Chapter 1

1. **THINK** In this problem we're given the radius of Earth, and asked to compute its circumference, surface area and volume.

EXPRESS Assuming Earth to be a sphere of radius

$$R_E = \left(6.37 \times 10^6 \text{ m}\right)\left(10^{-3} \text{ km/m}\right) = 6.37 \times 10^3 \text{ km},$$

the corresponding circumference, surface area and volume are:

$$C = 2\pi R_E, \quad A = 4\pi R_E^2, \quad V = \frac{4\pi}{3} R_E^3.$$

The geometric formulas are given in Appendix E.

ANALYZE (a) Using the formulas given above, we find the circumference to be

$$C = 2\pi R_E = 2\pi(6.37 \times 10^3 \text{ km}) = 4.00 \times 10^4 \text{ km}.$$

(b) Similarly, the surface area of Earth is

$$A = 4\pi R_E^2 = 4\pi\left(6.37 \times 10^3 \text{ km}\right)^2 = 5.10 \times 10^8 \text{ km}^2,$$

(c) and its volume is

$$V = \frac{4\pi}{3} R_E^3 = \frac{4\pi}{3}\left(6.37 \times 10^3 \text{ km}\right)^3 = 1.08 \times 10^{12} \text{ km}^3.$$

LEARN From the formulas given, we see that $C \sim R_E$, $A \sim R_E^2$, and $V \sim R_E^3$. The ratios of volume to surface area, and surface area to circumference are $V/A = R_E/3$ and $A/C = 2R_E$.

2. The conversion factors are: $1 \text{ gry} = 1/10 \text{ line}$, $1 \text{ line} = 1/12 \text{ inch}$ and 1 point = 1/72 inch. The factors imply that

$$1 \text{ gry} = (1/10)(1/12)(72 \text{ points}) = 0.60 \text{ point}.$$

Thus, 1 gry^2 = (0.60 point)^2 = 0.36 point^2, which means that $0.50 \text{ gry}^2 = 0.18 \text{ point}^2$.

3. The metric prefixes (micro, pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2).

(a) Since 1 km = 1 × 10^3 m and 1 m = 1 × 10^6 μm,

$$1\,\text{km} = 10^3\ \text{m} = \left(10^3\ \text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^9\ \mu\text{m}.$$

The given measurement is 1.0 km (two significant figures), which implies our result should be written as 1.0 × 10^9 μm.

(b) We calculate the number of microns in 1 centimeter. Since 1 cm = 10^{-2} m,

$$1\,\text{cm} = 10^{-2}\ \text{m} = \left(10^{-2}\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 10^4\ \mu\text{m}.$$

We conclude that the fraction of one centimeter equal to 1.0 μm is 1.0 × 10^{-4}.

(c) Since 1 yd = (3 ft)(0.3048 m/ft) = 0.9144 m,

$$1.0\,\text{yd} = \left(0.91\text{m}\right)\left(10^6\ \mu\text{m/m}\right) = 9.1 \times 10^5\ \mu\text{m}.$$

4. (a) Using the conversion factors 1 inch = 2.54 cm exactly and 6 picas = 1 inch, we obtain

$$0.80\ \text{cm} = \left(0.80\ \text{cm}\right)\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{6\ \text{picas}}{1\ \text{inch}}\right) \approx 1.9\ \text{picas}.$$

(b) With 12 points = 1 pica, we have

$$0.80\ \text{cm} = \left(0.80\ \text{cm}\right)\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{6\ \text{picas}}{1\ \text{inch}}\right)\left(\frac{12\ \text{points}}{1\ \text{pica}}\right) \approx 23\ \text{points}.$$

5. **THINK** This problem deals with conversion of furlongs to rods and chains, all of which are units for distance.

EXPRESS Given that $1\ \text{furlong} = 201.168\ \text{m}$, $1\ \text{rod} = 5.0292\ \text{m}$ and $1\ \text{chain} = 20.117\ \text{m}$, the relevant conversion factors are

$$1.0\ \text{furlong} = 201.168\ \text{m} = (201.168\ \cancel{\text{m}})\frac{1\ \text{rod}}{5.0292\ \cancel{\text{m}}} = 40\ \text{rods},$$

and

$$1.0\ \text{furlong} = 201.168\ \text{m} = (201.168\ \cancel{\text{m}})\frac{1\ \text{chain}}{20.117\ \cancel{\text{m}}} = 10\ \text{chains}.$$

Note the cancellation of m (meters), the unwanted unit.

ANALYZE Using the above conversion factors, we find

(a) the distance *d* in *rods* to be $d = 4.0\ \text{furlongs} = (4.0\ \text{fu}\cancel{\text{r}}\text{longs})\dfrac{40\ \text{rods}}{1\ \text{fu}\cancel{\text{r}}\text{long}} = 160\ \text{rods},$

(b) and in *chains* to be $d = 4.0 \text{ furlongs} = (4.0 \text{ fu}\cancel{r}\text{longs})\dfrac{10 \text{ chains}}{1 \text{ fu}\cancel{r}\text{long}} = 40 \text{ chains}.$

LEARN Since 4 furlongs is about 800 m, this distance is approximately equal to 160 rods ($1 \text{ rod} \approx 5 \text{ m}$) and 40 chains ($1 \text{ chain} \approx 20 \text{ m}$). So our results make sense.

6. We make use of Table 1-6.

(a) We look at the first ("cahiz") column: 1 fanega is equivalent to what amount of cahiz? We note from the already completed part of the table that 1 cahiz equals a dozen fanega. Thus, 1 fanega = $\frac{1}{12}$ cahiz, or 8.33×10^{-2} cahiz. Similarly, "1 cahiz = 48 cuartilla" (in the already completed part) implies that 1 cuartilla = $\frac{1}{48}$ cahiz, or 2.08×10^{-2} cahiz. Continuing in this way, the remaining entries in the first column are 6.94×10^{-3} and 3.47×10^{-3}.

(b) In the second ("fanega") column, we find 0.250, 8.33×10^{-2}, and 4.17×10^{-2} for the last three entries.

(c) In the third ("cuartilla") column, we obtain 0.333 and 0.167 for the last two entries.

(d) Finally, in the fourth ("almude") column, we get $\frac{1}{2}$ = 0.500 for the last entry.

(e) Since the conversion table indicates that 1 almude is equivalent to 2 medios, our amount of 7.00 almudes must be equal to 14.0 medios.

(f) Using the value (1 almude = 6.94×10^{-3} cahiz) found in part (a), we conclude that 7.00 almudes is equivalent to 4.86×10^{-2} cahiz.

(g) Since each decimeter is 0.1 meter, then 55.501 cubic decimeters is equal to 0.055501 m^3 or 55501 cm^3. Thus, 7.00 almudes = $\frac{7.00}{12}$ fanega = $\frac{7.00}{12}$(55501 cm^3) = 3.24×10^4 cm^3.

7. We use the conversion factors found in Appendix D.

$$1 \text{ acre} \cdot \text{ft} = (43{,}560 \text{ ft}^2) \cdot \text{ft} = 43{,}560 \text{ ft}^3$$

Since 2 in. = (1/6) ft, the volume of water that fell during the storm is

$$V = (26 \text{ km}^2)(1/6 \text{ ft}) = (26 \text{ km}^2)(3281 \text{ft/km})^2(1/6 \text{ ft}) = 4.66 \times 10^7 \text{ ft}^3.$$

Thus,

$$V = \frac{4.66 \times 10^7 \text{ ft}^3}{4.3560 \times 10^4 \text{ ft}^3 / \text{acre} \cdot \text{ft}} = 1.1 \times 10^3 \text{ acre} \cdot \text{ft}.$$

8. From Fig. 1-4, we see that 212 S is equivalent to 258 W and 212 – 32 = 180 S is equivalent to 216 – 60 = 156 Z. The information allows us to convert S to W or Z.

(a) In units of W, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{258 \text{ W}}{212 \text{ S}}\right) = 60.8 \text{ W}$$

(b) In units of Z, we have

$$50.0 \text{ S} = (50.0 \text{ S})\left(\frac{156 \text{ Z}}{180 \text{ S}}\right) = 43.3 \text{ Z}$$

9. The volume of ice is given by the product of the semicircular surface area and the thickness. The area of the semicircle is $A = \pi r^2/2$, where r is the radius. Therefore, the volume is

$$V = \frac{\pi}{2} r^2 z$$

where z is the ice thickness. Since there are 10^3 m in 1 km and 10^2 cm in 1 m, we have

$$r = (2000\,\text{km})\left(\frac{10^3\,\text{m}}{1\text{km}}\right)\left(\frac{10^2\,\text{cm}}{1\text{m}}\right) = 2000 \times 10^5 \text{ cm}.$$

In these units, the thickness becomes

$$z = 3000\,\text{m} = (3000\,\text{m})\left(\frac{10^2\,\text{cm}}{1\text{m}}\right) = 3000 \times 10^2 \text{ cm}$$

which yields $V = \frac{\pi}{2}\left(2000 \times 10^5 \text{ cm}\right)^2 \left(3000 \times 10^2 \text{ cm}\right) = 1.9 \times 10^{22} \text{ cm}^3$.

10. Since a change of longitude equal to $360°$ corresponds to a 24 hour change, then one expects to change longitude by $360°/24 = 15°$ before resetting one's watch by 1.0 h.

11. (a) Presuming that a French decimal day is equivalent to a regular day, then the ratio of weeks is simply 10/7 or (to 3 significant figures) 1.43.

(b) In a regular day, there are 86400 seconds, but in the French system described in the problem, there would be 10^5 seconds. The ratio is therefore 0.864.

12. A day is equivalent to 86400 seconds and a meter is equivalent to a million micrometers, so

$$\frac{(3.7\ \text{m})(10^6\ \mu\text{m/m})}{(14\ \text{day})(86400\ \text{s/day})} = 3.1\ \mu\text{m/s}.$$

13. The time on any of these clocks is a straight-line function of that on another, with slopes $\neq$ 1 and y-intercepts $\neq$ 0. From the data in the figure we deduce

$$t_C = \frac{2}{7}t_B + \frac{594}{7}, \quad t_B = \frac{33}{40}t_A - \frac{662}{5}.$$

These are used in obtaining the following results.

(a) We find

$$t'_B - t_B = \frac{33}{40}(t'_A - t_A) = 495\ \text{s}$$

when $t'_A - t_A = 600$ s.

(b) We obtain $t'_C - t_C = \frac{2}{7}(t'_B - t_B) = \frac{2}{7}(495) = 141$ s.

(c) Clock B reads $t_B = (33/40)(400) - (662/5) \approx 198$ s when clock A reads $t_A = 400$ s.

(d) From $t_C = 15 = (2/7)t_B + (594/7)$, we get $t_B \approx -245$ s.

14. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (also Table 1–2).

(a) $1\ \mu\text{century} = (10^{-6}\ \text{century})\left(\frac{100\ \text{y}}{1\ \text{century}}\right)\left(\frac{365\ \text{day}}{1\ \text{y}}\right)\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 52.6\ \text{min}.$

(b) The percent difference is therefore

$$\frac{52.6\ \text{min} - 50\ \text{min}}{52.6\ \text{min}} = 4.9\%.$$

15. A week is 7 days, each of which has 24 hours, and an hour is equivalent to 3600 seconds. Thus, two weeks (a fortnight) is 1209600 s. By definition of the micro prefix, this is roughly $1.21 \times 10^{12}\ \mu$s.

16. We denote the pulsar rotation rate f (for frequency).

$$f = \frac{1\ \text{rotation}}{1.55780644887275 \times 10^{-3}\ \text{s}}$$

(a) Multiplying f by the time-interval t = 7.00 days (which is equivalent to 604800 s, if we ignore *significant figure* considerations for a moment), we obtain the number of rotations:

$$N = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}\right)(604800 \text{ s}) = 388238218.4$$

which should now be rounded to 3.88×10^8 rotations since the time-interval was specified in the problem to three significant figures.

(b) We note that the problem specifies the *exact* number of pulsar revolutions (one million). In this case, our unknown is t, and an equation similar to the one we set up in part (a) takes the form $N = ft$, or

$$1 \times 10^6 = \left(\frac{1 \text{ rotation}}{1.55780644887275 \times 10^{-3} \text{ s}}\right) t$$

which yields the result t = 1557.80644887275 s (though students who do this calculation on their calculator might not obtain those last several digits).

(c) Careful reading of the problem shows that the time-uncertainty *per revolution* is $\pm 3\times 10^{-17}$s . We therefore expect that as a result of one million revolutions, the uncertainty should be $(\pm 3\times 10^{-17})(1\times 10^{6}) = \pm 3\times 10^{-11}$ s .

17. **THINK** In this problem we are asked to rank 5 clocks, based on their performance as timekeepers.

EXPRESS We first note that none of the clocks advance by exactly 24 h in a 24-h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important here is that the clock advance by the same (or nearly the same) amount in each 24-h period. The clock reading can then easily be adjusted to give the correct interval.

ANALYZE The chart below gives the corrections (in seconds) that must be applied to the reading on each clock for each 24-h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift (relative to WWF time); thus, C and D are easily made “perfect” with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17s. For clock B it is the range from –5 s to +10 s, for clock E it is in the range from –70 s to –2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best to worst, the ranking of the clocks is C, D, A, B, E.

CLOCK	Sun. -Mon.	Mon. -Tues.	Tues. -Wed.	Wed. -Thurs.	Thurs. -Fri.	Fri. -Sat.
A	–16	–16	–15	–17	–15	–15
B	–3	+5	–10	+5	+6	–7
C	–58	–58	–58	–58	–58	–58
D	+67	+67	+67	+67	+67	+67
E	+70	+55	+2	+20	+10	+10

LEARN Of the five clocks, the readings in clocks A, B and E jump around from one 24-h period to another, making it difficult to correct them.

18. The last day of the 20 centuries is longer than the first day by

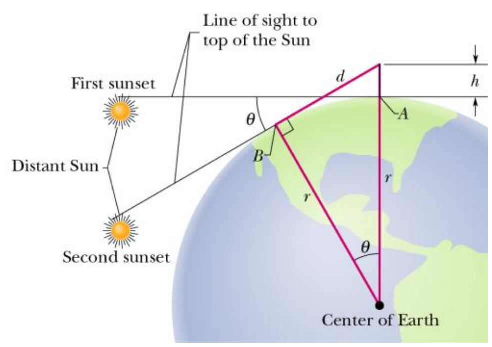
$$(20 \text{ century})\,(0.001 \text{ s/century}) = 0.02 \text{ s.}$$

The average day during the 20 centuries is (0 + 0.02)/2 = 0.01 s longer than the first day. Since the increase occurs uniformly, the cumulative effect T is

$$\begin{aligned} T &= (\text{average increase in length of a day})(\text{number of days}) \\ &= \left(\frac{0.01 \text{ s}}{\text{day}}\right)\left(\frac{365.25 \text{ day}}{\text{y}}\right)(2000 \text{ y}) \\ &= 7305 \text{ s} \end{aligned}$$

or roughly two hours.

19. When the Sun first disappears while lying down, your line of sight to the top of the Sun is tangent to the Earth's surface at point A shown in the figure. As you stand, elevating your eyes by a height h, the line of sight to the Sun is tangent to the Earth's surface at point B.



Let d be the distance from point B to your eyes. From the Pythagorean theorem, we have

$$d^2 + r^2 = (r+h)^2 = r^2 + 2rh + h^2$$

or $d^2 = 2rh + h^2$, where r is the radius of the Earth. Since $r \gg h$, the second term can be dropped, leading to $d^2 \approx 2rh$. Now the angle between the two radii to the two tangent points A and B is θ, which is also the angle through which the Sun moves about Earth during the time interval t = 11.1 s. The value of θ can be obtained by using

$$\frac{\theta}{360°} = \frac{t}{24\ \text{h}}.$$

This yields

$$\theta = \frac{(360°)(11.1\ \text{s})}{(24\ \text{h})(60\ \text{min/h})(60\ \text{s/min})} = 0.04625°.$$

Using $d = r\tan\theta$, we have $d^2 = r^2\tan^2\theta = 2rh$, or

$$r = \frac{2h}{\tan^2\theta}$$

Using the above value for θ and h = 1.7 m, we have $r = 5.2\times10^6$ m.

20. (a) We find the volume in cubic centimeters

$$193\ \text{gal} = (193\ \text{gal})\left(\frac{231\ \text{in}^3}{1\ \text{gal}}\right)\left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right)^3 = 7.31\times10^5\ \text{cm}^3$$

and subtract this from 1×10^6 cm^3 to obtain 2.69×10^5 cm^3. The conversion gal $\rightarrow$ in^3 is given in Appendix D (immediately below the table of Volume conversions).

(b) The volume found in part (a) is converted (by dividing by (100 cm/m)^3) to 0.731 m^3, which corresponds to a mass of

$$(1000\ \text{kg/m}^3)\ (0.731\ \text{m}^2) = 731\ \text{kg}$$

using the density given in the problem statement. At a rate of 0.0018 kg/min, this can be filled in

$$\frac{731\text{kg}}{0.0018\ \text{kg/min}} = 4.06\times10^5\ \text{min} = 0.77\ \text{y}$$

after dividing by the number of minutes in a year (365 days)(24 h/day) (60 min/h).

21. If M_E is the mass of Earth, m is the average mass of an atom in Earth, and N is the number of atoms, then $M_E = Nm$ or $N = M_E/m$. We convert mass m to kilograms using Appendix D (1 u = 1.661×10^{-27} kg). Thus,

$$N = \frac{M_E}{m} = \frac{5.98 \times 10^{24}\ \text{kg}}{(40\ \text{u})\left(1.661 \times 10^{-27}\ \text{kg/u}\right)} = 9.0 \times 10^{49}.$$

22. The density of gold is

$$\rho = \frac{m}{V} = \frac{19.32\ \text{g}}{1\ \text{cm}^3} = 19.32\ \text{g/cm}^3.$$

(a) We take the volume of the leaf to be its area A multiplied by its thickness z. With density $\rho = 19.32\ \text{g/cm}^3$ and mass $m = 27.63$ g, the volume of the leaf is found to be

$$V = \frac{m}{\rho} = 1.430\ \text{cm}^3.$$

We convert the volume to SI units:

$$V = \left(1.430\ \text{cm}^3\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = 1.430 \times 10^{-6}\ \text{m}^3.$$

Since $V = Az$ with $z = 1 \times 10^{-6}$ m (metric prefixes can be found in Table 1–2), we obtain

$$A = \frac{1.430 \times 10^{-6}\ \text{m}^3}{1 \times 10^{-6}\ \text{m}} = 1.430\ \text{m}^2.$$

(b) The volume of a cylinder of length ℓ is $V = A\ell$ where the cross-section area is that of a circle: $A = \pi r^2$. Therefore, with $r = 2.500 \times 10^{-6}$ m and $V = 1.430 \times 10^{-6}\ \text{m}^3$, we obtain

$$\ell = \frac{V}{\pi r^2} = 7.284 \times 10^4\ \text{m} = 72.84\ \text{km}.$$

23. **THINK** This problem consists of two parts: in the first part, we are asked to find the mass of water, given its volume and density; the second part deals with the mass flow rate of water, which is expressed as kg/s in SI units.

EXPRESS From the definition of density: $\rho = m/V$, we see that mass can be calculated as $m = \rho V$, the product of the volume of water and its density. With 1 g = 1×10^{-3} kg and 1 cm^3 = $(1 \times 10^{-2}\text{m})^3 = 1 \times 10^{-6}\text{m}^3$, the density of water in SI units (kg/m^3) is

$$\rho = 1\ \text{g/cm}^3 = \left(\frac{1\ \text{g}}{\text{cm}^3}\right)\left(\frac{10^{-3}\ \text{kg}}{\text{g}}\right)\left(\frac{\text{cm}^3}{10^{-6}\ \text{m}^3}\right) = 1 \times 10^3\ \text{kg/m}^3.$$

To obtain the flow rate, we simply divide the total mass of the water by the time taken to drain it.

ANALYZE (a) Using $m = \rho V$, the mass of a cubic meter of water is

$$m = \rho V = (1\times 10^3 \text{ kg/m}^3)(1\,\text{m}^3) = 1000\,\text{kg}.$$

(b) The total mass of water in the container is

$$M = \rho V = (1\times 10^3 \text{ kg}/\text{m}^3)(5700 \text{ m}^3) = 5.70\times 10^6 \text{ kg},$$

and the time elapsed is t = (10 h)(3600 s/h) = 3.6 × 10^4 s. Thus, the mass flow rate R is

$$R = \frac{M}{t} = \frac{5.70\times 10^6 \text{ kg}}{3.6\times 10^4 \text{ s}} = 158 \text{ kg/s}.$$

LEARN In terms of volume, the drain rate can be expressed as

$$R' = \frac{V}{t} = \frac{5700 \text{ m}^3}{3.6\times 10^4 \text{ s}} = 0.158 \text{ m}^3\text{/s} \approx 42 \text{ gal/s}.$$

The greater the flow rate, the less time required to drain a given amount of water.

24. The metric prefixes (micro (μ), pico, nano, …) are given for ready reference on the inside front cover of the textbook (see also Table 1–2). The surface area A of each grain of sand of radius r = 50 μm = 50 × 10^{-6} m is given by $A = 4\pi(50\times 10^{-6})^2 = 3.14\times 10^{-8}$ m^2 (Appendix E contains a variety of geometry formulas). We introduce the notion of density, $\rho = m/V$, so that the mass can be found from $m = \rho V$, where ρ = 2600 kg/m^3. Thus, using $V = 4\pi r^3/3$, the mass of each grain is

$$m = \rho V = \rho\left(\frac{4\pi r^3}{3}\right) = \left(2600 \frac{\text{kg}}{\text{m}^3}\right)\frac{4\pi\left(50\times 10^{-6} \text{ m}\right)^3}{3} = 1.36\times 10^{-9} \text{ kg}.$$

We observe that (because a cube has six equal faces) the indicated surface area is 6 m^2. The number of spheres (the grains of sand) N that have a total surface area of 6 m^2 is given by

$$N = \frac{6 \text{ m}^2}{3.14\times 10^{-8} \text{ m}^2} = 1.91\times 10^8.$$

Therefore, the total mass M is $M = Nm = \left(1.91\times 10^8\right)\left(1.36\times 10^{-9} \text{ kg}\right) = 0.260$ kg.

25. The volume of the section is (2500 m)(800 m)(2.0 m) = 4.0 × 10^6 m^3. Letting "d" stand for the thickness of the mud after it has (uniformly) distributed in the valley, then its volume there would be (400 m)(400 m)d. Requiring these two volumes to be equal, we can solve for d. Thus, d = 25 m. The volume of a small part of the mud over a patch of area of 4.0 m^2 is (4.0)d = 100 m^3. Since each cubic meter corresponds to a mass of

1900 kg (stated in the problem), then the mass of that small part of the mud is 1.9×10^5 kg.

26. (a) The volume of the cloud is (3000 m)π(1000 m)^2 = 9.4 × 10^9 m^3. Since each cubic meter of the cloud contains from 50 × 10^6 to 500 × 10^6 water drops, then we conclude that the entire cloud contains from 4.7 × 10^{18} to 4.7 × 10^{19} drops. Since the volume of each drop is $\frac{4}{3}\pi(10\times10^{-6}\text{ m})^3 = 4.2\times10^{-15}\text{ m}^3$, then the total volume of water in a cloud is from 2×10^3 to 2×10^4 m^3.

(b) Using the fact that $1\text{ L} = 1\times10^3\text{cm}^3 = 1\times10^{-3}\text{m}^3$, the amount of water estimated in part (a) would fill from 2×10^6 to 2×10^7 bottles.

(c) At 1000 kg for every cubic meter, the mass of water is from 2×10^6 to 2×10^7 kg. The coincidence in numbers between the results of parts (b) and (c) of this problem is due to the fact that each liter has a mass of one kilogram when water is at its normal density (under standard conditions).

27. We introduce the notion of density, $\rho = m/V$, and convert to SI units: 1000 g = 1 kg, and 100 cm = 1 m.

(a) The density ρ of a sample of iron is

$$\rho = \left(7.87\text{ g/cm}^3\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3 = 7870\text{ kg/m}^3.$$

If we ignore the empty spaces between the close-packed spheres, then the density of an individual iron atom will be the same as the density of any iron sample. That is, if M is the mass and V is the volume of an atom, then

$$V = \frac{M}{\rho} = \frac{9.27\times10^{-26}\text{ kg}}{7.87\times10^3\text{ kg/m}^3} = 1.18\times10^{-29}\text{ m}^3.$$

(b) We set $V = 4\pi R^3/3$, where R is the radius of an atom (Appendix E contains several geometry formulas). Solving for R, we find

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3\left(1.18\times10^{-29}\text{ m}^3\right)}{4\pi}\right)^{1/3} = 1.41\times10^{-10}\text{ m}.$$

The center-to-center distance between atoms is twice the radius, or 2.82 × 10^{-10} m.

28. If we estimate the "typical" large domestic cat mass as 10 kg, and the "typical" atom (in the cat) as 10 u ≈ 2 × 10^{-26} kg, then there are roughly (10 kg)/(2 × 10^{-26} kg) ≈ 5 × 10^{26} atoms. This is close to being a factor of a thousand greater than Avogadro's number. Thus this is roughly a kilomole of atoms.

29. The mass in kilograms is

$$(28.9 \text{ piculs})\left(\frac{100\,\text{gin}}{1\,\text{picul}}\right)\left(\frac{16\,\text{tahil}}{1\,\text{gin}}\right)\left(\frac{10\,\text{chee}}{1\,\text{tahil}}\right)\left(\frac{10\text{ hoon}}{1\text{ chee}}\right)\left(\frac{0.3779\,\text{g}}{1\,\text{hoon}}\right)$$

which yields 1.747 × 10^6 g or roughly 1.75× 10^3 kg.

30. To solve the problem, we note that the first derivative of the function with respect to time gives the rate. Setting the rate to zero gives the time at which an extreme value of the variable mass occurs; here that extreme value is a maximum.

(a) Differentiating $m(t) = 5.00t^{0.8} - 3.00t + 20.00$ with respect to t gives

$$\frac{dm}{dt} = 4.00t^{-0.2} - 3.00.$$

The water mass is the greatest when $dm/dt = 0$, or at $t = (4.00/3.00)^{1/0.2} = 4.21\,\text{s}$.

(b) At $t = 4.21\,\text{s}$, the water mass is

$$m(t = 4.21\,\text{s}) = 5.00(4.21)^{0.8} - 3.00(4.21) + 20.00 = 23.2\text{ g}.$$

(c) The rate of mass change at $t = 2.00\text{ s}$ is

$$\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} = \left[4.00(2.00)^{-0.2} - 3.00\right]\text{g/s} = 0.48\text{ g/s} = 0.48\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\text{ g}}\cdot\frac{60\text{ s}}{1\text{ min}}$$
$$= 2.89\times10^{-2}\text{ kg/min}.$$

(d) Similarly, the rate of mass change at $t = 5.00\text{ s}$ is

$$\left.\frac{dm}{dt}\right|_{t=2.00\,\text{s}} = \left[4.00(5.00)^{-0.2} - 3.00\right]\text{g/s} = -0.101\,\text{g/s} = -0.101\frac{\text{g}}{\text{s}}\cdot\frac{1\,\text{kg}}{1000\text{ g}}\cdot\frac{60\text{ s}}{1\text{ min}}$$
$$= -6.05\times10^{-3}\text{ kg/min}.$$

31. The mass density of the candy is

$$\rho = \frac{m}{V} = \frac{0.0200 \text{ g}}{50.0 \text{ mm}^3} = 4.00 \times 10^{-4} \text{ g/mm}^3 = 4.00 \times 10^{-4} \text{ kg/cm}^3.$$

If we neglect the volume of the empty spaces between the candies, then the total mass of the candies in the container when filled to height h is $M = \rho A h$, where $A = (14.0 \text{ cm})(17.0 \text{ cm}) = 238 \text{ cm}^2$ is the base area of the container that remains unchanged. Thus, the rate of mass change is given by

$$\begin{aligned}\frac{dM}{dt} &= \frac{d(\rho A h)}{dt} = \rho A \frac{dh}{dt} = (4.00 \times 10^{-4} \text{ kg/cm}^3)(238 \text{ cm}^2)(0.250 \text{ cm/s}) \\ &= 0.0238 \text{ kg/s} = 1.43 \text{ kg/min.}\end{aligned}$$

32. The total volume V of the real house is that of a triangular prism (of height h = 3.0 m and base area A = 20 × 12 = 240 m^2) in addition to a rectangular box (height h' = 6.0 m and same base). Therefore,

$$V = \frac{1}{2} hA + h'A = \left(\frac{h}{2} + h'\right) A = 1800 \text{ m}^3.$$

(a) Each dimension is reduced by a factor of 1/12, and we find

$$V_{\text{doll}} = \left(1800 \text{ m}^3\right)\left(\frac{1}{12}\right)^3 \approx 1.0 \text{ m}^3.$$

(b) In this case, each dimension (relative to the real house) is reduced by a factor of 1/144. Therefore,

$$V_{\text{miniature}} = \left(1800 \text{ m}^3\right)\left(\frac{1}{144}\right)^3 \approx 6.0 \times 10^{-4} \text{ m}^3.$$

33. **THINK** In this problem we are asked to differentiate between three types of tons: *displacement* ton, *freight* ton and *register* ton, all of which are units of volume.

EXPRESS The three different tons are defined in terms of *barrel bulk*, with $1 \text{ barrel bulk} = 0.1415 \text{ m}^3 = 4.0155 \text{ U.S. bushels}$ (using $1 \text{ m}^3 = 28.378 \text{ U.S. bushels}$). Thus, in terms of U.S. bushels, we have

$$1 \text{ displacement ton} = (7 \text{ barrels bulk}) \times \left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 28.108 \text{ U.S. bushels}$$

$$1 \text{ freight ton} = (8 \text{ barrels bulk}) \times \left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 32.124 \text{ U.S. bushels}$$

$$1 \text{ register ton} = (20 \text{ barrels bulk}) \times \left(\frac{4.0155 \text{ U.S. bushels}}{1 \text{ barrel bulk}}\right) = 80.31 \text{ U.S. bushels}$$

ANALYZE (a) The difference between 73 "freight" tons and 73 "displacement" tons is

$$\begin{aligned}\Delta V &= 73(\text{freight tons} - \text{displacement tons}) = 73(32.124 \text{ U.S. bushels} - 28.108 \text{ U.S. bushels}) \\ &= 293.168 \text{ U.S. bushels} \approx 293 \text{ U.S. bushels}\end{aligned}$$

(b) Similarly, the difference between 73 "register" tons and 73 "displacement" tons is

$$\begin{aligned}\Delta V &= 73(\text{register tons} - \text{displacement tons}) = 73(80.31 \text{ U.S. bushels} - 28.108 \text{ U.S. bushels}) \\ &= 3810.746 \text{ U.S. bushels} \approx 3.81\times10^3 \text{ U.S. bushels}\end{aligned}$$

LEARN With $1 \text{ register ton} > 1 \text{ freight ton} > 1 \text{ displacement ton},$ we expect the difference found in (b) to be greater than that in (a). This is indeed the case.

34. The customer expects a volume $V_1 = 20 \times 7056 \text{ in}^3$ and receives $V_2 = 20 \times 5826 \text{ in.}^3$, the difference being $\Delta V = V_1 - V_2 = 24600 \text{ in.}^3$, or

$$\Delta V = \left(24600 \text{ in.}^3\right)\left(\frac{2.54 \text{cm}}{1 \text{ inch}}\right)^3\left(\frac{1 \text{L}}{1000 \text{ cm}^3}\right) = 403 \text{L}$$

where Appendix D has been used.

35. The first two conversions are easy enough that a *formal* conversion is not especially called for, but in the interest of *practice makes perfect* we go ahead and proceed formally:

(a) $11 \text{ tuffets} = \left(11 \text{ tuffets}\right)\left(\frac{2 \text{ peck}}{1 \text{ tuffet}}\right) = 22 \text{ pecks}$.

(b) $11 \text{ tuffets} = \left(11 \text{ tuffets}\right)\left(\frac{0.50 \text{ Imperial bushel}}{1 \text{ tuffet}}\right) = 5.5 \text{ Imperial bushels}$.

(c) $11 \text{ tuffets} = \left(5.5 \text{ Imperial bushel}\right)\left(\frac{36.3687 \text{ L}}{1 \text{ Imperial bushel}}\right) \approx 200 \text{ L}$.

36. Table 7 can be completed as follows:

(a) It should be clear that the first column (under "wey") is the reciprocal of the first row – so that $\frac{9}{10} = 0.900$, $\frac{3}{40} = 7.50 \times 10^{-2}$, and so forth. Thus, 1 pottle = 1.56×10^{-3} wey and 1 gill = 8.32×10^{-6} wey are the last two entries in the first column.

(b) In the second column (under "chaldron"), clearly we have 1 chaldron = 1 chaldron (that is, the entries along the "diagonal" in the table must be 1's). To find out how many chaldron are equal to one bag, we note that 1 wey = 10/9 chaldron = 40/3 bag so that $\frac{1}{12}$ chaldron = 1 bag. Thus, the next entry in that second column is $\frac{1}{12}$ = 8.33×10^{-2}. Similarly, 1 pottle = 1.74×10^{-3} chaldron and 1 gill = 9.24×10^{-6} chaldron.

(c) In the third column (under "bag"), we have 1 chaldron = 12.0 bag, 1 bag = 1 bag, 1 pottle = 2.08×10^{-2} bag, and 1 gill = 1.11×10^{-4} bag.

(d) In the fourth column (under "pottle"), we find 1 chaldron = 576 pottle, 1 bag = 48 pottle, 1 pottle = 1 pottle, and 1 gill = 5.32×10^{-3} pottle.

(e) In the last column (under "gill"), we obtain 1 chaldron = 1.08×10^5 gill, 1 bag = 9.02×10^3 gill, 1 pottle = 188 gill, and, of course, 1 gill = 1 gill.

(f) Using the information from part (c), 1.5 chaldron = (1.5)(12.0) = 18.0 bag. And since each bag is 0.1091 m^3 we conclude 1.5 chaldron = (18.0)(0.1091) = 1.96 m^3.

37. The volume of one unit is 1 cm^3 = 1×10^{-6} m^3, so the volume of a mole of them is 6.02×10^{23} cm^3 = 6.02×10^{17} m^3. The cube root of this number gives the edge length: $8.4\times10^5\ \text{m}^3$. This is equivalent to roughly 8×10^2 km.

38. (a) Using the fact that the area A of a rectangle is (width) × (length), we find

$$\begin{aligned} A_{\text{total}} &= (3.00\,\text{acre}) + (25.0\,\text{perch})(4.00\,\text{perch}) \\ &= (3.00\ \text{acre})\left(\frac{(40\ \text{perch})(4\ \text{perch})}{1\text{acre}}\right) + 100\,\text{perch}^2 \\ &= 580\ \text{perch}^2. \end{aligned}$$

We multiply this by the perch^2 → rood conversion factor (1 rood/40 perch^2) to obtain the answer: A_{total} = 14.5 roods.

(b) We convert our intermediate result in part (a):

$$A_{\text{total}} = (580\ \text{perch}^2)\left(\frac{16.5\,\text{ft}}{1\,\text{perch}}\right)^2 = 1.58\times10^5\ \text{ft}^2.$$

Now, we use the feet → meters conversion given in Appendix D to obtain

$$A_{\text{total}} = (1.58\times10^5\ \text{ft}^2)\left(\frac{1\,\text{m}}{3.281\,\text{ft}}\right)^2 = 1.47\times10^4\ \text{m}^2.$$

39. **THINK** This problem compares the U.K. gallon with U.S. gallon, two non-SI units for volume. The interpretation of the type of gallons, whether U.K. or U.S., affects the amount of gasoline one calculates for traveling a given distance.

EXPRESS If the fuel consumption rate is R (in miles/gallon), then the amount of gasoline (in gallons) needed for a trip of distance d (in miles) would be

$$V(\text{gallon}) = \frac{d \ \ (\text{miles})}{R\,(\text{miles/gallon})}$$

Since the car was manufactured in U.K., the fuel consumption rate is calibrated based on U.K. gallon, and the correct interpretation should be "40 miles per U.K. gallon." In U.K., one would think of gallon as U.K. gallon; however, in the U.S., the word "gallon" would naturally be interpreted as U.S. gallon. Note also that since $1\text{ U.K. gallon} = 4.5460900\text{ L}$ and $1\text{ U.S. gallon} = 3.7854118\text{ L}$, the relationship between the two is

$$1\text{ U.K. gallon} = (4.5460900\text{ L})\left(\frac{1\text{ U.S. gallon}}{3.7854118\text{ L}}\right) = 1.20095\text{ U.S. gallons}$$

ANALYZE (a) The amount of gasoline actually required is

$$V' = \frac{750\text{ miles}}{40\text{ miles/U.K. gallon}} = 18.75\text{ U.K. gallons} \approx 18.8\text{ U.K. gallons}$$

This means that the driver mistakenly believes that the car should need 18.8 U.S. gallons.

(b) Using the conversion factor found above, this is equivalent to

$$V' = (18.75\text{ U.K. gallons}) \times \left(\frac{1.20095\text{ U.S. gallons}}{1\text{ U.K. gallon}}\right) \approx 22.5\text{ U.S. gallons}$$

LEARN One U.K. gallon is greater than one U.S gallon by roughly a factor of 1.2 in volume. Therefore, 40 mi/U.K. gallon is less fuel-efficient than 40 mi/U.S. gallon.

40. Equation 1-9 gives (to very high precision!) the conversion from atomic mass units to kilograms. Since this problem deals with the ratio of total mass (1.0 kg) divided by the mass of one atom (1.0 u, but converted to kilograms), then the computation reduces to simply taking the reciprocal of the number given in Eq. 1-9 and rounding off appropriately. Thus, the answer is 6.0×10^{26}.

41. **THINK** This problem involves converting *cord*, a non-SI unit for volume, to SI unit.

EXPRESS Using the (exact) conversion 1 in. = 2.54 cm = 0.0254 m for length, we have

$$1\text{ ft} = 12\text{ in} = (12\text{ in.}) \times \left(\frac{0.0254\text{ m}}{1\text{ in}}\right) = 0.3048\text{ m}.$$

Thus, $1\text{ ft}^3 = (0.3048\text{ m})^3 = 0.0283\text{ m}^3$ for volume (these results also can be found in Appendix D).

ANALYZE The volume of a cord of wood is $V = (8\text{ ft}) \times (4\text{ ft}) \times (4\text{ ft}) = 128\text{ ft}^3$. Using the conversion factor found above, we obtain

$$V = 1\text{ cord} = 128\text{ ft}^3 = (128\text{ ft}^3) \times \left(\frac{0.0283\text{ m}^3}{1\text{ ft}^3}\right) = 3.625\text{ m}^3$$

which implies that $1\text{ m}^3 = \left(\frac{1}{3.625}\right)\text{cord} = 0.276\text{ cord} \approx 0.3\text{ cord}$.

LEARN The unwanted units ft^3 all cancel out, as they should. In conversions, units obey the same algebraic rules as variables and numbers.

42. (a) In atomic mass units, the mass of one molecule is (16 + 1 + 1)u = 18 u. Using Eq. 1-9, we find

$$18\text{u} = (18\text{u})\left(\frac{1.6605402 \times 10^{-27}\text{ kg}}{1\text{u}}\right) = 3.0 \times 10^{-26}\text{ kg}.$$

(b) We divide the total mass by the mass of each molecule and obtain the (approximate) number of water molecules:

$$N \approx \frac{1.4 \times 10^{21}}{3.0 \times 10^{-26}} \approx 5 \times 10^{46}.$$

43. A million milligrams comprise a kilogram, so 2.3 kg/week is 2.3 × 10^6 mg/week. Figuring 7 days a week, 24 hours per day, 3600 second per hour, we find 604800 seconds are equivalent to one week. Thus, (2.3 × 10^6 mg/week)/(604800 s/week) = 3.8 mg/s.

44. The volume of the water that fell is

$$\begin{aligned} V &= (26\text{ km}^2)(2.0\text{ in.}) = (26\text{ km}^2)\left(\frac{1000\text{ m}}{1\text{ km}}\right)^2 (2.0\text{ in.})\left(\frac{0.0254\text{ m}}{1\text{ in.}}\right) \\ &= (26 \times 10^6\text{ m}^2)(0.0508\text{ m}) \\ &= 1.3 \times 10^6\text{ m}^3. \end{aligned}$$

We write the mass-per-unit-volume (density) of the water as: $\rho = \frac{m}{V} = 1 \times 10^3\text{ kg/m}^3$.

The mass of the water that fell is therefore given by $m = \rho V$:

$$m = (1 \times 10^3\text{ kg/m}^3)(1.3 \times 10^6\text{ m}^3) = 1.3 \times 10^9\text{ kg}.$$

45. The number of seconds in a year is 3.156×10^7. This is listed in Appendix D and results from the product

$$(365.25 \text{ day/y}) (24 \text{ h/day}) (60 \text{ min/h}) (60 \text{ s/min}).$$

(a) The number of shakes in a second is 10^8; therefore, there are indeed more shakes per second than there are seconds per year.

(b) Denoting the age of the universe as 1 u-day (or 86400 u-sec), then the time during which humans have existed is given by

$$\frac{10^6}{10^{10}} = 10^{-4} \text{ u-day},$$

which may also be expressed as $\left(10^{-4} \text{ u-day}\right)\left(\dfrac{86400 \text{ u-sec}}{1 \text{ u-day}}\right) = 8.6 \text{ u-sec}.$

46. The volume removed in one year is $V = (75 \times 10^4 \text{ m}^2)(26 \text{ m}) \approx 2 \times 10^7 \text{ m}^3$, which we convert to cubic kilometers: $V = \left(2 \times 10^7 \text{ m}^3\right)\left(\dfrac{1 \text{ km}}{1000 \text{ m}}\right)^3 = 0.020 \text{ km}^3.$

47. **THINK** This problem involves expressing the speed of light in astronomical units per minute.

EXPRESS We first convert meters to astronomical units (AU), and seconds to minutes, using

$$1000 \text{ m} = 1 \text{ km}, \quad 1 \text{ AU} = 1.50 \times 10^8 \text{ km}, \quad 60 \text{ s} = 1 \text{ min}.$$

ANALYZE Using the conversion factors above, the speed of light can be rewritten as

$$c = 3.0 \times 10^8 \text{ m/s} = \left(\frac{3.0 \times 10^8 \text{ m}}{\text{s}}\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)\left(\frac{\text{AU}}{1.50 \times 10^8 \text{ km}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 0.12 \text{ AU/min}.$$

LEARN When expressed the speed of light c in AU/min, we readily see that it takes about 8.3 (= 1/0.12) minutes for sunlight to reach the Earth (i.e., to travel a distance of 1 AU).

48. Since one atomic mass unit is $1 \text{ u} = 1.66 \times 10^{-24}$ g (see Appendix D), the mass of one mole of atoms is about $m = (1.66 \times 10^{-24} \text{ g})(6.02 \times 10^{23}) = 1 \text{ g}$. On the other hand, the mass of one mole of atoms in the common Eastern mole is

$$m' = \frac{75\text{ g}}{7.5} = 10\text{ g}$$

Therefore, in atomic mass units, the average mass of one atom in the common Eastern mole is

$$\frac{m'}{N_A} = \frac{10\text{ g}}{6.02\times10^{23}} = 1.66\times10^{-23}\text{ g} = 10\text{ u}.$$

49. (a) Squaring the relation 1 ken = 1.97 m, and setting up the ratio, we obtain

$$\frac{1\text{ ken}^2}{1\text{ m}^2} = \frac{1.97^2\text{ m}^2}{1\text{ m}^2} = 3.88.$$

(b) Similarly, we find

$$\frac{1\text{ ken}^3}{1\text{ m}^3} = \frac{1.97^3\text{ m}^3}{1\text{ m}^3} = 7.65.$$

(c) The volume of a cylinder is the circular area of its base multiplied by its height. Thus,

$$\pi r^2 h = \pi(3.00)^2(5.50) = 156\text{ ken}^3.$$

(d) If we multiply this by the result of part (b), we determine the volume in cubic meters: (155.5)(7.65) = 1.19×10^3 m^3.

50. According to Appendix D, a nautical mile is 1.852 km, so 24.5 nautical miles would be 45.374 km. Also, according to Appendix D, a mile is 1.609 km, so 24.5 miles is 39.4205 km. The difference is 5.95 km.

51. (a) For the minimum (43 cm) case, 9 cubits converts as follows:

$$9\text{cubits} = (9\text{cubits})\left(\frac{0.43\text{m}}{1\text{cubit}}\right) = 3.9\text{m}.$$

And for the maximum (53 cm) case we have $9\text{cubits} = (9\text{cubits})\left(\frac{0.53\text{m}}{1\text{cubit}}\right) = 4.8\text{m}.$

(b) Similarly, with 0.43 m $\to$ 430 mm and 0.53 m $\to$ 530 mm, we find 3.9×10^3 mm and 4.8×10^3 mm, respectively.

(c) We can convert length and diameter first and then compute the volume, or first compute the volume and then convert. We proceed using the latter approach (where d is diameter and ℓ is length).

$$V_{\text{cylinder, min}} = \frac{\pi}{4}\ell d^2 = 28 \text{ cubit}^3 = \left(28 \text{ cubit}^3\right)\left(\frac{0.43\,\text{m}}{1 \text{ cubit}}\right)^3 = 2.2 \text{ m}^3.$$

Similarly, with 0.43 m replaced by 0.53 m, we obtain $V_{\text{cylinder, max}} = 4.2 \text{ m}^3$.

52. Abbreviating wapentake as "wp" and assuming a hide to be 110 acres, we set up the ratio 25 wp/11 barn along with appropriate conversion factors:

$$\frac{(25 \text{ wp})\left(\frac{100 \text{ hide}}{1 \text{ wp}}\right)\left(\frac{110 \text{ acre}}{1 \text{ hide}}\right)\left(\frac{4047 \text{ m}^2}{1 \text{ acre}}\right)}{(11 \text{ barn})\left(\frac{1\times 10^{-28} \text{ m}^2}{1 \text{ barn}}\right)} \approx 1\times 10^{36}.$$

53. **THINK** The objective of this problem is to convert the Earth-Sun distance (1 AU) to parsecs and light-years.

EXPRESS To relate parsec (pc) to AU, we note that when θ is measured in radians, it is equal to the arc length s divided by the radius R. For a very large radius circle and small value of θ, the arc may be approximated as the straight line-segment of length 1 AU. Thus,

$$\theta = 1 \text{ arcsec} = (1 \text{ arcsec})\left(\frac{1 \text{ arcmin}}{60 \text{ arcsec}}\right)\left(\frac{1°}{60 \text{ arcmin}}\right)\left(\frac{2\pi \text{ radian}}{360°}\right) = 4.85\times 10^{-6} \text{ rad}.$$

Therefore, one parsec is

$$1 \text{ pc} = \frac{s}{\theta} = \frac{1 \text{ AU}}{4.85\times 10^{-6}} = 2.06\times 10^5 \text{ AU}.$$

Next, we relate AU to light-year (ly). Since a year is about 3.16×10^7 s,

$$1 \text{ ly} = (186{,}000 \text{ mi/s})(3.16\times 10^7 \text{ s}) = 5.9\times 10^{12} \text{ mi}.$$

ANALYZE (a) Since $1 \text{ pc} = 2.06\times 10^5 \text{ AU}$, inverting the relation gives

$$1 \text{ AU} = (1 \text{ AU})\left(\frac{1 \text{ pc}}{2.06\times 10^5 \text{ AU}}\right) = 4.9\times 10^{-6} \text{ pc}.$$

(b) Given that $1 \text{ AU} = 92.9\times 10^6 \text{ mi}$ and $1 \text{ ly} = 5.9\times 10^{12} \text{ mi}$, the two expressions together lead to

$$1 \text{ AU} = 92.9\times 10^6 \text{ mi} = (92.9\times 10^6 \text{ mi})\left(\frac{1 \text{ ly}}{5.9\times 10^{12} \text{ mi}}\right) = 1.57\times 10^{-5} \text{ ly}.$$

LEARN Our results can be further combined to give $1\,\text{pc} = 3.2\,\text{ly}$. From the above expression, we readily see that it takes $1.57\times10^{-5}\,\text{y}$, or about 8.3 min, for Sunlight to travel a distance of 1 AU to reach the Earth.

54. (a) Using Appendix D, we have 1 ft = 0.3048 m, 1 gal = 231 in.^{3}, and 1 in.^{3} = 1.639 × 10^{-2} L. From the latter two items, we find that 1 gal = 3.79 L. Thus, the quantity 460 ft^2/gal becomes

$$460\ \text{ft}^2/\text{gal} = \left(\frac{460\ \text{ft}^2}{\text{gal}}\right)\left(\frac{1\ \text{m}}{3.28\ \text{ft}}\right)^2\left(\frac{1\ \text{gal}}{3.79\ \text{L}}\right) = 11.3\ \text{m}^2/\text{L}.$$

(b) Also, since 1 m^3 is equivalent to 1000 L, our result from part (a) becomes

$$11.3\ \text{m}^2/\text{L} = \left(\frac{11.3\ \text{m}^2}{\text{L}}\right)\left(\frac{1000\ \text{L}}{1\ \text{m}^3}\right) = 1.13\times10^4\ \text{m}^{-1}.$$

(c) The inverse of the original quantity is $(460\ \text{ft}^2/\text{gal})^{-1} = 2.17\times10^{-3}$ gal/ft^2.

(d) The answer in (c) represents the volume of the paint (in gallons) needed to cover a square foot of area. From this, we could also figure the paint thickness [it turns out to be about a tenth of a millimeter, as one sees by taking the reciprocal of the answer in part (b)].

55. (a) The receptacle is a volume of (40 cm)(40 cm)(30 cm) = 48000 cm^3 = 48 L = (48)(16)/11.356 = 67.63 standard bottles, which is a little more than 3 nebuchadnezzars (the largest bottle indicated). The remainder, 7.63 standard bottles, is just a little less than 1 methuselah. Thus, the answer to part (a) is 3 nebuchadnezzars and 1 methuselah.

(b) Since 1 methuselah.= 8 standard bottles, then the extra amount is 8 − 7.63 = 0.37 standard bottle.

(c) Using the conversion factor 16 standard bottles = 11.356 L, we have

$$0.37\ \text{standard bottle} = (0.37\ \text{standard bottle})\left(\frac{11.356\ \text{L}}{16\ \text{standard bottles}}\right) = 0.26\ \text{L}.$$

56. The mass of the pig is 3.108 slugs, or (3.108)(14.59) = 45.346 kg. Referring now to the corn, a U.S. bushel is 35.238 liters. Thus, a value of 1 for the *corn-hog ratio* would be equivalent to 35.238/45.346 = 0.7766 in the indicated metric units. Therefore, a value of 5.7 for the *ratio* corresponds to 5.7(0.777) ≈ 4.4 in the indicated metric units.

57. Two jalapeño peppers have spiciness = 8000 SHU, and this amount multiplied by 400 (the number of people) is 3.2×10^6 SHU, which is roughly ten times the SHU value for a

single habanero pepper. More precisely, 10.7 habanero peppers will provide that total required SHU value.

58. In the simplest approach, we set up a ratio for the total increase in *horizontal depth x* (where Δx = 0.05 m is the increase in horizontal depth per step)

$$x = N_{\text{steps}}\,\Delta x = \left(\frac{4.57}{0.19}\right)(0.05\text{ m}) = 1.2\text{ m}.$$

However, we can approach this more carefully by noting that if there are $N = 4.57/.19 \approx 24$ rises then under normal circumstances we would expect $N - 1 = 23$ runs (horizontal pieces) in that staircase. This would yield (23)(0.05 m) = 1.15 m, which - to two significant figures - agrees with our first result.

59. The volume of the filled container is 24000 cm^3 = 24 liters, which (using the conversion given in the problem) is equivalent to 50.7 pints (U.S). The expected number is therefore in the range from 1317 to 1927 Atlantic oysters. Instead, the number received is in the range from 406 to 609 Pacific oysters. This represents a shortage in the range of roughly 700 to 1500 oysters (the answer to the problem). Note that the minimum value in our answer corresponds to the minimum Atlantic minus the maximum Pacific, and the maximum value corresponds to the maximum Atlantic minus the minimum Pacific.

60. (a) We reduce the stock amount to British teaspoons:

$$\begin{aligned}
1\text{ breakfastcup} &= 2\times 8\times 2\times 2 = 64\text{ teaspoons}\\
1\text{ teacup} &= 8\times 2\times 2 = 32\text{ teaspoons}\\
6\text{ tablespoons} &= 6\times 2\times 2 = 24\text{ teaspoons}\\
1\text{ dessertspoon} &= 2\text{ teaspoons}
\end{aligned}$$

which totals to 122 British teaspoons, or 122 U.S. teaspoons since liquid measure is being used. Now with one U.S cup equal to 48 teaspoons, upon dividing $122/48 \approx 2.54$, we find this amount corresponds to 2.5 U.S. cups plus a remainder of precisely 2 teaspoons. In other words,

$$122\text{ U.S. teaspoons} = 2.5\text{ U.S. cups} + 2\text{ U.S. teaspoons}.$$

(b) For the nettle tops, one-half quart is still one-half quart.

(c) For the rice, one British tablespoon is 4 British teaspoons which (since dry-goods measure is being used) corresponds to 2 U.S. teaspoons.

(d) A British saltspoon is $\frac{1}{2}$ British teaspoon which corresponds (since dry-goods measure is again being used) to 1 U.S. teaspoon.

Chapter 2

1. The speed (assumed constant) is v = (90 km/h)(1000 m/km) ⁄ (3600 s/h) = 25 m/s. Thus, in 0.50 s, the car travels a distance $d = vt$ = (25 m/s)(0.50 s) ≈ 13 m.

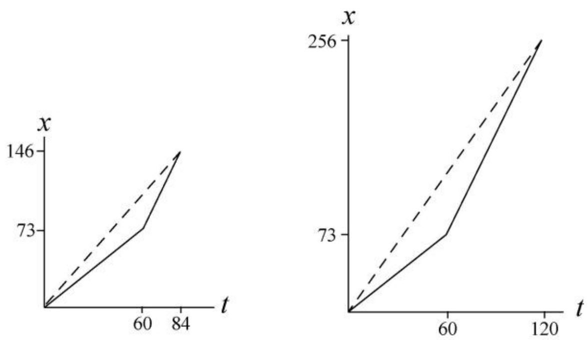
2. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

$$v_{\text{avg}} = \frac{73.2 \text{ m} + 73.2 \text{ m}}{\frac{73.2 \text{ m}}{1.22 \text{ m/s}} + \frac{73.2 \text{ m}}{3.05 \text{ m}}} = 1.74 \text{ m/s}.$$

(b) Using the fact that distance = vt while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22 \text{ m/s})(60 \text{ s}) + (3.05 \text{ m/s})(60 \text{ s})}{120 \text{ s}} = 2.14 \text{ m/s}.$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before — the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.



3. **THINK** This one-dimensional kinematics problem consists of two parts, and we are asked to solve for the average velocity and average speed of the car.

EXPRESS Since the trip consists of two parts, let the displacements during first and second parts of the motion be Δx_1 and Δx_2, and the corresponding time intervals be Δt_1 and Δt_2, respectively. Now, because the problem is one-dimensional and both displacements are in the same direction, the total displacement is simply $\Delta x = \Delta x_1 + \Delta x_2$, and the total time for the trip is $\Delta t = \Delta t_1 + \Delta t_2$. Using the definition of average velocity given in Eq. 2-2, we have

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}.$$

To find the average speed, we note that during a time Δt if the velocity remains a positive constant, then the speed is equal to the magnitude of velocity, and the distance is equal to the magnitude of displacement, with $d = |\Delta x| = v\Delta t$.

ANALYZE

(a) During the first part of the motion, the displacement is Δx_1 = 40 km and the time taken is

$$t_1 = \frac{(40\text{ km})}{(30\text{ km/h})} = 1.33\text{ h}.$$

Similarly, during the second part of the trip the displacement is Δx_2 = 40 km and the time interval is

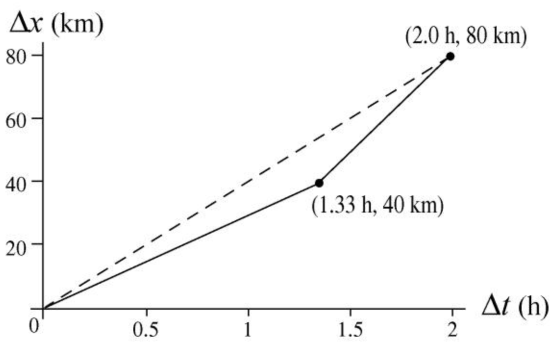
$$t_2 = \frac{(40\text{ km})}{(60\text{ km/h})} = 0.67\text{ h}.$$

The total displacement is $\Delta x = \Delta x_1 + \Delta x_2$ = 40 km + 40 km = 80 km, and the total time elapsed is $\Delta t = \Delta t_1 + \Delta t_2$ = 2.00 h. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{(80\text{ km})}{(2.0\text{ h})} = 40\text{ km/h}.$$

(b) In this case, the average speed is the same as the magnitude of the average velocity: $s_{\text{avg}} = 40$ km/h.

(c) The graph of the entire trip, shown below, consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to $(\Delta t_1, \Delta x_1)$ = (1.33 h, 40 km) and the second having a slope of 60 km/h and connecting $(\Delta t_1, \Delta x_1)$ to $(\Delta t, \Delta x)$ = (2.00 h, 80 km).



From the graphical point of view, the slope of the dashed line drawn from the origin to $(\Delta t, \Delta x)$ represents the average velocity.

LEARN The average velocity is a vector quantity that depends only on the net displacement (also a vector) between the starting and ending points.

4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the

motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in v_{up} = 40 km/h and v_{down} = 60 km/h, yields 48 km/h for the average speed.

5. **THINK** In this one-dimensional kinematics problem, we're given the position function $x(t)$, and asked to calculate the position and velocity of the object at a later time.

EXPRESS The position function is given as $x(t) = (3\text{ m/s})t - (4\text{ m/s}^2)t^2 + (1\text{ m/s}^3)t^3$. The position of the object at some instant t_0 is simply given by $x(t_0)$. For the time interval $t_1 \le t \le t_2$, the displacement is $\Delta x = x(t_2) - x(t_1)$. Similarly, using Eq. 2-2, the average velocity for this time interval is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

ANALYZE (a) Plugging in t = 1 s into $x(t)$ yields

$$x(1\text{ s}) = (3\text{ m/s})(1\text{ s}) - (4\text{ m/s}^2)(1\text{ s})^2 + (1\text{ m/s}^3)(1\text{ s})^3 = 0.$$

(b) With t = 2 s we get $x(2\text{ s}) = (3\text{ m/s})(2\text{ s}) - (4\text{ m/s}^2)(2\text{ s})^2 + (1\text{ m/s}^3)(2\text{ s})^3 = -2\text{ m}$.

(c) With t = 3 s we have $x(3\text{ s}) = (3\text{ m/s})(3\text{ s}) - (4\text{ m/s}^2)(3\text{ s})^2 + (1\text{ m/s}^3)(3\text{ s})^3 = 0\text{ m}$.

(d) Similarly, plugging in t = 4 s gives

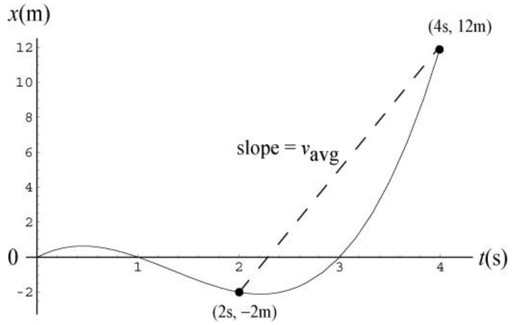
$$x(4\text{ s}) = (3\text{ m/s})(4\text{ s}) - (4\text{ m/s}^2)(4\text{ s})^2 + (1\text{ m/s}^3)(4\text{ s})^3 = 12\text{ m}.$$

(e) The position at t = 0 is x = 0. Thus, the displacement between t = 0 and t = 4 s is $\Delta x = x(4\text{ s}) - x(0) = 12\text{ m} - 0 = 12\text{ m}$.

(f) The position at t = 2 s is subtracted from the position at t = 4 s to give the displacement: $\Delta x = x(4\text{ s}) - x(2\text{ s}) = 12\text{ m} - (-2\text{ m}) = 14\text{ m}$. Thus, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14\text{ m}}{2\text{ s}} = 7\text{ m/s}.$$

(g) The position of the object for the interval $0 \le t \le 4$ is plotted below. The straight line drawn from the point at (t, x) = (2 s, –2 m) to (4 s, 12 m) would represent the average velocity, answer for part (f).



LEARN Our graphical representation illustrates once again that the average velocity for a time interval depends only on the net displacement between the starting and ending points.

6. Huber's speed is

$$v_0 = (200\text{ m})/(6.509\text{ s}) = 30.72\text{ m/s} = 110.6\text{ km/h},$$

where we have used the conversion factor 1 m/s = 3.6 km/h. Since Whittingham beat Huber by 19.0 km/h, his speed is v_1 = (110.6 km/h + 19.0 km/h) = 129.6 km/h, or 36 m/s (1 km/h = 0.2778 m/s). Thus, using Eq. 2-2, the time through a distance of 200 m for Whittingham is

$$\Delta t = \frac{\Delta x}{v_1} = \frac{200\text{ m}}{36\text{ m/s}} = 5.554\text{ s}.$$

7. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time that elapses before they crash is t = (60 km)/(60 km/h) = 1.0 h. During this time, the bird travels a distance of $x = vt$ = (60 km/h)(1.0 h) = 60 km.

8. The amount of time it takes for each person to move a distance L with speed v_s is $\Delta t = L/v_s$. With each additional person, the depth increases by one body depth d

(a) The rate of increase of the layer of people is

$$R = \frac{d}{\Delta t} = \frac{d}{L/v_s} = \frac{dv_s}{L} = \frac{(0.25\text{ m})(3.50\text{ m/s})}{1.75\text{ m}} = 0.50\text{ m/s}$$

(b) The amount of time required to reach a depth of $D = 5.0\text{ m}$ is

$$t = \frac{D}{R} = \frac{5.0\text{ m}}{0.50\text{ m/s}} = 10\text{ s}$$

9. Converting to seconds, the running times are t_1 = 147.95 s and t_2 = 148.15 s, respectively. If the runners were equally fast, then

$$s_{\text{avg}_1} = s_{\text{avg}_2} \quad \Rightarrow \quad \frac{L_1}{t_1} = \frac{L_2}{t_2}.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1\right) L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 = 0.00135 L_1 \approx 1.4 \text{ m}$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

10. Let v_w be the speed of the wind and v_c be the speed of the car.

(a) Suppose during time interval t_1, the car moves in the same direction as the wind. Then the effective speed of the car is given by $v_{eff,1} = v_c + v_w$, and the distance traveled is $d = v_{eff,1} t_1 = (v_c + v_w) t_1$. On the other hand, for the return trip during time interval t_2, the car moves in the opposite direction of the wind and the effective speed would be $v_{eff,2} = v_c - v_w$. The distance traveled is $d = v_{eff,2} t_2 = (v_c - v_w) t_2$. The two expressions can be rewritten as

$$v_c + v_w = \frac{d}{t_1} \quad \text{and} \quad v_c - v_w = \frac{d}{t_2}$$

Adding the two equations and dividing by two, we obtain $v_c = \frac{1}{2}\left(\frac{d}{t_1} + \frac{d}{t_2}\right)$. Thus, method 1 gives the car's speed v_c a in windless situation.

(b) If method 2 is used, the result would be

$$v_c' = \frac{d}{(t_1 + t_2)/2} = \frac{2d}{t_1 + t_2} = \frac{2d}{\dfrac{d}{v_c + v_w} + \dfrac{d}{v_c - v_w}} = \frac{v_c^2 - v_w^2}{v_c} = v_c \left[1 - \left(\frac{v_w}{v_c}\right)^2\right].$$

The fractional difference is

$$\frac{v_c - v_c'}{v_c} = \left(\frac{v_w}{v_c}\right)^2 = (0.0240)^2 = 5.76 \times 10^{-4}.$$

11. The values used in the problem statement make it easy to see that the first part of the trip (at 100 km/h) takes 1 hour, and the second part (at 40 km/h) also takes 1 hour. Expressed in decimal form, the time left is 1.25 hour, and the distance that remains is 160 km. Thus, a speed v = (160 km)/(1.25 h) = 128 km/h is needed.

12. (a) Let the fast and the slow cars be separated by a distance d at t = 0. If during the time interval $t = L/v_s = (12.0 \text{ m})/(5.0 \text{ m/s}) = 2.40 \text{ s}$ in which the slow car has moved a distance of $L = 12.0$ m, the fast car moves a distance of $vt = d + L$ to join the line of slow cars, then the shock wave would remain stationary. The condition implies a separation of

$$d = vt - L = (25 \text{ m/s})(2.4 \text{ s}) - 12.0 \text{ m} = 48.0 \text{ m}.$$

(b) Let the initial separation at $t = 0$ be $d = 96.0$ m. At a later time t, the slow and the fast cars have traveled $x = v_s t$ and the fast car joins the line by moving a distance $d + x$. From

$$t = \frac{x}{v_s} = \frac{d+x}{v},$$

we get

$$x = \frac{v_s}{v - v_s} d = \frac{5.00 \text{ m/s}}{25.0 \text{ m/s} - 5.00 \text{ m/s}}(96.0 \text{ m}) = 24.0 \text{ m},$$

which in turn gives $t = (24.0 \text{ m})/(5.00 \text{ m/s}) = 4.80 \text{ s}$. Since the rear of the slow-car pack has moved a distance of $\Delta x = x - L = 24.0 \text{ m} - 12.0 \text{ m} = 12.0 \text{ m}$ downstream, the speed of the rear of the slow-car pack, or equivalently, the speed of the shock wave, is

$$v_{\text{shock}} = \frac{\Delta x}{t} = \frac{12.0 \text{ m}}{4.80 \text{ s}} = 2.50 \text{ m/s}.$$

(c) Since $x > L$, the direction of the shock wave is downstream.

13. (a) Denoting the travel time and distance from San Antonio to Houston as *T* and *D*, respectively, the average speed is

$$s_{\text{avg1}} = \frac{D}{T} = \frac{(55 \text{ km/h})(T/2) + (90 \text{ km/h})(T/2)}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg2}} = \frac{D}{T} = \frac{D}{\frac{D/2}{55 \text{ km/h}} + \frac{D/2}{90 \text{ km/h}}} = 68.3 \text{ km/h}$$

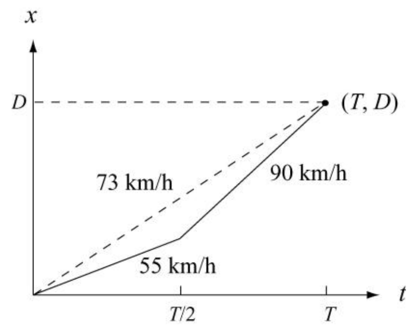
which should be rounded to 68 km/h.

(c) The total distance traveled (2*D*) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5 \text{ km/h}} + \frac{D}{68.3 \text{ km/h}}} = 70 \text{ km/h}.$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance *D* (the intent is *not* to make the student go to an atlas to look it up); the student can just as easily arbitrarily set *T* instead of *D*, as will be clear in the following discussion. We briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to (t_1, x_1) = ($T/2$, $55T/2$) and the second having a slope of 90 and connecting (t_1, x_1) to (T, D) where $D = (55 + 90)T/2$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (T, D). The graph (not drawn to scale) is depicted below:



14. Using the general property $\frac{d}{dx}\exp(bx) = b\exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d(19t)}{dt}\right)\cdot e^{-t} + (19t)\cdot\left(\frac{de^{-t}}{dt}\right) .$$

If a concern develops about the appearance of an argument of the exponential ($-t$) apparently having units, then an explicit factor of $1/T$ where T = 1 second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1-t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when t = 1 s. Now that we know *when* it stops, we find out *where* it stops by plugging our result t = 1 into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find x = 5.9 m.

15. We use Eq. 2-4 to solve the problem.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}\ (4 - 12t + 3t^2) \ = -12 + 6t.$$

Thus, at t = 1 s, the velocity is v = (–12 + (6)(1)) = –6 m/s.

(b) Since $v < 0$, it is moving in the $-x$ direction at t = 1 s.

(c) At t = 1 s, the *speed* is $|v|$ = 6 m/s.

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) Yes, since v smoothly changes from negative values (consider the t = 1 result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that v = 0 when $t = 2$ s.

(f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

16. We use the functional notation $x(t)$, $v(t)$, and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

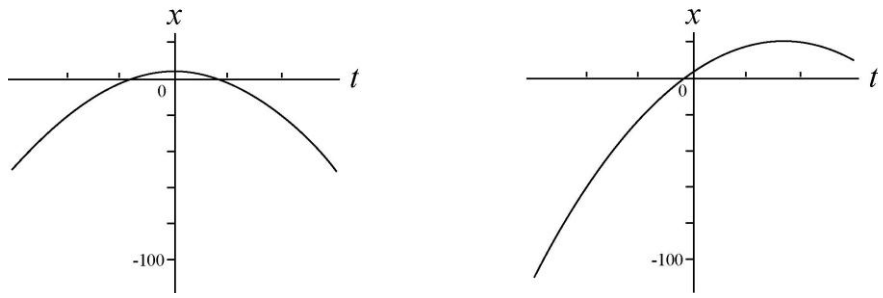
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m.

(c) and (d) Requiring $x(t) = 0$ in the expression $x(t) = 4.0 - 6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(e) We show both the asked-for graph (on the left) as well as the "shifted" graph that is relevant to part (f). In both cases, the time axis is given by $-3 \le t \le 3$ (SI units understood).



(f) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(g) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (e) is higher than that of the first).

17. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\ \text{cm} - 21.75\ \text{cm}}{3.00\ \text{s} - 2.00\ \text{s}}$$

which yields $v_{\text{avg}} = 28.5$ cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time $t = 2.00$ s, yields $v = (4.5)(2.00)^2 = 18.0$ cm/s.

(c) At t = 3.00 s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

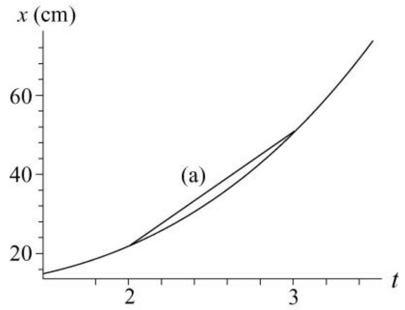
(d) At t = 2.50 s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \quad \Rightarrow \quad t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between t = 2 and t = 3 in this x-vs-t plot. The answers to parts (b), (c), (d), and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



18. (a) Taking derivatives of $x(t) = 12t^2 - 2t^3$ we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \qquad \text{and} \qquad a(t) = 24 - 12t$$

with length in meters and time in seconds. Plugging in the value t = 3 yields $x(3) = 54 \text{ m}$.

(b) Similarly, plugging in the value t = 3 yields $v(3)$ = 18 m/s.

(c) For t = 3, $a(3) = -12$ m/s^2.

(d) At the maximum x, we must have v = 0; eliminating the t = 0 root, the velocity equation reveals t = 24/6 = 4 s for the time of maximum x. Plugging t = 4 into the equation for x leads to x = 64 m for the largest x value reached by the particle.

(e) From (d), we see that the x reaches its maximum at t = 4.0 s.

(f) A maximum v requires a = 0, which occurs when t = 24/12 = 2.0 s. This, inserted into the velocity equation, gives v_{max} = 24 m/s.

(g) From (f), we see that the maximum of v occurs at t = 24/12 = 2.0 s.

(h) In part (e), the particle was (momentarily) motionless at t = 4 s. The acceleration at that time is readily found to be $24 - 12(4) = -24$ m/s^2.

(i) The *average velocity* is defined by Eq. 2-2, so we see that the values of x at $t = 0$ and $t = 3$ s are needed; these are, respectively, $x = 0$ and $x = 54$ m (found in part (a)). Thus,

$$v_{\text{avg}} = \frac{54-0}{3-0} = 18 \text{ m/s}.$$

19. **THINK** In this one-dimensional kinematics problem, we're given the speed of a particle at two instants and asked to calculate its average acceleration.

EXPRESS We represent the initial direction of motion as the $+x$ direction. The average acceleration over a time interval $t_1 \le t \le t_2$ is given by Eq. 2-7:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v(t_2) - v(t_1)}{t_2 - t_1}.$$

ANALYZE Let $v_1 = +18$ m/s at $t_1 = 0$ and $v_2 = -30$ m/s at $t_2 = 2.4$ s. Using Eq. 2-7 we find

$$a_{\text{avg}} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{(-30 \text{ m/s}) - (+1 \text{ m/s})}{2.4 \text{ s} - 0} = -20 \text{ m/s}^2 .$$

LEARN The average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity. This makes sense because the velocity of the particle is decreasing over the time interval. With $t_1 = 0$, the velocity of the particle as a function of time can be written as

$$v = v_0 + at = (18 \text{ m/s}) - (20 \text{ m/s}^2)t .$$

20. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

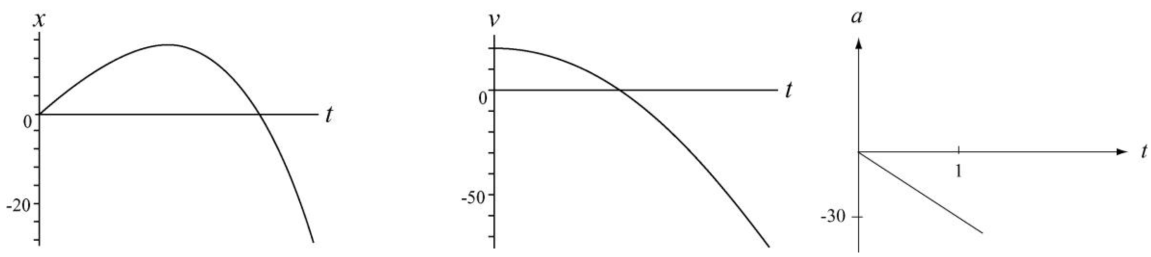
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$.

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) The graphs are shown below. SI units are understood.



21. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during 5 min ≤ t ≤ 10 min is taken to be the positive x direction. We also use the fact that $\Delta x = v\Delta t'$ when the velocity is constant during a time interval $\Delta t'$.

(a) The entire interval considered is Δt = 8 – 2 = 6 min, which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3\text{min} = 180$ s. His position at t = 2 min is x = 0 and his position at t = 8 min is $x = v\Delta t' = (2.2)(180) = 396\text{ m}$. Therefore,

$$v_{\text{avg}} = \frac{396\text{ m} - 0}{360\text{ s}} = 1.10\text{ m/s}.$$

(b) The man is at rest at t = 2 min and has velocity v = +2.2 m/s at t = 8 min. Thus, keeping the answer to 3 significant figures,

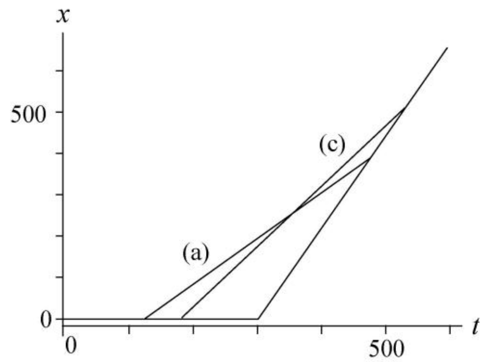
$$a_{\text{avg}} = \frac{2.2\text{ m/s} - 0}{360\text{ s}} = 0.00611\text{ m/s}^2 .$$

(c) Now, the entire interval considered is Δt = 9 – 3 = 6 min (360 s again), whereas the sub-interval in which he is moving is $\Delta t' = 9 - 5 = 4\text{min} = 240\text{ s}$). His position at $t = 3$ min is x = 0 and his position at t = 9 min is $x = v\Delta t' = (2.2)(240) = 528\text{ m}$. Therefore,

$$v_{\text{avg}} = \frac{528\text{ m} - 0}{360\text{ s}} = 1.47\text{ m/s}.$$

(d) The man is at rest at t = 3 min and has velocity v = +2.2 m/s at t = 9 min. Consequently, a_{avg} = 2.2/360 = 0.00611 m/s^2 just as in part (b).

(e) The horizontal line near the bottom of this x-vs-t graph represents the man standing at x = 0 for 0 ≤ t < 300 s and the linearly rising line for 300 ≤ t ≤ 600 s represents his constant-velocity motion. The lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v-vs-t is not shown here, but would consist of two horizontal "steps" (one at v = 0 for 0 ≤ t < 300 s and the next at v = 2.2 m/s for 300 ≤

$t \le 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of a_{avg}).

22. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

(a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system.

(b) Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

(c) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

$$t = \frac{2c}{3b} = \frac{2(3.0 \text{ m/s}^2)}{3(2.0 \text{ m/s}^3)} = 1.0 \text{ s} .$$

For $t = 0$, $x = x_0 = 0$ and for $t = 1.0$ s, $x = 1.0$ m $> x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1$s).

(d) In the first 4 s the particle moves from the origin to $x = 1.0$ m, turns around, and goes back to

$$x(4 \text{ s}) = (3.0 \text{ m/s}^2)(4.0 \text{ s})^2 - (2.0 \text{ m/s}^3)(4.0 \text{ s})^3 = -80 \text{ m} .$$

The total path length it travels is 1.0 m + 1.0 m + 80 m = 82 m.

(e) Its displacement is $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80$ m. Thus, $\Delta x = -80 \text{ m}$.

The velocity is given by $v = 2ct - 3bt^2 = (6.0 \text{ m/s}^2)t - (6.0 \text{ m/s}^3)t^2$.

(f) Plugging in $t = 1$ s, we obtain

$$v(1 \text{ s}) = (6.0 \text{ m/s}^2)(1.0 \text{ s}) - (6.0 \text{ m/s}^3)(1.0 \text{ s})^2 = 0.$$

(g) Similarly, $v(2 \text{ s}) = (6.0 \text{ m/s}^2)(2.0 \text{ s}) - (6.0 \text{ m/s}^3)(2.0 \text{ s})^2 = -12 \text{ m/s}$.

(h) $v(3 \text{ s}) = (6.0 \text{ m/s}^2)(3.0 \text{ s}) - (6.0 \text{ m/s}^3)(3.0 \text{ s})^2 = -36 \text{ m/s}$.

(i) $v(4 \text{ s}) = (6.0 \text{ m/s}^2)(4.0 \text{ s}) - (6.0 \text{ m/s}^3)(4.0 \text{ s})^2 = -72 \text{ m/s}$.

The acceleration is given by $a = dv/dt = 2c - 6b = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)t$.

(j) Plugging in $t = 1$ s, we obtain $a(1 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(1.0 \text{ s}) = -6.0 \text{ m/s}^2$.

(k) $a(2 \text{ s}) = 6.0 \text{ m/s}^2 - (12.0 \text{ m/s}^3)(2.0 \text{ s}) = -18 \text{ m/s}^2$.

(l) $a(3\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(3.0\text{ s}) = -30\text{ m/s}^2$.

(m) $a(4\text{ s}) = 6.0\text{ m/s}^2 - (12.0\text{ m/s}^3)(4.0\text{ s}) = -42\text{ m/s}^2$.

23. **THINK** The electron undergoes a constant acceleration. Given the final speed of the electron and the distance it has traveled, we can calculate its acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the electron can be readily analyzed using the equations given in Table 2-1:

$$v = v_0 + at \qquad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2-16)$$

The acceleration can be found by solving Eq. 2-16.

ANALYZE With $v_0 = 1.50\times10^5\text{ m/s}$, $v = 5.70\times10^6\text{ m/s}$, $x_0 = 0$ and $x = 0.010$ m, we find the average acceleration to be

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(5.7\times10^6\text{ m/s})^2 - (1.5\times10^5\text{ m/s})^2}{2(0.010\text{ m})} = 1.62\times10^{15}\text{ m/s}^2.$$

LEARN It is always a good idea to apply other equations in Table 2-1 not used for solving the problem as a consistency check. For example, since we now know the value of the acceleration, using Eq. 2-11, the time it takes for the electron to reach its final speed would be

$$t = \frac{v - v_0}{a} = \frac{5.70\times10^6\text{ m/s} - 1.5\times10^5\text{ m/s}}{1.62\times10^{15}\text{ m/s}^2} = 3.426\times10^{-9}\text{ s}$$

Substituting the value of t into Eq. 2-15, the distance the electron travels is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + (1.5\times10^5\text{ m/s})(3.426\times10^{-9}\text{ s}) + \frac{1}{2}(1.62\times10^{15}\text{ m/s}^2)(3.426\times10^{-9}\text{ s})^2$$
$$= 0.01\text{ m}$$

This is what was given in the problem statement. So we know the problem has been solved correctly.

24. In this problem we are given the initial and final speeds, and the displacement, and are asked to find the acceleration. We use the constant-acceleration equation given in Eq. 2-16, $v^2 = v_0^2 + 2a(x - x_0)$.

(a) Given that $v_0 = 0$, $v = 1.6$ m/s, and $\Delta x = 5.0\,\mu$m, the acceleration of the spores during the launch is

$$a=\frac{v^2-v_0^2}{2x}=\frac{(1.6\text{ m/s})^2}{2(5.0\times10^{-6}\text{ m})}=2.56\times10^5\text{ m/s}^2=2.6\times10^4 g$$

(b) During the speed-reduction stage, the acceleration is

$$a=\frac{v^2-v_0^2}{2x}=\frac{0-(1.6\text{ m/s})^2}{2(1.0\times10^{-3}\text{ m})}=-1.28\times10^3\text{ m/s}^2=-1.3\times10^2 g$$

The negative sign means that the spores are decelerating.

25. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0$; $v = 20$ m/s and $a = 2.0$ m/s^2. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20$ m/s; $v = 0$ and $a = -1.0$ m/s^2 (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20=0+2.0t_1$ yields $t_1 = 10$ s. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20 + (-1.0)t_2$ leads to $t_2 = 20$ s, and the total is $t = t_1 + t_2 = 30$ s.

(b) For part 1, taking $x_0 = 0$, we use the equation $v^2 = v_0^2 + 2a(x - x_0)$ from Table 2-1 and find

$$x=\frac{v^2-v_0^2}{2a}=\frac{(20\text{ m/s})^2-(0)^2}{2(2.0\text{ m/s}^2)}=100\text{ m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x-100\text{ m}=\frac{v^2-v_0^2}{2a}=\frac{(0)^2-(20\text{ m/s})^2}{2(-1.0\text{ m/s}^2)}.$$

Thus, the final position is $x = 300$ m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

26. The constant-acceleration condition permits the use of Table 2-1.

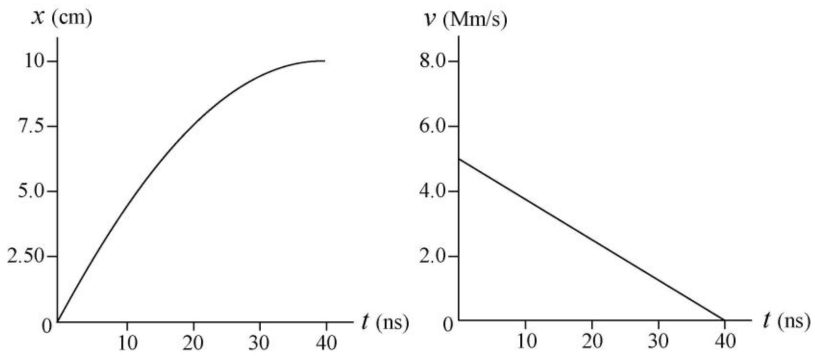
(a) Setting $v = 0$ and $x_0 = 0$ in $v^2=v_0^2+2a(x-x_0)$, we find

$$x=-\frac{1}{2}\frac{v_0^2}{a}=-\frac{1}{2}\frac{(5.00\times10^6)^2}{-1.25\times10^{14}}=0.100\text{ m}.$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so that other equations from Table 2-1 (such as $v=v_0+at$ and

$x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.



27. We use $v = v_0 + at$, with $t = 0$ as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$, we set $t = -2.5$ s. Thus, Eq. 2-11 gives

$$v = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(-2.5 \text{ s}) = 1.6 \text{ m/s}.$$

(b) Now, $t = +2.5$ s and we find $v = (9.6 \text{ m/s}) + \left(3.2 \text{ m/s}^2\right)(2.5 \text{ s}) = 18 \text{ m/s}.$

28. We take $+x$ in the direction of motion, so $v_0 = +24.6$ m/s and $a = -4.92$ m/s^2. We also take $x_0 = 0$.

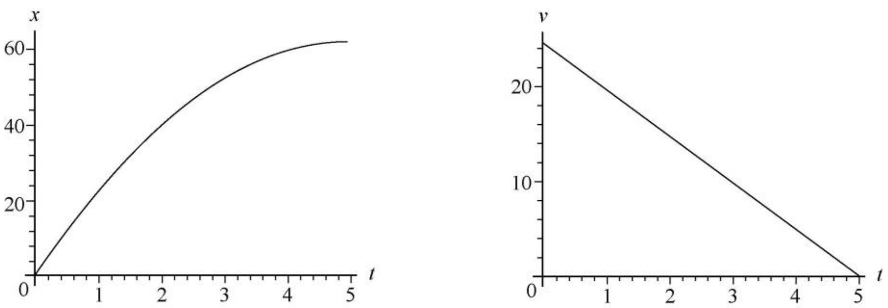
(a) The time to come to a halt is found using Eq. 2-11:

$$0 = v_0 + at \implies t = \frac{24.6 \text{ m/s}}{-4.92 \text{ m/s}^2} = 5.00 \text{ s}.$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \implies x = -\frac{(24.6 \text{ m/s})^2}{2\left(-4.92 \text{ m/s}^2\right)} = 61.5 \text{ m}.$$

(c) Using these results, we plot $v_0 t + \frac{1}{2}at^2$ (the x graph, shown next, on the left) and $v_0 + at$ (the v graph, on the right) over $0 \le t \le 5$ s, with SI units understood.



29. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then $a_1 = +1.22$ m/s^2 and $a_2 = -1.22$ m/s^2. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08$ m/s.

(a) We denote Δx as the distance moved during t_1, and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x \quad \Rightarrow \quad \Delta x = \frac{(5.08 \text{ m/s})^2}{2(1.22 \text{ m/s}^2)} = 10.59 \text{ m} \approx 10.6 \text{ m}.$$

(b) Using Eq. 2-11, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08 \text{ m/s}}{1.22 \text{ m/s}^2} = 4.17 \text{ s}.$$

The deceleration time t_2 turns out to be the same so that $t_1 + t_2 = 8.33$ s. The distances traveled during t_1 and t_2 are the same so that they total to 2(10.59 m) = 21.18 m. This implies that for a distance of 190 m – 21.18 m = 168.82 m, the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82 \text{ m}}{5.08 \text{ m/s}} = 33.21 \text{ s}.$$

Therefore, the total time is 8.33 s + 33.21 s ≈ 41.5 s.

30. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2-1.

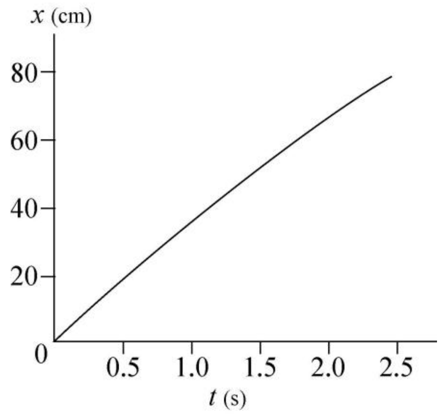
(a) Substituting $v_0 = 137$ km/h = 38.1 m/s, $v = 90$ km/h = 25 m/s, and $a = -5.2$ m/s^2 into $v = v_0 + at$, we obtain

$$t = \frac{25 \text{ m/s} - 38 \text{ m/s}}{-5.2 \text{ m/s}^2} = 2.5 \text{ s}.$$

(b) We take the car to be at $x = 0$ when the brakes are applied (at time $t = 0$). Thus, the coordinate of the car as a function of time is given by

$$x = \left(38 \text{ m/s}\right)t + \frac{1}{2}\left(-5.2 \text{ m/s}^2\right)t^2$$

in SI units. This function is plotted from $t = 0$ to $t = 2.5$ s on the graph to the right. We have not shown the v-vs-t graph here; it is a descending straight line from v_0 to v.



31. **THINK** The rocket ship undergoes a constant acceleration from rest, and we want to know the time elapsed and the distance traveled when the rocket reaches a certain speed.

EXPRESS Since the problem involves constant acceleration, the motion of the rocket can be readily analyzed using the equations in Table 2-1:

$$v = v_0 + at \qquad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2-16)$$

ANALYZE (a) Given that $a = 9.8\ \text{m/s}^2$, $v_0 = 0$ and $v = 0.1c = 3.0\times10^7\ \text{m/s}$, we can solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{3.0\times10^7\ \text{m/s} - 0}{9.8\ \text{m/s}^2} = 3.1\times10^6\ \text{s}$$

which is about 1.2 months. So it takes 1.2 months for the rocket to reach a speed of 0.1*c* starting from rest with a constant acceleration of 9.8 m/s^2.

(b) To calculate the distance traveled during this time interval, we evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with x_0 = 0 and $v_0 = 0$. The result is

$$x = \frac{1}{2}\left(9.8\ \text{m/s}^2\right)(3.1\times10^6\text{s})^2 = 4.6\times10^{13}\ \text{m}.$$

LEARN In solving parts (a) and (b), we did not use Eq. (2-16): $v^2 = v_0^2 + 2a(x - x_0)$. This equation can be used to check our answers. The final velocity based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{0 + 2(9.8\ \text{m/s}^2)(4.6\times10^{13}\ \text{m} - 0)} = 3.0\times10^7\ \text{m/s},$$

which is what was given in the problem statement. So we know the problems have been solved correctly.

32. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020\ \text{km/h})\left(\dfrac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)}{1.4\ \text{s}} = 202.4\ \text{m/s}^2\ .$$

In terms of the gravitational acceleration *g*, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = \left(\frac{202.4\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right)g = 21g\ .$$

33. **THINK** The car undergoes a constant negative acceleration to avoid impacting a barrier. Given its initial speed, we want to know the distance it has traveled and the time elapsed prior to the impact.

EXPRESS Since the problem involves constant acceleration, the motion of the car can be readily analyzed using the equations in Table 2-1:

$$v = v_0 + at \qquad (2-11)$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \qquad (2-15)$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad (2-16)$$

We take $x_0 = 0$ and $v_0 = 56.0$ km/h = 15.55 m/s to be the initial position and speed of the car. Solving Eq. 2-15 with $t = 2.00$ s gives the acceleration a. Once a is known, the speed of the car upon impact can be found by using Eq. 2-11.

ANALYZE (a) Using Eq. 2-15, we find the acceleration to be

$$a = \frac{2(x - v_0 t)}{t^2} = \frac{2\left[(24.0\text{ m}) - (15.55\text{ m/s})(2.00\text{ s})\right]}{(2.00\text{ s})^2} = -3.56\,\text{m/s}^2,$$

or $|a| = 3.56\text{ m/s}^2$. The negative sign indicates that the acceleration is opposite to the direction of motion of the car; the car is slowing down.

(b) The speed of the car at the instant of impact is

$$v = v_0 + at = 15.55\text{ m/s} + (-3.56\text{ m/s}^2)(2.00\text{ s}) = 8.43\text{ m/s}$$

which can also be converted to 30.3 km/h.

LEARN In solving parts (a) and (b), we did not use Eq. 1-16. This equation can be used as a consistency check. The final velocity based on this equation is

$$v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{(15.55\text{ m/s})^2 + 2(-3.56\text{ m/s}^2)(24\text{ m} - 0)} = 8.43\text{ m/s},$$

which is what was calculated in (b). This indicates that the problems have been solved correctly.

34. Let d be the 220 m distance between the cars at $t = 0$, and v_1 be the 20 km/h = 50/9 m/s speed (corresponding to a passing point of $x_1 = 44.5$ m) and v_2 be the 40 km/h =100/9 m/s speed (corresponding to a passing point of $x_2 = 76.6$ m) of the red car. We have two equations (based on Eq. 2-17):

$$d - x_1 = v_o t_1 + \tfrac{1}{2}a t_1^2 \qquad \text{where } t_1 = x_1/v_1$$

$$d - x_2 = v_o t_2 + \tfrac{1}{2}a t_2^2 \qquad \text{where } t_2 = x_2/v_2$$

We simultaneously solve these equations and obtain the following results:

(a) The initial velocity of the green car is $v_o = -\ 13.9$ m/s. or roughly – 50 km/h (the negative sign means that it's along the –x direction).

(b) The corresponding acceleration of the car is $a = -\ 2.0\ \text{m/s}^2$ (the negative sign means that it's along the –x direction).

35. The positions of the cars as a function of time are given by

$$x_r(t) = x_{r0} + \frac{1}{2}a_r t^2 = (-35.0\ \text{m}) + \frac{1}{2}a_r t^2$$
$$x_g(t) = x_{g0} + v_g t = (270\ \text{m}) - (20\ \text{m/s})t$$

where we have substituted the velocity and not the speed for the green car. The two cars pass each other at $t = 12.0\ \text{s}$ when the graphed lines cross. This implies that

$$(270\ \text{m}) - (20\ \text{m/s})(12.0\ \text{s}) = 30\ \text{m} = (-35.0\ \text{m}) + \frac{1}{2}a_r(12.0\ \text{s})^2$$

which can be solved to give $a_r = 0.90\ \text{m/s}^2$.

36. (a) Equation 2-15 is used for part 1 of the trip and Eq. 2-18 is used for part 2:

$\Delta x_1 = v_{o1}\, t_1 + \frac{1}{2}\, a_1\, t_1^2$ where $a_1 = 2.25\ \text{m/s}^2$ and $\Delta x_1 = \frac{900}{4}$ m

$\Delta x_2 = v_2\, t_2 - \frac{1}{2}\, a_2\, t_2^2$ where $a_2 = -0.75\ \text{m/s}^2$ and $\Delta x_2 = \frac{3(900)}{4}$ m

In addition, $v_{o1} = v_2 = 0$. Solving these equations for the times and adding the results gives $t = t_1 + t_2 = 56.6$ s.

(b) Equation 2-16 is used for part 1 of the trip:

$$v^2 = (v_{o1})^2 + 2a_1\Delta x_1 = 0 + 2(2.25)\left(\frac{900}{4}\right) = 1013\ \text{m}^2/\text{s}^2$$

which leads to $v = 31.8$ m/s for the maximum speed.

37. (a) From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

with $t = 1.0$ s, and then again with $t = 2.0$ s. This yields two equations for the two unknowns, v_0 and a:

$$0.0 - (-2.0\text{ m}) = v_0\,(1.0\text{ s}) + \frac{1}{2}a\,(1.0\text{ s})^2$$

$$6.0\text{ m} - (-2.0\text{ m}) = v_0\,(2.0\text{ s}) + \frac{1}{2}a\,(2.0\text{ s})^2\,.$$

Solving these simultaneous equations yields the results $v_0 = 0$ and $a = 4.0\text{ m/s}^2$.

(b) The fact that the answer is positive tells us that the acceleration vector points in the +*x* direction.

38. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34\text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34\text{ m/s}^2$ until it comes to a stop $(v_2 = 0)$ at the next station. The velocity at the midpoint is v_1, which occurs at $x_1 = 806/2 = 403\text{m}$.

(a) Equation 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \quad\Rightarrow\quad v_1 = \sqrt{2\left(1.34\text{ m/s}^2\right)\left(403\text{ m}\right)} = 32.9\text{ m/s}.$$

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

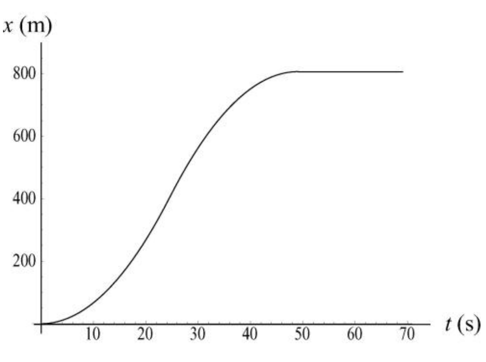
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \quad\Rightarrow\quad t_1 = \sqrt{\frac{2\left(403\text{ m}\right)}{1.34\text{ m/s}^2}} = 24.53\text{ s}\,.$$

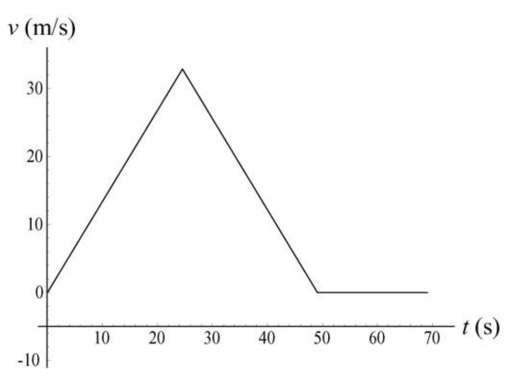
Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain $t = 49.1$ s for the travel time between stations.

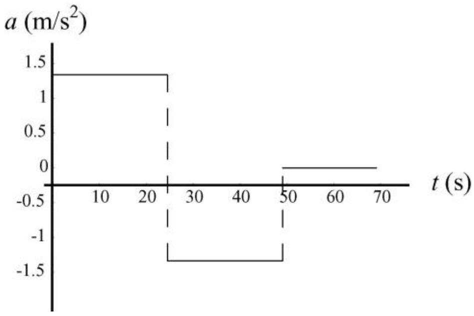
(c) With a "dead time" of 20 s, we have $T = t + 20 = 69.1$ s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806\text{ m}}{69.1\text{ s}} = 11.7\text{ m/s}\,.$$

(d) The graphs for *x*, *v* and *a* as a function of *t* are shown below. The third graph, $a(t)$, consists of three horizontal "steps" — one at 1.34 m/s^2 during $0 < t < 24.53$ s, and the next at -1.34 m/s^2 during $24.53\text{ s} < t < 49.1$ s and the last at zero during the "dead time" $49.1\text{ s} < t < 69.1$ s).







39. (a) We note that $v_A = 12/6 = 2$ m/s (with two significant figures understood). Therefore, with an initial x value of 20 m, car A will be at $x = 28$ m when $t = 4$ s. This must be the value of x for car B at that time; we use Eq. 2-15:

$$28 \text{ m} = (12 \text{ m/s})t + \tfrac{1}{2} a_B t^2 \qquad \text{where } t = 4.0 \text{ s} .$$

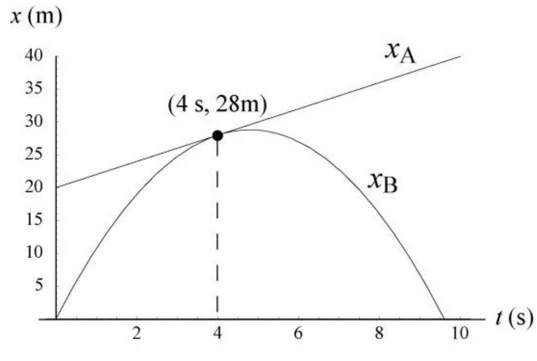
This yields $a_B = -2.5$ m/s^2.

(b) The question is: using the value obtained for a_B in part (a), are there other values of t (besides $t = 4$ s) such that $x_A = x_B$? The requirement is

$$20 + 2t = 12t + \tfrac{1}{2} a_B t^2$$

where $a_B = -5/2$. There are two distinct roots unless the discriminant $\sqrt{10^2 - 2(-20)(a_B)}$ is zero. In our case, it is zero – which means there is only one root. The cars are side by side only once at $t = 4$ s.

(c) A sketch is shown below. It consists of a straight line (x_A) tangent to a parabola (x_B) at $t = 4$.



(d) We only care about real roots, which means $10^2 - 2(-20)(a_B) \geq 0$. If $|a_B| > 5/2$ then there are no (real) solutions to the equation; the cars are never side by side.

(e) Here we have $10^2 - 2(-20)(a_B) > 0 \quad \Rightarrow \quad$ two real roots. The cars are side by side at two different times.

40. We take the direction of motion as $+x$, so $a = -5.18$ m/s^2, and we use SI units, so $v_0 = 55(1000/3600) = 15.28$ m/s.

(a) The velocity is constant during the reaction time T, so the distance traveled during it is

$$d_r = v_0T - (15.28 \text{ m/s})\,(0.75 \text{ s}) = 11.46 \text{ m}.$$

We use Eq. 2-16 (with $v = 0$) to find the distance d_b traveled during braking:

$$v^2 = v_0^2 + 2ad_b \;\Rightarrow\; d_b = -\frac{(15.28 \text{ m/s})^2}{2\left(-5.18 \text{ m/s}^2\right)}$$

which yields $d_b = 22.53$ m. Thus, the total distance is $d_r + d_b = 34.0$ m, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0, the car would enter the intersection in $t = (40 \text{ m})/(15.28 \text{ m/s}) = 2.6$ s, which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is 32/15.28 = 2.1 s, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

41. The displacement (Δx) for each train is the "area" in the graph (since the displacement is the integral of the velocity). Each area is triangular, and the area of a triangle is 1/2(base) × (height). Thus, the (absolute value of the) displacement for one train (1/2)(40 m/s)(5 s) = 100 m, and that of the other train is (1/2)(30 m/s)(4 s) = 60 m. The initial "gap" between the trains was 200 m, and according to our displacement computations, the gap has narrowed by 160 m. Thus, the answer is 200 – 160 = 40 m.

42. (a) Note that 110 km/h is equivalent to 30.56 m/s. During a two-second interval, you travel 61.11 m. The decelerating police car travels (using Eq. 2-15) 51.11 m. In light of the fact that the initial "gap" between cars was 25 m, this means the gap has narrowed by 10.0 m – that is, to a distance of 15.0 m between cars.

(b) First, we add 0.4 s to the considerations of part (a). During a 2.4 s interval, you travel 73.33 m. The decelerating police car travels (using Eq. 2-15) 58.93 m during that time. The initial distance between cars of 25 m has therefore narrowed by 14.4 m. Thus, at the start of your braking (call it t_0) the gap between the cars is 10.6 m. The speed of the police car at t_0 is 30.56 – 5(2.4) = 18.56 m/s. Collision occurs at time t when $x_{\text{you}} = x_{\text{police}}$ (we choose coordinates such that your position is $x = 0$ and the police car's position is $x = 10.6$ m at t_0). Eq. 2-15 becomes, for each car:

$$x_{\text{police}} - 10.6 = 18.56(t - t_0) - \tfrac{1}{2}(5)(t - t_0)^2$$
$$x_{\text{you}} = 30.56(t - t_0) - \tfrac{1}{2}(5)(t - t_0)^2 \quad .$$

Subtracting equations, we find

$$10.6 = (30.56 - 18.56)(t - t_0) \quad\Rightarrow\quad 0.883 \text{ s} = t - t_0.$$

At that time your speed is 30.56 + $a(t - t_0)$ = 30.56 – 5(0.883) ≈ 26 m/s (or 94 km/h).

43. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them, D, as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell .$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

which leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell\right)\left(\frac{v_\ell - v_t}{D}\right) = -\frac{1}{2D}(v_\ell - v_t)^2 .$$

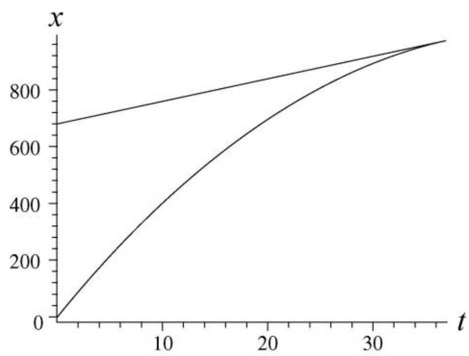
Hence,

$$a = -\frac{1}{2(0.676\text{ km})}\left(29\frac{\text{km}}{\text{h}} - 161\frac{\text{km}}{\text{h}}\right)^2 = -12888\text{ km/h}^2$$

which we convert as follows:

$$a = \left(-12888\text{ km/h}^2\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)^2 = -0.994\text{ m/s}^2$$

so that its *magnitude* is $|a|$ = 0.994 m/s². A graph is shown here for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.



The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.

44. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s² (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2}gt^2$, with y = 0.544 m and t = 0.200 s, we find

$$v_0 = \frac{y + gt^2/2}{t} = \frac{0.544\text{ m} + (9.8\text{ m/s}^2)(0.200\text{ s})^2/2}{0.200\text{ s}} = 3.70\text{ m/s}.$$

(b) The velocity at $y = 0.544$ m is

$$v = v_0 - gt = 3.70\text{ m/s} - (9.8\text{ m/s}^2)(0.200\text{ s}) = 1.74\text{ m/s}.$$

(c) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where $v = 0$).

$$y = \frac{v_0^2}{2g} = \frac{(3.7\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 0.698\text{ m}.$$

Thus, the armadillo goes 0.698 – 0.544 = 0.154 m higher.

45. **THINK** As the ball travels vertically upward, its motion is under the influence of gravitational acceleration. The kinematics is one-dimensional.

EXPRESS We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8\text{ m/s}^2$ (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is a = constant motion:

$$v = v_0 - gt \qquad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2}gt^2 \qquad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad (2-16)$$

We set $y_0 = 0$. Upon reaching the maximum height y, the speed of the ball is momentarily zero ($v = 0$). Therefore, we can relate its initial speed v_0 to y via the equation $0 = v^2 = v_0^2 - 2gy$. The time it takes for the ball to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0/g$. Therefore, for the entire trip (from the time it leaves the ground until the time it returns to the ground), the total flight time is $T = 2t = 2v_0/g$.

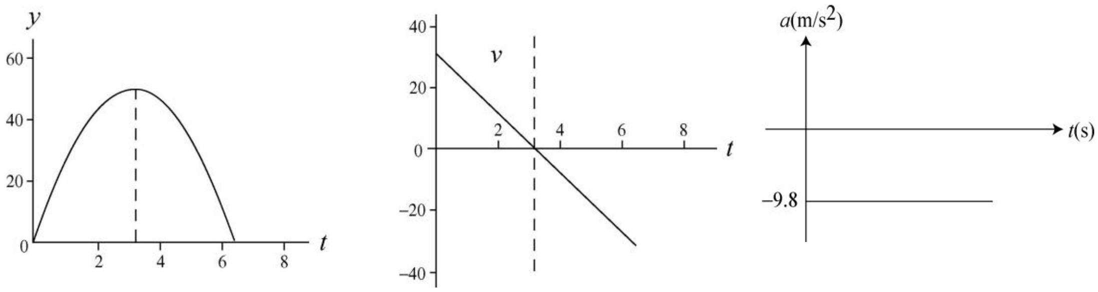
ANALYZE (a) At the highest point $v = 0$ and $v_0 = \sqrt{2gy}$. With $y = 50$ m, we find the initial speed of the ball to be

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8\text{ m/s}^2)(50\text{ m})} = 31.3\text{ m/s}.$$

(b) Using the result from (a) for v_0, the total flight time of the ball is

$$T = \frac{2v_0}{g} = \frac{2(31.3\text{ m/s})}{9.8\text{ m/s}^2} = 6.39\text{ s}$$

(c) The plots of y, v and a as a function of time are shown below. The acceleration graph is a horizontal line at –9.8 m/s^2. At t = 3.19 s, y = 50 m.



LEARN In calculating the total flight time of the ball, we could have used Eq. 2-15. At $t = T > 0$, the ball returns to its original position ($y = 0$). Therefore,

$$y = v_0 T - \frac{1}{2} g T^2 = 0 \;\Rightarrow\; T = \frac{2v_0}{g}$$

46. Neglect of air resistance justifies setting a = –g = –9.8 m/s^2 (where *down* is our –y direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8\text{ m/s}^2)(-1700\text{ m})} = -183\text{ m/s}.$$

Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

47. **THINK** The wrench is in free fall with an acceleration a = –g = –9.8 m/s^2.

EXPRESS We neglect air resistance, which justifies setting a = –g = –9.8 m/s^2 (taking *down* as the –y direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx):

$$v = v_0 - gt \qquad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \qquad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad (2-16)$$

Since the wrench had an initial speed v_0 = 0, knowing its speed of impact allows us to apply Eq. 2-16 to calculate the height from which it was dropped.

ANALYZE (a) Using $v^2 = v_0^2 + 2a\Delta y$, we find the initial height to be

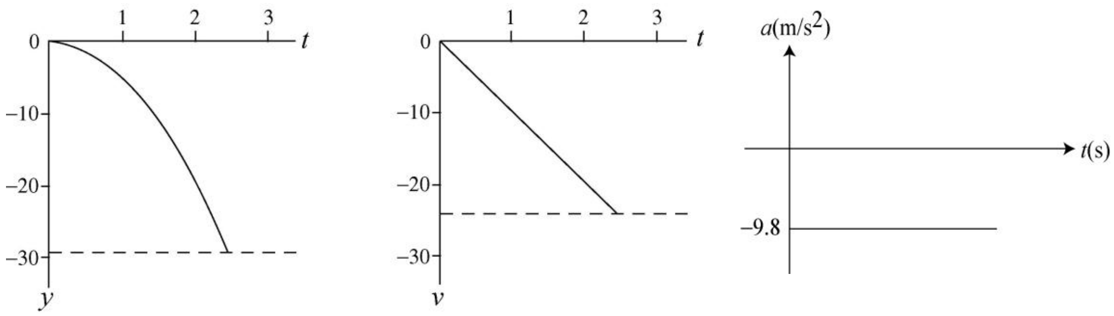
$$\Delta y = \frac{v_0^2 - v^2}{2a} = \frac{0-(-24\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 29.4\text{ m}.$$

So that it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we obtain a flight time of

$$t = \frac{v_0 - v}{g} = \frac{0-(-24\text{ m/s})}{9.8\text{ m/s}^2} = 2.45\text{ s}.$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. The acceleration graph is a horizontal line at –9.8 m/s^2.



LEARN As the wrench falls, with $a = -g < 0$, its speed increases but its velocity becomes more negative, as indicated by the second graph above.

48. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30$ m, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute t:

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with $v_0 = -12$ m/s since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-12\text{ m/s} + \sqrt{(-12\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(-30\text{ m})}}{9.8\text{ m/s}^2} = 1.54\text{ s}.$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain v; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1\text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

49. **THINK** In this problem a package is dropped from a hot-air balloon which is ascending vertically upward. We analyze the motion of the package under the influence of gravity.

EXPRESS We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. This allows us to use Table 2-1 (with Δy replacing Δx):

$$v = v_0 - gt \qquad (2-11)$$

$$y - y_0 = v_0 t - \frac{1}{2}gt^2 \qquad (2-15)$$

$$v^2 = v_0^2 - 2g(y - y_0) \qquad (2-16)$$

We place the coordinate origin on the ground and note that the initial velocity of the package is the same as the velocity of the balloon, v_0 = +12 m/s and that its initial coordinate is y_0 = +80 m. The time it takes for the package to hit the ground can be found by solving Eq. 2-15 with y = 0.

ANALYZE (a) We solve $0 = y = y_0 + v_0 t - \frac{1}{2}gt^2$ for time using the quadratic formula (choosing the positive root to yield a positive value for t):

$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12\text{ m/s} + \sqrt{(12\text{ m/s})^2 + 2\left(9.8\text{ m/s}^2\right)\left(80\text{ m}\right)}}{9.8\text{ m/s}^2} = 5.45\text{ s}.$$

(b) The speed of the package when it hits the ground can be calculated using Eq. 2-11. The result is

$$v = v_0 - gt = 12\text{ m/s} - (9.8\text{ m/s}^2)(5.447\text{ s}) = -41.38\text{ m/s}.$$

Its final *speed* is 41.38 m/s.

LEARN Our answers can be readily verified by using Eq. 2-16 which was not used in either (a) or (b). The equation leads to

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{(12\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(0 - 80\text{ m})} = -41.38\text{ m/s}$$

which agrees with that calculated in (b).

50. The y coordinate of Apple 1 obeys $y - y_{o1} = -\frac{1}{2}gt^2$ where y = 0 when t = 2.0 s. This allows us to solve for y_{o1}, and we find y_{o1} = 19.6 m.

The graph for the coordinate of Apple 2 (which is thrown apparently at t = 1.0 s with velocity v_2) is

$$y - y_{o2} = v_2(t - 1.0) - \frac{1}{2}\, g\, (t - 1.0)^2$$

where $y_{o2} = y_{o1} = 19.6$ m and where $y = 0$ when $t = 2.25$ s. Thus, we obtain $|v_2| = 9.6$ m/s, approximately.

51. (a) With upward chosen as the $+y$ direction, we use Eq. 2-11 to find the initial velocity of the package:

$$v = v_o + at \quad \Rightarrow \quad 0 = v_o - (9.8 \text{ m/s}^2)(2.0 \text{ s})$$

which leads to $v_o = 19.6$ m/s. Now we use Eq. 2-15:

$$\Delta y = (19.6 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(2.0 \text{ s})^2 \approx 20 \text{ m} .$$

We note that the “2.0 s” in this second computation refers to the time interval $2 < t < 4$ in the graph (whereas the “2.0 s” in the first computation referred to the $0 < t < 2$ time interval shown in the graph).

(b) In our computation for part (b), the time interval (“6.0 s”) refers to the $2 < t < 8$ portion of the graph:

$$\Delta y = (19.6 \text{ m/s})(6.0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(6.0 \text{ s})^2 \approx -59 \text{ m} ,$$

or $|\Delta y| = 59 \text{ m}$.

52. The full extent of the bolt’s fall is given by

$$y - y_0 = -\frac{1}{2}\, g\, t^2$$

where $y - y_0 = -90$ m (if upward is chosen as the positive y direction). Thus the time for the full fall is found to be $t = 4.29$ s. The first 80% of its free-fall distance is given by $-72 = -g\,\tau^2/2$, which requires time $\tau = 3.83$ s.

(a) Thus, the final 20% of its fall takes $t - \tau = 0.45$ s.

(b) We can find that speed using $v = -g\tau$. Therefore, $|v| = 38$ m/s, approximately.

(c) Similarly, $v_{final} = -\,g\,t \quad \Rightarrow \quad |v_{final}| = 42$ m/s.

53. **THINK** This problem involves two objects: a key dropped from a bridge, and a boat moving at a constant speed. We look for conditions such that the key will fall into the boat.

EXPRESS The speed of the boat is constant, given by $v_b = d/t$, where d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling.

To calculate t, we take the time to be zero at the instant the key is dropped, we compute the time t when $y = 0$ using $y = y_0 + v_0 t - \frac{1}{2}gt^2$, with $y_0 = 45$ m. Once t is known, the speed of the boat can be readily calculated.

ANALYZE Since the initial velocity of the key is zero, the coordinate of the key is given by $y_0 = \frac{1}{2}gt^2$. Thus, the time it takes for the key to drop into the boat is

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(45\text{ m})}{9.8\text{ m/s}^2}} = 3.03\text{ s}.$$

Therefore, the speed of the boat is $v_b = \dfrac{12\text{ m}}{3.03\text{ s}} = 4.0$ m/s.

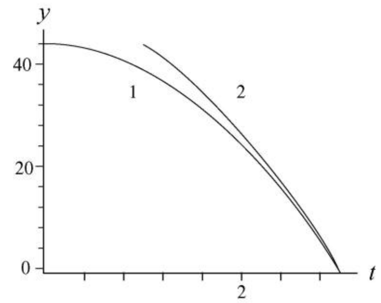
LEARN From the general expression $v_b = \dfrac{d}{t} = \dfrac{d}{\sqrt{2y_0/g}} = d\sqrt{\dfrac{g}{2y_0}}$, we see that $v_b \sim 1/\sqrt{y_0}$. This agrees with our intuition that the lower the height from which the key is dropped, the greater the speed of the boat in order to catch it.

54. (a) We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$

$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



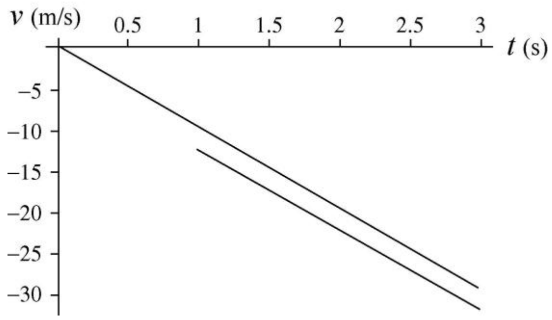
$$-43.9\text{ m} = (-v_0)(1.99\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(1.99\text{ s})^2$$

which leads to $v_0 = 12.3$ m/s.

(b) The velocity of the stones are given by

$$v'_y = \frac{d(\Delta y')}{dt} = -gt, \qquad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1)$$

The plot is shown below:



55. **THINK** The free-falling moist-clay ball strikes the ground with a non-zero speed, and it undergoes deceleration before coming to rest.

EXPRESS During contact with the ground its average acceleration is given by $a_{\text{avg}} = \dfrac{\Delta v}{\Delta t}$, where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0\times10^{-3}$ s is the duration of contact. Thus, we must first find the velocity of the ball just before it hits the ground (y = 0).

ANALYZE (a) Now, to find the velocity just *before* contact, we take t = 0 to be when it is dropped. Using Eq. 2-16 with $y_0 = 15.0$ m, we obtain

$$v = -\sqrt{v_0^2 - 2g(y-y_0)} = -\sqrt{0-2(9.8\text{ m/s}^2)(0-15\text{ m})} = -17.15\text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{0-(-17.1\text{ m/s})}{20.0\times10^{-3}\text{ s}} = 857\text{ m/s}^2.$$

(b) The fact that the result is positive indicates that this acceleration vector points upward.

LEARN Since Δt is very small, it is not surprising to have a very large acceleration to stop the motion of the ball. In later chapters, we shall see that the acceleration is directly related to the magnitude and direction of the force exerted by the ground on the ball during the course of collision.

56. We use Eq. 2-16,

$$v_B^2 = v_A^2 + 2a(y_B - y_A),$$

with a = –9.8 m/s^2, $y_B - y_A$ = 0.40 m, and $v_B = \frac{1}{3}v_A$. It is then straightforward to solve: v_A = 3.0 m/s, approximately.

57. The average acceleration during contact with the floor is $a_{\text{avg}} = (v_2 - v_1)/\Delta t$,

where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor (12×10^{-3} s).

(a) Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With $v_0 = 0$ and $y = -4.00$ m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8 \text{ m/s}^2)(-4.00 \text{ m})} = -8.85 \text{ m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with $v = 0$, $y = -2.00$ m (it ends up two meters *below* its initial drop height), and $y_0 = -4.00$ m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8 \text{ m/s}^2)(-2.00 \text{ m} + 4.00 \text{ m})} = 6.26 \text{ m/s} .$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26 \text{ m/s} - (-8.85 \text{ m/s})}{12.0 \times 10^{-3} \text{ s}} = 1.26 \times 10^3 \text{ m/s}^2.$$

(b) The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

58. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as $t - t'$ where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is $y - y' = 0.50h$, where y denotes the location of the ground. In these terms, y is the same as h, so we have $h - y' = 0.50h$ or $0.50h = y'$.

(a) We find t' and t from Eq. 2-15 (with $v_0 = 0$):

$$y' = \frac{1}{2} g t'^2 \;\Rightarrow\; t' = \sqrt{\frac{2y'}{g}}$$
$$y = \frac{1}{2} g t^2 \;\Rightarrow\; t = \sqrt{\frac{2y}{g}} .$$

Plugging in $y = h$ and $y' = 0.50h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)/g}{2h/g}} = \sqrt{0.50} .$$

Letting $t' = t - 1.00$ (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \;\Rightarrow\; t = \frac{1.00}{1 - \sqrt{0.50}}$$

which yields t = 3.41 s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$ we find h = 57 m.

(c) In our approach, we did not use the quadratic formula, but we did "choose a root" when we assumed (in the last calculation in part (a)) that $\sqrt{0.50}$ = +0.707 instead of –0.707. If we had instead let $\sqrt{0.50}$ = –0.707 then our answer for t would have been roughly 0.6 s, which would imply that t' = t – 1 would equal a negative number (indicating a time *before* it was dropped), which certainly does not fit with the physical situation described in the problem.

59. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the –y direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y-axis.

(a) The time drop 1 leaves the nozzle is taken as t = 0 and its time of landing on the floor t_1 can be computed from Eq. 2-15, with v_0 = 0 and y_1 = –2.00 m.

$$y_1 = -\frac{1}{2}gt_1^2 \;\Rightarrow\; t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00\text{ m})}{9.8\text{ m/s}^2}} = 0.639\text{ s}\,.$$

At that moment, the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at t = 0.639/3 = 0.213 s and drop 3 leaves the nozzle at t = 2(0.213 s) = 0.426 s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1$ – 0.213 s = 0.426 s. Its position at the moment drop 1 strikes the floor is

$$y_2 = -\frac{1}{2}gt_2^2 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.426\text{ s})^2 = -0.889\text{ m},$$

or about 89 cm below the nozzle.

(b) The time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1$ –0.426 s = 0.213 s. Its position at the moment drop 1 strikes the floor is

$$y_3 = -\frac{1}{2}gt_3^2 = -\frac{1}{2}(9.8\text{ m/s}^2)(0.213\text{ s})^2 = -0.222\text{ m},$$

or about 22 cm below the nozzle.

60. To find the "launch" velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \quad \Rightarrow \quad 0 = v_0 - \left(9.8\ \text{m/s}^2\right)\left(2.5\ \text{s}\right)$$

so that v_0 = 24.5 m/s (with +y up). Now we use Eq. 2-15 to find the height of the tower (taking y_0 = 0 at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad y - 0 = \left(24.5\ \text{m/s}\right)\left(1.5\ \text{s}\right) - \frac{1}{2}\left(9.8\ \text{m/s}^2\right)\left(1.5\ \text{s}\right)^2.$$

Thus, we obtain y = 26 m.

61. We choose *down* as the +y direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1, and passes the bottom (which is at y_2) at time t_2. We are told $y_2 - y_1$ = 1.20 m and $t_2 - t_1$ = 0.125 s. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1\left(t_2 - t_1\right) + \frac{1}{2}g\left(t_2 - t_1\right)^2$$

which immediately yields

$$v_1 = \frac{1.20\ \text{m} - \frac{1}{2}\left(9.8\ \text{m/s}^2\right)\left(0.125\ \text{s}\right)^2}{0.125\ \text{s}} = 8.99\ \text{m/s}.$$

From this, Eq. 2-16 (with v_0 = 0) reveals the value of y_1:

$$v_1^2 = 2gy_1 \quad \Rightarrow \quad y_1 = \frac{(8.99\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 4.12\ \text{m}.$$

It reaches the ground (y_3 = H) at t_3. Because of the symmetry expressed in the problem ("upward flight is a reverse of the fall'') we know that $t_3 - t_2$ = 2.00/2 = 1.00 s. And this means $t_3 - t_1$ = 1.00 s + 0.125 s = 1.125 s. Now Eq. 2-15 produces

$$y_3 - y_1 = v_1(t_3 - t_1) + \frac{1}{2}g\,(t_3 - t_1)^2$$

$$y_3 - 4.12\ \text{m} = (8.99\ \text{m/s})(1.125\ \text{s}) + \frac{1}{2}(9.8\ \text{m/s}^2)(1.125\ \text{s})^2$$

which yields $y_3 = H$ = 20.4 m.

62. The height reached by the player is y = 0.76 m (where we have taken the origin of the y axis at the floor and +y to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8\ \text{m/s}^2)(0.76\ \text{m})} = 3.86\ \text{m/s}\ .$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches y_1

= 0.76 m – 0.15 m = 0.61 m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.61\text{ m})} = 1.71\text{ m/s}.$$

The time t_1 that the player spends *ascending* in the top Δy_1 = 0.15 m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2}(v_1 + v)t_1 \;\Rightarrow\; t_1 = \frac{2(0.15\text{ m})}{1.71\text{ m/s} + 0} = 0.175\text{ s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is 2(0.175 s) = 0.35 s = 350 ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15\text{ m} = v_0 t_2 - \frac{1}{2}gt_2^2 = (3.86\text{ m/s})t_2 - \frac{1}{2}(9.8\text{ m/s}^2)t_2^2\,,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) t_2 = 0.041 s = 41 ms, which implies that the total time spent in that bottom 15 cm (both ascending and descending) is 2(41 ms) = 82 ms.

63. The time t the pot spends passing in front of the window of length L = 2.0 m is 0.25 s each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8$ m/s^2 (taking *down* to be the $-y$ direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2}gt^2 \quad\Rightarrow\quad v = \frac{L}{t} - \frac{1}{2}gt.$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{(L/t - gt/2)^2}{2g} = \frac{\left(2.00\text{ m}/0.25\text{ s} - (9.80\text{ m/s}^2)(0.25\text{ s})/2\right)^2}{2(9.80\text{ m/s}^2)} = 2.34\text{ m}.$$

64. The graph shows y = 25 m to be the highest point (where the speed momentarily vanishes). The neglect of “air friction” (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

(a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2-15 (with $+y$ up)

$$y - y_0 = vt + \frac{1}{2}g_p t^2 \quad\Rightarrow\quad 25\text{ m} - 0 = (0)(2.5\text{ s}) + \frac{1}{2}g_p(2.5\text{ s})^2$$

so that g_p = 8.0 m/s^2.

(b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)t \quad \Rightarrow \quad 25\text{ m} - 0 = \frac{1}{2}\,(v_0 + 0)(2.5\text{ s})$$

Therefore, v_0 = 20 m/s.

65. The key idea here is that the speed of the head (and the torso as well) at any given time can be calculated by finding the area on the graph of the head's acceleration versus time, as shown in Eq. 2-26:

$$v_1 - v_0 \;=\; \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

(a) From Fig. 2.15a, we see that the head begins to accelerate from rest (v_0 = 0) at t_0 = 110 ms and reaches a maximum value of 90 m/s^2 at t_1 = 160 ms. The area of this region is

$$\text{area} = \frac{1}{2}(160 - 110)\times 10^{-3}\text{s}\cdot\left(90\text{ m/s}^2\right) = 2.25\text{ m/s}$$

which is equal to v_1, the speed at t_1.

(b) To compute the speed of the torso at t_1=160 ms, we divide the area into 4 regions: From 0 to 40 ms, region A has zero area. From 40 ms to 100 ms, region B has the shape of a triangle with area

$$\text{area}_\text{B} = \frac{1}{2}(0.0600\text{ s})(50.0\text{ m/s}^2) = 1.50\text{ m/s}\,.$$

From 100 to 120 ms, region C has the shape of a rectangle with area

$$\text{area}_\text{C} \;=\; (0.0200\text{ s})\;(50.0\text{ m/s}^2) = 1.00\text{ m/s.}$$

From 110 to 160 ms, region D has the shape of a trapezoid with area

$$\text{area}_\text{D} \;=\; \frac{1}{2}(0.0400\text{ s})\;(50.0\;+\;20.0)\text{ m/s}^2 \;=\; 1.40\text{ m/s.}$$

Substituting these values into Eq. 2-26, with v_0 = 0 then gives

$$v_1 - 0 = 0 + 1.50\text{ m/s} + 1.00\text{ m/s} + 1.40\text{ m/s} = 3.90\text{ m/s},$$

or $v_1 = 3.90$ m/s.

66. The key idea here is that the position of an object at any given time can be calculated by finding the area on the graph of the object's velocity versus time, as shown in Eq. 2-30:

$$x_1 - \;x_0 \;=\; \begin{pmatrix} \text{area between the velocity curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}.$$

(a) To compute the position of the fist at t = 50 ms, we divide the area in Fig. 2-37 into two regions. From 0 to 10 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.010 \text{ s})(2 \text{ m/s}) = 0.01 \text{ m}.$$

From 10 to 50 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.040 \text{ s})(2+4) \text{ m/s} = 0.12 \text{ m}.$$

Substituting these values into Eq. 2-30 with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} = 0.13 \text{ m},$$

or $x_1 = 0.13$ m.

(b) The speed of the fist reaches a maximum at $t_1 = 120$ ms. From 50 to 90 ms, region *C* has the shape of a trapezoid with area

$$\text{area}_C = \frac{1}{2}(0.040 \text{ s})(4+5) \text{ m/s} = 0.18 \text{ m}.$$

From 90 to 120 ms, region D has the shape of a trapezoid with area

$$\text{area}_D = \frac{1}{2}(0.030 \text{ s})(5+7.5) \text{ m/s} = 0.19 \text{ m}.$$

Substituting these values into Eq. 2-30, with $x_0 = 0$ then gives

$$x_1 - 0 = 0 + 0.01 \text{ m} + 0.12 \text{ m} + 0.18 \text{ m} + 0.19 \text{ m} = 0.50 \text{ m},$$

or $x_1 = 0.50$ m.

67. The problem is solved using Eq. 2-31:

$$v_1 - v_0 = \begin{pmatrix} \text{area between the acceleration curve} \\ \text{and the time axis, from } t_0 \text{ to } t_1 \end{pmatrix}$$

To compute the speed of the unhelmeted, bare head at $t_1 = 7.0$ ms, we divide the area under the *a* vs. *t* graph into 4 regions: From 0 to 2 ms, region A has the shape of a triangle with area

$$\text{area}_A = \frac{1}{2}(0.0020 \text{ s})(120 \text{ m/s}^2) = 0.12 \text{ m/s}.$$

From 2 ms to 4 ms, region B has the shape of a trapezoid with area

$$\text{area}_B = \frac{1}{2}(0.0020 \text{ s})(120+140) \text{ m/s}^2 = 0.26 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_\text{C} = \frac{1}{2}(0.0020 \text{ s}) (140 + 200) \text{ m/s}^2 = 0.34 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_\text{D} = \frac{1}{2}(0.0010 \text{ s}) (200 \text{ m/s}^2) = 0.10 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with v_0=0 then gives

$$v_\text{unhelmeted} = 0.12 \text{ m/s} + 0.26 \text{ m/s} + 0.34 \text{ m/s} + 0.10 \text{ m/s} = 0.82 \text{ m/s}.$$

Carrying out similar calculations for the helmeted head, we have the following results: From 0 to 3 ms, region A has the shape of a triangle with area

$$\text{area}_\text{A} = \frac{1}{2}(0.0030 \text{ s}) (40 \text{ m/s}^2) = 0.060 \text{ m/s}.$$

From 3 ms to 4 ms, region B has the shape of a rectangle with area

$$\text{area}_\text{B} = (0.0010 \text{ s}) (40 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

From 4 to 6 ms, region C has the shape of a trapezoid with area

$$\text{area}_\text{C} = \frac{1}{2}(0.0020 \text{ s}) (40 + 80) \text{ m/s}^2 = 0.12 \text{ m/s}.$$

From 6 to 7 ms, region D has the shape of a triangle with area

$$\text{area}_\text{D} = \frac{1}{2}(0.0010 \text{ s}) (80 \text{ m/s}^2) = 0.040 \text{ m/s}.$$

Substituting these values into Eq. 2-31, with $v_0 = 0$ then gives

$$v_\text{helmeted} = 0.060 \text{ m/s} + 0.040 \text{ m/s} + 0.12 \text{ m/s} + 0.040 \text{ m/s} = 0.26 \text{ m/s}.$$

Thus, the difference in the speed is

$$\Delta v = v_\text{unhelmeted} - v_\text{helmeted} = 0.82 \text{ m/s} - 0.26 \text{ m/s} = 0.56 \text{ m/s}.$$

68. This problem can be solved by noting that velocity can be determined by the graphical integration of acceleration versus time. The speed of the tongue of the salamander is simply equal to the area under the acceleration curve:

$$\begin{aligned} v = \text{area} &= \frac{1}{2}(10^{-2} \text{ s})(100 \text{ m/s}^2) + \frac{1}{2}(10^{-2} \text{ s})(100 \text{ m/s}^2 + 400 \text{ m/s}^2) + \frac{1}{2}(10^{-2} \text{ s})(400 \text{ m/s}^2) \\ &= 5.0 \text{ m/s}. \end{aligned}$$

69. Since $v = dx/dt$ (Eq. 2-4), then $\Delta x = \int v\,dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base × height) and triangular $\left(\frac{1}{2}\,\text{base} \times \text{height}\right)$ areas, we have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4)\right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain Δx = 100 m.

70. To solve this problem, we note that velocity is equal to the time derivative of a position function, as well as the time integral of an acceleration function, with the integration constant being the initial velocity. Thus, the velocity of particle 1 can be written as

$$v_1 = \frac{dx_1}{dt} = \frac{d}{dt}\left(6.00t^2 + 3.00t + 2.00\right) = 12.0t + 3.00\,.$$

Similarly, the velocity of particle 2 is

$$v_2 = v_{20} + \int a_2 dt = 20.0 + \int (-8.00t)dt = 20.0 - 4.00t^2.$$

The condition that $v_1 = v_2$ implies

$$12.0t + 3.00 = 20.0 - 4.00t^2 \quad \Rightarrow \quad 4.00t^2 + 12.0t - 17.0 = 0$$

which can be solved to give (taking positive root) $t = (-3+\sqrt{26})/2 = 1.05\,\text{s}$. Thus, the velocity at this time is $v_1 = v_2 = 12.0(1.05) + 3.00 = 15.6$ m/s.

71. (a) The derivative (with respect to time) of the given expression for x yields the "velocity" of the spot:

$$v(t) = 9 - \tfrac{9}{4}\,t^2$$

with 3 significant figures understood. It is easy to see that v = 0 when t = 2.00 s.

(b) At t = 2 s, $x = 9(2) - ¾(2)^3 = 12$. Thus, the location of the spot when v = 0 is 12.0 cm from left edge of screen.

(c) The derivative of the velocity is $a = -\frac{9}{2}\,t$, which gives an acceleration of $-9.00\ \text{cm/m}^2$ (negative sign indicating leftward) when the spot is 12 cm from the left edge of screen.

(d) Since $v > 0$ for times less than t = 2 s, then the spot had been moving rightward.

(e) As implied by our answer to part (c), it moves leftward for times immediately after t = 2 s. In fact, the expression found in part (a) guarantees that for all $t > 2$, $v < 0$ (that is, until the clock is "reset" by reaching an edge).

(f) As the discussion in part (e) shows, the edge that it reaches at some $t > 2$ s cannot be the right edge; it is the left edge (x = 0). Solving the expression given in the problem statement (with x = 0) for positive t yields the answer: the spot reaches the left edge at $t = \sqrt{12}$ s ≈ 3.46 s.

72. We adopt the convention frequently used in the text: that "up" is the positive y direction.

(a) At the highest point in the trajectory v = 0. Thus, with t = 1.60 s, the equation $v = v_0 - gt$ yields v_0 = 15.7 m/s.

(b) One equation that is not dependent on our result from part (a) is $y - y_0 = vt + \frac{1}{2}gt^2$; this readily gives $y_{max} - y_0$ = 12.5 m for the highest ("max") point measured relative to where it started (the top of the building).

(c) Now we use our result from part (a) and plug into $y - y_0 = v_0t + \frac{1}{2}gt^2$ with t = 6.00 s and y = 0 (the ground level). Thus, we have

$$0 - y_0 = (15.68 \text{ m/s})(6.00 \text{ s}) - \tfrac{1}{2}\,(9.8 \text{ m/s}^2)(6.00 \text{ s})^2.$$

Therefore, y_0 (the height of the building) is equal to 82.3 m.

73. We denote the required time as t, assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as a and the (constant) speed of the truck as v then

$$\Delta x = \left(\frac{1}{2}at^2\right)_{\text{car}} = \left(vt\right)_{\text{truck}}$$

which leads to

$$t = \frac{2v}{a} = \frac{2\left(9.5 \text{ m/s}\right)}{2.2 \text{ m/s}^2} = 8.6 \text{ s} .$$

Therefore,

$$\Delta x = vt = \left(9.5 \text{ m/s}\right)\left(8.6 \text{ s}\right) = 82 \text{ m} .$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = at = \left(2.2 \text{ m/s}^2\right)\left(8.6 \text{ s}\right) = 19 \text{ m/s} .$$

74. If the plane (with velocity v) maintains its present course, and if the terrain continues its upward slope of 4.3°, then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35\text{ m}}{\tan 4.3°} = 465.5\text{ m} \approx 0.465\text{ km}.$$

This corresponds to a time of flight found from Eq. 2-2 (with $v = v_{avg}$ since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465\text{ km}}{1300\text{ km/h}} = 0.000358\text{ h} \approx 1.3\text{ s}.$$

This, then, estimates the time available to the pilot to make his correction.

75. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2} a t_b^2$$

where v_0 is the initial velocity and a is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a}.$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2}\frac{v_{01}^2}{a}, \qquad x_2 = v_{02} t_r - \frac{1}{2}\frac{v_{02}^2}{a}.$$

Solving these equations simultaneously for t_r and a we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}\left(v_{02} - v_{01}\right)}$$

and

$$a = -\frac{1}{2}\frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2}.$$

(a) Substituting x_1 = 56.7 m, v_{01} = 80.5 km/h = 22.4 m/s, x_2 = 24.4 m and v_{02} = 48.3 km/h = 13.4 m/s, we find

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}(v_{02} - v_{01})} = \frac{(13.4\text{ m/s})^2(56.7\text{ m}) - (22.4\text{ m/s})^2(24.4\text{ m})}{(22.4\text{ m/s})(13.4\text{ m/s})(13.4\text{ m/s} - 22.4\text{ m/s})}$$
$$= 0.74\text{ s}.$$

(b) Similarly, substituting x_1 = 56.7 m, v_{01} = 80.5 km/h = 22.4 m/s, x_2 = 24.4 m, and

v_{02} = 48.3 km/h = 13.4 m/s gives

$$a = -\frac{1}{2}\frac{v_{02}v_{01}^2 - v_{01}v_{02}^2}{v_{02}x_1 - v_{01}x_2} = -\frac{1}{2}\frac{(13.4\text{ m/s})(22.4\text{ m/s})^2 - (22.4\text{ m/s})(13.4\text{ m/s})^2}{(13.4\text{ m/s})(56.7\text{ m}) - (22.4\text{ m/s})(24.4\text{ m})}$$
$$= -6.2\text{ m/s}^2.$$

The *magnitude* of the deceleration is therefore 6.2 m/s^2. Although rounded-off values are displayed in the above substitutions, what we have input into our calculators are the "exact" values (such as $v_{02} = \frac{161}{12}$ m/s).

76. (a) A constant velocity is equal to the ratio of displacement to elapsed time. Thus, for the vehicle to be traveling at a constant speed v_p over a distance D_{23}, the time delay should be $t = D_{23}/v_p$.

(b) The time required for the car to accelerate from rest to a cruising speed v_p is $t_0 = v_p/a$. During this time interval, the distance traveled is $\Delta x_0 = at_0^2/2 = v_p^2/2a$. The car then moves at a constant speed v_p over a distance $D_{12} - \Delta x_0 - d$ to reach intersection 2, and the time elapsed is $t_1 = (D_{12} - \Delta x_0 - d)/v_p$. Thus, the time delay at intersection 2 should be set to

$$t_{\text{total}} = t_r + t_0 + t_1 = t_r + \frac{v_p}{a} + \frac{D_{12} - \Delta x_0 - d}{v_p} = t_r + \frac{v_p}{a} + \frac{D_{12} - (v_p^2/2a) - d}{v_p}$$
$$= t_r + \frac{1}{2}\frac{v_p}{a} + \frac{D_{12} - d}{v_p}$$

77. **THINK** The speed of the rod changes due to a nonzero acceleration.

EXPRESS Since the problem involves constant acceleration, the motion of the rod can be readily analyzed using the equations given in Table 2-1. We take +*x* to be in the direction of motion, so

$$v = (60\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = +16.7\text{ m/s}$$

and $a > 0$. The location where the rod starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

ANALYZE (a) Using Eq. 2-7, we find the average acceleration to be

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{16.7\text{ m/s} - 0}{5.4\text{ s} - 0} = 3.09\text{ m/s}^2.$$

(b) Assuming constant acceleration $a = a_{\text{avg}} = 3.09\text{ m/s}^2$, the total distance traveled during the 5.4-s time interval is

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(3.09\text{ m/s}^2)(5.4\text{ s})^2 = 45\text{ m}$$

(c) Using Eq. 2-15, the time required to travel a distance of $x = 250$ m is:

$$x = \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250\text{ m})}{3.1\text{ m/s}^2}} = 12.73\text{ s}$$

LEARN The displacement of the rod as a function of time can be written as $x(t) = \frac{1}{2}(3.09\text{ m/s}^2)t^2$. Note that we could have chosen Eq. 2-17 to solve for (b):

$$x = \frac{1}{2}(v_0 + v)\,t = \frac{1}{2}(16.7\text{ m/s})(5.4\text{ s}) = 45\text{ m}.$$

78. We take the moment of applying brakes to be $t = 0$. The deceleration is constant so that Table 2-1 can be used. Our primed variables (such as $v_0' = 72\text{ km/h} = 20\text{ m/s}$) refer to one train (moving in the $+x$ direction and located at the origin when $t = 0$) and unprimed variables refer to the other (moving in the $-x$ direction and located at x_0 = +950 m when $t = 0$). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is v_0 = –144 km/h = –40 m/s. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2-16, it should stop (meaning $v' = 0$) at

$$x' = \frac{(v')^2 - (v_0')^2}{2a'} = \frac{0 - (20\text{ m/s})^2}{-2\text{ m/s}^2} = 200\text{ m}.$$

The speed of the other train, when it reaches that location, is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(-40\text{ m/s})^2 + 2(1.0\text{ m/s}^2)(200\text{ m} - 950\text{ m})}$$
$$= 10\text{ m/s}$$

using Eq 2-16 again. Specifically, its velocity at that moment would be –10 m/s since it is still traveling in the $-x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-11 yields t = 20 s) and seeing where the unprimed train is at that moment (Eq. 2-18 yields x = 350 m, still a good distance away from contact).

79. The y coordinate of Piton 1 obeys $y - y_{01} = -\frac{1}{2}g\,t^2$ where $y = 0$ when $t = 3.0$ s. This allows us to solve for y_{o1}, and we find y_{01} = 44.1 m. The graph for the coordinate of Piton 2 (which is thrown apparently at $t = 1.0$ s with velocity v_1) is

$$y - y_{02} = v_1(t-1.0) - \tfrac{1}{2}\, g\, (t - 1.0)^2$$

where $y_{02} = y_{01} + 10 = 54.1$ m and where (again) $y = 0$ when $t = 3.0$ s. Thus we obtain $|v_1| = 17$ m/s, approximately.

80. We take $+x$ in the direction of motion. We use subscripts 1 and 2 for the data. Thus, $v_1 = +30$ m/s, $v_2 = +50$ m/s, and $x_2 - x_1 = +160$ m.

(a) Using these subscripts, Eq. 2-16 leads to

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{(50\text{ m/s})^2 - (30\text{ m/s})^2}{2(160\text{ m})} = 5.0\text{ m/s}^2\ .$$

(b) We find the time interval corresponding to the displacement $x_2 - x_1$ using Eq. 2-17:

$$t_2 - t_1 = \frac{2(x_2 - x_1)}{v_1 + v_2} = \frac{2(160\text{ m})}{30\text{ m/s} + 50\text{ m/s}} = 4.0\text{ s}\ .$$

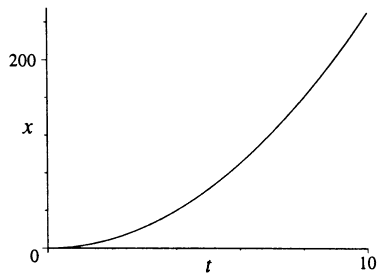
(c) Since the train is at rest ($v_0 = 0$) when the clock starts, we find the value of t_1 from Eq. 2-11:

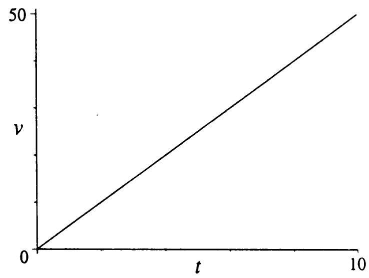
$$v_1 = v_0 + at_1 \ \Rightarrow\ t_1 = \frac{30\text{ m/s}}{5.0\text{ m/s}^2} = 6.0\text{ s}\ .$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so $x_0 = 0$). Thus, we are asked to find the value of x_1. Although any of several equations could be used, we choose Eq. 2-17:

$$x_1 = \frac{1}{2}(v_0 + v_1)t_1 = \frac{1}{2}(30\text{ m/s})(6.0\text{ s}) = 90\text{ m}\ .$$

(e) The graphs are shown below, with SI units understood.





81. **THINK** The particle undergoes a *non-constant* acceleration along the $+x$-axis. An integration is required to calculate velocity.

EXPRESS With a non-constant acceleration $a(t) = dv/dt$, the velocity of the particle at time t_1 is given by Eq. 2-27: $v_1 = v_0 + \int_{t_0}^{t_1} a(t)dt$, where v_0 is the velocity at time t_0. In our situation, we have $a = 5.0t$. In addition, we also know that $v_0 = 17$ m/s at $t_0 = 2.0$ s.

ANALYZE Integrating (from t = 2 s to variable t = 4 s) the acceleration to get the velocity and using the values given in the problem, leads to

$$v = v_0 + \int_{t_0}^{t} a dt = v_0 + \int_{t_0}^{t} (5.0t)dt = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \tfrac{1}{2}(5.0)(4^2 - 2^2) = 47 \text{ m/s}.$$

LEARN The velocity of the particle as a function of t is

$$v(t) = v_0 + \frac{1}{2}(5.0)(t^2 - t_0^2) = 17 + \frac{1}{2}(5.0)(t^2 - 4) = 7 + 2.5t^2$$

in SI units (m/s). Since the acceleration is linear in t, we expect the velocity to be quadratic in t, and the displacement to be cubic in t.

82. The velocity v at t = 6 (SI units and two significant figures understood) is $v_{\text{given}} + \int_{-2}^{6} a dt$. A quick way to implement this is to recall the area of a triangle ($\frac{1}{2}$ base × height). The result is v = 7 m/s + 32 m/s = 39 m/s.

83. The object, once it is dropped (v_0 = 0) is in free fall ($a = -g = -9.8$ m/s^2 if we take *down* as the $-y$ direction), and we use Eq. 2-15 repeatedly.

(a) The (positive) distance D from the lower dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2}gt^2$, or $D = gt^2/2$. Thus, for $t_1 = 50.0$ ms,

$$D_1 = \frac{\left(9.8\text{ m/s}^2\right)\left(50.0\times10^{-3}\text{ s}\right)^2}{2} = 0.0123\text{ m} = 1.23\text{ cm}.$$

(b) For t_2 = 100 ms, $D_2 = \dfrac{\left(9.8\text{m/s}^2\right)\left(100\times10^{-3}\text{ s}\right)^2}{2} = 0.049\text{m} = 4D_1.$

(c) For t_3 = 150 ms, $D_3 = \dfrac{\left(9.8\text{m/s}^2\right)\left(150\times10^{-3}\text{ s}\right)^2}{2} = 0.11\text{m} = 9D_1.$

(d) For t_4 = 200 ms, $D_4 = \dfrac{\left(9.8\text{m/s}^2\right)\left(200\times10^{-3}\text{ s}\right)^2}{2} = 0.196\text{m} = 16D_1.$

(e) For t_4 = 250 ms, $D_5 = \dfrac{\left(9.8\text{ m/s}^2\right)\left(250\times10^{-3}\text{ s}\right)^2}{2} = 0.306\text{ m} = 25D_1.$

84. We take the direction of motion as $+x$, take $x_0 = 0$ and use SI units, so v = 1600(1000/3600) = 444 m/s.

(a) Equation 2-11 gives 444 = a(1.8) or a = 247 m/s^2. We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{247 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) g = 25g.$$

(b) Equation 2-17 readily yields

$$x = \frac{1}{2}(v_0 + v)\, t = \frac{1}{2}(444 \text{ m/s})(1.8 \text{ s}) = 400 \text{ m}.$$

85. Let D be the distance up the hill. Then

$$\text{average speed} = \frac{\text{total distance traveled}}{\text{total time of travel}} = \frac{2D}{\dfrac{D}{20 \text{ km/h}} + \dfrac{D}{35 \text{ km/h}}} \approx 25 \text{ km/h}\,.$$

86. We obtain the velocity by integration of the acceleration:

$$v - v_0 = \int_0^t (6.1 - 1.2t')dt'\,.$$

Lengths are in meters and times are in seconds. The student is encouraged to look at the discussion in Section 2-7 to better understand the manipulations here.

(a) The result of the above calculation is $v = v_0 + 6.1t - 0.6t^2$, where the problem states that v_0 = 2.7 m/s. The maximum of this function is found by knowing when its derivative (the acceleration) is zero (a = 0 when t = 6.1/1.2 = 5.1 s) and plugging that value of t into the velocity equation above. Thus, we find $v = 18$ m/s.

(b) We integrate again to find x as a function of t:

$$x - x_0 = \int_0^t v\, dt' = \int_0^t (v_0 + 6.1t' - 0.6t'^2)\, dt' = v_0 t + 3.05t^2 - 0.2t^3\,.$$

With x_0 = 7.3 m, we obtain x = 83 m for t = 6. This is the correct answer, but one has the right to worry that it might not be; after all, the problem asks for the total distance traveled (and $x - x_0$ is just the *displacement*). If the cyclist backtracked, then his total distance would be greater than his displacement. Thus, we might ask, "did he backtrack?" To do so would require that his velocity be (momentarily) zero at some point (as he reversed his direction of motion). We could solve the above quadratic equation for velocity, for a positive value of t where v = 0; if we did, we would find that at t = 10.6 s, a reversal does indeed happen. However, in the time interval we are concerned with in our problem ($0 \le t \le 6$ s), there is no reversal and the displacement is the same as the total distance traveled.

87. **THINK** In this problem we're given two different speeds, and asked to find the difference in their travel times.

EXPRESS The time is takes to travel a distance d with a speed v_1 is $t_1 = d/v_1$. Similarly, with a speed v_2 the time would be $t_2 = d/v_2$. The two speeds in this problem are

$$v_1 = 55 \text{ mi/h} = (55 \text{ mi/h})\frac{1609 \text{ m/mi}}{3600 \text{ s/h}} = 24.58 \text{ m/s}$$

$$v_2 = 65 \text{ mi/h} = (65 \text{ mi/h})\frac{1609 \text{ m/mi}}{3600 \text{ s/h}} = 29.05 \text{ m/s}$$

ANALYZE With $d = 700 \text{ km} = 7.0\times10^5 \text{ m}$, the time difference between the two is

$$\Delta t = t_1 - t_2 = d\left(\frac{1}{v_1} - \frac{1}{v_2}\right) = (7.0\times10^5 \text{ m})\left(\frac{1}{24.58 \text{ m/s}} - \frac{1}{29.05 \text{ m/s}}\right) = 4383 \text{ s}$$
$$= 73 \text{ min}$$

or about 1.2 h.

LEARN The travel time was reduced from 7.9 h to 6.9 h. Driving at higher speed (within the legal limit) reduces travel time.

88. The acceleration is constant and we may use the equations in Table 2-1.

(a) Taking the first point as coordinate origin and time to be zero when the car is there, we apply Eq. 2-17:

$$x = \frac{1}{2}\left(v + v_0\right)t = \frac{1}{2}\left(15.0 \text{ m/s} + v_0\right)\left(6.00 \text{ s}\right).$$

With x = 60.0 m (which takes the direction of motion as the $+x$ direction) we solve for the initial velocity: v_0 = 5.00 m/s.

(b) Substituting v = 15.0 m/s, v_0 = 5.00 m/s, and t = 6.00 s into $a = (v - v_0)/t$ (Eq. 2-11), we find a = 1.67 m/s^2.

(c) Substituting v = 0 in $v^2 = v_0^2 + 2ax$ and solving for x, we obtain

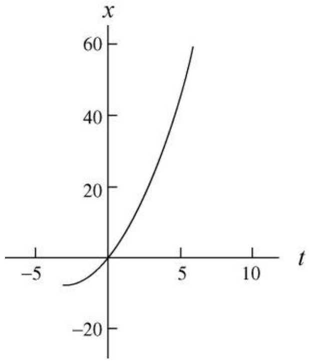
$$x = -\frac{v_0^2}{2a} = -\frac{(5.00 \text{ m/s})^2}{2\left(1.67 \text{ m/s}^2\right)} = -7.50 \text{ m},$$

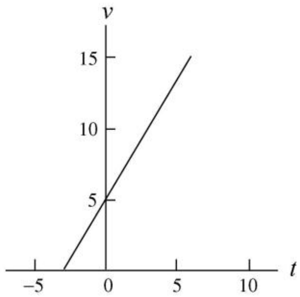
or $|x| = 7.50 \text{ m}$.

(d) The graphs require computing the time when v = 0, in which case, we use $v = v_0 + at' = 0$. Thus,

$$t' = \frac{-v_0}{a} = \frac{-5.00 \text{ m/s}}{1.67 \text{ m/s}^2} = -3.0 \text{ s}$$

indicates the moment the car was at rest. SI units are understood.





89. **THINK** In this problem we explore the connection between the maximum height an object reaches under the influence of gravity and the total amount of time it stays in air.

EXPRESS Neglecting air resistance and setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion, we analyze the motion of the ball using Table 2-1 (with Δy replacing Δx). We set $y_0 = 0$. Upon reaching the maximum height H, the speed of the ball is momentarily zero ($v = 0$). Therefore, we can relate its initial speed v_0 to H via the equation

$$0 = v^2 = v_0^2 - 2gH \quad \Rightarrow \quad v_0 = \sqrt{2gH} .$$

The time it takes for the ball to reach maximum height is given by $v = v_0 - gt = 0$, or $t = v_0 / g = \sqrt{2H/g}$.

ANALYZE If we want the ball to spend twice as much time in air as before, i.e., $t' = 2t$, then the new maximum height H' it must reach is such that $t' = \sqrt{2H'/g}$. Solving for H' we obtain

$$H' = \frac{1}{2}gt'^2 = \frac{1}{2}g(2t)^2 = 4\left(\frac{1}{2}gt^2\right) = 4H .$$

LEARN Since $H \sim t^2$, doubling t means that H must increase fourfold. Note also that for $t' = 2t$, the initial speed must be twice the original speed: $v_0' = 2v_0$.

90. (a) Using the fact that the area of a triangle is $\frac{1}{2}$(base) (height) (and the fact that the integral corresponds to the area under the curve) we find, from $t = 0$ through $t = 5$ s, the integral of v with respect to t is 15 m. Since we are told that $x_0 = 0$ then we conclude that $x = 15$ m when $t = 5.0$ s.

(b) We see directly from the graph that $v = 2.0$ m/s when $t = 5.0$ s.

(c) Since $a = dv/dt$ = slope of the graph, we find that the acceleration during the interval $4 < t < 6$ is uniformly equal to -2.0 m/s^2.

(d) Thinking of $x(t)$ in terms of accumulated area (on the graph), we note that $x(1) = 1$ m; using this and the value found in part (a), Eq. 2-2 produces

$$v_{avg} = \frac{x(5) - x(1)}{5 - 1} = \frac{15 \text{ m} - 1 \text{ m}}{4 \text{ s}} = 3.5 \text{ m/s}.$$

(e) From Eq. 2-7 and the values $v(t)$ we read directly from the graph, we find

$$a_{avg} = \frac{v(5) - v(1)}{5 - 1} = \frac{2 \text{ m/s} - 2 \text{ m/s}}{4 \text{ s}} = 0.$$

91. Taking the $+y$ direction *downward* and $y_0 = 0$, we have $y = v_0 t + \frac{1}{2} g t^2$, which (with $v_0 = 0$) yields $t = \sqrt{2y/g}$.

(a) For this part of the motion, $y_1 = 50$ m so that $t_1 = \sqrt{\frac{2(50 \text{ m})}{9.8 \text{ m/s}^2}} = 3.2 \text{ s}$.

(b) For this next part of the motion, we note that the total displacement is $y_2 = 100$ m. Therefore, the total time is

$$t_2 = \sqrt{\frac{2(100 \text{ m})}{9.8 \text{ m/s}^2}} = 4.5 \text{ s} .$$

The difference between this and the answer to part (a) is the time required to fall through that second 50 m distance: $\Delta t = t_2 - t_1 = 4.5$ s – 3.2 s = 1.3 s.

92. Direction of $+x$ is implicit in the problem statement. The initial position (when the clock starts) is $x_0 = 0$ (where $v_0 = 0$), the end of the speeding-up motion occurs at $x_1 = 1100/2 = 550$ m, and the subway train comes to a halt ($v_2 = 0$) at $x_2 = 1100$ m.

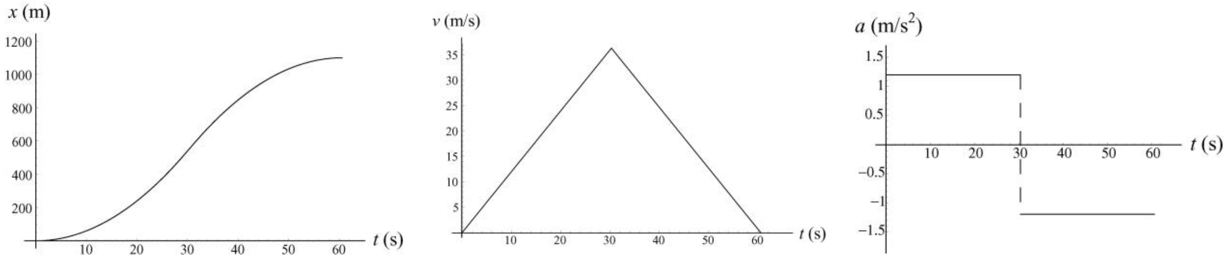
(a) Using Eq. 2-15, the subway train reaches x_1 at

$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550 \text{ m})}{1.2 \text{ m/s}^2}} = 30.3 \text{ s} .$$

The time interval $t_2 - t_1$ turns out to be the same value (most easily seen using Eq. 2-18 so the total time is $t_2 = 2(30.3) = 60.6$ s.

(b) Its maximum speed occurs at t_1 and equals $v_1 = v_0 + a_1 t_1 = 36.3 \text{ m/s}$.

(c) The graphs are shown below:



93. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with Δx replaced by y) because the ball has constant acceleration motion (and we choose $y_0 = 0$).

(a) We apply Eq. 2-16 to both measurements, with SI units understood.

$$v_B^2 = v_0^2 - 2gy_B \quad \Rightarrow \quad \left(\frac{1}{2}v\right)^2 + 2g\left(y_A + 3\right) = v_0^2$$
$$v_A^2 = v_0^2 - 2gy_A \quad \Rightarrow \quad v^2 + 2gy_A = v_0^2$$

We equate the two expressions that each equal v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \quad \Rightarrow \quad 2g(3) = \frac{3}{4}v^2$$

which yields $v = \sqrt{2g(4)} = 8.85\ \text{m/s}.$

(b) An object moving upward at A with speed v = 8.85 m/s will reach a maximum height $y - y_A = v^2/2g$ = 4.00 m above point A (this is again a consequence of Eq. 2-16, now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point B.

94. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y-axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root chosen. With $y = 0$, $v_0 = 0$, and $y_0 = h = 60$ m, we obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5\ \text{s}\ .$$

Thus, "1.2 s earlier" means we are examining where the rock is at t = 2.3 s:

$$y - h = v_0(2.3\,\text{s}) - \frac{1}{2}g(2.3\,\text{s})^2 \quad \Rightarrow \quad y = 34\ \text{m}$$

where we again use the fact that h = 60 m and $v_0 = 0$.

95. **THINK** This problem involves analyzing a plot describing the position of an iceboat as function of time. The boat has a nonzero acceleration due to the wind.

EXPRESS Since we are told that the acceleration of the boat is constant, the equations of Table 2-1 can be applied. However, the challenge here is that v_0, v, and a are not explicitly given. Our strategy to deduce these values is to apply the kinematic equation $x - x_0 = v_0 t + \frac{1}{2}at^2$ to a variety of points on the graph and solve for the unknowns from the simultaneous equations.

ANALYZE (a) From the graph, we pick two points on the curve: $(t, x) = (2.0\text{ s}, 16\text{ m})$ and $(3.0\text{ s}, 27\text{ m})$. The corresponding simultaneous equations are

$$16\text{ m} - 0 = v_0(2.0\text{ s}) + \frac{1}{2}a(2.0\text{ s})^2$$

$$27\text{ m} - 0 = v_0(3.0\text{ s}) + \frac{1}{2}a(3.0\text{ s})^2$$

Solving the equations lead to the values v_0 = 6.0 m/s and a = 2.0 m/s^2.

(b) From Table 2-1,

$$x - x_0 = vt - \frac{1}{2}at^2 \quad \Rightarrow \quad 27\text{ m} - 0 = v(3.0\text{ s}) - \frac{1}{2}(2.0\text{ m/s}^2)(3.0\text{ s})^2$$

which leads to v = 12 m/s.

(c) Assuming the wind continues during $3.0 \le t \le 6.0$, we apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ to this interval (where v_0 = 12.0 m/s from part (b)) to obtain

$$\Delta x = (12.0\text{ m/s})(3.0\text{ s}) + \frac{1}{2}(2.0\text{ m/s}^2)(3.0\text{ s})^2 = 45\text{ m}.$$

LEARN By using the results obtained in (a), the position and velocity of the iceboat as a function of time can be written as

$$x(t) = (6.0\text{ m/s})t + \frac{1}{2}(2.0\text{ m/s}^2)t^2 \quad \text{and} \quad v(t) = (6.0\text{ m/s}) + (2.0\text{ m/s}^2)t.$$

One can readily verify that the same answers are obtained for (b) and (c) using the above expressions for $x(t)$ and $v(t)$.

96. (a) Let the height of the diving board be h. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). Thus, $y = h$ designates the location where the ball strikes the water. Let the depth of the lake be D, and the total time for the ball to descend be T. The speed of the ball as it reaches the surface of the lake is then $v = \sqrt{2gh}$ (from Eq. 2-16), and the time for the ball to fall from the board to the lake surface is $t_1 = \sqrt{2h/g}$ (from Eq. 2-15). Now, the time it spends descending in the lake (at constant velocity v) is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}}.$$

Thus, $T = t_1 + t_2 = \sqrt{\dfrac{2h}{g}} + \dfrac{D}{\sqrt{2gh}}$, which gives

$$D = T\sqrt{2gh} - 2h = (4.80\text{ s})\sqrt{(2)(9.80\text{ m/s}^2)(5.20\text{ m})} - 2(5.20\text{ m}) = 38.1\text{ m}.$$

(b) Using Eq. 2-2, the magnitude of the average velocity is

$$v_{\text{avg}} = \frac{D+h}{T} = \frac{38.1\text{ m} + 5.20\text{ m}}{4.80\text{ s}} = 9.02\text{ m/s}$$

(c) In our coordinate choices, a positive sign for v_{avg} means that the ball is going downward. If, however, upward had been chosen as the positive direction, then this answer in (b) would turn out negative-valued.

(d) We find v_0 from $\Delta y = v_0 t + \frac{1}{2}gt^2$ with $t = T$ and $\Delta y = h + D$. Thus,

$$v_0 = \frac{h+D}{T} - \frac{gT}{2} = \frac{5.20\text{ m} + 38.1\text{ m}}{4.80\text{ s}} - \frac{(9.8\text{ m/s}^2)(4.80\text{ s})}{2} = 14.5\text{ m/s}$$

(e) Here in our coordinate choices the negative sign means that the ball is being thrown upward.

97. We choose *down* as the $+y$ direction and use the equations of Table 2-1 (replacing x with y) with $a = +g$, $v_0 = 0$, and $y_0 = 0$. We use subscript 2 for the elevator reaching the ground and 1 for the halfway point.

(a) Equation 2-16, $v_2^2 = v_0^2 + 2a(y_2 - y_0)$, leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2(9.8\text{ m/s}^2)(120\text{ m})} = 48.5\text{ m/s}.$$

(b) The time at which it strikes the ground is (using Eq. 2-15)

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2(120\text{ m})}{9.8\text{ m/s}^2}} = 4.95\text{ s}.$$

(c) Now Eq. 2-16, in the form $v_1^2 = v_0^2 + 2a(y_1 - y_0)$, leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2(9.8\text{ m/s}^2)(60\text{ m})} = 34.3\text{m/s}.$$

(d) The time at which it reaches the halfway point is (using Eq. 2-15)

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(60\text{ m})}{9.8\text{ m/s}^2}} = 3.50\text{ s}.$$

98. Taking $+y$ to be upward and placing the origin at the point from which the objects are dropped, then the location of diamond 1 is given by $y_1 = -\tfrac{1}{2}gt^2$ and the location of diamond 2 is given by $y_2 = -\tfrac{1}{2}g(t-1)^2$. We are starting the clock when the first object is dropped. We want the time for which $y_2 - y_1 = 10$ m. Therefore,

$$-\frac{1}{2}g(t-1)^2 + \frac{1}{2}gt^2 = 10 \quad\Rightarrow\quad t = (10/g) + 0.5 = 1.5\text{ s}.$$

99. With $+y$ upward, we have $y_0 = 36.6$ m and $y = 12.2$ m. Therefore, using Eq. 2-18 (the last equation in Table 2-1), we find

$$y - y_0 = vt + \tfrac{1}{2}gt^2 \quad\Rightarrow\quad v = -22.0\text{ m/s}$$

at $t = 2.00$ s. The term *speed* refers to the magnitude of the velocity vector, so the answer is $|v| = 22.0$ m/s.

100. During free fall, we ignore the air resistance and set $a = -g = -9.8$ m/s^2 where we are choosing *down* to be the $-y$ direction. The initial velocity is zero so that Eq. 2-15 becomes $\Delta y = -\tfrac{1}{2}gt^2$ where Δy represents the *negative* of the distance d she has fallen. Thus, we can write the equation as $d = \tfrac{1}{2}gt^2$ for simplicity.

(a) The time t_1 during which the parachutist is in free fall is (using Eq. 2-15) given by

$$d_1 = 50\text{ m} = \frac{1}{2}gt_1^2 = \frac{1}{2}\left(9.80\text{ m/s}^2\right)t_1^2$$

which yields $t_1 = 3.2$ s. The *speed* of the parachutist just before he opens the parachute is given by the positive root $v_1^2 = 2gd_1$, or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)\left(9.80\text{ m/s}^2\right)(50\text{ m})} = 31\text{ m/s}.$$

If the final speed is v_2, then the time interval t_2 between the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31\text{ m/s} - 3.0\text{ m/s}}{2\text{ m/s}^2} = 14\text{ s}.$$

This is a result of Eq. 2-11 where *speeds* are used instead of the (negative-valued) velocities (so that final-velocity minus initial-velocity turns out to equal initial-speed minus final-speed); we also note that the acceleration vector for this part of the motion is positive since it points upward (opposite to the direction of motion — which makes it a deceleration). The total time of flight is therefore $t_1 + t_2 = 17$ s.

(b) The distance through which the parachutist falls after the parachute is opened is given by

$$d = \frac{v_1^2 - v_2^2}{2a} = \frac{(31\,\text{m/s})^2 - (3.0\,\text{m/s})^2}{(2)(2.0\,\text{m/s}^2)} \approx 240\,\text{m}.$$

In the computation, we have used Eq. 2-16 with both sides multiplied by –1 (which changes the negative-valued Δy into the positive d on the left-hand side, and switches the order of v_1 and v_2 on the right-hand side). Thus the fall begins at a height of $h = 50 + d \approx 290$ m.

101. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to $y = 0$.

(a) With $y_0 = h$ and v_0 replaced with $-v_0$, Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 - 2g(y - y_0)} = \sqrt{v_0^2 + 2gh}\,.$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-15 for t, with v_0 replaced with $-v_0$,

$$\Delta y = -v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, this becomes

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g}.$$

(c) If it were thrown upward with that speed from height h then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation).

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_0$ in the equation where we had put in $-v_0$ in part (b). The details follow:

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root again chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, we obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g}.$$

102. We assume constant velocity motion and use Eq. 2-2 (with $v_{\text{avg}} = v > 0$). Therefore,

$$\Delta x = v\Delta t = \left(303\,\frac{\text{km}}{\text{h}}\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)\right)\left(100\times 10^{-3}\ \text{s}\right) = 8.4\ \text{m}.$$

103. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is $\Delta x = v\Delta t$, where Δx is the horizontal distance traveled, Δt is the time, and v is the (horizontal) velocity. Converting v to meters per second, we have 160 km/h = 44.4 m/s. Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4\ \text{m}}{44.4\ \text{m/s}} = 0.414\ \text{s}.$$

The velocity-unit conversion implemented above can be figured "from basics" (1000 m = 1 km, 3600 s = 1 h) or found in Appendix D.

104. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. Thus, $x(t) = 50t + 10t^2$ with SI units (meters and seconds) understood.

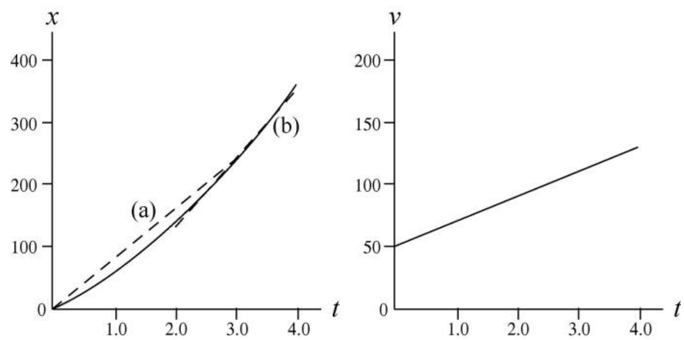
(a) The average velocity during the first 3 s is given by

$$v_{\text{avg}} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80\ \text{m/s}.$$

(b) The instantaneous velocity at time t is given by $v = dx/dt = 50 + 20t$, in SI units. At $t = 3.0$ s, $v = 50 + (20)(3.0) = 110$ m/s.

(c) The instantaneous acceleration at time t is given by $a = dv/dt = 20\ \text{m/s}^2$. It is constant, so the acceleration at any time is $20\ \text{m/s}^2$.

(d) and (e) The graphs that follow show the coordinate x and velocity v as functions of time, with SI units understood. The dashed line marked (a) in the first graph runs from $t = 0$, $x = 0$ to $t = 3.0$s, $x = 240$ m. Its slope is the average velocity during the first 3s of motion. The dashed line marked (b) is tangent to the x curve at $t = 3.0$ s. Its slope is the instantaneous velocity at $t = 3.0$ s.



105. We take $+x$ in the direction of motion, so $v_0 = +30$ m/s, $v_1 = +15$ m/s and $a < 0$. The acceleration is found from Eq. 2-11: $a = (v_1 - v_0)/t_1$ where $t_1 = 3.0$ s. This gives $a = -5.0$ m/s^2. The displacement (which in this situation is the same as the distance traveled) to the point it stops ($v_2 = 0$) is, using Eq. 2-16,

$$v_2^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad \Delta x = -\frac{(30\ \text{m/s})^2}{2(-5\ \text{m/s}^2)} = 90\ \text{m}.$$

106. The problem consists of two constant-acceleration parts: part 1 with $v_0 = 0$, $v = 6.0$ m/s, $x = 1.8$ m, and $x_0 = 0$ (if we take its original position to be the coordinate origin); and, part 2 with $v_0 = 6.0$ m/s, $v = 0$, and $a_2 = -2.5$ m/s^2 (negative because we are taking the positive direction to be the direction of motion).

(a) We can use Eq. 2-17 to find the time for the first part

$$x - x_0 = \frac{1}{2}(v_0 + v)\ t_1 \quad \Rightarrow \quad 1.8\ \text{m} - 0 = \frac{1}{2}(0 + 6.0\ \text{m/s})\ t_1$$

so that $t_1 = 0.6$ s. And Eq. 2-11 is used to obtain the time for the second part

$$v = v_0 + a_2 t_2 \quad \Rightarrow \quad 0 = 6.0\ \text{m/s} + (-2.5\ \text{m/s}^2)t_2$$

from which $t_2 = 2.4$ s is computed. Thus, the total time is $t_1 + t_2 = 3.0$ s.

(b) We already know the distance for part 1. We could find the distance for part 2 from several of the equations, but the one that makes no use of our part (a) results is Eq. 2-16

$$v^2 = v_0^2 + 2a_2\Delta x_2 \quad \Rightarrow \quad 0 = (6.0\ \text{m/s})^2 + 2(-2.5\ \text{m/s}^2)\Delta x_2$$

which leads to $\Delta x_2 = 7.2$ m. Therefore, the total distance traveled by the shuffleboard disk is $(1.8 + 7.2)$ m $= 9.0$ m.

107. The time required is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v = (100\ \text{km/h})\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right) = 27.8\ \text{m/s}\ .$$

Thus, $\Delta t = \Delta v/a = 27.8/50 = 0.556$ s.

108. From Table 2-1, $v^2 - v_0^2 = 2a\Delta x$ is used to solve for a. Its minimum value is

$$a_{\text{min}} = \frac{v_2 - v_0^2}{2\Delta x_{\text{max}}} = \frac{(360\ \text{km/h})^2}{2(1.80\ \text{km})} = 36000\ \text{km/h}^2$$

which converts to 2.78 m/s^2.

109. (a) For the automobile $\Delta v = 55 - 25 = 30$ km/h, which we convert to SI units:

$$a = \frac{\Delta v}{\Delta t} = \frac{(30 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)}{(0.50 \text{ min})(60 \text{ s/min})} = 0.28 \text{ m/s}^2 .$$

(b) The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also 0.28 m/s^2.

110. Converting to SI units, we have v = 3400(1000/3600) = 944 m/s (presumed constant) and Δt = 0.10 s. Thus, $\Delta x = v\Delta t$ = 94 m.

111. This problem consists of two parts: part 1 with constant acceleration (so that the equations in Table 2-1 apply), $v_0 = 0$, v = 11.0 m/s, x = 12.0 m, and $x_0 = 0$ (adopting the starting line as the coordinate origin); and, part 2 with constant velocity (so that $x - x_0 = vt$ applies) with v = 11.0 m/s, x_0 = 12.0, and x = 100.0 m.

(a) We obtain the time for part 1 from Eq. 2-17

$$x - x_0 = \frac{1}{2}(v_0 + v)\, t_1 \;\Rightarrow\; 12.0 - 0 = \frac{1}{2}(0 + 11.0)t_1$$

so that t_1 = 2.2 s, and we find the time for part 2 simply from 88.0 = (11.0)$t_2 \to t_2$ = 8.0 s. Therefore, the total time is $t_1 + t_2$ = 10.2 s.

(b) Here, the total time is required to be 10.0 s, and we are to locate the point x_p where the runner switches from accelerating to proceeding at constant speed. The equations for parts 1 and 2, used above, therefore become

$$x_p - 0 = \tfrac{1}{2}(0 + 11.0 \text{ m/s})t_1$$
$$100.0 \text{ m} - x_p = (11.0 \text{ m/s})(10.0 \text{ s} - t_1)$$

where in the latter equation, we use the fact that $t_2 = 10.0 - t_1$. Solving the equations for the two unknowns, we find that t_1 = 1.8 s and x_p = 10.0 m.

112. The bullet starts at rest (v_0 = 0) and after traveling the length of the barrel ($\Delta x = 1.2$ m) emerges with the given velocity (v = 640 m/s), where the direction of motion is the positive direction. Turning to the constant acceleration equations in Table 2-1, we use $\Delta x = \frac{1}{2}(v_0 + v)\, t$. Thus, we find t = 0.00375 s (or 3.75 ms).

113. There is no air resistance, which makes it quite accurate to set $a = -g = -9.8$ m/s^2 (where downward is the $-y$ direction) for the duration of the fall. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will again assume constant acceleration conditions; in this case, we have $a_2 = +25g = 245$ m/s^2.

(a) The time of fall is given by Eq. 2-15 with v_0 = 0 and y = 0. Thus,

$$t=\sqrt{\frac{2y_0}{g}}=\sqrt{\frac{2(145\text{ m})}{9.8\text{ m/s}^2}}=5.44\text{ s}.$$

(b) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other equations can be used but they would use the result from part (a))

$$v=-\sqrt{v_0^2-2g(y-y_0)}=-\sqrt{2gy_0}=-53.3\text{ m/s}$$

where the negative root is chosen since this is a downward velocity. Thus, the speed is $|v|=53.3$ m/s.

(c) For the catching process, the answer to part (b) plays the role of an *initial* velocity (v_0 = –53.3 m/s) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2=\frac{v^2-v_0^2}{2a_2}=\frac{-(-53.3\text{ m/s})^2}{2(245\text{ m/s}^2)}=-5.80\text{ m},$$

or $|\Delta y_2|=5.80$ m. The negative value of Δy_2 signifies that the distance traveled while arresting its motion is downward.

114. During T_r the velocity v_0 is constant (in the direction we choose as $+x$) and obeys $v_0 = D_r/T_r$ where we note that in SI units the velocity is v_0 = 200(1000/3600) = 55.6 m/s. During T_b the acceleration is opposite to the direction of v_0 (hence, for us, $a < 0$) until the car is stopped ($v = 0$).

(a) Using Eq. 2-16 (with Δx_b = 170 m) we find

$$v^2=v_0^2+2a\Delta x_b \quad\Rightarrow\quad a=-\frac{v_0^2}{2\Delta x_b}$$

which yields $|a|$ = 9.08 m/s^2.

(b) We express this as a multiple of g by setting up a ratio:

$$a=\left(\frac{9.08\text{ m/s}^2}{9.8\text{ m/s}^2}\right)g=0.926g\ .$$

(c) We use Eq. 2-17 to obtain the braking time:

$$\Delta x_b=\frac{1}{2}(v_0+v)T_b \quad\Rightarrow\quad T_b=\frac{2(170\text{ m})}{55.6\text{ m/s}}=6.12\text{ s}\ .$$

(d) We express our result for T_b as a multiple of the reaction time T_r by setting up a ratio:

$$T_b=\left(\frac{6.12\text{ s}}{400\times10^{-3}\text{ s}}\right)T_r=15.3T_r\ .$$

(e) Since $T_b > T_r$, most of the full time required to stop is spent in braking.

(f) We are only asked what the *increase* in distance D is, due to $\Delta T_r = 0.100$ s, so we simply have

$$\Delta D = v_0 \Delta T_r = (55.6 \text{ m/s})(0.100 \text{ s}) = 5.56 \text{ m} .$$

115. The total time elapsed is $\Delta t = 2 \text{ h } 41 \text{ min} = 161 \text{ min}$ and the center point is displaced by $\Delta x = 3.70 \text{ m} = 370 \text{ cm}$. Thus, the average velocity of the center point is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{370 \text{ cm}}{161 \text{ min}} = 2.30 \text{ cm/min}.$$

116. Using Eq. 2-11, $v = v_0 + at$, we find the initial speed to be

$$v_0 = v - at = 0 - (-3400)(9.8 \text{ m/s}^2)(6.5 \times 10^{-3} \text{ s}) = 216.6 \text{ m/s}$$

117. The total number of days walked is (including the first and the last day, and leap year)

$$N = 340 + 365 + 365 + 366 + 365 + 365 + 261 = 2427$$

Thus, the average speed of the walk is

$$s_{\text{avg}} = \frac{d}{\Delta t} = \frac{3.06 \times 10^7 \text{ m}}{(2427 \text{ days})(86400 \text{ s/day})} = 0.146 \text{ m/s}.$$

118. (a) Let d be the distance traveled. The average speed with and without wings set as sails are $v_s = d/t_s$ and $v_{ns} = d/t_{ns}$, respectively. Thus, the ratio of the two speeds is

$$\frac{v_s}{v_{ns}} = \frac{d/t_s}{d/t_{ns}} = \frac{t_{ns}}{t_s} = \frac{25.0 \text{ s}}{7.1 \text{ s}} = 3.52$$

(b) The difference in time expressed in terms of v_s is

$$\Delta t = t_{ns} - t_s = \frac{d}{v_{ns}} - \frac{d}{v_s} = \frac{d}{(v_s/3.52)} - \frac{d}{v_s} = 2.52 \frac{d}{v_s} = 2.52 \frac{(2.0 \text{ m})}{v_s} = \frac{5.04 \text{ m}}{v_s}$$

119. (a) Differentiating $y(t) = (2.0 \text{ cm}) \sin(\pi t/4)$ with respect to t, we obtain

$$v_y(t) = \frac{dy}{dt} = \left(\frac{\pi}{2} \text{ cm/s}\right) \cos(\pi t/4)$$

The average velocity between $t = 0$ and $t = 2.0$ s is

$$v_{\text{avg}} = \frac{1}{(2.0\text{ s})}\int_0^2 v_y dt = \frac{1}{(2.0\text{ s})}\left(\frac{\pi}{2}\text{ cm/s}\right)\int_0^2 \cos\left(\frac{\pi t}{4}\right) dt$$
$$= \frac{1}{(2.0\text{ s})}\left(2\text{ cm}\right)\int_0^{\pi/2} \cos x\, dx = 1.0\text{ cm/s}$$

(b) The instantaneous velocities of the particle at t = 0, 1.0 s, and 2.0 s are, respectively,

$$v_y(0) = \left(\frac{\pi}{2}\text{ cm/s}\right)\cos(0) = \frac{\pi}{2}\text{ cm/s}$$
$$v_y(1.0\text{ s}) = \left(\frac{\pi}{2}\text{ cm/s}\right)\cos(\pi/4) = \frac{\pi\sqrt{2}}{4}\text{ cm/s}$$
$$v_y(2.0\text{ s}) = \left(\frac{\pi}{2}\text{ cm/s}\right)\cos(\pi/2) = 0$$

(c) Differentiating $v_y(t)$ with respect to t, we obtain the following expression for acceleration:

$$a_y(t) = \frac{dv_y}{dt} = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(\pi t/4)$$

The average acceleration between t = 0 and t = 2.0 s is

$$a_{\text{avg}} = \frac{1}{(2.0\text{ s})}\int_0^2 a_y dt = \frac{1}{(2.0\text{ s})}\left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\int_0^2 \sin\left(\frac{\pi t}{4}\right) dt$$
$$= \frac{1}{(2.0\text{ s})}\left(-\frac{\pi}{2}\text{ cm/s}\right)\int_0^{\pi/2} \sin x\, dx = \frac{1}{(2.0\text{ s})}\left(-\frac{\pi}{2}\text{ cm/s}\right) = -\frac{\pi}{4}\text{ cm/s}^2$$

(d) The instantaneous accelerations of the particle at t = 0, 1.0 s, and 2.0 s are, respectively,

$$a_y(0) = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(0) = 0$$
$$a_y(1.0\text{ s}) = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(\pi/4) = -\frac{\pi^2\sqrt{2}}{16}\text{ cm/s}^2$$
$$a_y(2.0\text{ s}) = \left(-\frac{\pi^2}{8}\text{ cm/s}^2\right)\sin(\pi/2) = -\frac{\pi^2}{8}\text{ cm/s}^2$$

Chapter 3

1. **THINK** In this problem we're given the magnitude and direction of a vector in two dimensions, and asked to calculate its *x*- and *y*-components.

EXPRESS The *x*- and the *y*- components of a vector $\vec{a}$ lying in the *xy* plane are given by

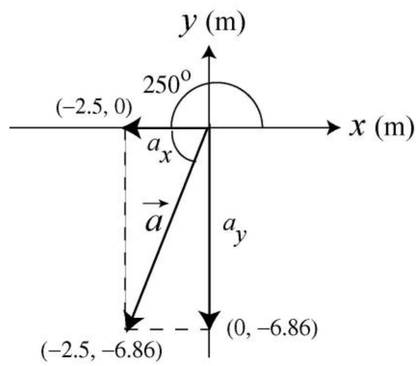
$$a_x = a\cos\theta, \qquad a_y = a\sin\theta$$

where $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$ is the magnitude and $\theta = \tan^{-1}(a_y / a_x)$ is the angle between $\vec{a}$ and the positive x axis. Given that $\theta = 250°$, we see that the vector is in the third quadrant, and we expect both the *x*- and the *y*-components of $\vec{a}$ to be negative.

ANALYZE (a) The *x* component of $\vec{a}$ is

$$a_x = a\cos\theta = (7.3 \text{ m})\cos 250° = -2.50 \text{ m},$$

(b) and the y component is $a_y = a\sin\theta = (7.3 \text{ m})\sin 250° = -6.86 \text{ m} \approx -6.9 \text{ m}.$ The results are depicted in the figure below:



LEARN In considering the variety of ways to compute these, we note that the vector is 70° below the – *x* axis, so the components could also have been found from

$$a_x = -(7.3 \text{ m})\cos 70° = -2.50 \text{ m}, \quad a_y = -(7.3 \text{ m})\sin 70° = -6.86 \text{ m}.$$

Similarly, we note that the vector is 20° to the left from the – *y* axis, so one could also achieve the same results by using

$$a_x = -(7.3 \text{ m})\sin 20° = -2.50 \text{ m}, \quad a_y = -(7.3 \text{ m})\cos 20° = -6.86 \text{ m}.$$

As a consistency check, we note that $\sqrt{a_x^2+a_y^2}=\sqrt{(-2.50\text{ m})^2+(-6.86\text{ m})^2}=7.3\text{ m}$ and $\tan^{-1}\left(a_y/a_x\right)=\tan^{-1}[(-6.86\text{ m})/(-2.50\text{ m})]=250°$, which are indeed the values given in the problem statement.

2. (a) With r = 15 m and θ = 30°, the x component of $\vec{r}$ is given by

$$r_x = r\cos\theta = (15\text{ m})\cos 30° = 13\text{ m}.$$

(b) Similarly, the y component is given by $r_y = r\sin\theta = (15\text{ m})\sin 30° = 7.5\text{ m}$.

3. **THINK** In this problem we're given the x- and y-components a vector $\vec{A}$ in two dimensions, and asked to calculate its magnitude and direction.

EXPRESS A vector $\vec{A}$ can be represented in the *magnitude-angle* notation (A, θ), where

$$A=\sqrt{A_x^2+A_y^2}$$

is the magnitude and

$$\theta=\tan^{-1}\left(\frac{A_y}{A_x}\right)$$

is the angle $\vec{A}$ makes with the positive x axis. Given that A_x = –25.0 m and A_y = 40.0 m, the above formulas can be readily used to calculate A and θ.

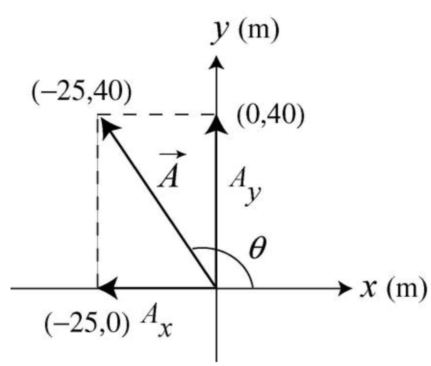
ANALYZE (a) The magnitude of the vector $\vec{A}$ is

$$A=\sqrt{A_x^2+A_y^2}=\sqrt{(-25.0\text{ m})^2+(40.0\text{ m})^2}=47.2\text{ m}$$

(b) Recalling that $\tan\theta = \tan(\theta + 180°)$,

$$\tan^{-1}[(40.0\text{ m})/(-25.0\text{ m})] = -58° \text{ or } 122°.$$

Noting that the vector is in the second quadrant (by the signs of its x and y components) we see that 122° is the correct answer. The results are depicted in the figure to the right.



LEARN We can check our answers by noting that the x- and the y- components of $\vec{A}$ can be written as

$$A_x = A\cos\theta, \qquad A_y = A\sin\theta.$$

Substituting the results calculated above, we obtain

$$A_x = (47.2\text{ m})\cos 122° = -25.0\text{ m}, \quad A_y = (47.2\text{ m})\sin 122° = +40.0\text{ m}$$

which indeed are the values given in the problem statement.

4. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a) $20.0° = (20.0°)\dfrac{2\pi\text{ rad}}{360°} = 0.349\text{ rad}$.

(b) $50.0° = (50.0°)\dfrac{2\pi\text{ rad}}{360°} = 0.873\text{ rad}$.

(c) $100° = (100°)\dfrac{2\pi\text{ rad}}{360°} = 1.75\text{ rad}$.

(d) $0.330\text{ rad} = (0.330\text{ rad})\dfrac{360°}{2\pi\text{ rad}} = 18.9°$.

(e) $2.10\text{ rad} = (2.10\text{ rad})\dfrac{360°}{2\pi\text{ rad}} = 120°$.

(f) $7.70\text{ rad} = (7.70\text{ rad})\dfrac{360°}{2\pi\text{ rad}} = 441°$.

5. The vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{new}}$ must give the same result as its originally intended displacement $\vec{d}_{\text{o}} = (120\text{ km})\hat{\mathrm{j}}$ where east is $\hat{\mathrm{i}}$, north is $\hat{\mathrm{j}}$. Thus, we write

$$\vec{d}_{\text{storm}} = (100\text{ km})\,\hat{\mathrm{i}}, \quad \vec{d}_{\text{new}} = A\hat{\mathrm{i}} + B\hat{\mathrm{j}}.$$

(a) The equation $\vec{d}_{\text{storm}} + \vec{d}_{\text{new}} = \vec{d}_{\text{o}}$ readily yields A = –100 km and B = 120 km. The magnitude of $\vec{d}_{\text{new}}$ is therefore equal to $|\vec{d}_{\text{new}}| = \sqrt{A^2 + B^2} = 156\text{ km}$.

(b) The direction is

$$\tan^{-1}(B/A) = -50.2° \text{ or } 180° + (-50.2°) = 129.8°.$$

We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

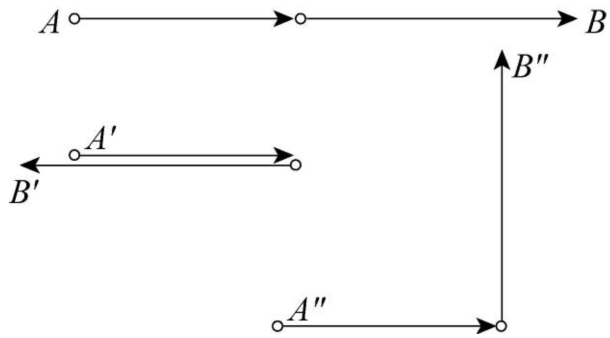
6. (a) The height is $h = d\sin\theta$, where d = 12.5 m and θ = 20.0°. Therefore, h = 4.28 m.

(b) The horizontal distance is $d\cos\theta$ = 11.7 m.

7. (a) The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below),

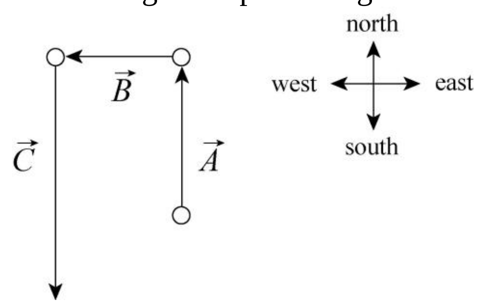
(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long (primed case shown),
(c) and perpendicular to achieve a resultant $\sqrt{3^2+4^2}=5\,\mathrm{m}$ long (the double-primed case shown).

In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by *A* (with or without primes, as the case may be) and the end is indicated by *B*.



8. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+*x* direction) and *north* as the $\hat{\mathrm{j}}$ direction (+*y* direction). All distances are understood to be in kilometers.

(a) The vector diagram representing the motion is shown next:



$$\vec{A}=(3.1\text{ km})\,\hat{\mathrm{j}}$$
$$\vec{B}=(-2.4\text{ km})\,\hat{\mathrm{i}}$$
$$\vec{C}=(-5.2\text{ km})\,\hat{\mathrm{j}}$$

(b) The final point is represented by

$$\vec{r}=\vec{A}+\vec{B}+\vec{C}=(-2.4\text{ km})\hat{\mathrm{i}}+(-2.1\text{ km})\hat{\mathrm{j}}$$

whose magnitude is

$$|\vec{r}|=\sqrt{(-2.4\text{ km})^2+(-2.1\text{ km})^2}\approx 3.2\text{ km}\ .$$

(c) There are two possibilities for the angle:

$$\theta = \tan^{-1}\left(\frac{-2.1\text{ km}}{-2.4\text{ km}}\right) = 41°, \text{or } 221°\,.$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar $\leftrightarrow$ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the +x axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

9. All distances in this solution are understood to be in meters.

(a) $\vec{a}+\vec{b} = [4.0+(-1.0)]\,\hat{\text{i}} + [(-3.0)+1.0]\,\hat{\text{j}} + (1.0+4.0)\hat{\text{k}} = (3.0\hat{\text{i}} - 2.0\hat{\text{j}} + 5.0\,\hat{\text{k}})$ m.

(b) $\vec{a}-\vec{b} = [4.0-(-1.0)]\hat{\text{i}} + [(-3.0)-1.0]\hat{\text{j}} + (1.0-4.0)\hat{\text{k}} = (5.0\,\hat{\text{i}} - 4.0\,\hat{\text{j}} - 3.0\,\hat{\text{k}})$ m.

(c) The requirement $\vec{a}-\vec{b}+\vec{c} = 0$ leads to $\vec{c} = \vec{b}-\vec{a}$, which we note is the opposite of what we found in part (b). Thus, $\vec{c} = (-5.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}})$ m.

10. The x, y, and z components of $\vec{r} = \vec{c}+\vec{d}$ are, respectively,

(a) $r_x = c_x + d_x = 7.4\text{ m} + 4.4\text{ m} = 12\text{ m}$,

(b) $r_y = c_y + d_y = -3.8\text{ m} - 2.0\text{ m} = -5.8\text{ m}$, and

(c) $r_z = c_z + d_z = -6.1\text{ m} + 3.3\text{ m} = -2.8\text{ m}$.

11. **THINK** This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$. We want to find the magnitude and direction of the resulting vector.

EXPRESS In two dimensions, a vector $\vec{a}$ can be written as, in unit vector notation,

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}}\,.$$

Similarly, a second vector $\vec{b}$ can be expressed as $\vec{b} = b_x\hat{\text{i}} + b_y\hat{\text{j}}$. Adding the two vectors gives

$$\vec{r} = \vec{a}+\vec{b} = (a_x + b_x)\hat{\text{i}} + (a_y + b_y)\hat{\text{j}} = r_x\hat{\text{i}} + r_y\hat{\text{j}}$$

ANALYZE (a) Given that $\vec{a} = (4.0\text{ m})\hat{\text{i}} + (3.0\text{ m})\hat{\text{j}}$ and $\vec{b} = (-13.0\text{ m})\hat{\text{i}} + (7.0\text{ m})\hat{\text{j}}$, we find the x and the y components of $\vec{r}$ to be

$$r_x = a_x + b_x = (4.0 \text{ m}) + (-13 \text{ m}) = -9.0 \text{ m}$$
$$r_y = a_y + b_y = (3.0 \text{ m}) + (7.0 \text{ m}) = 10.0 \text{ m}.$$

Thus $\vec{r} = (-9.0 \text{ m})\,\hat{\text{i}} + (10 \text{ m})\,\hat{\text{j}}$.

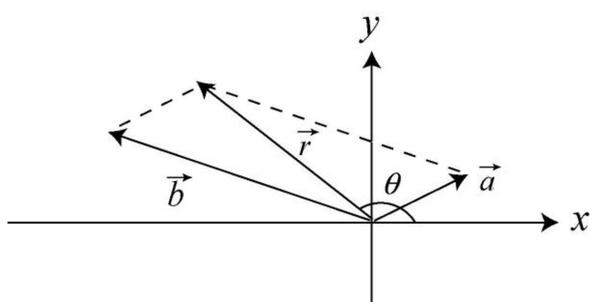
(b) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0 \text{ m})^2 + (10 \text{ m})^2} = 13 \text{ m}$.

(c) The angle between the resultant and the +*x* axis is given by

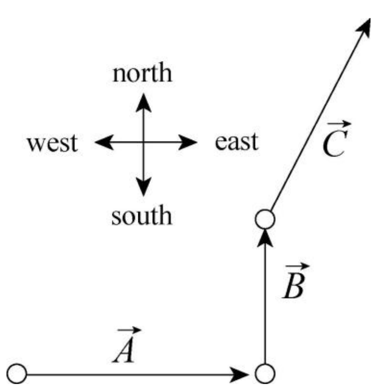
$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{10.0 \text{ m}}{-9.0 \text{ m}}\right) = -48^\circ \text{ or } 132^\circ.$$

Since the *x* component of the resultant is negative and the *y* component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from +*x* axis).

LEARN The addition of the two vectors is depicted in the figure below (not to scale). Indeed, since $r_x < 0$ and $r_y > 0$, we expect $\vec{r}$ to be in the second quadrant.



12. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\text{i}}$ direction (+*x* direction) and *north* as the $\hat{\text{j}}$ direction (+*y* direction). We note that the angle between $\vec{C}$ and the *x* axis is 60°. Thus,



$$\vec{A} = (50 \text{ km})\,\hat{\text{i}}$$
$$\vec{B} = (30 \text{ km})\,\hat{\text{j}}$$
$$\vec{C} = (25 \text{ km})\cos(60^\circ)\,\hat{\text{i}} + (25 \text{ km})\sin(60^\circ)\,\hat{\text{j}}$$

(a) The total displacement of the car from its initial position is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (62.5\ \text{km})\,\hat{\text{i}} + (51.7\ \text{km})\,\hat{\text{j}}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(62.5\,\text{km})^2 + (51.7\,\text{km})^2} = 81\ \text{km}.$$

(b) The angle (counterclockwise from +x axis) is $\tan^{-1}$ (51.7 km/62.5 km) = 40°, which is to say that it points 40° *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

13. We find the components and then add them (as scalars, not vectors). With d = 3.40 km and θ = 35.0° we find $d \cos\theta + d \sin\theta$ = 4.74 km.

14. (a) Summing the x components, we have

$$20\text{ m} + b_x - 20\text{ m} - 60\text{ m} = -140\text{ m},$$

which gives $b_x = -80$ m.

(b) Summing the y components, we have

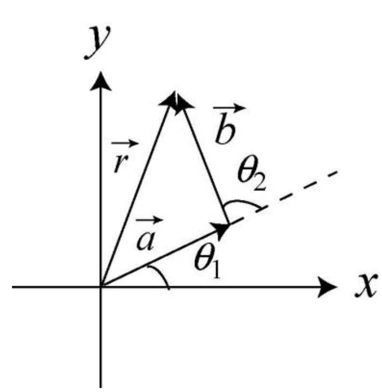
$$60\text{ m} - 70\text{ m} + c_y - 70\text{ m} = 30\text{ m},$$

which implies c_y =110 m.

(c) Using the Pythagorean theorem, the magnitude of the overall displacement is given by $\sqrt{(-140\text{ m})^2 + (30\text{ m})^2} \approx 143$ m.

(d) The angle is given by $\tan^{-1}(30/(-140)) = -12°$, (which would be 12° measured clockwise from the $-x$ axis, or 168° measured counterclockwise from the $+x$ axis).

15. **THINK** This problem involves the addition of two vectors $\vec{a}$ and $\vec{b}$ in two dimensions. We're asked to find the components, magnitude and direction of the resulting vector.



EXPRESS In two dimensions, a vector $\vec{a}$ can be written as, in unit vector notation,

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}} = (a\cos\alpha)\hat{\text{i}} + (a\sin\alpha)\hat{\text{j}}.$$

Similarly, a second vector $\vec{b}$ can be expressed as $\vec{b} = b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} = (b\cos\beta)\hat{\mathrm{i}} + (b\sin\beta)\hat{\mathrm{j}}$. From the figure, we have, $\alpha = \theta_1$ and $\beta = \theta_1 + \theta_2$ (since the angles are measured from the +*x*-axis) and the resulting vector is

$$\vec{r} = \vec{a} + \vec{b} = [a\cos\theta_1 + b\cos(\theta_1 + \theta_2)]\hat{\mathrm{i}} + [a\sin\theta_1 + b\sin(\theta_1 + \theta_2)]\hat{\mathrm{j}} = r_x\hat{\mathrm{i}} + r_y\hat{\mathrm{j}}$$

ANALYZE (a) Given that $a = b = 10$ m, $\theta_1 = 30°$ and $\theta_2 = 105°$, the *x* component of $\vec{r}$ is

$$r_x = a\cos\theta_1 + b\cos(\theta_1 + \theta_2) = (10\text{ m})\cos 30° + (10\text{ m})\cos(30° + 105°) = 1.59\text{ m}$$

(b) Similarly, the *y* component of $\vec{r}$ is

$$r_y = a\sin\theta_1 + b\sin(\theta_1 + \theta_2) = (10\text{ m})\sin 30° + (10\text{ m})\sin(30° + 105°) = 12.1\text{ m}.$$

(c) The magnitude of $\vec{r}$ is $r = |\vec{r}| = \sqrt{(1.59\text{ m})^2 + (12.1\text{ m})^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the +*x*-axis is

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{12.1\text{ m}}{1.59\text{ m}}\right) = 82.5°.$$

LEARN As depicted in the figure, the resultant $\vec{r}$ lies in the first quadrant. This is what we expect. Note that the magnitude of $\vec{r}$ can also be calculated by using law of cosine ($\vec{a}, \vec{b}$ and $\vec{r}$ form an isosceles triangle):

$$r = \sqrt{a^2 + b^2 - 2ab\cos(180 - \theta_2)} = \sqrt{(10\text{ m})^2 + (10\text{ m})^2 - 2(10\text{ m})(10\text{ m})\cos 75°}$$
$$= 12.2\text{ m}.$$

16. (a) $\vec{a} + \vec{b} = (3.0\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}})\text{ m} + (5.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}})\text{ m} = (8.0\text{ m})\hat{\mathrm{i}} + (2.0\text{ m})\hat{\mathrm{j}}$.

(b) The magnitude of $\vec{a} + \vec{b}$ is

$$|\vec{a} + \vec{b}| = \sqrt{(8.0\text{ m})^2 + (2.0\text{ m})^2} = 8.2\text{ m}.$$

(c) The angle between this vector and the +*x* axis is

$$\tan^{-1}[(2.0\text{ m})/(8.0\text{ m})] = 14°.$$

(d) $\vec{b} - \vec{a} = (5.0\hat{\mathrm{i}} - 2.0\hat{\mathrm{j}})\text{ m} - (3.0\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}})\text{ m} = (2.0\text{ m})\hat{\mathrm{i}} - (6.0\text{ m})\hat{\mathrm{j}}$.

(e) The magnitude of the difference vector $\vec{b} - \vec{a}$ is

$$|\vec{b} - \vec{a}| = \sqrt{(2.0\text{ m})^2 + (-6.0\text{ m})^2} = 6.3\text{ m}.$$

(f) The angle between this vector and the +*x* axis is tan^{-1}[(–6.0 m)/(2.0 m)] = –72°. The vector is 72° *clockwise* from the axis defined by $\hat{\text{i}}$.

17. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

(a) Using unit-vector notation,

$$\begin{aligned}
\vec{a} &= (50\text{ m})\cos(30°)\hat{\text{i}} + (50\text{ m})\sin(30°)\hat{\text{j}} \\
\vec{b} &= (50\text{ m})\cos(195°)\hat{\text{i}} + (50\text{ m})\sin(195°)\hat{\text{j}} \\
\vec{c} &= (50\text{ m})\cos(315°)\hat{\text{i}} + (50\text{ m})\sin(315°)\hat{\text{j}} \\
\vec{a} + \vec{b} + \vec{c} &= (30.4\text{ m})\hat{\text{i}} - (23.3\text{ m})\hat{\text{j}}.
\end{aligned}$$

The magnitude of this result is $\sqrt{(30.4\text{ m})^2 + (-23.3\text{ m})^2} = 38\text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the +*x* direction are tan^{-1}[(–23.2 m)/(30.4 m)] = –37.5°, and 180° + (–37.5°) = 142.5°. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is –37.5°, which is to say that it is 37.5° *clockwise* from the +*x* axis. This is equivalent to 322.5° counterclockwise from +*x*.

(c) We find

$$\vec{a} - \vec{b} + \vec{c} = [43.3 - (-48.3) + 35.4]\,\hat{\text{i}} - [25 - (-12.9) + (-35.4)]\,\hat{\text{j}} = (127\,\hat{\text{i}} + 2.60\,\hat{\text{j}})\text{ m}$$

in unit-vector notation. The magnitude of this result is

$$|\vec{a} - \vec{b} + \vec{c}| = \sqrt{(127\text{ m})^2 + (2.6\text{ m})^2} \approx 1.30 \times 10^2\text{ m}.$$

(d) The angle between the vector described in part (c) and the +*x* axis is $\tan^{-1}(2.6\text{ m}/127\text{ m}) \approx 1.2°$.

(e) Using unit-vector notation, $\vec{d}$ is given by $\vec{d} = \vec{a} + \vec{b} - \vec{c} = (-40.4\ \hat{\text{i}} + 47.4\ \hat{\text{j}})$ m, which has a magnitude of $\sqrt{(-40.4\text{ m})^2 + (47.4\text{ m})^2} = 62$ m.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the +*x* axis are $\tan^{-1}(47.4/(-40.4)) = -50.0°$, and $180° + (-50.0°) = 130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

18. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the +*x* axis is 180° + 20.0° = 200°.

(a) The *x* and *y* components of $\vec{B}$ are given by

$$B_x = C_x - A_x = (15.0\text{ m}) \cos 200° - (12.0\text{ m}) \cos 40° = -23.3\text{ m},$$
$$B_y = C_y - A_y = (15.0\text{ m}) \sin 200° - (12.0\text{ m}) \sin 40° = -12.8\text{ m}.$$

Consequently, its magnitude is $|\vec{B}| = \sqrt{(-23.3\text{ m})^2 + (-12.8\text{ m})^2} = 26.6\text{ m}$.

(b) The two possibilities presented by a simple calculation for the angle between $\vec{B}$ and the +*x* axis are $\tan^{-1}$[(–12.8 m)/(–23.3 m)] = 28.9°, and 180° + 28.9° = 209°. We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

19. (a) With $\hat{\text{i}}$ directed forward and $\hat{\text{j}}$ directed leftward, the resultant is $(5.00\ \hat{\text{i}} + 2.00\ \hat{\text{j}})$ m. The magnitude is given by the Pythagorean theorem: $\sqrt{(5.00\text{ m})^2 + (2.00\text{ m})^2}$ = 5.385 m ≈ 5.39 m.

(b) The angle is $\tan^{-1}(2.00/5.00) \approx 21.8°$ (left of forward).

20. The desired result is the displacement vector, in units of km, $\vec{A}$ = (5.6 km), 90º (measured counterclockwise from the +*x* axis), or $\vec{A} = (5.6\text{ km})\hat{\text{j}}$, where $\hat{\text{j}}$ is the unit vector along the positive *y* axis (north). This consists of the sum of two displacements: during the whiteout, $\vec{B} = (7.8\text{ km}),\ 50°$, or

$$\vec{B} = (7.8\text{ km})(\cos 50° \hat{\text{i}} + \sin 50°\ \hat{\text{j}}) = (5.01\text{ km})\hat{\text{i}} + (5.98\text{ km})\hat{\text{j}}$$

and the unknown $\vec{C}$. Thus, $\vec{A} = \vec{B} + \vec{C}$.

(a) The desired displacement is given by $\vec{C} = \vec{A} - \vec{B} = (-5.01\text{ km})\ \hat{\text{i}} - (0.38\text{ km})\ \hat{\text{j}}$. The magnitude is $\sqrt{(-5.01\text{ km})^2 + (-0.38\text{ km})^2} = 5.0\text{ km}$.

(b) The angle is $\tan^{-1}[(-0.38\text{ km})/(-5.01\text{ km})] = 4.3°,$ south of due west.

21. Reading carefully, we see that the (x, y) specifications for each "dart" are to be interpreted as (Δx, Δy) descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition.

(a) Along the x axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140,$$

which gives $b_x = -70.0$ cm.

(b) Along the y axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0$$

which yields $c_y = 80.0$ cm.

(c) The magnitude of the final location (–140 , –20.0) is $\sqrt{(-140)^2 + (-20.0)^2} = 141\text{ cm.}$

(d) Since the displacement is in the third quadrant, the angle of the overall displacement is given by $\pi + \tan^{-1}[(-20.0)/(-140)]$ or 188° counterclockwise from the $+x$ axis (or $-172°$ counterclockwise from the $+x$ axis).

22. Angles are given in 'standard' fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Wherever the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\vec{E} = 3.73\ \hat{\text{i}} + 4.70\ \hat{\text{j}}$$
$$\vec{F} = 1.29\ \hat{\text{i}} - 4.83\ \hat{\text{j}}$$
$$\vec{G} = 1.45\ \hat{\text{i}} + 3.73\ \hat{\text{j}}$$
$$\vec{H} = -5.20\ \hat{\text{i}} + 3.00\ \hat{\text{j}}$$
$$\vec{E} + \vec{F} + \vec{G} + \vec{H} = 1.28\ \hat{\text{i}}\ + 6.60\ \hat{\text{j}}.$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{(1.28\text{ m})^2 + (6.60\text{ m})^2} = 6.72\text{ m}$.

(c) Its angle measured counterclockwise from the +x axis is $\tan^{-1}(6.60/1.28) = 79.0°$.

(d) Using the conversion factor $\pi \text{ rad} = 180°$, 79.0° = 1.38 rad.

23. The resultant (along the y axis, with the same magnitude as $\vec{C}$) forms (along with $\vec{C}$) a side of an isosceles triangle (with $\vec{B}$ forming the base). If the angle between $\vec{C}$ and the y axis is $\theta = \tan^{-1}(3/4) = 36.87°$, then it should be clear that (referring to the magnitudes of the vectors) $B = 2C\sin(\theta/2)$. Thus (since C = 5.0) we find B = 3.2.

24. As a vector addition problem, we express the situation (described in the problem statement) as $\vec{A} + \vec{B} = (3A)\hat{\text{j}}$, where $\vec{A} = A\,\hat{\text{i}}$ and B = 7.0 m. Since $\hat{\text{i}} \perp \hat{\text{j}}$ we may use the Pythagorean theorem to express B in terms of the magnitudes of the other two vectors:

$$B = \sqrt{(3A)^2 + A^2} \qquad \Rightarrow \qquad A = \frac{1}{\sqrt{10}}B = 2.2 \text{ m}.$$

25. The strategy is to find where the camel is ($\vec{C}$) by adding the two consecutive displacements described in the problem, and then finding the difference between that location and the oasis ($\vec{B}$). Using the magnitude-angle notation

$$\vec{C} = (24 \angle -15°) + (8.0 \angle 90°) = (23.25 \angle 4.41°)$$

so

$$\vec{B} - \vec{C} = (25 \angle 0°) - (23.25 \angle 4.41°) = (2.5 \angle -45°)$$

which is efficiently implemented using a vector-capable calculator in polar mode. The distance is therefore 2.6 km.

26. The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $(1.69\hat{\text{i}} + 3.63\hat{\text{j}})\text{ m}$ and $(-2.87\hat{\text{i}} + 4.10\hat{\text{j}})\text{ m}$, respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Adding corresponding components, we obtain $\vec{R} = (-3.18 \text{ m})\hat{\text{i}} + (4.72 \text{ m})\hat{\text{j}}$.

(b) Using Eq. 3-6, the magnitude is

$$|\vec{R}| = \sqrt{(-3.18 \text{ m})^2 + (4.72 \text{ m})^2} = 5.69 \text{ m}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{4.72 \text{ m}}{-3.18 \text{ m}}\right) = -56.0° \ \ (\text{with } -x \text{ axis}).$$

If measured counterclockwise from +*x*-axis, the angle is then $180° - 56.0° = 124°$. Thus, converting the result to polar coordinates, we obtain

$$(-3.18, 4.72) \;\rightarrow\; (5.69 \angle 124°)$$

27. Solving the simultaneous equations yields the answers:

(a) $\vec{d}_1 = 4\,\vec{d}_3 = 8\,\hat{\text{i}} + 16\,\hat{\text{j}}$, and

(b) $\vec{d}_2 = \vec{d}_3 = 2\,\hat{\text{i}} + 4\,\hat{\text{j}}$.

28. Let $\vec{A}$ represent the first part of Beetle 1's trip (0.50 m east or $0.5\,\hat{\text{i}}$) and $\vec{C}$ represent the first part of Beetle 2's trip intended voyage (1.6 m at 50º north of east). For their respective second parts: $\vec{B}$ is 0.80 m at 30º north of east and $\vec{D}$ is the unknown. The final position of Beetle 1 is

$$\vec{A}+\vec{B} = (0.5\text{ m})\hat{\text{i}} + (0.8\text{ m})(\cos 30°\;\hat{\text{i}} + \sin 30°\;\hat{\text{j}}) = (1.19\text{ m})\;\hat{\text{i}} + (0.40\text{ m})\;\hat{\text{j}}.$$

The equation relating these is $\vec{A}+\vec{B} = \vec{C}+\vec{D}$, where

$$\vec{C} = (1.60\text{ m})(\cos 50.0°\hat{\text{i}} + \sin 50.0°\hat{\text{j}}) = (1.03\text{ m})\hat{\text{i}} + (1.23\text{ m})\hat{\text{j}}$$

(a) We find $\vec{D} = \vec{A}+\vec{B}-\vec{C} = (0.16\text{ m})\hat{\text{i}} + (-0.83\text{ m})\hat{\text{j}}$, and the magnitude is D = 0.84 m.

(b) The angle is $\tan^{-1}(-0.83/0.16) = -79°$, which is interpreted to mean 79º south of east (or 11º east of south).

29. Let $l_0 = 2.0\text{ cm}$ be the length of each segment. The nest is located at the endpoint of segment *w*.

(a) Using unit-vector notation, the displacement vector for point *A* is

$$\begin{aligned}\vec{d}_A &= \vec{w}+\vec{v}+\vec{i}+\vec{h} = l_0(\cos 60°\hat{\text{i}} + \sin 60°\;\hat{\text{j}}) + \left(l_0\;\hat{\text{j}}\right) + l_0(\cos 120°\hat{\text{i}} + \sin 120°\;\hat{\text{j}}) + \left(l_0\;\hat{\text{j}}\right)\\ &= (2+\sqrt{3})l_0\;\hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_A$ is $|\vec{d}_A| = (2+\sqrt{3})(2.0\text{ cm}) = 7.5\text{ cm}$.

(b) The angle of $\vec{d}_A$ is $\theta = \tan^{-1}(d_{A,y}/d_{A,x}) = \tan^{-1}(\infty) = 90°$.

(c) Similarly, the displacement for point *B* is

$$\begin{aligned}\vec{d}_B &= \vec{w}+\vec{v}+\vec{j}+\vec{p}+\vec{o}\\ &= l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+\left(l_0\,\hat{\text{j}}\right)+l_0(\cos 60°\hat{\text{i}}+\sin 60°\,\hat{\text{j}})+l_0(\cos 30°\hat{\text{i}}+\sin 30°\,\hat{\text{j}})+\left(l_0\,\hat{\text{i}}\right)\\ &= (2+\sqrt{3}/2)l_0\,\hat{\text{i}}+(3/2+\sqrt{3})l_0\,\hat{\text{j}}.\end{aligned}$$

Therefore, the magnitude of $\vec{d}_B$ is

$$|\vec{d}_B| = l_0\sqrt{(2+\sqrt{3}/2)^2+(3/2+\sqrt{3})^2} = (2.0\text{ cm})(4.3) = 8.6\text{ cm}.$$

(d) The direction of $\vec{d}_B$ is

$$\theta_B = \tan^{-1}\left(\frac{d_{B,y}}{d_{B,x}}\right) = \tan^{-1}\left(\frac{3/2+\sqrt{3}}{2+\sqrt{3}/2}\right) = \tan^{-1}(1.13) = 48°.$$

30. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6).

(a) The magnitude of $\vec{a}$ is $a = \sqrt{(4.0\text{ m})^2+(-3.0\text{ m})^2} = 5.0\text{ m}.$

(b) The angle between $\vec{a}$ and the $+x$ axis is $\tan^{-1}$ [(–3.0 m)/(4.0 m)] = –37°. The vector is 37° *clockwise* from the axis defined by $\hat{\text{i}}$.

(c) The magnitude of $\vec{b}$ is $b = \sqrt{(6.0\text{ m})^2+(8.0\text{ m})^2} = 10\text{ m}.$

(d) The angle between $\vec{b}$ and the $+x$ axis is $\tan^{-1}$[(8.0 m)/(6.0 m)] = 53°.

(e) $\vec{a}+\vec{b} = (4.0\text{ m}+6.0\text{ m})\,\hat{\text{i}}+[(-3.0\text{ m})+8.0\text{ m}]\hat{\text{j}} = (10\text{ m})\hat{\text{i}}+(5.0\text{ m})\hat{\text{j}}.$ The magnitude of this vector is $|\vec{a}+\vec{b}| = \sqrt{(10\text{ m})^2+(5.0\text{ m})^2} = 11\text{ m};$ we round to two significant figures in our results.

(f) The angle between the vector described in part (e) and the $+x$ axis is $\tan^{-1}$[(5.0 m)/(10 m)] = 27°.

(g) $\vec{b}-\vec{a} = (6.0\text{ m}-4.0\text{ m})\,\hat{\text{i}}+[8.0\text{ m}-(-3.0\text{ m})]\,\hat{\text{j}} = (2.0\text{ m})\,\hat{\text{i}}+(11\text{ m})\,\hat{\text{j}}.$ The magnitude of this vector is $|\vec{b}-\vec{a}| = \sqrt{(2.0\text{ m})^2+(11\text{ m})^2} = 11\text{ m},$ which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that $\vec{a}\perp\vec{b}$).

(h) The angle between the vector described in part (g) and the +x axis is $\tan^{-1}$[(11 m)/(2.0 m)] = 80°.

(i) $\vec{a}-\vec{b}=(4.0\text{ m}-6.0\text{ m})\,\hat{\text{i}}+[(-3.0\text{ m})-8.0\text{ m}]\,\hat{\text{j}}=(-2.0\text{ m})\,\hat{\text{i}}+(-11\text{ m})\,\hat{\text{j}}$. The magnitude of this vector is

$$|\vec{a}-\vec{b}|=\sqrt{(-2.0\text{ m})^2+(-11\text{ m})^2}=11\text{ m}.$$

(j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the +x direction are $\tan^{-1}$[(–11 m)/(–2.0 m)] = 80°, and 180° + 80° = 260°. The latter possibility is the correct answer (see part (k) for a further observation related to this result).

(k) Since $\vec{a}-\vec{b}=(-1)(\vec{b}-\vec{a})$, they point in opposite (anti-parallel) directions; the angle between them is 180°.

31. (a) With a = 17.0 m and θ = 56.0° we find $a_x = a\cos\theta$ = 9.51 m.

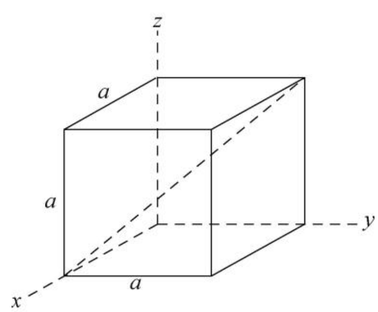
(b) Similarly, $a_y = a\sin\theta$ = 14.1 m.

(c) The angle relative to the new coordinate system is θ' = (56.0° – 18.0°) = 38.0°. Thus, $a'_x = a\cos\theta' = 13.4$ m.

(d) Similarly, $a'_y = a\sin\theta'$ = 10.5 m.

32. (a) As can be seen from Figure 3-30, the point diametrically opposite the origin (0,0,0) has position vector $a\,\hat{\text{i}}+a\,\hat{\text{j}}+a\,\hat{\text{k}}$ and this is the vector along the "body diagonal."

(b) From the point (a, 0, 0), which corresponds to the position vector $a\,\hat{\text{i}}$, the diametrically opposite point is (0, a, a) with the position vector $a\,\hat{\text{j}}+a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $-a\hat{\text{i}}+a\hat{\text{j}}+a\hat{\text{k}}$.



(c) If the starting point is (0, a, 0) with the corresponding position vector $a\,\hat{\text{j}}$, the diametrically opposite point is (a, 0, a) with the position vector $a\,\hat{\text{i}}+a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $a\,\hat{\text{i}}-a\,\hat{\text{j}}+a\,\hat{\text{k}}$.

(d) If the starting point is (a, a, 0) with the corresponding position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}}$, the diametrically opposite point is (0, 0, a) with the position vector $a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\,\hat{\mathrm{i}} - a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$.

(e) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. We may think of it as the sum of the vector $a\,\hat{\mathrm{i}}$ parallel to the x axis and the vector $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2+a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7°$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(f) The length of any of the diagonals is given by $\sqrt{a^2+a^2+a^2} = a\sqrt{3}$.

33. Examining the figure, we see that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $|\vec{a} \times \vec{b}| = (3.0)(4.0) = 12$ since the angle between them is 90º.

(b) Using the Right-Hand Rule, the vector $\vec{a}\times\vec{b}$ points in the $\hat{\mathrm{i}}\times\hat{\mathrm{j}} = \hat{\mathrm{k}}$, or the +$z$ direction.

(c) $|\vec{a} \times \vec{c}| = |\vec{a} \times (-\vec{a} - \vec{b})| = |-(\vec{a} \times \vec{b})| = 12$.

(d) The vector $-\vec{a}\times\vec{b}$ points in the $-\hat{\mathrm{i}}\times\hat{\mathrm{j}} = -\hat{\mathrm{k}}$, or the $-z$ direction.

(e) $|\vec{b} \times \vec{c}| = |\vec{b} \times (-\vec{a} - \vec{b})| = |-(\vec{b} \times \vec{a})| = |(\vec{a} \times \vec{b})| = 12$.

(f) The vector points in the +z direction, as in part (a).

34. We apply Eq. 3-23 and Eq. 3-27.

(a) $\vec{a}\times\vec{b} = (a_x b_y - a_y b_x)\,\hat{\mathrm{k}}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $[(3.0)(4.0)-(5.0)(2.0)]\hat{\mathrm{k}} = 2.0\,\hat{\mathrm{k}}$.

(b) $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$ yields (3.0)(2.0) + (5.0)(4.0) = 26.

(c) $\vec{a}+\vec{b} = (3.0 + 2.0)\,\hat{\mathrm{i}} + (5.0 + 4.0)\,\hat{\mathrm{j}} \Rightarrow (\vec{a}+\vec{b})\cdot\vec{b} = (5.0)\,(2.0) + (9.0)\,(4.0) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}}}{\sqrt{(2.0)^2 + (4.0)^2}}\,.$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(3.0)(2.0) + (5.0)(4.0)}{\sqrt{(2.0)^2 + (4.0)^2}} = 5.8.$$

35. (a) The scalar or dot product is (4.50)(7.30)cos(320° – 85.0°) = – 18.8 .

(b) The vector or cross product is in the $\hat{\mathrm{k}}$ direction (by the right-hand rule) with magnitude |(4.50)(7.30) sin(320° – 85.0°)| = 26.9 .

36. First, we rewrite the given expression as 4($\vec{d}_{\text{plane}} \cdot \vec{d}_{\text{cross}}$) where $\vec{d}_{\text{plane}} = \vec{d}_1 + \vec{d}_2$ and in the plane of $\vec{d}_1$ and $\vec{d}_2$, and $\vec{d}_{\text{cross}} = \vec{d}_1\times\vec{d}_2$. Noting that $\vec{d}_{\text{cross}}$ is perpendicular to the plane of $\vec{d}_1$ and $\vec{d}_2$, we see that the answer must be 0 (the scalar or dot product of perpendicular vectors is zero).

37. We apply Eq. 3-23 and Eq.3-27. If a vector-capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8.0\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}} + 6.0\,\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)\,(-8.0) + (3.0)(5.0) + (-2.0)\,(6.0) = -21.$$

(b) We note that $\vec{b} + \vec{c} = 1.0\,\hat{\mathrm{i}} - 2.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)\,(1.0) + (3.0)\,(-2.0) + (-2.0)\,(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0) - (-2.0)(-2.0)]\,\hat{\mathrm{i}} + [(-2.0)(1.0) - (3.0)(3.0)]\,\hat{\mathrm{j}} \\ &\quad + [(3.0)(-2.0) - (3.0)(1.0)]\,\hat{\mathrm{k}} \\ &= 5\hat{\mathrm{i}} - 11\hat{\mathrm{j}} - 9\hat{\mathrm{k}}\end{aligned}$$

38. Using the fact that

$$\hat{\mathrm{i}}\times\hat{\mathrm{j}} = \hat{\mathrm{k}},\quad \hat{\mathrm{j}}\times\hat{\mathrm{k}} = \hat{\mathrm{i}},\quad \hat{\mathrm{k}}\times\hat{\mathrm{i}} = \hat{\mathrm{j}}$$

we obtain

$$2\vec{A}\times\vec{B}=2\left(2.00\hat{\mathrm{i}}+3.00\hat{\mathrm{j}}-4.00\hat{\mathrm{k}}\right)\times\left(-3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}+2.00\hat{\mathrm{k}}\right)$$
$$=44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}}.$$

Next, making use of

$$\hat{\mathrm{i}}\cdot\hat{\mathrm{i}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{k}}=1$$
$$\hat{\mathrm{i}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{k}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{i}}=0$$

we have

$$3\vec{C}\cdot\left(2\vec{A}\times\vec{B}\right)=3\left(7.00\,\hat{\mathrm{i}}-8.00\hat{\mathrm{j}}\right)\cdot\left(44.0\,\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\,\hat{\mathrm{k}}\right)$$
$$=3[(7.00)(44.0)+(-8.00)(16.0)+(0)(34.0)]=540.$$

39. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B}=AB\cos\theta$, we have

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}$$

With $A=6.00$, $B=7.00$ and $\vec{A}\cdot\vec{B}=14.0$, $\cos\theta=0.333$, or $\theta=70.5°$.

40. The displacement vectors can be written as (in meters)

$$\vec{d}_1=(4.50\text{ m})(\cos 63°\,\hat{\mathrm{j}}+\sin 63°\,\hat{\mathrm{k}})=(2.04\text{ m})\,\hat{\mathrm{j}}+(4.01\text{ m})\,\hat{\mathrm{k}}$$
$$\vec{d}_2=(1.40\text{ m})(\cos 30°\,\hat{\mathrm{i}}+\sin 30°\,\hat{\mathrm{k}})=(1.21\text{ m})\,\hat{\mathrm{i}}+(0.70\text{ m})\,\hat{\mathrm{k}}\,.$$

(a) The dot product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1\cdot\vec{d}_2=(2.04\,\hat{\mathrm{j}}+4.01\hat{\mathrm{k}})\cdot(1.21\hat{\mathrm{i}}+0.70\,\hat{\mathrm{k}})=(4.01\hat{\mathrm{k}})\cdot(0.70\,\hat{\mathrm{k}})=2.81\,\mathrm{m}^2.$$

(b) The cross product of $\vec{d}_1$ and $\vec{d}_2$ is

$$\vec{d}_1\times\vec{d}_2=(2.04\,\hat{\mathrm{j}}+4.01\hat{\mathrm{k}})\times(1.21\hat{\mathrm{i}}+0.70\,\hat{\mathrm{k}})$$
$$=(2.04)(1.21)(-\hat{\mathrm{k}})+(2.04)(0.70)\hat{\mathrm{i}}+(4.01)(1.21)\hat{\mathrm{j}}$$
$$=(1.43\,\hat{\mathrm{i}}+4.86\,\hat{\mathrm{j}}-2.48\,\hat{\mathrm{k}})\,\mathrm{m}^2.$$

(c) The magnitudes of $\vec{d}_1$ and $\vec{d}_2$ are

$$d_1=\sqrt{(2.04\text{ m})^2+(4.01\text{ m})^2}=4.50\text{ m}$$
$$d_2=\sqrt{(1.21\text{ m})^2+(0.70\text{ m})^2}=1.40\text{ m}.$$

Thus, the angle between the two vectors is

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \vec{d}_2}{d_1 d_2}\right) = \cos^{-1}\left(\frac{2.81\ \text{m}^2}{(4.50\ \text{m})(1.40\ \text{m})}\right) = 63.5°.$$

41. **THINK** The angle between two vectors can be calculated using the definition of scalar product.

EXPRESS Since the scalar product of two vectors $\vec{a}$ and $\vec{b}$ is

$$\vec{a} \cdot \vec{b} = ab\cos\phi = a_x b_x + a_y b_y + a_z b_z\ ,$$

the angle between them is given by

$$\cos\phi = \frac{a_x b_x\ +\ a_y b_y\ +\ a_z b_z}{ab} \ \Rightarrow\ \phi = \cos^{-1}\left(\frac{a_x b_x\ +\ a_y b_y\ +\ a_z b_z}{ab}\right).$$

Once the magnitudes and components of the vectors are known, the angle ϕ can be readily calculated.

ANALYZE Given that $\vec{a} = (3.0)\hat{\text{i}} + (3.0)\hat{\text{j}} + (3.0)\hat{\text{k}}$ and $\vec{b} = (2.0)\hat{\text{i}} + (1.0)\hat{\text{j}} + (3.0)\hat{\text{k}}$, the magnitudes of the vectors are

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.20$$

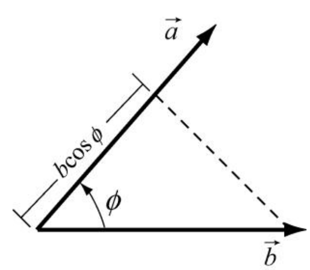
$$b = |\vec{b}| = \sqrt{b_x^2 + b_y^2 + b_z^2} = \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.74.$$

The angle between them is found to be

$$\cos\phi = \frac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.20)(3.74)} = 0.926,$$

or ϕ= 22°.

LEARN As the name implies, the scalar product (or dot product) between two vectors is a scalar quantity. It can be regarded as the product between the magnitude of one of the vectors and the scalar component of the second vector along the direction of the first one, as illustrated below (see also in Fig. 3-18 of the text):



$$\vec{a} \cdot \vec{b} = ab\cos\phi = (a)(b\cos\phi)$$

42. The two vectors are written as, in unit of meters,

$$\vec{d}_1 = 4.0\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}} = d_{1x}\hat{\mathrm{i}} + d_{1y}\hat{\mathrm{j}}, \qquad \vec{d}_2 = -3.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}} = d_{2x}\hat{\mathrm{i}} + d_{2y}\hat{\mathrm{j}}$$

(a) The vector (cross) product gives

$$\vec{d}_1 \times \vec{d}_2 = (d_{1x}d_{2y} - d_{1y}d_{2x})\hat{\mathrm{k}} = [(4.0)(4.0) - (5.0)(-3.0)]\hat{\mathrm{k}} = 31\ \hat{\mathrm{k}}$$

(b) The scalar (dot) product gives

$$\vec{d}_1 \cdot \vec{d}_2 = d_{1x}d_{2x} + d_{1y}d_{2y} = (4.0)(-3.0) + (5.0)(4.0) = 8.0.$$

(c)

$$(\vec{d}_1 + \vec{d}_2)\cdot\vec{d}_2 = \vec{d}_1\cdot\vec{d}_2 + d_2^2 = 8.0 + (-3.0)^2 + (4.0)^2 = 33.$$

(d) Note that the magnitude of the d_1 vector is $\sqrt{16+25}$ = 6.4. Now, the dot product is (6.4)(5.0)cosθ = 8. Dividing both sides by 32 and taking the inverse cosine yields θ = 75.5°. Therefore the component of the d_1 vector along the direction of the d_2 vector is 6.4cos$\theta \approx$ 1.6.

43. **THINK** In this problem we are given three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ on the *xy*-plane, and asked to calculate their components.

EXPRESS From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the +*x* axis is θ + 90°. In unit-vector notation, the three vectors can be written as

$$\begin{aligned}
\vec{a} &= a_x\hat{\mathrm{i}} \\
\vec{b} &= b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} = (b\cos\theta)\hat{\mathrm{i}} + (b\sin\theta)\hat{\mathrm{j}} \\
\vec{c} &= c_x\hat{\mathrm{i}} + c_y\hat{\mathrm{j}} = [c\cos(\theta + 90°)]\hat{\mathrm{i}} + [c\sin(\theta + 90°)]\hat{\mathrm{j}}.
\end{aligned}$$

The above expressions allow us to evaluate the components of the vectors.

ANALYZE (a) The *x*-component of $\vec{a}$ is $a_x = a \cos 0° = a$ = 3.00 m.

(b) Similarly, the *y*-componnet of $\vec{a}$ is $a_y = a \sin 0°$ = 0.

(c) The *x*-component of $\vec{b}$ is $b_x = b \cos 30°$ = (4.00 m) cos 30° = 3.46 m,

(d) and the *y*-component is $b_y = b \sin 30°$ = (4.00 m) sin 30° = 2.00 m.

(e) The *x*-component of $\vec{c}$ is $c_x = c \cos 120°$ = (10.0 m) cos 120° = –5.00 m,

(f) and the *y*-component is $c_y = c \sin 30° = (10.0 \text{ m}) \sin 120° = 8.66$ m.

(g) The fact that $\vec{c} = p\vec{a} + q\vec{b}$ implies

$$\vec{c} = c_x\hat{\text{i}} + c_y\hat{\text{j}} = p(a_x\hat{\text{i}}) + q(b_x\hat{\text{i}} + b_y\hat{\text{j}}) = (pa_x + qb_x)\hat{\text{i}} + qb_y\hat{\text{j}}$$

or

$$c_x = pa_x + qb_x, \qquad c_y = qb_y\,.$$

Substituting the values found above, we have

$$-5.00\text{ m} = p\,(3.00\text{ m}) + q\,(3.46\text{ m})$$
$$8.66\text{ m} = q\,(2.00\text{ m}).$$

Solving these equations, we find $p = -6.67$.

(h) Similarly, $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

LEARN This exercise shows that given two (non-parallel) vectors in two dimensions, the third vector can always be written as a linear combination of the first two.

44. Applying Eq. 3-23, $\vec{F} = q\vec{v} \times \vec{B}$ (where q is a scalar) becomes

$$F_x\,\hat{\text{i}} + F_y\,\hat{\text{j}} + F_z\,\hat{\text{k}} = q\left(v_yB_z - v_zB_y\right)\hat{\text{i}} + q\left(v_zB_x - v_xB_z\right)\hat{\text{j}} + q\left(v_xB_y - v_yB_x\right)\hat{\text{k}}$$

which — plugging in values — leads to three equalities:

$$4.0 = 2\,(4.0B_z - 6.0B_y)$$
$$-20 = 2\,(6.0B_x - 2.0B_z)$$
$$12 = 2\,(2.0B_y - 4.0B_x)$$

Since we are told that $B_x = B_y$, the third equation leads to $B_y = -3.0$. Inserting this value into the first equation, we find $B_z = -4.0$. Thus, our answer is

$$\vec{B} = -3.0\,\hat{\text{i}} - 3.0\,\hat{\text{j}} - 4.0\,\hat{\text{k}}.$$

45. The two vectors are given by

$$\vec{A} = 8.00(\cos 130°\,\hat{\text{i}} + \sin 130°\,\hat{\text{j}}) = -5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}}$$
$$\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}} = -7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}}.$$

(a) The dot product of $5\vec{A}\cdot\vec{B}$ is

$$5\vec{A}\cdot\vec{B} = 5(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\cdot(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 5[(-5.14)(-7.72) + (6.13)(-9.20)]$$
$$= -83.4.$$

(b) In unit vector notation

$$4\vec{A}\times 3\vec{B} = 12\vec{A}\times\vec{B} = 12(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\times(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 12(94.6\,\hat{\mathrm{k}}) = 1.14\times 10^3\,\hat{\mathrm{k}}$$

(c) We note that the azimuthal angle is undefined for a vector along the *z* axis. Thus, our result is "1.14×10^3, θ not defined, and $\phi = 0°$."

(d) Since $\vec{A}$ is in the *xy* plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90°.

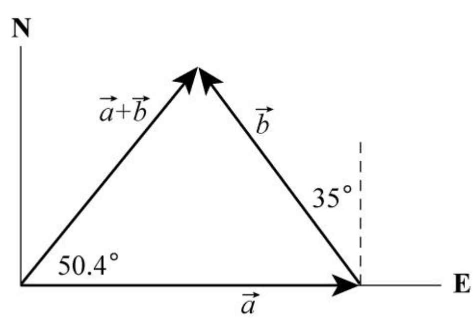
(e) Clearly, $\vec{A} + 3.00\,\hat{\mathrm{k}} = -5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}} + 3.00\,\hat{\mathrm{k}}$.

(f) The Pythagorean theorem yields magnitude $A = \sqrt{(5.14)^2 + (6.13)^2 + (3.00)^2} = 8.54$. The azimuthal angle is $\theta = 130°$, just as it was in the problem statement ($\vec{A}$ is the projection onto the *xy* plane of the new vector created in part (e)). The angle measured from the +*z* axis is

$$\phi = \cos^{-1}(3.00/8.54) = 69.4°.$$

46. The vectors are shown on the diagram. The *x* axis runs from west to east and the *y* axis runs from south to north. Then $a_x = 5.0$ m, $a_y = 0$,

$$b_x = -(4.0\text{ m})\sin 35° = -2.29\text{ m},\quad b_y = (4.0\text{ m})\cos 35° = 3.28\text{ m}.$$



(a) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x = 5.00\text{ m} - 2.29\text{ m} = 2.71\text{ m}$ and $c_y = a_y + b_y = 0 + 3.28\text{ m} = 3.28\text{ m}$. The magnitude of *c* is

$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71\text{m})^2 + (3.28\text{m})^2} = 4.2\text{ m}.$$

(b) The angle θ that $\vec{c} = \vec{a} + \vec{b}$ makes with the +*x* axis is

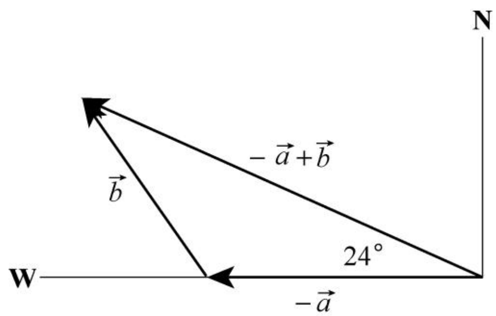
$$\theta = \tan^{-1}\left(\frac{c_y}{c_x}\right) = \tan^{-1}\left(\frac{3.28}{2.71}\right) = 50.5° \approx 50°.$$

The second possibility (θ = 50.4° + 180° = 230.4°) is rejected because it would point in a direction opposite to $\vec{c}$.

(c) The vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. The result is shown on the diagram to the right. Let $\vec{c} = \vec{b} - \vec{a}.$ The components are

$$c_x = b_x - a_x = -2.29 \text{ m} - 5.00 \text{ m} = -7.29 \text{ m}$$
$$c_y = b_y - a_y = 3.28 \text{ m}.$$

The magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0\,\text{m}$.



(d) The tangent of the angle θ that $\vec{c}$ makes with the +*x* axis (east) is

$$\tan\theta = \frac{c_y}{c_x} = \frac{3.28 \text{ m}}{-7.29 \text{ m}} = -4.50.$$

There are two solutions: –24.2° and 155.8°. As the diagram shows, the second solution is correct. The vector $\vec{c} = -\vec{a} + \vec{b}$ is 24° north of west.

47. Noting that the given 130° is measured counterclockwise from the +*x* axis, the two vectors can be written as

$$\vec{A} = 8.00(\cos 130°\,\hat{\text{i}} + \sin 130°\,\hat{\text{j}}) = -5.14\,\hat{\text{i}} + 6.13\,\hat{\text{j}}$$
$$\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}} = -7.72\,\hat{\text{i}} - 9.20\,\hat{\text{j}}.$$

(a) The angle between the negative direction of the *y* axis ($-\hat{\text{j}}$) and the direction of $\vec{A}$ is

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot(-\hat{\mathrm{j}})}{A}\right) = \cos^{-1}\left(\frac{-6.13}{\sqrt{(-5.14)^2+(6.13)^2}}\right) = \cos^{-1}\left(\frac{-6.13}{8.00}\right) = 140°.$$

Alternatively, one may say that the $-y$ direction corresponds to an angle of 270°, and the answer is simply given by 270°–130° = 140°.

(b) Since the y axis is in the xy plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90.0°.

(c) The vector can be simplified as

$$\begin{aligned}\vec{A}\times(\vec{B}+3.00\,\hat{\mathrm{k}}) &= (-5.14\,\hat{\mathrm{i}}+6.13\,\hat{\mathrm{j}})\times(-7.72\,\hat{\mathrm{i}}-9.20\,\hat{\mathrm{j}}+3.00\,\hat{\mathrm{k}}) \\ &= 18.39\,\hat{\mathrm{i}}+15.42\,\hat{\mathrm{j}}+94.61\hat{\mathrm{k}}\end{aligned}$$

Its magnitude is $|\vec{A}\times(\vec{B}+3.00\hat{\mathrm{k}})| = 97.6.$ The angle between the negative direction of the y axis ($-\hat{\mathrm{j}}$) and the direction of the above vector is

$$\theta = \cos^{-1}\left(\frac{-15.42}{97.6}\right) = 99.1°.$$

48. Where the length unit is not displayed, the unit meter is understood.

(a) We first note that the magnitudes of the vectors are $a = |\vec{a}| = \sqrt{(3.2)^2+(1.6)^2} = 3.58$ and $b = |\vec{b}| = \sqrt{(0.50)^2+(4.5)^2} = 4.53$. Now,

$$\begin{aligned}\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y &= ab\cos\phi \\ (3.2)(0.50)+(1.6)(4.5) &= (3.58)(4.53)\cos\phi\end{aligned}$$

which leads to ϕ = 57° (the inverse cosine is double-valued as is the inverse tangent, but we know this is the right solution since both vectors are in the same quadrant).

(b) Since the angle (measured from $+x$) for $\vec{a}$ is $\tan^{-1}(1.6/3.2)$ = 26.6°, we know the angle for $\vec{c}$ is 26.6° –90° = –63.4° (the other possibility, 26.6° + 90° would lead to a $c_x < 0$). Therefore,

$$c_x = c\cos(-63.4°) = (5.0)(0.45) = 2.2 \text{ m}.$$

(c) Also, $c_y = c\sin(-63.4°) = (5.0)(-0.89) = -4.5$ m.

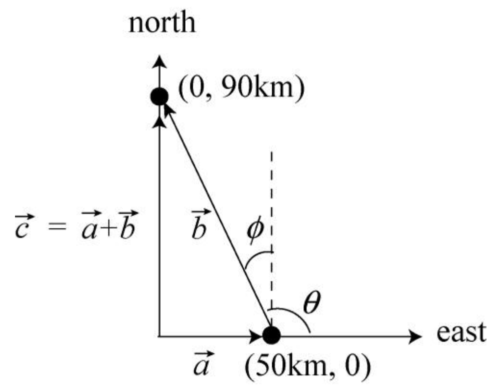
(d) And we know the angle for $\vec{d}$ to be 26.6° + 90° = 116.6°, which leads to

$$d_x = d\cos(116.6°) = (5.0)(-0.45) = -2.2 \text{ m}.$$

(e) Finally, $d_y = d \sin 116.6° = (5.0)(0.89) = 4.5$ m.

49. **THINK** This problem deals with the displacement of a sailboat. We want to find the displacement vector between two locations.

EXPRESS The situation is depicted in the figure below. Let $\vec{a}$ represent the first part of his actual voyage (50.0 km east) and $\vec{c}$ represent the intended voyage (90.0 km north). We look for a vector $\vec{b}$ such that $\vec{c} = \vec{a} + \vec{b}$.



ANALYZE (a) Using the Pythagorean theorem, the distance traveled by the sailboat is

$$b = \sqrt{(50.0 \text{ km})^2 + (90.0 \text{ km})^2} = 103 \text{ km}.$$

(b) The direction is

$$\phi = \tan^{-1}\left(\frac{50.0 \text{ km}}{90.0 \text{ km}}\right) = 29.1°$$

west of north (which is equivalent to 60.9° north of due west).

LEARN This problem could also be solved by first expressing the vectors in unit-vector notation: $\vec{a} = (50.0 \text{ km})\hat{\text{i}}, \vec{c} = (90.0 \text{ km})\hat{\text{j}}$. This gives

$$\vec{b} = \vec{c} - \vec{a} = -(50.0 \text{ km})\hat{\text{i}} + (90.0 \text{ km})\hat{\text{j}}.$$

The angle between $\vec{b}$ and the +x-axis is

$$\theta = \tan^{-1}\left(\frac{90.0 \text{ km}}{-50.0 \text{ km}}\right) = 119.1°.$$

The angle θ is related to ϕ by $\theta = 90° + \phi$.

50. The two vectors $\vec{d}_1$ and $\vec{d}_2$ are given by $\vec{d}_1 = -d_1\,\hat{\text{j}}$ and $\vec{d}_2 = d_2\,\hat{\text{i}}$.

(a) The vector $\vec{d}_2 / 4 = (d_2 / 4)\,\hat{\text{i}}$ points in the +*x* direction. The ¼ factor does not affect the result.

(b) The vector $\vec{d}_1 /(-4) = (d_1 / 4)\hat{\text{j}}$ points in the +*y* direction. The minus sign (with the "–4") does affect the direction: −(–*y*) = + *y*.

(c) $\vec{d}_1 \cdot \vec{d}_2 = 0$ since $\hat{\text{i}}\cdot\hat{\text{j}} = 0.$ The two vectors are perpendicular to each other.

(d) $\vec{d}_1 \cdot (\vec{d}_2 / 4) = (\vec{d}_1 \cdot \vec{d}_2) / 4 = 0$, as in part (c).

(e) $\vec{d}_1 \times \vec{d}_2 = -d_1 d_2 (\hat{\text{j}} \times \hat{\text{i}}) = d_1 d_2\,\hat{\text{k}}$, in the +*z*-direction.

(f) $\vec{d}_2 \times \vec{d}_1 = -d_2 d_1 (\hat{\text{i}} \times \hat{\text{j}}) = -d_1 d_2\,\hat{\text{k}}$, in the −*z*-direction.

(g) The magnitude of the vector in (e) is $d_1 d_2$.

(h) The magnitude of the vector in (f) is $d_1 d_2$.

(i) Since $d_1 \times (\vec{d}_2 / 4) = (d_1 d_2 / 4)\hat{\text{k}}$, the magnitude is $d_1 d_2 / 4$.

(j) The direction of $\vec{d}_1 \times (\vec{d}_2 / 4) = (d_1 d_2 / 4)\hat{\text{k}}$ is in the +*z*-direction.

51. Although we think of this as a three-dimensional movement, it is rendered effectively two-dimensional by referring measurements to its well-defined plane of the fault.

(a) The magnitude of the net displacement is

$$|\overrightarrow{AB}| = \sqrt{|AD|^2 + |AC|^2} = \sqrt{(17.0\text{ m})^2 + (22.0\text{ m})^2} = 27.8\text{m}.$$

(b) The magnitude of the vertical component of $\overrightarrow{AB}$ is $|AD|$ sin 52.0° = 13.4 m.

52. The three vectors are

$$\begin{aligned}\vec{d}_1 &= 4.0\,\hat{\text{i}} + 5.0\,\hat{\text{j}} - 6.0\,\hat{\text{k}}\\ \vec{d}_2 &= -1.0\,\hat{\text{i}} + 2.0\,\hat{\text{j}} + 3.0\,\hat{\text{k}}\\ \vec{d}_3 &= 4.0\,\hat{\text{i}} + 3.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}}\end{aligned}$$

(a) $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3 = (9.0 \text{ m})\hat{\text{i}} + (6.0 \text{ m})\hat{\text{j}} + (-7.0 \text{ m})\hat{\text{k}}$.

(b) The magnitude of $\vec{r}$ is $|\vec{r}| = \sqrt{(9.0 \text{ m})^2 + (6.0 \text{ m})^2 + (-7.0 \text{ m})^2} = 12.9$ m. The angle between $\vec{r}$ and the *z*-axis is given by

$$\cos\theta = \frac{\vec{r}\cdot\hat{\text{k}}}{|\vec{r}|} = \frac{-7.0 \text{ m}}{12.9 \text{ m}} = -0.543$$

which implies $\theta = 123°$.

(c) The component of $\vec{d}_1$ along the direction of $\vec{d}_2$ is given by $d_{\|} = \vec{d}_1 \cdot \hat{\text{u}} = d_1 \cos\varphi$ where φ is the angle between $\vec{d}_1$ and $\vec{d}_2$, and $\hat{\text{u}}$ is the unit vector in the direction of $\vec{d}_2$. Using the properties of the scalar (dot) product, we have

$$d_{\|} = d_1\left(\frac{\vec{d}_1\cdot\vec{d}_2}{d_1 d_2}\right) = \frac{\vec{d}_1\cdot\vec{d}_2}{d_2} = \frac{(4.0)(-1.0)+(5.0)(2.0)+(-6.0)(3.0)}{\sqrt{(-1.0)^2+(2.0)^2+(3.0)^2}} = \frac{-12}{\sqrt{14}} = -3.2 \text{ m}.$$

(d) Now we are looking for $d_\perp$ such that $d_1^2 = (4.0)^2 + (5.0)^2 + (-6.0)^2 = 77 = d_\|^2 + d_\perp^2$. From (c), we have

$$d_\perp = \sqrt{77 \text{ m}^2 - (-3.2 \text{ m})^2} = 8.2 \text{ m}.$$

This gives the magnitude of the perpendicular component (and is consistent with what one would get using Eq. 3-24), but if more information (such as the direction, or a full specification in terms of unit vectors) is sought then more computation is needed.

53. **THINK** This problem involves finding scalar and vector products between two vectors $\vec{a}$ and $\vec{b}$.

EXPRESS We apply Eqs. 3-20 and 3-24 to calculate the scalar and vector products between two vectors:

$$\vec{a}\cdot\vec{b} = ab\cos\phi$$
$$|\vec{a}\times\vec{b}| = ab\sin\phi.$$

ANALYZE (a) Given that $a = |\vec{a}| = 10$, $b = |\vec{b}| = 6.0$ and $\phi = 60°$, the scalar (dot) product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\cdot\vec{b} = ab\cos\phi = (10)\,(6.0)\cos 60° = 30.$$

(b) Similarly, the magnitude of the vector (cross) product of the two vectors is

$$|\vec{a}\times\vec{b}| = ab\sin\phi = (10)\,(6.0)\sin 60° = 52.$$

LEARN When two vectors $\vec{a}$ and $\vec{b}$ are parallel ($\phi = 0$), their scalar and vector products are $\vec{a}\cdot\vec{b} = ab\cos\phi = ab$ and $|\vec{a}\times\vec{b}| = ab\sin\phi = 0$, respectively. However, when they are perpendicular ($\phi = 90°$), we have $\vec{a}\cdot\vec{b} = ab\cos\phi = 0$ and $|\vec{a}\times\vec{b}| = ab\sin\phi = ab$.

54. From the figure, it is clear that $\vec{a} + \vec{b} + \vec{c} = 0$, where $\vec{a} \perp \vec{b}$.

(a) $\vec{a} \cdot \vec{b} = 0$ since the angle between them is 90º.

(b) $\vec{a} \cdot \vec{c} = \vec{a} \cdot (-\vec{a} - \vec{b}) = -|\vec{a}|^2 = -16$.

(c) Similarly, $\vec{b} \cdot \vec{c} = -9.0$.

55. We choose +x east and +y north and measure all angles in the "standard" way (positive ones are counterclockwise from +x). Thus, vector $\vec{d}_1$ has magnitude d_1 = 4.00 m (with the unit meter) and direction θ_1 = 225°. Also, $\vec{d}_2$ has magnitude d_2 = 5.00 m and direction θ_2 = 0°, and vector $\vec{d}_3$ has magnitude d_3 = 6.00 m and direction θ_3 = 60°.

(a) The x-component of $\vec{d}_1$ is $d_{1x} = d_1 \cos\theta_1 = -2.83$ m.

(b) The y-component of $\vec{d}_1$ is $d_{1y} = d_1 \sin\theta_1 = -2.83$ m.

(c) The x-component of $\vec{d}_2$ is $d_{2x} = d_2 \cos\theta_2 = 5.00$ m.

(d) The y-component of $\vec{d}_2$ is $d_{2y} = d_2 \sin\theta_2 = 0$.

(e) The x-component of $\vec{d}_3$ is $d_{3x} = d_3 \cos\theta_3 = 3.00$ m.

(f) The y-component of $\vec{d}_3$ is $d_{3y} = d_3 \sin\theta_3 = 5.20$ m.

(g) The sum of x-components is

$$d_x = d_{1x} + d_{2x} + d_{3x} = -2.83 \text{ m} + 5.00 \text{ m} + 3.00 \text{ m} = 5.17 \text{ m}.$$

(h) The sum of y-components is

$$d_y = d_{1y} + d_{2y} + d_{3y} = -2.83 \text{ m} + 0 + 5.20 \text{ m} = 2.37 \text{ m}.$$

(i) The magnitude of the resultant displacement is

$$d=\sqrt{d_x^2+d_y^2}=\sqrt{(5.17\text{ m})^2+(2.37\text{ m})^2}=5.69\text{ m}.$$

(j) And its angle is

$$\theta = \tan^{-1}(2.37/5.17) = 24.6°,$$

which (recalling our coordinate choices) means it points at about 25° north of east.

(k) and (l) This new displacement (the direct line home) when vectorially added to the previous (net) displacement must give zero. Thus, the new displacement is the negative, or opposite, of the previous (net) displacement. That is, it has the same magnitude (5.69 m) but points in the opposite direction (25° south of west).

56. If we wish to use Eq. 3-5 directly, we should note that the angles for $\vec{Q}, \vec{R}$, and $\vec{S}$ are 100°, 250°, and 310°, respectively, if they are measured counterclockwise from the +x axis.

(a) Using unit-vector notation, with the unit meter understood, we have

$$\begin{aligned}\vec{P}&=10.0\cos(25.0°)\,\hat{\text{i}}+10.0\sin(25.0°)\,\hat{\text{j}}\\ \vec{Q}&=12.0\cos(100°)\,\hat{\text{i}}+12.0\sin(100°)\,\hat{\text{j}}\\ \vec{R}&=8.00\cos(250°)\,\hat{\text{i}}+8.00\sin(250°)\,\hat{\text{j}}\\ \vec{S}&=9.00\cos(310°)\,\hat{\text{i}}+9.00\sin(310°)\,\hat{\text{j}}\\ \vec{P}+\vec{Q}+\vec{R}+\vec{S}&=(10.0\text{ m})\hat{\text{i}}+(1.63\text{ m})\hat{\text{j}}\end{aligned}$$

(b) The magnitude of the vector sum is $\sqrt{(10.0\text{ m})^2+(1.63\text{ m})^2}=10.2\text{ m}$.

(c) The angle is $\tan^{-1}$ (1.63 m/10.0 m) ≈ 9.24° measured counterclockwise from the +x axis.

57. **THINK** This problem deals with addition and subtraction of two vectors.

EXPRESS From the problem statement, we have

$$\vec{A}+\vec{B}=(6.0)\hat{\text{i}}+(1.0)\hat{\text{j}},\qquad \vec{A}-\vec{B}=-(4.0)\hat{\text{i}}+(7.0)\hat{\text{j}}$$

Solving the simultaneous equations gives $\vec{A}$ and $\vec{B}$.

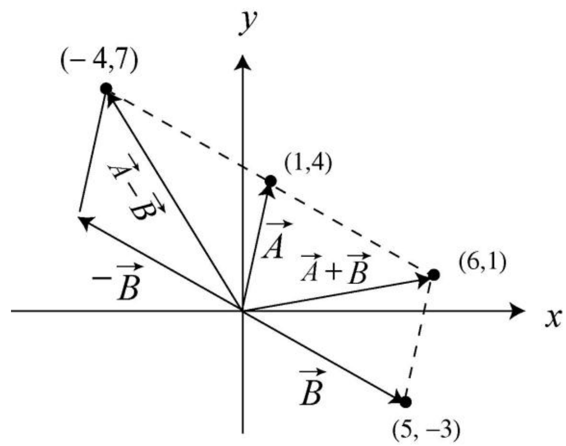
ANALYZE Adding the above equations and dividing by 2 leads to $\vec{A} = (1.0)\hat{\text{i}} + (4.0)\hat{\text{j}}$. The magnitude of $\vec{A}$ is

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$$

LEARN The vector $\vec{B}$ is $\vec{B} = (5.0)\hat{\text{i}} + (-3.0)\hat{\text{j}}$, and its magnitude is

$$B = |\vec{B}| = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.0)^2 + (-3.0)^2} = 5.8.$$

The results are summarized in the figure to the right.



58. The vector can be written as $\vec{d} = (2.5\text{ m})\hat{\text{j}}$, where we have taken $\hat{\text{j}}$ to be the unit vector pointing north.

(a) The magnitude of the vector $\vec{a} = 4.0\,\vec{d}$ is (4.0)(2.5 m) = 10 m.

(b) The direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ (north).

(c) The magnitude of the vector $\vec{c} = -3.0\vec{d}$ is (3.0)(2.5 m) = 7.5 m.

(d) The direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. Thus, the direction of $\vec{c}$ is south.

59. Reference to Figure 3-18 (and the accompanying material in that section) is helpful. If we convert $\vec{B}$ to the magnitude-angle notation (as $\vec{A}$ already is) we have $\vec{B} = (14.4 \angle 33.7°)$ (appropriate notation especially if we are using a vector capable calculator in polar mode). Where the length unit is not displayed in the solution, the unit meter should be understood. In the magnitude-angle notation, rotating the axis by +20° amounts to subtracting that angle from the angles previously specified. Thus, $\vec{A} = (12.0 \angle 40.0°)'$ and $\vec{B} = (14.4 \angle 13.7°)'$, where the 'prime' notation indicates that the description is in terms of the new coordinates. Converting these results to (*x*, *y*) representations, we obtain

(a) $\vec{A} = (9.19\text{ m})\,\hat{\text{i}}' + (7.71\text{ m})\,\hat{\text{j}}'$.

(b) Similarly, $\vec{B} = (14.0 \text{ m})\,\hat{\text{i}}' + (3.41 \text{ m})\,\hat{\text{j}}'$.

60. The two vectors can be found be solving the simultaneous equations.

(a) If we add the equations, we obtain $2\vec{a} = 6\vec{c}$, which leads to $\vec{a} = 3\vec{c} = 9\,\hat{\text{i}} + 12\,\hat{\text{j}}$.

(b) Plugging this result back in, we find $\vec{b} = \vec{c} = 3\hat{\text{i}} + 4\hat{\text{j}}$.

61. The three vectors given are

$$\begin{aligned} \vec{a} &= \ \ 5.0\ \hat{\text{i}} + 4.0\ \hat{\text{j}} - 6.0\ \hat{\text{k}} \\ \vec{b} &= -2.0\ \hat{\text{i}} + 2.0\ \hat{\text{j}} + 3.0\ \hat{\text{k}} \\ \vec{c} &= \ \ 4.0\ \hat{\text{i}} + 3.0\ \hat{\text{j}} + 2.0\ \hat{\text{k}} \end{aligned}$$

(a) The vector equation $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ is

$$\begin{aligned} \vec{r} &= [5.0 - (-2.0) + 4.0]\hat{\text{i}} + (4.0 - 2.0 + 3.0)\hat{\text{j}} + (-6.0 - 3.0 + 2.0)\hat{\text{k}} \\ &= 11\hat{\text{i}} + 5.0\hat{\text{j}} - 7.0\hat{\text{k}}. \end{aligned}$$

(b) We find the angle from +*z* by "dotting" (taking the scalar product) $\vec{r}$ with $\hat{\text{k}}$. Noting that

$$r = |\vec{r}\,| = \sqrt{(11.0)^2 + (5.0)^2 + (-7.0)^2} = 14,$$

Eq. 3-20 with Eq. 3-23 leads to

$$\vec{r} \cdot \vec{\text{k}} = -7.0 = (14)(1)\cos\phi \quad \Rightarrow \quad \phi = 120°.$$

(c) To find the component of a vector in a certain direction, it is efficient to "dot" it (take the scalar product of it) with a unit-vector in that direction. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}\,|} = \frac{-2.0\hat{\text{i}} + 2.0\hat{\text{j}} + 3.0\hat{\text{k}}}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(5.0)(-2.0) + (4.0)(2.0) + (-6.0)(3.0)}{\sqrt{(-2.0)^2 + (2.0)^2 + (3.0)^2}} = -4.9.$$

(d) One approach (if all we require is the magnitude) is to use the vector cross product, as the problem suggests; another (which supplies more information) is to subtract the result in part (c) (multiplied by $\hat{b}$) from $\vec{a}$. We briefly illustrate both methods. We note that if

a cos θ (where θ is the angle between $\vec{a}$ and $\vec{b}$) gives a_b (the component along $\hat{b}$) then we expect a sin θ to yield the orthogonal component:

$$a \sin\theta = \frac{|\vec{a} \times \vec{b}|}{b} = 7.3$$

(alternatively, one might compute θ form part (c) and proceed more directly). The second method proceeds as follows:

$$\begin{aligned}\vec{a} - a_b\,\hat{b} &= (5.0 - 2.35)\hat{\mathrm{i}} + (4.0 - (-2.35))\hat{\mathrm{j}} + ((-6.0) - (-3.53))\hat{\mathrm{k}} \\ &= 2.65\hat{\mathrm{i}} + 6.35\hat{\mathrm{j}} - 2.47\hat{\mathrm{k}}\end{aligned}$$

This describes the perpendicular part of $\vec{a}$ completely. To find the magnitude of this part, we compute

$$\sqrt{(2.65)^2 + (6.35)^2 + (-2.47)^2} = 7.3$$

which agrees with the first method.

62. We choose +x east and +y north and measure all angles in the "standard" way (positive ones counterclockwise from +x, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude d_1 = 3.66 (with the unit meter and three significant figures assumed) and direction θ_1 = 90°. Also, $\vec{d}_2$ has magnitude d_2 = 1.83 and direction θ_2 = –45°, and vector $\vec{d}_3$ has magnitude d_3 = 0.91 and direction θ_3 = –135°. We add the x and y components, respectively:

$$\begin{aligned}x&:\ d_1 \cos\theta_1 + d_2 \cos\theta_2 + d_3 \cos\theta_3 = 0.65 \text{ m} \\ y&:\ d_1 \sin\theta_1 + d_2 \sin\theta_2 + d_3 \sin\theta_3 = 1.7 \text{ m}.\end{aligned}$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1 + \vec{d}_2 + \vec{d}_3$) is $\sqrt{(0.65\text{ m})^2 + (1.7\text{ m})^2} = 1.8$ m.

(b) The angle (understood in the sense described above) is $\tan^{-1}$ (1.7/0.65) = 69°. That is, the first putt must aim in the direction 69° north of east.

63. The three vectors are

$$\begin{aligned}\vec{d}_1 &= -3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}} \\ \vec{d}_2 &= -2.0\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}} \\ \vec{d}_3 &= 2.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 1.0\,\hat{\mathrm{k}}.\end{aligned}$$

(a) Since $\vec{d}_2 + \vec{d}_3 = 0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}$, we have

$$\begin{aligned}\vec{d}_1 \cdot (\vec{d}_2 + \vec{d}_3) &= (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \cdot (0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}) \\ &= 0 - 3.0 + 6.0 = 3.0\ \mathrm{m}^2.\end{aligned}$$

(b) Using Eq. 3-27, we obtain $\vec{d}_2 \times \vec{d}_3 = -10\,\hat{\mathrm{i}} + 6.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}.$ Thus,

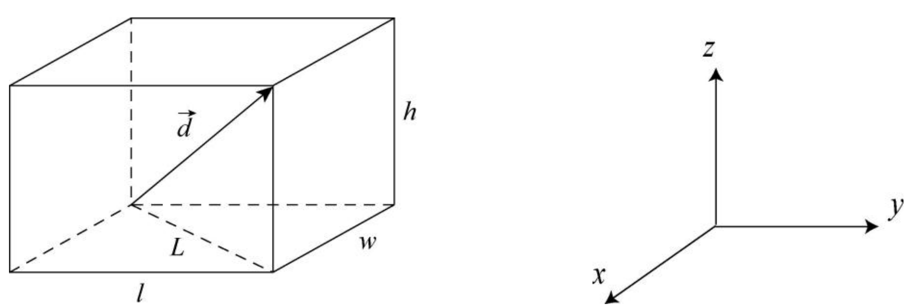
$$\begin{aligned}\vec{d}_1 \cdot (\vec{d}_2 \times \vec{d}_3) &= (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \cdot (-10\,\hat{\mathrm{i}} + 6.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \\ &= 30 + 18 + 4.0 = 52\ \mathrm{m}^3.\end{aligned}$$

(c) We found $\vec{d}_2 + \vec{d}_3$ in part (a). Use of Eq. 3-27 then leads to

$$\begin{aligned}\vec{d}_1 \times (\vec{d}_2 + \vec{d}_3) &= (-3.0\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}} + 2.0\,\hat{\mathrm{k}}) \times (0\,\hat{\mathrm{i}} - 1.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}) \\ &= (11\,\hat{\mathrm{i}} + 9.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}})\ \mathrm{m}^2\end{aligned}$$

64. **THINK** This problem deals with the displacement and distance traveled by a fly from one corner of a room to the diagonally opposite corner. The displacement vector is three-dimensional.

EXPRESS The displacement of the fly is illustrated in the figure below:



A coordinate system such as the one shown (above right) allows us to express the displacement as a three-dimensional vector.

ANALYZE (a) The magnitude of the displacement from one corner to the diagonally opposite corner is

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2}$$

Substituting the values given, we obtain

$$d = |\vec{d}| = \sqrt{w^2 + l^2 + h^2} = \sqrt{(3.70\ \mathrm{m})^2 + (4.30\ \mathrm{m})^2 + (3.00\ \mathrm{m})^2} = 6.42\ \mathrm{m}.$$

(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than *d*, the magnitude of the displacement.

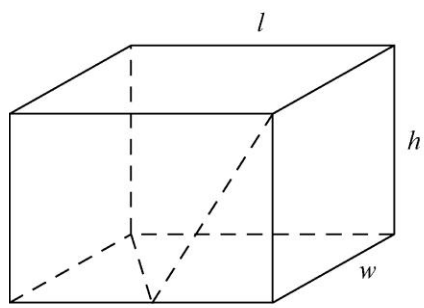
(c) The length of the path of the fly can be greater than *d*, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h = 11.0$ m.

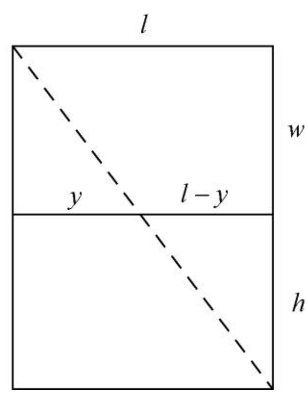
(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the *x* axis to be out of the page, the *y* axis to be to the right, and the *z* axis to be upward (as shown in the figure above). Then the *x* component of the displacement is w = 3.70 m, the *y* component of the displacement is 4.30 m, and the *z* component is 3.00 m. Thus, the displacement vector can be written as

$$\vec{d} = (3.70\text{ m})\hat{\text{i}} + (4.30\text{ m})\hat{\text{j}} + (3.00\text{ m})\hat{\text{k}}.$$

(f) Suppose the path of the fly is as shown by the dotted lines on the diagram (below left). Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown (above right).





The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$s_{\min} = \sqrt{(w+h)^2 + l^2} = \sqrt{(3.70\text{ m} + 3.00\text{ m})^2 + (4.30\text{ m})^2} = 7.96\text{ m}.$$

LEARN To show that the shortest path is indeed given by $s_{\min}$, we write the length of the path as

$$s = \sqrt{y^2 + w^2} + \sqrt{(l-y)^2 + h^2}\ .$$

The condition for minimum is given by

$$\frac{ds}{dy}=\frac{y}{\sqrt{y^2+w^2}}-\frac{l-y}{\sqrt{(l-y)^2+h^2}}=0\,.$$

A little algebra shows that the condition is satisfied when $y=lw/(w+h)$, which gives

$$s_{\min}=\sqrt{w^2\left(1+\frac{l^2}{(w+h)^2}\right)}+\sqrt{h^2\left(1+\frac{l^2}{(w+h)^2}\right)}=\sqrt{(w+h)^2+l^2}\,.$$

Any other path would be longer than 7.96 m.

65. (a) This is one example of an answer: $(-40\,\hat{\text{i}} - 20\,\hat{\text{j}} + 25\,\hat{\text{k}})$ m, with $\hat{\text{i}}$ directed anti-parallel to the first path, $\hat{\text{j}}$ directed anti-parallel to the second path, and $\hat{\text{k}}$ directed upward (in order to have a right-handed coordinate system). Other examples include $(40\,\hat{\text{i}} + 20\,\hat{\text{j}} + 25\,\hat{\text{k}})$ m and $(40\hat{\text{i}} - 20\,\hat{\text{j}} - 25\,\hat{\text{k}})$ m (with slightly different interpretations for the unit vectors). Note that the product of the components is positive in each example.

(b) Using the Pythagorean theorem, we have $\sqrt{(40\text{ m})^2+(20\text{ m})^2}$ = 44.7 m ≈ 45 m.

66. The vectors can be written as $\vec{a}=a\hat{\text{i}}$ and $\vec{b}=b\hat{\text{j}}$ where $a,\ b>0$.

(a) We are asked to consider

$$\frac{\vec{b}}{d}=\left(\frac{b}{d}\right)\hat{\text{j}}$$

in the case $d > 0$. Since the coefficient of $\hat{\text{j}}$ is positive, then the vector points in the $+y$ direction.

(b) If, however, $d < 0$, then the coefficient is negative and the vector points in the $-y$ direction.

(c) Since cos 90° = 0, then $\vec{a}\cdot\vec{b}=0$, using Eq. 3-20.

(d) Since $\vec{b}/d$ is along the y axis, then (by the same reasoning as in the previous part) $\vec{a}\cdot(\vec{b}/d)=0$.

(e) By the right-hand rule, $\vec{a}\times\vec{b}$ points in the $+z$-direction.

(f) By the same rule, $\vec{b}\times\vec{a}$ points in the $-z$-direction. We note that $\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}$ is true in this case and quite generally.

(g) Since sin 90° = 1, Eq. 3-24 gives $|\vec{a} \times \vec{b}| = ab$ where a is the magnitude of $\vec{a}$.

(h) Also, $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}| = ab$.

(i) With $d > 0$, we find that $\vec{a} \times (\vec{b}/d)$ has magnitude ab/d.

(j) The vector $\vec{a} \times (\vec{b}/d)$ points in the +z direction.

67. We note that the set of choices for unit vector directions has correct orientation (for a right-handed coordinate system). Students sometimes confuse "north" with "up", so it might be necessary to emphasize that these are being treated as the mutually perpendicular directions of our real world, not just some "on the paper" or "on the blackboard" representation of it. Once the terminology is clear, these questions are basic to the definitions of the scalar (dot) and vector (cross) products.

(a) $\hat{\mathrm{i}} \cdot \hat{\mathrm{k}} = 0$ since $\hat{\mathrm{i}} \perp \hat{\mathrm{k}}$

(b) $(-\hat{\mathrm{k}}) \cdot (-\hat{\mathrm{j}}) = 0$ since $\hat{\mathrm{k}} \perp \hat{\mathrm{j}}$.

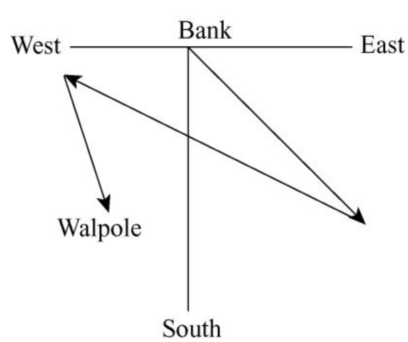
(c) $\hat{\mathrm{j}} \cdot (-\hat{\mathrm{j}}) = -1$.

(d) $\hat{\mathrm{k}} \times \hat{\mathrm{j}} = -\hat{\mathrm{i}}$ (west).

(e) $(-\hat{\mathrm{i}}) \times (-\hat{\mathrm{j}}) = +\hat{\mathrm{k}}$ (upward).

(f) $(-\hat{\mathrm{k}}) \times (-\hat{\mathrm{j}}) = -\hat{\mathrm{i}}$ (west).

68. A sketch of the displacements is shown. The resultant (not shown) would be a straight line from start (Bank) to finish (Walpole). With a careful drawing, one should find that the resultant vector has length 29.5 km at 35° west of south.



69. The point P is displaced vertically by $2R$, where R is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or πR. Since R = 0.450 m,

the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the *x* axis is horizontal and the *y* axis is vertical, the vector displacement (in meters) is $\vec{r} = \left(1.414\ \hat{\mathrm{i}} + 0.900\ \hat{\mathrm{j}}\right)$. The displacement has a magnitude of

$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68\,\mathrm{m}$$

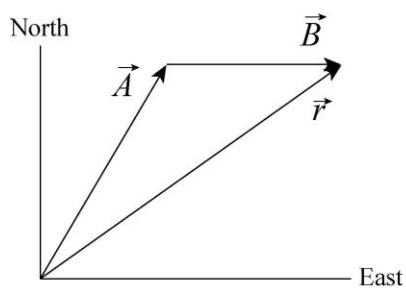
and an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. In physics there are no "exact" measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

70. The diagram shows the displacement vectors for the two segments of her walk, labeled $\vec{A}$ and $\vec{B}$, and the total ("final") displacement vector, labeled $\vec{r}$. We take east to be the +*x* direction and north to be the +*y* direction. We observe that the angle between $\vec{A}$ and the *x* axis is 60°. Where the units are not explicitly shown, the distances are understood to be in meters. Thus, the components of $\vec{A}$ are A_x = 250 cos60° = 125 and A_y = 250 sin60° = 216.5. The components of $\vec{B}$ are B_x = 175 and B_y = 0. The components of the total displacement are

$$r_x = A_x + B_x = 125 + 175 = 300$$
$$r_y = A_y + B_y = 216.5 + 0 = 216.5.$$



(a) The magnitude of the resultant displacement is

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(300\text{ m})^2 + (216.5\text{ m})^2} = 370\,\mathrm{m}.$$

(b) The angle the resultant displacement makes with the +*x* axis is

$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{216.5\text{ m}}{300\text{ m}}\right) = 36°.$$

The direction is $36°$ north of due east.

(c) The total *distance* walked is d = 250 m + 175 m = 425 m.

(d) The total distance walked is greater than the magnitude of the resultant displacement. The diagram shows why: $\vec{A}$ and $\vec{B}$ are not collinear.

71. The vector $\vec{d}$ (measured in meters) can be represented as $\vec{d} = (3.0\text{ m})(-\hat{\text{j}})$, where $-\hat{\text{j}}$ is the unit vector pointing south. Therefore, $5.0\vec{d} = 5.0(-3.0\text{ m }\hat{\text{j}}) = (-15\text{ m})\,\hat{\text{j}}$.

(a) The positive scalar factor (5.0) affects the magnitude but not the direction. The magnitude of $5.0\vec{d}$ is 15 m.

(b) The new direction of $5\vec{d}$ is the same as the old: south.

The vector $-2.0\vec{d}$ can be written as $-2.0\vec{d} = (6.0\text{ m})\,\hat{\text{j}}$.

(c) The absolute value of the scalar factor (|–2.0| = 2.0) affects the magnitude. The new magnitude is 6.0 m.

(d) The minus sign carried by this scalar factor reverses the direction, so the new direction is $+\hat{\text{j}}$, or north.

72. The ant's trip consists of three displacements:

$$\vec{d}_1 = (0.40\text{ m})(\cos 225°\,\hat{\text{i}} + \sin 225°\,\hat{\text{j}}) = (-0.28\text{ m})\,\hat{\text{i}} + (-0.28\text{ m})\,\hat{\text{j}}$$
$$\vec{d}_2 = (0.50\text{ m})\,\hat{\text{i}}$$
$$\vec{d}_3 = (0.60\text{ m})(\cos 60°\,\hat{\text{i}} + \sin 60°\,\hat{\text{j}}) = (0.30\text{ m})\,\hat{\text{i}} + (0.52\text{ m})\,\hat{\text{j}},$$

where the angle is measured with respect to the positive x axis. We have taken the positive x and y directions to correspond to east and north, respectively.

(a) The x component of $\vec{d}_1$ is $d_{1x} = (0.40\text{ m})\cos 225° = -0.28\text{ m}$.

(b) The y component of $\vec{d}_1$ is $d_{1y} = (0.40\text{ m})\sin 225° = -0.28\text{ m}$.

(c) The x component of $\vec{d}_2$ is $d_{2x} = 0.50\text{ m}$.

(d) The y component of $\vec{d}_2$ is $d_{2y} = 0\text{ m}$.

(e) The *x* component of $\vec{d}_3$ is $d_{3x} = (0.60 \text{ m})\cos 60° = 0.30 \text{ m}$.

(f) The *y* component of $\vec{d}_3$ is $d_{3y} = (0.60 \text{ m})\sin 60° = 0.52 \text{ m}$.

(g) The *x* component of the net displacement $\vec{d}_{\text{net}}$ is

$$d_{\text{net},x} = d_{1x} + d_{2x} + d_{3x} = (-0.28 \text{ m}) + (0.50 \text{ m}) + (0.30 \text{ m}) = 0.52 \text{ m}.$$

(h) The *y* component of the net displacement $\vec{d}_{\text{net}}$ is

$$d_{\text{net},y} = d_{1y} + d_{2y} + d_{3y} = (-0.28 \text{ m}) + (0 \text{ m}) + (0.52 \text{ m}) = 0.24 \text{ m}.$$

(i) The magnitude of the net displacement is

$$d_{\text{net}} = \sqrt{d_{\text{net},x}^2 + d_{\text{net},y}^2} = \sqrt{(0.52 \text{ m})^2 + (0.24 \text{ m})^2} = 0.57 \text{ m}.$$

(j) The direction of the net displacement is

$$\theta = \tan^{-1}\left(\frac{d_{\text{net},y}}{d_{\text{net},x}}\right) = \tan^{-1}\left(\frac{0.24 \text{ m}}{0.52 \text{ m}}\right) = 25° \text{ (north of east)}$$

If the ant has to return directly to the starting point, the displacement would be $-\vec{d}_{\text{net}}$.
(k) The distance the ant has to travel is $|-\vec{d}_{\text{net}}| = 0.57$ m.

(l) The direction the ant has to travel is $25°$ (south of west).

73. We apply Eq. 3-23 and Eq. 3-27.

(a) $\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\,\hat{k}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any *z* components. Consequently, we obtain $((3.0)(4.0) - (5.0)(2.0))\hat{k} = 2.0\hat{k}$.

(b) $\vec{a} \cdot b = a_x b_x + a_y b_y$ yields (3.0)(2.0) + (5.0)(4.0) = 26.

(c) $\vec{a} + \vec{b} = (3.0 + 2.0)\,\hat{i} + (5.0 + 4.0)\,\hat{j} \Rightarrow (\vec{a} + \vec{b}) \cdot \vec{b} = (5.0)(2.0) + (9.0)(4.0) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2.0\,\hat{\text{i}} + 4.0\,\hat{\text{j}}}{\sqrt{(2.0)^2 + (4.0)^2}}.$$

We therefore obtain

$$a_b = \vec{a} \cdot \hat{b} = \frac{(3.0)(2.0) + (5.0)(4.0)}{\sqrt{(2.0)^2 + (4.0)^2}} = 5.81.$$

74. The two vectors $\vec{a}$ and $\vec{b}$ are given by

$$\vec{a} = 3.20(\cos 63°\,\hat{\text{j}} + \sin 63°\,\hat{\text{k}}) = 1.45\,\hat{\text{j}} + 2.85\,\hat{\text{k}}$$

$$\vec{b} = 1.40(\cos 48°\,\hat{\text{i}} + \sin 48°\,\hat{\text{k}}) = 0.937\hat{\text{i}} + 1.04\,\hat{\text{k}}$$

The components of $\vec{a}$ are $a_x = 0$, $a_y = 3.20 \cos 63° = 1.45$, and $a_z = 3.20 \sin 63° = 2.85$. The components of $\vec{b}$ are $b_x = 1.40 \cos 48° = 0.937$, $b_y = 0$, and $b_z = 1.40 \sin 48° = 1.04$.

(a) The scalar (dot) product is therefore

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = (0)(0.937) + (1.45)(0) + (2.85)(1.04) = 2.97.$$

(b) The vector (cross) product is

$$\begin{aligned}\vec{a} \times \vec{b} &= \left(a_y b_z - a_z b_y\right)\hat{\text{i}} + \left(a_z b_x - a_x b_z\right)\hat{\text{j}} + \left(a_x b_y - a_y b_x\right)\hat{\text{k}} \\ &= \left((1.45)(1.04) - 0\right)\hat{\text{i}} + \left((2.85)(0.937) - 0\right)\hat{\text{j}} + \left(0 - (1.45)(0.937)\right)\hat{\text{k}} \\ &= 1.51\hat{\text{i}} + 2.67\,\hat{\text{j}} - 1.36\hat{\text{k}}.\end{aligned}$$

(c) The angle θ between $\vec{a}$ and $\vec{b}$ is given by

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{ab}\right) = \cos^{-1}\left(\frac{2.97}{(3.20)(1.40)}\right) = 48.5°.$$

75. We orient $\hat{\text{i}}$ eastward, $\hat{\text{j}}$ northward, and $\hat{\text{k}}$ upward, and use the following fundamental products:

$$\begin{aligned}\hat{\text{i}} \times \hat{\text{j}} &= -\hat{\text{j}} \times \hat{\text{i}} = \hat{\text{k}} \\ \hat{\text{j}} \times \hat{\text{k}} &= -\hat{\text{k}} \times \hat{\text{j}} = \hat{\text{i}} \\ \hat{\text{k}} \times \hat{\text{i}} &= -\hat{\text{i}} \times \hat{\text{k}} = \hat{\text{j}}\end{aligned}$$

(a) “north cross west” = $\hat{\text{j}} \times (-\hat{\text{i}}) = \hat{\text{k}}$ = “up.”

(b) "down dot south" = $(-\hat{k})\cdot(-\hat{j}) = 0$.

(c) "east cross up" = $\hat{i}\times(\hat{k}) = -\hat{j}$ = "south."

(d) "west dot west" = $(-\hat{i})\cdot(-\hat{i}) = 1$.

(e) "south cross south" = $(-\hat{j})\times(-\hat{j}) = 0$.

76. Let A denote the magnitude of $\vec{A}$; similarly for the other vectors. The vector equation is $\vec{A} + \vec{B} = \vec{C}$ where B = 8.0 m and $C = 2A$. We are also told that the angle (measured in the 'standard' sense) for $\vec{A}$ is 0° and the angle for $\vec{C}$ is 90°, which makes this a right triangle (when drawn in a "head-to-tail" fashion) where B is the size of the hypotenuse. Using the Pythagorean theorem,

$$B = \sqrt{A^2 + C^2} \Rightarrow 8.0 = \sqrt{A^2 + 4A^2}$$

which leads to $A = 8/\sqrt{5} = 3.6$ m.

77. We orient $\hat{i}$ eastward, $\hat{j}$ northward, and $\hat{k}$ upward.

(a) The displacement is $\vec{d} = (1300\text{ m})\hat{i} + (2200\text{ m})\hat{j} + (-410\text{ m})\hat{k}$.

(b) The displacement for the return portion is $\vec{d}' = -(1300\text{ m})\hat{i} - (2200\text{ m})\hat{j}$ and the magnitude is $d' = \sqrt{(-1300\text{ m})^2 + (-2200\text{ m})^2} = 2.56\times10^3\,\text{m}$.

The net displacement is zero since his final position matches his initial position.

78. Let $\vec{c} = \vec{b} \times \vec{a}$. Then the magnitude of $\vec{c}$ is $c = ab \sin \phi$. Since $\vec{c}$ is perpendicular to $\vec{a}$ the magnitude of $\vec{a} \times \vec{c}$ is ac. The magnitude of $\vec{a}\times(\vec{b}\times\vec{a})$ is consequently

$$|\vec{a}\times(\vec{b}\times\vec{a})| = ac = a^2 b \sin\phi .$$

Substituting the values given, we obtain

$$|\vec{a}\times(\vec{b}\times\vec{a})| = a^2 b \sin\phi = (3.90)^2(2.70)\sin 63.0° = 36.6 .$$

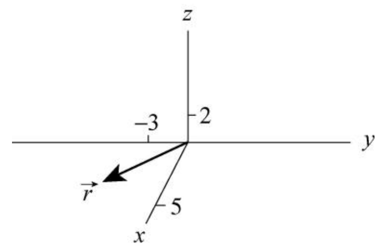
79. The area of a triangle is half the product of its base and altitude. The base is the side formed by vector $\vec{a}$. Then the altitude is $b \sin \phi$ and the area is $A = \frac{1}{2}ab\sin\phi = \frac{1}{2}|\vec{a}\times\vec{b}|$. Substituting the values given, we have

$$A = \frac{1}{2}ab\sin\phi = \frac{1}{2}(4.3)(5.4)\sin 46° \approx 8.4 .$$

Chapter 4

1. (a) The magnitude of $\vec{r}$ is

$$|\vec{r}| = \sqrt{(5.0\text{ m})^2 + (-3.0\text{ m})^2 + (2.0\text{ m})^2} = 6.2\text{ m}.$$



(b) A sketch is shown. The coordinate values are in meters.

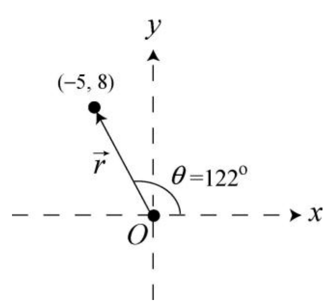
2. (a) The position vector, according to Eq. 4-1, is $\vec{r} = (-5.0\text{ m})\,\hat{\text{i}} + (8.0\text{ m})\hat{\text{j}}$.

(b) The magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-5.0\text{ m})^2 + (8.0\text{ m})^2 + (0\text{ m})^2} = 9.4\text{ m}.$

(c) Many calculators have polar ↔ rectangular conversion capabilities that make this computation more efficient than what is shown below. Noting that the vector lies in the *xy* plane and using Eq. 3-6, we obtain:

$$\theta = \tan^{-1}\left(\frac{8.0\text{ m}}{-5.0\text{ m}}\right) = -58° \text{ or } 122°$$

where the latter possibility (122° measured counterclockwise from the +*x* direction) is chosen since the signs of the components imply the vector is in the second quadrant.



(d) The sketch is shown to the right. The vector is 122° counterclockwise from the +*x* direction.

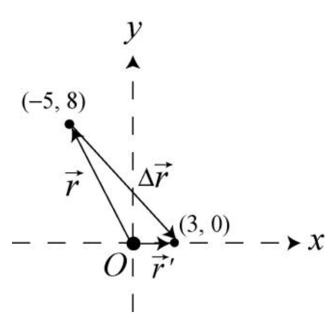
(e) The displacement is $\Delta\vec{r} = \vec{r}' - \vec{r}$ where $\vec{r}$ is given in part (a) and $\vec{r}' = (3.0\text{ m})\hat{\text{i}}$. Therefore, $\Delta\vec{r} = (8.0\text{ m})\hat{\text{i}} - (8.0\text{ m})\hat{\text{j}}$.

(f) The magnitude of the displacement is

$$|\Delta\vec{r}| = \sqrt{(8.0\text{ m})^2 + (-8.0\text{ m})^2} = 11\text{ m}.$$

(g) The angle for the displacement, using Eq. 3-6, is

$$\tan^{-1}\left(\frac{8.0\text{ m}}{-8.0\text{ m}}\right) = -45° \text{ or } 135°$$



where we choose the former possibility (–45°, or 45° measured *clockwise* from +*x*) since the signs of the components imply the vector is in the fourth quadrant. A sketch of $\Delta\vec{r}$ is shown on the right.

3. The initial position vector $\vec{r}_o$ satisfies $\vec{r}-\vec{r}_o=\Delta\vec{r}$, which results in

$$\vec{r}_o=\vec{r}-\Delta\vec{r}=(3.0\hat{\mathrm{j}}-4.0\hat{\mathrm{k}})\mathrm{m}-(2.0\hat{\mathrm{i}}-3.0\hat{\mathrm{j}}+6.0\hat{\mathrm{k}})\mathrm{m}=(-2.0\text{ m})\,\hat{\mathrm{i}}+(6.0\text{ m})\,\hat{\mathrm{j}}+(-10\text{ m})\,\hat{\mathrm{k}}.$$

4. We choose a coordinate system with origin at the clock center and +*x* rightward (toward the "3:00" position) and +*y* upward (toward "12:00").

(a) In unit-vector notation, we have $\vec{r}_1=(10\text{ cm})\hat{\mathrm{i}}$ and $\vec{r}_2=(-10\text{ cm})\hat{\mathrm{j}}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r}=\vec{r}_2-\vec{r}_1=(-10\text{ cm})\hat{\mathrm{i}}+(-10\text{ cm})\hat{\mathrm{j}}.$$

The magnitude is given by $|\Delta\vec{r}|=\sqrt{(-10\text{ cm})^2+(-10\text{ cm})^2}=14\text{ cm}.$

(b) Using Eq. 3-6, the angle is

$$\theta=\tan^{-1}\left(\frac{-10\text{ cm}}{-10\text{ cm}}\right)=45°\text{ or }-135°.$$

We choose $-135°$ since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write

$$\Delta\vec{r}=\vec{r}_2-\vec{r}_1=(-10\text{ cm})\hat{\mathrm{i}}+(-10\text{ cm})\hat{\mathrm{j}}\;\to(14\text{cm}\angle-135°).$$

(c) In this case, we have $\vec{r}_1=(-10\text{ cm})\hat{\mathrm{j}}$ and $\vec{r}_2=(10\text{ cm})\hat{\mathrm{j}}$, and $\Delta\vec{r}=(20\text{ cm})\hat{\mathrm{j}}$. Thus, $|\Delta\vec{r}|=20\text{ cm}.$

(d) Using Eq. 3-6, the angle is given by

$$\theta=\tan^{-1}\left(\frac{20\text{ cm}}{0\text{ cm}}\right)=90°.$$

(e) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

(f) The corresponding angle for a full-hour sweep is also zero.

5. **THINK** This problem deals with the motion of a train in two dimensions. The entire trip consists of three parts, and we're interested in the overall average velocity.

EXPRESS The average velocity of the entire trip is given by Eq. 4-8, $\vec{v}_{\text{avg}} = \Delta\vec{r}/\Delta t$, where the total displacement $\Delta\vec{r} = \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3$ is the sum of three displacements (each result of a constant velocity during a given time), and $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ is the total amount of time for the trip. We use a coordinate system with +x for East and +y for North.

ANALYZE (a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{40.0\ \text{min}}{60\ \text{min/h}}\right)\hat{\text{i}} = (40.0\ \text{km})\hat{\text{i}}.$$

The second displacement has a magnitude of $(60.0\ \frac{\text{km}}{\text{h}})\cdot(\frac{20.0\ \text{min}}{60\ \text{min/h}}) = 20.0\ \text{km}$, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0\ \text{km})\cos(40.0°)\,\hat{\text{i}} + (20.0\ \text{km})\sin(40.0°)\,\hat{\text{j}} = (15.3\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}.$$

Similarly, the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{50.0\ \text{min}}{60\ \text{min/h}}\right)\hat{\text{i}} = (-50.0\ \text{km})\,\hat{\text{i}}.$$

Thus, the total displacement is

$$\begin{aligned}\Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0\ \text{km})\hat{\text{i}} + (15.3\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}} - (50.0\ \text{km})\,\hat{\text{i}} \\ &= (5.30\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}.\end{aligned}$$

The time for the trip is $\Delta t = (40.0 + 20.0 + 50.0)$ min = 110 min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \frac{(5.30\ \text{km})\,\hat{\text{i}} + (12.9\ \text{km})\,\hat{\text{j}}}{1.83\ \text{h}} = (2.90\ \text{km/h})\,\hat{\text{i}} + (7.01\ \text{km/h})\,\hat{\text{j}}.$$

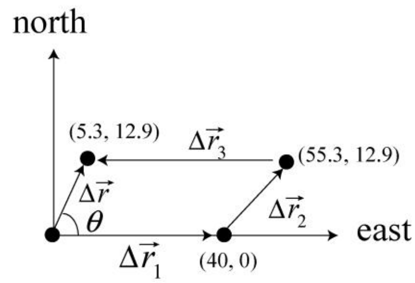
The magnitude of $\vec{v}_{\text{avg}}$ is $|\vec{v}_{\text{avg}}| = \sqrt{(2.90\ \text{km/h})^2 + (7.01\ \text{km/h})^2} = 7.59\ \text{km/h}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{v_{\text{avg},y}}{v_{\text{avg},x}}\right) = \tan^{-1}\left(\frac{7.01\ \text{km/h}}{2.90\ \text{km/h}}\right) = 67.5°\ \ \text{(north of east)},$$

or $22.5°$ east of due north.

LEARN The displacement of the train is depicted in the figure below:



Note that the net displacement $\Delta\vec{r}$ is found by adding $\Delta\vec{r}_1$, $\Delta\vec{r}_2$ and $\Delta\vec{r}_3$ vectorially.

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt.

(a) Equation 4-10 leads to

$$v(t) = \frac{d}{dt}\,(3.00t\hat{\mathrm{i}} - 4.00t^2\hat{\mathrm{j}} + 2.00\hat{\mathrm{k}}) = (3.00 \text{ m/s})\hat{\mathrm{i}} - (8.00 \text{ m/s}^2)t\,\hat{\mathrm{j}}$$

(b) Evaluating this result at t = 2.00 s produces $\vec{v} = (3.00\hat{\mathrm{i}} - 16.0\hat{\mathrm{j}})$ m/s.

(c) The speed at t = 2.00 s is $v = |\vec{v}| = \sqrt{(3.00 \text{ m/s})^2 + (-16.0 \text{ m/s})^2} = 16.3$ m/s.

(d) The angle of $\vec{v}$ at that moment is

$$\tan^{-1}\left(\frac{-16.0 \text{ m/s}}{3.00 \text{ m/s}}\right) = -79.4° \text{ or } 101°$$

where we choose the first possibility (79.4° measured *clockwise* from the +x direction, or 281° counterclockwise from +x) since the signs of the components imply the vector is in the fourth quadrant.

7. Using Eq. 4-3 and Eq. 4-8, we have

$$\vec{v}_{\text{avg}} = \frac{(-2.0\hat{\mathrm{i}} + 8.0\hat{\mathrm{j}} - 2.0\hat{\mathrm{k}}) \text{ m} - (5.0\hat{\mathrm{i}} - 6.0\hat{\mathrm{j}} + 2.0\hat{\mathrm{k}}) \text{ m}}{10 \text{ s}} = (-0.70\hat{\mathrm{i}} + 1.40\hat{\mathrm{j}} - 0.40\hat{\mathrm{k}}) \text{ m/s}.$$

8. Our coordinate system has $\hat{\mathrm{i}}$ pointed east and $\hat{\mathrm{j}}$ pointed north. The first displacement is $\vec{r}_{AB} = (483 \text{ km})\hat{\mathrm{i}}$ and the second is $\vec{r}_{BC} = (-966 \text{ km})\hat{\mathrm{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483\text{ km})\hat{\mathrm{i}} - (966\text{ km})\hat{\mathrm{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483\text{ km})^2 + (-966\text{ km})^2} = 1.08\times10^3\text{ km}.$

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{-966\text{ km}}{483\text{ km}}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483\text{ km})\hat{\mathrm{i}} - (966\text{ km})\hat{\mathrm{j}}}{2.25\text{ h}} = (215\text{ km/h})\hat{\mathrm{i}} - (429\text{ km/h})\hat{\mathrm{j}}$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(215\text{ km/h})^2 + (-429\text{ km/h})^2} = 480\text{ km/h}.$

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (480\text{ km/h} \angle -63.4°).$

(e) Assuming the *AB* trip was a straight one, and similarly for the *BC* trip, then $|\vec{r}_{AB}|$ is the distance traveled during the *AB* trip, and $|\vec{r}_{BC}|$ is the distance traveled during the *BC* trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483\text{ km} + 966\text{ km}}{2.25\text{ h}} = 644\text{ km/h}.$$

9. The (x,y) coordinates (in meters) of the points are A = (15, –15), B = (30, –45), C = (20, –15), and D = (45, 45). The respective times are t_A = 0, t_B = 300 s, t_C = 600 s, and t_D = 900 s. Average velocity is defined by Eq. 4-8. Each displacement $\Delta\vec{r}$ is understood to originate at point A.

(a) The average velocity having the least magnitude (5.0 m/600 s) is for the displacement ending at point C: $|\vec{v}_{\text{avg}}| = 0.0083$ m/s.

(b) The direction of $\vec{v}_{\text{avg}}$ is 0° (measured counterclockwise from the +x axis).

(c) The average velocity having the greatest magnitude ($\sqrt{(15\text{ m})^2 + (30\text{ m})^2}/300\text{ s}$) is for the displacement ending at point *B*: $|\vec{v}_{avg}| = 0.11$ m/s.

(d) The direction of $\vec{v}_{avg}$ is 297° (counterclockwise from +*x*) or −63° (which is equivalent to measuring 63° *clockwise* from the +*x* axis).

10. We differentiate $\vec{r} = 5.00t\,\hat{\text{i}} + (et + ft^2)\hat{\text{j}}$.

(a) The particle's motion is indicated by the derivative of $\vec{r}: \vec{v} = 5.00\,\hat{\text{i}} + (e + 2ft)\,\hat{\text{j}}$. The angle of its direction of motion is consequently

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(e + 2ft)/5.00].$$

The graph indicates $\theta_o = 35.0°$, which determines the parameter *e*:

$$e = (5.00\text{ m/s})\tan(35.0°) = 3.50\text{ m/s}.$$

(b) We note (from the graph) that $\theta = 0$ when $t = 14.0$ s. Thus, $e + 2ft = 0$ at that time. This determines the parameter *f* :

$$f = \frac{-e}{2t} = \frac{-3.5\text{ m/s}}{2(14.0\text{ s})} = -0.125\text{ m/s}^2.$$

11. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain

$$\vec{r}\Big|_{t=2.00} = [2.00(8) - 5.00(2)]\hat{\text{i}} + [6.00 - 7.00(16)]\hat{\text{j}} = (6.00\hat{\text{i}} - 106\hat{\text{j}})\text{ m}$$

(b) Taking the derivative of the given expression produces

$$\vec{v}(t) = (6.00t^2 - 5.00)\,\hat{\text{i}} - 28.0t^3\,\hat{\text{j}}$$

where we have written *v*(*t*) to emphasize its dependence on time. This becomes, at $t = 2.00$ s, $\vec{v} = (19.0\hat{\text{i}} - 224\hat{\text{j}})$ m/s.

(c) Differentiating the $\vec{v}(t)$ found above, with respect to *t* produces $12.0t\,\hat{\text{i}} - 84.0t^2\,\hat{\text{j}}$, which yields $\vec{a} = (24.0\hat{\text{i}} - 336\hat{\text{j}})\text{ m/s}^2$ at $t = 2.00$ s.

(d) The angle of $\vec{v}$, measured from +*x*, is either

$$\tan^{-1}\left(\frac{-224 \text{ m/s}}{19.0 \text{ m/s}}\right) = -85.2° \text{ or } 94.8°$$

where we settle on the first choice (–85.2°, which is equivalent to 275° measured counterclockwise from the +*x* axis) since the signs of its components imply that it is in the fourth quadrant.

12. We adopt a coordinate system with $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north; the coordinate origin is the flagpole. We "translate" the given information into unit-vector notation as follows:

$$\vec{r}_\text{o} = (40.0 \text{ m})\hat{\text{i}} \quad \text{and} \quad \vec{v}_\text{o} = (-10.0 \text{ m/s})\hat{\text{j}}$$
$$\vec{r} = (40.0 \text{ m})\hat{\text{j}} \quad \text{and} \quad \vec{v} = (10.0 \text{ m/s})\hat{\text{i}}.$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r} - \vec{r}_\text{o} = (-40.0 \text{ m})\hat{\text{i}} + (40.0 \text{ m})\hat{\text{j}}$$

with a magnitude $|\Delta\vec{r}| = \sqrt{(-40.0 \text{ m})^2 + (40.0 \text{ m})^2} = 56.6 \text{ m}.$

(b) The direction of $\Delta\vec{r}$ is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40.0 \text{ m}}{-40.0 \text{ m}}\right) = -45.0° \text{ or } 135°.$$

Since the desired angle is in the second quadrant, we pick $135°$ ($45°$ north of due west). Note that the displacement can be written as $\Delta\vec{r} = \vec{r} - \vec{r}_\text{o} = (56.6\angle 135°)$ in terms of the magnitude-angle notation.

(c) The magnitude of $\vec{v}_\text{avg}$ is simply the magnitude of the displacement divided by the time (Δt = 30.0 s). Thus, the average velocity has magnitude (56.6 m)/(30.0 s) = 1.89 m/s.

(d) Equation 4-8 shows that $\vec{v}_\text{avg}$ points in the same direction as $\Delta\vec{r}$, that is, $135°$ ($45°$ north of due west).

(e) Using Eq. 4-15, we have

$$\vec{a}_\text{avg} = \frac{\vec{v} - \vec{v}_\text{o}}{\Delta t} = (0.333 \text{ m/s}^2)\hat{\text{i}} + (0.333 \text{ m/s}^2)\hat{\text{j}}.$$

The magnitude of the average acceleration vector is therefore equal to $|\vec{a}_\text{avg}| = \sqrt{(0.333 \text{ m/s}^2)^2 + (0.333 \text{ m/s}^2)^2} = 0.471 \text{ m/s}^2$.

(f) The direction of $\vec{a}_{\text{avg}}$ is

$$\theta = \tan^{-1}\left(\frac{0.333 \text{ m/s}^2}{0.333 \text{ m/s}^2}\right) = 45° \text{ or } -135°.$$

Since the desired angle is now in the first quadrant, we choose $45°$, and $\vec{a}_{\text{avg}}$ points north of due east.

13. **THINK** Knowing the position of a particle as function of time allows us to calculate its corresponding velocity and acceleration by taking time derivatives.

EXPRESS From the position vector $\vec{r}(t)$, the velocity and acceleration of the particle can be found by differentiating $\vec{r}(t)$ with respect to time:

$$\vec{v} = \frac{d\vec{r}}{dt}, \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}.$$

ANALYZE (a) Taking the derivative of the position vector $\vec{r}(t) = \hat{\text{i}} + (4t^2)\hat{\text{j}} + t\hat{\text{k}}$ with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{\text{i}} + 4t^2\,\hat{\text{j}} + t\,\hat{\text{k}}) = 8t\,\hat{\text{j}} + \hat{\text{k}}.$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = \frac{d}{dt}(8t\,\hat{\text{j}} + \hat{\text{k}}) = 8\,\hat{\text{j}}.$$

LEARN The particle undergoes constant acceleration in the +*y*-direction. This can be seen by noting that the *y* component of $\vec{r}(t)$ is $4t^2$, which is quadratic in *t*.

14. We use Eq. 4-15 with $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

(a) The average acceleration during the $\Delta t = 4$ s interval is

$$\vec{a}_{\text{avg}} = \frac{(-2.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}} + 5.0\,\hat{\text{k}})\text{ m/s} - (4.0\,\hat{\text{i}} - 22\,\hat{\text{j}} + 3.0\,\hat{\text{k}})\text{ m/s}}{4\text{ s}} = (-1.5\text{ m/s}^2)\,\hat{\text{i}} + (0.5\,\text{m/s}^2)\,\hat{\text{k}}.$$

(b) The magnitude of $\vec{a}_{\text{avg}}$ is $\sqrt{(-1.5\text{ m/s}^2)^2 + (0.5\text{ m/s}^2)^2} = 1.6\,\text{m/s}^2$.

(c) Its angle in the *xz* plane (measured from the +*x* axis) is one of these possibilities:

$$\tan^{-1}\left(\frac{0.5\text{ m/s}^2}{-1.5\text{ m/s}^2}\right) = -18° \text{ or } 162°$$

where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

15. **THINK** Given the initial velocity and acceleration of a particle, we're interested in finding its velocity and position at a later time.

EXPRESS Since the acceleration, $\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}} = (-1.0\text{ m/s}^2)\hat{\text{i}} + (-0.50\text{ m/s}^2)\hat{\text{j}}$, is constant in both x and y directions, we may use Table 2-1 for the motion along each direction. This can be handled individually (for x and y) or together with the unit-vector notation (for $\Delta\vec{r}$).

Since the particle started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. Along the x-direction, we have

$$x(t) = v_{0x}t + \frac{1}{2}a_x t^2, \qquad v_x(t) = v_{0x} + a_x t$$

Similarly, along the y-direction, we get

$$y(t) = v_{0y}t + \frac{1}{2}a_y t^2, \qquad v_y(t) = v_{0y} + a_y t\ .$$

Known: $v_{0x} = 3.0\text{ m/s}, v_{0y} = 0,\ a_x = -1.0\text{ m/s}^2,\ a_y = -0.5\text{ m/s}^2$.

ANALYZE (a) Substituting the values given, the components of the velocity are

$$\begin{aligned} v_x(t) &= v_{0x} + a_x t = (3.0\text{ m/s}) - (1.0\text{ m/s}^2)t \\ v_y(t) &= v_{0y} + a_y t = -(0.50\text{ m/s}^2)t \end{aligned}$$

When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.0 - 1.0t_m = 0$ or $t_m = 3.0$ s. The y component of the velocity at this time is

$$v_y(t = 3.0\text{ s}) = -(0.50\text{ m/s}^2)(3.0) = -1.5\text{ m/s}$$

Thus, $\vec{v}_m = (-1.5\text{ m/s})\hat{\text{j}}$.

(b) At $t = 3.0$ s , the components of the position are

$$x(t = 3.0\text{ s}) = v_{0x}t + \frac{1}{2}a_x t^2 = (3.0\text{ m/s})(3.0\text{ s}) + \frac{1}{2}(-1.0\text{ m/s}^2)(3.0\text{ s})^2 = 4.5\text{ m}$$
$$y(t = 3.0\text{ s}) = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-0.5\text{ m/s}^2)(3.0\text{ s})^2 = -2.25\text{ m}$$

Using unit-vector notation, the results can be written as $\vec{r}_m = (4.50\text{ m})\hat{\text{i}} - (2.25\text{ m})\hat{\text{j}}$.

LEARN The motion of the particle in this problem is two-dimensional, and the kinematics in the *x*- and *y*-directions can be analyzed separately.

16. We make use of Eq. 4-16.

(a) The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\left(6.0t - 4.0t^2\right)\hat{\text{i}} + 8.0\,\hat{\text{j}}\right) = \left(6.0 - 8.0t\right)\hat{\text{i}}$$

in SI units. Specifically, we find the acceleration vector at $t = 3.0\text{ s}$ to be $\left(6.0 - 8.0(3.0)\right)\hat{\text{i}} = (-18\text{ m/s}^2)\hat{\text{i}}$.

(b) The equation is $\vec{a} = \left(6.0 - 8.0t\right)\hat{\text{i}} = 0$; we find t = 0.75 s.

(c) Since the y component of the velocity, v_y = 8.0 m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have

$$v = |\vec{v}| = \sqrt{\left(6.0t - 4.0t^2\right)^2 + \left(8.0\right)^2} = 10$$

in SI units (m/s). To solve for t, we first square both sides of the above equation, followed by some rearrangement:

$$\left(6.0t - 4.0t^2\right)^2 + 64 = 100 \quad \Rightarrow \quad \left(6.0t - 4.0t^2\right)^2 = 36$$

Taking the square root of the new expression and making further simplification lead to

$$6.0t - 4.0t^2 = \pm 6.0 \quad \Rightarrow \quad 4.0t^2 - 6.0t \pm 6.0 = 0$$

Finally, using the quadratic formula, we obtain

$$t = \frac{6.0 \pm \sqrt{36 - 4(4.0)(\pm 6.0)}}{2(8.0)}$$

where the requirement of a real positive result leads to the unique answer: t = 2.2 s.

17. We find t by applying Eq. 2-11 to motion along the y axis (with v_y = 0 characterizing $y = y_{\max}$):

$$0 = (12 \text{ m/s}) + (-2.0 \text{ m/s}^2)t \quad \Rightarrow \quad t = 6.0 \text{ s}.$$

Then, Eq. 2-11 applies to motion along the x axis to determine the answer:

$$v_x = (8.0 \text{ m/s}) + (4.0 \text{ m/s}^2)(6.0 \text{ s}) = 32 \text{ m/s}.$$

Therefore, the velocity of the cart, when it reaches $y = y_{\max}$, is (32 m/s)$\hat{\mathrm{i}}$.

18. We find t by solving $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$:

$$12.0 \text{ m} = 0 + (4.00 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^2)t^2$$

where we have used Δx = 12.0 m, v_x = 4.00 m/s, and a_x = 5.00 m/s^2 . We use the quadratic formula and find t = 1.53 s. Then, Eq. 2-11 (actually, its analog in two dimensions) applies with this value of t. Therefore, its velocity (when Δx = 12.00 m) is

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t = (4.00 \text{ m/s})\hat{\mathrm{i}} + (5.00 \text{ m/s}^2)(1.53 \text{ s})\hat{\mathrm{i}} + (7.00 \text{ m/s}^2)(1.53 \text{ s})\hat{\mathrm{j}} \\ &= (11.7 \text{ m/s})\,\hat{\mathrm{i}} + (10.7 \text{ m/s})\,\hat{\mathrm{j}}.\end{aligned}$$

Thus, the magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(11.7 \text{ m/s})^2 + (10.7 \text{ m/s})^2} = 15.8$ m/s.

(b) The angle of $\vec{v}$, measured from +x, is

$$\tan^{-1}\left(\frac{10.7 \text{ m/s}}{11.7 \text{ m/s}}\right) = 42.6°.$$

19. We make use of Eq. 4-16 and Eq. 4-10.

Using $\vec{a} = 3t\hat{\mathrm{i}} + 4t\hat{\mathrm{j}}$, we have (in m/s)

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}\; dt = (5.00\hat{\mathrm{i}} + 2.00\hat{\mathrm{j}}) + \int_0^t (3t\hat{\mathrm{i}} + 4t\hat{\mathrm{j}})\; dt = \left(5.00 + 3t^2/2\right)\hat{\mathrm{i}} + \left(2.00 + 2t^2\right)\hat{\mathrm{j}}$$

Integrating using Eq. 4-10 then yields (in meters)

$$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}dt = (20.0\hat{\mathrm{i}} + 40.0\hat{\mathrm{j}}) + \int_0^t [(5.00 + 3t^2/2)\hat{\mathrm{i}} + (2.00 + 2t^2)\hat{\mathrm{j}}]dt$$
$$= (20.0\hat{\mathrm{i}} + 40.0\hat{\mathrm{j}}) + (5.00t + t^3/2)\hat{\mathrm{i}} + (2.00t + 2t^3/3)\hat{\mathrm{j}}$$
$$= (20.0 + 5.00t + t^3/2)\hat{\mathrm{i}} + (40.0 + 2.00t + 2t^3/3)\hat{\mathrm{j}}$$

(a) At $t = 4.00$ s, we have $\vec{r}(t = 4.00\,\text{s}) = (72.0\ \text{m})\hat{\mathrm{i}} + (90.7\ \text{m})\hat{\mathrm{j}}$.

(b) $\vec{v}(t = 4.00\,\text{s}) = (29.0\ \text{m/s})\hat{\mathrm{i}} + (34.0\ \text{m/s})\hat{\mathrm{j}}$. Thus, the angle between the direction of travel and +x, measured counterclockwise, is $\theta = \tan^{-1}[(34.0\,\text{m/s})/(29.0\,\text{m/s})] = 49.5°$.

20. The acceleration is constant so that use of Table 2-1 (for both the x and y motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles A and B requires two things. First, the y motion of B must satisfy (using Eq. 2-15 and noting that θ is measured from the y axis)

$$y = \frac{1}{2}a_y t^2 \quad \Rightarrow \quad 30\ \text{m} = \frac{1}{2}\left[(0.40\ \text{m/s}^2)\cos\theta\right]t^2.$$

Second, the x motions of A and B must coincide:

$$vt = \frac{1}{2}a_x t^2 \quad \Rightarrow \ (3.0\ \text{m/s})t = \frac{1}{2}\left[(0.40\ \text{m/s}^2)\sin\theta\right]t^2.$$

We eliminate a factor of t in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0\ \text{m/s})}{(0.40\ \text{m/s}^2)\sin\theta}.$$

This is then plugged into the previous equation to produce

$$30\ \text{m} = \frac{1}{2}\left[(0.40\ \text{m/s}^2)\cos\theta\right]\left(\frac{2(3.0\ \text{m/s})}{(0.40\ \text{m/s}^2)\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1 - \cos^2\theta$, simplifies to

$$30 = \frac{9.0}{0.20}\,\frac{\cos\theta}{1-\cos^2\theta} \quad \Rightarrow \ 1 - \cos^2\theta = \frac{9.0}{(0.20)(30)}\cos\theta.$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

21. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the *y* coordinate of the dart is given by $y = -\frac{1}{2}gt^2$, so that with *y* = –*PQ* we have $PQ = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)\left(0.19 \text{ s}\right)^2 = 0.18 \text{ m}.$

(b) From $x = v_0t$ we obtain x = (10 m/s)(0.19 s) = 1.9 m.

22. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the *y* coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If *t* is the time of flight and *y* = –1.20 m indicates the level at which the ball hits the floor, then

$$t = \sqrt{\frac{2\left(-1.20 \text{ m}\right)}{-9.80 \text{ m/s}^2}} = 0.495 \text{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0\,\hat{\text{i}}$. Since *x* = 1.52 m is the horizontal position of its impact point with the floor, we have $x = v_0t$. Thus,

$$v_0 = \frac{x}{t} = \frac{1.52 \text{ m}}{0.495 \text{ s}} = 3.07 \text{ m/s}.$$

23. (a) From Eq. 4-22 (with $\theta_0 = 0$), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0 \text{ m})}{9.80 \text{ m/s}^2}} = 3.03 \text{ s}.$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0t = (250 \text{ m/s})(3.03 \text{ s}) = 758 \text{ m}.$$

(c) And from Eq. 4-23, we find

$$\left|v_y\right| = gt = (9.80 \text{ m/s}^2)(3.03 \text{ s}) = 29.7 \text{ m/s}.$$

24. We use Eq. 4-26

$$R_{\max} = \left(\frac{v_0^2}{g} \sin 2\theta_0 \right)_{\max} = \frac{v_0^2}{g} = \frac{\left(9.50\,\text{m/s}\right)^2}{9.80\,\text{m/s}^2} = 9.209\text{ m} \approx 9.21\text{m}$$

to compare with Powell's long jump; the difference from $R_{\max}$ is only ΔR =(9.21m – 8.95m) = 0.259 m.

25. Using Eq. (4-26), the take-off speed of the jumper is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80\text{ m/s}^2)(77.0\text{ m})}{\sin 2(12.0°)}} = 43.1\text{ m/s}$$

26. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is the throwing point (the stone's initial position). The x component of its initial velocity is given by $v_{0x} = v_0 \cos\theta_0$ and the y component is given by $v_{0y} = v_0 \sin\theta_0$, where v_0 = 20 m/s is the initial speed and θ_0 = 40.0° is the launch angle.

(a) At t = 1.10 s, its x coordinate is

$$x = v_0 t \cos\theta_0 = (20.0\text{ m/s})(1.10\text{ s}) \cos 40.0° = 16.9\text{ m}$$

(b) Its y coordinate at that instant is

$$y = v_0 t \sin\theta_0 - \frac{1}{2} g t^2 = (20.0\text{m/s})(1.10\text{s}) \sin 40.0° - \frac{1}{2}(9.80\text{m/s}^2)(1.10\text{s})^2 = 8.21\text{ m}.$$

(c) At t' = 1.80 s, its x coordinate is $x = (20.0\text{ m/s})(1.80\text{ s}) \cos 40.0° = 27.6\text{ m}.$

(d) Its y coordinate at t' is

$$y = (20.0\text{m/s})(1.80\text{s}) \sin 40.0° - \frac{1}{2}(9.80\text{m/s}^2)(1.80\text{s}^2) = 7.26\text{m}.$$

(e) The stone hits the ground earlier than t = 5.0 s. To find the time when it hits the ground solve $y = v_0 t \sin\theta_0 - \frac{1}{2} g t^2 = 0$ for t. We find

$$t = \frac{2v_0}{g} \sin\theta_0 = \frac{2(20.0\text{ m/s})}{9.8\text{ m/s}^2} \sin 40° = 2.62\text{ s}.$$

Its x coordinate on landing is

$$x = v_0 t \cos\theta_0 = (20.0 \text{ m/s})(2.62 \text{ s}) \cos 40° = 40.2 \text{ m}.$$

(f) Assuming it stays where it lands, its vertical component at t = 5.00 s is y = 0.

27. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write θ_0 = –30.0° since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release: v_0 = 290 km/h, which we convert to SI units: (290)(1000/3600) = 80.6 m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0 \cos\theta_0)\, t \quad \Rightarrow \quad t = \frac{700 \text{ m}}{(80.6 \text{ m/s})\cos(-30.0°)} = 10.0 \text{ s}.$$

(b) And we use Eq. 4-22 to solve for the initial height y_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - y_0 = (-40.3 \text{ m/s})(10.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(10.0 \text{ s})^2$$

which yields y_0 = 897 m.

28. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0 \sin\theta_0 t - \frac{1}{2}gt^2$$

which yields h = 51.8 m for y_0 = 0, v_0 = 42.0 m/s, θ_0 = 60.0°, and t = 5.50 s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0 \cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0 \cos\theta_0)^2 + (v_0 \sin\theta_0 - gt)^2} = 27.4 \text{ m/s}.$$

(c) We use Eq. 4-24 with v_y = 0 and $y = H$:

$$H = \frac{(v_0 \sin\theta_0)^2}{2g} = 67.5 \text{ m}.$$

29. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to v_{0x}). In this notation, we have $v_0 = 5v$. Next, we observe $v_0 \cos \theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \;\Rightarrow\; \theta_0 = \cos^{-1}\left(\frac{1}{5}\right) = 78.5°.$$

30. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With $\Delta y = 0$, we have

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \;\Rightarrow\; t = \frac{(19.5 \text{ m/s})\sin 45.0°}{(9.80 \text{ m/s}^2)/2} = 2.81 \text{ s}.$$

Then Eq. 4-21 yields $\Delta x = (v_0 \cos \theta_0)t = 38.7$ m. Thus, using Eq. 4-8, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(38.7 \text{ m})\,\hat{\text{i}} - (55 \text{ m})\hat{\text{i}}}{2.81 \text{ s}} = (-5.8 \text{ m/s})\,\hat{\text{i}}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

31. We first find the time it takes for the volleyball to hit the ground. Using Eq. 4-22, we have

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \;\Rightarrow\; 0 - 2.30 \text{ m} = (-20.0 \text{ m/s})\sin(18.0°)t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

which gives $t = 0.30$ s. Thus, the range of the volleyball is

$$R = \left(v_0 \cos\theta_0\right)t = (20.0 \text{ m/s})\cos 18.0°(0.30 \text{ s}) = 5.71 \text{ m}$$

On the other hand, when the angle is changed to $\theta_0' = 8.00°$, using the same procedure as shown above, we find

$$y - y_0 = (v_0 \sin\theta_0')\, t' - \frac{1}{2}gt'^2 \;\Rightarrow\; 0 - 2.30 \text{ m} = (-20.0 \text{ m/s})\sin(8.00°)t' - \frac{1}{2}(9.80 \text{ m/s}^2)t'^2$$

which yields $t' = 0.46$ s, and the range is

$$R' = \left(v_0 \cos\theta_0\right)t' = (20.0 \text{ m/s})\cos 18.0°(0.46 \text{ s}) = 9.06 \text{ m}$$

Thus, the ball travels an extra distance of

$$\Delta R = R' - R = 9.06\text{ m} - 5.71\text{ m} = 3.35\text{ m}$$

32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0\text{ m}}{(25.0\text{ m/s})\cos 40.0°} = 1.15\text{ s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = (25.0\text{ m/s})\sin 40.0°(1.15\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(1.15\text{ s})^2 = 12.0\text{ m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0° = 19.2$ m/s.

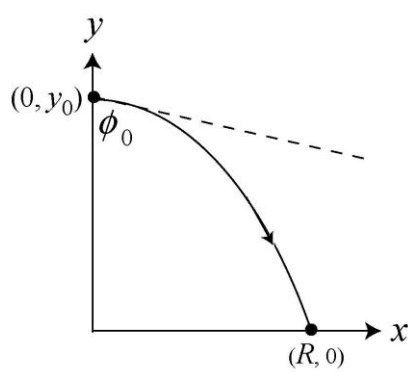
(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = (25.0\text{ m/s})\sin 40.0° - (9.80\text{ m/s}^2)(1.15\text{ s}) = 4.80\text{ m/s}.$$

(d) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

33. **THINK** This problem deals with projectile motion. We're interested in the horizontal displacement and velocity of the projectile before it strikes the ground.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37.0°$ for the angle measured from $+x$, since the angle $\phi_0 = 53.0°$ given in the problem is measured from the $-y$ direction. The initial setup of the problem is shown in the figure to the right (not to scale).



ANALYZE (a) The initial speed of the projectile is the plane's speed at the moment of release. Given that $y_0 = 730$ m and $y = 0$ at $t = 5.00$ s, we use Eq. 4-22 to find v_0:

$$y - y_0 = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad 0 - 730 \text{ m} = v_0 \sin(-37.0°)(5.00 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(5.00 \text{ s})^2$$

which yields v_0 = 202 m/s.

(b) The horizontal distance traveled is

$$R = v_x t = (v_0 \cos\theta_0)t = [(202 \text{ m/s})\cos(-37.0°)](5.00 \text{ s}) = 806 \text{ m}.$$

(c) The x component of the velocity (just before impact) is

$$v_x = v_0 \cos\theta_0 = (202 \text{ m/s})\cos(-37.0°) = 161 \text{ m/s}.$$

(d) The y component of the velocity (just before impact) is

$$v_y = v_0 \sin\theta_0 - gt = (202 \text{ m/s})\sin(-37.0°) - (9.80 \text{ m/s}^2)(5.00 \text{ s}) = -171 \text{ m/s}.$$

LEARN In this projectile problem we analyzed the kinematics in the vertical and horizontal directions separately since they do not affect each other. The x-component of the velocity, $v_x = v_0 \cos\theta_0$, remains unchanged throughout since there's no horizontal acceleration.

34. (a) Since the y-component of the velocity of the stone at the top of its path is zero, its speed is

$$v = \sqrt{v_x^2 + v_y^2} = v_x = v_0 \cos\theta_0 = (28.0 \text{ m/s})\cos 40.0° = 21.4 \text{ m/s}.$$

(b) Using the fact that $v_y = 0$ at the maximum height $y_{\max}$, the amount of time it takes for the stone to reach $y_{\max}$ is given by Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \ t = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g} \right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g} \right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time the stone descends to $y = y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t_{\pm} = \frac{(2 \pm \sqrt{2}) v_0 \sin\theta_0}{2g}.$$

Choosing $t = t_+$ (for descending), we have

$$v_x = v_0 \cos\theta_0 = (28.0 \text{ m/s})\cos 40.0° = 21.4 \text{ m/s}$$
$$v_y = v_0 \sin\theta_0 - g\,\frac{(2+\sqrt{2})v_0 \sin\theta_0}{2g} = -\frac{\sqrt{2}}{2}v_0 \sin\theta_0 = -\frac{\sqrt{2}}{2}(28.0 \text{ m/s})\sin 40.0° = -12.7 \text{ m/s}$$

Thus, the speed of the stone when $y = y_{\max}/2$ is

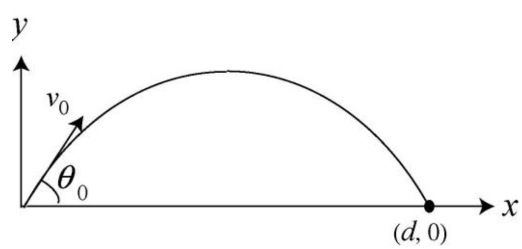
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.4 \text{ m/s})^2 + (-12.7 \text{ m/s})^2} = 24.9 \text{ m/s}\,.$$

(c) The percentage difference is

$$\frac{24.9 \text{ m/s} - 21.4 \text{ m/s}}{21.4 \text{ m/s}} = 0.163 = 16.3\%\,.$$

35. **THINK** This problem deals with projectile motion of a bullet. We're interested in the firing angle that allows the bullet to strike a target at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$.



The projectile motion equations lead to

$$d = (v_0 \cos\theta_0)t, \qquad 0 = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$$

where θ_0 is the firing angle. The setup of the problem is shown in the figure above (scale exaggerated).

ANALYZE The time at which the bullet strikes the target is given by $t = d/(v_0 \cos\theta_0)$. Eliminating t leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\ (2\theta_0) = gd \ \Rightarrow\ \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\ \text{m/s}^2)(45.7\ \text{m})}{(460\ \text{m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$, or θ_0 = 0.0606°. If the gun is aimed at a point a distance ℓ above the target, then $\tan\,\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\ \text{m})\tan(0.0606°) = 0.0484\ \text{m} = 4.84\ \text{cm}.$$

LEARN Due to the downward gravitational acceleration, in order for the bullet to strike the target, the gun must be aimed at a point slightly above the target.

36. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at x = 12.0 m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{12.0\ \text{m}}{(23.6\ \text{m/s})\cos 0°} = 0.508\ \text{s}.$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0\sin\theta_0)\,t - \frac{1}{2}gt^2 = 1.10\text{m}$$

which implies it does indeed clear the 0.90-m-high fence.

(b) At t = 0.508 s, the center of the ball is (1.10 m – 0.90 m) = 0.20 m above the net.

(c) Repeating the computation in part (a) with θ_0 = –5.0° results in t = 0.510 s and $y = 0.040$ m, which clearly indicates that it cannot clear the net.

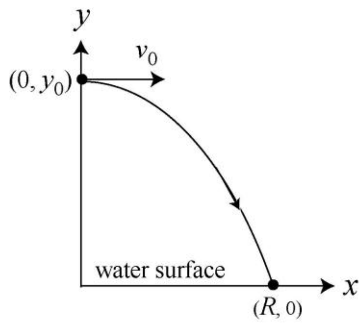
(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at t = 0.510 s is 0.90 m – 0.040 m = 0.86 m.

37. **THINK** The trajectory of the diver is a projectile motion. We are interested in the displacement of the diver at a later time.

EXPRESS The initial velocity has no vertical component ($\theta_0 = 0$), but only an x component. Eqs. 4-21 and 4-22 can be simplified to

$$x - x_0 = v_{0x} t$$
$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2 = -\frac{1}{2} g t^2.$$

where $x_0 = 0$, $v_{0x} = v_0 = +2.0\ \text{m/s}$ and y_0 = +10.0 m (taking the water surface to be at $y = 0$). The setup of the problem is shown in the figure below.



ANALYZE (a) At $t = 0.80\ \text{s}$, the horizontal distance of the diver from the edge is

$$x = x_0 + v_{0x} t = 0 + (2.0\ \text{m/s})(0.80\ \text{s}) = 1.60\ \text{m}.$$

(b) Similarly, using the second equation for the vertical motion, we obtain

$$y = y_0 - \frac{1}{2} g t^2 = 10.0\ \text{m} - \frac{1}{2}(9.80\ \text{m/s}^2)(0.80\ \text{s})^2 = 6.86\ \text{m}.$$

(c) At the instant the diver strikes the water surface, y = 0. Solving for t using the equation $y = y_0 - \frac{1}{2} g t^2 = 0$ leads to

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(10.0\ \text{m})}{9.80\ \text{m/s}^2}} = 1.43\ \text{s}.$$

During this time, the x-displacement of the diver is $R = x$ = (2.00 m/s)(1.43 s) = 2.86 m.

LEARN Using Eq. 4-25 with $\theta_0 = 0$, the trajectory of the diver can also be written as

$$y = y_0 - \frac{g x^2}{2 v_0^2}\ .$$

Part (c) can also be solved by using this equation:

$$y = y_0 - \frac{g x^2}{2 v_0^2} = 0 \ \Rightarrow\ x = R = \sqrt{\frac{2 v_0^2 y_0}{g}} = \sqrt{\frac{2(2.0\ \text{m/s})^2 (10.0\ \text{m})}{9.8\ \text{m/s}^2}} = 2.86\ \text{m}.$$

38. In this projectile motion problem, we have $v_0 = v_x$ = constant, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at t = 2.5 s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that v_x = 19 m/s.

(a) During t = 5 s, the horizontal motion is $x - x_0 = v_x t$ = 95 m.

(b) Since $\sqrt{(19\text{ m/s})^2 + v_{0y}^2} = 31\text{ m/s}$ (the first point on the graph), we find $v_{0y} = 24.5$ m/s. Thus, with t = 2.5 s, we can use $y_{\max} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^2 - 2g(y_{\max} - y_0)$, or $y_{\max} - y_0 = \frac{1}{2}\left(v_y + v_{0_y}\right)t$ to solve. Here we will use the latter:

$$y_{\max} - y_0 = \frac{1}{2}(v_y + v_{0y})\ t \Rightarrow y_{\max} = \frac{1}{2}(0 + 24.5\text{m/s})(2.5\text{ s}) = 31\text{ m}$$

where we have taken $y_0 = 0$ as the ground level.

39. Following the hint, we have the time-reversed problem with the ball thrown from the ground, toward the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with $+x$ as *rightward* and with positive angles measured counterclockwise.

(a) The x-equation (with $x_0 = 0$ and x = 25.0 m) leads to

$$25.0\text{ m} = (v_0 \cos 60.0°)(1.50\text{ s}),$$

so that v_0 = 33.3 m/s. And with $y_0 = 0$, and $y = h > 0$ at t = 1.50 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60.0°$. This leads to h = 32.3 m.

(b) We have

$$v_x = v_{0x} = (33.3\text{ m/s})\cos 60.0° = 16.7\text{ m/s}$$
$$v_y = v_{0y} - gt = (33.3\text{ m/s})\sin 60.0° - (9.80\text{ m/s}^2)(1.50\text{ s}) = 14.2\text{ m/s}.$$

The magnitude of $\vec{v}$ is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.7\text{ m/s})^2 + (14.2\text{ m/s})^2} = 21.9\text{ m/s}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2\text{ m/s}}{16.7\text{ m/s}}\right) = 40.4°.$$

(d) We interpret this result ("undoing" the time reversal) as an initial velocity (from the edge of the building) of magnitude 21.9 m/s with angle (down from leftward) of 40.4°.

40. (a) Solving the quadratic equation Eq. 4-22:

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - 2.160\text{ m} = (15.00\text{ m/s})\sin(45.00°)t - \frac{1}{2}(9.800\text{ m/s}^2)t^2$$

the total travel time of the shot in the air is found to be $t = 2.352\text{ s}$. Therefore, the horizontal distance traveled is

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\text{ m/s})\cos 45.00°(2.352\text{ s}) = 24.95\text{ m}.$$

(b) Using the procedure outlined in (a) but for $\theta_0 = 42.00°$, we have

$$y - y_0 = (v_0 \sin\theta_0)\ t - \frac{1}{2}gt^2 \quad \Rightarrow \quad 0 - 2.160\text{ m} = (15.00\text{ m/s})\sin(42.00°)t - \frac{1}{2}(9.800\text{ m/s}^2)t^2$$

and the total travel time is $t = 2.245\text{ s}$. This gives

$$R = \left(v_0 \cos\theta_0\right)t = (15.00\text{ m/s})\cos 42.00°(2.245\text{ s}) = 25.02\text{ m}.$$

41. With the Archer fish set to be at the origin, the position of the insect is given by (x, y) where $x = R/2 = v_0^2 \sin 2\theta_0 / 2g$, and y corresponds to the maximum height of the parabolic trajectory: $y = y_{\max} = v_0^2 \sin^2\theta_0 / 2g$. From the figure, we have

$$\tan\phi = \frac{y}{x} = \frac{v_0^2 \sin^2\theta_0 / 2g}{v_0^2 \sin 2\theta_0 / 2g} = \frac{1}{2}\tan\theta_0$$

Given that $\phi = 36.0°$, we find the launch angle to be

$$\theta_0 = \tan^{-1}\left(2\tan\phi\right) = \tan^{-1}\left(2\tan 36.0°\right) = \tan^{-1}\left(1.453\right) = 55.46° \approx 55.5°.$$

Note that θ_0 depends only on ϕ and is independent of d.

42. (a) Using the fact that the person (as the projectile) reaches the maximum height over the middle wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we can deduce the initial launch speed from Eq. 4-26:

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \quad \Rightarrow \quad v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2(9.8\text{ m/s}^2)(34.5\text{ m})}{\sin(2\cdot 53°)}} = 26.5\text{ m/s}.$$

Upon substituting the value to Eq. 4-25, we obtain

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (23\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(23\text{ m})^2}{2(26.5\text{ m/s})^2(\cos 53°)^2} = 23.3\text{ m}.$$

Since the height of the wheel is $h_w = 18$ m, the clearance over the first wheel is $\Delta y = y - h_w = 23.3\text{ m} - 18\text{ m} = 5.3\text{ m}$.

(b) The height of the person when he is directly above the second wheel can be found by solving Eq. 4-24. With the second wheel located at $x = 23\text{ m} + (23/2)\text{ m} = 34.5\text{ m}$, we have

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\text{ m} + (34.5\text{ m})\tan 53° - \frac{(9.8\text{ m/s}^2)(34.5\text{ m})^2}{2(26.52\text{ m/s})^2(\cos 53°)^2}$$
$$= 25.9\text{ m}.$$

Therefore, the clearance over the second wheel is $\Delta y = y - h_w = 25.9\text{ m} - 18\text{ m} = 7.9\text{ m}$.

(c) The location of the center of the net is given by

$$0 = y - y_0 = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} \quad\Rightarrow\quad x = \frac{v_0^2\sin 2\theta_0}{g} = \frac{(26.52\text{ m/s})^2\sin(2\cdot 53°)}{9.8\text{ m/s}^2} = 69\text{ m}.$$

43. We designate the given velocity $\vec{v} = (7.6\text{ m/s})\hat{\text{i}} + (6.1\text{ m/s})\hat{\text{j}}$ as $\vec{v}_1$, as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3$, and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial *y* velocity, that is how we will proceed. Using Eq. 2-16, we have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \quad\Rightarrow\quad (6.1\text{ m/s})^2 = v_{0y}^2 - 2(9.8\text{ m/s}^2)(9.1\text{ m})$$

which yields v_{0y} = 14.7 m/s. Knowing that v_{2y} must equal 0, we use Eq. 2-16 again but now with $\Delta y = h$ for the maximum height:

$$v_{2y}^2 = v_{0y}^2 - 2gh \quad\Rightarrow\quad 0 = (14.7\text{ m/s})^2 - 2(9.8\text{ m/s}^2)h$$

which yields h = 11 m.

(b) Recalling the derivation of Eq. 4-26, but using v_{0y} for $v_0 \sin\theta_0$ and v_{0x} for $v_0\cos\theta_0$, we have

$$0 = v_{0y}t - \frac{1}{2}gt^2, \quad R = v_{0x}t$$

which leads to $R = 2v_{0x}v_{0y}/g$. Noting that v_{0x} = v_{1x} = 7.6 m/s, we plug in values and obtain

$$R = 2(7.6 \text{ m/s})(14.7 \text{ m/s})/(9.8 \text{ m/s}^2) = 23 \text{ m}.$$

(c) Since v_{3x} = v_{1x} = 7.6 m/s and v_{3y} = – v_{0y} = –14.7 m/s, we have

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{(7.6 \text{ m/s})^2 + (-14.7 \text{ m/s})^2} = 17 \text{ m/s}.$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7 \text{ m}}{7.6 \text{ m}}\right) = -63° \text{ or } 117°$$

where we settle on the first choice (–63°, which is equivalent to 297°) since the signs of its components imply that it is in the fourth quadrant.

44. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that v_{0y} = 0 and $v_{0x} = v_0 = 161 \text{ km/h}$. Converting to SI units, this is v_0 = 44.7 m/s.

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the x coordinate is given by $x = v_0t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if x = 18.3/2 m, then t = (18.3/2 m)/(44.7 m/s) = 0.205 s.

(b) And the time to travel the next 18.3/2 m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0\Delta t$ in order that this result can be seen more clearly.

(c) Using the equation $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $|-\frac{1}{2}(9.80 \text{ m/s}^2)(0.205 \text{ s})^2| = 0.205 \text{ m}$ at the moment the ball is halfway to the batter.

(d) The ball's height when it reaches the batter is $-\frac{1}{2}(9.80 \text{ m/s}^2)(0.409 \text{ s})^2 = -0.820 \text{ m}$, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to t, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

45. (a) Let $m = \frac{d_2}{d_1} = 0.600$ be the slope of the ramp, so $y = mx$ there. We choose our coordinate origin at the point of launch and use Eq. 4-25. Thus,

$$y = \tan(50.0°)x - \frac{(9.80\text{ m/s}^2)x^2}{2(10.0\text{ m/s})^2(\cos 50.0°)^2} = 0.600x$$

which yields $x = 4.99$ m. This is less than d_1 so the ball *does* land on the ramp.

(b) Using the value of x found in part (a), we obtain $y = mx = 2.99$ m. Thus, the Pythagorean theorem yields a displacement magnitude of $\sqrt{x^2 + y^2} = 5.82$ m.

(c) The angle is, of course, the angle of the ramp: $\tan^{-1}(m) = 31.0°$.

46. Using the fact that $v_y = 0$ when the player is at the maximum height $y_{\max}$, the amount of time it takes to reach $y_{\max}$ can be solved by using Eq. 4-23:

$$0 = v_y = v_0 \sin\theta_0 - gt \quad \Rightarrow \quad t_{\max} = \frac{v_0 \sin\theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin\theta_0)\, t_{\max} - \frac{1}{2} g t_{\max}^2 = v_0 \sin\theta_0 \left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\theta_0}{g}\right)^2 = \frac{v_0^2 \sin^2\theta_0}{2g}.$$

To find the time when the player is at $y = y_{\max}/2$, we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2\theta_0}{4g} = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t_\pm = \frac{(2 \pm \sqrt{2}) v_0 \sin\theta_0}{2g}.$$

With $t = t_-$ (for ascending), the amount of time the player spends at a height $y \geq y_{\max}/2$ is

$$\Delta t = t_{\max} - t_- = \frac{v_0 \sin\theta_0}{g} - \frac{(2 - \sqrt{2}) v_0 \sin\theta_0}{2g} = \frac{v_0 \sin\theta_0}{\sqrt{2} g} = \frac{t_{\max}}{\sqrt{2}} \quad \Rightarrow \quad \frac{\Delta t}{t_{\max}} = \frac{1}{\sqrt{2}} = 0.707.$$

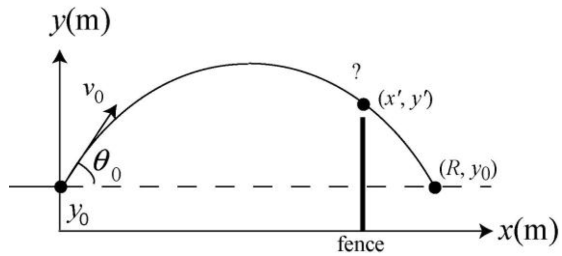
Therefore, the player spends about 70.7% of the time in the upper half of the jump. Note that the ratio $\Delta t / t_{\max}$ is independent of v_0 and θ_0, even though Δt and $t_{\max}$ depend on these quantities.

47. **THINK** The baseball undergoes projectile motion after being hit by the batter. We'd like to know if the ball clears a high fence at some distance away.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. In the absence of a fence, with $\theta_0 = 45°$, the horizontal range (same launch level) is $R = 107$ m. We want to know how high the ball is from the ground when it is at $x' = 97.5$ m, which requires knowing the initial velocity. The trajectory of the baseball can be described by Eq. 4-25:

$$y - y_0 = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}.$$

The setup of the problem is shown in the figure below (not to scale).



ANALYZE (a) We first solve for the initial speed v_0. Using the range information ($y = y_0$ when $x = R$) and θ_0 = 45°, Eq. 4-25 gives

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8\text{ m/s}^2)(107\text{ m})}{\sin(2\cdot 45°)}} = 32.4\text{ m/s}.$$

Thus, the time at which the ball flies over the fence is:

$$x' = (v_0\cos\theta_0)t' \quad\Rightarrow\quad t' = \frac{x'}{v_0\cos\theta_0} = \frac{97.5\text{ m}}{(32.4\text{ m/s})\cos 45°} = 4.26\text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$\begin{aligned} y' &= y_0 + (v_0\sin\theta_0)t' - \frac{1}{2}gt'^2 \\ &= 1.22\text{ m} + [(32.4\text{ m/s})\sin 45°](4.26\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(4.26\text{ s})^2 \\ &= 9.88\text{ m} \end{aligned}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At $t' = 4.26$ s, the center of the ball is 9.88 m – 7.32 m = 2.56 m above the fence.

LEARN Using the trajectory equation above, one can show that the minimum initial velocity required to clear the fence is given by

$$y' - y_0 = (\tan\theta_0)x' - \frac{gx'^2}{2(v_0\cos\theta_0)^2},$$

or about 31.9 m/s.

48. Following the hint, we have the time-reversed problem with the ball thrown from the roof, toward the left, at 60° measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take $+x$ as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

(a) With y_0 = 20.0 m, and y = 0 at t = 4.00 s, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to v_0 = 16.9 m/s. This plugs into the x-equation $x - x_0 = v_{0x}t$ (with x_0 = 0 and $x = d$) to produce

$$d = (16.9 \text{ m/s})\cos 60°(4.00 \text{ s}) = 33.7 \text{ m}.$$

(b) We have

$$v_x = v_{0x} = (16.9 \text{ m/s})\cos 60.0° = 8.43 \text{ m/s}$$
$$v_y = v_{0y} - gt = (16.9 \text{ m/s})\sin 60.0° - (9.80 \text{m/s}^2)(4.00 \text{ s}) = -24.6 \text{ m/s}.$$

The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(8.43 \text{ m/s})^2 + (-24.6 \text{ m/s})^2} = 26.0 \text{ m/s}.$

(c) The angle relative to horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-24.6 \text{ m/s}}{8.43 \text{ m/s}}\right) = -71.1°.$$

We may convert the result from rectangular components to magnitude-angle representation:

$$\vec{v} = (8.43, -24.6) \;\rightarrow\; (26.0 \angle -71.1°)$$

and we now interpret our result ("undoing" the time reversal) as an initial velocity of magnitude 26.0 m/s with angle (up from rightward) of 71.1°.

49. **THINK** In this problem a football is given an initial speed and it undergoes projectile motion. We'd like to know the smallest and greatest angles at which a field goal can be scored.

EXPRESS We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use x and y to denote the coordinates of the ball at the goalpost, and try to find the kicking angle(s) θ_0 so that y = 3.44 m when x = 50 m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos\theta_0, \quad y = v_0 t \sin\,\theta_0 - \tfrac{1}{2} g t^2,$$

we see the first equation gives $t = x/v_0 \cos\,\theta_0$, and when this is substituted into the second the result is

$$y = x \tan\theta_0 - \frac{g x^2}{2 v_0^2 \cos^2\,\theta_0}.$$

ANALYZE One may solve the above equation by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity

$$1 / \cos^2\,\theta_0 = 1 + \tan^2\,\theta_0,$$

we obtain

$$\frac{1}{2}\frac{g x^2}{v_0^2} \tan^2\,\theta_0 - x \tan\theta_0 + y + \frac{1}{2}\frac{g x^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\,\theta_0$. To simplify writing the solution, we denote

$$c = \frac{1}{2} g x^2 / v_0^2 = \frac{1}{2}\left(9.80\ \text{m/s}^2\right)\left(50\ \text{m}\right)^2 / \left(25\ \text{m/s}\right)^2 = 19.6\ \text{m}.$$

Then the second-order equation becomes $c\,\tan^2\,\theta_0 - x\,\tan\,\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\tan\,\theta_0 = \frac{x \pm \sqrt{x^2 - 4\left(y + c\right)c}}{2c} = \frac{50\ \text{m} \pm \sqrt{(50\ \text{m})^2 - 4\left(3.44\ \text{m} + 19.6\ \text{m}\right)\left(19.6\ \text{m}\right)}}{2\left(19.6\ \text{m}\right)}.$$

The two solutions are given by $\tan\theta_0 = 1.95$ and $\tan\theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63°$ and $\theta_0 = 31°$. Thus,

(a) The smallest elevation angle is $\theta_0 = 31°$, and

(b) The greatest elevation angle is $\theta_0 = 63°$.

LEARN If kicked at any angle between 31° and 63°, the ball will travel above the cross bar on the goalposts.

50. We apply Eq. 4-21, Eq. 4-22, and Eq. 4-23.

(a) From $\Delta x = v_{0x} t$, we find $v_{0x} = 40\ \text{m} / 2\ \text{s} = 20\ \text{m/s}$.

(b) From $\Delta y = v_{0y} t - \tfrac{1}{2} g t^2$, we find $v_{0y} = \left(53\ \text{m} + \tfrac{1}{2}(9.8\ \text{m/s}^2)(2\ \text{s})^2\right) / 2 = 36\ \text{m/s}$.

(c) From $v_y = v_{0y} - gt'$ with v_y = 0 as the condition for maximum height, we obtain $t' = (36 \text{ m/s})/(9.8 \text{ m/s}^2) = 3.7 \text{ s}$. During that time the x-motion is constant, so $x' - x_0 = (20 \text{ m/s})(3.7 \text{ s}) = 74 \text{ m}$.

51. (a) The skier jumps up at an angle of $\theta_0 = 11.3°$ up from the horizontal and thus returns to the launch level with his velocity vector 11.3° below the horizontal. With the snow surface making an angle of $\alpha = 9.0°$ (downward) with the horizontal, the angle between the slope and the velocity vector is $\phi = \theta_0 - \alpha = 11.3° - 9.0° = 2.3°$.

(b) Suppose the skier lands at a distance d down the slope. Using Eq. 4-25 with $x = d\cos\alpha$ and $y = -d\sin\alpha$ (the edge of the track being the origin), we have

$$-d\sin\alpha = d\cos\alpha\tan\theta_0 - \frac{g(d\cos\alpha)^2}{2v_0^2\cos^2\theta_0}.$$

Solving for d, we obtain

$$\begin{aligned} d &= \frac{2v_0^2\cos^2\theta_0}{g\cos^2\alpha}\left(\cos\alpha\tan\theta_0 + \sin\alpha\right) = \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\left(\cos\alpha\sin\theta_0 + \cos\theta_0\sin\alpha\right) \\ &= \frac{2v_0^2\cos\theta_0}{g\cos^2\alpha}\sin(\theta_0 + \alpha). \end{aligned}$$

Substituting the values given, we find

$$d = \frac{2(10 \text{ m/s})^2\cos(11.3°)}{(9.8 \text{ m/s}^2)\cos^2(9.0°)}\sin(11.3° + 9.0°) = 7.117 \text{ m}.$$

which gives

$$y = -d\sin\alpha = -(7.117 \text{ m})\sin(9.0°) = -1.11 \text{ m}.$$

Therefore, at landing the skier is approximately 1.1 m below the launch level.

(c) The time it takes for the skier to land is

$$t = \frac{x}{v_x} = \frac{d\cos\alpha}{v_0\cos\theta_0} = \frac{(7.117 \text{ m})\cos(9.0°)}{(10 \text{ m/s})\cos(11.3°)} = 0.72 \text{ s}.$$

Using Eq. 4-23, the x-and y-components of the velocity at landing are

$$\begin{aligned} v_x &= v_0\cos\theta_0 = (10 \text{ m/s})\cos(11.3°) = 9.81 \text{ m/s} \\ v_y &= v_0\sin\theta_0 - gt = (10 \text{ m/s})\sin(11.3°) - (9.8 \text{ m/s}^2)(0.72 \text{ s}) = -5.07 \text{ m/s} \end{aligned}$$

Thus, the direction of travel at landing is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.07\text{ m/s}}{9.81\text{ m/s}}\right) = -27.3°.$$

or 27.3° below the horizontal. The result implies that the angle between the skier's path and the slope is $\phi = 27.3° - 9.0° = 18.3°$, or approximately 18° to two significant figures.

52. From Eq. 4-21, we find $t = x/v_{0x}$. Then Eq. 4-23 leads to

$$v_y = v_{0y} - gt = v_{0y} - \frac{gx}{v_{0x}}.$$

Since the slope of the graph is –0.500, we conclude

$$\frac{g}{v_{0x}} = \frac{1}{2} \Rightarrow v_{ox} = 19.6 \text{ m/s}.$$

And from the "y intercept" of the graph, we find v_{oy} = 5.00 m/s. Consequently,

$$\theta_o = \tan^{-1}(v_{oy}/v_{ox}) = 14.3° \approx 14°.$$

53. Let $y_0 = h_0$ = 1.00 m at x_0 = 0 when the ball is hit. Let $y_1 = h$ (the height of the wall) and x_1 describe the point where it first rises above the wall one second after being hit; similarly, $y_2 = h$ and x_2 describe the point where it passes back down behind the wall four seconds later. And y_f = 1.00 m at $x_f = R$ is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that v_x is constant, we have $x_2 - x_1$ = 50.0 m = v_{1x} (4.00 s), which leads to v_{1x} = 12.5 m/s. Thus, applied to the full six seconds of motion:

$$x_f - x_0 = R = v_x(6.00\text{ s}) = 75.0\text{ m}.$$

(b) We apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the wall,

$$y_2 - y_1 = 0 = v_{1y}(4.00\text{ s}) - \frac{1}{2}g(4.00\text{ s})^2$$

and obtain v_{1y} = 19.6 m/s. One second earlier, using $v_{1y} = v_{0y} - g$(1.00 s), we find $v_{0y} = 29.4$ m/s. Therefore, the velocity of the ball just after being hit is

$$\vec{v} = v_{0x}\hat{\text{i}} + v_{0y}\hat{\text{j}} = (12.5\text{ m/s})\,\hat{\text{i}} + (29.4\text{ m/s})\,\hat{\text{j}}$$

Its magnitude is $|\vec{v}| = \sqrt{(12.5\text{ m/s})^2 + (29.4\text{ m/s})^2} = 31.9\text{ m/s}.$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.4\text{ m/s}}{12.5\text{ m/s}}\right) = 67.0°.$$

We interpret this result as a velocity of magnitude 31.9 m/s, with angle (up from rightward) of 67.0°.

(d) During the first 1.00 s of motion, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields

$$h = 1.0\text{ m} + (29.4\text{ m/s})(1.00\text{ s}) - \tfrac{1}{2}(9.8\text{ m/s}^2)(1.00\text{ s})^2 = 25.5\text{ m}.$$

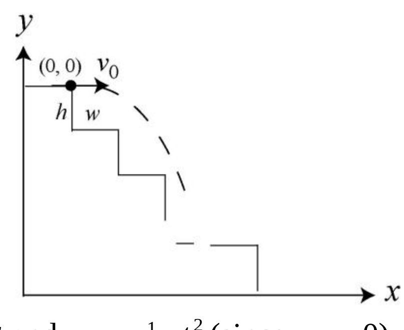
54. For $\Delta y = 0$, Eq. 4-22 leads to $t = 2v_o\sin\theta_o/g$, which immediately implies $t_{max} = 2v_o/g$ (which occurs for the "straight up" case: $\theta_o = 90°$). Thus,

$$\tfrac{1}{2}t_{max} = v_o/g \;\Rightarrow\; \tfrac{1}{2} = \sin\theta_o.$$

Therefore, the half-maximum-time flight is at angle θ_o = 30.0°. Since the least speed occurs at the top of the trajectory, which is where the velocity is simply the x-component of the initial velocity ($v_o\cos\theta_o = v_o\cos30°$ for the half-maximum-time flight), then we need to refer to the graph in order to find v_o – in order that we may complete the solution. In the graph, we note that the range is 240 m when θ_o = 45.0°. Equation 4-26 then leads to v_o = 48.5 m/s. The answer is thus (48.5 m/s)cos30.0° = 42.0 m/s.

55. **THINK** In this problem a ball rolls off the top of a stairway with an initial speed, and we'd like to know on which step it lands first.

EXPRESS We denote h as the height of a step and w as the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between $(n-1)w$ and nw. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction, as shown in the figure.



The coordinates of the ball at time t are given by $x = v_{0x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$).

ANALYZE We equate y to $-nh$ and solve for the time to reach the level of step n:

$$t = \sqrt{\frac{2nh}{g}}.$$

The x coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52 \text{ m/s})\sqrt{\frac{2n(0.203 \text{ m})}{9.8 \text{ m/s}^2}} = (0.309 \text{ m})\sqrt{n}.$$

The method is to try values of n until we find one for which x/w is less than n but greater than n – 1. For n = 1, x = 0.309 m and x/w = 1.52, which is greater than n. For n = 2, x = 0.437 m and x/w = 2.15, which is also greater than n. For n = 3, x = 0.535 m and x/w = 2.64. Now, this is less than n and greater than n – 1, so the ball hits the third step.

LEARN To check the consistency of our calculation, we can substitute n = 3 into the above equations. The results are t = 0.353 s, y = 0.609 m and x = 0.535 m. This indeed corresponds to the third step.

56. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find acceleration a.

(a) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is

$$r = (6.37 \times 10^6 + 640 \times 10^3) \text{ m} = 7.01 \times 10^6 \text{ m}.$$

Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(7.01\times 10^6 \text{ m})}{(98.0 \text{ min})(60 \text{ s/min})} = 7.49\times 10^3 \text{ m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(7.49\times 10^3 \text{ m/s})^2}{7.01\times 10^6 \text{ m}} = 8.00 \text{ m/s}^2.$$

57. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (toward the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83 \text{ m/s}^2$ east, then the center of the circle is *east* of this location. The distance is $r = v^2/a = (3.66 \text{ m/s})^2/(1.83 \text{ m/s}^2) = 7.32$ m.

(b) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(c) If the direction of $\vec{a}$ experienced by the passenger is now *south*—indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

58. (a) The circumference is $c = 2\pi r = 2\pi(0.15 \text{ m}) = 0.94$ m.

(b) With $T = (60 \text{ s})/1200 = 0.050$ s, the speed is $v = c/T = (0.94 \text{ m})/(0.050 \text{ s}) = 19$ m/s. This is equivalent to using Eq. 4-35.

(c) The magnitude of the acceleration is $a = v^2/r = (19 \text{ m/s})^2/(0.15 \text{ m}) = 2.4 \times 10^3 \text{ m/s}^2$.

(d) The period of revolution is $(1200 \text{ rev/min})^{-1} = 8.3 \times 10^{-4}$ min, which becomes, in SI units, $T = 0.050$ s = 50 ms.

59. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

(b) The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15 \text{ m})}{12 \text{ s}} = 7.85 \text{ m/s}$$

and his centripetal acceleration is $a = \dfrac{(7.85 \text{ m/s})^2}{15 \text{ m}} = 4.1 \text{ m/s}^2$.

(c) When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

(d) At the lowest point, the centripetal acceleration is $a = 4.1 \text{ m/s}^2$, same as part (b).

(e) The direction is up, toward the center of the orbit.

60. (a) During constant-speed circular motion, the velocity vector is perpendicular to the acceleration vector at every instant. Thus, $\vec{v} \cdot \vec{a} = 0$.

(b) The acceleration in this vector, at every instant, points toward the center of the circle, whereas the position vector points from the center of the circle to the object in motion. Thus, the angle between $\vec{r}$ and $\vec{a}$ is 180° at every instant, so $\vec{r} \times \vec{a} = 0$.

61. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find centripetal acceleration a.

(a) $v = 2\pi r/T = 2\pi(20 \text{ km})/1.0 \text{ s} = 126 \text{ km/s} = 1.3 \times 10^5$ m/s.

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(126 \text{ km/s})^2}{20 \text{ km}} = 7.9 \times 10^5 \text{ m/s}^2.$$

(c) Clearly, both *v* and *a* will increase if *T* is reduced.

62. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10 \text{ m/s})^2}{25 \text{ m}} = 4.0 \text{ m/s}^2.$$

63. We first note that $\vec{a}_1$ (the acceleration at t_1 = 2.00 s) is perpendicular to $\vec{a}_2$ (the acceleration at t_2=5.00 s), by taking their scalar (dot) product:

$$\vec{a}_1 \cdot \vec{a}_2 = [(6.00 \text{ m/s}^2)\hat{\text{i}} + (4.00 \text{ m/s}^2)\hat{\text{j}}] \cdot [(4.00 \text{ m/s}^2)\hat{\text{i}} + (-6.00 \text{ m/s}^2)\hat{\text{j}}] = 0.$$

Since the acceleration vectors are in the (negative) radial directions, then the two positions (at t_1 and t_2) are a quarter-circle apart (or three-quarters of a circle, depending on whether one measures clockwise or counterclockwise). A quick sketch leads to the conclusion that if the particle is moving counterclockwise (as the problem states) then it travels three-quarters of a circumference in moving from the position at time t_1 to the position at time t_2 . Letting *T* stand for the period, then $t_2 - t_1$ = 3.00 s = 3*T*/4. This gives *T* = 4.00 s. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(6.00 \text{ m/s}^2)^2 + (4.00 \text{ m/s})^2} = 7.21 \text{ m/s}^2.$$

Using Eqs. 4-34 and 4-35, we have $a = 4\pi^2 r / T^2$, which yields

$$r = \frac{aT^2}{4\pi^2} = \frac{(7.21 \text{ m/s}^2)(4.00 \text{ s})^2}{4\pi^2} = 2.92 \text{ m}.$$

64. When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points toward the center. Thus, the center is “straight up” from the cited point.

(a) Since the center is “straight up” from (4.00 m, 4.00 m), the *x* coordinate of the center is 4.00 m.

(b) To find out “how far up” we need to know the radius. Using Eq. 4-34 we find

$$r = \frac{v^2}{a} = \frac{(5.00\text{ m/s})^2}{12.5\ \text{ m/s}^2} = 2.00\text{ m}.$$

Thus, the y coordinate of the center is 2.00 m + 4.00 m = 6.00 m. Thus, the center may be written as (x, y) = (4.00 m, 6.00 m).

65. Since the period of a uniform circular motion is $T = 2\pi r / v$, where r is the radius and v is the speed, the centripetal acceleration can be written as

$$a = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2}.$$

Based on this expression, we compare the (magnitudes) of the wallet and purse accelerations, and find their ratio is the ratio of r values. Therefore, $a_{\text{wallet}} = 1.50\ a_{\text{purse}}$. Thus, the wallet acceleration vector is

$$a = 1.50[(2.00\text{ m/s}^2)\hat{\text{i}} + (4.00\text{ m/s}^2)\hat{\text{j}}] = (3.00\text{ m/s}^2)\hat{\text{i}} + (6.00\text{ m/s}^2)\hat{\text{j}}.$$

66. The fact that the velocity is in the $+y$ direction and the acceleration is in the $+x$ direction at t_1 = 4.00 s implies that the motion is clockwise. The position corresponds to the "9:00 position." On the other hand, the position at t_2 = 10.0 s is in the "6:00 position" since the velocity points in the $-x$ direction and the acceleration is in the $+y$ direction. The time interval $\Delta t = 10.0\text{ s} - 4.00\text{ s} = 6.00\text{ s}$ is equal to 3/4 of a period:

$$6.00\text{ s} = \frac{3}{4}T \quad \Rightarrow \quad T = 8.00\text{ s}.$$

Equation 4-35 then yields

$$r = \frac{vT}{2\pi} = \frac{(3.00\text{ m/s})(8.00\text{ s})}{2\pi} = 3.82\text{ m}.$$

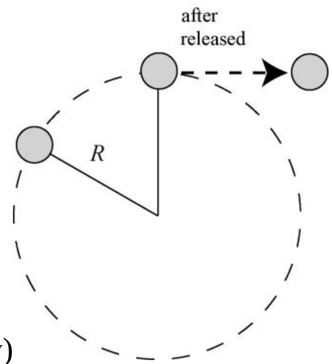
(a) The x coordinate of the center of the circular path is $x = 5.00\text{ m} + 3.82\text{ m} = 8.82\text{ m}.$

(b) The y coordinate of the center of the circular path is $y = 6.00\text{ m}.$

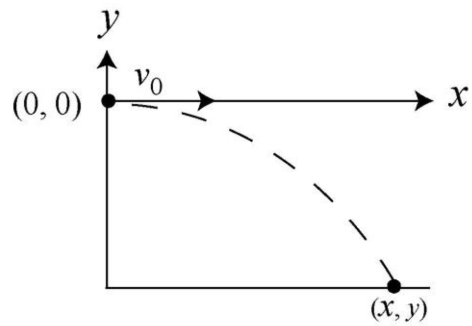
In other words, the center of the circle is at (x,y) = (8.82 m, 6.00 m).

67. **THINK** In this problem we have a stone whirled in a horizontal circle. After the string breaks, the stone undergoes projectile motion.

EXPRESS The stone moves in a circular path (top view shown below left) initially, but undergoes projectile motion after the string breaks (side view shown below right). Since $a = v^2 / R$, to calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed.



(top view)



(side view)

Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0y} = 0$). It hits the ground at $x = 10$ m and $y = -2.0$ m.

ANALYZE Formally solving the y-component equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\text{ m})\sqrt{-\frac{9.8\text{ m/s}^2}{2(-2.0\text{ m})}} = 15.7\text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v_0^2}{R} = \frac{(15.7\text{ m/s})^2}{1.5\text{ m}} = 160\text{ m/s}^2.$$

LEARN The above equations can be combined to give $a = \dfrac{gx^2}{-2yR}$. The equation implies that the greater the centripetal acceleration, the greater the initial speed of the projectile, and the greater the distance traveled by the stone. This is precisely what we expect.

68. We note that after three seconds have elapsed ($t_2 - t_1 = 3.00$ s) the velocity (for this object in circular motion of period T) is reversed; we infer that it takes three seconds to reach the opposite side of the circle. Thus, $T = 2(3.00\text{ s}) = 6.00$ s.

(a) Using Eq. 4-35, $r = vT/2\pi$, where $v = \sqrt{(3.00\text{ m/s})^2 + (4.00\text{ m/s})^2} = 5.00\text{ m/s}$, we obtain $r = 4.77$ m. The magnitude of the object's centripetal acceleration is therefore $a = v^2/r = 5.24\text{ m/s}^2$.

(b) The average acceleration is given by Eq. 4-15:

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{(-3.00\hat{\text{i}} - 4.00\hat{\text{j}})\ \text{m/s} - (3.00\hat{\text{i}} + 4.00\hat{\text{j}})\ \text{m/s}}{5.00\ \text{s} - 2.00\ \text{s}} = (-2.00\ \text{m/s}^2)\hat{\text{i}} + (-2.67\ \text{m/s}^2)\hat{\text{j}}$$

which implies $|\vec{a}_{\text{avg}}| = \sqrt{(-2.00\ \text{m/s}^2)^2 + (-2.67\ \text{m/s}^2)^2} = 3.33\ \text{m/s}^2$.

69. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so $20\ \text{km/h} \to 5.6\ \text{m/s}$, $30\ \text{km/h} \to 8.3\ \text{m/s}$, and $45\ \text{km/h} \to 12.5\ \text{m/s}$. We choose east as the $+\hat{\text{i}}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{\text{c t}} = \vec{v}_{\text{c g}} - \vec{v}_{\text{t g}} = (12.5\ \text{m/s})\ \hat{\text{i}} - (-5.6\ \text{m/s})\ \hat{\text{i}} = (18.1\ \text{m/s})\ \hat{\text{i}}$$

relative to the truck. Since the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is $(-8.3\ \text{m/s})\hat{\text{i}}$, the (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\text{avg}} = \frac{(18.1\ \text{m/s})\hat{\text{i}} - (-8.3\ \text{m/s})\hat{\text{i}}}{2.0\ \text{s}} = (13\ \text{m/s}^2)\hat{\text{i}},$$

or $|\vec{a}_{\text{avg}}| = 13\ \text{m/s}^2$.

(b) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

(c) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{0\text{cg}} = \vec{v}_{0\text{ct}} + \vec{v}_{0\text{tg}} = (-8.3\ \text{m/s})\hat{\text{i}} + (-5.6\ \text{m/s})\hat{\text{i}} = (-13.9\ \text{m/s})\hat{\text{i}}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\text{avg}} = \frac{(12.5\ \text{m/s})\hat{\text{i}} - (-13.9\ \text{m/s})\hat{\text{i}}}{2.0\ \text{s}} = (13\ \text{m/s}^2)\hat{\text{i}}, \quad |\vec{a}_{\text{avg}}| = 13\ \text{m/s}^2$$

identical to the result of part (a).

(d) The direction of $\vec{a}_{\text{avg}}$ is $+\hat{\text{i}}$, or eastward.

70. We use Eq. 4-44, noting that the upstream corresponds to the $+\hat{\text{i}}$ direction.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = (14 \text{ km/h})\ \hat{i} + (-9 \text{ km/h})\ \hat{i} = (5 \text{ km/h})\ \hat{i}.$$

Thus, the magnitude is $|\vec{v}_{bg}| = 5$ km/h.

(b) The direction of $\vec{v}_{bg}$ is $+x$, or upstream.

(c) We use the subscript c for the child, and obtain

$$\vec{v}_{c\,g} = \vec{v}_{c\,b} + \vec{v}_{b\,g} = (-6 \text{ km/h})\ \hat{i} + (5 \text{ km/h})\ \hat{i} = (-1 \text{ km/h})\ \hat{i}\ .$$

The magnitude is $|\vec{v}_{cg}| = 1$ km/h.

(d) The direction of $\vec{v}_{cg}$ is $-x$, or downstream.

71. While moving in the same direction as the sidewalk's motion (covering a distance d relative to the ground in time t_1 = 2.50 s), Eq. 4-44 leads to

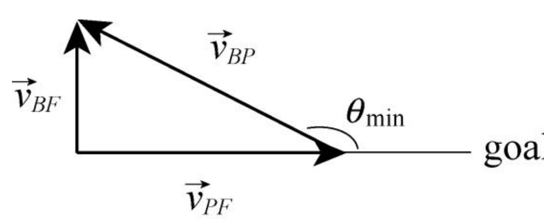
$$v_{\text{sidewalk}} + v_{\text{man running}} = \frac{d}{t_1}\ .$$

While he runs back (taking time t_2 = 10.0 s) we have

$$v_{\text{sidewalk}} - v_{\text{man running}} = -\frac{d}{t_2}\ .$$

Dividing these equations and solving for the desired ratio, we get $\frac{12.5}{7.5} = \frac{5}{3}$ = 1.67.

72. We denote the velocity of the player with $\vec{v}_{PF}$ and the relative velocity between the player and the ball be $\vec{v}_{BP}$. Then the velocity $\vec{v}_{BF}$ of the ball relative to the field is given by $\vec{v}_{BF} = \vec{v}_{PF} + \vec{v}_{BP}$. The smallest angle θ_{min} corresponds to the case when $\vec{v}_{BF} \perp \vec{v}_{PF}$. Hence,



$$\theta_{\text{min}} = 180° - \cos^{-1}\left(\frac{|\vec{v}_{PF}|}{|\vec{v}_{BP}|}\right) = 180° - \cos^{-1}\left(\frac{4.0 \text{ m/s}}{6.0 \text{ m/s}}\right) = 130°.$$

73. We denote the police and the motorist with subscripts p and m, respectively. The coordinate system is indicated in Fig. 4-46.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m\,p} = \vec{v}_m - \vec{v}_p = (-60 \text{ km/h})\hat{j} - (-80 \text{ km/h})\hat{i} = (80 \text{ km/h})\hat{i} - (60 \text{ km/h})\hat{j}.$$

(b) $\vec{v}_{mp}$ does happen to be along the line of sight. Referring to Fig. 4-46, we find the vector pointing from one car to another is $\vec{r} = (800\text{ m})\hat{\text{i}} - (600\text{ m})\hat{\text{j}}$ (from *M* to *P*). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{mp}$, they must point the same direction.

(c) No, they remain unchanged.

74. Velocities are taken to be constant; thus, the velocity of the plane relative to the ground is $\vec{v}_{PG} = (55\text{ km})/(1/4\text{ hour})\ \hat{\text{j}} = (220\text{ km/h})\hat{\text{j}}$. In addition,

$$\vec{v}_{AG} = (42\text{ km/h})(\cos 20°\hat{\text{i}} - \sin 20°\hat{\text{j}}) = (39\text{ km/h})\hat{\text{i}} - (14\text{ km/h})\hat{\text{j}}.$$

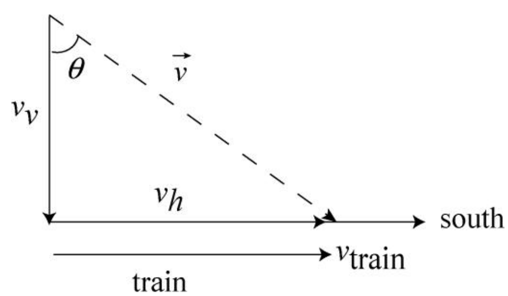
Using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$, we have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -(39\text{ km/h})\hat{\text{i}} + (234\text{ km/h})\hat{\text{j}}.$$

which implies $|\vec{v}_{PA}| = 237\text{ km/h}$, or 240 km/h (to two significant figures.)

75. **THINK** This problem deals with relative motion in two dimensions. Raindrops appear to fall vertically by an observer on a moving train.

EXPRESS Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop, v_h = 30 m/s, must be the same as the speed of the train, i.e., $v_h = v_{\text{train}}$ (see figure).



On the other hand, if v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then tan $\theta = v_h/v_v$. Knowing v_v and v_h allows us to determine the speed of the raindrops.

ANALYZE With $\theta = 70°$, we find the vertical component of the velocity to be

$$v_v = v_h/\tan\theta = (30\text{ m/s})/\tan 70° = 10.9\text{ m/s}.$$

Therefore, the speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30\text{ m/s})^2 + (10.9\text{ m/s})^2} = 32\text{ m/s}.$$

LEARN As long as the horizontal component of the velocity of the raindrops coincides with the speed of the train, the passenger on board will see the rain falling perfectly vertically.

76. The destination is $\vec{D}$ = 800 km $\hat{\text{j}}$ where we orient axes so that +*y* points north and +*x* points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is $\vec{v}_{\text{pg}}$ = (400 km/h) $\hat{\text{j}}$. This must be the vector sum of the plane's velocity with respect to the air which has (*x*,*y*) components (500cos70º, 500sin70º), and the velocity of the air (*wind*) relative to the ground $\vec{v}_{\text{ag}}$. Thus,

$$(400 \text{ km/h})\,\hat{\text{j}} = (500 \text{ km/h}) \cos 70^\circ\,\hat{\text{i}} + (500 \text{ km/h}) \sin 70^\circ\,\hat{\text{j}} + \vec{v}_{\text{ag}}$$

which yields

$$\vec{v}_{\text{ag}} = (-171 \text{ km/h})\hat{\text{i}} - (70.0 \text{ km/h})\hat{\text{j}}.$$

(a) The magnitude of $\vec{v}_{\text{ag}}$ is $|\vec{v}_{\text{ag}}| = \sqrt{(-171 \text{ km/h})^2 + (-70.0 \text{ km/h})^2} = 185 \text{ km/h}.$

(b) The direction of $\vec{v}_{\text{ag}}$ is

$$\theta = \tan^{-1}\left(\frac{-70.0 \text{ km/h}}{-171 \text{ km/h}}\right) = 22.3^\circ \ \ (\text{south of west}).$$

77. **THINK** This problem deals with relative motion in two dimensions. Snowflakes falling vertically downward are seen to fall at an angle by a moving observer.

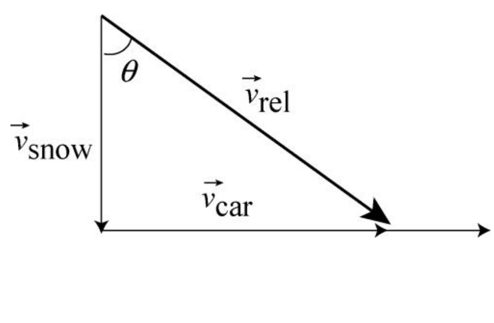
EXPRESS Relative to the car the velocity of the snowflakes has a vertical component of $v_v = 8.0 \text{ m/s}$ and a horizontal component of $v_h = 50 \text{ km/h} = 13.9 \text{ m/s}$.

ANALYZE The angle θ from the vertical is found from

$$\tan\theta = \frac{v_h}{v_v} = \frac{13.9 \text{ m/s}}{8.0 \text{ m/s}} = 1.74$$

which yields $\theta = 60°$.

LEARN The problem can also be solved by expressing the velocity relation in vector notation: $\vec{v}_{\text{rel}} = \vec{v}_{\text{car}} + \vec{v}_{\text{snow}}$, as shown in the figure.



78. We make use of Eq. 4-44 and Eq. 4-45.

The velocity of Jeep *P* relative to *A* at the instant is

$$\vec{v}_{PA} = (40.0\text{ m/s})(\cos 60°\hat{\text{i}} + \sin 60°\hat{\text{j}}) = (20.0\text{ m/s})\hat{\text{i}} + (34.6\text{ m/s})\hat{\text{j}}.$$

Similarly, the velocity of Jeep *B* relative to *A* at the instant is

$$\vec{v}_{BA} = (20.0\text{ m/s})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (17.3\text{ m/s})\hat{\text{i}} + (10.0\text{ m/s})\hat{\text{j}}.$$

Thus, the velocity of *P* relative to *B* is

$$\vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA} = (20.0\hat{\text{i}} + 34.6\hat{\text{j}})\text{ m/s} - (17.3\hat{\text{i}} + 10.0\hat{\text{j}})\text{ m/s} = (2.68\text{ m/s})\hat{\text{i}} + (24.6\text{ m/s})\hat{\text{j}}.$$

(a) The magnitude of $\vec{v}_{PB}$ is $|\vec{v}_{PB}| = \sqrt{(2.68\text{ m/s})^2 + (24.6\text{ m/s})^2} = 24.8\text{ m/s}.$

(b) The direction of $\vec{v}_{PB}$ is $\theta = \tan^{-1}[(24.6\text{ m/s})/(2.68\text{ m/s})] = 83.8°$ north of east (or 6.2° east of north).

(c) The acceleration of *P* is

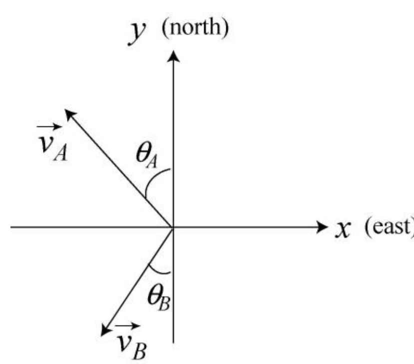
$$\vec{a}_{PA} = (0.400\text{ m/s}^2)(\cos 60.0°\hat{\text{i}} + \sin 60.0°\hat{\text{j}}) = (0.200\text{ m/s}^2)\hat{\text{i}} + (0.346\text{ m/s}^2)\hat{\text{j}},$$

and $\vec{a}_{PA} = \vec{a}_{PB}$. Thus, we have $|\vec{a}_{PB}| = 0.400\text{ m/s}^2$.

(d) The direction is 60.0° north of east (or 30.0° east of north).

79. **THINK** This problem involves analyzing the relative motion of two ships sailing in different directions.

EXPRESS Given that $\theta_A = 45°$, and $\theta_B = 40°$, as defined in the figure, the velocity vectors (relative to the shore) for ships *A* and *B* are given by



$$\vec{v}_A = -(v_A \cos 45°)\,\hat{\text{i}} + (v_A \sin 45°)\,\hat{\text{j}}$$
$$\vec{v}_B = -(v_B \sin 40°)\,\hat{\text{i}} - (v_B \cos 40°)\,\hat{\text{j}},$$

with v_A = 24 knots and v_B = 28 knots. We take east as $+\hat{\text{i}}$ and north as $\hat{\text{j}}$.

The velocity of ship *A* relative to ship *B* is simply given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.

ANALYZE (a) The relative velocity is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = (v_B \sin 40° - v_A \cos 45°)\hat{\mathrm{i}} + (v_B \cos 40° + v_A \sin 45°)\,\hat{\mathrm{j}}$$
$$= (1.03\ \text{knots})\hat{\mathrm{i}} + (38.4\ \text{knots})\hat{\mathrm{j}}$$

the magnitude of which is $|\vec{v}_{AB}| = \sqrt{(1.03\ \text{knots})^2 + (38.4\ \text{knots})^2} \approx 38.4$ knots.

(b) The angle θ_{AB} which $\vec{v}_{AB}$ makes with north is given by

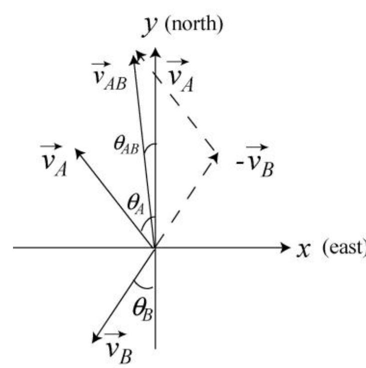
$$\theta_{AB} = \tan^{-1}\left(\frac{v_{AB,x}}{v_{AB,y}}\right) = \tan^{-1}\left(\frac{1.03\ \text{knots}}{38.4\ \text{knots}}\right) = 1.5°$$

which is to say that $\vec{v}_{AB}$ points 1.5° east of north.

(c) Since the two ships started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{AB}|}{|\vec{v}_{AB}|} = \frac{160\ \text{nautical miles}}{38.4\ \text{knots}} = 4.2\ \text{h}.$$

(d) The velocity $\vec{v}_{AB}$ does not change with time in this problem, and $\vec{r}_{AB}$ is in the same direction as $\vec{v}_{AB}$ since they started at the same time. Reversing the points of view, we have $\vec{v}_{AB} = -\vec{v}_{BA}$ so that $\vec{r}_{AB} = -\vec{r}_{BA}$ (i.e., they are 180° opposite to each other). Hence, we conclude that *B* stays at a bearing of 1.5° west of south relative to *A* during the journey (neglecting the curvature of Earth).



LEARN The relative velocity is depicted in the figure on the right. When analyzing relative motion in two dimensions, a vector diagram such as the one shown can be very helpful.

80. This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to +*x* and *north* corresponds to +*y*. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.0\angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.0\hat{\mathrm{i}}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0\angle 120°)$ where we have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the +*x* axis), or $\vec{v}_{BW} = (-4.0\hat{\mathrm{i}}+6.9\hat{\mathrm{j}})$ m/s.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = v_{BW} + \vec{v}_{WG} = (2.0\text{ m/s})\hat{\text{i}} + (-4.0\hat{\text{i}}+6.9\hat{\text{j}})\text{ m/s} = (-2.0\text{ m/s})\hat{\text{i}} + (6.9\text{ m/s})\hat{\text{j}}.$$

Thus, we find $|\vec{v}_{BG}| = 7.2\text{ m/s}$.

(b) The direction of $\vec{v}_{BG}$ is $\theta = \tan^{-1}[(6.9\text{ m/s})/(-2.0\text{ m/s})] = 106°$ (measured counterclockwise from the +*x* axis), or 16° west of north.

(c) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $(200\text{ m}) = (7.2\text{ m/s})\sin(106°)t \rightarrow t = 29\text{ s}$. Note: If a student obtains "28 s," then the student has probably neglected to take the *y* component properly (a common mistake).

81. Here, the subscript *W* refers to the water. Our coordinates are chosen with +*x* being *east* and +*y* being *north*. In these terms, the angle specifying *east* would be 0° and the angle specifying *south* would be –90° or 270°. Where the length unit is not displayed, km is to be understood.

(a) We have $\vec{v}_{AW} = \vec{v}_{AB} + \vec{v}_{BW}$, so that

$$\vec{v}_{AB} = (22\ \angle -90°) - (40\ \angle\ 37°) = (56\ \angle -125°)$$

in the magnitude-angle notation (conveniently done with a vector-capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{AB} = (-32\text{km/h})\ \hat{\text{i}} - (46\text{ km/h})\ \hat{\text{j}}\ .$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward $(\vec{r} - \vec{r}_0 = \int \vec{v}\ dt)$

$$\vec{r} = (2.5 - 32t)\ \hat{\text{i}}\ + (4.0 - 46t)\ \hat{\text{j}}$$

with lengths in kilometers and time in hours.

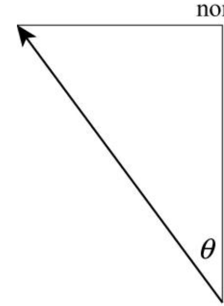
(c) The magnitude of this $\vec{r}$ is $r = \sqrt{(2.5-32t)^2 + (4.0-46t)^2}$. We minimize this by taking a derivative and requiring it to equal zero — which leaves us with an equation for *t*

$$\frac{dr}{dt} = \frac{1}{2}\frac{6286t - 528}{\sqrt{(2.5-32t)^2 + (4.0-46t)^2}} = 0$$

which yields *t* = 0.084 h.

(d) Plugging this value of t back into the expression for the distance between the ships (r), we obtain r = 0.2 km. Of course, the calculator offers more digits (r = 0.225…), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.

82. We construct a right triangle starting from the clearing on the south bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82\text{ m}) + (1.1\text{ m/s})t$, where the t-dependent contribution is the distance that the river will carry the boat downstream during time t.



The hypotenuse of this right triangle (the arrow in our sketch) also depends on t and on the boat's speed (relative to the water), and we set it equal to the Pythagorean "sum" of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for t

$$46724 + 180.4t - 14.8t^2 = 0.$$

(b) We solve for t first and find a positive value: t = 62.6 s.

(a) The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured "west of north") is then given by

$$\theta = \tan^{-1}\left(\frac{82 + 1.1t}{200}\right) = \tan^{-1}\left(\frac{151}{200}\right) = 37°.$$

83. We establish coordinates with $\hat{\text{i}}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\text{j}}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}|$ = 6.4 km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = (3.2\text{ km/h})\hat{\text{j}}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{\text{i}}$ where $v_{bg} > 0$ is unknown. Thus, all $\hat{\text{j}}$ components must cancel in the vector sum $\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$, which means the $\vec{v}_{bw}$ sin θ = (–3.2 km/h) $\hat{\text{j}}$, so

$$\theta = \sin^{-1}[(-3.2\text{ km/h})/(6.4\text{ km/h})] = -30°.$$

(b) Using the result from part (a), we find $v_{bg} = v_{bw} \cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of (6.4 km)/(5.5 km/h) = 1.15 h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with $v_{wg} =$ 3.2 km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where $D = 3.2$ km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = \frac{D}{v_{bw} - v_{wg}} + \frac{D}{v_{bw} + v_{wg}}$$

the answer is the same as in the previous part, that is, $t_{\text{total}} = 80 \text{ min}$.

(e) The shortest-time path should have $\theta = 0°$. This can also be shown by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta\ \hat{\mathrm{i}} + (v_{bw}\sin\theta + v_{wg})\ \hat{\mathrm{j}}$$

where the x component of $\vec{v}_{bg}$ must equal l/t. Thus,

$$t = \frac{l}{v_{bw}\cos\theta}$$

which can be minimized using $dt/d\theta = 0$.

(f) The above expression leads to $t = (6.4 \text{ km})/(6.4 \text{ km/h}) = 1.0$ h, or 60 min.

84. Relative to the sled, the launch velocity is $\vec{v}_{0\text{rel}} = v_{ox}\,\hat{\mathrm{i}} + v_{oy}\,\hat{\mathrm{j}}$. Since the sled's motion is in the negative direction with speed v_s (note that we are treating v_s as a positive number, so the sled's velocity is actually $-v_s\,\hat{\mathrm{i}}$), then the launch velocity relative to the ground is $\vec{v}_0 = (v_{ox} - v_s)\,\hat{\mathrm{i}} + v_{oy}\,\hat{\mathrm{j}}$. The horizontal and vertical displacement (relative to the ground) are therefore

$$x_{\text{land}} - x_{\text{launch}} = \Delta x_{\text{bg}} = (v_{ox} - v_s)\, t_{\text{flight}}$$

$$y_{\text{land}} - y_{\text{launch}} = 0 = v_{oy}\, t_{\text{flight}} + \tfrac{1}{2}(-g)(t_{\text{flight}})^2 .$$

Combining these equations leads to

$$\Delta x_{\rm bg} = \frac{2v_{0x}v_{0y}}{g} - \left(\frac{2v_{0y}}{g}\right)v_s.$$

The first term corresponds to the "y intercept" on the graph, and the second term (in parentheses) corresponds to the magnitude of the "slope." From the figure, we have

$$\Delta x_{bg} = 40 - 4v_s.$$

This implies v_{oy} = (4.0 s)(9.8 m/s^2)/2 = 19.6 m/s, and that furnishes enough information to determine v_{ox}.

(a) $v_{ox} = 40g/2v_{oy}$ = (40 m)(9.8 m/s^2)/(39.2 m/s) = 10 m/s.

(b) As noted above, v_{oy} = 19.6 m/s.

(c) Relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, so $\Delta x_{bs} = v_{ox}\, t_{\rm flight}$ = 40 m.

(d) As in (c), relative to the sled, the displacement Δx_{bs} does not depend on the sled's speed, and $\Delta x_{bs} = v_{ox}\, t_{\rm flight}$ = 40 m.

85. Using displacement = velocity × time (for each constant-velocity part of the trip), along with the fact that 1 hour = 60 minutes, we have the following vector addition exercise (using notation appropriate to many vector-capable calculators):

(1667 m ∠ 0º) + (1333 m ∠ –90º) + (333 m ∠ 180º) + (833 m ∠ –90º) + (667 m ∠ 180º) + (417 m ∠ –90º) = (2668 m ∠ –76º).

(a) Thus, the magnitude of the net displacement is 2.7 km.

(b) Its direction is 76° clockwise (relative to the initial direction of motion).

86. We use a coordinate system with $+x$ eastward and $+y$ upward.

(a) We note that 123° is the angle between the initial position and later position vectors, so that the angle from $+x$ to the later position vector is 40° + 123° = 163°. In unit-vector notation, the position vectors are

$$\vec{r}_1 = (360\text{ m})\cos(40°)\,\hat{\text{i}} + (360\text{ m})\sin(40°)\,\hat{\text{j}} = (276\text{ m})\hat{\text{i}} + (231\text{ m})\,\hat{\text{j}}$$
$$\vec{r}_2 = (790\text{ m})\cos(163°)\,\hat{\text{i}} + (790\text{ m})\sin(163°)\,\hat{\text{j}} = (-755\text{ m})\hat{\text{i}} + (231\text{ m})\hat{\text{j}}$$

respectively. Consequently, we plug into Eq. 4-3

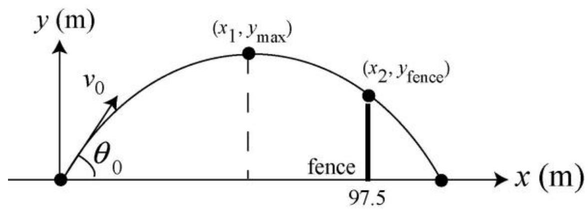
$$\Delta \vec{r} = [(-755 \text{ m}) - (276 \text{ m})]\hat{\text{i}} + (231 \text{ m} - 231 \text{ m})\hat{\text{j}} = -(1031 \text{ m})\,\hat{\text{i}}.$$

The magnitude of the displacement $\Delta\vec{r}$ is $|\Delta\vec{r}| = 1031 \text{ m}$.

(b) The direction of $\Delta\vec{r}$ is $-\hat{\text{i}}$, or westward.

87. **THINK** This problem deals with the projectile motion of a baseball. Given the information on the position of the ball at two instants, we are asked to analyze its trajectory.

EXPRESS The trajectory of the baseball is shown in the figure on the right. According to the problem statement, at $t_1 = 3.0\text{ s}$, the ball reaches it maximum height $y_{\max}$, and at $t_2 = t_1 + 2.5\text{ s} = 5.5\text{ s}$, it barely clears a fence at $x_2 = 97.5 \text{ m}$.



Eq. 2-15 can be applied to the vertical (y axis) motion related to reaching the maximum height (when t_1 = 3.0 s and v_y = 0):

$$y_{\max} - y_0 = v_y t - \frac{1}{2}gt^2 .$$

ANALYZE (a) With ground level chosen so y_0 = 0, this equation gives the result

$$y_{\max} = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(3.0 \text{ s})^2 = 44.1 \text{ m}$$

(b) After the moment it reached maximum height, it is falling; at $t_2 = t_1 + 2.5\text{ s} = 5.5\text{ s}$, it will have fallen an amount given by Eq. 2-18:

$$y_{\text{fence}} - y_{\max} = 0 - \frac{1}{2}g(t_2 - t_1)^2 .$$

Thus, the height of the fence is

$$y_{\text{fence}} = y_{\max} - \frac{1}{2}g(t_2 - t_1)^2 = 44.1 \text{ m} - \frac{1}{2}(9.8 \text{ m/s}^2)(2.5 \text{ s})^2 = 13.48 \text{ m} .$$

(c) Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find from 97.5 m = v_{0x}(5.5 s) that v_{0x} = 17.7 m/s. The total flight time of the ball is $T = 2t_1 = 2(3.0\text{ s}) = 6.0\text{ s}$. Thus, the range of the baseball is

$$R = v_{0x}T = (17.7 \text{ m/s})(6.0 \text{ s}) = 106.4 \text{ m}$$

which means that the ball travels an additional distance

$$\Delta x = R - x_2 = 106.4 \text{ m} - 97.5 \text{ m} = 8.86 \text{ m}$$

beyond the fence before striking the ground.

LEARN Part (c) can also be solved by noting that after passing the fence, the ball will strike the ground in 0.5 s (so that the total "fall-time" equals the "rise-time"). With v_{0x} = 17.7 m/s, we have Δx = (17.7 m/s)(0.5 s) = 8.86 m.

88. When moving in the same direction as the jet stream (of speed v_s), the time is

$$t_1 = \frac{d}{v_{ja} + v_s},$$

where d = 4000 km is the distance and v_{ja} is the speed of the jet relative to the air (1000 km/h). When moving against the jet stream, the time is

$$t_2 = \frac{d}{v_{ja} - v_s},$$

where $t_2 - t_1 = \frac{70}{60}$ h . Combining these equations and using the quadratic formula to solve gives v_s = 143 km/h.

89. **THINK** We have a particle moving in a two-dimensional plane with a constant acceleration. Since the x and y components of the acceleration are constants, we can use Table 2-1 for the motion along both axes.

EXPRESS Using vector notation with $\vec{r}_0 = 0$, the position and velocity of the particle as a function of time are given by $\vec{r}(t) = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$ and $\vec{v}(t) = \vec{v}_0 + \vec{a}t$, respectively. Where units are not shown, SI units are to be understood.

ANALYZE (a) Given the initial velocity $\vec{v}_0 = (8.0 \text{ m/s})\hat{\text{j}}$ and the acceleration $\vec{a} = (4.0 \text{ m/s}^2)\hat{\text{i}} + (2.0 \text{ m/s}^2)\hat{\text{j}}$, the position vector of the particle is

$$\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 = \left(8.0\,\hat{\text{j}}\right)t + \frac{1}{2}\left(4.0\,\hat{\text{i}} + 2.0\,\hat{\text{j}}\right)t^2 = \left(2.0t^2\right)\hat{\text{i}} + \left(8.0t + 1.0t^2\right)\hat{\text{j}}.$$

Therefore, the time that corresponds to x = 29 m can be found by solving the equation $2.0t^2 = 29$, which leads to t = 3.8 s. The y coordinate at that time is

$$y = (8.0 \text{ m/s})(3.8 \text{ s}) + (1.0 \text{ m/s}^2)(3.8 \text{ s})^2 = 45 \text{ m}.$$

(b) The velocity of the particle is given by $\vec{v} = \vec{v}_0 + \vec{a}t$. Thus, at t = 3.8 s, the velocity is

$$\vec{v} = (8.0 \text{ m/s})\,\hat{\text{j}} + \left((4.0 \text{ m/s}^2)\,\hat{\text{i}} + (2.0 \text{ m/s}^2)\,\hat{\text{j}}\right)(3.8 \text{ s}) = (15.2 \text{ m/s})\,\hat{\text{i}} + (15.6 \text{ m/s})\,\hat{\text{j}}$$

which has a magnitude of $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2 \text{ m/s})^2 + (15.6 \text{ m/s})^2} = 22 \text{ m/s}.$

LEARN Instead of using the vector notation, we can also deal with the x- and the y-components individually.

90. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields v_0 = 23 ft/s for g = 32 ft/s^2, x = 13 ft, y = 3 ft and θ_0 = 55°.

91. We make use of Eq. 4-25.

(a) By rearranging Eq. 4-25, we obtain the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2(x\tan\theta_0 - y)}}$$

which yields $v_0 = 255.5 \approx 2.6 \times 10^2$ m/s for x = 9400 m, y = –3300 m, and θ_0 = 35°.

(b) From Eq. 4-21, we obtain the time of flight:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{9400 \text{ m}}{(255.5 \text{ m/s})\cos 35°} = 45 \text{ s}.$$

(c) We expect the air to provide resistance but no appreciable lift to the rock, so we would need a greater launching speed to reach the same target.

92. We apply Eq. 4-34 to solve for speed v and Eq. 4-35 to find the period T.

(a) We obtain

$$v = \sqrt{ra} = \sqrt{(5.0 \text{ m})(7.0)(9.8 \text{ m/s}^2)} = 19 \text{ m/s}.$$

(b) The time to go around once (the period) is $T = 2\pi r/v$ = 1.7 s. Therefore, in one minute (t = 60 s), the astronaut executes

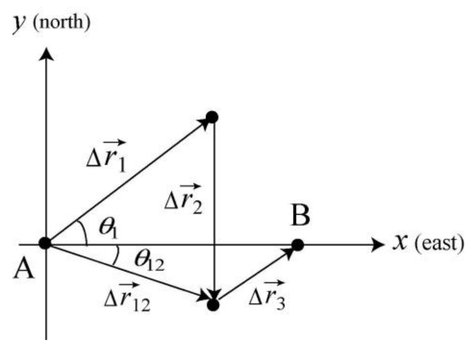
$$\frac{t}{T} = \frac{60\text{ s}}{1.7\text{ s}} = 35$$

revolutions. Thus, 35 rev/min is needed to produce a centripetal acceleration of 7*g* when the radius is 5.0 m.

(c) As noted above, T = 1.7 s.

93. **THINK** This problem deals with the two-dimensional kinematics of a desert camel moving from oasis A to oasis B.

EXPRESS The journey of the camel is illustrated in the figure on the right. We use a 'standard' coordinate system with +*x* East and +*y* North. Lengths are in kilometers and times are in hours. Using vector notation, we write the displacements for the first two segments of the trip as:



$$\Delta\vec{r}_1 = (75\text{ km})\cos(37°)\,\hat{\text{i}} + (75\text{ km})\sin(37°)\,\hat{\text{j}}$$

$$\Delta\vec{r}_2 = (-65\text{ km})\,\hat{\text{j}}$$

The net displacement is $\Delta\vec{r}_{12} = \Delta\vec{r}_1 + \Delta\vec{r}_2$. As can be seen from the figure, to reach oasis B requires an additional displacement $\Delta\vec{r}_3$.

ANALYZE (a) We perform the vector addition of individual displacements to find the net displacement of the camel: $\Delta\vec{r}_{12} = \Delta\vec{r}_1 + \Delta\vec{r}_2 = (60\text{ km})\,\hat{\text{i}} - (20\text{ km})\,\hat{\text{j}}$. Its corresponding magnitude is

$$|\Delta\vec{r}_{12}| = \sqrt{(60\text{ km})^2 + (-20\text{ km})^2} = 63\text{ km}.$$

(b) The direction of $\Delta\vec{r}_{12}$ is $\theta_{12} = \tan^{-1}[(-20\text{ km})/(60\text{ km})] = -18°$, or $18°$ south of east.

(c) To calculate the average velocity for the first two segments of the journey (including rest), we use the result from part (a) in Eq. 4-8 along with the fact that

$$\Delta t_{12} = \Delta t_1 + \Delta t_2 + \Delta t_{\text{rest}} = 50\text{ h} + 35\text{ h} + 5.0\text{ h} = 90\text{ h}.$$

In unit vector notation, we have $\vec{v}_{12,\text{avg}} = \dfrac{(60\,\hat{\text{i}} - 20\,\hat{\text{j}})\text{ km}}{90\text{ h}} = (0.67\,\hat{\text{i}} - 0.22\,\hat{\text{j}})\text{ km/h}.$

This leads to $|\vec{v}_{12,\text{avg}}| = 0.70\text{ km/h}.$

(d) The direction of $\vec{v}_{12,\text{avg}}$ is $\theta_{12} = \tan^{-1}[(-0.22\text{ km/h})/(0.67\text{ km/h})] = -18°$, or $18°$ south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain (140 km)/(90 h) = 1.56 km/h $\approx$1.6 km/h.

(f) The net displacement is required to be the 90 km East from *A* to *B*. The displacement from the resting place to *B* is denoted $\Delta\vec{r}_3$. Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90\text{ km})\,\hat{\text{i}}$$

which produces $\Delta\vec{r}_3 = (30\text{ km})\hat{\text{i}} + (20\text{ km})\hat{\text{j}}$ in unit-vector notation, or $(36\angle 33°)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{3,\text{avg}}| = \frac{36\,\text{km}}{(120-90)\text{ h}} = 1.2\,\text{km/h}.$$

(g) The direction of $\vec{v}_{3,\text{avg}}$ is the same as $\Delta\vec{r}_3$ (that is, 33° north of east).

LEARN With a vector-capable calculator in polar mode, we could perform the vector addition of the displacements as $(75\angle 37°) + (65\angle -90°) = (63\angle -18°)$. Note the distinction between average velocity and average speed.

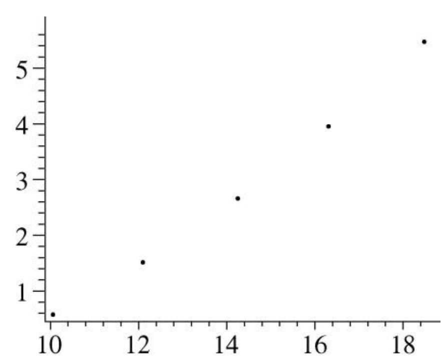
94. We compute the coordinate pairs (*x*, *y*) from $x = (v_0 \cos\theta)t$ and $y = v_0 \sin\theta t - \frac{1}{2}gt^2$ for t = 20 s and the speeds and angles given in the problem.

(a) We obtain

$$(x_A, y_A) = (10.1\text{ km}, 0.556\text{ km}) \qquad (x_B, y_B) = (12.1\text{ km}, 1.51\text{ km})$$
$$(x_C, y_C) = (14.3\text{ km}, 2.68\text{ km}) \qquad (x_D, y_D) = (16.4\text{ km}, 3.99\text{ km})$$

and (x_E, y_E) = (18.5 km, 5.53 km) which we plot in the next part.

(b) The vertical (*y*) and horizontal (*x*) axes are in kilometers. The graph does not start at the origin. The curve to "fit" the data is not shown, but is easily imagined (forming the "curtain of death").



95. (a) With Δx = 8.0 m, $t = \Delta t_1$, $a = a_x$, and v_{ox} = 0, Eq. 2-15 gives

$$8.0 \text{ m} = \tfrac{1}{2} a_x (\Delta t_1)^2 ,$$

and the corresponding expression for motion along the y axis leads to

$$\Delta y = 12 \text{ m} = \tfrac{1}{2} a_y (\Delta t_1)^2 .$$

Dividing the second expression by the first leads to $a_y / a_x = 3/2 = 1.5$.

(b) Letting $t = 2\Delta t_1$, then Eq. 2-15 leads to $\Delta x = (8.0 \text{ m})(2)^2 = 32$ m, which implies that its x coordinate is now (4.0 + 32) m = 36 m. Similarly, $\Delta y = (12 \text{ m})(2)^2 = 48$ m, which means its y coordinate has become (6.0 + 48) m = 54 m.

96. We assume the ball's initial velocity is perpendicular to the plane of the net. We choose coordinates so that $(x_0, y_0) = (0, 3.0)$ m, and $v_x > 0$ (note that $v_{0y} = 0$).

(a) To (barely) clear the net, we have

$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2 \;\; \Rightarrow \; 2.24 \text{ m} - 3.0 \text{ m} = 0 - \frac{1}{2}\left(9.8 \text{ m/s}^2\right) t^2$$

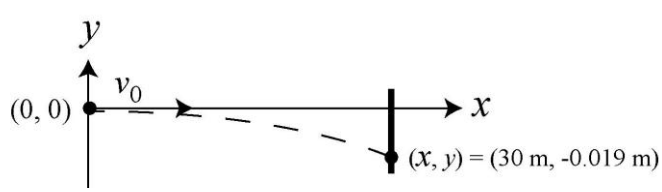
which gives $t = 0.39$ s for the time it is passing over the net. This is plugged into the x-equation to yield the (minimum) initial velocity $v_x = (8.0 \text{ m})/(0.39 \text{ s}) = 20.3$ m/s.

(b) We require $y = 0$ and find time t from the equation $y - y_0 = v_{0y} t - \tfrac{1}{2} g t^2$. This value $\left(t = \sqrt{2\left(3.0 \text{ m}\right)/(9.8 \text{ m/s}^2)} \; = 0.78 \text{ s}\right)$ is plugged into the x-equation to yield the (maximum) initial velocity

$$v_x = (17.0 \text{ m})/(0.78 \text{ s}) = 21.7 \text{ m/s}.$$

97. **THINK** A bullet fired horizontally from a rifle strikes the target at some distance below its aiming point. We're asked to find its total flight time and speed.

EXPRESS The trajectory of the bullet is shown in the figure on the right (not to scale). Note that the origin is chosen to be at the firing point. With this convention, the y coordinate of the bullet is given by $y = -\tfrac{1}{2} g t^2$. Knowing the coordinates (x, y) at the target allows us to calculate the total flight time and speed of the bullet.



ANALYZE (a) If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

$$t=\sqrt{\frac{-2y}{g}}=\sqrt{\frac{-2\left(-0.019\text{ m}\right)}{9.8\text{ m/s}^2}}=6.2\times10^{-2}\text{ s.}$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since x = 30 m is the horizontal position of the target, we have $x = v_0t$. Thus,

$$v_0=\frac{x}{t}=\frac{30\text{ m}}{6.3\times10^{-2}\text{ s}}=4.8\times10^{2}\text{ m/s.}$$

LEARN Alternatively, we may use Eq. 4-25 to solve for the initial velocity. With $\theta_0=0$ and $y_0=0$, the equation simplifies to $y=-\dfrac{gx^2}{2v_0^2}$, from which we find

$$v_0=\sqrt{-\frac{gx^2}{2y}}=\sqrt{-\frac{(9.8\text{ m/s}^2)(30\text{ m})^2}{2(-0.019\text{ m})}}=4.8\times10^{2}\text{ m/s},$$

in agreement with what we calculated in part (b).

98. For circular motion, we must have $\vec{v}$ with direction perpendicular to $\vec{r}$ and (since the speed is constant) magnitude $v=2\pi r/T$ where $r=\sqrt{(2.00\text{ m})^2+(-3.00\text{ m})^2}$ and $T=7.00\text{ s}$. The $\vec{r}$ (given in the problem statement) specifies a point in the fourth quadrant, and since the motion is clockwise then the velocity must have both components negative. Our result, satisfying these three conditions, (using unit-vector notation which makes it easy to double-check that $\vec{r}\cdot\vec{v}=0$) for $\vec{v}$ = (–2.69 m/s)$\hat{\text{i}}$ + (–1.80 m/s)$\hat{\text{j}}$.

99. Let v_o = 2π(0.200 m)/(0.00500 s) ≈ 251 m/s (using Eq. 4-35) be the speed it had in circular motion and θ_o = (1 hr)(360º/12 hr [for full rotation]) = 30.0º. Then Eq. 4-25 leads to

$$y=(2.50\text{ m})\tan 30.0°-\frac{(9.8\text{ m/s}^2)(2.50\text{ m})^2}{2(251\text{ m/s})^2(\cos 30.0°)^2}\approx1.44\text{ m}$$

which means its height above the floor is 1.44 m + 1.20 m = 2.64 m.

100. Noting that $\vec{v}_2=0$, then, using Eq. 4-15, the average acceleration is

$$\vec{a}_{\text{avg}}=\frac{\Delta\vec{v}}{\Delta t}=\frac{0-\left(6.30\,\hat{\text{i}}-8.42\,\hat{\text{j}}\right)\text{m/s}}{3\text{ s}}=\left(-2.1\hat{\text{i}}+2.8\,\hat{\text{j}}\right)\text{m/s}^2$$

101. Using Eq. 2-16, we obtain $v^2=v_0^2-2gh$, or $h=(v_0^2-v^2)/2g$.

(a) Since $v = 0$ at the maximum height of an upward motion, with $v_0 = 7.00$ m/s, we have

$$h = (7.00\ \text{m/s})^2 / 2(9.80\ \text{m/s}^2) = 2.50\ \text{m}.$$

(b) The relative speed is $v_r = v_0 - v_c = 7.00\ \text{m/s} - 3.00\ \text{m/s} = 4.00\ \text{m/s}$ with respect to the floor. Using the above equation we obtain $h = (4.00\ \text{m/s})^2 / 2(9.80\ \text{m/s}^2) = 0.82\ \text{m}.$

(c) The acceleration, or the rate of change of speed of the ball with respect to the ground is 9.80 m/s^2 (downward).

(d) Since the elevator cab moves at constant velocity, the rate of change of speed of the ball with respect to the cab floor is also 9.80 m/s^2 (downward).

102. (a) With r = 0.15 m and $a = 3.0 \times 10^{14}$ m/s^2, Eq. 4-34 gives

$$v = \sqrt{ra} = 6.7 \times 10^6\ \text{m/s}.$$

(b) The period is given by Eq. 4-35:

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7}\ \text{s}.$$

103. (a) The magnitude of the displacement vector $\Delta\vec{r}$ is given by

$$|\Delta\vec{r}| = \sqrt{(21.5\ \text{km})^2 + (9.7\ \text{km})^2 + (2.88\ \text{km})^2} = 23.8\ \text{km}.$$

Thus,

$$|\vec{v}_{\text{avg}}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{23.8\ \text{km}}{3.50\ \text{h}} = 6.79\ \text{km/h}.$$

(b) The angle θ in question is given by

$$\theta = \tan^{-1}\left(\frac{2.88\ \text{km}}{\sqrt{(21.5\ \text{km})^2 + (9.7\ \text{km})^2}}\right) = 6.96°.$$

104. The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

where $v_y = \sqrt{2gh}$ and we have used Eq. 2-16 with Δx replaced with h = 20 m. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = \left(3v_0\right)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8\text{ m/s}^2)(20\text{ m})}/2 = 7.0\text{ m/s}$.

105. We choose horizontal x and vertical y axes such that both components of $\vec{v}_0$ are positive. Positive angles are counterclockwise from +x and negative angles are clockwise from it. In unit-vector notation, the velocity at each instant during the projectile motion is

$$\vec{v} = v_0 \cos\theta_0\,\hat{\mathrm{i}} + \left(v_0 \sin\theta_0 - gt\right)\hat{\mathrm{j}}.$$

(a) With v_0 = 30 m/s and θ_0 = 60°, we obtain $\vec{v} = (15\hat{\mathrm{i}} + 6.4\,\hat{\mathrm{j}})$ m/s, for t = 2.0 s. The magnitude of $\vec{v}$ is $|\vec{v}| = \sqrt{(15\text{ m/s})^2 + (6.4\text{ m/s})^2} = 16$ m/s.

(b) The direction of $\vec{v}$ is

$$\theta = \tan^{-1}[(6.4\text{ m/s})/(15\text{ m/s})] = 23°,$$

measured counterclockwise from +x.

(c) Since the angle is positive, it is above the horizontal.

(d) With t = 5.0 s, we find $\vec{v} = (15\hat{\mathrm{i}} - 23\hat{\mathrm{j}})$ m/s, which yields

$$|\vec{v}| = \sqrt{(15\text{ m/s})^2 + (-23\text{ m/s})^2} = 27\text{ m/s}.$$

(e) The direction of $\vec{v}$ is $\theta = \tan^{-1}[(-23\text{ m/s})/(15\text{ m/s})] = -57°$, or 57° measured *clockwise* from +x.

(f) Since the angle is negative, it is below the horizontal.

106. We use Eq. 4-2 and Eq. 4-3.

(a) With the initial position vector as $\vec{r}_1$ and the later vector as $\vec{r}_2$, Eq. 4-3 yields

$$\Delta r = [(-2.0\text{ m}) - 5.0\text{ m}]\hat{\mathrm{i}} + [(6.0\text{ m}) - (-6.0\text{ m})]\hat{\mathrm{j}} + (2.0\text{ m} - 2.0\text{ m})\hat{\mathrm{k}} = (-7.0\text{ m})\hat{\mathrm{i}} + (12\text{ m})\hat{\mathrm{j}}$$

for the displacement vector in unit-vector notation.

(b) Since there is no z component (that is, the coefficient of $\hat{\mathrm{k}}$ is zero), the displacement vector is in the *xy* plane.

107. We write our magnitude-angle results in the form $(R \angle \theta)$ with SI units for the magnitude understood (m for distances, m/s for speeds, m/s^2 for accelerations). All angles θ are measured counterclockwise from +x, but we will occasionally refer to angles ϕ, which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is $v = 2\pi r/T$ where r = 3.00 m and T = 20.0 s; thus, v = 0.942 m/s. The particle is moving counterclockwise in Fig. 4-56.

(a) At t = 5.0 s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00\text{ s}}{20.0\text{ s}} = \frac{1}{4}$$

of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}(360°) = 90°$$

measured from vertical (as explained above). Referring to Fig. 4-56, we see that this position (which is the "3 o'clock" position on the circle) corresponds to x = 3.0 m and y = 3.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (4.2 \angle 45°)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations

$$x = r\sin\phi, \quad y = r - r\cos\phi.$$

Of course, $R = \sqrt{x^2 + y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector-capable calculators).

(b) At t = 7.5 s, the particle has traveled a fraction of 7.5/20 = 3/8 of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at ϕ = 3/8 (360°) = 135° measured from vertical in the manner discussed above. Referring to Fig. 4-56, we compute that this position corresponds to

$$x = (3.00\text{ m})\sin 135° = 2.1\text{ m}$$
$$y = (3.0\text{ m}) - (3.0\text{ m})\cos 135° = 5.1\text{ m}$$

relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (5.5 \angle 68°)$.

(c) At t = 10.0 s, the particle has traveled a fraction of 10/20 = 1/2 of a revolution around the circle. Relative to the circle-center, the particle is at ϕ = 180° measured from vertical (see explanation above). Referring to Fig. 4-56, we see that this position corresponds to x

= 0 and y = 6.0 m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R\angle\theta) = (6.0\angle 90°)$.

(d) We subtract the position vector in part (a) from the position vector in part (c):

$$(6.0\angle 90°) - (4.2\angle 45°) = (4.2\angle 135°)$$

using magnitude-angle notation (convenient when using vector-capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{R} = (0 - 3.0\text{ m})\,\hat{\text{i}} + (6.0\text{ m} - 3.0\text{ m})\,\hat{\text{j}} = (-3.0\text{ m})\hat{\text{i}} + (3.0\text{ m})\hat{\text{j}}$$

which leads to $|\Delta\vec{R}| = 4.2\text{ m}$ and $\theta = 135°$.

(e) From Eq. 4-8, we have $\vec{v}_{\text{avg}} = \Delta\vec{R}/\Delta t$. With $\Delta t = 5.0\text{ s}$, we have

$$\vec{v}_{\text{avg}} = (-0.60\text{ m/s})\,\hat{\text{i}} + (0.60\text{ m/s})\,\hat{\text{j}}$$

in unit-vector notation or (0.85 $\angle$ 135°) in magnitude-angle notation.

(f) The speed has already been noted (v = 0.94 m/s), but its direction is best seen by referring again to Fig. 4-56. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $(0.94\angle 90°)$.

(g) Again, the speed has been noted above (v = 0.94 m/s), but its direction is best seen by referring to Fig. 4-56. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $(0.94\angle 180°)$.

(h) The acceleration has magnitude $a = v^2/r$ = 0.30 m/s^2, and at this instant (see part (a)) it is horizontal (toward the center of the circle). Thus, our result is $(0.30\angle 180°)$.

(i) Again, $a = v^2/r$ = 0.30 m/s^2, but at this instant (see part (c)) it is vertical (toward the center of the circle). Thus, our result is $(0.30\angle 270°)$.

108. Equation 4-34 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r.

(a) The minimum turning radius of the train is given by

$$r_{\min} = \frac{v^2}{a_{\max}} = \frac{(216\ \text{km/h})^2}{(0.050)(9.8\ \text{m/s}^2)} = 7.3\times 10^3\ \text{m}.$$

(b) The speed of the train must be reduced to no more than

$$v = \sqrt{a_{\max} r} = \sqrt{0.050\left(9.8\ \text{m/s}^2\right)\left(1.00\times 10^3\ \text{m}\right)} = 22\ \text{m/s}$$

which is roughly 80 km/h.

109. (a) Using the same coordinate system assumed in Eq. 4-25, we find

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0\cos\theta_0)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0.$$

Thus, with $v_0 = 3.0\times 10^6$ m/s and $x = 1.0$ m, we obtain $y = -5.4\times 10^{-13}$ m, which is not practical to measure (and suggests why gravitational processes play such a small role in the fields of atomic and subatomic physics).

(b) It is clear from the above expression that $|y|$ decreases as v_0 is increased.

110. When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and t is the time the person takes to walk up it. This is v_p = (15 m)/(90 s) = 0.167 m/s. The escalator moves at v_e = (15 m)/(60 s) = 0.250 m/s. The speed of the person walking up the moving escalator is

$$v = v_p + v_e = 0.167\ \text{m/s} + 0.250\ \text{m/s} = 0.417\ \text{m/s}$$

and the time taken to move the length of the escalator is

$$t = \ell/v = (15\ \text{m})/(0.417\ \text{m/s}) = 36\ \text{s}.$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of ℓ (in meters) the speed (in meters per second) of the person walking on the stalled escalator is $\ell/90$, the speed of the moving escalator is $\ell/60$, and the speed of the person walking on the moving escalator is $v = (\ell/90) + (\ell/60) = 0.0278\ell$. The time taken is $t = \ell/v = \ell/0.0278\ell = 36\text{s}$ and is independent of ℓ.

111. The radius of Earth may be found in Appendix C.

(a) The speed of an object at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s):

$$v = 2\pi(6.37 \times 10^6 \text{ m})/(8.64 \times 10^4 \text{ s}) = 463 \text{ m/s}.$$

The magnitude of the acceleration is given by

$$a = \frac{v^2}{R} = \frac{(463\text{ m/s})^2}{6.37\times 10^6 \text{ m}} = 0.034 \text{ m/s}^2.$$

(b) If T is the period, then $v = 2\pi R/T$ is the speed and the magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}.$$

Thus,

$$T = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{6.37\times 10^6\text{ m}}{9.8\text{ m/s}^2}} = 5.1\times 10^3 \text{ s} = 84 \text{ min.}$$

112. With g_B = 9.8128 m/s^2 and g_M = 9.7999 m/s^2, we apply Eq. 4-26:

$$R_M - R_B = \frac{v_0^2 \sin 2\theta_0}{g_M} - \frac{v_0^2 \sin 2\theta_0}{g_B} = \frac{v_0^2 \sin 2\theta_0}{g_B}\left(\frac{g_B}{g_M} - 1\right)$$

which becomes

$$R_M - R_B = R_B\left(\frac{9.8128 \text{ m/s}^2}{9.7999 \text{ m/s}^2} - 1\right)$$

and yields (upon substituting R_B = 8.09 m) $R_M - R_B$ = 0.01 m = 1 cm.

113. From the figure, the three displacements can be written as

$$\vec d_1 = d_1(\cos\theta_1\hat{\text{i}} + \sin\theta_1\hat{\text{j}}) = (5.00 \text{ m})(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}) = (4.33 \text{ m})\hat{\text{i}} + (2.50 \text{ m})\hat{\text{j}}$$

$$\begin{aligned}\vec d_2 &= d_2[\cos(180° + \theta_1 - \theta_2)\hat{\text{i}} + \sin(180° + \theta_1 - \theta_2)\hat{\text{j}}] = (8.00 \text{ m})(\cos 160°\hat{\text{i}} + \sin 160°\hat{\text{j}}) \\ &= (-7.52 \text{ m})\hat{\text{i}} + (2.74 \text{ m})\hat{\text{j}}\end{aligned}$$

$$\begin{aligned}\vec d_3 &= d_3[\cos(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{i}} + \sin(360° - \theta_3 - \theta_2 + \theta_1)\hat{\text{j}}] = (12.0 \text{ m})(\cos 260°\hat{\text{i}} + \sin 260°\hat{\text{j}}) \\ &= (-2.08 \text{ m})\hat{\text{i}} - (11.8 \text{ m})\hat{\text{j}}\end{aligned}$$

where the angles are measured from the $+x$ axis. The net displacement is

$$\vec d = \vec d_1 + \vec d_2 + \vec d_3 = (-5.27 \text{ m})\hat{\text{i}} - (6.58 \text{ m})\hat{\text{j}}.$$

(a) The magnitude of the net displacement is

$$|\vec{d}| = \sqrt{(-5.27\text{ m})^2 + (-6.58\text{ m})^2} = 8.43\text{ m}.$$

(b) The direction of $\vec{d}$ is $\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-6.58\text{ m}}{-5.27\text{ m}}\right) = 51.3°\text{ or }231°.$

We choose $231°$ (measured counterclockwise from $+x$) since the desired angle is in the third quadrant. An equivalent answer is $-129°$ (measured clockwise from $+x$).

114. Taking derivatives of $\vec{r} = 2t\hat{\text{i}} + 2\sin(\pi t/4)\hat{\text{j}}$ (with lengths in meters, time in seconds, and angles in radians) provides expressions for velocity and acceleration:

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{\text{i}} + \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right)\hat{\text{j}}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{\pi^2}{8}\sin\left(\frac{\pi t}{4}\right)\hat{\text{j}}.$$

Thus, we obtain:

time t (s)			0.0	1.0	2.0	3.0	4.0
(a)	$\vec{r}$ position	x (m)	0.0	2.0	4.0	6.0	8.0
		y (m)	0.0	1.4	2.0	1.4	0.0
(b)	$\vec{v}$ velocity	v_x(m/s)		2.0	2.0	2.0	
		v_y (m/s)		1.1	0.0	–1.1	
(c)	$\vec{a}$ acceleration	a_x (m/s^2)		0.0	0.0	0.0	
		a_y (m/s^2)		–0.87	–1.2	–0.87	

115. Since this problem involves constant downward acceleration of magnitude a, similar to the projectile motion situation, we use the equations of §4-6 as long as we substitute a for g. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and

$$v_{0x} = v_0 = 1.00 \times 10^9\text{ cm/s}.$$

(a) If ℓ is the length of a plate and t is the time an electron is between the plates, then $\ell = v_0 t$, where v_0 is the initial speed. Thus

$$t = \frac{\ell}{v_0} = \frac{2.00\text{ cm}}{1.00 \times 10^9\text{ cm/s}} = 2.00 \times 10^{-9}\text{ s}.$$

(b) The vertical displacement of the electron is

$$y = -\frac{1}{2}at^2 = -\frac{1}{2}\left(1.00\times10^{17}\ \text{cm/s}^2\right)\left(2.00\times10^{-9}\ \text{s}\right)^2 = -0.20\,\text{cm} = -2.00\ \text{mm},$$

or $|\,y\,| = 2.00$ mm.

(c) The *x* component of velocity does not change:

$$v_x = v_0 = 1.00 \times 10^9 \text{ cm/s} = 1.00 \times 10^7 \text{ m/s}.$$

(d) The *y* component of the velocity is

$$\begin{aligned} v_y &= a_y t = \left(1.00\times10^{17}\ \text{cm/s}^2\right)\left(2.00\times10^{-9}\ \text{s}\right) = 2.00\times10^8\,\text{cm/s} \\ &= 2.00\times10^6\,\text{m/s}. \end{aligned}$$

116. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion of the shot ball. We are allowed to use Table 2-1 (with Δy replacing Δx) because the ball has constant acceleration motion. We use primed variables (except *t*) with the constant-velocity elevator (so $v' = 10$ m/s), and unprimed variables with the ball (with initial velocity $v_0 = v' + 20 = 30$ m/s, relative to the ground). SI units are used throughout.

(a) Taking the time to be zero at the instant the ball is shot, we compute its maximum height *y* (relative to the ground) with $v^2 = v_0^2 - 2g(y - y_0)$, where the highest point is characterized by $v = 0$. Thus,

$$y = y_o + \frac{v_0^2}{2g} = 76\ \text{m}$$

where $y_o = y'_o + 2 = 30$ m (where $y'_o = 28$ m is given in the problem) and v_0 = 30 m/s relative to the ground as noted above.

(b) There are a variety of approaches to this question. One is to continue working in the frame of reference adopted in part (a) (which treats the ground as motionless and "fixes" the coordinate origin to it); in this case, one describes the elevator motion with $y' = y'_o + v't$ and the ball motion with Eq. 2-15, and solves them for the case where they reach the same point at the same time. Another is to work in the frame of reference of the elevator (the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail:

$$\Delta y_e = v_{0_e} t - \frac{1}{2}gt^2 \quad\Rightarrow\quad t = \frac{v_{0_e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

where v_{0e} = 20 m/s is the initial velocity of the ball relative to the elevator and Δy_e = –2.0 m is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for t; the result is t = 4.2 s.

117. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the initial position for the football as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of its initial velocity measured from the +x axis.

(a) x = 46 m and y = –1.5 m are the coordinates for the landing point; it lands at time t = 4.5 s. Since $x = v_{0x}t$,

$$v_{0x} = \frac{x}{t} = \frac{46\text{ m}}{4.5\text{ s}} = 10.2\text{ m/s}.$$

Since $y = v_{0y}t - \frac{1}{2}gt^2$,

$$v_{0y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{(-1.5\text{ m}) + \frac{1}{2}(9.8\text{ m/s}^2)(4.5\text{ s})^2}{4.5\text{ s}} = 21.7\text{ m/s}.$$

The magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(10.2\text{ m/s})^2 + (21.7\text{ m/s})^2} = 24\text{ m/s}.$$

(b) The initial angle satisfies tan $\theta_0 = v_{0y}/v_{0x}$. Thus,

$$\theta_0 = \tan^{-1}[(21.7\text{ m/s})/(10.2\text{ m/s})] = 65°.$$

118. The velocity of Larry is v_1 and that of Curly is v_2. Also, we denote the length of the corridor by L. Now, Larry's time of passage is t_1 = 150 s (which must equal L/v_1), and Curly's time of passage is t_2 = 70 s (which must equal L/v_2). The time Moe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1/L + v_2/L} = \frac{1}{\frac{1}{150\text{ s}} + \frac{1}{70\text{ s}}} = 48\text{s}.$$

119. The boxcar has velocity $\vec{v}_{cg} = v_1\,\hat{\text{i}}$ relative to the ground, and the bullet has velocity

$$\vec{v}_{0bg} = v_2\cos\theta\,\hat{\text{i}} + v_2\sin\theta\,\hat{\text{j}}$$

relative to the ground before entering the car (we are neglecting the effects of gravity on the bullet). While in the car, its velocity relative to the outside ground is

$$\vec{v}_{bg} = 0.8v_2\cos\theta\,\hat{\text{i}} + 0.8v_2\sin\theta\,\hat{\text{j}}$$

(due to the 20% reduction mentioned in the problem). The problem indicates that the velocity of the bullet in the car *relative to the car* is (with v_3 unspecified) $\vec{v}_{b\,c} = v_3\,\hat{\mathrm{j}}$. Now, Eq. 4-44 provides the condition

$$\begin{aligned} \vec{v}_{b\,g} &= \vec{v}_{b\,c} + \vec{v}_{c\,g} \\ 0.8v_2\cos\theta\ \hat{\mathrm{i}} + 0.8v_2\sin\theta\ \hat{\mathrm{j}} &= v_3\ \hat{\mathrm{j}} + v_1\ \hat{\mathrm{i}} \end{aligned}$$

so that equating x components allows us to find θ. If one wished to find v_3 one could also equate the y components, and from this, if the car width were given, one could find the time spent by the bullet in the car, but this information is not asked for (which is why the width is irrelevant). Therefore, examining the x components in SI units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8v_2}\right) = \cos^{-1}\left(\frac{85\ \text{km/h}\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)}{0.8\ (650\ \text{m/s})}\right)$$

which yields 87° for the direction of $\vec{v}_{b\,g}$ (measured from $\hat{\mathrm{i}}$, which is the direction of motion of the car). The problem asks, "from what direction was it fired?" — which means the answer is not 87° but rather its supplement 93° (measured from the direction of motion). Stating this more carefully, in the coordinate system we have adopted in our solution, the bullet velocity vector is in the first quadrant, at 87° measured counterclockwise from the +x direction (the direction of train motion), which means that the direction from which the bullet came (where the sniper is) is in the third quadrant, at –93° (that is, 93° measured clockwise from +x).

120. (a) Using $a = v^2/R,$ the radius of the track is

$$R = \frac{v^2}{a} = \frac{(9.20\ \text{m/s})^2}{3.80\ \text{m/s}^2} = 22.3\ \text{m}\,.$$

(b) Using $T = 2\pi R/v,$ the period of the circular motion is

$$T = \frac{2\pi R}{v} = \frac{2\pi(22.3\ \text{m})}{9.20\ \text{m/s}} = 15.2\ \text{s}$$

121. (a) With $v = c/10 = 3\times10^7$ m/s and $a = 20g = 196\ \text{m/s}^2$, Eq. 4-34 gives

$$r = v^2/a = 4.6\times10^{12}\ \text{m}.$$

(b) The period is given by Eq. 4-35: $T = 2\pi r/v = 9.6\times10^5$ s. Thus, the time to make a quarter-turn is $T/4 = 2.4\times10^5$ s or about 2.8 days.

122. Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and v_y=0 at the target, we obtain

$$v_{0y} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(5.00\ \text{m}\right)} = 9.90\ \text{m/s}$$

(a) Since $v_0 \sin \theta_0 = v_{0y}$, with $v_0 = 12.0$ m/s, we find $\theta_0 = 55.6°$.

(b) Now, $v_y = v_{0y} - gt$ gives $t = (9.90\ \text{m/s})/(9.80\ \text{m/s}^2) = 1.01$ s. Thus,

$$\Delta x = (v_0 \cos \theta_0)t = 6.85\ \text{m}.$$

(c) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0 \cos \theta_0$ = 6.78 m/s.

123. With $v_0 = 30.0$ m/s and $R = 20.0$ m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218.$$

Because $\sin \phi = \sin (180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.58° \text{ and } 167.4°.$$

which correspond to the two possible launch angles that will hit the target (in the absence of air friction and related effects).

(a) The smallest angle is $\theta_0 = 6.29°$.

(b) The greatest angle is and $\theta_0 = 83.7°$.

An alternative approach to this problem in terms of Eq. 4-25 (with $y = 0$ and $1/\cos^2 = 1 + \tan^2$) is possible — and leads to a quadratic equation for $\tan\theta_0$ with the roots providing these two possible θ_0 values.

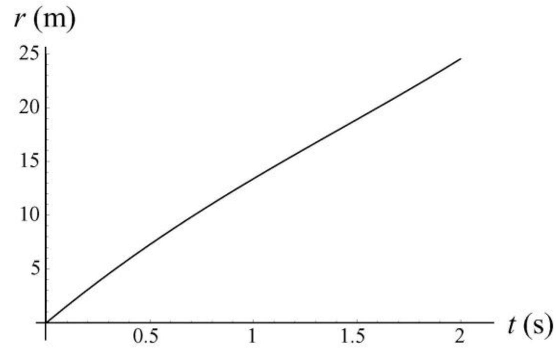
124. We make use of Eq. 4-21 and Eq.4-22.

(a) With $v_o = 16$ m/s, we square Eq. 4-21 and Eq. 4-22 and add them, then (using Pythagoras' theorem) take the square root to obtain r:

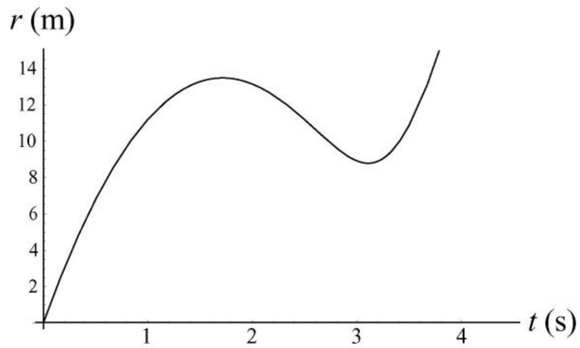
$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2} = \sqrt{(v_0 \cos\theta_0 t)^2 + (v_0 \sin\theta_0 t - gt^2/2)^2}$$

$$= t\sqrt{v_0^2 - v_0 g \sin\theta_0 t + g^2 t^2/4}$$

Below we plot r as a function of time for $\theta_o = 40.0°$:



(b) For this next graph for r versus t we set θ_0 = 80.0º.



(c) Differentiating r with respect to t, we obtain

$$\frac{dr}{dt} = \frac{v_0^2 - 3v_0 gt\sin\theta_0/2 + g^2t^2/2}{\sqrt{v_0^2 - v_0 g\sin\theta_0 t + g^2t^2/4}}$$

Setting $dr/dt = 0$, with $v_0 = 16.0$ m/s and $\theta_0 = 40.0°$, we have $256 - 151t + 48t^2 = 0$. The equation has no real solution. This means that the maximum is reached at the end of the flight, with

$$t_{total} = 2v_0 \sin\theta_0 / g = 2(16.0\text{ m/s})\sin(40.0°)/(9.80\text{ m/s}^2) = 2.10\text{ s}.$$

(d) The value of r is given by

$$r = (2.10)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 40.0°(2.10) + (9.80)^2(2.10)^2/4} = 25.7\text{ m}.$$

(e) The horizontal distance is $r_x = v_0 \cos\theta_0 t = (16.0\text{ m/s})\cos 40.0°(2.10\text{ s}) = 25.7\text{ m}.$

(f) The vertical distance is $r_y = 0$.

(g) For the θ_0 = 80º launch, the condition for maximum r is $256 - 232t + 48t^2 = 0$, or $t = 1.71$ s (the other solution, t = 3.13 s, corresponds to a minimum.)

(h) The distance traveled is

$$r = (1.71)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 80.0°(1.71) + (9.80)^2(1.71)^2/4} = 13.5 \text{ m}.$$

(i) The horizontal distance is

$$r_x = v_0 \cos\theta_0 t = (16.0 \text{ m/s})\cos 80.0°(1.71 \text{ s}) = 4.75 \text{ m}.$$

(j) The vertical distance is

$$r_y = v_0 \sin\theta_0 t - \frac{gt^2}{2} = (16.0 \text{ m/s})\sin 80°(1.71 \text{ s}) - \frac{(9.80 \text{ m/s}^2)(1.71 \text{ s})^2}{2} = 12.6 \text{ m}.$$

125. Using the same coordinate system assumed in Eq. 4-25, we find x for the elevated cannon from

$$y = x\tan\theta_0 - \frac{gx^2}{2(v_0 \cos\theta_0)^2} \quad \text{where } y = -30 \text{ m}.$$

Using the quadratic formula (choosing the positive root), we find

$$x = v_0 \cos\theta_0 \left(\frac{v_0 \sin\theta_0 + \sqrt{(v_0 \sin\theta_0)^2 - 2gy}}{g} \right)$$

which yields x = 715 m for v_0 = 82 m/s and θ_0 = 45°. This is 29 m longer than the distance of 686 m.

126. At maximum height, the y-component of a projectile's velocity vanishes, so the given 10 m/s is the (constant) x-component of velocity.

(a) Using v_{0y} to denote the y-velocity 1.0 s before reaching the maximum height, then (with v_y = 0) the equation $v_y = v_{0y} - gt$ leads to v_{0y} = 9.8 m/s. The magnitude of the velocity vector (or *speed*) at that moment is therefore

$$\sqrt{v_x^2 + v_{0y}{}^2} = \sqrt{(10 \text{ m/s})^2 + (9.8 \text{ m/s})^2} = 14 \text{ m/s}.$$

(b) It is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top, as it was 1.0 s before (14 m/s again). This may be verified by using $v_y = v_{0y} - gt$ again but now "starting the clock" at the highest point so that v_{0y} = 0 (and $t = 1.0$ s). This leads to v_y = –9.8 m/s and $\sqrt{(10 \text{ m/s})^2 + (-9.8 \text{ m/s})^2} = 14$ m/s.

(c) The x_0 value may be obtained from $x = 0 = x_0 + (10 \text{ m/s})(1.0\text{s})$, which yields $x_0 = -10\,\text{m}$.

(d) With $v_{0y} = 9.8$ m/s denoting the y-component of velocity one second before the top of the trajectory, then we have $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$ where $t = 1.0$ s. This yields $y_0 = -4.9$ m.

(e) By using $x - x_0 = (10 \text{ m/s})(1.0 \text{ s})$ where $x_0 = 0$, we obtain $x = 10$ m.

(f) Let $t = 0$ at the top with $y_0 = v_{0y} = 0$. From $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$, we have, for $t = 1.0$ s,

$$y = -(9.8 \text{ m/s}^2)(1.0 \text{ s})^2/2 = -4.9 \text{ m}.$$

127. With no acceleration in the x direction yet a constant acceleration of 1.40 m/s^2 in the y direction, the position (in meters) as a function of time (in seconds) must be

$$\vec{r} = (6.00t)\hat{\text{i}} + \left(\frac{1}{2}(1.40)t^2\right)\hat{\text{j}}$$

and $\vec{v}$ is its derivative with respect to t.

(a) At $t = 3.00$ s, therefore, $\vec{v} = (6.00\hat{\text{i}} + 4.20\hat{\text{j}})$ m/s.

(b) At $t = 3.00$ s, the position is $\vec{r} = (18.0\hat{\text{i}} + 6.30\hat{\text{j}})$ m.

128. We note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle, with one leg being $\vec{v}_{PG}$ (east), another leg being $\vec{v}_{AG}$ (magnitude = 20, direction = south), and the hypotenuse being $\vec{v}_{PA}$ (magnitude = 70). Lengths are in kilometers and time is in hours. Using the Pythagorean theorem, we have

$$|\vec{v}_{PA}| = \sqrt{|\vec{v}_{PG}|^2 + |\vec{v}_{AG}|^2} \;\Rightarrow\; 70 \text{ km/h} = \sqrt{|\vec{v}_{PG}|^2 + (20 \text{ km/h})^2}$$

which can be solved to give the ground speed: $|\vec{v}_{PG}| = 67$ km/h.

129. The figure offers many interesting points to analyze, and others are easily inferred (such as the point of maximum height). The focus here, to begin with, will be the final point shown (1.25 s after the ball is released) which is when the ball returns to its original height. In English units, $g = 32$ ft/s^2.

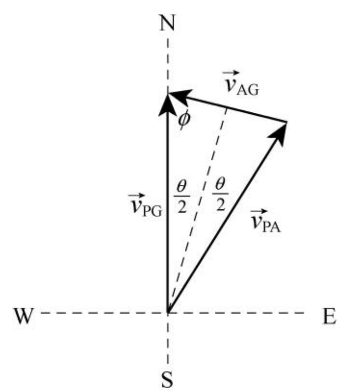
(a) Using $x - x_0 = v_x t$ we obtain v_x = (40 ft)/(1.25 s) = 32 ft/s. And $y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2$ yields $v_{0y} = \frac{1}{2}\left(32\ \text{ft/s}^2\right)\left(1.25\ \text{s}\right) = 20\ \text{ft/s}.$ Thus, the initial speed is

$$v_0 = |\vec{v}_0| = \sqrt{(32\ \text{ft/s})^2 + (20\ \text{ft/s})^2} = 38\ \text{ft/s}.$$

(b) Since $v_y = 0$ at the maximum height and the horizontal velocity stays constant, then the speed at the top is the same as v_x = 32 ft/s.

(c) We can infer from the figure (or compute from $v_y = 0 = v_{0y} - gt$) that the time to reach the top is 0.625 s. With this, we can use $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to obtain 9.3 ft (where y_0 = 3 ft has been used). An alternative approach is to use $v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right)$.

130. We denote $\vec{v}_{\text{PG}}$ as the velocity of the plane relative to the ground, $\vec{v}_{\text{AG}}$ as the velocity of the air relative to the ground, and $\vec{v}_{\text{PA}}$ as the velocity of the plane relative to the air.



(a) The vector diagram is shown on the right: $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. Since the magnitudes v_{PG} and v_{PA} are equal the triangle is isosceles, with two sides of equal length.

Consider either of the right triangles formed when the bisector of θ is drawn (the dashed line). It bisects $\vec{v}_{\text{AG}}$, so

$$\sin\left(\theta/2\right) = \frac{v_{\text{AG}}}{2v_{\text{PG}}} = \frac{70.0\ \text{mi/h}}{2\left(135\ \text{mi/h}\right)}$$

which leads to θ = 30.1°. Now $\vec{v}_{\text{AG}}$ makes the same angle with the E-W line as the dashed line does with the N-S line. The wind is blowing in the direction 15.0° north of west. Thus, it is blowing *from* 75.0° east of south.

(b) The plane is headed along $\vec{v}_{\text{PA}}$, in the direction 30.0° east of north. There is another solution, with the plane headed 30.0° west of north and the wind blowing 15° north of east (that is, from 75° west of south).

131. We make use of Eq. 4-24 and Eq. 4-25.

(a) With x = 180 m, θ_{o} = 30º, and v_{o} = 43 m/s, we obtain

$$y = \tan(30°)(180\ \text{m}) - \frac{(9.8\ \text{m/s}^2)(180\ \text{m})^2}{2(43\ \text{m/s})^2(\cos 30°)^2} = -11\ \text{m}$$

or $|y| = 11\text{ m}$. This implies the rise is roughly eleven meters above the fairway.

(b) The horizontal component (in the absence of air friction) is unchanged, but the vertical component increases (see Eq. 4-24). The Pythagorean theorem then gives the magnitude of final velocity (right before striking the ground): 45 m/s.

132. We let g_p denote the magnitude of the gravitational acceleration on the planet. A number of the points on the graph (including some "inferred" points — such as the max height point at $x = 12.5$ m and $t = 1.25$ s) can be analyzed profitably; for future reference, we label (with subscripts) the first $((x_0, y_0) = (0, 2)$ at $t_0 = 0)$ and last ("final") points $((x_f, y_f) = (25, 2)$ at $t_f = 2.5)$, with lengths in meters and time in seconds.

(a) The x-component of the initial velocity is found from $x_f - x_0 = v_{0x}\, t_f$. Therefore, $v_{0x} = 25/2.5 = 10$ m/s. We try to obtain the y-component from

$$y_f - y_0 = 0 = v_{0y}t_f - \tfrac{1}{2}g_p t_f^2.$$

This gives us $v_{0y} = 1.25g_p$, and we see we need another equation (by analyzing another point, say, the next-to-last one) $y - y_0 = v_{0y}t - \tfrac{1}{2}g_p t^2$ with $y = 6$ and $t = 2$; this produces our second equation $v_{0y} = 2 + g_p$. Simultaneous solution of these two equations produces results for v_{0y} and g_p (relevant to part (b)). Thus, our complete answer for the initial velocity is $\vec{v} = (10\text{ m/s})\hat{\text{i}} + (10\text{ m/s})\hat{\text{j}}$.

(b) As a by-product of the part (a) computations, we have $g_p = 8.0\text{ m/s}^2$.

(c) Solving for t_g (the time to reach the ground) in $y_g = 0 = y_0 + v_{0y}t_g - \tfrac{1}{2}g_p t_g^2$ leads to a positive answer: $t_g = 2.7$ s.

(d) With $g = 9.8\text{ m/s}^2$, the method employed in part (c) would produce the quadratic equation $-4.9t_g^2 + 10t_g + 2 = 0$ and then the positive result $t_g = 2.2$ s.

133. (a) The helicopter's speed is $v' = 6.2$ m/s, which implies that the speed of the package is $v_0 = 12 - v' = 5.8$ m/s, relative to the ground.

(b) Letting $+x$ be in the direction of $\vec{v}_0$ for the package and $+y$ be downward, we have (for the motion of the package)

$$\Delta x = v_0 t \quad \text{and} \quad \Delta y = \frac{1}{2}gt^2$$

where $\Delta y = 9.5$ m. From these, we find $t = 1.39$ s and $\Delta x = 8.08$ m for the package, while $\Delta x'$ (for the helicopter, which is moving in the opposite direction) is $-v'\,t = -8.63$ m. Thus, the horizontal separation between them is $8.08 - (-8.63) = 16.7\text{ m} \approx 17\text{ m}$.

(c) The components of $\vec{v}$ at the moment of impact are (v_x, v_y) = (5.8, 13.6) in SI units. The vertical component has been computed using Eq. 2-11. The angle (which is below horizontal) for this vector is $\tan^{-1}(13.6/5.8) = 67°$.

134. The type of acceleration involved in steady-speed circular motion is the centripetal acceleration $a = v^2/r$ which is at each moment directed towards the center of the circle. The radius of the circle is $r = (12)^2/3 = 48$ m.

(a) Thus, if at the instant the car is traveling *clockwise* around the circle, it is 48 m west of the center of its circular path.

(b) The same result holds here if at the instant the car is traveling *counterclockwise*. That is, it is 48 m west of the center of its circular path.

135. (a) Using the same coordinate system assumed in Eq. 4-21 and Eq. 4-22 (so that θ_0 = –20.0°), we use v_0 = 15.0 m/s and find the horizontal displacement of the ball at t = 2.30 s:

$$\Delta x = \left(v_0 \cos\theta_0\right)t = 32.4 \text{ m}.$$

(b) The vertical displacement is $\Delta y = \left(v_0 \sin\theta_0\right)t - \frac{1}{2}gt^2 = -37.7$ m.

136. We take the initial (x, y) specification to be (0.000, 0.762) m, and the positive x direction to be towards the "green monster." The components of the initial velocity are $(33.53 \angle 55°) \rightarrow (19.23,\ 27.47)\ \text{m/s}$.

(a) With t = 5.00 s, we have $x = x_0 + v_x t = 96.2$ m.

(b) At that time, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 15.59\ \text{m}$, which is 4.31 m above the wall.

(c) The moment in question is specified by t = 4.50 s. At that time, $x - x_0$ = (19.23)(4.50) = 86.5 m.

(d) The vertical displacement is $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 25.1$ m.

137. When moving in the same direction as the jet stream (of speed v_s), the time is $t = d/(v_{ja} + v_s),$ where d = 4350 km is the distance and $v_{ja} = 966\ \text{km/h}$ is the speed of the jet relative to the air. When moving against the jet stream, the time is $t' = d/(v_{ja} - v_s),$ with $t' - t = 50\ \text{min} = (5/6)\text{h}.$ Combining the expressions gives

$$t' - t = \frac{d}{v_{ja} - v_s} - \frac{d}{v_{ja} + v_s} = \frac{2dv_s}{v_{ja}^2 - v_s^2} = \frac{5}{6}\ \text{h}$$

Upon rearranging and using the quadratic formula to solve for v_s, we get v_s = 88.63 km/h.

138. We establish coordinates with $\hat{\text{i}}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\text{j}}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}|$ = 6.4 km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = 3.2\hat{\text{j}}$.

(a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{v}_{bg} = v_{bg}\hat{\text{i}}$ where $v > 0$ is unknown. Thus, all $\hat{\text{j}}$ components must cancel in the vector sum

$$\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$$

which means the $u \sin\theta = -3.2$, so $\theta = \sin^{-1}(-3.2/6.4) = -30°$.

(b) Using the result from part (a), we find $v_{bg} = v_{bw}\cos\theta = 5.5$ km/h. Thus, traveling a distance of ℓ = 6.4 km requires a time of 6.4/5.5 = 1.15 h or 69 min.

(c) If her motion is completely along the y axis (as the problem implies) then with v_{wg} = 3.2 km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where D = 3.2 km. This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = \frac{D}{v_{bw} - v_{wg}} + \frac{D}{v_{bw} + v_{wg}}$$

the answer is the same as in the previous part, i.e., $t_{\text{total}} = 80 \text{ min}$.

(e) The shortest-time path should have $\theta = 0$. This can also be shown by noting that the case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw}\cos\theta\,\hat{\text{i}} + (v_{bw}\sin\theta + v_{wg})\,\hat{\text{j}}$$

where the x component of $\vec{v}_{bg}$ must equal l/t. Thus, $t = \dfrac{l}{v_{bw}\cos\theta}$, which can be minimized using the condition $dt/d\theta = 0$. The above expression leads to t = 6.4/6.4 = 1.0 h, or 60 min.

Chapter 5

1. We are only concerned with horizontal forces in this problem (gravity plays no direct role). We take East as the +*x* direction and North as +*y*. This calculation is efficiently implemented on a vector-capable calculator, using magnitude-angle notation (with SI units understood).

$$\vec{a} = \frac{\vec{F}}{m} = \frac{(9.0 \angle 0°) + (8.0 \angle 118°)}{3.0} = (2.9 \angle 53°)$$

Therefore, the acceleration has a magnitude of 2.9 m/s^2.

2. We apply Newton's second law (Eq. 5-1 or, equivalently, Eq. 5-2). The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2\right) / m$.

(a) In the first case

$$\vec{F}_1 + \vec{F}_2 = \left[(3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right] + \left[(-3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right] = 0$$

so $\vec{a} = 0$.

(b) In the second case, the acceleration $\vec{a}$ equals

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((-3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right)}{2.0\,\text{kg}} = (4.0\,\text{m/s}^2)\hat{\text{j}}.$$

(c) In this final situation, $\vec{a}$ is

$$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\text{N})\hat{\text{i}} + (4.0\text{N})\hat{\text{j}}\right) + \left((3.0\text{N})\hat{\text{i}} + (-4.0\text{N})\hat{\text{j}}\right)}{2.0\,\text{kg}} = (3.0\,\text{m/s}^2)\hat{\text{i}}.$$

3. We apply Newton's second law (specifically, Eq. 5-2).

(a) We find the *x* component of the force is

$$F_x = ma_x = ma\cos 20.0° = (1.00\,\text{kg})\left(2.00\,\text{m/s}^2\right)\cos 20.0° = 1.88\text{N}.$$

(b) The *y* component of the force is

$$F_y = ma_y = ma \sin 20.0° = (1.0\,\text{kg})\left(2.00\,\text{m/s}^2\right) \sin 20.0° = 0.684\,\text{N}.$$

(c) In unit-vector notation, the force vector is

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (1.88\text{ N})\hat{\text{i}} + (0.684\text{ N})\hat{\text{j}}\,.$$

4. Since $\vec{v}$ = constant, we have $\vec{a}$ = 0, which implies

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = m\vec{a} = 0\,.$$

Thus, the other force must be

$$\vec{F}_2 = -\vec{F}_1 = (-2\text{ N})\,\hat{\text{i}} + (6\text{ N})\,\hat{\text{j}}.$$

5. The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right)/m$.

(a) The forces exerted by the three astronauts can be expressed in unit-vector notation as follows:

$$\begin{aligned}
\vec{F}_1 &= (32\text{ N})\left(\cos 30°\hat{\text{i}} + \sin 30°\hat{\text{j}}\right) = (27.7\text{ N})\,\hat{\text{i}} + (16\text{ N})\,\hat{\text{j}}\\
\vec{F}_2 &= (55\text{ N})\left(\cos 0°\hat{\text{i}} + \sin 0°\hat{\text{j}}\right) = (55\text{ N})\,\hat{\text{i}}\\
\vec{F}_3 &= (41\text{ N})\left(\cos(-60°)\,\hat{\text{i}} + \sin(-60°)\,\hat{\text{j}}\right) = (20.5\text{ N})\,\hat{\text{i}} - (35.5\text{ N})\,\hat{\text{j}}.
\end{aligned}$$

The resultant acceleration of the asteroid of mass m = 120 kg is therefore

$$\vec{a} = \frac{\left(27.7\,\hat{\text{i}} + 16\,\hat{\text{j}}\right)\text{N} + \left(55\,\hat{\text{i}}\right)\text{N} + \left(20.5\,\hat{\text{i}} - 35.5\,\hat{\text{j}}\right)\text{N}}{120\text{ kg}} = (0.86\text{ m/s}^2)\,\hat{\text{i}} - (0.16\text{ m/s}^2)\,\hat{\text{j}}\,.$$

(b) The magnitude of the acceleration vector is

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.86\text{ m/s}^2)^2 + \left(-0.16\text{ m/s}^2\right)^2} = 0.88\text{ m/s}^2\,.$$

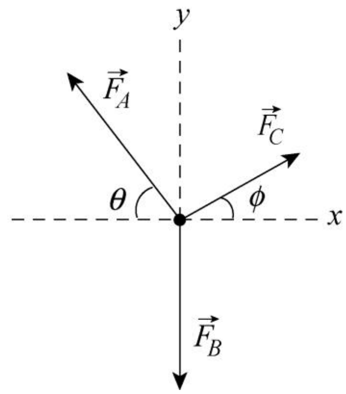
(c) The vector $\vec{a}$ makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.16\text{ m/s}^2}{0.86\text{ m/s}^2}\right) = -11°\,.$$

6. Since the tire remains stationary, by Newton's second law, the net force must be zero:

$$\vec{F}_{\text{net}} = \vec{F}_A + \vec{F}_B + \vec{F}_C = m\vec{a} = 0.$$

From the free-body diagram shown on the right, we have



$$0 = \sum F_{\text{net},x} = F_C\cos\phi - F_A\cos\theta$$
$$0 = \sum F_{\text{net},y} = F_A\sin\theta + F_C\sin\phi - F_B$$

To solve for F_B, we first compute ϕ. With $F_A = 220$ N, $F_C = 170$ N, and $\theta = 47°$, we get

$$\cos\phi = \frac{F_A\cos\theta}{F_C} = \frac{(220\ \text{N})\cos 47.0°}{170\ \text{N}} = 0.883 \quad \Rightarrow \ \phi = 28.0°$$

Substituting the value into the second force equation, we find

$$F_B = F_A\sin\theta + F_C\sin\phi = (220\ \text{N})\sin 47.0° + (170\ \text{N})\sin 28.0 = 241\ \text{N}.$$

7. **THINK** A box is under acceleration by two applied forces. We use Newton's second law to solve for the unknown second force.

EXPRESS We denote the two forces as $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so the second force is $\vec{F}_2 = m\vec{a} - \vec{F}_1$. Note that since the acceleration is in the third quadrant, we expect $\vec{F}_2$ to be in the third quadrant as well.

ANALYZE (a) In unit vector notation $\vec{F}_1 = (20.0\ \text{N})\hat{\text{i}}$ and

$$\vec{a} = -\left(12.0\sin 30.0°\,\text{m/s}^2\right)\hat{\text{i}} - \left(12.0\cos 30.0°\,\text{m/s}^2\right)\hat{\text{j}} = -\left(6.00\ \text{m/s}^2\right)\hat{\text{i}} - \left(10.4\ \text{m/s}^2\right)\hat{\text{j}}.$$

Therefore, we find the second force to be

$$\begin{aligned}\vec{F}_2 &= m\vec{a} - \vec{F}_1 \\ &= (2.00\ \text{kg})\left(-6.00\,\text{m/s}^2\right)\hat{\text{i}} + (2.00\ \text{kg})\left(-10.4\ \text{m/s}^2\right)\hat{\text{j}} - (20.0\ \text{N})\hat{\text{i}} \\ &= (-32.0\ \text{N})\hat{\text{i}} - (20.8\ \text{N})\hat{\text{j}}.\end{aligned}$$

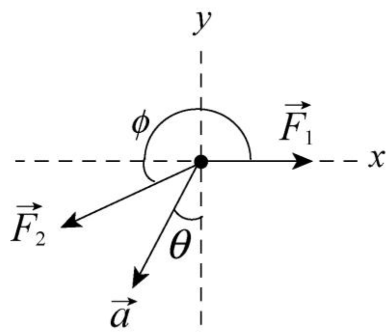
(b) The magnitude of $\vec{F}_2$ is $|\vec{F}_2| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32.0\ \text{N})^2 + (-20.8\ \text{N})^2} = 38.2\ \text{N}.$

(c) The angle that $\vec{F}_2$ makes with the positive *x*-axis is found from

$$\tan\phi = \left(\frac{F_{2y}}{F_{2x}}\right) = \frac{-20.8\text{ N}}{-32.0\text{ N}} = 0.656.$$

Consequently, the angle is either 33.0° or 33.0° + 180° = 213°. Since both the x and y components are negative, the correct result is ϕ = 213° from the +x-axis. An alternative answer is $213° - 360° = -147°$.

LEARN The result is shown in the figure on the right. The calculation confirms our expectation that $\vec{F}_2$ lies in the third quadrant (same as $\vec{a}$). The net force is



$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = (20.0\text{ N})\hat{\text{i}} + \left[(-32.0\text{ N})\hat{\text{i}} - (20.8\text{ N})\hat{\text{j}}\right]$$
$$= (-12.0\text{ N})\hat{\text{i}} - (20.8\text{ N})\hat{\text{j}}$$

which points in the same direction as $\vec{a}$.

8. We note that $m\vec{a}$ = (–16 N) $\hat{\text{i}}$ + (12 N) $\hat{\text{j}}$. With the other forces as specified in the problem, then Newton's second law gives the third force as

$$\vec{F}_3 = m\vec{a} - \vec{F}_1 - \vec{F}_2 = (-34\text{ N})\,\hat{\text{i}} - (12\text{ N})\,\hat{\text{j}}.$$

9. To solve the problem, we note that acceleration is the second time derivative of the position function; it is a vector and can be determined from its components. The net force is related to the acceleration via Newton's second law. Thus, differentiating $x(t) = -15.0 + 2.00t + 4.00t^3$ twice with respect to t, we get

$$\frac{dx}{dt} = 2.00 - 12.0t^2, \quad \frac{d^2x}{dt^2} = -24.0t$$

Similarly, differentiating $y(t) = 25.0 + 7.00t - 9.00t^2$ twice with respect to t yields

$$\frac{dy}{dt} = 7.00 - 18.0t, \quad \frac{d^2y}{dt^2} = -18.0$$

(a) The acceleration is

$$\vec{a} = a_x\hat{\text{i}} + a_y\hat{\text{j}} = \frac{d^2x}{dt^2}\hat{\text{i}} + \frac{d^2y}{dt^2}\hat{\text{j}} = (-24.0t)\hat{\text{i}} + (-18.0)\hat{\text{j}}.$$

At $t = 0.700\text{ s}$, we have $\vec{a} = (-16.8)\hat{\text{i}} + (-18.0)\hat{\text{j}}$ with a magnitude of

$$a = |\vec{a}| = \sqrt{(-16.8)^2 + (-18.0)^2} = 24.6\text{ m/s}^2.$$

Thus, the magnitude of the force is $F = ma = (0.34\text{ kg})(24.6\text{ m/s}^2) = 8.37\text{ N}.$

(b) The angle $\vec{F}$ or $\vec{a} = \vec{F}/m$ makes with $+x$ is

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-18.0\text{ m/s}^2}{-16.8\text{ m/s}^2}\right) = 47.0° \text{ or } -133°.$$

We choose the latter ($-133°$) since $\vec{F}$ is in the third quadrant.

(c) The direction of travel is the direction of a tangent to the path, which is the direction of the velocity vector:

$$\vec{v}(t) = v_x\hat{\text{i}} + v_y\hat{\text{j}} = \frac{dx}{dt}\hat{\text{i}} + \frac{dy}{dt}\hat{\text{j}} = (2.00 - 12.0t^2)\hat{\text{i}} + (7.00 - 18.0t)\hat{\text{j}}.$$

At $t = 0.700\text{ s}$, we have $\vec{v}(t = 0.700\text{ s}) = (-3.88\text{ m/s})\hat{\text{i}} + (-5.60\text{ m/s})\hat{\text{j}}.$ Therefore, the angle $\vec{v}$ makes with $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-5.60\text{ m/s}}{-3.88\text{ m/s}}\right) = 55.3° \text{ or } -125°.$$

We choose the latter ($-125°$) since $\vec{v}$ is in the third quadrant.

10. To solve the problem, we note that acceleration is the second time derivative of the position function, and the net force is related to the acceleration via Newton's second law. Thus, differentiating

$$x(t) = -13.00 + 2.00t + 4.00t^2 - 3.00t^3$$

twice with respect to t, we get

$$\frac{dx}{dt} = 2.00 + 8.00t - 9.00t^2, \quad \frac{d^2x}{dt^2} = 8.00 - 18.0t$$

The net force acting on the particle at $t = 3.40\text{ s}$ is

$$\vec{F} = m\frac{d^2x}{dt^2}\hat{\text{i}} = (0.150)\left[8.00 - 18.0(3.40)\right]\hat{\text{i}} = (-7.98\text{ N})\hat{\text{i}}$$

11. The velocity is the derivative (with respect to time) of given function x, and the acceleration is the derivative of the velocity. Thus, $a = 2c - 3(2.0)(2.0)t$, which we use in Newton's second law: $F = (2.0\text{ kg})a = 4.0c - 24t$ (with SI units understood). At $t = 3.0$ s, we are told that $F = -36$ N. Thus, $-36 = 4.0c - 24(3.0)$ can be used to solve for c. The result is $c = +9.0\text{ m/s}^2$.

12. From the slope of the graph we find $a_x = 3.0$ m/s^2. Applying Newton's second law to the x axis (and taking θ to be the angle between F_1 and F_2), we have

$$F_1 + F_2 \cos\theta = m a_x \quad \Rightarrow \quad \theta = 56°.$$

13. (a) From the fact that $T_3 = 9.8$ N, we conclude the mass of disk D is 1.0 kg. Both this and that of disk C cause the tension $T_2 = 49$ N, which allows us to conclude that disk C has a mass of 4.0 kg. The weights of these two disks plus that of disk B determine the tension $T_1 = 58.8$ N, which leads to the conclusion that $m_B = 1.0$ kg. The weights of all the disks must add to the 98 N force described in the problem; therefore, disk A has mass 4.0 kg.

(b) $m_B = 1.0$ kg, as found in part (a).

(c) $m_C = 4.0$ kg, as found in part (a).

(d) $m_D = 1.0$ kg, as found in part (a).

14. Three vertical forces are acting on the block: the earth pulls down on the block with gravitational force 3.0 N; a spring pulls up on the block with elastic force 1.0 N; and, the surface pushes up on the block with normal force F_N. There is no acceleration, so

$$\sum F_y = 0 = F_N + (1.0\text{ N}) + (-3.0\text{ N})$$

yields $F_N = 2.0$ N.

(a) By Newton's third law, the force exerted by the block on the surface has that same magnitude but opposite direction: 2.0 N.

(b) The direction is down.

15. **THINK** We have a piece of salami hung to a spring scale in various ways. The problem is to explore the concept of weight.

EXPRESS We first note that the reading on the spring scale is proportional to the weight of the salami. In all three cases (a) – (c) depicted in Fig. 5-34, the scale is not accelerating, which means that the two cords exert forces of equal magnitude on it. The scale reads the magnitude of either of these forces. In each case the tension force of the cord attached to the salami must be the same in magnitude as the weight of the salami because the salami is not accelerating. Thus the scale reading is mg, where m is the mass of the salami.

ANALYZE In all three cases (a) – (c), the reading on the scale is

$$w = mg = (11.0\text{ kg})(9.8\text{ m/s}^2) = 108\text{ N}.$$

LEARN The weight of an object is measured when the object is not accelerating vertically relative to the ground. If it is, then the weight measured is called the apparent weight.

16. (a) There are six legs, and the vertical component of the tension force in each leg is $T\sin\theta$ where $\theta = 40°$. For vertical equilibrium (zero acceleration in the *y* direction) then Newton's second law leads to

$$6T\sin\theta = mg \Rightarrow T = \frac{mg}{6\sin\theta}$$

which (expressed as a multiple of the bug's weight *mg*) gives roughly $T/mg \approx 0.260$.

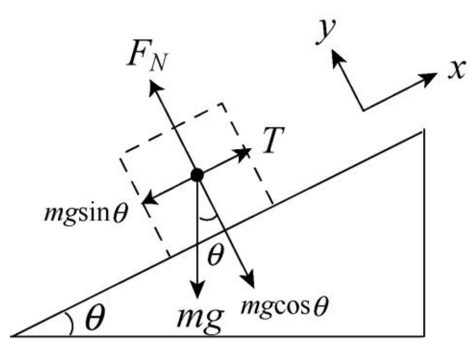
(b) The angle θ is measured from horizontal, so as the insect "straightens out the legs" θ will increase (getting closer to $90°$), which causes $\sin\theta$ to increase (getting closer to 1) and consequently (since $\sin\theta$ is in the denominator) causes *T* to decrease.

17. **THINK** A block attached to a cord is resting on an incline plane. We apply Newton's second law to solve for the tension in the cord and the normal force on the block.

EXPRESS The free-body diagram of the problem is shown to the right. Since the acceleration of the block is zero, the components of Newton's second law equation yield

$$T - mg\sin\theta = 0$$
$$F_N - mg\cos\theta = 0,$$

where T is the tension in the cord, and F_N is the normal force on the block.



ANALYZE (a) Solving the first equation for the tension in the string, we find

$$T = mg\sin\theta = (8.5\text{ kg})(9.8\text{ m/s}^2)\sin 30° = 42\text{ N}.$$

(b) We solve the second equation above for the normal force F_N:

$$F_N = mg\cos\theta = (8.5\text{ kg})(9.8\text{ m/s}^2)\cos 30° = 72\text{ N}.$$

(c) When the cord is cut, it no longer exerts a force on the block and the block accelerates. The *x* component of the second law becomes $-mg\sin\theta = ma$, so the acceleration becomes

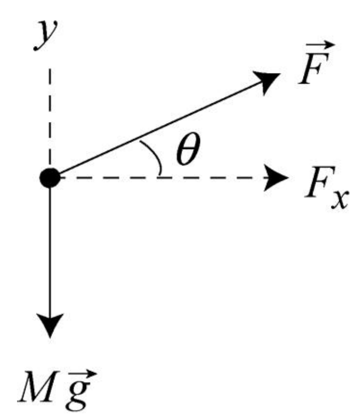
$$a = -g\sin\theta = -(9.8\text{ m/s}^2)\sin 30° = -4.9\text{ m/s}^2.$$

The negative sign indicates the acceleration is down the plane. The magnitude of the acceleration is 4.9 m/s^2.

LEARN The normal force F_N on the block must be equal to $mg\cos\theta$ so that the block is in contact with the surface of the incline at all time. When the cord is cut, the block has an acceleration $a = -g\sin\theta$, which in the limit $\theta \to 90°$ becomes $-g$, as in the case of a free fall.

18. The free-body diagram of the cars is shown on the right. The force exerted by John Massis is

$$F = 2.5mg = 2.5(80 \text{ kg})(9.8 \text{ m/s}^2) = 1960 \text{ N}.$$



Since the motion is along the horizontal x-axis, using Newton's second law, we have $Fx = F\cos\theta = Ma_x$, where M is the total mass of the railroad cars. Thus, the acceleration of the cars is

$$a_x = \frac{F\cos\theta}{M} = \frac{(1960 \text{ N})\cos 30°}{(7.0\times10^5 \text{ N}/9.8 \text{ m/s}^2)} = 0.024 \text{ m/s}^2.$$

Using Eq. 2-16, the speed of the car at the end of the pull is

$$v_x = \sqrt{2a_x\Delta x} = \sqrt{2(0.024 \text{ m/s}^2)(1.0 \text{ m})} = 0.22 \text{ m/s}.$$

19. **THINK** In this problem we're interested in the force applied to a rocket sled to accelerate it from rest to a given speed in a given time interval.

EXPRESS In terms of magnitudes, Newton's second law is $F = ma$, where $F = \left|\vec{F}_{\text{net}}\right|$, $a = |\vec{a}\,|$, and m is the (always positive) mass. The magnitude of the acceleration can be found using constant acceleration kinematics (Table 2-1). Solving $v = v_0 + at$ for the case where it starts from rest, we have $a = v/t$ (which we interpret in terms of magnitudes, making specification of coordinate directions unnecessary). Thus, the required force is $F = ma = mv/t$.

ANALYZE Expressing the velocity in SI units as

$$v = (1600 \text{ km/h})\,(1000 \text{ m/km})/(3600 \text{ s/h}) = 444 \text{ m/s},$$

we find the force to be

$$F = m\frac{v}{t} = (500\,\text{kg})\,\frac{444\,\text{m/s}}{1.8\,\text{s}} = 1.2\times10^5 \text{ N}.$$

LEARN From the expression $F = mv/t$, we see that the shorter the time to attain a given speed, the greater the force required.

20. The stopping force $\vec{F}$ and the path of the passenger are horizontal. Our +x axis is in the direction of the passenger's motion, so that the passenger's acceleration (''deceleration'') is negative-valued and the stopping force is in the –x direction: $\vec{F} = -F\,\hat{\text{i}}$. Using Eq. 2-16 with

$$v_0 = (53 \text{ km/h})(1000 \text{ m/km})/(3600 \text{ s/h}) = 14.7 \text{ m/s}$$

and $v = 0$, the acceleration is found to be

$$v^2 = v_0^2 + 2a\Delta x \;\Rightarrow\; a = -\frac{v_0^2}{2\Delta x} = -\frac{(14.7 \text{ m/s})^2}{2\left(0.65 \text{ m}\right)} = -167 \text{ m/s}^2.$$

Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

$$\vec{F} = m\vec{a} \;\Rightarrow\; -F = \left(41 \text{ kg}\right)\left(-167 \text{ m/s}^2\right)$$

which results in $F = 6.8 \times 10^3$ N.

21. (a) The slope of each graph gives the corresponding component of acceleration. Thus, we find $a_x = 3.00 \text{ m/s}^2$ and $a_y = -5.00 \text{ m/s}^2$. The magnitude of the acceleration vector is therefore

$$a = \sqrt{(3.00 \text{ m/s}^2)^2 + (-5.00 \text{ m/s}^2)^2} = 5.83 \text{ m/s}^2,$$

and the force is obtained from this by multiplying with the mass (m = 2.00 kg). The result is $F = ma$ =11.7 N.

(b) The direction of the force is the same as that of the acceleration:

$$\theta = \tan^{-1} [(-5.00 \text{ m/s}^2)/(3.00 \text{ m/s}^2)] = -59.0°.$$

22. (a) The coin undergoes free fall. Therefore, with respect to ground, its acceleration is

$$\vec{a}_{\text{coin}} = \vec{g} = (-9.8 \text{ m/s}^2)\hat{\text{j}}.$$

(b) Since the customer is being pulled down with an acceleration of $\vec{a}\,'_{\text{customer}} = 1.24\vec{g} = (-12.15 \text{ m/s}^2)\hat{\text{j}}$, the acceleration of the coin with respect to the customer is

$$\vec{a}_{\text{rel}} = \vec{a}_{\text{coin}} - \vec{a}\,'_{\text{customer}} = (-9.8 \text{ m/s}^2)\hat{\text{j}} - (-12.15 \text{ m/s}^2)\hat{\text{j}} = (+2.35 \text{ m/s}^2)\hat{\text{j}}.$$

(c) The time it takes for the coin to reach the ceiling is

$$t = \sqrt{\frac{2h}{a_{\text{rel}}}} = \sqrt{\frac{2(2.20\text{ m})}{2.35\text{ m/s}^2}} = 1.37\text{ s}.$$

(d) Since gravity is the only force acting on the coin, the actual force on the coin is

$$\vec{F}_{\text{coin}} = m\vec{a}_{\text{coin}} = m\vec{g} = (0.567\times10^{-3}\text{ kg})(-9.8\text{ m/s}^2)\hat{\text{j}} = (-5.56\times10^{-3}\text{ N})\hat{\text{j}}.$$

(e) In the customer's frame, the coin travels upward at a constant acceleration. Therefore, the apparent force on the coin is

$$\vec{F}_{\text{app}} = m\vec{a}_{\text{rel}} = (0.567\times10^{-3}\text{ kg})(+2.35\text{ m/s}^2)\hat{\text{j}} = (+1.33\times10^{-3}\text{ N})\hat{\text{j}}.$$

23. We note that the rope is 22.0° from vertical, and therefore 68.0° from horizontal.

(a) With T = 760 N, then its components are

$$\vec{T} = T\cos 68.0°\,\hat{\text{i}} + T\sin 68.0°\,\hat{\text{j}} = (285\text{N})\,\hat{\text{i}} + (705\text{N})\,\hat{\text{j}}.$$

(b) No longer in contact with the cliff, the only other force on Tarzan is due to earth's gravity (his weight). Thus,

$$\vec{F}_{\text{net}} = \vec{T} + \vec{W} = (285\text{ N})\,\hat{\text{i}} + (705\text{ N})\,\hat{\text{j}} - (820\text{ N})\,\hat{\text{j}} = (285\text{N})\,\hat{\text{i}} - (115\text{ N})\,\hat{\text{j}}.$$

(c) In a manner that is efficiently implemented on a vector-capable calculator, we convert from rectangular (x, y) components to magnitude-angle notation:

$$\vec{F}_{\text{net}} = (285,\ -115) \rightarrow (307\angle -22.0°)$$

so that the net force has a magnitude of 307 N.

(d) The angle (see part (c)) has been found to be −22.0°, or 22.0° below horizontal (away from the cliff).

(e) Since $\vec{a} = \vec{F}_{\text{net}}/m$ where $m = W/g$ = 83.7 kg, we obtain $\vec{a} = 3.67\text{ m/s}^2$.

(f) Eq. 5-1 requires that $\vec{a} \parallel \vec{F}_{\text{net}}$ so that the angle is also −22.0°, or 22.0° below horizontal (away from the cliff).

24. We take rightward as the $+x$ direction. Thus, $\vec{F}_1 = (20\text{ N})\hat{\text{i}}$. In each case, we use Newton's second law $\vec{F}_1 + \vec{F}_2 = m\vec{a}$ where m = 2.0 kg.

(a) If $\vec{a} = (+10 \text{ m/s}^2)\,\hat{\text{i}}$, then the equation above gives $\vec{F}_2 = 0$.

(b) If , $\vec{a} = (+20 \text{m/s}^2)\,\hat{\text{i}}$, then that equation gives $\vec{F}_2 = (20 \text{N})\hat{\text{i}}$.

(c) If $\vec{a} = 0$, then the equation gives $\vec{F}_2 = (-20 \text{N})\hat{\text{i}}$.

(d) If $\vec{a} = (-10 \text{ m/s}^2)\,\hat{\text{i}}$, the equation gives $\vec{F}_2 = (-40 \text{N})\hat{\text{i}}$.

(e) If $\vec{a} = (-20 \text{ m/s}^2)\,\hat{\text{i}}$, the equation gives $\vec{F}_2 = (-60 \text{N})\hat{\text{i}}$.

25. (a) The acceleration is

$$a = \frac{F}{m} = \frac{20 \text{ N}}{900 \text{kg}} = 0.022 \text{ m/s}^2 .$$

(b) The distance traveled in 1 day (= 86400 s) is

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(0.0222 \text{m/s}^2\right)(86400 \text{s})^2 = 8.3 \times 10^7 \text{ m} .$$

(c) The speed it will be traveling is given by

$$v = at = \left(0.0222 \text{ m/s}^2\right)\left(86400 \text{ s}\right) = 1.9 \times 10^3 \text{ m/s} .$$

26. Some assumptions (not so much for realism but rather in the interest of using the given information efficiently) are needed in this calculation: we assume the fishing line and the path of the salmon are horizontal. Thus, the weight of the fish contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 8.7$ kg). Our $+x$ axis is in the direction of the salmon's velocity (away from the fisherman), so that its acceleration (‘‘deceleration’’) is negative-valued and the force of tension is in the $-x$ direction: $\vec{T} = -T$. We use Eq. 2-16 and SI units (noting that $v = 0$).

$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(2.8 \text{ m/s})^2}{2\left(0.11 \text{ m}\right)} = -36 \text{ m/s}^2 .$$

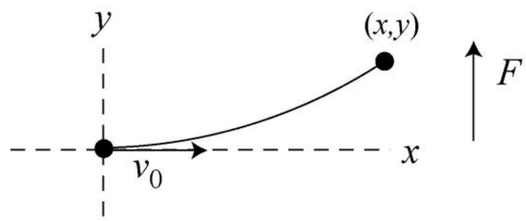
Assuming there are no significant horizontal forces other than the tension, Eq. 5-1 leads to

$$\vec{T} = m\vec{a} \quad \Rightarrow \quad -T = \left(8.7 \text{kg}\right)\left(-36 \text{m/s}^2\right)$$

which results in $T = 3.1 \times 10^2$ N.

27. **THINK** An electron moving horizontally is under the influence of a vertical force. Its path will be deflected toward the direction of the applied force.

EXPRESS The setup is shown in the figure below. The acceleration of the electron is vertical and for all practical purposes the only force acting on it is the electric force. The force of gravity is negligible. We take the $+x$ axis to be in the direction of the initial velocity v_0 and the $+y$ axis to be in the direction of the electrical force, and place the origin at the initial position of the electron.



Since the force and acceleration are constant, we use the equations from Table 2-1: $x = v_0 t$ and

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2.$$

ANALYZE The time taken by the electron to travel a distance x (= 30 mm) horizontally is $t = x/v_0$ and its deflection in the direction of the force is

$$y = \frac{1}{2}\frac{F}{m}\left(\frac{x}{v_0}\right)^2 = \frac{1}{2}\left(\frac{4.5\times10^{-16}\text{ N}}{9.11\times10^{-31}\text{ kg}}\right)\left(\frac{30\times10^{-3}\text{ m}}{1.2\times10^{7}\text{ m/s}}\right)^2 = 1.5\times10^{-3}\text{ m.}$$

LEARN Since the applied force is constant, the acceleration in the y-direction is also constant and the path is parabolic with $y \propto x^2$.

28. The stopping force $\vec{F}$ and the path of the car are horizontal. Thus, the weight of the car contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 1327$ kg). Our $+x$ axis is in the direction of the car's velocity, so that its acceleration (''deceleration'') is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F\,\hat{\text{i}}$.

(a) We use Eq. 2-16 and SI units (noting that $v = 0$ and $v_0 = 40(1000/3600) = 11.1$ m/s).

$$v^2 = v_0^2 + 2a\Delta x \quad\Rightarrow\quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(11.1\text{ m/s})^2}{2(15\text{ m})}$$

which yields $a = -4.12$ m/s^2. Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

$$\vec{F} = m\vec{a} \quad \Rightarrow \quad -F = (1327 \text{ kg})\left(-4.12 \text{ m/s}^2\right)$$

which results in $F = 5.5 \times 10^3$ N.

(b) Equation 2-11 readily yields $t = -v_0/a = 2.7$ s.

(c) Keeping F the same means keeping a the same, in which case (since $v = 0$) Eq. 2-16 expresses a direct proportionality between Δx and v_0^2. Therefore, doubling v_0 means quadrupling Δx. That is, the new over the old stopping distances is a factor of 4.0.

(d) Equation 2-11 illustrates a direct proportionality between t and v_0 so that doubling one means doubling the other. That is, the new time of stopping is a factor of 2.0 greater than the one found in part (b).

29. We choose up as the $+y$ direction, so $\vec{a} = (-3.00 \text{ m/s}^2)\hat{\text{j}}$ (which, without the unit-vector, we denote as a since this is a 1-dimensional problem in which Table 2-1 applies). From Eq. 5-12, we obtain the firefighter's mass: $m = W/g = 72.7$ kg.

(a) We denote the force exerted by the pole on the firefighter $\vec{F}_{\text{fp}} = F_{\text{fp}}\ \hat{\text{j}}$ and apply Eq. 5-1. Since $\vec{F}_{\text{net}} = m\vec{a}$, we have

$$F_{\text{fp}} - F_g = ma \quad \Rightarrow \quad F_{\text{fp}} - 712 \text{ N} = (72.7 \text{ kg})(-3.00 \text{ m/s}^2)$$

which yields $F_{\text{fp}} = 494$ N.

(b) The fact that the result is positive means $\vec{F}_{\text{fp}}$ points up.

(c) Newton's third law indicates $\vec{F}_{\text{fp}} = -\vec{F}_{\text{pf}}$, which leads to the conclusion that $|\vec{F}_{\text{pf}}| = 494$ N.

(d) The direction of $\vec{F}_{\text{pf}}$ is down.

30. The stopping force $\vec{F}$ and the path of the toothpick are horizontal. Our $+x$ axis is in the direction of the toothpick's motion, so that the toothpick's acceleration (''deceleration'') is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F\,\hat{\text{i}}$. Using Eq. 2-16 with $v_0 = 220$ m/s and $v = 0$, the acceleration is found to be

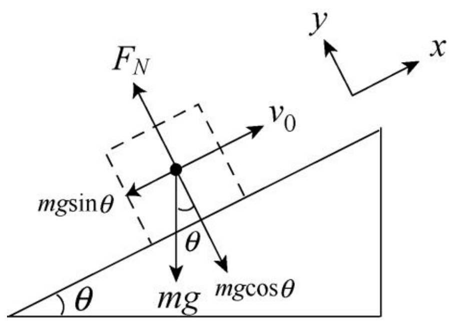
$$v^2 = v_0^2 + 2a\Delta x \quad \Rightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{(220 \text{ m/s})^2}{2(0.015 \text{ m})} = -1.61 \times 10^6 \text{ m/s}^2.$$

Thus, the magnitude of the force exerted by the branch on the toothpick is

$$F = m|a| = (1.3\times10^{-4}\text{ kg})(1.61\times10^{6}\text{ m/s}^2) = 2.1\times10^{2}\text{ N}.$$

31. **THINK** In this problem we analyze the motion of a block sliding up an inclined plane and back down.

EXPRESS The free-body diagram is shown below. $\vec{F}_N$ is the normal force of the plane on the block and $m\vec{g}$ is the force of gravity on the block. We take the $+x$ direction to be up the incline, and the $+y$ direction to be in the direction of the normal force exerted by the incline on the block.



The x component of Newton's second law is then $mg\sin\theta = -ma$; thus, the acceleration is $a = -g\sin\theta$. Placing the origin at the bottom of the plane, the kinematic equations (Table 2-1) for motion along the x axis which we will use are $v^2 = v_0^2 + 2ax$ and $v = v_0 + at$. The block momentarily stops at its highest point, where $v = 0$; according to the second equation, this occurs at time $t = -v_0/a$.

ANALYZE (a) The position where the block stops is

$$x = v_0t + \frac{1}{2}at^2 = v_0\left(\frac{-v_0}{a}\right) + \frac{1}{2}a\left(\frac{-v_0}{a}\right)^2 = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{(3.50\text{ m/s})^2}{-\left(9.8\text{ m/s}^2\right)\sin 32.0°}\right) = 1.18\text{ m}.$$

(b) The time it takes for the block to get there is

$$t = \frac{v_0}{a} = -\frac{v_0}{-g\sin\theta} = -\frac{3.50\text{ m/s}}{-(9.8\text{ m/s}^2)\sin 32.0°} = 0.674\text{ s}.$$

(c) That the return speed is identical to the initial speed is to be expected since there are no dissipative forces in this problem. In order to prove this, one approach is to set $x = 0$ and solve $x = v_0t + \frac{1}{2}at^2$ for the total time (up and back down) t. The result is

$$t=-\frac{2v_0}{a}=-\frac{2v_0}{-g\sin\theta}=-\frac{2(3.50\text{ m/s})}{-(9.8\text{ m/s}^2)\sin 32.0°}=1.35\text{ s}.$$

The velocity when it returns is therefore

$$v=v_0+at=v_0-gt\sin\theta=3.50\text{ m/s}-(9.8\text{ m/s}^2)(1.35\text{ s})\sin 32°=-3.50\text{ m/s}.$$

The negative sign indicates the direction is down the plane.

LEARN As expected, the speed of the block when it gets back to the bottom of the incline is the same as its initial speed. As we shall see in Chapter 8, this is a consequence of energy conservation. If friction is present, then the return speed will be smaller than the initial speed.

32. (a) Using notation suitable to a vector-capable calculator, the $\vec{F}_{\text{net}}$ = 0 condition becomes

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (6.00 \angle 150°) + (7.00 \angle -60.0°) + \vec{F}_3 = 0 .$$

Thus, $\vec{F}_3$ = (1.70 N) $\hat{\text{i}}$ + (3.06 N)$\hat{\text{j}}$.

(b) A constant velocity condition requires zero acceleration, so the answer is the same.

(c) Now, the acceleration is

$$\vec{a}=(13.0\text{ m/s}^2)\,\hat{\text{i}}-(14.0\text{ m/s}^2)\,\hat{\text{j}}.$$

Using $\vec{F}_{\text{net}} = m\,\vec{a}$ (with m = 0.025 kg) we now obtain

$$\vec{F}_3 = (2.02\text{ N})\,\hat{\text{i}} + (2.71\text{ N})\,\hat{\text{j}}.$$

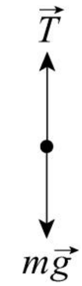
33. The free-body diagram is shown below. Let $\vec{T}$ be the tension of the cable and $m\vec{g}$ be the force of gravity. If the upward direction is positive, then Newton's second law is $T - mg = ma$, where a is the acceleration.

Thus, the tension is $T = m(g + a)$. We use constant acceleration kinematics (Table 2-1) to find the acceleration (where $v = 0$ is the final velocity, $v_0 = -12$ m/s is the initial velocity, and $y = -42$ m is the coordinate at the stopping point). Consequently, $v^2 = v_0^2 + 2ay$ leads to

$$a=-\frac{v_0^2}{2y}=-\frac{(-12\text{ m/s})^2}{2(-42\text{ m})}=1.71\text{ m/s}^2.$$

We now return to calculate the tension:

$$\begin{aligned} T &= m(g + a) \\ &= (1600 \text{ kg})(9.8 \text{ m/s}^2 + 1.71 \text{ m/s}^2) \\ &= 1.8 \times 10^4 \text{ N}. \end{aligned}$$

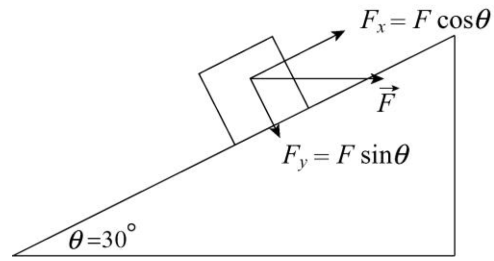


34. We resolve this horizontal force into appropriate components.

(a) Newton's second law applied to the x-axis produces

$$F\cos\theta - mg\sin\theta = ma.$$

For a = 0, this yields F = 566 N.



(b) Applying Newton's second law to the y axis (where there is no acceleration), we have

$$F_N - F\sin\theta - mg\cos\theta = 0$$

which yields the normal force $F_N = 1.13 \times 10^3$ N.

35. The acceleration vector as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(8.00t\,\hat{\text{i}} + 3.00t^2\,\hat{\text{j}}\right)\text{ m/s} = (8.00\,\hat{\text{i}} + 6.00t\,\hat{\text{j}})\text{ m/s}^2.$$

(a) The magnitude of the force acting on the particle is

$$F = ma = m|\vec{a}| = (3.00)\sqrt{(8.00)^2 + (6.00t)^2} = (3.00)\sqrt{64.0 + 36.0\,t^2}\text{ N}.$$

Thus, $F = 35.0$ N corresponds to $t = 1.415$ s, and the acceleration vector at this instant is

$$\vec{a} = [8.00\,\hat{\text{i}} + 6.00(1.415)\,\hat{\text{j}}]\text{ m/s}^2 = (8.00\text{ m/s}^2)\,\hat{\text{i}} + (8.49\text{ m/s}^2)\,\hat{\text{j}}.$$

The angle $\vec{a}$ makes with $+x$ is

$$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{8.49\text{ m/s}^2}{8.00\text{ m/s}^2}\right) = 46.7°.$$

(b) The velocity vector at $t = 1.415$ s is

$$\vec{v} = \left[8.00(1.415)\,\hat{\mathrm{i}} + 3.00(1.415)^2\,\hat{\mathrm{j}}\right]\mathrm{m/s} = (11.3\ \mathrm{m/s})\,\hat{\mathrm{i}} + (6.01\ \mathrm{m/s})\hat{\mathrm{j}}.$$

Therefore, the angle $\vec{v}$ makes with $+x$ is

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{6.01\ \mathrm{m/s}}{11.3\ \mathrm{m/s}}\right) = 28.0°.$$

36. (a) Constant velocity implies zero acceleration, so the "uphill" force must equal (in magnitude) the "downhill" force: $T = mg \sin \theta$. Thus, with $m = 50$ kg and $\theta = 8.0°$, the tension in the rope equals 68 N.

(b) With an uphill acceleration of 0.10 m/s^2, Newton's second law (applied to the x axis) yields

$$T - mg \sin \theta = ma \quad \Rightarrow \quad T - (50\ \mathrm{kg})\left(9.8\ \mathrm{m/s^2}\right) \sin 8.0° = (50\ \mathrm{kg})\left(0.10\ \mathrm{m/s^2}\right)$$

which leads to $T = 73$ N.

37. (a) Since friction is negligible the force of the girl is the only horizontal force on the sled. The vertical forces (the force of gravity and the normal force of the ice) sum to zero. The acceleration of the sled is

$$a_s = \frac{F}{m_s} = \frac{5.2\ \mathrm{N}}{8.4\ \mathrm{kg}} = 0.62\ \mathrm{m/s^2}\ .$$

(b) According to Newton's third law, the force of the sled on the girl is also 5.2 N. Her acceleration is

$$a_g = \frac{F}{m_g} = \frac{5.2\ \mathrm{N}}{40\ \mathrm{kg}} = 0.13\ \mathrm{m/s^2}\ .$$

(c) The accelerations of the sled and girl are in opposite directions. Assuming the girl starts at the origin and moves in the $+x$ direction, her coordinate is given by $x_g = \frac{1}{2}a_g t^2$. The sled starts at x_0 = 15 m and moves in the $-x$ direction. Its coordinate is given by $x_s = x_0 - \frac{1}{2}a_s t^2$. They meet when $x_g = x_s$, or

$$\frac{1}{2}a_g t^2 = x_0 - \frac{1}{2}a_s t^2.$$

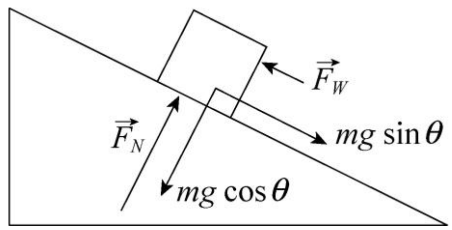
This occurs at time

$$t = \sqrt{\frac{2x_0}{a_g + a_s}}.$$

By then, the girl has gone the distance

$$x_g = \frac{1}{2} a_g t^2 = \frac{x_0 a_g}{a_g + a_s} = \frac{(15\text{ m})(0.13\text{ m/s}^2)}{0.13\text{ m/s}^2 + 0.62\text{ m/s}^2} = 2.6\text{ m}.$$

38. We label the 40 kg skier "*m*," which is represented as a block in the figure shown. The force of the wind is denoted $\vec{F}_w$ and might be either "uphill" or "downhill" (it is shown uphill in our sketch). The incline angle θ is 10°. The $-x$ direction is downhill.



(a) Constant velocity implies zero acceleration; thus, application of Newton's second law along the x axis leads to $mg \sin \theta - F_w = 0$. This yields F_w = 68 N (uphill).

(b) Given our coordinate choice, we have $a = |a| = 1.0\text{ m/s}^2$. Newton's second law

$$mg \sin \theta - F_w = ma$$

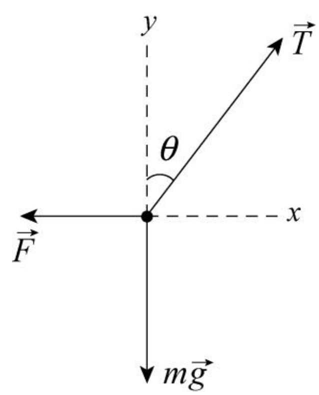
now leads to F_w = 28 N (uphill).

(c) Continuing with the forces as shown in our figure, the equation

$$mg \sin \theta - F_w = ma$$

will lead to $F_w = -12$ N when $|a| = 2.0\text{ m/s}^2$. This simply tells us that the wind is opposite to the direction shown in our sketch; in other words, $\vec{F}_w = 12\text{ N}$ *downhill.*

39. The solutions to parts (a) and (b) have been combined here. The free-body diagram is shown to the right, with the tension of the string $\vec{T}$, the force of gravity $m\vec{g}$, and the force of the air $\vec{F}$. Our coordinate system is shown. Since the sphere is motionless the net force on it is zero, and the x and the y components of the equations are:



$$T \sin \theta - F = 0$$
$$T \cos \theta - mg = 0,$$

where θ = 37°. We answer the questions in the reverse order. Solving $T \cos \theta - mg = 0$ for the tension, we obtain

$$T = mg/\cos\theta = (3.0 \times 10^{-4}\text{ kg})(9.8\text{ m/s}^2)/\cos 37° = 3.7 \times 10^{-3}\text{ N}.$$

Solving $T \sin\theta - F = 0$ for the force of the air:

$$F = T\sin\theta = (3.7 \times 10^{-3}\text{ N})\sin 37° = 2.2 \times 10^{-3}\text{ N}.$$

40. The acceleration of an object (neither pushed nor pulled by any force other than gravity) on a smooth inclined plane of angle θ is $a = -g\sin\theta$. The slope of the graph shown with the problem statement indicates $a = -2.50$ m/s^2. Therefore, we find $\theta = 14.8°$. Examining the forces perpendicular to the incline (which must sum to zero since there is no component of acceleration in this direction) we find $F_N = mg\cos\theta$, where $m = 5.00$ kg. Thus, the normal (perpendicular) force exerted at the box/ramp interface is 47.4 N.

41. The mass of the bundle is $m = (449\text{ N})/(9.80\text{ m/s}^2) = 45.8$ kg and we choose $+y$ upward.

(a) Newton's second law, applied to the bundle, leads to

$$T - mg = ma \;\Rightarrow\; a = \frac{387\text{ N} - 449\text{ N}}{45.8\text{ kg}}$$

which yields $a = -1.4$ m/s^2 (or $|a| = 1.4$ m/s^2) for the acceleration. The minus sign in the result indicates the acceleration vector points down. Any downward acceleration of magnitude greater than this is also acceptable (since that would lead to even smaller values of tension).

(b) We use Eq. 2-16 (with Δx replaced by $\Delta y = -6.1$ m). We assume $v_0 = 0$.

$$|v| = \sqrt{2a\Delta y} = \sqrt{2\left(-1.35\text{ m/s}^2\right)\left(-6.1\text{ m}\right)} = 4.1\text{ m/s}.$$

For downward accelerations greater than 1.4 m/s^2, the speeds at impact will be larger than 4.1 m/s.

42. The direction of motion (the direction of the barge's acceleration) is $+\hat{\mathrm{i}}$, and $+\vec{\mathrm{j}}$ is chosen so that the pull $\vec{F}_\mathrm{h}$ from the horse is in the first quadrant. The components of the unknown force of the water are denoted simply F_x and F_y.

(a) Newton's second law applied to the barge, in the x and y directions, leads to

$$\left(7900\,\text{N}\right)\cos 18° + F_x = ma$$
$$\left(7900\,\text{N}\right)\sin 18° + F_y = 0$$

respectively. Plugging in a = 0.12 m/s^2 and m = 9500 kg, we obtain F_x = − 6.4 × 10^3 N and F_y = − 2.4 × 10^3 N. The magnitude of the force of the water is therefore

$$F_{\text{water}} = \sqrt{F_x^2 + F_y^2} = 6.8 \times 10^3 \text{ N}.$$

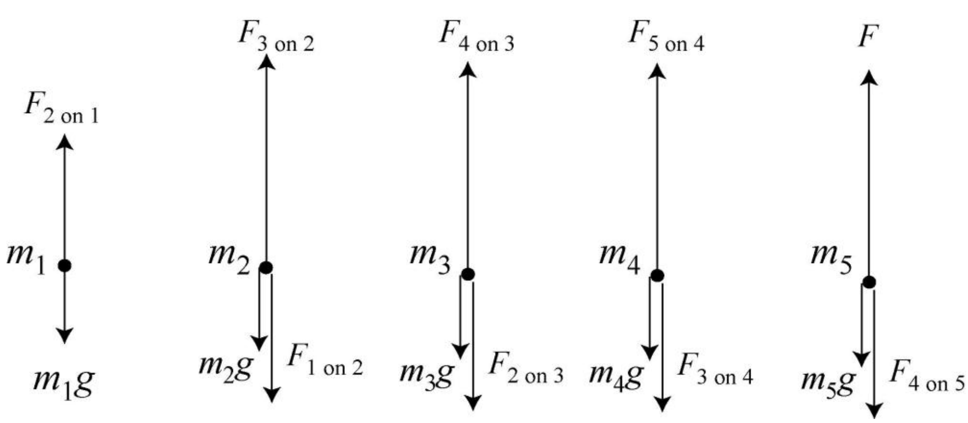
(b) Its angle measured from $+\hat{\mathrm{i}}$ is either

$$\tan^{-1}\left(\frac{F_y}{F_x}\right) = +21° \text{ or } 201°.$$

The signs of the components indicate the latter is correct, so $\vec{F}_{\text{water}}$ is at 201° measured counterclockwise from the line of motion (+x axis).

43. **THINK** A chain of five links is accelerated vertically upward by an external force. We are interested in the forces exerted by one link on its adjacent one.

EXPRESS The links are numbered from bottom to top. The forces on the first link are the force of gravity $m\vec{g}$, downward, and the force $\vec{F}_{2\text{on}1}$ of link 2, upward, as shown in the free-body diagram below (not drawn to scale). Take the positive direction to be upward. Then Newton's second law for the first link is $F_{2\text{on}1} - m_1 g = m_1 a$. The equations for the other links can be written in a similar manner (see below).



ANALYZE (a) Given that $a = 2.50\,\text{m/s}^2$, from $F_{2\text{on}1} - m_1 g = m_1 a$, the force exerted by link 2 on link 1 is

$$F_{2\text{on}1} = m_1(a+g) = (0.100\,\text{kg})(2.5\,\text{m/s}^2 + 9.80\,\text{m/s}^2) = 1.23\,\text{N}.$$

(b) From the free-body diagram above, we see that the forces on the second link are the force of gravity $m_2\vec{g}$, downward, the force $\vec{F}_{1\text{on}2}$ of link 1, downward, and the force $\vec{F}_{3\text{on}2}$

of link 3, upward. According to Newton's third law $\vec{F}_{1\text{on}2}$ has the same magnitude as $\vec{F}_{2\text{on}1}$. Newton's second law for the second link is

$$F_{3\text{on}2} - F_{1\text{on}2} - m_2 g = m_2 a$$

so

$$F_{3\text{on}2} = m_2(a + g) + F_{1\text{on}2} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 1.23 \text{ N} = 2.46 \text{ N}.$$

(c) Newton's second law equation for link 3 is $F_{4\text{on}3} - F_{2\text{on}3} - m_3 g = m_3 a$, so

$$F_{4\text{on}3} = m_3(a + g) + F_{2\text{on}3} = (0.100 \text{ N})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 2.46 \text{ N} = 3.69 \text{ N},$$

where Newton's third law implies $F_{2\text{on}3} = F_{3\text{on}2}$ (since these are magnitudes of the force vectors).

(d) Newton's second law for link 4 is

$$F_{5\text{on}4} - F_{3\text{on}4} - m_4 g = m_4 a,$$

so

$$F_{5\text{on}4} = m_4(a + g) + F_{3\text{on}4} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 3.69 \text{ N} = 4.92 \text{ N},$$

where Newton's third law implies $F_{3\text{on}4} = F_{4\text{on}3}$.

(e) Newton's second law for the top link is $F - F_{4\text{on}5} - m_5 g = m_5 a$, so

$$F = m_5(a + g) + F_{4\text{on}5} = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 4.92 \text{ N} = 6.15 \text{ N},$$

where $F_{4\text{on}5} = F_{5\text{on}4}$ by Newton's third law.

(f) Each link has the same mass ($m_1 = m_2 = m_3 = m_4 = m_5 = m$) and the same acceleration, so the same net force acts on each of them:

$$F_{\text{net}} = ma = (0.100 \text{ kg})\,(2.50 \text{ m/s}^2) = 0.250 \text{ N}.$$

LEARN In solving this problem we have used both Newton's second and third laws. Each pair of links constitutes a third-law force pair, with $\vec{F}_{\text{i on j}} = -\vec{F}_{\text{j on i}}$.

44. (a) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is downward). Thus (with $+y$ upward) the acceleration is $a = +2.4 \text{ m/s}^2$. Newton's second law leads to

$$T - mg = ma \;\Rightarrow\; m = \frac{T}{g + a}$$

which yields $m = 7.3$ kg for the mass.

(b) Repeating the above computation (now to solve for the tension) with a = +2.4 m/s^2 will, of course, lead us right back to T = 89 N. Since the direction of the velocity did not enter our computation, this is to be expected.

45. (a) The mass of the elevator is m = (27800/9.80) = 2837 kg and (with +y upward) the acceleration is a = +1.22 m/s^2. Newton's second law leads to

$$T - mg = ma \;\Rightarrow\; T = m(g + a)$$

which yields $T = 3.13 \times 10^4$ N for the tension.

(b) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is upward). Thus (with +y upward) the acceleration is now a = –1.22 m/s^2, so that the tension is

$$T = m\,(g + a) = 2.43 \times 10^4 \text{ N}.$$

46. With a_{ce} meaning "the acceleration of the coin relative to the elevator" and a_{eg} meaning "the acceleration of the elevator relative to the ground," we have

$$a_{ce} + a_{eg} = a_{cg} \quad \Rightarrow \quad -8.00 \text{ m/s}^2 + a_{eg} = -9.80 \text{ m/s}^2$$

which leads to a_{eg} = –1.80 m/s^2. We have chosen upward as the positive y direction. Then Newton's second law (in the "ground" reference frame) yields $T - m\,g = m\,a_{eg}$, or

$$T = m\,g + m\,a_{eg} = m(g + a_{eg}) = (2000 \text{ kg})(8.00 \text{ m/s}^2) = 16.0 \text{ kN}.$$

47. Using Eq. 4-26, the launch speed of the projectile is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(69 \text{ m})}{\sin 2(53^\circ)}} = 26.52 \text{ m/s}.$$

The horizontal and vertical components of the speed are

$$v_x = v_0 \cos\theta = (26.52 \text{ m/s})\cos 53^\circ = 15.96 \text{ m/s}$$
$$v_y = v_0 \sin\theta = (26.52 \text{ m/s})\sin 53^\circ = 21.18 \text{ m/s}.$$

Since the acceleration is constant, we can use Eq. 2-16 to analyze the motion. The component of the acceleration in the horizontal direction is

$$a_x = \frac{v_x^2}{2x} = \frac{(15.96 \text{ m/s})^2}{2(5.2 \text{ m})\cos 53^\circ} = 40.7 \text{ m/s}^2,$$

and the force component is

$$F_x = ma_x = (85\text{ kg})(40.7\text{ m/s}^2) = 3460\text{ N}.$$

Similarly, in the vertical direction, we have

$$a_y = \frac{v_y^2}{2y} = \frac{(21.18\text{ m/s})^2}{2(5.2\text{ m})\sin 53°} = 54.0\text{ m/s}^2.$$

and the force component is

$$F_y = ma_y + mg = (85\text{ kg})(54.0\text{ m/s}^2 + 9.80\text{ m/s}^2) = 5424\text{ N}.$$

Thus, the magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3460\text{ N})^2 + (5424\text{ N})^2} = 6434\text{ N} \approx 6.4\times10^3\text{ N},$$

to two significant figures.

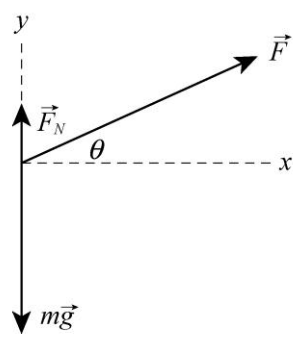
48. Applying Newton's second law to cab B (of mass m) we have

$$a = \tfrac{T}{m} - g = 4.89\text{ m/s}^2.$$

Next, we apply it to the box (of mass m_b) to find the normal force:

$$F_N = m_b(g + a) = 176\text{ N}.$$

49. The free-body diagram (not to scale) for the block is shown to the right. $\vec{F}_N$ is the normal force exerted by the floor and $m\vec{g}$ is the force of gravity.



(a) The x component of Newton's second law is $F\cos\theta = ma$, where m is the mass of the block and a is the x component of its acceleration. We obtain

$$a = \frac{F\cos\theta}{m} = \frac{(12.0\text{ N})\cos 25.0°}{5.00\text{ kg}} = 2.18\text{ m/s}^2.$$

This is its acceleration provided it remains in contact with the floor. Assuming it does, we find the value of F_N (and if F_N is positive, then the assumption is true but if F_N is negative then the block leaves the floor). The y component of Newton's second law becomes

$$F_N + F\sin\theta - mg = 0,$$

so

$$F_N = mg - F\sin\theta = (5.00\text{ kg})(9.80\text{ m/s}^2) - (12.0\text{ N})\sin 25.0° = 43.9\text{ N}.$$

Hence the block remains on the floor and its acceleration is a = 2.18 m/s^2.

(b) If F is the minimum force for which the block leaves the floor, then F_N = 0 and the y component of the acceleration vanishes. The y component of the second law becomes

$$F \sin\theta - mg = 0 \quad \Rightarrow \quad F = \frac{mg}{\sin\theta} = \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)}{\sin 25.0°} = 116\text{ N}.$$

(c) The acceleration is still in the x direction and is still given by the equation developed in part (a):

$$a = \frac{F\cos\theta}{m} = \frac{(116\text{ N})\cos 25.0°}{5.00\text{ kg}} = 21.0\text{ m/s}^2.$$

50. (a) The net force on the *system* (of total mass M = 80.0 kg) is the force of gravity acting on the total overhanging mass (m_{BC} = 50.0 kg). The magnitude of the acceleration is therefore $a = (m_{BC}\,g)/M$ = 6.125 m/s^2. Next we apply Newton's second law to block C itself (choosing *down* as the $+y$ direction) and obtain

$$m_C g - T_{BC} = m_C a.$$

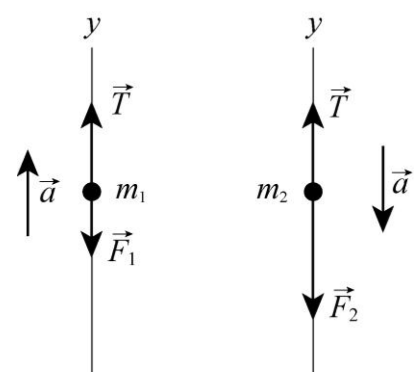
This leads to T_{BC} = 36.8 N.

(b) We use Eq. 2-15 (choosing *rightward* as the $+x$ direction): $\Delta x = 0 + \frac{1}{2}at^2$ = 0.191 m.

51. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$



which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

Substituting the result back, we have

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2}\right) g$$

(a) With $m_1 = 1.3$ kg and $m_2 = 2.8$ kg, the acceleration becomes

$$a = \left(\frac{2.80\text{ kg} - 1.30\text{ kg}}{2.80\text{ kg} + 1.30\text{ kg}}\right)(9.80\text{ m/s}^2) = 3.59\text{ m/s}^2 \approx 3.6\text{ m/s}^2.$$

(b) Similarly, the tension in the cord is

$$T = \frac{2(1.30\text{ kg})(2.80\text{ kg})}{1.30\text{ kg} + 2.80\text{ kg}}(9.80\text{ m/s}^2) = 17.4\text{ N} \approx 17\text{ N}.$$

52. Viewing the man-rope-sandbag as a system means that we should be careful to choose a consistent positive direction of motion (though there are other ways to proceed, say, starting with individual application of Newton's law to each mass). We take *down* as positive for the man's motion and *up* as positive for the sandbag's motion and, without ambiguity, denote their acceleration as a. The net force on the system is the different between the weight of the man and that of the sandbag. The system mass is m_{sys} = 85 kg + 65 kg = 150 kg. Thus, Eq. 5-1 leads to

$$(85\text{ kg})(9.8\text{ m/s}^2) - (65\text{ kg})(9.8\text{ m/s}^2) = m_{\text{sys}}\, a$$

which yields a = 1.3 m/s^2. Since the system starts from rest, Eq. 2-16 determines the speed (after traveling Δy = 10 m) as follows:

$$v = \sqrt{2a\Delta y} = \sqrt{2(1.3\text{ m/s}^2)(10\text{ m})} = 5.1\text{ m/s}.$$

53. We apply Newton's second law first to the three blocks as a single system and then to the individual blocks. The +x direction is to the right in Fig. 5-48.

(a) With $m_{sys} = m_1 + m_2 + m_3$ = 67.0 kg, we apply Eq. 5-2 to the x motion of the system, in which case, there is only one force $\vec{T}_3 = +\vec{T}_3\ \hat{\text{i}}$. Therefore,

$$T_3 = m_{\text{sys}} a \quad \Rightarrow \quad 65.0\text{ N} = (67.0\text{ kg})a$$

which yields a = 0.970 m/s^2 for the system (and for each of the blocks individually).

(b) Applying Eq. 5-2 to block 1, we find

$$T_1 = m_1 a = (12.0\text{ kg})\left(0.970\text{ m/s}^2\right) = 11.6\text{ N}.$$

(c) In order to find T_2, we can either analyze the forces on block 3 or we can treat blocks 1 and 2 as a system and examine its forces. We choose the latter.

$$T_2 = (m_1 + m_2)a = (12.0\text{ kg} + 24.0\text{ kg})\left(0.970\text{ m/s}^2\right) = 34.9\text{ N}.$$

54. First, we consider all the penguins (1 through 4, counting left to right) as one system, to which we apply Newton's second law:

$$T_4 = (m_1 + m_2 + m_3 + m_4)a \quad \Rightarrow \quad 222\,\text{N} = (12\,\text{kg} + m_2 + 15\,\text{kg} + 20\,\text{kg})a.$$

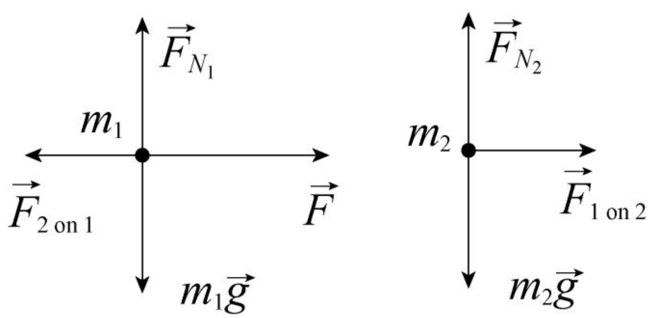
Second, we consider penguins 3 and 4 as one system, for which we have

$$T_4 - T_2 = (m_3 + m_4)a$$
$$111\,\text{N} = (15\text{ kg} + 20\,\text{kg})a \quad \Rightarrow \quad a = 3.2\text{ m/s}^2.$$

Substituting the value, we obtain m_2 = 23 kg.

55. **THINK** In this problem a horizontal force is applied to block 1 which then pushes against block 2. Both blocks move together as a rigid connected system.

EXPRESS The free-body diagrams for the two blocks in (a) are shown below. $\vec{F}$ is the applied force and $\vec{F}_{1\text{on}2}$ is the force exerted by block 1 on block 2. We note that $\vec{F}$ is applied directly to block 1 and that block 2 exerts a force $\vec{F}_{2\text{on}1} = -\vec{F}_{1\text{on}2}$ on block 1 (taking Newton's third law into account).

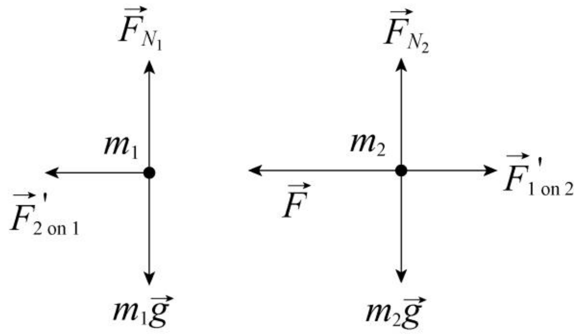


Newton's second law for block 1 is $F - F_{2\text{on}1} = m_1 a$, where a is the acceleration. The second law for block 2 is $F_{1\text{on}2} = m_2 a$. Since the blocks move together they have the same acceleration and the same symbol is used in both equations.

ANALYZE (a) From the second equation we obtain the expression $a = F_{1\text{on}2}/m_2$, which we substitute into the first equation to get $F - F_{2\text{on}1} = m_1 F_{1\text{on}2}/m_2$. Since $F_{2\text{on}1} = F_{1\text{on}2}$ (same magnitude for third-law force pair), we obtain

$$F_{2\text{on}1} = F_{1\text{on}2} = \frac{m_2}{m_1 + m_2} F = \frac{1.2\text{ kg}}{2.3\text{ kg} + 1.2\text{ kg}}(3.2\text{ N}) = 1.1\text{ N}.$$

(b) If $\vec{F}$ is applied to block 2 instead of block 1 (and in the opposite direction), the free-body diagrams would look like the following:



The corresponding force of contact between the blocks would be

$$F'_{2\text{on}1} = F'_{1\text{on}2} = \frac{m_1}{m_1 + m_2} F = \frac{2.3 \text{ kg}}{2.3 \text{ kg} + 1.2 \text{ kg}} (3.2 \text{ N}) = 2.1 \text{ N}.$$

(c) We note that the acceleration of the blocks is the same in the two cases. In part (a), the force $F_{1\text{on}2}$ is the only horizontal force on the block of mass m_2 and in part (b) $F'_{2\text{on}1}$ is the only horizontal force on the block with $m_1 > m_2$. Since $F_{1\text{on}2} = m_2 a$ in part (a) and $F'_{2\text{on}1} = m_1 a$ in part (b), then for the accelerations to be the same, $F'_{2\text{on}1} > F_{1\text{on}2}$, i.e., force between blocks must be larger in part (b).

LEARN This problem demonstrates that when two blocks are being accelerated together under an external force, the contact force between the two blocks is greater if the smaller mass is pushing against the bigger one, as in part (b). In the special case where the two masses are equal, $m_1 = m_2 = m$, $F'_{2\text{on}1} = F_{2\text{on}1} = F/2$.

56. Both situations involve the same applied force and the same total mass, so the accelerations must be the same in both figures.

(a) The (direct) force causing *B* to have this acceleration in the first figure is twice as big as the (direct) force causing *A* to have that acceleration. Therefore, *B* has the twice the mass of *A*. Since their total is given as 12.0 kg then *B* has a mass of m_B = 8.00 kg and *A* has mass m_A = 4.00 kg. Considering the first figure, (20.0 N)/(8.00 kg) = 2.50 m/s^2. Of course, the same result comes from considering the second figure ((10.0 N)/(4.00 kg) = 2.50 m/s^2).

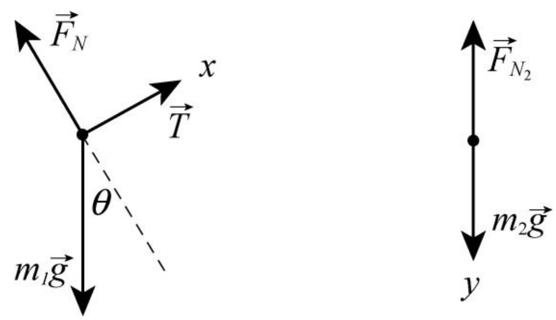
(b) F_a = (12.0 kg)(2.50 m/s^2) = 30.0 N

57. The free-body diagram for each block is shown below. *T* is the tension in the cord and θ = 30° is the angle of the incline. For block 1, we take the +*x* direction to be up the incline and the +*y* direction to be in the direction of the normal force $\vec{F}_N$ that the plane exerts on the block. For block 2, we take the +*y* direction to be down. In this way, the accelerations of the two blocks can be represented by the same symbol *a*, without

ambiguity. Applying Newton's second law to the x and y axes for block 1 and to the y axis of block 2, we obtain

$$\begin{aligned} T - m_1 g \sin\theta &= m_1 a \\ F_N - m_1 g \cos\theta &= 0 \\ m_2 g - T &= m_2 a \end{aligned}$$

respectively. The first and third of these equations provide a simultaneous set for obtaining values of a and T. The second equation is not needed in this problem, since the normal force is neither asked for nor is it needed as part of some further computation (such as can occur in formulas for friction).



(a) We add the first and third equations above:

$$m_2 g - m_1 g \sin\theta = m_1 a + m_2 a.$$

Consequently, we find

$$a = \frac{(m_2 - m_1\sin\theta)g}{m_1 + m_2} = \frac{[2.30\text{ kg} - (3.70\text{ kg})\sin 30.0°]\left(9.80\text{ m/s}^2\right)}{3.70\text{ kg} + 2.30\text{ kg}} = 0.735\text{m/s}^2.$$

(b) The result for a is positive, indicating that the acceleration of block 1 is indeed up the incline and that the acceleration of block 2 is vertically down.

(c) The tension in the cord is

$$T = m_1 a + m_1 g \sin\theta = (3.70\text{ kg})\left(0.735\text{ m/s}^2\right) + (3.70\text{ kg})\left(9.80\text{ m/s}^2\right)\sin 30.0° = 20.8\text{N}.$$

58. The motion of the man-and-chair is positive if upward.

(a) When the man is grasping the rope, pulling with a force equal to the tension T in the rope, the total upward force on the man-and-chair due its two contact points with the rope is $2T$. Thus, Newton's second law leads to

$$2T - mg = ma$$

so that when $a = 0$, the tension is $T = 466$ N.

(b) When $a = +1.30$ m/s^2 the equation in part (a) predicts that the tension will be $T = 527\ \text{N}$.

(c) When the man is not holding the rope (instead, the co-worker attached to the ground is pulling on the rope with a force equal to the tension T in it), there is only one contact point between the rope and the man-and-chair, and Newton's second law now leads to

$$T - mg = ma$$

so that when $a = 0$, the tension is $T = 931$ N.

(d) When $a = +1.30$ m/s^2, the equation in (c) yields $T = 1.05 \times 10^3$ N.

(e) The rope comes into contact (pulling down in each case) at the left edge and the right edge of the pulley, producing a total downward force of magnitude $2T$ on the ceiling. Thus, in part (a) this gives $2T = 931$ N.

(f) In part (b) the downward force on the ceiling has magnitude $2T = 1.05 \times 10^3$ N.

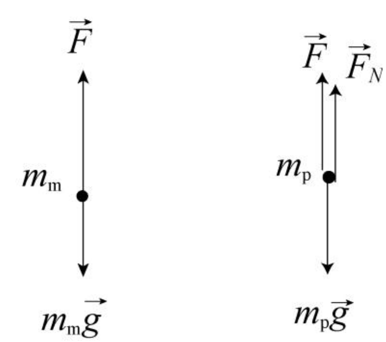
(g) In part (c) the downward force on the ceiling has magnitude $2T = 1.86 \times 10^3$ N.

(h) In part (d) the downward force on the ceiling has magnitude $2T = 2.11 \times 10^3$ N.

59. **THINK** This problem involves the application of Newton's third law. As the monkey climbs up a tree, it pulls downward on the rope, but the rope pulls upward on the monkey.

EXPRESS We take $+y$ to be up for both the monkey and the package. The force the monkey pulls downward on the rope has magnitude F.

The free-body diagrams for the monkey and the package are shown to the right (not to scale). According to Newton's third law, the rope pulls upward on the monkey with a force of the same magnitude, so Newton's second law for forces acting on the monkey leads to



$$F - m_m g = m_m a_m,$$

where m_m is the mass of the monkey and a_m is its acceleration.

Since the rope is massless, $F = T$ is the tension in the rope. The rope pulls upward on the package with a force of magnitude F, so Newton's second law for the package is

$$F + F_N - m_p g = m_p a_p,$$

where m_p is the mass of the package, a_p is its acceleration, and F_N is the normal force exerted by the ground on it. Now, if F is the minimum force required to lift the package, then $F_N = 0$ and $a_p = 0$. According to the second law equation for the package, this means $F = m_p g$.

ANALYZE (a) Substituting $m_p g$ for F in the equation for the monkey, we solve for a_m:

$$a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)g}{m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{10\text{ kg}} = 4.9\text{ m/s}^2.$$

(b) As discussed, Newton's second law leads to $F - m_p g = m_p a'_p$ for the package and $F - m_m g = m_m a'_m$ for the monkey. If the acceleration of the package is downward, then the acceleration of the monkey is upward, so $a'_m = -a'_p$. Solving the first equation for F

$$F = m_p(g + a'_p) = m_p(g - a'_m)$$

and substituting this result into the second equation:

$$m_p(g - a'_m) - m_m g = m_m a'_m,$$

we solve for a'_m:

$$a'_m = \frac{(m_p - m_m)g}{m_p + m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{15\text{ kg} + 10\text{ kg}} = 2.0\text{ m/s}^2.$$

(c) The result is positive, indicating that the acceleration of the monkey is upward.

(d) Solving the second law equation for the package, the tension in the rope is

$$F = m_p(g - a'_m) = (15\text{ kg})(9.8\text{ m/s}^2 - 2.0\text{ m/s}^2) = 120\text{N}.$$

LEARN The situations described in (b)-(d) are similar to that of an Atwood machine. With $m_p > m_m$, the package accelerates downward while the monkey accelerates upward.

60. The horizontal component of the acceleration is determined by the net horizontal force.

(a) If the rate of change of the angle is

$$\frac{d\theta}{dt} = (2.00\times10^{-2})°/\text{s} = (2.00\times10^{-2})°/\text{s}\cdot\left(\frac{\pi\text{ rad}}{180°}\right) = 3.49\times10^{-4}\text{rad/s},$$

then, using $F_x = F\cos\theta$, we find the rate of change of acceleration to be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{(20.0\text{ N})\sin 25.0°}{5.00\text{ kg}}\left(3.49\times10^{-4}\text{rad/s}\right)$$
$$= -5.90\times10^{-4}\text{ m/s}^3.$$

(b) If the rate of change of the angle is

$$\frac{d\theta}{dt} = -(2.00\times10^{-2})°/\text{s} = -(2.00\times10^{-2})°/\text{s}\cdot\left(\frac{\pi\text{ rad}}{180°}\right) = -3.49\times10^{-4}\text{rad/s},$$

then the rate of change of acceleration would be

$$\frac{da_x}{dt} = \frac{d}{dt}\left(\frac{F\cos\theta}{m}\right) = -\frac{F\sin\theta}{m}\frac{d\theta}{dt} = -\frac{(20.0\text{ N})\sin 25.0°}{5.00\text{ kg}}\left(-3.49\times10^{-4}\text{rad/s}\right)$$
$$= +5.90\times10^{-4}\text{ m/s}^3.$$

61. **THINK** As more mass is thrown out of the hot-air balloon, its upward acceleration increases.

EXPRESS The forces on the balloon are the force of gravity $m\vec{g}$ (down) and the force of the air $\vec{F}_a$ (up). We take the $+y$ to be up, and use a to mean the *magnitude* of the acceleration. When the mass is M (before the ballast is thrown out) the acceleration is downward and Newton's second law is

$$Mg - F_a = Ma$$

After the ballast is thrown out, the mass is $M - m$ (where m is the mass of the ballast) and the acceleration is now upward. Newton's second law leads to

$$F_a - (M - m)g = (M - m)a.$$

Combing the two equations allows us to solve for m.

ANALYZE The first equation gives $F_a = M(g - a)$, and this plugs into the new equation to give

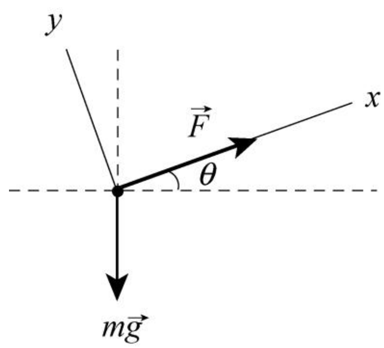
$$M(g-a) - (M-m)g = (M-m)a \;\Rightarrow\; m = \frac{2Ma}{g+a}.$$

LEARN More generally, if a ballast mass m' is tossed, the resulting acceleration is a' which is related to m' via:

$$m' = M\,\frac{a'+a}{g+a}\ ,$$

showing that the more mass thrown out, the greater is the upward acceleration. For $a' = a$, we get $m' = 2Ma/(g+a)$, which agrees with what was found above.

62. To solve the problem, we note that the acceleration along the slanted path depends on only the force components along the path, not the components perpendicular to the path.



(a) From the free-body diagram shown, we see that the net force on the putting shot along the +*x*-axis is

$$F_{\text{net},x} = F - mg\sin\theta = 380.0\text{ N} - (7.260\text{ kg})(9.80\text{ m/s}^2)\sin 30° = 344.4\text{ N},$$

which in turn gives

$$a_x = F_{\text{net},x}/m = (344.4\text{ N})/(7.260\text{ kg}) = 47.44\text{ m/s}^2.$$

Using Eq. 2-16 for constant-acceleration motion, the speed of the shot at the end of the acceleration phase is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{(2.500\text{ m/s})^2 + 2(47.44\text{ m/s}^2)(1.650\text{ m})} = 12.76\text{ m/s}.$$

(b) If $\theta = 42°,$ then

$$a_x = \frac{F_{\text{net},x}}{m} = \frac{F - mg\sin\theta}{m} = \frac{380.0\text{ N} - (7.260\text{ kg})(9.80\text{ m/s}^2)\sin 42.00°}{7.260\text{ kg}} = 45.78\text{ m/s}^2,$$

and the final (launch) speed is

$$v = \sqrt{v_0^2 + 2a_x\Delta x} = \sqrt{(2.500\text{ m/s})^2 + 2(45.78\text{ m/s}^2)(1.650\text{ m})} = 12.54\text{ m/s}.$$

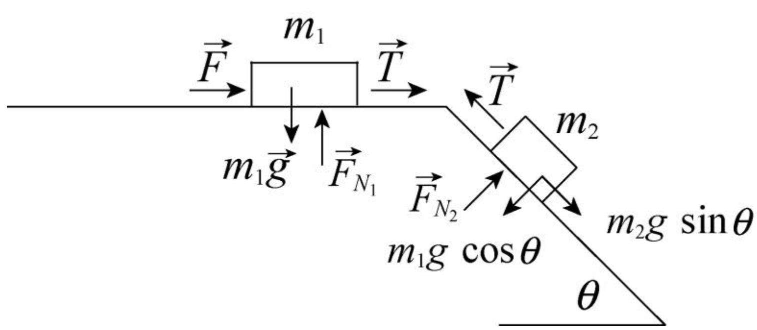
(c) The decrease in launch speed when changing the angle from $30.00°$ to $42.00°$ is

$$\frac{12.76\text{ m/s} - 12.54\text{ m/s}}{12.76\text{ m/s}} = 0.0169 = 1.69\%.$$

63. (a) The acceleration (which equals *F/m* in this problem) is the derivative of the velocity. Thus, the velocity is the integral of *F/m*, so we find the "area" in the graph (15 units) and divide by the mass (3) to obtain $v - v_o = 15/3 = 5$. Since $v_o = 3.0$ m/s, then $v = 8.0\text{ m/s}$.

(b) Our positive answer in part (a) implies $\vec{v}$ points in the $+x$ direction.

64. The $+x$ direction for $m_2 = 1.0$ kg is "downhill" and the $+x$ direction for $m_1 = 3.0$ kg is rightward; thus, they accelerate with the same sign.



(a) We apply Newton's second law to the *x* axis of each box:

$$\begin{aligned} m_2 g \sin\theta - T &= m_2 a \\ F + T &= m_1 a \end{aligned}$$

Adding the two equations allows us to solve for the acceleration:

$$a = \frac{m_2 g \sin\theta + F}{m_1 + m_2}$$

With $F = 2.3$ N and $\theta = 30°$, we have $a = 1.8$ m/s^2. We plug back in and find $T = 3.1$ N.

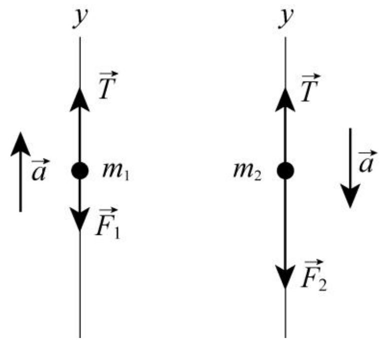
(b) We consider the "critical" case where the *F* has reached the *max* value, causing the tension to vanish. The first of the equations in part (a) shows that $a = g\sin 30°$ in this case; thus, $a = 4.9$ m/s^2. This implies (along with $T = 0$ in the second equation in part (a)) that

$$F = (3.0\text{ kg})(4.9\text{ m/s}^2) = 14.7\text{ N} \approx 15\text{ N}$$

in the critical case.

65. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:

$$T - m_1 g = m_1 a$$
$$m_2 g - T = m_2 a$$



which can be solved to give

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

(a) At $t = 0$, $m_{10} = 1.30$ kg. With $dm_1/dt = -0.200$ kg/s, we find the rate of change of acceleration to be

$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{(m_2 + m_{10})^2}\frac{dm_1}{dt} = -\frac{2(2.80\text{ kg})(9.80\text{ m/s}^2)}{(2.80\text{ kg} + 1.30\text{ kg})^2}\left(-0.200\text{ kg/s}\right) = 0.653\text{ m/s}^3.$$

(b) At $t = 3.00$ s, $m_1 = m_{10} + (dm_1/dt)t = 1.30\text{ kg} + (-0.200\text{ kg/s})(3.00\text{ s}) = 0.700\text{ kg}$, and the rate of change of acceleration is

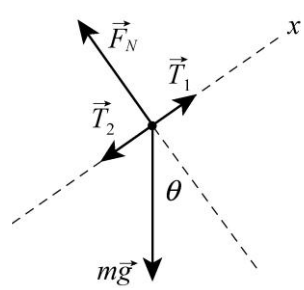
$$\frac{da}{dt} = \frac{da}{dm_1}\frac{dm_1}{dt} = -\frac{2m_2 g}{(m_2 + m_1)^2}\frac{dm_1}{dt} = -\frac{2(2.80\text{ kg})(9.80\text{ m/s}^2)}{(2.80\text{ kg} + 0.700\text{ kg})^2}\left(-0.200\text{ kg/s}\right) = 0.896\text{ m/s}^3.$$

(c) The acceleration reaches its maximum value when

$$0 = m_1 = m_{10} + (dm_1/dt)t = 1.30\text{ kg} + (-0.200\text{ kg/s})t,$$

or $t = 6.50$ s.

66. The free-body diagram is shown to the right. Newton's second law for the mass m for the x direction leads to



$$T_1 - T_2 - mg\sin\theta = ma,$$

which gives the difference in the tension in the pull cable:

$$T_1 - T_2 = m\left(g\sin\theta + a\right) = \left(2800\text{ kg}\right)\left[(9.8\text{ m/s}^2)\sin 35° + 0.81\text{ m/s}^2\right] = 1.8\times 10^4\text{ N}.$$

67. First we analyze the entire *system* with "clockwise" motion considered positive (that is, downward is positive for block C, rightward is positive for block B, and upward is positive for block A): $m_C g - m_A g = Ma$ (where M = mass of the *system* = 24.0 kg). This yields an acceleration of

$$a = g(m_C - m_A)/M = 1.63\ \text{m/s}^2.$$

Next we analyze the forces just on block C: $m_C\,g - T = m_C\,a$. Thus the tension is

$$T = m_C\,g(2m_A + m_B)/M = 81.7\ \text{N}.$$

68. We first use Eq. 4-26 to solve for the launch speed of the shot:

$$y - y_0 = (\tan\theta)x - \frac{gx^2}{2(v'\cos\theta)^2}.$$

With $\theta = 34.10°$, $y_0 = 2.11\ \text{m}$, and $(x, y) = (15.90\ \text{m}, 0)$, we find the launch speed to be $v' = 11.85\ \text{m/s}$. During this phase, the acceleration is

$$a = \frac{v'^2 - v_0^2}{2L} = \frac{(11.85\ \text{m/s})^2 - (2.50\ \text{m/s})^2}{2(1.65\ \text{m})} = 40.63\ \text{m/s}^2.$$

Since the acceleration along the slanted path depends on only the force components along the path, not the components perpendicular to the path, the average force on the shot during the acceleration phase is

$$F = m(a + g\sin\theta) = (7.260\ \text{kg})\left[40.63\ \text{m/s}^2 + (9.80\ \text{m/s}^2)\sin 34.10°\right] = 334.8\ \text{N}.$$

69. We begin by examining a slightly different problem: similar to this figure but without the string. The motivation is that if (without the string) block A is found to accelerate faster (or exactly as fast) as block B then (returning to the original problem) the tension in the string is trivially zero. In the absence of the string,

$$a_A = F_A/m_A = 3.0\ \text{m/s}^2$$
$$a_B = F_B/m_B = 4.0\ \text{m/s}^2$$

so the trivial case does not occur. We now (with the string) consider the net force on the *system*: $Ma = F_A + F_B = 36$ N. Since $M = 10$ kg (the total mass of the system) we obtain $a = 3.6\ \text{m/s}^2$. The two forces on block A are F_A and T (in the same direction), so we have

$$m_A\,a = F_A + T \quad\Rightarrow\quad T = 2.4\ \text{N}.$$

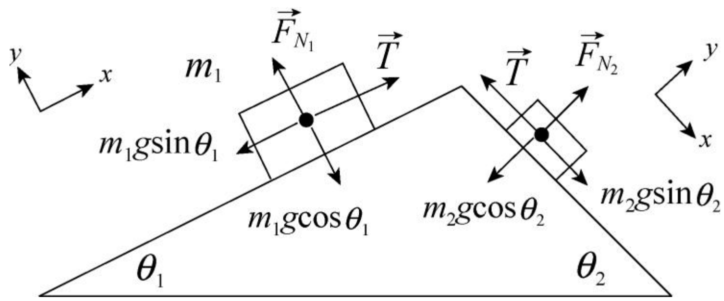
70. (a) For the 0.50 meter drop in "free fall," Eq. 2-16 yields a speed of 3.13 m/s. Using this as the "initial speed" for the final motion (over 0.02 meter) during which his motion slows at rate "a," we find the magnitude of his average acceleration from when his feet first touch the patio until the moment his body stops moving is $a = 245\ \text{m/s}^2$.

(b) We apply Newton's second law: $F_{\text{stop}} - mg = ma \;\Rightarrow\; F_{\text{stop}} = 20.4$ kN.

71. **THINK** We have two boxes connected together by a cord and placed on a wedge. The system accelerates together and we'd like to know the tension in the cord.

EXPRESS The $+x$ axis is "uphill" for m_1 = 3.0 kg and "downhill" for m_2 = 2.0 kg (so they both accelerate with the same sign). The x components of the two masses along the x axis are given by $m_1 g \sin\theta_1$ and $m_2 g \sin\theta_2$, respectively. The free-body diagram is shown below. Applying Newton's second law, we obtain

$$T - m_1 g \sin\theta_1 = m_1 a$$
$$m_2 g \sin\theta_2 - T = m_2 a$$



Adding the two equations allows us to solve for the acceleration:

$$a = \left(\frac{m_2 \sin\theta_2 - m_1 \sin\theta_1}{m_2 + m_1} \right) g$$

ANALYZE With $\theta_1 = 30°$ and $\theta_2 = 60°$, we have a = 0.45 m/s^2. This value is plugged back into either of the two equations to yield the tension

$$T = \frac{m_1 m_2 g}{m_2 + m_1} (\sin\theta_2 + \sin\theta_1) = 16.1\,\text{N}$$

LEARN In this problem we find $m_2 \sin\theta_2 > m_1 \sin\theta_1$, so that $a > 0$, indicating that m_2 slides down and m_1 slides up. The situation would reverse if $m_2 \sin\theta_2 < m_1 \sin\theta_1$. When $m_2 \sin\theta_2 = m_1 \sin\theta_1$, the acceleration is a = 0 and the two masses hang in balance. Notice also the symmetry between the two masses in the expression for T.

72. Since the velocity of the particle does not change, it undergoes no acceleration and must therefore be subject to zero net force. Therefore,

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0\,.$$

Thus, the third force $\vec{F}_3$ is given by

$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -\left(2\hat{\text{i}} + 3\hat{\text{j}} - 2\hat{\text{k}}\right)\text{N} - \left(-5\hat{\text{i}} + 8\hat{\text{j}} - 2\hat{\text{k}}\right)\text{N} = \left(3\hat{\text{i}} - 11\hat{\text{j}} + 4\hat{\text{k}}\right)\text{N}.$$

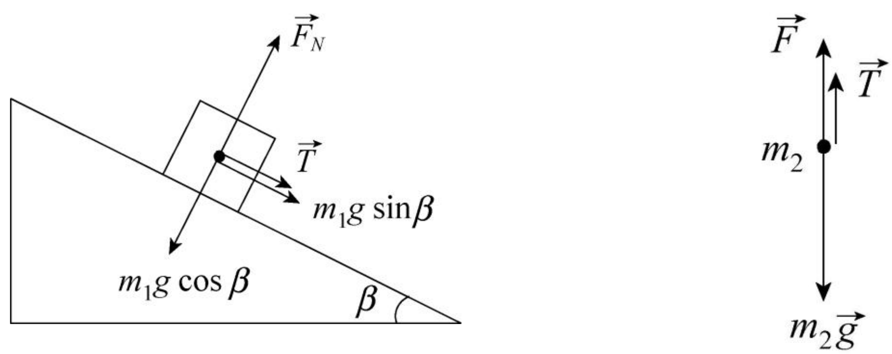
The specific value of the velocity is not used in the computation.

73. **THINK** We have two masses connected together by a cord. A force is applied to the second mass and the system accelerates together. We apply Newton's second law to solve the problem.

EXPRESS The free-body diagrams for the two masses are shown below (not to scale). We first analyze the forces on m_1=1.0 kg. The +x direction is "downhill" (parallel to $\vec{T}$). With an acceleration a = 5.5 m/s^2 in the positive x direction for m_1, Newton's second law applied to the x-axis gives

$$T + m_1 g \sin\beta = m_1 a\,.$$

On the other hand, for the second mass m_2=2.0 kg, we have $m_2 g - F - T = m_2 a$, where the tension comes in as an upward force (the cord can pull, not push). The two equations can be combined to solve for T and β.



ANALYZE We solve (b) first. By combining the two equations above, we obtain

$$\sin\beta = \frac{(m_1+m_2)a + F - m_2 g}{m_1 g} = \frac{(1.0\text{ kg} + 2.0\text{ kg})(5.5\text{ m/s}^2) + 6.0\text{ N} - (2.0\text{ kg})(9.8\text{ m/s}^2)}{(1.0\text{ kg})(9.8\text{ m/s}^2)}$$
$$= 0.296$$

which gives $\beta = 17.2°$.

(a) Substituting the value for β found in (a) into the first equation, we have

$$T = m_1(a - g\sin\beta) = (1.0\text{ kg})\left[5.5\text{ m/s}^2 - (9.8\text{ m/s}^2)\sin 17.2°\right] = 2.60\text{ N}.$$

LEARN For $\beta = 0$, the problem becomes the same as that discussed in Sample Problem "Block on table, block hanging." In this case, our results reduce to the familiar expressions: $a = m_2 g/(m_1+m_2)$, and $T = m_1 m_2 g/(m_1+m_2)$.

74. We are only concerned with horizontal forces in this problem (gravity plays no direct role). Without loss of generality, we take one of the forces along the $+x$ direction and the other at 80° (measured counterclockwise from the x axis). This calculation is efficiently implemented on a vector-capable calculator in polar mode, as follows (using magnitude-angle notation, with angles understood to be in degrees):

$$\vec{F}_{\text{net}} = (20 \angle 0) + (35 \angle 80) = (43 \angle 53) \Rightarrow |\vec{F}_{\text{net}}| = 43 \text{ N} .$$

Therefore, the mass is $m = (43 \text{ N})/(20 \text{ m/s}^2) = 2.2$ kg.

75. The goal is to arrive at the least magnitude of $\vec{F}_{\text{net}}$, and as long as the magnitudes of $\vec{F}_2$ and $\vec{F}_3$ are (in total) less than or equal to $|\vec{F}_1|$ then we should orient them opposite to the direction of $\vec{F}_1$ (which is the $+x$ direction).

(a) We orient both $\vec{F}_2$ and $\vec{F}_3$ in the $-x$ direction. Then, the magnitude of the net force is 50 – 30 – 20 = 0, resulting in zero acceleration for the tire.

(b) We again orient $\vec{F}_2$ and $\vec{F}_3$ in the negative x direction. We obtain an acceleration along the $+x$ axis with magnitude

$$a = \frac{F_1 - F_2 - F_3}{m} = \frac{50\,\text{N} - 30\,\text{N} - 10\,\text{N}}{12\,\text{kg}} = 0.83 \text{ m/s}^2 .$$

(c) The least value is $a = 0$. In this case, the forces $\vec{F}_2$ and $\vec{F}_3$ are collectively strong enough to have y components (one positive and one negative) that cancel each other and still have enough x contributions (in the $-x$ direction) to cancel $\vec{F}_1$. Since $|\vec{F}_2| = |\vec{F}_3|$, we see that the angle above the $-x$ axis to one of them should equal the angle below the $-x$ axis to the other one (we denote this angle θ). We require

$$-50 \text{ N} = F_{2x} + F_{3x} = -(30\text{N})\cos\theta - (30\text{N})\cos\theta$$

which leads to

$$\theta = \cos^{-1}\left(\frac{50\,\text{N}}{60\,\text{N}}\right) = 34° .$$

76. (a) A small segment of the rope has mass and is pulled down by the gravitational force of the Earth. Equilibrium is reached because neighboring portions of the rope pull up sufficiently on it. Since tension is a force *along* the rope, at least one of the neighboring portions must slope up away from the segment we are considering. Then, the tension has an upward component, which means the rope sags.

(b) The only force acting with a horizontal component is the applied force $\vec{F}$. Treating the block and rope as a single object, we write Newton's second law for it: $F = (M + m)a$, where a is the acceleration and the positive direction is taken to be to the right. The acceleration is given by $a = F/(M + m)$.

(c) The force of the rope F_r is the only force with a horizontal component acting on the block. Then Newton's second law for the block gives

$$F_r = Ma = \frac{MF}{M + m}$$

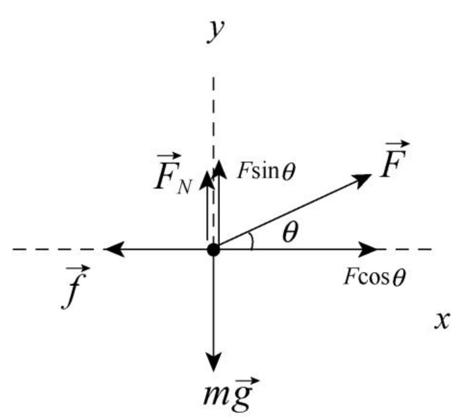
where the expression found above for a has been used.

(d) Treating the block and half the rope as a single object, with mass $M + \frac{1}{2}m$, where the horizontal force on it is the tension T_m at the midpoint of the rope, we use Newton's second law:

$$T_m = \left(M + \frac{1}{2}m\right)a = \frac{(M + m/2)F}{(M + m)} = \frac{(2M + m)F}{2(M + m)}.$$

77. **THINK** We have a crate that is being pulled at an angle. We apply Newton's second law to analyze the motion.

EXPRESS Although the full specification of $\vec{F}_{\text{net}} = m\vec{a}$ in this situation involves both x and y axes, only the x-application is needed to find what this particular problem asks for. We note that $a_y = 0$ so that there is no ambiguity denoting a_x simply as a. We choose $+x$ to the right and $+y$ up. The free-body diagram (not to scale) is shown to the right. The x component of the rope's tension (acting on the crate) is



$$F_x = F\cos\theta = (450\text{ N})\cos 38° = 355\text{ N},$$

and the resistive force (pointing in the $-x$ direction) has magnitude $f = 125$ N.

ANALYZE (a) Newton's second law leads to

$$F_x - f = ma \Rightarrow a = \frac{F\cos\theta - f}{m} = \frac{355\text{ N} - 125\text{ N}}{310\text{ kg}} = 0.74\text{m/s}^2.$$

(b) In this case, we use Eq. 5-12 to find the mass: $m' = W/g = 31.6\text{ kg}$. Newton's second law then leads to

$$F_x - f = m'a' \;\Rightarrow\; a' = \frac{F_x - f}{m'} = \frac{355\text{ N} - 125\text{ N}}{31.6\text{ kg}} = 7.3\text{ m/s}^2.$$

LEARN The resistive force opposing the motion is due to the friction between the crate and the floor. This topic is discussed in greater detail in Chapter 6.

78. We take $+x$ uphill for the m_2 = 1.0 kg box and $+x$ rightward for the m_1 = 3.0 kg box (so the accelerations of the two boxes have the same magnitude and the same sign). The uphill force on m_2 is F and the downhill forces on it are T and $m_2g \sin\theta$, where θ = 37°. The only horizontal force on m_1 is the rightward-pointed tension. Applying Newton's second law to each box, we find

$$\begin{aligned} F - T - m_2 g \sin\theta &= m_2 a \\ T &= m_1 a \end{aligned}$$

which can be added to obtain

$$F - m_2 g \sin\theta = (m_1 + m_2)a.$$

This yields the acceleration

$$a = \frac{12\text{ N} - (1.0\text{ kg})(9.8\text{ m/s}^2)\sin 37°}{1.0\text{ kg} + 3.0\text{ kg}} = 1.53\text{ m/s}^2.$$

Thus, the tension is $T = m_1 a$ = (3.0 kg)(1.53 m/s^2) = 4.6 N.

79. We apply Eq. 5-12.

(a) The mass is

$$m = W/g = (22\text{ N})/(9.8\text{ m/s}^2) = 2.2\text{ kg}.$$

At a place where g = 4.9 m/s^2, the mass is still 2.2 kg but the gravitational force is

$$F_g = mg = (2.2\text{ kg})\,(4.0\text{ m/s}^2) = 11\text{ N}.$$

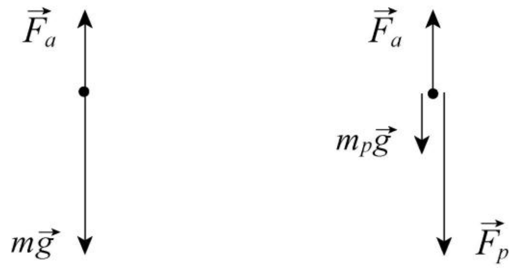
(b) As noted, m = 2.2 kg.

(c) At a place where g = 0 the gravitational force is zero.

(d) The mass is still 2.2 kg.

80. We take down to be the $+y$ direction.

(a) The first diagram (shown below left) is the free-body diagram for the person and parachute, considered as a single object with a mass of 80 kg + 5.0 kg = 85 kg.



$\vec{F}_a$ is the force of the air on the parachute and $m\vec{g}$ is the force of gravity. Application of Newton's second law produces $mg - F_a = ma$, where a is the acceleration. Solving for F_a we find

$$F_a = m(g - a) = (85\text{ kg})\left(9.8\text{ m/s}^2 - 2.5\text{ m/s}^2\right) = 620\text{ N}.$$

(b) The second diagram (above right) is the free-body diagram for the parachute alone. $\vec{F}_a$ is the force of the air, $m_p\vec{g}$ is the force of gravity, and $\vec{F}_p$ is the force of the person. Now, Newton's second law leads to

$$m_p g + F_p - F_a = m_p a.$$

Solving for F_p, we obtain

$$F_p = m_p(a - g) + F_a = (5.0\text{ kg})\left(2.5\text{ m/s}^2 - 9.8\text{ m/s}^2\right) + 620\text{ N} = 580\text{ N}.$$

81. The mass of the pilot is $m = 735/9.8 = 75$ kg. Denoting the upward force exerted by the spaceship (his seat, presumably) on the pilot as $\vec{F}$ and choosing upward as the $+y$ direction, then Newton's second law leads to

$$F - mg_{\text{moon}} = ma \quad \Rightarrow \quad F = (75\text{ kg})\left(1.6\text{ m/s}^2 + 1.0\text{ m/s}^2\right) = 195\text{ N}.$$

82. With SI units understood, the net force on the box is

$$\vec{F}_{\text{net}} = \left(3.0 + 14\cos 30° - 11\right)\hat{\text{i}} + \left(14\sin 30° + 5.0 - 17\right)\hat{\text{j}}$$

which yields $\vec{F}_{\text{net}} = (4.1\text{ N})\hat{\text{i}} - (5.0\text{ N})\hat{\text{j}}$.

(a) Newton's second law applied to the $m = 4.0$ kg box leads to

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = (1.0\text{ m/s}^2)\hat{\text{i}} - (1.3\text{ m/s}^2)\hat{\text{j}}.$$

(b) The magnitude of $\vec{a}$ is $a = \sqrt{(1.0\text{ m/s}^2)^2 + \left(-1.3\text{ m/s}^2\right)^2} = 1.6\text{ m/s}^2$.

(c) Its angle is $\tan^{-1}$ [(–1.3 m/s^2)/(1.0 m/s^2)] = –50° (that is, 50° measured clockwise from the rightward axis).

83. **THINK** This problem deals with the relationship between the three quantities: force, mass and acceleration in Newton's second law $F = ma$.

EXPRESS The "certain force," denoted as F, is assumed to be the net force on the object when it gives m_1 an acceleration a_1 = 12 m/s^2 and when it gives m_2 an acceleration a_2 = 3.3 m/s^2, i.e., $F = m_1a_1 = m_2a_2$. The accelerations for $m_2 + m_1$ and $m_2 - m_1$ can be solved by substituting $m_1 = F/a_1$ and $m_2 = F/a_2$.

ANALYZE (a) Now we seek the acceleration a of an object of mass $m_2 - m_1$ when F is the net force on it. The result is

$$a = \frac{F}{m_2 - m_1} = \frac{F}{(F/a_2) - (F/a_1)} = \frac{a_1a_2}{a_1 - a_2} = \frac{(12.0 \text{ m/s}^2)(3.30 \text{ m/s}^2)}{12.0 \text{ m/s}^2 - 3.30 \text{ m/s}^2} = 4.55 \text{ m/s}^2 .$$

(b) Similarly for an object of mass $m_2 + m_1$, we have:

$$a' = \frac{F}{m_2 + m_1} = \frac{F}{(F/a_2) + (F/a_1)} = \frac{a_1a_2}{a_1 + a_2} = \frac{(12.0 \text{ m/s}^2)(3.30 \text{ m/s}^2)}{12.0 \text{ m/s}^2 + 3.30 \text{ m/s}^2} = 2.59 \text{ m/s}^2 .$$

LEARN With the same applied force, the greater the mass the smaller the acceleration. In this problem, we have $a_1 > a > a_2 > a'$. This implies $m_1 < m_2 - m_1 < m_2 < m_2 + m_1$.

84. We assume the direction of motion is +x and assume the refrigerator starts from rest (so that the speed being discussed is the velocity $\vec{v}$ that results from the process). The only force along the x axis is the x component of the applied force $\vec{F}$.

(a) Since v_0 = 0, the combination of Eq. 2-11 and Eq. 5-2 leads simply to

$$F_x = m\left(\frac{v}{t}\right) \Rightarrow v_i = \left(\frac{F\cos\theta_i}{m}\right)t$$

for i = 1 or 2 (where we denote θ_1 = 0 and $\theta_2 = \theta$ for the two cases). Hence, we see that the ratio v_2 over v_1 is equal to cos θ.

(b) Since v_0 = 0, the combination of Eq. 2-16 and Eq. 5-2 leads to

$$F_x = m\left(\frac{v^2}{2\Delta x}\right) \Rightarrow v_i = \sqrt{2\left(\frac{F\cos\theta_i}{m}\right)\Delta x}$$

for i = 1 or 2 (again, $\theta_1 = 0$ and $\theta_2 = \theta$ is used for the two cases). In this scenario, we see that the ratio v_2 over v_1 is equal to $\sqrt{\cos\theta}$.

85. (a) Since the performer's weight is (52 kg)(9.8 m/s^2) = 510 N, the rope breaks.

(b) Setting T = 425 N in Newton's second law (with +y upward) leads to

$$T - mg = ma \;\Rightarrow\; a = \frac{T}{m} - g$$

which yields $|a|$ = 1.6 m/s^2.

86. We use $W_p = mg_p$, where W_p is the weight of an object of mass m on the surface of a certain planet p, and g_p is the acceleration of gravity on that planet.

(a) The weight of the space ranger on Earth is

$$W_e = mg_e = (75\text{ kg})\,(9.8\text{ m/s}^2) = 7.4 \times 10^2\text{ N}.$$

(b) The weight of the space ranger on Mars is

$$W_m = mg_m = (75\text{ kg})\,(3.7\text{ m/s}^2) = 2.8 \times 10^2\text{ N}.$$

(c) The weight of the space ranger in interplanetary space is zero, where the effects of gravity are negligible.

(d) The mass of the space ranger remains the same, m = 75 kg, at all the locations.

87. From the reading when the elevator was at rest, we know the mass of the object is m = (65 N)/(9.8 m/s^2) = 6.6 kg. We choose +y upward and note there are two forces on the object: mg downward and T upward (in the cord that connects it to the balance; T is the reading on the scale by Newton's third law).

(a) "Upward at constant speed" means constant velocity, which means no acceleration. Thus, the situation is just as it was at rest: T = 65 N.

(b) The term "deceleration" is used when the acceleration vector points in the direction opposite to the velocity vector. We're told the velocity is upward, so the acceleration vector points downward (a = –2.4 m/s^2). Newton's second law gives

$$T - mg = ma \;\Rightarrow\; T = (6.6\text{ kg})(9.8\text{ m/s}^2 - 2.4\text{ m/s}^2) = 49\text{ N}.$$

88. We use the notation g as the acceleration due to gravity near the surface of Callisto, m as the mass of the landing craft, a as the acceleration of the landing craft, and F as the rocket thrust. We take down to be the positive direction. Thus, Newton's second law takes the form $mg - F = ma$. If the thrust is F_1 (= 3260 N), then the acceleration is zero,

so $mg - F_1 = 0$. If the thrust is F_2 (= 2200 N), then the acceleration is a_2 (= 0.39 m/s^2), so $mg - F_2 = ma_2$.

(a) The first equation gives the weight of the landing craft: $mg = F_1 = 3260$ N.

(b) The second equation gives the mass:

$$m = \frac{mg - F_2}{a_2} = \frac{3260 \text{ N} - 2200 \text{ N}}{0.39 \text{ m/s}^2} = 2.7 \times 10^3 \text{ kg} .$$

(c) The weight divided by the mass gives the acceleration due to gravity:

$$g = (3260 \text{ N})/(2.7 \times 10^3 \text{ kg}) = 1.2 \text{ m/s}^2.$$

89. (a) When $\vec{F}_{\text{net}} = 3F - mg = 0$, we have

$$F = \frac{1}{3}mg = \frac{1}{3}(1400 \text{ kg})\left(9.8 \text{ m/s}^2\right) = 4.6 \times 10^3 \text{ N}$$

for the force exerted by each bolt on the engine.

(b) The force on each bolt now satisfies $3F - mg = ma$, which yields

$$F = \frac{1}{3}m(g + a) = \frac{1}{3}(1400 \text{ kg})\left(9.8 \text{ m/s}^2 + 2.6 \text{ m/s}^2\right) = 5.8 \times 10^3 \text{ N}.$$

90. We write the length unit light-month, the distance traveled by light in one month, as c·month in this solution.

(a) The magnitude of the required acceleration is given by

$$a = \frac{\Delta v}{\Delta t} = \frac{(0.10)(3.0 \times 10^8 \text{ m/s})}{(3.0 \text{ days})(86400 \text{ s/day})} = 1.2 \times 10^2 \text{ m/s}^2 .$$

(b) The acceleration in terms of g is $a = \left(\frac{a}{g}\right) g = \left(\frac{1.2 \times 10^2 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) g = 12g$.

(c) The force needed is

$$F = ma = \left(1.20 \times 10^6 \text{ kg}\right)\left(1.2 \times 10^2 \text{ m/s}^2\right) = 1.4 \times 10^8 \text{ N}.$$

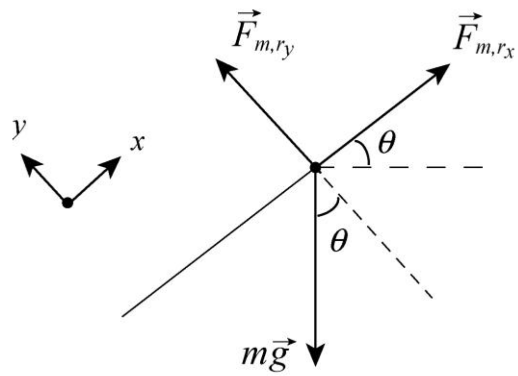
(d) The spaceship will travel a distance $d = 0.1\ c$·month during one month. The time it takes for the spaceship to travel at constant speed for 5.0 light-months is

$$t = \frac{d}{v} = \frac{5.0 \text{ c} \cdot \text{months}}{0.1c} = 50 \text{ months} \approx 4.2 \text{ years.}$$

91. **THINK** We have a motorcycle going up a ramp at a constant acceleration. We apply Newton's second law to calculate the net force on the rider and the force on the rider from the motorcycle.

EXPRESS The free-body diagram is shown to the right (not to scale). Note that F_{m,r_y} and F_{m,r_x}, respectively, denote the *y* and *x* components of the force $\vec{F}_{m,r}$ exerted by the motorcycle on the rider. The net force on the rider is

$$F_{\text{net}} = ma.$$



ANALYZE (a) Since the net force equals *ma*, then the magnitude of the net force on the rider is

$$F_{\text{net}} = ma = (60.0 \text{ kg})\,(3.0 \text{ m/s}^2) = 1.8 \times 10^2 \text{ N.}$$

(b) To calculate the force by the motorcycle on the rider, we apply Newton's second law to the *x*- and the *y*-axes separately. For the *x*-axis, we have:

$$F_{m,r_x} - mg \sin\theta = ma$$

where m = 60.0 kg, a = 3.0 m/s^2, and θ = 10°. Thus, $F_{m,r_x} = 282$ N. Applying it to the *y*-axis (where there is no acceleration), we have

$$F_{m,r_y} - mg\cos\theta = 0$$

which gives $F_{m,r_y} = 579$ N. Using the Pythagorean theorem, we find

$$F_{m,r} = \sqrt{F_{m,r_x}^2 + F_{m,r_y}^2} = \sqrt{(282 \text{ N})^2 + (579 \text{ N})^2} = 644 \text{ N.}$$

Now, the magnitude of the force exerted on the rider by the motorcycle is the same magnitude of force exerted by the rider on the motorcycle, so the answer is 644 N.

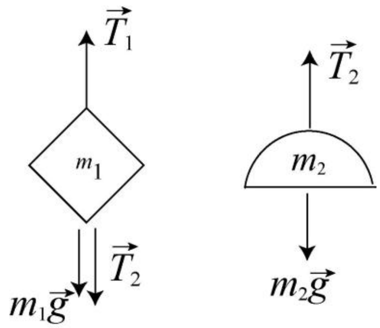
LEARN The force exerted by the motorcycle on the rider keeps the rider accelerating in the +*x*-direction, while maintaining contact with the inclines surface ($a_y = 0$).

92. We denote the thrust as *T* and choose +*y* upward. Newton's second law leads to

$$T - Mg = Ma \;\Rightarrow\; a = \frac{2.6\times10^5\ \text{N}}{1.3\times10^4\ \text{kg}} - 9.8\ \text{m/s}^2 = 10\ \text{m/s}^2.$$

93. **THINK** In this problem we have mobiles consisting of masses connected by cords. We apply Newton's second law to calculate the tensions in the cords.

EXPRESS The free-body diagrams for m_1 and m_2 for part (a) are shown to the right.



The bottom cord is only supporting m_2 = 4.5 kg against gravity, so its tension is $T_2 = m_2 g$. On the other hand, the top cord is supporting a total mass of $m_1 + m_2$ = (3.5 kg + 4.5 kg) = 8.0 kg against gravity. Applying Newton's second law gives

$$T_1 - T_2 - m_1 g = 0$$

so the tension is

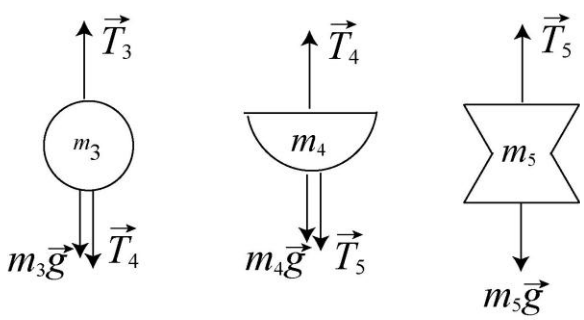
$$T_1 = m_1 g + T_2 = (m_1 + m_2)g.$$

ANALYZE (a) From the equations above, we find the tension in the bottom cord to be

$$T_2 = m_2 g = (4.5\ \text{kg})(9.8\ \text{m/s}^2) = 44\ \text{N}.$$

(b) Similarly, the tension in the top cord is $T_1 = (m_1 + m_2)g$ = (8.0 kg)(9.8 m/s^2) = 78 N.

(c) The free-body diagrams for m_3, m_4 and m_5 for part (b) are shown below (not to scale).



From the diagram, we see that the lowest cord supports a mass of m_5 = 5.5 kg against gravity and consequently has a tension of

$$T_5 = m_5 g = (5.5\text{ kg})(9.8\text{ m/s}^2) = 54\text{ N}.$$

(d) The top cord, as we are told, has a tension T_3 =199 N which supports a total of (199 N)/(9.80 m/s^2) = 20.3 kg, 10.3 kg of which is already accounted for in the figure. Thus, the unknown mass in the middle must be m_4 = 20.3 kg – 10.3 kg = 10.0 kg, and the tension in the cord above it must be enough to support

$$m_4 + m_5 = (10.0\text{ kg} + 5.50\text{ kg}) = 15.5\text{ kg},$$

so T_4 = (15.5 kg)(9.80 m/s^2) = 152 N.

LEARN Another way to calculate T_4 is to examine the forces on m_3 – one of the downward forces on it is T_4. From Newton's second law, we have $T_3 - m_3 g - T_4 = 0$, which can be solved to give

$$T_4 = T_3 - m_3 g = 199\text{ N} - (4.8\text{ kg})(9.8\text{ m/s}^2) = 152\text{ N}.$$

94. The coordinate choices are made in the problem statement.

(a) We write the velocity of the armadillo as $\vec{v} = v_x\,\hat{\text{i}} + v_y\,\hat{\text{j}}$. Since there is no net force exerted on it in the x direction, the x component of the velocity of the armadillo is a constant: v_x = 5.0 m/s. In the y direction at t = 3.0 s, we have (using Eq. 2-11 with $v_{0y} = 0$)

$$v_y = v_{0\,y} + a_y t = v_{0\,y} + \left(\frac{F_y}{m}\right)t = \left(\frac{17\text{ N}}{12\text{ kg}}\right)(3.0\text{ s}) = 4.3\text{ m/s}.$$

Thus, $\vec{v} = (5.0\text{ m/s})\,\hat{\text{i}} + (4.3\text{ m/s})\,\hat{\text{j}}$.

(b) We write the position vector of the armadillo as $\vec{r} = r_x\,\hat{\text{i}} + r_y\,\hat{\text{j}}$. At t = 3.0 s we have r_x = (5.0 m/s) (3.0 s) = 15 m and (using Eq. 2-15 with $v_{0\,y}$ = 0)

$$r_y = v_{0\,y}\,t + \frac{1}{2}a_y\,t^2 = \frac{1}{2}\left(\frac{F_y}{m}\right)t^2 = \frac{1}{2}\left(\frac{17\text{ N}}{12\text{ kg}}\right)(3.0\text{ s})^2 = 6.4\text{ m}.$$

The position vector at t = 3.0 s is therefore $\vec{r} = (15\text{ m})\hat{\text{i}} + (6.4\text{ m})\hat{\text{j}}$.

95. (a) Intuition readily leads to the conclusion that the heavier block should be the hanging one, for largest acceleration. The force that "drives" the system into motion is the weight of the hanging block (gravity acting on the block on the table has no effect on the dynamics, so long as we ignore friction). Thus, m = 4.0 kg.

The acceleration of the system and the tension in the cord can be readily obtained by solving

$$mg - T = ma, \quad T = Ma.$$

(b) The acceleration is given by $a = \left(\frac{m}{m+M}\right)g = 6.5\ \text{m/s}^2$.

(c) The tension is

$$T = Ma = \left(\frac{Mm}{m+M}\right)g = 13\ \text{N}.$$

96. According to Newton's second law, the magnitude of the force is given by $F = ma$, where a is the magnitude of the acceleration of the neutron. We use kinematics (Table 2-1) to find the acceleration that brings the neutron to rest in a distance d. Assuming the acceleration is constant, then $v^2 = v_0^2 + 2ad$ produces the value of a:

$$a = \frac{\left(v^2 - v_0^2\right)}{2d} = \frac{-\left(1.4 \times 10^7\ \text{m/s}\right)^2}{2\left(1.0 \times 10^{-14}\ \text{m}\right)} = -9.8 \times 10^{27}\ \text{m/s}^2.$$

The magnitude of the force is consequently

$$F = ma = \left(1.67 \times 10^{-27}\ \text{kg}\right)\left(9.8 \times 10^{27}\ \text{m/s}^2\right) = 16\ \text{N}.$$

97. (a) With SI units understood, the net force is

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = \left(3.0\ \text{N} + (-2.0\ \text{N})\right)\hat{\text{i}} + \left(4.0\ \text{N} + (-6.0\ \text{N})\right)\hat{\text{j}}$$

which yields $\vec{F}_{\text{net}} = (1.0\ \text{N})\hat{\text{i}} - (2.0\ \text{N})\hat{\text{j}}$.

(b) The magnitude of $\vec{F}_{\text{net}}$ is $F_{\text{net}} = \sqrt{(1.0\ \text{N})^2 + (-2.0\ \text{N})^2} = 2.2\ \text{N}$.

(c) The angle of $\vec{F}_{\text{net}}$ is $\theta = \tan^{-1}\left(\frac{-2.0\ \text{N}}{1.0\ \text{N}}\right) = -63°$.

(d) The magnitude of $\vec{a}$ is $a = F_{\text{net}}/m = (2.2\ \text{N})/(1.0\ \text{kg}) = 2.2\ \text{m/s}^2$.

(e) Since $\vec{F}_{\text{net}}$ is equal to $\vec{a}$ multiplied by mass m, which is a positive scalar that cannot affect the direction of the vector it multiplies, $\vec{a}$ has the same angle as the net force, i.e, $\theta = -63°$. In magnitude-angle notation, we may write $\vec{a} = \left(2.2\ \text{m/s}^2 \angle -63°\right)$.

Chapter 6

1. The greatest deceleration (of magnitude a) is provided by the maximum friction force (Eq. 6-1, with $F_N = mg$ in this case). Using Newton's second law, we find

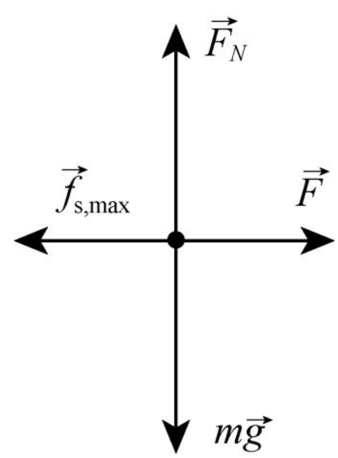
$$a = f_{s,\max}/m = \mu_s g.$$

Eq. 2-16 then gives the shortest distance to stop: $|\Delta x| = v^2/2a = 36$ m. In this calculation, it is important to first convert v to 13 m/s.

2. Applying Newton's second law to the horizontal motion, we have $F - \mu_k\, m\, g = ma$, where we have used Eq. 6-2, assuming that $F_N = mg$ (which is equivalent to assuming that the vertical force from the broom is negligible). Eq. 2-16 relates the distance traveled and the final speed to the acceleration: $v^2 = 2a\Delta x$. This gives $a = 1.4$ m/s^2. Returning to the force equation, we find (with $F = 25$ N and $m = 3.5$ kg) that $\mu_k = 0.58$.

3. **THINK** In the presence of friction between the floor and the bureau, a minimum horizontal force must be applied before the bureau would begin to move.

EXPRESS The free-body diagram for the bureau is shown to the right. We denote $\vec{F}$ as the horizontal force of the person, $\vec{f}_s$ is the force of static friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push $\vec{F}$ in the $+x$ direction). Applying Newton's second law to the x and y axes, we obtain



$$F - f_{s,\,\max} = ma$$
$$F_N - mg = 0$$

respectively.
The second equation yields the normal force $F_N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,\max} = \mu_s\, mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set $a = 0$ to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.
ANALYZE (a) With $\mu_s = 0.45$ and $m = 45$ kg, the equation above leads to

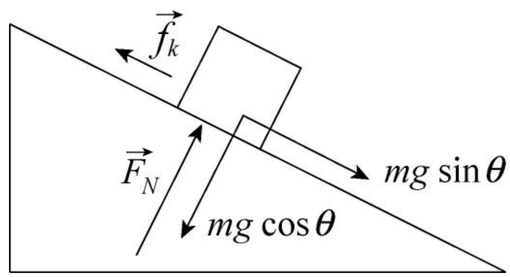
$$F = \mu_s mg = (0.45)(45\,\text{kg})(9.8\,\text{m/s}^2) = 198\,\text{N}.$$

To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

(b) Replacing m = 45 kg with m = 28 kg, the reasoning above leads to roughly $F = 1.2\times10^2$ N.

LEARN The values found above represent the minimum force required to overcome the friction. Applying a force greater than $\mu_s mg$ results in a net force in the +x-direction, and hence, non-zero acceleration.

4. We first analyze the forces on the pig of mass m. The incline angle is θ.



The +x direction is "downhill.'' Application of Newton's second law to the x- and y-axes leads to

$$mg\sin\theta - f_k = ma$$
$$F_N - mg\cos\theta = 0.$$

Solving these along with Eq. 6-2 ($f_k = \mu_k F_N$) produces the following result for the pig's downhill acceleration:

$$a = g\left(\sin\theta - \mu_k\cos\theta\right).$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2\ell}{a}}.$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

$$\frac{t}{t'} = \frac{\sqrt{2\ell/a}}{\sqrt{2\ell/a'}} = \sqrt{\frac{a'}{a}}$$

which leads us to conclude that if t/t' = 2 then a' = 4a. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4g\left(\sin\theta - \mu_k\cos\theta\right).$$

Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

5. In addition to the forces already shown in Fig. 6-17, a free-body diagram would include an upward normal force $\vec{F}_N$ exerted by the floor on the block, a downward $m\vec{g}$ representing the gravitational pull exerted by Earth, and an assumed-leftward $\vec{f}$ for the kinetic or static friction. We choose +x rightwards and +y upwards. We apply Newton's second law to these axes:

$$F - f = ma$$
$$P + F_N - mg = 0$$

where $F = 6.0$ N and $m = 2.5$ kg is the mass of the block.

(a) In this case, $P = 8.0$ N leads to

$$F_N = (2.5 \text{ kg})(9.8 \text{ m/s}^2) - 8.0 \text{ N} = 16.5 \text{ N}.$$

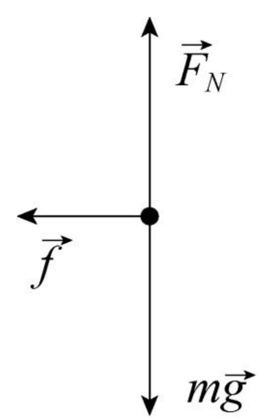
Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 6.6$ N, which is larger than the 6.0 N rightward force – so the block (which was initially at rest) does not move. Putting $a = 0$ into the first of our equations above yields a static friction force of $f = P = 6.0$ N.

(b) In this case, $P = 10$ N, the normal force is

$$F_N = (2.5 \text{ kg})(9.8 \text{ m/s}^2) - 10 \text{ N} = 14.5 \text{ N}.$$

Using Eq. 6-1, this implies $f_{s,\max} = \mu_s F_N = 5.8$ N, which is less than the 6.0 N rightward force – so the block does move. Hence, we are dealing not with static but with kinetic friction, which Eq. 6-2 reveals to be $f_k = \mu_k F_N = 3.6$ N.

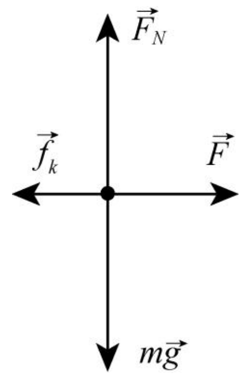
(c) In this last case, $P = 12$ N leads to $F_N = 12.5$ N and thus to $f_{s,\max} = \mu_s F_N = 5.0$ N, which (as expected) is less than the 6.0 N rightward force – so the block moves. The kinetic friction force, then, is $f_k = \mu_k F_N = 3.1$ N.



6. The free-body diagram for the player is shown to the right. $\vec{F}_N$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k F_N$. We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $F_N - mg = 0$; thus, $F_N = mg$. Consequently,

$$\mu_k = \frac{f}{F_N} = \frac{470\,\text{N}}{(79\,\text{kg})\,(9.8\,\text{m/s}^2)} = 0.61.$$

7. **THINK** A force is being applied to accelerate a crate in the presence of friction. We apply Newton's second law to solve for the acceleration.



EXPRESS The free-body diagram for the crate is shown to the right. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f}_k$ is the force of kinetic friction (in the $-x$ direction), F_N is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by Eq. 6-2: $f_k = \mu_k F_N$. Applying Newton's second law to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively.

ANALYZE (a) The second equation above yields the normal force $F_N = mg$, so that the friction is

$$f_k = \mu_k F_N = \mu_k mg = (0.35)(55\text{ kg})(9.8\text{ m/s}^2) = 1.9 \times 10^2\text{ N}.$$

(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with F = 220 N) we solve to find

$$a = \frac{F}{m} - \mu_k g = \frac{220\,\text{N}}{55\,\text{kg}} - (0.35)(9.8\,\text{m/s}^2) = 0.56\text{ m/s}^2.$$

LEARN For the crate to accelerate, the condition $F > f_k = \mu_k mg$ must be met. As can be seen from the equation above, the greater the value of μ_k, the smaller the acceleration under the same applied force.

8. To maintain the stone's motion, a horizontal force (in the $+x$ direction) is needed that cancels the retarding effect due to kinetic friction. Applying Newton's second to the x and y axes, we obtain

$$F - f_k = ma$$
$$F_N - mg = 0$$

respectively. The second equation yields the normal force $F_N = mg$, so that (using Eq. 6-2) the kinetic friction becomes $f_k = \mu_k\, mg$. Thus, the first equation becomes

$$F - \mu_k mg = ma = 0$$

where we have set a = 0 to be consistent with the idea that the horizontal velocity of the stone should remain constant. With m = 20 kg and μ_k = 0.80, we find $F = 1.6 \times 10^2$ N.

9. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

(a) We apply Newton's second law to the y axis:

$$F_N - F\sin\theta - mg = 0 \;\Rightarrow\; F_N = (15\text{ N})\sin 40° + (3.5\text{ kg})(9.8\text{ m/s}^2) = 44\text{ N}.$$

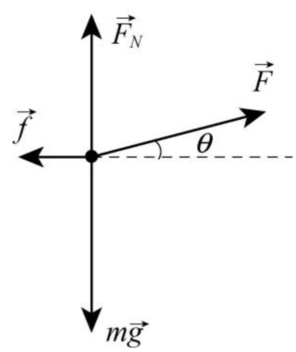
With μ_k = 0.25, Eq. 6-2 leads to f_k = 11 N.

(b) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma \;\Rightarrow\; a = \frac{(15\text{ N})\cos 40° - 11\text{ N}}{3.5\text{ kg}} = 0.14\text{ m/s}^2.$$

Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

10. (a) The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction.



We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. The equations for the x and the y components of the force according to Newton's second law are:

$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta + F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg - F\sin\theta$, which yields $f = \mu_k(mg - F\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$F\cos\theta - \mu_k(mg - F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}(\cos\theta + \mu_k\sin\theta) - \mu_k g.$$

(a) If $\mu_s = 0.600$ and $\mu_k = 0.500,$ then the magnitude of $\vec{f}$ has a maximum value of

$$f_{s,\max} = \mu_s F_N = (0.600)(mg - 0.500mg\sin 20°) = 0.497mg.$$

On the other hand, $F\cos\theta = 0.500mg\cos 20° = 0.470mg$. Therefore, $F\cos\theta < f_{s,\max}$ and the block remains stationary with $a = 0$.

(b) If $\mu_s = 0.400$ and $\mu_k = 0.300,$ then the magnitude of $\vec{f}$ has a maximum value of

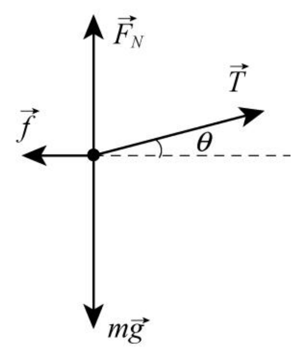
$$f_{s,\max} = \mu_s F_N = (0.400)(mg - 0.500mg\sin 20°) = 0.332mg.$$

In this case, $F\cos\theta = 0.500mg\cos 20° = 0.470mg > f_{s,\max}$. Therefore, the acceleration of the block is

$$\begin{aligned} a &= \frac{F}{m}\left(\cos\theta + \mu_k\sin\theta\right) - \mu_k g \\ &= (0.500)(9.80\ \text{m/s}^2)\left[\cos 20° + (0.300)\sin 20°\right] - (0.300)(9.80\ \text{m/s}^2) \\ &= 2.17\ \text{m/s}^2. \end{aligned}$$

11. **THINK** Since the crate is being pulled by a rope at an angle with the horizontal, we need to analyze the force components in both the *x* and *y*-directions.

EXPRESS The free-body diagram for the crate is shown to the right. Here $\vec{T}$ is the tension force of the rope on the crate, $\vec{F}_N$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. We assume the crate is motionless.



The equations for the *x* and the *y* components of the force according to Newton's second law are:

$$T\cos\theta - f = 0, \quad T\sin\theta + F_N - mg = 0$$

where θ = 15° is the angle between the rope and the horizontal. The first equation gives $f = T\cos\theta$ and the second gives $F_N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s F_N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu_s(mg - T\sin\theta)$.

ANALYZE (a) We solve for the tension:

$$T=\frac{\mu_s mg}{\cos\theta+\mu_s\sin\theta}=\frac{(0.50)\,(68\text{ kg})\,(9.8\text{ m/s}^2)}{\cos 15°+0.50\sin 15°}=304\text{ N}\approx 3.0\times10^2\text{ N}.$$

(b) The second law equations for the moving crate are

$$T\cos\theta - f = ma,\qquad T\sin\theta + F_N - mg = 0\,.$$

Now $f=\mu_k F_N$, and the second equation above gives $F_N = mg - T\sin\theta$, which then yields $f=\mu_k(mg-T\sin\theta)$. This expression is substituted for f in the first equation to obtain

$$T\cos\theta - \mu_k(mg - T\sin\theta) = ma,$$

so the acceleration is

$$\begin{aligned} a &= \frac{T(\cos\theta+\mu_k\sin\theta)}{m}-\mu_k g\\ &=\frac{(304\text{ N})(\cos 15°+0.35\sin 15°)}{68\text{ kg}}-(0.35)(9.8\text{ m/s}^2)=1.3\text{ m/s}^2. \end{aligned}$$

LEARN Let's check the limit where $\theta=0$. In this case, we recover the familiar expressions: $T=\mu_s mg$ and $a=(T-\mu_k mg)/m$.

12. There is no acceleration, so the (upward) static friction forces (there are four of them, one for each thumb and one for each set of opposing fingers) equals the magnitude of the (downward) pull of gravity. Using Eq. 6-1, we have

$$4\mu_s F_N = mg = (79\text{ kg})(9.8\text{ m/s}^2)$$

which, with $\mu_s = 0.70$, yields $F_N = 2.8\times10^2$ N.

13. We denote the magnitude of 110 N force exerted by the worker on the crate as F. The magnitude of the static frictional force can vary between zero and $f_{s,\max}=\mu_s F_N$.

(a) In this case, application of Newton's second law in the vertical direction yields $F_N = mg$. Thus,

$$f_{s,\,\max}=\mu_s F_N=\mu_s mg=(0.37)(35\text{kg})(9.8\text{m/s}^2)=1.3\times10^2\text{ N}$$

which is greater than F.

(b) The block, which is initially at rest, stays at rest since $F < f_{s,\,\max}$. Thus, it does not move.

(c) By applying Newton's second law to the horizontal direction, that the magnitude of the frictional force exerted on the crate is $f_s = 1.1\times10^2$ N.

(d) Denoting the upward force exerted by the second worker as F_2, then application of Newton's second law in the vertical direction yields $F_N = mg - F_2$, which leads to

$$f_{s,\max} = \mu_s F_N = \mu_s (mg - F_2) .$$

In order to move the crate, F must satisfy the condition $F > f_{s,\max} = \mu_s\ (mg - F_2)$,
or

$$110\,\mathrm{N} > \left(0.37\right)\left[(35\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) - F_2\right].$$

The minimum value of F_2 that satisfies this inequality is a value slightly bigger than 45.7 N, so we express our answer as $F_{2,\,\min} = 46$ N.

(e) In this final case, moving the crate requires a greater horizontal push from the worker than static friction (as computed in part (a)) can resist. Thus, Newton's law in the horizontal direction leads to

$$F + F_2 > f_{s,\,\max} \quad \Rightarrow \quad 110\ \mathrm{N} + F_2 > 126.9\ \mathrm{N}$$

which leads (after appropriate rounding) to $F_{2,\,\min} = 17$ N.

14. (a) Using the result obtained in Sample Problem – "Friction, applied force at an angle," the maximum angle for which static friction applies is

$$\theta_{\max} = \tan^{-1}\mu_s = \tan^{-1} 0.63 \approx 32° .$$

This is greater than the dip angle in the problem, so the block does not slide.

(b) Applying Newton's second law, we have

$$F + mg \sin\theta - f_{s,\,\max} = ma = 0$$
$$F_N - mg \cos\theta = 0.$$

Along with Eq. 6-1 ($f_{s,\,\max} = \mu_s F_N$) we have enough information to solve for F. With $\theta = 24°$ and $m = 1.8 \times 10^7$ kg, we find

$$F = mg\left(\mu_s \cos\theta - \sin\theta\right) = 3.0\times10^7\ \mathrm{N}.$$

15. An excellent discussion and equation development related to this problem is given in Sample Problem – "Friction, applied force at an angle." We merely quote (and apply) their main result:

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.04 \approx 2° .$$

16. (a) In this situation, we take $\vec{f}_s$ to point uphill and to be equal to its maximum value, in which case $f_{s,\,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$F_{\min 1} - mg\sin\theta + f_{s,\,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with $\theta = 20°$) leads to

$$F_{\min 1} - mg\left(\sin\theta + \mu_s\cos\theta\right) = 8.6\text{ N.}$$

(b) Now we take $\vec{f}_s$ to point downhill and to be equal to its maximum value, in which case $f_{s,\,\max} = \mu_s F_N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$F_{\min 2} = mg\sin\theta - f_{s,\,\max} = ma = 0$$
$$F_N - mg\cos\theta = 0$$

which (with $\theta = 20°$) leads to

$$F_{\min 2} = mg\left(\sin\theta + \mu_s\cos\theta\right) = 46\text{ N.}$$

A value slightly larger than the "exact" result of this calculation is required to make it accelerate uphill, but since we quote our results here to two significant figures, 46 N is a "good enough" answer.

(c) Finally, we are dealing with kinetic friction (pointing downhill), so that

$$0 = F - mg\sin\theta - f_k = ma$$
$$0 = F_N - mg\cos\theta$$

along with $f_k = \mu_k F_N$ (where $\mu_k = 0.15$) brings us to

$$F = mg\left(\sin\theta + \mu_k\cos\theta\right) = 39\text{ N}.$$

17. If the block is sliding then we compute the kinetic friction from Eq. 6-2; if it is not sliding, then we determine the extent of static friction from applying Newton's law, with zero acceleration, to the x axis (which is parallel to the incline surface). The question of whether or not it is sliding is therefore crucial, and depends on the maximum static friction force, as calculated from Eq. 6-1. The forces are resolved in the incline plane coordinate system in Figure 6-5 in the textbook. The acceleration, if there is any, is along the x axis, and we are taking uphill as $+x$. The net force along the y axis, then, is certainly zero, which provides the following relationship:

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

where $W = mg = 45$ N is the weight of the block, and $\theta = 15°$ is the incline angle. Thus, $F_N = 43.5$ N, which implies that the maximum static friction force should be

$$f_{s,\max} = (0.50)\,(43.5\text{ N}) = 21.7\text{ N}.$$

(a) For $\vec{P} = (-5.0\text{ N})\hat{\text{i}}$, Newton's second law, applied to the x axis becomes

$$f - |P| - mg\sin\theta = ma\,.$$

Here we are assuming $\vec{f}$ is pointing uphill, as shown in Figure 6-5, and if it turns out that it points downhill (which *is* a possibility), then the result for f_s will be negative. If $f = f_s$ then $a = 0$, we obtain

$$f_s = |P| + mg\sin\theta = 5.0\text{ N} + (43.5\text{ N})\sin 15° = 17\text{ N},$$

or $\vec{f}_s = (17\text{ N})\hat{\text{i}}$. This is clearly allowed since f_s is less than $f_{s,\max}$.

(b) For $\vec{P} = (-8.0\text{ N})\hat{\text{i}}$, we obtain (from the same equation) $\vec{f}_s = (20\text{ N})\hat{\text{i}}$, which is still allowed since it is less than $f_{s,\max}$.

(c) But for $\vec{P} = (-15\text{ N})\hat{\text{i}}$, we obtain (from the same equation) $f_s = 27$ N, which is not allowed since it is larger than $f_{s,\max}$. Thus, we conclude that it is the kinetic friction instead of the static friction that is relevant in this case. The result is

$$\vec{f}_k = \mu_k F_N \hat{\text{i}} = (0.34)(43.5\text{ N})\,\hat{\text{i}} = (15\text{ N})\,\hat{\text{i}}\,.$$

18. (a) We apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

where, using Eq. 6-11,

$$f = f_k = \mu_k F_N = \mu_k\, mg\cos\theta\,.$$

Thus, with $\mu_k = 0.600$, we have

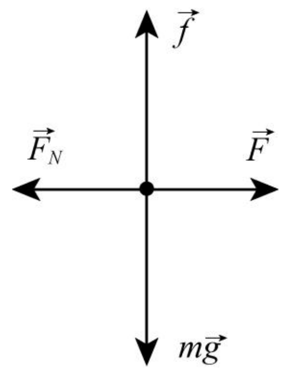
$$a = g\sin\theta - \mu_k\cos\theta = -3.72\text{ m/s}^2$$

which means, since we have chosen the positive direction in the direction of motion (down the slope) then the acceleration vector points "uphill"; it is decelerating. With $v_0 = 18.0$ m/s and $\Delta x = d = 24.0$ m, Eq. 2-16 leads to

$$v=\sqrt{v_0^2+2ad}=12.1 \text{ m/s}.$$

(b) In this case, we find a = +1.1 m/s^2, and the speed (when impact occurs) is 19.4 m/s.

19. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{F}_N$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block. We compare f and $\mu_s F_N$. If $f < \mu_s F_N$, the block does not slide on the wall but if $f > \mu_s F_N$, the block does slide. The horizontal component of Newton's second law is $F - F_N = 0$, so $F_N = F = 12$ N and



$$\mu_s F_N = (0.60)(12 \text{ N}) = 7.2 \text{ N}.$$

The vertical component is $f - mg = 0$, so $f = mg = 5.0$ N. Since $f < \mu_s F_N$ the block does not slide.

(b) Since the block does not move $f = 5.0$ N and $F_N = 12$ N. The force of the wall on the block is

$$\vec{F}_w = -F_N\,\hat{\mathrm{i}} + f\,\hat{\mathrm{j}} = -(12\text{N})\,\hat{\mathrm{i}} + (5.0\text{N})\,\hat{\mathrm{j}}$$

where the axes are as shown on Fig. 6-26 of the text.

20. Treating the two boxes as a single system of total mass $m_C + m_W$ =1.0 + 3.0 = 4.0 kg, subject to a total (leftward) friction of magnitude 2.0 N + 4.0 N = 6.0 N, we apply Newton's second law (with +x rightward):

$$F - f_{\text{total}} = m_{\text{total}}\, a \quad \Rightarrow \quad 12.0 \text{ N} - 6.0 \text{ N} = (4.0 \text{ kg})a$$

which yields the acceleration a = 1.5 m/s^2. We have treated F as if it were known to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. Turning our attention to the larger box (the Wheaties box of mass m_W = 3.0 kg) we apply Newton's second law to find the contact force F' exerted by the Cheerios box on it.

$$F' - f_W = m_W a \quad \Rightarrow \quad F' - 4.0 \text{ N} = (3.0 \text{ kg})(1.5 \text{ m/s}^2)\,.$$

From the above equation, we find the contact force to be F' = 8.5 N.

21. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$\begin{aligned} x:&\quad T\cos\phi - f = ma \\ y:&\quad T\sin\phi + F_N - mg = 0 \end{aligned}$$

Setting a = 0 and f = $f_{s,\max}$ = $\mu_s F_N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g}\left(\sin\phi + \frac{\cos\phi}{\mu_s}\right)$$

which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

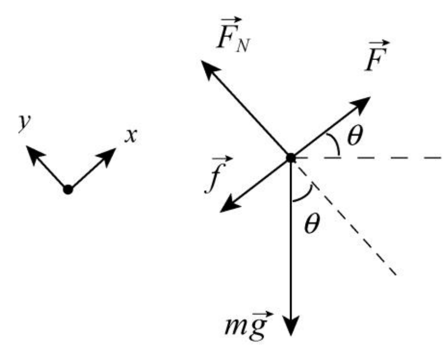
$$\frac{dm}{dt} = \frac{T}{g}\left(\cos\phi_m - \frac{\sin\phi_m}{\mu_s}\right) = 0$$

This leads to tan $\phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19°$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields m = 340 kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.

22. The free-body diagram for the sled is shown below, with $\vec{F}$ being the force applied to the sled, $\vec{F}_N$ the normal force of the inclined plane on the sled, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction.

We take the +x direction to be along the inclined plane and the +y direction to be in its normal direction. The equations for the x and the y components of the force according to Newton's second law are:



$$\begin{aligned} F_x &= F - f - mg\sin\theta = ma = 0 \\ F_y &= F_N - mg\cos\theta = 0 \end{aligned}$$

Now $f = \mu F_N$, and the second equation gives $F_N = mg\cos\theta$, which yields $f = \mu mg\cos\theta$. This expression is substituted for f in the first equation to obtain

$$F = mg(\sin\theta + \mu\cos\theta)$$

From the figure, we see that $F = 2.0$ N when $\mu = 0$. This implies $mg\sin\theta = 2.0$ N. Similarly, we also find $F = 5.0$ N when $\mu = 0.5$:

$$5.0\text{ N} = mg(\sin\theta + 0.50\cos\theta) = 2.0\text{ N} + 0.50mg\cos\theta$$

which yields $mg\cos\theta = 6.0\text{ N}.$ Combining the two results, we get

$$\tan\theta = \frac{2}{6} = \frac{1}{3} \quad\Rightarrow\quad \theta = 18°.$$

23. Let the tensions on the strings connecting m_2 and m_3 be $T_{23,}$ and that connecting m_2 and m_1 be T_{12}, respectively. Applying Newton's second law (and Eq. 6-2, with $F_N = m_2g$ in this case) to the *system* we have

$$\begin{aligned} m_3g - T_{23} &= m_3a \\ T_{23} - \mu_k m_2 g - T_{12} &= m_2 a \\ T_{12} - m_1 g &= m_1 a \end{aligned}$$

Adding up the three equations and using $m_1 = M, m_2 = m_3 = 2M$, we obtain

$$2Mg - 2\mu_k Mg - Mg = 5Ma\ .$$

With a = 0.500 m/s^2 this yields μ_k = 0.372. Thus, the coefficient of kinetic friction is roughly μ_k = 0.37.

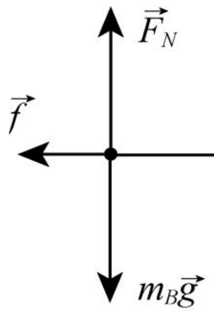
24. We find the acceleration from the slope of the graph (recall Eq. 2-11): a = 4.5 m/s^2. Thus, Newton's second law leads to

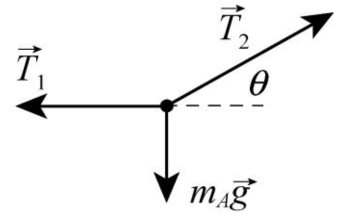
$$F - \mu_k mg = ma,$$

where F = 40.0 N is the constant horizontal force applied. With m = 4.1 kg, we arrive at μ_k =0.54.

25. **THINK** In order that the two blocks remain in equilibrium, friction must be present between block B and the surface.

EXPRESS The free-body diagrams for block B and for the knot just above block A are shown below. $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle θ = 30°) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{F}_N$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B (W_B = 711 N is the magnitude of $m_B\vec{g}$).





For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ F_N - W_B &= 0 \\ T_2 \cos\theta - T_1 &= 0 \\ T_2 \sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). The above equations yield $T_1 = \mu_s F_N$, $F_N = W_B$ and $T_1 = T_2 \cos\theta$.

ANALYZE Solving these equations with $\mu_s = 0.25$, we obtain

$$\begin{aligned} W_A &= T_2 \sin\theta = T_1 \tan\theta = \mu_s F_N \tan\theta = \mu_s W_B \tan\theta \\ &= (0.25)(711\,\text{N})\tan 30° = 1.0\times10^2\ \text{N} \end{aligned}$$

LEARN As expected, the maximum weight of A is proportional to the weight of B, as well as the coefficient of static friction. In addition, we see that W_A is proportional to $\tan\theta$ (the larger the angle, the greater the vertical component of T_2 that supports its weight).

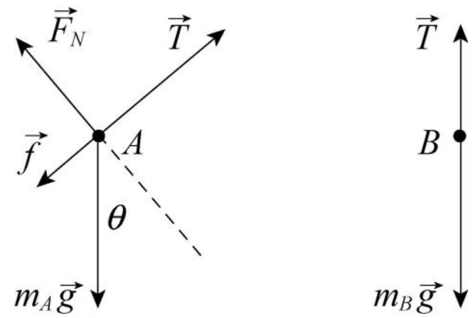
26. (a) Applying Newton's second law to the *system* (of total mass M = 60.0 kg) and using Eq. 6-2 (with $F_N = Mg$ in this case) we obtain

$$F - \mu_k Mg = Ma \;\Rightarrow\; a = 0.473\ \text{m/s}^2.$$

Next, we examine the forces just on m_3 and find $F_{32} = m_3(a + \mu_k g) = 147$ N. If the algebra steps are done more systematically, one ends up with the interesting relationship: $F_{32} = (m_3 / M)F$ (which is independent of the friction!).

(b) As remarked at the end of our solution to part (a), the result does not depend on the frictional parameters. The answer here is the same as in part (a).

27. First, we check to see if the bodies start to move. We assume they remain at rest and compute the force of (static) friction which holds them there, and compare its magnitude with the maximum value $\mu_s F_N$. The free-body diagrams are shown below.



T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on body A, F_N is the magnitude of the normal force of the plane on body A, $m_A\vec{g}$ is the force of gravity on body A (with magnitude W_A = 102 N), and $m_B\vec{g}$ is the force of gravity on body B (with magnitude W_B = 32 N). θ = 40° is the angle of incline. We are told the direction of $\vec{f}$ but we assume it is downhill. If we obtain a negative result for f, then we know the force is actually up the plane.

(a) For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force. The x and y components of Newton's second law become

$$T - f - W_A \sin\theta = 0$$
$$F_N - W_A \cos\theta = 0.$$

Taking the positive direction to be *downward* for body B, Newton's second law leads to $W_B - T = 0$. Solving these three equations leads to

$$f = W_B - W_A \sin\theta = 32\text{ N} - (102\text{ N})\sin 40° = -34\text{ N}$$

(indicating that the force of friction is *uphill*) and to

$$F_N = W_A \cos\theta = (102\text{ N})\cos 40° = 78\text{N}$$

which means that

$$f_{s,\max} = \mu_s F_N = (0.56)\,(78\text{ N}) = 44\text{ N}.$$

Since the magnitude f of the force of friction that holds the bodies motionless is less than $f_{s,\max}$ the bodies remain at rest. The acceleration is zero.

(b) Since A is moving up the incline, the force of friction is downhill with magnitude $f_k = \mu_k F_N$. Newton's second law, using the same coordinates as in part (a), leads to

$$\begin{aligned} T - f_k - W_A \sin\theta &= m_A a \\ F_N - W_A \cos\theta &= 0 \\ W_B - T &= m_B a \end{aligned}$$

for the two bodies. We solve for the acceleration:

$$a = \frac{W_B - W_A \sin\theta - \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° - (0.25)(102\text{N})\cos 40°}{(32\text{N}+102\text{N}) \,/\, (9.8\,\text{m/s}^2)}$$
$$= -3.9\ \text{m/s}^2.$$

The acceleration is down the plane, i.e., $\vec{a} = (-3.9\ \text{m/s}^2)\hat{\text{i}}$, which is to say (since the initial velocity was uphill) that the objects are slowing down. We note that $m = W/g$ has been used to calculate the masses in the calculation above.

(c) Now body A is initially moving down the plane, so the force of friction is uphill with magnitude $f_k = \mu_k F_N$. The force equations become

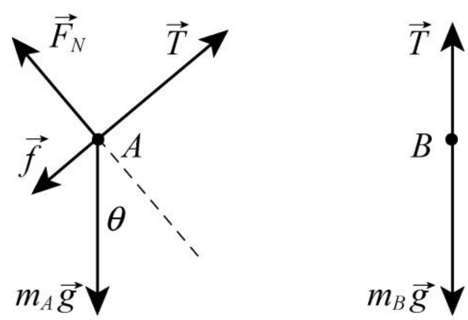
$$\begin{aligned} T + f_k - W_A \sin\theta &= m_A a \\ F_N - W_A \cos\theta &= 0 \\ W_B - T &= m_B a \end{aligned}$$

which we solve to obtain

$$a = \frac{W_B - W_A \sin\theta + \mu_k W_A \cos\theta}{m_B + m_A} = \frac{32\text{N} - (102\text{N})\sin 40° + (0.25)(102\text{N})\cos 40°}{(32\text{N}+102\text{N}) \,/\, (9.8\,\text{m/s}^2)}$$
$$= -1.0\ \text{m/s}^2.$$

The acceleration is again downhill the plane, i.e., $\vec{a} = (-1.0\ \text{m/s}^2)\hat{\text{i}}$. In this case, the objects are speeding up.

28. The free-body diagrams are shown to the right, where T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on block A, F_N is the magnitude of the normal force of the plane on block A, $m_A\vec{g}$ is the force of gravity on body A (where m_A = 10 kg), and $m_B\vec{g}$ is the force of gravity on block B. θ = 30° is the angle of incline. For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force; the positive direction is chosen *downward* for block B.



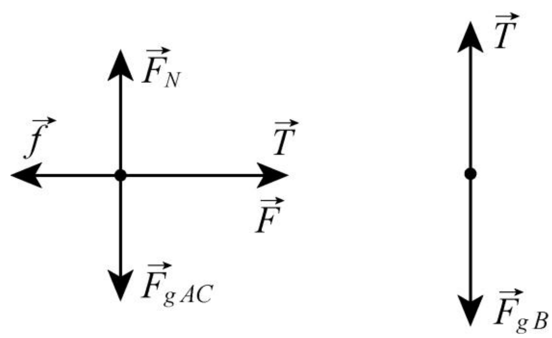
Since A is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k F_N$ (where μ_k = 0.20). Newton's second law leads to

$$\begin{aligned} T - f_k + m_A g \sin\theta &= m_A a = 0 \\ F_N - m_A g \cos\theta &= 0 \\ m_B g - T &= m_B a = 0 \end{aligned}$$

for the two bodies (where $a = 0$ is a consequence of the velocity being constant). We solve these for the mass of block B.

$$m_B = m_A \left(\sin\theta - \mu_k \cos\theta\right) = 3.3 \text{ kg}.$$

29. (a) Free-body diagrams for the blocks A and C, considered as a single object, and for the block B are shown below.



T is the magnitude of the tension force of the rope, F_N is the magnitude of the normal force of the table on block A, f is the magnitude of the force of friction, W_{AC} is the combined weight of blocks A and C (the magnitude of force $\vec{F}_{g\,AC}$ shown in the figure), and W_B is the weight of block B (the magnitude of force $\vec{F}_{g\,B}$ shown). Assume the blocks are not moving. For the blocks on the table we take the x axis to be to the right and the y axis to be upward. From Newton's second law, we have

$$x \text{ component:} \quad T - f = 0$$

$$y \text{ component:} \quad F_N - W_{AC} = 0.$$

For block B take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. The third equation gives $T = W_B$ and the first gives $f = T = W_B$. The second equation gives $F_N = W_{AC}$. If sliding is not to occur, f must be less than $\mu_s F_N$, or $W_B < \mu_s W_{AC}$. The smallest that W_{AC} can be with the blocks still at rest is

$$W_{AC} = W_B/\mu_s = (22 \text{ N})/(0.20) = 110 \text{ N}.$$

Since the weight of block A is 44 N, the least weight for C is (110 – 44) N = 66 N.

(b) The second law equations become

$$T - f = (W_A/g)a$$

$$F_N - W_A = 0$$
$$W_B - T = (W_B/g)a.$$

In addition, $f = \mu_k F_N$. The second equation gives $F_N = W_A$, so $f = \mu_k W_A$. The third gives $T = W_B - (W_B/g)a$. Substituting these two expressions into the first equation, we obtain

$$W_B - (W_B/g)a - \mu_k W_A = (W_A/g)a.$$

Therefore,

$$a = \frac{g\left(W_B - \mu_k W_A\right)}{W_A + W_B} = \frac{(9.8\text{ m/s}^2)\left(22\text{ N} - (0.15)(44\text{ N})\right)}{44\text{ N} + 22\text{ N}} = 2.3\text{ m/s}^2.$$

30. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y-axis, where there is presumed to be no acceleration, leads to

$$F_N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\max}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$F\cos\theta - f_s = ma \quad\Rightarrow\quad F\cos\theta - \mu_s(mg - F\sin\theta) = 0$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.42$, to obtain $F = 74$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s\sin\theta} = \frac{(0.42)(180\text{ N})}{\cos\theta + (0.42)\sin\theta} = \frac{76\text{ N}}{\cos\theta + (0.42)\sin\theta}.$$

(c) We minimize the above expression for F by working through the condition:

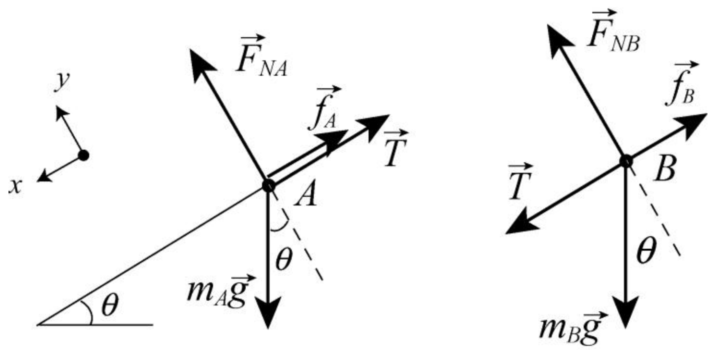
$$\frac{dF}{d\theta} = \frac{\mu_s W(\sin\theta - \mu_s\cos\theta)}{(\cos\theta + \mu_s\sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70\text{ N}$.

31. **THINK** In this problem we have two blocks connected by a string sliding down an inclined plane; the blocks have different coefficient of kinetic friction.

EXPRESS The free-body diagrams for the two blocks are shown below. T is the magnitude of the tension force of the string, $\vec{F}_{NA}$ is the normal force on block A (the leading block), $\vec{F}_{NB}$ is the normal force on block B, $\vec{f}_A$ is kinetic friction force on block A, $\vec{f}_B$ is kinetic friction force on block B. Also, m_A is the mass of block A (where $m_A = W_A/g$ and W_A = 3.6 N), and m_B is the mass of block B (where $m_B = W_B/g$ and W_B = 7.2 N). The angle of the incline is $\theta = 30°$.



For each block we take $+x$ downhill (which is toward the lower-left in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of both blocks A and B, we arrive at four equations:

$$W_A \sin\theta - f_A - T = m_A a$$
$$F_{NA} - W_A \cos\theta = 0$$
$$W_B \sin\theta - f_B + T = m_B a$$
$$F_{NB} - W_B \cos\theta = 0$$

which, when combined with Eq. 6-2 ($f_A = \mu_{kA} F_{NA}$ where $\mu_{k\,A}$ = 0.10 and $f_B = \mu_{kB} F_{NB}$ f_B where $\mu_{k\,B}$ = 0.20), fully describe the dynamics of the system so long as the blocks have the same acceleration and $T > 0$.

ANALYZE (a) From these equations, we find the acceleration to be

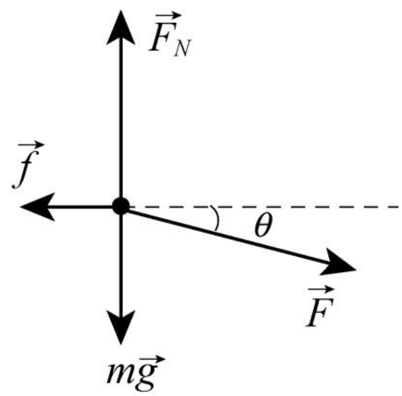
$$a = g\left(\sin\theta - \left(\frac{\mu_{k\,A} W_A + \mu_{k\,B} W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5\ \text{m/s}^2.$$

(b) We solve the above equations for the tension and obtain

$$T = \left(\frac{W_A W_B}{W_A + W_B}\right)\ (\mu_{k\,B} - \mu_{k\,A})\cos\theta = \frac{(3.6\ \text{N})(7.2\ \text{N})}{3.6\ \text{N} + 7.2\ \text{N}}(0.20 - 0.10)\cos 30° = 0.21\ \text{N}.$$

LEARN The tension in the string is proportional to $\mu_{kB} - \mu_{kA}$, the difference in coefficients of kinetic friction for the two blocks. When the coefficients are equal ($\mu_{kB} = \mu_{kA}$), the two blocks can be viewed as moving independent of one another and the tension is zero. Similarly, when $\mu_{kB} < \mu_{kA}$ (the leading block *A* has larger coefficient than the *B*), the string is slack, so the tension is also zero.

32. The free-body diagram for the block is shown below, with $\vec{F}$ being the force applied to the block, $\vec{F}_N$ the normal force of the floor on the block, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. The equations for the *x* and the *y* components of the force according to Newton's second law are:



$$F_x = F\cos\theta - f = ma$$
$$F_y = F_N - F\sin\theta - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = mg + F\sin\theta$, which yields

$$f = \mu_k(mg + F\sin\theta).$$

This expression is substituted for *f* in the first equation to obtain

$$F\cos\theta - \mu_k(mg + F\sin\theta) = ma,$$

so the acceleration is

$$a = \frac{F}{m}\left(\cos\theta - \mu_k\sin\theta\right) - \mu_k g.$$

From the figure, we see that $a = 3.0\ \text{m/s}^2$ when $\mu_k = 0$. This implies

$$3.0\ \text{m/s}^2 = \frac{F}{m}\cos\theta.$$

We also find $a = 0$ when $\mu_k = 0.20$:

$$0 = \frac{F}{m}\left(\cos\theta - (0.20)\sin\theta\right) - (0.20)(9.8\ \text{m/s}^2) = 3.00\ \text{m/s}^2 - 0.20\frac{F}{m}\sin\theta - 1.96\ \text{m/s}^2$$
$$= 1.04\ \text{m/s}^2 - 0.20\frac{F}{m}\sin\theta$$

which yields $5.2\ \text{m/s}^2 = \dfrac{F}{m}\sin\theta.$ Combining the two results, we get

$$\tan\theta = \left(\frac{5.2\ \text{m/s}^2}{3.0\ \text{m/s}^2}\right) = 1.73 \quad\Rightarrow\quad \theta = 60°.$$

33. **THINK** In this problem, the frictional force is not a constant, but instead proportional to the speed of the boat. Integration is required to solve for the speed.

EXPRESS We denote the magnitude of the frictional force as αv, where $\alpha = 70\ \text{N}\cdot\text{s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\frac{dv}{dt} \quad\Rightarrow\quad \frac{dv}{v} = -\frac{\alpha}{m}dt.$$

Integrating the equation gives

$$\int_{v_0}^{v}\frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. Solving the integral allows us to calculate the time it takes for the boat to slow down to 45 km/h, or $v = v_0/2$, where $v_0 = 90\ \text{km/h}$.

ANALYZE The integrals are evaluated with the result

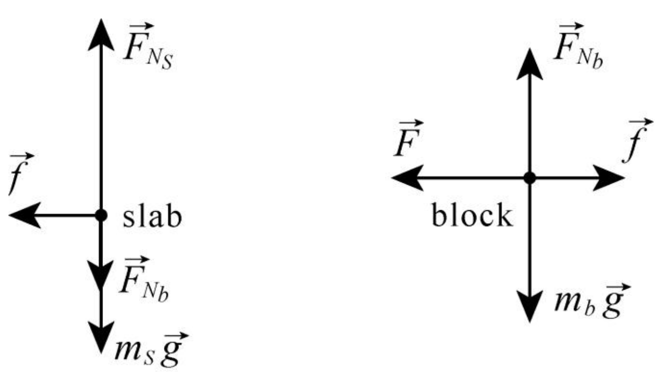
$$\ln\left(\frac{v}{v_0}\right) = -\frac{\alpha t}{m}$$

With $v = v_0/2$ and $m = 1000$ kg, we find the time to be

$$t = -\frac{m}{\alpha}\ln\left(\frac{v}{v_0}\right) = -\frac{m}{\alpha}\ln\left(\frac{1}{2}\right) = -\frac{1000\ \text{kg}}{70\ \text{N}\cdot\text{s/m}}\ln\left(\frac{1}{2}\right) = 9.9\ \text{s}.$$

LEARN The speed of the boat is given by $v(t) = v_0 e^{-\alpha t/m}$, showing exponential decay with time. The greater the value of α, the more rapidly the boat slows down.

34. The free-body diagrams for the slab and block are shown below.



$\vec{F}$ is the 100 N force applied to the block, $\vec{F}_{Ns}$ is the normal force of the floor on the slab, F_{Nb} is the magnitude of the normal force between the slab and the block, $\vec{f}$ is the force of friction between the slab and the block, m_s is the mass of the slab, and m_b is the mass of the block. For both objects, we take the $+x$ direction to be to the right and the $+y$ direction to be up.

Applying Newton's second law for the x and y axes for (first) the slab and (second) the block results in four equations:

$$\begin{aligned} -f &= m_s a_s \\ F_{Ns} - F_{Ns} - m_s g &= 0 \\ f - F &= m_b a_b \\ F_{Nb} - m_b g &= 0 \end{aligned}$$

from which we note that the maximum possible static friction magnitude would be

$$\mu_s F_{Nb} = \mu_s m_b g = (0.60)(10\text{ kg})(9.8\text{ m/s}^2) = 59\text{ N} .$$

We check to see if the block slides on the slab. Assuming it does not, then $a_s = a_b$ (which we denote simply as a) and we solve for f:

$$f = \frac{m_s F}{m_s + m_b} = \frac{(40\text{ kg})(100\text{ N})}{40\text{ kg} + 10\text{ kg}} = 80\text{ N}$$

which is greater than $f_{s,\max}$ so that we conclude the block is sliding across the slab (their accelerations are different).

(a) Using $f = \mu_k F_{Nb}$ the above equations yield

$$a_b = \frac{\mu_k m_b g - F}{m_b} = \frac{(0.40)(10\text{ kg})(9.8\text{ m/s}^2) - 100\text{ N}}{10\text{ kg}} = -6.1\text{ m/s}^2.$$

The negative sign means that the acceleration is leftward. That is, $\vec{a}_b = (-6.1\text{ m/s}^2)\hat{\mathrm{i}}$

(b) We also obtain

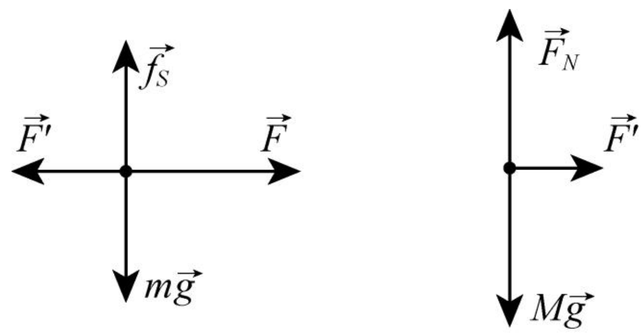
$$a_s = -\frac{\mu_k m_b g}{m_s} = -\frac{(0.40)(10\text{ kg})(9.8\text{ m/s}^2)}{40\text{ kg}} = -0.98\text{ m/s}^2.$$

As mentioned above, this means it accelerates to the left. That is, $\vec{a}_s = (-0.98\text{ m/s}^2)\hat{\mathrm{i}}$

35. The free-body diagrams for the two blocks, treated individually, are shown below (first m and then M). F' is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,\max} = \mu_s F'$ where $\mu_s = 0.38$).

Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newton's second law (with $+x$ rightward) to find an expression for the acceleration:

$$F = m_{\text{total}}\, a \quad \Rightarrow a = \frac{F}{m+M}$$



This is equivalent to having analyzed the two blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newton's second law to the x and y axes, substitute in the above expression for a, and use Eq. 6-1.

$$F - F' = ma \quad \Rightarrow \quad F' = F - m\left(\frac{F}{m+M}\right)$$
$$f_s - mg = 0 \quad \Rightarrow \quad \mu_s F' - mg = 0$$

These expressions are combined (to eliminate F') and we arrive at

$$F = \frac{mg}{\mu_s\left(1 - \dfrac{m}{m+M}\right)} = 4.9\times10^2\ \text{N}.$$

36. Using Eq. 6-16, we solve for the area $A\dfrac{2m\,g}{C\rho\, v_t^2}$ which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain

$$\frac{A_{\text{slow}}}{A_{\text{fast}}} = \left(\frac{310\ \text{km/h}}{160\ \text{km/h}}\right)^2 = 3.75.$$

37. In the solution to exercise 4, we found that the force provided by the wind needed to equal F = 157 N (where that last figure is not "significant'').

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v=\sqrt{\frac{2F}{C\rho A}}=\sqrt{\frac{2(157\text{ N})}{(0.80)(1.21\,\text{kg/m}^3)(0.040\text{ m}^2)}}=90\text{ m/s }=3.2\times10^2\text{ km/h.}$$

(b) Doubling our previous result, we find the reported speed to be 6.5×10^2 km/h.

(c) The result is not reasonable for a terrestrial storm. A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.

38. (a) From Table 6-1 and Eq. 6-16, we have

$$v_t=\sqrt{\frac{2F_g}{C\rho A}} \;\Rightarrow C\rho A=2\frac{mg}{v_t^2}$$

where v_t = 60 m/s. We estimate the pilot's mass at about m = 70 kg. Now, we convert v = 1300(1000/3600) ≈ 360 m/s and plug into Eq. 6-14:

$$D=\frac{1}{2}C\rho Av^2=\frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2=mg\left(\frac{v}{v_t}\right)^2$$

which yields $D = (70\text{ kg})(9.8\text{ m/s}^2)(360/60)^2 \approx 2 \times 10^4$ N.

(b) We assume the mass of the ejection seat is roughly equal to the mass of the pilot. Thus, Newton's second law (in the horizontal direction) applied to this system of mass 2m gives the magnitude of acceleration:

$$|a|=\frac{D}{2m}=\frac{g}{2}\left(\frac{v}{v_t}\right)^2=18g\ .$$

39. For the passenger jet $D_j=\frac{1}{2}C\rho_1Av_j^2$, and for the prop-driven transport $D_t=\frac{1}{2}C\rho_2Av_t^2$, where ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t}=\frac{\rho_1v_j^2}{\rho_2v_t^2}=\frac{(0.38\ \text{kg/m}^3)(1000\ \text{km/h})^2}{(0.67\ \text{kg/m}^3)(500\ \text{km/h})^2}=2.3.$$

40. This problem involves Newton's second law for motion along the slope.

(a) The force along the slope is given by

$$\begin{aligned}F_g &= mg\sin\theta - \mu F_N = mg\sin\theta - \mu mg\cos\theta = mg(\sin\theta - \mu\cos\theta)\\ &= (85.0\text{ kg})(9.80\text{ m/s}^2)\left[\sin 40.0° - (0.04000)\cos 40.0°\right]\\ &= 510\text{ N}.\end{aligned}$$

Thus, the terminal speed of the skier is

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{2(510\text{ N})}{(0.150)(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}} = 66.0\text{ m/s}.$$

(b) Differentiating v_t with respect to C, we obtain

$$\begin{aligned}dv_t &= -\frac{1}{2}\sqrt{\frac{2F_g}{\rho A}}\,C^{-3/2}dC = -\frac{1}{2}\sqrt{\frac{2(510\text{ N})}{(1.20\text{ kg/m}^3)(1.30\text{ m}^2)}}(0.150)^{-3/2}dC\\ &= -(2.20\times10^2\text{ m/s})dC.\end{aligned}$$

41. Perhaps surprisingly, the equations pertaining to this situation are exactly those in Sample Problem – "Car in flat circular turn," although the logic is a little different. In the Sample Problem, the car moves along a (stationary) road, whereas in this problem the cat is stationary relative to the merry-go-around platform. But the static friction plays the same role in both cases since the bottom-most point of the car tire is instantaneously at rest with respect to the race track, just as static friction applies to the contact surface between cat and platform. Using Eq. 6-23 with Eq. 4-35, we find

$$\mu_s = (2\pi R/T)^2/gR = 4\pi^2 R/gT^2.$$

With T = 6.0 s and R = 5.4 m, we obtain μ_s = 0.60.

42. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad v \le \sqrt{\mu_s Rg}\,.$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\max} = \sqrt{\mu_s Rg} = \sqrt{(0.60)(30.5\text{ m})(9.8\text{ m/s}^2)} = 13\text{ m/s} \approx 48\text{ km/h.}$$

43. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If F_N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is

$$f_{s,\max} = \mu_s F_N = \mu_s mg.$$

If the bicycle does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Rightarrow \quad R \ge \frac{v^2}{\mu_s g}.$$

Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is

$$R_{\min} = \frac{v^2}{\mu_s g} = \frac{(8.1\text{ m/s})^2}{(0.32)(9.8\text{ m/s}^2)} = 21\text{ m.}$$

44. With v = 96.6 km/h = 26.8 m/s, Eq. 6-17 readily yields

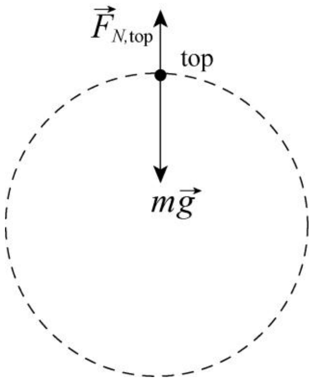
$$a = \frac{v^2}{R} = \frac{(26.8\text{ m/s})^2}{7.6\text{ m}} = 94.7\text{ m/s}^2$$

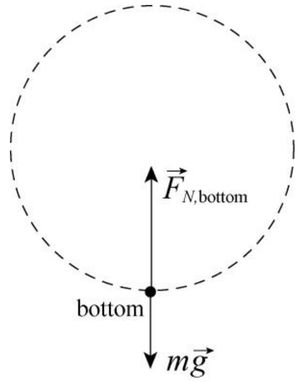
which we express as a multiple of g:

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7\text{ m/s}^2}{9.80\text{ m/s}^2}\right) g = 9.7g.$$

45. **THINK** Ferris wheel ride is a vertical circular motion. The apparent weight of the rider varies with his position.

EXPRESS The free-body diagrams of the student at the top and bottom of the Ferris wheel are shown next:





At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude $F_{N,\text{top}}$, while the Earth pulls down with a force of magnitude mg. Newton's second law for the radial direction gives

$$mg - F_{N,\text{top}} = \frac{mv^2}{R}.$$

At the bottom of the ride, $F_{N,\text{bottom}}$ is the magnitude of the upward force exerted by the seat. The net force toward the center of the circle is (choosing upward as the positive direction):

$$F_{N,\text{bottom}} - mg = \frac{mv^2}{R}.$$

The Ferris wheel is "steadily rotating" so the value $F_c = mv^2/R$ is the same everywhere. The apparent weight of the student is given by F_N.

ANALYZE (a) At the top, we are told that $F_{N,\text{top}}$ = 556 N and mg = 667 N. This means that the seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point. Thus, he feels "light."

(b) From (a), we find the centripetal force to be

$$F_c = \frac{mv^2}{R} = mg - F_{N,\text{top}} = 667\text{ N} - 556\text{ N} = 111\text{ N}.$$

Thus, the normal force at the bottom is

$$F_{N,\text{bottom}} = \frac{mv^2}{R} + mg = F_c + mg = 111\text{ N} + 667\text{ N} = 778\text{ N}.$$

(c) If the speed is doubled,

$$F_c' = \frac{m(2v)^2}{R} = 4(111\text{ N}) = 444\text{ N}.$$

Therefore, at the highest point we have

$$F'_{N,\text{top}} = mg - F'_c = 667\ \text{N} - 444\ \text{N} = 223\ \text{N}.$$

(d) Similarly, the normal force at the lowest point is now found to be

$$F'_{N,\text{bottom}} = F'_c + mg = 444\ \text{N} + 667\ \text{N} = 1111\ \text{N}.$$

LEARN The apparent weight of the student is the greatest at the bottom and smallest at the top of the ride. The speed $v = \sqrt{gR}$ would result in $F_{N,\text{top}} = 0$, giving the student a sudden sensation of "weightlessness" at the top of the ride.

46. (a) We note that the speed 80.0 km/h in SI units is roughly 22.2 m/s. The horizontal force that keeps her from sliding must equal the centripetal force (Eq. 6-18), and the upward force on her must equal *mg*. Thus,

$$F_{\text{net}} = \sqrt{(mg)^2 + (mv^2/R)^2} = 547\ \text{N}.$$

(b) The angle is

$$\tan^{-1}[(mv^2/R)/(mg)] = \tan^{-1}(v^2/gR) = 9.53^\circ$$

as measured from a vertical axis.

47. (a) Eq. 4-35 gives $T = 2\pi R/v = 2\pi(10\ \text{m})/(6.1\ \text{m/s}) = 10\ \text{s}$.

(b) The situation is similar to that of Sample Problem – "Vertical circular loop, Diavolo," but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R) = 486\ \text{N} \approx 4.9 \times 10^2\ \text{N}.$$

(c) Now we reverse both the normal force direction and the acceleration direction (from what is shown in Sample Problem – "Vertical circular loop, Diavolo") and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 1081\ \text{N} \approx 1.1\ \text{kN}.$$

48. We will start by assuming that the normal force (on the car from the rail) points up. Note that gravity points down, and the *y* axis is chosen positive upwards. Also, the direction to the center of the circle (the direction of centripetal acceleration) is down. Thus, Newton's second law leads to

$$F_N - mg = m\left(-\frac{v^2}{r}\right).$$

(a) When $v = 11$ m/s, we obtain $F_N = 3.7 \times 10^3$ N.

(b) $\vec{F}_N$ points upward.

(c) When v = 14 m/s, we obtain $F_N = -1.3 \times 10^3$ N, or $|F_N| = 1.3 \times 10^3$ N.

(d) The fact that this answer is negative means that $\vec{F}_N$ points opposite to what we had assumed. Thus, the magnitude of $\vec{F}_N$ is $|\vec{F}_N| = 1.3$ kN and its direction is *down*.

49. At the top of the hill, the situation is similar to that of Sample Problem – "Vertical circular loop, Diavolo," but with the normal force direction reversed. Adapting Eq. 6-19, we find

$$F_N = m(g - v^2/R).$$

Since $F_N = 0$ there (as stated in the problem) then $v^2 = gR$. Later, at the bottom of the valley, we reverse both the normal force direction and the acceleration direction (from what is shown in the Sample Problem) and adapt Eq. 6-19 accordingly. Thus we obtain

$$F_N = m(g + v^2/R) = 2mg = 1372 \text{ N} \approx 1.37 \times 10^3 \text{ N}.$$

50. The centripetal force on the passenger is $F = mv^2/r$.

(a) The slope of the plot at $v = 8.30$ m/s is

$$\left.\frac{dF}{dv}\right|_{v=8.30\text{ m/s}} = \left.\frac{2mv}{r}\right|_{v=8.30\text{ m/s}} = \frac{2(85.0\text{ kg})(8.30\text{ m/s})}{3.50\text{ m}} = 403\text{ N}\cdot\text{s/m}.$$

(b) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

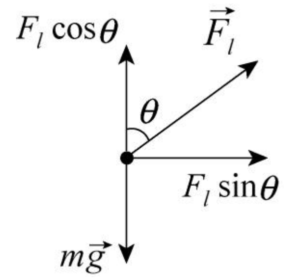
$$dF = -\frac{8\pi^2 mr}{T^3}\,dT.$$

The slope of the plot at $T = 2.50$ s is

$$\left.\frac{dF}{dT}\right|_{T=2.50\text{ s}} = -\left.\frac{8\pi^2 mr}{T^3}\right|_{T=2.50\text{ s}} = \frac{8\pi^2(85.0\text{ kg})(3.50\text{ m})}{(2.50\text{ s})^3} = -1.50\times10^3\text{ N/s}.$$

51. **THINK** An airplane with its wings tilted at an angle is in a circular motion. Centripetal force is involved in this problem.

EXPRESS The free-body diagram for the airplane of mass m is shown to the right. We note that $\vec{F}_l$ is the force of aerodynamic lift and $\vec{a}$ points rightwards in the figure. We also note that $|\vec{a}| = v^2 / R$. Applying Newton's law to the axes of the problem (+x rightward and +y upward) we obtain



$$F_l \sin\theta = m\frac{v^2}{R}$$
$$F_l \cos\theta = mg$$

Eliminating mass from these equations leads to $\tan\theta = \dfrac{v^2}{gR}$. The equation allows us to solve for the radius R.

ANALYZE With v = 480 km/h = 133 m/s and θ = 40°, we find

$$R = \frac{v^2}{g\tan\theta} = \frac{(133\,\text{m/s})^2}{(9.8\,\text{m/s}^2)\tan 40°} = 2151\ \text{m} \approx 2.2\times 10^3\ \text{m}.$$

LEARN Our approach to solving this problem is identical to that discussed in the Sample Problem – "Car in banked circular turn." Do you see the similarities?

52. The situation is somewhat similar to that shown in the "loop-the-loop" example done in the textbook (see Figure 6-10) except that, instead of a downward normal force, we are dealing with the force of the boom $\vec{F}_B$ on the car – which is capable of pointing any direction. We will assume it to be upward as we apply Newton's second law to the car (of total weight 5000 N): $F_B - W = ma$ where $m = W/g$ and $a = -v^2/r$. Note that the centripetal acceleration is downward (our choice for negative direction) for a body at the top of its circular trajectory.

(a) If r = 10 m and v = 5.0 m/s, we obtain $F_B = 3.7 \times 10^3$ N = 3.7 kN.

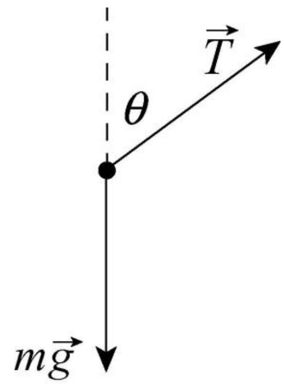
(b) The direction of $\vec{F}_B$ is up.

(c) If r = 10 m and v = 12 m/s, we obtain $F_B = -\,2.3 \times 10^3$ N = – 2.3 kN, or $|F_B|$ = 2.3 kN.

(d) The minus sign indicates that $\vec{F}_B$ points downward.

53. The free-body diagram (for the hand straps of mass m) is the view that a passenger might see if she was looking forward and the streetcar was curving towards the right (so $\vec{a}$ points rightwards in the figure). We note that $|\vec{a}| = v^2 / R$ where v = 16 km/h = 4.4 m/s.

Applying Newton's law to the axes of the problem (+x is rightward and +y is upward) we obtain



$$T\sin\theta = m\frac{v^2}{R}$$
$$T\cos\theta = mg.$$

We solve these equations for the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

which yields $\theta = 12°$.

54. The centripetal force on the passenger is $F = mv^2/r$.

(a) The variation of F with respect to r while holding v constant is $dF = -\frac{mv^2}{r^2}dr$.

(b) The variation of F with respect to v while holding r constant is $dF = \frac{2mv}{r}dv$.

(c) The period of the circular ride is $T = 2\pi r/v$. Thus,

$$F = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 mr}{T^2},$$

and the variation of F with respect to T while holding r constant is

$$dF = -\frac{8\pi^2 mr}{T^3}dT = -8\pi^2 mr\left(\frac{v}{2\pi r}\right)^3 dT = -\left(\frac{mv^3}{\pi r^2}\right)dT.$$

55. We note that the period T is eight times the time between flashes ($\frac{1}{2000}$ s), so T = 0.0040 s. Combining Eq. 6-18 with Eq. 4-35 leads to

$$F = \frac{4m\pi^2 R}{T^2} = \frac{4(0.030\text{ kg})\pi^2(0.035\text{ m})}{(0.0040\text{ s})^2} = 2.6 \times 10^3\text{ N} .$$

56. We refer the reader to Sample Problem – "Car in banked circular turn," and use the result Eq. 6-26:

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

with v = 60(1000/3600) = 17 m/s and R = 200 m. The banking angle is therefore $\theta = 8.1°$. Now we consider a vehicle taking this banked curve at v' = 40(1000/3600) = 11 m/s. Its (horizontal) acceleration is $a' = v'^2/R$, which has components parallel the incline and perpendicular to it:

$$a_{\|} = a'\cos\theta = \frac{v'^2\cos\theta}{R}$$
$$a_{\perp} = a'\sin\theta = \frac{v'^2\sin\theta}{R}.$$

These enter Newton's second law as follows (choosing downhill as the $+x$ direction and away-from-incline as $+y$):

$$mg\sin\theta - f_s = ma_{\|}$$
$$F_N - mg\cos\theta = ma_{\perp}$$

and we are led to

$$\frac{f_s}{F_N} = \frac{mg\sin\theta - mv'^2\cos\theta/R}{mg\cos\theta + mv'^2\sin\theta/R}.$$

We cancel the mass and plug in, obtaining $f_s/F_N = 0.078$. The problem implies we should set $f_s = f_{s,\max}$ so that, by Eq. 6-1, we have $\mu_s = 0.078$.

57. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed:

$$v = \sqrt{\frac{Mgr}{m}} = \sqrt{\frac{(2.50\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})}{1.50\text{ kg}}} = 1.81\text{ m/s}.$$

58. (a) Using the kinematic equation given in Table 2-1, the deceleration of the car is

$$v^2 = v_0^2 + 2ad \quad\Rightarrow\quad 0 = (35\text{ m/s})^2 + 2a(107\text{ m})$$

which gives $a = -5.72\text{ m/s}^2$. Thus, the force of friction required to stop by car is

$$f = m|a| = (1400\text{ kg})(5.72\text{ m/s}^2) \approx 8.0\times10^3\text{ N}.$$

(b) The maximum possible static friction is

$$f_{s,\max} = \mu_s mg = (0.50)(1400\text{ kg})(9.80\text{ m/s}^2) \approx 6.9\times10^3\text{ N}.$$

(c) If $\mu_k = 0.40$, then $f_k = \mu_k mg$ and the deceleration is $a = -\mu_k g$. Therefore, the speed of the car when it hits the wall is

$$v = \sqrt{v_0^2 + 2ad} = \sqrt{(35\text{ m/s})^2 - 2(0.40)(9.8\text{ m/s}^2)(107\text{ m})} \approx 20\text{ m/s}.$$

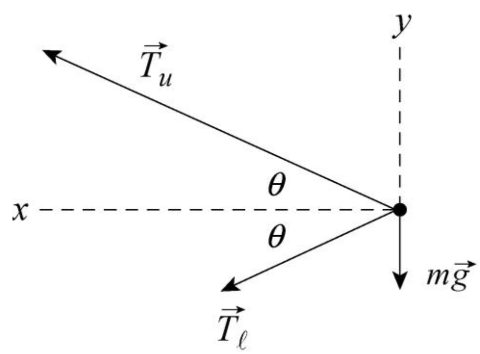
(d) The force required to keep the motion circular is

$$F_r = \frac{mv_0^2}{r} = \frac{(1400\text{ kg})(35.0\text{ m/s})^2}{107\text{ m}} = 1.6\times10^4\text{ N}.$$

(e) Since $F_r > f_{s,\max}$, no circular path is possible.

59. **THINK** As illustrated in Fig. 6-45, our system consists of a ball connected by two strings to a rotating rod. The tensions in the strings provide the source of centripetal force.

EXPRESS The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension in the lower string, and m is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



We take the $+x$ direction to be leftward (toward the center of the circular orbit) and $+y$ upward. Since the magnitude of the acceleration is $a = v^2/R$, the x component of Newton's second law is

$$T_u \cos\theta + T_\ell \cos\theta = \frac{mv^2}{R},$$

where v is the speed of the ball and R is the radius of its orbit. The y component is

$$T_u \sin\theta - T_\ell \sin\theta - mg = 0.$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$.

ANALYZE (a) Since the triangle is equilateral, the angle is $\theta = 30.0°$. Thus

$$T_\ell = T_u - \frac{mg}{\sin\theta} = 35.0\text{ N} - \frac{(1.34\text{ kg})(9.80\text{ m/s}^2)}{\sin 30.0°} = 8.74\text{ N}.$$

(b) The net force in the y-direction is zero. In the x-direction, the net force has magnitude

$$F_{\text{net,str}} = (T_u + T_\ell)\cos\theta = (35.0\text{ N} + 8.74\text{ N})\cos 30.0° = 37.9\text{ N}.$$

(c) The radius of the path is

$$R = L\cos\theta = (1.70\text{ m})\cos 30° = 1.47\text{ m}.$$

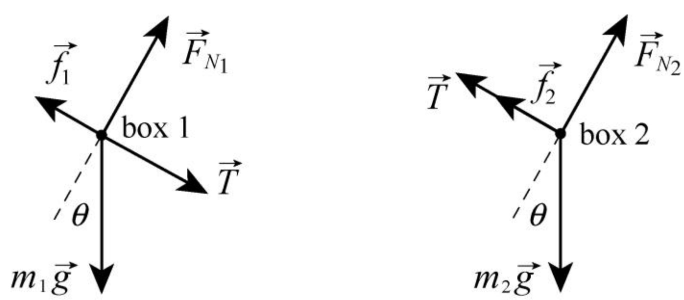
Using $F_{\text{net,str}} = mv^2/R$, we find the speed of the ball to be

$$v = \sqrt{\frac{RF_{\text{net,str}}}{m}} = \sqrt{\frac{(1.47\text{ m})(37.9\text{ N})}{1.34\text{ kg}}} = 6.45\text{ m/s}.$$

(d) The direction of $\vec{F}_{\text{net,str}}$ is leftward ("radially inward'').

LEARN The upper string, with a tension about 4 times that in the lower string ($T_u \approx 4T_\ell$), will break more easily than the lower one.

60. The free-body diagrams for the two boxes are shown below. T is the magnitude of the force in the rod (when $T > 0$ the rod is said to be in tension and when $T < 0$ the rod is under compression), $\vec{F}_{N2}$ is the normal force on box 2 (the uncle box), $\vec{F}_{N1}$ is the the normal force on the aunt box (box 1), $\vec{f}_1$ is kinetic friction force on the aunt box, and $\vec{f}_2$ is kinetic friction force on the uncle box. Also, m_1 = 1.65 kg is the mass of the aunt box and m_2 = 3.30 kg is the mass of the uncle box (which is a lot of ants!).



For each block we take +x downhill (which is toward the lower-right in these diagrams) and +y in the direction of the normal force. Applying Newton's second law to the x and y directions of first box 2 and next box 1, we arrive at four equations:

$$m_2 g\sin\theta - f_2 - T = m_2 a$$
$$F_{N2} - m_2 g\cos\theta = 0$$
$$m_1 g\sin\theta - f_1 + T = m_1 a$$
$$F_{N1} - m_1 g\cos\theta = 0$$

which, when combined with Eq. 6-2 ($f_1 = \mu_1 F_{N1}$ where $\mu_1 = 0.226$ and $f_2 = \mu_2 F_{N2}$ where $\mu_2 = 0.113$), fully describe the dynamics of the system.

(a) We solve the above equations for the tension and obtain

$$T = \left(\frac{m_2 m_1 g}{m_2 + m_1}\right)(\mu_1 - \mu_2)\cos\theta = 1.05 \text{ N}.$$

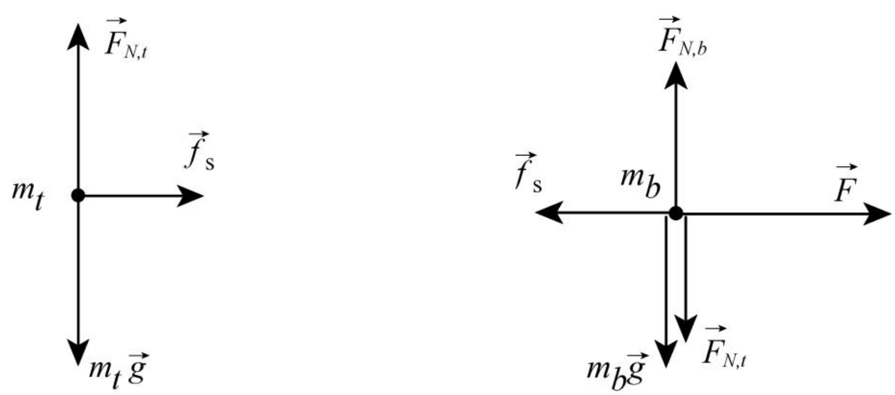
(b) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_2 m_2 + \mu_1 m_1}{m_2 + m_1}\right)\cos\theta\right) = 3.62 \text{ m/s}^2.$$

(c) Reversing the blocks is equivalent to switching the labels. We see from our algebraic result in part (a) that this gives a negative value for T (equal in magnitude to the result we got before). Thus, the situation is as it was before except that the rod is now in a state of compression.

61. **THINK** Our system consists of two blocks, one on top of the other. If we pull the bottom block too hard, the top block will slip on the bottom one. We're interested in the maximum force that can be applied such that the two will move together.

EXPRESS The free-body diagrams for the two blocks are shown below.



We first calculate the coefficient of static friction for the surface between the two blocks. When the force applied is at a maximum, the frictional force between the two blocks must also be a maximum. Since $F_t = 12$ N of force has to be applied to the top block for slipping to take place, using $F_t = f_{s,\max} = \mu_s F_{N,t} = \mu_s m_t g$, we have

$$\mu_s = \frac{F_t}{m_t g} = \frac{12 \text{ N}}{(4.0 \text{ kg})(9.8 \text{ m/s}^2)} = 0.31.$$

Using the same reasoning, for the two masses to move together, the maximum applied force would be

$$F = \mu_s(m_t + m_b)g$$

ANALYZE (a) Substituting the value of μ_s found above, the maximum horizontal force has a magnitude

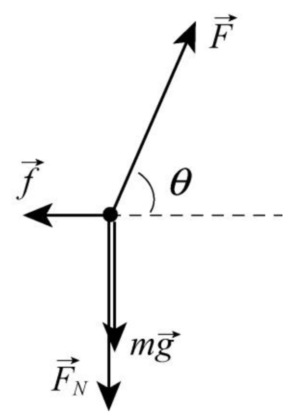
$$F = \mu_s(m_t + m_b)g = (0.31)(4.0\text{ kg} + 5.0\text{ kg})(9.8\text{ m/s}^2) = 27\text{ N}$$

(b) The maximum acceleration is

$$a_{\max} = \frac{F}{m_t + m_b} = \mu_s g = (0.31)(9.8\text{ m/s}^2) = 3.0\text{ m/s}^2\,.$$

LEARN Slipping will occur if the applied force exceeds 27.3 N. In the absence of friction ($\mu_s = 0$) between the two blocks, any amount of force would cause the top block to slip.

62. The free-body diagram for the stone is shown to the right, with $\vec{F}$ being the force applied to the stone, $\vec{F}_N$ the *downward* normal force of the ceiling on the stone, $m\vec{g}$ the force of gravity, and $\vec{f}$ the force of friction. We take the +*x* direction to be horizontal to the right and the +*y* direction to be up. The equations for the *x* and the *y* components of the force according to Newton's second law are:



$$F_x = F\cos\theta - f = ma$$
$$F_y = F\sin\theta - F_N - mg = 0$$

Now $f = \mu_k F_N$, and the second equation gives $F_N = F\sin\theta - mg$, which yields $f = \mu_k(F\sin\theta - mg)$. This expression is substituted for *f* in the first equation to obtain

$$F\cos\theta - \mu_k(F\sin\theta - mg) = ma.$$

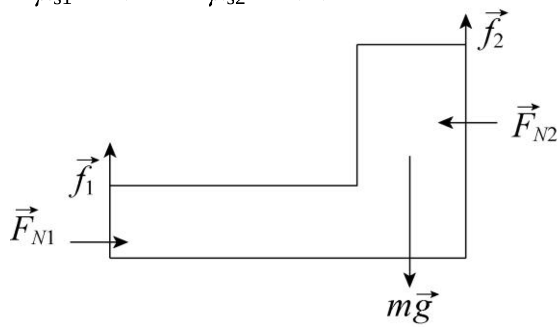
For $a = 0$, the force is

$$F = \frac{-\mu_k mg}{\cos\theta - \mu_k\sin\theta}.$$

With $\mu_k = 0.65$, $m = 5.0$ kg, and $\theta = 70°$, we obtain $F = 118$ N.

63. (a) The free-body diagram for the person (shown as an L-shaped block) is shown below. The force that she exerts on the rock slabs is not directly shown (since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces $\vec{F}_{N1}$ and $\vec{F}_{N2}$ exerted horizontally by the slabs onto her shoes and

back, respectively. We will show in part (b) that $F_{N1} = F_{N2}$ so that we there is no ambiguity in saying that the magnitude of her push is F_{N2}. The total upward force due to (maximum) static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ where $f_1 = \mu_{s1}F_{N1}$ and $f_2 = \mu_{s2}F_{N2}$. The problem gives the values $\mu_{s1} = 1.2$ and $\mu_{s2} = 0.8$.



(b) We apply Newton's second law to the *x* and *y* axes (with +*x* rightward and +*y* upward and there is no acceleration in either direction).

$$F_{N1} - F_{N2} = 0$$
$$f_1 + f_2 - mg = 0$$

The first equation tells us that the normal forces are equal $F_{N1} = F_{N2} = F_N$. Consequently, from Eq. 6-1,

$$f_1 = \mu_{s\,1}F_N$$
$$f_2 = \mu_{s\,2}F_N$$

we conclude that

$$f_1 = \left(\frac{\mu_{s\,1}}{\mu_{s\,2}}\right) f_2 \; .$$

Therefore, $f_1 + f_2 - mg = 0$ leads to

$$\left(\frac{\mu_{s\,1}}{\mu_{s\,2}} + 1\right) f_2 = mg$$

which (with $m = 49$ kg) yields $f_2 = 192$ N. From this we find $F_N = f_2 / \mu_{s2} = 240$ N. This is equal to the magnitude of the push exerted by the rock climber.

(c) From the above calculation, we find $f_1 = \mu_{s1}F_N = 288$ N which amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)\,(9.8)} = 0.60$$

or 60% of her weight.

64. (a) The upward force exerted by the car on the passenger is equal to the downward force of gravity (W = 500 N) on the passenger. So the *net* force does not have a vertical contribution; it only has the contribution from the horizontal force (which is necessary for maintaining the circular motion). Thus $|\vec{F}_{\text{net}}| = F = 210$ N.

(b) Using Eq. 6-18, we have

$$v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(210\text{ N})(470\text{ m})}{51.0\text{ kg}}} = 44.0\text{ m/s}.$$

65. The layer of ice has a mass of

$$m_{\text{ice}} = \left(917\text{ kg/m}^3\right)\ (400\text{ m}\times 500\text{ m}\times 0.0040\text{ m}) = 7.34\times 10^5\text{ kg}.$$

This added to the mass of the hundred stones (at 20 kg each) comes to $m = 7.36 \times 10^5$ kg.

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{\mu_k mg}{4C_{\text{ice}}\rho A_{\text{ice}}}} = \sqrt{\frac{(0.10)\left(7.36\times 10^5\text{ kg}\right)\left(9.8\text{ m/s}^2\right)}{4(0.002)\left(1.21\text{ kg/m}^3\right)\left(400\times 500\text{ m}^2\right)}} = 19\text{ m/s} \approx 69\text{ km/h}.$$

(b) Doubling our previous result, we find the reported speed to be 139 km/h.

(c) The result is reasonable for storm winds. (A category-5 hurricane has speeds on the order of 2.6×10^2 m/s.)

66. Note that since no static friction coefficient is mentioned, we assume f_s is not relevant to this computation. We apply Newton's second law to each block's x axis, which for m_1 is positive rightward and for m_2 is positive downhill:

$$\begin{aligned} T - f_k &= m_1 a \\ m_2 g\sin\theta - T &= m_2 a \end{aligned}$$

Adding the equations, we obtain the acceleration:

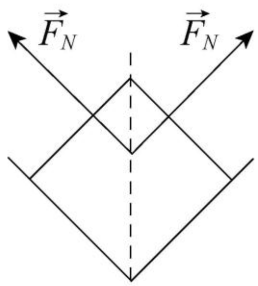
$$a = \frac{m_2 g\sin\theta - f_k}{m_1 + m_2}$$

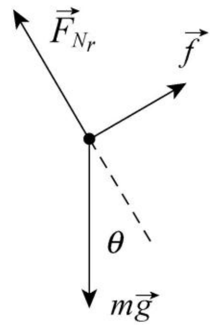
For $f_k = \mu_k F_N = \mu_k m_1 g$, we obtain

$$a = \frac{(3.0\text{ kg})(9.8\text{ m/s}^2)\sin 30° - (0.25)(2.0\text{ kg})(9.8\text{ m/s}^2)}{3.0\text{ kg} + 2.0\text{ kg}} = 1.96\text{ m/s}^2 .$$

Returning this value to either of the above two equations, we find T = 8.8 N.

67. Each side of the trough exerts a normal force on the crate. The first diagram shows the view looking in toward a cross section.





The net force is along the dashed line. Since each of the normal forces makes an angle of 45° with the dashed line, the magnitude of the resultant normal force is given by

$$F_{Nr} = 2F_N \cos 45° = \sqrt{2}F_N .$$

The second diagram is the free-body diagram for the crate (from a "side" view, similar to that shown in the first picture in Fig. 6-51). The force of gravity has magnitude mg, where m is the mass of the crate, and the magnitude of the force of friction is denoted by f. We take the $+x$ direction to be down the incline and $+y$ to be in the direction of $\vec{F}_{Nr}$. Then the x and the y components of Newton's second law are

$$x:\quad mg\sin\theta - f = ma$$
$$y:\quad F_{Nr} - mg\cos\theta = 0.$$

Since the crate is moving, each side of the trough exerts a force of kinetic friction, so the total frictional force has magnitude

$$f = 2\mu_k F_N = 2\mu_k F_{Nr}/\sqrt{2} = \sqrt{2}\mu_k F_{Nr}$$

Combining this expression with $F_{Nr} = mg\cos\theta$ and substituting into the x component equation, we obtain

$$mg\sin\theta - \sqrt{2}mg\cos\theta = ma .$$

Therefore $a = g(\sin\theta - \sqrt{2}\mu_k\cos\theta)$.

68. (a) To be on the verge of sliding out means that the force of static friction is acting "down the bank" (in the sense explained in the problem statement) with maximum

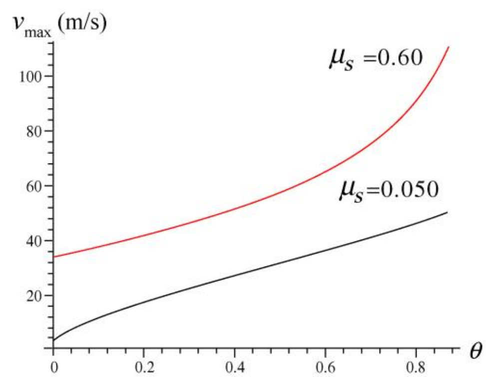
possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the *vertical* axis) $\phi = \theta + \theta_s$, where $\tan\theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle (as stated in the problem). Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg} \quad .$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}}$$

(b) The graph is shown below (with θ in radians):



(c) Either estimating from the graph (μ_s = 0.60, upper curve) or calculated it more carefully leads to v = 41.3 m/s = 149 km/h when θ = 10º = 0.175 radian.

(d) Similarly (for μ_s = 0.050, the lower curve) we find v = 21.2 m/s = 76.2 km/h when θ = 10º = 0.175 radian.

69. For simplicity, we denote the 70° angle as θ and the magnitude of the push (80 N) as P. The vertical forces on the block are the downward normal force exerted on it by the ceiling, the downward pull of gravity (of magnitude mg) and the vertical component of $\vec{P}$ (which is upward with magnitude P sin θ). Since there is no acceleration in the vertical direction, we must have

$$F_N = P\sin\theta - mg$$

in which case the leftward-pointed kinetic friction has magnitude

$$f_k = \mu_k(P\sin\theta - mg).$$

Choosing $+x$ rightward, Newton's second law leads to

$$P\cos\,\theta - f_k = ma \;\Rightarrow\; a = \frac{P\cos\theta - u_k(P\sin\theta - mg)}{m}$$

which yields $a = 3.4$ m/s^2 when $\mu_k = 0.40$ and $m = 5.0$ kg.

70. (a) We note that R (the horizontal distance from the bob to the axis of rotation) is the circumference of the circular path divided by 2π; therefore, $R = 0.94/2\pi = 0.15$ m. The angle that the cord makes with the horizontal is now easily found:

$$\theta = \cos^{-1}(R/L) = \cos^{-1}(0.15\text{ m}/0.90\text{ m}) = 80^\circ.$$

The vertical component of the force of tension in the string is $T\sin\theta$ and must equal the downward pull of gravity (mg). Thus,

$$T = \frac{mg}{\sin\theta} = 0.40\text{ N}.$$

Note that we are using T for tension (not for the period).

(b) The horizontal component of that tension must supply the centripetal force (Eq. 6-18), so we have $T\cos\theta = mv^2/R$. This gives speed $v = 0.49$ m/s. This divided into the circumference gives the time for one revolution: 0.94/0.49 = 1.9 s.

71. (a) To be "on the verge of sliding" means the applied force is equal to the maximum possible force of static friction (Eq. 6-1, with $F_N = mg$ in this case):

$$f_{s,\max} = \mu_s mg = 35.3\text{ N}.$$

(b) In this case, the applied force $\vec{F}$ indirectly decreases the maximum possible value of friction (since its y component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its x component). The normal force turns out to be

$$F_N = mg - F\sin\theta$$

where $\theta = 60^\circ$, so that the horizontal equation (the x application of Newton's second law) becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg - F\sin\theta) = 0 \quad\Rightarrow\; F = 39.7\text{ N}.$$

(c) Now, the applied force $\vec{F}$ indirectly increases the maximum possible value of friction (since its y component causes a reduction in the normal force) as well as directly opposing the friction force itself (because of its x component). The normal force in this case turns out to be

$$F_N = mg + F\sin\theta,$$

where $\theta = 60°$, so that the horizontal equation becomes

$$F\cos\theta - f_{s,\max} = F\cos\theta - \mu_s(mg + F\sin\theta) = 0 \quad \Rightarrow\ F = 320\ \text{N}.$$

72. With $\theta = 40°$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma,$$

$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

using Eq. 6-12. Thus,

$$a = 0.75\ \text{m/s}^2 = g(\sin\theta - \mu_k\cos\theta)$$

determines the coefficient of kinetic friction: $\mu_k = 0.74$.

73. (a) With $\theta = 60°$, we apply Newton's second law to the "downhill" direction:

$$mg\sin\theta - f = ma$$
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta.$$

Thus,

$$a = g(\sin\theta - \mu_k\cos\theta) = 7.5\ \text{m/s}^2.$$

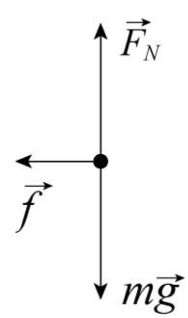
(b) The direction of the acceleration $\vec{a}$ is down the slope.

(c) Now the friction force is in the "downhill" direction (which is our positive direction) so that we obtain

$$a = g(\sin\theta + \mu_k\cos\theta) = 9.5\ \text{m/s}^2.$$

(d) The direction is down the slope.

74. The free-body diagram for the puck is shown on the right. $\vec{F}_N$ is the normal force of the ice on the puck, $\vec{f}$ is the force of friction (in the $-x$ direction), and $m\vec{g}$ is the force of gravity.



(a) The horizontal component of Newton's second law gives $-f = ma$, and constant acceleration kinematics (Table 2-1) can be used to find the acceleration.

Since the final velocity is zero, $v^2 = v_0^2 + 2ax$ leads to $a = -v_0^2/2x$. This is substituted into the Newton's law equation to obtain

$$f = \frac{mv_0^2}{2x} = \frac{(0.110 \text{ kg})(6.0 \text{ m/s})^2}{2(15 \text{ m})} = 0.13 \text{ N}.$$

(b) The vertical component of Newton's second law gives $F_N - mg = 0$, so $F_N = mg$ which implies (using Eq. 6-2) $f = \mu_k mg$. We solve for the coefficient:

$$\mu_k = \frac{f}{mg} = \frac{0.13 \text{ N}}{(0.110 \text{ kg})(9.8 \text{ m/s}^2)} = 0.12 .$$

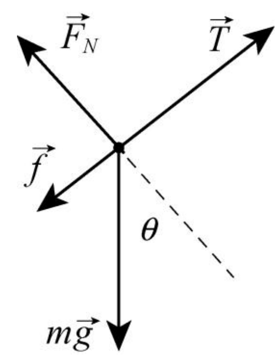
75. We may treat all 25 cars as a single object of mass $m = 25 \times 5.0 \times 10^4$ kg and (when the speed is 30 km/h = 8.3 m/s) subject to a friction force equal to

$$f = 25 \times 250 \times 8.3 = 5.2 \times 10^4 \text{ N}.$$

(a) Along the level track, this object experiences a "forward" force T exerted by the locomotive, so that Newton's second law leads to

$$T - f = ma \quad \Rightarrow \quad T = 5.2\times10^4 + (1.25\times10^6)(0.20) = 3.0\times10^5 \text{ N}.$$

(b) The free-body diagram is shown next, with θ as the angle of the incline. The $+x$ direction (which is the only direction to which we will be applying Newton's second law) is uphill (to the upper right in our sketch). Thus, we obtain



$$T - f - mg\sin\theta = ma$$

where we set $a = 0$ (implied by the problem statement) and solve for the angle. We obtain $\theta = 1.2°$.

76. An excellent discussion and equation development related to this problem is given in Sample Problem – "Friction, applied force at an angle." Using the result, we obtain

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.50 = 27°$$

which implies that the angle through which the slope should be *reduced* is

$$\phi = 45° - 27° \approx 20°.$$

77. We make use of Eq. 6-16 which yields

$$\sqrt{\frac{2mg}{C\rho\pi R^2}} = \sqrt{\frac{2(6)(9.8)}{(1.6)(1.2)\pi(0.03)^2}} = 147 \text{ m/s}.$$

78. (a) The coefficient of static friction is $\mu_s = \tan(\theta_{slip}) = 0.577 \approx 0.58$.

(b) Using

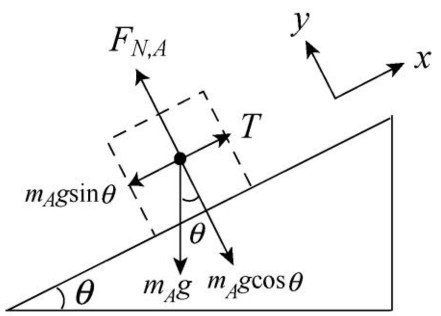
$$mg\sin\theta - f = ma$$

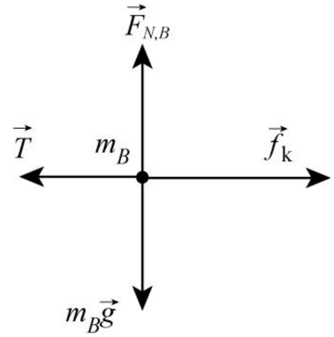
$$f = f_k = \mu_k F_N = \mu_k mg\cos\theta$$

and $a = 2d/t^2$ (with d = 2.5 m and t = 4.0 s), we obtain μ_k = 0.54.

79. **THINK** We have two blocks connected by a cord, as shown in Fig. 6-56. As block A slides down the frictionless inclined plane, it pulls block B, so there's a tension in the cord.

EXPRESS The free-body diagrams for blocks A and B are shown below:





Newton's law gives

$$m_A g\sin\theta - T = m_A a$$

for block A (where θ = 30º). For block B, we have

$$T - f_k = m_B a$$

Now the frictional force is given by $f_k = \mu_k F_{N,B} = \mu_k m_B g$. The equations allow us to solve for the tension T and the acceleration a.

ANALYZE (a) Combining the above equations to solve for T, we obtain

$$T = \frac{m_A m_B}{m_A + m_B}(\sin\theta + \mu_k)g = \frac{(4.0\text{ kg})(2.0\text{ kg})}{4.0\text{ kg} + 2.0\text{ kg}}(\sin 30° + 0.50)(9.80\text{ m/s}^2) = 13\text{ N}.$$

(b) Similarly, the acceleration of the two-block system is

$$a = \left(\frac{m_A\sin\theta - \mu_k m_B}{m_A + m_B}\right)g = \frac{(4.0\text{ kg})\sin 30° - (0.50)(2.0\text{ kg})}{4.0\text{ kg} + 2.0\text{ kg}}(9.80\text{ m/s}^2) = 1.6\text{ m/s}^2.$$

LEARN In the case where $\theta = 90°$ and $\mu_k = 0$, we have

$$T=\frac{m_A m_B}{m_A+m_B}g, \quad a=\frac{m_A}{m_A+m_B}g$$

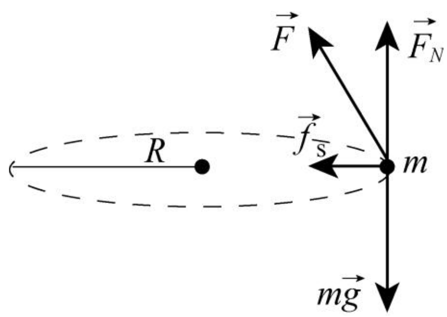
which correspond to the Sample Problem – "Block on table, block hanging," discussed in Chapter 5.

80. We use Eq. 6-14, $D=\frac{1}{2}C\rho Av^2$, where ρ is the air density, A is the cross-sectional area of the missile, v is the speed of the missile, and C is the drag coefficient. The area is given by $A = \pi R^2$, where R = 0.265 m is the radius of the missile. Thus

$$D=\frac{1}{2}(0.75)\left(1.2\ \text{kg}/\text{m}^3\right)\pi\left(0.265\ \text{m}\right)^2\left(250\ \text{m}/\text{s}\right)^2=6.2\times10^3\ \text{N}.$$

81. **THINK** How can a cyclist move in a circle? It is the force of friction that provides the centripetal force required for the circular motion.

EXPRESS The free-body diagram is shown below. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve.



The horizontal component of Newton's second law is $f_s = mv^2/R$, where f_s is the static friction exerted horizontally by the ground on the tires. Similarly, if F_N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $F_N = mg = 833\ \text{N}$.

ANALYZE (a) The frictional force is $f_s=\dfrac{mv^2}{R}=\dfrac{\left(85.0\ \text{kg}\right)\left(9.00\ \text{m/s}\right)^2}{25.0\ \text{m}}=275\ \text{N}.$

(b) Since the frictional force $\vec{f}_s$ and $\vec{F}_N$, the normal force exerted by the road, are perpendicular to each other, the magnitude of the force exerted by the ground on the bicycle is

$$F=\sqrt{f_s^2+F_N^2}=\sqrt{(275\ \text{N})^2+(833\ \text{N})^2}=877\ \text{N}.$$

LEARN The force exerted by the ground on the bicycle is at an angle $\theta = \tan^{-1}(275\text{ N}/833\text{ N}) = 18.3°$ with respect to the *vertical* axis.

82. At the top of the hill the vertical forces on the car are the upward normal force exerted by the ground and the downward pull of gravity. Designating +*y* downward, we have

$$mg - F_N = \frac{mv^2}{R}$$

from Newton's second law. To find the greatest speed without leaving the hill, we set $F_N = 0$ and solve for *v*:

$$v = \sqrt{gR} = \sqrt{(9.8\text{ m/s}^2)(250\text{ m})} = 49.5\text{ m/s} = 49.5(3600/1000)\text{ km/h} = 178\text{ km/h}.$$

83. (a) The push (to get it moving) must be at least as big as $f_{s,\max} = \mu_s F_N$ (Eq. 6-1, with $F_N = mg$ in this case), which equals (0.51)(165 N) = 84.2 N.

(b) While in motion, constant velocity (zero acceleration) is maintained if the push is equal to the kinetic friction force $f_k = \mu_k F_N = \mu_k mg = 52.8$ N.

(c) We note that the mass of the crate is 165/9.8 = 16.8 kg. The acceleration, using the push from part (a), is

$$a = (84.2\text{ N} - 52.8\text{ N})/(16.8\text{ kg}) \approx 1.87\text{ m/s}^2.$$

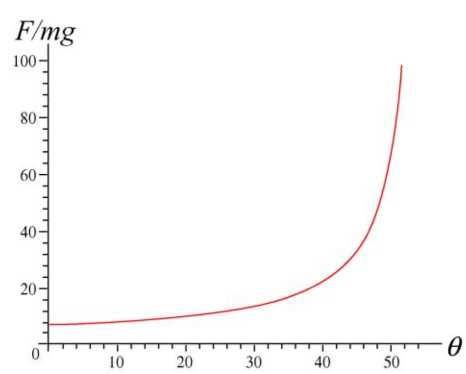
84. (a) The *x* component of $\vec{F}$ tries to move the crate while its *y* component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Newton's second law implies

$$x\text{ direction: } F\cos\theta - f_s = 0$$

$$y\text{ direction: } F_N - F\sin\theta - mg = 0.$$

To be "on the verge of sliding" means $f_s = f_{s,\max} = \mu_s F_N$ (Eq. 6-1). Solving these equations for *F* (actually, for the ratio of *F* to *mg*) yields

$$\frac{F}{mg} = \frac{\mu_s}{\cos\theta - \mu_s \sin\theta}\ .$$



This is plotted on the right (θ in degrees).

(b) The denominator of our expression (for *F/mg*) vanishes when

$$\cos\theta - \mu_s \sin\theta = 0 \quad \Rightarrow \quad \theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right)$$

For $\mu_s = 0.70$, we obtain $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 55°$.

(c) Reducing the coefficient means increasing the angle by the condition in part (b).

(d) For $\mu_s = 0.60$ we have $\theta_{\text{inf}} = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 59°$.

85. The car is in "danger of sliding" down when

$$\mu_s = \tan\theta = \tan 35.0° = 0.700.$$

This value represents a 3.4% decrease from the given 0.725 value.

86. (a) The tension will be the greatest at the lowest point of the swing. Note that there is no substantive difference between the tension T in this problem and the normal force F_N in Sample Problem – "Vertical circular loop, Diavolo." Eq. 6-19 of that Sample Problem examines the situation at the top of the circular path (where F_N is the least), and rewriting that for the bottom of the path leads to

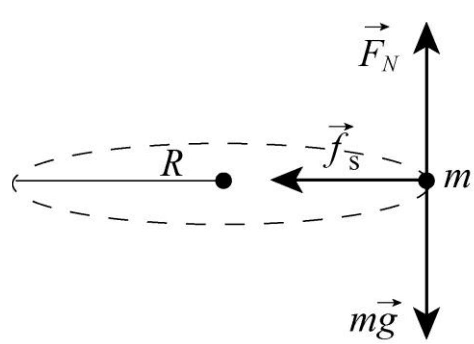
$$T = mg + mv^2/r$$

where F_N is at its greatest value.

(b) At the breaking point T = 33 N = $m(g + v^2/r)$ where m = 0.26 kg and r = 0.65 m. Solving for the speed, we find that the cord should break when the speed (at the lowest point) reaches 8.73 m/s.

87. **THINK** A car is making a turn on an unbanked curve. Friction is what provides the centripetal force needed for this circular motion.

EXPRESS The free-body diagram is shown to the right. The mass of the car is m = (10700/9.80) kg = 1.09×10^3 kg. We choose "inward" (horizontally toward the center of the circular path) as the positive direction. The normal force is $F_N = mg$ in this situation, and the required frictional force is $f_s = mv^2/R$.



ANALYZE (a) With a speed of v = 13.4 m/s and a radius R = 61 m, Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = \frac{(1.09\times10^3\ \text{kg})(13.4\ \text{m/s})^2}{61.0\ \text{m}} = 3.21\times10^3\ \text{N}\ .$$

(b) The maximum possible static friction is found to be

$$f_{s,\max} = \mu_s mg = (0.35)(10700\ \text{N}) = 3.75\times10^3\ \text{N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

LEARN From the above expressions, we see that with a coefficient of friction μ_s, the maximum speed of the car negotiating a curve of radius R is $v_{\max} = \sqrt{\mu_s gR}$. So in this case, the car can go up to a maximum speed of

$$v_{\max} = \sqrt{(0.35)(9.8\ \text{m/s}^2)(61\ \text{m})} = 14.5\ \text{m/s}$$

without skidding.

88. For the m_2 = 1.0 kg block, application of Newton's laws result in

$$\begin{aligned} F\cos\theta - T - f_k &= m_2 a \qquad x \text{ axis} \\ F_N - F\sin\theta - m_2 g &= 0 \qquad y \text{ axis} \end{aligned}$$

Since $f_k = \mu_k F_N$, these equations can be combined into an equation to solve for a:

$$F(\cos\theta - \mu_k \sin\theta) - T - \mu_k m_2 g = m_2 a$$

Similarly (but without the applied push) we analyze the m_1= 2.0 kg block:

$$\begin{aligned} T - f_k' &= m_1 a \qquad x \text{ axis} \\ F_N' - m_1 g &= 0 \qquad y \text{ axis} \end{aligned}$$

Using $f_k = \mu_k\ F_N'$, the equations can be combined:

$$T - \mu_k m_1 g = m_1 a$$

Subtracting the two equations for a and solving for the tension, we obtain

$$T = \frac{m_1(\cos\theta - \mu_k \sin\theta)}{m_1 + m_2} F = \frac{(2.0\ \text{kg})[\cos 35° - (0.20)\sin 35°]}{2.0\ \text{kg} + 1.0\ \text{kg}}(20\ \text{N}) = 9.4\ \text{N}.$$

89. **THINK** In order to move a filing cabinet, the force applied must be able to overcome the frictional force.

EXPRESS We apply Newton's second law (as $F_{\text{push}} - f = ma$). If we find the applied force F_{push} to be less than $f_{s,\max}$, the maximum static frictional force, our conclusion would then be "no, the cabinet does not move" (which means a is actually 0 and the frictional force is simply $f = F_{\text{push}}$). On the other hand, if we obtain $a > 0$ then the cabinet moves (so $f = f_k$). For $f_{s,\max}$ and f_k we use Eq. 6-1 and Eq. 6-2 (respectively), and in those formulas we set the magnitude of the normal force to the weight of the cabinet: $F_N = mg = 556\text{ N}$. Thus, the maximum static frictional force is

$$f_{s,\max} = \mu_s F_N = \mu_s mg = (0.68)(556\text{ N}) = 378\text{ N}.$$

and the kinetic frictional force is

$$f_k = \mu_k F_N = \mu_k mg = (0.56)(556\text{ N}) = 311\text{ N}.$$

ANALYZE (a) Here we find $F_{\text{push}} < f_{s,\max}$ which leads to $f = F_{\text{push}} = 222$ N. The cabinet does not move.

(b) Again we find $F_{\text{push}} < f_{s,\max}$ which leads to $f = F_{\text{push}} = 334$ N. The cabinet does not move.

(c) Now we have $F_{\text{push}} > f_{s,\max}$ which means the cabinet moves and $f = f_k = 311$ N.

(d) Again we have $F_{\text{push}} > f_{s,\max}$ which means the cabinet moves and $f = f_k = 311$ N.

(e) The cabinet moves in (c) and (d).

LEARN In summary, in order to make the cabinet move, the minimum applied force is equal to the maximum static frictional force $f_{s,\max}$.

90. Analysis of forces in the horizontal direction (where there can be no acceleration) leads to the conclusion that $F = F_N$; the magnitude of the normal force is 60 N. The maximum possible static friction force is therefore $\mu_s F_N = 33$ N, and the kinetic friction force (when applicable) is $\mu_k F_N = 23$ N.

(a) In this case, $\vec{P} = 34$ N upward. Assuming $\vec{f}$ points down, then Newton's second law for the y leads to

$$P - mg - f = ma .$$

if we assume $f = f_s$ and $a = 0$, we obtain $f = (34 - 22)$ N $= 12$ N. This is less than $f_{s,\max}$, which shows the consistency of our assumption. The answer is: $\vec{f_s} = 12$ N down.

(b) In this case, $\vec{P}$ = 12 N upward. The above equation, with the same assumptions as in part (a), leads to f = (12 – 22) N = –10 N. Thus, $|f_s| < f_{s,\,\max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f}_s$ = 10 N up.

(c) In this case, $\vec{P}$ = 48 N upward. The above equation, with the same assumptions as in part (a), leads to f = (48 – 22) N = 26 N. Thus, we again have $f_s < f_{s,\,\max}$, and our answer is: $\vec{f}_s$ = 26 N down.

(d) In this case, $\vec{P}$ = 62 N upward. The above equation, with the same assumptions as in part (a), leads to f = (62 – 22) N = 40 N, which is larger than $f_{s,\,\max}$, -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be positive, as we should expect. The answer is: $\vec{f}_k$ = 23 N down.

(e) In this case, $\vec{P}$ = 10 N downward. The above equation (but with P replaced with -P) with the same assumptions as in part (a), leads to f = (–10 – 22) N = –32 N. Thus, we have $|f_s| < f_{s,\,\max}$, justifying our assumption that the block is stationary, but its negative value tells us that our initial assumption about the direction of $\vec{f}$ is incorrect in this case. Thus, the answer is: $\vec{f}_s$ = 32 N up.

(f) In this case, $\vec{P}$ = 18 N downward. The above equation (but with P replaced with –P) with the same assumptions as in part (a), leads to f = (–18 – 22) N = –40 N, which is larger (in absolute value) than $f_{s,\,\max}$, -- invalidating our assumptions. Therefore, we take $f = f_k$ and $a \neq 0$ in the above equation; if we wished to find the value of a we would find it to be negative, as we should expect. The answer is: $\vec{f}_k$ = 23 N up.

(g) The block moves up the wall in case (d) where $a > 0$.

(h) The block moves down the wall in case (f) where $a < 0$.

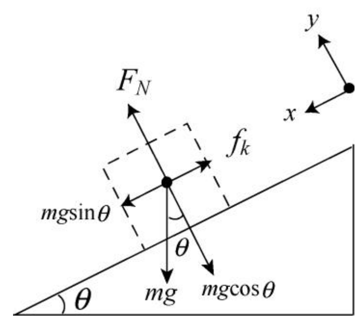
(i) The frictional force $\vec{f}_s$ is directed down in cases (a), (c) and (d).

91. **THINK** Whether the block is sliding down or up the incline, there is a frictional force in the opposite direction of the motion.

EXPRESS The free-body diagram for the first part of this problem (when the block is sliding downhill with zero acceleration) is shown next.

Newton's second law gives

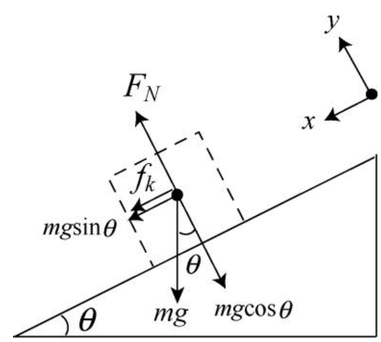
$$mg\sin\theta - f_k = mg\sin\theta - \mu_k F_N = ma_x = 0$$
$$mg\cos\theta - F_N = ma_y = 0$$



The two equations can be combined to give

$$\mu_k = \tan\theta.$$

Now (for the second part of the problem, with the block projected uphill) the friction direction is reversed (see figure to the right). Newton's second law for the uphill motion (and Eq. 6-12) leads to



$$mg\sin\theta + f_k = mg\sin\theta + \mu_k F_N = ma_x$$
$$mg\cos\theta - F_N = ma_y = 0$$

Note that by our convention, $a_x > 0$ means that the acceleration is downhill, and therefore, the speed of the block will decrease as it moves up the incline.

ANALYZE (a) Using $\mu_k = \tan\theta$ and $F_N = mg\cos\theta$, we find the *x*-component of the acceleration to be

$$a_x = g\sin\theta + \frac{\mu_k F_N}{m} = g\sin\theta + \frac{(\tan\theta)(mg\cos\theta)}{m} = 2g\sin\theta.$$

The distance the block travels before coming to a stop can be found by using Eq. 2-16: $v_f^2 = v_0^2 - 2a_x\Delta x$, which yields

$$\Delta x = \frac{v_0^2}{2a_x} = \frac{v_0^2}{2(2g\sin\theta)} = \frac{v_0^2}{4g\sin\theta}.$$

(b) We usually expect $\mu_s > \mu_k$ (see the discussion in Section 6-1). The "angle of repose" (the minimum angle necessary for a stationary block to start sliding downhill) is $\mu_s = \tan(\theta_{\text{repose}})$. Therefore, we expect $\theta_{\text{repose}} > \theta$ found in part (a). Consequently, when the block comes to rest, the incline is not steep enough to cause it to start slipping down the incline again.

LEARN An alternative way to see that the block will not slide down again is to note that the downward force of gravitation is not large enough to overcome the force of friction, i.e., $mg\sin\theta = f_k < f_{s,\max}$.

92. Consider that the car is "on the verge of sliding out" – meaning that the force of static friction is acting "down the bank" (or "downhill" from the point of view of an ant on the banked curve) with maximum possible magnitude. We first consider the vector sum $\vec{F}$ of the (maximum) static friction force and the normal force. Due to the facts that they are perpendicular and their magnitudes are simply proportional (Eq. 6-1), we find $\vec{F}$ is at angle (measured from the <u>vertical</u> axis) $\phi = \theta + \theta_s$ where $\tan\theta_s = \mu_s$ (compare with Eq. 6-13), and θ is the bank angle. Now, the vector sum of $\vec{F}$ and the vertically downward pull (mg) of gravity must be equal to the (horizontal) centripetal force (mv^2/R), which leads to a surprisingly simple relationship:

$$\tan\phi = \frac{mv^2/R}{mg} = \frac{v^2}{Rg} .$$

Writing this as an expression for the maximum speed, we have

$$v_{\max} = \sqrt{Rg\tan(\theta + \tan^{-1}\mu_s)} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}} .$$

(a) We note that the given speed is (in SI units) roughly 17 m/s. If we do not want the cars to "depend" on the static friction to keep from sliding out (that is, if we want the component "down the back" of gravity to be sufficient), then we can set $\mu_s = 0$ in the above expression and obtain $v = \sqrt{Rg\tan\theta}$. With $R = 150$ m, this leads to $\theta = 11°$.

(b) If, however, the curve is not banked (so $\theta = 0$) then the above expression becomes

$$v = \sqrt{Rg\tan(\tan^{-1}\mu_s)} = \sqrt{Rg\mu_s}$$

Solving this for the coefficient of static friction $\mu_s = 0.19$.

93. (a) The box doesn't move until $t = 2.8$ s, which is when the applied force $\vec{F}$ reaches a magnitude of $F = (1.8)(2.8) = 5.0$ N, implying therefore that $f_{s,\max} = 5.0$ N. Analysis of the vertical forces on the block leads to the observation that the normal force magnitude equals the weight $F_N = mg = 15$ N. Thus,

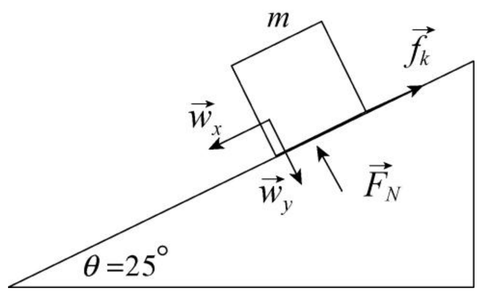
$$\mu_s = f_{s,\max}/F_N = 0.34.$$

(b) We apply Newton's second law to the horizontal x axis (positive in the direction of motion):

$$F - f_k = ma \;\Rightarrow\; 1.8t - f_k = (1.5)(1.2t - 2.4)$$

Thus, we find $f_k = 3.6$ N. Therefore, $\mu_k = f_k / F_N = 0.24$.

94. In the figure below, m = 140/9.8 = 14.3 kg is the mass of the child. We use $\vec{w}_x$ and $\vec{w}_y$ as the components of the gravitational pull of Earth on the block; their magnitudes are $w_x = mg \sin \theta$ and $w_y = mg \cos \theta$.



(a) With the x axis directed up along the incline (so that a = –0.86 m/s^2), Newton's second law leads to

$$f_k - 140 \sin 25° = m(-0.86)$$

which yields f_k = 47 N. We also apply Newton's second law to the y axis (perpendicular to the incline surface), where the acceleration-component is zero:

$$F_N - 140 \cos 25° = 0 \quad \Rightarrow \quad F_N = 127 \text{ N}.$$

Therefore, $\mu_k = f_k/F_N = 0.37$.

(b) Returning to our first equation in part (a), we see that if the downhill component of the weight force were insufficient to overcome static friction, the child would not slide at all. Therefore, we require 140 sin 25° > $f_{s,\max} = \mu_s F_N$, which leads to tan 25° = 0.47 > μ_s. The minimum value of μ_s equals μ_k and is more subtle; reference to §6-1 is recommended. If μ_k exceeded μ_s then when static friction were overcome (as the incline is raised) then it should start to move – which is impossible if f_k is large enough to cause deceleration! The bounds on μ_s are therefore given by 0.47 > μ_s > 0.37.

95. (a) The x component of $\vec{F}$ contributes to the motion of the crate while its y component indirectly contributes to the inhibiting effects of friction (by increasing the normal force). Along the y direction, we have $F_N - F\cos\theta - mg = 0$ and along the x direction we have $F\sin\theta - f_k = 0$ (since it is not accelerating, according to the problem). Also, Eq. 6-2 gives $f_k = \mu_k F_N$. Solving these equations for F yields

$$F = \frac{\mu_k mg}{\sin\theta - \mu_k \cos\theta} \ .$$

(b) When $\theta < \theta_0 = \tan^{-1} \mu_s$, F will not be able to move the mop head.

96. (a) The distance traveled in one revolution is $2\pi R = 2\pi(4.6 \text{ m}) = 29$ m. The (constant) speed is consequently $v = (29 \text{ m})/(30 \text{ s}) = 0.96$ m/s.

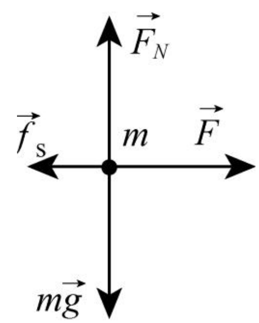
(b) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m(0.20)$$

in SI units. Noting that $F_N = mg$ in this situation, the maximum possible static friction is $f_{s,\text{max}} = \mu_s mg$ using Eq. 6-1. Equating this with $f_s = m(0.20)$ we find the mass m cancels and we obtain $\mu_s = 0.20/9.8 = 0.021$.

97. **THINK** In this problem a force is applied to accelerate a box. From the distance traveled and the speed at that instant, we can calculate the coefficient of kinetic friction between the box and the floor.

EXPRESS The free-body diagram is shown to the right. We adopt the familiar axes with +*x* rightward and +*y* upward, and refer to the 85 N horizontal push of the worker as F (and assume it to be rightward). Applying Newton's second law to the *x* axis and *y* axis, respectively, produces



$$F - f_k = ma_x, \qquad F_N - mg = 0.$$

On the other hand, using Eq. 2-16 ($v^2 = v_0^2 + 2a_x\Delta x$), we find the acceleration to be

$$a_x = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(1.0 \text{ m/s})^2 - 0}{2(1.4 \text{ m})} = 0.357 \text{ m/s}^2.$$

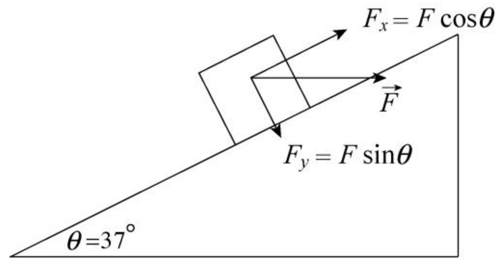
The above equations can be combined to give μ_k.

ANALYZE Using $f_k = \mu_k F_N$, we find the coefficient of kinetic friction between the box and the floor to be

$$\mu_k = \frac{f_k}{F_N} = \frac{F - ma_x}{mg} = \frac{85 \text{ N} - (40 \text{ kg})(0.357 \text{ m/s}^2)}{(40 \text{ kg})(9.8 \text{ m/s}^2)} = 0.18.$$

LEARN In general, the acceleration can be written as $a_x = (F/m) - \mu_k g$. We see that the smaller the value of μ_k, the greater the acceleration. In the limit $\mu_k = 0$, we simply have $a_x = F/m$.

98. We resolve this horizontal force into appropriate components.



(a) Applying Newton's second law to the x (directed uphill) and y (directed away from the incline surface) axes, we obtain

$$F\cos\theta - f_k - mg\sin\theta = ma$$
$$F_N - F\sin\theta - mg\cos\theta = 0.$$

Using $f_k = \mu_k F_N$, these equations lead to

$$a = \frac{F}{m}(\cos\theta - \mu_k \sin\theta) - g(\sin\theta + \mu_k \cos\theta)$$

which yields $a = -2.1\ \text{m/s}^2$, or $|a| = 2.1\ \text{m/s}^2$, for $\mu_k = 0.30$, $F = 50$ N and $m = 5.0$ kg.

(b) The direction of $\vec{a}$ is down the plane.

(c) With $v_0 = +4.0$ m/s and $v = 0$, Eq. 2-16 gives $\Delta x = -\dfrac{(4.0\ \text{m/s})^2}{2(-2.1\ \text{m/s}^2)} = 3.9\ \text{m}.$

(d) We expect $\mu_s \geq \mu_k$; otherwise, an object started into motion would immediately start decelerating (before it gained any speed)! In the minimal expectation case, where $\mu_s = 0.30$, the maximum possible (downhill) static friction is, using Eq. 6-1,

$$f_{s,\max} = \mu_s F_N = \mu_s (F\sin\theta + mg\cos\theta)$$

which turns out to be 21 N. But in order to have no acceleration along the x axis, we must have

$$f_s = F\cos\theta - mg\sin\theta = 10\ \text{N}$$

(the fact that this is positive reinforces our suspicion that $\vec{f}_s$ points downhill). Since the f_s needed to remain at rest is less than $f_{s,\max}$ then it stays at that location.

99. (a) We note that $F_N = mg$ in this situation, so

$$f_{s,\max} = \mu_s mg = (0.52)(11\ \text{kg})(9.8\ \text{m/s}^2) = 56\ \text{N}.$$

Consequently, the horizontal force $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than 56 N.

(b) Analyzing vertical forces when $\vec{F}$ is at nonzero θ yields

$$F\sin\theta + F_N = mg \Rightarrow f_{s,\max} = \mu_s (mg - F\sin\theta).$$

Now, the horizontal component of $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than this, so

$$F\cos\theta = \mu_s(mg - F\sin\,\theta) \;\Rightarrow\; F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\,\theta}$$

which yields F = 59 N when θ = 60°.

(c) We now set θ = –60° and obtain

$$F = \frac{(0.52)(11\,\text{kg})(9.8\,\text{m/s}^2)}{\cos(-60°) + (0.52)\,\sin\,(-60°)} = 1.1\times10^3\ \text{N}.$$

100. (a) If the skier covers a distance L during time t with zero initial speed and a constant acceleration a, then $L = at^2/2$, which gives the acceleration a_1 for the first (old) pair of skis:

$$a_1 = \frac{2L}{t_1^2} = \frac{2(200\,\text{m})}{(61\ \text{s})^2} = 0.11\ \text{m/s}^2.$$

(b) The acceleration a_2 for the second (new) pair is

$$a_2 = \frac{2L}{t_2^2} = \frac{2(200\,\text{m})}{(42\,\text{s})^2} = 0.23\ \text{m/s}^2\ .$$

(c) The net force along the slope acting on the skier of mass m is

$$F_{\text{net}} = mg\sin\theta - f_k = mg(\sin\theta - \mu_k\cos\theta) = ma$$

which we solve for μ_{k1} for the first pair of skis:

$$\mu_{k1} = \tan\theta - \frac{a_1}{g\cos\theta} = \tan 3.0° - \frac{0.11\,\text{m/s}^2}{(9.8\,\text{m/s}^2)\cos 3.0°} = 0.041$$

(d) For the second pair, we have

$$\mu_{k2} = \tan\theta - \frac{a_2}{g\cos\theta} = \tan 3.0° - \frac{0.23\,\text{m/s}^2}{(9.8\,\text{m/s}^2)\cos 3.0°} = 0.029\ .$$

101. If we choose "downhill" positive, then Newton's law gives

$$m\,g\sin\theta - f_k = m\,a$$

for the sliding child. Now using Eq. 6-12

$$f_k = \mu_k F_N = \mu_k m g,$$

so we obtain $a = g(\sin\theta - \mu_k \cos\theta) = -0.5$ m/s^2 (note that the problem gives the direction of the acceleration vector as uphill, even though the child is sliding downhill, so it is a deceleration). With $\theta = 35°$, we solve for the coefficient and find $\mu_k = 0.76$.

102. (a) Our $+x$ direction is horizontal and is chosen (as we also do with $+y$) so that the components of the 100 N force $\vec{F}$ are non-negative. Thus, $F_x = F \cos\theta = 100$ N, which the textbook denotes F_h in this problem.

(b) Since there is no vertical acceleration, application of Newton's second law in the y direction gives

$$F_N + F_y = mg \Rightarrow F_N = mg - F\sin\theta$$

where $m = 25.0$ kg. This yields $F_N = 245$ N in this case ($\theta = 0°$).

(c) Now, $F_x = F_h = F\cos\theta = 86.6$ N for $\theta = 30.0°$.

(d) And $F_N = mg - F\sin\theta = 195$ N.

(e) We find $F_x = F_h = F\cos\theta = 50.0$ N for $\theta = 60.0°$.

(f) And $F_N = mg - F\sin\theta = 158$ N.

(g) The condition for the chair to slide is

$$F_x > f_{s,\max} = \mu_s F_N \quad \text{where} \quad \mu_s = 0.42.$$

For $\theta = 0°$, we have

$$F_x = 100 \text{ N} < f_{s,\max} = (0.42)(245 \text{ N}) = 103 \text{ N}$$

so the crate remains at rest.

(h) For $\theta = 30.0°$, we find $F_x = 86.6 \text{ N} > f_{s,\max} = (0.42)(195 \text{ N}) = 81.9 \text{ N}$, so the crate slides.

(i) For $\theta = 60°$, we get $F_x = 50.0 \text{ N} < f_{s,\max} = (0.42)(158 \text{ N}) = 66.4 \text{ N}$, which means the crate must remain at rest.

103. (a) The intuitive conclusion, that the tension is greatest at the bottom of the swing, is certainly supported by application of Newton's second law there:

$$T - mg = \frac{mv^2}{R} \Rightarrow T = m\left(g + \frac{v^2}{R}\right)$$

where Eq. 6-18 has been used. Increasing the speed eventually leads to the tension at the bottom of the circle reaching that breaking value of 40 N.

(b) Solving the above equation for the speed, we find

$$v = \sqrt{R\left(\frac{T}{m} - g\right)} = \sqrt{(0.91\ \text{m})\left(\frac{40\ \text{N}}{0.37\ \text{kg}} - 9.8\ \text{m/s}^2\right)}$$

which yields v = 9.5 m/s.

104. (a) The component of the weight along the incline (with downhill understood as the positive direction) is $mg\sin\theta$ where m = 630 kg and θ = 10.2°. With f = 62.0 N, Newton's second law leads to $mg\sin\theta - f = ma$, which yields a = 1.64 m/s^2. Using Eq. 2-15, we have

$$80.0\ \text{m} = \left(6.20\ \frac{\text{m}}{\text{s}}\right)t + \frac{1}{2}\left(1.64\ \frac{\text{m}}{\text{s}^2}\right)t^2 \ .$$

This is solved using the quadratic formula. The positive root is t = 6.80 s.

(b) Running through the calculation of part (a) with f = 42.0 N instead of f = 62 N results in t = 6.76 s.

105. Except for replacing f_s with f_k, Fig 6-5 in the textbook is appropriate. With that figure in mind, we choose uphill as the +x direction. Applying Newton's second law to the x axis, we have

$$f_k - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g},$$

and where W = 40 N, a = +0.80 m/s^2 and θ = 25°. Thus, we find f_k = 20 N. Along the y-axis, we have

$$\sum \vec{F}_y = 0 \Rightarrow F_N = W\cos\theta$$

so that $\mu_k = f_k / F_N = 0.56$.

Chapter 7

1. **THINK** As the proton is being accelerated, its speed increases, and so does its kinetic energy.

EXPRESS To calculate the speed of the proton at a later time, we use the equation $v^2 = v_0^2 + 2a\Delta x$ from Table 2-1. The change in kinetic energy is then equal to

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) .$$

ANALYZE (a) With $\Delta x = 3.5 \text{ cm} = 0.035 \text{ m}$ and $a = 3.6\times10^{15} \text{ m/s}^2$, we find the proton speed to be

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{\left(2.4\times10^7 \text{ m/s}\right)^2 + 2\left(3.6\times10^{15} \text{ m/s}^2\right)\left(0.035 \text{ m}\right)} = 2.9\times10^7 \text{ m/s}.$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2}mv_0^2 = \frac{1}{2}\left(1.67\times10^{-27} \text{ kg}\right)\left(2.4\times10^7 \text{ m/s}\right)^2 = 4.8\times10^{-13} \text{ J},$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67\times10^{-27} \text{ kg}\right)\left(2.9\times10^7 \text{ m/s}\right)^2 = 6.9\times10^{-13} \text{ J}.$$

Thus, the change in kinetic energy is

$$\Delta K = K_f - K_i = 6.9\times10^{-13} \text{ J} - 4.8\times10^{-13} \text{ J} = 2.1\times10^{-13} \text{ J}.$$

LEARN The change in kinetic energy can be rewritten as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x = F\Delta x = W$$

which, according to the work-kinetic energy theorem, is simply the work done on the particle.

2. With speed v = 11200 m/s, we find

$$K=\frac{1}{2}mv^2=\frac{1}{2}(2.9\times10^5\ \text{kg})\ (11200\ \text{m/s})^2=1.8\times10^{13}\ \text{J}.$$

3. (a) The change in kinetic energy for the meteorite would be

$$\Delta K=K_f-K_i=-K_i=-\frac{1}{2}m_iv_i^2=-\frac{1}{2}\left(4\times10^6\ \text{kg}\right)\left(15\times10^3\ \text{m/s}\right)^2=-5\times10^{14}\ \text{J},$$

or $|\Delta K|=5\times10^{14}$ J. The negative sign indicates that kinetic energy is lost.

(b) The energy loss in units of megatons of TNT would be

$$-\Delta K=\left(5\times10^{14}\ \text{J}\right)\left(\frac{1\ \text{megaton TNT}}{4.2\times10^{15}\ \text{J}}\right)=0.1\text{megaton TNT}.$$

(c) The number of bombs N that the meteorite impact would correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio:

$$N=\frac{0.1\times1000\,\text{kiloton TNT}}{13\,\text{kiloton TNT}}=8.$$

4. (a) We set up the ratio

$$\frac{50\ \text{km}}{1\ \text{km}}=\left(\frac{E}{1\ \text{megaton}}\right)^{1/3}$$

and find $E = 50^3 \approx 1\times10^5$ megatons of TNT.

(b) We note that 15 kilotons is equivalent to 0.015 megatons. Dividing the result from part (a) by 0.013 yields about ten million (10^7) bombs.

5. We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i=\frac{1}{2}K_{\text{son}}$$

and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is $K_f=K_{\text{son}}$. We use these relations along with Eq. 7-1 in our solution.

(a) We see from the above that $K_i=\frac{1}{2}K_f$, which (with SI units understood) leads to

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left[\frac{1}{2}m\left(v_i + 1.0\text{ m/s}\right)^2\right].$$

The mass cancels and we find a second-degree equation for v_i : $\frac{1}{2}v_i^2 - v_i - \frac{1}{2} = 0$. The positive root (from the quadratic formula) yields v_i = 2.4 m/s.

(b) From the first relation above $\left(K_i = \frac{1}{2}K_{\text{son}}\right)$, we have

$$\frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{1}{2}(m/2)\,v_{\text{son}}^2\right)$$

and (after canceling m and one factor of 1/2) are led to $v_{\text{son}} = 2v_i = 4.8\text{ m/s}$.

6. We apply the equation $x(t) = x_0 + v_0 t + \frac{1}{2}at^2$, found in Table 2-1. Since at t = 0 s, x_0 = 0, and $v_0 = 12\text{ m/s}$, the equation becomes (in unit of meters)

$$x(t) = 12t + \tfrac{1}{2}at^2.$$

With $x = 10\text{ m}$ when $t = 1.0\text{ s}$, the acceleration is found to be $a = -4.0\text{ m/s}^2$. The fact that $a < 0$ implies that the bead is decelerating. Thus, the position is described by $x(t) = 12t - 2.0t^2$. Differentiating x with respect to t then yields

$$v(t) = \frac{dx}{dt} = 12 - 4.0t.$$

Indeed at t =3.0 s, $v(t = 3.0) = 0$ and the bead stops momentarily. The speed at $t = 10\text{ s}$ is $v(t = 10) = -28\text{ m/s}$, and the corresponding kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.8\times10^{-2}\text{kg})(-28\text{ m/s})^2 = 7.1\text{ J}.$$

7. Since this involves constant-acceleration motion, we can apply the equations of Table 2-1, such as $x = v_0 t + \frac{1}{2}at^2$ (where $x_0 = 0$). We choose to analyze the third and fifth points, obtaining

$$0.2\text{ m} = v_0(1.0\text{ s}) + \frac{1}{2}a\,(1.0\text{ s})^2$$
$$0.8\text{ m} = v_0(2.0\text{ s}) + \frac{1}{2}a\,(2.0\text{ s})^2.$$

Simultaneous solution of the equations leads to $v_0 = 0$ and $a = 0.40\,\text{m/s}^2$. We now have two ways to finish the problem. One is to compute force from $F = ma$ and then obtain the work from Eq. 7-7. The other is to find ΔK as a way of computing W (in accordance with Eq. 7-10). In this latter approach, we find the velocity at $t = 2.0\,\text{s}$ from $v = v_0 + at\,(\text{so } v = 0.80\,\text{m/s})$. Thus,

$$W = \Delta K = \frac{1}{2}(3.0\,\text{kg})(0.80\,\text{m/s})^2 = 0.96\ \text{J}.$$

8. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$W = \vec{F}\cdot\vec{d} = \left[(210\ \text{N})\,\hat{\text{i}} - (150\ \text{N})\,\hat{\text{j}}\right]\cdot\left[(15\ \text{m})\,\hat{\text{i}} - (12\ \text{m})\,\hat{\text{j}}\right] = (210\ \text{N})(15\ \text{m}) + (-150\ \text{N})(-12\ \text{m})$$
$$= 5.0\times 10^3\ \text{J}.$$

9. By the work-kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\,\text{kg})\left((6.0\,\text{m/s})^2 - (4.0\,\text{m/s})^2\right) = 20\ \text{J}.$$

We note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

10. Equation 7-8 readily yields

$$W = F_x\,\Delta x + F_y\,\Delta y = (2.0\ \text{N})\cos(100^\circ)(3.0\ \text{m}) + (2.0\ \text{N})\sin(100^\circ)(4.0\ \text{m}) = 6.8\ \text{J}.$$

11. Using the work-kinetic energy theorem, we have

$$\Delta K = W = \vec{F}\cdot\vec{d} = Fd\cos\phi.$$

In addition, $F = 12\ \text{N}$ and $d = \sqrt{(2.00\ \text{m})^2 + (-4.00\ \text{m})^2 + (3.00\ \text{m})^2} = 5.39\ \text{m}$.

(a) If $\Delta K = +30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 62.3^\circ.$$

(b) $\Delta K = -30.0\ \text{J}$, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-30.0\ \text{J}}{(12.0\ \text{N})(5.39\ \text{m})}\right) = 118^\circ.$$

12. (a) From Eq. 7-6, $F = W/x = 3.00$ N (this is the slope of the graph).

(b) Equation 7-10 yields $K = K_i + W$ = 3.00 J + 6.00 J = 9.00 J.

13. We choose $+x$ as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\text{kg})(-2.0\,\text{m/s}^2)$ so that

$$F = |\vec{F}| = 1.7\times 10^2\ \text{N}.$$

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \;\Rightarrow\; \Delta x = -\frac{(37\,\text{m/s})^2}{2\left(-2.0\,\text{m/s}^2\right)} = 3.4\times 10^2\ \text{m}.$$

Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8\times 10^4$ J.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\text{kg})\left(-4.0\,\text{m/s}^2\right)$ so that $F = |\vec{F}| = 3.4\times 10^2$ N.

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\text{m/s})^2}{2\left(-4.0\,\text{m/s}^2\right)} = 1.7\times 10^2\ \text{m}.$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8\times 10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

14. The forces are all constant, so the total work done by them is given by $W = F_{\text{net}}\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the x and y components of the net force:

$$\begin{aligned} F_{\text{net}\,x} &= -F_1 - F_2\sin 50.0° + F_3\cos 35.0° = -3.00\,\text{N} - (4.00\,\text{N})\sin 35.0° + (10.0\,\text{N})\cos 35.0° \\ &= 2.13\,\text{N} \end{aligned}$$

$$\begin{aligned} F_{\text{net}\,y} &= -F_2\cos 50.0° + F_3\sin 35.0° = -(4.00\,\text{N})\cos 50.0° + (10.0\,\text{N})\sin 35.0° \\ &= 3.17\,\text{N}. \end{aligned}$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{(2.13\text{ N})^2 + (3.17\text{ N})^2} = 3.82\text{ N}.$$

The work done by the net force is

$$W = F_{\text{net}}d = (3.82\text{ N})(4.00\text{ m}) = 15.3\text{ J}$$

where we have used the fact that $\vec{d} \parallel \vec{F}_{\text{net}}$ (which follows from the fact that the canister started from rest and moved horizontally under the action of horizontal forces — the resultant effect of which is expressed by $\vec{F}_{\text{net}}$).

15. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F}\cdot\vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d\cos\phi_1 = (5.00\text{ N})(3.00\text{ m})\cos 0° = 15.0\text{ J}.$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d\cos\phi_2 = (9.00\text{ N})(3.00\text{ m})\cos 120° = -13.5\text{ J}.$$

Force $\vec{F}_3$ is perpendicular to the displacement, so

$$W_3 = F_3 d\cos\phi_3 = 0 \text{ since } \cos 90° = 0.$$

The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\text{ J} - 13.5\text{ J} + 0 = +1.50\text{ J}.$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.50 J during the displacement.

16. The change in kinetic energy can be written as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x$$

where we have used $v_f^2 = v_i^2 + 2a\Delta x$ from Table 2-1. From the figure, we see that $\Delta K = (0-30)\text{ J} = -30\text{ J}$ when $\Delta x = +5\text{ m}$. The acceleration can then be obtained as

$$a = \frac{\Delta K}{m\Delta x} = \frac{(-30\text{ J})}{(8.0\text{ kg})(5.0\text{ m})} = -0.75\text{ m/s}^2.$$

The negative sign indicates that the mass is decelerating. From the figure, we also see that when $x = 5$ m the kinetic energy becomes zero, implying that the mass comes to rest momentarily. Thus,

$$v_0^2 = v^2 - 2a\Delta x = 0 - 2(-0.75 \text{ m/s}^2)(5.0 \text{ m}) = 7.5 \text{ m}^2/\text{s}^2,$$

or $v_0 = 2.7$ m/s. The speed of the object when $x = -3.0$ m is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{7.5 \text{ m}^2/\text{s}^2 + 2(-0.75 \text{ m/s}^2)(-3.0 \text{ m})} = \sqrt{12} \text{ m/s} = 3.5 \text{ m/s}.$$

17. **THINK** The helicopter does work to lift the astronaut upward against gravity. The work done on the astronaut is converted to the kinetic energy of the astronaut.

EXPRESS We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $a = g/10$ upward. According to Newton's second law, the force is given by

$$F - mg = ma \quad \Rightarrow \quad F = m(g + a) = \frac{11}{10}mg,$$

in the same direction as the displacement. On the other hand, the force of gravity has magnitude $F_g = mg$ and is opposite in direction to the displacement.

ANALYZE (a) Since the force of the cable $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

$$W_F = Fd = \frac{11mgd}{10} = \frac{11\,(72 \text{ kg})(9.8 \text{ m/s}^2)(15 \text{ m})}{10} = 1.164\times10^4 \text{ J} \approx 1.2\times10^4 \text{ J}.$$

(b) Using Eq. 7-7, the work done by gravity is

$$W_g = -F_g d = -mgd = -\,(72 \text{ kg})(9.8 \text{ m/s}^2)(15 \text{ m}) = -1.058\times10^4 \text{ J} \approx -1.1\times10^4 \text{ J}.$$

(c) The total work done is the sum of the two works:

$$W_{\text{net}} = W_F + W_g = 1.164\times10^4 \text{ J} - 1.058\times10^4 \text{ J} = 1.06\times10^3 \text{ J} \approx 1.1\times10^3 \text{ J}.$$

Since the astronaut started from rest, the work-kinetic energy theorem tells us that this is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(1.06\times10^3 \text{ J})}{72 \text{ kg}}} = 5.4 \text{ m/s}.$

LEARN For a general upward acceleration a, the net work done is

$$W_{\text{net}} = W_F + W_g = Fd - F_g d = m(g+a)d - mgd = mad.$$

Since $W_{\text{net}} = \Delta K = mv^2/2$ by the work-kinetic energy theorem, the speed of the astronaut would be $v = \sqrt{2ad}$, which is independent of the mass of the astronaut. In our case, $v = \sqrt{2(9.8\ \text{m/s}^2/10)(15\ \text{m})} = 5.4\ \text{m/s}$, which agrees with that calculated in (d).

18. In both cases, there is no acceleration, so the lifting force is equal to the weight of the object.

(a) Equation 7-8 leads to $W = \vec{F}\cdot\vec{d} = (360\,\text{kN})(0.10\,\text{m}) = 36\ \text{kJ}$.

(b) In this case, we find W = (4000 N)(0.050 m) $= 2.0\times10^2\ \text{J}$.

19. Equation 7-15 applies, but the wording of the problem suggests that it is only necessary to examine the contribution from the rope (which would be the "W_a" term in Eq. 7-15):

$$W_a = -(50\ \text{N})(0.50\ \text{m}) = -25\ \text{J}$$

(the minus sign arises from the fact that the pull from the rope is anti-parallel to the direction of motion of the block). Thus, the kinetic energy would have been 25 J greater if the rope had not been attached (given the same displacement).

20. From the figure, one may write the kinetic energy (in units of J) as a function of x as

$$K = K_s - 20x = 40 - 20x.$$

Since $W = \Delta K = \vec{F}_x \cdot \Delta x$, the component of the force along the force along $+x$ is $F_x = dK/dx = -20$ N. The normal force on the block is $F_N = F_y$, which is related to the gravitational force by

$$mg = \sqrt{F_x^2 + (-F_y)^2}.$$

(Note that F_N points in the opposite direction of the component of the gravitational force.) With an initial kinetic energy $K_s = 40.0\ \text{J}$ and $v_0 = 4.00\ \text{m/s}$, the mass of the block is

$$m = \frac{2K_s}{v_0^2} = \frac{2(40.0\ \text{J})}{(4.00\ \text{m/s})^2} = 5.00\ \text{kg}.$$

Thus, the normal force is

$$F_y = \sqrt{(mg)^2 - F_x^2} = \sqrt{(5.0\ \text{kg})^2(9.8\ \text{m/s}^2)^2 - (20\ \text{N})^2} = 44.7\ \text{N} \approx 45\ \text{N}.$$

21. **THINK** In this problem the cord is doing work on the block so that it does not undergo free fall.

EXPRESS We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude $F_g = Mg$), to prevent the block from undergoing free fall. The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\text{net}} = m\vec{a} \Rightarrow Mg - F = M\left(\frac{g}{4}\right),$$

so $F = 3Mg/4$, in the opposite direction of the displacement. On the other hand, the force of gravity $F_g = mg$ is in the same direction to the displacement.

ANALYZE (a) Since the displacement is downward, the work done by the cord's force is, using Eq. 7-7,

$$W_F = -Fd = -\frac{3}{4}Mgd.$$

(b) Similarly, the work done by the force of gravity is $W_g = F_g d = Mgd$.

(c) The total work done on the block is simply the sum of the two works:

$$W_{\text{net}} = W_F + W_g = -\frac{3}{4}Mgd + Mgd = \frac{1}{4}Mgd .$$

Since the block starts from rest, we use Eq. 7-15 to conclude that this $\left(M\,gd/4\right)$ is the block's kinetic energy K at the moment it has descended the distance d.

(d) With $K = \frac{1}{2}Mv^2$, the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2(Mgd/4)}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

LEARN For a general downward acceleration a, the force exerted by the cord is $F = m(g-a)$, so that the net work done on the block is $W_{\text{net}} = F_{\text{net}}d = mad$. The speed of the block after falling a distance d is $v = \sqrt{2ad}$. In the special case where the block hangs still, $a = 0$, $F = mg$ and $v = 0$. In our case, $a = g/4$, and $v = \sqrt{2(g/4)d} = \sqrt{gd/2}$, which agrees with that calculated in (d).

22. We use d to denote the magnitude of the spelunker's displacement during each stage. The mass of the spelunker is m = 80.0 kg. The work done by the lifting force is denoted W_i where i = 1, 2, 3 for the three stages. We apply the work-energy theorem, Eq. 17-15.

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00\ \text{m/s}$. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (80.0\ \text{kg})(9.80\ \text{m/s}^2)(10.0\ \text{m}) + \frac{1}{2}(80.0\ \text{kg})(5.00\ \text{m/s})^2 = 8.84\times 10^3\ \text{J}.$$

(b) For stage 2, $W_2 - mgd = \Delta K_2 = 0$, which leads to

$$W_2 = mgd = (80.0\ \text{kg})(9.80\ \text{m/s}^2)(10.0\ \text{m}) = 7.84\times 10^3\ \text{J}.$$

(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (80.0\ \text{kg})(9.80\ \text{m/s}^2)(10.0\ \text{m}) - \frac{1}{2}(80.0\ \text{kg})(5.00\ \text{m/s})^2 = 6.84\times 10^3\ \text{J}.$$

23. The fact that the applied force $\vec{F}_a$ causes the box to move up a frictionless ramp at a constant speed implies that there is no net change in the kinetic energy: $\Delta K = 0$. Thus, the work done by $\vec{F}_a$ must be equal to the negative work done by gravity: $W_a = -W_g$. Since the box is displaced vertically upward by $h = 0.150\ \text{m}$, we have

$$W_a = +mgh = (3.00\ \text{kg})(9.80\ \text{m/s}^2)(0.150\ \text{m}) = 4.41\ \text{J}$$

24. (a) Using notation common to many vector-capable calculators, we have (from Eq. 7-8) W = dot([20.0,0] + [0, –(3.00)(9.8)], [0.500 ∠ 30.0º]) = +1.31 J , where "dot" stands for dot product.

(b) Eq. 7-10 (along with Eq. 7-1) then leads to $v = \sqrt{2(1.31\ \text{J})/(3.00\ \text{kg})}$ = 0.935 m/s.

25. (a) The net upward force is given by

$$F + F_N - (m+M)g = (m+M)a$$

where m = 0.250 kg is the mass of the cheese, M = 900 kg is the mass of the elevator cab, F is the force from the cable, and $F_N = 3.00\ \text{N}$ is the normal force on the cheese. On the cheese alone, we have

$$F_N - mg = ma \quad\Rightarrow\quad a = \frac{3.00\ \text{N} - (0.250\ \text{kg})(9.80\ \text{m/s}^2)}{0.250\ \text{kg}} = 2.20\ \text{m/s}^2.$$

Thus the force from the cable is $F = (m+M)(a+g) - F_N = 1.08\times 10^4$ N, and the work done by the cable on the cab is

$$W = Fd_1 = (1.80\times 10^4 \text{ N})(2.40 \text{ m}) = 2.59\times 10^4 \text{ J}.$$

(b) If $W = 92.61$ kJ and $d_2 = 10.5$ m, the magnitude of the normal force is

$$F_N = (m+M)g - \frac{W}{d_2} = (0.250 \text{ kg} + 900 \text{ kg})(9.80 \text{ m/s}^2) - \frac{9.261\times 10^4 \text{ J}}{10.5 \text{ m}} = 2.45 \text{ N}.$$

26. We make use of Eq. 7-25 and Eq. 7-28 since the block is stationary before and after the displacement. The work done by the applied force can be written as

$$W_a = -W_s = \frac{1}{2}k(x_f^2 - x_i^2).$$

The spring constant is $k = (80 \text{ N})/(2.0 \text{ cm}) = 4.0\times 10^3 \text{N/m}$. With $W_a = 4.0$ J, and $x_i = -2.0$ cm, we have

$$x_f = \pm\sqrt{\frac{2W_a}{k} + x_i^2} = \pm\sqrt{\frac{2(4.0 \text{ J})}{(4.0\times 10^3 \text{ N/m})} + (-0.020 \text{ m})^2} = \pm 0.049 \text{ m} = \pm 4.9 \text{ cm}.$$

27. From Eq. 7-25, we see that the work done by the spring force is given by

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2).$$

The fact that 360 N of force must be applied to pull the block to x = + 4.0 cm implies that the spring constant is

$$k = \frac{360 \text{ N}}{4.0 \text{ cm}} = 90 \text{ N/cm} = 9.0\times 10^3 \text{ N/m}.$$

(a) When the block moves from $x_i = +5.0$ cm to $x = +3.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times 10^3 \text{ N/m})[(0.050 \text{ m})^2 - (0.030 \text{ m})^2] = 7.2 \text{ J}.$$

(b) Moving from $x_i = +5.0$ cm to $x = -3.0$ cm, we have

$$W_s = \frac{1}{2}(9.0\times 10^3 \text{ N/m})[(0.050 \text{ m})^2 - (-0.030 \text{ m})^2] = 7.2 \text{ J}.$$

(c) Moving from $x_i = +5.0\text{ cm}$ to $x = -5.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.050\text{ m})^2] = 0\text{ J}.$$

(d) Moving from $x_i = +5.0\text{ cm}$ to $x = -9.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(9.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.090\text{ m})^2] = -25\text{ J}.$$

28. The spring constant is k = 100 N/m and the maximum elongation is x_i = 5.00 m. Using Eq. 7-25 with x_f = 0, the work is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(100\text{ N/m})(5.00\text{ m})^2 = 1.25\times10^3\text{ J}.$$

29. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. The spring constant k can be deduced from the figure which shows the amount of work done to pull the block from 0 to x = 3.0 cm. The parabola $W_a = kx^2/2$ contains (0,0), (2.0 cm, 0.40 J) and (3.0 cm, 0.90 J). Thus, we may infer from the data that $k = 2.0\times10^3\text{ N/m}$.

(a) When the block moves from $x_i = +5.0\text{ cm}$ to $x = +4.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(2.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (0.040\text{ m})^2] = 0.90\text{ J}.$$

(b) Moving from $x_i = +5.0\text{ cm}$ to $x = -2.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(2.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.020\text{ m})^2] = 2.1\text{ J}.$$

(c) Moving from $x_i = +5.0\text{ cm}$ to $x = -5.0\text{ cm}$, we have

$$W_s = \frac{1}{2}(2.0\times10^3\text{ N/m})[(0.050\text{ m})^2 - (-0.050\text{ m})^2] = 0\text{ J}.$$

30. Hooke’s law and the work done by a spring is discussed in the chapter. We apply the work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in the figure at x = 1.0 m and x = 2.0 m, respectively. The “applied” work W_a is that due to the constant force $\vec{P}$.

$$4 \text{ J} = P(1.0 \text{ m}) - \frac{1}{2}k(1.0 \text{ m})^2$$
$$0 = P(2.0 \text{ m}) - \frac{1}{2}k(2.0 \text{ m})^2.$$

(a) Simultaneous solution leads to P = 8.0 N.

(b) Similarly, we find k = 8.0 N/m.

31. **THINK** The applied force varies with x, so an integration is required to calculate the work done on the body.

EXPRESS As the body moves along the x axis from x_i = 3.0 m to x_f = 4.0 m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} -6x \, dx = -3(x_f^2 - x_i^2) = -3\,(4.0^2 - 3.0^2) = -21 \text{ J}.$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). Given v_i, we can readily calculate v_f.

ANALYZE (a) The work-kinetic theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21 \text{ J})}{2.0 \text{ kg}} + (8.0 \text{ m/s})^2} = 6.6 \text{ m/s}.$$

(b) The velocity of the particle is v_f = 5.0 m/s when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3\left(x_f^2 - x_i^2\right)$, so the theorem leads to

$$W = \Delta K \quad \Rightarrow \quad -3\left(x_f^2 - x_i^2\right) = \frac{1}{2}m\,\left(v_f^2 - v_i^2\right).$$

Thus,

$$x_f = \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} = \sqrt{-\frac{2.0 \text{ kg}}{6 \text{ N/m}}\left((5.0 \text{ m/s})^2 - (8.0 \text{ m/s})^2\right) + (3.0 \text{ m})^2} = 4.7 \text{ m}.$$

LEARN Since $x_f > x_i$, $W = -3(x_f^2 - x_i^2) < 0$, i.e., the work done by the force is negative. From the work-kinetic energy theorem, this implies $\Delta K < 0$. Hence, the speed of the particle will continue to decrease as it moves in the +x-direction.

32. The work done by the spring force is given by Eq. 7-25: $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. Since $F_x = -kx$, the slope in Fig. 7-37 corresponds to the spring constant *k*. Its value is given by $k = 80\ \text{N/cm} = 8.0\times10^3\ \text{N/m}$.

(a) When the block moves from $x_i = +8.0\ \text{cm}$ to $x = +5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(b) Moving from $x_i = +8.0\ \text{cm}$ to $x = -5.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.050\ \text{m})^2] = 15.6\ \text{J} \approx 16\ \text{J}.$$

(c) Moving from $x_i = +8.0\ \text{cm}$ to $x = -8.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.080\ \text{m})^2] = 0\ \text{J}.$$

(d) Moving from $x_i = +8.0\ \text{cm}$ to $x = -10.0\ \text{cm}$, we have

$$W_s = \frac{1}{2}(8.0\times10^3\ \text{N/m})[(0.080\ \text{m})^2 - (-0.10\ \text{m})^2] = -14.4\ \text{J} \approx -14\ \text{J}.$$

33. (a) This is a situation where Eq. 7-28 applies, so we have

$$Fx = \tfrac{1}{2}kx^2 \ \Rightarrow\ (3.0\ \text{N})\,x = \tfrac{1}{2}(50\ \text{N/m})x^2$$

which (other than the trivial root) gives $x = (3.0/25)$ m = 0.12 m.

(b) The work done by the applied force is $W_a = Fx = (3.0\ \text{N})(0.12\ \text{m}) = 0.36$ J.

(c) Eq. 7-28 immediately gives $W_s = -W_a = -0.36$ J.

(d) With $K_f = K$ considered variable and $K_i = 0$, Eq. 7-27 gives $K = Fx - \tfrac{1}{2}kx^2$. We take the derivative of *K* with respect to *x* and set the resulting expression equal to zero, in order to find the position x_c taht corresponds to a maximum value of *K*:

$$x_c = \frac{F}{k} = (3.0/50)\ \text{m} = 0.060\ \text{m}.$$

We note that x_c is also the point where the applied and spring forces “balance.”

(e) At x_c we find $K = K_{max} = 0.090$ J.

34. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{20\ \text{m/s}^2}{8.0\ \text{m}} = 2.5\ \text{s}^{-2}.$$

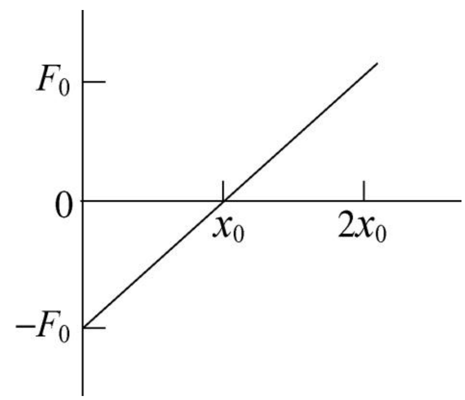
The force on the brick is in the positive x direction and, according to Newton's second law, its magnitude is given by $F = ma = m\alpha x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F\ dx = m\alpha \int_0^{x_f} x\ dx = \frac{m\alpha}{2} x_f^2 = \frac{(10\ \text{kg})(2.5\ \text{s}^{-2})}{2}(8.0\ \text{m})^2 = 8.0\times 10^2\ \text{J}.$$

35. **THINK** We have an applied force that varies with x. An integration is required to calculate the work done on the particle.

EXPRESS Given a one-dimensional force $F(x)$, the work done is simply equal to the area under the curve: $W = \int_{x_i}^{x_f} F(x)\ dx$.

ANALYZE (a) The plot of $F(x)$ is shown to the right. Here we take x_0 to be positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$.



Since the area of a triangle is (base)(altitude)/2, the work done from $x = 0$ to $x = x_0$ is $W_1 = -(x_0)(F_0)/2$ and the work done from $x = x_0$ to $x = 2x_0$ is

$$W_2 = (2x_0 - x_0)(F_0)/2 = (x_0)(F_0)/2$$

The total work is the sum of the two:

$$W = W_1 + W_2 = -\frac{1}{2}F_0 x_0 + \frac{1}{2}F_0 x_0 = 0.$$

(b) The integral for the work is

$$W = \int_0^{2x_0} F_0\left(\frac{x}{x_0} - 1\right)dx = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{2x_0} = 0.$$

LEARN If the particle starts out at $x = 0$ with an initial speed v_i, with a negative work $W_1 = -F_0 x_0/2 < 0$, its speed at $x = x_0$ will decrease to

$$v=\sqrt{v_i^2+\frac{2W_1}{m}}=\sqrt{v_i^2-\frac{F_0x_0}{m}}<v_i\,,$$

but return to v_i again at $x=2x_0$ with a positive work $W_2=F_0x_0/2>0$.

36. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas) we obtain

$$W=W_{0<x<2}+W_{2<x<4}+W_{4<x<6}+W_{6<x<8}=(20+10+0-5)\text{ J}=25\text{ J}.$$

37. (a) We first multiply the vertical axis by the mass, so that it becomes a graph of the applied force. Now, adding the triangular and rectangular "areas" in the graph (for $0 \le x \le 4$) gives 42 J for the work done.

(b) Counting the "areas" under the axis as negative contributions, we find (for $0 \le x \le 7$) the work to be 30 J at x = 7.0 m.

(c) And at x = 9.0 m, the work is 12 J.

(d) Equation 7-10 (along with Eq. 7-1) leads to speed v = 6.5 m/s at x = 4.0 m. Returning to the original graph (where a was plotted) we note that (since it started from rest) it has received acceleration(s) (up to this point) only in the $+x$ direction and consequently must have a velocity vector pointing in the $+x$ direction at x = 4.0 m.

(e) Now, using the result of part (b) and Eq. 7-10 (along with Eq. 7-1) we find the speed is 5.5 m/s at x = 7.0 m. Although it has experienced some deceleration during the $0 \le x \le 7$ interval, its velocity vector still points in the $+x$ direction.

(f) Finally, using the result of part (c) and Eq. 7-10 (along with Eq. 7-1) we find its speed v = 3.5 m/s at x = 9.0 m. It certainly has experienced a significant amount of deceleration during the $0 \le x \le 9$ interval; nonetheless, its velocity vector *still* points in the $+x$ direction.

38. (a) Using the work-kinetic energy theorem

$$K_f=K_i+\int_0^{2.0}(2.5-x^2)\,dx=0+(2.5)(2.0)-\frac{1}{3}(2.0)^3=2.3\text{ J}.$$

(b) For a variable end-point, we have K_f as a function of x, which could be differentiated to find the extremum value, but we recognize that this is equivalent to solving F = 0 for x:

$$F=0\;\Rightarrow\;2.5-x^2=0.$$

Thus, K is extremized at $x = \sqrt{2.5} \approx 1.6$ m and we obtain

$$K_f = K_i + \int_0^{\sqrt{2.5}} (2.5 - x^2)dx = 0 + (2.5)(\sqrt{2.5}) - \frac{1}{3}(\sqrt{2.5})^3 = 2.6 \text{ J}.$$

Recalling our answer for part (a), it is clear that this extreme value is a maximum.

39. As the body moves along the x axis from x_i = 0 m to x_f = 3.00 m the work done by the force is

$$W = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} (cx - 3.00x^2)dx = \left(\frac{c}{2}x^2 - x^3\right)_0^3 = \frac{c}{2}(3.00)^2 - (3.00)^3$$
$$= 4.50c - 27.0.$$

However, $W = \Delta K = (11.0 - 20.0) = -9.00$ J from the work-kinetic energy theorem. Thus,

$$4.50c - 27.0 = -9.00$$

or $c = 4.00$ N/m.

40. Using Eq. 7-32, we find

$$W = \int_{0.25}^{1.25} e^{-4x^2} \, dx = 0.21 \text{ J}$$

where the result has been obtained numerically. Many modern calculators have that capability, as well as most math software packages that a great many students have access to.

41. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is v_i = 3.0 m/s and the speed at t = 4 s is v_f = 19 m/s. The change in kinetic energy for the object of mass m = 3.0 kg is therefore

$$\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = 528 \text{ J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3 \times 10^2$ J.

42. We solve the problem using the work-kinetic energy theorem, which states that the change in kinetic energy is equal to the work done by the applied force, $\Delta K = W$. In our

problem, the work done is $W = Fd$, where F is the tension in the cord and d is the length of the cord pulled as the cart slides from x_1 to x_2. From the figure, we have

$$d = \sqrt{x_1^2 + h^2} - \sqrt{x_2^2 + h^2} = \sqrt{(3.00\text{ m})^2 + (1.20\text{ m})^2} - \sqrt{(1.00\text{ m})^2 + (1.20\text{ m})^2}$$
$$= 3.23\text{ m} - 1.56\text{ m} = 1.67\text{ m}$$

which yields $\Delta K = Fd = (25.0\text{ N})(1.67\text{ m}) = 41.7\text{ J}$.

43. **THINK** This problem deals with the power and work done by a constant force.

EXPRESS The power done by a constant force F is given by $P = Fv$ and the work done by F from time t_1 to time t_2 is

$$W = \int_{t_1}^{t_2} P\,dt = \int_{t_1}^{t_2} Fv\,dt$$

Since F is the magnitude of the net force, the magnitude of the acceleration is $a = F/m$. Thus, if the initial velocity is $v_0 = 0$, then the velocity of the body as a function of time is given by $v = v_0 + at = (F/m)t$. Substituting the expression for v into the equation above, the work done during the time interval (t_1, t_2) becomes

$$W = \int_{t_1}^{t_2} (F^2/m)t\,dt = \frac{F^2}{2m}\left(t_2^2 - t_1^2\right).$$

ANALYZE (a) For $t_1 = 0$ and $t_2 = 1.0\text{ s}$, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(1.0\text{ s})^2 - 0] = 0.83\text{ J}$.

(b) For $t_1 = 1.0\text{s}$, and $t_2 = 2.0\text{ s}$, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(2.0\text{ s})^2 - (1.0\text{ s})^2] = 2.5\text{ J}$.

(c) For $t_1 = 2.0\text{ s}$ and $t_2 = 3.0\text{ s}$, $W = \frac{1}{2}\left(\frac{(5.0\text{ N})^2}{15\text{ kg}}\right)[(3.0\text{ s})^2 - (2.0\text{ s})^2] = 4.2\text{ J}$.

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2 t/m$ for the power at any time t. At the end of the third second, the instantaneous power is

$$P = \left(\frac{(5.0\text{ N})^2\ (3.0\text{ s})}{15\text{ kg}}\right) = 5.0\text{ W}.$$

LEARN The work done here is quadratic in t. Therefore, from the definition $P = dW/dt$ for the instantaneous power, we see that P increases linearly with t.

44. (a) Since constant speed implies $\Delta K = 0$, we require $W_a = -W_g$, by Eq. 7-15. Since W_g is the same in both cases (same weight and same path), then $W_a = 9.0\times10^2$ J just as it was in the first case.

(b) Since the speed of 1.0 m/s is constant, then 8.0 meters is traveled in 8.0 seconds. Using Eq. 7-42, and noting that average power is *the* power when the work is being done at a steady rate, we have

$$P = \frac{W}{\Delta t} = \frac{900\text{ J}}{8.0\text{ s}} = 1.1\times10^2\text{ W}.$$

(c) Since the speed of 2.0 m/s is constant, 8.0 meters is traveled in 4.0 seconds. Using Eq. 7-42, with *average power* replaced by *power*, we have

$$P = \frac{W}{\Delta t} = \frac{900\text{ J}}{4.0\text{ s}} = 225\text{ W} \approx 2.3\times10^2\text{ W}.$$

45. **THINK** A block is pulled at a constant speed by a force directed at some angle with respect to the direction of motion. The quantity we're interested in is the power, or the time rate at which work is done by the applied force.

EXPRESS The power associated with force $\vec{F}$ is given by $P = \vec{F}\cdot\vec{v} = Fv\cos\phi$, where $\vec{v}$ is the velocity of the object on which the force acts, and ϕ is the angle between $\vec{F}$ and $\vec{v}$.

ANALYZE With $F = 122\text{ N}$, $v = 5.0\text{ m/s}$ and $\phi = 37.0°$, we find the power to be

$$P = Fv\cos\phi = (122\text{ N})(5.0\text{ m/s})\cos 37.0° = 4.9\times10^2\text{ W}.$$

LEARN From the expression $P = Fv\cos\phi$, we see that the power is at a maximum when $\vec{F}$ and $\vec{v}$ are in the same direction ($\phi = 0$), and is zero when they are perpendicular of each other. In addition, we're told that the block moves at a constant speed, so $\Delta K = 0$, and the net work done on it must also be zero by the work-kinetic energy theorem. Thus, the applied force here must be compensating another force (e.g., friction) for the net rate to be zero.

46. Recognizing that the force in the cable must equal the total weight (since there is no acceleration), we employ Eq. 7-47:

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

where we have used the fact that $\theta = 0°$ (both the force of the cable and the elevator's motion are upward). Thus,

$$P = (3.0\times10^3 \text{ kg})(9.8 \text{ m/s}^2)\left(\frac{210 \text{ m}}{23 \text{ s}}\right) = 2.7\times10^5 \text{ W}.$$

47. (a) Equation 7-8 yields

$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$
$= (2.00 \text{ N})(7.5 \text{ m} - 0.50 \text{ m}) + (4.00 \text{ N})(12.0 \text{ m} - 0.75 \text{ m}) + (6.00 \text{ N})(7.2\text{m} - 0.20 \text{ m})$
$= 101 \text{ J} \approx 1.0\times 10^2 \text{ J}.$

(b) Dividing this result by 12 s (see Eq. 7-42) yields $P = 8.4$ W.

48. (a) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero.

(b) The rate is given by $P = \vec{F}\cdot\vec{v} = -Fv$, where the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are anti-parallel to each other. The magnitude of the force is given by

$$F = kx = (500 \text{ N/m})(0.10 \text{ m}) = 50 \text{ N},$$

while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = 10 \text{ J} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.30 \text{ kg})v^2 + \frac{1}{2}(500 \text{ N/m})(0.10 \text{ m})^2$$

which gives $v = 7.1$ m/s. Thus,

$$P = -Fv = -(50 \text{ N})(7.1 \text{ m/s}) = -3.5 \times 10^2 \text{ W}.$$

49. **THINK** We have a loaded elevator moving upward at a constant speed. The forces involved are: gravitational force on the elevator, gravitational force on the counterweight, and the force by the motor via cable.

EXPRESS The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system:

$$W = W_e + W_c + W_m.$$

Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero, i.e., $W = \Delta K = 0$.

ANALYZE The elevator moves *upward* through 54 m, so the work done by gravity on it is

$$W_e = -m_e gd = -(1200 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = -6.35 \times 10^5 \text{ J}.$$

The counterweight moves *downward* the same distance, so the work done by gravity on it is

$$W_c = m_c gd = (950 \text{ kg})(9.80 \text{ m/s}^2)(54 \text{ m}) = 5.03\times10^5 \text{ J}.$$

Since $W = 0$, the work done by the motor on the system is

$$W_m = -W_e - W_c = 6.35\times10^5 \text{ J} - 5.03\times10^5 \text{ J} = 1.32\times10^5 \text{ J}.$$

This work is done in a time interval of $\Delta t = 3.0 \text{ min} = 180 \text{ s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_m}{\Delta t} = \frac{1.32 \times 10^5 \text{ J}}{180 \text{ s}} = 7.4 \times 10^2 \text{ W}.$$

LEARN In general, the work done by the motor is $W_m = (m_e - m_c)gd$. So when the counterweight mass balances the total mass, $m_c = m_e$, no work is required by the motor.

50. (a) Using Eq. 7-48 and Eq. 3-23, we obtain

$$P = \vec{F}\cdot\vec{v} = (4.0\text{ N})(-2.0 \text{ m/s}) + (9.0 \text{ N})(4.0 \text{ m/s}) = 28 \text{ W}.$$

(b) We again use Eq. 7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\text{j}}$.

$$P = \vec{F}\cdot\vec{v} \quad \Rightarrow -12 \text{ W} = (-2.0 \text{ N})v.$$

which yields $v = 6$ m/s.

51. (a) The object's displacement is

$$\vec{d} = \vec{d}_f - \vec{d}_i = (-8.00 \text{ m})\hat{\text{i}} + (6.00 \text{ m})\hat{\text{j}} + (2.00 \text{ m})\hat{\text{k}}\ .$$

Thus, Eq. 7-8 gives

$$W = \vec{F}\cdot\vec{d} = (3.00 \text{ N})(-8.00 \text{ m}) + (7.00 \text{ N})(6.00 \text{ m}) + (7.00 \text{ N})(2.00 \text{ m}) = 32.0 \text{ J}.$$

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32.0}{4.00} = 8.00 \text{ W}.$$

(c) The distance from the coordinate origin to the initial position is

$$d_i = \sqrt{(3.00\text{ m})^2 + (-2.00\text{ m})^2 + (5.00\text{ m})^2} = 6.16\text{ m},$$

and the magnitude of the distance from the coordinate origin to the final position is

$$d_f = \sqrt{(-5.00\text{ m})^2 + (4.00\text{ m})^2 + (7.00\text{ m})^2} = 9.49\text{ m}.$$

Their scalar (dot) product is

$$\vec{d}_i \cdot \vec{d}_f = (3.00\text{ m})(-5.00\text{ m}) + (-2.00\text{ m})(4.00\text{ m}) + (5.00\text{ m})(7.00\text{ m}) = 12.0\text{ m}^2.$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d}_i \cdot \vec{d}_f}{d_i d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right) = 78.2°.$$

52. According to the problem statement, the power of the car is

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = \text{constant}.$$

The condition implies $dt = mvdv/P$, which can be integrated to give

$$\int_0^T dt = \int_0^{v_T} \frac{mvdv}{P} \quad \Rightarrow \; T = \frac{mv_T^2}{2P}$$

where v_T is the speed of the car at $t = T$. On the other hand, the total distance traveled can be written as

$$L = \int_0^T vdt = \int_0^{v_T} v\frac{mvdv}{P} = \frac{m}{P}\int_0^{v_T} v^2 dv = \frac{mv_T^3}{3P}.$$

By squaring the expression for L and substituting the expression for T, we obtain

$$L^2 = \left(\frac{mv_T^3}{3P}\right)^2 = \frac{8P}{9m}\left(\frac{mv_T^2}{2P}\right)^3 = \frac{8PT^3}{9m}$$

which implies that

$$PT^3 = \frac{9}{8}mL^2 = \text{constant}.$$

Differentiating the above equation gives $dPT^3 + 3PT^2 dT = 0$, or $dT = -\dfrac{T}{3P}dP$.

53. (a) Noting that the x component of the third force is F_{3x} = (4.00 N)cos(60º), we apply Eq. 7-8 to the problem:

$$W = [5.00\text{ N} - 1.00\text{ N} + (4.00\text{ N})\cos 60^\circ](0.20\text{ m}) = 1.20\text{ J}.$$

(b) Equation 7-10 (along with Eq. 7-1) then yields $v = \sqrt{2W/m}$ = 1.10 m/s.

54. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be $x = 0$, where v_0 = 4.0 m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16\text{ J}$, we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4.0\text{ J}$$

so that K_3 (the kinetic energy when x = 3.0 m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x\Delta x = (-4.0\text{ N})(x_f - 3.0\text{ m})$ and apply the work-kinetic energy theorem:

$$K_{x_f} - K_3 = W_{3<x<x_f}$$
$$K_{x\,f} - 12 = (-4)(x_f - 3.0)$$

so that the requirement $K_{xf} = 8.0\text{ J}$ leads to $x_f = 4.0\text{ m}$.

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until x = 1.0 m. At that location, the kinetic energy is

$$K_1 = K_0 + W_{0<x<1} = 16\text{ J} + 2.0\text{ J} = 18\text{ J}.$$

55. **THINK** A horse is doing work to pull a cart at a constant speed. We'd like to know the work done during a time interval and the corresponding average power.

EXPRESS The horse pulls with a force $\vec{F}$. As the cart moves through a displacement $\vec{d}$, the work done by $\vec{F}$ is $W = \vec{F}\cdot\vec{d} = Fd\cos\phi,$ where ϕ is the angle between $\vec{F}$ and $\vec{d}$.

ANALYZE (a) In 10 min the cart moves a distance

$$d = v\Delta t = \left(6.0\ \frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft/mi}}{60\text{ min/h}}\right)(10\text{ min}) = 5280\text{ ft}$$

so that Eq. 7-7 yields

$$W = Fd\cos\phi = (40 \text{ lb})(5280 \text{ ft}) \cos 30° = 1.8\times10^5 \text{ ft}\cdot\text{lb}.$$

(b) The average power is given by Eq. 7-42. With $\Delta t = 10 \text{ min} = 600 \text{ s}$, we obtain

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{1.8\times10^5 \text{ ft}\cdot\text{lb}}{600 \text{ s}} = 305 \text{ ft}\cdot\text{lb/s},$$

which (using the conversion factor $1\,\text{hp} = 550\,\text{ft}\cdot\text{lb/s}$ found on the inside back cover) converts to P_{avg} = 0.55 hp.

LEARN The average power can also be calculate by using Eq. 7-48: $P_{\text{avg}} = Fv\cos\phi$. Converting the speed to $v = (6.0 \text{ mi/h})\left(\dfrac{5280 \text{ ft/mi}}{3600 \text{ s/h}}\right) = 8.8 \text{ ft/s}$, we get

$$P_{\text{avg}} = Fv\cos\phi = (40 \text{ lb})(8.8 \text{ ft/s})\cos 30° = 305 \text{ ft}\cdot\text{lb} = 0.55 \text{ hp}$$

which agrees with that found in (b).

56. The acceleration is constant, so we may use the equations in Table 2-1. We choose the direction of motion as $+x$ and note that the displacement is the same as the distance traveled, in this problem. We designate the force (assumed singular) along the x direction acting on the m = 2.0 kg object as F.

(a) With v_0 = 0, Eq. 2-11 leads to $a = v/t$. And Eq. 2-17 gives $\Delta x = \frac{1}{2}vt$. Newton's second law yields the force $F = ma$. Equation 7-8, then, gives the work:

$$W = F\Delta x = m\left(\frac{v}{t}\right)\left(\frac{1}{2}vt\right) = \frac{1}{2}mv^2$$

as we expect from the work-kinetic energy theorem. With v = 10 m/s, this yields $W = 1.0\times10^2$ J.

(b) Instantaneous power is defined in Eq. 7-48. With t = 3.0 s, we find

$$P = Fv = m\left(\frac{v}{t}\right)v = 67 \text{ W}.$$

(c) The velocity at $t' = 1.5\text{s}$ is $v' = at' = 5.0 \text{ m/s}$. Thus, $P' = Fv' = 33$ W.

57. (a) To hold the crate at equilibrium in the final situation, $\vec{F}$ must have the same magnitude as the horizontal component of the rope's tension $T\sin\theta$, where θ is the angle between the rope (in the final position) and vertical:

$$\theta = \sin^{-1}\left(\frac{4.00}{12.0}\right) = 19.5°.$$

But the vertical component of the tension supports against the weight: $T\cos\theta = mg$. Thus, the tension is

$$T = (230 \text{ kg})(9.80 \text{ m/s}^2)/\cos 19.5° = 2391 \text{ N}$$

and F = (2391 N) sin 19.5° = 797 N.

An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution.

(b) Since there is no change in kinetic energy, the net work on it is zero.

(c) The work done by gravity is $W_g = \vec{F}_g \cdot \vec{d} = -mgh$, where $h = L(1 - \cos\theta)$ is the vertical component of the displacement. With L = 12.0 m, we obtain W_g = –1547 J, which should be rounded to three significant figures: –1.55 kJ.

(d) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since cos 90° = 0).

(e) The implication of the previous three parts is that the work due to $\vec{F}$ is $-W_g$ (so the net work turns out to be zero). Thus, $W_F = -W_g$ = 1.55 kJ.

(f) Since $\vec{F}$ does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

58. (a) The force of the worker on the crate is constant, so the work it does is given by $W_F = \vec{F}\cdot\vec{d} = Fd\cos\phi$, where $\vec{F}$ is the force, $\vec{d}$ is the displacement of the crate, and ϕ is the angle between the force and the displacement. Here F = 210 N, d = 3.0 m, and ϕ = 20°. Thus,

$$W_F = (210 \text{ N})(3.0 \text{ m})\cos 20° = 590 \text{ J}.$$

(b) The force of gravity is downward, perpendicular to the displacement of the crate. The angle between this force and the displacement is 90° and cos 90° = 0, so the work done by the force of gravity is zero.

(c) The normal force of the floor on the crate is also perpendicular to the displacement, so the work done by this force is also zero.

(d) These are the only forces acting on the crate, so the total work done on it is 590 J.

59. The work done by the applied force $\vec{F}_a$ is given by $W = \vec{F}_a \cdot \vec{d} = F_a d \cos\phi$. From the figure, we see that $W = 25$ J when $\phi = 0$ and $d = 5.0$ cm. This yields the magnitude of $\vec{F}_a$:

$$F_a = \frac{W}{d} = \frac{25\text{ J}}{0.050\text{ m}} = 5.0\times10^2\text{ N}.$$

(a) For $\phi = 64°$, we have $W = F_a d\cos\phi = (5.0\times10^2\text{ N})(0.050\text{ m})\cos 64° = 11$ J.

(b) For $\phi = 147°$, we have $W = F_a d\cos\phi = (5.0\times10^2\text{ N})(0.050\text{ m})\cos 147° = -21$ J.

60. (a) In the work-kinetic energy theorem, we include both the work due to an applied force W_a and work done by gravity W_g in order to find the latter quantity.

$$\Delta K = W_a + W_g \quad \Rightarrow \quad 30\text{ J} = (100\text{ N})(1.8\text{ m})\cos 180° + W_g$$

leading to $W_g = 2.1\times10^2$ J.

(b) The value of W_g obtained in part (a) still applies since the weight and the path of the child remain the same, so $\Delta K = W_g = 2.1\times10^2$ J.

61. One approach is to assume a "path" from $\vec{r}_i$ to $\vec{r}_f$ and do the line-integral accordingly. Another approach is to simply use Eq. 7-36, which we demonstrate:

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy = \int_2^{-4} (2x)dx + \int_3^{-3} (3)\, dy$$

with SI units understood. Thus, we obtain W = 12 J – 18 J = – 6 J.

62. (a) The compression of the spring is d = 0.12 m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25\text{ kg})\left(9.8\text{ m/s}^2\right)(0.12\text{ m}) = 0.29\text{ J}.$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2}\,(250\text{ N/m})\,(0.12\text{ m})^2 = -1.8\text{ J}.$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15):

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29\text{ J} - 1.8\text{ J})}{0.25\text{ kg}}} = 3.5\text{ m/s}.$$

(d) If we instead had $v_i' = 7\,\text{m/s}$, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$0 - \frac{1}{2}mv_i'^2 = W_1' + W_2' = mgd' - \frac{1}{2}kd'^2$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^2}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this result, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048}\text{ m/s} = 3.471\text{ m/s}$ and v_i' = 6.942 m/s).

63. **THINK** A crate is being pushed up a frictionless inclined plane. The forces involved are: gravitational force on the crate, normal force on the crate, and the force applied by the worker.

EXPRESS The work done by a force $\vec{F}$ on an object as it moves through a displacement $\vec{d}$ is $W = \vec{F}\cdot\vec{d} = Fd\cos\phi,$ where ϕ is the angle between $\vec{F}$ and $\vec{d}$.

ANALYZE (a) The applied force is parallel to the inclined plane. Thus, using Eq. 7-6, the work done on the crate by the worker's applied force is

$$W_a = Fd\cos 0° = (209\text{ N})(1.50\text{ m}) \approx 314\text{ J}.$$

(b) Using Eq. 7-12, we find the work done by the gravitational force to be

$$\begin{aligned} W_g &= F_g d\cos(90° + 25°) = mgd\cos 115° \\ &= (25.0\text{ kg})(9.8\text{ m/s}^2)(1.50\text{ m})\cos 115° \\ &\approx -155\text{ J}. \end{aligned}$$

(c) The angle between the normal force and the direction of motion remains 90º at all times, so the work it does is zero:

$$W_N = F_N d\cos 90° = 0.$$

(d) The total work done on the crate is the sum of all three works:

$$W = W_a + W_g + W_N = 314 \text{ J} + (-155 \text{ J}) + 0 \text{ J} = 158 \text{ J}.$$

LEARN By work-kinetic energy theorem, if the crate is initially at rest ($K_i = 0$), then its kinetic energy after having moved 1.50 m up the incline would be $K_f = W = 158 \text{ J}$, and the speed of the crate at that instant is

$$v = \sqrt{2K_f / m} = \sqrt{2(158 \text{ J})/25.0 \text{ kg}} = 3.56 \text{ m/s}.$$

64. (a) The force $\vec{F}$ of the incline is a combination of normal and friction force, which is serving to "cancel" the tendency of the box to fall downward (due to its 19.6 N weight). Thus, $\vec{F} = mg$ upward. In this part of the problem, the angle ϕ between the belt and $\vec{F}$ is 80°. From Eq. 7-47, we have

$$P = Fv \cos\phi = (19.6 \text{ N})(0.50 \text{ m/s}) \cos 80° = 1.7 \text{ W}.$$

(b) Now the angle between the belt and $\vec{F}$ is 90°, so that $P = 0$.

(c) In this part, the angle between the belt and $\vec{F}$ is 100°, so that

$$P = (19.6 \text{ N})(0.50 \text{ m/s}) \cos 100° = -1.7 \text{ W}.$$

65. There is no acceleration, so the lifting force is equal to the weight of the object. We note that the person's pull $\vec{F}$ is equal (in magnitude) to the tension in the cord.

(a) As indicated in the *hint*, tension contributes twice to the lifting of the canister: $2T = mg$. Since $|\vec{F}| = T$, we find $|\vec{F}| = 98 \text{ N}$.

(b) To rise 0.020 m, two segments of the cord (see Fig. 7-47) must shorten by that amount. Thus, the amount of string pulled down at the left end (this is the magnitude of $\vec{d}$, the downward displacement of the hand) is $d = 0.040$ m.

(c) Since (at the left end) both $\vec{F}$ and $\vec{d}$ are downward, then Eq. 7-7 leads to

$$W = \vec{F} \cdot \vec{d} = (98 \text{ N})(0.040 \text{ m}) = 3.9 \text{ J}.$$

(d) Since the force of gravity $\vec{F}_g$ (with magnitude mg) is opposite to the displacement $\vec{d}_c = 0.020$ m (up) of the canister, Eq. 7-7 leads to

$$W = \vec{F}_g \cdot \vec{d}_c = -\,(196\text{ N})(0.020\text{ m}) = -3.9\text{ J}.$$

This is consistent with Eq. 7-15 since there is no change in kinetic energy.

66. After converting the speed: $v = 120\text{ km/h} = 33.33\text{ m/s}$, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1200\text{ kg})(33.33\text{m/s})^2 = 6.67\times10^5\text{ J}.$$

67. **THINK** In this problem we have packages hung from the spring. The extent of stretching can be determined from Hooke's law.

EXPRESS According to Hooke's law, the spring force is given by

$$F_x = -k(x - x_0) = -k\Delta x\,,$$

where Δx is the displacement from the equilibrium position. Thus, the first two situations in Fig. 7-48 can be written as

$$\begin{aligned} -110\text{ N} &= -k(40\text{ mm} - x_0) \\ -240\text{ N} &= -k(60\text{ mm} - x_0) \end{aligned}$$

The two equations allow us to solve for *k*, the spring constant, as well as x_0, the relaxed position when no mass is hung.

ANALYZE (a) The two equations can be added to give

$$240\text{ N} - 110\text{ N} \;=\; k(60\text{ mm} - 40\text{ mm})$$

which yields *k* = 6.5 N/mm. Substituting the result into the first equation, we find

$$x_0 = 40\text{ mm} - \frac{110\text{ N}}{k} = 40\text{ mm} - \frac{110\text{ N}}{6.5\text{ N/mm}} = 23\text{ mm}.$$

(b) Using the results from part (a) to analyze that last picture, we find the weight to be

$$W = k(30\text{mm} - x_o) = (6.5\text{ N/mm})(30\text{ mm} - 23\text{ mm}) = 45\text{ N}.$$

LEARN An alternative method to calculate *W* in the third picture is to note that since the amount of stretching is proportional to the weight hung, we have $\dfrac{W}{W'} = \dfrac{\Delta x}{\Delta x'}$. Applying this relation to the second and the third pictures, the weight *W* is

$$W = \left(\frac{\Delta x_3}{\Delta x_2}\right) W_2 = \left(\frac{30\text{ mm} - 23\text{ mm}}{60\text{ mm} - 23\text{ mm}}\right)(240\text{ N}) = 45\text{ N},$$

in agreement with the result shown in (b).

68. Using Eq. 7-7, we have $W = Fd\cos\phi = 1504\text{ J}$. Then, by the work-kinetic energy theorem, we find the kinetic energy $K_f = K_i + W = 0 + 1504$ J. The answer is therefore 1.5 kJ.

69. The total weight is (100)(660 N) = 6.60×10^4 N, and the words "raises … at constant speed" imply zero acceleration, so the lift-force is equal to the total weight. Thus

$$P = Fv = (6.60 \times 10^4)(150\text{ m}/60.0\text{ s}) = 1.65 \times 10^5\text{ W}.$$

70. With SI units understood, Eq. 7-8 leads to $W = (4.0)(3.0) - c(2.0) = 12 - 2c$.

(a) If $W = 0$, then $c = 6.0$ N.

(b) If $W = 17$ J, then $c = -2.5$ N.

(c) If $W = -18$ J, then $c = 15$ N.

71. Using Eq. 7-8, we find

$$W = \vec{F}\cdot\vec{d} = (F\cos\theta\,\hat{\text{i}} + F\sin\theta\,\hat{\text{j}})\cdot(x\hat{\text{i}} + y\hat{\text{j}}) = Fx\cos\theta + Fy\sin\theta$$

where $x = 2.0$ m, $y = -4.0$ m, $F = 10$ N, and $\theta = 150°$. Thus, we obtain $W = -37$ J. Note that the given mass value (2.0 kg) is not used in the computation.

72. (a) Eq. 7-10 (along with Eq. 7-1 and Eq. 7-7) leads to

$$v_f = (2\,\frac{d}{m}F\cos\theta)^{1/2} = (\cos\theta)^{1/2},$$

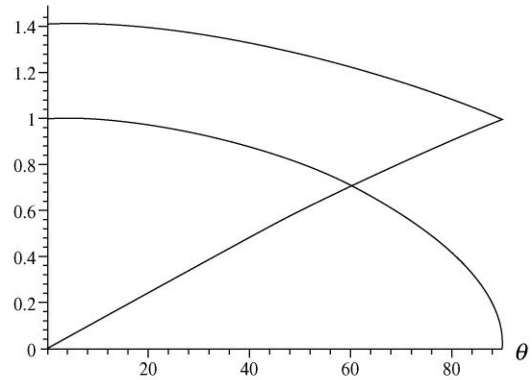
where we have substituted $F = 2.0$ N, $m = 4.0$ kg, and $d = 1.0$ m.

(b) With $v_i = 1$, those same steps lead to $v_f = (1 + \cos\theta)^{1/2}$.

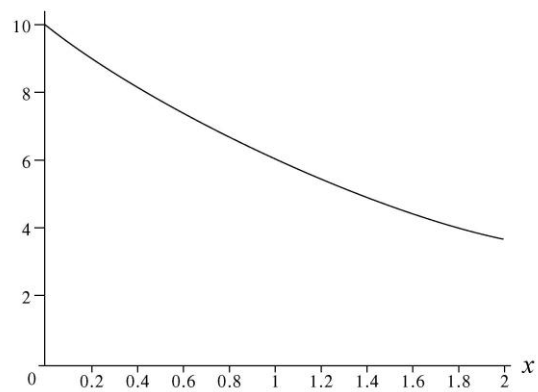
(c) Replacing θ with 180º – θ, and still using $v_i = 1$, we find

$$v_f = [1 + \cos(180° - \theta)]^{1/2} = (1 - \cos\theta)^{1/2}.$$

(d) The graphs are shown on the right. Note that as θ is increased in parts (a) and (b) the force provides less and less of a positive acceleration, whereas in part (c) the force provides less and less of a deceleration (as its θ value increases). The highest curve (which slowly decreases from 1.4 to 1) is the curve for part (b); the other decreasing curve (starting at 1 and ending at 0) is for part (a). The rising curve is for part (c); it is equal to 1 where $\theta = 90°$.



73. (a) The plot of the function (with SI units understood) is shown below.



Estimating the area under the curve allows for a range of answers. Estimates from 11 J to 14 J are typical.

(b) Evaluating the work analytically (using Eq. 7-32), we have

$$W = \int_0^2 10e^{-x/2}dx = -20e^{-x/2}\Big|_0^2 = 12.6 \text{ J} \approx 13 \text{ J}.$$

74. (a) Using Eq. 7-8 and SI units, we find

$$W = \vec{F} \cdot \vec{d} = (2\,\hat{\text{i}} - 4\,\hat{\text{j}}) \cdot (8\,\hat{\text{i}} + c\,\hat{\text{j}}) = 16 - 4c$$

which, if equal zero, implies c = 16/4 = 4 m.

(b) If $W > 0$ then $16 > 4c$, which implies $c < 4$ m.

(c) If $W < 0$ then $16 < 4c$, which implies $c > 4$ m.

75. **THINK** Power must be supplied in order to lift the elevator with load upward at a constant speed.

EXPRESS For the elevator-load system to move upward at a constant speed (zero acceleration), the applied force F must exactly balance the gravitational force on the system, i.e., $F = F_g = (m_{\text{elev}} + m_{\text{load}})g$. The power required can then be calculated using Eq. 17-48: $P = Fv$.

ANALYZE With $m_{\text{elev}} = 4500\,\text{kg}$, $m_{\text{load}} = 1800\,\text{kg}$ and $v = 3.80\,\text{m/s}$, we find the power to be

$$P = Fv = (m_{\text{elev}} + m_{\text{load}})gv = (4500\,\text{kg} + 1800\,\text{kg})(9.8\,\text{m/s}^2)(3.80\,\text{m/s}) = 235\,\text{kW}.$$

LEARN The power required is proportional to the speed at which the system moves; the greater the speed, the greater the power that must be supplied.

76. (a) The component of the force of gravity exerted on the ice block (of mass *m*) along the incline is *mg* sin θ, where $\theta = \sin^{-1}(0.91/1.5)$ gives the angle of inclination for the inclined plane. Since the ice block slides down with uniform velocity, the worker must exert a force $\vec{F}$ "uphill" with a magnitude equal to *mg* sin θ. Consequently,

$$F = mg\sin\theta = (45\text{ kg})(9.8\text{ m/s}^2)\left(\frac{0.91\text{m}}{1.5\text{m}}\right) = 2.7\times10^2\text{ N}.$$

(b) Since the "downhill" displacement is opposite to $\vec{F}$, the work done by the worker is

$$W_1 = -\left(2.7\times10^2\,\text{N}\right)(1.5\text{ m}) = -4.0\times10^2\,\text{J}.$$

(c) Since the displacement has a vertically downward component of magnitude 0.91 m (in the same direction as the force of gravity), we find the work done by gravity to be

$$W_2 = (45\text{ kg})\left(9.8\text{ m/s}^2\right)(0.91\text{ m}) = 4.0\times10^2\,\text{J}.$$

(d) Since $\vec{F}_N$ is perpendicular to the direction of motion of the block, and $\cos 90° = 0$, work done by the normal force is $W_3 = 0$ by Eq. 7-7.

(e) The resultant force $\vec{F}_{\text{net}}$ is zero since there is no acceleration. Thus, its work is zero, as can be checked by adding the above results $W_1 + W_2 + W_3 = 0$.

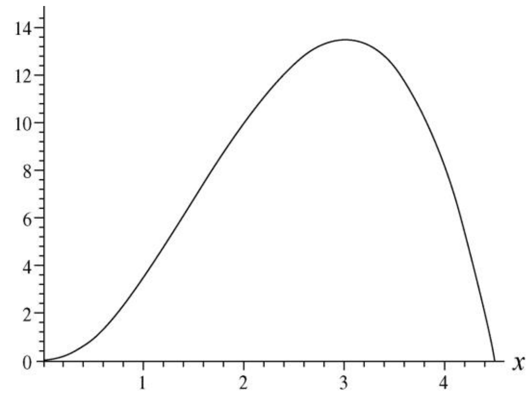
77. (a) To estimate the area under the curve between $x = 1$ m and $x = 3$ m (which should yield the value for the work done), one can try "counting squares" (or half-squares or thirds of squares) between the curve and the axis. Estimates between 5 J and 8 J are typical for this (crude) procedure.

(b) Equation 7-32 gives

$$\int_1^3 \frac{a}{x^2}\ dx = \frac{a}{3} - \frac{a}{1} = 6 \text{ J}$$

where a = –9 N·m^2 is given in the problem statement.

78. (a) Using Eq. 7-32, the work becomes $W = \frac{9}{2}x^2 - x^3$ (SI units understood). The plot is shown below:



(b) We see from the graph that its peak value occurs at x = 3.00 m. This can be verified by taking the derivative of W and setting equal to zero, or simply by noting that this is where the force vanishes.

(c) The maximum value is $W = \frac{9}{2}(3.00)^2 - (3.00)^3$ = 13.50 J.

(d) We see from the graph (or from our analytic expression) that W = 0 at x = 4.50 m.

(e) The case is at rest when $v = 0$. Since $W = \Delta K = mv^2/2$, the condition implies $W = 0$. This happens at x = 4.50 m.

79. **THINK** A box sliding in the +x-direction is slowed down by a steady wind in the –x-direction. The problem involves graphical analysis.

EXPRESS Fig. 7-51 represents $x(t)$, the position of the lunch box as a function of time. It is convenient to fit the curve to a concave-downward parabola:

$$x(t) = \frac{1}{10}t(10-t) = t - \frac{1}{10}t^2.$$

By taking one and two derivatives, we find the velocity and acceleration to be

$$v(t) = \frac{dx}{dt} = 1 - \frac{t}{5} \text{ (in m/s)}, \quad a = \frac{d^2x}{dt^2} = -\frac{1}{5} = -0.2 \text{ (in m/s}^2\text{)}.$$

The equations imply that the initial speed of the box is $v_i = v(0) = 1.0\text{ m/s}$, and the constant force by the wind is

$$F = ma = (2.0\text{ kg})(-0.2\text{ m/s}^2) = -0.40\text{ N}.$$

The corresponding work is given by (SI units understood)

$$W(t) = F \cdot x(t) = -0.04t(10-t).$$

The initial kinetic energy of the lunch box is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(2.0\text{ kg})(1.0\text{ m/s})^2 = 1.0\text{ J}.$$

With $\Delta K = K_f - K_i = W$, the kinetic energy at a later time is given by (in SI units)

$$K(t) = K_i + W = 1 - 0.04t(10-t)$$

ANALYZE (a) When t = 1.0 s, the above expression gives

$$K(1\text{ s}) = 1 - 0.04(1)(10-1) = 1 - 0.36 = 0.64 \approx 0.6\text{ J}$$

where the second significant figure is not to be taken too seriously.

(b) At t = 5.0 s, the above method gives $K(5.0\text{ s}) = 1 - 0.04(5)(10-5) = 1 - 1 = 0.$

(c) The work done by the force from the wind from t = 1.0 s to t = 5.0 s is

$$W = K(5.0) - K(1.0\text{ s}) = 0 - 0.6 \approx -0.6\text{ J}.$$

LEARN The result in (c) can also be obtained by evaluating $W(t) = -0.04t(10-t)$ directly at t = 5.0 s and t = 1.0 s, and subtracting:

$$W(5) - W(1) = -0.04(5)(10-5) - \left[-0.04(1)(10-1)\right] = -1 - (-0.36) = -0.64 \approx -0.6\text{ J}.$$

Note that at t = 5.0 s, K = 0, the box comes to a stop and then reverses its direction subsequently (with x decreasing).

80. The problem indicates that SI units are understood, so the result (of Eq. 7-23) is in joules. Done numerically, using features available on many modern calculators, the result is roughly 0.47 J. For the interested student it might be worthwhile to quote the “exact” answer (in terms of the “error function”):

$$\int_{.15}^{1.2} e^{-2x^2}\,dx = \tfrac{1}{4}\sqrt{2\pi}\,[\mathrm{erf}(6\sqrt{2}/5) - \mathrm{erf}(3\sqrt{2}/20)]\,.$$

81. (a) The work done by the spring force is $W_s = \frac{1}{2}k(x_i^2 - x_f^2)$. By energy conservation, when the block is at $x_f = 0$, the energy stored in the spring is completed converted to the kinetic energy of the block: $W_s = K = \frac{1}{2}mv^2$. Solving for v, we obtain

$$\frac{1}{2}k(x_i^2 - x_f^2) = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{500\text{ N/m}}{4.00\text{ kg}}}(0.300\text{ m}) = 3.35\text{ m/s}\,.$$

(b) The work done by the spring is

$$W_s = \frac{1}{2}kx_i^2 = \frac{1}{2}(500\text{ N/m})(0.300\text{ m})^2 = 22.5\text{ J}\,.$$

(c) The instantaneous power due to the spring can be written as

$$P = Fv = (kx)\sqrt{\frac{k}{m}\left(x_i^2 - x^2\right)}$$

At the release point x_i, the power is zero.

(d) Similarly, at $x = 0$, we also have $P = 0$.

(e) The position where the power is maximum can be found by differentiating P with respect to x, setting $dP/dx = 0$:

$$\frac{dP}{dx} = \frac{k^2(x_i^2 - 2x^2)}{\sqrt{\frac{k(x_i^2 - x^2)}{m}}} = 0$$

which gives $x = \dfrac{x_i}{\sqrt{2}} = \dfrac{(0.300\text{ m})}{\sqrt{2}} = 0.212\text{ m}$.

82. (a) Applying Newton's second law to the x (directed uphill) and y (normal to the inclined plane) axes, we obtain

$$F - mg\sin\theta = ma$$
$$F_N - mg\cos\theta = 0.$$

The second equation allows us to solve for the angle the inclined plane makes with the horizontal:

$$\theta = \cos^{-1}\left(\frac{F_N}{mg}\right) = \cos^{-1}\left(\frac{13.41\text{ N}}{(4.00\text{ kg})(9.8\text{ m/s}^2)}\right) = 70.0°$$

From the equation for the x-axis, we find the acceleration of the block to be

$$a=\frac{F}{m}-g\sin\theta=\frac{50\text{ N}}{4.00\text{ kg}}-(9.8\text{ m/s}^2)\sin 70.0°=3.29\text{ m/s}^2$$

Using the kinematic equation $v^2=v_0^2+2ad$, the speed of the block when d = 3.00 m is

$$v=\sqrt{2ad}=\sqrt{2(3.29\text{ m/s}^2)(3.00\text{ m})}=4.44\text{ m/s}$$

83. (a) The work done by the spring force (with spring constant $k=18\text{ N/cm}=1800\text{ N/m}$) is

$$W_s=\frac{1}{2}k(x_i^2-x_f^2)=-\frac{1}{2}kx_f^2=-\frac{1}{2}(1800\text{ N/m})(7.60\times10^{-3}\text{m})^2=-5.20\times10^{-2}\text{ J}$$

(b) For $x'_f=2x_f$, the work done by the spring force is $W'_s=-\frac{1}{2}kx'^2_f=-\frac{1}{2}k(2x_f)^2$, so the additional work done is

$$\Delta W=W'_s-W_s=-\frac{1}{2}k(2x_f)^2-\left(-\frac{1}{2}kx_f^2\right)=-\frac{3}{2}kx_f^2=3W_s=3(-5.20\times10^{-2}\text{ J})=-0.156\text{ J}$$

84. (a) The displacement of the object is

$$\Delta\vec{r}=\vec{r}_2-\vec{r}_1=(-4.10\hat{\text{i}}+3.30\hat{\text{j}}+5.40\,\hat{\text{k}})-(2.70\hat{\text{i}}-2.90\hat{\text{j}}+5.50\,\hat{\text{k}})=(-6.80\hat{\text{i}}+6.20\hat{\text{j}}-0.10\,\hat{\text{k}})$$

The work done by $\vec{F}=(2.00\hat{\text{i}}+9.00\hat{\text{j}}+5.30\,\hat{\text{k}})\text{N}$ is (in SI units)

$$W=\vec{F}\cdot\Delta\vec{r}=(2.00\hat{\text{i}}+9.00\hat{\text{j}}+5.30\,\hat{\text{k}})\cdot(-6.80\hat{\text{i}}+6.20\hat{\text{j}}-0.10\,\hat{\text{k}})=41.7\text{ J}$$

(b) The average power due to the force during the time interval is

$$P=\frac{W}{\Delta t}=\frac{41.7\text{ J}}{2.10\text{ s}}=19.8\text{ W}$$

(c) The magnitudes of the position vectors are (in SI units)

$$r_1=|\vec{r}_1|=\sqrt{(2.70)^2+(-2.90)^2+(5.50)^2}=6.78$$
$$r_2=|\vec{r}_2|=\sqrt{(-4.10)^2+(3.30)^2+(5.40)^2}=7.54$$

and their dot product is

$$\begin{aligned}\vec{r}_1 \cdot \vec{r}_2 &= (2.70\hat{\mathrm{i}} - 2.90\hat{\mathrm{j}} + 5.50\,\hat{\mathrm{k}}) \cdot (-4.10\hat{\mathrm{i}} + 3.30\hat{\mathrm{j}} + 5.40\,\hat{\mathrm{k}}) \\ &= (2.70)(-4.10) + (-2.90)(3.30) + (5.50)(5.40) = 9.06\end{aligned}$$

Using $\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos\theta,$, the angle between $\vec{r}_1$ and $\vec{r}_2$ is

$$\theta = \cos^{-1}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{r_1 r_2}\right) = \cos^{-1}\left(\frac{9.06}{(6.78)(7.54)}\right) = 79.8°$$

85. The work done by the force is (in SI units)

$$W = \vec{F} \cdot \vec{d} = (-5.00\hat{\mathrm{i}} + 5.00\hat{\mathrm{j}} + 4.00\,\hat{\mathrm{k}}) \cdot (2.00\hat{\mathrm{i}} + 2.00\hat{\mathrm{j}} + 7.00\,\hat{\mathrm{k}}) = 28\ \mathrm{J}$$

By energy conservation, $W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2).$ Thus, the final speed of the particle is

$$v_f = \sqrt{v_i^2 + \frac{2W}{m}} = \sqrt{(4.00\ \mathrm{m/s})^2 + \frac{2(28\ \mathrm{J})}{2.00\ \mathrm{kg}}} = 6.63\ \mathrm{m/s}\ .$$

Chapter 8

1. **THINK** A compressed spring stores potential energy. This exercise explores the relationship between the energy stored and the spring constant.

EXPRESS The potential energy stored by the spring is given by $U = kx^2/2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus, the spring constant is $k = 2U/x^2$.

ANALYZE Substituting $U = 25\text{ J}$ and $x = 7.5\text{ m} = 0.075\text{ cm}$ into the above expression, we find the spring constant to be

$$k = \frac{2U}{x^2} = \frac{2(25\text{ J})}{(0.075\text{ m})^2} = 8.9\times 10^3\text{ N/m}.$$

LEARN The spring constant k has units N/m. The quantity provides a measure of stiffness of the spring, for a given x, the greater the value of k, the greater the potential energy U.

2. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and A is horizontal, so ϕ = 90.0° and $W_g = 0$ (since cos 90.0° = 0).

(b) The displacement between the initial point and B has a vertical component of $h/2$ downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = \frac{1}{2}mgh = \frac{1}{2}(825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 1.70\times 10^5\text{ J}.$$

(c) The displacement between the initial point and C has a vertical component of h downward (same direction as $\vec{F}_g$), so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = (825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 3.40\times 10^5\text{ J}.$$

(d) With the reference position at C, we obtain

$$U_B = \frac{1}{2}mgh = \frac{1}{2}(825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 1.70\times 10^5\text{ J}.$$

(e) Similarly, we find

$$U_A = mgh = (825\text{ kg})(9.80\text{ m/s}^2)(42.0\text{ m}) = 3.40\times10^5\text{ J}.$$

(f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.

3. (a) Noting that the vertical displacement is 10.0 m – 1.50 m = 8.50 m downward (same direction as $\vec{F}_g$), Eq. 7-12 yields

$$W_g = mgd\cos\phi = (2.00\text{ kg})(9.80\text{ m/s}^2)(8.50\text{ m})\cos 0° = 167\text{ J}.$$

(b) One approach (which is fairly trivial) is to use Eq. 8-1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upward understood to be the $+y$ direction). The result is

$$\Delta U = mg(y_f - y_i) = (2.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m} - 10.0\text{ m}) = -167\text{ J}.$$

(c) In part (b) we used the fact that $U_i = mgy_i$ =196 J.

(d) In part (b), we also used the fact $U_f = mgy_f$ = 29 J.

(e) The computation of W_g does not use the new information (that U = 100 J at the ground), so we again obtain W_g = 167 J.

(f) As a result of Eq. 8-1, we must again find $\Delta U = -W_g$ = –167 J.

(g) With this new information (that U_0 = 100 J where y = 0) we have

$$U_i = mgy_i + U_0 = 296\text{ J}.$$

(h) With this new information (that U_0 = 100 J where y = 0) we have

$$U_f = mgy_f + U_0 = 129\text{ J}.$$

We can check part (f) by subtracting the new U_i from this result.

4. (a) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length L of the rod, so the work done by the force of gravity is

$$W = mgL = (0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = 1.51\text{ J}.$$

(b) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to *L*, but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is

$$W = -mgL = -(0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = -1.51\text{ J}.$$

(c) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement.

(d) The force of gravity is conservative. The change in the gravitational potential energy of the ball-Earth system is the negative of the work done by gravity:

$$\Delta U = -mgL = -(0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = -1.51\text{ J}$$

as the ball goes to the lowest point.

(e) Continuing this line of reasoning, we find

$$\Delta U = +mgL = (0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = 1.51\text{ J}$$

as it goes to the highest point.

(f) Continuing this line of reasoning, we have $\Delta U = 0$ as it goes to the point at the same height.

(g) The change in the gravitational potential energy depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the potential energy is the *same* since the initial and final positions are the same.

5. **THINK** As the ice flake slides down the frictionless bowl, its potential energy changes due to the work done by the gravitational force.

EXPRESS The force of gravity is constant, so the work it does is given by $W = \vec{F}\cdot\vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude *mg*, where *m* is the mass of the flake, so this reduces to $W = mgh$, where *h* is the height from which the flake falls. The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: $\Delta U = -W$.

ANALYZE (a) The ice flake falls a distance $h = r = 22.0\text{ cm} = 0.22\text{ m}$. Therefore, the work done by gravity is

$$W = mgr = (2.00\times10^{-3}\text{ kg})(9.8\text{ m/s}^2)(22.0\times10^{-2}\text{ m}) = 4.31\times10^{-3}\text{J}.$$

(b) The change in gravitational potential energy is $\Delta U = -W = -4.31 \times 10^{-3}$ J.

(c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = +4.31 \times 10^{-3}$ J at the top.

(d) If $U = 0$ at the top, then $U = -4.31 \times 10^{-3}$ J at the bottom.

(e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

LEARN While the potential energy depends on the reference point (location where $U = 0$), the change in potential energy, ΔU, does not. In both (c) and (d), we find $\Delta U = -4.31 \times 10^{-3}$ J.

6. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and Q has a vertical component of $h - R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 4mgR = 4(3.20 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.15\ \text{J}.$$

(b) The displacement between the initial point and the top of the loop has a vertical component of $h - 2R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = 3mgR = 3(3.20 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.11\ \text{J}.$$

(c) With $y = h = 5R$, at P we find

$$U = 5mgR = 5(3.20 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.19\ \text{J}.$$

(d) With $y = R$, at Q we have

$$U = mgR = (3.20 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.038\ \text{J}.$$

(e) With $y = 2R$, at the top of the loop, we find

$$U = 2mgR = 2(3.20 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.12\ \text{m}) = 0.075\ \text{J}.$$

(f) The new information $(v_i \neq 0)$ is not involved in any of the preceding computations; the above results are unchanged.

7. The main challenge for students in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the

swing) $h = L - L\cos\theta$ (for angle θ measured from vertical as shown in Fig. 8-34). Once this relation (which we will not derive here since we have found this to be most easily illustrated at the blackboard) is established, then the principal results of this problem follow from Eq. 7-12 (for W_g) and Eq. 8-9 (for U).

(a) The vertical component of the displacement vector is downward with magnitude h, so we obtain

$$\begin{aligned} W_g &= \vec{F}_g \cdot \vec{d} = mgh = mgL(1-\cos\theta) \\ &= (5.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})(1-\cos 30°) = 13.1\text{ J.} \end{aligned}$$

(b) From Eq. 8-1, we have $\Delta U = -W_g = -mgL(1 - \cos\theta) = -13.1$ J.

(c) With $y = h$, Eq. 8-9 yields $U = mgL(1 - \cos\theta) = 13.1$ J.

(d) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos\theta$ decreases so that $1 - \cos\theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b) also increases.

8. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F}\cdot\vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the snowball. The expression for the work reduces to $W = mgh$, where h is the height through which the snowball drops. Thus

$$W = mgh = (1.50\text{ kg})(9.80\text{ m/s}^2)(12.5\text{ m}) = 184\text{ J}.$$

(b) The force of gravity is conservative, so the change in the potential energy of the snowball-Earth system is the negative of the work it does: $\Delta U = -W = -184$ J.

(c) The potential energy when it reaches the ground is less than the potential energy when it is fired by $|\Delta U|$, so $U = -184$ J when the snowball hits the ground.

9. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In Problem 9-2, we found $U_A = mgh$ (with the reference position at C). Referring again to Fig. 8-29, we see that this is the same as U_0, which implies that $K_A = K_0$ and thus that

$$v_A = v_0 = 17.0\text{ m/s}.$$

(b) In the solution to Problem 9-2, we also found $U_B = mgh/2$. In this case, we have

$$K_0 + U_0 = K_B + U_B$$
$$\frac{1}{2}mv_0^2 + mgh = \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right)$$

which leads to

$$v_B = \sqrt{v_0^2 + gh} = \sqrt{(17.0 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(42.0 \text{ m})} = 26.5 \text{ m/s}.$$

(c) Similarly, $v_C = \sqrt{v_0^2 + 2gh} = \sqrt{(17.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(42.0 \text{ m})} = 33.4 \text{ m/s}.$

(d) To find the "final" height, we set $K_f = 0$. In this case, we have

$$K_0 + U_0 = K_f + U_f$$
$$\frac{1}{2}mv_0^2 + mgh = 0 + mgh_f$$

which yields $h_f = h + \frac{v_0^2}{2g} = 42.0 \text{ m} + \frac{(17.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 56.7 \text{ m}.$

(e) It is evident that the above results do not depend on mass. Thus, a different mass for the coaster must lead to the same results.

10. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to Problem 9-3 (to which this problem refers), we found $U_i = mgy_i$ = 196 J and $U_f = mgy_f$ = 29.0 J (assuming the reference position is at the ground). Since $K_i = 0$ in this case, we have

$$0 + 196 \text{ J} = K_f + 29.0 \text{ J}$$

which gives $K_f = 167 \text{ J}$ and thus leads to $v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(167 \text{ J})}{2.00 \text{ kg}}} = 12.9 \text{ m/s}.$

(b) If we proceed algebraically through the calculation in part (a), we find $K_f = -\Delta U = mgh$ where $h = y_i - y_f$ and is positive-valued. Thus,

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as we might also have derived from the equations of Table 2-1 (particularly Eq. 2-16). The fact that the answer is independent of mass means that the answer to part (b) is identical to that of part (a), that is, $v = 12.9 \text{ m/s}$.

(c) If $K_i \neq 0$, then we find $K_f = mgh + K_i$ (where K_i is necessarily positive-valued). This represents a larger value for K_f than in the previous parts, and thus leads to a larger value for *v*.

11. **THINK** As the ice flake slides down the frictionless bowl, its potential energy decreases (discussed in Problem 8-5). By conservation of mechanical energy, its kinetic energy must increase.

EXPRESS If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then

$$K_f + U_f = K_i + U_i .$$

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220$ m is the radius of the bowl and *m* is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $K_f = mv^2/2$.

ANALYZE (a) Energy conservation leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad \frac{1}{2}mv^2 + 0 = 0 + mgr .$$

The speed is $v = \sqrt{2gr} = 2.08$ m/s.

(b) Since the expression for speed is $v = \sqrt{2gr}$, which does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. If K_i is greater than before, then K_f will also be greater. This means the final speed of the flake is greater.

LEARN The mechanical energy conservation principle can also be expressed as $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$, which implies $\Delta K = -\Delta U$, i.e., the increase in kinetic energy is equal to the negative of the change in potential energy.

12. We use Eq. 8-18, representing the conservation of mechanical energy. We choose the reference position for computing *U* to be at the ground below the cliff; it is also regarded as the “final” position in our calculations.

(a) Using Eq. 8-9, the initial potential energy is given by $U_i = mgh$ where $h = 12.5$ m and $m = 1.50$ kg. Thus, we have

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv^2 + 0$$

which leads to the speed of the snowball at the instant before striking the ground:

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_i^2 + mgh\right)} = \sqrt{v_i^2 + 2gh}$$

where v_i = 14.0 m/s is the magnitude of its initial velocity (not just one component of it). Thus we find v = 21.0 m/s.

(b) As noted above, v_i is the magnitude of its initial velocity and not just one component of it; therefore, there is no dependence on launch angle. The answer is again 21.0 m/s.

(c) It is evident that the result for v in part (a) does not depend on mass. Thus, changing the mass of the snowball does not change the result for v.

13. **THINK** As the marble moves vertically upward, its gravitational potential energy increases. This energy comes from the release of elastic potential energy stored in the spring.

EXPRESS We take the reference point for gravitational potential energy to be at the position of the marble when the spring is compressed. The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$. On the other had, the energy stored in the spring is $U_s = kx^2/2$. Applying mechanical energy conservation principle allows us to solve the problem.

ANALYZE (a) The height of the highest point is h = 20 m. With initial gravitational potential energy set to zero, we find

$$\Delta U_g = mgh = (5.0\times10^{-3}\text{ kg})(9.8\text{ m/s}^2)(20\text{ m}) = 0.98\text{ J}.$$

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is U_s = 0.98 J. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{0.98\text{ J}}{(0.080\text{ m})^2} = 3.1\times10^2\text{ N/m} = 3.1\text{ N/cm}.$$

LEARN In general, the marble has both kinetic and potential energies:

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + mgy$$

At the maximum height $y_{\max} = h, \quad v = 0$ and $mgh = kx^2/2$, or $h = \dfrac{kx^2}{2mg}$.

14. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) The change in potential energy is $\Delta U = mgL$ as it goes to the highest point. Thus, we have

$$\Delta K + \Delta U = 0$$
$$K_{\text{top}} - K_0 + mgL = 0$$

which, upon requiring $K_{\text{top}} = 0$, gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL} = \sqrt{2(9.80 \text{ m/s}^2)(0.452 \text{ m})} = 2.98 \text{ m/s}.$$

(b) We also found in Problem 9-4 that the potential energy change is $\Delta U = -mgL$ in going from the initial point to the lowest point (the bottom). Thus,

$$\Delta K + \Delta U = 0$$
$$K_{\text{bottom}} - K_0 - mgL = 0$$

which, with $K_0 = mgL$, leads to $K_{\text{bottom}} = 2mgL$. Therefore,

$$v_{\text{bottom}} = \sqrt{\frac{2K_{\text{bottom}}}{m}} = \sqrt{4gL} = \sqrt{4(9.80 \text{ m/s}^2)(0.452 \text{ m})} = 4.21 \text{ m/s}.$$

(c) Since there is no change in height (going from initial point to the rightmost point), then $\Delta U = 0$, which implies $\Delta K = 0$. Consequently, the speed is the same as what it was initially,

$$v_{\text{right}} = v_0 = 2.98 \text{ m/s}.$$

(d) It is evident from the above manipulations that the results do not depend on mass. Thus, a different mass for the ball must lead to the same results.

15. **THINK** The truck with failed brakes is moving up an escape ramp. In order for it to come to a complete stop, all of its kinetic energy must be converted into gravitational potential energy.

EXPRESS We ignore any work done by friction. In SI units, the initial speed of the truck just before entering the escape ramp is v_i = 130(1000/3600) = 36.1 m/s. When the truck comes to a stop, its kinetic and potential energies are K_f = 0 and $U_f = mgh$. We apply mechanical energy conservation to solve the problem.

ANALYZE (a) Energy conservation implies $K_f + U_f = K_i + U_i$. With U_i = 0, and $K_i = \frac{1}{2}mv_i^2$, we obtain

$$\frac{1}{2}mv_i^2 + 0 = 0 + mgh \quad \Rightarrow \quad h = \frac{v_i^2}{2g} = \frac{(36.1\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 66.5\text{ m}.$$

If L is the minimum length of the ramp, then $L\sin\theta = h$, or L sin 15° = 66.5 m so that $L = (66.5\text{ m})/\sin 15° = 257\text{ m}$. That is, the ramp must be about 2.6×10^2 m long if friction is negligible.

(b) The minimum length is $L = \dfrac{h}{\sin\theta} = \dfrac{v_i^2}{2g\sin\theta}$ which does not depend on the mass of the truck. Thus, the answer remains the same if the mass is reduced.

(c) If the speed is decreased, then h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

LEARN The greater the speed of the truck, the longer the ramp required. This length can be shortened considerably if the friction between the tires and the ramp surface is factored in.

16. We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use x for the spring's compression, measured positively downward (so $x > 0$ means it is compressed).

(a) With x = 0.190 m, Eq. 7-26 gives

$$W_s = -\frac{1}{2}kx^2 = -7.22\text{ J} \approx -7.2\text{ J}$$

for the work done by the spring force. Using Newton's third law, we see that the work done on the spring is 7.2 J.

(b) As noted above, W_s = –7.2 J.

(c) Energy conservation leads to

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + mgh_0 = \frac{1}{2}kx^2 - mgx$$

which (with m = 0.70 kg) yields h_0 = 0.86 m.

(d) With a new value for the height $h_0' = 2h_0 = 1.72\ \text{m}$, we solve for a new value of x using the quadratic formula (taking its positive root so that $x > 0$).

$$mgh_0' = -mgx + \frac{1}{2}kx^2 \;\Rightarrow\; x = \frac{mg + \sqrt{(mg)^2 + 2mgkh_0'}}{k}$$

which yields x = 0.26 m.

17. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$K_P + U_P = K_Q + U_Q$$
$$0 + mgh = \frac{1}{2}mv^2 + mgR$$

Using the fact that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is

$$F = mv^2/R = 8mg = 8(0.032\ \text{kg})(9.8\ \text{m/s}^2) = 2.5\ \text{N}.$$

The direction is to the left (in the same direction as $\vec{a}$).

(b) The downward component of the net force on the block at Q is the downward force of gravity

$$F = mg = (0.032\ \text{kg})(9.8\ \text{m/s}^2) = 0.31\ \text{N}.$$

(c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \;\Rightarrow\; mv_t^2 = mgR.$$

This requires a different value of h than what was used above.

$$K_P + U_P = K_t + U_t$$
$$0 + mgh = \frac{1}{2}mv_t^2 + mgh_t$$
$$mgh = \frac{1}{2}(mgR) + mg(2R)$$

Consequently, $h = 2.5R$ = (2.5)(0.12 m) = 0.30 m.

(d) The normal force F_N, for speeds v_t greater than $\sqrt{gR}$ (which are the only possibilities for nonzero F_N — see the solution in the previous part), obeys

$$F_N = \frac{mv_t^2}{R} - mg$$

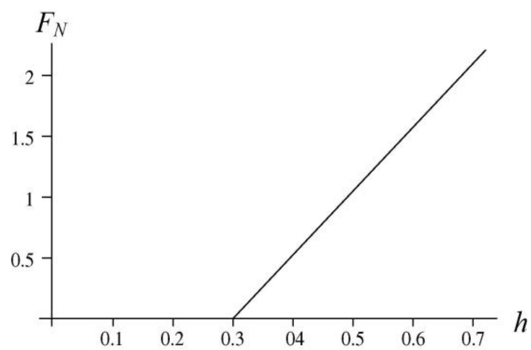
from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \Rightarrow gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ — see the solution in the previous part), becomes

$$F_N = \frac{2mgh}{R} - 5mg\ .$$

Thus, the graph for $h \geq 2.5R = 0.30$ m consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes). Note that for $h \leq 2.5R$, the normal force is zero.



18. We use Eq. 8-18, representing the conservation of mechanical energy. The reference position for computing U is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) The potential energy is $U = mgL(1 - \cos\theta)$ at the position shown in Fig. 8-34 (which we consider to be the initial position). Thus, we have

$$K_i + U_i = K_f + U_f$$
$$0 + mgL(1-\cos\theta) = \frac{1}{2}mv^2 + 0$$

which leads to

$$v = \sqrt{\frac{2mgL(1-\cos\theta)}{m}} = \sqrt{2gL(1-\cos\theta)}.$$

Plugging in L = 2.00 m and θ = 30.0° we find v = 2.29 m/s.

(b) It is evident that the result for v does not depend on mass. Thus, a different mass for the ball must not change the result.

19. We convert to SI units and choose upward as the $+y$ direction. Also, the relaxed position of the top end of the spring is the origin, so the initial compression of the spring (defining an equilibrium situation between the spring force and the force of gravity) is y_0 = –0.100 m and the additional compression brings it to the position y_1 = –0.400 m.

(a) When the stone is in the equilibrium (a = 0) position, Newton's second law becomes

$$\begin{aligned} \vec{F}_{\text{net}} &= ma \\ F_{\text{spring}} - mg &= 0 \\ -k(-0.100) - (8.00)(9.8) &= 0 \end{aligned}$$

where Hooke's law (Eq. 7-21) has been used. This leads to a spring constant equal to $k = 784\ \text{N/m}$.

(b) With the additional compression (and release) the acceleration is no longer zero, and the stone will start moving upward, turning some of its elastic potential energy (stored in the spring) into kinetic energy. The amount of elastic potential energy at the moment of release is, using Eq. 8-11,

$$U = \frac{1}{2}ky_1^2 = \frac{1}{2}(784\ \text{N/m})(-0.400)^2 = 62.7\ \text{J}\,.$$

(c) Its maximum height y_2 is beyond the point that the stone separates from the spring (entering free-fall motion). As usual, it is characterized by having (momentarily) zero speed. If we choose the y_1 position as the reference position in computing the gravitational potential energy, then

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + \frac{1}{2}ky_1^2 &= 0 + mgh \end{aligned}$$

where $h = y_2 - y_1$ is the height above the release point. Thus, mgh (the gravitational potential energy) is seen to be equal to the previous answer, 62.7 J, and we proceed with the solution in the next part.

(d) We find $h = ky_1^2/2mg = 0.800\ \text{m}$, or 80.0 cm.

20. (a) We take the reference point for gravitational energy to be at the lowest point of the swing. Let θ be the angle measured from vertical. Then the height y of the pendulum "bob" (the object at the end of the pendulum, which in this problem is the stone) is given by $L(1 - \cos\theta) = y$. Hence, the gravitational potential energy is

$$mg\,y = mgL(1 - \cos\theta).$$

When $\theta = 0°$ (the string at its lowest point) we are told that its speed is 8.0 m/s; its kinetic energy there is therefore 64 J (using Eq. 7-1). At $\theta = 60°$ its mechanical energy is

$$E_{\text{mech}} = \frac{1}{2}\, mv^2 + mgL(1 - \cos\theta)\ .$$

Energy conservation (since there is no friction) requires that this be equal to 64 J. Solving for the speed, we find $v = 5.0$ m/s.

(b) We now set the above expression again equal to 64 J (with θ being the unknown) but with zero speed (which gives the condition for the maximum point, or "turning point" that it reaches). This leads to $\theta_{\max} = 79°$.

(c) As observed in our solution to part (a), the total mechanical energy is 64 J.

21. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects). The reference position for computing U (and height h) is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) Careful examination of the figure leads to the trigonometric relation $h = L - L\cos\theta$ when the angle is measured from vertical as shown. Thus, the gravitational potential energy is $U = mgL(1 - \cos\theta_0)$ at the position shown in Fig. 8-34 (the initial position). Thus, we have

$$K_0 + U_0 = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + mgL\,(1 - \cos\theta_0) = \frac{1}{2}mv^2 + 0$$

which leads to

$$v = \sqrt{\frac{2}{m}\left[\frac{1}{2}mv_0^2 + mgL(1 - \cos\theta_0)\right]} = \sqrt{v_0^2 + 2gL(1 - \cos\theta_0)}$$

$$= \sqrt{(8.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(1.25\text{ m})(1 - \cos 40°)} = 8.35\text{ m/s}.$$

(b) We look for the initial speed required to barely reach the horizontal position — described by $v_h = 0$ and $\theta = 90°$ (or $\theta = -90°$, if one prefers, but since $\cos(-\phi) = \cos\phi$, the sign of the angle is not a concern).

$$K_0 + U_0 = K_h + U_h$$

$$\frac{1}{2}mv_0^2 + mgL\,(1 - \cos\theta_0) = 0 + mgL$$

which yields

$$v_0 = \sqrt{2gL\cos\theta_0} = \sqrt{2(9.80\text{ m/s}^2)(1.25\text{ m})\cos 40°} = 4.33\text{ m/s}.$$

(c) For the cord to remain straight, then the centripetal force (at the top) must be (at least) equal to gravitational force:

$$\frac{mv_t^2}{r} = mg \Rightarrow mv_t^2 = mgL$$

where we recognize that $r = L$. We plug this into the expression for the kinetic energy (at the top, where $\theta = 180°$).

$$K_0 + U_0 = K_t + U_t$$

$$\frac{1}{2}mv_0^2 + mgL\left(1-\cos\theta_0\right) = \frac{1}{2}mv_t^2 + mg\left(1-\cos 180°\right)$$

$$\frac{1}{2}mv_0^2 + mgL\left(1-\cos\theta_0\right) = \frac{1}{2}(mgL) + mg(2L)$$

which leads to

$$v_0 = \sqrt{gL(3+2\cos\theta_0)} = \sqrt{(9.80\text{ m/s}^2)(1.25\text{ m})(3+2\cos 40°)} = 7.45\text{ m/s}.$$

(d) The more initial potential energy there is, the less initial kinetic energy there needs to be, in order to reach the positions described in parts (b) and (c). Increasing θ_0 amounts to increasing U_0, so we see that a greater value of θ_0 leads to smaller results for v_0 in parts (b) and (c).

22. From Chapter 4, we know the height h of the skier's jump can be found from $v_y^2 = 0 = v_{0y}^2 - 2gh$ where $v_{0y} = v_0 \sin 28°$ is the upward component of the skier's "launch velocity." To find v_0 we use energy conservation.

(a) The skier starts at rest $y = 20$ m above the point of "launch" so energy conservation leads to

$$mgy = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy} = 20\text{ m/s}$$

which becomes the initial speed v_0 for the launch. Hence, the above equation relating h to v_0 yields

$$h = \frac{\left(v_0 \sin 28°\right)^2}{2g} = 4.4\text{ m}.$$

(b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.

23. (a) As the string reaches its lowest point, its original potential energy $U = mgL$ (measured relative to the lowest point) is converted into kinetic energy. Thus,

$$mgL = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gL} \; .$$

With L = 1.20 m we obtain $v = \sqrt{2gL} = \sqrt{2(9.80\,\text{m/s}^2)(1.20\text{ m})} = 4.85\,\text{m/s}$.

(b) In this case, the total mechanical energy is shared between kinetic $\frac{1}{2}mv_b^2$ and potential mgy_b. We note that $y_b = 2r$ where $r = L - d = 0.450$ m. Energy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

which yields $v_b = \sqrt{2gL - 2g(2r)} = 2.42\text{ m/s}$.

24. We denote m as the mass of the block, h = 0.40 m as the height from which it dropped (measured from the relaxed position of the spring), and x as the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$K_i + U_i = K_f + U_f$$

$$0 = -mg(h+x) + \frac{1}{2}kx^2$$

which is a second degree equation in x. Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k} \; .$$

Now mg = 19.6 N, h = 0.40 m, and $k = 1960\text{ N/m}$, and we choose the positive root so that $x > 0$.

$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10\text{ m} \; .$$

25. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this t = 6.0 s flight.

$$\Delta y = v_{0y}t - \frac{1}{2}gt^2$$

This leads to $\Delta y = -32\text{ m}$. Therefore $\Delta U = mg\Delta y = -318\text{ J} \approx -3.2\times10^{-2}\text{ J}$.

26. (a) With energy in joules and length in meters, we have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)dx' \ .$$

Therefore, with U (0) = 27 J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 \ .$$

(b) We can maximize the above function by working through the $dU/dx = 0$ condition, or we can treat this as a force equilibrium situation — which is the approach we show.

$$F = 0 \Rightarrow 6x_{eq} - 12 = 0$$

Thus, x_{eq} = 2.0 m, and the above expression for the potential energy becomes U = 39 J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the negative value of x for which U = 0 to be x = –1.6 m.

(d) Similarly, we find the positive value of x for which U = 0 to be x = 5.6 m.

27. (a) To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine — if it didn't break — would have the greatest tension. Choosing upward positive, Newton's second law leads to

$$T - mg = m\frac{v^2}{r}$$

where r = 18.0 m and $m = W/g = 688/9.8 = 70.2$ kg. We find the v^2 from energy conservation (where the reference position for the potential energy is at the lowest point).

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = 2gh$$

where h = 3.20 m. Combining these results, we have

$$T = mg + m\frac{2gh}{r} = mg\left(1 + \frac{2h}{r}\right)$$

which yields 933 N. Thus, the vine does not break.

(b) Rounding to an appropriate number of significant figures, we see the maximum tension is roughly 9.3×10^2 N.

28. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10\,\mathrm{N/cm} = 10\,\mathrm{N/m}.$$

(a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}}$$

which yields v = 2.8 m/s for m = 0.0038 kg and x = 0.055 m.

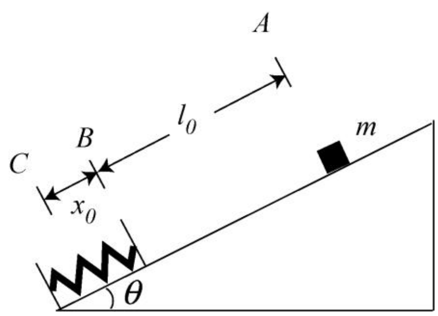
(b) The new scenario involves some potential energy at the moment of release. With d = 0.015 m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \Rightarrow v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

which yields v = 2.7 m/s.

29. **THINK** As the block slides down the inclined plane, it compresses the spring, then stops momentarily before sliding back up again.

EXPRESS We refer to its starting point as A, the point where it first comes into contact with the spring as B, and the point where the spring is compressed by $x_0 = 0.055$ m as C (see the figure below). Point C is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed.



Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270\ \mathrm{N}}{0.02\ \mathrm{m}} = 1.35 \times 10^4\ \mathrm{N/m}\ .$$

The distance between points *A* and *B* is l_0 and we note that the total sliding distance $l_0 + x_0$ is related to the initial height h_A of the block (measured relative to *C*) by $\sin\theta = \dfrac{h_A}{l_0 + x_0}$, where the incline angle θ is 30°.

ANALYZE (a) Mechanical energy conservation leads to

$$K_A + U_A = K_C + U_C \quad \Rightarrow \quad 0 + mgh_A = \frac{1}{2}kx_0^2$$

which yields

$$h_A = \frac{kx_0^2}{2mg} = \frac{(1.35\times10^4 \text{ N/m})(0.055 \text{ m})^2}{2(12 \text{ kg})(9.8 \text{ m/s}^2)} = 0.174 \text{ m}.$$

Therefore, the total distance traveled by the block before coming to a stop is

$$l_0 + x_0 = \frac{h_A}{\sin 30°} = \frac{0.174 \text{ m}}{\sin 30°} = 0.347 \text{ m} \approx 0.35 \text{ m}.$$

(b) From this result, we find $l_0 = x_0 = 0.347 \text{ m} - 0.055 \text{ m} = 0.292 \text{ m}$, which means that the block has descended a vertical distance

$$|\Delta y| = h_A - h_B = l_0 \sin\theta = (0.292 \text{ m})\sin 30° = 0.146 \text{ m}$$

in sliding from point *A* to point *B*. Thus, using Eq. 8-18, we have

$$0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B \quad \Rightarrow \quad \frac{1}{2}mv_B^2 = mg\,|\Delta y|$$

which yields $v_B = \sqrt{2g\,|\Delta y|} = \sqrt{2(9.8 \text{ m/s}^2)(0.146 \text{ m})} = 1.69 \text{ m/s} \approx 1.7 \text{ m/s}$.

LEARN Energy is conserved in the process. The total energy of the block at position *B* is

$$E_B = \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}(12 \text{ kg})(1.69 \text{ m/s})^2 + (12 \text{ kg})(9.8 \text{ m/s}^2)(0.028 \text{ m}) = 20.4 \text{ J},$$

which is equal to the elastic potential energy in the spring:

$$\frac{1}{2}kx_0^2 = \frac{1}{2}(1.35\times10^4 \text{ N/m})(0.055 \text{ m})^2 = 20.4 \text{ J}.$$

30. We take the original height of the box to be the $y = 0$ reference level and observe that, in general, the height of the box (when the box has moved a distance d downhill) is $y = -d\sin 40°$.

(a) Using the conservation of energy, we have

$$K_i + U_i = K + U \Rightarrow 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2.$$

Therefore, with $d = 0.10$ m, we obtain $v = 0.81$ m/s.

(b) We look for a value of $d \neq 0$ such that $K = 0$.

$$K_i + U_i = K + U \Rightarrow 0 + 0 = 0 + mgy + \frac{1}{2}kd^2.$$

Thus, we obtain $mgd\ \sin 40° = \frac{1}{2}kd^2$ and find $d = 0.21$ m.

(c) The uphill force is caused by the spring (Hooke's law) and has magnitude $kd = 25.2$ N. The downhill force is the component of gravity $mg\sin 40° = 12.6$ N. Thus, the net force on the box is (25.2 – 12.6) N = 12.6 N uphill, with

$$a = F/m = (12.6\ \text{N})/(2.0\ \text{kg}) = 6.3\ \text{m/s}^2.$$

(d) The acceleration is up the incline.

31. The reference point for the gravitational potential energy U_g (and height h) is at the block when the spring is maximally compressed. When the block is moving to its highest point, it is first accelerated by the spring; later, it separates from the spring and finally reaches a point where its speed v_f is (momentarily) zero. The x axis is along the incline, pointing uphill (so x_0 for the initial compression is negative-valued); its origin is at the relaxed position of the spring. We use SI units, so $k = 1960$ N/m and $x_0 = -0.200$ m.

(a) The elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2\ \text{J}$.

(b) Since initially $U_g = 0$, the change in U_g is the same as its final value mgh where m = 2.00 kg. That this must equal the result in part (a) is made clear in the steps shown in the next part. Thus, $\Delta U_g = U_g = 39.2$ J.

(c) The principle of mechanical energy conservation leads to

$$K_0 + U_0 = K_f + U_f$$

$$0 + \frac{1}{2}kx_0^2 = 0 + mgh$$

which yields h = 2.00 m. The problem asks for the distance *along the incline*, so we have $d = h/\sin 30° = 4.00$ m.

32. The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy, we note that the mass of a segment is $(m/L)\,dy$ and the change in potential energy of a segment when it is a distance $|y|$ below the table top is

$$dU = (m/L)g|y|\,dy = -(m/L)gy\,dy$$

since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total potential energy change is

$$\Delta U = -\frac{mg}{L}\int_{-L/4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}(L/4)^2 = mgL/32.$$

The work required to pull the chain onto the table is therefore

$$W = \Delta U = mgL/32 = (0.012 \text{ kg})(9.8 \text{ m/s}^2)(0.28 \text{ m})/32 = 0.0010 \text{ J}.$$

33. All heights h are measured from the lower end of the incline (which is our reference position for computing gravitational potential energy mgh). Our x axis is along the incline, with $+x$ being uphill (so spring compression corresponds to $x > 0$) and its origin being at the relaxed end of the spring. The height that corresponds to the canister's initial position (with spring compressed amount x = 0.200 m) is given by $h_1 = (D+x)\sin\theta$, where $\theta = 37°$.

(a) Energy conservation leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_2^2 + mgD\sin\theta$$

which yields, using the data m = 2.00 kg and k = 170 N/m,

$$v_2 = \sqrt{2gx\sin\theta + kx^2/m} = 2.40 \text{ m/s}.$$

(b) In this case, energy conservation leads to

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mg(D+x)\sin\theta + \frac{1}{2}kx^2 = \frac{1}{2}mv_3^2 + 0$$

which yields $v_3 = \sqrt{2g(D+x)\sin\theta + kx^2/m} = 4.19$ m/s.

34. Let $\vec{F}_N$ be the normal force of the ice on him and m is his mass. The net inward force is $mg\cos\theta - F_N$ and, according to Newton's second law, this must be equal to mv^2/R, where v is the speed of the boy. At the point where the boy leaves the ice $F_N = 0$, so $g\cos\theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is

$$U = -mgR(1 - \cos\theta).$$

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives

$$0 = \tfrac{1}{2}mv^2 - mgR(1-\cos\theta),$$

or $v^2 = 2gR(1 - \cos\theta)$. We substitute this expression into the equation developed from the second law to obtain $g\cos\theta = 2g(1 - \cos\theta)$. This gives $\cos\theta = 2/3$. The height of the boy above the bottom of the mound is

$$h = R\cos\theta = \frac{2}{3}R = \frac{2}{3}(13.8\text{ m}) = 9.20\text{ m}.$$

35. (a) The (final) elastic potential energy is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(431\text{ N/m})(0.210\text{ m})^2 = 9.50\text{ J}.$$

Ultimately this must come from the original (gravitational) energy in the system mgy (where we are measuring y from the lowest "elevation" reached by the block, so

$$y = (d + x)\sin(30^\circ).$$

Thus,

$$mg(d + x)\sin(30^\circ) = 9.50\text{ J} \quad\Rightarrow\quad d = 0.396\text{ m}.$$

(b) The block is still accelerating (due to the component of gravity along the incline, $mg\sin(30^\circ)$) for a few moments after coming into contact with the spring (which exerts the Hooke's law force kx), until the Hooke's law force is strong enough to cause the block to begin decelerating. This point is reached when

$$kx = mg\sin 30^\circ$$

which leads to $x = 0.0364$ m = 3.64 cm; this is long before the block finally stops (36.0 cm before it stops).

36. The distance the marble travels is determined by its initial speed (and the methods of Chapter 4), and the initial speed is determined (using energy conservation) by the original compression of the spring. We denote h as the height of the table, and x as the horizontal

distance to the point where the marble lands. Then $x = v_0 t$ and $h = \frac{1}{2}gt^2$ (since the vertical component of the marble's "launch velocity" is zero). From these we find $x = v_0\sqrt{2h/g}$. We note from this that the distance to the landing point is directly proportional to the initial speed. We denote v_{01} be the initial speed of the first shot and D_1 = (2.20 – 0.27) m = 1.93 m be the horizontal distance to its landing point; similarly, v_{02} is the initial speed of the second shot and D = 2.20 m is the horizontal distance to its landing spot. Then

$$\frac{v_{02}}{v_{01}} = \frac{D}{D_1} \quad \Rightarrow \quad v_{02} = \frac{D}{D_1}v_{01}$$

When the spring is compressed an amount ℓ, the elastic potential energy is $\frac{1}{2}k\ell^2$. When the marble leaves the spring its kinetic energy is $\frac{1}{2}mv_0^2$. Mechanical energy is conserved: $\frac{1}{2}mv_0^2 = \frac{1}{2}k\ell^2$, and we see that the initial speed of the marble is directly proportional to the original compression of the spring. If ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second, then $v_{02} = (\ell_2/\ell_1)v_{01}$. Relating this to the previous result, we obtain

$$\ell_2 = \frac{D}{D_1}\ell_1 = \left(\frac{2.20\text{ m}}{1.93\text{ m}}\right)(1.10\text{ cm}) = 1.25\text{ cm}.$$

37. Consider a differential element of length dx at a distance x from one end (the end that remains stuck) of the cord. As the cord turns vertical, its change in potential energy is given by

$$dU = -(\lambda dx)gx$$

where $\lambda = m/h$ is the mass/unit length and the negative sign indicates that the potential energy decreases. Integrating over the entire length, we obtain the total change in the potential energy:

$$\Delta U = \int dU = -\int_0^h \lambda gx\,dx = -\frac{1}{2}\lambda gh^2 = -\frac{1}{2}mgh.$$

With m = 15 g and h = 25 cm, we have $\Delta U = -0.018$ J.

38. In this problem, the mechanical energy (the sum of K and U) remains constant as the particle moves.

(a) Since mechanical energy is conserved, $U_B + K_B = U_A + K_A$, the kinetic energy of the particle in region A ($3.00\text{ m} \le x \le 4.00\text{ m}$) is

$$K_A = U_B - U_A + K_B = 12.0\text{ J} - 9.00\text{ J} + 4.00\text{ J} = 7.00\text{ J}.$$

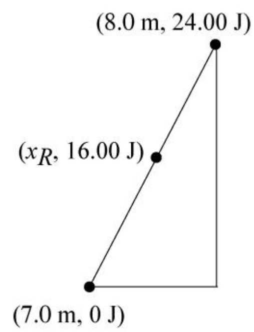
With $K_A = mv_A^2/2$, the speed of the particle at $x = 3.5$ m (within region A) is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(7.00\text{ J})}{0.200\text{ kg}}} = 8.37\text{ m/s}.$$

(b) At $x = 6.5$ m, $U = 0$ and $K = U_B + K_B = 12.0\text{ J} + 4.00\text{ J} = 16.0\text{ J}$ by mechanical energy conservation. Therefore, the speed at this point is

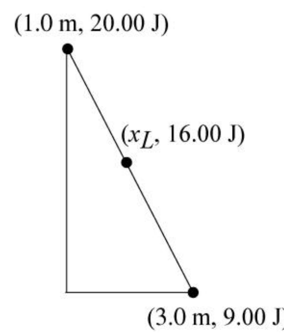
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(16.0\text{ J})}{0.200\text{ kg}}} = 12.6\text{ m/s}.$$

(c) At the turning point, the speed of the particle is zero. Let the position of the right turning point be x_R. From the figure shown on the right, we find x_R to be



$$\frac{16.00\text{ J} - 0}{x_R - 7.00\text{ m}} = \frac{24.00\text{ J} - 16.00\text{ J}}{8.00\text{ m} - x_R} \quad \Rightarrow \quad x_R = 7.67\text{ m}.$$

(d) Let the position of the left turning point be x_L. From the figure shown, we find x_L to be



$$\frac{16.00\text{ J} - 20.00\text{ J}}{x_L - 1.00\text{ m}} = \frac{9.00\text{ J} - 16.00\text{ J}}{3.00\text{ m} - x_L} \quad \Rightarrow \quad x_L = 1.73\text{ m}.$$

39. From the figure, we see that at x = 4.5 m, the potential energy is U_1 = 15 J. If the speed is v = 7.0 m/s, then the kinetic energy is

$$K_1 = mv^2/2 = (0.90\text{ kg})(7.0\text{ m/s})^2/2 = 22\text{ J}.$$

The total energy is $E_1 = U_1 + K_1 = (15 + 22)\text{ J} = 37\text{ J}$.

(a) At x = 1.0 m, the potential energy is U_2 = 35 J. By energy conservation, we have K_2 = 2.0 J > 0. This means that the particle can reach there with a corresponding speed

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(2.0\text{ J})}{0.90\text{ kg}}} = 2.1\text{ m/s}.$$

(b) The force acting on the particle is related to the potential energy by the negative of the slope:

$$F_x = -\frac{\Delta U}{\Delta x}$$

From the figure we have $F_x = -\dfrac{35\text{ J} - 15\text{ J}}{2\text{ m} - 4\text{ m}} = +10\text{ N}$.

(c) Since the magnitude $F_x > 0$, the force points in the +*x* direction.

(d) At x = 7.0 m, the potential energy is U_3 = 45 J, which exceeds the initial total energy E_1. Thus, the particle can never reach there. At the turning point, the kinetic energy is zero. Between x = 5 and 6 m, the potential energy is given by

$$U(x) = 15 + 30(x - 5), \qquad 5 \le x \le 6.$$

Thus, the turning point is found by solving $37 = 15 + 30(x - 5)$, which yields x = 5.7 m.

(e) At x = 5.0 m, the force acting on the particle is

$$F_x = -\frac{\Delta U}{\Delta x} = -\frac{(45 - 15)\text{ J}}{(6 - 5)\text{ m}} = -30\text{ N}.$$

The magnitude is $|F_x| = 30\text{ N}$.

(f) The fact that $F_x < 0$ indicated that the force points in the –*x* direction.

40. (a) The force at the equilibrium position $r = r_{\text{eq}}$ is

$$F = -\left.\frac{dU}{dr}\right|_{r = r_{\text{eq}}} = 0 \quad \Rightarrow \quad -\frac{12A}{r_{\text{eq}}^{13}} + \frac{6B}{r_{\text{eq}}^{7}} = 0$$

which leads to the result

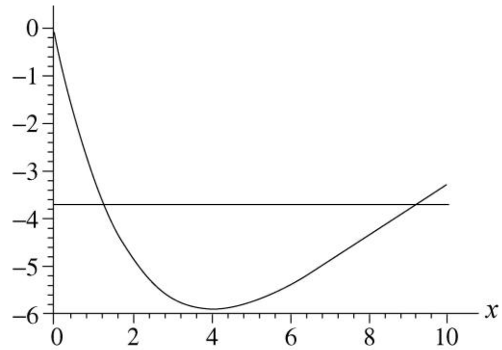
$$r_{\text{eq}} = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12\left(\frac{A}{B}\right)^{\frac{1}{6}}.$$

(b) This defines a minimum in the potential energy curve (as can be verified either by a graph or by taking another derivative and verifying that it is concave upward at this point), which means that for values of r slightly smaller than r_{eq} the slope of the curve is negative (so the force is positive, repulsive).

(c) And for values of r slightly larger than r_{eq} the slope of the curve must be positive (so the force is negative, attractive).

41. (a) The energy at x = 5.0 m is $E = K + U$ = 2.0 J – 5.7 J = –3.7 J.

(b) A plot of the potential energy curve (SI units understood) and the energy E (the horizontal line) is shown for $0 \le x \le 10$ m.



(c) The problem asks for a graphical determination of the turning points, which are the points on the curve corresponding to the total energy computed in part (a). The result for the smallest turning point (determined, to be honest, by more careful means) is $x = 1.3$ m.

(d) And the result for the largest turning point is $x = 9.1$ m.

(e) Since $K = E - U$, then maximizing K involves finding the minimum of U. A graphical determination suggests that this occurs at $x = 4.0$ m, which plugs into the expression $E - U = -3.7 - (-4xe^{-x/4})$ to give $K = 2.16 \text{ J} \approx 2.2 \text{ J}$. Alternatively, one can measure from the graph from the minimum of the U curve up to the level representing the total energy E and thereby obtain an estimate of K at that point.

(f) As mentioned in the previous part, the minimum of the U curve occurs at $x = 4.0$ m.

(g) The force (understood to be in newtons) follows from the potential energy, using Eq. 8-20 (and Appendix E if students are unfamiliar with such derivatives).

$$F = \frac{dU}{dx} = (4 - x)e^{-x/4}$$

(h) This revisits the considerations of parts (d) and (e) (since we are returning to the minimum of $U(x)$) — but now with the advantage of having the analytic result of part (g). We see that the location that produces $F = 0$ is exactly $x = 4.0$ m.

42. Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F \cos \theta$ (where $\theta = 32°$) must equal the friction force magnitude $f_k = \mu_k F_N$. Also, the vertical forces must cancel, implying

$$W_{\text{applied}} = (8.0\text{N})(0.70\text{m}) = 5.6 \text{ J}$$

which is solved to find $F = 71$ N.

(a) The work done on the block by the worker is, using Eq. 7-7,

$$W = Fd\cos\theta = (71\text{ N})(9.2\text{ m})\cos 32° = 5.6\times10^2\text{ J}.$$

(b) Since $f_k = \mu_k\,(mg + F\sin\theta)$, we find $\Delta E_{\text{th}} = f_k d = (60\,\text{N})(9.2\,\text{m}) = 5.6\times10^2$ J.

43. (a) Using Eq. 7-8, we have $W_{\text{applied}} = (8.0\,\text{N})(0.70\,\text{m}) = 5.6$ J.

(b) Using Eq. 8-31, the thermal energy generated is $\Delta E_{\text{th}} = f_k d = (5.0\,\text{N})(0.70\,\text{m}) = 3.5$ J.

44. (a) The work is $W = Fd$ = (35.0 N)(3.00 m) = 105 J.

(b) The total amount of energy that has gone to thermal forms is (see Eq. 8-31 and Eq. 6-2)

$$\Delta E_{\text{th}} = \mu_k mgd = (0.600)(4.00\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 70.6\text{ J}.$$

If 40.0 J has gone to the block then (70.6 – 40.0) J = 30.6 J has gone to the floor.

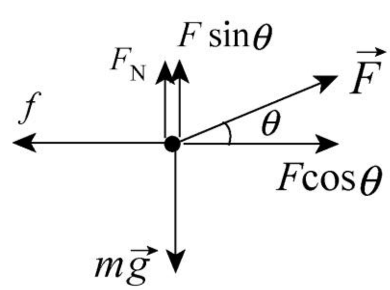
(c) Much of the work (105 J) has been "wasted" due to the 70.6 J of thermal energy generated, but there still remains (105 – 70.6) J = 34.4 J that has gone into increasing the kinetic energy of the block. (It has not gone into increasing the potential energy of the block because the floor is presumed to be horizontal.)

45. **THINK** Work is done against friction while pulling a block along the floor at a constant speed.

EXPRESS Place the *x*-axis along the path of the block and the *y*-axis normal to the floor. The free-body diagram is shown below. The *x* and the *y* component of Newton's second law are

$$\begin{aligned} x&: \quad F\cos\theta - f = 0 \\ y&: \quad F_N + F\sin\theta - mg = 0, \end{aligned}$$

where m is the mass of the block, F is the force exerted by the rope, f is the magnitude of the kinetic friction force, and θ is the angle between that force and the horizontal.



The work done on the block by the force in the rope is $W = Fd\cos\theta$. Similarly, the increase in thermal energy of the block-floor system due to the frictional force is given by Eq. 8-29, $\Delta E_{\text{th}} = fd$.

ANALYZE (a) Substituting the values given, we find the work done on the block by the rope's force to be

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0° = 30.1 \text{ J}.$$

(b) The increase in thermal energy is $\Delta E_{\text{th}} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1$ J.

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/F_N$ to obtain the coefficient of friction. The x-component of Newton's law gives

$$f = F\cos\theta = (7.68 \text{ N})\cos 15.0° = 7.42 \text{ N}.$$

Similarly, the y-component yields

$$F_N = mg - F\sin\theta = (3.57 \text{ kg})(9.8 \text{ m/s}^2) - (7.68 \text{ N})\sin 15.0° = 33.0 \text{ N}.$$

Thus, the coefficient of kinetic friction is

$$\mu_k = \frac{f}{F_N} = \frac{7.42 \text{ N}}{33.0 \text{ N}} = 0.225.$$

LEARN In this problem, the block moves at a constant speed so that $\Delta K = 0$, i.e., no change in kinetic energy. The work done by the external force is converted into thermal energy of the system, $W = \Delta E_{\text{th}}$.

46. We work this using English units (with g = 32 ft/s), but for consistency we convert the weight to pounds

$$mg = (9.0)\,\text{oz}\left(\frac{1 \text{ 1b}}{16\,\text{oz}}\right) = 0.56\,\text{lb}$$

which implies $m = 0.018 \text{ lb}\cdot\text{s}^2/\text{ft}$ (which can be phrased as 0.018 slug as explained in Appendix D). And we convert the initial speed to feet-per-second

$$v_i = (81.8\,\text{mi/h})\left(\frac{5280 \text{ ft/mi}}{3600 \text{ s/h}}\right) = 120 \text{ ft/s}$$

or a more "direct" conversion from Appendix D can be used. Equation 8-30 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\Delta E_{\text{th}} = \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f) = \frac{1}{2}(0.018)(120^2 - 110^2) + 0 = 20 \text{ ft}\cdot\text{lb}.$$

47. We use SI units so m = 0.075 kg. Equation 8-33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f)\\ &= \frac{1}{2}(0.075\text{ kg})[(12\text{ m/s})^2 - (10.5\text{ m/s})^2] + (0.075\text{ kg})(9.8\text{ m/s}^2)(1.1\text{ m} - 2.1\text{ m})\\ &= 0.53\text{ J}.\end{aligned}$$

48. We use Eq. 8-31 to obtain $\Delta E_{\text{th}} = f_k d = (10\,\text{N})(5.0\,\text{m}) = 50\ \text{J}$, and Eq. 7-8 to get

$$W = Fd = (2.0\,\text{N})(5.0\,\text{m}) = 10\ \text{J}.$$

Similarly, Eq. 8-31 gives

$$\begin{aligned}W &= \Delta K + \Delta U + \Delta E_{\text{th}}\\ 10 &= 35 + \Delta U + 50\end{aligned}$$

which yields $\Delta U = -75$ J. By Eq. 8-1, then, the work done by gravity is $W = -\Delta U = 75$ J.

49. **THINK** As the bear slides down the tree, its gravitational potential energy is converted into both kinetic energy and thermal energy.

EXPRESS We take the initial gravitational potential energy to be $U_i = mgL$, where L is the length of the tree, and final gravitational potential energy at the bottom to be $U_f = 0$. To solve this problem, we note that the changes in the mechanical and thermal energies must sum to zero.

ANALYZE (a) Substituting the values given, the change in gravitational potential energy is

$$\Delta U = U_f - U_i = -mgL = -(25\,\text{kg})(9.8\,\text{m/s}^2)(12\text{ m}) = -2.9\times10^3\ \text{J}.$$

(b) The final speed is $v_f = 5.6\,\text{m/s}$. Therefore, the kinetic energy is

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(25\text{ kg})(5.6\text{ m/s})^2 = 3.9\times10^2\ \text{J}.$$

(c) The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore, from $\Delta E_{\text{th}} + \Delta K + \Delta U = 0$, we find f to be

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.9\times10^2\ \text{J} - 2.9\times10^3\ \text{J}}{12\text{ m}} = 2.1\times10^2\ \text{N}.$$

LEARN In this problem, no external work is done to the bear. Therefore,

$$W = \Delta E_{\text{th}} + \Delta E_{\text{mech}} = \Delta E_{\text{th}} + \Delta K + \Delta U = 0,$$

which implies $\Delta K = -\Delta U - \Delta E_{\text{th}} = -\Delta U - fL$. Thus, $\Delta E_{\text{th}} = fL$ can be interpreted as the additional change (decrease) in kinetic energy due to frictional force.

50. Equation 8-33 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\text{th}} &= \frac{1}{2}m(v_i^2 - v_f^2) + mg(y_i - y_f)\\ &= \frac{1}{2}(60\text{ kg})[(24\text{ m/s})^2 - (22\text{ m/s})^2] + (60\text{ kg})(9.8\text{ m/s}^2)(14\text{ m})\\ &= 1.1\times 10^4\text{ J}.\end{aligned}$$

That the angle of 25° is nowhere used in this calculation is indicative of the fact that energy is a scalar quantity.

51. (a) The initial potential energy is

$$U_i = mgy_i = (520\text{ kg})\left(9.8\,\text{m/s}^2\right)(300\text{ m}) = 1.53\times 10^6\text{ J}$$

where $+y$ is upward and $y = 0$ at the bottom (so that $U_f = 0$).

(b) Since $f_k = \mu_k F_N = \mu_k mg \cos\theta$ we have $\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta$ from Eq. 8-31. Now, the hillside surface (of length d = 500 m) is treated as an hypotenuse of a 3-4-5 triangle, so $\cos\theta = x/d$ where x = 400 m. Therefore,

$$\Delta E_{\text{th}} = \mu_k mgd\,\frac{x}{d} = \mu_k mgx = (0.25)(520)(9.8)(400) = 5.1\times 10^5\text{ J}.$$

(c) Using Eq. 8-31 (with $W = 0$) we find

$$K_f = K_i + U_i - U_f - \Delta E_{\text{th}} = 0 + (1.53\times 10^6\text{ J}) - 0 - (5.1\times 10^6\text{ J}) = 1.02\times 10^6\text{ J}.$$

(d) From $K_f = mv^2/2$, we obtain v = 63 m/s.

52. (a) An appropriate picture (once friction is included) for this problem is Figure 8-3 in the textbook. We apply Eq. 8-31, $\Delta E_{\text{th}} = f_k d$, and relate initial kinetic energy K_i to the "resting" potential energy U_r:

$$K_i + U_i = f_k d + K_r + U_r \quad\Rightarrow\quad 20.0\text{ J} + 0 = f_k d + 0 + \frac{1}{2}kd^2$$

where f_k = 10.0 N and k = 400 N/m. We solve the equation for d using the quadratic formula or by using the polynomial solver on an appropriate calculator, with d = 0.292 m being the only positive root.

(b) We apply Eq. 8-31 again and relate U_r to the "second" kinetic energy K_s it has at the unstretched position.

$$K_r + U_r = f_k d + K_s + U_s \quad \Rightarrow \quad \frac{1}{2}kd^2 = f_k d + K_s + 0$$

Using the result from part (a), this yields K_s = 14.2 J.

53. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $F_N = mg$, where m is the mass of the block. Thus $f = \mu_k F_N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{\text{th}} = fd = \mu_k mgD$, where D is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{\text{th}} = (0.25)(3.5\,\text{kg})\left(9.8\,\text{m/s}^2\right)(7.8\,\text{m}) = 67\,\text{J}.$$

(b) The block has its maximum kinetic energy K_{max} just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus, $K_{\text{max}} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{\text{max}}}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46\,\text{m}.$$

54. (a) Using the force analysis shown in Chapter 6, we find the normal force $F_N = mg\cos\theta$ (where mg = 267 N) which means

$$f_k = \mu_k F_N = \mu_k mg \cos\theta.$$

Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta = (0.10)(267)(6.1)\cos 20° = 1.5\times 10^2\,\text{J}.$$

(b) The potential energy change is

$$\Delta U = mg(-d\sin\theta) = (267\text{ N})(-6.1\text{ m})\sin 20° = -5.6\times 10^2\text{ J}.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{267\,\text{N}}{9.8\,\text{m/s}^2}\right)(0.457\,\text{m/s}^2) = 2.8\ \text{J}.$$

Therefore, using Eq. 8-33 (with W = 0), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - \left(-5.6\times 10^2\right) - 1.5\times 10^2 = 4.1\times 10^2\,\text{J}.$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5\ \text{m/s}$.

55. (a) With x = 0.075 m and $k = 320\,\text{N/m}$, Eq. 7-26 yields $W_s = -\frac{1}{2}kx^2 = -0.90$ J. For later reference, this is equal to the negative of ΔU.

(b) Analyzing forces, we find $F_N = mg$, which means $f_k = \mu_k F_N = \mu_k mg$. With $d = x$, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgx = (0.25)(2.5)(9.8)(0.075) = 0.46\ \text{J}.$$

(c) Equation 8-33 (with W = 0) indicates that the initial kinetic energy is

$$K_i = \Delta U + \Delta E_{\text{th}} = 0.90 + 0.46 = 1.36\ \text{J}$$

which leads to $v_i = \sqrt{2K_i/m} = 1.0\ \text{m/s}$.

56. Energy conservation, as expressed by Eq. 8-33 (with W = 0) leads to

$$\Delta E_{\text{th}} = K_i - K_f + U_i - U_f \;\Rightarrow\; f_k d = 0 - 0 + \frac{1}{2}kx^2 - 0$$
$$\Rightarrow\; \mu_k mgd = \frac{1}{2}(200\,\text{N/m})(0.15\,\text{m})^2 \;\Rightarrow\; \mu_k(2.0\,\text{kg})(9.8\,\text{m/s}^2)(0.75\,\text{m}) = 2.25\ \text{J}$$

which yields μ_k = 0.15 as the coefficient of kinetic friction.

57. Since the valley is frictionless, the only reason for the speed being less when it reaches the higher level is the gain in potential energy $\Delta U = mgh$ where h = 1.1 m. Sliding along the rough surface of the higher level, the block finally stops since its remaining kinetic energy has turned to thermal energy $\Delta E_{\text{th}} = f_k d = \mu mgd$, where $\mu = 0.60$. Thus, Eq. 8-33 (with W = 0) provides us with an equation to solve for the distance d:

$$K_i = \Delta U + \Delta E_{\text{th}} = mg\left(h + \mu d\right)$$

where $K_i = mv_i^2/2$ and v_i = 6.0 m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2\,\text{m}.$$

58. This can be worked entirely by the methods of Chapters 2–6, but we will use energy methods in as many steps as possible.

(a) By a force analysis of the style done in Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$ (where $\theta = 40°$), which means $f_k = \mu_k F_N = \mu_k mg\cos\theta$ where $\mu_k = 0.15$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = mgd\sin\theta$. Eq. 8-33 (with $W = 0$ and $K_f = 0$) provides an equation for determining d:

$$\begin{aligned} K_i &= \Delta U + \Delta E_{\text{th}} \\ \frac{1}{2}mv_i^2 &= mgd\left(\sin\theta + \mu_k\cos\theta\right) \end{aligned}$$

where $v_i = 1.4\,\text{m/s}$. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)} = 0.13\,\text{m}.$$

(b) Now that we know where on the incline it stops ($d' = 0.13 + 0.55 = 0.68$ m from the bottom), we can use Eq. 8-33 again (with $W = 0$ and now with $K_i = 0$) to describe the final kinetic energy (at the bottom):

$$\begin{aligned} K_f &= -\Delta U - \Delta E_{\text{th}} \\ \frac{1}{2}mv^2 &= mgd'\left(\sin\theta - \mu_k\cos\theta\right) \end{aligned}$$

which — after dividing by the mass and rearranging — yields

$$v = \sqrt{2gd'\left(\sin\theta - \mu_k\cos\theta\right)} = 2.7\,\text{m/s}.$$

(c) In part (a) it is clear that d increases if μ_k decreases — both mathematically (since it is a positive term in the denominator) and intuitively (less friction — less energy "lost"). In part (b), there are two terms in the expression for v that imply that it should increase if μ_k were smaller: the increased value of $d' = d_0 + d$ and that last factor $\sin\theta - \mu_k\cos\theta$, which indicates that less is being subtracted from $\sin\theta$ when μ_k is less (so the factor itself increases in value).

59. (a) The maximum height reached is h. The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{th} = fh$ by Eq. 8-31. We use energy conservation in the form of Eq. 8-33 (with $W = 0$):

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2}mv_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$, where $w = mg$ is the weight of the stone. Thus, $wh + fh = \frac{1}{2}mv_0^2$, and we solve for the height:

$$h = \frac{mv_0^2}{2(w+f)} = \frac{v_0^2}{2g(1+f/w)}.$$

Numerically, we have, with $m = (5.29 \text{ N})/(9.80 \text{ m/s}^2) = 0.54$ kg,

$$h = \frac{(20.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(1+0.265/5.29)} = 19.4 \text{ m}.$$

(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{th} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2.$$

We substitute the expression found for h to obtain

$$\frac{2fv_0^2}{2g(1+f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2fv_0^2}{mg(1+f/w)} = v_0^2 - \frac{2fv_0^2}{w(1+f/w)} = v_0^2\left(1 - \frac{2f}{w+f}\right) = v_0^2\,\frac{w-f}{w+f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0\sqrt{\frac{w-f}{w+f}} = (20.0\text{ m/s})\sqrt{\frac{5.29\text{ N} - 0.265\text{ N}}{5.29\text{ N} + 0.265\text{ N}}} = 19.0\text{ m/s}.$$

60. We look for the distance along the incline d, which is related to the height ascended by $\Delta h = d \sin\theta$. By a force analysis of the style done in Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$, which means $f_k = \mu_k mg \cos\theta$. Thus, Eq. 8-33 (with W = 0) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\text{th}} \\ &= 0 - K_i + mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which leads to

$$d = \frac{K_i}{mg(\sin\theta + \mu_k\cos\theta)} = \frac{128}{(4.0)(9.8)(\sin 30° + 0.30\cos 30°)} = 4.3\text{ m}.$$

61. Before the launch, the mechanical energy is $\Delta E_{\text{mech},0} = 0$. At the maximum height h where the speed of the beetle vanishes, the mechanical energy is $\Delta E_{\text{mech},1} = mgh$. The change of the mechanical energy is related to the external force by

$$\Delta E_{\text{mech}} = \Delta E_{\text{mech},1} - \Delta E_{\text{mech},0} = mgh = F_{avg} d\cos\phi,$$

where F_{avg} is the average magnitude of the external force on the beetle.

(a) From the above equation, we have

$$F_{avg} = \frac{mgh}{d\cos\phi} = \frac{(4.0\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)(0.30\text{ m})}{(7.7\times10^{-4}\text{ m})(\cos 0°)} = 1.5\times10^{-2}\text{ N}.$$

(b) Dividing the above result by the mass of the beetle, we obtain

$$a = \frac{F_{avg}}{m} = \frac{h}{d\cos\phi}\, g = \frac{(0.30\text{ m})}{(7.7\times10^{-4}\text{ m})(\cos 0°)}\, g = 3.8\times10^2 g.$$

62. We will refer to the point where it first encounters the "rough region" as point C (this is the point at a height h above the reference level). From Eq. 8-17, we find the speed it has at point C to be

$$v_C = \sqrt{v_A^2 - 2gh} = \sqrt{(8.0)^2 - 2(9.8)(2.0)} = 4.980 \approx 5.0\text{ m/s}.$$

Thus, we see that its kinetic energy right at the beginning of its "rough slide" (heading uphill towards B) is

$$K_C = \frac{1}{2}m(4.980\text{ m/s})^2 = 12.4m$$

(with SI units understood). Note that we "carry along" the mass (as if it were a known quantity); as we will see, it will cancel out, shortly. Using Eq. 8-37 (and Eq. 6-2 with $F_N = mg\cos\theta$) and $y = d\sin\theta$, we note that if $d < L$ (the block does not reach point B), this kinetic energy will turn entirely into thermal (and potential) energy

$$K_C = mgy + f_k d \quad \Rightarrow \quad 12.4m = mgd\sin\theta \; + \mu_k mgd\cos\theta.$$

With $\mu_k = 0.40$ and $\theta = 30°$, we find $d = 1.49$ m, which is greater than L (given in the problem as 0.75 m), so our assumption that $d < L$ is incorrect. What is its kinetic energy as it reaches point B? The calculation is similar to the above, but with d replaced by L and the final v^2 term being the unknown (instead of assumed zero):

$$\frac{1}{2}\, m v^2 = K_C - (mgL\sin\theta + \mu_k mgL\cos\theta)\;.$$

This determines the speed with which it arrives at point B:

$$\begin{aligned} v_B &= \sqrt{v_C^2 - 2gL(\sin\theta + \mu_k\cos\theta)} \\ &= \sqrt{(4.98\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.75\text{ m})(\sin 30° + 0.4\cos 30°)} = 3.5\text{ m/s}. \end{aligned}$$

63. We observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. The friction force of magnitude f = 4400 N mentioned in the problem is kinetic friction and (as mentioned) is constant (and directed upward), and the thermal energy change associated with it is $\Delta E_{\text{th}} = fd$ (Eq. 8-31) where d = 3.7 m in part (a) (but will be replaced by x, the spring compression, in part (b)).

(a) With W = 0 and the reference level for computing $U = mgy$ set at the top of the (relaxed) spring, Eq. 8-33 leads to

$$U_i = K + \Delta E_{\text{th}} \Rightarrow v = \sqrt{2d\left(g - \frac{f}{m}\right)}$$

which yields $v = 7.4\text{ m/s}$ for m = 1800 kg.

(b) We again utilize Eq. 8-33 (with W = 0), now relating its kinetic energy at the moment it makes contact with the spring to the system energy at the bottom-most point. Using the same reference level for computing $U = mgy$ as we did in part (a), we end up with gravitational potential energy equal to $mg(-x)$ at that bottom-most point, where the spring (with spring constant $k = 1.5\times10^5$ N/m) is fully compressed.

$$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

where $K = \frac{1}{2}mv^2 = 4.9 \times 10^4$ J using the speed found in part (a). Using the abbreviation $\xi = mg - f = 1.3 \times 10^4$ N, the quadratic formula yields

$$x = \frac{\xi \pm \sqrt{\xi^2 + 2kK}}{k} = 0.90\,\text{m}$$

where we have taken the positive root.

(c) We relate the energy at the bottom-most point to that of the highest point of rebound (a distance d' above the relaxed position of the spring). We assume $d' > x$. We now use the bottom-most point as the reference level for computing gravitational potential energy.

$$\frac{1}{2}kx^2 = mgd' + fd' \Rightarrow d' = \frac{kx^2}{2(mg+d)} = 2.8\,\text{m}.$$

(d) The non-conservative force (§8-1) is friction, and the energy term associated with it is the one that keeps track of the total distance traveled (whereas the potential energy terms, coming as they do from conservative forces, depend on positions — but not on the paths that led to them). We assume the elevator comes to final rest at the equilibrium position of the spring, with the spring compressed an amount d_{eq} given by

$$mg = kd_{\text{eq}} \Rightarrow d_{\text{eq}} = \frac{mg}{k} = 0.12\,\text{m}.$$

In this part, we use that final-rest point as the reference level for computing gravitational potential energy, so the original $U = mgy$ becomes $mg(d_{\text{eq}} + d)$. In that final position, then, the gravitational energy is zero and the spring energy is $kd_{\text{eq}}^2/2$. Thus, Eq. 8-33 becomes

$$mg\left(d_{\text{eq}} + d\right) = \frac{1}{2}kd_{\text{eq}}^2 + fd_{\text{total}}$$
$$(1800)(9.8)(0.12 + 3.7) = \frac{1}{2}\left(1.5\times 10^5\right)(0.12)^2 + (4400)\,d_{\text{total}}$$

which yields $d_{\text{total}} = 15$ m.

64. In the absence of friction, we have a simple conversion (as it moves along the inclined ramps) of energy between the kinetic form (Eq. 7-1) and the potential form (Eq. 8-9). Along the horizontal plateaus, however, there is friction that causes some of the kinetic energy to dissipate in accordance with Eq. 8-31 (along with Eq. 6-2 where $\mu_k = 0.50$ and $F_N = mg$ in this situation). Thus, after it slides down a (vertical) distance d it has gained $K = \frac{1}{2}mv^2 = mgd$, some of which ($\Delta E_{\text{th}} = \mu_k\, mgd$) is dissipated, so that the value of kinetic energy at the end of the first plateau (just before it starts descending towards the lowest plateau) is

$$K = mgd - \mu_k mgd = \frac{1}{2}mgd\ .$$

In its descent to the lowest plateau, it gains $mgd/2$ more kinetic energy, but as it slides across it "loses" $\mu_k\, mgd/2$ of it. Therefore, as it starts its climb up the right ramp, it has kinetic energy equal to

$$K = \frac{1}{2}mgd + \frac{1}{2}mgd - \frac{1}{2}\mu_k mgd = \frac{3}{4}mgd\ .$$

Setting this equal to Eq. 8-9 (to find the height to which it climbs) we get $H = ¾d$. Thus, the block (momentarily) stops on the inclined ramp at the right, at a height of

$$H = 0.75d = 0.75\ (40\text{ cm}) = 30\text{ cm}$$

measured from the lowest plateau.

65. The initial and final kinetic energies are zero, and we set up energy conservation in the form of Eq. 8-33 (with $W = 0$) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{th} = f_k d$ where $d \le L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total thermal energy is $\Delta E_{th} = \mu_k\, mg(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \le d \le L$). Generalizing to the n^{th} pass through, we see that

$$\Delta E_{th} = \mu_k\, mg[(n-1)L + d].$$

In this way, we have

$$mgh = \mu_k mg\big((n-1)L + d\big)$$

which simplifies (when $h = L/2$ is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$$

The first two terms give $1 + 1/2\mu_k = 3.5,$ so that the requirement $0 \le d/L \le 1$ demands that $n = 3$. We arrive at the conclusion that $d/L = \dfrac{1}{2}$, or

$$d = \frac{1}{2}L = \frac{1}{2}(40\text{ cm}) = 20\text{ cm}$$

and that this occurs on its third pass through the flat region.

66. (a) Equation 8-9 gives $U = mgh = (3.2\text{ kg})(9.8\text{ m/s}^2)(3.0\text{ m}) = 94\text{ J}$.

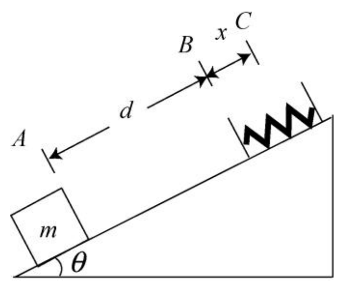
(b) The mechanical energy is conserved, so $K = 94$ J.

(c) The speed (from solving Eq. 7-1) is

$$v = \sqrt{2K/m} = \sqrt{2(94\text{ J})/(32\text{ kg})} = 7.7\text{ m/s}.$$

67. **THINK** As the block is projected up the inclined plane, its kinetic energy is converted into gravitational potential energy and elastic potential energy of the spring. The block compresses the spring, stopping momentarily before sliding back down again.

EXPRESS Let *A* be the starting point and the reference point for computing gravitational potential energy ($U_A = 0$). The block first comes into contact with the spring at *B*. The spring is compressed by an additional amount x at *C*, as shown in the figure below.



By energy conservation, $K_A + U_A = K_B + U_B = K_C + U_C$. Note that

$$U = U_g + U_s = mgy + \frac{1}{2}kx^2,$$

i.e., the total potential energy is the sum of gravitational potential energy and elastic potential energy of the spring.

ANALYZE (a) At the instant when $x_C = 0.20\text{ m}$, the vertical height is

$$y_C = (d + x_C)\sin\theta = (0.60\text{ m} + 0.20\text{ m})\sin 40° = 0.514\text{ m}.$$

Applying energy conservation principle gives

$$K_A + U_A = K_C + U_C \;\Rightarrow\; 16\text{ J} + 0 = K_C + mgy_C + \frac{1}{2}kx_C^2$$

from which we obtain

$$\begin{aligned}K_C &= K_A - mgy_C - \frac{1}{2}kx_C^2\\ &= 16\ \text{J} - (1.0\ \text{kg})(9.8\ \text{m/s}^2)(0.514\ \text{m}) - \frac{1}{2}(200\ \text{N/m})(0.20\ \text{m})^2 = 6.96\ \text{J} \approx 7.0\ \text{J}.\end{aligned}$$

(b) At the instant when $x'_C = 0.40\ \text{m}$, the vertical height is

$$y'_C = (d + x'_C)\sin\theta = (0.60\ \text{m} + 0.40\ \text{m})\sin 40° = 0.64\ \text{m}.$$

Applying energy conservation principle, we have $K'_A + U'_A = K'_C + U'_C$. Since $U'_A = 0$, the initial kinetic energy that gives $K'_C = 0$ is

$$\begin{aligned}K'_A = U'_C &= mgy'_C + \frac{1}{2}kx'^2_C\\ &= (1.0\ \text{kg})(9.8\ \text{m/s}^2)(0.64\ \text{m}) + \frac{1}{2}(200\ \text{N/m})(0.40\ \text{m})^2\\ &= 22\ \text{J}.\end{aligned}$$

LEARN Comparing the results found in (a) and (b), we see that more kinetic energy is required to move the block higher in the inclined plane to achieve a greater spring compression.

68. (a) At the point of maximum height, where y = 140 m, the vertical component of velocity vanishes but the horizontal component remains what it was when it was launched (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(0.55\,\text{kg})v_x^2.$$

Also, its potential energy (with the reference level chosen at the level of the cliff edge) at that moment is $U = mgy$ = 755 J. Thus, by mechanical energy conservation,

$$K = K_i - U = 1550 - 755 \Rightarrow v_x = \sqrt{\frac{2(1550 - 755)}{0.55}} = 54\ \text{m/s}.$$

(b) As mentioned, $v_x = v_{ix}$ so that the initial kinetic energy

$$K_i = \frac{1}{2}m\left(v_{ix}^2 + v_{iy}^2\right)$$

can be used to find v_{iy}. We obtain $v_{iy} = 52\ \text{m/s}$.

(c) Applying Eq. 2-16 to the vertical direction (with $+y$ upward), we have

$$v_y^2 = v_{iy}^2 - 2g\Delta y \quad \Rightarrow \ (65 \text{ m/s})^2 = (52 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)\Delta y$$

which yields $\Delta y = -76$ m. The minus sign tells us it is below its launch point.

69. **THINK** The two blocks are connected by a cord. As block *B* falls, block *A* moves up the incline.

EXPRESS If the larger mass (block *B*, m_B = 2.0 kg) falls a vertical distance $d = 0.25$ m, then the smaller mass (blocks *A*, m_A = 1.0 kg) must increase its height by $h = d\sin 30°$. The change in gravitational potential energy is

$$\Delta U = -m_B g d + m_A g h\,.$$

By mechanical energy conservation, $\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$, the change in kinetic energy of the system is $\Delta K = -\Delta U$.

ANALYZE Since the initial kinetic energy is zero, the final kinetic energy is

$$\begin{aligned} K_f &= \Delta K = m_B g d - m_A g h = m_B g d - m_A g d \sin\theta \\ &= (m_B - m_A \sin\theta) g d = [2.0 \text{ kg} - (1.0 \text{ kg})\sin 30°](9.8 \text{ m/s}^2)(0.25 \text{ m}) \\ &= 3.7 \text{ J}. \end{aligned}$$

LEARN From the above expression, we see that in the special case where $m_B = m_A \sin\theta$, the two-block system would remain stationary. On the other hand, if $m_A \sin\theta > m_B$, block *A* will slide down the incline, with block *B* moving vertically upward.

70. We use conservation of mechanical energy: the mechanical energy must be the same at the top of the swing as it is initially. Newton's second law is used to find the speed, and hence the kinetic energy, at the top. There the tension force *T* of the string and the force of gravity are both downward, toward the center of the circle. We notice that the radius of the circle is $r = L - d$, so the law can be written

$$T + mg = mv^2/(L-d),$$

where *v* is the speed and *m* is the mass of the ball. When the ball passes the highest point with the least possible speed, the tension is zero. Then

$$mg = m\frac{v^2}{L-d} \ \Rightarrow \ v = \sqrt{g(L-d)}\ .$$

We take the gravitational potential energy of the ball-Earth system to be zero when the ball is at the bottom of its swing. Then the initial potential energy is *mgL*. The initial kinetic energy is zero since the ball starts from rest. The final potential energy, at the top of the swing, is $2mg(L-d)$ and the final kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mg(L-d)$ using the above result for *v*. Conservation of energy yields

$$mgL = 2mg(L-d) + \frac{1}{2}mg(L-d) \Rightarrow d = 3L/5 .$$

With L = 1.20 m, we have d = 0.60(1.20 m) = 0.72 m.

Notice that if *d* is greater than this value, so the highest point is lower, then the speed of the ball is greater as it reaches that point and the ball passes the point. If *d* is less, the ball cannot go around. Thus the value we found for *d* is a lower limit.

71. **THINK** As the block slides down the frictionless incline, its gravitational potential energy is converted to kinetic energy, so the speed of the block increases.

EXPRESS By energy conservation, $K_A + U_A = K_B + U_B$. Thus, the change in kinetic energy as the block moves from points *A* to *B* is

$$\Delta K = K_B - K_A = -\Delta U = -(U_B - U_A).$$

In both circumstances, we have the same potential energy change. Thus, $\Delta K_1 = \Delta K_2$.

ANALYZE With $\Delta K_1 = \Delta K_2$, the speed of the block at B the second time is given by

$$\frac{1}{2}mv_{B,1}^2 - \frac{1}{2}mv_{A,1}^2 = \frac{1}{2}mv_{B,2}^2 \, \frac{1}{2}mv_{A,2}^2$$

or

$$v_{B,2} = \sqrt{v_{B,1}^2 - v_{A,1}^2 + v_{A,2}^2} = \sqrt{(2.60\text{ m/s})^2 - (2.00\text{ m/s})^2 + (4.00\text{ m/s})^2} = 4.33\text{ m/s} .$$

LEARN The speed of the block at *A* is greater the second time, $v_{A,2} > v_{A,1}$. This can happen if the block slides down from a higher position with greater initial gravitational potential energy.

72. (a) We take the gravitational potential energy of the skier-Earth system to be zero when the skier is at the bottom of the peaks. The initial potential energy is $U_i = mgH$, where *m* is the mass of the skier, and *H* is the height of the higher peak. The final potential energy is $U_f = mgh$, where *h* is the height of the lower peak. The skier initially has a kinetic energy of $K_i = 0$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where *v* is the speed of the skier at the top of the lower peak. The normal force of the slope on the skier does no work and friction is negligible, so mechanical energy is conserved:

$$U_i + K_i = U_f + K_f \quad \Rightarrow \quad mgH = mgh + \frac{1}{2}mv^2.$$

Thus,

$$v = \sqrt{2g(H-h)} = \sqrt{2(9.8\,\text{m/s}^2)(850\ \text{m} - 750\ \text{m})} = 44\ \text{m/s}.$$

(b) We recall from analyzing objects sliding down inclined planes that the normal force of the slope on the skier is given by $F_N = mg \cos \theta$, where θ is the angle of the slope from the horizontal, 30° for each of the slopes shown. The magnitude of the force of friction is given by $f = \mu_k F_N = \mu_k mg \cos \theta$. The thermal energy generated by the force of friction is $fd = \mu_k mgd \cos \theta$, where d is the total distance along the path. Since the skier gets to the top of the lower peak with no kinetic energy, the increase in thermal energy is equal to the decrease in potential energy. That is, $\mu_k mgd \cos \theta = mg(H - h)$. Consequently,

$$\mu_k = \frac{H-h}{d\cos\theta} = \frac{(850\ \text{m} - 750\ \text{m})}{(3.2\times 10^3\ \text{m})\cos 30°} = 0.036.$$

73. **THINK** As the cube is pushed across the floor, both the thermal energies of floor and the cube increase because of friction.

EXPRESS By law of conservation of energy, we have $W = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$ for the floor-cube system. Since the speed is constant, $\Delta K = 0$, Eq. 8-33 (an application of the energy conservation concept) implies

$$W = \Delta E_{\text{mech}} + \Delta E_{\text{th}} = \Delta E_{\text{th}} = \Delta E_{\text{th (cube)}} + \Delta E_{\text{th (floor)}}.$$

ANALYZE With $W = (15\ \text{N})(3.0\ \text{m}) = 45$ J, and we are told that $\Delta E_{\text{th (cube)}} = 20$ J, then we conclude that $\Delta E_{\text{th (floor)}} = 25$ J.

LEARN The applied work here has all been converted into thermal energies of the floor and the cube. The amount of thermal energy transferred to a material depends on its thermal properties, as we shall discuss in Chapter 18.

74. We take her original elevation to be the $y = 0$ reference level and observe that the top of the hill must consequently have $y_A = R(1 - \cos 20°) = 1.2$ m, where R is the radius of the hill. The mass of the skier is $m = (600\ \text{N})/(9.8\,\text{m/s}^2) = 61\,\text{kg}$.

(a) Applying energy conservation, Eq. 8-17, we have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = K_A + mgy_A.$$

Using $K_B = \frac{1}{2}(61\,\text{kg})(8.0\,\text{m/s})^2$, we obtain $K_A = 1.2 \times 10^3$ J. Thus, we find the speed at the hilltop is

$$v_A = \sqrt{\frac{2K_A}{m}} = \sqrt{\frac{2(1.2\times10^3\ \text{J})}{61\,\text{kg}}} = 6.4\ \text{m/s}.$$

Note: One might wish to check that the skier stays in contact with the hill — which is indeed the case here. For instance, at A we find $v^2/r \approx 2$ m/s^2, which is considerably less than g.

(b) With $K_A = 0$, we have

$$K_B + U_B = K_A + U_A \Rightarrow K_B + 0 = 0 + mgy_A$$

which yields $K_B = 724$ J, and the corresponding speed is

$$v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(724\ \text{J})}{61\,\text{kg}}} = 4.9\ \text{m/s}.$$

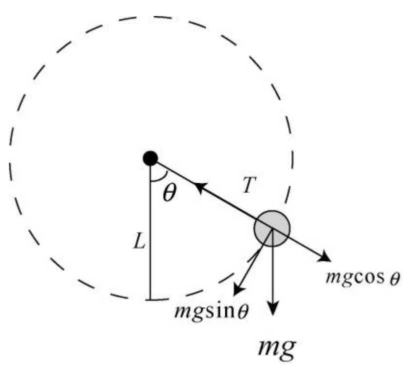
(c) Expressed in terms of mass, we have

$$K_B + U_B = K_A + U_A \Rightarrow$$
$$\frac{1}{2}mv_B^2 + mgy_B = \frac{1}{2}mv_A^2 + mgy_A\ .$$

Thus, the mass m cancels, and we observe that solving for speed does not depend on the value of mass (or weight).

75. **THINK** This problem deals with pendulum motion. The kinetic and potential energies of the ball attached to the rod change with position, but the mechanical energy remains conserved throughout the process.

EXPRESS Let L be the length of the pendulum. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy mgh) is given by $h = L(1 - \cos\theta)$.



The free-body diagram is shown above. The initial height is at $h_1 = 2L$, and at the lowest point, we have $h_2 = 0$. The total mechanical energy is conserved throughout.

ANALYZE (a) Initially the ball is at $h_1 = 2L$ with $K_1 = 0$ and $U_1 = mgh_1 = mg(2L)$. At the lowest point $h_2 = 0$, we have $K_2 = \frac{1}{2}mv_2^2$ and $U_2 = 0$. Using energy conservation in the form of Eq. 8-17 leads to

$$K_1 + U_1 = K_2 + U_2 \quad \Rightarrow \quad 0 + 2mgL = \frac{1}{2}mv_2^2 + 0$$

This leads to $v_2 = 2\sqrt{gL}$. With $L = 0.62$ m, we have

$$v_2 = 2\sqrt{(9.8\text{ m/s}^2)(0.62\text{ m})} = 4.9\text{ m/s}.$$

(b) At the lowest point, the ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2/r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m\frac{v^2}{r} \Rightarrow T = m\left(g + \frac{4gL}{L}\right) = 5\,mg.$$

With $m = 0.092$ kg, the tension is $T = 4.5$ N.

(c) The pendulum is now started (with zero speed) at $\theta_i = 90°$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ, as can be seen from the free-body diagram shown above, Newton's second law applied to the axis along the rod yields

$$\frac{mv^2}{r} = T - mg\cos\theta = mg(1 - \cos\theta)$$

which (since $r = L$) implies $v^2 = gL(1 - \cos\theta)$ at the position we are looking for. Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 + mgL &= \frac{1}{2}mv^2 + mgL\,(1 - \cos\theta) \\ gL &= \frac{1}{2}(gL(1 - \cos\theta)) + gL\,(1 - \cos\theta) \end{aligned}$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 71°.$$

(d) Since the angle found in (c) is independent of the mass, the result remains the same if the mass of the ball is changed.

LEARN At a given angle θ with respect to the vertical, the tension in the rod is

$$T = m\left(\frac{v^2}{r} + g\cos\theta\right)$$

The tangential acceleration, $a_t = g\sin\theta$, is what causes the speed and, therefore, the kinetic energy to change with time. Nonetheless, mechanical energy is conserved.

76. (a) The table shows that the force is +(3.0 N)$\hat{\mathrm{i}}$ while the displacement is in the +*x* direction ($\vec{d}$ = +(3.0 m)$\hat{\mathrm{i}}$), and it is –(3.0 N)$\hat{\mathrm{i}}$ while the displacement is in the –*x* direction. Using Eq. 7-8 for each part of the trip, and adding the results, we find the work done is 18 J. This is not a conservative force field; if it had been, then the net work done would have been zero (since it returned to where it started).

(b) This, however, is a conservative force field, as can be easily verified by calculating that the net work done here is zero.

(c) The two integrations that need to be performed are each of the form $\int 2x\,dx$ so that we are adding two equivalent terms, where each equals x^2 (evaluated at $x = 4$, minus its value at $x = 1$). Thus, the work done is $2(4^2 – 1^2) = 30$ J.

(d) This is another conservative force field, as can be easily verified by calculating that the net work done here is zero.

(e) The forces in (b) and (d) are conservative.

77. **THINK** This problem involves graphical analyses. From the graph of potential energy as a function of position, the conservative force can de deduced.

EXPRESS The connection between the potential energy function $U(x)$ and the conservative force $F(x)$ is given by Eq. 8-22: $F(x) = -dU/dx$. A positive slope of $U(x)$ at a point means that $F(x)$ is negative, and vice versa.

ANALYZE (a) The force at $x = 2.0$ m is

$$F = -\frac{dU}{dx} \approx -\frac{\Delta U}{\Delta x} = -\frac{U(x=4\text{ m}) - U(x=1\text{ m})}{4.0\text{ m} - 1.0\text{ m}} = -\frac{-(17.5\text{ J}) - (-2.8\text{ J})}{4.0\text{ m} - 1.0\text{ m}} = 4.9\,\text{N}.$$

(b) Since the slope of $U(x)$ at $x = 2.0$ m is negative, the force points in the +*x* direction (but there is some uncertainty in reading the graph which makes the last digit not very significant).

(c) At x = 2.0 m, we estimate the potential energy to be

$$U(x=2.0\text{ m}) \approx U(x=1.0\text{ m}) + (-4.9\text{ J/m})(1.0\text{ m}) = -7.7\text{ J}$$

Thus, the total mechanical energy is

$$E = K + U = \frac{1}{2}mv^2 + U = \frac{1}{2}(2.0\text{ kg})(-1.5\text{ m/s})^2 + (-7.7\text{ J}) = -5.5\text{ J}.$$

Again, there is some uncertainty in reading the graph which makes the last digit not very significant. At that level (–5.5 J) on the graph, we find two points where the potential energy curve has that value — at $x \approx 1.5$ m and $x \approx 13.5$ m. Therefore, the particle remains in the region $1.5 < x < 13.5$ m. The left boundary is at x = 1.5 m.

(d) From the above results, the right boundary is at x = 13.5 m.

(e) At x = 7.0 m, we read $U \approx -17.5$ J. Thus, if its total energy (calculated in the previous part) is $E \approx -5.5$ J, then we find

$$\frac{1}{2}mv^2 = E - U \approx 12\text{ J} \Rightarrow v = \sqrt{\frac{2}{m}(E-U)} \approx 3.5\text{ m/s}$$

where there is certainly room for disagreement on that last digit for the reasons cited above.

LEARN Since the total mechanical energy is negative, the particle is bounded by the potential, with its motion confined to the region 1.5 m < x < 13.5 m. At the turning points (1.5 m and 13.5 m), kinetic energy is zero and the particle is momentarily at rest.

78. (a) Since the speed of the crate of mass m increases from 0 to 1.20 m/s relative to the factory ground, the kinetic energy supplied to it is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(300\text{ kg})(120\text{ m/s})^2 = 216\text{ J}.$$

(b) The magnitude of the kinetic frictional force is

$$f = \mu F_N = \mu mg = (0.400)(300\text{ kg})(9.8\text{ m/s}^2) = 1.18\times10^3\text{ N}.$$

(c) Let the distance the crate moved relative to the conveyor belt before it stops slipping be d. Then from Eq. 2-16 ($v^2 = 2ad = 2(f/m)d$) we find

$$\Delta E_{\text{th}} = fd = \frac{1}{2}mv^2 = K.$$

Thus, the total energy that must be supplied by the motor is

$$W = K + \Delta E_{\text{th}} = 2K = (2)(216\,\text{J}) = 432\ \text{J}.$$

(d) The energy supplied by the motor is the work W it does on the system, and must be greater than the kinetic energy gained by the crate computed in part (b). This is due to the fact that part of the energy supplied by the motor is being used to compensate for the energy dissipated ΔE_{th} while it was slipping.

79. **THINK** As the car slides down the incline, due to the presence of frictional force, some of its mechanical energy is converted into thermal energy.

EXPRESS The incline angle is $\theta = 5.0°$. Thus, the change in height between the car's highest and lowest points is Δy = –(50 m) sin θ = – 4.4 m. We take the lowest point (the car's final reported location) to correspond to the y = 0 reference level. The change in potential energy is given by $\Delta U = mg\Delta y$.

As for the kinetic energy, we first convert the speeds to SI units, $v_0 = 8.3\ \text{m/s}$ and $v = 11.1\ \text{m/s}$. The change in kinetic energy is $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$. The total change in mechanical energy is $\Delta E_{\text{mech}} = \Delta K + \Delta U$.

ANALYZE (a) Substituting the values given, we find ΔE_{mech} to be

$$\begin{aligned}\Delta E_{\text{mech}} &= \Delta K + \Delta U = \frac{1}{2}m(v_f^2 - v_i^2) + mg\Delta y \\ &= \frac{1}{2}(1500\ \text{kg})\left[(11.1\,\text{m/s})^2 - (8.3\,\text{m/s})^2\right] + (1500\ \text{kg})(9.8\,\text{m/s}^2)(-4.4\ \text{m}) \\ &= -23940\ \text{J} \approx -2.4\times 10^4\ \text{J}\end{aligned}$$

That is, the mechanical energy decreases (due to friction) by 2.4 × 10^4 J.

(b) Using Eq. 8-31 and Eq. 8-33, we find $\Delta E_{\text{th}} = f_k d = -\Delta E_{\text{mech}}$. With d = 50 m, we solve for f_k and obtain

$$f_k = \frac{-\Delta E_{\text{mech}}}{d} = \frac{-(-2.4\times 10^4\ \text{J})}{50\ \text{m}} = 4.8\times 10^2\ \text{N}.$$

LEARN The amount of mechanical energy lost is proportional to the frictional force; in the absence of friction, mechanical energy would have been conserved.

80. We note that in one second, the block slides d = 1.34 m up the incline, which means its height increase is $h = d \sin\theta$ where

$$\theta = \tan^{-1}\left(\frac{30}{40}\right) = 37°.$$

We also note that the force of kinetic friction in this inclined plane problem is $f_k = \mu_k mg\cos\theta$, where μ_k = 0.40 and m = 1400 kg. Thus, using Eq. 8-31 and Eq. 8-33, we find

$$W = mgh + f_k d = mgd\left(\sin\theta + \mu_k \cos\theta\right)$$

or $W = 1.69 \times 10^4$ J for this one-second interval. Thus, the power associated with this is

$$P = \frac{1.69\times10^4\ \text{J}}{1\ \text{s}} = 1.69\times10^4\ \text{W} \approx 1.7\times10^4\ \text{W}.$$

81. (a) The remark in the problem statement that the forces can be associated with potential energies is illustrated as follows: the work from x = 3.00 m to x = 2.00 m is

$$W = F_2\,\Delta x = (5.00\ \text{N})(–1.00\ \text{m}) = –5.00\ \text{J},$$

so the potential energy at x = 2.00 m is U_2 = +5.00 J.

(b) Now, it is evident from the problem statement that E_{max} = 14.0 J, so the kinetic energy at x = 2.00 m is

$$K_2 = E_{\text{max}} – U_2 = 14.0 – 5.00 = 9.00\ \text{J}.$$

(c) The work from x = 2.00 m to x = 0 is $W = F_1\,\Delta x$ =(3.00 N)(–2.00 m) = – 6.00 J, so the potential energy at x = 0 is

$$U_0 = 6.00\ \text{J} + U_2 = (6.00 + 5.00)\ \text{J} = 11.0\ \text{J}.$$

(d) Similar reasoning to that presented in part (a) then gives

$$K_0 = E_{\text{max}} – U_0 = (14.0 – 11.0)\ \text{J} = 3.00\ \text{J}.$$

(e) The work from x = 8.00 m to x = 11.0 m is $W = F_3\,\Delta x$ =(– 4.00 N)(3.00 m) = –12.0 J, so the potential energy at x = 11.0 m is U_{11} = 12.0 J.

(f) The kinetic energy at x = 11.0 m is therefore

$$K_{11} = E_{\text{max}} – U_{11} = (14.0 – 12.0)\ \text{J} = 2.00\ \text{J}.$$

(g) Now we have $W = F_4\,\Delta x$ =(–1.00 N)(1.00 m) = –1.00 J, so the potential energy at $x = 12.0$ m is

$$U_{12} = 1.00\ \text{J} + U_{11} = (1.00 + 12.0)\ \text{J} = 13.0\ \text{J}.$$

(h) Thus, the kinetic energy at x = 12.0 m is

$$K_{12} = E_{max} - U_{12} = (14.0 - 13.0) = 1.00 \text{ J}.$$

(i) There is no work done in this interval (from x = 12.0 m to x = 13.0 m) so the answers are the same as in part (g): U_{12} = 13.0 J.

(j) There is no work done in this interval (from x = 12.0 m to x = 13.0 m) so the answers are the same as in part (h): K_{12} = 1.00 J.

(k) Although the plot is not shown here, it would look like a "potential well" with piecewise-sloping sides: from x = 0 to x = 2 (SI units understood) the graph of U is a decreasing line segment from 11 to 5, and from x = 2 to x = 3, it then heads down to zero, where it stays until x = 8, where it starts increasing to a value of 12 (at x = 11), and then in another positive-slope line segment it increases to a value of 13 (at x = 12). For $x > 12$ its value does not change (this is the "top of the well").

(l) The particle can be thought of as "falling" down the $0 < x < 3$ slopes of the well, gaining kinetic energy as it does so, and certainly is able to reach x = 5. Since U = 0 at x = 5, then its initial potential energy (11 J) has completely converted to kinetic: now $K = 11.0$ J.

(m) This is not sufficient to climb up and out of the well on the large x side (x > 8), but does allow it to reach a "height" of 11 at x = 10.8 m. As discussed in section 8-5, this is a "turning point" of the motion.

(n) Next it "falls" back down and rises back up the small x slope until it comes back to its original position. Stating this more carefully, when it is (momentarily) stopped at x = 10.8 m it is accelerated to the left by the force $\vec{F}_3$; it gains enough speed as a result that it eventually is able to return to x = 0, where it stops again.

82. (a) At x = 5.00 m the potential energy is zero, and the kinetic energy is

$$K = \frac{1}{2} mv^2 = \frac{1}{2} (2.00 \text{ kg})(3.45 \text{ m/s})^2 = 11.9 \text{ J}.$$

The total energy, therefore, is great enough to reach the point x = 0 where U = 11.0 J, with a little "left over" (11.9 J – 11.0 J = 0.9025 J). This is the kinetic energy at x = 0, which means the speed there is

$$v = \sqrt{2(0.9025 \text{ J})/(2 \text{ kg})} = 0.950 \text{ m/s}.$$

It has now come to a stop, therefore, so it has not encountered a turning point.

(b) The total energy (11.9 J) is equal to the potential energy (in the scenario where it is initially moving rightward) at x = 10.9756 ≈ 11.0 m. This point may be found by interpolation or simply by using the work-kinetic energy theorem:

$$K_f = K_i + W = 0 \quad \Rightarrow \quad 11.9025 + (-4)d = 0 \quad \Rightarrow \quad d = 2.9756 \approx 2.98$$

(which when added to x = 8.00 [the point where F_3 begins to act] gives the correct result). This provides a turning point for the particle's motion.

83. **THINK** Energy is transferred from an external agent to the block so that its speed continues to increase.

EXPRESS According to Eq. 8-25, the work done by the external force is $W = \Delta E_{\text{mech}} = \Delta K + \Delta U$. When there is no change in potential energy, $\Delta U = 0$, the expression simplifies to

$$W = \Delta E_{\text{mech}} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2).$$

The average power, or average rate of work done, is given by $P_{\text{avg}} = W/\Delta t$.

ANALYZE (a) Substituting the values given, the change in mechanical energy is

$$\Delta E_{\text{mech}} = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(15\text{ kg})[(30\text{ m/s})^2 - (10\text{ m/s})^2] = 6000\text{ J} = 6.0\times10^3\text{ J}$$

(b) From the above, we have W = 6.0 × 10^3 J. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10$ s. Thus, using Eq. 7-42, the average rate at which energy is transferred to the block is

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0\times10^3\text{ J}}{10.0\text{ s}} = 600\text{ W}.$$

(c) and (d) The constant applied force is $F = ma$ = 30 N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power:

$$P = \vec{F}\cdot\vec{v} = \begin{cases} 300\text{ W} & \text{for } v = 10\text{ m/s} \\ 900\text{ W} & \text{for } v = 30\text{ m/s} \end{cases}$$

LEARN The average of these two values found in (c) and (d) agrees with the result in part (b). Note that the expression for the instantaneous rate used above can be derived from:

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = m\vec{v}\cdot\frac{d\vec{v}}{dt} = m\vec{v}\cdot\vec{a} = \vec{F}\cdot\vec{v}$$

84. (a) To stretch the spring an external force, equal in magnitude to the force of the spring but opposite to its direction, is applied. Since a spring stretched in the positive x direction exerts a force in the negative x direction, the applied force must be $F = 52.8x + 38.4x^2$, in the $+x$ direction. The work it does is

$$W=\int_{0.50}^{1.00}(52.8x+38.4x^2)dx=\left(\frac{52.8}{2}x^2+\frac{38.4}{3}x^3\right)\Bigg|_{0.50}^{1.00}=31.0\text{ J}.$$

(b) The spring does 31.0 J of work and this must be the increase in the kinetic energy of the particle. Its speed is then

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(31.0\text{ J})}{2.17\text{ kg}}}=5.35\text{ m/s}.$$

(c) The force is conservative since the work it does as the particle goes from any point x_1 to any other point x_2 depends only on x_1 and x_2, not on details of the motion between x_1 and x_2.

85. **THINK** This problem deals with the concept of hydroelectric generator – kinetic energy of water can be converted into electrical energy.

EXPRESS By energy conservation, the change in kinetic energy of water in one second is

$$\Delta K=-\Delta U=mgh=\rho Vgh=(10^3\text{ kg/m}^3)(1200\text{ m}^3)(9.8\text{ m/s}^2)(100\text{ m})=1.176\times10^9\text{ J}$$

Only 3/4 of this amount is transferred to electrical energy.

ANALYZE The power generation (assumed constant, so average power is the same as instantaneous power) is

$$P_{\text{avg}}=\frac{(3/4)\Delta K}{t}=\frac{(3/4)(1.176\times10^9\text{ J})}{1.0\text{ s}}=8.82\times10^8\text{ W}.$$

LEARN Hydroelectricity is the most widely used renewable energy; it accounts for almost 20% of the world's electricity supply.

86. (a) At B the speed is (from Eq. 8-17)

$$v=\sqrt{v_0^2+2gh_1}=\sqrt{(7.0\text{ m/s})^2+2(9.8\text{ m/s}^2)(6.0\text{ m})}=13\text{ m/s}.$$

(a) Here what matters is the difference in heights (between A and C):

$$v=\sqrt{v_0^2+2g(h_1-h_2)}=\sqrt{(7.0\text{ m/s})^2+2(9.8\text{ m/s}^2)(4.0\text{ m})}=11.29\text{ m/s}\approx11\text{ m/s}.$$

(c) Using the result from part (b), we see that its kinetic energy right at the beginning of its "rough slide" (heading horizontally toward D) is $\frac{1}{2}m(11.29\text{ m/s})^2=63.7m$ (with SI units understood). Note that we "carry along" the mass (as if it were a known quantity);

as we will see, it will cancel out, shortly. Using Eq. 8-31 (and Eq. 6-2 with $F_N = mg$) we note that this kinetic energy will turn entirely into thermal energy

$$63.7m = \mu_k mgd$$

if $d < L$. With $\mu_k = 0.70$, we find $d = 9.3$ m, which is indeed less than L (given in the problem as 12 m). We conclude that the block stops before passing out of the "rough" region (and thus does not arrive at point D).

87. **THINK** We have a ball attached to a rod that moves in a vertical circle. The total mechanical energy of the system is conserved.

EXPRESS Let position A be the reference point for potential energy, $U_A = 0$. The total mechanical energies at A, B and C are:

$$E_A = \frac{1}{2}mv_A^2 + U_A = \frac{1}{2}mv_0^2$$
$$E_B = \frac{1}{2}mv_B^2 + U_B = \frac{1}{2}mv_B^2 - mgL$$
$$E_D = \frac{1}{2}mv_D^2 + U_D = mgL$$

where $v_D = 0$. The problem can be analyzed by applying energy conservation: $E_A = E_B = E_D$.

ANALYZE (a) The condition $E_A = E_D$ gives

$$\frac{1}{2}mv_0^2 = mgL \quad \Rightarrow \quad v_0 = \sqrt{2gL}$$

(b) To find the tension in the rod when the ball passes through B, we first calculate the speed at B. Using $E_B = E_D$, we find

$$\frac{1}{2}mv_B^2 - mgL = mgL$$

or $v_B = \sqrt{4gL}$. The direction of the centripetal acceleration is upward (at that moment), as is the tension force. Thus, Newton's second law gives

$$T - mg = \frac{mv_B^2}{r} = \frac{m(4gL)}{L} = 4mg$$

or $T = 5mg$.

(c) The difference in height between C and D is L, so the "loss" of mechanical energy (which goes into thermal energy) is $-mgL$.

(d) The difference in height between B and D is $2L$, so the total "loss" of mechanical energy (which all goes into thermal energy) is $-2mgL$.

LEARN An alternative way to calculate the energy loss in (d) is to note that

$$E'_B = \frac{1}{2}mv'^2_B + U_B = 0 - mgL = -mgL$$

which gives

$$\Delta E = E'_B - E_A = -mgL - mgL = -2mgL.$$

88. (a) The initial kinetic energy is $K_i = \frac{1}{2}(1.5)(3)^2 = 6.75\text{ J}$.

(b) The work of gravity is the negative of its change in potential energy. At the highest point, all of K_i has converted into U (if we neglect air friction) so we conclude the work of gravity is –6.75 J.

(c) And we conclude that $\Delta U = 6.75\text{ J}$.

(d) The potential energy there is $U_f = U_i + \Delta U = 6.75\text{ J}$.

(e) If $U_f = 0$, then $U_i = U_f - \Delta U = -6.75\text{ J}$.

(f) Since $mg\Delta y = \Delta U$, we obtain $\Delta y = 0.459\text{ m}$.

89. (a) By mechanical energy conversation, the kinetic energy as it reaches the floor (which we choose to be the $U = 0$ level) is the sum of the initial kinetic and potential energies:

$$K = K_i + U_i = \frac{1}{2}\,(2.50\text{ kg})(3.00\text{ m/s})^2 + (2.50\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m}) = 109\text{ J}.$$

For later use, we note that the speed with which it reaches the ground is

$$v = \sqrt{2K/m} = 9.35\text{ m/s}.$$

(b) When the drop in height is 2.00 m instead of 4.00 m, the kinetic energy is

$$K = \frac{1}{2}\,(2.50\text{ kg})(3.00\text{ m/s})^2 + (2.50\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 60.3\text{ J}.$$

(c) A simple way to approach this is to imagine the can being *launched* from the ground at $t = 0$ with a speed 9.35 m/s (see above) and calculate the height and speed at t = 0.200 s, using Eq. 2-15 and Eq. 2-11:

$$y = (9.35 \text{ m/s})(0.200 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(0.200 \text{ s})^2 = 1.67 \text{ m},$$

$$v = 9.35 \text{ m/s} - (9.80 \text{ m/s}^2)(0.200 \text{ s}) = 7.39 \text{ m/s}.$$

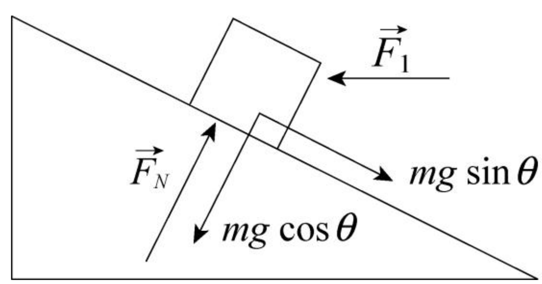
The kinetic energy is $K = \frac{1}{2}(2.50 \text{ kg})(7.39 \text{ m/s})^2 = 68.2 \text{ J}$.

(d) The gravitational potential energy is

$$U = mgy = (2.5 \text{ kg})(9.8 \text{ m/s}^2)(1.67 \text{ m}) = 41.0 \text{ J}.$$

90. The free-body diagram for the trunk is shown below. The x and y applications of Newton's second law provide two equations:

$$F_1 \cos\theta - f_k - mg\sin\theta = ma$$
$$F_N - F_1 \sin\theta - mg\cos\theta = 0.$$



(a) The trunk is moving up the incline at constant velocity, so $a = 0$. Using $f_k = \mu_k F_N$, we solve for the push-force F_1 and obtain

$$F_1 = \frac{mg(\sin\theta + \mu_k\cos\theta)}{\cos\theta - \mu_k \sin\theta}.$$

The work done by the push-force $\vec{F}_1$ as the trunk is pushed through a distance ℓ up the inclined plane is therefore

$$\begin{aligned} W_1 = F_1\ell\cos\theta &= \frac{(mg\ell\cos\theta)(\sin\theta + \mu_k\cos\theta)}{\cos\theta - \mu_k\sin\theta} \\ &= \frac{(50 \text{ kg})(9.8 \text{ m/s}^2)(6.0 \text{ m})(\cos 30°)(\sin 30° + (0.20)\cos 30°)}{\cos 30° - (0.20)\sin 30°} \\ &= 2.2\times10^3 \text{ J}. \end{aligned}$$

(b) The increase in the gravitational potential energy of the trunk is

$$\Delta U = mg\ell \sin\theta = (50\,\text{kg})(9.8\,\text{m/s}^2)(6.0\,\text{m})\sin 30° = 1.5\times 10^3\ \text{J}.$$

Since the speed (and, therefore, the kinetic energy) of the trunk is unchanged, Eq. 8-33 leads to

$$W_1 = \Delta U + \Delta E_{\text{th}}.$$

Thus, using more precise numbers than are shown above, the increase in thermal energy (generated by the kinetic friction) is 2.24×10^3 J – 1.47×10^3 J = 7.7×10^2 J. An alternate way to this result is to use $\Delta E_{\text{th}} = f_k \ell$ (Eq. 8-31).

91. The initial height of the $2M$ block, shown in Fig. 8-69, is the $y = 0$ level in our computations of its value of U_g. As that block drops, the spring stretches accordingly. Also, the kinetic energy K_{sys} is evaluated for the *system*, that is, for a total moving mass of $3M$.

(a) The conservation of energy, Eq. 8-17, leads to

$$K_i + U_i = K_{sys} + U_{sys} \quad\Rightarrow\quad 0 + 0 = K_{sys} + (2M)g(-0.090) + \frac{1}{2}\,k(0.090)^2 .$$

Thus, with M = 2.0 kg, we obtain K_{sys} = 2.7 J.

(b) The kinetic energy of the $2M$ block represents a fraction of the total kinetic energy:

$$\frac{K_{2M}}{K_{sys}} = \frac{(2M)v^2/2}{(3M)v^2/2} = \frac{2}{3}.$$

Therefore, $K_{2M} = \frac{2}{3}(2.7\ \text{J}) = 1.8$ J.

(c) Here we let $y = -d$ and solve for d.

$$K_i + U_i = K_{sys} + U_{sys} \quad\Rightarrow\quad 0 + 0 = 0 + (2M)g(-d) + \frac{1}{2}\,kd^2 .$$

Thus, with M = 2.0 kg, we obtain d = 0.39 m.

92. By energy conservation, $mgh = mv^2/2$, the speed of the volcanic ash is given by $v = \sqrt{2gh}$. In our present problem, the height is related to the distance (on the θ = 10º slope) d = 920 m by the trigonometric relation $h = d\sin\theta$. Thus,

$$v = \sqrt{2(9.8\,\text{m/s}^2)(920\ \text{m})\sin 10°} = 56\ \text{m/s}.$$

93. (a) The assumption is that the slope of the bottom of the slide is horizontal, like the ground. A useful analogy is that of the pendulum of length R = 12 m that is pulled

leftward to an angle θ (corresponding to being at the top of the slide at height h = 4.0 m) and released so that the pendulum swings to the lowest point (zero height) gaining speed $v = 6.2\ \text{m/s}$. Exactly as we would analyze the trigonometric relations in the pendulum problem, we find

$$h = R(1-\cos\theta) \Rightarrow \theta = \cos^{-1}\left(1-\frac{h}{R}\right) = 48°$$

or 0.84 radians. The slide, representing a circular arc of length $s = R\theta$, is therefore (12 m)(0.84) = 10 m long.

(b) To find the magnitude f of the frictional force, we use Eq. 8-31 (with W = 0):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + fs \end{aligned}$$

so that (with m = 25 kg) we obtain f = 49 N.

(c) The assumption is no longer that the slope of the bottom of the slide is horizontal, but rather that the slope of the top of the slide is vertical (and 12 m to the left of the center of curvature). Returning to the pendulum analogy, this corresponds to releasing the pendulum from horizontal (at θ_1 = 90° measured from vertical) and taking a snapshot of its motion a few moments later when it is at angle θ_2 with speed v = 6.2 m/s. The difference in height between these two positions is (just as we would figure for the pendulum of length R)

$$\Delta h = R(1-\cos\theta_2) - R(1-\cos\theta_1) = -R\cos\theta_2$$

where we have used the fact that cos θ_1 = 0. Thus, with Δh = –4.0 m, we obtain θ_2 = 70.5° which means the arc subtends an angle of $|\Delta\theta|$ = 19.5° or 0.34 radians. Multiplying this by the radius gives a slide length of s' = 4.1 m.

(d) We again find the magnitude $f\,'$ of the frictional force by using Eq. 8-31 (with W = 0):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + f\,'s' \end{aligned}$$

so that we obtain $f\,' = 1.2 \times 10^2$ N.

94. We use $P = Fv$ to compute the force:

$$F = \frac{P}{v} = \frac{92\times10^6\ \text{W}}{(32.5\ \text{knot})\left(1.852\ \frac{\text{km/h}}{\text{knot}}\right)\left(\frac{1000\ \text{m/km}}{3600\ \text{s/h}}\right)} = 5.5\times10^6\ \text{N}.$$

95. This can be worked entirely by the methods of Chapters 2–6, but we will use energy methods in as many steps as possible.

(a) By a force analysis in the style of Chapter 6, we find the normal force has magnitude $F_N = mg\cos\theta$ (where $\theta = 39°$), which means $f_k = \mu_k mg\cos\theta$ where $\mu_k = 0.28$. Thus, Eq. 8-31 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta.$$

Also, elementary trigonometry leads us to conclude that $\Delta U = -mgd\sin\theta$ where $d = 3.7\text{ m}$. Since $K_i = 0$, Eq. 8-33 (with $W = 0$) indicates that the final kinetic energy is

$$K_f = -\Delta U - \Delta E_{\text{th}} = mgd\,(\sin\theta - \mu_k\cos\theta)$$

which leads to the speed at the bottom of the ramp

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gd\,(\sin\theta - \mu_k\cos\theta)} = 5.5\text{ m/s}.$$

(b) This speed begins its horizontal motion, where $f_k = \mu_k mg$ and $\Delta U = 0$. It slides a distance d' before it stops. According to Eq. 8-31 (with $W = 0$),

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= 0 - \frac{1}{2}mv^2 + 0 + \mu_k mgd' \\ &= -\frac{1}{2}\left(2gd\,(\sin\theta - \mu_k\cos\theta)\right) + \mu_k gd' \end{aligned}$$

where we have divided by mass and substituted from part (a) in the last step. Therefore,

$$d' = \frac{d(\sin\theta - \mu_k\cos\theta)}{\mu_k} = 5.4\text{ m}.$$

(c) We see from the algebraic form of the results, above, that the answers do not depend on mass. A 90 kg crate should have the same speed at the bottom and sliding distance across the floor, to the extent that the friction relations in Chapter 6 are accurate. Interestingly, since g does not appear in the relation for d', the sliding distance would seem to be the same if the experiment were performed on Mars!

96. (a) The loss of the initial $K = \frac{1}{2}mv^2 = \frac{1}{2}(70\text{ kg})(10\text{ m/s})^2$ is 3500 J, or 3.5 kJ.

(b) This is dissipated as thermal energy; $\Delta E_{\text{th}} = 3500\text{ J} = 3.5\text{ kJ}$.

97. Eq. 8-33 gives $mgy_f = K_i + mgy_i - \Delta E_{\text{th}}$, or

$$(0.50 \text{ kg})(9.8 \text{ m/s}^2)(0.80 \text{ m}) = \frac{1}{2}(0.50 \text{ kg})(4.00 \text{ /s})^2 + (0.50 \text{ kg})(9.8 \text{ m/s}^2)(0) - \Delta E_{\text{th}}$$

which yields ΔE_{th} = 4.00 J – 3.92 J = 0.080 J.

98. Since the period T is (2.5 rev/s)$^{-1}$ = 0.40 s, then Eq. 4-33 leads to v = 3.14 m/s. The frictional force has magnitude (using Eq. 6-2)

$$f = \mu_k F_N = (0.320)(180 \text{ N}) = 57.6 \text{ N}.$$

The power dissipated by the friction must equal that supplied by the motor, so Eq. 7-48 gives P = (57.6 N)(3.14 m/s) = 181 W.

99. To swim at constant velocity the swimmer must push back against the water with a force of 110 N. Relative to him the water is going at 0.22 m/s toward his rear, in the same direction as his force. Using Eq. 7-48, his power output is obtained:

$$P = \vec{F} \cdot \vec{v} = Fv = (110 \text{ N})(0.22 \text{ m/s}) = 24 \text{ W}.$$

100. The initial kinetic energy of the automobile of mass m moving at speed v_i is $K_i = \frac{1}{2}mv_i^2$, where m = 16400/9.8 = 1673 kg. Using Eq. 8-31 and Eq. 8-33, this relates to the effect of friction force f in stopping the auto over a distance d by $K_i = fd$, where the road is assumed level (so ΔU = 0). With

$$v_i = (113 \text{ km/h}) = (113 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 31.4 \text{ m/s},$$

we obtain

$$d = \frac{K_i}{f} = \frac{mv_i^2}{2f} = \frac{(1673 \text{ kg})(31.4 \text{ m/s})^2}{2(8230 \text{ N})} = 100 \text{ m}.$$

101. With the potential energy reference level set at the point of throwing, we have (with SI units understood)

$$\Delta E = mgh - \frac{1}{2}mv_0^2 = m\left((9.8)(8.1) - \frac{1}{2}(14)^2\right)$$

which yields ΔE = –12 J for m = 0.63 kg. This “loss” of mechanical energy is presumably due to air friction.

102. (a) The (internal) energy the climber must convert to gravitational potential energy is

$$\Delta U = mgh = (90\text{ kg})(9.80\text{ m/s}^2)(8850\text{ m}) = 7.8\times10^6\text{ J}.$$

(b) The number of candy bars this corresponds to is

$$N = \frac{7.8\times10^6\text{ J}}{1.25\times10^6\text{ J/bar}} \approx 6.2\,\text{bars}.$$

103. (a) The acceleration of the sprinter is (using Eq. 2-15)

$$a = \frac{2\Delta x}{t^2} = \frac{(2)(7.0\,\text{m})}{(1.6\,\text{s})^2} = 5.47\,\text{m/s}^2.$$

Consequently, the speed at t = 1.6s is $v = at = (5.47\,\text{m/s}^2)(1.6\,\text{s}) = 8.8\,\text{m/s}$. Alternatively, Eq. 2-17 could be used.

(b) The kinetic energy of the sprinter (of weight w and mass $m = w/g$) is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{1}{2}\left(670\text{ N/(9.8 m/s}^2)\right)(8.8\text{ m/s})^2 = 2.6\times10^3\text{ J}.$$

(c) The average power is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{2.6\times10^3\text{ J}}{1.6\text{ s}} = 1.6\times10^3\text{ W}.$$

104. From Eq. 8-6, we find (with SI units understood)

$$U(\xi) = -\int_0^{\xi}\left(-3x - 5x^2\right)dx = \frac{3}{2}\xi^2 + \frac{5}{3}\xi^3.$$

(a) Using the above formula, we obtain $U(2) \approx 19$ J.

(b) When its speed is v = 4 m/s, its mechanical energy is $\frac{1}{2}mv^2 + U(5)$. This must equal the energy at the origin:

$$\frac{1}{2}mv^2 + U(5) = \frac{1}{2}mv_o^2 + U(0)$$

so that the speed at the origin is

$$v_o = \sqrt{v^2 + \frac{2}{m}\left(U(5) - U(0)\right)}.$$

Thus, with $U(5)$ = 246 J, $U(0)$ = 0 and m = 20 kg, we obtain v_o = 6.4 m/s.

(c) Our original formula for U is changed to

$$U(x) = -8 + \frac{3}{2}x^2 + \frac{5}{3}x^3$$

in this case. Therefore, $U(2) = 11$ J. But we still have $v_o = 6.4$ m/s since that calculation only depended on the difference of potential energy values (specifically, $U(5) - U(0)$).

105. (a) Resolving the gravitational force into components and applying Newton's second law (as well as Eq. 6-2), we find

$$F_{\text{machine}} - mg\sin\theta - \mu_k mg\cos\theta = ma.$$

In the situation described in the problem, we have $a = 0$, so

$$F_{\text{machine}} = mg\sin\theta + \mu_k mg\cos\theta = 372 \text{ N}.$$

Thus, the work done by the machine is $F_{\text{machine}}d = 744 \text{ J} = 7.4 \times 10^2$ J.

(b) The thermal energy generated is $(\mu_k mg\cos\theta)\, d = 240 \text{ J} = 2.4 \times 10^2$ J.

106. (a) At the highest point, the velocity $v = v_x$ is purely horizontal and is equal to the horizontal component of the launch velocity (see section 4-6): $v_{ox} = v_o\cos\theta$, where $\theta = 30°$ in this problem. Equation 8-17 relates the kinetic energy at the highest point to the launch kinetic energy:

$$K_o = mgy + \frac{1}{2}mv^2 = \frac{1}{2}mv_{ox}^2 + \frac{1}{2}mv_{oy}^2,$$

with $y = 1.83$ m. Since the $mv_{ox}^2/2$ term on the left-hand side cancels the $mv^2/2$ term on the right-hand side, this yields $v_{oy} = \sqrt{2gy} \approx 6$ m/s. With $v_{oy} = v_o\sin\theta$, we obtain

$$v_o = 11.98 \text{ m/s} \approx 12 \text{ m/s}.$$

(b) Energy conservation (including now the energy stored elastically in the spring, Eq. 8-11) also applies to the motion along the muzzle (through a distance d that corresponds to a vertical height increase of $d\sin\theta$):

$$\frac{1}{2}kd^2 = K_o + mgd\sin\theta \quad \Rightarrow \quad d = 0.11 \text{ m}.$$

107. The work done by $\vec{F}$ is the negative of its potential energy change (see Eq. 8-6), so $U_B = U_A - 25 = 15$ J.

108. (a) We assume his mass is between m_1 = 50 kg and m_2 = 70 kg (corresponding to a weight between 110 lb and 154 lb). His increase in gravitational potential energy is therefore in the range

$$m_1 gh \le \Delta U \le m_2 gh \quad \Rightarrow \quad 2\times10^5 \le \Delta U \le 3\times10^5$$

in SI units (J), where h = 443 m.

(b) The problem only asks for the amount of internal energy that converts into gravitational potential energy, so this result is the same as in part (a). But if we were to consider his *total* internal energy "output" (much of which converts to heat) we can expect that external climb is quite different from taking the stairs.

109. (a) We implement Eq. 8-37 as

$$K_f \quad = K_i + mgy_i - f_k\, d = 0 + (60 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) - 0 = 2.35 \times 10^3 \text{ J}.$$

(b) Now it applies with a nonzero thermal term:

$$K_f = K_i + mgy_i - f_k\, d = 0 + (60 \text{ kg})(9.8 \text{ m/s}^2)(4.0 \text{ m}) - (500 \text{ N})(4.0 \text{ m}) = 352 \text{ J}.$$

110. We take the bottom of the incline to be the y = 0 reference level. The incline angle is $\theta = 30°$. The distance along the incline d (measured from the bottom) is related to height y by the relation $y = d \sin\theta$.

(a) Using the conservation of energy, we have

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} \Rightarrow \frac{1}{2}mv_0^2 + 0 = 0 + mgy$$

with $v_0 = 5.0 \text{ m/s}$. This yields y = 1.3 m, from which we obtain d = 2.6 m.

(b) An analysis of forces in the manner of Chapter 6 reveals that the magnitude of the friction force is $f_k = \mu_k mg \cos\theta$. Now, we write Eq. 8-33 as

$$\begin{aligned} K_0 + U_0 &= K_{\text{top}} + U_{\text{top}} + f_k d \\ \frac{1}{2}mv_0^2 + 0 &= 0 + mgy + f_k d \\ \frac{1}{2}mv_0^2 &= mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which — upon canceling the mass and rearranging — provides the result for d:

$$d = \frac{v_0^2}{2g\left(\mu_k \cos\theta + \sin\theta\right)} = 1.5 \text{ m} .$$

(c) The thermal energy generated by friction is $f_k d = \mu_k mgd \cos\theta = 26$ J.

(d) The slide back down, from the height $y = 1.5 \sin 30°$, is also described by Eq. 8-33. With ΔE_{th} again equal to 26 J, we have

$$K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}} + f_k d \Rightarrow 0 + mgy = \frac{1}{2} m v_{\text{bot}}^2 + 0 + 26$$

from which we find $v_{\text{bot}} = 2.1 \text{ m/s}$.

111. Equation 8-8 leads directly to $\Delta y = \dfrac{68000 \text{ J}}{(9.4 \text{ kg})(9.8 \text{ m/s}^2)} = 738$ m.

112. We assume his initial kinetic energy (when he jumps) is negligible. Then, his initial gravitational potential energy measured relative to where he momentarily stops is what becomes the elastic potential energy of the stretched net (neglecting air friction). Thus,

$$U_{\text{net}} = U_{\text{grav}} = mgh$$

where h = 11.0 m + 1.5 m = 12.5 m. With m = 70 kg, we obtain U_{net} = 8580 J.

113. We use SI units so m = 0.030 kg and d = 0.12 m.

(a) Since there is no change in height (and we assume no changes in elastic potential energy), then $\Delta U = 0$ and we have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2} m v_0^2 = -3.8 \times 10^3 \text{ J}$$

where v_0 = 500 m/s and the final speed is zero.

(b) By Eq. 8-33 (with $W = 0$) we have $\Delta E_{\text{th}} = 3.8 \times 10^3$ J, which implies

$$f = \frac{\Delta E_{\text{th}}}{d} = 3.1 \times 10^4 \text{ N}$$

using Eq. 8-31 with f_k replaced by f (effectively generalizing that equation to include a greater variety of dissipative forces than just those obeying Eq. 6-2).

114. (a) The kinetic energy K of the automobile of mass m at t = 30 s is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1500\,\text{kg})\left((72\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right)^2 = 3.0\times 10^5\,\text{J}.$$

(b) The average power required is

$$P_{\text{avg}} = \frac{\Delta K}{\Delta t} = \frac{3.0\times 10^5\,\text{J}}{30\,\text{s}} = 1.0\times 10^4\,\text{W}.$$

(c) Since the acceleration a is constant, the power is $P = Fv = mav = ma(at) = ma^2t$ using Eq. 2-11. By contrast, from part (b), the average power is $P_{\text{avg}} = \frac{mv^2}{2t}$, which becomes $\frac{1}{2}ma^2t$ when $v = at$ is again utilized. Thus, the instantaneous power at the end of the interval is twice the average power during it:

$$P = 2P_{\text{avg}} = (2)(1.0\times 10^4\,\text{W}) = 2.0\times 10^4\,\text{W}.$$

115. (a) The initial kinetic energy is $K_i = (1.5\text{ kg})(20\text{ m/s})^2/2 = 300\text{ J}$.

(b) At the point of maximum height, the vertical component of velocity vanishes but the horizontal component remains what it was when it was “shot” (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(1.5\text{ kg})\left[(20\text{ m/s})\cos 34°\right]^2 = 206\text{ J}.$$

Thus, $\Delta U = K_i - K = 300\text{ J} - 206\text{ J} = 93.8\text{ J}$.

(c) Since $\Delta U = mg\,\Delta y$, we obtain $\Delta y = \dfrac{94\text{ J}}{(1.5\text{ kg})(9.8\text{ m/s}^2)} = 6.38\text{ m}$.

116. (a) The rate of change of the gravitational potential energy is

$$\frac{dU}{dt} = mg\frac{dy}{dt} = -mg|v| = -(68)(9.8)(59) = -3.9\times 10^4\,\text{J/s}.$$

Thus, the gravitational energy is being reduced at the rate of 3.9×10^4 W.

(b) Since the velocity is constant, the rate of change of the kinetic energy is zero. Thus the rate at which the mechanical energy is being dissipated is the same as that of the gravitational potential energy (3.9×10^4 W).

117. (a) The effect of (sliding) friction is described in terms of energy dissipated as shown in Eq. 8-31. We have

$$\Delta E = K + \frac{1}{2}k(0.08)^2 - \frac{1}{2}k(0.10)^2 = -f_k(0.02)$$

where distances are in meters and energies are in joules. With k = 4000 N/m and $f_k = 80$ N, we obtain K = 5.6 J.

(b) In this case, we have d = 0.10 m. Thus,

$$\Delta E = K + 0 - \frac{1}{2}k(0.10)^2 = -f_k(0.10)$$

which leads to K = 12 J.

(c) We can approach this two ways. One way is to examine the dependence of energy on the variable d:

$$\Delta E = K + \frac{1}{2}k(d_0 - d)^2 - \frac{1}{2}kd_0^2 = -f_k d$$

where d_0 = 0.10 m, and solving for K as a function of d:

$$K = -\frac{1}{2}kd^2 + (kd_0)d - f_k d .$$

In this first approach, we could work through the $dK/d(d) = 0$ condition (or with the special capabilities of a graphing calculator) to obtain the answer $K_{\max} = \frac{1}{2k}(kd_0 - f_k)^2$. In the second (and perhaps easier) approach, we note that K is maximum where v is maximum — which is where $a = 0 \Rightarrow$ equilibrium of forces. Thus, the second approach simply solves for the equilibrium position

$$|F_{\text{spring}}| = f_k \Rightarrow kx = 80.$$

Thus, with k = 4000 N/m we obtain x = 0.02 m. But $x = d_0 - d$ so this corresponds to d = 0.08 m. Then the methods of part (a) lead to the answer $K_{\max}$ = 12.8 J ≈ 13 J.

118. We work this in SI units and convert to horsepower in the last step. Thus,

$$v = (80\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 22.2\,\text{m/s}.$$

The force F_P needed to propel the car (of weight w and mass $m = w/g$) is found from Newton's second law:

$$F_{\text{net}} = F_P - F = ma = \frac{wa}{g}$$

where $F = 300 + 1.8v^2$ in SI units. Therefore, the power required is

$$P = \vec{F}_P \cdot \vec{v} = \left(F + \frac{wa}{g}\right)v = \left(300 + 1.8(22.2)^2 + \frac{(12000)(0.92)}{9.8}\right)(22.2) = 5.14\times10^4 \text{ W}$$
$$= \left(5.14\times10^4 \text{ W}\right)\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 69 \text{ hp}.$$

119. **THINK** We apply energy method to analyze the projectile motion of a ball.

EXPRESS We choose the initial position at the window to be our reference point for calculating the potential energy. The initial energy of the ball is $E_0 = \frac{1}{2}mv_0^2$. At the top of its flight, the vertical component of the velocity is zero, and the horizontal component (neglecting air friction) is the same as it was when it was thrown: $v_x = v_0\cos\theta$. At a position h below the window, the energy of the ball is

$$E = K + U = \frac{1}{2}mv^2 - mgh$$

where v is the speed of the ball.

ANALYZE (a) The kinetic energy of the ball at the top of the flight is

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}m(v_0\cos\theta)^2 = \frac{1}{2}(0.050 \text{ kg})[(8.0 \text{ m/s})\cos 30°]^2 = 1.2 \text{ J}.$$

(b) When the ball is h = 3.0 m below the window, by energy conservation, we have

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 - mgh$$

or

$$v = \sqrt{v_0^2 + 2gh} = \sqrt{(8.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(3.0 \text{ m})} = 11.1 \text{ m/s}.$$

(c) As can be seen from our expression above, $v = \sqrt{v_0^2 + 2gh}$, which is independent of the mass m.

(d) Similarly, the speed v is independent of the initial angle θ.

LEARN Our results demonstrate that the quantity v in the kinetic energy formula is the magnitude of the velocity vector; it does not depend on direction. In addition, mass cancels out in the energy conservation equation, so that v is independent of m.

120. (a) In the initial situation, the elongation was (using Eq. 8-11)

$$x_i = \sqrt{2(1.44)/3200} = 0.030 \text{ m (or 3.0 cm)}.$$

In the next situation, the elongation is only 2.0 cm (or 0.020 m), so we now have less stored energy (relative to what we had initially). Specifically,

$$\Delta U = \frac{1}{2}(3200 \text{ N/m})(0.020 \text{ m})^2 - 1.44 \text{ J} = -0.80 \text{ J}.$$

(b) The elastic stored energy for $|x| = 0.020$ m does not depend on whether this represents a stretch or a compression. The answer is the same as in part (a), $\Delta U = -0.80$ J.

(c) Now we have $|x| = 0.040$ m, which is greater than x_i, so this represents an increase in the potential energy (relative to what we had initially). Specifically,

$$\Delta U = \frac{1}{2}(3200 \text{ N/m})(0.040 \text{ m})^2 - 1.44 \text{ J} = +1.12 \text{ J} \approx 1.1 \text{ J}.$$

121. (a) With $P = 1.5$ MW $= 1.5 \times 10^6$ W (assumed constant) and $t = 6.0$ min $= 360$ s, the work-kinetic energy theorem becomes

$$W = Pt = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right).$$

The mass of the locomotive is then

$$m = \frac{2Pt}{v_f^2 - v_i^2} = \frac{(2)\left(1.5\times10^6 \text{ W}\right)(360 \text{ s})}{(25 \text{ m/s})^2 - (10 \text{ m/s})^2} = 2.1\times10^6 \text{ kg}.$$

(b) With t arbitrary, we use $Pt = \frac{1}{2}m\left(v^2 - v_i^2\right)$ to solve for the speed $v = v(t)$ as a function of time and obtain

$$v(t) = \sqrt{v_i^2 + \frac{2Pt}{m}} = \sqrt{(10)^2 + \frac{(2)\left(1.5\times10^6\right)t}{2.1\times10^6}} = \sqrt{100 + 1.5t}$$

in SI units (v in m/s and t in s).

(c) The force $F(t)$ as a function of time is

$$F(t)=\frac{P}{v(t)}=\frac{1.5\times10^{6}}{\sqrt{100+1.5t}}$$

in SI units (F in N and t in s).

(d) The distance d the train moved is given by

$$d=\int_0^t v(t')dt'=\int_0^{360}\left(100+\frac{3}{2}t\right)^{1/2}dt=\frac{4}{9}\left(100+\frac{3}{2}t\right)^{3/2}\Bigg|_0^{360}=6.7\times10^{3}\text{ m.}$$

122. **THINK** A shuffleboard disk is accelerated over some distance by an external force, but it eventually comes to rest due to the frictional force.

EXPRESS In the presence of frictional force, the work done on a system is $W=\Delta E_{\text{mech}}+\Delta E_{\text{th}}$, where $\Delta E_{\text{mech}}=\Delta K+\Delta U$ and $\Delta E_{\text{th}}=f_k d$. In our situation, work has been done by the cue only to the first 2.0 m, and not to the subsequent 12 m of distance traveled.

ANALYZE (a) During the final d = 12 m of motion, $W=0$ and we use

$$K_1+U_1=K_2+U_2+f_k d$$
$$\frac{1}{2}mv^2+0=0+0+f_k d$$

where $m=0.42$ kg and v = 4.2 m/s. This gives f_k = 0.31 N. Therefore, the thermal energy change is $\Delta E_{\text{th}}=f_k d=3.7$ J.

(b) Using f_k = 0.31 N for the entire distance d_{total} = 14 m, we obtain

$$\Delta E_{\text{th,total}}=f_k d_{\text{total}}=(0.31\text{ N})(14\text{ m})=4.3\text{ J}$$

for the thermal energy generated by friction.

(c) During the initial d' = 2 m of motion, we have

$$W=\Delta E_{\text{mech}}+\Delta E'_{\text{th}}=\Delta K+\Delta U+f_k d'=\frac{1}{2}mv^2+0+f_k d'$$

which essentially combines Eq. 8-31 and Eq. 8-33. Thus, the work done on the disk by the cue is

$$W=\frac{1}{2}mv^2+f_k d'=\frac{1}{2}(0.42\text{ kg})(4.2\text{ m/s})^2+(0.31\text{ N})(2.0\text{ m})=4.3\text{ J}.$$

LEARN Our answer in (c) is the same as that in (b). This is expected because all the work done becomes thermal energy at the end.

123. The water has gained

$$\Delta K = \frac{1}{2}\ (10\ \text{kg})(13\ \text{m/s})^2 - \frac{1}{2}\ (10\ \text{kg})(3.2\ \text{m/s})^2 = 794\ \text{J}$$

of kinetic energy, and it has lost ΔU = (10 kg)(9.8 m/s^2)(15 m) = 1470 J .

of potential energy (the lack of agreement between these two values is presumably due to transfer of energy into thermal forms). The ratio of these values is 0.54 = 54%. The mass of the water cancels when we take the ratio, so that the assumption (stated at the end of the problem: m = 10 kg) is not needed for the final result.

124. (a) The integral (see Eq. 8-6, where the value of U at $x = \infty$ is required to vanish) is straightforward. The result is $U(x) = -Gm_1m_2/x$.

(b) One approach is to use Eq. 8-5, which means that we are effectively doing the integral of part (a) all over again. Another approach is to use our result from part (a) (and thus use Eq. 8-1). Either way, we arrive at

$$W = \frac{G\,m_1\,m_2}{x_1} - \frac{G\,m_1\,m_2}{x_1 + d} = \frac{G\,m_1\,m_2\,d}{x_1(x_1 + d)}\ .$$

125. (a) During one second, the decrease in potential energy is

$$-\Delta U = mg(-\Delta y) = (5.5\times10^6\ \text{kg})\left(9.8\,\text{m/s}^2\right)(50\ \text{m}) = 2.7\times10^9\ \text{J}$$

where $+y$ is upward and $\Delta y = y_f - y_i$.

(b) The information relating mass to volume is not needed in the computation. By Eq. 8-40 (and the SI relation W = J/s), the result follows:

$$P = (2.7\times10^9\ \text{J})/(1\ \text{s}) = 2.7\times10^9\ \text{W}.$$

(c) One year is equivalent to 24 × 365.25 = 8766 h which we write as 8.77 kh. Thus, the energy supply rate multiplied by the cost and by the time is

$$(2.7\times10^9\,\text{W})(8.77\ \text{kh})\left(\frac{1\ \text{cent}}{1\ \text{kWh}}\right) = 2.4\times10^{10}\,\text{cents} = \$2.4\times10^8.$$

126. The connection between angle θ (measured from vertical) and height h (measured from the lowest point, which is our choice of reference position in computing the

gravitational potential energy) is given by $h = L(1 - \cos\theta)$ where L is the length of the pendulum.

(a) We use energy conservation in the form of Eq. 8-17.

$$K_1 + U_1 = K_2 + U_2$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_2^2 + mgL(1 - \cos\theta_2)$$

With $L = 1.4$ m, $\theta_1 = 30°$, and $\theta_2 = 20°$, we have

$$v_2 = \sqrt{2gL(\cos\theta_2 - \cos\theta_1)} = 1.4 \text{ m/s}.$$

(b) The maximum speed v_3 is at the lowest point. Our formula for h gives $h_3 = 0$ when $\theta_3 = 0°$, as expected. From

$$K_1 + U_1 = K_3 + U_3$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_3^2 + 0$$

we obtain $v_3 = 1.9$ m/s.

(c) We look for an angle θ_4 such that the speed there is $v_4 = v_3/3$. To be as accurate as possible, we proceed algebraically (substituting $v_3^2 = 2gL(1 - \cos\theta_1)$ at the appropriate place) and plug numbers in at the end. Energy conservation leads to

$$K_1 + U_1 = K_4 + U_4$$
$$0 + mgL(1 - \cos\theta_1) = \frac{1}{2}mv_4^2 + mgL(1 - \cos\theta_4)$$
$$mgL(1 - \cos\theta_1) = \frac{1}{2}m\frac{v_3^2}{9} + mgL(1 - \cos\theta_4)$$
$$-gL\cos\theta_1 = \frac{1}{2}\frac{2gL(1 - \cos\theta_1)}{9} - gL\cos\theta_4$$

where in the last step we have subtracted out mgL and then divided by m. Thus, we obtain

$$\theta_4 = \cos^{-1}\left(\frac{1}{9} + \frac{8}{9}\cos\theta_1\right) = 28.2° \approx 28°.$$

127. Equating the mechanical energy at his initial position (as he emerges from the canon, where we set the reference level for computing potential energy) to his energy as he lands, we obtain

$$K_i = K_f + U_f$$

$$\frac{1}{2}(60\,\text{kg})(16\,\text{m/s})^2 = K_f + (60\,\text{kg})\left(9.8\,\text{m/s}^2\right)(3.9\,\text{m})$$

which leads to $K_f = 5.4 \times 10^3$ J.

128. (a) This part is essentially a free-fall problem, which can be easily done with Chapter 2 methods. Instead, choosing energy methods, we take $y = 0$ to be the ground level.

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0$$

Therefore $v = \sqrt{2gy_i} = 9.2\,\text{m/s}$, where $y_i = 4.3$ m.

(b) Eq. 8-29 provides $\Delta E_{\text{th}} = f_k d$ for thermal energy generated by the kinetic friction force. We apply Eq. 8-31:

$$K_i + U_i = K + U \Rightarrow 0 + mgy_i = \frac{1}{2}mv^2 + 0 + f_k d\,.$$

With $d = y_i$, $m = 70$ kg and $f_k = 500$ N, this yields $v = 4.8$ m/s.

129. We want to convert (at least in theory) the water that falls through $h = 500$ m into electrical energy. The problem indicates that in one year, a volume of water equal to $A\Delta z$ lands in the form of rain on the country, where $A = 8 \times 10^{12}$ m^2 and $\Delta z = 0.75$ m. Multiplying this volume by the density $\rho = 1000$ kg/m^3 leads to

$$m_{\text{total}} = \rho A \Delta z = (1000)\left(8 \times 10^{12}\right)(0.75) = 6 \times 10^{15}\,\text{kg}$$

for the mass of rainwater. One-third of this "falls" to the ocean, so it is $m = 2 \times 10^{15}$ kg that we want to use in computing the gravitational potential energy mgh (which will turn into electrical energy during the year). Since a year is equivalent to 3.2×10^7 s, we obtain

$$P_{\text{avg}} = \frac{\left(2 \times 10^{15}\right)(9.8)(500)}{3.2 \times 10^7} = 3.1 \times 10^{11}\,\text{W}.$$

130. The spring is relaxed at $y = 0$, so the elastic potential energy (Eq. 8-11) is $U_{\text{el}} = \frac{1}{2}ky^2$. The total energy is conserved, and is zero (determined by evaluating it at its initial position). We note that U is the same as ΔU in these manipulations. Thus, we have

$$0 = K + U_g + U_e \quad \Rightarrow \quad K = -U_g - U_e$$

where $U_g = mgy = (20\text{ N})y$ with y in meters (so that the energies are in Joules). We arrange the results in a table:

position y	–0.05	–0.10	–0.15	–0.20
K	(a) 0.75	(d) 1.0	(g) 0.75	(j) 0
U_g	(b) –1.0	(e) –2.0	(h) –3.0	(k) –4.0
U_e	(c) 0.25	(f) 1.0	(i) 2.25	(l) 4.0

131. Let the amount of stretch of the spring be x. For the object to be in equilibrium

$$kx - mg = 0 \Rightarrow x = mg/k .$$

Thus the gain in elastic potential energy for the spring is

$$\Delta U_e = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

while the loss in the gravitational potential energy of the system is

$$-\Delta U_g = mgx = mg\left(\frac{mg}{k}\right) = \frac{m^2g^2}{k}$$

which we see (by comparing with the previous expression) is equal to $2\Delta U_e$. The reason why $\left|\Delta U_g\right| \neq \Delta U_e$ is that, since the object is slowly lowered, an upward external force (e.g., due to the hand) must have been exerted on the object during the lowering process, preventing it from accelerating downward. This force does *negative* work on the object, reducing the total mechanical energy of the system.

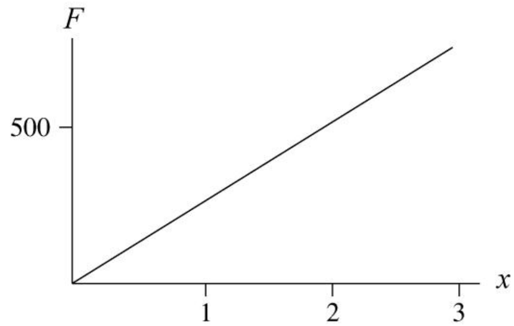
132. (a) The compression is "spring-like" so the maximum force relates to the distance x by Hooke's law:

$$F_x = kx \Rightarrow x = \frac{750}{2.5\times 10^5} = 0.0030\,\text{m}.$$

(b) The work is what produces the "spring-like" potential energy associated with the compression. Thus, using Eq. 8-11,

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(2.5\times 10^5)(0.0030)^2 = 1.1\,\text{J}.$$

(c) By Newton's third law, the force F exerted by the tooth is equal and opposite to the "spring-like" force exerted by the licorice, so the graph of F is a straight line of slope k. We plot F (in newtons) versus x (in millimeters); both are taken as positive.

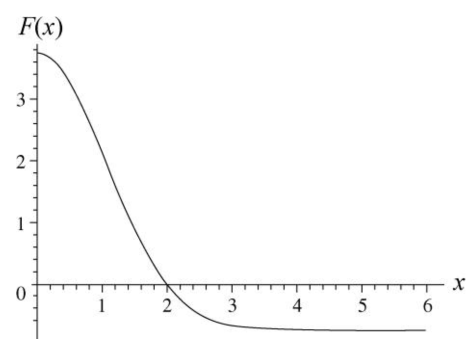


(d) As mentioned in part (b), the spring potential energy expression is relevant. Now, whether or not we can ignore dissipative processes is a deeper question. In other words, it seems unlikely that — if the tooth at any moment were to reverse its motion — that the licorice could "spring back" to its original shape. Still, to the extent that $U = \frac{1}{2}kx^2$ applies, the graph is a parabola (not shown here) which has its vertex at the origin and is either concave upward or concave downward depending on how one wishes to define the sign of F (the connection being $F = -dU/dx$).

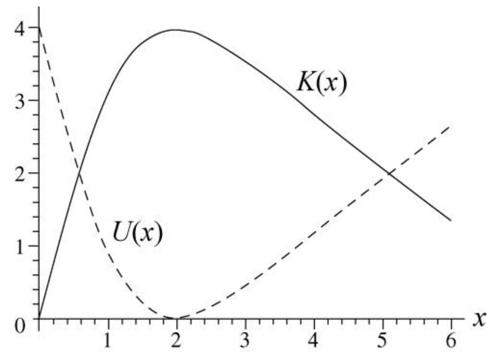
(e) As a crude estimate, the area under the curve is roughly half the area of the entire plotting-area (8000 N by 12 mm). This leads to an approximate work of $\frac{1}{2}$ (8000 N) (0.012 m) ≈ 50 J. Estimates in the range $40 \le W \le 50$ J are acceptable.

(f) Certainly dissipative effects dominate this process, and we cannot assign it a meaningful potential energy.

133. (a) The force (SI units understood) from Eq. 8-20 is plotted in the graph below.



(b) The potential energy U(x) and the kinetic energy $K(x)$ are shown in the next. The potential energy curve begins at 4 and drops (until about $x = 2$); the kinetic energy curve is the one that starts at zero and rises (until about $x = 2$).



134. The style of reasoning used here is presented in Section 8-5.

(a) The horizontal line representing E_1 intersects the potential energy curve at a value of $r \approx 0.07$ nm and seems not to intersect the curve at larger r (though this is somewhat unclear since $U(r)$ is graphed only up to $r = 0.4$ nm). Thus, if m were propelled towards M from large r with energy E_1 it would "turn around" at 0.07 nm and head back in the direction from which it came.

(b) The line representing E_2 has two intersection points $r_1 \approx 0.16$ nm and $r_2 \approx 0.28$ nm with the $U(r)$ plot. Thus, if m starts in the region $r_1 < r < r_2$ with energy E_2 it will bounce back and forth between these two points, presumably forever.

(c) At $r = 0.3$ nm, the potential energy is roughly $U = -1.1 \times 10^{-19}$ J.

(d) With $M >> m$, the kinetic energy is essentially just that of m. Since $E = 1 \times 10^{-19}$ J, its kinetic energy is $K = E - U \approx 2.1 \times 10^{-19}$ J.

(e) Since force is related to the slope of the curve, we must (crudely) estimate $|F| \approx 1\times 10^{-9}\,\text{N}$ at this point. The sign of the slope is positive, so by Eq. 8-20, the force is negative-valued. This is interpreted to mean that the atoms are attracted to each other.

(f) Recalling our remarks in the previous part, we see that the sign of F is positive (meaning it's repulsive) for $r < 0.2$ nm.

(g) And the sign of F is negative (attractive) for $r > 0.2$ nm.

(h) At $r = 0.2$ nm, the slope (hence, F) vanishes.

135. The distance traveled up the incline can be calculated using the kinematic equations discussed in Chapter 2:

$$v^2 = v_0^2 + 2a\Delta x \;\; \rightarrow \Delta x = 200 \text{ m}.$$

This corresponds to an increase in height equal to $y = (200 \text{ m})\sin\theta = 17$ m, where $\theta = 5.0°$. We take its initial height to be $y = 0$.

(a) Eq. 8-24 leads to

$$W_{\text{app}} = \Delta E = \frac{1}{2}m\left(v^2 - v_0^2\right) + mgy \ .$$

Therefore, $\Delta E = 8.6 \times 10^3$ J .

(b) From the above manipulation, we see $W_{\text{app}} = 8.6 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10$ s. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{8.6 \times 10^3}{10} = 860 \text{ W}$$

where the answer has been rounded off (from the 856 value that is provided by the calculator).

(c) and (d) Taking into account the component of gravity along the incline surface, the applied force is $ma + mg \sin\theta = 43$ N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 430 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 1300 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

where these answers have been rounded off (from 428 and 1284, respectively). We note that the average of these two values agrees with the result in part (b).

136. (a) Conservation of mechanical energy leads to

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad 0 + \frac{1}{2}ky_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}k(y_f - y_i)^2 + mgy_f$$

where $y_i = 0.25$ m is the initial depression of the spring, and $y_f - y_i$ is the displacement of the spring from its equilibrium position when the block is at y_f. Thus, the kinetic energy of the block can be written as

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f \ .$$

For $y_f = 0,$ the kinetic energy is $K_f = 0,$ as expected, since this corresponds to the initial release point.

(b) At $y_f = 0.050$ m, we have

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.050\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.050\text{ m}) = 4.48\text{ J}$$

(c) At $y_f = 0.100$ m, we have

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.100\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.100\text{ m}) = 7.40\text{ J}$$

(d) Similarly, the kinetic energy at $y_f = 0.150$ m is

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.150\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.150\text{ m}) = 8.78\text{ J}$$

(e) At $y_f = 0.200$ m, the kinetic energy of the block is

$$K_f = \frac{1}{2}k\left[y_i^2 - (y_f - y_i)^2\right] - mgy_f$$
$$= \frac{1}{2}(620\text{ N/m})\left[(0.250\text{ m})^2 - (0.200\text{ m} - 0.250\text{ m})^2\right] - (50\text{ N})(0.200\text{ m}) = 8.60\text{ J}$$

(f) The spring returns to its uncompressed state once $y_f \geq y_i$. Since the block becomes detached from the spring beyond that point, at its maximum height, $K = 0$, and we have

$$\frac{1}{2}ky_i^2 = mgy_{\text{max}} \quad \Rightarrow \quad y_{\text{max}} = \frac{ky_i^2}{2mg} = \frac{(620\text{ N/m})(0.250\text{ m})^2}{2(50\text{ N})} = 0.388\text{ m}.$$

Chapter 9

1. We use Eq. 9-5 to solve for (x_3, y_3).

(a) The x coordinate of the system's center of mass is:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(-1.20\text{ m}) + (4.00\text{ kg})(0.600\text{ m}) + (3.00\text{ kg})\,x_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.500\text{ m}.$$

Solving the equation yields $x_3 = -1.50$ m.

(b) The y coordinate of the system's center of mass is:

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2.00\text{ kg})(0.500\text{ m}) + (4.00\text{ kg})(-0.750\text{ m}) + (3.00\text{ kg})\,y_3}{2.00\text{ kg} + 4.00\text{ kg} + 3.00\text{ kg}}$$
$$= -0.700\text{ m}.$$

Solving the equation yields $y_3 = -1.43$ m.

2. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0$ kg particle; $x_2 = 2.0$ m and $y_2 = 1.0$ m are the coordinates of the $m_2 = 4.0$ kg particle; and $x_3 = 1.0$ m and $y_3 = 2.0$ m are the coordinates of the $m_3 = 8.0$ kg particle.

(a) The x coordinate of the center of mass is

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(2.0\text{ m}) + (8.0\text{ kg})(1.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.1\text{ m}.$$

(b) The y coordinate of the center of mass is

$$y_{\text{com}} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (4.0\text{ kg})(1.0\text{ m}) + (8.0\text{ kg})(2.0\text{ m})}{3.0\text{ kg} + 4.0\text{ kg} + 8.0\text{ kg}} = 1.3\text{ m}.$$

(c) As the mass of m_3, the topmost particle, is increased, the center of mass shifts toward that particle. As we approach the limit where m_3 is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of m_3.

3. We use Eq. 9-5 to locate the coordinates.

(a) By symmetry $x_{\text{com}} = -d_1/2 = -(13\text{ cm})/2 = -6.5$ cm. The negative value is due to our choice of the origin.

(b) We find y_{com} as

$$y_{\text{com}} = \frac{m_i y_{\text{com},i} + m_a y_{\text{com},a}}{m_i + m_a} = \frac{\rho_i V_i y_{\text{com},i} + \rho_a V_a y_{\text{cm},a}}{\rho_i V_i + \rho_a V_a}$$

$$= \frac{(11\text{ cm}/2)(7.85\text{ g/cm}^3) + 3(11\text{ cm}/2)(2.7\text{ g/cm}^3)}{7.85\text{ g/cm}^3 + 2.7\text{ g/cm}^3} = 8.3\text{ cm.}$$

(c) Again by symmetry, we have $z_{\text{com}} = (2.8\text{ cm})/2 = 1.4$ cm.

4. We will refer to the arrangement as a "table." We locate the coordinate origin at the left end of the tabletop (as shown in Fig. 9-37). With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x,y) = (+L, -L/2)$, the center of mass of the left leg is at $(x,y) = (0, -L/2)$, and the center of mass of the tabletop is at $(x,y) = (L/2, 0)$.

(a) The x coordinate of the (whole table) center of mass is

$$x_{\text{com}} = \frac{M(+L) + M(0) + 3M(+L/2)}{M + M + 3M} = \frac{L}{2}\,.$$

With $L = 22$ cm, we have $x_{\text{com}} = (22\text{ cm})/2 = 11$ cm.

(b) The y coordinate of the (whole table) center of mass is

$$y_{\text{com}} = \frac{M(-L/2) + M(-L/2) + 3M(0)}{M + M + 3M} = -\frac{L}{5}\,,$$

or $y_{\text{com}} = -(22\text{ cm})/5 = -4.4$ cm.

From the coordinates, we see that the whole table center of mass is a small distance 4.4 cm directly below the middle of the tabletop.

5. Since the plate is uniform, we can split it up into three rectangular pieces, with the mass of each piece being proportional to its area and its center of mass being at its geometric center. We'll refer to the large 35 cm × 10 cm piece (shown to the left of the y axis in Fig. 9-38) as section 1; it has 63.6% of the total area and its center of mass is at $(x_1, y_1) = (-5.0\text{ cm}, -2.5\text{ cm})$. The top 20 cm × 5 cm piece (section 2, in the first quadrant) has 18.2% of the total area; its center of mass is at $(x_2, y_2) = (10\text{ cm}, 12.5\text{ cm})$. The bottom 10 cm x 10 cm piece (section 3) also has 18.2% of the total area; its center of mass is at $(x_3, y_3) = (5\text{ cm}, -15\text{ cm})$.

(a) The x coordinate of the center of mass for the plate is

$$x_{com} = (0.636)x_1 + (0.182)x_2 + (0.182)x_3 = -0.45 \text{ cm} .$$

(b) The y coordinate of the center of mass for the plate is

$$y_{com} = (0.636)y_1 + (0.182)y_2 + (0.182)y_3 = -2.0 \text{ cm} .$$

6. The centers of mass (with centimeters understood) for each of the five sides are as follows:

$(x_1, y_1, z_1) = (0, 20, 20)$	for the side in the yz plane
$(x_2, y_2, z_2) = (20, 0, 20)$	for the side in the xz plane
$(x_3, y_3, z_3) = (20, 20, 0)$	for the side in the xy plane
$(x_4, y_4, z_4) = (40, 20, 20)$	for the remaining side parallel to side 1
$(x_5, y_5, z_5) = (20, 40, 20)$	for the remaining side parallel to side 2

Recognizing that all sides have the same mass m, we plug these into Eq. 9-5 to obtain the results (the first two being expected based on the symmetry of the problem).

(a) The x coordinate of the center of mass is

$$x_{com} = \frac{mx_1 + mx_2 + mx_3 + mx_4 + mx_5}{5m} = \frac{0 + 20 + 20 + 40 + 20}{5} = 20 \text{ cm}$$

(b) The y coordinate of the center of mass is

$$y_{com} = \frac{my_1 + my_2 + my_3 + my_4 + my_5}{5m} = \frac{20 + 0 + 20 + 20 + 40}{5} = 20 \text{ cm}$$

(c) The z coordinate of the center of mass is

$$z_{com} = \frac{mz_1 + mz_2 + mz_3 + mz_4 + mz_5}{5m} = \frac{20 + 20 + 0 + 20 + 20}{5} = 16 \text{ cm}$$

7. (a) By symmetry the center of mass is located on the axis of symmetry of the molecule – the y axis. Therefore $x_{com} = 0$.

(b) To find y_{com}, we note that $3m_H y_{com} = m_N(y_N - y_{com})$, where y_N is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:

$$y_N = \sqrt{\left(10.14\times10^{-11}\text{ m}\right)^2 - \left(9.4\times10^{-11}\text{ m}\right)^2} = 3.803\times10^{-11}\text{ m}.$$

Thus,

$$y_{\text{com}} = \frac{m_{\text{N}} y_{\text{N}}}{m_{\text{N}} + 3m_{\text{H}}} = \frac{(14.0067)(3.803\times10^{-11}\ \text{m})}{14.0067 + 3(1.00797)} = 3.13\times10^{-11}\ \text{m}$$

where Appendix F has been used to find the masses.

8. (a) Since the can is uniform, its center of mass is at its geometrical center, a distance $H/2$ above its base. The center of mass of the soda alone is at its geometrical center, a distance $x/2$ above the base of the can. When the can is full this is $H/2$. Thus the center of mass of the can and the soda it contains is a distance

$$h = \frac{M(H/2) + m(H/2)}{M + m} = \frac{H}{2}$$

above the base, on the cylinder axis. With H = 12 cm, we obtain h = 6.0 cm.

(b) We now consider the can alone. The center of mass is $H/2$ = 6.0 cm above the base, on the cylinder axis.

(c) As x decreases the center of mass of the soda in the can at first drops, then rises to $H/2$ = 6.0 cm again.

(d) When the top surface of the soda is a distance x above the base of the can, the mass of the soda in the can is $m_p = m(x/H)$, where m is the mass when the can is full ($x = H$). The center of mass of the soda alone is a distance $x/2$ above the base of the can. Hence

$$h = \frac{M(H/2) + m_p(x/2)}{M + m_p} = \frac{M(H/2) + m(x/H)(x/2)}{M + (mx/H)} = \frac{MH^2 + mx^2}{2(MH + mx)}.$$

We find the lowest position of the center of mass of the can and soda by setting the derivative of h with respect to x equal to 0 and solving for x. The derivative is

$$\frac{dh}{dx} = \frac{2mx}{2(MH + mx)} - \frac{(MH^2 + mx^2)m}{2(MH + mx)^2} = \frac{m^2x^2 + 2MmHx - MmH^2}{2(MH + mx)^2}.$$

The solution to $m^2x^2 + 2MmHx - MmH^2 = 0$ is

$$x = \frac{MH}{m}\left(-1 + \sqrt{1 + \frac{m}{M}}\right).$$

The positive root is used since x must be positive. Next, we substitute the expression found for x into $h = (MH^2 + mx^2)/2(MH + mx)$. After some algebraic manipulation we obtain

$$h = \frac{HM}{m}\left(\sqrt{1+\frac{m}{M}} - 1\right) = \frac{(12 \text{ cm})(0.14 \text{ kg})}{0.354 \text{ kg}}\left(\sqrt{1+\frac{0.354 \text{ kg}}{0.14 \text{ kg}}} - 1\right) = 4.2 \text{ cm}.$$

9. We use the constant-acceleration equations of Table 2-1 (with $+y$ downward and the origin at the release point), Eq. 9-5 for y_{com} and Eq. 9-17 for $\vec{v}_{com}$.

(a) The location of the first stone (of mass m_1) at $t = 300 \times 10^{-3}$ s is

$$y_1 = (1/2)gt^2 = (1/2)(9.8 \text{ m/s}^2)\,(300 \times 10^{-3} \text{ s})^2 = 0.44 \text{ m},$$

and the location of the second stone (of mass $m_2 = 2m_1$) at $t = 300 \times 10^{-3}$ s is

$$y_2 = (1/2)gt^2 = (1/2)(9.8 \text{ m/s}^2)(300 \times 10^{-3} \text{ s} - 100 \times 10^{-3} \text{ s})^2 = 0.20 \text{ m}.$$

Thus, the center of mass is at

$$y_{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(0.44 \text{ m}) + 2m_1(0.20 \text{ m})}{m_1 + 2m_2} = 0.28 \text{ m}.$$

(b) The speed of the first stone at time t is $v_1 = gt$, while that of the second stone is

$$v_2 = g(t - 100 \times 10^{-3} \text{ s}).$$

Thus, the center-of-mass speed at $t = 300 \times 10^{-3}$ s is

$$\begin{aligned} v_{com} &= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{m_1\left(9.8 \text{ m/s}^2\right)\left(300\times10^{-3} \text{ s}\right) + 2m_1\left(9.8 \text{ m/s}^2\right)\left(300\times10^{-3} \text{ s} - 100\times10^{-3} \text{ s}\right)}{m_1 + 2m_1} \\ &= 2.3 \text{ m/s}. \end{aligned}$$

10. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for x_{com} and Eq. 9-17 for $\vec{v}_{com}$. At $t = 3.0$ s, the location of the automobile (of mass m_1) is

$$x_1 = \tfrac{1}{2}at^2 = \tfrac{1}{2}\left(4.0 \text{ m/s}^2\right)\left(3.0 \text{ s}\right)^2 = 18 \text{ m},$$

while that of the truck (of mass m_2) is $x_2 = vt = (8.0 \text{ m/s})(3.0\text{s}) = 24$ m. The speed of the automobile then is $v_1 = at = \left(4.0 \text{ m/s}^2\right)\left(3.0 \text{ s}\right) = 12$ m/s, while the speed of the truck remains $v_2 = 8.0$ m/s.

(a) The location of their center of mass is

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1000\text{ kg})(18\text{ m}) + (2000\text{ kg})(24\text{ m})}{1000\text{ kg} + 2000\text{ kg}} = 22\text{ m}.$$

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1000\text{ kg})(12\text{ m/s}) + (2000\text{ kg})(8.0\text{ m/s})}{1000\text{ kg} + 2000\text{ kg}} = 9.3\text{ m/s}.$$

11. The implication in the problem regarding $\vec{v}_0$ is that the olive and the nut start at rest. Although we could proceed by analyzing the forces on each object, we prefer to approach this using Eq. 9-14. The total force on the nut-olive system is $\vec{F}_\text{o} + \vec{F}_\text{n} = (-\hat{\text{i}} + \hat{\text{j}})\text{ N}$. Thus, Eq. 9-14 becomes

$$(-\hat{\text{i}} + \hat{\text{j}})\text{ N} = M\vec{a}_{\text{com}}$$

where M = 2.0 kg. Thus, $\vec{a}_{\text{com}} = (-\tfrac{1}{2}\hat{\text{i}} + \tfrac{1}{2}\hat{\text{j}})\text{ m/s}^2$. Each component is constant, so we apply the equations discussed in Chapters 2 and 4 and obtain

$$\Delta\vec{r}_{\text{com}} = \frac{1}{2}\vec{a}_{\text{com}}t^2 = (-4.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}$$

when t = 4.0 s. It is perhaps instructive to work through this problem the *long way* (separate analysis for the olive and the nut and then application of Eq. 9-5) since it helps to point out the computational advantage of Eq. 9-14.

12. Since the center of mass of the two-skater system does not move, both skaters will end up at the center of mass of the system. Let the center of mass be a distance x from the 40-kg skater, then

$$(65\text{ kg})(10\text{ m} - x) = (40\text{ kg})x \Rightarrow x = 6.2\text{ m}.$$

Thus the 40-kg skater will move by 6.2 m.

13. **THINK** A shell explodes into two segments at the top of its trajectory. Knowing the motion of one segment allows us to analyze the motion of the other using the momentum conservation principle.

EXPRESS We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0y} - gt$ and this is zero at time $t = v_{0y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin 60°\cos 60° = 17.7\text{ m}$$

and

$$y = v_{0y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin^2 60° = 15.3\text{ m}.$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. In addition, since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then

$$Mv_0\cos\theta_0 = MV_0/2,$$

since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\text{ m/s})\cos 60° = 20\text{ m/s}.$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands.

ANALYZE Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\text{ m} + (20\text{ m/s})\sqrt{\frac{2(15.3\text{ m})}{9.8\text{ m/s}^2}} = 53\text{ m}.$$

LEARN In the absence of explosion, the shell with a mass M would have landed at

$$R = 2x_0 = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{(20\text{ m/s})^2}{9.8\text{ m/s}^2}\sin[2(60°)] = 35.3\text{ m}$$

which is shorter than $x = 53$ m found above. This makes sense because the broken fragment, having a smaller mass but greater horizontal speed, can travel much farther than the original shell.

14. (a) The phrase (in the problem statement) "such that it [particle 2] always stays directly above particle 1 during the flight" means that the shadow (as if a light were directly above the particles shining down on them) of particle 2 coincides with the position of particle 1, at each moment. We say, in this case, that they are vertically

aligned. Because of that alignment, $v_{2x} = v_1 = 10.0$ m/s. Because the initial value of v_2 is given as 20.0 m/s, then (using the Pythagorean theorem) we must have

$$v_{2y} = \sqrt{v_2^2 - v_{2x}^2} = \sqrt{300} \text{ m/s}$$

for the initial value of the y component of particle 2's velocity. Equation 2-16 (or conservation of energy) readily yields $y_{\max} = 300/19.6 = 15.3$ m. Thus, we obtain

$$H_{\max} = m_2 y_{\max} / m_{\text{total}} = (3.00 \text{ g})(15.3 \text{ m})/(8.00 \text{ g}) = 5.74 \text{ m}.$$

(b) Since both particles have the same horizontal velocity, and particle 2's vertical component of velocity vanishes at that highest point, then the center of mass velocity then is simply $(10.0 \text{ m/s})\hat{\text{i}}$ (as one can verify using Eq. 9-17).

(c) Only particle 2 experiences any acceleration (the free fall acceleration downward), so Eq. 9-18 (or Eq. 9-19) leads to

$$a_{\text{com}} = m_2 g / m_{\text{total}} = (3.00 \text{ g})(9.8 \text{ m/s}^2)/(8.00 \text{ g}) = 3.68 \text{ m/s}^2$$

for the magnitude of the downward acceleration of the center of mass of this system. Thus, $\vec{a}_{\text{com}} = (-3.68 \text{ m/s}^2)\,\hat{\text{j}}$.

15. (a) The net force on the *system* (of total mass $m_1 + m_2$) is $m_2 g$. Thus, Newton's second law leads to $a = g(m_2/(m_1 + m_2)) = 0.4g$. For block 1, this acceleration is to the right (the $\hat{\text{i}}$ direction), and for block 2 this is an acceleration downward (the $-\hat{\text{j}}$ direction). Therefore, Eq. 9-18 gives

$$\vec{a}_{\text{com}} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2} = \frac{(0.6)(0.4g\hat{\text{i}}) + (0.4)(-0.4g\hat{\text{j}})}{0.6 + 0.4} = (2.35\,\hat{\text{i}} - 1.57\,\hat{\text{j}}) \text{ m/s}^2 .$$

(b) Integrating Eq. 4-16, we obtain

$$\vec{v}_{\text{com}} = (2.35\,\hat{\text{i}} - 1.57\hat{\text{j}})\, t$$

(with SI units understood), since it started at rest. We note that the *ratio* of the y-component to the x-component (for the velocity vector) does not change with time, and it is that ratio which determines the angle of the velocity vector (by Eq. 3-6), and thus the direction of motion for the center of mass of the system.

(c) The last sentence of our answer for part (b) implies that the path of the center-of-mass is a straight line.

(d) Equation 3-6 leads to $\theta = -34°$. The path of the center of mass is therefore straight, at downward angle 34°.

16. We denote the mass of Ricardo as M_R and that of Carmelita as M_C. Let the center of mass of the two-person system (assumed to be closer to Ricardo) be a distance x from the middle of the canoe of length L and mass m. Then

$$M_R(L/2 - x) = mx + M_C(L/2 + x).$$

Now, after they switch positions, the center of the canoe has moved a distance $2x$ from its initial position. Therefore, x = 40 cm/2 = 0.20 m, which we substitute into the above equation to solve for M_C:

$$M_C = \frac{M_R(L/2 - x) - mx}{L/2 + x} = \frac{(80)(\frac{3.0}{2} - 0.20) - (30)(0.20)}{(3.0/2) + 0.20} = 58 \text{ kg}.$$

17. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16,

$$M\Delta x_{\text{com}} = 0 = m_b \Delta x_b + m_d \Delta x_d ,$$

which implies

$$|\Delta x_b| = \frac{m_d}{m_b}|\Delta x_d|.$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance d = 2.4 m:

$$|\Delta x_b| + |\Delta x_d| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}|(\Delta x_d)| + |\Delta x_d| = d$$

which leads to $|\Delta x_d| = \dfrac{d}{1 + m_d/m_b} = \dfrac{2.4 \text{ m}}{1 + (4.5/18)} = 1.92 \text{ m}.$

The dog is therefore 1.9 m closer to the shore than initially (where it was D = 6.1 m from it). Thus, it is now $D - |\Delta x_d|$ = 4.2 m from the shore.

18. The magnitude of the ball's momentum change is

$$\Delta p = m|v_i - v_f| = (0.70 \text{ kg})|(5.0 \text{ m/s}) - (-2.0 \text{ m/s})| = 4.9 \text{ kg}\cdot\text{m/s}.$$

19. (a) The change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2100 \text{ kg})\left((51 \text{ km/h})^2 - (41 \text{ km/h})^2\right) \\ &= 9.66\times10^4 \text{ kg}\cdot(\text{km/h})^2\left((10^3 \text{ m/km})(1 \text{ h/3600 s})\right)^2 \\ &= 7.5\times10^4 \text{ J.}\end{aligned}$$

(b) The magnitude of the change in velocity is

$$|\Delta\vec{v}| = \sqrt{(-v_i)^2 + (v_f)^2} = \sqrt{(-41 \text{ km/h})^2 + (51 \text{ km/h})^2} = 65.4 \text{ km/h}$$

so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m|\Delta\vec{v}| = (2100 \text{ kg})(65.4 \text{ km/h})\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right) = 3.8\times10^4 \text{ kg}\cdot\text{m/s.}$$

(c) The vector $\Delta\vec{p}$ points at an angle θ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41 \text{ km/h}}{51 \text{ km/h}}\right) = 39°.$$

20. We infer from the graph that the horizontal component of momentum p_x is 4.0 $\text{kg}\cdot\text{m/s}$. Also, its initial magnitude of momentum p_o is 6.0 $\text{kg}\cdot\text{m/s}$. Thus,

$$\cos\theta_o = \frac{p_x}{p_o} \quad\Rightarrow\quad \theta_o = 48° .$$

21. We use coordinates with +x horizontally toward the pitcher and +y upward. Angles are measured counterclockwise from the +x axis. Mass, velocity, and momentum units are SI. Thus, the initial momentum can be written $\vec{p}_0 = (4.5\angle 215°)$ in magnitude-angle notation.

(a) In magnitude-angle notation, the momentum change is

$$(6.0 \angle -90°) - (4.5 \angle 215°) = (5.0 \angle -43°)$$

(efficiently done with a vector-capable calculator in polar mode). The magnitude of the momentum change is therefore 5.0 $\text{kg}\cdot\text{m/s}$.

(b) The momentum change is $(6.0 \angle 0°) - (4.5 \angle 215°) = (10 \angle 15°)$. Thus, the magnitude of the momentum change is 10 $\text{kg}\cdot\text{m/s}$.

22. (a) Since the force of impact on the ball is in the y direction, p_x is conserved:

$$p_{xi} = p_{xf} \quad\Rightarrow\quad mv_i\sin\theta_1 = mv_i\sin\theta_2 .$$

With θ_1 = 30.0°, we find θ_2 = 30.0°.

(b) The momentum change is

$$\begin{aligned}\Delta\vec{p} &= mv_i\cos\theta_2\left(-\hat{\text{j}}\right) - mv_i\cos\theta_2\left(+\hat{\text{j}}\right) = -2\left(0.165\text{ kg}\right)\left(2.00\text{ m/s}\right)\left(\cos 30°\right)\hat{\text{j}}\\ &= (-0.572\text{ kg}\cdot\text{m/s})\hat{\text{j}}.\end{aligned}$$

23. We estimate his mass in the neighborhood of 70 kg and compute the upward force F of the water from Newton's second law: $F - mg = ma$, where we have chosen $+y$ upward, so that $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the water). His speed when he arrives at the surface of the water is found either from Eq. 2-16 or from energy conservation: $v = \sqrt{2gh}$, where $h = 12$ m, and since the deceleration a reduces the speed to zero over a distance d = 0.30 m we also obtain $v = \sqrt{2ad}$. We use these observations in the following.

Equating our two expressions for v leads to $a = gh/d$. Our force equation, then, leads to

$$F = mg + m\left(g\frac{h}{d}\right) = mg\left(1 + \frac{h}{d}\right)$$

which yields $F \approx 2.8 \times 10^4$ kg. Since we are not at all certain of his mass, we express this as a guessed-at range (in kN) $25 < F < 30$.

Since $F \gg mg$, the impulse $\vec{J}$ due to the net force (while he is in contact with the water) is overwhelmingly caused by the upward force of the water: $\int F\,dt = \vec{J}$ to a good approximation. Thus, by Eq. 9-29,

$$\int F dt = \vec{p}_f - \vec{p}_i = 0 - m\left(-\sqrt{2gh}\right)$$

(the minus sign with the initial velocity is due to the fact that downward is the negative direction), which yields $(70\text{ kg})\sqrt{2\left(9.8\text{ m/s}^2\right)\left(12\text{ m}\right)} = 1.1\times10^3\text{kg}\cdot\text{m/s}$. Expressing this as a range we estimate

$$1.0\times10^3\text{kg}\cdot\text{m/s} < \int F\,dt < 1.2\times10^3\text{kg}\cdot\text{m/s}.$$

24. We choose $+y$ upward, which implies $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

(a) The maximum deceleration a_{max} of the paratrooper (of mass m and initial speed v = 56 m/s) is found from Newton's second law

$$F_{\text{snow}} - mg = ma_{\text{max}}$$

where we require $F_{\text{snow}} = 1.2 \times 10^5$ N. Using Eq. 2-15 $v^2 = 2a_{\text{max}}d$, we find the minimum depth of snow for the man to survive:

$$d = \frac{v^2}{2a_{\text{max}}} = \frac{mv^2}{2\left(F_{\text{snow}} - mg\right)} \approx \frac{(85\text{kg})(56\,\text{m/s})^2}{2\left(1.2\times10^5\,\text{N}\right)} = 1.1 \text{ m}.$$

(b) His short trip through the snow involves a change in momentum

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - (85\,\text{kg})(-56\,\text{m/s}) = -4.8\times10^3\,\text{kg}\cdot\text{m/s},$$

or $|\Delta\vec{p}| = 4.8\times10^3\,\text{kg}\cdot\text{m/s}$. The negative value of the initial velocity is due to the fact that downward is the negative direction. By the impulse-momentum theorem, this equals the impulse due to the net force $F_{\text{snow}} - mg$, but since $F_{\text{snow}} \gg mg$ we can approximate this as the impulse on him just from the snow.

25. We choose $+y$ upward, which means $\vec{v}_i = -25\,\text{m/s}$ and $\vec{v}_f = +10\,\text{m/s}$. During the collision, we make the reasonable approximation that the net force on the ball is equal to F_{avg}, the average force exerted by the floor up on the ball.

(a) Using the impulse momentum theorem (Eq. 9-31) we find

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10) - (1.2)(-25) = 42\,\text{kg}\cdot\text{m/s}.$$

(b) From Eq. 9-35, we obtain

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{42}{0.020} = 2.1\times10^3\,\text{N}.$$

26. (a) By energy conservation, the speed of the victim when he falls to the floor is

$$\frac{1}{2}mv^2 = mgh \quad\Rightarrow\quad v = \sqrt{2gh} = \sqrt{2(9.8\text{ m/s}^2)(0.50\text{ m})} = 3.1\,\text{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (70\text{ kg})(3.1\,\text{m/s}) \approx 2.2\times10^2\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 0.082\,\text{s}$ for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.2\times10^2\ \text{N}\cdot\text{s}}{0.082\,\text{s}} \approx 2.7\times10^3\ \text{N}.$$

27. **THINK** The velocity of the ball is changing because of the external force applied. Impulse-linear momentum theorem is involved.

EXPRESS The initial direction of motion is in the +x direction. The magnitude of the average force F_{avg} is given by

$$F_{avg} = \frac{J}{\Delta t} = \frac{32.4 \text{ N}\cdot\text{s}}{2.70\times10^{-2} \text{ s}} = 1.20\times10^{3} \text{ N}.$$

The force is in the negative direction. Using the linear momentum-impulse theorem stated in Eq. 9-31, we have

$$-F_{avg}\Delta t = J = \Delta p = m(v_f - v_i).$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. The equation can be used to solve for v_f.

ANALYZE (a) Using the above expression, we find

$$v_f = \frac{mv_i - F_{avg}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27\times10^{-3}\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s}.$$

The final speed of the ball is $|v_f| = 67$ m/s.

(b) The negative sign in v_f indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{avg} = 1.20\times10^{3}$ N.

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

LEARN In vector notation, $\vec{F}_{avg}\Delta t = \vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$, which gives

$$\vec{v}_f = \vec{v}_i + \frac{\vec{J}}{m} = \vec{v}_i + \frac{\vec{F}_{avg}\Delta t}{m}$$

Since $\vec{J}$ or $\vec{F}_{avg}$ is in the opposite direction of $\vec{v}_i$, the velocity of the ball will decrease under the applied force. The ball first moves in the $+x$-direction, but then slows down and comes to a stop, and then reverses its direction of travel.

28. (a) The magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (0.70\text{ kg})(13\text{ m/s}) \approx 9.1\,\text{kg}\cdot\text{m/s} = 9.1\,\text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.1\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 1.8\times10^{3}\ \text{N}.$$

29. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed *v*, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass *m* is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)(3\times10^{-3}\,\text{kg})(500\,\text{m/s})}{(1\,\text{min})(60\,\text{s/min})} \approx 5\,\text{N}.$$

30. (a) By Eq. 9-30, impulse can be determined from the "area" under the *F*(*t*) curve. Keeping in mind that the area of a triangle is $\frac{1}{2}$(base)(height), we find the impulse in this case is 1.00 N·s.

(b) By definition (of the average of function, in the calculus sense) the average force must be the result of part (a) divided by the time (0.010 s). Thus, the average force is found to be 100 N.

(c) Consider ten hits. Thinking of ten hits as 10 *F*(*t*) triangles, our total time interval is 10(0.050 s) = 0.50 s, and the total area is 10(1.0 N·s). We thus obtain an average force of 10/0.50 = 20.0 N. One could consider 15 hits, 17 hits, and so on, and still arrive at this same answer.

31. (a) By energy conservation, the speed of the passenger when the elevator hits the floor is

$$\frac{1}{2}mv^2 = mgh \quad\Rightarrow\quad v = \sqrt{2gh} = \sqrt{2(9.8\ \text{m/s}^2)(36\ \text{m})} = 26.6\ \text{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (90\ \text{kg})(26.6\ \text{m/s}) \approx 2.39\times10^{3}\ \text{N}\cdot\text{s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{2.39\times10^{3}\ \text{N}\cdot\text{s}}{5.0\times10^{-3}\ \text{s}} \approx 4.78\times10^{5}\ \text{N}.$$

(c) If the passenger were to jump upward with a speed of $v' = 7.0\text{ m/s}$, then the resulting downward velocity would be

$$v'' = v - v' = 26.6\text{ m/s} - 7.0\text{ m/s} = 19.6\text{ m/s},$$

and the magnitude of the impulse becomes

$$J'' = |\Delta p''| = m|\Delta v''| = mv'' = (90\text{ kg})(19.6\text{ m/s}) \approx 1.76\times10^3\text{ N}\cdot\text{s}.$$

(d) The corresponding average force would be

$$F''_{\text{avg}} = \frac{J''}{\Delta t} = \frac{1.76\times10^3\text{ N}\cdot\text{s}}{5.0\times10^{-3}\text{ s}} \approx 3.52\times10^5\text{ N}.$$

32. (a) By the impulse-momentum theorem (Eq. 9-31) the change in momentum must equal the "area" under the $F(t)$ curve. Using the facts that the area of a triangle is $\frac{1}{2}$ (base)(height), and that of a rectangle is (height)(width), we find the momentum at t = 4 s to be (30 kg·m/s)$\hat{\text{i}}$.

(b) Similarly (but keeping in mind that areas beneath the axis are counted negatively) we find the momentum at t = 7 s is (38 kg·m/s)$\hat{\text{i}}$.

(c) At t = 9 s, we obtain $\vec{v}$ = (6.0 m/s)$\hat{\text{i}}$.

33. We use coordinates with +x rightward and +y upward, with the usual conventions for measuring the angles (so that the initial angle becomes 180 + 35 = 215°). Using SI units and magnitude-angle notation (efficient to work with when using a vector-capable calculator), the change in momentum is

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3.00\angle 90°) - (3.60\angle 215°) = (5.86\angle 59.8°).$$

(a) The magnitude of the impulse is $J = \Delta p = 5.86\text{ kg}\cdot\text{m/s} = 5.86\text{ N}\cdot\text{s}$.

(b) The direction of $\vec{J}$ is 59.8° measured counterclockwise from the +x axis.

(c) Equation 9-35 leads to

$$J = F_{\text{avg}}\Delta t = 5.86\text{ N}\cdot\text{s} \quad\Rightarrow\quad F_{\text{avg}} = \frac{5.86\text{ N}\cdot\text{s}}{2.00\times10^{-3}\text{ s}} \approx 2.93\times10^3\text{N}.$$

We note that this force is very much larger than the weight of the ball, which justifies our (implicit) assumption that gravity played no significant role in the collision.

(d) The direction of $\vec{F}_{\text{avg}}$ is the same as $\vec{J}$, 59.8° measured counterclockwise from the +*x* axis.

34. (a) Choosing upward as the positive direction, the momentum change of the foot is

$$\Delta\vec{p} = 0 - m_{\text{foot}}\vec{v}_i = -(0.003\text{ kg})\,(-1.50\text{ m/s}) = 4.50\times10^{-3}\text{ N}\cdot\text{s}.$$

(b) Using Eq. 9-35 and now treating *downward* as the positive direction, we have

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = m_{\text{lizard}}g\,\Delta t = (0.090\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ s}) = 0.529\text{ N}\cdot\text{s}.$$

(c) Push is what provides the primary support.

35. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued). We evaluate the integral $J = \int F\,dt$ by adding the appropriate areas (of a triangle, a rectangle, and another triangle) shown in the graph (but with the *t* converted to seconds). With *m* = 0.058 kg and *v* = 34 m/s, we apply the impulse-momentum theorem:

$$\begin{aligned}\int F_{\text{wall}}\,dt = m\vec{v}_f - m\vec{v}_i \quad &\Rightarrow \quad \int_0^{0.002} F\,dt + \int_{0.002}^{0.004} F\,dt + \int_{0.004}^{0.006} F\,dt = m(+v) - m(-v)\\ &\Rightarrow \quad \frac{1}{2}F_{\max}(0.002\text{s}) + F_{\max}(0.002\text{s}) + \frac{1}{2}F_{\max}(0.002\text{s}) = 2mv\end{aligned}$$

which yields $F_{\max}(0.004\text{s}) = 2(0.058\text{kg})(34\text{m/s}) = 9.9\times10^2$ N.

36. (a) Performing the integral (from time *a* to time *b*) indicated in Eq. 9-30, we obtain

$$\int_a^b (12-3t^2)\,dt = 12(b-a) - (b^3 - a^3)$$

in SI units. If *b* = 1.25 s and *a* = 0.50 s, this gives $7.17\text{ N}\cdot\text{s}$.

(b) This integral (the impulse) relates to the change of momentum in Eq. 9-31. We note that the force is zero at *t* = 2.00 s. Evaluating the above expression for *a* = 0 and *b* = 2.00 gives an answer of $16.0\text{ kg}\cdot\text{m/s}$.

37. **THINK** We're given the force as a function of time, and asked to calculate the corresponding impulse, the average force and the maximum force.

EXPRESS Since the motion is one-dimensional, we work with the magnitudes of the vector quantities. The impulse J due to a force $F(t)$ exerted on a body is

$$J = \int_{t_i}^{t_f} F(t)dt = F_{\text{avg}}\Delta t\,,$$

where F_{avg} is the average force and $\Delta t = t_f - t_i$. To find the time at which the maximum force occurs, we set the derivative of F with respect to time equal to zero, and solve for t.

ANALYZE (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$J = \int_0^{3.0\times10^{-3}} F dt = \int_0^{3.0\times10^{-3}} \left[(6.0\times10^6)\,t - (2.0\times10^9)t^2\right]dt$$

$$= \left[\frac{1}{2}(6.0\times10^6)t^2 - \frac{1}{3}(2.0\times10^9)t^3\right]\Bigg|_0^{3.0\times10^{-3}} = 9.0\,\text{N}\cdot\text{s}.$$

(b) Using $J = F_{\text{avg}}\,\Delta t$, we find the average force to be

$$F_{\text{avg}} \frac{J}{\Delta t} = \frac{9.0\ \text{N}\cdot\text{s}}{3.0\times10^{-3}\ \text{s}} = 3.0\times10^3\ \text{N}.$$

(c) Differentiating $F(t)$ with respect to t and setting it to zero, we have

$$\frac{dF}{dt} = \frac{d}{dt}\left[(6.0\times10^6)\,t - (2.0\times10^9)t^2\right] = (6.0\times10^6) - (4.0\times10^9)t = 0\,,$$

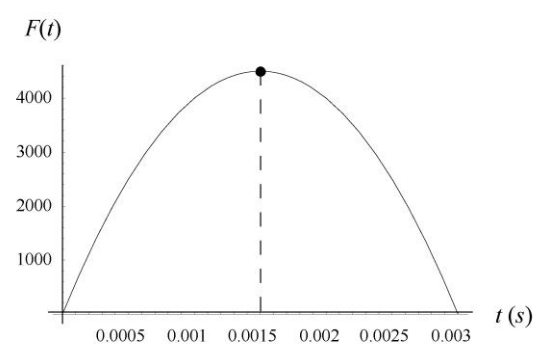
which can be solved to give $t = 1.5\times10^{-3}$ s. At that time the force is

$$F_{\text{max}} = \left(6.0\times10^6\right)\left(1.5\times10^{-3}\right) - \left(2.0\times10^9\right)\left(1.5\times10^{-3}\right)^2 = 4.5\times10^3\ \text{N}.$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let m be the mass of the ball and v its speed as it leaves the foot. The speed of the ball immediately after it loses contact with the player's foot is

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0\ \text{N}\cdot\text{s}}{0.45\ \text{kg}} = 20\ \text{m/s}.$$

LEARN The force as function of time is shown below. The area under the curve is the impulse J. From the plot, we readily see that $F(t)$ is a maximum at $t = 0.0015\,\text{s}$, with $F_{\text{max}} = 4500$ N.



38. From Fig. 9-54, +*y* corresponds to the direction of the rebound (directly away from the wall) and +*x* toward the right. Using unit-vector notation, the ball's initial and final velocities are

$$\vec{v}_i = v\cos\theta\,\hat{\mathrm{i}} - v\sin\theta\,\hat{\mathrm{j}} = 5.2\,\hat{\mathrm{i}} - 3.0\,\hat{\mathrm{j}}$$
$$\vec{v}_f = v\cos\theta\,\hat{\mathrm{i}} + v\sin\theta\,\hat{\mathrm{j}} = 5.2\,\hat{\mathrm{i}} + 3.0\,\hat{\mathrm{j}}$$

respectively (with SI units understood).

(a) With m = 0.30 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = 2\left(0.30 \text{ kg}\right)(3.0 \text{ m/s } \hat{\mathrm{j}}) = (1.8 \text{ N}\cdot\text{s})\hat{\mathrm{j}}.$$

(b) Using Eq. 9-35, the force on the ball by the wall is $\vec{J}/\Delta t = (1.8/0.010)\hat{\mathrm{j}} = (180\,\text{N})\,\hat{\mathrm{j}}$. By Newton's third law, the force on the wall by the ball is $(-180\text{ N})\hat{\mathrm{j}}$ (that is, its magnitude is 180 N and its direction is directly into the wall, or "down" in the view provided by Fig. 9-54).

39. **THINK** This problem deals with momentum conservation. Since no external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, the total momentum of the system is conserved.

EXPRESS Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked. Also, let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then, by momentum conservation,

$$m_s v_s + m_m v_m = 0 \;\;\Rightarrow\;\; v_m = -\frac{m_s}{m_m} v_s.$$

ANALYZE We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{m_s}{m_m} v_s = -\frac{0.068 \text{ kg}}{91 \text{ kg}}\left(4.0 \text{ m/s}\right) = -3.0\times10^{-3} \text{ m/s}$$

or $|v_m| = 3.0\times10^{-3}$ m/s.

LEARN The negative sign in v_m indicates that the man moves in the direction opposite to the motion of the stone. Note that his speed is much smaller (by a factor of m_s/m_m) compared to the speed of the stone.

40. Our notation is as follows: the mass of the motor is M; the mass of the module is m; the initial speed of the system is v_0; the relative speed between the motor and the module

is v_r; and, the speed of the module relative to the Earth is v after the separation. Conservation of linear momentum requires

$$(M + m)v_0 = mv + M(v - v_r).$$

Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300 \text{ km/h} + \frac{(4m)(82 \text{ km/h})}{4m+m} = 4.4\times10^3 \text{ km/h}.$$

41. (a) With SI units understood, the velocity of block L (in the frame of reference indicated in the figure that goes with the problem) is $(v_1 - 3)\hat{\text{i}}$. Thus, momentum conservation (for the explosion at $t = 0$) gives

$$m_L(v_1 - 3) + (m_C + m_R)v_1 = 0$$

which leads to

$$v_1 = \frac{3\,m_L}{m_L + m_C + m_R} = \frac{3(2\text{ kg})}{10\text{ kg}} = 0.60\text{ m/s}.$$

Next, at $t = 0.80$ s, momentum conservation (for the second explosion) gives

$$m_C\,v_2 + m_R(v_2 + 3) = (m_C + m_R)v_1 = (8\text{ kg})(0.60\text{ m/s}) = 4.8\ \text{kg}\cdot\text{m/s}.$$

This yields $v_2 = -0.15$. Thus, the velocity of block C after the second explosion is

$$v_2 = -(0.15\text{ m/s})\hat{\text{i}}.$$

(b) Between $t = 0$ and $t = 0.80$ s, the block moves $v_1\Delta t = (0.60\text{ m/s})(0.80\text{ s}) = 0.48$ m. Between $t = 0.80$ s and $t = 2.80$ s, it moves an additional

$$v_2\Delta t = (-0.15\text{ m/s})(2.00\text{ s}) = -0.30\text{ m}.$$

Its net displacement since $t = 0$ is therefore 0.48 m – 0.30 m = 0.18 m.

42. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is m; its initial velocity is $\vec{v}_0 = v\,\hat{\text{i}}$; the mass of the less massive piece is m_1; its velocity is $\vec{v}_1 = 0$; and, the mass of the more massive piece is m_2. We note that the conditions $m_2 = 3m_1$ (specified in the problem) and $m_1 + m_2 = m$ generally assumed in classical physics (before Einstein) lead us to conclude

$$m_1 = \frac{1}{4}m \text{ and } m_2 = \frac{3}{4}m.$$

Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 \;\Rightarrow\; mv\,\hat{\text{i}} = 0 + \frac{3}{4}m\vec{v}_2$$

which leads to $\vec{v}_2 = \frac{4}{3}v\,\hat{\text{i}}$. The increase in the system's kinetic energy is therefore

$$\Delta K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}mv_0^2 = 0 + \frac{1}{2}\left(\frac{3}{4}m\right)\left(\frac{4}{3}v\right)^2 - \frac{1}{2}mv^2 = \frac{1}{6}mv^2.$$

43. With $\vec{v}_0 = (9.5\,\hat{\text{i}} + 4.0\,\hat{\text{j}})$ m/s, the initial speed is

$$v_0 = \sqrt{v_{x0}^2 + v_{y0}^2} = \sqrt{(9.5\text{ m/s})^2 + (4.0\text{ m/s})^2} = 10.31\text{ m/s}$$

and the takeoff angle of the athlete is

$$\theta_0 = \tan^{-1}\left(\frac{v_{y0}}{v_{x0}}\right) = \tan^{-1}\left(\frac{4.0}{9.5}\right) = 22.8°.$$

Using Equation 4-26, the range of the athlete without using halteres is

$$R_0 = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(10.31\text{ m/s})^2 \sin 2(22.8°)}{9.8\text{ m/s}^2} = 7.75\text{ m.}$$

On the other hand, if two halteres of mass m = 5.50 kg were thrown at the maximum height, then, by momentum conservation, the subsequent speed of the athlete would be

$$(M + 2m)v_{x0} = Mv'_x \quad \Rightarrow v'_x = \frac{M + 2m}{M}v_{x0}$$

Thus, the change in the x-component of the velocity is

$$\Delta v_x = v'_x - v_{x0} = \frac{M + 2m}{M}v_{x0} - v_{x0} = \frac{2m}{M}v_{x0} = \frac{2(5.5\text{ kg})}{78\text{ kg}}(9.5\text{ m/s}) = 1.34\text{ m/s.}$$

The maximum height is attained when $v_y = v_{y0} - gt = 0$, or

$$t = \frac{v_{y0}}{g} = \frac{4.0\text{ m/s}}{9.8\text{ m/s}^2} = 0.41\text{ s.}$$

Therefore, the increase in range with use of halteres is

$$\Delta R = (\Delta v'_x)t = (1.34\text{ m/s})(0.41\text{ s}) = 0.55\text{ m.}$$

44. We can think of the sliding-until-stopping as an example of kinetic energy converting into thermal energy (see Eq. 8-29 and Eq. 6-2, with $F_N = mg$). This leads to $v^2 = 2\mu gd$ being true separately for each piece. Thus we can set up a ratio:

$$\left(\frac{v_L}{v_R}\right)^2 = \frac{2\mu_L g d_L}{2\mu_R g d_R} = \frac{12}{25}.$$

But (by the conservation of momentum) the ratio of speeds must be inversely proportional to the ratio of masses (since the initial momentum before the explosion was zero). Consequently,

$$\left(\frac{m_R}{m_L}\right)^2 = \frac{12}{25} \;\Rightarrow\; m_R = \tfrac{2}{5}\sqrt{3}\, m_L = 1.39 \text{ kg}.$$

Therefore, the total mass is $m_R + m_L \approx 3.4$ kg.

45. **THINK** The moving body is an isolated system with no external force acting on it. When it breaks up into three pieces, momentum remains conserved, both in the *x*- and the *y*-directions.

EXPRESS Our notation is as follows: the mass of the original body is $M = 20.0$ kg; its initial velocity is $\vec{v}_0 = (200\text{ m/s})\hat{\text{i}}$; the mass of one fragment is $m_1 = 10.0$ kg; its velocity is $\vec{v}_1 = (100\text{ m/s})\hat{\text{j}}$; the mass of the second fragment is $m_2 = 4.0$ kg; its velocity is $\vec{v}_2 = (-500\text{ m/s})\hat{\text{i}}$; and, the mass of the third fragment is $m_3 = 6.00$ kg. Conservation of linear momentum requires

$$M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3.$$

The energy released in the explosion is equal to ΔK, the change in kinetic energy.

ANALYZE (a) The above momentum-conservation equation leads to

$$\begin{aligned}\vec{v}_3 &= \frac{M\vec{v}_0 - m_1\vec{v}_1 - m_2\vec{v}_2}{m_3} \\ &= \frac{(20.0\text{ kg})(200\text{ m/s})\hat{\text{i}} - (10.0\text{ kg})(100\text{ m/s})\hat{\text{j}} - (4.0\text{ kg})(-500\text{ m/s})\hat{\text{i}}}{6.00\text{ kg}}. \\ &= (1.00\times10^3\text{ m/s})\,\hat{\text{i}} - (0.167\times10^3\text{ m/s})\,\hat{\text{j}}\end{aligned}$$

The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{(1000\text{ m/s})^2 + (-167\text{ m/s})^2} = 1.01\times10^3\text{ m/s}$. It points at $\theta = \tan^{-1}(-167/1000) = -9.48°$ (that is, at 9.5° measured clockwise from the +*x* axis).

(b) The energy released is ΔK:

$$\Delta K = K_f - K_i = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2\right) - \frac{1}{2}Mv_0^2 = 3.23\times10^6 \text{ J}.$$

LEARN The energy released in the explosion, of chemical nature, is converted into the kinetic energy of the fragments.

46. Our $+x$ direction is east and $+y$ direction is north. The linear momenta for the two m = 2.0 kg parts are then

$$\vec{p}_1 = m\vec{v}_1 = mv_1\,\hat{\text{j}}$$

where v_1 = 3.0 m/s, and

$$\vec{p}_2 = m\vec{v}_2 = m\left(v_{2x}\,\hat{\text{i}} + v_{2y}\,\hat{\text{j}}\right) = mv_2\left(\cos\theta\,\hat{\text{i}} + \sin\theta\,\hat{\text{j}}\right)$$

where v_2 = 5.0 m/s and θ = 30°. The combined linear momentum of both parts is then

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 = mv_1\,\hat{\text{j}} + mv_2\left(\cos\theta\,\hat{\text{i}} + \sin\theta\,\hat{\text{j}}\right) = \left(mv_2\cos\theta\right)\hat{\text{i}} + \left(mv_1 + mv_2\sin\theta\right)\hat{\text{j}} \\ &= (2.0\text{ kg})(5.0\text{ m/s})(\cos 30°)\,\hat{\text{i}} + (2.0\text{ kg})\left(3.0\text{ m/s} + (5.0\text{ m/s})(\sin 30°)\right)\hat{\text{j}} \\ &= \left(8.66\,\hat{\text{i}} + 11\,\hat{\text{j}}\right)\text{ kg}\cdot\text{m/s}.\end{aligned}$$

From conservation of linear momentum we know that this is also the linear momentum of the whole kit before it splits. Thus the speed of the 4.0-kg kit is

$$v = \frac{P}{M} = \frac{\sqrt{P_x^2 + P_y^2}}{M} = \frac{\sqrt{(8.66\text{ kg}\cdot\text{m/s})^2 + (11\text{ kg}\cdot\text{m/s})^2}}{4.0\text{ kg}} = 3.5\text{ m/s}.$$

47. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of one piece is $m_1 = m$; its velocity is $\vec{v}_1 = (-30\text{ m/s})\hat{\text{i}}$; the mass of the second piece is $m_2 = m$; its velocity is $\vec{v}_2 = (-30\text{ m/s})\hat{\text{j}}$; and, the mass of the third piece is $m_3 = 3m$.

(a) Conservation of linear momentum requires

$$m\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \quad\Rightarrow\quad 0 = m\left(-30\hat{\text{i}}\right) + m\left(-30\hat{\text{j}}\right) + 3m\vec{v}_3$$

which leads to $\vec{v}_3 = (10\hat{\text{i}} + 10\hat{\text{j}})\text{ m/s}$. Its magnitude is $v_3 = 10\sqrt{2} \approx 14\text{ m/s}$.

(b) The direction is 45° *counterclockwise* from $+x$ (in this system where we have m_1 flying off in the $-x$ direction and m_2 flying off in the $-y$ direction).

48. This problem involves both mechanical energy conservation $U_i = K_1 + K_2$, where U_i = 60 J, and momentum conservation

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $m_2 = 2m_1$. From the second equation, we find $|\vec{v}_1| = 2|\vec{v}_2|$, which in turn implies (since $v_1 = |\vec{v}_1|$ and likewise for v_2)

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\left(\frac{1}{2}m_2\right)(2v_2)^2 = 2\left(\frac{1}{2}m_2v_2^2\right) = 2K_2.$$

(a) We substitute $K_1 = 2K_2$ into the energy conservation relation and find

$$U_i = 2K_2 + K_2 \Rightarrow K_2 = \frac{1}{3}U_i = 20 \text{ J}.$$

(b) And we obtain $K_1 = 2(20) = 40$ J.

49. We refer to the discussion in the textbook (see Sample Problem – "Conservation of momentum, ballistic pendulum," which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{2.010}{0.010}\sqrt{2(9.8)(0.12)} = 3.1\times10^2 \text{ m/s}.$$

50. (a) We choose $+x$ along the initial direction of motion and apply momentum conservation:

$$m_{\text{bullet}}\vec{v}_i = m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2$$
$$(5.2 \text{ g})(672 \text{ m/s}) = (5.2 \text{ g})(428 \text{ m/s}) + (700 \text{ g})\vec{v}_2$$

which yields $v_2 = 1.81$ m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2 \text{ g})(672 \text{ m/s})}{5.2 \text{ g} + 700 \text{ g}} = 4.96 \text{ m/s}.$$

51. In solving this problem, our $+x$ direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

$$(0.0035\text{ kg})v = (1.8035\text{ kg})(1.4\text{ m/s}) \;\Rightarrow\; v = 721\text{ m/s}.$$

(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed v found in part (a)).

$$(0.0035\text{ kg})v_0 = (1.20\text{ kg})(0.630\text{ m/s}) + (0.00350\text{ kg})(721\text{ m/s})$$

which yields v_0 = 937 m/s.

52. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height h measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01\text{kg})(1000\text{m/s}) = (5.0\text{kg})\vec{v} + (0.01\text{kg})(400\text{m/s})$$

which yields $\vec{v} = 1.2\text{ m/s}$. The second part involves either the free-fall equations from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch. 8. Choosing the latter approach, we have

$$\frac{1}{2}(5.0\text{kg})(1.2\text{m/s})^2 = (5.0\text{kg})\left(9.8\text{m/s}^2\right)h$$

which gives the result h = 0.073 m.

53. With an initial speed of v_i, the initial kinetic energy of the car is $K_i = m_c v_i^2/2$. After a totally inelastic collision with a moose of mass m_m, by momentum conservation, the speed of the combined system is

$$m_c v_i = (m_c + m_m)v_f \;\;\Rightarrow\;\; v_f = \frac{m_c v_i}{m_c + m_m},$$

with final kinetic energy

$$K_f = \frac{1}{2}(m_c + m_m)v_f^2 = \frac{1}{2}(m_c + m_m)\left(\frac{m_c v_i}{m_c + m_m}\right)^2 = \frac{1}{2}\frac{m_c^2}{m_c + m_m}v_i^2.$$

(a) The percentage loss of kinetic energy due to collision is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{m_c}{m_c + m_m} = \frac{m_m}{m_c + m_m} = \frac{500\text{ kg}}{1000\text{ kg} + 500\text{ kg}} = \frac{1}{3} = 33.3\%.$$

(b) If the collision were with a camel of mass $m_{\text{camel}} = 300\text{ kg}$, then the percentage loss of kinetic energy would be

$$\frac{\Delta K}{K_i} = \frac{m_{\text{camel}}}{m_c + m_{\text{camel}}} = \frac{300\text{ kg}}{1000\text{ kg} + 300\text{ kg}} = \frac{3}{13} = 23\%.$$

(c) As the animal mass decreases, the percentage loss of kinetic energy also decreases.

54. The total momentum immediately before the collision (with $+x$ upward) is

$$p_i = (3.0\text{ kg})(20\text{ m/s}) + (2.0\text{ kg})(-12\text{ m/s}) = 36\ \text{kg}\cdot\text{m/s}.$$

Their momentum immediately after, when they constitute a combined mass of M = 5.0 kg, is $p_f = (5.0\text{ kg})\vec{v}$. By conservation of momentum, then, we obtain $\vec{v}$ = 7.2 m/s, which becomes their "initial" velocity for their subsequent free-fall motion. We can use Ch. 2 methods or energy methods to analyze this subsequent motion; we choose the latter. The level of their collision provides the reference (y = 0) position for the gravitational potential energy, and we obtain

$$K_0 + U_0 = K + U \quad \Rightarrow \quad \frac{1}{2}Mv_0^2 + 0 = 0 + Mgy_{\text{max}}.$$

Thus, with v_0 = 7.2 m/s, we find y_{max} = 2.6 m.

55. We choose $+x$ in the direction of (initial) motion of the blocks, which have masses m_1 = 5 kg and m_2 = 10 kg. Where units are not shown in the following, SI units are to be understood.

(a) Momentum conservation leads to

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$
$$(5\text{ kg})(3.0\text{ m/s}) + (10\text{ kg})(2.0\text{ m/s}) = (5\text{ kg})\vec{v}_{1f} + (10\text{ kg})(2.5\text{ m/s})$$

which yields $\vec{v}_{1f} = 2.0\text{ m/s}$. Thus, the speed of the 5.0 kg block immediately after the collision is 2.0 m/s.

(b) We find the reduction in total kinetic energy:

$$\begin{aligned} K_i - K_f &= \frac{1}{2}(5\text{ kg})(3\text{ m/s})^2 + \frac{1}{2}(10\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(5\text{ kg})(2\text{ m/s})^2 - \frac{1}{2}(10\text{ kg})(2.5\text{ m/s})^2 \\ &= -1.25\text{ J} \approx -1.3\text{ J}. \end{aligned}$$

(c) In this new scenario where $\vec{v}_{2f} = 4.0\text{ m/s}$, momentum conservation leads to $\vec{v}_{1f} = -1.0\text{ m/s}$ and we obtain $\Delta K = +40\text{ J}$.

(d) The creation of additional kinetic energy is possible if, say, some gunpowder were on the surface where the impact occurred (initially stored chemical energy would then be contributing to the result).

56. (a) The magnitude of the deceleration of each of the cars is $a = f/m = \mu_k mg/m = \mu_k g$. If a car stops in distance d, then its speed v just after impact is obtained from Eq. 2-16:

$$v^2 = v_0^2 + 2ad \Rightarrow v = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

since $v_0 = 0$ (this could alternatively have been derived using Eq. 8-31). Thus,

$$v_A = \sqrt{2\mu_k gd_A} = \sqrt{2(0.13)(9.8\ \text{m/s}^2)(8.2\ \text{m})} = 4.6\ \text{m/s}.$$

(b) Similarly, $v_B = \sqrt{2\mu_k gd_B} = \sqrt{2(0.13)(9.8\ \text{m/s}^2)(6.1\ \text{m})} = 3.9\ \text{m/s}.$

(c) Let the speed of car B be v just before the impact. Conservation of linear momentum gives $m_B v = m_A v_A + m_B v_B$, or

$$v = \frac{(m_A v_A + m_B v_B)}{m_B} = \frac{(1100)(4.6) + (1400)(3.9)}{1400} = 7.5\ \text{m/s}.$$

(d) The conservation of linear momentum during the impact depends on the fact that the only significant force (during impact of duration Δt) is the force of contact between the bodies. In this case, that implies that the force of friction exerted by the road on the cars is neglected during the brief Δt. This neglect would introduce some error in the analysis. Related to this is the assumption we are making that the transfer of momentum occurs at one location, that the cars do not slide appreciably during Δt, which is certainly an approximation (though probably a good one). Another source of error is the application of the friction relation Eq. 6-2 for the sliding portion of the problem (after the impact); friction is a complex force that Eq. 6-2 only partially describes.

57. (a) Let v be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m + M)v$. Therefore,

$$v = \frac{mv_i}{m+M} = \frac{(60\ \text{g})(22\ \text{m/s})}{60\ \text{g} + 240\ \text{g}} = 4.4\ \text{m/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy is

$$K_f = \tfrac{1}{2}(m+M)v^2 = \tfrac{1}{2}m^2 v_i^2 \big/ (m+M).$$

The problem indicates $\Delta E_{\text{th}} = 0$, so the difference $K_i - K_f$ must equal the energy U_s stored in the spring:

$$U_s = \frac{1}{2} m v_i^2 - \frac{1}{2} \frac{m^2 v_i^2}{(m+M)} = \frac{1}{2} m v_i^2 \left(1 - \frac{m}{m+M}\right) = \frac{1}{2} m v_i^2 \, \frac{M}{m+M}.$$

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is

$$\frac{U_s}{K_i} = \frac{M}{m+M} = \frac{240}{60+240} = 0.80 \,.$$

58. We think of this as having two parts: the first is the collision itself, where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet, and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount x_m. The first part involves momentum conservation (with $+x$ rightward):

$$m_1 v_1 = (m_1 + m_2) v \quad \Rightarrow \quad (2.0 \text{ kg})(4.0 \text{ m/s}) = (3.0 \text{ kg})\vec{v}$$

which yields $\vec{v} = 2.7 \text{ m/s}$. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0 \text{ kg})\,(2.7 \text{ m/s})^2 = \frac{1}{2}\,(200 \text{ N/m}) x_{\text{m}}^2$$

which gives the result $x_m = 0.33$ m.

59. As hinted in the problem statement, the velocity v of the system as a whole, when the spring reaches the maximum compression x_m, satisfies

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v.$$

The change in kinetic energy of the system is therefore

$$\Delta K = \frac{1}{2}(m_1 + m_2) v^2 - \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_2 v_{2i}^2 = \frac{(m_1 v_{1i} + m_2 v_{2i})^2}{2(m_1 + m_2)} - \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} m_2 v_{2i}^2$$

which yields $\Delta K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right) v_{\text{rel}}^2$ where $v_{\text{rel}} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2} k x_{\text{m}}^2 = -\Delta K \Rightarrow x_{\text{m}} = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35 \text{ J})}{1120 \text{ N/m}}} = 0.25 \text{ m}.$$

60. (a) Let m_A be the mass of the block on the left, v_{Ai} be its initial velocity, and v_{Af} be its final velocity. Let m_B be the mass of the block on the right, v_{Bi} be its initial velocity, and v_{Bf} be its final velocity. The momentum of the two-block system is conserved, so

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

and

$$v_{Af} = \frac{m_A v_{Ai} + m_B v_{Bi} - m_B v_{Bf}}{m_A} = \frac{(1.6\text{ kg})(5.5\text{ m/s}) + (2.4\text{ kg})(2.5\text{ m/s}) - (2.4\text{ kg})(4.9\text{ m/s})}{1.6\text{ kg}}$$
$$= 1.9\text{ m/s}.$$

(b) The block continues going to the right after the collision.

(c) To see whether the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \frac{1}{2} m_A v_{Ai}^2 + \frac{1}{2} m_B v_{Bi}^2 = \frac{1}{2}(1.6\text{ kg})(5.5\text{ m/s})^2 + \frac{1}{2}(2.4\text{ kg})(2.5\text{ m/s})^2 = 31.7\text{ J}.$$

The total kinetic energy after is

$$K_f = \frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 = \frac{1}{2}(1.6\text{ kg})(1.9\text{ m/s})^2 + \frac{1}{2}(2.4\text{ kg})(4.9\text{ m/s})^2 = 31.7\text{ J}.$$

Since $K_i = K_f$ the collision is found to be elastic.

61. **THINK** We have a moving cart colliding with a stationary cart. Since the collision is elastic, the total kinetic energy of the system remains unchanged.

EXPRESS Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Conservation of linear momentum gives $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$. Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 .$$

Solving for v_{1f} and v_{2f}, we obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

The speed of the center of mass is $v_{\text{com}} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$.

ANALYZE (a) With $m_1 = 0.34$ kg, $v_{1i} = 1.2$ m/s and $v_{1f} = 0.66$ m/s, we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{1.2 \text{ m/s} - 0.66 \text{ m/s}}{1.2 \text{ m/s} + 0.66 \text{ m/s}} \right) (0.34 \text{ kg}) = 0.0987 \text{ kg} \approx 0.099 \text{ kg}.$$

(b) The velocity of the second cart is:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(0.34 \text{ kg})}{0.34 \text{ kg} + 0.099 \text{ kg}} \right) (1.2 \text{ m/s}) = 1.9 \text{ m/s}.$$

(c) From the above, we find the speed of the center of mass to be

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34 \text{ kg})(1.2 \text{ m/s}) + 0}{0.34 \text{ kg} + 0.099 \text{ kg}} = 0.93 \text{ m/s}.$$

LEARN In solving for v_{com}, values for the initial velocities were used. Since the system is isolated with no external force acting on it, v_{com} remains the same after the collision, so the same result is obtained if values for the final velocities are used. That is,

$$v_{\text{com}} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2} = \frac{(0.34 \text{ kg})(0.66 \text{ m/s}) + (0.099 \text{ kg})(1.9 \text{ m/s})}{0.34 \text{ kg} + 0.099 \text{ kg}} = 0.93 \text{ m/s}.$$

62. (a) Let m_1 be the mass of one sphere, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the other sphere, v_{2i} be its velocity before the collision, and v_{2f} be its velocity after the collision. Then, according to Eq. 9-75,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}.$$

Suppose sphere 1 is originally traveling in the positive direction and is at rest after the collision. Sphere 2 is originally traveling in the negative direction. Replace v_{1i} with v, v_{2i} with $-v$, and v_{1f} with zero to obtain $0 = m_1 - 3m_2$. Thus,

$$m_2 = m_1 / 3 = (300 \text{ g}) / 3 = 100 \text{ g}.$$

(b) We use the velocities before the collision to compute the velocity of the center of mass:

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(300 \text{ g})(2.00 \text{ m/s}) + (100 \text{ g})(-2.00 \text{ m/s})}{300 \text{ g} + 100 \text{ g}} = 1.00 \text{ m/s}.$$

63. (a) The center of mass velocity does not change in the absence of external forces. In this collision, only forces of one block on the other (both being part of the same system) are exerted, so the center of mass velocity is 3.00 m/s before and after the collision.

(b) We can find the velocity v_{1i} of block 1 before the collision (when the velocity of block 2 is known to be zero) using Eq. 9-17:

$$(m_1 + m_2)v_{com} = m_1 v_{1i} + 0 \quad \Rightarrow \quad v_{1i} = 12.0 \text{ m/s} .$$

Now we use Eq. 9-68 to find v_{2f}:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = 6.00 \text{ m/s} .$$

64. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with h = 0.700 m) leads to

$$m_1 gh = \frac{1}{2} m_1 v^2 \Rightarrow v = \sqrt{2gh} = 3.7 \text{ m/s}.$$

(a) We now treat the elastic collision using Eq. 9-67:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 \text{ kg} - 2.5 \text{ kg}}{0.5 \text{ kg} + 2.5 \text{ kg}} (3.7 \text{ m/s}) = -2.47 \text{ m/s}$$

which means the final speed of the ball is 2.47 m/s.

(b) Finally, we use Eq. 9-68 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5 \text{ kg})}{0.5 \text{ kg} + 2.5 \text{ kg}} (3.7 \text{ m/s}) = 1.23 \text{ m/s}.$$

65. **THINK** We have a mass colliding with another stationary mass. Since the collision is elastic, the total kinetic energy of the system remains unchanged.

EXPRESS Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Conservation of linear momentum gives

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} .$$

Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 .$$

The solution to v_{1f} is given by Eq. 9-67: $v_{1f} = \dfrac{m_1 - m_2}{m_1 + m_2} v_{1i}$. We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 .$$

The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} .$$

ANALYZE (a) given that $v_{1f} = v_{1i}/4$, we find the second mass to be

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}} m_1 = \left(\frac{v_{1i} - v_{1i}/4}{v_{1i} + v_{1i}/4}\right) m_1 = \frac{3}{5} m_1 = \frac{3}{5}(2.0\text{ kg}) = 1.2\text{ kg} .$$

(b) The speed of the center of mass is $v_{\text{com}} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \dfrac{(2.0\text{ kg})(4.0\text{ m/s})}{2.0\text{ kg} + 1.2\text{ kg}} = 2.5\text{ m/s}$.

LEARN The final speed of the second mass is

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \left(\frac{2(2.0\text{ kg})}{2.0\text{ kg} + 1.2\text{ kg}}\right)(4.0\text{ m/s}) = 5.0\text{ m/s}.$$

Since the system is isolated with no external force acting on it, v_{com} remains the same after the collision, so the same result is obtained if values for the final velocities are used:

$$v_{\text{com}} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2} = \frac{(2.0\text{ kg})(1.0\text{ m/s}) + (1.2\text{ kg})(5.0\text{ kg})}{2.0\text{ kg} + 1.2\text{ kg}} = 2.5\text{ m/s} .$$

66. Using Eq. 9-67 and Eq. 9-68, we have after the collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 0.40 m_1}{m_1 + 0.40 m_1}(4.0\text{ m/s}) = 1.71\text{ m/s}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 0.40 m_1}(4.0\text{ m/s}) = 5.71\text{ m/s}.$$

(a) During the (subsequent) sliding, the kinetic energy of block 1 $K_{1f} = \frac{1}{2} m_1 v_{1f}^2$ is converted into thermal form ($\Delta E_{\text{th}} = \mu_k\, m_1\, g\, d_1$). Solving for the sliding distance d_1 we obtain $d_1 = 0.2999$ m ≈ 30 cm.

(b) A very similar computation (but with subscript 2 replacing subscript 1) leads to block 2's sliding distance $d_2 = 3.332$ m ≈ 3.3 m.

67. We use Eq 9-67 and 9-68 to find the velocities of the particles after their first collision (at $x = 0$ and $t = 0$):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{0.30\text{ kg} - 0.40\text{ kg}}{0.30\text{ kg} + 0.40\text{ kg}} (2.0\text{ m/s}) = -0.29\text{ m/s}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2(0.30\text{ kg})}{0.30\text{ kg} + 0.40\text{ kg}} (2.0\text{ m/s}) = 1.7\text{ m/s}.$$

At a rate of motion of 1.7 m/s, $2x_w$ = 140 cm (the distance to the wall and back to $x = 0$) will be traversed by particle 2 in 0.82 s. At t = 0.82 s, particle 1 is located at

$$x = (–2/7)(0.82) = –23 \text{ cm},$$

and particle 2 is "gaining" at a rate of (10/7) m/s leftward; this is their relative velocity at that time. Thus, this "gap" of 23 cm between them will be closed after an additional time of (0.23 m)/(10/7 m/s) = 0.16 s has passed. At this time (t = 0.82 + 0.16 = 0.98 s) the two particles are at $x = (–2/7)(0.98) = –28$ cm.

68. (a) If the collision is perfectly elastic, then Eq. 9-68 applies

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + (2.00)m_1} \sqrt{2gh} = \frac{2}{3}\sqrt{2gh}$$

where we have used the fact (found most easily from energy conservation) that the speed of block 1 at the bottom of the frictionless ramp is $\sqrt{2gh}$ (where h = 2.50 m). Next, for block 2's "rough slide" we use Eq. 8-37:

$$\tfrac{1}{2} m_2 v_2^2 = \Delta E_{\text{th}} = f_k d = \mu_k m_2 g d$$

where μ_k = 0.500. Solving for the sliding distance d, we find that m_2 cancels out and we obtain d = 2.22 m.

(b) In a completely inelastic collision, we apply Eq. 9-53: $v_2 = \dfrac{m_1}{m_1 + m_2} v_{1i}$ (where, as above, $v_{1i} = \sqrt{2gh}$). Thus, in this case we have $v_2 = \sqrt{2gh}/3$. Now, Eq. 8-37 (using the total mass since the blocks are now joined together) leads to a sliding distance of $d = 0.556$ m (one-fourth of the part (a) answer).

69. (a) We use conservation of mechanical energy to find the speed of either ball after it has fallen a distance h. The initial kinetic energy is zero, the initial gravitational potential energy is $M\,gh$, the final kinetic energy is $\tfrac{1}{2}Mv^2$, and the final potential energy is zero. Thus $Mgh = \tfrac{1}{2}Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of M with the floor is an elastic collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is

traveling upward with a speed of $\sqrt{2gh}$. The ball of mass *m* is traveling downward with the same speed. We use Eq. 9-75 to find an expression for the velocity of the ball of mass *M* after the collision:

$$v_{Mf} = \frac{M-m}{M+m}v_{Mi} + \frac{2m}{M+m}v_{mi} = \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh} = \frac{M-3m}{M+m}\sqrt{2gh} .$$

For this to be zero, $m = M/3$. With $M = 0.63$ kg, we have $m = 0.21$ kg.

(b) We use the same equation to find the velocity of the ball of mass *m* after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}$$

which becomes (upon substituting $M = 3m$) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height h' to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is mgh'. Thus,

$$\frac{1}{2}mv_{mf}^2 = mgh' \Rightarrow h' = \frac{v_{mf}^2}{2g} = 4h .$$

With $h = 1.8$ m, we have $h' = 7.2\text{ m}$.

70. We use Eqs. 9-67, 9-68, and 4-21 for the elastic collision and the subsequent projectile motion. We note that both pucks have the same time-of-fall *t* (during their projectile motions). Thus, we have

$$\Delta x_2 = v_2 t \quad \text{where } \Delta x_2 = d \text{ and } v_2 = \frac{2m_1}{m_1 + m_2}v_{1i}$$

$$\Delta x_1 = v_1 t \quad \text{where } \Delta x_1 = -2d \text{ and } v_1 = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} .$$

Dividing the first equation by the second, we arrive at

$$\frac{d}{-2d} = \frac{\frac{2m_1}{m_1 + m_2}v_{1i}\,t}{\frac{m_1 - m_2}{m_1 + m_2}v_{1i}\,t} .$$

After canceling v_{1i}, t, and d, and solving, we obtain $m_2 = 1.0$ kg.

71. We apply the conservation of linear momentum to the *x* and *y* axes respectively.

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$
$$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2.$$

We are given $v_{2f} = 1.20\times10^5$ m/s, $\theta_1 = 64.0°$ and $\theta_2 = 51.0°$. Thus, we are left with two unknowns and two equations, which can be readily solved.

(a) We solve for the final alpha particle speed using the *y*-momentum equation:

$$v_{1f} = \frac{m_2 v_{2f} \sin\theta_2}{m_1 \sin\theta_1} = \frac{(16.0)\left(1.20\times10^5\right)\sin(51.0°)}{(4.00)\sin(64.0°)} = 4.15\times10^5 \text{ m/s} .$$

(b) Plugging our result from part (a) into the *x*-momentum equation produces the initial alpha particle speed:

$$\begin{aligned} v_{1i} &= \frac{m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2}{m_{1i}} \\ &= \frac{(4.00)\left(4.15\times10^5\right) \cos(64.0°) + (16.0)\left(1.2\times10^5\right) \cos(51.0°)}{4.00} \\ &= 4.84\times10^5 \text{ m/s} . \end{aligned}$$

72. We orient our +*x* axis along the initial direction of motion, and specify angles in the "standard" way — so θ = –90° for the particle *B*, which is assumed to scatter "downward" and ϕ > 0 for particle *A*, which presumably goes into the first quadrant. We apply the conservation of linear momentum to the *x* and *y* axes, respectively.

$$m_B v_B = m_B v'_B \cos\theta + m_A v'_A \cos\phi$$
$$0 = m_B v'_B \sin\theta + m_A v'_A \sin\phi$$

(a) Setting $v_B = v$ and $v'_B = v/2$, the *y*-momentum equation yields

$$m_A v'_A \sin\phi = m_B \frac{v}{2}$$

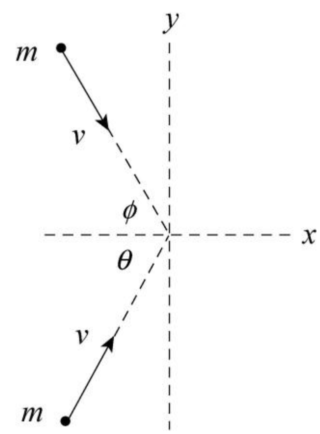
and the *x*-momentum equation yields $m_A v'_A \cos\phi = m_B v$. Dividing these two equations, we find $\tan\phi = \frac{1}{2}$, which yields $\phi = 27°$.

(b) We can *formally* solve for v'_A (using the *y*-momentum equation and the fact that $\phi = 1/\sqrt{5}$)

$$v'_A = \frac{\sqrt{5}}{2}\frac{m_B}{m_A} v$$

but lacking numerical values for v and the mass ratio, we cannot fully determine the final speed of A. Note: substituting $\cos\phi = 2/\sqrt{5}$, into the x-momentum equation leads to exactly this same relation (that is, no new information is obtained that might help us determine an answer).

73. Suppose the objects enter the collision along lines that make the angles $\theta > 0$ and $\phi > 0$ with the x axis, as shown in the diagram that follows. Both have the same mass m and the same initial speed v. We suppose that after the collision the combined object moves in the positive x direction with speed V.



Since the y component of the total momentum of the two-object system is conserved,

$$mv \sin\theta - mv \sin\phi = 0.$$

This means $\phi = \theta$. Since the x component is conserved,

$$2mv \cos\theta = 2mV.$$

We now use $V = v/2$ to find that $\cos\theta = 1/2$. This means $\theta = 60°$. The angle between the initial velocities is 120°.

74. (a) Conservation of linear momentum implies

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B .$$

Since $m_A = m_B = m = 2.0$ kg, the masses divide out and we obtain

$$\begin{aligned}\vec{v}\,'_B &= \vec{v}_A + \vec{v}_B - \vec{v}\,'_A = (15\hat{\text{i}} + 30\hat{\text{j}})\text{ m/s} + (-10\hat{\text{i}} + 5\hat{\text{j}})\text{ m/s} - (-5\hat{\text{i}} + 20\hat{\text{j}})\text{ m/s} \\ &= (10\hat{\text{i}} + 15\hat{\text{j}})\text{ m/s} .\end{aligned}$$

(b) The final and initial kinetic energies are

$$K_f = \frac{1}{2}mv'^2_A + \frac{1}{2}mv'^2_B = \frac{1}{2}(2.0)\left((-5)^2 + 20^2 + 10^2 + 15^2\right) = 8.0\times 10^2 \text{ J}$$

$$K_i = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}(2.0)\left(15^2 + 30^2 + (-10)^2 + 5^2\right) = 1.3\times 10^3 \text{ J} .$$

The change kinetic energy is then $\Delta K = -5.0 \times 10^2$ J (that is, 500 J of the initial kinetic energy is lost).

75. We orient our +x axis along the initial direction of motion, and specify angles in the "standard" way — so θ = +60° for the proton (1), which is assumed to scatter into the first quadrant and ϕ = –30° for the target proton (2), which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to θ). We apply the conservation of linear momentum to the x and y axes, respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v_1' \cos\theta + m_2 v_2' \cos\phi \\ 0 &= m_1 v_1' \sin\theta + m_2 v_2' \sin\phi. \end{aligned}$$

We are given v_1 = 500 m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for v_2' we obtain

$$v_2' = \frac{v_1 \sin\theta}{\sin(\theta - \phi)} = \frac{(500\ \text{m/s})\sin(60°)}{\sin(90°)} = 433\ \text{m/s}.$$

We used the identity sin $\theta\cos\phi$ – $\cos\theta$ $\sin\phi$ = sin (θ– ϕ) in simplifying our final expression.

(b) In a similar manner, we find

$$v_1' = \frac{v_1 \sin\theta}{\sin(\phi - \theta)} = \frac{(500\ \text{m/s})\sin(-30°)}{\sin(-90°)} = 250\ \text{m/s}\ .$$

76. We use Eq. 9-88. Then

$$v_f = v_i + v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) = 105\ \text{m/s} + (253\ \text{m/s}) \ln\left(\frac{6090\ \text{kg}}{6010\ \text{kg}}\right) = 108\ \text{m/s}.$$

77. **THINK** The mass of the faster barge is increasing at a constant rate. Additional force must be provided in order to maintain a constant speed.

EXPRESS We consider what must happen to the coal that lands on the faster barge during a time interval Δt. In that time, a total of Δm of coal must experience a change of velocity (from slow to fast) $\Delta v = v_{\text{fast}} - v_{\text{slow}}$, where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta p}{\Delta t} = \frac{(\Delta m)}{\Delta t}\Delta v = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}})$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$. Note that we ignore the transverse speed of the coal as it is shoveled from the slower barge to the faster one.

ANALYZE (a) With $v_{\text{fast}} = 20\,\text{km/h} = 5.56\,\text{m/s}$, $v_{\text{slow}} = 10\,\text{km/h} = 2.78\,\text{m/s}$ and the rate of mass change $(\Delta m/\Delta t) = 1000\,\text{kg/min} = (16.67\,\text{kg/s})$, the force that must be applied to the faster barge is

$$F_{\text{fast}} = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}}) = (16.67\text{ kg/s})(5.56\text{ m/s} - 2.78\text{ m/s}) = 46.3\,\text{N}$$

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

LEARN The force that must be applied to the faster barge in order to maintain a constant speed is equal to the rate of change of momentum of the coal.

78. We use Eq. 9-88 and simplify with $v_i = 0$, $v_f = v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln\frac{M_i}{M_f} \Rightarrow \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$ we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7$.

(b) If $v = 2u$ we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4$.

79. **THINK** As fuel is consumed, both the mass and the speed of the rocket will change.

EXPRESS The thrust of the rocket is given by $T = Rv_{\text{rel}}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. On the other hand, the mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$, where Δt is the time interval of the burn. Thus, the mass of the rocket after the burn is

$$M_f = M_i - M_{\text{fuel}}\,.$$

ANALYZE (a) Given that $R = 480$ kg/s and $v_{\text{rel}} = 3.27 \times 10^3$ m/s, we find the thrust to be

$$T = Rv_{\text{rel}} = (480\,\text{kg/s})(3.27\times10^3\,\text{m/s}) = 1.57\times10^6\,\text{N}.$$

(b) With the mass of fuel ejected given by $M_{\text{fuel}} = R\Delta t =$ (480 kg/s)(250 s) = 1.20×10^5 kg, the final mass of the rocket is

$$M_f = M_i - M_{\text{fuel}} = (2.55 \times 10^5 \text{ kg}) - (1.20 \times 10^5 \text{ kg}) = 1.35 \times 10^5 \text{ kg}.$$

(c) Since the initial speed is zero, the final speed of the rocket is

$$v_f = v_{\text{rel}} \ln\frac{M_i}{M_f} = \left(3.27\times10^3 \text{ m/s}\right)\ln\left(\frac{2.55\times10^5 \text{ kg}}{1.35\times10^5 \text{ kg}}\right) = 2.08\times10^3 \text{ m/s}.$$

LEARN The speed of the rocket continues to rise as the fuel is consumed. From the first rocket equation given in Eq. 9-87, the thrust of the rocket is related to the acceleration by $T = Ma$. Using this equation, we find the initial acceleration to be

$$a_i = \frac{T}{M_i} = \frac{1.57\times10^6 \text{ N}}{2.55\times10^5 \text{ kg}} = 6.16 \text{ m/s}^2.$$

80. The velocity of the object is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left((3500 - 160t)\,\hat{\text{i}} + 2700\,\hat{\text{j}} + 300\,\hat{\text{k}}\right) = -(160 \text{ m/s})\hat{\text{i}}.$$

(a) The linear momentum is $\vec{p} = m\vec{v} = \left(250 \text{ kg}\right)\left(-160 \text{ m/s}\,\hat{\text{i}}\right) = (-4.0\times10^4 \text{ kg}\cdot\text{m/s})\,\hat{\text{i}}.$

(b) The object is moving west (our $-\hat{\text{i}}$ direction).

(c) Since the value of $\vec{p}$ does not change with time, the net force exerted on the object is zero, by Eq. 9-23.

81. We assume no external forces act on the system composed of the two parts of the last stage. Hence, the total momentum of the system is conserved. Let m_c be the mass of the rocket case and m_p the mass of the payload. At first they are traveling together with velocity v. After the clamp is released m_c has velocity v_c and m_p has velocity v_p. Conservation of momentum yields

$$(m_c + m_p)v = m_c v_c + m_p v_p.$$

(a) After the clamp is released the payload, having the lesser mass, will be traveling at the greater speed. We write $v_p = v_c + v_{\text{rel}}$, where v_{rel} is the relative velocity. When this expression is substituted into the conservation of momentum condition, the result is

$$\left(m_c + m_p\right)v = m_c v_c + m_p v_c + m_p v_{\text{rel}}.$$

Therefore,

$$v_c = \frac{(m_c + m_p)v - m_p v_{\text{rel}}}{m_c + m_p} = \frac{(290.0 \text{ kg} + 150.0 \text{ kg})(7600 \text{ m/s}) - (150.0 \text{ kg})(910.0 \text{ m/s})}{290.0 \text{ kg} + 150.0 \text{ kg}}$$
$$= 7290 \text{ m/s}.$$

(b) The final speed of the payload is $v_p = v_c + v_{\text{rel}}$ = 7290 m/s + 910.0 m/s = 8200 m/s.

(c) The total kinetic energy before the clamp is released is

$$K_i = \frac{1}{2}(m_c + m_p)v^2 = \frac{1}{2}(290.0 \text{ kg} + 150.0 \text{ kg})(7600 \text{ m/s})^2 = 1.271 \times 10^{10} \text{ J}.$$

(d) The total kinetic energy after the clamp is released is

$$K_f = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}(290.0 \text{ kg})(7290 \text{ m/s})^2 + \frac{1}{2}(150.0 \text{ kg})(8200 \text{ m/s})^2$$
$$= 1.275 \times 10^{10} \text{ J}.$$

The total kinetic energy increased slightly. Energy originally stored in the spring is converted to kinetic energy of the rocket parts.

82. Let m be the mass of the higher floors. By energy conservation, the speed of the higher floors just before impact is

$$mgd = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gd}.$$

The magnitude of the impulse during the impact is

$$J = |\Delta p| = m|\Delta v| = mv = m\sqrt{2gd} = mg\sqrt{\frac{2d}{g}} = W\sqrt{\frac{2d}{g}}$$

where $W = mg$ represents the weight of the higher floors. Thus, the average force exerted on the lower floor is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{W}{\Delta t}\sqrt{\frac{2d}{g}}$$

With $F_{\text{avg}} = sW$, where s is the safety factor, we have

$$s = \frac{1}{\Delta t}\sqrt{\frac{2d}{g}} = \frac{1}{1.5 \times 10^{-3} \text{ s}}\sqrt{\frac{2(4.0 \text{ m})}{9.8 \text{ m/s}^2}} = 6.0 \times 10^2.$$

83. (a) Momentum conservation gives

$$m_R v_R + m_L v_L = 0 \;\Rightarrow\; (0.500 \text{ kg})\, v_R + (1.00 \text{ kg})(-1.20 \text{ m/s}) = 0$$

which yields v_R = 2.40 m/s. Thus, $\Delta x = v_R\, t$ = (2.40 m/s)(0.800 s) = 1.92 m.

(b) Now we have $m_R v_R + m_L (v_R - 1.20 \text{ m/s}) = 0$, which yields

$$v_R = \frac{(1.2 \text{ m/s})m_L}{m_L + m_R} = \frac{(1.20 \text{ m/s})(1.00 \text{ kg})}{1.00 \text{ kg} + 0.500 \text{ kg}} = 0.800 \text{ m/s}.$$

Consequently, $\Delta x = v_R\, t$ = 0.640 m.

84. (a) This is a highly symmetric collision, and when we analyze the y-components of momentum we find their net value is zero. Thus, the stuck-together particles travel along the x axis.

(b) Since it is an elastic collision with identical particles, the final speeds are the same as the initial values. Conservation of momentum along each axis then assures that the angles of approach are the same as the angles of scattering. Therefore, one particle travels along line 2, the other along line 3.

(c) Here the final speeds are less than they were initially. The total x-component cannot be less, however, by momentum conservation, so the loss of speed shows up as a decrease in their y-velocity-components. This leads to smaller angles of scattering. Consequently, one particle travels through region B, the other through region C; the paths are symmetric about the x-axis. We note that this is intermediate between the final states described in parts (b) and (a).

(d) Conservation of momentum along the x-axis leads (because these are identical particles) to the simple observation that the x-component of each particle remains constant:

$$v_{fx} = v\cos\theta = 3.06 \text{ m/s}.$$

(e) As noted above, in this case the speeds are unchanged; both particles are moving at 4.00 m/s in the final state.

85. Using Eq. 9-67 and Eq. 9-68, we have after the first collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{m_1 - 2m_1}{m_1 + 2m_1} v_{1i} = -\frac{1}{3} v_{1i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + 2m_1} v_{1i} = \frac{2}{3} v_{1i}.$$

After the second collision, the velocities are

$$v_{2ff} = \frac{m_2 - m_3}{m_2 + m_3} v_{2f} = \frac{-m_2}{3m_2}\frac{2}{3} v_{1i} = -\frac{2}{9} v_{1i}$$

$$v_{3ff} = \frac{2m_2}{m_2 + m_3} v_{2f} = \frac{2m_2}{3m_2}\frac{2}{3} v_{1i} = \frac{4}{9} v_{1i} \ .$$

(a) Setting $v_{1i} = 4$ m/s, we find $v_{3ff} \approx 1.78$ m/s.

(b) We see that v_{3ff} is less than v_{1i} .

(c) The final kinetic energy of block 3 (expressed in terms of the initial kinetic energy of block 1) is

$$K_{3ff} = \frac{1}{2} m_3 v_3^2 = \frac{1}{2}(4m_1)\left(\frac{4}{9}\right)^2 v_{1i}^2 = \frac{64}{81} K_{1i} \ .$$

We see that this is less than K_{1i} .

(d) The final momentum of block 3 is $p_{3ff} = m_3 v_{3ff} = (4m_1)\left(\frac{16}{9}\right) v_1 > m_1 v_1$.

86. (a) We use Eq. 9-68 twice:

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{1.5m_1}(4.00 \text{ m/s}) = \frac{16}{3} \text{ m/s}$$

$$v_3 = \frac{2m_2}{m_2 + m_3} v_2 = \frac{2m_2}{1.5m_2}(16/3 \text{ m/s}) = \frac{64}{9} \text{ m/s} = 7.11 \text{ m/s} \ .$$

(b) Clearly, the speed of block 3 is greater than the (initial) speed of block 1.

(c) The kinetic energy of block 3 is

$$K_{3f} = \frac{1}{2} m_3 v_3^2 = \left(\frac{1}{2}\right)^3 m_1 \left(\frac{16}{9}\right)^2 v_{1i}^2 = \frac{64}{81} K_{1i} \ .$$

We see the kinetic energy of block 3 is less than the (initial) K of block 1. In the final situation, the initial K is being shared among the three blocks (which are all in motion), so this is not a surprising conclusion.

(d) The momentum of block 3 is

$$p_{3f} = m_3 v_3 = \left(\frac{1}{2}\right)^2 m_1 \left(\frac{16}{9}\right) v_{1i} = \frac{4}{9} p_{1i}$$

and is therefore less than the initial momentum (both of these being considered in magnitude, so questions about ± sign do not enter the discussion).

87. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued $\vec{v}_i = -5.2 \text{ m/s}$).

(a) The speed of the ball right after the collision is

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(K_i/2)}{m}} = \sqrt{\frac{mv_i^2/2}{m}} = \frac{v_i}{\sqrt{2}} \approx 3.7 \text{ m/s}.$$

(b) With m = 0.15 kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (0.15 \text{ kg})(3.7 \text{ m/s}) - (0.15 \text{ kg})(-5.2 \text{ m/s}) = 1.3 \text{ N}\cdot\text{s}.$$

(c) Equation 9-35 leads to $F_{avg} = J/\Delta t = 1.3/0.0076 = 1.8 \times 10^2$ N.

88. We first consider the 1200 kg part. The impulse has magnitude J and is (by our choice of coordinates) in the positive direction. Let m_1 be the mass of the part and v_1 be its velocity after the bolts are exploded. We assume both parts are at rest before the explosion. Then $J = m_1 v_1$, so

$$v_1 = \frac{J}{m_1} = \frac{300 \text{ N}\cdot\text{s}}{1200 \text{ kg}} = 0.25 \text{ m/s}.$$

The impulse on the 1800 kg part has the same magnitude but is in the opposite direction, so $-J = m_2 v_2$, where m_2 is the mass and v_2 is the velocity of the part. Therefore,

$$v_2 = -\frac{J}{m_2} = -\frac{300 \text{N}\cdot\text{s}}{1800 \text{kg}} = -0.167 \text{ m/s}.$$

Consequently, the relative speed of the parts after the explosion is

$$u = 0.25 \text{ m/s} - (-0.167 \text{ m/s}) = 0.417 \text{ m/s}.$$

89. **THINK** The momentum of the car changes as it turns and collides with a tree.

EXPRESS Let the initial and final momenta of the car be $\vec{p}_i = m\vec{v}_i$ and $\vec{p}_f = m\vec{v}_f$, respectively. The impulse on it equals the change in its momentum:

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(\vec{v}_f - \vec{v}_i).$$

The average force over the duration Δt is given by $\vec{F}_{avg} = \vec{J}/\Delta t$.

ANALYZE (a) The initial momentum of the car is

$$\vec{p}_i = m\vec{v}_i = (1400\,\text{kg})(5.3\,\text{m/s})\hat{\text{j}} = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{j}}$$

and the final momentum after making the turn is $\vec{p}_f = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ (note that the magnitude remains the same, only the direction is changed). Thus, the impulse is

$$\vec{J} = \vec{p}_f - \vec{p}_i = (7.4\times10^3\ \text{N}\cdot\text{s})(\hat{\text{i}} - \hat{\text{j}}).$$

(b) The initial momentum of the car after the turn is $\vec{p}\,'_i = (7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}$ and the final momentum after colliding with a tree is $\vec{p}\,'_f = 0$. The impulse acting on it is

$$\vec{J}\,' = \vec{p}\,'_f - \vec{p}\,'_i = (-7.4\times10^3\,\text{N}\cdot\text{s})\hat{\text{i}}.$$

(c) The average force on the car during the turn is

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400\,\text{kg}\cdot\text{m/s})(\hat{\text{i}} - \hat{\text{j}})}{4.6\ \text{s}} = (1600\,\text{N})(\hat{\text{i}} - \hat{\text{j}})$$

and its magnitude is

$$F_{\text{avg}} = (1600\,\text{N})\sqrt{2} = 2.3\times10^3\,\text{N}.$$

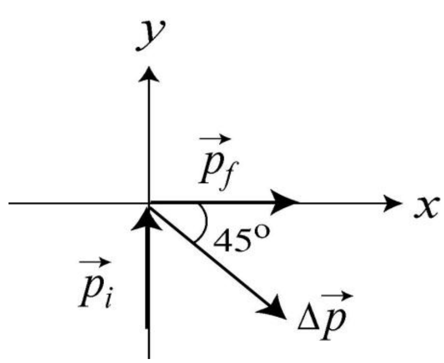
(d) The average force during the collision with the tree is

$$\vec{F}\,'_{\text{avg}} = \frac{\vec{J}\,'}{\Delta t} = \frac{(-7400\,\text{kg}\cdot\text{m/s})\hat{\text{i}}}{350\times10^{-3}\text{s}} = (-2.1\times10^4\,\text{N})\hat{\text{i}}$$

and its magnitude is $F'_{\text{avg}} = 2.1\times10^4\,\text{N}$.

(e) As shown in (c), the average force during the turn, in unit vector notation, is $\vec{F}_{\text{avg}} = (1600\,\text{N})(\hat{\text{i}} - \hat{\text{j}})$. The force is 45° below the positive x axis.

LEARN During the turn, the average force $\vec{F}_{\text{avg}}$ is in the same direction as $\vec{J}$, or $\Delta\vec{p}$. Its x and y components have equal magnitudes. The x component is positive and the y component is negative, so the force is 45° below the positive x axis.



90. (a) We find the momentum $\vec{p}_{n\,r}$ of the residual nucleus from momentum conservation.

$$\vec{p}_{n\,i} = \vec{p}_e + \vec{p}_\nu + \vec{p}_{n\,r} \quad\Rightarrow\quad 0 = (-1.2\times10^{-22}\ \text{kg}\cdot\text{m/s})\,\hat{\text{i}} + (-6.4\times10^{-23}\ \text{kg}\cdot\text{m/s})\,\hat{\text{j}} + \vec{p}_{n\,r}$$

Thus, $\vec{p}_{n\,r} = (1.2\times10^{-22}\,\text{kg}\cdot\text{m/s})\,\hat{\text{i}} + (6.4\times10^{-23}\,\text{kg}\cdot\text{m/s})\,\hat{\text{j}}$. Its magnitude is

$$|\,\vec{p}_{n\,r}\,| = \sqrt{\left(1.2\times10^{-22}\ \text{kg}\cdot\text{m/s}\right)^2 + \left(6.4\times10^{-23}\ \text{kg}\cdot\text{m/s}\right)^2} = 1.4\times10^{-22}\ \text{kg}\cdot\text{m/s}.$$

(b) The angle measured from the +x axis to $\vec{p}_{n\,r}$ is

$$\theta = \tan^{-1}\left(\frac{6.4\times10^{-23}\ \text{kg}\cdot\text{m/s}}{1.2\times10^{-22}\ \text{kg}\cdot\text{m/s}}\right) = 28°.$$

(c) Combining the two equations $p = mv$ and $K = \frac{1}{2}mv^2$, we obtain (with $p = p_{n\,r}$ and $m = m_{n\,r}$)

$$K = \frac{p^2}{2m} = \frac{\left(1.4\times10^{-22}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(5.8\times10^{-26}\ \text{kg}\right)} = 1.6\times10^{-19}\ \text{J}.$$

91. No external forces with horizontal components act on the cart-man system and the vertical forces sum to zero, so the total momentum of the system is conserved. Let m_c be the mass of the cart, v be its initial velocity, and v_c be its final velocity (after the man jumps off). Let m_m be the mass of the man. His initial velocity is the same as that of the cart and his final velocity is zero. Conservation of momentum yields $(m_m + m_c)v = m_c v_c$. Consequently, the final speed of the cart is

$$v_c = \frac{v(m_m + m_c)}{m_c} = \frac{(2.3\ \text{m/s})(75\ \text{kg} + 39\ \text{kg})}{39\ \text{kg}} = 6.7\ \text{m/s}.$$

The cart speeds up by 6.7 m/s – 2.3 m/s = + 4.4 m/s. In order to slow himself, the man gets the cart to push backward on him by pushing forward on it, so the cart speeds up.

92. The fact that they are connected by a spring is not used in the solution. We use Eq. 9-17 for $\vec{v}_{\text{com}}$:

$$M\vec{v}_{\text{com}} = m_1\vec{v}_1 + m_2\vec{v}_2 = (1.0\ \text{kg})(1.7\ \text{m/s}) + (3.0\ \text{kg})\vec{v}_2$$

which yields $|\vec{v}_2| = 0.57\ \text{m/s}$. The direction of $\vec{v}_2$ is opposite that of $\vec{v}_1$ (that is, they are both headed toward the center of mass, but from opposite directions).

93. **THINK** A completely inelastic collision means that the railroad freight car and the caboose car move together after the collision. The motion is one-dimensional.

EXPRESS Let m_F be the mass of the freight car and v_F be its initial velocity. Let m_C be the mass of the caboose and v be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to

$$m_F v_F = (m_F + m_C)v \;\Rightarrow\; v = \frac{m_F v_F}{m_F + m_C}.$$

The initial kinetic energy of the system is $K_i = \frac{1}{2} m_F v_F^2$ and the final kinetic energy is

$$K_f = \frac{1}{2}(m_F + m_C)v^2 = \frac{1}{2}(m_F + m_C)\frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)}.$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73K_i$. Combining with the two equations above allows us to solve for m_C, the mass of the caboose.

ANALYZE With $K_f = 0.73K_i$, or

$$\frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73)\left(\frac{1}{2} m_F v_F^2\right)$$

we obtain $m_F/(m_F + m_C) = 0.73,$ which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73} m_F = 0.37 m_F = (0.37)(3.18 \times 10^4 \text{ kg}) = 1.18 \times 10^4 \text{ kg}.$$

LEARN Energy is lost during an inelastic collision, but momentum is still conserved because there's no external force acting on the two-car system.

94. Let m_c be the mass of the Chrysler and v_c be its velocity. Let m_f be the mass of the Ford and v_f be its velocity. Then the velocity of the center of mass is

$$v_{\text{com}} = \frac{m_c v_c + m_f v_f}{m_c + m_f} = \frac{(2400 \text{ kg})(80 \text{ km/h}) + (1600 \text{ kg})(60 \text{ km/h})}{2400 \text{ kg} + 1600 \text{ kg}} = 72 \text{ km/h}.$$

We note that the two velocities are in the same direction, so the two terms in the numerator have the same sign.

95. **THINK** A billiard ball undergoes glancing collision with another identical billiard ball. The collision is two-dimensional.

EXPRESS The mass of each ball is *m*, and the initial speed of one of the balls is $v_{1i} = 2.2\,\mathrm{m/s}$. We apply the conservation of linear momentum to the *x* and *y* axes respectively:

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$
$$0 = mv_{1f}\sin\theta_1 - mv_{2f}\sin\theta_2$$

The mass *m* cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

ANALYZE (a) Solving the simultaneous equations leads to

$$v_{1f} = \frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}v_{1i}, \quad v_{2f} = \frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}v_{1i}$$

Since $v_{2f} = v_{1i}/2 = 1.1\,\mathrm{m/s}$ and $\theta_2 = 60°$, we have

$$\frac{\sin\theta_1}{\sin(\theta_1+60°)} = \frac{1}{2} \quad \Rightarrow \quad \tan\theta_1 = \frac{1}{\sqrt{3}}$$

or $\theta_1 = 30°$. Thus, the speed of ball 1 after collision is

$$v_{1f} = \frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}v_{1i} = \frac{\sin 60°}{\sin(30°+60°)}v_{1i} = \frac{\sqrt{3}}{2}v_{1i} = \frac{\sqrt{3}}{2}(2.2\,\mathrm{m/s}) = 1.9\,\mathrm{m/s}\,.$$

(b) From the above, we have θ_1 = 30°, measured *clockwise* from the +*x*-axis, or equivalently, –30°, measured *counterclockwise* from the +*x*-axis.

(c) The kinetic energy before collision is $K_i = \frac{1}{2}mv_{1i}^2$. After the collision, we have

$$K_f = \frac{1}{2}m\left(v_{1f}^2 + v_{2f}^2\right)$$

Substituting the expressions for v_{1f} and v_{2f} found above gives

$$K_f = \frac{1}{2}m\left[\frac{\sin^2\theta_2}{\sin^2(\theta_1+\theta_2)} + \frac{\sin^2\theta_1}{\sin^2(\theta_1+\theta_2)}\right]v_{1i}^2$$

Since $\theta_1 = 30°$ and $\theta_2 = 60°$, $\sin(\theta_1+\theta_2) = 1$ and $\sin^2\theta_1 + \sin^2\theta_2 = \sin^2\theta_1 + \cos^2\theta_1 = 1$, and indeed, we have $K_f = \frac{1}{2}mv_{1i}^2 = K_i$, which means that energy is conserved.

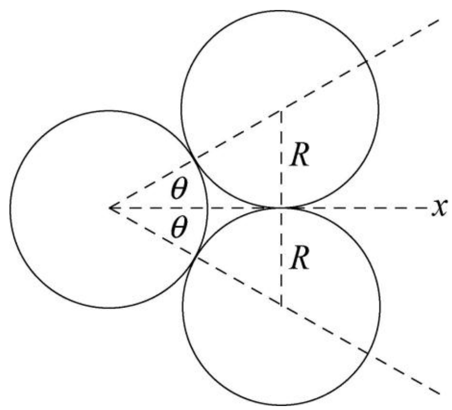
LEARN One may verify that when two identical masses collide elastically, they will move off perpendicularly to each other with $\theta_1 + \theta_2 = 90°$.

96. (a) We use Eq. 9-87. The thrust is

$$Rv_{\text{rel}} = Ma = \left(4.0\times10^4\ \text{kg}\right)\left(2.0\,\text{m/s}^2\right) = 8.0\times10^4\ \text{N}.$$

(b) Since v_{rel} = 3000 m/s, we see from part (a) that $R \approx 27$ kg/s.

97. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two.



It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the x axis. The three dashed lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked θ are 30°. Let v_0 be the velocity of the incident ball before the collision and V be its velocity afterward. The two target balls leave the collision with the same speed. Let v represent that speed. Each ball has mass m. Since the x component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta$$

and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right).$$

We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum equation gives $V = v_0 - 2v\cos\theta$, so

$$V^2 = v_0^2 - 4v_0v\cos\theta + 4v^2\cos^2\theta$$

and the energy equation becomes $v_0^2 = v_0^2 - 4v_0 v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1+2\cos^2\theta} = \frac{2(10\ \text{m/s})\cos 30°}{1+2\cos^2 30°} = 6.93\ \text{m/s}.$$

(a) The discussion and computation above determines the final speed of ball 2 (as labeled in Fig. 9-76) to be 6.9 m/s.

(b) The direction of ball 2 is at 30° counterclockwise from the +x axis.

(c) Similarly, the final speed of ball 3 is 6.9 m/s.

(d) The direction of ball 3 is at –30° counterclockwise from the +x axis.

(e) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10\ \text{m/s} - 2(6.93\ \text{m/s})\ \cos 30° = -2.0\,\text{m/s}.$$

So the speed of ball 1 is $|V| = 2.0$ m/s.

(f) The minus sign indicates that it bounces back in the – x direction. The angle is –180°.

98. (a) The momentum change for the 0.15 kg object is

$$\Delta\vec{p} = (0.15)[2\,\hat{\text{i}} + 3.5\,\hat{\text{j}} - 3.2\,\hat{\text{k}} - (5\,\hat{\text{i}} + 6.5\,\hat{\text{j}} + 4\,\hat{\text{k}})] = (-0.450\hat{\text{i}} - 0.450\hat{\text{j}} - 1.08\hat{\text{k}})\ \text{kg}\cdot\text{m/s}.$$

(b) By the impulse-momentum theorem (Eq. 9-31), $\vec{J} = \Delta\vec{p}$, we have

$$\vec{J} = (-0.450\hat{\text{i}} - 0.450\hat{\text{j}} - 1.08\hat{\text{k}})\ \text{N}\cdot\text{s}.$$

(c) Newton's third law implies $\vec{J}_{\text{wall}} = -\vec{J}_{\text{ball}}$ (where $\vec{J}_{\text{ball}}$ is the result of part (b)), so

$$\vec{J}_{\text{wall}} = (0.450\hat{\text{i}} + 0.450\hat{\text{j}} + 1.08\hat{\text{k}})\ \text{N}\cdot\text{s}.$$

99. (a) We place the origin of a coordinate system at the center of the pulley, with the x axis horizontal and to the right and with the y axis downward. The center of mass is halfway between the containers, at $x = 0$ and $y = \ell$, where ℓ is the vertical distance from the pulley center to either of the containers. Since the diameter of the pulley is 50 mm, the center of mass is at a horizontal distance of 25 mm from each container.

(b) Suppose 20 g is transferred from the container on the left to the container on the right. The container on the left has mass m_1 = 480 g and is at x_1 = –25 mm. The container on the right has mass m_2 = 520 g and is at x_2 = +25 mm. The x coordinate of the center of mass is then

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(480 \text{ g})(-25 \text{ mm}) + (520 \text{ g})(25 \text{ mm})}{480 \text{ g} + 520 \text{ g}} = 1.0 \text{ mm}.$$

The y coordinate is still ℓ. The center of mass is 26 mm from the lighter container, along the line that joins the bodies.

(c) When they are released the heavier container moves downward and the lighter container moves upward, so the center of mass, which must remain closer to the heavier container, moves downward.

(d) Because the containers are connected by the string, which runs over the pulley, their accelerations have the same magnitude but are in opposite directions. If a is the acceleration of m_2, then $-a$ is the acceleration of m_1. The acceleration of the center of mass is

$$a_{\text{com}} = \frac{m_1(-a) + m_2 a}{m_1 + m_2} = a\,\frac{m_2 - m_1}{m_1 + m_2}.$$

We must resort to Newton's second law to find the acceleration of each container. The force of gravity $m_1 g$, down, and the tension force of the string T, up, act on the lighter container. The second law for it is $m_1 g - T = -m_1 a$. The negative sign appears because a is the acceleration of the heavier container. The same forces act on the heavier container and for it the second law is $m_2 g - T = m_2 a$. The first equation gives $T = m_1 g + m_1 a$. This is substituted into the second equation to obtain $m_2 g - m_1 g - m_1 a = m_2 a$, so

$$a = (m_2 - m_1)g/(m_1 + m_2).$$

Thus,

$$a_{\text{com}} = \frac{g(m_2 - m_1)^2}{(m_1 + m_2)^2} = \frac{(9.8 \text{ m/s}^2)(520 \text{ g} - 480 \text{ g})^2}{(480 \text{ g} + 520 \text{ g})^2} = 1.6 \times 10^{-2} \text{ m/s}^2.$$

The acceleration is downward.

100. (a) We use Fig. 9-21 of the text (which treats both angles as positive-valued, even though one of them is in the fourth quadrant; this is why there is an explicit minus sign in Eq. 9-80 as opposed to it being implicitly in the angle). We take the cue ball to be body 1 and the other ball to be body 2. Conservation of the x and the components of the total momentum of the two-ball system leads to:

$$mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$

$$0 = -mv_{1f}\sin\theta_1 + mv_{2f}\sin\theta_2.$$

The masses are the same and cancel from the equations. We solve the second equation for $\sin\theta_2$:

$$\sin\theta_2 = \frac{v_{1f}}{v_{2f}}\sin\theta_1 = \left(\frac{3.50 \text{ m/s}}{2.00 \text{ m/s}}\right)\sin 22.0° = 0.656 \text{ .}$$

Consequently, the angle between the second ball and the initial direction of the first is θ_2 = 41.0°.

(b) We solve the first momentum conservation equation for the initial speed of the cue ball.

$$v_{1i} = v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 = (3.50 \text{ m/s})\cos 22.0° + (2.00 \text{ m/s})\cos 41.0° = 4.75 \text{ m/s .}$$

(c) With SI units understood, the initial kinetic energy is

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}m(4.75)^2 = 11.3m$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left((3.50)^2 + (2.00)^2\right) = 8.1m.$$

Kinetic energy is not conserved.

101. This is a completely inelastic collision, followed by projectile motion. In the collision, we use momentum conservation.

$$\vec{p}_{\text{shoes}} = \vec{p}_{\text{together}} \quad \Rightarrow \quad (3.2 \text{ kg})(3.0 \text{ m/s}) = (5.2 \text{ kg})\vec{v}$$

Therefore, $\vec{v} = 1.8 \text{ m/s}$ toward the right as the combined system is projected from the edge of the table. Next, we can use the projectile motion material from Ch. 4 or the energy techniques of Ch. 8; we choose the latter.

$$K_{\text{edge}} + U_{\text{edge}} = K_{\text{floor}} + U_{\text{floor}}$$

$$\frac{1}{2}(5.2 \text{ kg})(1.8 \text{ m/s})^2 + (5.2 \text{ kg})(9.8 \text{ m/s}^2)(0.40 \text{ m}) = K_{\text{floor}} + 0$$

Therefore, the kinetic energy of the system right before hitting the floor is K_{floor} = 29 J.

102. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed u relative to the ground as the man climbs up the ladder.

(b) The speed of the man relative to the ground is $v_g = v - u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{mv_g - Mu}{M+m} = \frac{m(v-u) - Mu}{M+m} = 0.$$

This yields

$$u = \frac{mv}{M+m} = \frac{(80\text{ kg})(2.5\text{ m/s})}{320\text{ kg} + 80\text{ kg}} = 0.50\text{ m/s}.$$

(c) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to v_{com}, which is zero. So the balloon will again be stationary.

103. The velocities of m_1 and m_2 just after the collision with each other are given by Eq. 9-75 and Eq. 9-76 (setting $v_{1i} = 0$):

$$v_{1f} = \frac{2m_2}{m_1+m_2}v_{2i}, \quad v_{2f} = \frac{m_2-m_1}{m_1+m_2}v_{2i}$$

After bouncing off the wall, the velocity of m_2 becomes $-v_{2f}$. In these terms, the problem requires $v_{1f} = -v_{2f}$, or

$$\frac{2m_2}{m_1+m_2}v_{2i} = -\frac{m_2-m_1}{m_1+m_2}v_{2i}$$

which simplifies to

$$2m_2 = -(m_2 - m_1) \Rightarrow m_2 = \frac{m_1}{3}\ .$$

With m_1 = 6.6 kg, we have m_2 = 2.2 kg.

104. We treat the car (of mass m_1) as a "point-mass" (which is initially 1.5 m from the right end of the boat). The left end of the boat (of mass m_2) is initially at x = 0 (where the dock is), and its left end is at x = 14 m. The boat's center of mass (in the absence of the car) is initially at x = 7.0 m. We use Eq. 9-5 to calculate the center of mass of the system:

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500\text{ kg})(14\text{ m} - 1.5\text{ m}) + (4000\text{ kg})(7\text{ m})}{1500\text{ kg} + 4000\text{ kg}} = 8.5\text{ m}.$$

In the absence of *external* forces, the center of mass of the system does not change. Later, when the car (about to make the jump) is near the left end of the boat (which has moved from the shore an amount δx), the value of the system center of mass is still 8.5 m. The car (at this moment) is thought of as a "point-mass" 1.5 m from the left end, so we must have

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(1500\text{ kg})(\delta x + 1.5\text{ m}) + (4000\text{ kg})(7\text{ m} + \delta x)}{1500\text{ kg} + 4000\text{ kg}} = 8.5\text{ m}.$$

Solving this for δx, we find δx = 3.0 m.

105. **THINK** Both momentum and energy are conserved during an elastic collision.

EXPRESS Let m_1 be the mass of the object that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let $m_2 = M$ be the mass of the object that is originally at rest and v_{2f} be its velocity after the collision. Conservation of linear momentum gives $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$. Similarly, the total kinetic energy is conserved and we have

$$\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 .$$

Solving for v_{1f} and v_{2f}, we obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

The second equation can be inverted to give $m_2 = m_1\left(\frac{2v_{1i}}{v_{2f}} - 1\right)$.

ANALYZE With m_1 = 3.0 kg, v_{1i} = 8.0 m/s and v_{2f} = 6.0 m/s, the above expression leads to

$$m_2 = M = m_1\left(\frac{2v_{1i}}{v_{2f}} - 1\right) = (3.0\text{ kg})\left(\frac{2(8.0\text{ m/s})}{6.0\text{ m/s}} - 1\right) = 5.0\text{ kg}$$

LEARN Our analytic expression for m_2 shows that if the two masses are equal, then $v_{2f} = v_{1i}$, and the pool player's result is recovered.

106. We denote the mass of the car as M and that of the sumo wrestler as m. Let the initial velocity of the sumo wrestler be $v_0 > 0$ and the final velocity of the car be v. We apply the momentum conservation law.

(a) From $mv_0 = (M + m)v$ we get

$$v = \frac{mv_0}{M+m} = \frac{(242\text{ kg})(5.3\text{ m/s})}{2140\text{ kg} + 242\text{ kg}} = 0.54\text{ m/s}.$$

(b) Since $v_{\text{rel}} = v_0$, we have

$$mv_0 = Mv + m(v + v_{\text{rel}}) = mv_0 + (M+m)v,$$

and obtain $v = 0$ for the final speed of the flatcar.

(c) Now $mv_0 = Mv + m(v - v_{\text{rel}})$, which leads to

$$v = \frac{m(v_0 + v_{\text{rel}})}{m+M} = \frac{(242\text{ kg})(5.3\text{ m/s} + 5.3\text{ m/s})}{242\text{ kg} + 2140\text{ kg}} = 1.1\text{ m/s}.$$

107. **THINK** To successfully launch a rocket from the ground, fuel is consumed at a rate that results in a thrust big enough to overcome the gravitational force.

EXPRESS The thrust of the rocket is given by $T = Rv_{\rm rel}$ where R is the rate of fuel consumption and $v_{\rm rel}$ is the speed of the exhaust gas relative to the rocket.

ANALYZE (a) The exhaust speed is $v_{\rm rel}$ = 1200 m/s. For the thrust to equal the weight Mg where M = 6100 kg, we must have

$$T = Rv_{\rm rel} = Mg \quad \Rightarrow \quad R = \frac{Mg}{v_{\rm rel}} = \frac{(6100\,\text{kg})(9.8\,\text{m/s}^2)}{1200\,\text{m/s}} = 49.8\,\text{kg/s} \approx 50\,\text{kg/s}\,.$$

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{\rm rel} - Mg = Ma$$

so that requiring a = 21 m/s^2 leads to

$$R = \frac{M(g+a)}{v_{\rm rel}} = \frac{(6100\,\text{kg})(9.8\,\text{m/s}^2 + 21\,\text{m/s}^2)}{1200\,\text{m/s}} = 156.6\,\text{kg/s} \approx 1.6\times 10^2\ \text{kg/s}\,.$$

LEARN A greater upward acceleration requires a greater fuel consumption rate. To be launched from Earth's surface, the initial acceleration of the rocket must exceed $g = 9.8\,\text{m/s}^2$. This means that the rate R must be greater than 50 kg/s.

108. Conservation of momentum leads to

$$(900\ \text{kg})(1000\ \text{m/s}) = (500\ \text{kg})(v_{\rm shuttle} - 100\ \text{m/s}) + (400\ \text{kg})(v_{\rm shuttle})$$

which yields $v_{\rm shuttle}$ = 1055.6 m/s for the shuttle speed and $v_{\rm shuttle}$ – 100 m/s = 955.6 m/s for the module speed (all measured in the frame of reference of the stationary main spaceship). The fractional increase in the kinetic energy is

$$\frac{\Delta K}{K_i} = \frac{K_f}{K_i} - 1 = \frac{(500\ \text{kg})(955.6\ \text{m/s})^2/2 + (400\ \text{kg})(1055.6\ \text{m/s})^2/2}{(900\ \text{kg})(1000\ \text{m/s})^2/2} = 2.5\times 10^{-3}.$$

109. **THINK** In this problem, we are asked to locate the center of mass of the Earth-Moon system.

EXPRESS We locate the coordinate origin at the center of Earth. Then the distance $r_{\rm com}$ of the center of mass of the Earth-Moon system is given by

$$r_{\rm com} = \frac{m_M r_{ME}}{m_M + m_E}$$

where m_M is the mass of the Moon, m_E is the mass of Earth, and r_{ME} is their separation.

ANALYZE (a) With $m_E = 5.98\times10^{24}$ kg , $m_M = 7.36\times10^{22}$ kg and $r_{ME} = 3.82\times10^8$ m (these values are given in Appendix C), we find the center of mass to be at

$$r_{\text{com}} = \frac{(7.36\times10^{22}\ \text{kg})(3.82\times10^8\ \text{m})}{7.36\times10^{22}\ \text{kg}+5.98\times10^{24}\ \text{kg}} = 4.64\times10^6\ \text{m} \approx 4.6\times10^3\ \text{km}.$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6$ m, so $r_{\text{com}} / R_E = 0.73 = 73\%$.

LEARN The center of mass of the Earth-Moon system is located inside the Earth!

110. (a) The magnitude of the impulse is equal to the change in momentum:

$$J = mv - m(-v) = 2mv = 2(0.140\ \text{kg})(7.80\ \text{m/s}) = 2.18\ \text{kg}\cdot\text{m/s}$$

(b) Since in the calculus sense the average of a function is the integral of it divided by the corresponding interval, then the average force is the impulse divided by the time Δt. Thus, our result for the magnitude of the average force is $2mv/\Delta t$. With the given values, we obtain

$$F_{\text{avg}} = \frac{2(0.140\ \text{kg})(7.80\ \text{m/s})}{0.00380\ \text{s}} = 575\ \text{N}\ .$$

111. **THINK** The water added to the sled will move at the same speed as the sled.

EXPRESS Let the mass of the sled be m_s and its initial speed be v_i. If the total mass of water being scooped up is m_w, then by momentum conservation, $m_s v_i = (m_s + m_w)v_f$, where v_f is the final speed of the sled-water system.

ANALYZE With $m_s = 2900\,\text{kg}$, $m_w = 920\,\text{kg}$ and $v_i = 250\,\text{m/s}$, we obtain

$$v_f = \frac{m_s v_i}{m_s + m_w} = \frac{(2900\ \text{kg})(250\ \text{m/s})}{2900\ \text{kg}+920\ \text{kg}} = 189.8\,\text{m/s} \approx 190\,\text{m/s}\ .$$

LEARN The water added to the sled can be regarded as undergoing completely inelastic collision with the sled. Some kinetic energy is converted into other forms of energy (thermal, sound, etc.) and the final speed of the sled-water system is smaller than the initial speed of the sled alone.

112. **THINK** The pellets that were fired carry both kinetic energy and momentum. Force is exerted by the rigid wall in stopping the pellets.

EXPRESS Let *m* be the mass of a pellet and *v* be its velocity as it hits the wall, then its momentum is $p = mv$, toward the wall. The kinetic energy of a pellet is $K = mv^2/2$. The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers p. If ΔN pellets hit in time Δt, then the average rate at which momentum is transferred would be $F_{\text{avg}} = p(\Delta N/\Delta t)$.

ANALYZE (a) With $m = 2.0 \times 10^{-3}$ kg and $v = 500$ m/s, the momentum of a pellet is

$$p = mv = (2.0 \times 10^{-3}\ \text{kg})(500\ \text{m/s}) = 1.0\ \text{kg} \cdot \text{m/s}.$$

(b) The kinetic energy of a pellet is $K = \frac{1}{2}mv^2 = \frac{1}{2}(2.0\times10^{-3}\,\text{kg})(500\,\text{m/s})^2 = 2.5\times10^2\,\text{J}$.

(c) With $(\Delta N/\Delta t) = 10/\text{s}$, the average force on the wall from the stream of pellets is

$$F_{\text{avg}} = p\left(\frac{\Delta N}{\Delta t}\right) = (1.0\,\text{kg}\cdot\text{m/s})(10\,\text{s}^{-1}) = 10\,\text{N}.$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If $\Delta t'$ is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F'_{\text{avg}} = \frac{p}{\Delta t'} = \frac{1.0\,\text{kg}\cdot\text{m/s}}{0.6\times10^{-3}\text{s}} = 1.7\times10^3\,\text{N}.$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. Hence, $F'_{\text{avg}} \neq F_{\text{avg}}$.

LEARN During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

113. We convert mass rate to SI units: R = (540 kg/min)/(60 s/min) = 9.00 kg/s. In the absence of the asked-for additional force, the car would decelerate with a magnitude given by Eq. 9-87: $Rv_{\text{rel}} = M|a|$, so that if a = 0 is desired then the additional force must have a magnitude equal to $R\,v_{\text{rel}}$ (so as to cancel that effect):

$$F = Rv_{\text{rel}} = (9.00\,\text{kg/s})(3.20\,\text{m/s}) = 28.8\,\text{N}.$$

114. First, we imagine that the small square piece (of mass m) that was cut from the large plate is returned to it so that the large plate is again a complete 6 m × 6 m (d =1.0 m) square plate (which has its center of mass at the origin). Then we “add” a square piece of

"negative mass" ($-m$) at the appropriate location to obtain what is shown in the figure. If the mass of the whole plate is M, then the mass of the small square piece cut from it is obtained from a simple ratio of areas:

$$m=\left(\frac{2.0\,\text{m}}{6.0\,\text{m}}\right)^2 M \Rightarrow M=9m.$$

(a) The x coordinate of the small square piece is x = 2.0 m (the middle of that square "gap" in the figure). Thus the x coordinate of the center of mass of the remaining piece is

$$x_{\text{com}}=\frac{(-m)x}{M+(-m)}=\frac{-m(2.0\text{ m})}{9m-m}=-0.25\text{ m}.$$

(b) Since the y coordinate of the small square piece is zero, we have y_{com} = 0.

115. **THINK** We have two forces acting on two masses separately. The masses will move according to Newton's second law.

EXPRESS Let $\vec{F}_1$ be the force acting on m_1, and $\vec{F}_2$ the force acting on m_2. According to Newton's second law, their displacements are

$$\vec{d}_1=\frac{1}{2}\vec{a}_1t^2=\frac{1}{2}\left(\frac{\vec{F}_1}{m_1}\right)t^2,\quad \vec{d}_2=\frac{1}{2}\vec{a}_2t^2=\frac{1}{2}\left(\frac{\vec{F}_2}{m_2}\right)t^2$$

The corresponding displacement of the center of mass is

$$\vec{d}_{\text{cm}}=\frac{m_1\vec{d}_1+m_2\vec{d}_2}{m_1+m_2}=\frac{1}{2}\frac{m_1}{m_1+m_2}\left(\frac{\vec{F}_1}{m_1}\right)t^2+\frac{1}{2}\frac{m_2}{m_1+m_2}\left(\frac{\vec{F}_2}{m_2}\right)t^2=\frac{1}{2}\left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t^2.$$

ANALYZE (a) The two masses are $m_1=2.00\times10^{-3}$ kg and $m_2=4.00\times10^{-3}$ kg. With the forces given by $\vec{F}_1=(-4.00\text{ N})\hat{\text{i}}+(5.00\text{ N})\hat{\text{j}}$ and $\vec{F}_2=(2.00\text{ N})\hat{\text{i}}-(4.00\text{ N})\hat{\text{j}}$, and $t=2.00\times10^{-3}$ s, we obtain

$$\begin{aligned}\vec{d}_{\text{cm}}&=\frac{1}{2}\left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t^2=\frac{1}{2}\frac{(-4.00\text{ N}+2.00\text{ N})\hat{\text{i}}+(5.00\text{ N}-4.00\text{ N})\hat{\text{j}}}{2.00\times10^{-3}\text{ kg}+4.00\times10^{-3}\text{ kg}}(2.00\times10^{-3}\text{ s})^2\\&=(-6.67\times10^{-4}\text{ m})\hat{\text{i}}+(3.33\times10^{-4}\text{ m})\hat{\text{j}}.\end{aligned}$$

The magnitude of $\vec{d}_{\text{cm}}$ is

$$d_{\text{cm}}=\sqrt{(-6.67\times10^{-4}\text{ m})^2+(3.33\times10^{-4}\text{ m})^2}=7.45\times10^{-4}\text{ m}$$

or 0.745 mm.

(b) The angle of $\vec{d}_{\rm cm}$ is given by

$$\theta = \tan^{-1}\left(\frac{3.33\times10^{-4}\ \text{m}}{-6.67\times10^{-4}\ \text{m}}\right) = \tan^{-1}\left(-\frac{1}{2}\right) = 153°,$$

measured counterclockwise from $+x$-axis.

(c) The velocities of the two masses are

$$\vec{v}_1 = \vec{a}_1 t = \frac{\vec{F}_1 t}{m_1}, \quad \vec{v}_2 = \vec{a}_2 t = \frac{\vec{F}_2 t}{m_2},$$

and the velocity of the center of mass is

$$\vec{v}_{\rm cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{m_1}{m_1+m_2}\left(\frac{\vec{F}_1 t}{m_1}\right) + \frac{m_2}{m_1+m_2}\left(\frac{\vec{F}_2 t}{m_2}\right) = \left(\frac{\vec{F}_1+\vec{F}_2}{m_1+m_2}\right)t\,.$$

The corresponding kinetic energy of the center of mass is

$$K_{\rm cm} = \frac{1}{2}(m_1+m_2)v_{\rm cm}^2 = \frac{1}{2}\frac{|\vec{F}_1+\vec{F}_2|^2}{m_1+m_2}t^2$$

With $|\vec{F}_1+\vec{F}_2| = |(-2.00\ \text{N})\hat{\text{i}} + (1.00\ \text{N})\hat{\text{j}}| = \sqrt{5}\ \text{N}$, we get

$$K_{\rm cm} = \frac{1}{2}\frac{|\vec{F}_1+\vec{F}_2|^2}{m_1+m_2}t^2 = \frac{1}{2}\frac{(\sqrt{5}\ \text{N})^2}{2.00\times10^{-3}\ \text{kg} + 4.00\times10^{-3}\ \text{kg}}(2.00\times10^{-3}\ \text{s})^2 = 1.67\times10^{-3}\ \text{J}.$$

LEARN The motion of the center of the mass could be analyzed as though a force $\vec{F} = \vec{F}_1 + \vec{F}_2$ is acting on a mass $M = m_1 + m_2$. Thus, the acceleration of the center of the mass is $\vec{a}_{\rm cm} = \dfrac{\vec{F}_1+\vec{F}_2}{m_1+m_2}$.

116. (a) The center of mass does not move in the absence of external forces (since it was initially at rest).

(b) They collide at their center of mass. If the initial coordinate of P is x = 0 and the initial coordinate of Q is x = 1.0 m, then Eq. 9-5 gives

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{0 + (0.30 \text{ kg})(1.0 \text{ m})}{0.1 \text{ kg} + 0.3 \text{ kg}} = 0.75 \text{ m}.$$

Thus, they collide at a point 0.75 m from *P*'s original position.

117. This is a completely inelastic collision, but Eq. 9-53 ($V = \frac{m_1}{m_1 + m_2} v_{1i}$) is not easily applied since that equation is designed for use when the struck particle is initially stationary. To deal with this case (where particle 2 is already in motion), we return to the principle of momentum conservation:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{V} \quad \Rightarrow \quad \vec{V} = \frac{2(4\hat{\text{i}} - 5\hat{\text{j}}) + 4(6\hat{\text{i}} - 2\hat{\text{j}})}{2 + 4}.$$

(a) In unit-vector notation, then, $\vec{V} = (2.67 \text{ m/s})\hat{\text{i}} + (-3.00 \text{ m/s})\hat{\text{j}}$.

(b) The magnitude of $\vec{V}$ is $|\vec{V}| = 4.01$ m/s.

(c) The direction of $\vec{V}$ is 48.4° (measured *clockwise* from the $+x$ axis).

118. We refer to the discussion in the textbook (Sample Problem – "Elastic collision, two pendulums," which uses the same notation that we use here) for some important details in the reasoning. We choose rightward in Fig. 9-20 as our $+x$ direction. We use the notation $\vec{v}$ when we refer to velocities and v when we refer to speeds (which are necessarily positive). Since the algebra is fairly involved, we find it convenient to introduce the notation $\Delta m = m_2 - m_1$ (which, we note for later reference, is a positive-valued quantity).

(a) Since $\vec{v}_{1i} = +\sqrt{2gh_1}$ where $h_1 = 9.0$ cm, we have

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}$$

which is to say that the *speed* of sphere 1 immediately after the collision is

$$v_{1f} = \left(\Delta m / (m_1 + m_2)\right)\sqrt{2gh_1}$$

and that $\vec{v}_{1f}$ points in the $-x$ direction. This leads (by energy conservation $m_1gh_{1f} = \frac{1}{2}m_1v_{1f}^2$) to

$$h_{1f} = \frac{v_{1f}^2}{2g} = \left(\frac{\Delta m}{m_1 + m_2}\right)^2 h_1 .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{1f} \approx 0.60 \text{ cm}$.

(b) Equation 9-68 gives

$$v_{2f} = \frac{2m_1}{m_1+m_2} v_{1i} = \frac{2m_1}{m_1+m_2} \sqrt{2gh_1}$$

which leads (by energy conservation $m_2 g h_{2f} = \frac{1}{2} m_2 v_{2f}^2$) to

$$h_{2f} = \frac{v_{2f}^2}{2g} = \left(\frac{2m_1}{m_1+m_2}\right)^2 h_1 \ .$$

With m_1 = 50 g and m_2 = 85 g, this becomes $h_{2f} \approx 4.9 \text{ cm}$.

(c) Fortunately, they hit again at the lowest point (as long as their amplitude of swing was "small," this is further discussed in Chapter 16). At the risk of using cumbersome notation, we refer to the *next* set of heights as h_{1ff} and h_{2ff}. At the lowest point (before this second collision) sphere 1 has velocity $+\sqrt{2gh_{1f}}$ (rightward in Fig. 9-20) and sphere 2 has velocity $-\sqrt{2gh_{1f}}$ (that is, it points in the $-x$ direction). Thus, the velocity of sphere 1 immediately after the second collision is, using Eq. 9-75,

$$\begin{aligned}
\vec{v}_{1ff} &= \frac{m_1-m_2}{m_1+m_2}\sqrt{2gh_{1f}} + \frac{2m_2}{m_1+m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{-\Delta m}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) - \frac{2m_2}{m_1+m_2}\left(\frac{2m_1}{m_1+m_2}\sqrt{2gh_1}\right) \\
&= -\frac{(\Delta m)^2 + 4m_1m_2}{(m_1+m_2)^2}\sqrt{2gh_1} \ .
\end{aligned}$$

This can be greatly simplified (by expanding $(\Delta m)^2$ and $(m_1 + m_2)^2$) to arrive at the conclusion that the speed of sphere 1 immediately after the second collision is simply $v_{1ff} = \sqrt{2gh_1}$ and that $\vec{v}_{1ff}$ points in the $-x$ direction. Energy conservation $\left(m_1 g h_{1ff} = \frac{1}{2} m_1 v_{1ff}^2\right)$ leads to

$$h_{1ff} = \frac{v_{1ff}^2}{2g} = h_1 = 9.0 \text{ cm} .$$

(d) One can reason (energy-wise) that h_{1ff} = 0 simply based on what we found in part (c). Still, it might be useful to see how this shakes out of the algebra. Equation 9-76 gives the velocity of sphere 2 immediately after the second collision:

$$v_{2ff} = \frac{2m_1}{m_1+m_2}\sqrt{2gh_{1f}} + \frac{m_2-m_1}{m_1+m_2}\left(-\sqrt{2gh_{2f}}\right)$$
$$= \frac{2m_1}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) + \frac{\Delta m}{m_1+m_2}\left(\frac{-2m_1}{m_1+m_2}\sqrt{2gh_1}\right)$$

which vanishes since $(2m_1)(\Delta m)-(\Delta m)(2m_1)=0$. Thus, the second sphere (after the second collision) stays at the lowest point, which basically recreates the conditions at the start of the problem (so all subsequent swings-and-impacts, neglecting friction, can be easily predicted, as they are just replays of the first two collisions).

119. (a) Each block is assumed to have uniform density, so that the center of mass of each block is at its geometric center (the positions of which are given in the table [see problem statement] at $t = 0$). Plugging these positions (and the block masses) into Eq. 9-29 readily gives $x_{\text{com}} = -0.50$ m (at $t = 0$).

(b) Note that the left edge of block 2 (the middle of which is still at $x = 0$) is at $x = -2.5$ cm, so that at the moment they touch the right edge of block 1 is at $x = -2.5$ cm and thus the middle of block 1 is at $x = -5.5$ cm. Putting these positions (for the middles) and the block masses into Eq. 9-29 leads to $x_{\text{com}} = -1.83$ cm or -0.018 m (at $t = (1.445 \text{ m})/(0.75 \text{ m/s}) = 1.93$ s).

(c) We could figure where the blocks are at $t = 4.0$ s and use Eq. 9-29 again, but it is easier (and provides more insight) to note that in the absence of *external* forces on the system the center of mass should move at constant velocity:

$$\vec{v}_{\text{com}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = 0.25 \text{ m/s } \hat{\text{i}}$$

as can be easily verified by putting in the values at $t = 0$. Thus,

$$x_{\text{com}} = x_{\text{com}\,initial} + \vec{v}_{\text{com}}\,t = (-0.50 \text{ m}) + (0.25 \text{ m/s})(4.0 \text{ s}) = +0.50 \text{ m}.$$

120. One approach is to choose a *moving* coordinate system that travels the center of mass of the body, and another is to do a little extra algebra analyzing it in the original coordinate system (in which the speed of the $m = 8.0$ kg mass is $v_0 = 2$ m/s, as given). Our solution is in terms of the latter approach since we are assuming that this is the approach most students would take. Conservation of linear momentum (along the direction of motion) requires

$$mv_0 = m_1v_1 + m_2v_2 \quad \Rightarrow \quad (8.0)(2.0) = (4.0)v_1 + (4.0)v_2$$

which leads to $v_2 = 4 - v_1$ in SI units (m/s). We require

$$\Delta K = \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}mv_0^2 \quad \Rightarrow \quad 16 = \left(\frac{1}{2}(4.0)v_1^2 + \frac{1}{2}(4.0)v_2^2\right) - \frac{1}{2}(8.0)(2.0)^2$$

which simplifies to $v_2^2 = 16 - v_1^2$ in SI units. If we substitute for v_2 from above, we find

$$(4 - v_1)^2 = 16 - v_1^2$$

which simplifies to $2v_1^2 - 8v_1 = 0$, and yields either $v_1 = 0$ or $v_1 = 4$ m/s. If $v_1 = 0$ then $v_2 = 4 - v_1 = 4$ m/s, and if $v_1 = 4$ m/s then $v_2 = 0$.

(a) Since the forward part continues to move in the original direction of motion, the speed of the rear part must be zero.

(b) The forward part has a velocity of 4.0 m/s along the original direction of motion.

121. We use m_1 for the mass of the electron and $m_2 = 1840m_1$ for the mass of the hydrogen atom. Using Eq. 9-68,

$$v_{2f} = \frac{2m_1}{m_1 + 1840m_1} v_{1i} = \frac{2}{1841} v_{1i}$$

we compute the final kinetic energy of the hydrogen atom:

$$K_{2f} = \frac{1}{2}(1840m_1)\left(\frac{2v_{1i}}{1841}\right)^2 = \frac{(1840)(4)}{1841^2}\left(\frac{1}{2}(1840m_1)v_{1i}^2\right)$$

so we find the fraction to be $(1840)(4)/1841^2 \approx 2.2 \times 10^{-3}$, or 0.22%.

122. Denoting the new speed of the car as v, then the new speed of the man relative to the ground is $v - v_{\text{rel}}$. Conservation of momentum requires

$$\left(\frac{W}{g} + \frac{w}{g}\right)v_0 = \left(\frac{W}{g}\right)v + \left(\frac{w}{g}\right)(v - v_{\text{rel}}).$$

Consequently, the change of velocity is

$$\Delta\vec{v} = v - v_0 = \frac{w\, v_{\text{rel}}}{W + w} = \frac{(915 \text{ N})(4.00 \text{ m/s})}{(2415 \text{ N}) + (915 \text{ N})} = 1.10 \text{ m/s}.$$

123. Conservation of linear momentum gives $mv + MV_J = mv_f + MV_{Jf}$. Similarly, the total kinetic energy is conserved:

$$\frac{1}{2}mv^2+\frac{1}{2}MV_J^2=\frac{1}{2}mv_f^2+\frac{1}{2}MV_{J\,f}^2\ .$$

Solving for v_f and $V_{J\,f}$, we obtain:

$$v_{1f}=\frac{m-M}{m+M}v+\frac{2M}{m+M}V_J,\quad V_{J\,f}=\frac{2m}{m+M}v+\frac{M-m}{m+M}V_J$$

Since $m\ll M,$ the above expressions can be simplified to

$$v_{1f}\approx -v+2V_J,\quad V_{J\,f}\approx V_J$$

The velocity of the probe relative to the Sun is

$$v_{1f}\approx -v+2V_J=-(10.5\text{ km/s})+2(-13.0\text{ km/s})=-36.5\text{ km/s}\ .$$

The speed is $|v_{1f}|=36.5\text{ km/s}.$

124. (a) The change in momentum (taking upwards to be the positive direction) is

$$\Delta\vec{p}\ =\ (0.550\text{ kg})[\ (3\text{ m/s})\hat{\mathrm{j}}-(-12\text{ m/s})\hat{\mathrm{j}}\ ]=\ (+8.25\text{ kg}\cdot\text{m/s})\,\hat{\mathrm{j}}\ .$$

(b) By the impulse-momentum theorem (Eq. 9-31) $\vec{J}=\Delta\vec{p}=\ (+8.25\text{ N}\cdot\text{s})\,\hat{\mathrm{j}}$.

(c) By Newton's third law, $\vec{J}_\mathrm{c}\ =\ -\vec{J}_\mathrm{b}\ =(-8.25\text{ N}\cdot\text{s})\,\hat{\mathrm{j}}$.

125. (a) Since the initial momentum is zero, then the final momenta must add (in the vector sense) to 0. Therefore, with SI units understood, we have

$$\begin{aligned}\vec{p}_3&=-\vec{p}_1-\vec{p}_2=-m_1\vec{v}_1-m_2\vec{v}_2\\&=-\left(16.7\times10^{-27}\right)\left(6.00\times10^6\,\hat{\mathrm{i}}\right)-\left(8.35\times10^{-27}\right)\left(-8.00\times10^6\,\hat{\mathrm{j}}\right)\\&=\left(-1.00\times10^{-19}\,\hat{\mathrm{i}}+0.67\times10^{-19}\,\hat{\mathrm{j}}\right)\text{kg}\cdot\text{m/s}.\end{aligned}$$

(b) Dividing by m_3 = 11.7 × 10^{-27} kg and using the Pythagorean theorem we find the speed of the third particle to be v_3 = 1.03 × 10^7 m/s. The total amount of kinetic energy is

$$\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{1}{2}m_3v_3^2=1.19\times10^{-12}\ \text{J}.$$

126. Using Eq. 9-67, we have after the elastic collision

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = \frac{-200\text{ g}}{600\text{ g}} v_{1i} = -\tfrac{1}{3}(3.00\text{ m/s}) = -\,1.00\text{ m/s}\,.$$

(a) The impulse is therefore

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200\text{ kg})(-1.00\text{ m/s}) - (0.200\text{ kg})(3.00\text{ m/s}) = -\,0.800\text{ N}\cdot\text{s}$$
$$= -\,0.800\text{ kg}\cdot\text{m/s},$$

or $|J| = -0.800$ kg·m/s.

(b) For the completely inelastic collision Eq. 9-75 applies

$$v_{1f} = V = \frac{m_1}{m_1 + m_2} v_{1i} = +\,1.00\text{ m/s}\,.$$

Now the impulse is

$$J = m_1 v_{1f} - m_1 v_{1i} = (0.200\text{ kg})(1.00\text{ m/s}) - (0.200\text{ kg})(3.00\text{ m/s}) = 0.400\text{ N}\cdot\text{s}$$
$$= 0.400\text{ kg}\cdot\text{m/s}.$$

127. We use Eq. 9-88 and simplify with $v_f - v_i = \Delta v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) \Rightarrow \frac{M_f}{M_i} = e^{-\Delta v/u}$$

If $\Delta v = 2.2$ m/s and $u = 1000$ m/s, we obtain $\dfrac{M_i - M_f}{M_i} = 1 - e^{-0.0022} \approx 0.0022.$

128. Using the linear momentum-impulse theorem, we have

$$J = F_{\text{avg}}\Delta t = \Delta p = m(v_f - v_i)\,.$$

where m is the mass, v_i the initial velocity, and v_f the final velocity of the ball. With $v_i = 0$, we obtain

$$v_f = \frac{F_{\text{avg}}\Delta t}{m} = \frac{(32\text{ N})(14\times 10^{-3}\text{s})}{0.20\,\text{kg}} = 2.24\,\text{m/s}.$$

Chapter 10

1. The problem asks us to assume v_{com} and ω are constant. For consistency of units, we write

$$v_{\text{com}} = (85\,\text{mi/h})\left(\frac{5280\,\text{ft/mi}}{60\,\text{min/h}}\right) = 7480\,\text{ft/min}\ .$$

Thus, with $\Delta x = 60\,\text{ft}$, the time of flight is

$$t = \Delta x / v_{\text{com}} = (60\ \text{ft})/(7480\ \text{ft/min}) = 0.00802\,\text{min}\,.$$

During that time, the angular displacement of a point on the ball's surface is

$$\theta = \omega t = (1800\,\text{rev/min})(0.00802\,\text{min}) \approx 14\,\text{rev}\ .$$

2. (a) The second hand of the smoothly running watch turns through 2π radians during $60\ \text{s}$. Thus,

$$\omega = \frac{2\pi}{60} = 0.105\ \text{rad/s.}$$

(b) The minute hand of the smoothly running watch turns through 2π radians during $3600\ \text{s}$. Thus,

$$\omega = \frac{2\pi}{3600} = 1.75 \times 10^{-3}\ \text{rad / s.}$$

(c) The hour hand of the smoothly running 12-hour watch turns through 2π radians during 43200 s. Thus,

$$\omega = \frac{2\pi}{43200} = 1.45 \times 10^{-4}\ \text{rad / s.}$$

3. The falling is the type of constant-acceleration motion you had in Chapter 2. The time it takes for the buttered toast to hit the floor is

$$\Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.76\ \text{m})}{9.8\ \text{m/s}^2}} = 0.394\,\text{s.}$$

(a) The smallest angle turned for the toast to land butter-side down is $\Delta\theta_{\text{min}} = 0.25\ \text{rev} = \pi/2\ \text{rad}.$ This corresponds to an angular speed of

$$\omega_{\min} = \frac{\Delta\theta_{\min}}{\Delta t} = \frac{\pi/2 \text{ rad}}{0.394 \text{ s}} = 4.0 \text{ rad/s.}$$

(b) The largest angle (less than 1 revolution) turned for the toast to land butter-side down is $\Delta\theta_{\max} = 0.75 \text{ rev} = 3\pi/2 \text{ rad.}$ This corresponds to an angular speed of

$$\omega_{\max} = \frac{\Delta\theta_{\max}}{\Delta t} = \frac{3\pi/2 \text{ rad}}{0.394 \text{ s}} = 12.0 \text{ rad/s.}$$

4. If we make the units explicit, the function is

$$\theta = 2.0 \text{ rad} + \left(4.0 \text{ rad/s}^2\right)t^2 + \left(2.0 \text{ rad/s}^3\right)t^3$$

but in some places we will proceed as indicated in the problem—by letting these units be understood.

(a) We evaluate the function θ at t = 0 to obtain θ_0 = 2.0 rad.

(b) The angular velocity as a function of time is given by Eq. 10-6:

$$\omega = \frac{d\theta}{dt} = \left(8.0 \text{ rad/s}^2\right)t + \left(6.0 \text{ rad/s}^3\right)t^2$$

which we evaluate at t = 0 to obtain ω_0 = 0.

(c) For t = 4.0 s, the function found in the previous part is

$$\omega_4 = (8.0)(4.0) + (6.0)(4.0)^2 = 128 \text{ rad/s.}$$

If we round this to two figures, we obtain $\omega_4 \approx 1.3\times10^2$ rad/s.

(d) The angular acceleration as a function of time is given by Eq. 10-8:

$$\alpha = \frac{d\omega}{dt} = 8.0 \text{ rad/s}^2 + \left(12 \text{ rad/s}^3\right)t$$

which yields α_2 = 8.0 + (12)(2.0) = 32 rad/s^2 at t = 2.0 s.

(e) The angular acceleration, given by the function obtained in the previous part, depends on time; it is not constant.

5. Applying Eq. 2-15 to the vertical axis (with +y downward) we obtain the free-fall time:

$$\Delta y = v_{0y} t + \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2(10 \text{ m})}{9.8 \text{ m/s}^2}} = 1.4 \text{ s}.$$

Thus, by Eq. 10-5, the magnitude of the average angular velocity is

$$\omega_{\text{avg}} = \frac{(2.5 \text{ rev})(2\pi \text{ rad/rev})}{1.4 \text{ s}} = 11 \text{ rad/s}.$$

6. If we make the units explicit, the function is

$$\theta = (4.0 \text{ rad/s})t - (3.0 \text{ rad/s}^2)t^2 + (1.0 \text{ rad/s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Equation 10-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2.$$

Evaluating this at t = 2 s yields ω_2 = 4.0 rad/s.

(b) Evaluating the expression in part (a) at t = 4 s gives ω_4 = 28 rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t.$$

Evaluating this at t = 2 s produces α_2 = 6.0 rad/s^2.

(e) Evaluating the expression in part (d) at t = 4 s yields α_4 = 18 rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

7. (a) To avoid touching the spokes, the arrow must go through the wheel in not more than

$$\Delta t = \frac{1/8 \text{ rev}}{2.5 \text{ rev/s}} = 0.050 \text{ s}.$$

The minimum speed of the arrow is then $v_{\min} = \dfrac{20 \text{ cm}}{0.050 \text{ s}} = 400 \text{ cm/s} = 4.0 \text{ m/s}.$

(b) No—there is no dependence on radial position in the above computation.

8. (a) We integrate (with respect to time) the $\alpha = 6.0t^4 - 4.0t^2$ expression, taking into account that the initial angular velocity is 2.0 rad/s. The result is

$$\omega = 1.2\, t^5 - 1.33\, t^3 + 2.0.$$

(b) Integrating again (and keeping in mind that $\theta_o = 1$) we get

$$\theta = 0.20t^6 - 0.33\, t^4 + 2.0\, t + 1.0\ .$$

9. (a) With $\omega = 0$ and $\alpha = -4.2 \text{ rad/s}^2$, Eq. 10-12 yields $t = -\omega_o/\alpha = 3.00$ s.

(b) Eq. 10-4 gives $\theta - \theta_o = -\,\omega_o^{\,2} / 2\alpha = 18.9$ rad.

10. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

(a) The angular acceleration satisfies Eq. 10-13:

$$25 \text{ rad} = \frac{1}{2}\alpha (5.0 \text{ s})^2 \;\;\Rightarrow\;\; \alpha = 2.0 \text{ rad/s}^2.$$

(b) The average angular velocity is given by Eq. 10-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25 \text{ rad}}{5.0 \text{ s}} = 5.0 \text{ rad/s}.$$

(c) Using Eq. 10-12, the instantaneous angular velocity at $t = 5.0$ s is

$$\omega = \left(2.0 \text{ rad/s}^2\right)(5.0 \text{ s}) = 10 \text{ rad/s}\ .$$

(d) According to Eq. 10-13, the angular displacement at $t = 10$ s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0 \text{ rad/s}^2)(10 \text{ s})^2 = 100 \text{ rad}.$$

Thus, the displacement between $t = 5$ s and $t = 10$ s is $\Delta\theta = 100$ rad – 25 rad = 75 rad.

11. We assume the sense of initial rotation is positive. Then, with ω_0 = +120 rad/s and ω = 0 (since it stops at time t), our angular acceleration (''deceleration'') will be negative-valued: $\alpha = -4.0$ rad/s^2.

(a) We apply Eq. 10-12 to obtain t.

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{0 - 120 \text{ rad/s}}{-4.0 \text{ rad/s}^2} = 30 \text{ s}.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\,t = \frac{1}{2}(120 \text{ rad/s} + 0)(30 \text{ s}) = 1.8\times 10^3 \text{ rad}.$$

Alternatively, Eq. 10-14 could be used if it is desired to only use the given information (as opposed to using the result from part (a)) in obtaining θ. If using the result of part (a) is acceptable, then any angular equation in Table 10-1 (except Eq. 10-12) can be used to find θ.

12. (a) We assume the sense of rotation is positive. Applying Eq. 10-12, we obtain

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{(3000 - 1200) \text{ rev/min}}{(12/60) \text{ min}} = 9.0\times 10^3 \text{ rev/min}^2.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\,t = \frac{1}{2}(1200 \text{ rev/min} + 3000 \text{ rev/min})\left(\frac{12}{60} \text{ min}\right) = 4.2\times 10^2 \text{ rev}.$$

13. The wheel has angular velocity ω_0 = +1.5 rad/s = +0.239 rev/s at t = 0, and has constant value of angular acceleration $\alpha < 0$, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at t = 0) is θ_1 = +20 rev, and at t_2 its angular displacement is θ_2 = +40 rev and its angular velocity is $\omega_2 = 0$.

(a) We obtain t_2 using Eq. 10-15:

$$\theta_2 = \frac{1}{2}\left(\omega_0 + \omega_2\right)t_2 \quad \Rightarrow \quad t_2 = \frac{2(40 \text{ rev})}{0.239 \text{ rev/s}} = 335 \text{ s}$$

which we round off to $t_2 \approx 3.4\times 10^2$ s.

(b) Any equation in Table 10-1 involving α can be used to find the angular acceleration; we select Eq. 10-16.

$$\theta_2 = \omega_2 t_2 - \frac{1}{2}\alpha t_2^2 \quad \Rightarrow \quad \alpha = -\frac{2(40\text{ rev})}{(335\text{ s})^2} = -7.12\times10^{-4}\text{ rev/s}^2$$

which we convert to $\alpha = -4.5\times10^{-3}$ rad/s^2.

(c) Using $\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$ (Eq. 10-13) and the quadratic formula, we have

$$t_1 = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta_1\alpha}}{\alpha} = \frac{-(0.239\text{ rev/s}) \pm \sqrt{(0.239\text{ rev/s})^2 + 2(20\text{ rev})(-7.12\times10^{-4}\text{ rev/s}^2)}}{-7.12\times10^{-4}\text{ rev/s}^2}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if $t_1 < t_2$ we conclude the correct result is t_1 = 98 s.

14. The wheel starts turning from rest (ω_0 = 0) at t = 0, and accelerates uniformly at $\alpha > 0$, which makes our choice for positive sense of rotation. At t_1 its angular velocity is ω_1 = +10 rev/s, and at t_2 its angular velocity is ω_2 = +15 rev/s. Between t_1 and t_2 it turns through $\Delta\theta$ = 60 rev, where $t_2 - t_1 = \Delta t$.

(a) We find α using Eq. 10-14:

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta \quad \Rightarrow \quad \alpha = \frac{(15\text{ rev/s})^2 - (10\text{ rev/s})^2}{2(60\text{ rev})} = 1.04\text{ rev/s}^2$$

which we round off to 1.0 rev/s^2.

(b) We find Δt using Eq. 10-15: $\Delta\theta = \frac{1}{2}\left(\omega_1 + \omega_2\right)\Delta t \Rightarrow \Delta t = \frac{2(60\text{ rev})}{10\text{ rev/s} + 15\text{ rev/s}} = 4.8\text{ s.}$

(c) We obtain t_1 using Eq. 10-12: $\omega_1 = \omega_0 + \alpha t_1 \Rightarrow t_1 = \frac{10\text{ rev/s}}{1.04\text{ rev/s}^2} = 9.6\text{ s.}$

(d) Any equation in Table 10-1 involving θ can be used to find θ_1 (the angular displacement during $0 \le t \le t_1$); we select Eq. 10-14.

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta_1 \Rightarrow \theta_1 = \frac{(10\text{ rev/s})^2}{2(1.04\text{ rev/s}^2)} = 48\text{ rev.}$$

15. **THINK** We have a wheel rotating with constant angular acceleration. We can apply the equations given in Table 10-1 to analyze the motion.

EXPRESS Since the wheel starts from rest, its angular displacement as a function of time is given by $\theta = \frac{1}{2}\alpha t^2$. We take t_1 to be the start time of the interval so that $t_2 = t_1 + 4.0\text{ s}$. The corresponding angular displacements at these times are

$$\theta_1 = \frac{1}{2}\alpha t_1^2, \quad \theta_2 = \frac{1}{2}\alpha t_2^2$$

Given $\Delta\theta = \theta_2 - \theta_1$, we can solve for t_1, which tells us how long the wheel has been in motion up to the beginning of the 4.0 s-interval.

ANALYZE The above expressions can be combined to give

$$\Delta\theta = \theta_2 - \theta_1 = \frac{1}{2}\alpha\left(t_2^2 - t_1^2\right) = \frac{1}{2}\alpha(t_2 + t_1)(t_2 - t_1)$$

With $\Delta\theta = 120\text{ rad}$, $\alpha = 3.0\text{ rad/s}^2$, and $t_2 - t_1 = 4.0\text{ s}$, we obtain

$$t_2 + t_1 = \frac{2(\Delta\theta)}{\alpha(t_2 - t_1)} = \frac{2(120\text{ rad})}{(3.0\text{ rad/s}^2)(4.0\text{ s})} = 20\text{ s},$$

which can be further solved to give $t_2 = 12.0\text{ s}$ and $t_1 = 8.0\text{ s}$. So, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

LEARN We can readily verify the results by calculating θ_1 and θ_2 explicitly:

$$\theta_1 = \frac{1}{2}\alpha t_1^2 = \frac{1}{2}(3.0\text{ rad/s}^2)(8.0\text{ s})^2 = 96\text{ rad}$$
$$\theta_2 = \frac{1}{2}\alpha t_2^2 = \frac{1}{2}(3.0\text{ rad/s}^2)(12.0\text{ s})^2 = 216\text{ rad}.$$

Indeed the difference is $\Delta\theta = \theta_2 - \theta_1 = 120\text{ rad}$.

16. (a) Eq. 10-13 gives

$$\theta - \theta_o = \omega_o t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}(1.5\text{ rad/s}^2)\,t_1^2$$

where $\theta - \theta_o$ = (2 rev)(2π rad/rev). Therefore, t_1 = 4.09 s.

(b) We can find the time to go through a full 4 rev (using the same equation to solve for a new time t_2) and then subtract the result of part (a) for t_1 in order to find this answer.

$$(4\text{ rev})(2\pi\text{ rad/rev}) = 0 + \tfrac{1}{2}(1.5\text{ rad/s}^2)\,t_2^2 \quad \Rightarrow \quad t_2 = 5.789\text{ s}.$$

Thus, the answer is 5.789 s – 4.093 s ≈ 1.70 s.

17. The problem has (implicitly) specified the positive sense of rotation. The angular acceleration of magnitude 0.25 rad/s^2 in the negative direction is assumed to be constant over a large time interval, including negative values (for t).

(a) We specify θ_{max} with the condition $\omega = 0$ (this is when the wheel reverses from positive rotation to rotation in the negative direction). We obtain θ_{max} using Eq. 10-14:

$$\theta_{max} = -\frac{\omega_o^2}{2\alpha} = -\frac{(4.7 \text{ rad/s})^2}{2(-0.25 \text{ rad/s}^2)} = 44 \text{ rad.}$$

(b) We find values for t_1 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = 22$ rad (or 22.09 rad if we wish to keep track of accurate values in all intermediate steps and only round off on the final answers). Using Eq. 10-13 and the quadratic formula, we have

$$\theta_1 = \omega_o t_1 + \frac{1}{2}\alpha t_1^2 \Rightarrow t_1 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_1\alpha}}{\alpha}$$

which yields the two roots 5.5 s and 32 s. Thus, the first time the reference line will be at $\theta_1 = 22$ rad is $t = 5.5$ s.

(c) The second time the reference line will be at $\theta_1 = 22$ rad is $t = 32$ s.

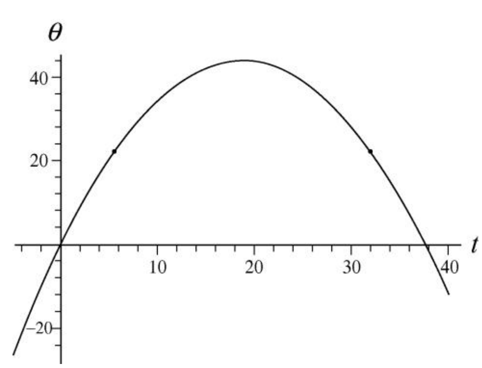
(d) We find values for t_2 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_2 = -10.5$ rad. Using Eq. 10-13 and the quadratic formula, we have

$$\theta_2 = \omega_o t_2 + \frac{1}{2}\alpha t_2^2 \Rightarrow t_2 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_2\alpha}}{\alpha}$$

which yields the two roots –2.1 s and 40 s. Thus, at $t = -2.1$ s the reference line will be at $\theta_2 = -10.5$ rad.

(e) At $t = 40$ s the reference line will be at $\theta_2 = -10.5$ rad.

(f) With radians and seconds understood, the graph of θ versus t is shown below (with the points found in the previous parts indicated as small dots).



18. (a) A complete revolution is an angular displacement of $\Delta\theta = 2\pi$ rad, so the angular velocity in rad/s is given by $\omega = \Delta\theta/T = 2\pi/T$. The angular acceleration is given by

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}.$$

For the pulsar described in the problem, we have

$$\frac{dT}{dt} = \frac{1.26\times 10^{-5}\ \text{s/y}}{3.16\times 10^{7}\ \text{s/y}} = 4.00\times 10^{-13}.$$

Therefore,

$$\alpha = -\left(\frac{2\pi}{(0.033\ \text{s})^2}\right)(4.00\times 10^{-13}) = -2.3\times 10^{-9}\ \text{rad/s}^2.$$

The negative sign indicates that the angular acceleration is opposite the angular velocity and the pulsar is slowing down.

(b) We solve $\omega = \omega_0 + \alpha t$ for the time t when $\omega = 0$:

$$t = -\frac{\omega_0}{\alpha} = -\frac{2\pi}{\alpha T} = -\frac{2\pi}{(-2.3\times 10^{-9}\ \text{rad/s}^2)(0.033\ \text{s})} = 8.3\times 10^{10}\ \text{s} \approx 2.6\times 10^{3}\ \text{ years}$$

(c) The pulsar was born 1992–1054 = 938 years ago. This is equivalent to (938 y)(3.16×10^7 s/y) = 2.96×10^{10} s. Its angular velocity at that time was

$$\omega = \omega_0 + \alpha t + \frac{2\pi}{T} + \alpha t = \frac{2\pi}{0.033\ \text{s}} + (-2.3\times 10^{-9}\ \text{rad/s}^2)(-2.96\times 10^{10}\ \text{s}) = 258\ \text{rad/s}.$$

Its period was

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{258\ \text{rad/s}} = 2.4\times 10^{-2}\ \text{s}.$$

19. (a) Converting from hours to seconds, we find the angular velocity (assuming it is positive) from Eq. 10-18:

$$\omega = \frac{v}{r} = \frac{\left(2.90\times 10^{4}\ \text{km/h}\right)\left(1.000\ \text{h}/3600\ \text{s}\right)}{3.22\times 10^{3}\ \text{km}} = 2.50\times 10^{-3}\ \text{rad/s}.$$

(b) The radial (or centripetal) acceleration is computed according to Eq. 10-23:

$$a_r = \omega^2 r = \left(2.50\times 10^{-3}\ \text{rad/s}\right)^2\left(3.22\times 10^{6}\ \text{m}\right) = 20.2\ \text{m/s}^2.$$

(c) Assuming the angular velocity is constant, then the angular acceleration and the tangential acceleration vanish, since

$$\alpha = \frac{d\omega}{dt} = 0 \text{ and } a_t = r\alpha = 0.$$

20. The function $\theta = \xi e^{\beta t}$ where ξ = 0.40 rad and β = 2 s^{-1} is describing the angular coordinate of a line (which is marked in such a way that all points on it have the same value of angle at a given time) on the object. Taking derivatives with respect to time leads to $\frac{d\theta}{dt} = \xi\beta e^{\beta t}$ and $\frac{d^2\theta}{dt^2} = \xi\beta^2 e^{\beta t}$.

(a) Using Eq. 10-22, we have $a_t = \alpha r = \frac{d^2\theta}{dt^2} r = 6.4 \text{ cm/s}^2$.

(b) Using Eq. 10-23, we get $a_r = \omega^2 r = \left(\frac{d\theta}{dt}\right)^2 r = 2.6 \text{ cm/s}^2$.

21. We assume the given rate of 1.2 × 10^{-3} m/y is the linear speed of the top; it is also possible to interpret it as just the horizontal component of the linear speed but the difference between these interpretations is arguably negligible. Thus, Eq. 10-18 leads to

$$\omega = \frac{1.2 \times 10^{-3} \text{ m/y}}{55 \text{ m}} = 2.18 \times 10^{-5} \text{ rad/y}$$

which we convert (since there are about 3.16 × 10^7 s in a year) to ω = 6.9 × 10^{-13} rad/s.

22. (a) Using Eq. 10-6, the angular velocity at t = 5.0s is

$$\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \left.\frac{d}{dt}\left(0.30t^2\right)\right|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad/s}.$$

(b) Equation 10-18 gives the linear speed at t = 5.0s: $v = \omega r = (3.0 \text{ rad/s})(10 \text{ m}) = 30 \text{ m/s}$.

(c) The angular acceleration is, from Eq. 10-8,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad/s}^2.$$

Then, the tangential acceleration at t = 5.0s is, using Eq. 10-22,

$$a_t = r\alpha = (10 \text{ m})\left(0.60 \text{ rad/s}^2\right) = 6.0 \text{ m/s}^2.$$

(d) The radial (centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = (3.0\ \text{rad/s})^2 (10\ \text{m}) = 90\ \text{m/s}^2.$$

23. **THINK** A positive angular acceleration is required in order to increase the angular speed of the flywheel.

EXPRESS The linear speed of the flywheel is related to its angular speed by $v = \omega r$, where r is the radius of the wheel. As the wheel is accelerated, its angular speed at a later time is $\omega = \omega_0 + \alpha t$.

ANALYZE (a) The angular speed of the wheel, expressed in rad/s, is

$$\omega_0 = \frac{(200\ \text{rev/min})(2\pi\ \text{rad/rev})}{60\ \text{s/min}} = 20.9\ \text{rad/s}.$$

(b) With r = (1.20 m)/2 = 0.60 m, using Eq. 10-18, we find the linear speed to be

$$v = r\omega_0 = (0.60\ \text{m})(20.9\ \text{rad/s}) = 12.5\ \text{m/s}.$$

(c) With t = 1 min, ω = 1000 rev/min and ω_0 = 200 rev/min, Eq. 10-12 gives the required acceleration:

$$\alpha = \frac{\omega - \omega_0}{t} = 800\ \text{rev/min}^2.$$

(d) With the same values used in part (c), Eq. 10-15 becomes

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(200\ \text{rev/min} + 1000\ \text{rev/min})(1.0\ \text{min}) = 600\ \text{rev}.$$

LEARN An alternative way to solve for (d) is to use Eq. 10-13:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + (200\ \text{rev/min})(1.0\ \text{min}) + \frac{1}{2}(800\ \text{rev/min}^2)(1.0\ \text{min})^2 = 600\ \text{rev}.$$

24. Converting $33\frac{1}{3}$ rev/min to radians-per-second, we get ω = 3.49 rad/s. Combining $v = \omega r$ (Eq. 10-18) with $\Delta t = d/v$ where Δt is the time between bumps (a distance d apart), we arrive at the rate of striking bumps:

$$\frac{1}{\Delta t} = \frac{\omega r}{d} \approx 199/\text{s}.$$

25. **THINK** The linear speed of a point on Earth's surface depends on its distance from the Earth's axis of rotation.

EXPRESS To solve for the linear speed, we use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R\cos 40°$, where R is the radius of Earth (6.4×10^6 m). On the other hand, $r = R$ at the equator.

ANALYZE (a) Earth makes one rotation per day and 1 d is (24 h) (3600 s/h) = 8.64×10^4 s, so the angular speed of Earth is

$$\omega = \frac{2\pi\,\text{rad}}{8.64\times10^4\ \text{s}} = 7.3\times10^{-5}\ \text{rad/s}.$$

(b) At latitude of 40°, the linear speed is

$$v = \omega(R\cos 40°) = (7.3\times10^{-5}\ \text{rad/s})(6.4\times10^6\,\text{m})\cos 40° = 3.5\times10^2\ \text{m/s}.$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.3×10^{-5} rad/s).

(d) The latitude at the equator is 0° and the speed is

$$v = \omega R = (7.3\times10^{-5}\ \text{rad/s})(6.4\times10^6\,\text{m}) = 4.6\times10^2\ \text{m/s}.$$

LEARN The linear speed at the poles is zero since $r = R\cos 90° = 0$.

26. (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0-150\ \text{rev/min}}{(2.2\ \text{h})(60\ \text{min/1h})} = -1.14\ \text{rev/min}^2.$$

(b) Using Eq. 10-13 with t = (2.2) (60) = 132 min, the number of revolutions is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (150\ \text{rev/min})(132\ \text{min}) + \frac{1}{2}\left(-1.14\ \text{rev/min}^2\right)\left(132\,\text{min}\right)^2 = 9.9\times10^3\ \text{rev}.$$

(c) With r = 500 mm, the tangential acceleration is

$$a_t = \alpha r = \left(-1.14\ \text{rev/min}^2\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 (500\ \text{mm})$$

which yields $a_t = -0.99$ mm/s^2.

(d) The angular speed of the flywheel is

$$\omega = (75\ \text{rev/min})(2\pi\,\text{rad/rev})(1\ \text{min/}\ 60\ \text{s}) = 7.85\ \text{rad/s}.$$

With r = 0.50 m, the radial (or centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = (7.85 \text{ rad/s})^2 \, (0.50 \text{ m}) \approx 31 \, \text{m/s}^2$$

which is much bigger than a_t. Consequently, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} \approx a_r = 31 \text{ m/s}^2.$$

27. (a) The angular speed in rad/s is

$$\omega = \left(33\frac{1}{3} \text{rev/min}\right)\left(\frac{2\pi \text{ rad/rev}}{60 \text{ s/min}}\right) = 3.49 \text{ rad/s}.$$

Consequently, the radial (centripetal) acceleration is (using Eq. 10-23)

$$a = \omega^2 r = \left(3.49 \text{ rad/s}\right)^2 (6.0 \times 10^{-2} \text{ m}) = 0.73 \text{ m/s}^2.$$

(b) Using Ch. 6 methods, we have $ma = f_s \leq f_{s,\max} = \mu_s mg$, which is used to obtain the (minimum allowable) coefficient of friction:

$$\mu_{s,\min} = \frac{a}{g} = \frac{0.73}{9.8} = 0.075.$$

(c) The radial acceleration of the object is $a_r = \omega^2 r$, while the tangential acceleration is $a_t = \alpha r$. Thus,

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2} = r\sqrt{\omega^4 + \alpha^2}.$$

If the object is not to slip at any time, we require

$$f_{s,\max} = \mu_s mg = ma_{\max} = mr\sqrt{\omega_{\max}^4 + \alpha^2}.$$

Thus, since $\alpha = \omega/t$ (from Eq. 10-12), we find

$$\mu_{s,\min} = \frac{r\sqrt{\omega_{\max}^4 + \alpha^2}}{g} = \frac{r\sqrt{\omega_{\max}^4 + (\omega_{\max}/t)^2}}{g} = \frac{(0.060)\sqrt{3.49^4 + (3.4/0.25)^2}}{9.8} = 0.11.$$

28. Since the belt does not slip, a point on the rim of wheel C has the same tangential acceleration as a point on the rim of wheel A. This means that $\alpha_A r_A = \alpha_C r_C$, where α_A is the angular acceleration of wheel A and α_C is the angular acceleration of wheel C. Thus,

$$\alpha_C = \left(\frac{r_A}{r_C}\right)\alpha_C = \left(\frac{10\text{ cm}}{25\text{ cm}}\right)(1.6\text{ rad/s}^2) = 0.64\text{ rad/s}^2.$$

With the angular speed of wheel C given by $\omega_C = \alpha_C t$, the time for it to reach an angular speed of ω = 100 rev/min = 10.5 rad/s starting from rest is

$$t = \frac{\omega_C}{\alpha_C} = \frac{10.5\text{ rad/s}}{0.64\text{ rad/s}^2} = 16\text{ s}.$$

29. (a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle of $\theta = 2\pi/500 = 1.26 \times 10^{-2}$ rad. That time is

$$t = \frac{2\ell}{c} = \frac{2(500\text{ m})}{2.998\times10^8\text{ m/s}} = 3.34\times10^{-6}\text{ s}$$

so the angular velocity of the wheel is

$$\omega = \frac{\theta}{t} = \frac{1.26\times10^{-2}\text{ rad}}{3.34\times10^{-6}\text{ s}} = 3.8\times10^3\text{ rad/s}.$$

(b) If r is the radius of the wheel, the linear speed of a point on its rim is

$$v = \omega r = \left(3.8\times10^3\text{ rad/s}\right)\left(0.050\text{ m}\right) = 1.9\times10^2\text{ m/s}.$$

30. (a) The tangential acceleration, using Eq. 10-22, is

$$a_t = \alpha r = \left(14.2\text{ rad/s}^2\right)(2.83\text{ cm}) = 40.2\text{ cm/s}^2.$$

(b) In rad/s, the angular velocity is ω = (2760)(2π/60) = 289 rad/s, so

$$a_r = \omega^2 r = (289\text{ rad/s})^2(0.0283\text{ m}) = 2.36\times10^3\text{ m/s}^2.$$

(c) The angular displacement is, using Eq. 10-14,

$$\theta = \frac{\omega^2}{2\alpha} = \frac{(289\text{ rad/s})^2}{2(14.2\text{ rad/s}^2)} = 2.94\times10^3\text{ rad}.$$

Then, using Eq. 10-1, the distance traveled is

$$s = r\theta = (0.0283\text{ m})(2.94\times10^3\text{ rad}) = 83.2\text{ m}.$$

31. (a) The upper limit for centripetal acceleration (same as the radial acceleration – see Eq. 10-23) places an upper limit of the rate of spin (the angular velocity ω) by considering a point at the rim (r = 0.25 m). Thus, $\omega_{max} = \sqrt{a/r}$ = 40 rad/s. Now we apply Eq. 10-15 to first half of the motion (where ω_o = 0):

$$\theta - \theta_o = \tfrac{1}{2}(\omega_o + \omega)t \;\Rightarrow\; 400 \text{ rad} = \tfrac{1}{2}(0 + 40 \text{ rad/s})t$$

which leads to t = 20 s. The second half of the motion takes the same amount of time (the process is essentially the reverse of the first); the total time is therefore 40 s.

(b) Considering the first half of the motion again, Eq. 10-11 leads to

$$\omega = \omega_o + \alpha t \;\Rightarrow\; \alpha = \frac{40 \text{ rad/s}}{20 \text{ s}} = 2.0 \text{ rad/s}^2 .$$

32. (a) The linear speed at t = 15.0 s is

$$v = a_t t = \left(0.500\,\text{m/s}^2\right)(15.0\,\text{s}) = 7.50\,\text{m/s} .$$

The radial (centripetal) acceleration at that moment is

$$a_r = \frac{v^2}{r} = \frac{(7.50\,\text{m/s})^2}{30.0\,\text{m}} = 1.875\,\text{m/s}^2 .$$

Thus, the net acceleration has magnitude:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(0.500\,\text{m/s}^2\right)^2 + \left(1.875\,\text{m/s}^2\right)^2} = 1.94\,\text{m/s}^2 .$$

(b) We note that $\vec{a}_t \parallel \vec{v}$. Therefore, the angle between $\vec{v}$ and $\vec{a}$ is

$$\tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.875}{0.5}\right) = 75.1°$$

so that the vector is pointing more toward the center of the track than in the direction of motion.

33. **THINK** We want to calculate the rotational inertia of a wheel, given its rotational energy and rotational speed.

EXPRESS The kinetic energy (in J) is given by $K = \frac{1}{2}I\omega^2$, where I is the rotational inertia (in $\text{kg}\cdot\text{m}^2$) and ω is the angular velocity (in rad/s).

ANALYZE Expressing the angular speed as

$$\omega = \frac{(602 \text{ rev/min})(2\pi \text{rad/rev})}{60 \text{ s/min}} = 63.0 \text{ rad/s},$$

we find the rotational inertia to be $I = \dfrac{2K}{\omega^2} = \dfrac{2(24400 \text{ J})}{(63.0 \text{ rad / s})^2} = 12.3 \text{ kg} \cdot \text{m}^2$.

LEARN Note the analogy between rotational kinetic energy $\frac{1}{2}I\omega^2$ and $\frac{1}{2}mv^2$, the kinetic energy associated with linear motion.

34. (a) Equation 10-12 implies that the angular acceleration α should be the slope of the ω vs t graph. Thus, α = 9/6 = 1.5 rad/s^2.

(b) By Eq. 10-34, K is proportional to ω^2. Since the angular velocity at t = 0 is –2 rad/s (and this value squared is 4) and the angular velocity at t = 4 s is 4 rad/s (and this value squared is 16), then the ratio of the corresponding kinetic energies must be

$$\frac{K_o}{K_4} = \frac{4}{16} \quad \Rightarrow \quad K_o = K_4/4 = 0.40 \text{ J} .$$

35. **THINK** The rotational inertia of a rigid body depends on how its mass is distributed.

EXPRESS Since the rotational inertia of a cylinder is $I = \frac{1}{2}MR^2$ (Table 10-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\omega^2.$$

ANALYZE (a) For the smaller cylinder, we have

$$K_1 = \frac{1}{4}(1.25 \text{ kg})(0.25 \text{ m})^2(235 \text{ rad/s})^2 = 1.08\times 10^3 \text{ J}.$$

(b) For the larger cylinder, we obtain

$$K_2 = \frac{1}{4}(1.25 \text{ kg})(0.75 \text{ m})^2(235 \text{ rad/s})^2 = 9.71\times 10^3 \text{ J}.$$

LEARN The ratio of the rotational kinetic energies of the two cylinders having the same mass and angular speed is

$$\frac{K_2}{K_1} = \left(\frac{R_2}{R_1}\right)^2 = \left(\frac{0.75 \text{ m}}{0.25 \text{ m}}\right)^2 = (3)^2 = 9.$$

36. The parallel axis theorem (Eq. 10-36) shows that I increases with h. The phrase "out to the edge of the disk" (in the problem statement) implies that the maximum h in the graph is, in fact, the radius R of the disk. Thus, R = 0.20 m. Now we can examine, say, the h = 0 datum and use the formula for I_{com} (see Table 10-2(c)) for a solid disk, or (which might be a little better, since this is independent of whether it is really a solid disk) we can the difference between the h = 0 datum and the $h = h_{max} = R$ datum and relate that difference to the parallel axis theorem (thus the difference is $M(h_{max})^2 = 0.10\ \text{kg}\cdot\text{m}^2$). In either case, we arrive at M = 2.5 kg.

37. **THINK** We want to calculate the rotational inertia of a meter stick about an axis perpendicular to the stick but not through its center.

EXPRESS We use the parallel-axis theorem: $I = I_{com} + Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 10-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies h = 0.50 m – 0.20 m = 0.30 m.

ANALYZE With $M = 0.56\,\text{kg}$ and $L = 1.0\ \text{m}$, we have

$$I_{com} = \frac{1}{12}ML^2 = \frac{1}{12}(0.56\ \text{kg})(1.0\ \text{m})^2 = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

Consequently, the parallel-axis theorem yields

$$I = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2 + (0.56\ \text{kg})(0.30\ \text{m})^2 = 9.7\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

LEARN A greater moment of inertia $I > I_{com}$ means that it is more difficult to rotate the meter stick about this axis than the case where the axis passes through the center.

38. (a) Equation 10-33 gives

$$I_{total} = md^2 + m(2d)^2 + m(3d)^2 = 14\,md^2.$$

If the innermost one is removed then we would only obtain $m(2d)^2 + m(3d)^2 = 13\,md^2$. The percentage difference between these is (13 – 14)/14 = 0.0714 ≈ 7.1%.

(b) If, instead, the outermost particle is removed, we would have $md^2 + m(2d)^2 = 5\,md^2$. The percentage difference in this case is 0.643 ≈ 64%.

39. (a) Using Table 10-2(c) and Eq. 10-34, the rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}(500\,\text{kg})(200\pi\ \text{rad/s})^2(1.0\,\text{m})^2 = 4.9\times10^7\ \text{J}.$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9 \times 10^7 \text{ J}}{8.0 \times 10^3 \text{ W}} = 6.2 \times 10^3 \text{ s}$$

which we rewrite as $t \approx 1.0 \times 10^2$ min.

40. (a) Consider three of the disks (starting with the one at point O): ⊕OO . The first one (the one at point O, shown here with the plus sign inside) has rotational inertial (see item (c) in Table 10-2) $I = \frac{1}{2}mR^2$. The next one (using the parallel-axis theorem) has

$$I = \tfrac{1}{2}mR^2 + mh^2$$

where $h = 2R$. The third one has $I = \frac{1}{2}mR^2 + m(4R)^2$. If we had considered five of the disks OO⊕OO with the one at O in the middle, then the total rotational inertia is

$$I = 5(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2).$$

The pattern is now clear and we can write down the total I for the collection of fifteen disks:

$$I = 15(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2 + m(6R)^2 + \ldots + m(14R)^2) = \tfrac{2255}{2}mR^2.$$

The generalization to N disks (where N is assumed to be an odd number) is

$$I = \tfrac{1}{6}(2N^2 + 1)NmR^2.$$

In terms of the total mass ($m = M/15$) and the total length ($R = L/30$), we obtain

$$I = 0.083519ML^2 \approx (0.08352)(0.1000 \text{ kg})(1.0000 \text{ m})^2 = 8.352 \times 10^{-3} \text{ kg} \cdot \text{m}^2.$$

(b) Comparing to the formula (e) in Table 10-2 (which gives roughly $I = 0.08333\ ML^2$), we find our answer to part (a) is 0.22% lower.

41. The particles are treated "point-like" in the sense that Eq. 10-33 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 10-2(e) and the parallel-axis theorem (Eq. 10-36).

(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$I = I_1 + I_2 + I_3 + I_4 = \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2 + \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2\right) + m(2d)^2$$
$$= \frac{8}{3}Md^2 + 5md^2 = \frac{8}{3}(1.2\text{ kg})(0.056\text{ m})^2 + 5(0.85\text{ kg})(0.056\text{ m})^2$$
$$= 0.023\text{ kg}\cdot\text{m}^2.$$

(b) Using Eq. 10-34, we have

$$K = \frac{1}{2}I\omega^2 = \left(\frac{4}{3}M + \frac{5}{2}m\right)d^2\omega^2 = \left[\frac{4}{3}(1.2\text{ kg}) + \frac{5}{2}(0.85\text{ kg})\right](0.056\text{ m})^2(0.30\text{ rad/s})^2$$
$$= 1.1\times10^{-3}\text{ J}.$$

42. (a) We apply Eq. 10-33:

$$I_x = \sum_{i=1}^{4} m_i y_i^2 = \left[50(2.0)^2 + (25)(4.0)^2 + 25(-3.0)^2 + 30(4.0)^2\right]\text{g}\cdot\text{cm}^2 = 1.3\times10^3\text{ g}\cdot\text{cm}^2.$$

(b) For rotation about the *y* axis we obtain

$$I_y = \sum_{i=1}^{4} m_i x_i^2 = 50(2.0)^2 + (25)(0)^2 + 25(3.0)^2 + 30(2.0)^2 = 5.5\times10^2\text{ g}\cdot\text{cm}^2.$$

(c) And about the *z* axis, we find (using the fact that the distance from the *z* axis is $\sqrt{x^2 + y^2}$)

$$I_z = \sum_{i=1}^{4} m_i\left(x_i^2 + y_i^2\right) = I_x + I_y = 1.3\times10^3 + 5.5\times10^2 = 1.9\times10^2\text{ g}\cdot\text{cm}^2.$$

(d) Clearly, the answer to part (c) is $A + B$.

43. **THINK** Since the rotation axis does not pass through the center of the block, we use the parallel-axis theorem to calculate the rotational inertia.

EXPRESS According to Table 10-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by $I_{\text{com}} = \frac{M}{12}\left(a^2 + b^2\right)$. A parallel axis through the corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center. Therefore,

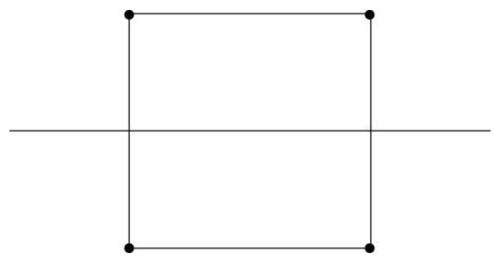
$$I = I_{\text{com}} + Mh^2 = \frac{M}{12}\left(a^2 + b^2\right) + \frac{M}{4}\left(a^2 + b^2\right) = \frac{M}{3}\left(a^2 + b^2\right).$$

ANALYZE With $M = 0.172\text{ kg}$, $a = 3.5\text{ cm}$ and $b = 8.4\text{ cm}$, we have

$$I = \frac{M}{3}\left(a^2 + b^2\right) = \frac{0.172 \text{ kg}}{3}[(0.035 \text{ m})^2 + (0.084 \text{ m})^2] = 4.7\times10^{-4} \text{ kg}\cdot\text{m}^2.$$

LEARN A greater moment of inertia $I > I_{\text{com}}$ means that it is more difficult to rotate the block about the axis through the corner than the case where the axis passes through the center.

44. (a) We show the figure with its axis of rotation (the thin horizontal line).



We note that each mass is r = 1.0 m from the axis. Therefore, using Eq. 10-26, we obtain

$$I = \sum m_i r_i^2 = 4\ (0.50 \text{ kg})\ (1.0 \text{ m})^2 = 2.0 \text{ kg}\cdot\text{m}^2.$$

(b) In this case, the two masses nearest the axis are r = 1.0 m away from it, but the two furthest from the axis are $r = \sqrt{(1.0 \text{ m})^2 + (2.0 \text{ m})^2}$ from it. Here, then, Eq. 10-33 leads to

$$I = \sum m_i r_i^2 = 2(0.50 \text{ kg})\left(1.0 \text{ m}^2\right) + 2(0.50 \text{ kg})\left(5.0 \text{ m}^2\right) = 6.0 \text{ kg}\cdot\text{m}^2.$$

(c) Now, two masses are on the axis (with r = 0) and the other two are a distance $r = \sqrt{(1.0 \text{ m})^2 + (1.0 \text{ m})^2}$ away. Now we obtain $I = 2.0 \text{ kg}\cdot\text{m}^2$.

45. **THINK** Torque is the product of the force applied and the moment arm. When two torques act on a body, the net torque is their vector sum.

EXPRESS We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1 F_1 \sin \theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2F_2 \sin \theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1F_1 \sin\theta_1 - r_2F_2 \sin\theta_2.$$

ANALYZE Substituting the given values, we obtain

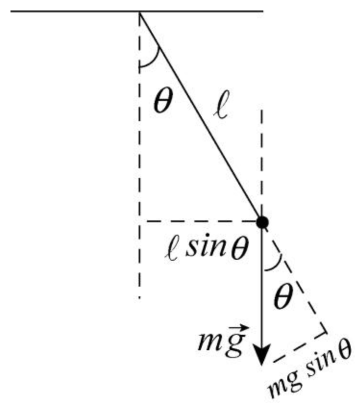
$$\begin{aligned}\tau &= r_1F_1 \sin\theta_1 - r_2F_2 \sin\theta_2 = (1.30 \text{ m})(4.20 \text{ N})\sin 75° - (2.15 \text{ m})(4.90 \text{ N})\sin 60° \\ &= -3.85 \text{ N}\cdot\text{m}.\end{aligned}$$

LEARN Since $\tau < 0$, the body will rotate clockwise about the pivot point.

46. The net torque is

$$\begin{aligned}\tau &= \tau_A + \tau_B + \tau_C = F_A r_A \sin\phi_A - F_B r_B \sin\phi_B + F_C r_C \sin\phi_C \\ &= (10)(8.0)\sin 135° - (16)(4.0)\sin 90° + (19)(3.0)\sin 160° \\ &= 12\ \text{N}\cdot\text{m}.\end{aligned}$$

47. **THINK** In this problem we have a pendulum made up of a ball attached to a massless rod. There are two forces acting on the ball, the force of the rod and the force of gravity.



EXPRESS No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is $mg \sin\theta$. If ℓ is the length of the rod, then the torque associated with this force has magnitude

$$\tau = mg\ell \sin\theta .$$

ANALYZE With $m = 0.75\ \text{kg}$, $\ell = 1.25\ \text{m}$ and $\theta = 30°$, we find the torque to be

$$\tau = mg\ell \sin\theta = (0.75)(9.8)(1.25)\sin 30° = 4.6\ \text{N}\cdot\text{m} .$$

LEARN The moment arm of the gravitational force mg is $\ell \sin\theta$. Alternatively, we may say that ℓ is the moment arm of $mg\sin\theta$, the tangential component of the gravitational force. Both interpretations lead to the same result: $\tau = (mg)(\ell\sin\theta) = (mg\sin\theta)(\ell)$.

48. We compute the torques using $\tau = rF \sin\phi$.

(a) For $\phi = 30°$, $\tau_a = (0.152\ \text{m})(111\ \text{N})\sin 30° = 8.4\ \text{N}\cdot\text{m}$.

(b) For $\phi = 90°$, $\tau_b = (0.152\ \text{m})(111\ \text{N})\sin 90° = 17\ \text{N}\cdot\text{m}$.

(c) For $\phi = 180°$, $\tau_c = (0.152\ \text{m})(111\ \text{N})\sin 180° = 0$.

49. **THINK** Since the angular velocity of the diver changes with time, there must be a non-vanishing angular acceleration.

EXPRESS To calculate the angular acceleration α, we use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity and t is the time. If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is $\tau = I\alpha$.

ANALYZE (a) Using the values given, the angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20\ \text{rad/s}}{220 \times 10^{-3}\ \text{s}} = 28.2\ \text{rad/s}^2.$$

(b) Similarly, we find the magnitude of the torque on the diver to be

$$\tau = I\alpha = \left(12.0\ \text{kg}\cdot\text{m}^2\right)\left(28.2\ \text{rad/s}^2\right) = 3.38 \times 10^2\ \text{N}\cdot\text{m}.$$

LEARN A net toque results in an angular acceleration that changes angular velocity. The equation $\tau = I\alpha$ implies that the greater the rotational inertia I, the greater the torque required for a given angular acceleration α.

50. The rotational inertia is found from Eq. 10-45.

$$I = \frac{\tau}{\alpha} = \frac{32.0}{25.0} = 1.28\ \text{kg}\cdot\text{m}^2$$

51. (a) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block m_2, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750\ \text{m})}{(5.00\ \text{s})^2} = 6.00 \times 10^{-2}\ \text{m/s}^2.$$

Block 1 has an acceleration of 6.00×10^{-2} m/s^2 upward.

(b) Newton's second law for block 2 is $m_2 g - T_2 = m_2 a$, where m_2 is its mass and T_2 is the tension force on the block. Thus,

$$T_2 = m_2(g - a) = (0.500\ \text{kg})\left(9.8\ \text{m/s}^2 - 6.00\times10^{-2}\ \text{m/s}^2\right) = 4.87\ \text{N}.$$

(c) Newton's second law for block 1 is $m_1 g - T_1 = -m_1 a,$ where T_1 is the tension force on the block. Thus,

$$T_1 = m_1(g + a) = (0.460\ \text{kg})\left(9.8\ \text{m/s}^2 + 6.00\times10^{-2}\ \text{m/s}^2\right) = 4.54\ \text{N}.$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00\times10^{-2}\ \text{m/s}^2}{5.00\times10^{-2}\ \text{m}} = 1.20\ \text{rad/s}^2.$$

(e) The net torque acting on the pulley is $\tau = (T_2 - T_1)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$I = \frac{(T_2 - T_1)R}{\alpha} = \frac{(4.87\ \text{N} - 4.54\ \text{N})(5.00\times10^{-2}\ \text{m})}{1.20\ \text{rad/s}^2} = 1.38\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

52. According to the sign conventions used in the book, the magnitude of the net torque exerted on the cylinder of mass m and radius R is

$$\tau_{\text{net}} = F_1R - F_2R - F_3r = (6.0\ \text{N})(0.12\ \text{m}) - (4.0\ \text{N})(0.12\ \text{m}) - (2.0\ \text{N})(0.050\ \text{m}) = 71\,\text{N}\cdot\text{m}.$$

(a) The resulting angular acceleration of the cylinder (with $I = \frac{1}{2}MR^2$ according to Table 10-2(c)) is

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{71\,\text{N}\cdot\text{m}}{\frac{1}{2}(2.0\,\text{kg})(0.12\ \text{m})^2} = 9.7\,\text{rad/s}^2.$$

(b) The direction is counterclockwise (which is the positive sense of rotation).

53. Combining Eq. 10-45 ($\tau_{\text{net}} = I\ \alpha$) with Eq. 10-38 gives $RF_2 - RF_1 = I\alpha$, where $\alpha = \omega/t$ by Eq. 10-12 (with $\omega_o = 0$). Using item (c) in Table 10-2 and solving for F_2 we find

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(0.02)(0.02)(250)}{2(1.25)} + 0.1 = 0.140\ \text{N}.$$

54. (a) In this case, the force is mg = (70 kg)(9.8 m/s^2), and the “lever arm” (the perpendicular distance from point O to the line of action of the force) is 0.28 m. Thus, the torque (in absolute value) is (70 kg)(9.8 m/s^2)(0.28 m). Since the moment-of-inertia is $I = 65\ \text{kg}\cdot\text{m}^2$, then Eq. 10-45 gives $|\alpha| = 2.955 \approx 3.0$ rad/s^2.

(b) Now we have another contribution (1.4 m × 300 N) to the net torque, so

$$|\tau_{\text{net}}| = (70\ \text{kg})(9.8\ \text{m/s}^2)(0.28\ \text{m}) + (1.4\ \text{m})(300\ \text{N}) = (65\ \text{kg}\cdot\text{m}^2)\ |\alpha|$$

which leads to $|\alpha|$ = 9.4 rad/s^2.

55. Combining Eq. 10-34 and Eq. 10-45, we have $RF = I\alpha$, where α is given by ω/t (according to Eq. 10-12, since $\omega_o = 0$ in this case). We also use the fact that

$$I = I_{\text{plate}} + I_{\text{disk}}$$

where $I_{\text{disk}} = \frac{1}{2}MR^2$ (item (c) in Table 10-2). Therefore,

$$I_{\text{plate}} = \frac{RFt}{\omega} - \frac{1}{2}MR^2 = 2.51 \times 10^{-4}\ \text{kg}\cdot\text{m}^2.$$

56. With counterclockwise positive, the angular acceleration α for both masses satisfies

$$\tau = mgL_1 - mgL_2 = I\alpha = \left(mL_1^2 + mL_2^2\right)\alpha,$$

by combining Eq. 10-45 with Eq. 10-39 and Eq. 10-33. Therefore, using SI units,

$$\alpha = \frac{g\left(L_1 - L_2\right)}{L_1^2 + L_2^2} = \frac{\left(9.8\ \text{m/s}^2\right)\left(0.20\ \text{m} - 0.80\ \text{m}\right)}{(0.20\ \text{m})^2 + (0.80\ \text{m})^2} = -\,8.65\ \text{rad/s}^2$$

where the negative sign indicates the system starts turning in the clockwise sense. The magnitude of the acceleration vector involves no radial component (yet) since it is evaluated at $t = 0$ when the instantaneous velocity is zero. Thus, for the two masses, we apply Eq. 10-22:

(a) $\left|\vec{a}_1\right| = |\alpha|L_1 = \left(8.65\ \text{rad/s}^2\right)\left(0.20\ \text{m}\right) = 1.7\ \text{m/s}.$

(b) $\left|\vec{a}_2\right| = |\alpha|L_2 = \left(8.65\ \text{rad/s}^2\right)\left(0.80\ \text{m}\right) = 6.9\ \text{m/s}^2.$

57. Since the force acts tangentially at $r = 0.10$ m, the angular acceleration (presumed positive) is

$$\alpha = \frac{\tau}{I} = \frac{Fr}{I} = \frac{\left(0.5t + 0.3t^2\right)\left(0.10\right)}{1.0 \times 10^{-3}} = 50t + 30t^2$$

in SI units (rad/s^2).

(a) At $t = 3$ s, the above expression becomes $\alpha = 4.2 \times 10^2$ rad/s^2.

(b) We integrate the above expression, noting that $\omega_0 = 0$, to obtain the angular speed at t = 3 s:

$$\omega = \int_0^3 \alpha\, dt = \left(25t^2 + 10t^3\right)\Big|_0^3 = 5.0\times 10^2\ \text{rad/s}.$$

58. (a) The speed of v of the mass m after it has descended $d = 50$ cm is given by $v^2 = 2ad$ (Eq. 2-16). Thus, using $g = 980$ cm/s^2, we have

$$v = \sqrt{2ad} = \sqrt{\frac{2(2mg)d}{M+2m}} = \sqrt{\frac{4(50)(980)(50)}{400+2(50)}} = 1.4 \times 10^2 \text{ cm/s}.$$

(b) The answer is still 1.4×10^2 cm/s = 1.4 m/s, since it is independent of R.

59. With ω = (1800)(2π/60) = 188.5 rad/s, we apply Eq. 10-55:

$$P = \tau\omega \quad \Rightarrow \quad \tau = \frac{74600 \text{ W}}{188.5 \text{ rad/s}} = 396 \text{ N} \cdot \text{m} .$$

60. (a) We apply Eq. 10-34:

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{3} mL^2\right)\omega^2 = \frac{1}{6} mL^2\omega^2$$
$$= \frac{1}{6}(0.42 \text{ kg})(0.75 \text{ m})^2 (4.0 \text{ rad/s})^2 = 0.63 \text{ J}.$$

(b) Simple conservation of mechanical energy leads to $K = mgh$. Consequently, the center of mass rises by

$$h = \frac{K}{mg} = \frac{mL^2\omega^2}{6mg} = \frac{L^2\omega^2}{6g} = \frac{(0.75 \text{ m})^2(4.0 \text{ rad/s})^2}{6(9.8 \text{ m/s}^2)} = 0.153 \text{ m} \approx 0.15 \text{ m}.$$

61. The initial angular speed is ω = (280 rev/min)(2π/60) = 29.3 rad/s.

(a) Since the rotational inertia is (Table 10-2(a)) $I = (32 \text{ kg})(1.2 \text{ m})^2 = 46.1 \text{ kg} \cdot \text{m}^2$, the work done is

$$W = \Delta K = 0 - \frac{1}{2} I\omega^2 = -\frac{1}{2}(46.1 \text{ kg} \cdot \text{m}^2)(29.3 \text{ rad/s})^2 = -1.98 \times 10^4 \text{ J} .$$

(b) The average power (in absolute value) is therefore

$$|P| = \frac{|W|}{\Delta t} = \frac{19.8 \times 10^3}{15} = 1.32 \times 10^3 \text{ W}.$$

62. (a) Eq. 10-33 gives

$$I_{\text{total}} = md^2 + m(2d)^2 + m(3d)^2 = 14\, md^2,$$

where d = 0.020 m and m = 0.010 kg. The work done is

$$W = \Delta K = \tfrac{1}{2} I\omega_f^2 - \tfrac{1}{2} I\omega_i^2,$$

where ω_f = 20 rad/s and ω_i = 0. This gives W = 11.2 mJ.

(b) Now, ω_f = 40 rad/s and ω_i = 20 rad/s, and we get W = 33.6 mJ.

(c) In this case, ω_f = 60 rad/s and ω_i = 40 rad/s. This gives W = 56.0 mJ.

(d) Equation 10-34 indicates that the slope should be $\frac{1}{2}I$. Therefore, it should be

$$7md^2 = 2.80 \times 10^{-5}\ \mathrm{J{\cdot}s^2/\,rad^2}.$$

63. **THINK** As the meter stick falls by rotating about the axis passing through one end of the stick, its potential energy is converted into rotational kinetic energy.

EXPRESS We use ℓ to denote the length of the stick. The meter stick is initially at rest so its initial kinetic energy is zero. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $U_g = \frac{1}{2}mg\ell$, where m is its mass. Just before the stick hits the floor, its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{mg\ell}{I}}.$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 10-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}.$$

ANALYZE Using Table 10-2 and the parallel-axis theorem, the rotational inertial is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3\left(9.8\ \mathrm{m/s^2}\right)\left(1.00\ \mathrm{m}\right)} = 5.42\ \mathrm{m/s}.$$

LEARN The linear speed of a point on the meter stick depends on its distance from the axis of rotation. One may show that the speed of the center of mass is

$$v_{\mathrm{cm}} = \omega(\ell/2) = \frac{1}{2}\sqrt{3g\ell}.$$

64. (a) We use the parallel-axis theorem to find the rotational inertia:

$$I = I_{\mathrm{com}} + Mh^2 = \frac{1}{2}MR^2 + Mh^2 = \frac{1}{2}\left(20\ \mathrm{kg}\right)\left(0.10\ \mathrm{m}\right)^2 + \left(20\ \mathrm{kg}\right)\left(0.50\ \mathrm{m}\right)^2 = 0.15\ \mathrm{kg\cdot m^2}.$$

(b) Conservation of energy requires that $Mgh = \frac{1}{2}I\omega^2$, where ω is the angular speed of the cylinder as it passes through the lowest position. Therefore,

$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20\text{ kg})(9.8\text{ m/s}^2)(0.050\text{ m})}{0.15\text{ kg}\cdot\text{m}^2}} = 11\text{ rad/s.}$$

65. (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2)\cos\theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2}I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2)\cos\theta + \frac{1}{2}I\omega^2 \Rightarrow \omega^2 = (MgH/I)(1-\cos\theta).$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 10-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1-\cos\theta)} = \sqrt{\frac{3(9.80\text{ m/s}^2)}{55.0\text{ m}}(1-\cos 35.0°)} = 0.311\text{ rad/s.}$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so

$$a_r = 3g(1-\cos\theta) = 3(9.80\text{ m/s}^2)(1-\cos 35.0°) = 5.32\text{ m/s}^2.$$

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 10-1 since the acceleration is not uniform. Hence, we differentiate

$$\omega^2 = (3g/H)(1-\cos\theta)$$

with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \Rightarrow \alpha = (3g/2H)\sin\theta.$$

Consequently,

$$a_t = H\alpha = \frac{3g}{2}\sin\theta = \frac{3(9.80\text{ m/s}^2)}{2}\sin 35.0° = 8.43\text{ m/s}^2.$$

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, $\sin\theta = 2/3$ and we obtain $\theta = 41.8°$.

66. From Table 10-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega_{\text{sphere}}^2 + \frac{1}{2}I\omega_{\text{pulley}}^2 + \frac{1}{2}mv^2.$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$
$$= \sqrt{\frac{2(9.8)(0.82)}{1 + 3.0\times10^{-3}/((0.60)(0.050)^2) + 2(4.5)/3(0.60)}} = 1.4 \text{ m/s}.$$

67. Using the parallel axis theorem and items (e) and (h) in Table 10-2, the rotational inertia is

$$I = \frac{1}{12}mL^2 + m(L/2)^2 + \frac{1}{2}mR^2 + m(R+L)^2 = 10.83mR^2,$$

where $L = 2R$ has been used. If we take the base of the rod to be at the coordinate origin ($x = 0$, $y = 0$) then the center of mass is at

$$y = \frac{mL/2 + m(L+R)}{m+m} = 2R.$$

Comparing the position shown in the textbook figure to its upside down (inverted) position shows that the change in center of mass position (in absolute value) is $|\Delta y| = 4R$. The corresponding loss in gravitational potential energy is converted into kinetic energy. Thus,

$$K = (2m)g(4R) \quad \Rightarrow \quad \omega = 9.82 \text{ rad/s}$$

where Eq. 10-34 has been used.

68. We choose ± directions such that the initial angular velocity is $\omega_0 = -317$ rad/s and the values for α, τ, and F are positive.

(a) Combining Eq. 10-12 with Eq. 10-45 and Table 10-2(f) (and using the fact that $\omega = 0$) we arrive at the expression

$$\tau = \left(\frac{2}{5}MR^2\right)\left(-\frac{\omega_0}{t}\right) = -\frac{2}{5}\frac{MR^2\omega_0}{t}.$$

With t = 15.5 s, R = 0.226 m, and M = 1.65 kg, we obtain τ = 0.689 N · m.

(b) From Eq. 10-40, we find $F = \tau/R$ = 3.05 N.

(c) Using again the expression found in part (a), but this time with R = 0.854 m, we get $\tau = 9.84\ \text{N}\cdot\text{m}$.

(d) Now, $F = \tau / R$ = 11.5 N.

69. The volume of each disk is $\pi r^2 h$ where we are using h to denote the thickness (which equals 0.00500 m). If we use R (which equals 0.0400 m) for the radius of the larger disk and r (which equals 0.0200 m) for the radius of the smaller one, then the mass of each is $m = \rho\pi r^2 h$ and $M = \rho\pi R^2 h$ where ρ = 1400 kg/m^3 is the given density. We now use the parallel axis theorem as well as item (c) in Table 10-2 to obtain the rotation inertia of the two-disk assembly:

$$I = \tfrac{1}{2}MR^2 + \tfrac{1}{2}mr^2 + m(r + R)^2 = \rho\pi h[\tfrac{1}{2}R^4 + \tfrac{1}{2}r^4 + r^2(r + R)^2] = 6.16 \times 10^{-5}\ \text{kg}\cdot\text{m}^2.$$

70. The wheel starts turning from rest (ω_0 = 0) at t = 0, and accelerates uniformly at $\alpha = 2.00\ \text{rad/s}^2$. Between t_1 and t_2 the wheel turns through $\Delta\theta$ = 90.0 rad, where $t_2 - t_1 = \Delta t$ = 3.00 s. We solve (b) first.

(b) We use Eq. 10-13 (with a slight change in notation) to describe the motion for $t_1 \le t \le t_2$:

$$\Delta\theta = \omega_1\Delta t + \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow \omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2}$$

which we plug into Eq. 10-12, set up to describe the motion during $0 \le t \le t_1$:

$$\omega_1 = \omega_0 + \alpha t_1 \;\Rightarrow\; \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \alpha t_1 \;\Rightarrow\; \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = (2.00)t_1$$

yielding t_1 = 13.5 s.

(a) Plugging into our expression for ω_1 (in previous part) we obtain

$$\omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = 27.0\ \text{rad/s}.$$

71. **THINK** Since the string that connects the two blocks does not slip, the pulley rotates about its axel as the blocks move.

EXPRESS We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2):

$$\begin{aligned} m_1 g - T_1 &= m_1 a_1 \\ T_2 - f_2 &= m_2 a_2 \\ T_1 R - T_2 R &= I\alpha \end{aligned}$$

ANALYZE (a) From Eq. 10-13 (with $\omega_0 = 0$) we find the magnitude of the pulley's angular acceleration to be

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \;\;\Rightarrow\;\; \alpha = \frac{2\theta}{t^2} = \frac{2(0.130\text{ rad})}{(0.0910\text{ s})^2} = 31.4\text{ rad/s}^2.$$

(b) From the fact that $a = R\alpha$ (noted above), the acceleration of the blocks is

$$a = \frac{2R\theta}{t^2} = \frac{2(0.024\text{ m})(0.130\text{ rad})}{(0.0910\text{ s})^2} = 0.754\text{ m/s}^2.$$

(c) From the first of the above equations, we find the string tension T_1 to be

$$T_1 = m_1\left(g - a_1\right) = M\left(g - \frac{2R\theta}{t^2}\right) = (6.20\text{ kg})\left(9.80\text{ m/s}^2 - \frac{2(0.024\text{ m})(0.130\text{ rad})}{(0.0910\text{ s})^2}\right) = 56.1\text{ N}.$$

(d) From the last of the above equations, we obtain the second tension:

$$T_2 = T_1 - \frac{I\alpha}{R} = 56.1\text{ N} - \frac{(7.40\times10^{-4}\text{ kg}\cdot\text{m}^2)(31.4\text{ rad/s}^2)}{0.024\text{ m}} = 55.1\text{ N}.$$

LEARN The torque acting on the pulley is $\tau = I\alpha = (T_1 - T_2)R$. If the pulley becomes massless, then $I = 0$ and we recover the expected result: $T_1 = T_2$.

72. (a) Constant angular acceleration kinematics can be used to compute the angular acceleration α. If ω_0 is the initial angular velocity and t is the time to come to rest, then $0 = \omega_0 + \alpha t$, which gives

$$\alpha = -\frac{\omega_0}{t} = -\frac{39.0\,\text{rev/s}}{32.0\,\text{s}} = -1.22\,\text{rev/s}^2 = -7.66\,\text{rad/s}^2 \ .$$

(b) We use τ = *I*α, where τ is the torque and *I* is the rotational inertia. The contribution of the rod to *I* is $M\ell^2/12$ (Table 10-2(e)), where *M* is its mass and ℓ is its length. The contribution of each ball is $m(\ell/2)^2$, where *m* is the mass of a ball. The total rotational inertia is

$$I = \frac{M\ell^2}{12} + 2\frac{m\ell^2}{4} = \frac{(6.40\text{ kg})(1.20\text{ m})^2}{12} + \frac{(1.06\text{ kg})(1.20\text{ m})^2}{2}$$

which yields *I* = 1.53 kg·m^2. The torque, therefore, is

$$\tau = (1.53\text{ kg}\cdot\text{m}^2)(-7.66\text{ rad/s}^2) = -11.7\text{ N}\cdot\text{m}.$$

(c) Since the system comes to rest the mechanical energy that is converted to thermal energy is simply the initial kinetic energy

$$K_i = \frac{1}{2}I\omega_0^2 = \frac{1}{2}(1.53\text{ kg}\cdot\text{m}^2)\left((2\pi)(39)\text{ rad/s}\right)^2 = 4.59\times10^4\text{ J}.$$

(d) We apply Eq. 10-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \left((2\pi)(39)\text{ rad/s}\right)(32.0\text{ s}) + \frac{1}{2}(-7.66\text{ rad/s}^2)(32.0\text{ s})^2$$

which yields 3920 rad or (dividing by 2π) 624 rev for the value of angular displacement *θ*.

(e) Only the mechanical energy that is converted to thermal energy can still be computed without additional information. It is 4.59 × 10^4 J no matter how τ varies with time, as long as the system comes to rest.

73. The *Hint* given in the problem would make the computation in part (a) very straightforward (without doing the integration as we show here), but we present this further level of detail in case that hint is not obvious or — simply — in case one wishes to see how the calculus supports our intuition.

(a) The (centripetal) force exerted on an infinitesimal portion of the blade with mass *dm* located a distance *r* from the rotational axis is (Newton's second law) *dF* = (*dm*)$\omega^2 r$, where *dm* can be written as (*M*/*L*)*dr* and the angular speed is

$$\omega = (320)(2\pi/60) = 33.5\text{ rad/s}\ .$$

Thus for the entire blade of mass *M* and length *L* the total force is given by

$$F=\int dF=\int \omega^2 r dm=\frac{M}{L}\int_0^L \omega^2 r dr=\frac{M\omega^2 L}{2}=\frac{(110\,\text{kg})\,(33.5\ \text{rad/s})^2\,(7.80\,\text{m})}{2}$$
$$=4.81\times10^5\,\text{N}.$$

(b) About its center of mass, the blade has $I = ML^2/12$ according to Table 10-2(e), and using the parallel-axis theorem to "move" the axis of rotation to its end-point, we find the rotational inertia becomes $I = ML^2/3$. Using Eq. 10-45, the torque (assumed constant) is

$$\tau = I\alpha = \left(\frac{1}{3}ML^2\right)\left(\frac{\Delta\omega}{\Delta t}\right)=\frac{1}{3}(110\,\text{kg})(7.8\ \text{m})^2\left(\frac{33.5\,\text{rad/s}}{6.7\,\text{s}}\right)=1.12\times10^4\,\text{N}\cdot\text{m}.$$

(c) Using Eq. 10-52, the work done is

$$W=\Delta K=\frac{1}{2}I\omega^2-0=\frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2=\frac{1}{6}(110\,\text{kg})(7.80\,\text{m})^2(33.5\,\text{rad/s})^2=1.25\times10^6\,\text{J}.$$

74. The angular displacements of disks *A* and *B* can be written as:

$$\theta_A=\omega_A t,\quad \theta_B=\frac{1}{2}\alpha_B t^2.$$

(a) The time when $\theta_A=\theta_B$ is given by

$$\omega_A t=\frac{1}{2}\alpha_B t^2 \;\Rightarrow\; t=\frac{2\omega_A}{\alpha_B}=\frac{2(9.5\ \text{rad/s})}{(2.2\ \text{rad/s}^2)}=8.6\ \text{s}.$$

(b) The difference in the angular displacement is

$$\Delta\theta=\theta_A-\theta_B=\omega_A t-\frac{1}{2}\alpha_B t^2=9.5t-1.1t^2.$$

For their reference lines to align momentarily, we only require $\Delta\theta=2\pi N$, where N is an integer. The quadratic equation can be readily solve to yield

$$t_N=\frac{9.5\pm\sqrt{(9.5)^2-4(1.1)(2\pi N)}}{2(1.1)}=\frac{9.5\pm\sqrt{90.25-27.6N}}{2.2}.$$

The solution $t_0=8.63\ \text{s}$ (taking the positive root) coincides with the result obtained in (a), while $t_0=0$ (taking the negative root) is the moment when both disks begin to rotate. In fact, two solutions exist for N = 0, 1, 2, and 3.

75. The magnitude of torque is the product of the force magnitude and the distance from the pivot to the line of action of the force. In our case, it is the gravitational force that passes through the walker's center of mass. Thus,

$$\tau = I\alpha = rF = rmg.$$

(a) Without the pole, with $I = 15\ \mathrm{kg\cdot m^2}$, the angular acceleration is

$$\alpha = \frac{rF}{I} = \frac{rmg}{I} = \frac{(0.050\ \mathrm{m})(70\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{15\ \mathrm{kg\cdot m^2}} = 2.3\ \mathrm{rad/s^2}.$$

(b) When the walker carries a pole, the torque due to the gravitational force through the pole's center of mass opposes the torque due to the gravitational force that passes through the walker's center of mass. Therefore,

$$\tau_{\mathrm{net}} = \sum_i r_i F_i = (0.050\ \mathrm{m})(70\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) - (0.10\ \mathrm{m})(14\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) = 20.58\ \mathrm{N\cdot m},$$

and the resulting angular acceleration is

$$\alpha = \frac{\tau_{\mathrm{net}}}{I} = \frac{20.58\ \mathrm{N\cdot m}}{15\ \mathrm{kg\cdot m^2}} \approx 1.4\ \mathrm{rad/s^2}.$$

76. The motion consists of two stages. The first, the interval $0 \le t \le 20$ s, consists of constant angular acceleration given by

$$\alpha = \frac{5.0\ \mathrm{rad/s}}{2.0\ \mathrm{s}} = 2.5\ \mathrm{rad/s^2}.$$

The second stage, $20 < t \le 40$ s, consists of constant angular velocity $\omega = \Delta\theta / \Delta t$. Analyzing the first stage, we find

$$\theta_1 = \frac{1}{2}\alpha t^2\Big|_{t=20} = 500\ \mathrm{rad}, \qquad \omega = \alpha t\big|_{t=20} = 50\ \mathrm{rad/s}.$$

Analyzing the second stage, we obtain

$$\theta_2 = \theta_1 + \omega\Delta t = 500\ \mathrm{rad} + (50\ \mathrm{rad/s})(20\ \mathrm{s}) = 1.5\times10^3\ \mathrm{rad}.$$

77. **THINK** The record turntable comes to a stop due to a constant angular acceleration. We apply equations given in Table 10-1 to analyze the rotational motion.

EXPRESS We take the sense of initial rotation to be positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued. The angular acceleration is constant, so we can apply Eq. 10-12 ($\omega = \omega_0 + \alpha t$), which gives $\alpha = (\omega - \omega_0)/t$. Similarly, the angular displacement can be found by using Eq. 10-13: $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$.

ANALYZE (a) To obtain the requested units, we use t = 30 s = 0.50 min. With $\omega_0 = 33.33\ \text{rev/min}$, we find the angular acceleration to be

$$\alpha = -\frac{33.33\ \text{rev/min}}{0.50\,\text{min}} = -66.7\ \text{rev/min}^2 \approx -67\ \text{rev/min}^2.$$

(b) Substituting the value of α obtained above into Eq. 10-13, we get

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (33.33\ \text{rev/min})(0.50\ \text{min}) + \frac{1}{2}(-66.7\text{rev/min}^2)(0.50\ \text{min})^2 = 8.33\ \text{rev}.$$

LEARN To solve for the angular displacement in (b), we may also use Eq. 10-15:

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(33.33\ \text{rev/min} + 0)(0.50\ \text{min}) = 8.33\ \text{rev}.$$

78. We use conservation of mechanical energy. The center of mass is at the midpoint of the cross bar of the **H** and it drops by $L/2$, where L is the length of any one of the rods. The gravitational potential energy decreases by $MgL/2$, where M is the mass of the body. The initial kinetic energy is zero and the final kinetic energy may be written $\frac{1}{2}I\omega^2$, where I is the rotational inertia of the body and ω is its angular velocity when it is vertical. Thus,

$$0 = -MgL/2 + \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{MgL/I}.$$

Since the rods are thin the one along the axis of rotation does not contribute to the rotational inertia. All points on the other leg are the same distance from the axis of rotation, so that leg contributes $(M/3)L^2$, where $M/3$ is its mass. The cross bar is a rod that rotates around one end, so its contribution is $(M/3)L^2/3 = ML^2/9$. The total rotational inertia is

$$I = (ML^2/3) + (ML^2/9) = 4ML^2/9.$$

Consequently, the angular velocity is

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{4ML^2/9}} = \sqrt{\frac{9g}{4L}} = \sqrt{\frac{9(9.800\ \text{m/s}^2)}{4(0.600\ \text{m})}} = 6.06\ \text{rad/s}.$$

79. **THINK** In this problem we compare the rotational inertia between a solid cylinder and a hoop.

EXPRESS According to Table 10-2, the rotational inertia formulas for a cylinder of radius *R* and mass *M*, and a hoop of radius *r* and mass *M* are

$$I_C = \frac{1}{2}MR^2, \quad I_H = Mr^2.$$

Equating $I_C = I_H$ allows us to deduce the relationship between *r* and *R*.

ANALYZE (a) Since both the cylinder and the hoop have the same mass, then they will have the same rotational inertia ($I_C = I_H$) if $R^2/2 = r^2 \;\rightarrow\; r = R/\sqrt{2}$.

(b) We require the rotational inertia of any given body to be written as $I = Mk^2$, where *M* is the mass of the given body and *k* is the radius of the "equivalent hoop." It follows directly that $k = \sqrt{I/M}$.

LEARN Listed below are some examples of equivalent hoop and their radii:

$$I_C = \frac{1}{2}MR^2 = M(R/\sqrt{2})^2 \quad \Rightarrow \quad k_C = R/\sqrt{2}$$
$$I_S = \frac{2}{5}MR^2 = M\left(\sqrt{\frac{2}{5}}R\right)^2 \quad \Rightarrow \quad k_S = \sqrt{\frac{2}{5}}R$$

80. (a) Using Eq. 10-15, we have 60.0 rad = $\frac{1}{2}(\omega_1 + \omega_2)$(6.00 s) . With ω_2 = 15.0 rad/s, then ω_1 = 5.00 rad/s.

(b) Eq. 10-12 gives α = (15.0 rad/s – 5.0 rad/s)/(6.00 s) = 1.67 rad/s^2.

(c) Interpreting ω now as ω_1 and θ as θ_1 = 10.0 rad (and ω_o = 0) Eq. 10-14 leads to

$$\theta_o = -\frac{\omega_1^2}{2\alpha} + \theta_1 = 2.50 \text{ rad} .$$

81. The center of mass is initially at height $h = \frac{L}{2}\sin 40°$ when the system is released (where L = 2.0 m). The corresponding potential energy *Mgh* (where *M* = 1.5 kg) becomes rotational kinetic energy $\frac{1}{2}I\omega^2$ as it passes the horizontal position (where *I* is the rotational inertia about the pin). Using Table 10-2 (e) and the parallel axis theorem, we find

$$I = \frac{1}{12}ML^2 + M(L/2)^2 = \frac{1}{3}ML^2.$$

Therefore,

$$Mg\frac{L}{2}\sin 40° = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \quad\Rightarrow\quad \omega = \sqrt{\frac{3g\sin 40°}{L}} = 3.1 \text{ rad/s.}$$

82. The rotational inertia of the passengers is (to a good approximation) given by Eq. 10-53: $I = \sum mR^2 = NmR^2$ where N is the number of people and m is the (estimated) mass per person. We apply Eq. 10-52:

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}NmR^2\omega^2$$

where R = 38 m and $N = 36 \times 60 = 2160$ persons. The rotation rate is constant so that $\omega = \theta/t$ which leads to $\omega = 2\pi/120 = 0.052$ rad/s. The mass (in kg) of the average person is probably in the range $50 \le m \le 100$, so the work should be in the range

$$\frac{1}{2}(2160)(50)(38)^2(0.052)^2 \le W \le \frac{1}{2}(2160)(100)(38)^2(0.052)^2$$
$$2\times10^5 \text{ J} \le W \le 4\times10^5 \text{ J.}$$

83. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_1 = a_2 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose upward positive for m_1, downward positive for m_2, and (somewhat unconventionally) clockwise for positive sense of disk rotation. Applying Newton's second law to m_1m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations.

$$T_1 - m_1 g = m_1 a_1$$
$$m_2 g - T_2 = m_2 a_2$$
$$T_2 R - T_1 R = I\alpha$$

(a) The rotational inertia of the disk is $I = \frac{1}{2}MR^2$ (Table 10-2(c)), so we divide the third equation (above) by R, add them all, and use the earlier equality among accelerations — to obtain:

$$m_2 g - m_1 g = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$

which yields $a = \frac{4}{25}g = 1.57\ \text{m/s}^2$.

(b) Plugging back in to the first equation, we find

$$T_1 = \frac{29}{25}m_1 g = 4.55 \text{ N}$$

where it is important in this step to have the mass in SI units: m_1 = 0.40 kg.

(c) Similarly, with m_2 = 0.60 kg, we find $T_2 = \frac{5}{6} m_2 g = 4.94\ \text{N}$.

84. (a) The longitudinal separation between Helsinki and the explosion site is $\Delta\theta = 102° - 25° = 77°$. The spin of the Earth is constant at

$$\omega = \frac{1\ \text{rev}}{1\ \text{day}} = \frac{360°}{24\ \text{h}}$$

so that an angular displacement of $\Delta\theta$ corresponds to a time interval of

$$\Delta t = (77°)\left(\frac{24\ \text{h}}{360°}\right) = 5.1\ \text{h}.$$

(b) Now $\Delta\theta = 102° - (-20°) = 122°$ so the required time shift would be

$$\Delta t = (122°)\left(\frac{24\ \text{h}}{360°}\right) = 8.1\ \text{h}.$$

85. To get the time to reach the maximum height, we use Eq. 4-23, setting the left-hand side to zero. Thus, we find

$$t = \frac{(60\ \text{m/s})\sin(20^{\circ})}{9.8\ \text{m/s}^2} = 2.094\ \text{s}.$$

Then (assuming α = 0) Eq. 10-13 gives

$$\theta - \theta_o = \omega_o t = (90\ \text{rad/s})(2.094\ \text{s}) = 188\ \text{rad},$$

which is equivalent to roughly 30 rev.

86. In the calculation below, M_1 and M_2 are the ring masses, R_{1i} and R_{2i} are their inner radii, and R_{1o} and R_{2o} are their outer radii. Referring to item (b) in Table 10-2, we compute

$$I = \tfrac{1}{2} M_1 (R_{1i}{}^2 + R_{1o}{}^2) + \tfrac{1}{2} M_2 (R_{2i}{}^2 + R_{2o}{}^2) = 0.00346\ \text{kg}\cdot\text{m}^2 .$$

Thus, with Eq. 10-38 ($\tau = rF$ where $r = R_{2o}$) and $\tau = I\alpha$ (Eq. 10-45), we find

$$\alpha = \frac{(0.140)(12.0)}{0.00346} = 485\ \text{rad/s}^2 .$$

Then Eq. 10-12 gives $\omega = \alpha t$ = 146 rad/s.

87. We choose positive coordinate directions so that each is accelerating positively, which will allow us to set $a_{\text{box}} = R\alpha$ (for simplicity, we denote this as a). Thus, we choose downhill positive for the m = 2.0 kg box and (as is conventional) counterclockwise for positive sense of wheel rotation. Applying Newton's second law to the box and (in the form of Eq. 10-45) to the wheel, respectively, we arrive at the following two equations (using θ as the incline angle 20°, not as the angular displacement of the wheel).

$$mg\sin\theta - T = ma$$
$$TR = I\alpha$$

Since the problem gives a = 2.0 m/s^2, the first equation gives the tension $T = m(g\sin\theta - a)$ = 2.7 N. Plugging this and R = 0.20 m into the second equation (along with the fact that $\alpha = a/R$) we find the rotational inertia

$$I = TR^2/a = 0.054\ \text{kg}\cdot\text{m}^2.$$

88. (a) We use $\tau = I\alpha$, where τ is the net torque acting on the shell, I is the rotational inertia of the shell, and α is its angular acceleration. Therefore,

$$I = \frac{\tau}{\alpha} = \frac{960\ \text{N}\cdot\text{m}}{6.20\ \text{rad/s}^2} = 155\ \text{kg}\cdot\text{m}^2.$$

(b) The rotational inertia of the shell is given by $I = (2/3)\,MR^2$ (see Table 10-2 of the text). This implies

$$M = \frac{3I}{2R^2} = \frac{3\left(155\ \text{kg}\cdot\text{m}^2\right)}{2\left(1.90\ \text{m}\right)^2} = 64.4\ \text{kg}.$$

89. Equation 10-40 leads to $\tau = mgr$ = (70 kg) (9.8 m/s^2) (0.20 m) = 1.4×10^2 N·m.

90. (a) Equation 10-12 leads to $\alpha = -\omega_o/t = -(25.0\ \text{rad/s})/(20.0\ \text{s}) = -1.25\ \text{rad/s}^2$.

(b) Equation 10-15 leads to $\theta = \frac{1}{2}\omega_o t = \frac{1}{2}(25.0\ \text{rad/s})(20.0\ \text{s}) = 250\ \text{rad}$.

(c) Dividing the previous result by 2π we obtain θ = 39.8 rev.

91. **THINK** As the box falls, gravitational force gives rise to a torque that causes the wheel to rotate.

EXPRESS We employ energy methods to solve this problem; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, where $K_{\text{box}} = m_{\text{box}}v^2/2$. The rotational kinetic energy of the wheel is $K_{\text{rot}} = I\omega^2/2$.

ANALYZE (a) With $K_{\text{box}} = 0.60\ \text{J}$, we find the speed of the box to be

$$K_{\text{box}} = \frac{1}{2}m_{\text{box}}v^2 \Rightarrow v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = \sqrt{\frac{2(6.0\ \text{J})}{6.0\ \text{kg}}} = 1.41\ \text{m/s},$$

implying that the angular speed is ω = (1.41 m/s)/(0.20 m) = 7.07 rad/s. Thus, the kinetic energy of rotation is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.40\ \text{kg}\cdot\text{m}^2)(7.07\ \text{rad/s})^2 = 10.0\ \text{J}.$$

(b) Since it was released from rest, we will take the initial position to be our reference point for gravitational potential. Energy conservation requires

$$K_0 + U_0 = K + U \quad\Rightarrow\quad 0 + 0 = (6.0\ \text{J} + 10.0\ \text{J}) + m_{\text{box}}g(-h).$$

Therefore,

$$h = \frac{K}{m_{\text{box}}g} = \frac{6.0\ \text{J} + 10.0\ \text{J}}{(6.0\ \text{kg})(9.8\ \text{m/s}^2)} = 0.27\ \text{m}.$$

LEARN As the box falls, its gravitational potential energy gets converted into kinetic energy of the box as well as rotational kinetic energy of the wheel; the total energy remains conserved.

92. (a) The time for one revolution is the circumference of the orbit divided by the speed v of the Sun: $T = 2\pi R/v$, where R is the radius of the orbit. We convert the radius:

$$R = (2.3\times10^4\ \text{ly})(9.46\times10^{12}\ \text{km/ly}) = 2.18\times10^{17}\ \text{km}$$

where the ly $\leftrightarrow$ km conversion can be found in Appendix D or figured “from basics” (knowing the speed of light). Therefore, we obtain

$$T = \frac{2\pi(2.18\times10^{17}\ \text{km})}{250\ \text{km/s}} = 5.5\times10^{15}\ \text{s}.$$

(b) The number of revolutions N is the total time t divided by the time T for one revolution; that is, $N = t/T$. We convert the total time from years to seconds and obtain

$$N = \frac{(4.5\times10^9\ \text{y})(3.16\times10^7\ \text{s/y})}{5.5\times10^{15}\ \text{s}} = 26.$$

93. **THINK** The applied force P accelerates the block. In addition, it gives rise to a torque that causes the wheel to undergo angular acceleration.

EXPRESS We take rightward to be positive for the block and clockwise negative for the wheel (as is conventional). With this convention, we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P - T = ma$, where T is the tension in the cord. Similarly, applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$. Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \Rightarrow T = a\frac{I}{R^2}.$$

Adding this to the above equation (for the block) leads to $P = (m + I/R^2)a$. Thus, the angular acceleration is

$$\alpha = -\frac{a}{R} = -\frac{P}{(m + I/R^2)R}$$

ANALYZE With $m = 2.0$ kg, $I = 0.050\ \text{kg}\cdot\text{m}^2$, $P = 3.0$ N and $R = 0.20$ m, we find

$$\alpha = -\frac{P}{(m + I/R^2)R} = -\frac{3.0\ \text{N}}{[2.0\ \text{kg} + (0.050\ \text{kg}\cdot\text{m}^2)/(0.20\ \text{m})^2](0.20\ \text{m})} = -4.62\ \text{rad/s}^2\ ,$$

or $|\alpha| = 4.62\ \text{rad/s}^2$.

LEARN The greater the applied force P, the greater the (magnitude of) angular acceleration. Note that the negative sign in α should not be mistaken for a deceleration; it simply indicates the clockwise sense to the motion.

94. First, we convert the angular velocity: ω = (2000 rev/min)(2π/60) = 209 rad/s. Also, we convert the plane's speed to SI units: (480)(1000/3600) = 133 m/s. We use Eq. 10-18 in part (a) and (implicitly) Eq. 4-39 in part (b).

(a) The speed of the tip as seen by the pilot is $v_t = \omega r = (209\ \text{rad/s})(1.5\ \text{m}) = 314\ \text{m/s}$, which (since the radius is given to only two significant figures) we write as $v_t = 3.1\times10^2\ \text{m/s}$.

(b) The plane's velocity $\vec{v}_p$ and the velocity of the tip $\vec{v}_t$ (found in the plane's frame of reference), in any of the tip's positions, must be perpendicular to each other. Thus, the speed as seen by an observer on the ground is

$$v = \sqrt{v_p^2 + v_t^2} = \sqrt{(133\ \text{m/s})^2 + (314\ \text{m/s})^2} = 3.4\times10^2\ \text{m/s}\ .$$

95. The distances from P to the particles are as follows:

$$r_1 = a \text{ for } m_1 = 2M(\text{lower left})$$
$$r_2 = \sqrt{b^2 - a^2} \text{ for } m_2 = M(\text{top})$$
$$r_3 = a \text{ for } m_1 = 2M(\text{lower right})$$

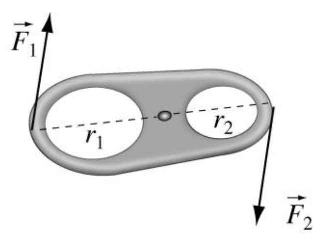
The rotational inertia of the system about P is

$$I = \sum_{i=1}^{3} m_i r_i^2 = \left(3a^2 + b^2\right)M,$$

which yields $I = 0.208\,\text{kg}\cdot\text{m}^2$ for M = 0.40 kg, a = 0.30 m, and b = 0.50 m. Applying Eq. 10-52, we find

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(0.208\,\text{kg}\cdot\text{m}^2\right)\left(5.0\text{ rad/s}\right)^2 = 2.6\text{ J}.$$

96. In the figure below, we show a pull tab of a beverage can. Since the tab is pivoted, when pulling on one end upward with a force $\vec{F}_1$, a force $\vec{F}_2$ will be exerted on the other end. The torque produced by $\vec{F}_1$ must be balanced by the torque produced by $\vec{F}_2$ so that the tab does not rotate.



The two forces are related by

$$r_1F_1 = r_2F_2$$

where $r_1 \approx 1.8$ cm and $r_2 \approx 0.73$ cm. Thus, if F_1 = 10 N,

$$F_2 = \left(\frac{r_1}{r_2}\right)F_1 \approx \left(\frac{1.8\text{ cm}}{0.73\text{ cm}}\right)(10\text{ N}) \approx 25\text{ N}.$$

97. The centripetal acceleration at a point P that is r away from the axis of rotation is given by Eq. 10-23: $a = v^2/r = \omega^2 r$, where $v = \omega r$, with $\omega = 2000$ rev/min $\approx$ 209.4 rad/s.

(a) If points A and P are at a radial distance r_A = 1.50 m and r = 0.150 m from the axis, the difference in their acceleration is

$$\Delta a = a_A - a = \omega^2(r_A - r) = (209.4\text{ rad/s})^2(1.50\text{ m} - 0.150\text{ m}) \approx 5.92\times10^4\,\text{m/s}^2.$$

(b) The slope is given by $a/r = \omega^2 = 4.39\times10^4 /\text{s}^2$.

98. Let T be the tension on the rope. From Newton's second law, we have

$$T - mg = ma \;\;\Rightarrow\;\; T = m(g+a).$$

Since the box has an upward acceleration a = 0.80 m/s^2, the tension is given by

$$T = (30\text{ kg})(9.8\text{ m/s}^2 + 0.8\text{ m/s}^2) = 318\text{ N}.$$

The rotation of the device is described by $F_{\text{app}}R - Tr = I\alpha = Ia/r$. The moment of inertia can then be obtained as

$$I = \frac{r(F_{\text{app}}R - Tr)}{a} = \frac{(0.20\text{ m})[(140\text{ N})(0.50\text{ m}) - (318\text{ N})(0.20\text{ m})]}{0.80\text{ m/s}^2} = 1.6\text{ kg}\cdot\text{m}^2$$

99. (a) With r = 0.780 m, the rotational inertia is

$$I = Mr^2 = (1.30\text{ kg})(0.780\text{ m})^2 = 0.791\text{ kg}\cdot\text{m}^2.$$

(b) The torque that must be applied to counteract the effect of the drag is

$$\tau = rf = (0.780\text{ m})(2.30\times10^{-2}\text{ N}) = 1.79\times10^{-2}\text{ N}\cdot\text{m}.$$

100. We make use of Table 10-2(e) as well as the parallel-axis theorem, Eq. 10-34, where needed. We use ℓ (as a subscript) to refer to the long rod and s to refer to the short rod.

(a) The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12}m_sL_s^2 + \frac{1}{3}m_\ell L_\ell^2 = 0.019\text{ kg}\cdot\text{m}^2.$$

(b) We note that the center of the short rod is a distance of h = 0.25 m from the axis. The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12}m_sL_s^2 + m_sh^2 + \frac{1}{12}m_\ell L_\ell^2$$

which again yields I = 0.019 kg·m^2.

101. (a) The linear speed of a point on belt 1 is

$$v_1 = r_A\omega_A = (15\text{ cm})(10\text{ rad/s}) = 1.5\times10^2\text{ cm/s}.$$

(b) The angular speed of pulley *B* is

$$r_B\omega_B = r_A\omega_A \quad \Rightarrow \quad \omega_B = \frac{r_A\omega_A}{r_B} = \left(\frac{15\ \text{cm}}{10\ \text{cm}}\right)(10\ \text{rad/s}) = 15\ \text{rad/s}\,.$$

(c) Since the two pulleys are rigidly attached to each other, the angular speed of pulley B' is the same as that of pulley *B*, that is, $\omega_B' = 15\ \text{rad/s}$.

(d) The linear speed of a point on belt 2 is

$$v_2 = r_{B'}\omega_B' = (5\ \text{cm})(15\ \text{rad/s}) = 75\ \text{cm/s}\,.$$

(e) The angular speed of pulley *C* is

$$r_C\omega_C = r_{B'}\omega_B' \quad \Rightarrow \quad \omega_C = \frac{r_{B'}\omega_B'}{r_C} = \left(\frac{5\ \text{cm}}{25\ \text{cm}}\right)(15\ \text{rad/s}) = 3.0\ \text{rad/s}$$

102. (a) The rotational inertia relative to the specified axis is

$$I = \sum m_i r_i^2 = (2M)L^2 + (2M)L^2 + M(2L)^2$$

which is found to be I = 4.6 $\text{kg}\cdot\text{m}^2$. Then, with ω = 1.2 rad/s, we obtain the kinetic energy from Eq. 10-34:

$$K = \frac{1}{2}I\omega^2 = 3.3\ \text{J}.$$

(b) In this case the axis of rotation would appear as a standard *y* axis with origin at *P*. Each of the 2*M* balls are a distance of $r = L\cos 30°$ from that axis. Thus, the rotational inertia in this case is

$$I = \sum m_i r_i^2 = (2M)r^2 + (2M)r^2 + M(2L)^2$$

which is found to be I = 4.0 $\text{kg}\cdot\text{m}^2$. Again, from Eq. 10-34 we obtain the kinetic energy

$$K = \frac{1}{2}I\omega^2 = 2.9\ \text{J}.$$

103. We make use of Table 10-2(e) and the parallel-axis theorem in Eq. 10-36.

(a) The moment of inertia is

$$I = \frac{1}{12}ML^2 + Mh^2 = \frac{1}{12}(3.0\ \text{kg})(4.0\ \text{m})^2 + (3.0\ \text{kg})(1.0\ \text{m})^2 = 7.0\ \text{kg}\cdot\text{m}^2.$$

(b) The rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2} I\omega^2 \;\;\Rightarrow\;\; \omega = \sqrt{\frac{2K_{\text{rot}}}{I}} = \sqrt{\frac{2(20\text{ J})}{7\text{ kg}\cdot\text{m}^2}} = 2.4\text{ rad/s}.$$

The linear speed of the end *B* is given by $v_B = \omega r_{AB} = (2.4\text{ rad/s})(3.00\text{ m}) = 7.2\text{ m/s}$, where r_{AB} is the distance between *A* and *B*.

(c) The maximum angle θ is attained when all the rotational kinetic energy is transformed into potential energy. Moving from the vertical position ($\theta = 0$) to the maximum angle θ, the center of mass is elevated by $\Delta y = d_{AC}(1-\cos\theta)$, where d_{AC} = 1.00 m is the distance between *A* and the center of mass of the rod. Thus, the change in potential energy is

$$\Delta U = mg\Delta y = mgd_{AC}(1-\cos\theta) \quad\Rightarrow\quad 20\text{ J} = (3.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m})(1-\cos\theta)$$

which yields $\cos\theta = 0.32$, or $\theta \approx 71°$.

104. (a) The particle at *A* has $r = 0$ with respect to the axis of rotation. The particle at *B* is $r = L = 0.50$ m from the axis; similarly for the particle directly above *A* in the figure. The particle diagonally opposite *A* is a distance $r = \sqrt{2}L = 0.71\text{ m}$ from the axis. Therefore,

$$I = \sum m_i r_i^2 = 2mL^2 + m\left(\sqrt{2}L\right)^2 = 0.20\text{ kg}\cdot\text{m}^2.$$

(b) One imagines rotating the figure (about point *A*) clockwise by 90° and noting that the center of mass has fallen a distance equal to *L* as a result. If we let our reference position for gravitational potential be the height of the center of mass at the instant *AB* swings through vertical orientation, then

$$K_0 + U_0 = K + U \;\;\Rightarrow\;\; 0 + (4m)gh_0 = K + 0.$$

Since $h_0 = L = 0.50$ m, we find $K = 3.9$ J. Then, using Eq. 10-34, we obtain

$$K = \frac{1}{2} I_A \omega^2 \Rightarrow \omega = 6.3\text{ rad/s}.$$

105. (a) We apply Eq. 10-18, using the subscript J for the Jeep.

$$\omega = \frac{v_J}{r_J} = \frac{114\text{ km/h}}{0.100\text{ km}}$$

which yields 1140 rad/h or (dividing by 3600) 0.32 rad/s for the value of the angular speed ω.

(b) Since the cheetah has the same angular speed, we again apply Eq. 10-18, using the subscript c for the cheetah.

$$v_c = r_c\omega = (92\,\text{m})\,(1140\ \text{rad/h}) = 1.048 \times 10^5\ \text{m/h} \approx 1.0 \times 10^2\ \text{km/h}$$

for the cheetah's speed.

106. Using Eq. 10-7 and Eq. 10-18, the average angular acceleration is

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{r\Delta t} = \frac{25-12}{(0.75/2)(6.2)} = 5.6\,\text{rad/s}^2\ .$$

107. (a) Using Eq. 10-1, the angular displacement is

$$\theta = \frac{5.6\,\text{m}}{8.0\times 10^{-2}\ \text{m}} = 1.4\times 10^2\ \text{rad}\ .$$

(b) We use $\theta = \frac{1}{2}\alpha t^2$ (Eq. 10-13) to obtain t:

$$t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2\left(1.4\times 10^2\ \text{rad}\right)}{1.5\,\text{rad/s}^2}} = 14\,\text{s}\ .$$

108. (a) We obtain

$$\omega = \frac{(33.33\ \text{rev/min})\ (2\pi\ \text{rad/rev})}{60\ \text{s/min}} = 3.5\ \text{rad/s}.$$

(b) Using Eq. 10-18, we have $v = r\omega = (15)(3.49) = 52\ \text{cm/s}$.

(c) Similarly, when r = 7.4 cm we find $v = r\omega$ = 26 cm/s. The goal of this exercise is to observe what is and is not the same at different locations on a body in rotational motion (ω is the same, v is not), as well as to emphasize the importance of radians when working with equations such as Eq. 10-18.

Chapter 11

1. The velocity of the car is a constant

$$\vec{v} = +\left(80 \text{ km/h}\right)(1000 \text{ m/km})(1 \text{ h/3600 s})\,\hat{\text{i}} = (+22\,\text{m/s})\hat{\text{i}},$$

and the radius of the wheel is $r = 0.66/2 = 0.33$ m.

(a) In the car's reference frame (where the lady perceives herself to be at rest) the road is moving toward the rear at $\vec{v}_{\text{road}} = -v = -22\,\text{m/s}$, and the motion of the tire is purely rotational. In this frame, the center of the tire is "fixed" so $v_{\text{center}} = 0$.

(b) Since the tire's motion is only rotational (not translational) in this frame, Eq. 10-18 gives $\vec{v}_{\text{top}} = (+22\,\text{m/s})\hat{\text{i}}$.

(c) The bottom-most point of the tire is (momentarily) in firm contact with the road (not skidding) and has the same velocity as the road: $\vec{v}_{\text{bottom}} = (-22\,\text{m/s})\hat{\text{i}}$. This also follows from Eq. 10-18.

(d) This frame of reference is not accelerating, so "fixed" points within it have zero acceleration; thus, $a_{\text{center}} = 0$.

(e) Not only is the motion purely rotational in this frame, but we also have ω = constant, which means the only acceleration for points on the rim is radial (centripetal). Therefore, the magnitude of the acceleration is

$$a_{\text{top}} = \frac{v^2}{r} = \frac{(22 \text{ m/s})^2}{0.33 \text{ m}} = 1.5\times 10^3 \text{ m/s}^2 .$$

(f) The magnitude of the acceleration is the same as in part (d): $a_{\text{bottom}} = 1.5 \times 10^3$ m/s^2.

(g) Now we examine the situation in the road's frame of reference (where the road is "fixed" and it is the car that appears to be moving). The center of the tire undergoes purely translational motion while points at the rim undergo a combination of translational and rotational motions. The velocity of the center of the tire is $\vec{v} = (+22\,\text{m/s})\hat{\text{i}}$.

(h) In part (b), we found $\vec{v}_{\text{top,car}} = +v$ and we use Eq. 4-39:

$$\vec{v}_{\text{top, ground}} = \vec{v}_{\text{top, car}} + \vec{v}_{\text{car, ground}} = v\,\hat{\text{i}} + v\,\hat{\text{i}} = 2v\,\hat{\text{i}}$$

which yields $2v$ = +44 m/s.

(i) We can proceed as in part (h) or simply recall that the bottom-most point is in firm contact with the (zero-velocity) road. Either way, the answer is zero.

(j) The translational motion of the center is constant; it does not accelerate.

(k) Since we are transforming between constant-velocity frames of reference, the accelerations are unaffected. The answer is as it was in part (e): 1.5×10^3 m/s^2.

(1) As explained in part (k), $a = 1.5 \times 10^3$ m/s^2.

2. The initial speed of the car is

$$v = (80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 22.2 \text{ m/s}.$$

The tire radius is R = 0.750/2 = 0.375 m.

(a) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 11-2 leads to

$$\omega_0 = \frac{v_{\text{com}0}}{R} = \frac{22.2 \text{ m/s}}{0.375 \text{ m}} = 59.3 \text{ rad/s}.$$

(b) With θ = (30.0)(2π) = 188 rad and ω = 0, Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \implies |\alpha| = \frac{(59.3 \text{ rad/s})^2}{2(188 \text{ rad})} = 9.31 \text{ rad/s}^2.$$

(c) Equation 11-1 gives $R\theta$ = 70.7 m for the distance traveled.

3. **THINK** The work required to stop the hoop is the negative of the initial kinetic energy of the hoop.

EXPRESS From Eq. 11-5, the initial kinetic energy of the hoop is $K_i = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, where $I = mR^2$ is its rotational inertia about the center of mass. Eq. 11-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K_i = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2 + \frac{1}{2}mv^2 = mv^2$$

ANALYZE With m = 140 kg, and the speed of its center of mass v = 0.150 m/s, we find the initial kinetic energy to be

$$K_i = mv^2 = (140\ \text{kg})(0.150\ \text{m/s})^2 = 3.15\ \text{J}$$

which implies that the work required is $W = \Delta K = K_f - K_i = -K_i = -3.15$ J .

LEARN By the work-kinetic energy theorem, the work done is negative since it decreases the kinetic energy. A rolling body has two types of kinetic energy: rotational and translational.

4. We use the results from section 11.3.

(a) We substitute $I = \frac{2}{5} M R^2$ (Table 10-2(f)) and $a = -0.10g$ into Eq. 11-10:

$$-0.10g = -\frac{g\sin\theta}{1+\left(\frac{2}{5}MR^2\right)/MR^2} = -\frac{g\sin\theta}{7/5}$$

which yields $\theta = \sin^{-1}(0.14) = 8.0°$.

(b) The acceleration would be more. We can look at this in terms of forces or in terms of energy. In terms of forces, the uphill static friction would then be absent so the downhill acceleration would be due only to the downhill gravitational pull. In terms of energy, the rotational term in Eq. 11-5 would be absent so that the potential energy it started with would simply become $\frac{1}{2}mv^2$ (without it being "shared" with another term) resulting in a greater speed (and, because of Eq. 2-16, greater acceleration).

5. Let M be the mass of the car (presumably including the mass of the wheels) and v be its speed. Let I be the rotational inertia of one wheel and ω be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2}I\omega^2\right),$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by

$$K = \tfrac{1}{2}Mv^2 + 4(\tfrac{1}{2}I\omega^2)\ .$$

The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}\ .$$

For a uniform disk (relative to its center of mass) $I = \frac{1}{2}mR^2$ (Table 10-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 11-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50} = 0.020.$$

The wheel radius cancels from the equations and is not needed in the computation.

6. We plug $a = -3.5$ m/s^2 (where the magnitude of this number was estimated from the "rise over run" in the graph), $\theta = 30°$, $M = 0.50$ kg, and $R = 0.060$ m into Eq. 11-10 and solve for the rotational inertia. We find $I = 7.2 \times 10^{-4}$ kg·m^2.

7. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0 \sin 30° = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves the roof) is (Eq. 11-5)

$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2.$$

Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \tfrac{1}{2}MR^2$ (Table 10-2(c)), conservation of energy leads to

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 = \frac{3}{4}MR^2\omega^2.$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10\text{ m}}\sqrt{\frac{4}{3}(9.8\text{ m/s}^2)(3.0\text{ m})} = 63\text{ rad/s}.$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$v_{0x} = v_0 \cos 30° = 5.4\text{ m/s}$$
$$v_{0y} = v_0 \sin 30° = 3.1\text{ m/s}.$$

The projectile motion equations become

$$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 .$$

We first find the time when $y = H = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gH}}{g} = 0.74\,\text{s}.$$

Then we substitute this into the x equation and obtain $x = (5.4 \text{ m/s})(0.74 \text{ s}) = 4.0 \text{ m}.$

8. (a) Let the turning point be designated P. By energy conservation, the mechanical energy at $x = 7.0$ m is equal to the mechanical energy at P. Thus, with Eq. 11-5, we have

$$75 \text{ J} = \tfrac{1}{2}mv_p^2 + \tfrac{1}{2}I_{com}\,\omega_p^2 + U_p .$$

Using item (*f*) of Table 10-2 and Eq. 11-2 (which means, if this is to be a turning point, that $\omega_p = v_p = 0$), we find $U_p = 75$ J. On the graph, this seems to correspond to $x = 2.0$ m, and we conclude that there is a turning point (and this is it). The ball, therefore, does not reach the origin.

(b) We note that there is no point (on the graph, to the right of $x = 7.0$ m) taht is shown "higher" than 75 J, so we suspect that there is no turning point in this direction, and we seek the velocity v_p at $x = 13$ m. If we obtain a real, nonzero answer, then our suspicion is correct (that it does reach this point P at $x = 13$ m). By energy conservation, the mechanical energy at $x = 7.0$ m is equal to the mechanical energy at P. Therefore,

$$75 \text{ J} = \tfrac{1}{2}mv_p^2 + \tfrac{1}{2}I_{com}\,\omega_p^2 + U_p .$$

Again, using item (*f*) of Table 11-2, Eq. 11-2 (less trivially this time) and $U_p = 60$ J (from the graph), as well as the numerical data given in the problem, we find $v_p = 7.3$ m/s.

9. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = M\,gH$. Its final kinetic energy (as it leaves the track) is given by Eq. 11-5:

$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2$$

and its final potential energy is $M\,gh$. Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set $\omega = v/R$. Using $I = \tfrac{2}{5}MR^2$ (Table 10-2(f)), conservation of energy leads to

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh$$
$$= \frac{7}{10}Mv^2 + Mgh.$$

The mass M cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7}g(H-h)} = \sqrt{\frac{10}{7}\left(9.8\ \text{m/s}^2\right)\left(6.0\ \text{m} - 2.0\ \text{m}\right)} = 7.48\ \text{m/s}.$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take +x rightward and +y downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt, \quad y = -\frac{1}{2}gt^2.$$

Solving for x at the time when y = h, the second equation gives $t = \sqrt{2h/g}$. Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48\ \text{m/s})\sqrt{\frac{2(2.0\ \text{m})}{9.8\ \text{m/s}^2}} = 4.8\ \text{m}.$$

10. From $I = \frac{2}{3}MR^2$ (Table 10-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3\left(0.040\ \text{kg}\cdot\text{m}^2\right)}{2\left(0.15\ \text{m}\right)^2} = 2.7\ \text{kg}.$$

It also follows from the rotational inertia expression that $\frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{com} = R\omega$, and we find

$$\frac{K_{rot}}{K_{com} + K_{rot}} = \frac{\frac{1}{3}MR^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{3}MR^2\omega^2}.$$

(a) Simplifying the above ratio, we find $K_{rot}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or

$$K_{rot} = (0.4)(20\ \text{J}) = 8.0\ \text{J}.$$

(b) From $K_{rot} = \frac{1}{3}M\,R^2\omega^2 = 8.0\,\text{J}$ (and using the above result for M) we find

$$\omega = \frac{1}{0.15 \text{ m}}\sqrt{\frac{3(8.0 \text{ J})}{2.7 \text{ kg}}} = 20 \text{ rad/s}$$

which leads to v_{com} = (0.15 m)(20 rad/s) = 3.0 m/s.

(c) We note that the inclined distance of 1.0 m corresponds to a height h = 1.0 sin 30° = 0.50 m. Mechanical energy conservation leads to

$$K_i = K_f + U_f \quad \Rightarrow \quad 20\,\text{J} = K_f + Mgh$$

which yields (using the values of M and h found above) K_f = 6.9 J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3}MR^2\omega_f^2 = (0.40)K_f \quad \Rightarrow \quad \omega_f = \frac{1}{0.15 \text{ m}}\sqrt{\frac{3(0.40)(6.9 \text{ J})}{2.7 \text{ kg}}}$$

which yields ω_f = 12 rad/s and leads to

$$v_{com\,f} = R\omega_f = (0.15 \text{ m})(12 \text{ rad/s}) = 1.8 \text{ m/s.}$$

11. With $\vec{F}_{app} = (10 \text{ N})\hat{\text{i}}$, we solve the problem by applying Eq. 9-14 and Eq. 11-37.

(a) Newton's second law in the x direction leads to

$$F_{app} - f_s = ma \quad \Rightarrow f_s = 10\,\text{N} - (10\,\text{kg})\left(0.60 \text{ m/s}^2\right) = 4.0 \text{ N.}$$

In unit vector notation, we have $\vec{f}_s = (-4.0 \text{ N})\hat{\text{i}}$, which points leftward.

(b) With R = 0.30 m, we find the magnitude of the angular acceleration to be

$$|\alpha| = |a_{com}| / R = 2.0 \text{ rad/s}^2,$$

from Eq. 11-6. The only force not directed toward (or away from) the center of mass is $\vec{f}_s$, and the torque it produces is clockwise:

$$|\tau| = I\,|\alpha| \quad \Rightarrow \quad (0.30\,\text{m})(4.0\,\text{N}) = I\left(2.0\,\text{rad/s}^2\right)$$

which yields the wheel's rotational inertia about its center of mass: $I = 0.60\,\text{kg}\cdot\text{m}^2$.

12. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$U_{\text{release}} = K_{\text{top}} + U_{\text{top}} \quad \Rightarrow \quad mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg\left(2R\right).$$

Substituting $I = \frac{2}{5}mr^2$ (Table 10-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 11-2), we obtain

$$mgh = \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \quad \Rightarrow \quad gh = \frac{7}{10}v_{\text{com}}^2 + 2gR$$

where we have canceled out mass *m* in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is nearly zero. In this case, Newton's second law along the vertical direction (+*y* downward) leads to

$$mg = ma_r \Rightarrow g = \frac{v_{\text{com}}^2}{R-r}$$

where we have used Eq. 10-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R - r$ from the center of the loop). Plugging the result $v_{\text{com}}^2 = g\left(R-r\right)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}\left(g\right)\left(R-r\right) + 2gR$$

which leads to $h = 2.7R - 0.7r \approx 2.7R.$ With R = 14.0 cm , we have

$$h = (2.7)(14.0 \text{ cm}) = 37.8 \text{ cm}.$$

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point *Q* (which, however, is only at a height of *R*) yielding the condition

$$g\left(6R\right) = \frac{7}{10}v_{\text{com}}^2 + gR$$

which gives us $v_{\text{com}}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at *Q* leads to

$$N = m\frac{v_{\text{com}}^2}{R-r} = m\frac{50gR}{7\left(R-r\right)}$$

which (for $R \gg r$) gives

$$N \approx \frac{50mg}{7} = \frac{50(2.80\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{7} = 1.96\times10^{-2}\ \text{N}.$$

(b) The direction is toward the center of the loop.

13. The physics of a rolling object usually requires a separate and very careful discussion (above and beyond the basics of rotation discussed in Chapter 10); this is done in the first three sections of Chapter 11. Also, the normal force on something (which is here the center of mass of the ball) following a circular trajectory is discussed in Section 6-6. Adapting Eq. 6-19 to the consideration of forces at the *bottom* of an arc, we have

$$F_N - Mg = Mv^2/r$$

which tells us (since we are given $F_N = 2Mg$) that the center of mass speed (squared) is $v^2 = gr$, where r is the arc radius (0.48 m) Thus, the ball's angular speed (squared) is

$$\omega^2 = v^2/R^2 = gr/R^2,$$

where R is the ball's radius. Plugging this into Eq. 10-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{\text{com}} = 2MhR^2/r - MR^2 = MR^2[2(0.36/0.48) - 1]\ .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2(0.36/0.48) - 1 = 0.50.$$

14. To find the center of mass speed v on the plateau, we use the projectile motion equations of Chapter 4. With $v_{oy} = 0$ (and using "h" for h_2) Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Now, to find the speed v_p at point P, we apply energy conservation, that is, mechanical energy on the plateau is equal to the mechanical energy at P. With Eq. 11-5, we obtain

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{\text{com}}\,\omega^2 + mgh_1 = \tfrac{1}{2}mv_p^{\,2} + \tfrac{1}{2}I_{\text{com}}\,\omega_p^{\,2}\ .$$

Using item (*f*) of Table 10-2, Eq. 11-2, and our expression (above) $v^2 = gd^2/2h$, we obtain

$$gd^2/2h + 10gh_1/7 = v_p^{\,2}$$

which yields (using the values stated in the problem) $v_p = 1.34$ m/s.

15. (a) We choose clockwise as the negative rotational sense and rightward as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 11-2 becomes

$$v_{\text{com}} = -R\omega = (-0.11\ \text{m})\omega.$$

This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in the figure.

(b) The force of friction exerted on the ball of mass m is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{com} leads to

$$a_{\text{com}} = -\mu g = -(0.21)\left(9.8 \text{ m/s}^2\right) = -2.1 \text{ m/s}^2$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating.

(c) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu mgR$. Using Table 10-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 10-45)

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)\left(9.8 \text{ m/s}^2\right)}{2(0.11 \text{ m})} = -47 \text{ rad/s}^2$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is “speeding up’’).

(d) The center of mass of the sliding ball decelerates from $v_{\text{com},0}$ to v_{com} during time t according to Eq. 2-11: $v_{\text{com}} = v_{\text{com},0} - \mu gt$. During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 10-12:

$$|\omega| = |\alpha| t = \frac{5\mu gt}{2R} = \frac{v_{\text{com}}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{\text{com},0}}{7\mu g} = \frac{2(8.5 \text{ m/s})}{7(0.21)\left(9.8 \text{ m/s}^2\right)} = 1.2 \text{ s}.$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{\text{com},0}t - \frac{1}{2}(\mu g)t^2 = (8.5 \text{ m/s})(1.2 \text{ s}) - \frac{1}{2}(0.21)\left(9.8 \text{ m/s}^2\right)(1.2 \text{ s})^2 = 8.6 \text{ m}.$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{\text{com}} = v_{\text{com},0} - \mu g t = 8.5\ \text{m/s} - (0.21)\left(9.8\ \text{m/s}^2\right)(1.2\ \text{s}) = 6.1\ \text{m/s}.$$

16. Using energy conservation with Eq. 11-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{\text{com}} = 2MhR^2/r - MR^2 = MR^2[2g(H-h)/v^2 - 1]\ .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2g(H-h)/v^2 - 1.$$

To proceed further, we need to find the center of mass speed v, which we do using the projectile motion equations of Chapter 4. With $v_{oy} = 0$, Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Plugging this into our expression for β gives

$$2g(H-h)/v^2 - 1 = 4h(H-h)/d^2 - 1.$$

Therefore, with the values given in the problem, we find β = 0.25.

17. **THINK** The yo-yo has both translational and rotational types of motion.

EXPRESS The derivation of the acceleration is given by Eq. 11-13:

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}} / MR_0^2}$$

where M is the mass of the yo-yo, I_{cm} is the rotational inertia and R_0 is the radius of the axel. The positive direction is upward. The time it takes for the yo-yo to reach the end of the string can be found by solving the kinematic equation $y_{\text{com}} = \frac{1}{2}a_{\text{com}}t^2$.

ANALYZE (a) With $I_{\text{com}} = 950\ \text{g}\cdot\text{cm}^2$, M =120 g, R_0 = 0.320 cm and g = 980 cm/s^2, we obtain

$$|a_{\text{com}}| = \frac{980\ \text{cm/s}^2}{1 + \left(950\ \text{g}\cdot\text{cm}^2\right)/(120\ \text{g})(0.32\ \text{cm})^2} = 12.5\ \text{cm/s}^2 \approx 13\ \text{cm/s}^2.$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{\text{com}} = \frac{1}{2}a_{\text{com}}t^2$. Thus, we set y_{com} = – 120 cm and find

$$t = \sqrt{\frac{2y_{\text{com}}}{a_{\text{com}}}} = \sqrt{\frac{2(-120\,\text{cm})}{-12.5\ \text{cm/s}^2}} = 4.38\ \text{s} \approx 4.4\ \text{s}.$$

(c) As the yo-yo reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{com} = a_{com}t = \left(-12.5\ \text{cm/s}^2\right)\left(4.38\,\text{s}\right) = -54.8\ \text{cm/s},$$

so its linear speed then is approximately $|v_{com}| = 55$ cm/s.

(d) The translational kinetic energy of the yo-yo is

$$K_{trans} = \frac{1}{2}mv_{com}^2 = \frac{1}{2}\left(0.120\ \text{kg}\right)\left(0.548\ \text{m/s}\right)^2 = 1.8\times10^{-2}\ \text{J}.$$

(e) The angular velocity is $\omega = -v_{com}/R_0$, so the rotational kinetic energy is

$$K_{rot} = \frac{1}{2}I_{com}\omega^2 = \frac{1}{2}I_{com}\left(\frac{v_{com}}{R_0}\right)^2 = \frac{1}{2}(9.50\times10^{-5}\ \text{kg}\cdot\text{m}^2)\left(\frac{0.548\ \text{m/s}}{3.2\times10^{-3}\,\text{m}}\right)^2$$
$$= 1.393\ \text{J} \approx 1.4\ \text{J}$$

(f) The angular speed is

$$\omega = \frac{|v_{com}|}{R_0} = \frac{0.548\ \text{m/s}}{3.2\times10^{-3}\ \text{m}} = 1.7\times10^2\ \text{rad/s} \ = 27\,\text{rev/s}.$$

LEARN As the yo-yo rolls down, its gravitational potential energy gets converted into both translational kinetic energy as well as rotational kinetic energy of the wheel. To show that the total energy remains conserved, we note that the initial energy is

$$U_i = Mgy_i = (0.120\,\text{kg})(9.80\,\text{m/s}^2)(1.20\ \text{m}) = 1.411\ \text{J}$$

which is equal to the sum of K_{trans} (= 0.018 J) and K_{rot} (= 1.393 J).

18. (a) The derivation of the acceleration is found in § 11-4; Eq. 11-13 gives

$$a_{com} = -\frac{g}{1 + I_{com}/MR_0^2}$$

where the positive direction is upward. We use $I_{com} = MR^2/2$ where the radius is R = 0.32 m and M = 116 kg is the *total* mass (thus including the fact that there are two disks) and obtain

$$a = -\frac{g}{1 + (MR^2/2)/MR_0^2} = \frac{g}{1 + \left(R/R_0\right)^2/2}$$

which yields $a = -g/51$ upon plugging in $R_0 = R/10 = 0.032$ m. Thus, the magnitude of the center of mass acceleration is 0.19 m/s^2.

(b) As observed in §11-4, our result in part (a) applies to both the descending and the rising yo-yo motions.

(c) The external forces on the center of mass consist of the cord tension (upward) and the pull of gravity (downward). Newton's second law leads to

$$T - Mg = ma \Rightarrow T = M\left(g - \frac{g}{51}\right) = 1.1 \times 10^3 \text{ N}.$$

(d) Our result in part (c) indicates that the tension is well below the ultimate limit for the cord.

(e) As we saw in our acceleration computation, all that mattered was the ratio R/R_0 (and, of course, g). So if it's a scaled-up version, then such ratios are unchanged and we obtain the same result.

(f) Since the tension also depends on mass, then the larger yo-yo will involve a larger cord tension.

19. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}},$ then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

With (using SI units) $x = 0$, $y = -4.0$, $z = 5.0$, $F_x = 0$, $F_y = -2.0$, and $F_z = 3.0$ (these latter terms being the individual forces that contribute to the net force), the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = (-2.0\,\text{N}\cdot\text{m})\hat{\text{i}}.$$

20. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}},$ then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

(a) In the above expression, we set (with SI units understood) $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 6.0$, $F_y = 0$, and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (24\,\text{N}\cdot\text{m})\hat{\text{j}}.$

(b) The values are just as in part (a) with the exception that now $F_x = -6.0$. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-24\,\text{N}\cdot\text{m})\hat{\text{j}}.$

(c) In the above expression, we set $x = -2.0$, $y = 0$, $z = 4.0$, $F_x = 0$, $F_y = 0$, and $F_z = 6.0$. We get $\vec{\tau} = \vec{r} \times \vec{F} = (12\,\text{N}\cdot\text{m})\hat{\text{j}}.$

(d) The values are just as in part (c) with the exception that now F_z = –6.0. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-12\,\mathrm{N\cdot m})\hat{\mathrm{j}}$.

21. If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

(a) In the above expression, we set (with SI units understood) x = 0, y = – 4.0, z = 3.0, F_x = 2.0, F_y = 0, and F_z = 0. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(6.0\hat{\mathrm{j}} + 8.0\hat{\mathrm{k}}\right)\mathrm{N\cdot m}.$$

This has magnitude $\sqrt{(6.0\ \mathrm{N\cdot m})^2 + (8.0\ \mathrm{N\cdot m})^2} = 10\ \mathrm{N\cdot m}$ and is seen to be parallel to the yz plane. Its angle (measured counterclockwise from the +y direction) is $\tan^{-1}\left(8/6\right) = 53°$.

(b) In the above expression, we set x = 0, y = – 4.0, z = 3.0, F_x = 0, F_y = 2.0, and F_z = 4.0. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (-22\,\mathrm{N\cdot m})\hat{\mathrm{i}}$. The torque has magnitude $22\ \mathrm{N\cdot m}$ and points in the –x direction.

22. Equation 11-14 (along with Eq. 3-30) gives

$$\vec{\tau} = \vec{r} \times \vec{F} = 4.00\hat{\mathrm{i}} + (12.0 + 2.00F_x)\hat{\mathrm{j}} + (14.0 + 3.00F_x)\hat{\mathrm{k}}$$

with SI units understood. Comparing this with the known expression for the torque (given in the problem statement), we see that F_x must satisfy two conditions:

$$12.0 + 2.00F_x = 2.00 \quad \text{and} \quad 14.0 + 3.00F_x = -1.00.$$

The answer (F_x = –5.00 N) satisfies both conditions.

23. We use the notation $\vec{r}\,'$ to indicate the vector pointing from the axis of rotation directly to the position of the particle. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\mathrm{i}} + \left(z'F_x - x'F_z\right)\hat{\mathrm{j}} + \left(x'F_y - y'F_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$. Dropping the primes in the above expression, we set (with SI units understood) x = 0, y = 0.5, z = –2.0, F_x = 2.0, F_y = 0, and F_z = –3.0. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(-1.5\hat{\text{i}} - 4.0\hat{\text{j}} - 1.0\hat{\text{k}}\right) \text{N}\cdot\text{m}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\hat{\text{i}} - 3.0\hat{\text{k}}$. Therefore, in the above expression, we set $x' = -2.0, y' = 0.5, z' = 1.0, F_x = 2.0, F_y = 0$, and $F_z = -3.0$. Thus, we obtain

$$\vec{\tau} = \vec{r}\,' \times \vec{F} = \left(-1.5\,\hat{\text{i}} - 4.0\,\hat{\text{j}} - 1.0\hat{\text{k}}\right) \text{N}\cdot\text{m}.$$

24. If we write $\vec{r}\,' = x'\,\hat{\text{i}} + y'\,\hat{\text{j}} + z'\,\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$\left(y'F_z - z'F_y\right)\hat{\text{i}} + \left(z'F_x - x'F_z\right)\hat{\text{j}} + \left(x'F_y - y'F_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\hat{\text{i}} - 2.0\hat{\text{j}} + 4.0\hat{\text{k}}$, and $\vec{F} = \vec{F}_1$. Thus, dropping the prime in the above expression, we set (with SI units understood) $x = 3.0$, $y = -2.0$, $z = 4.0$, $F_x = 3.0$, $F_y = -4.0$, and $F_z = 5.0$. Then we obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_1 = \left(6.0\,\hat{\text{i}} - 3.0\,\hat{\text{j}} - 6.0\,\hat{\text{k}}\right) \text{N}\cdot\text{m}.$$

(b) This is like part (a) but with $\vec{F} = \vec{F}_2$. We plug in $F_x = -3.0$, $F_y = -4.0$, and $F_z = -5.0$ and obtain

$$\vec{\tau} = \vec{r} \times \vec{F}_2 = \left(26\,\hat{\text{i}} + 3.0\,\hat{\text{j}} - 18\,\hat{\text{k}}\right) \text{N}\cdot\text{m}.$$

(c) We can proceed in either of two ways. We can add (vectorially) the answers from parts (a) and (b), or we can first add the two force vectors and then compute $\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$ (these total force components are computed in the next part). The result is

$$\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right) = \left(32\,\hat{\text{i}} - 24\,\hat{\text{k}}\right) \text{N}\cdot\text{m}.$$

(d) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 3.0\hat{\text{i}} + 2.0\hat{\text{j}} + 4.0\hat{\text{k}}$. Therefore, in the above expression, we set $x' = 0, y' = -4.0, z' = 0$, and

$$F_x = 3.0 - 3.0 = 0$$
$$F_y = -4.0 - 4.0 = -8.0$$
$$F_z = 5.0 - 5.0 = 0.$$

We get $\vec{\tau} = \vec{r}\,' \times \left(\vec{F}_1 + \vec{F}_2\right) = 0.$

25. **THINK** We take the cross product of $\vec{r}$ and $\vec{F}$ to find the torque $\vec{\tau}$ on a particle.

EXPRESS If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$ and $\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} + F_z\hat{\text{k}}$, then (using Eq. 3-30) the general expression for torque can be written as

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(yF_z - zF_y\right)\hat{\text{i}} + \left(zF_x - xF_z\right)\hat{\text{j}} + \left(xF_y - yF_x\right)\hat{\text{k}}.$$

ANALYZE (a) With $\vec{r} = (3.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}$ and $\vec{F} = (-8.0\text{ N})\hat{\text{i}} + (6.0\text{ N})\hat{\text{j}}$, we have

$$\vec{\tau} = \left[(3.0\text{m})(6.0\text{N}) - (4.0\text{m})(-8.0\text{N})\right]\hat{\text{k}} = (50\,\text{N}\cdot\text{m})\,\hat{\text{k}}.$$

(b) To find the angle ϕ between $\vec{r}$ and $\vec{F}$, we use Eq. 3-27: $|\vec{r} \times \vec{F}| = rF\sin\phi$. Now $r = \sqrt{x^2 + y^2} = 5.0\text{ m}$ and $F = \sqrt{F_x^2 + F_y^2} = 10\text{ N}$. Thus,

$$rF = (5.0\text{ m})(10\text{ N}) = 50\text{ N}\cdot\text{m},$$

the same as the magnitude of the vector product calculated in part (a). This implies sin ϕ = 1 and ϕ = 90°.

LEARN Our result (ϕ = 90°) implies that $\vec{r}$ and $\vec{F}$ are perpendicular to each other. A useful check is to show that their dot product is zero. This is indeed the case:

$$\begin{aligned}\vec{r}\cdot\vec{F} &= [(3.0\text{ m})\hat{\text{i}} + (4.0\text{ m})\hat{\text{j}}]\cdot[(-8.0\text{ N})\hat{\text{i}} + (6.0\text{ N})\hat{\text{j}}]\\ &= (3.0\text{ m})(-8.0\text{ N}) + (4.0\text{ m})(6.0\text{ N}) = 0.\end{aligned}$$

26. We note that the component of $\vec{v}$ perpendicular to $\vec{r}$ has magnitude v sin θ_2 where θ_2= 30°. A similar observation applies to $\vec{F}$.

(a) Eq. 11-20 leads to

$$\ell = rmv_{\perp} = (3.0\text{ m})(2.0\text{ kg})(4.0\text{ m/s})\sin 30° = 12\text{ kg}\cdot\text{m}^2/\text{s}.$$

(b) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{p}$ points out of the page, or along the +z axis, perpendicular to the plane of the figure.

(c) Similarly, Eq. 10-38 leads to

$$\tau = rF\sin\theta_2 = (3.0\text{ m})(2.0\text{ N})\sin 30° = 3.0\,\text{N}\cdot\text{m}.$$

(d) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{F}$ is also out of the page, or along the +z axis, perpendicular to the plane of the figure.

27. **THINK** We evaluate the cross product $\vec{\ell} = m\vec{r}\times\vec{v}$ to find the angular momentum $\vec{\ell}$ on the object, and the cross product of $\vec{r}\times\vec{F}$ for the torque $\vec{\tau}$.

EXPRESS Let $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$ be the position vector of the object, $\vec{v} = v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}}$ its velocity vector, and m its mass. The cross product of $\vec{r}$ and $\vec{v}$ is (using Eq. 3-30)

$$\vec{r}\times\vec{v} = \left(yv_z - zv_y\right)\hat{\mathrm{i}} + \left(zv_x - xv_z\right)\hat{\mathrm{j}} + \left(xv_y - yv_x\right)\hat{\mathrm{k}}.$$

Since only the x and z components of the position and velocity vectors are nonzero (i.e., $y = 0$ and $v_y = 0$), the above expression becomes $\vec{r}\times\vec{v} = \left(-xv_z + zv_z\right)\hat{\mathrm{j}}$. As for the torque, writing $\vec{F} = F_x\hat{\mathrm{i}} + F_y\hat{\mathrm{j}} + F_z\hat{\mathrm{k}}$, we find $\vec{r}\times\vec{F}$ to be

$$\vec{\tau} = \vec{r}\times\vec{F} = \left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

ANALYZE (a) With $\vec{r} = (2.0\text{ m})\hat{\mathrm{i}} - (2.0\text{ m})\hat{\mathrm{k}}$ and $\vec{v} = (-5.0\text{ m/s})\hat{\mathrm{i}} + (5.0\text{ m/s})\hat{\mathrm{k}}$, in unit-vector notation, the angular momentum of the object is

$$\vec{\ell} = m\left(-xv_z + zv_x\right)\hat{\mathrm{j}} = \left(0.25\text{ kg}\right)\left(-\left(2.0\text{ m}\right)\left(5.0\text{ m/s}\right) + \left(-2.0\text{ m}\right)\left(-5.0\text{ m/s}\right)\right)\hat{\mathrm{j}} = 0.$$

(b) With $x = 2.0$ m, $z = -2.0$ m, $F_y = 4.0$ N and all other components zero, the expression above yields

$$\vec{\tau} = \vec{r}\times\vec{F} = (8.0\text{ N}\cdot\text{m})\hat{\mathrm{i}} + (8.0\text{ N}\cdot\text{m})\hat{\mathrm{k}}.$$

LEARN The fact that $\vec{\ell} = 0$ implies that $\vec{r}$ and $\vec{v}$ are parallel to each other ($\vec{r}\times\vec{v} = 0$). Using $\tau = |\vec{r}\times\vec{F}| = rF\sin\phi$, we find the angle between $\vec{r}$ and $\vec{F}$ to be

$$\sin\phi = \frac{\tau}{rF} = \frac{8\sqrt{2}\text{ N}\cdot\text{m}}{(2\sqrt{2}\text{ m})(4.0\text{ N})} = 1 \quad\Rightarrow\quad \phi = 90°$$

That is, $\vec{r}$ and $\vec{F}$ are perpendicular to each other.

28. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' = \vec{v}$ is equal to

$$\left(y'v_z - z'v_y\right)\hat{\mathrm{i}} + \left(z'v_x - x'v_z\right)\hat{\mathrm{j}} + \left(x'v_y - y'v_x\right)\hat{\mathrm{k}}.$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3.0\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}}$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3.0, y = -4.0, z = 0, v_x = 30, v_y = 60$, and $v_z = 0$. Then (with $m = 2.0$ kg) we obtain

$$\vec{\ell} = m(\vec{r}\times\vec{v}) = (6.0\times10^2\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_\text{o}$ where $\vec{r}_\text{o} = -2.0\hat{\text{i}} - 2.0\hat{\text{j}}$. Therefore, in the above expression, we set $x' = 5.0, y' = -2.0, z' = 0, v_x = 30, v_y = 60$, and $v_z = 0$. We get

$$\vec{\ell} = m(\vec{r}\,'\times\vec{v}) = (7.2\times10^2\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

29. For the 3.1 kg particle, Eq. 11-21 yields

$$\ell_1 = r_{\perp 1} m v_1 = (2.8\ \text{m})(3.1\ \text{kg})(3.6\ \text{m/s}) = 31.2\ \text{kg}\cdot\text{m}^2/\text{s}.$$

Using the right-hand rule for vector products, we find this $(\vec{r}_1\times\vec{p}_1)$ is out of the page, or along the +*z* axis, perpendicular to the plane of Fig. 11-41. And for the 6.5 kg particle, we find

$$\ell_2 = r_{\perp 2} m v_2 = (1.5\ \text{m})(6.5\ \text{kg})(2.2\ \text{m/s}) = 21.4\ \text{kg}\cdot\text{m}^2/\text{s}.$$

And we use the right-hand rule again, finding that this $(\vec{r}_2\times\vec{p}_2)$ is into the page, or in the –*z* direction.

(a) The two angular momentum vectors are in opposite directions, so their vector sum is the *difference* of their magnitudes: $L = \ell_1 - \ell_2 = 9.8\ \text{kg}\cdot\text{m}^2/\text{s}$.

(b) The direction of the net angular momentum is along the +*z* axis.

30. (a) The acceleration vector is obtained by dividing the force vector by the (scalar) mass:

$$\vec{a} = \vec{F}/m = (3.00\ \text{m/s}^2)\hat{\text{i}} - (4.00\ \text{m/s}^2)\hat{\text{j}} + (2.00\ \text{m/s}^2)\hat{\text{k}}.$$

(b) Use of Eq. 11-18 leads directly to

$$\vec{L} = (42.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{i}} + (24.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{j}} + (60.0\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

(c) Similarly, the torque is

$$\vec{\tau} = \vec{r}\times\vec{F} = (-8.00\ \text{N}\cdot\text{m})\hat{\text{i}} - (26.0\ \text{N}\cdot\text{m})\hat{\text{j}} - (40.0\ \text{N}\cdot\text{m})\hat{\text{k}}.$$

(d) We note (using the Pythagorean theorem) that the magnitude of the velocity vector is 7.35 m/s and that of the force is 10.8 N. The dot product of these two vectors is $\vec{\text{v}}\cdot\vec{F} = -48$ (in SI units). Thus, Eq. 3-20 yields

$$\theta = \cos^{-1}[-48.0/(7.35\times10.8)] = 127°.$$

31. (a) Since the speed is (momentarily) zero when it reaches maximum height, the angular momentum is zero then.

(b) With the convention (used in several places in the book) that clockwise sense is to be associated with the negative sign, we have $L = -\, r_\perp m v$ where $r_\perp = 2.00$ m, $m = 0.400$ kg, and v is given by free-fall considerations (as in Chapter 2). Specifically, y_{max} is determined by Eq. 2-16 with the speed at max height set to zero; we find $y_{max} = v_o^2/2g$ where $v_o = 40.0$ m/s. Then with $y = \frac{1}{2} y_{max}$, Eq. 2-16 can be used to give $v = v_o/\sqrt{2}$. In this way we arrive at $L = -22.6 \text{ kg}\cdot\text{m}^2/\text{s}$.

(c) As mentioned in the previous part, we use the minus sign in writing $\tau = -\, r_\perp F$ with the force F being equal (in magnitude) to mg. Thus, $\tau = -7.84 \text{ N}\cdot\text{m}$.

(d) Due to the way $r_\perp$ is defined it does not matter how far up the ball is. The answer is the same as in part (c), $\tau = -7.84 \text{ N}\cdot\text{m}$.

32. The rate of change of the angular momentum is

$$\frac{d\vec{\ell}}{dt} = \vec{\tau}_1 + \vec{\tau}_2 = (2.0 \text{ N}\cdot\text{m})\hat{\text{i}} - (4.0 \text{ N}\cdot\text{m})\hat{\text{j}}.$$

Consequently, the vector $d\vec{\ell}/dt$ has a magnitude $\sqrt{(2.0 \text{ N}\cdot\text{m})^2 + \left(-4.0 \text{ N}\cdot\text{m}\right)^2} = 4.5 \text{ N}\cdot\text{m}$ and is at an angle θ (in the xy plane, or a plane parallel to it) measured from the positive x axis, where

$$\theta = \tan^{-1}\left(\frac{-4.0 \text{ N}\cdot\text{m}}{2.0 \text{ N}\cdot\text{m}}\right) = -63^\circ ,$$

the negative sign indicating that the angle is measured clockwise as viewed "from above" (by a person on the $+z$ axis).

33. **THINK** We evaluate the cross product $\vec{\ell} = m\vec{r}\times\vec{v}$ to find the angular momentum $\vec{\ell}$ on the particle, and the cross product of $\vec{r}\times\vec{F}$ for the torque $\vec{\tau}$.

EXPRESS Let $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$ be the position vector of the object, $\vec{v} = v_x\hat{\text{i}} + v_y\hat{\text{j}} + v_z\hat{\text{k}}$ its velocity vector, and m its mass. The cross product of $\vec{r}$ and $\vec{v}$ is

$$\vec{r}\times\vec{v} = \left(yv_z - zv_y\right)\hat{\text{i}} + \left(zv_x - xv_z\right)\hat{\text{j}} + \left(xv_y - yv_x\right)\hat{\text{k}}.$$

The angular momentum is given by the vector product $\vec{\ell} = m\vec{r}\times\vec{v}$. As for the torque, writing $\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} + F_z\hat{\text{k}}$, then we find $\vec{r}\times\vec{F}$ to be

$$\vec{\tau} = \vec{r} \times \vec{F} = \left(yF_z - zF_y\right)\hat{\mathrm{i}} + \left(zF_x - xF_z\right)\hat{\mathrm{j}} + \left(xF_y - yF_x\right)\hat{\mathrm{k}}.$$

ANALYZE (a) Substituting m = 3.0 kg, x = 3.0 m, y = 8.0 m, z = 0, v_x = 5.0 m/s, $v_y = -6.0$ m/s and v_z = 0 into the above expression, we obtain

$$\vec{\ell} = \left(3.0\ \text{kg}\right)[(3.0\ \text{m})(-6.0\ \text{m/s}) - (8.0\ \text{m})(5.0\ \text{m/s})]\hat{\mathrm{k}} = (-174\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\mathrm{k}}.$$

(b) Given that $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}}$ and $\vec{F} = F_x\hat{\mathrm{i}}$, the corresponding torque is

$$\vec{\tau} = \left(x\hat{\mathrm{i}} + y\hat{\mathrm{j}}\right) \times \left(F_x\hat{\mathrm{i}}\right) = -yF_x\hat{\mathrm{k}}.$$

Substituting the values given, we find

$$\vec{\tau} = -\left(8.0\,\text{m}\right)\left(-7.0\,\text{N}\right)\hat{\mathrm{k}} = (56\,\text{N}\cdot\text{m})\hat{\mathrm{k}}.$$

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is 56 $\text{kg}\cdot\text{m}^2/\text{s}^2$, in the positive z direction.

LEARN The direction of $\vec{\ell}$ is in the –z-direction, which is perpendicular to both $\vec{r}$ and $\vec{v}$. Similarly, the torque $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$ (i.e, $\vec{\tau}$ is in the direction normal to the plane formed by $\vec{r}$ and $\vec{F}$).

34. We use a right-handed coordinate system with $\hat{\mathrm{k}}$ directed out of the xy plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{\mathrm{k}}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{\mathrm{k}}$ in SI units. We use Eq. 11-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\frac{dt^2}{dt} = (-8.0t\ \text{N}\cdot\text{m})\hat{\mathrm{k}}\ .$$

This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all t > 0.

(c) With $\vec{\ell} = (-4.0\sqrt{t})\hat{\mathrm{k}}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{1}{2\sqrt{t}}\right) = \left(-\frac{2.0}{\sqrt{t}}\hat{k}\right)\text{N}\cdot\text{m}.$$

This vector points in the $-\hat{k}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{k}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{k}\right)\left(\frac{-2}{t^3}\right) = \left(\frac{8.0}{t^3}\hat{k}\right)\text{N}\cdot\text{m}.$$

This vector points in the $+\hat{k}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.

35. (a) We note that

$$\vec{v} = \frac{d\vec{r}}{dt} = 8.0t\,\hat{i} - (2.0 + 12t)\hat{j}$$

with SI units understood. From Eq. 11-18 (for the angular momentum) and Eq. 3-30, we find the particle's angular momentum is $8t^2\hat{k}$. Using Eq. 11-23 (relating its time-derivative to the (single) torque) then yields $\vec{\tau} = (48t\,\hat{k})\ \text{N}\cdot\text{m}$.

(b) From our (intermediate) result in part (a), we see the angular momentum increases in proportion to t^2.

36. We relate the motions of the various disks by examining their linear speeds (using Eq. 10-18). The fact that the linear speed at the rim of disk A must equal the linear speed at the rim of disk C leads to $\omega_A = 2\omega_C$. The fact that the linear speed at the hub of disk A must equal the linear speed at the rim of disk B leads to $\omega_A = \frac{1}{2}\omega_B$. Thus, $\omega_B = 4\omega_C$. The ratio of their angular momenta depend on these angular velocities as well as their rotational inertias (see item (c) in Table 11-2), which themselves depend on their masses. If h is the thickness and ρ is the density of each disk, then each mass is $\rho\pi R^2 h$. Therefore,

$$\frac{L_C}{L_B} = \frac{(\tfrac{1}{2})\rho\pi R_C^2\, h\, R_C^2\, \omega_C}{(\tfrac{1}{2})\rho\pi R_B^2\, h\, R_B^2\, \omega_B} = 1024.$$

37. (a) A particle contributes mr_2 to the rotational inertia. Here r is the distance from the origin O to the particle. The total rotational inertia is

$$I = m(3d)^2 + m(2d)^2 + m(d)^2 = 14md^2 = 14(2.3\times10^{-2}\,\text{kg})(0.12\ \text{m})^2$$
$$= 4.6\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

(b) The angular momentum of the middle particle is given by $L_m = I_m\omega$, where $I_m = 4md^2$ is its rotational inertia. Thus

$$L_m = 4md^2\omega = 4(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 1.1\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

(c) The total angular momentum is

$$I\omega = 14md^2\omega = 14(2.3\times10^{-2}\,\text{kg})(0.12\text{ m})^2(0.85\text{ rad/s}) = 3.9\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

38. (a) Equation 10-34 gives $\alpha = \tau/I$ and Eq. 10-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau\, t = (16\,\text{N}\cdot\text{m})(0.033\text{s}) = 0.53\,\text{kg}\cdot\text{m}^2/\text{s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau\, t}{I} = \frac{(16\text{ N}\cdot\text{m})(0.033\,\text{s})}{1.2\times10^{-3}\ \text{kg}\cdot\text{m}^2} = 440\,\text{rad/s}$$

which we convert as follows:

$$\omega = (440\text{ rad/s})(60\text{ s/min})(1\text{ rev}/2\pi\text{ rad}) \approx 4.2\times10^3\text{ rev/min}.$$

39. **THINK** A non-zero torque is required to change the angular momentum of the flywheel. We analyze the rotational motion of the wheel using the equations given in Table 10-1.

EXPRESS Since the torque is equal to the rate of change of angular momentum, $\tau = dL/dt$, the average torque acting during any interval Δt is simply given by $\tau_{\text{avg}} = (L_f - L_i)/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. For uniform angular acceleration, the angle turned is $\theta = \omega_0 t + \alpha t^2/2$, and the work done on the wheel is $W = \tau\theta$.

ANALYZE (a) Substituting the values given, the average torque is

$$\tau_{\text{avg}} = \frac{L_f - L_i}{\Delta t} = \frac{(0.800\text{ kg}\cdot\text{m}^2/\text{s}) - (3.00\text{ kg}\cdot\text{m}^2/\text{s})}{1.50\text{ s}} = -1.47\text{ N}\cdot\text{m},$$

or $|\tau_{\text{avg}}| = 1.47\text{ N}\cdot\text{m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) If the angular acceleration α is uniform, so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\theta = \frac{L_i t + \tau\, t^2/2}{I} = \frac{\left(3.00\,\text{kg}\cdot\text{m}^2/\text{s}\right)\left(1.50\ \text{s}\right) + \left(-1.467\,\text{N}\cdot\text{m}\right)\left(1.50\ \text{s}\right)^2/2}{0.140\,\text{kg}\cdot\text{m}^2} = 20.4\,\text{rad}.$$

(c) Using the values of τ and θ found above, we find the work done on the wheel to be

$$W = \tau\theta = \left(-1.47\,\text{N}\cdot\text{m}\right)\left(20.4\,\text{rad}\right) = -29.9\ \text{J}.$$

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{\text{avg}} = -\frac{W}{\Delta t} = -\frac{-29.9\ \text{J}}{1.50\,\text{s}} = 19.9\,\text{W}.$$

LEARN An alternative way to calculate the work done on the wheel is to apply the work-kinetic energy theorem:

$$W = \Delta K = K_f - K_i = \frac{1}{2}I(\omega_f^2 - \omega_i^2) = \frac{1}{2}I\left[\left(\frac{L_f}{I}\right)^2 - \left(\frac{L_i}{I}\right)^2\right] = \frac{L_f^2 - L_i^2}{2I}$$

Substituting the values given, we have

$$W = \frac{L_f^2 - L_i^2}{2I} = \frac{(0.800\ \text{kg}\cdot\text{m}^2/\text{s})^2 - (3.00\ \text{kg}\cdot\text{m}^2/\text{s})^2}{2(0.140\,\text{kg}\cdot\text{m}^2)} = -29.9\ \text{J}$$

which agrees with that calculated in part (c).

40. Torque is the time derivative of the angular momentum. Thus, the change in the angular momentum is equal to the time integral of the torque. With $\tau = (5.00 + 2.00t)\ \text{N}\cdot\text{m}$, the angular momentum (in units $\text{kg}\cdot\text{m}^2/\text{s}$) as a function of time is

$$L(t) = \int \tau\, dt = \int (5.00 + 2.00t)\,dt = L_0 + 5.00t + 1.00t^2.$$

Since $L = 5.00\ \text{kg}\cdot\text{m}^2/\text{s}$ when $t = 1.00\,\text{s}$, the integration constant is $L_0 = -1$. Thus, the complete expression of the angular momentum is

$$L(t) = -1 + 5.00t + 1.00t^2.$$

At $t = 3.00\,\text{s}$, we have $L(t = 3.00) = -1 + 5.00(3.00) + 1.00(3.00)^2 = 23.0\ \text{kg}\cdot\text{m}^2/\text{s}.$

41. (a) For the hoop, we use Table 10-2(h) and the parallel-axis theorem to obtain

$$I_1 = I_{\text{com}} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 .$$

Of the thin bars (in the form of a square), the member along the rotation axis has (approximately) no rotational inertia about that axis (since it is thin), and the member farthest from it is very much like it (by being parallel to it) except that it is displaced by a distance *h*; it has rotational inertia given by the parallel axis theorem:

$$I_2 = I_{\text{com}} + mh^2 = 0 + mR^2 = mR^2 .$$

Now the two members of the square perpendicular to the axis have the same rotational inertia (that is $I_3 = I_4$). We find I_3 using Table 10-2(e) and the parallel-axis theorem:

$$I_3 = I_{\text{com}} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2 .$$

Therefore, the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6\,\text{kg}\cdot\text{m}^2 .$$

(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5\,\text{rad/s}.$$

Thus, $L = I_{\text{total}}\omega = 4.0\,\text{kg}\cdot\text{m}^2/\text{s}$.

42. The results may be found by integrating Eq. 11-29 with respect to time, keeping in mind that $\vec{L}_i = 0$ and that the integration may be thought of as "adding the areas" under the line-segments (in the plot of the torque versus time, with "areas" under the time axis contributing negatively). It is helpful to keep in mind, also, that the area of a triangle is $\frac{1}{2}$ (base)(height).

(a) We find that $\vec{L} = 24\ \text{kg}\cdot\text{m}^2/\text{s}$ at t = 7.0 s.

(b) Similarly, $\vec{L} = 1.5\ \text{kg}\cdot\text{m}^2/\text{s}$ at t = 20 s.

43. We assume that from the moment of grabbing the stick onward, they maintain rigid postures so that the system can be analyzed as a symmetrical rigid body with center of mass midway between the skaters.

(a) The total linear momentum is zero (the skaters have the same mass and equal and opposite velocities). Thus, their center of mass (the middle of the 3.0 m long stick) remains fixed and they execute circular motion (of radius r = 1.5 m) about it.

(b) Using Eq. 10-18, their angular velocity (counterclockwise as seen in Fig. 11-47) is

$$\omega = \frac{v}{r} = \frac{1.4 \text{ m/s}}{1.5 \text{ m}} = 0.93 \text{ rad/s}.$$

(c) Their rotational inertia is that of two particles in circular motion at r = 1.5 m, so Eq. 10-33 yields

$$I = \sum mr^2 = 2(50 \text{ kg})(1.5 \text{ m})^2 = 225 \text{ kg}\cdot\text{m}^2.$$

Therefore, Eq. 10-34 leads to

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(225 \text{ kg}\cdot\text{m}^2\right)(0.93 \text{ rad/s})^2 = 98 \text{ J}.$$

(d) Angular momentum is conserved in this process. If we label the angular velocity found in part (a) ω_i and the rotational inertia of part (b) as I_i, we have

$$I_i\omega_i = \left(225 \text{ kg}\cdot\text{m}^2\right)(0.93 \text{ rad/s}) = I_f\omega_f.$$

The final rotational inertia is $\sum mr_f^2$ where r_f = 0.5 m so $I_f = 25 \text{ kg}\cdot\text{m}^2$. Using this value, the above expression gives $\omega_f = 8.4 \text{ rad/s}$.

(e) We find

$$K_f = \frac{1}{2} I_f\omega_f^2 = \frac{1}{2}\left(25 \text{ kg}\cdot\text{m}^2\right)(8.4 \text{ rad/s})^2 = 8.8\times10^2 \text{ J}.$$

(f) We account for the large increase in kinetic energy (part (e) minus part (c)) by noting that the skaters do a great deal of work (converting their internal energy into mechanical energy) as they pull themselves closer — "fighting" what appears to them to be large "centrifugal forces" trying to keep them apart.

44. So that we don't get confused about ± signs, we write the angular *speed* to the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach "stops" we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = \left(mR^2 + I\right)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I|\omega_0| = -(mR^2 + I)|\omega_f|.$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I|\omega_0|}{mR^2 + I} = \frac{(0.17\ \text{kg})(2.0\ \text{m/s})(0.15\ \text{m}) - (5.0\times10^{-3}\ \text{kg}\cdot\text{m}^2)(2.8\ \text{rad/s})}{(5.0\times10^{-3}\ \text{kg}\cdot\text{m}^2) + (0.17\ \text{kg})(0.15\ \text{m})^2}$$
$$= 4.2\ \text{rad/s}.$$

(b) No, $K_f \neq K_i$ and — if desired — we can solve for the difference:

$$K_i - K_f = \frac{mI}{2}\frac{v^2 + \omega_0^2 R^2 + 2Rv|\omega_0|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" — that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

45. **THINK** No external torque acts on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved.

EXPRESS Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ by angular momentum conservation. The kinetic energy (of rotational nature) is given by $K = I\omega^2/2$.

ANALYZE (a) The final angular momentum of the system is

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{6.0\ \text{kg}\cdot\text{m}^2}{2.0\ \text{kg}\cdot\text{m}^2}\right)(1.2\,\text{rev/s}) = 3.6\,\text{rev/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, and the final kinetic energy is $K_f = \frac{1}{2}I_f\omega_f^2$, so that their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{(2.0\,\text{kg}\cdot\text{m}^2)(3.6\,\text{rev/s})^2/2}{(6.0\,\text{kg}\cdot\text{m}^2)(1.2\,\text{rev/s})^2/2} = 3.0.$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's internal energy.

LEARN The work done by the person is equal to the change in kinetic energy:

$$W = K_f - K_i = 3K_i - K_i = 2K_i = I_i\omega_i^2 = (6.0\,\text{kg}\cdot\text{m}^2)(2\pi\cdot 1.2\,\text{rad/s})^2 = 341\text{ J}.$$

46. Angular momentum conservation $I_i\omega_i = I_f\omega_f$ leads to

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}\omega_i = 3$$

which implies

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2/2}{I_i\omega_i^2/2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = 3.$$

47. **THINK** No external torque acts on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero.

EXPRESS Let $I = MR^2$ be the rotational inertia of the wheel (which we treat as a hoop). Its angular momentum is

$$\vec{L}_{\text{wheel}} = (I\omega)\hat{\text{k}} = -M\,R^2\,|\omega|\,\hat{\text{k}},$$

where $\hat{\text{k}}$ is *up* in Fig. 11-48 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is $-|\omega|R$ and the speed of the train (going counterclockwise in Fig. 11-48 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $\vec{L}_{\text{train}} = m\left(v - |\omega|R\right)R\,\hat{\text{k}}$. Conservation of angular momentum yields

$$0 = \vec{L}_{\text{wheel}} + \vec{L}_{\text{train}} = -MR^2\,|\omega|\,\hat{\text{k}} + m\left(v - |\omega|R\right)R\hat{\text{k}}$$

which we can use to solve for $|\omega|$.

ANALYZE Solving for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{v}{(M/m+1)R} = \frac{0.15\text{ m/s}}{(1.1+1)(0.43\text{ m})} = 0.17\text{ rad/s}.$$

LEARN By angular momentum conservation, we must have $\vec{L}_{\text{wheel}} = -\vec{L}_{\text{train}}$, which means that train and the wheel must have opposite senses of rotation.

48. Using Eq. 11-31 with angular momentum conservation, $\vec{L}_i = \vec{L}_f$ (Eq. 11-33) leads to the ratio of rotational inertias being inversely proportional to the ratio of angular velocities. Thus, $I_f/I_i = 6/5 = 1.0 + 0.2$. We interpret the "1.0" as the ratio of disk rotational inertias (which does not change in this problem) and the "0.2" as the ratio of the roach rotational inertial to that of the disk. Thus, the answer is 0.20.

49. (a) We apply conservation of angular momentum:

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega.$$

The angular speed after coupling is therefore

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{(3.3\,\text{kg}\cdot\text{m}^2)(450\ \text{rev/min}) + (6.6\,\text{kg}\cdot\text{m}^2)(900\ \text{rev/min})}{3.3\,\text{kg}\cdot\text{m}^2 + 6.6\,\text{kg}\cdot\text{m}^2}$$
$$= 750\ \text{rev/min}.$$

(b) In this case, we obtain

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{(3.3\ \text{kg}\cdot\text{m}^2)(450\ \text{rev/min}) + (6.6\ \text{kg}\cdot\text{m}^2)(-900\ \text{rev/min})}{3.3\ \text{kg}\cdot\text{m}^2 + 6.6\ \text{kg}\cdot\text{m}^2}$$
$$= -450\ \ \text{rev/min}$$

or $|\omega| = 450\ \ \text{rev/min}$.

(c) The minus sign indicates that $\vec{\omega}$ is clockwise, that is, in the direction of the second disk's initial angular velocity.

50. We use conservation of angular momentum:

$$I_m\omega_m = I_p\omega_p.$$

The respective angles θ_m and θ_p by which the motor and probe rotate are therefore related by

$$\int I_m\omega_m dt = I_m\theta_m = \int I_p\omega_p dt = I_p\theta_p$$

which gives

$$\theta_m = \frac{I_p\theta_p}{I_m} = \frac{(12\,\text{kg}\cdot\text{m}^2)(30°)}{2.0\times10^{-3}\ \text{kg}\cdot\text{m}^2} = 180000°.$$

The number of revolutions for the rotor is then

$$N = (1.8 \times 10^5)°/(360°/\text{rev}) = 5.0 \times 10^2\ \text{rev}.$$

51. **THINK** No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved.

EXPRESS Let I_1 be the rotational inertia of the wheel that is originally spinning $(\text{at } \omega_i)$ and I_2 be the rotational inertia of the wheel that is initially at rest. Then by angular momentum conservation, $L_i = L_f$, or $I_1\omega_i = (I_1 + I_2)\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i$$

where ω_f is the common final angular velocity of the wheels.

ANALYZE (a) Substituting $I_2 = 2I_1$ and $\omega_i = 800 \text{ rev/min}$, we obtain

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i = \frac{I_1}{I_1 + 2(I_1)}(800 \text{ rev/min}) = \frac{1}{3}(800 \text{ rev/min}) = 267 \text{ rev/min}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2.$$

Therefore, the fraction lost is

$$\frac{\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{I\omega_i^2/6}{I\omega_i^2/2} = \frac{2}{3} = 0.667.$$

LEARN The situation here is analogous to the case of completely inelastic collision, in which some energy is lost but momentum remains conserved.

52. We denote the cockroach with subscript 1 and the disk with subscript 2. The cockroach has a mass $m_1 = m$, while the mass of the disk is $m_2 = 4.00\ m$.

(a) Initially the angular momentum of the system consisting of the cockroach and the disk is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R^2 + \frac{1}{2}m_2\omega_0 R^2.$$

After the cockroach has completed its walk, its position (relative to the axis) is $r_{1f} = R/2$ so the final angular momentum of the system is

$$L_f = m_1\omega_f\left(\frac{R}{2}\right)^2 + \frac{1}{2}m_2\omega_f R^2.$$

Then from $L_f = L_i$ we obtain

$$\omega_f\left(\frac{1}{4}m_1R^2+\frac{1}{2}m_2R\right)=\omega_0\left(m_1R^2+\frac{1}{2}m_2R^2\right).$$

Thus,

$$\omega_f=\left(\frac{m_1R^2+m_2R^2/2}{m_1R^2/4+m_2R^2/2}\right)\omega_0=\left(\frac{1+(m_2/m_1)/2}{1/4+(m_2/m_1)/2}\right)\omega_0=\left(\frac{1+2}{1/4\ +\ 2}\right)\omega_0=1.33\omega_0.$$

With ω_0 = 0.260 rad/s, we have ω_f =0.347 rad/s.

(b) We substitute $I = L/\omega$ into $K=\frac{1}{2}I\omega^2$ and obtain $K=\frac{1}{2}L\omega$. Since we have $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K}{K_0}=\frac{L_f\omega_f/2}{L_i\omega_i/2}=\frac{\omega_f}{\omega_0}=1.33.$$

(c) The cockroach does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.

53. The axis of rotation is in the middle of the rod, with r = 0.25 m from either end. By Eq. 11-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv\sin\theta$ where m = 0.003 kg and θ = 60°. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is

$$I = I_{\text{rod}} + mr^2$$

where $I_{\text{rod}} = ML^2/12$ by Table 10-2(e), with M = 4.0 kg and L = 0.5 m. Angular momentum conservation leads to

$$rmv\sin\theta=\left(\frac{1}{12}ML^2+mr^2\right)\omega.$$

Thus, with ω = 10 rad/s, we obtain

$$v=\frac{\left(\frac{1}{12}(4.0\text{ kg})(0.5\text{ m})^2+(0.003\text{ kg})(0.25\text{ m})^2\right)(10\text{ rad/s})}{(0.25\text{ m})(0.003\text{ kg})\sin 60°}=1.3\times10^3\text{ m/s}.$$

54. We denote the cat with subscript 1 and the ring with subscript 2. The cat has a mass $m_1 = M/4$, while the mass of the ring is $m_2 = M$ = 8.00 kg. The moment of inertia of the ring is $I_2=m_2(R_1^2+R_2^2)/2$ (Table 10-2), and $I_1 = m_1r^2$ for the cat, where r is the perpendicular distance from the axis of rotation.

Initially the angular momentum of the system consisting of the cat (at $r = R_2$) and the ring is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_0 = m_1 R_2^2\,\omega_0 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right].$$

After the cat has crawled to the inner edge at $r = R_1$ the final angular momentum of the system is

$$L_f = m_1\omega_f R_1^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_f = m_1 R_1^2\,\omega_f \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)\right].$$

Then from $L_f = L_i$ we obtain

$$\frac{\omega_f}{\omega_0} = \left(\frac{R_2}{R_1}\right)^2 \frac{1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)}{1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)} = (2.0)^2 \frac{1 + 2(0.25 + 1)}{1 + 2(1 + 4)} = 1.273.$$

Thus, $\omega_f = 1.273\omega_0$. Using ω_0 =8.00 rad/s, we have ω_f =10.2 rad/s. By substituting $I = L/\omega$ into $K = I\omega^2/2$, we obtain $K = L\omega/2$. Since $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K_f}{K_i} = \frac{L_f\omega_f/2}{L_i\omega_i/2} = \frac{\omega_f}{\omega_0} = 1.273.$$

which implies $\Delta K = K_f - K_i = 0.273 K_i$. The cat does positive work while walking toward the center of the ring, increasing the total kinetic energy of the system.

Since the initial kinetic energy is given by

$$\begin{aligned}K_i &= \frac{1}{2}\left[m_1 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\right]\omega_0^2 = \frac{1}{2} m_1 R_2^2 \omega_0^2 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right] \\ &= \frac{1}{2}(2.00\text{ kg})(0.800\text{ m})^2(8.00\text{ rad/s})^2[1+(1/2)(4)(0.5^2+1)] \\ &= 143.36\text{ J},\end{aligned}$$

the increase in kinetic energy is

$$\Delta K = (0.273)(143.36\text{ J}) = 39.1\text{ J}.$$

55. For simplicity, we assume the record is turning freely, without any work being done by its motor (and without any friction at the bearings or at the stylus trying to slow it down). Before the collision, the angular momentum of the system (presumed positive) is $I_i\omega_i$ where $I_i = 5.0\times10^{-4}\ \text{kg}\cdot\text{m}^2$ and $\omega_i = 4.7$ rad/s. The rotational inertia afterward is

$$I_f = I_i + mR^2$$

where m = 0.020 kg and R = 0.10 m. The mass of the record (0.10 kg), although given in the problem, is not used in the solution. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \frac{I_i\omega_i}{I_i + mR^2} = 3.4 \text{ rad/s.}$$

56. Table 10-2 gives the rotational inertia of a thin rod rotating about a perpendicular axis through its center. The angular speeds of the two arms are, respectively,

$$\omega_1 = \frac{(0.500 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 4.49 \text{ rad/s}$$
$$\omega_2 = \frac{(1.00 \text{ rev})(2\pi \text{ rad/rev})}{0.700 \text{ s}} = 8.98 \text{ rad/s.}$$

Treating each arm as a thin rod of mass 4.0 kg and length 0.60 m, the angular momenta of the two arms are

$$L_1 = I\omega_1 = mr^2\omega_1 = (4.0 \text{ kg})(0.60 \text{ m})^2(4.49 \text{ rad/s}) = 6.46 \text{ kg}\cdot\text{m}^2\text{/s}$$
$$L_2 = I\omega_2 = mr^2\omega_2 = (4.0 \text{ kg})(0.60 \text{ m})^2(8.98 \text{ rad/s}) = 12.92 \text{ kg}\cdot\text{m}^2\text{/s.}$$

From the athlete's reference frame, one arm rotates clockwise, while the other rotates counterclockwise. Thus, the total angular momentum about the common rotation axis though the shoulders is

$$L = L_2 - L_1 = 12.92 \text{ kg}\cdot\text{m}^2\text{/s} - 6.46 \text{ kg}\cdot\text{m}^2\text{/s} = 6.46 \text{ kg}\cdot\text{m}^2\text{/s.}$$

57. Their angular velocities, when they are stuck to each other, are equal, regardless of whether they share the same central axis. The initial rotational inertia of the system is, using Table 10-2(c),

$$I_0 = I_{\text{bigdisk}} + I_{\text{smalldisk}}$$

where $I_{\text{bigdisk}} = MR^2/2$. Similarly, since the small disk is initially concentric with the big one, $I_{\text{smalldisk}} = \frac{1}{2}mr^2$. After it slides, the rotational inertia of the small disk is found from the parallel axis theorem (using $h = R - r$). Thus, the new rotational inertia of the system is

$$I = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m(R-r)^2.$$

(a) Angular momentum conservation, $I_0\omega_0 = I\omega$, leads to the new angular velocity:

$$\omega = \omega_0 \frac{(MR^2/2) + (mr^2/2)}{(MR^2/2) + (mr^2/2) + m\left(R-r\right)^2}.$$

Substituting $M = 10m$ and $R = 3r$, this becomes $\omega = \omega_0(91/99)$. Thus, with $\omega_0 = 20$ rad/s, we find $\omega = 18$ rad/s.

(b) From the previous part, we know that

$$\frac{I_0}{I} = \frac{91}{99}, \ \frac{\omega}{\omega_0} = \frac{91}{99}.$$

Plugging these into the ratio of kinetic energies, we have

$$\frac{K}{K_0} = \frac{I\omega^2/2}{I_0\omega_0^2/2} = \frac{I}{I_0}\left(\frac{\omega}{\omega_0}\right)^2 = \frac{99}{91}\left(\frac{91}{99}\right)^2 = 0.92.$$

58. The initial rotational inertia of the system is $I_i = I_{\text{disk}} + I_{\text{student}}$, where $I_{\text{disk}} = 300\ \text{kg}\cdot\text{m}^2$ (which, incidentally, does agree with Table 10-2(c)) and $I_{\text{student}} = mR^2$ where $m = 60\ \text{kg}$ and $R = 2.0$ m.

The rotational inertia when the student reaches $r = 0.5$ m is $I_f = I_{\text{disk}} + mr^2$. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \omega_i \frac{I_{\text{disk}} + mR^2}{I_{\text{disk}} + mr^2}$$

which yields, for $\omega_i = 1.5$ rad/s, a final angular velocity of $\omega_f = 2.6$ rad/s.

59. By angular momentum conservation (Eq. 11-33), the total angular momentum after the explosion must be equal to that before the explosion:

$$L'_p + L'_r = L_p + L_r$$

$$\left(\tfrac{L}{2}\right)mv_{\text{p}} + \tfrac{1}{12}ML^2\,\omega' = I_{\text{p}}\,\omega + \tfrac{1}{12}ML^2\,\omega$$

where one must be careful to avoid confusing the length of the rod ($L = 0.800$ m) with the angular momentum symbol. Note that $I_{\text{p}} = m(L/2)^2$ by Eq.10-33, and

$$\omega' = v_{\text{end}}/r = (v_{\text{p}} - 6)/(L/2),$$

where the latter relation follows from the penultimate sentence in the problem (and “6” stands for “6.00 m/s” here). Since $M = 3m$ and $\omega = 20$ rad/s, we end up with enough information to solve for the particle speed: $v_{\text{p}} = 11.0$ m/s.

60. (a) With r = 0.60 m, we obtain $I = 0.060 + (0.501)r^2 = 0.24$ kg · m^2.

(b) Invoking angular momentum conservation, with SI units understood,

$$\ell_0 = L_f \;\;\Rightarrow\;\; mv_0 r = I\omega \;\;\Rightarrow\;\; (0.001)v_0(0.60) = (0.24)(4.5)$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.

61. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities in this problem are positive. With r = 0.60 m and I_0 = 0.12 kg · m^2, the rotational inertia of the putty-rod system (after the collision) is

$$I = I_0 + (0.20)r^2 = 0.19 \text{ kg} \cdot \text{m}^2.$$

Invoking angular momentum conservation $L_0 = L_f$ or $I_0\omega_0 = I\omega$, we have

$$\omega = \frac{I_0}{I}\omega_0 = \frac{0.12 \text{ kg} \cdot \text{m}^2}{0.19 \text{ kg} \cdot \text{m}^2}(2.4 \text{ rad/s}) = 1.5 \text{ rad/s}.$$

62. The aerialist is in extended position with $I_1 = 19.9 \text{ kg} \cdot \text{m}^2$ during the first and last quarter of the turn, so the total angle rotated in t_1 is $\theta_1 = 0.500$ rev. In t_2 he is in a tuck position with $I_2 = 3.93 \text{ kg} \cdot \text{m}^2$, and the total angle rotated is $\theta_2 = 3.500$ rev. Since there is no external torque about his center of mass, angular momentum is conserved, $I_1\omega_1 = I_2\omega_2$. Therefore, the total flight time can be written as

$$t = t_1 + t_2 = \frac{\theta_1}{\omega_1} + \frac{\theta_2}{\omega_2} = \frac{\theta_1}{I_2\omega_2 / I_1} + \frac{\theta_2}{\omega_2} = \frac{1}{\omega_2}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right).$$

Substituting the values given, we find ω_2 to be

$$\omega_2 = \frac{1}{t}\left(\frac{I_1}{I_2}\theta_1 + \theta_2\right) = \frac{1}{1.87 \text{ s}}\left(\frac{19.9 \text{ kg} \cdot \text{m}^2}{3.93 \text{ kg} \cdot \text{m}^2}(0.500 \text{ rev}) + 3.50 \text{ rev}\right) = 3.23 \text{ rev/s}.$$

63. This is a completely inelastic collision, which we analyze using angular momentum conservation. Let m and v_0 be the mass and initial speed of the ball and R the radius of the merry-go-round. The initial angular momentum is

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 \Rightarrow \ell_0 = R(mv_0)\cos 37°$$

where ϕ = 37° is the angle between $\vec{v}_0$ and the line tangent to the outer edge of the merry-go-around. Thus, $\ell_0 = 19 \text{ kg} \cdot \text{m}^2/\text{s}$. Now, with SI units understood,

$$\ell_0 = L_f \quad \Rightarrow \quad 19\ \text{kg}\cdot\text{m}^2 = I\omega = \left(150 + (30)R^2 + (1.0)R^2\right)\omega$$

so that ω = 0.070 rad/s.

64. We treat the ballerina as a rigid object rotating around a fixed axis, initially and then again near maximum height. Her initial rotational inertia (trunk and one leg extending outward at a $90°$ angle) is

$$I_i = I_{\text{trunk}} + I_{\text{leg}} = 0.660\ \text{kg}\cdot\text{m}^2 + 1.44\ \text{kg}\cdot\text{m}^2 = 2.10\ \text{kg}\cdot\text{m}^2.$$

Similarly, her final rotational inertia (trunk and *both* legs extending outward at a $\theta = 30°$ angle) is

$$I_f = I_{\text{trunk}} + 2I_{\text{leg}}\sin^2\theta = 0.660\ \text{kg}\cdot\text{m}^2 + 2(1.44\ \text{kg}\cdot\text{m}^2)\sin^2 30° = 1.38\ \text{kg}\cdot\text{m}^2,$$

where we have used the fact that the effective length of the extended leg at an angle θ is $L_\perp = L\sin\theta$ and $I \sim L_\perp^2$. Once airborne, there is no external torque about the ballerina's center of mass and her angular momentum cannot change. Therefore, $L_i = L_f$ or $I_i\omega_i = I_f\omega_f$, and the ratio of the angular speeds is

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f} = \frac{2.10\ \text{kg}\cdot\text{m}^2}{1.38\ \text{kg}\cdot\text{m}^2} = 1.52.$$

65. **THINK** If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad.

EXPRESS The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where d is the length of the rod. The angular momentum of the wad is $mvd/2$ where m and v are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls (each having a mass M) and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where $I = (2M + m)(d/2)^2$. The equation allows us to solve for ω.

ANALYZE (a) With M = 2.00 kg, d = 0.500 m, m = 0.0500 kg, and v = 3.00 m/s, we find the angular speed to be

$$\omega = \frac{mvd}{2I} = \frac{2mv}{(2M + m)d} = \frac{2(0.0500\ \text{kg})(3.00\ \text{m/s})}{\left(2(2.00\ \text{kg}) + 0.0500\ \text{kg}\right)(0.500\ \text{m})}$$
$$= 0.148\ \text{rad/s}.$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is

$$K_f/K_i = I\omega^2/mv^2.$$

When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, the ratio becomes

$$\frac{K_f}{K_i} = \frac{m}{2M+m} = \frac{0.0500\text{ kg}}{2(2.00\text{ kg}) + 0.0500\text{ kg}} = 0.0123.$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance *h*, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mg(d/2)$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is

$$U_f = mg(d/2)(1 - \cos\theta).$$

The initial kinetic energy is the sum of that of the balls and wad:

$$K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M+m)(d/2)^2\,\omega^2.$$

At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$U_i + K_i = U_f + K_f \quad \Rightarrow \quad mg\frac{d}{2} + \frac{1}{2}(2M+m)\left(\frac{d}{2}\right)^2\omega^2 = mg\frac{d}{2}(1-\cos\theta).$$

When this equation is solved for $\cos\theta$, the result is

$$\begin{aligned}\cos\theta &= -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 \\ &= -\frac{1}{2}\left(\frac{2(2.00\text{ kg}) + 0.0500\text{ kg}}{(0.0500\text{ kg})(9.8\text{ m/s}^2)}\right)\left(\frac{0.500\text{ m}}{2}\right)(0.148\text{ rad/s})^2 \\ &= -0.0226.\end{aligned}$$

Consequently, the result for θ is 91.3°. The total angle through which it has swung is 90° + 91.3° = 181°.

LEARN This problem is rather involved. To summarize, we calculated ω using angular momentum conservation. Some energy is lost due to the inelastic collision between the putty wad and one of the balls. However, in the subsequent motion, energy is conserved, and we apply energy conservation to find the angle at which the system comes to rest momentarily.

66. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = \left(I_{\text{rod}} + md^2\right)\omega$$

where I_{rod} is found using Table 10-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2.$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{(Md^2/3) + md^2}$$

which means the system has kinetic energy $\left(I_{\text{rod}} + md^2\right)\omega^2/2$, which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 = mgH + Mg\frac{H}{2} \quad \Rightarrow \quad \frac{1}{2}\frac{m^2d^2(2gh)}{(Md^2/3) + md^2} = \left(m + \frac{M}{2}\right)gd\left(1 - \cos\theta\right)$$

from which we obtain

$$\theta = \cos^{-1}\left(1 - \frac{m^2 h}{\left(m + M/2\right)\left(m + M/3\right)}\right) = \cos^{-1}\left(1 - \frac{h/d}{\left(1 + M/2m\right)\left(1 + M/3m\right)}\right)$$
$$= \cos^{-1}\left(1 - \frac{(20\text{ cm}/\ 40\text{ cm})}{(1+1)(1+2/3)}\right) = \cos^{-1}(0.85)$$
$$= 32°.$$

67. (a) We consider conservation of angular momentum (Eq. 11-33) about the center of the rod:

$$L_i = L_f \ \Rightarrow\ -dmv + \frac{1}{12}ML^2\omega = 0$$

where negative is used for "clockwise." Item (*e*) in Table 11-2 and Eq. 11-21 (with $r_\perp = d$) have also been used. This leads to

$$d = \frac{ML^2\,\omega}{12\,m\,\text{v}} = \frac{M(0.60\text{ m})^2\,(80\text{ rad/s})}{12(M/3)(40\text{ m/s})} = 0.180\text{ m}\,.$$

(b) Increasing *d* causes the magnitude of the negative (clockwise) term in the above equation to increase. This would make the total angular momentum negative before the collision, and (by Eq. 11-33) also negative afterward. Thus, the system would rotate clockwise if *d* were greater.

68. (a) The angular speed of the top is $\omega = 30\text{ rev/s} = 30(2\pi)\text{ rad/s}$. The precession rate of the top can be obtained by using Eq. 11-46:

$$\Omega = \frac{Mgr}{I\omega} = \frac{(0.50\text{ kg})(9.8\text{ m/s}^2)(0.040\text{ m})}{(5.0\times10^{-4}\text{ kg}\cdot\text{m}^2)(60\pi\text{ rad/s})} = 2.08\text{ rad/s} \approx 0.33\text{ rev/s}.$$

(b) The direction of the precession is clockwise as viewed from overhead.

69. The precession rate can be obtained by using Eq. 11-46 with r = (11/2) cm = 0.055 m. Noting that $I_{\text{disk}} = MR^2/2$ and its angular speed is

$$\omega = 1000\text{ rev/min} = \frac{2\pi(1000)}{60}\text{ rad/s} \approx 1.0\times10^2\text{ rad/s},$$

we have

$$\Omega = \frac{Mgr}{(MR^2/2)\omega} = \frac{2gr}{R^2\omega} = \frac{2(9.8\text{ m/s}^2)(0.055\text{ m})}{(0.50\text{ m})^2(1.0\times10^2\text{ rad/s})} \approx 0.041\text{ rad/s}.$$

70. Conservation of energy implies that mechanical energy at maximum height up the ramp is equal to the mechanical energy on the floor. Thus, using Eq. 11-5, we have

$$\frac{1}{2}mv_f^2 + \frac{1}{2}I_{\text{com}}\omega_f^2 + mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_{\text{com}}\omega^2$$

where $v_f = \omega_f = 0$ at the point on the ramp where it (momentarily) stops. We note that the height h relates to the distance traveled along the ramp d by $h = d\sin(15°)$. Using item (*f*) in Table 10-2 and Eq. 11-2, we obtain

$$mgd\sin 15° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2.$$

After canceling m and plugging in $d = 1.5$ m, we find $v = 2.33$ m/s.

71. **THINK** The applied force gives rise to a torque that causes the cylinder to rotate to the right at a constant angular acceleration.

EXPRESS We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration is positive (as is the linear acceleration of the center of mass, since we take rightwards as positive). We approach this in the manner of Eq. 11-3 (*pure rotation* about point *P*) but use torques instead of energy. The torque (relative to point *P*) is $\tau = I_P\alpha$, where

$$I_P = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

with the use of the parallel-axis theorem and Table 10-2(c). The torque is due to the F_{app} force and can be written as $\tau = F_{\text{app}}(2R)$. In this way, we find

$$\tau = I_P\alpha = \left(\frac{3}{2}MR^2\right)\alpha = 2RF_{\text{app}}.$$

The equation allows us to solve for the angular acceleration α, which is related to the acceleration of the center of mass as $\alpha = a_{\text{com}}/R$.

ANALYZE (a) With $M = 10\,\text{kg}$, $R = 0.10$ m and $F_{\text{app}} = 12$ N, we obtain

$$a_{\text{com}} = \alpha R = \frac{2R^2F_{\text{app}}}{3MR^2/2} = \frac{4F_{\text{app}}}{3M} = \frac{4(12\text{ N})}{3(10\text{ kg})} = 1.6\text{ m/s}^2.$$

(b) The magnitude of the angular acceleration is

$$\alpha = a_{\text{com}}/R = (1.6\text{ m/s}^2)/(0.10\text{ m}) = 16\,\text{rad/s}^2.$$

(c) Applying Newton's second law in its linear form yields $(12\,\text{N}) - f = Ma_{\text{com}}$. Therefore, f = –4.0 N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* with magnitude 4.0 N, i.e., $\vec{f} = (4.0\ \text{N})\hat{\text{i}}$.

LEARN As the cylinder rolls to the right, the frictional force also points to the right to oppose the tendency to slip.

72. The rotational kinetic energy is $K = \frac{1}{2} I\omega^2$, where $I = mR^2$ is its rotational inertia about the center of mass (Table 10-2(a)), m = 140 kg, and $\omega = v_{\text{com}}/R$ (Eq. 11-2). The ratio is

$$\frac{K_{\text{transl}}}{K_{\text{rot}}} = \frac{\frac{1}{2} m v_{\text{com}}^2}{\frac{1}{2}\left(mR^2\right)\left(v_{\text{com}}/R\right)^2} = 1.00.$$

73. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r}\,' = x'\hat{\text{i}} + y'\hat{\text{j}}$, then (using Eq. 3-30) we find

$$\vec{r}\,' \times \vec{v} = \left(x'v_y - y'v_x\right)\hat{\text{k}}.$$

(a) Here, $\vec{r}\,'$ points in either the $+\hat{\text{i}}$ or the $-\hat{\text{i}}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\text{i}} \times \hat{\text{i}} = 0$) that $\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = 0$ in this case.

(b) The net force is in the $-\hat{\text{i}}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_{\text{o}}$ where $\vec{r}_{\text{o}} = 2.0\hat{\text{i}} + 5.0\hat{\text{j}}$ (with SI units understood) and points from (2.0, 5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$, which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_{\text{o}} \times \vec{v}\right) = -(3.0)\Big((2.0)(0) - (5.0)\left(-2.0t^3\right)\Big)\hat{\text{k}}$$

which yields $\vec{\ell} = (-30t^3\hat{\text{k}})\ \text{kg}\cdot\text{m/s}^2$.

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\hat{\text{i}}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_{\text{o}} \times \vec{a}\right) = -(3.0)\Big((2.0)(0) - (5.0)\left(-6.0t^2\right)\Big)\hat{\text{k}}$$

which yields $\vec{\tau} = (-90t^2\hat{k})$ N·m.

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\hat{i} - 5.0\hat{j}$ (with SI units understood) and points from (2.0, –5.0, 0) to the instantaneous position of the car (indicated by $\vec{r}$, which points in either the +*x* or –*x* directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = -m(\vec{r}_o \times \vec{v}) = -(3.0)\Big((2.0)(0) - (-5.0)(-2.0t^3)\Big)\hat{k}$$

which yields $\vec{\ell} = (30t^3\hat{k})$ kg·m²/s.

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\hat{i}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m(\vec{r}\,' \times \vec{a}) = -m(\vec{r}_o \times \vec{a}) = -(3.0)\Big((2.0)(0) - (-5.0)(-6.0t^2)\Big)\hat{k}$$

which yields $\vec{\tau} = (90t^2\hat{k})$ N·m.

74. For a constant (single) torque, Eq. 11-29 becomes

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{\Delta\vec{L}}{\Delta t}.$$

Thus, we obtain

$$\Delta t = \frac{\Delta L}{\tau} = \frac{600\ \text{kg}\cdot\text{m}^2/\text{s}}{50\ \text{N}\cdot\text{m}} = 12\ \text{s}.$$

75. **THINK** No external torque acts on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved.

EXPRESS An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 11-21, *mvR*, where *R* is the radius of the merry-go-round.

ANALYZE (a) In terms of the radius of gyration *k*, the rotational inertia of the merry-go-round is $I = Mk^2$. With M = 180 kg and k = 0.91 m, we obtain

$$I = (180\ \text{kg})\,(0.910\ \text{m})^2 = 149\ \text{kg}\cdot\text{m}^2.$$

(b) The magnitude of angular momentum of the running child about the axis of rotation of the merry-go-round is

$$L_{\text{child}} = mvR = (44.0\ \text{kg})(3.00\ \text{m/s})(1.20\ \text{m}) = 158\ \text{kg}\cdot\text{m}^2/\text{s}.$$

(c) The initial angular momentum is given by $L_i = L_{\text{child}} = mvR$; the final angular momentum is given by $L_f = (I + mR^2)\ \omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = (I + mR^2)\omega$ and

$$\omega = \frac{mvR}{I + mR^2} = \frac{158\ \text{kg}\cdot\text{m}^2/\text{s}}{149\ \text{kg}\cdot\text{m}^2 + (44.0\ \text{kg})(1.20\ \text{m})^2} = 0.744\ \text{rad/s}.$$

LEARN The child initially had an angular velocity of

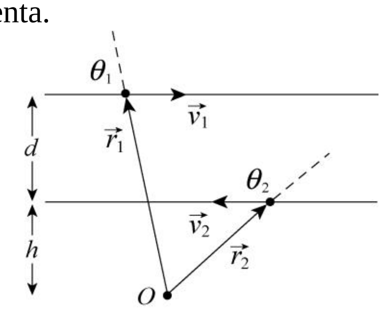
$$\omega_0 = \frac{v}{R} = \frac{3.00\ \text{m/s}}{1.20\ \text{m}} = 2.5\ \text{rad/s}.$$

After he jumped onto the merry-go-round, the rotational inertia of the system (merry-go-round + child) increases, so the angular velocity decreases by angular momentum conservation.

76. Item (*i*) in Table 10-2 gives the moment of inertia about the center of mass in terms of width *a* (0.15 m) and length *b* (0.20 m). In using the parallel axis theorem, the distance from the center to the point about which it spins (as described in the problem) is $\sqrt{(a/4)^2 + (b/4)^2}$. If we denote the thickness as *h* (0.012 m) then the volume is *abh*, which means the mass is ρabh (where $\rho = 2640\ \text{kg/m}^3$ is the density). We can write the kinetic energy in terms of the angular momentum by substituting $\omega = L/I$ into Eq. 10-34:

$$K = \frac{1}{2}\frac{L^2}{I} = \frac{1}{2}\frac{(0.104)^2}{\rho abh((a^2 + b^2)/12 + (a/4)^2 + (b/4)^2)} = 0.62\ \text{J}.$$

77. **THINK** Our system consists of two particles moving in opposite directions along parallel lines. The angular momentum of the system about a point is the vector sum of the two individual angular momenta.



EXPRESS The diagram above shows the particles and their lines of motion. The origin is marked *O* and may be anywhere. We set up our coordinate system in such a way that

+x is to the right, +y up and +z out of the page. The angular momentum of the system about O is

$$\vec{\ell} = \vec{\ell}_1 + \vec{\ell}_2 = \vec{r}_1 \times \vec{p}_1 + \vec{r}_2 \times \vec{p}_2 = m(\vec{r}_1 \times \vec{v}_1 + \vec{r}_2 \times \vec{v}_2)$$

since $m_1 = m_2 = m$.

ANALYZE (a) With $\vec{v}_1 = v_1\hat{\text{i}}$, the angular momentum of particle 1 has magnitude

$$\ell_1 = mvr_1 \sin\theta_1 = mv\left(d+h\right)$$

and is in the –z-direction, or into the page. On the other hand, with $\vec{v}_2 = -v_2\hat{\text{i}}$, the angular momentum of particle 2 has magnitude $\ell_2 = mvr_2 \sin\theta_2 = mvh$, and is in the +$z$-direction, or out of the page. The net angular momentum has magnitude

$$\ell = mv(d+h) - mvh = mvd$$

which depends only on the separation between the two lines and not on the location of the origin. Thus, if O is midway between the two lines, the total angular momentum is

$$\ell = mvd = (2.90\times10^{-4}\text{ kg})(5.46\text{ m/s})(0.042\text{ m}) = 6.65\times10^{-5}\text{ kg}\cdot\text{m}^2/\text{s}$$

and is into the page.

(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then

$$\ell = mv(d+h) + mvh = mv(d+2h).$$

This result now depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $\ell = 0$.

(d) As we have seen in part (c), the result depends on the choice of origin.

LEARN Angular momentum is a vector quantity. For a system of many particles, the total angular momentum about a point is

$$\vec{\ell} = \vec{\ell}_1 + \vec{\ell}_2 + \cdots = \sum_i \vec{\ell}_i = \sum_i m_i \vec{r}_i \times \vec{v}_i .$$

78. (a) Using Eq. 2-16 for the translational (center-of-mass) motion, we find

$$v^2 = v_0^2 + 2a\Delta x \Rightarrow a = -\frac{v_0^2}{2\Delta x}$$

which yields a = –4.11 for v_0 = 43 and $\Delta x = 225$ (SI units understood). The magnitude of the linear acceleration of the center of mass is therefore 4.11 m/s^2.

(b) With R = 0.250 m, Eq. 11-6 gives

$$|\alpha| = |a|/R = 16.4\,\text{rad/s}^2.$$

If the wheel is going rightward, it is rotating in a clockwise sense. Since it is slowing down, this angular acceleration is counterclockwise (opposite to ω) so (with the usual convention that counterclockwise is positive) there is no need for the absolute value signs for α.

(c) Equation 11-8 applies with Rf_s representing the magnitude of the frictional torque. Thus,

$$Rf_s = I\alpha = (0.155\ \text{kg}\cdot\text{m}^2)\,(16.4\ \text{rad/s}^2) = 2.55\ \text{N}\cdot\text{m}.$$

79. We use $L = I\omega$ and $K = \frac{1}{2}I\omega^2$ and observe that the speed of points on the rim (corresponding to the speed of points on the belt) of wheels A and B must be the same (so $\omega_A R_A = \omega_B r_B$).

(a) If $L_A = L_B$ (call it L) then the ratio of rotational inertias is

$$\frac{I_A}{I_B} = \frac{L/\omega_A}{L/\omega_B} = \frac{\omega_A}{\omega_B} = \frac{R_A}{R_B} = \frac{1}{3} = 0.333.$$

(b) If we have $K_A = K_B$ (call it K) then the ratio of rotational inertias becomes

$$\frac{I_A}{I_B} = \frac{2K/\omega_A^2}{2K/\omega_B^2} = \left(\frac{\omega_B}{\omega_A}\right)^2 = \left(\frac{R_A}{R_B}\right)^2 = \frac{1}{9} = 0.111.$$

80. The total angular momentum (about the origin) before the collision (using Eq. 11-18 and Eq. 3-30 for each particle and then adding the terms) is

$$\vec{L}_i = [(0.5\ \text{m})(2.5\ \text{kg})(3.0\ \text{m/s}) + (0.1\ \text{m})(4.0\ \text{kg})(4.5\ \text{m/s})]\hat{\text{k}}.$$

The final angular momentum of the stuck-together particles (after the collision) measured relative to the origin is (using Eq. 11-33)

$$\vec{L}_f = \vec{L}_i = (5.55\ \text{kg}\cdot\text{m}^2/\text{s})\hat{\text{k}}.$$

81. **THINK** As the wheel rolls without slipping down an inclined plane, its gravitational potential energy is converted into translational and rotational kinetic energies.

EXPRESS As the wheel-axel system rolls down the inclined plane by a distance *d*, the change in potential energy is $\Delta U = -mgd\sin\theta$. By energy conservation, the total kinetic energy gained is

$$-\Delta U = \Delta K = \Delta K_{\text{trans}} + \Delta K_{\text{rot}} \quad \Rightarrow \quad mgd\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 .$$

Since the axel rolls without slipping, the angular speed is given by $\omega = v/r$, where r is the radius of the axel. The above equation then becomes

$$mgd\sin\theta = \frac{1}{2}I\omega^2\left(\frac{mr^2}{I}+1\right) = \Delta K_{\text{rot}}\left(\frac{mr^2}{I}+1\right).$$

ANALYZE (a) With m=10.0 kg, d = 2.00 m, r = 0.200 m, and $I = 0.600\ \text{kg}\cdot\text{m}^2$, the rotational kinetic energy may be obtained as

$$\Delta K_{\text{rot}} = \frac{mgd\sin\theta}{\dfrac{mr^2}{I}+1} = \frac{(10.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\sin 30.0^\circ}{\dfrac{(10.0\ \text{kg})(0.200\ \text{m})^2}{0.600\ \text{kg}\cdot\text{m}^2}+1} = 58.8\ \text{J} .$$

(b) The translational kinetic energy is $\Delta K_{\text{trans}} = \Delta K - \Delta K_{\text{rot}} = 98\ \text{J} - 58.8\ \text{J} = 39.2\ \text{J}$.

LEARN One may show that $mr^2/I = 2/3$, which implies that $\Delta K_{\text{trans}}/\Delta K_{\text{rot}} = 2/3$. Equivalently, we may write $\Delta K_{\text{trans}}/\Delta K = 2/5$ and $\Delta K_{\text{rot}}/\Delta K = 3/5$. So as the wheel rolls down, 40% of the kinetic energy is translational while the other 60% is rotational.

82. (a) We use Table 10-2(e) and the parallel-axis theorem to obtain the rod's rotational inertia about an axis through one end:

$$I = I_{\text{com}} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

where L = 6.00 m and M = 10.0/9.8 = 1.02 kg. Thus, the inertia is $I = 12.2\ \text{kg}\cdot\text{m}^2$.

(b) Using ω = (240)(2π/60) = 25.1 rad/s, Eq. 11-31 gives the magnitude of the angular momentum as

$$I\omega = \left(12.2\ \text{kg}\cdot\text{m}^2\right)\left(25.1\text{rad/s}\right) = 308\ \ \text{kg}\cdot\text{m}^2\text{/s} .$$

Since it is rotating clockwise as viewed from above, then the right-hand rule indicates that its direction is down.

83. We note that its mass is M = 36/9.8 = 3.67 kg and its rotational inertia is $I_{\text{com}} = \frac{2}{5}MR^2$ (Table 10-2(f)).

(a) Using Eq. 11-2, Eq. 11-5 becomes

$$K = \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{com}}^2 = \frac{7}{10}Mv_{\text{com}}^2$$

which yields K = 61.7 J for v_{com} = 4.9 m/s.

(b) This kinetic energy turns into potential energy Mgh at some height $h = d \sin \theta$ where the sphere comes to rest. Therefore, we find the distance traveled up the $\theta = 30°$ incline from energy conservation:

$$\frac{7}{10}Mv_{\text{com}}^2 = Mgd\sin\theta \quad \Rightarrow \quad d = \frac{7v_{\text{com}}^2}{10g\sin\theta} = 3.43\,\text{m}.$$

(c) As shown in the previous part, M cancels in the calculation for d. Since the answer is independent of mass, then it is also independent of the sphere's weight.

84. (a) The acceleration is given by Eq. 11-13:

$$a_{\text{com}} = \frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where upward is the positive translational direction. Taking the coordinate origin at the initial position, Eq. 2-15 leads to

$$y_{\text{com}} = v_{\text{com},0}t + \frac{1}{2}a_{\text{com}}t^2 = v_{\text{com},0}t - \frac{\frac{1}{2}gt^2}{1 + I_{\text{com}}/MR_0^2}$$

where y_{com} = – 1.2 m and $v_{\text{com},0}$ = – 1.3 m/s. Substituting $I_{\text{com}} = 0.000095\ \text{kg}\cdot\text{m}^2$, M = 0.12 kg, R_0 = 0.0032 m, and g = 9.8 m/s^2, we use the quadratic formula and find

$$\begin{aligned} t &= \frac{\left(1 + \frac{I_{\text{com}}}{MR_0^2}\right)\left(v_{\text{com},0} \mp \sqrt{v_{\text{com},0}^2 - \frac{2gy_{\text{com}}}{1 + I_{\text{com}}/MR_0^2}}\right)}{g} \\ &= \frac{\left(1 + \frac{0.000095}{(0.12)(0.0032)^2}\right)\left(-1.3 \mp \sqrt{(1.3)^2 - \frac{2(9.8)(-1.2)}{1 + 0.000095/(0.12)(0.0032)^2}}\right)}{9.8} \\ &= -21.7 \text{ or } 0.885 \end{aligned}$$

where we choose t = 0.89 s as the answer.

(b) We note that the initial potential energy is $U_i = Mgh$ and h = 1.2 m (using the bottom as the reference level for computing U). The initial kinetic energy is as shown in Eq. 11-5, where the initial angular and linear speeds are related by Eq. 11-2. Energy conservation leads to

$$\begin{aligned}K_f &= K_i + U_i = \frac{1}{2}mv_{\text{com},0}^2 + \frac{1}{2}I\left(\frac{v_{\text{com},0}}{R_0}\right)^2 + Mgh \\ &= \frac{1}{2}(0.12\text{ kg})(1.3\text{ m/s})^2 + \frac{1}{2}\left(9.5\times10^{-5}\text{ kg}\cdot\text{m}^2\right)\left(\frac{1.3\text{ m/s}}{0.0032\text{ m}}\right)^2 + (0.12\text{ kg})\left(9.8\text{ m/s}^2\right)(1.2\text{ m}) \\ &= 9.4\text{ J.}\end{aligned}$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}}t = v_{\text{com},0} - \frac{gt}{1 + I_{\text{com}}/MR_0^2}.$$

Thus, we obtain

$$v_{\text{com}} = -1.3\text{ m/s} - \frac{\left(9.8\text{ m/s}^2\right)(0.885\text{ s})}{1 + \dfrac{0.000095\text{ kg}\cdot\text{m}^2}{(0.12\text{ kg})(0.0032\text{ m})^2}} = -1.41\text{ m/s}$$

so its linear speed at that moment is approximately 1.4 m/s.

(d) The translational kinetic energy is

$$\tfrac{1}{2}mv_{\text{com}}^2 = \tfrac{1}{2}(0.12\text{ kg})(-1.41\text{ m/s})^2 = 0.12\text{ J.}$$

(e) The angular velocity at that moment is given by

$$\omega = -\frac{v_{\text{com}}}{R_0} = -\frac{-1.41\text{ m/s}}{0.0032\text{ m}} = 441\text{rad/s} \approx 4.4\times10^2\text{ rad/s}.$$

(f) And the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}\left(9.50\times10^{-5}\text{ kg}\cdot\text{m}^2\right)(441\text{ rad/s})^2 = 9.2\text{ J.}$$

85. The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $(I + MR^2)\,\omega$, which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground.

(a) Angular momentum conservation leads to

$$0 = \left(I + MR^2\right)\omega - mRv \quad \Rightarrow \quad \omega = \frac{mRv}{I + MR^2}.$$

(b) The girl's linear speed is given by Eq. 10-18:

$$R\omega = \frac{mvR^2}{I + MR^2}.$$

86. (a) Interpreting *h* as the height increase for the center of mass of the body, then (using Eq. 11-5) mechanical energy conservation, $K_i = U_f$, leads to

$$\frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 = mgh \quad \Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = mg\left(\frac{3v^2}{4g}\right)$$

from which *v* cancels and we obtain $I = \frac{1}{2}mR^2$.

(b) From Table 10-2(c), we see that the body could be a solid cylinder.

Chapter 12

1. (a) The center of mass is given by

$$x_{\text{com}} = \frac{0+0+0+(m)(2.00\text{ m})+(m)(2.00\text{ m})+(m)(2.00\text{ m})}{6m} = 1.00\text{ m}.$$

(b) Similarly, we have

$$y_{\text{com}} = \frac{0+(m)(2.00\text{ m})+(m)(4.00\text{ m})+(m)(4.00\text{ m})+(m)(2.00\text{ m})+0}{6m} = 2.00\text{ m}.$$

(c) Using Eq. 12-14 and noting that the gravitational effects are different at the different locations in this problem, we have

$$x_{\text{cog}} = \frac{\sum_{i=1}^{6} x_i m_i g_i}{\sum_{i=1}^{6} m_i g_i} = \frac{x_1 m_1 g_1 + x_2 m_2 g_2 + x_3 m_3 g_3 + x_4 m_4 g_4 + x_5 m_5 g_5 + x_6 m_6 g_6}{m_1 g_1 + m_2 g_2 + m_3 g_3 + m_4 g_4 + m_5 g_5 + m_6 g_6} = 0.987\text{ m}.$$

(d) Similarly, we have

$$\begin{aligned} y_{\text{cog}} &= \frac{\sum_{i=1}^{6} y_i m_i g_i}{\sum_{i=1}^{6} m_i g_i} = \frac{y_1 m_1 g_1 + y_2 m_2 g_2 + y_3 m_3 g_3 + y_4 m_4 g_4 + y_5 m_5 g_5 + y_6 m_6 g_6}{m_1 g_1 + m_2 g_2 + m_3 g_3 + m_4 g_4 + m_5 g_5 + m_6 g_6} \\ &= \frac{0+(2.00)(7.80m)+(4.00)(7.60m)+(4.00)(7.40m)+(2.00)(7.60m)+0}{8.0m+7.80m+7.60m+7.40m+7.60m+7.80m} \\ &= 1.97\text{ m}. \end{aligned}$$

2. Our notation is as follows: M = 1360 kg is the mass of the automobile; L = 3.05 m is the horizontal distance between the axles; $\ell = (3.05 - 1.78)\text{ m} = 1.27\text{ m}$ is the horizontal distance from the rear axle to the center of mass; F_1 is the force exerted on each front wheel; and F_2 is the force exerted on each back wheel.

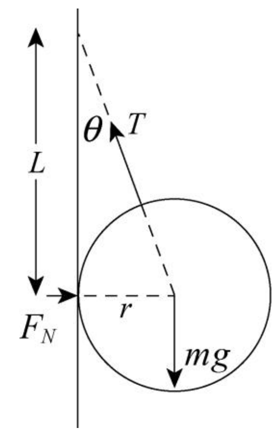
(a) Taking torques about the rear axle, we find

$$F_1 = \frac{Mg\ell}{2L} = \frac{(1360\text{ kg})(9.80\text{ m/s}^2)(1.27\text{ m})}{2(3.05\text{ m})} = 2.77\times10^3\text{ N}.$$

(b) Equilibrium of forces leads to $2F_1 + 2F_2 = Mg$, from which we obtain $F_2 = 3.89 \times 10^3$ N .

3. **THINK** Three forces act on the sphere: the tension force $\vec{T}$ of the rope, the force of the wall $\vec{F}_N$, and the force of gravity $m\vec{g}$.

EXPRESS The free-body diagram is shown to the right. The tension force $\vec{T}$ acts along the rope, the force of the wall $\vec{F}_N$ acts horizontally away from the wall, and the force of gravity $m\vec{g}$ acts downward. Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then Newton's second law gives



vertical component : $T \cos\theta - mg = 0$
horizontal component : $F_N - T \sin\theta = 0$.

ANALYZE (a) We solve the first equation for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L^2 + r^2}$ to obtain

$$T = \frac{mg\sqrt{L^2+r^2}}{L} = \frac{(0.85\ \text{kg})(9.8\ \text{m/s}^2)\sqrt{(0.080\ \text{m})^2 + (0.042\ \text{m})^2}}{0.080\ \text{m}} = 9.4\ \text{N}.$$

(b) We solve the second equation for the normal force: $F_N = T\sin\theta$. Using $\sin\theta = r/\sqrt{L^2+r^2}$, we obtain

$$F_N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\frac{r}{\sqrt{L^2+r^2}} = \frac{mgr}{L} = \frac{(0.85\ \text{kg})(9.8\ \text{m/s}^2)(0.042\ \text{m})}{(0.080\ \text{m})} = 4.4\ \text{N}.$$

LEARN Since the sphere is in static equilibrium, the vector sum of all external forces acting on it must be zero.

4. The situation is somewhat similar to that depicted for problem 10 (see the figure that accompanies that problem in the text). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T \sin\theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are collinear). Setting $T = F$ therefore yields $\theta = 30°$. Since $\alpha = 180° - 2\theta$ is the angle between the two segments, then we find $\alpha = 120°$.

5. The object exerts a downward force of magnitude $F = 3160$ N at the midpoint of the rope, causing a "kink" similar to that shown for problem 10 (see the figure that accompanies that problem in the text). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T \sin\theta = F$, where θ is the angle (taken

positive) between each segment of the string and its "relaxed" position (when the two segments are collinear). In this problem, we have

$$\theta = \tan^{-1}\left(\frac{0.35\,\text{m}}{1.72\,\text{m}}\right) = 11.5°.$$

Therefore, $T = F/(2\sin\theta) = 7.92 \times 10^3$ N.

6. Let $\ell_1 = 1.5\,\text{m}$ and $\ell_2 = (5.0 - 1.5)\ \text{m} = 3.5\ \text{m}$. We denote tension in the cable closer to the window as F_1 and that in the other cable as F_2. The force of gravity on the scaffold itself (of magnitude $m_s g$) is at its midpoint, $\ell_3 = 2.5\,\text{m}$ from either end.

(a) Taking torques about the end of the plank farthest from the window washer, we find

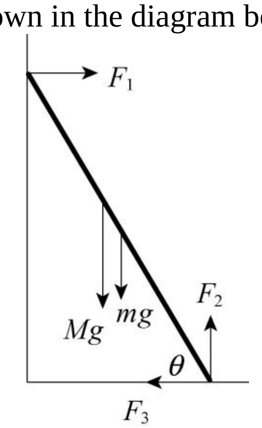
$$F_1 = \frac{m_w g \ell_2 + m_s g \ell_3}{\ell_1 + \ell_2} = \frac{(80\,\text{kg})(9.8\,\text{m/s}^2)(3.5\,\text{m}) + (60\,\text{kg})(9.8\,\text{m/s}^2)(2.5\,\text{m})}{5.0\,\text{m}}$$
$$= 8.4 \times 10^2\ \text{N}.$$

(b) Equilibrium of forces leads to

$$F_1 + F_2 = m_s g + m_w g = (60\,\text{kg} + 80\,\text{kg})(9.8\,\text{m/s}^2) = 1.4 \times 10^3\ \text{N}$$

which (using our result from part (a)) yields $F_2 = 5.3 \times 10^2\,\text{N}$.

7. The forces on the ladder are shown in the diagram below.



F_1 is the force of the window, horizontal because the window is frictionless. F_2 and F_3 are components of the force of the ground on the ladder. M is the mass of the window cleaner and m is the mass of the ladder.

The force of gravity on the man acts at a point 3.0 m up the ladder and the force of gravity on the ladder acts at the center of the ladder. Let θ be the angle between the ladder and the ground. We use $\cos\theta = d/L$ or $\sin\theta = \sqrt{L^2 - d^2}/L$ to find θ = 60º. Here L

is the length of the ladder (5.0 m) and d is the distance from the wall to the foot of the ladder (2.5 m).

(a) Since the ladder is in equilibrium the sum of the torques about its foot (or any other point) vanishes. Let ℓ be the distance from the foot of the ladder to the position of the window cleaner. Then,

$$Mg\ell\cos\theta + mg\left(L/2\right)\cos\theta - F_1 L\sin\theta = 0\,,$$

and

$$F_1 = \frac{(M\ell + mL/2)g\cos\theta}{L\sin\theta} = \frac{[(75\,\text{kg})(3.0\,\text{m})+(10\,\text{kg})(2.5\,\text{m})](9.8\,\text{m/s}^2)\cos 60°}{(5.0\,\text{m})\sin 60°}$$
$$= 2.8\times10^2\ \text{N}.$$

This force is outward, away from the wall. The force of the ladder on the window has the same magnitude but is in the opposite direction: it is approximately 280 N, inward.

(b) The sum of the horizontal forces and the sum of the vertical forces also vanish:

$$F_1 - F_3 = 0$$
$$F_2 - Mg - mg = 0$$

The first of these equations gives $F_3 = F_1 = 2.8\times10^2\,\text{N}$ and the second gives

$$F_2 = (M+m)g = (75\,\text{kg} + 10\,\text{kg})(9.8\,\text{m/s}^2) = 8.3\times10^2\,\text{N}\,.$$

The magnitude of the force of the ground on the ladder is given by the square root of the sum of the squares of its components:

$$F = \sqrt{F_2^{\,2} + F_3^{\,2}} = \sqrt{(2.8\times10^2\,\text{N})^2 + (8.3\times10^2\,\text{N})^2} = 8.8\times10^2\,\text{N}.$$

(c) The angle ϕ between the force and the horizontal is given by

$$\tan\phi = F_3/F_2 = (830\text{ N})/(280\text{ N}) = 2.94,$$

so $\phi = 71°$. The force points to the left and upward, 71° above the horizontal. We note that this force is not directed along the ladder.

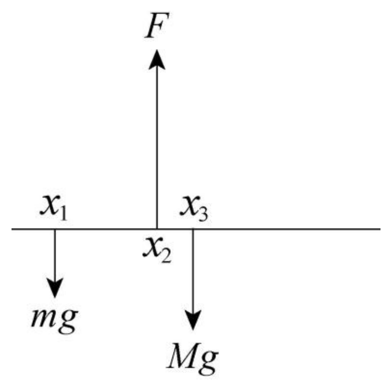
8. From $\vec{\tau} = \vec{r}\times\vec{F}$, we note that persons 1 through 4 exert torques pointing out of the page (relative to the fulcrum), and persons 5 through 8 exert torques pointing into the page.

(a) Among persons 1 through 4, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 2.

(b) Among persons 5 through 8, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 7.

9. **THINK** In order for the meter stick to remain in equilibrium, the net force acting on it must be zero. In addition, the net torque about any point must also be zero.

EXPRESS Let the x axis be along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown to the right. The coins are at $x = x_1 = 0.120$ m, and $m = 10.0$ g is their total mass. The knife edge is at $x = x_2 = 0.455$ m and exerts force $\vec{F}$. The mass of the meter stick is M, and the force of gravity acts at the center of the stick, $x = x_3 = 0.500$ m.



Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish:

$$Mg(x_3 - x_2) - mg(x_2 - x_1) = 0.$$

ANALYZE Solving the equation above for M, we find the mass of the meter stick to be

$$M = \left(\frac{x_2 - x_1}{x_3 - x_2}\right)m = \left(\frac{0.455\,\text{m} - 0.120\,\text{m}}{0.500\,\text{m} - 0.455\,\text{m}}\right)(10.0\text{g}) = 74.4\ \text{g}.$$

LEARN Since the torque about any point is zero, we could have chosen x_1. In this case, balance of torques requires that

$$F(x_2 - x_1) - Mg(x_3 - x_1) = 0$$

The fact that the net force is zero implies $F = (M + m)g$. Substituting this into the above equation gives the same result as before:

$$M = \left(\frac{x_2 - x_1}{x_3 - x_2}\right)m.$$

10. (a) Analyzing vertical forces where string 1 and string 2 meet, we find

$$T_1 = \frac{w_A}{\cos\phi} = \frac{40\text{N}}{\cos 35°} = 49\text{N}.$$

(b) Looking at the horizontal forces at that point leads to

$$T_2 = T_1 \sin 35° = (49\text{N})\sin 35° = 28\,\text{N}.$$

(c) We denote the components of T_3 as T_x (rightward) and T_y (upward). Analyzing horizontal forces where string 2 and string 3 meet, we find $T_x = T_2 = 28$ N. From the vertical forces there, we conclude $T_y = w_B = 50$ N. Therefore,

$$T_3 = \sqrt{T_x^2 + T_y^2} = 57\,\text{N}.$$

(d) The angle of string 3 (measured from vertical) is

$$\theta = \tan^{-1}\left(\frac{T_x}{T_y}\right) = \tan^{-1}\left(\frac{28}{50}\right) = 29°.$$

11. **THINK** The diving board is in equilibrium, so the net force and net torque must be zero.

EXPRESS We take the force of the left pedestal to be F_1 at $x = 0$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = d$. Upward direction is taken to be positive and W is the weight of the diver, located at $x = L$. The following two equations result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

$$F_1 + F_2 - W = 0$$
$$F_1 d + W(L - d) = 0$$

ANALYZE (a) The second equation gives

$$F_1 = -\left(\frac{L-d}{d}\right)W = -\left(\frac{3.0\,\text{m}}{1.5\,\text{m}}\right)(580\,\text{N}) = -1160\,\text{N}$$

which should be rounded off to $F_1 = -1.2\times10^3$ N. Thus, $|F_1| = 1.2\times10^3$ N.

(b) Since F_1 is negative, this force is downward.

(c) The first equation gives $F_2 = W - F_1 = 580\,\text{N} + 1160\,\text{N} = 1740\,\text{N}$.

which should be rounded off to $F_2 = 1.7\times10^3$ N. Thus, $|F_2| = 1.7\times10^3$ N.

(d) The result is positive, indicating that this force is upward.

(e) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched.

(f) The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

LEARN We can relate F_1 and F_2 via $F_1 = -\left(\frac{L-d}{L}\right)F_2$. The expression makes it clear that the two forces must be of opposite signs, i.e., one acting downward and the other upward.

12. The angle of each half of the rope, measured from the dashed line, is

$$\theta = \tan^{-1}\left(\frac{0.30\,\text{m}}{9.0\,\text{m}}\right) = 1.9°.$$

Analyzing forces at the "kink" (where $\vec{F}$ is exerted) we find

$$T = \frac{F}{2\sin\theta} = \frac{550\,\text{N}}{2\sin 1.9°} = 8.3\times10^3\ \text{N}.$$

13. The (vertical) forces at points A, B, and P are F_A, F_B, and F_P, respectively. We note that $F_P = W$ and is upward. Equilibrium of forces and torques (about point B) lead to

$$F_A + F_B + W = 0$$
$$bW - aF_A = 0.$$

(a) From the second equation, we find

$$F_A = bW/a = (15/5)W = 3W = 3(900\ \text{N}) = 2.7\times10^3\ \text{N}.$$

(b) The direction is upward since $F_A > 0$.

(c) Using this result in the first equation above, we obtain

$$F_B = W - F_A = -4W = -4(900\ \text{N}) = -3.6\times10^3\,\text{N},$$

or $|F_B| = 3.6\times10^3\,\text{N}$.

(d) F_B points downward, as indicated by the negative sign.

14. With pivot at the left end, Eq. 12-9 leads to

$$-m_s g\frac{L}{2} - Mgx + T_R L = 0$$

where m_s is the scaffold's mass (50 kg) and M is the total mass of the paint cans (75 kg). The variable x indicates the center of mass of the paint can collection (as measured from the left end), and T_R is the tension in the right cable (722 N). Thus we obtain $x = 0.702$ m.

15. (a) Analyzing the horizontal forces (which add to zero) we find $F_h = F_3 = 5.0$ N.

(b) Equilibrium of vertical forces leads to $F_v = F_1 + F_2 = 30$ N.

(c) Computing torques about point O, we obtain

$$F_v d = F_2 b + F_3 a \Rightarrow d = \frac{(10\ \mathrm{N})(3.0\,\mathrm{m}) + (5.0\ \mathrm{N})(2.0\,\mathrm{m})}{30\ \mathrm{N}} = 1.3\,\mathrm{m}.$$

16. The forces exerted horizontally by the obstruction and vertically (upward) by the floor are applied at the bottom front corner C of the crate, as it verges on tipping. The center of the crate, which is where we locate the gravity force of magnitude $mg = 500$ N, is a horizontal distance $\ell = 0.375\,\mathrm{m}$ from C. The applied force of magnitude $F = 350$ N is a vertical distance h from C. Taking torques about C, we obtain

$$h = \frac{mg\ell}{F} = \frac{(500\,\mathrm{N})(0.375\,\mathrm{m})}{350\,\mathrm{N}} = 0.536\,\mathrm{m}.$$

17. (a) With the pivot at the hinge, Eq. 12-9 gives

$$TL\cos\theta - mg\,\tfrac{L}{2} = 0.$$

This leads to $\theta = 78°$. Then the geometric relation $\tan\theta = L/D$ gives $D = 0.64$ m.

(b) A higher (steeper) slope for the cable results in a smaller tension. Thus, making D greater than the value of part (a) should prevent rupture.

18. With pivot at the left end of the lower scaffold, Eq. 12-9 leads to

$$-m_2 g \frac{L_2}{2} - mgd + T_R L_2 = 0$$

where m_2 is the lower scaffold's mass (30 kg) and L_2 is the lower scaffold's length (2.00 m). The mass of the package ($m = 20$ kg) is a distance $d = 0.50$ m from the pivot, and T_R is the tension in the rope connecting the right end of the lower scaffold to the larger scaffold above it. This equation yields $T_R = 196$ N. Then Eq. 12-8 determines T_L (the tension in the cable connecting the right end of the lower scaffold to the larger scaffold above it): $T_L = 294$ N. Next, we analyze the larger scaffold (of length $L_1 = L_2 + 2d$ and mass m_1, given in the problem statement) placing our pivot at its left end and using Eq. 12-9:

$$-m_1 g \frac{L_1}{2} - T_L d - T_R (L_1 - d) + T L_1 = 0.$$

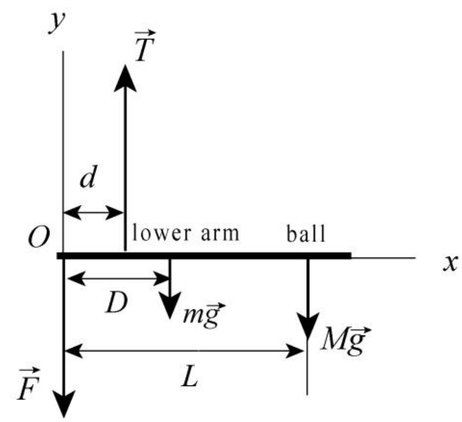
This yields $T = 457$ N.

19. Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_{\perp} = (40\,\text{N})\frac{d}{L} = (40\,\text{N})\frac{2.6\,\text{cm}}{12\,\text{cm}} = 8.7\,\text{N}.$$

20. Our system consists of the lower arm holding a bowling ball. As shown in the free-body diagram, the forces on the lower arm consist of $\vec{T}$ from the biceps muscle, $\vec{F}$ from the bone of the upper arm, and the gravitational forces, $m\vec{g}$ and $M\vec{g}$. Since the system is in static equilibrium, the net force acting on the system is zero:

$$0 = \sum F_{\text{net},y} = T - F - (m+M)g\ .$$



In addition, the net torque about O must also vanish:

$$0 = \sum_{O} \tau_{\text{net}} = (d)(T) + (0)F - (D)(mg) - L(Mg)\ .$$

(a) From the torque equation, we find the force on the lower arms by the biceps muscle to be

$$T = \frac{(mD+ML)g}{d} = \frac{[(1.8\text{ kg})(0.15\text{ m}) + (7.2\text{ kg})(0.33\text{ m})](9.8\text{ m/s}^2)}{0.040\text{ m}}$$
$$= 648\text{ N} \approx 6.5\times 10^2\text{ N}.$$

(b) Substituting the above result into the force equation, we find F to be

$$F = T - (M+m)g = 648\text{ N} - (7.2\text{ kg} + 1.8\text{ kg})(9.8\text{ m/s}^2) = 560\text{ N} = 5.6\times 10^2\text{ N}.$$

21. (a) We note that the angle between the cable and the strut is

$$\alpha = \theta - \phi = 45^\circ - 30^\circ = 15^\circ.$$

The angle between the strut and any vertical force (like the weights in the problem) is β = 90º – 45º = 45º. Denoting M = 225 kg and m = 45.0 kg, and ℓ as the length of the boom, we compute torques about the hinge and find

$$T = \frac{Mg\ell\sin\beta + mg\left(\frac{\ell}{2}\right)\sin\beta}{\ell\sin\alpha} = \frac{Mg\sin\beta + mg\sin\beta/2}{\sin\alpha}.$$

The unknown length ℓ cancels out and we obtain $T = 6.63 \times 10^3$ N.

(b) Since the cable is at 30º from horizontal, then horizontal equilibrium of forces requires that the horizontal hinge force be

$$F_x = T\cos 30° = 5.74\times 10^3\,\text{N}.$$

(c) And vertical equilibrium of forces gives the vertical hinge force component:

$$F_y = Mg + mg + T\sin 30° = 5.96\times 10^3\,\text{N}.$$

22. (a) The problem asks for the person's pull (his force exerted on the rock) but since we are examining forces and torques *on the person*, we solve for the reaction force F_{N1} (exerted leftward on the hands by the rock). At that point, there is also an upward force of static friction on his hands, f_1, which we will take to be at its maximum value $\mu_1 F_{N1}$. We note that equilibrium of horizontal forces requires $F_{N1} = F_{N2}$ (the force exerted leftward on his feet); on his feet there is also an upward static friction force of magnitude $\mu_2 F_{N2}$. Equilibrium of vertical forces gives

$$f_1 + f_2 - mg = 0 \Rightarrow F_{N1} = \frac{mg}{\mu_1 + \mu_2} = 3.4\times 10^2\,\text{N}.$$

(b) Computing torques about the point where his feet come in contact with the rock, we find

$$mg\left(d+w\right) - f_1 w - F_{N1}h = 0 \;\Rightarrow\; h = \frac{mg\left(d+w\right) - \mu_1 F_{N1} w}{F_{N1}} = 0.88\text{ m}.$$

(c) Both intuitively and mathematically (since both coefficients are in the denominator) we see from part (a) that F_{N1} would increase in such a case.

(d) As for part (b), it helps to plug part (a) into part (b) and simplify:

$$h = (d+w)\mu_2 + d\mu_1$$

from which it becomes apparent that *h* should decrease if the coefficients decrease.

23. The beam is in equilibrium: the sum of the forces and the sum of the torques acting on it each vanish. As shown in the figure, the beam makes an angle of 60º with the vertical and the wire makes an angle of 30º with the vertical.

(a) We calculate the torques around the hinge. Their sum is

$$TL\sin 30° - W(L/2)\sin 60° = 0.$$

Here W is the force of gravity acting at the center of the beam, and T is the tension force of the wire. We solve for the tension:

$$T = \frac{W\sin 60°}{2\sin 30°} = \frac{(222\,\text{N})\sin 60°}{2\sin 30°} = 192\,\text{N}.$$

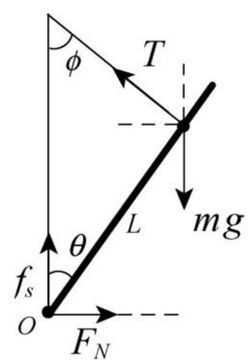
(b) Let F_h be the horizontal component of the force exerted by the hinge and take it to be positive if the force is outward from the wall. Then, the vanishing of the horizontal component of the net force on the beam yields $F_h - T\sin 30^{\circ} = 0$ or

$$F_h = T\sin 30° = (192.3\,\text{N})\sin 30° = 96.1\,\text{N}.$$

(c) Let F_v be the vertical component of the force exerted by the hinge and take it to be positive if it is upward. Then, the vanishing of the vertical component of the net force on the beam yields $F_v + T\cos 30^{\circ} - W = 0$ or

$$F_v = W - T\cos 30° = 222\,\text{N} - (192.3\,\text{N})\cos 30° = 55.5\,\text{N}.$$

24. As shown in the free-body diagram, the forces on the climber consist of $\vec{T}$ from the rope, normal force $\vec{F}_N$ on her feet, upward static frictional force $\vec{f}_s$, and downward gravitational force $m\vec{g}$.



Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have

$$0 = \sum F_{\text{net},x} = F_N - T\sin\phi$$
$$0 = \sum F_{\text{net},y} = T\cos\phi + f_s - mg\,.$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = mgL\sin\theta - TL\sin(180° - \theta - \phi)$$

From the torque equation, we obtain

$$T = mg\sin\theta / \sin(180° - \theta - \phi).$$

Substituting the expression into the force equations, and noting that $f_s = \mu_s F_N$, we find the coefficient of static friction to be

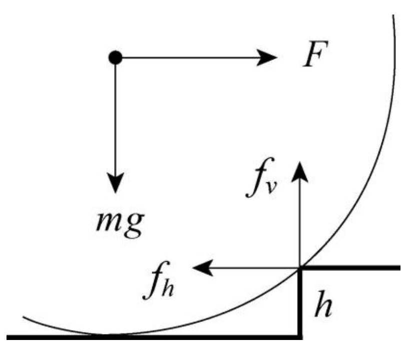
$$\mu_s = \frac{f_s}{F_N} = \frac{mg - T\cos\phi}{T\sin\phi} = \frac{mg - mg\sin\theta\cos\phi/\sin(180° - \theta - \phi)}{mg\sin\theta\sin\phi/\sin(180° - \theta - \phi)}$$
$$= \frac{1 - \sin\theta\cos\phi/\sin(180° - \theta - \phi)}{\sin\theta\sin\phi/\sin(180° - \theta - \phi)}.$$

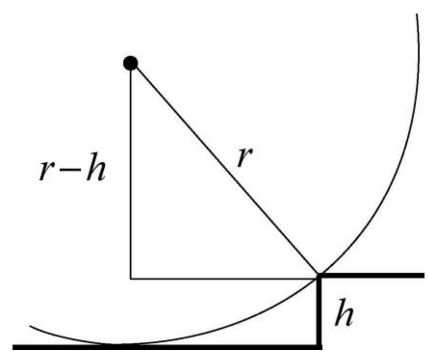
With $\theta = 40°$ and $\phi = 30°$, the result is

$$\mu_s = \frac{1 - \sin\theta\cos\phi/\sin(180° - \theta - \phi)}{\sin\theta\sin\phi/\sin(180° - \theta - \phi)} = \frac{1 - \sin 40°\cos 30°/\sin(180° - 40° - 30°)}{\sin 40°\sin 30°/\sin(180° - 40° - 30°)}$$
$$= 1.19.$$

25. **THINK** At the moment when the wheel leaves the lower floor, the floor no longer exerts a force on it.

EXPRESS As the wheel is raised over the obstacle, the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v.





If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.

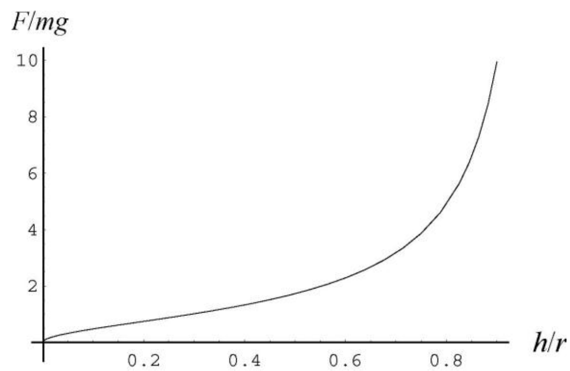
We calculate the torque around the step corner. The second diagram (above right) indicates that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step. The distance from the line of mg to the corner is $\sqrt{r^2 + (r-h)^2} = \sqrt{2rh - h^2}$. Thus,

$$F(r-h) - mg\sqrt{2rh - h^2} = 0.$$

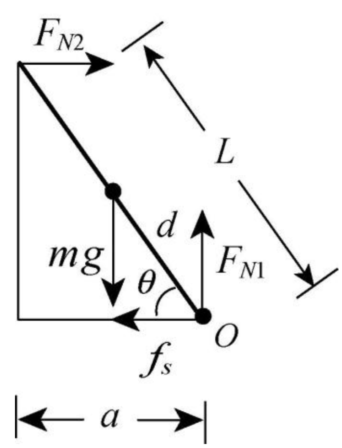
ANALYZE The solution for F is

$$F=\frac{\sqrt{2rh-h^2}}{r-h}mg=\frac{\sqrt{2(6.00\times10^{-2}\,\text{m})(3.00\times10^{-2}\,\text{m})-(3.00\times10^{-2}\,\text{m})^2}}{(6.00\times10^{-2}\,\text{m})-(3.00\times10^{-2}\,\text{m})}(0.800\text{ kg})(9.80\text{ m/s}^2)$$
$$=13.6\text{ N.}$$

LEARN The applied force here is about 1.73 times the weight of the wheel. If the height is increased, the force that must be applied also goes up. Below we plot F/mg as a function of the ratio h/r. The required force increases rapidly as $h/r\to1$.



26. As shown in the free-body diagram, the forces on the climber consist of the normal forces F_{N1} on his hands from the ground and F_{N2} on his feet from the wall, static frictional force f_s, and downward gravitational force mg. Since the climber is in static equilibrium, the net force acting on him is zero.



Applying Newton's second law to the vertical and horizontal directions, we have

$$0=\sum F_{\text{net},x}=F_{N2}-f_s$$
$$0=\sum F_{\text{net},y}=F_{N1}-mg\,.$$

In addition, the net torque about O (contact point between his feet and the wall) must also vanish:

$$0=\sum_O\tau_{\text{net}}=mgd\cos\theta-F_{N2}L\sin\theta\,.$$

The torque equation gives

$$F_{N2}=mgd\cos\theta/L\sin\theta=mgd\cot\theta/L\,.$$

On the other hand, from the force equation we have $F_{N2}=f_s$ and $F_{N1}=mg$. These expressions can be combined to yield

$$f_s=F_{N2}=F_{N1}\cot\theta\frac{d}{L}\,.$$

On the other hand, the frictional force can also be written as $f_s = \mu_s F_{N1}$, where μ_s is the coefficient of static friction between his feet and the ground. From the above equation and the values given in the problem statement, we find μ_s to be

$$\mu_s = \cot\theta \frac{d}{L} = \frac{a}{\sqrt{L^2 - a^2}} \frac{d}{L} = \frac{0.914 \text{ m}}{\sqrt{(2.10 \text{ m})^2 - (0.914 \text{ m})^2}} \frac{0.940 \text{ m}}{2.10 \text{ m}} = 0.216\,.$$

27. (a) All forces are vertical and all distances are measured along an axis inclined at θ = 30º. Thus, any trigonometric factor cancels out and the application of torques about the contact point (referred to in the problem) leads to

$$F_{\text{tricep}} = \frac{(15\,\text{kg})(9.8\,\text{m/s}^2)(35\,\text{cm}) - (2.0\,\text{kg})(9.8\,\text{m/s}^2)(15\,\text{cm})}{2.5\,\text{cm}} = 1.9\times10^3\text{ N.}$$

(b) The direction is upward since $F_{\text{tricep}} > 0$.

(c) Equilibrium of forces (with upward positive) leads to

$$F_{\text{tricep}} + F_{\text{humer}} + (15\,\text{kg})(9.8\,\text{m/s}^2) - (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 0$$

and thus to $F_{\text{humer}} = -2.1\times10^3\,\text{N}$, or $|F_{\text{humer}}| = 2.1\times10^3\,\text{N}$.

(d) The negative sign implies that F_{humer} points downward.

28. (a) Computing torques about point *A*, we find

$$T_{\max} L\sin\theta = Wx_{\max} + W_b\left(\frac{L}{2}\right).$$

We solve for the maximum distance:

$$x_{\max} = \left(\frac{T_{\max}\sin\theta - W_b/2}{W}\right)L = \left(\frac{(500\text{ N})\sin 30.0° - (200\text{ N})/2}{300\text{ N}}\right)(3.00\text{ m}) = 1.50\,\text{m.}$$

(b) Equilibrium of horizontal forces gives $F_x = T_{\max}\cos\theta = 433\,\text{N}$.

(c) And equilibrium of vertical forces gives $F_y = W + W_b - T_{\max}\sin\theta = 250\,\text{N}$.

29. The problem states that each hinge supports half the door's weight, so each vertical hinge force component is $F_y = mg/2 = 1.3 \times 10^2$ N. Computing torques about the top hinge, we find the horizontal hinge force component (at the bottom hinge) is

$$F_h = \frac{(27\,\text{kg})(9.8\,\text{m/s}^2)\left(0.91\,\text{m/2}\right)}{2.1\,\text{m} - 2(0.30\,\text{m})} = 80\,\text{N}.$$

Equilibrium of horizontal forces demands that the horizontal component of the top hinge force has the same magnitude (though opposite direction).

(a) In unit-vector notation, the force on the door at the top hinge is

$$F_{\text{top}} = (-80\text{ N})\hat{\text{i}} + (1.3\times 10^2\,\text{N})\hat{\text{j}}.$$

(b) Similarly, the force on the door at the bottom hinge is

$$F_{\text{bottom}} = (+80\text{ N})\hat{\text{i}} + (1.3\times 10^2\,\text{N})\hat{\text{j}}.$$

30. (a) The sign is attached in two places: at x_1 = 1.00 m (measured rightward from the hinge) and at x_2 = 3.00 m. We assume the downward force due to the sign's weight is equal at these two attachment points, each being *half* the sign's weight of mg. The angle where the cable comes into contact (also at x_2) is

$$\theta = \tan^{-1}(d_v/d_h) = \tan^{-1}(4.00\text{ m}/3.00\text{ m})$$

and the force exerted there is the tension T. Computing torques about the hinge, we find

$$T = \frac{\frac{1}{2}mgx_1 + \frac{1}{2}mgx_2}{x_2\sin\theta} = \frac{\frac{1}{2}\left(50.0\text{ kg}\right)\left(9.8\text{ m/s}^2\right)\left(1.00\text{ m}\right) + \frac{1}{2}(50.0\text{ kg})(9.8\text{m/s}^2)(3.00\text{ m})}{\left(3.00\text{ m}\right)\left(0.800\right)}$$
$$= 408\text{ N}.$$

(b) Equilibrium of horizontal forces requires that the horizontal hinge force be

$$F_x = T\cos\theta = 245\text{ N}.$$

(c) The direction of the horizontal force is rightward.

(d) Equilibrium of vertical forces requires that the vertical hinge force be

$$F_y = mg - T\sin\theta = 163\text{ N}.$$

(e) The direction of the vertical force is upward.

31. The bar is in equilibrium, so the forces and the torques acting on it each sum to zero. Let T_l be the tension force of the left-hand cord, T_r be the tension force of the right-hand cord, and m be the mass of the bar. The equations for equilibrium are:

$$\begin{aligned}\text{vertical force components:}\quad & T_l\cos\theta + T_r\cos\phi - mg = 0\\ \text{horizontal force components:}\quad & -T_l\sin\theta + T_r\sin\phi = 0\\ \text{torques:}\quad & mgx - T_r L\cos\phi = 0.\end{aligned}$$

The origin was chosen to be at the left end of the bar for purposes of calculating the torque. The unknown quantities are T_l, T_r, and x. We want to eliminate T_l and T_r, then solve for x. The second equation yields $T_l = T_r \sin\phi/\sin\theta$ and when this is substituted into the first and solved for T_r the result is

$$T_r = \frac{mg\sin\theta}{\sin\phi\cos\theta + \cos\phi\sin\theta}\,.$$

This expression is substituted into the third equation and the result is solved for x:

$$x = L\frac{\sin\theta\cos\phi}{\sin\phi\cos\theta + \cos\phi\sin\theta} = L\frac{\sin\theta\cos\phi}{\sin(\theta+\phi)}.$$

The last form was obtained using the trigonometric identity

$$\sin(A + B) = \sin A\cos B + \cos A\sin B.$$

For the special case of this problem $\theta + \phi = 90°$ and $\sin(\theta + \phi) = 1$. Thus,

$$x = L\sin\theta\cos\phi = (6.10\text{ m})\sin 36.9°\cos 53.1° = 2.20\text{ m}.$$

32. (a) With $F = ma = -\mu_k mg$ the magnitude of the deceleration is

$$|a| = \mu_k g = (0.40)(9.8\text{ m/s}^2) = 3.92\text{ m/s}^2.$$

(b) As hinted in the problem statement, we can use Eq. 12-9, evaluating the torques about the car's center of mass, and bearing in mind that the friction forces are acting horizontally at the bottom of the wheels; the total friction force there is $f_k = \mu_k gm = 3.92m$ (with SI units understood, and m is the car's mass), a vertical distance of 0.75 meter below the center of mass. Thus, torque equilibrium leads to

$$(3.92m)(0.75) + F_{Nr}(2.4) - F_{Nf}(1.8) = 0\,.$$

Equation 12-8 also holds (the acceleration is horizontal, not vertical), so we have $F_{Nr} + F_{Nf} = mg$, which we can solve simultaneously with the above torque equation. The mass is obtained from the car's weight: $m = 11000/9.8$, and we obtain $F_{Nr} = 3929 \approx 4000$ N. Since each involves <u>two</u> wheels then we have (roughly) 2.0×10^3 N on each rear wheel.

(c) From the above equation, we also have F_{Nf} = 7071 ≈ 7000 N, or 3.5×10^3 N on each front wheel, as the values of the individual normal forces.

(d) For friction on each rear wheel, Eq. 6-2 directly yields

$$f_{r1} = \mu_k (F_{Nr}/2) = (0.40)(3929\text{ N}/2) = 7.9\times10^2\text{ N} .$$

(e) Similarly, for friction on the front rear wheel, Eq. 6-2 gives

$$f_{f1} = \mu_k (F_{Nf}/2) = (0.40)(7071\text{ N}/2) = 1.4\times10^3\text{ N} .$$

33. (a) With the pivot at the hinge, Eq. 12-9 yields

$$TL\cos\theta - F_a y = 0 .$$

This leads to $T = (F_a/\cos\theta)(y/L)$ so that we can interpret $F_a/\cos\theta$ as the slope on the tension graph (which we estimate to be 600 in SI units). Regarding the F_h graph, we use Eq. 12-7 to get

$$F_h = T\cos\theta - F_a = (-F_a)(y/L) - F_a$$

after substituting our previous expression. The result implies that the slope on the F_h graph (which we estimate to be –300) is equal to $-F_a$, or F_a = 300 N and (plugging back in) θ = 60.0°.

(b) As mentioned in the previous part, F_a = 300 N.

34. (a) Computing torques about the hinge, we find the tension in the wire:

$$TL\sin\theta - Wx = 0 \Rightarrow T = \frac{Wx}{L\sin\theta}.$$

(b) The horizontal component of the tension is $T\cos\theta$, so equilibrium of horizontal forces requires that the horizontal component of the hinge force is

$$F_x = \left(\frac{Wx}{L\sin\theta}\right)\cos\theta = \frac{Wx}{L\tan\theta}.$$

(c) The vertical component of the tension is $T\sin\theta$, so equilibrium of vertical forces requires that the vertical component of the hinge force is

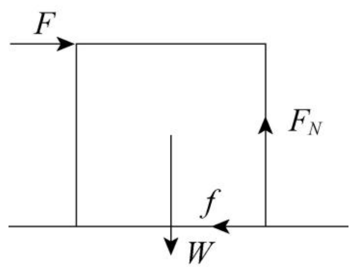
$$F_y = W - \left(\frac{Wx}{L\sin\theta}\right)\sin\theta = W\left(1 - \frac{x}{L}\right).$$

35. **THINK** We examine the box when it is about to tip. Since it will rotate about the lower right edge, this is where the normal force of the floor is exerted.

EXPRESS The free-body diagram is shown below. The normal force is labeled F_N, the force of friction is denoted by *f*, the applied force by *F*, and the force of gravity by *W*. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes:

$$FL - WL/2 = 0.$$

Here *L* is the length of a side of the box and the origin was chosen to be at the lower right edge.



ANALYZE (a) From the torque equation, we find $F = \frac{W}{2} = \frac{890\,\text{N}}{2} = 445\,\text{N}.$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the equations of equilibrium

$$F_N = W = 890\text{ N}$$
$$f = F = 445\text{ N},$$

so

$$\mu_s = \frac{f}{F_N} = \frac{445\,\text{N}}{890\,\text{N}} = 0.50.$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes

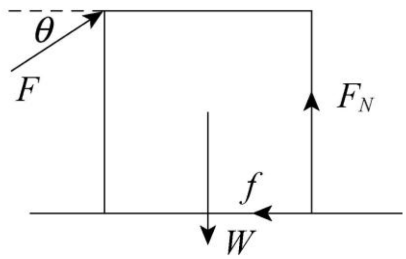
$$FL\cos\theta + FL\sin\theta - WL/2 = 0,$$

with the solution

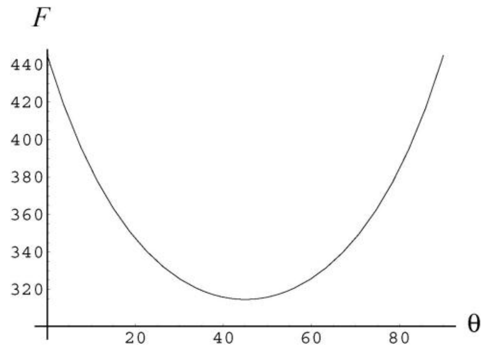
$$F = \frac{W}{2(\cos\theta + \sin\theta)}.$$

We want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45^\circ$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

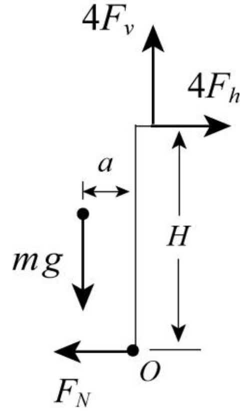
$$F = \frac{W}{2(\cos 45° + \sin 45°)} = \frac{890\,\text{N}}{2(\cos 45° + \sin 45°)} = 315\,\text{N}.$$



LEARN The applied force as a function of θ is plotted below. From the figure, we readily see that $\theta = 0°$ corresponds to a maximum and $\theta = 45°$ a minimum.



36. As shown in the free-body diagram, the forces on the climber consist of the normal force from the wall, the vertical component F_v and the horizontal component F_h of the force acting on her four fingertips, and the downward gravitational force mg.



Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have

$$0 = \sum F_{\text{net},x} = 4F_h - F_N$$
$$0 = \sum F_{\text{net},y} = 4F_v - mg\ .$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = (mg)a - (4F_h)H .$$

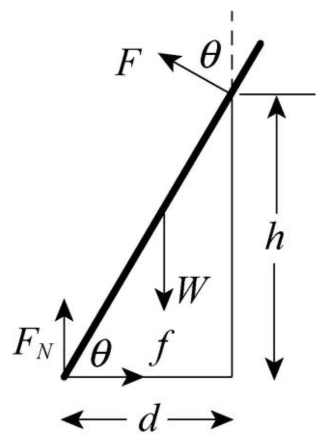
(a) From the torque equation, we find the horizontal component of the force on her fingertip to be

$$F_h = \frac{mga}{4H} = \frac{(70\text{ kg})(9.8\text{ m/s}^2)(0.20\text{ m})}{4(2.0\text{ m})} \approx 17\text{ N}.$$

(b) From the y-component of the force equation, we obtain

$$F_v = \frac{mg}{4} = \frac{(70\text{ kg})(9.8\text{ m/s}^2)}{4} \approx 1.7\times10^2\text{ N}.$$

37. The free-body diagram below shows the forces acting on the plank. Since the roller is frictionless, the force it exerts is normal to the plank and makes the angle θ with the vertical.



Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. F_N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$.

The equations of equilibrium are:

horizontal force components:	$F\sin\theta - f = 0$
vertical force components:	$F\cos\theta - W + F_N = 0$
torques:	$F_N d - fh - W\left(d - \frac{L}{2}\cos\theta\right) = 0.$

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When θ = 70º the plank just begins to slip and $f = \mu_s F_N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute F_N and f for θ = 70º, then use $\mu_s = f/F_N$ to compute the coefficient of friction.

The second equation gives $F = (W - F_N)/\cos\theta$ and this is substituted into the first to obtain

$$f = (W - F_N)\sin\theta/\cos\theta = (W - F_N)\tan\theta.$$

This is substituted into the third equation and the result is solved for F_N:

$$F_N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta}W = \frac{h(1+\tan^2\theta) - (L/2)\sin\theta}{h(1+\tan^2\theta)}W,$$

where we have used $d = h/\tan\theta$ and multiplied both numerator and denominator by $\tan\theta$. We use the trigonometric identity $1+\tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$F_N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right).$$

Now we use this expression for F_N in $f = (W - F_N)\tan\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta.$$

Substituting these expressions for f and F_N into $\mu_s = f/F_N$ leads to

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}.$$

Evaluating this expression for θ = 70º, L = 6.10 m and h = 3.05 m gives

$$\mu_s = \frac{(6.1\,\text{m})\sin^2 70°\cos 70°}{2(3.05\,\text{m}) - (6.1\,\text{m})\sin 70°\cos^2 70°} = 0.34.$$

38. The phrase "loosely bolted" means that there is no torque exerted by the bolt at that point (where A connects with B). The force exerted on A at the hinge has x and y components F_x and F_y. The force exerted on A at the bolt has components G_x and G_y, and those exerted on B are simply $-G_x$ and $-G_y$ by Newton's third law. The force exerted on B at its hinge has components H_x and H_y. If a horizontal force is positive, it points rightward, and if a vertical force is positive it points upward.

(a) We consider the combined A∪B system, which has a total weight of Mg where M = 122 kg and the line of action of that downward force of gravity is x = 1.20 m from the wall. The vertical distance between the hinges is y = 1.80 m. We compute torques about the bottom hinge and find

$$F_x = -\frac{Mgx}{y} = -797\,\text{N}.$$

If we examine the forces on A alone and compute torques about the bolt, we instead find

$$F_y = \frac{m_A gx}{\ell} = 265\,\text{N}$$

where m_A = 54.0 kg and ℓ = 2.40 m (the length of beam A). Thus, in unit-vector notation, we have

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}.$$

(b) Equilibrium of horizontal and vertical forces on beam A readily yields

$$G_x = -F_x = 797\text{ N}, \quad G_y = m_A g - F_y = 265\text{ N}.$$

In unit-vector notation, we have

$$\vec{G} = G_x\hat{\text{i}} + G_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}.$$

(c) Considering again the combined A∪B system, equilibrium of horizontal and vertical forces readily yields $H_x = -F_x$ = 797 N and $H_y = Mg - F_y$ = 931 N. In unit-vector notation, we have

$$\vec{H} = H_x\hat{\text{i}} + H_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (931\text{ N})\hat{\text{j}}.$$

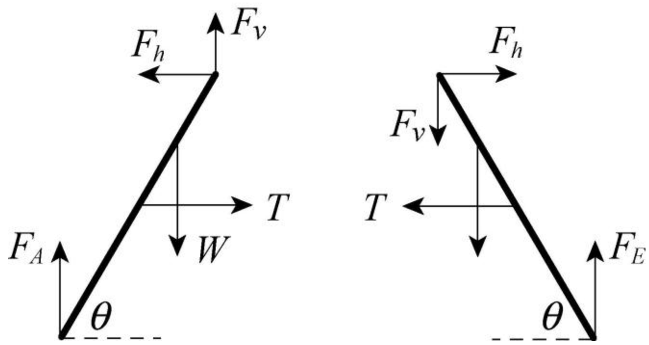
(d) As mentioned above, Newton's third law (and the results from part (b)) immediately provide $-G_x = -797$ N and $-G_y = -265$ N for the force components acting on B at the bolt. In unit-vector notation, we have

$$-\vec{G} = -G_x\hat{\text{i}} - G_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} - (265\text{ N})\hat{\text{j}}.$$

39. The diagrams show the forces on the two sides of the ladder, separated. F_A and F_E are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), F_h is the horizontal component of the force exerted by one side of the ladder on the other, and F_v is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction. Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero:

$$F_v + F_A - W = 0.$$

The horizontal components must sum to zero: $T - F_h = 0$.



The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then

$$F_A L \cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta = 0.$$

Here we recognize that the man is a distance d from the bottom of the ladder (or $L - d$ from the top), and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $F_E - F_v = 0$, $F_h - T = 0$, and $F_E L\cos\theta - T(L/2)\sin\theta = 0$. There are 5 different equations:

$$\begin{aligned} F_v + F_A - W &= 0, \\ T - F_h &= 0 \\ F_A L\cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta &= 0 \\ F_E - F_v &= 0 \\ F_E L\cos\theta - T(L/2)\sin\theta &= 0. \end{aligned}$$

The unknown quantities are F_A, F_E, F_v, F_h, and T.

(a) First we solve for T by systematically eliminating the other unknowns. The first equation gives $F_A = W - F_v$ and the fourth gives $F_v = F_E$. We use these to substitute into the remaining three equations to obtain

$$\begin{aligned} T - F_h &= 0 \\ WL\cos\theta - F_E L\cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta &= 0 \\ F_E L\cos\theta - T(L/2)\sin\theta &= 0. \end{aligned}$$

The last of these gives $F_E = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is

$$T = \frac{Wd}{L\tan\theta}.$$

To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half the tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of $\sqrt{(1.22\,\text{m})^2-(0.381\,\text{m})^2}=1.16\,\text{m}$. This means

$$\tan\theta = (1.16\text{m})/(0.381\text{m}) = 3.04.$$

Thus,

$$T=\frac{(854\,\text{N})(1.80\,\text{m})}{(2.44\,\text{m})(3.04)}=207\,\text{N}.$$

(b) We now solve for F_A. We substitute $F_v=F_E=(T/2)\tan\theta=Wd/2L$ into the equation $F_v + F_A - W = 0$ and solve for F_A. The solution is

$$F_A=W-F_v=W\left(1-\frac{d}{2L}\right)=(854\text{ N})\left(1-\frac{1.80\text{ m}}{2(2.44\text{ m})}\right)=539\text{ N}.$$

(c) Similarly, $F_E=W\dfrac{d}{2L}=(854\text{ N})\dfrac{1.80\text{ m}}{2(2.44\text{ m})}=315\text{ N}$.

40. (a) Equation 12-9 leads to

$$TL\sin\theta - m_p g x - m_b g\left(\tfrac{L}{2}\right) = 0\,.$$

This can be written in the form of a straight line (in the graph) with

$$T = (\text{“slope”})\,\tfrac{x}{L}\;+\;\text{“}y\text{-intercept”}$$

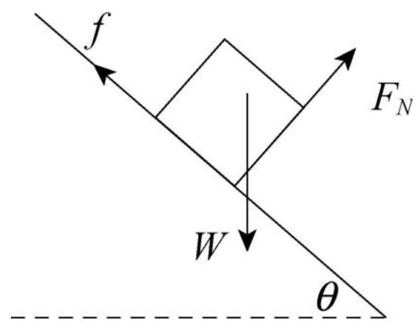
where “slope” = $m_p g/\sin\theta$ and “y-intercept” = $m_b g/2\sin\theta$. The graph suggests that the slope (in SI units) is 200 and the y-intercept is 500. These facts, combined with the given $m_p + m_b = 61.2$ kg datum, lead to the conclusion:

$$\sin\theta = 61.22g/1200 \Rightarrow \theta = 30.0^\circ.$$

(b) It also follows that $m_p = 51.0$ kg.

(c) Similarly, $m_b = 10.2$ kg.

41. The force diagram shown depicts the situation just before the crate tips, when the normal force acts at the front edge. However, it may also be used to calculate the angle for which the crate begins to slide. W is the force of gravity on the crate, F_N is the normal force of the plane on the crate, and f is the force of friction. We take the x-axis to be down the plane and the y-axis to be in the direction of the normal force. We assume the acceleration is zero but the crate is on the verge of sliding.



(a) The x and y components of Newton's second law are

$$W\sin\theta - f = 0 \text{ and } F_N - W\cos\theta = 0$$

respectively. The y equation gives $F_N = W\cos\theta$. Since the crate is about to slide

$$f = \mu_s F_N = \mu_s W\cos\theta,$$

where μ_s is the coefficient of static friction. We substitute into the x equation and find

$$W\sin\theta - \mu_s W\cos\theta = 0 \quad \Rightarrow \quad \tan\theta = \mu_s.$$

This leads to $\theta = \tan^{-1}\mu_s = \tan^{-1}(0.60) = 31.0°$.

In developing an expression for the total torque about the center of mass when the crate is about to tip, we find that the normal force and the force of friction act at the front edge. The torque associated with the force of friction tends to turn the crate clockwise and has magnitude fh, where h is the perpendicular distance from the bottom of the crate to the center of gravity. The torque associated with the normal force tends to turn the crate counterclockwise and has magnitude $F_N\ell/2$, where ℓ is the length of an edge. Since the total torque vanishes, $fh = F_N\ell/2$. When the crate is about to tip, the acceleration of the center of gravity vanishes, so $f = W\sin\theta$ and $F_N = W\cos\theta$. Substituting these expressions into the torque equation, we obtain

$$\theta = \tan^{-1}\frac{\ell}{2h} = \tan^{-1}\frac{1.2\,\text{m}}{2(0.90\,\text{m})} = 33.7°.$$

As θ is increased from zero the crate slides before it tips.

(b) It starts to slide when $\theta = 31°$.

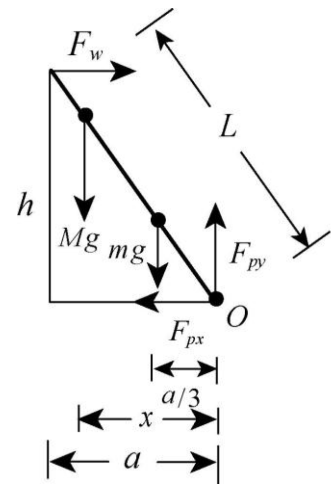
(c) The crate begins to slide when

$$\theta = \tan^{-1}\mu_s = \tan^{-1}(0.70) = 35.0°$$

and begins to tip when $\theta = 33.7°$. Thus, it tips first as the angle is increased.

(d) Tipping begins at $\theta = 33.7° \approx 34°$.

42. Let x be the horizontal distance between the firefighter and the origin O (see the figure) that makes the ladder on the verge of sliding. The forces on the firefighter + ladder system consist of the horizontal force F_w from the wall, the vertical component F_{py} and the horizontal component F_{px} of the force $\vec{F}_p$ on the ladder from the pavement, and the downward gravitational forces Mg and mg, where M and m are the masses of the firefighter and the ladder, respectively.



Since the system is in static equilibrium, the net force acting on the system is zero. Applying Newton's second law to the vertical and horizontal directions, we have

$$0 = \sum F_{\text{net},x} = F_w - F_{px}$$
$$0 = \sum F_{\text{net},y} = F_{py} - (M+m)g\ .$$

Since the ladder is on the verge of sliding, $F_{px} = \mu_s F_{py}$. Therefore, we have

$$F_w = F_{px} = \mu_s F_{py} = \mu_s (M+m)g\ .$$

In addition, the net torque about O (contact point between the ladder and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = -h(F_w) + x(Mg) + \frac{a}{3}(mg) = 0\,.$$

Solving for x, we obtain

$$x = \frac{hF_w - (a/3)mg}{Mg} = \frac{h\mu_s(M+m)g - (a/3)mg}{Mg} = \frac{h\mu_s(M+m) - (a/3)m}{M}$$

Substituting the values given in the problem statement (with $a = \sqrt{L^2 - h^2} = 7.58\text{ m}$), the fraction of ladder climbed is

$$\frac{x}{a} = \frac{h\mu_s(M+m) - (a/3)m}{Ma} = \frac{(9.3\text{ m})(0.53)(72\text{ kg} + 45\text{ kg}) - (7.58\text{ m}/3)(45\text{ kg})}{(72\text{ kg})(7.58\text{ m})}$$
$$= 0.848 \approx 85\%.$$

43. **THINK** The weight of the object hung on the end provides the source of shear stress.

EXPRESS The shear stress is given by F/A, where F is the magnitude of the force applied parallel to one face of the aluminum rod and A is the cross–sectional area of the rod. In this case $F = mg$, where m is the mass of the object. The cross-sectional area is $A = \pi r^2$ where r is the radius of the rod.

ANALYZE (a) Substituting the values given, we find the shear stress to be

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1200\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(0.024\,\text{m})^2} = 6.5\times10^6\,\text{N/m}^2.$$

(b) The shear modulus G is given by

$$G = \frac{F/A}{\Delta x/L},$$

where L is the protrusion of the rod and Δx is its vertical deflection at its end. Thus,

$$\Delta x = \frac{(F/A)L}{G} = \frac{(6.5\times10^6\ \text{N/m}^2)(0.053\,\text{m})}{3.0\times10^{10}\ \text{N/m}^2} = 1.1\times10^{-5}\ \text{m}.$$

LEARN As expected, the extent of vertical deflection Δx is proportional to F, the weight of the object hung from the end. On the other hand, it is inversely proportional to the shear modulus G.

44. (a) The Young's modulus is given by

$$E = \frac{\text{stress}}{\text{strain}} = \text{slope of the stress-strain curve} = \frac{150\times10^6\,\text{N/m}^2}{0.002} = 7.5\times10^{10}\,\text{N/m}^2.$$

(b) Since the linear range of the curve extends to about 2.9×10^8 N/m^2, this is approximately the yield strength for the material.

45. (a) Since the brick is now horizontal and the cylinders were initially the same length ℓ, then both have been compressed an equal amount $\Delta\ell$. Thus,

$$\frac{\Delta\ell}{\ell} = \frac{FA}{A_A E_A} \quad \text{and} \quad \frac{\Delta\ell}{\ell} = \frac{F_B}{A_B E_B}$$

which leads to

$$\frac{F_A}{F_B} = \frac{A_A E_A}{A_B E_B} = \frac{(2A_B)(2E_B)}{A_B E_B} = 4.$$

When we combine this ratio with the equation $F_A + F_B = W$, we find $F_A/W = 4/5 = 0.80$.

(b) This also leads to the result $F_B/W = 1/5 = 0.20$.

(c) Computing torques about the center of mass, we find $F_A d_A = F_B d_B$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{1}{4} = 0.25.$$

46. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (eq. 7-32) as

$$W = \int F dx \;=\; \int (\text{stress})\, A\, (\text{differential strain}) L \;= AL \int (\text{stress})\, (\text{differential strain})$$

which means the work is (thread cross-sectional area) × (thread length) × (graph area under curve). The area under the curve is

$$\begin{aligned}\text{graph area} &= \frac{1}{2} a s_1 + \frac{1}{2}(a+b)(s_2 - s_1) + \frac{1}{2}(b+c)(s_3 - s_2) = \frac{1}{2}\left[a s_2 + b(s_3 - s_1) + c(s_3 - s_2)\right] \\ &= \frac{1}{2}\left[(0.12\times10^9 \text{ N/m}^2)(1.4) + (0.30\times10^9 \text{ N/m}^2)(1.0) + (0.80\times10^9 \text{ N/m}^2)(0.60)\right] \\ &= 4.74\times10^8 \text{ N/m}^2.\end{aligned}$$

(a) The kinetic energy that would put the thread on the verge of breaking is simply equal to W:

$$\begin{aligned}K &= W = AL(\text{graph area}) = (8.0\times10^{-12} \text{ m}^2)(8.0\times10^{-3} \text{ m})(4.74\times10^8 \text{ N/m}^2) \\ &= 3.03\times10^{-5} \text{ J}.\end{aligned}$$

(b) The kinetic energy of the fruit fly of mass 6.00 mg and speed 1.70 m/s is

$$K_f = \frac{1}{2} m_f v_f^2 = \frac{1}{2}(6.00\times10^{-6} \text{ kg})(1.70 \text{ m/s})^2 = 8.67\times10^{-6} \text{ J}.$$

(c) Since $K_f < W$, the fruit fly will not be able to break the thread.

(d) The kinetic energy of a bumble bee of mass 0.388 g and speed 0.420 m/s is

$$K_b = \frac{1}{2} m_b v_b^2 = \frac{1}{2}(3.99\times10^{-4} \text{ kg})0.420 \text{ m/s})^2 = 3.42\times10^{-5} \text{ J}.$$

(e) On the other hand, since $K_b > W$, the bumble bee will be able to break the thread.

47. The flat roof (as seen from the air) has area A = 150 m × 5.8 m = 870 m^2. The volume of material directly above the tunnel (which is at depth d = 60 m) is therefore

$$V = A \times d = (870 \text{ m}^2) \times (60 \text{ m}) = 52200 \text{ m}^3.$$

Since the density is $\rho = 2.8$ g/cm^3 = 2800 kg/m^3, we find the mass of material supported by the steel columns to be $m = \rho V = 1.46 \times 10^8$ kg.

(a) The weight of the material supported by the columns is $mg = 1.4 \times 10^9$ N.

(b) The number of columns needed is

$$n = \frac{1.43\times10^9\text{ N}}{\frac{1}{2}(400\times10^6\text{ N/m}^2)(960\times10^{-4}\text{m}^2)} = 75.$$

48. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (Eq. 7-32) as

$$W = \int F dx \;=\; \int (\text{stress})\, A\, (\text{differential strain}) L \;= AL \int (\text{stress})\, (\text{differential strain})$$

which means the work is (wire area) × (wire length) × (graph area under curve). Since the area of a triangle (see the graph in the problem statement) is $\frac{1}{2}$(base)(height) then we determine the work done to be

$$W = (2.00 \times 10^{-6} \text{ m}^2)(0.800 \text{ m})\left(\tfrac{1}{2}\right)(1.0 \times 10^{-3})(7.0 \times 10^7 \text{ N/m}^2) = 0.0560 \text{ J}.$$

49. (a) Let F_A and F_B be the forces exerted by the wires on the log and let m be the mass of the log. Since the log is in equilibrium, $F_A + F_B - mg = 0$. Information given about the stretching of the wires allows us to find a relationship between F_A and F_B. If wire A originally had a length L_A and stretches by ΔL_A, then $\Delta L_A = F_A L_A / AE$, where A is the cross-sectional area of the wire and E is Young's modulus for steel (200 × 10^9 N/m^2). Similarly, $\Delta L_B = F_B L_B / AE$. If ℓ is the amount by which B was originally longer than A then, since they have the same length after the log is attached, $\Delta L_A = \Delta L_B + \ell$. This means

$$\frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + \ell.$$

We solve for F_B:

$$F_B = \frac{F_A L_A}{L_B} - \frac{AE\ell}{L_B}.$$

We substitute into $F_A + F_B - mg = 0$ and obtain

$$F_A = \frac{mgL_B + AE\ell}{L_A + L_B}.$$

The cross-sectional area of a wire is

$$A = \pi r^2 = \pi\left(1.20\times10^{-3}\,\text{m}\right)^2 = 4.52\times10^{-6}\,\text{m}^2 .$$

Both L_A and L_B may be taken to be 2.50 m without loss of significance. Thus

$$F_A = \frac{(103\,\text{kg})(9.8\,\text{m/s}^2)(2.50\,\text{m}) + (4.52\times10^{-6}\,\text{m}^2)(200\times10^9\,\text{N/m}^2)(2.0\times10^{-3}\,\text{m})}{2.50\,\text{m}+2.50\,\text{m}}$$
$$= 866\,\text{N}.$$

(b) From the condition $F_A + F_B - mg = 0$, we obtain

$$F_B = mg - F_A = (103\,\text{kg})(9.8\,\text{m/s}^2) - 866\,\text{N} = 143\,\text{N}.$$

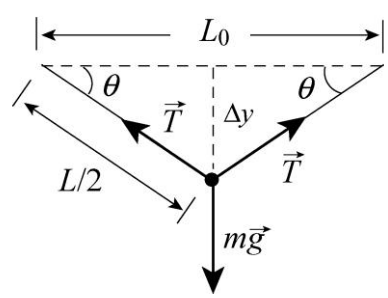
(c) The net torque must also vanish. We place the origin on the surface of the log at a point directly above the center of mass. The force of gravity does not exert a torque about this point. Then, the torque equation becomes $F_A d_A - F_B d_B = 0$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143\,\text{N}}{866\,\text{N}} = 0.165.$$

50. On the verge of breaking, the length of the thread is

$$L = L_0 + \Delta L = L_0(1+\Delta L/L_0) = L_0(1+2) = 3L_0 ,$$

where $L_0 = 0.020$ m is the original length, and $\text{strain} = \Delta L / L_0 = 2$, as given in the problem. The free-body diagram of the system is shown below.



The condition for equilibrium is $mg = 2T\sin\theta$, where m is the mass of the insect and $T = A(\text{stress})$. Since the volume of the thread remains constant as it is being stretched, we have $V = A_0 L_0 = AL$, or $A = A_0(L_0/L) = A_0/3$. The vertical distance Δy is

$$\Delta y = \sqrt{(L/2)^2 - (L_0/2)^2} = \sqrt{\frac{9L_0^2}{4} - \frac{L_0^2}{4}} = \sqrt{2}L_0 .$$

Thus, the mass of the insect is

$$m=\frac{2T\sin\theta}{g}=\frac{2(A_0/3)(\text{stress})\sin\theta}{g}=\frac{2A_0(\text{stress})}{3g}\frac{\Delta y}{3L_0/2}=\frac{4\sqrt{2}A_0(\text{stress})}{9g}$$
$$=\frac{4\sqrt{2}(8.00\times10^{-12}\ \text{m}^2)(8.20\times10^{8}\ \text{N/m}^2)}{9(9.8\ \text{m/s}^2)}=4.21\times10^{-4}\ \text{kg}$$

or 0.421 g.

51. Let the forces that compress stoppers A and B be F_A and F_B, respectively. Then equilibrium of torques about the axle requires

$$FR = r_AF_A + r_BF_B.$$

If the stoppers are compressed by amounts $|\Delta y_A|$ and $|\Delta y_B|$, respectively, when the rod rotates a (presumably small) angle θ (in radians), then $|\Delta y_A| = r_A\theta$ and $|\Delta y_B| = r_B\theta$.

Furthermore, if their "spring constants" k are identical, then $k = |F/\Delta y|$ leads to the condition $F_A/r_A = F_B/r_B$, which provides us with enough information to solve.

(a) Simultaneous solution of the two conditions leads to

$$F_A=\frac{Rr_A}{r_A^2+r_B^2}F=\frac{(5.0\ \text{cm})(7.0\ \text{cm})}{(7.0\ \text{cm})^2+(4.0\ \text{cm})^2}(220\ \text{N})=118\ \text{N}\approx1.2\times10^2\ \text{N}.$$

(b) It also yields

$$F_B=\frac{Rr_B}{r_A^2+r_B^2}F=\frac{(5.0\ \text{cm})(4.0\ \text{cm})}{(7.0\ \text{cm})^2+(4.0\ \text{cm})^2}(220\ \text{N})=68\ \text{N}.$$

52. (a) If L (= 1500 cm) is the unstretched length of the rope and $\Delta L = 2.8\,\text{cm}$ is the amount it stretches, then the strain is

$$\Delta L/L=(2.8\,\text{cm})/(1500\,\text{cm})=1.9\times10^{-3}.$$

(b) The stress is given by F/A where F is the stretching force applied to one end of the rope and A is the cross-sectional area of the rope. Here F is the force of gravity on the rock climber. If m is the mass of the rock climber then $F = mg$. If r is the radius of the rope then $A=\pi r^2$. Thus the stress is

$$\frac{F}{A}=\frac{mg}{\pi r^2}=\frac{(95\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(4.8\times10^{-3}\ \text{m})^2}=1.3\times10^7\ \text{N/m}^2.$$

(c) Young's modulus is the stress divided by the strain:

$$E = (1.3 \times 10^7\ \text{N/m}^2) / (1.9 \times 10^{-3}) = 6.9 \times 10^9\ \text{N/m}^2.$$

53. **THINK** The slab can remain in static equilibrium if the combined force of the friction and the bolts is greater than the component of the weight of the slab along the incline.

EXPRESS We denote the mass of the slab as *m*, its density as ρ, and volume as $V = LTW$. The angle of inclination is $\theta = 26°$. The component of the weight of the slab along the incline is $F_1 = mg\sin\theta = \rho Vg\sin\theta$, and the static force of friction is

$$f_s = \mu_s F_N = \mu_s mg\cos\theta = \mu_s \rho Vg\cos\theta.$$

ANALYZE (a) Substituting the values given, we find F_1 to be

$$F_1 = \rho Vg\sin\theta = (3.2\times10^3\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\sin 26° \approx 1.8\times10^7\ \text{N}.$$

(b) Similarly, the static force of friction is

$$f_s = \mu_s \rho Vg\cos\theta = (0.39)(3.2\times10^3\ \text{kg/m}^3)(43\,\text{m})(2.5\,\text{m})(12\,\text{m})(9.8\,\text{m/s}^2)\cos 26° \approx 1.4\times10^7\ \text{N}.$$

(c) The minimum force needed from the bolts to stabilize the slab is

$$F_2 = F_1 - f_s = 1.77\times10^7\ \text{N} - 1.42\times10^7\ \text{N} = 3.5\times10^6\ \text{N}.$$

If the minimum number of bolts needed is *n*, then $F_2 / nA \le S_G$, where $S_G = 3.6\times10^8\ \text{N/m}^2$ is the shear stress. Solving for *n*, we find

$$n \ge \frac{3.5\times10^6\ \text{N}}{(3.6\times10^8\ \text{N/m}^2)(6.4\times10^{-4}\ \text{m}^2)} = 15.2$$

Therefore, 16 bolts are needed.

LEARN In general, the number of bolts needed to maintain static equilibrium of the slab is

$$n = \frac{F_1 - f_s}{S_G A}.$$

Thus, no bolt would be necessary if $f_s > F_1$.

54. The notation and coordinates are as shown in Fig. 12-7 in the textbook. Here, the ladder's center of mass is halfway up the ladder (unlike in the textbook figure). Also, we label the *x* and *y* forces at the ground f_s and F_N, respectively. Now, balancing forces, we have

$$\Sigma F_x = 0 \quad \Rightarrow \quad f_s = F_w$$
$$\Sigma F_y = 0 \quad \Rightarrow \quad F_N = mg\,.$$

Since $f_s = f_{s,\max}$, we divide the equations to obtain

$$\frac{f_{s,\max}}{F_N} = \mu_s = \frac{F_w}{mg}\,.$$

Now, from $\Sigma\, \tau_z = 0$ (with axis at the ground) we have $mg(a/2) - F_w h = 0$. But from the Pythagorean theorem, $h = \sqrt{L^2 - a^2}$, where L is the length of the ladder. Therefore,

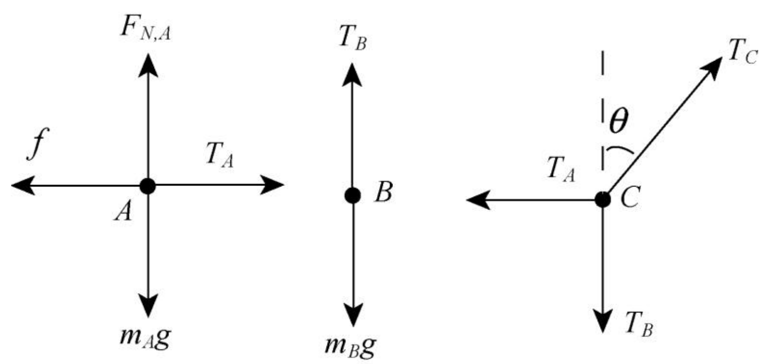
$$\frac{F_w}{mg} = \frac{a/2}{h} = \frac{a}{2\sqrt{L^2 - a^2}}\,.$$

In this way, we find

$$\mu_s = \frac{a}{2\sqrt{L^2 - a^2}} \quad \Rightarrow \quad a = \frac{2\mu_s L}{\sqrt{1 + 4\mu_s^2}} = 3.4 \text{ m}.$$

55. **THINK** Block A can be in equilibrium if friction is present between the block and the surface in contact.

EXPRESS The free-body diagrams for blocks A, B and the knot (denoted as C) are shown below.



The tensions in the three strings are denoted as T_A, T_B and T_C Analyzing forces at C, the conditions for static equilibrium are

$$T_C \cos\theta = T_B, \qquad T_C \sin\theta = T_A$$

which can be combined to give $\tan\theta = T_A / T_B$. On the other hand, equilibrium condition for block B implies $T_B = m_B g$. Similarly, for block A, the conditions are

$$F_{N,A} = m_A g, \quad f = T_A$$

For the static force to be at its maximum value, we have $f = \mu_s F_{N,A} = \mu_s m_A g$. Combining all the equations leads to

$$\tan\theta = \frac{T_A}{T_B} = \frac{\mu_s m_A g}{m_B g} = \frac{\mu_s m_A}{m_B}.$$

ANALYZE Solving for μ_s, we get

$$\mu_s = \left(\frac{m_B}{m_A}\right)\tan\theta = \left(\frac{5.0\text{ kg}}{10\text{ kg}}\right)\tan 30° = 0.29$$

LEARN The greater the mass of block *B*, the greater the static coefficient μ_s would be required for block *A* to be in equilibrium.

56. (a) With pivot at the hinge (at the left end), Eq. 12-9 gives

$$-mgx - Mg\tfrac{L}{2} + F_h h = 0$$

where *m* is the man's mass and *M* is that of the ramp; F_h is the leftward push of the right wall onto the right edge of the ramp. This equation can be written in the form (for a straight line in a graph)

$$F_h = (\text{"slope"})x + (\text{"y-intercept"}),$$

where the "slope" is *mg/h* and the "y-intercept" is $MgD/2h$. Since h = 0.480 m and $D = 4.00$ m, and the graph seems to intercept the vertical axis at 20 kN, then we find M = 500 kg.

(b) Since the "slope" (estimated from the graph) is (5000 N)/(4 m), then the man's mass must be m = 62.5 kg.

57. With the *x* axis parallel to the incline (positive uphill), then

$$\Sigma F_x = 0 \quad \Rightarrow \quad T\cos 25° - mg\sin 45° = 0.$$

Therefore,

$$T = mg\frac{\sin 45°}{\cos 25°} = (10\text{ kg})(9.8\text{ m/s}^2)\frac{\sin 45°}{\cos 25°} \approx 76\text{ N}.$$

58. The beam has a mass M = 40.0 kg and a length L = 0.800 m. The mass of the package of tamale is m = 10.0 kg.

(a) Since the system is in static equilibrium, the normal force on the beam from roller *A* is equal to half of the weight of the beam:

$$F_A = Mg/2 = (40.0\text{ kg})(9.80\text{ m/s}^2)/2 = 196\text{ N}.$$

(b) The normal force on the beam from roller *B* is equal to half of the weight of the beam plus the weight of the tamale:

$$F_B = Mg/2 + mg = (40.0\text{ kg})(9.80\text{ m/s}^2)/2 + (10.0\text{ kg})(9.80\text{ m/s}^2) = 294\text{ N}.$$

(c) When the right-hand end of the beam is centered over roller *B*, the normal force on the beam from roller *A* is equal to the weight of the beam plus half of the weight of the tamale:

$$F_A = Mg + mg/2 = (40.0\text{ kg})(9.8\text{ m/s}^2) + (10.0\text{ kg})(9.80\text{ m/s}^2)/2 = 441\text{ N}.$$

(d) Similarly, the normal force on the beam from roller *B* is equal to half of the weight of the tamale:

$$F_B = mg/2 = (10.0\text{ kg})(9.80\text{ m/s}^2)/2 = 49.0\text{ N}.$$

(e) We choose the rotational axis to pass through roller *B*. When the beam is on the verge of losing contact with roller *A*, the net torque is zero. The balancing equation may be written as

$$mgx = Mg(L/4 - x) \quad \Rightarrow \quad x = \frac{L}{4}\frac{M}{M+m}.$$

Substituting the values given, we obtain $x = 0.160$ m.

59. **THINK** The bucket is in static equilibrium. The forces acting on it are the downward force of gravity and the upward tension force of cable A.

EXPRES Since the bucket is in equilibrium, the tension force of cable A is equal to the weight of the bucket: $T_A = W = mg$. To solve for T_B and T_C, we use the coordinates axes defined in the diagram. Cable A makes an angle of θ_2 = 66.0º with the negative *y* axis, cable B makes an angle of 27.0º with the positive *y* axis, and cable C is along the *x* axis. The *y* components of the forces must sum to zero since the knot is in equilibrium. This means

$$T_B \cos 27.0^\circ - T_A \cos 66.0^\circ = 0.$$

Similarly, the fact that the *x* components of forces must also sum to zero implies

$$T_C + T_B \sin 27.0^\circ - T_A \sin 66.0^\circ = 0.$$

ANALYZE (a) Substituting the values given, we find the tension force of cable A to be

$$T_A = mg = (817\text{ kg})(9.80\text{ m/s}^2) = 8.01\times10^3\text{ N}.$$

(b) Equilibrium condition for the *y*-components gives

$$T_B = \left(\frac{\cos 66.0°}{\cos 27.0°}\right) T_A = \left(\frac{\cos 66.0°}{\cos 27.0°}\right)(8.01\times10^3\,\text{N}) = 3.65\times10^3\text{N}.$$

(c) Using the equilibrium condition for the *x*-components, we have

$$\begin{aligned} T_C &= T_A \sin 66.0° - T_B \sin 27.0° = (8.01\times10^3\,\text{N})\sin 66.0° - (3.65\times10^3\,\text{N})\sin 27.0° \\ &= 5.66\times10^3\ \text{N}. \end{aligned}$$

LEARN One may verify that the tensions obey law of sine:

$$\frac{T_A}{\sin(180°-\theta_1-\theta_2)} = \frac{T_B}{\sin(90°+\theta_2)} = \frac{T_C}{\sin(90°+\theta_1)} \;.$$

60. (a) Equation 12-8 leads to $T_1 \sin 40^\circ + T_2 \sin\theta = mg$. Also, Eq. 12-7 leads to

$$T_1 \cos 40^\circ - T_2 \cos\theta = 0.$$

Combining these gives the expression

$$T_2 = \frac{mg}{\cos\theta \tan 40° + \sin\theta}\,.$$

To minimize this, we can plot it or set its derivative equal to zero. In either case, we find that it is at its minimum at $\theta = 50°$.

(b) At $\theta = 50°$, we find $T_2 = 0.77mg$.

61. The cable that goes around the lowest pulley is cable 1 and has tension $T_1 = F$. That pulley is supported by the cable 2 (so $T_2 = 2T_1 = 2F$) and goes around the middle pulley. The middle pulley is supported by cable 3 (so $T_3 = 2T_2 = 4F$) and goes around the top pulley. The top pulley is supported by the upper cable with tension T, so $T = 2T_3 = 8F$. Three cables are supporting the block (which has mass $m = 6.40$ kg):

$$T_1 + T_2 + T_3 = mg \Rightarrow F = \frac{mg}{7} = 8.96\,\text{N}.$$

Therefore, $T = 8(8.96\text{ N}) = 71.7$ N.

62. To support a load of $W = mg = (670\text{ kg})(9.8\text{ m/s}^2) = 6566$ N, the steel cable must stretch an amount proportional to its "free" length:

$$\Delta L = \left(\frac{W}{AY}\right)L \quad \text{where } A = \pi r^2$$

and r = 0.0125 m.

(a) If L = 12 m, then $\Delta L = \left(\dfrac{6566 \text{ N}}{\pi(0.0125 \text{ m})^2\,(2.0\times10^{11} \text{ N/m}^2)}\right)(12 \text{ m}) = 8.0\times10^{-4}\,\text{m}.$

(b) Similarly, when L = 350 m, we find $\Delta L = 0.023\,\text{m}$.

63. (a) The center of mass of the top brick cannot be further (to the right) with respect to the brick below it (brick 2) than $L/2$; otherwise, its center of gravity is past any point of support and it will fall. So $a_1 = L/2$ in the maximum case.

(b) With brick 1 (the top brick) in the maximum situation, then the combined center of mass of brick 1 and brick 2 is halfway between the middle of brick 2 and its right edge. That point (the combined com) must be supported, so in the maximum case, it is just above the right edge of brick 3. Thus, $a_2 = L/4$.

(c) Now the total center of mass of bricks 1, 2, and 3 is one-third of the way between the middle of brick 3 and its right edge, as shown by this calculation:

$$x_{\text{com}} = \frac{2m(0) + m(-L/2)}{3m} = -\frac{L}{6}$$

where the origin is at the right edge of brick 3. This point is above the right edge of brick 4 in the maximum case, so $a_3 = L/6$.

(d) A similar calculation,

$$x'_{\text{com}} = \frac{3m(0) + m(-L/2)}{4m} = -\frac{L}{8}$$

shows that $a_4 = L/8$.

(e) We find $h = \sum_{i=1}^{4} a_i = 25L/24$.

64. Since all surfaces are frictionless, the contact force $\vec{F}$ exerted by the lower sphere on the upper one is along that 45° line, and the forces exerted by walls and floors are "normal" (perpendicular to the wall and floor surfaces, respectively). Equilibrium of forces on the top sphere leads to the two conditions

$$F_{\text{wall}} = F\cos 45° \quad \text{and} \quad F\sin 45° = mg.$$

And (using Newton's third law) equilibrium of forces on the bottom sphere leads to the two conditions

$$F'_{\text{wall}} = F\cos 45° \quad \text{and} \quad F'_{\text{floor}} = F\sin 45° + mg.$$

(a) Solving the above equations, we find F'_{floor} = 2*mg*.

(b) We obtain for the left side of the container, $F'_{\text{wall}} = mg$.

(c) We obtain for the right side of the container, $F_{\text{wall}} = mg$.

(d) We get $F = mg/\sin 45° = \sqrt{2}mg$.

65. (a) Choosing an axis through the hinge, perpendicular to the plane of the figure and taking torques that would cause counterclockwise rotation as positive, we require the net torque to vanish:

$$FL\sin 90° - Th\sin 65° = 0$$

where the length of the beam is L = 3.2 m and the height at which the cable attaches is h = 2.0 m. Note that the weight of the beam does not enter this equation since its line of action is directed towards the hinge. With F = 50 N, the above equation yields

$$T = \frac{FL}{h\sin 65°} = \frac{(50\text{ N})(3.2\text{ m})}{(2.0\text{ m})\sin 65°} = 88\text{ N}.$$

(b) To find the components of $\vec{F}_p$ we balance the forces:

$$\sum F_x = 0 \quad \Rightarrow \quad F_{px} = T\cos 25° - F$$
$$\sum F_y = 0 \quad \Rightarrow \quad F_{py} = T\sin 25° + W$$

where W is the weight of the beam (60 N). Thus, we find that the hinge force components are F_{px} = 30 N pointing rightward, and F_{py} = 97 N pointing upward. In unit-vector notation, $\vec{F}_p = (30\text{ N})\hat{\text{i}} + (97\text{ N})\hat{\text{j}}$.

66. Adopting the usual convention that torques that would produce counterclockwise rotation are positive, we have (with axis at the hinge)

$$\sum \tau_z = 0 \quad \Rightarrow \quad TL\sin 60° - Mg\left(\frac{L}{2}\right) = 0$$

where L = 5.0 m and M = 53 kg. Thus, T = 300 N. Now (with F_p for the force of the hinge)

$$\sum F_x = 0 \Rightarrow F_{px} = -T\cos\theta = -150\text{N}$$
$$\sum F_y = 0 \Rightarrow F_{py} = Mg - T\sin\theta = 260\text{N}$$

where $\theta = 60°$. Therefore, $\vec{F}_p = (-1.5\times10^2\ \text{N})\hat{\text{i}} + (2.6\times10^2\ \text{N})\hat{\text{j}}$.

67. The cube has side length l and volume $V = l^3$. We use $p = B\Delta V / V$ for the pressure p. We note that

$$\frac{\Delta V}{V} = \frac{\Delta l^3}{l^3} = \frac{(l+\Delta l)^3 - l^3}{l^3} \approx \frac{3l^2\Delta l}{l^3} = 3\frac{\Delta l}{l}.$$

Thus, the pressure required is

$$p = \frac{3B\Delta l}{l} = \frac{3(1.4\times10^{11}\ \text{N/m}^2)(85.5\,\text{cm} - 85.0\,\text{cm})}{85.5\,\text{cm}} = 2.4\times10^9\ \text{N/m}^2.$$

68. (a) The angle between the beam and the floor is

$$\sin^{-1}(d/L) = \sin^{-1}(1.5/2.5) = 37°,$$

so that the angle between the beam and the weight vector $\vec{W}$ of the beam is 53°. With L = 2.5 m being the length of the beam, and choosing the axis of rotation to be at the base,

$$\sum \tau_z = 0 \;\Rightarrow\; PL - W\left(\frac{L}{2}\right)\sin 53° = 0$$

Thus, $P = ½\, W \sin 53° = 200$ N.

(b) Note that

$$\vec{P} + \vec{W} = (200 \angle 90°) + (500 \angle -127°) = (360 \angle -146°)$$

using magnitude-angle notation (with angles measured relative to the beam, where "uphill" along the beam would correspond to 0°) with the unit newton understood. The "net force of the floor" $\vec{F}_f$ is equal and opposite to this (so that the total net force on the beam is zero), so that $|\vec{F}_f|$ = 360 N and is directed 34° counterclockwise from the beam.

(c) Converting that angle to one measured from true horizontal, we have $\theta = 34° + 37° = 71°$. Thus, $f_s = F_f \cos\theta$ and $F_N = F_f \sin\theta$. Since $f_s = f_{s,\max}$, we divide the equations to obtain

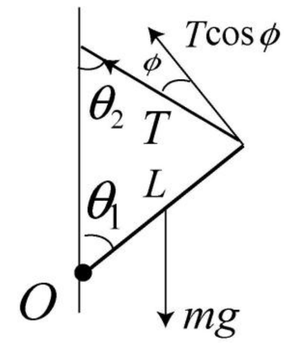
$$\frac{F_N}{f_{s,\max}} = \frac{1}{\mu_s} = \tan\theta.$$

Therefore, $\mu_s = 0.35$.

69. **THINK** Since the rod is in static equilibrium, the net torque about the hinge must be zero.

EXPRESS The free-body diagram is shown below (not to scale). The tension in the rope is denoted as *T*. Since the rod is in rotational equilibrium, the net torque about the hinge, denoted as *O*, must be zero. This implies

$$-mg\sin\theta_1\frac{L}{2} + TL\cos\phi = 0,$$



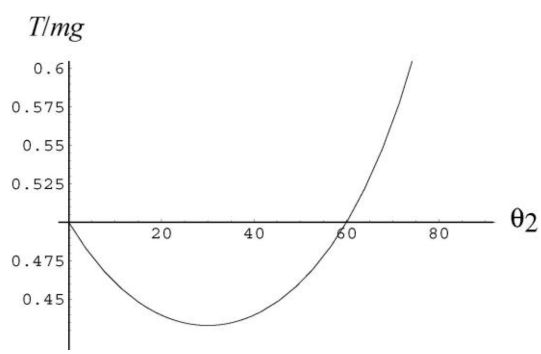
where $\phi = \theta_1 + \theta_2 - 90°$.

ANALYZE Solving for *T* gives

$$T = \frac{mg}{2}\frac{\sin\theta_1}{\cos(\theta_1+\theta_2-90°)} = \frac{mg}{2}\frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}.$$

With θ_1 = 60° and $T = mg/2$, we have $\sin 60° = \sin(60° + \theta_2)$, which yields θ_2 = 60°.

LEARN A plot of T/mg as a function of θ_2 is shown below. The other solution, θ_2 = 0°, is rejected since it corresponds to the limit where the rope becomes infinitely long.



70. (a) Setting up equilibrium of torques leads to

$$F_{\text{far end}}L = (73\,\text{kg})(9.8\,\text{m/s}^2)\frac{L}{4} + (2700\,\text{N})\frac{L}{2}$$

which yields $F_{\text{far end}} = 1.5 \times 10^3$ N.

(b) Then, equilibrium of vertical forces provides

$$F_{\text{near end}} = (73)(9.8) + 2700 - F_{\text{far end}} = 1.9 \times 10^3\,\text{N}.$$

71. **THINK** Upon applying a horizontal force, the cube may tip or slide, depending on the friction between the cube and the floor.

EXPRESS When the cube is about to move, we are still able to apply the equilibrium conditions, but (to obtain the critical condition) we set static friction equal to its maximum value and picture the normal force $\vec{F}_N$ as a concentrated force (upward) at the bottom corner of the cube, directly below the point *O* where *P* is being applied. Thus, the line of action of $\vec{F}_N$ passes through point *O* and exerts no torque about *O* (of course, a similar observation applied to the pull *P*). Since $F_N = mg$ in this problem, we have $f_{s\max} = \mu_c mg$ applied a distance *h* away from *O*. And the line of action of force of gravity (of magnitude *mg*), which is best pictured as a concentrated force at the center of the cube, is a distance *L*/2 away from *O*. Therefore, equilibrium of torques about *O* produces

$$\mu_c mgh = mg\left(\frac{L}{2}\right) \Rightarrow \mu_c = \frac{L}{2h} = \frac{(8.0\text{ cm})}{2(7.0\text{ cm})} = 0.57$$

for the critical condition we have been considering. We now interpret this in terms of a range of values for μ.

ANALYZE (a) For it to slide but not tip, a value of μ *less* than μ_c is needed, since then — static friction will be exceeded for a smaller value of *P*, before the pull is strong enough to cause it to tip. Thus, the required condition is

$$\mu < \mu_c = L/2h = 0.57.$$

(b) And for it to tip but not slide, we need μ *greater* than μ_c is needed, since now — static friction will not be exceeded even for the value of *P* which makes the cube rotate about its front lower corner. That is, we need to have $\mu > \mu_c = L/2h = 0.57$ in this case.

LEARN Note that the value μ_c depends only on the ratio L/h. The cube will tend to slide when μ is mall (think about the limit of a frictionless floor), and tend to tip over when the friction is sufficiently large.

72. We denote the tension in the upper left string (*bc*) as *T´* and the tension in the lower right string (*ab*) as *T*. The supported weight is $W = Mg = (2.0\text{ kg})(9.8\text{ m/s}^2) = 19.6$ N. The force equilibrium conditions lead to

$$T'\cos 60° = T\cos 20° \qquad \text{horizontal forces}$$
$$T'\sin 60° = W + T\sin 20° \qquad \text{vertical forces.}$$

(a) We solve the above simultaneous equations and find

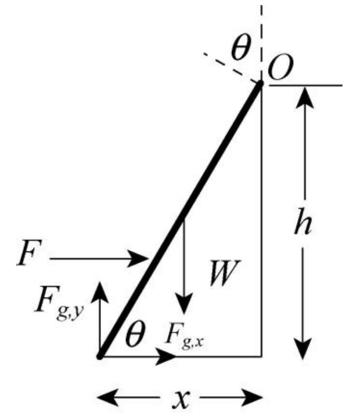
$$T = \frac{W}{\tan 60°\cos 20° - \sin 20°} = \frac{19.6\text{ N}}{\tan 60°\cos 20° - \sin 20°} = 15\text{ N}.$$

(b) Also, we obtain

$$T' = T\cos 20° / \cos 60° = 29\text{ N}.$$

73. **THINK** The force of the ground prevents the ladder from sliding.

EXPRESS The free-body diagram for the ladder is shown to the right. We choose an axis through O, the top (where the ladder comes into contact with the wall), perpendicular to the plane of the figure and take torques that would cause counterclockwise rotation as positive. The length of the ladder is $L = 10\text{ m}$. Given that $h = 8.0\text{ m}$, the horizontal distance to the wall is



$$x = \sqrt{L^2 - h^2} = \sqrt{(10\text{ m})^2 - (8\text{ m})^2} = 6.0\text{m}.$$

Note that the line of action of the applied force $\vec{F}$ intersects the wall at a height of $(8.0\text{ m})/5 = 1.6\text{m}$.

In other words, the *moment arm* for the applied force (in terms of where we have chosen the axis) is

$$r_\perp = (L-d)\sin\theta = (L-d)(h/L) = (8.0\text{ m})(8.0\text{ m}/10.0\text{ m}) = 6.4\text{m}.$$

The moment arm for the weight is $x/2 = 3.0\text{m}$, half the horizontal distance from the wall to the base of the ladder. Similarly, the moment arms for the x and y components of the force at the ground $\left(\vec{F}_g\right)$ are h = 8.0 m and x = 6.0 m, respectively. Thus, we have

$$\begin{aligned}\sum\tau_z &= Fr_\perp + W(x/2) + F_{g,x}h - F_{g,y}x \\ &= F(6.4\text{ m}) + W(3.0\text{ m}) + F_{g,x}(8.0\text{ m}) - F_{g,y}(6.0\text{ m}) = 0.\end{aligned}$$

In addition, from balancing the vertical forces we find that $W = F_{g,y}$ (keeping in mind that the wall has no friction). Therefore, the above equation can be written as

$$\sum\tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{g,x}(8.0\text{ m}) - W(6.0\text{ m}) = 0.$$

ANALYZE (a) With F = 50 N and W = 200 N, the above equation yields $F_{g,x} = 35\text{ N}$. Thus, in unit vector notation we obtain

$$\vec{F}_g = (35\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(b) Similarly, with F = 150 N and W = 200 N, the above equation yields $F_{g,x} = -45\text{ N}$. Therefore, in unit vector notation we obtain

$$\vec{F}_g = (-45\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(c) Note that the phrase "start to move towards the wall" implies that the friction force is pointed away from the wall (in the $-\hat{\text{i}}$ direction). Now, if $f = -F_{g,x}$ and

$F_N = F_{g,y} = 200\text{ N}$ are related by the (maximum) static friction relation ($f = f_{s,\max} = \mu_s F_N$) with μ_s = 0.38, then we find $F_{g,x} = -76\text{ N}$. Returning this to the above equation, we obtain

$$F = \frac{W(x/2) + \mu_s W h}{r_\perp} = \frac{(200\text{ N})(3.0\text{ m}) + (0.38)(200\text{N})(8.0\text{ m})}{6.4\text{ m}} = 1.9\times10^2\text{ N}.$$

LEARN The force needed to move the ladder toward the wall would decrease with a larger $r_\perp$ or a smaller μ_s.

74. One arm of the balance has length ℓ_1 and the other has length ℓ_2. The two cases described in the problem are expressed (in terms of torque equilibrium) as

$$m_1\ell_1 = m\ell_2 \quad \text{and} \quad m\ell_1 = m_2\ell_2.$$

We divide equations and solve for the unknown mass: $m = \sqrt{m_1 m_2}$.

75. Since *GA* exerts a leftward force *T* at the corner *A*, then (by equilibrium of horizontal forces at that point) the force F_{diag} in *CA* must be pulling with magnitude

$$F_{\text{diag}} = \frac{T}{\sin 45°} = T\sqrt{2}.$$

This analysis applies equally well to the force in *DB*. And these diagonal bars are pulling on the bottom horizontal bar exactly as they do to the top bar, so the bottom bar *CD* is the "mirror image" of the top one (it is also under tension *T*). Since the figure is symmetrical (except for the presence of the turnbuckle) under 90° rotations, we conclude that the side bars (*DA* and *BC*) also are under tension *T* (a conclusion that also follows from considering the vertical components of the pull exerted at the corners by the diagonal bars).

(a) Bars that are in tension are *BC*, *CD*, and *DA*.

(b) The magnitude of the forces causing tension is $T = 535\text{ N}$.

(c) The magnitude of the forces causing compression on *CA* and *DB* is

$$F_{\text{diag}} = \sqrt{2}T = (1.41)535\text{ N} = 757\text{ N}.$$

76. (a) For computing torques, we choose the axis to be at support 2 and consider torques that encourage counterclockwise rotation to be positive. Let *m* = mass of gymnast and *M* = mass of beam. Thus, equilibrium of torques leads to

$$Mg(1.96\text{ m}) - mg(0.54\text{ m}) - F_1(3.92\text{ m}) = 0.$$

Therefore, the upward force at support 1 is F_1 = 1163 N (quoting more figures than are significant — but with an eye toward using this result in the remaining calculation). In unit-vector notation, we have $\vec{F}_1 \approx (1.16\times10^3\ \text{N})\hat{\text{j}}$.

(b) Balancing forces in the vertical direction, we have $F_1 + F_2 - Mg - mg = 0$, so that the upward force at support 2 is F_2 =1.74×10^3 N. In unit-vector notation, we have $\vec{F}_2 \approx (1.74\times10^3\ \text{N})\hat{\text{j}}$.

77. (a) Let d = 0.00600 m. In order to achieve the same final lengths, wires 1 and 3 must stretch an amount d more than wire 2 stretches:

$$\Delta L_1 = \Delta L_3 = \Delta L_2 + d\ .$$

Combining this with Eq. 12-23 we obtain

$$F_1 = F_3 = F_2 + \frac{dAE}{L}\ .$$

Now, Eq. 12-8 produces $F_1 + F_3 + F_2 - mg = 0$. Combining this with the previous relation (and using Table 12-1) leads to $F_1 = 1380\ \text{N} \approx 1.38\times10^3\,\text{N}$.

(b) Similarly, F_2 = 180 N.

78. (a) Computing the torques about the hinge, we have

$$TL\sin 40° = W\frac{L}{2}\sin 50°,$$

where the length of the beam is L = 12 m and the tension is T = 400 N. Therefore, the weight is $W = 671\ \text{N}$, which means that the gravitational force on the beam is $\vec{F}_w = (-671\ \text{N})\hat{\text{j}}$.

(b) Equilibrium of horizontal and vertical forces yields, respectively,

$$F_{\text{hinge}\,x} = T = 400\ \text{N}$$
$$F_{\text{hinge}\,y} = W = 671\ \text{N}$$

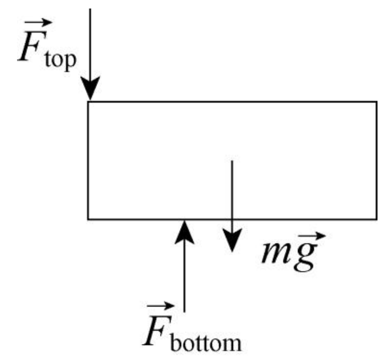
where the hinge force components are rightward (for x) and upward (for y). In unit-vector notation, we have $\vec{F}_{\text{hinge}} = (400\ \text{N})\hat{\text{i}} + (671\ \text{N})\hat{\text{j}}$.

79. We locate the origin of the x axis at the edge of the table and choose rightward positive. The criterion (in part (a)) is that the center of mass of the block above another must be no further than the edge of the one below; the criterion in part (b) is more subtle

and is discussed below. Since the edge of the table corresponds to $x = 0$ then the total center of mass of the blocks must be zero.

(a) We treat this as three items: one on the upper left (composed of two bricks, one directly on top of the other) of mass $2m$ whose center is above the left edge of the bottom brick; a single brick at the upper right of mass m, which necessarily has its center over the right edge of the bottom brick (so $a_1 = L/2$ trivially); and, the bottom brick of mass m. The total center of mass is

$$\frac{(2m)(a_2 - L) + ma_2 + m(a_2 - L/2)}{4m} = 0$$



which leads to $a_2 = 5L/8$. Consequently, $h = a_2 + a_1 = 9L/8$.

(b) We have four bricks (each of mass m) where the center of mass of the top one and the center of mass of the bottom one have the same value, $x_{cm} = b_2 - L/2$. The middle layer consists of two bricks, and we note that it is possible for each of their centers of mass to be beyond the respective edges of the bottom one! This is due to the fact that the top brick is exerting downward forces (each equal to half its weight) on the middle blocks — and in the extreme case, this may be thought of as a pair of concentrated forces exerted at the innermost edges of the middle bricks. Also, in the extreme case, the support force (upward) exerted on a middle block (by the bottom one) may be thought of as a concentrated force located at the edge of the bottom block (which is the point about which we compute torques, in the following).

If (as indicated in our sketch, where $\vec{F}_{\text{top}}$ has magnitude $mg/2$) we consider equilibrium of torques on the rightmost brick, we obtain

$$mg\left(b_1 - \frac{1}{2}L\right) = \frac{mg}{2}(L - b_1)$$

which leads to $b_1 = 2L/3$. Once we conclude from symmetry that $b_2 = L/2$, then we also arrive at $h = b_2 + b_1 = 7L/6$.

80. The assumption stated in the problem (that the density does not change) is not meant to be realistic; those who are familiar with Poisson's ratio (and other topics related to the strengths of materials) might wish to think of this problem as treating a fictitious material (which happens to have the same value of E as aluminum, given in Table 12-1) whose density does not significantly change during stretching. Since the mass does not change either, then the constant-density assumption implies the volume (which is the circular area times its length) stays the same:

$$(\pi r^2 L)_{\text{new}} = (\pi r^2 L)_{\text{old}} \quad \Rightarrow \quad \Delta L = L\left[(1000/999.9)^2 - 1\right].$$

Now, Eq. 12-23 gives

$$F = \pi r^2 E\, \Delta L/L \;=\; \pi r^2 (7.0 \text{ x } 10^9 \text{ N/m}^2)[(1000/999.9)^2 - 1]\ .$$

Using either the new or old value for r gives the answer F = 44 N.

81. Where the crosspiece comes into contact with the beam, there is an upward force of $2F$ (where F is the upward force exerted by each man). By equilibrium of vertical forces, $W = 3F$ where W is the weight of the beam. If the beam is uniform, its center of gravity is a distance $L/2$ from the man in front, so that computing torques about the front end leads to

$$W\frac{L}{2} = 2Fx = 2\left(\frac{W}{3}\right)x$$

which yields $x = 3L/4$ for the distance from the crosspiece to the front end. It is therefore a distance $L/4$ from the rear end (the "free" end).

82. The force F exerted on the beam is F = 7900 N, as computed in the Sample Problem. Let $F/A = S_u/6$, where $S_u = 50\times10^6 \text{ N/m}^2$ is the ultimate strength (see Table 12-1). Then

$$A = \frac{6F}{S_u} = \frac{6(7900 \text{ N})}{50\times10^6 \text{ N/m}^2} = 9.5\times10^{-4}\,\text{m}^2.$$

Thus the thickness is $\sqrt{A} = \sqrt{9.5\times10^{-4} \text{ m}^2} = 0.031\,\text{m}$.

83. (a) Because of Eq. 12-3, we can write

$$\vec{T} \;+\; (m_B\, g \;\angle -90^\circ) + (m_A\, g \;\angle -150^\circ) \;=\; 0\ .$$

Solving the equation, we obtain $\vec{T}$ = (106.34 $\angle$ 63.963º). Thus, the magnitude of the tension in the upper cord is 106 N,

(b) and its angle (measured counterclockwise from the $+x$ axis) is 64.0°.

84. (a) and (b) With $+x$ rightward and $+y$ upward (we assume the adult is pulling with force $\vec{P}$ to the right), we have

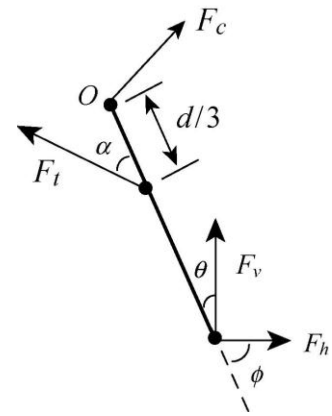
$$\sum F_y = 0 \quad\Rightarrow\quad W = T\cos\theta = 270 \text{ N}$$
$$\sum F_x = 0 \quad\Rightarrow\quad P = T\sin\theta = 72 \text{ N}$$

where θ = 15°.

(c) Dividing the above equations leads to

$$\frac{P}{W} = \tan\theta \;.$$

Thus, with $W = 270$ N and $P = 93$ N, we find $\theta = 19°$.



85. Our system is the second finger bone. Since the system is in static equilibrium, the net force acting on it is zero. In addition, the torque about any point must be zero. We set up the torque equation about point O where $\vec{F}_c$ act:

$$0 = \sum_O \tau_{\text{net}} = -\left(\frac{d}{3}\right) F_t \sin\alpha + (d) F_v \sin\theta + (d) F_h \sin\phi \,.$$

Solving for F_t and substituting the values given, we obtain

$$F_t = \frac{3(F_v \sin\theta + F_h \sin\phi)}{\sin\alpha} = \frac{3[(162.4\text{ N})\sin 10° + (13.4\text{ N})\sin 80°]}{\sin 45°} = 175.6\text{ N}$$
$$\approx 1.8\times 10^2\text{ N}.$$

86. (a) Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_{\text{catch}} = (11\,\text{kg})(9.8\,\text{m/s}^2)\frac{(91/2 - 10)\,\text{cm}}{91\,\text{cm}} = 42\,\text{N}.$$

(b) Then, equilibrium of vertical forces provides

$$F_{\text{hinge}} = (11\,\text{kg})(9.8\,\text{m/s}^2) - F_{\text{catch}} = 66\,\text{N}.$$

87. (a) For the net force to be zero, $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0,$ we require

$$\begin{aligned}\vec{F}_3 &= -\vec{F}_1 - \vec{F}_2 = -\left[(8.40\text{ N})\hat{\text{i}} - (5.70\text{ N})\hat{\text{j}}\right] - \left[(16.0\text{ N})\hat{\text{i}} + (4.10\text{ N})\hat{\text{j}}\right] \\ &= (-24.4\text{ N})\hat{\text{i}} + (1.60\text{ N})\hat{\text{j}}\end{aligned}$$

Thus, $F_{3x} = -24.4\text{ N}$.

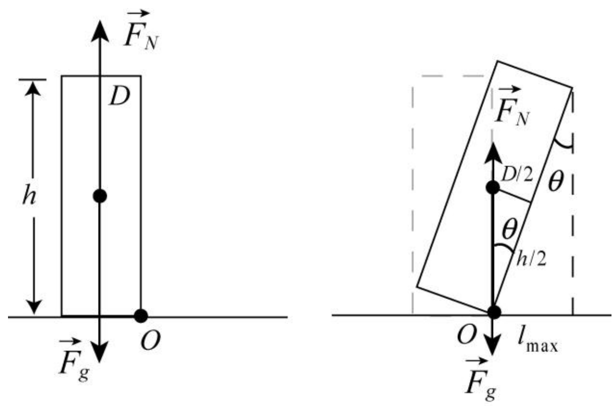
(b) Similarly, $F_{3y} = 1.60\text{ N}$.

(c) The angle $\vec{F}_3$ makes relative to the +x-axis is

$$\theta = \tan^{-1}\left(\frac{F_{3y}}{F_{3x}}\right) = \tan^{-1}\left(\frac{1.60\ \text{N}}{-24.4\ \text{N}}\right) = 176.25°.$$

88. We solve part (b) first.

(b) The critical tilt angle corresponds to the situation where the line of action of $\vec{F}_g$ passes through the supporting edge (point O in the figure).



At this state, the normal force also passes through the supporting edge, so the net torque is zero and the Tower is in static equilibrium. However, this equilibrium is unstable and the Tower is on the verge of falling over. From the figure, we find the critical angle to be

$$\tan\theta = \frac{D/2}{h/2} = \frac{D}{h} \qquad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{D}{h}\right)\tan^{-1}\left(\frac{7.44\ \text{m}}{59.1\ \text{m}}\right) = 7.18°$$

(a) From the figure, the maximum displacement is

$$l_{\text{max}} = h\sin\theta = (59.1\ \text{m})\sin 7.18° = 7.38\ \text{m}$$

Thus, the additional displacement to put the Tower on the verge of toppling is

$$\Delta l = l_{\text{max}} - l = 7.38\ \text{m} - 4.01\ \text{m} = 3.37\ \text{m}$$