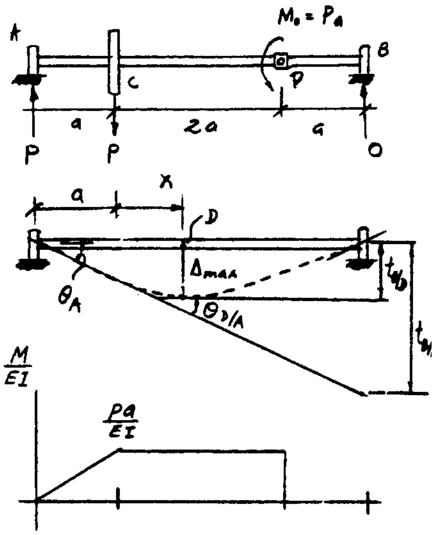
$$\Delta_C = (\frac{b}{a+b})\, t_{A/B} - t_{C/B}$$

$$= \frac{M_0 b\,(b^3 + 3ab^2 - 2a^3)}{6EI(a + b)^2} - \frac{M_0 b^3}{6EI(a + b)}$$

$$= \frac{M_0 a\, b(b-a)}{3EI(a + b)} \qquad \textbf{Ans}$$

12–79 If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at A and the maximum deflection.





$$t_{B/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a)\left(3a + \frac{a}{3}\right) + \left(\frac{Pa}{EI}\right)(2a)(a + a) = \frac{17Pa^3}{3EI}$$

$$\theta_A = \frac{|t_{B/A}|}{4a} = \frac{17Pa^2}{12EI} \qquad \textbf{Ans}$$

Assume Δ_{max} is at point D located at $0 < x < 2a$

$$\theta_{D/A} = \frac{1}{2}\left(\frac{Pa}{EI}\right)(a) + \left(\frac{Pa}{EI}\right)(x) = \frac{Pa^2}{2EI} + \frac{Pax}{EI}$$

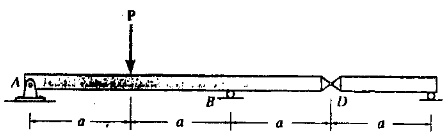
$$\theta_D = 0 = \theta_{D/A} + \theta_A$$

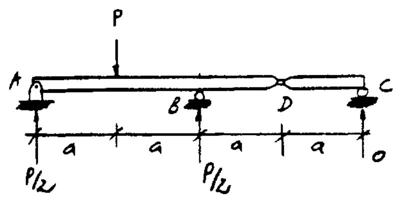
$$0 = \frac{Pa^2}{2EI} + \frac{Pax}{EI} + \left(\frac{-17Pa^2}{12EI}\right)$$

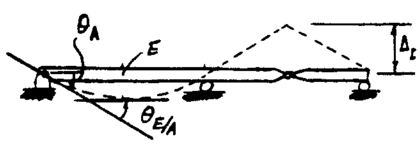
$$x = \frac{11}{12}a$$

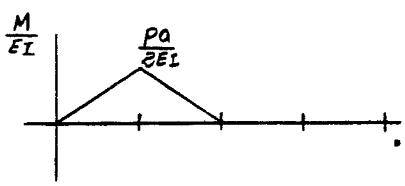
$$\Delta_{max} = |t_{B/D}| = \left(\frac{Pa}{EI}\right)\left(2a - \frac{11}{12}a\right)\left[\frac{(2a - \frac{11}{12}a)}{2} + a\right] = \frac{481Pa^3}{288EI} \qquad \textbf{Ans}$$

•12–80 The two bars are pin connected at D. Determine the slope at A and the deflection at D. EI is constant.









$$\theta_{E/A} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{Pa^2}{4EI}$$

$$\theta_E = \theta_{E/A} + \theta_A$$

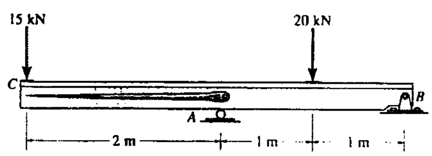
$$0 = \frac{-Pa^2}{4EI} + \theta_A$$

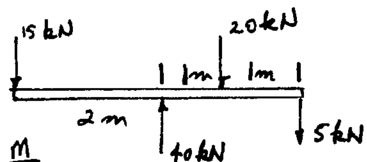
$$\theta_A = \frac{Pa^2}{4EI}$$ **Ans**

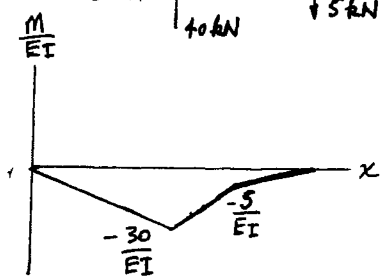
$$\theta_B = \theta_A = \frac{Pa^2}{4EI}$$

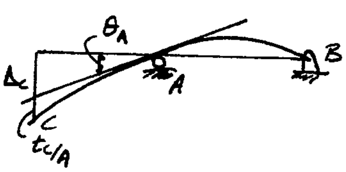
$$\Delta_D = \theta_B a = \frac{Pa^3}{4EI}$$ **Ans**

12–81 A beam having a constant EI is supported as shown. Attached to the beam at A is a pointer, free of load. Both the beam and pointer are originally horizontal when no load is applied to the beam. Determine the distance between the end of the beam and the pointer after each has been displaced by the loading shown.







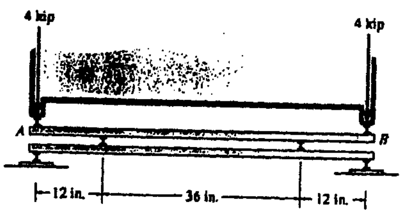


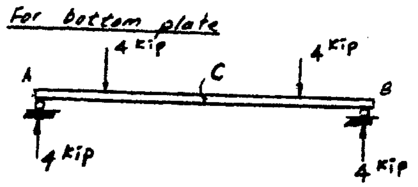
Determine $t_{C/A}$

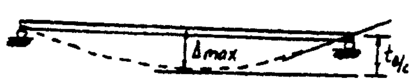
$$t_{C/A} = \frac{1}{2}\left(\frac{30}{EI}\right)(2)\left(\frac{2}{3}\right)(2)$$

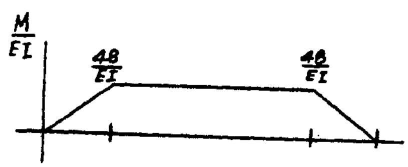
$$t_{C/A} = \frac{40}{EI} \quad \textbf{Ans}$$

12-82. The two A-36 steel bars have a thickness of 1 in. and a width of 4 in. They are designed to act as a spring for the machine which exerts a force of 4 kip on them at A and B. If the supports exert only vertical forces on the bars, determine the maximum deflection of the bottom bar.





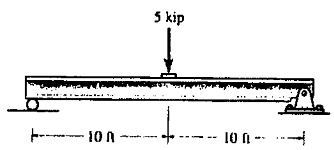


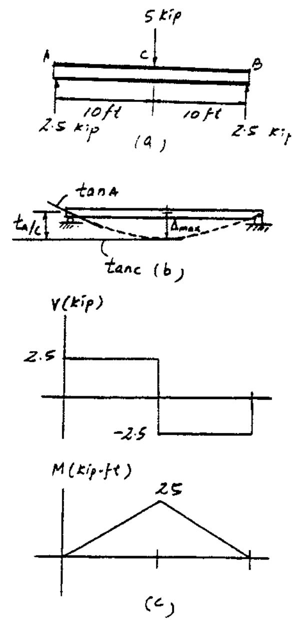


$$\Delta_{max} = t_{B/C} = \left(\frac{48}{EI}\right)(18)(9+12) + \frac{1}{2}\left(\frac{48}{EI}\right)(12)(8)$$

$$= \frac{20448}{EI} = \frac{20448}{29(10^3)(\frac{1}{12})(4)(1^3)} = 2.12 \text{ in.} \qquad \textbf{Ans}$$

12–83 Beams made of fiber-reinforced plastic may one day replace many of those made of A-36 steel since they are one-fourth the weight of steel and are corrosion resistant. Using the table in Appendix *B*, with $\sigma_{allow} = 22$ ksi and $\tau_{allow} = 12$ ksi, select the lightest-weight steel wide-flange beam that will safely support the 5-kip load, then compute its maximum deflection. What would be the maximum deflection of this beam if it were made of a fiber-reinforced plastic with $E_p = 18(10^3)$ ksi and had the same moment of inertia as the steel beam?





$$M_{max} = 25 \text{ kip}\cdot\text{ft}$$

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{25(12)}{22} = 13.63 \text{ in}^3$$

Select W 12 x 14

($S_x = 14.9 \text{ in}^3$ $\quad I_x = 88.6 \text{ in}^4$ $\quad d = 11.91$ in. $\quad t_w = 0.200$ in.)

Check shear :

$$\tau_{max} = \frac{V_{max}}{A_w} = \frac{2.5}{11.91(0.200)} = 1.05 \text{ ksi} < \tau_{allow} = 12 \text{ ksi} \quad \text{OK}$$

Use W 12 x 14 **Ans**

$$\Delta_{max} = |t_{A/C}| = \frac{1}{2}\left(\frac{25}{EI}\right)(10)\left(\frac{2}{3}\right)(10) = \frac{833.33 \text{ kip}\cdot\text{ft}^3}{EI}$$

For the $A-36$ steel beam :

$$\Delta_{max} = \frac{833.33(12^3)}{29(10^3)(88.6)} = 0.560 \text{ in.}$$ **Ans**

For fiber-reinforced plastic beam :

$$\Delta_{max} = \frac{833.33(12^3)}{18(10^3)(88.6)} = 0.903 \text{ in.}$$ **Ans**

***12-84.** Determine the slope at C and deflection at B. EI is constant.

Support Reactions and Elastic Curve : As shown.

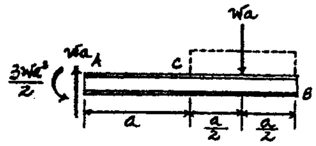
M/EI Diagram : As shown.

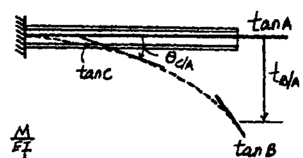
Moment-Area Theorems : The slope at support A is zero. The slope at C is

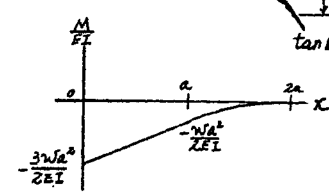
$$\theta_C = |\theta_{C/A}| = \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a) + \left(-\frac{wa^2}{2EI}\right)(a)$$
$$= \frac{wa^3}{EI} \qquad \textbf{Ans}$$

The displacement at B is

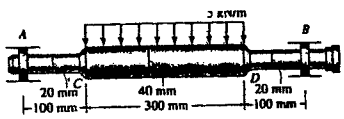
$$\Delta_B = |t_{B/A}| = \frac{1}{2}\left(-\frac{wa^2}{EI}\right)(a)\left(a+\frac{2}{3}a\right) + \left(-\frac{wa^2}{2EI}\right)(a)\left(a+\frac{a}{2}\right)$$
$$+\frac{1}{3}\left(-\frac{wa^2}{2EI}\right)(a)\left(\frac{3}{4}a\right)$$
$$= \frac{41wa^4}{24EI} \downarrow \qquad \textbf{Ans}$$

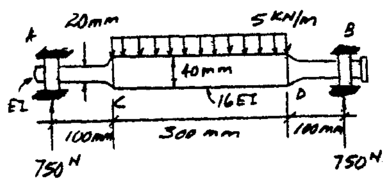


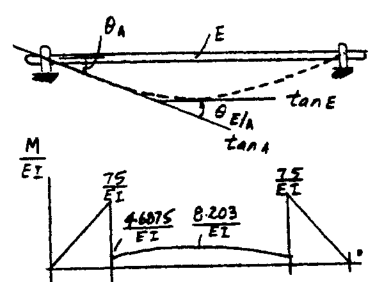




12-85. The A-36 steel shaft is used to support a rotor that exerts a uniform load of 5 kN/m within the region CD of the shaft. Determine the slope of the shaft at the bearings A and B. The bearings exert only vertical reactions on the shaft.





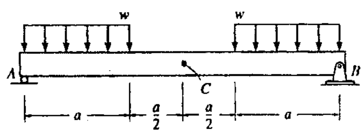


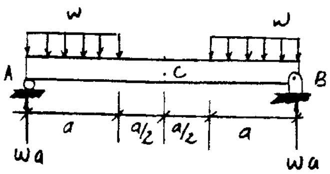
$$\theta_{E/A} = \frac{1}{2}\left(\frac{75}{EI}\right)(0.1) + \left(\frac{4.6875}{EI}\right)(0.15) + \frac{2}{3}\left(\frac{3.5156}{EI}\right)(0.15) = \frac{4.805}{EI}$$

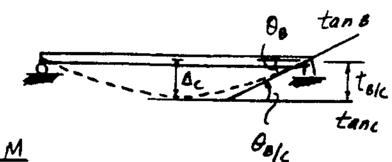
$$\theta_A = \theta_{E/A} = \frac{4.805}{EI} = \frac{4.805}{\quad^{9})\frac{\pi}{4}(0.01)^4} = 0.00306 \text{ rad} = 0.175^\circ$$

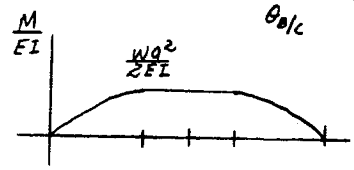
Ans

12-86 The beam is subjected to the loading shown. Determine the slope at B and deflection at C. EI is constant.









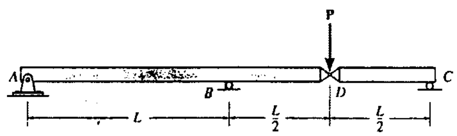
$$\theta_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a) = \frac{7wa^3}{12EI}$$

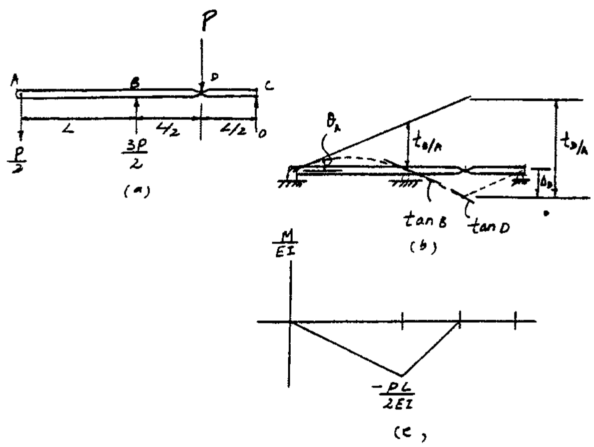
$$\theta_B = \theta_{B/C} = \frac{7wa^3}{12EI}$$ **Ans**

$$\Delta_C = t_{B/C} = \frac{wa^2}{2EI}\left(\frac{a}{2}\right)\left(a + \frac{a}{4}\right) + \frac{2}{3}\left(\frac{wa^2}{2EI}\right)(a)\left(\frac{5}{8}a\right)$$

$$= \frac{25wa^4}{48EI}$$ **Ans**

12-87 The two bars are pin connected at D. Determine the slope at A and the deflection at D. EI is constant.





$$t_{B/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{3}\right) = \frac{-PL^3}{12EI}$$

$$\theta_A = \frac{|t_{B/A}|}{L} = \frac{PL^2}{12EI} \qquad \textbf{Ans}$$

The deflection :

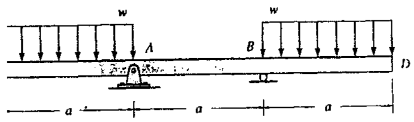
$$t_{D/A} = \frac{1}{2}\left(\frac{-PL}{2EI}\right)(L)\left(\frac{L}{2} + \frac{L}{3}\right) + \frac{1}{2}\left(\frac{-PL}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)$$

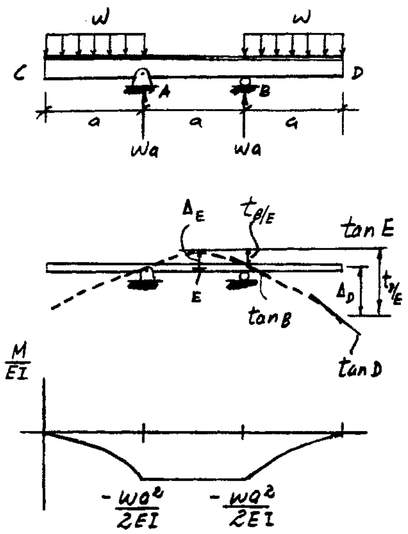
$$= -\frac{PL^3}{4EI}$$

$$\Delta_D = |t_{D/A}| - \left(\frac{\frac{3}{2}L}{L}\right)|t_{B/A}|$$

$$= \frac{PL^3}{4EI} - \frac{3}{2}\left(\frac{PL^3}{12EI}\right) = \frac{PL^3}{8EI} \qquad \textbf{Ans}$$

•12–88 Determine the maximum deflection of the beam. EI is constant.





$$t_{B/E} = \left(\frac{-wa^2}{2EI}\right)\left(\frac{a}{2}\right)\left(\frac{a}{4}\right) = \frac{-wa^4}{16EI}$$

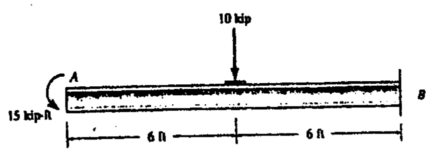
$$\Delta_E = |t_{B/E}| = \frac{wa^4}{16EI} \uparrow$$

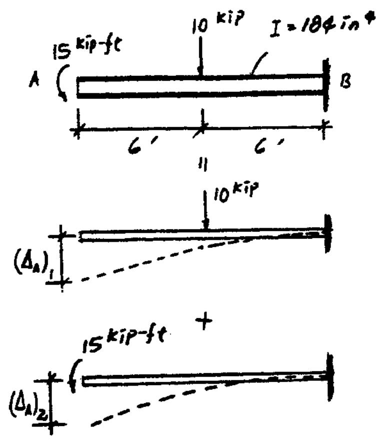
$$t_{D/E} = \left(\frac{-wa^2}{2EI}\right)\left(\frac{a}{2}\right)\left(a + \frac{a}{4}\right) + \frac{1}{3}\left(\frac{-wa^2}{2EI}\right)(a)\left(\frac{3a}{4}\right) = -\frac{7wa^4}{16EI}$$

$$\Delta_D = |t_{D/E}| - |t_{B/E}| = \frac{7wa^4}{16EI} - \frac{wa^4}{16EI} = \frac{3wa^4}{8EI}$$

$$\Delta_{max} = \Delta_D = \frac{3wa^4}{8EI}$$ **Ans**

12-89. The $W8 \times 48$ cantilevered beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its end A.





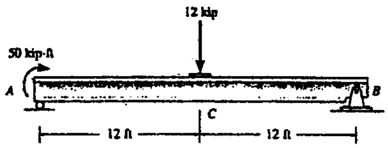
$$(\Delta_A)_1 = \frac{5PL^3}{48EI} = \frac{5(10)(12^3)}{48EI} = \frac{1800}{EI} \downarrow$$

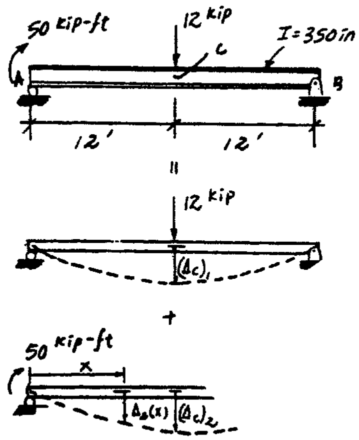
$$(\Delta_A)_2 = \frac{ML^2}{2EI} = \frac{15(12^2)}{2EI} = \frac{1080}{EI} \downarrow$$

$$+\downarrow \ \Delta_A = (\Delta_A)_1 + (\Delta_A)_2 = \frac{1800}{EI} + \frac{1080}{EI} = \frac{2880}{EI}$$

$$= \frac{2880(1728)}{29(10^3)(184)} = 0.933 \text{ in.} \qquad \textbf{Ans}$$

12-90. The $W12 \times 45$ simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C.





$$(\Delta_C)_1 = \frac{PL^3}{48EI} = \frac{12(24^3)}{48EI} = \frac{3456}{EI} \downarrow$$

$$\Delta_2(x) = \frac{Mx}{6LEI}(x^2 - 3Lx + 2L^2)$$

At point C, $\quad x = \frac{L}{2}$

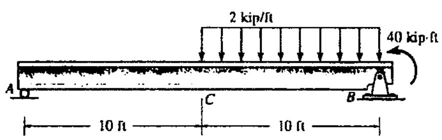
$$(\Delta_C)_2 = \frac{M(\frac{L}{2})}{6LEI}\left(\frac{L^2}{4} - 3L\left(\frac{L}{2}\right) + 2L^2\right)$$

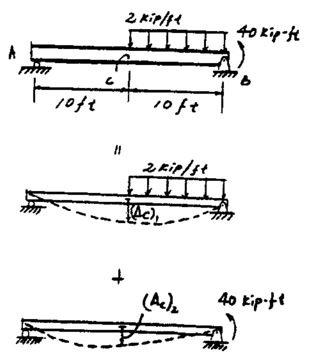
$$= \frac{ML^2}{16EI} = \frac{50(24^2)}{16EI} = \frac{1800}{EI} \downarrow$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{3456}{EI} + \frac{1800}{EI} = \frac{5256}{EI}$$

$$= \frac{5256(1728)}{29(10^3)(350)} = 0.895 \text{ in.} \downarrow \qquad \textbf{Ans}$$

12-91 The $W14 \times 43$ simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C.





$$(\Delta_C)_1 = \frac{5wL^4}{768EI} = \frac{5(2)(20^4)}{768EI} = \frac{2083.33}{EI} \downarrow$$

$$(\Delta_C)_2 = \frac{Mx}{6EIL}\left(x^2 - 3Lx + 2L^2\right) = \frac{40(10)}{6(20)EI}[10^2 - 3(20)(10) + 2(20)^2]$$

$$= \frac{1000}{EI} \downarrow$$

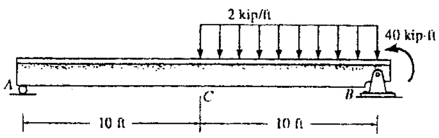
$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{2083.33}{EI} + \frac{1000}{EI}$$

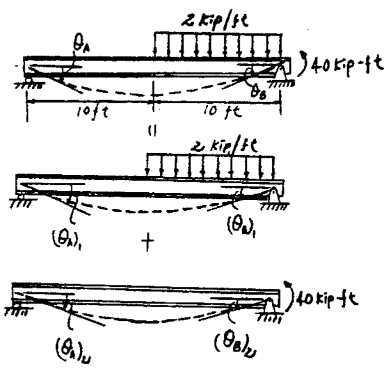
$$= \frac{3083.33}{EI} \text{ kip} \cdot \text{ft}^3$$

Numerical substitution for W 14 x 43, $I_x = 428 \text{ in}^4$

$$\Delta_C = \frac{3083.33(12^3)}{29(10^3)(428)} = 0.429 \text{ in.}$$ **Ans**

•12–92 The $W14 \times 43$ simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the slope at A and B.





$$\theta_A = \theta_{A_1} + \theta_{A_2}$$

$$= \frac{7wL^3}{384\,EI} + \frac{ML}{6\,EI}$$

$$= \frac{\frac{7(2)}{12}(240^3)}{384\,EI} + \frac{40(12)(240)}{6EI} = \frac{61{,}200}{29(10^3)(428)}$$

$$= 0.00493 \text{ rad} = 0.283° \qquad \textbf{Ans}$$

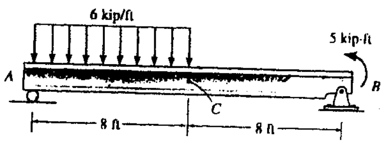
$$\theta_B = \theta_{B_1} + \theta_{B_2}$$

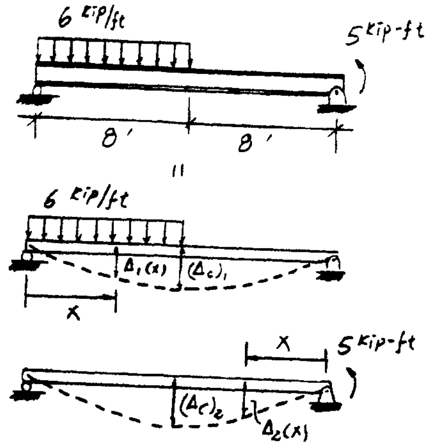
$$= \frac{3wL^3}{128\,EI} + \frac{ML}{3\,EI}$$

$$= \frac{\frac{3(2)}{12}(240^3)}{128\,EI} + \frac{40(12)(240)}{3EI} = \frac{92{,}400}{29(10^3)(428)}$$

$$= 0.007444 \text{ rad} = 0.427° \qquad \textbf{Ans}$$

12-93 The $W8 \times 24$ simply supported beam is made of A-36 steel and is subjected to the loading shown. Determine the deflection at its center C.





$I = 82.8\ \text{in}^4$

$$(\Delta_C)_1 = \frac{5wL^4}{768EI} = \frac{5(6)(16^4)}{768EI} = \frac{2560}{EI} \downarrow$$

$$\Delta_2(x) = \frac{Mx}{6LEI}(x^2 - 3Lx + 2L^2)$$

At point C, $\quad x = \frac{L}{2}$

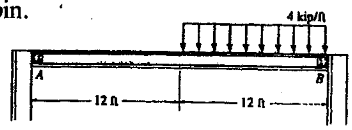
$$(\Delta_C)_2 = \frac{M(\frac{L}{2})}{6LEI}\left(\frac{L^2}{4} - 3L\left(\frac{L}{2}\right) + 2L^2\right)$$

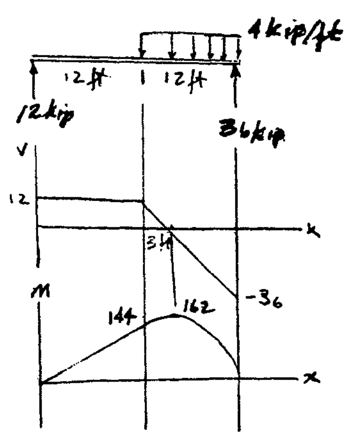
$$= \frac{ML^2}{16EI} = \frac{5(16^2)}{16EI} = \frac{80}{EI} \downarrow$$

$$\Delta_C = (\Delta_C)_1 + (\Delta_C)_2 = \frac{2560}{EI} + \frac{80}{EI} = \frac{2640}{EI}$$

$$= \frac{2640(1728)}{29(10^3)(82.8)} = 1.90\ \text{in.} \qquad \textbf{Ans}$$

12-94. The beam supports the loading shown. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. Assume A is a roller and B is a pin.





$V_{max} = 36$ kip

$M_{max} = 162$ kip · ft

Strength criterion :

$$\sigma_{allow} = \frac{M}{S_{req'd}}$$

$$24 = \frac{162(12)}{S_{req'd}}$$

$$S_{req'd} = 81 \text{ in}^3$$

Choose W 16 x 50, $S = 81.0 \text{ in}^3$, $t_w = 0.380$ in., $d = 16.26$ in., $I_x = 659 \text{ in}^4$

Check shear :

$$\tau_{allow} = \frac{V}{A_{web}}$$

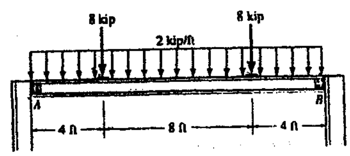
$$14 \geq \frac{36}{(16.26)(0.380)} = 5.83 \text{ ksi} \quad \text{OK}$$

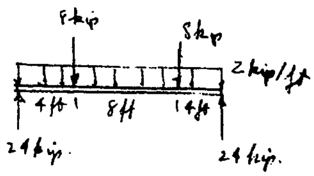
Deflection Criterion;

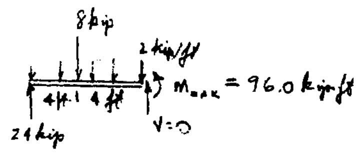
$$v_{max} = 0.006563\frac{wL^4}{EI} = 0.006563\left(\frac{(4)(24)^4(12)^3}{29(10^3)(659)}\right) = 0.7875 \text{ in.} < \frac{1}{360}(24)(12) = 0.800 \quad \text{OK}$$

Use W16 x50 **Ans**

12-95. The simply supported beam carries a uniform load of 2 kip/ft. Code restrictions, due to a plaster ceiling, require the maximum deflection not to exceed 1/360 of the span length. Select the lightest-weight A-36 steel wide-flange beam from Appendix B that will satisfy this requirement and safely support the load. The allowable bending stress is $\sigma_{allow} = 24$ ksi and the allowable shear stress is $\tau_{allow} = 14$ ksi. Assume A is a pin and B a roller support.







$M_{max} = 96$ kip · ft

Strength criterion :

$$\sigma_{allow} = \frac{M}{S_{req'd}}$$

$$24 = \frac{96(12)}{S_{req'd}}$$

$$S_{req'd} = 48 \text{ in}^3$$

Choose W 14 x 34 , $S = 48.6 \text{ in}^3$, $t_w = 0.285$ in., $d = 13.98$ in., $I = 340 \text{ in}^4$

$$\tau_{allow} = \frac{V}{A_{web}}$$

$$14 \geq \frac{24}{(13.98)(0.285)} = 6.02 \text{ ksi} \qquad \text{OK}$$

Deflection criterion;

Maximum is at center.

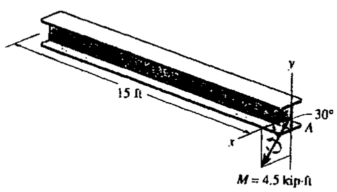
$$v_{max} = \frac{5wL^4}{384EI} + (2)\frac{P(4)(8)}{6EI(16)}[(16)^2 - (4)^2 - (8)^2)](12)^3$$

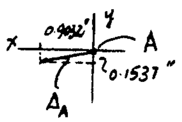
$$= [\frac{5(2)(16)^4}{384EI} + \frac{117.33(8)}{EI}](12)^3$$

$$= \frac{4.571(10^6)}{29(10^6)(340)} = 0.000464 \text{ in.} < \frac{1}{360}(16)(12) = 0.533 \text{ in.} \qquad \text{OK}$$

Jse W 14 x34 Ans

•12–96 The $W10 \times 30$ steel cantilevered beam is made of A-36 steel and is subjected to unsymmetrical bending caused by the applied moment. Determine the deflection of the centroid at its end A due to the loading. *Hint:* Resolve the moment into components and use superposition.





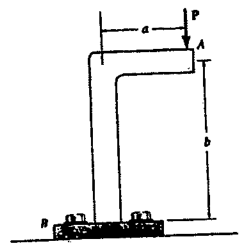
$I_x = 170 \text{ in}^4, \quad I_y = 16.7 \text{ in}^4$

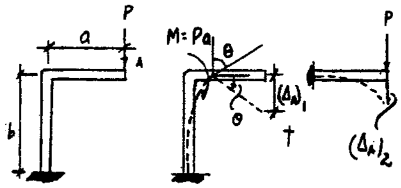
$$x_{max} = \frac{(M \sin\theta)L^2}{2EI_y} = \frac{4.5(\sin 30°)(15^2)(12)^3}{2(29)(10^3)(16.7)} = 0.9032 \text{ in.}$$

$$y_{max} = \frac{(M \cos\theta)L^2}{2EI_x} = \frac{4.5(\cos 30°)(15^2)(12)^3}{2(29)(10^3)(170)} = 0.1537 \text{ in.}$$

$$\Delta_A = \sqrt{0.9032^2 - 0.1537^2} = 0.916 \text{ in.} \qquad \textbf{Ans}$$

12-97. Determine the vertical deflection at the end A of the bracket. Assume that the bracket is fixed supported at its base B and neglect axial deflection. EI is constant.





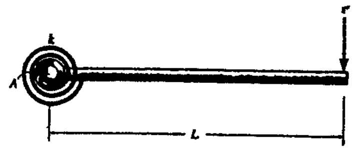
$$\theta = \frac{ML}{EI} = \frac{Pab}{EI}$$

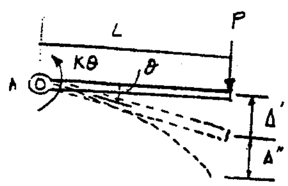
$$(\Delta_A)_1 = \theta(a) = \frac{Pa^2b}{EI}$$

$$(\Delta_A)_2 = \frac{PL^3}{3EI} = \frac{Pa^3}{3EI}$$

$$\Delta_A = (\Delta_A)_1 + (\Delta_A)_2 = \frac{Pa^2b}{EI} + \frac{Pa^3}{3EI} = \frac{Pa^2(3b+a)}{3EI}$$ **Ans**

12-98. The rod is pinned at its end A and attached to a torsional spring having a stiffness k, which measures the torque per radian of rotation of the spring. If a force **P** is always applied perpendicular to the end of the rod, determine the displacement of the force. EI is constant.





In order to maintain equilibrium, the rod has to rotate through an angle θ.

$\circlearrowleft + \Sigma M_A = 0;\quad k\theta - PL = 0;\quad \theta = \dfrac{PL}{k}$

Hence,

$$\Delta' = L\theta = L\left(\frac{PL}{k}\right) = \frac{PL^2}{k}$$

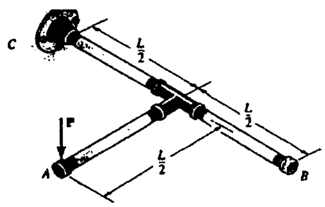
Elastic deformation :

$$\Delta'' = \frac{PL^3}{3EI}$$

Therefore,

$$\Delta = \Delta' + \Delta'' = \frac{PL^2}{k} + \frac{PL^3}{3EI} = PL^2\left(\frac{1}{k} + \frac{L}{3EI}\right)$$ **Ans**

12-99. The pipe assembly consists of three equal-sized pipes with flexibility stiffness EI and torsional stiffness GJ. Determine the vertical deflection at point A.



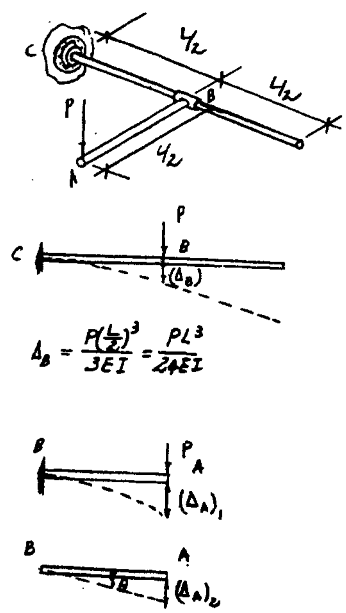
$$\Delta_B = \frac{P(\frac{L}{2})^3}{3EI} = \frac{PL^3}{24EI}$$

$$(\Delta_A)_1 = \frac{P(\frac{L}{2})^3}{3EI} = \frac{PL^3}{24EI}$$

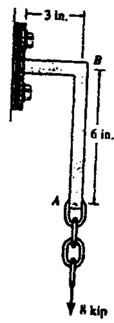
$$\theta = \frac{TL}{JG} = \frac{(PL/2)(\frac{L}{2})}{JG} = \frac{PL^2}{4JG}$$

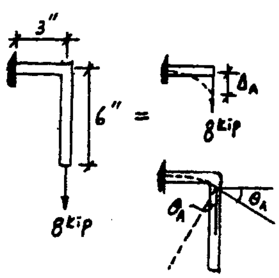
$$(\theta_A)_2 = \theta(\frac{L}{2}) = \frac{PL^3}{8JG}$$

$$\Delta_A = \Delta_B + (\Delta_A)_1 + (\Delta_A)_2$$
$$= \frac{PL^3}{24EI} + \frac{PL^3}{24EI} + \frac{PL^3}{8JG}$$
$$= PL^3(\frac{1}{12EI} + \frac{1}{8JG}) \qquad \textbf{Ans}$$



*12-100.** Determine the vertical deflection and slope at the end A of the bracket. Assume that the bracket is fixed supported at its base, and neglect the axial deformation of segment AB. EI is constant.

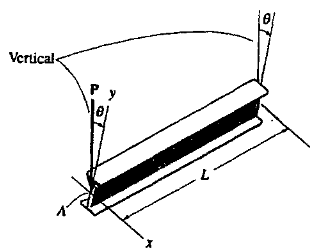




$$\Delta_A = \frac{PL^3}{3EI} = \frac{8(3)^3}{3EI} = \frac{72}{EI} \quad \textbf{Ans}$$

$$\theta_A = \frac{PL^2}{2EI} = \frac{8(3^2)}{2EI} = \frac{36}{EI} \quad \textbf{Ans}$$

12–101 The wide-flange beam acts as a cantilever. Due to an error it is installed at an angle θ with the vertical. Determine the ratio of its deflection in the x direction to its deflection in the y direction at A when a load **P** is applied at this point. The moments of inertia are I_x and I_y. For the solution, resolve **P** into components and use the method of superposition. *Note:* The result indicates that large lateral deflections (x direction) can occur in narrow beams, $I_y \ll I_x$, when they are improperly installed in this manner. To show this numerically, compute the deflections in the x and y directions for an A-36 steel $W10 \times 15$, with $P = 1.5$ kip, $\theta = 10°$, and $L = 12$ ft.



$$y_{max} = \frac{P\cos\theta\, L^3}{3EI_x}; \qquad x_{max} = \frac{P\sin\theta\, L^3}{3EI_y}$$

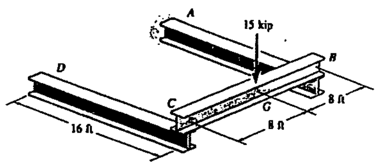
$$\frac{x_{max}}{y_{max}} = \frac{\frac{P\sin\theta\, L^3}{3EI_y}}{\frac{P\cos\theta\, L^3}{3EI_x}} = \frac{I_x}{I_y}\tan\theta$$ **Ans**

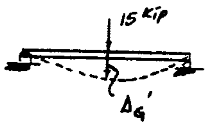
$W\,10 \times 15 \qquad I_x = 68.9\ \text{in}^4 \qquad I_y = 2.89\ \text{in}^4$

$$y_{max} = \frac{1.5(\cos 10°)(144)^3}{3(29)(10^3)(68.9)} = 0.736\ \text{in.}$$ **Ans**

$$x_{max} = \frac{1.5(\sin 10°)(144)^3}{3(29)(10^3)(2.89)} = 3.09\ \text{in.}$$ **Ans**

12-102. The framework consists of two A-36 steel cantilevered beams *CD* and *BA* and a simply supported beam *CB*. If each beam is made of steel and has a moment of inertia about its principal axis of $I_x = 118\ \text{in}^4$, determine the deflection at the center *G* of beam *CB*.





$$\Delta_C = \frac{PL^3}{3EI} = \frac{7.5(16^3)}{3EI} = \frac{10{,}240}{EI} \downarrow$$

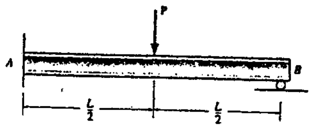
$$\Delta'_G = \frac{PL^3}{48EI} = \frac{15(16^3)}{48EI} = \frac{1{,}280}{EI} \downarrow$$

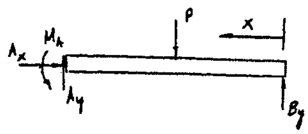
$$\Delta_G = \Delta_C + \Delta'_G$$

$$= \frac{10{,}240}{EI} + \frac{1{,}280}{EI} = \frac{11{,}520}{EI}$$

$$= \frac{11{,}520(1{,}768)}{29(10^3)(118)} = 5.82\ \text{in.} \downarrow \qquad \textbf{Ans}$$

12-103. Determine the reactions at the supports A and B, then draw the shear and moment diagrams. Use discontinuity functions. EI is constant.





$\xrightarrow{+} \Sigma F_x = 0 \qquad A_x = 0 \qquad$ **Ans**

$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y - P = 0$

$$A_y = P - B_y \qquad (1)$$

$\circlearrowleft + \Sigma M_A = 0 \qquad M_A + B_y(L) - P(L/2) = 0$

$$M_A = \frac{PL}{2} - B_y L \qquad (2)$$

Bending Moment $M(x)$:

$$M(x) = -(-B_y) < x - 0 > - P < x - \frac{L}{2} > \; = B_y x - P < x - \frac{L}{2} >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - P < x - \frac{L}{2} >$$

$$EI\frac{dv}{dx} = \frac{B_y x^2}{2} - \frac{P}{2} < x - \frac{L}{2} >^2 + C_1 \qquad (3)$$

$$EIv = \frac{B_y x^3}{6} - \frac{P}{6} < x - \frac{L}{2} >^3 + C_1 x + C_2 \qquad (4)$$

Boundary conditions:

$v = 0 \quad$ at $\quad x = 0$

From Eq. (4)

$C_2 = 0$

$v = 0 \quad$ at $\quad x = L$

From Eq. (4)

$$0 = \frac{B_y L^3}{6} - \frac{PL^3}{48} + C_1 L \qquad (5)$$

$\frac{dv}{dx} = 0 \quad$ at $\quad x = L$

From Eq. (3),

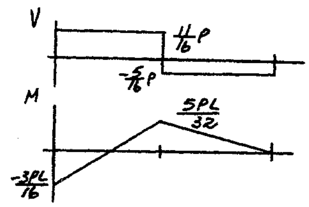
$$0 = \frac{B_y L^2}{2} - \frac{PL^2}{8} + C_1 \qquad (6)$$

Solving Eqs.(5) and (6) yields,

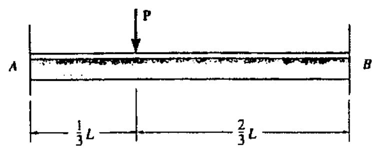
$B_y = \frac{5}{16}P \qquad$ **Ans** $\qquad C_1 = \frac{-PL^2}{32}$

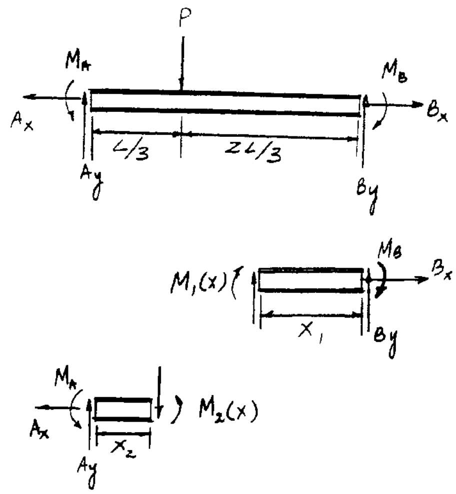
Substitute $B_y = \frac{5}{16}P$ into Eqs. (1) and (2),

$A_y = \frac{11}{16}P \qquad$ **Ans** $\qquad M_A = \frac{3PL}{16} \qquad$ **Ans**



•12–104 Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant. Neglect the effect of axial load.





$\curvearrowleft + \Sigma M_A = 0; \quad M_A + B_y L - P(\frac{L}{3}) - M_B = 0$ (1)

$+\uparrow \Sigma F_y = 0; \quad A_y + B_y - P = 0$ (2)

Moment functions :

$M_1(x) = B_y x_1 - M_B$

$M_2(x) = A_y x_2 - M_A$

Slope and elastic curve :

$EI\frac{d^2 v}{dx^2} = M(x)$

For $M_1(x) = B_y x_1 - M_B$; $\quad EI\frac{d^2 v_1}{dx_1{}^2} = B_y x_1 - M_B$

$EI\frac{dv_1}{dx_1} = \frac{B_y x_1^2}{2} - M_B x_1 + C_1$

$EIv_1 = \frac{B_y x_1{}^3}{6} - \frac{M_B x_1^2}{2} + C_1 x + C_2$

For $M_2(x) = A_y x_2 - M_A$

$EI\frac{d^2 v_2}{dx_2{}^2} = A_y x_2 - M_A$

$EI\frac{dv_2}{dx_2} = \frac{A_y x_2{}^2}{2} - M_A x_2 + C_3$

$EI\, v_2 = \frac{A_y x_2{}^3}{6} - \frac{M_A x_2{}^3}{6} + C_3 x_2 + C_4$

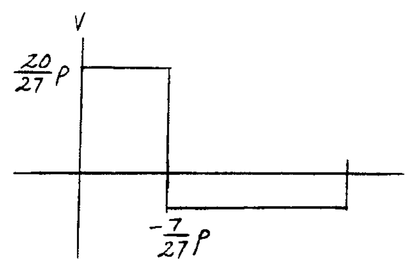
Boundary conditions :

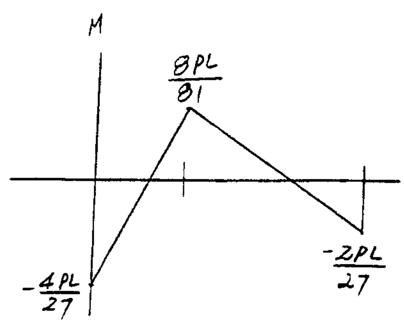
At $x_1 = 0$, $\frac{dv_1}{dx_1} = 0$

From Eq. (3),

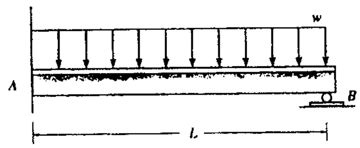
$0 = 0 - 0 + C_1; \quad C_1 = 0$

At $x_1 = 0$, $v_1 = 0$





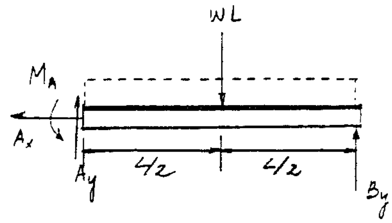
12–105 Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



$\leftarrow \Sigma F_x = 0; \quad A_x = 0$ **Ans**

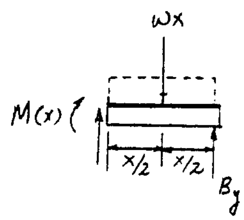
$+\uparrow \Sigma F_y = 0; \quad A_y + B_y - wL = 0 \qquad (1)$

$\curvearrowleft + \Sigma M_A = 0; \quad M_A + B_y L - wL\left(\frac{L}{2}\right) = 0 \qquad (2)$



$\curvearrowleft + \Sigma M_{NA} = 0; \quad B_y(x) - wx\left(\frac{x}{2}\right) - M(x) = 0$

$$M(x) = B_y x - \frac{wx^2}{2}$$



$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = B_y x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{B_y x^2}{2} - \frac{wx^3}{6} + C_1 \qquad (3)$$

$$EI\,v = \frac{B_y x^3}{6} - \frac{wx^4}{24} + C_1 x + C_2 \qquad (4)$$

Boundary conditions :

At $x = 0$, $v = 0$

From Eq. (4),

$0 = 0 - 0 + 0 + C_2; \quad C_2 = 0$

At $x = L$, $\frac{dv}{dx} = 0$

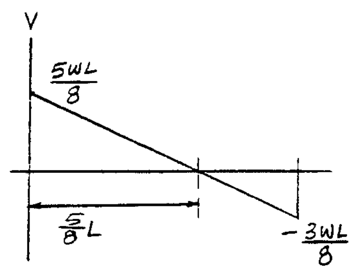
From Eq. (3),

$$0 = \frac{B_y L^2}{2} - \frac{wL^3}{6} + C_1 \qquad (5)$$

At $x = L$, $v = 0$

From Eq. (4),

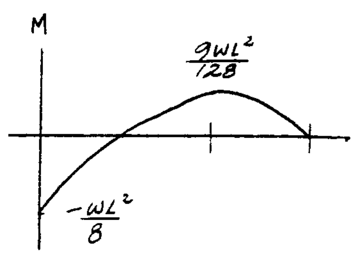
$$0 = \frac{B_y L^3}{6} - \frac{wL^4}{24} + C_1 L \qquad (6)$$



Solving Eqs. (5) and (6) yields :

$B_y = \frac{3wL}{8}$ **Ans**

$C_1 = -\frac{wL^3}{48}$

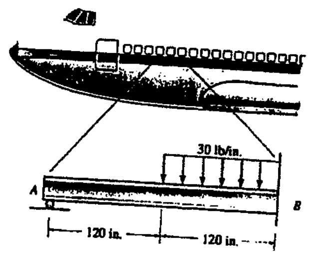


Substituting B_y into Eqs. (1) and (2) yields :

$A_y = \frac{5wL}{8}$ **Ans**

$M_A = \frac{wL^2}{8}$ **Ans**

12-106. The loading on a floor beam used in the airplane is shown. Use discontinuity functions and determine the reactions at the supports A and B, and then draw the moment diagram for the beam. The beam is made of aluminum and has a moment of inertia of $I = 320\ \text{in}^4$.



$\xrightarrow{+} \Sigma F_x = 0 \qquad B_x = 0 \qquad$ **Ans**

$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y - \dfrac{wL}{2} = 0; \qquad B_y = \dfrac{wL}{2} - A_y \qquad (1)$

$\curvearrowleft + \Sigma M_A = 0 \qquad M_B + A_y L - \dfrac{wL^2}{8} = 0 \qquad M_B = \dfrac{wL^2}{8} - A_y L \qquad (2)$

Bending Moment $M(x)$:

$$M(x) = -(-A_y) < x - 0 > -\frac{w}{2} < x - \frac{L}{2} >^2 = A_y x - \frac{w}{2} < x - \frac{L}{2} >^2$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - \frac{w}{2} < x - \frac{L}{2} >^2$$

$$EI\frac{dv}{dx} = \frac{A_y x^2}{2} - \frac{w}{6} < x - \frac{L}{2} >^3 + C_1 \qquad (3)$$

$$EIv = \frac{A_y x^3}{6} - \frac{w}{24} < x - \frac{L}{2} >^4 + C_1 x + C_2 \qquad (4)$$

Boundary conditions:

$v = 0 \quad$ at $\quad x = 0$

From Eq. (4)

$C_2 = 0$

$v = 0 \quad$ at $\quad x = L$

From Eq. (4)

$$0 = \frac{A_y L^3}{6} - \frac{wL^4}{384} + C_1 L \qquad (5)$$

$\dfrac{dv}{dx} = 0 \quad$ at $\quad x = L$

From Eq. (3)

$$0 = \frac{A_y L^2}{2} - \frac{wL^3}{48} + C_1 \qquad (6)$$

Solving Eqs. (5) and (6) yield:

$$A_y = \frac{7}{128}wL \qquad C_1 = \frac{-5}{768}wL^3$$

Substitute A_y into Eqs. (1) and (2):

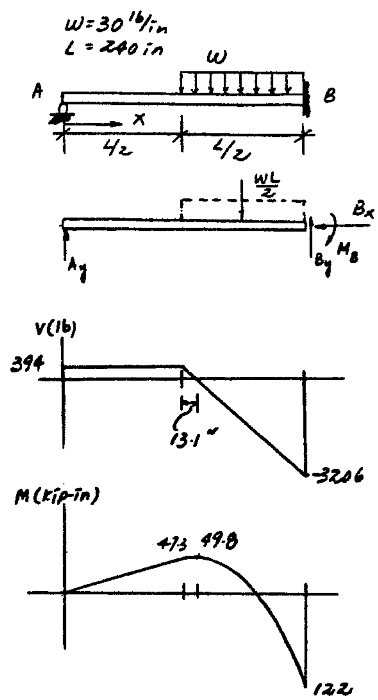
$$B_y = \frac{57}{128}wL \qquad M_B = \frac{9}{128}wL^2$$

Substitute numerical values:

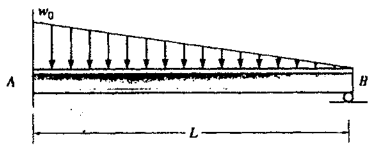
$A_y = \dfrac{7}{128}(30)(240) = 394\ \text{lb} \qquad$ **Ans**

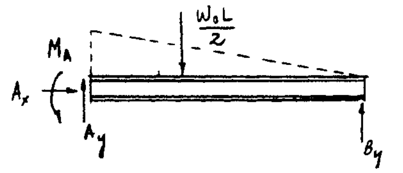
$B_y = \dfrac{57}{128}(30)(240) = 3206\ \text{lb} = 3.21\ \text{kip} \qquad$ **Ans**

$M_B = \dfrac{9}{128}(30)(240)^2 = 121500\ \text{lb}\cdot\text{in.} = 122\ \text{kip}\cdot\text{in.} \qquad$ **Ans**



12–107 Determine the reactions at the supports A and B. EI is constant.





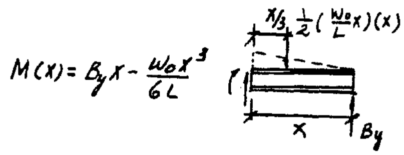
$\rightarrow \Sigma F_x = 0 \qquad A_x = 0 \qquad$ **Ans**

$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad A_y = \frac{w_0 L}{2} - B_y \qquad (1)$

$\circlearrowleft + \Sigma M_A = 0 \qquad M_A + B_y L - \frac{w_0 L}{2}(\frac{L}{3}) = 0 \qquad M_A = \frac{w_0 L^2}{6} - B_y L \qquad (2)$

Bending Moment $M(x)$:

$$M(x) = B_y x = \frac{w_0 x^3}{6L}$$



Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = B_y x - \frac{w_0 x^3}{6L}$$

$$EI\frac{dv}{dx} = \frac{B_y x^2}{2} - \frac{w_0 x^4}{24L} + C_1 \qquad (3)$$

$$EIv = \frac{B_y x^3}{6} - \frac{w_0 x^5}{120L} + C_1 x + C_2 \qquad (4)$$

Boundary conditions :

$v = 0 \quad$ at $\quad x = 0$

From Eq. (4)

$C_2 = 0$

$v = 0 \quad$ at $\quad x = L$

From Eq. (4)

$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} + C_1 L \qquad (5)$$

$\frac{dv}{dx} = 0 \quad$ at $\quad x = L$

From Eq. (3)

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} + C_1 \qquad (6)$$

Solving Eqs. (5) and (6) yield :

$B_y = \frac{1}{10} w_0 L \qquad$ **Ans**

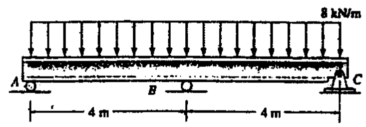
$C_1 = -\frac{w_0 L^3}{120}$

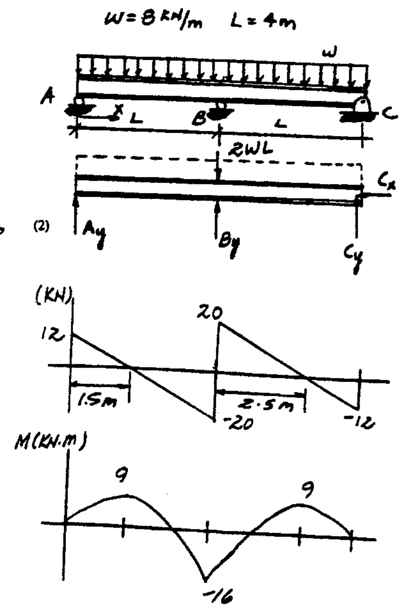
Substitute B_y into Eqs. (1) and (2) :

$A_y = \frac{2}{5} w_0 L \qquad$ **Ans**

$M_A = \frac{1}{15} w_0 L^2 \qquad$ **Ans**

***12-108.** Use discontinuity functions and determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.





$\xrightarrow{+} \Sigma F_x = 0; \quad C_x = 0 \quad$ **Ans**

$+\uparrow \Sigma F_y = 0; \quad A_y + B_y + C_y - 2wL = 0 \quad (1)$

$\circlearrowleft + \Sigma M_C = 0; \quad A_y(2L) + B_y(L) - 2wL(L) = 0; \quad B_y = 2wL - 2A_y \quad (2)$

Bending moment $M(x)$:

$$M(x) = -(-A_y) < x-0 > -\frac{w}{2} < x-0 >^2 - (-B_y) < x-L >$$

$$= A_y x - \frac{w}{2}x^2 + B_y < x-L >$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = A_y x - \frac{w}{2}x^2 + B_y < x-L >$$

$$EI\frac{dv}{dx} = \frac{A_y x^2}{2} - \frac{wx^2}{6} + \frac{B_y}{2} < x-L >^2 + C_1 \quad (3)$$

$$EIv = \frac{A_y x^3}{6} - \frac{wx^4}{24} + \frac{B_y}{6} < x-L >^3 + C_1 x + C_2 \quad (4)$$

Boundary Conditions :

$v = 0 \quad$ at $\quad x = 0$

From Eq. (4)

$C_2 = 0$

$v = 0 \quad$ at $\quad x = L$

From Eq. (4)

$$0 = \frac{A_y L^3}{6} - \frac{wL^4}{24} + C_1 L \quad (5)$$

$\frac{dv}{dx} = 0 \quad$ at $\quad x = L$

From Eq. (3)

$$0 = \frac{A_y L^2}{2} - \frac{wL^3}{6} + C_1 \quad (6)$$

Solving for Eqs. (5) and (6) yield :

$$C_1 = -\frac{wL^3}{48}$$

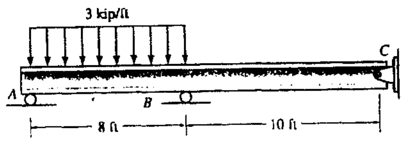
$A_y = \frac{3}{8}wL = \frac{3}{8}(8)(4) = 12.0 \text{ kN} \quad$ **Ans**

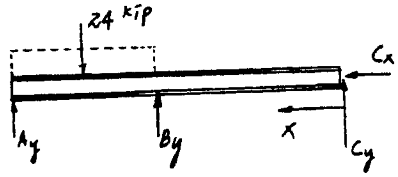
Substitute A_y into Eqs. (1) and (2)

$B_y = \frac{5}{4}wL = \frac{5}{4}(8)(4) = 40.0 \text{ kN} \quad$ **Ans**

$C_y = \frac{3}{8}wL = \frac{3}{8}(8)(4) = 12.0 \text{ kN} \quad$ **Ans**

12-109. Use discontinuity functions and determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.





$\xrightarrow{+} \Sigma F_x = 0 \qquad C_x = 0$ **Ans**

$+\uparrow \Sigma F_y = 0 \qquad A_y + B_y + C_y - 24 = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_A = 0 \qquad 18\,C_y + 8B_y - 24(4) = 0 \qquad (2)$

Bending Moment $M(x)$:

$$M(x) = -(-C_y) < x - 0 > -(-B_y) < x - 10 > -\frac{3}{2} < x - 10 >^2$$

$$= C_y x + B_y < x - 10 > -\frac{3}{2} < x - 10 >^2$$

Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x) = C_y x + B_y < x - 10 > -\frac{3}{2} < x - 10 >^2$$

$$EI\frac{dv}{dx} = \frac{C_y x^2}{2} + \frac{B_y}{2} < x - 10 >^2 -\frac{1}{2} < x - 10 >^3 + C_1 \qquad (3)$$

$$EIv = \frac{C_y x^3}{6} + \frac{B_y}{6} < x - 10 >^3 -\frac{1}{8} < x - 10 >^4 + C_1 x + C_2 \qquad (4)$$

Boundary conditions :

$v = 0 \quad$ at $\quad x = 0$

From Eq. (4)

$C_2 = 0$

$v = 0 \quad$ at $\quad x = 10$ ft.

From Eq. (4)

$0 = 166.67\,C_y + 10C_1 \qquad (5)$

$v = 0 \quad$ at $\quad x = 18$ ft.

$0 = 972C_y + 85.33B_y - 512 + 18C_1 \qquad (6)$

Solving Eqs. (2),(5), and (6) yields :

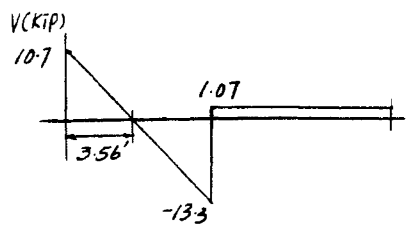
$B_y = 14.4$ kip **Ans**

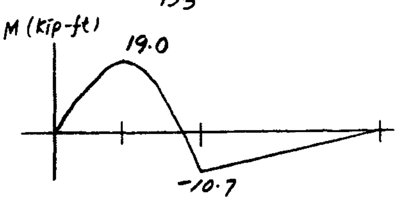
$C_y = -1.07$ kip $= 1.07$ kip $\downarrow$ **Ans**

$C_1 = 17.78$

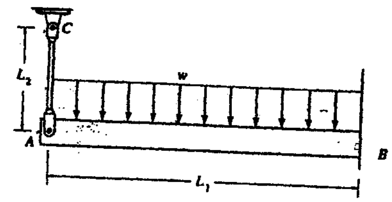
From Eq. (1) :

$A_y = 10.7$ kip **Ans**





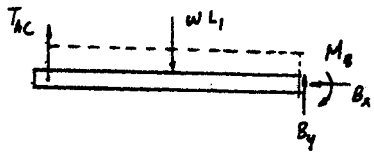
12-110. The beam has a constant $E_1 I_1$ and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.



$$+\uparrow \Sigma F_y = 0 \qquad T_{AC} + B_y - wL_1 = 0 \qquad (1)$$

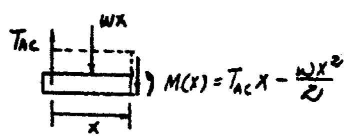
$$\circlearrowleft + \Sigma M_B = 0 \qquad T_{AC}(L_1) + M_B - \frac{wL_1^2}{2} = 0$$

$$M_B = \frac{wL_1^2}{2} - T_{AC}L_1 \qquad (2)$$



Bending Moment $M(x)$:

$$M(x) = T_{AC}x - \frac{wx^2}{2}$$



Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M(x) = T_{AC}x - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{T_{AC}x^2}{2} - \frac{wx^3}{6} + C_1 \qquad (3)$$

$$EIv = \frac{T_{AC}x^3}{6} - \frac{wx^4}{24} + C_1 x + C_2 \qquad (4)$$

Boundary conditions:

$$v = -\frac{T_{AC}L_2}{A_2 E_2} \qquad x = 0$$

rom Eq. (4)

$$-E_1 I_1\left(\frac{T_{AC}L_2}{A_2 E_2}\right) = 0 - 0 + 0 + C_2$$

$$C_2 = \left(\frac{-E_1 I_1 L_2}{A_2 E_2}\right)T_{AC}$$

$$v = 0 \quad \text{at} \quad x = L_1$$

Fı m Eq. (4)

$$0 = \frac{T_{AC}L_1^3}{6} - \frac{wL_1^4}{24} + C_1 L_1 - \frac{E_1 I_1 L_2}{A_2 E_2}T_{AC} \qquad (5)$$

$$\frac{dv}{dx} = 0 \quad \text{at} \quad x = L_1$$

Fı Eq. (3)

Fro..

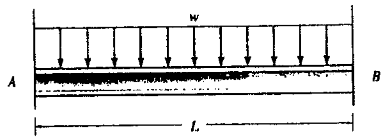
$$0 = \frac{T_{AC}L_1^2}{2} - \frac{wL_1^3}{6} + C_1 \qquad (6)$$

Solving Eqs. (5) and (6) yields:

$$T_{AC} = \frac{3A_2 E_2 w L_1^4}{8(A_2 E_2 L_1^3 + 3E_1 I_1 L_2)} \qquad \textbf{Ans}$$

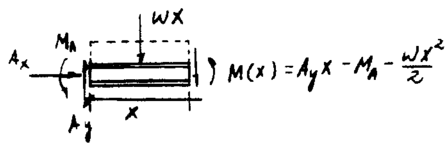
rom

12-111. Determine the moment reactions at the supports A and B, and then draw the shear and moment diagrams. Solve by expressing the internal moment in the beam in terms of A_y and M_A. EI is constant.



$$M(x) = A_y x - M_A - \frac{wx^2}{2}$$

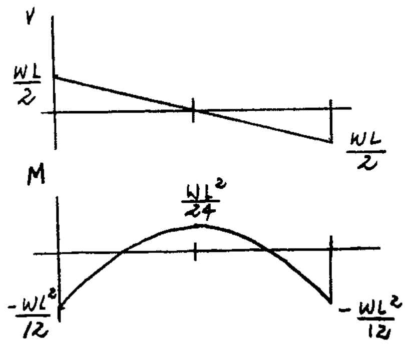
Elastic curve and slope :



$$EI\frac{d^2v}{dx^2} = M(x) = A_y x - M_A - \frac{wx^2}{2}$$

$$EI\frac{dv}{dx} = \frac{A_y x^2}{2} - M_A x - \frac{wx^3}{6} + C_1 \qquad (1)$$

$$EIv = \frac{A_y x^3}{6} - \frac{M_A x^2}{2} - \frac{wx^4}{24} + C_1 x + C_2 \qquad (2)$$



Boundary conditions :

$\frac{dv}{dx} = 0$ at $x = 0$

From Eq. (1)

$C_1 = 0$

$v = 0$ at $x = 0$

From Eq. (2)

$C_2 = 0$

$\frac{dv}{dx} = 0$ at $x = L$

From Eq. (1)

$$0 = \frac{A_y L^2}{2} - M_A L - \frac{wL^3}{6} \qquad (3)$$

$v = 0$ at $x = L$

From Eq. (2)

$$0 = \frac{A_y L^3}{6} - \frac{M_A L^2}{2} - \frac{wL^4}{24} \qquad (4)$$

Solving Eqs. (3) and (4) yields :

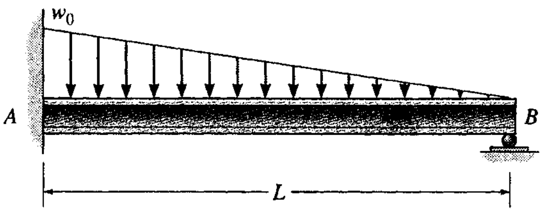
$$A_y = \frac{wL}{2}$$

$$M_A = \frac{wL^2}{12} \quad \textbf{Ans}$$

Due to symmetry :

$$M_B = \frac{wL^2}{12} \quad \textbf{Ans}$$

***12-112.** Determine the moment reactions at the supports A and B. EI is constant.

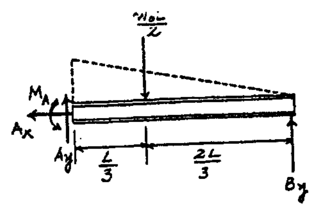


Support Reactions : FBD(a).

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w_0 L}{2} = 0 \qquad [1]$$

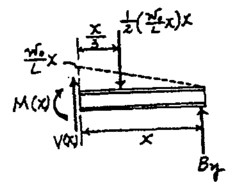
$$\curvearrowleft + \Sigma M_A = 0; \qquad B_y L + M_A - \frac{w_0 L}{2}\left(\frac{L}{3}\right) = 0 \qquad [2]$$



Moment Function : FBD(b).

$$\curvearrowleft + \Sigma M_{NA} = 0; \qquad -M(x) - \frac{1}{2}\left(\frac{w_0}{L}x\right)x\left(\frac{x}{3}\right) + B_y x = 0$$

$$M(x) = B_y x - \frac{w_0}{6L}x^3$$



Slope and Elastic Curve :

$$EI\frac{d^2 v}{dx^2} = M(x)$$

$$EI\frac{d^2 v}{dx^2} = B_y x - \frac{w_0}{6L}x^3$$

$$EI\frac{dv}{dx} = \frac{B_y}{2}x^2 - \frac{w_0}{24L}x^4 + C_1 \qquad [3]$$

$$EI\,v = \frac{B_y}{6}x^3 - \frac{w_0}{120L}x^5 + C_1 x + C_2 \qquad [4]$$

Boundary Conditions :

At $x = 0$, $v = 0$. From Eq. [4]. $\qquad C_2 = 0$

At $x = L$, $\frac{dv}{dx} = 0$. From Eq. [3].

$$0 = \frac{B_y L^2}{2} - \frac{w_0 L^3}{24} + C_1$$

$$C_1 = -\frac{B_y L^2}{2} + \frac{w_0 L^3}{24}$$

At $x = L$, $v = 0$. From Eq. [4].

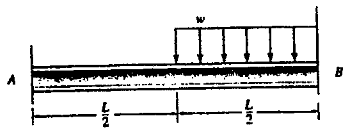
$$0 = \frac{B_y L^3}{6} - \frac{w_0 L^4}{120} + \left(-\frac{B_y L^2}{2} + \frac{w_0 L^3}{24}\right)L$$

$$B_y = \frac{w_0 L}{10} \qquad \textbf{Ans}$$

Substituting B_y into Eq. [1] and [2] yields,

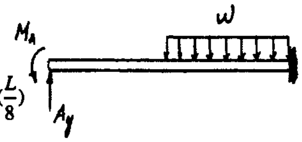
$$A_y = \frac{2w_0 L}{5} \qquad M_A = \frac{w_0 L^2}{15} \qquad \textbf{Ans}$$

12-113. Determine the moment reactions at the supports *A* and *B*. *EI* is constant.



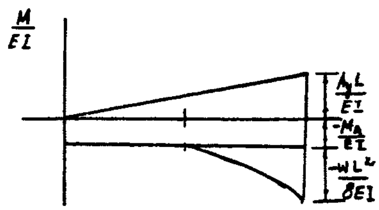
$$\theta_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L) + \left(\frac{-M_A}{EI}\right)(L) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)$$

$$0 = \frac{A_y L}{2} - M_A - \frac{wL^2}{48} \qquad (2)$$



$$t_{B/A} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{L}{3}\right) + \left(\frac{-M_A}{EI}\right)(L)\left(\frac{L}{2}\right) + \frac{1}{3}\left(\frac{-wL^2}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{8}\right)$$

$$0 = \frac{A_y L}{6} - \frac{M_A}{2} - \frac{wL^2}{384} \qquad (3)$$



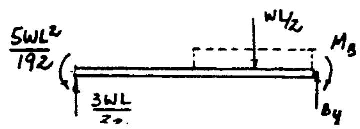
Solving Eqs. (2) and (3) yields :

$$A_y = \frac{3wL}{32}$$

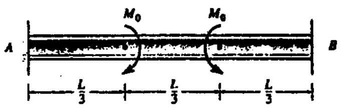
$$M_A = \frac{5wL^2}{192} \qquad \textbf{Ans}$$

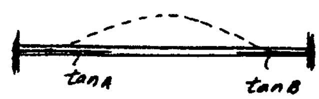
$$\curvearrowleft + \Sigma M_B = 0; \qquad M_B + \frac{3wL}{32}(L) - \frac{5wL^2}{192} - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$M_B = \frac{11wL^2}{192} \qquad \textbf{Ans}$$



12-114. Determine the moment reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.

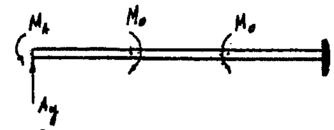




$t_{B/A} = 0$ $\theta_{B/A} = 0$

$$\theta_{B/A} = 0 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{3}\right) + \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L) + \left(\frac{-M_A}{EI}\right)(L)$$

$$0 = \frac{M_0}{3} + \frac{1}{2}A_y L - M_A \qquad (1)$$



$$t_{B/A} = 0 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{3}\right)\left(\frac{L}{3} + \frac{L}{6}\right) + \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{L}{3}\right) + \left(\frac{-M_A}{EI}\right)(L)\left(\frac{L}{2}\right)$$

$$0 = \frac{M_0}{6} + \frac{A_y L}{6} - \frac{M_A}{2} \qquad (2)$$

Solving Eqs. (1) and (2) yields :

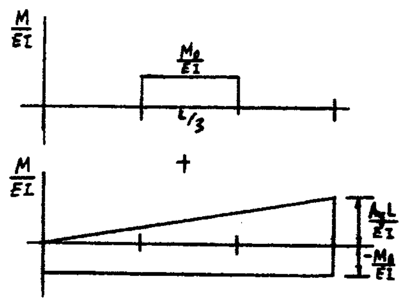
$A_y = 0$

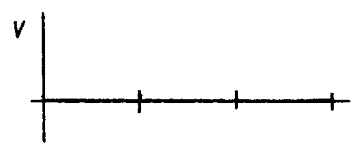
$M_A = \frac{M_0}{3}$ **Ans**

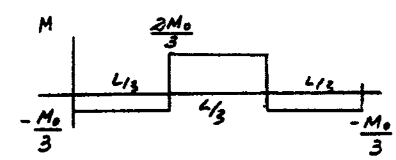
Due to symmetry :

$B_y = 0$

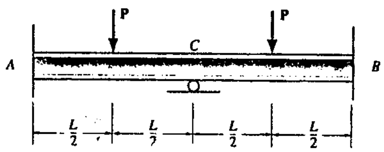
$M_B = \frac{M_0}{3}$ **Ans**

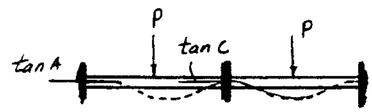


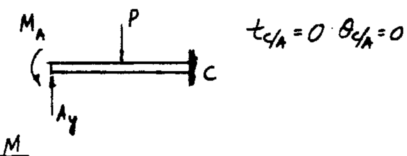


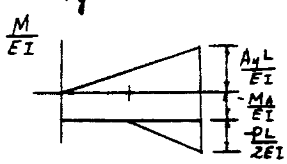


12–115 Determine the reactions at the supports, and then draw the shear and moment diagrams. EI is constant.









$$\theta_{C/A} = 0 = \frac{1}{2}(\frac{A_y L}{EI})(L) + (\frac{-M_A}{EI})(L) + \frac{1}{2}(\frac{-PL}{2EI})(\frac{L}{2})$$

$$0 = \frac{1}{2}A_y L - M_A - \frac{PL}{8} \qquad (1)$$

$$t_{C/A} = 0 = \frac{1}{2}(\frac{A_y L}{EI})(L)(\frac{L}{3}) + (\frac{-M_A}{EI})(L)(\frac{L}{2}) + \frac{1}{2}(\frac{-PL}{2EI})(\frac{L}{2})(\frac{L}{6})$$

$$0 = \frac{A_y L}{6} - \frac{M_A}{2} - \frac{PL}{48} \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$A_y = \frac{P}{2}$ **Ans**

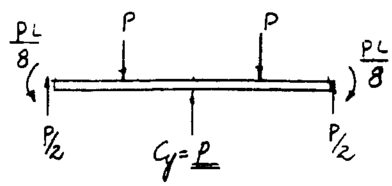
$M_A = \frac{PL}{8}$ **Ans**

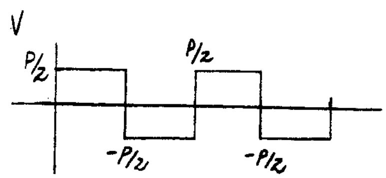
Due to symmetry :

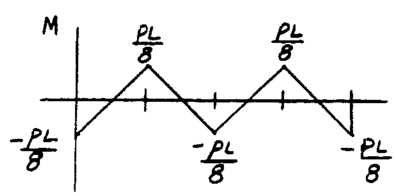
$B_y = \frac{P}{2}$ **Ans**

$M_B = \frac{PL}{8}$ **Ans**

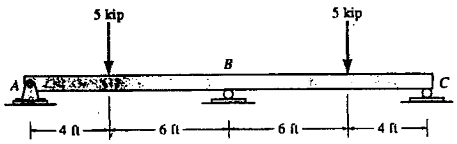
$C_y = P$ **Ans**

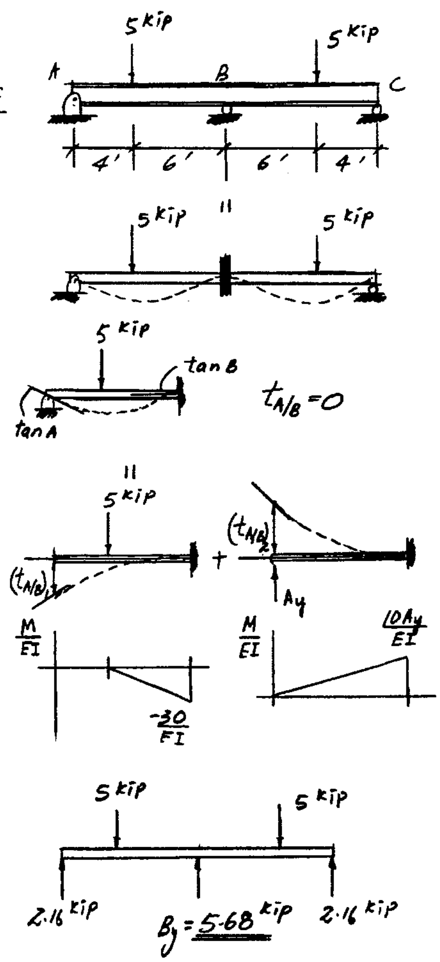






•12–116 Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.





$$(t_{A/B})_1 = \frac{1}{2}\left(\frac{-30}{EI}\right)(6)(4+4) = \frac{-720}{EI}$$

$$(t_{A/B})_2 = \frac{1}{2}\left(\frac{10A_y}{EI}\right)(10)\left(\frac{20}{3}\right) = \frac{333.33\,A_y}{EI}$$

$$t_{A/B} = 0 = (t_{A/B})_1 + (t_{A/B})_2$$

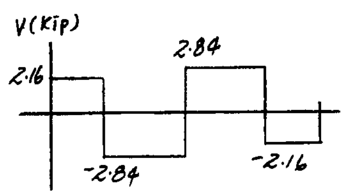
$$0 = \frac{-720}{EI} + \frac{333.33\,A_y}{EI}$$

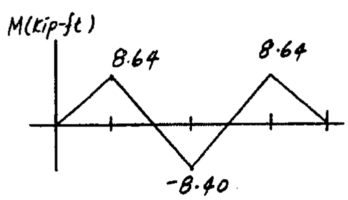
$A_y = 2.16$ kip **Ans**

Due to symmetry :

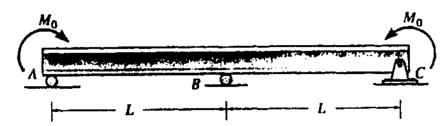
$C_y = 2.16$ kip **Ans**

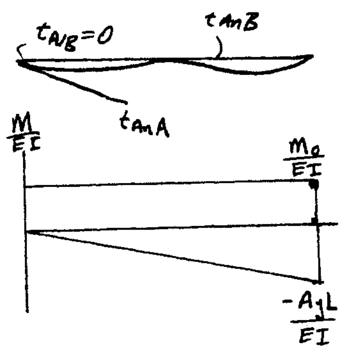
$B_y = 5.68$ kip **Ans**





12–117 Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant.





Require :

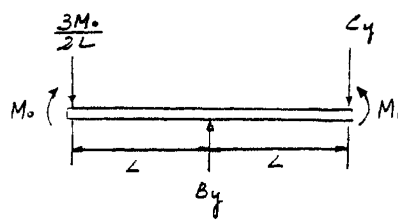
$$t_{A/B} = 0 = \left(\frac{M_0}{EI}\right)(L)\left(\frac{L}{2}\right) + \frac{1}{2}\left(\frac{-A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right)$$

$$0 = \frac{M_0 L^2}{2EI} - \frac{A_y L^3}{3EI}; \qquad A_y = \frac{3M_0}{2L} \qquad \textbf{Ans}$$

Equilibrium :

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{3M_0}{2L}(L) - C_y(L) = 0$$

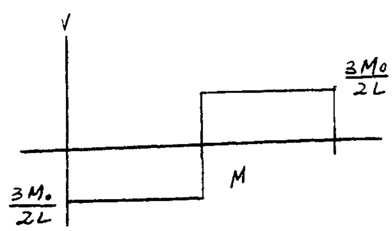
$$C_y = \frac{3M_0}{2L} \qquad \textbf{Ans}$$

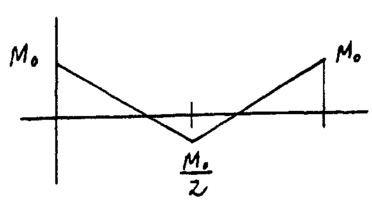


$$+\uparrow \Sigma F_y = 0; \qquad B_y - \frac{3M_0}{2L} - \frac{3M_0}{2L} = 0$$

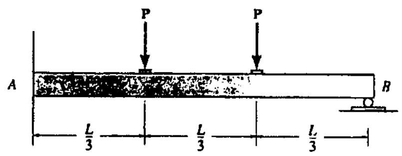
$$B_y = \frac{3M_0}{L} \qquad \textbf{Ans}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x = 0 \qquad \textbf{Ans}$$



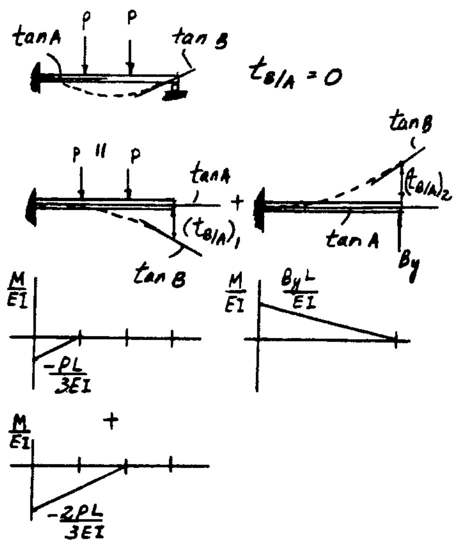


12–118 Determine the reactions at the supports A and B, then draw the shear and moment diagrams. EI is constant.



$$(t_{B/A})_1 = \frac{1}{2}\left(\frac{-PL}{3EI}\right)\left(\frac{L}{3}\right)\left(\frac{2L}{3}+\frac{2L}{9}\right)+\frac{1}{2}\left(\frac{-2PL}{3EI}\right)\left(\frac{2L}{3}\right)\left(\frac{L}{3}+\frac{4L}{9}\right) = -\frac{2PL^3}{9EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(\frac{B_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{B_y L^3}{3EI}$$



$$t_{B/A} = 0 = (t_{B/A})_1 + (t_{B/A})_2$$

$$0 = -\frac{2PL^3}{9EI} + \frac{B_y L^3}{3EI}$$

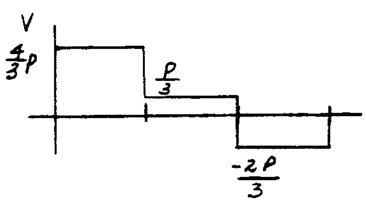
$$B_y = \frac{2}{3}P \qquad \textbf{Ans}$$

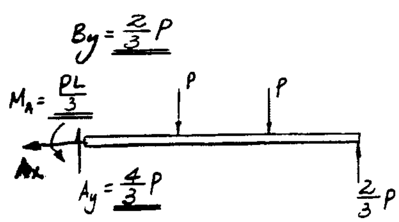
From the free-body diagram,

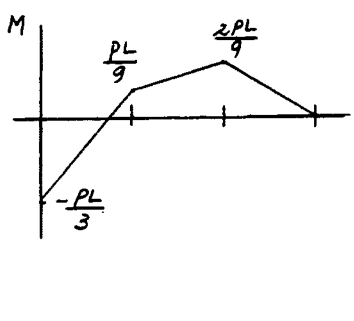
$$M_A = \frac{PL}{3} \qquad \textbf{Ans}$$

$$A_y = \frac{4}{3}P \qquad \textbf{Ans}$$

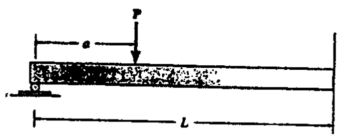
$$A_x = 0 \qquad \textbf{Ans}$$







12-119. Determine the value of a for which the maximum positive moment has the same magnitude as the maximum negative moment. EI is constant.



$$(t_{A/B})_1 = \frac{1}{2}\left(\frac{-P(L-a)}{EI}\right)(L-a)\left(a+\frac{2(L-a)}{3}\right) = \frac{-P(L-a)^2(2L+a)}{6EI}$$

$$(t_{A/B})_2 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) = \frac{A_y L^3}{3EI}$$

$$t_{A/B} = 0 = (t_{A/B})_1 + (t_{A/B})_2$$

$$0 = \frac{-P(L-a)^2(2L+a)}{6EI} + \frac{A_y L^3}{3EI}$$

$$A_y = \frac{P(L-a)^2(2L+a)}{2L^3}$$

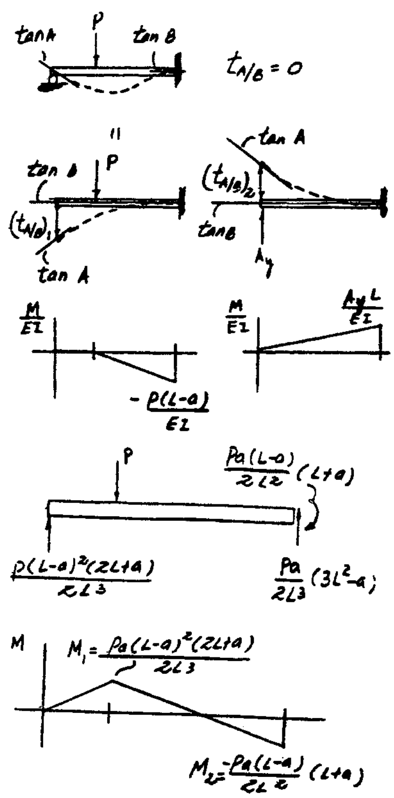
Require :

$$|M_1| = |M_2|$$

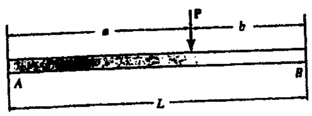
$$\frac{Pa(L-a)^2(2L+a)}{2L^3} = \frac{Pa(L-a)(L+a)}{2L^2}$$

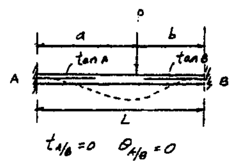
$$a^2 + 2La - L^2 = 0$$

$$a = 0.414L \qquad \textbf{Ans}$$



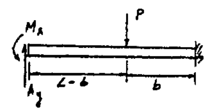
***12-120.** Determine the moment reactions at the supports A and B. EI is constant.





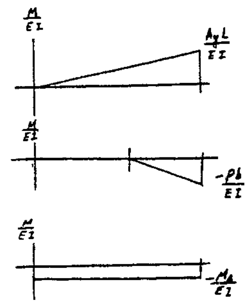
Require :

$$\theta_{A/B} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)L + \frac{1}{2}\left(-\frac{Pb}{EI}\right)(b) + \left(-\frac{M_A}{EI}\right)(L)$$



$$A_y L^2 - Pb^2 - 2M_A L = 0 \qquad (1)$$

$$t_{A/B} = 0 = \frac{1}{2}\left(\frac{A_y L}{EI}\right)(L)\left(\frac{2L}{3}\right) + \frac{1}{2}\left(-\frac{Pb}{EI}\right)(b)\left(L - \frac{b}{3}\right) + \left(-\frac{M_A}{EI}\right)(L)\left(\frac{L}{2}\right)$$

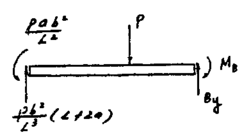


$$2A_y L^3 - 3Pb^2 L + Pb^3 - 3M_A L^2 = 0 \qquad (2)$$

Solving Eqs. (1) and (2) yields :

$$M_A = \frac{Pb^2(L-b)}{L^2}; \qquad L - b = a$$

$$M_A = \frac{Pab^2}{L^2} \qquad \textbf{Ans}$$



$$A_y = \frac{Pb^2}{L^3}(L + 2a)$$

Equilibrium :

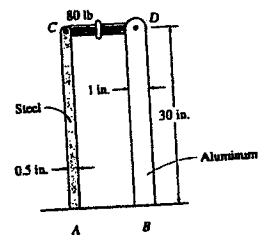
$$+\Sigma M_B = 0; \qquad \frac{Pab^2}{L^2} - \left[\frac{Pb^2}{L^3}(L + 2a)\right]L + Pb - M_B = 0$$

$$M_B = \frac{PbL^2 - Pb^2L - Pab^2}{L^2}$$

$$= \frac{Pb(a+b)^2 - Pb^2(a+b) - Pab^2}{L^2}$$

$$= \frac{Pa^2b}{L^2} \qquad \textbf{Ans}$$

12-121. The assembly consists of a steel and an aluminum bar, each of which is 1 in. thick, fixed at its ends A and B, and pin connected to the *rigid* short link CD. If a horizontal force of 80 lb is applied to the link as shown, determine the moments created at A and B. $E_{st} = 29(10^3)$ ksi, $E_{al} = 10(10^3)$ ksi.



$\leftarrow \Sigma F_x = 0 \qquad P_{al} + P_{st} - 80 = 0 \qquad (1)$

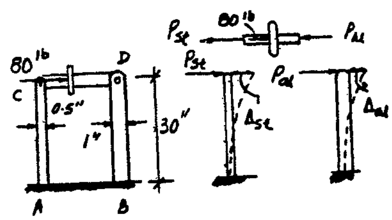
Compatibility condition :

$$\Delta_{st} = \Delta_{al}$$

$$\frac{P_{st} L^3}{3E_{st} I_{st}} = \frac{P_{al} L^3}{3E_{al} I_{al}}$$

$$P_{st} = \left(\frac{E_{st} I_{st}}{E_{al} I_{al}}\right)(P_{al}) = \frac{(29)(10^3)(\frac{1}{12})(1)(0.5^3)}{(10)(10^3)(\frac{1}{12})(1)(1^3)} P_{al}$$

$$P_{st} = 0.3625\, P_{al} \qquad (2)$$



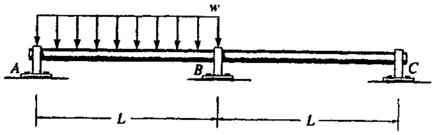
Solving Eqs. (1) and (2) yields :

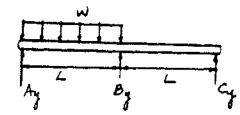
$P_{al} = 58.72$ lb $\qquad P_{st} = 21.28$ lb

$M_A = P_{st}(30) = 639$ lb · in. $= 0.639$ kip · in. **Ans**

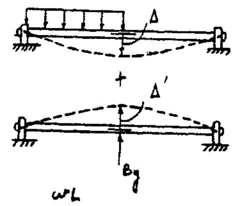
$M_B = P_{al}(30) = 1761$ lb · in. $= 1.76$ kip · in. **Ans**

12–122 Determine the reactions at the supports, then draw the shear and moment diagrams. EI is constant. The bearings exert only vertical reactions on the shaft.





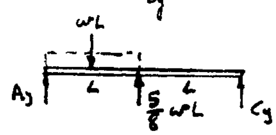
$$\Delta = \frac{5w(2L)^4}{768EI} = \frac{5wL^4}{48EI} \downarrow$$



$$\Delta' = \frac{B_y(2L)^3}{48EI} = \frac{B_y L^3}{6EI} \uparrow$$

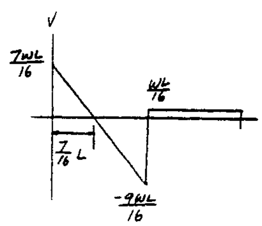
Require :

$$(+\downarrow) \qquad 0 = \Delta - \Delta'$$



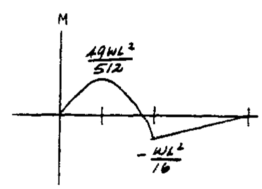
$$0 = \frac{5wL^4}{48EI} - \frac{B_y L^3}{6EI}$$

$$B_y = \frac{5}{8}wL \uparrow \qquad \textbf{Ans}$$



$$\circlearrowleft + \Sigma M_A = 0; \qquad wL\left(\frac{L}{2}\right) - \frac{5}{8}wL(L) - C_y(2L) = 0$$

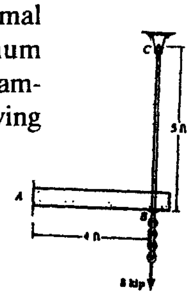
$$C_y = -\frac{wL}{16} = \frac{wL}{16} \downarrow \qquad \textbf{Ans}$$



$$+\uparrow \Sigma F_y = 0; \qquad -wL - \frac{wL}{16} + \frac{5}{8}wL + A_y = 0$$

$$A_y = \frac{7}{16}wL \qquad \textbf{Ans}$$

12-123. The A-36 steel beam and rod are used to support the load of 8 kip. If it is required that the allowable normal stress for the steel is $\sigma_{\text{allow}} = 18$ ksi, and the maximum deflection not exceed 0.05 in., determine the smallest diameter rod that should be used. The beam is rectangular, having a height of 5 in. and a thickness of 3 in.



$$\delta_r = \delta_b$$

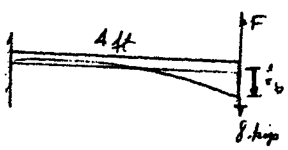
$$\frac{F(5)(12)}{AE} = \frac{(8-F)(48)^3}{3E(\frac{1}{12})(3)(5)^3}$$

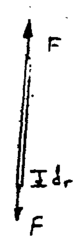
Assume rod reaches its maximum stress.

$$\sigma = \frac{F}{A} = 18(10^3)$$

$$\frac{18(5)(12)}{E} = \frac{1179.648(8-F)}{E}$$

$$F = 7.084 \text{ kip}$$





Maximum stress in beam,

$$\sigma = \frac{Mc}{I} = \frac{(8-7.084)(48)(2.5)}{\frac{1}{12}(3)(5)^3} = 3.52 \text{ ksi} < 18 \text{ ksi} \qquad \text{OK}$$

Maximum deflection

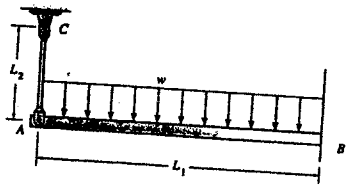
$$\delta = \frac{PL^3}{3EI} = \frac{(8-7.084)(48)^3}{3(29)(10^3)(\frac{1}{12})(3)(5)^3} = 0.0372 \text{ in.} < 0.05 \text{ in.} \qquad \text{OK}$$

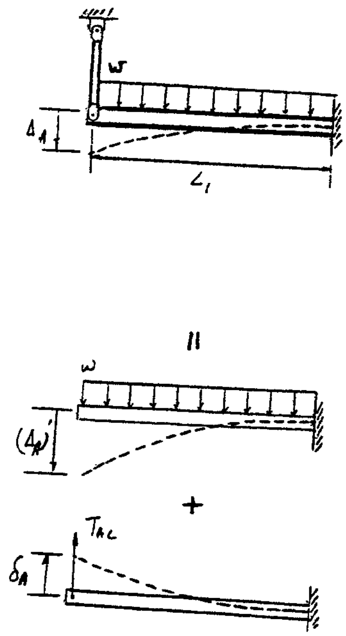
Thus,

$$A = \frac{7.084}{18} = 0.39356 \text{ in}^2 = \frac{1}{4}\pi d^2$$

$$d = 0.708 \text{ in.} \qquad \textbf{Ans}$$

***12-124.** The beam has a constant $E_1 I_1$ and is supported by the fixed wall at B and the rod AC. If the rod has a cross-sectional area A_2 and the material has a modulus of elasticity E_2, determine the force in the rod.





$$(\Delta_A)' = \frac{wL_1^4}{8E_1 I_1}; \qquad \Delta_A = \frac{T_{AC}L_2}{A_2 E_2}$$

$$\delta_A = \frac{T_{AC}L_1^3}{3E_1 I_1}$$

By superposition :

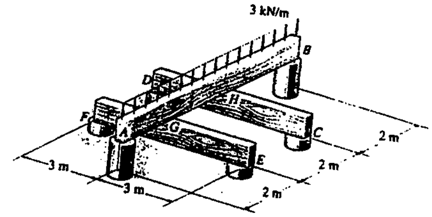
$(+\downarrow) \qquad \Delta_A = (\Delta_A)' - \delta_A$

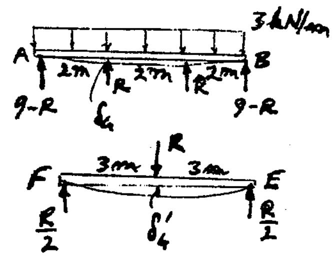
$$\frac{T_{AC}L_2}{A_2 E_2} = \frac{wL_1^4}{8E_1 I_1} - \frac{T_{AC}L_1^3}{3E_1 I_1}$$

$$T_{AC}\left(\frac{L_2}{A_2 E_2} + \frac{L_1^3}{3E_1 I_1}\right) = \frac{wL_1^4}{8E_1 I_1}$$

$$T_{AC} = \frac{3wA_2 E_2 L_1^4}{8[3E_1 I_1 L_2 + A_2 E_2 L_1^3]}$$ **Ans**

12-125. The assembly consists of three simply supported beams for which the bottom of the top beam rests on the top of the bottom two. If a uniform load of 3 kN/m is applied to the top beam, determine the vertical reactions at each of the supports. EI is constant.





$$\delta_G = \delta_G'$$

$$\delta_G = \delta_1 + \delta_2 + \delta_3$$

$$\delta_1 = \frac{w(L/3)}{24EI}((L/3)^3 - 2L(L/3)^2 + L^3)$$

Set $L = 8\text{m}$, $w = 3\text{ kN/m}$

$$\delta_1 = \frac{139.062}{EI}\ \downarrow$$

$$\delta_2 = \frac{P(\frac{1}{3}L)(\frac{1}{3}L)}{6EI(L)}(L^2 - (\frac{1}{3}L)^2 - (\frac{1}{3}L)^2)$$

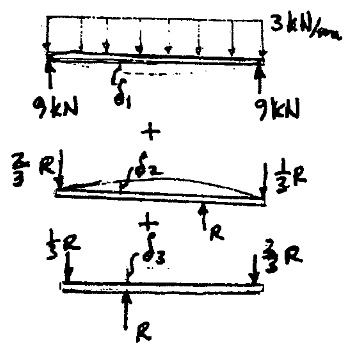
Set $L = 8\text{m}$, $P = R$

$$\delta_2 = \frac{7.374\,R}{EI}\ \uparrow$$

$$\delta_3 = \frac{P(\frac{2}{3}L)(\frac{1}{3}L)}{6EIL}(L^2 - (\frac{2}{3}L)^2 - (\frac{1}{3}L)^2)$$

Set $L = 8\text{m}$, $P = R$

$$\delta_3 = \frac{8.428\,R}{EI}\ \uparrow$$



Thus,

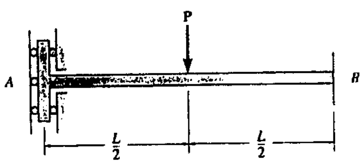
$$\frac{139.062}{EI} - \frac{7.374\,R}{EI} - \frac{8.428\,R}{EI} = \frac{R(6)^3}{48\,EI}$$

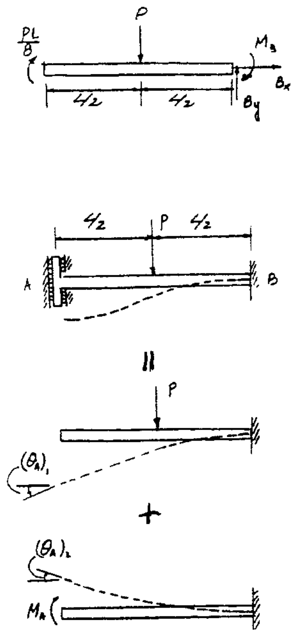
$$R = 6.850\text{ kN}$$

Thus,

$A_y = B_y = 9 - 6.850 = 2.15\text{ kN}$ **Ans**

12–126 Determine the reactions at A and B. Assume the support at A only exerts a moment on the beam. EI is constant.





$$(\theta_A)_1 = \frac{PL^2}{8EI}; \qquad (\theta_A)_2 = \frac{M_A L}{EI}$$

By superposition :

$$0 = (\theta_A)_1 - (\theta_A)_2$$

$$0 = \frac{PL^2}{8EI} - \frac{M_A L}{EI}$$

$$M_A = \frac{PL}{8} \qquad \textbf{Ans}$$

Equilibrium :

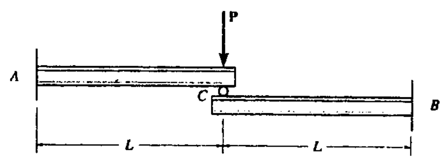
$$\circlearrowleft + \Sigma M_B = 0; \qquad -\frac{PL}{8} + \frac{PL}{2} - M_B = 0$$

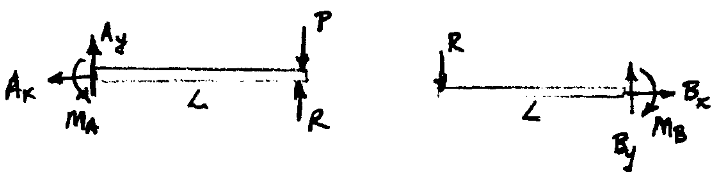
$$M_B = \frac{3PL}{8} \qquad \textbf{Ans}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0 \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y = P \qquad \textbf{Ans}$$

12–127 The compound beam segments meet in the center using a smooth contact (roller). Determine the reactions at the fixed supports A and B when the load **P** is applied. EI is constant.





$$\Delta_C = \frac{(P-R)L^3}{3EI} = \frac{RL^3}{3EI}$$

$$R = \frac{P}{2}$$

Member AC:

$\Sigma F_y = 0;\quad A_y = \frac{P}{2}$ **Ans**

$\Sigma F_x = 0;\quad A_x = 0$ **Ans**

$\Sigma M_A = 0;\quad M_A = \frac{PL}{2}$ **Ans**

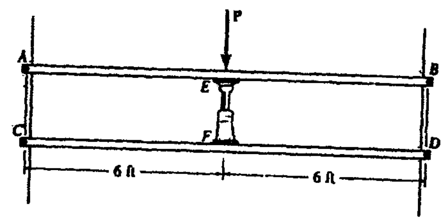
Member BC:

$\Sigma F_y = 0;\quad B_y = \frac{P}{2}$ **Ans**

$\Sigma F_x = 0;\quad B_x = 0$ **Ans**

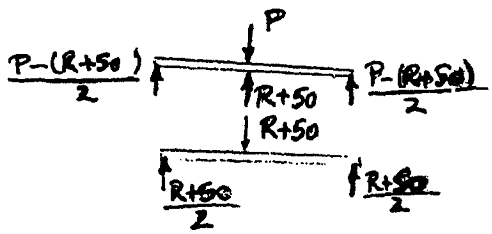
$\Sigma M_B = 0;\quad M_B = \frac{PL}{2}$ **Ans**

***12-128.** Each of the two members is made from 6061-T6 aluminum and has a square cross section 1 in. × 1 in. They are pin connected at their ends and a jack is placed between them and opened until the force it exerts on each member is 500 lb. Determine the greatest force P that can be applied to the center of the top member without causing either of the two members to yield. For the analysis neglect the axial force in each member. Assume the jack is rigid.



$$\delta_E = \delta_F$$

$$\frac{[P-(R+50\ )]L^3}{48\,EI} = \frac{(R+50\ )L^3}{48\,EI}$$



$$P = 2R + 100$$

$$R = \frac{P}{2} - 50$$

Maximum moment occurs at center of each member.

Top member :

$$M_{max} = \frac{1}{2}[(P - (\frac{P}{2} - 50\ \ + 50\ )](6)(12) = 18\,P$$

Bottom member :

$$M_{max} = \frac{1}{2}[(\frac{P}{2} - 50\ \ + 50\ )](6)(12) = 18\,P$$

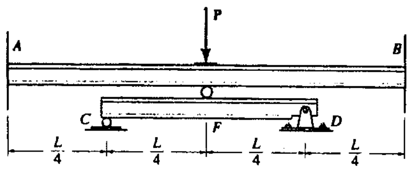
Both memberss will yield at the same time.

$$\sigma_{max} = \frac{Mc}{I}$$

$$37(10^3) = \frac{18P(\frac{1}{2})}{\frac{1}{12}(1)(1)^3}$$

$P = 343$ lb **Ans**

12–129 The fixed supported beam AB is strengthened using the simply supported beam CD and the roller at F which is set in place just before application of the load **P**. Determine the reactions at the supports if EI is constant.



δ_F = Deflection of top beam at F

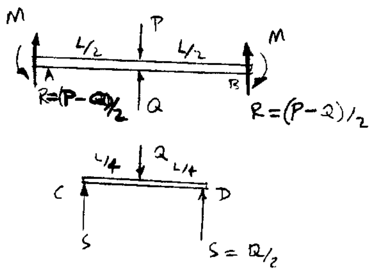
δ'_F = Deflection of bottom beam at F

$$\delta_F = \delta'_F$$

$$(+\downarrow)\ \frac{(P-Q)(L^3)}{48\,EI} - \frac{2M(\frac{L}{2})}{6\,EIL}((\frac{L}{2})^2 - 3L(\frac{L}{2}) + 2L^2) = \frac{Q(\frac{L}{2})^3}{48EI}$$

$$\frac{(P-Q)L}{48} - \frac{1}{6}M(\frac{1}{4} - \frac{3}{2} + 2) = \frac{QL}{48(8)}$$

$$8PL - 48M = 9QL \qquad (1)$$



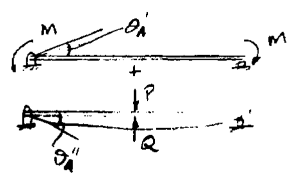
$$\theta_A = \theta'_A + \theta''_A = 0$$

$$(\curvearrowleft +)\quad -\frac{ML}{6EI} - \frac{ML}{3EI} + \frac{(P-Q)L^2}{16EI} = 0$$

$$8M = (P-Q)L \qquad (2)$$

Solving Eqs. (1) and (2) :

$M = QL/16$
$Q = 2P/3$
$S = P/3$
$R = P/6$
$M = PL/24$



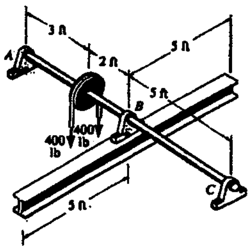
Thus,

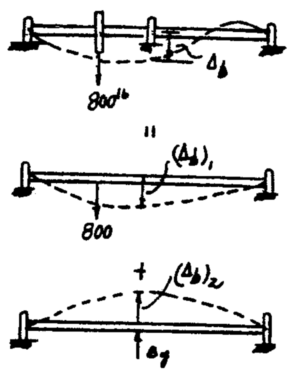
$$M_A = M_B = \frac{1}{24}PL \qquad \textbf{Ans}$$

$$A_y = B_y = \frac{1}{6}P \qquad \textbf{Ans}$$

$$C_y = D_y = \frac{1}{3}P \qquad \textbf{Ans}$$

12-130. The 1-in.-diameter A-36 steel shaft is supported by unyielding bearings at A and C. The bearing at B rests on a simply supported steel wide-flange beam having a moment of inertia of $I = 500\ \text{in}^4$. If the belt loads on the pulley are 400 lb each, determine the vertical reactions at A, B, and C.





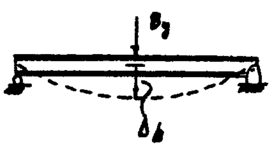
For the shaft :

$$(\Delta_b)_1 = \frac{800(3)(5)}{6EI_s(10)}(-5^2 - 3^2 + 10^2) = \frac{13200}{EI_s}$$

$$(\Delta_b)_2 = \frac{B_y(10^3)}{48EI_s} = \frac{20.833B_y}{EI_s}$$

For the beam :

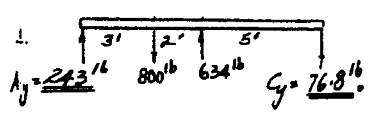
$$\Delta_b = \frac{B_y(10^3)}{48EI_b} = \frac{20.833B_y}{EI_b}$$



Compatibility condition :

$$+\downarrow\ \Delta_b = (\Delta_b)_1 - (\Delta_b)_2$$

$$\frac{20.833B_y}{EI_b} = \frac{13200}{EI_s} - \frac{20.833B_y}{EI_s}$$



$$I_S = \frac{\pi}{4}(0.5)^4 = 0.04909\ \text{in}^4$$

$$\frac{20.833B_y(0.04909)}{500} = 13200 - 20.833B_y$$

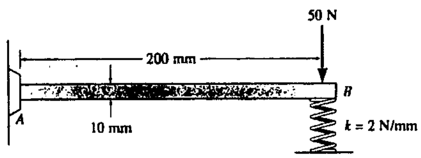
$B_y = 634$ lb **Ans**

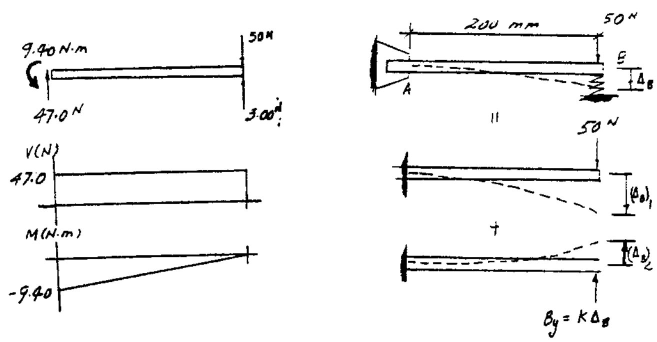
From the free-body diagram,

$A_y = 243$ lb **Ans**

$C_y = 76.8$ lb **Ans**

•12–132 Determine the deflection at the end B of the clamped A-36 steel strip. The spring has a stiffness of $k = 2$ N/mm. The strip is 5 mm wide and 10 mm high. Also, draw the shear and moment diagrams for the strip.





$$I = \frac{1}{12}(0.005)(0.01)^3 = 0.4166\,(10^{-9})\text{ m}^4$$

$$(\Delta_B)_1 = \frac{PL^3}{3EI} = \frac{50(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.0016\text{ m}$$

$$(\Delta_B)_2 = \frac{PL^3}{3EI} = \frac{2000\Delta_B(0.2^3)}{3(200)(10^9)(0.4166)(10^{-9})} = 0.064\,\Delta_B$$

Compatibility condition :

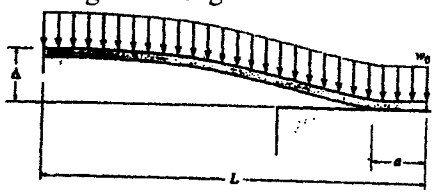
$+\downarrow \qquad \Delta_B = (\Delta_B)_1 - (\Delta_B)_2$

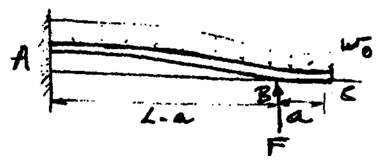
$\Delta_B = 0.0016 - 0.064\Delta_B$

$\Delta_B = 0.001503\text{ m} = 1.50\text{ mm}$ **Ans**

$B_y = k\Delta_B = 2(1.5) = 3.00\text{ N}$

12-133. The beam is made from a soft elastic material having a constant EI. If it is originally a distance Δ from the surface of its end support, determine the distance a at which it rests on this support when it is subjected to the uniform load w_0, which is great enough to cause this to happen.





The curvature of the beam in region BC is zero, therefore there is no bending moment in the region BC. The reaction F is at B where it touches the support. The slope is zero at this point and the deflection is Δ where

$$\Delta = \frac{w_0(L-a)^4}{8EI} - \frac{R(L-a)^3}{3EI}$$

$$\theta = \frac{w_0(L-a)^3}{6EI} - \frac{R(L-a)^2}{2EI}$$

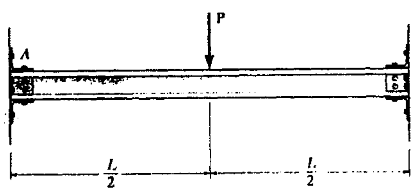
Thus,

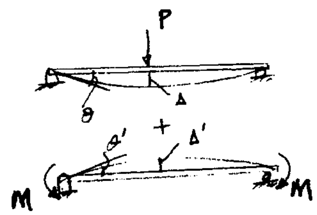
$$R = \left(\frac{8\Delta EI}{9w_0}\right)^{1/4} \qquad \textbf{Ans}$$

$$L-a = \left(\frac{72\Delta EI}{w_0}\right)^{1/4}$$

$$a = L - \left(\frac{72\Delta EI}{w_0}\right)^{\frac{1}{4}} \qquad \textbf{Ans}$$

12–134 The beam is supported by the bolted supports at its ends. When loaded these supports do not provide an actual fixed connection, but instead allow a slight rotation α before becoming fixed. Determine the moment at the connections and the maximum deflection of the beam.





$$\theta - \theta' = \alpha$$

$$\frac{PL^2}{16EI} - \frac{ML}{3EI} - \frac{ML}{6EI} = \alpha$$

$$M = (\frac{PL^2}{16EI} - \alpha)(2EI)$$

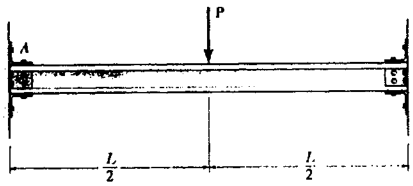
$$M = (\frac{PL}{8} - \frac{2EI}{L}\alpha) \qquad \textbf{Ans}$$

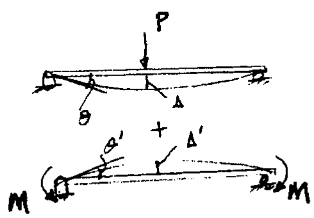
$$\Delta_{max} = \Delta - \Delta' = \frac{PL^3}{48EI} - 2[\frac{M(\frac{L}{2})}{6EIL}[(L/2)^2 - \frac{3L^2}{2} + 2L^2]]$$

$$\Delta_{max} = \frac{PL^3}{48EI} - \frac{L^2}{8EI}(\frac{PL}{8} - \frac{2EI\alpha}{L})$$

$$\Delta_{max} = \frac{PL^3}{192EI} + \frac{\alpha L}{4} \qquad \textbf{Ans}$$

12–134 The beam is supported by the bolted supports at its ends. When loaded these supports do not provide an actual fixed connection, but instead allow a slight rotation α before becoming fixed. Determine the moment at the connections and the maximum deflection of the beam.





$$\theta - \theta' = \alpha$$

$$\frac{PL^2}{16EI} - \frac{ML}{3EI} - \frac{ML}{6EI} = \alpha$$

$$M = \left(\frac{PL^2}{16EI} - \alpha\right)(2EI)$$

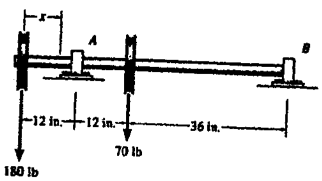
$$M = \left(\frac{PL}{8} - \frac{2EI}{L}\alpha\right) \qquad \textbf{Ans}$$

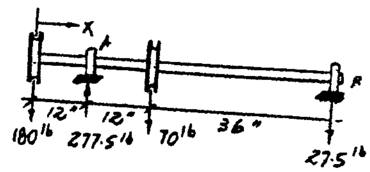
$$\Delta_{max} = \Delta - \Delta' = \frac{PL^3}{48EI} - 2\left[\frac{M(\frac{L}{2})}{6EIL}\left[(L/2)^2 - \frac{3L^2}{2} + 2L^2\right]\right]$$

$$\Delta_{max} = \frac{PL^3}{48EI} - \frac{L^2}{8EI}\left(\frac{PL}{8} - \frac{2EI\alpha}{L}\right)$$

$$\Delta_{max} = \frac{PL^3}{192EI} + \frac{\alpha L}{4} \qquad \textbf{Ans}$$

12-135. The shaft supports the two pulley loads shown. Using discontinuity functions, determine the equation of the elastic curve. The bearings at A and B exert only vertical reactions on the shaft. EI is constant.





$$M = -180 < x-0 > -(-277.5) < x-12 > -70 < x-24 >$$
$$M = -180x + 277.5 < x-12 > -70 < x-24 >$$

Elastic curve and slope:

$$EI\frac{d^2v}{dx^2} = M = -180x + 277.5 < x-12 > -70 < x-24 >$$

$$EI\frac{dv}{dx} = -90x^2 + 138.75 < x-12 >^2 - 35 < x-24 >^2 + C_1$$

$$EIv = -30x^3 + 46.25 < x-12 >^3 - 11.67 < x-24 >^3 + C_1x + C_2 \qquad (1)$$

Boundary conditions:

$v = 0$ at $x = 12$ in.

From Eq. (1)

$$0 = -51,840 + 12C_1 + C_2$$
$$12C_1 + C_2 = 51\,840 \qquad (2)$$

$v = 0$ at $x = 60$ in.

From Eq. (1)

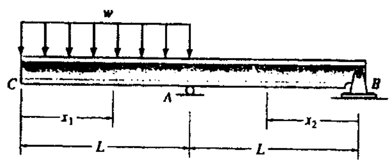
$$0 = -6\,480\,000 + 5\,114\,880 - 544\,320 + 60C_1 + C_2$$
$$60C_1 + C_2 = 1909440 \qquad (3)$$

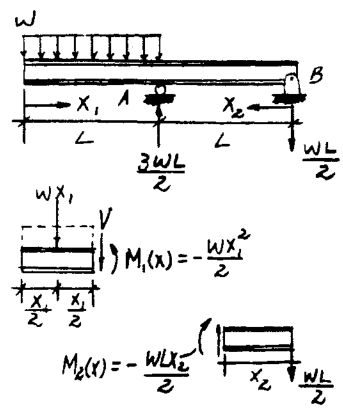
Solving Eqs. (2) and (3) yields:

$$C_1 = 38\,700 \qquad C_2 = -412\,560$$

$$v = \frac{1}{EI}[-30x^3 + 46.25 < x-12 >^3 - 11.7 < x-24 >^3 + 38\,700x - 412\,560] \qquad \textbf{Ans}$$

•12–136 Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. Specify the slope at A and the maximum deflection. EI is constant. Use the method of integration.





Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \frac{-wx_1^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad (1)$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = \frac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_2 = 0$ at $x_2 = 0$

From Eq. (4) :

$C_4 = 0$

$v_2 = 0$ at $x_2 = L$

From Eq. (4) :

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2)

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad (5)$$

Continuity conditions :

$$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2} \quad \text{at} \quad x_1 = x_2 = L$$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -\left(-\frac{wL^3}{4} + \frac{wL^3}{12}\right)$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad (6)$$

$$\theta_A = \left.\frac{dv_1}{dx_1}\right|_{x_1=L} = -\left.\frac{dv_2}{dx_2}\right|_{x_2=L} = \frac{wL^3}{6EI} \qquad \textbf{Ans}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4) \qquad \textbf{Ans}$$

$$(v_1)_{max} = \frac{-7wL^4}{24EI} \qquad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad (7) \qquad \textbf{Ans}$$

$(v_2)_{max}$ occurs when $\frac{dv_2}{dx_2} = 0$

From Eq. (6)

$$L^3 - 3Lx_2^2 = 0$$

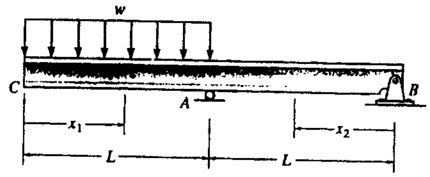
$$x_2 = \frac{L}{\sqrt{3}}$$

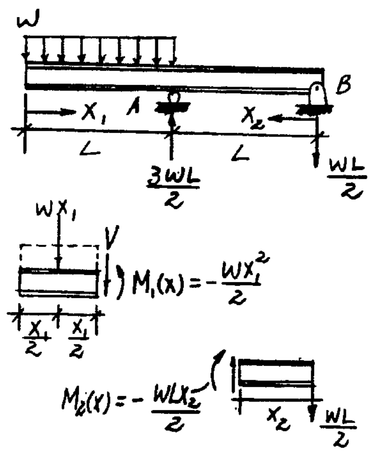
Substitute x_2 into Eq (7),

$$(v_2)_{max} = \frac{wL^4}{18\sqrt{3}\,EI} \qquad (8)$$

$$v_{max} = (v_1)_{max} = \frac{7wL^4}{24EI} \qquad (9) \qquad \textbf{Ans}$$

12–137 Determine the maximum deflection between the supports *A* and *B*. *EI* is constant. Use the method of integration.





Elastic curve and slope :

$$EI\frac{d^2v}{dx^2} = M(x)$$

For $M_1(x) = \frac{-wx_1^2}{2}$

$$EI\frac{d^2v_1}{dx_1^2} = \frac{-wx_1^2}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{-wx_1^3}{6} + C_1 \qquad (1)$$

$$EIv_1 = \frac{-wx_1^4}{24} + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = \frac{-wLx_2}{2}$

$$EI\frac{d^2v_2}{dx_2^2} = \frac{-wLx_2}{2}$$

$$EI\frac{dv_2}{dx_2} = \frac{-wLx_2^2}{4} + C_3 \qquad (3)$$

$$EIv_2 = \frac{-wLx_2^3}{12} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions :

$v_2 = 0$ at $x_2 = 0$

From Eq. (4) :

$C_4 = 0$

$v_2 = 0$ at $x_2 = L$

From Eq. (4) :

$$0 = \frac{-wL^4}{12} + C_3L$$

$$C_3 = \frac{wL^3}{12}$$

$v_1 = 0$ at $x_1 = L$

From Eq. (2)

$$0 = -\frac{wL^4}{24} + C_1L + C_2 \qquad (5)$$

Continuity conditions :

$\frac{dv_1}{dx_1} = \frac{dv_2}{-dx_2}$ at $x_1 = x_2 = L$

From Eqs. (1) and (3)

$$-\frac{wL^3}{6} + C_1 = -(-\frac{wL^3}{4} + \frac{wL^3}{12})$$

$$C_1 = \frac{wL^3}{3}$$

Substitute C_1 into Eq. (5)

$$C_2 = -\frac{7wL^4}{24}$$

$$\frac{dv_1}{dx_1} = \frac{w}{6EI}(2L^3 - x_1^3)$$

$$\frac{dv_2}{dx_2} = \frac{w}{12EI}(L^3 - 3Lx_2^2) \qquad (6)$$

$$\theta_A = \frac{dv_1}{dx_1}\bigg|_{x_1=L} = -\frac{dv_2}{dx_2}\bigg|_{x_2=L} = \frac{wL^3}{6EI} \qquad \textbf{Ans}$$

$$v_1 = \frac{w}{24EI}(-x_1^4 + 8L^3x_1 - 7L^4) \qquad \textbf{Ans}$$

$$(v_1)_{max} = \frac{-7wL^4}{24EI} \qquad (x_1 = 0)$$

The negative sign indicates downward displacement.

$$v_2 = \frac{wL}{12EI}(L^2x_2 - x_2^3) \qquad (7) \qquad \textbf{Ans}$$

$(v_2)_{max}$ occurs when $\frac{dv_2}{dx_2} = 0$

From Eq. (6)

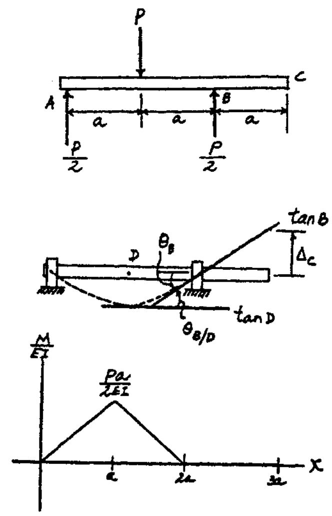
$$L^3 - 3Lx_2^2 = 0$$

$$x_2 = \frac{L}{\sqrt{3}}$$

Substitute x_2 into Eq (7),

$$(v_2)_{max} = \frac{wL^4}{18\sqrt{3}\,EI} \qquad \textbf{Ans}$$

12-138. If the bearings at A and B exert only vertical reactions on the shaft, determine the slope at B and the deflection at C. EI is constant. Use the moment-area theorems.



Support Reaction and Elastic Curve : As shown.

M/EI Diagram : As shown.

Moment-Area Theorems :

$$\theta_{B/D} = \frac{1}{2}\left(\frac{Pa}{2EI}\right)(a) = \frac{Pa^2}{4EI}$$

Due to symmetry, the slope at point D is zero. Hence, the slope at B is

$$\theta_B = |\theta_{B/D}| = \frac{Pa^2}{4EI} \qquad \textbf{Ans}$$

The displacement at C is

$$\Delta_C = \theta_B L_{BC} = \frac{Pa^2}{4EI}(a) = \frac{Pa^3}{4EI} \uparrow \qquad \textbf{Ans}$$

12-139. The bearing supports A, B, and C exert only vertical reactions on the shaft. Determine these reactions, then draw the shear and moment diagrams. EI is constant. Use the moment-area theorems.

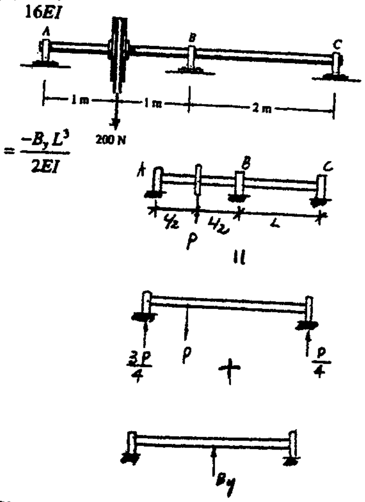
$$(t_{B/A})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{2}+\frac{L}{6}\right)+\frac{1}{2}\left(\frac{PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{3}\right)+\frac{PL}{4EI}\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)=\frac{5PL^3}{48EI}$$

$$(t_{C/A})_1 = \frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{L}{2}\right)\left(\frac{3L}{2}+\frac{L}{6}\right)+\frac{1}{2}\left(\frac{3PL}{8EI}\right)\left(\frac{3L}{2}\right)(L)=\frac{7PL^3}{16EI}$$

$$(t_{B/A})_2 = \frac{1}{2}\left(\frac{-B_yL}{2EI}\right)(L)\left(\frac{L}{3}\right)=\frac{-B_yL^3}{12EI}$$

$$(t_{C/A})_2 = \frac{1}{2}\left(\frac{-B_yL}{2EI}\right)(L)\left(L+\frac{L}{3}\right)+\frac{1}{2}\left(\frac{-B_yL}{2EI}\right)(L)\left(\frac{2}{3}L\right)=\frac{-B_yL^3}{2EI}$$

$$2t_{B/A} = t_{C/A}$$



$$2[(t_{B/A})_1 + (t_{B/A})_2] = (t_{C/A})_1 + (t_{C/A})_2$$

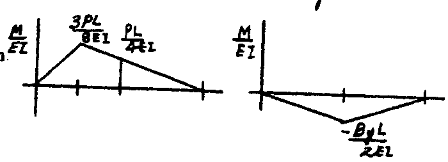
$$2\left[\frac{5PL^3}{48EI}+\left(\frac{-B_yL^3}{12EI}\right)\right]=\frac{7PL^3}{16EI}+\left(\frac{-B_yL^3}{2EI}\right)$$

$$B_y = \frac{11}{16}P$$

Thus,

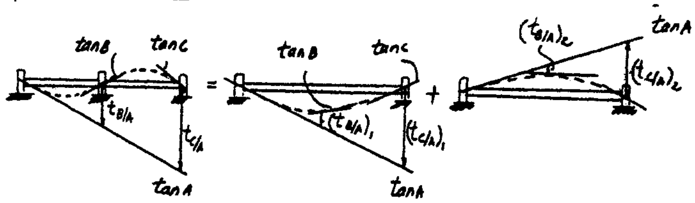
$$B_y = \frac{11}{16}(200) = 138 \text{ N} \qquad \textbf{Ans}$$

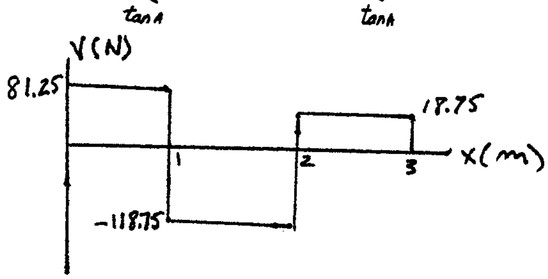
As shown on the free-body diagram

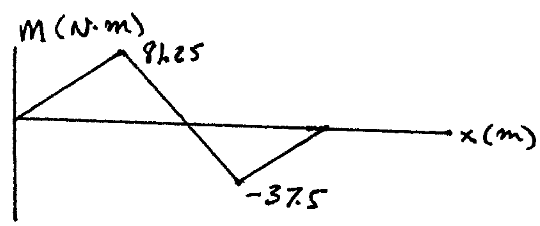


$A_y = 81.3$ N **Ans**

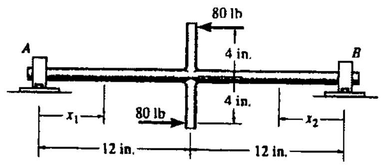
$C_y = 18.8$ N **Ans**

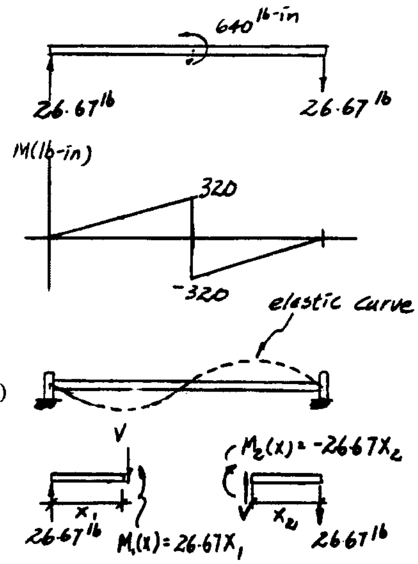






•12–140 The shaft is supported by a journal bearing at A, which exerts only vertical reactions on the shaft, and by a thrust bearing at B, which exerts both horizontal and vertical reactions on the shaft. Draw the bending-moment diagram for the shaft and then, from this diagram, sketch the deflection or elastic curve for the shaft's centerline. Determine the equations of the elastic curve using the coordinates x_1 and x_2. EI is constant.





For $M_1(x) = 26.67\,x_1$

$$EI\frac{d^2v_1}{dx_1^2} = 26.67x_1$$

$$EI\frac{dv_1}{dx_1} = 13.33x_1^2 + C_1 \qquad (1)$$

$$EIv_1 = 4.44x_1^3 + C_1x_1 + C_2 \qquad (2)$$

For $M_2(x) = -26.67x_2$

$$EI\frac{d^2v_2}{dx_2^2} = -26.67x_2$$

$$EI\frac{dv_2}{dx_2} = -13.33x_2^2 + C_3 \qquad (3)$$

$$EIv_2 = -4.44x_2^3 + C_3x_2 + C_4 \qquad (4)$$

Boundary conditions :

$v_1 = 0 \quad$ at $\quad x_1 = 0$

From Eq. (2)

$C_2 = 0$

$v_2 = 0 \quad$ at $\quad x_2 = 0$

$C_4 = 0$

Continuity conditions :

$$\frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2} \quad \text{at} \quad x_1 = x_2 = 12$$

From Eqs. (1) and (3)

$$1920 + C_1 = -(-1920 + c_3)$$

$$C_1 = -C_3 \qquad (5)$$

$v_1 = v_2 \quad$ at $\quad x_1 = x_2 = 12$

$$7680 + 12C_1 = -7680 + 12C_3$$

$$C_3 - C_1 = 1280 \qquad (6)$$

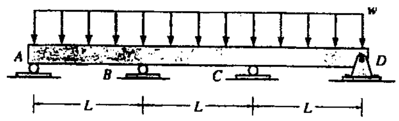
Solving Eqs. (5) and (6) yields :

$C_3 = 640 \qquad C_1 = -640$

$$v_1 = \frac{1}{EI}(4.44x_1^3 - 640x_1)\,\text{lb}\cdot\text{in}^3 \qquad \textbf{Ans}$$

$$v_2 = \frac{1}{EI}(-4.44x_2^3 + 640x_2)\,\text{lb}\cdot\text{in}^3 \qquad \textbf{Ans}$$

12–141 Determine the reactions at the supports. EI is constant. Use the method of superposition.



$$\Delta_B = \Delta_C = \frac{wL}{24EI}[L^3 - 2(3L)L^2 + (3L)^3]$$

$$= \frac{11wL^4}{12EI}$$

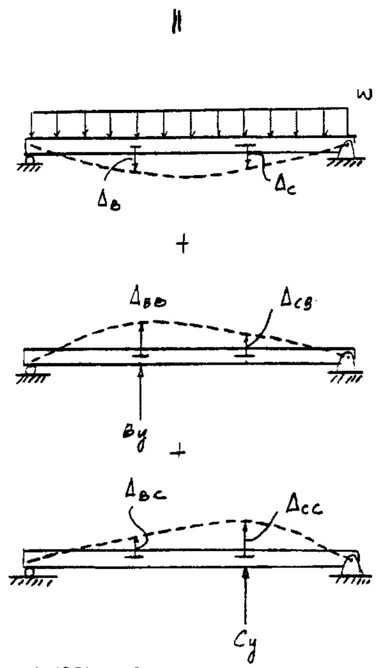
Due to symmetry, $B_y = C_y$

$$\Delta_{BB} = \Delta_{CC} = \frac{B_y(L)(2L)}{6EI(3L)}[(3L)^2 - (2L)^2 - L^2]$$

$$= \frac{4B_yL^3}{9EI}$$

$$\Delta_{BC} = \Delta_{CB} = \frac{B_y(L)(L)}{6EI(3L)}[-L^2 - L^2 + (3L)^2]$$

$$= \frac{7B_yL^3}{18EI}$$



By superposition :

$+\downarrow \quad 0 = \Delta_B - \Delta_{BB} - \Delta_{BC}$

$$0 = \frac{11wL^4}{12EI} - \frac{4B_yL^3}{9EI} - \frac{7B_yL^3}{18EI}$$

$$B_y = C_y = \frac{11wL}{10} \qquad \textbf{Ans}$$

Equilibrium :

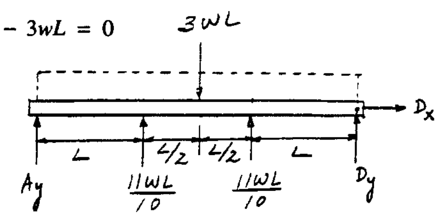
$$\circlearrowleft + \Sigma M_D = 0; \quad 3wL\left(\frac{3L}{2}\right) - \frac{11wL}{10}(L) - \frac{11wL}{10}(2L) - A_y(3L) = 0$$

$$A_y = \frac{2wL}{5} \qquad \textbf{Ans}$$

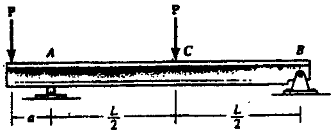
$$\uparrow + \Sigma F_y = 0; \quad \frac{2wL}{5} + \frac{11wL}{10} + \frac{11wL}{10} + D_y - 3wL = 0$$

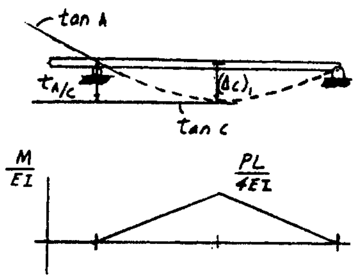
$$D_y = \frac{2wL}{5} \qquad \textbf{Ans}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad D_x = 0 \qquad \textbf{Ans}$$



12-142. Determine the value of a so that the deflection at C is equal to zero. EI is constant. Use the moment-area theorems.





$$(\Delta_C)_1 = t_{A/C} = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right) = \frac{PL^3}{48EI}\downarrow$$

$$t_{B/A} = \frac{1}{2}\left(\frac{-Pa}{EI}\right)(L)\left(\frac{2}{3}\right)(L) = -\frac{PaL^2}{3EI}$$

$$t_{C/A} = \left(\frac{-Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + \frac{1}{2}\left(\frac{-Pa}{2EI}\right)\left(\frac{L}{2}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right) = \frac{-5PaL^2}{48EI}$$

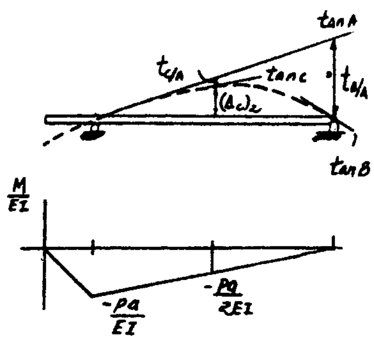
$$(\Delta_C)_2 = \frac{1}{2}|t_{B/A}| - |t_{C/A}| = \frac{PaL^2}{6EI} - \frac{5PaL^2}{48EI} = \frac{PaL^2}{16EI}\uparrow$$

Require:

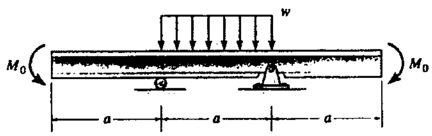
$+\uparrow \quad 0 = (\Delta_C)_2 - (\Delta_C)_1$

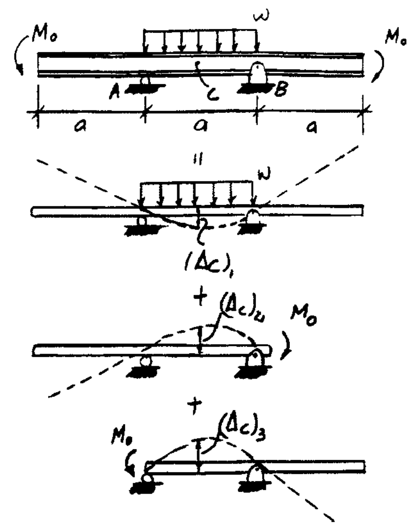
$$0 = \frac{PaL^2}{16EI} - \frac{PL^3}{48EI}$$

$$a = \frac{L}{3} \qquad \textbf{Ans}$$



*12-143. Using the method of superposition, determine the magnitude of M_0 in terms of the distributed load w and dimension a so that the deflection at the center of the beam is zero. EI is constant.





$$(\Delta_C)_1 = \frac{5wa^4}{384EI} \downarrow$$

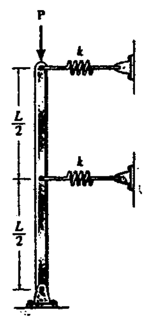
$$(\Delta_C)_2 = (\Delta_C)_3 = \frac{M_0 a^2}{16EI} \uparrow$$

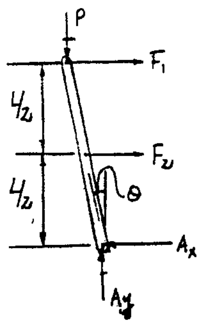
$$\Delta_C = 0 = (\Delta_C)_1 + (\Delta_C)_2 + (\Delta_C)_3$$

$$+\uparrow \qquad 0 = \frac{-5wa^4}{384EI} + \frac{M_0 a^2}{8EI}$$

$$M_0 = \frac{5wa^2}{48} \qquad \textbf{Ans}$$

13–1. Determine the critical buckling load for the column. The material can be assumed rigid.





$F_1 = k(L\,\theta)\,; \qquad F_2 = k(\frac{L}{2}\,\theta)$

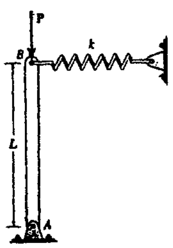
$\curvearrowleft + \Sigma M_A = 0; \qquad P\,(\theta)(L) - (F_1 L) - F_2(\frac{L}{2}) = 0$

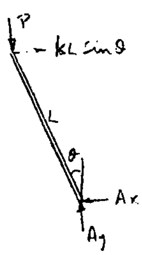
$P(\theta)(L) - kL^2\,\theta - k(\frac{L}{2})^2\theta = 0$

Require :

$P_{cr} = kL + \frac{kL}{4} = \frac{5kL}{4}$ **Ans**

13-2. The column consists of a rigid member that is pinned at its bottom and attached to a spring at its top. If the spring is unstretched when the column is in the vertical position, determine the critical load that can be placed on the column.





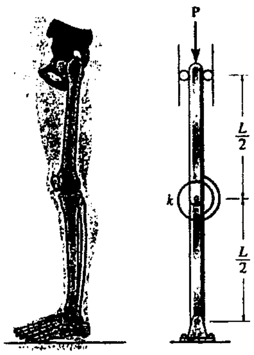
$\circlearrowleft + \Sigma M_A = 0;\quad PL\sin\theta - (kL\sin\theta)(L\cos\theta) = 0$

$$P = kL\cos\theta$$

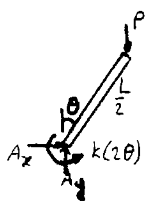
Since θ is small $\cos\theta \approx 1$

$$P_{cr} = kL \quad \textbf{Ans}$$

13-3 The leg in (a) acts as a column and can be modeled (b) by the two pin-connected members that are attached to a torsional spring having a stiffness k (torque/rad). Determine the critical buckling load. Assume the bone material is rigid.



(a) (b)

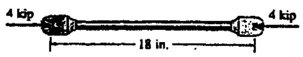


$$\curvearrowleft + \Sigma M_A = 0; \qquad -P(\theta)\left(\frac{L}{2}\right) + 2k\theta = 0$$

Require :

$$P_{cr} = \frac{4k}{L} \qquad \textbf{Ans}$$

***13-4.** The aircraft link is made from an A-36 steel rod. Determiné the smallest diameter of the rod, to the nearest $\frac{1}{16}$ in., that will support the load of 4 kip without buckling. The ends are pin connected.



$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

$$4 = \frac{\pi^2(29)(10^3)\left(\frac{\pi d^4}{64}\right)}{((1.0)(18))^2}$$

$$d = 0.551 \text{ in.}$$

$$\text{Use } d = \frac{9}{16} \text{in.} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4}{\frac{\pi}{4}(0.551^2)} = 16.7 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

13-5 A square bar is made from PVC plastic that has a modulus of elasticity of $E = 1.25(10^6)$ psi and a yield strain of $\epsilon_Y = 0.001$ in./in. Determine its smallest cross-sectional dimensions a so it does not fail from elastic buckling. It is pinned at its ends and has a length of 50 in.

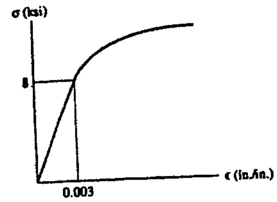
$$\sigma_Y = E\varepsilon_Y = 1.25(10^6)(0.001) = 1.25(10^3)\text{ psi}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$1.25(10^3)(a)^2 = \frac{\pi^2(1.25)(10^6)(\frac{1}{12}a^4)}{(1.0(50))^2}$$

$$a = 1.74\text{ in.} \qquad \textbf{Ans}$$

13-6. A rod made from polyurethane has a stress-strain diagram in compression as shown. If the rod is pinned at its ends and is 37 in. long, determine its smallest diameter so it does not fail from elastic buckling.



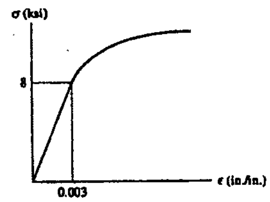
$$E = \frac{\sigma}{\varepsilon} = \frac{8(10^3)}{0.003} = 2.667(10^6)\text{ psi}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$8(10^3)\pi\,(d/2)^2 = \frac{\pi^2(2.667)(10^6)(\frac{\pi}{4})(\frac{d}{2})^4}{(1.0(37))^2}$$

$d = 2.58$ in. **Ans**

13-7. A rod made from polyurethane has a stress-strain diagram in compression as shown. If the rod is pinned at its top and fixed at its base, and is 37 in. long, determine its smallest diameter so it does not fail from elastic buckling.



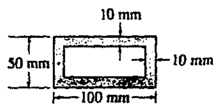
$$E = \frac{8(10^3)}{0.003} = 2.667(10^6) \text{ psi}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$8(10^3)\pi\,(d/2)^2 = \frac{\pi^2(2.667)(10^6)(\frac{\pi}{4})(\frac{d}{2})^4}{[(0.7)(37)]^2}$$

$d = 1.81$ in. **Ans**

•13–8 An A-36 steel column has a length of 5 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I = \frac{1}{12}(0.1)(0.05^3) - \frac{1}{12}(0.08)(0.03^3) = 0.86167\,(10^{-6})\text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(0.86167)\,(10^{-6})}{[(0.5)(5)]^2}$$

$$= 272\,138\text{ N}$$

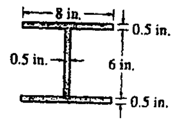
$$= 272\text{ kN} \quad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A}; \qquad A = (0.1)(0.05) - (0.08)(0.03) = 2.6\,(10^{-3})\text{ m}^2$$

$$= \frac{272\,138}{2.6\,(10^{-3})} = 105\text{ MPa} < \sigma_Y$$

Therefore, Euler's formula is valid.

13-9 An A-36 steel column has a length of 15 ft and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



$$I_x = \frac{1}{12}(8)(7^3) - \frac{1}{12}(7.5)(6^3) = 93.67 \text{ in}^4$$

$$I_y = 2\,(\frac{1}{12})(0.5)(8^3) + \frac{1}{12}(6)(0.5^3) = 42.729 \text{ in}^4 \text{ (controls)}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(42.729)}{[(1.0)(15)(12)]^2}$$

$$= 377 \text{ kip} \qquad \textbf{Ans}$$

Check :

$$A = (2)(8)(0.5) + 6(0.5) = 11 \text{ in}^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{377}{11} = 34.3 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid

13-10 Solve Prob. 13-9 if the column is fixed at its bottom and free at its top.

$$I_x = \frac{1}{12}(8)(7^3) - \frac{1}{12}(7.5)(6^3) = 93.67 \text{ in}^4$$

$$I_y = 2\left(\frac{1}{12}\right)(0.5)(8^3) + \frac{1}{12}(6)(0.5^3) = 42.729 \text{ in}^4 \text{ (controls)}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(42.729)}{(2.0(15)(12))^2} = 94.4 \text{ kip} \qquad \textbf{Ans}$$

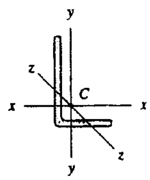
Check :

$$A = 2(8)(0.5) + 6(0.5) = 11 \text{ in}^2$$

$$\sigma_{cr} = \frac{P}{A} = \frac{94.4}{11} = 8.58 \text{ ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

13–11 The A-36 steel angle has a cross-sectional area of $A = 2.48\ \text{in}^2$ and a radius of gyration about the x axis of $r_x = 1.26$ in. and about the y axis of $r_y = 0.879$ in. The smallest radius of gyration occurs about the z axis and is $r_z = 0.644$ in. If the angle is to be used as a pin-connected 10-ft-long column, determine the largest axial load that can be applied through its centroid C without causing it to buckle.



The least radius of gyration :

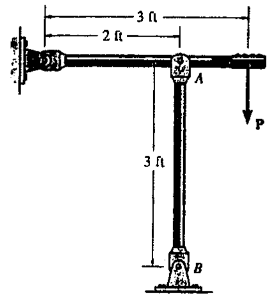
$r_z = 0.644$ in. controls.

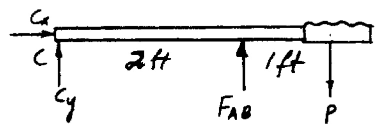
$$\sigma_{cr} = \frac{\pi^2 E}{(\frac{KL}{r})^2}; \qquad K = 1.0$$

$$= \frac{\pi^2 (29)(10^3)}{\left[\frac{1.0\ (120)}{0.644}\right]^2} = 8.243\ \text{ksi} < \sigma_Y \qquad \textbf{OK}$$

$$P_{cr} = \sigma_{cr} A = 8.243\ (2.48) = 20.4\ \text{kip} \qquad \textbf{Ans}$$

•13-12 Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod AB does not buckle. The rod has a diameter of 1.25 in. It is pin connected at its ends.





$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}(2) - P(3) = 0$

$$P = \frac{2}{3}F_{AB} \qquad (1)$$

Bucking load for rod AB :

$$I = \frac{\pi}{4}(0.625^4) = 0.1198 \text{ in}^4$$

$$A = \pi(0.625^2) = 1.2272 \text{ in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = P_{cr} = \frac{\pi^2(29)(10^3)(0.1198)}{[1.0(3)(12)]^2} = 26.46 \text{ kip}$$

From Eq. (1)

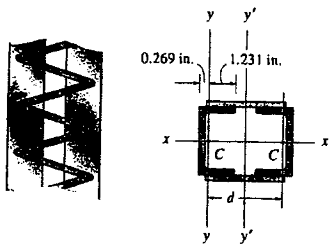
$$P = \frac{2}{3}(26.46) = 17.6 \text{ kip} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{26.46}{1.2272} = 21.6 \text{ ksi} < \sigma_Y \text{ OK}$$

Therefore, Euler's formula is valid.

13–13 The two steel channels are to be laced together to form a 30-ft-long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 3.10\ \text{in}^2$ and moments of inertia $I_x = 55.4\ \text{in}^4$, $I_y = 0.382\ \text{in}^4$. The centroid C of its area is located in the figure. Determine the proper distance d between the centroids of the channels so that buckling occurs about the $x-x$ and $y'-y'$ axes due to the same load. What is the value of this critical load? Neglect the effect of the lacing. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



$$I_x = 2(55.4) = 110.8\ \text{in}^4$$

$$I_y = 2(0.382) + 2(3.10)\left(\frac{d}{2}\right)^2 = 0.764 + 1.55\,d^2$$

In order for the column to buckle about $x-x$ and $y-y$ axes at the same time, I_y must be equal to I_x

$$I_y = I_x$$

$$0.764 + 1.55\,d^2 = 110.8$$

$d = 8.43$ in. **Ans**

Check :

$d > 2(1.231) = 2.462$ in. OK

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2(29)(10^3)(110.8)}{[1.0(360)]^2}$$

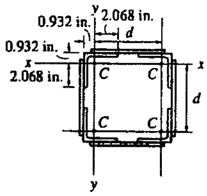
$= 245$ kip **Ans**

Check stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{245}{2(3.10)} = 39.5\ \text{ksi} < \sigma_Y$$

Therefore, Euler's formula is valid.

13–14 A column is constructed using four A-36 steel angles that are laced together as shown in Prob. 13–13. The length of the column is to be 25 ft and the ends are assumed to be pin connected. Each angle shown below has an area of $A = 2.75\ \text{in}^2$ and moments of inertia of $I_{x'} = I_{y'} = 2.22\ \text{in}^4$. Determine the distance d between the centroids C of the angles so that the column can support an axial load of $P = 350$ kip without buckling. Neglect the effect of the lacing.



$$I_x = I_y = 4[\,2.22 + 2.75\left(\frac{d}{2}\right)^2] = 8.88 + 2.75\,d^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{350}{4(2.75)} = 31.8\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

Therefore, Euler's formula is valid.

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

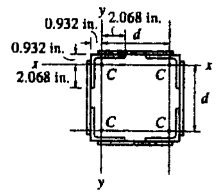
$$350 = \frac{\pi^2 (29)(10^3)(8.88 + 2.75\,d^2)}{[1.0\,(300)]^2}$$

$d = 6.07$ in. **Ans**

Check dimension :

$d > 2\,(2.068) = 4.136$ in. OK

13–15. A column is constructed using four A-36 steel angles that are laced together as shown in Prob. 13-13. The length of the column is to be 40 ft and the ends are assumed to be fixed connected. Each angle shown below has an area of $A = 2.75\ \text{in}^2$ and moments of inertia of $I_x = I_y = 2.22\ \text{in}^4$. Determine the distance d between the centroids C of the angles so that the column can support an axial load of $P = 350$ kip without buckling. Neglect the effect of the lacing.



$$I_x = I_y = 4\left[2.22 + 2.75\left(\frac{d}{2}\right)^2\right] = 8.88 + 2.75\,d^2$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{350}{4(2.75)} = 31.8\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

Therefore, Euler's formula is valid.

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}\ ; \qquad 350 = \frac{\pi^2 (29)(10^3)(8.88 + 2.75\,d^2)}{[0.5(12)(40)]^2}$$

$d = 4.73$ in. **Ans**

Check dimension :

$d > 2(2.068) = 4.136$ in. OK

***13-16.** The $W12 \times 87$ structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, and it is subjected to an axial load of $P = 380$ kip, determine the factor of safety with respect to buckling.

$W\,12 \times 87 \quad A = 25.6\ \text{in}^2 \quad I_x = 740\ \text{in}^4 \quad I_y = 241\ \text{in}^4$ (controls)

$K = 2.0$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(241)}{[(2.0)(12)(12)]^2} = 831.63\ \text{kip}$$

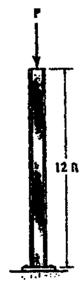
$$\text{F.S.} = \frac{P_{cr}}{P} = \frac{831.63}{380} = 2.19 \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A}$$

$$= \frac{831.63}{25.6} = 32.5\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

13-17. The $W12 \times 87$ structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, determine the largest axial load it can support. Use a factor of safety with respect to buckling of 1.75.



$W\ 12 \times 87 \qquad A = 25.6\ \text{in}^2 \qquad I_x = 740\ \text{in}^4 \qquad I_y = 241\ \text{in}^4 \qquad \text{(controls)}$

$K = 2.0$

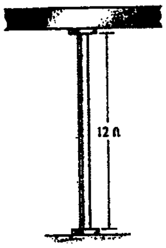
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(241)}{(2.0(12)(12))^2} = 831.63\ \text{kip}$$

$$P = \frac{P_{cr}}{F.S.} = \frac{831.63}{1.75} = 475\ \text{kip} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P}{A} = \frac{475}{25.6} = 18.6\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

13-18. The 12-ft A-36 steel pipe column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the ends are assumed to be pin connected.



$$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$$

$$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$$

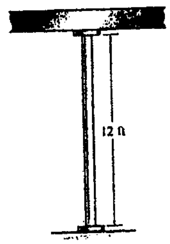
$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(1.0)(12)(12)]^2} = 28.4 \text{ kip} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{28.4}{2.1598} = 13.1 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

13-19. The 12-ft A-36 steel column has an outer diameter of 3 in. and a thickness of 0.25 in. Determine the critical load if the bottom is fixed and the top is pinned.



$A = \pi(1.5^2 - 1.25^2) = 2.1598 \text{ in}^2$

$I = \frac{\pi}{4}(1.5^4 - 1.25^4) = 2.0586 \text{ in}^4$

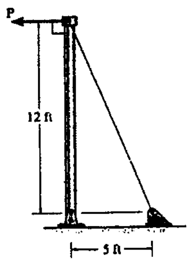
$K = 0.7$

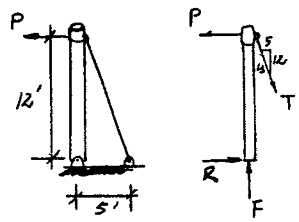
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(2.0586)}{[(0.7)(12)(12)]^2} = 58.0 \text{ kip} \quad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{58.0}{2.1598} = 26.8 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

***13-20.** The A-36 steel pipe has an outer diameter of 2 in. and a thickness of 0.5 in. If it is held in place by a guywire, determine the largest horizontal force P that can be applied without causing the pipe to buckle. Assume that the ends of the pipe are pin connected.





$\xrightarrow{+} \Sigma F_x = 0; \quad \frac{5}{13}T - P = 0$

$$T = \frac{13}{5}P$$

$+\uparrow \Sigma F_y = 0; \quad F - \frac{12}{13}T = 0$

$$F = \frac{12}{13}\left(\frac{13}{5}P\right) = \frac{12}{5}P$$

Bucking Load :

$$A = \pi(1^2 - 0.5^2) = 2.356 \text{ in}^2$$

$$I = \frac{\pi}{4}(1^4 - 0.5^4) = 0.7363 \text{ in}^4$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F = \frac{12}{5}P = \frac{\pi^2(29)(10^3)(0.7363)}{[(1.0)(12)(12)]^2}$$

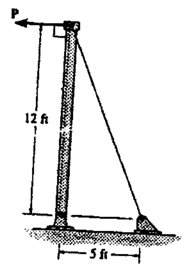
$$P = 4.23 \text{ kip} \qquad \textbf{Ans}$$

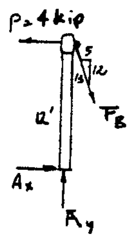
$$P_{cr} = F = 10.16 \text{ kip}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{10.16}{2.356} = 4.31 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

13-21. The A-36 steel pipe has an outer diameter of 2 in. If it is held in place by a guywire, determine the pipe's required inner diameter to the nearest $\frac{1}{8}$ in., so that it can support a maximum horizontal load of $P = 4$ kip without causing the pipe to buckle. Assume the ends of the pipe are pin connected





$\circlearrowleft + \Sigma M_A = 0; \quad 4(12) - F_B(\frac{5}{13})(12) = 0$

$F_B = 10.4$ kip

$+\uparrow \Sigma F_y = 0; \quad A_y - 10.4(\frac{12}{13}) = 0$

$A_y = 9.60$ kip

Section properties :

$A = \frac{\pi}{4}(2^2 - d_i^2); \quad I = \frac{\pi}{4}(1^4 - \frac{d_i^4}{16})$

Critical load :

$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \quad$ where $K = 1.0$

$$9.60 = \frac{\pi^2(29)(10^3)(\frac{\pi}{4})(1^4 - \frac{d_i^4}{16})}{(1.0(12)(12))^2}$$

$d_i = 1.163$ in.

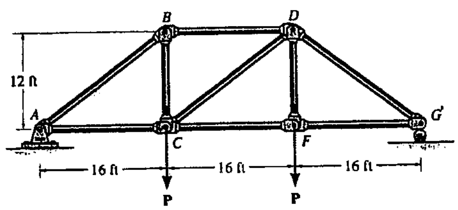
Check critical stress :

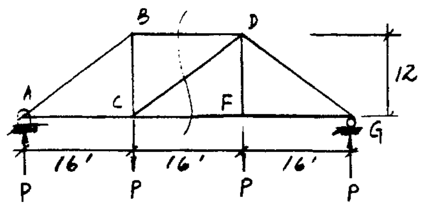
$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{9.60}{\frac{\pi}{4}(2^2 - (1.163)^2)} = 4.62 \text{ ksi} < \sigma_Y \qquad$ OK

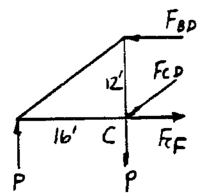
Hence, Euler's equation is still valid.

Use $d_i = 1\frac{1}{8}$ in. **Ans**

13–22 The members of the truss are assumed to be pin connected. If member *BD* is an A-36 steel rod of radius 2 in., determine the maximum load *P* that can be supported by the truss without causing the member to buckle.







$$\curvearrowleft + \Sigma M_C = 0; \qquad F_{BD}(12) - P(16) = 0$$

$$F_{BD} = \frac{4}{3}P$$

Bucking Load :

$$A = \pi(2^2) = 12.56 \text{ in}^2$$

$$I = \frac{\pi}{4}(2^4) = 4\pi \text{ in}^4$$

$$L = 16(12) = 192 \text{ in.}$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{BD} = \frac{4}{3}P = \frac{\pi^2(29)(10^3)(4\pi)}{[(1.0)(192)]^2}$$

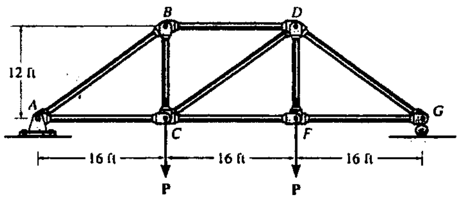
$$P = 73.2 \text{ kip} \qquad \textbf{Ans}$$

$$P_{cr} = F_{BD} = 97.56 \text{ kip}$$

Check :

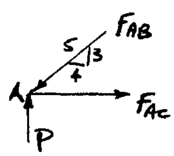
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{97.56}{12.56} = 7.76 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

13–23 Solve Prob. 13–22 in the case of member *AB*, which has a radius of 2 in.



$+\uparrow \Sigma F_y = 0; \qquad P - \frac{3}{5}F_{AB} = 0$

$F_{AB} = 1.667\,P$



Buckling load :

$A = \pi\,(2)^2 = 12.57\ \text{in}^2$

$I = \frac{\pi}{4}(2)^4 = 4\pi\ \text{in}^4$

$L = 20(12) = 240\ \text{in.}$

$K = 1.0$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(4\pi)}{(1.0(240))^2} = 62.443\ \text{kip}$$

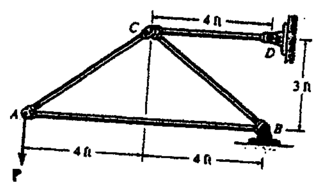
$P_{cr} = F_{AB} = 1.667\,P = 62.443$

$P = 37.5\ \text{kip}$ **Ans**

Check :

$$\sigma_{cr} = \frac{P}{A} = \frac{37.5}{12.57} = 2.98\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

***13-24.** The truss is made from A-36 steel bars, each of which has a circular cross section with a diameter of 1.5 in. Determine the maximum force P that can be applied without causing any of the members to buckle. The members are pin connected at their ends.

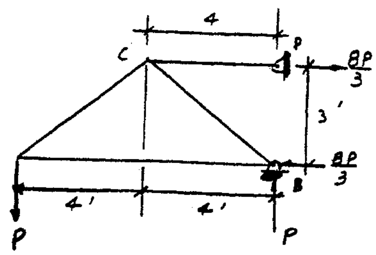


$$I = \frac{\pi}{4}(0.75^4) = 0.2485 \text{ in}^4$$

$$A = \pi(0.75^2) = 1.7671 \text{ in}^2$$

Members AB and BC are in compression :

Joint A :

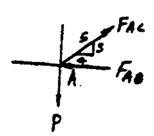


$$+\uparrow \Sigma F_y = 0; \quad \frac{3}{5}F_{AC} - P = 0$$

$$F_{AC} = \frac{5P}{3}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_{AB} - \frac{4}{5}(\frac{5P}{3}) = 0$$

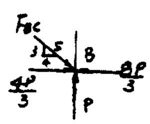
$$F_{AB} = \frac{4P}{3}$$



Joint B :

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad \frac{4}{5}F_{BC} + \frac{4P}{3} - \frac{8P}{3} = 0$$

$$F_{BC} = \frac{5P}{3}$$



Failure of rod AB :

$K = 1.0 \qquad L = 8(12) = 96$ in.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = \frac{4P}{3} = \frac{\pi^2(29)(10^3)(0.2485)}{((1.0)(96))^2}$$

$P = 5.79$ kip (controls) **Ans**

Check :

$$P_{cr} = F_{AB} = 7.72 \text{ kip}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{7.72}{1.7671} = 4.36 \text{ ksi} < \sigma_Y \quad \text{OK}$$

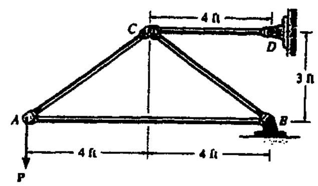
·f rod BC :

$$L = 5(12) = 60 \text{ in.}$$

$$\cdot = \frac{\pi^2(29)(10^3)(0.2485)}{[(1.0)(60)]^2}$$

,9 kip

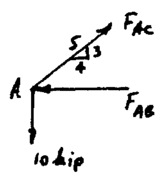
13-25. The truss is made from A-36 steel bars, each of which has a circular cross section. If the applied load $P = 10$ kip, determine the diameter of member AB to the nearest $\frac{1}{8}$ in. that will prevent this member from buckling. The members are pin supported at their ends.



Joint A:

$+\uparrow \Sigma F_y = 0; \quad -10 + F_{AC}(\frac{3}{5}) = 0; \quad F_{AC} = 16.667$ kip

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{AB} + 16.667(\frac{4}{5}) = 0; \quad F_{AB} = 13.33$ kip



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$13.33 = \frac{\pi^2(29)(10^3)(\frac{\pi}{4})(r)^4}{(1.0(8)(12))^2}$$

$r = 0.8599$ in.

$d = 2r = 1.72$ in.

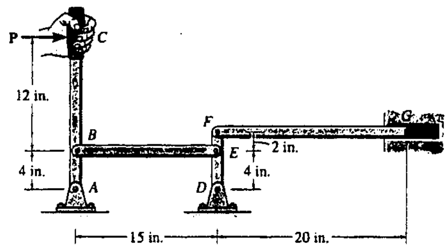
Use:

$d = 1\frac{3}{4}$ in. **Ans**

Check:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{13.33}{\frac{\pi}{4}(1.75)^2} = 5.54 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

13-26 The control linkage for a machine consists of two L2 steel rods *BE* and *FG*, each with a diameter of 1 in. If a device at *G* causes the end *G* to freeze up and become pin-connected, determine the maximum horizontal force *P* that can be applied to the handle without causing either of the two rods to buckle. The members are pin connected at *A*, *B*, *D*, *E*, and *F*.

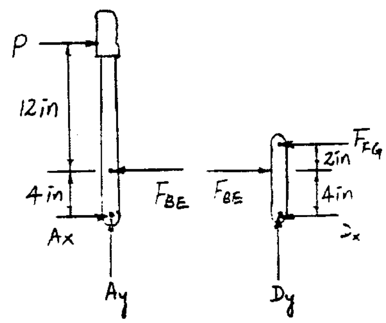


$$\curvearrowleft + \Sigma M_A = 0; \qquad F_{BE}(4) - P(16) = 0$$

$$F_{BE} = 4P$$

$$\curvearrowleft + \Sigma M_D = 0; \qquad F_{FG}(6) - 4P(4) = 0$$

$$F_{FG} = 2.6667\,P$$



For rod *BE*,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad K = 1.0$$

$$4P = \frac{\pi^2 (29)(10^3)(\frac{\pi}{4})(0.5^4)}{[1.0(15)]^2}$$

$P = 15.6$ kip

Check stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4\,(15.6)}{\frac{\pi}{4}(1^2)} = 79.5 \text{ ksi} < \sigma_Y = 102 \text{ ksi} \qquad \textbf{OK}$$

For rod *FG* :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}; \qquad K = 1.0$$

$$2.6667\,P = \frac{\pi^2 [(29)(10^3)]\frac{\pi}{4}(0.5^4)}{[1.0(20)]^2}$$

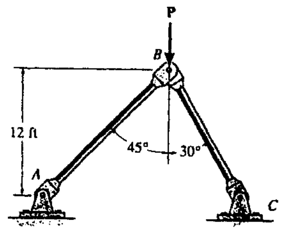
$P = 13.2$ kip (controls) **Ans**

Check stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{2.6667\,(13.2)}{\frac{\pi}{4}(1^2)} = 44.7 \text{ ksi} < \sigma_Y = 102 \text{ ksi} \qquad \textbf{OK}$$

Hence, Euler's equation is still valid.

13-27 The linkage is made using two A-36 steel rods, each having a circular cross section. Determine the diameter of each rod to the nearest $\frac{1}{8}$ in. that will support a load of P = 6 kip. Assume that the rods are pin connected at their ends. Use a factor of safety with respect to buckling of 1.8.



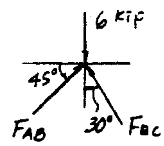
$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

Joint B :

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{AB}\cos 45^\circ - F_{BC}\sin 30^\circ = 0$

$F_{AB} = 0.7071\, F_{BC}$ (1)

$+\uparrow \Sigma F_y = 0;\quad F_{AB}\sin 45^\circ + F_{BC}\cos 30^\circ - 6 = 0$ (2)



Solving Eqs. (1) and (2) yields :

$F_{BC} = 4.392$ kip $\quad F_{AB} = 3.106$ kip

For rod AB :

$P_{cr} = 3.106\,(1.8) = 5.591$ kip

$K = 1.0 \quad L_{AB} = \frac{12(12)}{\cos 45^\circ} = 203.64$ in.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$5.591 = \frac{\pi^2(29)(10^3)\left(\frac{\pi d_{AB}^4}{64}\right)}{[(1.0)(203.64)]^2}$$

$d_{AB} = 2.015$ in. $\quad$ Use $d_{AB} = 2\frac{1}{8}$ in. **Ans**

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{5.591}{\frac{\pi}{4}(2.125^2)} = 1.58 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

For rod BC :

$P_{cr} = 4.392\,(1.8) = 7.9056$ kip

$K = 1.0 \quad L_{BC} = \frac{12(12)}{\cos 30^\circ} = 166.28$ in.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$7.9056 = \frac{\pi^2(29)(10^3)\left(\frac{\pi d_{BC}^4}{64}\right)}{[(1.0)(166.28)]^2}$$

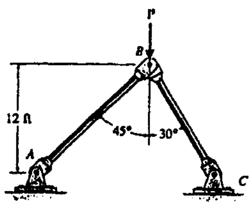
$d_{BC} = 1.986$ in.

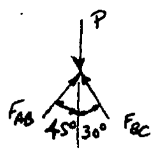
Use $d_{BC} = 2$ in. **Ans**

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{7.9056}{\frac{\pi}{4}(2^2)} = 2.52 \text{ ksi} < \sigma_Y \qquad \text{OK}$$

***13-28.** The linkage is made using two A-36 steel rods, each having a circular cross section. If each rod has a diameter of $\frac{3}{4}$ in., determine the largest load it can support without causing any rod to buckle. Assume that the rods are pin-connected at their ends.





$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} \sin 45° - F_{BC} \sin 30° = 0$

$+\uparrow \Sigma F_y = 0; \quad F_{AB} \cos 45° - F_{BC} \cos 30° - P = 0$

$F_{AB} = 0.5176\,P$

$F_{BC} = 0.73205\,P$

$$L_{AB} = \frac{12}{\cos 45°} = 16.971 \text{ ft}$$

$$L_{BC} = \frac{12}{\cos 30°} = 13.856 \text{ ft}$$

Assume rod AB buckles :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$0.5176\,P = \frac{\pi^2 (29)(10^6)(\frac{\pi}{4})(\frac{3}{8})^4}{(1.0(16.971)(12))^2}$$

$P = 207$ lb (controls) **Ans**

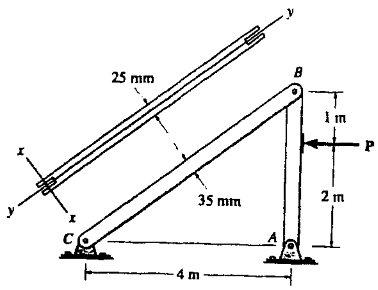
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{207}{\pi(\frac{3}{8})^2} = 469 \text{ psi} < \sigma_Y \qquad \text{OK}$$

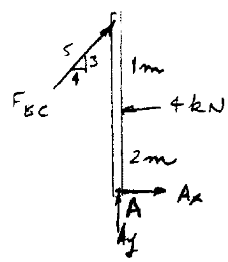
Assume rod BC buckles :

$$0.73205\,P = \frac{\pi^2 (29)(10^6)(\frac{\pi}{4})(\frac{3}{8})^4}{(1.0(13.856)(12))^2}$$

$P = 220$ lb

13-29 The frame supports the load of $P = 4$ kN. As a result, the A-36 steel member BC is subjected to a compressive load. Due to the forked ends on this member, consider the supports at B and C to act as pins for $x-x$ axis buckling and as fixed supports for $y-y$ axis buckling. Determine the factor of safety with respect to buckling about each of these axes.





$$\curvearrowleft + \Sigma M_A = 0; \qquad 4(2) - F_{BC}\left(\frac{4}{5}\right)(3) = 0$$

$$F_{BC} = 3.333 \text{ kN}$$

x - x axis buckling :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (200)(10^9)(\frac{1}{12})(0.025)(0.035)^3}{(1.0(5))^2} = 7.053 \text{ kN}$$

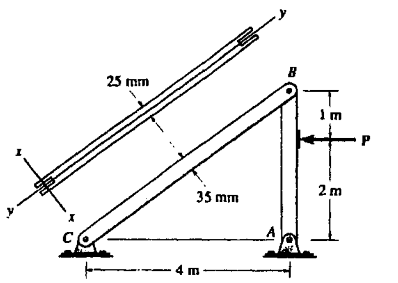
$$\text{F.S.} = \frac{7.053}{3.333} = 2.12 \qquad \textbf{Ans}$$

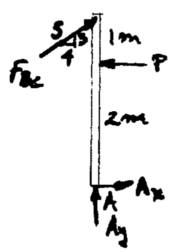
y - y axis buckling :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (200)(10^9)(\frac{1}{12})(0.035)(0.025)^3}{(0.5(5))^2} = 14.39 \text{ kN}$$

$$\text{F.S.} = \frac{14.39}{3.333} = 4.32 \qquad \textbf{Ans}$$

13–30 Determine the greatest load P the frame will support without causing the A-36 steel member BC to buckle. Due to the forked ends on the member, consider the supports at B and C to act as pins for $x-x$ axis buckling and as fixed supports for $y-y$ axis buckling.





$$\circlearrowleft + \Sigma M_A = 0; \qquad P(2) - 3(\frac{4}{5})F_{BC} = 0$$

$$F_{BC} = 0.8333\,P$$

x - x axis buckling :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(\frac{1}{12})(0.025)(0.035)^3}{(1.0(5))^2} = 7.053\text{ kN}$$

y - y axis buckling :

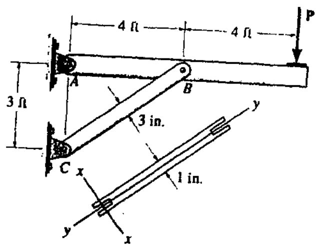
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(\frac{1}{12})(0.035)(0.025)^3}{(0.5(5))^2} = 14.39\text{ kN}$$

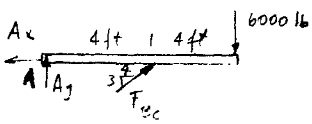
Thus,

$0.8333\,P = 7.053$

$P = 8.46$ kN **Ans**

13-31 The beam supports the load of $P = 6$ kip. As a result, the A-36 steel member BC is subjected to a compressive load. Due to the forked ends on the member, consider the supports at B and C to act as pins for $x-x$ axis buckling and as fixed supports for $y-y$ axis buckling. Determine the factor of safety with respect to buckling about each of these axes.





$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}(\frac{3}{5})(4) - 6000(8) = 0$$

$$F_{BC} = 20 \text{ kip}$$

x-x axis buckling :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(1)(3)^3}{(1.0(5)(12))^2} = 178.9 \text{ kip}$$

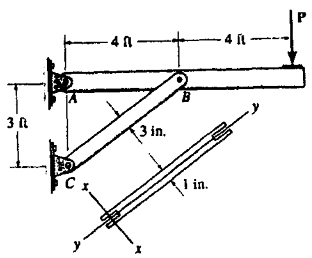
$$\text{F.S.} = \frac{178.9}{20} = 8.94 \qquad \textbf{Ans}$$

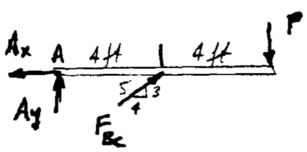
y-y axis buckling :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(3)(1)^3}{(0.5(5)(12))^2} = 79.51$$

$$\text{F.S.} = \frac{79.51}{20} = 3.98 \qquad \textbf{Ans}$$

•13-32 Determine the greatest load P the frame will support without causing the A-36 steel member BC to buckle. Due to the forked ends on the member, consider the supports at B and C to act as pins for $x-x$ axis buckling and as fixed supports for $y-y$ axis buckling.





$\curvearrowleft + \Sigma M_A = 0; \qquad F_{BC}(\frac{3}{5})(4) - P(8) = 0$

$F_{BC} = 3.33\,P$

x-x axis buckling :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(1)(3)^3}{(1.0(5)(12))^2} = 178.9 \text{ kip}$$

y-y axis buckling :

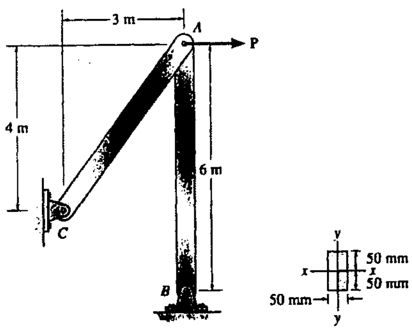
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(\frac{1}{12})(3)(1)^3}{(0.5(5)(12))^2} = 79.51 \text{ kip}$$

Thus,

$3.33\,P = 79.51$

$P = 23.9$ kip **Ans**

13–33 The steel bar AB of the frame is pin-connected at its ends. If $P = 18$ kN, determine the factor of safety with respect to buckling about the $y-y$ axis due to the applied loading. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



$$I_y = \frac{1}{12}(0.10)(0.05^3) = 1.04167\,(10^{-6})\ \text{m}^4$$

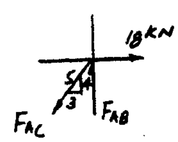
Joint A :

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad \frac{3}{5}F_{AC} - 18 = 0$$

$$F_{AC} = 30\ \text{kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{4}{5}(30) = 0$$

$$F_{AB} = 24\ \text{kN}$$



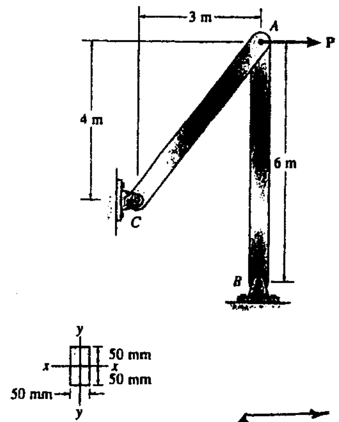
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(1.04167)\,(10^{-6})}{[(1.0)(6)]^2} = 57116\ \text{N} = 57.12\ \text{kN}$$

$$\text{F.S.} = \frac{P_{cr}}{F_{AB}} = \frac{57.12}{24} = 2.38 \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{57.12\,(10^3)}{0.1\,(0.05)} = 11.4\ \text{MPa} < \sigma_Y \qquad \text{OK}$$

13–34 Determine the maximum load P the frame can support without buckling member AB. Assume that AB is made of steel and is pinned at its ends for $y-y$ axis buckling and fixed at its ends for $x-x$ axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

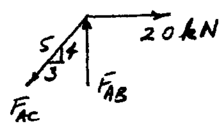


$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AC}\left(\frac{3}{5}\right) + P = 0$$

$$F_{AC} = \frac{5}{3}P$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} - \frac{5}{3}P\left(\frac{4}{5}\right) = 0$$

$$F_{AB} = \frac{4}{3}P$$



$$I_y = \frac{1}{12}(0.10)(0.05)^3 = 1.04167(10^{-6})\text{m}^4$$

$$I_x = \frac{1}{12}(0.05)(0.10)^3 = 4.16667(10^{-6})\text{m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

x-x axis buckling :

$$P_{cr} = \frac{\pi^2(200)(10^9)(4.16667)(10^{-6})}{(0.5(6))^2} = 914 \text{ kN}$$

y-y axis buckling :

$$P_{cr} = \frac{\pi^2 (200)(10^9)(1.04167)(10^{-6})}{(1(6))^2} = 57.12 \text{ kN}$$

y-y axis buckling controls

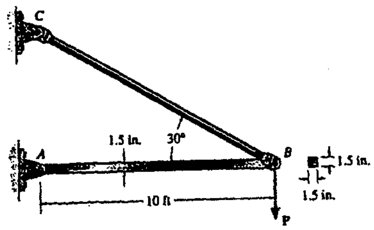
$$\frac{4}{3}P = 57.12$$

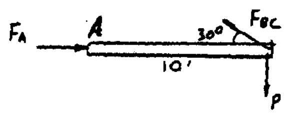
$$P = 42.8 \text{ kN} \qquad \textbf{Ans}$$

Check :

$$\sigma_{cr} = \frac{P}{A} = \frac{57.12(10^3)}{(0.1)(0.05)} = 11.4 \text{ MPa} < \sigma_Y \qquad \text{OK}$$

13-35. The A-36 steel bar AB has a square cross section. If it is pin-connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of 2.





$\circlearrowleft + \Sigma M_A = 0;\quad F_{BC}\sin 30^\circ(10) - P(10) = 0$

$F_{BC} = 2P$

$\xrightarrow{+} \Sigma F_x = 0;\quad F_A - 2P\cos 30^\circ = 0$

$F_A = 1.732P$

Buckling load :

$P_{cr} = F_A(\text{F.S.}) = 1.732P(2) = 3.464P$

$L = 10(12) = 120 \text{ in.}$

$I = \frac{1}{12}(1.5)(1.5)^3 = 0.421875 \text{ in}^4$

$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

$3.464P = \frac{\pi^2(29)(10^3)(0.421875)}{[(1.0)(120)]^2}$

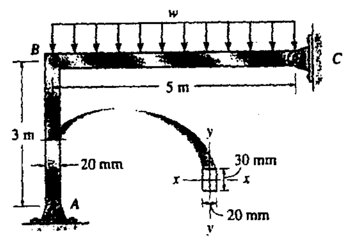
$P = 2.42 \text{ kip}$ **Ans**

$P_{cr} = F_A(\text{F.S.}) = 1.732(2.42)(2) = 8.38 \text{ kip}$

Check :

$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{8.38}{1.5(1.5)} = 3.72 \text{ ksi} < \sigma_Y$ **OK**

•13-36 The steel bar AB has a rectangular cross section. If it is pin connected at its ends, determine the maximum allowable intensity w of the distributed load that can be applied to BC without causing bar AB to buckle. Use a factor of safety with respect to buckling of 1.5. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Buckling load :

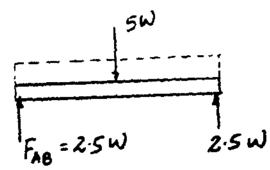
$$P_{cr} = F_{AB}(\text{F.S.}) = 2.5\,w\,(1.5) = 3.75\,w$$

$$I = \frac{1}{12}(0.03)(0.02)^3 = 20\,(10^{-9})\text{ m}^4$$

$$K = 1.0$$

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2}$$

$$3.75\,w = \frac{\pi^2\,(200)(10^9)(20)(10^{-9})}{[(1.0)(3)]^2}$$



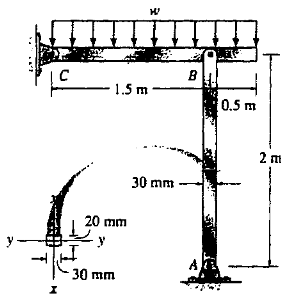
$$w = 1170\text{ N/m} = 1.17\text{ kN/m} \qquad \textbf{Ans}$$

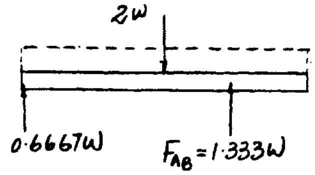
$$P_{cr} = 4.38\text{ kN}$$

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4.38\,(10^3)}{0.02\,(0.03)} = 7.31\text{ MPa} < \sigma_Y \qquad \text{OK}$$

13–37 Determine the maximum allowable intensity w of the distributed load that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for $x-x$ axis buckling and fixed at its ends for $y-y$ axis buckling. Use a factor of safety with respect to buckling of 3. E_{st} = 200 GPa, σ_Y = 360 MPa.





Moment of inertia :

$$I_x = \frac{1}{12}(0.02)(0.03^3) = 45.0(10^{-9})\text{m}^4$$

$$I_y = \frac{1}{12}(0.03)(0.02^3) = 20(10^{-9})\text{ m}^4$$

x - x axis :

$$P_{cr} = F_{AB}(\text{F.S.}) = 1.333w(3) = 4.0\ w$$

$$K = 1.0, \qquad L = 2\text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$4.0w = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{[(1.0)(2)]^2}$$

$w = 5552$ N/m $= 5.55$ kN/m (controls) **Ans**

y - y axis

$$K = 0.5, \qquad L = 2\text{m}$$

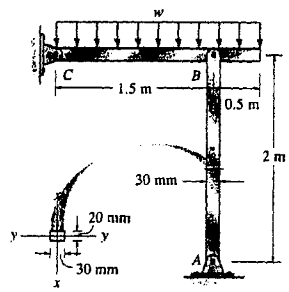
$$4.0w = \frac{\pi^2(200)(10^9)(20)(10^{-9})}{[(0.5)(2)]^2}$$

$w = 9870$ N/m $= 9.87$ kN/m

Check :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4(5552)}{(0.02)(0.03)} = 37.0\text{ MPa} < \sigma_Y \qquad \text{OK}$$

13–38 Determine if the frame can support a load of w = 6 kN/m if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for $x-x$ axis buckling and fixed at its ends for $y-y$ axis buckling. E_{st} = 200 GPa, σ_Y = 360 MPa.



Check x - x axis buckling :

$$I_x = \frac{1}{12}(0.02)(0.03)^3 = 45.0(10^{-9})\text{ m}^4$$

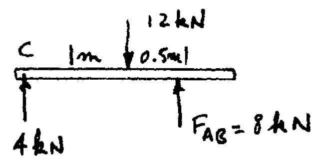
$K = 1.0 \qquad L = 2\text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200)(10^9)(45.0)(10^{-9})}{((1.0)(2))^2}$$

$P_{cr} = 22.2\text{ kN}$

$\curvearrowleft + \Sigma M_C = 0; \qquad F_{AB}(1.5) - 6(2)(1) = 0$

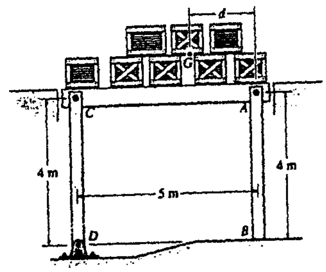
$F_{AB} = 8\text{ kN}$

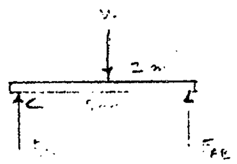


$P_{req'd} = 8(3) = 24\text{ kN} > 22.0\text{ kN}$

No, AB will fail. **Ans**

13-39. The deck is supported by the two 40-mm-square columns. Column *AB* is pinned at *A* and fixed at *B*, whereas *CD* is pinned at *C* and *D*. If the deck is prevented from sidesway, determine the greatest weight of the load that can be applied without causing the deck to collapse. The center of gravity of the load is located at $d = 2$ m. Both columns are made from Douglas Fir.





$\circlearrowleft + \Sigma M_C = 0;\quad F_{AB}(5) - W(3) = 0$

$$F_{AB} = 0.6\,W$$

$+\uparrow \Sigma F_y = 0;\quad F_{CD} + 0.6\,W - W = 0$

$$F_{CD} = 0.4\,W$$

Column *CD* :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(13.1)(10^9)(\frac{1}{12})(0.04)^4}{(1(4))^2} = 0.4\,W$$

$$W = 4.31 \text{ kN}$$

Column *AB* :

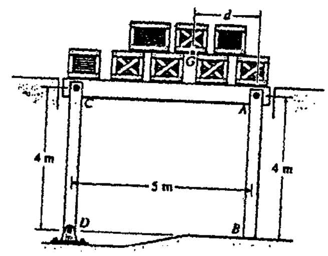
$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(13.1)(10^9)(\frac{1}{12})(0.04)^4}{(0.7(4))^2} = 0.6\,W$$

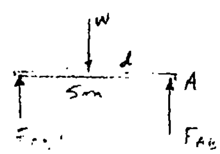
$$W = 5.86 \text{ kN}$$

Thus,

$W = 4.31$ kN **Ans**

***13-40.** The deck is supported by the two 40-mm-square columns. Column AB is pinned at A and fixed at B, whereas CD is pinned at C and D. If the deck is prevented from sidesway, determine the position d of the center of gravity of the load and the load's greatest magnitude without causing the deck to collapse. Both columns are made from Douglas Fir.





Column CD :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{CD} = \frac{\pi^2(13.1)(10^9)(\frac{1}{12})(0.04)^4}{(1.0(4))^2} = 1.7239(10^3)\text{ N}$$

Column AB :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$F_{AB} = \frac{\pi^2(13.1)(10^9)(\frac{1}{12})(0.04)^4}{(0.7(4))^2} = 3.5181(10^3)\text{ N}$$

Thus,

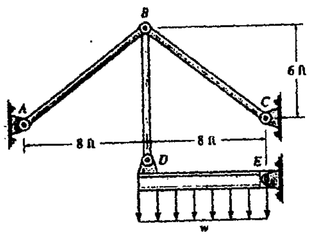
$+\uparrow \Sigma F_y = 0;\quad 1.7239(10^3) + 3.5181(10^3) - W = 0$

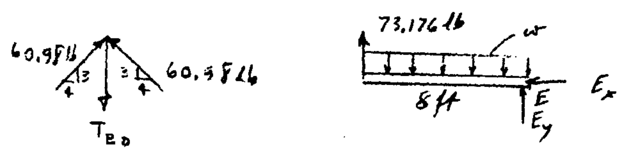
$W = 5.2420(10^3)\text{ N} = 5.24\text{ kN}$ **Ans**

$\circlearrowleft + \Sigma M_A = 0;\quad 5.2420(10^3)(d) - 1.7239(10^3)(5) = 0$

$d = 1.64\text{ m}$ **Ans**

13-41. The beam is supported by the three pin-connected suspender bars, each having a diameter of 0.5 in. and made from A-36 steel. Determine the greatest uniform load w that can be applied to the beam without causing AB and CB to buckle.





$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^6)(\frac{\pi}{4})(0.25)^4}{(1.0(10)(12))^2} = 60.98 \text{ lb}$$

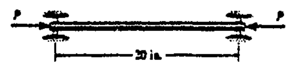
$+\uparrow \Sigma F_y = 0;\quad 2((60.98)(\frac{3}{5})) - T_{BD} = 0$

$T_{BD} = 73.176$ lb

$\circlearrowleft + \Sigma M_E = 0;\quad -73.176(8) + w(8)(4) = 0$

$w = 18.3$ lb/ft **Ans**

13-42. The rod is made from a 1-in.-diameter steel rod. Determine the critical buckling load if the ends are roller supported. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



Critical Buckling Load : $I = \frac{\pi}{4}\left(0.5^4\right) = 0.015625\pi$ in^4

and $K = 1$ for roller supported ends column. Applying *Euler's* formula,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

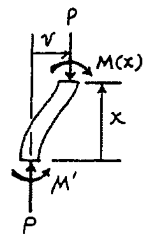
$$= \frac{\pi^2(29)(10^3)(0.015625\pi)}{[1(20)]^2}$$

$$= 35.12 \text{ kip} = 35.1 \text{ kip} \qquad \textbf{Ans}$$

Critical Stress : *Euler's* formula is only valid if $\sigma_{cr} < \sigma_Y$.

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{35.12}{\frac{\pi}{4}(1^2)} = 44.72 \text{ ksi} < \sigma_Y = 50 \text{ ksi } (O.K!)$$

13-43. Consider an ideal column as in Fig. 13–12c, having both ends fixed. Show that the critical load on the column is given by $P_{cr} = 4\pi^2 EI/L^2$. Hint: Due to the vertical deflection of the top of the column, a constant moment **M′** will be developed at the supports. Show that $d^2v/dx^2 + (P/EI)v = M'/EI$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + M'/P$.



Moment Functions :

$$M(x) = M' - Pv$$

Differential Equation of The Elastic Curve :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = M' - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{M'}{EI} \qquad (Q.E.D.)$$

The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{M'}{P} \qquad [1]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) \qquad [2]$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions :

At $x = 0$, $v = 0$. From Eq.[1], $C_2 = -\frac{M'}{P}$

At $x = 0$, $\frac{dv}{dx} = 0$. From Eq.[2], $C_1 = 0$

Elastic Curve :

$$v = \frac{M'}{P}\left[1 - \cos\left(\sqrt{\frac{P}{EI}}x\right)\right]$$

and

$$\frac{dv}{dx} = \frac{M'}{P}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

However, due to symmetry $\frac{dv}{dx} = 0$ at $x = \frac{L}{2}$. Then,

$$\sin\left[\sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right)\right] = 0 \quad \text{or} \quad \sqrt{\frac{P}{EI}}\left(\frac{L}{2}\right) = n\pi \quad \text{where } n = 1,2,3.$$

The smallest critical load occurs when $n = 1$.

$$P_{cr} = \frac{4\pi^2 EI}{L^2} \qquad (Q.E.D.)$$

*13-44. Consider an ideal column as in Fig. 13–12d, having one end fixed and the other pinned. Show that the critical load on the column is given by $P_{cr} = 20.19\, EI/L^2$. *Hint*: Due to the vertical deflection at the top of the column, a constant moment **M′** will be developed at the fixed support and horizontal reactive forces **R′** will be developed at both supports. Show that $d^2v/dx^2 + (P/EI)v = (R'/EI)(L - x)$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}\,x) + C_2 \cos(\sqrt{P/EI}\,x) + (R'/P)(L - x)$. After application of the boundary conditionsshow that $\tan(\sqrt{P/EI}\,L) = \sqrt{P/EI}\,L$. Solve by trial and error for the smallest root.

Equilibrium : FBD(a).

Moment Functions : FBD(b).

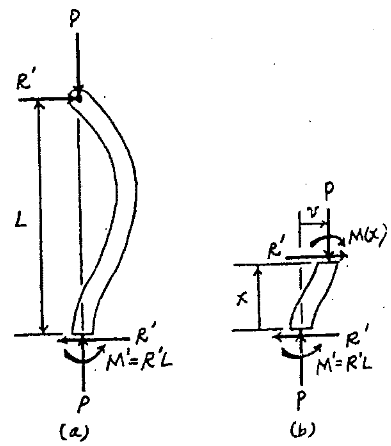
$$M(x) = R'(L-x) - Pv$$

Differential Equation of The Elastic Curve :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = R'(L-x) - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{R'}{EI}(L-x)$$



The solution of the above differential equation is of the form

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L-x) \qquad [1]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'}{P} \qquad [2]$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions :

At $x = 0$, $v = 0$. From Eq. [1], $C_2 = -\frac{R'L}{P}$

At $x = 0$, $\frac{dv}{dx} = 0$. From Eq. [2], $C_1 = \frac{R'}{P}\sqrt{\frac{EI}{P}}$

Elastic Curve :

$$v = \frac{R'}{P}\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{R'L}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{R'}{P}(L-x)$$

$$= \frac{R'}{P}\left[\sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - L\cos\left(\sqrt{\frac{P}{EI}}x\right) + (L-x)\right]$$

However, $v = 0$ at $x = L$. Then,

$$0 = \sqrt{\frac{EI}{P}}\sin\left(\sqrt{\frac{P}{EI}}L\right) - L\cos\left(\sqrt{\frac{P}{EI}}L\right)$$

$$\tan\left(\sqrt{\frac{P}{EI}}L\right) = \sqrt{\frac{P}{EI}}L \qquad (Q.E.D.)$$

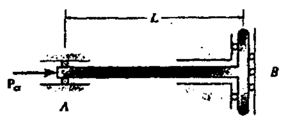
By trial and error and choosing the smallest root, we have

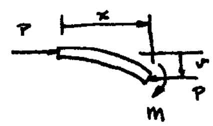
$$\sqrt{\frac{P}{EI}}L = 4.49341$$

Then,

$$P_{cr} = \frac{20.19EI}{L^2} \qquad (Q.E.D.)$$

13-45. The column is supported at B by a support that does not permit rotation but allows vertical deflection. Determine the critical load P_{cr}. EI is constant.





Elastic curve :

$$EI\frac{d^2v}{dx^2} = M = -Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = 0$$

$$v = C_1\sin\left[\sqrt{\frac{P}{EI}}x\right] + C_2\cos\left[\sqrt{\frac{P}{EI}}x\right]$$

Boundary conditions :

At $x = 0$; $\quad v = 0$

$$0 = 0 + C_2; \qquad C_2 = 0$$

At $x = L$; $\quad \frac{dv}{dx} = 0$

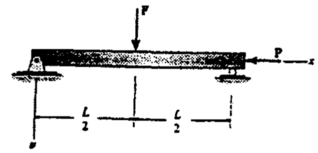
$$\frac{dv}{dx} = C_2\sqrt{\frac{P}{EI}}\cos\left[\sqrt{\frac{P}{EI}}L\right] = 0; \qquad C_2\sqrt{\frac{P}{EI}} \neq 0$$

$$\cos\left[\sqrt{\frac{P}{EI}}L\right] = 0; \qquad \sqrt{\frac{P}{EI}}L = n\left(\frac{\pi}{2}\right)$$

For $n = 1$; $\quad \frac{P}{EI} = \frac{\pi^2}{4L^2}$

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad \textbf{Ans}$$

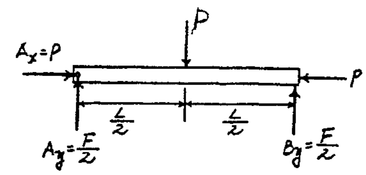
13-46. The ideal column is subjected to the force **F** at its midpoint and the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection Eq. 13–1. The general solution is $v = A\sin kx + B\cos kx - c^2x/k^2$, where $c^2 = F/2EI$, $k^2 = P/EI$.

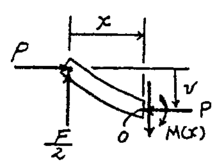


Moment Functions : FBD(b).

$$\curvearrowleft +\Sigma M_O = 0; \quad M(x) + \frac{F}{2}x + P(v) = 0$$

$$M(x) = -\frac{F}{2}x - Pv \qquad [1]$$





Differential Equation of The Elastic Curve :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = -\frac{F}{2}x - Pv$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{F}{2EI}x$$

The solution of the above differential equation is of the form,

$$v = C_1\sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2\cos\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x \qquad [2]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P} \qquad [3]$$

The integration constants can be determined from the boundary conditions.

Boundary Conditions :

At $x = 0$, $v = 0$. From Eq.[2], $\quad C_2 = 0$

At $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$. From Eq.[3],

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{F}{2P}$$

$$C_1 = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Elastic Curve :

$$v = \frac{F}{2P}\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{F}{2P}x$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) - x\right]$$

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]$$

$$= \frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right] \qquad \textbf{Ans}$$

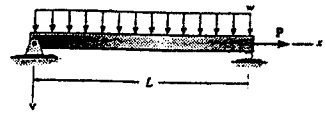
Maximum Moment : The maximum moment occurs at $x = \frac{L}{2}$. From Eq.[1],

$$M_{max} = -\frac{F}{2}\left(\frac{L}{2}\right) - Pv_{max}$$

$$= -\frac{FL}{4} - P\left\{\frac{F}{2P}\left[\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L}{2}\right]\right\}$$

$$= -\frac{F}{2}\sqrt{\frac{EI}{P}}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \qquad \textbf{Ans}$$

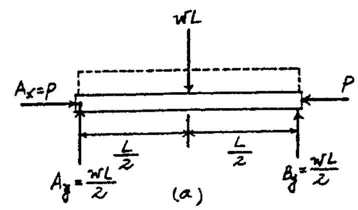
13-47. The ideal column has a weight w (force/length) and rests in the horizontal position when it is subjected to the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection Eq. 13–1, with the origin at the midspan. The general solution is $v = A\sin kx + B\cos kx + C_1 + C_2x + C_3x^2$, where $k^2 = P/EI$.



Moment Functions : FBD(b).

$$\circlearrowleft + \Sigma M_O = 0; \quad wx\left(\frac{x}{2}\right) - M(x) - \frac{wL}{2}x - Pv = 0$$

$$M(x) = \frac{w}{2}\left(x^2 - Lx\right) - Pv \qquad [1]$$



Differential Equation of The Elastic Curve :

$$EI\frac{d^2v}{dx^2} = M(x)$$

$$EI\frac{d^2v}{dx^2} = \frac{w}{2}\left(x^2 - Lx\right) - Pv$$

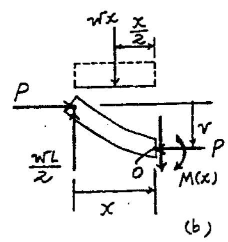
$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{w}{2EI}\left(x^2 - Lx\right)$$

The solution of the above differential equation is of the form

$$v = C_1\sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{2P}x^2 - \frac{wL}{2P}x - \frac{wEI}{P^2} \qquad [2]$$

and

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{w}{P}x - \frac{wL}{2P} \qquad [3]$$



The integration constants can be determined from the boundary conditions.

Boundary Conditions :

At $x = 0$, $v = 0$. From Eq. [2],

$$0 = C_2 - \frac{wEI}{P^2} \qquad C_2 = \frac{wEI}{P^2}$$

At $x = \frac{L}{2}$, $\frac{dv}{dx} = 0$. From Eq. [3],

$$0 = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{wEI}{P^2}\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{w}{P}\left(\frac{L}{2}\right) - \frac{wL}{2P}$$

$$C_1 = \frac{wEI}{P^2}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Elastic Curve :

$$v = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}x\right) + \frac{x^2}{2} - \frac{L}{2}x - \frac{EI}{P}\right]$$

However, $v = v_{max}$ at $x = \frac{L}{2}$. Then,

$$v_{max} = \frac{w}{P}\left[\frac{EI}{P}\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \frac{EI}{P}\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{L^2}{8} - \frac{EI}{P}\right]$$

$$= \frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right] \qquad \textbf{Ans}$$

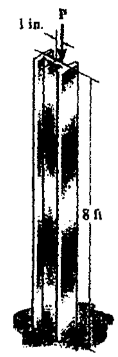
Maximum Moment : The maximum moment occurs at $x = \frac{L}{2}$. From Eq. [1],

$$M_{max} = \frac{w}{2}\left[\frac{L^2}{4} - L\left(\frac{L}{2}\right)\right] - Pv_{max}$$

$$= -\frac{wL^2}{8} - P\left\{\frac{wEI}{P^2}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - \frac{PL^2}{8EI} - 1\right]\right\}$$

$$= -\frac{wEI}{P}\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - 1\right] \qquad \textbf{Ans}$$

•13–48 Determine the load P required to cause the A-36 steel $W8 \times 15$ column to fail either by buckling or by yielding. The column is fixed at its base and free at its top.



Section properties for $W\,8 \times 15$:

$A = 4.44\ \text{in}^2 \quad I_x = 48.0\ \text{in}^4 \quad I_y = 3.41\ \text{in}^4$

$r_x = 3.29\ \text{in.} \quad d = 8.11\ \text{in.}$

Buckling about y - y axis :

$K = 2.0 \quad L = 8(12) = 96\ \text{in.}$

$$P = P_{cr} = \frac{\pi^2 EI_y}{(KL)^2} = \frac{\pi^2(29)(10^3)(3.41)}{[(2.0)(96)]^2} = 26.5\ \text{kip} \qquad \text{controls} \qquad \textbf{Ans}$$

Check : $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{26.5}{4.44} = 5.96\ \text{ksi} < \sigma_Y$ OK

Check yielding about x - x axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

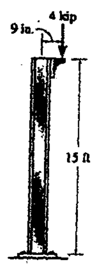
$$\frac{P}{A} = \frac{26.5}{4.44} = 5.963\ \text{ksi}$$

$$\frac{ec}{r^2} = \frac{(1)(\frac{8.11}{2})}{(3.29)^2} = 0.37463$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(96)}{2(3.29)}\sqrt{\frac{26.5}{29(10^3)(4.44)}} = 0.4184$$

$$\sigma_{max} = 5.963[1 + 0.37463\sec(0.4184)] = 8.41\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \quad \text{OK}$$

13-49. The $W10 \times 12$ structural A-36 steel column is used to support a load of 4 kip. If the column is fixed at the base and free at the top, determine the deflection at the top of the column due to the loading.



Section properties for $W\,10 \times 12$

$A = 3.54\ \text{in}^2 \qquad I_x = 53.8\ \text{in}^4 \qquad r_x = 3.90\ \text{in.} \qquad d = 9.89\ \text{in.}$

Maximum deflection :

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\,\frac{KL}{2}\right) - 1\right] \qquad K = 2.0$$

$$\sqrt{\frac{P}{EI}}\,\frac{KL}{2} = \sqrt{\frac{4}{29\,(10^3)53.8}}\left(\frac{2.0\,(15)(12)}{2}\right) = 0.2882$$

$$v_{max} = 9[\sec(0.2882) - 1] = 0.387\ \text{in.} \qquad \textbf{Ans}$$

13-50. The $W10 \times 12$ structural A-36 steel column is used to support a load of 4 kip. If the column is fixed at its base and free at its top, determine the maximum stress in the column due to this loading.

Section properties for W 10 x 12

$A = 3.54\ \text{in}^2 \quad I_x = 53.8\ \text{in}^4 \quad r_x = 3.90\ \text{in.} \quad d = 9.89\ \text{in.}$

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

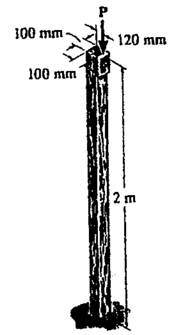
$$\frac{P}{A} = \frac{4}{3.54} = 1.13\ \text{ksi}$$

$$\frac{ec}{r^2} = \frac{9(\frac{9.89}{2})}{3.90^2} = 2.926$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(15)(12)}{2(3.90)}\sqrt{\frac{4}{29(10^3)(3.54)}} = 0.2881$$

$\sigma_{max} = 1.13[1 + 2.926\sec(0.2881)] = 4.57\ \text{ksi} < \sigma_Y$ OK **Ans**

13–51 The wood column has a square cross section with dimensions 100 mm by 100 mm. It is fixed at its base and free at its top. Determine the load P that can be applied to the edge of the column without causing the column to fail either by buckling or by yielding. $E_w = 12$ GPa, $\sigma_Y = 55$ MPa.



Section properties :

$$A = 0.1(0.1) = 0.01 \text{ m}^2 \qquad I = \frac{1}{12}(0.1)(0.1^3) = 8.333(10^{-6}) \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{8.333(10^{-6})}{0.01}} = 0.02887 \text{ m}$$

Buckling :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(12)(10^9)(8.333)(10^{-6})}{[2.0(2)]^2} = 61.7 \text{ kN}$$

Check : $\quad \sigma_{cr} = \frac{P_{cr}}{A} = \frac{61.7(10^3)}{0.01} = 6.17 \text{ MPa} < \sigma_Y \quad$ OK

Yielding :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{0.12(0.05)}{(0.02887)^2} = 7.20$$

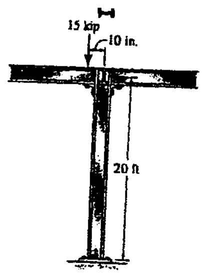
$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(2)}{2(0.02887)}\sqrt{\frac{P}{12(10^9)(0.01)}} = 0.006324\sqrt{P}$$

$$55(10^6)(0.01) = P[1 + 7.20 \sec(0.006324\sqrt{P})]$$

By trial and error :

$P = 31400 \text{ N} = 31.4 \text{ kN}$ controls **Ans**

*13-52. The $W14 \times 26$ structural A-36 steel member is used as a 20-ft-long column that is assumed to be fixed at its top and fixed at its bottom. If the 15-kip load is applied at an eccentric distance of 10 in., determine the maximum stress in the column.



Section properties for W 14 x 26

$A = 7.69\text{ in}^2 \quad d = 13.91\text{ in.} \quad I_x = 245\text{ in}^4 \quad r_x = 5.65\text{ in.}$

Yielding about $x-x$ axis :

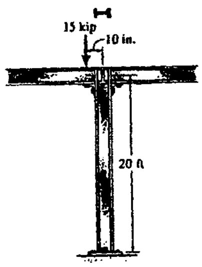
$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]; \quad K = 0.5$$

$$\frac{P}{A} = \frac{15}{7.69} = 1.9506\text{ ksi}; \quad \frac{ec}{r^2} = \frac{10\left(\frac{13.91}{2}\right)}{(5.65)^2} = 2.178714$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{0.5(20)(12)}{2(5.65)}\sqrt{\frac{15}{29(10^3)(7.69)}} = 0.087094$$

$$\sigma_{max} = 1.9506[1 + 2.178714\sec(0.087094)]$$
$$= 6.22\text{ ksi} < \sigma_Y = 36\text{ ksi} \quad \text{OK} \qquad \textbf{Ans}$$

13-53. The $W14 \times 26$ structural A-36 steel member is used as a column that is assumed to be fixed at its top and pinned at its bottom. If the 15-kip load is applied at an eccentric distance of 10 in., determine the maximum stress in the column.



Section properties for W 14 x 26

$A = 7.69\ \text{in}^2 \quad d = 13.91\ \text{in.} \quad I_x = 245\ \text{in}^4 \quad r_x = 5.65\ \text{in.}$

Yielding about $x-x$ axis :

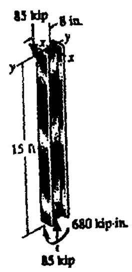
$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]; \quad K = 0.7$$

$$\frac{P}{A} = \frac{15}{7.69} = 1.9506\ \text{ksi}; \quad \frac{ec}{r^2} = \frac{10\left(\frac{13.91}{2}\right)}{(5.65)^2} = 2.178714$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{0.7(20)(12)}{2(5.65)}\sqrt{\frac{15}{29(10^3)(7.69)}} = 0.121931$$

$$\sigma_{max} = 1.9506[1 + 2.178714\sec(0.121931)]$$
$$= 6.24\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \quad \text{OK} \qquad \textbf{Ans}$$

13-54. The $W10 \times 30$ structural A-36 steel column is pinned at its top and bottom. If it is subjected to the eccentric load of 85 kip, determine the factor of safety with respect to yielding.



Section properties for $W\,10\text{x}30$:

$A = 8.84\ \text{in}^2 \quad I_x = 170\ \text{in}^4 \quad r_x = 4.38\ \text{in.}$

$d = 10.47\ \text{in.} \quad I_y = 16.7\ \text{in}^4$

Yielding about x-x axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{8(\frac{10.47}{2})}{4.38^2} = 2.1830$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{1.0(15)(12)}{2(4.38)}\sqrt{\frac{P}{29(10^3)(8.84)}} = 0.040583\sqrt{P}$$

$$36(8.84) = P[1 + 2.1830\sec(0.040583\sqrt{P})]$$

By trial and error :

$P = 94.8$ kip controls

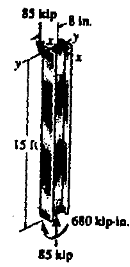
Buckling about y-y axis :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(16.7)}{[(1.0)(15)(12)]^2} = 147.5\ \text{kip}$$

Check : $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{147.5}{8.84} = 16.7\ \text{ksi} < \sigma_Y$ OK

$$\text{F.S.} = \frac{94.8}{85} = 1.12 \quad \textbf{Ans}$$

13-55. The $W10 \times 30$ structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to the eccentric load of 85 kip, determine if the column fails by yielding. The column is braced so that it does not buckle about the y–y axis.



Section properties for W 10x30 :

$A = 8.84\text{ in}^2 \quad I_x = 170\text{ in}^4 \quad r_x = 4.38\text{ in.}$

$d = 10.47\text{ in.} \quad I_y = 16.7\text{ in}^4$

Yielding about x - x axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{8(\frac{10.47}{2})}{4.38^2} = 2.1830$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{(2)(15)(12)}{2(4.38)}\sqrt{\frac{P}{29(10^3)(8.84)}} = 0.081166\sqrt{P}$$

$$36(8.84) = P[1 + 2.1830\sec(0.8166\sqrt{P})]$$

By trial and error :

$P = 81.0$ kip

Since 81.0 kip $<$ 85 kip the column fails by yielding.

Yes. **Ans**

***13-56.** A $W12 \times 26$ structural A-36 steel column is fixed connected at its ends and has a length $L = 23$ ft. Determine the maximum eccentric load P that can be applied so the column does not buckle or yield. Compare this value with an axial critical load P' applied through the centroid of the column.

Section properties for $W12x26$:

$A = 7.65 \text{ in}^2 \quad I_x = 204 \text{ in}^4 \quad I_y = 17.3 \text{ in}^4$

$r_x = 5.17 \text{ in.} \quad d = 12.22 \text{ in.}$

Buckling about y - y axis :

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$$P_{cr} = P_{cr} = \frac{\pi^2(29)(10^3)(17.3)}{[(0.5)(23)(12)]^2} = 260 \text{ kip}$$

Check : $\quad \sigma_{cr} = \frac{P_{cr}}{A} = \frac{260}{7.65} = 34.0 \text{ ksi} < \sigma_Y \qquad$ OK

Yielding about x - x axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{6(\frac{12.22}{2})}{5.17^2} = 1.37155$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{(0.5)(23)(12)}{2(5.17)}\sqrt{\frac{P}{29(10^3)(7.65)}} = 0.028335\sqrt{P}$$

$$36(7.65) = P[1 + 1.37155 \sec(0.028335\sqrt{P})]$$

By trial and error :

$P = 112.7 \text{ kip} = 113 \text{ kip} \qquad$ controls $\qquad$ **Ans**

13-57. A $W14 \times 30$ structural A-36 steel column is fixed connected at its ends and has a length $L = 20$ ft. Determine the maximum eccentric load P that can be applied so the column does not buckle or yield. Compare this value with an axial critical load P' applied through the centroid of the column.

Section properties for W 14 x 30

$A = 8.85 \text{ in}^2 \qquad d = 13.84 \text{ in.} \qquad I_x = 291 \text{ in}^4 \qquad r_x = 5.73 \text{ in.} \qquad I_y = 19.6 \text{ in}^4$

Buckling about $y-y$ axis :

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2} \qquad K = 0.5$$

$$P' = \frac{\pi^2 (29)(10^3)(19.6)}{[\, 0.5\,(20)(12)\,]^2} = 390 \text{ kip} \qquad \textbf{Ans}$$

Yielding about $x-x$ axis :

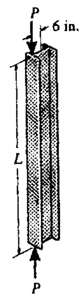
$$\sigma_{max} = \frac{P}{A}[1 + \frac{e c}{r^2}\sec(\frac{K L}{2 r}\sqrt{\frac{P}{E A}})]$$

$$36 = \frac{P}{8.85}[1 + \frac{6\,(\frac{13.84}{2})}{5.73^2}\sec(\frac{0.5\,(20)(12)}{2\,(5.73)}\sqrt{\frac{P}{29(10^3)(8.85)}})]$$

Solving by trial and error :

$P = 139$ kip controls **Ans**

13-58 Solve Prob. 13–57 if the column is fixed at its bottom and free at its top.



Section properties : For W 14 x 30

$A = 8.85\ \text{in}^2 \qquad d = 13.84\ \text{in.} \qquad I_x = 291\ \text{in}^4 \qquad r_x = 5.73\ \text{in.} \qquad I_y = 19.6\ \text{in}^4$

Buckling about $y-y$ axis :

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2} \qquad K = 2$$

$$P' = P_{cr} = \frac{\pi^2 (29)(10^3)(19.6)}{[\ 2\,(20)(12)\ ]^2} = 24.3\ \text{kip} \qquad \text{(controls)}$$ **Ans**

Yielding about $x-x$ axis :

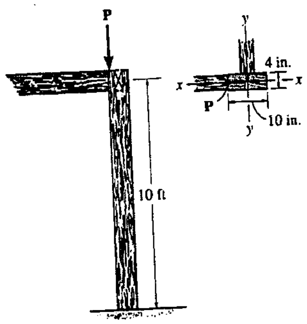
$$\sigma_{max} = \frac{P}{A}[1 + \frac{e\,c}{r^2}\sec(\frac{K L}{2 r}\sqrt{\frac{P}{E A}})]$$

$$36 = \frac{P}{8.85}[1 + \frac{6\,(\frac{13.84}{2})}{5.73^2}\sec(\frac{2\,(20)(12)}{2\,(5.73)}\sqrt{\frac{P}{29(10^3)(8.85)}})]$$

Solving by trial and error,

$P = 108.4\ \text{kip}$ **Ans**

13–59 The wood column is fixed at its base and can be assumed pin connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



Section Properties :

$$A = 10(4) = 40 \text{ in}^2 \qquad I_x = \frac{1}{12}(4)(10^3) = 333.33 \text{ in}^4 \qquad I_y = \frac{1}{12}(10)(4^3) = 53.33 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868 \text{ in.}$$

Buckling about y - y axis :

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(0.7)(10)(12)]^2} = 134 \text{ kip}$$

Check : $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{134}{40} = 3.36 \text{ ksi} < \sigma_Y$ OK

Yielding about x - x axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r})\sqrt{\frac{P}{EA}}]$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

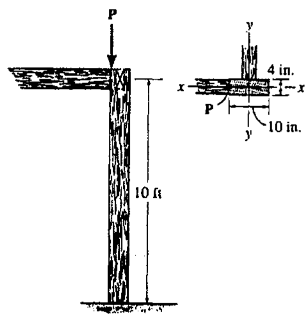
$$(\frac{KL}{2r})\sqrt{\frac{P}{EA}} = \frac{0.7(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.054221\sqrt{P}$$

$$8(40) = P[1 + 3.0\sec(0.054221\sqrt{P})]$$

By trial and error :

$P = 73.5$ kip controls **Ans**

•13–60 The wood column is fixed at its base and can be assumed fixed connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



Section Properties :

$$A = 10(4) = 40 \text{ in}^2 \qquad I_x = \frac{1}{12}(4)(10^3) = 333.33 \text{ in}^4 \qquad I_y = \frac{1}{12}(10)(4^3) = 53.33 \text{ in}^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{333.33}{40}} = 2.8868 \text{ in.}$$

Buckling about y - y axis :

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(1.8)(10^3)(53.33)}{[(0.5)(10)(12)]^2} = 263 \text{ kip}$$

Check : $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{263}{40} = 6.58 \text{ ksi} < \sigma_Y$ OK

Yielding about x - x axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{5(5)}{2.8868^2} = 3.0$$

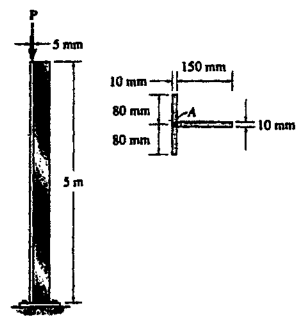
$$(\frac{KL}{2r})\sqrt{\frac{P}{EA}} = \frac{0.5(10)(12)}{2(2.8868)}\sqrt{\frac{P}{1.8(10^3)(40)}} = 0.038729\sqrt{P}$$

$$8(40) = P[1 + 3.0 \sec(0.038729\sqrt{P})]$$

By trial and error :

$P = 76.6$ kip (controls) **Ans**

13-61. The aluminum column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding. $E_{al} = 70$ GPa, $\sigma_Y = 95$ MPa.



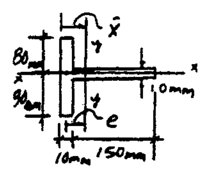
$$\bar{x} = \frac{(0.005)(0.16)(0.01) + (0.085)(0.15)(0.01)}{0.16(0.01) + 0.15(0.01)} = 0.04371 \text{ m}$$

$$I_y = \frac{1}{12}(0.16)(0.01)^3 + (0.16)(0.01)(0.04371 - 0.005)^2 + \frac{1}{12}(0.01)(0.15)^3 + (0.15)(0.01)(0.085 - 0.04371)^2 = 7.7807(10^{-6}) \text{ m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.16^3) + \frac{1}{12}(0.15)(0.01^3) = 3.42583(10^{-6}) \text{ m}^4$$

$$A = (0.16)(0.01) + (0.15)(0.01) = 3.1(10^{-3}) \text{ m}^2$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7.7807(10^{-6})}{3.1(10^{-3})}} = 0.0501 \text{ m}$$



Buckling about x-x axis :

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(70)(10^9)(3.42583)(10^{-6})}{[(2.0)(5)]^2} = 23668 \text{ N}$$

$$P_{allow} = \frac{P_{cr}}{3} = 7.89 \text{ kN} \qquad \text{(controls)} \qquad \text{Ans}$$

Check : $\sigma_{cr} = \dfrac{P_{cr}}{A} = \dfrac{23668 \text{ N}}{3.1(10^{-3})} = 7.63 \text{ MPa} < \sigma_Y$ OK

Yielding about y-y axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r})\sqrt{\frac{P}{EA}}]$$

$$\frac{ec}{r^2} = \frac{(0.03871)(0.04371)}{0.0501^2} = 0.6741$$

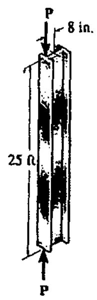
$$(\frac{KL}{2r})\sqrt{\frac{P}{EA}} = \frac{2.0(5)}{2(0.0501)}\sqrt{\frac{P}{70(10^9)(3.1)(10^{-3})}} = 6.7749(10^{-3})\sqrt{P}$$

$$95(10^6)(3.1)(10^{-3}) = P[1 + 0.6741 \sec(6.7749(10^{-3})\sqrt{P}]$$

By trial and error :

$$P = 45.61 \text{ kN} \qquad P_{allow} = \frac{45.61}{3} = 15.2 \text{ kN}$$

13-62 A $W\ 10 \times 15$ structural A-36 steel member is used as a fixed-connected column. Determine the maximum eccentric load P that can be applied so the column does not buckle or yield. Compare this value with an axial critical load P' applied through the centroid of the column.



Section properties for $W\ 10 \times 15$

$A = 4.41\ \text{in}^2 \quad d = 9.99\ \text{in.} \quad I_x = 68.9\ \text{in}^4 \quad r_x = 3.95\ \text{in.} \quad I_y = 2.89\ \text{in}^4$

Buckling about $y-y$ axis :

$$P' = P = P_{cr} = \frac{\pi^2 E I}{(K L)^2} = \frac{\pi^2 (29)(10^3)(2.89)}{[\,0.5\,(25)(12)\,]^2}$$

$$= 36.8\ \text{kip} \quad \text{(controls)} \qquad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{36.8}{4.41} = 8.34\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{OK}$$

Yielding about $x-x$ axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{e c}{r^2}\sec(\frac{K L}{2 r}\sqrt{\frac{P}{E A}})]$$

$$\frac{e c}{r^2} = \frac{8\,(\frac{9.99}{2})}{(3.95)^2} = 2.561128$$

$$\frac{K L}{2 r}\sqrt{\frac{P}{E A}} = \frac{0.5\,(25)(12)}{2\,(3.95)}\sqrt{\frac{P}{29\,(10^3)(4.41)}} = 0.05309\sqrt{P}$$

$$36\,(4.41) = P\,[1 + 2.561128\sec(0.05309\sqrt{P})]$$

By trial and error,

$$P = 42.6\ \text{kip}$$

13-63 Solve Prob. 13-62 if the column is pin-connected at its ends.

Section properties for W 10 x 15

$A = 4.41 \text{ in}^2 \quad d = 9.99 \text{ in.} \quad I_x = 68.9 \text{ in}^4 \quad r_x = 3.95 \text{ in.} \quad I_y = 2.89 \text{ in}^4$

Buckling about $y-y$ axis :

$$P' = P = P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2 (29)(10^3)(2.89)}{[\,1.0\,(25)(12)\,]^2}$$

$$= 9.19 \text{ kip} \qquad \text{(controls)}$$ **Ans**

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{9.19}{4.41} = 2.08 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \text{OK}$$

Yielding about $x-x$ axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{e\,c}{r^2}\sec(\frac{K\,L}{2\,r}\sqrt{\frac{P}{E\,A}})]$$

$$\frac{e\,c}{r^2} = \frac{8\,(\frac{9.99}{2})}{(3.95)^2} = 2.561128024$$

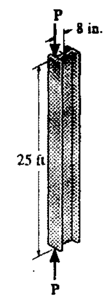
$$\frac{K\,L}{2\,r}\sqrt{\frac{P}{E\,A}} = \frac{1.0\,(25)(12)}{2\,(3.95)}\sqrt{\frac{P}{29\,(10^3)(4.41)}} = 0.106188104\,\sqrt{P}$$

$$36\,(4.41) = P\,[1 + 2.561128 \sec(0.106188104\,\sqrt{P})]$$

By trial and error,

$$P = 37.6 \text{ kip}$$

*13-64 Solvė Prob. 13-62 if the column is fixed at its bottom and pinned at its top.



Section properties for $W\,10 \times 15$

$A = 4.41 \text{ in}^2 \qquad d = 9.99 \text{ in.} \qquad I_x = 68.9 \text{ in}^4 \qquad r_x = 3.95 \text{ in.} \qquad I_y = 2.89 \text{ in}^4$

Buckling about $y-y$ axis :

$$P' = P = P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2 (29)(10^3)(2.89)}{[\,0.7\,(25)(12)\,]^2}$$

$$= 18.8 \text{ kip} \qquad \text{(controls)} \qquad \textbf{Ans}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{18.8}{4.41} = 4.25 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \qquad \text{OK}$$

Yielding about $x-x$ axis :

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e c}{r^2} \sec\left(\frac{K L}{2 r}\sqrt{\frac{P}{E A}}\right)\right]$$

$$\frac{e c}{r^2} = \frac{8\,(\frac{9.99}{2})}{(3.95)^2} = 2.561128024$$

$$\frac{K L}{2 r}\sqrt{\frac{P}{E A}} = \frac{0.7\,(25)(12)}{2\,(3.95)}\sqrt{\frac{P}{29\,(10^3)(4.41)}} = 0.074331673\,\sqrt{P}$$

$$36\,(4.41) = P\,[1 + 2.561128024 \sec(0.074331673\,\sqrt{P})]$$

By trial and error,

$$P = 40.9 \text{ kip}$$

13–65 The W 14 × 53 structural A-36 steel column is fixed at its base and free at its top. If $P = 75$ kip, determine the sidesway deflection at its top and the maximum stress in the column.

Section properties for a W 14x53 :

$A = 15.6\text{ in}^2 \quad I_x = 541\text{ in}^4 \quad I_y = 57.7\text{ in}^4$

$r_x = 5.89\text{ in.} \quad d = 13.92\text{ in.}$

Maximum deflection :

$$v_{max} = e[\sec(\sqrt{\frac{P}{EI}}\frac{KL}{2}) - 1]$$

$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{75}{29(10^3)541}}(\frac{2.0(18)(12)}{2}) = 0.472267$$

$$v_{max} = 10[\sec(0.472267) - 1] = 1.23\text{ in.} \qquad \textbf{Ans}$$

Maximum stress :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{P}{A} = \frac{75}{15.6} = 4.808\text{ ksi}$$

$$\frac{ec}{r^2} = \frac{10(\frac{13.92}{2})}{5.89^2} = 2.0062$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(18)(12)}{2(5.89)}\sqrt{\frac{75}{29(10^3)(15.6)}} = 0.47218$$

$$\sigma_{max} = 4.808[1 + 2.0062\sec(0.47218)] = 15.6\text{ ksi} < \sigma_Y \qquad \textbf{Ans}$$

13-66 The W 14 × 53 steel column is fixed at its base and free at its top. Determine the maximum eccentric load P that it can support without causing it to buckle or yield. E_{st} = 29(10³) ksi, σ_Y = 50 ksi.

Section properties for a W 14x53 :

$A = 15.6\ \text{in}^2 \quad I_x = 541\ \text{in}^4 \quad I_y = 57.7\ \text{in}^4$

$r_x = 5.89\ \text{in.} \quad d = 13.92\ \text{in.}$

Buckling about y - y axis :

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(29)(10^3)(57.7)}{[(2.0)(18)(12)]^2} = 88.5\ \text{kip} \qquad \text{controls} \qquad \textbf{Ans}$$

Check : $$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{88.5}{15.6} = 5.67\ \text{ksi} < \sigma_Y \qquad \text{OK}$$

Yielding about x - x axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{10(\frac{13.92}{2})}{5.89^2} = 2.0062$$

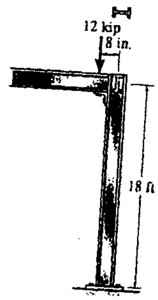
$$(\frac{KL}{2r})\sqrt{\frac{P}{EA}} = \frac{2.0(18)(12)}{2(5.89)}\sqrt{\frac{P}{29(10^3)(15.6)}} = 0.054523\sqrt{P}$$

$$50(15.6) = P[1 + 2.0062\sec(0.054523\sqrt{P})]$$

By trial and error :

$P = 204$ kip

13-67. The W10 × 45 structural A-36 steel column is assumed to be pinned at its top and fixed at its bottom. If the 12-kip load is applied at an eccentric distance of 8 in., determine the maximum stress in the column.



Section properties for W 10x45 :

$A = 13.3\ \text{in}^2 \quad I_x = 248\ \text{in}^4 \quad I_y = 53.4\ \text{in}^4$

$r_x = 4.32\ \text{in.} \quad d = 10.10\ \text{in.}$

Secant formula :

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

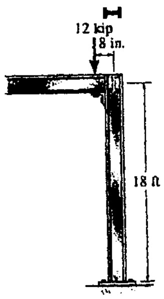
$$\frac{P}{A} = \frac{12}{13.3} = 0.90226\ \text{ksi}$$

$$\frac{ec}{r^2} = \frac{8(\frac{10.10}{2})}{4.32^2} = 2.16478$$

$$\left(\frac{KL}{2r}\right)\sqrt{\frac{P}{EA}} = \frac{0.7(18)(12)}{2(4.32)}\sqrt{\frac{12}{29(10^3)(13.3)}} = 0.097612$$

$\sigma_{max} = 0.90226[1 + 2.16478\sec(0.097612)] = 2.86\ \text{ksi}$ **Ans**

•13-68 The W 10 × 45 structural A-36 steel column is assumed to be fixed at its top and bottom. If the 12-kip load is applied at an eccentric distance of 8 in., determine the maximum stress in the column.



Section properties for W 10x45 :

$A = 13.3\ \text{in}^2 \quad I_x = 248\ \text{in}^4 \quad I_y = 53.4\ \text{in}^4$

$r_x = 4.32\ \text{in.} \quad d = 10.10\ \text{in.}$

Secant formula :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

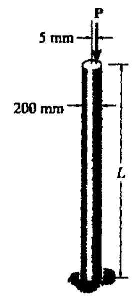
$$\frac{P}{A} = \frac{12}{13.3} = 0.90226\ \text{ksi}$$

$$\frac{ec}{r^2} = \frac{8(\frac{10.10}{2})}{4.32^2} = 2.16478$$

$$(\frac{KL}{2r})\sqrt{\frac{P}{EA}} = \frac{0.5(18)(12)}{2(4.32)}\sqrt{\frac{12}{29(10^3)(13.3)}} = 0.069723$$

$$\sigma_{max} = 0.90226[1 + 2.16478\sec(0.069723)] = 2.86\ \text{ksi} \qquad \textbf{Ans}$$

13-69 The aluminum rod is fixed at its base and free at its top. If the eccentric load $P = 200$ kN is applied, determine the greatest allowable length L of the rod so that it does not buckle or yield. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.



Section properties :

$A = \pi(0.1^2) = 0.031416\ \text{m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6})\ \text{m}^4$

$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05\ \text{m}$

Yielding :

$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$

$\frac{P}{A} = \frac{200(10^3)}{0.31416} = 6.3662(10^6)\ \text{Pa}$

$\frac{ec}{r^2} = \frac{0.005(0.1)}{(0.05)^2} = 0.2$

$(\frac{KL}{2r})\sqrt{\frac{P}{EA}} = \frac{2.0(L)}{2(0.5)}\sqrt{\frac{200(10^3)}{72(10^9)(0.031416)}} = 0.188063L$

$410(10^6) = 6.3662(10^6)[1 + 0.2\sec(0.188063\ L)]$

$L = 8.34$ m (controls) **Ans**

Buckling about x - x axis :

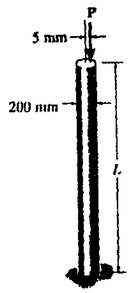
$\frac{P}{A} = 6.36\ \text{MPa} < \sigma_Y$ Euler formula is valid.

$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$

$200(10^3) = \frac{\pi^2(72)(10^9)(78.54)(10^{-6})}{[(2.0)(L)]^2}$

$L = 8.35$ m

13-70 The aluminum rod is fixed at its base and free at its top. If the length of the rod is $L = 2$ m, determine the greatest allowable load P that can be applied so that the rod does not buckle or yield. Also, determine the largest sidesway deflection of the rod due to the loading. $E_{al} = 72$ GPa, $\sigma_Y = 410$ MPa.



Section properties :

$$A = \pi(0.1^2) = 0.031416 \text{ m}^2 \qquad I = \frac{\pi}{4}(0.1^4) = 78.54(10^{-6}) \text{ m}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{78.54(10^{-6})}{0.031416}} = 0.05 \text{ m}$$

Yielding :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]$$

$$\frac{ec}{r^2} = \frac{(0.005)(0.1)}{0.05^2} = 0.2$$

$$(\frac{KL}{2r})\sqrt{\frac{P}{EA}} = \frac{2(2)}{2(0.05)}\sqrt{\frac{P}{72(10^9)(0.031416)}} = 0.8410(10^{-3})\sqrt{P}$$

$$410(10^6)(0.031416) = P[1 + 0.2\sec(0.8410(10^{-3})\sqrt{P})]$$

By trial and error :

$P = 3.20$ MN (controls) **Ans**

Buckling :

$$P = P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(72)(10^9)(78.54)(10^{-6})}{[(2.0)(2)]^2} = 3488 \text{ kN}$$

Check : $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{3488(10^3)}{0.031416} = 111 \text{ MPa} < \sigma_Y$ OK

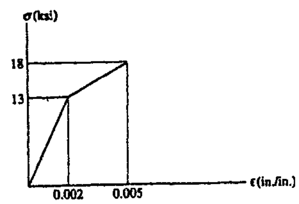
Maximum deflection :

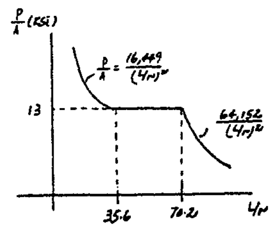
$$v_{max} = e[\sec(\sqrt{\frac{P}{EI}}\frac{KL}{2}) - 1]$$

$$\sqrt{\frac{P}{EI}}\frac{KL}{2} = \sqrt{\frac{3.20(10^6)}{72(10^9)(78.54)(10^{-6})}}(\frac{2.0(2)}{2}) = 1.5045$$

$v_{max} = 5[\sec(1.5045) - 1] = 70.5$ mm **Ans**

13-71. Construct the buckling curve, P/A versus L/r, for a column that has a bilinear stress–strain curve in compression as shown.





$$E_1 = \frac{13}{0.002} = 6.5\,(10^3)\text{ ksi}$$

$$E_2 = \frac{18 - 13}{0.005 - 0.002} = 1.6667\,(10^3)\text{ ksi}$$

For $E_t = E_1$

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 E_t}{(\frac{L}{r})^2} = \frac{\pi^2 (6.5)(10^3)}{(\frac{L}{r})^2} = \frac{64152}{(\frac{L}{r})^2}$$

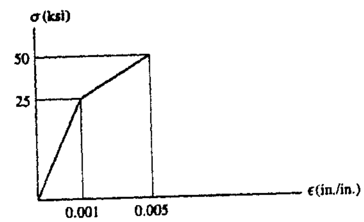
$$\sigma_{cr} = 13 = \frac{\pi^2 (6.5)(10^3)}{(\frac{L}{r})^2}; \quad \frac{L}{r} = 70.2$$

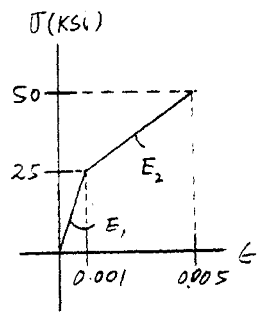
For $E_t = E_2$

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 (1.6667)(10^3)}{(\frac{L}{r})^2} = \frac{16449}{(\frac{L}{r})^2}$$

$$\sigma_{cr} = 13 = \frac{\pi^2 (1.6667)(10^3)}{(\frac{L}{r})^2}; \quad \frac{L}{r} = 35.6$$

•13–72 Construct the buckling curve, P/A versus L/r, for a column that has a bilinear stress–strain curve in compression as shown.





From Fig. (a) :

$$E_1 = \frac{25}{0.001} = 25\,(10^3)\text{ ksi}$$

$$E_2 = \frac{50-25}{0.005-0.001} = 6.25\,(10^3)\text{ ksi}$$

For $E_t = E_1$

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 E_t}{(\frac{L}{r})^2}$$

$$= \frac{\pi^2 (25)(10^3)}{(\frac{L}{r})^2} = \frac{247\,(10^3)}{(\frac{L}{r})^2}$$

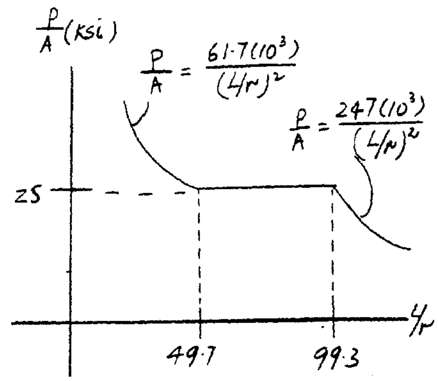
$$\sigma_{cr} = 25 = \frac{\pi^2 (25)(10^3)}{(\frac{L}{r})^2}; \qquad \frac{L}{r} = 99.3$$

For $E_t = E_2$

$$\sigma_{cr} = \frac{P}{A} = \frac{\pi^2 (6.25)(10^3)}{(\frac{L}{r})^2}$$

$$= \frac{61.7\,(10^3)}{(\frac{L}{r})^2}$$

$$\sigma_{cr} = 25 = \frac{\pi^2 (6.25)(10^3)}{(\frac{L}{r})^2}; \qquad \frac{L}{r} = 49.7$$



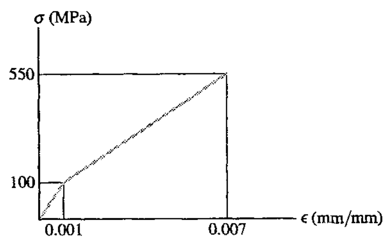
13–73 A column of intermediate length buckles when the compressive strength is 40 ksi. If the slenderness ratio is 60, determine the tangent modulus.

$$\sigma_{cr} = \frac{\pi^2 E_t}{(\frac{KL}{r})^2}; \qquad (\frac{KL}{r}) = 60$$

$$40 = \frac{\pi^2 E_t}{(60)^2}$$

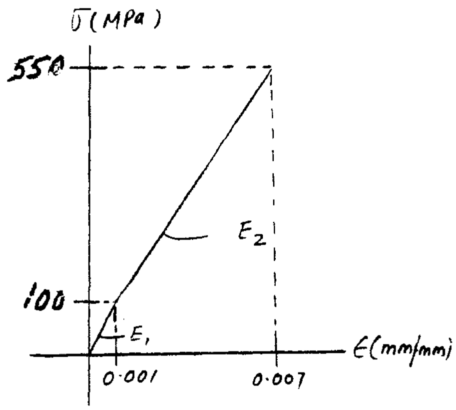
$$E_t = 14590 \text{ ksi} = 14.6\,(10^3) \text{ ksi}$$ **Ans**

13–74 The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.



$$E_1 = \frac{100\ (10^6)}{0.001} = 100 \text{ GPa}$$

$$E_2 = \frac{550\ (10^6) - 100\ (10^6)}{0.007 - 0.001} = 75 \text{ GPa}$$



Section properties :

$$I = \frac{\pi}{4} c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4} c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02 \text{ m}$$

Engesser's equation :

$$\frac{KL}{r} = \frac{1.0\,(1.5)}{0.02} = 75$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{(\frac{KL}{r})^2} = \frac{\pi^2 E_t}{(75)^2} = 1.7546\,(10^{-3})\, E_t$$

Assume $E_t = E_1 = 100$ GPa

$\sigma_{cr} = 1.7546\,(10^{-3})(200)(10^9) = 175 \text{ MPa} > 100 \text{ MPa}$

Therefore, inelastic buckling occurs :

Assume $E_t = E_2 = 75$ GPa

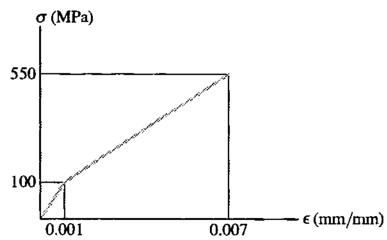
$\sigma_{cr} = 1.7546\,(10^{-3})\,(75)(10^9) = 131.6 \text{ MPa}$

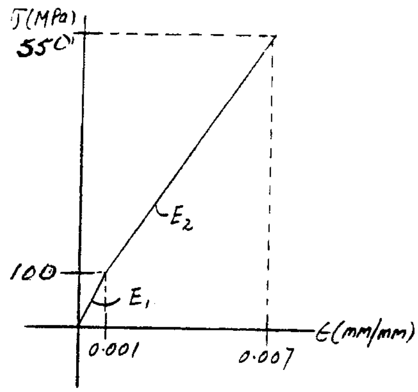
$100 \text{ MPa} < \sigma_{cr} < 550 \text{ MPa}$ OK

Critical load :

$P_{cr} = \sigma_{cr} A = 131.6\,(10^6)(\pi)(0.04^2) = 661 \text{ kN}$ **Ans**

13–75 The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.





$$E_1 = \frac{100\,(10^6)}{0.001} = 100\text{ GPa}$$

$$E_2 = \frac{550\,(10^6) - 100\,(10^6)}{0.007 - 0.001} = 75\text{ GPa}$$

Section properties :

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation :

$$\frac{KL}{r} = \frac{0.5\,(1.5)}{0.02} = 37.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{(\frac{KL}{r})^2} = \frac{\pi^2 E_t}{(37.5)^2} = 7.018385\,(10^{-3})\,E_t$$

Assume $E_t = E_1 = 100$ GPa

$\sigma_{cr} = 7.018385\,(10^{-3})(100)(10^9) = 701.8\text{ MPa} > 100\text{ MPa}$ NG

Assume $E_t = E_2 = 75$ Gpa

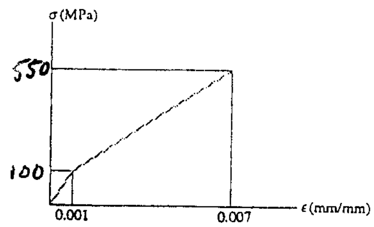
$\sigma_{cr} = 7.018385\,(10^{-3})(75)(10^9) = 526.4\text{ MPa}$

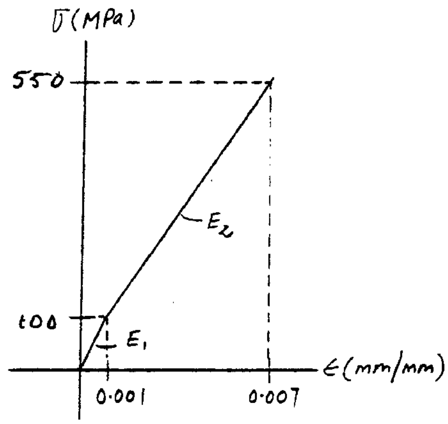
$100\text{ MPa} < \sigma_{cr} < 550\text{ MPa}$ OK

Critical load :

$P_{cr} = \sigma_{cr} A = 526.4\,(10^6)(\pi)(0.04^2) = 2645.9\text{ kN}$ **Ans**

•13–76 The stress–strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and length of 1.5 m is made from this material, determine the critical load provided one end is pinned and the other is fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.





$$E_1 = \frac{100\,(10^6)}{0.001} = 100\text{ GPa}$$

$$E_2 = \frac{550\,(10^6) - 100\,(10^6)}{0.007 - 0.001} = 75\text{ GPa}$$

Section properties :

$$I = \frac{\pi}{4}c^4; \qquad A = \pi c^2$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{4}c^4}{\pi c^2}} = \frac{c}{2} = \frac{0.04}{2} = 0.02\text{ m}$$

Engesser's equation :

$$\frac{KL}{r} = \frac{0.7\,(1.5)}{0.02} = 52.5$$

$$\sigma_{cr} = \frac{\pi^2 E_t}{(\frac{KL}{r})^2} = \frac{\pi^2 E_t}{(52.5)^2} = 3.58081\,(10^{-3})\,E_t$$

Assume $E_t = E_1 = 100$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(100)(10^9) = 358.1\text{ MPa} > 100\text{ MPa}$ NG

Assume $E_t = E_2 = 75$ GPa

$\sigma_{cr} = 3.58081\,(10^{-3})(75)(10^9) = 268.6\text{ MPa}$

$100\text{ MPa} < \sigma_{cr} < 550\text{ MPa}$ OK

Critical load :

$P_{cr} = \sigma_{cr} A = 268.6\,(10^6)(\pi)(0.04^2) = 1350\text{ kN}$ **Ans**

13-77. Determine the largest length of a $W10 \times 12$ structural A-36 steel section if it is pin supported and is subjected to an axial load of 28 kip. Use the AISC equations.

For a W 10x12, $r_y = 0.785$ in. $A = 3.54\ \text{in}^2$

$$\sigma = \frac{P}{A} = \frac{28}{3.54} = 7.91\ \text{ksi}$$

Assume a long column:

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\pi^2 E}{23\,\sigma_{allow}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(7.91)}} = 137.4$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \qquad \frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

Long column.

$$\frac{KL}{r} = 137.4$$

$$L = 137.4\left(\frac{r}{K}\right) = 137.4\left(\frac{0.785}{1}\right) = 107.86\ \text{in.}$$ **Ans.**

$= 8.99$ ft **Ans.**

13–78 Determine the largest length of a $W\ 10 \times 12$ structural A-36 steel section if it is fixed supported and is subjected to an axial load of 28 kip. Use the AISC equations.

For a W 10x12, $r_y = 0.785$ in. $A = 3.54\text{ in}^2$

$$\sigma = \frac{P}{A} = \frac{28}{3.54} = 7.91\text{ ksi}$$

Assume a long column :

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$(\frac{KL}{r})^2 = \frac{12\pi^2 E}{23\,\sigma_{\text{allow}}} = \sqrt{\frac{12\pi^2 E}{23\,\sigma_{\text{allow}}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(7.91)}} = 137.4$$

$$(\frac{KL}{r})_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \qquad \frac{KL}{r} > (\frac{KL}{r})_c$$

Long column.

$$\frac{KL}{r} = 137.4$$

$$L = 137.4(\frac{r}{K}) = 137.4(\frac{0.785}{0.5}) = 215.72\text{ in.}$$

$L = 18.0$ ft **Ans**

13-79. Determine the largest length of a $W8 \times 31$ structural A-36 steel section if it is pin supported and is subjected to an axial load of 130 kip. Use the AISC equations.

For a $W\,8\text{x}31$, $A = 9.13\ \text{in}^2$ $r_y = 2.02$ in.

$$\sigma = \frac{P}{A} = \frac{130}{9.13} = 14.239\ \text{ksi}$$

Assume a long column :

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\,\pi^2 E}{23\,\sigma_{allow}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(14.239)}} = 102.4$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]}$$

$$14.239 = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{126.1}\right) - \frac{1}{8}\left(\frac{KL/r}{126.1}\right)^3\right]}$$

$$1.132(10^{-3})\left(\frac{KL}{r}\right)^2 + 0.042344\left(\frac{KL}{r}\right) - 0.887655(10^{-6})\left(\frac{KL}{r}\right)^3 = 12.268$$

By trial and error :

$$\frac{KL}{r} = 89.71$$

$$L = 89.71\left(\frac{2.02}{1.0}\right) = 181.21\ \text{in.} = 15.1\ \text{ft} \qquad \textbf{Ans}$$

•13–80 Determine the largest length of a $W\,8 \times 31$ structural A-36 steel section if it is pin supported and is subjected to an axial load of 80 kip. Use the AISC equations.

For a W 8x31 $\quad A = 9.13\ \text{in}^2 \qquad r_y = 2.02$ in.

$$\sigma = \frac{P}{A} = \frac{80}{9.13} = 8.762\ \text{ksi}$$

Assume a long column :

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\left(\frac{KL}{r}\right) = \sqrt{\frac{12\,\pi^2 E}{23\,\sigma_{\text{allow}}}} = \sqrt{\frac{12\pi^2(29)(10^3)}{23(8.762)}} = 130.54$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r} > \left(\frac{KL}{r}\right)_c \qquad \text{(Assumption OK)}$$

$$\frac{KL}{r} = 130.54$$

$$L = 130.54\left(\frac{2.02}{1.0}\right) = 263.7\ \text{in.} = 22.0\ \text{ft} \qquad \textbf{Ans}$$

13–81 Using the AISC equations, select from Appendix *B* the lightest-weight structural A-36 steel column that is 12 ft long and supports an axial load of 20 kip. The ends are pinned.

Try $W\,6\text{x}12$ $A = 3.55\ \text{in}^2$ $r_y = 0.918$ in.

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\left(\frac{KL}{r_y}\right) = \frac{(1.0)(12)(12)}{0.918} = 156.9, \qquad \left(\frac{KL}{r_y}\right) > \left(\frac{KL}{r}\right)_c$$

Long column

$$\sigma_{\text{allow}} = \frac{12\,\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(156.9)^2} = 6.069\ \text{ksi}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$
$$= 6.069(3.55) = 21.5\ \text{kip} > 20\ \text{kip} \qquad \textbf{OK}$$

Use $W\,6\text{x}12$ **Ans**

13-82. Using the AISC equations, select from Appendix B the lightest-weight structural steel column that is 14 ft long and supports an axial load of 40 kip. The ends are pinned. Take $\sigma_y = 50$ ksi.

Try, $W\,6\text{x}15$ ($A = 4.43\ \text{in}^2$ $\quad r_y = 1.46$ in.)

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\left(\frac{KL}{r_y}\right) = \frac{(1.0)(14)(12)}{1.46} = 115.1, \qquad \left(\frac{KL}{r_y}\right) > \left(\frac{KL}{r}\right)_c$$

Long column

$$\sigma_{\text{allow}} = \frac{12\,\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(115.1)^2} = 11.28\ \textbf{ksi}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A$$
$$= 11.28(4.43) = 50.0\ \text{kip} > 40\ \text{kip} \qquad \text{OK}$$

Use $W\,6\text{x}15$ **Ans**

13-83. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 12 ft long and supports an axial load of 40 kip. The ends are fixed. Take $\sigma_y = 50$ ksi.

Try $W\,6\text{x}9 \qquad A = 2.68\ \text{in}^2 \qquad r_y = 0.905$ in.

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} = \frac{0.5(12)(12)}{0.905} = 79.56$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{79.56}{126.1}\right)^2\right]36\ \text{ksi}}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{79.56}{126.1}\right) - \frac{1}{8}\left(\frac{79.56}{12.61}\right)^3\right]} = 15.40\ \text{ksi}$$

$$P_{allow} = \sigma_{allow} A$$
$$= 15.40(2.68)$$
$$= 41.3\ \text{kip} > 40\ \text{kip} \qquad \text{OK}$$

Use $W6\text{x}9$ **Ans**

*13-84. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 14 ft long and supports an axial load of 40 kip. The ends are fixed.

Try $W\,6\text{x}9$ $\quad A = 2.68\ \text{in}^2 \qquad r_y = 0.905\ \text{in.}$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\frac{KL}{r_y} = \frac{0.5(14)(12)}{0.905} = 92.82$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Intermediate column

$$\sigma_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{92.82}{107}\right)^2\right]50}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{92.82}{107}\right) - \frac{1}{8}\left(\frac{92.82}{107}\right)^3\right]} = 16.33\ \text{ksi}$$

$$P_{allow} = \sigma_{allow} A$$
$$= 16.33(2.68)$$
$$= 43.8\ \text{kip} > 40\ \text{kip} \qquad \text{OK}$$

Use $W6\text{x}9$ **Ans**

13-85. Determine the largest length of a $W8 \times 48$ structural A-36 steel section if it is pin supported and is subjected to an axial load of 55 kip. Use the AISC equations.

Section Properties **:** For a W8 × 48 wide flange section,

$$A = 14.1\ \text{in}^2 \qquad r_y = 2.08\ \text{in}$$

Slenderness Ratio **:** For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(L)}{2.08} = 0.4808L$$

AISC Column Formula **:** Assume it is a *long* column.

$$\sigma_{allow} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}$$

$$\frac{55}{14.1} = \frac{12\pi^2[29(10^3)]}{23(0.4808L)^2}$$

$$L = 407.0\ \text{in.}$$

Here, $\frac{KL}{r} = 0.4808(407.0) = 195.7$ and for A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \leq \frac{KL}{r} \leq 200$, the assumption is correct. Thus,

$$L = 407.0\ \text{in.} = 33.9\ \text{ft}$$ **Ans**

13–86 Determine the largest length of a $W8 \times 31$ structural A-36 steel section if it is pin supported and is subjected to an axial load of 18 kip. Use the AISC equations.

Section properties : For $W\,8 \times 31$ $\quad r_y = 2.02$ in. $\quad A = 9.13\ \text{in}^2$

Assume it as a long column :

$$\sigma_{\text{allow}} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}; \qquad \left(\frac{KL}{r}\right)^2 = \frac{12\pi^2 E}{23\,\sigma_{\text{allow}}}$$

$$\frac{KL}{r} = \sqrt{\frac{12\pi^2 E}{23\,\sigma_{\text{allow}}}}$$

Here $\sigma_{\text{allow}} = \dfrac{P}{A} = \dfrac{18}{9.13} = 1.9715\ \text{ksi}$

$$\frac{KL}{r} = \sqrt{\frac{12\pi^2 (29)(10^3)}{23\,(1.9715)}} = 275.2 > 200$$

Thus use $\dfrac{KL}{r} = 200$

$$\frac{1.0\,(L)}{2.02} = 200$$

$L = 404\ \text{in.} = 33.7\ \text{ft}$ **Ans**

13–87 Using the AISC equations, select from Appendix *B* the lightest-weight structural A-36 steel column that is 30 ft long and supports an axial load of 200 kip. The ends are fixed.

Try $W\,8 \times 48$ $\quad r_y = 2.08\text{ in.} \quad A = 14.1\text{ in}^2$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} = \frac{0.5(30)(12)}{2.08} = 86.54$$

$\left(\frac{KL}{r_y}\right) < \left(\frac{KL}{r}\right)_c$ intermediate column.

$$\sigma_{allow} = \frac{\left\{1 - \frac{1}{2}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^2\right\}\sigma_Y}{\left\{\frac{5}{3} + \frac{3}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right] - \frac{1}{8}\left[\frac{\frac{KL}{r}}{\left(\frac{KL}{r}\right)_c}\right]^3\right\}}$$

$$= \frac{\left\{1 - \frac{1}{2}\left[\frac{86.54}{126.1}\right]^2\right\}36}{\left\{\frac{5}{3} + \frac{3}{8}\left[\frac{86.54}{126.1}\right] - \frac{1}{8}\left[\frac{86.54}{126.1}\right]^3\right\}} = 14.611\text{ ksi}$$

$P_{allow} = \sigma_{allow} A = 14.611(14.1) = 206\text{ kip} > P = 200\text{ kip}$ **OK**

Use $W\,8 \times 48$ **Ans**

•13–88 Determine the largest length of a $W\,8 \times 31$ structural A-36 steel column if it is to support an axial load of 10 kip. The ends are pinned.

$W\,8 \times 31 \qquad r_y = 2.02 \text{ in.} \qquad A = 9.13 \text{ in}^2$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} = \frac{1.0L}{2.02}$$

Assume $\dfrac{KL}{r_y} > \left(\dfrac{KL}{r}\right)_c$

$$\sigma_{allow} = \frac{12\pi^2 E}{23\left(\frac{KL}{r}\right)^2}; \qquad \frac{KL}{r} = \sqrt{\frac{12\pi^2 E}{23\,\sigma_{allow}}}$$

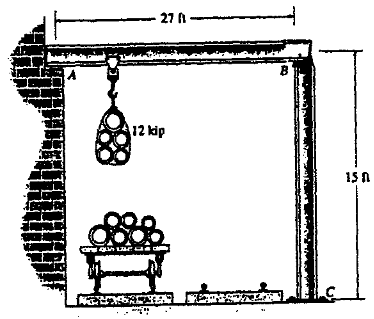
Here $\sigma_{allow} = \dfrac{P}{A} = \dfrac{10}{9.13} = 1.10$

$$\frac{KL}{r} = \sqrt{\frac{12\pi^2\,29(10^3)}{23(1.10)}} = 369.2 > \left(\frac{KL}{r}\right)_c \qquad \text{Assumption OK}$$

$$\frac{1.0(L)}{2.02} = 369.2$$

$L = 745.9 \text{ in.} = 62.2 \text{ ft}$ **Ans**

13-89. The beam and column arrangement is used in a railroad yard for loading and unloading cars. If the maximum anticipated hoist load is 12 kip, determine if the $W8 \times 31$ structural A-36 steel column is adequate for supporting the load. The hoist travels along the bottom flange of the beam, 1 ft $\leq x \leq$ 25 ft, and has negligible size. Assume the beam is pinned to the column at B and pin-supported at A. The column is also pinned at C and is braced so it will not buckle out of the plane of the loading.

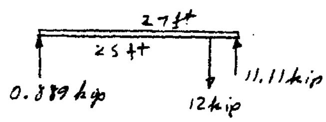


For $W\,8 \times 31$, $r_x = 3.47$ in., $A = 9.13$ in^2

Maximum axial load occurs when $x = 25$ ft

$$\frac{KL}{r} = \frac{(1.0)(15)(12)}{3.47} = 51.87$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$



Here $0 < 51.87 < 126.1$

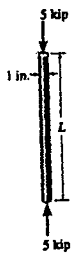
Intermediate column :

$$\sigma_{allow} = \frac{[1 - \frac{1}{2}(\frac{KL/r}{(KL/r)_c})^2]\sigma_Y}{[\frac{5}{3} + \frac{3}{8}(\frac{KL/r}{(KL/r)_c}) - \frac{1}{8}(\frac{KL/r}{(KL/r)_c})^3]}$$

$$= \frac{(1 - \frac{1}{2}(51.87/126.1)^2)36}{\{\frac{5}{3} + [(\frac{3}{8})(51.87/126.1)] - [\frac{1}{8}(51.87/126.1)^3]\}} = 18.2 \text{ ksi}$$

$$\sigma = \frac{P}{A} = \frac{11.11}{9.13} = 1.22 \text{ ksi} < 18.2 \text{ ksi} \qquad \text{OK} \qquad \text{Ans}$$

13-90. The 1-in.-diameter rod is used to support an axial load of 5 kip. Determine its greatest allowable length L if it is made of 2014-T6 aluminum. Assume that the ends are pin connected.



Section Properties :

$$A = \pi(0.5^2) = 0.7854 \text{ in}^2$$

$$r = \frac{0.5}{2} = 0.25 \text{ in.}$$

Allowable Stress :

$$\sigma_{allow} = \frac{P}{A} = \frac{5}{0.7854} = 6.366 \text{ ksi}$$

Assume long column :

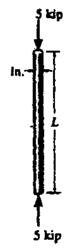
$$\sigma_{allow} = \frac{54000}{(KL/r)^2}$$

$$\frac{KL}{r} = \sqrt{\frac{54000}{\sigma_{allow}}} = \sqrt{\frac{54000}{6.366}} = 92.1 > 55 \qquad \text{Assumption OK}$$

$$\frac{KL}{r} = 92.1$$

$$L = 92.1\left(\frac{0.25}{1.0}\right) = 23.02 \text{ in.} = 1.92 \text{ ft} \qquad \textbf{Ans}$$

13-91. The 1-in.-diameter rod is used to support an axial load of 5 kip. Determine its greatest allowable length L if it is made of 2014-T6 aluminum. Assume that the ends are fixed connected.



Section Properties :

$A = \pi(0.5^2) = 0.7854\ \text{in}^2$

$r = \dfrac{0.5}{2} = 0.25\ \text{in.}$

Allowable Stress :

$$\sigma_{allow} = \frac{P}{A} = \frac{5}{0.7854} = 6.366\ \text{ksi}$$

Assume long column :

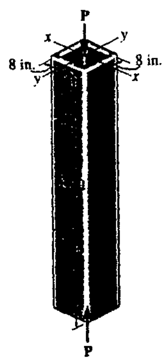
$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$

$$\frac{KL}{r} = \sqrt{\frac{54\,000}{\sigma_{allow}}} = \sqrt{\frac{54\,000}{6.366}} = 92.1 > 55 \quad \text{Assumption OK.}$$

$$\frac{KL}{r} = 92.1$$

$$L = 92.1\left(\frac{0.25}{0.5}\right) = 46.05\ \text{in.} = 3.84\ \text{ft} \qquad \textbf{Ans}$$

•13-92 The tube is 0.5 in. thick, is made from aluminum alloy 2014-T6, and is pin connected at its ends. Determine the largest axial load that it can support.



Section properties :

$$A = (8)(8) - (7)(7) = 15\ \text{in}^2$$

$$I_x = I_y = \frac{1}{12}(8)(8^3) - \frac{1}{12}(7)(7^3) = 141.25\ \text{in}^4$$

$$r_x = r_y = \sqrt{\frac{I}{A}} = \sqrt{\frac{141.25}{15}} = 3.069\ \text{in.}$$

Allowable stress :

$$\frac{KL}{r} = \frac{1.0(12)(12)}{3.069} = 46.93,\ 12 < \frac{KL}{r} < 55$$

Intermediate column

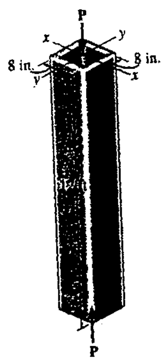
$$\sigma_{allow} = 30.7 - 0.23(\frac{KL}{r})$$

$$= 30.7 - 0.23(46.93) = 19.91\ \text{ksi}$$

$$P_{allow} = \sigma_{allow} A$$

$$= 19.91(15) = 299\ \text{kip} \qquad \textbf{Ans}$$

13-93 The tube is 0.5 in. thick, is made of aluminum alloy 2014-T6, and is fixed connected at its ends. Determine the largest axial load that it can support.



Section Properties :

$$A = (8)(8) - (7)(7) = 15 \text{ in}^2$$

$$I_x = I_y = \frac{1}{12}(8)(8^3) - \frac{1}{12}(7)(7^3) = 141.25 \text{ in}^4$$

$$r_x = r_y = \sqrt{\frac{I}{A}} = \sqrt{\frac{141.25}{15}} = 3.069 \text{ in.}$$

Allowable stress :

$$\frac{KL}{r} = \frac{0.5(12)(12)}{3.069} = 23.46, \quad 12 < \frac{KL}{r} < 55$$

Intermediate column

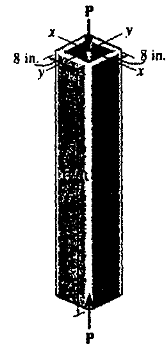
$$\sigma_{allow} = 30.7 - 0.23\left(\frac{KL}{r}\right)$$

$$= 30.7 - 0.23(23.46) = 25.30 \text{ ksi}$$

$$P_{allow} = \sigma_{allow} A$$

$$= 25.30(15) = 380 \text{ kip} \qquad \textbf{Ans}$$

13–94 The tube is 0.5 in. thick, is made from aluminum alloy 2014-T6, and is fixed at its bottom and pinned at its top. Determine the largest axial load that it can support.



Section Properties :

$$A = (8)(8) - (7)(7) = 15 \text{ in}^2$$

$$I_x = I_y = \frac{1}{12}(8)(8^3) - \frac{1}{12}(7)(7^3) = 141.25 \text{ in}^4$$

$$r_x = r_y = \sqrt{\frac{I}{A}} = \sqrt{\frac{141.25}{15}} = 3.069 \text{ in.}$$

Allowable stress :

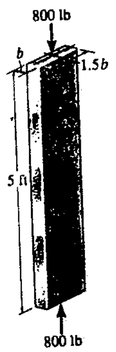
$$\frac{KL}{r} = \frac{0.7(12)(12)}{3.069} = 32.8446, \quad 12 < \frac{KL}{r} < 55$$

Intermediate column

$$\sigma_{allow} = 30.7 - 0.23(\frac{KL}{r}) = 30.7 - 0.23(32.8446) = 23.15 \text{ ksi}$$

$$P_{allow} = \sigma_{allow} A = 23.15(15) = 347 \text{ kip} \qquad \textbf{Ans}$$

13–95 The bar is made of aluminum alloy 2014-T6. Determine its thickness b if its width is $1.5b$. Assume that it is pin connected at its ends.



Section properties :

$$A = 1.5\,b^2 \qquad I_y = \frac{1}{12}(1.5b)(b^3) = 0.125\,b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.125\,b^4}{1.5\,b^2}} = 0.2887\,b$$

$$\sigma_{allow} = \frac{P}{A} = \frac{0.8}{1.5\,b^2} = \frac{0.5333}{b^2}$$

Assume long column :

$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$

$$\frac{0.5333}{b^2} = \frac{54\,000}{[\frac{(1.0)(5)(12)}{0.2887\,b}]^2}$$

$b = 0.808$ in.

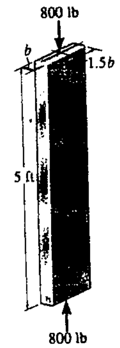
$r_y = 0.2887(0.808) = 0.2333$ in.

$$\frac{KL}{r_y} = \frac{(1.0)(5)(12)}{0.2333} = 257$$

$\frac{KL}{r_y} > 55$ Assumption OK

Use $b = 0.808$ in. **Ans**

•13-96 The bar is made of aluminum alloy 2014-T6. Determine its thickness b if its width is $1.5b$. Assume that it is fixed connected at its ends.



Section properties :

$$A = 1.5\,b^2$$

$$I_y = \frac{1}{12}(1.5b)(b^3) = 0.125\,b^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{0.125\,b^4}{1.5\,b^2}} = 0.2887\,b$$

$$\sigma_{allow} = \frac{P}{A} = \frac{0.8}{1.5\,b^2} = \frac{0.5333}{b^2}$$

Assume long column :

$$\sigma_{allow} = \frac{54\,000}{(KL/r)^2}$$

$$\frac{0.5333}{b^2} = \frac{54\,000}{[\frac{(0.5)(5)(12)}{0.2887\,b}]^2}$$

$$b = 0.571 \text{ in.}$$

$$r_y = 0.2887(0.571) = 0.1650 \text{ in.}$$

$$\frac{KL}{r_y} = \frac{(0.5)(5)(12)}{0.1650} = 181.8, \quad \frac{KL}{r_y} > 55 \qquad \text{Assumption OK}$$

Use $b = 0.571$ in. **Ans**

13-97 A 5-ft-long rod is used in a machine to transmit an axial compressive load of 3 kip. Determine its diameter if it is pin connected at its ends and is made of a 2014-T6 aluminum alloy.

Section properties :

$$A = \frac{\pi}{4} d^2 ; \qquad I = \frac{\pi}{4} \left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4} d^2}} = \frac{d}{4}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{3}{\frac{\pi}{4} d^2} = \frac{3.820}{d^2}$$

Assume long column :

$$\frac{KL}{r} = \frac{1.0\,(5)(12)}{\frac{d}{4}} = \frac{240}{d}$$

$$\sigma_{allow} = \frac{54\,000}{\left(\frac{KL}{r}\right)^2} ; \qquad \frac{3.820}{d^2} = \frac{54\,000}{\left[\frac{240}{d}\right]^2}$$

$d = 1.42$ in. **Ans**

$$\frac{KL}{r} = \frac{240}{1.42} = 169 > 55 \qquad \text{(OK)}$$

13–98 Solve Prob. 13–97 if the rod is fixed connected at its ends.

Section properties :

$$A = \frac{\pi}{4} d^2 ; \qquad I = \frac{\pi}{4} (\frac{d}{2})^4 = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi d^4}{64}}{\frac{\pi}{4} d^2}} = \frac{d}{4}$$

$$\sigma_{allow} = \frac{P}{A} = \frac{3}{\frac{\pi}{4} d^2} = \frac{3.820}{d^2}$$

Assume a long column :

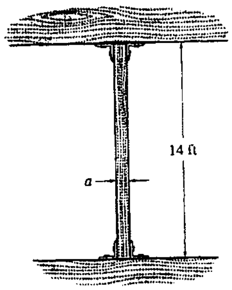
$$\frac{KL}{r} = \frac{0.5(5)(12)}{\frac{d}{4}} = \frac{120}{d}$$

$$\sigma_{allow} = \frac{54\,000}{(\frac{KL}{r})^2} ; \qquad \frac{3.820}{d^2} = \frac{54\,000}{[\frac{120}{d}]^2}$$

$d = 1.00$ in. **Ans**

$$\frac{KL}{r} = \frac{120}{1.79} = 67.2 > 55 \quad \text{OK}$$

13-99 The timber column has a square cross section and is assumed to be pin connected at its top and bottom. If it supports an axial load of 50 kip, determine its side dimensions a to the nearest $\frac{1}{2}$ in. Use the NFPA formulas.



Section properties :

$A = a^2 \qquad \sigma_{allow} = \sigma = \frac{P}{A} = \frac{50}{a^2}$

Assume long column :

$$\sigma_{allow} = \frac{540}{(\frac{KL}{d})^2}$$

$$\frac{50}{a^2} = \frac{540}{[\frac{(1.0)(14)(12)}{a}]^2}$$

$a = 7.15$ in.

$$\frac{KL}{d} = \frac{(1.0)(14)(12)}{7.15} = 23.5, \quad \frac{KL}{d} < 26 \qquad \text{Assumption NG}$$

Assume intermediate column :

$$\sigma_{allow} = 1.20[1 - \frac{1}{3}(\frac{KL/d}{26.0})^2$$

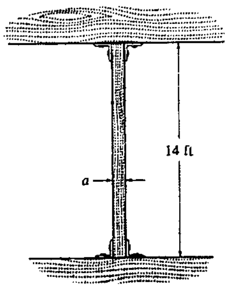
$$\frac{50}{a^2} = 1.20[1 - \frac{1}{3}(\frac{\frac{1.0(14)(12)}{a}}{26.0})^2$$

$a = 7.45$ in.

$$\frac{KL}{d} = \frac{1.0(14)(12)}{7.45} = 22.53, \quad 11 < \frac{KL}{d} < 26 \qquad \text{Assumption OK}$$

Use $a = 7\frac{1}{2}$ in. **Ans**

•13–100 Solve Prob. 13–99 if the column is assumed to be fixed connected at its top and bottom.



$$\sigma_{allow} = \sigma = \frac{P}{A} = \frac{50}{a^2}$$

Assume long column :

$$\sigma_{allow} = \frac{540}{(KL/d)^2}$$

$$\frac{50}{a^2} = \frac{540}{(\frac{0.5(14)(12)}{a})^2}$$

$a = 5.056$ in.

$$\frac{KL}{d} = \frac{0.5(14)(12)}{5.056} = 16.615, \qquad \frac{KL}{d} < 26 \qquad \text{Assumption N.G.}$$

Assume intermediate column :

$$\sigma_{allow} = 1.20[1 - \frac{1}{3}(\frac{KL/d}{26.0})^2]$$

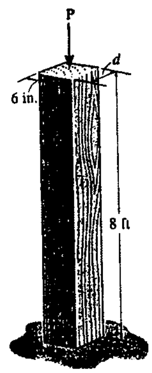
$$\frac{50}{a^2} = 1.20[1 - \frac{1}{3}(\frac{\frac{0.5(14)(12)}{a}}{26.0})^2]$$

$a = 6.72$ in.

$$\frac{KL}{d} = \frac{0.5(14)(12)}{6.72} = 12.5, \qquad 11 < \frac{KL}{d} < 26 \qquad \text{Assumption OK}$$

Use $a = 7.00$ in. **Ans**

13–101 The wood column is used to support an axial load of $P = 30$ kip. If it is fixed at the bottom and free at the top, determine the minimum width of the column based on the NFPA formulas.



Section properties :

$$A = 6d, \qquad \sigma_{allow} = \frac{P}{A} = \frac{30}{6d} = \frac{5}{d}$$

Buckling about x - x axis :

$d < 6$ in.

Assume long column :

$$\sigma_{allow} = \frac{540}{(\frac{KL}{d})^2}$$

$$\frac{5}{d} = \frac{540}{(\frac{2(8)(12)}{d})^2}$$

$d = 6.99$ in > 6 in. Assumption N.G.

Buckling about y - y axis :

$d > 6$ in.

$$\frac{KL}{d} = \frac{2.0(12)8}{6} = 32, \quad 26 < \frac{KL}{d} < 50$$

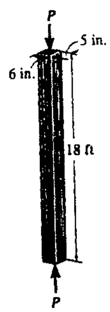
Long column

$$\sigma_{allow} = \frac{540}{(KL/d)^2}$$

$$\frac{5}{d} = \frac{540}{32^2}$$

$d = 9.48$ in. > 6 in. OK **Ans.**

13–102 The timber column has a length of 18 ft and is pin connected at its ends. Use the NFPA formulas to determine the largest axial force *P* that it can support.



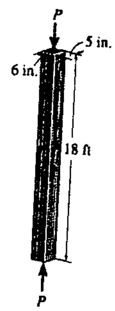
$$\frac{KL}{d} = \frac{(1.0)(18)(12)}{5} = 43.2, \quad 26 < \frac{KL}{d} < 50$$

Long column

$$\sigma_{allow} = \frac{540}{(KL/d)^2}$$
$$= \frac{540}{43.2^2} = 0.28935 \text{ ksi}$$

$$P_{allow} = \sigma_{allow}A$$
$$= 0.28935(6)(5)$$
$$= 8.68 \text{ kip} \qquad \textbf{Ans}$$

13–103 The timber column has a length of 18 ft and is fixed connected at its ends. Use the NFPA formulas to determine the largest axial force P that it can support.



$$\frac{KL}{d} = \frac{(0.5)(18)(12)}{5} = 21.6, \quad 11 < \frac{KL}{d} < 26$$

Intermediate column

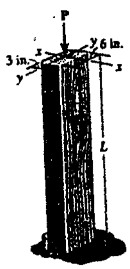
$$\sigma_{allow} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26}\right)^2\right]$$

$$= 1.20\left[1 - \frac{1}{3}\left(\frac{21.6}{26}\right)^2\right] = 0.92393 \text{ ksi}$$

$$P_{allow} = \sigma_{allow} A$$

$$= 0.92393(6)(5) = 27.7 \text{ kip} \qquad \textbf{Ans}$$

***13-104.** The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine its greatest allowable length if it supports an axial load of $P = 6$ kip.



Assume long column :

$$\sigma_{allow} = \sigma = \frac{P}{A} = \frac{6}{6(3)} = 0.3333 \text{ ksi}$$

$$\sigma_{allow} = \frac{540}{(KL/d)^2} \qquad K = 2.0 \qquad d = 3 \text{ in.}$$

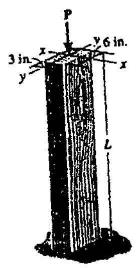
$$0.3333 = \frac{540}{[2.0(L)/3]^2}$$

$$L = 60.37 \text{ in.} = 5.03 \text{ ft} \qquad \textbf{Ans}$$

Check :

$$\frac{KL}{d} = \frac{2.0(60.37)}{3} = 40.25, \quad 26 < \frac{KL}{d} < 50 \quad \text{Assumption OK.}$$

13-105. The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine the largest allowable axial load P that it can support if it has a length $L = 6$ ft.



$K = 2.0 \qquad L = 6(12) = 72 \text{ in.} \qquad d = 3 \text{ in.}$

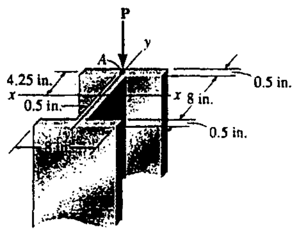
$$\frac{KL}{d} = \frac{2.0(72)}{3} = 48, \quad 26 < \frac{KL}{d} < 50$$

Long column

$$\sigma_{allow} = \frac{540}{(KL/d)^2} = \frac{540}{(48)^2} = 0.2344 \text{ ksi}$$

$$P_{allow} = \sigma_{allow} A$$
$$= 0.2344(6)(3) = 4.22 \text{ kip} \qquad \textbf{Ans}$$

13–106 A 16-ft-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and Eq. 13–30.



Section properties :

$$A = 2(0.5)(8) + 8(0.5) = 12 \text{ in}^2$$

$$I_x = \frac{1}{12}(8)(9^3) - \frac{1}{12}(7.5)(8^3) = 166 \text{ in}^4$$

$$I_y = 2(\frac{1}{12})(0.5)(8^3) + \frac{1}{12}(8)(0.5^3) = 42.75 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{42.75}{12}} = 1.8875 \text{ in.}$$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{1.8875} = 50.86, \; 12 < \frac{KL}{r_y} < 55$$

$$\sigma_{allow} = [30.7 - 0.23(\frac{KL}{r})]$$

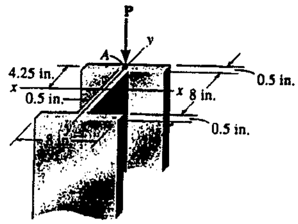
$$= [30.7 - 0.23(50.86)] = 19.00 \text{ ksi}$$

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$19.00 = \frac{P}{12} + \frac{P(4.25)(4.5)}{166}$$

$P = 95.7$ kip **Ans**

13–107 A 16-ft-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{allow} = 20$ ksi.



Section properties :

$$A = 2(0.5)(8) + 8(0.5) = 12 \text{ in}^2$$

$$I_x = \frac{1}{12}(8)(9^3) - \frac{1}{12}(7.5)(8^3) = 166 \text{ in}^4$$

$$I_y = 2(\frac{1}{12})(0.5)(8^3) + \frac{1}{12}(8)(0.5^3) = 42.75 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{42.75}{12}} = 1.8875 \text{ in.}$$

Interaction method :

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{1.8875} = 50.86, \quad 12 < \frac{KL}{r_y} < 55$$

$$\sigma_{allow} = [30.7 - 0.23(\frac{KL}{r})]$$

$$= [30.7 - 0.23(50.86)]$$

$$= 19.00 \text{ ksi}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{12} = 0.08333P$$

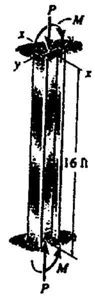
$$\sigma_b = \frac{Mc}{I_x} = \frac{P(4.25)(4.50)}{166} = 0.1152P$$

$$\frac{\sigma_a}{(\sigma_a)_{allow}} + \frac{\sigma_b}{(\sigma_b)_{allow}} = 1.0$$

$$\frac{0.08333P}{19.00} + \frac{0.1152P}{20} = 1$$

$P = 98.6$ kip **Ans**

***13-108.** The $W8 \times 15$ structural A-36 steel column is fixed at its top and bottom. If it supports end moments of $M = 5$ kip·ft, determine the axial force P that can be applied. Bending is about the x–x axis. Use the AISC equations of Sec. 13.6 and Eq. 13–30.



Section properties for W 8x15 :

$A = 4.44\ \text{in}^2 \quad I_x = 48.0\ \text{in}^4 \quad r_y = 0.876\ \text{in.} \quad d = 8.11\ \text{in.}$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{0.876} = 109.59$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{109.59}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{109.59}{126.1}\right) - \frac{1}{8}\left(\frac{109.59}{126.1}\right)^3\right]} = 11.727\ \text{ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$11.727 = \frac{P}{4.44} + \frac{5(12)\left(\frac{8.11}{2}\right)}{48}$$

$$P = 29.6\ \text{kip} \qquad \textbf{Ans}$$

13-109. The $W8 \times 15$ structural A-36 steel column is fixed at its top and bottom. If it supports end moments of $M = 23$ kip·ft, determine the axial force P that can be applied. Bending is about the x–x axis. Use the interaction formula with $(\sigma_b)_{allow} = 24$ ksi.

Section properties for W 8x15 :

$A = 4.44\ \text{in}^2 \qquad I_x = 48.0\ \text{in}^4 \qquad r_y = 0.876\ \text{in.} \qquad d = 8.11\ \text{in.}$

Interaction method :

$$\frac{KL}{r_y} = \frac{0.5(16)(12)}{0.876} = 109.59$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{(KL/r)_c}\right)^2\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{(KL/r)_c}\right) - \frac{1}{8}\left(\frac{KL/r}{(KL/r)_c}\right)^3\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{109.59}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{109.59}{126.1}\right) - \frac{1}{8}\left(\frac{109.59}{126.1}\right)^3\right]} = 11.727\ \text{ksi}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{4.44} = 0.2252\text{P}$$

$$\sigma_b = \frac{Mc}{I} = \frac{23(12)(\frac{8.11}{2})}{48} = 23.316\ \text{ksi}$$

$$\frac{\sigma_a}{(\sigma_a)_{allow}} + \frac{\sigma_b}{(\sigma_b)_{allow}} = 1$$

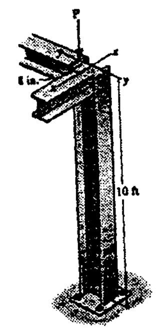
$$\frac{0.2252P}{11.727} + \frac{23.316}{24} = 1$$

$P = 1.48$ kip **Ans**

Note : $\dfrac{\sigma_a}{(\sigma_a)_{allow}} = \dfrac{0.2252(1.48)}{11.727} = 0.0285 < 0.15$

Therefore the method is allowed.

13-110. The $W8 \times 15$ structural A-36 steel column is assumed to be pinned at its top and bottom. Determine the largest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.



Section Properties : For a W8 × 15 wide flange section,

$A = 4.44\ \text{in}^2 \quad d = 8.11\ \text{in.} \quad I_x = 48.0\ \text{in}^4 \quad r_x = 3.29\ \text{in.}$
$r_y = 0.876\ \text{in.}$

Slenderness Ratio : By observation, the largest slenderness ratio is about $y-y$ axis. For a column pinned at both ends, $K = 1$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{1(10)(12)}{0.876} = 137.0$$

Allowable Stress : The allowable stress can be determined using *AISC Column Formulas*. For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\left(\frac{KL}{r}\right)_c \le \frac{KL}{r} \le 200$, the column is a *long* column. Applying Eq. 13−21,

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29.0)(10^3)}{23(137.0^2)} = 7.958\ \text{ksi}$$

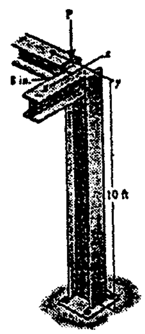
Maximum Stress : Bending is about $x-x$ axis. Applying Eq. 13 − 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$7.958 = \frac{P}{4.44} + \frac{P(8)\left(\frac{8.11}{2}\right)}{48}$$

$$P = 8.83\ \text{kip} \qquad \textbf{Ans}$$

13-111. Solve Prob. 13–110 if the column is fixed at its top and bottom.



Section Properties : For a W8 × 15 wide flange section,

$A = 4.44\ \text{in}^2 \quad d = 8.11\ \text{in.} \quad I_x = 48.0\ \text{in}^4 \quad r_x = 3.29\ \text{in.}$

$r_y = 0.876\ \text{in.}$

Slenderness Ratio : By observation, the largest slenderness ratio is about y − y axis. For a column fixed at both ends, $K = 0.7$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.7(10)(12)}{0.876} = 95.89$$

Allowable Stress : The allowable stress can be determined using *AISC Column Formulas*. For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$

$= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13 − 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(95.89^2)}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(95.89)}{8(126.1)} - \frac{(95.89^3)}{8(126.1^3)}}$$

$$= 13.491\ \text{ksi}$$

Maximum Stress : Bending is about x − x axis. Applying Eq. 13 − 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$13.491 = \frac{P}{4.44} + \frac{P(8)\left(\frac{8.11}{2}\right)}{48}$$

$$P = 15.0\ \text{kip} \qquad \textbf{Ans}$$

***13-112.** Solve Prob. 13–110 if the column is fixed at its bottom and pinned at its top.

Section Properties : For a W8× 15 wide flange section,

$A = 4.44\ \text{in}^2 \quad d = 8.11\ \text{in.} \quad I_x = 48.0\ \text{in}^4 \quad r_x = 3.29\ \text{in.}$
$r_y = 0.876\ \text{in.}$

Slenderness Ratio : By observation, the largest slenderness ratio is about $y-y$ axis. For a column fixed at both ends, $K = 0.5$. Thus,

$$\left(\frac{KL}{r}\right)_y = \frac{0.5(10)(12)}{0.876} = 68.49$$

Allowable Stress : The allowable stress can be determined using *AISC Column Formulas*. For A−36 steel, $\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$ $= \sqrt{\frac{2\pi^2[29(10^3)]}{36}} = 126.1$. Since $\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$, the column is an *intermediate* column. Applying Eq. 13 − 23,

$$\sigma_{allow} = \frac{\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}}$$

$$= \frac{\left[1 - \frac{(68.49^2)}{2(126.1^2)}\right](36)}{\frac{5}{3} + \frac{3(68.49)}{8(126.1)} - \frac{(68.49^3)}{8(126.1^3)}}$$

$$= 16.586\ \text{ksi}$$

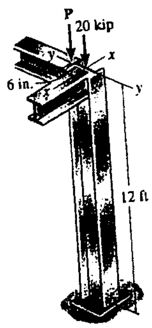
Maximum Stress : Bending is about $x-x$ axis. Applying Eq. 13 − 30, we have

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$16.586 = \frac{P}{4.44} + \frac{P(8)\left(\frac{8.11}{2}\right)}{48}$$

$$P = 18.4\ \text{kip} \qquad \textbf{Ans}$$

13–113 The W 10 × 19 structural A-36 steel column is assumed to be pinned at its top and bottom. Determine the largest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.



Section properties for W 10× 19 :

$A = 5.62\ \text{in}^2 \quad d = 10.24\ \text{in.} \quad I_x = 96.3\ \text{in}^4$

$r_x = 4.14\ \text{in.} \quad r_y = 0.874\ \text{in.}$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{1.0(12)(12)}{0.874} = 164.76$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r_y} > \left(\frac{KL}{r}\right)_c$$

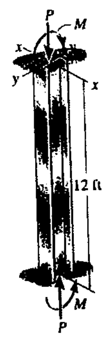
$$(\sigma_a)_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(164.76)^2} = 5.501\ \text{ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$5.501 = \frac{P+20}{5.62} + \frac{P(6)(\frac{10.24}{2})}{96.3}$$

$P = 3.91$ kip **Ans**

13–114 The $W\,14 \times 22$ structural A-36 steel column is fixed at its top and bottom. If it supports end moments of M = 10 kip · ft, determine the maximum allowable axial force P that can be applied. Bending is about the $x-x$ axis. Use the AISC equations of Sec. 13.6 and Eq. 13–30.



Section properties for $W\,14 \times 22$:

$A = 6.49\ \text{in}^2 \qquad d = 13.74\ \text{in}^2 \qquad I_x = 199\ \text{in}^4 \qquad r_y = 1.04\ \text{in.}$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{0.5(12)(12)}{1.04} = 69.231$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\left(\frac{(\frac{KL}{r})^2}{(\frac{KL}{r})_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{(\frac{KL}{r})_c} - \frac{1}{8}\frac{(\frac{KL}{r})^3}{(\frac{KL}{r})_c^3}\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{69.231}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{69.231}{126.1}\right) - \frac{1}{8}\left(\frac{69.231}{126.1}\right)^3\right]} = 16.510\ \text{ksi}$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_x}$$

$$16.510 = \frac{P}{6.49} + \frac{10(12)(\frac{13.74}{2})}{199}$$

$P = 80.3\ \text{kip}$ **Ans**

13-115 The W 14 × 22 column is fixed at its top and bottom. If it suppprts end moments of M = 15 kip · ft, determine the maximum allowable axial force P that can be applied. Bending is about the $x-x$ axis. Use the interaction formula with $(\sigma_b)_{allow}$ = 24 ksi.

Section Properties for W 14 x 22 :

$A = 6.49\ \text{in}^2 \qquad d = 13.74\ \text{in}^2 \qquad I_x = 199\ \text{in}^4 \qquad r_y = 1.04\ \text{in.}$

Interaction method :

$$\frac{KL}{r_y} = \frac{0.5(12)(12)}{1.04} = 69.231$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1, \quad \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

Hence,

$$(\sigma_a)_{allow} = \frac{\left[1 - \frac{1}{2}\left(\frac{(\frac{KL}{r})^2}{(\frac{KL}{r})_c^2}\right)\right]\sigma_Y}{\left[\frac{5}{3} + \frac{3}{8}\frac{\frac{KL}{r}}{(\frac{KL}{r})_c} - \frac{1}{8}\frac{(\frac{KL}{r})^3}{(\frac{KL}{r})_c^3}\right]} = \frac{\left[1 - \frac{1}{2}\left(\frac{69.231}{126.1}\right)^2\right]36}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{69.231}{126.1}\right) - \frac{1}{8}\left(\frac{69.231}{126.1}\right)^3\right]} = 16.510\ \text{ksi}$$

$$\sigma_a = \frac{P}{A} = \frac{P}{6.49} = 0.15408\,P$$

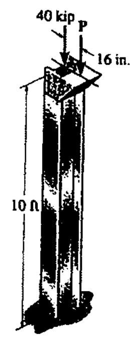
$$\sigma_b = \frac{M_x c}{I_x} = \frac{15(12)(\frac{13.74}{2})}{199} = 6.214\ \text{ksi}$$

$$\frac{\sigma_a}{(\sigma_a)_{allow}} + \frac{\sigma_b}{(\sigma_b)_{allow}} = 1.0$$

$$\frac{0.15408\,P}{16.510} + \frac{6.2141}{24} = 1.0$$

P = 79.4 kip **Ans**

•13-116 The W 12 × 50 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load P that can be applied using Eq. 13-30 and the AISC equations of Sec. 13.6.



Section properties for $W\,12\times 50$:

$A = 14.7\text{ in}^2 \quad d = 12.19\text{ in.} \quad I_y = 56.3\text{ in}^4$

$r_y = 1.96\text{ in.} \quad b_f = 8.08\text{ in.}$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{2.0(10)(12)}{1.96} = 122.45$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{allow} = \frac{[1-\frac{1}{2}(\frac{KL/r}{(KL/r)_c})^2]\sigma_Y}{[\frac{5}{3}+\frac{3}{8}(\frac{KL/r}{(KL/r)_c})-\frac{1}{8}(\frac{KL/r}{(KL/r)_c})^3]}$$

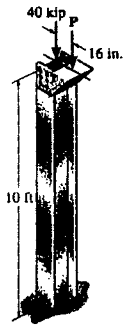
$$= \frac{[1-\frac{1}{2}(\frac{122.45}{126.1})^2]36}{\frac{5}{3}+\frac{3}{8}(\frac{122.45}{126.1})-\frac{1}{8}(\frac{122.45}{126.1})^3} = 9.929\text{ ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$9.929 = \frac{P+40}{14.7} + \frac{P(16)(\frac{8.08}{2})}{56.3}$$

$P = 5.93\text{ kip}$ **Ans**

13–117 The $W\,12 \times 87$ structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.



Section properties for $W\,12\times 87$:

$A = 25.6\text{ in}^2 \qquad d = 12.53\text{ in.}$

$I_y = 241\text{ in}^4 \qquad r_y = 3.07\text{ in.}$

$b = 12.125$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{2.0(10)(12)}{3.07} = 78.176$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1,\ \frac{KL}{r_y} < \left(\frac{KL}{r}\right)_c$$

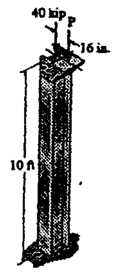
$$(\sigma_a)_{\text{allow}} = \frac{\left[1 - \frac{1}{2}\frac{(KL/r)^2}{(KL/r)_c^2}\right]\sigma_y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{KL/r_c}\right) - \frac{1}{8}\frac{(KL/r)^3}{(KL/r_c)^3}} = \frac{\left[1 - \frac{1}{2}\left(\frac{78.176}{126.1}\right)^2\right]36}{\frac{5}{3} + \frac{3}{8}\left(\frac{78.176}{126.1}\right) - \frac{1}{8}\left(\frac{78.176}{126.1}\right)^3} = 15.56\text{ ksi}$$

$$\sigma_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$15.56 = \frac{P+40}{25.6} + \frac{P(16)\left(\frac{12.125}{2}\right)}{241}$$

$P = 31.7\text{ kip}$ **Ans**

13-118. The $W14 \times 43$ structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.



Section properties for $W\,14\times43$:

$A = 12.6\text{ in}^2 \qquad d = 13.66\text{ in.}$

$I_y = 45.2\text{ in}^4 \qquad r_y = 1.89\text{ in.}$

$b = 7.995$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{2(10)(12)}{1.89} = 126.98$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\,\pi^2\,(29)(10^3)}{36}} = 126.1,\ 200 > \frac{KL}{r_y} > \left(\frac{KL}{r}\right)_c$$

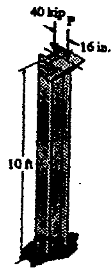
$$(\sigma_a)_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(126.98)^2} = 9.26\text{ ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$9.26 = \frac{P+40}{12.6} + \frac{P(16)(\frac{7.995}{2})}{45.2}$$

$P = 4.07\text{ kip}$ **Ans**

13-119. The $W10 \times 45$ structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 2$ kip, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.



Section properties for $W\,10 \times 45$:

$A = 13.3\ \text{in}^2 \qquad d = 10.10\ \text{in.}$

$I_y = 53.4\ \text{in}^4 \qquad r_y = 2.01\ \text{in.}$

$b = 8.020\ \text{in.}$

Allowable stress method :

$$\frac{KL}{r_y} = \frac{2.0(10)(12)}{2.01} = 119.4$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\frac{KL}{r} < \left(\frac{KL}{r}\right)_c$$

$$(\sigma_a)_{allow} \geq \frac{\left[1 - \frac{1}{2}\frac{(KL/r)^2}{(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{KL/r_c}\right) - \frac{1}{8}\frac{(KL/r)^3}{(KL/r_c)^3}} = \frac{\left[1 - \frac{1}{2}\left(\frac{119.4}{126.1}\right)^2\right]36}{\frac{5}{3} + \frac{3}{8}\left(\frac{119.4}{126.1}\right) - \frac{1}{8}\left(\frac{119.4}{126.1}\right)^3} = 10.37\ \text{ksi}$$

$$(\sigma_a)_{allow} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$10.37 \geq \frac{42}{13.3} + \frac{2(16)\left(\frac{8.020}{2}\right)}{53.4}$$

$10.37 \geq 5.56$ OK Column is safe.

Yes. **Ans**

***13-120.** Check if the wood column is adequate for supporting the eccentric load of $P = 800$ lb applied at its top. It is fixed at its base and free at its top. Use the NFPA equations of Sec. 13.6 and Eq. 13–30.

Section properties :

$A = 3(6) = 18 \text{ in}^2 \qquad I_x = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4$

$d = 3$ in.

Allowable stress method :

$$\frac{KL}{d} = \frac{2.0(6)(12)}{3} = 48 \text{ in.}, \; 26 < \frac{KL}{d} < 50$$

$$(\sigma_a)_{allow} = \frac{540}{(KL/d)^2} = \frac{540}{(48)^2} = 0.2344 \text{ ksi}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$= \frac{0.8}{18} + \frac{0.8(5)(3)}{54} = 0.2667 \text{ ksi}$$

$\sigma_{max} > (\sigma_a)_{allow}$

The column is inadequate

No. **Ans**

13-121. Determine the maximum allowable eccentric load P that can be applied to the wood column. The column is fixed at its base and free at its top. Use the NFPA equations of Sec. 13.6 and Eq. 13–30.

Section properties :

$A = 6(3) = 18 \text{ in}^2 \qquad I_x = \frac{1}{12}(3)(6)^3 = 54 \text{ in}^4$

$d = 3$ in.

Allowable stress method :

$$\frac{KL}{d} = \frac{2.0(6)(12)}{3} = 48 \text{ in.}, \; 26 < \frac{KL}{d} < 50$$

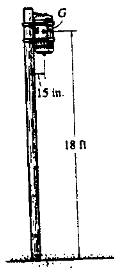
$$(\sigma_a)_{allow} = \frac{540}{(KL/d)^2} = \frac{540}{(48)^2} = 0.2344 \text{ ksi}$$

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{M_x c}{I_x}$$

$$0.2344 = \frac{P}{18} + \frac{P(5)(3)}{54}$$

$P = 0.703$ kip $= 703$ lb **Ans**

13–122 The 10-in.-diameter utility pole supports the transformer that has a weight of 600 lb and center of gravity at G. If the pole is fixed to the ground and free at its top, determine if it is adequate according to the NFPA equations of Sec. 13.6 and Eq. 13–30.



$$\frac{KL}{d} = \frac{2(18)(12)}{10} = 43.2 \text{ in.}$$

$26 < 43.2 \le 50$

Use Eq. 13 - 29,

$$\sigma_{allow} = \frac{540}{(KL/d)} = \frac{540}{(43.2)^2} = 0.2894 \text{ ksi}$$

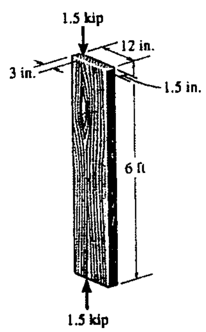
$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I}$$

$$\sigma_{max} = \frac{600}{\pi(5)^2} + \frac{(600)(15)(5)}{(\frac{\pi}{4})(5)^4}$$

$\sigma_{max} = 99.31 \text{ psi} < 0.289 \text{ ksi}$ OK

Yes. **Ans**

13–123 Determine if the column can support the eccentric compressive load of 1.5 kip. Assume that the ends are pin connected. Use the NFPA equations in Sec. 13.6 and Eq. 13–30.



$A = 12(1.5) = 18 \text{ in}^2 ; \quad I_x = \frac{1}{12}(1.5)(12)^3 = 216 \text{ in}^4$

$d = 1.5 \text{ in.}$

$\frac{KL}{d} = \frac{1.0(6)(12)}{1.5} = 48$

$26 < \frac{KL}{d} < 50$

$(\sigma_a)_{\text{allow}} = \frac{540}{(\frac{KL}{d})^2} = \frac{540}{(48)^2} = 0.2344$

$\sigma_{\max} = \frac{P}{A} + \frac{M_x c}{I_x}$

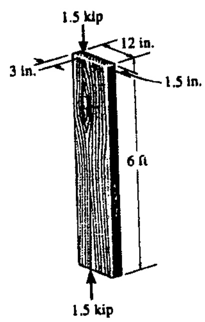
$= \frac{1.5}{18} + \frac{1.5(3)(6)}{216} = 0.208 \text{ ksi}$

$(\sigma_a)_{\text{allow}} > \sigma_{\max}$

The column is adequate.

Yes. **Ans**

•13–124 Determine if the column can support the eccentric compressive load of 1.5 kip. Assume that the bottom is fixed and the top is pinned. Use the NFPA equations in Sec. 13.6 and Eq. 13–30.



$A = 12\,(1.5) = 18\text{ in}^2\,; \qquad I_x = \frac{1}{12}(1.5)(12)^3 = 216\text{ in}^4$

$d = 1.5\text{ in.}$

$\frac{KL}{d} = \frac{0.7\,(6)(12)}{1.5} = 33.6$

$26 < \frac{KL}{d} < 50$

$(\sigma_a)_{allow} = \frac{540}{(\frac{KL}{d})^2} = \frac{540}{(33.6)^2} = 0.4783$

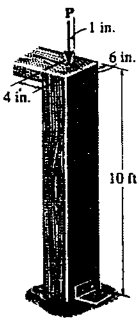
$\sigma_{max} = \frac{P}{A} + \frac{M_x c}{I_x} = \frac{1.5}{18} + \frac{1.5(3)(6)}{216} = 0.208\text{ ksi}$

$(\sigma_a)_{allow} > \sigma_{max}$

The column is adequate.

Yes. **Ans**

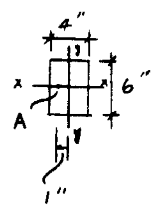
13–125 The wood column has a thickness of 4 in. and a width of 6 in. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied. Assume that the column is pinned at both its top and bottom.



Section properties :

$$A = 6(4) = 24 \text{ in}^2 \qquad I_y = \frac{1}{12}(6)(4^3) = 32 \text{ in}^4$$

$$d = 4 \text{ in.}$$



Allowable stress method :

$$\frac{KL}{d} = \frac{1.0(10)(12)}{4} = 30 \text{ in.}$$

$$26 < \frac{KL}{d} < 50$$

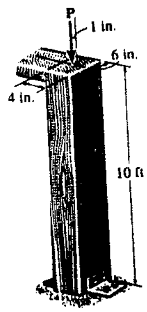
$$(\sigma_a)_{allow} = \frac{540}{(KL/d)^2} = \frac{540}{30^2} = 0.6 \text{ ksi}$$

$$\sigma_{max} = (\sigma_a)_{allow} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$0.6 = \frac{P}{24} + \frac{P(1)(2)}{32}$$

$$P = 5.76 \text{ kip} \qquad \textbf{Ans}$$

13–126 The wood column has a thickness of 4 in. and a width of 6 in. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied. Assume that the column is pinned at the top and fixed at the bottom.



Section properties :

$$A = 6(4) = 24 \text{ in}^2 \qquad I_y = \frac{1}{12}(6)(4^3) = 32 \text{ in}^4$$

$$d = 4 \text{ in.}$$

Allowable stress method :

$$\frac{KL}{d} = \frac{0.7(10)(12)}{4} = 21$$

$$11 < \frac{KL}{d} < 26$$

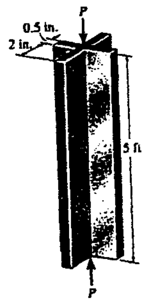
$$(\sigma_a)_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26}\right)^2\right] = 1.20\left[1 - \frac{1}{3}\left(\frac{21}{26}\right)^2\right] = 0.9391 \text{ ksi}$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{M_y c}{I_y}$$

$$0.9391 = \frac{P}{24} + \frac{P(1)(2)}{32}$$

$$P = 9.01 \text{ kip} \qquad \textbf{Ans}$$

13-127. The member has a symmetric cross section. If it is pin connected at its ends, determine the largest force it can support. It is made of 2014-T6 aluminum alloy.



Section properties :

$A = 4.5(0.5) + 4(0.5) = 4.25\ \text{in}^2$

$I = \frac{1}{12}(0.5)(4.5^3) + \frac{1}{12}(4)(0.5)^3 = 3.839\ \text{in}^4$

$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{3.839}{4.25}} = 0.9504\ \text{in.}$

Allowable stress :

$\frac{KL}{r} = \frac{1.0(5)(12)}{0.9504} = 63.13$

$\frac{KL}{r} > 55$

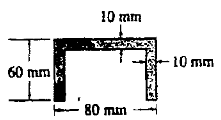
Long column

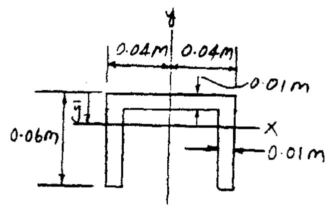
$\sigma_{allow} = \frac{54000}{(KL/r)^2} = \frac{54000}{63.13^2} = 13.55\ \text{ksi}$

$P_{allow} = \sigma_{allow}A$

$= 13.55(4.25) = 57.6\ \text{kip}$ **Ans**

•13–128 A steel column has a length of 5 m and is free at one end and fixed at the other end. If the cross-sectional area has the dimensions shown, determine the critical load. E_{st} = 200 GPa, σ_Y = 360 MPa.





Section properties :

$$A = 0.06\,(0.01) + 2\,(0.06)(0.01) = 1.80\left(10^{-3}\right)\,\text{m}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{0.005\,(0.06)(0.01) + 2[0.03\,(0.06)(0.01)]}{0.06\,(0.01) + 2\,(0.06)(0.01)} = 0.02167\ \text{m}$$

$$I_x = \frac{1}{12}(0.06)(0.01)^3 + 0.06\,(0.01)(0.02167 - 0.005)^2$$

$$+\, 2[\frac{1}{12}(0.01)(0.06)^3 + 0.01\,(0.06)(0.03 - 0.02167)^2] = 0.615\,(10^{-6})\ \text{m}^4 \qquad \text{(controls)}$$

$$I_y = \frac{1}{12}(0.06)(0.08)^3 - \frac{1}{12}(0.05)(0.06)^3 = 1.66\,(10^{-6})\ \text{m}^4$$

Critical load :

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2}; \qquad K = 2.0$$

$$= \frac{\pi^2\,(200)(10^9)(0.615)(10^{-6})}{[2.0\,(5)\,]^2}$$

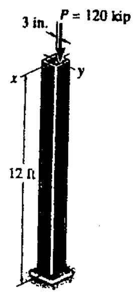
$$= 12140\ \text{N} = 12.1\ \text{kN} \qquad \textbf{Ans}$$

Check stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12140}{1.80\,(10^{-3})} = 6.74\ \text{MPa} < \sigma_Y = 360\ \text{MPa}$$

Hence, Euler's equation is still valid.

13–129 The square structural A-36 steel tubing has outer dimensions of 8 in. by 8 in. Its cross-sectional area is 14.40 in² and its moments of inertia are $I_x = I_y = 131\ \text{in}^4$. If a load of 120 kip is applied at its top as shown, determine the factor of safety of the tube with respect to yielding. The column can be assumed fixed at its base and free at its top.



Section properties :

$$A = 14.4\ \text{in}^2\ ; \qquad I_x = I_y = 131\ \text{in}^4$$

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{131}{14.4}} = 3.01616\ \text{in.}$$

Yielding :

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e\,c}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right]; \qquad K = 2.0$$

$$\frac{e\,c}{r^2} = \frac{3\,(4)}{(3.01616)^2} = 1.319084$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2(12)(12)}{2(3.01616)}\sqrt{\frac{P}{29\,(10^3)(14.40)}} = 0.073880\sqrt{P}$$

$$36\,(14.4) = P\,[1 + 1.319084\sec(0.073880\sqrt{P})]$$

By trial and error :

$P = 160.70$ kip (controls)

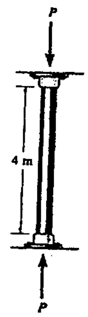
Buckling :

$$P = P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2(29)(10^3)(131)}{[2\,(12)(12)\,]^2} = 452\ \text{kip}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{452}{14.4} = 31.4\ \text{ksi} < \sigma_Y = 36\ \text{ksi} \qquad \text{(OK)}$$

$$\text{F.S.} = \frac{160.70}{120} = 1.34$$ **Ans**

13-130. The steel pipe is fixed supported at its ends. If it is 4 m long and has an outer diameter of 50 mm, determine its required thickness so that it can support an axial load of $P = 100$ kN without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



$$I = \frac{\pi}{4}(0.025^4 - r_i^4)$$

Critical load :

$$P_{cr} = \frac{\pi^2 E I}{(K L)^2}; \qquad K = 0.5$$

$$100(10^3) = \frac{\pi^2 (200)(10^9)[\frac{\pi}{4}(0.025^4 - r_i^4)]}{[0.5(4)]^2}$$

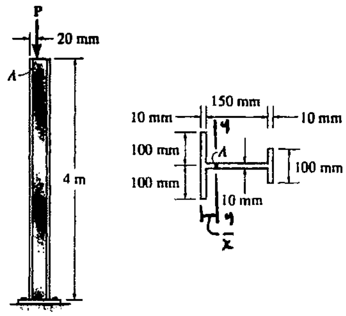
$$r_i = 0.01908 \text{ m} = 19.1 \text{ mm}$$

$$t = 25 \text{ mm} - 19.1 \text{ mm} = 5.92 \text{ mm}$$ **Ans**

Check stress :

$$\sigma = \frac{P_{cr}}{A} = \frac{100(10^3)}{\pi(0.025^2 - 0.0191^2)} = 122 \text{ MPa} < \sigma_Y = 250 \text{ MPa} \text{ (OK)}$$

13–131 The structural A-36 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding.



Section properties :

$$\Sigma A = 0.2(0.01) + 0.15(0.01) + 0.1(0.01) = 4.5(10^{-3})\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.005(0.2)(0.01) + 0.085(0.15)(0.01) + 0.165(0.1)(0.01)}{4.5(10^{-3})} = 0.06722\ \text{m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2(0.01)(0.06722-0.005)^2 + \frac{1}{12}(0.01)(0.15^3) + 0.01(0.15)(0.085-0.06722)^2 + \frac{1}{12}(0.1)(0.01^3) + 0.1(0.01)(0.165-0.06722)^2$$
$$= 20.615278(10^{-6})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3) = 7.5125(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278(10^{-6})}{4.5(10^{-3})}} = 0.06783648\ \text{m}$$

Buckling about $x-x$ axis :

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0(4)]^2} = 231.70\ \text{kN} \quad \text{(controls)}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{231.7(10^3)}{4.5(10^{-3})} = 51.5\ \text{MPa} < \sigma_Y = 250\ \text{MPa}$$

Yielding about $y-y$ axis :

$$\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{KL}{2r}\sqrt{\frac{P}{EA}})]; \qquad e = 0.06722 - 0.02 = 0.04722\ \text{m}$$

$$\frac{ec}{r^2} = \frac{0.04722(0.06722)}{(0.06783648)^2} = 0.689815$$

$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0(4)}{2(0.06783648)}\sqrt{\frac{P}{200(10^9)(4.5)(10^{-3})}} = 1.965511(10^{-3})\sqrt{P}$$

$$250(10^6)(4.5)(10^{-3}) = P[1 + 0.689815\sec(1.965511(10^{-3})\sqrt{P})]$$

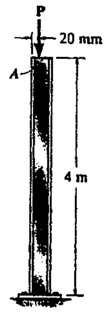
By trial and error :

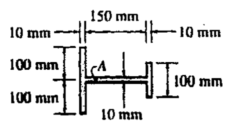
$P = 379.8$ kN

Hence,

$$P_{allow} = \frac{231.70}{3} = 77.2\ \text{kN} \qquad \textbf{Ans}$$

•13–132 The structural A-36 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine if the column will buckle or yield when the load $P = 10$ kN. Use a factor of safety of 3 with respect to buckling and yielding.





Section properties :

$$\Sigma A = 0.2\,(0.01) + 0.15\,(0.01) + 0.1\,(0.01) = 4.5\,(10^{-3})\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{0.005\,(0.2)(0.01) + 0.085\,(0.15)(0.01) + 0.165\,(0.1)(0.01)}{4.5\,(10^{-3})} = 0.06722\ \text{m}$$

$$I_y = \frac{1}{12}(0.2)(0.01^3) + 0.2\,(0.01)(0.06722 - 0.005)^2 + \frac{1}{12}(0.01)(0.15^3) + 0.01\,(0.15)(0.085 - 0.06722)^2 + \frac{1}{12}(0.1)(0.01^3) + 0.1\,(0.01)(0.165 - 0.06722)^2 = 20.615278\,(10^{-6})\ \text{m}^4$$

$$I_x = \frac{1}{12}(0.01)(0.2^3) + \frac{1}{12}(0.15)(0.01^3) + \frac{1}{12}(0.01)(0.1^3) = 7.5125\,(10^{-6})\ \text{m}^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{20.615278\,(10^{-6})}{4.5\,(10^{-3})}} = 0.06783648\ \text{m}$$

Buckling about $x-x$ axis :

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2(200)(10^9)(7.5125)(10^{-6})}{[2.0\,(4)]^2} = 231.70\ \text{kN}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{231.7\,(10^3)}{4.5\,(10^{-3})} = 51.5\ \text{MPa} < \sigma_Y = 250\ \text{MPa} \quad \text{(OK)}$$

$$P_{allow} = \frac{P_{cr}}{FS} = \frac{231.7}{3} = 77.2\ \text{kN} > P = 10\ \text{kN}$$

Hence the column does not buckle !

Yielding about $y-y$ axis :

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{KL}{2r}\sqrt{\frac{P}{EA}}\right)\right] \qquad e = 0.06722 - 0.02 = 0.04722\ \text{m}$$

$$P = \frac{10}{3} = 3.333\ \text{kN}$$

$$\frac{P}{A} = \frac{3.333\,(10^3)}{4.5\,(10^{-3})} = 0.7407\ \text{MPa}$$

$$\frac{ec}{r^2} = \frac{0.04722\,(0.06722)}{(0.06783648)^2} = 0.689815$$

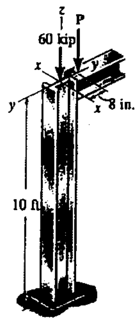
$$\frac{KL}{2r}\sqrt{\frac{P}{EA}} = \frac{2.0\,(4)}{2(0.06783648)}\sqrt{\frac{3.333\,(10^3)}{200\,(10^9)(4.5)(10^{-3})}} = 0.1134788$$

$$\sigma_{max} = 0.7407\,[1 + 0.689815\sec(0.1134788)] = 1.25\ \text{MPa} < \sigma_Y = 250\ \text{MPa}$$

Hence the column does not yield!

No. **Ans**

13–133 The $W\,10 \times 45$ steel column supports an axial load of 60 kip in addition to an eccentric load **P**. Determine the maximum allowable value of **P** based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that in the $x-z$ plane $K_x = 1.0$ and in the $y-z$ plane $K_y = 2.0$. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



Section properties for $W\,10 \times 45$:

$A = 13.3\ \text{in}^2 \qquad d = 10.10\ \text{in.} \qquad I_x = 248\ \text{in}^4$

$r_x = 4.32\ \text{in.} \qquad r_y = 2.01\ \text{in.}$

Allowable stress method :

$$\left(\frac{KL}{r}\right)_x = \frac{1.0(10)(12)}{4.32} = 27.8$$

$$\left(\frac{KL}{r}\right)_y = \frac{2.0(10)(12)}{2.01} = 119.4 \qquad \text{(controls)}$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{50}} = 107$$

$$\frac{KL}{r} > \left(\frac{KL}{r}\right)_c$$

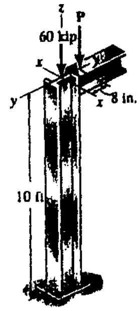
$$(\sigma_a)_{\text{allow}} = \frac{12\pi^2 E}{23(KL/r)^2} = \frac{12\pi^2(29)(10^3)}{23(119.4)^2} = 10.47\ \text{ksi}$$

$$\sigma_{\max} = (\sigma_a)_{\text{allow}} = \frac{P}{A} + \frac{Mc}{I}$$

$$10.47 = \frac{P + 60}{13.3} + \frac{P(8)(\frac{10.10}{2})}{248}$$

$P = 25.0$ kip **Ans**

13–134 The $W_r 14 \times 53$ structural A-36 steel column supports an axial load of 60 kip in addition to an eccentric load **P**. Determine the maximum allowable value of **P** based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that in the $x-z$ plane $K_x = 1.0$ and in the $y-z$ plane $K_y = 2.0$.



Section properties for $W\,14 \times 53$:

$A = 15.6\ \text{in}^2 \qquad d = 13.92\ \text{in.}$

$I_x = 541\ \text{in}^4 \qquad r_x = 5.89\ \text{in.} \qquad r_y = 1.92\ \text{in.}$

Allowable stress method :

$$\left(\frac{KL}{r}\right)_x = \frac{1.0(10)(12)}{5.89} = 20.37$$

$$\left(\frac{KL}{r}\right)_y = \frac{2.0(10)(12)}{1.92} = 125$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{r_y}} = \sqrt{\frac{2\pi^2(29)(10^3)}{36}} = 126.1$$

$$\left(\frac{KL}{r}\right)_y < \left(\frac{KL}{r}\right)_c$$

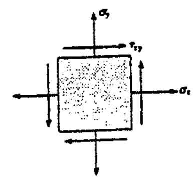
$$(\sigma_a)_{allow} = \frac{\left[1 - \frac{1}{2}\frac{(KL/r)^2}{(KL/r)_c^2}\right]\sigma_Y}{\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{KL/r_c}\right) - \frac{1}{8}\frac{(KL/r)^3}{(KL/r_c)^3}} = \frac{\left[1 - \frac{1}{2}\left(\frac{125}{126.1}\right)^2\right]36}{\frac{5}{3} + \frac{3}{8}\left(\frac{125}{126.1}\right) - \frac{1}{8}\left(\frac{125}{126.1}\right)^3} = 9.55\ \text{ksi}$$

$$\sigma_{max} = \sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$9.55 = \frac{P + 60}{15.6} + \frac{P(8)\left(\frac{13.92}{2}\right)}{541}$$

$P = 34.2$ ksi **Ans**

14–1. A material is subjected to a general state of plane stress. Express the strain energy density in terms of the elastic constants E, G, and ν and the stress components σ_x, σ_y, and τ_{xy}.



Strain Energy Due to Normal Stresses : We will consider the application of normal stresses on the element in two successive stages. For the first stage, we apply only σ_x on the element. Since σ_x is a constant, from Eq. 14 – 8, we have

$$(U_i)_1 = \int_V \frac{\sigma_x^2}{2E} dV = \frac{\sigma_x^2 V}{2E}$$

When σ_y is applied in the second stage, the normal strain ε_x will be strained by $\varepsilon_x' = -\nu\varepsilon_y = -\frac{\nu\sigma_y}{E}$. Therefore, the strain energy for the second stage is

$$(U_i)_2 = \int_V \left(\frac{\sigma_y^2}{2E} + \sigma_x \varepsilon_x'\right) dV$$

$$= \int_V \left[\frac{\sigma_y^2}{2E} + \sigma_x\left(-\frac{\nu\sigma_y}{E}\right)\right] dV$$

Since σ_x and σ_y are constants,

$$(U_i)_2 = \frac{V}{2E}\left(\sigma_y^2 - 2\nu\sigma_x\sigma_y\right)$$

Strain Energy Due to Shear Stress : The application of τ_{xy} does not strain the element in normal direction. Thus, from Eq. 14 – 11, we have

$$(U_i)_3 = \int_V \frac{\tau_{xy}^2}{2G} dV = \frac{\tau_{xy}^2 V}{2G}$$

The total strain energy is

$$U_i = (U_i)_1 + (U_i)_2 + (U_i)_3$$

$$= \frac{\sigma_x^2 V}{2E} + \frac{V}{2E}\left(\sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{\tau_{xy}^2 V}{2G}$$

$$= \frac{V}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{\tau_{xy}^2 V}{2G}$$

and the strain energy density is

$$\frac{U_i}{V} = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y\right) + \frac{\tau_{xy}^2}{2G} \qquad \textbf{Ans}$$

14–2 The strain-energy density must be the same whether the state of stress is represented by σ_x, σ_y, and τ_{xy}, or by the principal stresses σ_1 and σ_2. This being the case, equate the strain-energy expressions for each of these two cases and show that $G = E/[2(1 + \nu)]$.

$$U = \int_V \left[\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2\right] dV$$

$$U = \int_V \left[\frac{1}{2E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\sigma_2\right] dV$$

Equating the above two equations yields.

$$\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2 = \frac{1}{2E}(\sigma_1^2 + \sigma_2^2) - \frac{\nu}{E}\sigma_1\sigma_2 \qquad (1)$$

However, $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Thus, $(\sigma_1^2 + \sigma_2^2) = \sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2$

$$\sigma_1\sigma_2 = \sigma_x\sigma_y - \tau_{xy}^2$$

Substitute into Eq.(1)

$$\frac{1}{2E}(\sigma_x^2 + \sigma_y^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{1}{2G}\tau_{xy}^2 = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + 2\tau_{xy}^2) - \frac{\nu}{E}\sigma_x\sigma_y + \frac{\nu}{E}\tau_{xy}^2$$

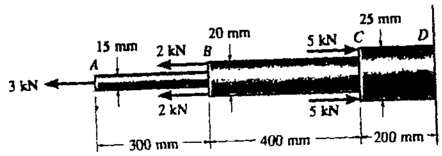
$$\frac{1}{2G}\tau_{xy}^2 = \frac{\tau_{xy}^2}{E} + \frac{\nu}{E}\tau_{xy}^2$$

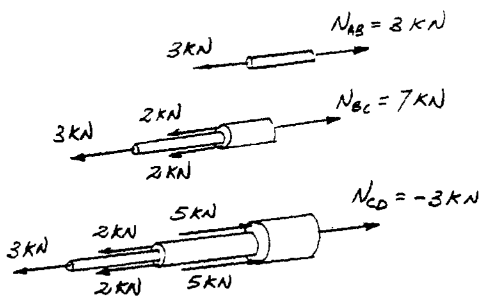
$$\frac{1}{2G} = \frac{1}{E} + \frac{\nu}{E}$$

$$\frac{1}{2G} = \frac{1}{E}(1 + \nu)$$

$$G = \frac{E}{2(1 + \nu)} \qquad \textbf{QED}$$

14–3 Determine the strain energy in the rod assembly. Portion AB is steel, BC is brass, and CD is aluminum. E_{st} = 200 GPa, E_{br} = 101 GPa, E_{al} = 73.1 GPa.



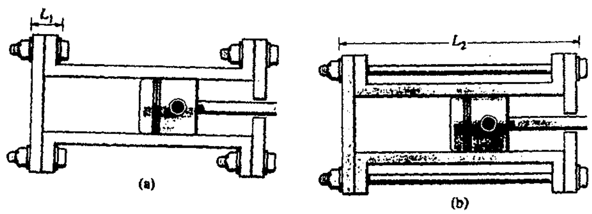


$$U_i = \Sigma \frac{N^2 L}{2AE}$$

$$= \frac{[3(10^3)]^2 (0.3)}{2(\frac{\pi}{4})(0.015^2)(200)(10^9)} + \frac{[7(10^3)]^2 (0.4)}{2(\frac{\pi}{4})(0.02^2)(101)(10^9)} + \frac{[-3(10^3)]^2 (0.2)}{2(\frac{\pi}{4})(0.025^2)(73.1)(10^9)}$$

$= 0.372$ N.m $= 0.372$ J **Ans**

•14–4 Using bolts of the same material and cross-sectional area, two possible attachments for a cylinder head are shown. Compare the strain energy developed in each case, and then explain which design is better for resisting an axial shock or impact load.



Case (a)

$$U_A = \frac{N^2 L_1}{2AE}$$

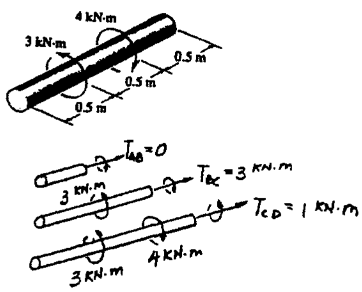
Case (b)

$$U_B = \frac{N^2 L_2}{2AE}$$

Since $U_B > U_A$,i.e., $L_2 > L_1$ the design for case (*b*) is better able to absorb energy.

Case (b) **Ans**

14-5. Determine the torsional strain energy in the A-36 steel shaft. The shaft has a radius of 30 mm.

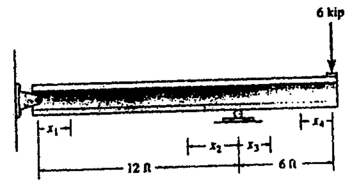


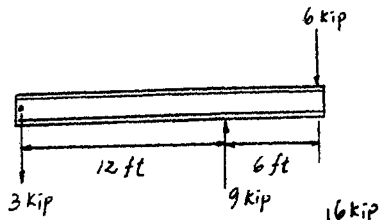
$$U_i = \Sigma\frac{T^2L}{2JG} = \frac{1}{2JG}[0^2(0.5) + ((3)(10^3))^2(0.5) + ((1)(10^3))^2(0.5)]$$

$$= \frac{2.5(10^6)}{JG}$$

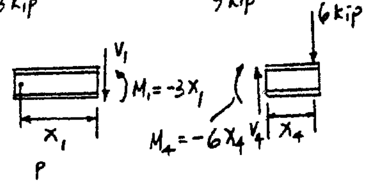
$$= \frac{2.5(10^6)}{75(10^9)(\frac{\pi}{2})(0.03)^4} = 26.2\text{ N}\cdot\text{m} = 26.2\text{ J} \qquad \textbf{Ans}$$

14-6. Determine the bending strain energy in the A-36 structural steel $W10 \times 12$ beam. Obtain the answer using the coordinates (a) x_1 and x_4, and (b) x_2 and x_3.





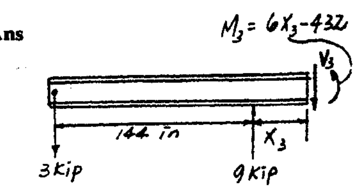
a) $$U_i = \int_0^L \frac{M^2\,dx}{2EI}$$



$$= \int_0^{144} \frac{(-3x_1)^2\,dx_1}{2EI} + \int_0^{72} \frac{(-6x_4)^2\,dx_4}{2EI}$$

$$= \frac{9}{2EI}\frac{144^3}{3} + \frac{36}{2EI}\frac{72^3}{3}$$

$$= \frac{6718464}{29\quad(53.8)} = 4306\ \text{lb}\cdot\text{in.}$$ **Ans**



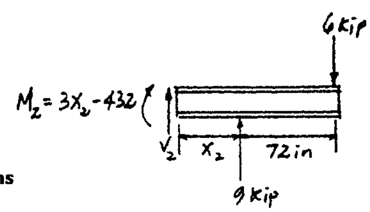
b) $M_3 = 6x_3 - 432$

$M_2 = 9x_2 - 6(x_2 + 72) = 3x_2 - 432$

$$U_i = \int_0^{144} \frac{(3x_2 - 432)^2\,dx_2}{2EI} + \int_0^{72} \frac{(6x_3 - 432)^2\,dx_3}{2EI}$$

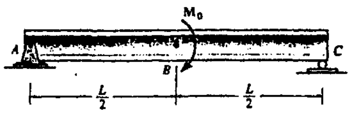
$$= \int_0^{144} \frac{(9x_2^2 - 2592x_2 + 186624)\,dx_2}{2EI} + \int_0^{72} \frac{(36x_3^2 - 5184x_3 + 186624)\,dx_3}{2EI}$$

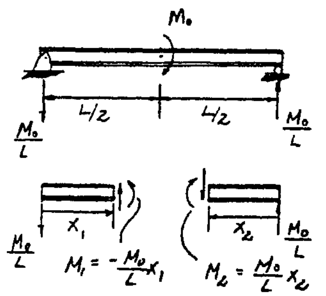
$$= \frac{1}{2EI}[3(144)^3 - \frac{2592}{2}(144)^2 + 186624(144)$$
$$+ 12(72)^3 - \frac{5184}{2}(72)^2 + 186624(72)]$$



$$= \frac{6718464}{29\quad(53.8)} = 4306\ \text{lb}\cdot\text{in.}$$ **Ans**

14-7. Determine the bending strain energy in the beam due to the loading shown. EI is constant.





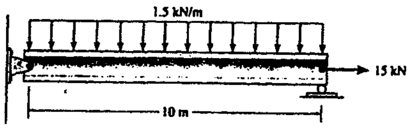
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

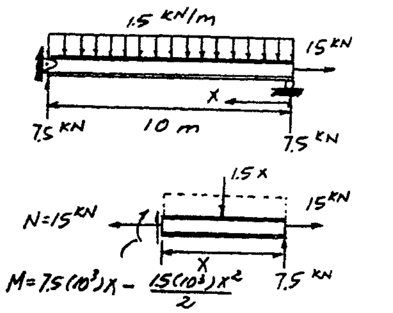
$$= \frac{1}{2EI}\left[\int_0^{L/2} \left(\frac{-M_0}{L}x_1\right)^2 dx_1 + \int_0^{L/2} \left(\frac{M_0}{L}x_2\right)^2 dx_2\right]$$

$$= \frac{M_0^2 L}{24EI} \quad \textbf{Ans}$$

Note : Strain energy is always positive regardless of the sign of the moment function.

***14-8.** Determine the total axial and bending strain energy in the A-36 steel beam. $A = 2300 \text{ mm}^2$, $I = 9.5(10^6) \text{ mm}^4$,





Axial load :

$$(U_a)_i = \int_0^L \frac{N^2 dx}{2EA} = \frac{N^2 L}{2EA}$$

$$(U_a)_i = \frac{((15)(10^3))^2(10)}{2(200)(10^9)(2.3)(10^{-3})} = 2.4456 \text{ J}$$

Bending :

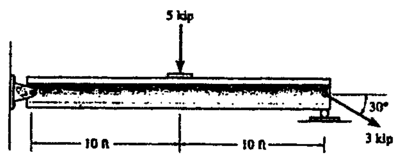
$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^{10} [(7.5)(10^3)x - 0.75(10^3)x^2]^2 dx$$

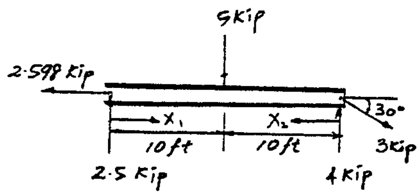
$$= \frac{1}{2EI}\int_0^{10} [56.25(10^6)x^2 + 562.5(10^3)x^4 - 11.25(10^6)x^3] dx$$

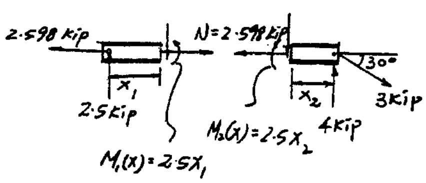
$$(U_b)_i = \frac{0.9375(10^9)}{200(10^9)(9.5)(10^{-6})} = 493.4210 \text{ J}$$

$U_i = (U_a)_i + (U_b)_i = 2.4456 + 493.4210 = 496 \text{ J}$ **Ans**

14-9. Determine the total axial and bending strain energy in the A-36 structural steel $W8 \times 58$ beam.







Axial load :

$$(U_a)_i = \int_0^L \frac{N^2\,dx}{2AE} = \frac{N^2 L}{2AE}$$

$$= \frac{[2.598]^2 (20)(12)}{2(17.1)(29)(10^3)} = 1.6334\,(10^{-3})\ \text{in}\cdot\text{kip}$$

$$= 0.1361\,(10^{-3})\ \text{ft}\cdot\text{kip}$$

Bending :

$$(U_b)_i = \int_0^L \frac{M^2\,dx}{2EI} = \frac{2}{2EI}\int_0^{120\text{ in.}} (2.5\,x)^2\,dx$$

$$= \frac{3.6\,(10^6)}{EI} = \frac{3.6\,(10^6)}{29\,(10^3)(228)}$$

$$= 0.5446\ \text{in.}\cdot\text{kip} = 0.04537\ \text{ft}\cdot\text{kip}$$

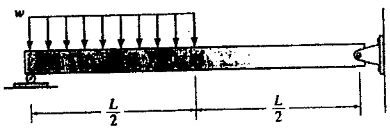
Total strain energy :

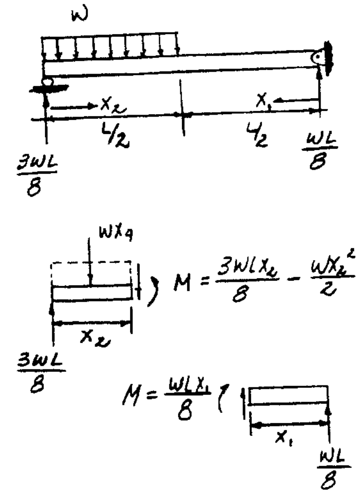
$$U_i = (U_a)_i + (U_b)_i$$

$$= 0.1361\,(10^{-3}) + 0.04537$$

$$= 0.0455\ \text{ft}\cdot\text{kip} = 45.5\ \text{ft}\cdot\text{lb} \qquad \textbf{Ans}$$

14–10 The simply supported beam is subjected to the loading shown. Determine the bending strain energy in the beam.



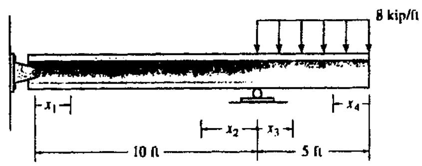


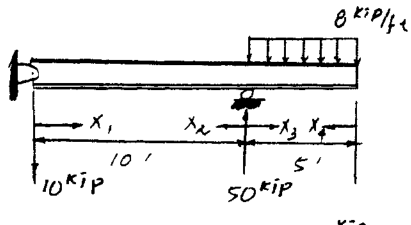
$$U_i = \int_0^L \frac{M^2\,dx}{2EI} = \frac{1}{2EI}\left[\int_0^{L/2}\left(\frac{3wLx_2}{8} - \frac{wx_2^2}{2}\right)^2 dx_2 + \int_0^{L/2}\left(\frac{wLx_1}{8}\right)^2 dx_1\right]$$

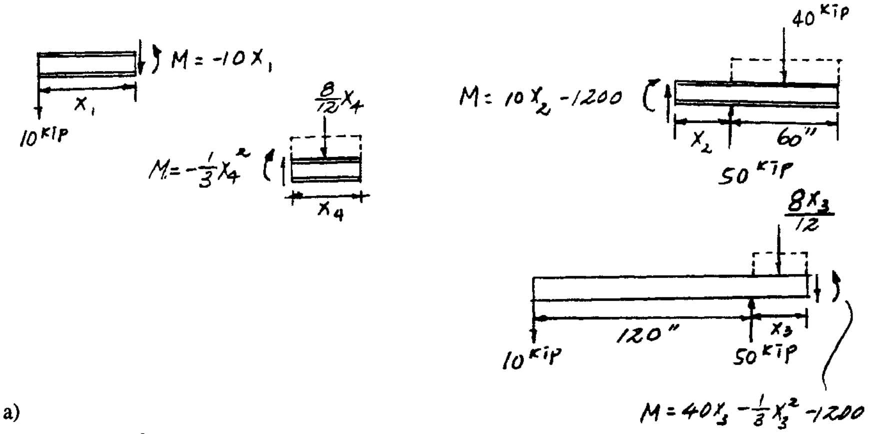
$$= \frac{1}{2EI}\left[\int_0^{L/2}\left(\frac{9w^2L^2x_2^2}{64} + \frac{w^2x_2^4}{4} - \frac{3w^2Lx_2^3}{8}\right)dx_2 + \int_0^{L/2}\frac{w^2L^2x_1^2}{64}dx_1\right]$$

$$= \frac{0.00111w^2L^5}{EI}$$ **Ans**

14–11 Determine the bending strain energy in the A-36 steel beam due to the loading shown. Obtain the answer using the coordinates (*a*) x_1 and x_4, and (*b*) x_2 and x_3. $I = 53.8\ \text{in}^4$.







a)

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{120\text{ in.}} (-10x_1)^2 dx_1 + \int_0^{60\text{ in.}} (-\frac{1}{3}x_4^2)^2 dx_4\right]$$

$$= \frac{37.44(10^6)}{EI} = \frac{37.44(10^6)}{29(10^3)(53.8)} = 24.00\text{ in.}\cdot\text{kip} = 2.00\text{ ft}\cdot\text{kip} \qquad \textbf{Ans}$$

b)

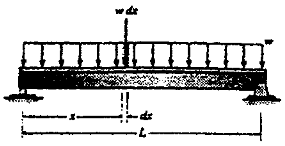
$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{60\text{ in.}} (40x_3 - \frac{1}{3}x_3^2 - 1200)^2 dx_3 + \int_0^{120\text{ in.}} (10x_2 - 1200)^2 dx_2\right]$$

$$= \frac{1}{2EI}\left[\int_0^{60\text{in.}} (\frac{1}{9}x_3^4 - \frac{80}{3}x_3^3 + 2400x_3^2 - 96000x_3 + 1440000)dx_3 + \right.$$

$$\left.\int_0^{120\text{ in.}} (100x_2^2 - 24000x_2 + 1440000)dx_2\right]$$

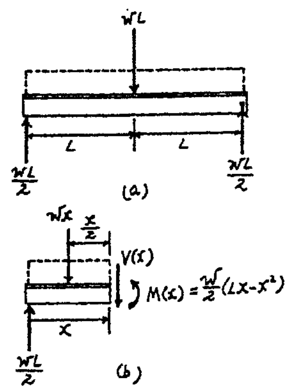
$$= \frac{37.44(10^6)}{EI} = \frac{37.44(10^6)}{29(10^3)(53.8)} = 24.00\text{ in.}\cdot\text{kip} = 2.00\text{ ft}\cdot\text{kip} \qquad \textbf{Ans}$$

*14-12. Determine the bending strain energy in the simply supported beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on the segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 2Lx^3 - L^3x)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.



Support Reactions : As shown on FBD(a).

Internal Moment Function : As shown on FBD(b).



Bending Strain Energy : a) Applying Eq. 14 – 17 gives

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

$$= \frac{1}{2EI}\left[\int_0^L \left[\frac{w}{2}(Lx - x^2)\right]^2 dx\right]$$

$$= \frac{w^2}{8EI}\left[\int_0^L (L^2x^2 + x^4 - 2Lx^3)\, dx\right]$$

$$= \frac{w^2L^5}{240EI} \qquad \textbf{Ans}$$

b) Integrating $dU_i = \frac{1}{2}(w dx)(-y)$

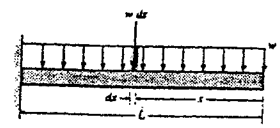
$$dU_i = \frac{1}{2}(w dx)\left[-\frac{w}{24EI}(-x^4 + 2Lx^3 - L^3x)\right]$$

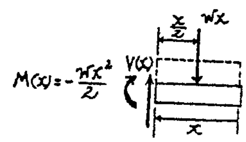
$$dU_i = \frac{w^2}{48EI}(x^4 - 2Lx^3 + L^3x)\, dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L (x^4 - 2Lx^3 + L^3x)\, dx$$

$$= \frac{w^2L^5}{240EI} \qquad \textbf{Ans}$$

*** 14-13.** Determine the bending strain energy in the cantilevered beam due to a uniform load w. Solve the problem two ways. (a) Apply Eq. 14–17. (b) The load $w\,dx$ acting on a segment dx of the beam is displaced a distance y, where $y = w(-x^4 + 4L^3x - 3L^4)/(24EI)$, the equation of the elastic curve. Hence the internal strain energy in the differential segment dx of the beam is equal to the external work, i.e., $dU_i = \frac{1}{2}(w\,dx)(-y)$. Integrate this equation to obtain the total strain energy in the beam. EI is constant.





Internal Moment Function : As shown on FBD.

Bending Strain Energy : **a)** Applying Eq. 14 – 17 gives

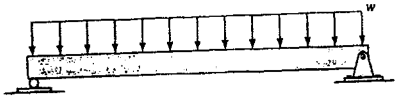
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI}\left[\int_0^L \left[-\frac{w}{2}x^2\right]^2 dx\right]$$
$$= \frac{w^2}{8EI}\left[\int_0^L x^4 dx\right]$$
$$= \frac{w^2 L^5}{40EI} \qquad \textbf{Ans}$$

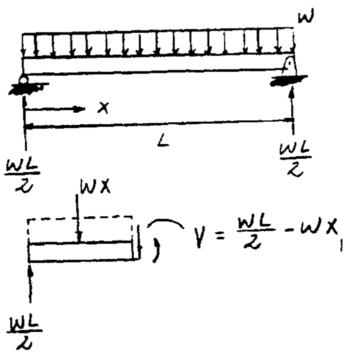
b) Integrating $dU_i = \frac{1}{2}(wdx)(-y)$

$$dU_i = \frac{1}{2}(wdx)\left[-\frac{w}{24EI}\left(-x^4+4L^3x-3L^4\right)\right]$$
$$dU_i = \frac{w^2}{48EI}\left(x^4-4L^3x+3L^4\right)dx$$

$$U_i = \frac{w^2}{48EI}\int_0^L \left(x^4-4L^3x+3L^4\right)dx$$
$$= \frac{w^2 L^5}{40EI} \qquad \textbf{Ans}$$

14–14 Determine the shear strain energy in the beam. The beam has a rectangular cross section of area A, and the shear modulus is G.





$$U_i = \int_0^L \frac{f_s V^2\,dx}{2GA} = \frac{f_s}{2GA}\int_0^L \left(\frac{wL}{2} - wx\right)^2 dx$$

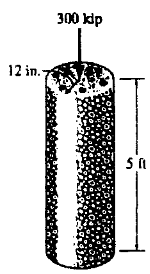
$$= \frac{f_s}{2GA}\int_0^L \left(\frac{w^2L^2}{4} + w^2x^2 - w^2Lx\right)dx$$

$$= \frac{f_s\,w^2L^3}{24\,GA}$$

For a rectangular section $f_s = \frac{6}{5}$

$$U_i = \frac{w^2L^3}{20\,GA}$$ **Ans**

14–15 The concrete column contains six 1-in.-diameter steel reinforcing rods. If the column supports a load of 300 kip, determine the strain energy in the column. $E_{st} = 29(10^3)$ ksi, $E_c = 3.6(10^3)$ ksi.



Equilibrium :

$$+\uparrow \Sigma F_y = 0; \qquad P_{conc} + P_{st} - 300 = 0 \qquad (1)$$

Compatibility condition :

$$\Delta_{conc} = \Delta_{st}$$

$$\frac{P_{conc}L}{[\pi(12^2) - 6\pi(0.5^2)](3.6)(10^3)} = \frac{P_{st}L}{6\pi(0.5^2)(29)(10^3)}$$

$$P_{conc} = 11.7931\, P_{st} \qquad (2)$$

Solving Eqs. (1) and (2) yields :

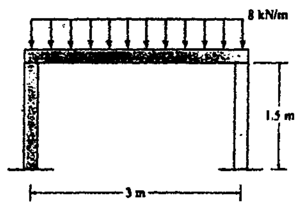
$P_{st} = 23.45$ kip

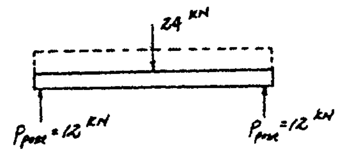
$P_{conc} = 276.55$ kip

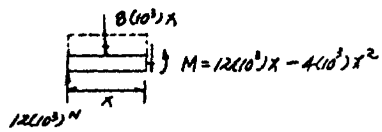
$$U_i = \Sigma\frac{N^2L}{2AE} = \frac{(23.45)^2(5)(12)}{2(6)(\pi)(0.5)^2(29)(10^3)} + \frac{(276.55)^2(5)(12)}{2[(\pi)(12^2) - 6\pi(0.5^2)](3.6)(10^3)}$$

$= 1.544$ in. · kip $= 0.129$ ft · kip **Ans**

***14–16.** Determine the bending strain energy in the beam and the axial strain energy in each of the two posts. All members are made of aluminum and have a square cross section 50 mm by 50 mm. Assume the posts only support an axial load. $E_{al} = 70$ GPa.







Section properties :

$A = (0.05)(0.05) = 2.5(10^{-3})\text{m}^2$

$I = \frac{1}{12}(0.05)(0.05)^3 = 0.52083(10^{-6})\text{m}^4$

Bending strain energy :

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}[\int_0^3 (12(10^3)x - 4(10^3)x^2)^2 dx]$$

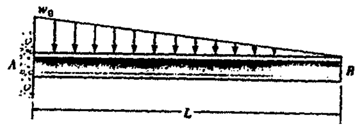
$$= \frac{1}{2EI}[\int_0^3 (144(10^6)x^2 + 16(10^6)x^4 - 96(10^6)x^3)dx]$$

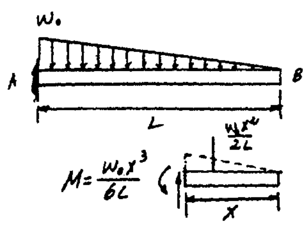
$$= \frac{64.8(10^6)}{EI} = \frac{64.8(10^6)}{70(10^9)(0.52083)(10^{-6})} = 1777 \text{ J} = 1.78 \text{ kJ} \quad \textbf{Ans}$$

Axial strain energy :

$$U_i = \int_0^L \frac{N^2 dx}{2EA} = \frac{N^2 L}{2EA} = \frac{[12(10^3)]^2(1.5)}{2(70)(10^9)(2.5)(10^{-3})} = 0.617 \text{ J} \quad \textbf{Ans}$$

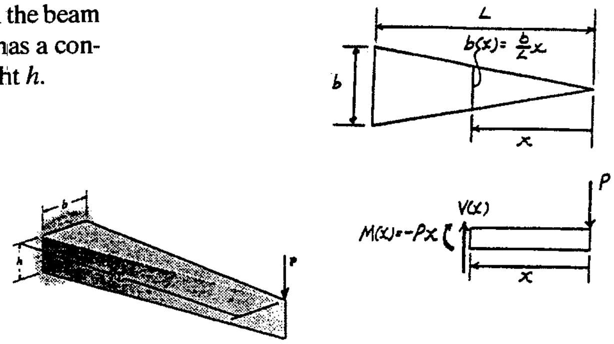
14–17. Determine the bending strain energy in the beam due to the distributed load. *EI* is constant.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L \left(\frac{w_0 x^3}{6L}\right)^2 dx = \frac{w_0^2 L^5}{504\,EI} \qquad \textbf{Ans}$$

14-18. The beam shown is tapered along its width. If a force **P** is applied to its end, determine the strain energy in the beam and compare this result with that of a beam that has a constant rectangular cross section of width b and height h.



Moment of Inertia : For the beam with the uniform section,

$$I = \frac{bh^3}{12} = I_0$$

For the beam with the tapered section,

$$I = \frac{1}{12}\left(\frac{b}{L}x\right)\left(h^3\right) = \frac{bh^3}{12L}x = \frac{I_0}{L}x$$

Internal Moment Function : As shown on FBD.

Bending Strain Energy : For the beam with the tapered section, applying Eq. 14 – 17 gives

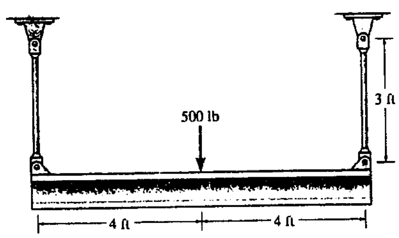
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2E}\int_0^L \frac{(-Px)^2}{\frac{I_0}{L}x}dx$$
$$= \frac{P^2 L}{2EI_0}\int_0^L x\,dx$$
$$= \frac{P^2L^3}{4EI_0} = \frac{3P^2L^3}{bh^3E} \quad \textbf{Ans}$$

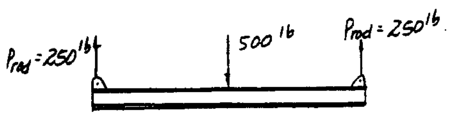
For the beam with the uniform section,

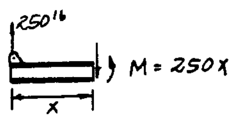
$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$
$$= \frac{1}{2EI_0}\int_0^L (-Px)^2 dx$$
$$= \frac{P^2L^3}{6EI_0}$$

The strain energy in the tapered beam is **1.5 times** as great as that in the beam having a uniform cross section. **Ans**

14–19 Determine the total strain energy in the steel assembly. Consider the axial strain energy in the two 0.5-in.-diameter rods and the bending strain energy in the beam, which has a moment of inertia of $I = 43.4\ \text{in}^4$ about its neutral axis. $E_{st} = 29(10^3)$ ksi.







Bending strain energy :

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^{4(12)} (250x)^2 dx$$

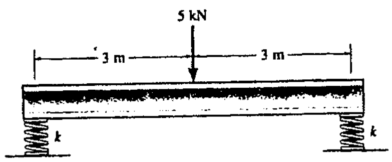
$$= \frac{2.304(10^9)}{EI} = \frac{2.304(10^9)}{29(10^6)(43.4)} = 1.831\ \text{in.}\cdot\text{lb}$$

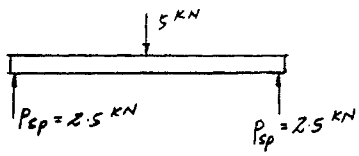
Rods strain energy :

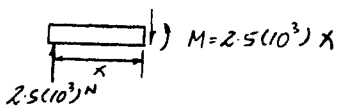
$$(U_a)_i = \int_0^L \frac{N^2 dx}{2EA} = (2)\frac{N^2 L}{2EA} = \frac{(250)^2(3)(12)}{29(10^6)\frac{\pi}{4}(0.5^2)} = 0.395\ \text{in.}\cdot\text{lb}$$

$$U_i = (U_b)_i + (U_a)_i = 1.831 + 0.395 = 2.23\ \text{in.}\cdot\text{lb} \qquad \textbf{Ans}$$

•14–20 A load of 5 kN is applied to the center of the A-36 steel beam, for which $I = 4.5(10^6)\ \text{mm}^4$. If the beam is supported on two springs, each having a stiffness of $k = 8$ MN/m, determine the strain energy in each of the springs and the bending strain energy in the beam.







Bending strain energy :

$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^3 (2.5(10^3)x)^2 dx$$

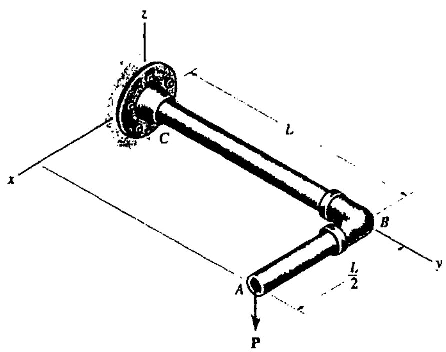
$$= \frac{56.25(10^6)}{EI} = \frac{56.25(10^6)}{200(10^9)(4.5)(10^{-6})} = 62.5\ \text{J} \qquad \textbf{Ans}$$

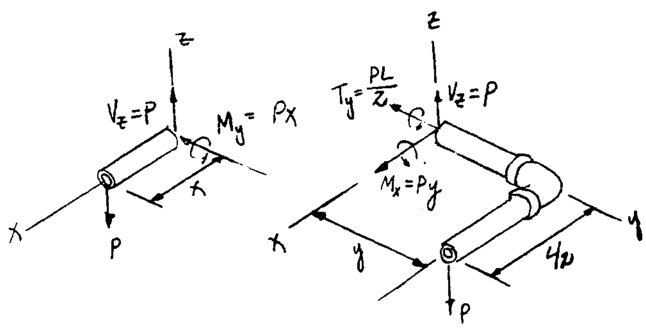
Spring strain energy :

$$\Delta_{sp} = \frac{P_{sp}}{k} = \frac{2.5(10^3)}{8(10^6)} = 0.3125(10^{-3})\ \text{m}$$

$$(U_i)_{sp} = (U_e)_{sp} = \frac{1}{2}k\Delta_{sp}^2 = \frac{1}{2}(8)(10^6)[0.3125(10^{-3})]^2 = 0.391\ \text{J} \qquad \textbf{Ans}$$

14–21 The pipe lies in the horizontal plane. If it is subjected to a vertical force **P** at its end, determine the strain energy due to bending and torsion. Express the results in terms of the cross-sectional properties *I* and *J*, and the material properties *E* and *G*.





$$U_i = \int \frac{M^2\,dx}{2EI} + \int \frac{T^2\,dx}{2JG}$$

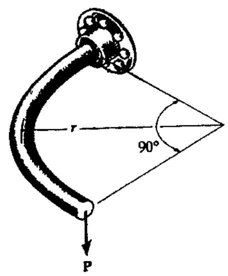
$$= \int_0^{\frac{L}{2}} \frac{(P\,x)^2\,dx}{2EI} + \int_0^{L} \frac{(P\,x)^2\,dx}{2EI} + \int_0^{L} \frac{(\frac{PL}{2})^2\,dx}{2JG}$$

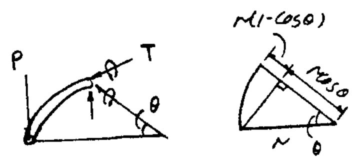
$$= \frac{P^2}{2EI}\left(\frac{L}{2}\right)^3\frac{1}{3} + \frac{P^2}{2EI}\frac{L^3}{3} + \frac{P^2L^2}{8JG}(L)$$

$$= \frac{9P^2L^3}{48EI} + \frac{P^2L^3}{8JG}$$

$$= P^2L^3\left[\frac{9}{48EI} + \frac{1}{8JG}\right] \qquad \textbf{Ans}$$

14–22 Determine the strain energy in the *horizontal* curved bar due to torsion. There is a *vertical* force **P** acting at its end. JG is constant.





$T = Pr(1 - \cos\theta)$

Strain energy :

$$U_i = \int_0^L \frac{T^2 ds}{2JG}$$

However,

$s = r\theta; \qquad ds = r d\theta$

$$U_i = \int_0^\theta \frac{T^2 r d\theta}{2JG} = \frac{r}{2JG}\int_0^{\pi/2}[Pr(1-\cos\theta)]^2 d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2}(1-\cos\theta)^2 d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2}(1+\cos^2\theta - 2\cos\theta)d\theta$$

$$= \frac{P^2 r^3}{2JG}\int_0^{\pi/2}(1+\frac{\cos 2\theta + 1}{2} - 2\cos\theta)d\theta$$

$$= \frac{P^2 r^3}{JG}(\frac{3\pi}{8} - 1) \qquad \textbf{Ans}$$

14–23 Consider the thin-walled tube of Fig. 5–30. Use the formula for shear stress, $\tau_{avg} = T/2tA_m$, Eq. 5–18, and the general equation of shear strain energy, Eq. 14–11, to show that the twist of the tube is given by Eq. 5–20. *Hint:* Equate the work done by the torque T to the strain energy in the tube, determined from integrating the strain energy for a differential element, Fig. 14–4, over the volume of material.

$$U_i = \int_V \frac{\tau^2\,dV}{2\,G} \qquad \text{but } \tau = \frac{T}{2\,t A_m}$$

Thus,

$$U_i = \int_V \frac{T^2}{8\,t^2 A_m^2 G}\,dV$$

$$= \frac{T^2}{8\,A_m^2 G}\int_V \frac{dV}{t^2} = \frac{T^2}{8\,A_m^2 G}\int_A \frac{dV}{t^2}\int_0^L dx = \frac{T^2 L}{8\,A_m^2 G}\int_A \frac{dA}{t^2}$$

However, $dA = t\,ds$. Thus,

$$U_i = \frac{T^2 L}{8\,A_m^2 G}\int \frac{ds}{t}$$

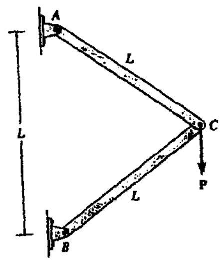
$$U_e = \frac{1}{2}T\phi$$

$$U_e = U_i$$

$$\frac{1}{2}T\,\phi = \frac{T^2 L}{8\,A_m^2 G}\int \frac{ds}{t}$$

$$\phi = \frac{T\,L}{4\,A_m^2 G}\int \frac{ds}{t} \qquad \textbf{QED}$$

***14-24.** Determine the vertical displacement of joint C. AE is constant.



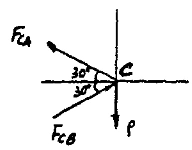
Joint C:

$\xrightarrow{+} \Sigma F_x = 0 \qquad F_{CB}\cos 30° - F_{CA}\cos 30° = 0$

$$F_{CB} = F_{CA}$$

$+\uparrow \Sigma F_y = 0 \qquad F_{CA}\sin 30° + F_{CB}\sin 30° - P = 0$

$$F_{CB} = F_{CA} = P$$



Conservation of energy :

$U_e = U_i$

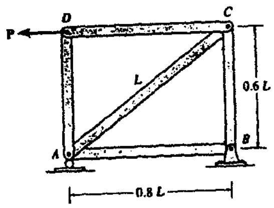
$$\frac{1}{2}P\Delta_C = \Sigma\frac{N^2L}{2EA}$$

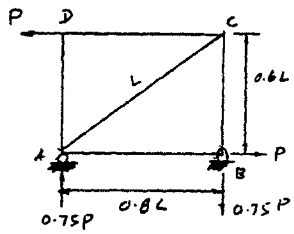
$$\frac{1}{2}P\Delta_C = \frac{L}{2EA}[F_{CB}^2 + F_{CA}^2]$$

$$P\Delta_C = \frac{L}{EA}(P^2 + P^2)$$

$$\Delta_C = \frac{2PL}{AE} \qquad \textbf{Ans}$$

14-25. Determine the horizontal displacement of joint D. AE is constant.

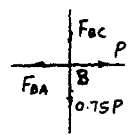




Joint B:

$+\uparrow \Sigma F_y = 0; \quad F_{BC} = 0.75P$

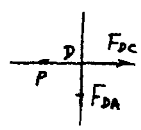
$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_{BA} = P$



Joint D:

$+\downarrow \Sigma F_y = 0; \quad F_{DA} = 0$

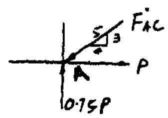
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{DC} = P$



Joint A:

$+\downarrow \Sigma F_y = 0; \quad \frac{3}{5}F_{AC} - 0.75P = 0$

$F_{AC} = 1.25P$



Conservation of energy:

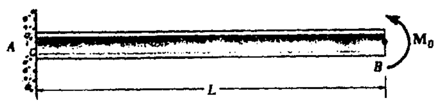
$$U_e = U_i$$

$$\frac{1}{2}P\Delta_D = \Sigma \frac{N^2 L}{2AE}$$

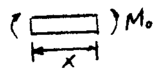
$$\frac{1}{2}P\Delta_D = \frac{1}{2AE}[(0.75P)^2(0.6L) + (P)^2(0.8L) + (0^2)(0.6L) + (P^2)(0.8L) + (1.25P)^2(L)]$$

$$\Delta_D = \frac{3.50PL}{AE} \quad \textbf{Ans}$$

14-26 The cantilevered beam is subjected to a couple moment M_0 applied at its end. Determine the slope of the beam at B. EI is constant.



$M = M_0$



$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^L M_0^2 dx = \frac{M_0^2 L}{2EI}$$

$$U_e = \frac{1}{2}(M_0\,\theta_B)$$

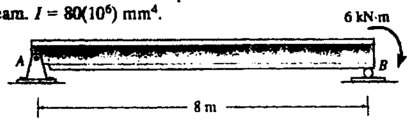
Conservation of energy :

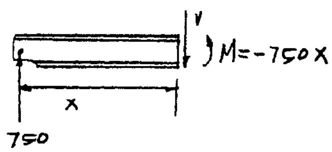
$$U_e = U_i$$

$$\frac{1}{2}M_0\,\theta_B = \frac{M_0{}^2 L}{2EI}$$

$$\theta_B = \frac{M_0 L}{EI} \qquad \textbf{Ans}$$

14-27 Determine the slope at the end B of the A-36 steel beam. $I = 80(10^6)\ \text{mm}^4$.





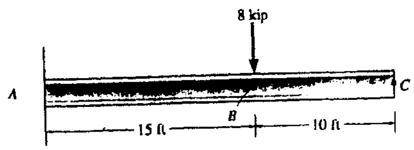
$$M = -750x$$

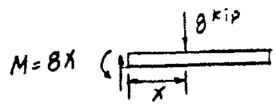
$$\frac{1}{2} M\,\theta_B = \int_0^L \frac{M^2\,dx}{2EI}$$

$$\frac{1}{2}(6\,(10^3))\,\theta_B = \int_0^8 \frac{(-750x)^2\,dx}{2EI}$$

$$\theta_B = \frac{16000}{200\,(10^9)(80)(10^{-6})} = 1\,(10^{-3})\ \text{rad}$$ **Ans**

•14–28 Determine the displacement of point B on the A-36 steel beam. $I = 250\ \text{in}^4$.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\int_0^{15(12)} (8x)^2 dx = \frac{62208000}{EI}$$

$$U_e = \frac{1}{2}P\Delta_B = \frac{1}{2}(8)\Delta_B = 4\Delta_B$$

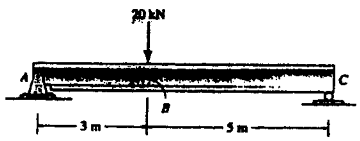
Conservation of energy :

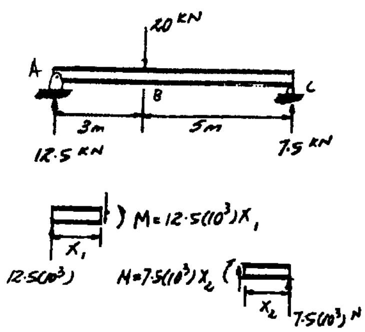
$$U_e = U_i$$

$$4\Delta_B = \frac{62208000}{EI}$$

$$\Delta_B = \frac{15552000}{EI} = \frac{15552000}{29(10^3)(250)} = 2.15 \text{ in.}$$ **Ans**

14-29. Determine the displacement of point B on the A-36 steel beam. $I = 80(10^6)\ \text{mm}^4$.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^3 [(12.5)(10^3)(x_1)]^2 dx_1 + \int_0^5 [(7.5)(10^3)(x_2)]^2 dx_2\right] = \frac{1.875(10^9)}{EI}$$

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(20)(10^3)\Delta_B = 10(10^3)\Delta_B$$

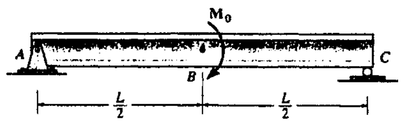
Conservation of energy :

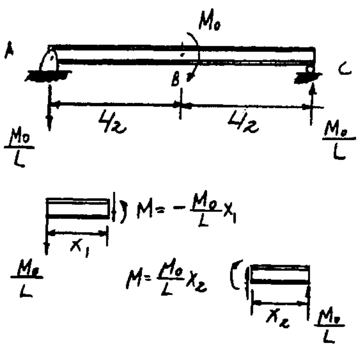
$U_e = U_i$

$$10(10^3)\Delta_B = \frac{1.875(10^9)}{EI}$$

$$\Delta_B = \frac{187500}{EI} = \frac{187500}{200(10^9)(80)(10^{-6})} = 0.0117\ \text{m} = 11.7\ \text{mm} \qquad \textbf{Ans}$$

14–30 Use the method of work and energy and determine the slope of the beam at point B in Prob. 14–7. EI is constant.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^{L/2}\left(-\frac{M_0}{L}x_1\right)^2 dx_1 + \int_0^{L/2}\left(\frac{M_0}{L}x_2\right)dx_2\right]$$

$$= \frac{M_0^2 L}{24EI}$$

$$U_e = \frac{1}{2}M'\theta = \frac{1}{2}M_0\,\theta_B$$

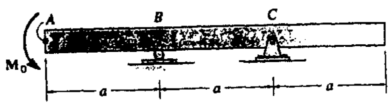
Conservation of energy :

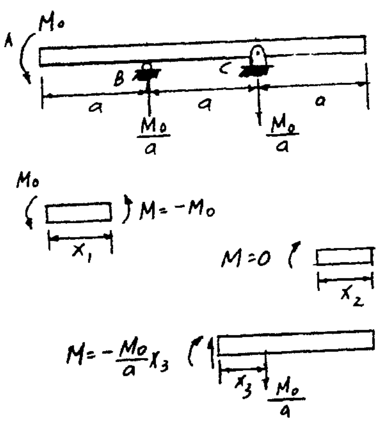
$$U_e = U_i$$

$$\frac{1}{2}M_0\,\theta_B = \frac{M_0{}^2 L}{24EI}$$

$$\theta_B = \frac{M_0 L}{12EI} \qquad \textbf{Ans}$$

14–31 Determine the slope at point A of the beam. EI is constant.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^a (-M_0)^2 dx_1 + \int_0^a (0) dx_2 + \int_0^a \left(-\frac{M_0}{a}x_3\right)^2 dx_3\right]$$

$$= \frac{2M_0^2 a}{3EI}$$

$$U_e = \frac{1}{2}M'\theta = \frac{1}{2}M_0\,\theta_A$$

Conservation of energy :

$$U_e = U_i$$

$$\frac{1}{2}M_0\,\theta_A = \frac{2M_0^2 a}{3EI}$$

$$\theta_A = \frac{4M_0 a}{3EI}$$ **Ans**

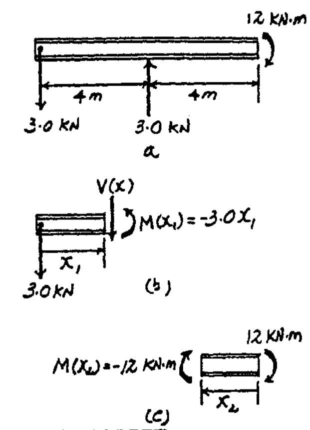
***14-32.** Determine the slope at point C of the A-36 steel beam. $I = 9.50(10^6)\ \text{mm}^4$.

Support Reactions : As shown on FBD(a).

Moment Functions : As shown on FBD(b) and (c).

Bending Strain Energy : Applying 14–17, we have

$$U_i = \int_0^L \frac{M^2dx}{2EI}$$
$$= \frac{1}{2EI}\left[\int_0^{4\text{m}} (-3.00x_1)^2 dx_1 + \int_0^{4\text{m}} (-12.0)^2 dx_2\right]$$
$$= \frac{384\ \text{kN}^2\cdot\text{m}^3}{EI}$$
$$= \frac{384(10^6)}{200(10^9)[9.50(10^{-6})]} = 202.11\ \text{N}\cdot\text{m}$$



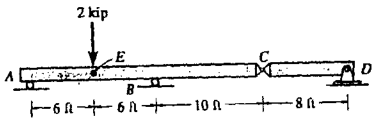
External Work : The external work done by 12 kN · m couple moment is

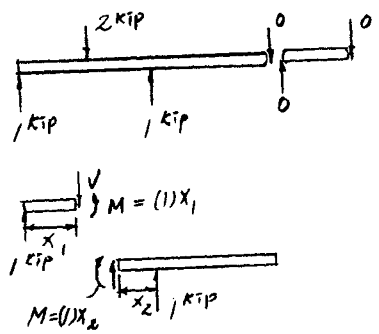
$$U_e = \frac{1}{2}\left[12(10^3)\right](\theta_C) = 6.00(10^3)\ \theta_C$$

Conservation of Energy :

$$U_e = U_i$$
$$6.00(10^3)\ \theta_C = 202.11$$
$$\theta_C = 0.0337\ \text{rad} = 1.93^\circ \qquad \textbf{Ans}$$

14–33 The A-36 steel bars are pin connected at C. If they each have a diameter of 2 in., determine the displacement at E.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^{6(12)} (x_1)^2 dx_1 = \frac{124416}{EI}$$

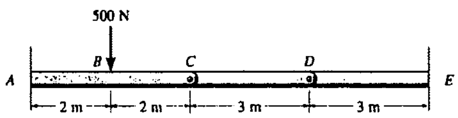
$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(2)\Delta_E = \Delta_E$$

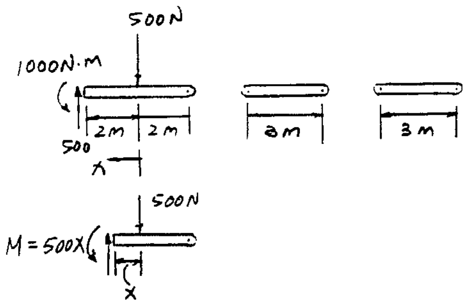
Conservation of energy :

$$U_e = U_i$$

$$\Delta_E = \frac{124416}{EI} = \frac{124416}{29(10^3)(\frac{\pi}{4})(1^4)} = 5.46 \text{ in.} \qquad \textbf{Ans}$$

14-34 The A-36 steel bars are pin connected at C and D. If they each have the same rectangular cross section, with a height of 200 mm and a width of 100 mm, determine the vertical displacement at B. Neglect the axial load in the bars.





Internal strain energy :

$$U_i = \int_0^L \frac{M^2\,dx}{2EI} = \frac{1}{2EI}\int_0^{2\text{ m}} [500x]^2\,dx = \frac{0.3333\,(10^6)}{EI}$$

External work :

$$U_e = \frac{1}{2}P\,\Delta_B = \frac{1}{2}(500)\,\Delta_B = 250\,\Delta_B$$

Conservation of energy :

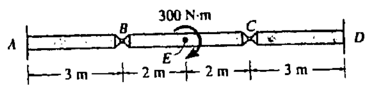
$$U_e = U_i$$

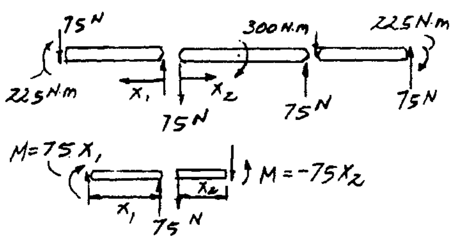
$$250\,\Delta_B = \frac{0.3333\,(10^6)}{EI}$$

$$\Delta_B = \frac{1333.33}{EI} = \frac{1333.33}{200\,(10^9)(\frac{1}{12})(0.1)(0.2^3)}$$

$$= 0.1\,(10^{-3})\text{ m} = 0.100\text{ mm}$$ **Ans**

14-35 The A-36 steel bars are pin connected at B and C. If they each have a diameter of 30 mm, determine the slope at E.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^3 (75x_1)^2 dx_1 + (2)\frac{1}{2EI}\int_0^2 (-75x_2)^2 dx_2 = \frac{65625}{EI}$$

$$U_e = \tfrac{1}{2}(M')\theta = \tfrac{1}{2}(300)\theta_E = 150\theta_E$$

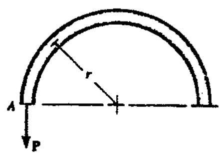
Conservation of energy :

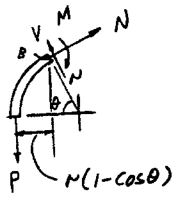
$$U_e = U_i$$

$$150\theta_E = \frac{65625}{EI}$$

$$\theta_E = \frac{473.5}{EI} = \frac{473.5}{(200)(10^9)(\frac{\pi}{4})(0.015^4)} = 0.0550 \text{ rad} = 3.15° \qquad \textbf{Ans}$$

***14-36.** The rod has a circular cross section with a moment of inertia I. If a vertical force **P** is applied at A, determine the vertical displacement at this point. Only consider the strain energy due to bending. The modulus of elasticity is E.





Moment function :

$\curvearrowleft + \Sigma M_B = 0; \quad P[r(1-\cos\theta)] - M = 0; \quad M = Pr(1-\cos\theta)$

Bending strain energy :

$$U_i = \int_0^s \frac{M^2\,ds}{2EI} \qquad ds = r\,d\theta$$

$$= \int_0^\theta \frac{M^2\,r\,d\theta}{2EI} = \frac{r}{2EI}\int_0^\pi [Pr(1-\cos\theta)]^2\,d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi (1+\cos^2\theta - 2\cos\theta)\,d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi \left(1+\frac{1}{2}+\frac{\cos 2\theta}{2} - 2\cos\theta\right)d\theta$$

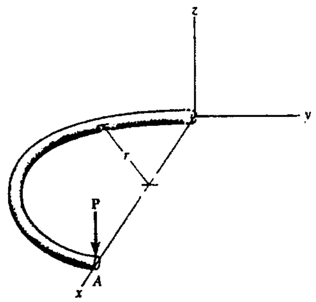
$$= \frac{P^2r^3}{2EI}\int_0^\pi \left(\frac{3}{2}+\frac{\cos 2\theta}{2} - 2\cos\theta\right)d\theta = \frac{P^2r^3}{2EI}\left(\frac{3}{2}\pi\right) = \frac{3\pi P^2 r^3}{4EI}$$

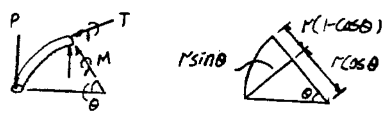
Conservation of energy :

$$U_e = U_i; \quad \frac{1}{2}P\Delta_A = \frac{3\pi P^2 r^3}{4EI}$$

$$\Delta_A = \frac{3\pi P r^3}{2EI} \qquad \textbf{Ans}$$

14–37 The rod has a circular cross section with a polar moment of inertia J and moment of inertia I. If a vertical force **P** is applied at A, determine the vertical displacement at this point. Consider the strain energy due to bending and torsion. The material constants are E and G.





$T = Pr(1-\cos\theta)$
$M = Pr\sin\theta$

$$T = Pr(1-\cos\theta); \qquad M = Pr\sin\theta$$

Torsion strain energy :

$$U_i = \int_0^s \frac{T^2 ds}{2GJ} = \int_0^\theta \frac{T^2 r d\theta}{2GJ}$$

$$= \frac{r}{2GJ}\int_0^\pi [Pr(1-\cos\theta)]^2 d\theta$$

$$= \frac{P^2r^3}{2GJ}\int_0^\pi (1+\cos^2\theta - 2\cos\theta)d\theta$$

$$= \frac{P^2r^3}{2GJ}\int_0^\pi (1+\frac{\cos 2\theta + 1}{2} - 2\cos\theta)d\theta$$

$$= \frac{3P^2r^3\pi}{4GJ}$$

Bending strain energy :

$$U_i = \int_0^s \frac{M^2 ds}{2EI}$$

$$= \int_0^\theta \frac{M^2 r\, d\theta}{2EI} = \frac{r}{2EI}\int_0^\pi [Pr\sin\theta]^2 d\theta$$

$$= \frac{P^2r^3}{2EI}\int_0^\pi (\frac{1-\cos 2\theta}{2})d\theta = \frac{P^2r^3\pi}{4EI}$$

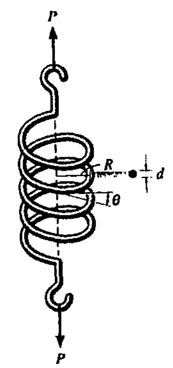
Conservation of energy :

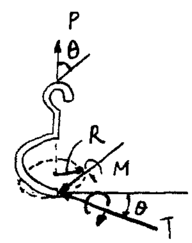
$$U_e = U_i$$

$$\frac{1}{2}P\Delta = \frac{3P^2r^3\pi}{4GJ} + \frac{P^2r^3\pi}{4EI}$$

$$\Delta = \frac{Pr^3\pi}{2}(\frac{3}{GJ} + \frac{1}{EI}) \qquad \textbf{Ans}$$

14–38 The load **P** causes the open coils of the spring to make an angle θ with the horizontal when the spring is stretched. Show that for this position this causes a torque $T = PR\cos\theta$ and a bending moment $M = PR\sin\theta$ at the cross section. Use these results to determine the maximum normal stress in the material.





$$T = PR\cos\theta; \qquad M = PR\sin\theta$$

Bending :

$$\sigma_{max} = \frac{Mc}{I} = \frac{PR\sin\theta\, d}{2\left(\frac{\pi}{4}\right)\left(\frac{d^4}{16}\right)}$$

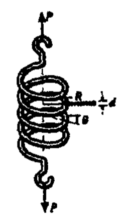
$$\tau_{max} = \frac{Tc}{J} = \frac{PR\cos\theta\,\frac{d}{2}}{\frac{\pi}{2}\left(\frac{d^4}{16}\right)}$$

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16PR\sin\theta}{\pi d^3} \pm \sqrt{\left(\frac{16PR\sin\theta}{\pi d^3}\right)^2 + \left(\frac{16PR\cos\theta}{\pi d^3}\right)^2}$$

$$= \frac{16PR}{\pi d^3}[\sin\theta + 1] \qquad \textbf{Ans}$$

14-39. The coiled spring has n coils and is made from a material having a shear modulus G. Determine the stretch of the spring when it is subjected to the load **P**. Assume that the coils are close to each other so that $\theta \approx 0°$ and the deflection is caused entirely by the torsional stress in the coil.

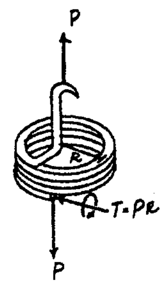


Bending Strain Energy : Applying 14 – 22, we have

$$U_i = \frac{T^2L}{2GJ} = \frac{P^2R^2L}{2G\left[\frac{\pi}{32}(d^4)\right]} = \frac{16P^2R^2L}{\pi d^4G}$$

However, $L = n(2\pi R) = 2n\pi R$. Then

$$U_i = \frac{32nP^2R^3}{d^4G}$$



External Work : The external work done by force P is

$$U_e = \frac{1}{2}P\Delta$$

Conservation of Energy :

$$U_e = U_i$$

$$\frac{1}{2}P\Delta = \frac{32nP^2R^3}{d^4G}$$

$$\Delta = \frac{64nPR^3}{d^4G} \qquad \textbf{Ans}$$

•14–40 A bar is 4 m long and has a diameter of 30 mm. If it is to be used to absorb energy in tension from an impact loading, determine the total amount of elastic energy that it can absorb if (a) it is made of steel for which E_{st} = 200 GPa, σ_Y = 800 MPa, and (b) it is made from an aluminum alloy for which E_{al} = 70 GPa, σ_Y = 405 MPa.

a) $$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{800(10^6)}{200(10^9)} = 4(10^{-3})\ \text{m/m}$$

$$u_r = \frac{1}{2}(\sigma_Y)(\varepsilon_Y) = \frac{1}{2}(800)(10^6)(\text{N/m}^2)(4)(10^{-3})\text{m/m} = 1.6\ \textbf{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^2$$

$$u_i = 1.6(10^6)(0.9)(10^{-3})\pi = 4.52\ \text{kJ} \qquad \textbf{Ans}$$

b)

$$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{405(10^6)}{70(10^9)} = 5.786(10^{-3})\ \text{m/m}$$

$$u_r = \frac{1}{2}(\sigma_Y)(\varepsilon_Y) = \frac{1}{2}(405)(10^6)(\text{N/m}^2)(5.786)(10^{-3})\text{m/m} = 1.172\ \textbf{MJ/m}^3$$

$$V = \frac{\pi}{4}(0.03)^2(4) = 0.9(10^{-3})\pi\ \text{m}^3$$

$$u_i = 1.172(10^6)(0.9)(10^{-3})\pi = 3.31\ \textbf{kJ} \qquad \textbf{Ans}$$

14-41 Determine the diameter of a red brass C83400 bar that is 8 ft long if it is to be used to absorb 800 ft · lb of energy in tension from an impact loading.

$$\varepsilon_y = \frac{\sigma_Y}{E} = \frac{10}{14.6\,(10^3)} = 0.68493\,(10^{-3})\ \text{in./in.}$$

$$u_r = \frac{1}{2}\,\sigma_Y \varepsilon_Y = \frac{1}{2}\,(10)(10^3)\,\frac{\text{lb}}{\text{in}^2}\,(0.68493)(10^{-3})\ \text{in./in.}$$

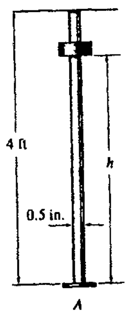
$$= 3.4247\,\frac{\text{in.}\cdot\text{lb}}{\text{in}^3}$$

$$V = \frac{\pi}{4}\,(d^2)(8)(12) = 75.398\,d^2$$

$$800(12) = 3.4247\,(75.398\,d^2)$$

$d = 6.10$ in. **Ans**

14–42 The collar has a weight of 50 lb and falls down the titanium bar. If the bar has a diameter of 0.5 in, determine the maximum stress developed in the bar if the weight is (*a*) dropped from a height of h = 1 ft, (*b*) released from a height $h \approx 0$, and (*c*) placed slowly on the flange at *A*. $E_{ti} = 16(10^3)$ ksi, $\sigma_Y = 60$ ksi.



a)

$$\Delta_{st} = \frac{WL}{AE} = \frac{50(4)(12)}{\frac{\pi}{4}(0.5)^2(16)(10^6)} = 0.7639(10^{-3})\text{ in.}$$

$$P_{max} = W[1+\sqrt{1+2(\frac{h}{\Delta_{st}})}\,] = 50[1+\sqrt{1+2(\frac{(1)(12)}{0.7639(10^{-3})})}\,] = 8912\text{ lb}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{8912}{\frac{\pi}{4}(0.5)^2} = 45390\text{ psi} = 45.4\text{ ksi} < \sigma_Y \qquad \textbf{Ans}$$

b)

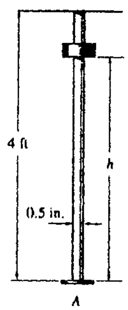
$$P_{max} = W[1+\sqrt{1+2(\frac{h}{\Delta_{st}})}\,] = 50[1+\sqrt{1+2(0)}] = 100\text{ lb}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{100}{\frac{\pi}{4}(0.5)^2} = 509\text{ psi} < \sigma_Y \qquad \textbf{Ans}$$

c)

$$\sigma_{max} = \frac{W}{A} = \frac{50}{\frac{\pi}{4}(0.5)^2} = 254\text{ psi} < \sigma_Y \qquad \textbf{Ans}$$

14–43 The collar has a weight of 50 lb and falls down the titanium bar. If the bar has a diameter of 0.5 in., determine the largest height h at which the weight can be released and not permanently damage the bar after striking the flange at A. $E_{ti} = 16(10^3)$ ksi, $\sigma_Y = 60$ ksi.



$$\Delta_{st} = \frac{WL}{AE} = \frac{50(4)(12)}{\frac{\pi}{4}(0.5)^2(16)(10^6)} = 0.7639(10^{-3})\text{ in.}$$

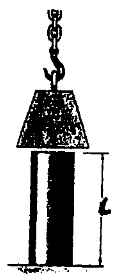
$$P_{max} = W[1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\ ]$$

$$60(10^3)(\frac{\pi}{4})(0.5^2) = 50[1 + \sqrt{1 + 2(\frac{h}{0.7639(10^{-3})})}\ ]$$

$$235.62 = 1 + \sqrt{1 + 2618h}$$

$$h = 21.02\text{ in.} = 1.75\text{ ft}\qquad \textbf{Ans}$$

•14-44 The mass of 50 Mg is held just over the top of the steel post having a length of $L = 2$ m and a cross-sectional area of 0.01 m². If the mass is released, determine the maximum stress developed in the bar and its maximum deflection. $E_{st} = 200$ GPa, $\sigma_Y = 600$ MPa.



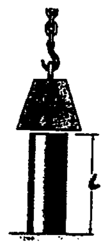
$$n = [1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\ ] = 1 + \sqrt{1 + 2(0)} = 2$$

$$\sigma_{max} = n\sigma_{st} = (2)(\frac{50(10^3)(9.81)}{0.01}) = 98.1 \text{ MPa} < \sigma_Y \qquad \textbf{Ans}$$

$$\Delta_{st} = \frac{WL}{AE} = \frac{50(10^3)(9.81)(2)}{(0.01)(200)(10^9)} = 0.4905(10^{-3})\text{m}$$

$$\Delta_{max} = n\Delta_{st} = 2(0.4905)(10^{-3}) = 0.981(10^{-3}) \text{ m} = 0.981 \text{ mm} \qquad \textbf{Ans}$$

14-45. Determine the speed v of the 50-Mg mass when it is just over the top of the steel post, if after impact, the maximum stress developed in the post is 550 MPa. The post has a length of $L = 1$ m and a cross-sectional area of 0.01 m^2. $E_{st} = 200$ GPa, $\sigma_Y = 600$ MPa.



The maximum stress :

$$\sigma_{max} = \frac{P_{max}}{A}$$

$$550(10^6) = \frac{P_{max}}{0.01}; \quad P_{max} = 5500 \text{ kN}$$

$$\Delta_{max} = \frac{P_{max}}{k} \quad \text{Here } k = \frac{AE}{L} = \frac{0.01(200)(10^9)}{1} = 2(10^9) \text{ N/m}$$

$$= \frac{5500(10^3)}{2(10^9)} = 2.75(10^{-3}) \text{ m}$$

Conservation of energy :

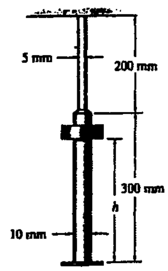
$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}(50)(10^3)(v^2) + 50(10^3)(9.81)[2.75(10^{-3})] = \frac{1}{2}(2)(10^9)[2.75(10^{-3})]^2$$

$v = 0.499$ m/s **Ans**

14-46. The composite aluminum bar is made from two segments having diameters of 5 mm and 10 mm. Determine the maximum axial stress developed in the bar if the 5-kg collar is dropped from a height of $h = 100$ mm. $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



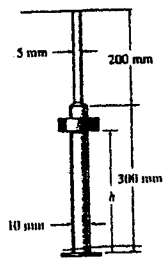
$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{5(9.81)(0.2)}{\frac{\pi}{4}(0.005^2)(70)(10^9)} + \frac{5(9.81)(0.3)}{\frac{\pi}{4}(0.01^2)(70)(10^9)} = 9.8139(10^{-6})\text{ m}$$

$$P_{max} = W[1+\sqrt{1+2(\frac{h}{\Delta_{st}})}\,]$$

$$= 5(9.81)[1+\sqrt{1+2(\frac{0.1}{9.8139(10^{-6})})}\,] = 7051\text{ N}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{7051}{\frac{\pi}{4}(0.005^2)} = 359\text{ MPa} < \sigma_Y \qquad \text{OK} \qquad \text{Ans}$$

14-47. The composite aluminum bar is made from two segments having diameters of 5 mm and 10 mm. Determine the maximum height h from which the 5-kg collar should be dropped so that it produces a maximum axial stress in the bar of $\sigma_{max} = 300$ MPa. $E_{al} = 70$ GPa, $\sigma_Y = 410$ MPa.



$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{5(9.81)(0.2)}{\frac{\pi}{4}(0.005^2)(70)(10^9)} + \frac{5(9.81)(0.3)}{\frac{\pi}{4}(0.01^2)(70)(10^9)} = 9.8139(10^{-6})\text{ m}$$

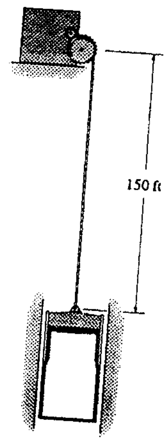
$$P_{max} = W[1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\,]$$

$$300(10^6)(\frac{\pi}{4})(0.005^2) = 5(9.81)[1 + \sqrt{1 + 2(\frac{h}{9.8139(10^{-6})})}\,]$$

$$120.1 = 1 + \sqrt{1 + 203791.6\,h}$$

$h = 0.0696$ m $= 69.6$ mm **Ans**

•14–48 A steel cable having a diameter of 0.4 in. wraps over a drum and is used to lower an elevator having a weight of 800 lb. The elevator is 150 ft below the drum and is descending at the constant rate of 2 ft/s when the drum suddenly stops. Determine the maximum stress developed in the cable when this occurs. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



$$k = \frac{AE}{L} = \frac{\frac{\pi}{4}(0.4^2)(29)(10^3)}{150\,(12)} = 2.0246 \text{ kip/in.}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}\left[\frac{800}{32.2\,(12)}\right][(12)(2)]^2 + 800\,\Delta_{max} = \frac{1}{2}(2.0246)(10^3)\Delta_{max}^2$$

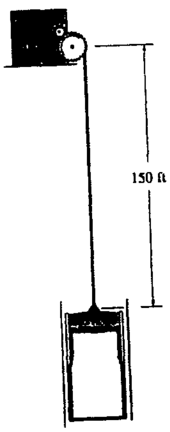
$$596.27 + 800\,\Delta_{max} = 1012.29\,\Delta_{max}^2$$

$$\Delta_{max} = 1.2584 \text{ in.}$$

$$P_{max} = k\Delta_{max} = 2.0246\,(1.2584) = 2.5477 \text{ kip}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{2.5477}{\frac{\pi}{4}(0.4)^2} = 20.3 \text{ ksi} < \sigma_Y \qquad \text{OK} \qquad \textbf{Ans}$$

14–49 Solve Prob. 14–48 if the elevator is descending at the constant rate of 3 ft/s.



$$k = \frac{AE}{L} = \frac{\frac{\pi}{4}(0.4^2)(29)(10^3)}{150\,(12)} = 2.0246 \text{ kip/in.}$$

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta_{max}^2$$

$$\frac{1}{2}\left[\frac{800}{32.2\,(12)}\right][(12)(3)]^2 + 800\,\Delta_{max} = \frac{1}{2}(2.0246)(10^3)\Delta_{max}^2$$

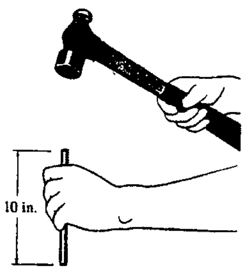
$$1341.61 + 800\,\Delta_{max} = 1012.29\,\Delta_{max}^2$$

$$\Delta_{max} = 1.6123 \text{ in.}$$

$$P_{max} = k\,\Delta_{max} = 2.0246(1.6123) = 3.2643 \text{ kip}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{3.2643}{\frac{\pi}{4}(0.4)^2} = 26.0 \text{ ksi} < \sigma_Y \qquad \text{OK} \qquad \textbf{Ans}$$

14–50 The steel chisel has a diameter of 0.5 in. and a length of 10 in. It is struck by a hammer that weighs 3 lb, and at the instant of impact it is moving at 12 ft/s. Determine the maximum compressive stress in the chisel, assuming that 80% of the impacting energy goes into the chisel. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 100$ ksi.



$$k = \frac{AE}{L} = \frac{\frac{\pi}{4}(0.5^2)(29)(10^3)}{10} = 569.41 \text{ kip/in.}$$

$$0.8U_e = U_i$$

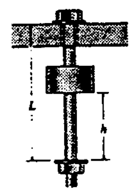
$$0.8[\frac{1}{2}(\frac{3}{(32.2)(12)})((12)(12))^2 + 3\Delta_{max}] = \frac{1}{2}(569.41)(10^3)\Delta_{max}^2$$

$$\Delta_{max} = 0.015044 \text{ in.}$$

$$P = k\Delta_{max} = 569.41(0.015044) = 8.566 \text{ kip}$$

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{8.566}{\frac{\pi}{4}(0.5)^2} = 43.6 \text{ ksi} < \sigma_Y \qquad \text{OK} \qquad \textbf{Ans}$$

14-51. The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm, determine its required length L so the stress in the bolt does not exceed 150 MPa.



Maximum Stress : With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{2(9.81)(L)}{\frac{\pi}{4}(0.004^2)[200(10^9)]}$

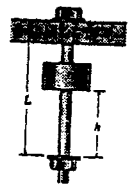
$= 7.80655(10^{-6})\,L$ and $\sigma_{st} = \dfrac{W}{A} = \dfrac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa, we have

$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}$$

$$150(10^6) = \left[1+\sqrt{1+2\left(\frac{0.03}{7.80655(10^{-6})L}\right)}\right]\left[1.56131(10^6)\right]$$

$$L = 0.8504 \text{ m} = 850 \text{ mm} \qquad \textbf{Ans}$$

***14-52.** The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls $h = 30$ mm. If the bolt has a diameter of 4 mm and a length of $L = 200$ mm, determine if the stress in the bolt will exceed 175 MPa.



Maximum Stress : With

$$\Delta_{st} = \frac{WL}{AE} = \frac{2(9.81)(0.2)}{\frac{\pi}{4}(0.004^2)[200(10^9)]} = 1.56131(10^{-6})\ \text{m}$$

$$\sigma_{st} = \frac{W}{A} = \frac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131\ \text{MPa}$$

Applying Eq. 14–34, we have

$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{0.03}{1.56131(10^{-6})}\right)} = 197.04$$

Thus,

$$\sigma_{max} = n\sigma_{st} = 197.04(1.56131) = 307.6\ \text{MPa}$$

Yes, σ_{max} exceeded 175 MPa. **Ans**

14-53. The A-36 steel bolt is required to absorb the energy of a 2-kg mass that falls along the 4-mm-diameter bolt shank that is 150 mm long. Determine the maximum height h of release so the stress in the bolt does no exceed 150 MPa.

Maximum Stress : With $\Delta_{st} = \dfrac{WL}{AE} = \dfrac{2(9.81)(0.15)}{\frac{\pi}{4}(0.004^2)[200(10^9)]}$

$= 1.17098(10^{-6})$ m and $\sigma_{st} = \dfrac{W}{A} = \dfrac{2(9.81)}{\frac{\pi}{4}(0.004^2)} = 1.56131$ MPa,

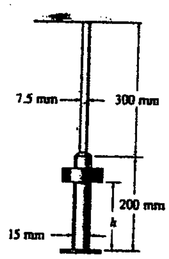
we have

$$\sigma_{max} = n\sigma_{st} \quad \text{where } n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)}$$

$$150(10^6) = \left[1 + \sqrt{1 + 2\left(\frac{h}{1.17098(10^{-6})}\right)}\right]\left[1.56131(10^6)\right]$$

$$h = 5.292(10^{-3}) \text{ m} = 5.29 \text{ mm} \qquad \textbf{Ans}$$

14-54. The composite aluminum 2014-T6 bar is made from two segments having diameters of 7.5 mm and 15 mm. Determine the maximum axial stress developed in the bar if the 10-kg collar is dropped from a height of $h = 100$ mm.



$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{10\,(9.81)(0.3)}{\frac{\pi}{4}(0.0075)^2(73.1)(10^9)} + \frac{10\,(9.81)(0.2)}{\frac{\pi}{4}(0.015)^2(73.1)(10^9)}$$

$$= 10.63181147\,(10^{-6})\text{ m}$$

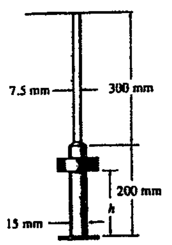
$$n = [1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\,] = [1 + \sqrt{1 + 2(\frac{0.1}{10.63181147\,(10^{-6})})}\,] = 138.16$$

$\sigma_{max} = n\,\sigma_{st}$ Here $\sigma_{st} = \frac{W}{A} = \frac{10\,(9.81)}{\frac{\pi}{4}(0.0075^2)} = 2.22053$ MPa

$$\sigma_{max} = 138.16\,(2.22053)$$

$$= 307\text{ MPa} < \sigma_Y = 414\text{ MPa} \qquad \text{OK} \qquad \text{Ans}$$

14-55. The composite aluminum 2014-T6 bar is made from two segments having diameters of 7.5 mm and 15 mm. Determine the maximum height h from which the 10-kg collar should be dropped so that it produces a maximum axial stress in the bar of $\sigma_{max} = 300$ MPa.



$$\Delta_{st} = \Sigma\frac{WL}{AE} = \frac{10\,(9.81)(0.3)}{\frac{\pi}{4}(0.0075^2)(73.1)(10^9)} + \frac{10\,(9.81)(0.2)}{\frac{\pi}{4}(0.015^2)(73.1)(10^9)}$$

$$= 10.63181147\,(10^{-6})\text{ m}$$

$$n = [1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\,] = [1 + \sqrt{1 + 2(\frac{h}{10.63181147\,(10^{-6})})}\,]$$

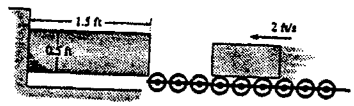
$$= [1 + \sqrt{1 + 188114.7\,h}\,]$$

$$\sigma_{max} = n\,\sigma_{st} \qquad \text{Here } \sigma_{st} = \frac{W}{A} = \frac{10\,(9.81)}{\frac{\pi}{4}(0.0075^2)} = 2.22053 \text{ MPa}$$

$$300\,(10^6) = [1 + \sqrt{1 + 188114.7\,h}\,]\,(2220530)$$

$h = 0.09559$ m $= 95.6$ mm **Ans**

***14-56.** A cylinder having the dimensions shown is made from magnesium Am 1004-T61. If it is struck by a rigid block having a weight of 800 lb and traveling at 2 ft/s, determine the maximum stress in the cylinder. Neglect the mass of the cylinder.



Conservation of Energy : The equivalent spring constant for the post is $k = \frac{AE}{L} = \frac{\frac{\pi}{4}(6^2)[6.48(10^6)]}{1.5(12)} = 10.1788(10^6)$ lb/in..

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k\Delta_{max}^2$$

$$\left[\frac{1}{2}\left(\frac{800}{32.2}\right)(2^2)\right](12) = \frac{1}{2}\left[10.1788(10^6)\right]\Delta_{max}^2$$

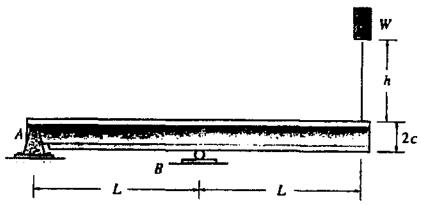
$$\Delta_{max} = 0.01082 \text{ in.}$$

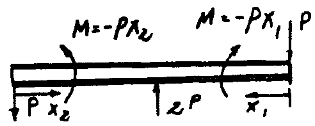
Maximum Stress : The maximum axial force is $P_{max} = k\Delta_{max} = 10.1788(10^6)(0.01082) = 110175.5$ lb.

$$\sigma_{max} = \frac{P_{max}}{A} = \frac{110175.5}{\frac{\pi}{4}(6^2)} = 3897 \text{ psi} = 3.90 \text{ ksi} \qquad \textbf{Ans}$$

Since $\sigma_{max} < \sigma_Y = 22$ ksi, the above analysis is valid.

14-57 The wide-flange beam has a length of $2L$, a depth $2c$, and a constant EI. Determine the maximum height h at which a weight W can be dropped on its end without exceeding a maximum elastic stress σ_{max} in the beam.





$$\frac{1}{2}P\Delta_C = 2\left(\frac{1}{2EI}\right)\int_0^L(-Px)^2dx$$

$$\Delta_C = \frac{2PL^3}{3EI}$$

$$\Delta_{st} = \frac{2WL^3}{3EI}$$

$$n = 1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}$$

$$\sigma_{max} = n(\sigma_{st})_{max} \qquad (\sigma_{st})_{max} = \frac{WLc}{I}$$

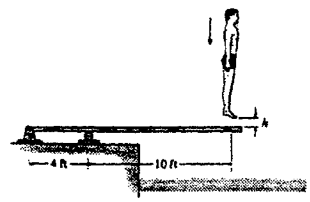
$$\sigma_{max} = \left[1+\sqrt{1+2\left(\frac{h}{\Delta_{st}}\right)}\,\right]\frac{WLc}{I}$$

$$\left(\frac{\sigma_{max}I}{WLc}-1\right)^2 = 1+\frac{2h}{\Delta_{st}}$$

$$h = \frac{\Delta_{st}}{2}\left[\left(\frac{\sigma_{max}I}{WLc}-1\right)^2-1\right]$$

$$= \frac{WL^3}{3EI}\left[\left(\frac{\sigma_{max}I}{WLc}\right)^2-\frac{2\sigma_{max}I}{WLc}\right] = \frac{\sigma_{max}L^2}{3Ec}\left[\frac{\sigma_{max}I}{WLc}-2\right] \qquad \textbf{Ans}$$

14-58. The diver weighs 150 lb and, while holding himself rigid, strikes the end of a wooden diving board ($h = 0$) with a downward velocity of 2 ft/s. Determine the maximum bending stress developed in the board. The board has a thickness of 1.5 in. and width of 1.5 ft. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



Static Displacment : The static displacement at the end of the diving board can be determined using the conservation of energy.

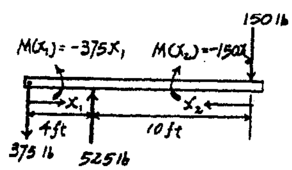
$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}(150)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ft}}(-375x_1)^2 dx_1 + \int_0^{10\text{ft}}(-150x_2)dx_2\right]$$

$$\Delta_{st} = \frac{70.0(10^3)\text{ lb}\cdot\text{ft}^3}{EI}$$

$$= \frac{70.0(10^3)(12^3)}{1.8(10^6)\left[\frac{1}{12}(18)(1.5^3)\right]}$$

$$= 13.274 \text{ in.}$$



Conservation of Energy : The equivalent spring constant for the board is $k = \frac{W}{\Delta_{st}} = \frac{150}{13.274} = 11.30$ lb/in..

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W\Delta_{max} = \frac{1}{2}k\Delta^2_{max}$$

$$\left[\frac{1}{2}\left(\frac{150}{32.2}\right)(4^2)\right](12) + 150\Delta_{max} = \frac{1}{2}(11.30)\Delta^2_{max}$$

Solving for the positive root, we have

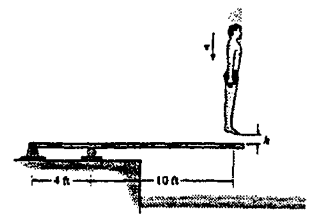
$$\Delta_{max} = 29.2538 \text{ in.}$$

Maximum Stress : The maximum force on the beam is $P_{max} = k\Delta_{max}$ $= 11.30(29.2538) = 330.57$ lb. The maximum moment occurs at the middle support. $M_{max} = 330.57(10)(12) = 39668.90$ lb · in.

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{39668.90(0.75)}{\frac{1}{12}(18)(1.5^3)} = 5877 \text{ psi} = 5.88 \text{ ksi} \qquad \textbf{Ans}$$

Note : The result will be somewhat inaccurate since the static displacement is so large.

14-59. The diver weighs 150 lb and, while holding himself rigid, strikes the end of the wooden diving board. Determine the maximum height h from which he can jump onto the board so that the maximum bending stress in the wood does not exceed 6 ksi. The board has a thickness of 1.5 in. and width of 1.5 ft. $E_w = 1.8(10^3)$ ksi.



Static Displacment : The static displacement at the end of the diving board can be determined using the conservation of energy.

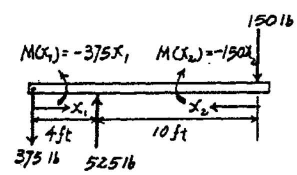
$$\frac{1}{2}P\Delta = \int_0^L \frac{M^2 dx}{2EI}$$

$$\frac{1}{2}(150)\Delta_{st} = \frac{1}{2EI}\left[\int_0^{4\text{ft}}(-375x_1)^2 dx_1 + \int_0^{10\text{ft}}(-150x_2)dx_2\right]$$

$$\Delta_{st} = \frac{70.0(10^3)\ \text{lb}\cdot\text{ft}^3}{EI}$$

$$= \frac{70.0(10^3)(12^3)}{1.8(10^6)\left[\frac{1}{12}(18)(1.5^3)\right]}$$

$$= 13.274\ \text{in.}$$



Maximum Stress : The maximum force on the beam is P_{max}. The maximum moment occurs at the middle support. $M_{max} = P_{max}(10)(12) = 120P_{max}$.

$$\sigma_{max} = \frac{M_{max}c}{I}$$

$$6(10^3) = \frac{120P_{max}(0.75)}{\frac{1}{12}(18)(1.5^3)}$$

$$P_{max} = 337.5\ \text{lb}$$

Conservation of Energy : The equivalent spring constant for the board is $k = \frac{W}{\Delta_{st}} = \frac{150}{13.274} = 11.30$ lb/in.. The maximum displacement at the end of the board is $\Delta_{max} = \frac{P_{max}}{k} = \frac{337.5}{11.30} = 29.867$ in.

$$U_e = U_i$$

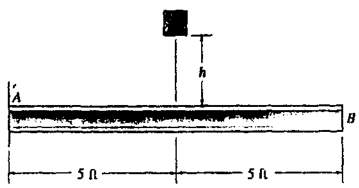
$$W(h + \Delta_{max}) = \frac{1}{2}k\Delta_{max}^2$$

$$150(h + 29.867) = \frac{1}{2}(11.30)(29.867^2)$$

$$h = 3.73\ \text{in.} \qquad \textbf{Ans}$$

Note : The result will be somewhat inaccurate since the static displacement is so large.

*14-60 A 40-lb weight is dropped from a height of $h = 2$ ft onto the center of the cantilevered A-36 steel beam. If the beam is a $W\,10 \times 15$, determine the maximum bending stress developed in the beam.



From Appendix C :

$$\Delta_{st} = \frac{P L^3}{3 E I} = \frac{40\,[5(12)\,]^3}{3\,(29)(10^6)(68.9)} = 1.44137\,(10^{-3})\text{ in.}$$

$$n = [1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\,] = [1 + \sqrt{1 + 2(\frac{2(12)}{1.44137\,(10^{-3})})}\,] = 183.49$$

$$\sigma_{st} = \frac{M c}{I}; \qquad \text{Here } M = 40(5)(12) = 2400 \text{ lb} \cdot \text{in.}$$

For $W\,10 \times 15$: $\quad I = 68.9 \text{ in}^4 \quad d = 9.99$ in.

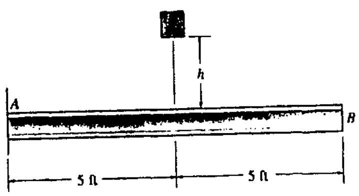
$$\sigma_{st} = \frac{2400\,(4.995)}{68.9}, \qquad c = \frac{9.99}{2} = 4.995 \text{ in.}$$

$$= 174.0 \text{ psi}$$

$$\sigma_{max} = n\,\sigma_{st} = 183.49\,(174.0)$$

$$= 31926 \text{ psi} = 31.9 \text{ ksi} < \sigma_Y = 36 \text{ ksi} \quad \text{OK} \qquad \text{Ans}$$

14-61 If the maximum allowable bending stress for the W 10 × 15 structural A-36 steel beam is $\sigma_{allow} = 20$ ksi, determine the maximum height h from which a 50-lb weight can be released from rest and strike the center of the beam.



From Appendix C :

$$\Delta_{st} = \frac{P L^3}{3 E I} = \frac{50\,[5(12)\,]^3}{3\,(29)(10^6)(68.9)} = 1.80171\,(10^{-3})\text{ in.}$$

$$\sigma_{st} = \frac{M c}{I}; \qquad \text{Here } M = 50(5)(12) = 3000\text{ lb}\cdot\text{in.}$$

For W 10×15 : $I = 68.9\text{ in}^4$ $d = 9.99$ in.

$$\sigma_{st} = \frac{3000\,(4.995)}{68.9}, \qquad c = \frac{9.99}{2} = 4.995\text{ in.}$$

$$= 217.49\text{ psi}$$

$$\sigma_{max} = n\,\sigma_{st}$$

$$20\,(10^3) = n(217.49); \qquad n = 91.96$$

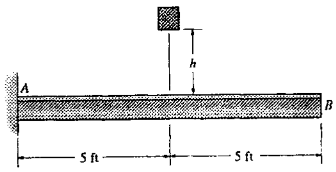
$$n = [1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\,]$$

$$91.96 = [1 + \sqrt{1 + 2(\frac{h}{1.80171\,(10^{-3})})}\,]$$

$$91.96 = [1 + \sqrt{1 + 1110.06\,h}\,]$$

$h = 7.45$ in. **Ans**

14-62 A 40-lb weight is dropped from a height of $h = 2$ ft onto the center of the cantilevered A-36 steel beam. If the beam is a $W\,10 \times 15$, determine the vertical displacement of its end B due to the impact.



From Appendix C :

$$\Delta_{st} = \frac{P L^3}{3 E I} = \frac{40\,[5(12)\,]^3}{3\,(29)(10^6)(68.9)} = 1.44137\,(10^{-3})\text{ in.}$$

$$n = [1 + \sqrt{1 + 2(\frac{h}{\Delta_{st}})}\,] = [1 + \sqrt{1 + 2(\frac{24}{1.44137\,(10^{-3})})}\,] = 183.49$$

From Appendix C :

$$\theta_{st} = \frac{P L^2}{2 E I} = \frac{40\,[5(12)\,]^2}{2\,(29)(10^6)(68.9)} = 36.034\,(10^{-6})\text{ rad}$$

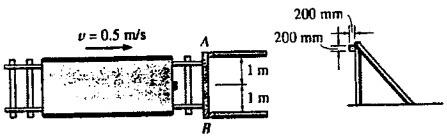
$$\theta_{max} = n\,\theta_{st} = 183.49\,[36.034\,(10^{-6})] = 6.612\,(10^{-3})\text{ rad}$$

$$\Delta_{max} = n\,\Delta_{st} = 183.49\,[1.44137\,(10^{-3})] = 0.26448\text{ in.}$$

$$(\Delta_B)_{max} = \Delta_{max} + \theta_{max} L = 0.26448 + 6.612\,(10^{-3})(5)(12)$$

$$= 0.661\text{ in.}$$ **Ans**

14-63 The steel beam AB acts to stop the oncoming railroad car, which has a mass of 10 Mg and is coasting towards it at $v = 0.5$ m/s. Determine the maximum stress developed in the beam if it is struck at its center by the car. The beam is simply supported and only horizontal forces occur at A and B. Assume that the railroad car and the supporting framework for the beam remains rigid. Also, compute the maximum deflection of the beam. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.



From Appendix C :

$$\Delta_{st} = \frac{PL^3}{48EI} = \frac{10(10^3)(9.81)(2^3)}{48(200)(10^9)(\frac{1}{12})(0.2)(0.2^3)} = 0.613125(10^{-3})\ \text{m}$$

$$k = \frac{W}{\Delta_{st}} = \frac{10(10^3)(9.81)}{0.613125(10^{-3})} = 160(10^6)\ \text{N/m}$$

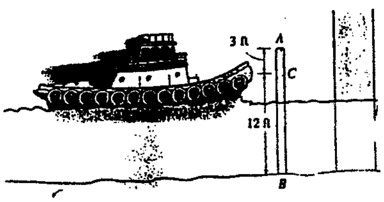
$$W' = k\Delta_{max} = 160(10^6)(3.953)(10^{-3}) = 632455.53\ \text{N}$$

$$M' = \frac{w'L}{4} = \frac{632455.53(2)}{4} = 316228\ \text{N}\cdot\text{m}$$

$$\sigma_{max} = \frac{M'c}{I} = \frac{316228(0.1)}{\frac{1}{12}(0.2)(0.2^3)} = 237\ \text{MPa} < \sigma_Y \qquad \text{OK} \qquad \textbf{Ans}$$

$$\Delta_{max} = \sqrt{\frac{\Delta_{st}v^2}{g}} = \sqrt{\frac{0.613125(10^{-3})(0.5^2)}{9.81}} = 3.953(10^{-3})\ \text{m} = 3.95\ \text{mm} \qquad \textbf{Ans}$$

***14-64.** The tugboat has a weight of 120 000 lb and is traveling forward at 2 ft/s when it strikes the 12-in.-diameter fender post AB used to protect a bridge pier. If the post is made from treated white spruce and is assumed fixed at the river bed, determine the maximum horizontal distance the top of the post will move due to the impact. Assume the tugboat is rigid and neglect the effect of the water.



From Appendix C :

$$P_{max} = \frac{3EI(\Delta_C)_{max}}{(L_{BC})^3}$$

Conservation of energy :

$$\frac{1}{2}mv^2 = \frac{1}{2}P_{max}(\Delta_C)_{max}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{3EI(\Delta_C)^2_{max}}{(L_{BC})^3}\right)$$

$$(\Delta_C)_{max} = \sqrt{\frac{mv^2 L^3_{BC}}{3EI}}$$

$$(\Delta_C)_{max} = \sqrt{\frac{(120\,000/32.2)(2)^2(12)^3}{(3)(1.40)(10^6)(144)(\frac{\pi}{4})(0.5)^4}} = 0.9315 \text{ ft} = 11.177 \text{ in.}$$

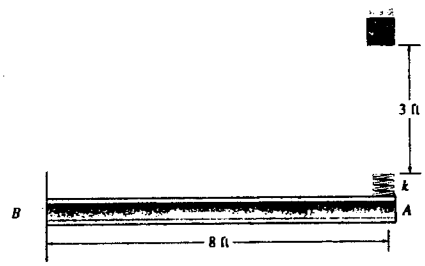
$$P_{max} = \frac{3[1.40(10^6)](\frac{\pi}{4})(6)^4(11.177)}{(144)^3} = 16.00 \text{ kip}$$

$$\theta_C = \frac{P_{max}L^2_{BC}}{2EI} = \frac{16.00(10^3)(144)^2}{2(1.40)(10^6)(\frac{\pi}{4})(6)^4} = 0.11644 \text{ rad}$$

$$(\Delta_A)_{max} = (\Delta_C)_{max} + \theta_C(L_{CA})$$

$$(\Delta_A)_{max} = 11.177 + 0.11644(36) = 15.4 \text{ in.} \quad \textbf{Ans}$$

14-65 The W 10 × 12 beam is made from A-36 steel and is cantilevered from the wall at B. The spring mounted on the beam has a stiffness of k = 1000 lb/in. If a weight of 8 lb is dropped onto the spring from a height of 3 ft, determine the maximum bending stress developed in the beam.



For W 10×12 : $I = 53.8\text{ in}^4$ $d = 9.87$ in.

From Appendix C :

$$\Delta_{beam} = \frac{PL^3}{3EI}$$

$$k_{beam} = \frac{3EI}{L^3} = \frac{3(29)(10^3)(53.8)}{[8(12)]^3} = 5.2904 \text{ kip/in.}$$

Equilibrium (equivalent system) :

$$F_{sp} = F_{beam}$$

$$k_{sp}\Delta_{sp} = k_{beam}\Delta_{beam}$$

$$\Delta_{sp} = \frac{5.2904(10^3)}{1000}\Delta_{beam}$$

$$\Delta_{sp} = 5.2904\,\Delta_{beam} \qquad (1)$$

Conservation of energy :

$$U_e = U_i$$

$$W(h + \Delta_{sp} + \Delta_{beam}) = \frac{1}{2}k_{beam}\,\Delta_{beam}^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2$$

From Eq. (1)

$$8[(3)(12) + 5.2904\,\Delta_{beam} + \Delta_{beam}]$$
$$= \frac{1}{2}(5.2904)(10^3)\Delta_{beam}^2 + \frac{1}{2}(1000)(5.2904\,\Delta_{beam})^2$$

$$16639.37\,\Delta_{beam}^2 - 50.32\,\Delta_{beam} - 288 = 0$$

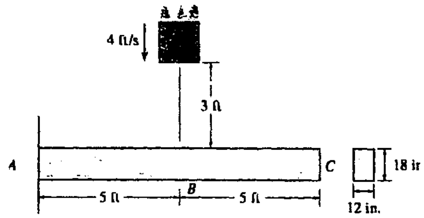
$$\Delta_{beam} = 0.13308 \text{ in.}$$

$$F_{beam} = k_{beam}\Delta_{beam} = 5.2904\,(0.13308) = 0.70406 \text{ kip}$$

$$M_{max} = 0.70406(8)(12) = 67.59 \text{ kip}\cdot\text{in.}$$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{67.59\,(\frac{9.87}{2})}{53.8}$$
$$= 6.20 \text{ ksi} < \sigma_Y \qquad \text{OK} \qquad \textbf{Ans}$$

14-66 The 200-lb block has a downward velocity of 4 ft/s when it is 3 ft from the top of the wooden beam. Determine the maximum stress in the beam due to the impact and compute the maximum deflection of its end C. $E_w = 1.9(10^3)$ ksi, $\sigma_Y = 6$ ksi.



From Appendix C :

$$\Delta_{st} = \frac{PL^3}{3EI}$$

$$k = \frac{3EI}{L^3} = \frac{3(1.9)(10^3)(\frac{1}{12})(12)(18^3)}{(5(12))^3} = 153.9 \text{ kip/in.}$$

Conservation of energy :

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{max}) = \frac{1}{12}k\Delta_{max}^2$$

$$\frac{1}{2}(\frac{200}{32.2(12)})(4(12))^2 + 200(3(12)) + \Delta_{max}) = \frac{1}{2}(153.9)(10^3)\Delta_{max}^2$$

$$7796.27 + 200\Delta_{max} = 76950\Delta_{max}^2$$

$$\Delta_{max} = 0.31960 \text{ in.}$$

$$W' = k\Delta_{max} = 153.9(0.31960) = 49.187 \text{ kip}$$

$$M' = 49.187(5)(12) = 2951.22 \text{ kip} \cdot \text{in.}$$

$$\sigma_{max} = \frac{M'c}{I} = \frac{2951.22(9)}{\frac{1}{12}(12)(18^3)} = 4.55 \text{ ksi} < \sigma_Y \quad \text{OK} \quad \textbf{Ans}$$

From Appendix C :

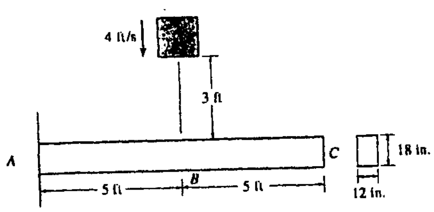
$$\theta_B = \frac{W'L^2}{2EI} = \frac{49.187(5(12))^2}{2(1.9)(10^3)(\frac{1}{12})(12)(18^3)} = 7.990(10^{-3}) \text{ rad}$$

$$\Delta_B = \Delta_{max} = 0.31960 \text{ in.}$$

$$\Delta_C = \Delta_B + \theta_B(5)(12)$$

$$= 0.31960 + 7.990(10^{-3})(60) = 0.799 \text{ in.} \quad \textbf{Ans}$$

14–67 The 100-lb block has a downward velocity of 4 ft/s when it is 3 ft from the top of the wooden beam. Determine the maximum stress in the beam due to the impact and compute the maximum deflection of point B. $E_w = 1.9(10^3)$ ksi, $\sigma_Y = 8$ ksi.



From Appendix C :

$$\Delta_{st} = \frac{PL^3}{3EI}$$

$$k = \frac{3EI}{L^3} = \frac{3(1.9)(10^3)(\frac{1}{12})(12)(18^3)}{(5(12))^3} = 153.9 \text{ kip/in.}$$

Conservation of energy :

$$U_e = U_i$$

$$\frac{1}{2}mv^2 + W(h + \Delta_{max}) = \frac{1}{12}k\Delta_{max}^2$$

$$\frac{1}{2}(\frac{100}{32.2(12)})(4(12))^2 + 100(3(12)) + \Delta_{max}) = \frac{1}{2}(153.9)(10^3)\Delta_{max}^2$$

$$3898.14 + 100\Delta_{max} = 76950\Delta_{max}^2$$

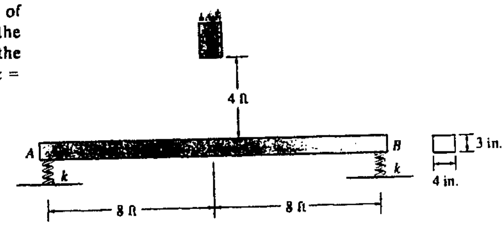
$$\Delta_{max} = 0.2257 \text{ in.} = 0.226 \text{ in.}$$ **Ans**

$$W' = k\Delta_{max} = 153.9(0.2257) = 34.739 \text{ kip}$$

$$M' = 34.739(5)(12) = 2084.33 \text{ kip} \cdot \text{in.}$$

$$\sigma_{max} = \frac{M'c}{I} = \frac{2084.33(9)}{\frac{1}{12}(12)(18^3)} = 3.22 \text{ ksi} < \sigma_Y$$ OK **Ans**

•14–68 The weight of 175 lb is dropped from a height of 4 ft from the top of the A-36 steel beam. Determine the maximum deflection and maximum stress in the beam if the supporting springs at A and B each have a stiffness of k = 500 lb/in. The beam is 3 in. thick and 4 in. wide.

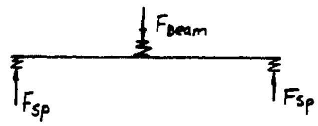


From Appendix C :

$$\Delta_{beam} = \frac{PL^3}{48EI}$$

$$k_{beam} = \frac{48EI}{L^3} = \frac{48(29)(10^3)(\frac{1}{12})(4)(3^3)}{(16(12))^3} = 1.7700 \text{ kip/in.}$$

From equilibrium (equivalent system) :



$$2F_{sp} = F_{beam}$$

$$2k_{sp}\Delta_{sp} = k_{beam}\Delta_{beam}$$

$$\Delta_{sp} = \frac{1.7700(10^3)}{2(500)}\Delta_{beam}$$

$$\Delta_{sp} = 1.7700\Delta_{beam} \qquad (1)$$

Conservation of energy :

$$U_e = U_i$$

$$W(h + \Delta_{sp} + \Delta_{beam}) = \frac{1}{2}k_{beam}\Delta_{beam}^2 + 2(\frac{1}{2})k_{sp}\Delta_{sp}^2$$

From Eq. (1) :

$$175[(4)(12) + 1.770\Delta_{beam} + \Delta_{beam}] = \frac{1}{2}(1.7700)(10^3)\Delta_{beam}^2 + 500(1.7700\Delta_{beam})^2$$

$$2451.5\Delta_{beam}^2 - 484.75\Delta_{beam} - 8400 = 0$$

$$\Delta_{beam} = 1.9526 \text{ in.}$$

From Eq. (1) :

$$\Delta_{sp} = 3.4561 \text{ in.}$$

$$\Delta_{max} = \Delta_{sp} + \Delta_{beam}$$

$$= 3.4561 + 1.9526 = 5.41 \text{ in.} \qquad \textbf{Ans}$$

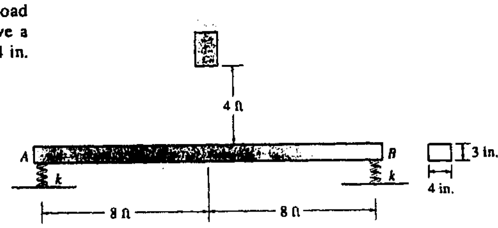
$$F_{beam} = k_{beam}\Delta_{beam}$$

$$= 1.7700(1.9526) = 3.4561 \text{ kip}$$

$$M_{max} = \frac{F_{beam}L}{4} = \frac{3.4561(16)(12)}{4} = 165.893 \text{ kip} \cdot \text{in.}$$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{165.893(1.5)}{\frac{1}{12}(4)(3^3)} = 27.6 \text{ ksi} < \sigma_Y \qquad \text{OK} \qquad \textbf{Ans}$$

14-69 The weight of 175 lb is dropped from a height of 4 ft from the top of the A-36 steel beam. Determine the load factor n if the supporting springs at A and B each have a stiffness of k = 300 lb/in. The beam is 3 in. thick and 4 in. wide.

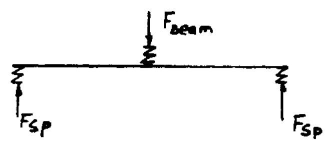


From Appendix C :

$$\Delta_{beam} = \frac{PL^3}{48EI}$$

$$k_{beam} = \frac{48EI}{L^3} = \frac{48(29)(10^3)(\frac{1}{12})(4)(3^3)}{(16(12))^3} = 1.7700 \text{ kip/in.}$$

From equilibrium (equivalent system) :



$$2F_{sp} = F_{beam}$$

$$2F_{sp}\Delta_{sp} = k_{beam}\Delta_{beam}$$

$$\Delta_{sp} = \frac{1.7700(10^3)}{2(300)}\Delta_{beam}$$

$$\Delta_{sp} = 2.95\Delta_{beam} \qquad (1)$$

Conservation of energy :

$$U_e = U_i$$

$$W(h + \Delta_{beam} + \Delta_{sp}) = \frac{1}{2}k_{beam}\Delta_{beam}^2 + 2(\frac{1}{2})k_{sp}\,\Delta_{sp}^2$$

From Eq. (1) :

$$175[(4)(12) + \Delta_{beam} + 2.95\Delta_{beam}] = \frac{1}{2}(1.7700)(10^3)\Delta_{beam}^2 + 300(2.95\Delta_{beam})^2$$

$$3495.75\Delta_{beam}^2 - 691.25\Delta_{beam} - 8400 = 0$$

$$\Delta_{beam} = 1.6521 \text{ in.}$$

$$F_{beam} = k_{beam}\Delta_{beam}$$

$$= 1.7700(1.6521) = 2.924 \text{ kip}$$

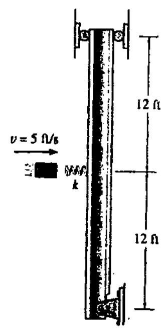
$$n = \frac{2.924(10^3)}{175} = 16.7 \qquad \textbf{Ans}$$

$$\sigma_{max} = n(\sigma_{st})_{max} = n(\frac{Mc}{I})$$

$$M = \frac{175(16)(12)}{4} = 8.40 \text{ kip} \cdot \text{in.}$$

$$\sigma_{max} = 16.7\,(\frac{8.40(1.5)}{\frac{1}{12}(4)(3^3)}) = 23.4 \text{ ksi} \; < \; \sigma_Y \qquad \text{OK}$$

14-70 The simply supported W 10 × 15 structural A-36 steel beam lies in the horizontal plane and acts as a shock absorber for the 500-lb block which is traveling toward it at 5 ft/s. Determine the maximum deflection of the beam and the maximum stress in the beam during the impact. The spring has a stiffness of $k = 1000$ lb/in.



For $W\,10\times15$: $\quad I = 68.9\ \text{in}^4 \qquad d = 9.99$ in.

From Appendix C :

$$\Delta_{beam} = \frac{PL^3}{48EI}$$

$$k_{beam} = \frac{48EI}{L^3} = \frac{48(29)(10^3)(68.9)}{(24(12))^3} = 4.015\ \text{kip/in.}$$

Equilibrium (equivalent system) :

$$F_{sp} = F_{beam}$$

$$k_{sp}\Delta_{sp} = k_{beam}\Delta_{beam}$$

$$\Delta_{sp} = \frac{4.015(10^3)}{1000}\Delta_{beam}$$

$$\Delta_{sp} = 4.015\Delta_{beam} \qquad (1)$$

Conservation of energy :

$$U_e = U_i$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k_{beam}\Delta_{beam}^2 + \frac{1}{2}k_{sp}\Delta_{sp}^2$$

From Eq. (1) :

$$\frac{1}{2}\left(\frac{500}{32.2(12)}\right)(5(12))^2 = \frac{1}{2}(4.015)(10^3)\Delta_{beam}^2 + \frac{1}{2}(1000)(4.015\Delta_{beam}^2)^2$$

$$10067.6\Delta_{beam}^2 = 2329.2$$

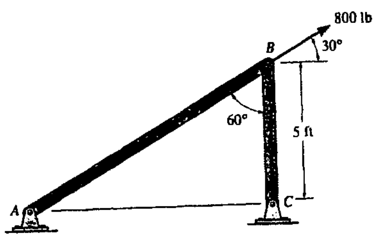
$$\Delta_{beam} = 0.481\ \text{in.} \qquad \textbf{Ans}$$

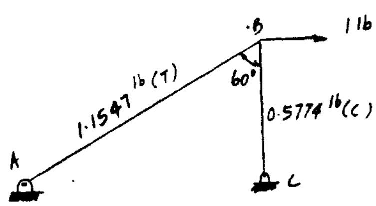
$$F_{beam} = k_{beam}\Delta_{beam}$$
$$= 4.015(0.481) = 1.931\ \text{kip}$$

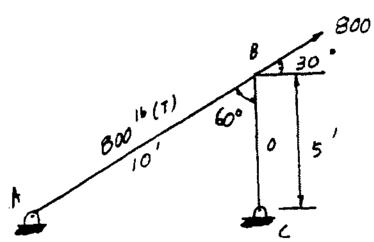
$$M_{max} = \left(\frac{1.931}{2}\right)(12)(12) = 139.05\ \text{kip}\cdot\text{in.}$$

$$\sigma_{max} = \frac{M_{max}c}{I} = \frac{139.05(\frac{9.99}{2})}{68.9} = 10.1\ \text{ksi} < \sigma_Y \quad \text{OK} \qquad \textbf{Ans}$$

14–71 Determine the horizontal displacement of point B on the two-member frame. Each A-36 steel member has a cross-sectional area of 2 in^2.





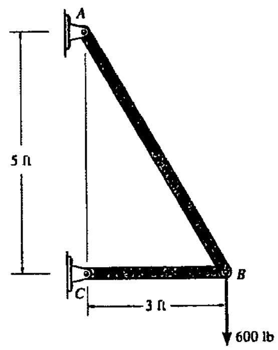


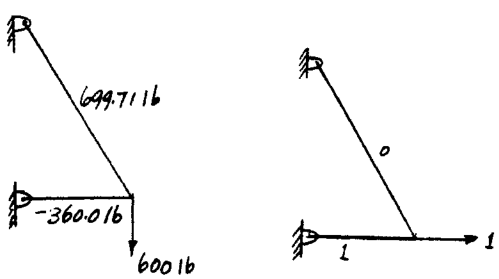
Member	n	N	L	nNL
AB	1.1547	800	120	11085.25
BC	−0.5774	0	60	0
				$\Sigma = 110\,851.25$

$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{110851.25}{AE} = \frac{110851.25}{29(10^6)(2)} = 0.00191 \text{ in.} \qquad \textbf{Ans}$$

•14–72 Determine the horizontal displacement of point *B*. Each A-36 steel member has a cross-sectional area of 2 in².



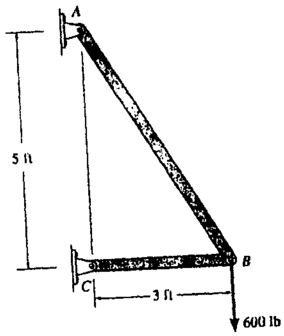


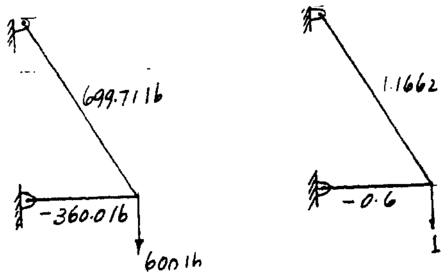
$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{1\,(-360)(3)(12)}{2\,(29)(10^6)} = -\,0.223\,(10^{-3})\text{ in.}$$

$$= 0.223(10^{-3})\text{ in.} \leftarrow$$ **Ans**

14–73 Determine the vertical displacement of point *B*. Each A-36 steel member has a cross-sectional area of 2 in².



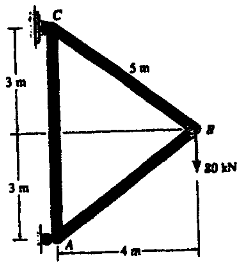


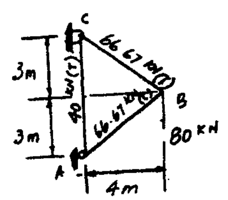
$$1 \cdot \Delta_{B_v} = \Sigma \frac{nNL}{AE}$$

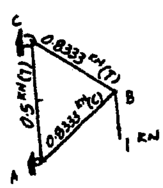
$$\Delta_{B_v} = \frac{1.1662\,(699.71)(5.831)(12)}{AE} + \frac{-0.60\,(-360)(3)(12)}{AE}$$

$$= \frac{64872.807}{2\,(29)(10^6)} = 0.00112 \text{ in.} \downarrow \qquad \textbf{Ans}$$

14-74. Determine the vertical displacement of joint *B* of the truss. Each A-36 steel member has a cross-sectional area of 300 mm^2.





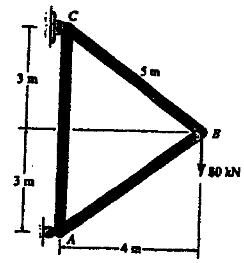


Member	n	N	L	nNL
AB	−0.8333	−66.67	5	277.78
BC	0.8333	66.67	5	277.78
AC	0.5	40	6	120.00
			Σ =	675.56

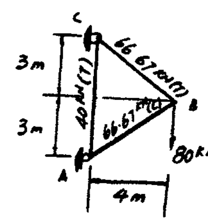
$$1 \cdot \Delta_{B_v} = \Sigma \frac{nNL}{AE}$$

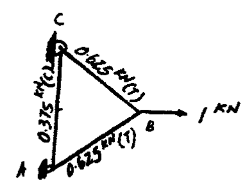
$$\Delta_{B_v} = \frac{675.56(10^3)}{300(10^{-6})(200)(10^9)} = 0.01126\text{ m} = 11.3\text{ mm} \qquad \textbf{Ans}$$

14-75. Determine the horizontal displacement of joint B of the truss. Each A-36 steel member has a cross-sectional area of 300 mm^2.



Member	n	N	L	nNL
AB	0.625	−66.67	5	−208.33
BC	0.625	66.67	5	208.33
AC	−0.375	40	6	−90.00
				$\Sigma = -90.00$

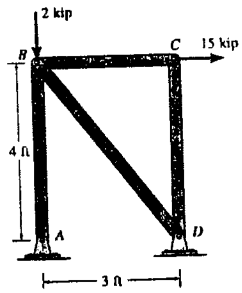


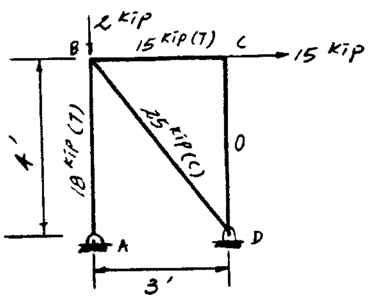


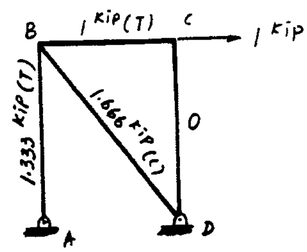
$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{-90(10^3)}{300(10^{-6})(200)(10^9)} = -1.50(10^{-3})\text{ m} = -1.50\text{ mm} = 1.50\text{ mm} \leftarrow \qquad \textbf{Ans}$$

*14–76 Determine the horizontal displacement of joint C on the truss. Each A-36 steel member has a cross-sectional area of 3 in^2.







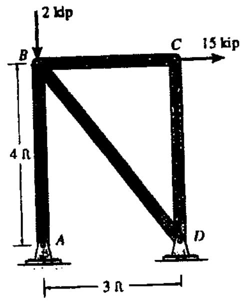
Member	n	N	L	nNL
AB	1.333	18.00	48	1152
BC	1.000	15.00	36	540
BD	−1.666	−25.00	60	2500
CD	0	0	48	0

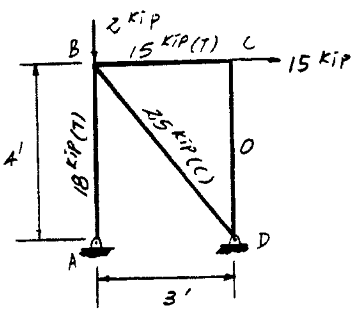
$\Sigma = 4192$

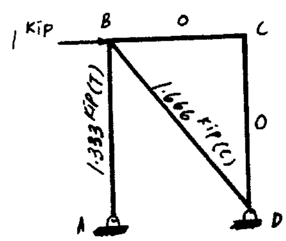
$$1 \cdot \Delta_{C_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{C_h} = \frac{4192}{(3)(29)(10^3)} = 0.0482 \text{ in.}$$ **Ans**

14–77 Determine the horizontal displacement of joint B on the truss. Each A-36 steel member has a cross-sectional area of 3 in^2.







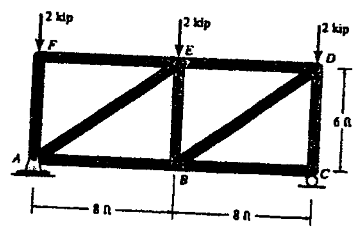
Member	n	N	L (in.)	nNL
AB	1.333	18.00	48	1152
BC	0	15.00	36	0
BD	−1.666	−25.00	60	2500
CD	0	0	48	0

$\Sigma = 3652$

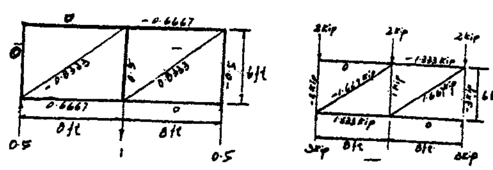
$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{3652}{(3)(29)(10^3)} = 0.0420 \text{ in.} \qquad \textbf{Ans}$$

14-78. Determine the vertical displacement of joint B. For each A-36 steel member $A = 1.5\ \text{in}^2$.



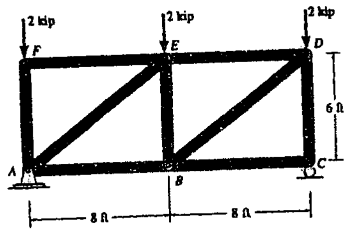
$$1 \cdot \Delta_{B_v} = \Sigma \frac{nNL}{AE}$$

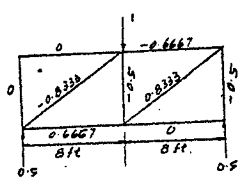


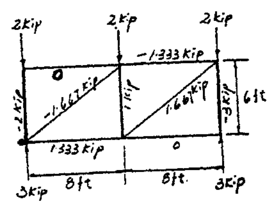
$$\Delta_{B_v} = \frac{1}{AE}\{(-1.667)(-0.8333)(10) + (1.667)(0.8333)(10) + (0.6667)(1.333)(8) + (-0.6667)(-1.333)(8) + (-1)(0.5)(6) + (-0.5)(-3)(6)\}(12)$$

$$= \frac{576}{1.5\,(29)(10^3)} = 0.0132\ \text{in.} \qquad \textbf{Ans}$$

14-79. Determine the vertical displacement of joint E. For each A-36 steel member $A = 1.5\ \text{in}^2$.





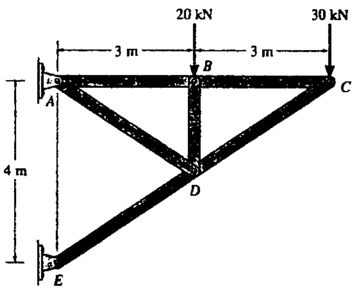


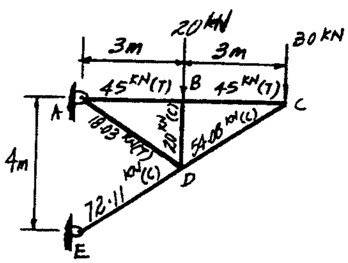
$$1 \cdot \Delta_{E_v} = \Sigma \frac{nNL}{AE}$$

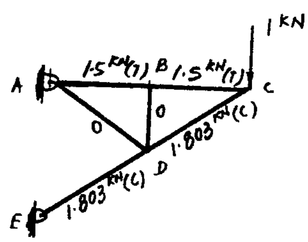
$$\Delta_{E_v} = \frac{1}{AE}[(-1.667)(-0.833)(10) + (1.667)(0.8333)(10) + (0.667)(1.33)(8) + (-0.667)(-1.33)(8) + (-1)(-0.5)(6) + (-0.5)(-3)(6)](12)$$

$$= \frac{648}{1.5\,(29)(10^3)} = 0.0149\ \text{in.} \qquad \textbf{Ans}$$

•14–80 Determine the vertical displacement of joint C on the truss. Each A-36 steel member has a cross-sectional area of $A = 300\ \text{mm}^2$.







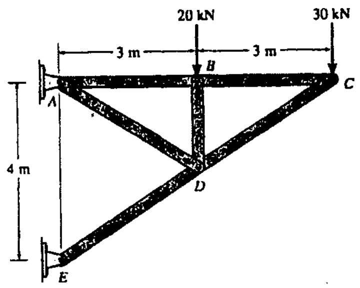
Member	n	N	L	nNL
AB	1.50	45.0	3	202.5
AD	0	18.03	$\sqrt{13}$	0
BC	1.50	45.0	3	202.5
BD	0	−20.0	2	0
CD	−1.803	−54.08	$\sqrt{13}$	351.56
DE	−1.803	−72.11	$\sqrt{13}$	468.77

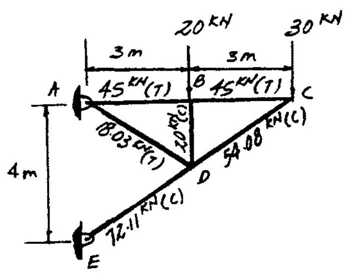
$\Sigma = 1225.33$

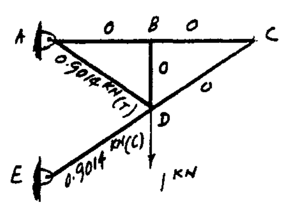
$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{C_v} = \frac{1225.33(10^3)}{300(10^{-6})(200)(10^9)} = 0.0204\ \text{m} = 20.4\ \text{mm} \qquad \textbf{Ans}$$

14-81 Determine the vertical displacement of joint D on the truss. Each A-36 steel member has a cross-sectional area of $A = 300\ \text{mm}^2$.







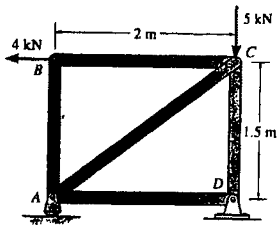
Member	n	N	L	nNL
AB	0	45.0	3	0
AD	0.9014	18.03	$\sqrt{13}$	58.60
BC	0	45.0	3	0
BD	0	−20.0	2	0
CD	0	−54.08	$\sqrt{13}$	0
DE	−0.9014	−72.11	$\sqrt{13}$	234.36

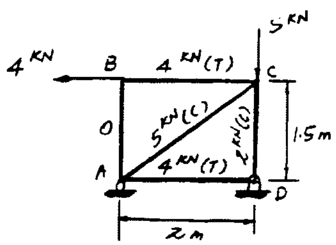
$\Sigma = 292.96$

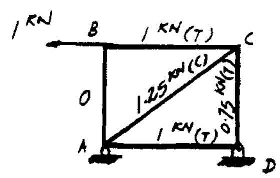
$$1 \cdot \Delta_{D_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{D_v} = \frac{292.96(10^3)}{300(10^{-6})(200)(10^9)} = 4.88(10^{-3})\ \text{m} = 4.88\ \text{mm} \qquad \textbf{Ans}$$

14–82 Determine the horizontal displacement of joint B of the truss. Each A-36 steel member has a cross-sectional area of 400 mm².







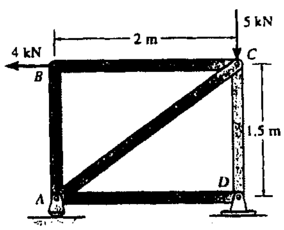
Member	n	N	L	nNL
AB	0	0	1.5	0
AC	−1.25	−5.00	2.5	15.625
AD	1.00	4.00	2.0	8.000
BC	1.00	4.00	2.0	8.000
CD	0.75	−2.00	1.5	−2.25

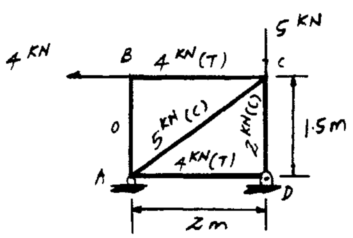
$\Sigma = 29.375$

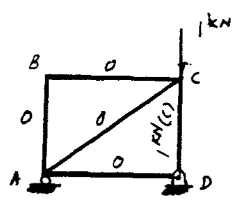
$$1 \cdot \Delta_{B_h} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_h} = \frac{29.375(10^3)}{400(10^{-6})(200)(10^9)} = 0.3672(10^{-3})\text{m} = 0.367 \text{ mm} \qquad \textbf{Ans}$$

14–83 Determine the vertical displacement of joint C of the truss. Each A-36 steel member has a cross-sectional area of $400\ mm^2$.







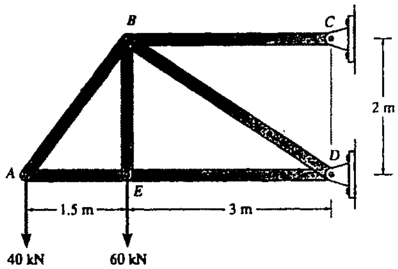
Member	n	N	L	nNL
AB	0	0	1.5	0
AC	0	−5.00	2.5	0
AD	0	4.00	2.0	0
BC	0	4.00	2.0	0
CD	−1.00	−2.00	1.5	3.00

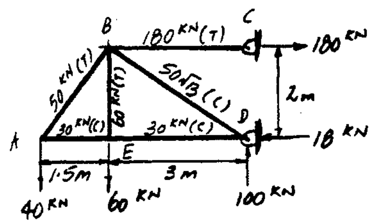
$\Sigma = 3.00$

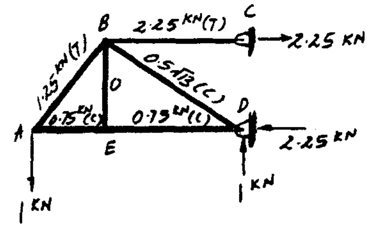
$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{C_v} = \frac{3.00\,(10^3)}{400(10^{-6})(200)(10^9)} = 37.5(10^{-6})\text{m} = 0.0375\ \text{mm} \qquad \textbf{Ans}$$

•14–84 Determine the vertical displacement of joint A. Each A-36 steel member has a cross-sectional area of $400\ \text{mm}^2$.







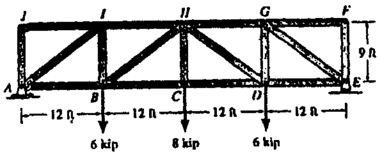
Member	n	N	L	nNL
AB	1.25	50	2.5	156.25
AE	−0.75	−30	1.5	33.75
BC	2.25	180	3.0	1215.00
BD	$-0.5\sqrt{13}$	$-50\sqrt{13}$	$\sqrt{13}$	1171.80
BE	0	60	2.0	0
DE	−0.75	−30	3.0	67.5

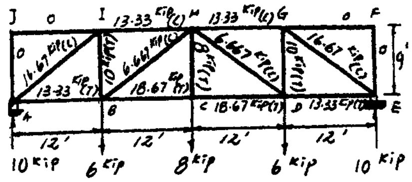
$\Sigma = 2644.30$

$$1 \cdot \Delta_{A_v} = \Sigma \frac{nNL}{AE}$$

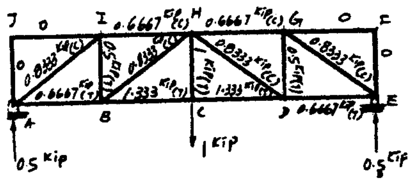
$$\Delta_{A_v} = \frac{2644.30(10^3)}{400(10^{-6})(200)(10^9)} = 0.0331\text{m} = 33.1\text{ mm} \qquad \textbf{Ans}$$

14-85. Determine the vertical displacement of joint C. Each A-36 steel member has a cross-sectional area of 4.5 in^2.





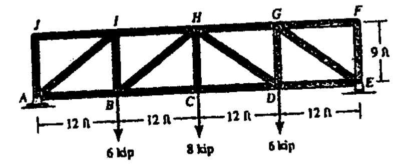
Member	n	N	L	nNL
AJ	0	0	108	0
AI	-16.67	-0.8333	180	2500
AB	13.33	0.6667	144	1280
BI	10.0	0.500	108	540
BH	-6.667	-0.8333	180	1000
BC	18.67	1.333	144	3584
CH	8.00	1.00	108	864
CD	18.67	1.333	144	3584
DH	-6.667	-0.8333	180	1000
DG	10.00	0.50	108	540
DE	13.33	0.6667	144	1280
EG	-16.67	-0.8333	180	2500
EF	0	0	108	0
FG	0	0	144	0
GH	-13.33	-0.6667	144	1280
HI	[illegible]	[illegible]	[illegible]	1280



$$1 \cdot \Delta_{C_v} = \Sigma \frac{nNL}{AE}$$

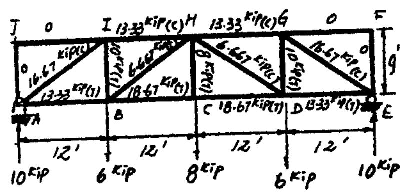
$$\Delta_{C_v} = \frac{21\,232}{4.5\,(29\,(10^3))} = 0.163 \text{ in.} \qquad \textbf{Ans}$$

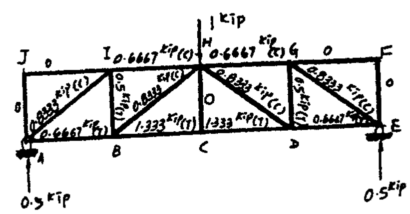
14-86. Determine the vertical displacement of joint H. Each A-36 steel member has a cross-sectional area of 4.5 in^2.



Member	n	N	L	nNL
AJ	0	0	108	0
AI	-16.67	-0.8333	180	2500
AB	13.33	0.6667	144	1280
BI	10.00	0.500	108	540
BH	-6.67	-0.8333	180	1000
BC	18.67	1.333	144	3584
CH	8.00	0	108	0
CD	18.67	1.333	144	3584
DH	-6.67	-0.8333	180	1000
DG	10.00	0.500	108	540
DE	13.33	0.6667	144	1280
EG	-16.67	-0.8333	180	2500
EF	0	0	108	0
FG	0	0	144	0
GH	-13.33	-0.6667	144	1280
HI	-13.33	-0.6667	144	1280
IJ	0	0	144	0

Σ 20368

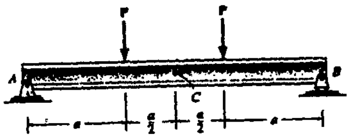


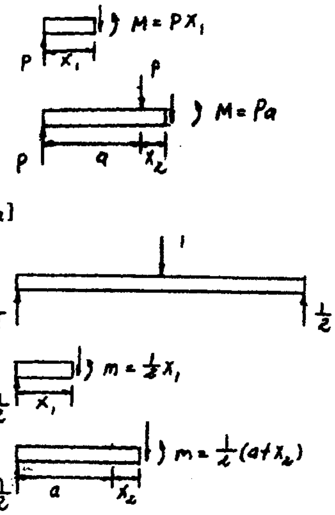


$$1 \cdot \Delta_{H_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{H_v} = \frac{20\,368}{4.5\,(29\,(10^3))} = 0.156 \text{ in.} \quad \textbf{Ans}$$

14-87. Determine the displacement at point C. EI is constant.



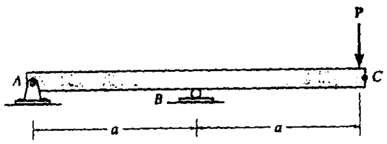


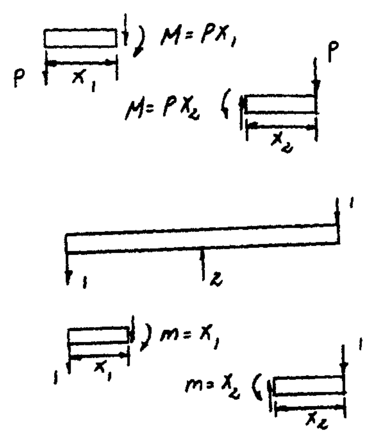
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = 2\left(\frac{1}{EI}\right)\left[\int_0^a \left(\frac{1}{2}x_1\right)(Px_1)dx_1 + \int_0^{a/2} \frac{1}{2}(a+x_2)(Pa)dx_2\right]$$

$$= \frac{23Pa^3}{24EI} \quad \textbf{Ans}$$

*14-88 Determine the displacement at point C. EI is constant.



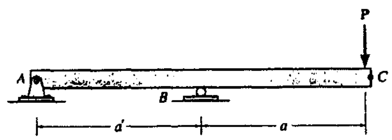


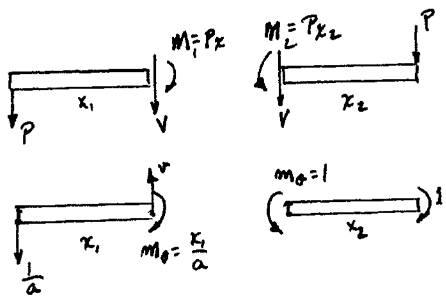
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}[\int_0^a (x_1)(Px_1)dx_1 + \int_0^a (x_2)(Px_2)dx_2$$

$$= \frac{2Pa^3}{3EI} \quad \textbf{Ans}$$

14-89 Determine the slope at point C. EI is constant.



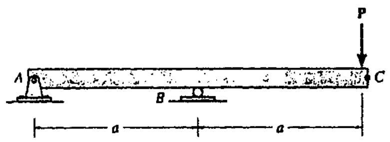


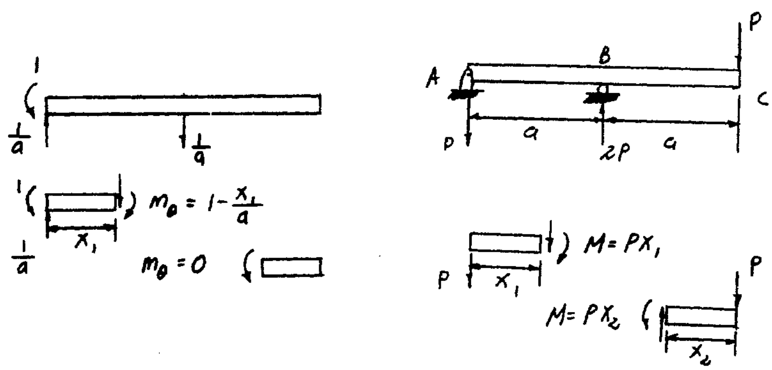
$$1 \cdot \theta_C = \int_0^L \frac{m_\theta M dx}{EI}$$

$$\theta_C = \int_0^a \frac{(\frac{x_1}{a}) P x_1 \, dx_1}{EI} + \int_0^a \frac{(1) P x_2 \, dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI} \qquad \textbf{Ans}$$

14–90 Determine the slope at point *A*. *EI* is constant.

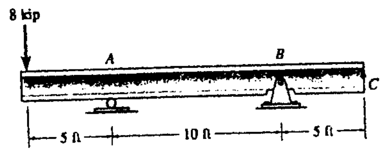


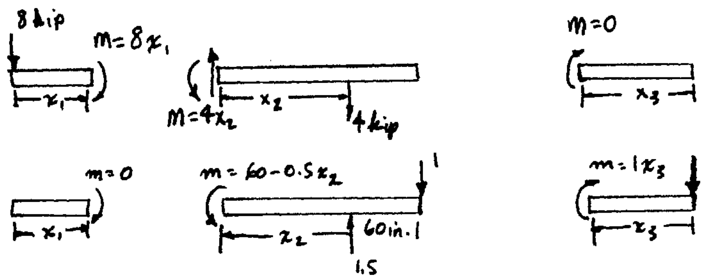


$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^a \left(1 - \frac{x_1}{a}\right)(Px_1)\,dx_1 + \int_0^a (0)(Px_2)\,dx_2\right] = \frac{Pa^2}{6EI} \qquad \textbf{Ans}$$

14–91 Determine the displacement of point C of the beam made from A-36 steel and having a moment of inertia of $I = 53.8\ \text{in}^4$



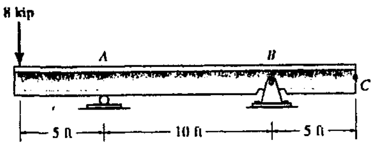


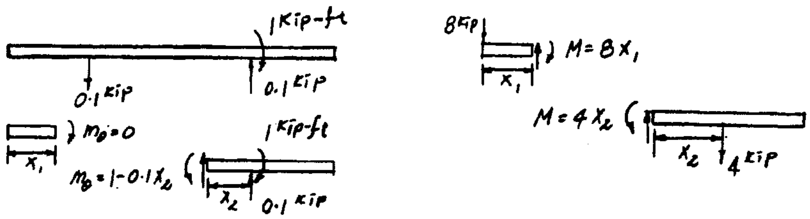
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_C = \frac{1}{EI}[0 + \int_0^{120} (60 - 0.5)(4x_2)\, dx_2 + 0]$$

$$= \frac{576\,000}{EI} = \frac{576\,000}{29(10^3)(53.8)} = 0.369\ \text{in.} \qquad \textbf{Ans}$$

•14–92 Determine the slope at B of the beam made from A-36 steel and having a moment of inertia of $I = 53.8\ \text{in}^4$.



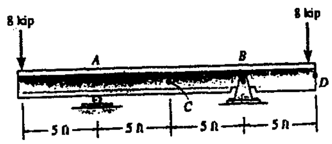


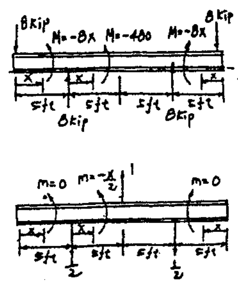
$$1\cdot\theta_B = \int_0^L \frac{m_\theta M}{EI}dx$$

$$\theta_B = \frac{1}{EI}\left[\int_0^5 (0)(8x_1)dx_1 + \int_0^{10}(1-0.1x_2)4x_2\,dx_2\right]$$

$$= \frac{66.67\ \text{kip}\cdot\text{ft}^2}{EI} = \frac{66.67(12^2)}{29(10^3)(53.8)} = 6.153(10^{-3})\text{rad} = 0.353^\circ \qquad \textbf{Ans}$$

14-93. Determine the displacement of point C of the $W14 \times 26$ beam made from A-36 steel.



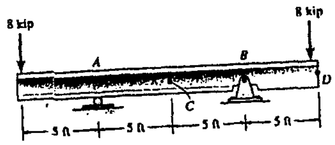


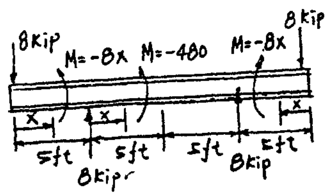
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

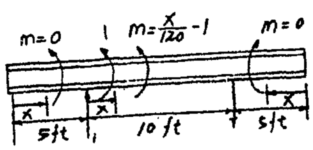
$$\Delta_C = 0 + 2\int_0^{60} \frac{(-\frac{x}{2})(-480)}{EI} dx$$

$$= \frac{864\,000}{29\,(10^3)(245)} = 0.122 \text{ in.} \quad \textbf{Ans}$$

14-94. Determine the slope at A of the $W14 \times 26$ beam made from A-36 steel.





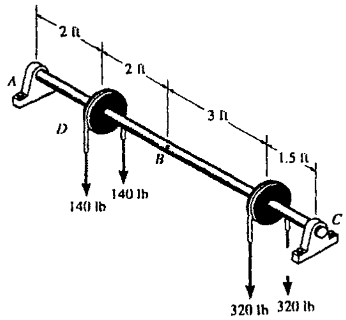


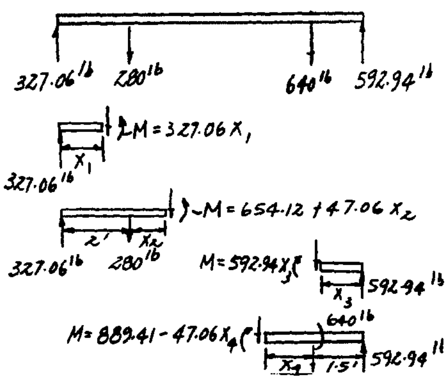
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

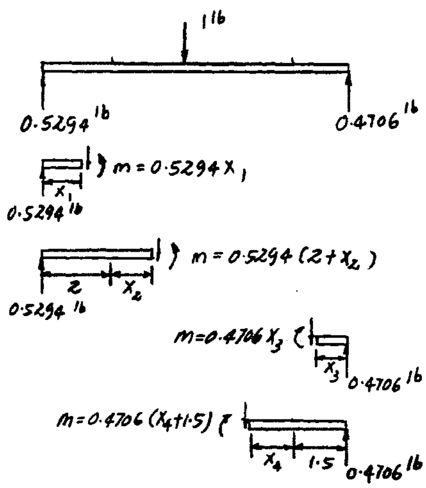
$$\theta_A = 0 + \int_0^{120} \frac{(\frac{x}{120} - 1)(-480)}{EI} dx$$

$$= \frac{28\,800}{29\,(10^3)(245)} = 4.05\,(10^{-3}) \text{ rad} \qquad \textbf{Ans}$$

14–95 Determine the displacement at *B* of the 1.5-in-diameter A-36 steel shaft.







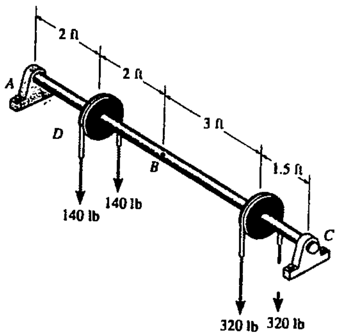
$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}dx$$

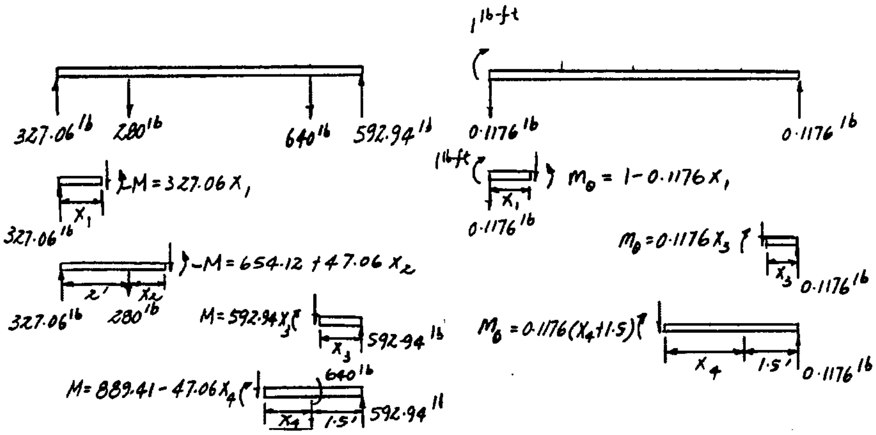
$$\Delta_B = \frac{1}{EI}[\int_0^2 (0.5294x_1)(327.06x_1)dx_1 + \int_0^2 0.5294(2+x_1)(654.12 + 47.06x_2)dx_2 +$$

$$\int_0^{1.5} (0.4706x_3)(592.94x_3)dx_3 + \int_0^3 0.4706(x_4 + 1.5)(889.41 - 47.06x_4)dx_4]$$

$$= \frac{6437.67 \text{ lb} \cdot \text{ft}^3}{EI} = \frac{6437.67(12^3)}{29(10^6)\frac{\pi}{4}(0.75)^4} = 1.54 \text{ in.} \qquad \textbf{Ans}$$

•14–96 Determine the slope of the 1.5-in-diameter A-36 steel shaft at the bearing support *A*.



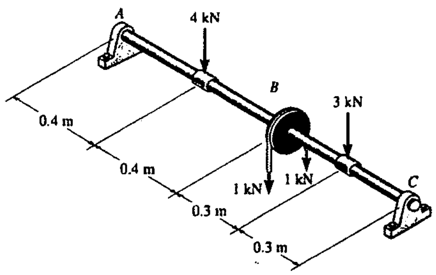


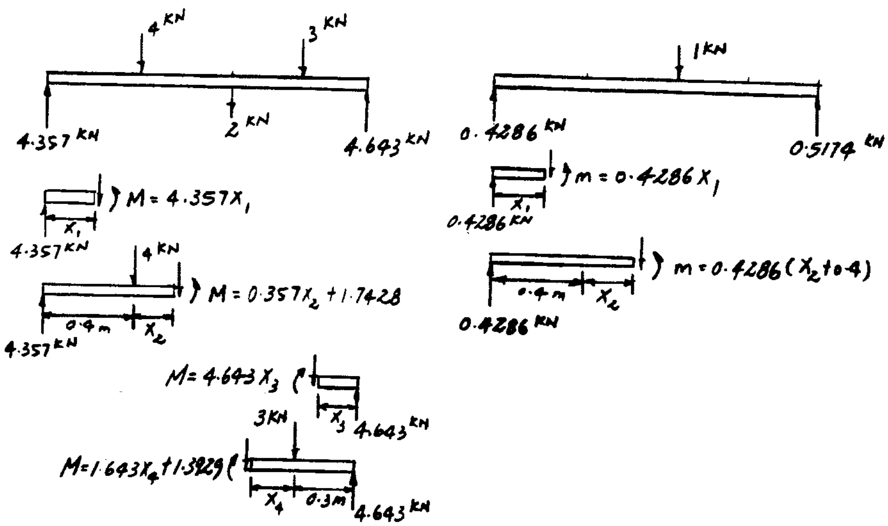
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI}\,dx$$

$$\theta_A = \frac{1}{EI}\Big[\int_0^2 (1 - 0.1176x_1)(327.06x_1)\,dx_1 + \int_0^{1.5} (0.1176x_3)(592.94x_3)\,dx_3 + \int_0^5 0.1176(x_4 + 1.5)(889.41 - 47.06x_4)\,dx_4\Big]$$

$$= \frac{2387.53 \text{ lb}\cdot\text{ft}^2}{EI} = \frac{2387.53(12^2)}{29(10^6)(\frac{\pi}{4})(0.75^4)} = 0.0477 \text{ rad} = 2.73^\circ \qquad \textbf{Ans}$$

14-97 Determine the displacement at pulley *B*. The A-36 steel shaft has a diameter of 30 mm.



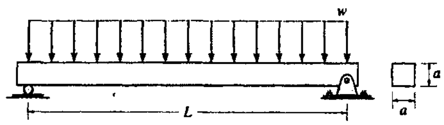


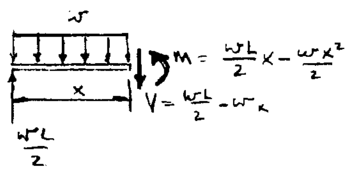
$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI}dx$$

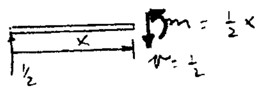
$$\Delta_B = \frac{1}{EI}\left[\int_0^{0.4}(0.4286x_1)(4.357x_1)dx_1 + \int_0^{0.4} 0.4286(x_2 + 0.4)(0.357x_2 + 1.7428)dx_2\right.$$
$$\left. + \int_0^{0.3}(0.5714x_3)(4.643x_3)dx_3 + \int_0^{0.3} 0.5714(x_4 + 0.3)(1.643x_4 + 1.3929)dx_4\right]$$

$$= \frac{0.37972\ \text{kN}\cdot\text{m}^3}{EI} = \frac{0.37972(10^3)}{200(10^9)(\frac{\pi}{4})(0.015^4)} = 0.0478\text{m} = 47.8\ \text{mm} \qquad \textbf{Ans}$$

14–98 The simply supported beam having a square cross section is subjected to a uniform load w. Determine the maximum deflection of the beam caused only by bending, and caused by bending and shear. Take $E = 3G$.







For bending and shear,

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}dx + \int_0^L \frac{f_s vV}{GA}dx$$

$$\Delta = 2\int_0^{L/2} \frac{(\frac{1}{2}x)(\frac{wL}{2}x - w\frac{x^2}{2})dx}{EI} + 2\int_0^{L/2} \frac{(\frac{6}{5})(\frac{1}{2})(\frac{wL}{2} - wx)dx}{GA}$$

$$= \frac{1}{EI}(\frac{wL}{6}x^3 - \frac{wx^4}{8})\Big|_0^{L/2} + \frac{(\frac{6}{5})}{GA}(\frac{wL}{2}x - \frac{wx^2}{2})\Big|_0^{L/2}$$

$$= \frac{5wL^4}{384EI} + \frac{3wL^2}{20\,GA}$$

$$\Delta = \frac{5wL^4}{384(3G)(\frac{1}{12})a^4} + \frac{3wL^2}{20(G)a^2}$$

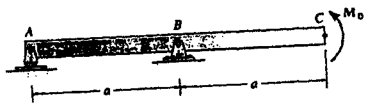
$$= \frac{20wL^4}{384Ga^4} + \frac{3wL^2}{20Ga^2}$$

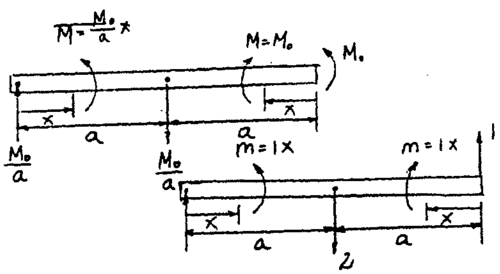
$$= \left(\frac{w}{G}\right)\left(\frac{L}{a}\right)^2\left[\left(\frac{20}{384}\right)\left(\frac{L}{a}\right)^2 + \frac{3}{20}\right] \quad \textbf{Ans}$$

For bending only,

$$\Delta = \frac{5w}{96G}\left(\frac{L}{a}\right)^4 \quad \textbf{Ans.}$$

14–99. Determine the displacement at point C. EI is constant.



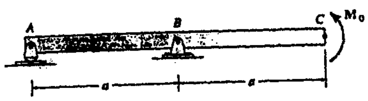


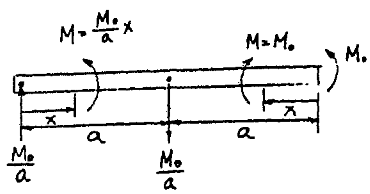
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI}dx$$

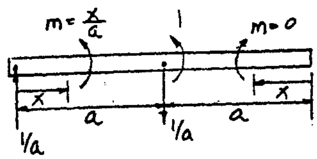
$$\Delta_C = \int_0^a \frac{(1x)(\frac{M_0}{a}x)}{EI}dx + \int_0^a \frac{(1x)M_0}{EI}dx$$

$$= \frac{5M_0a^2}{6EI} \qquad \textbf{Ans}$$

***14–100.** Determine the slope at B. EI is constant.





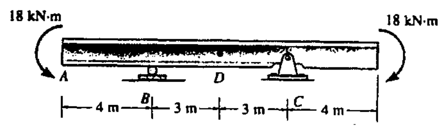


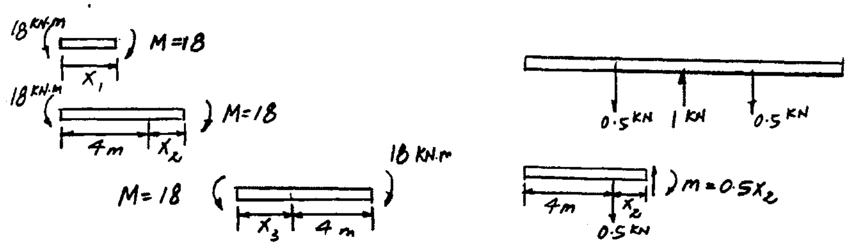
$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_B = \int_0^a \frac{\left(\frac{x}{a}\right)\left(\frac{M_0}{a} x\right)}{EI} dx$$

$$= \frac{M_0 a}{3EI} \qquad \textbf{Ans}$$

14-101 The A-36 steel beam has a moment of inertia of $I = 125(10^6)$ mm^4. Determine the displacement at D.



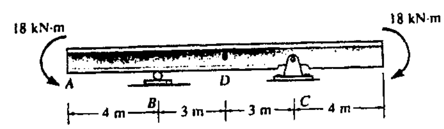


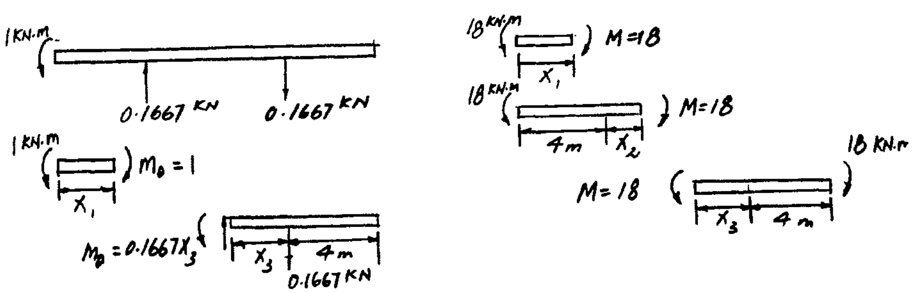
$$1 \cdot \Delta_D = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_D = (2)\frac{1}{EI}\left[\int_0^3 (0.5x_2)(18)dx_2\right] = \frac{81 \text{ kN} \cdot \text{m}^3}{EI} = \frac{81(10^3)}{200(10^9)(125)(10^{-6})}$$

$$= 3.24(10^{-3}) = 3.24 \text{ mm}$$ **Ans**

14–102 The A-36 steel beam has a moment of inertia of $I = 125(10^6)\ \text{mm}^4$. Determine the slope at A.



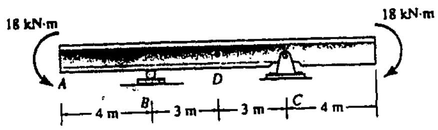


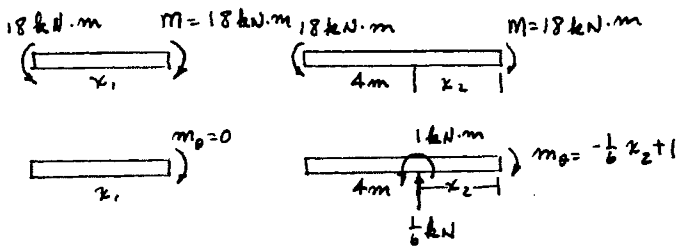
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_A = \frac{1}{EI}\left[\int_0^4 (1)(18)(dx_1) + \int_0^6 (0.1667x_3)(18)dx_3\right] = \frac{126\ \text{kN}\cdot\text{m}^2}{EI}$$

$$= \frac{126(10^3)}{200(10^9)(125)(10^{-6})} = 5.04(10^{-3})\ \text{rad} = 0.289° \qquad \textbf{Ans}$$

14–103 The A-36 structural steel beam has a moment of inertia of $I = 125(10^6)$ mm^4. Determine the slope of the beam at B.



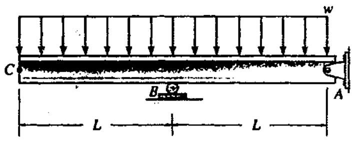


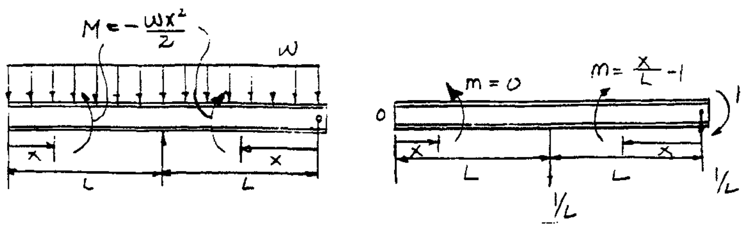
$$1 \cdot \theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$\theta_B = 0 + \frac{1}{EI}\int_0^6 \frac{(-\frac{1}{6}x_2 + 1)(18)dx}{EI}$$

$$= \frac{54}{EI} = \frac{54(10^3)}{200(10^9)(125(10^{-6}))} = 0.00216 \text{ rad} \quad = 0.124°$$ **Ans.**

•14–104 Determine the slope at A. EI is constant.



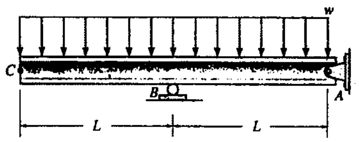


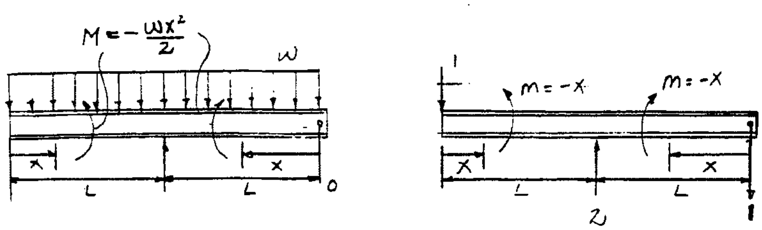
$$\theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= 0 + \int_0^L \frac{(\frac{x}{L} - 1)(\frac{-w x^2}{2})}{EI} dx$$

$$= \frac{-\frac{w L^4}{8 L} + \frac{w L^3}{6}}{EI} = \frac{w L^3}{24 EI}$$ **Ans**

14-105 Determine the displacement at C. EI is constant.



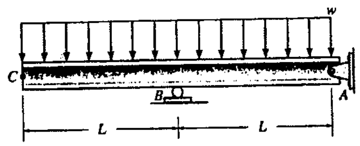


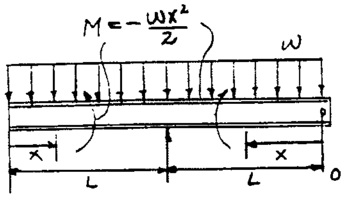
$$\Delta_C = \int_0^L \frac{m M}{E I} dx$$

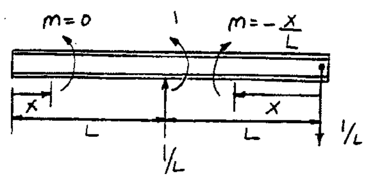
$$= 2\int_0^L \frac{(-1x)\,(\frac{-w\,x^2}{2})}{E I} dx$$

$$= 2\,\frac{w}{2 E I}\,(\frac{L^4}{4}) = \frac{w L^4}{4 E I}$$ **Ans**

14-106 Determine the slope at *B*. *EI* is constant.





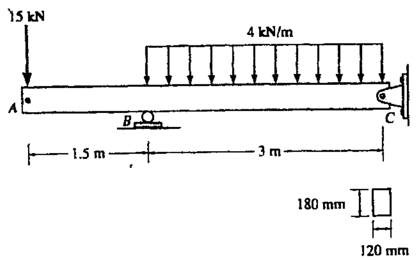


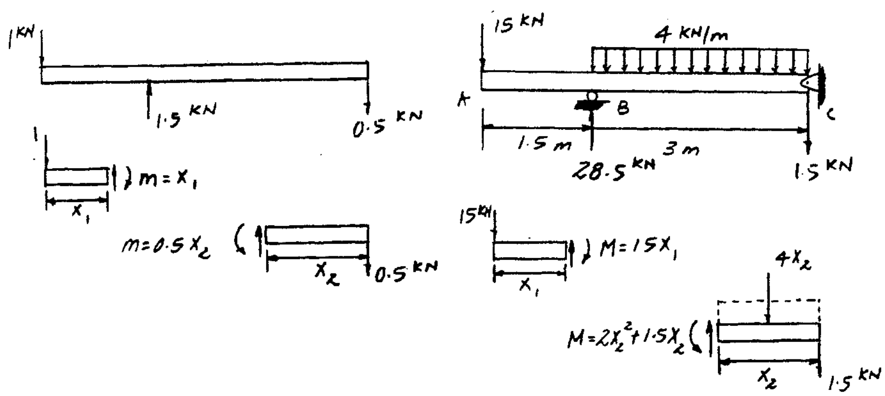
$$\theta_B = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \int_0^L \frac{(\frac{x}{L})(\frac{-w\,x^2}{2})}{EI} dx$$

$$= \frac{w L^4}{8 L E I} = \frac{w L^3}{8 E I}$$ **Ans**

14–107 The beam is made of southern pine for which E_p = 13 GPa. Determine the displacement at A.



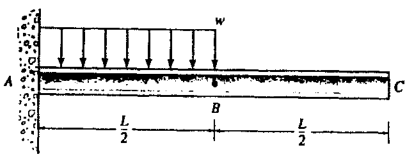


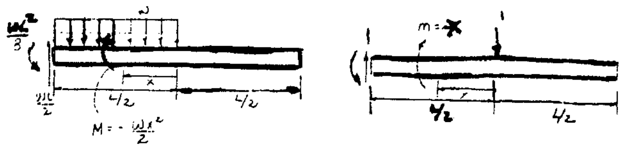
$$1 \cdot \Delta_A = \int_0^L \frac{mM}{EI}$$

$$\Delta_A = \frac{1}{EI}\left[\int_0^{1.5} (x_1)(15x_1)dx_1 + \int_0^{3} (0.5x_2)(2x_2^2 + 1.5x_2)dx_2\right]$$

$$= \frac{43.875 \text{ kN}\cdot\text{m}^3}{EI} = \frac{43.875(10^3)}{13(10^9)(\frac{1}{12})(0.12)(0.18)^3} = 0.0579 \text{ m} = 57.9 \text{ mm} \qquad \textbf{Ans}$$

•14–108 Determine the displacement at *B*. *EI* is constant.



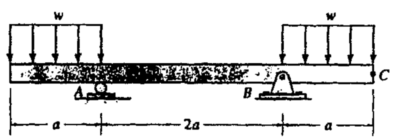


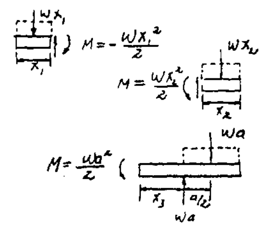
$$1 \cdot \Delta_B = \int_0^L \frac{mM}{EI} dx$$

$$\Delta_B = \int_0^{\frac{L}{2}} \frac{(-1\,x)(\frac{-w\,x^2}{2})}{EI} dx = \frac{w\,(\frac{L}{2})^4}{8EI}$$

$$= \frac{wL^4}{128EI} \qquad \textbf{Ans}$$

14-109 Determine the slope and displacement at point C. EI is constant.

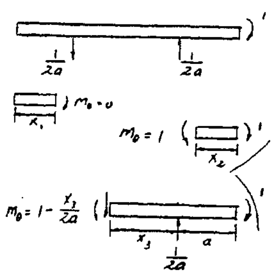




$$\theta_C = \int_0^L \frac{m_\theta M}{EI} dx$$

$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (1)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(1 - \frac{x_3}{2a}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

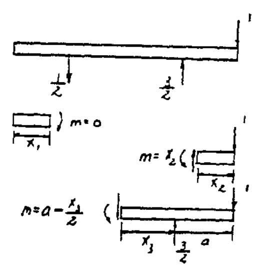
$$= \frac{2wa^3}{3EI} \qquad \textbf{Ans}$$



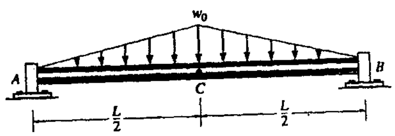
$$\Delta_C = \int_0^L \frac{mM}{EI} dx$$

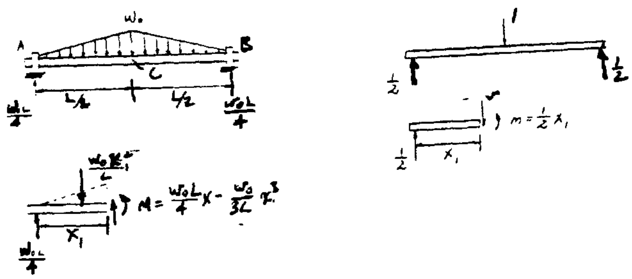
$$= \frac{1}{EI}\left[\int_0^a (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^a (x_2)\left(\frac{wx_2^2}{2}\right)dx_2 + \int_0^{2a}\left(a - \frac{x_3}{2}\right)\left(\frac{wa^2}{2}\right)dx_3\right]$$

$$= \frac{5wa^4}{8EI} \qquad \textbf{Ans}$$



14–110 Determine the displacement of the shaft at *C*. *EI* is constant.



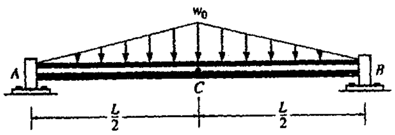


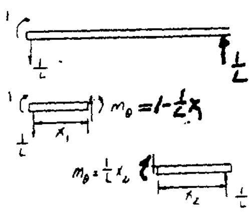
$$1 \cdot \Delta_C = \int_0^L \frac{mM}{EI} dx$$

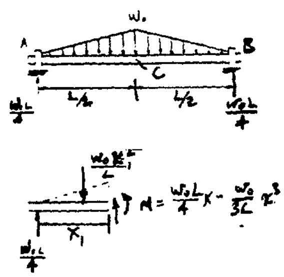
$$\Delta_C = 2\left(\frac{1}{EI}\right)\int_0^{\frac{L}{2}} \left(\frac{1}{2}x_1\right)\left(\frac{w_0 L}{4}x_1 - \frac{w_0}{3L}x_1^3\right)dx_1$$

$$= \frac{w_0 L^4}{120EI} \qquad \textbf{Ans}$$

14–111 Determine the slope of the shaft at the bearing support A. EI is constant.







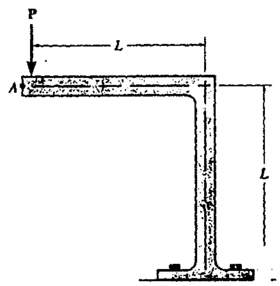
$$1 \cdot \theta_A = \int_0^L \frac{m_\theta M}{EI} dx$$

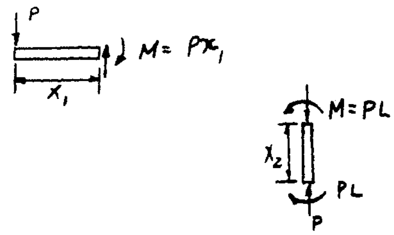
$$\theta_A = \frac{1}{EI}\left[\int_0^{\frac{L}{2}} \left(1 - \frac{1}{L}x_1\right)\left(\frac{w_0 L}{4}x_1 - \frac{w_0}{3L}x_1^3\right)dx_1 \right.$$

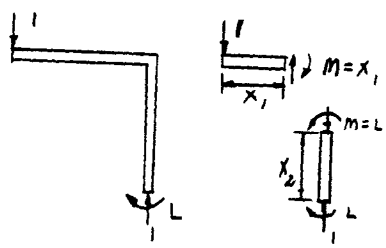
$$\left. + \int_0^{\frac{L}{2}} \left(\frac{1}{L}x_2\right)\left(\frac{w_0 L}{4}x_2 - \frac{w_0}{3L}x_2^3\right)dx_2 \right]$$

$$= \frac{5\, w_0 L^3}{192\, EI} \qquad \textbf{Ans}$$

•14–112 Determine the vertical displacement of point A on the angle bracket due to the concentrated force **P**. The bracket is fixed connected to its support. EI is constant. Consider only the effect of bending.





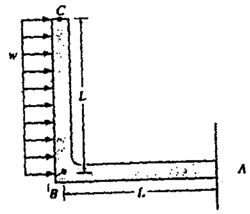


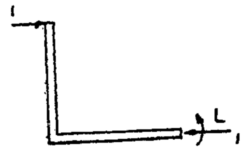
$$1 \cdot \Delta_{A_v} = \int_0^L \frac{mM}{EI} dx$$

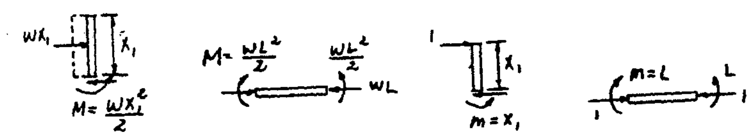
$$\Delta_{A_v} = \frac{1}{EI}[\int_0^L (x_1)(Px_1)dx_1 + \int_0^L (1L)(PL)dx_2]$$

$$= \frac{4PL^3}{3EI} \qquad \textbf{Ans}$$

14-113. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the horizontal displacement of the end C.





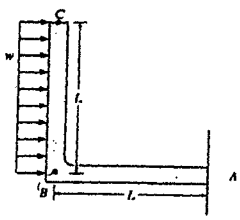


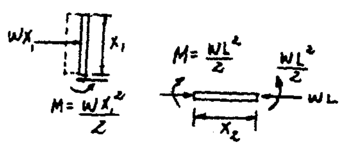
$$1 \cdot \Delta_{C_h} = \int_0^L \frac{mM}{EI}dx$$

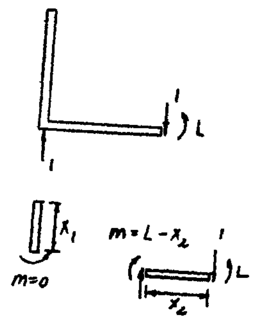
$$\Delta_{C_h} = \frac{1}{EI}\left[\int_0^L (1x_1)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (1L)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{5wL^4}{8EI} \qquad \textbf{Ans}$$

14-114. The L-shaped frame is made from two segments, each of length L and flexural stiffness EI. If it is subjected to the uniform distributed load, determine the vertical displacement of point B.





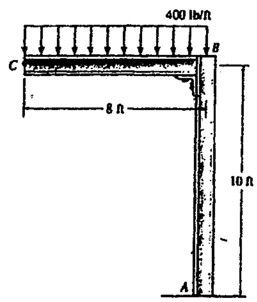


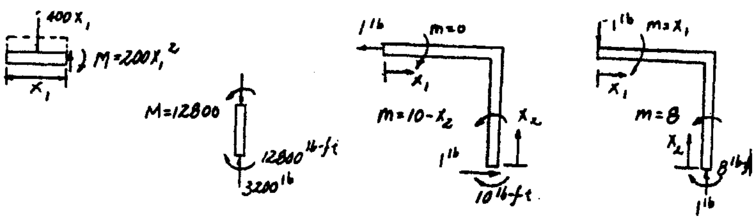
$$1\cdot\Delta_{B_v} = \int_0^L \frac{mM}{EI}dx$$

$$\Delta_{B_v} = \frac{1}{EI}\left[\int_0^L (0)\left(\frac{wx_1^2}{2}\right)dx_1 + \int_0^L (L-x_2)\left(\frac{wL^2}{2}\right)dx_2\right]$$

$$= \frac{wL^4}{4EI} \quad \textbf{Ans}$$

14-115. Determine the horizontal and vertical displacements of point *C*. There is a fixed support at *A*. *EI* is constant.



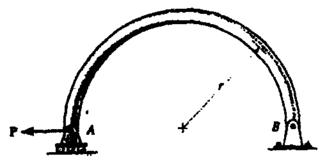


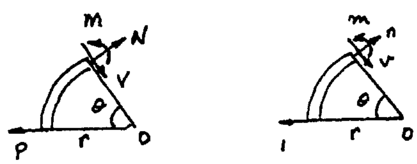
$$1\cdot\Delta = \int_0^L \frac{mM}{EI}dx$$

$$\Delta_{C_h} = \frac{1}{EI}\left[\int_0^8 (0)(200x_1^2)dx_1 + \int_0^{10}(10-x_2)(12,800)dx_2\right] = \frac{640\,000\ \text{lb}\cdot\text{ft}^3}{EI}$$ **Ans**

$$\Delta_{C_v} = \frac{1}{EI}\left[\int_0^8 (x_1)(200x_1^2)dx_1 + \int_0^{10}(8)(12,800)dx_2\right] = \frac{1\,228\,800\ \text{lb}\cdot\text{ft}^3}{EI}$$ **Ans**

***14-116.** The semi-circular rod has a cross-sectional area A and modulus of elasticity E. Determine the horizontal deflection at the roller due to the loading.





$\Sigma F_x = 0; \quad N - P\sin\theta = 0$

$N = P\sin\theta$

$\Sigma M_O = 0; \quad M - P\sin\theta\, r = 0$

$M = Pr\sin\theta$

$\Sigma F_x = 0; \quad n - 1\sin\theta = 0$

$n = 1\sin\theta$

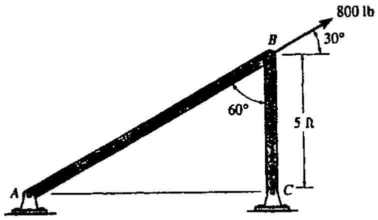
$\circlearrowleft + \Sigma M_O = 0; \quad m - (1\sin\theta)r = 0$

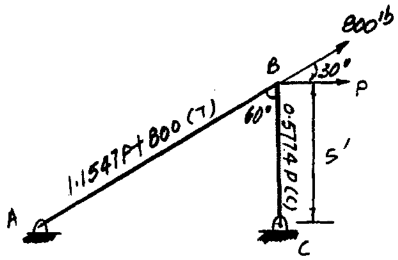
$m = r\sin\theta$

$$1 \cdot \Delta = \int_0^L \frac{mM}{EI}dx = \frac{Pr^2}{EI}\int_0^\pi \sin^2\theta\,(r\,d\theta) = \frac{Pr^3}{EI}\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Big|_0^\pi$$

$$\Delta = \frac{\pi Pr^3}{2EI} \quad \textbf{Ans}$$

14–117 Solve Prob. 14–71 using Castigliano's theorem.



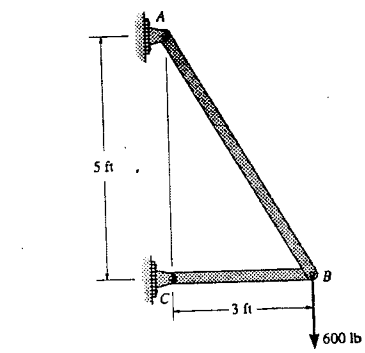


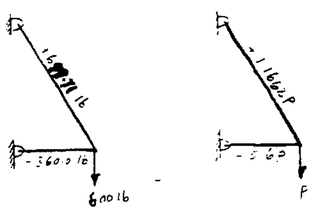
Member	N	$\partial N/\partial P$	$N(P=0)$	L	$N(\partial N/\partial P)L$
AB	$1.1547P + 800$	1.1547	800	120	110 851.25
BC	$-0.5774P$	-0.5774	0	60	0

$\Sigma = 110\,851.25$

$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{110851.25}{AE} = \frac{110851.25}{(2)(29)(10^6)} = 0.00191 \text{ in.} \qquad \textbf{Ans}$$

14–118 Solve Prob. 14–73 using Castigliano's theorem.

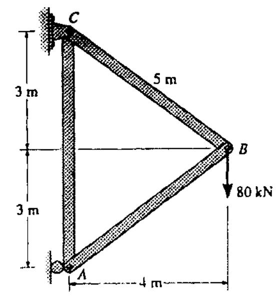


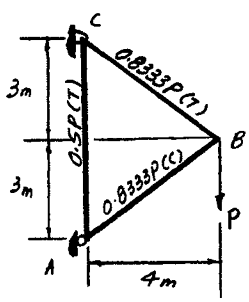


$$\Delta_{B_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{699.71\,(1.166)(5.831)(12)}{2\,(29)(10^6)} + \frac{-360\,(-0.6)(3)(12)}{2\,(29)(10^6)}$$

$$= 0.00112 \text{ in.} \downarrow$$ **Ans**

14–119 Solve Prob. 14–74 using Castigliano's theorem.



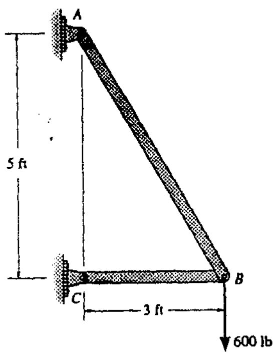


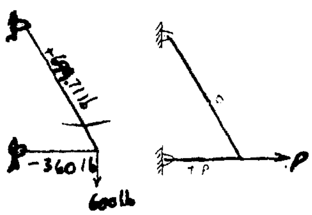
Member	N	$\partial N/\partial P$	$N(P = 0)$	L	$N(\partial N/\partial P)L$
AB	$-0.8333P$	-0.8333	-66.67	5	277.78
A*C*	$-0.5P$	0.5	40	6	120.00
BC	$0.8333P$	0.8333	66.67	5	277.78

$\Sigma = 675.56$

$$\Delta_{B_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{675.56}{AE} = \frac{675.56(10^3)}{300(10^{-6})(200)(10^9)} = 0.0113 \text{ m} = 11.3 \text{ mm} \qquad \textbf{Ans}$$

•14–120 Solve Prob. 14–72 using Castigliano's theorem.

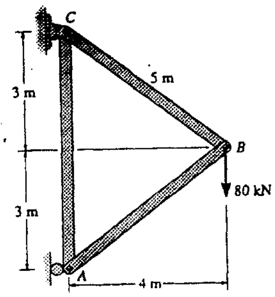


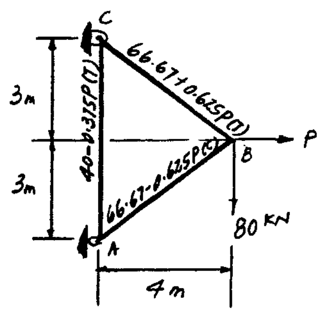


$$\Delta_{Bh} = \Sigma N(\frac{\partial N}{\partial P})\frac{L}{AE} = \frac{-360\,(1)(3)(12)}{2\,(29)(10^6)} + 0 = -0.223\,(10^{-3})\text{ in.}$$

$$= 0.223\,(10^{-3})\text{ in.} \leftarrow$$ **Ans**

14–121 Solve Prob. 14–75 using Castigliano's theorem.





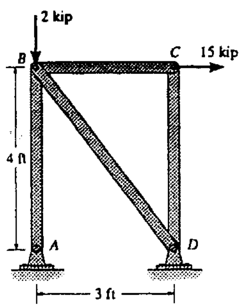
Member	N	$\partial N/\partial P$	$N(P=0)$	L	$N(\partial N/\partial P)L$
AB	$-(66.67-0.625P)$	0.625	−66.67	5	−208.33
AC	$40-0.375P$	−0.375	40	6	−90.00
BC	$66.67+0.625P$	0.625	66.67	5	208.33

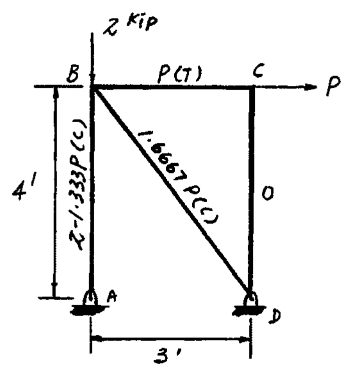
$\Sigma = -90.00$

$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{-90}{AE} = \frac{-90(10^3)}{300(10^{-6})(200)(10^9)} = -1.50(10^{-3})\text{m}$$

$= -1.50\text{mm} = 1.50\text{ mm} \leftarrow$ **Ans**

14–122 Solve Prob. 14–76 using Castigliano's theorem.



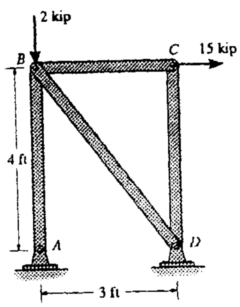


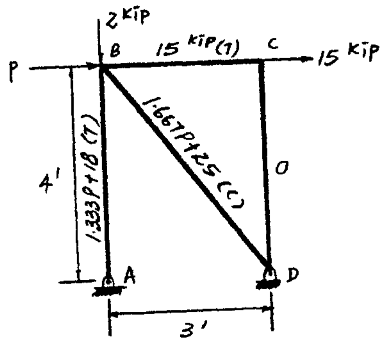
Member	N	$\partial N/\partial P$	$N(P=15)$	L	$N(\partial N/\partial P)L$
AB	$-(2-1.333P)$	1.333	18	48	1152
AC	P	1.0	15	36	540
BC	$-1.6667P$	-1.6667	25	60	2500
CD	0	0	0	0	0

$\Sigma = 4192$

$$\Delta_{C_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{4192}{3(29)(10^3)} = 0.0482 \text{ in.}$$ **Ans**

14–123 Solve Prob. 14–77 using Castigliano's theorem.

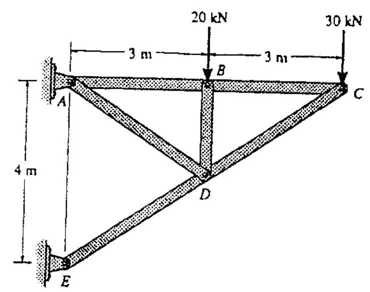


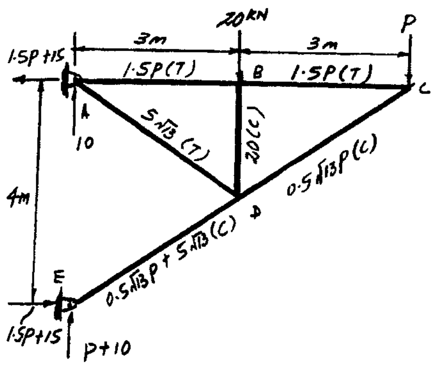


Member	N	$\partial N/\partial P$	$N(P = 15)$	L	$N(\partial N/\partial P)L$
AB	$1.333P + 18$	1.333	18	48	1152
AC	15	1.0	15	36	0
BC	$-(1.667P + 25)$	-1.6667	-25	60	2500
CD	0	0	0	0	0
					$\Sigma = 3652$

$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{3652}{AE} = \frac{3652}{(3)(29)(10^3)} = 0.0420 \text{ in.}$$ **Ans**

*14–124 Solve Prob. 14–80 using Castigliano's theorem.





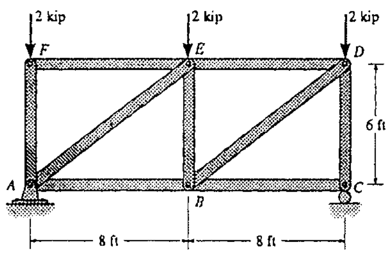
Member	N	$\partial N/\partial P$	$N(P=30)$	L	$N(\partial N/\partial P)L$
AB	$1.50P$	1.50	45.00	3.0	202.50
AD	$5\sqrt{13}$	0	$5\sqrt{13}$	$\sqrt{13}$	0
BD	-20	0	-20	2.0	0
BC	$1.5P$	1.5	45.00	3.0	202.50
CD	$-0.5\sqrt{13}P$	$-0.5\sqrt{13}$	$-15\sqrt{13}$	$\sqrt{13}$	351.54
DE	$-(0.5\sqrt{13}P+5\sqrt{13})$	$-0.5\sqrt{13}$	$-20\sqrt{13}$	$\sqrt{13}$	468.72

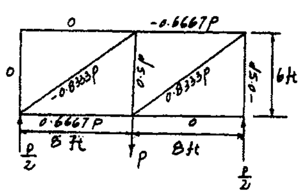
$\Sigma = 1225.26$

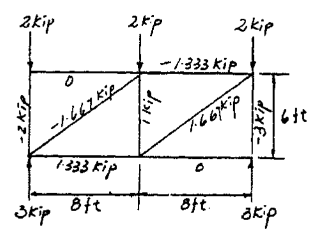
$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{1225.26(10^3)}{300(10^{-6})(200)(10^9)}$$

$= 0.02.04\ \text{m} = 20.4\ \text{mm}$ **Ans**

14-125 Solve Prob. 14-78 using Castigliano's theorem.





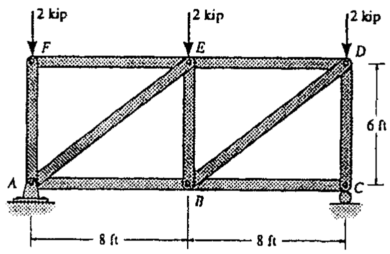


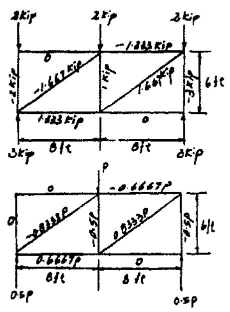
$$\Delta_B = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$= [(-1.333)(-0.667)(8) + (1.33)(0.667)(8) + (-1)(0.5)(6) + (-1.667)(-0.833)(10) + (1.667)(0.8333)(10) + (-3)(-0.5)(6)]\frac{12}{AE}$$

$$= \frac{576}{1.5\,(29)(10^3)} = 0.0132 \text{ in.} \qquad \textbf{Ans}$$

14–126 Solve Prob. 14–79 using Castigliano's theorem.



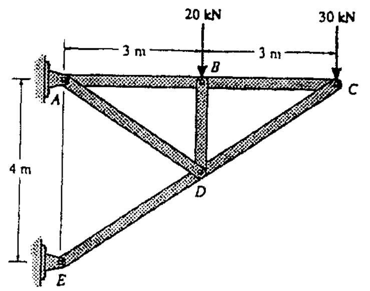


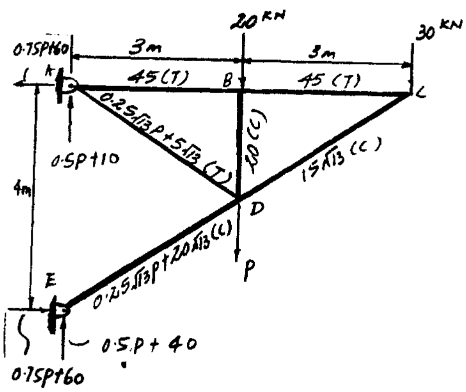
$$\Delta_E = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE}$$

$$= \{ (-1.667)(-0.833)(10) + (1.667)(0.833)(10) + (-1)(-0.5)(6) + (-0.5)(-3)(6) + (0.667)(1.33)(8) + (-0.667)(-1.33)(8) \}\frac{12}{AE}$$

$$= \frac{648}{1.5\,(29)(10^3)} = 0.0149 \text{ in.}$$ **Ans**

14–127 Solve Prob. 14–81 using Castigliano's theorem.



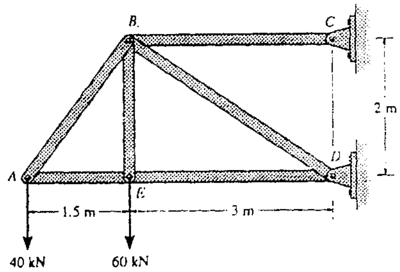


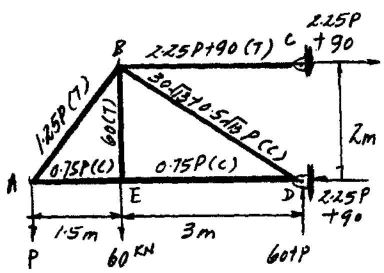
Member	N	$\partial N/\partial P$	$N(P=0)$	L	$N(\partial N/\partial P)L$
AB	45	0	45	3	0
AD	$0.25\sqrt{13}P+5\sqrt{13}$	$0.25\sqrt{13}$	$5\sqrt{13}$	$\sqrt{13}$	58.59
BC	45	0	45	3	0
BD	-20	0	-20	2	0
CD	$-15\sqrt{13}$	0	$-15\sqrt{13}$	$\sqrt{13}$	0
DE	$-(0.25\sqrt{13}P+20\sqrt{13})$	$-0.25\sqrt{13}$	$-20\sqrt{13}$	$\sqrt{13}$	234.36
					$\Sigma = 292.95$

$$\Delta_{D_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{292.95}{AE} = \frac{292.95(10^3)}{300(10^{-6})(200)(10^9)}$$

$$= 4.88(10^{-3})\text{ m} = 4.88\text{ mm} \qquad \textbf{Ans}$$

•14–128 Solve Prob. 14–84 using Castigliano's theorem.



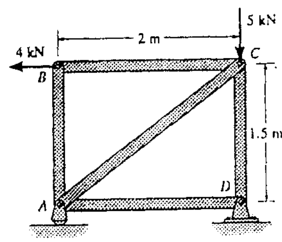


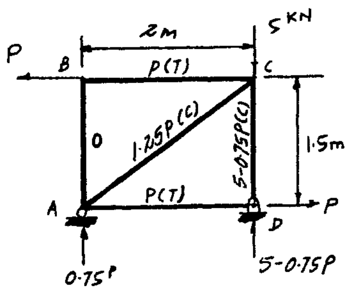
Member	N	$\partial N/\partial P$	$N(P=40)$	L	$N(\partial N/\partial P)L$
AB	$1.25P$	1.25	50	2.5	156.25
AE	$-0.75P$	-0.75	-30	1.5	33.75
BC	$2.25P+90$	2.25	180	3.0	1215.00
BD	$-(30\sqrt{13}+0.5\sqrt{13}P)$	$-0.5\sqrt{13}$	$-50\sqrt{13}$	$\sqrt{13}$	1171.80
BE	60	0	60	2.0	0
DE	$-0.75P$	-0.75	-30	3.0	67.5

$\Sigma = 2644.30$

$$\Delta_{A_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{2644.30(10^3)}{400(10^{-6})(200)(10^9)} = 0.0331 \text{ m} = 33.1 \text{ mm} \qquad \textbf{Ans}$$

14–129 Solve Prob. 14–82 using Castigliano's theorem.





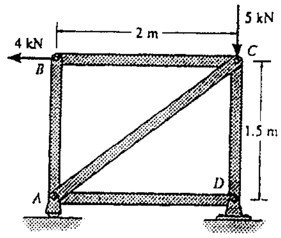
Member	N	$\partial N/\partial P$	$N(P=0)$	L	$N(\partial N/\partial P)L$
AB	0	0	0	1.5	0
AC	$-1.25P$	-1.25	-5	2.5	15.625
AD	P	1	4	2.0	8.00
BC	P	1	4	2.0	8.00
CD	$-(5-0.75P)$	0.75	-2	1.5	-2.25

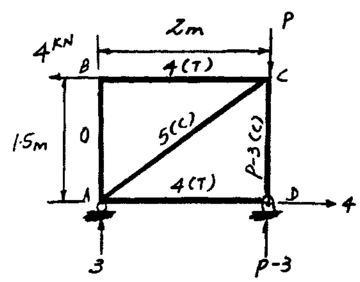
$\Sigma = 29.375$

$$\Delta_{B_h} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\left(\frac{L}{AE}\right) = \frac{29.375(10^3)}{400(10^{-6})(200)(10^9)} = 0.367(10^{-3})\text{ m}$$

$= 0.367$ mm **Ans**

14–130 Solve Prob. 14–83 using Castigliano's theorem.



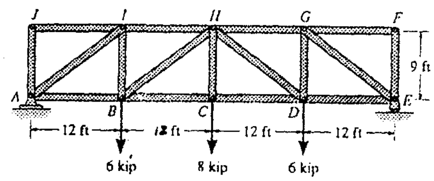


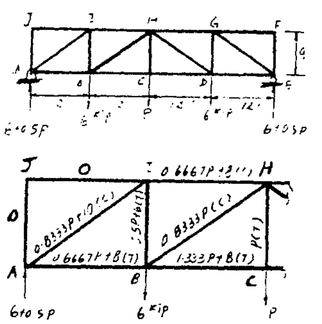
Member	N	$\partial N/\partial P$	$N(P = 5)$	L	$N(\partial N/\partial P)L$
AB	0	0	0	1.5	0
AC	−5	0	−5	2.5	0
AD	4	0	4	2.0	0
BC	4	0	4	2.0	0
CD	$-(P-3)$	−1	−2	1.5	3

$\Sigma = 3$

$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{3}{AE} = \frac{3(10^3)}{400(10^{-6})(200)(10^9)} = 37.5(10^{-6})\text{ m} = 0.0375\text{ mm}$$ **Ans**

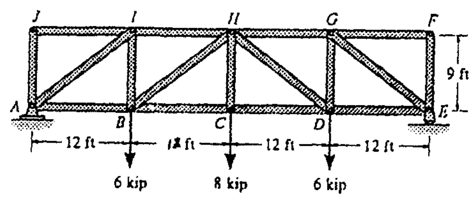
14–131 Solve Prob. 14–85 using Castigliano's theorem.

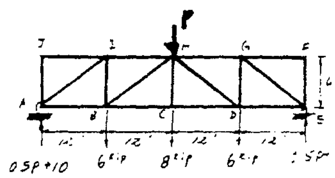


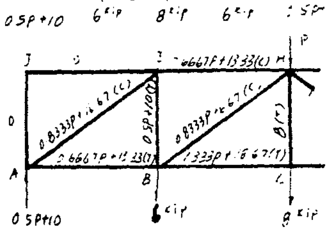


$$\Delta_{C_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{21232}{AE} = \frac{21232}{4.5\,(29)(10^3)} = 0.163 \text{ in.} \qquad \textbf{Ans}$$

•14–132 Solve Prob. 14–86 using Castigliano's theorem.



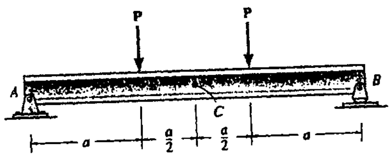


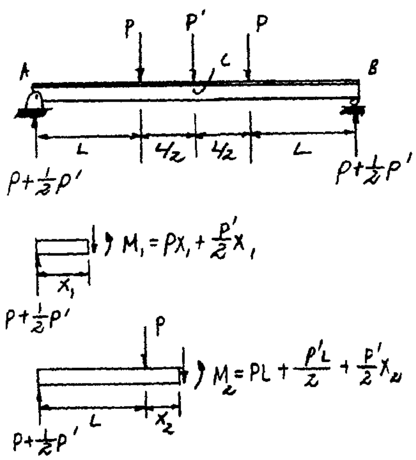


$$\Delta_{Hv} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{20368}{AE} = \frac{20368}{4.5\,(29)(10^3)}$$

$$= 0.156 \text{ in.} \qquad \textbf{Ans}$$

14–133 Solve Prob. 14–87 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P'} = \frac{x_1}{2} \qquad \frac{\partial M_2}{\partial P'} = \frac{L}{2} + \frac{x_2}{2}$$

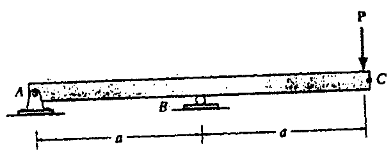
Set $P' = 0$

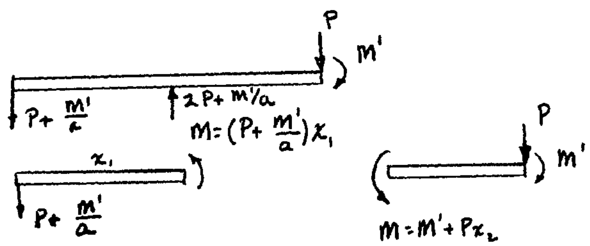
$$M_1 = Px_1 \qquad M_2 = PL$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = (2)\frac{1}{EI}\left[\int_0^L (Px_1)\left(\frac{1}{2}x_1\right)dx + \int_0^{L/2} (PL)\left(\frac{L}{2} + \frac{1}{2}x_2\right)dx_2\right]$$

$$= \frac{23PL^3}{24\,EI} \qquad \textbf{Ans}$$

14–134 Solve Prob. 14–89 using Castigliano's theorem.





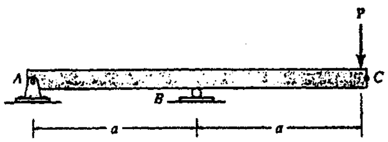
Set $M' = 0$

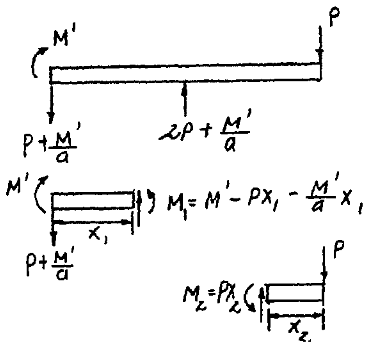
$$\theta_C = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \int_0^a \frac{(Px_1)(\frac{1}{a}x_1)dx_1}{EI} + \int_0^a \frac{(Px_2)(1)dx_2}{EI}$$

$$= \frac{Pa^2}{3EI} + \frac{Pa^2}{2EI} = \frac{5Pa^2}{6EI}$$ **Ans**

14–135 Solve Prob. 14–90 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 1 - \frac{x_1}{a} \qquad \frac{\partial M_2}{\partial M'} = 0$$

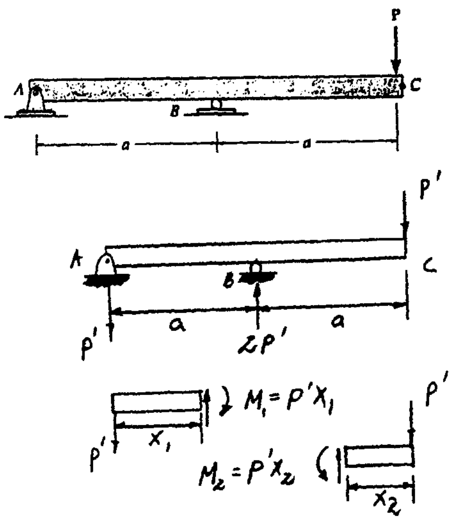
Set $M' = 0$

$$M_1 = -Px_1 \qquad M_2 = Px_2$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^a (-Px_1)\left(1 - \frac{x_1}{a}\right)dx_1 + \int_0^a (Px_2)(0)dx_2\right] = \frac{-Pa^2}{6EI}$$

$$= \frac{Pa^2}{6EI} \qquad \textbf{Ans.}$$

•14–136 Solve Prob. 14–88 using Castigliano's theorem.



$$\frac{\partial M_1}{\partial P'} = x_1 \qquad \frac{\partial M_2}{\partial P'} = x_2$$

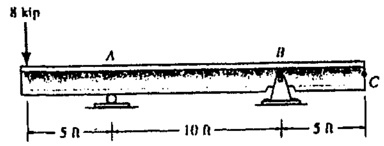
Set $P = P'$

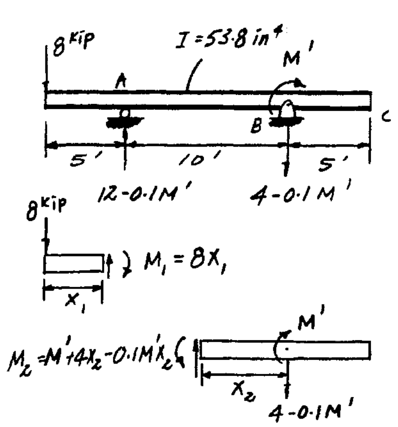
$$M_1 = Px_1 \qquad M_2 = Px_2$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P'}\right)dx = \frac{1}{EI}\left[\int_0^a (Px_1)(x_1)dx_1 + \int_0^a (Px_2)(x_2)dx_2\right]$$

$$= \frac{2Pa^3}{3EI} \qquad \textbf{Ans}$$

14–137 Solve Prob. 14–91 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 0 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1x_2$$

Set $M' = 0$

$$M_1 = 8x_1 \qquad M_2 = 4x_2$$

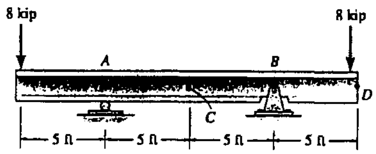
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

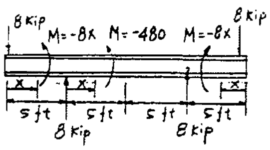
$$= \frac{1}{EI}\left[\int_0^5 (8x_1)(0)dx_1 + \int_0^{10}(4x_2)(1 - 0.1x_2)dx_2\right]$$

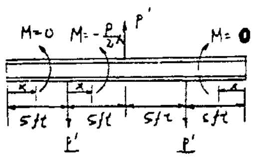
$$= \frac{66.67 \text{ kip}\cdot\text{ft}^2}{EI} = \frac{66.67(12^2)}{29(10^3)(53.8)} = 6.15(10^{-3}) \text{ rad} = 0.353° \qquad \textbf{Ans}$$

$$\Delta_C = \theta_B(5)(12) = 6.15(10^{-3})(60) = 0.369 \text{ in.} \qquad \textbf{Ans}$$

14–138 Solve Prob. 14–93 using Castigliano's theorem.



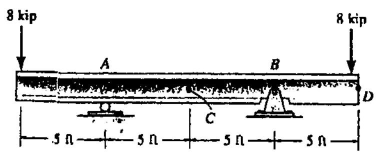


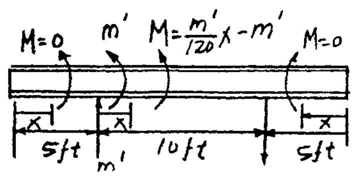


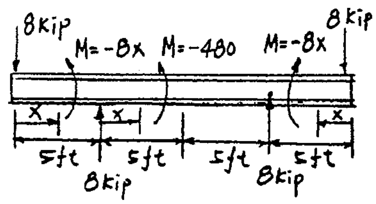
$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{E\,I} = 0 + 2\int_0^{60} \frac{-480\,(-\frac{x}{2})}{E\,I}dx$$

$$= \frac{2\,(\frac{480}{2})\frac{(60^2)}{2}}{E\,I} = \frac{864\,000}{29\,(10^3)(245)} = 0.122 \text{ in.} \qquad \textbf{Ans}$$

14–139 Solve Prob. 14–94 using Castigliano's theorem.





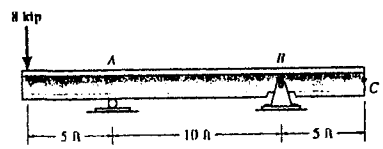


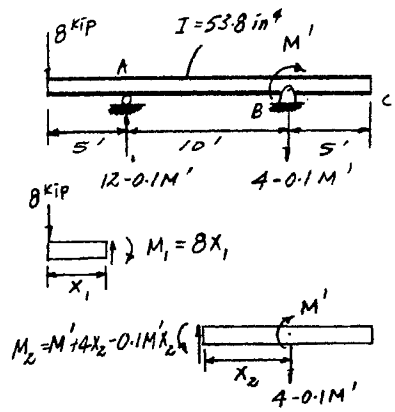
$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= 0 + \int_0^{120} \frac{-480\left(\frac{x}{120} - 1\right)}{EI}dx = 0 + \frac{-480\left[\frac{1}{2}\left(\frac{120^2}{120}\right) - 120\right]}{EI}$$

$$= \frac{28\,800}{29\,(10^3)(245)} = 4.05\,(10^{-3})\text{ rad} \qquad \textbf{Ans}$$

14–140 Solve Prob. 14–92 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 0 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1\,x_2$$

Set $M' = 0$

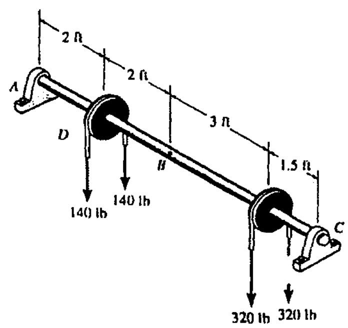
$$M_1 = 8x_1 \qquad M_2 = 4x_2$$

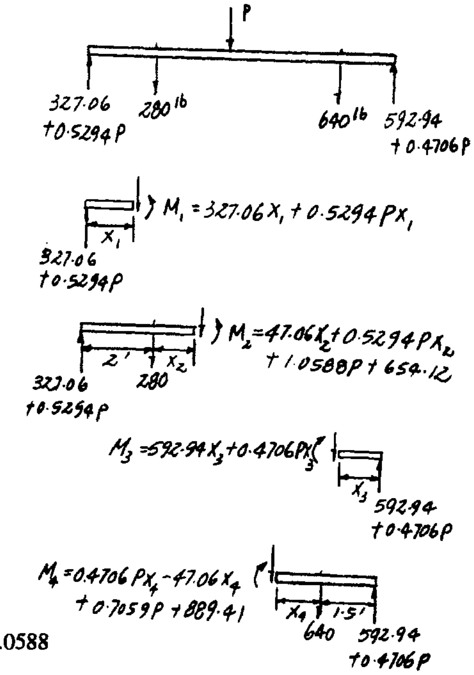
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^5 (8x_1)(0)dx_1 + \int_0^{10}(4x_2)(1 - 0.1x_2)dx_2\right] = \frac{66.67\ \text{kip}\cdot\text{ft}^2}{EI}$$

$$= \frac{66.67(12)^2}{(29)(10^3)(53.8)} = 6.15(10^{-3})\ \text{rad} = 0.353° \qquad \textbf{Ans}$$

14–141 Solve Prob. 14–95 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P} = 0.5294x_1 \qquad \frac{\partial M_2}{\partial P} = 0.5294x_2 + 1.0588$$

$$\frac{\partial M_3}{\partial P} = 0.4706x_3 \qquad \frac{\partial M_4}{\partial P} = 0.4706x_4 + 0.7059$$

Set $P = 0$

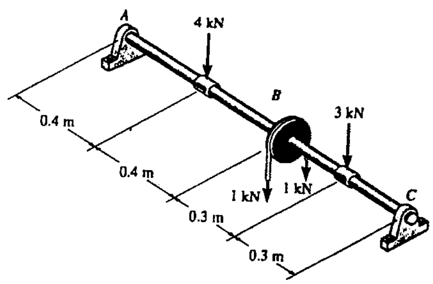
$$M_1 = 327.06x_1 \qquad M_2 = 47.06x_2 + 654.12$$
$$M_3 = 592.94x_3 \qquad M_4 = 889.41 - 47.06x_4$$

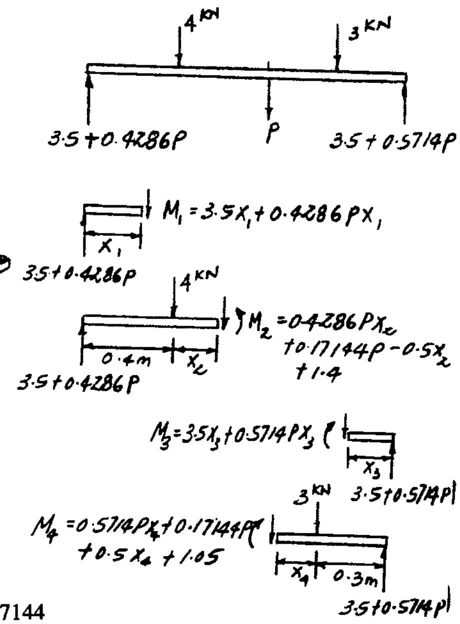
$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\Big[\int_0^2 (327.06x_1)(0.5294x_1)dx_1 + \int_0^2 (47.06x_2 + 654.12)(0.5294x_2 + 1.0588)dx_2 +$$
$$\int_0^{1.5} (592.94x_3)(0.4706x_3)dx_3 + \int_0^3 (889.41 - 47.06x_4)(0.4706x_4 + 0.7059)dx_4\Big]$$

$$= \frac{6437.69 \text{ lb}\cdot\text{ft}^3}{EI} = \frac{6437.69(12^3)}{29(10^6)(\frac{\pi}{4})(0.75^4)} = 1.54 \text{ in.} \qquad \textbf{Ans}$$

14–142 Solve Prob. 14–97 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P} = 0.4286x_1 \qquad \frac{\partial M_2}{\partial P} = 0.4286x_2 + 0.17144$$

$$\frac{\partial M_3}{\partial P} = 0.5714x_3 \qquad \frac{\partial M_4}{\partial P} = 0.5714x_4 + 0.17144$$

Set $P = 2$ kN

$$M_1 = 4.3572\,x_1 \qquad M_2 = 0.3572x_2 + 1.7429$$

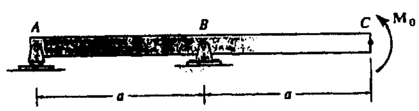
$$M_3 = 4.6428x_3 \qquad M_4 = 1.6428x_4 + 1.3929$$

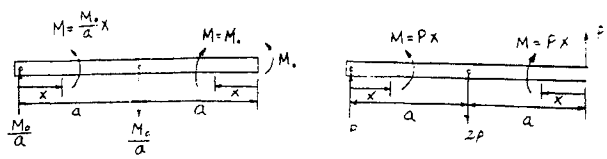
$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\Big[\int_0^{0.4}(4.3572x_1)(0.4286x_1)dx_1 + \int_0^{0.4}(0.3572x_2 + 1.7429)(0.4286x_2 + 0.17144)dx_2 + \int_0^{0.3}(4.6428x_3)(0.5714x_3)dx_3 + \int_0^{0.3}(1.6428x_4 + 1.3929)(0.5714x_4 + 0.17144)dx_4\Big]$$

$$= \frac{0.37944\text{ kN}\cdot\text{m}^3}{EI} = \frac{0.37944(10^3)}{200(10^9)\frac{\pi}{4}(0.015)^4} = 0.0478\text{ m} = 47.8\text{ mm} \qquad \textbf{Ans}$$

14–143 Solve Prob. 14–99 using Castigliano's theorem.

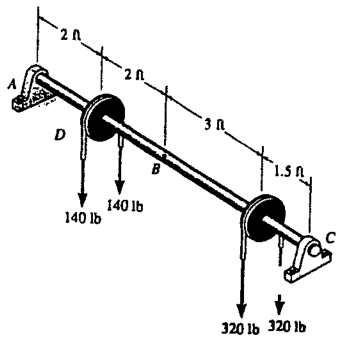


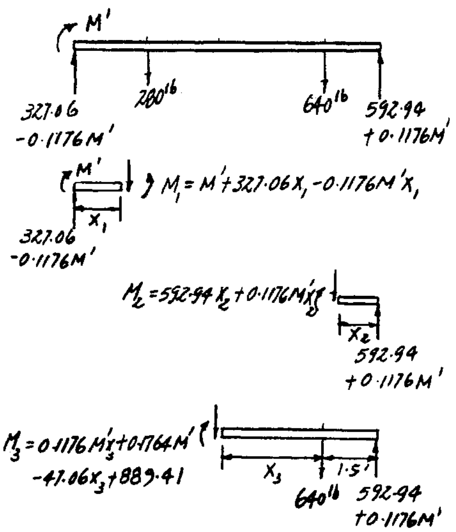


$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^a \frac{\left(\frac{M_0}{a}x\right)(1x)}{EI}dx + \int_0^a \frac{M_0\,(1x)}{EI}dx$$

$$= \frac{5\,M_0\,a^2}{6EI} \qquad \textbf{Ans}$$

•14–144 Solve Prob. 14–96 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 1 - 0.1176\,x_1 \qquad \frac{\partial M_2}{\partial M'} = 0.1176\,x_2 \qquad \frac{\partial M_3}{\partial M'} = 0.1176x_3 + 0.1764$$

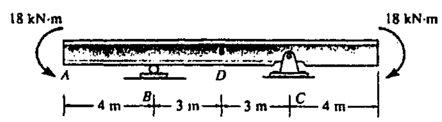
Set $M' = 0$

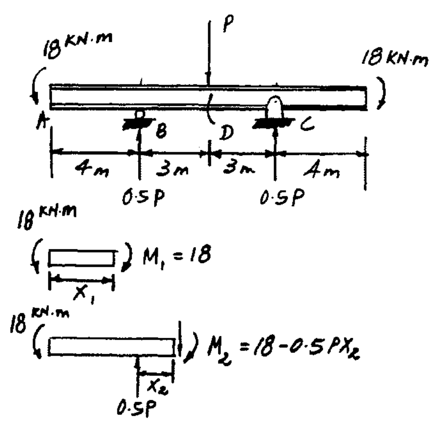
$$M_1 = 327.06x_1 \qquad M_2 = 592.94x_2 \qquad M_3 = 889.41 - 47.06x_3$$

$$\theta_A = \int M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^2 (327.06x_1)(1 - 0.1176x_1)dx_1 + \int_0^{1.5}(592.94x_2)(0.1176x_2)dx_2 + \int_0^3 (889.41 - 47.06x_3)(0.1176x_3 + 0.1764)dx_3\right]$$

$$= \frac{2387.54\ \text{lb}\cdot\text{ft}^2}{EI} = \frac{2387.54(12^2)}{29(10^6)(\frac{\pi}{4})(0.75^4)} = 0.0477\ \text{rad} = 2.73° \qquad \textbf{Ans}$$

14–145 Solve Prob. 14–101 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P} = 0 \qquad \frac{\partial M_2}{\partial P} = -0.5x_2$$

Set $P = 0$

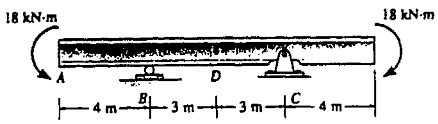
$$M_1 = 18 \qquad M_2 = 18$$

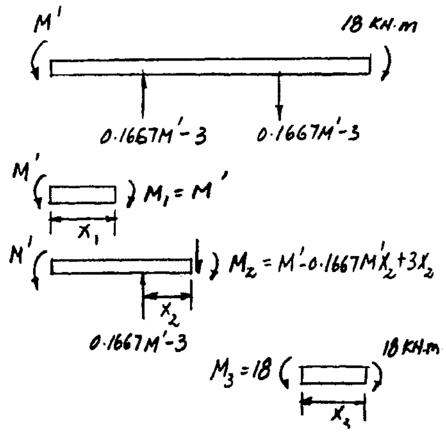
$$\Delta_D = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= (2)\frac{1}{EI}\left[\int_0^4 (18)(0)dx_1 + \int_0^3 (18)(-0.5x_2)dx_2\right]$$

$$= \frac{81\ \text{kN}\cdot\text{m}^3}{EI} = \frac{81(10^3)}{200(10^9)(125)(10^{-6})} = 3.24(10^{-3})\ \text{m} = 3.24\ \text{mm} \qquad \textbf{Ans}$$

14–146 Solve Prob. 14–102 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 1 \qquad \frac{\partial M_2}{\partial M'} = 1 - 0.1667x_2 \qquad \frac{\partial M_3}{\partial M'} = 0$$

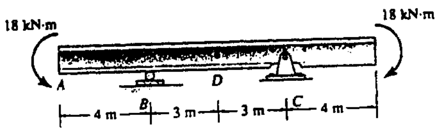
Set $M' = 18\ \text{kN}\cdot\text{m}$

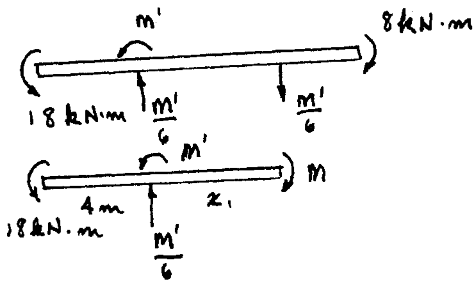
$$M_1 = 18\ \text{kN}\cdot\text{m} \qquad M_2 = 18\ \text{kN}\cdot\text{m} \qquad M_3 = 18\ \text{kN}\cdot\text{m}$$

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^4 (18)(1)dx_1 + \int_0^6 18(1 - 0.1667x_2)dx_2 + \int_0^4 (18)(0)dx_3\right]$$

$$= \frac{126\ \text{kN}\cdot\text{m}^2}{EI} = \frac{126(10^3)}{200(10^9)(125)(10^{-6})} = 5.04(10^{-3})\ \text{rad} = 0.289^\circ \qquad \textbf{Ans}$$

14–147 Solve Prob. 14–103 using Castigliano's theorem.

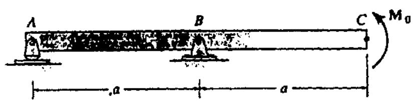


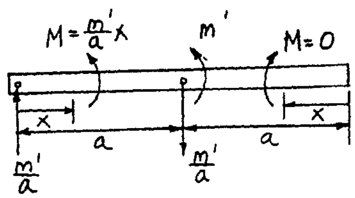


$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^6 \frac{(-18)\,(-\frac{1}{6}x)dx(10^3)}{EI}$$

$$= \frac{18\,(6^2)(10^3)}{6\,(2)(200)(10^9)(125)(10^{-6})} = 0.00216 \text{ rad} \qquad \textbf{Ans}$$

•14–148 Solve Prob. 14–100 using Castigliano's theorem.

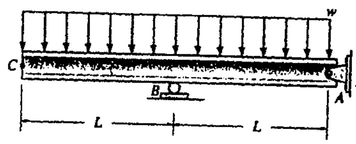


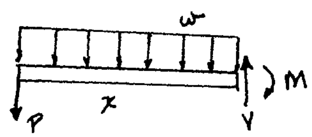


$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^a \frac{\left(\frac{M_0}{a}x\right)\left(\frac{x}{a}\right)}{EI}dx$$

$$= \frac{M_0\, a}{3EI} \qquad \textbf{Ans}$$

14–149 Solve Prob. 14–105 using Castigliano's theorem.





$M = -Px - \frac{wx^2}{2}$

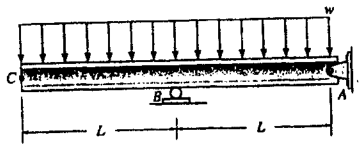
$\frac{\partial M}{\partial P} = -x$

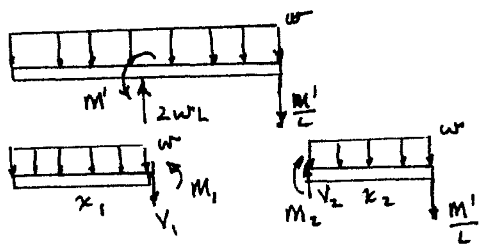
Set $P = 0$

$\Delta_C = \int_0^L M \frac{\partial M}{\partial P} \frac{dx}{EI} = 2\int_0^L \frac{(\frac{-w\,x^2}{2})(-1x)}{EI} dx$

$= 2\frac{w}{2EI}(\frac{L^4}{4}) = \frac{wL^4}{4EI}$ **Ans**

14–150 Solve Prob. 14–106 using Castigliano's theorem.





$$M_1 = -\frac{wx_1^2}{2}$$

$$\frac{\partial M_1}{\partial M'} = 0$$

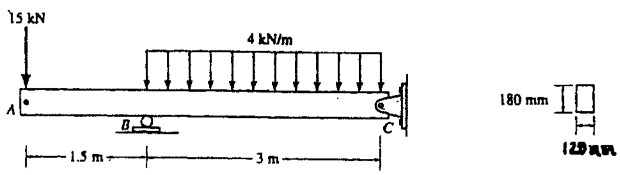
$$M_2 = -\frac{M'}{L}x_2 - \frac{wx_2^2}{2}$$

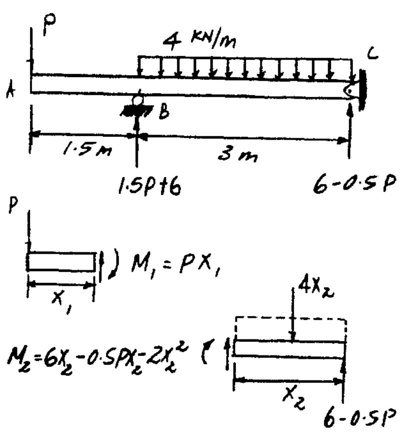
$$\frac{\partial M_2}{\partial M'} = -\frac{x_2}{L}$$

Set $M' = 0$

$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = 0 + \int_0^L \left(\frac{-wx_2^2}{2}\right)\left(-\frac{x_2}{L}\right)\frac{dx}{EI} = \frac{wL^3}{8EI}$$ **Ans**

14–151 Solve Prob. 14–107 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial P} = x_1 \qquad \frac{\partial M_2}{\partial P} = -0.5\,x_2$$

Set $P = 15$ kN

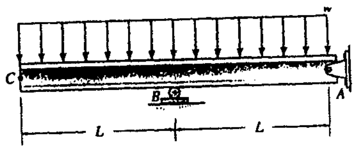
$M_1 = 15x_1 \qquad M_2 = -1.5x_2 - 2x_2^2$

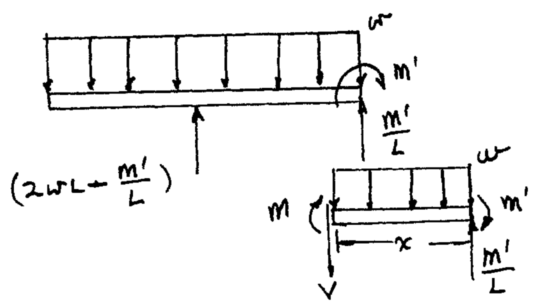
$$\Delta_A = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^{1.5}(15x_1)(x_1)dx + \int_0^3(-1.5x_2 - 2x_2^2)(-0.5x_2)dx_2\right]$$

$$= \frac{43.875\ \text{kN}\cdot\text{m}^3}{EI} = \frac{43.875(10^3)}{13(10^9)\frac{1}{12}(0.12)(0.18)^3} = 0.0579\ \text{m}$$

$= 57.9$ mm **Ans**

•14–152 Solve Prpb. 14–104 using Castigliano's theorem.





M' does not influence the moment within the overhang.

$$M = \frac{M'}{L}x - M' - \frac{wx^2}{2}$$

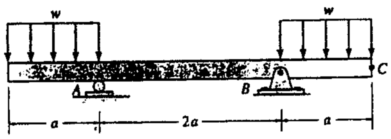
$$\frac{\partial M}{\partial M'} = \frac{x}{L} - 1$$

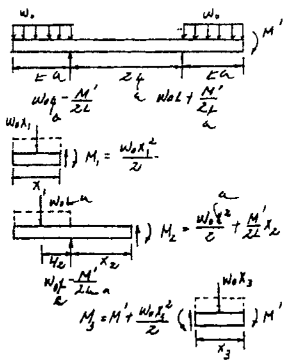
Setting $M' = 0$,

$$\theta_A = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \frac{1}{EI}\int_0^L \left(-\frac{wx^2}{2}\right)\left(\frac{x}{L} - 1\right)dx = \frac{-w}{2EI}\left[\frac{L^3}{4} - \frac{L^3}{3}\right]$$

$$= \frac{wL^3}{24\,EI} \qquad \textbf{Ans}$$

14–153 Solve Prob. 14–109 using Castigliano's theorem.





$$\frac{\partial M_1}{\partial M'} = 0 \qquad \frac{\partial M_2}{\partial M'} = \frac{x_2}{2a} \qquad \frac{\partial M_3}{\partial M'} = 1$$

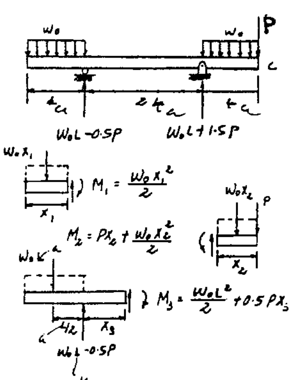
Setting $M' = 0$;

$$M_1 = \frac{wx_1^2}{2}; \qquad M_2 = \frac{wa^2}{2}; \qquad M_3 = \frac{wx_3^2}{2}$$

$$\theta_C = \int_0^a M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^a \left(\frac{w\,x_1^2}{2}\right)(0)dx_1 + \int_0^{2a}\left(\frac{w\,a^2}{2}\right)\left(\frac{x_2}{2\,a}\right)dx_2 + \int_0^a\left(\frac{wx_3^2}{2}\right)(1)dx_3\right]$$

$$= \frac{2\,w\,a^3}{3\,E\,I} \qquad \textbf{Ans}$$



$$\frac{\partial M_1}{\partial P} = 0; \qquad \frac{\partial M_2}{\partial P} = x_2; \qquad \frac{\partial M_3}{\partial P} = 0.5x_3$$

Setting $P = 0$;

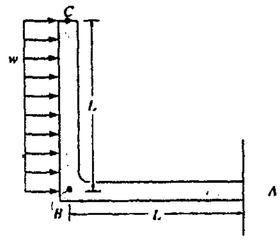
$$M_1 = \frac{wx_1^2}{2} \qquad M_2 = \frac{wx_2^2}{2} \qquad M_3 = \frac{wa^2}{2}$$

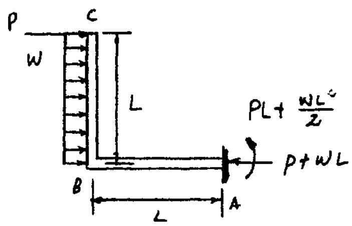
$$\Delta_C = \int_0^a M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

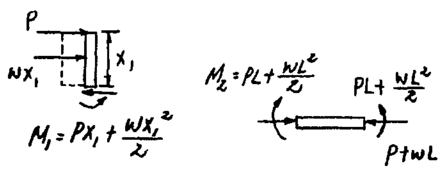
$$= \frac{1}{EI}\left[\int_0^a\left(\frac{w\,x_1^2}{2}\right)(0)dx_1 + \int_0^a\left(\frac{w\,x_2^2}{2}\right)(x_2)dx_2 + \int_0^{2a}\left(\frac{w\,a^2}{2}\right)(0.5x_3)dx_3\right]$$

$$= \frac{5\,wa^4}{8\,E\,I} \qquad \textbf{Ans}$$

14-154 Solve Prob. 14-113 using Castigliano's theorem.







$$M_1 = Px_1 + \frac{wx_1^2}{2}$$

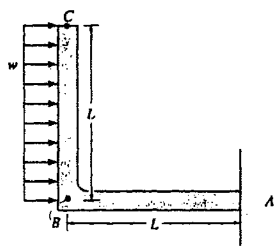
$$\frac{\partial M_1}{\partial P} = x_1 \qquad \frac{\partial M_2}{\partial P} = L$$

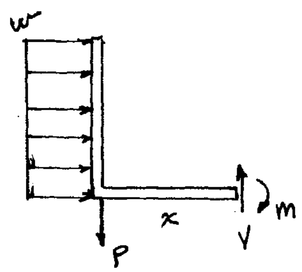
Setting $P = 0$

$$M_1 = \frac{wx_1^2}{2} \qquad M_2 = \frac{wL^2}{2}$$

$$\Delta_C = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \frac{1}{EI}\left[\int_0^L \frac{wx_1^2}{2}(x_1)dx_1 + \int_0^L \frac{wL^2}{2}L dx_2\right] = \frac{5wL^4}{8EI} \qquad \textbf{Ans}$$

14–155 Solve Prob. 14–114 using Castigliano's theorem.





P does not influence moment within segment.

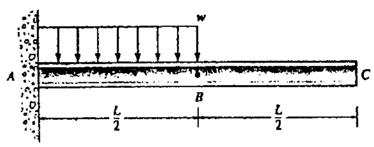
$$M = Px = \frac{wL^2}{2}$$

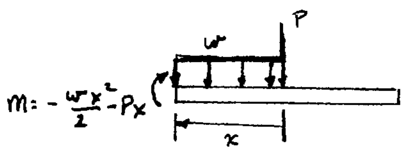
$$\frac{\partial M}{\partial P} = x$$

Set $P = 0$

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L \left(-\frac{wL^2}{2}\right)(x)\frac{dx}{EI} = \frac{wL^4}{4EI} \qquad \textbf{Ans}$$

•14–156 Solve Prob. 14–108 using Castigliano's theorem.





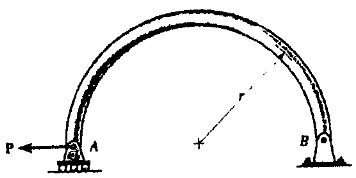
$$M = \frac{-wx^2}{2} - Px$$

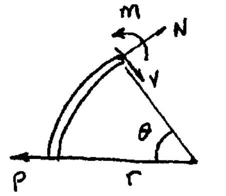
$$\frac{\partial M}{\partial P} = -x$$

$$\Delta_B = \int_0^L \left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{\frac{L}{2}} \frac{\left(\frac{-w\,x^2}{2}\right)(-1\,x)}{EI}dx = \frac{w\left(\frac{L}{2}\right)^4}{8\,EI}$$

$$= \frac{w\,L^4}{128\,EI}$$ **Ans**

14-157 Solve Prob. 14-116 using Castigliano's theorem.





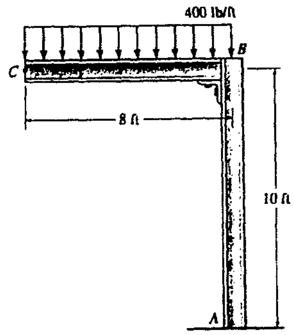
$$M = Pr\sin\theta$$

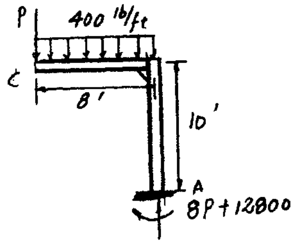
$$\frac{\partial M}{\partial P} = r\sin\theta$$

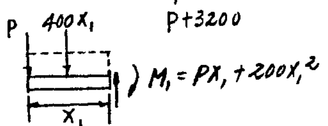
$$\Delta_A = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^\pi Pr^2\sin^2\theta\,\frac{(rd\theta)}{EI}$$

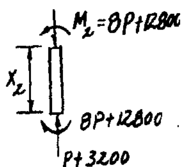
$$= \frac{Pr^3}{EI}\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]\Bigg|_0^\pi = \frac{\pi Pr^3}{2EI} \qquad \textbf{Ans}$$

14–158 Solve Prob. 14–115 using Castigliano's theorem.

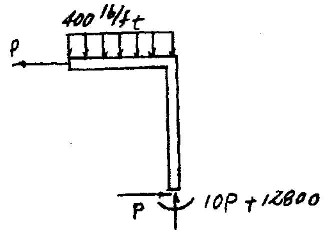


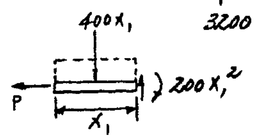


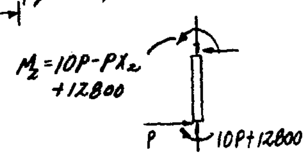




$$\frac{\partial M_1}{\partial P} = 0 \qquad \frac{\partial M_2}{\partial P} = 10 - x_2$$







Setting $P = 0$

$$M_1 = 200x_1^2 \qquad M_2 = 12\,800$$

$$\Delta_{C_h} = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^8 (200x_1^2)(0)dx_1 + \int_0^{10} 12\,800(10 - x_2)dx_2\right] = \frac{640\,000\ \text{lb}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans}$$

$$\frac{\partial M_1}{\partial P} = x_1 \qquad \frac{\partial M_2}{\partial P} = 8$$

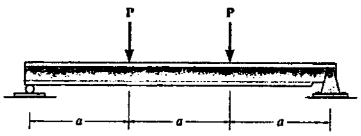
Setting $P = 0$

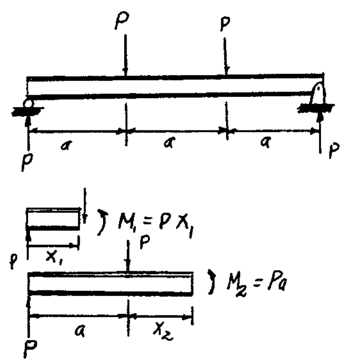
$$M_1 = 200x_1^2 \qquad M_2 = 12\,800$$

$$\Delta_{C_v} = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

$$= \frac{1}{EI}\left[\int_0^8 (200x_1^2)(x_1)dx_1 + \int_0^{10} (12\,800)(8)dx_2\right] = \frac{1\,228\,800\ \text{lb}\cdot\text{ft}^3}{EI} \qquad \textbf{Ans}$$

14–159 Determine the bending strain energy in the beam due to the loading shown. EI is constant.

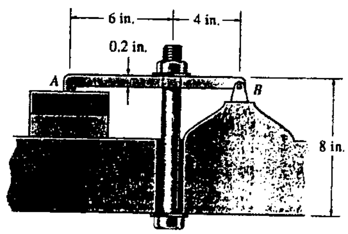


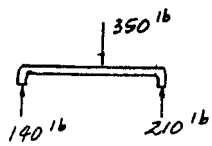


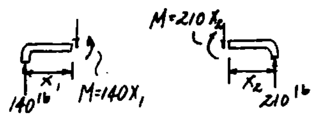
$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[2\int_0^a (Px_1)^2 dx_1 + \int_0^a (Pa)^2 dx_2\right]$$

$$= \frac{5P^2a^3}{6EI} \quad \textbf{Ans}$$

•14–160 The $L2$ steel bolt has a diameter of 0.25 in., and the link AB has a rectangular cross section that is 0.5 in. wide by 0.2 in. thick. Determine the strain energy in the link AB due to bending, and in the bolt due to axial force. The bolt is tightened so that it has a tension of 350 lb. Neglect the hole in the link.







Bending strain energy :

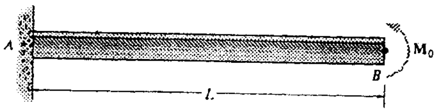
$$(U_b)_i = \int_0^L \frac{M^2 dx}{2EI} = \frac{1}{2EI}\left[\int_0^6 (140x_1)^2 dx_1 + \int_0^4 (210x_2)^2 dx_2\right]$$

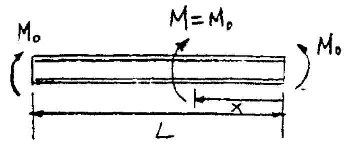
$$= \frac{1.176(10^6)}{EI} = \frac{1.176(10^6)}{29(10^6)(\frac{1}{12})(0.5)(0.2^3)} = 122 \text{ in}\cdot\text{lb} = 10.1 \text{ ft}\cdot\text{lb} \qquad \textbf{Ans}$$

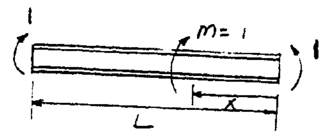
Axial force strain energy :

$$(U_a)_i = \int_0^L \frac{N^2 dx}{2EA} = \frac{N^2 L}{2AE} = \frac{(350)^2(8)}{2(29)(10^6)(\frac{\pi}{4})(0.25^2)} = 0.344 \text{ in}\cdot\text{lb} \qquad \textbf{Ans}$$

14–161 The cantilevered beam is subjected to a couple moment M_0 applied at its end. Determine the slope of the beam at B. EI is constant. Use the method of virtual work.



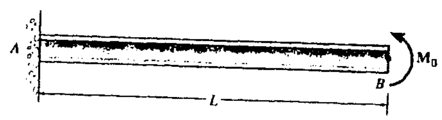


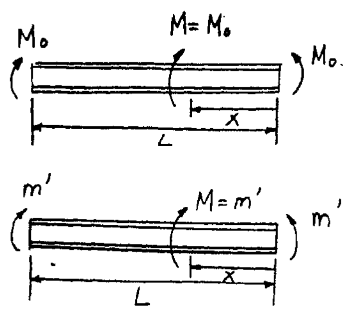


$$\theta_B = \int_0^L \frac{m_\theta M}{EI} dx = \int_0^L \frac{(1)\,M_0}{EI} dx$$

$$= \frac{M_0\,L}{EI} \qquad \textbf{Ans}$$

14–162 Solve Prob. 14–161 using Castigliano's theorem.

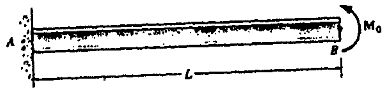


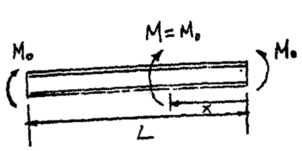


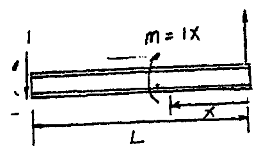
$$\theta_B = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI} = \int_0^L \frac{M_0\,(1)}{EI}dx$$

$$= \frac{M_0\,L}{EI} \qquad \textbf{Ans}$$

14-163. The cantilevered beam is subjected to a couple moment $\mathbf{M}_0$ applied at its end. Determine the displacement of the beam at B. EI is constant. Use the method of virtual work.



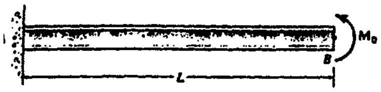




$$\Delta_B = \int_0^L \frac{mM}{EI}dx = \int_0^L \frac{(1x)\,M_0}{EI}dx$$

$$= \frac{M_0 L^2}{2EI} \qquad \textbf{Ans}$$

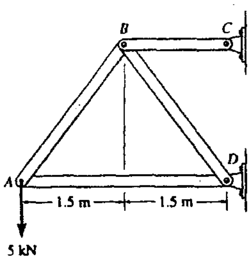
***14-164.** Solve Prob. 14–163 using Castigliano's theorem.

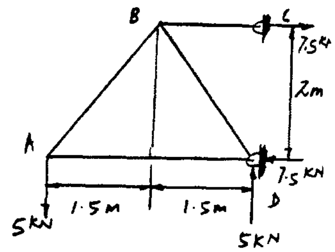


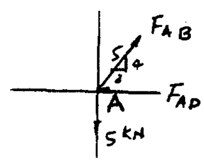
$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^L \frac{M_0\,(1x)}{EI}dx$$

$$= \frac{M_0\,L^2}{2EI} \qquad \textbf{Ans}$$

14-165 Determine the vertical displacement of joint *A*. Each bar is made of A-36 steel and has a cross-sectional area of 600 mm². Use the conservation of energy.





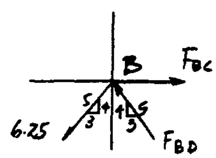


Joint *A* :

$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}F_{AB} - 5 = 0 \qquad F_{AB} = 6.25\text{ kN}$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_{AD} - \frac{3}{5}(6.25) = 0 \qquad F_{AD} = 3.75\text{ kN}$

Joint *B* :



$+\uparrow \Sigma F_y = 0; \quad \frac{4}{5}F_{BD} - \frac{4}{5}(6.25) = 0 \qquad F_{BD} = 6.25\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BC} - 2(\frac{3}{5})(6.25) = 0 \qquad F_{BC} = 7.5\text{ kN}$

Conservation of energy :

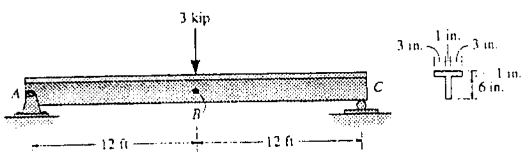
$$U_e = U_i$$

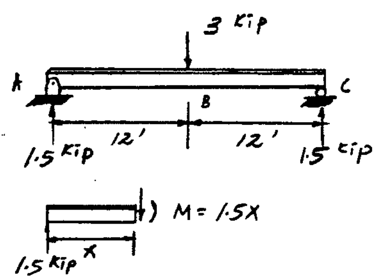
$$\frac{1}{2}P\Delta = \Sigma\frac{N^2L}{2AE}$$

$$\frac{1}{2}(5)(10^3)\Delta_A = \frac{1}{2AE} = [(6.25(10^3))^2(2.5) + (3.75(10^3))^2(3) + (6.25(10^3))^2(2.5) + (7.5(10^3))^2(1.5)]$$

$$\Delta_A = \frac{64\,375}{AE} = \frac{64\,375}{600(10^{-6})(200)(10^9)} = 0.5364(10^{-3})\text{ m} = 0.536\text{ mm} \qquad \textbf{Ans}$$

14-166 Determine the displacement of point B on the aluminum beam. $E_{al} = 10.6(10^3)$ ksi. Use the conservation of energy.





$$U_i = \int_0^L \frac{M^2 dx}{2EI} = (2)\frac{1}{2EI}\int_0^{12(12)} (1.5x)^2 dx = \frac{2239488}{EI}$$

$$U_e = \frac{1}{2}P\Delta = \frac{1}{2}(3)\Delta_B = 1.5\Delta_B$$

Conservation of energy :

$$U_e = U_i$$

$$1.5\Delta_B = \frac{2\,239\,488}{EI}$$

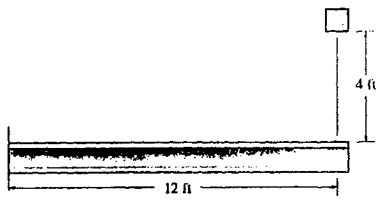
$$\Delta_B = \frac{1\,492\,992}{EI}$$

$$\bar{y} = \frac{0.5(7)(1) + (4)(6)(1)}{7(1) + 6(1)} = 2.1154 \text{ in.}$$

$$I = \frac{1}{12}(7)(1^3) + (7)(1)(2.1154 - 0.5)^2 + \frac{1}{12}(1)(6^3) + (1)(6)(4 - 2.1154)^2 = 58.16 \text{ in}^4$$

$$\Delta_B = \frac{1\,492\,992}{(10.6)(10^3)(58.16)} = 2.42 \text{ in.} \qquad \textbf{Ans}$$

***14–167** A 20-lb weight is dropped from a height of 4 ft onto the end of a cantilevered A-36 steel beam. If the beam is a W 12 × 50, determine the maximum stress developed in the beam.



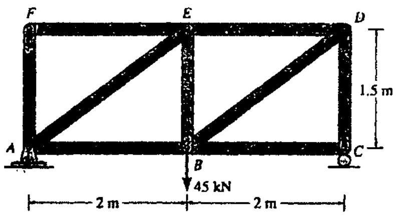
From Appendix C :

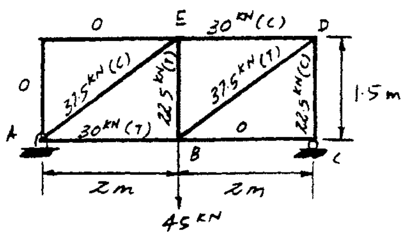
$$\Delta_{st} = \frac{PL^3}{3EI} = \frac{20(12(12))^3}{3(29)(10^6)(394)} = 1.742216(10^{-3})\ \text{in.}$$

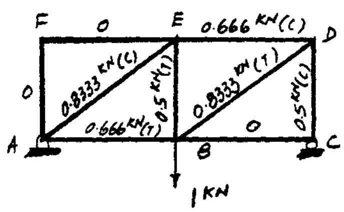
$$n = 1 + \sqrt{1 + 2\left(\frac{h}{\Delta_{st}}\right)} = 1 + \sqrt{1 + 2\left(\frac{4(12)}{1.742216(10^{-3})}\right)} = 235.74$$

$$\sigma_{max} = n\sigma_{st} = 235.74\left(\frac{20(12)(12)\left(\frac{12.19}{2}\right)}{394}\right) = 10503\ \text{psi} = 10.5\ \text{ksi} < \sigma_Y \quad \text{OK} \qquad \textbf{Ans}$$

14–168 Determine the vertical displacement of joint B. For each member $A = 400\ mm^2$, $E = 200$ GPa. Use the method of virtual work.





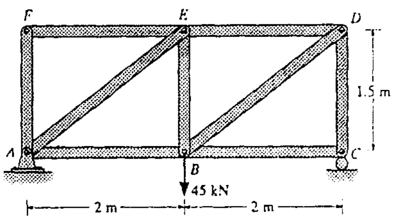


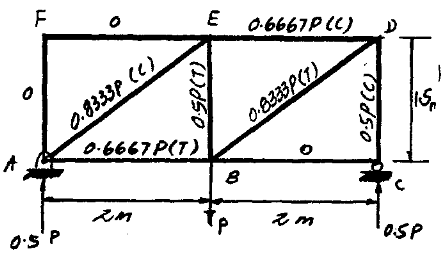
Member	n	N	L	nNL
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	0.50	22.5	1.5	16.875
ED	−0.6667	−30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875
				$\Sigma = 270$

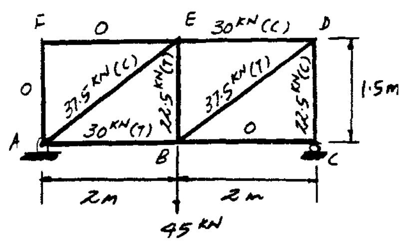
$$1 \cdot \Delta_{B_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{B_v} = \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\text{m} = 3.38\text{ mm} \qquad \textbf{Ans}$$

14–169 Solve Prob. 14–168 using Castigliano's theorem.







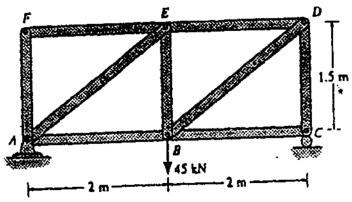
Member	N	$\partial N/\partial P$	$N\ (P = 45)$	L	$N(\partial N/\partial P)L$
AF	0	0	0	1.5	0
AE	$-0.8333P$	-0.8333	-37.5	2.5	78.125
AB	$0.6667P$	0.6667	30.0	2.0	40.00
BE	$0.5P$	0.5	22.5	1.5	16.875
BD	$0.8333P$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-0.5P$	-0.5	-22.5	1.5	16.875
DE	$-0.6667P$	-0.6667	-30.0	2.0	40.00
EF	0	0	0	2.0	0

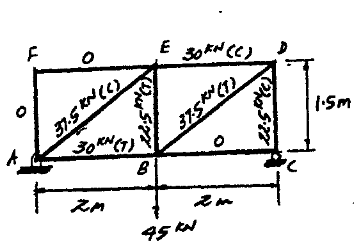
$\Sigma = 270$

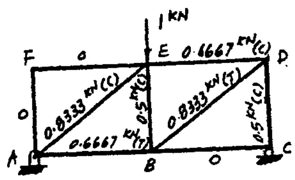
$$\Delta_{B_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{270}{AE}$$

$$= \frac{270(10^3)}{400(10^{-6})(200)(10^9)} = 3.375(10^{-3})\ \text{m} = 3.38\ \text{mm} \qquad \textbf{Ans}$$

14-170. Determine the vertical displacement of joint E. For each member $A = 400\ \text{mm}^2$, $E = 200\ \text{GPa}$. Use the method of virtual work.







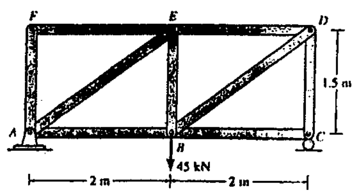
Member	n	N	L	nNL
AF	0	0	1.5	0
AE	−0.8333	−37.5	2.5	78.125
AB	0.6667	30.0	2.0	40.00
EF	0	0	2.0	0
EB	−0.50	30.0	1.5	−16.875
ED	−0.6667	30.0	2.0	40.00
BC	0	0	2.0	0
BD	0.8333	37.5	2.5	78.125
CD	−0.5	−22.5	1.5	16.875

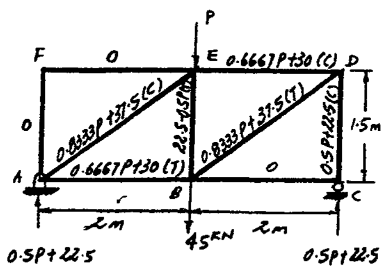
$\Sigma = 236.25$

$$1 \cdot \Delta_{E_v} = \Sigma \frac{nNL}{AE}$$

$$\Delta_{E_v} = \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3}) = 2.95\ \text{mm} \qquad \textbf{Ans}$$

14-171. Solve Prob. 14–170 using Castigliano's theorem.



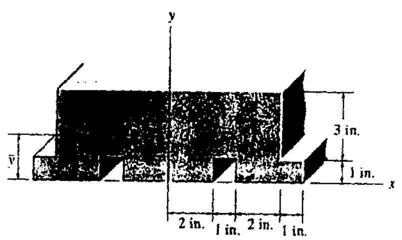


Member	N	$\partial N/\partial P$	$N(P=45)$	L	$N(\partial N/\partial P)L$
AF	0	0	0	1.5	0
AE	$-(0.8333P+37.5)$	−0.8333	−37.5	2.5	78.125
AB	$0.6667P+30$	0.6667	30.0	2.0	40.00
BE	$22.5-0.5P$	−0.5	22.5	1.5	−16.875
BD	$0.8333P+37.5$	0.8333	37.5	2.5	78.125
BC	0	0	0	2.0	0
CD	$-(0.5P+22.5)$	−0.5	−22.5	1.5	16.875
DE	$-(0.6667P+30)$	−0.6667	−30.0	2.0	40.00
EF	0	0	0	2.0	0
					$\Sigma = 236.25$

$$\Delta_{E_v} = \Sigma N\left(\frac{\partial N}{\partial P}\right)\frac{L}{AE} = \frac{236.25}{AE}$$

$$= \frac{236.25(10^3)}{400(10^{-6})(200)(10^9)} = 2.95(10^{-3})\text{m} = 2.95\text{ mm} \qquad \textbf{Ans}$$

A–1 Determine the location $\bar{y}$ of the centroid C for the beam's cross-sectional area. The beam is symmetric with respect to the y axis.

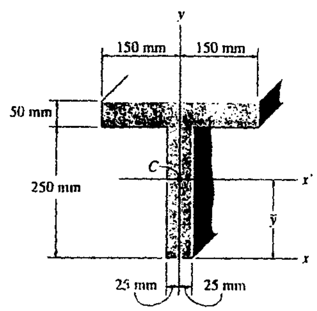


$$\Sigma\tilde{y}A = (2)(6)(4) - (0.5)(1)(1) - (2.5)(3)(1) = 40 \text{ in}^3$$

$$\Sigma A = 6(4) - 1(1) - 3(1) = 20 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{40}{20} = 2.0 \text{ in.} \qquad \textbf{Ans}$$

A-2 Determine $\bar{y}$, which locates the centroid, and then find the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_y$ for the T-beam.



$$\Sigma\, \tilde{y}A = 125(50)(250) + 275(300)(50) = 5687500 \text{ mm}^3$$

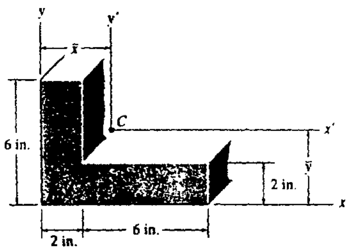
$$\Sigma\, A = 50(250) + 300(50) = 27500 \text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{5687500}{27500} = 206.82 \text{ mm} = 207 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \frac{1}{12}(50)(250)^3 + 50(250)(206.82 - 125)^2 +$$
$$\frac{1}{12}(300((50)^3 + 300(50)(275 - 206.82)^2 = 222(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

$$\bar{I}_y = \frac{1}{12}(250)(50^3) + \frac{1}{12}(50)(300^3) = 115(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

A-3 Determine the location ($\bar{x}$, $\bar{y}$) of the centroid C, then find the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_{y'}$.



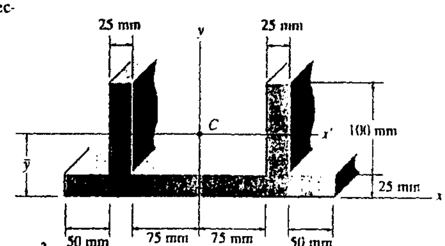
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(1)(2)(4) + (4)(2)(8)}{2(4) + (2)(8)} = 3 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(1)(6)(2) + (3)(2)(6)}{(6)(2) + (2)(6)} = 2 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \frac{1}{12}(6)(2^3) + (6)(2)(2-1)^2 + \frac{1}{12}(2)(6^3) + 2(6)(3-2)^2 = 64 \text{ in}^4 \qquad \textbf{Ans}$$

$$\bar{I}_{y'} = \frac{1}{12}(4)(2^3) + (4)(2)(3-1)^2 + \frac{1}{12}(2)(8^3) + (2)(8)(4-3)^2 = 136 \text{ in}^4 \qquad \textbf{Ans}$$

*A-4 Determine the centroid $\bar{y}$ for the beam's cross-sectional area, then find $\bar{I}_{x'}$.



$$\Sigma \tilde{y} A = 12.5(150)(25) + (75)(100)(25) = 234\,375 \text{ mm}^3$$

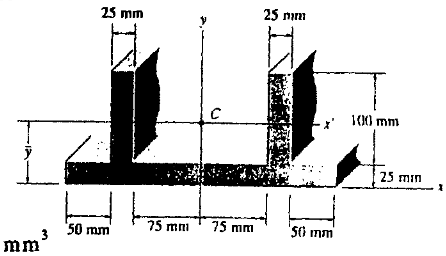
$$\Sigma A = 150(25) + 100(25) = 6250 \text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{234\,375}{6250} = 37.5 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \frac{1}{12}(300)(25^3) + 300(25)(37.5-12.5)^2 + 2[\frac{1}{12}(25)(100^3) + 25(100)(75-37.5)^2]$$

$$= 16.3(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

•A–4 Determine the centroid $\bar{y}$ for the beam's cross-sectional area, then find $\bar{I}_{x'}$.



$$\Sigma\tilde{y}A = 12.5(150)(25) + (75)(100)(25) = 234\,375\ \text{mm}^3$$

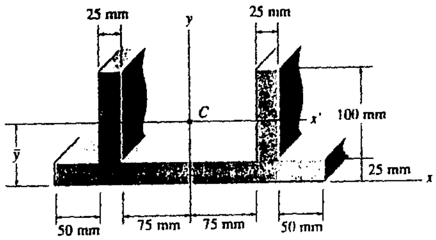
$$\Sigma A = 150(25) + 100(25) = 6250\ \text{mm}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{234\,375}{6250} = 37.5\ \text{mm} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \frac{1}{12}(300)(25^3) + 300(25)(37.5 - 12.5)^2 + 2[\frac{1}{12}(25)(100^3) + 25(100)(75 - 37.5)^2]$$

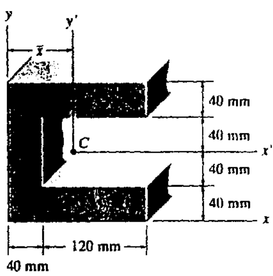
$$= 16.3(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

A-5 Determine I_y for the beam having the cross-sectional area shown.



$$I_y = \frac{1}{12}(25)(300^3) + 2[\frac{1}{12}(100)(25^3) + 100(25)(87.5^2)] = 94.8(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

A-6 Determine $\bar{x}$ which locates the centroid C, and then find the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_{y'}$ for the shaded area.



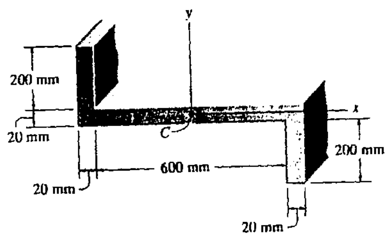
$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{(20)(160)(40) + 2[100(120)(40)]}{160(40) + 2[120(40)]} = 68.0 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{I}_{x'} = \tfrac{1}{12}(160)(160)^3 - \tfrac{1}{12}(120)(80)^3 = 49.5(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

$$\bar{I}_{y'} = \left[\tfrac{1}{12}(160)(160)^3 + (160)(160)(80 - 68.0)^2\right] - \left[\tfrac{1}{12}(80)(120)^3 + 80(120)(100 - 68.0)^2\right]$$

$$= 36.9(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

A–7 Determine the moments of inertia I_x and I_y of the Z-section. The origin of coordinates is at the centroid C.



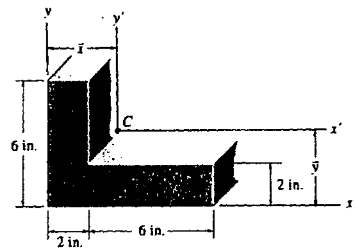
$$I_x = \frac{1}{12}(600)(20)^3 + 2[\frac{1}{12}(20)(220^3) + 20(220)(100^2)]$$

$$= 123.89(10^6)\ \text{mm}^4 = 124\ (10^6)\text{mm}^4 \qquad \textbf{Ans}$$

$$I_y = \frac{1}{12}(20)(600^3) + 2[\frac{1}{12}(220)(20)^3 + 220(20)(310)^2]$$

$$= 1205.97(10^6)\ \text{mm}^4 = 1.21(10^9)\ \text{mm}^4 \qquad \textbf{Ans}$$

*A–8 Determine the location ($\bar{x}, \bar{y}$) of the centroid C of the cross-sectional area for the angle, then find the product of inertia with respect to the x and y axes and with respect to the x' and y' axes.

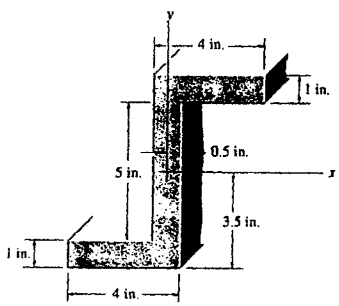


$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(1)(2)(4) + (4)(2)(8)}{2(4) + (2)(8)} = 3 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{(1)(6)(2) + (3)(2)(6)}{(6)(2) + (2)(6)} = 2 \text{ in.} \qquad \textbf{Ans}$$

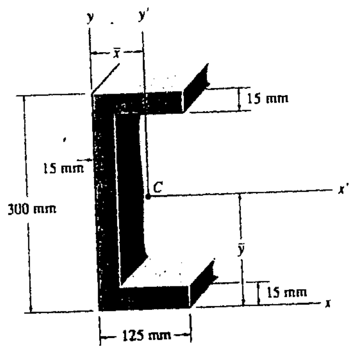
$$I_{x'y'} = \Sigma \tilde{x}\tilde{y} A = (-2)(1)(6)(2) + (2)(-1)(6)(2) = -48 \text{ in}^4 \qquad \textbf{Ans}$$

A–9 Determine the product of inertia of the cross-sectional area with respect to the x and y axes that have their origin located at the centroid C.



$I_{xy} = \Sigma \tilde{x}\tilde{y}A = (1.5)(3)(4)(1) + (0)(0)(5)(1) + (-1.5)(-3)(4)(1) = 36\text{ in}^4$ **Ans**

A-10 Locate the centroid ($\bar{x}$, $\bar{y}$) of the channel section and then determine the moments of inertia $\bar{I}_{x'}$ and $\bar{I}_{y'}$.



$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{(62.5)(2)(125)(15) + (7.5)(270)(15)}{2(125)(15) + 270(15)} = 33.942 \text{ mm} = 33.9 \text{ mm} \qquad \textbf{Ans}$$

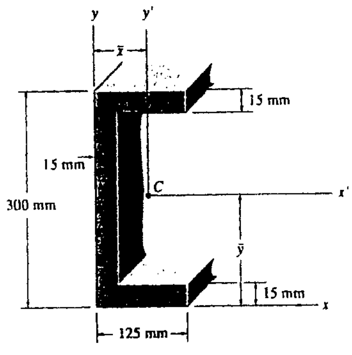
Due to symmetry $\bar{y} = 150$ mm **Ans**

$$\bar{I}_{x'} = \frac{1}{12}(125)(300^3) - \frac{1}{12}(110)(270^3) = 101(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

$$\bar{I}_{y'} = 2[\frac{1}{12}(15)(125^3) + 15(125)(62.5 - 33.942)^2] + \frac{1}{12}(270)(15^3)$$

$$+ 270(15)(33.942 - 7.5)^2 = 10.8(10^6) \text{ mm}^4 \qquad \textbf{Ans}$$

A–11 Locate the centroid ($\bar{x}$, $\bar{y}$) of the channel section and then determine the product of inertia $I_{x'y'}$ with respect to the x', y' axes.



$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{(62.5)(2)(125)(15) + (7.5)(270)(15)}{2(125)(15) + 270(15)} = 33.942 \text{ mm} = 33.9 \text{ mm} \qquad \textbf{Ans}$$

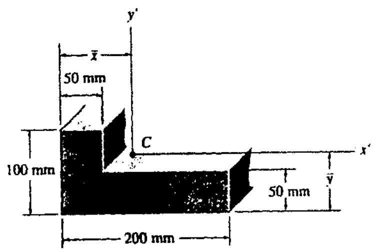
Due to symmetry $\bar{y} = 150$ mm **Ans**

$$I_{x'y'} = \Sigma\tilde{x}\tilde{y}A = (62.5 - 33.942)(150 - 7.5)(15)(125)$$

$$+ (7.5 - 33.942)(0)(270)(15)$$

$$+ (62.5 - 33.942)(7.5 - 150)(15)(125) = 0 \qquad \textbf{Ans}$$

*A–12 Locate the position ($\bar{x}$, $\bar{y}$) for the centroid C of the cross sectional area and then determine the product of inertia with respect to the x' and y' axes.

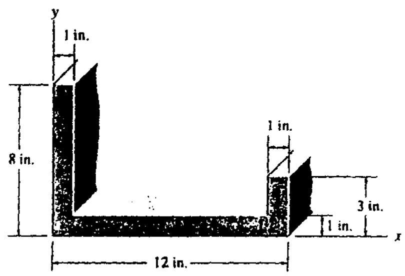


$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{25(50)(50) + 100(50)(200)}{50(50) + (50)(200)} = 85 \text{ mm} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{(50)(100)(50) + 25(150)(50)}{100(50) + 150(50)} = 35 \text{ mm} \qquad \textbf{Ans}$$

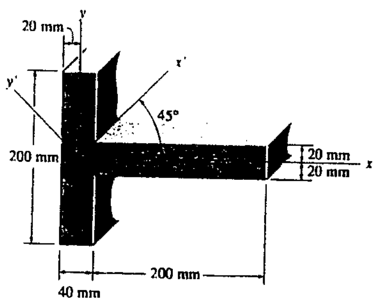
$$I_{x'y'} = \Sigma\tilde{x}\tilde{y}A = (-85+25)(75-35)(50)(50) + (100-85)(-35+25)(200)(50) = -7.50(10^6)\text{ mm}^4 \qquad \textbf{Ans}$$

A–13 Determine the product of inertia of the area with respect to the *x* and *y* axes.



$$I_{xy} = \Sigma \tilde{x}\tilde{y}A = (0.5)(4)(8)(1) + (6)(0.5)(10)(1) + (11.5)(1.5)(3)(1) = 97.75 \text{ in}^4 \qquad \textbf{Ans}$$

A–14 Determine the moments of inertia $I_{x'}$ and $I_{y'}$ of the shaded area.



$$I_x = \frac{1}{12}(40)(200^3) + \frac{1}{12}(200)(40^3) = 27.733(10^6)\ \text{mm}^4$$

$$I_y = \frac{1}{12}(200)(40^3) + \frac{1}{12}(40)(200^3) + (40)(200)(120^2) = 142.933(10^6)\ \text{mm}^4$$

$I_{xy} = 0$ (Symmetry about x axis)

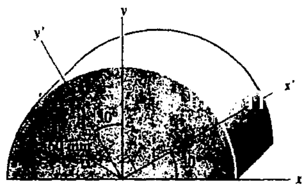
$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= [\frac{27.733 + 142.933}{2} + \frac{27.733 - 142.933}{2}\cos 90° + 0](10^6) = 85.3(10^6)\ \text{mm}^4$$ **Ans**

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= [\frac{27.733 + 142.933}{2} - \frac{27.733 - 142.933}{2}\cos 90° + 0](10^6) = 85.3(10^6)\ \text{mm}^4$$ **Ans**

A–15 Determine the moments of inertia $I_{x'}$ and $I_{y'}$ and the product of inertia $I_{x'y'}$ for the semicircular area.



$$I_x = I_y = \frac{1}{8}(\pi)(60^4) = 5.0894(10^6)\ \text{mm}^4$$

$I_{xy} = 0$ (symmetry about y axis)

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= [\frac{5.0894 + 5.0894}{2} + \frac{5.0894 - 5.0894}{2}\cos 60° - 0](10^6) = 5.09(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

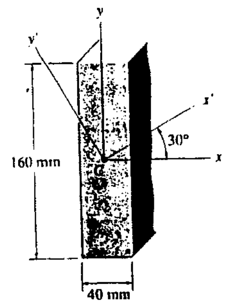
$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= [\frac{5.0894 + 5.0894}{2} - \frac{5.0894 - 5.0894}{2}\cos 60° + 0](10^6) = 5.09(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= [\frac{5.0894 - 5.0894}{2}\sin 60° + 0](10^6) = 0 \qquad \textbf{Ans}$$

•A–16 Determine the moments of inertia $I_{x'}$ and $I_{y'}$ and the product of inertia $I_{x'y'}$ for the rectangular area. The x' and y' axes pass through the centroid C.



$$I_x = \frac{1}{12}(40)(160^3) = 13.653(10^6)\ \text{mm}^4$$

$$I_y = \frac{1}{12}(160)(40^3) = 0.853(10^6)\ \text{mm}^4$$

$$I_{xy} = 0 \qquad \text{(symmetry)}$$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$= [\frac{13.653 + 0.853}{2} + \frac{13.653 - 0.853}{2}\cos 60° - 0](10^6) = 10.5(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

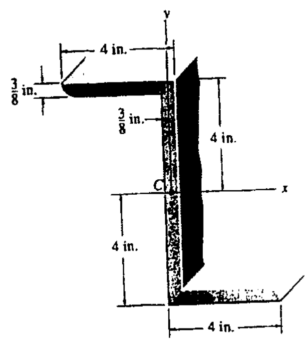
$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$= [\frac{13.653 + 0.853}{2} - \frac{13.653 - 0.853}{2}\cos 60° + 0](10^6) = 4.05(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

$$I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

$$= [\frac{13.653 - 0.853}{2}\sin 60° + 0] = 5.54(10^6)\ \text{mm}^4 \qquad \textbf{Ans}$$

A-17 Determine the principal moments of inertia of the cross-sectional area about the principal axes that have their origin located at the centroid C. Use the equations developed in Sec. A.4. For the calculation, assume all corners to be square.



$$I_x = 2[\frac{1}{12}(4)(0.375)^3 + (4)(0.375)(4-0.1875)^2] + \frac{1}{12}(0.375)(7.25)^3 = 55.55 \text{ in}^4$$

$$I_y = 2[\frac{1}{12}(0.375)(4^3) + (0.375)(4)(2-0.1875)^2] + \frac{1}{12}(7.25)(0.375)^3 = 13.89 \text{ in}^4$$

$$I_{xy} = \Sigma\bar{x}\bar{y}A = (-2+0.1875)(4-0.1875)(4)(0.375) + (0)(0)(7.25)(0.375) + (1.8125)(-3.1825)(4)(0.375) = -20.73 \text{ in}^4$$

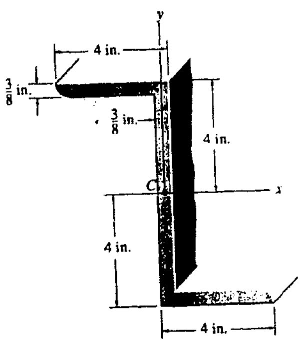
$$I_{\substack{max\\min}} = \frac{I_x + I_y}{2} \pm \sqrt{(\frac{I_x - I_y}{2})^2 + I_{xy}^2}$$

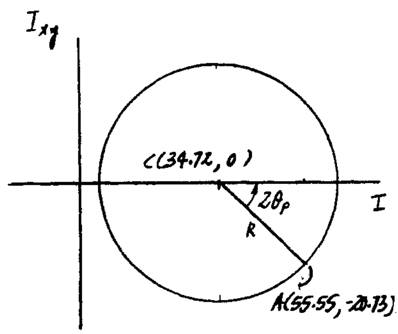
$$= \frac{55.55+13.89}{2} \pm \sqrt{(\frac{55.55-13.89}{2})^2 + (-20.73)^2}$$

$I_{max} = 64.1 \text{ in}^4$ **Ans**

$I_{min} = 5.33 \text{ in}^4$ **Ans**

A–18 Solve Prob. A–17 using Mohr's circle.





$$I_x = 2[\frac{1}{12}(4)(0.375)^3 + (4)(0.375)(4-0.1875)^2] + \frac{1}{12}(0.375)(7.25)^3 = 55.55 \text{ in}^4$$

$$I_y = 2[\frac{1}{12}(0.375)(4^3) + (0.375)(4)(2-0.1875)^2] + \frac{1}{12}(7.25)(0.375)^3 = 13.89 \text{ in}^4$$

$$I_{xy} = \Sigma\bar{x}\bar{y}A = (-2+0.1875)(4-0.1875)(4)(0.375) + (0)(0)(7.25)(0.375) + (1.8125)(-3.1825)(4)(0.375) = -20.73 \text{ in}^4$$

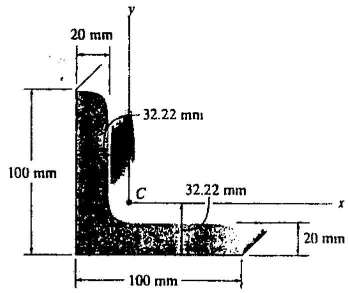
$$\frac{I_x + I_y}{2} = \frac{55.55 + 13.89}{2} = 34.72$$

$A(55.55, -20.73)$
$C(34.72, 0)$

$$R = \sqrt{(55.55-34.72)^2 + (20.73)^2} = 29.387$$

$I_{max} = 34.72 + 29.387 = 64.1 \text{ in}^4$ **Ans**
$I_{min} = 34.72 - 29.387 = 5.33 \text{ in}^4$ **Ans**

A–19 Determine the principal moments of inertia for the angle's cross-sectional area with respect to a set of principal axes that have their origin located at the centroid *C*. Use the equations developed in Sec. *A*.4. For the calculation, assume all corners to be square.



$$I_x = I_y = \frac{1}{12}(80)(20^3) + 80(20)(32.22-10)^2 + \frac{1}{12}(20)(100^3) + 20(100)(50-32.22)^2$$

$$I_x = I_y = 3.1422(10^6)\ \text{mm}^4$$

$$I_{xy} = \Sigma\tilde{x}\tilde{y}A = (-32.22+10)(50-32.22)(100)(20) + (60-32.22)(-32.22+10)(80)(20)$$
$$= 1.7778(10^6)\ \text{mm}^4$$

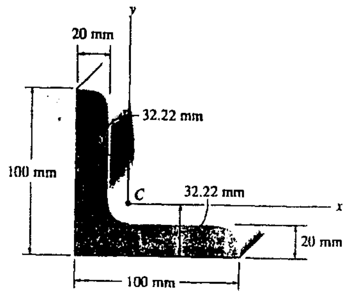
$$I_{\substack{max \\ min}} = \frac{I_x+I_y}{2} \pm \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}^2}$$

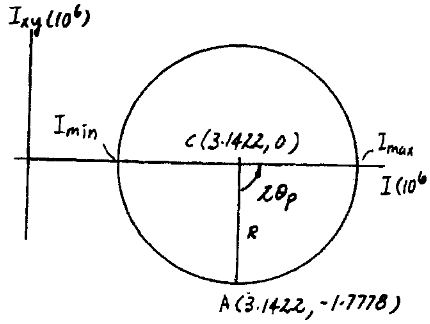
$$= \left[\frac{3.1422+3.1422}{2} \pm \sqrt{\left(\frac{3.1422-3.1422}{2}\right)^2 + (1.7778)^2}\,\right](10^6)$$

$I_{max} = 4.92(10^6)\ \text{mm}^4$ **Ans**

$I_{min} = 1.36(10^6)\ \text{mm}^4$ **Ans**

•A–20 Solve Prob. A–19 using Mohr's circle.





$$I_x = I_y = \frac{1}{12}(80)(20^3) + 80(20)(32.22-10)^2 + \frac{1}{12}(20)(100^3) + 20(100)(50-32.22)^2$$

$$I_x = I_y = 3.1422(10^6)\ \text{mm}^4$$

$$I_{xy} = \Sigma \tilde{x}\tilde{y}A = (-32.22+10)(50-32.22)(100)(20) + (60-32.22)(-32.22+10)(80)(20)$$
$$= 1.7778(10^6)\ \text{mm}^4$$

$A(3.1422, -1.7778)(10^6)$
$C(3.1422, 0)$
$R = 1.7778$

$I_{max} = (3.1422 + 1.7778)(10^6) = 4.92(10^6)\ \text{mm}^4$ **Ans**
$I_{min} = (3.1422 - 1.7778)(10^6) = 1.36(10^6)\ \text{mm}^4$ **Ans**