# Chapter 13

1. The gravitational force between the two parts is

$$F = \frac{Gm\left(M-m\right)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to *m* and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}\left(M - 2m\right) \Rightarrow M = 2m\,.$$

This leads to the result $m/M = 1/2$.

2. The gravitational force between you and the moon at its initial position (directly opposite of Earth from you) is

$$F_0 = \frac{GM_m m}{(R_{ME} + R_E)^2}$$

where $M_m$ is the mass of the moon, $R_{ME}$ is the distance between the moon and the Earth, and $R_E$ is the radius of the Earth. At its final position (directly above you), the gravitational force between you and the moon is

$$F_1 = \frac{GM_m m}{(R_{ME} - R_E)^2}\,.$$

(a) The ratio of the moon's gravitational pulls at the two different positions is

$$\frac{F_1}{F_0} = \frac{GM_m m/(R_{ME} - R_E)^2}{GM_m m/(R_{ME} + R_E)^2} = \left(\frac{R_{ME} + R_E}{R_{ME} - R_E}\right)^2 = \left(\frac{3.82\times 10^8\ \text{m} + 6.37\times 10^6\ \text{m}}{3.82\times 10^8\ \text{m} - 6.37\times 10^6\ \text{m}}\right)^2 = 1.06898.$$

Therefore, the increase is 0.06898, or approximately 6.9%.

(b) The change of the gravitational pull may be approximated as

$$F_1 - F_0 = \frac{GM_m m}{(R_{ME} - R_E)^2} - \frac{GM_m m}{(R_{ME} + R_E)^2} \approx \frac{GM_m m}{R_{ME}^2}\left(1 + 2\frac{R_E}{R_{ME}}\right) - \frac{GM_m m}{R_{ME}^2}\left(1 - 2\frac{R_E}{R_{ME}}\right)$$
$$= \frac{4GM_m m R_E}{R_{ME}^3}.$$

On the other hand, your weight, as measured on a scale on Earth, is

$$F_g = mg_E = \frac{GM_E m}{R_E^2}.$$

Since the moon pulls you "up," the percentage decrease of weight is

$$\frac{F_1 - F_0}{F_g} = 4\left(\frac{M_m}{M_E}\right)\left(\frac{R_E}{R_{ME}}\right)^3 = 4\left(\frac{7.36\times10^{22}\text{ kg}}{5.98\times10^{24}\text{ kg}}\right)\left(\frac{6.37\times10^{6}\text{ m}}{3.82\times10^{8}\text{ m}}\right)^3 = 2.27\times10^{-7} \approx (2.3\times10^{-5})\%.$$

3. **THINK** The magnitude of gravitational force between two objects depends on their distance of separation.

**EXPRESS** The magnitude of the gravitational force of one particle on the other is given by $F = Gm_1m_2/r^2$, where $m_1$ and $m_2$ are the masses, $r$ is their separation, and $G$ is the universal gravitational constant.

**ANALYZE** Solve for $r$ using the values given, we obtain

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.2\text{kg}\right)\left(2.4\text{kg}\right)}{2.3\times10^{-12}\,\text{N}}} = 19\,\text{m}.$$

**LEARN** The force of gravitation is inversely proportional to $r^2$.

4. We use subscripts $s$, $e$, and $m$ for the Sun, Earth and Moon, respectively. Plugging in the numerical values (say, from Appendix C) we find

$$\frac{F_{sm}}{F_{em}} = \frac{Gm_sm_m/r_{sm}^2}{Gm_em_m/r_{em}^2} = \frac{m_s}{m_e}\left(\frac{r_{em}}{r_{sm}}\right)^2 = \frac{1.99\times10^{30}\text{ kg}}{5.98\times10^{24}\text{ kg}}\left(\frac{3.82\times10^{8}\text{ m}}{1.50\times10^{11}\text{ m}}\right)^2 = 2.16.$$

5. The gravitational force from Earth on you (with mass $m$) is

$$F_g = \frac{GM_E m}{R_E^2} = mg$$

where $g = GM_E/R_E^2 = 9.8\,\text{m/s}^2$. If $r$ is the distance between you and a tiny black hole of mass $M_b = 1\times10^{11}$ kg that has the same gravitational pull on you as the Earth, then

$$F_g = \frac{GM_b m}{r^2} = mg.$$

Combining the two equations, we obtain

$$mg = \frac{GM_E m}{R_E^2} = \frac{GM_b m}{r^2} \Rightarrow r = \sqrt{\frac{GM_b}{g}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1\times10^{11}\ \text{kg})}{9.8\ \text{m/s}^2}} \approx 0.8\ \text{m}.$$

6. The gravitational forces on $m_5$ from the two 5.00 g masses $m_1$ and $m_4$ cancel each other. Contributions to the net force on $m_5$ come from the remaining two masses:

$$F_{\text{net}} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.50\times10^{-3}\ \text{kg}\right)\left(3.00\times10^{-3}\ \text{kg} - 1.00\times10^{-3}\ \text{kg}\right)}{\left(\sqrt{2}\times10^{-1}\ \text{m}\right)^2}$$
$$= 1.67\times10^{-14}\ \text{N}.$$

The force is directed along the diagonal between $m_2$ and $m_3$, toward $m_2$. In unit-vector notation, we have

$$\vec{F}_{\text{net}} = F_{\text{net}}(\cos 45°\,\hat{\text{i}} + \sin 45°\,\hat{\text{j}}) = (1.18\times10^{-14}\ \text{N})\,\hat{\text{i}} + (1.18\times10^{-14}\ \text{N})\,\hat{\text{j}}.$$

7. We require the magnitude of force (given by Eq. 13-1) exerted by particle $C$ on $A$ be equal to that exerted by $B$ on $A$. Thus,

$$\frac{Gm_A m_C}{r^2} = \frac{Gm_A m_B}{d^2}\ .$$

We substitute in $m_B = 3m_A$ and $m_B = 3m_A$, and (after canceling "$m_A$") solve for $r$. We find $r = 5d$. Thus, particle $C$ is placed on the $x$ axis, to the left of particle $A$ (so it is at a negative value of $x$), at $x = -5.00d$.

8. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with $1.9 \times 10^{-8}$ N, and the rightmost mass pulls rightward on the one at the origin with $1.0 \times 10^{-8}$ N. Thus, the $(x, y)$ components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\text{net}} = \left(1.04\times10^{-8}, 1.85\times10^{-8}\right) \Rightarrow \left(2.13\times10^{-8}\ \angle\ 60.6°\right).$$

(a) The magnitude of the force is $2.13 \times 10^{-8}$ N.

(b) The direction of the force relative to the $+x$ axis is $60.6°$.

9. **THINK** Both the Sun and the Earth exert a gravitational pull on the space probe. The net force can be calculated by using superposition principle.

**EXPRESS** At the point where the two forces balance, we have $GM_E m/r_1^2 = GM_S m/r_2^2$, where $M_E$ is the mass of Earth, $M_S$ is the mass of the Sun, $m$ is the mass of the space probe, $r_1$ is the distance from the center of Earth to the probe, and $r_2$ is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where $d$ is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_E}{r_1^2} = \frac{M_S}{(d - r_1)^2}.$$

**ANALYZE** Using the values for $M_E$, $M_S$, and $d$ given in Appendix C, we take the positive square root of both sides to solve for $r_1$. A little algebra yields

$$r_1 = \frac{d}{1+\sqrt{M_S/M_E}} = \frac{1.50\times10^{11}\ \text{m}}{1+\sqrt{(1.99\times10^{30}\ \text{kg})/(5.98\times10^{24}\ \text{kg})}} = 2.60\times10^{8}\ \text{m}.$$

**LEARN** The fact that $r_1 \ll d$ indicates that the probe is much closer to the Earth than the Sun.

10. Using Eq. 13-1, we find

$$\vec{F}_{AB} = \frac{2Gm_A^2}{d^2}\,\hat{\text{j}}, \quad \vec{F}_{AC} = -\frac{4Gm_A^2}{3d^2}\,\hat{\text{i}}\ .$$

Since the vector sum of all three forces must be zero, we find the third force (using magnitude-angle notation) is

$$\vec{F}_{AD} = \frac{Gm_A^2}{d^2}(2.404\ \angle\ -56.3^\circ)\ .$$

This tells us immediately the direction of the vector $\vec{r}$ (pointing from the origin to particle $D$), but to find its magnitude we must solve (with $m_D = 4m_A$) the following equation:

$$2.404\left(\frac{Gm_A^2}{d^2}\right) = \frac{Gm_A m_D}{r^2}\ .$$

This yields $r = 1.29d$. In magnitude-angle notation, then, $\vec{r} = (1.29\ \angle\ -56.3^\circ)$, with SI units understood. The "exact" answer without regard to significant figure considerations is

$$\vec{r} = \left(2\sqrt{\frac{6}{13\sqrt{13}}},\ -3\sqrt{\frac{6}{13\sqrt{13}}}\right).$$

(a) In $(x, y)$ notation, the $x$ coordinate is $x = 0.716d$.

(b) Similarly, the $y$ coordinate is $y = -1.07d$.

11. (a) The distance between any of the spheres at the corners and the sphere at the center is

$$r = \ell/2\cos 30^\circ = \ell/\sqrt{3}$$

where $\ell$ is the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass $m$) to the total force on $m_4$ has magnitude

$$2F_y = 2\left(\frac{Gm_4m}{r^2}\right)\sin 30° = 3\frac{Gm_4m}{\ell^2}.$$

This must equal the magnitude of the pull from $M$, so

$$3\frac{Gm_4m}{\ell^2} = \frac{Gm_4m}{(\ell/\sqrt{3})^2}$$

which readily yields $m = M$.

(b) Since $m_4$ cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

12. (a) We are told the value of the force when particle $C$ is removed (that is, as its position $x$ goes to infinity), which is a situation in which any force caused by $C$ vanishes (because Eq. 13-1 has $r^2$ in the denominator). Thus, this situation only involves the force exerted by $A$ on $B$:

$$F_{\text{net},x} = F_{AB} = \frac{Gm_Am_B}{r_{AB}^2} = 4.17 \times 10^{-10}\ \text{N}.$$

Since $m_B$ = 1.0 kg and $r_{AB} = 0.20\ \text{m}$, then this yields

$$m_A = \frac{r_{AB}^2F_{AB}}{Gm_B} = \frac{(0.20\ \text{m})^2(4.17\times10^{-10}\ \text{N})}{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.0\ \text{kg})} = 0.25\ \text{kg}.$$

(b) We note (from the graph) that the net force on $B$ is zero when $x$ = 0.40 m. Thus, at that point, the force exerted by $C$ must have the same magnitude (but opposite direction) as the force exerted by $A$ (which is the one discussed in part (a)). Therefore

$$\frac{Gm_Cm_B}{(0.40\ \text{m})^2} = 4.17 \times 10^{-10}\ \text{N} \quad \Rightarrow m_C = 1.00\ \text{kg}.$$

13. If the lead sphere were not hollowed the magnitude of the force it exerts on $m$ would be $F_1 = GMm/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on $m$ by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere.

The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere, that is, $M_c/r^3 = M/R^3$, where $M_c$ is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = \left(\frac{r^3}{R^3}\right)M = \left(\frac{R^3}{8R^3}\right)M = \frac{M}{8}.$$

The center of the cavity is $d - r = d - R/2$ from $m$, so the force it exerts on $m$ is

$$F_2 = \frac{G(M/8)m}{(d - R/2)^2}.$$

The force of the hollowed sphere on $m$ is

$$F = F_1 - F_2 = GMm\left(\frac{1}{d^2} - \frac{1}{8(d-R/2)^2}\right) = \frac{GMm}{d^2}\left(1 - \frac{1}{8(1-R/2d)^2}\right)$$
$$= \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(2.95\ \text{kg})(0.431\ \text{kg})}{(9.00\times10^{-2}\,\text{m})^2}\left(1 - \frac{1}{8[1-(4\times10^{-2}\,\text{m})/(2\cdot9\times10^{-2}\,\text{m})]^2}\right)$$
$$= 8.31\times10^{-9}\,\text{N}.$$

14. All the forces are being evaluated at the origin (since particle $A$ is there), and all forces (except the net force) are along the location vectors $\vec{r}$, which point to particles $B$ and $C$. We note that the angle for the location-vector pointing to particle $B$ is 180º – 30.0º = 150º (measured counterclockwise from the +$x$ axis). The component along, say, the $x$ axis of one of the force vectors $\vec{F}$ is simply $Fx/r$ in this situation (where $F$ is the magnitude of $\vec{F}$). Since the force itself (see Eq. 13-1) is inversely proportional to $r^2$, then the aforementioned $x$ component would have the form $GmMx/r^3$; similarly for the other components. With $m_A$ = 0.0060 kg, $m_B$ = 0.0120 kg, and $m_C$ = 0.0080 kg, we therefore have

$$F_{\text{net}\,x} = \frac{Gm_Am_Bx_B}{r_B^3} + \frac{Gm_Am_Cx_C}{r_C^3} = (2.77\times10^{-14}\,\text{N})\cos(-163.8^\circ)$$

and

$$F_{\text{net}\,y} = \frac{Gm_Am_By_B}{r_B^3} + \frac{Gm_Am_Cy_C}{r_C^3} = (2.77\times10^{-14}\,\text{N})\sin(-163.8^\circ)$$

where $r_B = d_{AB}$ = 0.50 m, and $(x_B, y_B) = (r_B\cos(150^\circ), r_B\sin(150^\circ))$ (with SI units understood). A fairly quick way to solve for $r_C$ is to consider the vector difference between the net force and the force exerted by $A$, and then employ the Pythagorean theorem. This yields $r_C$ = 0.40 m.

(a) By solving the above equations, the $x$ coordinate of particle $C$ is $x_C = -0.20$ m.

(b) Similarly, the $y$ coordinate of particle $C$ is $y_C = -0.35$ m.

15. All the forces are being evaluated at the origin (since particle $A$ is there), and all forces are along the location vectors $\vec{r}$, which point to particles $B$, $C$, and $D$. In three dimensions, the Pythagorean theorem becomes $r = \sqrt{x^2 + y^2 + z^2}$ . The component along, say, the $x$ axis of one of the force-vectors $\vec{F}$ is simply $Fx/r$ in this situation (where $F$ is the magnitude of $\vec{F}$ ). Since the force itself (see Eq. 13-1) is inversely proportional to $r^2$ then the aforementioned $x$ component would have the form $GmMx/r^3$; similarly for the other components. For example, the $z$ component of the force exerted on particle $A$ by particle $B$ is

$$\frac{Gm_A m_B z_B}{r_B^{\,3}} = \frac{Gm_A(2m_A)(2d)}{((2d)^2 + d^2 + (2d)^2)^3} = \frac{4Gm_A^2}{27d^2} .$$

In this way, each component can be written as some multiple of $Gm_A^{\,2}/d^2$. For the $z$ component of the force exerted on particle $A$ by particle $C$, that multiple is $-9\sqrt{14}/196$. For the $x$ components of the forces exerted on particle $A$ by particles $B$ and $C$, those multiples are 4/27 and $-3\sqrt{14}/196$, respectively. And for the $y$ components of the forces exerted on particle $A$ by particles $B$ and $C$, those multiples are 2/27 and $3\sqrt{14}/98$, respectively. To find the distance $r$ to particle $D$ one method is to solve (using the fact that the vector add to zero)

$$\left(\frac{Gm_A m_D}{r^2}\right)^2 = \left[\left(\frac{4}{27} - \frac{3\sqrt{14}}{196}\right)^2 + \left(\frac{2}{27} + \frac{3\sqrt{14}}{98}\right)^2 + \left(\frac{4}{27} - \frac{9\sqrt{14}}{196}\right)^2\right]\left(\frac{Gm_A^2}{d^2}\right)^2 = 0.4439\left(\frac{Gm_A^2}{d^2}\right)^2$$

With $m_D = 4m_A$, we obtain

$$\left(\frac{4}{r^2}\right)^2 = \frac{0.4439}{(d^2)^2} \quad\Rightarrow\quad r = \left(\frac{16}{0.4439}\right)^{1/4} d = 4.357d .$$

The individual values of $x$, $y$, and $z$ (locating the particle $D$) can then be found by considering each component of the $Gm_Am_D/r^2$ force separately.

(a) The $x$ component of $\vec{r}$ would be

$$\frac{Gm_A m_D x}{r^3} = -\left(\frac{4}{27} - \frac{3\sqrt{14}}{196}\right)^2 \frac{Gm_A^2}{d^2} = -0.0909\frac{Gm_A^2}{d^2} ,$$

which yields $x = -0.0909\dfrac{m_A r^3}{m_D d^2} = -0.0909\dfrac{m_A(4.357d)^3}{(4m_A)d^2} = -1.88d$ .

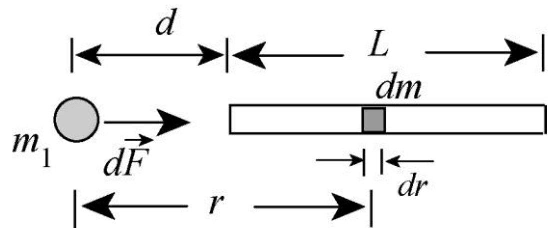
(b) Similarly, $y = -3.90d$,

(c) and $z = 0.489d$.

In this way we are able to deduce that $(x, y, z) = (1.88d, 3.90d, 0.489d)$.

16. Since the rod is an extended object, we cannot apply Equation 13-1 directly to find the force. Instead, we consider a small differential element of the rod, of mass $dm$ of thickness $dr$ at a distance $r$ from $m_1$. The gravitational force between $dm$ and $m_1$ is

$$dF = \frac{Gm_1 dm}{r^2} = \frac{Gm_1 (M/L)dr}{r^2},$$




where we have substituted $dm = (M/L)dr$ since mass is uniformly distributed. The direction of $d\vec{F}$ is to the right (see figure). The total force can be found by integrating over the entire length of the rod:

$$F = \int dF = \frac{Gm_1 M}{L}\int_d^{L+d} \frac{dr}{r^2} = -\frac{Gm_1 M}{L}\left(\frac{1}{L+d} - \frac{1}{d}\right) = \frac{Gm_1 M}{d(L+d)}.$$

Substituting the values given in the problem statement, we obtain

$$F = \frac{Gm_1 M}{d(L+d)} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(0.67\ \text{kg})(5.0\ \text{kg})}{(0.23\ \text{m})(3.0\ \text{m}+0.23\ \text{m})} = 3.0\times10^{-10}\ \text{N}.$$

17. (a) The gravitational acceleration at the surface of the Moon is $g_{\text{moon}}$ = 1.67 m/s$^2$ (see Appendix C). The ratio of weights (for a given mass) is the ratio of $g$-values, so

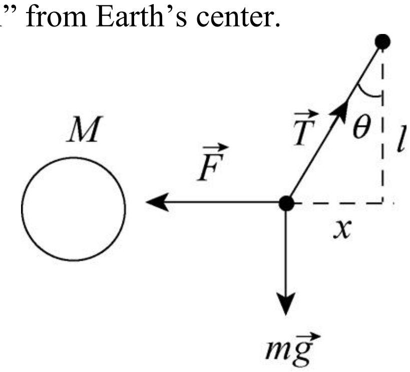
$$W_{\text{moon}} = (100\ \text{N})(1.67/9.8) = 17\ \text{N}.$$

(b) For the force on that object caused by Earth's gravity to equal 17 N, then the free-fall acceleration at its location must be $a_g$ = 1.67 m/s$^2$. Thus,

$$a_g = \frac{Gm_E}{r^2} \Rightarrow r = \sqrt{\frac{Gm_E}{a_g}} = 1.5\times10^7\,\text{m}$$

so the object would need to be a distance of $r/R_E$ = 2.4 "radii" from Earth's center.

18. The free-body diagram of the force acting on the plumb line is shown to the right. The mass of the sphere is




$$M = \rho V = \rho\left(\frac{4\pi}{3}R^3\right) = \frac{4\pi}{3}(2.6\times10^3\ \text{kg/m}^3)(2.00\times10^3\ \text{m})^3$$
$$= 8.71\times10^{13}\ \text{kg}.$$

The force between the "spherical" mountain and the plumb line is $F = GMm/r^2$. Suppose at equilibrium the line makes an angle $\theta$ with the vertical and the net force acting on the line is zero. Therefore,

$$0 = \sum F_{\text{net},\,x} = T\sin\theta - F = T\sin\theta - \frac{GMm}{r^2}$$
$$0 = \sum F_{\text{net},\,y} = T\cos - mg$$

The two equations can be combined to give $\tan\theta = \dfrac{F}{mg} = \dfrac{GM}{gr^2}$. The distance the lower end moves toward the sphere is

$$x = l\tan\theta = l\frac{GM}{gr^2} = (0.50\text{ m})\frac{(6.67\times10^{-11}\text{ m}^3/\text{kg}\cdot\text{s}^2)(8.71\times10^{13}\text{ kg})}{(9.8)(3\times2.00\times10^3\text{ m})^2}$$
$$= 8.2\times10^{-6}\text{ m}.$$

19. **THINK** Earth's gravitational acceleration varies with altitude.

**EXPRESS** The acceleration due to gravity is given by $a_g = GM/r^2$, where $M$ is the mass of Earth and $r$ is the distance from Earth's center. We substitute $r = R + h$, where $R$ is the radius of Earth and $h$ is the altitude, to obtain

$$a_g = \frac{GM}{r^2} = \frac{GM}{(R_E + h)^2}.$$

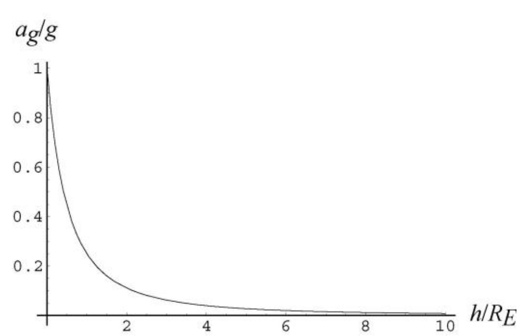
**ANALYZE** Solving for $h$, we obtain $h = \sqrt{GM/a_g} - R_E$. From Appendix C, $R_E = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{\left(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\text{kg}\right)}{\left(4.9\text{m/s}^2\right)}} - 6.37\times10^6\text{m} = 2.6\times10^6\text{m}.$$

**LEARN** We may rewrite $a_g$ as

$$a_g = \frac{GM}{r^2} = \frac{GM/R_E^2}{(1+h/R_E)^2} = \frac{g}{(1+h/R_E)^2}$$

where $g = 9.83\text{ m/s}^2$ is the gravitational acceleration on the Surface of the Earth. The plot below depicts how $a_g$ decreases with increasing altitude.



20. We follow the method shown in Sample Problem 13.2 – "Difference in acceleration at head and feet." Thus,

$$a_g = \frac{GM_E}{r^2} \Rightarrow da_g = -2\frac{GM_E}{r^3}\,dr$$

which implies that the change in weight is

$$W_{\text{top}} - W_{\text{bottom}} \approx m\left(da_g\right).$$

However, since $W_{\text{bottom}} = GmM_E/R^2$ (where $R$ is Earth's mean radius), we have

$$mda_g = -2\frac{GmM_E}{R^3}\,dr = -2W_{\text{bottom}}\frac{dr}{R} = -2\left(600\text{ N}\right)\frac{1.61\times10^3\text{ m}}{6.37\times10^6\text{ m}} = -0.303\text{ N}$$

for the weight change (the minus sign indicating that it is a decrease in $W$). We are not including any effects due to the Earth's rotation (as treated in Eq. 13-13).

21. From Eq. 13-14, we see the extreme case is when "$g$" becomes zero, and plugging in Eq. 13-15 leads to

$$0 = \frac{GM}{R^2} - R\omega^2 \Rightarrow M = \frac{R^3\omega^2}{G}.$$

Thus, with $R$ = 20000 m and $\omega = 2\pi$ rad/s, we find $M = 4.7 \times 10^{24}$ kg $\approx 5 \times 10^{24}$ kg.

22. (a) Plugging $R_h = 2GM_h/c^2$ into the indicated expression, we find

$$a_g = \frac{GM_h}{\left(1.001R_h\right)^2} = \frac{GM_h}{\left(1.001\right)^2\left(2GM_h/c^2\right)^2} = \frac{c^4}{\left(2.002\right)^2 G}\frac{1}{M_h}$$

which yields $a_g = (3.02 \times 10^{43}\ \text{kg}\cdot\text{m/s}^2)/M_h$.

(b) Since $M_h$ is in the denominator of the above result, $a_g$ decreases as $M_h$ increases.

(c) With $M_h = (1.55 \times 10^{12})(1.99 \times 10^{30}$ kg), we obtain $a_g = 9.82$ m/s$^2$.

(d) This part refers specifically to the very large black hole treated in the previous part. With that mass for $M$ in Eq. 13-16, and $r = 2.002GM/c^2$, we obtain

$$da_g = -2\frac{GM}{\left(2.002GM/c^2\right)^3}\,dr = -\frac{2c^6}{\left(2.002\right)^3\left(GM\right)^2}\,dr$$

where $dr \to 1.70$ m as in Sample Problem 13.2 – "Difference in acceleration at head and feet." This yields (in absolute value) an acceleration difference of $7.30 \times 10^{-15}$ m/s$^2$.

(e) The miniscule result of the previous part implies that, in this case, any effects due to the differences of gravitational forces on the body are negligible.

23. (a) The gravitational acceleration is $a_g = \dfrac{GM}{R^2}$ = 7.6 m/s$^2$.

(b) Note that the total mass is 5$M$. Thus, $a_g = \dfrac{G(5M)}{(3R)^2}$ = 4.2 m/s$^2$.

24. (a) What contributes to the $GmM/r^2$ force on $m$ is the (spherically distributed) mass $M$ contained within $r$ (where $r$ is measured from the center of $M$). At point $A$ we see that $M_1 + M_2$ is at a smaller radius than $r = a$ and thus contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{G(M_1 + M_2)m}{a^2}.$$

(b) In the case $r = b$, only $M_1$ is contained within that radius, so the force on $m$ becomes $GM_1m/b^2$.

(c) If the particle is at $C$, then no other mass is at smaller radius and the gravitational force on it is zero.

25. Using the fact that the volume of a sphere is $4\pi R^3/3$, we find the density of the sphere:

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0\times10^4\,\text{kg}}{\frac{4}{3}\pi(1.0\text{ m})^3} = 2.4\times10^3\,\text{kg/m}^3.$$

When the particle of mass $m$ (upon which the sphere, or parts of it, are exerting a gravitational force) is at radius $r$ (measured from the center of the sphere), then whatever mass $M$ is at a radius less than $r$ must contribute to the magnitude of that force ($GMm/r^2$).

(a) At $r$ = 1.5 m, all of $M_{\text{total}}$ is at a smaller radius and thus all contributes to the force:

$$\left|F_{\text{on } m}\right| = \frac{GmM_{\text{total}}}{r^2} = m\left(3.0\times10^{-7}\,\text{N/kg}\right).$$

(b) At $r$ = 0.50 m, the portion of the sphere at radius smaller than that is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.3\times10^3\ \text{kg}.$$

Thus, the force on $m$ has magnitude $GMm/r^2 = m\,(3.3 \times 10^{-7}$ N/kg).

(c) Pursuing the calculation of part (b) algebraically, we find

$$\left|F_{\text{on } m}\right| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr\left(6.7\times10^{-7}\,\frac{\text{N}}{\text{kg}\cdot\text{m}}\right).$$

26. (a) Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{3M_{\text{total}}}{4\pi R^3}.$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius $r$ (measured from the center of the sphere), the mass $M$, which is at radius less than $r$, is what contributes to the reading ($GM/r^2$). Since $M = \rho(4\pi r^3/3)$ for $r \le R$, then we can write this result as

$$\frac{G\left(\frac{3M_{\text{total}}}{4\pi R^3}\right)\left(\frac{4\pi r^3}{3}\right)}{r^2} = \frac{GM_{\text{total}}r}{R^3}$$

when we are considering points on or inside the sphere. Thus, the value $a_g$ referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2},$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}r}{R^3} \;\Rightarrow\; r = \frac{R}{3}.$$

(b) Now we treat the case of an external test point. For points with $r > R$ the acceleration is $GM_{\text{total}}/r^2$, so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \;\Rightarrow\; r = \sqrt{3}R.$$

27. (a) The magnitude of the force on a particle with mass $m$ at the surface of Earth is given by $F = GMm/R^2$, where $M$ is the total mass of Earth and $R$ is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(6.37\times10^{6}\ \text{m}\right)^2} = 9.83\ \text{m/s}^2.$$

(b) Now $a_g = GM/R^2$, where $M$ is the total mass contained in the core and mantle together and $R$ is the outer radius of the mantle ($6.345 \times 10^6$ m, according to the figure). The total mass is

$$M = (1.93 \times 10^{24}\ \text{kg} + 4.01 \times 10^{24}\ \text{kg}) = 5.94 \times 10^{24}\ \text{kg}.$$

The first term is the mass of the core and the second is the mass of the mantle. Thus,

$$a_g = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.94\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.84\ \text{m/s}^2.$$

(c) A point 25 km below the surface is at the mantle–crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6$ m. Since the mass is now assumed to be uniformly distributed, the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3/R_e^3)M_e$, where $M_e$ is the total mass of Earth and $R_e$ is the radius of Earth. Thus,

$$M = \left(\frac{6.345\times10^{6}\ \text{m}}{6.37\times10^{6}\ \text{m}}\right)^3\left(5.98\times10^{24}\ \text{kg}\right) = 5.91\times10^{24}\ \text{kg}.$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.91\times10^{24}\ \text{kg}\right)}{\left(6.345\times10^{6}\ \text{m}\right)^2} = 9.79\ \text{m/s}^2.$$

28. (a) Using Eq. 13-1, we set $GmM/r^2$ equal to $\frac{1}{2}GmM/R^2$, and we find $r = R\sqrt{2}$. Thus, the distance from the surface is $(\sqrt{2} - 1)R = 0.414R$.

(b) Setting the density $\rho$ equal to $M/V$ where $V = \frac{4}{3}\pi R^3$, we use Eq. 13-19:

$$F = \frac{4\pi Gmr\rho}{3} = \frac{4\pi Gmr}{3}\left(\frac{M}{4\pi R^3/3}\right) = \frac{GMmr}{R^3} = \frac{1}{2}\frac{GMm}{R^2} \quad\Rightarrow\quad r = R/2.$$

29. The equation immediately preceding Eq. 13-28 shows that $K = -U$ (with $U$ evaluated at the planet's surface: $-5.0 \times 10^9$ J) is required to "escape." Thus, $K = 5.0 \times 10^9$ J.

30. The gravitational potential energy is

$$U = -\frac{Gm(M-m)}{r} = -\frac{G}{r}\left(Mm - m^2\right)$$

which we differentiate with respect to $m$ and set equal to zero (in order to minimize). Thus, we find $M - 2m = 0$, which leads to the ratio $m/M = 1/2$ to obtain the least potential energy.

Note that a second derivative of $U$ with respect to $m$ would lead to a positive result regardless of the value of $m$, which means its graph is everywhere concave upward and thus its extremum is indeed a minimum.

31. **THINK** Given the mass and diameter of Mars, we can calculate its mean density, gravitational acceleration and escape speed.

**EXPRESS** The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where $M$ is its mass and $R$ is its radius. On the other hand, the value of gravitational acceleration $a_g$ at the surface of a planet is given by $a_g = GM/R^2$. for a particle of mass $m$, its escape speed is given by

$$\frac{1}{2}mv^2 = G\frac{mM}{R} \quad \Rightarrow \quad v = \sqrt{\frac{2GM}{R}}.$$

**ANALYZE** (a) From the definition of density above, we find the ratio of the density of Mars to the density of Earth to be

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^3 = 0.74.$$

(b) The value of gravitational acceleration for Mars is

$$a_{gM} = \frac{GM_M}{R_M^2} = \frac{M_M}{R_M^2}\cdot\frac{R_E^2}{M_E}\cdot\frac{GM_E}{R_E^2} = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2}a_{gE} = 0.11\left(\frac{0.65\times10^4\ \text{km}}{3.45\times10^3\ \text{km}}\right)^2\left(9.8\ \text{m/s}^2\right) = 3.8\ \text{m/s}^2.$$

(c) For Mars, the escape speed is

$$v_M = \sqrt{\frac{2GM_M}{R_M}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.11)\left(5.98\times10^{24}\ \text{kg}\right)}{3.45\times10^6\ \text{m}}} = 5.0\times10^3\ \text{m/s}.$$

**LEARN** The ratio of the escape speeds on Mars and on Earth is

$$\frac{v_M}{v_E} = \frac{\sqrt{2GM_M/R_M}}{\sqrt{2GM_E/R_E}} = \sqrt{\frac{M_M}{M_E}\cdot\frac{R_E}{R_M}} = \sqrt{(0.11)\cdot\frac{6.5\times10^3\ \text{km}}{3.45\times10^3\ \text{km}}} = 0.455.$$

32. (a) The gravitational potential energy is

$$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(5.2\ \text{kg}\right)\left(2.4\ \text{kg}\right)}{19\ \text{m}} = -\ 4.4\times10^{-11}\ \text{J}.$$

(b) Since the change in potential energy is

$$\Delta U = -\frac{GMm}{3r} - \left(-\frac{GMm}{r}\right) = -\frac{2}{3}\left(-4.4\times10^{-11}\ \text{J}\right) = 2.9\times10^{-11}\ \text{J},$$

the work done by the gravitational force is $W = -\ \Delta U = -2.9 \times 10^{-11}$ J.

(c) The work done by you is $W' = \Delta U = 2.9 \times 10^{-11}$ J.

33. The amount of (kinetic) energy needed to escape is the same as the (absolute value of the) gravitational potential energy at its original position. Thus, an object of mass $m$ on a planet of mass $M$ and radius $R$ needs $K = GmM/R$ in order to (barely) escape.
(a) Setting up the ratio, we find

$$\frac{K_m}{K_E} = \frac{M_m}{M_E}\frac{R_E}{R_m} = 0.0451$$

using the values found in Appendix C.

(b) Similarly, for the Jupiter escape energy (divided by that for Earth) we obtain

$$\frac{K_J}{K_E} = \frac{M_J}{M_E}\frac{R_E}{R_J} = 28.5.$$

34. (a) The potential energy $U$ at the surface is $U_s = -5.0 \times 10^9$ J according to the graph, since $U$ is inversely proportional to $r$ (see Eq. 13-21), at an $r$-value a factor of 5/4 times what it was at the surface then $U$ must be 4 $U_s/5$. Thus, at $r = 1.25R_s$, $U = -\ 4.0 \times 10^9$ J. Since mechanical energy is assumed to be conserved in this problem, we have

$$K + U = -2.0 \times 10^9\ \text{J}$$

at this point. Since $U = -\ 4.0 \times 10^9$ J here, then $K = 2.0\times10^9$ J at this point.

(b) To reach the point where the mechanical energy equals the potential energy (that is, where $U = -\ 2.0 \times 10^9$ J) means that $U$ must reduce (from its value at $r = 1.25R_s$) by a factor of 2, which means the $r$ value must increase (relative to $r = 1.25R_s$) by a corresponding factor of 2. Thus, the turning point must be at $r = 2.5R_s$.

35. Let $m$ = 0.020 kg and $d$ = 0.600 m (the original edge-length, in terms of which the final edge-length is $d$/3). The total initial gravitational potential energy (using Eq. 13-21 and some elementary trigonometry) is

$$U_i = -\frac{4Gm^2}{d} - \frac{2Gm^2}{\sqrt{2}\,d} .$$

Since $U$ is inversely proportional to $r$ then reducing the size by 1/3 means increasing the magnitude of the potential energy by a factor of 3, so

$$U_f = 3U_i \;\Rightarrow\; \Delta U = 2U_i = 2(4 + \sqrt{2}\,)\left(-\frac{Gm^2}{d}\right) = -4.82 \times 10^{-13}\text{ J} .$$

36. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad\Rightarrow\quad K_1 - \frac{GmM}{r_1} = K_2 - \frac{GmM}{r_2}$$

where $M = 5.0 \times 10^{23}$ kg, $r_1 = R = 3.0 \times 10^6$ m and $m$ = 10 kg.

(a) If $K_1 = 5.0 \times 10^7$ J and $r_2 = 4.0 \times 10^6$ m, then the above equation leads to

$$K_2 = K_1 + GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 2.2 \times 10^7\text{ J.}$$

(b) In this case, we require $K_2 = 0$ and $r_2 = 8.0 \times 10^6$ m, and solve for $K_1$:

$$K_1 = K_2 + GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 6.9 \times 10^7\text{ J.}$$

37. (a) The work done by you in moving the sphere of mass $m_B$ equals the change in the potential energy of the three-sphere system. The initial potential energy is

$$U_i = -\frac{Gm_Am_B}{d} - \frac{Gm_Am_C}{L} - \frac{Gm_Bm_C}{L-d}$$

and the final potential energy is

$$U_f = -\frac{Gm_Am_B}{L-d} - \frac{Gm_Am_C}{L} - \frac{Gm_Bm_C}{d} .$$

The work done is

$$W = U_f - U_i = Gm_B\left[m_A\left(\frac{1}{d} - \frac{1}{L-d}\right) + m_C\left(\frac{1}{L-d} - \frac{1}{d}\right)\right]$$
$$= Gm_B\left[m_A\frac{L-2d}{d(L-d)} + m_C\frac{2d-L}{d(L-d)}\right] = Gm_B(m_A - m_C)\frac{L-2d}{d(L-d)}$$
$$= (6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(0.010\ \text{kg})(0.080\ \text{kg} - 0.020\ \text{kg})\frac{0.12\ \text{m} - 2(0.040\ \text{m})}{(0.040\ \text{m})(0.12 - 0.040\ \text{m})}$$
$$= +5.0\times10^{-13}\ \text{J}.$$

(b) The work done by the force of gravity is $-(U_f - U_i) = -5.0\times10^{-13}$ J.

38. (a) The initial gravitational potential energy is

$$U_i = -\frac{GM_AM_B}{r_i} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.80\ \text{m}}$$
$$= -1.67\times10^{-8}\ \text{J} \approx -1.7\times10^{-8}\ \text{J}.$$

(b) We use conservation of energy (with $K_i = 0$):

$$U_i = K + U \quad\Rightarrow\quad -1.7\times10^{-8} = K - \frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(20\ \text{kg})(10\ \text{kg})}{0.60\ \text{m}}$$

which yields $K = 5.6\times10^{-9}$ J. Note that the value of $r$ is the difference between 0.80 m and 0.20 m.

39. **THINK** The escape speed on the asteroid is related to the gravitational acceleration at the surface of the asteroid and its size.

**EXPRESS** We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where $M$ is the mass of the asteroid, $R$ is its radius, and $m$ is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields

$$-GMm/R + \tfrac{1}{2}mv^2 = 0.$$

We replace $GM/R$ with $a_gR$, where $a_g$ is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. Solving for $v$, we have

$$v = \sqrt{2a_gR}\ .$$

**ANALYZE** (a) Given that $R = 500\,\text{km}$ and $a_g = 3.0\,\text{m/s}^2$, we find the escape speed to be

$$v = \sqrt{2a_g R} = \sqrt{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})} = 1.7\times10^3\ \text{m/s}.$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance $h$ above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R + h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}.$$

We replace $GM$ with $a_gR^2$ and cancel $m$ in the energy equation to obtain

$$-a_g R + \frac{1}{2}v^2 = -\frac{a_g R^2}{(R+h)}.$$

The solution for $h$ is

$$h = \frac{2a_g R^2}{2a_g R - v^2} - R = \frac{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})^2}{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m}) - (1000\ \text{m/s})^2} - (500\times10^3\ \text{m})$$
$$= 2.5\times10^5\ \text{m}.$$

(c) Initially the particle is a distance $h$ above the surface and is at rest. Its potential energy is $U_i = -GMm/(R + h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2.$$

We substitute $a_gR^2$ for $GM$ and cancel $m$, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2.$$

The solution for $v$ is

$$v = \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} = \sqrt{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m}) - \frac{2(3.0\ \text{m/s}^2)(500\times10^3\ \text{m})^2}{(500\times10^3\ \text{m}) + (1000\times10^3\ \text{m})}}$$
$$= 1.4\times10^3\ \text{m/s}.$$

**LEARN** The key idea in this problem is to realize that energy is conserved in the process:

$$K_i + U_i = K_f + U_f \quad \Rightarrow \quad \Delta K + \Delta U = 0.$$

The decrease in potential energy is equal to the gain in kinetic energy, and vice versa.

40. (a) From Eq. 13-28, we see that $v_0 = \sqrt{GM/2R_E}$ in this problem. Using energy conservation, we have

$$\frac{1}{2} m v_0^2 - GMm/R_E = - GMm/r$$

which yields $r = 4R_E/3$. So the multiple of $R_E$ is 4/3 or 1.33.

(b) Using the equation in the textbook immediately preceding Eq. 13-28, we see that in this problem we have $K_i = GMm/2R_E$, and the above manipulation (using energy conservation) in this case leads to $r = 2R_E$. So the multiple of $R_E$ is 2.00.

(c) Again referring to the equation in the textbook immediately preceding Eq. 13-28, we see that the mechanical energy = 0 for the "escape condition."

41. **THINK** The two neutron stars are attracted toward each other due to their gravitational interaction.

**EXPRESS** The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where $M$ is the mass of either star and $r_i$ is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -GM^2/r_f$, where the final separation is $r_f = r_i/2$. We write $Mv^2$ for the final kinetic energy of the system. This is the sum of two terms, each of which is ½$Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2.$$

**ANALYZE** (a) The solution for $v$ is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{m}^3/\text{s}^2 \cdot \text{kg})(10^{30}\ \text{kg})}{10^{10}\ \text{m}}} = 8.2 \times 10^4\ \text{m/s}.$$

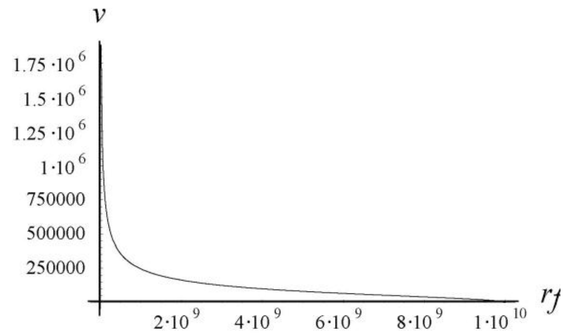
(b) Now the final separation of the centers is $r_f = 2R = 2 \times 10^5$ m, where $R$ is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes

$$-GM^2/r_i = -GM^2/r_f + Mv^2.$$

The solution for $v$ is

$$v = \sqrt{GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} = \sqrt{(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(10^{30}\,\text{kg})\left(\frac{1}{2\times10^5\ \text{m}} - \frac{1}{10^{10}\ \text{m}}\right)}$$
$$= 1.8\times10^7\ \text{m/s}.$$

**LEARN** The speed of the stars as a function of their final separation is plotted below. The decrease in gravitational potential energy is accompanied by an increase in kinetic energy, so that the total energy of the two-star system remains conserved.




42. (a) Applying Eq. 13-21 and the Pythagorean theorem leads to

$$U = -\left(\frac{GM^2}{2D} + \frac{2GmM}{\sqrt{y^2 + D^2}}\right)$$

where $M$ is the mass of particle $B$ (also that of particle $C$) and $m$ is the mass of particle $A$. The value given in the problem statement (for infinitely large $y$, for which the second term above vanishes) determines $M$, since $D$ is given. Thus $M = 0.50$ kg.

(b) We estimate (from the graph) the $y = 0$ value to be $U_o = -3.5 \times 10^{-10}$ J. Using this, our expression above determines $m$. We obtain $m = 1.5$ kg.

43. (a) If $r$ is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by $GMm/r^2$, where $M$ is the mass of Earth and $m$ is the mass of the satellite. The magnitude of the acceleration of the satellite is given by $v^2/r$, where $v$ is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is $6.37 \times 10^6$ m, the orbit radius is $r = (6.37 \times 10^6\ \text{m} + 160 \times 10^3\ \text{m}) = 6.53 \times 10^6$ m. The solution for $v$ is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})(5.98\times10^{24}\,\text{kg})}{6.53\times10^6\ \text{m}}} = 7.82\times10^3\ \text{m/s}.$$

(b) Since the circumference of the circular orbit is $2\pi r$, the period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.53\times10^6\,\text{m})}{7.82\times10^3\ \text{m/s}} = 5.25\times10^3\,\text{s}.$$

This is equivalent to 87.5 min.

44. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{r_s}{r_m}\right)^3 = \left(\frac{T_s}{T_m}\right)^2 \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{T_s}{1 \text{ lunar month}}\right)^2$$

which yields $T_s$ = 0.35 lunar month for the period of the satellite.

45. The period $T$ and orbit radius $r$ are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where $M$ is the mass of Mars. The period is 7 h 39 min, which is $2.754 \times 10^4$ s. We solve for $M$:

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (9.4 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})\left(2.754 \times 10^4 \text{ s}\right)^2} = 6.5 \times 10^{23} \text{ kg}.$$

46. From Eq. 13-37, we obtain $v = \sqrt{GM/r}$ for the speed of an object in circular orbit (of radius $r$) around a planet of mass $M$. In this case, $M = 5.98 \times 10^{24}$ kg and

$$r = (700 + 6370)\text{m} = 7070 \text{ km} = 7.07 \times 10^6 \text{ m}.$$

The speed is found to be $v = 7.51 \times 10^3$ m/s. After multiplying by 3600 s/h and dividing by 1000 m/km this becomes $v = 2.7 \times 10^4$ km/h.

(a) For a head-on collision, the relative speed of the two objects must be $2v = 5.4 \times 10^4$ km/h.

(b) A perpendicular collision is possible if one satellite is, say, orbiting above the equator and the other is following a longitudinal line. In this case, the relative speed is given by the Pythagorean theorem: $\sqrt{v^2 + v^2} = 3.8 \times 10^4$ km/h.

47. **THINK** The centripetal force on the Sun is due to the gravitational attraction between the Sun and the stars at the center of the Galaxy.

**EXPRESS** Let $N$ be the number of stars in the galaxy, $M$ be the mass of the Sun, and $r$ be the radius of the galaxy. The total mass in the galaxy is $N\,M$ and the magnitude of the gravitational force acting on the Sun is

$$F_g = \frac{GM(NM)}{R^2} = \frac{GNM^2}{R^2}.$$

The force, pointing toward the galactic center, is the centripetal force on the Sun. Thus,

$$F_c = F_g \quad \Rightarrow \quad \frac{Mv^2}{R} = \frac{GNM^2}{R^2}.$$

The magnitude of the Sun's acceleration is $a = v^2/R$, where $v$ is its speed. If $T$ is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields

$$GNM^2/R^2 = 4\pi^2 MR/T^2.$$

The solution for $N$ is

$$N = \frac{4\pi^2 R^3}{GT^2 M}.$$

**ANALYZE** The period is $2.5 \times 10^8$ y, which is $7.88 \times 10^{15}$ s, so

$$N = \frac{4\pi^2 (2.2 \times 10^{20}\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{m}^3/\text{s}^2 \cdot \text{kg})(7.88 \times 10^{15}\ \text{s})^2 (2.0 \times 10^{30}\ \text{kg})} = 5.1 \times 10^{10}.$$

**LEARN** The number of stars in the Milky Way is between $10^{11}$ to $4\times 10^{11}$. Our simplified model provides a reasonable estimate.

48. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{a_M}{a_E}\right)^3 = \left(\frac{T_M}{T_E}\right)^2 \Rightarrow (1.52)^3 = \left(\frac{T_M}{1\,\text{y}}\right)^2$$

where we have substituted the mean-distance (from Sun) ratio for the semi-major axis ratio. This yields $T_M$ = 1.87 y. The value in Appendix C (1.88 y) is quite close, and the small apparent discrepancy is not significant, since a more precise value for the semi-major axis ratio is $a_M/a_E$ = 1.523, which does lead to $T_M$ = 1.88 y using Kepler's law. A question can be raised regarding the use of a ratio of mean distances for the ratio of semi-major axes, but this requires a more lengthy discussion of what is meant by a "mean distance" than is appropriate here.

49. (a) The period of the comet is 1420 years (and one month), which we convert to $T = 4.48 \times 10^{10}$ s. Since the mass of the Sun is $1.99 \times 10^{30}$ kg, then Kepler's law of periods gives

$$(4.48 \times 10^{10}\ \text{s})^2 = \left(\frac{4\pi^2}{(6.67\times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99 \times 10^{30}\ \text{kg})}\right) a^3 \Rightarrow a = 1.89 \times 10^{13}\ \text{m}.$$

(b) Since the distance from the focus (of an ellipse) to its center is $ea$ and the distance from center to the aphelion is $a$, then the comet is at a distance of

$$ea + a = (0.9932 + 1)\,(1.89\times 10^{13}\ \text{m}) = 3.767\times 10^{13}\ \text{m}$$

when it is farthest from the Sun. To express this in terms of Pluto's orbital radius (found in Appendix C), we set up a ratio:

$$\left(\frac{3.767\times10^{13}}{5.9\times10^{12}}\right)R_P \approx 6,4R_P.$$

50. To "hover" above Earth ($M_E$ = 5.98 × $10^{24}$ kg) means that it has a period of 24 hours (86400 s). By Kepler's law of periods,

$$(86400)^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 \Rightarrow r = 4.225\times10^7 \text{ m}.$$

Its altitude is therefore $r - R_E$ (where $R_E$ = 6.37 × $10^6$ m), which yields 3.58 × $10^7$ m.

51. **THINK** The satellite moves in an elliptical orbit about Earth. An elliptical orbit can be characterized by its semi-major axis and eccentricity.

**EXPRESS** The greatest distance between the satellite and Earth's center (the apogee distance) and the least distance (perigee distance) are, respectively,

$$R_a = R_E + d_a = 6.37\times10^6 \text{ m} + 360\times10^3 \text{ m} = 6.73\times10^6 \text{ m}$$
$$R_p = R_E + d_p = 6.37\times10^6 \text{ m} + 180\times10^3 \text{ m} = 6.55\times10^6 \text{ m}.$$

Here $R_E$ = 6.37 × $10^6$ m is the radius of Earth.

**ANALYZE** The semi-major axis is given by

$$a = \frac{R_a + R_p}{2} = \frac{6.73\times10^6 \text{ m} + 6.55\times10^6 \text{ m}}{2} = 6.64\times10^6 \text{ m}.$$

(b) The apogee and perigee distances are related to the eccentricity $e$ by $R_a = a(1 + e)$ and $R_p = a(1 - e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73\times10^6 \text{ m} - 6.55\times10^6 \text{ m}}{6.73\times10^6 \text{ m} + 6.55\times10^6 \text{ m}} = 0.0136.$$

**LEARN** Since $e$ is very small, the orbit is nearly circular. On the other hand, if $e$ is close to unity, then the orbit would be a long, thin ellipse.

52. (a) The distance from the center of an ellipse to a focus is $ae$ where $a$ is the semi-major axis and $e$ is the eccentricity. Thus, the separation of the foci (in the case of Earth's orbit) is

$$2ae = 2\left(1.50\times10^{11} \text{ m}\right)\left(0.0167\right) = 5.01\times10^9 \text{ m}.$$

(b) To express this in terms of solar radii (see Appendix C), we set up a ratio:

$$\frac{5.01\times10^9\ \text{m}}{6.96\times10^8\ \text{m}} = 7.20.$$

53. From Kepler's law of periods (where $T$ = (2.4 h)(3600 s/h) = 8640 s), we find the planet's mass $M$:

$$(8640\,\text{s})^2 = \left(\frac{4\pi^2}{GM}\right)(8.0\times10^6\ \text{m})^3 \Rightarrow M = 4.06\times10^{24}\ \text{kg}.$$

However, we also know $a_g = GM/R^2 = 8.0\ \text{m/s}^2$ so that we are able to solve for the planet's radius:

$$R = \sqrt{\frac{GM}{a_g}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(4.06\times10^{24}\ \text{kg})}{8.0\ \text{m/s}^2}} = 5.8\times10^6\ \text{m}.$$

54. The two stars are in circular orbits, not about each other, but about the two-star system's center of mass (denoted as $O$), which lies along the line connecting the centers of the two stars. The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, for the visible, Newton's second law gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{m_1v^2}{r_1}$$

where $r$ is the distance between the centers of the stars. To find the relation between $r$ and $r_1$, we locate the center of mass relative to $m_1$. Using Equation 9-1, we obtain

$$r_1 = \frac{m_1(0)+m_2r}{m_1+m_2} = \frac{m_2r}{m_1+m_2} \quad \Rightarrow \quad r = \frac{m_1+m_2}{m_2}r_1.$$

On the other hand, since the orbital speed of $m_1$ is $v = 2\pi r_1/T$, then $r_1 = vT/2\pi$ and the expression for $r$ can be rewritten as

$$r = \frac{m_1+m_2}{m_2}\frac{vT}{2\pi}.$$

Substituting $r$ and $r_1$ into the force equation, we obtain

$$F = \frac{4\pi^2Gm_1m_2^3}{(m_1+m_2)^2v^2T^2} = \frac{2\pi m_1v}{T}$$

or

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{v^3T}{2\pi G} = \frac{(2.7\times10^5\ \text{m/s})^3(1.70\ \text{days})(86400\ \text{s/day})}{2\pi(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)} = 6.90\times10^{30}\ \text{kg}$$
$$= 3.467M_s,$$

where $M_s = 1.99\times10^{30}$ kg is the mass of the sun. With $m_1 = 6M_s$, we write $m_2 = \alpha M_s$ and solve the following cubic equation for $\alpha$:

$$\frac{\alpha^3}{(6+\alpha)^2} - 3.467 = 0 .$$

The equation has one real solution: $\alpha = 9.3$, which implies $m_2 / M_s \approx 9$.

55. (a) If we take the logarithm of Kepler's law of periods, we obtain

$$2 \log(T) = \log (4\pi^2/GM) + 3 \log (a) \Rightarrow \log (a) = \frac{2}{3} \log (T) - \frac{1}{3} \log (4\pi^2/GM)$$

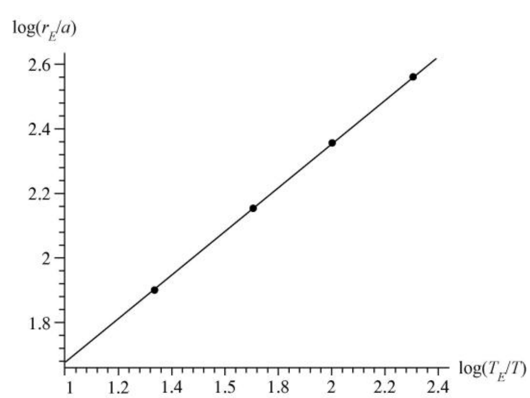
where we are ignoring an important subtlety about units (the arguments of logarithms cannot have units, since they are transcendental functions). Although the problem can be continued in this way, we prefer to set it up without units, which requires taking a ratio. If we divide Kepler's law (applied to the Jupiter–moon system, where $M$ is mass of Jupiter) by the law applied to Earth orbiting the Sun (of mass $M_o$), we obtain

$$(T/T_E)^2 = \left(\frac{M_o}{M}\right)\left(\frac{a}{r_E}\right)^3$$

where $T_E$ = 365.25 days is Earth's orbital period and $r_E = 1.50 \times 10^{11}$ m is its mean distance from the Sun. In this case, it is perfectly legitimate to take logarithms and obtain

$$\log\left(\frac{r_E}{a}\right) = \frac{2}{3}\log\left(\frac{T_E}{T}\right) + \frac{1}{3}\log\left(\frac{M_o}{M}\right)$$

(written to make each term positive), which is the way we plot the data (log ($r_E/a$) on the vertical axis and log ($T_E/T$) on the horizontal axis).




(b) When we perform a least-squares fit to the data, we obtain

$$\log(r_E/a) = 0.666 \log(T_E/T) + 1.01,$$

which confirms the expectation of slope = 2/3 based on the above equation.

(c) And the 1.01 intercept corresponds to the term 1/3 log ($M_o/M$), which implies

$$\frac{M_o}{M} = 10^{3.03} \Rightarrow M = \frac{M_o}{1.07 \times 10^3}.$$

Plugging in $M_o = 1.99 \times 10^{30}$ kg (see Appendix C), we obtain $M = 1.86 \times 10^{27}$ kg for Jupiter's mass. This is reasonably consistent with the value $1.90 \times 10^{27}$ kg found in Appendix C.

56. (a) The period is $T = 27(3600) = 97200$ s, and we are asked to assume that the orbit is circular (of radius $r = 100000$ m). Kepler's law of periods provides us with an approximation to the asteroid's mass:

$$(97200)^2 = \left(\frac{4\pi^2}{GM}\right)(100000)^3 \Rightarrow M = 6.3 \times 10^{16} \text{ kg}.$$

(b) Dividing the mass $M$ by the given volume yields an average density equal to

$$\rho = (6.3 \times 10^{16} \text{ kg})/(1.41 \times 10^{13} \text{ m}^3) = 4.4 \times 10^3 \text{ kg/m}^3,$$

which is about 20% less dense than Earth.

57. In our system, we have $m_1 = m_2 = M$ (the mass of our Sun, $1.99 \times 10^{30}$ kg). With $r = 2r_1$ in this system (so $r_1$ is one-half the Earth-to-Sun distance $r$), and $v = \pi r/T$ for the speed, we have

$$\frac{Gm_1m_2}{r^2} = m_1\frac{(\pi r/T)^2}{r/2} \Rightarrow T = \sqrt{\frac{2\pi^2r^3}{GM}}.$$

With $r = 1.5 \times 10^{11}$ m, we obtain $T = 2.2 \times 10^7$ s. We can express this in terms of Earth-years, by setting up a ratio:

$$T = \left(\frac{T}{1\text{y}}\right)(1\text{y}) = \left(\frac{2.2 \times 10^7 \text{ s}}{3.156 \times 10^7 \text{ s}}\right)(1 \text{ y}) = 0.71 \text{ y}.$$

58. (a) We make use of

$$\frac{m_2^3}{(m_1 + m_2)^2} = \frac{v^3T}{2\pi G}$$

where $m_1 = 0.9M_{\text{Sun}}$ is the estimated mass of the star. With $v = 70$ m/s and $T = 1500$ days (or $1500 \times 86400 = 1.3 \times 10^8$ s), we find

$$\frac{m_2^3}{(0.9M_{\text{Sun}}+m_2)^2} = 1.06\times 10^{23}\text{ kg} .$$

Since $M_{\text{Sun}} \approx 2.0 \times 10^{30}$ kg, we find $m_2 \approx 7.0 \times 10^{27}$ kg. Dividing by the mass of Jupiter (see Appendix C), we obtain $m \approx 3.7m_J$.

(b) Since $v = 2\pi r_1/T$ is the speed of the star, we find

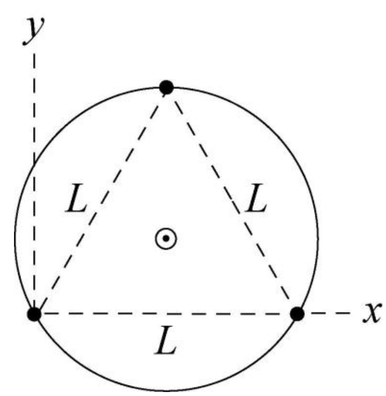
$$r_1 = \frac{vT}{2\pi} = \frac{(70\text{ m/s})(1.3\times 10^8\text{ s})}{2\pi} = 1.4\times 10^9\text{ m}$$

for the star's orbital radius. If $r$ is the distance between the star and the planet, then $r_2 = r - r_1$ is the orbital radius of the planet, and is given by

$$r_2 = r_1\left(\frac{m_1+m_2}{m_2}-1\right) = r_1\frac{m_1}{m_2} = 3.7\times 10^{11}\text{ m} .$$

Dividing this by $1.5 \times 10^{11}$ m (Earth's orbital radius, $r_E$) gives $r_2 = 2.5r_E$.

59. Each star is attracted toward each of the other two by a force of magnitude $GM^2/L^2$, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If $R$ is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.




The stars rotate about their center of mass (marked by a circled dot on the diagram above) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $\left(\sqrt{3}/2\right)L$, so the stars are located at $x = 0$, $y = 0$; $x = L$, $y = 0$; and $x = L/2$, $y = \sqrt{3}L/2$. The $x$ coordinate of the center of mass is $x_c = (L +$

$L$/2)/3 = $L$/2 and the $y$ coordinate is $y_c = \left(\sqrt{3}L/2\right)/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is

$$R = \sqrt{x_c^2 + y_c^2} = \sqrt{\left(L^2/4\right) + \left(L^2/12\right)} = L/\sqrt{3}.$$

Once the substitution for $R$ is made, Newton's second law then becomes $\left(2GM^2/L^2\right)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified further by recognizing that $\cos 30° = \sqrt{3}/2$. Divide the equation by $M$ then gives $GM/L^2 = v^2/L$, or $v = \sqrt{GM/L}$.

60. (a) From Eq. 13-40, we see that the energy of each satellite is $-GM_E m/2r$. The total energy of the two satellites is twice that result:

$$E = E_A + E_B = -\frac{GM_E m}{r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})(125\ \text{kg})}{7.87\times10^6\ \text{m}}$$
$$= -6.33\times10^9\,\text{J}.$$

(b) We note that the speed of the wreckage will be zero (immediately after the collision), so it has no kinetic energy at that moment. Replacing $m$ with $2m$ in the potential energy expression, we therefore find the total energy of the wreckage at that instant is

$$E = -\frac{GM_E(2m)}{2r} = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\text{kg})2(125\ \text{kg})}{2(7.87\times10^6\ \text{m})} = -6.33\times10^9\,\text{J}.$$

(c) An object with zero speed at that distance from Earth will simply fall toward the Earth, its trajectory being toward the center of the planet.

61. The energy required to raise a satellite of mass $m$ to an altitude $h$ (at rest) is given by

$$E_1 = \Delta U = GM_E m\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right),$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2}mv_{\text{orb}}^2 = \frac{GM_E m}{2\left(R_E + h\right)}.$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_E m\left[\frac{1}{R_E} - \frac{3}{2(R_E + h)}\right].$$

(a) Solving the above equation, the height $h_0$ at which $\Delta E = 0$ is given by

$$\frac{1}{R_E} - \frac{3}{2(R_E + h_0)} = 0 \quad \Rightarrow h_0 = \frac{R_E}{2} = 3.19\times10^6 \text{ m}.$$

(b) For greater height $h > h_0$, $\Delta E > 0$, implying $E_1 > E_2$. Thus, the energy of lifting is greater.

62. Although altitudes are given, it is the orbital radii that enter the equations. Thus, $r_A$ = (6370 + 6370) km = 12740 km, and $r_B$ = (19110 + 6370) km = 25480 km.

(a) The ratio of potential energies is

$$\frac{U_B}{U_A} = \frac{-GmM / r_B}{-GmM / r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(b) Using Eq. 13-38, the ratio of kinetic energies is

$$\frac{K_B}{K_A} = \frac{GmM / 2r_B}{GmM / 2r_A} = \frac{r_A}{r_B} = \frac{1}{2}.$$

(c) From Eq. 13-40, it is clear that the satellite with the largest value of *r* has the smallest value of $|E|$ (since *r* is in the denominator). And since the values of *E* are negative, then the smallest value of $|E|$ corresponds to the largest energy *E*. Thus, satellite *B* has the largest energy.

(d) The difference is

$$\Delta E = E_B - E_A = -\frac{GmM}{2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Being careful to convert the *r* values to meters, we obtain $\Delta E = 1.1 \times 10^8$ J. The mass *M* of Earth is found in Appendix C.

63. **THINK** We apply Kepler's laws to analyze the motion of the asteroid.

**EXPRESS** We use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where *M* is the mass of the Sun ($1.99 \times 10^{30}$ kg) and *r* is the radius of the orbit. On the other hand, the kinetic energy of any asteroid or planet in a circular orbit of radius *r* is given by $K = GMm/2r$, where *m* is the mass of the asteroid or planet. We note that it is proportional to *m* and inversely proportional to *r*.

**ANALYZE** (a) The radius of the orbit is twice the radius of Earth's orbit: $r = 2r_{SE} = 2(150 \times 10^9 \text{ m}) = 300 \times 10^9$ m. Thus, the period of the asteroid is

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (300\times10^9 \text{ m})^3}{(6.67\times10^{-11}\text{m}^3/\text{s}^2\cdot\text{kg})(1.99\times10^{30}\text{kg})}} = 8.96\times10^7 \text{ s.}$$

Dividing by (365 d/y) (24 h/d) (60 min/h) (60 s/min), we obtain $T$ = 2.8 y.

(b) The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is

$$\frac{K}{K_E} = \frac{GMm/(2r)}{GMM_E/(2r_{SE})} = \frac{m}{M_E}\cdot\frac{r_{SE}}{r} = (2.0\times10^{-4})\left(\frac{1}{2}\right) = 1.0\times10^{-4}.$$

**LEARN** An alternative way to calculate the ratio of kinetic energies is to use $K = mv^2/2$ and note that $v = 2\pi r/T$. This gives

$$\frac{K}{K_E} = \frac{mv^2/2}{M_E v_E^2/2} = \frac{m}{M_E}\left(\frac{v}{v_E}\right)^2 = \frac{m}{M_E}\left(\frac{r/T}{r_{SE}/T_E}\right)^2 = \frac{m}{M_E}\left(\frac{r}{r_{SE}}\cdot\frac{T_E}{T}\right)^2$$
$$= (2.0\times10^{-4})\left(2\cdot\frac{1.0\text{ y}}{2.8\text{ y}}\right)^2 = 1.0\times10^{-4}$$

in agreement with what we found in (b).

64. (a) Circular motion requires that the force in Newton's second law provide the necessary centripetal acceleration:

$$\frac{GmM}{r^2} = m\frac{v^2}{r}.$$

Since the left-hand side of this equation is the force given as 80 N, then we can solve for the combination $mv^2$ by multiplying both sides by $r = 2.0\times10^7$ m. Thus, $mv^2 = (2.0\times10^7$ m) (80 N) $= 1.6\times10^9$ J. Therefore,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.6\times10^9 \text{ J}\right) = 8.0\times10^8 \text{ J}.$$

(b) Since the gravitational force is inversely proportional to the square of the radius, then

$$\frac{F'}{F} = \left(\frac{r}{r'}\right)^2.$$

Thus, $F'$ = (80 N) $(2/3)^2$ = 36 N.

65. (a) From Kepler's law of periods, we see that $T$ is proportional to $r^{3/2}$.

(b) Equation 13-38 shows that $K$ is inversely proportional to $r$.

(c) and (d) From the previous part, knowing that $K$ is proportional to $v^2$, we find that $v$ is proportional to $1/\sqrt{r}$. Thus, by Eq. 13-31, the angular momentum (which depends on the product $rv$) is proportional to $r/\sqrt{r} = \sqrt{r}$.

66. (a) The pellets will have the same speed $v$ but opposite direction of motion, so the *relative speed* between the pellets and satellite is $2v$. Replacing $v$ with $2v$ in Eq. 13-38 is equivalent to multiplying it by a factor of 4. Thus,

$$K_{\text{rel}} = 4\left(\frac{GM_E m}{2r}\right) = \frac{2(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)\left(5.98\times10^{24}\ \text{kg}\right)\left(0.0040\ \text{kg}\right)}{(6370+500)\times10^3\ \text{m}}$$
$$= 4.6\times10^5\ \text{J}.$$

(b) We set up the ratio of kinetic energies:

$$\frac{K_{\text{rel}}}{K_{\text{bullet}}} = \frac{4.6\times10^5\ \text{J}}{\frac{1}{2}\left(0.0040\,\text{kg}\right)\left(950\,\text{m/s}\right)^2} = 2.6\times10^2.$$

67. (a) The force acting on the satellite has magnitude $GMm/r^2$, where $M$ is the mass of Earth, $m$ is the mass of the satellite, and $r$ is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is $v^2/r$, where $v$ is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite:

$$r = (6.37\times10^6 + 640\times10^3)\ \text{m} = 7.01\times10^6\ \text{m}.$$

Thus,

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)}{7.01\times10^6\ \text{m}}} = 7.54\times10^3\ \text{m/s}.$$

(b) The period is

$$T = 2\pi r/v = 2\pi(7.01\times10^6\ \text{m})/(7.54\times10^3\ \text{m/s}) = 5.84\times10^3\ \text{s} \approx 97\ \text{min}.$$

(c) If $E_0$ is the initial energy then the energy after $n$ orbits is $E = E_0 - nC$, where $C = 1.4\times10^5$ J/orbit. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after $n$ orbits is given by $r = -GMm/2E$.
The initial energy is

$$E_0 = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)\left(220\ \text{kg}\right)}{2(7.01\times10^6\ \text{m})} = -6.26\times10^9\ \text{J},$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26\times10^9\ \text{J} - \left(1500\ \text{orbit}\right)\left(1.4\times10^5\ \text{J/orbit}\right) = -6.47\times10^9\ \text{J},$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)\left(220\ \text{kg}\right)}{2(-6.47\times10^9\ \text{J})} = 6.78\times10^6\ \text{m}.$$

The altitude is

$$h = r - R = (6.78\times10^6\ \text{m} - 6.37\times10^6\ \text{m}) = 4.1\times10^5\ \text{m}.$$

Here $R$ is the radius of Earth. This torque is internal to the satellite–Earth system, so the angular momentum of that system is conserved.

(d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{m}^3/\text{s}^2\cdot\text{kg})\left(5.98\times10^{24}\ \text{kg}\right)}{6.78\times10^6\ \text{m}}} = 7.67\times10^3\ \text{m/s} \approx 7.7\ \text{km/s}.$$

(e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78\times10^6\ \text{m})}{7.67\times10^3\ \text{m/s}} = 5.6\times10^3\ \text{s} \approx 93\ \text{min}.$$

(f) Let $F$ be the magnitude of the average force and $s$ be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01\times10^6\ \text{m}) = 4.40\times10^7\ \text{m}$, and $\Delta E = -1.4\times10^5$ J. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4\times10^5\ \text{J}}{4.40\times10^7\ \text{m}} = 3.2\times10^{-3}\ \text{N}.$$

(g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.

(h) The satellite–Earth system is essentially isolated, so its momentum is very nearly conserved.

68. The orbital radius is $r = R_E + h = 6370\ \text{km} + 400\ \text{km} = 6770\ \text{km} = 6.77\times10^6\ \text{m}.$

(a) Using Kepler's law given in Eq. 13-34, we find the period of the ships to be

$$T_0 = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (6.77 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})}} = 5.54 \times 10^3 \text{ s} \approx 92.3 \text{ min.}$$

(b) The speed of the ships is

$$v_0 = \frac{2\pi r}{T_0} = \frac{2\pi(6.77\times 10^6 \text{ m})}{5.54\times 10^3 \text{ s}} = 7.68\times 10^3 \text{ m/s}^2 .$$

(c) The new kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(0.99v_0)^2 = \frac{1}{2}(2000 \text{ kg})(0.99)^2(7.68\times 10^3 \text{ m/s})^2 = 5.78\times 10^{10} \text{ J.}$$

(d) Immediately after the burst, the potential energy is the same as it was before the burst. Therefore,

$$U = -\frac{GMm}{r} = -\frac{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})(2000 \text{ kg})}{6.77 \times 10^6 \text{ m}} = -1.18\times 10^{11} \text{ J.}$$

(e) In the new elliptical orbit, the total energy is

$$E = K + U = 5.78\times 10^{10} \text{ J} + (-1.18\times 10^{11} \text{ J}) = -6.02\times 10^{10} \text{ J.}$$

(f) For elliptical orbit, the total energy can be written as (see Eq. 13-42) $E = -GMm/2a$, where $a$ is the semi-major axis. Thus,

$$a = -\frac{GMm}{2E} = -\frac{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})(2000 \text{ kg})}{2(-6.02\times 10^{10} \text{ J})} = 6.63 \times 10^6 \text{ m.}$$

(g) To find the period, we use Eq. 13-34 but replace $r$ with $a$. The result is

$$T = \sqrt{\frac{4\pi^2 a^3}{GM}} = \sqrt{\frac{4\pi^2 (6.63 \times 10^6 \text{ m})^3}{(6.67 \times 10^{-11} \text{m}^3/\text{s}^2 \cdot \text{kg})(5.98 \times 10^{24} \text{kg})}} = 5.37 \times 10^3 \text{ s} \approx 89.5 \text{ min.}$$

(h) The orbital period $T$ for Picard's elliptical orbit is shorter than Igor's by

$$\Delta T = T_0 - T = 5540 \text{ s} - 5370 \text{ s} = 170 \text{ s} .$$

Thus, Picard will arrive back at point $P$ ahead of Igor by 170 s – 90 s = 80 s.

69. We define the "effective gravity" in his environment as $g_{eff}$ = 220/60 = 3.67 m/s$^2$. Thus, using equations from Chapter 2 (and selecting downward as the positive direction), we find the "fall-time" to be

$$\Delta y = v_0 t + \frac{1}{2} g_{eff} t^2 \Rightarrow t = \sqrt{\frac{2(2.1\text{ m})}{3.67\text{ m/s}^2}} = 1.1\text{ s.}$$

70. (a) The gravitational acceleration $a_g$ is defined in Eq. 13-11. The problem is concerned with the difference between $a_g$ evaluated at $r = 50R_h$ and $a_g$ evaluated at $r = 50R_h + h$ (where $h$ is the estimate of your height). Assuming $h$ is much smaller than $50R_h$ then we can approximate $h$ as the $dr$ that is present when we consider the differential of Eq. 13-11:

$$|da_g| = \frac{2GM}{r^3}\,dr \approx \frac{2GM}{50^3 R_h^3}\,h = \frac{2GM}{50^3(2GM/c^2)^3}\,h\,.$$

If we approximate $|da_g| = 10$ m/s$^2$ and $h \approx 1.5$ m, we can solve this for $M$. Giving our results in terms of the Sun's mass means dividing our result for $M$ by $2 \times 10^{30}$ kg. Thus, admitting some tolerance into our estimate of $h$ we find the "critical" black hole mass should in the range of 105 to 125 solar masses.

(b) Interestingly, this turns out to be lower limit (which will surprise many students) since the above expression shows $|da_g|$ is inversely proportional to $M$. It should perhaps be emphasized that a distance of $50R_h$ from a small black hole is much smaller than a distance of $50R_h$ from a large black hole.

71. (a) All points on the ring are the same distance ($r = \sqrt{x^2 + R^2}$) from the particle, so the gravitational potential energy is simply $U = -GMm/\sqrt{x^2 + R^2}$, from Eq. 13-21. The corresponding force (by symmetry) is expected to be along the $x$ axis, so we take a (negative) derivative of $U$ (with respect to $x$) to obtain it (see Eq. 8-20). The result for the magnitude of the force is $GMmx(x^2 + R^2)^{-3/2}$.

(b) Using our expression for $U$, the change in potential energy as the particle falls to the center is

$$\Delta U = -GMm\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right)$$

By conservation of mechanical energy, this must "turn into" kinetic energy, $\Delta K = -\Delta U = mv^2/2$. We solve for the speed and obtain

$$\frac{1}{2}mv^2 = GMm\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right) \Rightarrow v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}}\right)}.$$

72. (a) With $M = 2.0\times10^{30}$ kg and $r = 10000$ m, we find $a_g = \dfrac{GM}{r^2} = 1.3\times10^{12}\text{ m/s}^2$.

(b) Although a close answer may be gotten by using the constant acceleration equations of Chapter 2, we show the more general approach (using energy conservation):

$$K_o + U_o = K + U$$

where $K_o = 0$, $K = ½mv^2$, and $U$ is given by Eq. 13-21. Thus, with $r_o = 10001$ m, we find

$$v = \sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_o}\right)} = 1.6 \times 10^6 \text{ m/s} .$$

73. Using energy conservation (and Eq. 13-21) we have

$$K_1 - \frac{GMm}{r_1} = K_2 - \frac{GMm}{r_2} \quad .$$

(a) Plugging in two pairs of values (for $(K_1, r_1)$ and $(K_2, r_2)$) from the graph and using the value of $G$ and $M$ (for Earth) given in the book, we find $m \approx 1.0 \times 10^3$ kg.

(b) Similarly, $v = (2K/m)^{1/2} \approx 1.5 \times 10^3$ m/s (at $r = 1.945 \times 10^7$ m).

74. We estimate the planet to have radius $r = 10$ m. To estimate the mass $m$ of the planet, we require its density equal that of Earth (and use the fact that the volume of a sphere is $4\pi r^3/3$):

$$\frac{m}{4\pi r^3/3} = \frac{M_E}{4\pi R_E^3/3} \quad \Rightarrow \quad m = M_E\left(\frac{r}{R_E}\right)^3$$

which yields (with $M_E \approx 6 \times 10^{24}$ kg and $R_E \approx 6.4 \times 10^6$ m) $m = 2.3 \times 10^7$ kg.

(a) With the above assumptions, the acceleration due to gravity is

$$a_g = \frac{Gm}{r^2} = \frac{\left(6.7\times10^{-11} \text{ m}^3/\text{s}^2\cdot\text{kg}\right)\left(2.3\times10^7 \text{ kg}\right)}{(10 \text{ m})^2} = 1.5\times10^{-5} \text{ m/s}^2 \approx 2\times10^{-5} \text{ m/s}^2 .$$

(b) Equation 13-28 gives the escape speed: $v = \sqrt{\frac{2Gm}{r}} \approx 0.02$ m/s .

75. We use $m_1$ for the 20 kg of the sphere at $(x_1, y_1) = (0.5, 1.0)$ (SI units understood), $m_2$ for the 40 kg of the sphere at $(x_2, y_2) = (–1.0, –1.0)$, and $m_3$ for the 60 kg of the sphere at $(x_3, y_3) = (0, –0.5)$. The mass of the 20 kg object at the origin is simply denoted $m$. We note that $r_1 = \sqrt{1.25}$, $r_2 = \sqrt{2}$, and $r_3 = 0.5$ (again, with SI units understood). The force $\vec{F}_n$ that the $n^{\text{th}}$ sphere exerts on $m$ has magnitude $Gm_n m/r_n^2$ and is directed from the origin toward $m_n$, so that it is conveniently written as

$$\vec{F}_n = \frac{Gm_n m}{r_n^2}\left(\frac{x_n}{r_n}\,\hat{\mathrm{i}} + \frac{y_n}{r_n}\,\hat{\mathrm{j}}\right) = \frac{Gm_n m}{r_n^3}\left(x_n\hat{\mathrm{i}} + y_n\hat{\mathrm{j}}\right).$$

Consequently, the vector addition to obtain the net force on $m$ becomes

$$\vec{F}_{\text{net}} = \sum_{n=1}^{3}\vec{F}_n = Gm\left(\left(\sum_{n=1}^{3}\frac{m_n x_n}{r_n^3}\right)\hat{\mathrm{i}} + \left(\sum_{n=1}^{3}\frac{m_n y_n}{r_n^3}\right)\hat{\mathrm{j}}\right) = (-9.3\times10^{-9}\text{ N})\hat{\mathrm{i}} - (3.2\times10^{-7}\text{ N})\hat{\mathrm{j}}\ .$$

Therefore, we find the net force magnitude is $\left|\vec{F}_{\text{net}}\right| = 3.2\times10^{-7}\,\text{N}$ .

76. **THINK** We apply Newton's law of gravitation to calculate the force between the meteor and the satellite.

**EXPRESS** We use $F = Gm_s m_m/r^2$, where $m_s$ is the mass of the satellite, $m_m$ is the mass of the meteor, and $r$ is the distance between their centers. The distance between centers is $r = R + d$ = 15 m + 3 m = 18 m. Here $R$ is the radius of the satellite and $d$ is the distance from its surface to the center of the meteor.

**ANALYZE** The gravitational force between the meteor and the satellite is

$$F = \frac{Gm_s m_s}{r^2} = \frac{\left(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(20\text{kg}\right)\left(7.0\text{kg}\right)}{\left(18\text{m}\right)^2} = 2.9\times10^{-11}\,\text{N}.$$

**LEARN** The force of gravitation is inversely proportional to $r^2$ .

77. We note that $r_A$ (the distance from the origin to sphere $A$, which is the same as the separation between $A$ and $B$) is 0.5, $r_C$ = 0.8, and $r_D$ = 0.4 (with SI units understood). The force $\vec{F}_k$ that the $k^{\text{th}}$ sphere exerts on $m_B$ has magnitude $Gm_k m_B/r_k^2$ and is directed from the origin toward $m_k$ so that it is conveniently written as

$$\vec{F}_k = \frac{Gm_k m_B}{r_k^2}\left(\frac{x_k}{r_k}\,\hat{\mathrm{i}} + \frac{y_k}{r_k}\,\hat{\mathrm{j}}\right) = \frac{Gm_k m_B}{r_k^3}\left(x_k\hat{\mathrm{i}} + y_k\hat{\mathrm{j}}\right).$$

Consequently, the vector addition (where $k$ equals $A$, $B$, and $D$) to obtain the net force on $m_B$ becomes

$$\vec{F}_{\text{net}} = \sum_k \vec{F}_k = Gm_B\left(\left(\sum_k \frac{m_k x_k}{r_k^3}\right)\hat{\mathrm{i}} + \left(\sum_k \frac{m_k y_k}{r_k^3}\right)\hat{\mathrm{j}}\right) = (3.7\times10^{-5}\,\text{N})\hat{\mathrm{j}}.$$

78. (a) We note that $r_C$ (the distance from the origin to sphere $C$, which is the same as the separation between $C$ and $B$) is 0.8, $r_D$ = 0.4, and the separation between spheres $C$ and $D$ is $r_{CD}$ = 1.2 (with SI units understood). The total potential energy is therefore

$$-\frac{GM_BM_C}{r_C^2}-\frac{GM_BM_D}{r_D^2}-\frac{GM_CM_D}{r_{CD}^2}=-1.3\times10^{-4}\ \text{J}$$

using the mass-values given in the previous problem.

(b) Since any gravitational potential energy term (of the sort considered in this chapter) is necessarily negative ($-GmM/r^2$ where all variables are positive) then having another mass to include in the computation can only lower the result (that is, make the result more negative).

(c) The observation in the previous part implies that the work I do in removing sphere *A* (to obtain the case considered in part (a)) must lead to an increase in the system energy; thus, I do positive work.

(d) To put sphere *A* back in, I do negative work, since I am causing the system energy to become more negative.

79. **THINK** Since the orbit is circular, the net gravitational force on the smaller star is equal to the centripetal force.

**EXPRESS** The magnitude of the net gravitational force on one of the smaller stars (of mass *m*) is

$$F=\frac{GMm}{r^2}+\frac{Gmm}{(2r)^2}=\frac{Gm}{r^2}\left(M+\frac{m}{4}\right).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2}\left(M+\frac{m}{4}\right)=m\frac{v^2}{r}$$

where $v=2\pi r/T$. Combining the two expressions allows us to solve for *T*.

**ANALYZE** Plugging in for speed *v*, we arrive at an equation for the period *T*:

$$T=\frac{2\pi r^{3/2}}{\sqrt{G(M+m/4)}}.$$

**LEARN** In the limit where $m\ll M$, we recover the expected result $T=\dfrac{2\pi r^{3/2}}{\sqrt{GM}}$ for two bodies.

80. If the angular velocity were any greater, loose objects on the surface would not go around with the planet but would travel out into space.

(a) The magnitude of the gravitational force exerted by the planet on an object of mass $m$ at its surface is given by $F = GmM / R^2$, where $M$ is the mass of the planet and $R$ is its radius. According to Newton's second law this must equal $mv^2 / R$, where $v$ is the speed of the object. Thus,

$$\frac{GM}{R^2} = \frac{v^2}{R}.$$

With $M = 4\pi\rho R^3/3$ where ρ is the density of the planet, and $v = 2\pi R/T$, where $T$ is the period of revolution, we find

$$\frac{4\pi}{3}G\rho R = \frac{4\pi^2 R}{T^2}.$$

We solve for $T$ and obtain

$$T = \sqrt{\frac{3\pi}{G\rho}}.$$

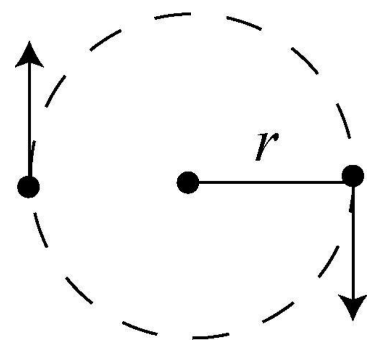
(b) The density is $3.0 \times 10^3$ kg/m$^3$. We evaluate the equation for $T$:

$$T = \sqrt{\frac{3\pi}{\left(6.67\times10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg}\right)\left(3.0\times10^3\,\text{kg/m}^3\right)}} = 6.86\times10^3\,\text{s} = 1.9\,\text{h}.$$

81. **THINK** In a two-star system, the stars rotate about their common center of mass.

**EXPRESS** The situation is depicted on the right. The gravitational force between the two stars (each having a mass $M$) is

$$F_g = \frac{GM^2}{(2r)^2} = \frac{GM^2}{4r^2}$$




The gravitational force between the stars provides the centripetal force necessary to keep their orbits circular. Thus, writing the centripetal acceleration as $r\omega^2$ where $\omega$ is the angular speed, we have

$$F_g = F_c \quad\Rightarrow\quad \frac{GM^2}{4r^2} = Mr\omega^2.$$

**ANALYZE** (a) Substituting the values given, we find the common angular speed to be

$$\omega = \frac{1}{2}\sqrt{\frac{GM}{r^3}} = \frac{1}{2}\sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.0\times10^{30}\ \text{kg})}{(1.0\times10^{11}\ \text{m})^3}} = 2.2\times10^{-7}\,\text{rad/s}.$$

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 13-28). If $m$ is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \;\Rightarrow\; v = \sqrt{\frac{4GM}{r}} = 8.9\times10^4 \text{ m/s} .$$

**LEARN** Comparing with Eq. 13-28, we see that the escape speed of the two-star system is the same as that of a star with mass 2$M$.

82. The key point here is that angular momentum is conserved:

$$I_p\omega_p = I_a\omega_a$$

which leads to $\omega_p = (r_a/r_p)^2\omega_a$, but $r_p = 2a - r_a$ where $a$ is determined by Eq. 13-34 (particularly, see the paragraph after that equation in the textbook). Therefore,

$$\omega_p = \frac{r_a^{\,2}\,\omega_a}{(2(GMT^{\,2}/4\pi^2)^{1/3} - r_a)^2} = 9.24\times10^{-5} \text{ rad/s} .$$

83. **THINK** The orbit of the shuttle goes from circular to elliptical after changing its speed by firing the thrusters.

**EXPRESS** We first use the law of periods: $T^2 = (4\pi^2/GM)r^3$, where $M$ is the mass of the planet and $r$ is the radius of the orbit. After the orbit of the shuttle turns elliptical by firing the thrusters to reduce its speed, the semi-major axis is $a = -GMm/2E$, where $E = K + U$ is the mechanical energy of the shuttle and its new period becomes $T' = \sqrt{4\pi^2a^3/GM}$.

**ANALYZE** (a) Using Kepler's law of periods, we find the period to be

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right)r^3} = \sqrt{\frac{4\pi^2(4.20\times10^7\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\text{ kg})}} = 2.15\times10^4 \text{ s} .$$

(b) The speed is constant (before she fires the thrusters), so

$$v_0 = \frac{2\pi r}{T} = \frac{2\pi(4.20\times10^7\text{ m})}{2.15\times10^4\text{ s}} = 1.23\times10^4 \text{ m/s} .$$

(c) A two percent reduction in the previous value gives

$$v = 0.98v_0 = 0.98(1.23\times10^4\text{ m/s}) = 1.20\times10^4 \text{ m/s} .$$

(d) The kinetic energy is $K=\frac{1}{2}mv^2=\frac{1}{2}(3000\,\text{kg})(1.20\times10^4\ \text{m/s})^2=2.17\times10^{11}\ \text{J}$.

(e) Immediately after the firing, the potential energy is the same as it was before firing the thruster:

$$U=-\frac{GMm}{r}=-\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\ \text{kg})(3000\,\text{kg})}{4.20\times10^7\ \text{m}}=-4.53\times10^{11}\ \text{J}.$$

(f) Adding these two results gives the total mechanical energy:

$$E=K+U=2.17\times10^{11}\ \text{J}+(-4.53\times10^{11}\ \text{J})=-2.35\times10^{11}\ \text{J}.$$

(g) Using Eq. 13-42, we find the semi-major axis to be

$$a=-\frac{GMm}{2E}=-\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\ \text{kg})(3000\,\text{kg})}{2(-2.35\times10^{11}\ \text{J})}=4.04\times10^7\ \text{m}.$$

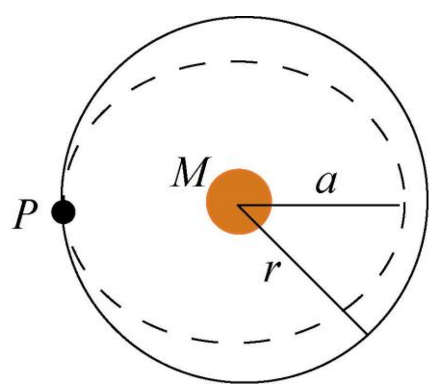
(h) Using Kepler's law of periods for elliptical orbits (using *a* instead of *r*) we find the new period to be

$$T'=\sqrt{\left(\frac{4\pi^2}{GM}\right)a^3}=\sqrt{\frac{4\pi^2(4.04\times10^7\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.50\times10^{25}\ \text{kg})}}=2.03\times10^4\ \text{s}.$$

This is smaller than our result for part (a) by $T - T' = 1.22 \times 10^3$ s.

(i) Comparing the results in (a) and (h), we see that elliptical orbit has a smaller period.

**LEARN** The orbits of the shuttle before and after firing the thruster are shown below. Point P corresponds to the location where the thruster was fired.




84. The difference between free-fall acceleration *g* and the gravitational acceleration $a_g$ at the equator of the star is (see Equation 13.14):

$$a_g - g = \omega^2 R$$

where

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.041\,\text{s}} = 153\,\text{rad/s}$$

is the angular speed of the star. The gravitational acceleration at the equator is

$$a_g = \frac{GM}{R^2} = \frac{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.98\times10^{30}\ \text{kg})}{(1.2\times10^4\ \text{m})^2} = 9.17\times10^{11}\ \text{m/s}^2.$$

Therefore, the percentage difference is

$$\frac{a_g - g}{a_g} = \frac{\omega^2 R}{a_g} = \frac{(153\,\text{rad/s})^2(1.2\times10^4\ \text{m})}{9.17\times10^{11}\ \text{m/s}^2} = 3.06\times10^{-4} \approx 0.031\%.$$

85. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \quad\Rightarrow\quad \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 5.98 \times 10^{24}$ kg, $r_1 = R = 6.37 \times 10^6$, m and $v_1 = 10000$ m/s. Setting $v_2 = 0$ to find the maximum of its trajectory, we solve the above equation (noting that $m$ cancels in the process) and obtain $r_2 = 3.2 \times 10^7$ m. This implies that its *altitude* is

$$h = r_2 - R = 2.5 \times 10^7\ \text{m}.$$

86. We note that, since $v = 2\pi r/T$, the centripetal acceleration may be written as $a = 4\pi^2 r/T^2$. To express the result in terms of $g$, we divide by 9.8 m/s$^2$.

(a) The acceleration associated with Earth's spin ($T$ = 24 h = 86400 s) is

$$a = g\,\frac{4\pi^2\,(6.37\times10^6\ \text{m})}{(86400\,\text{s})^2\,(9.8\,\text{m/s}^2)} = 3.4\times10^{-3}\,g\ .$$

(b) The acceleration associated with Earth's motion around the Sun ($T = 1\ \text{y} = 3.156 \times 10^7$ s) is

$$a = g\,\frac{4\pi^2\,(1.5\times10^{11}\ \text{m})}{(3.156\times10^7\ \text{s})^2\,(9.8\,\text{m/s}^2)} = 6.1\times10^{-4}\,g\ .$$

(c) The acceleration associated with the Solar System's motion around the galactic center ($T = 2.5 \times 10^8\ \text{y} = 7.9 \times 10^{15}$ s) is

$$a = g\,\frac{4\pi^2\,(2.2\times 10^{20}\ \text{m})}{(7.9\times 10^{15}\ \text{s})^2\,(9.8\,\text{m/s}^2)} = 1.4\times 10^{-11} g\ .$$

87. (a) It is possible to use $v^2 = v_0^2 + 2a\,\Delta\,y$ as we did for free-fall problems in Chapter 2 because the acceleration can be considered approximately constant over this interval. However, our approach will not assume constant acceleration; we use energy conservation:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{r} \;\Rightarrow\; v = \sqrt{\frac{2GM(r_0 - r)}{r_0\,r}}$$

which yields $v = 1.4\times 10^6$ m/s.

(b) We estimate the height of the apple to be $h$ = 7 cm = 0.07 m. We may find the answer by evaluating Eq. 13-11 at the surface (radius $r$ in part (a)) and at radius $r + h$, being careful not to round off, and then taking the difference of the two values, or we may take the differential of that equation — setting $dr$ equal to $h$. We illustrate the latter procedure:

$$|\,da_g\,| = \left|-2\frac{GM}{r^3}\,dr\right| \approx 2\frac{GM}{r^3}\,h = 3\times 10^6\ \text{m/s}^2\,.$$

88. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed $v_f$. The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F\,dr = \int(-Kr)dr$. Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F\,dr = \int_R^0 (-Kr)\,dr = \frac{1}{2}KR^2$$

where $R$ is the radius of Earth. Solving for the final speed, we obtain $v_f = R\,\sqrt{K/m}$. We note that the acceleration of gravity $a_g = g$ = 9.8 m/s$^2$ on the surface of Earth is given by

$$a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2,$$

where $\rho$ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$v_f = R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} = \sqrt{(9.8\ \text{m/s}^2)(6.37\times 10^6\ \text{m})} = 7.9\times 10^3\ \text{m/s}\ .$$

89. **THINK** To compare the kinetic energy, potential energy, and the speed of the Earth at aphelion (farthest distance) and perihelion (closest distance), we apply both conservation of energy and conservation of angular momentum.

**EXPRESS** As Earth orbits about the Sun, its total energy is conserved:

$$\frac{1}{2}mv_a^2 - \frac{GM_SM_E}{R_a} = \frac{1}{2}mv_p^2 - \frac{GM_SM_E}{R_p}.$$

In addition, angular momentum conservation implies $v_aR_a = v_pR_p$.

**ANALYZE** (a) The total energy is conserved, so there is no difference between its values at aphelion and perihelion.

(b) The difference in potential energy is

$$\begin{aligned}\Delta U = U_a - U_p &= -GM_SM_E\left(\frac{1}{R_a} - \frac{1}{R_p}\right)\\ &= -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(5.98\times10^{24}\ \text{kg})\left(\frac{1}{1.52\times10^{11}\ \text{m}} - \frac{1}{1.47\times10^{11}\ \text{m}}\right)\\ &\approx 1.8\times10^{32}\ \text{J}.\end{aligned}$$

(c) Since $\Delta K + \Delta U = 0$, $\Delta K = K_a - K_p = -\Delta U \approx -1.8\times10^{32}\ \text{J}$.

(d) With $v_aR_a = v_pR_p$, the change in kinetic energy may be written as

$$\Delta K = K_a - K_p = \frac{1}{2}M_E\left(v_a^2 - v_p^2\right) = \frac{1}{2}M_Ev_a^2\left(1 - \frac{R_a^2}{R_p^2}\right)$$

from which we find the speed at the aphelion to be

$$v_a = \sqrt{\frac{2(\Delta K)}{M_E(1 - R_a^2/R_p^2)}} = 2.95\times10^4\ \text{m/s}.$$

Thus, the variation in speed is

$$\begin{aligned}\Delta v = v_a - v_p &= \left(1 - \frac{R_a}{R_p}\right)v_a = \left(1 - \frac{1.52\times10^{11}\ \text{m}}{1.47\times10^{11}\ \text{m}}\right)(2.95\times10^4\ \text{m/s})\\ &= -0.99\times10^3\ \text{m/s} = -0.99\ \text{km/s}.\end{aligned}$$

The speed at the aphelion is smaller than that at the perihelion.

**LEARN** Since the changes are small, the problem could also be solved by using differentials:

$$dU = \left(\frac{GM_E M_S}{r^2}\right) dr \approx \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(1.5\times10^{11}\ \text{m}\right)^2}\left(5\times10^{9}\ \text{m}\right).$$

This yields $\Delta U \approx 1.8 \times 10^{32}$ J. Similarly, with $\Delta K \approx dK = M_E v\, dv$, where $v \approx 2\pi R/T$, we have

$$1.8\times10^{32}\ \text{J} \approx \left(5.98\times10^{24}\ \text{kg}\right)\left(\frac{2\pi\left(1.5\times10^{11}\ \text{m}\right)}{3.156\times10^{7}\ \text{s}}\right)\Delta v$$

which yields a difference of $\Delta v \approx 0.99$ km/s in Earth's speed (relative to the Sun) between aphelion and perihelion.

90. (a) Because it is moving in a circular orbit, $F/m$ must equal the centripetal acceleration:

$$\frac{80\ \text{N}}{50\ \text{kg}} = \frac{v^2}{r}.$$

However, $v = 2\pi r/T$, where $T = 21600$ s, so we are led to

$$1.6\,\text{m/s}^2 = \frac{4\pi^2}{T^2}r$$

which yields $r = 1.9 \times 10^7$ m.

(b) From the above calculation, we infer $v^2 = (1.6\ \text{m/s}^2)r$, which leads to $v^2 = 3.0 \times 10^7$ $\text{m}^2/\text{s}^2$. Thus, $K = ½mv^2 = 7.6 \times 10^8$ J.

(c) As discussed in Section 13-4, $F/m$ also tells us the gravitational acceleration:

$$a_g = 1.6\ \text{m/s}^2 = \frac{GM}{r^2}.$$

We therefore find $M = 8.6 \times 10^{24}$ kg.

91. (a) Their initial potential energy is $-Gm^2/R_i$ and they started from rest, so energy conservation leads to

$$-\frac{Gm^2}{R_i} = K_{\text{total}} - \frac{Gm^2}{0.5R_i} \Rightarrow K_{\text{total}} = \frac{Gm^2}{R_i}\ .$$

(b) They have equal mass, and this is being viewed in the center-of-mass frame, so their speeds are identical and their kinetic energies are the same. Thus,

$$K = \frac{1}{2}K_{\text{total}} = \frac{Gm^2}{2R_i}.$$

(c) With $K = ½\, mv^2$, we solve the above equation and find $v = \sqrt{Gm/R_i}$ .

(d) Their relative speed is $2v = 2\sqrt{Gm/R_i}$ . This is the (instantaneous) rate at which the gap between them is closing.

(e) The premise of this part is that we assume we are not moving (that is, that body *A* acquires no kinetic energy in the process). Thus, $K_{\text{total}} = K_B$, and the logic of part (a) leads to $K_B = Gm^2/R_i$.

(f) And $\frac{1}{2}mv_B^2 = K_B$ yields $v_B = \sqrt{2Gm/R_i}$ .

(g) The answer to part (f) is incorrect, due to having ignored the accelerated motion of "our" frame (that of body *A*). Our computations were therefore carried out in a noninertial frame of reference, for which the energy equations of Chapter 8 are not directly applicable.

92. (a) We note that the altitude of the rocket is $h = R - R_E$ where $R_E = 6.37\times 10^6$ m. With $M = 5.98\times 10^{24}$ kg, $R_0 = R_E + h_0 = 6.57 \times 10^6$ m and $R = 7.37 \times 10^6$ m, we have

$$K_i + U_i = K + U \Rightarrow \frac{1}{2}m\,(3.70\times 10^3 \text{ m/s})^2 - \frac{GmM}{R_0} = K - \frac{GmM}{R},$$

which yields $K = 3.83 \times 10^7$ J.

(b) Again, we use energy conservation.

$$K_i + U_i = K_f + U_f \Rightarrow \frac{1}{2}m\,(3.70\times 10^3)^2 - \frac{GmM}{R_0} = 0 - \frac{GmM}{R_f}$$

Therefore, we find $R_f = 7.40 \times 10^6$ m. This corresponds to a distance of 1034.9 km ≈ $1.03 \times 10^3$ km above the Earth's surface.

93. Energy conservation for this situation may be expressed as follows:

$$K_1 + U_1 = K_2 + U_2 \;\Rightarrow\; \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} = \frac{1}{2}mv_2^2 - \frac{GmM}{r_2}$$

where $M = 7.0 \times 10^{24}$ kg, $r_2 = R = 1.6 \times 10^6$ m, and $r_1 = \infty$ (which means that $U_1 = 0$). We are told to assume the meteor starts at rest, so $v_1 = 0$. Thus, $K_1 + U_1 = 0$, and the above equation is rewritten as

$$\frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \quad \Rightarrow \quad v_2 = \sqrt{\frac{2GM}{R}} = 2.4 \times 10^4 \text{ m/s}.$$

94. The initial distance from each fixed sphere to the ball is $r_0 = \infty$, which implies the initial gravitational potential energy is zero. The distance from each fixed sphere to the ball when it is at $x = 0.30$ m is $r = 0.50$ m, by the Pythagorean theorem.

(a) With $M = 20$ kg and $m = 10$ kg, energy conservation leads to

$$K_i + U_i = K + U \Rightarrow 0 + 0 = K - 2\frac{GmM}{r}$$

which yields $K = 2GmM/r = 5.3 \times 10^{-8}$ J.

(b) Since the $y$-component of each force will cancel, the net force points in the $-x$ direction, with a magnitude

$$2F_x = 2\,(GmM/r^2)\cos\theta,$$

where $\theta = \tan^{-1}(4/3) = 53°$. Thus, the result is $\vec{F}_{\text{net}} = (-6.4 \times 10^{-8} \text{ N})\hat{\text{i}}$.

95. The magnitudes of the individual forces (acting on $m_C$, exerted by $m_A$ and $m_B$, respectively) are

$$F_{AC} = \frac{Gm_A\, m_C}{r_{AC}^2} = 2.7 \times 10^{-8} \text{ N} \quad \text{and} \quad F_{BC} = \frac{Gm_B\, m_C}{r_{BC}^2} = 3.6 \times 10^{-8} \text{ N}$$

where $r_{AC} = 0.20$ m and $r_{BC} = 0.15$ m. With $r_{AB} = 0.25$ m, the angle $\vec{F}_A$ makes with the $x$ axis can be obtained as

$$\theta_A = \pi + \cos^{-1}\left(\frac{r_{AC}^2 + r_{AB}^2 - r_{BC}^2}{2r_{AC}r_{AB}}\right) = \pi + \cos^{-1}(0.80) = 217°.$$

Similarly, the angle $\vec{F}_B$ makes with the $x$ axis can be obtained as

$$\theta_B = -\cos^{-1}\left(\frac{r_{AB}^2 + r_{BC}^2 - r_{AC}^2}{2r_{AB}r_{BC}}\right) = -\cos^{-1}(0.60) = -53°.$$

The net force acting on $m_C$ then becomes

$$\begin{aligned}\vec{F}_C &= F_{AC}(\cos\theta_A\,\hat{\text{i}} + \sin\theta_A\,\hat{\text{j}}) + F_{BC}(\cos\theta_B\,\hat{\text{i}} + \sin\theta_B\,\hat{\text{j}}) \\ &= (F_{AC}\cos\theta_A + F_{BC}\cos\theta_B)\hat{\text{i}} + (F_{AC}\sin\theta_A + F_{BC}\sin\theta_B)\hat{\text{j}} \\ &= (-4.4\times10^{-8}\ \text{N})\hat{\text{j}}.\end{aligned}$$

96. (a) From Chapter 2, we have $v^2 = v_0^2 + 2a\Delta x$, where $a$ may be interpreted as an average acceleration in cases where the acceleration is not uniform. With $v_0 = 0$, $v$ = 11000 m/s, and $\Delta x$ = 220 m, we find $a = 2.75 \times 10^5$ m/s$^2$. Therefore,

$$a = \left(\frac{2.75\times10^5\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) g = 2.8\times10^4\, g\,.$$

(b) The acceleration is certainly deadly enough to kill the passengers.

(c) Again using $v^2 = v_0^2 + 2a\Delta x$, we find

$$a = \frac{(7000\ \text{m/s})^2}{2(3500\ \text{m})} = 7000\ \text{m/s}^2 = 714g\ .$$

(d) Energy conservation gives the craft's speed $v$ (in the absence of friction and other dissipative effects) at altitude $h$ = 700 km after being launched from $R = 6.37 \times 10^6$ m (the surface of Earth) with speed $v_0$ = 7000 m/s. That altitude corresponds to a distance from Earth's center of $r = R + h = 7.07 \times 10^6$ m.

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}\ .$$

With $M = 5.98 \times 10^{24}$ kg (the mass of Earth) we find $v = 6.05 \times 10^3$ m/s. However, to orbit at that radius requires (by Eq. 13-37)

$$v' = \sqrt{GM/r} = 7.51\times10^3\ \text{m/s}.$$

The difference between these two speeds is $v' - v = 1.46 \times 10^3$ m/s $\approx 1.5\times10^3$ m/s, which presumably is accounted for by the action of the rocket engine.

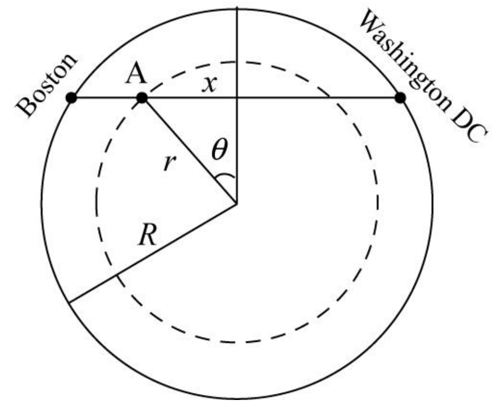
97. We integrate Eq. 13-1 with respect to $r$ from $3R_E$ to $4R_E$ and obtain the work equal to

$$W = -\Delta U = -GM_E m\left(\frac{1}{4R_E} - \frac{1}{3R_E}\right) = \frac{GM_E m}{12R_E}\ .$$

98. The gravitational force at a radial distance $r$ inside Earth (e.g., point A in the figure) is

$$F_g = -\frac{GMm}{R^3} r$$

The component of the force along the tunnel is




$$F_x = F_g \sin\theta = \left(-\frac{GMm}{R^3} r\right)\frac{x}{r} = -\frac{GMm}{R^3} x$$

which can be rewritten as

$$a_x = \frac{d^2x}{dt^2} - \frac{GM}{R^3} x = -\omega^2 x$$

where $\omega^2 = GM/R^3$. The equation is similar to Hooke's law, in that the force on the train is proportional to the displacement of the train but oppositely directed. Without exiting the tunnel, the motion of the train would be periodic would a period given by $T = 2\pi/\omega$. The travel time required from Boston to Washington DC is only half that (one-way):

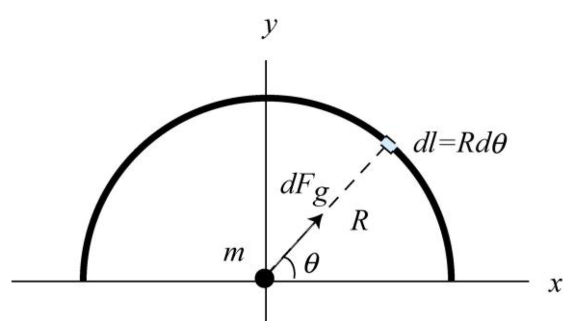
$$\Delta t = \frac{T}{2} = \frac{\pi}{\omega} = \pi\sqrt{\frac{R^3}{GM}} = \pi\sqrt{\frac{(6.37\times10^6\,\text{m})^3}{(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.98\times10^{24}\ \text{kg})}} = 2529\ \text{s} = 42.1\ \text{min}$$

Note that the result is independent of the distance between the two cities.

99. The gravitational force exerted on $m$ due to a mass element $dM$ from the thin rod is

$$dF_g = \frac{Gm(dM)}{R^2}$$

By symmetry, the force is along the $y$-direction. With




$$dM = \lambda dl = \left(\frac{M}{\pi R}\right) R d\theta = \frac{M}{\pi} d\theta$$

where $\lambda = M/\pi R$ is the mass density (mass per unit length), we have

$$dF_{g,y} = dF_g \sin\theta = \frac{Gm}{R^2}\left(\frac{Md\theta}{\pi}\right)\sin\theta = \frac{GMm}{\pi R^2}\sin\theta d\theta$$

Integrating over $\theta$ gives

$$F_{g,y} = \int_0^{\pi} \frac{GMm}{\pi R^2}\sin\theta d\theta = \frac{GMm}{\pi R^2}\int_0^{\pi}\sin\theta d\theta = \frac{2GMm}{\pi R^2}$$

Substituting the values given leads to

$$F_{g,y} = \frac{2GMm}{\pi R^2} = \frac{2(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(5.0\ \text{kg})(3.0\times10^{-3}\ \text{kg})}{\pi(0.650\ \text{m})^2} = 1.51\times10^{-12}\ \text{N}$$

If the rod were a complete circle, by symmetry, the net force on the particle would be zero.

100. The gravitational acceleration at a distance r from the center of Earth is

$$a_g = \frac{GM}{r^2}$$

Thus, the weight difference between the two objects is

$$\Delta w = m(g - a_g) = \frac{GMm}{R^2} - \frac{GMm}{(R+h)^2} = \frac{GMm}{R^2}\left[1-(1+h/R)^{-2}\right] \approx \frac{GMm}{R^2}\cdot\frac{2h}{R} = \frac{2GMmh}{R^3}$$

With $M = \frac{4}{3}\pi R^3\rho$, the above expression can be rewritten as

$$\Delta w = \frac{2GMmh}{R^3} = \frac{2Gmh}{R^3}\cdot\left(\frac{4\pi}{3}R^3\rho\right) = \frac{8\pi\rho Gmh}{3}$$

Substituting the values given, we obtain

$$\Delta w = \frac{8\pi\rho Gmh}{3} = \frac{8\pi}{3}(5.5\times10^3\ \text{kg/m}^3)(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(2.00\ \text{kg})(0.050\ \text{m})$$
$$= 3.07\times10^{-7}\ \text{N}$$

101. Let the distance from Earth to the spaceship be $r$. $R_{em} = 3.82 \times 10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{\left(R_{em} - r\right)^2} = F_E = \frac{GM_e m}{r^2},$$

where $m$ is the mass of the spaceship. Solving for $r$, we obtain

$$r = \frac{R_{em}}{\sqrt{M_m/M_e}+1} = \frac{3.82\times10^8\,\text{m}}{\sqrt{(7.36\times10^{22}\,\text{kg})/(5.98\times10^{24}\,\text{kg})}+1} = 3.44\times10^8\,\text{m}.$$

# Chapter 14

1. Let the volume of the expanded air sacs be $V_a$ and that of the fish with its air sacs collapsed be $V$. Then

$$\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V} = 1.08 \text{ g/cm}^3 \quad \text{and} \quad \rho_w = \frac{m_{\text{fish}}}{V + V_a} = 1.00 \text{ g/cm}^3$$

where $\rho_w$ is the density of the water. This implies

$$\rho_{\text{fish}} V = \rho_w (V + V_a) \text{ or } (V + V_a)/V = 1.08/1.00,$$

which gives $V_a/(V + V_a) = 0.074 = 7.4\%$.

2. The magnitude $F$ of the force required to pull the lid off is $F = (p_o - p_i)A$, where $p_o$ is the pressure outside the box, $p_i$ is the pressure inside, and $A$ is the area of the lid. Recalling that $1\text{N/m}^2 = 1$ Pa, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0 \times 10^5 \text{ Pa} - \frac{480 \text{ N}}{77 \times 10^{-4} \text{ m}^2} = 3.8 \times 10^4 \text{ Pa}.$$

3. **THINK** The increase in pressure is equal to the applied force divided by the area.

**EXPRESS** The change in pressure is given by $\Delta p = F/A = F/\pi r^2$, where $r$ is the radius of the piston.

**ANALYZE** substituting the values given, we obtain

$$\Delta p = (42 \text{ N})/\pi(0.011 \text{ m})^2 = 1.1 \times 10^5 \text{ Pa}.$$

This is equivalent to 1.1 atm.

**LEARN** The increase in pressure is proportional to the force applied. In addition, since $\Delta p \sim 1/A$, the smaller the cross-sectional area of the syringe, the greater the pressure increase under the same applied force.

4. We note that the container is cylindrical, the important aspect of this being that it has a uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquids. Using the fact that 1L = 1000 $\text{cm}^3$, we find the weight of the first liquid to be

$$W_1 = m_1 g = \rho_1 V_1 g = (2.6 \text{ g/cm}^3)(0.50 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 1.27\times 10^6 \text{ g}\cdot\text{cm/s}^2$$
$$= 12.7 \text{ N}.$$

In the last step, we have converted grams to kilograms and centimeters to meters. Similarly, for the second and the third liquids, we have

$$W_2 = m_2 g = \rho_2 V_2 g = (1.0 \text{ g/cm}^3)(0.25 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 2.5 \text{ N}$$

and

$$W_3 = m_3 g = \rho_3 V_3 g = (0.80 \text{ g/cm}^3)(0.40 \text{ L})(1000 \text{ cm}^3/\text{L})(980 \text{ cm/s}^2) = 3.1 \text{ N}.$$

The total force on the bottom of the container is therefore $F = W_1 + W_2 + W_3 = 18$ N.

5. **THINK** The pressure difference between two sides of the window results in a net force acting on the window.

**EXPRESS** The air inside pushes outward with a force given by $p_iA$, where $p_i$ is the pressure inside the room and $A$ is the area of the window. Similarly, the air on the outside pushes inward with a force given by $p_oA$, where $p_o$ is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$.

**ANALYZE** Since 1 atm = $1.013 \times 10^5$ Pa, the net force is

$$F = (p_i - p_o)A = (1.0 \text{ atm} - 0.96 \text{ atm})(1.013\times 10^5 \text{ Pa/atm})(3.4 \text{ m})(2.1 \text{ m})$$
$$= 2.9 \times 10^4 \text{ N}.$$

**LEARN** The net force on the window vanishes when the pressure inside the office is equal to the pressure outside.

6. Knowing the standard air pressure value in several units allows us to set up a variety of conversion factors:

(a) $P = \left(28 \text{ lb/in.}^2\right)\left(\dfrac{1.01\times 10^5 \text{ Pa}}{14.7 \text{ lb/in}^2}\right) = 190 \text{ kPa}$.

(b) $(120 \text{ mmHg})\left(\dfrac{1.01\times 10^5 \text{Pa}}{760 \text{ mmHg}}\right) = 15.9 \text{ kPa}, \quad (80 \text{ mmHg})\left(\dfrac{1.01\times 10^5 \text{ Pa}}{760 \text{ mmHg}}\right) = 10.6 \text{ kPa}.$

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.

We consider a force vector at angle $\theta$. Its leftward component is $\Delta p \cos\theta dA$, where $dA$ is the area element for where the force is applied. We make use of the symmetry of the problem and let $dA$ be that of a ring of constant $\theta$ on the surface. The radius of the ring is $r = R \sin\theta$, where $R$ is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\,d\theta$ and its area is $dA = 2\pi R^2 \sin\theta\,d\theta$. Thus the net horizontal component of the force of the air is given by

$$F_h = 2\pi R^2 \Delta p \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi R^2 \Delta p \left. \sin^2\theta \right|_0^{\pi/2} = \pi R^2 \Delta p.$$

(b) We use 1 atm = $1.01 \times 10^5$ Pa to show that $\Delta p$ = 0.90 atm = $9.09 \times 10^4$ Pa. The sphere radius is $R$ = 0.30 m, so

$$F_h = \pi(0.30\text{ m})^2(9.09 \times 10^4\text{ Pa}) = 2.6 \times 10^4\text{ N}.$$

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

8. Using Eq. 14-7, we find the gauge pressure to be $p_{\text{gauge}} = \rho gh$, where $\rho$ is the density of the fluid medium, and $h$ is the vertical distance to the point where the pressure is equal to the atmospheric pressure.

The gauge pressure at a depth of 20 m in seawater is

$$p_1 = \rho_{\text{sw}} gd = (1024\text{ kg/m}^3)(9.8\text{ m/s}^2)(20\text{ m}) = 2.00\times10^5\text{ Pa}.$$

On the other hand, the gauge pressure at an altitude of 7.6 km is

$$p_2 = \rho_{\text{air}} gh = (0.87\text{ kg/m}^3)(9.8\text{ m/s}^2)(7600\text{ m}) = 6.48\times10^4\text{ Pa}.$$

Therefore, the change in pressure is

$$\Delta p = p_1 - p_2 = 2.00\times10^5\text{ Pa} - 6.48\times10^4\text{ Pa} \approx 1.4\times10^5\text{ Pa}.$$

9. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the heart of the *Argentinosaurus* is

$$p_{\text{heart}} = p_{\text{brain}} + \rho gh = 80\text{ torr} + (1.06\times10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(21\text{ m} - 9.0\text{ m})\left(\frac{1\text{ torr}}{133.33\text{ Pa}}\right)$$
$$= 1.0\times10^3\text{ torr}.$$

(b) The gauge pressure at the feet of the *Argentinosaurus* is

$$p_{\text{feet}} = p_{\text{brain}} + \rho g h' = 80 \text{ torr} + (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(21 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right)$$
$$= 80 \text{ torr} + 1642 \text{ torr} = 1722 \text{ torr} \approx 1.7\times10^3 \text{ torr}.$$

10. With $A$ = 0.000500 m$^2$ and $F = pA$ (with $p$ given by Eq. 14-9), then we have $\rho g h A$ = 9.80 N. This gives $h \approx 2.0$ m, which means $d + h = 2.80$ m.

11. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the brain of the giraffe is

$$p_{\text{brain}} = p_{\text{heart}} - \rho g h = 250 \text{ torr} - (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.0 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right)$$
$$= 94 \text{ torr}.$$

(b) The gauge pressure at the feet of the giraffe is

$$p_{\text{feet}} = p_{\text{heart}} + \rho g h = 250 \text{ torr} + (1.06\times10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.0 \text{ m})\left(\frac{1 \text{ torr}}{133.33 \text{ Pa}}\right) = 406 \text{ torr}$$
$$\approx 4.1\times10^2 \text{ torr}.$$

(c) The increase in the blood pressure at the brain as the giraffe lowers its head to the level of its feet is

$$\Delta p = p_{\text{feet}} - p_{\text{brain}} = 406 \text{ torr} - 94 \text{ torr} = 312 \text{ torr} \approx 3.1\times10^2 \text{ torr}.$$

12. Note that 0.05 atm equals 5065 Pa. Application of Eq. 14-7 with the notation in this problem leads to

$$d_{\max} = \frac{p}{\rho_{\text{liquid}} g} = \frac{0.05 \text{ atm}}{\rho_{\text{liquid}} g} = \frac{5065 \text{ Pa}}{\rho_{\text{liquid}} g}.$$

Thus the difference of this quantity between fresh water (998 kg/m$^3$) and Dead Sea water (1500 kg/m$^3$) is

$$\Delta d_{\max} = \frac{5065 \text{ Pa}}{g}\left(\frac{1}{\rho_{\text{fw}}} - \frac{1}{\rho_{\text{sw}}}\right) = \frac{5065 \text{ Pa}}{9.8 \text{ m/s}^2}\left(\frac{1}{998 \text{ kg/m}^3} - \frac{1}{1500 \text{ kg/m}^3}\right) = 0.17 \text{ m}.$$

13. Recalling that 1 atm = $1.01 \times 10^5$ Pa, Eq. 14-8 leads to

$$\rho gh = (1024 \text{ kg/m}^3)\ (9.80 \text{ m/s}^2)\ (10.9 \times 10^3 \text{ m}) \left(\frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}}\right) \approx 1.08 \times 10^3 \text{ atm}.$$

14. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho gh = (1.06 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.83 \text{ m}) = 1.90 \times 10^4 \text{ Pa}.$$

15. In this case, Bernoulli's equation reduces to Eq. 14-10. Thus,

$$p_g = \rho g(-h) = -(1800 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.5 \text{ m}) = -2.6 \times 10^4 \text{ Pa}.$$

16. At a depth $h$ without the snorkel tube, the external pressure on the diver is $p = p_0 + \rho gh$, where $p_0$ is the atmospheric pressure. Thus, with a snorkel tube of length $h$, the pressure difference between the internal air pressure and the water pressure against the body is

$$\Delta p = p = p_0 = \rho gh.$$

(a) If $h = 0.20$ m, then

$$\Delta p = \rho gh = (998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.20 \text{ m}) \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} = 0.019 \text{ atm}.$$

(b) Similarly, if $h = 4.0$ m, then

$$\Delta p = \rho gh = (998 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(4.0 \text{ m}) \frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}} \approx 0.39 \text{ atm}.$$

17. **THINK** The minimum force that must be applied to open the hatch is equal to the gauge pressure times the area of the hatch.

**EXPRESS** The pressure $p$ at the depth $d$ of the hatch cover is $p_0 + \rho gd$, where $\rho$ is the density of ocean water and $p_0$ is atmospheric pressure. Thus, the gauge pressure is $p_{\text{gauge}} = \rho gd$, and the minimum force that must be applied by the crew to open the hatch has magnitude $F = p_{\text{gauge}}A = (\rho gd)A$, where $A$ is the area of the hatch.

Substituting the values given, we find the force to be

$$\begin{aligned} F = p_{\text{gauge}}A &= (\rho gd)A = (1024 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(100 \text{ m})(1.2 \text{ m})(0.60 \text{ m}) \\ &= 7.2 \times 10^5 \text{ N}. \end{aligned}$$

**LEARN** The downward force of the water on the hatch cover is $(p_0 + \rho gd)A$, and the air in the submarine exerts an upward force of $p_0A$. The greater the depth of the submarine, the greater the force required to open the hatch.

18. Since the pressure (caused by liquid) at the bottom of the barrel is doubled due to the presence of the narrow tube, so is the hydrostatic force. The ratio is therefore equal to 2.0. The difference between the hydrostatic force and the weight is accounted for by the additional upward force exerted by water on the top of the barrel due to the increased pressure introduced by the water in the tube.

19. We can integrate the pressure (which varies linearly with depth according to Eq. 14-7) over the area of the wall to find out the net force on it, and the result turns out fairly intuitive (because of that linear dependence): the force is the "average" water pressure multiplied by the area of the wall (or at least the part of the wall that is exposed to the water), where "average" pressure is taken to mean $\frac{1}{2}$(pressure at surface + pressure at bottom). Assuming the pressure at the surface can be taken to be zero (in the gauge pressure sense explained in section 14-4), then this means the force on the wall is $\frac{1}{2}\rho gh$ multiplied by the appropriate area. In this problem the area is $hw$ (where $w$ is the 8.00 m width), so the force is $\frac{1}{2}\rho gh^2 w$, and the change in force (as $h$ is changed) is

$$\frac{1}{2}\rho gw\,(h_f^{\,2} - h_i^{\,2}) = \frac{1}{2}(998\text{ kg/m}^3)(9.80\text{ m/s}^2)(8.00\text{ m})(4.00^2 - 2.00^2)\text{m}^2 = 4.69\times10^5\text{ N}.$$

20. (a) The force on face $A$ of area $A_A$ due to the water pressure alone is

$$\begin{aligned}F_A &= p_A A_A = \rho_w g h_A A_A = \rho_w g(2d)d^2 = 2\left(1.0\times10^3\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)\left(5.0\text{ m}\right)^3\\ &= 2.5\times10^6\text{ N}.\end{aligned}$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0\times10^5\text{ Pa})(5.0\text{ m})^2 = 2.5\times10^6\text{ N},$$

we have

$$F_A' = F_0 + F_A = 2.5\times10^6\text{ N} + 2.5\times10^6\text{ N} = 5.0\times10^6\text{ N}.$$

(b) The force on face $B$ due to water pressure alone is

$$\begin{aligned}F_B &= p_{\text{avg}B} A_B = \rho_\omega g\left(\frac{5d}{2}\right)d^2 = \frac{5}{2}\rho_w g d^3 = \frac{5}{2}\left(1.0\times10^3\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)\left(5.0\text{ m}\right)^3\\ &= 3.1\times10^6\text{ N}.\end{aligned}$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0\times10^5\text{ Pa})(5.0\text{ m})^2 = 2.5\times10^6\text{ N},$$

we obtain

$$F_B' = F_0 + F_B = 2.5\times10^6\text{ N} + 3.1\times10^6\text{ N} = 5.6\times10^6\text{ N}.$$

21. **THINK** Work is done to remove liquid from one vessel to another.

**EXPRESS** When the levels are the same, the height of the liquid is $h = (h_1 + h_2)/2$, where $h_1$ and $h_2$ are the original heights. Suppose $h_1$ is greater than $h_2$. The final situation can then be achieved by taking liquid from the first vessel with volume $V = A(h_1 - h)$ and mass $m = \rho V = \rho A(h_1 - h)$, and lowering it a distance $\Delta y = h - h_2$. The work done by the force of gravity is

$$W_g = mg\Delta y = \rho A(h_1 - h)g(h - h_2).$$

**ANALYZE** We substitute $h = (h_1 + h_2)/2$ to obtain

$$\begin{aligned} W_g &= \frac{1}{4}\rho g A\left(h_1 - h_2\right)^2 = \frac{1}{4}(1.30\times10^3\,\text{kg/m}^3)(9.80\text{ m/s}^2)(4.00\times10^{-4}\,\text{m}^2)(1.56\text{ m} - 0.854\text{ m})^2 \\ &= 0.635\text{ J} \end{aligned}$$

**LEARN** Since gravitational force is conservative, the work done only depends on the initial and final heights of the vessels, and not on how the liquid is transferred.

22. To find the pressure at the brain of the pilot, we note that the inward acceleration can be treated from the pilot's reference frame as though it is an outward gravitational acceleration against which the heart must push the blood. Thus, with $a = 4g$, we have

$$\begin{aligned} p_{\text{brain}} &= p_{\text{heart}} - \rho a r = 120\text{ torr} - (1.06\times10^3\text{ kg/m}^3)(4\times9.8\text{ m/s}^2)(0.30\text{ m})\left(\frac{1\text{ torr}}{133\text{ Pa}}\right) \\ &= 120\text{ torr} - 94\text{ torr} = 26\text{ torr.} \end{aligned}$$

23. Letting $p_a = p_b$, we find

$$\rho_c g(6.0\text{ km} + 32\text{ km} + D) + \rho_m(y - D) = \rho_c g(32\text{ km}) + \rho_m y$$

and obtain

$$D = \frac{(6.0\,\text{km})\rho_c}{\rho_m - \rho_c} = \frac{(6.0\,\text{km})\left(2.9\,\text{g/cm}^3\right)}{3.3\,\text{g/cm}^3 - 2.9\,\text{g/cm}^3} = 44\,\text{km.}$$

24. (a) At depth $y$ the gauge pressure of the water is $p = \rho g y$, where $\rho$ is the density of the water. We consider a horizontal strip of width $W$ at depth $y$, with (vertical) thickness $dy$, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho g y W\,dy$. The total force of the water on the dam is

$$\begin{aligned} F &= \int_0^D \rho g y W\,dy = \frac{1}{2}\rho g W D^2 = \frac{1}{2}\left(1.00\times10^3\,\text{kg/m}^3\right)\left(9.80\,\text{m/s}^2\right)\left(314\,\text{m}\right)\left(35.0\,\text{m}\right)^2 \\ &= 1.88\times10^9\,\text{N.} \end{aligned}$$

(b) Again we consider the strip of water at depth $y$. Its moment arm for the torque it exerts about $O$ is $D - y$ so the torque it exerts is

$$d\tau = dF(D - y) = \rho g y W (D - y) dy$$

and the total torque of the water is

$$\tau = \int_0^D \rho g y W (D - y) dy = \rho g W \left( \frac{1}{2} D^3 - \frac{1}{3} D^3 \right) = \frac{1}{6} \rho g W D^3$$
$$= \frac{1}{6} (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(314 \text{ m})(35.0 \text{ m})^3 = 2.20 \times 10^{10} \text{ N} \cdot \text{m}.$$

(c) We write $\tau = rF$, where $r$ is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6} \rho g W D^3}{\frac{1}{2} \rho g W D^2} = \frac{D}{3} = \frac{35.0 \text{ m}}{3} = 11.7 \text{ m}.$$

25. As shown in Eq. 14-9, the atmospheric pressure $p_0$ bearing down on the barometer's mercury pool is equal to the pressure $\rho g h$ at the base of the mercury column: $p_0 = \rho g h$. Substituting the values given in the problem statement, we find the atmospheric pressure to be

$$p_0 = \rho g h = (1.3608 \times 10^4 \text{ kg/m}^3)(9.7835 \text{ m/s}^2)(0.74035 \text{ m}) \left( \frac{1 \text{ torr}}{133.33 \text{ Pa}} \right)$$
$$= 739.26 \text{ torr}.$$

26. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(4.0 \times 10^{-2} \text{ m})}{1.01 \times 10^5 \text{ Pa/atm}} = -3.9 \times 10^{-3} \text{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

27. **THINK** The atmospheric pressure at a given height depends on the density distribution of air.

**EXPRESS** If the air density were uniform, $\rho$ =const., then the variation of pressure with height may be written as: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take $y_1$ to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and $y_2$ to be at the top of the atmosphere, where the pressure is $p_2 = 0$. On the other hand, if the density varies with altitude, then

$$p_2 = p_1 - \int_0^h \rho g \; dy .$$

For the case where the density decreases linearly with height, $\rho = \rho_0\,(1 - y/h)$, where $\rho_0$ is the density at Earth's surface and $g$ = 9.8 m/s$^2$ for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 gh.$$

**ANALYZE** (a) For uniform density with $\rho$ = 1.3 kg/m$^3$, we find the height of the atmosphere to be

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01\times10^5 \text{ Pa}}{(1.3 \text{ kg/m}^3)(9.8\,\text{m/s}^2)} = 7.9\times10^3 \text{ m} = 7.9 \text{ km}.$$

(b) With density decreasing linearly with height, $p_2 = p_1 - \rho_0 gh/2$. The condition $p_2 = 0$ implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01\times10^5 \text{ Pa})}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 16\times10^3 \text{ m} = 16 \text{ km}.$$

**LEARN** Actually the decrease in air density is approximately exponential, with pressure halved at a height of about 5.6 km.

28. (a) According to Pascal's principle, $F/A = f/a \rightarrow F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A}F = \frac{(3.80 \text{ cm})^2}{(53.0 \text{ cm})^2}(20.0\times10^3 \text{ N}) = 103 \text{ N}.$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

29. Equation 14-13 combined with Eq. 5-8 and Eq. 7-21 (in absolute value) gives

$$mg = kx\frac{A_1}{A_2}.$$

With $A_2 = 18A_1$ (and the other values given in the problem) we find $m$ = 8.50 kg.

30. Taking "down" as the positive direction, then using Eq. 14-16 in Newton's second law, we have (5.00 kg)$g$ – (3.00 kg)$g$ = 5$a$. This gives $a = \frac{2}{5}g$ = 3.92 m/s$^2$, where $g$ = 9.8 m/s$^2$. Then (see Eq. 2-15) $\frac{1}{2}at^2$ = 0.0784 m (in the downward direction).

31. **THINK** The block floats in both water and oil. We apply Archimedes' principle to analyze the problem.

**EXPRESS** Let $V$ be the volume of the block. Then, the submerged volume in water is $V_s = 2V/3$. Since the block is floating, by Archimedes' principle the weight of the displaced water is equal to the weight of the block, i.e., $\rho_w V_s = \rho_b V$, where $\rho_w$ is the density of water, and $\rho_b$ is the density of the block.

**ANALYZE** (a) We substitute $V_s = 2V/3$ to obtain the density of the block:

$$\rho_b = 2\rho_w/3 = 2(1000 \text{ kg/m}^3)/3 \approx 6.7 \times 10^2 \text{ kg/m}^3.$$

(b) Now, if $\rho_o$ is the density of the oil, then Archimedes' principle yields $\rho_o V_s' = \rho_b V$. Since the volume submerged in oil is $V_s' = 0.90V$, the density of the oil is

$$\rho_o = \rho_b\left(\frac{V}{V'}\right) = (6.7\times10^2 \text{ kg/m}^3)\frac{V}{0.90V} = 7.4\times10^2 \text{ kg/m}^3.$$

**LEARN** Another way to calculate the density of the oil is to note that the mass of the block can be written as

$$m = \rho_b V = \rho_o V_s' = \rho_w V_s.$$

Therefore,

$$\rho_o = \rho_w\left(\frac{V_s}{V_s'}\right) = (1000 \text{ kg/m}^3)\frac{2V/3}{0.90V} = 7.4\times10^2 \text{ kg/m}^3.$$

That is, by comparing the fraction submerged with that in water (or another liquid with known density), the density of the oil can be deduced.

32. (a) The pressure (including the contribution from the atmosphere) at a depth of $h_{\text{top}} = L/2$ (corresponding to the top of the block) is

$$p_{\text{top}} = p_{\text{atm}} + \rho g h_{\text{top}} = 1.01\times10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.300 \text{ m}) = 1.04\times10^5 \text{ Pa}$$

where the unit Pa (pascal) is equivalent to N/m$^2$. The force on the top surface (of area $A = L^2 = 0.36$ m$^2$) is

$$F_{\text{top}} = p_{\text{top}} A = 3.75 \times 10^4 \text{ N}.$$

(b) The pressure at a depth of $h_{\text{bot}} = 3L/2$ (that of the bottom of the block) is

$$\begin{aligned} p_{\text{bot}} &= p_{\text{atm}} + \rho g h_{\text{bot}} = 1.01\times10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.900 \text{ m}) \\ &= 1.10\times10^5 \text{ Pa} \end{aligned}$$

where we recall that the unit Pa (pascal) is equivalent to N/m$^2$. The force on the bottom surface is

$$F_{\text{bot}} = p_{\text{bot}} A = 3.96 \times 10^4 \text{ N}.$$

(c) Taking the difference $F_{\text{bot}} - F_{\text{top}}$ cancels the contribution from the atmosphere (including any numerical uncertainties associated with that value) and leads to

$$F_{\text{bot}} - F_{\text{top}} = \rho g\,(h_{\text{bot}} - h_{\text{top}})\; A = \rho g L^3 = 2.18\times10^3 \text{ N}$$

which is to be expected on the basis of Archimedes' principle. Two other forces act on the block: an upward tension $T$ and a downward pull of gravity $mg$. To remain stationary, the tension must be

$$T = mg - (F_{\text{bot}} - F_{\text{top}}) = (450 \text{ kg})(9.80 \text{ m/s}^2) - 2.18\times10^3 \text{ N} = 2.23\times10^3 \text{ N}.$$

(d) This has already been noted in the previous part: $F_b = 2.18\times10^3$ N, and $T + F_b = mg$.

33. **THINK** The iron anchor is submerged in water, so we apply Archimedes' principle to calculate its volume and weight in air.

**EXPRESS** The anchor is completely submerged in water of density $\rho_w$. Its apparent weight is $W_{\text{app}} = W - F_b$, where $W = mg$ is its actual weight and $F_b = \rho_w gV$ is the buoyant force.

**ANALYZE** (a) Substituting the values given, we find the volume of the anchor to be

$$V = \frac{W - W_{\text{app}}}{\rho_w g} = \frac{F_b}{\rho_w g} = \frac{200 \text{ N}}{\left(1000 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)} = 2.04\times10^{-2} \text{ m}^3.$$

(b) The mass of the anchor is $m = \rho_{\text{Fe}} g$, where $\rho_{\text{Fe}}$ is the density of iron (found in Table 14-1). Therefore, its weight in air is

$$W = mg = \rho_{\text{Fe}} V g = (7870 \text{ kg/m}^3)(2.04\times10^{-2} \text{ m}^3)(9.80 \text{ m/s}^2) = 1.57\times10^3 \text{ N}.$$

**LEARN** In general, the apparent weight of an object of density $\rho$ that is completely submerged in a fluid of density $\rho_f$ can be written as $W_{\text{app}} = (\rho - \rho_f)Vg$.

34. (a) Archimedes' principle makes it clear that a body, in order to float, displaces an amount of the liquid that corresponds to the weight of the body. The problem (indirectly) tells us that the weight of the boat is $W = 35.6$ kN. In salt water of density $\rho' = 1100$ kg/m$^3$, it must displace an amount of liquid having weight equal to 35.6 kN.

(b) The displaced volume of salt water is equal to

$$V' = \frac{W}{\rho' g} = \frac{3.56\times10^3 \text{ N}}{(1.10\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 3.30 \text{ m}^3.$$

In freshwater, it displaces a volume of $V = W/\rho g$ = 3.63 m$^3$, where $\rho$ = 1000 kg/m$^3$. The difference is $V - V'$ = 0.330 m$^3$.

35. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV$$

where $N$ is the (minimum) number of logs needed to keep them afloat and $V$ is the volume of each log:

$$V = \pi(0.15\text{ m})^2\,(1.80\text{ m}) = 0.13\text{ m}^3.$$

The buoyant force is $F_b = \rho_{\text{water}}gV_{\text{submerged}}$, where we require $V_{\text{submerged}} \le NV$. The density of water is 1000 kg/m$^3$. To obtain the minimum value of $N$, we set $V_{\text{submerged}} = NV$ and then round our "answer" for $N$ up to the nearest integer:

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV = \rho_{\text{water}}gNV \;\Rightarrow\; N = \frac{3(356\,\text{N})}{gV(\rho_{\text{water}} - \rho_{\text{wood}})}$$

which yields $N = 4.28 \to 5$ logs.

36. From the "kink" in the graph it is clear that $d$ = 1.5 cm. Also, the $h$ = 0 point makes it clear that the (true) weight is 0.25 N. We now use Eq. 14-19 at $h = d$ = 1.5 cm to obtain

$$F_b = (0.25\text{ N} - 0.10\text{ N}) = 0.15\text{ N}.$$

Thus, $\rho_{\text{liquid}}\,g\,V = 0.15$, where

$$V = (1.5\text{ cm})(5.67\text{ cm}^2) = 8.5 \times 10^{-6}\text{ m}^3.$$

Thus, $\rho_{\text{liquid}}$ = 1800 kg/m$^3$ = 1.8 g/cm$^3$.

37. For our estimate of $V_{\text{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\text{submerged}} \approx \frac{4}{3}\pi r_o^3 \quad \text{where } r_o = 60\text{ cm}.$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\text{iron}}g \;\Rightarrow\; \rho_{\text{water}}gV_{\text{submerged}} = \rho_{\text{iron}}g\left(\frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3\right)$$

where $r_i$ is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\text{submerged}}$ as well as the densities of water (1.0 g/cm$^3$) and iron (7.87 g/cm$^3$), we obtain the inner diameter:

$$2r_i = 2r_o\left(1 - \frac{1.0\ \text{g/cm}^3}{7.87\ \text{g/cm}^3}\right)^{1/3} = 57.3\ \text{cm}.$$

38. (a) An object of the same density as the surrounding liquid (in which case the "object" could just be a packet of the liquid itself) is not going to accelerate up or down (and thus won't gain any kinetic energy). Thus, the point corresponding to zero $K$ in the graph must correspond to the case where the density of the object equals $\rho_{\text{liquid}}$. Therefore, $\rho_{\text{ball}}$ = 1.5 g/cm$^3$ (or 1500 kg/m$^3$).

(b) Consider the $\rho_{\text{liquid}}$ = 0 point (where $K_{\text{gained}}$ = 1.6 J). In this case, the ball is falling through perfect vacuum, so that $v^2 = 2gh$ (see Eq. 2-16) which means that $K = \frac{1}{2}mv^2 = 1.6$ J can be used to solve for the mass. We obtain $m_{\text{ball}}$ = 4.082 kg. The volume of the ball is then given by

$$m_{\text{ball}}/\rho_{\text{ball}} = 2.72 \times 10^{-3}\ \text{m}^3.$$

39. **THINK** The hollow sphere is half submerged in a fluid. We apply Archimedes' principle to calculate its mass and density.

**EXPRESS** The downward force of gravity $mg$ is balanced by the upward buoyant force of the liquid: $mg = \rho g V_s$. Here $m$ is the mass of the sphere, $\rho$ is the density of the liquid, and $V_s$ is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3)r_o^3$, where $r_o$ is the outer radius.

**ANALYZE** (a) Substituting the values given, we find the mass of the sphere to be

$$m = \rho V_s = \rho\left(\frac{1}{2}\cdot\frac{4\pi}{3}r_o^3\right) = \frac{2\pi}{3}\rho r_o^3 = \left(\frac{2\pi}{3}\right)(800\ \text{kg/m}^3)(0.090\ \text{m})^3 = 1.22\ \text{kg}.$$

(b) The density $\rho_m$ of the material, assumed to be uniform, is given by $\rho_m = m/V$, where $m$ is the mass of the sphere and $V$ is its volume. If $r_i$ is the inner radius, the volume is

$$V = \frac{4\pi}{3}(r_o^3 - r_i^3) = \frac{4\pi}{3}\left((0.090\ \text{m})^3 - (0.080\ \text{m})^3\right) = 9.09 \times 10^{-4}\ \text{m}^3.$$

The density is

$$\rho_m = \frac{1.22\ \text{kg}}{9.09 \times 10^{-4}\ \text{m}^3} = 1.3 \times 10^3\ \text{kg/m}^3.$$

**LEARN** Note that $\rho_m > \rho$, i.e., the density of the material is greater that of the fluid. However, the sphere floats (and displaces its own weight of fluid) because it's hollow.

40. If the alligator floats, by Archimedes' principle the buoyancy force is equal to the alligator's weight (see Eq. 14-17). Therefore,

$$F_b = F_g = m_{H_2O}g = (\rho_{H_2O}Ah)g .$$

If the mass is to increase by a small amount $m \to m' = m + \Delta m$, then

$$F_b \to F_b' = \rho_{H_2O}A(h+\Delta h)g .$$

With $\Delta F_b = F_b' - F_b = 0.010mg$, the alligator sinks by

$$\Delta h = \frac{\Delta F_b}{\rho_{H_2O}Ag} = \frac{0.01mg}{\rho_{H_2O}Ag} = \frac{0.010(130\text{ kg})}{(998\text{ kg/m}^3)(0.20\text{ m}^2)} = 6.5\times 10^{-3}\text{ m} = 6.5\text{ mm} .$$

41. Let $V_i$ be the total volume of the iceberg. The non-visible portion is below water, and thus the volume of this portion is equal to the volume $V_f$ of the fluid displaced by the iceberg. The fraction of the iceberg that is visible is

$$\text{frac} = \frac{V_i - V_f}{V_i} = 1 - \frac{V_f}{V_i} .$$

Since iceberg is floating, Eq. 14-18 applies:

$$F_g = m_i g = m_f g \quad \Rightarrow \quad m_i = m_f .$$

Since $m = \rho V$, the above equation implies

$$\rho_i V_i = \rho_f V_f \quad \Rightarrow \quad \frac{V_f}{V_i} = \frac{\rho_i}{\rho_f} .$$

Thus, the visible fraction is

$$\text{frac} = 1 - \frac{V_f}{V_i} = 1 - \frac{\rho_i}{\rho_f} .$$

(a) If the iceberg ($\rho_i = 917\text{ kg/m}^3$) floats in salt water with $\rho_f = 1024\text{ kg/m}^3$, then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\text{ kg/m}^3}{1024\text{ kg/m}^3} = 0.10 = 10\% .$$

(b) On the other hand, if the iceberg floats in fresh water ($\rho_f = 1000\text{ kg/m}^3$), then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\text{ kg/m}^3}{1000\text{ kg/m}^3} = 0.083 = 8.3\% .$$

42. Work is the integral of the force over distance (see Eq. 7-32). Referring to the equation immediately preceding Eq. 14-7, we see the work can be written as

$$W = \int \rho_{\text{water}}\, gA(-y)\, dy$$

where we are using $y = 0$ to refer to the water surface (and the $+y$ direction is upward). Let $h = 0.500$ m. Then, the integral has a lower limit of $-h$ and an upper limit of $y_f$, with

$$y_f / h = -\rho_{\text{cylinder}} / \rho_{\text{water}} = -0.400.$$

The integral leads to

$$W = \tfrac{1}{2}\, \rho_{\text{water}} gAh^2(1 - 0.4^2) = 4.11 \text{ kJ}.$$

43. (a) When the model is suspended (in air) the reading is $F_g$ (its true weight, neglecting any buoyant effects caused by the air). When the model is submerged in water, the reading is lessened because of the buoyant force: $F_g - F_b$. We denote the difference in readings as $\Delta m$. Thus,

$$F_g - (F_g - F_b) = \Delta mg$$

which leads to $F_b = \Delta mg$. Since $F_b = \rho_w g V_m$ (the weight of water displaced by the model) we obtain

$$V_m = \frac{\Delta m}{\rho_w} = \frac{0.63776\,\text{kg}}{1000\ \text{kg/m}} \approx 6.378 \times 10^{-4}\ \text{m}^3.$$

(b) The $\frac{1}{20}$ scaling factor is discussed in the problem (and for purposes of significant figures is treated as exact). The actual volume of the dinosaur is

$$V_{\text{dino}} = 20^3\ V_m = 5.102\ \text{m}^3\ .$$

(c) Using $\rho = \dfrac{m_{\text{dino}}}{V_{\text{dino}}} \approx \rho_w = 1000\,\text{kg/m}^3$, we find the mass of the *T. rex* to be

$$m_{\text{dino}} \approx \rho_w V_{\text{dino}} = (1000\,\text{kg/m}^3)\ (5.102\ \text{m}^3) = 5.102 \times 10^3\ \text{kg}.$$

44. (a) Since the lead is not displacing any water (of density $\rho_w$), the lead's volume is not contributing to the buoyant force $F_b$. If the immersed volume of wood is $V_i$, then

$$F_b = \rho_w V_i g = 0.900\,\rho_w V_{\text{wood}} g = 0.900\,\rho_w g \left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right),$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.900\,\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) = (m_{\text{wood}} + m_{\text{lead}})g\,.$$

Thus,

$$m_{\text{lead}} = 0.900\rho_w\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) - m_{\text{wood}} = \frac{(0.900)(1000\,\text{kg/m}^3)(3.67\,\text{kg})}{600\ \text{kg/m}^3} - 3.67\ \text{kg}$$
$$= 1.84\ \text{kg}.$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to $F_b$. Consequently,

$$F_b = 0.900\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) + \left(\frac{\rho_w}{\rho_{\text{lead}}}\right)m_{\text{lead}}g = (m_{\text{wood}} + m_{\text{lead}})g,$$

which leads to

$$m_{\text{lead}} = \frac{0.900(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}} = \frac{1.84\ \text{kg}}{1 - (1.00\times10^3\ \text{kg/m}^3/1.13\times10^4\ \text{kg/m}^3)}$$
$$= 2.01\ \text{kg}.$$

45. The volume $V_{\text{cav}}$ of the cavities is the difference between the volume $V_{\text{cast}}$ of the casting as a whole and the volume $V_{\text{iron}}$ contained: $V_{\text{cav}} = V_{\text{cast}} - V_{\text{iron}}$. The volume of the iron is given by $V_{\text{iron}} = W/g\rho_{\text{iron}}$, where $W$ is the weight of the casting and $\rho_{\text{iron}}$ is the density of iron. The effective weight in water (of density $\rho_w$) is $W_{\text{eff}} = W - g\rho_w V_{\text{cast}}$. Thus, $V_{\text{cast}} = (W - W_{\text{eff}})/g\rho_w$ and

$$V_{\text{cav}} = \frac{W - W_{\text{eff}}}{g\rho_w} - \frac{W}{g\rho_{\text{iron}}} = \frac{6000\ \text{N} - 4000\ \text{N}}{(9.8\ \text{m/s}^2)(1000\,\text{kg/m}^3)} - \frac{6000\ \text{N}}{(9.8\ \text{m/s}^2)(7.87\times10^3\ \text{kg/m}^3)}$$
$$= 0.126\ \text{m}^3\ .$$

46. Due to the buoyant force, the ball accelerates upward (while in the water) at rate $a$ given by Newton's second law: $\rho_{\text{water}}Vg - \rho_{\text{ball}}Vg = \rho_{\text{ball}}Va$, which yields

$$\rho_{\text{water}} = \rho_{\text{ball}}(1 + a/g)\,.$$

With $\rho_{\text{ball}} = 0.300\ \rho_{\text{water}}$, we find that

$$a = g\left(\frac{\rho_{\text{water}}}{\rho_{\text{ball}}} - 1\right) = (9.80\ \text{m/s}^2)\left(\frac{1}{0.300} - 1\right) = 22.9\ \text{m/s}^2\,.$$

Using Eq. 2-16 with $\Delta y = 0.600$ m, the speed of the ball as it emerges from the water is

$$v = \sqrt{2a\Delta y} = \sqrt{2(22.9\ \text{m/s}^2)(0.600\ \text{m})} = 5.24\ \text{m/s}\,.$$

This causes the ball to reach a maximum height $h_{max}$ (measured above the water surface) given by $h_{max} = v^2/2g$ (see Eq. 2-16 again). Thus,

$$h_{max} = \frac{v^2}{2g} = \frac{(5.24 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 1.40 \text{ m}.$$

47. (a) If the volume of the car below water is $V_1$ then $F_b = \rho_w V_1 g = W_{car}$, which leads to

$$V_1 = \frac{W_{car}}{\rho_w g} = \frac{(1800 \text{ kg})(9.8 \text{ m/s}^2)}{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 1.80 \text{ m}^3.$$

(b) We denote the total volume of the car as $V$ and that of the water in it as $V_2$. Then

$$F_b = \rho_w V g = W_{car} + \rho_w V_2 g$$

which gives

$$V_2 = V - \frac{W_{car}}{\rho_w g} = \left(0.750 \text{ m}^3 + 5.00 \text{ m}^3 + 0.800 \text{ m}^3\right) - \frac{1800 \text{ kg}}{1000 \text{ kg/m}^3} = 4.75 \text{ m}^3.$$

48. Let $\rho$ be the density of the cylinder (0.30 g/cm$^3$ or 300 kg/m$^3$) and $\rho_{Fe}$ be the density of the iron (7.9 g/cm$^3$ or 7900 kg/m$^3$). The volume of the cylinder is

$$V_c = (6\times 12) \text{ cm}^3 = 72 \text{ cm}^3 = 0.000072 \text{ m}^3,$$

and that of the ball is denoted $V_b$. The part of the cylinder that is submerged has volume

$$V_s = (4 \times 12) \text{ cm}^3 = 48 \text{ cm}^3 = 0.000048 \text{ m}^3.$$

Using the ideas of section 14-7, we write the equilibrium of forces as

$$\rho g V_c + \rho_{Fe} g V_b = \rho_w g V_s + \rho_w g V_b \quad \Rightarrow \quad V_b = 3.8 \text{ cm}^3$$

where we have used $\rho_w$ = 998 kg/m$^3$ (for water, see Table 14-1). Using $V_b = \frac{4}{3}\pi r^3$ we find $r$ = 9.7 mm.

49. This problem involves use of continuity equation (Eq. 14-23): $A_1 v_1 = A_2 v_2$.

(a) Initially the flow speed is $v_i = 1.5$ m/s and the cross-sectional area is $A_i = HD$. At point $a$, as can be seen from the figure, the cross-sectional area is

$$A_a = (H-h)D - (b-h)d.$$

Thus, by continuity equation, the speed at point *a* is

$$v_a = \frac{A_i v_i}{A_a} = \frac{HDv_i}{(H-h)D-(b-h)d} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m}-0.80\text{ m})(55\text{ m})-(12\text{ m}-0.80\text{ m})(30\text{ m})}$$
$$= 2.96\text{ m/s} \approx 3.0\text{ m/s}.$$

(b) Similarly, at point *b*, the cross-sectional area is $A_b = HD - bd$, and therefore, by continuity equation, the speed at point *b* is

$$v_b = \frac{A_i v_i}{A_b} = \frac{HDv_i}{HD-bd} = \frac{(14\text{ m})(55\text{ m})(1.5\text{ m/s})}{(14\text{ m})(55\text{ m})-(12\text{ m})(30\text{ m})} = 2.8\text{ m/s}.$$

50. The left and right sections have a total length of 60.0 m, so (with a speed of 2.50 m/s) it takes 60.0/2.50 = 24.0 seconds to travel through those sections. Thus it takes (88.8 – 24.0) s = 64.8 s to travel through the middle section. This implies that the speed in the middle section is

$$v_{\text{mid}} = (50\text{ m})/(64.8\text{ s}) = 0.772\text{ m/s}.$$

Now Eq. 14-23 (plus that fact that $A = \pi r^2$) implies $r_{\text{mid}} = r_A\sqrt{(2.5\text{ m/s})/(0.772\text{ m/s})}$ where $r_A$ = 2.00 cm. Therefore, $r_{\text{mid}} = 3.60\text{ cm}$.

51. **THINK** We use the equation of continuity to solve for the speed of water as it leaves the sprinkler hole.

**EXPRESS** Let $v_1$ be the speed of the water in the hose and $v_2$ be its speed as it leaves one of the holes. The cross-sectional area of the hose is $A_1 = \pi R^2$. If there are $N$ holes and $A_2$ is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2 (NA_2) \quad \Rightarrow \quad v_2 = \frac{A_1}{NA_2} v_1 = \frac{R^2}{Nr^2} v_1$$

where $R$ is the radius of the hose and $r$ is the radius of a hole.

**ANALYZE** Noting that $R/r = D/d$ (the ratio of diameters) we find the speed to be

$$v_2 = \frac{D^2}{Nd^2} v_1 = \frac{(1.9\text{ cm})^2}{24(0.13\text{ cm})^2}(0.91\text{ m/s}) = 8.1\text{ m/s}.$$

**LEARN** The equation of continuity implies that the smaller the cross-sectional area of the sprinkler hole, the greater the speed of water as it emerges from the hole.

52. We use the equation of continuity and denote the depth of the river as $h$. Then,

$$(8.2\,\text{m})(3.4\,\text{m})(2.3\,\text{m/s})+(6.8\,\text{m})(3.2\,\text{m})(2.6\,\text{m/s})=h(10.5\,\text{m})(2.9\,\text{m/s})$$

which leads to $h$ = 4.0 m.

53. **THINK** The power of the pump is the rate of work done in lifting the water.

**EXPRESS** Suppose that a mass $\Delta m$ of water is pumped in time $\Delta t$. The pump increases the potential energy of the water by $\Delta U = (\Delta m)gh$, where $h$ is the vertical distance through which it is lifted, and increases its kinetic energy by $\Delta K = \frac{1}{2}(\Delta m)v^2$, where $v$ is its final speed. The work it does is

$$\Delta W = \Delta U + \Delta K = (\Delta m)gh + \frac{1}{2}(\Delta m)v^2$$

and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right).$$

The rate of mass flow is $\Delta m/\Delta t = \rho_w A v$, where $\rho_w$ is the density of water and $A$ is the area of the hose.

**ANALYZE** The area of the hose is $A = \pi r^2 = \pi(0.010\text{ m})^2 = 3.14 \times 10^{-4}\text{ m}^2$ and

$$\rho_w A v = (1000\text{ kg/m}^3)\,(3.14 \times 10^{-4}\text{ m}^2)\,(5.00\text{ m/s}) = 1.57\text{ kg/s}.$$

Thus, the power of the pump is

$$P = \rho A v\left(gh + \frac{1}{2}v^2\right) = (1.57\,\text{kg/s})\left((9.8\,\text{m/s}^2)(3.0\,\text{m}) + \frac{(5.0\,\text{m/s})^2}{2}\right) = 66\,\text{W}.$$

**LEARN** The work done by the pump is converted into both the potential energy and kinetic energy of the water.

54. (a) The equation of continuity provides (26 + 19 + 11) L/min = 56 L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{56/\pi(1.9)^2/4}{26/\pi(1.3)^2/4} \approx 1.0.$$

55. We rewrite the formula for work $W$ (when the force is constant in a direction parallel to the displacement $d$) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where $V$ is the volume of the water being forced through, and $p$ is to be interpreted as the pressure difference between the two ends of the pipe. Thus,

$$W = (1.0 \times 10^5 \text{ Pa})\ (1.4 \text{ m}^3) = 1.4 \times 10^5 \text{ J}.$$

56. (a) The speed $v$ of the fluid flowing out of the hole satisfies $\frac{1}{2}\rho v^2 = \rho g h$ or $v = \sqrt{2gh}$. Thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$, which leads to

$$\rho_1 \sqrt{2gh} A_1 = \rho_2 \sqrt{2gh} A_2 \quad \Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{A_2}{A_1} = 2.$$

(b) The ratio of volume flow is

$$\frac{R_1}{R_2} = \frac{v_1 A_1}{v_2 A_2} = \frac{A_1}{A_2} = \frac{1}{2}.$$

(c) Letting $R_1/R_2 = 1$, we obtain $v_1/v_2 = A_2/A_1 = 2 = \sqrt{h_1/h_2}$. Thus,

$$h_2 = h_1/4 = (12.0 \text{ cm})/4 = 3.00 \text{ cm}.$$

57. **THINK** We use the Bernoulli equation to solve for the flow rate, and the continuity equation to relate cross-sectional area to the vertical distance from the hole.

**EXPRESS** According to the Bernoulli equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2,$$

where $\rho$ is the density of water, $h_1$ is the height of the water in the tank, $p_1$ is the pressure there, and $v_1$ is the speed of the water there; $h_2$ is the altitude of the hole, $p_2$ is the pressure there, and $v_2$ is the speed of the water there. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then simplifies to $\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$.

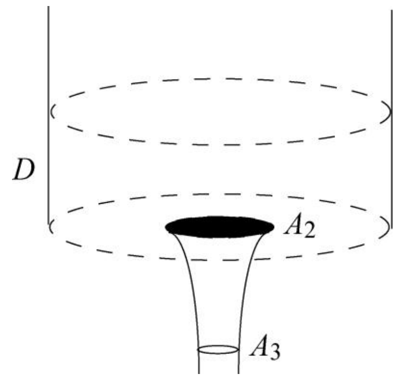
**ANALYZE** (a) With $D = h_1 - h_2 = 0.30$ m, the speed of water as it emerges from the hole is

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2\left(9.8 \text{ m/s}^2\right)(0.30 \text{ m})} = 2.42 \text{ m/s}.$$

Thus, the flow rate is

$$A_2v_2 = (6.5 \times 10^{-4}\ \text{m}^2)(2.42\ \text{m/s}) = 1.6 \times 10^{-3}\ \text{m}^3/\text{s}.$$

(b) We use the equation of continuity: $A_2v_2 = A_3v_3$, where $A_3 = \frac{1}{2}A_2$ and $v_3$ is the water speed where the area of the stream is half its area at the hole (see diagram below).




Thus,

$$v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\ \text{m/s}.$$

The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2 + \rho g h_2 = \frac{1}{2}\rho v_3^2 + \rho g h_3$. Thus,

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84\,\text{m/s})^2 - (2.42\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 0.90\ \text{m}.$$

**LEARN** By combing the two expressions obtained from Bernoulli's equation and equation of continuity, the cross-sectional area of the stream may be related to the vertical height fallen as

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{v_2^2}{2g}\left[\left(\frac{A_2}{A_3}\right)^2 - 1\right] = \frac{v_3^2}{2g}\left[1 - \left(\frac{A_3}{A_2}\right)^2\right].$$

58. We use Bernoulli's equation:

$$p_2 - p_i = \rho g D + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

where $\rho = 1000\ \text{kg/m}^3$, $D = 180$ m, $v_1 = 0.40$ m/s, and $v_2 = 9.5$ m/s. Therefore, we find $\Delta p = 1.7 \times 10^6$ Pa, or 1.7 MPa. The SI unit for pressure is the pascal (Pa) and is equivalent to $\text{N/m}^2$.

59. **THINK** The elevation and cross-sectional area of the pipe are changing, so we apply the Bernoulli equation and continuity equation to analyze the flow of water through the pipe.

**EXPRESS** To calculate the flow speed at the lower level, we use the equation of continuity: $A_1v_1 = A_2v_2$. Here $A_1$ is the area of the pipe at the top and $v_1$ is the speed of the water there; $A_2$ is the area of the pipe at the bottom and $v_2$ is the speed of the water there. As for the pressure at the lower level, we use the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 ,$$

where $\rho$ is the density of water, $h_1$ is its initial altitude, and $h_2$ is its final altitude.

**ANALYZE** (a) From the continuity equation, we find the speed at the lower level to be

$$v_2 = (A_1/A_2)v_1 = [(4.0 \text{ cm}^2)/(8.0 \text{ cm}^2)]\,(5.0 \text{ m/s}) = 2.5\text{m/s}.$$

(b) Similarly, from the Bernoulli equation, the pressure at the lower level is

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right) \\ &= 1.5\times10^5\,\text{Pa} + \frac{1}{2}(1000\,\text{kg/m}^3)\left[(5.0\,\text{m/s})^2 - (2.5\,\text{m/s})^2\right] + (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(10\text{ m}) \\ &= 2.6\times10^5\text{ Pa.} \end{aligned}$$

**LEARN** The water at the lower level has a smaller speed ($v_2 < v_1$) but higher pressure ($p_2 > p_1$).

60. (a) We use $Av$ = const. The speed of water is

$$v = \frac{(25.0\,\text{cm})^2 - (5.00\,\text{cm})^2}{(25.0\,\text{cm})^2}\left(2.50\,\text{m/s}\right) = 2.40\,\text{m/s}.$$

(b) Since $p + \tfrac{1}{2}\rho v^2 = \text{const.},$ the pressure difference is

$$\Delta p = \frac{1}{2}\rho\Delta v^2 = \frac{1}{2}\left(1000\,\text{kg/m}^3\right)\left[\left(2.50\,\text{m/s}\right)^2 - \left(2.40\,\text{m/s}\right)^2\right] = 245\,\text{Pa}.$$

61. (a) The equation of continuity leads to

$$v_2A_2 = v_1A_1 \quad\Rightarrow\quad v_2 = v_1\left(\frac{r_1^2}{r_2^2}\right)$$

which gives $v_2$ = 3.9 m/s.

(b) With $h$ = 7.6 m and $p_1 = 1.7 \times 10^5$ Pa, Bernoulli's equation reduces to

$$p_2 = p_1 - \rho gh + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 8.8\times10^4\,\text{Pa}.$$

62. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{\text{air}}v^2\cdot$ However, $\Delta p = p_A - p_B = \rho gh$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho gh = \frac{1}{2}\rho_{\text{air}}v^2$, or

$$v = \sqrt{\frac{2\rho gh}{\rho_{\text{air}}}}.$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho gh}{\rho_{\text{air}}}} = \sqrt{\frac{2\left(810\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(0.260\,\text{m})}{1.03\,\text{kg/m}^3}} = 63.3\,\text{m/s}.$$

63. We use the formula for $v$ obtained in the previous problem:

$$v = \sqrt{\frac{2\Delta p}{\rho_{\text{air}}}} = \sqrt{\frac{2(180\,\text{Pa})}{0.031\,\text{kg/m}^3}} = 1.1\times10^2\,\text{m/s}.$$

64. (a) The volume of water (during 10 minutes) is

$$V = (v_1t)A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\left(\frac{\pi}{4}\right)(0.03\,\text{m})^2 = 6.4\,\text{m}^3.$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

(c) Since

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho gh_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho gh_2$$

and $h_1 = h_2$, $p_1 = p_0$, which is the atmospheric pressure,

$$p_2 = p_0 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 1.01\times10^5\,\text{Pa} + \frac{1}{2}\left(1.0\times10^3\,\text{kg/m}^3\right)\left[(15\,\text{m/s})^2 - (5.4\,\text{m/s})^2\right]$$
$$= 1.99\times10^5\,\text{Pa} = 1.97\,\text{atm}.$$

Thus, the gauge pressure is (1.97 atm – 1.00 atm) = 0.97 atm = $9.8\times10^4$ Pa.

65. **THINK** The design principles of the Venturi meter, a device that measures the flow speed of a fluid in a pipe, involve both the continuity equation and Bernoulli's equation.

**EXPRESS** The continuity equation yields $AV = av$, and Bernoulli's equation yields $\frac{1}{2}\rho V^2 = \Delta p + \frac{1}{2}\rho v^2$, where $\Delta p = p_2 - p_1$ with $p_2$ equal to the pressure in the throat and $p_1$ the pressure in the pipe. The first equation gives $v = (A/a)V$. We use this to substitute for $v$ in the second equation and obtain

$$\tfrac{1}{2}\rho V^2 = \Delta p + \tfrac{1}{2}\rho\left(A/a\right)^2 V^2.$$

The equation can be used to solve for $V$.

**ANALYZE** (a) The above equation gives the following expression for $V$:

$$V = \sqrt{\frac{2\Delta p}{\rho\left(1-(A/a)^2\right)}} = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}}.$$

(b) We substitute the values given to obtain

$$V = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}} = \sqrt{\frac{2(32\times10^{-4}\,\text{m}^2)^2(41\times10^3\,\text{Pa} - 55\times10^3\,\text{Pa})}{(1000\,\text{kg/m}^3)\left((32\times10^{-4}\,\text{m}^2)^2 - (64\times10^{-4}\,\text{m}^2)^2\right)}} = 3.06\,\text{m/s}.$$

Consequently, the flow rate is

$$R = AV = (64\times10^{-4}\,\text{m}^2)(3.06\text{ m/s}) = 2.0\times10^{-2}\,\text{m}^3/\text{s}.$$

**LEARN** The pressure difference $\Delta p$ between points 1 and 2 is what causes the height difference of the fluid in the two arms of the manometer. Note that $\Delta p = p_2 - p_1 < 0$ (pressure in throat less than that in the pipe), but $a < A$, so the expression inside the square root is positive.

66. We use the result of part (a) in the previous problem.

(a) In this case, we have $\Delta p = p_1 = 2.0$ atm. Consequently,

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{4(1.01\times10^5\text{ Pa})}{(1000\text{ kg/m}^3)\,[(5a/a)^2 - 1]}} = 4.1\text{m/s}.$$

(b) And the equation of continuity yields $V = (A/a)v = (5a/a)v = 5v = 21$ m/s.

(c) The flow rate is given by

$$Av = \frac{\pi}{4}\,(5.0\times10^{-4}\text{ m}^2)\,(4.1\text{ m/s}) = 8.0\times10^{-3}\text{m}^3/\text{s}.$$

67. (a) The friction force is

$$f = A\Delta p = \rho_\omega g d A = (1.0\times10^3 \text{ kg/m}^3)\ (9.8 \text{ m/s}^2)\ (6.0\text{m})\left(\frac{\pi}{4}\right)(0.040 \text{ m})^2 = 74 \text{ N}.$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gd}$. Thus, the volume of water flowing out of the pipe in $t$ = 3.0 h is

$$V = Avt = \frac{\pi^2}{4}(0.040 \text{ m})^2\sqrt{2(9.8 \text{ m/s}^2)\ (6.0 \text{ m})}\ (3.0 \text{ h})\ (3600 \text{ s/h}) = 1.5\times10^2 \text{m}^3.$$

68. (a) We note (from the graph) that the pressures are equal when the value of inverse-area-squared is 16 (in SI units). This is the point at which the areas of the two pipe sections are equal. Thus, if $A_1 = 1/\sqrt{16}$ when the pressure difference is zero, then $A_2$ is 0.25 m$^2$.

(b) Using Bernoulli's equation (in the form Eq. 14-30) we find the pressure difference may be written in the form of a straight line: $mx + b$ where $x$ is inverse-area-squared (the horizontal axis in the graph), $m$ is the slope, and $b$ is the intercept (seen to be –300 kN/m$^2$). Specifically, Eq. 14-30 predicts that $b$ should be $-\frac{1}{2}\rho v_2^2$. Thus, with $\rho$ = 1000 kg/m$^3$ we obtain $v_2 = \sqrt{600}$ m/s. Then the volume flow rate (see Eq. 14-24) is

$$R = A_2\, v_2 = (0.25 \text{ m}^2)(\sqrt{600} \text{ m/s}) = 6.12 \text{ m}^3\text{/s}.$$

If the more accurate value (see Table 14-1) $\rho$ = 998 kg/m$^3$ is used, then the answer is 6.13 m$^3$/s.

69. (a) Combining Eq. 14-35 and Eq. 14-36 in a manner very similar to that shown in the textbook, we find

$$R = A_1 A_2\sqrt{\frac{2\Delta p}{\rho\left(A_1^2 - A_2^2\right)}}$$

for the flow rate expressed in terms of the pressure difference and the cross-sectional areas. Note that $\Delta p = p_1 - p_2 = -7.2 \times 10^3$ Pa and $A_1^2 - A_2^2 = -8.66\times10^{-3} \text{ m}^4$, so that the square root is well defined. Therefore, we obtain $R$ = 0.0776 m$^3$/s.

(b) The mass rate of flow is $\rho R = (900 \text{ kg/m}^3)(0.0776 \text{ m}^3\text{/s}) = 69.8 \text{ kg/s}$.

70. By Eq. 14-23, the speeds in the left and right sections are $\frac{1}{4}v_{\text{mid}}$ and $\frac{1}{9}v_{\text{mid}}$, respectively, where $v_{\text{mid}}$ = 0.500 m/s. We also note that 0.400 m$^3$ of water has a mass of 399 kg (see Table 14-1). Then Eq. 14-31 (and the equation below it) gives

$$W = \frac{1}{2} m v_{\text{mid}}^2 \left( \frac{1}{9^2} - \frac{1}{4^2} \right) = \frac{1}{2} (399 \text{ kg})(0.50 \text{ m/s})^2 \left( \frac{1}{9^2} - \frac{1}{4^2} \right) = -2.50 \text{ J}.$$

71. (a) The stream of water emerges horizontally ($\theta_0$ = 0° in the notation of Chapter 4) with $v_0 = \sqrt{2gh}$ . Setting $y - y_0 = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)}.$$

Using this in Eq. 4-21, where $x_0 = 0$ by choice of coordinate origin, we find

$$x = v_0 t = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)} = 2\sqrt{(10 \text{ cm})(40 \text{ cm} - 10 \text{ cm})} = 35 \text{ cm}.$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for $h$ once $x$ and $H$ are specified. Two solutions for $h$ are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than $H$? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \quad \Rightarrow \quad h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root $h_1$ (where the plus sign is chosen) and the smaller root as $h_2$ (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H.$$

Thus, one root is related to the other (generically labeled $h'$ and $h$) by $h' = H - h$. Its numerical value is $h' = 40\text{cm} - 10 \text{ cm} = 30 \text{ cm}$.

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to $h$ and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \Rightarrow h = \frac{H}{2}$$

or $h$ = (40 cm)/2 = 20 cm, as the depth from which an emerging stream of water will travel the maximum horizontal distance.

72. We use Bernoulli's equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 .$$

When the water level rises to height $h_2$, just on the verge of flooding, $v_2$, the speed of water in pipe $M$ is given by

$$\rho g(h_1 - h_2) = \frac{1}{2}\rho v_2^2 \;\Rightarrow\; v_2 = \sqrt{2g(h_1 - h_2)} = 13.86 \text{ m/s}.$$

By the continuity equation, the corresponding rainfall rate is

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2 = \frac{\pi(0.030 \text{ m})^2}{(30 \text{ m})(60 \text{ m})}(13.86 \text{ m/s}) = 2.177\times10^{-5} \text{ m/s} \approx 7.8 \text{ cm/h}.$$

73. Equilibrium of forces (on the floating body) is expressed as

$$F_b = m_{\text{body}}\, g \Rightarrow \rho_{\text{liquid}}\, g V_{\text{submerged}} = \rho_{\text{body}} g V_{\text{total}}$$

which leads to

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}.$$

We are told (indirectly) that two-thirds of the body is below the surface, so the fraction above is 2/3. Thus, with $\rho_{\text{body}}$ = 0.98 g/cm$^3$, we find $\rho_{\text{liquid}} \approx$ 1.5 g/cm$^3$ — certainly much more dense than normal seawater (the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region).

74. If the mercury level in one arm of the tube is lowered by an amount $x$, it will rise by $x$ in the other arm. Thus, the net difference in mercury level between the two arms is $2x$, causing a pressure difference of $\Delta p = 2\rho_{\text{Hg}} g x$, which should be compensated for by the water pressure $p_w = \rho_w g h$, where $h$ = 11.2 cm. In these units, $\rho_w$ = 1.00 g/cm$^3$ and $\rho_{\text{Hg}}$ = 13.6 g/cm$^3$ (see Table 14-1). We obtain

$$x = \frac{\rho_w g h}{2\rho_{\text{Hg}} g} = \frac{(1.00 \text{ g/cm}^3)\,(11.2 \text{ cm})}{2(13.6 \text{ g/cm}^3)} = 0.412 \text{ cm}.$$

75. Using $m = \rho V$, Newton's second law becomes

$$\rho_{\text{water}} V g - \rho_{\text{bubble}} V g = \rho_{\text{bubble}} V a,$$

or

$$\rho_{\text{water}} = \rho_{\text{bubble}}(1 + a/g)$$

With $\rho_{\text{water}}$ = 998 kg/m$^3$ (see Table 14-1), we find

$$\rho_{\text{bubble}} = \frac{\rho_{\text{water}}}{1 + a/g} = \frac{998\text{ kg/m}^3}{1 + (0.225\text{ m/s}^2)/(9.80\text{ m/s}^2)} = 975.6\text{ kg/m}^3.$$

Using volume $V = \frac{4}{3}\pi r^3$ with $r = 5.00\times10^{-4}$ m for the bubble, we then find its mass: $m_{\text{bubble}} = 5.11 \times 10^{-7}$ kg.

76. To be as general as possible, we denote the ratio of body density to water density as $f$ (so that $f = \rho/\rho_w = 0.95$ in this problem). Floating involves equilibrium of vertical forces acting on the body (Earth's gravity pulls down and the buoyant force pushes up). Thus,

$$F_b = F_g \Rightarrow \rho_w g V_w = \rho g V$$

where $V$ is the total volume of the body and $V_w$ is the portion of it that is submerged.

(a) We rearrange the above equation to yield

$$\frac{V_w}{V} = \frac{\rho}{\rho_w} = f$$

which means that 95% of the body is submerged and therefore 5.0% is above the water surface.

(b) We replace $\rho_w$ with $1.6\rho_w$ in the above equilibrium of forces relationship, and find

$$\frac{V_w}{V} = \frac{\rho}{1.6\rho_w} = \frac{f}{1.6}$$

which means that 59% of the body is submerged and thus 41% is above the quicksand surface.

(c) The answer to part (b) suggests that a person in that situation is able to breathe.

77. The normal force $\vec{F}_N$ exerted (upward) on the glass ball of mass $m$ has magnitude 0.0948 N. The buoyant force exerted by the milk (upward) on the ball has magnitude

$$F_b = \rho_{\text{milk}}\, g\, V$$

where $V = \frac{4}{3}\pi r^3$ is the volume of the ball. Its radius is $r = 0.0200$ m. The milk density is $\rho_{\text{milk}} = 1030\text{ kg/m}^3$. The (actual) weight of the ball is, of course, downward, and has magnitude $F_g = m_{\text{glass}}\, g$. Application of Newton's second law (in the case of zero acceleration) yields

$$F_N + \rho_{\text{milk}}\, g\, V - m_{\text{glass}}\, g = 0$$

which leads to $m_{\text{glass}} = 0.0442$ kg.

78. Since $F_g = mg = \rho_{\text{skier}}\, g\, V$ and the buoyant force is $F_b = \rho_{\text{snow}}\, g\, V$, then their ratio is

$$\frac{F_b}{F_g} = \frac{\rho_{\text{snow}}\, g\, V}{\rho_{\text{skier}}\, g\, V} = \frac{\rho_{\text{snow}}}{\rho_{\text{skier}}} = \frac{96}{1020} = 0.094 \text{ (or 9.4\%)}.$$

79. Neglecting the buoyant force caused by air, then the 30 N value is interpreted as the true weight $W$ of the object. The buoyant force of the water on the object is therefore (30 – 20) N = 10 N, which means

$$F_b = \rho_w V g \Rightarrow V = \frac{10\,\text{N}}{(1000\ \text{kg/m}^3)(9.8\,\text{m/s}^2)} = 1.02 \times 10^{-3}\ \text{m}^3$$

is the volume of the object. When the object is in the second liquid, the buoyant force is (30 – 24) N = 6.0 N, which implies

$$\rho_2 = \frac{6.0\ \text{N}}{(9.8\ \text{m/s}^2)(1.02 \times 10^{-3}\ \text{m}^3)} = 6.0\times10^2\ \text{kg/m}^3\ .$$

80. An object of mass $m = \rho V$ floating in a liquid of density $\rho_{\text{liquid}}$ is able to float if the downward pull of gravity $mg$ is equal to the upward buoyant force $F_b = \rho_{\text{liquid}} g V_{\text{sub}}$ where $V_{\text{sub}}$ is the portion of the object that is submerged. This readily leads to the relation:

$$\frac{\rho}{\rho_{\text{liquid}}} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged of a floating object. When the liquid is water, as described in this problem, this relation leads to

$$\frac{\rho}{\rho_w} = 1$$

since the object "floats fully submerged" in water (thus, the object has the same density as water). We assume the block maintains an "upright" orientation in each case (which is not necessarily realistic).

(a) For liquid $A$, $\dfrac{\rho}{\rho_A} = \dfrac{1}{2}$, so that, in view of the fact that $\rho = \rho_w$, we obtain $\rho_A/\rho_w = 2$.

(b) For liquid $B$, noting that two-thirds *above* means one-third *below*, $\dfrac{\rho}{\rho_B} = \dfrac{1}{3}$, so that $\rho_B/\rho_w = 3$.

(c) For liquid *C*, noting that one-fourth *above* means three-fourths *below*, $\dfrac{\rho}{\rho_C} = \dfrac{3}{4}$, so that $\rho_C/\rho_w = 4/3$.

81. **THINK** The U-tube contains two types of liquid in static equilibrium. The pressures at the interface level on both sides of the tube must be the same.

**EXPRESS** If we examine both sides of the U-tube at the level where the low-density liquid (with $\rho = 0.800\ \text{g/cm}^3 = 800\ \text{kg/m}^3$) meets the water (with $\rho_w = 0.998\ \text{g/cm}^3 = 998\ \text{kg/m}^3$), then the pressures there on either side of the tube must agree:

$$\rho g h = \rho_w g h_w$$

where $h = 8.00$ cm = 0.0800 m, and Eq. 14-9 has been used. Thus, the height of the water column (as measured from that level) is $h_w = (800/998)(8.00\ \text{cm}) = 6.41$ cm.

**ANALYZE** The volume of water in that column is

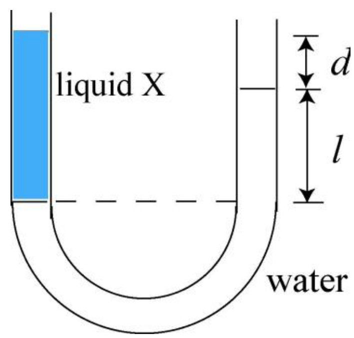
$$V = \pi r^2 h_w = \pi(1.50\ \text{cm})^2(6.41\ \text{cm}) = 45.3\ \text{cm}^3.$$

This is the amount of water that flows out of the right arm.

**LEARN** As discussed in the Sample Problem 14.3 – Balancing of pressure in a U-tube, the relationship between the densities of the two liquids can be written as

$$\rho_X = \rho_w \frac{l}{l+d}$$

The liquid in the left arm is higher than the water in the right because the liquid is less dense than water $\rho_X < \rho_w$.




82. The downward force on the balloon is $mg$ and the upward force is $F_b = \rho_{out} V g$. Newton's second law (with $m = \rho_{in} V$) leads to

$$\rho_{out} Vg - \rho_{in} Vg = \rho_{in} Va \quad \Rightarrow \left(\frac{\rho_{out}}{\rho_{in}} - 1\right) g = a.$$

The problem specifies $\rho_{out} / \rho_{in} = 1.39$ (the outside air is cooler and thus more dense than the hot air inside the balloon). Thus, the upward acceleration is

$$a = (1.39 - 1.00)(9.80\ \text{m/s}^2) = 3.82\ \text{m/s}^2.$$

83. (a) We consider a point $D$ on the surface of the liquid in the container, in the same tube of flow with points $A$, $B$, and $C$. Applying Bernoulli's equation to points $D$ and $C$, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \approx \sqrt{2g\,(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{\text{air}}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_C = \sqrt{2(9.8\text{ m/s}^2)(0.40\text{ m} + 0.12\text{ m})} = 3.2\text{ m/s}.$$

(b) We now consider points $B$ and $C$:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C\ .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{\text{air}}$, Bernoulli's equation becomes

$$\begin{aligned} p_B &= p_C + \rho g(h_C - h_B) = p_{\text{air}} - \rho g\,(h_1 + h_2 + d) \\ &= 1.0\times10^5\text{ Pa} - (1.0\times10^3\text{kg/m}^3)(9.8\text{ m/s}^2)(0.25\text{ m} + 0.40\text{ m} + 0.12\text{ m}) \\ &= 9.2\times10^4\text{ Pa}. \end{aligned}$$

(c) Since $p_B \geq 0$, we must let

$$p_{\text{air}} - \rho g(h_1 + d + h_2) \geq 0,$$

which yields

$$h_1 \leq h_{1,\max} = \frac{p_{\text{air}}}{\rho} - d - h_2 \leq \frac{p_{\text{air}}}{\rho} = 10.3\text{ m}.$$

84. The volume rate of flow is $R = vA$ where $A = \pi r^2$ and $r = d/2$. Solving for speed, we obtain

$$v = \frac{R}{A} = \frac{R}{\pi(d/2)^2} = \frac{4R}{\pi d^2}.$$

(a) With $R = 7.0 \times 10^{-3}$ m$^3$/s and $d = 14 \times 10^{-3}$ m, our formula yields $v = 45$ m/s, which is about 13% of the speed of sound (which we establish by setting up a ratio: $v/v_s$ where $v_s$ = 343 m/s).

(b) With the contracted trachea ($d = 5.2 \times 10^{-3}$ m) we obtain $v = 330$ m/s, or 96% of the speed of sound.

85. We consider the can with nearly its total volume submerged, and just the rim above water. For calculation purposes, we take its submerged volume to be $V$ = 1200 cm$^3$. To float, the total downward force of gravity (acting on the tin mass $m_t$ and the lead mass $m_\ell$) must be equal to the buoyant force upward:

$$(m_t + m_\ell)\, g = \rho_w V g \Rightarrow m_\ell = (1\text{g/cm}^3)\ (1200\ \text{cm}^3) - 130\ \text{g}$$

which yields 1.07×10$^3$ g for the (maximum) mass of the lead (for which the can still floats). The given density of lead is not used in the solution.

86. Before undergoing acceleration, the net force exerted on the block is zero, and Newton's second law gives

$$F_b - mg - T_0 = 0 \quad \Rightarrow \quad T_0 = F_b - mg$$

where $F_b = \rho V g$ is the buoyant force from the fluid of density $\rho$. When the container is given an upward acceleration $a$, the apparent weight of the block becomes $m(g+a)$, and the corresponding buoyant force is $F_b' = \rho V(g+a)$. In this case, Newton's second-law equation is

$$F_b' - m(g+a) - T = 0$$

which gives

$$T = F_b' - m(g+a) = \rho V(g+a) - m(g+a) = (\rho V - m) g (1 + a/g) = T_0 (1 + a/g)\,.$$

With $a = 0.25g$, we have $T/T_0 = 1 + a/g = 1.25$.

87. We assume that the top surface of the slab is at the surface of the water and that the automobile is at the center of the ice surface. Let $M$ be the mass of the automobile, $\rho_i$ be the density of ice, and $\rho_w$ be the density of water. Suppose the ice slab has area $A$ and thickness $h$. Since the volume of ice is $Ah$, the downward force of gravity on the automobile and ice is $(M + \rho_i Ah)g$. The buoyant force of the water is $\rho_w Ahg$, so the condition of equilibrium is $(M + \rho_i Ah)g - \rho_w Ahg = 0$ and

$$A = \frac{M}{(\rho_w - \rho_i)h} = \frac{938\,\text{kg}}{\left(998\,\text{kg/m}^3 - 917\,\text{kg/m}^3\right)(0.441\text{m})} = 26.3\ \text{m}^2.$$

88. (a) Using Eq. 14-10, we have

$$p_g = \rho g h = (1025\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(2.22\times10^3\text{m}) = 2.23\times10^7\,\text{Pa}\,.$$

(b) By definition, the total pressure is

$$p = p_0 + p_g = 1.01\times10^5\,\text{Pa} + 2.23\times10^7\,\text{Pa} = 2.24\times10^7\,\text{Pa}\,.$$

(c) The net force compressing the sphere's surface is

$$F = pA = p(4\pi R^2) = (2.24\times10^7\,\text{Pa})4\pi(6.22\times10^{-2}\,\text{m})^2 = 1.09\times10^6\,\text{N}.$$

(d) The upward buoyant force exerted on the sphere by the seawater is

$$F_b = \rho g V = \rho g\left(\frac{4\pi}{3}R^3\right) = (1025\,\text{kg/m}^3)(9.8\,\text{m/s}^2)\frac{4\pi}{3}(6.22\times10^{-2}\,\text{m})^3 = 10.1\,\text{N}.$$

(e) Newton's second law applied to the sphere of mass $m$ = 6.80 kg yields

$$F_b - mg = ma \;\;\Rightarrow\;\; a = \frac{F_b}{m} - g = \frac{10.1\,\text{N}}{8.60\,\text{kg}} - 9.8\,\text{m/s}^2 = -8.62\,\text{m/s}^2.$$

The acceleration vector has a magnitude of 8.62 m/s$^2$ and the direction is downward.

89. (a) The total weight is

$$W = \rho g V = \rho g h A = (1030\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(255\,\text{m})(2200\,\text{m}^2) = 5.66\times10^9\,\text{N}.$$

(b) The gauge pressure at this depth is

$$p_g = \rho g h = (1030\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(255\,\text{m})\left(\frac{1\,\text{atm}}{1.01\times10^5\,\text{Pa}}\right) = 25.5\,\text{atm}.$$

90. Using Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2,$$

we find the minimum pressure to be (setting $v_1 = v_2$)

$$\Delta p = p_2 - p_1 = \rho g(y_1 - y_2) = (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(6.59\,\text{m} - 2.16\,\text{m}) = 4.34\times10^4\ \text{Pa}.$$

# Chapter 15

1. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a "turning point" (that is, when $x = +x_m$ or $x = -x_m$). Consider that it starts at $x = +x_m$ and we are told that $t = 0.25$ second elapses until the object reaches $x = -x_m$. To execute a full cycle of the motion (which takes a period $T$ to complete), the object which started at $x = +x_m$, must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = -x_m$). Thus, $T = 2t = 0.50$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T = 2.0$ Hz.

(c) The 36 cm distance between $x = +x_m$ and $x = -x_m$ is $2x_m$. Thus, $x_m = 36/2 = 18$ cm.

2. (a) The acceleration amplitude is related to the maximum force by Newton's second law: $F_{\max} = ma_m$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega$ is the angular frequency ($\omega = 2\pi f$ since there are $2\pi$ radians in one cycle). The frequency is the reciprocal of the period: $f = 1/T = 1/0.20 = 5.0$ Hz, so the angular frequency is $\omega = 10\pi$ (understood to be valid to two significant figures). Therefore,

$$F_{\max} = m\omega^2 x_m = (0.12\text{ kg})(10\pi\text{ rad/s})^2(0.085\text{ m}) = 10\text{ N}.$$

(b) Using Eq. 15-12, we obtain

$$\omega = \sqrt{\frac{k}{m}} \;\Rightarrow\; k = m\omega^2 = (0.12\text{kg})(10\pi\text{ rad/s})^2 = 1.2\times10^2\text{ N/m}.$$

3. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega$ is the angular frequency ($\omega = 2\pi f$ since there are $2\pi$ radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = \left(2\pi(6.60\text{ Hz})\right)^2(0.0220\text{ m}) = 37.8\text{ m/s}^2.$$

4. (a) Since the problem gives the frequency $f = 3.00$ Hz, we have $\omega = 2\pi f = 6\pi$ rad/s (understood to be valid to three significant figures). Each spring is considered to support one fourth of the mass $m_{\text{car}}$ so that Eq. 15-12 leads to

$$\omega = \sqrt{\frac{k}{m_{\text{car}}/4}} \;\Rightarrow\; k = \frac{1}{4}(1450\text{kg})(6\pi\text{ rad/s})^2 = 1.29\times10^5\text{ N/m}.$$

(b) If the new mass being supported by the four springs is $m_{\text{total}}$ = [1450 + 5(73)] kg = 1815 kg, then Eq. 15-12 leads to

$$\omega_{\text{new}} = \sqrt{\frac{k}{m_{\text{total}}/4}} \;\Rightarrow\; f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{1.29\times10^5 \text{ N/m}}{(1815/4)\text{ kg}}} = 2.68\text{Hz}.$$

5. **THINK** The blade of the shaver undergoes simple harmonic motion. We want to find its amplitude, maximum speed and maximum acceleration.

**EXPRESS** The amplitude $x_m$ is half the range of the displacement $D$. Once the amplitude is known, the maximum speed $v_m$ is related to the amplitude by $v_m = \omega x_m$, where $\omega$ is the angular frequency. Similarly, the maximum acceleration is $a_m = \omega^2 x_m$.

**ANALYZE** (a) The amplitude is $x_m = D/2$ = (2.0 mm)/2 = 1.0 mm.

(b) The maximum speed $v_m$ is related to the amplitude $x_m$ by $v_m = \omega x_m$, where $\omega$ is the angular frequency. Since $\omega = 2\pi f$, where $f$ is the frequency,

$$v_m = 2\pi f x_m = 2\pi(120\text{ Hz})(1.0\times10^{-3}\text{ m}) = 0.75\text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(120\text{ Hz}))^2(1.0\times10^{-3}\text{ m}) = 5.7\times10^2\text{ m/s}^2.$$

**LEARN** In SHM, acceleration is proportional to the displacement $x_m$.

6. (a) The angular frequency $\omega$ is given by $\omega = 2\pi f = 2\pi/T$, where $f$ is the frequency and $T$ is the period. The relationship $f = 1/T$ was used to obtain the last form. Thus

$$\omega = 2\pi/(1.00 \times 10^{-5}\text{ s}) = 6.28 \times 10^5\text{ rad/s}.$$

(b) The maximum speed $v_m$ and maximum displacement $x_m$ are related by $v_m = \omega x_m$, so

$$x_m = \frac{v_m}{\omega} = \frac{1.00\times10^3\text{ m/s}}{6.28\times10^5\text{ rad/s}} = 1.59\times10^{-3}\text{ m}.$$

7. **THINK** This problem compares the magnitude of the acceleration of an oscillating diaphragm in a loudspeaker to gravitational acceleration $g$.

**EXPRESS** The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where $\omega$ is the angular frequency and $x_m$ is the amplitude.

**ANALYZE** (a) The angular frequency for which the maximum acceleration has a magnitude $g$ is given by $\omega = \sqrt{g / x_m}$, so the corresponding frequency is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2}{1.0\times10^{-6}\text{m}}} = 498 \text{ Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds $g$ for some part of the motion.

**LEARN** The acceleration $a_m$ of the diaphragm in a loudspeaker increases with $\omega^2$, or equivalently, with $f^2$.

8. We note (from the graph in the text) that $x_m$ = 6.00 cm. Also the value at $t$ = 0 is $x_o$ = – 2.00 cm. Then Eq. 15-3 leads to

$$\phi = \cos^{-1}(-2.00/6.00) = +1.91 \text{ rad or } -4.37 \text{ rad}.$$

The other "root" (+4.37 rad) can be rejected on the grounds that it would lead to a positive slope at $t$ = 0.

9. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0\cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = 3.0 \text{ m}.$$

(b) Differentiating with respect to time and evaluating at $t$ = 2.0 s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left(3\pi(2.0) + \frac{\pi}{3}\right) = -49 \text{ m/s}.$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = -2.7\times10^2 \text{ m/s}^2.$$

(d) In the second paragraph after Eq. 15-3, the textbook defines the phase of the motion. In this case (with $t$ = 2.0 s) the phase is $3\pi(2.0) + \pi/3 \approx 20$ rad.

(e) Comparing with Eq. 15-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

10. (a) The problem describes the time taken to execute one cycle of the motion. The period is $T = 0.75$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T \approx 1.3$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second.

(c) Since $2\pi$ radians are equivalent to a cycle, the angular frequency $\omega$ (in radians-per-second) is related to frequency $f$ by $\omega = 2\pi f$ so that $\omega \approx 8.4$ rad/s.

11. When displaced from equilibrium, the net force exerted by the springs is $-2kx$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -2kx.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find $\omega^2 = 2k/m$, where $\omega$ is in radians per unit time. Since there are $2\pi$ radians in a cycle, and frequency $f$ measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2(7580\text{ N/m})}{0.245\text{ kg}}} = 39.6\text{ Hz}.$$

12. We note (from the graph) that $v_m = \omega x_m = 5.00$ cm/s. Also the value at $t = 0$ is $v_o = 4.00$ cm/s. Then Eq. 15-6 leads to

$$\phi = \sin^{-1}(-4.00/5.00) = -0.927 \text{ rad or } +5.36 \text{ rad}.$$

The other "root" (+4.07 rad) can be rejected on the grounds that it would lead to a positive slope at $t = 0$.

13. **THINK** The mass-spring system undergoes simple harmonic motion. Given the amplitude and the period, we can determine the corresponding frequency, angular frequency, spring constant, maximum speed and maximum force.

**EXPRESS** The angular frequency $\omega$ is given by $\omega = 2\pi f = 2\pi/T$, where $f$ is the frequency and $T$ is the period, with $f = 1/T$. The angular frequency is related to the spring constant $k$ and the mass $m$ by $\omega = \sqrt{k/m}$. The maximum speed $v_m$ is related to the amplitude $x_m$ by $v_m = \omega x_m$.

**ANALYZE** (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500\text{ s}) = 2.00$ Hz.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(2.00\text{ Hz}) = 12.6$ rad/s.

(d) We solve for the spring constant $k$ and obtain

$$k = m\omega^2 = (0.500 \text{ kg})(12.6 \text{ rad/s})^2 = 79.0 \text{ N/m}.$$

(e) The amplitude is $x_m$=35.0 cm = 0.350 m, so the maximum speed is

$$v_m = \omega x_m = (12.6 \text{ rad/s})(0.350 \text{ m}) = 4.40 \text{ m/s}.$$

(f) The maximum force is exerted when the displacement is a maximum. Thus, we have

$$F_m = kx_m = (79.0 \text{ N/m})(0.350 \text{ m}) = 27.6 \text{ N}.$$

**LEARN** With the maximum acceleration given by $a_m = \omega^2 x_m$, we see that the magnitude of the maximum force can also be written as $F_m = kx_m = m\omega^2 x_m = ma_m$ . Maximum acceleration occurs at the endpoints of the path of the block.

14. Equation 15-12 gives the angular velocity:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m}}{2.00 \text{ kg}}} = 7.07 \text{rad/s}.$$

Energy methods (discussed in Section 15-4) provide one method of solution. Here, we use trigonometric techniques based on Eq. 15-3 and Eq. 15-6.

(a) Dividing Eq. 15-6 by Eq. 15-3, we obtain

$$\frac{v}{x} = -\omega \tan(\omega t + \phi)$$

so that the phase ($\omega t + \phi$) is found from

$$\omega t + \phi = \tan^{-1}\left(\frac{-v}{\omega x}\right) = \tan^{-1}\left(\frac{-3.415 \text{ m/s}}{(7.07 \text{ rad/s})(0.129 \text{ m})}\right).$$

With the calculator in radians mode, this gives the phase equal to –1.31 rad. Plugging this back into Eq. 15-3 leads to $0.129 \text{ m} = x_m \cos(-1.31) \;\Rightarrow\; x_m = 0.500 \text{ m}.$

(b) Since $\omega t + \phi = -1.31$ rad at $t$ = 1.00 s, we can use the above value of $\omega$ to solve for the phase constant $\phi$. We obtain $\phi = -8.38$ rad (though this, as well as the previous result, can have 2π or 4π (and so on) added to it without changing the physics of the situation). With this value of $\phi$, we find $x_o = x_m \cos \phi = -0.251$ m.

(c) And we obtain $v_o = -x_m \omega \sin\phi = 3.06$ m/s.

15. **THINK** Our system consists of two particles undergoing SHM along a common straight-line segment. Their oscillations are out of phase.

**EXPRESS** Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here $T$ is the period. Note that since the range of the motion is $A$, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period.

**ANALYZE** (a) The coordinates of the particles 0.50 s later (that is, at $t = 0.50$ s) are

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\text{ s}}{1.5\text{ s}}\right) = -0.25A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\text{ s}}{1.5\text{ s}} + \frac{\pi}{6}\right) = -0.43A.$$

Their separation at that time is $\Delta x = x_1 - x_2 = -0.25A + 0.43A = 0.18A$.

(b) The velocities of the particles are given by

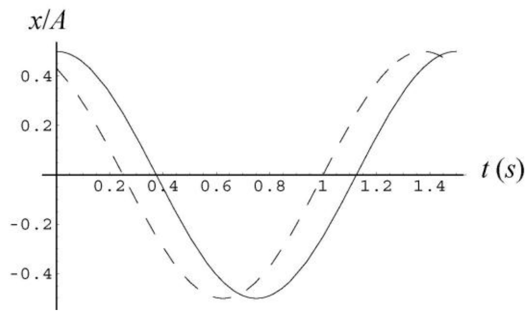
$$v_1 = \frac{dx_1}{dt} = -\frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

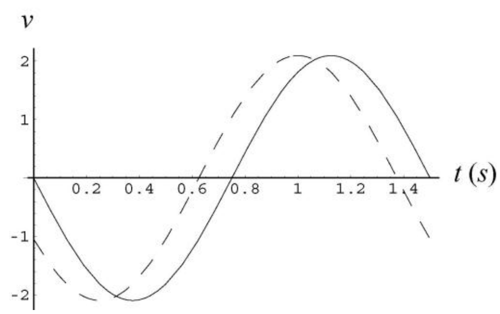
and

$$v_2 = \frac{dx_2}{dt} = -\frac{\pi A}{T}\sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right).$$

We evaluate these expressions for $t = 0.50$ s and find they are both negative-valued, indicating that the particles are moving in the same direction.

**LEARN** The plots of $x$ and $v$ as a function of time for particle 1 (so1id) and particle 2 (dashed line) are given below.




16. They pass each other at time *t*, at $x_1 = x_2 = \frac{1}{2} x_m$ where

$$x_1 = x_m \cos(\omega t + \phi_1) \quad \text{and} \quad x_2 = x_m \cos(\omega t + \phi_2).$$

From this, we conclude that $\cos(\omega t + \phi_1) = \cos(\omega t + \phi_2) = \frac{1}{2}$, and therefore that the phases (the arguments of the cosines) are either both equal to $\pi/3$ or one is $\pi/3$ while the other is $-\pi/3$. Also at this instant, we have $v_1 = -v_2 \neq 0$ where

$$v_1 = -x_m \omega \sin(\omega t + \phi_1) \quad \text{and} \quad v_2 = -x_m \omega \sin(\omega t + \phi_2).$$

This leads to $\sin(\omega t + \phi_1) = -\sin(\omega t + \phi_2)$. This leads us to conclude that the phases have opposite sign. Thus, one phase is $\pi/3$ and the other phase is $-\pi/3$; the *wt* term cancels if we take the phase difference, which is seen to be $\pi/3 - (-\pi/3) = 2\pi/3$.

17. (a) Equation 15-8 leads to

$$a = -\omega^2 x \;\Rightarrow\; \omega = \sqrt{\frac{-a}{x}} = \sqrt{\frac{123\text{ m/s}^2}{0.100\text{ m}}} = 35.07\text{ rad/s}.$$

Therefore, $f = \omega/2\pi = 5.58$ Hz.

(b) Equation 15-12 provides a relation between $\omega$ (found in the previous part) and the mass:

$$\omega = \sqrt{\frac{k}{m}} \;\Rightarrow\; m = \frac{400\text{ N/m}}{(35.07\text{ rad/s})^2} = 0.325\text{kg}.$$

(c) By energy conservation, $\frac{1}{2}kx_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time *t* described in the problem.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow x_m = \frac{m}{k}v^2 + x^2.$$

Consequently, $x_m = \sqrt{(0.325\text{ kg}/400\text{ N/m})(13.6\text{ m/s})^2 + (0.100\text{ m})^2} = 0.400$ m.

18. From highest level to lowest level is twice the amplitude $x_m$ of the motion. The period is related to the angular frequency by Eq. 15-5. Thus, $x_m = \frac{1}{2}d$ and $\omega = 0.503$ rad/h. The phase constant $\phi$ in Eq. 15-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for $t$ when $x$ is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek $t$ when the ocean surface is at $x = \frac{1}{2}x_m = \frac{1}{4}d$. With $x = x_m \cos(\omega t + \phi)$, we obtain

$$\frac{1}{4}d = \left(\frac{1}{2}d\right)\cos\left(0.503t + 0\right) \Rightarrow \frac{1}{2} = \cos(0.503t)$$

which has $t = 2.08$ h as the smallest positive root. The calculator is in radians mode during this calculation.

19. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega$ is the angular frequency; this is the expression we set equal to $g = 9.8$ m/s$^2$.

(a) Using Eq. 15-5 and $T = 1.0$ s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m = g \Rightarrow x_m = \frac{gT^2}{4\pi^2} = 0.25 \text{ m.}$$

(b) Since $\omega = 2\pi f$, and $x_m = 0.050$ m is given, we find

$$\left(2\pi f\right)^2 x_m = g \quad \Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2 \text{ Hz.}$$

20. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where $\omega = 1.20$ rad/s in this problem. Evaluating this at $t = 0$ and using the values from the graphs shown in the problem, we find

$$\phi = \tan^{-1}\left(\frac{-v_0}{x_0\omega}\right) = \tan^{-1}\left(\frac{+4.00 \text{ cm/s}}{(2.0 \text{ cm})(1.20 \text{ rad/s})}\right) = 1.03 \text{ rad (or } -5.25 \text{ rad).}$$

One can check that the other "root" (4.17 rad) is unacceptable since it would give the wrong signs for the individual values of $v_0$ and $x_0$.

21. Let the spring constants be $k_1$ and $k_2$. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1 x + k_2 x|$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/d^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -k_1x - k_2x.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{k_1 + k_2}{m}$$

where $\omega$ is in radians per unit time. Since there are $2\pi$ radians in a cycle, and frequency $f$ measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}.$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = 30 \text{ Hz}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = 45 \text{ Hz},$$

respectively. Comparing these expressions, it is clear that

$$f = \sqrt{f_1^2 + f_2^2} = \sqrt{(30 \text{ Hz})^2 + (45 \text{ Hz})^2} = 54 \text{ Hz}.$$

22. The statement that "the spring does not affect the collision" justifies the use of elastic collision formulas in section 10-5. We are told the period of SHM so that we can find the mass of block 2:

$$T = 2\pi\sqrt{\frac{m_2}{k}} \quad \Rightarrow \quad m_2 = \frac{kT^2}{4\pi^2} = 0.600 \text{ kg}.$$

At this point, the rebound speed of block 1 can be found from Eq. 10-30:

$$|v_{1f}| = \left|\frac{0.200 \text{ kg} - 0.600 \text{ kg}}{0.200 \text{ kg} + 0.600 \text{ kg}}\right|(8.00 \text{ m/s}) = 4.00 \text{ m/s}.$$

This becomes the initial speed $v_0$ of the projectile motion of block 1. A variety of choices for the positive axis directions are possible, and we choose left as the $+x$ direction and down as the $+y$ direction, in this instance. With the "launch" angle being zero, Eq. 4-21 and Eq. 4-22 (with $-g$ replaced with $+g$) lead to

$$x - x_0 = v_0 t = v_0\sqrt{\frac{2h}{g}} = (4.00 \text{ m/s})\sqrt{\frac{2(4.90 \text{ m})}{9.8 \text{ m/s}^2}}.$$

Since $x - x_0 = d$, we arrive at $d = 4.00$ m.

23. **THINK** The maximum force that can be exerted by the surface must be less than the static frictional force or else the block will not follow the surface in its motion.

**EXPRESS** The static frictional force is given by $f_s = \mu_s F_N$, where $\mu_s$ is the coefficient of static friction and $F_N$ is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where $m$ is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by

$$F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m,$$

where $a_m$ is the magnitude of the maximum acceleration, $\omega$ is the angular frequency, and $f$ is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form.

**ANALYZE** We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\ \text{m/s}^2)}{(2\pi \times 2.0\ \text{Hz})^2} = 0.031\ \text{m}.$$

**LEARN** A larger amplitude would require a larger force at the end points of the motion. The block slips if the surface cannot supply a larger force.

24. We wish to find the effective spring constant for the combination of springs shown in the figure. We do this by finding the magnitude $F$ of the force exerted on the mass when the total elongation of the springs is $\Delta x$. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by $\Delta x_\ell$ and the right-hand spring is elongated by $\Delta x_r$. The left-hand spring exerts a force of magnitude $k\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same, and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k\Delta x_\ell$. Thus,

$$k_{\text{eff}} = k\Delta x_\ell / 2\Delta x_r = k/2.$$

The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion, replace $k_{\text{eff}}$ in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}.$$

With $m = 0.245$ kg and $k = 6430$ N/m, the frequency is $f = 18.2$ Hz.

25. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is *x*, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \;\;\Rightarrow\;\; x = \frac{mg\sin\theta}{k} = \frac{(14.0\text{ N})\sin 40.0°}{120\text{ N/m}} = 0.0750\text{ m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore (0.450 + 0.75) m = 0.525 m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 15-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0\text{ N}/9.80\text{ m/s}^2}{120\text{ N/m}}} = 0.686\text{ s.}$$

26. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{max} = \mu_s mg$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega = \sqrt{k/(m+M)}$ is the angular frequency (from Eq. 15-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \Rightarrow \frac{k}{m+M}x_m = \mu_s g$$

which leads to

$$x_m = \frac{\mu_s g(m+M)}{k} = \frac{(0.40)(9.8\text{ m/s}^2)(1.8\text{ kg} + 10\text{ kg})}{200\text{ N/m}} = 0.23\text{ m} = 23\text{ cm.}$$

27. **THINK** This problem explores the relationship between energies, both kinetic and potential, with amplitude in SHM.

**EXPRESS** In simple harmonic motion, let the displacement be

$$x(t) = x_m\cos(\omega t + \phi).$$

The corresponding velocity is

$$v(t) = dx/dt = -\omega x_m\sin(\omega t + \phi)\,.$$

Using the expressions for $x(t)$ and $v(t)$, we find the potential and kinetic energies to be

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_m^2\cos^2(\omega t + \phi)$$
$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t + \phi) = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

where $k = m\omega^2$ is the spring constant and $x_m$ is the amplitude. The total energy is

$$E = U(t) + K(t) = \frac{1}{2}kx_m^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right] = \frac{1}{2}kx_m^2.$$

**ANALYZE** (a) The condition $x(t) = x_m/2$ implies that $\cos(\omega t + \phi) = 1/2$, or $\sin(\omega t + \phi) = \sqrt{3}/2$. Thus, the fraction of energy that is kinetic is

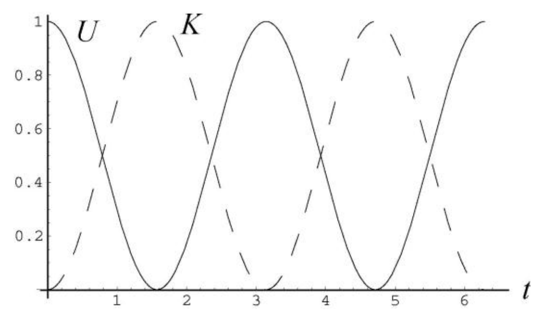
$$\frac{K}{E} = \sin^2(\omega t + \phi) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}.$$

(b) Similarly, we have $\frac{U}{E} = \cos^2(\omega t + \phi) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

(c) Since $E = \frac{1}{2}kx_m^2$ and $U = \frac{1}{2}kx(t)^2$, $U/E = x^2/x_m^2$. Solving $x^2/x_m^2 = 1/2$ for $x$, we get $x = x_m/\sqrt{2}$.

**LEARN** The figure to the right depicts the potential energy (solid line) and kinetic energy (dashed line) as a function of time, assuming $x(0) = x_m$. The curves intersect when $K = U = E/2$, or equivalently,

$$\cos^2 \omega t = \sin^2 \omega t = 1/2.$$




28. The total mechanical energy is equal to the (maximum) kinetic energy as it passes through the equilibrium position ($x$ = 0):

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}(2.0\text{ kg})(0.85\text{ m/s})^2 = 0.72\text{ J}.$$

Looking at the graph in the problem, we see that $U(x = 10) = 0.5$ J. Since the potential function has the form $U(x) = bx^2$, the constant is $b = 5.0\times10^{-3}\text{ J/cm}^2$. Thus, $U(x) = 0.72$ J when $x$ = 12 cm.

(a) Thus, the mass does turn back before reaching $x$ = 15 cm.

(b) It turns back at $x$ = 12 cm.

29. **THINK** Knowing the amplitude and the spring constant, we can calculate the mechanical energy of the mass-spring system in simple harmonic motion.

**EXPRESS** In simple harmonic motion, let the displacement be $x(t) = x_m \cos(\omega t + \phi)$. The corresponding velocity is

$$v(t) = dx/dt = -\omega x_m \sin(\omega t + \phi).$$

Using the expressions for $x(t)$ and $v(t)$, we find the potential and kinetic energies to be

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$$
$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2 x_m^2 \sin^2(\omega t + \phi) = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

where $k = m\omega^2$ is the spring constant and $x_m$ is the amplitude. The total energy is

$$E = U(t) + K(t) = \frac{1}{2}kx_m^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right] = \frac{1}{2}kx_m^2.$$

**ANALYZE** With $k = 1.3\ \text{N/cm} = 130\ \text{N/m}$ and $x_m = 2.4\ \text{cm} = 0.024\ \text{m}$, the mechanical energy is

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}\left(1.3\times10^2\ \text{N/m}\right)\left(0.024\ \text{m}\right)^2 = 3.7\times10^{-2}\ \text{J}.$$

**LEARN** An alternative to calculate $E$ is to note that when the block is at the end of its path and is momentarily stopped ($v = 0 \Rightarrow K = 0$), its displacement is equal to the amplitude and all the energy is potential in nature ($E = U + K = U$). With the spring potential energy taken to be zero when the block is at its equilibrium position, we recover the expression $E = kx_m^2/2$.

30. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where $E$ = 1.00 J and $x_m$ = 0.100 m. Thus,

$$k = \frac{2E}{x_m^2} = 200\ \text{N/m}.$$

(b) The energy as the block passes through the equilibrium position (with speed $v_m$ = 1.20 m/s) is purely kinetic:

$$E = \frac{1}{2}mv_m^2 \quad \Rightarrow \quad m = \frac{2E}{v_m^2} = 1.39\ \text{kg}.$$

(c) Equation 15-12 (divided by $2\pi$) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.91\,\text{Hz}.$$

31. (a) Equation 15-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000\text{ N/m}}{5.00\text{ kg}}} = 2.25\,\text{Hz}.$$

(b) With $x_0$ = 0.500 m, we have $U_0 = \frac{1}{2}kx_0^2 = 125\text{ J}$.

(c) With $v_0$ = 10.0 m/s, the initial kinetic energy is $K_0 = \frac{1}{2}mv_0^2 = 250\text{ J}$.

(d) Since the total energy $E = K_0 + U_0$ = 375 J is conserved, then consideration of the energy at the turning point leads to

$$E = \frac{1}{2}kx_m^2 \Rightarrow x_m = \sqrt{\frac{2E}{k}} = 0.866\text{ m}.$$

32. We infer from the graph (since mechanical energy is conserved) that the *total* energy in the system is 6.0 J; we also note that the amplitude is apparently $x_m$ = 12 cm = 0.12 m. Therefore we can set the maximum *potential* energy equal to 6.0 J and solve for the spring constant $k$:

$$\tfrac{1}{2}k\,x_m^2 = 6.0\text{ J} \quad\Rightarrow\quad k = 8.3\times10^2\text{ N/m}.$$

33. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m + M$ attached to a spring of spring constant $k$).

(a) Momentum conservation readily yields $v' = mv/(m + M)$. With $m$ = 9.5 g, $M$ = 5.4 kg, and $v$ = 630 m/s, we obtain $v' = 1.1\text{ m/s}$.

(b) Since $v'$ occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for $x_m$, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\frac{1}{2}(m+M)v'^2 = \frac{1}{2}kx_m^2 \quad\Rightarrow\quad \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \frac{1}{2}kx_m^2$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} = \frac{(9.5\times10^{-3}\text{kg})(630\text{ m/s})}{\sqrt{(6000\text{ N/m})(9.5\times10^{-3}\text{kg} + 5.4\text{kg})}} = 3.3\times10^{-2}\text{ m}.$$

34. We note that the spring constant is

$$k = 4\pi^2 m_1/T^2 = 1.97 \times 10^5 \text{ N/m}.$$

It is important to determine where in its simple harmonic motion (which "phase" of its motion) block 2 is when the impact occurs. Since $\omega = 2\pi/T$ and the given value of $t$ (when the collision takes place) is one-fourth of $T$, then $\omega t = \pi/2$ and the location then of block 2 is $x = x_m\cos(\omega t + \phi)$ where $\phi = \pi/2$ which gives

$$x = x_m\cos(\pi/2 + \pi/2) = -x_m.$$

This means block 2 is at a turning point in its motion (and thus has zero speed right before the impact occurs); this means, too, that the spring is stretched an amount of 1 cm = 0.01 m at this moment. To calculate its after-collision speed (which will be the same as that of block 1 right after the impact, since they stick together in the process) we use momentum conservation and obtain

$$v = (4.0 \text{ kg})(6.0 \text{ m/s})/(6.0 \text{ kg}) = 4.0 \text{ m/s}.$$

Thus, at the end of the impact itself (while block 1 is still at the same position as before the impact) the system (consisting now of a total mass $M = 6.0$ kg) has kinetic energy

$$K = \frac{1}{2}(6.0 \text{ kg})(4.0 \text{ m/s})^2 = 48 \text{ J}$$

and potential energy

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(1.97 \times 10^5 \text{ N/m})(0.010 \text{ m})^2 \approx 10 \text{ J},$$

meaning the total mechanical energy in the system at this stage is approximately $E = K + U = 58$ J. When the system reaches its new turning point (at the new amplitude $X$ ) then this amount must equal its (maximum) potential energy there: $E = \frac{1}{2}(1.97 \times 10^5 \text{ N/m})\, X^2$. Therefore, we find

$$X = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(58 \text{ J})}{1.97\times10^5 \text{ N/m}}} = 0.024 \text{ m}.$$

35. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega$ is the angular frequency and $x_m = 0.0020$ m is the amplitude. Thus, $a_m = 8000 \text{ m/s}^2$ leads to $\omega = 2000$ rad/s. Using Newton's second law with $m = 0.010$ kg, we have

$$F = ma = m\left(-a_m\cos(\omega t + \phi)\right) = -(80 \text{ N})\cos\left(2000t - \frac{\pi}{3}\right)$$

where $t$ is understood to be in seconds.

(a) Equation 15-5 gives $T = 2\pi/\omega = 3.1 \times 10^{-3}$ s.

(b) The relation $v_m = \omega x_m$ can be used to solve for $v_m$, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter. By Eq. 15-12, the spring constant is $k = \omega^2 m = 40000$ N/m. Then, energy conservation leads to

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \quad \Rightarrow \quad v_m = x_m\sqrt{\frac{k}{m}} = 4.0 \text{ m/s}.$$

(c) The total energy is $\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 = 0.080$ J.

(d) At the maximum displacement, the force acting on the particle is

$$F = kx = (4.0\times10^4 \text{ N/m})(2.0\times10^{-3}\text{m}) = 80 \text{ N}.$$

(e) At half of the maximum displacement, $x = 1.0$ mm, and the force is

$$F = kx = (4.0\times10^4 \text{ N/m})(1.0\times10^{-3}\text{m}) = 40 \text{ N}.$$

36. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where $\omega$ is given by Eq. 15-12. Since the kinetic energy is $\frac{1}{2}mv^2$ and the potential energy is $\frac{1}{2}kx^2$ (which may be conveniently written as $\frac{1}{2}m\omega^2x^2$) then the ratio of kinetic to potential energy is simply

$$(v/x)^2/\omega^2 = \tan^2(\omega t + \phi),$$

which at $t = 0$ is $\tan^2\phi$. Since $\phi = \pi/6$ in this problem, then the ratio of kinetic to potential energy at $t = 0$ is $\tan^2(\pi/6) = 1/3$.

37. (a) The object oscillates about its equilibrium point, where the downward force of gravity is balanced by the upward force of the spring. If $\ell$ is the elongation of the spring at equilibrium, then $k\ell = mg$, where $k$ is the spring constant and $m$ is the mass of the object. Thus $k/m = g/\ell$ and

$$f = \omega/2\pi = (1/2\pi)\sqrt{k/m} = (1/2\pi)\sqrt{g/\ell}.$$

Now the equilibrium point is halfway between the points where the object is momentarily at rest. One of these points is where the spring is unstretched and the other is the lowest point, 10 cm below. Thus $\ell = 5.0\,\text{cm} = 0.050\,\text{m}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2}{0.050 \text{ m}}} = 2.2 \text{ Hz}.$$

(b) Use conservation of energy. We take the zero of gravitational potential energy to be at the initial position of the object, where the spring is unstretched. Then both the initial potential and kinetic energies are zero. We take the *y*-axis to be positive in the downward direction and let $y$ = 0.080 m. The potential energy when the object is at this point is $U = \frac{1}{2}ky^2 - mgy$. The energy equation becomes

$$0 = \tfrac{1}{2}ky^2 - mgy + \tfrac{1}{2}mv^2 .$$

We solve for the speed:

$$v = \sqrt{2gy - \frac{k}{m}y^2} = \sqrt{2gy - \frac{g}{\ell}y^2} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.080\,\text{m}\right) - \left(\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}\right)\left(0.080\,\text{m}\right)^2}$$
$$= 0.56\,\text{m/s}$$

(c) Let $m$ be the original mass and $\Delta m$ be the additional mass. The new angular frequency is $\omega' = \sqrt{k/(m+\Delta m)}$. This should be half the original angular frequency, or $\frac{1}{2}\sqrt{k/m}$. We solve

$$\sqrt{k/(m+\Delta m)} = \tfrac{1}{2}\sqrt{k/m}$$

for $m$. Square both sides of the equation, then take the reciprocal to obtain $m + \Delta m = 4m$. This gives

$$m = \Delta m/3 = (300\text{ g})/3 = 100\text{ g} = 0.100\text{ kg}.$$

(d) The equilibrium position is determined by the balancing of the gravitational and spring forces: $ky = (m + \Delta m)g$. Thus $y = (m + \Delta m)g/k$. We will need to find the value of the spring constant $k$ using $k = m\omega^2 = m(2\pi f)^2$. Then

$$y\frac{\left(m+\Delta m\right)g}{m\left(2\pi f\right)^2} = \frac{\left(0.100\text{ kg}+0.300\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(0.100\text{ kg}\right)\left(2\pi\times 2.24\text{ Hz}\right)^2} = 0.200\text{ m.}$$

This is measured from the initial position.

38. From Eq. 15-23 (in absolute value) we find the torsion constant:

$$\kappa = \left|\frac{\tau}{\theta}\right| = \frac{0.20\text{ N}\cdot\text{m}}{0.85\text{ rad}} = 0.235\ \text{N}\cdot\text{m/rad}.$$

With $I = 2mR^2/5$ (the rotational inertia for a solid sphere — from Chapter 11), Eq. 15–23 leads to

$$T = 2\pi\sqrt{\frac{\frac{2}{5}mR^2}{\kappa}} = 2\pi\sqrt{\frac{\frac{2}{5}\left(95\text{ kg}\right)\left(0.15\text{ m}\right)^2}{0.235\ \text{N}\cdot\text{m/rad}}} = 12\text{ s.}$$

39. **THINK** The balance wheel in the watch undergoes angular simple harmonic oscillation. From the amplitude and period, we can calculate the corresponding angular velocity and angular acceleration.

**EXPRESS** We take the angular displacement of the wheel to be $\theta(t) = \theta_m \cos(2\pi t/T)$, where $\theta_m$ is the amplitude and $T$ is the period. We differentiate with respect to time to find the angular velocity:

$$\Omega = d\theta/dt = -(2\pi/T)\theta_m \sin(2\pi t/T).$$

The symbol $\Omega$ is used for the angular velocity of the wheel so it is not confused with the angular frequency.

**ANALYZE** (a) The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi \text{ rad})}{0.500 \text{ s}} = 39.5 \text{ rad/s}.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1-\cos^2(2\pi t/T)} = \sqrt{1-(1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500 \text{ s}}\right)(\pi \text{ rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2 \text{ rad/s}.$$

During another portion of the cycle its angular speed is +34.2 rad/s when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

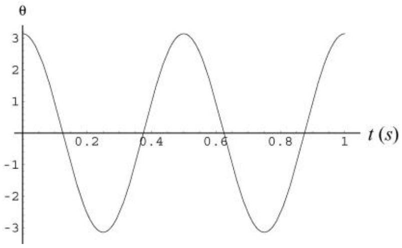
$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta.$$

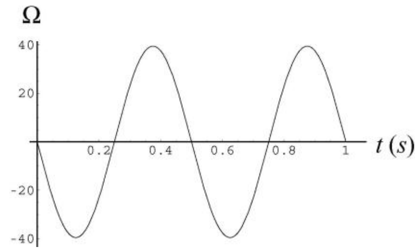
When $\theta = \pi/4$,

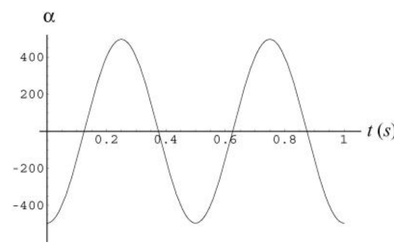
$$\alpha = -\left(\frac{2\pi}{0.500 \text{ s}}\right)^2 \left(\frac{\pi}{4}\right) = -124 \text{ rad/s}^2,$$

or $|\alpha| = 124 \text{ rad/s}^2$.

**LEARN** The angular displacement, angular velocity and angular acceleration as a function of time are plotted next.




40. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$, the unknown. For a meter stick of mass $m$, the rotational inertia about its center of mass is $I_{cm} = mL^2/12$ where $L$ = 1.0 m. Thus, for $T$ = 2.5 s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}}.$$

Squaring both sides and solving for $d$ leads to the quadratic formula:

$$d = \frac{g(T/2\pi)^2 \pm \sqrt{d^2(T/2\pi)^4 - L^2/3}}{2}.$$

Choosing the plus sign leads to an impossible value for $d$ ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: $d$ = 0.056 m.

41. **THINK** Our physical pendulum consists of a disk and a rod. To find the period of oscillation, we first calculate the moment of inertia and the distance between the center-of-mass of the disk-rod system to the pivot.

**EXPRESS** A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}Mr^2$, where $M$ is its mass and $r$ is its radius. The disk of this problem rotates about a point that is displaced from its center by $r + L$, where $L$ is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}Mr^2 + \frac{1}{2}M(L+r)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where $m$ is its mass.

**ANALYZE** (a) The total rotational inertia of the disk and rod is

$$\begin{aligned} I &= \frac{1}{2}Mr^2 + M(L+r)^2 + \frac{1}{3}mL^2 \\ &= \frac{1}{2}(0.500\,\text{kg})(0.100\,\text{m})^2 + (0.500\,\text{kg})(0.500\,\text{m} + 0.100\,\text{m})^2 + \frac{1}{3}(0.270\,\text{kg})(0.500\,\text{m})^2 \\ &= 0.205\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + r = 0.500\text{ m} + 0.100\text{ m} = 0.600\text{ m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\text{ m})/2 = 0.250\text{ m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M+m} = \frac{(0.500\text{ kg})(0.600\text{ m}) + (0.270\text{ kg})(0.250\text{ m})}{0.500\text{ kg} + 0.270\text{ kg}} = 0.477\text{ m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\text{ kg}\cdot\text{m}^2}{(0.500\text{ kg} + 0.270\text{ kg})(9.80\text{ m/s}^2)(0.447\text{ m})}} = 1.50\text{ s}.$$

**LEARN** Consider the limit where $M \to 0$ (i.e., uniform disk removed). In this case, $I = mL^2/3$, $d = \ell_r = L/2$ and the period of oscillation becomes

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{mL^2/3}{mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which is the result given in Eq. 15-32.

42. (a) Comparing the given expression to Eq. 15-3 (after changing notation $x \to \theta$), we see that $\omega = 4.43$ rad/s. Since $\omega = \sqrt{g/L}$ then we can solve for the length: $L = 0.499$ m.

(b) Since $v_m = \omega x_m = \omega L\theta_m = (4.43\text{ rad/s})(0.499\text{ m})(0.0800\text{ rad})$ and $m = 0.0600$ kg, then we can find the maximum kinetic energy: $\frac{1}{2}mv_m^2 = 9.40 \times 10^{-4}$ J.

43. (a) Referring to Sample Problem 15.5 – "Physical pendulum, period and length," we see that the distance between $P$ and $C$ is $h = \frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. The parallel axis theorem (see Eq. 15–30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = \left(\frac{1}{12} + \frac{1}{36}\right)mL^2 = \frac{1}{9}mL^2.$$

Equation 15-29 then gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L^2/9}{gL/6}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which yields $T = 1.64$ s for $L = 1.00$ m.

(b) We note that this $T$ is identical to that computed in Sample Problem 15.5 – "Physical pendulum, period and length." As far as the characteristics of the periodic motion are concerned, the center of oscillation provides a pivot that is equivalent to that chosen in the Sample Problem (pivot at the edge of the stick).

44. To use Eq. 15-29 we need to locate the center of mass and we need to compute the rotational inertia about $A$. The center of mass of the stick shown horizontal in the figure is at $A$, and the center of mass of the other stick is 0.50 m below $A$. The two sticks are of equal mass, so the center of mass of the system is $h = \frac{1}{2}(0.50\text{ m}) = 0.25\text{m}$ below $A$, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia $I_1$ of the stick shown horizontal in the figure and the rotational inertia $I_2$ of the stick shown vertical. Thus, we have

$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where $L$ = 1.00 m and $M$ is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 15-29 yields

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, $T$ = 1.83 s.

45. From Eq. 15-28, we find the length of the pendulum when the period is $T$ = 8.85 s:

$$L = \frac{gT^2}{4\pi^2}.$$

The new length is $L' = L - d$ where $d$ = 0.350 m. The new period is

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{L}{g} - \frac{d}{g}} = 2\pi\sqrt{\frac{T^2}{4\pi^2} - \frac{d}{g}}$$

which yields $T'$ = 8.77 s.

46. We require

$$T = 2\pi\sqrt{\frac{L_o}{g}} = 2\pi\sqrt{\frac{I}{mgh}}$$

similar to the approach taken in part (b) of Sample Problem 15.5 – "Physical pendulum, period and length," but treating in our case a more general possibility for $I$. Canceling 2π, squaring both sides, and canceling $g$ leads directly to the result; $L_o = I/mh$.

47. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$. For a solid disk of mass $m$, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + md^2}{mgd}} = 2\pi\sqrt{\frac{R^2 + 2d^2}{2gd}} = 2\pi\sqrt{\frac{(2.35\text{ cm})^2 + 2(1.75\text{ cm})^2}{2(980\text{ cm/s}^2)(1.75\text{ cm})}} = 0.366\text{ s.}$$

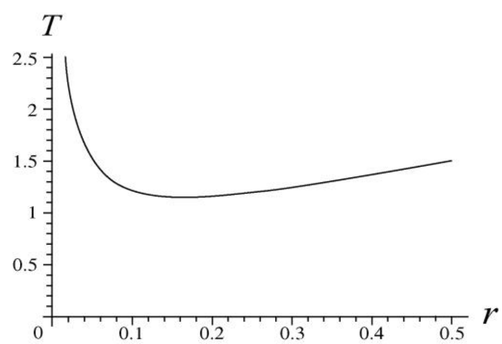
48. (a) For the "physical pendulum" we have

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{com} + mh^2}{mgh}} .$$

If we substitute $r$ for $h$ and use item (i) in Table 10-2, we have

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{a^2 + b^2}{12r} + r} .$$

In the figure below, we plot $T$ as a function of $r$, for $a = 0.35$ m and $b = 0.45$ m.




(b) The minimum of $T$ can be located by setting its derivative to zero, $dT/dr = 0$. This yields

$$r = \sqrt{\frac{a^2 + b^2}{12}} = \sqrt{\frac{(0.35\text{ m})^2 + (0.45\text{ m})^2}{12}} = 0.16\text{ m.}$$

(c) The direction from the center does not matter, so the locus of points is a circle around the center, of radius $[(a^2 + b^2)/12]^{1/2}$.

49. Replacing $x$ and $v$ in Eq. 15-3 and Eq. 15-6 with $\theta$ and $d\theta/dt$, respectively, we identify 4.44 rad/s as the angular frequency $\omega$. Then we evaluate the expressions at $t = 0$ and divide the second by the first:

$$\left(\frac{d\theta/dt}{\theta}\right)_{\text{at } t=0} = -\,\omega\tan\phi .$$

(a) The value of $\theta$ at $t = 0$ is 0.0400 rad, and the value of $d\theta/dt$ then is –0.200 rad/s, so we are able to solve for the phase constant:

$$\phi = \tan^{-1}[0.200/(0.0400 \text{ x } 4.44)] = 0.845 \text{ rad.}$$

(b) Once $\phi$ is determined we can plug back in to $\theta_o = \theta_m \cos\phi$ to solve for the angular amplitude. We find $\theta_m = 0.0602$ rad.

50. (a) The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = 1/3ML^2$. Therefore, Eq. 15-29 leads to

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} \quad \Rightarrow \quad L = \frac{3gT^2}{8\pi^2} = \frac{3(9.8 \text{ m/s}^2)(1.5 \text{ s})^2}{8\pi^2} = 0.84 \text{ m.}$$

(b) By energy conservation

$$E_{\text{bottom of swing}} = E_{\text{end of swing}} \quad \Rightarrow \quad K_m = U_m$$

where $U = Mg\ell(1-\cos\theta)$ with $\ell$ being the distance from the axis of rotation to the center of mass. If we use the small-angle approximation ($\cos\theta \approx 1 - \frac{1}{2}\theta^2$ with $\theta$ in radians (Appendix E)), we obtain

$$U_m = (0.5 \text{ kg})(9.8 \text{ m/s}^2)\left(\frac{L}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

where $\theta_m = 0.17$ rad. Thus, $K_m = U_m = 0.031$ J. If we calculate $(1 - \cos\theta)$ directly (without using the small angle approximation) then we obtain within 0.3% of the same answer.

51. This is similar to the situation treated in Sample Problem 15.5 — "Physical pendulum, period and length," except that $O$ is no longer at the end of the stick. Referring to the center of mass as $C$ (assumed to be the geometric center of the stick), we see that the distance between $O$ and $C$ is $h = x$. The parallel axis theorem (see Eq. 15-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right).$$

Equation 15-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi\sqrt{\frac{\left(L^2 + 12x^2\right)}{12gx}}.$$

(a) Minimizing $T$ by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with $12gT^2/2\pi$ instead of $T$ (it should be clear that $12gT^2/2\pi$ is a minimum whenever $T$ is a minimum). The result is

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x}+12x\right)}{dx} = -\frac{L^2}{x^2}+12$$

which yields $x = L/\sqrt{12} = (1.85\text{ m})/\sqrt{12} = 0.53\text{ m}$ as the value of $x$ that should produce the smallest possible value of $T$.

(b) With $L$ = 1.85 m and $x$ = 0.53 m, we obtain $T$ = 2.1 s from the expression derived in part (a).

52. Consider that the length of the spring as shown in the figure (with one of the block's corners lying directly above the block's center) is some value $L$ (its rest length). If the (constant) distance between the block's center and the point on the wall where the spring attaches is a distance $r$, then $r\cos\theta = d/\sqrt{2}$, and $r\cos\theta = L$ defines the angle $\theta$ measured from a line on the block drawn from the center to the top corner to the line of $r$ (a straight line from the center of the block to the point of attachment of the spring on the wall). In terms of this angle, then, the problem asks us to consider the dynamics that results from increasing $\theta$ from its original value $\theta_o$ to $\theta_o + 3^o$ and then releasing the system and letting it oscillate. If the new (stretched) length of spring is $L'$ (when $\theta = \theta_o + 3^o$), then it is a straightforward trigonometric exercise to show that

$$(L')^2 = r^2 + (d/\sqrt{2}\,)^2 - 2r(d/\sqrt{2}\,)\cos(\theta_o + 3^o) = L^2 + d^2 - d^2\cos(3^o) + \sqrt{2}\,Ld\sin(3^o)$$

since $\theta_o$ = 45º. The difference between $L'$ (as determined by this expression) and the original spring length $L$ is the amount the spring has been stretched (denoted here as $x_m$). If one plots $x_m$ versus $L$ over a range that seems reasonable considering the figure shown in the problem (say, from $L$ = 0.03 m to $L$ = 0.10 m) one quickly sees that $x_m \approx 0.00222$ m is an excellent approximation (and is very close to what one would get by approximating $x_m$ as the arc length of the path made by that upper block corner as the block is turned through 3º, even though this latter procedure should in principle overestimate $x_m$). Using this value of $x_m$ with the given spring constant leads to a potential energy of $U = \frac{1}{2}k\,x_m^{\,2}$ = 0.00296 J. Setting this equal to the kinetic energy the block has as it passes back through the initial position, we have

$$K = 0.00296\text{ J} = \tfrac{1}{2}\,I\,\omega_m^{\,2}$$

where $\omega_m$ is the maximum angular speed of the block (and is not to be confused with the angular frequency $\omega$ of the oscillation, though they are related by $\omega_m = \theta_o\omega$ if $\theta_o$ is expressed in radians). The rotational inertia of the block is $I = \frac{1}{6}Md^2$ = 0.0018 kg·m$^2$. Thus, we can solve the above relation for the maximum angular speed of the block:

$$\omega_m = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(0.00296 \text{ J})}{0.0018 \text{ kg}\cdot\text{m}^2}} = 1.81 \text{ rad/s}.$$

Therefore the angular frequency of the oscillation is $\omega = \omega_m/\theta_o$ = 34.6 rad/s. Using Eq. 15-5, then, the period is $T$ = 0.18 s.

53. **THINK** By assuming that the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and that the torque tends to pull the rod toward its equilibrium orientation, we see that the rod will oscillate in simple harmonic motion.

**EXPRESS** Let $\tau = -C\theta$, where $\tau$ is the torque, $\theta$ is the angle of rotation, and $C$ is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}},$$

where $I$ is the rotational inertia of the rod. The plan is to find the torque as a function of $\theta$ and identify the constant $C$ in terms of given quantities. This immediately gives the period in terms of given quantities. Let $\ell_0$ be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle $\theta$, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved a distance $(L/2)(1 - \cos\theta)$ to the right. The length of the spring is now

$$\ell = \sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}.$$

If the angle $\theta$ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with $\theta$ in radians. Then the length of the spring is given by $\ell \approx \ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\Delta x = kL\theta/2$. Since $\theta$ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus, $\tau = -(kL^2/4)\theta$. The constant of proportionality $C$ that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$ (see Table 10-2), where $m$ is its mass.

**ANALYZE** Substituting the expressions for $C$ and $I$, we find the period of oscillation to be

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}.$$

With $m$ = 0.600 kg and $k$ = 1850 N/m, we obtain $T$ = 0.0653 s.

**LEARN** As in the case of a simple linear harmonic oscillator formed by a mass and a spring, the period of the rotating rod is inversely proportional to $\sqrt{k}$. Our result indicates that the rod oscillates very rapidly, with a frequency $f = 1/T = 15.3\,\text{Hz}$, i.e., about 15 times in one second.

54. We note that the initial angle is $\theta_o$ = 7º = 0.122 rad (though it turns out this value will cancel in later calculations). If we approximate the initial stretch of the spring as the arc-length that the corresponding point on the plate has moved through ($x = r\,\theta_o$ where $r$ = 0.025 m) then the initial potential energy is approximately $\frac{1}{2}kx^2$ = 0.0093 J. This should equal to the kinetic energy of the plate ($\frac{1}{2}I\,\omega_m^2$ where this $\omega_m$ is the maximum angular speed of the plate, not the angular frequency ω). Noting that the maximum angular speed of the plate is $\omega_m = \omega\theta_o$ where $\omega = 2\pi/T$ with $T$ = 20 ms = 0.02 s as determined from the graph, then we can find the rotational inertial from $\frac{1}{2}\,I\omega_m^2$ = 0.0093 J. Thus, $I = 1.3\times10^{-5}\ \text{kg}\cdot\text{m}^2$.

55. (a) The period of the pendulum is given by $T = 2\pi\sqrt{I/mgd}$, where $I$ is its rotational inertia, $m$ = 22.1 g is its mass, and $d$ is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ with $L$ = 2.20 m. According to the parallel-axis theorem, its rotational inertia when it is pivoted a distance $d$ from the center is $I = mL^2/12 + md^2$. Thus,

$$T = 2\pi\sqrt{\frac{m(L^2/12 + d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}}.$$

Minimizing $T$ with respect to $d$, $dT/d(d) = 0$, we obtain $d = L/\sqrt{12}$. Therefore, the minimum period $T$ is

$$T_{\min} = 2\pi\sqrt{\frac{L^2 + 12(L/\sqrt{12})^2}{12g(L/\sqrt{12})}} = 2\pi\sqrt{\frac{2L}{\sqrt{12}g}} = 2\pi\sqrt{\frac{2(2.20\text{ m})}{\sqrt{12}(9.80\text{ m/s}^2)}} = 2.26\text{ s.}$$

(b) If $d$ is chosen to minimize the period, then as $L$ is increased the period will increase as well.

(c) The period does not depend on the mass of the pendulum, so $T$ does not change when $m$ increases.

56. The table of moments of inertia in Chapter 11, plus the parallel axis theorem found in that chapter, leads to

$$I_P = \tfrac{1}{2}MR^2 + Mh^2 = \tfrac{1}{2}(2.5\text{ kg})(0.21\text{ m})^2 + (2.5\text{ kg})(0.97\text{ m})^2 = 2.41\text{ kg}\cdot\text{m}^2$$

where $P$ is the hinge pin shown in the figure (the point of support for the physical pendulum), which is a distance $h$ = 0.21 m + 0.76 m away from the center of the disk.

(a) Without the torsion spring connected, the period is

$$T = 2\pi\sqrt{\frac{I_P}{Mgh}} = 2.00 \text{ s} .$$

(b) Now we have two "restoring torques" acting in tandem to pull the pendulum back to the vertical position when it is displaced. The magnitude of the torque-sum is $(Mgh + \kappa)\theta = I_P\,\alpha$, where the small-angle approximation ($\sin\theta \approx \theta$ in radians) and Newton's second law (for rotational dynamics) have been used. Making the appropriate adjustment to the period formula, we have

$$T' = 2\pi\sqrt{\frac{I_P}{Mgh + \kappa}} .$$

The problem statement requires $T = T' + 0.50$ s. Thus, $T' = (2.00 - 0.50)\text{s} = 1.50$ s. Consequently,

$$\kappa = \frac{4\pi^2}{T'^2} I_P - Mgh = 18.5 \text{ N·m/rad} .$$

57. Since the energy is proportional to the amplitude squared (see Eq. 15-21), we find the fractional change (assumed small) is

$$\frac{E' - E}{E} \approx \frac{dE}{E} = \frac{dx_m^2}{x_m^2} = \frac{2x_m dx_m}{x_m^2} = 2\frac{dx_m}{x_m}.$$

Thus, if we approximate the fractional change in $x_m$ as $dx_m/x_m$, then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if $x_m$ decreases by 3%, then $E$ must decrease by 6.0%.

58. Referring to the numbers in Sample Problem 15.6 – "Damped harmonic oscillator, time to decay, energy," we have $m = 0.25$ kg, $b = 0.070$ kg/s, and $T = 0.34$ s. Thus, when $t = 20T$, the damping factor becomes

$$e^{-bt/2m} = e^{-(0.070)(20)(0.34)/2(0.25)} = 0.39.$$

59. **THINK** In the presence of a damping force, the amplitude of oscillation of the mass-spring system decreases with time.

**EXPRESS** As discussed in 15-8, when a damping force is present, we have

$$x(t) = x_m e^{-bt/2m}\cos(\omega' t + \phi)$$

where $b$ is the damping constant and the angular frequency is given by

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} .$$

**ANALYZE** (a) We want to solve $e^{-bt/2m} = 1/3$ for $t$. We take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Therefore,

$$t = -(2m/b)\ln(1/3) = (2m/b)\ln 3.$$

Thus,

$$t = \frac{2(1.50\text{ kg})}{0.230\text{ kg/s}}\ln 3 = 14.3\text{ s}.$$

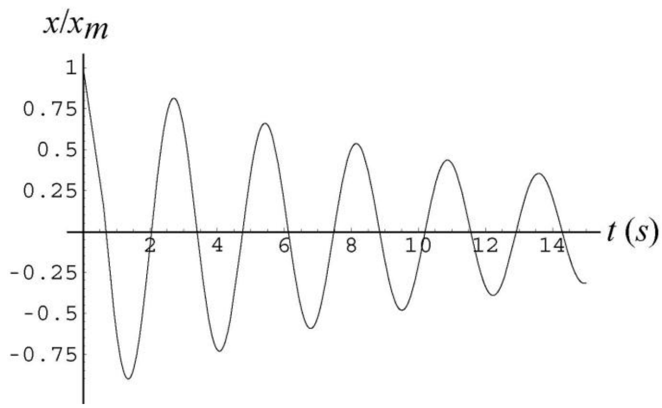
(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\text{ N/m}}{1.50\text{ kg}} - \frac{(0.230\text{ kg/s})^2}{4(1.50\text{ kg})^2}} = 2.31\text{ rad/s}.$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31\text{ rad/s}) = 2.72$ s and the number of oscillations is

$$t/T = (14.3\text{ s})/(2.72\text{ s}) = 5.27.$$

**LEARN** The displacement $x(t)$ as a function of time is shown below. The amplitude, $x_m e^{-bt/2m}$, decreases exponentially with time.




60. (a) From Hooke's law, we have

$$k = \frac{(500\text{ kg})(9.8\text{ m/s}^2)}{10\text{ cm}} = 4.9\times10^2\text{ N/cm}.$$

(b) The amplitude decreasing by 50% during one period of the motion implies

$$e^{-bT/2m} = \frac{1}{2} \quad\text{where}\quad T = \frac{2\pi}{\omega'}.$$

Since the problem asks us to estimate, we let $\omega' \approx \omega = \sqrt{k/m}$. That is, we let

$$\omega' \approx \sqrt{\frac{49000\text{ N/m}}{500\text{ kg}}} \approx 9.9\text{ rad/s},$$

so that $T \approx 0.63$ s. Taking the (natural) log of both sides of the above equation, and rearranging, we find

$$b = \frac{2m}{T}\ln 2 \approx \frac{2(500\text{ kg})}{0.63\text{ s}}(0.69) = 1.1\times10^3 \text{ kg/s}.$$

Note: if one worries about the $\omega' \approx \omega$ approximation, it is quite possible (though messy) to use Eq. 15-43 in its full form and solve for $b$. The result would be (quoting more figures than are significant)

$$b = \frac{2\ln 2\sqrt{mk}}{\sqrt{(\ln 2)^2 + 4\pi^2}} = 1086 \text{ kg / s}$$

which is in good agreement with the value gotten "the easy way" above.

61. (a) We set $\omega = \omega_d$ and find that the given expression reduces to $x_m = F_m/b\omega$ at resonance.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. Thus, at resonance, we have $v_m = \omega F_m/b\omega = F_m/b$.

62. With $\omega = 2\pi/T$ then Eq. 15-28 can be used to calculate the angular frequencies for the given pendulums. For the given range of $2.00 < \omega < 4.00$ (in rad/s), we find only two of the given pendulums have appropriate values of $\omega$: pendulum (d) with length of 0.80 m (for which $\omega$ = 3.5 rad/s) and pendulum (e) with length of 1.2 m (for which $\omega$ = 2.86 rad/s).

63. With $M$ = 1000 kg and $m$ = 82 kg, we adapt Eq. 15-12 to this situation by writing

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{M+4m}} .$$

If $d$ = 4.0 m is the distance traveled (at constant car speed $v$) between impulses, then we may write $T = v/d$, in which case the above equation may be solved for the spring constant:

$$\frac{2\pi v}{d} = \sqrt{\frac{k}{M+4m}} \;\Rightarrow\; k = (M+4m)\left(\frac{2\pi v}{d}\right)^2 .$$

Before the people got out, the equilibrium compression is $x_i = (M + 4m)g/k$, and afterward it is $x_f = Mg/k$. Therefore, with $v$ = 16000/3600 = 4.44 m/s, we find the rise of the car body on its suspension is

$$x_i - x_f = \frac{4mg}{k} = \frac{4mg}{M+4m}\left(\frac{d}{2\pi v}\right)^2 = 0.050 \text{ m}.$$

64. Since $\omega = 2\pi f$ where $f = 2.2$ Hz, we find that the angular frequency is $\omega = 13.8$ rad/s. Thus, with $x = 0.010$ m, the acceleration amplitude is $a_m = x_m\,\omega^2 = 1.91$ m/s$^2$. We set up a ratio:

$$a_m = \left(\frac{a_m}{g}\right)g = \left(\frac{1.91}{9.8}\right)g = 0.19g.$$

65. (a) The problem gives the frequency $f = 440$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second. The angular frequency $\omega$ is similar to frequency except that $\omega$ is in radians-per-second. Recalling that $2\pi$ radians are equivalent to a cycle, we have $\omega = 2\pi f \approx 2.8\times10^3$ rad/s.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. With $x_m = 0.00075$ m and the above value for $\omega$, this expression yields $v_m = 2.1$ m/s.

(c) In the discussion immediately after Eq. 15-7, the book introduces the acceleration amplitude $a_m = \omega^2 x_m$, which (if the more precise value $\omega = 2765$ rad/s is used) yields $a_m$ = 5.7 km/s.

66. (a) First consider a single spring with spring constant $k$ and unstretched length $L$. One end is attached to a wall and the other is attached to an object. If it is elongated by $\Delta x$ the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants $k_1$ and $k_2$, arranged so spring 1 is attached to the object. If spring 1 is elongated by $\Delta x_1$ then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between $\Delta x$ and $\Delta x_1$. The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where $C$ is a constant of proportionality. The total elongation is

$$\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n + 1),$$

where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n + 1)/n$. We substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n + 1)/n$ into $k\,\Delta x = k_1\,\Delta x_1$ and solve for $k_1$. With $k = 8600$ N/m and $n = L_1/L_2 = 0.70$, we obtain

$$k_1 = \left(\frac{n+1}{n}\right)k = \left(\frac{0.70+1.0}{0.70}\right)(8600\text{ N/m}) = 20886\text{ N/m} \approx 2.1\times10^4\text{N/m}.$$

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it. Now $k\,\Delta x = k_2\,\Delta x_2$. We use $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n + 1)$, then solve for $k_2$. The result is $k_2 = k(n + 1)$.

$$k_2 = (n+1)k = (0.70+1.0)(8600 \text{ N/m}) = 14620 \text{ N/m} \approx 1.5\times10^4 \text{N/m}$$

(c) To find the frequency when spring 1 is attached to mass $m$, we replace $k$ in $(1/2\pi)\sqrt{k/m}$ with $k(n + 1)/n$. With $f = (1/2\pi)\sqrt{k/m}$, we obtain, for $f = 200$ Hz and $n$ = 0.70,

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}} = \sqrt{\frac{n+1}{n}}f = \sqrt{\frac{0.70+1.0}{0.70}}(200 \text{ Hz}) = 3.1\times10^2 \text{ Hz.}$$

(d) To find the frequency when spring 2 is attached to the mass, we replace $k$ with $k(n + 1)$ to obtain

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{n+1}\,f = \sqrt{0.70+1.0}(200 \text{ Hz}) = 2.6\times10^2 \text{Hz.}$$

67. The magnitude of the downhill component of the gravitational force acting on each ore car is

$$w_x = (10000 \text{ kg})(9.8 \text{ m/s}^2)\sin\theta$$

where $\theta$ = 30° (and it is important to have the calculator in degrees mode during this problem). We are told that a downhill pull of $3\omega_x$ causes the cable to stretch $x$ = 0.15 m. Since the cable is expected to obey Hooke's law, its spring constant is

$$k = \frac{3w_x}{x} = 9.8\times10^5 \text{ N/m.}$$

(a) Noting that the oscillating mass is that of *two* of the cars, we apply Eq. 15-12 (divided by 2π).

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\times10^5 \text{ N/m}}{20000 \text{ kg}}} = 1.1 \text{ Hz.}$$

(b) The difference between the equilibrium positions of the end of the cable when supporting two as opposed to three cars is

$$\Delta x = \frac{3w_x - 2w_x}{k} = 0.050 \text{ m.}$$

68. (a) Hooke's law readily yields (0.300 kg)(9.8 m/s$^2$)/(0.0200 m) = 147 N/m.

(b) With $m$ = 2.00 kg, the period is $T = 2\pi\sqrt{\frac{m}{k}} = 0.733\,\text{s}$.

69. **THINK** The piston undergoes simple harmonic motion. Given the amplitude and frequency of oscillation, its maximum speed can be readily calculated.

**EXPRESS** Let the amplitude be $x_m$. The maximum speed $v_m$ is related to the amplitude by $v_m = \omega x_m$, where $\omega$ is the angular frequency.

**ANALYZE** We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is $f$ = (180 rev)/(60 s) = 3.0 Hz and the amplitude is half the stroke, or $x_m$ = 0.38 m. Thus,

$$v_m = 2\pi(3.0\text{ Hz})(0.38\text{ m}) = 7.2\text{ m/s}.$$

**LEARN** In a similar manner, the maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(3.0\text{ Hz}))^2 (0.38\text{ m}) = 135\text{ m/s}^2.$$

Acceleration is proportional to the displacement $x_m$ in SHM.

70. (a) The rotational inertia of a hoop is $I = mR^2$, and the energy of the system becomes

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

and $\theta$ is in radians. We note that $r\omega = v$ (where $v = dx/dt$). Thus, the energy becomes

$$E = \frac{1}{2}\left(\frac{mR^2}{r^2}\right)v^2 + \frac{1}{2}kx^2$$

which looks like the energy of the simple harmonic oscillator discussed in Section15-4 if we identify the mass $m$ in that section with the term $mR^2/r^2$ appearing in this problem. Making this identification, Eq. 15-12 yields

$$\omega = \sqrt{\frac{k}{mR^2/r^2}} = \frac{r}{R}\sqrt{\frac{k}{m}}.$$

(b) If $r = R$ the result of part (a) reduces to $\omega = \sqrt{k/m}$.

(c) And if $r = 0$ then $\omega = 0$ (the spring exerts no restoring torque on the wheel so that it is not brought back toward its equilibrium position).

71. Since $T$ = 0.500 s, we note that $\omega = 2\pi/T = 4\pi$ rad/s. We work with SI units, so $m$ = 0.0500 kg and $v_m$ = 0.150 m/s.

(a) Since $\omega = \sqrt{k/m}$, the spring constant is

$$k = \omega^2 m = (4\pi\text{ rad/s})^2(0.0500\text{ kg}) = 7.90\text{ N/m}.$$

(b) We use the relation $v_m = x_m\omega$ and obtain

$$x_m = \frac{v_m}{\omega} = \frac{0.150}{4\pi} = 0.0119 \text{ m}.$$

(c) The frequency is $f = \omega/2\pi = 2.00$ Hz (which is equivalent to $f = 1/T$).

72. (a) We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = R = 0.126$ m. For a solid disk of mass $m$, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} = 0.873\text{s}.$$

(b) We seek a value of $r \neq R$ such that

$$2\pi\sqrt{\frac{R^2 + 2r^2}{2gr}} = 2\pi\sqrt{\frac{3R}{2g}}$$

and are led to the quadratic formula:

$$r = \frac{3R \pm \sqrt{(3R)^2 - 8R^2}}{4} = R \quad \text{or} \quad \frac{R}{2}.$$

Thus, our result is $r = 0.126/2 = 0.0630$ m.

73. **THINK** A mass attached to the end of a vertical spring undergoes simple harmonic motion. Energy is conserved in the process.

**EXPRESS** The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky_0 = mg$, where $y_0 = 0.096$ m is the elongation of the spring at equilibrium, $k$ is the spring constant, and $m = 1.3$ kg is the mass of the block. As the block oscillate, its speed is a maximum as it passes the equilibrium point, and zero at the endpoints.

**ANALYZE** (a) The spring constant is

$$k = mg/y_0 = (1.3 \text{ kg})(9.8 \text{ m/s}^2)/(0.096 \text{ m}) = 1.33 \times 10^2 \text{ N/m}.$$

(b) The period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.3 \text{ kg}}{133 \text{ N/m}}} = 0.62 \text{ s}.$$

(c) The frequency is $f = 1/T = 1/0.62 \text{ s} = 1.6$ Hz.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest $\Delta y$ = 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) At the initial position,

$$y_i = y_0 + \Delta y = 9.6\text{ cm} + 5.0\text{ cm} = 14.6\text{ cm} = 0.146\text{ m},$$

the block is not moving but it has potential energy

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3\text{ kg})(9.8\text{ m/s}^2)(0.146\text{ m}) + \frac{1}{2}(133\text{ N/m})(0.146\text{ m})^2 = -0.44\text{ J}.$$

When the block is at the equilibrium point, the elongation of the spring is $y_0$ = 9.6 cm and the potential energy is

$$U_f = -mgy_0 + \frac{1}{2}ky_0^2 = -(1.3\text{ kg})(9.8\text{ m/s}^2)(0.096\text{ m}) + \frac{1}{2}(133\text{ N/m})(0.096\text{ m})^2$$
$$= -0.61\text{ J}.$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for $v$:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44\text{ J} + 0.61\text{ J})}{1.3\text{ kg}}} = 0.51\text{ m/s}.$$

**LEARN** Both the gravitational force and the spring force are conservative, so the work done by the forces is independent of path. By energy conservation, the kinetic energy of the block is equal to the negative of the change in potential energy of the system:

$$\begin{aligned}\Delta K &= -\Delta U = -(U_f - U_i) = U_i - U_f = -mg(y_i - y_0) + \frac{1}{2}k(y_i^2 - y_0^2)\\ &= -mg\Delta y + \frac{1}{2}k\left[(y_0 + \Delta y)^2 - y_0^2\right] = -mg\Delta y + \frac{1}{2}k\left[(\Delta y)^2 + 2y_0\Delta y\right]\\ &= \Delta y(-mg + ky_0) + \frac{1}{2}k(\Delta y)^2\\ &= \frac{1}{2}k(\Delta y)^2\end{aligned}$$

where the relation $ky_0 = mg$ was used.

74. The distance from the relaxed position of the bottom end of the spring to its equilibrium position when the body is attached is given by Hooke's law:

$$\Delta x = F/k = (0.20 \text{ kg})(9.8 \text{ m/s}^2)/(19 \text{ N/m}) = 0.103 \text{ m}.$$

(a) The body, once released, will not only fall through the $\Delta x$ distance but continue through the equilibrium position to a “turning point” equally far on the other side. Thus, the total descent of the body is $2\Delta x = 0.21$ m.

(b) Since $f = \omega/2\pi$, Eq. 15-12 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.6 \text{ Hz}.$$

(c) The maximum distance from the equilibrium position gives the amplitude:

$$x_m = \Delta x = 0.10 \text{ m}.$$

75. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet–block system is conserved during the collision. Let $m$ be the mass of the bullet, $M$ be the mass of the block, $v_0$ be the initial speed of the bullet, and $v$ be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m + M)v$, so

$$v = \frac{mv_0}{m+M} = \frac{(0.050 \text{ kg})(150 \text{ m/s})}{0.050 \text{ kg} + 4.0 \text{ kg}} = 1.85 \text{ m/s}.$$

When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by $Mg/k$. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance

$$\ell = (M+m)g/k,$$

somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance $y_m$ above the position where the spring and gravitational forces balance. Note that $y_m$ is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$ . The gravitational potential energy is $(M + m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k(y_m - \ell)^2 + (M+m)gy_m.$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$y_m = \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)}$$
$$= \sqrt{\frac{(0.050\text{ kg}+4.0\text{ kg})(1.85\text{ m/s})^2}{500\text{ N/m}} - \frac{(0.050\text{ kg})\left(9.8\text{ m/s}^2\right)^2}{(500\text{ N/m})^2}\ 2(4.0\text{ kg})+0.050\text{ kg}}$$
$$= 0.166\text{ m}.$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050\text{ kg})(150\text{ m/s})^2 = 563\text{ J}$. The kinetic energy of the bullet–block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050\text{ kg}+4.0\text{ kg})(1.85\text{ m/s})^2 = 6.94\text{ J}.$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, $E$ is the energy that is transferred. The ratio is

$$E/E_0 = (6.94\text{ J})/(563\text{ J}) = 0.0123 \text{ or } 1.23\%.$$

76. (a) We note that

$$\omega = \sqrt{k/m} = \sqrt{1500/0.055} = 165.1\text{ rad/s}.$$

We consider the most direct path in each part of this problem. That is, we consider in part (a) the motion directly from $x_1 = +0.800x_m$ at time $t_1$ to $x_2 = +0.600x_m$ at time $t_2$ (as opposed to, say, the block moving from $x_1 = +0.800x_m$ through $x = +0.600x_m$, through $x = 0$, reaching $x = -x_m$ and after returning back through $x = 0$ then getting to $x_2 = +0.600x_m$). Equation 15-3 leads to

$$\omega t_1 + \phi = \cos^{-1}(0.800) = 0.6435\text{ rad}$$

$$\omega t_2 + \phi = \cos^{-1}(0.600) = 0.9272\text{ rad}.$$

Subtracting the first of these equations from the second leads to

$$\omega(t_2 - t_1) = 0.9272 - 0.6435 = 0.2838\text{ rad}.$$

Using the value for ω computed earlier, we find $t_2 - t_1 = 1.72 \times 10^{-3}$ s.

(b) Let $t_3$ be when the block reaches $x = -0.800x_m$ in the direct sense discussed above. Then the reasoning used in part (a) leads here to

$$\omega(t_3 - t_1) = (2.4981 - 0.6435)\text{ rad} = 1.8546\text{ rad}$$

and thus to $t_3 - t_1 = 11.2 \times 10^{-3}$ s.

77. (a) From the graph, we find $x_m$ = 7.0 cm = 0.070 m, and $T$ = 40 ms = 0.040 s. Thus, the angular frequency is $\omega = 2\pi/T$ = 157 rad/s. Using $m$ = 0.020 kg, the maximum kinetic energy is then $\frac{1}{2}mv^2 = \frac{1}{2}m\,\omega^2 x_m^2$ = 1.2 J.

(b) Using Eq. 15-5, we have $f = \omega/2\pi$ = 50 oscillations per second. Of course, Eq. 15-2 can also be used for this.

78. (a) From the graph we see that $x_m$ = 7.0 cm = 0.070 m and $T$ = 40 ms = 0.040 s. The maximum speed is $x_m\omega = x_m 2\pi/T$ = 11 m/s.

(b) The maximum acceleration is $x_m\omega^2 = x_m(2\pi/T)^2 = 1.7 \times 10^3$ m/s$^2$.

79. Setting 15 mJ (0.015 J) equal to the maximum kinetic energy leads to $v_{max}$ = 0.387 m/s. Then one can use either an "exact" approach using $v_{max} = \sqrt{2gL(1-\cos\theta_{max})}$ or the "SHM" approach where

$$v_{max} = L\,\omega_{max} = L\,\omega\theta_{max} = L\sqrt{g/L}\;\theta_{max}$$

to find $L$. Both approaches lead to $L$ = 1.53 m.

80. Its total mechanical energy is equal to its maximum potential energy $\frac{1}{2}kx_m^2$, and its potential energy at $t$ = 0 is $\frac{1}{2}kx_o^2$ where $x_o = x_m\cos(\pi/5)$ in this problem. The ratio is therefore $\cos^2(\pi/5)$ = 0.655 = 65.5%.

81. (a) From the graph, it is clear that $x_m$ = 0.30 m.

(b) With $F = -kx$, we see $k$ is the (negative) slope of the graph — which is 75/0.30 = 250 N/m. Plugging this into Eq. 15-13 yields

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.50\text{ kg}}{250\text{ N/m}}} = 0.28\text{ s}.$$

(c) As discussed in Section 15-2, the maximum acceleration is

$$a_m = \omega^2 x_m = \left(\frac{k}{m}\right)x_m = \left(\frac{250\text{ N/m}}{0.50\text{ kg}}\right)(0.30\text{ m}) = 1.5\times10^2\text{ m/s}^2.$$

Alternatively, we could arrive at this result using $a_m = (2\pi/T)^2\,x_m$.

(d) Also in Section 15-2 is $v_m = \omega x_m$ so that the maximum kinetic energy is

$$K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\omega^2 x_m^2 = \frac{1}{2}kx_m^2 = \frac{1}{2}(250\text{ N/m})(0.30\text{ m})^2 = 11.3\text{ J} \approx 11\text{ J}.$$

We note that the above manipulation reproduces the notion of energy conservation for this system (maximum kinetic energy being equal to the maximum potential energy).

82. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{eff} = \sqrt{g^2 + (v^2/R)^2}.$$

Then, since frequency is the reciprocal of the period, Eq. 15-28 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{eff}}{L}} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + v^4/R^2}}{L}}.$$

With $v$ = 70 m/s, $R$ = 50 m, and $L$ = 0.20 m, we have $f \approx 3.5\text{ s}^{-1} = 3.5\text{ Hz}$.

83. (a) Hooke's law readily yields

$$k = (15\text{ kg})(9.8\text{ m/s}^2)/(0.12\text{ m}) = 1225\text{ N/m}.$$

Rounding to three significant figures, the spring constant is therefore 1.23 kN/m.

(b) We are told $f$ = 2.00 Hz = 2.00 cycles/sec. Since a cycle is equivalent to $2\pi$ radians, we have $\omega = 2\pi(2.00) = 4\pi$ rad/s (understood to be valid to three significant figures). Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{1225\text{ N/m}}{(4\pi\text{ rad/s})^2} = 7.76\text{ kg}.$$

Consequently, the weight of the package is $mg$ = 76.0 N.

84. (a) Comparing with Eq. 15-3, we see $\omega$ = 10 rad/s in this problem. Thus, $f = \omega/2\pi$ = 1.6 Hz.

(b) Since $v_m = \omega x_m$ and $x_m$ = 10 cm (see Eq. 15-3), then $v_m$ = (10 rad/s)(10 cm) = 100 cm/s or 1.0 m/s.

(c) The maximum occurs at $t$ = 0.

(d) Since $a_m = \omega^2 x_m$, then $v_m = (10\text{ rad/s})^2(10\text{ cm}) = 1000\text{ cm/s}^2$ or 10 m/s$^2$.

(e) The acceleration extremes occur at the displacement extremes: $x = \pm x_m$ or $x = \pm 10$ cm.

(f) Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = (0.10\text{ kg})(10\text{ rad/s})^2 = 10\text{ N/m}.$$

Thus, Hooke's law gives $F = -kx = -10x$ in SI units.

85. Using $\Delta m = 2.0$ kg, $T_1 = 2.0$ s and $T_2 = 3.0$ s, we write

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m+\Delta m}{k}}.$$

Dividing one relation by the other, we obtain

$$\frac{T_2}{T_1} = \sqrt{\frac{m+\Delta m}{m}}$$

which (after squaring both sides) simplifies to $m = \dfrac{\Delta m}{(T_2/T_1)^2 - 1} = 1.6\text{kg}.$

86. (a) The amplitude of the acceleration is given by $a_m = \omega^2 x_m$, where $\omega$ is the angular frequency ($\omega = 2\pi f$ since there are $2\pi$ radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = (2\pi(1000\text{ Hz}))^2(0.00040\text{ m}) = 1.6\times10^4\text{ m/s}^2.$$

(b) Similarly, in the discussion after Eq. 15-6, we find $v_m = \omega x_m$ so that

$$v_m = (2\pi(1000\text{ Hz}))(0.00040\text{ m}) = 2.5\text{ m/s}.$$

(c) From Eq. 15-8, we have (in absolute value)

$$|a| = (2\pi(1000\text{ Hz}))^2(0.00020\text{ m}) = 7.9\times10^3\text{ m/s}^2.$$

(d) This can be approached with the energy methods of Section 15-4, but here we will use trigonometric relations along with Eq. 15-3 and Eq. 15-6. Thus, allowing for both roots stemming from the square root,

$$\sin(\omega t+\phi) = \pm\sqrt{1-\cos^2(\omega t+\phi)} \quad \Rightarrow \quad -\frac{v}{\omega x_m} = \pm\sqrt{1-\frac{x^2}{x_m^2}}.$$

Taking absolute values and simplifying, we obtain

$$|v| = 2\pi f\sqrt{x_m^2 - x^2} = 2\pi(1000)\sqrt{0.00040^2 - 0.00020^2} = 2.2 \text{ m/s}.$$

87. (a) The rotational inertia is $I = \frac{1}{2}MR^2 = \frac{1}{2}(3.00 \text{ kg})(0.700 \text{ m})^2 = 0.735 \text{ kg}\cdot\text{m}^2$.

(b) Using Eq. 15-22 (in absolute value), we find

$$\kappa = \frac{\tau}{\theta} = \frac{0.0600 \text{ N}\cdot\text{m}}{2.5 \text{ rad}} = 0.0240 \text{ N}\cdot\text{m/rad}.$$

(c) Using Eq. 15-5, Eq. 15-23 leads to

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{0.024 \text{N}\cdot\text{m/rad}}{0.735 \text{kg}\cdot\text{m}^2}} = 0.181 \text{ rad/s}.$$

88. (a) The Hooke's law force (of magnitude (100)(0.30) = 30 N) is directed upward and the weight (20 N) is downward. Thus, the net force is 10 N upward.

(b) The equilibrium position is where the upward Hooke's law force balances the weight, which corresponds to the spring being stretched (from unstretched length) by 20 N/100 N/m = 0.20 m. Thus, relative to the equilibrium position, the block (at the instant described in part (a)) is at what one might call *the bottom turning point* (since $v$ = 0) at $x = -x_m$ where the amplitude is $x_m$ = 0.30 – 0.20 = 0.10 m.

(c) Using Eq. 15-13, we have

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(20 \text{ N})/(9.8 \text{ m/s}^2)}{100 \text{ N/m}}} = 0.90 \text{ s}.$$

(d) The maximum kinetic energy is equal to the maximum potential energy $\frac{1}{2}kx_m^2$. Thus,

$$K_m = U_m = \frac{1}{2}(100 \text{ N / m})(0.10 \text{ m})^2 = 0.50 \text{ J}.$$

89. (a) We require $U = \frac{1}{2}E$ at some value of $x$. Using Eq. 15-21, this becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kx_m^2\right) \Rightarrow x = \frac{x_m}{\sqrt{2}}.$$

We compare the given expression $x$ as a function of $t$ with Eq. 15-3 and find $x_m$ = 5.0 m. Thus, the value of $x$ we seek is $x = 5.0/\sqrt{2} \approx 3.5 \text{ m}$.

(b) We solve the given expression (with $x = 5.0/\sqrt{2}$), making sure our calculator is in radians mode:

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 1.54 \text{ s}.$$

Since we are asked for the interval $t_{eq} - t$ where $t_{eq}$ specifies the instant the particle passes through the equilibrium position, then we set $x = 0$ and find

$$t_{eq} = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}(0) = 2.29 \text{ s}.$$

Consequently, the time interval is $t_{eq} - t = 0.75$ s.

90. Since the particle has zero speed (momentarily) at $x \neq 0$, then it must be at its turning point; thus, $x_o = x_m = 0.37$ cm. It is straightforward to infer from this that the phase constant $\phi$ in Eq. 15-2 is zero. Also, $f = 0.25$ Hz is given, so we have $\omega = 2\pi f = \pi/2$ rad/s. The variable $t$ is understood to take values in seconds.

(a) The period is $T = 1/f = 4.0$ s.

(b) As noted above, $\omega = \pi/2$ rad/s.

(c) The amplitude, as observed above, is 0.37 cm.

(d) Equation 15-3 becomes $x = (0.37 \text{ cm}) \cos(\pi t/2)$.

(e) The derivative of $x$ is $v = -(0.37 \text{ cm/s})(\pi/2) \sin(\pi t/2) \approx (-0.58 \text{ cm/s}) \sin(\pi t/2)$.

(f) From the previous part, we conclude $v_m = 0.58$ cm/s.

(g) The acceleration-amplitude is $a_m = \omega^2 x_m = 0.91 \text{ cm/s}^2$.

(h) Making sure our calculator is in radians mode, we find $x = (0.37) \cos(\pi(3.0)/2) = 0$. It is important to avoid rounding off the value of $\pi$ in order to get precisely zero, here.

(i) With our calculator still in radians mode, we obtain $v = -(0.58 \text{ cm/s})\sin(\pi(3.0)/2) = 0.58$ cm/s.

91. **THINK** This problem explores the oscillation frequency of a pendulum under various accelerating conditions.

**EXPRESS** In a room, the frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where $L$ is the length of the pendulum. Inside an elevator, the forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where $m$ is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g} - \vec{a}_e) + \vec{T} = m\vec{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}_e$.

**ANALYZE** (a) With $L = 2.0$ m, we find the frequency of the pendulum in a room to be

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\,\text{m/s}^2}{2.0\,\text{m}}} = 0.35\,\text{Hz}.$$

(b) With the elevator accelerating upward, $\vec{g}$ and $\vec{a}_e$ are along the same line but in opposite directions, we can find the frequency for small amplitude oscillations by replacing $g$ with the effective gravitational acceleration $g_{\text{eff}} = g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus,

$$f = \frac{1}{2\pi}\sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2 + 2.0\,\text{m/s}^2}{2.0\,\text{m}}} = 0.39\,\text{Hz}.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g} - \vec{a}_e = 0$. To find the frequency for small amplitude oscillations, replace $g$ with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

**LEARN** The frequency of the pendulum increases as $g_{\text{eff}}$ increases.

92. The period formula, Eq. 15-29, requires knowing the distance $h$ from the axis of rotation and the center of mass of the system. We also need the rotational inertia $I$ about the axis of rotation. From the figure, we see $h = L + R$ where $R = 0.15$ m. Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M\left(L + R\right)^2,$$

where $M = 1.0\,\text{kg}$. Thus, Eq. 15-29, with $T = 2.0$ s, leads to

$$2.0 = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + M\left(L+R\right)^2}{Mg\left(L+R\right)}}$$

which leads to $L = 0.8315$ m.

93. (a) Hooke's law provides the spring constant:

$$k = (4.00\text{ kg})(9.8\text{ m/s}^2)/(0.160\text{ m}) = 245\text{ N/m}.$$

(b) The attached mass is $m$ = 0.500 kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.500}{245}} = 0.284\text{ s.}$$

94. We note (from the graph) that $a_m = \omega^2 x_m$ = 4.00 cm/s$^2$. Also, the value at $t$ = 0 is $a_o$ = 1.00 cm/s$^2$. Then Eq. 15-7 leads to

$$\phi = \cos^{-1}(-1.00/4.00) = +1.82\text{ rad or } -4.46\text{ rad.}$$

The other "root" (+4.46 rad) can be rejected on the grounds that it would lead to a negative slope at $t$ = 0.

95. The time for one cycle is $T$ = (50 s)/20 = 2.5 s. Thus, from Eq. 15-23, we find

$$I = \kappa\left(\frac{T}{2\pi}\right)^2 = (0.50)\left(\frac{2.5}{2\pi}\right)^2 = 0.079\text{ kg}\cdot\text{m}^2.$$

96. The angular frequency of the simple harmonic oscillation is given by Eq. 15-13:

$$\omega = \sqrt{\frac{k}{m}}\,.$$

Thus, for two different masses $m_1$ and $m_2$, with the same spring constant $k$, the ratio of the frequencies would be

$$\frac{\omega_1}{\omega_2} = \frac{\sqrt{k/m_1}}{\sqrt{k/m_2}} = \sqrt{\frac{m_2}{m_1}}\,.$$

In our case, with $m_1 = m$ and $m_2 = 2.5m$, the ratio is $\frac{\omega_1}{\omega_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{2.5} = 1.58$.

97. (a) The graphs suggest that $T$ = 0.40 s and $\kappa$ = 4/0.2 = 0.02 N·m/rad. With these values, Eq. 15-23 can be used to determine the rotational inertia:

$$I = \kappa T^2/4\pi^2 = 8.11\times10^{-5}\text{ kg}\cdot\text{m}^2.$$

(b) We note (from the graph) that $\theta_{max}$ = 0.20 rad. Setting the maximum kinetic energy ($\frac{1}{2}I\omega_{max}{}^2$) equal to the maximum potential energy (see the hint in the problem) leads to $\omega_{max} = \theta_{max}\sqrt{\kappa/I}$ = 3.14 rad/s.

98. (a) Hooke's law provides the spring constant: $k$ = (20 N)/(0.20 m) = 1.0×10$^2$ N/m.

(b) The attached mass is $m$ = (5.0 N)/(9.8 m/s$^2$) = 0.51 kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.51\,\text{kg}}{100\ \text{N/m}}} = 0.45\,\text{s}.$$

99. For simple harmonic motion, Eq. 15-24 must reduce to

$$\tau = -L\left(F_g \sin\theta\right) \rightarrow -L\left(F_g\theta\right)$$

where $\theta$ is in radians. We take the percent difference (in absolute value)

$$\left|\frac{\left(-LF_g\sin\theta\right)-\left(-LF_g\theta\right)}{-LF_g\sin\theta}\right| = \left|1-\frac{\theta}{\sin\theta}\right|$$

and set this equal to 0.010 (corresponding to 1.0%). In order to solve for $\theta$ (since this is not possible "in closed form"), several approaches are available. Some calculators have built-in numerical routines to facilitate this, and most math software packages have this capability. Alternatively, we could expand $\sin\theta \approx \theta - \theta^3/6$ (valid for small $\theta$) and thereby find an approximate solution (which, in turn, might provide a seed value for a numerical search). Here we show the latter approach:

$$\left|1-\frac{\theta}{\theta-\theta^3/6}\right| \approx 0.010 \Rightarrow \frac{1}{1-\theta^2/6} \approx 1.010$$

which leads to $\theta \approx \sqrt{6(0.01/1.01)} = 0.24\ \text{rad} = 14.0°$. A more accurate value (found numerically) for $\theta$ that results in a 1.0% deviation is 13.986°.

100. (a) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\begin{aligned}\frac{1}{2}kx_m^2 &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}^2\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2 \\ &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{4}Mv_{\text{cm}}^2 = \frac{3}{4}Mv_{\text{cm}}^2\end{aligned}$$

which leads to $Mv_{\text{cm}}^2 = 2kx_m^2/3$ = 0.125 J. The translational kinetic energy is therefore $\frac{1}{2}Mv_{\text{cm}}^2 = kx_m^2/3 = 0.0625\,\text{J}$.

(b) And the rotational kinetic energy is $\frac{1}{4}Mv_{\text{cm}}^2 = kx_m^2/6 = 0.03125\text{J} \approx 3.13\times10^{-2}$ J .

(c) In this part, we use $v_{cm}$ to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{3}{4}Mv_{cm}^2\right) + \frac{d}{dt}\left(\frac{1}{2}kx^2\right) = \frac{3}{2}Mv_{cm}a_{cm} + kxv_{cm} = 0$$

which leads to

$$a_{cm} = -\left(\frac{2k}{3M}\right)x.$$

Comparing with Eq. 15-8, we see that $\omega = \sqrt{2k/3M}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi\sqrt{3M/2k}$.

101. **THINK** The block is in simple harmonic motion, so its position relative to the equilibrium position can be written as $x(t) = x_m \cos(\omega t + \phi)$.

**EXPRESS** The speed of the block is

$$v(t) = dx/dt = -\omega x_m \sin(\omega t + \phi).$$

For a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given $v$ = 5.2 m/s at $x$ = 0 is the maximum value $v_m = \omega x_m$ where

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{480 \text{ N/m}}{1.2 \text{ kg}}} = 20 \text{ rad/s}.$$

**ANALYZE** (a) Since $\omega = 2\pi f$, we find $f$ = 3.2 Hz.

(b) We have $v_m$ = 5.2 m/s = $\omega x_m$ = (20 rad/s)$x_m$, which leads to $x_m$ = 0.26 m.

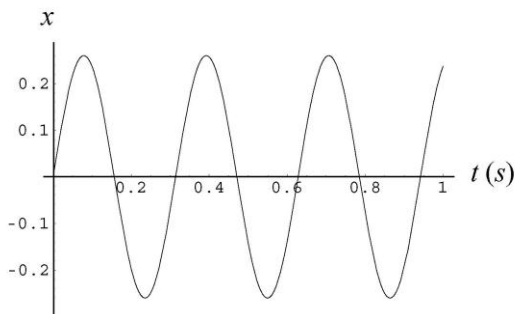
(c) With meters, seconds and radians understood,

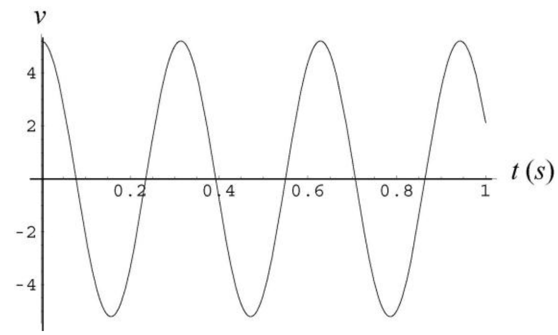
$$x = (0.26 \text{ m})\cos(20t + \phi)$$
$$v = -(5.2 \text{ m/s})\sin(20t + \phi).$$

The requirement that $x$ = 0 at $t$ = 0 implies (from the first equation above) that either $\phi$ = +π/2 or $\phi$ = –π/2. Only one of these choices meets the further requirement that $v$ > 0 when $t$ = 0; that choice is $\phi$ = –π/2. Therefore,

$$x = (0.26 \text{ m})\cos\left(20t - \frac{\pi}{2}\right) = (0.26 \text{ m})\sin(20t).$$

**LEARN** The plots of $x$ and $v$ as a function of time are given next:




102. (a) Equation 15-21 leads to

$$E = \frac{1}{2}kx_m^2 \quad \Rightarrow \quad x_m = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(4.0\text{ J})}{200\text{ N/m}}} = 0.20\text{ m}.$$

(b) Since $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.80\text{ kg}/200\text{ N/m}} \approx 0.4\text{ s}$, then the block completes 10/0.4 = 25 cycles during the specified interval.

(c) The maximum kinetic energy is the total energy, 4.0 J.

(d) This can be approached more than one way; we choose to use energy conservation:

$$E = K + U \Rightarrow 4.0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

Therefore, when $x$ = 0.15 m, we find $v$ = 2.1 m/s.

103. (a) By Eq. 15-13, the mass of the block is

$$m_b = \frac{kT_0^2}{4\pi^2} = 2.43\text{ kg}.$$

Therefore, with $m_p$ = 0.50 kg, the new period is

$$T = 2\pi\sqrt{\frac{m_p + m_b}{k}} = 0.44\text{ s}.$$

(b) The speed before the collision (since it is at its maximum, passing through equilibrium) is $v_0 = x_m\omega_0$ where $\omega_0 = 2\pi/T_0$; thus, $v_0$ = 3.14 m/s. Using momentum conservation (along the horizontal direction) we find the speed after the collision:

$$V = v_0\frac{m_b}{m_p + m_b} = 2.61\text{ m/s}.$$

The equilibrium position has not changed, so (for the new system of greater mass) this represents the maximum speed value for the subsequent harmonic motion: $V = x'_m\omega$ where $\omega = 2\pi/T = 14.3$ rad/s. Therefore, $x'_m = 0.18$ m.

104. (a) We are told that when $t = 4T$, with $T = 2\pi/\omega' \approx 2\pi\sqrt{m/k}$ (neglecting the second term in Eq. 15-43),

$$e^{-bt/2m} = \frac{3}{4}.$$

Thus,

$$T \approx 2\pi\sqrt{(2.00\text{kg})/(10.0\,\text{N/m})} = 2.81\,\text{s}$$

and we find

$$\frac{b(4T)}{2m} = \ln\left(\frac{4}{3}\right) = 0.288 \quad \Rightarrow \quad b = \frac{2(2.00\text{ kg})(0.288)}{4(2.81\,\text{s})} = 0.102\text{ kg/s.}$$

(b) Initially, the energy is $E_o = \frac{1}{2}kx_{mo}^2 = \frac{1}{2}(10.0)(0.250)^2 = 0.313\,\text{J}$. At $t = 4T$,

$$E = \tfrac{1}{2}k(\tfrac{3}{4}x_{mo})^2 = 0.176\,\text{J}.$$

Therefore, $E_o - E = 0.137$ J.

105. (a) From Eq. 16-12, $T = 2\pi\sqrt{m/k} = 0.45\,\text{s}$.

(b) For a vertical spring, the distance between the unstretched length and the equilibrium length (with a mass $m$ attached) is $mg/k$, where in this problem $mg = 10$ N and $k = 200$ N/m (so that the distance is 0.05 m). During simple harmonic motion, the convention is to establish $x = 0$ at the equilibrium length (the middle level for the oscillation) and to write the total energy without any gravity term; that is, $E = K + U$, where $U = kx^2/2$. Thus, as the block passes through the unstretched position, the energy is

$$E = 2.0 + \tfrac{1}{2}k(0.05)^2 = 2.25\,\text{J}.$$

At its topmost and bottommost points of oscillation, the energy (using this convention) is all elastic potential: $\frac{1}{2}kx_m^2$. Therefore, by energy conservation,

$$2.25 = \frac{1}{2}kx_m^2 \Rightarrow x_m = \pm 0.15\text{ m}.$$

This gives the amplitude of oscillation as 0.15 m, but how far are these points from the *unstretched* position? We add (or subtract) the 0.05 m value found above and obtain 0.10 m for the top-most position and 0.20 m for the bottom-most position.

(c) As noted in part (b), $x_m = \pm 0.15$ m.

(d) The maximum kinetic energy equals the maximum potential energy (found in part (b)) and is equal to 2.25 J.

106. (a) The graph makes it clear that the period is $T = 0.20$ s.

(b) The period of the simple harmonic oscillator is given by Eq. 15-13: $T = 2\pi\sqrt{\frac{m}{k}}$.

Thus, using the result from part (a) with $k = 200$ N/m, we obtain $m = 0.203 \approx 0.20$ kg.

(c) The graph indicates that the speed is (momentarily) zero at $t = 0$, which implies that the block is at $x_0 = \pm x_m$. From the graph we also note that the slope of the velocity curve (hence, the acceleration) is positive at $t = 0$, which implies (from $ma = -kx$) that the value of $x$ is negative. Therefore, with $x_m = 0.20$ m, we obtain $x_0 = -0.20$ m.

(d) We note from the graph that $v = 0$ at $t = 0.10$ s, which implied $a = \pm a_m = \pm\omega^2 x_m$. Since acceleration is the instantaneous slope of the velocity graph, then (looking again at the graph) we choose the negative sign. Recalling $\omega^2 = k/m$ we obtain $a = -197 \approx -2.0 \times 10^2$ m/s$^2$.

(e) The graph shows $v_m = 6.28$ m/s, so $K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}(0.20\text{ kg})(6.28\text{ m/s})^2 = 4.0\text{ J}.$

107. The mass is $m = \frac{0.108\text{ kg}}{6.02\times 10^{23}} = 1.8\times 10^{-25}$ kg. Using Eq. 15-12 and the fact that $f = \omega/2\pi$, we have

$$1\times 10^{13}\text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = \left(2\pi\times 10^{13}\right)^2\left(1.8\times 10^{-25}\right) \approx 7\times 10^2\text{ N/m}.$$

108. Using Hooke's law, we have $mg = k\Delta y = kh.$ The frequency of oscillation for the mass-spring system is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Similarly, the frequency of oscillation for a simple pendulum is

$$f' = \frac{1}{T'} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

If $f = f'$, then $\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, which gives

$$L = \frac{mg}{k} = \frac{kh}{k} = h = 2.00\text{ cm}.$$

109. The rotational inertia for an axis through $A$ is $I_A = I_{cm} + mh_A^2$ and that for an axis through $B$ is $I_B = I_{cm} + mh_B^2$, where $h_A$ and $h_B$ are distances from $A$ and $B$ to the center of mass. Using Eq. 15-29, $T = 2\pi\sqrt{I/mgh}$, we require

$$T_A = T_B \quad \Rightarrow \quad 2\pi\sqrt{\frac{I_{cm} + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{I_{cm} + mh_B^2}{mgh_B}}$$

which (after canceling $2\pi$ and squaring both sides) becomes

$$\frac{I_{cm} + mh_A^2}{mgh_A} = \frac{I_{cm} + mh_B^2}{mgh_B}.$$

Cross-multiplying and rearranging, we obtain

$$I_{cm}(h_B - h_A) = m(h_A h_B^2 - h_B h_A^2) = mh_A h_B(h_B - h_A)$$

which simplifies to $I_{cm} = mh_A h_B$. We plug this back into the first period formula above and obtain

$$T = 2\pi\sqrt{\frac{mh_A h_B + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{h_B + h_A}{g}}.$$

From the figure, we see that $h_B + h_A = L$, and (after squaring both sides) we can solve the above equation for $L$:

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.8\ \text{m/s}^2)(1.80\ \text{s})^2}{4\pi^2} = 0.804\ \text{m}.$$

110. Since $d_m$ is the amplitude of oscillation, then the maximum acceleration being set to $0.2g$ provides the condition: $\omega^2 d_m = 0.2g$. Since $d_s$ is the amount the spring stretched in order to achieve vertical equilibrium of forces, then we have the condition $kd_s = mg$. Since we can write this latter condition as $m\omega^2 d_s = mg$, then $\omega^2 = g/d_s$. Plugging this into our first condition, we obtain

$$d_s = d_m/0.2 = (10\ \text{cm})/0.2 = 50\ \text{cm}.$$

111. Using Eq. 15-12, we find $\omega = \sqrt{k/m} = 10\ \text{rad/s}$. We also use $v_m = x_m\omega$ and $a_m = x_m\omega^2$.

(a) The amplitude (meaning "displacement amplitude") is $x_m = v_m/\omega = 3/10 = 0.30$ m.

(b) The acceleration-amplitude is $a_m = (0.30\ \text{m})(10\ \text{rad/s})^2 = 30\ \text{m/s}^2$.

(c) One interpretation of this question is "what is the most negative value of the acceleration?" in which case the answer is $-a_m = -30$ m/s$^2$. Another interpretation is "what is the smallest value of the absolute-value of the acceleration?" in which case the answer is zero.

(d) Since the period is $T = 2\pi/\omega = 0.628$ s. Therefore, seven cycles of the motion requires $t = 7T = 4.4$ s.

112. (a) Eq. 15-28 gives

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{17m}{9.8\,\text{m/s}^2}} = 8.3\,\text{s}.$$

(b) Plugging $I = mL^2$ into Eq. 15-25, we see that the mass $m$ cancels out. Thus, the characteristics (such as the period) of the periodic motion do not depend on the mass.

113. (a) The net horizontal force is $F$ since the batter is assumed to exert no horizontal force on the bat. Thus, the horizontal acceleration (which applies as long as $F$ acts on the bat) is $a = F/m$.

(b) The only torque on the system is that due to $F$, which is exerted at $P$, at a distance $L_o - \frac{1}{2}L$ from $C$. Since $L_o = 2L/3$ (see Sample Problem 15-5), then the distance from $C$ to $P$ is $\frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. Since the net torque is equal to the rotational inertia ($I = 1/12mL^2$ about the center of mass) multiplied by the angular acceleration, we obtain

$$\alpha = \frac{\tau}{I} = \frac{F\left(\frac{1}{6}L\right)}{\frac{1}{12}mL^2} = \frac{2F}{mL}.$$

(c) The distance from $C$ to $O$ is $r = L/2$, so the contribution to the acceleration at $O$ stemming from the angular acceleration (in the counterclockwise direction of Fig. 15-13) is $\alpha r = \frac{1}{2}\alpha L$ (leftward in that figure). Also, the contribution to the acceleration at $O$ due to the result of part (a) is $F/m$ (rightward in that figure). Thus, if we choose rightward as positive, then the net acceleration of $O$ is

$$a_O = \frac{F}{m} - \frac{1}{2}\alpha L = \frac{F}{m} - \frac{1}{2}\left(\frac{2F}{mL}\right)L = 0.$$

(d) Point $O$ stays relatively stationary in the batting process, and that might be possible due to a force exerted by the batter or due to a finely tuned cancellation such as we have shown here. We assumed that the batter exerted no force, and our first expectation is that the impulse delivered by the impact would make all points on the bat go into motion, but for this particular choice of impact point, we have seen that the point being held by the batter is naturally stationary and exerts no force on the batter's hands which would otherwise have to "fight" to keep a good hold of it.

114. (a) By energy conservation, the required elastic potential energy stored in the spring is $\frac{1}{2}k(\Delta y)^2 = \frac{1}{2}mv_{\text{esc}}^2$. Solving for *k*, we obtain

$$k = \frac{mv_{\text{esc}}^2}{(\Delta y)^2} = \frac{(0.170\text{ kg})(11.2\times10^3\text{ m/s})}{(2.30\text{ m})^2} = 4.03\times10^6\text{ N/m}.$$

(b) The total applied force on the spring is

$$F_a = k(\Delta y) = (4.03\times10^6\text{ N/m})(2.30\text{ m}) = 9.27\times10^6\text{ N}.$$

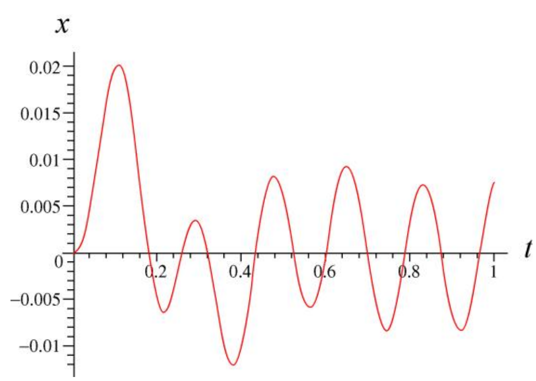
Thus, the number of people needed to exert this force is

$$\frac{F_a}{F_1} = \frac{9.27\times10^6\text{ N}}{490\text{ N}} = 1.89\times10^4.$$

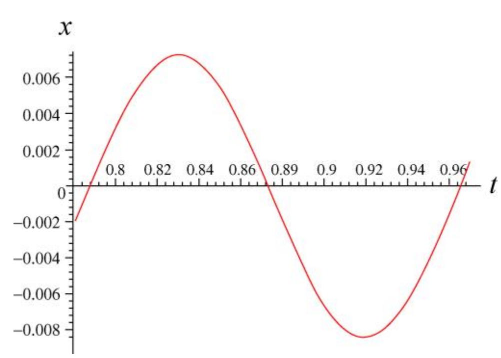
115. The period of oscillation is $T = 2\pi\sqrt{L/g} = 3.2\text{ s}$. Thus, the length for this simple pendulum is

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\text{ m})(3.20\text{ s})^2}{4\pi^2} = 2.54\text{ m}.$$

116. (a) A plot of *x* versus *t* (in SI units) is shown below:

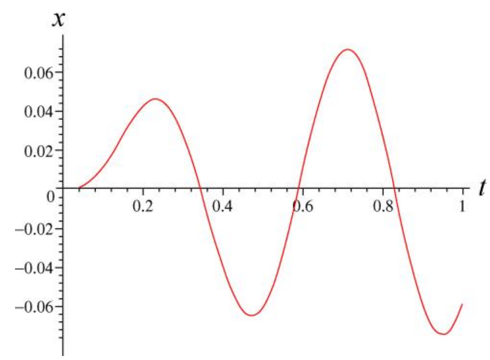



If we expand the plot near the end of that time interval we have



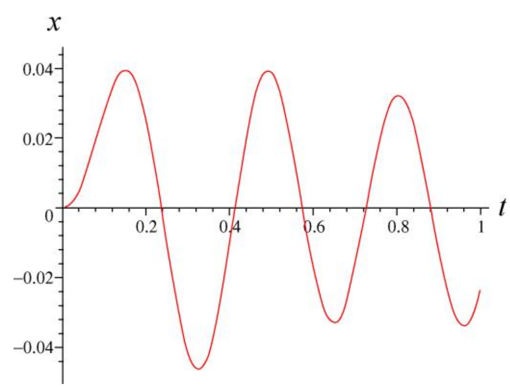
This is close enough to a regular sine wave cycle that we can estimate its period ($T$ = 0.18 s, so $\omega$ = 35 rad/s) and its amplitude ($y_m$ = 0.008 m).

(b) Now, with the new driving frequency ($\omega_d$ = 13.2 rad/s), the $x$ versus $t$ graph (for the first one second of motion) is as shown below:




It is a little more difficult in this case to estimate a regular sine-curve-like amplitude and period (for the part of the above graph near the end of that time interval), but we arrive at roughly $y_m$ = 0.07 m, $T$ = 0.48 s, and $\omega$ = 13 rad/s.

(c) Now, with $\omega_d$ = 20 rad/s, we obtain (for the behavior of the graph, below, near the end of the interval) the estimates: $y_m$ = 0.03 m, $T$ = 0.31 s, and $\omega$ = 20 rad/s.



# Chapter 16

1. Let $y_1$ = 2.0 mm (corresponding to time $t_1$) and $y_2$ = –2.0 mm (corresponding to time $t_2$). Then we find

$$kx + 600t_1 + \phi = \sin^{-1}(2.0/6.0)$$

and

$$kx + 600t_2 + \phi = \sin^{-1}(-2.0/6.0)\ .$$

Subtracting equations gives

$$600(t_1 - t_2) = \sin^{-1}(2.0/6.0) - \sin^{-1}(-2.0/6.0).$$

Thus we find $t_1 - t_2$ = 0.011 s (or 1.1 ms).

2. (a) The speed of the wave is the distance divided by the required time. Thus,

$$v = \frac{853\ \text{seats}}{39\ \text{s}} = 21.87\ \text{seats/s} \approx 22\ \text{seats/s}\ .$$

(b) The width $w$ is equal to the distance the wave has moved during the average time required by a spectator to stand and then sit. Thus,

$$w = vt = (21.87\ \text{seats/s})(1.8\ \text{s}) \approx 39\ \text{seats}\ .$$

3. (a) The angular wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}$.

(b) The speed of the wave is $v = \lambda f = \frac{\lambda\omega}{2\pi} = \frac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}$.

4. The distance $d$ between the beetle and the scorpion is related to the transverse speed $v_t$ and longitudinal speed $v_\ell$ as

$$d = v_t t_t = v_\ell t_\ell$$

where $t_t$ and $t_\ell$ are the arrival times of the wave in the transverse and longitudinal directions, respectively. With $v_t = 50$ m/s and $v_\ell = 150$ m/s, we have

$$\frac{t_t}{t_\ell} = \frac{v_\ell}{v_t} = \frac{150 \text{ m/s}}{50 \text{ m/s}} = 3.0 .$$

Thus, if

$$\Delta t = t_t - t_\ell = 3.0 t_\ell - t_\ell = 2.0 t_\ell = 4.0 \times 10^{-3} \text{ s} \;\; \Rightarrow \;\; t_\ell = 2.0 \times 10^{-3} \text{ s} ,$$

then $d = v_\ell t_\ell = (150 \text{ m/s})(2.0 \times 10^{-3} \text{ s}) = 0.30 \text{ m} = 30 \text{ cm}.$

5. (a) The motion from maximum displacement to zero is one-fourth of a cycle. One-fourth of a period is 0.170 s, so the period is $T$ = 4(0.170 s) = 0.680 s.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680 \text{ s}} = 1.47 \text{ Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40 \text{ m}}{0.680 \text{ s}} = 2.06 \text{ m/s}.$$

6. The slope that they are plotting is the physical slope of the sinusoidal waveshape (not to be confused with the more abstract "slope" of its time development; the physical slope is an $x$-derivative, whereas the more abstract "slope" would be the $t$-derivative). Thus, where the figure shows a maximum slope equal to 0.2 (with no unit), it refers to the maximum of the following function:

$$\frac{dy}{dx} = \frac{d}{dx}\left[ y_m \sin(kx - \omega t) \right] = y_m k \cos(kx - \omega t) .$$

The problem additionally gives $t$ = 0, which we can substitute into the above expression if desired. In any case, the maximum of the above expression is $y_m k$, where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40 \text{ m}} = 15.7 \text{ rad/m} .$$

Therefore, setting $y_m k$ equal to 0.20 allows us to solve for the amplitude $y_m$. We find

$$y_m = \frac{0.20}{15.7 \text{ rad/m}} = 0.0127 \text{ m} \approx 1.3 \text{ cm} .$$

7. (a) From the simple harmonic motion relation $u_m = y_m \omega$, we have

$$\omega = \frac{16 \text{ m/s}}{0.040 \text{ m}} = 400 \text{ rad/s}.$$

Since $\omega = 2\pi f$, we obtain $f$ = 64 Hz.

(b) Using $v = f\lambda$, we find $\lambda$ = (80 m/s)/(64 Hz) = 1.26 m $\approx 1.3$ m.

(c) The amplitude of the transverse displacement is $y_m = 4.0 \text{ cm} = 4.0\times10^{-2}$ m.

(d) The wave number is $k = 2\pi/\lambda$ = 5.0 rad/m.

(e) As shown in (a), the angular frequency is $\omega = (16 \text{ m/s})/(0.040 \text{ m}) = 4.0\times10^{2}$ rad/s.

(f) The function describing the wave can be written as

$$y = 0.040\sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant $\phi$ to satisfy the condition $y$ = 0.040 at $x = t = 0$. Therefore, sin $\phi$ = 1, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040\sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of $\omega$ is minus.

8. Setting $x = 0$ in $u = -\omega y_m \cos(kx - \omega t + \phi)$ (see Eq. 16-21 or Eq. 16-28) gives

$$u = -\omega y_m \cos(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a positive "slope" (referring to its $t$-derivative) at $t$ = 0, or

$$\frac{du}{dt} = \frac{d}{dt}\left[-\omega y_m \cos(-\omega t + \phi)\right] = -y_m\omega^2 \sin(-\omega t + \phi) > 0$$

at $t$ = 0. This implies that – sin$\phi$ > 0 and consequently that $\phi$ is in either the third or fourth quadrant. The graph shows (at $t$ = 0) $u$ = –4 m/s, and (at some later $t$) $u_{max}$ = 5 m/s. We note that $u_{max} = y_m\,\omega$. Therefore,

$$u = -u_{max}\cos(-\omega t + \phi)\Big|_{t=0} \;\Rightarrow\; \phi = \cos^{-1}\left(\tfrac{4}{5}\right) = \pm 0.6435 \text{ rad}$$

(bear in mind that $\cos\theta = \cos(-\theta)$), and we must choose $\phi = -0.64$ rad (since this is about $-37°$ and is in fourth quadrant). Of course, this answer added to $2n\pi$ is still a valid answer (where $n$ is any integer), so that, for example, $\phi = -0.64 + 2\pi = 5.64$ rad is also an acceptable result.

9. (a) The amplitude $y_m$ is half of the 6.00 mm vertical range shown in the figure, that is, $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t = 15$ m/s, where $d = 0.060$ m and $t = 0.0040$ s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda = 0.40$ m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = kv = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between $kx$ and $\omega t$) in the argument of the sine function because the wave is shown traveling to the right (in the $+x$ direction, see Section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16x - 2.4\times10^2\, t) .$$

10. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find $\lambda$ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0\times10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of $y$) is

$$u_{\max} = 2\pi f y_m = \left(4.0\,\pi\,\text{s}^{-1}\right)\left(6.0\,\text{cm}\right) = 75\,\text{cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0$ cm.

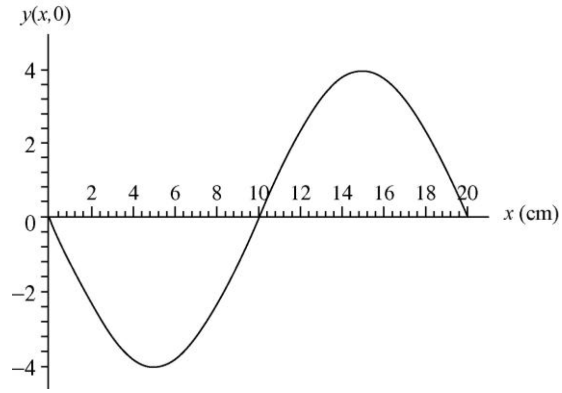
11. From Eq. 16-10, a general expression for a sinusoidal wave traveling along the +*x* direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi) .$$

(a) The figure shows that at $x = 0$, $y(0,t) = y_m \sin(-\omega t + \phi)$ is a positive sine function, that is, $y(0,t) = +y_m \sin \omega t$. Therefore, the phase constant must be $\phi = \pi$. At $t = 0$, we then have

$$y(x,0) = y_m \sin(kx + \pi) = -y_m \sin kx$$

which is a negative sine function. A plot of $y(x, 0)$ is depicted on the right.




(b) From the figure we see that the amplitude is $y_m$ = 4.0 cm.

(c) The angular wave number is given by $k = 2\pi/\lambda = \pi/10 = 0.31$ rad/cm.

(d) The angular frequency is $\omega = 2\pi/T = \pi/5 = 0.63$ rad/s.

(e) As found in part (a), the phase is $\phi = \pi$.

(f) The sign is minus since the wave is traveling in the +*x* direction.

(g) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(h) From the results above, the wave may be expressed as

$$y(x,t) = 4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0 \sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right).$$

Taking the derivative of *y* with respect to *t*, we find

$$u(x,t) = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which yields $u(0, 5.0) = -2.5$ cm/s.

12. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = (225\pi) \sin (\pi x - 15\pi t) .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2(\pi x - 15\pi t) + \cos^2(\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi\sqrt{15^2 - y^2}.$$

Therefore, where $y = 12$, $u$ must be $\pm 135\pi$. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

13. Using $v = f\lambda$, we find the length of one cycle of the wave is

$$\lambda = 350/500 = 0.700 \text{ m} = 700 \text{ mm}.$$

From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}$ s = 2.00 ms.

(a) A cycle is equivalent to $2\pi$ radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6$ = (700 mm)/6 = 117 mm.

(b) The interval 1.00 ms is half of $T$ and thus corresponds to half of one cycle, or half of $2\pi$ rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

14. (a) Comparing with Eq. 16-2, we see that $k$ = 20/m and $\omega$ = 600 rad/s. Therefore, the speed of the wave is (see Eq. 16-13) $v = \omega/k$ = 30 m/s.

(b) From Eq. 16–26, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017 \text{ kg/m} = 17 \text{ g/m}.$$

15. **THINK** Numerous physical properties of a traveling wave can be deduced from its wave function.

**EXPRESS** We first recall that from Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

where $y_m$ is the amplitude, $k = 2\pi/\lambda$ is the angular wave number, $\omega = 2\pi/T$ is the angular frequency and $\phi$ is the phase constant. The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string.

**ANALYZE** (a) The amplitude of the wave is $y_m$=0.120 mm.

(b) The wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is $f$ = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative $x$ direction.

**LEARN** In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

16. We use $v = \sqrt{\tau/\mu} \propto \sqrt{\tau}$ to obtain

$$\tau_2 = \tau_1\left(\frac{v_2}{v_1}\right)^2 = (120\,\text{N})\left(\frac{180\,\text{m/s}}{170\,\text{m/s}}\right)^2 = 135\text{N}.$$

17. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where $\lambda$ is the wavelength, $T$ is the period, $\omega$ is the angular frequency ($2\pi/T$), and $k$ is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\text{ m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\text{ rad/s})/(2.0\text{ m}^{-1}) = 15\text{ m/s}.$$

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6\times10^{-4}\,\text{kg/m}\right)(15\,\text{m/s})^2 = 0.036\,\text{N}.$$

18. The volume of a cylinder of height $\ell$ is $V = \pi r^2 \ell = \pi d^2 \ell/4$. The strings are long, narrow cylinders, one of diameter $d_1$ and the other of diameter $d_2$ (and corresponding linear densities $\mu_1$ and $\mu_2$). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho\pi d^2\,\ell/4}{\ell} = \frac{\pi\rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho\, d_1^2/4}{\pi\rho\, d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2.$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

19. **THINK** The speed of a transverse wave in a rope is related to the tension in the rope and the linear mass density of the rope.

**EXPRESS** The wave speed $v$ is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the rope and $\mu$ is the rope's linear mass density, which is defined as the mass per unit length of rope $\mu = m/L$.

**ANALYZE** With a linear mass density of

$$\mu = m/L = (0.0600 \text{ kg})/(2.00 \text{ m}) = 0.0300 \text{ kg/m},$$

we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\,\text{m/s}.$$

**LEARN** Since $v \sim 1/\sqrt{\mu}$, the thicker the rope (larger $\mu$), the slower the speed of the rope under the same tension $\tau$.

20. From $v = \sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2}.$$

21. The pulses have the same speed $v$. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time $t$ is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where $L$ is the length of the wire, at time $t = 30$ ms. If this time is denoted by $t_0$, then the coordinate of this wave at time $t$ is $x_2 = L - v(t - t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t - t_0)$. We solve for the time they meet: $t = (L + vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L + vt_0)/2$. Now, we calculate the wave speed:

$$v=\sqrt{\frac{\tau L}{m}}=\sqrt{\frac{(250\,\text{N})(10.0\,\text{m})}{0.100\,\text{kg}}}=158\,\text{m/s}.$$

Here $\tau$ is the tension in the wire and $L/m$ is the linear mass density of the wire. The coordinate of the meeting point is

$$x=\frac{10.0\,\text{m}+(158\,\text{m/s})(30.0\times10^{-3}\text{s})}{2}=7.37\,\text{m}.$$

This is the distance from the left end of the wire. The distance from the right end is $L - x$ = (10.0 m – 7.37 m ) = 2.63 m.

22. (a) The general expression for $y(x, t)$ for the wave is $y(x, t) = y_m \sin(kx - \omega t)$, which, at $x$ = 10 cm, becomes $y(x = 10 \text{ cm}, t) = y_m \sin[k(10 \text{ cm} - \omega t)]$. Comparing this with the expression given, we find $\omega$ = 4.0 rad/s, or $f = \omega/2\pi$ = 0.64 Hz.

(b) Since $k$(10 cm) = 1.0, the wave number is $k$ = 0.10/cm. Consequently, the wavelength is $\lambda = 2\pi/k$ = 63 cm.

(c) The amplitude is $y_m = 5.0$ cm.

(d) In part (b), we have shown that the angular wave number is $k$ = 0.10/cm.

(e) The angular frequency is $\omega$ = 4.0 rad/s.

(f) The sign is minus since the wave is traveling in the +$x$ direction.

Summarizing the results obtained above by substituting the values of $k$ and $\omega$ into the general expression for $y(x, t)$, with centimeters and seconds understood, we obtain

$$y(x,t)=5.0\sin(0.10x-4.0t).$$

(g) Since $v=\omega/k=\sqrt{\tau/\mu}$, the tension is

$$\tau=\frac{\omega^2\mu}{k^2}=\frac{(4.0\,\text{g/cm})(4.0\,\text{s}^{-1})^2}{(0.10\,\text{cm}^{-1})^2}=6400\,\text{g}\cdot\text{cm/s}^2=0.064\,\text{N}.$$

23. **THINK** Various properties of the sinusoidal wave can be deduced from the plot of its displacement as a function of position.

**EXPRESS** In analyzing the properties of the wave, we first recall that from Eq. 16-10, a general expression for a sinusoidal wave traveling along the +$x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

where $y_m$ is the amplitude, $k = 2\pi/\lambda$ is the angular wave number, $\omega = 2\pi/T$ is the angular frequency and $\phi$ is the phase constant. The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string.

**ANALYZE** (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55 \text{ cm} - 15 \text{ cm}) = 40 \text{ cm} = 0.40 \text{ m}.$$

(c) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{3.6\,\text{N}}{25\times10^{-3}\,\text{kg/m}}} = 12 \text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12 \text{ m/s})/(0.40 \text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30 \text{ Hz}) = 0.033 \text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30 \text{ Hz})\,(5.0 \text{ cm}) = 940 \text{ cm/s} = 9.4 \text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40 \text{ m}) = 16 \text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30 \text{ Hz}) = 1.9\times10^2$ rad/s .

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is $4.0 \times 10^{-2}$ m. The formula for the displacement gives $y(0, 0) = y_m \sin \phi$. We wish to select $\phi$ so that

$$(5.0 \times 10^{-2} \text{ m})\sin \phi = (4.0 \times 10^{-2} \text{ m}).$$

The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y\,(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative $x$ direction.

**LEARN** Summarizing the results obtained above, the wave function of the traveling wave can be written as

$$y(x,t) = \left(5.0\times10^{-2}\,\text{m}\right)\sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right].$$

24. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1 = \sqrt{\frac{\tau}{\mu_1}} = \sqrt{\frac{Mg}{2\mu_1}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(3.00\,\text{g/m})}} = 28.6\,\text{m/s}.$$

(b) And the wave speed in string 2 is

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(5.00\,\text{g/m})}} = 22.1\,\text{m/s}.$$

(c) Let $v_1 = \sqrt{M_1 g/(2\mu_1)} = v_2 = \sqrt{M_2 g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for $M_1$ and obtain

$$M_1 = \frac{M}{1+\mu_2/\mu_1} = \frac{500\,\text{g}}{1+5.00/3.00} = 187.5\,\text{g} \approx 188\,\text{g}.$$

(d) And we solve for the second mass: $M_2 = M - M_1 = (500\text{ g} - 187.5\text{ g}) \approx 313$ g.

25. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension at that point and $\mu$ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance $y$ from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by $\mu gy$, so the tension is $\tau = \mu gy$. The wave speed is $v = \sqrt{\mu gy/\mu} = \sqrt{gy}.$

(b) The time $dt$ for the wave to move past a length $dy$, a distance $y$ from the bottom end, is $dt = dy/v = dy\big/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\Bigg|_0^L = 2\sqrt{\frac{L}{g}}\;.$$

26. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$f=\frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}}=\frac{1}{2\pi(7.70\times10^{-3}\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}}=198\text{ Hz.}$$

27. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2\text{ ms}} = 500$ Hz, and that the wavelength $\lambda$ = 0.20 m. We also note from the graph that the maximum value of *dK/dt* is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\tfrac{1}{2}\mu v\,\omega^2 y_m{}^2 = 10$$

with SI units understood. Substituting in $\mu = 0.002$ kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m=\sqrt{\frac{10}{2\pi^2\mu\lambda f^3}}=0.0032\text{ m}.$$

28. Comparing

$$y(x,t)=(3.00\text{ mm})\sin[(4.00\text{ m}^{-1})x-(7.00\text{ s}^{-1})t]$$

to the general expression $y(x,t)=y_m\sin(kx-\omega t)$, we see that $k=4.00\text{ m}^{-1}$ and $\omega=7.00$ rad/s. The speed of the wave is

$$v=\omega/k=(7.00\text{ rad/s})/(4.00\text{ m}^{-1})=1.75\text{ m/s.}$$

29. The wave

$$y(x,t)=(2.00\text{ mm})[(20\text{ m}^{-1})x-(4.0\text{ s}^{-1})t]^{1/2}$$

is of the form $h(kx-\omega t)$ with angular wave number $k=20\text{ m}^{-1}$ and angular frequency $\omega=4.0$ rad/s. Thus, the speed of the wave is

$$v=\omega/k=(4.0\text{ rad/s})/(20\text{ m}^{-1})=0.20\text{ m/s.}$$

30. The wave $y(x,t)=(4.00\text{ mm})\,h[(30\text{ m}^{-1})x+(6.0\text{ s}^{-1})t]$ is of the form $h(kx-\omega t)$ with angular wave number $k=30\text{ m}^{-1}$ and angular frequency $\omega=6.0$ rad/s. Thus, the speed of the wave is

$$v=\omega/k=(6.0\text{ rad/s})/(30\text{ m}^{-1})=0.20\text{ m/s.}$$

31. **THINK** By superposition principle, the resultant wave is the algebraic sum of the two interfering waves.

**EXPRESS** The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \;= 2y_m \cos\left(\tfrac{1}{2}\phi\right)\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right),$$

where we have used

$$\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta).$$

**ANALYZE** The two waves are out of phase by $\phi = \pi/2$, so the amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41y_m .$$

**LEARN** The interference between two waves can be constructive or destructive, depending on their phase difference.

32. (a) Let the phase difference be $\phi$. Then from Eq. 16-52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8°.$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to $2\pi$ rad), this is equivalent to 1.45 rad/$2\pi$ = 0.230 wavelength.

33. (a) The amplitude of the second wave is $y_m = 9.00$ mm, as stated in the problem.

(b) The figure indicates that $\lambda$ = 40 cm = 0.40 m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t$ = (56.0 cm)/(8.0 ms) = 70 m/s. This, in turn, implies that the angular frequency is

$$\omega = kv = 1100 \text{ rad/s} = 1.1\times10^3 \text{ rad/s}.$$

(d) The figure depicts two traveling waves (both going in the $-x$ direction) of equal amplitude $y_m$. The amplitude of their resultant wave, as shown in the figure, is $y'_m$ = 4.00 mm. Equation 16-52 applies:

$$y'_m = 2y_m \cos(\tfrac{1}{2}\phi_2) \quad\Rightarrow\quad \phi_2 = 2\cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it was shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, $\phi$ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16\,x + 1100\,t) \quad \text{and} \quad y_2 = (0.00900)\sin(16\,x + 1100\,t + 2.7\,)\,.$$

34. (a) We use Eq. 16-26 and Eq. 16-33 with μ = 0.00200 kg/m and $y_m = 0.00300$ m. These give $v = \sqrt{\tau/\mu} = 775\ \text{m/s}$ and

$$P_{\text{avg}} = \tfrac{1}{2}\ \mu v\ \omega^2 y_m^{\ 2} = 10\ \text{W}.$$

(b) In this situation, the waves are two separate string (no superposition occurs). The answer is clearly twice that of part (a); $P = 20$ W.

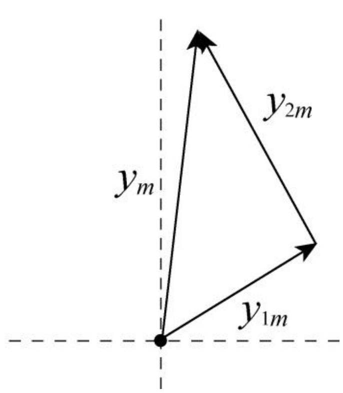
(c) Now they are on the same string. If they are interfering constructively (as in Fig. 16-13(a)) then the amplitude $y_m$ is doubled, which means its square $y_m^{\ 2}$ increases by a factor of 4. Thus, the answer now is four times that of part (a); $P = 40$ W.

(d) Equation 16-52 indicates in this case that the amplitude (for their superposition) is $2\,y_m\cos(0.2\pi) = 1.618$ times the original amplitude $y_m$. Squared, this results in an increase in the power by a factor of 2.618. Thus, $P = 26$ W in this case.

(e) Now the situation depicted in Fig. 16-13(b) applies, so $P = 0$.

35. **THINK** We use phasors to add the two waves and calculate the amplitude of the resultant wave.

**EXPRESS** The phasor diagram is shown below: $y_{1m}$ and $y_{2m}$ represent the original waves and $y_m$ represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle.




**ANALYZE** The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus, the amplitude of the resultant wave is $y_m$ = 5.0 cm.

**LEARN** When adding two waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. The same result, however, could also be obtained as follows: Writing the two waves as $y_1 = 3\sin(kx - \omega t)$ and $y_2 = 4\sin(kx - \omega t + \pi/2) = 4\cos(kx - \omega t)$, we have, after a little algebra,

$$\begin{aligned} y = y_1 + y_2 &= 3\sin(kx - \omega t) + 4\cos(kx - \omega t) = 5\left[\frac{3}{5}\sin(kx - \omega t) + \frac{4}{5}\cos(kx - \omega t)\right] \\ &= 5\sin(kx - \omega t + \phi) \end{aligned}$$

where $\phi = \tan^{-1}(4/3)$. In deducing the phase $\phi$, we set $\cos\phi = 3/5$ and $\sin\phi = 4/5$, and use the relation $\cos\phi\sin\theta + \sin\phi\cos\theta = \sin(\theta + \phi)$.

36. We see that $y_1$ and $y_3$ cancel (they are 180º) out of phase, and $y_2$ cancels with $y_4$ because their phase difference is also equal to π rad (180º). There is no resultant wave in this case.

37. (a) Using the phasor technique, we think of these as two "vectors" (the first of "length" 4.6 mm and the second of "length" 5.60 mm) separated by an angle of $\phi = 0.8\pi$ radians (or 144º). Standard techniques for adding vectors then lead to a resultant vector of length 3.29 mm.

(b) The angle (relative to the first vector) is equal to 88.8º (or 1.55 rad).

(c) Clearly, it should in "in phase" with the result we just calculated, so its phase angle relative to the first phasor should be also 88.8º (or 1.55 rad).

38. (a) As shown in Figure 16-13(b) in the textbook, the least-amplitude resultant wave is obtained when the phase difference is π rad.

(b) In this case, the amplitude is (8.0 mm – 5.0 mm) = 3.0 mm.

(c) As shown in Figure 16-13(a) in the textbook, the greatest-amplitude resultant wave is obtained when the phase difference is 0 rad.

(d) In the part (c) situation, the amplitude is (8.0 mm + 5.0 mm) = 13 mm.

(e) Using phasor terminology, the angle "between them" in this case is π/2 rad (90º), so the Pythagorean theorem applies:

$$\sqrt{(8.0 \text{ mm})^2 + (5.0 \text{ mm})^2} = 9.4 \text{ mm} .$$

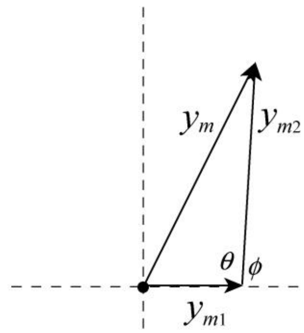
39. The phasor diagram is shown to the right. We use the cosine theorem:

$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi.$$

We solve for cos $\phi$:

$$\cos\phi = \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}\,y_{m2}} = \frac{(9.0\,\text{mm})^2 - (5.0\,\text{mm})^2 - (7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} = 0.10.$$

The phase constant is therefore $\phi = 84°$.




40. The string is flat each time the particle passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude $T = 2(0.50 \text{ s}) = 1.0$ s. Thus, $f = 1/T = 1.0$ Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\text{cm/s}}{1.0\,\text{Hz}} = 10 \text{ cm}.$$

41. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

**EXPRESS** The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where $M$ is the mass of the string and $L$ is its length. The possible wavelengths of a standing wave are given by $\lambda_n = 2L/n$, where $L$ is the length of the string and $n$ is an integer.

**ANALYZE** (a) The wave speed is

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0 \text{ N})\,(8.40 \text{ m})}{0.120 \text{ kg}}} = 82.0 \text{ m/s}.$$

(b) The longest possible wavelength $\lambda$ for a standing wave is related to the length of the string by $L = \lambda_1/2$ ($n = 1$), so $\lambda_1 = 2L = 2(8.40 \text{ m}) = 16.8$ m.

(c) The corresponding frequency is $f_1 = v/\lambda_1 = (82.0 \text{ m/s})/(16.8 \text{ m}) = 4.88$ Hz.

**LEARN** The resonant frequencies are given by

$$f_n = \frac{v}{\lambda} = \frac{v}{2L/n} = n\frac{v}{2L} = nf_1,$$

where $f_1 = v/\lambda_1 = v/2L$. The oscillation mode with $n = 1$ is called the fundamental mode or the first harmonic.

42. Use Eq. 16-66 (for the resonant frequencies) and Eq. 16-26 $(v=\sqrt{\tau/\mu})$ to find $f_n$:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2f_3.$$

(b) And we get the new wavelength $\lambda'_3 = \dfrac{v'}{f_3'} = \dfrac{2L}{3} = \lambda_3.$

43. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

**EXPRESS** Possible wavelengths are given by $\lambda_n = 2L/n$, where $L$ is the length of the wire and $n$ is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where $v$ is the wave speed. The wave speed is given by $v=\sqrt{\tau/\mu}=\sqrt{\tau L/M}$, where $\tau$ is the tension in the wire, $\mu$ is the linear mass density of the wire, and $M$ is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus,

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\ \text{N}}{(10.0\ \text{m})\ (0.100\ \text{kg})}} = n\ (7.91\ \text{Hz}).$$

**ANALYZE** (a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91\ \text{Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91\ \text{Hz}) = 23.7$ Hz.

**LEARN** The frequencies are integer multiples of the fundamental frequency $f_1$. This means that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency $f_1$.

44. (a) The wave speed is given by $v=\sqrt{\frac{\tau}{\mu}}=\sqrt{\frac{7.00\,\text{N}}{2.00\times 10^{-3}\,\text{kg}/1.25\,\text{m}}}=66.1\,\text{m/s}.$

(b) The wavelength of the wave with the lowest resonant frequency $f_1$ is $\lambda_1 = 2L$, where $L = 125$ cm. Thus,

$$f_1=\frac{v}{\lambda_1}=\frac{66.1\text{ m/s}}{2(1.25\text{ m})}=26.4\text{ Hz.}$$

45. **THINK** The difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency.

**EXPRESS** The resonant wavelengths are given by $\lambda_n = 2L/n$, where $L$ is the length of the string and $n$ is an integer, and the resonant frequencies are

$$f_n = v/\lambda = nv/2L = nf_1,$$

where $v$ is the wave speed. Suppose the lower frequency is associated with the integer $n$. Then, since there are no resonant frequencies between, the higher frequency is associated with $n + 1$. The frequency difference between successive modes is

$$\Delta f = f_{n+1}-f_n=\frac{v}{2L}=f_1.$$

**ANALYZE** (a) The lowest possible resonant frequency is

$$f_1=\Delta f = f_{n+1}-f_n=420\text{ Hz}-315\text{ Hz}=105\text{ Hz}.$$

(b) The longest possible wavelength is $\lambda_1 = 2L$. If $f_1$ is the lowest possible frequency then

$$v = \lambda_1 f_1 = (2L)f_1 = 2(0.75\text{ m})(105\text{ Hz}) = 158\text{ m/s}.$$

**LEARN** Since 315 Hz = 3(105 Hz) and 420 Hz = 4(105 Hz), the two frequencies correspond to $n = 3$ and $n = 4$, respectively.

46. The $n$th resonant frequency of string $A$ is

$$f_{n,A}=\frac{v_A}{2l_A}n=\frac{n}{2L}\sqrt{\frac{\tau}{\mu}},$$

while for string $B$ it is

$$f_{n,B}=\frac{v_B}{2l_B}n=\frac{n}{8L}\sqrt{\frac{\tau}{\mu}}=\frac{1}{4}f_{n,A}.$$

(a) Thus, we see $f_{1,A} = f_{4,B}$. That is, the fourth harmonic of *B* matches the frequency of *A*'s first harmonic.

(b) Similarly, we find $f_{2,A} = f_{8,B}$.

(c) No harmonic of *B* would match $f_{3,A} = \frac{3v_A}{2l_A} = \frac{3}{2L}\sqrt{\frac{\tau}{\mu}}$.

47. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus,

$$f_1 = (390 \text{ Hz} - 325 \text{ Hz}) = 65 \text{ Hz}.$$

This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

48. Using Eq. 16-26, we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{65.2\times10^6 \text{ N}}{3.35 \text{ kg/m}}} = 4412 \text{ m/s}.$$

The corresponding resonant frequencies are

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The wavelength of the wave with the lowest (fundamental) resonant frequency $f_1$ is $\lambda_1 = 2L$, where $L$ = 347 m. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{4412 \text{ m/s}}{2(347 \text{ m})} = 6.36 \text{ Hz}.$$

(b) The frequency difference between successive modes is

$$\Delta f = f_n - f_{n-1} = \frac{v}{2L} = \frac{4412 \text{ m/s}}{2(347 \text{ m})} = 6.36 \text{ Hz}.$$

49. (a) Equation 16-26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150 \text{ N}}{7.20\times10^{-3} \text{ kg/m}}} = 144.34 \text{ m/s} \approx 1.44\times10^2 \text{ m/s}.$$

(b) From the figure, we find the wavelength of the standing wave to be

$$\lambda = (2/3)(90.0 \text{ cm}) = 60.0 \text{ cm}.$$

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^2 \text{ m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

50. From the $x = 0$ plot (and the requirement of an anti-node at $x = 0$), we infer a standing wave function of the form

$$y(x,t) = -(0.04)\cos(kx)\sin(\omega t),$$

where $\omega = 2\pi/T = \pi$ rad/s, with length in meters and time in seconds. The parameter $k$ is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive $x$ direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y(0.20 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0.040\,\text{m}\ .$$

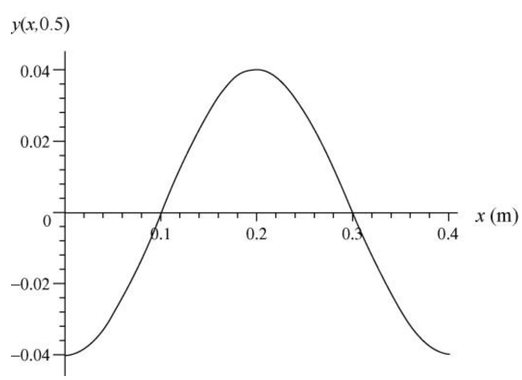
(b) The above equation yields $y(0.30 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0$ .

(c) We take the derivative with respect to time and obtain, at $t = 0.50$ s and $x = 0.20$ m,

$$u = \frac{dy}{dt} = -0.04\omega\ \cos(kx)\cos(\omega t) = 0\,.$$

d) The above equation yields $u = -0.13$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown next:



51. **THINK** In this problem, in order to produce the standing wave pattern, the two waves must have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions.

**EXPRESS** We take the two waves to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The superposition principle gives

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) = \left[2y_m \sin kx\right]\cos \omega t\ .$$

**ANALYZE** (a) The amplitude $y_m$ is half the maximum displacement of the standing wave, or (0.01 m)/2 = $5.0 \times 10^{-3}$ m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With $L$ = 3.0m, $\lambda$ = 2.0 m. The angular wave number is

$$k = 2\pi/\lambda = 2\pi/(2.0 \text{ m}) = 3.1 \text{ m}^{-1}.$$

(c) If $v$ is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\text{m/s})}{2(3.0\,\text{m})} = 50 \text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or

$$\omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ rad/s}.$$

(d) If one of the waves has the form $y_2(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y_1(x,t) = y_m \sin(kx - \omega t)$. The sign in front of $\omega$ for $y'(x,t)$ is minus.

**LEARN** Using the results above, the two waves can be written as

$$y_1 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x - \left(314\,\text{s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x + \left(314\,\text{s}^{-1}\right)t\right].$$

52. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 16-20(b), where (see Eq. 16-65)

$$\lambda = L, \quad f = \frac{v}{L}.$$

(a) Comparing the given function with Eq. 16-60, we obtain $k = \pi/2$ and $\omega = 12\pi$ rad/s. Since $k = 2\pi/\lambda$, then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \quad \Rightarrow \quad \lambda = 4.0\,\text{m} \quad \Rightarrow \quad L = 4.0\,\text{m}.$$

(b) Since $\omega = 2\pi f$, then $2\pi f = 12\pi$ rad/s, which yields

$$f = 6.0\,\text{Hz} \quad \Rightarrow \quad v = f\lambda = 24\,\text{m/s}.$$

(c) Using Eq. 16-26, we have

$$v = \sqrt{\frac{\tau}{\mu}} \quad \Rightarrow \quad 24\ \text{m/s} = \sqrt{\frac{200\ \text{N}}{m/(4.0\ \text{m})}}$$

which leads to $m = 1.4$ kg.

(d) With

$$f = \frac{3v}{2L} = \frac{3(24\ \text{m/s})}{2(4.0\ \text{m})} = 9.0\,\text{Hz}$$

the period is $T = 1/f = 0.11$ s.

53. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\ \text{cm}^{-1}$. The wave speed is

$$v = \omega/k = (40\pi\ \text{rad/s})/(\pi/3\ \text{cm}^{-1}) = 1.2\times10^2\ \text{cm/s}.$$

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\ \text{cm}^{-1}) = 3.0$ cm. Here $2\pi/k$ was substituted for $\lambda$.

(d) The string speed is given by

$$u(x, t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t).$$

For the given coordinate and time,

$$u = -(40\pi\ \text{rad/s})\ (0.50\,\text{cm})\ \sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\ \left[\left(40\pi\ \text{s}^{-1}\right)\left(\frac{9}{8}\ \text{s}\right)\right] = 0.$$

54. Reference to point $A$ as an anti-node suggests that this is a standing wave pattern and thus that the waves are traveling in opposite directions. Thus, we expect one of them to be of the form $y = y_m \sin(kx + \omega t)$ and the other to be of the form $y = y_m \sin(kx - \omega t)$.

(a) Using Eq. 16-60, we conclude that $y_m = \frac{1}{2}(9.0 \text{ mm}) = 4.5$ mm, due to the fact that the amplitude of the standing wave is $\frac{1}{2}(1.80 \text{ cm}) = 0.90 \text{ cm} = 9.0$ mm.

(b) Since one full cycle of the wave (one wavelength) is 40 cm, $k = 2\pi/\lambda \approx 16 \text{ m}^{-1}$.

(c) The problem tells us that the time of half a full period of motion is 6.0 ms, so $T = 12$ ms and Eq. 16-5 gives $\omega = 5.2 \times 10^2$ rad/s.

(d) The two waves are therefore

$$y_1(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x + (520 \text{ s}^{-1})t]$$

and

$$y_2(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x - (520 \text{ s}^{-1})t] .$$

If one wave has the form $y(x,t) = y_m \sin(kx + \omega t)$ as in $y_1$, then the other wave must be of the form $y'(x,t) = y_m \sin(kx - \omega t)$ as in $y_2$. Therefore, the sign in front of $\omega$ is minus.

55. Recalling the discussion in section 16-12, we observe that this problem presents us with a standing wave condition with amplitude 12 cm. The angular wave number and frequency are noted by comparing the given waves with the form $y = y_m \sin(k\,x \pm \omega\,t)$. The anti-node moves through 12 cm in simple harmonic motion, just as a mass on a vertical spring would move from its upper turning point to its lower turning point, which occurs during a half-period. Since the period $T$ is related to the angular frequency by Eq. 15-5, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.00\pi} = 0.500 \text{ s}.$$

Thus, in a time of $t = \frac{1}{2}T = 0.250$ s, the wave moves a distance $\Delta x = vt$ where the speed of the wave is $v = \omega / k = 1.00$ m/s. Therefore, $\Delta x = (1.00 \text{ m/s})(0.250 \text{ s}) = 0.250$ m.

56. The nodes are located from vanishing of the spatial factor $\sin 5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \cdots$$

(a) The smallest value of $x$ that corresponds to a node is $x = 0$.

(b) The second smallest value of $x$ that corresponds to a node is $x = 0.20$ m.

(c) The third smallest value of $x$ that corresponds to a node is $x = 0.40$ m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16-58 through Eq. 16-60, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -(0.040)(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all $x$) at times such as $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \cdots$$

Thus, the first time in which all points on the string have zero transverse velocity is when $t = 0$ s.

(h) The second time in which all points on the string have zero transverse velocity is when $t = 1/40$ s = 0.025 s.

(i) The third time in which all points on the string have zero transverse velocity is when $t = 2/40$ s = 0.050 s.

57. (a) The angular frequency is $\omega = 8.00\pi/2 = 4.00\pi$ rad/s, so the frequency is

$$f = \omega/2\pi = (4.00\pi \text{ rad/s})/2\pi = 2.00 \text{ Hz}.$$

(b) The angular wave number is $k = 2.00\pi/2 = 1.00\pi \text{ m}^{-1}$, so the wavelength is

$$\lambda = 2\pi/k = 2\pi/(1.00\pi \text{ m}^{-1}) = 2.00 \text{ m}.$$

(c) The wave speed is

$$v = \lambda f = (2.00 \text{ m})(2.00 \text{ Hz}) = 4.00 \text{ m/s}.$$

(d) We need to add two cosine functions. First convert them to sine functions using cos $\alpha$ = sin ($\alpha$ + π/2), then apply

$$\cos\alpha+\cos\beta=\sin\left(\alpha+\frac{\pi}{2}\right)+\sin\left(\beta+\frac{\pi}{2}\right)=2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right).$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m\cos(kx+\omega t)+y_m\cos(kx-\omega t)=2y_m\cos(kx)\cos(\omega t).$$

Nodes occur where cos($kx$) = 0 or $kx = n\pi + \pi/2$, where $n$ is an integer (including zero). Since $k$ = 1.0π m$^{-1}$, this means $x=\left(n+\frac{1}{2}\right)(1.00\,\text{m})$. Thus, the smallest value of $x$ that corresponds to a node is $x$ = 0.500 m ($n$ = 0).

(e) The second smallest value of $x$ that corresponds to a node is $x$ = 1.50 m ($n$ = 1).

(f) The third smallest value of $x$ that corresponds to a node is $x$ = 2.50 m ($n$ = 2).

(g) The displacement is a maximum where cos($kx$) = ±1. This means $kx = n\pi$, where $n$ is an integer. Thus, $x = n$(1.00 m). The smallest value of $x$ that corresponds to an anti-node (maximum) is $x$ = 0 ($n$ = 0).

(h) The second smallest value of $x$ that corresponds to an anti-node (maximum) is $x=1.00$ m ($n$ = 1).

(i) The third smallest value of $x$ that corresponds to an anti-node (maximum) is $x=2.00$ m ($n$ = 2).

58. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f=\frac{nv}{2L}=\frac{n}{2L}\sqrt{\frac{\tau}{\mu}}=\frac{n}{2L}\sqrt{\frac{mg}{\mu}},\qquad n=1,2,3,\ldots$$

(a) The mass that allows the oscillator to set up the 4th harmonic ($n=4$) on the string is

$$m=\left.\frac{4L^2f^2\mu}{n^2g}\right|_{n=4}=\frac{4(1.20\text{ m})^2(120\text{ Hz})^2(0.00160\text{ kg/m})}{(4)^2(9.80\text{ m/s}^2)}=0.846\text{ kg}$$

(b) If the mass of the block is $m=1.00$ kg, the corresponding $n$ is

$$n=\sqrt{\frac{4L^2 f^2\mu}{g}}=\sqrt{\frac{4(1.20\text{ m})^2(120\text{ Hz})^2(0.00160\text{ kg/m})}{9.80\text{ m/s}^2}}=3.68$$

which is not an integer. Therefore, the mass cannot set up a standing wave on the string.

59. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are $n_1$ loops in the aluminum section of the wire. Then,

$$L_1 = n_1\lambda_1/2 = n_1v_1/2f,$$

where $\lambda_1$ is the wavelength and $v_1$ is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where $f$ is the frequency. Thus $f = n_1v_1/2L_1$. A similar expression holds for the steel section: $f = n_2v_2/2L_2$. Since the frequency is the same for the two sections, $n_1v_1/L_1 = n_2v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1=\sqrt{\tau/\mu_1}$, where $\mu_1$ is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1AL_1$, where $\rho_1$ is the mass density (mass per unit volume) for aluminum and $A$ is the cross-sectional area of the wire. Thus

$$\mu_1 = \rho_1AL_1/L_1 = \rho_1A$$

and $v_1=\sqrt{\tau/\rho_1A}$. A similar expression holds for the wave speed in the steel section: $v_2=\sqrt{\tau/\rho_2A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1}=n_2/L_2\sqrt{\rho_2}$, where $A$ has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1}=\frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}}=\frac{(0.866\,\text{m})\sqrt{7.80\times10^3\text{ kg/m}^3}}{(0.600\,\text{m})\sqrt{2.60\times10^3\text{ kg/m}^3}}=2.50.$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is

$$f=n_1v_1/2L_1=(n_1/2L_1)\sqrt{\tau/\rho_1A}.$$

The tension is provided by the hanging block and is $\tau = mg$, where $m$ is the mass of the block. Thus,

$$f=\frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1A}}=\frac{2}{2(0.600\,\text{m})}\sqrt{\frac{(10.0\,\text{kg})(9.80\,\text{m/s}^2)}{(2.60\times10^3\text{kg/m}^3)(1.00\times10^{-6}\text{m}^2)}}=324\,\text{Hz}.$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

60. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

The mass that allows the oscillator to set up the *n*th harmonic on the string is

$$m = \frac{4L^2 f^2 \mu}{n^2 g}.$$

Thus, we see that the block mass is inversely proportional to the harmonic number squared. Thus, if the 447 gram block corresponds to harmonic number $n$, then

$$\frac{447}{286.1} = \frac{(n+1)^2}{n^2} = \frac{n^2 + 2n + 1}{n^2} = 1 + \frac{2n+1}{n^2}.$$

Therefore, $\frac{447}{286.1} - 1 = 0.5624$ must equal an odd integer ($2n + 1$) divided by a squared integer ($n^2$). That is, multiplying 0.5624 by a square (such as 1, 4, 9, 16, etc.) should give us a number very close (within experimental uncertainty) to an odd number (1, 3, 5, …). Trying this out in succession (starting with multiplication by 1, then by 4, …), we find that multiplication by 16 gives a value very close to 9; we conclude $n = 4$ (so $n^2 = 16$ and $2n + 1 = 9$). Plugging in $m = 0.447$ kg, $n = 4$, and the other values given in the problem, we find

$$\mu = 0.000845 \text{ kg/m} = 0.845 \text{ g/m}.$$

61. To oscillate in four loops means $n = 4$ in Eq. 16-65 (treating both ends of the string as effectively "fixed"). Thus, $\lambda = 2(0.90 \text{ m})/4 = 0.45$ m. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is

$$\mu = m/L = (0.044 \text{ kg})/(0.90 \text{ m}) = 0.049 \text{ kg/m}.$$

Thus, using Eq. 16-26, we obtain the tension:

$$\tau = v^2 \mu = (27 \text{ m/s})^2(0.049 \text{ kg/m}) = 36 \text{ N}.$$

62. We write the expression for the displacement in the form $y(x, t) = y_m \sin(kx - \omega t)$.

(a) The amplitude is $y_m = 2.0$ cm $= 0.020$ m, as given in the problem.

(b) The angular wave number $k$ is $k = 2\pi/\lambda = 2\pi/(0.10\text{ m}) = 63\text{ m}^{-1}$.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(400\text{ Hz}) = 2510\text{ rad/s} = 2.5\times10^3\text{ rad/s}$.

(d) A minus sign is used before the $\omega t$ term in the argument of the sine function because the wave is traveling in the positive $x$ direction.

Using the results above, the wave may be written as

$$y(x,t) = (2.00\,\text{cm})\sin\left(\left(62.8\,\text{m}^{-1}\right)x - \left(2510\,\text{s}^{-1}\right)t\right).$$

(e) The (transverse) speed of a point on the cord is given by taking the derivative of $y$:

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

which leads to a maximum speed of $u_m = \omega y_m = (2510\text{ rad/s})(0.020\text{ m}) = 50\text{ m/s}$.

(f) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\text{rad/s}}{62.8\,\text{rad/m}} = 40\,\text{m/s}.$$

63. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\text{m/s}}{3.2\,\text{m}} = 75\text{ Hz}.$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\text{Hz}} = 0.0133\,\text{s} \approx 13\,\text{ms}.$$

64. (a) At $x = 2.3$ m and $t = 0.16$ s the displacement is

$$y(x,t) = 0.15\sin\left[(0.79)(2.3) - 13(0.16)\right]\text{m} = -0.039\,\text{m}.$$

(b) We choose $y_m = 0.15$ m, so that there would be nodes (where the wave amplitude is zero) in the string as a result.

(c) The second wave must be traveling with the same speed and frequency. This implies $k = 0.79\text{ m}^{-1}$,

(d) and $\omega = 13\text{ rad/s}$.

(e) The wave must be traveling in the –$x$ direction, implying a plus sign in front of $\omega$.

Thus, its general form is $y'(x,t)$ = (0.15 m)sin(0.79$x$ + 13$t$).

(f) The displacement of the standing wave at $x$ = 2.3 m and $t$ = 0.16 s is

$$y(x,t) = -0.039\,\text{m} + (0.15\,\text{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14\,\text{m}.$$

65. We use Eq. 16-2, Eq. 16-5, Eq. 16-9, Eq. 16-13, and take the derivative to obtain the transverse speed $u$.

(a) The amplitude is $y_m$ = 2.0 mm.

(b) Since $\omega$ = 600 rad/s, the frequency is found to be $f$ = 600/2π ≈ 95 Hz.

(c) Since $k$ = 20 rad/m, the velocity of the wave is $v = \omega/k$ = 600/20 = 30 m/s in the +$x$ direction.

(d) The wavelength is λ = 2π/$k$ ≈ 0.31 m, or 31 cm.

(e) We obtain

$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \Rightarrow u_m = \omega y_m$$

so that the maximum transverse speed is $u_m$ = (600)(2.0) = 1200 mm/s, or 1.2 m/s.

66. Setting $x$ = 0 in $y = y_m \sin(kx - \omega t + \phi)$ gives $y = y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a positive "slope" (referring to its $t$-derivative) at $t$ = 0, or

$$\frac{dy}{dt} = \frac{d}{dt}\left[y_m \sin(-\omega t + \phi)\right] = -y_m \omega \cos(-\omega t + \phi) > 0$$

at $t$ = 0. This implies that $-\cos\phi > 0$ and consequently that $\phi$ is in either the second or third quadrant. The graph shows (at $t$ = 0) $y$ = 2.00 mm, and (at some later $t$) $y_m$ = 6.00 mm. Therefore,

$$y = y_m \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}\left(\tfrac{1}{3}\right) = 0.34 \text{ rad} \quad \text{or} \quad 2.8 \text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi$ = 2.8 rad because this is about 161° and is in second quadrant. Of course, this answer added to 2$n$π is still a valid answer (where $n$ is any integer), so that, for example, $\phi$ = 2.8 – 2π = –3.48 rad is also an acceptable result.

67. We compare the resultant wave given with the standard expression (Eq. 16–52) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda, 2y_m\cos\left(\frac{1}{2}\phi\right) = 3.0\,\text{mm}$, and $\frac{1}{2}\phi = 0.820\,\text{rad}$.

(a) Therefore, $\lambda = 2\pi/k = 0.31$ m.

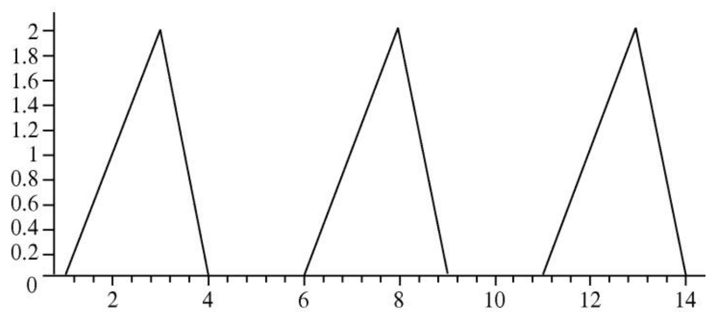
(b) The phase difference is $\phi = 1.64$ rad.

(c) And the amplitude is $y_m = 2.2$ mm.

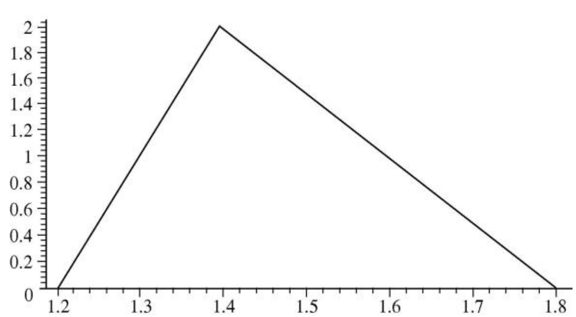
68. (a) Recalling the discussion in Section 16-5, we see that the speed of the wave given by a function with argument $x - 5.0t$ (where $x$ is in centimeters and $t$ is in seconds) must be $5.0$ cm/s.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 16-44 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0$ s. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2.0 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.

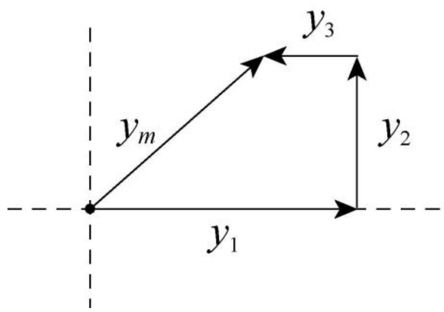



(d) The leading edge of the pulse reaches $x = 10$ cm at $t = (10 - 4.0)/5 = 1.2$ s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h = 2$ cm at $t = (10 - 3.0)/5 = 1.4$ s. Finally, the trailing edge of the pulse departs from $x = 10$ cm at $t = (10 - 1.0)/5 = 1.8$ s. Thus, we find for $h(t)$ at $x = 10$ cm (with the horizontal axis, $t$, in seconds):



69. **THINK** We use phasors to add the three waves and calculate the amplitude of the resultant wave.

**EXPRESS** The phasor diagram is shown here: $y_1$, $y_2$, and $y_3$ represent the original waves and $y_m$ represents the resultant wave.




The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$.

**ANALYZE** (a) The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1.$$

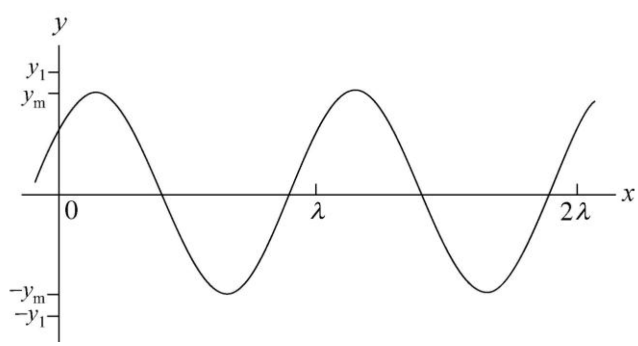
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\left(\frac{y_{mv}}{y_{mh}}\right) = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 0.644 \text{ rad} = 37°.$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



**LEARN** In adding the three sinusoidal waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. However, adding the three terms explicitly gives, after a little algebra,

$$\begin{aligned} y_1+y_2+y_3 &= y_1\sin(kx-\omega t)+\frac{1}{2}y_1\sin(kx-\omega t+\pi/2)+\frac{1}{3}y_1\sin(kx-\omega t+\pi) \\ &= y_1\sin(kx-\omega t)+\frac{1}{2}y_1\cos(kx-\omega t)-\frac{1}{3}y_1\sin(kx-\omega t) \\ &= \frac{2}{3}y_1\sin(kx-\omega t)+\frac{1}{2}y_1\cos(kx-\omega t) \\ &= \frac{5}{6}y_1\left[\frac{4}{5}\sin(kx-\omega t)+\frac{3}{5}\cos(kx-\omega t)\right] \\ &= \frac{5}{6}y_1\sin(kx-\omega t+\phi) \end{aligned}$$

where $\phi=\tan^{-1}(3/4)=0.644$ rad. In deducing the phase $\phi$, we set $\cos\phi=4/5$ and $\sin\phi=3/5$, and use the relation $\cos\phi\sin\theta+\sin\phi\cos\theta=\sin(\theta+\phi)$. The result indeed agrees with that obtained in (c).

70. Setting $x = 0$ in $a_y = -\omega^2 y$, where $y = y_m\sin(kx - \omega t + \phi)$ gives

$$a_y = -\omega^2 y_m \sin(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a negative "slope" (referring to its $t$-derivative) at $t = 0$, or

$$\frac{da_y}{dt}=\frac{d}{dt}\left[-\omega^2 y_m\sin(-\omega t+\phi)\right]=\omega^3 y_m\cos(-\omega t+\phi)<0$$

at $t = 0$. This implies that $\cos\phi < 0$ and consequently that $\phi$ is in either the second or third quadrant. The graph shows (at $t = 0$) $a_y = -100$ m/s², and (at another $t$) $a_{max} = 400$ m/s². Therefore,

$$a_y = -a_{max}\sin(-\omega t+\phi)\Big|_{t=0} \quad\Rightarrow\quad \phi = \sin^{-1}(\tfrac{1}{4}) = 0.25 \text{ rad} \quad\text{or}\quad 2.9 \text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.9$ rad because this is about 166° and is in the second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where $n$ is any integer), so that, for example, $\phi = 2.9 - 2\pi = -3.4$ rad is also an acceptable result.

71. (a) Let the displacement of the string be of the form $y(x, t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is

$$u(x, t) = \partial y/\partial t = -\omega\, y_m \cos(kx - \omega t)$$

and its maximum value is $u_m = \omega y_m$. For this wave the frequency is $f$ = 120 Hz and the angular frequency is $\omega = 2\pi f = 2\pi$ (120 Hz) = 754 rad/s. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00 \times 10^{-3}$ m. The maximum speed is

$$u_m = (754 \text{ rad/s})\,(5.00 \times 10^{-3} \text{ m}) = 3.77 \text{ m/s}.$$

(b) Consider the string at coordinate $x$ and at time $t$ and suppose it makes the angle $\theta$ with the $x$ axis. The tension is along the string and makes the same angle with the $x$ axis. Its transverse component is $\tau_{\text{trans}} = \tau \sin\theta$. Now $\theta$ is given by $\tan\theta = \partial y/\partial x = ky_m \cos(kx - \omega t)$ and its maximum value is given by $\tan\theta_m = ky_m$. We must calculate the angular wave number $k$. It is given by $k = \omega/v$, where $v$ is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the rope and $\mu$ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1}.$$

Thus,

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00\times10^{-3}\text{m}) = 0.138$$

and $\theta$ = 7.83°. The maximum value of the transverse component of the tension in the string is

$$\tau_{\text{trans}} = (90.0 \text{ N}) \sin 7.83° = 12.3 \text{ N}.$$

We note that $\sin\theta$ is nearly the same as $\tan\theta$ because $\theta$ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at $x$. The transverse component of the tension pulling on it due to the string to the left is $-\tau(\partial y/\partial x) = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is

$$u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement $\Delta y$, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}}\frac{\Delta y}{\Delta t} = \tau_{\text{trans}} u.$$

$P$ has its maximum value when the transverse component $\tau_{\text{trans}}$ of the tension and the string speed $u$ have their maximum values. Hence the maximum power is (12.3 N)(3.77 m/s) = 46.4 W.

(e) As shown above, $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50$ cm.

72. We use Eq. 16-52 in interpreting the figure.

(a) Since $y' = 6.0$ mm when $\phi = 0$, then Eq. 16-52 can be used to determine $y_m = 3.0$ mm.

(b) We note that $y' = 0$ when the shift distance is 10 cm; this occurs because $\cos(\phi/2) = 0$ there $\Rightarrow$ $\phi = \pi$ rad or ½ cycle. Since a full cycle corresponds to a distance of one full wavelength, this ½ cycle shift corresponds to a distance of $\lambda/2$. Therefore, $\lambda = 20$ cm $\Rightarrow$ $k = 2\pi/\lambda = 31\ \text{m}^{-1}$.

(c) Since $f = 120$ Hz, $\omega = 2\pi f = 754 \text{ rad/s} \approx 7.5\times10^2$ rad/s.

(d) The sign in front of $\omega$ is minus since the waves are traveling in the $+x$ direction.

The results may be summarized as $y = (3.0\ \text{mm})\sin[(31.4\ \text{m}^{-1})x - (754\ \text{s}^{-1})t]]$ (this applies to each wave when they are in phase).

73. We note that

$$dy/dt = -\omega\cos(kx - \omega t + \phi),$$

which we will refer to as $u(x,t)$. so that the ratio of the function $y(x,t)$ divided by $u(x,t)$ is $-\tan(kx - \omega t + \phi)/\omega$. With the given information (for $x = 0$ and $t = 0$) then we can take the inverse tangent of this ratio to solve for the phase constant:

$$\phi = \tan^{-1}\left(\frac{-\omega y(0,0)}{u(0,0)}\right) = \tan^{-1}\left(\frac{-(440)(0.0045)}{-0.75}\right) = 1.2 \text{ rad}.$$

74. We use $P = \frac{1}{2}\mu v\omega^2 y_m^2 \propto vf^2 \propto \sqrt{\tau}\, f^2$.

(a) If the tension is quadrupled, then $P_2 = P_1\sqrt{\frac{\tau_2}{\tau_1}} = P_1\sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1$.

(b) If the frequency is halved, then $P_2 = P_1\left(\frac{f_2}{f_1}\right)^2 = P_1\left(\frac{f_1/2}{f_1}\right)^2 = \frac{1}{4}P_1$.

75. (a) Let the cross-sectional area of the wire be $A$ and the density of steel be $\rho$. The tensile stress is given by $\tau/A$ where $\tau$ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$v_{\max} = \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} = \sqrt{\frac{7.00\times10^8\ \text{N/m}^2}{7800\,\text{kg/m}^3}} = 3.00\times10^2\ \text{m/s}\ .$$

(b) The result does not depend on the diameter of the wire.

76. Repeating the steps of Eq. 16-47 $\rightarrow$ Eq. 16-53, but applying

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 16-50, we obtain $y' = [0.10\cos\pi x]\cos 4\pi t$, with SI units understood.

(a) For non-negative $x$, the smallest value to produce cos $\pi x = 0$ is $x = 1/2$, so the answer is $x = 0.50$ m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = [0.10\cos\pi x](-4\pi\sin 4\pi t).$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ Thus, the value of the first time the particle at $x = 0$ has zero velocity is $t = 0$.

(c) Using the result obtained in (b), the second time where the velocity at $x = 0$ vanishes would be $t = 0.25$ s,

(d) and the third time is $t = 0.50$ s.

77. **THINK** The speed of a transverse wave in the stretched rubber band is related to the tension in the band and the linear mass density of the band.

**EXPRESS** The wave speed $v$ is given by $v = \sqrt{F/\mu}$, where $F$ is the tension in the rubber band and $\mu$ is the band's linear mass density, which is defined as the mass per unit length $\mu = m/L$. The fact that the band obeys Hooke's law implies $F = k\Delta\ell$, where $k$ is the spring constant and $\Delta\ell$ is the elongation. Thus, when a force $F$ is applied, the rubber band has a length $L = \ell + \Delta\ell$, where $\ell$ is the unstretched length, resulting in a linear mass density $\mu = m/(\ell + \Delta\ell)$.

**ANALYZE** (a) The wave speed is $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta\ell}{m/(\ell+\Delta\ell)}} = \sqrt{\frac{k\Delta\ell(\ell+\Delta\ell)}{m}}$.

(b) The time required for the pulse to travel the length of the rubber band is

$$t = \frac{2\pi(\ell+\Delta\ell)}{v} = \frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}.$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$. On the other hand, if $\ell/\Delta\ell \ll 1$, then we have $t \simeq 2\pi\sqrt{m/k} = \text{const}$.

**LEARN** When $\Delta\ell \ll \ell$, the applied force $F = k\Delta\ell$ is small while $\mu \approx m/\ell = \text{constant}$, leading to a small wave speed. On the other hand, when $\Delta\ell \gg \ell$, $\mu \approx m/\Delta\ell$ and $v = \sqrt{F/\mu} \propto \Delta\ell$, so that $t \simeq 2\pi\sqrt{m/k}$, which is a constant.

78. (a) For visible light

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{700\times10^{-9}\ \text{m}} = 4.3\times10^{14}\ \text{Hz}$$

and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{400\times10^{-9}\ \text{m}} = 7.5\times10^{14}\ \text{Hz}.$$

(b) For radio waves

$$\lambda_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{300\times10^{6}\ \text{Hz}} = 1.0\ \text{m}$$

and

$$\lambda_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.5\times10^{6}\ \text{Hz}} = 2.0\times10^{2}\ \text{m}.$$

(c) For X rays

$$f_{min} = \frac{c}{\lambda_{max}} = \frac{3.0\times10^8 \text{ m/s}}{5.0\times10^{-9}\text{m}} = 6.0\times10^{16}\text{Hz}$$

and

$$f_{max} = \frac{c}{\lambda_{min}} = \frac{3.0\times10^8 \text{ m/s}}{1.0\times10^{-11}\text{m}} = 3.0\times10^{19}\text{ Hz.}$$

79. **THINK** A wire held rigidly at both ends can be made to oscillate in standing wave patterns.

**EXPRESS** Possible wavelengths are given by $\lambda_n = 2L/n$, where $L$ is the length of the wire and $n$ is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where $v$ is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$ where $\tau$ is the tension in the wire and $\mu$ is the linear mass density of the wire.

**ANALYZE** (a) The wave speed is $v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{120\text{ N}}{8.70\times10^{-3}\text{ kg}/1.50\text{ m}}} = 144\text{ m/s}.$

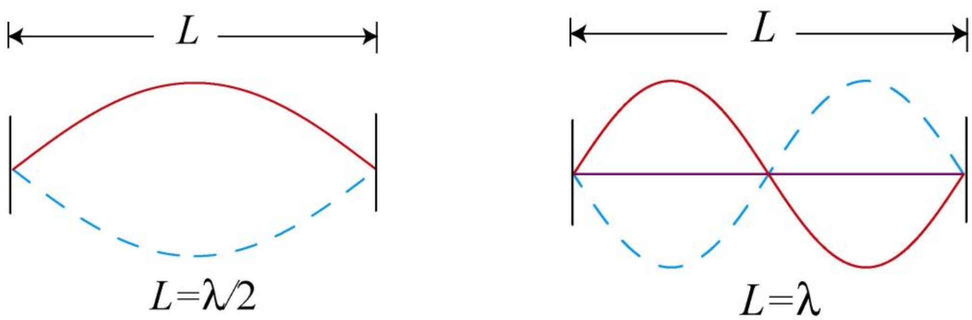
(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\text{ m}) = 3.00\text{ m}$.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50\text{ m}$.

(d) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\text{ m/s})/(3.00\text{ m}) = 48.0\text{ Hz}$.

(e) The frequency for the two-loop wave is $f_2 = v/\lambda_2 = (144\text{ m/s})/(1.50\text{ m}) = 96.0\text{ Hz}$.

**LEARN** The one-loop and two-loop standing wave patterns are plotted below:




80. By Eq. 16–66, the higher frequencies are integer multiples of the lowest (the fundamental).

(a) The frequency of the second harmonic is $f_2 = 2(440) = 880$ Hz.

(b) The frequency of the third harmonic is $f_3 = 3(440) = 1320$ Hz.

81. (a) The amplitude is $y_m$ = 1.00 cm = 0.0100 m, as given in the problem.

(b) Since the frequency is $f$ = 550 Hz, the angular frequency is $\omega = 2\pi f = 3.46\times10^3$ rad/s.

(c) The angular wave number is $k = \omega / v = (3.46\times10^3 \text{ rad/s})/(330 \text{ m/s}) = 10.5 \text{ rad/m}$.

(d) Since the wave is traveling in the –$x$ direction, the sign in front of $\omega$ is plus and the argument of the trig function is $kx + \omega t$.

The results may be summarized as

$$y(x,t) = y_\text{m}\sin(kx+\omega t) = y_\text{m}\sin\left[2\pi f\left(\frac{x}{v}+t\right)\right]$$
$$= (0.010\,\text{m})\sin\left[2\pi(550\,\text{Hz})\left(\frac{x}{330\,\text{m/s}}+t\right)\right]$$
$$= (0.010\,\text{m})\sin[(10.5\text{ rad/s})\,x + (3.46\times10^3\text{ rad/s})t].$$

82. We orient one phasor along the $x$ axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

$$(3.0\text{ mm}) + (5.0\text{ mm})\cos(70°) = 4.71\text{mm along } x\text{ axis}$$
$$(5.0\text{ mm})\sin(70°) = 4.70\text{ mm along } y\text{ axis.}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{(4.71\text{ mm})^2 + (4.70\text{ mm})^2} = 6.7\,\text{mm}.$

(b) And the angle (phase constant) is $\tan^{-1}$ (4.70/4.71) = 45°.

83. **THINK** The speed of a point on the cord is given by $u(x, t) = \partial y/\partial t$, where $y(x, t)$ is displacement.

**EXPRESS** We take the form of the displacement to be

$$y(x, t) = y_m \sin(kx - \omega t).$$

The speed of a point on the cord is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t),$$

and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$.

(a) The ratio of the maximum particle speed to the wave speed is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on $y_m/\lambda$, the ratio of the amplitude to the wavelength.

**LEARN** Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

84. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where $\lambda$ is the wavelength and $L$ is the length of the string. The wavelength is related to the frequency $f$ and wave speed $v$ by $\lambda = v/f$, so $L/2 = v/f$ and

$$L = 2v/f = 2(400 \text{ m/s})/(600 \text{ Hz}) = 1.3 \text{ m}.$$

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx) \cos(\omega t)$, where $y_m$ is the maximum displacement, $k$ is the angular wave number, and $\omega$ is the angular frequency. The angular wave number is

$$k = 2\pi/\lambda = 2\pi f/v = 2\pi(600 \text{ Hz})/(400 \text{ m/s}) = 9.4\text{m}^{-1}$$

and the angular frequency is

$$\omega = 2\pi f = 2\pi(600 \text{ Hz}) = 3800 \text{ rad/s}.$$

With $y_m$ = 2.0 mm, the displacement is given by

$$y(x,t) = (2.0\,\text{mm})\sin[(9.4\,\text{m}^{-1})x]\cos[(3800\,\text{s}^{-1})t].$$

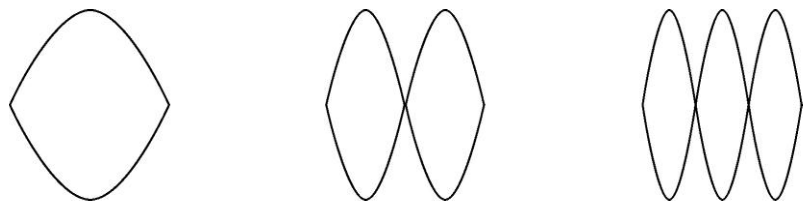
85. We make use of Eq. 16-65 with $L$ = 120 cm.

(a) The longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_1 = 2L/1 = 240$ cm.

(b) The second longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_2 = 2L/2 = 120$ cm.

(c) The third longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_3 = 2L/3 = 80.0$ cm.

The three standing waves are shown next:



86. (a) Let the displacements of the wave at ($y,t$) be $z(y,t)$. Then

$$z(y,t) = z_m \sin(ky - \omega t),$$

where $z_m$ = 3.0 mm, $k$ = 60 cm$^{-1}$, and $\omega = 2\pi/T = 2\pi/0.20$ s = $10\pi$ s$^{-1}$. Thus

$$z(y,t) = (3.0\,\text{mm})\sin\left[\left(60\,\text{cm}^{-1}\right)y - \left(10\pi\,\text{s}^{-1}\right)t\right].$$

(b) The maximum transverse speed is $u_m = \omega z_m = (2\pi/0.20\,\text{s})(3.0\,\text{mm}) = 94\,\text{mm/s}$.

87. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi\cos\left(\frac{\pi x}{8} - 4\pi t\right).$$

Thus, when $x$ = 6 and $t = \frac{1}{4}$, we obtain

$$u = -60\pi\cos\frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for $u$ is $-60\pi$. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2\sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when $x$ = 6 and $t = \frac{1}{4}$ we obtain $a = -240\pi^2\sin(-\pi/4)$, which yields $a$ = 16.7 m/s$^2$.

(d) The numerical coefficient of the sine in the expression for $a$ is $-240\pi^2$. Thus, the maximum acceleration is 23.7 m/s$^2$.

88. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = \left(2000\,\text{m/s}\right)\left(40\times10^{-6}\,\text{s}\right) = 8.0\times10^{-2}\,\text{m}.$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve $a = 300/40 \times 10^{-6}$) we find the stopping distance $d$:

$$v^2 = v_o^2 + 2ad \Rightarrow d = \frac{(300)^2 \left(40\times10^{-6}\right)}{2(300)}$$

which gives $d = 6.0\times10^{-3}$ m. This and the radius $r$ form the legs of a right triangle (where $r$ is opposite from $\theta = 60°$). Therefore,

$$\tan 60° = \frac{r}{d} \Rightarrow r = d\,\tan 60° = 1.0\times10^{-2}\,\text{m}.$$

89. Using Eq. 16-50, we have

$$y' = \left(0.60\cos\frac{\pi}{6}\right)\sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 16-55 for comparison).

(a) The amplitude is seen to be $0.60\cos\frac{\pi}{6} = 0.3\sqrt{3} = 0.52\,\text{m}$.

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 16-12), $v = \frac{\omega}{k} = 40\,\text{m/s}$.

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

90. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as $u_m$) is 5.0 cm/s. Using the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \quad \Rightarrow \quad y_m = 3.2 \text{ cm}.$$

(c) As already noted, $f = 0.25$ Hz.

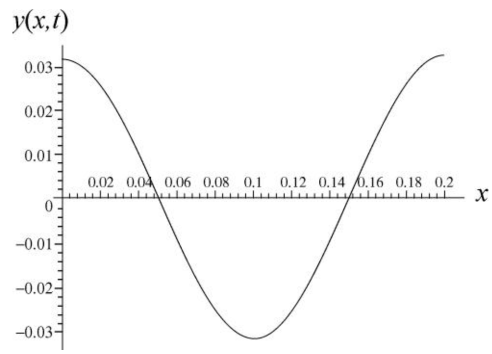
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the $t$ and $x$ terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is $0.050\ \sin\pi t/2$. Therefore, the function $u(x,t)$ is

$$u(x,t) = 0.050\sin\left(\frac{\pi}{2}t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y(x,t) = -\frac{2(0.050)}{\pi}\cos\left(\frac{\pi}{2}t - 10\pi x\right) + C$$

where $C$ is an integration constant (which we will assume to be zero). The sketch of this function at $t$ = 2.0 s for $0 \le x \le 0.20$ m is shown below.




91. **THINK** The rope with both ends fixed and made to oscillate in fundamental mode has wavelength $\lambda = 2L$, where $L$ is the length of the rope.

**EXPRESS** We first observe that the anti-node at $x$ = 1.0 m having zero displacement at $t$ = 0 suggests the use of sine instead of cosine for the simple harmonic motion factor. We take the form of the displacement to be

$$y(x, t) = y_m \sin(kx)\sin(\omega t).$$

A point on the rope undergoes simple harmonic motion with a speed

$$u(x, t) = \partial y/\partial t = \omega y_m \sin(kx)\cos(\omega t).$$

It has maximum speed $u_m = \omega y_m$ as it passes through its "middle" point. On the other hand, the wave speed is $v = \sqrt{\tau/\mu}$ where $\tau$ is the tension in the rope and $\mu$ is the linear mass density of the rope. For standing waves, possible wavelengths are given by $\lambda_n = 2L/n$, where $L$ is the length of the rope and $n$ is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where $v$ is the wave speed. For fundamental mode, we set $n$ = 1.

**ANALYZE** (a) With $f$ = 5.0 Hz, we find the angular frequency to be $\omega = 2\pi f = 10\pi$ rad/s. Thus, if the maximum speed of a point on the rope is $u_m$ = 5.0 m/s, then its amplitude is

$$y_m = \frac{u_m}{\omega} = \frac{5.0\text{ m/s}}{10\pi\text{ rad/s}} = 0.16\text{ m}.$$

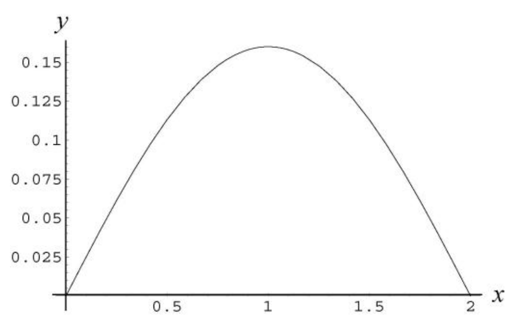
(b) Since the oscillation is in the *fundamental* mode, we have λ = 2$L$ = 4.0 m. Therefore, the speed of waves along the rope is $v = f\lambda$ = 20 m/s. Then, with $\mu = m/L$ = 0.60 kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \;\Rightarrow\; \tau = \mu v^2 = 240 \text{ N} \approx 2.4\times10^2 \text{ N}.$$

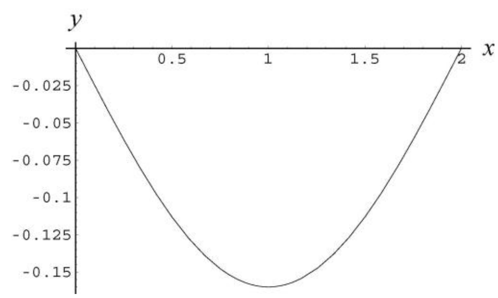
(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$. Now, *if* the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16 \text{ m}) \sin\left(\frac{\pi x}{2}\right) \sin(10\pi t) = (0.16 \text{ m})\sin[(1.57 \text{ m}^{-1})x]\sin[(31.4 \text{ rad/s})t]$$

**LEARN** The period of oscillation is $T = 1/f = 0.20\,\text{s}$. The snapshots of the patterns at $t = T/4 = 0.05\,\text{s}$ and $t = 3T/4 = 0.15\,\text{s}$ are given below. At $t = T/2$ and $T$, the displacement is zero everywhere.

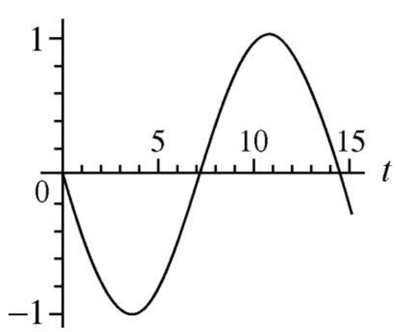


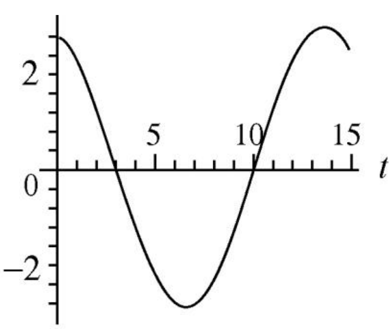

$t = T/4 = 0.05\,\text{s}$



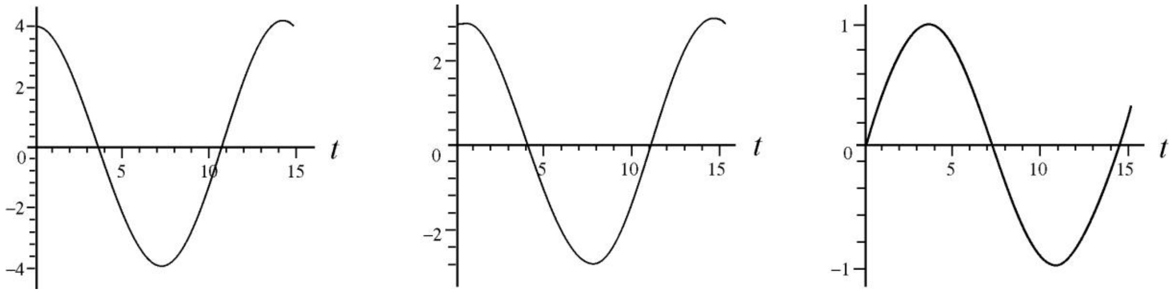

$t = 3T/4 = 0.15\,\text{s}$

92. (a) The wave number for each wave is $k$ = 25.1/m, which means $\lambda = 2\pi/k$ = 250.3 mm. The angular frequency is $\omega$ = 440/s; therefore, the period is $T = 2\pi/\omega$ = 14.3 ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x$ = 0 (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis ($y$) is in millimeters.



The following three graphs show the oscillation at $x = \lambda/4$ =62.6 mm ≈ 63 mm (graph on the left), at $x = 3\lambda/8$ ≈ 94 mm (middle graph), and at $x = \lambda/2$ ≈ 125 mm.




(b) We can think of wave $y_1$ as being made of two smaller waves going in the same direction, a wave $y_{1a}$ of amplitude 1.50 mm (the same as $y_2$) and a wave $y_{1b}$ of amplitude 1.00 mm. It is made clear in Section 16-12 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves $y_{1a}$ and $y_2$ form a standing wave, which leaves $y_{1b}$ as the remaining traveling wave. Since the argument of $y_{1b}$ involves the subtraction $kx - \omega t$, then $y_{1b}$ travels in the $+x$ direction.

(c) If $y_2$ (which travels in the $-x$ direction, which for simplicity will be called "leftward") had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is $y_{max}$ = 4.0 mm and occurs at $x = \lambda/4$ = 62.6 mm.

(e) The smallest amplitude of oscillation is $y_{min}$ = 1.0 mm and occurs at $x$ = 0 and at

$$x = \lambda/2 = 125 \text{ mm}.$$

(f) The largest amplitude can be related to the amplitudes of $y_1$ and $y_2$ in a simple way:

$$y_{max} = y_{1m} + y_{2m},$$

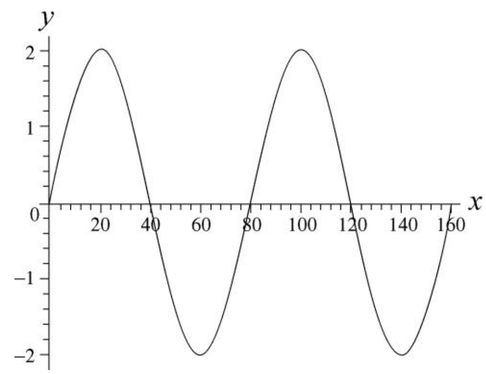
where $y_{1m}$ = 2.5 mm and $y_{2m}$ = 1.5 mm are the amplitudes of the original traveling waves.

(g) The smallest amplitudes is

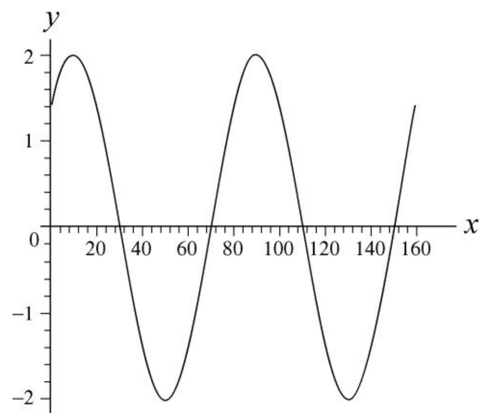
$$y_{min} = y_{1m} - y_{2m},$$

where $y_{1m}$ = 2.5 mm and $y_{2m}$ = 1.5 mm are the amplitudes of the original traveling waves.

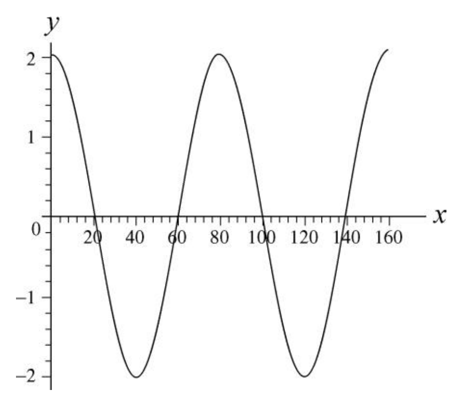
93. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find




(b) The previous graph is at $t$ = 0, and this next one is at $t$ = 0.050 s.




And the final one, shown below, is at $t$ = 0.010 s.




(c) The wave can be written as $y(x,t) = y_m \sin(kx + \omega t)$, where $v = \omega / k$ is the speed of propagation. From the problem statement, we see that $\omega = 2\pi / 0.40 = 5\pi$ rad/s and $k = 2\pi / 80 = \pi / 40$ rad/cm. This yields $v = 2.0\times10^2$ cm/s $= 2.0$ m/s.

(d) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the –$x$ direction.

94. The speed of the transverse wave along the string is given by Eq. 16-26: $v = \sqrt{\tau / \mu}$, where $\tau$ is the tension and $\mu$ is the linear mass density of the string. Applying Newton's second law to a small segment of the string, the radial restoring force is (see Eq. 16-23)

$$F = 2(\tau \sin\theta) \approx \tau \frac{\Delta l}{R}$$

Since $F = (\Delta m) v_T^2 / R$, where $v_T$ is the tangential speed of the segment of mass $\Delta m = \mu \Delta l$, and $R$ is the radius of the circle, we have

$$\tau \frac{\Delta l}{R} = (\mu \Delta l) \frac{v_T^2}{R} \quad \Rightarrow \quad \tau = \mu v_T^2$$

On the other hand, the fact that $v = \sqrt{\tau / \mu}$ implies $\tau = \mu v^2$. Thus, we must have $v = v_T$, which in this case, is equal to 5.00 cm/s. Note that $v$ is independent of the radius of the circular loop.

95. (a) With total reflection, $A = B$, and $\text{SWR} = \dfrac{A+B}{A-B} \to \infty$.

(b) With no reflection, $B = 0$, and $\text{SWR} = \dfrac{A+B}{A-B} = \dfrac{A}{A} = 1$.

(c) In terms of $R = (B/A)^2$, we can rewrite SWR as

$$\text{SWR} = \frac{A+B}{A-B} = \frac{1+(B/A)}{1-(B/A)} = \frac{1+\sqrt{R}}{1-\sqrt{R}} \quad \Rightarrow \quad R = \left(\frac{\text{SWR}-1}{\text{SWR}+1}\right)^2$$

With SWR = 1.50, we obtain

$$R = \left(\frac{\text{SWR}-1}{\text{SWR}+1}\right)^2 = \left(\frac{1.50-1}{1.50+1}\right)^2 = 0.040 = 4.0\%.$$

96. (a) The speed of each individual wave is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{40\text{ N}}{(0.125\text{ kg})/(2.25\text{ m})}} = 26.83\text{ m/s}.$$

The average rate at which energy is transmitted from one side is

$$P_{\text{avg},1} = \frac{1}{2}\mu v\omega^2 y_m^2 = \frac{1}{2}\left(\frac{0.125\text{ kg}}{2.25\text{ m}}\right)(26.83\text{ m/s})(2\pi\times 120\text{ Hz})^2(5.0\times 10^{-3}\text{ m})^2 = 10.6\text{ W}.$$

(b) From both sides, $P_{\text{avg}} = 2P_{\text{avg},1} = 2(10.6\text{ W}) = 21.2\text{ W}$.

(c) The rate of change of kinetic energy from one side is given by Eq. 16-30:

$$\frac{dK_1}{dt} = \frac{1}{2}\mu v\omega^2 y_m^2 \cos^2(kx-\omega t).$$

Integrating over one period for both sides, we obtain

$$K = \int\left(2\frac{dK_1}{dt}\right)dt = \mu v\omega^2 y_m^2 \int_0^T \cos^2(kx-\omega t)dt = \frac{T}{2}\mu v\omega^2 y_m^2 = \frac{P_{\text{avg}}}{2f}$$
$$= \frac{21.2\text{ W}}{2(120\text{ Hz})} = 8.83\times 10^{-2}\text{ J}.$$

# Chapter 17

1. (a) The time for the sound to travel from the kicker to a spectator is given by *d*/*v*, where *d* is the distance and *v* is the speed of sound. The time for light to travel the same distance is given by *d*/*c*, where *c* is the speed of light. The delay between seeing and hearing the kick is $\Delta t = (d/v) - (d/c)$. The speed of light is so much greater than the speed of sound that the delay can be approximated by $\Delta t = d/v$. This means $d = v\,\Delta t$. The distance from the kicker to spectator *A* is

$$d_A = v\,\Delta t_A = (343\text{ m/s})(0.23\text{ s}) = 79\text{ m}.$$

(b) The distance from the kicker to spectator *B* is $d_B = v\,\Delta t_B = (343\text{ m/s})(0.12\text{ s}) = 41\text{ m}$.

(c) Lines from the kicker to each spectator and from one spectator to the other form a right triangle with the line joining the spectators as the hypotenuse, so the distance between the spectators is

$$D = \sqrt{d_A^2 + d_B^2} = \sqrt{(79\,\text{m})^2 + (41\,\text{m})^2} = 89\,\text{m}\,.$$

2. The density of oxygen gas is

$$\rho = \frac{0.0320\,\text{kg}}{0.0224\,\text{m}^3} = 1.43\,\text{kg/m}^3.$$

From $v = \sqrt{B/\rho}$ we find

$$B = v^2\rho = (317\,\text{m/s})^2\left(1.43\,\text{kg/m}^3\right) = 1.44\times 10^5\text{ Pa}.$$

3. (a) When the speed is constant, we have $v = d/t$ where $v$ = 343 m/s is assumed. Therefore, with $t$ = 15/2 s being the time for sound to travel to the far wall we obtain $d$ = (343 m/s) × (15/2 s), which yields a distance of 2.6 km.

(b) Just as the $\frac{1}{2}$ factor in part (a) was $1/(n + 1)$ for $n$ = 1 reflection, so also can we write

$$d = (343\,\text{m/s})\left(\frac{15\,\text{s}}{n+1}\right) \quad\Rightarrow\quad n = \frac{(343)(15)}{d} - 1$$

for multiple reflections (with *d* in meters). For *d* = 25.7 m, we find *n* = 199 $\approx 2.0\times 10^2$ .

4. The time it takes for a soldier in the rear end of the column to switch from the left to the right foot to stride forward is *t* = 1 min/120 = 1/120 min = 0.50 s. This is also the time

for the sound of the music to reach from the musicians (who are in the front) to the rear end of the column. Thus the length of the column is

$$l = vt = (343\,\text{m/s})(0.50\,\text{s}) = 1.7 \times 10^2\text{m}.$$

5. **THINK** The S and P waves generated by the earthquake travel at different speeds. Knowing the speeds of the waves and the time difference of their arrival to the seismograph allows us to determine the location of the earthquake.

**EXPRESS** Let $d$ be the distance from the location of the earthquake to the seismograph. If $v_s$ is the speed of the S waves, then the time for these waves to reach the seismograph is $t_s$. = $d/v_s$. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is

$$\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p,$$

**ANALYZE** With $v_s = 4.5\,\text{km/s}$, $v_p = 8.0\,\text{km/s}$ and $\Delta t = 3.0\ \text{min} = 180\,\text{s}$, we find the distance to be

$$d = \frac{v_s v_p \Delta t}{(v_p - v_s)} = \frac{(4.5\ \text{km/s})(8.0\,\text{km/s})(180\ \text{s})}{8.0\,\text{km/s} - 4.5\,\text{km/s}} = 1.9 \times 10^3\,\text{km}.$$

**LEARN** The distance to the earthquake is proportional to the difference in the arrival times of the P and S waves.

6. Let $\ell$ be the length of the rod. Then the time of travel for sound in air (speed $v_s$) will be $t_s = \ell / v_s$. And the time of travel for compression waves in the rod (speed $v_r$) will be $t_r = \ell / v_r$. In these terms, the problem tells us that

$$t_s - t_r = 0.12\,\text{s} = \ell\left(\frac{1}{v_s} - \frac{1}{v_r}\right).$$

Thus, with $v_s$ = 343 m/s and $v_r = 15v_s$ = 5145 m/s, we find $\ell = 44\,\text{m}$.

7. **THINK** The time elapsed before hearing the splash is the sum of the time it takes for the stone to hit the water in the well, and the time it takes for the sound wave to travel back to the listener.

**EXPRESS** Let $t_f$ be the time for the stone to fall to the water and $t_s$ be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If $d$ is the depth of the well, then the kinematics of free fall gives

$$d = \frac{1}{2} g t_f^2 \quad \Rightarrow \quad t_f = \sqrt{2d/g}.$$

The sound travels at a constant speed $v_s$, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for $d$.

**ANALYZE** Rewriting the above expression as $\sqrt{2d/g} = t - d/v_s$ and squaring both sides, we obtain

$$2d/g = t^2 - 2(t/v_s)d + (1 + v_s^2)d^2.$$

Now multiply by $g v_s^2$ and rearrange to get

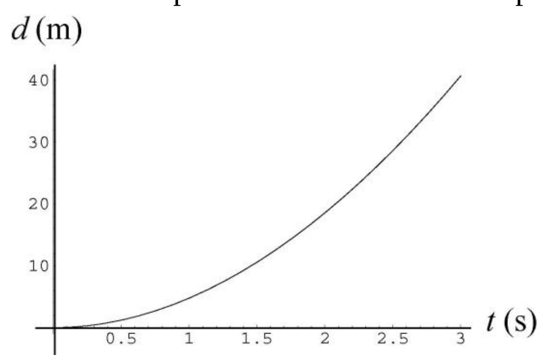
$$gd^2 - 2v_s(gt + v_s)d + g v_s^2 t^2 = 0.$$

This is a quadratic equation for $d$. Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2 v_s^2 t^2}}{2g}.$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7$ m is obtained.

**LEARN** The relation between the depth of the well and time is plotted below:




8. Using Eqs. 16-13 and 17-3, the speed of sound can be expressed as

$$v = \lambda f = \sqrt{\frac{B}{\rho}},$$

where $B = -(dp/dV)/V$. Since $V, \lambda,$ and $\rho$ are not changed appreciably, the frequency ratio becomes

$$\frac{f_s}{f_i} = \frac{v_s}{v_i} = \sqrt{\frac{B_s}{B_i}} = \sqrt{\frac{(dp/dV)_s}{(dp/dV)_i}}\,.$$

Thus, we have

$$\frac{(dV/dp)_s}{(dV/dp)_i} = \frac{B_i}{B_s} = \left(\frac{f_i}{f_s}\right)^2 = \left(\frac{1}{0.333}\right)^2 = 9.00\,.$$

9. Without loss of generality we take $x = 0$, and let $t = 0$ be when $s = 0$. This means the phase is $\phi = -\pi/2$ and the function is $s = (6.0\text{ nm})\sin(\omega t)$ at $x = 0$. Noting that $\omega = 3000$ rad/s, we note that at $t = \sin^{-1}(1/3)/\omega = 0.1133$ ms the displacement is $s = +2.0$ nm. Doubling that time (so that we consider the excursion from –2.0 nm to +2.0 nm) we conclude that the time required is 2(0.1133 ms) = 0.23 ms.

10. The key idea here is that the time delay $\Delta t$ is due to the distance $d$ that each wavefront must travel to reach your left ear ($L$) after it reaches your right ear ($R$).

(a) From the figure, we find $\Delta t = \dfrac{d}{v} = \dfrac{D\sin\theta}{v}$.

(b) Since the speed of sound in water is now $v_w$, with $\theta = 90°$, we have

$$\Delta t_w = \frac{D\sin 90°}{v_w} = \frac{D}{v_w}\,.$$

(c) The apparent angle can be found by substituting $D/v_w$ for $\Delta t$:

$$\Delta t = \frac{D\sin\theta}{v} = \frac{D}{v_w}\,.$$

Solving for $\theta$ with $v_w = 1482$ m/s (see Table 17-1), we obtain

$$\theta = \sin^{-1}\left(\frac{v}{v_w}\right) = \sin^{-1}\left(\frac{343\text{ m/s}}{1482\text{ m/s}}\right) = \sin^{-1}(0.231) = 13°\,.$$

11. **THINK** The speed of sound in a medium is the product of the wavelength and frequency.

**EXPRESS** The wavelength of the sound wave is given by $\lambda = v/f$, where $v$ is the speed of sound in the medium and $f$ is the frequency,

**ANALYZE** (a) The speed of sound in air (at $20°\text{C}$) is $v = 343\text{ m/s}$. Thus, we find

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{4.50\times10^{6}\ \text{Hz}} = 7.62\times10^{-5}\ \text{m}.$$

(b) The frequency of sound is the same for air and tissue. Now the speed of sound in tissue is $v = 1500\ \text{m/s}$, the corresponding wavelength is

$$\lambda = \frac{v}{f} = \frac{1500\,\text{m/s}}{4.50\times10^{6}\ \text{Hz}} = 3.33\times10^{-4}\ \text{m}.$$

**LEARN** The speed of sound depends on the medium through which it propagates. Table 17-1 provides a list of sound speed in various media.

12. (a) The amplitude of a sinusoidal wave is the numerical coefficient of the sine (or cosine) function: $p_m$ = 1.50 Pa.

(b) We identify $k = 0.9\pi$ and $\omega = 315\pi$ (in SI units), which leads to $f = \omega/2\pi$ = 158 Hz.

(c) We also obtain $\lambda = 2\pi/k$ = 2.22 m.

(d) The speed of the wave is $v = \omega/k$ = 350 m/s.

13. The problem says "At one instant..." and we choose that instant (without loss of generality) to be $t = 0$. Thus, the displacement of "air molecule $A$" at that instant is

$$s_A = +s_m = s_m\cos(kx_A - \omega t + \phi)\big|_{t=0} = s_m\cos(kx_A + \phi),$$

where $x_A$ = 2.00 m. Regarding "air molecule $B$" we have

$$s_B = +\tfrac{1}{3}s_m = s_m\cos(kx_B - \omega t + \phi)\big|_{t=0} = s_m\cos(kx_B + \phi).$$

These statements lead to the following conditions:

$$kx_A + \phi = 0$$
$$kx_B + \phi = \cos^{-1}(1/3) = 1.231$$

where $x_B$ = 2.07 m. Subtracting these equations leads to

$$k(x_B - x_A) = 1.231 \quad\Rightarrow\quad k = 17.6\ \text{rad/m}.$$

Using the fact that $k = 2\pi/\lambda$ we find $\lambda$ = 0.357 m, which means

$$f = v/\lambda = 343/0.357 = 960\ \text{Hz}.$$

Another way to complete this problem (once $k$ is found) is to use $kv = \omega$ and then the fact that $\omega = 2\pi f$.

14. (a) The period is $T = 2.0$ ms (or 0.0020 s) and the amplitude is $\Delta p_m = 8.0$ mPa (which is equivalent to 0.0080 N/m$^2$). From Eq. 17-15 we get

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi/T)} = 6.1\times10^{-9} \text{ m}$$

where $\rho = 1.21$ kg/m$^3$ and $v = 343$ m/s.

(b) The angular wave number is $k = \omega/v = 2\pi/vT = 9.2$ rad/m.

(c) The angular frequency is $\omega = 2\pi/T = 3142$ rad/s $\approx 3.1\times10^3$ rad/s.

The results may be summarized as $s(x, t) = (6.1 \text{ nm}) \cos[(9.2 \text{ m}^{-1})x - (3.1 \times 10^3 \text{ s}^{-1})t]$.

(d) Using similar reasoning, but with the new values for density ($\rho' = 1.35$ kg/m$^3$) and speed ($v' = 320$ m/s), we obtain

$$s_m = \frac{\Delta p_m}{v'\rho'\omega} = \frac{\Delta p_m}{v'\rho'(2\pi/T)} = 5.9\times10^{-9} \text{ m}.$$

(e) The angular wave number is $k = \omega/v' = 2\pi/v'T = 9.8$ rad/m.

(f) The angular frequency is $\omega = 2\pi/T = 3142$ rad/s $\approx 3.1\times10^3$ rad/s.

The new displacement function is $s(x, t) = (5.9 \text{ nm}) \cos[(9.8 \text{ m}^{-1})x - (3.1 \times 10^3 \text{ s}^{-1})t]$.

15. (a) Consider a string of pulses returning to the stage. A pulse that came back just before the previous one has traveled an extra distance of $2w$, taking an extra amount of time $\Delta t = 2w/v$. The frequency of the pulse is therefore

$$f = \frac{1}{\Delta t} = \frac{v}{2w} = \frac{343 \text{ m/s}}{2(0.75 \text{ m})} = 2.3\times10^2 \text{ Hz}.$$

(b) Since $f \propto 1/w$, the frequency would be higher if $w$ were smaller.

16. Let the separation between the point and the two sources (labeled 1 and 2) be $x_1$ and $x_2$, respectively. Then the phase difference is

$$\Delta\phi = \phi_1 - \phi_2 = 2\pi\left(\frac{x_1}{\lambda} + ft\right) - 2\pi\left(\frac{x_2}{\lambda} + ft\right) = \frac{2\pi(x_1 - x_2)}{\lambda} = \frac{2\pi(4.40\,\text{m} - 4.00\,\text{m})}{(330\,\text{m/s})/540\,\text{Hz}}$$
$$= 4.12\,\text{rad}.$$

17. Building on the theory developed in Section 17-5, we set $\Delta L/\lambda = n - 1/2,\ n = 1, 2, ...$ in order to have destructive interference. Since $v = f\lambda$, we can write this in terms of frequency:

$$f_{\min,n} = \frac{(2n-1)v}{2\Delta L} = (n - 1/2)(286\text{ Hz})$$

where we have used $v$ = 343 m/s (note the remarks made in the textbook at the beginning of the exercises and problems section) and $\Delta L$ = (19.5 – 18.3 ) m = 1.2 m.

(a) The lowest frequency that gives destructive interference is ($n$ = 1)

$$f_{\min,1} = (1 - 1/2)(286\text{ Hz}) = 143\text{ Hz}.$$

(b) The second lowest frequency that gives destructive interference is ($n$ = 2)

$$f_{\min,2} = (2 - 1/2)(286\text{ Hz}) = 429\text{ Hz} = 3(143\text{ Hz}) = 3f_{\min,1}.$$

So the factor is 3.

(c) The third lowest frequency that gives destructive interference is ($n$ = 3)

$$f_{\min,3} = (3 - 1/2)(286\text{ Hz}) = 715\text{ Hz} = 5(143\text{ Hz}) = 5f_{\min,1}.$$

So the factor is 5.

Now we set $\Delta L/\lambda = \frac{1}{2}$ (even numbers) — which can be written more simply as "(all integers $n$ = 1, 2,…)" — in order to establish constructive interference. Thus,

$$f_{\max,n} = \frac{nv}{\Delta L} = n(286\text{ Hz}).$$

(d) The lowest frequency that gives constructive interference is ($n$ =1) $f_{\max,1} = (286\text{ Hz})$.

(e) The second lowest frequency that gives constructive interference is ($n$ = 2)

$$f_{\max,2} = 2(286\text{ Hz}) = 572\text{ Hz} = 2f_{\max,1}.$$

Thus, the factor is 2.

(f) The third lowest frequency that gives constructive interference is ($n$ = 3)

$$f_{\max,3} = 3(286 \text{ Hz}) = 858 \text{ Hz} = 3 f_{\max,1}.$$

Thus, the factor is 3.

18. (a) To be out of phase (and thus result in destructive interference if they superpose) means their path difference must be $\lambda/2$ (or $3\lambda/2$ or $5\lambda/2$ or …). Here we see their path difference is $L$, so we must have (in the least possibility) $L = \lambda/2$, or $q = L/\lambda = 0.5$.

(b) As noted above, the next possibility is $L = 3\lambda/2$, or $q = L/\lambda = 1.5$.

19. (a) The problem is asking at how many angles will there be "loud" resultant waves, and at how many will there be "quiet" ones? We note that at all points (at large distance from the origin) along the *x* axis there will be quiet ones; one way to see this is to note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value 3.5, implying a half-wavelength (180°) phase difference (destructive interference) between the waves. To distinguish the destructive interference along the $+x$ axis from the destructive interference along the $-x$ axis, we label one with +3.5 and the other –3.5. This labeling is useful in that it suggests that the complete enumeration of the quiet directions in the upper-half plane (including the *x* axis) is: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5. Similarly, the complete enumeration of the loud directions in the upper-half plane is: –3, –2, –1, 0, +1, +2, +3. Counting also the "other" –3, –2, –1, 0, +1, +2, +3 values for the *lower*-half plane, then we conclude there are a total of 7 + 7 = 14 "loud" directions.

(b) The discussion about the "quiet" directions was started in part (a). The number of values in the list: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5 along with –2.5, –1.5, –0.5, +0.5, +1.5, +2.5 (for the lower-half plane) is 14. There are 14 "quiet" directions.

20. (a) The problem indicates that we should ignore the decrease in sound amplitude, which means that all waves passing through point *P* have equal amplitude. Their superposition at *P* if $d = \lambda/4$ results in a net effect of zero there since there are four sources (so the first and third are $\lambda/2$ apart and thus interfere destructively; similarly for the second and fourth sources).

(b) Their superposition at *P* if $d = \lambda/2$ also results in a net effect of zero there since there are an even number of sources (so the first and second being $\lambda/2$ apart will interfere destructively; similarly for the waves from the third and fourth sources).

(c) If $d = \lambda$ then the waves from the first and second sources will arrive at *P* in phase; similar observations apply to the second and third, and to the third and fourth sources. Thus, four waves interfere constructively there with net amplitude equal to $4s_m$.

21. **THINK** The sound waves from the two speakers undergo interference. Whether the interference is constructive or destructive depends on the path length difference, or the phase difference.

**EXPRESS** From the figure, we see that the distance from the closer speaker to the listener is $L = d_2$, and the distance from the other speaker to the listener is $L' = \sqrt{d_1^2 + d_2^2}$, where $d_1$ is the distance between the speakers. The phase difference at the location of the listener is $\phi = 2\pi(L' - L)/\lambda$, where $\lambda$ is the wavelength. For a minimum in intensity at the listener, $\phi = (2n + 1)\pi$, where $n$ is an integer. Thus,

$$\phi = \frac{2\pi(L'-L)}{\lambda_{\text{min}}} = (2n+1)\pi \quad \Rightarrow \quad \lambda_{\text{min}} = \frac{2(L'-L)}{2n+1},$$

and the frequency is

$$f_{\text{min}} = \frac{v}{\lambda_{\text{min}}} = \frac{(2n+1)v}{2\left(\sqrt{d_1^2+d_2^2}-d_2\right)} = \frac{(2n+1)(343\,\text{m/s})}{2\left(\sqrt{(2.00\,\text{m})^2+(3.75\,\text{m})^2}-3.75\,\text{m}\right)} = (2n+1)(343\,\text{Hz}).$$

Now 20,000/343 = 58.3, so $2n + 1$ must range from 0 to 57 for the frequency to be in the audible range (20 Hz to 20 kHz). This means $n$ ranges from 0 to 28.

On the other hand, for a maximum in intensity at the listener, $\phi = 2n\pi$, where $n$ is any positive integer. Thus $\lambda_{\text{max}} = (1/n)\left(\sqrt{d_1^2+d_2^2}-d_2\right)$ and

$$f_{\text{max}} = \frac{v}{\lambda_{\text{max}}} = \frac{nv}{\sqrt{d_1^2+d_2^2}-d_2} = \frac{n(343\,\text{m/s})}{\sqrt{(2.00\,\text{m})^2+(3.75\,\text{m})^2}-3.75\,\text{m}} = n(686\,\text{Hz}).$$

Since 20,000/686 = 29.2, $n$ must be in the range from 1 to 29 for the frequency to be audible.

**ANALYZE** (a) The lowest frequency that gives minimum signal is ($n$ = 0) $f_{\text{min},1} = 343$ Hz.

(b) The second lowest frequency is ($n$ = 1) $f_{\text{min},2} = [2(1)+1](343\text{ Hz}) = 1029\text{ Hz} = 3f_{\text{min},1}$. Thus, the factor is 3.

(c) The third lowest frequency is ($n$ = 2) $f_{\text{min},3} = [2(2)+1](343\text{ Hz}) = 1715\text{ Hz} = 5f_{\text{min},1}$. Thus, the factor is 5.

(d) The lowest frequency that gives maximum signal is ($n$ =1) $f_{\text{max},1} = 686$ Hz.

(e) The second lowest frequency is ($n$ = 2) $f_{\text{max},2} = 2(686\text{ Hz}) = 1372\text{ Hz} = 2f_{\text{max},1}$. Thus, the factor is 2.

(f) The third lowest frequency is ($n$ = 3) $f_{\max,3} = 3(686\text{ Hz}) = 2058\text{ Hz} = 3f_{\max,1}$. Thus, the factor is 3.

**LEARN** We see that the interference of the two sound waves depends on their phase difference $\phi = 2\pi(L' - L)/\lambda$. The interference is fully constructive when $\phi$ is a multiple of $2\pi$, but fully destructive when $\phi$ is an odd multiple of $\pi$.

22. At the location of the detector, the phase difference between the wave that traveled straight down the tube and the other one, which took the semi-circular detour, is

$$\Delta\phi = k\Delta d = \frac{2\pi}{\lambda}(\pi r - 2r).$$

For $r = r_{\min}$ we have $\Delta\phi = \pi$, which is the smallest phase difference for a destructive interference to occur. Thus,

$$r_{\min} = \frac{\lambda}{2(\pi - 2)} = \frac{40.0\,\text{cm}}{2(\pi - 2)} = 17.5\,\text{cm}.$$

23. (a) If point $P$ is infinitely far away, then the small distance $d$ between the two sources is of no consequence (they seem effectively to be the same distance away from $P$). Thus, there is no perceived phase difference.

(b) Since the sources oscillate in phase, then the situation described in part (a) produces fully constructive interference.

(c) For finite values of $x$, the difference in source positions becomes significant. The path lengths for waves to travel from $S_1$ and $S_2$ become now different. We interpret the question as asking for the behavior of the absolute value of the phase difference $|\Delta\phi|$, in which case any change from zero (the answer for part (a)) is certainly an increase.

The path length difference for waves traveling from $S_1$ and $S_2$ is

$$\Delta\ell = \sqrt{d^2 + x^2} - x \qquad \text{for} \quad x > 0.$$

The phase difference in “cycles” (in absolute value) is therefore

$$|\Delta\phi| = \frac{\Delta\ell}{\lambda} = \frac{\sqrt{d^2 + x^2} - x}{\lambda}.$$

Thus, in terms of $\lambda$, the phase difference is identical to the path length difference: $|\Delta\phi| = \Delta\ell > 0$. Consider $\Delta\ell = \lambda/2$. Then $\sqrt{d^2 + x^2} = x + \lambda/2$. Squaring both sides, rearranging, and solving, we find

$$x = \frac{d^2}{\lambda} - \frac{\lambda}{4}.$$

In general, if $\Delta\ell = \xi\lambda$ for some multiplier $\xi > 0$, we find

$$x = \frac{d^2}{2\xi\lambda} - \frac{1}{2}\xi\lambda = \frac{64.0}{\xi} - \xi$$

where we have used $d$ = 16.0 m and $\lambda$ = 2.00 m.

(d) For $\Delta\ell = 0.50\lambda$, or $\xi = 0.50$, we have $x = (64.0/0.50 - 0.50)\text{ m} = 127.5\text{ m} \approx 128\text{ m}$.

(e) For $\Delta\ell = 1.00\lambda$, or $\xi = 1.00$, we have $x = (64.0/1.00 - 1.00)\text{ m} = 63.0\text{ m}$.

(f) For $\Delta\ell = 1.50\lambda$, or $\xi = 1.50$, we have $x = (64.0/1.50 - 1.50)\text{ m} = 41.2\text{ m}$.

Note that since whole cycle phase differences are equivalent (as far as the wave superposition goes) to zero phase difference, then the $\xi$ = 1, 2 cases give constructive interference. A shift of a half-cycle brings "troughs" of one wave in superposition with "crests" of the other, thereby canceling the waves; therefore, the $\xi = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ cases produce destructive interference.

24. (a) Equation 17-29 gives the relation between sound level $\beta$ and intensity $I$, namely

$$I = I_0 10^{(\beta/10\text{dB})} = (10^{-12}\text{ W/m}^2)10^{(\beta/10\text{dB})} = 10^{-12+(\beta/10\text{dB})}\text{ W/m}^2$$

Thus we find that for a $\beta$ = 70 dB level we have a high intensity value of $I_{\text{high}}$ = 10 $\mu$W/m$^2$.

(b) Similarly, for a $\beta$ = 50 dB level we have a low intensity value of $I_{\text{low}}$ = 0.10 $\mu$W/m$^2$.

(c) Equation 17-27 gives the relation between the displacement amplitude and $I$. Using the values for density and wave speed, we find $s_m$ = 70 nm for the high intensity case.

(d) Similarly, for the low intensity case we have $s_m$ = 7.0 nm.

We note that although the intensities differed by a factor of 100, the amplitudes differed by only a factor of 10.

25. The intensity is given by $I = \frac{1}{2}\rho v \omega^2 s_m^2$, where $\rho$ is the density of air, $v$ is the speed of sound in air, $\omega$ is the angular frequency, and $s_m$ is the displacement amplitude for the sound wave. Replace $\omega$ with $2\pi f$ and solve for $s_m$:

$$s_m = \sqrt{\frac{I}{2\pi^2 \rho v f^2}} = \sqrt{\frac{1.00\times10^{-6}\ \text{W/m}^2}{2\pi^2(1.21\,\text{kg/m}^3)(343\,\text{m/s})(300\,\text{Hz})^2}} = 3.68\times10^{-8}\ \text{m}.$$

26. (a) Since intensity is power divided by area, and for an isotropic source the area may be written $A = 4\pi r^2$ (the area of a sphere), then we have

$$I = \frac{P}{A} = \frac{1.0\,\text{W}}{4\pi(1.0\,\text{m})^2} = 0.080\,\text{W/m}^2.$$

(b) This calculation may be done exactly as shown in part (a) (but with $r$ = 2.5 m instead of $r$ = 1.0 m), or it may be done by setting up a ratio. We illustrate the latter approach. Thus,

$$\frac{I'}{I} = \frac{P/4\pi(r')^2}{P/4\pi r^2} = \left(\frac{r}{r'}\right)^2$$

leads to $I'$ = (0.080 W/m$^2$)(1.0/2.5)$^2$ = 0.013 W/m$^2$.

27. **THINK** The sound level increases by 10 dB when the intensity increases by a factor of 10.

**EXPRESS** The sound level $\beta$ is defined as (see Eq. 17-29):

$$\beta = (10\ \text{dB})\log\frac{I}{I_0}$$

where $I_0 = 10^{-12}\ \text{W/m}^2$ is the standard reference intensity. In this problem, let $I_1$ be the original intensity and $I_2$ be the final intensity. The original sound level is $\beta_1 = (10\ \text{dB})\log(I_1/I_0)$ and the final sound level is $\beta_2$ = (10 dB) log($I_2/I_0$). With $\beta_2 = \beta_1 + 30\,\text{dB}$, we have

$$(10\ \text{dB})\log(I_2/I_0) = (10\ \text{dB})\log(I_1/I_0) + 30\ \text{dB},$$

or

$$(10\ \text{dB})\log(I_2/I_0) - (10\ \text{dB})\log(I_1/I_0) = 30\ \text{dB}.$$

The above equation allows us to solve for the ratio $I_2/I_1$. On the other hand, combing Eqs. 17-15 and 17-27 leads to the following relation between the intensity $I$ and the pressure amplitude $\Delta p_m$: $I = \frac{1}{2}\frac{(\Delta p_m)^2}{\rho v}$.

**ANALYZE** (a) Divide by 10 dB and use log($I_2/I_0$) – log($I_1/I_0$) = log($I_2/I_1$) to obtain log($I_2/I_1$) = 3. Now use each side as an exponent of 10 and recognize that

$10^{\log(I_2/I_1)} = I_2/I_1$. The result is $I_2/I_1 = 10^3$. The intensity is increased by a factor of $1.0 \times 10^3$.

(b) The pressure amplitude is proportional to the square root of the intensity so it is increased by a factor of $\sqrt{1000} \approx 32$.

**LEARN** From the definition of $\beta$, we see that doubling sound intensity increases the sound level by $\Delta\beta = (10 \text{ dB})\log 2 = 3.01 \text{ dB}$.

28. The sound level $\beta$ is defined as (see Eq. 17-29):

$$\beta = (10 \text{ dB})\log\frac{I}{I_0}$$

where $I_0 = 10^{-12} \text{ W/m}^2$ is the standard reference intensity. In this problem, let the two intensities be $I_1$ and $I_2$ such that $I_2 > I_1$. The sound levels are $\beta_1 = (10 \text{ dB})\log(I_1/I_0)$ and $\beta_2 = (10 \text{ dB}) \log(I_2/I_0)$. With $\beta_2 = \beta_1 + 1.0 \text{ dB}$, we have

$$(10 \text{ dB}) \log(I_2/I_0) = (10 \text{ dB}) \log(I_1/I_0) + 1.0 \text{ dB},$$

or

$$(10 \text{ dB}) \log(I_2/I_0) - (10 \text{ dB}) \log(I_1/I_0) = 1.0 \text{ dB}.$$

Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 0.1$. Now use each side as an exponent of 10 and recognize that $10^{\log(I_2/I_1)} = I_2/I_1$. The result is

$$\frac{I_2}{I_1} = 10^{0.1} = 1.26.$$

29. **THINK** Power is the time rate of energy transfer, and intensity is the rate of energy flow per unit area perpendicular to the flow.

**EXPRESS** The rate at which energy flow across every sphere centered at the source is the same, regardless of the sphere radius, and is the same as the power output of the source. If $P$ is the power output and $I$ is the intensity a distance $r$ from the source, then $P = IA = 4\pi r^2 I$, where $A = 4\pi r^2$ is the surface area of a sphere of radius $r$.

**ANALYZE** With $r = 2.50$ m and $I = 1.91 \times 10^{-4} \text{ W/m}^2$, we find the power of the source to be

$$P = 4\pi(2.50 \text{ m})^2 (1.91 \times 10^{-4} \text{ W/m}^2) = 1.50 \times 10^{-2} \text{ W}.$$

**LEARN** Since intensity falls off as $1/r^2$, the further away from the source, the weaker the intensity.

30. (a) The intensity is given by $I = P/4\pi r^2$ when the source is "point-like." Therefore, at $r$ = 3.00 m,

$$I = \frac{1.00\times10^{-6}\ \text{W}}{4\pi(3.00\,\text{m})^2} = 8.84\times10^{-9}\ \text{W/m}^2.$$

(b) The sound level there is

$$\beta = 10\log\left(\frac{8.84\times10^{-9}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 39.5\,\text{dB}.$$

31. We use $\beta = 10\log(I/I_o)$ with $I_o = 1\times10^{-12}$ W/m$^2$ and $I = P/4\pi r^2$ (an assumption we are asked to make in the problem). We estimate $r \approx 0.3$ m (distance from knuckle to ear) and find

$$P \approx 4\pi(0.3\,\text{m})^2\left(1\times10^{-12}\ \text{W/m}^2\right)10^{6.2} = 2\times10^{-6}\ \text{W} = 2\ \mu\text{W}.$$

32. (a) Since $\omega = 2\pi f$, Eq. 17-15 leads to

$$\Delta p_m = v\rho(2\pi f)s_m \;\Rightarrow\; s_m = \frac{1.13\times10^{-3}\ \text{Pa}}{2\pi(1665\,\text{Hz})(343\,\text{m/s})(1.21\ \text{kg/m}^3)}$$

which yields $s_m$ = 0.26 nm. The nano prefix represents $10^{-9}$. We use the speed of sound and air density values given at the beginning of the exercises and problems section in the textbook.

(b) We can plug into Eq. 17–27 or into its equivalent form, rewritten in terms of the pressure amplitude:

$$I = \frac{1}{2}\frac{(\Delta p_m)^2}{\rho v} = \frac{1}{2}\frac{\left(1.13\times10^{-3}\ \text{Pa}\right)^2}{\left(1.21\,\text{kg/m}^3\right)(343\,\text{m/s})} = 1.5\ \text{nW/m}^2.$$

33. We use $\beta = 10\log(I/I_o)$ with $I_o = 1\times10^{-12}$ W/m$^2$ and Eq. 17–27 with

$$\omega = 2\pi f = 2\pi(260\ \text{Hz}),$$

$v$ = 343 m/s and $\rho$ = 1.21 kg/m$^3$.

$$I = I_o\left(10^{8.5}\right) = \frac{1}{2}\rho v(2\pi f)^2 s_m^2 \quad\Rightarrow\quad s_m = 7.6\times10^{-7}\ \text{m} = 0.76\ \mu\text{m}.$$

34. Combining Eqs.17-28 and 17-29 we have $\beta = 10\log\left(\frac{P}{I_o 4\pi r^2}\right)$. Taking differences (for sounds $A$ and $B$) we find

$$\Delta\beta = 10\log\left(\frac{P_A}{I_o4\pi r^2}\right) - 10\log\left(\frac{P_B}{I_o4\pi r^2}\right) = 10\log\left(\frac{P_A}{P_B}\right)$$

using well-known properties of logarithms. Thus, we see that $\Delta\beta$ is independent of $r$ and can be evaluated anywhere.

(a) We can solve the above relation (once we know $\Delta\beta$ = 5.0) for the ratio of powers; we find $P_A/P_B \approx 3.2$.

(b) At $r$ = 1000 m it is easily seen (in the graph) that $\Delta\beta$ = 5.0 dB. This is the same Δβ we expect to find, then, at $r$ = 10 m.

35. (a) The intensity is

$$I = \frac{P}{4\pi r^2} = \frac{30.0\,\text{W}}{(4\pi)(200\,\text{m})^2} = 5.97\times 10^{-5}\ \text{W/m}^2.$$

(b) Let $A$ (= 0.750 cm$^2$) be the cross-sectional area of the microphone. Then the power intercepted by the microphone is

$$P' = IA = 0 = (6.0\times 10^{-5}\ \text{W/m}^2)(0.750\,\text{cm}^2)(10^{-4}\ \text{m}^2/\text{cm}^2) = 4.48\times 10^{-9}\ \text{W}.$$

36. The difference in sound level is given by Eq. 17-37:

$$\Delta\beta = \beta_f - \beta_i = (10\ \text{db})\log\left(\frac{I_f}{I_i}\right).$$

Thus, if $\Delta\beta = 5.0$ db, then $\log(I_f/I_i) = 1/2$, which implies that $I_f = \sqrt{10}I_i$. On the other hand, the intensity at a distance $r$ from the source is $I = \dfrac{P}{4\pi r^2}$, where $P$ is the power of the source. A fixed $P$ implies that $I_i r_i^2 = I_f r_f^2$. Thus, with $r_i = 1.2$ m, we obtain

$$r_f = \left(\frac{I_i}{I_f}\right)^{1/2} r_i = \left(\frac{1}{10}\right)^{1/4}(1.2\ \text{m}) = 0.67\ \text{m}.$$

37. (a) The average potential energy transport rate is the same as that of the kinetic energy. This implies that the (average) rate for the total energy is

$$\left(\frac{dE}{dt}\right)_{\text{avg}} = 2\left(\frac{dK}{dt}\right)_{\text{avg}} = 2\,(\tfrac{1}{4}\,\rho A v \omega^2 s_m^{\,2})$$

using Eq. 17-44. In this equation, we substitute $\rho$ = 1.21 kg/m$^3$, $A = \pi r^2 = \pi(0.020\ \text{m})^2$, $v$ = 343 m/s, $\omega$ = 3000 rad/s, $s_m = 12\times 10^{-9}$ m, and obtain the answer $3.4\times 10^{-10}$ W.

(b) The second string is in a separate tube, so there is no question about the waves superposing. The total rate of energy, then, is just the addition of the two: $2(3.4 \times 10^{-10}$ W) $= 6.8 \times 10^{-10}$ W.

(c) Now we *do* have superposition, with $\phi = 0$, so the resultant amplitude is twice that of the individual wave, which leads to the energy transport rate being four times that of part (a). We obtain $4(3.4 \times 10^{-10}$ W) $= 1.4 \times 10^{-9}$ W.

(d) In this case $\phi = 0.4\pi$, which means (using Eq. 17-39)

$$s_m' = 2\, s_m \cos(\phi/2) = 1.618 s_m.$$

This means the energy transport rate is $(1.618)^2 = 2.618$ times that of part (a). We obtain $2.618(3.4 \times 10^{-10}$ W) $= 8.8 \times 10^{-10}$ W.

(e) The situation is as shown in Fig. 17-14(b). The answer is zero.

38. The frequency is $f$ = 686 Hz and the speed of sound is $v_{\text{sound}}$ = 343 m/s. If $L$ is the length of the air-column, then using Eq. 17–41, the water height is (in unit of meters)

$$h = 1.00 - L = 1.00 - \frac{nv}{4f} = 1.00 - \frac{n(343)}{4(686)} = (1.00 - 0.125n) \text{ m}$$

where $n$ = 1, 3, 5,… with only one end closed.

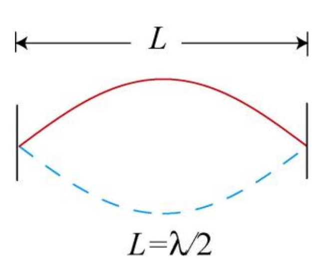
(a) There are 4 values of $n$ ($n$ = 1, 3, 5, 7) which satisfies $h > 0$.

(b) The smallest water height for resonance to occur corresponds to $n = 7$ with $h = 0.125$ m.

(c) The second smallest water height corresponds to $n = 5$ with $h = 0.375$ m.

39. **THINK** Violin strings are fixed at both ends. A string clamped at both ends can be made to oscillate in standing wave patterns.

**EXPRESS** When the string is fixed at both ends and set to vibrate at its fundamental, lowest resonant frequency, exactly one-half of a wavelength fits between the ends (see figure to the right). The wave speed is given by $v = \lambda f = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string.




**ANALYZE** (a) With $\lambda = 2L$ , we find the wave speed to be

$$v = f\lambda = 2Lf = 2(0.220\text{ m})(920\text{ Hz}) = 405\text{ m/s}.$$

(b) If $M$ is the mass of the (uniform) string, then $\mu = M/L$. Thus, the string tension is

$$\tau = \mu v^2 = (M/L)v^2 = [(800 \times 10^{-6}\text{ kg})/(0.220\text{ m})]\,(405\text{ m/s})^2 = 596\text{ N}.$$

(c) The wavelength is $\lambda = 2L = 2(0.220\text{ m}) = 0.440\text{ m}$.

(d) If $v_a$ is the speed of sound in air, then the wavelength in air is

$$\lambda_a = v_a/f = (343\text{ m/s})/(920\text{ Hz}) = 0.373\text{ m}.$$

**LEARN** The frequency of the sound wave in air is the same as the frequency of oscillation of the string. However, the wavelengths of the wave on the string and the sound waves emitted by the string are different because their wave speeds are not the same.

40. At the beginning of the exercises and problems section in the textbook, we are told to assume $v_{\text{sound}} = 343$ m/s unless told otherwise. The second harmonic of pipe $A$ is found from Eq. 17-39 with $n = 2$ and $L = L_A$, and the third harmonic of pipe $B$ is found from Eq. 17-41 with $n = 3$ and $L = L_B$. Since these frequencies are equal, we have

$$\frac{2v_{\text{sound}}}{2L_A} = \frac{3v_{\text{sound}}}{4L_B} \Rightarrow L_B = \frac{3}{4}L_A.$$

(a) Since the fundamental frequency for pipe $A$ is 300 Hz, we immediately know that the second harmonic has $f = 2(300\text{ Hz}) = 600$ Hz. Using this, Eq. 17-39 gives

$$L_A = (2)(343\text{ m/s})/2(600\text{ s}^{-1}) = 0.572\text{ m}.$$

(b) The length of pipe $B$ is $L_B = \frac{3}{4}L_A = 0.429\,\text{m}.$

41. (a) From Eq. 17–53, we have

$$f = \frac{nv}{2L} = \frac{(1)(250\,\text{m/s})}{2(0.150\,\text{m})} = 833\,\text{Hz}.$$

(b) The frequency of the wave on the string is the same as the frequency of the sound wave it produces during its vibration. Consequently, the wavelength in air is

$$\lambda = \frac{v_{\text{sound}}}{f} = \frac{348\,\text{m/s}}{833\,\text{Hz}} = 0.418\,\text{m}.$$

42. The distance between nodes referred to in the problem means that $\lambda/2$ = 3.8 cm, or $\lambda$ = 0.076 m. Therefore, the frequency is

$$f = v/\lambda = (1500 \text{ m/s})/(0.076 \text{ m}) \approx 20 \times 10^3 \text{ Hz}.$$

43. **THINK** The pipe is open at both ends so there are displacement antinodes at both ends.

**EXPRESS** If $L$ is the pipe length and $\lambda$ is the wavelength then $\lambda = 2L/n$, where $n$ is an integer. That is, an integer number of half-wavelengths fit into the length of the pipe. If $v$ is the speed of sound then the resonant frequencies are given by $f = v/\lambda = nv/2L$. Now $L$ = 0.457 m, so

$$f = \frac{nv}{2L} = \frac{n(344 \text{ m/s})}{2(0.457 \text{ m})} = (376.4 \text{ Hz})n .$$

**ANALYZE** (a) To find the resonant frequencies that lie between 1000 Hz and 2000 Hz, first set $f$ = 1000 Hz and solve for $n$, then set $f$ = 2000 Hz and again solve for $n$. The results are 2.66 and 5.32, which imply that $n$ = 3, 4, and 5 are the appropriate values of $n$. Thus, there are 3 frequencies.

(b) The lowest frequency at which resonance occurs corresponds to $n$ = 3, or

$$f = 3(376.4 \text{ Hz}) = 1129 \text{ Hz} .$$

(c) The second lowest frequency at which resonance occurs corresponds to $n$ = 4, or

$$f = 4(376.4 \text{ Hz}) = 1506 \text{ Hz} .$$

**LEARN** The third lowest frequency at which resonance occurs corresponds to $n$ = 5, or

$$f = 5(376.4 \text{ Hz}) = 1882 \text{ Hz} .$$

Changing the length of the pipe can affect the number of resonant frequencies.

44. (a) Using Eq. 17-39 with $v$ = 343 m/s and $n$ = 1, we find $f = nv/2L$ = 86 Hz for the fundamental frequency in a nasal passage of length $L$ = 2.0 m (subject to various assumptions about the nature of the passage as a "bent tube open at both ends").

(b) The sound would be perceptible as *sound* (as opposed to just a general vibration) of very low frequency.

(c) Smaller $L$ implies larger $f$ by the formula cited above. Thus, the female's sound is of higher pitch (frequency).

45. (a) We note that 1.2 = 6/5. This suggests that both even and odd harmonics are present, which means the pipe is open at both ends (see Eq. 17-39).

(b) Here we observe 1.4 = 7/5. This suggests that only odd harmonics are present, which means the pipe is open at only one end (see Eq. 17-41).

46. We observe that "third lowest … frequency" corresponds to harmonic number $n_A = 3$ for pipe *A*, which is open at both ends. Also, "second lowest … frequency" corresponds to harmonic number $n_B = 3$ for pipe *B*, which is closed at one end.

(a) Since the frequency of *B* matches the frequency of A, using Eqs. 17-39 and 17-41, we have

$$f_A = f_B \quad \Rightarrow \frac{3v}{2L_A} = \frac{3v}{4L_B}$$

which implies $L_B = L_A/2 = (1.20 \text{ m})/2 = 0.60 \text{ m}$. Using Eq. 17-40, the corresponding wavelength is

$$\lambda = \frac{4L_B}{3} = \frac{4(0.60 \text{ m})}{3} = 0.80 \text{ m}.$$

The change from node to anti-node requires a distance of λ/4 so that every increment of 0.20 m along the *x*-axis involves a switch between node and anti-node. Since the closed end is a node, the next node appears at *x* = 0.40 m, so there are 2 nodes. The situation corresponds to that illustrated in Fig. 17-14(b) with $n = 3$.

(b) The smallest value of *x* where a node is present is *x* = 0.

(c) The second smallest value of *x* where a node is present is *x* = 0.40 m.

(d) Using *v* = 343 m/s, we find $f_3 = v/\lambda = 429$ Hz. Now, we find the fundamental resonant frequency by dividing by the harmonic number, $f_1 = f_3/3 = 143$ Hz.

47. The top of the water is a displacement node and the top of the well is a displacement anti-node. At the lowest resonant frequency exactly one-fourth of a wavelength fits into the depth of the well. If *d* is the depth and λ is the wavelength, then λ = 4*d*. The frequency is *f* = *v*/λ = *v*/4*d*, where *v* is the speed of sound. The speed of sound is given by $v = \sqrt{B/\rho}$, where *B* is the bulk modulus and $\rho$ is the density of air in the well. Thus $f = (1/4d)\sqrt{B/\rho}$ and

$$d = \frac{1}{4f}\sqrt{\frac{B}{\rho}} = \frac{1}{4(7.00\,\text{Hz})}\sqrt{\frac{1.33\times10^5 \text{ Pa}}{1.10\,\text{kg/m}^3}} = 12.4 \text{ m}.$$

48. (a) Since the difference between consecutive harmonics is equal to the fundamental frequency (see section 17-6) then $f_1$ = (390 – 325) Hz = 65 Hz. The next harmonic after 195 Hz is therefore (195 + 65) Hz = 260 Hz.

(b) Since $f_n = nf_1$, then $n$ = 260/65 = 4.

(c) Only *odd* harmonics are present in tube *B*, so the difference between consecutive harmonics is equal to *twice* the fundamental frequency in this case (consider taking differences of Eq. 17-41 for various values of *n*). Therefore,

$$f_1 = \tfrac{1}{2}(1320 - 1080)\text{ Hz} = 120\text{ Hz}.$$

The next harmonic after 600 Hz is consequently [600 + 2(120)] Hz = 840 Hz.

(d) Since $f_n = nf_1$ (for *n* odd), then $n$ = 840/120 = 7.

49. **THINK** Violin strings are fixed at both ends. A string clamped at both ends can be made to oscillate in standing wave patterns.

**EXPRESS** The resonant wavelengths are given by $\lambda = 2L/n$, where $L$ is the length of the string and $n$ is an integer. The resonant frequencies are given by $f_n = v/\lambda = nv/2L$, where $v$ is the wave speed on the string. Now $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string. Thus $f_n = (n/2L)\sqrt{\tau/\mu}$.

**ANALYZE** Suppose the lower frequency is associated with $n_1$ and the higher frequency is associated with $n_2 = n_1 + 1$. There are no resonant frequencies between so you know that the integers associated with the given frequencies differ by 1. Thus, $f_{n_1} = (n_1/2L)\sqrt{\tau/\mu}$ and

$$f_{n_2} = \frac{n_1+1}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L}\sqrt{\frac{\tau}{\mu}} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}} = f_{n_1} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}.$$

This means $f_{n_2} - f_{n_1} = (1/2L)\sqrt{\tau/\mu}$ and

$$\begin{aligned}\tau &= 4L^2\mu(f_{n_2} - f_{n_1})^2 = 4(0.300\,\text{m})^2(0.650\times10^{-3}\text{ kg/m})(1320\,\text{Hz} - 880\,\text{Hz})^2 \\ &= 45.3\,\text{N}.\end{aligned}$$

**LEARN** Since the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency: $\Delta f = f_{n+1} - f_n = \dfrac{v}{2L} = f_1$, we find

$$f_1 = 1320\,\text{Hz} - 880\,\text{Hz} = 440\,\text{Hz}.$$

Since 880 Hz = 2(440 Hz) and 1320 Hz = 3(440 Hz), the two frequencies correspond to $n_1 = 2$ and $n_2 = 3$, respectively.

50. (a) Using Eq. 17-39 with $n$ = 1 (for the fundamental mode of vibration) and 343 m/s for the speed of sound, we obtain

$$f = \frac{(1)v_{\text{sound}}}{4L_{\text{tube}}} = \frac{343\,\text{m/s}}{4(1.20\,\text{m})} = 71.5\,\text{Hz}.$$

(b) For the wire (using Eq. 17-53) we have

$$f' = \frac{nv_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{\tau}{\mu}}$$

where $\mu = m_{\text{wire}}/L_{\text{wire}}$. Recognizing that $f = f'$ (both the wire and the air in the tube vibrate at the same frequency), we solve this for the tension $\tau$:

$$\tau = (2L_{\text{wire}}\,f)^2\left(\frac{m_{\text{wire}}}{L_{\text{wire}}}\right) = 4f^2 m_{\text{wire}}L_{\text{wire}} = 4(71.5\,\text{Hz})^2(9.60\times10^{-3}\,\text{kg})(0.330\,\text{m}) = 64.8\,\text{N}.$$

51. Let the period be $T$. Then the beat frequency is $1/T - 440\,\text{Hz} = 4.00\,\text{beats/s}.$ Therefore, $T = 2.25 \times 10^{-3}$ s. The string that is "too tightly stretched" has the higher tension and thus the higher (fundamental) frequency.

52. Since the beat frequency equals the difference between the frequencies of the two tuning forks, the frequency of the first fork is either 381 Hz or 387 Hz. When mass is added to this fork its frequency decreases (recall, for example, that the frequency of a mass–spring oscillator is proportional to $1/\sqrt{m}$ ). Since the beat frequency also decreases, the frequency of the first fork must be greater than the frequency of the second. It must be 387 Hz.

53. **THINK** Beat arises when two waves detected have slightly different frequencies: $f_{\text{beat}} = f_2 - f_1$.

**EXPRESS** Each wire is vibrating in its fundamental mode so the wavelength is twice the length of the wire ($\lambda = 2L$) and the frequency is

$$f = v/\lambda = (1/2L)\sqrt{\tau/\mu}\,,$$

where $v = \sqrt{\tau/\mu}$ is the wave speed for the wire, $\tau$ is the tension in the wire, and $\mu$ is the linear mass density of the wire. Suppose the tension in one wire is $\tau$ and the oscillation frequency of that wire is $f_1$. The tension in the other wire is $\tau + \Delta\tau$ and its frequency is $f_2$. You want to calculate $\Delta\tau/\tau$ for $f_1$ = 600 Hz and $f_2$ = 606 Hz. Now, $f_1 = (1/2L)\sqrt{\tau/\mu}$ and $f_2 = (1/2L)\sqrt{(\tau+\Delta\tau)/\mu}$, so

$$f_2/f_1 = \sqrt{(\tau+\Delta\tau)/\tau} = \sqrt{1+(\Delta\tau/\tau)}.$$

**ANALYZE** The fractional increase in tension is

$$\Delta\tau/\tau = (f_2/f_1)^2 - 1 = [(606\,\text{Hz})/(600\,\text{Hz})]^2 - 1 = 0.020.$$

**LEARN** Beat frequency $f_{\text{beat}} = f_2 - f_1$ is zero when $\Delta\tau = 0$. The beat phenomenon is used by musicians to tune musical instruments. The instrument tuned is sounded against a standard frequency until beat disappears.

54. (a) The number of different ways of picking up a pair of tuning forks out of a set of five is 5!/(2!3!) = 10. For each of the pairs selected, there will be one beat frequency. If these frequencies are all different from each other, we get the maximum possible number of 10.

(b) First, we note that the minimum number occurs when the frequencies of these forks, labeled 1 through 5, increase in equal increments: $f_n = f_1 + n\Delta f$, where $n$ = 2, 3, 4, 5. Now, there are only 4 different beat frequencies: $f_{\text{beat}} = n\Delta f$, where $n$ = 1, 2, 3, 4.

55. We use $v_S = r\omega$ (with $r$ = 0.600 m and $\omega$ = 15.0 rad/s) for the linear speed during circular motion, and Eq. 17-47 for the Doppler effect (where $f$ = 540 Hz, and $v$ = 343 m/s for the speed of sound).

(a) The lowest frequency is

$$f' = f\left(\frac{v+0}{v+v_S}\right) = 526\ \text{Hz}.$$

(b) The highest frequency is

$$f' = f\left(\frac{v+0}{v-v_S}\right) = 555\ \text{Hz}.$$

56. The Doppler effect formula, Eq. 17-47, and its accompanying rule for choosing ± signs, are discussed in Section 17-10. Using that notation, we have $v$ = 343 m/s, $v_D$ = 2.44 m/s, $f'$ = 1590 Hz, and $f$ = 1600 Hz. Thus,

$$f' = f\left(\frac{v+v_D}{v+v_S}\right) \Rightarrow v_S = \frac{f}{f'}\,(v+v_D) - v = 4.61\,\text{m/s}.$$

57. In the general Doppler shift equation, the trooper's speed is the source speed and the speeder's speed is the detector's speed. The Doppler effect formula, Eq. 17-47, and its accompanying rule for choosing ± signs, are discussed in Section 17-10. Using that notation, we have $v$ = 343 m/s,

$$v_D = v_S = 160 \text{ km/h} = (160000 \text{ m})/(3600 \text{ s}) = 44.4 \text{ m/s},$$

and $f$ = 500 Hz. Thus,

$$f' = (500 \text{ Hz})\left(\frac{343 \text{ m/s} - 44.4 \text{ m/s}}{343 \text{ m/s} - 44.4 \text{ m/s}}\right) = 500 \text{ Hz} \Rightarrow \Delta f = 0.$$

58. We use Eq. 17-47 with $f$ = 1200 Hz and $v$ = 329 m/s.

(a) In this case, $v_D$ = 65.8 m/s and $v_S$ = 29.9 m/s, and we choose signs so that $f'$ is larger than $f$:

$$f' = f\left(\frac{329 \text{ m/s} + 65.8 \text{ m/s}}{329 \text{ m/s} - 29.9 \text{ m/s}}\right) = 1.58 \times 10^3 \text{ Hz}.$$

(b) The wavelength is $\lambda = v/f'$ = 0.208 m.

(c) The wave (of frequency $f'$) "emitted" by the moving reflector (now treated as a "source," so $v_S$ = 65.8 m/s) is returned to the detector (now treated as a detector, so $v_D$ = 29.9 m/s) and registered as a new frequency $f''$:

$$f'' = f'\left(\frac{329 \text{ m/s} + 29.9 \text{ m/s}}{329 \text{ m/s} - 65.8 \text{ m/s}}\right) = 2.16 \times 10^3 \text{ Hz}.$$

(d) This has wavelength $v/f''$ = 0.152 m.

59. We denote the speed of the French submarine by $u_1$ and that of the U.S. sub by $u_2$.

(a) The frequency as detected by the U.S. sub is

$$f_1' = f_1\left(\frac{v + u_2}{v - u_1}\right) = (1.000 \times 10^3 \text{ Hz})\left(\frac{5470 \text{ km/h} + 70.00 \text{ km/h}}{5470 \text{ km/h} - 50.00 \text{ km/h}}\right) = 1.022 \times 10^3 \text{ Hz}.$$

(b) If the French sub were stationary, the frequency of the reflected wave would be

$$f_r = f_1(v + u_2)/(v - u_2).$$

Since the French sub is moving toward the reflected signal with speed $u_1$, then

$$f_r' = f_r\left(\frac{v+u_1}{v}\right) = f_1\frac{(v+u_1)(v+u_2)}{v(v-u_2)} = \frac{(1.000\times10^3\text{ Hz})(5470+50.00)(5470+70.00)}{(5470)(5470-70.00)}$$
$$=1.045\times10^3\text{ Hz.}$$

60. We are combining two effects: the reception of a moving object (the truck of speed $u$ = 45.0 m/s) of waves emitted by a stationary object (the motion detector), and the subsequent emission of those waves by the moving object (the truck), which are picked up by the stationary detector. This could be figured in two steps, but is more compactly computed in one step as shown here:

$$f_{\text{final}} = f_{\text{initial}}\left(\frac{v+u}{v-u}\right) = (0.150\,\text{MHz})\left(\frac{343\,\text{m/s} + 45\,\text{m/s}}{343\,\text{m/s} - 45\,\text{m/s}}\right) = 0.195\,\text{MHz.}$$

61. As a result of the Doppler effect, the frequency of the reflected sound as heard by the bat is

$$f_r = f'\left(\frac{v+u_{\text{bat}}}{v-u_{\text{bat}}}\right) = (3.9\times10^4\text{ Hz})\left(\frac{v+v/40}{v-v/40}\right) = 4.1\times10^4\text{ Hz.}$$

62. The "third harmonic" refers to a resonant frequency $f_3$ = 3 $f_1$, where $f_1$ is the fundamental lowest resonant frequency. When the source is stationary, with respect to the air, then Eq. 17-47 gives

$$f' = f\left(1-\frac{v_d}{v}\right)$$

where $v_d$ is the speed of the detector (assumed to be moving away from the source, in the way we've written it, above). The problem, then, wants us to find $v_d$ such that $f' = f_1$ when the emitted frequency is $f = f_3$. That is, we require $1 - v_d/v = 1/3$. Clearly, the solution to this is $v_d/v = 2/3$ (independent of length and whether one or both ends are open [the latter point being due to the fact that the odd harmonics occur in both systems]). Thus,

(a) For tube 1, $v_d = 2v/3$.

(b) For tube 2, $v_d = 2v/3$.

(c) For tube 3, $v_d = 2v/3$.

(d) For tube 4, $v_d = 2v/3$.

63. In this case, the intruder is moving *away* from the source with a speed $u$ satisfying $u/v \ll 1$. The Doppler shift (with $u$ = –0.950 m/s) leads to

$$f_{\text{beat}} = |f_r - f_s| \approx \frac{2|u|}{v} f_s = \frac{2(0.95\,\text{m/s})(28.0\,\text{kHz})}{343\,\text{m/s}}) = 155\,\text{Hz}\,.$$

64. When the detector is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = \frac{f}{1 - v_s / v}$$

where $v_s$ is the speed of the source (assumed to be approaching the detector in the way we've written it, above). The difference between the approach and the recession is

$$f' - f'' = f\left(\frac{1}{1 - v_s/v} - \frac{1}{1 + v_s/v}\right) = f\left(\frac{2v_s/v}{1 - (v_s/v)^2}\right)$$

which, after setting $(f' - f'')/f = 1/2$, leads to an equation that can be solved for the ratio $v_s/v$. The result is $\sqrt{5} - 2 = 0.236$. Thus, $v_s/v = 0.236$.

65. The Doppler shift formula, Eq. 17-47, is valid only when both $u_S$ and $u_D$ are measured with respect to a stationary medium (i.e., no wind). To modify this formula in the presence of a wind, we switch to a new reference frame in which there is no wind.

(a) When the wind is blowing from the source to the observer with a speed $w$, we have $u'_S = u'_D = w$ in the new reference frame that moves together with the wind. Since the observer is now approaching the source while the source is backing off from the observer, we have, in the new reference frame,

$$f' = f\left(\frac{v + u'_D}{v + u'_S}\right) = f\left(\frac{v + w}{v + w}\right) = 2.0 \times 10^3\ \text{Hz}.$$

In other words, there is no Doppler shift.

(b) In this case, all we need to do is to reverse the signs in front of both $u'_D$ and $u'_S$. The result is that there is still no Doppler shift:

$$f' = f\left(\frac{v - u'_D}{v - u'_S}\right) = f\left(\frac{v - w}{v - w}\right) = 2.0 \times 10^3\ \text{Hz}.$$

In general, there will always be no Doppler shift as long as there is no relative motion between the observer and the source, regardless of whether a wind is present or not.

66. We use Eq. 17-47 with $f$ = 500 Hz and $v$ = 343 m/s. We choose signs to produce $f' > f$.

(a) The frequency heard in still air is

$$f' = (500\ \text{Hz})\left(\frac{343\ \text{m/s} + 30.5\ \text{m/s}}{343\ \text{m/s} - 30.5\ \text{m/s}}\right) = 598\,\text{Hz}.$$

(b) In a frame of reference where the air seems still, the velocity of the detector is 30.5 – 30.5 = 0, and that of the source is 2(30.5). Therefore,

$$f' = (500\ \text{Hz})\left(\frac{343\ \text{m/s} + 0}{343\ \text{m/s} - 2(30.5\ \text{m/s})}\right) = 608\,\text{Hz}.$$

(c) We again pick a frame of reference where the air seems still. Now, the velocity of the source is 30.5 – 30.5 = 0, and that of the detector is 2(30.5). Consequently,

$$f' = (500\ \text{Hz})\left(\frac{343\ \text{m/s} + 2(30.5\ \text{m/s})}{343\ \text{m/s} - 0}\right) = 589\,\text{Hz}.$$

67. **THINK** The girl and her uncle hear different frequencies because of Doppler effect.

**EXPRESS** The Doppler shifted frequency is given by

$$f' = f\,\frac{v \pm v_D}{v \mp v_S},$$

where $f$ is the unshifted frequency, $v$ is the speed of sound, $v_D$ is the speed of the detector (the uncle), and $v_S$ is the speed of the source (the locomotive). All speeds are relative to the air.

**ANALYZE** (a) The uncle is at rest with respect to the air, so $v_D = 0$. The speed of the source is $v_S$ = 10 m/s. Since the locomotive is moving away from the uncle the frequency decreases and we use the plus sign in the denominator. Thus

$$f' = f\,\frac{v}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} + 10.00\,\text{m/s}}\right) = 485.8\,\text{Hz}.$$

(b) The girl is now the detector. Relative to the air she is moving with speed $v_D$ = 10.00 m/s toward the source. This tends to increase the frequency and we use the plus sign in the numerator. The source is moving at $v_S$ = 10.00 m/s away from the girl. This tends to decrease the frequency and we use the plus sign in the denominator. Thus, $(v + v_D) =$

$(v + v_S)$ and $f' = f$ = 500.0 Hz.

(c) Relative to the air the locomotive is moving at $v_S$ = 20.00 m/s away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at $v_D$ = 10.00 m/s toward the locomotive. Use the plus sign in the numerator. Thus

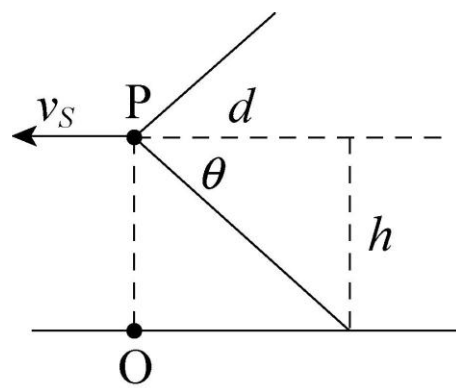
$$f' = f\,\frac{v+v_D}{v+v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s} + 10.00\,\text{m/s}}{343\,\text{m/s} + 20.00\,\text{m/s}}\right) = 486.2\,\text{Hz}.$$

(d) Relative to the air the locomotive is moving at $v_S$ = 20.00 m/s away from the girl and the girl is moving at $v_D$ = 20.00 m/s toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus, $(v + v_D) = (v + v_S)$ and $f' = f$ = 500.0 Hz.

**LEARN** The uncle, standing near the track, hears different frequencies, depending on the direction of the wind. On other hand, since the girl (a detector) is sitting in the train and there's no relative motion between her and the source (locomotive whistle), she hears the same frequency as the source regardless of the wind direction.

68. We note that 1350 km/h is $v_S$ = 375 m/s. Then, with $\theta$ = 60º, Eq. 17-57 gives $v = 3.3\times10^2$ m/s.

69. **THINK** Mach number is the ratio $v_S/v$, where $v_S$ is the speed of the source and $v$ is the sound speed. A mach number of 1.5 means that the jet plane moves at a supersonic speed.




**EXPRESS** The half angle $\theta$ of the Mach cone is given by $\sin\theta = v/v_S$, where $v$ is the speed of sound and $v_S$ is the speed of the plane. To calculate the time it takes for the shock wave to each you after the plane has passed directly overhead, let $h$ be the altitude of the plane and suppose the Mach cone intersects Earth's surface a distance $d$ behind the plane. The situation is shown in the diagram below, with P indicating the plane and O indicating the observer.

The cone angle is related to $h$ and $d$ by $\tan\theta = h/d$, so $d = h/\tan\theta$. The shock wave reaches O in the time the plane takes to fly the distance $d$.

**ANALYZE** (a) Since $v_S = 1.5v$, $\sin\theta = v/1.5v = 1/1.5$. This means $\theta = 42°$.

(b) The time required for the shock wave to reach you is

$$t = \frac{d}{v} = \frac{h}{v\tan\theta} = \frac{5000\text{ m}}{1.5(331\text{ m/s})\tan 42°} = 11\,\text{s}.$$

**LEARN** The shock wave generated by the supersonic jet produces an explosive sound called sonic boom, in which the air pressure first increases suddenly, and then drops suddenly below normal before returning to normal.

70. The altitude $H$ and the horizontal distance $x$ for the legs of a right triangle, so we have

$$H = x\tan\theta = v_p t\tan\theta = 1.25vt\sin\theta$$

where $v$ is the speed of sound, $v_p$ is the speed of the plane, and

$$\theta = \sin^{-1}\left(\frac{v}{v_p}\right) = \sin^{-1}\left(\frac{v}{1.25v}\right) = 53.1°.$$

Thus the altitude is

$$H = x\tan\theta = (1.25)(330\,\text{m/s})(60\,\text{s})(\tan 53.1°) = 3.30\times10^4\ \text{m}.$$

71. The source being a "point source" means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2.$$

From the discussion in Section 17-5, we know that the intensity ratio between "barely audible" and the "painful threshold" is $10^{-12} = I_2/I_1$. Thus, with $r_2 = 10000$ m, we find

$$r_1 = r_2\sqrt{10^{-12}} = 0.01\,\text{m} = 1\ \text{cm}.$$

72. The angle is $\sin^{-1}(v/v_s) = \sin^{-1}(343/685) = 30°$.

73. The round-trip time is $t = 2L/v$, where we estimate from the chart that the time between clicks is 3 ms. Thus, with $v = 1372$ m/s, we find $L = \frac{1}{2}vt = 2.1\ \text{m}$.

74. We use $v = \sqrt{B/\rho}$ to find the bulk modulus $B$:

$$B = v^2\rho = \left(5.4\times10^3\ \text{m/s}\right)^2\left(2.7\times10^3\ \text{kg/m}^3\right) = 7.9\times10^{10}\ \text{Pa}.$$

75. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

(a) With $I_1 = 9.60 \times 10^{-4}$ W/m$^2$, $r_1 = 6.10$ m, and $r_2 = 30.0$ m, we find

$$I_2 = (9.60 \times 10^{-4}\ \text{W/m}^2)(6.10/30.0)^2 = 3.97 \times 10^{-5}\ \text{W/m}^2.$$

(b) Using Eq. 17-27 with $I_1 = 9.60 \times 10^{-4}$ W/m$^2$, $\omega = 2\pi(2000$ Hz), $v = 343$ m/s, and $\rho = 1.21$ kg/m$^3$, we obtain

$$s_m = \sqrt{\frac{2I}{\rho v \omega^2}} = 1.71 \times 10^{-7}\ \text{m}.$$

(c) Equation 17-15 gives the pressure amplitude:

$$\Delta p_m = \rho v \omega s_m = 0.893\ \text{Pa}.$$

76. We use $\Delta\beta_{12} = \beta_1 - \beta_2 = (10\ \text{dB}) \log(I_1/I_2)$.

(a) Since $\Delta\beta_{12} = (10\ \text{dB}) \log(I_1/I_2) = 37$ dB, we get

$$I_1/I_2 = 10^{37\ \text{dB}/10\ \text{dB}} = 10^{3.7} = 5.0 \times 10^3.$$

(b) Since $\Delta p_m \propto s_m \propto \sqrt{I}$, we have $\Delta p_{m1}/\Delta p_{m2} = \sqrt{I_1/I_2} = \sqrt{5.0 \times 10^3} = 71.$

(c) The displacement amplitude ratio is $s_{m1}/s_{m2} = \sqrt{I_1/I_2} = 71.$

77. Any phase changes associated with the reflections themselves are rendered inconsequential by the fact that there is an even number of reflections. The additional path length traveled by wave $A$ consists of the vertical legs in the zig-zag path: $2L$. To be (minimally) out of phase means, therefore, that $2L = \lambda/2$ (corresponding to a half-cycle, or 180°, phase difference). Thus, $L = \lambda/4$, or $L/\lambda = 1/4 = 0.25$.

78. Since they are approaching each other, the sound produced (of emitted frequency $f$) by the flatcar-trumpet received by an observer on the ground will be of higher pitch $f'$. In these terms, we are told $f' - f = 4.0$ Hz, and consequently that $f'/f = 444/440 = 1.0091$. With $v_S$ designating the speed of the flatcar and $v = 343$ m/s being the speed of sound, the Doppler equation leads to

$$\frac{f'}{f} = \frac{v+0}{v-v_S} \Rightarrow v_S = (343\ \text{m/s})\frac{1.0091-1}{1.0091} = 3.1\text{m/s}.$$

79. (a) Incorporating a term ($\lambda$/2) to account for the phase shift upon reflection, then the path difference for the waves (when they come back together) is

$$\sqrt{L^2 + (2d)^2} - L + \lambda/2 = \Delta(\text{path}) .$$

Setting this equal to the condition needed to destructive interference ($\lambda/2$, $3\lambda/2$, $5\lambda/2$ …) leads to $d$ = 0, 2.10 m, … Since the problem explicitly excludes the $d$ = 0 possibility, then our answer is $d$ = 2.10 m.

(b) Setting this equal to the condition needed to constructive interference ($\lambda$, $2\lambda$, $3\lambda$ …) leads to $d$ = 1.47 m, … Our answer is $d$ = 1.47 m.

80. When the source is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = f\left(1 - \frac{v_d}{v}\right),$$

where $v_d$ is the speed of the detector (assumed to be moving away from the source, in the way we've written it, above). The difference between the approach and the recession is

$$f'' - f' = f\left[\left(1 + \frac{v_d}{v}\right) - \left(1 - \frac{v_d}{v}\right)\right] = f\left(2\frac{v_d}{v}\right)$$

which, after setting $(f'' - f')/f = 1/2$, leads to an equation that can be solved for the ratio $v_d/v$. The result is 1/4. Thus, $v_d/v$ = 0.250.

81. **THINK** The pressure amplitude of the sound wave depends on the medium it propagates through.

**EXPRESS** The intensity of a sound wave is given by $I = \frac{1}{2}\rho v \omega^2 s_m^2$, where $\rho$ is the density of the medium, $v$ is the speed of sound, $\omega$ is the angular frequency, and $s_m$ is the displacement amplitude. The displacement and pressure amplitudes are related by $\Delta p_m = \rho v \omega s_m$, so $s_m = \Delta p_m/\rho v \omega$ and $I = (\Delta p_m)^2/2\rho v$. For waves of the same frequency the ratio of the intensity for propagation in water to the intensity for propagation in air is

$$\frac{I_w}{I_a} = \left(\frac{\Delta p_{mw}}{\Delta p_{ma}}\right)^2 \frac{\rho_a v_a}{\rho_w v_w},$$

where the subscript $a$ denotes air and the subscript $w$ denotes water.

**ANALYZE** (a) In case where the intensities are equal, $I_a = I_w$, the ratio of the pressure amplitude is

$$\frac{\Delta p_{mw}}{\Delta p_{ma}} = \sqrt{\frac{\rho_w v_w}{\rho_a v_a}} = \sqrt{\frac{(0.998\times 10^3\ \text{kg/m}^3)(1482\ \text{m/s})}{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}} = 59.7.$$

The speeds of sound are given in Table 17-1 and the densities are given in Table 14-1.

(b) Now, if the pressure amplitudes are equal: $\Delta p_{mw} = \Delta p_{ma}$, then the ratio of the intensities is

$$\frac{I_w}{I_a} = \frac{\rho_a v_a}{\rho_w v_w} = \frac{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}{(0.998\times 10^3\ \text{kg/m}^3)(1482\ \text{m/s})} = 2.81\times 10^{-4}.$$

**LEARN** The pressure amplitude of sound wave and the intensity depend on the density of the medium and the sound speed in the medium.

82. The wave is written as $s(x,t) = s_m \cos(kx \pm \omega t)$.

(a) The amplitude $s_m$ is equal to the maximum displacement: $s_m = 0.30$ cm.

(b) Since λ = 24 cm, the angular wave number is $k = 2\pi/\lambda = 0.26\ \text{cm}^{-1}$.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(25\ \text{Hz}) = 1.6\times 10^2$ rad/s.

(d) The speed of the wave is $v = \lambda f = (24\ \text{cm})(25\ \text{Hz}) = 6.0 \times 10^2$ cm/s.

(e) Since the direction of propagation is $-x$, the sign is plus, so $s(x,t) = s_m \cos(kx + \omega t)$.

83. **THINK** This problem deals with the principle of Doppler ultrasound. The technique can be used to measure blood flow and blood pressure by reflecting high-frequency, ultrasound sound waves off blood cells.

**EXPRESS** The direction of blood flow can be determined by the Doppler shift in frequency. The reception of the ultrasound by the blood and the subsequent remitting of the signal by the blood back toward the detector is a two-step process which may be compactly written as

$$f + \Delta f = f\left(\frac{v + v_x}{v - v_x}\right)$$

where $v_x = v_{\text{blood}} \cos\theta$. If we write the ratio of frequencies as $R = (f + \Delta f)/f$, then the solution of the above equation for the speed of the blood is

$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta}.$$

**ANALYZE** (a) The blood is moving towards the right (towards the detector), because the Doppler shift in frequency is an *increase*: $\Delta f > 0$.

(b) With $v$ = 1540 m/s, $\theta$ = 20°, and $R$ = 1 + (5495 Hz)/(5 × 10$^6$ Hz) =1.0011, using the expression above, we find the speed of the blood to be

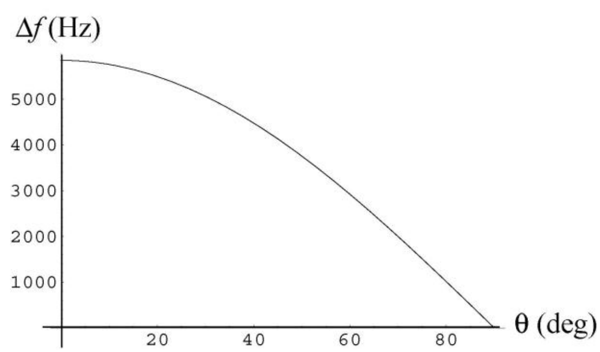
$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta} = 0.90\,\text{m/s}.$$

(c) We interpret the question as asking how $\Delta f$ (still taken to be positive, since the detector is in the "forward" direction) changes as the detection angle $\theta$ changes. Since larger $\theta$ means smaller horizontal component of velocity $v_x$ then we expect $\Delta f$ to decrease towards zero as $\theta$ is increased towards 90°.

**LEARN** The expression for $v_{\text{blood}}$ can be inverted to give

$$\Delta f = \left(\frac{2v_{\text{blood}}\cos\theta}{v - v_{\text{blood}}\cos\theta}\right) f\ .$$

The plot of the frequency shift $\Delta f$ as a function of $\theta$ is given below. Indeed we find $\Delta f$ to decrease with increasing $\theta$.




84. (a) The time it takes for sound to travel in air is $t_a = L/v$, while it takes $t_m = L/v_m$ for the sound to travel in the metal. Thus,

$$\Delta t = t_a - t_m = \frac{L}{v} - \frac{L}{v_m} = \frac{L(v_m - v)}{v_m v}.$$

(b) Using the values indicated (see Table 17-1), we obtain

$$L = \frac{\Delta t}{1/v - 1/v_m} = \frac{1.00\,\text{s}}{1/(343\,\text{m/s})\ -\ 1/(5941\,\text{m/s})} = 364\,\text{m}.$$

85. (a) The period is the reciprocal of the frequency: $T = 1/f = 1/(90 \text{ Hz}) = 1.1 \times 10^{-2}$ s.

(b) Using $v$ = 343 m/s, we find $\lambda = v/f = 3.8$ m.

86. Let $r$ stand for the ratio of the source speed to the speed of sound. Then, Eq. 17-55 (plus the fact that frequency is inversely proportional to wavelength) leads to

$$2\left(\frac{1}{1+r}\right) = \frac{1}{1-r} \ .$$

Solving, we find $r = 1/3$. Thus, $v_s/v = 0.33$.

87. **THINK** The siren is between you and the cliff, moving away from you and towards the cliff. You hear two frequencies, one directly from the siren and the other from the sound reflected off the cliff.

**EXPRESS** The Doppler shifted frequency is given by

$$f' = f\,\frac{v \pm v_D}{v \mp v_S},$$

where $f$ is the unshifted frequency, $v$ is the speed of sound, $v_D$ is the speed of the detector, and $v_S$ is the speed of the source. All speeds are relative to the air. Both "detectors" (you and the cliff) are stationary, so $v_D = 0$ in Eq. 17–47. The source is the siren with $v_S = 10 \text{ m/s}$. The problem asks us to use $v$ = 330 m/s for the speed of sound.

**ANALYZE** (a) With $f$ = 1000 Hz, the frequency $f_y$ you hear becomes

$$f_y = f\left(\frac{v+0}{v+v_S}\right) = (1000 \text{ Hz})\left(\frac{330 \text{ m/s}}{330 \text{ m/s} + 10 \text{ m/s}}\right) = 970.6 \text{ Hz} \approx 9.7\times10^2 \text{ Hz}.$$

(b) The frequency heard by an observer at the cliff (and thus the frequency of the sound reflected by the cliff, ultimately reaching your ears at some distance from the cliff) is

$$f_c = f\left(\frac{v+0}{v-v_S}\right) = (1000 \text{ Hz})\left(\frac{330 \text{ m/s}}{330 \text{ m/s} - 10 \text{ m/s}}\right) = 1031.3 \text{ Hz} \approx 1.0\times10^3 \text{ Hz}.$$

(c) The beat frequency is $f_{\text{beat}} = f_c - f_y$ = 60 beats/s (which, due to specific features of the human ear, is too large to be perceptible).

**LEARN** The beat frequency in this case can be written as

$$f_{\text{beat}} = f_c - f_y = f\left(\frac{v}{v - v_S}\right) - f\left(\frac{v}{v + v_S}\right) = \frac{2vv_S}{v^2 - v_S^2} f$$

Solving for the source speed, we obtain

$$v_S = \left(\frac{-f + \sqrt{f^2 + f_{\text{beat}}^2}}{f_{\text{beat}}}\right) v$$

For the beat frequency to be perceptible ( $f_{\text{beat}} < 20\text{ Hz}$ ), the source speed would have to be less than 3.3 m/s.

88. When $\phi = 0$ it is clear that the superposition wave has amplitude $2\Delta p_m$. For the other cases, it is useful to write

$$\Delta p_1 + \Delta p_2 = \Delta p_m \left(\sin\left(\omega t\right) + \sin\left(\omega t - \phi\right)\right) = \left(2\Delta p_m \cos\frac{\phi}{2}\right)\sin\left(\omega t - \frac{\phi}{2}\right).$$

The factor in front of the sine function gives the amplitude $\Delta p_r$. Thus, $\Delta p_r / \Delta p_m = 2\cos(\phi/2)$.

(a) When $\phi = 0$, $\Delta p_r / \Delta p_m = 2\cos(0) = 2.00$.

(b) When $\phi = \pi/2$, $\Delta p_r / \Delta p_m = 2\cos(\pi/4) = \sqrt{2} = 1.41$.

(c) When $\phi = \pi/3$, $\Delta p_r / \Delta p_m = 2\cos(\pi/6) = \sqrt{3} = 1.73$.

(d) When $\phi = \pi/4$, $\Delta p_r / \Delta p_m = 2\cos(\pi/8) = 1.85$.

89. (a) Adapting Eq. 17-39 to the notation of this chapter, we have

$$s_m' = 2\, s_m \cos(\phi/2) = 2(12\text{ nm})\cos(\pi/6) = 20.78\text{ nm}.$$

Thus, the amplitude of the resultant wave is roughly 21 nm.

(b) The wavelength ($\lambda$ = 35 cm) does not change as a result of the superposition.

(c) Recalling Eq. 17-47 (and the accompanying discussion) from the previous chapter, we conclude that the standing wave amplitude is 2(12 nm) = 24 nm when they are traveling in opposite directions.

(d) Again, the wavelength ($\lambda$ = 35 cm) does not change as a result of the superposition.

90. (a) The separation distance between points *A* and *B* is one-quarter of a wavelength; therefore, $\lambda$ = 4(0.15 m) = 0.60 m. The frequency, then, is

$$f = v/\lambda = (343 \text{ m/s})/(0.60 \text{ m}) = 572 \text{ Hz}.$$

(b) The separation distance between points *C* and *D* is one-half of a wavelength; therefore, $\lambda$ = 2(0.15 m) = 0.30 m. The frequency, then, is

$$f = v/\lambda = (343 \text{ m/s})/(0.30 \text{ m}) = 1144 \text{ Hz},$$

or approximately 1.14 kHz.

91. Let the frequencies of sound heard by the person from the left and right forks be $f_l$ and $f_r$, respectively.

92. If the speeds of both forks are $u$, then $f_{l,r} = fv/(v \pm u)$ and

$$f_{\text{beat}} = |f_r - f_l| = fv\left(\frac{1}{v-u} - \frac{1}{v+u}\right) = \frac{2fuv}{v^2 - u^2} = \frac{2(440\,\text{Hz})(3.00\,\text{m/s})(343\,\text{m/s})}{(343\,\text{m/s})^2 - (3.00\,\text{m/s})^2} = 7.70\,\text{Hz}.$$

(b) If the speed of the listener is $u$, then $f_{l,r} = f(v \pm u)/v$ and

$$f_{\text{beat}} = |f_l - f_r| = 2f\left(\frac{u}{v}\right) = 2(440\,\text{Hz})\left(\frac{3.00\,\text{m/s}}{343\,\text{m/s}}\right) = 7.70\,\text{Hz}.$$

92. The rule: if you divide the time (in seconds) by 3, then you get (approximately) the straight-line distance *d*. We note that the speed of sound we are to use is given at the beginning of the problem section in the textbook, and that the speed of light is very much larger than the speed of sound. The proof of our rule is as follows:

$$t = t_{\text{sound}} - t_{\text{light}} \approx t_{\text{sound}} = \frac{d}{v_{\text{sound}}} = \frac{d}{343\,\text{m/s}} = \frac{d}{0.343\,\text{km/s}}.$$

Cross-multiplying yields (approximately) (0.3 km/s)$t$ = $d$, which (since 1/3 $\approx$ 0.3) demonstrates why the rule works fairly well.

93. **THINK** Acoustic interferometer can be used to demonstrate the interference of sound waves.

**EXPRESS** When the right side of the instrument is pulled out a distance *d* the path length for sound waves increases by 2*d*. Since the interference pattern changes from a minimum to the next maximum, this distance must be half a wavelength of the sound. So $2d = \lambda/2$, where $\lambda$ is the wavelength. Thus $\lambda = 4d$.

On the other hand, the intensity is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where $\rho$ is the density of the medium, $v$ is the speed of sound, $\omega$ is the angular frequency, and $s_m$ is the displacement amplitude. Thus, $s_m$ is proportional to the square root of the intensity, and we write $\sqrt{I} = Cs_m$, where $C$ is a constant of proportionality. At the minimum, interference is destructive and the displacement amplitude is the difference in the amplitudes of the individual waves: $s_m = s_{SAD} - s_{SBD}$, where the subscripts indicate the paths of the waves. At the maximum, the waves interfere constructively and the displacement amplitude is the sum of the amplitudes of the individual waves: $s_m = s_{SAD} + s_{SBD}$.

**ANALYZE** (a) The speed of sound is $v$ = 343 m/s, so the frequency is

$$f = v/\lambda = v/4d = (343 \text{ m/s})/4(0.0165 \text{ m}) = 5.2 \times 10^3 \text{ Hz}.$$

(b) At intensity minimum, we have $\sqrt{100} = C(s_{SAD} - s_{SBD})$, and $\sqrt{900} = C(s_{SAD} + s_{SBD})$ at the maximum. Adding the equations give

$$s_{SAD} = (\sqrt{100} + \sqrt{900} / 2C = 20/C,$$

while subtracting them yields

$$s_{SBD} = (\sqrt{900} - \sqrt{100}) / 2C = 10/C.$$

Thus, the ratio of the amplitudes is $s_{SAD}/s_{SBD} = 2$.

(c) Any energy losses, such as might be caused by frictional forces of the walls on the air in the tubes, result in a decrease in the displacement amplitude. Those losses are greater on path B since it is longer than path A.

**LEARN** We see that the sound waves propagated along the two paths in the interferometer can interfere constructively or destructively, depending on their path length difference.

94. (a) Using $m = 7.3 \times 10^7$ kg, the initial gravitational potential energy is $U = mgy = 3.9 \times 10^{11}$ J, where $h$ = 550 m. Assuming this converts primarily into kinetic energy during the fall, then $K = 3.9 \times 10^{11}$ J just before impact with the ground. Using instead the mass estimate $m = 1.7 \times 10^8$ kg, we arrive at $K = 9.2 \times 10^{11}$ J.

(b) The process of converting this kinetic energy into other forms of energy (during the impact with the ground) is assumed to take $\Delta t$ = 0.50 s (and in the average sense, we take the "power" $P$ to be wave-energy/$\Delta t$). With 20% of the energy going into creating a seismic wave, the intensity of the body wave is estimated to be

$$I = \frac{P}{A_{\text{hemisphere}}} = \frac{(0.20)K/\Delta t}{\frac{1}{2}(4\pi r^2)} = 0.63\,\text{W/m}^2$$

using $r = 200 \times 10^3$ m and the smaller value for $K$ from part (a). Using instead the larger estimate for $K$, we obtain $I = 1.5$ W/m$^2$.

(c) The surface area of a cylinder of "height" $d$ is $2\pi rd$, so the intensity of the surface wave is

$$I = \frac{P}{A_{\text{cylinder}}} = \frac{(0.20)K/\Delta t}{(2\pi rd)} = 25\times 10^3\ \text{W/m}^2$$

using $d = 5.0$ m, $r = 200 \times 10^3$ m, and the smaller value for $K$ from part (a). Using instead the larger estimate for $K$, we obtain $I = 58$ kW/m$^2$.

(d) Although several factors are involved in determining which seismic waves are most likely to be detected, we observe that on the basis of the above findings we should expect the more intense waves (the surface waves) to be more readily detected.

95. **THINK** Intensity is power divided by area. For an isotropic source the area is the surface area of a sphere.

**EXPRESS** If $P$ is the power output and $I$ is the intensity a distance $r$ from the source, then $P = IA = 4\pi r^2 I$, where $A = 4\pi r^2$ is the surface area of a sphere of radius $r$. On the other hand, the sound level $\beta$ can be calculated using Eq. 17-29:

$$\beta = (10\ \text{dB})\log\frac{I}{I_0}$$

where $I_0 = 10^{-12}\ \text{W/m}^2$ is the standard reference intensity.

**ANALYZE** (a) With $r = 10$ m and $I = 8.0\times 10^{-3}\ \text{W/m}^2$, we have

$$P = 4\pi r^2 I = 4\pi(10)^2(8.0\times 10^{-3}\ \text{W/m}^2) = 10\,\text{W}.$$

(b) Using the value of $P$ obtained in (a), we find the intensity at $r' = 5.0$ m to be

$$I' = \frac{P}{4\pi r'^2} = \frac{10\ \text{W}}{4\pi(5.0\ \text{m})^2} = 0.032\ \text{W/m}^2.$$

(c) Using Eq. 17–29 with $I = 0.0080$ W/m$^2$, we find the sound level to be

$$\beta = (10\ \text{dB})\log\left(\frac{8.0\times 10^{-3}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = 99\,\text{dB}.$$

**LEARN** The ratio of the sound intensities at two different locations can be written as

$$\frac{I}{I'} = \frac{P/4\pi r^2}{P/4\pi r'^2} = \left(\frac{r'}{r}\right)^2 .$$

Similarly, the difference in sound level is given by $\Delta\beta = \beta - \beta' = (10\text{ dB})\log\left(\frac{I}{I'}\right)$.

96. We note that waves 1 and 3 differ in phase by $\pi$ radians (so they cancel upon superposition). Waves 2 and 4 also differ in phase by $\pi$ radians (and also cancel upon superposition). Consequently, there is no resultant wave.

97. Since they oscillate out of phase, then their waves will cancel (producing a node) at a point exactly midway between them (the midpoint of the system, where we choose $x = 0$). We note that Figure 17-13, and the $n = 3$ case of Figure 17-14(a) have this property (of a node at the midpoint). The distance $\Delta x$ between nodes is $\lambda/2$, where $\lambda = v/f$ and $f = 300$ Hz and $v = 343$ m/s. Thus, $\Delta x = v/2f = 0.572$ m.

Therefore, nodes are found at the following positions:

$$x = n\Delta x = n(0.572\text{ m}),\ \ n = 0, \pm 1, \pm 2, ...$$

(a) The shortest distance from the midpoint where nodes are found is $\Delta x = 0$.

(b) The second shortest distance from the midpoint where nodes are found is $\Delta x$ =0.572 m.

(c) The third shortest distance from the midpoint where nodes are found is $2\Delta x = 1.14$ m.

98. (a) With $f = 686$ Hz and $v = 343$ m/s, then the "separation between adjacent wavefronts" is $\lambda = v/f = 0.50$ m.

(b) This is one of the effects that are part of the Doppler phenomena. Here, the wavelength shift (relative to its "true" value in part (a)) equals the source speed $v_s$ (with appropriate ± sign) relative to the speed of sound $v$:

$$\frac{\Delta\lambda}{\lambda} = \pm\frac{v_s}{v} .$$

In front of the source, the shift in wavelength is –(0.50 m)(110 m/s)/(343 m/s) = –0.16 m, and the wavefront separation is 0.50 m – 0.16 m = 0.34 m.

(c) Behind the source, the shift in wavelength is +(0.50 m)(110 m/s)/(343 m/s) = +0.16 m, and the wavefront separation is 0.50 m + 0.16 m = 0.66 m.

99. We use $I \propto r^{-2}$ appropriate for an isotropic source. We have

$$\frac{I_{r=d}}{I_{r=D-d}} = \frac{(D-d)^2}{D^2} = \frac{1}{2},$$

where $d$ = 50.0 m. We solve for

$$D: D = \sqrt{2}d/\left(\sqrt{2}-1\right) = \sqrt{2}\,(50.0\,\text{m})/\left(\sqrt{2}-1\right) = 171\,\text{m}.$$

100. Pipe $A$ (which can only support odd harmonics – see Eq. 17-41) has length $L_A$. Pipe $B$ (which supports both odd and even harmonics [any value of $n$] – see Eq. 17-39) has length $L_B = 4L_A$ . Taking ratios of these equations leads to the condition:

$$\left(\frac{n}{2}\right)_B = (n_{\text{odd}})_A \ .$$

Solving for $n_B$ we have $n_B = 2n_{\text{odd}}$.

(a) Thus, the smallest value of $n_B$ at which a harmonic frequency of $B$ matches that of $A$ is $n_B = 2(1) = 2$.

(b) The second smallest value of $n_B$ at which a harmonic frequency of $B$ matches that of $A$ is $n_B = 2(3) = 6$.

(c) The third smallest value of $n_B$ at which a harmonic frequency of $B$ matches that of $A$ is $n_B = 2(5) = 10$.

101. (a) We observe that "third lowest … frequency" corresponds to harmonic number $n$ = 5 for such a system. Using Eq. 17-41, we have

$$f = \frac{nv}{4L} \quad \Rightarrow \quad 750\,\text{Hz} = \frac{5v}{4(0.60\text{ m})}$$

so that $v = 3.6\times10^2$ m/s.

(b) As noted, $n$ = 5; therefore, $f_1$ = 750/5 = 150 Hz.

102. (a) Let $P$ be the power output of the source. This is the rate at which energy crosses the surface of any sphere centered at the source and is therefore equal to the product of the intensity $I$ at the sphere surface and the area of the sphere. For a sphere of radius $r$, $P = 4\pi r^2 I$ and $I = P/4\pi r^2$. The intensity is proportional to the square of the displacement amplitude $s_m$. If we write $I = Cs_m^2$ , where $C$ is a constant of proportionality, then $Cs_m^2 = P/4\pi r^2$ . Thus,

$$s_m = \sqrt{P/4\pi r^2 C} = \left(\sqrt{P/4\pi C}\right)(1/r).$$

The displacement amplitude is proportional to the reciprocal of the distance from the source. We take the wave to be sinusoidal. It travels radially outward from the source, with points on a sphere of radius $r$ in phase. If $\omega$ is the angular frequency and $k$ is the angular wave number, then the time dependence is sin($kr - \omega t$). Letting $b = \sqrt{P/4\pi C}$, the displacement wave is then given by

$$s(r,t) = \sqrt{\frac{P}{4\pi C}}\frac{1}{r}\sin(kr - \omega t) = \frac{b}{r}\sin(kr - \omega t).$$

(b) Since $s$ and $r$ both have dimensions of length and the trigonometric function is dimensionless, the dimensions of $b$ must be length squared.

103. Using Eq. 17-47 with great care (regarding its ± sign conventions), we have

$$f' = (440\text{ Hz})\left(\frac{340\text{ m/s } - 80.0\text{ m/s}}{340\text{ m/s} - 54.0\text{ m/s}}\right) = 400\text{ Hz}.$$

104. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$. Since intensity is proportional to the square of the amplitude (see Eq. 17-27), this further implies

$$\frac{I_2}{I_1} = \left(\frac{s_{m2}}{s_{m1}}\right)^2 = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2$$

or $s_{m2}/s_{m1} = r_1/r_2$.

(a) $I = P/4\pi r^2 = (10\text{ W})/4\pi(3.0\text{ m})^2 = 0.088\text{ W/m}^2$.

(b) Using the notation $A$ instead of $s_m$ for the amplitude, we find

$$\frac{A_4}{A_3} = \frac{3.0\,\text{m}}{4.0\,\text{m}} = 0.75.$$

105. (a) The problem is asking at how many angles will there be "loud" resultant waves, and at how many will there be "quiet" ones? We consider the resultant wave (at large distance from the origin) along the $+x$ axis; we note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value $n$ = 3.2, implying a sort of intermediate condition between constructive interference (which would follow if, say, $n$ = 3) and destructive interference (such as the $n$ = 3.5 situation found in the solution to the previous problem) between the waves. To distinguish this resultant along the $+x$ axis from the similar one along the $-x$ axis, we label one with $n$ = +3.2 and the other $n$ = –3.2. This labeling facilitates the complete enumeration of the loud directions in the upper-half plane: $n$ = –3, –2, –1, 0, +1,

+2, +3. Counting also the "other" –3, –2, –1, 0, +1, +2, +3 values for the *lower*-half plane, then we conclude there are a total of 7 + 7 = 14 "loud" directions.

(b) The labeling also helps us enumerate the quiet directions. In the upper-half plane we find: $n =$ –2.5, –1.5, –0.5, +0.5, +1.5, +2.5. This is duplicated in the lower half plane, so the total number of quiet directions is 6 + 6 = 12.

106. We are combining two effects: the reception by a moving target with speed $u$ of waves emitted by the stationary transmitter/detector, and the subsequent emission of those waves by the moving target, which are picked up by the stationary transmitter/detector. The first step gives

$$f_S' = f_s \frac{v+u}{v}$$

and the second step leads to

$$f_r = f_S' \frac{v}{v-u} = f_s \frac{v+u}{v} \cdot \frac{v}{v-u} = f_s \left(\frac{v+u}{v-u}\right)$$

Solving for $u$, we get

$$u = \left(\frac{f_r - f_s}{f_r + f_s}\right) v = \left(\frac{22.2\text{ kHz} - 18.0\text{ kHz}}{22.2\text{ kHz} + 18.0\text{ kHz}}\right)(343\text{ m/s}) = 35.84\text{ m/s}$$

107. The cork fillings are collected at the pressure anti-nodes when the standing waves are set up. The anti-nodes are separated by half a wavelength, $d = \lambda/2$. Thus, the speed of the sound in the gas is

$$v = f\lambda = f(2d) = 2fd = 2(4.46\times10^3\text{Hz})(0.0920\text{ m}) = 821\text{ m/s}$$

108. When the layer is at height $H$, a constructive interference implies that the path length difference must be an integer multiple of the wavelength:

$$n\lambda = L_1 - d = 2\sqrt{H^2 + (d/2)^2} - d = \sqrt{4H^2 + d^2} - d$$

On the other hand, when the layer is at height $H + h$, a destructive interference implies that the path length difference must be an odd multiple of half the wavelength:

$$\left(n + \frac{1}{2}\right)\lambda = L_2 - d = 2\sqrt{(H+h)^2 + (d/2)^2} - d = \sqrt{4(H+h)^2 + d^2} - d$$

Subtracting the first equation from the second, we obtain

$$\frac{1}{2}\lambda = \sqrt{4(H+h)^2 + d^2} - \sqrt{4H^2 + d^2}$$

or

$$\lambda = 2\left(\sqrt{4(H+h)^2+d^2}-\sqrt{4H^2+d^2}\right).$$

109. The difference between the sound waves that travel along $R_1$ and thus that bounce and travel along $R_2$ is

$$\Delta d = \sqrt{25.0^2 + 12.5^2} - \sqrt{20.0^2 + 12.5^2} + \tfrac{1}{2}\lambda$$

where the last term is included for the reflection effect (mentioned in the problem). To produce constructive interference at $D$ then we require $\Delta d = m\lambda$ where $m$ is an integer. Since $\lambda$ relates to frequency by the relation $\lambda = v/f$ (with $v$ = 343 m/s) then we have an equation for a set of values (depending on $m$) for the frequency. We find

$$\begin{aligned} f &= 39.3\text{ Hz} \quad \text{for} \quad m = 1 \\ f &= 118\text{ Hz} \quad \text{for} \quad m = 2 \\ f &= 196\text{ Hz} \quad \text{for} \quad m = 3 \\ f &= 275\text{ Hz} \quad \text{for} \quad m = 4 \end{aligned}$$

and so on.

(a) The lowest frequency is $f$ = 39.3 Hz.

(b) The second lowest frequency is $f$ = 118 Hz.

110. (a) Since the source is moving toward the wall, the frequency of the sound as received at the wall is

$$f' = f\left(\frac{v}{v-v_S}\right) = (440\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} - 20.0\,\text{m/s}}\right) = 467\,\text{Hz}.$$

(b) Since the person is moving with a speed $u$ toward the reflected sound with frequency $f'$, the frequency registered at the source is

$$f_r = f'\left(\frac{v+u}{v}\right) = (467\,\text{Hz})\left(\frac{343\,\text{m/s} + 20.0\,\text{m/s}}{343\,\text{m/s}}\right) = 494\,\text{Hz}.$$

111. We find the difference in the two applications of the Doppler formula:

$$f_2 - f_1 = 37\text{ Hz} = f\left(\frac{340\text{ m/s} + 25\text{ m/s}}{340\,\text{m/s} - 15\text{ m/s}} - \frac{340\text{ m/s}}{340\text{ m/s} - 15\text{ m/s}}\right) = f\left(\frac{25\text{ m/s}}{340\text{ m/s} - 15\text{ m/s}}\right)$$

which leads to $f = 4.8\times10^2\text{ Hz}$.

# Chapter 18

1. From Eq. 18-6, we see that the limiting value of the pressure ratio is the same as the absolute temperature ratio: (373.15 K)/(273.16 K) = 1.366.

2. We take $p_3$ to be 80 kPa for both thermometers. According to Fig. 18-6, the nitrogen thermometer gives 373.35 K for the boiling point of water. Use Eq. 18-5 to compute the pressure:

$$p_N = \frac{T}{273.16\,\text{K}}\,p_3 = \left(\frac{373.35\,\text{K}}{273.16\,\text{K}}\right)(80\,\text{kPa}) = 109.343\,\text{kPa}.$$

The hydrogen thermometer gives 373.16 K for the boiling point of water and

$$p_H = \left(\frac{373.16\,\text{K}}{273.16\,\text{K}}\right)(80\,\text{kPa}) = 109.287\,\text{kPa}.$$

(a) The difference is $p_N - p_H = 0.056$ kPa $\approx 0.06$ kPa.

(b) The pressure in the nitrogen thermometer is higher than the pressure in the hydrogen thermometer.

3. Let $T_L$ be the temperature and $p_L$ be the pressure in the left-hand thermometer. Similarly, let $T_R$ be the temperature and $p_R$ be the pressure in the right-hand thermometer. According to the problem statement, the pressure is the same in the two thermometers when they are both at the triple point of water. We take this pressure to be $p_3$. Writing Eq. 18-5 for each thermometer,

$$T_L = (273.16\,\text{K})\left(\frac{p_L}{p_3}\right) \quad \text{and} \quad T_R = (273.16\,\text{K})\left(\frac{p_R}{p_3}\right),$$

we subtract the second equation from the first to obtain

$$T_L - T_R = (273.16\,\text{K})\left(\frac{p_L - p_R}{p_3}\right).$$

First, we take $T_L$ = 373.125 K (the boiling point of water) and $T_R$ = 273.16 K (the triple point of water). Then, $p_L - p_R$ = 120 torr. We solve

$$373.125\,\text{K} - 273.16\,\text{K} = (273.16\,\text{K})\left(\frac{120\,\text{torr}}{p_3}\right)$$

for $p_3$. The result is $p_3$ = 328 torr. Now, we let $T_L$ = 273.16 K (the triple point of water) and $T_R$ be the unknown temperature. The pressure difference is $p_L - p_R$ = 90.0 torr. Solving the equation

$$273.16\,\text{K} - T_R = (273.16\,\text{K})\left(\frac{90.0\,\text{torr}}{328\,\text{torr}}\right)$$

for the unknown temperature, we obtain $T_R$ = 348 K.

4. (a) Let the reading on the Celsius scale be $x$ and the reading on the Fahrenheit scale be $y$. Then $y = \frac{9}{5}x + 32$. For $x$ = –71°C, this gives $y$ = –96°F.

(b) The relationship between $y$ and $x$ may be inverted to yield $x = \frac{5}{9}(y - 32)$. Thus, for $y$ = 134 we find $x \approx 56.7$ on the Celsius scale.

5. (a) Let the reading on the Celsius scale be $x$ and the reading on the Fahrenheit scale be $y$. Then $y = \frac{9}{5}x + 32$. If we require $y = 2x$, then we have

$$2x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = (5)(32) = 160°\text{C}$$

which yields $y = 2x$ = 320°F.

(b) In this case, we require $y = \frac{1}{2}x$ and find

$$\frac{1}{2}x = \frac{9}{5}x + 32 \quad \Rightarrow \quad x = -\frac{(10)(32)}{13} \approx -24.6°\text{C}$$

which yields $y = x/2$ = –12.3°F.

6. We assume scales X and Y are linearly related in the sense that reading $x$ is related to reading $y$ by a linear relationship $y = mx + b$. We determine the constants $m$ and $b$ by solving the simultaneous equations:

$$-70.00 = m(-125.0) + b$$
$$-30.00 = m(375.0) + b$$

which yield the solutions $m$ = 40.00/500.0 = $8.000 \times 10^{-2}$ and $b$ = –60.00. With these values, we find $x$ for $y$ = 50.00:

$$x = \frac{y - b}{m} = \frac{50.00 + 60.00}{0.08000} = 1375°\text{X}.$$

7. We assume scale X is a linear scale in the sense that if its reading is $x$ then it is related to a reading $y$ on the Kelvin scale by a linear relationship $y = mx + b$. We determine the constants $m$ and $b$ by solving the simultaneous equations:

$$\begin{aligned} 373.15 &= m(-53.5) + b \\ 273.15 &= m(-170) + b \end{aligned}$$

which yield the solutions $m = 100/(170 - 53.5) = 0.858$ and $b = 419$. With these values, we find $x$ for $y = 340$:

$$x = \frac{y-b}{m} = \frac{340-419}{0.858} = -92.1°\text{X}.$$

8. The increase in the surface area of the brass cube (which has six faces), which had side length $L$ at 20°, is

$$\begin{aligned} \Delta A &= 6(L+\Delta L)^2 - 6L^2 \approx 12L\Delta L = 12\alpha_{\text{b}} L^2 \Delta T = 12\ (19\times10^{-6}/\text{C°})\ (30\,\text{cm})^2(75°\text{C} - 20°\text{C}) \\ &= 11\,\text{cm}^2. \end{aligned}$$

9. The new diameter is

$$D = D_0(1+\alpha_{Al}\Delta T) = (2.725\,\text{cm})[1+(23\times10^{-6}/\text{C°})(100.0°\text{C} - 0.000°\text{C})] = 2.731\,\text{cm}.$$

10. The change in length for the aluminum pole is

$$\Delta\ell = \ell_0 \alpha_{Al}\Delta T = (33\,\text{m})(23\times10^{-6}/\text{C°})(15\ °\text{C}) = 0.011\,\text{m}.$$

11. The volume at 30°C is given by

$$\begin{aligned} V' &= V(1+\beta\Delta T) = V(1+3\alpha\Delta T) = (50.00\,\text{cm}^3)[1+3(29.00\times10^{-6}/\text{C°})\ (30.00°\text{C} - 60.00°\text{C})] \\ &= 49.87\,\text{cm}^3 \end{aligned}$$

where we have used $\beta = 3\alpha$.

12. (a) The coefficient of linear expansion $\alpha$ for the alloy is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{10.015\,\text{cm} - 10.000\,\text{cm}}{(10.01\,\text{cm})(100°\text{C} - 20.000°\text{C})} = 1.88\times10^{-5}/\text{C°}.$$

Thus, from 100°C to 0°C we have

$$\Delta L = L\alpha\Delta T = (10.015\,\text{cm})(1.88\times10^{-5}/\text{C°})(0°\text{C} - 100°\text{C}) = -1.88\ \times\ 10^{-2}\,\text{cm}.$$

The length at 0°C is therefore $L' = L + \Delta L = (10.015\text{ cm} - 0.0188\text{ cm}) = 9.996\text{ cm}$.

(b) Let the temperature be $T_x$. Then from 20°C to $T_x$ we have

$$\Delta L = 10.009\,\text{cm} - 10.000\,\text{cm} = \alpha L \Delta T = (1.88\times10^{-5}/\text{C}°)(10.000\,\text{cm})\,\Delta T,$$

giving $\Delta T$ = 48 °C. Thus, $T_x$ = (20°C + 48 °C ) = 68°C.

13. **THINK** The aluminum sphere expands thermally when being heated, so its volume increases.

**EXPRESS** Since a volume is the product of three lengths, the change in volume due to a temperature change $\Delta T$ is given by $\Delta V = 3\alpha V\,\Delta T$, where $V$ is the original volume and $\alpha$ is the coefficient of linear expansion (see Eq. 18-11).

**ANALYZE** With the volume of the sphere given by $V = (4\pi/3)R^3$, where $R$= 10 cm is the original radius of the sphere and $\alpha = 23\times10^{-6}/\text{C}°$, then

$$\Delta V = 3\alpha\left(\frac{4\pi}{3}R^3\right)\Delta T = \left(23\times10^{-6}/\text{C}°\right)(4\pi)(10\,\text{cm})^3(100\,°\text{C}) = 29\,\text{cm}^3.$$

The value for the coefficient of linear expansion is found in Table 18-2.

**LEARN** The change in volume can be expressed as $\Delta V/V = \beta\Delta T$, where $\beta = 3\alpha$ is the coefficient of volume expansion. For aluminum, we have $\beta = 3\alpha = 69\times10^{-6}/\text{C}°$.

14. (a) Since $A = \pi D^2/4$, we have the differential $dA = 2(\pi D/4)dD$. Dividing the latter relation by the former, we obtain $dA/A = 2\,dD/D$. In terms of Δ's, this reads

$$\frac{\Delta A}{A} = 2\frac{\Delta D}{D} \quad \text{for} \quad \frac{\Delta D}{D} \ll 1.$$

We can think of the factor of 2 as being due to the fact that area is a two-dimensional quantity. Therefore, the area increases by 2(0.18%) = 0.36%.

(b) Assuming that all dimensions are allowed to freely expand, then the thickness increases by 0.18%.

(c) The volume (a three-dimensional quantity) increases by 3(0.18%) = 0.54%.

(d) The mass does not change.

(e) The coefficient of linear expansion is

$$\alpha = \frac{\Delta D}{D\Delta T} = \frac{0.18\times10^{-2}}{100°\text{C}} = 1.8\times10^{-5}/\text{C}°.$$

15. After the change in temperature the diameter of the steel rod is $D_s = D_{s0} + \alpha_s D_{s0}\,\Delta T$ and the diameter of the brass ring is $D_b = D_{b0} + \alpha_b D_{b0}\,\Delta T$, where $D_{s0}$ and $D_{b0}$ are the original diameters, $\alpha_s$ and $\alpha_b$ are the coefficients of linear expansion, and $\Delta T$ is the change in temperature. The rod just fits through the ring if $D_s = D_b$. This means

$$D_{s0} + \alpha_s D_{s0}\,\Delta T = D_{b0} + \alpha_b D_{b0}\,\Delta T.$$

Therefore,

$$\Delta T = \frac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}} = \frac{3.000\,\text{cm} - 2.992\,\text{cm}}{\left(19.00\ \times 10^{-6}/\text{C}°\right)\left(2.992\,\text{cm}\right) - \left(11.00\times 10^{-6}/\text{C}°\right)\left(3.000\,\text{cm}\right)}$$
$$= 335.0\,°\text{C}.$$

The temperature is $T$ = (25.00°C + 335.0 °C) = 360.0°C.

16. (a) We use $\rho = m/V$ and

$$\Delta\rho = \Delta(m/V) = m\Delta(1/V) \simeq -m\Delta V/V^2 = -\rho(\Delta V/V) = -3\rho(\Delta L/L).$$

The percent change in density is

$$\frac{\Delta\rho}{\rho} = -3\frac{\Delta L}{L} = -3(0.23\%) = -0.69\%.$$

(b) Since $\alpha = \Delta L/(L\Delta T\,) = (0.23 \times 10^{-2}) / (100°\text{C} - 0.0°\text{C}) = 23 \times 10^{-6}$ /C°, the metal is aluminum (using Table 18-2).

17. **THINK** Since the aluminum cup and the glycerin have different coefficients of thermal expansion, their volumes would change by a different amount under the same $\Delta T$.

**EXPRESS** If $V_c$ is the original volume of the cup, $\alpha_a$ is the coefficient of linear expansion of aluminum, and $\Delta T$ is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a\, V_c\,\Delta T$ (See Eq. 18-11).

On the other hand, if $\beta$ is the coefficient of volume expansion for glycerin, then the change in the volume of glycerin is $\Delta V_g = \beta V_c\,\Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\Delta V_g - \Delta V_c = \left(\beta - 3\alpha_a\right)V_c\Delta T = \left[\left(5.1\times 10^{-4}/\text{C}°\right) - 3\left(23\times 10^{-6}/\text{C}°\right)\right]\left(100\,\text{cm}^3\right)\left(6.0\,°\text{C}\right)$$
$$= 0.26\,\text{cm}^3.$$

**LEARN** Glycerin spills over because $\beta > 3\alpha$, which gives $\Delta V_g - \Delta V_c > 0$. Note that since liquids in general have greater coefficients of thermal expansion than solids, heating a cup filled with liquid generally will cause the liquid to spill out.

18. The change in length for the section of the steel ruler between its 20.05 cm mark and 20.11 cm mark is

$$\Delta L_s = L_s \alpha_s \Delta T = (20.11\text{cm})(11\times10^{-6}/\text{C}°)(270°\text{C} - 20°\text{C}) = 0.055\text{cm}.$$

Thus, the actual change in length for the rod is

$$\Delta L = (20.11 \text{ cm} - 20.05 \text{ cm}) + 0.055 \text{ cm} = 0.115 \text{ cm}.$$

The coefficient of thermal expansion for the material of which the rod is made is then

$$\alpha = \frac{\Delta L}{\Delta T} = \frac{0.115 \text{ cm}}{270°\text{C} - 20°\text{C}} = 23\times10^{-6}/\text{C}°.$$

19. The initial volume $V_0$ of the liquid is $h_0A_0$ where $A_0$ is the initial cross-section area and $h_0$ = 0.64 m. Its final volume is $V = hA$ where $h - h_0$ is what we wish to compute. Now, the area expands according to how the glass expands, which we analyze as follows. Using $A = \pi r^2$, we obtain

$$dA = 2\pi r\, dr = 2\pi r\left(r\alpha\, dT\right) = 2\alpha(\pi r^2)dT = 2\alpha A\, dT.$$

Therefore, the height is

$$h = \frac{V}{A} = \frac{V_0\left(1+\beta_{\text{liquid}}\Delta T\right)}{A_0\left(1+2\alpha_{\text{glass}}\Delta T\right)}.$$

Thus, with $V_0/A_0 = h_0$ we obtain

$$h - h_0 = h_0\left(\frac{1+\beta_{\text{liquid}}\Delta T}{1+2\alpha_{\text{glass}}\Delta T} - 1\right) = (0.64)\left(\frac{1+\left(4\times10^{-5}\right)\left(10°\right)}{1+2\left(1\times10^{-5}\right)\left(10°\right)}\right) = 1.3\times10^{-4} \text{ m}.$$

20. We divide Eq. 18-9 by the time increment $\Delta t$ and equate it to the (constant) speed $v$ = 100 × $10^{-9}$ m/s.

$$v = \alpha L_0 \frac{\Delta T}{\Delta t}$$

where $L_0$ = 0.0200 m and $\alpha$ = 23 × $10^{-6}$/C°. Thus, we obtain

$$\frac{\Delta T}{\Delta t} = 0.217\frac{\text{C}°}{\text{s}} = 0.217\frac{\text{K}}{\text{s}}.$$

21. **THINK** The bar expands thermally when heated. Since its two ends are held fixed, the bar buckles upward.

**EXPRESS** Consider half the bar. Its original length is $\ell_0 = L_0/2$ and its length after the temperature increase is $\ell = \ell_0 + \alpha\ell_0\Delta T$. The old position of the half-bar, its new position, and the distance *x* that one end is displaced form a right triangle, with a hypotenuse of length $\ell$, one side of length $\ell_0$, and the other side of length *x*. The Pythagorean theorem yields

$$x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1+\alpha\Delta T)^2 - \ell_0^2.$$

Since the change in length is small we may approximate $(1 + \alpha\,\Delta T)^2$ by $1 + 2\alpha\,\Delta T$, where the small term $(\alpha\,\Delta T\,)^2$ was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2\alpha\,\Delta T - \ell_0^2 = 2\ell_0^2\alpha\,\Delta T$$

and $x \approx \ell_0\sqrt{2\alpha\,\Delta T}$.

**ANALYZE** Substituting the values given, we obtain

$$x = \ell_0\sqrt{2\alpha\,\Delta T} = \frac{3.77\,\text{m}}{2}\sqrt{2\left(25\times10^{-6}/\text{C}^\circ\right)\left(32^\circ\,\text{C}\right)} = 7.5\times10^{-2}\,\text{m}.$$

**LEARN** The length of the bar changes by $\Delta\ell = \alpha\ell_0\Delta T \sim \alpha\Delta T$. However, to the leading order, the vertical distance the bar has risen is proportional to $(\alpha\Delta T)^{1/2}$.

22. (a) The water (of mass *m*) releases energy in two steps, first by lowering its temperature from 20°C to 0°C, and then by freezing into ice. Thus the total energy transferred from the water to the surroundings is

$$Q = c_w m\Delta T + L_F m = \left(4190\,\text{J/kg}\cdot\text{K}\right)\left(125\,\text{kg}\right)\left(20^\circ\text{C}\right) + \left(333\,\text{kJ/kg}\right)\left(125\,\text{kg}\right) = 5.2\times10^7\,\text{J}.$$

(b) Before all the water freezes, the lowest temperature possible is 0°C, below which the water must have already turned into ice.

23. **THINK** Electrical energy is supplied and converted into thermal energy to raise the water temperature.

**EXPRESS** The water has a mass $m$ = 0.100 kg and a specific heat $c$ = 4190 J/kg·K. When raised from an initial temperature $T_i$ = 23°C to its boiling point $T_f$ = 100°C, the heat input is given by $Q = cm(T_f - T_i)$. This must be the power output of the heater $P$ multiplied by the time $t$: $Q = Pt$.

**ANALYZE** The time it takes to heat up the water is

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(100°\text{C} - 23°\text{C})}{200\,\text{J/s}} = 160\,\text{s}.$$

**LEARN** With a fixed power output, the time required is proportional to $Q$, which is proportional to $\Delta T = T_f - T_i$. In real life, it would take longer because of heat loss.

24. (a) The specific heat is given by $c = Q/m(T_f - T_i)$, where $Q$ is the heat added, $m$ is the mass of the sample, $T_i$ is the initial temperature, and $T_f$ is the final temperature. Thus, recalling that a change in Celsius degrees is equal to the corresponding change on the Kelvin scale,

$$c = \frac{314\,\text{J}}{(30.0\times10^{-3}\,\text{kg})(45.0°\text{C} - 25.0°\text{C})} = 523\,\text{J/kg}\cdot\text{K}.$$

(b) The molar specific heat is given by

$$c_m = \frac{Q}{N(T_f - T_i)} = \frac{314\,\text{J}}{(0.600\,\text{mol})(45.0°\text{C} - 25.0°\text{C})} = 26.2\,\text{J/mol}\cdot\text{K}.$$

(c) If $N$ is the number of moles of the substance and $M$ is the mass per mole, then $m = NM$, so

$$N = \frac{m}{M} = \frac{30.0\times10^{-3}\,\text{kg}}{50\times10^{-3}\,\text{kg/mol}} = 0.600\,\text{mol}.$$

25. We use $Q = cm\Delta T$. The textbook notes that a nutritionist's "Calorie" is equivalent to 1000 cal. The mass $m$ of the water that must be consumed is

$$m = \frac{Q}{c\Delta T} = \frac{3500\times10^{3}\,\text{cal}}{(1\,\text{g/cal}\cdot\text{C°})(37.0°\text{C} - 0.0°\text{C})} = 94.6\times10^{4}\,\text{g},$$

which is equivalent to $9.46 \times 10^4$ g/(1000 g/liter) = 94.6 liters of water. This is certainly too much to drink in a single day!

26. The work the man has to do to climb to the top of Mt. Everest is given by

$$W = mgy = (73.0\text{ kg})(9.80\text{ m/s}^2)(8840\text{ m}) = 6.32 \times 10^6\text{ J}.$$

Thus, the amount of butter needed is

$$m = \frac{(6.32\times10^{6}\text{ J})\left(\frac{1.00\,\text{cal}}{4.186\,\text{J}}\right)}{6000\,\text{cal/g}} \approx 250\,\text{g} = 0.25\,\text{kg}.$$

27. **THINK** Silver is solid at 15.0° C. To melt the sample, we must first raise its temperature to the melting point, and then supply heat of fusion.

**EXPRESS** The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0° C (= 288 K) to 1235 K. This requires heat

$$Q_1 = cm(T_f - T_i) = (236\,\text{J/kg}\cdot\text{K})(0.130\,\text{kg})(1235°\text{C} - 288°\text{C}) = 2.91\times10^4\ \text{J}.$$

Now the silver at its melting point must be melted. If $L_F$ is the heat of fusion for silver this requires

$$Q_2 = mL_F = (0.130\,\text{kg})(105\times10^3\ \text{J/kg}) = 1.36\times10^4\ \text{J}.$$

**ANALYZE** The total heat required is

$$Q = Q_1 + Q_2 = 2.91\times10^4\ \text{J} + 1.36\times10^4\ \text{J} = 4.27\times10^4\ \text{J}.$$

**LEARN** The heating process is associated with the specific heat of silver, while the melting process involves heat of fusion. Both the specific heat and the heat of fusion are chemical properties of the material itself.

28. The amount of water $m$ that is frozen is

$$m = \frac{Q}{L_F} = \frac{50.2\,\text{kJ}}{333\,\text{kJ/kg}} = 0.151\,\text{kg} = 151\,\text{g}.$$

Therefore the amount of water that remains unfrozen is 260 g – 151 g = 109 g.

29. The power consumed by the system is

$$P = \left(\frac{1}{20\%}\right)\frac{cm\Delta T}{t} = \left(\frac{1}{20\%}\right)\frac{(4.18\,\text{J/g}\cdot°\text{C})(200\times10^3\,\text{cm}^3)(1\,\text{g/cm}^3)(40°\text{C} - 20°\text{C})}{(1.0\,\text{h})(3600\,\text{s/h})}$$
$$= 2.3\times10^4\ \text{W}.$$

The area needed is then $A = \dfrac{2.3\times10^4\ \text{W}}{700\,\text{W/m}^2} = 33\,\text{m}^2.$

30. While the sample is in its liquid phase, its temperature change (in absolute values) is $|\Delta T|$ = 30 °C. Thus, with $m$ = 0.40 kg, the absolute value of Eq. 18-14 leads to

$$|Q| = c\,m\,|\Delta T| = (3000\ \text{J/kg}\cdot°\text{C})(0.40\ \text{kg})(30\,°\text{C}) = 36000\ \text{J}.$$

The rate (which is constant) is

$$P = |Q|/t = (36000\ \text{J})/(40\ \text{min}) = 900\ \text{J/min},$$

which is equivalent to 15 W.

(a) During the next 30 minutes, a phase change occurs that is described by Eq. 18-16:

$$|Q| = P\,t = (900 \text{ J/min})(30 \text{ min}) = 27000 \text{ J} = L\,m\,.$$

Thus, with $m$ = 0.40 kg, we find $L$ = 67500 J/kg $\approx$ 68 kJ/kg.

(b) During the final 20 minutes, the sample is solid and undergoes a temperature change (in absolute values) of $|\Delta T| = 20\text{ C}°$. Now, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m\,|\Delta T|} = \frac{P\,t}{m\,|\Delta T|} = \frac{(900)(20)}{(0.40)(20)} = 2250\ \frac{\text{J}}{\text{kg}\cdot\text{C}°} \approx 2.3\ \frac{\text{kJ}}{\text{kg}\cdot\text{C}°}\ .$$

31. Let the mass of the steam be $m_s$ and that of the ice be $m_i$. Then

$$L_F m_c + c_w m_c (T_f - 0.0°\text{C}) = m_s L_s + m_s c_w (100°\text{C} - T_f)\,,$$

where $T_f$ = 50°C is the final temperature. We solve for $m_s$:

$$m_s = \frac{L_F m_c + c_w m_c (T_f - 0.0°\text{C})}{L_s + c_w(100°\text{C} - T_f)} = \frac{(79.7\,\text{cal/g})(150\,\text{g}) + (1\,\text{cal/g}\cdot°\text{C})(150\text{g})(50°\text{C} - 0.0°\text{C})}{539\,\text{cal/g} + (1\,\text{cal/g}\cdot\text{C}°)(100°\text{C} - 50°\text{C})}$$
$$= 33\,\text{g}.$$

32. The heat needed is found by integrating the heat capacity:

$$Q = \int_{T_i}^{T_f} cm\ dT = m\int_{T_i}^{T_f} c dT = (2.09)\int_{5.0°\text{C}}^{15.0°\text{C}} (0.20 + 0.14T + 0.023T^2)\,dT$$
$$= (2.0)\,(0.20T + 0.070T^2 + 0.00767T^3)\Big|_{5.0}^{15.0}\ (\text{cal})$$
$$= 82\,\text{cal}.$$

33. We note from Eq. 18-12 that 1 Btu = 252 cal. The heat relates to the power, and to the temperature change, through

$$Q = Pt = cm\Delta T.$$

Therefore, the time $t$ required is

$$t = \frac{cm\Delta T}{P} = \frac{(1000\,\text{cal/kg}\cdot\text{C}°)(40\,\text{gal})(1000\,\text{kg}/264\,\text{gal})(100°\text{F} - 70°\text{F})(5°\text{C}/9°\text{F})}{(2.0\times10^5\ \text{Btu/h})(252.0\ \text{cal/Btu})(1\,\text{h}/60\,\text{min})}$$
$$= 3.0\,\text{min}.$$

The metric version proceeds similarly:

$$t = \frac{c\rho V\Delta T}{P} = \frac{(4190\ \text{J/kg}\cdot\text{C°})(1000\ \text{kg/m}^3)(150\ \text{L})(1\ \text{m}^3/1000\ \text{L})(38°\text{C} - 21°\text{C})}{(59000\ \text{J/s})(60\ \text{s}/1\,\text{min})}$$
$$= 3.0\,\text{min}.$$

34. We note that the heat capacity of sample *B* is given by the reciprocal of the slope of the line in Figure 18-34(b) (compare with Eq. 18-14). Since the reciprocal of that slope is 16/4 = 4 kJ/kg·C°, then $c_B$ = 4000 J/kg·C° = 4000 J/kg·K (since a change in Celsius is equivalent to a change in Kelvins). Now, following the same procedure as shown in Sample Problem 18.03 —"Hot slug in water, coming to equilibrium," we find

$$c_A\, m_A\, (T_f - T_A) + c_B\, m_B\, (T_f - T_B) = 0$$

$$c_A\, (5.0\ \text{kg})(40°\text{C} - 100°\text{C}) + (4000\ \text{J/kg}\cdot\text{C°})(1.5\ \text{kg})(40°\text{C} - 20°\text{C}) = 0$$

which leads to $c_A = 4.0\times10^2$ J/kg·K.

35. We denote the ice with subscript *I* and the coffee with *c*, respectively. Let the final temperature be $T_f$. The heat absorbed by the ice is

$$Q_I = \lambda_F m_I + m_I c_w\, (T_f - 0°\text{C}),$$

and the heat given away by the coffee is $|Q_c| = m_w c_w\, (T_I - T_f)$. Setting $Q_I = |Q_c|$, we solve for $T_f$:

$$T_f = \frac{m_w c_w T_I - \lambda_F m_I}{(m_I + m_c)c_w} = \frac{(130\,\text{g})(4190\,\text{J/kg}\cdot\text{C°})\,(80.0°\text{C}) - (333\times10^3\ \text{J/g})\,(12.0\,\text{g})}{(12.0\,\text{g} + 130\,\text{g})\,(4190\,\text{J/kg}\cdot\text{C°})}$$
$$= 66.5°\text{C}.$$

Note that we work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. Therefore, the temperature of the coffee will cool by $|\Delta T|$ = 80.0°C – 66.5°C = 13.5C°.

36. (a) Using Eq. 18-17, the heat transferred to the water is

$$Q_w = c_w m_w \Delta T + L_V m_s = (1\,\text{cal/g}\cdot\text{C°})(220\,\text{g})(100°\text{C} - 20.0°\text{C}) + (539\,\text{cal/g})(5.00\,\text{g})$$
$$= 20.3\,\text{kcal}.$$

(b) The heat transferred to the bowl is

$$Q_b = c_b m_b \Delta T = (0.0923\,\text{cal/g}\cdot\text{C°})(150\,\text{g})(100°\,\text{C} - 20.0°\,\text{C}) = 1.11\,\text{kcal}.$$

(c) If the original temperature of the cylinder be $T_i$, then $Q_w + Q_b = c_c m_c(T_i - T_f)$, which leads to

$$T_i = \frac{Q_w + Q_b}{c_c m_c} + T_f = \frac{20.3\,\text{kcal} + 1.11\,\text{kcal}}{(0.0923\,\text{cal/g}\cdot\text{C°})(300\,\text{g})} + 100°\text{C} = 873°\text{C}.$$

37. We compute with Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. If the equilibrium temperature is $T_f$, then the energy absorbed as heat by the ice is

$$Q_I = L_F m_I + c_w m_I (T_f - 0°\text{C}),$$

while the energy transferred as heat from the water is $Q_w = c_w m_w (T_f - T_i)$. The system is insulated, so $Q_w + Q_I = 0$, and we solve for $T_f$ :

$$T_f = \frac{c_w m_w T_i - L_F m_I}{(m_I + m_C)c_w}.$$

(a) Now $T_i$ = 90°C so

$$T_f = \frac{(4190\,\text{J/kg}\cdot\text{C°})(0.500\,\text{kg})(90°\text{C}) - (333\times10^3\,\text{J/kg})(0.500\,\text{kg})}{(0.500\,\text{kg} + 0.500\,\text{kg})(4190\,\text{J/kg}\cdot\text{C°})} = 5.3°\text{C}.$$

(b) Since no ice has remained at $T_f = 5.3°C$ , we have $m_f = 0$ .

(c) If we were to use the formula above with $T_i$ = 70°C, we would get $T_f < 0$, which is impossible. In fact, not all the ice has melted in this case, and the equilibrium temperature is $T_f$ = 0°C.

(d) The amount of ice that melts is given by

$$m_I' = \frac{c_w m_w (T_i - 0°\text{C})}{L_F} = \frac{(4190\,\text{J/kg}\cdot\text{C°})(0.500\,\text{kg})(70\,\text{C°})}{333\times10^3\,\text{J/kg}} = 0.440\,\text{kg}.$$

Therefore, the amount of (solid) ice remaining is $m_f = m_I - m'_I$ = 500 g – 440 g = 60.0 g, and (as mentioned) we have $T_f$ = 0°C (because the system is an ice-water mixture in thermal equilibrium).

38. (a) Equation 18-14 (in absolute value) gives

$$|Q| = (4190\ \text{J/kg}\cdot°\text{C})(0.530\ \text{kg})(40\ °\text{C}) = 88828\ \text{J}.$$

Since $dQ/dt$ is assumed constant (we will call it $P$) then we have

$$P = \frac{88828\text{ J}}{40\text{ min}} = \frac{88828\text{ J}}{2400\text{ s}} = 37\text{ W}.$$

(b) During that same time (used in part (a)) the ice warms by 20 C°. Using Table 18-3 and Eq. 18-14 again we have

$$m_{\text{ice}} = \frac{Q}{c_{\text{ice}}\,\Delta T} = \frac{88828}{(2220)(20°)} = 2.0\text{ kg}.$$

(c) To find the ice produced (by freezing the water that has already reached 0°C, so we concerned with the $40\text{ min} < t < 60\text{ min}$ time span), we use Table 18-4 and Eq. 18-16:

$$m_{\text{water becoming ice}} = \frac{Q_{\text{20 min}}}{L_F} = \frac{44414}{333000} = 0.13\text{ kg}.$$

39. To accomplish the phase change at 78°C,

$$Q = L_V m = (879\text{ kJ/kg})\,(0.510\text{ kg}) = 448.29\text{ kJ}$$

must be removed. To cool the liquid to –114°C,

$$Q = cm|\Delta T| = (2.43\text{ kJ/kg}\cdot\text{K})\,(0.510\text{ kg})\,(192\text{ K}) = 237.95\text{ kJ}$$

must be removed. Finally, to accomplish the phase change at –114°C,

$$Q = L_F m = (109\text{ kJ/kg})\,(0.510\text{ kg}) = 55.59\text{ kJ}$$

must be removed. The grand total of heat removed is therefore (448.29 + 237.95 + 55.59) kJ = 742 kJ.

40. Let $m_w$ = 14 kg, $m_c$ = 3.6 kg, $m_m$ = 1.8 kg, $T_{i1}$ = 180°C, $T_{i2}$ = 16.0°C, and $T_f$ = 18.0°C. The specific heat $c_m$ of the metal then satisfies

$$\left(m_w c_w + m_c c_m\right)\left(T_f - T_{i2}\right) + m_m c_m\left(T_f - T_{i1}\right) = 0$$

which we solve for $c_m$:

$$c_m = \frac{m_w c_w\left(T_{i2} - T_f\right)}{m_c\left(T_f - T_{i2}\right) + m_m\left(T_f - T_{i1}\right)} = \frac{(14\text{ kg})(4.18\text{ kJ/kg}\cdot\text{K})(16.0°\text{C} - 18.0°\text{C})}{(3.6\text{ kg})(18.0°\text{C} - 16.0°\text{C}) + (1.8\text{ kg})(18.0°\text{C} - 180°\text{C})}$$
$$= 0.41\text{ kJ/kg}\cdot\text{C°} = 0.41\text{ kJ/kg}\cdot\text{K}.$$

41. **THINK** Our system consists of both water and ice cubes. Initially the ice cubes are at $-15°\text{C}$ (below freezing temperatures), so they must first absorb heat until $0°\text{C}$ is reached. The final equilibrium temperature reached is related to the amount of ice melted.

**EXPRESS** There are three possibilities:

• None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.

• The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.

• All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale.

First, suppose that no ice melts. The temperature of the water decreases from $T_{Wi}$ = 25°C to some final temperature $T_f$ and the temperature of the ice increases from $T_{Ii}$ = –15°C to $T_f$. If $m_W$ is the mass of the water and $c_W$ is its specific heat then the water rejects heat

$$|Q| = c_W m_W (T_{Wi} - T_f).$$

If $m_I$ is the mass of the ice and $c_I$ is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii}).$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (T_f - T_{Ii}).$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I} \\ &= \frac{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(-15°\text{C})}{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})} \\ &= 16.6°\text{C}. \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at $T_f$ = 0°C, with mass $m$ (< $m_I$) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi},$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + mL_F ,$$

where $L_F$ is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass $m$ of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + mL_F .$$

This equation can be solved for the mass $m$ of ice melted.

**ANALYZE** (a) Solving for $m$ and substituting the values given, we find the amount of ice melted to be

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190\,\text{J/kg}\cdot\text{K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg}\cdot\text{K})(0.100\,\text{kg})(-15°\text{C})}{333\times 10^3\,\text{J/kg}} \\ &= 5.3\times 10^{-2}\,\text{kg} = 53\,\text{g}. \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F .$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to $T_f$, and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F .$$

The solution for $T_f$ is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)} .$$

Inserting the given values, we obtain $T_f$ = 2.5°C.

**LEARN** In order to melt some ice, the energy released by the water must be sufficient to first raise the temperature of the ice to the melting point ($-c_I m_I T_{Ii}$ required, $T_{Ii} < 0$), with the remaining energy contributing to the heat of fusion. If the remaining energy is greater than $m_I L_F$, then all ice will be melted and the final temperature will be above 0°C.

42. If the ring diameter at 0.000°C is $D_{r0}$, then its diameter when the ring and sphere are in thermal equilibrium is

$$D_r = D_{r0}(1+\alpha_c T_f),$$

where $T_f$ is the final temperature and $\alpha_c$ is the coefficient of linear expansion for copper. Similarly, if the sphere diameter at $T_i$ (= 100.0°C) is $D_{s0}$, then its diameter at the final temperature is

$$D_s = D_{s0}[1+\alpha_a(T_f - T_i)],$$

where $\alpha_a$ is the coefficient of linear expansion for aluminum. At equilibrium the two diameters are equal, so

$$D_{r0}(1+\alpha_c T_f) = D_{s0}[1+\alpha_a(T_f - T_i)].$$

The solution for the final temperature is

$$\begin{aligned} T_f &= \frac{D_{r0} - D_{s0} + D_{s0}\alpha_a T_i}{D_{s0}\alpha_a - D_{r0}\alpha_c} \\ &= \frac{2.54000\,\text{cm} - 2.54508\,\text{cm} + (2.54508\,\text{cm})(23\times10^{-6}/\text{C}°)(100.0°\text{C})}{(2.54508\,\text{cm})(23\times10^{-6}/\text{C}°) - (2.54000\,\text{cm})(17\times10^{-6}/\text{C}°)} \\ &= 50.38°\text{C}. \end{aligned}$$

The expansion coefficients are from Table 18-2 of the text. Since the initial temperature of the ring is 0°C, the heat it absorbs is $Q = c_c m_r T_f$, where $c_c$ is the specific heat of copper and $m_r$ is the mass of the ring. The heat released by the sphere is

$$|Q| = c_a m_s (T_i - T_f)$$

where $c_a$ is the specific heat of aluminum and $m_s$ is the mass of the sphere. Since these two heats are equal,

$$c_c m_r T_f = c_a m_s (T_i - T_f),$$

we use specific heat capacities from the textbook to obtain

$$m_s = \frac{c_c m_r T_f}{c_a(T_i - T_f)} = \frac{(386\,\text{J/kg}\cdot\text{K})(0.0200\,\text{kg})(50.38°\text{C})}{(900\,\text{J/kg}\cdot\text{K})(100°\text{C} - 50.38°\text{C})} = 8.71\times10^{-3}\,\text{kg}.$$

43. (a) One part of path *A* represents a constant pressure process. The volume changes from 1.0 m$^3$ to 4.0 m$^3$ while the pressure remains at 40 Pa. The work done is

$$W_A = p\Delta V = (40\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 1.2\times 10^2\ \text{J}.$$

(b) The other part of the path represents a constant volume process. No work is done during this process. The total work done over the entire path is 120 J. To find the work done over path *B* we need to know the pressure as a function of volume. Then, we can evaluate the integral $W = \int p\, dV$. According to the graph, the pressure is a linear function of the volume, so we may write $p = a + bV$, where $a$ and $b$ are constants. In order for the pressure to be 40 Pa when the volume is 1.0 m$^3$ and 10 Pa when the volume is 4.00 m$^3$ the values of the constants must be $a$ = 50 Pa and $b$ = –10 Pa/m$^3$. Thus,

$$p = 50\ \text{Pa} - (10\ \text{Pa/m}^3)V$$

and

$$W_B = \int_1^4 p\, dV = \int_1^4 (50 - 10V)\, dV = \left(50V - 5V^2\right)\Big|_1^4 = 200\,\text{J} - 50\,\text{J} - 80\,\text{J}\ +\ 5.0\,\text{J} = 75\,\text{J}.$$

(c) One part of path *C* represents a constant pressure process in which the volume changes from 1.0 m$^3$ to 4.0 m$^3$ while $p$ remains at 10 Pa. The work done is

$$W_C = p\Delta V = (10\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 30\,\text{J}.$$

The other part of the process is at constant volume and no work is done. The total work is 30 J. We note that the work is different for different paths.

44. During process $A \to B$, the system is expanding, doing work on its environment, so $W > 0$, and since $\Delta E_{\text{int}} > 0$ is given then $Q = W + \Delta E_{\text{int}}$ must also be positive.

(a) $Q > 0$.

(b) $W > 0$.

During process $B \to C$, the system is neither expanding nor contracting. Thus,

(c) $W = 0$.

(d) The sign of $\Delta E_{\text{int}}$ must be the same (by the first law of thermodynamics) as that of $Q$, which is given as positive. Thus, $\Delta E_{\text{int}} > 0$.

During process $C \to A$, the system is contracting. The environment is doing work on the system, which implies $W < 0$. Also, $\Delta E_{\text{int}} < 0$ because $\sum \Delta E_{\text{int}} = 0$ (for the whole cycle) and the other values of $\Delta E_{\text{int}}$ (for the other processes) were positive. Therefore, $Q = W + \Delta E_{\text{int}}$ must also be negative.

(e) $Q < 0$.

(f) $W < 0$.

(g) $\Delta E_{\text{int}} < 0$.

(h) The area of a triangle is $\frac{1}{2}$ (base)(height). Applying this to the figure, we find

$$|W_{\text{net}}| = \tfrac{1}{2}(2.0\,\text{m}^3)(20\,\text{Pa}) = 20\,\text{J}\,.$$

Since process $C \to A$ involves larger negative work (it occurs at higher average pressure) than the positive work done during process $A \to B$, then the net work done during the cycle must be negative. The answer is therefore $W_{\text{net}} = -20$ J.

45. **THINK** Over a complete cycle, the internal energy is the same at the beginning and end, so the heat $Q$ absorbed equals the work done: $Q = W$.

**EXPRESS** Over the portion of the cycle from $A$ to $B$ the pressure $p$ is a linear function of the volume $V$ and we may write $p = a + bV$. The work done over this portion of the cycle is

$$W_{AB} = \int_{V_A}^{V_B} p\,dV = \int_{V_A}^{V_B} (a + bV)\,dV = a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right).$$

The $BC$ portion of the cycle is at constant pressure and the work done by the gas is

$$W_{BC} = p_B \Delta V_{BC} = p_B(V_C - V_B)\,.$$

The $CA$ portion of the cycle is at constant volume, so no work is done. The total work done by the gas is

$$W = W_{AB} + W_{BC} + W_{CA}\,.$$

**ANALYZE** The pressure function can be written as

$$p = \frac{10}{3}\ \text{Pa} + \left(\frac{20}{3}\ \text{Pa/m}^3\right)V,$$

where the coefficients $a$ and $b$ were chosen so that $p = 10$ Pa when $V = 1.0$ m$^3$ and $p = 30$ Pa when $V = 4.0$ m$^3$. Therefore, the work done going from $A$ to $B$ is

$$W_{AB} = a(V_B - V_A) + \frac{1}{2}b\left(V_B^2 - V_A^2\right)$$
$$= \left(\frac{10}{3}\text{ Pa}\right)\left(4.0\text{ m}^3 - 1.0\text{ m}^3\right) + \frac{1}{2}\left(\frac{20}{3}\text{ Pa/m}^3\right)\left[(4.0\text{ m}^3)^2 - (1.0\text{ m}^3)^2\right]$$
$$= 10\text{ J} + 50\text{ J} = 60\text{ J}$$

Similarly, with $p_B = p_C = 30\text{ Pa}$, $V_C = 1.0\text{ m}^3$ and $V_B = 4.0\text{ m}^3$, we have

$$W_{BC} = p_B(V_C - V_B) = (30\text{ Pa})(1.0\text{ m}^3 – 4.0\text{ m}^3) = –90\text{ J}.$$

Adding up all contributions, we find the total work done by the gas to be

$$W = W_{AB} + W_{BC} + W_{CA} = 60\text{ J} – 90\text{ J} + 0 = –30\text{ J}.$$

Thus, the total heat absorbed is $Q = W = –30$ J. This means the gas loses 30 J of energy in the form of heat.

**LEARN** Notice that in calculating the work done by the gas, we always start with Eq. 18-25: $W = \int p dV$. For isobaric process where $p$ = constant, $W = p\Delta V$, and for isochoric process where $V$ = constant, $W = 0$.

46. (a) Since work is done *on* the system (perhaps to compress it) we write $W = –200$ J.

(b) Since heat leaves the system, we have $Q = –70.0$ cal $= –293$ J.

(c) The change in internal energy is $\Delta E_{\text{int}} = Q – W = –293\text{ J} – (–200\text{ J}) = –93\text{ J}$.

47. **THINK** Since the change in internal energy $\Delta E_{\text{int}}$ only depends on the initial and final states, it is the same for path *iaf* and path *ibf*.

**EXPRESS** According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q – W$, where $Q$ is the heat absorbed and $W$ is the work done by the system. Along *iaf* , we have

$$\Delta E_{\text{int}} = Q – W = 50\text{ cal} – 20\text{ cal} = 30\text{ cal}.$$

**ANALYZE** (a) The work done along path *ibf* is given by

$$W = Q – \Delta E_{\text{int}} = 36\text{ cal} – 30\text{ cal} = 6.0\text{ cal}.$$

(b) Since the curved path is traversed from $f$ to $i$ the change in internal energy is $\Delta E_{\text{int}} = –30\text{ cal}$, and

$$Q = \Delta E_{\text{int}} + W = –30\text{ cal} – 13\text{ cal} = – 43\text{ cal}.$$

(c) Let $\Delta E_{\text{int}} = E_{\text{int},\,f} - E_{\text{int},\,i}$. We then have

$$E_{\text{int},\,f} = \Delta E_{\text{int}} + E_{\text{int},\,i} = 30 \text{ cal} + 10 \text{ cal} = 40 \text{ cal}.$$

(d) The work $W_{bf}$ for the path *bf* is zero, so

$$Q_{bf} = E_{\text{int},\,f} - E_{\text{int},\,b} = 40 \text{ cal} - 22 \text{ cal} = 18 \text{ cal}.$$

(e) For the path *ibf*, $Q$ = 36 cal so $Q_{ib} = Q - Q_{bf}$ = 36 cal – 18 cal = 18 cal.

**LEARN** Work $W$ and heat $Q$ in general are path-dependent quantities, i.e., they depend on how the finial state is reached. However, the combination $\Delta E_{\text{int}} = Q - W$ is path independent; it is a *state function*.

48. Since the process is a complete cycle (beginning and ending in the same thermodynamic state) the change in the internal energy is zero, and the heat absorbed by the gas is equal to the work done by the gas: $Q = W$. In terms of the contributions of the individual parts of the cycle $Q_{AB} + Q_{BC} + Q_{CA} = W$ and

$$Q_{CA} = W - Q_{AB} - Q_{BC} = +15.0 \text{ J} - 20.0 \text{ J} - 0 = -5.0 \text{ J}.$$

This means 5.0 J of energy leaves the gas in the form of heat.

49. We note that there is no work done in the process going from *d* to *a*, so $Q_{da} = \Delta E_{\text{int}\ da}$ = 80 J. Also, since the total change in internal energy around the cycle is zero, then

$$\Delta E_{\text{int}\ ac} + \Delta E_{\text{int}\ cd} + \Delta E_{\text{int}\ da} = 0$$

$$-200 \text{ J} + \Delta E_{\text{int}\ cd} + 80 \text{ J} = 0$$

which yields $\Delta E_{\text{int}\ cd}$ = 120 J. Thus, applying the first law of thermodynamics to the *c* to *d* process gives the work done as

$$W_{cd} = Q_{cd} - \Delta E_{\text{int}\ cd} = 180 \text{ J} - 120 \text{ J} = 60 \text{ J}.$$

50. (a) We note that process *a* to *b* is an expansion, so $W > 0$ for it. Thus, $W_{ab}$ = +5.0 J. We are told that the change in internal energy during that process is +3.0 J, so application of the first law of thermodynamics for that process immediately yields $Q_{ab}$ = +8.0 J.

(b) The net work (+1.2 J) is the same as the net heat ($Q_{ab} + Q_{bc} + Q_{ca}$), and we are told that $Q_{ca}$ = +2.5 J. Thus we readily find $Q_{bc}$ = (1.2 – 8.0 – 2.5) J = –9.3 J.

51. We use Eqs. 18-38 through 18-40. Note that the surface area of the sphere is given by $A = 4\pi r^2$, where $r$ = 0.500 m is the radius.

(a) The temperature of the sphere is $T$ = (273.15 + 27.00) K = 300.15 K. Thus

$$P_r = \sigma\varepsilon AT^4 = \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(0.850)(4\pi)(0.500\,\text{m})^2\,(300.15\,\text{K})^4 = 1.23\times10^3\ \text{W}.$$

(b) Now, $T_{\text{env}}$ = 273.15 + 77.00 = 350.15 K so

$$P_a = \sigma\varepsilon AT_{\text{env}}^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.850)(4\pi)(0.500\,\text{m})^2\,(350.15\,\text{K})^4 = 2.28\times10^3\ \text{W}.$$

(c) From Eq. 18-40, we have

$$P_n = P_a - P_r = 2.28\times10^3\ \text{W} - 1.23\times10^3\ \text{W} = 1.05\times10^3\ \text{W}.$$

52. We refer to the polyurethane foam with subscript *p* and silver with subscript *s*. We use Eq. 18-32 to find $L = kR$.

(a) From Table 18-6 we find $k_p$ = 0.024 W/m·K, so

$$\begin{aligned} L_p &= k_p R_p \\ &= (0.024\,\text{W/m}\cdot\text{K})\left(30\,\text{ft}^2\cdot\text{F}^\circ\cdot\text{h/Btu}\right)(1\text{m}/3.281\text{ft})^2\,(5\text{C}^\circ/9\text{F}^\circ)(3600\,\text{s/h})(1\text{Btu}/1055\,\text{J}) \\ &= 0.13\,\text{m}. \end{aligned}$$

(b) For silver $k_s$ = 428 W/m·K, so

$$L_s = k_s R_s = \left(\frac{k_s R_s}{k_p R_p}\right) L_p = \left[\frac{428(30)}{0.024(30)}\right](0.13\,\text{m}) = 2.3\times10^3\ \text{m}.$$

53. **THINK** Energy is transferred as heat from the hot reservoir at temperature $T_H$ to the cold reservoir at temperature $T_C$. The conduction rate is the amount of energy transferred per unit time.

**EXPRESS** The rate of heat flow is given by

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L},$$

where $k$ is the thermal conductivity of copper (401 W/m·K), $A$ is the cross-sectional area (in a plane perpendicular to the flow), $L$ is the distance along the direction of flow between the points where the temperature is $T_H$ and $T_C$. The thermal conductivity is found in Table 18-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

**ANALYZE** Substituting the values given, we find the rate to be

$$P_{\text{cond}} = \frac{(401\,\text{W/m}\cdot\text{K})(90.0\times10^{-4}\,\text{m}^2)(125°\text{C}-10.0°\text{C})}{0.250\,\text{m}} = 1.66\times10^3\,\text{J/s}.$$

**LEARN** The thermal resistance ($R$-value) of the copper slab is

$$R = \frac{L}{k} = \frac{0.250\,\text{m}}{401\,\text{W/m}\cdot\text{K}} = 6.23\times10^{-4}\,\text{m}^2\cdot\text{K/W}.$$

The low value of $R$ is an indication that the copper slab is a good conductor.

54. (a) We estimate the surface area of the average human body to be about 2 m$^2$ and the skin temperature to be about 300 K (somewhat less than the internal temperature of 310 K). Then from Eq. 18-37

$$P_r = \sigma\varepsilon A T^4 \approx (5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(0.9)(2.0\,\text{m}^2)(300\,\text{K})^4 = 8\times10^2\,\text{W}.$$

(b) The energy lost is given by $\Delta E = P_r\Delta t = (8\times10^2\,\text{W})(30\,\text{s}) = 2\times10^4\,\text{J}.$

55. (a) Recalling that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale, we find that the rate of heat conduction is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(401\,\text{W/m}\cdot\text{K})(4.8\times10^{-4}\,\text{m}^2)(100\,°\text{C})}{1.2\,\text{m}} = 16\ \text{J/s}.$$

(b) Using Table 18-4, the rate at which ice melts is

$$\left|\frac{dm}{dt}\right| = \frac{P_{\text{cond}}}{L_F} = \frac{16\,\text{J/s}}{333\,\text{J/g}} = 0.048\,\text{g/s}.$$

56. The surface area of the ball is $A = 4\pi R^2 = 4\pi(0.020\text{ m})^2 = 5.03\times10^{-3}\text{ m}^2$. Using Eq. 18-37 with $T_i = 35+273 = 308\text{ K}$ and $T_f = 47+273 = 320\text{ K}$, the power required to maintain the temperature is

$$P_r = \sigma\varepsilon A(T_f^4 - T_i^4) \approx (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.80)(5.03\times10^{-3}\text{ m}^2)\left[(320\text{ K})^4 - (308\text{ K})^4\right]$$
$$= 0.34\,\text{W}.$$

Thus, the heat each bee must produce during the 20-minute interval is

$$\frac{Q}{N} = \frac{P_r t}{N} = \frac{(0.34\text{ W})(20\text{ min})(60\text{ s/min})}{500} = 0.81\text{ J}.$$

57. (a) We use

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L}$$

with the conductivity of glass given in Table 18-6 as 1.0 W/m·K. We choose to use the Celsius scale for the temperature: a temperature difference of

$$T_H - T_C = 72°\text{F} - (-20°\text{F}) = 92\ °\text{F}$$

is equivalent to $\frac{5}{9}(92) = 51.1\text{C}°$. This, in turn, is equal to 51.1 K since a change in Kelvin temperature is entirely equivalent to a Celsius change. Thus,

$$\frac{P_{\text{cond}}}{A} = k\frac{T_H - T_C}{L} = (1.0\ \text{W/m}\cdot\text{K})\left(\frac{51.1°\text{C}}{3.0\times10^{-3}\text{m}}\right) = 1.7\times10^4\ \text{W/m}^2.$$

(b) The energy now passes in succession through 3 layers, one of air and two of glass. The heat transfer rate $P$ is the same in each layer and is given by

$$P_{\text{cond}} = \frac{A(T_H - T_C)}{\sum L/k}$$

where the sum in the denominator is over the layers. If $L_g$ is the thickness of a glass layer, $L_a$ is the thickness of the air layer, $k_g$ is the thermal conductivity of glass, and $k_a$ is the thermal conductivity of air, then the denominator is

$$\sum\frac{L}{k} = \frac{2L_g}{k_g} + \frac{L_a}{k_a} = \frac{2L_g k_a + L_a k_g}{k_a k_g}.$$

Therefore, the heat conducted per unit area occurs at the following rate:

$$\frac{P_{\text{cond}}}{A} = \frac{(T_H - T_C)k_a k_g}{2L_g k_a + L_a k_g} = \frac{(51.1°\text{C})(0.026\,\text{W/m}\cdot\text{K})(1.0\,\text{W/m}\cdot\text{K})}{2(3.0\times10^{-3}\text{m})(0.026\,\text{W/m}\cdot\text{K}) + (0.075\,\text{m})(1.0\,\text{W/m}\cdot\text{K})}$$
$$= 18\,\text{W/m}^2.$$

58. (a) The surface area of the cylinder is given by

$$A_1 = 2\pi r_1^2 + 2\pi r_1 h_1 = 2\pi(2.5\times10^{-2}\,\text{m})^2 + 2\pi(2.5\times10^{-2}\,\text{m})(5.0\times10^{-2}\,\text{m}) = 1.18\times10^{-2}\,\text{m}^2,$$

its temperature is $T_1$ = 273 + 30 = 303 K, and the temperature of the environment is $T_{\text{env}}$ = 273 + 50 = 323 K. From Eq. 18-39 we have

$$P_1 = \sigma\varepsilon A_1\left(T_{\text{env}}^4 - T^4\right) = (0.85)\left(1.18\times10^{-2}\ \text{m}^2\right)\left((323\,\text{K})^4 - (303\,\text{K})^4\right) = 1.4\,\text{W}.$$

(b) Let the new height of the cylinder be $h_2$. Since the volume $V$ of the cylinder is fixed, we must have $V = \pi r_1^2 h_1 = \pi r_2^2 h_2$. We solve for $h_2$:

$$h_2 = \left(\frac{r_1}{r_2}\right)^2 h_1 = \left(\frac{2.5\,\text{cm}}{0.50\,\text{cm}}\right)^2 (5.0\,\text{cm}) = 125\,\text{cm} = 1.25\,\text{m}.$$

The corresponding new surface area $A_2$ of the cylinder is

$$A_2 = 2\pi r_2^2 + 2\pi r_2 h_2 = 2\pi(0.50\times10^{-2}\ \text{m})^2 + 2\pi(0.50\times10^{-2}\ \text{m})(1.25\ \text{m}) = 3.94\times10^{-2}\ \text{m}^2.$$

Consequently,

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{3.94\times10^{-2}\ \text{m}^2}{1.18\times10^{-2}\ \text{m}^2} = 3.3.$$

59. We use $P_{\text{cond}} = kA\Delta T/L \propto A/L$. Comparing cases ($a$) and ($b$) in Fig. 18-45, we have

$$P_{\text{cond}\ b} = \left(\frac{A_b L_a}{A_a L_b}\right) P_{\text{cond}\ a} = 4P_{\text{cond}\ a}.$$

Consequently, it would take 2.0 min/4 = 0.50 min for the same amount of heat to be conducted through the rods welded as shown in Fig. 18-45($b$).

60. (a) As in Sample Problem 18.06 — "Thermal conduction through a layered wall," we take the rate of conductive heat transfer through each layer to be the same. Thus, the rate of heat transfer across the entire wall $P_w$ is equal to the rate across layer 2 ($P_2$). Using Eq. 18-37 and canceling out the common factor of area $A$, we obtain

$$\frac{T_H - T_c}{(L_1/k_1 + L_2/k_2 + L_3/k_3)} = \frac{\Delta T_2}{(L_2/k_2)} \quad\Rightarrow\quad \frac{45\ \text{C°}}{(1 + 7/9 + 35/80)} = \frac{\Delta T_2}{(7/9)}$$

which leads to $\Delta T_2$ = 15.8 °C.

(b) We expect (and this is supported by the result in the next part) that greater conductivity should mean a larger rate of conductive heat transfer.

(c) Repeating the calculation above with the new value for $k_2$, we have

$$\frac{45\ \text{C°}}{(1 + 7/11 + 35/80)} = \frac{\Delta T_2}{(7/11)}$$

which leads to $\Delta T_2$ = 13.8 °C.  This is less than our part (a) result, which implies that the temperature gradients across layers 1 and 3 (the ones where the parameters did not change) are greater than in part (a); those larger temperature gradients lead to larger conductive heat currents (which is basically a statement of "Ohm's law as applied to heat conduction").

61. **THINK** As heat continues to leave the water via conduction, more ice is formed and the ice slab gets thicker.

**EXPRESS** Let *h* be the thickness of the ice slab and *A* be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA\left(T_H - T_C\right)}{h},$$

where *k* is the thermal conductivity of ice, $T_H$ is the temperature of the water (0°C), and $T_C$ is the temperature of the air above the ice (–10°C). The heat leaving the water freezes it, the heat required to freeze mass *m* of water being $Q = L_F m$, where $L_F$ is the heat of fusion for water. Differentiate with respect to time and recognize that $dQ/dt = P_{\text{cond}}$ to obtain

$$P_{\text{cond}} = L_F \frac{dm}{dt}.$$

Now, the mass of the ice is given by $m = \rho A h$, where $\rho$ is the density of ice and *h* is the thickness of the ice slab, so $dm/dt = \rho A(dh/dt)$ and

$$P_{\text{cond}} = L_F \rho A \frac{dh}{dt}.$$

We equate the two expressions for $P_{\text{cond}}$ and solve for $dh/dt$:

$$\frac{dh}{dt} = \frac{k\left(T_H - T_C\right)}{L_F \rho h}.$$

**ANALYZE** Since 1 cal = 4.186 J and 1 cm = 1 × $10^{-2}$ m, the thermal conductivity of ice has the SI value

$$k = (0.0040 \text{ cal/s·cm·K})\,(4.186 \text{ J/cal})/(1 \times 10^{-2} \text{ m/cm}) = 1.674 \text{ W/m·K}.$$

The density of ice is $\rho = 0.92 \text{ g/cm}^3 = 0.92 \times 10^3 \text{ kg/m}^3$. Thus, we obtain

$$\frac{dh}{dt} = \frac{\left(1.674\,\text{W/m}\cdot\text{K}\right)\left(0°\text{C} + 10°\text{C}\right)}{\left(333\times10^3\,\text{J/kg}\right)\left(0.92\times10^3\,\text{kg/m}^3\right)\left(0.050\,\text{m}\right)} = 1.1\times10^{-6}\ \text{m/s} = 0.40\,\text{cm/h}.$$

**LEARN** The rate of ice formation is proportional to the conduction rate – the faster the energy leaves the water, the faster the water freezes.

62. (a) Using Eq. 18-32, the rate of energy flow through the surface is

$$P_{\text{cond}} = \frac{kA(T_s - T_w)}{L} = (0.026 \text{ W/m}\cdot\text{K})(4.00\times10^{-6} \text{ m}^2)\frac{300°\text{C} - 100°\text{C}}{1.0\times10^{-4} \text{ m}} = 0.208\text{W} \approx 0.21 \text{ W}.$$

(Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.)

(b) With $P_{\text{cond}} t = L_V m = L_V(\rho V) = L_V(\rho A h)$, the drop will last a duration of

$$t = \frac{L_V \rho A h}{P_{\text{cond}}} = \frac{(2.256\times10^6 \text{ J/kg})(1000 \text{ kg/m}^3)(4.00\times10^{-6} \text{ m}^2)(1.50\times10^{-3} \text{ m})}{0.208\text{W}} = 65 \text{ s}.$$

63. We divide both sides of Eq. 18-32 by area $A$, which gives us the (uniform) rate of heat conduction per unit area:

$$\frac{P_{\text{cond}}}{A} = k_1 \frac{T_H - T_1}{L_1} = k_4 \frac{T - T_C}{L_4}$$

where $T_H$ = 30°C, $T_1$ = 25°C and $T_C$ = –10°C. We solve for the unknown $T$.

$$T = T_C + \frac{k_1 L_4}{k_4 L_1}(T_H - T_1) = -4.2°\text{C}.$$

64. (a) For each individual penguin, the surface area that radiates is the sum of the top surface area and the sides:

$$A_r = a + 2\pi r h = a + 2\pi\sqrt{\frac{a}{\pi}}h = a + 2h\sqrt{\pi a},$$

where we have used $r = \sqrt{a/\pi}$ (from $a = \pi r^2$) for the radius of the cylinder. For the huddled cylinder, the radius is $r' = \sqrt{Na/\pi}$ (since $Na = \pi r'^2$), and the total surface area is

$$A_h = Na + 2\pi r' h = Na + 2\pi\sqrt{\frac{Na}{\pi}}h = Na + 2h\sqrt{N\pi a}.$$

Since the power radiated is proportional to the surface area, we have

$$\frac{P_h}{NP_r} = \frac{A_h}{NA_r} = \frac{Na + 2h\sqrt{N\pi a}}{N(a + 2h\sqrt{\pi a})} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}}.$$

With $N = 1000$, $a = 0.34\ \text{m}^2$, and $h = 1.1\ \text{m}$, the ratio is

$$\frac{P_h}{NP_r} = \frac{1+2h\sqrt{\pi / Na}}{1+2h\sqrt{\pi / a}} = \frac{1+2(1.1\ \text{m})\sqrt{\pi /(1000\cdot 0.34\ \text{m}^2)}}{1+2(1.1\ \text{m})\sqrt{\pi /(0.34\ \text{m}^2)}} = 0.16\,.$$

(b) The total radiation loss is reduced by $1.00 - 0.16 = 0.84$, or 84%.

65. We assume (although this should be viewed as a “controversial” assumption) that the top surface of the ice is at $T_C$ = –5.0°C. Less controversial are the assumptions that the bottom of the body of water is at $T_H$ = 4.0°C and the interface between the ice and the water is at $T_X$ = 0.0°C. The primary mechanism for the heat transfer through the total distance $L$ = 1.4 m is assumed to be conduction, and we use Eq. 18-34:

$$\frac{k_{\text{water}}A(T_H - T_X)}{L - L_{\text{ice}}} = \frac{k_{\text{ice}}A(T_X - T_C)}{L_{\text{ice}}} \quad \Rightarrow \quad \frac{(0.12)A\left(4.0° - 0.0°\right)}{1.4 - L_{\text{ice}}} = \frac{(0.40)A\left(0.0° + 5.0°\right)}{L_{\text{ice}}}.$$

We cancel the area $A$ and solve for thickness of the ice layer: $L_{\text{ice}}$ = 1.1 m.

66. The condition that the energy lost by the beverage can be due to evaporation equals the energy gained via radiation exchange implies

$$L_V \frac{dm}{dt} = P_{\text{rad}} = \sigma\varepsilon A(T_{\text{env}}^4 - T^4)\,.$$

The total area of the top and side surfaces of the can is

$$A = \pi r^2 + 2\pi rh = \pi(0.022\ \text{m})^2 + 2\pi(0.022\ \text{m})(0.10\ \text{m}) = 1.53\times10^{-2}\ \text{m}^2\,.$$

With $T_{\text{env}} = 32°\text{C} = 305\ \text{K}$, $T = 15°\text{C} = 288\ \text{K}$, and $\varepsilon = 1$, the rate of water mass loss is

$$\frac{dm}{dt} = \frac{\sigma\varepsilon A}{L_V}(T_{\text{env}}^4 - T^4) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.0)(1.53\times10^{-2}\ \text{m}^2)}{2.256\times10^6\ \text{J/kg}}\left[(305\ \text{K})^4 - (288\ \text{K})^4\right]$$
$$= 6.82\times10^{-7}\ \text{kg/s} \approx 0.68\ \text{mg/s}.$$

67. We denote the total mass $M$ and the melted mass $m$. The problem tells us that work/$M$ = $p/\rho$, and that all the work is assumed to contribute to the phase change $Q = Lm$ where $L$ = 150 × 10$^3$ J/kg. Thus,

$$\frac{p}{\rho}M = Lm \quad \Rightarrow \quad m = \frac{5.5\times10^6}{1200}\,\frac{M}{150\times10^3}$$

which yields $m = 0.0306M$. Dividing this by 0.30 M (the mass of the fats, which we are told is equal to 30% of the total mass), leads to a percentage 0.0306/0.30 = 10%.

68. The heat needed is

$$Q = (10\%)mL_F = \left(\frac{1}{10}\right)(200{,}000\,\text{metric tons})\ (1000\,\text{kg / metric ton})\ (333\,\text{kJ/kg}) = 6.7\times10^{12}\ \text{J}.$$

69. (a) Regarding part (a), it is important to recognize that the problem is asking for the total work done during the two-step "path": $a \to b$ followed by $b \to c$. During the latter part of this "path" there is no volume change and consequently no work done. Thus, the answer to part (b) is also the answer to part (a). Since $\Delta U$ for process $c \to a$ is –160 J, then $U_c - U_a$ = 160 J. Therefore, using the First Law of Thermodynamics, we have

$$\begin{aligned}160 &= U_c - U_b + U_b - U_a \\ &= Q_{b\to c} - W_{b\to c} + Q_{a\to b} - W_{a\to b} \\ &= 40 - 0 + 200 - W_{a\to b}.\end{aligned}$$

Therefore, $W_{a\to b\to c} = W_{a\to b}$ = 80 J.

(b) $W_{a\to b}$ = 80 J.

70. We use $Q = cm\Delta T$ and $m = \rho V$. The volume of water needed is

$$V = \frac{m}{\rho} = \frac{Q}{\rho C\Delta T} = \frac{\left(1.00\times10^6\ \text{kcal/day}\right)\left(5\,\text{days}\right)}{\left(1.00\times10^3\ \text{kg/m}^3\right)\left(1.00\,\text{kcal/kg}\right)\left(50.0°\text{C} - 22.0°\text{C}\right)} = 35.7\,\text{m}^3.$$

71. The graph shows that the absolute value of the temperature change is $|\Delta T|$ = 25 °C. Since a watt is a joule per second, we reason that the energy removed is

$$|Q| = (2.81\ \text{J/s})(20\ \text{min})(60\ \text{s/min}) = 3372\ \text{J}.$$

Thus, with $m$ = 0.30 kg, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m\,|\Delta T|} = 4.5\times10^2\ \text{J/kg}\cdot\text{K}.$$

72. We use $P_{\text{cond}} = kA(T_H - T_C)/L$. The temperature $T_H$ at a depth of 35.0 km is

$$T_H = \frac{P_{\text{cond}}L}{kA} + T_C = \frac{\left(54.0\times10^{-3}\ \text{W/m}^2\right)\left(35.0\times10^3\ \text{m}\right)}{2.50\,\text{W/m}\cdot\text{K}} + 10.0°\text{C} = 766°\text{C}.$$

73. Its initial volume is $5^3 = 125$ cm$^3$, and using Table 18-2, Eq. 18-10, and Eq. 18-11, we find

$$\Delta V = (125\,\text{m}^3)\,(3\times 23\times 10^{-6}\,/\,\text{C}°)\,(50.0\,\text{C}°) = 0.432\,\text{cm}^3.$$

74. As is shown Sample Problem 18.03 — "Hot slug in water, coming to equilibrium," we can express the final temperature in the following way:

$$T_f = \frac{m_A c_A T_A + m_B c_B T_B}{m_A c_A + m_B c_B} = \frac{c_A T_A + c_B T_B}{c_A + c_B}$$

where the last equality is made possible by the fact that $m_A = m_B$. Thus, in a graph of $T_f$ versus $T_A$, the "slope" must be $c_A/(c_A + c_B)$, and the "$y$ intercept" is $c_B/(c_A + c_B)T_B$. From the observation that the "slope" is equal to 2/5 we can determine, then, not only the ratio of the heat capacities but also the coefficient of $T_B$ in the "$y$ intercept"; that is,

$$c_B/(c_A + c_B)T_B = (1 - \text{"slope"})T_B.$$

(a) We observe that the "$y$ intercept" is 150 K, so

$$T_B = 150/(1 - \text{"slope"}) = 150/(3/5)$$

which yields $T_B = 2.5\times10^2$ K.

(b) As noted already, $c_A/(c_A + c_B) = \frac{2}{5}$, so $5\,c_A = 2c_A + 2c_B$, which leads to $c_B/c_A = \frac{3}{2} = 1.5$.

75. We note that there is no work done in process $c \to b$, since there is no change of volume. We also note that the *magnitude* of work done in process $b \to c$ is given, but not its sign (which we identify as negative as a result of the discussion in Section 18-8). The total (or *net*) heat transfer is $Q_{\text{net}} = [(-40) + (-130) + (+400)]$ J = 230 J. By the First Law of Thermodynamics (or, equivalently, conservation of energy), we have $Q_{\text{net}} = W_{\text{net}}$, or

$$230\,\text{J} = W_{a\to c} + W_{c\to b} + W_{b\to a} = W_{a\to c} + 0 + (-80\,\text{J}).$$

Therefore, $W_{a\to c} = 3.1\times10^2$ J.

76. From the law of cosines, with $\phi = 59.95°$, we have

$$L_{\text{Invar}}^2 = L_{\text{alum}}^2 + L_{\text{steel}}^2 - 2L_{\text{alum}}L_{\text{steel}}\cos\phi$$

Plugging in $L = L_0\,(1 + \alpha\Delta T)$, dividing by $L_0$ (which is the same for all sides) and ignoring terms of order $(\Delta T)^2$ or higher, we obtain

$$1 + 2\alpha_{\text{Invar}}\Delta T = 2 + 2\,(\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T - 2\,(1 + (\alpha_{\text{alum}} + \alpha_{\text{steel}})\,\Delta T)\cos\phi\ .$$

This is rearranged to yield

$$\Delta T = \frac{\cos\phi - ½}{(\alpha_{\text{alum}} + \alpha_{\text{steel}})(1 - \cos\phi) - \alpha_{\text{Invar}}} = \approx 46\,°\text{C},$$

so that the final temperature is $T = 20.0° + \Delta T = 66°$ C. Essentially the same argument, but arguably more elegant, can be made in terms of the differential of the above cosine law expression.

77. **THINK** The heat absorbed by the ice not only raises its temperature but could also change its phase – to water.

**EXPRESS** Let $m_I$ be the mass of the ice cube and $c_I$ be its specific heat. The energy required to bring the ice cube to the melting temperature (0 C°) is

$$Q_1 = c_I m_I (0\ \text{C°} - T_{Ii}) = (2220\ \text{J/kg}\cdot\text{K})(0.700\ \text{kg})(150\ \text{K}) = 2.331\times10^5\ \text{J}.$$

Since the total amount of energy transferred to the ice is $Q = 6.993\times10^5$ J, and $Q_1 < Q$, some or all the ice will melt. The energy required to melt all the ice is

$$Q_2 = m_I L_F = (0.700\,\text{kg})(3.33\times10^5\ \text{J/kg}) = 2.331\times10^5\ \text{J}.$$

However, since

$$Q_1 + Q_2 = 4.662\times10^5\ \text{J} < Q = 6.993\times10^5\ \text{J},$$

this means that all the ice will melt and the extra energy

$$\Delta Q = Q - (Q_1 + Q_2) = 6.993\times10^5\ \text{J} - 4.662\times10^5\ \text{J} = 2.331\times10^5\ \text{J}$$

would be used to raise the temperature of the water.

**ANALYZE** The final temperature of the water is given by $\Delta Q = m_I c_{\text{water}} T_f$. Substituting the values given, we have

$$T_f = \frac{\Delta Q}{m_I c_{\text{water}}} = \frac{2.331\times10^5\ \text{J}}{(0.700\ \text{kg})(4186.8\ \text{J/kg}\cdot\text{K})} = 79.5°\text{C}$$

**LEARN** The key concepts in this problem are outlined in the Sample Problem 18.04 – “Heat to change temperature and state.” An important difference with part (b) of the sample problem is that, in our case, the final state of the $H_2O$ is *all liquid* at $T_f > 0$. As discussed in part (a) of that sample problem, there are three steps to the total process.

78. (a) Using Eq. 18-32, we find the rate of energy conducted upward to be

$$P_{\text{cond}} = \frac{Q}{t} = kA\frac{T_H - T_C}{L} = (0.400\ \text{W/m}\cdot°\text{C})A\frac{5.0\ °\text{C}}{0.12\ \text{m}} = (16.7A)\ \text{W}.$$

Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.

(b) The heat of fusion in this process is $Q = L_F m$, where $L_F = 3.33\times10^5$ J/kg. Differentiating the expression with respect to *t* and equating the result with $P_{\text{cond}}$, we have

$$P_{\text{cond}} = \frac{dQ}{dt} = L_F\,\frac{dm}{dt}\,.$$

Thus, the rate of mass converted from liquid to ice is

$$\frac{dm}{dt} = \frac{P_{\text{cond}}}{L_F} = \frac{16.7A\text{ W}}{3.33\times10^5\text{ J/kg}} = (5.02\times10^{-5}A)\text{ kg/s}\,.$$

(c) Since $m = \rho V = \rho A h$, differentiating both sides of the expression gives

$$\frac{dm}{dt} = \frac{d}{dt}\left(\rho A h\right) = \rho A\frac{dh}{dt}\,.$$

Thus, the rate of change of the icicle length is

$$\frac{dh}{dt} = \frac{1}{\rho A}\frac{dm}{dt} = \frac{5.02\times10^{-5}\,\text{kg/m}^2\cdot\text{s}}{1000\text{ kg/m}^3} = 5.02\times10^{-8}\text{ m/s}$$

79. **THINK** The work done by the expanding gas is given by Eq. 18-24: $W = \int p\,dV$.

**EXPRESS** Let $V_i$ and $V_f$ be the initial and final volumes, respectively. With $p = aV^2$, the work done by the gas is

$$W = \int_{V_i}^{V_f} p\,dV = \int_{V_i}^{V_f} aV^2 dV = \frac{1}{3}a\left(V_f^3 - V_i^3\right).$$

**ANALYZE** With $a = 10\text{ N/m}^8$, $V_i = 1.0\text{ m}^3$ and $V_f = 2.0\text{ m}^3$, we obtain

$$W = \frac{1}{3}a\left(V_f^3 - V_i^3\right) = \frac{1}{3}\left(10\text{ N/m}^8\right)\left[(2.0\text{ m}^3)^3 - (1.0\text{ m}^3)^3\right] = 23\text{ J}\,.$$

**LEARN** In this problem, the initial and final pressures are

$$p_i = aV_i^2 = (10\text{ N/m}^8)(1.0\text{ m}^3)^2 = 10\text{ N/m}^2 = 10\text{ Pa}$$
$$p_f = aV_f^2 = (10\text{ N/m}^8)(2.0\text{ m}^3)^2 = 40\text{ N/m}^2 = 40\text{ Pa}$$

In this case, since $p \sim V^2$, the work done would be proportional to $V^3$ after volume integration.

80. We use $Q = -\lambda_F m_{ice} = W + \Delta E_{\text{int}}$. In this case $\Delta E_{\text{int}} = 0$. Since $\Delta T = 0$ for the ideal gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333\,\text{J/g})(100\,\text{g}) = 33.3\,\text{kJ}.$$

81. **THINK** The work done is the "area under the curve:" $W = \int p\, dV$.

**EXPRESS** According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, where $Q$ is the heat absorbed and $W$ is the work done by the system. For process 1,

$$W_1 = p_i(V_b - V_i) = p_i(5.0V_i - V_i) = 4.0\,p_iV_i$$

so that

$$\Delta E_{\text{int}} = Q - W_1 = 10\,p_iV_i - 4.0\,p_iV_i = 6.0\,p_iV_i\ .$$

Path 2 involves more work than path 1 (note the triangle in the figure of area $\frac{1}{2}(4V_i)(p_i/2) = p_iV_i$). Thus, $W_2 = W_1 + p_iV_i = 5.0\,p_iV_i$. Note that $\Delta E_{\text{int}} = 6.0\,p_iV_i$ is the same for all three paths.

**ANALYZE** (a) The energy transferred to the gas as heat in process 2 is

$$Q_2 = \Delta E_{\text{int}} + W_2 = 6.0\,p_iV_i + 5.0\,p_iV_i = 11\,p_iV_i.$$

(b) Path 3 starts at $a$ and ends at $b$ (same as paths 1 and 2), so $\Delta E_{\text{int}} = 6.0\,p_iV_i$.

**LEARN** Work $W$ and heat $Q$ in general are path-dependent quantities, i.e., they depend on how the finial state is reached. However, the combination $\Delta E_{\text{int}} = Q - W$ is path independent; it is a *state function*.

82. (a) We denote $T_H = 100°\text{C}$, $T_C = 0°\text{C}$, the temperature of the copper–aluminum junction by $T_1$. and that of the aluminum-brass junction by $T_2$. Then,

$$P_{\text{cond}} = \frac{k_cA}{L}(T_H - T_1) = \frac{k_aA}{L}(T_1 - T_2) = \frac{k_bA}{L}(T_2 - T_c).$$

We solve for $T_1$ and $T_2$ to obtain

$$T_1 = T_H + \frac{T_C - T_H}{1 + k_c(k_a + k_b)/k_ak_b} = 100°\text{C} + \frac{0.00°\text{C} - 100°\text{C}}{1 + 401(235+109)/[(235)(109)]} = 84.3°\text{C}$$

(b) and

$$T_2 = T_c + \frac{T_H - T_C}{1 + k_b(k_c + k_a)/k_c k_a} = 0.00°\text{C} + \frac{100°\text{C} - 0.00°\text{C}}{1 + 109(235 + 401)/[(235)(401)]}$$
$$= 57.6°\text{C}.$$

83. **THINK** The Pyrex disk expands as a result of heating, so we expect $\Delta V > 0$.

**EXPRESS** The initial volume of the disk (thought of as a short cylinder) is $V_0 = \pi r^2 L$ where $L$ = 0.50 cm is its thickness and $r$ = 8.0 cm is its radius. After heating, the volume becomes

$$V = \pi(r + \Delta r)^2(L + \Delta L) = \pi r^2 L + \pi r^2 \Delta L + 2\pi r L \Delta r + ...$$

where we ignore higher-order terms. Thus, the change in volume of the disk is

$$\Delta V = V - V_0 \approx \pi r^2 \Delta L + 2\pi r L \Delta r$$

**ANALYZE** With $\Delta L = L\alpha\Delta T$ and $\Delta r = r\alpha\Delta T$, the above expression becomes

$$\Delta V = \pi r^2 L\alpha\Delta T + 2\pi r^2 L\alpha\Delta T = 3\pi r^2 L\alpha\Delta T\ .$$

Substituting the values given ($\alpha = 3.2 \times 10^{-6}$/C° from Table 18-2), we obtain

$$\Delta V = 3\pi r^2 L\alpha\Delta T = 3\pi(0.080\text{ m})^2(0.0050\text{ m})(3.2\times10^{-6}/°\text{C})(60°\text{C} - 10°\text{C})$$
$$= 4.83\times10^{-8}\text{ m}^3$$

**LEARN** All dimensions of the disk expand when heated. So we must take into consideration the change in radius as well as the thickness.

84. (a) The rate of heat flow is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(0.040\text{ W/m}\cdot\text{K})(1.8\text{ m}^2)(33°\text{C} - 1.0°\text{C})}{1.0\times10^{-2}\text{ m}} = 2.3\times10^2\text{ J/s.}$$

(b) The new rate of heat flow is

$$P'_{\text{cond}} = \frac{k'P_{\text{cond}}}{k} = \frac{(0.60\text{ W/m}\cdot\text{K})(230\text{ J/s})}{0.040\text{ W/m}\cdot\text{K}} = 3.5\times10^3\text{ J/s,}$$

which is about 15 times as fast as the original heat flow.

85. **THINK** Since the system remains thermally insulated, the total energy remains unchanged. The energy released by the aluminum lump raises the water temperature.

**EXPRESS** Let $T_f$ be the final temperature of the aluminum lump-water system. The energy transferred from the aluminum is $Q_{Al} = m_{Al}c_{Al}(T_{i,Al} - T_f)$. Similarly, the energy transferred as heat into water is $Q_{\text{water}} = m_{\text{water}}c_{\text{water}}(T_f - T_{i,\text{ water}})$. Equating $Q_{Al}$ with $Q_{\text{water}}$ allows us to solve for $T_f$.

**ANALYZE** With

$$m_{Al}c_{Al}(T_{i,Al} - T_f) = m_{\text{water}}c_{\text{water}}(T_f - T_{i,\text{ water}}),$$

we find the final equilibrium temperature to be

$$\begin{aligned} T_f &= \frac{m_{Al}c_{Al}T_{i,Al} + m_{\text{water}}c_{\text{water}}T_{i,\text{water}}}{m_{Al}c_{Al} + m_{\text{water}}c_{\text{water}}} \\ &= \frac{(2.50\text{ kg})(900\text{ J/kg}\cdot\text{K})(92°\text{C}) + (8.00\text{ kg})(4186.8\text{ J/kg}\cdot\text{K})(5.0°\text{C})}{(2.50\text{ kg})(900\text{ J/kg}\cdot\text{K}) + (8.00\text{ kg})(4186.8\text{ J/kg}\cdot\text{K})} \\ &= 10.5°\text{C}. \end{aligned}$$

**LEARN** No phase change is involved in this problem, so the thermal energy transferred from the aluminum can only change the water temperature.

86. If the window is $L_1$ high and $L_2$ wide at the lower temperature and $L_1 + \Delta L_1$ high and $L_2 + \Delta L_2$ wide at the higher temperature, then its area changes from $A_1 = L_1L_2$ to

$$A_2 = (L_1 + \Delta L_1)(L_2 + \Delta L_2) \approx L_1L_2 + L_1\ \Delta L_2 + L_2\ \Delta L_1$$

where the term $\Delta L_1\ \Delta L_2$ has been omitted because it is much smaller than the other terms, if the changes in the lengths are small. Consequently, the change in area is

$$\Delta A = A_2 - A_1 = L_1\ \Delta L_2 + L_2\ \Delta L_1.$$

If $\Delta T$ is the change in temperature then $\Delta L_1 = \alpha L_1\ \Delta T$ and $\Delta L_2 = \alpha L_2\ \Delta T$, where $\alpha$ is the coefficient of linear expansion. Thus

$$\Delta A = \alpha(L_1L_2 + L_1L_2)\ \Delta T = 2\alpha L_1L_2\Delta T = 2\left(9\times10^{-6}/\text{C°}\right)(30\,\text{cm})\,(20\,\text{cm})\,(30°\text{C}) = 0.32\,\text{cm}^2.$$

87. For a cylinder of height $h$, the surface area is $A_c = 2\pi rh$, and the area of a sphere is $A_o = 4\pi R^2$. The net radiative heat transfer is given by Eq. 18-40.

(a) We estimate the surface area $A$ of the body as that of a cylinder of height 1.8 m and radius $r$ = 0.15 m plus that of a sphere of radius $R$ = 0.10 m. Thus, we have $A \approx A_c + A_o$ = 1.8 m$^2$. The emissivity $\varepsilon$ = 0.80 is given in the problem, and the Stefan-Boltzmann constant is found in Section 18-11: $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K$^4$. The "environment" temperature is $T_{\text{env}}$ = 303 K, and the skin temperature is $T = \frac{5}{9}(102 - 32) + 273 = 312$ K. Therefore,

$$P_{\text{net}} = \sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right) = -86\,\text{W}.$$

The corresponding sign convention is discussed in the textbook immediately after Eq. 18-40. We conclude that heat is being lost by the body at a rate of roughly 90 W.

(b) Half the body surface area is roughly $A$ = 1.8/2 = 0.9 m$^2$. Now, with $T_{\text{env}}$ = 248 K, we find

$$|P_{\text{net}}| = |\sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right)| \approx 2.3\times10^2\ \text{W}.$$

(c) Finally, with $T_{\text{env}}$ = 193 K (and still with $A$ = 0.9 m$^2$) we obtain $|P_{\text{net}}| = 3.3\times10^2$ W.

88. We take absolute values of Eq. 18-9 and Eq. 12-25:

$$|\Delta L| = L\alpha\,|\Delta T| \quad \text{and} \quad \left|\frac{F}{A}\right| = E\left|\frac{\Delta L}{L}\right|.$$

The ultimate strength for steel is $(F/A)_{\text{rupture}} = S_u = 400 \times 10^6$ N/m$^2$ from Table 12-1. Combining the above equations (eliminating the ratio $\Delta L/L$), we find the rod will rupture if the temperature change exceeds

$$|\Delta T| = \frac{S_u}{E\alpha} = \frac{400\times10^6\ \text{N/m}^2}{\left(200\times10^9\ \text{N/m}^2\right)\left(11\times10^{-6}/\text{C}°\right)} = 182°\text{C}.$$

Since we are dealing with a temperature decrease, then, the temperature at which the rod will rupture is $T$ = 25.0°C – 182°C = –157°C.

89. (a) Let the number of weight lift repetitions be $N$. Then $Nmgh = Q$, or (using Eq. 18-12 and the discussion preceding it)

$$N = \frac{Q}{mgh} = \frac{(3500\,\text{Cal})(4186\,\text{J/Cal})}{(80.0\,\text{kg})\left(9.80\,\text{m/s}^2\right)(1.00\,\text{m})} \approx 1.87\times10^4.$$

(b) The time required is

$$t = (18700)(2.00\,\text{s})\left(\frac{1.00\,\text{h}}{3600\,\text{s}}\right) = 10.4\,\text{h}.$$

90. For isotropic materials, the coefficient of linear expansion $\alpha$ is related to that for volume expansion by $\alpha = \frac{1}{3}\beta$ (Eq. 18-11). The radius of Earth may be found in the Appendix. With these assumptions, the radius of the Earth should have increased by approximately

$$\Delta R_E = R_E \alpha \Delta T = \left(6.4\times10^3\text{ km}\right)\left(\frac{1}{3}\right)\left(3.0\times10^{-5}/\text{K}\right)\ (3000\,\text{K}-300\,\text{K}) = 1.7\times10^2\text{ km.}$$

91. We assume the ice is at 0°C to begin with, so that the only heat needed for melting is that described by Eq. 18-16 (which requires information from Table 18-4). Thus,

$$Q = Lm = (333\text{ J/g})(1.00\text{ g}) = 333\text{ J.}$$

92. One method is to simply compute the change in length in each edge ($x_0$ = 0.200 m and $y_0$ = 0.300 m) from Eq. 18-9 ($\Delta x = 3.6 \times 10^{-5}$ m and $\Delta y = 5.4 \times 10^{-5}$ m) and then compute the area change:

$$A - A_0 = \left(x_0 + \Delta x\right)\left(y_0 + \Delta y\right) - x_0 y_0 = 2.16\times10^{-5}\,\text{m}^2.$$

Another (though related) method uses $\Delta A = 2\alpha A_0 \Delta T$ (valid for $\Delta A/A \ll 1$) which can be derived by taking the differential of $A = xy$ and replacing $d$ 's with $\Delta$'s.

93. The problem asks for 0.5% of $E$, where $E = Pt$ with $t$ = 120 s and $P$ given by Eq. 18-38. Therefore, with $A = 4\pi r^2 = 5.0 \times 10^{-3}\text{ m}^2$, we obtain

$$\left(0.005\right)Pt = \left(0.005\right)\sigma\varepsilon A T^4 t = 8.6\text{ J.}$$

94. Let the initial water temperature be $T_{wi}$ and the initial thermometer temperature be $T_{ti}$. Then, the heat absorbed by the thermometer is equal (in magnitude) to the heat lost by the water:

$$c_t m_t \left(T_f - T_{ti}\right) = c_w m_w \left(T_{wi} - T_f\right).$$

We solve for the initial temperature of the water:

$$T_{wi} = \frac{c_t m_t \left(T_f - T_{ti}\right)}{c_w m_w} + T_f = \frac{\left(0.0550\,\text{kg}\right)\left(0.837\,\text{kJ/kg}\cdot\text{K}\right)\left(44.4-15.0\right)\text{K}}{\left(4.18\,\text{kJ/kg}\cdot\text{C}^\circ\right)\left(0.300\,\text{kg}\right)} + 44.4^\circ\text{C} = 45.5^\circ\text{C.}$$

95. The net work may be computed as a sum of works (for the individual processes involved) or as the "area" (with appropriate ± sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in $pV$ diagrams:

$$\text{for a straight line: Work} = \frac{p_i + p_f}{2}\Delta V$$

which is easily verified using the definition Eq. 18-25. The cycle represented by the "triangle" $BC$ consists of three processes:

• "tilted" straight line from (1.0 m$^3$, 40 Pa) to (4.0 m$^3$, 10 Pa), with

$$\text{Work} = \frac{40\,\text{Pa} + 10\,\text{Pa}}{2}\left(4.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 75\,\text{J}$$

• horizontal line from (4.0 m$^3$, 10 Pa) to (1.0 m$^3$, 10 Pa), with

$$\text{Work} = \left(10\,\text{Pa}\right)\left(1.0\,\text{m}^3 - 4.0\,\text{m}^3\right) = -30\,\text{J}$$

• vertical line from (1.0 m$^3$, 10 Pa) to (1.0 m$^3$, 40 Pa), with

$$\text{Work} = \frac{10\,\text{Pa} + 40\,\text{Pa}}{2}\left(1.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 0$$

(a) and (b) Thus, the total work during the *BC* cycle is (75 – 30) J = 45 J. During the *BA* cycle, the "tilted" part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with Work = (40 Pa)(–3.0 m$^3$) = –120 J. Therefore, the total work during the *BA* cycle is (75 – 120) J = – 45 J.

96. (a) The total length change of the composite bar is

$$\Delta L = \Delta L_1 + \Delta L_2 = \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T = (\alpha_1 L_1 + \alpha_2 L_2)\Delta T.$$

Writing $\Delta L = \alpha L \Delta T$ and equating the two expressions leads to $\alpha = \dfrac{\alpha_1 L_1 + \alpha_2 L_2}{L}$.

(b) The coefficients of thermal expansions are $\alpha_1 = 11\times 10^{-6}\,/\,\text{C}°$ for steel and $\alpha_2 = 19\times 10^{-6}\,/\,\text{C}°$ for brass. We solve the system of equations

$$\alpha = 13\times 10^{-6}\,/\,\text{C}° = \frac{(11\times 10^{-6}\,/\,\text{C}°)L_1 + (19\times 10^{-6}\,/\,\text{C}°)L_2}{L_1 + L_2}$$
$$L = L_1 + L_2 = 52.4\text{ cm}$$

and obtain $L_1 = 39.3\text{ cm},$ and

(c) $L_2 = 13.1\text{ cm}.$

97. The heat required to raise the water of mass *m* from an initial temperature $T_i$ to final temperature $T_f$ is $Q = cm(T_f - T_i)$, where *c* is the specific heat of water. On the other hand, each shake supplies an energy $\Delta U_1 = mgh,$ where *h* is the vertical distance the water has moved during each shake. Thus, with 27 shakes/min, the time required to raise the water temperature to $T_f$ is

$$\Delta t = \frac{Q}{R(\Delta U_1)} = \frac{cm(T_f - T_i)}{Rmgh} = \frac{c(T_f - T_i)}{Rgh} = \frac{(4186.8\,\text{J/kg}\cdot\text{C}^\circ)(100^\circ\text{C} - 19^\circ\text{C})}{(27\,\text{shakes/min})(9.8\,\text{m/s}^2)(0.32\,\text{m})}$$
$$= 4.0\times 10^3\ \text{min}.$$

98. Since the combination "$p_1V_1$" appears frequently in this derivation we denote it as "$x$. Thus for process 1, the heat transferred is $Q_1 = 5x = \Delta E_{\text{int }1} + W_1$ , and for path 2 (which consists of two steps, one at constant volume followed by an expansion accompanied by a linear pressure decrease) it is $Q_2 = 5.5x = \Delta E_{\text{int }2} + W_2$. If we subtract these two expressions and make use of the fact that internal energy is state function (and thus has the same value for path 1 as for path 2) then we have

$$5.5x - 5x = W_2 - W_1 = \text{"area" inside the triangle} = \frac{1}{2}(2\,V_1)(p_2 - p_1)\,.$$

Thus, dividing both sides by $x$ (= $p_1V_1$), we find $0.5 = (p_2/p_1) - 1$, which leads immediately to the result: $p_2/p_1 = 1.5$ .

99. The cube has six faces, each of which has an area of $(6.0\times 10^{-6}\ \text{m})^2$. Using Kelvin temperatures and Eq. 18-40, we obtain

$$P_{\text{net}} = \sigma\varepsilon A\,(T_{\text{env}}^4 - T^4)$$
$$= \left(5.67\times 10^{-8}\,\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(0.75)\left(2.16\times 10^{-10}\ \text{m}^2\right)\left((123.15\ \text{K})^4 - (173.15\ \text{K})^4\right)$$
$$= -6.1\times 10^{-9}\ \text{W}.$$

100. We denote the density of the liquid as $\rho$, the rate of liquid flowing in the calorimeter as $\mu$, the specific heat of the liquid as $c$, the rate of heat flow as $P$, and the temperature change as $\Delta T$. Consider a time duration $dt$, during this time interval, the amount of liquid being heated is $dm = \mu\rho dt$. The energy required for the heating is

$$dQ = Pdt = c(dm)\,\Delta T = c\mu\Delta T dt.$$

Thus,

$$c = \frac{P}{\rho\mu\Delta T} = \frac{250\,\text{W}}{(8.0\times 10^{-6}\ \text{m}^3/\text{s})(0.85\times 10^3\ \text{kg/m}^3)(15^\circ\text{C})}$$
$$= 2.5\times 10^3\ \text{J/kg}\cdot\text{C}^\circ = 2.5\times 10^3\ \text{J/kg}\cdot\text{K}.$$

101. Consider the object of mass $m_1$ falling through a distance $h$. The loss of its mechanical energy is $\Delta E = m_1gh$. This amount of energy is then used to heat up the temperature of water of mass $m_2$: $\Delta E = m_1gh = Q = m_2c\Delta T$. Thus, the maximum possible rise in water temperature is

$$\Delta T = \frac{m_1 gh}{m_2 c} = \frac{(6.00\,\text{kg})(9.8\,\text{m/s}^2)(50.0\,\text{m})}{(0.600\,\text{kg})(4190\,\text{J/kg}\cdot\text{C°})} = 1.17\,\text{°C}.$$

102. When the temperature changes from $T$ to $T + \Delta T$ the diameter of the mirror changes from $D$ to $D + \Delta D$, where $\Delta D = \alpha D\,\Delta T$. Here $\alpha = 3.2 \times 10^{-6}$/C° is the coefficient of linear expansion for Pyrex glass. The range of values for the diameters can be found by setting $\Delta T$ equal to the temperature range. Thus

$$\Delta L = \alpha D \Delta T = (3.2\times10^{-6}\,/\text{C°})\left(170\text{ in.}\cdot\frac{0.0254\text{ m}}{1\text{ in.}}\right)\left(32\,\text{°C} - (-16\,\text{°C})\right)$$
$$= 6.63\times10^{-4}\,\text{m} \approx 660\ \mu\text{m}.$$

103. The change in area for the plate is

$$\Delta A = (a+\Delta a)(b+\Delta b) - ab \approx a\Delta b + b\Delta a = 2ab\alpha\Delta T = 2\alpha A\Delta T$$
$$= 2(32\times10^{-6}\,/\text{C°})(1.4\text{ m}^2)(89\,\text{°C}) = 7.97\times10^{-3}\,\text{m}^2 \approx 8.0\times10^{-3}\,\text{m}^2.$$

104. The relative volume change is

$$\frac{\Delta V}{V} = \beta\Delta T = (6.6\times10^{-4}\,/\text{C°})(12\,\text{°C}) = 7.92\times10^{-3}.$$

Since the expansion the glass tube can be ignored, the cross-sectional area of the liquid remains unchanged, and we have $\frac{\Delta h}{h} = \frac{\Delta V}{V} = 7.92\times10^{-3}$.

105. (a) We note that if the pendulum shortens, its frequency of oscillation will increase, thereby causing it to record more units of time ("ticks") than have actually passed during an interval. Thus, as the pendulum contracts (this problem involves cooling the brass wire), the pendulum will "run fast."

(b) The period of the pendulum is $\tau = 2\pi\sqrt{L/g}$ (so not to be confused with temperature $T$). Differentiating $\tau$ with respect to $L$ gives

$$\frac{d\tau}{dL} = \frac{d}{dL}\left(2\pi\sqrt{\frac{L}{g}}\right) = \pi\frac{1}{\sqrt{Lg}} = \frac{1}{2L}\left(2\pi\sqrt{\frac{L}{g}}\right) = \frac{\tau}{2L}.$$

Thus,

$$\Delta\tau = \frac{\tau\Delta L}{2L} = \frac{1}{2}\tau\alpha\Delta T.$$

Substituting the values given, the change in period is

$$\Delta\tau = \frac{1}{2}\tau\alpha\Delta T = \frac{1}{2}\left(\frac{3600\text{ s}}{1\text{ h}}\right)(19\times10^{-6}/\text{C}°)(23\text{C}°) = 0.787\text{ s/h}.$$

106. Recalling that 1 W = 1 J/s, the heat $Q$ which is added to the room in 6.9 h is

$$Q = 4(100\text{ W})(0.73)(6.9\text{ h})\left(\frac{3600\text{ s}}{1.00\text{ h}}\right) = 7.25\times10^{6}\text{ J}.$$

107. With 1 Calorie = 1000 cal, we find the athlete's rate of dissipating energy to be

$$P = 4000\text{ Cal/day} = \frac{\left(4000\times10^{3}\text{ cal}\right)\left(4.1868\text{ J/cal}\right)}{\left(1\text{day}\right)\left(86400\text{ s/day}\right)} = 193.83\text{ W},$$

which is about 1.9 times as much as the power of a 100 W light bulb.

108. The initial speed of the car is $v_i = 83\text{ km/h} = (83\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = 23.056\text{ m/s}$. The deceleration $a$ of the car is given by $v_f^2 - v_i^2 = -v_i^2 = 2ad$, or

$$a = -\frac{(23.056\text{ m/s})^2}{2(93\text{ m})} = -2.86\text{ m/s}^2.$$

The time $\Delta t$ it takes for the car to stop is then

$$\Delta t = \frac{v_f - v_i}{a} = \frac{-23.056\text{ m/s}}{-2.86\text{ m/s}^2} = 8.07\text{ s}.$$

The change in kinetic energy of the car is

$$\Delta K = -\frac{1}{2}mv_i^2 = -\frac{1}{2}(1700\text{ kg})(23.056\text{ m/s})^2 = -4.52\times10^{5}\text{ J}.$$

Thus, the average rate at which mechanical energy is transferred to thermal energy is

$$P = \frac{\Delta E_{\text{th}}}{\Delta t} = \frac{-\Delta K}{\Delta t} = \frac{4.52\times10^{5}\text{ J}}{8.07\text{ s}} = 5.6\times10^{4}\text{ W}.$$

# Chapter 19

1. Each atom has a mass of $m = M/N_A$, where $M$ is the molar mass and $N_A$ is the Avogadro constant. The molar mass of arsenic is 74.9 g/mol or $74.9\times10^{-3}$ kg/mol. Therefore, $7.50 \times 10^{24}$ arsenic atoms have a total mass of

$$(7.50 \times 10^{24})\,(74.9 \times 10^{-3}\ \text{kg/mol})/(6.02 \times 10^{23}\ \text{mol}^{-1}) = 0.933\ \text{kg}.$$

2. (a) Equation 19-3 yields $n = M_{sam}/M = 2.5/197 = 0.0127$ mol.

(b) The number of atoms is found from Eq. 19-2:

$$N = nN_A = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}.$$

3. **THINK** We treat the oxygen gas in this problem as ideal and apply the ideal-gas law.

**EXPRESS** In solving the ideal-gas law equation $pV = nRT$ for $n$, we first convert the temperature to the Kelvin scale: $T_i = (40.0 + 273.15)\,\text{K} = 313.15\ \text{K}$, and the volume to SI units: $V_i = 1000\ \text{cm}^3 = 10^{-3}\,\text{m}^3$.

**ANALYZE** (a) The number of moles of oxygen present is

$$n = \frac{pV_i}{RT_i} = \frac{(1.01 \times 10^5\ \text{Pa})(1.000 \times 10^{-3}\ \text{m}^3)}{(8.31\,\text{J/mol}\cdot\text{K})(313.15\,\text{K})} = 3.88 \times 10^{-2}\,\text{mol}.$$

(b) Similarly, the ideal gas law $pV = nRT$ leads to

$$T_f = \frac{pV_f}{nR} = \frac{(1.06 \times 10^5\ \text{Pa})(1.500 \times 10^{-3}\ \text{m}^3)}{(3.88 \times 10^{-2}\ \text{mol})(8.31\,\text{J/mol}\cdot\text{K})} = 493\,\text{K}.$$

We note that the final temperature may be expressed in degrees Celsius as 220°C.

**LEARN** The final temperature can also be calculated by noting that $\dfrac{p_iV_i}{T_i} = \dfrac{p_fV_f}{T_f}$ , or

$$T_f = \left(\frac{p_f}{p_i}\right)\left(\frac{V_f}{V_i}\right)T_i = \left(\frac{1.06\times10^5\ \text{Pa}}{1.01\times10^5\ \text{Pa}}\right)\left(\frac{1500\ \text{cm}^3}{1000\ \text{cm}^3}\right)(313.15\ \text{K}) = 493\,\text{K}.$$

4. (a) With $T$ = 283 K, we obtain

$$n = \frac{pV}{RT} = \frac{100 \times 10^3 \text{ Pa} \quad 2.50\,\text{m}^3}{8.31\,\text{J/mol}\cdot\text{K} \quad 283\,\text{K}} = 106\,\text{mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and temperature, and solve the ideal gas law for the new volume, or we could set up the gas law in ratio form as:

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

(where $n_i = n_f$ and thus cancels out), which yields a final volume of

$$V_f = V_i\left(\frac{p_i}{p_f}\right)\left(\frac{T_f}{T_i}\right) = \left(2.50\,\text{m}^3\right)\left(\frac{100\,\text{kPa}}{300\,\text{kPa}}\right)\left(\frac{303\,\text{K}}{283\,\text{K}}\right) = 0.892 \text{ m}^3.$$

5. With $V = 1.0 \times 10^{-6}$ m$^3$, $p = 1.01 \times 10^{-13}$ Pa, and $T$ = 293 K, the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^{-13} \text{ Pa}\right)\left(1.0 \times 10^{-6} \text{ m}^3\right)}{\left(8.31 \text{ J/mol}\cdot\text{K}\right)\left(293 \text{ K}\right)} = 4.1 \times 10^{-23}\,\text{mole}.$$

Consequently, Eq. 19-2 yields $N = nN_A$ = 25 molecules. We can express this as a ratio (with $V$ now written as 1 cm$^3$) $N/V$ = 25 molecules/cm$^3$.

6. The initial and final temperatures are $T_i = 5.00°\text{C} = 278 \text{ K}$ and $T_f = 75.0°\text{C} = 348 \text{ K}$, respectively. Using the ideal gas law with $V_i = V_f$, we find the final pressure to be

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad \Rightarrow \quad p_f = \frac{T_f}{T_i} p_i = \left(\frac{348\,\text{K}}{278\,\text{K}}\right)\left(1.00 \text{ atm}\right) = 1.25 \text{ atm}.$$

7. (a) Equation 19-45 (which gives 0) implies $Q = W$. Then Eq. 19-14, with $T$ = (273 + 30.0)K leads to gives $Q = -3.14 \times 10^3$ J, or $|Q| = 3.14 \times 10^3$ J.

(b) That negative sign in the result of part (a) implies the transfer of heat is *from* the gas.

8. (a) We solve the ideal gas law $pV = nRT$ for $n$:

$$n = \frac{pV}{RT} = \frac{\left(100\,\text{Pa}\right)\left(1.0\times10^{-6}\,\text{m}^3\right)}{\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(220\,\text{K}\right)} = 5.47 \times 10^{-8}\,\text{mol}.$$

(b) Using Eq. 19-2, the number of molecules $N$ is

$$N = nN_A = \left(5.47\times10^{-6}\text{ mol}\right)\left(6.02\times10^{23}\text{ mol}^{-1}\right) = 3.29\times10^{16}\text{ molecules.}$$

9. Since (standard) air pressure is 101 kPa, then the initial (absolute) pressure of the air is $p_i$ = 266 kPa. Setting up the gas law in ratio form (where $n_i = n_f$ and thus cancels out), we have

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i}$$

which yields

$$p_f = p_i\left(\frac{V_i}{V_f}\right)\left(\frac{T_f}{T_i}\right) = (266\text{ kPa})\left(\frac{1.64\times10^{-2}\text{ m}^3}{1.67\times10^{-2}\text{ m}^3}\right)\left(\frac{300\text{ K}}{273\text{ K}}\right) = 287\text{ kPa}\,.$$

Expressed as a gauge pressure, we subtract 101 kPa and obtain 186 kPa.

10. The pressure $p_1$ due to the first gas is $p_1 = n_1RT/V$, and the pressure $p_2$ due to the second gas is $p_2 = n_2RT/V$. So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1RT}{V} + \frac{n_2RT}{V} = (n_1 + n_2)\,\frac{RT}{V}.$$

The fraction of $P$ due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2RT/V}{(n_1+n_2)(RT/V)} = \frac{n_2}{n_1+n_2} = \frac{0.5}{2+0.5} = 0.2.$$

11. **THINK** The process consists of two steps: isothermal expansion, followed by isobaric (constant-pressure) compression. The total work done by the air is the sum of the works done for the two steps.

**EXPRESS** Suppose the gas expands from volume $V_i$ to volume $V_f$ during the isothermal portion of the process. The work it does is

$$W_1 = \int_{V_i}^{V_f} p\,dV = nRT\int_{V_i}^{V_f}\frac{dV}{V} = nRT\,\ln\frac{V_f}{V_i},$$

where the ideal gas law $pV = nRT$ was used to replace $p$ with $nRT/V$. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace $nRT$ with $p_iV_i$ to obtain

$$W_1 = p_iV_i\,\ln\frac{p_i}{p_f}.$$

During the constant-pressure portion of the process the work done by the gas is $W_2 = p_f(V_i - V_f)$. The gas starts in a state with pressure $p_f$, so this is the pressure throughout this portion of the process. We also note that the volume decreases from $V_f$ to $V_i$. Now $V_f = p_iV_i/p_f$, so

$$W_2 = p_f\left(V_i - \frac{p_iV_i}{p_f}\right) = \left(p_f - p_i\right)V_i .$$

**ANALYZE** For the first portion, since the initial gauge pressure is $1.03 \times 10^5$ Pa,

$$p_i = 1.03 \times 10^5 \text{ Pa} + 1.013 \times 10^5 \text{ Pa} = 2.04 \times 10^5 \text{ Pa}.$$

The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus,

$$W_1 = \left(2.04 \times 10^5 \text{ Pa}\right)\left(0.14 \text{ m}^3\right) \ln\left(\frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}}\right) = 2.00 \times 10^4 \text{ J}.$$

Similarly, for the second portion, we have

$$W_2 = (p_f - p_i)V_i = (1.013\times10^5 \text{ Pa} - 2.04\times10^5 \text{ Pa})(0.14 \text{ m}^3) = -1.44\times10^4 \text{ J}.$$

The total work done by the gas over the entire process is

$$W = W_1 + W_2 = 2.00 \times 10^4 \text{ J} + (-1.44 \times 10^4 \text{ J}) = 5.60 \times 10^3 \text{ J}.$$

**LEARN** The work done by the gas is positive when it expands, and negative when it contracts.

12. (a) At the surface, the air volume is

$$V_1 = Ah = \pi(1.00 \text{ m})^2(4.00 \text{ m}) = 12.57 \text{ m}^3 \approx 12.6 \text{ m}^3.$$

(b) The temperature and pressure of the air inside the submarine at the surface are $T_1 = 20°\text{C} = 293 \text{ K}$ and $p_1 = p_0 = 1.00 \text{ atm}$. On the other hand, at depth $h = 80$ m, we have $T_2 = -30°\text{C} = 243 \text{ K}$ and

$$p_2 = p_0 + \rho gh = 1.00 \text{ atm} + (1024 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(80.0 \text{ m})\frac{1.00 \text{ atm}}{1.01\times10^5 \text{ Pa}}$$
$$= 1.00 \text{ atm} + 7.95 \text{ atm} = 8.95 \text{ atm}.$$

Therefore, using the ideal gas law, $pV = NkT$, the air volume at this depth would be

$$\frac{p_1V_1}{p_2V_2}=\frac{T_1}{T_2} \quad\Rightarrow\quad V_2=\left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)V_1=\left(\frac{1.00\text{ atm}}{8.95\text{ atm}}\right)\left(\frac{243\,\text{K}}{293\,\text{K}}\right)\left(12.57\text{ m}^3\right)=1.16\text{ m}^3.$$

(c) The decrease in volume is $\Delta V = V_1 - V_2 = 11.44\text{ m}^3$. Using Eq. 19-5, the amount of air this volume corresponds to is

$$n=\frac{p\Delta V}{RT}=\frac{(8.95\text{ atm})\left(1.01\times10^5\text{ Pa/atm}\right)\left(11.44\,\text{m}^3\right)}{\left(8.31\text{ J/mol}\cdot\text{K}\right)\left(243\,\text{K}\right)}=5.10\times10^3\,\text{mol}.$$

Thus, in order for the submarine to maintain the original air volume in the chamber, $5.10\times10^3\,\text{mol}$ of air must be released.

13. (a) At point *a*, we know enough information to compute *n*:

$$n=\frac{pV}{RT}=\frac{\left(2500\,\text{Pa}\right)\left(1.0\,\text{m}^3\right)}{\left(8.31\text{ J/mol}\cdot\text{K}\right)\left(200\,\text{K}\right)}=1.5\,\text{mol}.$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law in terms of ratios (note: $n_a = n_b$ and cancels out):

$$\frac{p_bV_b}{p_aV_a}=\frac{T_b}{T_a}\Rightarrow T_b=\left(200\,\text{K}\right)\left(\frac{7.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at *b* of $T_b = 1.8\times10^3$ K.

(c) As in the previous part, we choose to approach this using the gas law in ratio form:

$$\frac{p_cV_c}{p_aV_a}=\frac{T_c}{T_a}\Rightarrow T_c=\left(200\,\text{K}\right)\left(\frac{2.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at *c* of $T_c = 6.0\times10^2$ K.

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the *pV* diagram shown in Fig. 19-20). This, in turn, is related to ± "area" inside that triangle (with $\text{area} = \frac{1}{2}(\text{base})(\text{height})$), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_{\text{net}} = W_{\text{net}} = \frac{1}{2}\left(2.0\,\text{m}^3\right)\left(5.0\times10^3\ \text{Pa}\right) = 5.0\times10^3\ \text{J}.$$

14. Since the pressure is constant the work is given by $W = p(V_2 - V_1)$. The initial volume is $V_1 = (AT_1 - BT_1^2)/p$, where $T_1 = 315$ K is the initial temperature, $A$ =24.9 J/K and $B$ = 0.00662 J/K$^2$. The final volume is $V_2 = (AT_2 - BT_2^2)/p$, where $T_2 = 315$ K. Thus

$$\begin{aligned} W &= A(T_2 - T_1) - B(T_2^2 - T_1^2) \\ &= (24.9\ \text{J/K})(325\ \text{K} - 315\ \text{K}) - (0.00662\ \text{J/K}^2)[(325\ \text{K})^2 - (315\ \text{K})^2] = 207\ \text{J}. \end{aligned}$$

15. Using Eq. 19-14, we note that since it is an isothermal process (involving an ideal gas) then $Q = W = nRT\ln(V_f/V_i)$ applies at any point on the graph. An easy one to read is $Q$ = 1000 J and $V_f = 0.30$ m$^3$, and we can also infer from the graph that $V_i = 0.20$ m$^3$. We are told that $n = 0.825$ mol, so the above relation immediately yields $T = 360$ K.

16. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If $d$ is the depth of the lake and $\rho$ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho gd$, where $p_0$ is atmospheric pressure. Since $p_1V_1 = nRT_1$, the number of moles of gas in the bubble is

$$n = p_1V_1/RT_1 = (p_0 + \rho gd)V_1/RT_1,$$

where $V_1$ is the volume of the bubble at the bottom of the lake and $T_1$ is the temperature there. At the surface of the lake the pressure is $p_0$ and the volume of the bubble is $V_2 = nRT_2/p_0$. We substitute for $n$ to obtain

$$\begin{aligned} V_2 &= \frac{T_2}{T_1}\,\frac{p_0 + \rho gd}{p_0}\,V_1 \\ &= \left(\frac{293\,\text{K}}{277\,\text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + \left(0.998\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(40\,\text{m}\right)}{1.013\times10^5\ \text{Pa}}\right)\left(20\,\text{cm}^3\right) \\ &= 1.0\times10^2\ \text{cm}^3. \end{aligned}$$

17. When the valve is closed the number of moles of the gas in container $A$ is $n_A = p_AV_A/RT_A$ and that in container $B$ is $n_B = 4p_BV_A/RT_B$. The total number of moles in both containers is then

$$n = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right) = \text{const.}$$

After the valve is opened, the pressure in container $A$ is $p'_A = Rn'_AT_A/V_A$ and that in container $B$ is $p'_B = Rn'_BT_B/4V_A$. Equating $p'_A$ and $p'_B$, we obtain $Rn'_AT_A/V_A = Rn'_BT_B/4V_A$, or $n'_B = (4T_A/T_B)n'_A$. Thus,

$$n = n'_A + n'_B = n'_A\left(1 + \frac{4T_A}{T_B}\right) = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right).$$

We solve the above equation for $n'_A$:

$$n'_A = \frac{V}{R}\,\frac{p_A/T_A + 4p_B/T_B}{1 + 4T_A/T_B}.$$

Substituting this expression for $n'_A$ into $p'V_A = n'_A R T_A$, we obtain the final pressure:

$$p' = \frac{n'_A R T_A}{V_A} = \frac{p_A + 4p_B T_A/T_B}{1 + 4T_A/T_B} = 2.0 \times 10^5 \text{ Pa}.$$

18. First we rewrite Eq. 19-22 using Eq. 19-4 and Eq. 19-7:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(kN_{\text{A}})T}{(mN_{\text{A}})}} = \sqrt{\frac{3kT}{m}}.$$

The mass of the electron is given in the problem, and $k = 1.38 \times 10^{-23}$ J/K is given in the textbook. With $T = 2.00 \times 10^6$ K, the above expression gives $v_{\text{rms}} = 9.53 \times 10^6$ m/s. The pressure value given in the problem is not used in the solution.

19. Table 19-1 gives $M = 28.0$ g/mol for nitrogen. This value can be used in Eq. 19-22 with $T$ in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 19-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using $v$ for $v_{\text{rms}}$:

$$\frac{v_2}{v_1} = \frac{\sqrt{3RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{T_2}{T_1}}.$$

(a) With $T_2 = (20.0 + 273.15)$ K ≈ 293 K, we obtain $v_2 = (517\text{ m/s})\sqrt{\frac{293\text{ K}}{300\text{ K}}} = 511\text{ m/s}.$

(b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for $T_3$:

$$T_3 = T_2\left(\frac{v_3}{v_2}\right)^2 = (293\text{ K})\left(\frac{1}{2}\right)^2 = 73.0\text{ K}$$

which we write as 73.0 – 273 = – 200°C.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2\left(\frac{v_4}{v_2}\right)^2 = (293\,\text{K})(4) = 1.17 \times 10^3\ \text{K}$$

which is equivalent to 899°C.

20. Appendix F gives $M$ = 4.00 × $10^{-3}$ kg/mol (Table 19-1 gives this to fewer significant figures). Using Eq. 19-22, we obtain

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\ \ 8.31\ \text{J/mol}\cdot\text{K}\ \ \ 1000\,\text{K}}{4.00\times10^{-3}\ \text{kg/mol}}} = 2.50\times10^3\ \text{m/s}.$$

21. **THINK** According to kinetic theory, the rms speed is (see Eq. 19-34) $v_{\text{rms}} = \sqrt{3RT/M}$ , where $T$ is the temperature and $M$ is the molar mass.

**EXPRESS** The rms speed is defined as $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$ , where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v)dv$ , with the Maxwell's speed distribution function given by

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT}\ .$$

According to Table 19-1, the molar mass of molecular hydrogen is 2.02 g/mol = 2.02 × $10^{-3}$ kg/mol.

**ANALYZE** At $T = 2.7\ \text{K}$, we find the rms speed to be

$$v_{\text{rms}} = \sqrt{\frac{3\,(8.31\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02\times10^{-3}\ \text{kg/mol}}} = 1.8\times10^2\ \text{m/s}.$$

**LEARN** The corresponding average speed and most probable speed are

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.31\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{\pi(2.02\times10^{-3}\ \text{kg/mol})}} = 1.7\times10^2\ \text{m/s}$$

and

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2(8.31\text{J/mol}\cdot\text{K})(2.7\,\text{K})}{2.02\times10^{-3}\ \text{kg/mol}}} = 1.5\times10^2\ \text{m/s}\,,$$

respectively.

22. The molar mass of argon is 39.95 g/mol. Eq. 19-22 gives

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.31\,\text{J/mol}\cdot\text{K}\right)\left(313\,\text{K}\right)}{39.95\times10^{-3}\,\text{kg/mol}}} = 442\,\text{m/s}.$$

23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv\cos\theta$, where $m$ is the mass of the molecule, $v$ is its speed, and $\theta$ is the angle between its velocity and the normal to the wall. If $N$ molecules collide with the wall, then the change in their total momentum is $2Nmv\cos\theta$, and if the total time taken for the collisions is $\Delta t$, then the average rate of change of the total momentum is $2(N/\Delta t)mv\cos\theta$. This is the average force exerted by the $N$ molecules on the wall, and the pressure is the average force per unit area:

$$p = \frac{2}{A}\left(\frac{N}{\Delta t}\right) mv\cos\theta = \left(\frac{2}{2.0\times10^{-4}\,\text{m}^2}\right)\left(1.0\times10^{23}\,\text{s}^{-1}\right)\left(3.3\times10^{-27}\,\text{kg}\right)\left(1.0\times10^{3}\,\text{m/s}\right)\cos 55^\circ$$
$$= 1.9\times10^{3}\,\text{Pa}.$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to $\text{m}^2$.

24. We can express the ideal gas law in terms of density using $n = M_{\text{sam}}/M$:

$$pV = \frac{M_{\text{sam}}RT}{M} \Rightarrow \rho = \frac{pM}{RT}.$$

We can also use this to write the rms speed formula in terms of density:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(pM/\rho)}{M}} = \sqrt{\frac{3p}{\rho}}.$$

(a) We convert to SI units: $\rho = 1.24\times10^{-2}\,\text{kg/m}^3$ and $p = 1.01\times10^{3}$ Pa. The rms speed is $\sqrt{3(1010)/0.0124} = 494$ m/s.

(b) We find $M$ from $\rho = pM/RT$ with $T = 273$ K.

$$M = \frac{\rho RT}{p} = \frac{(0.0124\,\text{kg/m}^3)\ \ 8.31\,\text{J/mol}\cdot\text{K}\ \ (273\,\text{K})}{1.01\times10^{3}\,\text{Pa}} = 0.0279\ \text{kg/mol} = 27.9\ \text{g/mol}.$$

(c) From Table 19.1, we identify the gas to be $N_2$.

25. (a) Equation 19-24 gives $K_{\text{avg}} = \frac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)(273\,\text{K}) = 5.65\times10^{-21}$ J .

(b) For $T = 373$ K, the average translational kinetic energy is $K_{\text{avg}} = 7.72 \times 10^{-21}$ J .

(c) The unit mole may be thought of as a (large) collection: $6.02 \times 10^{23}$ molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

$$K_{\text{mole}} = N_A K_{\text{avg}} = (6.02\times10^{23})(5.65\times10^{-21}\text{ J}) = 3.40\times10^{3}\text{ J}.$$

(d) Similarly, the result from part (b) leads to

$$K_{\text{mole}} = (6.02\times10^{23})(7.72\times10^{-21}\text{ J}) = 4.65\times10^{3}\text{ J}.$$

26. The average translational kinetic energy is given by $K_{\text{avg}} = \frac{3}{2}kT$ , where $k$ is the Boltzmann constant ($1.38 \times 10^{-23}$ J/K) and $T$ is the temperature on the Kelvin scale. Thus

$$K_{\text{avg}} = \frac{3}{2}(1.38\times10^{-23}\text{ J/K})(1600\text{ K}) = 3.31\times10^{-20}\text{ J}.$$

27. (a) We use $\varepsilon = L_V/N$, where $L_V$ is the heat of vaporization and $N$ is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol, so the molar mass of $H_2O$ is (1.0 + 1.0 + 16) = 18 g/mol. There are $N_A = 6.02 \times 10^{23}$ molecules in a mole, so the number of molecules in a gram of water is $(6.02 \times 10^{23}\text{ mol}^{-1})/(18\text{ g/mol}) = 3.34 \times 10^{22}$ molecules/g. Thus

$$\varepsilon = (539\text{ cal/g})/(3.34 \times 10^{22}/\text{g}) = 1.61 \times 10^{-20}\text{ cal} = 6.76 \times 10^{-20}\text{ J}.$$

(b) The average translational kinetic energy is

$$K_{\text{avg}} = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23}\text{ J/K})[(32.0+273.15)\text{ K}] = 6.32\times10^{-21}\text{ J}.$$

The ratio $\varepsilon/K_{\text{avg}}$ is $(6.76 \times 10^{-20}\text{ J})/(6.32 \times 10^{-21}\text{ J}) = 10.7$.

28. Using $v = f\lambda$ with $v = 331$ m/s (see Table 17-1) with Eq. 19-2 and Eq. 19-25 leads to

$$\begin{aligned} f &= \frac{v}{\left(\dfrac{1}{\sqrt{2}\pi d^2 (N/V)}\right)} = (331\text{ m/s})\pi\sqrt{2}\,(3.0\times10^{-10}\text{ m})^2\left(\frac{nN_A}{V}\right) \\ &= \left(8.0\times10^{7}\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{n}{V}\right) = \left(8.0\times10^{7}\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{1.01\times10^{5}\text{ Pa}}{(8.31\text{ J/mol}\cdot\text{K})(273.15\text{ K})}\right) \\ &= 3.5\times10^{9}\text{ Hz} \end{aligned}$$

where we have used the ideal gas law and substituted $n/V = p/RT$. If we instead use $v$ = 343 m/s (the "default value" for speed of sound in air, used repeatedly in Ch. 17), then the answer is $3.7 \times 10^9$ Hz.

29. **THINK** Mean free path is the average distance traveled by a molecule between successive collisions.

**EXPRESS** According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where $d$ is the diameter of a molecule and $N$ is the number of molecules in volume $V$.

**ANALYZE** (a) Substituting $d = 2.0 \times 10^{-10}$ m and $N/V = 1 \times 10^6$ molecules/m$^3$, we obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10}\ \text{m})^2 (1 \times 10^6\ \text{m}^{-3})} = 6 \times 10^{12}\ \text{m}.$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

**LEARN** Mean free path is inversely proportional to the number density, $N/V$. The typical value of $N/V$ at room temperature and atmospheric pressure for ideal gas is

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01\times10^5\ \text{Pa}}{(1.38\times10^{-23}\ \text{J/K})(298\ \text{K})} = 2.46\times10^{25}\ \text{molecules/m}^3 = 2.46\times10^{19}\ \text{molecules/cm}^3.$$

This is much higher than that in the outer space.

30. We solve Eq. 19-25 for $d$:

$$d = \sqrt{\frac{1}{\lambda\pi\sqrt{2}\,(N/V)}} = \sqrt{\frac{1}{(0.80 \times 10^5\ \text{cm})\ \pi\sqrt{2}\,(2.7 \times 10^{19}/\text{cm}^3)}}$$

which yields $d = 3.2 \times 10^{-8}$ cm, or 0.32 nm.

31. (a) We use the ideal gas law $pV = nRT = NkT$, where $p$ is the pressure, $V$ is the volume, $T$ is the temperature, $n$ is the number of moles, and $N$ is the number of molecules. The substitutions $N = nN_A$ and $k = R/N_A$ were made. Since 1 cm of mercury = 1333 Pa, the pressure is $p = (10^{-7})(1333\ \text{Pa}) = 1.333 \times 10^{-4}$ Pa. Thus,

$$\frac{N}{V}=\frac{p}{kT}=\frac{1.333\times10^{-4}\ \text{Pa}}{(1.38\times10^{-23}\ \text{J/K})(295\,\text{K})}=3.27\times10^{16}\ \text{molecules/m}^3=3.27\times10^{10}\ \text{molecules/cm}^3.$$

(b) The molecular diameter is $d = 2.00 \times 10^{-10}$ m, so, according to Eq. 19-25, the mean free path is

$$\lambda=\frac{1}{\sqrt{2}\pi d^2 N/V}=\frac{1}{\sqrt{2}\pi(2.00\times10^{-10}\ \text{m})^2(3.27\times10^{16}\ \text{m}^{-3})}=172\,\text{m}.$$

32. (a) We set up a ratio using Eq. 19-25:

$$\frac{\lambda_{\text{Ar}}}{\lambda_{\text{N}_2}}=\frac{1/\left(\pi\sqrt{2}d_{\text{Ar}}^2\,(N/V)\right)}{1/\left(\pi\sqrt{2}d_{\text{N}_2}^2\,(N/V)\right)}=\left(\frac{d_{\text{N}_2}}{d_{\text{Ar}}}\right)^2.$$

Therefore, we obtain

$$\frac{d_{\text{Ar}}}{d_{\text{N}_2}}=\sqrt{\frac{\lambda_{\text{N}_2}}{\lambda_{\text{Ar}}}}=\sqrt{\frac{27.5\times10^{-6}\ \text{cm}}{9.9\times10^{-6}\ \text{cm}}}=1.7.$$

(b) Using Eq. 19-2 and the ideal gas law, we substitute $N/V = N_A n/V = N_A p/RT$ into Eq. 19-25 and find

$$\lambda=\frac{RT}{\pi\sqrt{2}d^2 pN_A}.$$

Comparing (for the same species of molecule) at two different pressures and temperatures, this leads to

$$\frac{\lambda_2}{\lambda_1}=\left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right).$$

With $\lambda_1 = 9.9 \times 10^{-6}$ cm, $T_1$ = 293 K (the same as $T_2$ in this part), $p_1$ = 750 torr, and $p_2$ = 150 torr, we find $\lambda_2 = 5.0 \times 10^{-5}$ cm.

(c) The ratio set up in part (b), using the same values for quantities with subscript 1, leads to $\lambda_2 = 7.9 \times 10^{-6}$ cm for $T_2$ = 233 K and $p_2$ = 750 torr.

33. **THINK** We're given the speeds of 10 molecules. The speed distribution is discrete.

**EXPRESS** The average speed is $\bar{v}=\frac{\sum v}{N}$, where the sum is over the speeds of the particles and $N$ is the number of particles. Similarly, the rms speed is given by $v_{\text{rms}}=\sqrt{\frac{\sum v^2}{N}}$.

**ANALYZE** (a) From the equation above, we find the average speed to be

$$\bar{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0)\,\text{km/s}}{10} = 6.5\,\text{km/s}.$$

(b) With

$$\sum v^2 = [(2.0)^2+(3.0)^2+(4.0)^2+(5.0)^2+(6.0)^2 + (7.0)^2+(8.0)^2+(9.0)^2+(10.0)^2+(11.0)^2]\ \text{km}^2/\text{s}^2 = 505\ \text{km}^2/\text{s}^2$$

the rms speed is

$$v_{\text{rms}} = \sqrt{\frac{505\ \text{km}^2/\text{s}^2}{10}} = 7.1\,\text{km/s}.$$

**LEARN** Each speed is weighted equally in calculating the average and the rms values.

34. (a) The average speed is

$$v_{\text{avg}} = \frac{\sum n_i v_i}{\sum n_i} = \frac{[2(1.0)+4(2.0)+6(3.0)+8(4.0)+2(5.0)]\ \text{cm/s}}{2+4+6+8+2} = 3.2\,\text{cm/s}.$$

(b) From $v_{\text{rms}} = \sqrt{\sum n_i v_i^{\,2} / \sum n_i}$ we get

$$v_{\text{rms}} = \sqrt{\frac{2(1.0)^2+4(2.0)^2+6(3.0)^2+8(4.0)^2+2(5.0)^2}{2+4+6+8+2}}\ \text{cm/s} = 3.4\,\text{cm/s}.$$

(c) There are eight particles at $v$ = 4.0 cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

35. (a) The average speed is

$$v_{\text{avg}} = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200\ \text{m/s})+2(500\ \text{m/s})+4(600\ \text{m/s})] = 420\ \text{m/s}.$$

(b) The rms speed is

$$v_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200\ \text{m/s})^2+2(500\ \text{m/s})^2+4(600\ \text{m/s})^2]} = 458\ \text{m/s}$$

(c) Yes, $v_{\text{rms}} > v_{\text{avg}}$.

36. We divide Eq. 19-35 by Eq. 19-22:

$$\frac{v_P}{v_{\rm rms}} = \frac{\sqrt{2RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{2T_2}{3T_1}}$$

which, for $v_P = v_{\rm rms}$, leads to

$$\frac{T_2}{T_1} = \frac{3}{2}\left(\frac{v_P}{v_{\rm rms}}\right)^2 = \frac{3}{2} .$$

37. **THINK** From the distribution function $P(v)$, we can calculate the average and rms speeds.

**EXPRESS** The distribution function gives the fraction of particles with speeds between $v$ and $v + dv$, so its integral over all speeds is unity: $\int P(v)\, dv = 1$. The average speed is defined as $v_{\rm avg} = \int_0^\infty vP(v)dv$. Similarly, the rms speed is given by $v_{\rm rms} = \sqrt{(v^2)_{\rm avg}}$, where $(v^2)_{\rm avg} = \int_0^\infty v^2 P(v)dv$.

**ANALYZE** (a) Evaluate the integral by calculating the area under the curve in Fig. 19-23. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or $av_0$. Thus,

$$\int P(v)dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0 ,$$

so $\frac{3}{2}av_0 = 1$ and $av_0 = 2/3 = 0.67$.

(b) For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2 dv = \frac{a}{3v_0}v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0,$$

where $2/3v_0$ was substituted for $a$. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\, dv = \frac{a}{2}\left(4v_0^2 - v_0^2\right) = \frac{3a}{2}v_0^2 = v_0.$$

Therefore, we have

$$v_{\rm avg} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \quad \Rightarrow \quad \frac{v_{\rm avg}}{v_0} = 1.22 .$$

(c) In calculating $v_{\text{avg}}^2 = \int v^2 P(v)\, dv$, we note that the contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3 dv = \frac{a}{4v_0} v_0^4 = \frac{1}{6} v_0^2.$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2 dv = \frac{a}{3}\left(8v_0^3 - v_0^3\right) = \frac{7a}{3} v_0^3 = \frac{14}{9} v_0^2.$$

Thus,

$$v_{\text{rms}} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31 v_0 \;\Rightarrow\; \frac{v_{\text{rms}}}{v_0} = 1.31\,.$$

(d) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is

$$Na(2.0v_0 - 1.5v_0) = 0.5N\, av_0 = N/3,$$

where $2/3v_0$ was substituted for $a$. In other words, the fraction of particles in this range is 1/3 or 0.33.

**LEARN** From the distribution function shown in Fig. 19-23, it is clear that there are more particles with a speed in the range $v_0 < v < 2v_0$ than $0 < v < v_0$. In fact, straightforward calculation shows that the fraction of particles with speeds between $1.0v_0$ and $2v_0$ is

$$\int_{1.0v_0}^{2v_0} P(v)dv = a(2v_0 - 1.0v_0) = av_0 = \frac{2}{3}.$$

38. (a) From the graph we see that $v_p$ = 400 m/s. Using the fact that $M$ = 28 g/mol = 0.028 kg/mol for nitrogen ($N_2$) gas, Eq. 19-35 can then be used to determine the absolute temperature. We obtain $T = \frac{1}{2} M v_p^2 / R = 2.7\times10^2$ K.

(b) Comparing with Eq. 19-34, we conclude $v_{\text{rms}} = \sqrt{3/2}\, v_p = 4.9\times10^2$ m/s.

39. The rms speed of molecules in a gas is given by $v_{rms} = \sqrt{3RT/M}$, where $T$ is the temperature and $M$ is the molar mass of the gas. See Eq. 19-34. The speed required for escape from Earth's gravitational pull is $v = \sqrt{2gr_e}$, where $g$ is the acceleration due to gravity at Earth's surface and $r_e$ (= $6.37\times10^6$ m) is the radius of Earth. To derive this

expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass $m$ at Earth's surface is

$$U = -GMm/r_e^2 = -mgr_e,$$

where $g = GM/r_e^2$ was used. If $v$ is the speed of the particle, then its total energy is $E = -mgr_e + \frac{1}{2}mv^2$. If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means $E$ = 0 and $v = \sqrt{2gr_e}$. We equate the expressions for the speeds to obtain $\sqrt{3RT/M} = \sqrt{2gr_e}$. The solution for $T$ is $T = 2gr_eM/3R$.

(a) The molar mass of hydrogen is $2.02 \times 10^{-3}$ kg/mol, so for that gas

$$T = \frac{2(9.8\,\text{m/s}^2)(6.37\times10^6\,\text{m})(2.02\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 1.0\times10^4\ \text{K}.$$

(b) The molar mass of oxygen is $32.0 \times 10^{-3}$ kg/mol, so for that gas

$$T = \frac{2(9.8\,\text{m/s}^2)(6.37\times10^6\,\text{m})(32.0\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 1.6\times10^5\ \text{K}.$$

(c) Now, $T = 2g_mr_mM / 3R$, where $r_m = 1.74 \times 10^6$ m is the radius of the Moon and $g_m = 0.16g$ is the acceleration due to gravity at the Moon's surface. For hydrogen, the temperature is

$$T = \frac{2(0.16)(9.8\,\text{m/s}^2)(1.74\times10^6\,\text{m})(2.02\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 4.4\times10^2\ \text{K}.$$

(d) For oxygen, the temperature is

$$T = \frac{2(0.16)(9.8\,\text{m/s}^2)(1.74\times10^6\,\text{m})(32.0\times10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 7.0\times10^3\ \text{K}.$$

(e) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result, the atmosphere is depleted of hydrogen.

(f) On the other hand, very few oxygen atoms escape. So there should be much oxygen high in Earth’s upper atmosphere.

40. We divide Eq. 19-31 by Eq. 19-22:

$$\frac{v_{\text{avg2}}}{v_{\text{rms1}}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which, for $v_{\text{avg2}} = 2v_{\text{rms1}}$, leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\text{avg2}}}{v_{\text{rms1}}}\right)^2 = \frac{3\pi}{2} = 4.7\,.$$

41. (a) The root-mean-square speed is given by $v_{\text{rms}} = \sqrt{3RT/M}$ . See Eq. 19-34. The molar mass of hydrogen is $2.02 \times 10^{-3}$ kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31\,\text{J/mol}\cdot\text{K})(4000\,\text{K})}{2.02\times10^{-3}\,\text{kg/mol}}} = 7.0\times10^3\,\text{m/s}.$$

(b) When the surfaces of the spheres that represent an $H_2$ molecule and an Ar atom are touching, the distance between their centers is the sum of their radii:

$$d = r_1 + r_2 = 0.5 \times 10^{-8}\text{ cm} + 1.5 \times 10^{-8}\text{cm} = 2.0 \times 10^{-8}\text{ cm}.$$

(c) The argon atoms are essentially at rest so in time $t$ the hydrogen atom collides with all the argon atoms in a cylinder of radius $d$, and length $vt$, where $v$ is its speed. That is, the number of collisions is $\pi d^2 vtN/V$, where $N/V$ is the concentration of argon atoms. The number of collisions per unit time is

$$\frac{\pi d^2 vN}{V} = \pi\left(2.0\times10^{-10}\,\text{m}\right)^2\left(7.0\times10^3\,\text{m/s}\right)\left(4.0\times10^{25}\,\text{m}^{-3}\right) = 3.5\times10^{10}\,\text{collisions/s}.$$

42. The internal energy is

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(1.0\,\text{mol})(8.31\text{ J/mol}\cdot\text{K})(273\,\text{K}) = 3.4\times10^3\text{ J}.$$

43. (a) From Table 19-3, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Thus, Eq. 19-46 yields

$$Q = nC_p\Delta T = (3.00)\left(\frac{7}{2}(8.31)\right)(40.0) = 3.49\times10^3\text{ J}.$$

(b) Equation 19-45 leads to

$$\Delta E_{\text{int}} = nC_V\Delta T = (3.00)\left(\frac{5}{2}(8.31)\right)(40.0) = 2.49\times10^3 \text{ J.}$$

(c) From either $W = Q - \Delta E_{\text{int}}$ or $W = p\Delta T = nR\Delta T$, we find $W = 997$ J.

(d) Equation 19-24 is written in more convenient form (for this problem) in Eq. 19-38. Thus, the increase in kinetic energy is

$$\Delta K_{\text{trans}} = \Delta\left(NK_{\text{avg}}\right) = n\left(\frac{3}{2}R\right)\Delta T \approx 1.49\times10^3 \text{ J.}$$

Since $\Delta E_{\text{int}} = \Delta K_{\text{trans}} + \Delta K_{\text{rot}}$, the increase in rotational kinetic energy is

$$\Delta K_{\text{rot}} = \Delta E_{\text{int}} - \Delta K_{\text{trans}} = 2.49\times10^3 \text{ J} - 1.49\times10^3 \text{ J} = 1.00\times10^3 \text{ J .}$$

Note that had there been no rotation, all the energy would have gone into the translational kinetic energy.

44. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the $pV$ diagram, the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in joules, pressure should be in pascals ($N/m^2$) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$E_{\text{int}\,c} - E_{\text{int}\,a} = \frac{5}{2}\left(p_cV_c - p_aV_a\right) = \frac{5}{2}\left(\left(2.0\times10^3\text{ Pa}\right)\left(4.0\text{ m}^3\right) - \left(5.0\times10^3\text{ Pa}\right)\left(2.0\text{ m}^3\right)\right)$$
$$= -5.0\times10^3 \text{ J.}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = \left(3.5\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right)$$

which yields $W_{\text{diag}} = 7.0\times10^3$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0\times10^3 + 7.0\times10^3)\ \text{J} = 2.0\times10^3\ \text{J}.$$

(c) The fact that $\Delta E_{\text{int}}$ only depends on the initial and final states, and not on the details of the "path" between them, means we can write $\Delta E_{\text{int}} = E_{\text{int}\,c} - E_{\text{int}\,a} = -5.0\times10^3$ J for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = \left(5.0\times10^3\ \text{Pa}\right)\left(2.0\,\text{m}^3\right) + 0 = 1.0\times10^4\ \text{J}.$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0\times10^3 + 1.0\times10^4)\ \text{J} = 5.0\times10^3\ \text{J}.$$

45. Argon is a monatomic gas, so $f = 3$ in Eq. 19-51, which provides

$$C_V = \frac{3}{2}R = \frac{3}{2}\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(\frac{1\ \text{cal}}{4.186\ \text{J}}\right) = 2.98\frac{\text{cal}}{\text{mol}\cdot\text{C}^\circ}$$

where we have converted joules to calories, and taken advantage of the fact that a Celsius degree is equivalent to a unit change on the Kelvin scale. Since (for a given substance) $M$ is effectively a conversion factor between grams and moles, we see that $c_V$ (see units specified in the problem statement) is related to $C_V$ by $C_V = c_V\ M$ where $M = mN_A$, and $m$ is the mass of a single atom (see Eq. 19-4).

(a) From the above discussion, we obtain

$$m = \frac{M}{N_A} = \frac{C_V / c_V}{N_A} = \frac{2.98/0.075}{6.02\times10^{23}} = 6.6\times10^{-23}\ \text{g} = 6.6\times10^{-26}\ \text{kg}.$$

(b) The molar mass is found to be

$$M = C_V/c_V = 2.98/0.075 = 39.7\ \text{g/mol}$$

which should be rounded to 40 g/mol since the given value of $c_V$ is specified to only two significant figures.

46. (a) Since the process is a constant-pressure expansion,

$$W = p\Delta V = nR\Delta T = (2.02\,\text{mol})(8.31\ \text{J/mol}\cdot\text{K})(15\,\text{K}) = 249\,\text{J}.$$

(b) Now, $C_p = \frac{5}{2}R$ in this case, so $Q = nC_p\Delta T =$ +623 J by Eq. 19-46.

(c) The change in the internal energy is $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) The change in the average kinetic energy per atom is

$$\Delta K_{\text{avg}} = \Delta E_{\text{int}}/N = +3.11 \times 10^{-22}\ \text{J}.$$

47. (a) The work is zero in this process since volume is kept fixed.

(b) Since $C_V = \frac{3}{2}R$ for an ideal monatomic gas, then Eq. 19-39 gives $Q = +374$ J.

(c) $\Delta E_{\text{int}} = Q - W = +374$ J.

(d) Two moles are equivalent to $N = 12 \text{ x } 10^{23}$ particles. Dividing the result of part (c) by $N$ gives the average translational kinetic energy change per atom: $3.11 \times 10^{-22}$ J.

48. (a) According to the first law of thermodynamics $Q = \Delta E_{\text{int}} + W$. When the pressure is a constant $W = p\,\Delta V$. So

$$\Delta E_{\text{int}} = Q - p\Delta V = 20.9\ \text{J} - (1.01\times10^5\ \text{Pa})(100\ \text{cm}^3 - 50\ \text{cm}^3)\left(\frac{1\times10^{-6}\ \text{m}^3}{1\ \text{cm}^3}\right) = 15.9\ \text{J}.$$

(b) The molar specific heat at constant pressure is

$$C_p = \frac{Q}{n\Delta T} = \frac{Q}{n(p\Delta V/nR)} = \frac{R}{p}\frac{Q}{\Delta V} = \frac{(8.31\ \text{J/mol}\cdot\text{K})(20.9\,\text{J})}{(1.01\times10^5\ \text{Pa})(50\times10^{-6}\text{m}^3)} = 34.4\,\text{J/mol}\cdot\text{K}.$$

(c) Using Eq. 19-49, $C_V = C_p - R = 26.1$ J/mol·K.

49. **THINK** The molar specific heat at constant volume for a gas is given by Eq. 19-41: $C_V = \Delta E_{\text{int}}/n\Delta T$ . Our system consists of three non-interacting gases.

**EXPRESS** When the temperature changes by $\Delta T$ the internal energy of the first gas changes by $n_1 C_{V1}\,\Delta T$, the internal energy of the second gas changes by $n_2C_{V2}\,\Delta T$, and the internal energy of the third gas changes by $n_3C_{V3}\,\Delta T$. The change in the internal energy of the composite gas is

$$\Delta E_{\text{int}} = (n_1 C_{V1} + n_2 C_{V2} + n_3 C_{V3})\,\Delta T.$$

This must be $(n_1 + n_2 + n_3)\, C_V\, \Delta T$, where $C_V$ is the molar specific heat of the mixture. Thus,

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2} + n_3 C_{V3}}{n_1 + n_2 + n_3}.$$

**ANALYZE** With $n_1$=2.40 mol, $C_{V1}$=12.0 J/mol·K for gas 1, $n_2$=1.50 mol, $C_{V2}$=12.8 J/mol·K for gas 2, and $n_3$=3.20 mol, $C_{V3}$=20.0 J/mol·K for gas 3, we obtain

$$C_V = \frac{(2.40\text{ mol})(12.0\text{ J/mol}\cdot\text{K}) + (1.50\text{ mol})(12.8\text{ J/mol}\cdot\text{K}) + (3.20\text{ mol})(20.0\text{ J/mol}\cdot\text{K})}{2.40\text{ mol} + 1.50\text{ mol} + 3.20\text{ mol}}$$
$$= 15.8\text{ J/mol}\cdot\text{K}$$

for the mixture.

**LEARN** The molar specific heat of the mixture $C_V$ is the sum of each individual $C_{Vi}$ weighted by the molar fraction.

50. Referring to Table 19-3, Eq. 19-45 and Eq. 19-46, we have

$$\Delta E_{\text{int}} = nC_V\Delta T = \frac{5}{2}nR\Delta T, \quad Q = nC_p\Delta T = \frac{7}{2}nR\Delta T.$$

Dividing the equations, we obtain

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{5}{7}.$$

Thus, the given value $Q$ = 70 J leads to $\Delta E_{\text{int}} = 50$ J.

51. The fact that they rotate but do not oscillate means that the value of $f$ given in Table 19-3 is relevant. Thus, Eq. 19-46 leads to

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)\left(T_f - T_i\right) = nRT_i\left(\frac{7}{2}\right)\left(\frac{T_f}{T_i} - 1\right)$$

where $T_i$ = 273 K and $n$ = 1.0 mol. The ratio of absolute temperatures is found from the gas law in ratio form. With $p_f = p_i$ we have

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2.$$

Therefore, the energy added as heat is

$$Q = (1.0\,\text{mol})(8.31\text{ J/mol}\cdot\text{K})(273\,\text{K})\left(\frac{7}{2}\right)(2-1) \approx 8.0\times10^{3}\text{ J}.$$

52. (a) Using $M$ = 32.0 g/mol from Table 19-1 and Eq. 19-3, we obtain

$$n = \frac{M_{\text{sam}}}{M} = \frac{12.0\text{ g}}{32.0\text{ g/mol}} = 0.375\text{ mol}.$$

(b) This is a constant pressure process with a diatomic gas, so we use Eq. 19-46 and Table 19-3. We note that a change of Kelvin temperature is numerically the same as a change of Celsius degrees.

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)\Delta T = (0.375\text{ mol})\left(\frac{7}{2}\right)(8.31\text{ J/mol}\cdot\text{K})(100\,\text{K}) = 1.09\times10^{3}\text{ J}.$$

(c) We could compute a value of $\Delta E_{\text{int}}$ from Eq. 19-45 and divide by the result from part (b), or perform this manipulation algebraically to show the generality of this answer (that is, many factors will be seen to cancel). We illustrate the latter approach:

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{n\left(\frac{5}{2}R\right)\,\Delta T}{n\left(\frac{7}{2}R\right)\,\Delta T} = \frac{5}{7} \approx 0.714.$$

53. **THINK** The molecules are diatomic, with translational and rotational degrees of freedom. The temperature change is under constant pressure.

**EXPRESS** Since the process is at constant pressure, energy transferred as heat to the gas is given by $Q = nC_p\,\Delta T$, where $n$ is the number of moles in the gas, $C_p$ is the molar specific heat at constant pressure, and $\Delta T$ is the increase in temperature. Similarly, the change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where $C_V$ is the specific heat at constant volume. For a diatomic ideal gas, $C_p = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$ (see Table 19-3).

**ANALYZE** (a) The heat transferred is

$$Q = nC_p\Delta T = n\left(\frac{7R}{2}\right)\Delta T = \frac{7}{2}(4.00\,\text{mol})(8.31\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 6.98\times10^{3}\text{ J}.$$

(b) From the above, we find the change in the internal energy to be

$$\Delta E_{\text{int}} = nC_V\Delta T = n\left(\frac{5R}{2}\right)\Delta T = \frac{5}{2}(4.00\,\text{mol})(8.31\text{J/mol.K})(60.0\,\text{K}) = 4.99\times10^{3}\text{ J}.$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = 6.98\times10^3\,\text{J} - 4.99\times10^3\,\text{J} = 1.99\times10^3\,\text{J}.$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2}nR\Delta T = \frac{3}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 2.99\times10^3\,\text{J}.$$

**LEARN** The diatomic gas has three translational and two rotational degrees of freedom (making $f = 3+2 = 5$). By equipartition theorem, each degree of freedom accounts for an energy of $RT/2$ per mole. Thus, $C_V = (f/2)R = 5R/2$ and $C_p = C_V + R = 7R/2$.

54. The fact that they rotate but do not oscillate means that the value of $f$ given in Table 19-3 is relevant. In Section 19-11, it is noted that $\gamma = C_p/C_V$ so that we find $\gamma = 7/5$ in this case. In the state described in the problem, the volume is

$$V = \frac{nRT}{p} = \frac{(2.0\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{1.01\times10^5\,\text{N/m}^2} = 0.049\,\text{m}^3.$$

Consequently,

$$pV^{\gamma} = (1.01\times10^5\,\text{N/m}^2)(0.049\,\text{m}^3)^{1.4} = 1.5\times10^3\,\text{N}\cdot\text{m}^{2.2}.$$

55. (a) Let $p_i$, $V_i$, and $T_i$ represent the pressure, volume, and temperature of the initial state of the gas. Let $p_f$, $V_f$, and $T_f$ represent the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_iV_i^{\gamma} = p_fV_f^{\gamma}$, so

$$p_f = \left(\frac{V_i}{V_f}\right)^{\gamma} p_i = \left(\frac{4.3\,\text{L}}{0.76\,\text{L}}\right)^{1.4}(1.2\,\text{atm}) = 13.6\,\text{atm} \approx 14\,\text{atm}.$$

We note that since $V_i$ and $V_f$ have the same units, their units cancel and $p_f$ has the same units as $p_i$.

(b) The gas obeys the ideal gas law $pV = nRT$, so $p_iV_i/p_fV_f = T_i/T_f$ and

$$T_f = \frac{p_fV_f}{p_iV_i}T_i = \left[\frac{(13.6\,\text{atm})(0.76\,\text{L})}{(1.2\,\text{atm})(4.3\,\text{L})}\right](310\,\text{K}) = 6.2\times10^2\,\text{K}.$$

56. (a) We use Eq. 19-54 with $V_f/V_i = \frac{1}{2}$ for the gas (assumed to obey the ideal gas law).

$$p_i V_i^\gamma = p_f V_f^\gamma \Rightarrow \frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma = (2.00)^{1.3}$$

which yields $p_f$ = (2.46)(1.0 atm) = 2.46 atm.

(b) Similarly, Eq. 19-56 leads to

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = (273\,\text{K})(1.23) = 336\,\text{K}.$$

(c) We use the gas law in ratio form and note that when $p_1 = p_2$ then the ratio of volumes is equal to the ratio of (absolute) temperatures. Consequently, with the subscript 1 referring to the situation (of small volume, high pressure, and high temperature) the system is in at the end of part (a), we obtain

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273\,\text{K}}{336\,\text{K}} = 0.813.$$

The volume $V_1$ is half the original volume of one liter, so

$$V_2 = 0.813(0.500\,\text{L}) = 0.406\,\text{L}.$$

57. (a) Equation 19-54, $p_i V_i^\gamma = p_f V_f{}^\gamma$, leads to

$$p_f = p_i \left(\frac{V_i}{V_f}\right)^\gamma \Rightarrow \quad 4.00\text{ atm} = (1.00\,\text{atm})\left(\frac{200\,\text{L}}{74.3\,\text{L}}\right)^\gamma$$

which can be solved to yield

$$\gamma = \frac{\ln(p_f/p_i)}{\ln(V_i/V_f)} = \frac{\ln(4.00\,\text{atm}/1.00\,\text{atm})}{\ln(200\,\text{L}/74.3\,\text{L})} = 1.4 = \frac{7}{5}.$$

This implies that the gas is diatomic (see Table 19-3).

(b) One can now use either Eq. 19-56 or use the ideal gas law itself. Here we illustrate the latter approach:

$$\frac{P_f V_f}{P_i V_i} = \frac{nRT_f}{nRT_i} \quad \Rightarrow \quad T_f = 446\text{ K}.$$

(c) Again using the ideal gas law: $n = P_i V_i / RT_i$ = 8.10 moles. The same result would, of course, follow from $n = P_f V_f / RT_f$.

58. Let $p_i$, $V_i$, and $T_i$ represent the pressure, volume, and temperature of the initial state of the gas, and let $p_f$, $V_f$, and $T_f$ be the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_i V_i^\gamma = p_f V_f^{\,\gamma}$. Combining with the ideal gas law, $pV = NkT$, we obtain

$$p_i V_i^\gamma = p_i (T_i / p_i)^\gamma = p_i^{1-\gamma} T_i^\gamma = \text{constant} \quad \Rightarrow \quad p_i^{1-\gamma} T_i^\gamma = p_f^{1-\gamma} T_f^\gamma$$

With $\gamma = 4/3$, which gives $(1-\gamma)/\gamma = -1/4$, the temperature at the end of the adiabatic expansion is

$$T_f = \left(\frac{p_i}{p_f}\right)^{\frac{1-\gamma}{\gamma}} T_i = \left(\frac{5.00 \text{ atm}}{1.00 \text{ atm}}\right)^{-1/4} (278 \text{ K}) = 186 \text{ K} = -87°\text{C}.$$

59. Since $\Delta E_{\text{int}}$ does not depend on the type of process,

$$\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = \left(\Delta E_{\text{int}}\right)_{\text{path 1}}.$$

Also, since (for an ideal gas) it only depends on the temperature variable (so $\Delta E_{\text{int}} = 0$ for isotherms), then

$$\left(\Delta E_{\text{int}}\right)_{\text{path1}} = \sum \left(\Delta E_{\text{int}}\right)_{\text{adiabat}}.$$

Finally, since $Q = 0$ for adiabatic processes, then (for path 1)

$$\begin{aligned} \left(\Delta E_{\text{int}}\right)_{\text{adiabatic expansion}} &= -W = -40 \text{ J} \\ \left(\Delta E_{\text{int}}\right)_{\text{adiabatic compression}} &= -W = -(-25)\text{ J} = 25 \text{ J}. \end{aligned}$$

Therefore, $\left(\Delta E_{\text{int}}\right)_{\text{path 2}} = -40 \text{ J} + 25 \text{ J} = -15 \text{ J}$.

60. Let $p_1, V_1$, and $T_1$ represent the pressure, volume, and temperature of the air at $y_1 = 4267$ m. Similarly, let $p, V$, and $T$ be the pressure, volume, and temperature of the air at $y = 1567$ m. Since the process is adiabatic, $p_1 V_1^\gamma = pV^\gamma$. Combining with the ideal gas law, $pV = NkT$, we obtain

$$pV^\gamma = p(T/p)^\gamma = p^{1-\gamma} T^\gamma = \text{constant} \quad \Rightarrow \quad p^{1-\gamma} T^\gamma = p_1^{1-\gamma} T_1^\gamma.$$

With $p = p_0 e^{-ay}$ and $\gamma = 4/3$ (which gives $(1-\gamma)/\gamma = -1/4$), the temperature at the end of the descent is

$$T = \left(\frac{p_1}{p}\right)^{\frac{1-\gamma}{\gamma}} T_1 = \left(\frac{p_0 e^{-ay_1}}{p_0 e^{-ay}}\right)^{\frac{1-\gamma}{\gamma}} T_1 = e^{-a(y-y_1)/4} T_1 = e^{-(1.16\times10^{-4}/\text{m})(1567\text{ m}-4267\text{ m})/4}(268\text{ K})$$
$$= (1.08)(268\text{ K}) = 290\text{ K} = 17°\text{C}.$$

61. The aim of this problem is to emphasize what it means for the internal energy to be a state function. Since path 1 and path 2 start and stop at the same places, then the internal energy change along path 1 is equal to that along path 2. Now, during isothermal processes (involving an ideal gas) the internal energy change is zero, so the only step in path 1 that we need to examine is step 2. Equation 19-28 then immediately yields –20 J as the answer for the internal energy change.

62. Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_i V_i^{\gamma} \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma}.$$

Using Eq. 19-54 we can write this as

$$W = p_i V_i \frac{1-(p_f / p_i)^{1-1/\gamma}}{1-\gamma}$$

In this problem, $\gamma$ = 7/5 (see Table 19-3) and $P_f/P_i$ = 2. Converting the initial pressure to pascals we find $P_i\ V_i$ = 24240 J. Plugging in, then, we obtain $W = -1.33 \times 10^4$ J.

63. In the following, $C_V = \frac{3}{2}R$ is the molar specific heat at constant volume, $C_p = \frac{5}{2}R$ is the molar specific heat at constant pressure, $\Delta T$ is the temperature change, and $n$ is the number of moles.

The process 1 $\to$ 2 takes place at constant volume.

(a) The heat added is

$$Q = nC_V\,\Delta T = \frac{3}{2} nR\,\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 3.74\times10^3\ \text{J}.$$

(b) Since the process takes place at constant volume, the work $W$ done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74\times10^3\ \text{J}.$$

(c) The work *W* done by the gas is zero.

The process 2 $\rightarrow$ 3 is adiabatic.

(d) The heat added is zero.

(e) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K}-600\,\text{K}) = -1.81\times 10^3\,\text{J}.$$

(f) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81\times 10^3\,\text{J}.$$

The process 3 $\rightarrow$ 1 takes place at constant pressure.

(g) The heat added is

$$Q = nC_p\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}(1.00\text{ mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K}-455\,\text{K}) = -3.22\times 10^3\text{ J}.$$

(h) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K}-455\,\text{K}) = -1.93\times 10^3\text{ J}.$$

(i) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22\times 10^3\text{ J} + 1.93\times 10^3\text{ J} = -1.29\times 10^3\text{ J}.$$

(j) For the entire process the heat added is

$$Q = 3.74\times 10^3\text{ J} + 0 - 3.22\times 10^3\text{ J} = 520\text{ J}.$$

(k) The change in the internal energy is

$$\Delta E_{\text{int}} = 3.74\times 10^3\text{ J} - 1.81\times 10^3\text{ J} - 1.93\times 10^3\text{ J} = 0.$$

(l) The work done by the gas is

$$W = 0 + 1.81\times 10^3\text{ J} - 1.29\times 10^3\text{ J} = 520\text{ J}.$$

(m) We first find the initial volume. Use the ideal gas law $p_1V_1 = nRT_1$ to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(1.013\times10^5\,\text{Pa})} = 2.46\times10^{-2}\,\text{m}^3.$$

(n) Since 1 → 2 is a constant volume process, $V_2 = V_1 = 2.46 \times 10^{-2}$ m$^3$. The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K})}{2.46\times10^{-2}\,\text{m}^3} = 2.02\times10^5\,\text{Pa}.$$

This is approximately equal to 2.00 atm.

(o) 3 → 1 is a constant pressure process. The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K})}{1.013\times10^5\,\text{Pa}} = 3.73\times10^{-2}\,\text{m}^3.$$

(p) The pressure for state 3 is the same as the pressure for state 1: $p_3 = p_1 = 1.013 \times 10^5$ Pa (1.00 atm)

64. We write $T$ = 273 K and use Eq. 19-14:

$$W = (1.00\,\text{mol})\,(8.31\;\;\text{J/mol}\cdot\text{K})\,(273\,\text{K})\;\ln\left(\frac{16.8}{22.4}\right)$$

which yields $W$ = –653 J. Recalling the sign conventions for work stated in Chapter 18, this means an external agent does 653 J of work *on* the ideal gas during this process.

65. (a) We use $p_iV_i^\gamma = p_fV_f^{\gamma}$ to compute $\gamma$:

$$\gamma = \frac{\ln(p_i/p_f)}{\ln(V_f/V_i)} = \frac{\ln(1.0\,\text{atm}/1.0\times10^5\,\text{atm})}{\ln(1.0\times10^3\,\text{L}/1.0\times10^6\,\text{L})} = \frac{5}{3}.$$

Therefore the gas is monatomic.

(b) Using the gas law in ratio form, the final temperature is

$$T_f = T_i\frac{p_fV_f}{p_iV_i} = (273\,\text{K})\frac{(1.0\times10^5\,\text{atm})(1.0\times10^3\,\text{L})}{(1.0\,\text{atm})(1.0\times10^6\,\text{L})} = 2.7\times10^4\,\text{K}.$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{\left(1.01\times10^5 \text{ Pa}\right)\left(1.0\times10^3 \text{ cm}^3\right)}{\left(8.31 \text{ J/mol}\cdot\text{K}\right)\left(273\text{K}\right)} = 4.5\times10^4 \text{ mol.}$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2}RT_i = \frac{3}{2}\left(8.31 \text{ J/mol}\cdot\text{K}\right)\left(273\text{K}\right) = 3.4\times10^3 \text{ J.}$$

(e) After the compression,

$$K_f = \frac{3}{2}RT_f = \frac{3}{2}\left(8.31 \text{ J/mol}\cdot\text{K}\right)\left(2.7\times10^4 \text{ K}\right) = 3.4\times10^5 \text{ J.}$$

(f) Since $v_{\text{rms}}^2 \propto T$ , we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273\text{K}}{2.7\times10^4 \text{ K}} = 0.010.$$

66. Equation 19-25 gives the mean free path:

$$\lambda = \frac{1}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, (N/V)} = \frac{n\, R\, T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, P\, N}$$

where we have used the ideal gas law in that last step. Thus, the change in the mean free path is

$$\Delta\lambda = \frac{n\, R\, \Delta T}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, P\, N} = \frac{R\, Q}{\sqrt{2}\, d^2\, \pi\, \varepsilon_o\, P\, N\, C_p}$$

where we have used Eq. 19-46. The constant pressure molar heat capacity is (7/2)$R$ in this situation, so (with $N = 9 \times 10^{23}$ and $d = 250 \times 10^{-12}$ m) we find

$$\Delta\lambda = 1.52 \times 10^{-9} \text{ m} = 1.52 \text{ nm.}$$

67. (a) The volume has increased by a factor of 3, so the pressure must decrease accordingly (since the temperature does not change in this process). Thus, the final pressure is one-third of the original 6.00 atm. The answer is 2.00 atm.

(b) We note that Eq. 19-14 can be written as $P_i V_i \ln(V_f/V_i)$. Converting “atm” to “Pa” (a pascal is equivalent to a N/m$^2$) we obtain $W$ = 333 J.

(c) The gas is monatomic so $\gamma = 5/3$. Equation 19-54 then yields $P_f$ = 0.961 atm.

(d) Using Eq. 19-53 in Eq. 18-25 gives

$$W = p_i V_i^{\gamma} \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} = \frac{p_f V_f - p_i V_i}{1-\gamma}$$

where in the last step Eq. 19-54 has been used. Converting "atm" to "Pa," we obtain $W = 236$ J.

68. Using the ideal gas law, one mole occupies a volume equal to

$$V = \frac{nRT}{p} = \frac{(1)(8.31)(50.0)}{1.00\times10^{-8}} = 4.16\times10^{10}\ \text{m}^3.$$

Therefore, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_\text{A}}{V} = \frac{(1)\left(6.02\times10^{23}\right)}{4.16\times10^{10}} = 1.45\times10^{13}\ \frac{\text{molecules}}{\text{m}^3}.$$

Using $d = 20.0 \times 10^{-9}$ m, Eq. 19-25 yields

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 \left(\frac{N}{V}\right)} = 38.8\ \text{m}.$$

69. **THINK** The net upward force is the difference between the buoyant force and the weight of the balloon with air inside.

**EXPRESS** Let $\rho_\text{c}$ be the density of the cool air surrounding the balloon and $\rho_h$ be the density of the hot air inside the balloon. The magnitude of the buoyant force on the balloon is $F_b = \rho_c g V$, where $V$ is the volume of the envelope. The force of gravity is $Fg = W + \rho_h g V$, where $W$ is the combined weight of the basket and the envelope. Thus, the net upward force is

$$F_\text{net} = F_b - F_g = \rho_c g V - W - \rho_h g V\ .$$

**ANALYZE** With $F_\text{net}$ = 2.67 × 10$^3$ N, $W$ = 2.45 × 10$^3$ N, $V$ = 2.18 × 10$^3$ m$^3$, and $\rho_c g = 11.9$ N/m$^3$, we obtain

$$\rho_h g = \frac{\rho_c g V - W - F_\text{net}}{V} = \frac{(11.9\ \text{N/m}^3)(2.18\times10^3\ \text{m}^3) - 2.45\times10^3\ \text{N} - 2.67\times10^3\ \text{N}}{2.18\times10^3\ \text{m}^3} = 9.55\ \text{N/m}^3$$

The ideal gas law gives $p/RT = n/V$. Multiplying both sides by the "molar weight" $Mg$ then leads to

$$\frac{pMg}{RT} = \frac{nMg}{V} = \rho_h g \ .$$

With $p = 1.01\times10^5$ Pa and $M$ = 0.028 kg/m$^3$, we find the temperature to be

$$T = \frac{pMg}{R\rho_h g} = \frac{(1.01\times10^5 \text{ Pa})(0.028\text{ kg/mol})(9.8\text{ m/s}^2)}{(8.31\text{ J/mol}\cdot\text{K})(9.55\text{ N/m}^3)} = 349\text{ K} \ .$$

**LEARN** As can be seen from the results above, increasing the temperature of the gas inside the balloon increases the value of $F_{\text{net}}$, i.e., the lifting capacity.

70. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with $V_2$ = 4 m$^3$; then continues on to state 3 isothermally, with $V_3$ = 10 m$^3$; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1\to2$ $p_1V_1^\gamma = p_2V_2^\gamma$, for the isothermal process 2 → 3 $p_2V_2 = p_3V_3$, and finally for the adiabatic process $3\to4$ $p_3V_3^\gamma = p_4V_4^\gamma$. These equations yield

$$p_4 = p_3\left(\frac{V_3}{V_4}\right)^\gamma = p_2\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma = p_1\left(\frac{V_1}{V_2}\right)^\gamma\left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma .$$

We substitute this expression for $p_4$ into the equation $p_1V_1 = p_4V_4$ (since $T_1 = T_4$) to obtain $V_1V_3 = V_2V_4$. Solving for $V_4$ we obtain

$$V_4 = \frac{V_1V_3}{V_2} = \frac{\left(2.0\text{ m}^3\right)\left(10\text{ m}^3\right)}{4.0\text{ m}^3} = 5.0\text{ m}^3.$$

71. **THINK** An adiabatic process is a process in which the energy transferred as heat is zero.

**EXPRESS** The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where $C_V$ is the specific heat at constant volume, $n$ is the number of moles in the gas, and $\Delta T$ is the change in temperature. According to the first law of thermodynamics, the work done by the gas is $W = Q - \Delta E_{\text{int}}$. For an adiabatic process, $Q = 0$, and $W = -\Delta E_{\text{int}}$.

**ANALYZE** (a) The work done by the gas is

$$W = -\Delta E_{\text{int}} = -nC_V\Delta T = -\frac{3}{2}nR\Delta T = -\frac{3}{2}(2.0\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(15.0\text{ K}) = -374\text{ J} \ .$$

(b) $Q = 0$ since the process is adiabatic.

(c) The change in internal energy is $\Delta E_{\text{int}} = \frac{3}{2} nR\Delta T = 374 \text{ J}$.

(d) The number of atoms in the gas is $N = nN_A$, where $N_A$ is the Avogadro's number. Thus, the change in the average kinetic energy per atom is

$$\Delta K_1 = \frac{\Delta E_{\text{int}}}{N} = \frac{\Delta E_{\text{int}}}{nN_A} = \frac{374 \text{ J}}{(2.00)(6.02\times 10^{23}/\text{mol})} = 3.11\times 10^{-22} \text{ J}.$$

**LEARN** The work done *on* the system is the negative of the work done *by* the system: $W_{on} = -W = \Delta E_{\text{int}} = +374 \text{ J}$. By work-kinetic energy theorem: $\Delta K = \Delta W_{\text{on}} = \Delta E_{\text{int}}$.

72. We solve

$$\sqrt{\frac{3RT}{M_{\text{helium}}}} = \sqrt{\frac{3R(293\,\text{K})}{M_{\text{hydrogen}}}}$$

for $T$. With the molar masses found in Table 19-1, we obtain

$$T = (293\,\text{K})\left(\frac{4.0}{2.02}\right) = 580\,\text{K}$$

which is equivalent to 307°C.

73. **THINK** The collision frequency is related to the mean free path and average speed of the molecules.

**EXPRESS** According to Eq. 19-25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V},$$

where $d$ is the diameter of a molecule and $N$ is the number of molecules in volume $V$. Using ideal gas law, the number density can be written as $N/V = p/kT$, where $p$ is the pressure, $T$ is the temperature on the Kelvin scale and $k$ is the Boltzmann constant. The average time between collisions is $\tau = \lambda / v_{\text{avg}}$, where $v_{\text{avg}} = \sqrt{8RT/\pi M}$, where $R$ is the universal gas constant and $M$ is the molar mass. The collision frequency is simply given by $f = 1/\tau$.

**ANALYZE** With $p = 2.02 \times 10^3$ Pa and $d = 290 \times 10^{-12}$ m, we find the mean free path to be

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 (p/kT)} = \frac{kT}{\sqrt{2}\pi d^2 p} = \frac{(1.38\times10^{-23}\text{ J/K})(400\text{ K})}{\sqrt{2}\pi(290\times10^{-12}\text{ m})^2(1.01\times10^5\text{ Pa})} = 7.31\times10^{-8}\text{ m}.$$

Similarly, with $M$ = 0.032 kg/mol, we find the average speed to be

$$v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.31\text{J/mol}\cdot\text{K})(400\text{K})}{\pi(32\times10^{-3}\text{ kg/mol})}} = 514\text{m/s}.$$

Thus, the collision frequency is $f = \dfrac{v_{\text{avg}}}{\lambda} = \dfrac{514\text{ m/s}}{7.31\times10^{-8}\text{ m}} = 7.04\times10^9\text{ collisions/s}$.

**LEARN** This is very similar to the Sample Problem 19.04 – "Mean free path, average speed and collision frequency." A general expression for $f$ is

$$f = \frac{\text{speed}}{\text{distance}} = \frac{v_{\text{avg}}}{\lambda} = \frac{pd^2}{k}\sqrt{\frac{16\pi R}{MT}}.$$

74. (a) Since $n/V = p/RT$, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_{\text{A}}}{V} = N_{\text{A}}\left(\frac{p}{RT}\right)(6.02\times10^{23})\frac{1.01\times10^5\text{ Pa}}{\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(293\text{K})} = 2.5\times10^{25}\frac{\text{molecules}}{\text{m}^3}.$$

(b) Three-fourths of the $2.5 \times 10^{25}$ value found in part (a) are nitrogen molecules with $M$ = 28.0 g/mol (using Table 19-1), and one-fourth of that value are oxygen molecules with $M$ = 32.0 g/mol. Consequently, we generalize the $M_{\text{sam}} = NM/N_{\text{A}}$ expression for these two species of molecules and write

$$\frac{3}{4}(2.5\times10^{25})\frac{28.0}{6.02\times10^{23}} + \frac{1}{4}(2.5\times10^{25})\frac{32.0}{6.02\times10^{23}} = 1.2\times10^3\text{ g} = 1.2\text{ kg}.$$

75. We note that $\Delta K = n\left(\frac{3}{2}R\right)\Delta T$ according to the discussion in Sections 19-5 and 19-9. Also, $\Delta E_{\text{int}} = nC_V\Delta T$ can be used for each of these processes (since we are told this is an ideal gas). Finally, we note that Eq. 19-49 leads to $C_p = C_V + R \approx 8.0$ cal/mol·K after we convert joules to calories in the ideal gas constant value (Eq. 19-6): $R \approx 2.0$ cal/mol·K. The first law of thermodynamics $Q = \Delta E_{\text{int}} + W$ applies to each process.

• Constant volume process with $\Delta T$ = 50 K and $n$ = 3.0 mol.

(a) Since the change in the internal energy is $\Delta E_{\text{int}}$ = (3.0)(6.00)(50) = 900 cal, and the work done by the gas is $W$ = 0 for constant volume processes, the first law gives $Q$ = 900 + 0 = 900 cal.

(b) As shown in part (a), $W = 0$.

(c) The change in the internal energy is, from part (a), $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal.

(d) The change in the total translational kinetic energy is

$$\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$$

• Constant pressure process with $\Delta T = 50$ K and $n = 3.0$ mol.

(e) $W = p\Delta V$ for constant pressure processes, so (using the ideal gas law)

$$W = nR\Delta T = (3.0)(2.0)(50) = 300 \text{ cal}.$$

The first law gives $Q = (900 + 300)$ cal = 1200 cal.

(f) From (e), we have $W = 300$ cal.

(g) The change in the internal energy is $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal.

(h) The change in the translational kinetic energy is $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$

• Adiabiatic process with $\Delta T = 50$ K and $n = 3.0$ mol.

(i) $Q = 0$ by definition of "adiabatic."

(j) The first law leads to $W = Q - E_{int} = 0 - 900$ cal = –900 cal.

(k) The change in the internal energy is $\Delta E_{int} = (3.0)(6.00)(50) = 900$ cal.

(l) As in part (d) and (h), $\Delta K = (3.0)\left(\tfrac{3}{2}(2.0)\right)(50) = 450\,\text{cal}.$

76. (a) With work being given by

$$W = p\Delta V = (250)(-0.60)\text{ J} = -150\text{ J},$$

and the heat transfer given as –210 J, then the change in internal energy is found from the first law of thermodynamics to be [–210 – (–150)] J = –60 J.

(b) Since the pressures (and also the number of moles) don't change in this process, then the volume is simply proportional to the (absolute) temperature. Thus, the final temperature is ¼ of the initial temperature. The answer is 90 K.

77. **THINK** From the distribution function $P(v)$, we can calculate the average and rms speeds of the gas.

**EXPRESS** The distribution function gives the fraction of particles with speeds between *v* and *v* + *dv*, so its integral over all speeds is unity: ∫ *P*(*v*) *dv* = 1. The average speed is defined as $v_{\text{avg}} = \int_0^\infty vP(v)dv$. Similarly, the rms speed is given by $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$, where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v)dv$.

**ANALYZE** (a) By normalizing the distribution function:

$$1 = \int_0^{v_o} P(v)\,dv = \int_0^{v_o} Cv^2\,dv = \frac{C}{3}v_0^3$$

we find the constant *C* to be $C = 3/v_0^3$.

(b) The average speed is

$$v_{\text{avg}} = \int_0^{v_o} vP(v)\,dv = \int_0^{v_o} v\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{v_o^3}\int_0^{v_o} v^3 dv = \frac{3}{4}v_o\,.$$

(c) Similarly, the rms speed is the square root of

$$\int_0^{v_o} v^2 P(v)\,dv = \int_0^{v_o} v^2\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{v_o^3}\int_0^{v_o} v^4 dv = \frac{3}{5}v_o^2\,.$$

Therefore, $v_{\text{rms}} = \sqrt{3/5}\,v_\circ \approx 0.775 v_\circ$.

**LEARN** The maximum speed of the gas is $v_{\max} = v_0$, as indicated by the distribution function. Using Eq. 19-29, we find the fraction of molecules with speed between $v_1$ and $v_2$ to be

$$\text{frac} = \int_{v_1}^{v_2} P(v)\,dv = \int_{v_1}^{v_2}\left(\frac{3v^2}{v_o^3}\right)dv = \frac{3}{v_o^3}\int_{v_1}^{v_2} v^2 dv = \frac{v_2^3 - v_1^3}{v_o^3}.$$

78. (a) In the free expansion from state 0 to state 1 we have *Q* = *W* = 0, so $\Delta E_{\text{int}} = 0$, which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3.00 V_0} = \frac{1}{3.00}p_0 \;\Rightarrow\; \frac{p_1}{p_0} = \frac{1}{3.00} = 0.333.$$

(b) For the adiabatic process from state 1 to 2 we have $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$, that is,

$$\frac{1}{3.00}\,p_0\left(3.00V_0\right)^{\gamma} = \left(3.00\right)^{\frac{1}{3}}\,p_0V_0^{\gamma}$$

which gives $\gamma = 4/3$. The gas is therefore polyatomic.

(c) From $T = pV/nR$ we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = \left(3.00\right)^{1/3} = 1.44.$$

79. **THINK** The compression is isothermal so $\Delta T = 0$. In addition, since the gas is ideal, we can use the ideal gas law: $pV = nRT$.

**EXPRESS** The work done by the gas during the isothermal compression process from volume $V_i$ to volume $V_f$ is given by

$$W = \int_{V_i}^{V_f} p\,dV = nRT\int_{V_i}^{V_f}\frac{dV}{V} = nRT\ln\left(\frac{V_f}{V_i}\right),$$

where we use the ideal gas law to replace $p$ with $nRT/V$.

**ANALYZE** (a) The temperature is $T$ = 10.0°C = 283 K. Then, with $n$ = 3.50 mol, we obtain

$$W = nRT\ln\left(\frac{V_f}{V_0}\right) = (3.50\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(283\text{ K})\ln\left(\frac{3.00\text{ m}^3}{4.00\text{ m}^3}\right) = -2.37\times10^3\text{ J}.$$

(b) The internal energy change $\Delta E_{\text{int}}$ vanishes (for an ideal gas) when $\Delta T$ = 0 so that the First Law of Thermodynamics leads to $Q = W$ = –2.37 kJ.

**LEARN** The work done by the gas is negative since $V_f < V_i$. Also, the negative value in $Q$ implies that the heat transfer is from the sample to its environment.

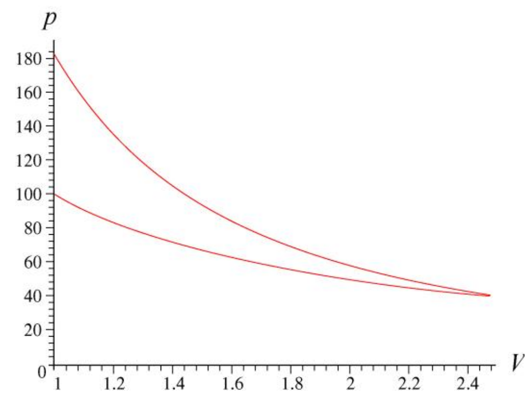
80. The ratio is

$$\frac{mgh}{mv_{\text{rms}}^2/2} = \frac{2gh}{v_{\text{rms}}^2} = \frac{2Mgh}{3RT}$$

where we have used Eq. 19-22 in that last step. With $T$ = 273 K, $h$ = 0.10 m and $M$ = 32 g/mol = 0.032 kg/mol, we find the ratio equals $9.2 \times 10^{-6}$.

81. (a) The $p$-$V$ diagram is shown next. Note that to obtain the graph, we have chosen $n$ = 0.37 moles for concreteness, in which case the horizontal axis (which we note starts not at zero but at 1) is to be interpreted in units of cubic centimeters, and the vertical axis (the absolute pressure) is in kilopascals. However, the constant volume temperature-increase

process described in the third step (see the problem statement) is difficult to see in this graph since it coincides with the pressure axis.




(b) We note that the change in internal energy is zero for an ideal gas isothermal process, so (since the net change in the internal energy must be zero for the entire cycle) the increase in internal energy in step 3 must equal (in magnitude) its decease in step 1. By Eq. 19-28, we see this number must be 125 J.

(c) As implied by Eq. 19-29, this is equivalent to heat being added *to the gas*.

82. (a) The ideal gas law leads to

$$V=\frac{nRT}{p}=\frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})\,(273\,\text{K})}{1.01\times10^{5}\,\text{Pa}}$$

which yields $V$ = 0.0225 m$^3$ = 22.5 L. If we use the standard pressure value given in Appendix D, 1 atm = 1.013 × 10$^5$ Pa, then our answer rounds more properly to 22.4 L.

(b) From Eq. 19-2, we have $N$ = 6.02 × 10$^{23}$ molecules in the volume found in part (a) (which may be expressed as $V$ = 2.24 × 10$^4$ cm$^3$), so that

$$\frac{N}{V}=\frac{6.02\times10^{23}}{2.24\times10^{4}\,\text{cm}^3}=2.69\times10^{19}\ \text{molecules/cm}^3\,.$$

83. **THINK** For an isothermal expansion, $\Delta T=0$. However, if the expansion is adiabatic, then $\Delta Q=0$.

**EXPRESS** Using ideal gas law: $pV=nRT$, we have $\dfrac{p_fV_f}{p_iV_i}=\dfrac{T_f}{T_i}$. For isothermal process, $T_f=T_i$, which gives $p_f=\dfrac{p_iV_i}{V_f}$. The work done by the gas is

$$W = \int_{V_i}^{V_f} p\, dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln\left(\frac{V_f}{V_i}\right).$$

Now, for an adiabatic process, $p_i V_i^{\gamma} = p_f V_f^{\gamma}$. The final pressures and temperatures are

$$p_f = p_i \left(\frac{V_i}{V_f}\right)^{\gamma}, \qquad T_f = \frac{p_f V_f T_i}{p_i V_i}$$

The work done is $W = Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}}$.

**ANALYZE** (a) For the isothermal process, the final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32\,\text{atm})(1.0\,\text{L})}{4.0\,\text{L}} = 8.0\,\text{atm}.$$

(b) The final temperature of the gas is the same as the initial temperature: $T_f = T_i = 300$ K.

(c) The work done is

$$W = nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) = (32\,\text{atm})(1.01\times10^5\,\text{Pa/atm})(1.0\times10^{-3}\,\text{m}^3)\ln\left(\frac{4.0\,\text{L}}{1.0\,\text{L}}\right)$$
$$= 4.4\times10^3\,\text{J}.$$

(d) For the adiabatic process, the final pressure is ( $\gamma = 5/3$ for monatomic gas)

$$p_f = p_i \left(\frac{V_i}{V_f}\right)^{\gamma} = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{5/3} = 3.2\,\text{atm}.$$

(e) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 120\,\text{K}\ .$$

(f) The work done is

$$W = -\Delta E_{\text{int}} = -\frac{3}{2} nR\Delta T = -\frac{3}{2}(p_f V_f - p_i V_i)$$
$$= -\frac{3}{2}\left[(3.2\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right](1.01\times10^5\,\text{Pa/atm})(10^{-3}\,\text{m}^3/\text{L})$$
$$= 2.9\times10^3\,\text{J}.$$

(g) If the gas is diatomic, then $\gamma = 1.4$, and the final pressure is

$$p_f = p_i\left(\frac{V_i}{V_f}\right)^{\gamma} = (32\,\text{atm})\left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{1.4} = 4.6\,\text{atm}\,.$$

(h) The final temperature is

$$T_f = \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 170\,\text{K}\,.$$

(i) The work done is

$$\begin{aligned} W &= Q - \Delta E_{\text{int}} = -\frac{5}{2}nR\Delta T = -\frac{5}{2}\left(p_f V_f - p_i V_i\right) \\ &= -\frac{5}{2}\left[(4.6\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})\right]\left(1.01\times10^5\ \text{Pa/atm}\right)\left(10^{-3}\ \text{m}^3/\text{L}\right) \\ &= 3.4\times10^3\ \text{J}. \end{aligned}$$

**LEARN** Comparing (c) with (f), we see that more work is done by the gas if the expansion is isothermal rather than adiabatic.

84. (a) With $P_1 = (20.0)(1.01 \times 10^5$ Pa) and $V_1 = 0.0015$ m$^3$, the ideal gas law gives

$$P_1V_1 = nRT_1 \quad \Rightarrow \quad T_1 = 121.54\ \text{K} \approx 122\ \text{K}.$$

(b) From the information in the problem, we deduce that $T_2 = 3T_1 = 365$ K.

(c) We also deduce that $T_3 = T_1$, which means $\Delta T = 0$ for this process. Since this involves an ideal gas, this implies the change in internal energy is zero here.

85. (a) We use $pV = nRT$. The volume of the tank is

$$V = \frac{nRT}{p} = \frac{\left(\frac{300\text{g}}{17\ \text{g/mol}}\right)(8.31\ \text{J/mol}\cdot\text{K})(350\,\text{K})}{1.35\times10^6\ \text{Pa}} = 3.8\times10^{-2}\ \text{m}^3 = 38\,\text{L}.$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{\left(8.7\times10^5\ \text{Pa}\right)\left(3.8\times10^{-2}\ \text{m}^3\right)}{(8.31\ \text{J/mol}\cdot\text{K})(293\,\text{K})} = 13.5\,\text{mol}.$$

The mass of the gas that leaked out is then

$$\Delta m = 300\ \text{g} - (13.5\ \text{mol})(17\ \text{g/mol}) = 71\ \text{g}.$$

86. To model the "uniform rates" described in the problem statement, we have expressed the volume and the temperature functions as follows:

$$V = V_i + \left(\frac{V_f - V_i}{\tau_f}\right)t\,, \qquad T = T_i + \left(\frac{T_f - T_i}{\tau_f}\right)t$$

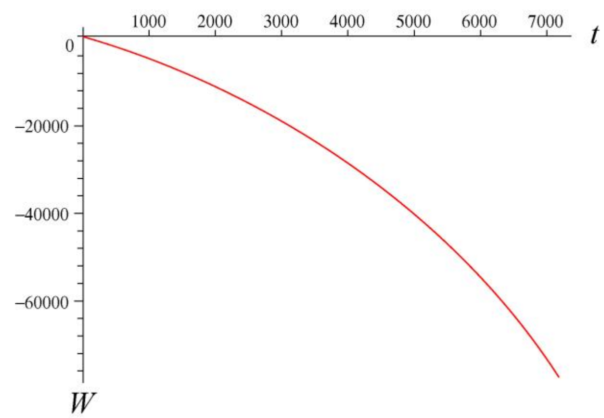
where $V_i$ = 0.616 m$^3$, $V_f$ = 0.308 m$^3$, $\tau_f$ = 7200 s, $T_i$ = 300 K, and $T_f$ = 723 K.

(a) We can take the derivative of $V$ with respect to $t$ and use that to evaluate the cumulative work done (from $t$ = 0 until $t$ = $\tau$):

$$W = \int p\,dV = \int\left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right)dt = 12.2\,\tau + 238113\ln(14400 - \tau) - 2.28\times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $W$ = –77169 J ≈ –77.2 kJ, or $|W|$ ≈ 77.2 kJ.

The graph of cumulative work is shown below. The graph for work done is purely negative because the gas is being compressed (work is being done *on* the gas).

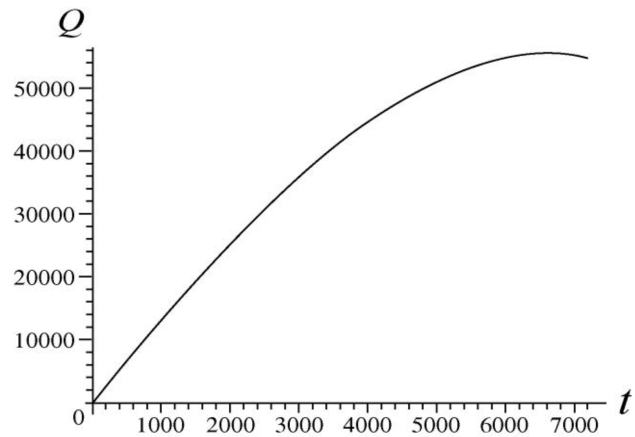



(b) With $C_V = \frac{3}{2}R$ (since it's a monatomic ideal gas) then the (infinitesimal) change in internal energy is $nC_V\,dT = \frac{3}{2}nR\left(\frac{dT}{dt}\right)dt$, which involves taking the derivative of the temperature expression listed above. Integrating this and adding this to the work done gives the cumulative heat absorbed (from $t$ = 0 until $t$ = $\tau$):

$$Q = \int \left(\frac{nRT}{V}\right)\left(\frac{dV}{dt}\right) + \frac{3}{2}nR\left(\frac{dT}{dt}\right)dt = 30.5\,\tau + 238113\ln(14400 - \tau) - 2.28\times 10^6$$

with SI units understood. With $\tau = \tau_f$ our result is $Q_{\text{total}}$ = 54649 J ≈ 5.46×10$^4$ J.

The graph cumulative heat is shown below. We see that $Q > 0$, since the gas is absorbing heat.




(c) Defining $C = \dfrac{Q_{\text{total}}}{n(T_f - T_i)}$, we obtain $C$ = 5.17 J/mol·K. We note that this is considerably smaller than the constant-volume molar heat $C_{V.}$

We are now asked to consider this to be a two-step process (time dependence is no longer an issue) where the first step is isothermal and the second step occurs at constant volume (the ending values of pressure, volume, and temperature being the same as before).

(d) Equation 19-14 readily yields $W = -43222$ J $\approx -4.32 \times 10^4$ J (or $|W| \approx 4.32 \times 10^4$ J), where it is important to keep in mind that no work is done in a process where the volume is held constant.

(e) In step 1 the heat is equal to the work (since the internal energy does not change during an isothermal ideal gas process), and in step 2 the heat is given by Eq. 19-39. The total heat is therefore 88595 $\approx 8.86 \times 10^4$ J.

(f) Defining a molar heat capacity in the same manner as we did in part (c), we now arrive at $C$ = 8.38 J/ mol·K.

87. For convenience, the "int" subscript for the internal energy will be omitted in this solution. Recalling Eq. 19-28, we note that $\sum_{\text{cycle}} E = 0$, which gives

$$\Delta E_{A\to B} + \Delta E_{B\to C} + \Delta E_{C\to D} + \Delta E_{D\to E} + \Delta E_{E\to A} = 0.$$

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A\to B} = \Delta E_{D\to E} = 0.$$

Now, with $\Delta E_{E\to A}$ = 8.0 J given in the problem statement, we have

$$\Delta E_{B\to C} + \Delta E_{C\to D} + 8.0\ \text{J} = 0.$$

In an adiabatic process, $\Delta E = -W$, which leads to

$$-5.0\ \text{J} + \Delta E_{C\to D} + 8.0\ \text{J} = 0,$$

and we obtain $\Delta E_{C\to D} = -3.0$ J.

88. (a) The work done in a constant-pressure process is $W = p\Delta V$. Therefore,

$$W = \left(25\,\text{N/m}^2\right)(1.8\,\text{m}^3 - 3.0\,\text{m}^3) = -30\,\text{J}.$$

The sign conventions discussed in the textbook for $Q$ indicate that we should write –75 J for the energy that leaves the system in the form of heat. Therefore, the first law of thermodynamics leads to

$$\Delta E_{\text{int}} = Q - W = (-75\ \text{J}) - (-30\ \text{J}) = -45\ \text{J}.$$

(b) Since the pressure is constant (and the number of moles is presumed constant), the ideal gas law in ratio form leads to

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (300\,\text{K})\left(\frac{1.8\,\text{m}^3}{3.0\,\text{m}^3}\right) = 1.8\times10^2\ \text{K}.$$

It should be noted that this is consistent with the gas being monatomic (that is, if one assumes $C_V = \frac{3}{2}R$ and uses Eq. 19-45, one arrives at this same value for the final temperature).

89. Consider the open end of the pipe. The balance of the pressures inside and outside the pipe requires that $p + \rho_w g(L/2) = p_0 + \rho_w gh$, where $p_0$ is the atmospheric pressure, and $p$ is the pressure of the air inside the pipe, which satisfies $p(L/2) = p_0L$, or $p = 2p_0$. We solve for $h$:

$$h = \frac{p - p_0}{\rho_w g} + \frac{L}{2} = \frac{1.01\times10^5\ \text{Pa}}{\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)} + \frac{25.0\,\text{m}}{2} = 22.8\,\text{m}.$$

90. (a) For diatomic gas, $\gamma = 7/5$. Using $pV^\gamma = \text{constant}$, we find the final gas pressure to be

$$p_f = \left(\frac{V_i}{V_f}\right)^\gamma p_i = \left(\frac{50\ \text{cm}^3}{250\ \text{cm}^3}\right)^{7/5}(15\ \text{atm}) = 1.58\ \text{atm}.$$

The work done by the gas during the adiabatic expansion process is

$$W = p_i V_i^{\gamma} \int_{V_i}^{V_f} V^{-\gamma} dV = p_i V_i^{\gamma} \frac{V_f^{1-\gamma} - V_i^{1-\gamma}}{1-\gamma} = \frac{p_f V_f - p_i V_i}{1-\gamma}$$

$$= \frac{(1.58 \text{ atm})(1.01\times10^5 \text{ Pa/atm})(250\times10^{-6}\text{m}^3) - (15 \text{ atm})(1.01\times10^5 \text{ Pa/atm})(50\times10^{-6}\text{m}^3)}{1-(7/5)}$$

$$= 89.64 \text{ J}$$

The period for each cycle is $\tau = (60 \text{ s})/(4000) = 0.015 \text{ s}.$ Since the time involved in the expansion is one-half of the total cycle: $\Delta t = \tau/2 = 7.5\times10^{-3}\text{s},$ the average power for the expansion is

$$P = \frac{W}{\Delta t} = \frac{89.64 \text{ J}}{7.5\times10^{-3}\text{s}} = 1.2\times10^4 \text{ W}.$$

(b) Using the conversion factor $1 \text{ hp} = 746 \text{ W},$ the power can also be expressed as 16 hp.

91. (a) For adiabatic process, $pV^{\gamma} = \text{constant},$ or $p = CV^{-\gamma}$. Thus,

$$B = -V\frac{dp}{dV} = -V\frac{d}{dV}\left(CV^{-\gamma}\right) = \gamma CV^{-\gamma} = \gamma p\,.$$

(b) Using $p = nRT/V = (m/M)RT/V$ with $\rho = m/V$, we find the speed of sound in an ideal gas to be

$$v_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma(m/M)RT/V}{m/V}} = \sqrt{\frac{\gamma RT}{M}}.$$

92. With $p = 1.01 \times 10^5$ Pa and $\rho = 1.29$ kg/m$^3$, we use the result of part (b) of the previous problem to obtain

$$\gamma = \frac{\rho v^2}{p} = \frac{\left(1.29\,\text{kg/m}^3\right)\left(331\text{m/s}\right)^2}{1.01\times10^5 \text{ Pa}} = 1.40.$$

93. Using $v_s = \sqrt{\gamma RT/M}$ , the result obtained in part (b) of problem 91, we find the ratio to be

$$\frac{v_1}{v_2} = \frac{\sqrt{\gamma RT/M_1}}{\sqrt{\gamma RT/M_2}} = \sqrt{\frac{M_2}{M_1}}.$$

94. The speed of sound in the gas is $v_s = \sqrt{\gamma RT/M}$ , and the rms speed of the gas is $v_{\text{rms}} = \sqrt{3RT/M}$ . Thus, the ratio is

$$\frac{v_s}{v_{\text{rms}}}=\frac{\sqrt{\gamma RT/M}}{\sqrt{3RT/M}}=\sqrt{\frac{\gamma}{3}}=\sqrt{\frac{C_p}{3C_V}}=\sqrt{\frac{C_V+R}{3C_V}}=\sqrt{\frac{5.0R+R}{3(5.0R)}}=\sqrt{\frac{2}{5}}=0.63.$$

95. The speed of sound is an ideal gas is $v_s=\sqrt{\gamma RT/M}$ , which gives

$$\gamma=\frac{Mv_s^2}{RT}.$$

Since the nodes of the standing waves are separated by half a wavelength, we have $\lambda=2(9.57\text{ cm})=19.14\text{ cm}=0.1914\text{ m},$ and the corresponding speed of sound is

$$v_s=\lambda f=(0.1914\text{ m})(1000\text{ Hz})=191.4\text{ m/s}.$$

Thus,

$$\gamma=\frac{Mv_s^2}{RT}=\frac{(0.127\text{ kg/mol})(191.4\text{ m/s})^2}{(8.314\text{ J/mol}\cdot\text{K})(400\text{ K})}=1.40.$$

96. The speed of sound is an ideal gas is $v_s=\sqrt{\gamma RT/M}$ . Differentiating $v_s$ with respect to $T$, we obtain

$$\frac{dv_s}{dT}=\frac{1}{2}\sqrt{\frac{\gamma R}{M}}T^{-1/2}=\frac{1}{2T}\sqrt{\frac{\gamma RT}{M}}=\frac{v_s}{2T}$$

Near $T=0\,°\text{C}=273\text{ K}$, the speed of sound is $331\text{ m/s}$. Thus, with $\Delta T=1\,°\text{C}=1\text{ K}$, the change in speed is

$$\Delta v_s=\frac{\Delta T}{2T}v_s=\frac{1\text{ K}}{2(273\text{ K})}(331\text{ m/s})=0.606\text{ m/s}\approx 0.61\text{ m/s}.$$

97. The average speed and rms speed of an ideal gas are given by $v_{\text{avg}}=\sqrt{8RT/\pi M}$ and $v_{\text{rms}}=\sqrt{3RT/M}$ , respectively. Thus,

$$\frac{v_{\text{avg}\,2}}{v_{\text{rms}\,1}}=\frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}}=\sqrt{\frac{8M_1}{3\pi M_2}}.$$

If $v_{\text{avg2}}=2v_{\text{rms1}}$, then

$$\frac{m_1}{m_2}=\frac{M_1}{M_2}=\frac{3\pi}{8}\left(\frac{v_{\text{avg}\,2}}{v_{\text{rms}\,1}}\right)^2=\frac{3\pi}{2}=4.71.$$

# Chapter 20

1. **THINK** If the expansion of the gas is reversible and isothermal, then there's no change in internal energy. However, if the process is reversible and adiabatic, then there would be no change in entropy.

**EXPRESS** Since the gas is ideal, its pressure $p$ is given in terms of the number of moles $n$, the volume $V$, and the temperature $T$ by $p = nRT/V$. If the expansion is isothermal, the work done by the gas is

$$W = \int_{V_1}^{V_2} p\,dV = nRT\int_{V_1}^{V_2}\frac{dV}{V} = nRT\ln\frac{V_2}{V_1}\ ,$$

and the corresponding change in entropy is $\Delta S = \int (1/T)\ dQ = Q/T$, where $Q$ is the heat absorbed (see Eq. 20-2).

**ANALYZE** (a) With $V_2 = 2.00V_1$ and $T = 400$ K, we obtain

$$W = nRT\ln 2.00 = (4.00\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(400\ \text{K})\ln 2.00 = 9.22\times10^3\ \text{J}.$$

(b) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22\times10^3\ \text{J}}{400\ \text{K}} = 23.1\ \text{J/K}.$$

(c) The change in entropy $\Delta S$ is zero for all reversible adiabatic processes.

**LEARN** The general expression for $\Delta S$ for reversible processes is given by Eq. 20-4:

$$\Delta S = S_f - S_i = nR\ln\left(\frac{V_f}{V_i}\right) + nC_V\ln\left(\frac{T_f}{T_i}\right).$$

Note that $\Delta S$ does not depend on how the gas changes from its initial state $i$ to the final state $f$.

2. An isothermal process is one in which $T_i = T_f$, which implies $\ln(T_f/T_i) = 0$. Therefore, Eq. 20-4 leads to

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) \Rightarrow n = \frac{22.0}{(8.31)\ln(3.4/1.3)} = 2.75\ \text{mol}.$$

3. An isothermal process is one in which $T_i = T_f$, which implies $\ln(T_f/T_i) = 0$. Therefore, with $V_f/V_i = 2.00$, Eq. 20-4 leads to

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (2.50\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})\ln(2.00) = 14.4\ \text{J/K}.$$

4. From Eq. 20-2, we obtain $Q = T\Delta S = (405\ \text{K})(46.0\ \text{J/K}) = 1.86\times10^4\ \text{J}$.

5. We use the following relation derived in Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium:"

$$\Delta S = mc\ln(T_f/T_i).$$

(a) The energy absorbed as heat is given by Eq. 19-14. Using Table 19-3, we find

$$Q = cm\Delta T = \left(386\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(2.00\ \text{kg})(75\ \text{K}) = 5.79\times10^4\ \text{J}$$

where we have used the fact that a change in Kelvin temperature is equivalent to a change in Celsius degrees.

(b) With $T_f = 373.15$ K and $T_i = 298.15$ K, we obtain

$$\Delta S = (2.00\ \text{kg})\left(386\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{373.15}{298.15}\right) = 173\ \text{J/K}.$$

6. (a) This may be considered a reversible process (as well as isothermal), so we use $\Delta S = Q/T$ where $Q = Lm$ with $L = 333$ J/g from Table 19-4. Consequently,

$$\Delta S = \frac{(333\ \text{J/g})(12.0\ \text{g})}{273\ \text{K}} = 14.6\ \text{J/K}.$$

(b) The situation is similar to that described in part (a), except with $L = 2256$ J/g, $m = 5.00$ g, and $T = 373$ K. We therefore find $\Delta S = 30.2$ J/K.

7. (a) We refer to the copper block as block 1 and the lead block as block 2. The equilibrium temperature $T_f$ satisfies $m_1c_1(T_f - T_{i,1}) + m_2c_2(T_f - T_{i2}) = 0$, which we solve for $T_f$:

$$T_f = \frac{m_1c_1T_{i,1} + m_2c_2T_{i,2}}{m_1c_1 + m_2c_2} = \frac{(50.0\ \text{g})(386\ \text{J/kg}\cdot\text{K})(400\ \text{K}) + (100\ \text{g})(128\ \text{J/kg}\cdot\text{K})(200\ \text{K})}{(50.0\ \text{g})(386\ \text{J/kg}\cdot\text{K}) + (100\ \text{g})(128\ \text{J/kg}\cdot\text{K})}$$

$$= 320\ \text{K}.$$

(b) Since the two-block system in thermally insulated from the environment, the change in internal energy of the system is zero.

(c) The change in entropy is

$$\Delta S = \Delta S_1 + \Delta S_2 = m_1 c_1 \ln\left(\frac{T_f}{T_{i,1}}\right) + m_2 c_2 \ln\left(\frac{T_f}{T_{i,2}}\right)$$
$$= (50.0 \text{ g})(386 \text{ J/kg}\cdot\text{K})\ln\left(\frac{320 \text{ K}}{400 \text{ K}}\right) + (100 \text{ g})(128 \text{ J/kg}\cdot\text{K})\ln\left(\frac{320 \text{ K}}{200 \text{ K}}\right)$$
$$= +1.72 \text{ J/K}.$$

8. We use Eq. 20-1:

$$\Delta S = \int \frac{nC_V \, dT}{T} = nA\int_{5.00}^{10.0} T^2 dT = \frac{nA}{3}\left[(10.0)^3 - (5.00)^3\right] = 0.0368 \text{ J/K}.$$

9. The ice warms to 0°C, then melts, and the resulting water warms to the temperature of the lake water, which is 15°C. As the ice warms, the energy it receives as heat when the temperature changes by $dT$ is $dQ = mc_I \, dT$, where $m$ is the mass of the ice and $c_I$ is the specific heat of ice. If $T_i$ (= 263 K) is the initial temperature and $T_f$ (= 273 K) is the final temperature, then the change in its entropy is

$$\Delta S = \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln\frac{T_f}{T_i} = (0.010 \text{ kg})(2220 \text{ J/kg}\cdot\text{K})\ln\left(\frac{273 \text{ K}}{263 \text{ K}}\right) = 0.828 \text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is $mL_F$, where $L_F$ is the heat of fusion for ice. Thus,

$$\Delta S = Q/T = mL_F/T = (0.010 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = 12.20 \text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln\frac{T_f}{T_i},$$

where $c_w$ is the specific heat of water (4190 J/kg · K). Thus,

$$\Delta S = (0.010 \text{ kg})(4190 \text{ J/kg}\cdot\text{K}) \ln\left(\frac{288 \text{ K}}{273 \text{ K}}\right) = 2.24 \text{ J/K}.$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828 \text{ J/K} + 12.20 \text{ J/K} + 2.24 \text{ J/K} = 15.27 \text{ J/K}.$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where $Q$ is the energy it receives as heat (the negative of the energy it supplies the ice) and $T$ is its temperature. When the ice warms to 0°C,

$$Q = -mc_I\left(T_f - T_i\right) = -\left(0.010 \text{ kg}\right)\left(2220 \text{ J/kg}\cdot\text{ K}\right)\left(10 \text{ K}\right) = -222 \text{ J}.$$

When the ice melts,

$$Q = -mL_F = -(0.010 \text{ kg})\left(333\times 10^3 \text{ J / kg}\right) = -3.33\times 10^3 \text{ J}.$$

When the water from the ice warms,

$$Q = -mc_w\left(T_f - T_i\right) = -(0.010 \text{ kg})(4190 \text{ J / kg}\cdot\text{ K})(15 \text{ K}) = -629 \text{ J}.$$

The total energy leaving the lake water is

$$Q = -222 \text{ J} - 3.33 \times 10^3 \text{ J} - 6.29 \times 10^2 \text{ J} = -4.18 \times 10^3 \text{ J}.$$

The change in entropy is

$$\Delta S = -\frac{4.18\times 10^3 \text{ J}}{288 \text{ K}} = -14.51 \text{ J/K}.$$

The change in the entropy of the ice–lake system is $\Delta S$ = (15.27 – 14.51) J/K = 0.76 J/K.

10. We follow the method shown in Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium." Since

$$\Delta S = mc\int_{T_i}^{T_f} \frac{dT}{T} = mc \ln(T_f /T_i) ,$$

then with $\Delta S$ = 50 J/K, $T_f$ = 380 K, $T_i$ = 280 K, and $m$ = 0.364 kg, we obtain $c = 4.5\times 10^2$ J/kg·K.

11. **THINK** The aluminum sample gives off energy as heat to water. Thermal equilibrium is reached when both the aluminum and the water come to a common final temperature $T_f$.

**EXPRESS** The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a(T_{ai} - T_f)$, where $m_a$ is the mass of the aluminum, $c_a$ is the specific heat of aluminum, $T_{ai}$ is the initial temperature of the aluminum, and $T_f$ is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w(T_f - T_{wi})$, where $m_w$ is the mass of the water, $c_w$ is the specific heat of water, and $T_{wi}$ is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a \left(T_{ai} - T_f\right) = m_w c_w \left(T_f - T_{wi}\right) \Rightarrow T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The change in entropy is $\Delta S = \int dQ/T$ .

**ANALYZE** (a) The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$T_f = \frac{(0.200\text{ kg})(900\text{ J/kg}\cdot\text{K})(100°\text{C}) + (0.0500\text{ kg})(4190\text{ J/kg}\cdot\text{K})(20°\text{C})}{(0.200\text{ kg})(900\text{ J/kg}\cdot\text{K}) + (0.0500\text{ kg})(4190\text{ J/kg}\cdot\text{K})}$$
$$= 57.0°\text{C} = 330\text{ K}.$$

(b) Now temperatures must be given in Kelvins: $T_{ai}$ = 393 K, $T_{wi}$ = 293 K, and $T_f$ = 330 K. For the aluminum, $dQ = m_a c_a dT$ and the change in entropy is

$$\Delta S_a = \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln\left(\frac{T_f}{T_{ai}}\right) = (0.200\text{ kg})(900\text{ J/kg}\cdot\text{K})\ln\left(\frac{330\text{ K}}{373\text{ K}}\right)$$
$$= -22.1\text{ J/K}.$$

(c) The entropy change for the water is

$$\Delta S_w = \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln\left(\frac{T_f}{T_{wi}}\right) = (0.0500\text{ kg})(4190\text{ J/kg.K})\ln\left(\frac{330\text{K}}{293\text{K}}\right)$$
$$= +24.9\text{ J/K}.$$

(d) The change in the total entropy of the aluminum-water system is

$$\Delta S = \Delta S_a + \Delta S_w = -22.1\text{ J/K} + 24.9\text{ J/K} = +2.8\text{ J/K}.$$

**LEARN** The system is closed and the process is irreversible. For aluminum the entropy change is negative ( $\Delta S_a < 0$ ) since $T_f < T_{ai}$ . However, for water, entropy increases because $T_f > T_{wi}$ . The overall entropy change for the aluminum-water system is positive, in accordance with the second law of thermodynamics.

12. We concentrate on the first term of Eq. 20-4 (the second term is zero because the final and initial temperatures are the same, and because ln(1) = 0). Thus, the entropy change is

$$\Delta S = nR\ln(V_f/V_i).$$

Noting that $\Delta S$ = 0 at $V_f$ = 0.40 m$^3$, we are able to deduce that $V_i$ = 0.40 m$^3$. We now examine the point in the graph where $\Delta S$ = 32 J/K and $V_f$ = 1.2 m$^3$; the above expression can now be used to solve for the number of moles. We obtain $n$ = 3.5 mol.

13. This problem is similar to Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium." The only difference is that we need to find the mass $m$ of each of the blocks. Since the two blocks are identical, the final temperature $T_f$ is the average of the initial temperatures:

$$T_f = \frac{1}{2}\left(T_i + T_f\right) = \frac{1}{2}(305.5\text{ K} + 294.5\text{ K}) = 300.0\text{ K}.$$

Thus from $Q = mc\Delta T$ we find the mass $m$:

$$m = \frac{Q}{c\Delta T} = \frac{215\text{ J}}{(386\text{ J / kg}\cdot\text{K})(300.0\text{ K} - 294.5\text{ K})} = 0.101\text{ kg}.$$

(a) The change in entropy for block $L$ is

$$\Delta S_L = mc\ \ln\left(\frac{T_f}{T_{iL}}\right) = \left(0.101\text{ kg}\right)\left(386\text{ J/kg}\cdot\text{K}\right)\ln\left(\frac{300.0\text{ K}}{305.5\text{ K}}\right) = -0.710\text{ J/K}.$$

(b) Since the temperature of the reservoir is virtually the same as that of the block, which gives up the same amount of heat as the reservoir absorbs, the change in entropy $\Delta S'_L$ of the reservoir connected to the left block is the opposite of that of the left block: $\Delta S'_L$ = $-\Delta S_L$ = +0.710 J/K.

(c) The entropy change for block $R$ is

$$\Delta S_R = mc\ln\left(\frac{T_f}{T_{iR}}\right) = \left(0.101\text{ kg}\right)\left(386\text{ J/kg}\cdot\text{K}\right)\ln\left(\frac{300.0\text{ K}}{294.5\text{ K}}\right) = +0.723\text{ J/K}.$$

(d) Similar to the case in part (b) above, the change in entropy $\Delta S'_R$ of the reservoir connected to the right block is given by $\Delta S'_R$ = $-\Delta S_R$ = $-0.723$ J/K.

(e) The change in entropy for the two-block system is

$$\Delta S_L + \Delta S_R = -0.710\text{ J/K} + 0.723\text{ J/K} = +0.013\text{ J/K}.$$

(f) The entropy change for the entire system is given by

$$\Delta S = \Delta S_L + \Delta S'_L + \Delta S_R + \Delta S'_R = \Delta S_L - \Delta S_L + \Delta S_R - \Delta S_R = 0,$$

which is expected of a reversible process.

14. (a) Work is done only for the $ab$ portion of the process. This portion is at constant pressure, so the work done by the gas is

$$W = \int_{V_0}^{4V_0} p_0\, dV = p_0(4.00V_0 - 1.00V_0) = 3.00\, p_0 V_0 \;\Rightarrow\; \frac{W}{p_0 V} = 3.00.$$

(b) We use the first law: $\Delta E_{\text{int}} = Q - W$. Since the process is at constant volume, the work done by the gas is zero and $E_{\text{int}} = Q$. The energy $Q$ absorbed by the gas as heat is $Q = nC_V\, \Delta T$, where $C_V$ is the molar specific heat at constant volume and $\Delta T$ is the change in temperature. Since the gas is a monatomic ideal gas, $C_V = 3R/2$. Use the ideal gas law to find that the initial temperature is

$$T_b = \frac{p_b V_b}{nR} = \frac{4\, p_0 V_0}{nR}$$

and that the final temperature is

$$T_c = \frac{p_c V_c}{nR} = \frac{(2\, p_0)(4V_0)}{nR} = \frac{8\, p_0 V_0}{nR}.$$

Thus,

$$Q = \frac{3}{2} nR\left(\frac{8\, p_0 V_0}{nR} - \frac{4\, p_0 V_0}{nR}\right) = 6.00\, p_0 V_0.$$

The change in the internal energy is $\Delta E_{\text{int}} = 6p_0V_0$ or $\Delta E_{\text{int}}/p_0V_0 = 6.00$. Since $n = 1$ mol, this can also be written $Q = 6.00RT_0$.

(c) For a complete cycle, $\Delta E_{\text{int}} = 0$.

(d) Since the process is at constant volume, use $dQ = nC_V\, dT$ to obtain

$$\Delta S = \int \frac{dQ}{T} = nC_V \int_{T_b}^{T_c} \frac{dT}{T} = nC_V \ln \frac{T_c}{T_b}.$$

Substituting $C_V = \frac{3}{2}R$ and using the ideal gas law, we write

$$\frac{T_c}{T_b} = \frac{p_c V_c}{p_b V_b} = \frac{(2\, p_0)(4V_0)}{p_0(4V_0)} = 2.$$

Thus, $\Delta S = \frac{3}{2} nR \ln 2$. Since $n = 1$, this is $\Delta S = \frac{3}{2} R \ln 2 = 8.64$ J/K.

(e) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $\Delta S = 0$.

15. (a) The final mass of ice is (1773 g + 227 g)/2 = 1000 g. This means 773 g of water froze. Energy in the form of heat left the system in the amount $mL_F$, where $m$ is the mass of the water that froze and $L_F$ is the heat of fusion of water. The process is isothermal, so the change in entropy is

$$\Delta S = Q/T = -mL_F/T = -(0.773 \text{ kg})(333 \times 10^3 \text{ J/kg})/(273 \text{ K}) = -943 \text{ J/K}.$$

(b) Now, 773 g of ice is melted. The change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = +943 \text{ J/K}.$$

(c) Yes, they are consistent with the second law of thermodynamics. Over the entire cycle, the change in entropy of the water–ice system is zero even though part of the cycle is irreversible. However, the system is not closed. To consider a closed system, we must include whatever exchanges energy with the ice and water. Suppose it is a constant-temperature heat reservoir during the freezing portion of the cycle and a Bunsen burner during the melting portion. During freezing the entropy of the reservoir increases by 943 J/K. As far as the reservoir–water–ice system is concerned, the process is adiabatic and reversible, so its total entropy does not change. The melting process is irreversible, so the total entropy of the burner–water–ice system increases. The entropy of the burner either increases or else decreases by less than 943 J/K.

16. In coming to equilibrium, the heat lost by the 100 cm$^3$ of liquid water (of mass $m_w$ = 100 g and specific heat capacity $c_w$ = 4190 J/kg·K) is absorbed by the ice (of mass $m_i$, which melts and reaches $T_f > 0$ °C). We begin by finding the equilibrium temperature:

$$\sum Q = 0$$

$$Q_{\text{warm water cools}} + Q_{\text{ice warms to } 0^\circ} + Q_{\text{ice melts}} + Q_{\text{melted ice warms}} = 0$$

$$c_w m_w \left(T_f - 20^\circ\right) + c_i m_i \left(0^\circ - (-10^\circ)\right) + L_F m_i + c_w m_i \left(T_f - 0^\circ\right) = 0$$

which yields, after using $L_F$ = 333000 J/kg and values cited in the problem, $T_f$ = 12.24 ° which is equivalent to $T_f$ = 285.39 K. Sample Problem 20.01 — "Entropy change of two blocks coming to equilibrium" shows that

$$\Delta S_{\text{temp change}} = mc \ln\left(\frac{T_2}{T_1}\right)$$

for processes where $\Delta T = T_2 - T_1$, and Eq. 20-2 gives $\Delta S_{\text{melt}} = L_F\, m / T_{\text{o}}$ for the phase change experienced by the ice (with $T_{\text{o}}$ = 273.15 K). The total entropy change is (with $T$ in Kelvins)

$$\Delta S_{\text{system}} = m_w c_w \ln\left(\frac{285.39}{293.15}\right) + m_i c_i \ln\left(\frac{273.15}{263.15}\right) + m_i c_w \ln\left(\frac{285.39}{273.15}\right) + \frac{L_F\, m_i}{273.15}$$
$$= (-11.24 + 0.66 + 1.47 + 9.75)\text{J/K} = 0.64 \text{ J/K}.$$

17. The connection between molar heat capacity and the degrees of freedom of a diatomic gas is given by setting $f$ = 5 in Eq. 19-51. Thus, $C_V = 5R/2$, $C_p = 7R/2$, and

$\gamma = 7/5$. In addition to various equations from Chapter 19, we also make use of Eq. 20-4 of this chapter. We note that we are asked to use the ideal gas constant as $R$ and not plug in its numerical value. We also recall that isothermal means constant temperature, so $T_2 = T_1$ for the $1 \to 2$ process. The statement (at the end of the problem) regarding "per mole" may be taken to mean that $n$ may be set identically equal to 1 wherever it appears.

(a) The gas law in ratio form is used to obtain

$$p_2 = p_1\left(\frac{V_1}{V_2}\right) = \frac{p_1}{3} \;\Rightarrow\; \frac{p_2}{p_1} = \frac{1}{3} = 0.333\,.$$

(b) The adiabatic relations Eq. 19-54 and Eq. 19-56 lead to

$$p_3 = p_1\left(\frac{V_1}{V_3}\right)^{\gamma} = \frac{p_1}{3^{1.4}} \;\Rightarrow\; \frac{p_3}{p_1} = \frac{1}{3^{1.4}} = 0.215\,.$$

(c) Similarly, we find

$$T_3 = T_1\left(\frac{V_1}{V_3}\right)^{\gamma-1} = \frac{T_1}{3^{0.4}} \;\Rightarrow\; \frac{T_3}{T_1} = \frac{1}{3^{0.4}} = 0.644.$$

• process $1 \to 2$

(d) The work is given by Eq. 19-14:

$$W = nRT_1 \ln (V_2/V_1) = RT_1 \ln 3 = 1.10RT_1.$$

Thus, $W/ nRT_1 = \ln 3 = 1.10$.

(e) The internal energy change is $\Delta E_{\text{int}} = 0$, since this is an ideal gas process without a temperature change (see Eq. 19-45). Thus, the energy absorbed as heat is given by the first law of thermodynamics: $Q = \Delta E_{\text{int}} + W \approx 1.10RT_1$, or $Q/ nRT_1 = \ln 3 = 1.10$.

(f) $\Delta E_{\text{int}} = 0$ or $\Delta E_{\text{int}} / nRT_1 = 0$

(g) The entropy change is $\Delta S = Q/T_1 = 1.10R$, or $\Delta S/R = 1.10$.

• process $2 \to 3$

(h) The work is zero, since there is no volume change. Therefore, $W/nRT_1 = 0$.

(i) The internal energy change is

$$\Delta E_{\text{int}} = nC_V\left(T_3 - T_2\right) = (1)\left(\frac{5}{2}R\right)\left(\frac{T_1}{3^{0.4}} - T_1\right) \approx -0.889\,RT_1 \;\Rightarrow\; \frac{\Delta E_{\text{int}}}{nRT_1} \approx -0.889.$$

This ratio (–0.889) is also the value for $Q/nRT_1$ (by either the first law of thermodynamics or by the definition of $C_V$).

(j) $\Delta E_{\text{int}}/nRT_1 = -0.889$.

(k) For the entropy change, we obtain

$$\frac{\Delta S}{R} = n\ln\left(\frac{V_3}{V_1}\right) + n\frac{C_V}{R}\ln\left(\frac{T_3}{T_1}\right) = (1)\ \ln(1) + (1)\left(\frac{5}{2}\right)\ln\left(\frac{T_1/3^{0.4}}{T_1}\right) = 0 + \frac{5}{2}\ln(3^{-0.4}) \approx -1.10\ .$$

• process 3 → 1

(l) By definition, $Q = 0$ in an adiabatic process, which also implies an absence of entropy change (taking this to be a reversible process). The internal change must be the negative of the value obtained for it in the previous process (since all the internal energy changes must add up to zero, for an entire cycle, and its change is zero for process 1 → 2), so $\Delta E_{\text{int}} = +0.889RT_1$. By the first law of thermodynamics, then,

$$W = Q - \Delta E_{\text{int}} = -0.889RT_1,$$

or $W/nRT_1 = -0.889$.

(m) $Q = 0$ in an adiabatic process.

(n) $\Delta E_{\text{int}}/nRT_1 = +0.889$.

(o) $\Delta S/nR = 0$.

18. (a) It is possible to motivate, starting from Eq. 20-3, the notion that heat may be found from the integral (or "area under the curve") of a curve in a *TS* diagram, such as this one. Either from calculus, or from geometry (area of a trapezoid), it is straightforward to find the result for a "straight-line" path in the *TS* diagram:

$$Q_{\text{straight}} = \left(\frac{T_i + T_f}{2}\right)\Delta S$$

which could, in fact, be *directly* motivated from Eq. 20-3 (but it is important to bear in mind that this is rigorously true only for a process that forms a straight line in a graph that plots $T$ versus $S$). This leads to

$$Q = (300\text{ K})\,(15\text{ J/K}) = 4.5\times10^3\text{ J}$$

for the energy absorbed as heat by the gas.

(b) Using Table 19-3 and Eq. 19-45, we find

$$\Delta E_{\text{int}} = n\left(\frac{3}{2}R\right)\Delta T = (2.0 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(200 \text{ K} - 400 \text{ K}) = -5.0\times10^3 \text{ J}.$$

(c) By the first law of thermodynamics, $W = Q - \Delta E_{\text{int}} = 4.5 \text{ kJ} - (-5.0 \text{ kJ}) = 9.5 \text{ kJ}.$

19. We note that the connection between molar heat capacity and the degrees of freedom of a monatomic gas is given by setting $f$ = 3 in Eq. 19-51. Thus, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$.

(a) Since this is an ideal gas, Eq. 19-45 holds, which implies $\Delta E_{\text{int}} = 0$ for this process. Equation 19-14 also applies, so that by the first law of thermodynamics,

$$Q = 0 + W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln 2 \quad \rightarrow \quad Q/p_1V_1 = \ln 2 = 0.693.$$

(b) The gas law in ratio form implies that the pressure decreased by a factor of 2 during the isothermal expansion process to $V_2 = 2.00V_1$, so that it needs to increase by a factor of 4 in this step in order to reach a final pressure of $p_2 = 2.00p_1$. That same ratio form now applied to this constant-volume process, yielding $4.00 = T_2T_1$, which is used in the following:

$$Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)(T_2 - T_1) = \frac{3}{2}nRT_1\left(\frac{T_2}{T_1} - 1\right) = \frac{3}{2}p_1V_1(4-1) = \frac{9}{2}p_1V_1$$

or $Q/p_1V_1 = 9/2 = 4.50$.

(c) The work done during the isothermal expansion process may be obtained by using Eq. 19-14:

$$W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln 2.00 \ \rightarrow \ W/p_1V_1 = \ln 2 = 0.693.$$

(d) In step 2 where the volume is kept constant, $W = 0$.

(e) The change in internal energy can be calculated by combining the above results and applying the first law of thermodynamics:

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(p_1V_1 \ln 2 + \frac{9}{2}p_1V_1\right) - (p_1V_1 \ln 2 + 0) = \frac{9}{2}p_1V_1$$

or $\Delta E_{\text{int}}/p_1V_1 = 9/2 = 4.50$.

(f) The change in entropy may be computed by using Eq. 20-4:

$$\Delta S = R\ln\left(\frac{2.00V_1}{V_1}\right) + C_V \ln\left(\frac{4.00T_1}{T_1}\right) = R\ln 2.00 + \left(\frac{3}{2}R\right)\ln(2.00)^2$$

$$= R\ln 2.00 + 3R\ln 2.00 = 4R\ln 2.00 = 23.0 \text{ J/K}.$$

The second approach consists of an isothermal (constant $T$) process in which the volume halves, followed by an isobaric (constant $p$) process.

(g) Here the gas law applied to the first (isothermal) step leads to a volume half as big as the original. Since $\ln(1/2.00) = -\ln 2.00$, the reasoning used above leads to

$$Q = -p_1V_1 \ln 2.00 \Rightarrow Q/p_1V_1 = -\ln 2.00 = -0.693.$$

(h) To obtain a final volume twice as big as the original, in this step we need to increase the volume by a factor of 4.00. Now, the gas law applied to this isobaric portion leads to a temperature ratio $T_2/T_1 = 4.00$. Thus,

$$Q = C_p\Delta T = \frac{5}{2}R(T_2 - T_1) = \frac{5}{2}RT_1\left(\frac{T_2}{T_1} - 1\right) = \frac{5}{2}p_1V_1(4-1) = \frac{15}{2}p_1V_1$$

or $Q/p_1V_1 = 15/2 = 7.50$.

(i) During the isothermal compression process, Eq. 19-14 gives

$$W = nRT_1 \ln V_2/V_1 = p_1V_1 \ln(-1/2.00) = -p_1V_1 \ln 2.00 \Rightarrow W/p_1V_1 = -\ln 2 = -0.693.$$

(j) The initial value of the volume, for this part of the process, is $V_i = V_1/2$, and the final volume is $V_f = 2V_1$. The pressure maintained during this process is $p' = 2.00p_1$. The work is given by Eq. 19-16:

$$W = p'\Delta V = p'(V_f - V_i) = (2.00p_1)\left(2.00V_1 - \frac{1}{2}V_1\right) = 3.00p_1V_1 \Rightarrow W/p_1V_1 = 3.00.$$

(k) Using the first law of thermodynamics, the change in internal energy is

$$\Delta E_{\text{int}} = Q_{\text{total}} - W_{\text{total}} = \left(\frac{15}{2}p_1V_1 - p_1V_1\ln 2.00\right) - (3p_1V_1 - p_1V_1\ln 2.00) = \frac{9}{2}p_1V_1$$

or $\Delta E_{\text{int}}/p_1V_1 = 9/2 = 4.50$. The result is the same as that obtained in part (e).

(l) Similarly, $\Delta S = 4R\ln 2.00 = 23.0$ J/K. the same as that obtained in part (f).

20. (a) The final pressure is

$$p_f = (5.00 \text{ kPa})e^{(V_i - V_f)/a} = (5.00 \text{ kPa})e^{(1.00 \text{ m}^3 - 2.00 \text{ m}^3)/1.00 \text{ m}^3} = 1.84 \text{ kPa}.$$

(b) We use the ratio form of the gas law to find the final temperature of the gas:

$$T_f = T_i\left(\frac{p_f V_f}{p_i V_i}\right) = (600\text{ K})\frac{(1.84\text{ kPa})(2.00\text{ m}^3)}{(5.00\text{ kPa})(1.00\text{ m}^3)} = 441\text{ K}.$$

For later purposes, we note that this result can be written "exactly" as $T_f = T_i\,(2e^{-1})$. In our solution, we are avoiding using the "one mole" datum since it is not clear how precise it is.

(c) The work done by the gas is

$$W = \int_i^f p\,dV = \int_{V_i}^{V_f}(5.00\text{ kPa})e^{(V_i-V)/a}dV = (5.00\text{ kPa})e^{V_i/a}\cdot\left[-ae^{-V/a}\right]_{V_i}^{V_f}$$
$$= (5.00\text{ kPa})e^{1.00}(1.00\text{ m}^3)\left(e^{-1.00} - e^{-2.00}\right)$$
$$= 3.16\text{ kJ}.$$

(d) Consideration of a two-stage process, as suggested in the hint, brings us simply to Eq. 20-4. Consequently, with $C_V = \frac{3}{2}R$ (see Eq. 19-43), we find

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) + n\left(\frac{3}{2}R\right)\ln\left(\frac{T_f}{T_i}\right) = nR\left(\ln 2 + \frac{3}{2}\ln\left(2e^{-1}\right)\right) = \frac{p_i V_i}{T_i}\left(\ln 2 + \frac{3}{2}\ln 2 + \frac{3}{2}\ln e^{-1}\right)$$
$$= \frac{(5000\text{ Pa})(1.00\text{ m}^3)}{600\text{ K}}\left(\frac{5}{2}\ln 2 - \frac{3}{2}\right)$$
$$= 1.94\text{ J/K}.$$

21. We consider a three-step reversible process as follows: the supercooled water drop (of mass $m$) starts at state 1 ($T_1$ = 268 K), moves on to state 2 (still in liquid form but at $T_2$ = 273 K), freezes to state 3 ($T_3 = T_2$), and then cools down to state 4 (in solid form, with $T_4 = T_1$). The change in entropy for each of the stages is given as follows:

$$\Delta S_{12} = mc_w \ln (T_2/T_1),$$

$$\Delta S_{23} = -mL_F/T_2,$$

$$\Delta S_{34} = mc_I \ln (T_4/T_3) = mc_I \ln (T_1/T_2) = -mc_I \ln (T_2/T_1).$$

Thus the net entropy change for the water drop is

$$\Delta S = \Delta S_{12} + \Delta S_{23} + \Delta S_{34} = m(c_w - c_I)\ln\left(\frac{T_2}{T_1}\right) - \frac{mL_F}{T_2}$$
$$= (1.00\text{ g})(4.19\text{ J/g}\cdot\text{K} - 2.22\text{ J/g}\cdot\text{K})\ln\left(\frac{273\text{ K}}{268\text{ K}}\right) - \frac{(1.00\text{ g})(333\text{ J/g})}{273\text{ K}}$$
$$= -1.18\text{ J/K}.$$

22. (a) We denote the mass of the ice (which turns to water and warms to $T_f$) as $m$ and the mass of original water (which cools from 80º down to $T_f$) as $m'$. From $\Sigma Q = 0$ we have

$$L_F\, m + cm\,(T_f - 0^\circ) + cm'\,(T_f - 80^\circ) = 0.$$

Since $L_F = 333 \times 10^3$ J/kg, $c = 4190$ J/(kg·Cº), $m' = 0.13$ kg, and $m = 0.012$ kg, we find $T_f$ = 66.5ºC, which is equivalent to 339.67 K.

(b) Using Eq. 20-2, the process of ice at 0º C turning to water at 0º C involves an entropy change of

$$\frac{Q}{T} = \frac{L_F\, m}{273.15\ \text{K}} = 14.6\ \text{J/K}.$$

(c) Using Eq. 20-1, the process of $m = 0.012$ kg of water warming from 0º C to 66.5º C involves an entropy change of

$$\int_{273.15}^{339.67} \frac{cm\,dT}{T} = cm \ln\left(\frac{339.67}{273.15}\right) = 11.0\ \text{J/K}\,.$$

(d) Similarly, the cooling of the original water involves an entropy change of

$$\int_{353.15}^{339.67} \frac{cm'\,dT}{T} = cm' \ln\left(\frac{339.67}{353.15}\right) = -21.2\ \text{J/K}\,.$$

(e) The net entropy change in this calorimetry experiment is found by summing the previous results; we find (by using more precise values than those shown above) $\Delta S_{\text{net}}$ = 4.39 J/K.

23. With $T_L$ = 290 k, we find

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow T_H = \frac{T_L}{1-\varepsilon} = \frac{290\ \text{K}}{1-0.40}$$

which yields the (initial) temperature of the high-temperature reservoir: $T_H$ = 483 K. If we replace $\varepsilon$ = 0.40 in the above calculation with $\varepsilon$ = 0.50, we obtain a (final) high temperature equal to $T_H' = 580\ \text{K}$. The difference is

$$T_H' - T_H = 580\ \text{K} - 483\ \text{K} = 97\ \text{K}.$$

24. The answers to this exercise do not depend on the engine being of the Carnot design. Any heat engine that intakes energy as heat (from, say, consuming fuel) equal to $|Q_H|$ = 52 kJ and exhausts (or discards) energy as heat equal to $|Q_L|$ = 36 kJ will have these values of efficiency $\varepsilon$ and net work $W$.

(a) Equation 20-12 gives $\varepsilon = 1 - \left|\dfrac{Q_L}{Q_H}\right| = 0.31 = 31\%$ .

(b) Equation 20-8 gives $W = |Q_H| - |Q_L| = 16$ kJ .

25. We solve (b) first.

(b) For a Carnot engine, the efficiency is related to the reservoir temperatures by Eq. 20-13. Therefore,

$$T_H = \frac{T_H - T_L}{\varepsilon} = \frac{75 \text{ K}}{0.22} = 341 \text{ K}$$

which is equivalent to 68°C.

(a) The temperature of the cold reservoir is $T_L = T_H - 75 = 341 \text{ K} - 75 \text{ K} = 266$ K.

26. Equation 20-13 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{373 \text{ K}}{7\times10^8 \text{ K}} = 0.9999995$$

quoting more figures than are significant. As a percentage, this is $\varepsilon = 99.99995\%$.

27. **THINK** The thermal efficiency of the Carnot engine depends on the temperatures of the reservoirs.

**EXPRESS** The efficiency of the Carnot engine is given by

$$\varepsilon_C = \frac{T_H - T_L}{T_H},$$

where $T_H$ is the temperature of the higher-temperature reservoir, and $T_L$ the temperature of the lower-temperature reservoir, in kelvin scale. The work done by the engine is $|W| = \varepsilon|Q_H|$ .

**ANALYZE** (a) The efficiency of the engine is

$$\varepsilon_c = \frac{T_H - T_L}{T_H} = \frac{(235-115)\,\text{K}}{(235+273)\,\text{K}} = 0.236 = 23.6\% \,.$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_H|$, the work done is given by

$$|W| = \varepsilon |Q_{\rm H}| = 0.236(6.30\times10^4\ {\rm J}) = 1.49\times10^4\ {\rm J}\ .$$

**LEARN** Expressing the efficiency as $\varepsilon_c = 1 - T_{\rm L}/T_{\rm H}$, we see that $\varepsilon_c$ approaches unity (100% efficiency) in the limit $T_L/T_H \to 0$. This is an impossible dream. An alternative version of the second law of thermodynamics is: *there are no perfect engines*.

28. All terms are assumed to be positive. The total work done by the two-stage system is $W_1 + W_2$. The heat-intake (from, say, consuming fuel) of the system is $Q_1$, so we have (by Eq. 20-11 and Eq. 20-8)

$$\varepsilon = \frac{W_1 + W_2}{Q_1} = \frac{(Q_1 - Q_2) + (Q_2 - Q_3)}{Q_1} = 1 - \frac{Q_3}{Q_1}.$$

Now, Eq. 20-10 leads to

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}$$

where we assume $Q_2$ is absorbed by the second stage at temperature $T_2$. This implies the efficiency can be written

$$\varepsilon = 1 - \frac{T_3}{T_1} = \frac{T_1 - T_3}{T_1}.$$

29. (a) The net work done is the rectangular "area" enclosed in the $pV$ diagram:

$$W = (V - V_0)(p - p_0) = (2V_0 - V_0)(2p_0 - p_0) = V_0 p_0.$$

Inserting the values stated in the problem, we obtain $W = 2.27$ kJ.

(b) We compute the energy added as heat during the "heat-intake" portions of the cycle using Eq. 19-39, Eq. 19-43, and Eq. 19-46:

$$\begin{aligned} Q_{abc} &= nC_V(T_b - T_a) + nC_p(T_c - T_b) = n\left(\frac{3}{2}R\right)T_a\left(\frac{T_b}{T_a} - 1\right) + n\left(\frac{5}{2}R\right)T_a\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right) \\ &= nRT_a\left(\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right) = p_0V_0\left(\frac{3}{2}(2-1) + \frac{5}{2}(4-2)\right) \\ &= \frac{13}{2}p_0V_0 \end{aligned}$$

where, to obtain the last line, the gas law in ratio form has been used. Therefore, since $W = p_0V_0$, we have $Q_{abc} = 13W/2 = 14.8$ kJ.

(c) The efficiency is given by Eq. 20-11:

$$\varepsilon = \frac{W}{|Q_H|} = \frac{2}{13} = 0.154 = 15.4\%.$$

(d) A Carnot engine operating between $T_c$ and $T_a$ has efficiency equal to

$$\varepsilon = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.750 = 75.0\%$$

where the gas law in ratio form has been used.

(e) This is greater than our result in part (c), as expected from the second law of thermodynamics.

30. (a) Equation 20-13 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{333\text{ K}}{373\text{ K}} = 0.107.$$

We recall that a watt is joule-per-second. Thus, the (net) work done by the cycle per unit time is the given value 500 J/s. Therefore, by Eq. 20-11, we obtain the heat input per unit time:

$$\varepsilon = \frac{W}{|Q_H|} \quad \Rightarrow \quad \frac{0.500\text{ kJ/s}}{0.107} = 4.67\text{ kJ/s} .$$

(b) Considering Eq. 20-8 on a per unit time basis, we find (4.67 – 0.500) kJ/s = 4.17 kJ/s for the rate of heat exhaust.

31. (a) We use $\varepsilon = |W/Q_H|$. The heat absorbed is $|Q_H| = \frac{|W|}{\varepsilon} = \frac{8.2\,\text{kJ}}{0.25} = 33\,\text{kJ}.$

(b) The heat exhausted is then $|Q_L| = |Q_H| - |W| = 33\,\text{kJ} - 8.2\,\text{kJ} = 25\,\text{kJ}.$

(c) Now we have $|Q_H| = \frac{|W|}{\varepsilon} = \frac{8.2\,\text{kJ}}{0.31} = 26\,\text{kJ}.$

(d) Similarly, $|Q_C| = |Q_H| - |W| = 26\,\text{kJ} - 8.2\,\text{kJ} = 18\,\text{kJ}$.

32. From Fig. 20-28, we see $Q_H$ = 4000 J at $T_H$ = 325 K. Combining Eq. 20-11 with Eq. 20-13, we have

$$\frac{W}{Q_H} = 1 - \frac{T_C}{T_H} \quad \Rightarrow \quad W = 923\text{ J} .$$

Now, for $T'_H$ = 550 K, we have

$$\frac{W}{Q'_H} = 1 - \frac{T_C}{T'_H} \quad \Rightarrow \quad Q'_H = 1692 \text{ J} \approx 1.7 \text{ kJ} .$$

33. **THINK** Our engine cycle consists of three steps: isochoric heating (*a* to *b*), adiabatic expansion (*b* to *c*), and isobaric compression (*c* to *a*).

**EXPRESS** Energy is added as heat during the portion of the process from *a* to *b*. This portion occurs at constant volume ($V_b$), so $Q_H = nC_V \Delta T$. The gas is a monatomic ideal gas, so $C_V = 3R/2$ and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a)V_b.$$

Thus, $Q_H = \frac{3}{2}(p_b - p_a)V_b$. On the other hand, energy leaves the gas as heat during the portion of the process from *c* to *a*. This is a constant pressure process, so

$$Q_L = nC_p\Delta T = nC_p(T_a - T_c) = nC_p\left(\frac{p_aV_a}{nR} - \frac{p_cV_c}{nR}\right) = \frac{C_p}{R} p_a (V_a - V_c) .$$

where $C_p$ is the molar specific heat for constant-pressure process.

**ANALYZE** (a) $V_b$ and $p_b$ are given. We need to find $p_a$. Now $p_a$ is the same as $p_c$ and points *c* and *b* are connected by an adiabatic process. With $p_cV_c^\gamma = p_bV_b^\gamma$ for the adiabat, we have ($\gamma = 5/3$ for monatomic gas)

$$p_a = p_c = \left(\frac{V_b}{V_c}\right)^\gamma p_b = \left(\frac{1}{8.00}\right)^{5/3}\left(1.013\times10^6 \text{ Pa}\right) = 3.167\times10^4 \text{ Pa}.$$

Thus, the energy added as heat is

$$Q_H = \frac{3}{2}(p_b - p_a)V_b = \frac{3}{2}\left(1.013\times10^6 \text{ Pa} - 3.167\times10^4 \text{ Pa}\right)\left(1.00\times10^{-3} \text{ m}^3\right) = 1.47\times10^3 \text{ J}.$$

(b) The energy leaving the gas as heat going from *c* to *a* is

$$Q_L = \frac{5}{2} p_a (V_a - V_c) = \frac{5}{2}\left(3.167\times10^4 \text{ Pa}\right)(-7.00)\left(1.00\times10^{-3} \text{ m}^3\right) = -5.54\times10^2 \text{ J} ,$$

or $|Q_L| = 5.54\times10^2$ J. The substitutions $V_a - V_c = V_a - 8.00\, V_a = -7.00\, V_a$ and $C_p = \frac{5}{2}R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = Q_{\rm H} - Q_{\rm L} = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_{\rm H} = (9.18 \times 10^2 \text{ J})/(1.47 \times 10^3 \text{ J}) = 0.624 = 62.4\%.$$

**LEARN** To summarize, the heat engine in this problem intakes energy as heat (from, say, consuming fuel) equal to $|Q_{\rm H}| = 1.47$ kJ and exhausts energy as heat equal to $|Q_{\rm L}| = 554$ J; its efficiency and net work are $\varepsilon = 1 - |Q_{\rm L}|/|Q_{\rm H}|$ and $W = |Q_{\rm H}| - |Q_{\rm L}|$. The less the exhaust heat $|Q_L|$, the more efficient is the engine.

34. (a) Using Eq. 19-54 for process $D \to A$ gives

$$p_D V_D^{\gamma} = p_A V_A^{\gamma} \quad \Rightarrow \quad \frac{p_0}{32}(8V_0)^{\gamma} = p_0 V_0^{\gamma}$$

which leads to $8^{\gamma} = 32 \Rightarrow \gamma = 5/3$. The result (see Sections 19-9 and 19-11) implies the gas is monatomic.

(b) The input heat is that absorbed during process $A \to B$:

$$Q_{\rm H} = nC_p \Delta T = n\left(\frac{5}{2}R\right) T_A \left(\frac{T_B}{T_A} - 1\right) = nRT_A\left(\frac{5}{2}\right)(2-1) = p_0 V_0 \left(\frac{5}{2}\right)$$

and the exhaust heat is that liberated during process $C \to D$:

$$Q_{\rm L} = nC_p \Delta T = n\left(\frac{5}{2}R\right) T_D \left(1 - \frac{T_{\rm L}}{T_D}\right) = nRT_D\left(\frac{5}{2}\right)(1-2) = -\frac{1}{4} p_0 V_0 \left(\frac{5}{2}\right)$$

where in the last step we have used the fact that $T_D = \frac{1}{4} T_A$ (from the gas law in ratio form). Therefore, Eq. 20-12 leads to

$$\varepsilon = 1 - \left|\frac{Q_{\rm L}}{Q_{\rm H}}\right| = 1 - \frac{1}{4} = 0.75 = 75\%.$$

35. (a) The pressure at 2 is $p_2 = 3.00p_1$, as given in the problem statement. The volume is $V_2 = V_1 = nRT_1/p_1$. The temperature is

$$T_2 = \frac{p_2 V_2}{nR} = \frac{3.00\, p_1 V_1}{nR} = 3.00 T_1 \;\Rightarrow\; \frac{T_2}{T_1} = 3.00.$$

(b) The process 2 → 3 is adiabatic, so $T_2V_2^{\gamma-1} = T_3V_3^{\gamma-1}$. Using the result from part (a), $V_3$ = $4.00V_1$, $V_2 = V_1$, and $\gamma$ =1.30, we obtain

$$\frac{T_3}{T_1} = \frac{T_3}{T_2/3.00} = 3.00\left(\frac{V_2}{V_3}\right)^{\gamma-1} = 3.00\left(\frac{1}{4.00}\right)^{0.30} = 1.98\,.$$

(c) The process 4 → 1 is adiabatic, so $T_4V_4^{\gamma-1} = T_1V_1^{\gamma-1}$. Since $V_4 = 4.00V_1$, we have

$$\frac{T_4}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} = \left(\frac{1}{4.00}\right)^{0.30} = 0.660.$$

(d) The process 2 → 3 is adiabatic, so $p_2V_2^{\gamma} = p_3V_3^{\gamma}$ or $p_3 = \left(V_2/V_3\right)^{\gamma} p_2$. Substituting $V_3$ = $4.00V_1$, $V_2 = V_1$, $p_2 = 3.00p_1$, and $\gamma$ = 1.30, we obtain

$$\frac{p_3}{p_1} = \frac{3.00}{(4.00)^{1.30}} = 0.495.$$

(e) The process 4 → 1 is adiabatic, so $p_4V_4^{\gamma} = p_1V_1^{\gamma}$ and

$$\frac{p_4}{p_1} = \left(\frac{V_1}{V_4}\right)^{\gamma} = \frac{1}{(4.00)^{1.30}} = 0.165,$$

where we have used $V_4 = 4.00V_1$.

(f) The efficiency of the cycle is $\varepsilon = W/Q_{12}$, where $W$ is the total work done by the gas during the cycle and $Q_{12}$ is the energy added as heat during the 1 → 2 portion of the cycle, the only portion in which energy is added as heat. The work done during the portion of the cycle from 2 to 3 is $W_{23} = \int p\,dV$. Substitute $p = p_2V_2^{\gamma}/V^{\gamma}$ to obtain

$$W_{23} = p_2V_2^{\gamma}\int_{V_2}^{V_3} V^{-\gamma}\,dV = \left(\frac{p_2V_2^{\gamma}}{\gamma-1}\right)\left(V_2^{1-\gamma} - V_3^{1-\gamma}\right).$$

Substitute $V_2 = V_1$, $V_3 = 4.00V_1$, and $p_3 = 3.00p_1$ to obtain

$$W_{23} = \left(\frac{3p_1V_1}{1-\gamma}\right)\left(1-\frac{1}{4^{\gamma-1}}\right) = \left(\frac{3nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

Similarly, the work done during the portion of the cycle from 4 to 1 is

$$W_{41}=\left(\frac{p_1V_1^{\gamma}}{\gamma-1}\right)\left(V_4^{1-\gamma}-V_1^{1-\gamma}\right)=-\left(\frac{p_1V_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right)=-\left(\frac{nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

No work is done during the 1 $\rightarrow$ 2 and 3 $\rightarrow$ 4 portions, so the total work done by the gas during the cycle is

$$W=W_{23}+W_{41}=\left(\frac{2nRT_1}{\gamma-1}\right)\left(1-\frac{1}{4^{\gamma-1}}\right).$$

The energy added as heat is

$$Q_{12}=nC_V\,(T_2-T_1)=nC_V\,(3T_1-T_1)=2nC_VT_1,$$

where $C_V$ is the molar specific heat at constant volume. Now

$$\gamma=C_p/C_V=(C_V+R)/C_V=1+(R/C_V),$$

so $C_V = R/(\gamma - 1)$. Here $C_p$ is the molar specific heat at constant pressure, which for an ideal gas is $C_p = C_V + R$. Thus, $Q_{12} = 2nRT_1/(\gamma - 1)$. The efficiency is

$$\varepsilon=\frac{2nRT_1}{\gamma-1}\left(1-\frac{1}{4^{\gamma-1}}\right)\frac{\gamma-1}{2nRT_1}=1-\frac{1}{4^{\gamma-1}}.$$

With $\gamma = 1.30$, the efficiency is $\varepsilon = 0.340$ or 34.0%.

36. (a) Using Eq. 20-14 and Eq. 20-16, we obtain

$$|W|=\frac{|Q_L|}{K_C}=(1.0\text{ J})\left(\frac{300\,\text{K}-280\,\text{K}}{280\,\text{K}}\right)=0.071\text{J}.$$

(b) A similar calculation (being sure to use absolute temperature) leads to 0.50 J in this case.

(c) With $T_L = 100$ K, we obtain $|W| = 2.0$ J.

(d) Finally, with the low temperature reservoir at 50 K, an amount of work equal to $|W| = 5.0$ J is required.

37. **THINK** The performance of the refrigerator is related to its rate of doing work.

**EXPRESS** The coefficient of performance for a refrigerator is given by

$$K=\frac{\text{what we want}}{\text{what we pay for}}=\frac{|Q_L|}{|W|},$$

where $Q_L$ is the energy absorbed from the cold reservoir as heat and $W$ is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields

$Q_H + Q_L - W = 0$ for an integer number of cycles. Here $Q_H$ is the energy ejected to the hot reservoir as heat. Thus, $Q_L = W - Q_H$. $Q_H$ is negative and greater in magnitude than $W$, so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|W|$ is $|W| = |Q_H|/(K + 1)$.

**ANALYZE** In one hour, $|Q_H| = 7.54\ \text{MJ}$. With $K = 3.8$, the work done is

$$|W| = \frac{7.54\,\text{MJ}}{3.8+1} = 1.57\,\text{MJ}.$$

The rate at which work is done is $P = |W|/\Delta t = (1.57 \times 10^6\ \text{J})/(3600\ \text{s}) = 440\ \text{W}$.

**LEARN** The greater the value of $K$, the less the amount of work $|W|$ required to transfer the heat.

38. Equation 20-10 still holds (particularly due to its use of absolute values), and energy conservation implies $|W| + Q_L = Q_H$. Therefore, with $T_L = 268.15$ K and $T_H = 290.15$ K, we find

$$|Q_H| = |Q_L|\left(\frac{T_H}{T_L}\right) = \left(|Q_H| - |W|\right)\left(\frac{290.15}{268.15}\right)$$

which (with $|W| = 1.0$ J) leads to $|Q_H| = |W|\left(\dfrac{1}{1 - 268.15/290.15}\right) = 13\,\text{J}.$

39. **THINK** A large (small) value of coefficient of performance $K$ means that less (more) work would be required to transfer the heat

**EXPRESS** A Carnot refrigerator working between a hot reservoir at temperature $T_H$ and a cold reservoir at temperature $T_L$ has a coefficient of performance $K$ that is given by

$$K = \frac{T_L}{T_H - T_L},$$

where $T_H$ is the temperature of the higher-temperature reservoir, and $T_L$ the temperature of the lower-temperature reservoir, in Kelvin scale. Equivalently, the coefficient of performance is the energy $Q_L$ drawn from the cold reservoir as heat divided by the work done: $K = |Q_L|/|W|$.

**ANALYZE** For the refrigerator of this problem, $T_H = 96°$ F = 309 K and $T_L = 70°$ F = 294 K, so

$$K = (294\ \text{K})/(309\ \text{K} - 294\ \text{K}) = 19.6.$$

Thus, with $|W|=1.0$ J, the amount of heat removed from the room is

$$|Q_L| = K|W| = (19.6)(1.0\text{ J}) = 20\text{ J}.$$

**LEARN** The Carnot air conditioner in this problem (with $K$ = 19.6) are much more efficient than that of the typical room air conditioners ($K \approx 2.5$).

40. (a) Equation 20-15 provides

$$K_C = \frac{|Q_L|}{|Q_H|-|Q_L|} \Rightarrow |Q_H| = |Q_L|\left(\frac{1+K_C}{K_C}\right)$$

which yields $|Q_H|$ = 49 kJ when $K_C$ = 5.7 and $|Q_L|$ = 42 kJ.

(b) From Section 20-5 we obtain

$$|W| = |Q_H| - |Q_L| = 49.4\text{ kJ} - 42.0\text{ kJ} = 7.4\text{ kJ}$$

if we take the initial 42 kJ datum to be accurate to three figures. The given temperatures are not used in the calculation; in fact, it is possible that the given room temperature value is not meant to be the high temperature for the (reversed) Carnot cycle — since it does not lead to the given $K_C$ using Eq. 20-16.

41. We are told $K = 0.27K_C$, where

$$K_C = \frac{T_L}{T_H - T_L} = \frac{294\text{ K}}{307\text{ K} - 294\text{ K}} = 23$$

where the Fahrenheit temperatures have been converted to Kelvins. Expressed on a per unit time basis, Eq. 20-14 leads to

$$\frac{|W|}{t} = \frac{|Q_L|/t}{K} = \frac{4000\text{ Btu/h}}{(0.27)(23)} = 643\text{ Btu/h}.$$

Appendix D indicates 1 But/h = 0.0003929 hp, so our result may be expressed as $|W|/t$ = 0.25 hp.

42. The work done by the motor in $t$ = 10.0 min is $|W| = Pt$ = (200 W)(10.0 min)(60 s/min) = $1.20 \times 10^5$ J. The heat extracted is then

$$|Q_L| = K|W| = \frac{T_L|W|}{T_H - T_L} = \frac{(270\text{ K})(1.20\times 10^5\text{ J})}{300\text{ K} - 270\text{ K}} = 1.08\times 10^6\text{ J}.$$

43. The efficiency of the engine is defined by $\varepsilon = W/Q_1$ and is shown in the text to be

$$\varepsilon = \frac{T_1 - T_2}{T_1} \Rightarrow \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}.$$

The coefficient of performance of the refrigerator is defined by $K = Q_4/W$ and is shown in the text to be

$$K = \frac{T_4}{T_3 - T_4} \Rightarrow \frac{Q_4}{W} = \frac{T_4}{T_3 - T_4}.$$

Now $Q_4 = Q_3 - W$, so

$$(Q_3 - W)/W = T_4/(T_3 - T_4).$$

The work done by the engine is used to drive the refrigerator, so $W$ is the same for the two. Solve the engine equation for $W$ and substitute the resulting expression into the refrigerator equation. The engine equation yields $W = (T_1 - T_2)Q_1/T_1$ and the substitution yields

$$\frac{T_4}{T_3 - T_4} = \frac{Q_3}{W} - 1 = \frac{Q_3 T_1}{Q_1(T_1 - T_2)} - 1.$$

Solving for $Q_3/Q_1$, we obtain

$$\frac{Q_3}{Q_1} = \left(\frac{T_4}{T_3 - T_4} + 1\right)\left(\frac{T_1 - T_2}{T_1}\right) = \left(\frac{T_3}{T_3 - T_4}\right)\left(\frac{T_1 - T_2}{T_1}\right) = \frac{1 - (T_2/T_1)}{1 - (T_4/T_3)}.$$

With $T_1$ = 400 K, $T_2$ = 150 K, $T_3$ = 325 K, and $T_4$ = 225 K, the ratio becomes $Q_3/Q_1$ = 2.03.

44. (a) Equation 20-13 gives the Carnot efficiency as $1 - T_L/T_H$. This gives 0.222 in this case. Using this value with Eq. 20-11 leads to $W$ = (0.222)(750 J) = 167 J.

(b) Now, Eq. 20-16 gives $K_C$ = 3.5. Then, Eq. 20-14 yields $|W|$ = 1200/3.5 = 343 J.

45. We need nine labels:

| Label | Number of molecules on side 1 | Number of molecules on side 2 |
|---|---|---|
| I | 8 | 0 |
| II | 7 | 1 |
| III | 6 | 2 |
| IV | 5 | 3 |
| V | 4 | 4 |
| VI | 3 | 5 |
| VII | 2 | 6 |
| VIII | 1 | 7 |
| IX | 0 | 8 |

The multiplicity $W$ is computing using Eq. 20-20. For example, the multiplicity for label IV is

$$W = \frac{8!}{(5!)(3!)} = \frac{40320}{(120)(6)} = 56$$

and the corresponding entropy is (using Eq. 20-21)

$$S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln(56) = 5.6\times10^{-23}\text{ J/K}.$$

In this way, we generate the following table:

| Label | $W$ | $S$ |
|---|---|---|
| I | 1 | 0 |
| II | 8 | $2.9 \times 10^{-23}$ J/K |
| III | 28 | $4.6 \times 10^{-23}$ J/K |
| IV | 56 | $5.6 \times 10^{-23}$ J/K |
| V | 70 | $5.9 \times 10^{-23}$ J/K |
| VI | 56 | $5.6 \times 10^{-23}$ J/K |
| VII | 28 | $4.6 \times 10^{-23}$ J/K |
| VIII | 8 | $2.9 \times 10^{-23}$ J/K |
| IX | 1 | 0 |

46. (a) We denote the configuration with $n$ heads out of $N$ trials as ($n$; $N$). We use Eq. 20-20:

$$W(25;50) = \frac{50!}{(25!)(50-25)!} = 1.26\times10^{14}.$$

(b) There are 2 possible choices for each molecule: it can either be in side 1 or in side 2 of the box. If there are a total of $N$ independent molecules, the total number of available states of the $N$-particle system is

$$N_{\text{total}} = 2\times2\times2\times\cdots\times2 = 2^N.$$

With $N = 50$, we obtain $N_{\text{total}} = 2^{50} = 1.13 \times 10^{15}$.

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration:

$$p(25;50) = \frac{W(25;50)}{2^{50}} = \frac{1.26\times10^{14}}{1.13\times10^{15}} = 11.1\%.$$

With $N = 100$, we obtain

(d) $W(N/2, N) = N!/[(N/2)!]^2 = 1.01 \times 10^{29}$,

(e) $N_{\text{total}} = 2^N = 1.27 \times 10^{30}$,

(f) and $p(N/2; N) = W(N/2, N)/ N_{\text{total}} = 8.0\%$.

Similarly, for $N = 200$, we obtain

(g) $W(N/2, N) = 9.25 \times 10^{58}$,

(h) $N_{\text{total}} = 1.61 \times 10^{60}$,

(i) and $p(N/2; N) = 5.7\%$.

(j) As $N$ increases, the number of available microscopic states increases as $2^N$, so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration. Thus, the time spent there decreases with an increase in $N$.

47. **THINK** The gas molecules inside a box can be distributed in many different ways. The number of microstates associated with each distinct configuration is called the multiplicity.

**EXPRESS** Given $N$ molecules, if the box is divided into $m$ equal parts, with $n_1$ molecules in the first, $n_2$ in the second,…, such that $n_1 + n_2 + ... n_m = N$. There are $N!$ arrangements of the $N$ molecules, but $n_1!$ are simply rearrangements of the $n_1$ molecules in the first part, $n_2!$ are rearrangements of the $n_2$ molecules in the second,… These rearrangements do not produce a new configuration. Therefore, the multiplicity factor associated with this is

$$W = \frac{N!}{n_1!n_2!n_3!\ldots n_m!}.$$

**ANALYZE** (a) Suppose there are $n_L$ molecules in the left third of the box, $n_C$ molecules in the center third, and $n_R$ molecules in the right third. Using the argument above, we find the multiplicity to be

$$W = \frac{N!}{n_L!n_C!n_R!}.$$

Note that $n_L + n_C + n_R = N$.

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!(N/2)!}.$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!(N/3)!(N/3)!}.$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!(N/2)!}{(N/3)!(N/3)!(N/3)!}.$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!\,50!}{33!\,33!\,34!} = 4.2\times10^{16}.$$

**LEARN** The more parts the box is divided into, the greater the number of configurations. This exercise illustrates the statistical view of entropy, which is related to $W$ as $S = k\ln W$.

48. (a) A good way to (mathematically) think of this is to consider the terms when you expand:

$$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4.$$

The coefficients correspond to the multiplicities. Thus, the smallest coefficient is 1.

(b) The largest coefficient is 6.

(c) Since the logarithm of 1 is zero, then Eq. 20-21 gives $S = 0$ for the least case.

(d) $S = k \ln(6) = 2.47 \times 10^{-23}$ J/K.

49. From the formula for heat conduction, Eq. 19-32, using Table 19-6, we have

$$H = kA\frac{T_H - T_C}{L} = (401)\left(\pi(0.02)^2\right) 270/1.50$$

which yields $H$ = 90.7 J/s. Using Eq. 20-2, this is associated with an entropy rate-of-decrease of the high temperature reservoir (at 573 K) equal to

$$\Delta S/t = -90.7/573 = -0.158 \text{ (J/K)/s}.$$

And it is associated with an entropy rate-of-increase of the low temperature reservoir (at 303 K) equal to

$$\Delta S/t = +90.7/303 = 0.299 \text{ (J/K)/s}.$$

The net result is (0.299 – 0.158) (J/K)/s = 0.141 (J/K)/s.

50. For an isothermal ideal gas process, we have $Q = W = nRT \ln(V_f/V_i)$. Thus,

$$\Delta S = Q/T = W/T = nR \ln(V_f/V_i)$$

(a) $V_f/V_i$ = (0.800)/(0.200) = 4.00, $\Delta S$ = (0.55)(8.31)ln(4.00) = 6.34 J/K.

(b) $V_f/V_i$ = (0.800)/(0.200) = 4.00, $\Delta S$ = (0.55)(8.31)ln(4.00) = 6.34 J/K.

(c) $V_f/V_i$ = (1.20)/(0.300) = 4.00, $\Delta S$ = (0.55)(8.31)ln(4.00) = 6.34 J/K.

(d) $V_f/V_i$ = (1.20)/(0.300) = 4.00, $\Delta S$ = (0.55)(8.31)ln(4.00) = 6.34 J/K.

51. **THINK** Increasing temperature causes a shift of the probability distribution function $P(v)$ toward higher speed.

**EXPRESS** According to kinetic theory, the rms speed and the most probable speed are (see Eqs. 19-34 and 19035) $v_{\text{rms}} = \sqrt{3RT/M}$ , $v_P = \sqrt{2RT/M}$ and where $T$ is the temperature and $M$ is the molar mass. The rms speed is defined as $v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}}$ , where $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v) dv$ , with the Maxwell's speed distribution function given by

$$P(v) = 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{-Mv^2/2RT} .$$

Thus, the difference between the two speeds is

$$\Delta v = v_{\text{rms}} - v_P = \sqrt{\frac{3RT}{M}} - \sqrt{\frac{2RT}{M}} = \left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{RT}{M}} .$$

**ANALYZE** (a) With $M$ = 28 g/mol = 0.028 kg/mol (see Table 19-1), and $T_i$ = 250 K, we have

$$\Delta v_i = \left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{RT_i}{M}} = \left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{(8.31\text{ J/mol}\cdot\text{K})(250\text{ K})}{0.028\text{ kg/mol}}} = 87\text{ m/s} .$$

(b) Similarly, at $T_f$ = 500 K,

$$\Delta v_f = \left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{RT_f}{M}} = \left(\sqrt{3}-\sqrt{2}\right)\sqrt{\frac{(8.31\text{ J/mol}\cdot\text{K})(500\text{ K})}{0.028\text{ kg/mol}}} = 122\text{ m/s} \approx 1.2\times 10^2\text{ m/s} .$$

(c) From Table 19-3 we have $C_V = 5R/2$ (see also Table 19-2). For $n$ = 1.5 mol, using Eq. 20-4, we find the change in entropy to be

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) + nC_V\ln\left(\frac{T_f}{T_i}\right) = 0 + (1.5\text{ mol})(5/2)(8.31\text{ J/mol}\cdot\text{K})\ln\left(\frac{500\text{ K}}{250\text{ K}}\right)$$
$$= 22\text{ J/K}$$

**LEARN** Notice that the expression for $\Delta v$ implies $T=\frac{M}{R(\sqrt{3}-\sqrt{2})^2}(\Delta v)^2$. Thus, one may also express $\Delta S$ as

$$\Delta S = nC_V \ln\left(\frac{T_f}{T_i}\right) = nC_V \ln\left(\frac{(\Delta v_f)^2}{(\Delta v_i)^2}\right) = 2nC_V \ln\left(\frac{\Delta v_f}{\Delta v_i}\right).$$

The entropy of the gas increases as the result of temperature increase.

52. (a) The most obvious input-heat step is the constant-volume process. Since the gas is monatomic, we know from Chapter 19 that $C_V = \frac{3}{2}R$. Therefore,

$$Q_V = nC_V\Delta T = (1.0 \text{ mol})\left(\frac{3}{2}\right)\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600 \text{ K} - 300 \text{ K}) = 3740 \text{ J}.$$

Since the heat transfer during the isothermal step is positive, we may consider it also to be an input-heat step. The isothermal $Q$ is equal to the isothermal work (calculated in the next part) because $\Delta E_{\text{int}}$ = 0 for an ideal gas isothermal process (see Eq. 19-45). Borrowing from the part (b) computation, we have

$$Q_{\text{isotherm}} = nRT_{\text{H}}\ln 2 = (1 \text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600 \text{ K})\ln 2 = 3456 \text{ J}.$$

Therefore, $Q_{\text{H}} = Q_V + Q_{\text{isotherm}} = 7.2 \times 10^3$ J.

(b) We consider the sum of works done during the processes (noting that no work is done during the constant-volume step). Using Eq. 19-14 and Eq. 19-16, we have

$$W = nRT_{\text{H}}\ln\left(\frac{V_{\text{max}}}{V_{\text{min}}}\right) + p_{\text{min}}(V_{\text{min}} - V_{\text{max}})$$

where, by the gas law in ratio form, the volume ratio is $\frac{V_{\text{max}}}{V_{\text{min}}} = \frac{T_{\text{H}}}{T_{\text{L}}} = \frac{600 \text{ K}}{300 \text{ K}} = 2.$

Thus, the net work is

$$\begin{aligned} W &= nRT_{\text{H}}\ln 2 + p_{\text{min}}V_{\text{min}}\left(1 - \frac{V_{\text{max}}}{V_{\text{min}}}\right) = nRT_{\text{H}}\ln 2 + nRT_{\text{L}}(1-2) = nR(T_{\text{H}}\ln 2 - T_{\text{L}}) \\ &= (1 \text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)((600 \text{ K})\ln 2 - (300 \text{ K})) \\ &= 9.6\times 10^2 \text{ J}. \end{aligned}$$

(c) Equation 20-11 gives $\varepsilon = \dfrac{W}{Q_H} = 0.134 \approx 13\%.$

53. (a) If $T_H$ is the temperature of the high-temperature reservoir and $T_L$ is the temperature of the low-temperature reservoir, then the maximum efficiency of the engine is

$$\varepsilon = \frac{T_H - T_L}{T_H} = \frac{(800+40)\ \text{K}}{(800+273)\ \text{K}} = 0.78 \ \text{ or } \ 78\%.$$

(b) The efficiency is defined by $\varepsilon = |W|/|Q_H|$, where $W$ is the work done by the engine and $Q_H$ is the heat input. $W$ is positive. Over a complete cycle, $Q_H = W + |Q_L|$, where $Q_L$ is the heat output, so $\varepsilon = W/(W + |Q_L|)$ and $|Q_L| = W[(1/\varepsilon) - 1]$. Now $\varepsilon = (T_H - T_L)/T_H$, where $T_H$ is the temperature of the high-temperature heat reservoir and $T_L$ is the temperature of the low-temperature reservoir. Thus,

$$\frac{1}{\varepsilon} - 1 = \frac{T_L}{T_H - T_L} \ \text{ and } \ |Q_L| = \frac{WT_L}{T_H - T_L}.$$

The heat output is used to melt ice at temperature $T_i = -40°\text{C}$. The ice must be brought to 0°C, then melted, so

$$|Q_L| = mc(T_f - T_i) + mL_F,$$

where $m$ is the mass of ice melted, $T_f$ is the melting temperature (0°C), $c$ is the specific heat of ice, and $L_F$ is the heat of fusion of ice. Thus,

$$WT_L/(T_H - T_L) = mc(T_f - T_i) + mL_F.$$

We differentiate with respect to time and replace $dW/dt$ with $P$, the power output of the engine, and obtain

$$PT_L/(T_H - T_L) = (dm/dt)[c(T_f - T_i) + L_F].$$

Therefore,

$$\frac{dm}{dt} = \left(\frac{PT_L}{T_H - T_L}\right)\left(\frac{1}{c(T_f - T_i) + L_F}\right).$$

Now, $P = 100 \times 10^6$ W, $T_L = 0 + 273 = 273$ K, $T_H = 800 + 273 = 1073$ K, $T_i = -40 + 273 = 233$ K, $T_f = 0 + 273 = 273$ K, $c = 2220$ J/kg·K, and $L_F = 333 \times 10^3$ J/kg, so

$$\frac{dm}{dt} = \left[\frac{(100\times10^6\ \text{J/s})(273\ \text{K})}{1073\ \text{K} - 273\ \text{K}}\right]\left[\frac{1}{(2220\ \text{J/kg}\cdot\text{K})(273\ \text{K} - 233\ \text{K}) + 333\times10^3\ \text{J/kg}}\right]$$
$$= 82\,\text{kg/s}.$$

We note that the engine is now operated between 0°C and 800°C.

54. Equation 20-4 yields

$$\Delta S = nR\ln(V_f/V_i) + nC_V\ln(T_f/T_i) = 0 + nC_V\ln(425/380)$$

where $n$ = 3.20 and $C_V = \frac{3}{2}R$ (Eq. 19-43). This gives 4.46 J/K.

55. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = 40.9°\text{C},$$

which is equivalent to 314 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{copper}} = \int_{353}^{314} \frac{cm\,dT}{T} = (386)(0.600)\ln\left(\frac{314}{353}\right) = -27.1 \text{ J/K}.$$

(c) For water, the change in entropy is

$$\Delta S_{\text{water}} = \int_{283}^{314} \frac{cm\,dT}{T} = (4187)(0.0700)\ln\left(\frac{314}{283.15}\right) = 30.3 \text{ J/K}.$$

(d) The net result for the system is (30.3 – 27.1) J/K = 3.2 J/K. (Note: These calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

56. Using Hooke's law, we find the spring constant to be

$$k = \frac{F_s}{x_s} = \frac{1.50 \text{ N}}{0.0350 \text{ m}} = 42.86 \text{ N/m}.$$

To find the rate of change of entropy with a small additional stretch, we use Eq. 20-7 and obtain

$$\left|\frac{dS}{dx}\right| = \frac{k\,|\,x\,|}{T} = \frac{(42.86 \text{ N/m})(0.0170 \text{ m})}{275 \text{ K}} = 2.65\times10^{-3} \text{ J/K}\cdot\text{m}.$$

57. Since the volume of the monatomic ideal gas is kept constant, it does not do any work in the heating process. Therefore the heat $Q$ it absorbs is equal to the change in its inertial energy: $dQ = dE_{\text{int}} = \frac{3}{2}nR\,dT$ . Thus,

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{(3nR/2)\,dT}{T} = \frac{3}{2}nR\ln\left(\frac{T_f}{T_i}\right) = \frac{3}{2}(1.00\text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{400\text{ K}}{300\text{ K}}\right)$$
$$= 3.59\text{ J/K}.$$

58. With the pressure kept constant,

$$dQ = nC_p dT = n(C_V + R)dT = \left(\frac{3}{2}nR + nR\right)dT = \frac{5}{2}nRdT,$$

so we need to replace the factor 3/2 in the last problem by 5/2. The rest is the same. Thus the answer now is

$$\Delta S = \frac{5}{2}nR\ln\left(\frac{T_f}{T_i}\right) = \frac{5}{2}(1.00\text{ mol})\left(8.31\frac{\text{J}}{\text{mol}\cdot\text{ K}}\right)\ln\left(\frac{400\text{ K}}{300\text{ K}}\right) = 5.98\text{ J/K}.$$

59. **THINK** The temperature of the ice is first raised to $0°\text{C}$, then the ice melts and the temperature of the resulting water is raised to $40°\text{C}$. We want to calculate the entropy change in this process.

**EXPRESS** As the ice warms, the energy it receives as heat when the temperature changes by $dT$ is $dQ = mc_I\,dT$, where $m$ is the mass of the ice and $c_I$ is the specific heat of ice. If $T_i$ ($= -20°\text{C} = 253$ K) is the initial temperature and $T_f$ (= 273 K) is the final temperature, then the change in its entropy is

$$\Delta S_1 = \int \frac{dQ}{T} = mc_I\int_{T_i}^{T_f}\frac{dT}{T} = mc_I\ln\left(\frac{T_f}{T_i}\right) = (0.60\text{ kg})(2220\text{ J/kg}\cdot\text{K})\ln\left(\frac{273\text{ K}}{253\text{ K}}\right) = 101\text{ J/K}.$$

Melting is an isothermal process. The energy leaving the ice as heat is $mL_F$, where $L_F$ is the heat of fusion for ice. Thus,

$$\Delta S_2 = \frac{Q}{T} = \frac{mL_F}{T} = \frac{(0.60\text{ kg})(333\times10^3\text{ J/kg})}{273\text{ K}} = 732\text{ J/K}.$$

For the warming of the water from the melted ice, the change in entropy is

$$\Delta S_3 = mc_w\ln\left(\frac{T_f}{T_i}\right) = (0.600\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\frac{313\text{ K}}{273\text{ K}}\right) = 344\text{ J/K},$$

where $c_w$ = 4190 J/kg · K is the specific heat of water.

**ANALYZE** The total change in entropy for the ice and the water it becomes is

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 = 101 \text{ J/K} + 732 \text{ J/K} + 344 \text{ J/K} = 1.18\times10^3 \text{ J/K}.$$

**LEARN** From the above, we readily see that the biggest increase in entropy comes from $\Delta S_2$, which accounts for the melting process.

60. The net work is figured from the (positive) isothermal expansion (Eq. 19-14) and the (negative) constant-pressure compression (Eq. 19-48). Thus,

$$W_{net} = nRT_H \ln(V_{max}/V_{min}) + nR(T_L - T_H)$$

where $n$ = 3.4, $T_H$ = 500 K, $T_L$ = 200 K, and $V_{max}/V_{min}$ = 5/2 (same as the ratio $T_H/T_L$). Therefore, $W_{net}$ = 4468 J. Now, we identify the "input heat" as that transferred in steps 1 and 2:

$$Q_{in} = Q_1 + Q_2 = nC_V(T_H - T_L) + nRT_H \ln(V_{max}/V_{min})$$

where $C_V$ = 5$R$/2 (see Table 19-3). Consequently, $Q_{in}$ = 34135 J. Dividing these results gives the efficiency: $W_{net}/Q_{in}$ = 0.131 (or about 13.1%).

61. Since the inventor's claim implies that less heat (typically from burning fuel) is needed to operate his engine than, say, a Carnot engine (for the same magnitude of net work), then $Q_H' < Q_H$ (see Fig. 20-34(a)) which implies that the Carnot (ideal refrigerator) unit is delivering more heat to the high temperature reservoir than engine X draws from it. This (using also energy conservation) immediately implies Fig. 20-34(b), which violates the second law.

62. (a) From Eq. 20-1, we infer $Q = \int T\,dS$, which corresponds to the "area under the curve" in a $T$-$S$ diagram. Thus, since the area of a rectangle is (height)×(width), we have

$$Q_{1\to2} = (350)(2.00) = 700 \text{ J}.$$

(b) With no "area under the curve" for process 2 → 3, we conclude $Q_{2\to3} = 0$.

(c) For the cycle, the (net) heat should be the "area inside the figure," so using the fact that the area of a triangle is ½ (base) × (height), we find

$$Q_{net} = \frac{1}{2}(2.00)(50) = 50 \text{ J}.$$

(d) Since we are dealing with an ideal gas (so that $\Delta E_{int} = 0$ in an isothermal process), then

$$W_{1\to2} = Q_{1\to2} = 700 \text{ J}.$$

(e) Using Eq. 19-14 for the isothermal work, we have

$$W_{1\to 2} = nRT \ln \frac{V_2}{V_1} .$$

where $T$ = 350 K. Thus, if $V_1$ = 0.200 m$^3$, then we obtain

$$V_2 = V_1 \exp(W/nRT) = (0.200)\, e^{0.12} = 0.226 \text{ m}^3.$$

(f) Process 2 → 3 is adiabatic; Eq. 19-56 applies with $\gamma$ = 5/3 (since only translational degrees of freedom are relevant here):

$$T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1} .$$

This yields $V_3$ = 0.284 m$^3$.

(g) As remarked in part (d), $\Delta E_{\text{int}}$ = 0 for process 1 → 2.

(h) We find the change in internal energy from Eq. 19-45 (with $C_V = \frac{3}{2}R$):

$$\Delta E_{\text{int}} = nC_V (T_3 - T_2) = -1.25 \times 10^3 \text{ J}.$$

(i) Clearly, the net change of internal energy for the entire cycle is zero. This feature of a closed cycle is as true for a *T*-*S* diagram as for a *p*-*V* diagram.

(j) For the adiabatic (2 → 3) process, we have $W = -\Delta E_{\text{int}}$. Therefore, $W = 1.25 \times 10^3$ J. Its positive value indicates an expansion.

63. (a) It is a reversible set of processes returning the system to its initial state; clearly, $\Delta S_{\text{net}}$ = 0.

(b) Process 1 is adiabatic and reversible (as opposed to, say, a free expansion) so that Eq. 20-1 applies with $dQ$ = 0 and yields $\Delta S_1$ = 0.

(c) Since the working substance is an ideal gas, then an isothermal process implies $Q = W$, which further implies (regarding Eq. 20-1) $dQ = p\, dV$. Therefore,

$$\int \frac{dQ}{T} = \int \frac{p\, dV}{\left(\frac{pV}{nR}\right)} = nR \int \frac{dV}{V}$$

which leads to $\Delta S_3 = nR \ln(1/2) = -23.0 \text{ J/K}$.

(d) By part (a), $\Delta S_1 + \Delta S_2 + \Delta S_3 = 0$. Then, part (b) implies $\Delta S_2 = -\Delta S_3$. Therefore, $\Delta S_2$ = 23.0 J/K.

64. (a) Combining Eq. 20-11 with Eq. 20-13, we obtain

$$|W| = |Q_{\rm H}|\left(1 - \frac{T_{\rm L}}{T_{\rm H}}\right) = (500\,{\rm J})\left(1 - \frac{260\,{\rm K}}{320\,{\rm K}}\right) = 93.8\,{\rm J}.$$

(b) Combining Eq. 20-14 with Eq. 20-16, we find

$$|W| = \frac{|Q_{\rm L}|}{\left(\frac{T_{\rm L}}{T_{\rm H}-T_{\rm L}}\right)} = \frac{1000\,{\rm J}}{\left(\frac{260\,{\rm K}}{320\,{\rm K}-260\,{\rm K}}\right)} = 231\ {\rm J}.$$

65. (a) Processes 1 and 2 both require the input of heat, which is denoted $Q_{\rm H}$. Noting that rotational degrees of freedom are not involved, then, from the discussion in Chapter 19, $C_V = 3R/2$, $C_p = 5R/2$, and $\gamma = 5/3$. We further note that since the working substance is an ideal gas, process 2 (being isothermal) implies $Q_2 = W_2$. Finally, we note that the volume ratio in process 2 is simply 8/3. Therefore,

$$Q_{\rm H} = Q_1 + Q_2 = nC_V\,(T' - T) + nRT'\,\ln\frac{8}{3}$$

which yields (for $T$ = 300 K and $T'$ = 800 K) the result $Q_{\rm H} = 25.5 \times 10^3$ J.

(b) The net work is the net heat ($Q_1 + Q_2 + Q_3$). We find $Q_3$ from

$$nC_p\,(T - T') = -20.8 \times 10^3\ {\rm J}.$$

Thus, $W = 4.73 \times 10^3$ J.

(c) Using Eq. 20-11, we find that the efficiency is

$$\varepsilon = \frac{|W|}{|Q_{\rm H}|} = \frac{4.73\times10^3}{25.5\times10^3} = 0.185 \text{ or } 18.5\%.$$

66. (a) Equation 20-14 gives $K = 560/150 = 3.73$.

(b) Energy conservation requires the exhaust heat to be 560 + 150 = 710 J.

67. The change in entropy in transferring a certain amount of heat $Q$ from a heat reservoir at $T_1$ to another one at $T_2$ is $\Delta S = \Delta S_1 + \Delta S_2 = Q(1/T_2 - 1/T_1)$.

(a) $\Delta S$ = (260 J)(1/100 K – 1/400 K) = 1.95 J/K.

(b) $\Delta S$ = (260 J)(1/200 K – 1/400 K) = 0.650 J/K.

(c) $\Delta S$ = (260 J)(1/300 K – 1/400 K) = 0.217 J/K.

(d) $\Delta S$ = (260 J)(1/360 K – 1/400 K) = 0.072 J/K.

(e) We see that as the temperature difference between the two reservoirs decreases, so does the change in entropy.

68. Equation 20-10 gives

$$\left|\frac{Q_{\text{to}}}{Q_{\text{from}}}\right| = \frac{T_{\text{to}}}{T_{\text{from}}} = \frac{300\,\text{K}}{4.0\,\text{K}} = 75.$$

69. (a) Equation 20-2 gives the entropy change for each reservoir (each of which, by definition, is able to maintain constant temperature conditions within itself). The net entropy change is therefore

$$\Delta S = \frac{+|Q|}{273 + 24} + \frac{-|Q|}{273 + 130} = 4.45 \text{ J/K}$$

where we set $|Q|$ = 5030 J.

(b) We have assumed that the conductive heat flow in the rod is "steady-state"; that is, the situation described by the problem has existed and will exist for "long times." Thus there are no entropy change terms included in the calculation for elements of the rod itself.

70. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = -44.2°\text{C},$$

which is equivalent to 229 K.

(b) From Eq. 20-1, we have

$$\Delta S_{\text{tungsten}} = \int_{303}^{229} \frac{cm\,dT}{T} = (134)(0.045)\ln\left(\frac{229}{303}\right) = -1.69 \text{ J/K}.$$

(c) Also,

$$\Delta S_{\text{silver}} = \int_{153}^{229} \frac{cm\,dT}{T} = (236)(0.0250)\ln\left(\frac{229}{153}\right) = 2.38 \text{ J/K}.$$

(d) The net result for the system is (2.38 – 1.69) J/K = 0.69 J/K. (Note: These calculations are fairly sensitive to round-off errors. To arrive at this final answer, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

71. (a) We use Eq. 20-16. For configuration *A*

$$W_A = \frac{N!}{(N/2)!(N/2)!} = \frac{50!}{(25!)(25!)} = 1.26\times10^{14}.$$

(b) For configuration $B$

$$W_B = \frac{N!}{(0.6N)!(0.4N)!} = \frac{50!}{[0.6(50)]![0.4(50)]!} = 4.71\times10^{13}.$$

(c) Since all microstates are equally probable,

$$f = \frac{W_B}{W_A} = \frac{1265}{3393} \approx 0.37.$$

We use these formulas for $N$ = 100. The results are

(d) $W_A = \dfrac{N!}{(N/2)!(N/2)!} = \dfrac{100!}{(50!)(50!)} = 1.01\times10^{29}$,

(e) $W_B = \dfrac{N!}{(0.6N)!(0.4N)!} = \dfrac{100!}{[0.6(100)]![0.4(100)]!} = 1.37\times10^{28}$,

(f) and $f$  $W_B/W_A \approx 0.14$.

Similarly, using the same formulas for $N$ = 200, we obtain

(g) $W_A = 9.05 \times 10^{58}$,

(h) $W_B = 1.64 \times 10^{57}$,

(i) and $f$ = 0.018.

(j) We see from the calculation above that $f$ decreases as $N$ increases, as expected.

72. A metric ton is 1000 kg, so that the heat generated by burning 380 metric tons during one hour is $(380000 \text{ kg})(28 \text{ MJ/kg}) = 10.6\times10^6 \text{ MJ}$. The work done in one hour is

$$W = (750 \text{ MJ/s})(3600 \text{ s}) = 2.7\times10^6 \text{ MJ}$$

where we use the fact that a watt is a joule-per-second. By Eq. 20-11, the efficiency is

$$\varepsilon = \frac{2.7\times10^6 \text{ MJ}}{10.6\times10^6 \text{ MJ}} = 0.253 = 25\%.$$

73. **THINK** The performance of the Carnot refrigerator is related to its rate of doing work.

**EXPRESS** The coefficient of performance for a refrigerator is defined as

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|},$$

where $Q_L$ is the energy absorbed from the cold reservoir (interior of refrigerator) as heat and $W$ is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields $Q_H + Q_L - W = 0$ for an integer number of cycles. Here $Q_H$ is the energy ejected as heat to the hot reservoir (the room). Thus, $Q_L = W - Q_H$. $Q_H$ is negative and greater in magnitude than $W$, so $|Q_L| = |Q_H| - |W|$. Thus,

$$K = \frac{|Q_H| - |W|}{|W|}.$$

The solution for $|Q_H| = |W|(1+K) = |Q_L|(1+K)/K$.

**ANALYZE** (a) From the expression above, the energy per cycle transferred as heat to the room is

$$|Q_H| = |Q_L|\frac{1+K}{K} = (35.0\text{ kJ})\frac{1+4.60}{4.60} = 42.6\text{ kJ}.$$

(b) Similarly, the work done per cycle is $|W| = \frac{|Q_L|}{K} = \frac{35.0\text{ kJ}}{4.60} = 7.61\text{ kJ}$.

**LEARN** A Carnot refrigerator is a Carnot engine operating in reverse. Its coefficient of performance can also be written as

$$K = \frac{T_L}{T_H - T_L}$$

The value of $K$ is higher when the temperatures of the two reservoirs are close to each other.

74. The Carnot efficiency (Eq. 20-13) depends linearly on $T_L$ so that we can take a derivative

$$\varepsilon = 1 - \frac{T_L}{T_H} \Rightarrow \frac{d\varepsilon}{dT_L} = -\frac{1}{T_H}$$

and quickly get to the result. With $d\varepsilon \to \Delta\varepsilon = 0.100$ and $T_H$ = 400 K, we find $dT_L \to \Delta T_L$ = –40 K.

75. **THINK** The gas molecules inside a box can be distributed in many different ways. The number of microstates associated with each distinct configuration is called the multiplicity.

**EXPRESS** In general, if there are $N$ molecules and if the box is divided into two halves, with $n_L$ molecules in the left half and $n_R$ in the right half, such that $n_L + n_R = N$. There are $N$! arrangements of the $N$ molecules, but $n_L$! are simply rearrangements of the $n_L$ molecules in the left half, and $n_R$! are rearrangements of the $n_R$ molecules in the right half. These rearrangements do not produce a new configuration. Therefore, the multiplicity factor associated with this is

$$W = \frac{N!}{n_L!n_R!}.$$

The entropy is given by $S = k\ln W$.

**ANALYZE** (a) The least multiplicity configuration is when all the particles are in the same half of the box. In this case, for system $A$ with with $N = 3$, we have

$$W = \frac{3!}{3!0!} = 1.$$

(b) Similarly for box $B$, with $N = 5$, $W = 5!/(5!0!) = 1$ in the "least" case.

(c) The most likely configuration in the 3 particle case is to have 2 on one side and 1 on the other. Thus,

$$W = \frac{3!}{2!1!} = 3.$$

(d) The most likely configuration in the 5 particle case is to have 3 on one side and 2 on the other. Therefore,

$$W = \frac{5!}{3!2!} = 10.$$

(e) We use Eq. 20-21 with our result in part (c) to obtain

$$S = k\ln W = \left(1.38\times10^{-23}\right)\ln 3 = 1.5\times10^{-23}\text{ J/K}.$$

(f) Similarly for the 5 particle case (using the result from part (d)), we find

$$S = k\ln 10 = 3.2\times10^{-23}\text{ J/K}.$$

**LEARN** The least multiplicity is $W = 1$; this happens when $n_L = N$ or $n_L = 0$. On the other hand, the greatest multiplicity occurs when $n_L = (N-1)/2$ or $n_L = (N+1)/2$.

76. (a) Using $Q = T\Delta S$, we note that heat enters the cycle along the top path at 400 K, and leaves along the bottom path at 250 K. Thus,

$$Q_{\text{in}} = (400\text{ K})(0.60\text{ J/K} - 0.10\text{ J/K}) = 200\text{ J}$$
$$Q_{\text{out}} = (250\text{ K})(0.10\text{ J/K} - 0.60\text{ J/K}) = -125\text{ J}$$

and the net heat transfer is $Q = Q_{\text{in}} + Q_{\text{out}} = 200\text{ J} - 125\text{ J} = 75\text{ J}$.

(b) For cyclic path, $\Delta E_{\text{int}} = Q - W = 0$. Therefore, the work done by the system is $W = Q = 75\text{ J}$.

77. The efficiency of an ideal heat engine and coefficient of performance of a reversible refrigerator are

$$\varepsilon = \frac{|W|}{|Q_{\text{H}}|}, \qquad K = \frac{|Q_{\text{H}}| - |W|}{|W|}.$$

Thus,

$$K = \frac{|Q_{\text{H}}| - |W|}{|W|} = \frac{|Q_{\text{H}}|}{|W|} - 1 = \frac{1}{\varepsilon} - 1 \quad \Rightarrow \quad \varepsilon = \frac{1}{K+1}$$

78. (a) The efficiency is defined by $\varepsilon = |W|/|Q_{\text{H}}|$, where $W$ is the work done by the engine and $Q_{\text{H}}$ is the heat input. In our case, the temperatures of the hot and cold reservoirs are $T_{\text{H}} = 100\,°\text{C} = 373\text{ K}$ and $T_{\text{L}} = 60\,°\text{C} = 333\text{ K}$, respectively. The maximum efficiency of the engine is

$$\varepsilon = \frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{333\text{ K}}{373\text{ K}} = 0.107.$$

Thus, the rate of heat input is

$$\frac{dQ_{\text{H}}}{dt} = \frac{1}{\varepsilon}\frac{dW}{dt} = \frac{1}{0.107}(500\text{ W}) = 4.66\times10^{3}\text{ W}.$$

(b) The rate of exhaust heat output is

$$\frac{dQ_{\text{L}}}{dt} = \frac{dQ_{\text{H}}}{dt} - \frac{dW}{dt} = 4.66\times10^{3}\text{ W} - 500\text{ W} = 4.16\times10^{3}\text{ W}.$$

79. The temperatures of the hot and cold reservoirs are $T_{\text{H}} = 26\,°\text{C} = 299\text{ K}$ and $T_{\text{L}} = -13\,°\text{C} = 260\text{ K}$, respectively. Therefore, the theoretical coefficient of performance of the refrigerator is

$$K = \frac{T_{\text{L}}}{T_{\text{H}} - T_{\text{L}}} = \frac{260\text{ K}}{299\text{ K} - 260\text{ K}} = 6.67.$$

# Chapter 21

1. **THINK** After the transfer, the charges on the two spheres are $Q-q$ and $q$.

**EXPRESS** The magnitude of the electrostatic force between two charges $q_1$ and $q_2$ separated by a distance $r$ is given by the Coulomb's law (see Eq. 21-1):

$$F = k\frac{q_1 q_2}{r^2},$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\mathrm{N\cdot m^2/C^2}$. In our case, $q_1 = Q-q$ and $q_2 = q$, so the magnitude of the force of either of the charges on the other is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q-q)}{r^2}.$$

We want the value of $q$ that maximizes the function $f(q) = q(Q-q)$.

**ANALYZE** Setting the derivative $df/dq$ equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.500$.

**LEARN** The force between the two spheres is a maximum when charges are distributed evenly between them.

2. The fact that the spheres are identical allows us to conclude that when two spheres are in contact, they share equal charge. Therefore, when a charged sphere ($q$) touches an uncharged one, they will (fairly quickly) each attain half that charge ($q/2$). We start with spheres 1 and 2, each having charge $q$ and experiencing a mutual repulsive force $F = kq^2/r^2$. When the neutral sphere 3 touches sphere 1, sphere 1's charge decreases to $q/2$. Then sphere 3 (now carrying charge $q/2$) is brought into contact with sphere 2; a total amount of $q/2 + q$ becomes shared equally between them. Therefore, the charge of sphere 3 is $3q/4$ in the final situation. The repulsive force between spheres 1 and 2 is finally

$$F' = k\frac{(q/2)(3q/4)}{r^2} = \frac{3}{8}k\frac{q^2}{r^2} = \frac{3}{8}F \quad\Rightarrow\quad \frac{F'}{F} = \frac{3}{8} = 0.375.$$

3. **THINK** The magnitude of the electrostatic force between two charges $q_1$ and $q_2$ separated by a distance $r$ is given by Coulomb's law.

**EXPRESS** Equation 21-1 gives Coulomb's law, $F = k\frac{|q_1||q_2|}{r^2}$, which can be used to solve for the distance:

$$r = \sqrt{\frac{k\,|q_1||q_2|}{F}}\,.$$

**ANALYZE** With $F = 5.70\text{ N}$, $q_1 = 2.60\times10^{-6}\text{ C}$ and $q_2 = -47.0\times10^{-6}$ C, the distance between the two charges is

$$r = \sqrt{\frac{k\,|q_1||q_2|}{F}} = \sqrt{\frac{\left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(26.0\times10^{-6}\,\text{C}\right)\left(47.0\times10^{-6}\,\text{C}\right)}{5.70\,\text{N}}} = 1.39\text{ m}.$$

**LEARN** The electrostatic force between two charges falls as $1/r^2$. The same inverse-square nature is also seen in the gravitational force between two masses.

4. The unit ampere is discussed in Section 21-4. Using $i$ for current, the charge transferred is

$$q = it = \left(2.5\times10^{4}\,\text{A}\right)\left(20\times10^{-6}\,\text{s}\right) = 0.50\text{ C}.$$

5. The magnitude of the mutual force of attraction at $r$ = 0.120 m is

$$F = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times10^{-6}\,\text{C}\right)\left(1.50\times10^{-6}\,\text{C}\right)}{(0.120\text{ m})^2} = 2.81\,\text{N}.$$

6. (a) With $a$ understood to mean the magnitude of acceleration, Newton's second and third laws lead to

$$m_2a_2 = m_1a_1 \Rightarrow m_2 = \frac{\left(6.3\times10^{-7}\,\text{kg}\right)\left(7.0\,\text{m/s}^2\right)}{9.0\,\text{m/s}^2} = 4.9\times10^{-7}\,\text{kg}.$$

(b) The magnitude of the (only) force on particle 1 is

$$F = m_1a_1 = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{|q|^2}{(0.0032\text{ m})^2}.$$

Inserting the values for $m_1$ and $a_1$ (see part (a)) we obtain $|q| = 7.1\times10^{-11}$ C.

7. With rightward positive, the net force on $q_3$ is

$$F_3 = F_{13} + F_{23} = k\frac{q_1q_3}{\left(L_{12}+L_{23}\right)^2} + k\frac{q_2q_3}{L_{23}^2}\,.$$

We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on $q_3$ by $q_1$) is negative if they are unlike charges, indicating that $q_3$ is being pulled toward $q_1$, and it is positive if they are like charges (so $q_3$ would be repelled from $q_1$). Setting the net force equal to zero $L_{23}= L_{12}$ and canceling $k$, $q_3$, and $L_{12}$ leads to

$$\frac{q_1}{4.00} + q_2 = 0 \quad \Rightarrow \quad \frac{q_1}{q_2} = -4.00.$$

8. In experiment 1, sphere $C$ first touches sphere $A$, and they divided up their total charge ($Q$/2 plus $Q$) equally between them. Thus, sphere $A$ and sphere $C$ each acquired charge 3$Q$/4. Then, sphere $C$ touches $B$ and those spheres split up their total charge (3$Q$/4 plus –$Q$/4) so that $B$ ends up with charge equal to $Q$/4. The force of repulsion between $A$ and $B$ is therefore

$$F_1 = k\frac{(3Q/4)(Q/4)}{d^2}$$

at the end of experiment 1. Now, in experiment 2, sphere $C$ first touches $B$, which leaves each of them with charge $Q$/8. When $C$ next touches $A$, sphere $A$ is left with charge 9$Q$/16. Consequently, the force of repulsion between $A$ and $B$ is

$$F_2 = k\frac{(9Q/16)(Q/8)}{d^2}$$

at the end of experiment 2. The ratio is

$$\frac{F_2}{F_1} = \frac{(9/16)(1/8)}{(3/4)(1/4)} = 0.375.$$

9. **THINK** Since opposite charges attract, the initial charge configurations must be of opposite signs. Similarly, since like charges repel, the final charge configurations must carry the same sign.

**EXPRESS** We assume that the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let $q_1$ and $q_2$ be the original charges. We choose the coordinate system so the force on $q_2$ is positive if it is repelled by $q_1$. Then the force on $q_2$ is

$$F_a = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = -k\frac{q_1 q_2}{r^2}$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$ and $r$ = 0.500 m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is ($q_1$ + $q_2$)/2. The force is now repulsive and is given by

$$F_b = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k\,\frac{(q_1+q_2)^2}{4r^2}.$$

We solve the two force equations simultaneously for $q_1$ and $q_2$.

**ANALYZE** The first equation gives the product

$$q_1 q_2 = -\frac{r^2 F_a}{k} = -\frac{(0.500\,\text{m})^2(0.108\,\text{N})}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = -3.00\times10^{-12}\,\text{C}^2,$$

and the second gives the sum

$$q_1 + q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\,\text{m})\sqrt{\frac{0.0360\,\text{N}}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}} = 2.00\times10^{-6}\,\text{C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-(3.00\times10^{-12}\,\text{C}^2)}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00\times10^{-12}\,\text{C}^2}{q_1} = 2.00\times10^{-6}\,\text{C}.$$

Multiplying by $q_1$ and rearranging, we obtain a quadratic equation

$$q_1^2 - (2.00\times10^{-6}\,\text{C})q_1 - 3.00\times10^{-12}\,\text{C}^2 = 0.$$

The solutions are

$$q_1 = \frac{2.00\times10^{-6}\,\text{C} \pm \sqrt{(-2.00\times10^{-6}\,\text{C})^2 - 4(-3.00\times10^{-12}\,\text{C}^2)}}{2}.$$

If the positive sign is used, $q_1 = 3.00 \times 10^{-6}$ C, and if the negative sign is used, $q_1 = -1.00\times10^{-6}\,\text{C}$.

(a) Using $q_2 = (-3.00 \times 10^{-12})/q_1$ with $q_1 = 3.00 \times 10^{-6}$ C, we get $q_2 = -1.00\times10^{-6}\,\text{C}$.

(b) If we instead work with the $q_1 = -1.00 \times 10^{-6}$ C root, then we find $q_2 = 3.00\times10^{-6}\,\text{C}$.

**LEARN** Note that since the spheres are identical, the solutions are essentially the same: one sphere originally had charge $-1.00 \times 10^{-6}$ C and the other had charge $+3.00 \times 10^{-6}$ C. What happens if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00 \times 10^{-6}$ C on one sphere and a charge of $-3.00 \times 10^{-6}$ C on the other also satisfies the conditions of the problem.

10. For ease of presentation (of the computations below) we assume $Q > 0$ and $q < 0$ (although the final result does not depend on this particular choice).

(a) The $x$-component of the force experienced by $q_1 = Q$ is

$$F_{1x} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{(Q)(Q)}{\left(\sqrt{2}a\right)^2}\cos 45° + \frac{(|q|)\,(Q)}{a^2}\right) = \frac{Q|q|}{4\pi\varepsilon_0 a^2}\left(-\frac{Q/|q|}{2\sqrt{2}}+1\right)$$

which (upon requiring $F_{1x} = 0$) leads to $Q/|q| = 2\sqrt{2}$, or $Q/q = -2\sqrt{2} = -2.83$.

(b) The $y$-component of the net force on $q_2 = q$ is

$$F_{2y} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q|^2}{\left(\sqrt{2}a\right)^2}\sin 45° - \frac{(|q|)\,(Q)}{a^2}\right) = \frac{|q|^2}{4\pi\varepsilon_0 a^2}\left(\frac{1}{2\sqrt{2}} - \frac{Q}{|q|}\right)$$

which (if we demand $F_{2y} = 0$) leads to $Q/q = -1/2\sqrt{2}$. The result is inconsistent with that obtained in part (a). Thus, we are unable to construct an equilibrium configuration with this geometry, where the only forces present are given by Eq. 21-1.

11. The force experienced by $q_3$ is

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{|q_3||q_1|}{a^2}\hat{\mathrm{j}} + \frac{|q_3||q_2|}{(\sqrt{2}a)^2}(\cos 45°\hat{\mathrm{i}} + \sin 45°\hat{\mathrm{j}}) + \frac{|q_3||q_4|}{a^2}\hat{\mathrm{i}}\right)$$

(a) Therefore, the $x$-component of the resultant force on $q_3$ is

$$F_{3x} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(\frac{|q_2|}{2\sqrt{2}} + |q_4|\right) = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(\frac{1}{2\sqrt{2}} + 2\right) = 0.17\ \mathrm{N}.$$

(b) Similarly, the $y$-component of the net force on $q_3$ is

$$F_{3y} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(-|q_1| + \frac{|q_2|}{2\sqrt{2}}\right) = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(-1 + \frac{1}{2\sqrt{2}}\right)$$
$$= -0.046\ \mathrm{N}.$$

12. (a) For the net force to be in the $+x$ direction, the $y$ components of the individual forces must cancel. The angle of the force exerted by the $q_1$ = 40 $\mu$C charge on $q_3 = 20\mu C$ is 45°, and the angle of force exerted on $q_3$ by $Q$ is at $-\theta$ where

$$\theta = \tan^{-1}\left(\frac{2.0 \text{ cm}}{3.0 \text{ cm}}\right) = 33.7°.$$

Therefore, cancellation of $y$ components requires

$$k\frac{q_1 q_3}{\left(0.02\sqrt{2} \text{ m}\right)^2}\sin 45° = k\frac{|Q|\, q_3}{\left(\sqrt{(0.030 \text{ m})^2 + (0.020 \text{ m})^2}\right)^2}\sin\theta$$

from which we obtain $|Q|$ = 83 $\mu$C. Charge $Q$ is "pulling" on $q_3$, so (since $q_3 > 0$) we conclude $Q$ = –83 $\mu$C.

(b) Now, we require that the $x$ components cancel, and we note that in this case, the angle of force on $q_3$ exerted by $Q$ is $+\theta$ (it is repulsive, and $Q$ is positive-valued). Therefore,

$$k\frac{q_1\, q_3}{\left(0.02\sqrt{2} \text{ m}\right)^2}\cos 45° = k\frac{Qq_3}{\left(\sqrt{(0.030 \text{ m})^2 + (0.020 \text{ m})^2}\right)^2}\cos\theta$$

from which we obtain $Q$ = 55.2 $\mu$C $\approx 55$ $\mu$C.

13. (a) There is no equilibrium position for $q_3$ *between* the two fixed charges, because it is being pulled by one and pushed by the other (since $q_1$ and $q_2$ have different signs); in this region this means the two force arrows on $q_3$ are in the same direction and cannot cancel. It should also be clear that off-axis (with the axis defined as that which passes through the two fixed charges) there are no equilibrium positions. On the semi-infinite region of the axis that is nearest $q_2$ and furthest from $q_1$ an equilibrium position for $q_3$ cannot be found because $|q_1| < |q_2|$ and the magnitude of force exerted by $q_2$ is everywhere (in that region) stronger than that exerted by $q_1$ on $q_3$. Thus, we must look in the semi-infinite region of the axis which is nearest $q_1$ and furthest from $q_2$, where the net force on $q_3$ has magnitude

$$\left| k\frac{|q_1 q_3|}{L_0^2} - k\frac{|q_2 q_3|}{\left(L + L_0\right)^2} \right|$$

with $L$ = 10 cm and $L_0$ is assumed to be *positive*. We set this equal to zero, as required by the problem, and cancel $k$ and $q_3$. Thus, we obtain

$$\frac{|q_1|}{L_0^2}-\frac{|q_2|}{(L+L_0)^2}=0 \;\Rightarrow\; \left(\frac{L+L_0}{L_0}\right)^2=\left|\frac{q_2}{q_1}\right|=\left|\frac{-3.0\ \mu\text{C}}{+1.0\ \mu\text{C}}\right|=3.0$$

which yields (after taking the square root)

$$\frac{L+L_0}{L_0}=\sqrt{3} \;\Rightarrow\; L_0=\frac{L}{\sqrt{3}-1}=\frac{10\text{ cm}}{\sqrt{3}-1}\approx 14\,\text{cm}$$

for the distance between $q_3$ and $q_1$. That is, $q_3$ should be placed at $x=-14$ cm along the $x$-axis.

(b) As stated above, $y = 0$.

14. (a) The individual force magnitudes (acting on $Q$) are, by Eq. 21-1,

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{(-a-a/2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{(a-a/2)^2}$$

which leads to $|q_1| = 9.0\ |q_2|$. Since $Q$ is located between $q_1$ and $q_2$, we conclude $q_1$ and $q_2$ are like-sign. Consequently, $q_1/q_2 = 9.0$.

(b) Now we have

$$\frac{1}{4\pi\varepsilon_0}\frac{|q_1|Q}{(-a-3a/2)^2}=\frac{1}{4\pi\varepsilon_0}\frac{|q_2|Q}{(a-3a/2)^2}$$

which yields $|q_1| = 25\ |q_2|$. Now, $Q$ is not located between $q_1$ and $q_2$; one of them must push and the other must pull. Thus, they are unlike-sign, so $q_1/q_2 = -25$.

15. (a) The distance between $q_1$ and $q_2$ is

$$r_{12}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-0.020\text{ m}-0.035\text{ m})^2+(0.015\text{ m}-0.005\text{ m})^2}=0.056\text{ m}.$$

The magnitude of the force exerted by $q_1$ on $q_2$ is

$$F_{21}=k\frac{|q_1q_2|}{r_{12}^2}=\frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.0\times10^{-6}\text{C}\right)\left(4.0\times10^{-6}\text{C}\right)}{(0.056\text{ m})^2}=35\text{ N}.$$

(b) The vector $\vec{F}_{21}$ is directed toward $q_1$ and makes an angle $\theta$ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \tan^{-1}\left(\frac{1.5 \text{ cm} - 0.5 \text{ cm}}{-2.0 \text{ cm} - 3.5 \text{ cm}}\right) = -10.3° \approx -10°.$$

(c) Let the third charge be located at ($x_3$, $y_3$), a distance $r$ from $q_2$. We note that $q_1$, $q_2$, and $q_3$ must be collinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place $q_3$ on the same side of $q_2$ where we also find $q_1$, since in that region both forces (exerted on $q_2$ by $q_3$ and $q_1$) would be in the same direction (since $q_2$ is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r\cos\theta$ and $y_3 = y_2 - r\sin\theta$ (which means $y_3 > y_2$ since $\theta$ is negative). The magnitude of force exerted on $q_2$ by $q_3$ is $F_{23} = k|q_2 q_3|/r^2$, which must equal that of the force exerted on it by $q_1$ (found in part (a)). Therefore,

$$k\frac{|q_2 q_3|}{r^2} = k\frac{|q_1 q_2|}{r_{12}^2} \Rightarrow r = r_{12}\sqrt{\frac{q_3}{q_1}} = 0.0645 \text{ m} = 6.45 \text{ cm}.$$

Consequently, $x_3 = x_2 - r\cos\theta = -2.0 \text{ cm} - (6.45 \text{ cm})\cos(-10°) = -8.4 \text{ cm}$,

(d) and $y_3 = y_2 - r\sin\theta = 1.5 \text{ cm} - (6.45 \text{ cm})\sin(-10°) = 2.7 \text{ cm}$.

16. (a) According to the graph, when $q_3$ is very close to $q_1$ (at which point we can consider the force exerted by particle 1 on 3 to dominate) there is a (large) force in the positive $x$ direction. This is a repulsive force, then, so we conclude $q_1$ has the same sign as $q_3$. Thus, $q_3$ is a positive-valued charge.

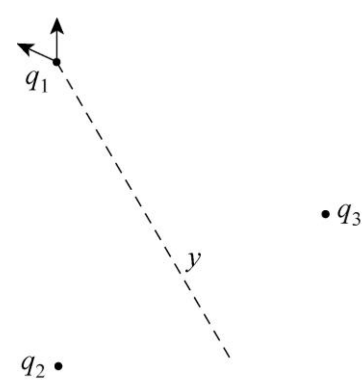
(b) Since the graph crosses zero and particle 3 is *between* the others, $q_1$ must have the same sign as $q_2$, which means it is also positive-valued. We note that it crosses zero at $r$ = 0.020 m (which is a distance $d$ = 0.060 m from $q_2$). Using Coulomb's law at that point, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_3 q_2}{d^2} \Rightarrow q_2 = \left(\frac{d}{r}\right)^2 q_1 = \left(\frac{0.060 \text{ m}}{0.020 \text{ m}}\right)^2 q_1 = 9.0\, q_1,$$

or $q_2/q_1 = 9.0$.

17. (a) Equation 21-1 gives

$$F_{12} = k\frac{q_1 q_2}{d^2} = \left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(20.0\times10^{-6}\text{ C}\right)^2}{(1.50\text{m})^2} = 1.60 \text{ N}.$$




(b) On the right, a force diagram is shown as well as our choice of $y$ axis (the dashed line).

The $y$ axis is meant to bisect the line between $q_2$ and $q_3$ in order to make use of the symmetry in the problem (equilateral triangle of side length $d$, equal-magnitude charges $q_1 = q_2 = q_3 = q$). We see that the resultant force is along this symmetry axis, and we obtain

$$\left|F_y\right| = 2\left(k\frac{q^2}{d^2}\right)\cos 30° = 2\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(20.0\times10^{-6}\,\text{C}\right)^2}{\left(1.50\,\text{m}\right)^2}\cos 30° = 2.77\,\text{N}\,.$$

18. Since the forces involved are proportional to $q$, we see that the essential difference between the two situations is $F_a \propto q_B + q_C$ (when those two charges are on the same side) versus $F_b \propto -q_B + q_C$ (when they are on opposite sides). Setting up ratios, we have

$$\frac{F_a}{F_b} = \frac{q_B + q_C}{-q_B + q_C} \quad\Rightarrow\quad \frac{2.014\times10^{-23}\ \text{N}}{-2.877\times10^{-24}\ \text{N}} = \frac{1 + q_C/q_B}{-1 + q_C/q_B}\ .$$

After noting that the ratio on the left hand side is very close to – 7, then, after a couple of algebra steps, we are led to

$$\frac{q_C}{q_B} = \frac{7+1}{7-1} = \frac{8}{6} = 1.333.$$

19. **THINK** Our system consists of two charges in a straight line. We'd like to place a third charge so that all three charges are in equilibrium.

**EXPRESS** If the system of three charges is to be in equilibrium, the force on each charge must be zero. The third charge $q_3$ must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and $q_3$ could not be in equilibrium. Suppose $q_3$ is at a distance $x$ from $q$, and $L – x$ from $4.00q$. The force acting on it is then given by

$$F_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} - \frac{4qq_3}{\left(L-x\right)^2}\right)$$

where the positive direction is rightward. We require $F_3 = 0$ and solve for $x$.

**ANALYZE** (a) Canceling common factors yields $1/x^2 = 4/(L – x)^2$ and taking the square root yields $1/x = 2/(L – x)$. The solution is $x = L/3$. With $L = 9.00$ cm, we have $x = 3.00$ cm.

(b) Similarly, the $y$ coordinate of $q_3$ is $y = 0$.

(c) The force on $q$ is

$$F_q = \frac{-1}{4\pi\varepsilon_0}\left(\frac{qq_3}{x^2} + \frac{4.00q^2}{L^2}\right).$$

The signs are chosen so that a negative force value would cause $q$ to move leftward. We require $F_q = 0$ and solve for $q_3$:

$$q_3 = -\frac{4qx^2}{L^2} = -\frac{4}{9}q \;\Rightarrow\; \frac{q_3}{q} = -\frac{4}{9} = -0.444$$

where $x = L/3$ is used.

**LEARN** We may also verify that the force on $4.00q$ also vanishes:

$$F_{4q} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) = 0\,.$$

20. We note that the problem is examining the force <u>on</u> charge *A*, so that the respective distances (involved in the Coulomb force expressions) between *B* and *A*, and between *C* and *A*, do not change as particle *B* is moved along its circular path. We focus on the endpoints ($\theta = 0°$ and 180°) of each graph, since they represent cases where the forces (on *A*) due to *B* and *C* are either parallel or anti-parallel (yielding maximum or minimum force magnitudes, respectively). We note, too, that since Coulomb's law is inversely proportional to $r^2$ then (if, say, the charges were all the same) the force due to *C* would be one-fourth as big as that due to *B* (since *C* is twice as far away from *A*). The charges, it turns out, are not the same, so there is also a factor of the charge ratio $\xi$ (the charge of *C* divided by the charge of *B*), as well as the aforementioned ¼ factor. That is, the force exerted by *C* is, by Coulomb's law, equal to $\pm\frac{1}{4}\xi$ multiplied by the force exerted by *B*.

(a) The maximum force is $2F_0$ and occurs when $\theta = 180°$ (*B* is to the left of *A*, while *C* is the right of *A*). We choose the minus sign and write

$$2F_0 = (1 - \tfrac{1}{4}\xi)\,F_0 \quad\Rightarrow\quad \xi = -4\,.$$

One way to think of the minus sign choice is cos(180°) = –1. This is certainly consistent with the minimum force ratio (zero) at $\theta = 0°$ since that would also imply

$$0 = 1 + \tfrac{1}{4}\xi \quad\Rightarrow\quad \xi = -4\,.$$

(b) The ratio of maximum to minimum forces is 1.25/0.75 = 5/3 in this case, which implies

$$\frac{5}{3} = \frac{1 + \frac{1}{4}\xi}{1 - \frac{1}{4}\xi} \quad\Rightarrow\quad \xi = 16\,.$$

Of course, this could also be figured as illustrated in part (a), looking at the maximum force ratio by itself and solving, or looking at the minimum force ratio (¾) at $\theta = 180°$ and solving for $\xi$.

21. The charge $dq$ within a thin shell of thickness $dr$ is $dq = \rho dV = \rho A dr$ where $A = 4\pi r^2$. Thus, with $\rho = b/r$, we have

$$q = \int dq = 4\pi b\int_{r_1}^{r_2} r\,dr = 2\pi b\left(r_2^2 - r_1^2\right).$$

With $b$ = 3.0 $\mu$C/m$^2$, $r_2$ = 0.06 m, and $r_1$ = 0.04 m, we obtain $q$ = 0.038 $\mu$C = 3.8 × 10$^{-8}$ C.

22. (a) We note that cos(30º) $= \frac{1}{2}\sqrt{3}$, so that the dashed line distance in the figure is $r = 2d/\sqrt{3}$. The net force on $q_1$ due to the two charges $q_3$ and $q_4$ (with $|q_3| = |q_4|$ = 1.60 × 10$^{-19}$ C) on the $y$ axis has magnitude

$$2\frac{|q_1q_3|}{4\pi\varepsilon_0 r^2}\cos(30°) = \frac{3\sqrt{3}\,|q_1q_3|}{16\pi\varepsilon_0 d^2}.$$

This must be set equal to the magnitude of the force exerted on $q_1$ by $q_2$ = 8.00 × 10$^{-19}$ C = 5.00 $|q_3|$ in order that its net force be zero:

$$\frac{3\sqrt{3}\,|q_1q_3|}{16\pi\varepsilon_0 d^2} = \frac{|q_1q_2|}{4\pi\varepsilon_0 (D+d)^2} \quad \Rightarrow \quad D = d\left(2\sqrt{\frac{5}{3\sqrt{3}}} - 1\right) = 0.9245\ d.$$

Given $d$ = 2.00 cm, this then leads to $D$ = 1.92 cm.

(b) As the angle decreases, its cosine increases, resulting in a larger contribution from the charges on the $y$ axis. To offset this, the force exerted by $q_2$ must be made stronger, so that it must be brought closer to $q_1$ (keep in mind that Coulomb's law is *inversely* proportional to distance-squared). Thus, $D$ must be decreased.

23. If $\theta$ is the angle between the force and the $x$-axis, then

$$\cos\theta = \frac{x}{\sqrt{x^2 + d^2}}.$$

We note that, due to the symmetry in the problem, there is no $y$ component to the net force on the third particle. Thus, $F$ represents the magnitude of force exerted by $q_1$ or $q_2$ on $q_3$. Let $e$ = +1.60 × 10$^{-19}$ C, then $q_1 = q_2 = +2e$ and $q_3 = 4.0e$ and we have

$$F_{\text{net}} = 2F\cos\theta = \frac{2(2e)(4e)}{4\pi\varepsilon_o (x^2 + d^2)}\,\frac{x}{\sqrt{x^2 + d^2}} = \frac{4e^2 x}{\pi\varepsilon_o (x^2 + d^2)^{3/2}}.$$

(a) To find where the force is at an extremum, we can set the derivative of this expression equal to zero and solve for $x$, but it is good in any case to graph the function for a fuller understanding of its behavior, and as a quick way to see whether an extremum point is a maximum or a miminum. In this way, we find that the value coming from the derivative procedure is a maximum (and will be presented in part (b)) and that the minimum is found at the lower limit of the interval. Thus, the net force is found to be zero at $x$ = 0, which is the smallest value of the net force in the interval 5.0 m ≥ $x$ ≥ 0.

(b) The maximum is found to be at $x = d/\sqrt{2}$ or roughly 12 cm.

(c) The value of the net force at $x = 0$ is $F_{\text{net}} = 0$.

(d) The value of the net force at $x = d/\sqrt{2}$ is $F_{\text{net}} = 4.9 \times 10^{-26}$ N.

24. (a) Equation 21-1 gives

$$F = \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00 \times 10^{-16}\,\text{C}\right)^2}{\left(1.00 \times 10^{-2}\ \text{m}\right)^2} = 8.99 \times 10^{-19}\,\text{N}.$$

(b) If $n$ is the number of excess electrons (of charge $-e$ each) on each drop then

$$n = -\frac{q}{e} = -\frac{-1.00 \times 10^{-16}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 625.$$

25. Equation 21-11 (in absolute value) gives $n = \dfrac{|q|}{e} = \dfrac{1.0 \times 10^{-7}\,\text{C}}{1.6 \times 10^{-19}\,\text{C}} = 6.3 \times 10^{11}$.

26. The magnitude of the force is

$$F = k\frac{e^2}{r^2} = \left(8.99 \times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(2.82 \times 10^{-10}\ \text{m}\right)^2} = 2.89 \times 10^{-9}\ \text{N}.$$

27. **THINK** The magnitude of the electrostatic force between two charges $q_1$ and $q_2$ separated by a distance $r$ is given by Coulomb's law.

**EXPRESS** Let the charge of the ions be $q$. With $q_1 = q_2 = +q$, the magnitude of the force between the (positive) ions is given by

$$F = \frac{(q)(q)}{4\pi\varepsilon_0 r^2} = k\frac{q^2}{r^2},$$

where $k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$.

**ANALYZE** (a) We solve for the charge:

$$q = r\sqrt{\frac{F}{k}} = \left(5.0 \times 10^{-10}\ \text{m}\right)\sqrt{\frac{3.7 \times 10^{-9}\ \text{N}}{8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 3.2 \times 10^{-19}\ \text{C}.$$

(b) Let $n$ be the number of electrons missing from each ion. Then, $ne = q$, or

$$n = \frac{q}{e} = \frac{3.2\times10^{-9}\text{ C}}{1.6\times10^{-19}\text{ C}} = 2.$$

**LEARN** Electric charge is quantized. This means that any charge can be written as $q = ne$, where $n$ is an integer (positive or negative), and $e = 1.6\times10^{-19}$ C is the elementary charge.

28. Keeping in mind that an ampere is a coulomb per second (1 A = 1 C/s), and that a minute is 60 seconds, the charge (in absolute value) that passes through the chest is

$$|\,q\,| = (0.300\text{ C/s})(120\text{ s}) = 36.0\text{ C}.$$

This charge consists of $n$ electrons (each of which has an absolute value of charge equal to $e$). Thus,

$$n = \frac{|\,q\,|}{e} = \frac{36.0\text{ C}}{1.60\times10^{-19}\text{ C}} = 2.25\times10^{20}.$$

29. (a) We note that tan(30°) = $1/\sqrt{3}$. In the initial (highly symmetrical) configuration, the net force on the central bead is in the $-y$ direction and has magnitude $3F$ where $F$ is the Coulomb's law force of one bead on another at distance $d$ = 10 cm. This is due to the fact that the forces exerted on the central bead (in the initial situation) by the beads on the $x$ axis cancel each other; also, the force exerted "downward" by bead 4 on the central bead is four times larger than the "upward" force exerted by bead 2. This net force along the $y$ axis does not change as bead 1 is now moved, though there is now a nonzero $x$-component $F_x$. The components are now related by

$$\tan(30°) = \frac{F_x}{F_y} \Rightarrow \frac{1}{\sqrt{3}} = \frac{F_x}{3F}$$

which implies $F_x = \sqrt{3}\,F$. Now, bead 3 exerts a "leftward" force of magnitude $F$ on the central bead, while bead 1 exerts a "rightward" force of magnitude $F'$. Therefore,

$$F' - F = \sqrt{3}\,F. \quad \Rightarrow \quad F' = (\sqrt{3} + 1)\,F.$$

The fact that Coulomb's law depends inversely on distance-squared then implies

$$r^2 = \frac{d^2}{\sqrt{3}+1} \Rightarrow r = \frac{d}{\sqrt{\sqrt{3}+1}} = \frac{10\text{ cm}}{\sqrt{\sqrt{3}+1}} = \frac{10\text{ cm}}{1.65} = 6.05\text{ cm}$$

where $r$ is the distance between bead 1 and the central bead. This corresponds to $x = -6.05$ cm.

(b) To regain the condition of high symmetry (in particular, the cancellation of $x$-components) bead 3 must be moved closer to the central bead so that it, too, is the distance $r$ (as calculated in part (a)) away from it.

30. (a) Let $x$ be the distance between particle 1 and particle 3. Thus, the distance between particle 3 and particle 2 is $L - x$. Both particles exert leftward forces on $q_3$ (so long as it is on the line between them), so the magnitude of the net force on $q_3$ is

$$F_{net} = |\vec{F}_{13}| + |\vec{F}_{23}| = \frac{|q_1 q_3|}{4\pi\varepsilon_0 x^2} + \frac{|q_2 q_3|}{4\pi\varepsilon_0 (L-x)^2} = \frac{e^2}{\pi\varepsilon_0}\left(\frac{1}{x^2} + \frac{27}{(L-x)^2}\right)$$

with the values of the charges (stated in the problem) plugged in. Finding the value of $x$ that minimizes this expression leads to $x = \frac{1}{4} L$. Thus, $x = 2.00$ cm.

(b) Substituting $x = \frac{1}{4} L$ back into the expression for the net force magnitude and using the standard value for $e$ leads to $F_{net} = 9.21 \times 10^{-24}$ N.

31. The unit ampere is discussed in Section 21-4. The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi(6.37 \times 10^6 \text{ m})^2 = 5.1 \times 10^{14} \text{ m}^2$ would be

$$i = \left(5.1\times10^{14}\,\text{m}^2\right)\left(1500\,\frac{\text{protons}}{\text{s}\cdot\text{m}^2}\right)\left(1.6\times10^{-19}\text{ C/proton}\right) = 0.122\,\text{A}.$$

32. Since the graph crosses zero, $q_1$ must be positive-valued: $q_1 = +8.00e$. We note that it crosses zero at $r = 0.40$ m. Now the asymptotic value of the force yields the magnitude and sign of $q_2$:

$$\frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = F \quad\Rightarrow\quad q_2 = \left(\frac{1.5\times10^{-25}}{kq_1}\right) r^2 = 2.086 \times 10^{-18}\,\text{C} = 13e.$$

33. The volume of 250 cm$^3$ corresponds to a mass of 250 g since the density of water is 1.0 g/cm$^3$. This mass corresponds to 250/18 = 14 moles since the molar mass of water is 18. There are ten protons (each with charge $q = +e$) in each molecule of $H_2O$, so

$$Q = 14N_A q = 14\left(6.02\times10^{23}\right)(10)\left(1.60\times10^{-19}\text{C}\right) = 1.3\times10^{7}\,\text{C}.$$

34. Let $d$ be the vertical distance from the coordinate origin to $q_3 = -q$ and $q_4 = -q$ on the $+y$ axis, where the symbol $q$ is assumed to be a positive value. Similarly, $d$ is the (positive) distance from the origin $q_4 = -$ on the $-y$ axis. If we take each angle $\theta$ in the figure to be positive, then we have

$$\tan\theta = d/R \text{ and } \cos\theta = R/r,$$

where $r$ is the dashed line distance shown in the figure. The problem asks us to consider $\theta$ to be a variable in the sense that, once the charges on the $x$ axis are fixed in place (which determines $R$), $d$ can then be arranged to some multiple of $R$, since $d = R\tan\theta$. The aim of this exploration is to show that if $q$ is bounded then $\theta$ (and thus $d$) is also bounded.

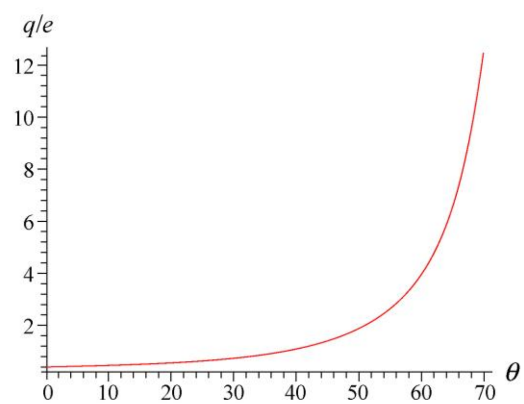
From symmetry, we see that there is no net force in the vertical direction on $q_2 = -e$ sitting at a distance $R$ to the left of the coordinate origin. We note that the net $x$ force caused by $q_3$ and $q_4$ on the $y$-axis will have a magnitude equal to

$$2\frac{qe}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{2qe\cos\theta}{4\pi\varepsilon_0 (R/\cos\theta)^2} = \frac{2qe\cos^3\theta}{4\pi\varepsilon_0 R^2} \ .$$

Consequently, to achieve a zero net force along the $x$ axis, the above expression must equal the magnitude of the repulsive force exerted on $q_2$ by $q_1 = -e$. Thus,

$$\frac{2qe\cos^3\theta}{4\pi\varepsilon_0 R^2} = \frac{e^2}{4\pi\varepsilon_0 R^2} \quad \Rightarrow \quad q = \frac{e}{2\cos^3\theta} \, .$$

Below we plot $q/e$ as a function of the angle (in degrees):




The graph suggests that $q/e < 5$ for $\theta < 60^{\circ}$, roughly. We can be more precise by solving the above equation. The requirement that $q \le 5e$ leads to

$$\frac{e}{2\cos^3\theta} \le 5e \quad \Rightarrow \quad \frac{1}{(10)^{1/3}} \le \cos\theta$$

which yields $\theta \le 62.34^{\circ}$. The problem asks for "physically possible values," and it is reasonable to suppose that only positive-integer-multiple values of $e$ are allowed for $q$. If we let $q = ne$, for $n = 1 \ldots 5$, then $\theta_N$ will be found by taking the inverse cosine of the cube root of $(1/2n)$.

(a) The smallest value of angle is $\theta_1$ = 37.5º (or 0.654 rad).

(b) The second smallest value of angle is $\theta_2$ = 50.95º (or 0.889 rad).

(c) The third smallest value of angle is $\theta_3$ = 56.6º (or 0.988 rad).

35. **THINK** Our system consists of 8 $Cs^+$ ions at the corners of a cube and a $Cl^-$ ion at the cube's center. To calculate the electrostatic force on the $Cl^-$ ion, we apply the superposition principle and make use of the symmetry property of the configuration.

**EXPRESS** In (a) where all 8 $Cs^+$ ions are present, every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is attractive and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube.

In (b) where one $Cs^+$ ion is missing at the corner, rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

**ANALYZE** (a) Since the two $Cs^+$ ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) The length of a body diagonal of a cube is $\sqrt{3}a$, where $a$ is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = \left(\sqrt{3}/2\right)a$. The force has magnitude

$$F = k\frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(3/4)\left(0.40\times10^{-9}\ \text{m}\right)^2} = 1.9\times10^{-9}\ \text{N}.$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

**LEARN** When solving electrostatic problems involving a discrete number of charges, symmetry argument can often be applied to simplify the problem.

36. (a) Since the proton is positively charged, the emitted particle must be a positron (as opposed to the negatively charged electron) in accordance with the law of charge conservation.

(b) In this case, the initial state had zero charge (the neutron is neutral), so the sum of charges in the final state must be zero. Since there is a proton in the final state, there should also be an electron (as opposed to a positron) so that $\Sigma q = 0$.

37. **THINK** Charges are conserved in nuclear reactions.

**EXPRESS** We note that none of the reactions given include a beta decay (see Chapter 42), so the number of protons (*Z*), the number of neutrons (*N*), and the number of electrons are each conserved. The mass number (total number of nucleons) is defined as $A = N + Z$. Atomic numbers (number of protons) and molar masses can be found in Appendix F of the text.

**ANALYZE** (a) $^{1}$H has 1 proton, 1 electron, and 0 neutrons and $^{9}$Be has 4 protons, 4 electrons, and 9 – 4 = 5 neutrons, so X has 1 + 4 = 5 protons, 1 + 4 = 5 electrons, and 0 + 5 – 1 = 4 neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of 5 + 4 = 9 g/mol: $^{9}$B.

(b) $^{12}$C has 6 protons, 6 electrons, and 12 – 6 = 6 neutrons and $^{1}$H has 1 proton, 1 electron, and 0 neutrons, so X has 6 + 1 = 7 protons, 6 + 1 = 7 electrons, and 6 + 0 = 6 neutrons. It must be nitrogen with a molar mass of 7 + 6 = 13 g/mol: $^{13}$N.

(c) $^{15}$N has 7 protons, 7 electrons, and 15 – 7 = 8 neutrons; $^{1}$H has 1 proton, 1 electron, and 0 neutrons; and $^{4}$He has 2 protons, 2 electrons, and 4 – 2 = 2 neutrons; so X has 7 + 1 – 2 = 6 protons, 6 electrons, and 8 + 0 – 2 = 6 neutrons. It must be carbon with a molar mass of 6 + 6 = 12 g/mol: $^{12}$C.

**LEARN** A general expression for the reaction can be expressed as

$$^{A_1}_{Z_1}X1_{N_1} + {}^{A_2}_{Z_2}X2_{N_2} \rightarrow {}^{A_3}_{Z_3}X3_{N_3} + {}^{A_4}_{Z_4}X4_{N_4}$$

where $A_i = Z_i + N_i$. Since the number of protons (*Z*), the number of neutrons (*N*), and the number of nucleons (*A*) are each conserved, we have $A_1 + A_2 = A_3 + A_4$, $Z_1 + Z_2 = Z_3 + Z_4$ and $N_1 + N_2 = N_3 + N_4$.

38. As a result of the first action, both sphere *W* and sphere *A* possess charge $\frac{1}{2}q_A$, where $q_A$ is the initial charge of sphere *A*. As a result of the second action, sphere *W* has charge

$$\frac{1}{2}\left(\frac{q_A}{2} - 32e\right).$$

As a result of the final action, sphere *W* now has charge equal to

$$\frac{1}{2}\left[\frac{1}{2}\left(\frac{q_A}{2}-32e\right)+48e\right].$$

Setting this final expression equal to +18*e* as required by the problem leads (after a couple of algebra steps) to the answer: $q_A$ = +16*e*.

39. **THINK** We have two discrete charges in the *xy*-plane. The electrostatic force on particle 2 due to particle 1 has both *x* and *y* components.

**EXPRESS** Using Coulomb's law, the magnitude of the force of particle 1 on particle 2 is $F_{21}=k\frac{q_1q_2}{r^2}$, where $r=\sqrt{d_1^2+d_2^2}$ and $k=1/4\pi\varepsilon_0=8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$. Since both $q_1$ and $q_2$ are positively charged, particle 2 is repelled by particle 1, so the direction of $\vec{F}_{21}$ is away from particle 1 and toward 2. In unit-vector notation, $\vec{F}_{21}=F_{21}\hat{\text{r}}$, where

$$\hat{\text{r}}=\frac{\vec{r}}{r}=\frac{d_2\hat{\text{i}}-d_1\hat{\text{j}}}{\sqrt{d_1^2+d_2^2}}.$$

The *x* component of $\vec{F}_{21}$ is $F_{21,x}=F_{21}d_2/\sqrt{d_1^2+d_2^2}$.

**ANALYZE** Combining the expressions above, we obtain

$$\begin{aligned}F_{21,x}&=k\frac{q_1q_2d_2}{r^3}=k\frac{q_1q_2d_2}{(d_1^2+d_2^2)^{3/2}}\\&=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4\cdot1.60\times10^{-19}\text{ C})(6\cdot1.60\times10^{-19}\text{ C})(6.00\times10^{-3}\text{ m})}{\left[(2.00\times10^{-3}\text{ m})^2+(6.00\times10^{-3}\text{ m})^2\right]^{3/2}}\\&=1.31\times10^{-22}\text{ N}\end{aligned}$$

**LEARN** In a similar manner, we find the *y* component of $\vec{F}_{21}$ to be

$$\begin{aligned}F_{21,y}&=-k\frac{q_1q_2d_1}{r^3}=-k\frac{q_1q_2d_1}{(d_1^2+d_2^2)^{3/2}}\\&=-\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4\cdot1.60\times10^{-19}\text{ C})(6\cdot1.60\times10^{-19}\text{ C})(2.00\times10^{-3}\text{ m})}{\left[(2.00\times10^{-3}\text{ m})^2+(6.00\times10^{-3}\text{ m})^2\right]^{3/2}}\\&=-0.437\times10^{-22}\text{ N}.\end{aligned}$$

Thus, $\vec{F}_{21}=(1.31\times10^{-22}\text{ N})\hat{\text{i}}-(0.437\times10^{-22}\text{ N})\hat{\text{j}}$.

40. Regarding the forces on $q_3$ exerted by $q_1$ and $q_2$, one must “push” and the other must “pull” in order that the net force is zero; hence, $q_1$ and $q_2$ have opposite signs. For individual forces to cancel, their magnitudes must be equal:

$$k\frac{|q_1||q_3|}{(L_{12}+L_{23})^2} = k\frac{|q_2||q_3|}{(L_{23})^2}.$$

With $L_{23} = 2.00L_{12}$, the above expression simplifies to $\frac{|q_1|}{9} = \frac{|q_2|}{4}$. Therefore, $q_1 = -9q_2/4$, or $q_1/q_2 = -2.25$.

41. (a) The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = G\frac{mM}{r^2}$$

where $q$ is the charge on either body, $r$ is the center-to-center separation of Earth and Moon, $G$ is the universal gravitational constant, $M$ is the mass of Earth, and $m$ is the mass of the Moon. We solve for $q$:

$$q = \sqrt{4\pi\varepsilon_0 GmM}.$$

According to Appendix C of the text, $M = 5.98 \times 10^{24}$ kg, and $m = 7.36 \times 10^{22}$ kg, so (using $4\pi\varepsilon_0 = 1/k$) the charge is

$$q = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\ \text{kg})(5.98\times10^{24}\ \text{kg})}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 5.7\times10^{13}\ \text{C}.$$

(b) The distance $r$ cancels because both the electric and gravitational forces are proportional to $1/r^2$.

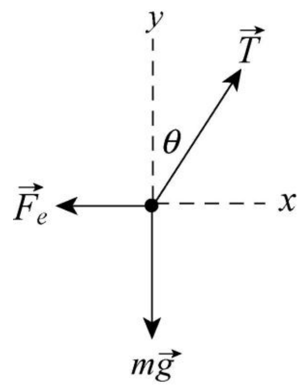
(c) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19}$ C, so there must be

$$n = \frac{q}{e} = \frac{5.7\times10^{13}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 3.6\times10^{32}\ \text{ions}.$$

Each ion has a mass of $m_i = 1.67 \times 10^{-27}$ kg, so the total mass needed is

$$m = nm_i = (3.6\times10^{32})(1.67\times10^{-27}\ \text{kg}) = 6.0\times10^{5}\ \text{kg}.$$

42. (a) A force diagram for one of the balls is shown below. The force of gravity $m\vec{g}$ acts downward, the electrical force $\vec{F}_e$ of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle $\theta$ to the vertical. The ball is in equilibrium, so its acceleration is zero. The $y$ component of Newton's second law yields $T\cos\theta - mg = 0$ and the $x$ component yields $T\sin\theta - F_e = 0$. We solve the first equation for $T$ and obtain $T = mg/\cos\theta$. We substitute the result into the second to obtain $mg\tan\theta - F_e = 0$.




Examination of the geometry of the figure shown leads to $\tan\theta = \dfrac{x/2}{\sqrt{L^2 - (x/2)^2}}$.

If $L$ is much larger than $x$ (which is the case if $\theta$ is very small), we may neglect $x/2$ in the denominator and write $\tan\theta \approx x/2L$. This is equivalent to approximating $\tan\theta$ by $\sin\theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$$

by Eq. 21-4. When these two expressions are used in the equation $mg\tan\theta = F_e$, we obtain

$$\frac{mgx}{2L} \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \Rightarrow x \approx \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}.$$

(b) We solve $x^3 = 2kq^2L/mg$ for the charge (using Eq. 21-5):

$$q = \sqrt{\frac{mgx^3}{2kL}} = \sqrt{\frac{(0.010\,\text{kg})(9.8\,\text{m/s}^2)(0.050\,\text{m})^3}{2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.20\,\text{m})}} = \pm 2.4\times10^{-8}\,\text{C}.$$

Thus, the magnitude is $|q| = 2.4\times10^{-8}\,\text{C}$.

43. (a) If one of them is discharged, there would no electrostatic repulsion between the two balls and they would both come to the position $\theta = 0$, making contact with each other.

(b) A redistribution of the remaining charge would then occur, with each of the balls getting $q/2$. Then they would again be separated due to electrostatic repulsion, which results in the new equilibrium separation

$$x' = \left[\frac{(q/2)^2 L}{2\pi\varepsilon_0 mg}\right]^{1/3} = \left(\frac{1}{4}\right)^{1/3} x = \left(\frac{1}{4}\right)^{1/3} (5.0 \text{ cm}) = 3.1 \text{ cm}.$$

44. **THINK** The problem compares the electrostatic force between two protons and the gravitational force by Earth on a proton.

**EXPRESS** The magnitude of the gravitational force on a proton near the surface of the Earth is $F_g = mg$, where $m = 1.67\times10^{-27}$ kg is the mass of the proton. On the other hand, the electrostatic force between two protons separated by a distance $r$ is $F_e = kq^2/r$. When the two forces are equal, we have $kq^2/r^2 = mg$.

**ANALYZE** Solving for $r$, we obtain

$$r = q\sqrt{\frac{k}{mg}} = (1.60\times10^{-19}\text{ C})\sqrt{\frac{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{(1.67\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)}} = 0.119 \text{ m}.$$

**LEARN** The electrostatic force at this distance is $F_e = F_g = 1.64\times10^{-26}$ N.

45. There are two protons (each with charge $q = +e$) in each molecule, so

$$Q = N_A q = (6.02\times10^{23})(2)(1.60\times10^{-19}\text{ C}) = 1.9\times10^5 \text{ C} = 0.19 \text{ MC}.$$

46. Let $\vec{F}_{12}$ denotes the force <u>on</u> $q_1$ exerted <u>by</u> $q_2$ and $F_{12}$ be its magnitude.

(a) We consider the net force on $q_1$. $\vec{F}_{12}$ points in the $+x$ direction since $q_1$ is attracted to $q_2$. $\vec{F}_{13}$ and $\vec{F}_{14}$ both point in the $-x$ direction since $q_1$ is repelled by $q_3$ and $q_4$. Thus, using $d = 0.0200$ m, the net force is

$$F_1 = F_{12} - F_{13} - F_{14} = \frac{2e|-e|}{4\pi\varepsilon_0 d^2} - \frac{(2e)(e)}{4\pi\varepsilon_0 (2d)^2} - \frac{(2e)(4e)}{4\pi\varepsilon_0 (3d)^2} = \frac{11}{18}\frac{e^2}{4\pi\varepsilon_0 d^2}$$
$$= \frac{11}{18}\frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(2.00\times10^{-2}\text{ m})^2} = 3.52\times10^{-25} \text{ N}$$

or $\vec{F}_1 = (3.52\times10^{-25} \text{ N})\hat{\text{i}}$.

(b) We now consider the net force on $q_2$. We note that $\vec{F}_{21} = -\vec{F}_{12}$ points in the –$x$ direction, and $\vec{F}_{23}$ and $\vec{F}_{24}$ both point in the +$x$ direction. The net force is

$$F_{23} + F_{24} - F_{21} = \frac{4e|-e|}{4\pi\varepsilon_0 (2d)^2} + \frac{e|-e|}{4\pi\varepsilon_0 d^2} - \frac{2e|-e|}{4\pi\varepsilon_0 d^2} = 0 .$$

47. We are looking for a charge $q$ that, when placed at the origin, experiences $\vec{F}_{\text{net}} = 0$, where

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .$$

The magnitude of these individual forces are given by Coulomb's law, Eq. 21-1, and without loss of generality we assume $q > 0$. The charges $q_1$ (+6 $\mu$C), $q_2$ (–4 $\mu$C), and $q_3$ (unknown), are located on the +$x$ axis, so that we know $\vec{F}_1$ points toward –$x$, $\vec{F}_2$ points toward +$x$, and $\vec{F}_3$ points toward –$x$ if $q_3 > 0$ and points toward +$x$ if $q_3 < 0$. Therefore, with $r_1$ = 8 m, $r_2$ = 16 m and $r_3$ = 24 m, we have

$$0 = -k\frac{q_1 q}{r_1^2} + k\frac{|q_2| q}{r_2^2} - k\frac{q_3 q}{r_3^2} .$$

Simplifying, this becomes

$$0 = -\frac{6}{8^2} + \frac{4}{16^2} - \frac{q_3}{24^2}$$

where $q_3$ is now understood to be in $\mu$C. Thus, we obtain $q_3$ = –45 $\mu$C.

48. (a) Since $q_A$ = –2.00 nC and $q_C$ = +8.00 nC, Eq. 21-4 leads to

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-2.00\times10^{-9}\ \text{C})(8.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 3.60\times10^{-6}\ \text{N}.$$

(b) After making contact with each other, both $A$ and $B$ have a charge of

$$\frac{q_A + q_B}{2} = \left(\frac{-2.00 + (-4.00)}{2}\right)\ \text{nC} = -3.00\ \text{nC}.$$

When $B$ is grounded its charge is zero. After making contact with $C$, which has a charge of +8.00 nC, $B$ acquires a charge of [0 + (–8.00 nC)]/2 = –4.00 nC, which charge $C$ has as well. Finally, we have $Q_A$ = –3.00 nC and $Q_B = Q_C$ = –4.00 nC. Therefore,

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-3.00\times10^{-9}\ \text{C})(-4.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 2.70\times10^{-6}\ \text{N}.$$

(c) We also obtain

$$|\vec{F}_{BC}| = \frac{|q_B q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00\times10^{-9}\ \text{C})(-4.00\times10^{-9}\ \text{C})|}{(0.200\ \text{m})^2} = 3.60\times10^{-6}\ \text{N}.$$

49. Coulomb's law gives

$$F = \frac{|q|^2}{4\pi\varepsilon_0 r^2} = \frac{k(e/3)^2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(1.60\times10^{-19}\ \text{C})^2}{9(2.6\times10^{-15}\,\text{m})^2} = 3.8\,\text{N}.$$

50. (a) Since the rod is in equilibrium, the net force acting on it is zero, and the net torque about any point is also zero. We write an expression for the net torque about the bearing, equate it to zero, and solve for $x$. The charge $Q$ on the left exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\ (qQ/h^2)$, at a distance $L/2$ from the bearing. We take the torque to be negative. The attached weight exerts a downward force of magnitude $W$, at a distance $x - L/2$ from the bearing. This torque is also negative. The charge $Q$ on the right exerts an upward force of magnitude $(1/4\pi\varepsilon_0)\ (2qQ/h^2)$, at a distance $L/2$ from the bearing. This torque is positive. The equation for rotational equilibrium is

$$\frac{-1}{4\pi\varepsilon_0}\frac{qQ}{h^2}\frac{L}{2} - W\left(x - \frac{L}{2}\right) + \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}\frac{L}{2} = 0.$$

The solution for $x$ is

$$x = \frac{L}{2}\left(1 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2 W}\right).$$

(b) If $F_N$ is the magnitude of the upward force exerted by the bearing, then Newton's second law (with zero acceleration) gives

$$W - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2} - \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2} - F_N = 0.$$

We solve for $h$ so that $F_N = 0$. The result is

$$h = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{3qQ}{W}}.$$

51. The charge $dq$ within a thin section of the rod (of thickness $dx$) is $\rho A dx$ where $A = 4.00\times10^{-4}\,\text{m}^2$ and $\rho$ is the charge per unit volume. The number of (excess) electrons in the rod (of length $L$ = 2.00 m) is $n = q/(-e)$ where $e$ is given in Eq. 21-12.

(a) In the case where $\rho = -4.00 \times 10^{-6}$ C/m$^3$, we have

$$n = \frac{q}{-e} = \frac{\rho A}{-e}\int_0^L dx = \frac{|\rho| AL}{e} = \; 2.00\times 10^{10} \; .$$

(b) With $\rho = bx^2$ ($b$ = –2.00 × 10$^{-6}$ C/m$^5$) we obtain

$$n = \frac{b\,A}{-e}\int_0^L x^2\,dx = \frac{|b| AL^3}{3e} = 1.33\times 10^{10}.$$

52. For the Coulomb force to be sufficient for circular motion at that distance (where $r$ = 0.200 m and the acceleration needed for circular motion is $a = v^2/r$) the following equality is required:

$$\frac{Qq}{4\pi\varepsilon_0 r^2} = -\frac{mv^2}{r} \; .$$

With $q$ = 4.00 × 10$^{-6}$ C, $m$ = 0.000800 kg, $v$ = 50.0 m/s, this leads to

$$Q = -\frac{4\pi\varepsilon_0 rmv^2}{q} = -\frac{(0.200\text{ m})(8.00\times 10^{-4}\text{ kg})(50.0\text{ m/s})^2}{(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(4.00\times 10^{-6}\text{C})} = -1.11\times 10^{-5}\text{C} \; .$$

53. (a) Using Coulomb's law, we obtain

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00\text{C}\right)^2}{\left(1.00\text{m}\right)^2} = 8.99\times 10^9\,\text{N}.$$

(b) If $r$ = 1000 m, then

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00\text{C}\right)^2}{\left(1.00\times 10^3\text{m}\right)^2} = 8.99\times 10^3\,\text{N}.$$

54. Let $q_1$ be the charge of one part and $q_2$ that of the other part; thus, $q_1 + q_2 = Q$ = 6.0 μC. The repulsive force between them is given by Coulomb's law:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{q_1(Q - q_1)}{4\pi\varepsilon_0 r^2} \; .$$

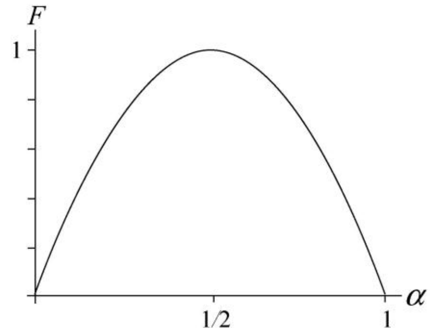
If we maximize this expression by taking the derivative with respect to $q_1$ and setting equal to zero, we find $q_1 = Q/2$ , which might have been anticipated (based on symmetry arguments). This implies $q_2 = Q/2$ also. With $r$ = 0.0030 m and $Q$ = 6.0 × 10$^{-6}$ C, we find

$$F = \frac{(Q/2)(Q/2)}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{Q^2}{4\pi\varepsilon_0 r^2} = \frac{1}{4}\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\,\text{C}\right)^2}{\left(3.00\times10^{-3}\,\text{m}\right)^2} \approx 9.0\times10^3\ \text{N}.$$

55. The two charges are $q = \alpha Q$ (where $\alpha$ is a pure number presumably less than 1 and greater than zero) and $Q - q = (1 - \alpha)Q$. Thus, Eq. 21-4 gives

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(\alpha Q)\left((1-\alpha)Q\right)}{d^2} = \frac{Q^2\alpha(1-\alpha)}{4\pi\varepsilon_0 d^2}.$$

The graph below, of $F$ versus $\alpha$, has been scaled so that the maximum is 1. In actuality, the maximum value of the force is $F_{max} = Q^2/16\pi\varepsilon_0 d^2$.




(a) It is clear that $\alpha = 1/2 = 0.5$ gives the maximum value of $F$.

(b) Seeking the half-height points on the graph is difficult without grid lines or some of the special tracing features found in a variety of modern calculators. It is not difficult to algebraically solve for the half-height points (this involves the use of the quadratic formula). The results are

$$\alpha_1 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right) \approx 0.15 \quad \text{and} \quad \alpha_2 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) \approx 0.85.$$

Thus, the smaller value of $\alpha$ is $\alpha_1 = 0.15$,

(c) and the larger value of $\alpha$ is $\alpha_2 = 0.85$.

56. (a) Equation 21-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{2.00\times10^{-6}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 1.25\times10^{13}\ \text{electrons}.$$

(b) Since you have the excess electrons (and electrons are lighter and more mobile than protons) then the electrons "leap" from you to the faucet instead of protons moving from the faucet to you (in the process of neutralizing your body).

(c) Unlike charges attract, and the faucet (which is grounded and is able to gain or lose any number of electrons due to its contact with Earth's large reservoir of mobile charges) becomes positively charged, especially in the region closest to your (negatively charged) hand, just before the spark.

(d) The cat is positively charged (before the spark), and by the reasoning given in part (b) the flow of charge (electrons) is from the faucet to the cat.

(e) If we think of the nose as a conducting sphere, then the side of the sphere closest to the fur is of one sign (of charge) and the side furthest from the fur is of the opposite sign (which, additionally, is oppositely charged from your bare hand, which had stroked the cat's fur). The charges in your hand and those of the furthest side of the "sphere" therefore attract each other, and when close enough, manage to neutralize (due to the "jump" made by the electrons) in a painful spark.

57. If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be $q_p - |q_e| = 1.6 \times 10^{-25}$ C. Amplified by a factor of 29 × 3 × $10^{22}$ as indicated in the problem, this amounts to a deviation from perfect neutrality of

$$\Delta q = \left(29 \times 3 \times 10^{22}\right)\left(1.6 \times 10^{-25}\ \text{C}\right) = 0.14\ \text{C}$$

in a copper penny. Two such pennies, at $r$ = 1.0 m, would therefore experience a very large force. Equation 21-1 gives

$$F = k\frac{(\Delta q)^2}{r^2} = 1.7 \times 10^8\ \text{N}.$$

58. Charge $q_1 = -80 \times 10^{-6}$ C is at the origin, and charge $q_2 = +40 \times 10^{-6}$ C is at $x$ = 0.20 m. The force on $q_3 = +20 \times 10^{-6}$ C is due to the attractive and repulsive forces from $q_1$ and $q_2$, respectively. In symbols, $\vec{F}_{3\,\text{net}} = \vec{F}_{31} + \vec{F}_{32}$, where

$$\left|\vec{F}_{31}\right| = k\frac{q_3\,|\,q_1\,|}{r_{31}^2}, \quad |\vec{F}_{32}\,| = k\frac{q_3 q_2}{r_{32}^2}.$$

(a) In this case $r_{31}$ = 0.40 m and $r_{32}$ = 0.20 m, with $\vec{F}_{31}$ directed toward $-x$ and $\vec{F}_{32}$ directed in the $+x$ direction. Using the value of $k$ in Eq. 21-5, we obtain

$$\vec{F}_{3\,\text{net}} = -\left|\vec{F}_{31}\right|\hat{\text{i}} + |\vec{F}_{32}|\,\hat{\text{i}} = \left(-k\frac{q_3|q_1|}{r_{31}^2} + k\frac{q_3 q_2}{r_{32}^2}\right)\hat{\text{i}} = kq_3\left(-\frac{|q_1|}{r_{31}^2} + \frac{q_2}{r_{32}^2}\right)\hat{\text{i}}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(20\times10^{-6}\,\text{C})\left(\frac{-80\times10^{-6}\,\text{C}}{(0.40\text{m})^2} + \frac{+40\times10^{-6}\,\text{C}}{(0.20\text{m})^2}\right)\hat{\text{i}}$$

$$= (89.9\ \text{N})\hat{\text{i}}\ .$$

(b) In this case $r_{31}$ = 0.80 m and $r_{32}$ = 0.60 m, with $\vec{F}_{31}$ directed toward $-x$ and $\vec{F}_{32}$ toward $+x$. Now we obtain

$$\vec{F}_{3\,\text{net}} = -\left|\vec{F}_{31}\right|\hat{\text{i}} + |\vec{F}_{32}|\,\hat{\text{i}} = \left(-k\frac{q_3|q_1|}{r_{31}^2} + k\frac{q_3 q_2}{r_{32}^2}\right)\hat{\text{i}} = kq_3\left(-\frac{|q_1|}{r_{31}^2} + \frac{q_2}{r_{32}^2}\right)\hat{\text{i}}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(20\times10^{-6}\,\text{C})\left(\frac{-80\times10^{-6}\,\text{C}}{(0.80\text{m})^2} + \frac{+40\times10^{-6}\,\text{C}}{(0.60\text{m})^2}\right)\hat{\text{i}}$$

$$= -(2.50\ \text{N})\hat{\text{i}}\ .$$

(c) Between the locations treated in parts (a) and (b), there must be one where $\vec{F}_{3\,\text{net}} = 0$. Writing $r_{31}$ = $x$ and $r_{32}$ = $x$ – 0.20 m, we equate $\left|\vec{F}_{31}\right|$ and $\left|\vec{F}_{32}\right|$, and after canceling common factors, arrive at

$$\frac{|q_1|}{x^2} = \frac{q_2}{\left(x - 0.20\ \text{m}\right)^2}.$$

This can be further simplified to

$$\frac{(x-0.20\ \text{m})^2}{x^2} = \frac{q_2}{|q_1|} = \frac{1}{2}.$$

Taking the (positive) square root and solving, we obtain $x$ = 0.683 m. If one takes the negative root and 'solves', one finds the location where the net force *would* be zero *if* $q_1$ and $q_2$ were of like sign (which is not the case here).

(d) From the above, we see that $y$ = 0.

59. The mass of an electron is $m = 9.11 \times 10^{-31}$ kg, so the number of electrons in a collection with total mass $M$ = 75.0 kg is

$$n = \frac{M}{m} = \frac{75.0\,\text{kg}}{9.11\times10^{-31}\,\text{kg}} = 8.23\times10^{31}\ \text{electrons}.$$

The total charge of the collection is

$$q = -\,ne = -\left(8.23\times10^{31}\right)\left(1.60\times10^{-19}\text{ C}\right) = -1.32\times10^{13}\text{ C}.$$

60. We note that, as result of the fact that the Coulomb force is inversely proportional to $r^2$, a particle of charge $Q$ that is distance $d$ from the origin will exert a force on some charge $q_o$ at the origin of equal strength as a particle of charge $4Q$ at distance $2d$ would exert on $q_o$. Therefore, $q_6 = +8e$ on the $-y$ axis could be replaced with a $+2e$ closer to the origin (at half the distance); this would add to the $q_5 = +2e$ already there and produce $+4e$ below the origin, which exactly cancels the force due to $q_2 = +4e$ above the origin.

Similarly, $q_4 = +4e$ to the far right could be replaced by a $+e$ at half the distance, which would add to $q_3 = +e$ already there to produce a $+2e$ at distance $d$ to the right of the central charge $q_7$. The horizontal force due to this $+2e$ is cancelled exactly by that of $q_1 = +2e$ on the $-x$ axis, so that the net force on $q_7$ is zero.

61. (a) Charge $Q_1 = +80 \times 10^{-9}$ C is on the $y$ axis at $y = 0.003$ m, and charge $Q_2 = +80\times10^{-9}$ C is on the $y$ axis at $y = -0.003$ m. The force on particle 3 (which has a charge of $q = +18 \times 10^{-9}$ C) is due to the vector sum of the repulsive forces from $Q_1$ and $Q_2$. In symbols, $\vec{F}_{31} + \vec{F}_{32} = \vec{F}_3$, where

$$|\vec{F}_{31}| = k\frac{q_3|q_1|}{r_{31}^2}\,,\quad |\vec{F}_{32}| = k\frac{q_3 q_2}{r_{32}^2}.$$

Using the Pythagorean theorem, we have $r_{31} = r_{32} = 0.005$ m. In magnitude-angle notation (particularly convenient if one uses a vector-capable calculator in polar mode), the indicated vector addition becomes

$$\vec{F}_3 = \left(0.518\angle -37°\right) + \left(0.518\angle 37°\right) = \left(0.829\angle 0°\right).$$

Therefore, the net force is $\vec{F}_3 = (0.829\text{ N})\hat{\text{i}}$ .

(b) Switching the sign of $Q_2$ amounts to reversing the direction of its force on $q$. Consequently, we have

$$\vec{F}_3 = \left(0.518\angle -37°\right) + \left(0.518\angle -143°\right) = \left(0.621\angle -90°\right).$$

Therefore, the net force is $\vec{F}_3 = -(0.621\text{ N})\hat{\text{j}}$.

62. **THINK** We have four discrete charges in the $xy$-plane. We use superposition principle to calculate the net electrostatic force on particle 4 due to the other three particles.

**EXPRESS** Using Coulomb's law, the magnitude of the force on particle 4 by particle *i* is $F_{4i} = k\dfrac{q_4 q_i}{r_{4i}^2}$. For example, the magnitude of $\vec{F}_{41}$ is

$$F_{41} = k\frac{|q_4||q_1|}{r_{41}^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\text{ C})(3.20\times10^{-19}\text{ C})}{(0.0300\text{ m})^2} = 1.02\times10^{-24}\text{ N}.$$

Since the force is attractive, $\hat{r}_{41} = -\cos\theta_1\hat{\text{i}} - \sin\theta_1\hat{\text{j}} = -\cos35°\hat{\text{i}} - \sin35°\hat{\text{j}} = -0.82\hat{\text{i}} - 0.57\hat{\text{j}}$. In unit-vector notation, we have

$$\vec{F}_{41} = F_{41}\hat{r}_{41} = (1.02\times10^{-24}\text{ N})(-0.82\hat{\text{i}} - 0.57\hat{\text{j}}) = -(8.36\times10^{-25}\text{ N})\hat{\text{i}} - (5.85\times10^{-24}\text{ N})\hat{\text{j}}.$$

Similarly,

$$\vec{F}_{42} = -k\frac{|q_4||q_2|}{r_{42}^2}\hat{\text{j}} = -\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\text{ C})(3.20\times10^{-19}\text{ C})}{(0.0200\text{ m})^2}\hat{\text{j}} = -(2.30\times10^{-24}\text{ N})\hat{\text{j}}$$

and

$$\vec{F}_{43} = -k\frac{|q_4||q_3|}{r_{43}^2}\hat{\text{i}} = -\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(6.40\times10^{-19}\text{ C})(3.20\times10^{-19}\text{ C})}{(0.0200\text{ m})^2}\hat{\text{i}} = -(4.60\times10^{-24}\text{ N})\hat{\text{j}}.$$

**ANALYZE** (a) The net force on particle 4 is

$$\vec{F}_{4,\text{net}} = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43} = -(5.44\times10^{-24}\text{ N})\hat{\text{i}} - (2.89\times10^{-24}\text{ N})\hat{\text{j}}.$$

The magnitude of the force is

$$F_{4,\text{net}} = \sqrt{(-5.44\times10^{-24}\text{ N})^2 + (-2.89\times10^{-24}\text{ N})^2} = 6.16\times10^{-24}\text{ N}.$$

(b) The direction of the net force is at an angle of

$$\varphi = \tan^{-1}\left(\frac{F_{4y,\text{net}}}{F_{4x,\text{net}}}\right) = \tan^{-1}\left(\frac{-2.89\times10^{-24}\text{ N}}{-5.44\times10^{-24}\text{ N}}\right) = 208°,$$

measured counterclockwise from the +*x* axis.

**LEARN** A nonzero net force indicates that particle 4 will be accelerated in the direction of the force.

63. The magnitude of the net force on the $q = 42 \times 10^{-6}$ C charge is

$$k\frac{q_1 q}{0.28^2} + k\frac{|q_2| q}{0.44^2}$$

where $q_1 = 30 \times 10^{-9}$ C and $|q_2| = 40 \times 10^{-9}$ C. This yields 0.22 N. Using Newton's second law, we obtain

$$m = \frac{F}{a} = \frac{0.22\,\text{N}}{100 \times 10^3\,\text{m/s}^2} = 2.2 \times 10^{-6}\,\text{kg}.$$

64. Let the two charges be $q_1$ and $q_2$. Then $q_1 + q_2 = Q = 5.0 \times 10^{-5}$ C. We use Eq. 21-1:

$$1.0\ \text{N} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) q_1 q_2}{\left(2.0\ \text{m}\right)^2}.$$

We substitute $q_2 = Q - q_1$ and solve for $q_1$ using the quadratic formula. The two roots obtained are the values of $q_1$ and $q_2$, since it does not matter which is which. We get $1.2 \times 10^{-5}$ C and $3.8 \times 10^{-5}$ C. Thus, the charge on the sphere with the smaller charge is $1.2 \times 10^{-5}$ C.

65. When sphere $C$ touches sphere $A$, they divide up their total charge ($Q/2$ plus $Q$) equally between them. Thus, sphere $A$ now has charge $3Q/4$, and the magnitude of the force of attraction between $A$ and $B$ becomes

$$F = k\frac{(3Q/4)(Q/4)}{d^2} = 4.68 \times 10^{-19}\,\text{N}.$$

66. With $F = m_e g$, Eq. 21-1 leads to

$$y^2 = \frac{ke^2}{m_e g} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\,\text{C}\right)^2}{\left(9.11 \times 10^{-31}\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right)}$$

which leads to $y = \pm 5.1$ m. We choose $y = -5.1$ m since the second electron must be below the first one, so that the repulsive force (acting on the first) is in the direction opposite to the pull of Earth's gravity.

67. **THINK** Our system consists of two charges along a straight line. We'd like to place a third charge so that the net force on it due to charges 1 and 2 vanishes.

**EXPRESS** The net force on particle 3 is the vector sum of the forces due to particles 1 and 2: $\vec{F}_{3,\text{net}} = \vec{F}_{31} + \vec{F}_{32}$. In order that $\vec{F}_{3,\text{net}} = 0$, particle 3 must be on the $x$ axis and be attracted by one and repelled by another. As the result, it cannot be between particles 1 and 2, but instead either to the left of particle 1 or to the right of particle 2. Let $q_3$ be placed a distance $x$ to the right of $q_1 = -5.00q$. Then its attraction to $q_1$ particle will be exactly balanced by its repulsion from $q_2 = +2.00q$:

$$F_{3x,\text{net}} = k\left[\frac{q_1 q_3}{x^2} + \frac{q_2 q_3}{(x-L)^2}\right] = kq_3 q\left[\frac{-5}{x^2} + \frac{2}{(x-L)^2}\right] = 0\,.$$

**ANALYZE** (a) Cross-multiplying and taking the square root, we obtain

$$\frac{x}{x-L} = \sqrt{\frac{5}{2}}$$

which can be rearranged to produce

$$x = \frac{L}{1-\sqrt{2/5}} \approx 2.72\,L\,.$$

(b) The $y$ coordinate of particle 3 is $y = 0$.

**LEARN** We can use the result obtained above for consistency check. We find the force on particle 3 due to particle 1 to be

$$F_{31} = k\frac{q_1 q_3}{x^2} = k\frac{(-5.00q)(q_3)}{(2.72L)^2} = -0.675\frac{kqq_3}{L^2}\,.$$

Similarly, the force on particle 3 due to particle 2 is

$$F_{32} = k\frac{q_2 q_3}{x^2} = k\frac{(+2.00q)(q_3)}{(2.72L-L)^2} = +0.675\frac{kqq_3}{L^2}\,.$$

Indeed, the sum of the two forces is zero.

68. The net charge carried by John whose mass is $m$ is roughly

$$\begin{aligned} q &= (0.0001)\frac{mN_A Ze}{M} \\ &= (0.0001)\frac{(90\,\text{kg})(6.02\times10^{23}\text{ molecules/mol})(18\text{ electron proton pairs/molecule})(1.6\times10^{-19}\text{ C})}{0.018\,\text{kg/mol}} \\ &= 8.7\times10^{5}\,\text{C}, \end{aligned}$$

and the net charge carried by Mary is half of that. So the electrostatic force between them is estimated to be

$$F \approx k\frac{q(q/2)}{d^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(8.7\times10^5\ \text{C})^2}{2(30\text{m})^2} \approx 4\times10^{18}\ \text{N}.$$

Thus, the order of magnitude of the electrostatic force is $10^{18}$ N .

69. We are concerned with the charges in the nucleus (not the "orbiting" electrons, if there are any). The nucleus of Helium has 2 protons and that of thorium has 90.

(a) Equation 21-1 gives

$$F = k\frac{q^2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2(1.60\times10^{-19}\ \text{C}))(90(1.60\times10^{-19}\ \text{C}))}{(9.0\times10^{-15}\ \text{m})^2} = 5.1\times10^2\ \text{N}.$$

(b) Estimating the helium nucleus mass as that of 4 protons (actually, that of 2 protons and 2 neutrons, but the neutrons have approximately the same mass), Newton's second law leads to

$$a = \frac{F}{m} = \frac{5.1\times10^2\ \text{N}}{4\left(1.67\times10^{-27}\ \text{kg}\right)} = 7.7\times10^{28}\ \text{m/s}^2 .$$

70. For the net force on $q_1 = +Q$ to vanish, the $x$ force component due to $q_2 = q$ must exactly cancel the force of attraction caused by $q_4 = -2Q$. Consequently,

$$\frac{Qq}{4\pi\varepsilon_0 a^2} = \frac{Q|2Q|}{4\pi\varepsilon_0(\sqrt{2}a)^2}\cos 45° = \frac{Q^2}{4\pi\varepsilon_0\sqrt{2}a^2}$$

or $q = Q/\sqrt{2}$ . This implies that $q/Q = 1/\sqrt{2} = 0.707$.

71. (a) The second shell theorem states that a charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell. Thus, inside the spherical metal shell at $r = 0.500R < R,$ the net force on the electron is zero, and therefore, $a = 0.$

(b) The first shell theorem states that a charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center. Thus, the magnitude of the Coulomb force on the electron at $r = 2.00R$ is

$$\begin{aligned} F &= k\frac{Q|e|}{r^2} = k\frac{(4\pi R^2\sigma)|e|}{(2.0R)^2} = k\pi\sigma|e| \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\pi(6.90\times10^{-13}\ \text{C/m}^2)(1.60\times10^{-19}\ \text{C}) \\ &= 3.12\times10^{-21}\ \text{N}, \end{aligned}$$

and the corresponding acceleration is

$$a = \frac{F}{m} = \frac{3.12\times10^{-21}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = 3.43\times10^{9}\ \text{m/s}^2 .$$

72. Since the total energy is conserved,

$$\frac{1}{2}m_e v_i^2 = \frac{1}{2}m_e v_f^2 - \frac{ke^2}{r_f}$$

where $r_f$ is the distance between the electron and the proton. For $v_f = 2v_i$, we solve for $r_f$ and obtain

$$r_f = \frac{2ke^2}{m_e(v_f^2 - v_i^2)} = \frac{2ke^2}{3m_e v_i^2} = \frac{2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{3(9.11\times10^{-31}\ \text{kg})(3.2\times10^{5}\,\text{m/s})^2}$$
$$= 1.64\times10^{-9}\ \text{m}$$

or about 1.6 nm.

73. (a) The Coulomb force between the electron and the proton provides the centripetal force that keeps the electron in circular orbit about the proton:

$$\frac{k|e|^2}{r^2} = \frac{m_e v^2}{r}$$

The smallest orbital radius is $r_1 = a_0 = 52.9\times10^{-12}$m. The corresponding speed of the electron is

$$v_1 = \sqrt{\frac{k|e|^2}{m_e r_1}} = \sqrt{\frac{k|e|^2}{m_e a_0}} = \sqrt{\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(52.9\times10^{-12}\,\text{m})}}$$
$$= 2.19\times10^{6}\ \text{m/s}.$$

(b) The radius of the second smallest orbit is $r_2 = (2)^2 a_0 = 4a_0$. Thus, the speed of the electron is

$$v_2 = \sqrt{\frac{k|e|^2}{m_e r_2}} = \sqrt{\frac{k|e|^2}{m_e(4a_0)}} = \frac{1}{2}v_1 = \frac{1}{2}(2.19\times10^{6}\ \text{m/s})$$
$$= 1.09\times10^{6}\ \text{m/s}.$$

(c) Since the speed is inversely proportional to $r^{1/2}$, the speed of the electron will decrease if it moves to larger orbits.

74. Electric current $i$ is the rate $dq/dt$ at which charge passes a point. With $i$ = 0.83A, the time it takes for one mole of electron to pass through the lamp is

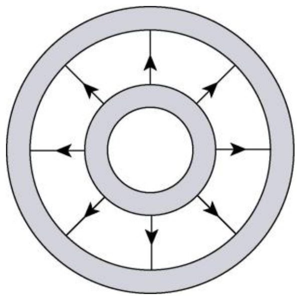
$$\Delta t = \frac{\Delta q}{i} = \frac{N_A e}{i} = \frac{(6.02\times10^{23})(1.6\times10^{-19}\text{ C})}{0.83\text{ A}} = 1.16\times10^{5}\text{ s} \approx 1.3\text{ days.}$$

75. The electrical force between an electron and a positron separated by a distance $r$ is $F_e = ke^2/r^2$. On the other hand, the gravitational force between the two charges is $F_g = Gm_e^2/r^2$. Thus, the ratio of the two forces is

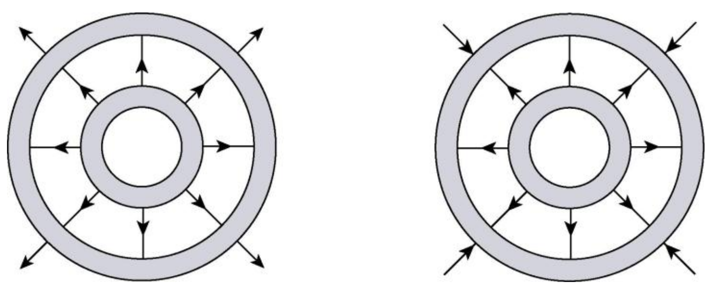
$$\frac{F_e}{F_g} = \frac{ke^2/r^2}{Gm_e^2/r^2} = \frac{ke^2}{Gm_e^2} = \frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})^2}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.11\times10^{-31}\text{ kg})^2} = 4.16\times10^{42}.$$

# Chapter 22

1. We note that the symbol $q_2$ is used in the problem statement to mean the absolute value of the negative charge that resides on the larger shell. The following sketch is for $q_1 = q_2$.



The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).



2. (a) We note that the electric field points leftward at both points. Using $\vec{F} = q_0\vec{E}$, and orienting our $x$ axis rightward (so $\hat{\text{i}}$ points right in the figure), we find

$$\vec{F} = \left(+1.6\times10^{-19}\,\text{C}\right)\left(-40\frac{\text{N}}{\text{C}}\hat{\text{i}}\right) = (-6.4\times10^{-18}\,\text{N})\hat{\text{i}}$$

which means the magnitude of the force on the proton is $6.4 \times 10^{-18}$ N and its direction $(-\hat{\text{i}})$ is leftward.

(b) As the discussion in Section 22-2 makes clear, the field strength is proportional to the "crowdedness" of the field lines. It is seen that the lines are twice as crowded at $A$ than at $B$, so we conclude that $E_A = 2E_B$. Thus, $E_B$ = 20 N/C.

3. **THINK** Since the nucleus is treated as a sphere with uniform surface charge distribution, the electric field at the surface is exactly the same as it would be if the charge were all at the center.

**EXPRESS** The nucleus has a radius $R$ = 6.64 fm and a total charge $q = Ze$, where $Z = 94$ for Pu. Thus, the magnitude of the electric field at the nucleus surface is

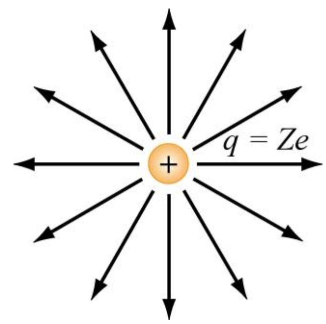
$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{Ze}{4\pi\varepsilon_0 R^2}.$$

**ANALYZE** (a) Substituting the values given, we find the field to be

$$E = \frac{Ze}{4\pi\varepsilon_0 R^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(94)(1.60\times10^{-19}\text{C})}{(6.64\times10^{-15}\text{m})^2} = 3.07\times10^{21}\ \text{N/C}.$$

(b) The field is normal to the surface. In addition, since the charge is positive, it points outward from the surface.

**LEARN** The direction of electric field lines is radially outward for a positive charge, and radially inward for a negative charge. The field lines of our nucleus are shown on the right.




4. With $x_1$ = 6.00 cm and $x_2$ = 21.00 cm, the point midway between the two charges is located at $x$ = 13.5 cm. The values of the charge are

$$q_1 = -q_2 = -2.00\times10^{-7}\ \text{C},$$

and the magnitudes and directions of the individual fields are given by:

$$\vec{E}_1 = -\frac{|q_1|}{4\pi\varepsilon_0 (x-x_1)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)|-2.00\times10^{-7}\text{C}|}{(0.135\ \text{m}-0.060\ \text{m})^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

$$\vec{E}_2 = -\frac{q_2}{4\pi\varepsilon_0 (x-x_2)^2}\hat{\text{i}} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-7}\text{C})}{(0.135\ \text{m}-0.210\ \text{m})^2}\hat{\text{i}} = -(3.196\times10^5\ \text{N/C})\hat{\text{i}}$$

Thus, the net electric field is $\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = -(6.39\times10^5\ \text{N/C})\hat{\text{i}}$.

5. **THINK** The magnitude of the electric field produced by a point charge $q$ is given by $E = |q|/4\pi\varepsilon_0 r^2$, where $r$ is the distance from the charge to the point where the field has magnitude $E$.

**EXPRESS** From $E = |q|/4\pi\varepsilon_0 r^2$, the magnitude of the charge is $|q| = 4\pi\varepsilon_0 r^2 E$.

**ANALYZE** With $E = 2.0$ N/C at $r = 50\text{ cm} = 0.50\text{ m}$, we obtain

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\text{m})^2(2.0\,\text{N/C})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 5.6\times10^{-11}\text{C}.$$

**LEARN** To determine the sign of the charge, we would need to know the direction of the field. The field lines extend away from a positive charge and toward a negative charge.

6. We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(1.00\,\text{N/C})(1.00\,\text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.11\times10^{-10}\text{C}.$$

7. **THINK** Our system consists of four point charges that are placed at the corner of a square. The total electric field at a point is the vector sum of the electric fields of individual charges.

**EXPRESS** Applying the superposition principle, the net electric field at the center of the square is

$$\vec{E} = \sum_{i=1}^{4}\vec{E}_i = \sum_{i=1}^{4}\frac{1}{4\pi\varepsilon_0}\frac{q_i}{r_i^2}\hat{r}_i.$$

With $q_1 = +10$ nC, $q_2 = -20$ nC, $q_3 = +20$ nC, and $q_4 = -10$ nC, the $x$ component of the electric field at the center of the square is given by, taking the signs of the charges into consideration,

$$\begin{aligned}E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} - \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45^\circ \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1| + |q_2| - |q_3| - |q_4|\right)\frac{1}{\sqrt{2}}.\end{aligned}$$

Similarly, the $y$ component of the electric field is

$$\begin{aligned}E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} + \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45^\circ \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1| + |q_2| + |q_3| - |q_4|\right)\frac{1}{\sqrt{2}}.\end{aligned}$$

The magnitude of the net electric field is $E = \sqrt{E_x^2 + E_y^2}$.

**ANALYZE** Substituting the values given, we obtain

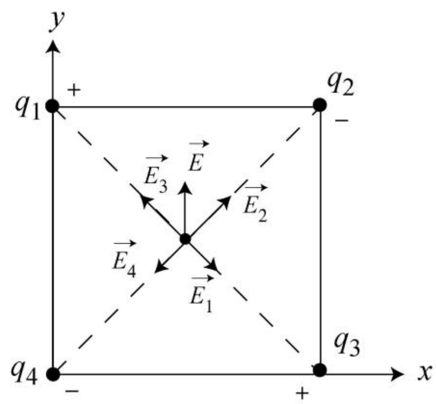
$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(10\text{ nC}+20\text{ nC}-20\text{ nC}-10\text{ nC}\right)=0$$

and

$$\begin{aligned} E_y &= \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(-10\text{ nC}+20\text{ nC}+20\text{ nC}-10\text{ nC}\right) \\ &= \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(2.0\times10^{-8}\text{ C})\sqrt{2}}{(0.050\text{ m})^2} \\ &= 1.02\times10^5\text{ N/C}. \end{aligned}$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\text{j}} = (1.02\times10^5\text{ N/C})\hat{\text{j}}$.

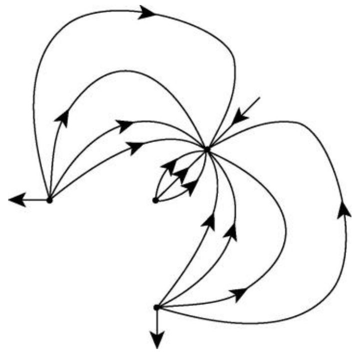
**LEARN** The net electric field at the center of the square is depicted in the figure below (not to scale). The field, pointing to the +*y* direction, is the vector sum of the electric fields of individual charges.




8. We place the origin of our coordinate system at point *P* and orient our *y* axis in the direction of the $q_4$ = –12*q* charge (passing through the $q_3$ = +3*q* charge). The *x* axis is perpendicular to the *y* axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $|\vec{E}_1|$, $|\vec{E}_2|$, $|\vec{E}_3|$, and $|\vec{E}_4|$ are figured from Eq. 22-3, where the absolute value signs for $q_1$, $q_2$, and $q_3$ are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from $q_1$ cancels that of $q_2$ (that is, $|\vec{E}_1| = |\vec{E}_2|$), and the net field (if there is any) should be along the *y* axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2}\right)\hat{\text{j}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4d^2} - \frac{3q}{d^2}\right)\hat{\text{j}}$$

which is seen to be zero. A rough sketch of the field lines is shown next:



9. (a) The vertical components of the individual fields (due to the two charges) cancel, by symmetry. Using $d$ = 3.00 m and $y$ = 4.00 m, the horizontal components (both pointing to the $-x$ direction) add to give a magnitude of

$$E_{x,\text{net}} = \frac{2|q|d}{4\pi\varepsilon_0(d^2+y^2)^{3/2}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.20\times10^{-19}\ \text{C})(3.00\ \text{m})}{[(3.00\ \text{m})^2+(4.00\ \text{m})^2]^{3/2}} .$$
$$= 1.38\times10^{-10}\ \text{N/C} .$$

(b) The net electric field points in the $-x$ direction, or 180° counterclockwise from the $+x$ axis.

10. For it to be possible for the net field to vanish at some $x > 0$, the two individual fields (caused by $q_1$ and $q_2$) must point in opposite directions for $x > 0$. Given their locations in the figure, we conclude they are therefore oppositely charged. Further, since the net field points more strongly leftward for the small positive $x$ (where it is very close to $q_2$) then we conclude that $q_2$ is the negative-valued charge. Thus, $q_1$ is a positive-valued charge. We write each charge as a multiple of some positive number $\xi$ (not determined at this point). Since the problem states the absolute value of their ratio, and we have already inferred their signs, we have $q_1 = 4\,\xi$ and $q_2 = -\xi$. Using Eq. 22-3 for the individual fields, we find

$$E_{\text{net}} = E_1 + E_2 = \frac{4\,\xi}{4\pi\varepsilon_0\,(L+x)^2} - \frac{\xi}{4\pi\varepsilon_0\,x^2}$$

for points along the positive $x$ axis. Setting $E_{\text{net}} = 0$ at $x$ = 20 cm (see graph) immediately leads to $L$ = 20 cm.

(a) If we differentiate $E_{\text{net}}$ with respect to $x$ and set equal to zero (in order to find where it is maximum), we obtain (after some simplification) that location:

$$x = \left(\frac{2}{3}\sqrt[3]{2} + \frac{1}{3}\sqrt[3]{4} + \frac{1}{3}\right)L = 1.70(20\ \text{cm}) = 34\ \text{cm}.$$

We note that the result for part (a) does not depend on the particular value of $\xi$.

(b) Now we are asked to set $\xi = 3e$, where $e = 1.60 \times 10^{-19}$ C, and evaluate $E_{\text{net}}$ at the value of $x$ (converted to meters) found in part (a). The result is $2.2 \times 10^{-8}$ N/C .

11. **THINK** Our system consists of two point charges of opposite signs fixed to the $x$ axis. Since the net electric field at a point is the vector sum of the electric fields of individual charges, there exists a location where the net field is zero.

**EXPRESS** At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge $q_2 = -4.00\, q_1$ located at $x_2$ = 70 cm has a greater magnitude than $q_1 = 2.1 \times 10^{-8}$ C located at $x_1$ = 20 cm, a point of zero field must be closer to $q_1$ than to $q_2$. It must be to the left of $q_1$.

Let $x$ be the coordinate of $P$, the point where the field vanishes. Then, the total electric field at $P$ is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_2|}{(x-x_2)^2} - \frac{|q_1|}{(x-x_1)^2}\right).$$

**ANALYZE** If the field is to vanish, then

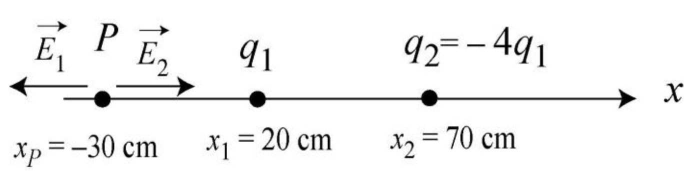
$$\frac{|q_2|}{(x-x_2)^2} = \frac{|q_1|}{(x-x_1)^2} \quad\Rightarrow\quad \frac{|q_2|}{|q_1|} = \frac{(x-x_2)^2}{(x-x_1)^2}.$$

Taking the square root of both sides, noting that $|q_2|/|q_1| = 4$, we obtain

$$\frac{x - 70\text{ cm}}{x - 20\text{ cm}} = \pm 2.0\,.$$

Choosing –2.0 for consistency, the value of $x$ is found to be $x$ = –30 cm.

**LEARN** The results are depicted in the figure below. At $P$, the field $\vec{E}_1$ due to $q_1$ points to the left, while the field $\vec{E}_2$ due to $q_2$ points to the right. Since $|\vec{E}_1| = |\vec{E}_2|$, the net field at $P$ is zero.




12. The field of each charge has magnitude

$$E = \frac{kq}{r^2} = k\frac{e}{(0.020\,\text{m})^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{1.60\times10^{-19}\,\text{C}}{(0.020\ \text{m})^2} = 3.6\times10^{-6}\ \text{N/C}.$$

The directions are indicated in standard format below. We use the magnitude-angle notation (convenient if one is using a vector-capable calculator in polar mode) and write (starting with the proton on the left and moving around clockwise) the contributions to $\vec{E}_{\text{net}}$ as follows:

$$\left(E\angle-20°\right)+\left(E\angle 130°\right)+\left(E\angle-100°\right)+\left(E\angle-150°\right)+\left(E\angle 0°\right).$$

This yields $\left(3.93\times10^{-6}\angle-76.4°\right)$, with the N/C unit understood.

(a) The result above shows that the magnitude of the net electric field is $|\vec{E}_{\text{net}}|=3.93\times10^{-6}$ N/C.

(b) Similarly, the direction of $\vec{E}_{\text{net}}$ is –76.4° from the *x*-axis.

13. (a) The electron $e_c$ is a distance $r = z = 0.020$ m away. Thus,

$$E_C=\frac{e}{4\pi\varepsilon_0 r^2}=\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})}{(0.020\ \text{m})^2}=3.60\times10^{-6}\ \text{N/C}.$$

(b) The horizontal components of the individual fields (due to the two $e_s$ charges) cancel, and the vertical components add to give

$$E_{\text{s,net}}=\frac{2ez}{4\pi\varepsilon_0(R^2+z^2)^{3/2}}=\frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})(0.020\ \text{m})}{[(0.020\ \text{m})^2+(0.020\ \text{m})^2]^{3/2}}$$
$$=2.55\times10^{-6}\ \text{N/C}.$$

(c) Calculation similar to that shown in part (a) now leads to a stronger field $E_c=3.60\times10^{-4}$ N/C from the central charge.

(d) The field due to the side charges may be obtained from calculation similar to that shown in part (b). The result is $E_{\text{s, net}} = 7.09\times10^{-7}$ N/C.

(e) Since $E_c$ is inversely proportional to $z^2$, this is a simple result of the fact that *z* is now much smaller than in part (a). For the net effect due to the side charges, it is the "trigonometric factor" for the *y* component (here expressed as $z/\sqrt{r}$ ) that shrinks almost linearly (as *z* decreases) for very small *z*, plus the fact that the *x* components cancel, which leads to the decreasing value of $E_{\text{s, net}}$ .

14. (a) The individual magnitudes $|\vec{E}_1|$ and $|\vec{E}_2|$ are figured from Eq. 22-3, where the absolute value signs for $q_2$ are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on whether $\vec{E}_1$ is in the same, or opposite,

direction as $\vec{E}_2$. At points left of $q_1$ (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $|\vec{E}_1|$ is everywhere bigger than $|\vec{E}_2|$ in this region. In the region between the charges ($0 < x < L$) both fields point leftward and there is no possibility of cancellation. At points to the right of $q_2$ (where $x > L$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left(|\vec{E}_2| - |\vec{E}_1|\right)\hat{\text{i}}\,.$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to $q_2$ than to $q_1$. Thus, we look for the zero net field point in the $x > L$ region:

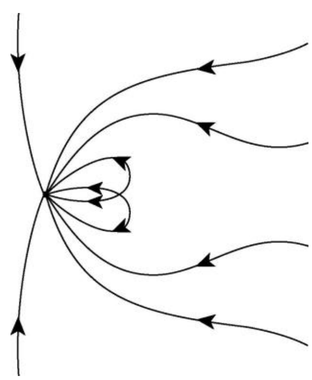
$$|\vec{E}_1| = |\vec{E}_2| \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-L)^2}$$

which leads to

$$\frac{x-L}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}}.$$

Thus, we obtain $x = \dfrac{L}{1-\sqrt{2/5}} \approx 2.72L$.

(b) A sketch of the field lines is shown in the figure below:



15. By symmetry we see that the contributions from the two charges $q_1 = q_2 = +e$ cancel each other, and we simply use Eq. 22-3 to compute magnitude of the field due to $q_3 = +2e$.

(a) The magnitude of the net electric field is

$$|\vec{E}_{\text{net}}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$

$$= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.60\times10^{-19}\ \text{C})}{(6.00\times10^{-6}\ \text{m})^2} = 160\ \text{N/C}.$$

(b) This field points at 45.0°, counterclockwise from the *x* axis.

16. The net field components along the *x* and *y* axes are

$$E_{\text{net},x} = \frac{q_1}{4\pi\varepsilon_0 R^2} - \frac{q_2\cos\theta}{4\pi\varepsilon_0 R^2}, \quad E_{\text{net},y} = -\frac{q_2\sin\theta}{4\pi\varepsilon_0 R^2}.$$

The magnitude is the square root of the sum of the components squared. Setting the magnitude equal to $E = 2.00 \times 10^5$ N/C, squaring and simplifying, we obtain

$$E^2 = \frac{q_1^2 + q_1^2 - 2q_1q_2\cos\theta}{(4\pi\varepsilon_0 R^2)^2}.$$

With $R = 0.500$ m, $q_1 = 2.00 \times 10^{-6}$ C, and $q_2 = 6.00 \times 10^{-6}$ C, we can solve this expression for cos $\theta$ and then take the inverse cosine to find the angle:

$$\theta = \cos^{-1}\left(\frac{q_1^2 + q_1^2 - (4\pi\varepsilon_0 R^2)^2 E^2}{2q_1q_2}\right).$$

There are two answers.

(a) The positive value of angle is $\theta = 67.8°$.

(b) The positive value of angle is $\theta = -67.8°$.

17. We make the assumption that bead 2 is in the lower half of the circle, partly because it would be awkward for bead 1 to "slide through" bead 2 if it were in the path of bead 1 (which is the upper half of the circle) and partly to eliminate a second solution to the problem (which would have opposite angle and charge for bead 2). We note that the net *y* component of the electric field evaluated at the origin is negative (points *down*) for all positions of bead 1, which implies (with our assumption in the previous sentence) that bead 2 is a negative charge.

(a) When bead 1 is on the +*y* axis, there is no *x* component of the net electric field, which implies bead 2 is on the –*y* axis, so its angle is –90°.

(b) Since the downward component of the net field, when bead 1 is on the +*y* axis, is of largest magnitude, then bead 1 must be a positive charge (so that its field is in the same direction as that of bead 2, in that situation). Comparing the values of $E_y$ at 0° and at 90° we see that the absolute values of the charges on beads 1 and 2 must be in the ratio of 5 to 4. This checks with the 180° value from the $E_x$ graph, which further confirms our belief that bead 1 is positively charged. In fact, the 180° value from the $E_x$ graph allows us to solve for its charge (using Eq. 22-3):

$$q_1 = 4\pi\varepsilon_0 r^2 E = 4\pi(8.854 \times 10^{-12} \tfrac{\text{C}^2}{\text{N m}^2})(0.60 \text{ m})^2 (5.0 \times 10^4 \tfrac{\text{N}}{\text{C}}) = 2.0 \times 10^{-6} \text{ C}.$$

(c) Similarly, the 0° value from the $E_y$ graph allows us to solve for the charge of bead 2:

$$q_2 = 4\pi\varepsilon_0 r^2 E = 4\pi(8.854 \times 10^{-12} \tfrac{\text{C}^2}{\text{N m}^2})(0.60 \text{ m})^2 (-4.0 \times 10^4 \tfrac{\text{N}}{\text{C}}) = -1.6 \times 10^{-6} \text{ C}.$$

18. Referring to Eq. 22-6, we use the binomial expansion (see Appendix E) but keeping higher order terms than are shown in Eq. 22-7:

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\left(1 + \frac{d}{z} + \frac{3}{4}\frac{d^2}{z^2} + \frac{1}{2}\frac{d^3}{z^3} + \ldots\right) - \left(1 - \frac{d}{z} + \frac{3}{4}\frac{d^2}{z^2} - \frac{1}{2}\frac{d^3}{z^3} + \ldots\right)\right)$$

$$= \frac{q\,d}{2\pi\varepsilon_0 z^3} + \frac{q\,d^3}{4\pi\varepsilon_0 z^5} + \ldots$$

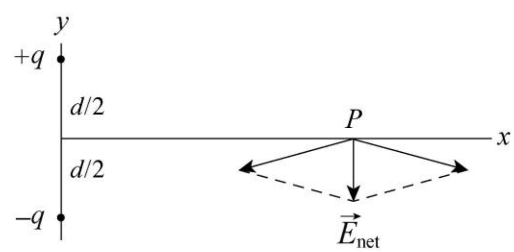
Therefore, in the terminology of the problem, $E_{\text{next}} = q\,d^3/ 4\pi\varepsilon_0 z^5$.

19. (a) Consider the figure below. The magnitude of the net electric field at point $P$ is

$$\left|\vec{E}_{\text{net}}\right| = 2E_1 \sin\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(d/2)^2 + r^2}\right]\frac{d/2}{\sqrt{(d/2)^2 + r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[(d/2)^2 + r^2\right]^{3/2}}$$

For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$|\vec{E}_{\text{net}}| \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}.$$




(b) From the figure, it is clear that the net electric field at point $P$ points in the $-\hat{\text{j}}$ direction, or –90° from the +$x$ axis.

20. According to the problem statement, $E_{\text{act}}$ is Eq. 22-5 (with $z = 5d$)

$$E_{\text{act}} = \frac{q}{4\pi\varepsilon_0 (4.5d)^2} - \frac{q}{4\pi\varepsilon_0 (5.5d)^2} = \frac{160}{9801}\cdot\frac{q}{4\pi\varepsilon_0 d^2}$$

and $E_{\text{approx}}$ is

$$E_{\text{approx}} = \frac{2qd}{4\pi\varepsilon_0 (5d)^3} = \frac{2}{125}\cdot\frac{q}{4\pi\varepsilon_0 d^2}.$$

The ratio is $\dfrac{E_{\text{approx}}}{E_{\text{act}}} = 0.9801 \approx 0.98$.

21. **THINK** The electric quadrupole is composed of two dipoles, each with a dipole moment of magnitude $p = qd$. The dipole moments point in the opposite directions and produce fields in the opposite directions at points on the quadrupole axis.

**EXPRESS** Consider the point $P$ on the axis, a distance $z$ to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is given by $qd/2\pi\varepsilon_0(z - d/2)^3$ while the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$.

**ANALYZE** Use the binomial expansions

$$(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$$

$$(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$$

we obtain

$$E = \frac{qd}{2\pi\varepsilon_0(z-d/2)^3} - \frac{qd}{2\pi\varepsilon_0(z+d/2)^3} \approx \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}.$$

Since the quadrupole moment is $Q = 2qd^2$, we have $E = \dfrac{3Q}{4\pi\varepsilon_0 z^4}$.

**LEARN** For a quadrupole moment $Q$, the electric field varies with $z$ as $E \sim Q/z^4$. For a point charge $q$, the dependence is $E \sim q/z^2$, and for a dipole $p$, we have $E \sim p/z^3$.

22. (a) We use the usual notation for the linear charge density: $\lambda = q/L$. The arc length is $L = r\theta$ with $\theta$ is expressed in radians. Thus,

$$L = (0.0400 \text{ m})(0.698 \text{ rad}) = 0.0279 \text{ m}.$$

With $q = -300(1.602 \times 10^{-19}$ C), we obtain $\lambda = -1.72 \times 10^{-15}$ C/m.

(b) We consider the same charge distributed over an area $A = \pi r^2 = \pi(0.0200 \text{ m})^2$ and obtain

$$\sigma = q/A = -3.82 \times 10^{-14} \text{ C/m}^2.$$

(c) Now the area is four times larger than in the previous part ($A_{\text{sphere}} = 4\pi r^2$) and thus obtain an answer that is one-fourth as big:

$$\sigma = q/A_{\text{sphere}} = -9.56 \times 10^{-15} \text{ C/m}^2.$$

(d) Finally, we consider that same charge spread throughout a volume of $V = 4\pi r^3/3$ and obtain the charge density $\rho = q/V = -1.43 \times 10^{-12}$ C/m$^3$.

23. We use Eq. 22-3, assuming both charges are positive. At $P$, we have

$$E_{\text{left ring}} = E_{\text{right ring}} \quad \Rightarrow \quad \frac{q_1 R}{4\pi\varepsilon_0 \left(R^2+R^2\right)^{3/2}} = \frac{q_2 (2R)}{4\pi\varepsilon_0 [(2R)^2 + R^2]^{3/2}}$$

Simplifying, we obtain

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.506.$$

24. (a) It is clear from symmetry (also from Eq. 22-16) that the field vanishes at the center.

(b) The result ($E$ = 0) for points infinitely far away can be reasoned directly from Eq. 22-16 (it goes as $1/z^2$ as $z \to \infty$) or by recalling the starting point of its derivation (Eq. 22-11, which makes it clearer that the field strength decreases as $1/r^2$ at distant points).

(c) Differentiating Eq. 22-16 and setting equal to zero (to obtain the location where it is maximum) leads to

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}} = 0 \quad \Rightarrow \quad z = +\frac{R}{\sqrt{2}} = 0.707R\,.$$

(d) Plugging this value back into Eq. 22-16 with the values stated in the problem, we find $E_{\max} = 3.46 \times 10^7$ N/C.

25. The smallest arc is of length $L_1 = \pi r_1/2 = \pi R/2$; the middle-sized arc has length $L_2 = \pi r_2/2 = \pi(2R)/2 = \pi R$; and, the largest arc has $L_3 = \pi(3R)/2$. The charge per unit length for each arc is $\lambda = q/L$ where each charge $q$ is specified in the figure. Thus, we find the net electric field to be

$$E_{\text{net}} = \frac{\lambda_1(2\sin 45°)}{4\pi\varepsilon_0 r_1} + \frac{\lambda_2(2\sin 45°)}{4\pi\varepsilon_0 r_2} + \frac{\lambda_3(2\sin 45°)}{4\pi\varepsilon_0 r_3} = \frac{Q}{\sqrt{2}\pi^2\varepsilon_0 R^2}$$

which yields $E_{\text{net}} = 1.62 \times 10^6$ N/C .

(b) The direction is – 45°, measured counterclockwise from the +$x$ axis.

26. Studying Sample Problem 22.03 — "Electric field of a charged circular rod," we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\theta}^{\theta}$$

along the symmetry axis, with $\lambda = q/r\theta$ with $\theta$ in radians. In this problem, each charged quarter-circle produces a field of magnitude

$$|\vec{E}| = \frac{|q|}{r\pi/2}\frac{1}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\pi/4}^{\pi/4} = \frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}|q|}{\pi r^2}.$$

That produced by the positive quarter-circle points at –45°, and that of the negative quarter-circle points at +45°.

(a) The magnitude of the net field is

$$E_{\text{net},x} = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}|q|}{\pi r^2}\right)\cos 45° = \frac{1}{4\pi\varepsilon_0}\frac{4|q|}{\pi r^2}$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)4(4.50\times10^{-12}\,\text{C})}{\pi(5.00\times10^{-2}\ \text{m})^2} = 20.6\ \text{N/C}.$$

(b) By symmetry, the net field points vertically downward in the $-\hat{\text{j}}$ direction, or –90° counterclockwise from the +*x* axis.

27. From symmetry, we see that the net field at *P* is twice the field caused by the upper semicircular charge $+q = \lambda(\pi R)$ (and that it points downward). Adapting the steps leading to Eq. 22-21, we find

$$\vec{E}_{\text{net}} = 2(-\hat{\text{j}})\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_{-90°}^{90°} = -\left(\frac{q}{\varepsilon_0\pi^2 R^2}\right)\hat{\text{j}}.$$

(a) With $R = 8.50\times10^{-2}$ m and $q = 1.50\times10^{-8}$ C, $|\vec{E}_{\text{net}}| = 23.8$ N/C.

(b) The net electric field $\vec{E}_{\text{net}}$ points in the $-\hat{\text{j}}$ direction, or –90° counterclockwise from the +*x* axis.

28. We find the maximum by differentiating Eq. 22-16 and setting the result equal to zero.

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{(z^2+R^2)^{5/2}} = 0$$

which leads to $z = R/\sqrt{2}$. With $R$ = 2.40 cm, we have $z$ = 1.70 cm.

29. First, we need a formula for the field due to the arc. We use the notation $\lambda$ for the charge density, $\lambda = Q/L$. Sample Problem 22.03 — "Electric field of a charged circular

rod" illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle $\theta$) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ if $\theta$ is expressed in radians. Thus, using $R$ instead of $r$, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 r} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}.$$

The problem asks for the ratio $E_{\text{particle}}/E_{\text{arc}}$, where $E_{\text{particle}}$ is given by Eq. 22-3:

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{Q/4\pi\varepsilon_0 R^2}{2Q\sin(\theta/2)/4\pi\varepsilon_0 R^2\theta} = \frac{\theta}{2\sin(\theta/2)}.$$

With $\theta = \pi$, we have

$$\frac{E_{\text{particle}}}{E_{\text{arc}}} = \frac{\pi}{2} \approx 1.57.$$

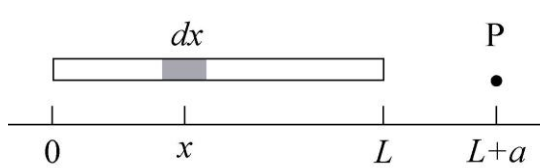
30. We use Eq. 22-16, with "$q$" denoting the charge on the larger ring:

$$\frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} + \frac{qz}{4\pi\varepsilon_0[z^2+(3R)^2]^{3/2}} = 0 \;\Rightarrow\; q = -Q\left(\frac{13}{5}\right)^{3/2} = -4.19Q.$$

Note: We set $z = 2R$ in the above calculation.

31. **THINK** Our system is a non-conducting rod with uniform charge density. Since the rod is an extended object and not a point charge, the calculation of electric field requires an integration.

**EXPRESS** The linear charge density $\lambda$ is the charge per unit length of rod. Since the total charge $-q$ is uniformly distributed on the rod of length $L$, we have $\lambda = -q/L$. To calculate the electric at the point $P$ shown in the figure, we position the $x$-axis along the rod with the origin at the left end of the rod, as shown in the diagram below.



Let *dx* be an infinitesimal length of rod at *x*. The charge in this segment is $dq = \lambda\, dx$. The charge *dq* may be considered to be a point charge. The electric field it produces at point *P* has only an *x* component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2}.$$

The total electric field produced at *P* by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(L+a-x)^2} = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{L+a-x}\bigg|_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{L+a}\right)$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a(L+a)} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)},$$

upon substituting $-q = \lambda L$.

**ANALYZE** (a) With $q$ = 4.23 × $10^{-15}$ C, $L$ = 0.0815 m, and $a$ = 0.120 m, the linear charge density of the rod is

$$\lambda = \frac{-q}{L} = \frac{-4.23\times10^{-15}\ \text{C}}{0.0815\ \text{m}} = -5.19\times10^{-14}\ \text{C/m}.$$

(b) Similarly, we obtain

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)} = -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.23\times10^{-15}\ \text{C})}{(0.120\ \text{m})(0.0815\ \text{m}+0.120\ \text{m})} = -1.57\times10^{-3}\ \text{N/C},$$

or $|E_x| = 1.57\times10^{-3}\ \text{N/C}$.

(c) The negative sign in $E_x$ indicates that the field points in the –*x* direction, or –180° counterclockwise from the +*x* axis.

(d) If *a* is much larger than *L*, the quantity *L* + *a* in the denominator can be approximated by *a*, and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2}.$$

Since $a = 50\ \text{m} \gg L = 0.0815\ \text{m}$, the above approximation applies and we have $E_x = -1.52\times10^{-8}\ \text{N/C}$, or $|E_x| = 1.52\times10^{-8}\ \text{N/C}$.

(e) For a particle of charge $-q = -4.23\times10^{-15}\ \text{C}$, the electric field at a distance *a* = 50 m away has a magnitude $|E_x| = 1.52\times10^{-8}\ \text{N/C}$.

**LEARN** At a distance much greater than the length of the rod ($a \gg L$), the rod can be effectively regarded as a point charge $-q$, and the electric field can be approximated as $E_x \approx \dfrac{-q}{4\pi\varepsilon_0 a^2}$.

32. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element $dx$ of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the $dq$'s) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = R/r$ where $r = \sqrt{x^2 + R^2}$.

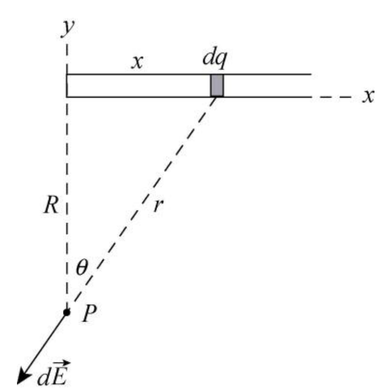
(a) Using Eq. 22-3 (with the 2 and $\sin\theta$ factors just discussed) the magnitude is

$$\begin{aligned}|\vec{E}| &= 2\int_0^{L/2}\left(\frac{dq}{4\pi\varepsilon_0 r^2}\right)\sin\theta = \frac{2}{4\pi\varepsilon_0}\int_0^{L/2}\left(\frac{\lambda\, dx}{x^2+R^2}\right)\left(\frac{y}{\sqrt{x^2+R^2}}\right)\\ &= \frac{\lambda R}{2\pi\varepsilon_0}\int_0^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{(q/L)R}{2\pi\varepsilon_0}\cdot\left.\frac{x}{R^2\sqrt{x^2+R^2}}\right|_0^{L/2}\\ &= \frac{q}{2\pi\varepsilon_0 LR}\frac{L/2}{\sqrt{(L/2)^2+R^2}} = \frac{q}{2\pi\varepsilon_0 R}\frac{1}{\sqrt{L^2+4R^2}}\end{aligned}$$

where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals). With $q = 7.81\times10^{-12}$ C, $L = 0.145$ m, and $R = 0.0600$ m, we have $|\vec{E}| = 12.4$ N/C.

(b) As noted above, the electric field $\vec{E}$ points in the $+y$ direction, or $+90°$ counterclockwise from the $+x$ axis.

33. Consider an infinitesimal section of the rod of length $dx$, a distance $x$ from the left end, as shown in the following diagram. It contains charge $dq = \lambda dx$ and is a distance $r$ from $P$. The magnitude of the field it produces at $P$ is given by




$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}.$$

The $x$ and the y components are

$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{r^2}\sin\theta$$

and

$$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{r^2}\cos\theta\,,$$

respectively. We use $\theta$ as the variable of integration and substitute $r = R/\cos\theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\theta)\,d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta\bigg|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\bigg|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}.$$

We notice that $E_x = E_y$ no matter what the value of $R$. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of $R$.

34. From Eq. 22-26, we obtain

$$E = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right) = \frac{5.3\times10^{-6}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)}\left[1-\frac{12\,\text{cm}}{\sqrt{(12\,\text{cm})^2+(2.5\,\text{cm})^2}}\right] = 6.3\times10^3\ \text{N/C}.$$

35. **THINK** Our system is a uniformly charged disk of radius $R$. We compare the field strengths at different points on its axis of symmetry.

**EXPRESS** At a point on the axis of a uniformly charged disk a distance $z$ above the center of the disk, the magnitude of the electric field is given by Eq. 22-26:

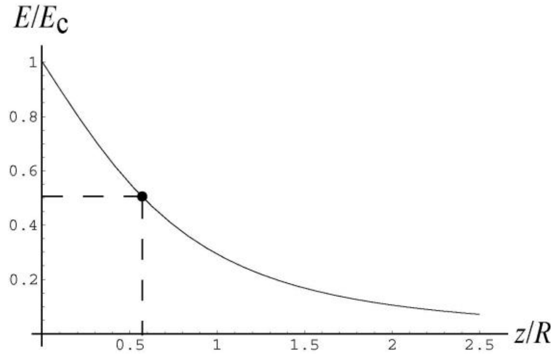
$$E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]$$

where $R$ is the radius of the disk and $\sigma$ is the surface charge density on the disk. The magnitude of the field at the center of the disk ($z$ = 0) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of $z$ such that $E/E_c = 1/2$. This means

$$1-\frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \Rightarrow \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}.$$

**ANALYZE** Squaring both sides, then multiplying them by $z^2 + R^2$, we obtain $z^2 = (z^2/4) + (R^2/4)$. Thus, $z^2 = R^2/3$, or $z = R/\sqrt{3}$. With $R$ = 0.600 m, we have $z$ = 0.346 m.

**LEARN** The ratio of the electric field strengths, $E/E_c = 1-(z/R)/\sqrt{(z/R)^2+1}$, as a function of $z/R$, is plotted below. From the plot, we readily see that at $z/R = (0.346\text{ m})/(0.600\text{ m}) = 0.577$, the ratio indeed is 1/2.




36. From $dA = 2\pi r\, dr$ (which can be thought of as the differential of $A = \pi r^2$) and $dq = \sigma\, dA$ (from the definition of the surface charge density σ), we have

$$dq = \left(\frac{Q}{\pi R^2}\right) 2\pi r\, dr$$

where we have used the fact that the disk is uniformly charged to set the surface charge density equal to the total charge ($Q$) divided by the total area ($\pi R^2$). We next set $r$ = 0.0050 m and make the approximation $dr \approx 30 \times 10^{-6}$ m. Thus we get $dq \approx 2.4 \times 10^{-16}$ C.

37. We use Eq. 22-26, noting that the disk in Figure 22-57(*b*) is effectively equivalent to the disk in Figure 22-57(*a*) plus a concentric smaller disk (of radius *R*/2) with the opposite value of σ. That is,

$$E_{(b)} = E_{(a)} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{2R}{\sqrt{(2R)^2 + (R/2)^2}}\right)$$

where

$$E_{(a)} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{2R}{\sqrt{(2R)^2 + R^2}}\right).$$

We find the relative difference and simplify:

$$\frac{E_{(a)} - E_{(b)}}{E_{(a)}} = \frac{1-2/\sqrt{4+1/4}}{1-2/\sqrt{4+1}} = \frac{1-2/\sqrt{17/4}}{1-2/\sqrt{5}} = \frac{0.0299}{0.1056} = 0.283$$

or approximately 28%.

38. We write Eq. 22-26 as

$$\frac{E}{E_{max}} = 1 - \frac{z}{(z^2+R^2)^{1/2}}$$

and note that this ratio is $\frac{1}{2}$ (according to the graph shown in the figure) when $z$ = 4.0 cm. Solving this for $R$ we obtain $R = z\sqrt{3}$ = 6.9 cm.

39. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = -qE$, where $m$ is the mass of the drop, $q$ is the charge on the drop, and $E$ is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where $r$ is its radius and $\rho$ is its mass density. Thus,

$$q = -\frac{mg}{E} = -\frac{4\pi r^3 \rho g}{3E} = -\frac{4\pi\left(1.64\times10^{-6}\,\text{m}\right)^3\left(851\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)}{3\left(1.92\times10^5\ \text{N/C}\right)} = -8.0\times10^{-19}\,\text{C}$$

and $q/e = (-8.0\times10^{-19}\ \text{C})/(1.60\times10^{-19}\ \text{C}) = -5$, or $q = -5e$.

40. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$ ). We use $v_f^2 - v_i^2 = 2a\Delta x$ with $v_f$ = 0 and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance $\Delta x$:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.00\times10^6\ \text{m/s}\right)^2}{-2\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^3\ \text{N/C}\right)} = 7.12\times10^{-2}\ \text{m}.$$

(b) Equation 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2\left(7.12\times10^{-2}\ \text{m}\right)}{5.00\times10^6\ \text{m/s}} = 2.85\times10^{-8}\ \text{s}.$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of $\Delta x$, we find

$$\frac{\Delta K}{K_i} = \frac{\Delta\left(\frac{1}{2}m_e v^2\right)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2}$$
$$= \frac{-2\left(1.60\times10^{-19}\,\text{C}\right)\left(1.00\times10^3\ \text{N/C}\right)\left(8.00\times10^{-3}\text{m}\right)}{\left(9.11\times10^{-31}\text{kg}\right)\left(5.00\times10^6\ \text{m/s}\right)^2} = -0.112.$$

Thus, the fraction of the initial kinetic energy lost in the region is 0.112 or 11.2%.

41. **THINK** In this problem we compare the strengths between the electrostatic force and the gravitational force.

**EXPRESS** The magnitude of the electrostatic force on a point charge of magnitude $q$ is given by $F = qE$, where $E$ is the magnitude of the electric field at the location of the particle. On the other hand, the force of gravity on a particle of mass $m$ is $F_g = mg$.

**ANALYZE** (a) With $q = -2.0\times10^{-9}$ C and $F = 3.0\times10^{-6}$ N, the magnitude of the electric field strength is

$$E = \frac{F}{q} = \frac{3.0\times10^{-6}\text{ N}}{2.0\times10^{-9}\text{ C}} = 1.5\times10^{3}\text{ N/C}.$$

In vector notation, $\vec{F} = q\vec{E}$. Since the force points downward and the charge is negative, the field $\vec{E}$ must points upward (in the opposite direction of $\vec{F}$ ).

(b) The magnitude of the electrostatic force on a proton is

$$F_{el} = eE = \left(1.60\times10^{-19}\text{ C}\right)\left(1.5\times10^{3}\text{ N/C}\right) = 2.4\times10^{-16}\text{N}.$$

(c) A proton is positively charged, so the force is in the same direction as the field, upward.

(d) The magnitude of the gravitational force on the proton is

$$F_g = mg = \left(1.67\times10^{-27}\text{ kg}\right)\left(9.8\text{ m/s}^2\right) = 1.6\times10^{-26}\text{N}.$$

The force is downward.

(e) The ratio of the forces is

$$\frac{F_{el}}{F_g} = \frac{2.4\times10^{-16}\text{ N}}{1.64\times10^{-26}\text{ N}} = 1.5\times10^{10}.$$

**LEARN** The force of gravity on the proton is much smaller than the electrostatic force on the proton due to the field of strength $E = 1.5\times10^{3}$ N/C. For the two forces to have equal strength, the electric field would have to be very small:

$$E = \frac{mg}{q} = \frac{(1.67\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)}{1.6\times10^{-19}\text{ C}} = 1.02\times10^{-7}\text{ N/C}.$$

42. (a) $F_e = Ee = (3.0 \times 10^6\text{ N/C})(1.6 \times 10^{-19}\text{ C}) = 4.8 \times 10^{-13}$ N.

(b) $F_i = Eq_{\text{ion}} = Ee = (3.0 \times 10^6\text{ N/C})(1.6 \times 10^{-19}\text{ C}) = 4.8 \times 10^{-13}$ N.

43. **THINK** The acceleration of the electron is given by Newton's second law: $F = ma$, where $F$ is the electrostatic force.

**EXPRESS** The magnitude of the force acting on the electron is $F = eE$, where $E$ is the magnitude of the electric field at its location. Using Newton's second law, the acceleration of the electron is

$$a = \frac{F}{m} = \frac{eE}{m}.$$

**ANALYZE** With $e = 1.6\times10^{-19}$ C, $E = 2.00\times10^{4}$ N/C, and $m = 9.11\times10^{-31}$ kg, we find the acceleration to be

$$a = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{4}\text{ N/C}\right)}{9.11\times10^{-31}\text{ kg}} = 3.51\times10^{15}\text{ m/s}^2.$$

**LEARN** In vector notation, $\vec{a} = \vec{F}/m = -e\vec{E}/m$, so $\vec{a}$ is in the opposite direction of $\vec{E}$. The magnitude of electron's acceleration is proportional to the field strength $E$: the greater the value of $E$, the greater the acceleration.

44. (a) Vertical equilibrium of forces leads to the equality

$$q\left|\vec{E}\right| = mg \;\Rightarrow\; \left|\vec{E}\right| = \frac{mg}{2e}.$$

Substituting the values given in the problem, we obtain

$$\left|\vec{E}\right| = \frac{mg}{2e} = \frac{(6.64\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)}{2(1.6\times10^{-19}\text{C})} = 2.03\times10^{-7}\text{ N/C}.$$

(b) Since the force of gravity is downward, then $q\vec{E}$ must point upward. Since $q > 0$ in this situation, this implies $\vec{E}$ must itself point upward.

45. We combine Eq. 22-9 and Eq. 22-28 (in absolute values).

$$F = |q|E = |q|\left(\frac{p}{2\pi\varepsilon_0 z^3}\right) = \frac{2kep}{z^3}$$

where we have used Eq. 21-5 for the constant $k$ in the last step. Thus, we obtain

$$F = \frac{2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)\left(3.6\times10^{-29}\text{C}\cdot\text{m}\right)}{\left(25\times10^{-9}\text{m}\right)^3} = 6.6\times10^{-15}\text{ N}.$$

If the dipole is oriented such that $\vec{p}$ is in the $+z$ direction, then $\vec{F}$ points in the $-z$ direction.

46. Equation 22-28 gives

$$\vec{E} = \frac{\vec{F}}{q} = \frac{m\vec{a}}{(-e)} = -\left(\frac{m}{e}\right)\vec{a}$$

using Newton's second law.

(a) With *east* being the $\hat{\text{i}}$ direction, we have

$$\vec{E} = -\left(\frac{9.11\times10^{-31}\text{ kg}}{1.60\times10^{-19}\text{ C}}\right)\left(1.80\times10^{9}\text{ m/s}^2\,\hat{\text{i}}\right) = (-0.0102\,\text{N/C})\,\hat{\text{i}}$$

which means the field has a magnitude of 0.0102 N/C .

(b) The result shows that the field $\vec{E}$ is directed in the $-x$ direction, or westward.

47. **THINK** The acceleration of the proton is given by Newton's second law: $F = ma$, where $F$ is the electrostatic force.

**EXPRESS** The magnitude of the force acting on the proton is $F = eE$, where $E$ is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where $m$ is the mass of the proton. Thus,

$$a = \frac{F}{m} = \frac{eE}{m}.$$

We assume that the proton starts from rest ($v_0 = 0$) and apply the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$). Thus, the speed of the proton after having traveling a distance $x$ is $v = \sqrt{2ax}$.

**ANALYZE** (a) With $e = 1.6\times10^{-19}$ C, $E = 2.00\times10^{4}$ N/C, and $m = 1.67\times10^{-27}$ kg, we find the acceleration to be

$$a = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{4}\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 1.92\times10^{12}\text{ m/s}^2.$$

(b) With $x = 1.00\text{ cm} = 1.0\times10^{-2}\text{ m}$, the speed of the proton is

$$v = \sqrt{2ax} = \sqrt{2\left(1.92\times10^{12}\ \text{m/s}^2\right)\left(0.0100\,\text{m}\right)} = 1.96\times10^{5}\ \text{m/s}.$$

**LEARN** The time it takes for the proton to attain the final speed is

$$t = \frac{v}{a} = \frac{1.96\times10^{5}\ \text{m/s}}{1.92\times10^{12}\ \text{m/s}^2} = 1.02\times10^{-7}\ \text{s}.$$

The distance the proton travels can be written as

$$x = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{eE}{m}\right)t^2.$$

48. We are given $\sigma = 4.00 \times 10^{-6}$ C/m$^2$ and various values of $z$ (in the notation of Eq. 22-26, which specifies the field $E$ of the charged disk). Using this with $F = eE$ (the magnitude of Eq. 22-28 applied to the electron) and $F = ma$, we obtain $a = F/m = eE/m$.

(a) The magnitude of the acceleration at a distance $R$ is

$$a = \frac{e\,\sigma\,(2-\sqrt{2}\,)}{4\,m\,\varepsilon_o} = 1.16 \times 10^{16}\ \text{m/s}^2\ .$$

(b) At a distance $R$/100, $a = \dfrac{e\,\sigma\,(10001-\sqrt{10001}\,)}{20002\,m\,\varepsilon_o} = 3.94 \times 10^{16}\ \text{m/s}^2$ .

(c) At a distance $R$/1000, $a = \dfrac{e\,\sigma\,(1000001-\sqrt{1000001}\,)}{2000002\,m\,\varepsilon_o} = 3.97 \times 10^{16}\ \text{m/s}^2$ .

(d) The field due to the disk becomes more uniform as the electron nears the center point. One way to view this is to consider the forces exerted on the electron by the charges near the edge of the disk; the net force on the electron caused by those charges will decrease due to the fact that their contributions come closer to canceling out as the electron approaches the middle of the disk.

49. (a) Using Eq. 22-28, we find

$$\begin{aligned}\vec{F} &= \left(8.00\times10^{-5}\text{C}\right)\left(3.00\times10^{3}\ \text{N/C}\right)\hat{\text{i}} + \left(8.00\times10^{-5}\text{C}\right)\left(-600\,\text{N/C}\right)\hat{\text{j}}\\ &= \left(0.240\ \text{N}\right)\hat{\text{i}} - \left(0.0480\ \text{N}\right)\hat{\text{j}}.\end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(0.240\ \text{N}\right)^2 + \left(-0.0480\ \text{N}\right)^2} = 0.245\ \text{N}.$$

(b) The angle the force $\vec{F}$ makes with the +*x* axis is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.0480\text{ N}}{0.240\text{ N}}\right) = -11.3°$$

measured counterclockwise from the +*x* axis.

(c) With $m$ = 0.0100 kg, the ($x$, $y$) coordinates at $t$ = 3.00 s can be found by combining Newton's second law with the kinematics equations of Chapters 2–4. The *x* coordinate is

$$x = \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{(0.240\text{ N})(3.00\text{ s})^2}{2(0.0100\text{ kg})} = 108\text{ m}.$$

(d) Similarly, the *y* coordinate is

$$y = \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{(-0.0480\text{ N})(3.00\text{ s})^2}{2(0.0100\text{ kg})} = -21.6\text{ m}.$$

50. We assume there are no forces or force-components along the *x* direction. We combine Eq. 22-28 with Newton's second law, then use Eq. 4-21 to determine time $t$ followed by Eq. 4-23 to determine the final velocity (with $-g$ replaced by the $a_y$ of this problem); for these purposes, the velocity components *given* in the problem statement are re-labeled as $v_{0x}$ and $v_{0y}$, respectively.

(a) We have $\vec{a} = q\vec{E}/m = -(e/m)\vec{E},$ which leads to

$$\vec{a} = -\left(\frac{1.60\times10^{-19}\text{C}}{9.11\times10^{-31}\text{kg}}\right)\left(120\frac{\text{N}}{\text{C}}\right)\hat{\text{j}} = -(2.1\times10^{13}\text{ m/s}^2)\,\hat{\text{j}}.$$

(b) Since $v_x = v_{0x}$ in this problem (that is, $a_x = 0$), we obtain

$$t = \frac{\Delta x}{v_{0x}} = \frac{0.020\text{ m}}{1.5\times10^5\text{ m/s}} = 1.3\times10^{-7}\text{ s}$$

$$v_y = v_{0y} + a_y t = 3.0\times10^3\text{ m/s} + \left(-2.1\times10^{13}\text{ m/s}^2\right)\left(1.3\times10^{-7}\text{ s}\right)$$

which leads to $v_y = -2.8 \times 10^6$ m/s. Therefore, the final velocity is

$$\vec{v} = (1.5\times10^5\text{ m/s})\,\hat{\text{i}} - (2.8\times10^6\text{ m/s})\,\hat{\text{j}}.$$

51. We take the charge $Q = 45.0$ pC of the bee to be concentrated as a particle at the center of the sphere. The magnitude of the induced charges on the sides of the grain is $|q| = 1.000$ pC.

(a) The electrostatic force on the grain by the bee is

$$F = \frac{kQq}{(d+D/2)^2} + \frac{kQ(-q)}{(D/2)^2} = -kQ|q|\left[\frac{1}{(D/2)^2} - \frac{1}{(d+D/2)^2}\right]$$

where $D = 1.000$ cm is the diameter of the sphere representing the honeybee, and $d = 40.0\mu\text{m}$ is the diameter of the grain. Substituting the values, we obtain

$$F = -\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\,\text{C})(1.000\times10^{-12}\,\text{C})\left[\frac{1}{(5.00\times10^{-3}\ \text{m})^2} - \frac{1}{(5.04\times10^{-3}\ \text{m})^2}\right]$$
$$= -2.56\times10^{-10}\ \text{N}.$$

The negative sign implies that the force between the bee and the grain is attractive. The magnitude of the force is $|F| = 2.56\times10^{-10}$ N.

(b) Let $|Q'| = 45.0$ pC be the magnitude of the charge on the tip of the stigma. The force on the grain due to the stigma is

$$F' = \frac{k|Q'|q}{(d+D')^2} + \frac{k|Q'|(-q)}{(D')^2} = -k|Q'||q|\left[\frac{1}{(D')^2} - \frac{1}{(d+D')^2}\right]$$

where $D' = 1.000$ mm is the distance between the grain and the tip of the stigma. Substituting the values given, we have

$$F' = -\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(45.0\times10^{-12}\,\text{C})(1.000\times10^{-12}\,\text{C})\left[\frac{1}{(1.000\times10^{-3}\ \text{m})^2} - \frac{1}{(1.040\times10^{-3}\ \text{m})^2}\right]$$
$$= -3.06\times10^{-8}\ \text{N}.$$

The negative sign implies that the force between the grain and the stigma is attractive. The magnitude of the force is $|F'| = 3.06\times10^{-8}$ N.

(c) Since $|F'| > |F|$, the grain will move to the stigma.

52. (a) Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the same direction as the velocity leads to deceleration. Thus, with $t = 1.5 \times 10^{-9}$ s, we find

$$v = v_0 - |a|t = v_0 - \frac{eE}{m}t = 4.0\times10^4 \text{ m/s} - \frac{(1.6\times10^{-19}\text{ C})(50\text{ N/C})}{9.11\times10^{-31}\text{ kg}}(1.5\times10^{-9}\text{ s})$$
$$= 2.7\times10^4 \text{ m/s}.$$

(b) The displacement is equal to the distance since the electron does not change its direction of motion. The field is uniform, which implies the acceleration is constant. Thus,

$$d = \frac{v+v_0}{2}t = 5.0\times10^{-5}\text{m}.$$

53. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where $E$ is the magnitude of the electric field, $m_p$ is the mass of the proton, and $m_e$ is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time $t$ is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or

$$\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2.$$

This means $t^2 = 2L/(a_p - a_e)$ and

$$x = \frac{a_p}{a_p - a_e}L = \frac{eE/m_p}{(eE/m_p)+(eE/m_e)}L = \left(\frac{m_e}{m_e+m_p}\right)L$$
$$= \left(\frac{9.11\times10^{-31}\text{kg}}{9.11\times10^{-31}\text{kg}+1.67\times10^{-27}\text{kg}}\right)(0.050\text{ m})$$
$$= 2.7\times10^{-5}\text{m}.$$

54. Due to the fact that the electron is negatively charged, then (as a consequence of Eq. 22-28 and Newton's second law) the field $\vec{E}$ pointing in the $+y$ direction (which we will call "upward") leads to a downward acceleration. This is exactly like a projectile motion problem as treated in Chapter 4 (but with $g$ replaced with $a = eE/m = 8.78\times10^{11}\text{ m/s}^2$). Thus, Eq. 4-21 gives

$$t = \frac{x}{v_0\cos\theta_0} = \frac{3.00\text{ m}}{(2.00\times10^6\text{ m/s})\cos 40.0°} = 1.96\times10^{-6}\text{ s}.$$

This leads (using Eq. 4-23) to

$$v_y = v_0\sin\theta_0 - at = (2.00\times10^6\text{ m/s})\sin 40.0° - (8.78\times10^{11}\text{ m/s}^2)(1.96\times10^{-6}\text{ s})$$
$$= -4.34\times10^5\text{ m/s}.$$

Since the $x$ component of velocity does not change, then the final velocity is

$$\vec{v} = (1.53 \times 10^6 \text{ m/s})\,\hat{\text{i}} - (4.34 \times 10^5 \text{ m/s})\,\hat{\text{j}}\,.$$

55. (a) We use $\Delta x = v_{avg}t = vt/2$:

$$v = \frac{2\Delta x}{t} = \frac{2\left(2.0\times10^{-2}\,\text{m}\right)}{1.5\times10^{-8}\,\text{s}} = 2.7\times10^{6}\ \text{m/s}.$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta xm}{et^2} = \frac{2\left(2.0\times10^{-2}\,\text{m}\right)\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.5\times10^{-8}\,\text{s}\right)^2} = 1.0\times10^{3}\ \text{N/C}.$$

56. (a) Equation 22-33 leads to $\tau = pE\sin 0° = 0.$

(b) With $\theta = 90°$, the equation gives

$$\tau = pE = \left(2\left(1.6\times10^{-19}\ \text{C}\right)\left(0.78\times10^{-9}\ \text{m}\right)\right)\left(3.4\times10^{6}\ \text{N/C}\right) = 8.5\times10^{-22}\ \text{N}\cdot\text{m}.$$

(c) Now the equation gives $\tau = pE\sin 180° = 0.$

57. **THINK** The potential energy of the electric dipole placed in an electric field depends on its orientation relative to the electric field.

**EXPRESS** The magnitude of the electric dipole moment is $p = qd$, where $q$ is the magnitude of the charge, and $d$ is the separation between the two charges. When placed in an electric field, the potential energy of the dipole is given by Eq. 22-38:

$$U(\theta) = -\vec{p}\cdot\vec{E} = -pE\cos\theta\,.$$

Therefore, if the initial angle between $\vec{p}$ and $\vec{E}$ is $\theta_0$ and the final angle is $\theta$, then the change in potential energy would be

$$\Delta U = U(\theta) - U_0(\theta) = -pE\left(\cos\theta - \cos\theta_0\right).$$

**ANALYZE** (a) With $q = 1.50\times10^{-9}$ C and $d = 6.20\times10^{-6}$ m, we find the magnitude of the dipole moment to be

$$p = qd = \left(1.50\times10^{-9}\ \text{C}\right)\left(6.20\times10^{-6}\ \text{m}\right) = 9.30\times10^{-15}\ \text{C}\cdot\text{m}.$$

(b) The initial and the final angles are $\theta_0 = 0$ (parallel) and $\theta = 180°$ (anti-parallel), so we find $\Delta U$ to be

$$\Delta U = U(180°) - U(0) = 2pE = 2\left(9.30\times10^{-15}\,\text{C}\cdot\text{m}\right)\left(1100\ \text{N/C}\right) = 2.05\times10^{-11}\,\text{J}.$$

**LEARN** The potential energy is a maximum ($U_{max} = +pE$) when the dipole is oriented antiparallel to $\vec{E}$, and is a minimum ($U_{min} = -pE$) when it is parallel to $\vec{E}$.

58. Examining the lowest value on the graph, we have (using Eq. 22-38)

$$U = -\vec{p}\cdot\vec{E} = -1.00\times10^{-28}\,\text{J}.$$

If $E$ = 20 N/C, we find $p = 5.0\times10^{-28}$ C·m.

59. Following the solution to part (c) of Sample Problem 22.05 — "Torque and energy of an electric dipole in an electric field," we find

$$\begin{aligned} W &= U(\theta_0+\pi) - U(\theta_0) = -pE\left(\cos(\theta_0+\pi) - \cos(\theta_0)\right) = 2pE\cos\theta_0 \\ &= 2(3.02\times10^{-25}\,\text{C}\cdot\text{m})(46.0\ \text{N/C})\cos 64.0° \\ &= 1.22\times10^{-23}\ \text{J}. \end{aligned}$$

60. Using Eq. 22-35, considering $\theta$ as a variable, we note that it reaches its maximum value when $\theta$ = –90°: $\tau_{max} = pE$. Thus, with $E$ = 40 N/C and $\tau_{max} = 100\times10^{-28}$ N·m (determined from the graph), we obtain the dipole moment: $p = 2.5\times10^{-28}$ C·m.

61. Equation 22-35 $(\tau = -pE\sin\theta)$ captures the sense as well as the magnitude of the effect. That is, this is a restoring torque, trying to bring the tilted dipole back to its aligned equilibrium position. If the amplitude of the motion is small, we may replace sin $\theta$ with $\theta$ in radians. Thus, $\tau \approx -pE\theta$. Since this exhibits a simple negative proportionality to the angle of rotation, the dipole oscillates in simple harmonic motion, like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency $\omega$ is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

where $I$ is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}.$$

62. (a) We combine Eq. 22-28 (in absolute value) with Newton's second law:

$$a=\frac{|q|E}{m}=\left(\frac{1.60\times10^{-19}\,\text{C}}{9.11\times10^{-31}\,\text{kg}}\right)\left(1.40\times10^{6}\,\frac{\text{N}}{\text{C}}\right)=2.46\times10^{17}\ \text{m/s}^2\,.$$

(b) With $v=\frac{c}{10}=3.00\times10^{7}\ \text{m/s}$, we use Eq. 2-11 to find

$$t=\frac{v-v_o}{a}=\frac{3.00\times10^{7}\ \text{m/s}}{2.46\times10^{17}\ \text{m/s}^2}=1.22\times10^{-10}\ \text{s}.$$

(c) Equation 2-16 gives

$$\Delta x=\frac{v^2-v_o^2}{2a}=\frac{\left(3.00\times10^{7}\ \text{m/s}\right)^2}{2\left(2.46\times10^{17}\ \text{m/s}^2\right)}=1.83\times10^{-3}\ \text{m}.$$

63. (a) Using the density of water ($\rho$ = 1000 kg/m$^3$), the weight $mg$ of the spherical drop (of radius $r = 6.0 \times 10^{-7}$ m) is

$$W=\rho Vg=\left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}\left(6.0\times10^{-7}\ \text{m}\right)^3\right)\left(9.8\,\text{m/s}^2\right)=8.87\times10^{-15}\ \text{N}.$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for $n$, the number of excess electrons:

$$n=\frac{mg}{eE}=\frac{8.87\times10^{-15}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(462\,\text{N/C}\right)}=120.$$

64. The two closest charges produce fields at the midpoint that cancel each other out. Thus, the only significant contribution is from the furthest charge, which is a distance $r=\sqrt{3}d/2$ away from that midpoint. Plugging this into Eq. 22-3 immediately gives the result:

$$E=\frac{Q}{4\pi\varepsilon_0 r^2}=\frac{Q}{4\pi\varepsilon_0(\sqrt{3}d/2)^2}=\frac{4}{3}\frac{Q}{4\pi\varepsilon_0 d^2}\,.$$

65. First, we need a formula for the field due to the arc. We use the notation λ for the charge density, $\lambda = Q/L$. Sample Problem 22.03 — "Electric field of a charged circular rod," illustrates the simplest approach to circular arc field problems. Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle $\theta$) is

$$E_{\text{arc}}=\frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2)-\sin(-\theta/2)\right]=\frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}\,.$$

Now, the arc length is $L = r\theta$ with $\theta$ expressed in radians. Thus, using $R$ instead of $r$, we obtain

$$E_{\text{arc}} = \frac{2(Q/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(Q/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} \ .$$

Thus, the problem requires $E_{\text{arc}} = \frac{1}{2}\ E_{\text{particle}}$, where $E_{\text{particle}}$ is given by Eq. 22-3. Hence,

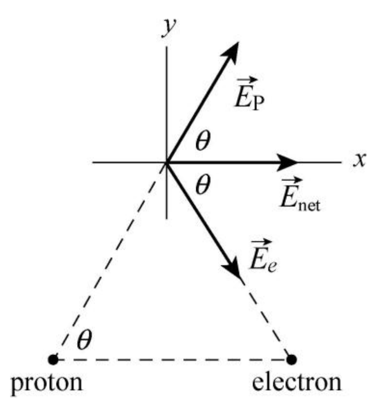
$$\frac{2Q\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2} \ \Rightarrow \ \sin\frac{\theta}{2} = \frac{\theta}{4}$$

where we note, again, that the angle is in radians. The approximate solution to this equation is $\theta = 3.791$ rad $\approx 217°$.

66. We denote the electron with subscript $e$ and the proton with $p$. From the figure below we see that

$$\left|\vec{E}_e\right| = \left|\vec{E}_p\right| = \frac{e}{4\pi\varepsilon_0 d^2}$$

where $d = 2.0 \times 10^{-6}$ m. We note that the components along the $y$ axis cancel during the vector summation. With $k = 1/4\pi\varepsilon_0$ and $\theta = 60°$, the magnitude of the net electric field is obtained as follows:




$$\left|\vec{E}_{\text{net}}\right| = E_x = 2E_e\cos\theta = 2\left(\frac{e}{4\pi\varepsilon_0 d^2}\right)\cos\theta = 2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(1.6\times10^{-19}\ \text{C}\right)}{\left(2.0\times10^{-6}\ \text{m}\right)^2}\cos 60°$$

$$= 3.6\times10^2\ \ \text{N/C}.$$

67. A small section of the distribution that has charge $dq$ is $\lambda\, dx$, where $\lambda = 9.0 \times 10^{-9}$ C/m. Its contribution to the field at $x_P = 4.0$ m is

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0\left(x - x_P\right)^2}$$

pointing in the $+x$ direction. Thus, we have

$$\vec{E}=\int_0^{3.0\,\mathrm{m}}\frac{\lambda\,dx}{4\pi\varepsilon_0\left(x-x_P\right)^2}\hat{\mathrm{i}}$$

which becomes, using the substitution $u = x - x_P$,

$$\vec{E}=\frac{\lambda}{4\pi\varepsilon_0}\int_{-4.0\,\mathrm{m}}^{-1.0\,\mathrm{m}}\frac{du}{u^2}\hat{\mathrm{i}}=\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{-1}{-1.0\,\mathrm{m}}-\frac{-1}{-4.0\,\mathrm{m}}\right)\hat{\mathrm{i}}$$

which yields 61 N/C in the $+x$ direction.

68. Most of the individual fields, caused by diametrically opposite charges, will cancel, except for the pair that lie on the $x$ axis passing through the center. This pair of charges produces a field pointing to the right

$$\vec{E}=\frac{3q}{4\pi\varepsilon_0 d^2}\hat{\mathrm{i}}=\frac{3e}{4\pi\varepsilon_0 d^2}\hat{\mathrm{i}}=\frac{3\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(0.020\mathrm{m}\right)^2}\hat{\mathrm{i}}=(1.08\times10^{-5}\ \mathrm{N/C})\hat{\mathrm{i}}\,.$$

69. (a) From symmetry, we see the net field component along the $x$ axis is zero; the net field component along the $y$ axis points upward. With $\theta = 60°$,

$$E_{\mathrm{net},y}=2\frac{Q\sin\theta}{4\pi\varepsilon_0 a^2}\ .$$

Since $\sin(60°) = \sqrt{3}/2$, we can write this as $E_{\mathrm{net}} = kQ\sqrt{3}/a^2$ (using the notation of the constant $k$ defined in Eq. 21-5). Numerically, this gives roughly 47 N/C.

(b) From symmetry, we see in this case that the net field component along the $y$ axis is zero; the net field component along the $x$ axis points rightward. With $\theta = 60°$,

$$E_{\mathrm{net},x}=2\frac{Q\cos\theta}{4\pi\varepsilon_0 a^2}\,.$$

Since $\cos(60°) = 1/2$, we can write this as $E_{\mathrm{net}} = kQ/a^2$ (using the notation of Eq. 21-5). Thus, $E_{\mathrm{net}} \approx 27$ N/C.

70. Our approach (based on Eq. 22-29) consists of several steps. The first is to find an *approximate* value of $e$ by taking differences between all the given data. The smallest difference is between the fifth and sixth values:

$$18.08\times10^{-19}\ \mathrm{C}-16.48\times10^{-19}\ \mathrm{C}=1.60\times10^{-19}\ \mathrm{C}$$

which we denote $e_{\text{approx}}$. The goal at this point is to assign integers $n$ using this approximate value of $e$:

| | | | |
|---|---|---|---|
| datum1 | $\dfrac{6.563\times10^{-19}\text{C}}{e_{\text{approx}}}=4.10\Rightarrow n_1=4$ | datum6 | $\dfrac{18.08\times10^{-19}\text{C}}{e_{\text{appeox}}}=11.30\Rightarrow n_6=11$ |
| datum2 | $\dfrac{8.204\times10^{-19}\text{C}}{e_{\text{approx}}}=5.13\Rightarrow n_2=5$ | datum7 | $\dfrac{19.71\times10^{-19}\text{C}}{e_{\text{approx}}}=12.32\Rightarrow n_7=12$ |
| datum3 | $\dfrac{11.50\times10^{-19}\text{C}}{e_{\text{approx}}}=7.19\Rightarrow n_3=7$ | datum8 | $\dfrac{22.89\times10^{-19}\text{C}}{e_{\text{approx}}}=14.31\Rightarrow n_8=14$ |
| datum4 | $\dfrac{13.13\times10^{-19}\text{C}}{e_{\text{approx}}}=8.21\Rightarrow n_4=8$ | datum9 | $\dfrac{26.13\times10^{-19}\text{C}}{e_{\text{approx}}}=16.33\Rightarrow n_9=16$ |
| datum5 | $\dfrac{16.48\times10^{-19}\text{C}}{e_{\text{approx}}}=10.30\Rightarrow n_5=10$ | | |

Next, we construct a new data set ($e_1$, $e_2$, $e_3$, …) by dividing the given data by the respective exact integers $n_i$ (for $i$ = 1, 2, 3, …):

$$(e_1,e_2,e_3,\ldots)=\left(\frac{6.563\times10^{-19}\,\text{C}}{n_1},\frac{8.204\times10^{-19}\,\text{C}}{n_2},\frac{11.50\times10^{-19}\,\text{C}}{n_3},\ldots\right)$$

which gives (carrying a few more figures than are significant)

$$\left(1.64075\times10^{-19}\,\text{C},\ 1.6408\times10^{-19}\text{C},\ 1.64286\times10^{-19}\text{C},\ldots\right)$$

as the new data set (our experimental values for $e$). We compute the average and standard deviation of this set, obtaining

$$e_{\text{exptal}}=e_{\text{avg}}\pm\Delta e=(1.641\pm0.004)\times10^{-19}\,\text{C}$$

which does not agree (to within one standard deviation) with the modern accepted value for $e$. The lower bound on this spread is $e_{\text{avg}}-\Delta e = 1.637\times10^{-19}$ C, which is still about 2% too high.

71. Studying Sample Problem 22.03 — "Electric field of a charged circular rod," we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E}=\frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\bigg|_{-\theta}^{\theta}$$

along the symmetry axis, where $\lambda = q/\ell = q/r\theta$ with $\theta$ in radians. Here $\ell$ is the length of the arc, given as $\ell = 4.0\,\text{m}$. Therefore, the angle is $\theta = \ell/r = 4.0/2.0 = 2.0\,\text{rad}$. Thus, with $q = 20 \times 10^{-9}$ C, we obtain

$$|\vec{E}| = \frac{(q/\ell)}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-1.0\text{ rad}}^{1.0\text{ rad}} = 38\text{ N/C}.$$

72. The electric field at a point on the axis of a uniformly charged ring, a distance $z$ from the ring center, is given by

$$E = \frac{qz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}$$

where $q$ is the charge on the ring and $R$ is the radius of the ring (see Eq. 22-16). For $q$ positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\varepsilon_0\left(z^2 + R^2\right)^{3/2}}.$$

For small amplitude oscillations $z \ll R$ and $z$ can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\varepsilon_0 R^3}.$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to $z$, just as if the electron were attached to a spring with spring constant $k = eq/4\pi\varepsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 m R^3}}$$

where $m$ is the mass of the electron.

73. **THINK** We have a positive charge in the $xy$ plane. From the electric fields it produces at two different locations, we can determine the position and the magnitude of the charge.

**EXPRESS** Let the charge be placed at $(x_0, y_0)$. In Cartesian coordinates, the electric field at a point $(x, y)$ can be written as

$$\vec{E} = E_x\hat{\text{i}} + E_y\hat{\text{j}} = \frac{q}{4\pi\varepsilon_0}\frac{(x-x_0)\hat{\text{i}} + (y-y_0)\hat{\text{j}}}{\left[(x-x_0)^2 + (y-y_0)^2\right]^{3/2}}.$$

The ratio of the field components is

$$\frac{E_y}{E_x} = \frac{y-y_0}{x-x_0}.$$

**ANALYZE** (a) The fact that the second measurement at the location (2.0 cm, 0) gives $\vec{E} = (100 \text{ N/C})\hat{\text{i}}$ indicates that $y_0 = 0$, that is, the charge must be somewhere on the *x* axis. Thus, the above expression can be simplified to

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{(x-x_0)\hat{\text{i}} + y\hat{\text{j}}}{\left[(x-x_0)^2 + y^2\right]^{3/2}}.$$

On the other hand, the field at (3.0 cm, 3.0 cm) is $\vec{E} = (7.2 \text{ N/C})(4.0\hat{\text{i}} + 3.0\hat{\text{j}})$, which gives $E_y / E_x = 3/4$. Thus, we have

$$\frac{3}{4} = \frac{3.0 \text{ cm}}{3.0 \text{ cm} - x_0}$$

which implies $x_0 = -1.0$ cm.

(b) As shown above, the *y* coordinate is $y_0 = 0$.

(c) To calculate the magnitude of the charge, we note that the field magnitude measured at (2.0 cm, 0) (which is $r = 0.030$ m from the charge) is

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = 100 \text{ N/C}.$$

Therefore,

$$q = 4\pi\varepsilon_0\left|\vec{E}\right|r^2 = \frac{(100 \text{ N/C})(0.030 \text{ m})^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 1.0\times10^{-11}\text{C}.$$

**LEARN** Alternatively, we may calculate *q* by noting that at (3.0 cm, 3.00 cm)

$$E_x = 28.8 \text{ N/C} = \frac{q}{4\pi\varepsilon_0}\frac{0.040 \text{ m}}{\left[(0.040 \text{ m})^2 + (0.030 \text{ m})^2\right]^{3/2}} = \frac{q}{4\pi\varepsilon_0}\left(320/\text{m}^2\right).$$

This gives

$$q = \frac{28.8 \text{ N/C}}{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(320/\text{m}^2)} = 1.0\times10^{-11} \text{ C},$$

in agreement with that calculated above.

74. (a) Let $E = \sigma/2\varepsilon_0 = 3 \times 10^6$ N/C. With $\sigma = |q|/A$, this leads to

$$|q| = \pi R^2\sigma = 2\pi\varepsilon_0 R^2 E = \frac{R^2E}{2k} = \frac{\left(2.5\times10^{-2}\,\text{m}\right)^2\left(3.0\times10^6\ \text{N/C}\right)}{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)} = 1.0\times10^{-7}\ \text{C},$$

where $k = 1/4\pi\varepsilon_0 = 8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

(b) Setting up a simple proportionality (with the areas), the number of atoms is estimated to be

$$n = \frac{\pi\left(2.5\times10^{-2}\,\text{m}\right)^2}{0.015\times10^{-18}\,\text{m}^2} = 1.3\times10^{17}.$$

(c) The fraction is

$$\frac{q}{Ne} = \frac{1.0\times10^{-7}\,\text{C}}{\left(1.3\times10^{17}\right)\left(1.6\times10^{-19}\,\text{C}\right)} \approx 5.0\times10^{-6}.$$

75. On the one hand, the conclusion (that $Q = +1.00\ \mu$C) is clear from symmetry. If a more in-depth justification is desired, one should use Eq. 22-3 for the electric field magnitudes of the three charges (each at the same distance $r = a/\sqrt{3}$ from $C$) and then find field components along suitably chosen axes, requiring each component-sum to be zero. If the $y$ axis is vertical, then (assuming $Q > 0$) the component-sum along that axis leads to $2kq\sin 30°/r^2 = kQ/r^2$ where $q$ refers to either of the charges at the bottom corners. This yields $Q = 2q \sin 30° = q$ and thus to the conclusion mentioned above.

76. Equation 22-38 gives $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. We note that $\theta_i = 110°$ and $\theta_f = 70.0°$. Therefore,

$$\Delta U = -pE\left(\cos 70.0° - \cos 110°\right) = -3.28\times10^{-21}\,\text{J}.$$

77. (a) Since the two charges in question are of the same sign, the point $x = 2.0$ mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be $x'$ ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at $x$ is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x' - x)^2$. We set the net field equal to zero:

$$\vec{E}_{\text{net}} = 0 \Rightarrow E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0\left(x'-x\right)^2}.$$

Thus, we obtain $x' = 3x = 3(2.0\text{ mm}) = 6.0$ mm.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative $x$ direction, when evaluated at $x$ = 2.0 mm. Therefore, the net field points in the negative $x$ direction, or 180°, measured counterclockwise from the $+x$ axis.

78. Let $q_1$ denote the charge at $y = d$ and $q_2$ denote the charge at $y = -d$. The individual magnitudes $|\vec{E}_1|$ and $|\vec{E}_2|$ are figured from Eq. 22-3, where the absolute value signs for $q$ are unnecessary since these charges are both positive. The distance from $q_1$ to a point on the $x$ axis is the same as the distance from $q_2$ to a point on the $x$ axis: $r = \sqrt{x^2 + d^2}$. By symmetry, the $y$ component of the net field along the $x$ axis is zero. The $x$ component of the net field, evaluated at points on the positive $x$ axis, is

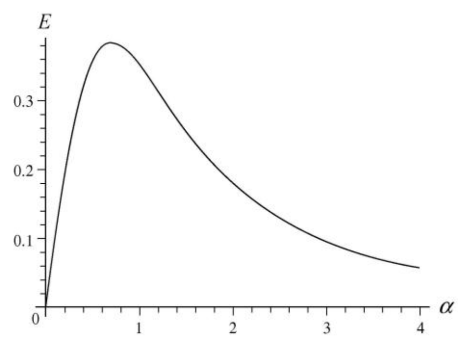
$$E_x = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{x^2+d^2}\right)\left(\frac{x}{\sqrt{x^2+d^2}}\right)$$

where the last factor is $\cos\theta = x/r$ with $\theta$ being the angle for each individual field as measured from the $x$ axis.

(a) If we simplify the above expression, and plug in $x = \alpha d$, we obtain

$$E_x = \frac{q}{2\pi\varepsilon_0 d^2}\frac{\alpha}{(\alpha^2+1)^{3/2}}.$$

(b) The graph of $E = E_x$ versus $\alpha$ is shown below. For the purposes of graphing, we set $d$ = 1 m and $q = 5.56 \times 10^{-11}$ C.




(c) From the graph, we estimate $E_{max}$ occurs at about $\alpha$ = 0.71. More accurate computation shows that the maximum occurs at $\alpha = 1/\sqrt{2}$.

(d) The graph suggests that "half-height" points occur at $\alpha \approx 0.2$ and $\alpha \approx 2.0$. Further numerical exploration leads to the values: $\alpha$ = 0.2047 and $\alpha$ = 1.9864.

79. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock (–$q$) and seven o'clock (–7$q$) positions is clearly equivalent to that of a single –6$q$ charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock (–6$q$) and twelve o'clock (–12$q$) positions is the same as that due to a single –6$q$ charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock, … twelve o'clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points toward the nine-thirty position.

80. The magnitude of the dipole moment is given by $p = qd$, where $q$ is the positive charge in the dipole and $d$ is the separation of the charges. For the dipole described in the problem,

$$p = \left(1.60\times10^{-19}\ \text{C}\right)\left(4.30\times10^{-9}\ \text{m}\right) = 6.88\times10^{-28}\ \text{C}\cdot\text{m}.$$

The dipole moment is a vector that points from the negative toward the positive charge.

81. (a) Since $\vec{E}$ points down and we need an upward electric force (to cancel the downward pull of gravity), then we require the charge of the sphere to be negative. The magnitude of the charge is found by working with the absolute value of Eq. 22-28:

$$|q| = \frac{F}{E} = \frac{mg}{E} = \frac{4.4\text{N}}{150\ \text{N/C}} = 0.029\text{C},$$

or $q = -0.029$ C.

(b) The feasibility of this experiment may be studied by using Eq. 22-3 (using $k$ for $1/4\pi\varepsilon_0$). We have $E = k|q|/r^2$ with

$$\rho_{\text{sulfur}}\left(\frac{4}{3}\pi r^3\right) = m_{\text{sphere}}$$

Since the mass of the sphere is 4.4/9.8 ≈ 0.45 kg and the density of sulfur is about $2.1\times10^3$ kg/m$^3$ (see Appendix F), then we obtain

$$r = \left(\frac{3m_{\text{sphere}}}{4\pi\rho_{\text{sulfur}}}\right)^{1/3} = 0.037\,\text{m} \;\Rightarrow\; E = k\frac{|q|}{r^2} \approx 2\times10^{11}\ \text{N/C}$$

which is much too large a field to maintain in air.

82. We interpret the linear charge density, $\lambda = |Q|/L$, to indicate a positive quantity (so we can relate it to the magnitude of the field). Sample Problem 22.03 — "Electric field of a charged circular rod" illustrates the simplest approach to circular arc field problems.

Following the steps leading to Eq. 22-21, we see that the general result (for arcs that subtend angle $\theta$) is

$$E_{\text{arc}} = \frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(\theta/2) - \sin(-\theta/2)\right] = \frac{2\lambda\sin(\theta/2)}{4\pi\varepsilon_0 r}.$$

Now, the arc length is $L = r\theta$ with $\theta$ is expressed in radians. Thus, using $R$ instead of $r$, we obtain

$$E_{\text{arc}} = \frac{2(|Q|/L)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2(|Q|/R\theta)\sin(\theta/2)}{4\pi\varepsilon_0 R} = \frac{2|Q|\sin(\theta/2)}{4\pi\varepsilon_0 R^2\theta}.$$

With $|Q| = 6.25\times10^{-12}$ C, $\theta = 2.40$ rad $= 137.5°$, and $R = 9.00\times10^{-2}$ m, the magnitude of the electric field is $E = 5.39$ N/C.

83. **THINK** The potential energy of the electric dipole placed in an electric field depends on its orientation relative to the electric field. The field causes a torque that tends to align the dipole with the field.

**EXPRESS** When placed in an electric field $\vec{E}$, the potential energy of the dipole $\vec{p}$ is given by Eq. 22-38:

$$U(\theta) = -\vec{p}\cdot\vec{E} = -pE\cos\theta.$$

The torque caused by the electric field is (see Eq. 22-34) $\vec{\tau} = \vec{p}\times\vec{E}$.

**ANALYZE** (a) From Eq. 22-38 (and the facts that $\hat{i}\cdot\hat{i} = 1$ and $\hat{j}\cdot\hat{i} = 0$), the potential energy is

$$\begin{aligned} U &= -\vec{p}\cdot\vec{E} = -\left[\left(3.00\hat{i} + 4.00\hat{j}\right)\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\right]\cdot\left[\left(4000\,\text{N/C}\right)\hat{i}\right] \\ &= -1.49\times10^{-26}\ \text{J}. \end{aligned}$$

(b) From Eq. 22-34 (and the facts that $\hat{i}\times\hat{i} = 0$ and $\hat{j}\times\hat{i} = -\hat{k}$), the torque is

$$\vec{\tau} = \vec{p}\times\vec{E} = \left[\left(3.00\hat{i} + 4.00\hat{j}\right)\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\right]\times\left[\left(4000\,\text{N/C}\right)\hat{i}\right] = \left(-1.98\times10^{-26}\,\text{N}\cdot\text{m}\right)\hat{k}.$$

(c) The work done is

$$\begin{aligned} W &= \Delta U = \Delta\left(-\vec{p}\cdot\vec{E}\right) = \left(\vec{p}_i - \vec{p}_f\right)\cdot\vec{E} \\ &= \left(3.00\hat{i} + 4.00\hat{j}\right) - \left(-4.00\hat{i} + 3.00\hat{j}\right)\left(1.24\times10^{-30}\,\text{C}\cdot\text{m}\right)\cdot\left(4000\,\text{N/C}\right)\hat{i} \\ &= 3.47\times10^{-26}\ \text{J}. \end{aligned}$$

**LEARN** The work done by the agent is equal to the change in the potential energy of the dipole.

84. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m$, where $E$ is the magnitude of the field and $m$ is the mass of the electron. Its numerical value is

$$a = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.00\times10^{3}\,\text{N/C}\right)}{9.11\times10^{-31}\,\text{kg}} = 3.51\times10^{14}\ \text{m/s}^2 .$$

We put the origin of a coordinate system at the initial position of the electron. We take the $x$ axis to be horizontal and positive to the right; take the $y$ axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_0 t\cos\theta,\quad y = v_0 t\sin\theta - \frac{1}{2}at^2,\qquad \text{and}\qquad v_y = v_0\sin\theta - at.$$

First, we find the greatest $y$ coordinate attained by the electron. If it is less than $d$, the electron does not hit the upper plate. If it is greater than $d$, it will hit the upper plate if the corresponding $x$ coordinate is less than $L$. The greatest $y$ coordinate occurs when $v_y = 0$. This means $v_0 \sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$y_{\max} = \frac{v_0^2\sin^2\theta}{a} - \frac{1}{2}a\frac{v_0^2\sin^2\theta}{a^2} = \frac{1}{2}\frac{v_0^2\sin^2\theta}{a} = \frac{\left(6.00\times10^{6}\ \text{m/s}\right)^2\sin^2 45^\circ}{2\left(3.51\times10^{14}\ \text{m/s}^2\right)}$$
$$= 2.56\times10^{-2}\,\text{m}.$$

Since this is greater than $d$ = 2.00 cm, the electron might hit the upper plate.

(b) Now, we find the $x$ coordinate of the position of the electron when $y = d$. Since

$$v_0\sin\theta = \left(6.00\times10^{6}\ \text{m/s}\right)\sin 45^\circ = 4.24\times10^{6}\ \text{m/s}$$

and

$$2ad = 2\left(3.51\times10^{14}\ \text{m/s}^2\right)\left(0.0200\,\text{m}\right) = 1.40\times10^{13}\ \text{m}^2/\text{s}^2$$

the solution to $d = v_0 t\sin\theta - \frac{1}{2}at^2$ is

$$t = \frac{v_0\sin\theta - \sqrt{v_0^2\sin^2\theta - 2ad}}{a} = \frac{(4.24\times10^{6}\ \text{m/s}) - \sqrt{\left(4.24\times10^{6}\ \text{m/s}\right)^2 - 1.40\times10^{13}\ \text{m}^2/\text{s}^2}}{3.51\times10^{14}\ \text{m/s}^2}$$
$$= 6.43\times10^{-9}\,\text{s}.$$

The negative root was used because we want the *earliest* time for which $y = d$. The $x$ coordinate is

$$x = v_0 t\cos\theta = \left(6.00\times10^{6}\ \text{m/s}\right)\left(6.43\times10^{-9}\,\text{s}\right)\cos 45° = 2.72\times10^{-2}\,\text{m}.$$

This is less than $L$ so the electron hits the upper plate at $x = 2.72$ cm.

85. (a) If we subtract each value from the next larger value in the table, we find a set of numbers that are suggestive of a basic unit of charge: $1.64 \times 10^{-19}$, $3.3 \times 10^{-19}$, $1.63 \times 10^{-19}$, $3.35 \times 10^{-19}$, $1.6 \times 10^{-19}$, $1.63 \times 10^{-19}$, $3.18 \times 10^{-19}$, $3.24 \times 10^{-19}$, where the SI unit Coulomb is understood. These values are either close to a common $e \approx 1.6\times10^{-19}\,\text{C}$ value or are double that. Taking this, then, as a crude approximation to our experimental $e$ we divide it into all the values in the original data set and round to the nearest integer, obtaining $n$ = 4, 5, 7, 8,10, 11, 12, 14, and 16.

(b) When we perform a least squares fit of the original data set versus these values for $n$ we obtain the linear equation:

$$q = 7.18 \times 10^{-21} + 1.633 \times 10^{-19} n\ .$$

If we dismiss the constant term as unphysical (representing, say, systematic errors in our measurements) then we obtain $e = 1.63 \times 10^{-19}$ when we set $n = 1$ in this equation.

86. (a) From symmetry, we see the net force component along the $y$ axis is zero.

(b) The net force component along the $x$ axis points rightward. With $\theta = 60°$,

$$F_3 = 2\frac{q_3 q_1 \cos\theta}{4\pi\varepsilon_0 a^2}\ .$$

Since cos(60°) =1/2, we can write this as

$$F_3 = \frac{k q_3 q_1}{a^2} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-12}\,\text{C})(2.00\times10^{-12}\,\text{C})}{(0.0950\ \text{m})^2} = 9.96\times10^{-12}\,\text{N}.$$

87. (a) For point $A$, we have (in SI units)

$$\vec{E}_A = \left[\frac{q_1}{4\pi\varepsilon_0 r_1^2} + \frac{q_2}{4\pi\varepsilon_0 r_2^2}\right](-\hat{\text{i}}) = \frac{(8.99\times10^{9})(1.00\times10^{-12}\,\text{C})}{\left(5.00\times10^{-2}\right)^2}(-\hat{\text{i}}) + \frac{(8.99\times10^{9})\,|-2.00\times10^{-12}\,\text{C}|}{\left(2\times5.00\times10^{-2}\right)^2}(+\hat{\text{i}})$$
$$= (-1.80\ \text{N/C})\hat{\text{i}}.$$

(b) Similar considerations leads to

$$\vec{E}_B=\left[\frac{q_1}{4\pi\varepsilon_0 r_1^2}+\frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\text{i}}=\frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\text{C}\right)}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\text{i}}+\frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\text{C}|}{\left(0.500\times5.00\times10^{-2}\right)^2}\hat{\text{i}}$$

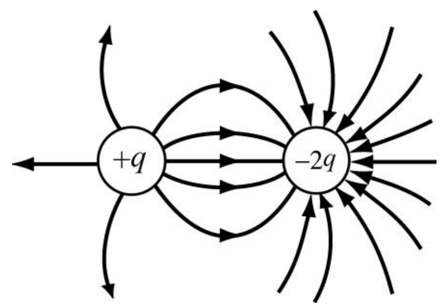
$$=(43.2\,\text{N/C})\hat{\text{i}}.$$

(c) For point C, we have

$$\vec{E}_C=\left[\frac{q_1}{4\pi\varepsilon_0 r_1^2}-\frac{|q_2|}{4\pi\varepsilon_0 r_2^2}\right]\hat{\text{i}}=\frac{\left(8.99\times10^9\right)\left(1.00\times10^{-12}\,\text{C}\right)}{\left(2.00\times5.00\times10^{-2}\right)^2}\hat{\text{i}}-\frac{\left(8.99\times10^9\right)|-2.00\times10^{-12}\,\text{C}|}{\left(5.00\times10^{-2}\right)^2}\hat{\text{i}}$$

$$=-(6.29\,\text{N/C})\hat{\text{i}}.$$

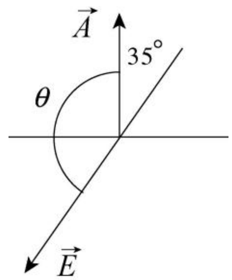
(d) The field lines are shown to the right. Note that there are twice as many field lines "going into" the negative charge $-2q$ as compared to that flowing out from the positive charge $+q$.



# Chapter 23

1. **THINK** This exercise deals with electric flux through a square surface.

**EXPRESS** The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram below.




The electric flux through the surface is given by $\Phi = \vec{E} \cdot \vec{A} = EA\cos\theta.$

**EXPRESS** The angle $\theta$ between $\vec{A}$ and $\vec{E}$ is 180° – 35° = 145°, so the electric flux through the area is

$$\Phi = EA\cos\theta = (1800 \text{ N/C})(3.2\times10^{-3} \text{ m})^2 \cos 145° = -1.5\times10^{-2} \text{ N}\cdot\text{m}^2/\text{C}.$$

**LEARN** The flux is a maximum when $\vec{A}$ and $\vec{E}$ points in the same direction ($\theta = 0$), and is zero when the two vectors are perpendicular to each other ($\theta = 90$).

2. We use $\Phi = \int \vec{E} \cdot d\vec{A}$ and note that the side length of the cube is (3.0 m–1.0 m) = 2.0 m.

(a) On the top face of the cube $y$ = 2.0 m and $d\vec{A} = (dA)\hat{\text{j}}$. Therefore, we have $\vec{E} = 4\hat{\text{i}} - 3\left((2.0)^2 + 2\right)\hat{\text{j}} = 4\hat{\text{i}} - 18\hat{\text{j}}$. Thus the flux is

$$\Phi = \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} \left(4\hat{\text{i}} - 18\hat{\text{j}}\right)\cdot(dA)\hat{\text{j}} = -18\int_{\text{top}} dA = (-18)(2.0)^2 \text{ N}\cdot\text{m}^2/\text{C} = -72 \text{ N}\cdot\text{m}^2/\text{C}.$$

(b) On the bottom face of the cube $y$ = 0 and $d\vec{A} = (dA)\left(-\hat{\text{j}}\right)$. Therefore, we have $E = 4\hat{\text{i}} - 3\left(0^2 + 2\right)\hat{\text{j}} = 4\hat{\text{i}} - 6\hat{\text{j}}$. Thus, the flux is

$$\Phi = \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} \left(4\hat{\text{i}} - 6\hat{\text{j}}\right)\cdot(dA)\left(-\hat{\text{j}}\right) = 6\int_{\text{bottom}} dA = 6(2.0)^2 \text{ N}\cdot\text{m}^2/\text{C} = +24 \text{ N}\cdot\text{m}^2/\text{C}.$$

(c) On the left face of the cube $d\vec{A} = (dA)(-\hat{\text{i}})$. So

$$\Phi = \int_{\text{left}} \hat{E} \cdot d\vec{A} = \int_{\text{left}} \left(4\hat{\text{i}} + E_y \hat{\text{j}}\right) \cdot (dA)\left(-\hat{\text{i}}\right) = -4\int_{\text{bottom}} dA = -4(2.0)^2 \text{ N}\cdot\text{m}^2/\text{C} = -16\,\text{N}\cdot\text{m}^2/\text{C}.$$

(d) On the back face of the cube $d\vec{A} = (dA)(-\hat{\text{k}})$. But since $\vec{E}$ has no $z$ component $\vec{E} \cdot d\vec{A} = 0$. Thus, $\Phi = 0$.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or +16 $\text{N}\cdot\text{m}^2/\text{C}$. Thus the net flux through the cube is

$$\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\ \text{N}\cdot\text{m}^2/\text{C} = -48\ \text{N}\cdot\text{m}^2/\text{C}.$$

3. We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\hat{\text{j}} = (1.40\text{m})^2\,\hat{\text{j}}$.

(a) $\Phi = (6.00\ \text{N/C})\hat{\text{i}} \cdot (1.40\ \text{m})^2\,\hat{\text{j}} = 0.$

(b) $\Phi = (-2.00\ \text{N/C})\hat{\text{j}} \cdot (1.40\ \text{m})^2\,\hat{\text{j}} = -3.92\ \text{N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi = \left[(-3.00\ \text{N/C})\hat{\text{i}} + (400\,\text{N/C})\hat{\text{k}}\right] \cdot (1.40\ \text{m})^2\,\hat{\text{j}} = 0$.

(d) The total flux of a uniform field through a closed surface is always zero.

4. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E.$ Thus, the flux through the netting is

$$\Phi' = -\Phi = -\pi a^2 E = -\pi(0.11\ \text{m})^2(3.0\times10^{-3}\ \text{N/C}) = -1.1\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C}.$$

5. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length $d$, with a proton of charge $q = +1.6\times10^{-19}$ C situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{\text{net}} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus,

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{1.6\times10^{-19}\ \text{C}}{6(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.01\times10^{-9}\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

6. There is no flux through the sides, so we have two "inward" contributions to the flux, one from the top (of magnitude $(34)(3.0)^2$) and one from the bottom (of magnitude

(20)(3.0)$^2$). With "inward" flux being negative, the result is $\Phi$ = – 486 N·m$^2$/C. Gauss' law then leads to

$$q_{\text{enc}} = \varepsilon_0 \Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-486\ \ \text{N}\cdot\text{m}^2/\text{C}) = -4.3\times10^{-9}\,\text{C}.$$

7. We use Gauss' law: $\varepsilon_0\Phi = q$, where $\Phi$ is the total flux through the cube surface and $q$ is the net charge inside the cube. Thus,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.8\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \ \text{C}^2/\text{N}\cdot\text{m}^2} = 2.0\times10^5\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

8. (a) The total surface area bounding the bathroom is

$$A = 2(2.5\times3.0) + 2(3.0\times2.0) + 2(2.0\times2.5) = 37\ \text{m}^2.$$

The absolute value of the total electric flux, with the assumptions stated in the problem, is

$$|\Phi| = |\sum\vec{E}\cdot\vec{A}| = |\vec{E}|\,A = (600\ \text{N/C})(37\ \text{m}^2) = 22\times10^3\ \text{N}\cdot\text{m}^2/\text{C}.$$

By Gauss' law, we conclude that the enclosed charge (in absolute value) is $|q_{\text{enc}}| = \varepsilon_0|\Phi| = 2.0\times10^{-7}$ C. Therefore, with volume $V$ = 15 m$^3$, and recognizing that we are dealing with negative charges, the charge density is

$$\rho = \frac{q_{\text{enc}}}{V} = \frac{-2.0\times10^{-7}\ \text{C}}{15\ \text{m}^3} = -1.3\times10^{-8}\ \text{C/m}^3.$$

(b) We find $(|q_{\text{enc}}|/e)/V$ = (2.0 × 10$^{-7}$ C/1.6 × 10$^{-19}$ C)/15 m$^3$ = 8.2 × 10$^{10}$ excess electrons per cubic meter.

9. (a) Let $A$ = (1.40 m)$^2$. Then

$$\Phi = \left(3.00y\,\hat{\text{j}}\right)\cdot\left(-A\,\hat{\text{j}}\right)\Big|_{y=0} + \left(3.00y\,\hat{\text{j}}\right)\cdot\left(\text{A}\,\hat{\text{j}}\right)\Big|_{y=1.40} = (3.00)(1.40)(1.40)^2 = 8.23\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

(b) The charge is given by

$$q_{\text{enc}} = \varepsilon_0\Phi = \left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

(c) The electric field can be re-written as $\vec{E} = 3.00y\,\hat{\text{j}} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\text{i}} + 6.00\hat{\text{j}}$ is a constant field which does not contribute to the net flux through the cube. Thus $\Phi$ is still 8.23 N·m$^2$/C.

(d) The charge is again given by

$$q_{enc} = \varepsilon_0 \Phi = \left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(8.23\ \text{N}\cdot\text{m}^2/\text{C}\right) = 7.29\times10^{-11}\ \text{C}.$$

10. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the $x$ dependent term only. In Si units, we have

$$E_{\text{nonconstant}} = 3x\,\hat{\text{i}}\ .$$

The face of the cube located at $x = 0$ (in the $yz$ plane) has area $A = 4\ \text{m}^2$ (and it "faces" the $+\hat{\text{i}}$ direction) and has a "contribution" to the flux equal to $E_{\text{nonconstant}}A = (3)(0)(4) = 0$. The face of the cube located at $x = -2$ m has the same area $A$ (and this one "faces" the $-\hat{\text{i}}$ direction) and a contribution to the flux:

$$-E_{\text{nonconstant}}A = -(3)(-2)(4) = 24\ \text{N}\cdot\text{m/C}^2.$$

Thus, the net flux is $\Phi = 0 + 24 = 24\ \text{N}\cdot\text{m/C}^2$. According to Gauss' law, we therefore have $q_{enc} = \varepsilon_o\Phi = 2.13 \times 10^{-10}$ C.

11. None of the constant terms will result in a nonzero contribution to the flux (see Eq. 23-4 and Eq. 23-7), so we focus on the $x$ dependent term only:

$$E_{\text{nonconstant}} = (-4.00y^2)\ \hat{\text{i}}\ \text{(in SI units)}\ .$$

The face of the cube located at $y = 4.00$ has area $A = 4.00\ \text{m}^2$ (and it "faces" the $+\hat{\text{j}}$ direction) and has a "contribution" to the flux equal to

$$E_{\text{nonconstant}}A = (-4)(4^2)(4) = -256\ \text{N}\cdot\text{m/C}^2.$$

The face of the cube located at $y = 2.00$ m has the same area $A$ (however, this one "faces" the $-\hat{\text{j}}$ direction) and a contribution to the flux:

$$-E_{\text{nonconstant}}A = -\ (-4)(2^2)(4) = 64\ \text{N}\cdot\text{m/C}^2.$$

Thus, the net flux is $\Phi = (-256 + 64)\ \text{N}\cdot\text{m/C}^2 = -192\ \text{N}\cdot\text{m/C}^2$. According to Gauss's law, we therefore have

$$q_{enc} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-192\ \text{N}\cdot\text{m}^2/\text{C}) = -1.70\times10^{-9}\text{C}.$$

12. We note that only the smaller shell contributes a (nonzero) field at the designated point, since the point is inside the radius of the large sphere (and $E = 0$ inside of a spherical charge), and the field points toward the $-x$ direction. Thus, with $R = 0.020$ m (the radius of the smaller shell), $L = 0.10$ m and $x = 0.020$ m, we obtain

$$\vec{E}=E(-\hat{\text{j}})=-\frac{q}{4\pi\varepsilon_0 r^2}\,\hat{\text{j}}=-\frac{4\pi R^2\sigma_2}{4\pi\varepsilon_0(L-x)^2}\,\hat{\text{j}}=-\frac{R^2\sigma_2}{\varepsilon_0(L-x)^2}\,\hat{\text{j}}$$

$$=-\frac{(0.020\text{ m})^2(4.0\times10^{-6}\text{C/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.10\text{ m}-0.020\text{ m})^2}\,\hat{\text{j}}=\left(-2.8\times10^4\text{ N/C}\right)\hat{\text{j}}.$$

13. **THINK** A cube has six surfaces. The total flux through the cube is the sum of fluxes through each individual surface. We use Gauss' law to find the net charge inside the cube.

**EXPRESS** Let $A$ be the area of one face of the cube, $E_u$ be the magnitude of the electric field at the upper face, and $E_l$ be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero (because their area vectors are parallel to the field), so the total flux through the cube surface is

$$\Phi = A(E_\ell - E_u).$$

The net charge inside the cube is given by Gauss' law: $q=\varepsilon_0\Phi.$

**ANALYZE** Substituting the values given, we find the net charge to be

$$q=\varepsilon_0\Phi=\varepsilon_0 A(E_\ell-E_u)=(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(100\text{ m})^2(100\text{ N/C}-60.0\text{ N/C})$$
$$=3.54\times10^{-6}\text{ C}=3.54\,\mu\text{C}.$$

**LEARN** Since $\Phi>0,$ we conclude that the cube encloses a net positive charge.

14. Equation 23-6 (Gauss' law) gives $\varepsilon_0\Phi = q_{\text{enc}}$ .

(a) Thus, the value $\Phi=2.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$ for small $r$ leads to

$$q_{\text{central}}=\varepsilon_0\Phi=(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.0\times10^5\text{ N}\cdot\text{m}^2/\text{C})=1.77\times10^{-6}\text{C}\approx1.8\times10^{-6}\text{C}.$$

(b) The next value that $\Phi$ takes is $\Phi=-4.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$ , which implies that $q_{\text{enc}}=-3.54\times10^{-6}\text{C}.$ But we have already accounted for some of that charge in part (a), so the result for part (b) is

$$q_A = q_{\text{enc}} - q_{\text{central}} = -5.3\times10^{-6}\text{ C}.$$

(c) Finally, the large $r$ value for $\Phi$ is $\Phi=6.0\times10^5\text{ N}\cdot\text{m}^2/\text{C}$ , which implies that $q_{\text{total enc}}=5.31\times10^{-6}\text{C}.$ Considering what we have already found, then the result is $q_{\text{total enc}}-q_A-q_{\text{central}}=+8.9\mu C.$

15. The total flux through any surface that completely surrounds the point charge is $q/\varepsilon_0$.

(a) If we stack identical cubes side by side and directly on top of each other, we will find that eight cubes meet at any corner. Thus, one-eighth of the field lines emanating from the point charge pass through a cube with a corner at the charge, and the total flux through the surface of such a cube is $q/8\varepsilon_0$. Now the field lines are radial, so at each of the three cube faces that meet at the charge, the lines are parallel to the face and the flux through the face is zero.

(b) The fluxes through each of the other three faces are the same, so the flux through each of them is one-third of the total. That is, the flux through each of these faces is $(1/3)(q/8\varepsilon_0) = q/24\varepsilon_0$. Thus, the multiple is 1/24 = 0.0417.

16. The total electric flux through the cube is $\Phi = \oint \vec{E} \cdot d\vec{A}$. The net flux through the two faces parallel to the *yz* plane is

$$\Phi_{yz} = \iint \left[ E_x(x=x_2) - E_x(x=x_1) \right] dydz = \int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz \left[ 10 + 2(4) - 10 - 2(1) \right]$$
$$= 6 \int_{y_1=0}^{y_2=1} dy \int_{z_1=1}^{z_2=3} dz = 6(1)(2) = 12.$$

Similarly, the net flux through the two faces parallel to the *xz* plane is

$$\Phi_{xz} = \iint \left[ E_y(y=y_2) - E_y(y=y_1) \right] dxdz = \int_{x_1=1}^{x_2=4} dy \int_{z_1=1}^{z_2=3} dz[-3-(-3)] = 0 ,$$

and the net flux through the two faces parallel to the *xy* plane is

$$\Phi_{xy} = \iint \left[ E_z(z=z_2) - E_z(z=z_1) \right] dxdy = \int_{x_1=1}^{x_2=4} dx \int_{y_1=0}^{y_2=1} dy \left( 3b - b \right) = 2b(3)(1) = 6b.$$

Applying Gauss' law, we obtain

$$q_{\text{enc}} = \varepsilon_0 \Phi = \varepsilon_0 (\Phi_{xy} + \Phi_{xz} + \Phi_{yz}) = \varepsilon_0 (6.00b + 0 + 12.0) = 24.0\varepsilon_0$$

which implies that $b = 2.00 \ \text{N/C} \cdot \text{m}$.

17. **THINK** The system has spherical symmetry, so our Gaussian surface is a sphere of radius $R$ with a surface area $A = 4\pi R^2$.

**EXPRESS** The charge on the surface of the sphere is the product of the surface charge density $\sigma$ and the surface area of the sphere: $q = \sigma A = \sigma(4\pi R^2)$. We calculate the total electric flux leaving the surface of the sphere using Gauss' law: $q = \varepsilon_0 \Phi$.

**ANALYZE** (a) With $R = (1.20 \text{ m})/2 = 0.60 \text{ m}$ and $\sigma = 8.1 \times 10^{-6} \text{ C/m}^2$, the charge on the surface is

$$q = 4\pi R^2\sigma = 4\pi\left(0.60 \text{ m}\right)^2 (8.1\times10^{-6} \text{ C/m}^2) = 3.7\times10^{-5} \text{ C}.$$

(b) We choose a Gaussian surface in the form of a sphere, concentric with the conducting sphere and with a slightly larger radius. By Gauss's law, the flux is

$$\Phi = \frac{q}{\varepsilon_0} = \frac{3.66\times10^{-5} \text{ C}}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = 4.1\times10^6 \text{ N}\cdot\text{m}^2/\text{C}.$$

**LEARN** Since there is no charge inside the conducting sphere, the total electric flux through the surface of the sphere only depends on the charge residing on the surface of the sphere.

18. Using Eq. 23-11, the surface charge density is

$$\sigma = E\varepsilon_0 = \left(2.3\times10^5 \text{ N/C}\right)\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2\right) = 2.0\times10^{-6} \text{ C/m}^2.$$

19. (a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4\times10^{-6} \text{ C}}{\pi\left(1.3 \text{ m}\right)^2} = 4.5\times10^{-7} \text{ C/m}^2.$$

(b) Equation 23-11 gives

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.5\times10^{-7} \text{ C/m}^2}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = 5.1\times10^4 \text{ N/C}.$$

20. Equation 23-6 (Gauss' law) gives $\varepsilon_o\Phi = q_{enc}$.

(a) The value $\Phi = -9.0\times10^5 \text{ N}\cdot\text{m}^2/\text{C}$ for small $r$ leads to $q_{central} = -7.97\times10^{-6}$ C or roughly – 8.0 $\mu$C.

(b) The next (nonzero) value that Φ takes is $\Phi = +4.0\times10^5 \text{ N}\cdot\text{m}^2/\text{C}$, which implies $q_{enc} = 3.54\times10^{-6}$C. But we have already accounted for some of that charge in part (a), so the result is

$$q_A = q_{enc} - q_{central} = 11.5\times10^{-6} \text{ C} \approx 12 \ \mu\text{C}.$$

(c) Finally, the large $r$ value for Φ is $\Phi = -2.0\times10^5 \text{ N}\cdot\text{m}^2/\text{C}$, which implies $q_{total\ enc} = -1.77\times10^{-6}$C. Considering what we have already found, then the result is

$$q_{total\ enc} - q_A - q_{central} = -5.3\ \mu\text{C}.$$

21. (a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it

encloses is zero. The net charge is the sum of the charge $q$ in the cavity and the charge $q_w$ on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0 \times 10^{-6}$C.

(b) The net charge $Q$ of the conductor is the sum of the charge on the cavity wall and the charge $q_s$ on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_{\omega} = \left(10\times10^{-6}\ \text{C}\right) - \left(-3.0\times10^{-6}\ \text{C}\right) = +1.3\times10^{-5}\ \text{C}.$$

22. We combine Newton's second law ($F = ma$) with the definition of electric field ($F = qE$) and with Eq. 23-12 (for the field due to a line of charge). In terms of magnitudes, we have (if $r = 0.080$ m and $\lambda = 6.0\times10^{-6}\,\text{C/m}$)

$$ma = eE = \frac{e\lambda}{2\pi\varepsilon_0 r} \quad\Rightarrow\quad a = \frac{e\lambda}{2\pi\varepsilon_0 r\, m} = 2.1\times10^{17}\ \text{m/s}^2\ .$$

23. (a) The side surface area $A$ for the drum of diameter $D$ and length $h$ is given by $A = \pi Dh$. Thus,

$$\begin{aligned} q &= \sigma A = \sigma\pi Dh = \pi\varepsilon_0 EDh \\ &= \pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.3\times10^5\ \text{N/C}\right)\left(0.12\ \text{m}\right)\left(0.42\ \text{m}\right) \\ &= 3.2\times10^{-7}\ \text{C}. \end{aligned}$$

(b) The new charge is

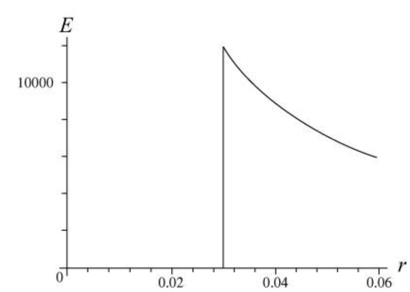
$$q' = q\left(\frac{A'}{A}\right) = q\left(\frac{\pi D'h'}{\pi Dh}\right) = \left(3.2\times10^{-7}\text{C}\right)\left[\frac{(8.0\ \text{cm})(28\ \text{cm})}{(12\ \text{cm})(42\ \text{cm})}\right] = 1.4\times10^{-7}\ \text{C}.$$

24. We imagine a cylindrical Gaussian surface $A$ of radius $r$ and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E}\cdot d\vec{A} = 2\pi rE = \frac{q_{\text{enc}}}{\varepsilon_0}$.

(a) For $r < R$, $q_{\text{enc}} = 0$, so $E = 0$.

(b) For $r > R$, $q_{\text{enc}} = \lambda$, so $E(r) = \lambda / 2\pi r\varepsilon_0$. With $\lambda = 2.00\times10^{-8}\ \text{C/m}$ and $r = 2.00R = 0.0600$ m, we obtain

$$E = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.0600\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 5.99\times10^3\ \text{N/C}.$$




(c) The plot of $E$ vs. $r$ is shown to the right. Here, the maximum value is

$$E_{\max} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{\left(2.0\times10^{-8}\ \text{C/m}\right)}{2\pi\left(0.030\ \text{m}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.2\times10^4\ \text{N/C}.$$

25. **THINK** Our system is an infinitely long line of charge. Since the system possesses cylindrical symmetry, we may apply Gauss' law and take the Gaussian surface to be in the form of a closed cylinder.

**EXPRESS** We imagine a cylindrical Gaussian surface $A$ of radius $r$ and length $h$ concentric with the metal tube. Then by symmetry,

$$\oint_A \vec{E}\cdot d\vec{A} = 2\pi rhE = \frac{q}{\varepsilon_0},$$

where $q$ is the amount of charge enclosed by the Gaussian cylinder. Thus, the magnitude of the electric field produced by a uniformly charged infinite line is

$$E = \frac{q/h}{2\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where $\lambda$ is the linear charge density and $r$ is the distance from the line to the point where the field is measured.

**ANALYZE** Substituting the values given, we have

$$\lambda = 2\pi\varepsilon_0 Er = 2\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.5\times10^4\ \text{N/C}\right)\left(2.0\ \text{m}\right)$$
$$= 5.0\times10^{-6}\ \text{C/m}.$$

**LEARN** Since $\lambda > 0$, the direction of $\vec{E}$ is radially outward from the line of charge. Note that the field varies with $r$ as $E \sim 1/r$, in contrast to the $1/r^2$ dependence due to a point charge.

26. As we approach $r$ = 3.5 cm from the inside, we have

$$E_{\text{internal}} = \frac{2\lambda}{4\pi\varepsilon_0 r} = 1000\ \text{N/C}.$$

And as we approach $r$ = 3.5 cm from the outside, we have

$$E_{\text{external}} = \frac{2\lambda}{4\pi\varepsilon_0 r} + \frac{2\lambda'}{4\pi\varepsilon_0 r} = -3000\ \text{N/C}\ .$$

Considering the difference ($E_{\text{external}} - E_{\text{internal}}$) allows us to find $\lambda'$ (the charge per unit length on the larger cylinder). Using $r$ = 0.035 m, we obtain $\lambda' = -5.8 \times 10^{-9}$ C/m.

27. We denote the radius of the thin cylinder as $R$ = 0.015 m. Using Eq. 23-12, the net electric field for $r > R$ is given by

$$E_{\text{net}} = E_{\text{wire}} + E_{\text{cylinder}} = \frac{-\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda'}{2\pi\varepsilon_0 r}$$

where $-\lambda$ = –3.6 nC/m is the linear charge density of the wire and $\lambda'$ is the linear charge density of the thin cylinder. We note that the surface and linear charge densities of the thin cylinder are related by

$$q_{\text{cylinder}} = \lambda' L = \sigma(2\pi RL) \Rightarrow \lambda' = \sigma(2\pi R).$$

Now, $E_{\text{net}}$ outside the cylinder will equal zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6\times10^{-6}\ \text{C/m}}{(2\pi)(0.015\ \text{m})} = 3.8\times10^{-8}\ \text{C/m}^2.$$

28. (a) In Eq. 23-12, $\lambda = q/L$ where $q$ is the net charge enclosed by a cylindrical Gaussian surface of radius $r$. The field is being measured outside the system (the charged rod coaxial with the neutral cylinder) so that the net enclosed charge is only that which is on the rod. Consequently,

$$\left|\vec{E}\right| = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{2(2.0\times10^{-9}\text{C/m})}{4\pi\varepsilon_0(0.15\ \text{m})} = 2.4\times10^{2}\ \text{N/C}.$$

(b) Since the field is zero inside the conductor (in an electrostatic configuration), then there resides on the inner surface charge $-q$, and on the outer surface, charge $+q$ (where $q$ is the charge on the rod at the center). Therefore, with $r_i$ = 0.05 m, the surface density of charge is

$$\sigma_{\text{inner}} = \frac{-q}{2\pi r_i L} = -\frac{\lambda}{2\pi r_i} = -\frac{2.0\times10^{-9}\text{C/m}}{2\pi(0.050\ \text{m})} = -6.4\times10^{-9}\ \text{C/m}^2$$

for the inner surface.

(c) With $r_o$ = 0.10 m, the surface charge density of the outer surface is

$$\sigma_{\text{outer}} = \frac{+q}{2\pi r_o L} = \frac{\lambda}{2\pi r_o} = +3.2\times10^{-9}\ \text{C/m}^2.$$

29. **THINK** The charge densities of both the conducting cylinder and the shell are uniform, and we neglect fringing effect. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

**EXPRESS** We take the Gaussian surface to be a cylinder of length $L$, coaxial with the given cylinders and of radius $r$. The flux through this surface is $\Phi = 2\pi rLE$, where $E$ is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Gauss' law yields $q_{\text{enc}} = \varepsilon_0 \Phi = 2\pi r \varepsilon_0 LE$, where $q_{\text{enc}}$ is the charge enclosed by the Gaussian surface.

**ANALYZE** (a) In this case, we take the radius of our Gaussian cylinder to be

$$r = 2.00R_2 = 20.0R_1 = (20.0)(1.3\times10^{-3}\text{ m}) = 2.6\times10^{-2}\text{ m}.$$

The charge enclosed is

$$q_{\text{enc}} = Q_1 + Q_2 = -Q_1 = -3.40\times10^{-12}\text{ C}.$$

Consequently, Gauss' law yields

$$E = \frac{q_{\text{enc}}}{2\pi\varepsilon_0 Lr} = \frac{-3.40\times10^{-12}\text{ C}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(11.0\text{ m})(2.60\times10^{-2}\text{m})} = -0.214\text{ N/C},$$

or $|E| = 0.214$ N/C.

(b) The negative sign in $E$ indicates that the field points inward.

(c) Next, for $r$ = 5.00 $R_1$, the charge enclosed by the Gaussian surface is $q_{\text{enc}} = Q_1 = 3.40\times10^{-12}$ C. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = q_{\text{enc}}$, or

$$E = \frac{q_{\text{enc}}}{2\pi\varepsilon_0 Lr} = \frac{3.40\times10^{-12}\text{ C}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(11.0\text{ m})(5.00\times1.30\times10^{-3}\text{m})} = 0.855\text{ N/C}.$$

(d) The positive sign indicates that the field points outward.

(e) We consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod has charge $Q_1$, the inner surface of the shell must have charge $Q_{\text{in}} = -Q_1 = -3.40\times10^{-12}$ C.

(f) Since the shell is known to have total charge $Q_2 = -2.00Q_1$, it must have charge $Q_{\text{out}} = Q_2 - Q_{\text{in}} = -Q_1 = -3.40\times10^{-12}$ C on its outer surface.

**LEARN** Cylindrical symmetry of the system allows us to apply Gauss' law to the problem. Since electric field is zero inside the conducting shell, by Gauss' law, any net charge must be distributed on the surfaces of the shells.

30. We reason that point *P* (the point on the *x* axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that *P* is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for *P*. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{2\lambda_1}{4\pi\varepsilon_0(x+L/2)} + \frac{2\lambda_2}{4\pi\varepsilon_0(x-L/2)} \ .$$

Setting this equal to zero and solving for *x* we find

$$x = \left(\frac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2}\right)\frac{L}{2} = \left(\frac{6.0\mu\text{C/m}-(-2.0\mu\text{C/m})}{6.0\mu\text{C/m}+(-2.0\mu\text{C/m})}\right)\frac{8.0\text{ cm}}{2} = 8.0\text{ cm}\,.$$

31. We denote the inner and outer cylinders with subscripts *i* and *o*, respectively.

(a) Since $r_i < r = 4.0$ cm $< r_o$,

$$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\text{ C/m}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(4.0\times10^{-2}\text{ m})} = 2.3\times10^6\text{ N/C.}$$

(b) The electric field $\vec{E}(r)$ points radially outward.

(c) Since $r > r_o$,

$$E(r=8.0\text{ cm}) = \frac{\lambda_i+\lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times10^{-6}\text{ C/m}-7.0\times10^{-6}\text{ C/m}}{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(8.0\times10^{-2}\text{ m})} = -4.5\times10^5\text{ N/C,}$$

or $|E(r=8.0\text{ cm})| = 4.5\times10^5$ N/C.

(d) The minus sign indicates that $\vec{E}(r)$ points radially inward.

32. To evaluate the field using Gauss' law, we employ a cylindrical surface of area $2\pi r L$ where *L* is very large (large enough that contributions from the ends of the cylinder become irrelevant to the calculation). The volume within this surface is $V = \pi r^2 L$, or expressed more appropriate to our needs: $dV = 2\pi rLdr$. The charge enclosed is, with $A = 2.5\times10^{-6}\text{C/m}^5$,

$$q_{\text{enc}} = \int_0^r Ar^2\, 2\pi r\, L\, dr = \frac{\pi}{2}ALr^4.$$

By Gauss' law, we find $\Phi = |\vec{E}|(2\pi rL) = q_{\text{enc}}/\varepsilon_0$; we thus obtain $\left|\vec{E}\right| = \dfrac{Ar^3}{4\varepsilon_0}$.

(a) With $r$ = 0.030 m, we find $|\vec{E}| = 1.9$ N/C.

(b) Once outside the cylinder, Eq. 23-12 is obeyed. To find λ = $q/L$ we must find the total charge $q$. Therefore,

$$\frac{q}{L} = \frac{1}{L}\int_0^{0.04} Ar^2\, 2\pi r\, L\, dr = 1.0\times10^{-11}\ \text{C/m}.$$

And the result, for $r$ = 0.050 m, is $|\vec{E}| = \lambda/2\pi\varepsilon_0 r = 3.6$ N/C.

33. We use Eq. 23-13.

(a) To the left of the plates:

$$\vec{E} = \left(\sigma/2\varepsilon_0\right)(-\hat{\text{i}})\ \text{(from the right plate)} + (\sigma/2\varepsilon_0)\hat{\text{i}}\ \text{(from the left one)} = 0.$$

(b) To the right of the plates:

$$\vec{E} = \left(\sigma/2\varepsilon_0\right)\hat{\text{i}}\ \text{(from the right plate)} + \left(\sigma/2\varepsilon_0\right)(-\hat{\text{i}})\ \text{(from the left one)} = 0.$$

(c) Between the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\text{i}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)\left(-\hat{\text{i}}\right) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\hat{\text{i}}) = -\left(\frac{7.00\times10^{-22}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)\hat{\text{i}}$$
$$= \left(-7.91\times10^{-11}\ \text{N/C}\right)\hat{\text{i}}.$$

34. The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density $\sigma = 4.50\times10^{-12}$ C/m$^2$ plus a small circular pad of radius $R$ = 1.80 cm located at the middle of the sheet with charge density $-\sigma$. We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. Using Eq. 22-26 for $\vec{E}_2$, the net electric field $\vec{E}$ at a distance $z$ = 2.56 cm along the central axis is then

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\text{k}} + \frac{(-\sigma)}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{\text{k}} = \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}\,\hat{\text{k}}$$
$$= \frac{(4.50\times10^{-12}\ \text{C/m}^2)(2.56\times10^{-2}\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\sqrt{(2.56\times10^{-2}\ \text{m})^2+(1.80\times10^{-2}\ \text{m})^2}}\,\hat{\text{k}} = (0.208\ \text{N/C})\,\hat{\text{k}}.$$

35. In the region between sheets 1 and 2, the net field is $E_1 - E_2 + E_3 = 2.0 \times 10^5$ N/C .

In the region between sheets 2 and 3, the net field is at its greatest value:

$$E_1 + E_2 + E_3 = 6.0 \times 10^5 \text{ N/C} .$$

The net field vanishes in the region to the right of sheet 3, where $E_1 + E_2 = E_3$ . We note the implication that $\sigma_3$ is negative (and is the largest surface-density, in magnitude). These three conditions are sufficient for finding the fields:

$$E_1 = 1.0 \times 10^5 \text{ N/C}, \; E_2 = 2.0 \times 10^5 \text{ N/C}, \; E_3 = 3.0 \times 10^5 \text{ N/C} .$$

From Eq. 23-13, we infer (from these values of $E$)

$$\frac{|\sigma_3|}{|\sigma_2|} = \frac{3.0 \text{ x } 10^5 \text{ N/C}}{2.0 \text{ x } 10^5 \text{ N/C}} = 1.5.$$

Recalling our observation, above, about $\sigma_3$, we conclude that $\frac{\sigma_3}{\sigma_2} = -1.5$.

36. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density $\sigma = 1.77\times 10^{-22}$ C/m$^2$ is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0 = (1.77 \times 10^{-22}\text{ C/m}^2)/(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 2.00\times10^{-11}$ N/C, pointing in the upward direction, or $\vec{E} = (2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$;

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.00\times10^{-11}\text{ N/C})\hat{\text{j}}$.

37. **THINK** To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we replace the finite plate with an infinite plate having the same charge density. Planar symmetry then allows us to apply Gauss' law to calculate the electric field.

**EXPRESS** Using Gauss' law, we find the magnitude of the field to be $E = \sigma/\varepsilon_0$, where $\sigma$ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus, $\sigma = q/2A$.

**ANALYZE** (a) With $q = 6.0\times10^{-6}$ C and $A = (0.080 \text{ m})^2$, we obtain

$$\sigma = \frac{q}{2A} = \frac{6.0\times10^{-6}\ \text{C}}{2(0.080\ \text{m})^2} = 4.69\times10^{-4}\ \text{C/m}^2.$$

The magnitude of the field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.69\times10^{-4}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.3\times10^7\ \text{N/C}.$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where $r$ is the distance from the plate. Thus,

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\times10^{-6}\ \text{C}\right)}{\left(30\ \text{m}\right)^2} = 60\ \text{N/C}.$$

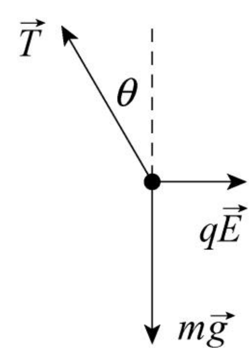
**LEARN** In summary, the electric field is nearly uniform ($E = \sigma/\varepsilon_0$) close to the plate, but resembles that of a point charge far away from the plate.

38. The field due to the sheet is $E = \frac{\sigma}{2\varepsilon_o}$. The force (in magnitude) on the electron (due to that field) is $F = eE$, and assuming it's the *only* force then the acceleration is

$$a = \frac{e\sigma}{2\varepsilon_o m} = \text{slope of the graph}\ \ (= 2.0\times10^5\ \text{m/s divided by } 7.0\times10^{-12}\ \text{s})\ .$$

Thus we obtain $\sigma = 2.9\times10^{-6}\ \text{C/m}^2$.

39. **THINK** Since the non-conducting charged ball is in equilibrium with the non-conducting charged sheet (see Fig. 23-49), both the vertical and horizontal components of the net force on the ball must be zero.




**EXPRESS** The forces acting on the ball are shown in the diagram to the right. The gravitational force has magnitude $mg$, where $m$ is the mass of the ball; the electrical force has magnitude $qE$, where $q$ is the charge on the ball and $E$ is the magnitude of the electric field at the position of the ball; and the tension in the thread is denoted by $T$. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle $\theta$ (= 30°) with the vertical. Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields

$$qE - T\sin\theta = 0$$

and the sum of the vertical components yields

$$T\cos\theta - mg = 0\,.$$

We solve for the electric field $E$ and deduce $\sigma$, the charge density of the sheet, from $E = \sigma/2\varepsilon_0$ (see Eq. 23-13).

**ANALYZE** The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform sheet of charge is given by $E = \sigma/2\varepsilon_0$, so

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and we have

$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^{-6}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\tan 30°}{2.0\times10^{-8}\ \text{C}}$$

$$= 5.0\times10^{-9}\ \text{C/m}^2.$$

**LEARN** Since both the sheet and the ball are positively charged, the force between them is repulsive. This is balanced by the horizontal component of the tension in the thread. The angle the thread makes with the vertical direction increases with the charge density of the sheet.

40. The point where the individual fields cancel cannot be in the region between the sheet and the particle ($-d < x < 0$) since the sheet and the particle have opposite-signed charges. The point(s) could be in the region to the right of the particle ($x > 0$) and in the region to the left of the sheet ($x < d$); this is where the condition

$$\frac{|\sigma|}{2\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

must hold. Solving this with the given values, we find $r = x = \pm\sqrt{3/2\pi} \approx \pm\ 0.691$ m.

If $d = 0.20$ m (which is less than the magnitude of $r$ found above), then neither of the points ($x \approx \pm\ 0.691$ m) is in the "forbidden region" between the particle and the sheet. Thus, both values are allowed. Thus, we have

(a) $x = 0.691$ m on the positive axis, and

(b) $x = -\,0.691$ m on the negative axis.

(c) If, however, $d$ = 0.80 m (greater than the magnitude of $r$ found above), then one of the points ($x \approx -0.691$ m) is in the "forbidden region" between the particle and the sheet and is disallowed. In this part, the fields cancel only at the point $x \approx +0.691$ m.

41. The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where $\sigma$ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where $m$ is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If $v_0$ is the initial velocity of the electron, $v$ is the final velocity, and $x$ is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace $a$ with $-e\sigma/\varepsilon_0 m$, then solve for $x$. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}.$$

Now $\frac{1}{2}mv_0^2$ is the initial kinetic energy $K_0$, so

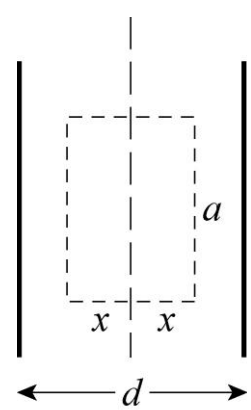
$$x = \frac{\varepsilon_0 K_0}{e\sigma} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.60\times10^{-17}\ \text{J}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.0\times10^{-6}\ \text{C/m}^2\right)} = 4.4\times10^{-4}\ \text{m}.$$

42. The surface charge density is given by

$$E = \sigma/\varepsilon_0 \Rightarrow \sigma = \varepsilon_0 E = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(55\ \text{N/C}) = 4.9\times10^{-10}\ \text{C/m}^2.$$

Since the area of the plates is $A = 1.0\ \text{m}^2$, the magnitude of the charge on the plate is $Q = \sigma A = 4.9\times10^{-10}$ C.

43. We use a Gaussian surface in the form of a box with rectangular sides. The cross section is shown with dashed lines in the diagram to the right. It is centered at the central plane of the slab, so the left and right faces are each a distance $x$ from the central plane. We take the thickness of the rectangular solid to be $a$, the same as its length, so the left and right faces are squares.



The electric field is normal to the left and right faces and is uniform over them. Since $\rho$ = 5.80 fC/m$^3$ is positive, it points outward at both faces: toward the left at the left face and toward the right at the right face. Furthermore, the magnitude is the same at both faces. The electric flux through each of these faces is $Ea^2$. The field is parallel to the other faces of the Gaussian surface and the flux through them is zero. The total flux through the Gaussian surface is $\Phi = 2Ea^2$. The volume enclosed by the Gaussian surface is $2a^2x$ and the charge contained within it is $q = 2a^2x\rho$. Gauss' law yields

$$2\varepsilon_0 Ea^2 = 2a^2x\rho.$$

We solve for the magnitude of the electric field: $E = \rho x/\varepsilon_0$.

(a) For $x = 0$, $E = 0$.

(b) For $x$ = 2.00 mm = $2.00 \times 10^{-3}$ m,

$$E = \frac{\rho x}{\varepsilon_0} = \frac{(5.80\times10^{-15}\text{ C/m}^3)(2.00\times10^{-3}\text{ m})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 1.31\times10^{-6}\text{ N/C.}$$

(c) For $x = d/2$ = 4.70 mm = $4.70 \times 10^{-3}$ m,

$$E = \frac{\rho x}{\varepsilon_0} = \frac{(5.80\times10^{-15}\text{ C/m}^3)(4.70\times10^{-3}\text{ m})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 3.08\times10^{-6}\text{ N/C.}$$

(d) For $x$ = 26.0 mm = $2.60 \times 10^{-2}$ m, we take a Gaussian surface of the same shape and orientation, but with $x > d/2$, so the left and right faces are outside the slab. The total flux through the surface is again $\Phi = 2Ea^2$ but the charge enclosed is now $q = a^2d\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = a^2d\rho$, so

$$E = \frac{\rho d}{2\varepsilon_0} = \frac{(5.80\times10^{-15}\text{ C/m}^3)(9.40\times10^{-3}\text{ m})}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)} = 3.08\times10^{-6}\text{ N/C.}$$

44. We determine the (total) charge on the ball by examining the maximum value ($E$ = $5.0 \times 10^7$ N/C) shown in the graph (which occurs at $r$ = 0.020 m). Thus, from $E = q/4\pi\varepsilon_0 r^2$, we obtain

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.020\text{ m})^2(5.0\times10^7\text{ N/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.2\times10^{-6}\text{C} .$$

45. (a) Since $r_1$ = 10.0 cm < $r$ = 12.0 cm < $r_2$ = 15.0 cm,

$$E(r)=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times10^{-8}\ \text{C}\right)}{\left(0.120\ \text{m}\right)^2}=2.50\times10^4\ \text{N/C}.$$

(b) Since $r_1 < r_2 < r = 20.0$ cm,

$$E(r)=\frac{1}{4\pi\varepsilon_0}\frac{q_1+q_2}{r^2}=\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00+2.00\right)\left(1\times10^{-8}\ \text{C}\right)}{\left(0.200\ \text{m}^2\right)}=1.35\times10^4\ \text{N/C}.$$

46. The field at the proton's location (but not caused by the proton) has magnitude $E$. The proton's charge is $e$. The ball's charge has magnitude $q$. Thus, as long as the proton is at $r \ge R$ then the force on the proton (caused by the ball) has magnitude

$$F = eE = e\left(\frac{q}{4\pi\varepsilon_o r^2}\right) = \frac{e\,q}{4\pi\varepsilon_o r^2}$$

where $r$ is measured from the center of the ball (to the proton). This agrees with Coulomb's law from Chapter 22. We note that if $r = R$ then this expression becomes

$$F_R = \frac{e\,q}{4\pi\varepsilon_o R^2}.$$

(a) If we require $F = \frac{1}{2}F_R$, and solve for $r$, we obtain $r = \sqrt{2}\,R$. Since the problem asks for the measurement from the surface then the answer is $\sqrt{2}\,R - R = 0.41R$.

(b) Now we require $F_{\text{inside}} = \frac{1}{2}F_R$ where $F_{\text{inside}} = eE_{\text{inside}}$ and $E_{\text{inside}}$ is given by Eq. 23-20. Thus,

$$e\left(\frac{q}{4\pi\varepsilon_o R^2}\right)r = \frac{1}{2}\,\frac{e\,q}{4\pi\varepsilon_o R^2} \quad\Rightarrow\quad r = \frac{1}{2}R = 0.50\,R.$$

47. **THINK** The unknown charge is distributed uniformly over the surface of the conducting solid sphere.

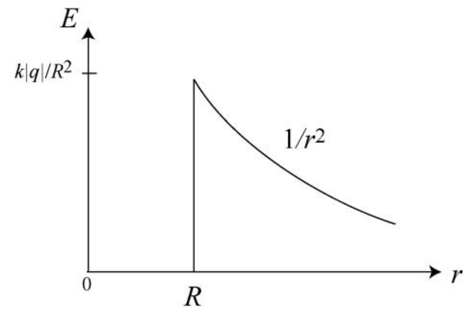
**EXPRESS** The electric field produced by the unknown charge at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = |q|/4\pi\varepsilon_0 r^2$, where $|q|$ is the magnitude of the charge on the sphere and $r$ is the distance from the center of the sphere to the point where the field is measured.

**ANALYZE** Thus, we have

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{\left(0.15\ \text{m}\right)^2\left(3.0\times10^3\ \text{N/C}\right)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}=7.5\times10^{-9}\ \text{C}.$$

The field points inward, toward the sphere center, so the charge is negative, i.e., $q = -7.5\times10^{-9}$ C.

**LEARN** The electric field strength as a function of $r$ is shown to the right. Inside the metal sphere, $E$ = 0; outside the sphere, $E = k\,|q|/r^2$, where $k = 1/4\pi\varepsilon_0$.




48. Let $E_A$ designate the magnitude of the field at $r$ = 2.4 cm. Thus $E_A$ = 2.0 × $10^7$ N/C, and is totally due to the particle. Since $E_{\text{particle}} = q/4\pi\varepsilon_0 r^2$, then the field due to the particle at any other point will relate to $E_A$ by a ratio of distances squared. Now, we note that at $r$ = 3.0 cm the total contribution (from particle and shell) is 8.0 × $10^7$ N/C. Therefore,

$$E_{\text{shell}} + E_{\text{particle}} = E_{\text{shell}} + (2.4/3)^2 E_A = 8.0\times10^7\ \text{N/C}\ .$$

Using the value for $E_A$ noted above, we find $E_{\text{shell}}$ = 6.6 × $10^7$ N/C. Thus, with $r$ = 0.030 m, we find the charge $Q$ using $E_{\text{shell}} = Q/4\pi\varepsilon_0 r^2$:

$$Q = 4\pi\varepsilon_0 r^2 E_{\text{shell}} = \frac{r^2 E_{\text{shell}}}{k} = \frac{(0.030\ \text{m})^2(6.6\times10^7\ \text{N/C})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 6.6\times10^{-6}\,\text{C}$$

49. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E$, where $r$ is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r$ = 0, the above equation implies $E$ = 0.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{2(2.00\times10^{-2}\,\text{m})^2} = 5.62\times10^{-2}\ \text{N/C}.$$

(c) For $r = a$, we have

$$E=\frac{q_1}{4\pi\varepsilon_0 a^2}=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(2.00\times10^{-2}\,\text{m})^2}=0.112\text{ N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is $q_1$, so Gauss' law leads to

$$4\pi r^2 E=\frac{q_1}{\varepsilon_0}\quad\Rightarrow\quad E=\frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E=\frac{q_1}{4\pi\varepsilon_0 r^2}=\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\,\text{C})}{(1.50\times2.00\times10^{-2}\,\text{m})^2}=0.0499\text{ N/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E=0 \Rightarrow E=0.$ Thus, $E = 0$ at $r = 3.50a$.

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E}\cdot d\vec{A}=0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If $Q_i$ is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -5.00$ fC.

(h) Let $Q_o$ be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0.$$

50. The point where the individual fields cancel cannot be in the region between the shells since the shells have opposite-signed charges. It cannot be inside the radius $R$ of one of the shells since there is only one field contribution there (which would not be canceled by another field contribution and thus would not lead to zero net field). We note shell 2 has greater magnitude of charge ($|\sigma_2|A_2$) than shell 1, which implies the point is not to the right of shell 2 (any such point would always be closer to the larger charge and thus no possibility for cancellation of equal-magnitude fields could occur). Consequently, the point should be in the region to the left of shell 1 (at a distance $r > R_1$ from its center); this is where the condition

$$E_1=E_2\quad\Rightarrow\quad\frac{|q_1|}{4\pi\varepsilon_0 r^2}=\frac{|q_2|}{4\pi\varepsilon_0 (r+L)^2}$$

or

$$\frac{\sigma_1 A_1}{4\pi\varepsilon_0 r^2}=\frac{|\sigma_2|A_2}{4\pi\varepsilon_0 (r+L)^2}\ .$$

Using the fact that the area of a sphere is $A = 4\pi R^2$, this condition simplifies to

$$r = \frac{L}{(R_2/R_1)\sqrt{|\sigma_2|/\sigma_1} - 1} = 3.3 \text{ cm} .$$

We note that this value satisfies the requirement $r > R_1$. The answer, then, is that the net field vanishes at $x = -r = -3.3$ cm.

51. **THINK** Since our system possesses spherical symmetry, to calculate the electric field strength, we may apply Gauss' law and take the Gaussian surface to be in the form of a sphere of radius $r$.

**EXPRESS** To find an expression for the electric field inside the shell in terms of $A$ and the distance from the center of the shell, choose $A$ so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius $r_g$, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance $r_g$ from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take $dV$ to be the volume of a spherical shell with radius $r$ and infinitesimal thickness $dr$: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 dr = 4\pi A \int_a^{r_g} r \, dr = 2\pi A \left(r_g^2 - a^2\right).$$

The total charge inside the Gaussian surface is

$$q_{\text{enc}} = q + q_s = q + 2\pi A(r_g^2 - a^2).$$

The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E,$ where $E$ is the magnitude of the field. Gauss' law yields

$$\Phi = q_{\text{enc}} / \varepsilon_0 \quad \Rightarrow \quad 4\pi\varepsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2).$$

We solve for $E$:

$$E = \frac{1}{4\pi\varepsilon_0} \left[ \frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2} \right].$$

**ANALYZE** For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$. With $a = 2.00 \times 10^{-2}$ m and $q = 45.0 \times 10^{-15}$ C, we have $A = 1.79 \times 10^{-11} \text{C/m}^2$.

**LEARN** The value we have found for $A$ ensures the uniformity of the field strength inside the shell. Using the result found above, we can readily show that the electric field in the region $a \le r \le b$ is

$$E = \frac{2\pi A}{4\pi\varepsilon_0} = \frac{A}{2\varepsilon_0} = \frac{1.79\times10^{-11}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.01\ \text{N/C}.$$

52. The field is zero for $0 \le r \le a$ as a result of Eq. 23-16. Thus,

(a) $E = 0$ at $r = 0$,

(b) $E = 0$ at $r = a/2.00$, and

(c) $E = 0$ at $r = a$.

For $a \le r \le b$ the enclosed charge $q_{\text{enc}}$ (for $a \le r \le b$) is related to the volume by

$$q_{\text{enc}} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right).$$

Therefore, the electric field is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$.

(d) For $r = 1.50a$, we have

$$E = \frac{\rho}{3\varepsilon_0}\frac{(1.50a)^3 - a^3}{(1.50a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{2.375}{2.25}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{2.375}{2.25}\right) = 7.32\ \text{N/C}.$$

(e) For $r = b = 2.00a$, the electric field is

$$E = \frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3 - a^3}{(2.00a)^2} = \frac{\rho a}{3\varepsilon_0}\left(\frac{7}{4}\right) = \frac{(1.84\times10^{-9}\ \text{C/m}^3)(0.100\ \text{m})}{3(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{4}\right) = 12.1\ \text{N/C}.$$

(f) For $r \ge b$ we have $E = q_{\text{total}} / 4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2}.$$

Thus, for $r = 3.00b = 6.00a$, the electric field is

$$E=\frac{\rho}{3\varepsilon_0}\frac{(2.00a)^3-a^3}{(6.00a)^2}=\frac{\rho a}{3\varepsilon_0}\left(\frac{7}{36}\right)=\frac{(1.84\times10^{-9}\text{ C/m}^3)(0.100\text{ m})}{3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{7}{36}\right)=1.35\text{ N/C.}$$

53. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx\int dy\int dz\,\rho=4\pi\int_0^R dr\;r^2\;\rho=Q.$$

Substituting the expression $\rho=\rho_s r/R$, with $\rho_s$= 14.1 pC/m$^3$, and performing the integration leads to

$$4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{R^4}{4}\right)=Q$$

or

$$Q=\pi\rho_s R^3=\pi(14.1\times10^{-12}\text{ C/m}^3)(0.0560\text{ m})^3=7.78\times10^{-15}\text{ C.}$$

(b) At $r$ = 0, the electric field is zero ($E$ = 0) since the enclosed charge is zero.

At a certain point within the sphere, at some distance $r$ from the center, the field (see Eq. 23-8 through Eq. 23-10) is given by Gauss' law:

$$E=\frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2}$$

where $q_{\text{enc}}$ is given by an integral similar to that worked in part (a):

$$q_{\text{enc}}=4\pi\int_0^r dr\,r^2\rho=4\pi\left(\frac{\rho_s}{R}\right)\left(\frac{r^4}{4}\right).$$

Therefore,

$$E=\frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^4}{Rr^2}=\frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r^2}{R}.$$

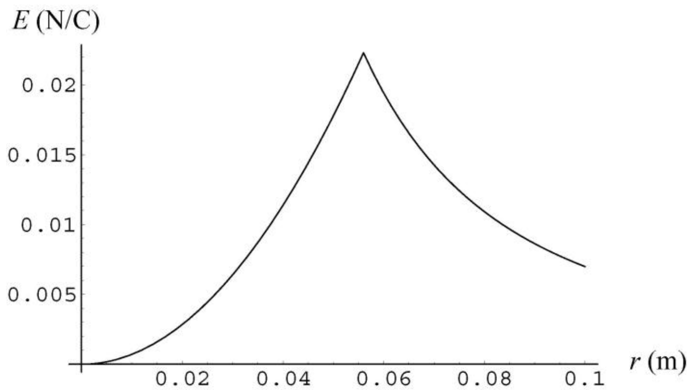
(c) For $r = R/2.00$, where $R$ = 5.60 cm, the electric field is

$$\begin{aligned}E&=\frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s(R/2.00)^2}{R}=\frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R}{4.00}\\&=\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\pi(14.1\times10^{-12}\text{ C/m}^3)(0.0560\text{ m})}{4.00}\\&=5.58\times10^{-3}\text{ N/C.}\end{aligned}$$

(d) For $r = R$, the electric field is

$$E=\frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s R^2}{R}=\frac{\pi\rho_s R}{4\pi\varepsilon_0}=(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\pi(14.1\times10^{-12}\ \mathrm{C/m^3})(0.0560\ \mathrm{m})$$
$$=2.23\times10^{-2}\ \mathrm{N/C}.$$

(e) The electric field strength as a function of *r* is depicted below:




54. Applying Eq. 23-20, we have

$$E_1=\frac{|q_1|}{4\pi\varepsilon_0 R^3}r_1=\frac{|q_1|}{4\pi\varepsilon_0 R^3}\left(\frac{R}{2}\right)=\frac{1}{2}\frac{|q_1|}{4\pi\varepsilon_0 R^2}\ .$$

Also, outside sphere 2 we have

$$E_2=\frac{|q_2|}{4\pi\varepsilon_0 r^2}=\frac{|q_2|}{4\pi\varepsilon_0(1.50R)^2}\ .$$

Equating these and solving for the ratio of charges, we arrive at $\frac{q_2}{q_1}=\frac{9}{8}=1.125$.

55. We use

$$E(r)=\frac{q_{\text{enc}}}{4\pi\varepsilon_0 r^2}=\frac{1}{4\pi\varepsilon_0 r^2}\int_0^r \rho(r)4\pi r^2dr$$

to solve for $\rho(r)$ and obtain

$$\rho(r)=\frac{\varepsilon_0}{r^2}\frac{d}{dr}\ r^2E(r)\ =\frac{\varepsilon_0}{r^2}\frac{d}{dr}\left(Kr^6\right)=6K\varepsilon_0 r^3.$$

56. (a) There is no flux through the sides, so we have two contributions to the flux, one from the $x$ = 2 end (with $\Phi_2 = +(2 + 2)(\pi(0.20)^2) = 0.50\ \mathrm{N\cdot m^2/C}$) and one from the $x$ = 0 end (with $\Phi_0 = -(2)(\pi(0.20)^2)$).

(b) By Gauss' law we have $q_{\text{enc}} = \varepsilon_0(\Phi_2 + \Phi_0) = 2.2\times10^{-12}$ C.

57. (a) For $r < R$, $E = 0$ (see Eq. 23-16).

(b) For $r$ slightly greater than $R$,

$$E_R = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-7}\,\text{C}\right)}{\left(0.250\text{m}\right)^2} = 2.88\times10^4\ \text{N/C}.$$

(c) For $r > R$, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = E_R\left(\dfrac{R}{r}\right)^2 = \left(2.88\times10^4\ \text{N/C}\right)\left(\dfrac{0.250\ \text{m}}{3.00\ \text{m}}\right)^2 = 200\,\text{N/C}.$

58. From Gauss's law, we have

$$\Phi = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\sigma\pi r^2}{\varepsilon_0} = \frac{(8.0\times10^{-9}\text{C/m}^2)\pi(0.050\ \text{m})^2}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 7.1\,\text{N}\cdot\text{m}^2/\text{C}\ .$$

59. (a) At $x = 0.040$ m, the net field has a rightward (+$x$) contribution (computed using Eq. 23-13) from the charge lying between $x = -0.050$ m and $x = 0.040$ m, and a leftward (–$x$) contribution (again computed using Eq. 23-13) from the charge in the region from $x = 0.040$ m to $x = 0.050$ m. Thus, since $\sigma = q/A = \rho V/A = \rho\Delta x$ in this situation, we have

$$\left|\vec{E}\right| = \frac{\rho(0.090\,\text{m})}{2\varepsilon_0} - \frac{\rho(0.010\,\text{m})}{2\varepsilon_0} = \frac{(1.2\times10^{-9}\,\text{C/m}^3)(0.090\,\text{m}-0.010\,\text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 5.4\,\text{N/C}.$$

(b) In this case, the field contributions from all layers of charge point rightward, and we obtain

$$\left|\vec{E}\right| = \frac{\rho(0.100\,\text{m})}{2\varepsilon_0} = \frac{(1.2\times10^{-9}\,\text{C/m}^3)(0.100\,\text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 6.8\,\text{N/C}.$$

60. (a) We consider the radial field produced at points within a uniform cylindrical distribution of charge. The volume enclosed by a Gaussian surface in this case is $L\pi r^2$. Thus, Gauss' law leads to

$$E = \frac{|q_{\text{enc}}|}{\varepsilon_0 A_{\text{cylinder}}} = \frac{|\rho|\left(L\pi r^2\right)}{\varepsilon_0(2\pi rL)} = \frac{|\rho| r}{2\varepsilon_0}.$$

(b) We note from the above expression that the magnitude of the radial field grows with $r$.

(c) Since the charged powder is negative, the field points radially inward.

(d) The largest value of $r$ that encloses charged material is $r_{\max} = R$. Therefore, with $|\rho| = 0.0011\ \text{C/m}^3$ and $R = 0.050$ m, we obtain

$$E_{\max} = \frac{|\rho| R}{2\varepsilon_0} = \frac{(0.0011\ \text{C/m}^3)(0.050\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.1\times10^6\ \text{N/C}.$$

(e) According to condition 1 mentioned in the problem, the field is high enough to produce an electrical discharge (at $r = R$).

61. **THINK** Our system consists of two concentric metal shells. We apply the superposition principle and Gauss' law to calculate the electric field everywhere.

**EXPRESS** At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the metal shells of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so

$$\Phi = \oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = \frac{q_{\text{enc}}}{\varepsilon_0},$$

where $r$ is the radius of the Gaussian surface.

**ANALYZE** (a) For $r < a$, the charge enclosed is $q_{\text{enc}} = 0$, so $E = 0$ in the region inside the shell.

(b) For $a < r < b$, the charged enclosed by the Gaussian surface is $q_{\text{enc}} = q_a$, so the field strength is $E = q_a / 4\pi\varepsilon_0 r^2$.

(c) For $r > b$, the charged enclosed by the Gaussian surface is $q_{\text{enc}} = q_a + q_b$, so the field strength is $E = (q_a + q_b)/4\pi\varepsilon_0 r^2$.

(d) Since $E = 0$ for $r < a$ the charge on the inner surface of the inner shell is always zero. The charge on the outer surface of the inner shell is therefore $q_a$. Since $E = 0$ inside the metallic outer shell the net charge enclosed in a Gaussian surface that lies in between the inner and outer surfaces of the outer shell is zero. Thus the inner surface of the outer shell must carry a charge $-q_a$, leaving the charge on the outer surface of the outer shell to be $q_b + q_a$.

**LEARN** The concepts involved in this problem are discussed in Section 23-9 of the text. In the case of a single shell of radius $R$ and charge $q$, the field strength is $E = 0$ for $r < R$, and $E = q/4\pi\varepsilon_0 r^2$ for $r > R$ (see Eqs. 23-15 and 23-16).

62. (a) The direction of the electric field at $P_1$ is away from $q_1$ and its magnitude is

$$\left|\vec{E}\right| = \frac{q}{4\pi\varepsilon_0 r_1^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-7}\ \text{C})}{(0.015\,\text{m})^2} = 4.0\times10^6\ \text{N/C}.$$

(b) $\vec{E} = 0$, since $P_2$ is inside the metal.

63. The proton is in uniform circular motion, with the electrical force of the sphere on the proton providing the centripetal force. According to Newton's second law, $F = mv^2/r$, where $F$ is the magnitude of the force, $v$ is the speed of the proton, and $r$ is the radius of its orbit, essentially the same as the radius of the sphere. The magnitude of the force on the proton is $F = e|q|/4\pi\varepsilon_0 r^2$, where $|q|$ is the magnitude of the charge on the sphere. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{e\,|\,q\,|}{r^2} = \frac{mv^2}{r}$$

so

$$|\,q\,| = \frac{4\pi\varepsilon_0 mv^2 r}{e} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.00\times10^5\ \text{m/s}\right)^2\left(0.0100\ \text{m}\right)}{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-9}\ \text{C}\right)} = 1.04\times10^{-9}\ \text{C}.$$

The force must be inward, toward the center of the sphere, and since the proton is positively charged, the electric field must also be inward. The charge on the sphere is negative: $q = -1.04 \times 10^{-9}$ C.

64. We interpret the question as referring to the field *just* outside the sphere (that is, at locations roughly equal to the radius $r$ of the sphere). Since the area of a sphere is $A = 4\pi r^2$ and the surface charge density is $\sigma = q/A$ (where we assume $q$ is positive for brevity), then

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

which we recognize as the field of a point charge (see Eq. 22-3).

65. (a) Since the volume contained within a radius of $\frac{1}{2}R$ is one-eighth the volume contained within a radius of $R$, the charge at $0 < r < R/2$ is $Q/8$. The fraction is 1/8 = 0.125.

(b) At $r = R/2$, the magnitude of the field is

$$E = \frac{Q/8}{4\pi\varepsilon_0 (R/2)^2} = \frac{1}{2}\frac{Q}{4\pi\varepsilon_0 R^2}$$

and is equivalent to *half* the field at the surface. Thus, the ratio is 0.500.

66. (a) The flux is still $-750\ \mathrm{N\cdot m^2/C}$, since it depends only on the amount of charge enclosed.

(b) We use $\Phi = q/\varepsilon_0$ to obtain the charge $q$:

$$q = \varepsilon_0 \Phi = \left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(-750\ \mathrm{N\cdot m^2/C}\right) = -6.64\times10^{-9}\ \mathrm{C}.$$

67. **THINK** The electric field at $P$ is due to the charge on the surface of the metallic conductor and the point charge $Q$.

**EXPRESS** The initial field (evaluated "just outside the outer surface" which means it is evaluated at $R_2$ = 0.20 m, the outer radius of the conductor) is related to the charge $q$ on the hollow conductor by Eq. 23-15: $E_{\text{initial}} = q/4\pi\varepsilon_0 R_2^2$. After the point charge $Q$ is placed at the geometric center of the hollow conductor, the final field at that point is a combination of the initial and that due to $Q$ (determined by Eq. 22-3):

$$E_{\text{final}} = E_{\text{initial}} + \frac{Q}{4\pi\varepsilon_0 R_2^2}.$$

**ANALYZE** (a) The charge on the spherical shell is

$$q = 4\pi\varepsilon_0 R_2^2 E_{\text{initial}} = \frac{(0.20\ \mathrm{m})^2(450\ \mathrm{N/C})}{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}} = 2.0\times10^{-9}\ \mathrm{C}.$$

(b) Similarly, using the equation above, we find the point charge to be

$$Q = 4\pi\varepsilon_0 R_2^2\left(E_{\text{final}} - E_{\text{initial}}\right) = \frac{(0.20\ \mathrm{m})^2(180\ \mathrm{N/C} - 450\ \mathrm{N/C})}{8.99\times10^9\ \mathrm{N\cdot m^2/C^2}} = -1.2\times10^{-9}\ \mathrm{C}.$$

(c) In order to cancel the field (due to $Q$) within the conducting material, there must be an amount of charge equal to $-Q$ distributed uniformly on the inner surface (of radius $R_1$). Thus, the answer is $+1.2 \times 10^{-9}$ C.

(d) Since the total excess charge on the conductor is $q$ and is located on the surfaces, then the outer surface charge must equal the total minus the inner surface charge. Thus, the answer is $2.0 \times 10^{-9}\ \mathrm{C} - 1.2 \times 10^{-9}\ \mathrm{C} = +0.80 \times 10^{-9}$ C.

**LEARN** The key idea here is to realize that the electric field inside the conducting shell ($R_1 < r < R_2$) must be zero, so the charge must be distributed in such a way that the charge enclosed by a Gaussian sphere of radius $r$ ($R_1 < r < R_2$) is zero.

68. Let $\Phi_0 = 10^3\ \mathrm{N\cdot m^2/C}$. The net flux through the entire surface of the dice is given by

$$\Phi = \sum_{n=1}^{6} \Phi_n = \sum_{n=1}^{6} (-1)^n n \Phi_0 = \Phi_0 (-1+2-3+4-5+6) = 3\Phi_0 .$$

Thus, the net charge enclosed is

$$q = \varepsilon_0 \Phi = 3\varepsilon_0 \Phi_0 = 3\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(10^3\,\text{N}\cdot\text{m}^2/\text{C}\right) = 2.66\times10^{-8}\,\text{C}.$$

69. Since the fields involved are uniform, the precise location of $P$ is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward $(+\hat{\text{j}})$, and (from Eq. 23-13) its magnitude is

$$|\vec{E}| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0\times10^{-6}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 5.65\times10^4\ \text{N/C}.$$

In unit-vector notation, we have $\vec{E} = (5.65\times10^4\ \text{N/C})\hat{\text{j}}$.

70. Since the charge distribution is uniform, we can find the total charge $q$ by multiplying $\rho$ by the spherical volume ( $\frac{4}{3}\pi r^3$ ) with $r = R =$ 0.050 m. This gives $q$ = 1.68 nC.

(a) Applying Eq. 23-20 with $r$ = 0.035 m, we have $E_{\text{internal}} = \dfrac{|q|\,r}{4\pi\varepsilon_0 R^3} = 4.2\times10^3\ \text{N/C}$.

(b) Outside the sphere we have (with $r$ = 0.080 m)

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.68\times10^{-9}\,\text{C})}{(0.080\ \text{m})^2} = 2.4\times10^3\ \text{N/C}\ .$$

71. We choose a coordinate system whose origin is at the center of the flat base, such that the base is in the $xy$ plane and the rest of the hemisphere is in the $z > 0$ half space.

(a) $\Phi = \pi R^2 \left(-\hat{\text{k}}\right)\cdot E\hat{\text{k}} = -\pi R^2 E = -\pi(0.0568\ \text{m})^2(2.50\ \text{N/C}) = -0.0253\ \text{N}\cdot\text{m}^2/\text{C}.$

(b) Since the flux through the entire hemisphere is zero, the flux through the curved surface is $\vec{\Phi}_c = -\Phi_{\text{base}} = \pi R^2 E = 0.0253\ \text{N}\cdot\text{m}^2/\text{C}.$

72. The net enclosed charge $q$ is given by

$$q = \varepsilon_0 \Phi = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(-48\ \text{N}\cdot\text{m}^2/\text{C}\right) = -4.2\times10^{-10}\,\text{C}.$$

73. (a) From Gauss' law, we get $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{\left(4\pi\rho r^3/3\right)\vec{r}}{r^3} = \frac{\rho\vec{r}}{3\varepsilon_0}.$

(b) The charge distribution in this case is equivalent to that of a whole sphere of charge density $\rho$ plus a smaller sphere of charge density $-\rho$ that fills the void. By superposition

$$\vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\varepsilon_0} + \frac{(-\rho)(\vec{r}-\vec{a})}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0}.$$

74. (a) The cube is totally within the spherical volume, so the charge enclosed is

$$q_{\text{enc}} = \rho\, V_{\text{cube}} = (500 \times 10^{-9}\ \text{C/m}^3)(0.0400\ \text{m})^3 = 3.20 \times 10^{-11}\ \text{C}.$$

By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 3.62\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) Now the sphere is totally contained within the cube (note that the radius of the sphere is less than half the side-length of the cube). Thus, the total charge is

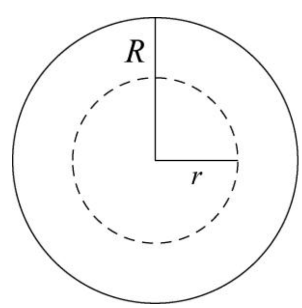
$$q_{\text{enc}} = \rho\, V_{\text{sphere}} = 4.5 \times 10^{-10}\ \text{C}.$$

By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 51.1\ \text{N}\cdot\text{m}^2/\text{C}$.

75. The electric field is radially outward from the central wire. We want to find its magnitude in the region between the wire and the cylinder as a function of the distance $r$ from the wire. Since the magnitude of the field at the cylinder wall is known, we take the Gaussian surface to coincide with the wall. Thus, the Gaussian surface is a cylinder with radius $R$ and length $L$, coaxial with the wire. Only the charge on the wire is actually enclosed by the Gaussian surface; we denote it by $q$. The area of the Gaussian surface is $2\pi RL$, and the flux through it is $\Phi = 2\pi RLE$. We assume there is no flux through the ends of the cylinder, so this $\Phi$ is the total flux. Gauss' law yields $q = 2\pi\varepsilon_0 RLE$. Thus,

$$q = 2\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.014\ \text{m})(0.16\ \text{m})\ (2.9\times10^4\ \text{N/C}) = 3.6\times10^{-9}\ \text{C}.$$

76. (a) The diagram shows a cross section (or, perhaps more appropriately, "end view") of the charged cylinder (solid circle).




Consider a Gaussian surface in the form of a cylinder with radius $r$ and length $\ell$, coaxial with the charged cylinder. An "end view" of the Gaussian surface is shown as a dashed circle. The charge enclosed by it is $q = \rho V = \pi r^2\,\ell\rho$, where $V = \pi r^2\ell$ is the volume of the cylinder. If $\rho$ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r\ell)$. Now, Gauss' law leads to

$$2\pi\varepsilon_0 r\ell E = \pi r^2\ell\rho \;\;\Rightarrow\;\; E = \frac{\rho r}{2\varepsilon_0}.$$

(b) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field $E_{\text{ext}}$ then the flux is $\Phi = 2\pi r\ell E_{\text{ext}}$. The charge enclosed is the total charge in a section of the charged cylinder with length $\ell$. That is, $q = \pi R^2\ell\rho$. In this case, Gauss' law yields

$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2\ell\rho \;\;\Rightarrow\;\; E_{\text{ext}} = \frac{R^2\rho}{2\varepsilon_0 r}.$$

77. **THINK** The total charge on the conducting shell is equal to the sum of the charges on the shell's inner surface and the outer surface.

**EXPRESS** Let $q_{\text{in}}$ be the charge on the inner surface and $q_{\text{out}}$ the charge on the outer surface. The net charge on the shell is $Q = q_{\text{in}} + q_{\text{out}}$.

**ANALYZE** (a) In order to have net charge $Q$ = –10 $\mu$C when the charge on the outer surface is $q_{\text{out}} = -14\ \mu\text{C}$, then there must be

$$q_{\text{in}} = Q - q_{\text{out}} = -10\ \mu\text{C} - (-14\ \mu\text{C}) = +4\ \mu\text{C}$$

on the inner surface (since charges reside on the surfaces of a conductor in electrostatic situations).

(b) Let $q$ be the charge of the particle. In order to cancel the electric field inside the conducting material, the contribution from the $q_{\text{in}} = +4\ \mu\text{C}$ on the inner surface must be canceled by that of the charged particle in the hollow, that is, $q_{\text{enc}} = q + q_{\text{in}} = 0$. Thus, the particle's charge is $q = -q_{\text{in}} = -4\ \mu\text{C}$.

**LEARN** The key idea here is to realize that the electric field inside the conducting shell must be zero. Thus, in the presence of a point charge in the hollow, the charge on the shell must be redistributed between its inner and outer surfaces in such a way that the net charge enclosed by a Gaussian sphere of radius $r$ ( $R_1 < r < R_2$ , where $R_1$ is the inner radius and $R_2$ is the outer radius) remains zero.

78. (a) Outside the sphere, we use Eq. 23-15 and obtain

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-12}\ \text{C})}{(0.0600\ \text{m})^2} = 15.0\ \text{N/C}.$$

(b) With $q$ = +6.00 × $10^{-12}$ C, Eq. 23-20 leads to $E = 25.3\,\text{N/C}$.

79. (a) The mass flux is $wd\rho v$ = (3.22 m) (1.04 m) (1000 kg/m$^3$) (0.207 m/s) = 693 kg/s.

(b) Since water flows only through area $wd$, the flux through the larger area is still 693 kg/s.

(c) Now the mass flux is $(wd/2)\rho v$ = (693 kg/s)/2 = 347 kg/s.

(d) Since the water flows through an area ($wd$/2), the flux is 347 kg/s.

(e) Now the flux is $\left(wd\cos\theta\right)\rho v = \left(693\,\text{kg/s}\right)\left(\cos 34°\right) = 575\ \text{kg/s}$.

80. The field due to a sheet of charge is given by Eq. 23-13. Both sheets are horizontal (parallel to the $xy$ plane), producing vertical fields (parallel to the $z$ axis). At points above the $z$ = 0 sheet (sheet $A$), its field points upward (toward +z); at points above the $z$ = 2.0 sheet (sheet $B$), its field does likewise. However, below the $z$ = 2.0 sheet, its field is oriented downward.

(a) The magnitude of the net field in the region between the sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0} = \frac{8.00\times10^{-9}\,\text{C/m}^2 - 3.00\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.82\times10^2\ \text{N/C}.$$

(b) The magnitude of the net field at points above both sheets is

$$|\vec{E}| = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0} = \frac{8.00\times10^{-9}\,\text{C/m}^2 + 3.00\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 6.21\times10^2\ \text{N/C}.$$

81. (a) The field maximum occurs at the outer surface:

$$E_{\text{max}} = \left(\frac{|q|}{4\pi\varepsilon_o r^2}\right)_{\text{at } r = R} = \frac{|q|}{4\pi\varepsilon_o R^2}$$

Applying Eq. 23-20, we have

$$E_{\text{internal}} = \frac{|q|}{4\pi\varepsilon_o R^3}\, r = \frac{1}{4} E_{\text{max}} \quad\Rightarrow\quad r = \frac{R}{4} = 0.25\, R.$$

(b) Outside sphere 2 we have

$$E_{\text{external}} = \frac{|q|}{4\pi\varepsilon_o r^2} = \frac{1}{4} E_{\text{max}} \quad\Rightarrow\quad r = 2.0R.$$

# Chapter 24

1. **THINK** Ampere is the SI unit for current. An ampere is one coulomb per second.

**EXPRESS** To calculate the total charge through the circuit, we note that $1\text{ A} = 1\text{ C/s}$ and $1\text{ h} = 3600\text{ s}$.

**ANALYZE** (a) Thus,

$$84\text{ A}\cdot\text{h} = \left(84\frac{\text{C}\cdot\text{h}}{\text{s}}\right)\left(3600\frac{\text{s}}{\text{h}}\right) = 3.0\times10^{5}\text{ C}.$$

(b) The change in potential energy is $\Delta U = q\,\Delta V = (3.0\times10^{5}\text{ C})(12\text{ V}) = 3.6\times10^{6}\text{ J}$.

**LEARN** Potential difference is the change of potential energy per unit charge. Unlike electric field, potential difference is a scalar quantity.

2. The magnitude is $\Delta U = e\Delta V = 1.2\times10^{9}\text{ eV} = 1.2\text{ GeV}$.

3. (a) The change in energy of the transferred charge is

$$\Delta U = q\,\Delta V = (30\text{ C})(1.0\times10^{9}\text{ V}) = 3.0\times10^{10}\text{ J}.$$

(b) If all this energy is used to accelerate a 1000-kg car from rest, then $\Delta U = K = \frac{1}{2}mv^2$, and we find the car's final speed to be

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\Delta U}{m}} = \sqrt{\frac{2(3.0\times10^{10}\text{ J})}{1000\text{ kg}}} = 7.7\times10^{3}\text{ m/s}.$$

4. (a) $E = F/e = \left(3.9\times10^{-15}\text{ N}\right)/\left(1.60\times10^{-19}\text{ C}\right) = 2.4\times10^{4}\text{ N/C} = 2.4\times10^{4}\text{ V/m}.$

(b) $\Delta V = E\Delta s = \left(2.4\times10^{4}\text{ N/C}\right)\left(0.12\text{ m}\right) = 2.9\times10^{3}\text{ V}.$

5. **THINK** The electric field produced by an infinite sheet of charge is normal to the sheet and is uniform.

**EXPRESS** The magnitude of the electric field produced by the infinite sheet of charge is $E = \sigma/2\varepsilon_0$, where $\sigma$ is the surface charge density. Place the origin of a coordinate system at the sheet and take the $x$ axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex,$$

where $V_s$ is the potential at the sheet. The equipotential surfaces are surfaces of constant $x$; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by $\Delta x$ then their potentials differ in magnitude by

$$\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x.$$

**ANALYZE** Thus, for $\sigma = 0.10\times10^{-6}\ \text{C/m}^2$ and $\Delta V = 50\ \text{V}$, we have

$$\Delta x = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(50\ \text{V})}{0.10\times10^{-6}\ \text{C/m}^2} = 8.8\times10^{-3}\,\text{m}.$$

**LEARN** Equipotential surfaces are always perpendicular to the electric field lines. Figure 24-5(a) depicts the electric field lines and equipotential surfaces for a uniform electric field.

6. (a) $V_B - V_A = \Delta U/q = -W/(-e) = -(3.94\times10^{-19}\ \text{J})/(-1.60\times10^{-19}\ \text{C}) = 2.46\ \text{V}$.

(b) $V_C - V_A = V_B - V_A = 2.46\ \text{V}$.

(c) $V_C - V_B = 0$ (since $C$ and $B$ are on the same equipotential line).

7. We connect $A$ to the origin with a line along the $y$ axis, along which there is no change of potential (Eq. 24-18: $\int \vec{E}\cdot d\vec{s} = 0$). Then, we connect the origin to $B$ with a line along the $x$ axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E}\cdot d\vec{s} = -4.00\int_0^4 x\,dx = -4.00\left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32.0\ \text{V}$.

8. (a) By Eq. 24-18, the change in potential is the negative of the "area" under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_0^{x=2} \vec{E}\cdot d\vec{s} = \frac{1}{2}(2)(20)$$

which yields $V = 30\ \text{V}$.

(b) For any region within $0 < x < 3\text{m}$, $-\int \vec{E}\cdot d\vec{s}$ is positive, but for any region for which $x > 3$ m it is negative. Therefore, $V = V_{\max}$ occurs at $x = 3$ m.

$$V - 10 = -\int_0^{x=3} \vec{E} \cdot d\vec{s} = \frac{1}{2}(3)(20)$$

which yields $V_{max}$ = 40 V.

(c) In view of our result in part (b), we see that now (to find $V$ = 0) we are looking for some $X$ > 3 m such that the "area" from $x$ = 3 m to $x = X$ is 40 V. Using the formula for a triangle (3 < $x$ < 4) and a rectangle (4 < $x$ < $X$), we require

$$\frac{1}{2}(1)(20) + (X - 4)(20) = 40.$$

Therefore, $X$ = 5.5 m.

9. (a) The work done by the electric field is

$$W = \int_i^f q_0 \vec{E} \cdot d\vec{s} = \frac{q_0\sigma}{2\varepsilon_0}\int_0^d dz = \frac{q_0\sigma d}{2\varepsilon_0} = \frac{(1.60\times10^{-19}\,\text{C})(5.80\times10^{-12}\ \text{C/m}^2)(0.0356\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}$$
$$= 1.87\times10^{-21}\,\text{J}.$$

(b) Since

$$V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0,$$

with $V_0$ set to be zero on the sheet, the electric potential at $P$ is

$$V = -\frac{\sigma z}{2\varepsilon_0} = -\frac{(5.80\times10^{-12}\ \text{C/m}^2)(0.0356\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = -1.17\times10^{-2}\ \text{V}.$$

10. In the "inside" region between the plates, the individual fields (given by Eq. 24-13) are in the same direction ($-\hat{\text{i}}$):

$$\vec{E}_{\text{in}} = -\left(\frac{50\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} + \frac{25\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\right)\hat{\text{i}} = -(4.2\times10^3\ \text{N/C})\hat{\text{i}}\,.$$

In the "outside" region where $x$ > 0.5 m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50\times10^{-9}\,\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} + \frac{25\times10^{-9}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\hat{\text{i}} = -(1.4\times10^3\ \text{N/C})\hat{\text{i}}.$$

Therefore, by Eq. 24-18, we have

$$\Delta V = -\int_0^{0.8} \vec{E} \cdot d\vec{s} = -\int_0^{0.5} \left|\vec{E}_{\text{in}}\right| dx - \int_{0.5}^{0.8} \left|\vec{E}_{\text{out}}\right| dx = -\left(4.2\times10^3\right)(0.5) - \left(1.4\times10^3\right)(0.3)$$
$$= 2.5\times10^3\ \text{V}.$$

11. (a) The potential as a function of $r$ is

$$V(r)=V(0)-\int_0^r E(r)dr=0-\int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}dr=-\frac{qr^2}{8\pi\varepsilon_0 R^3}$$
$$=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\text{C})(0.0145\ \text{m})^2}{2(0.0231\ \text{m})^3}=-2.68\times10^{-4}\ \text{V}.$$

(b) Since $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$, we have

$$V(R)=-\frac{q}{8\pi\varepsilon_0 R}=-\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.50\times10^{-15}\text{C})}{2(0.0231\ \text{m})}=-6.81\times10^{-4}\ \text{V}.$$

12. The charge is

$$q=4\pi\varepsilon_0 RV=\frac{(10\text{m})(-1.0\text{V})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}=-1.1\times10^{-9}\text{C}.$$

13. (a) The charge on the sphere is

$$q=4\pi\varepsilon_0 VR=\frac{(200\ \text{V})(0.15\ \text{m})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}=3.3\times10^{-9}\ \text{C}.$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma=\frac{q}{4\pi R^2}=\frac{3.3\times10^{-9}\ \text{C}}{4\pi(0.15\ \text{m})^2}=1.2\times10^{-8}\ \text{C/m}^2.$$

14. (a) The potential difference is

$$V_A-V_B=\frac{q}{4\pi\varepsilon_0 r_A}-\frac{q}{4\pi\varepsilon_0 r_B}=(1.0\times10^{-6}\ \text{C})(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{1}{2.0\ \text{m}}-\frac{1}{1.0\ \text{m}}\right)$$
$$=-4.5\times10^3\ \text{V}.$$

(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

15. **THINK** The electric potential for a spherically symmetric charge distribution falls off as $1/r$, where $r$ is the radial distance from the center of the charge distribution.

**EXPRESS** The electric potential $V$ at the surface of a drop of charge $q$ and radius $R$ is given by $V = q/4\pi\varepsilon_0 R$.

**ANALYZE** (a) With $V$ = 500 V and $q = 30\times10^{-12}$ C, we find the radius to be

$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(30\times10^{-12}\ \text{C}\right)}{500\ \text{V}} = 5.4\times10^{-4}\ \text{m}.$$

(b) After the two drops combine to form one big drop, the total volume is twice the volume of an original drop, so the radius $R'$ of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of the original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\ \text{V}) \approx 790\ \text{V}.$$

**LEARN** A positively charged configuration produces a positive electric potential, and a negatively charged configuration produces a negative electric potential. Adding more charge increases the electric potential.

16. In applying Eq. 24-27, we are assuming $V \to 0$ as $r \to \infty$. All corner particles are equidistant from the center, and since their total charge is

$$2q_1 - 3q_1 + 2\,q_1 - q_1 = 0,$$

then their contribution to Eq. 24-27 vanishes. The net potential is due, then, to the two $+4q_2$ particles, each of which is a distance of $a/2$ from the center:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} + \frac{1}{4\pi\varepsilon_0}\frac{4q_2}{a/2} = \frac{16q_2}{4\pi\varepsilon_0 a} = \frac{16(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-12}\text{C})}{0.39\ \text{m}}$$
$$= 2.21\ \text{V}.$$

17. A charge $-5q$ is a distance $2d$ from $P$, a charge $-5q$ is a distance $d$ from $P$, and two charges $+5q$ are each a distance $d$ from $P$, so the electric potential at $P$ is

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{2d} - \frac{1}{d} + \frac{1}{d} + \frac{1}{d}\right] = \frac{q}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-15}\text{C})}{2(4.00\times10^{-2}\ \text{m})}$$
$$= 5.62\times10^{-4}\,\text{V}.$$

The zero of the electric potential was taken to be at infinity.

18. When the charge $q_2$ is infinitely far away, the potential at the origin is due only to the charge $q_1$ :

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 d} = 5.76\times10^{-7}\ \text{V}.$$

Thus, $q_1/d = 6.41 \times 10^{-17}$ C/m. Next, we note that when $q_2$ is located at $x = 0.080$ m, the net potential vanishes ($V_1 + V_2 = 0$). Therefore,

$$0 = \frac{kq_2}{0.08\text{ m}} + \frac{kq_1}{d}$$

Thus, we find $q_2 = -(q_1/d)(0.08\text{ m}) = -5.13 \times 10^{-18}\text{ C} = -32\,e$.

19. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the $x$ axis: $x < 0$ and $0 < x < d$.

(a) For $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$. With $d = 24.0$ cm, we have $x = 6.00$ cm.

(b) Similarly, for $x < 0$ the separation between $q_1$ and a point on the $x$ axis whose coordinate is $x$ is given by $d_1 = -x$; while the corresponding separation for $q_2$ is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. With $d = 24.0$ cm, we have $x = -12.0$ cm.

20. Since according to the problem statement there is a point in between the two charges on the $x$ axis where the net electric field is zero, the fields at that point due to $q_1$ and $q_2$ must be directed opposite to each other. This means that $q_1$ and $q_2$ must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

21. We use Eq. 24-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.47\times3.34\times10^{-30}\text{ C}\cdot\text{m}\right)}{\left(52.0\times10^{-9}\text{ m}\right)^2} = 1.63\times10^{-5}\text{ V}.$$

22. From Eq. 24-30 and Eq. 24-14, we have (for $\theta_i = 0°$)

$$W_a = q\Delta V = e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2} - \frac{p\cos\theta_i}{4\pi\varepsilon_0 r^2}\right) = \frac{ep\cos\theta}{4\pi\varepsilon_0 r^2}(\cos\theta - 1)$$

with $r$ = 20 × $10^{-9}$ m. For $\theta$ = 180º the graph indicates $W_a$ = –4.0 × $10^{-30}$ J, from which we can determine $p$. The magnitude of the dipole moment is therefore $5.6\times10^{-37}$ C·m.

23. (a) From Eq. 24-35, we find the potential to be

$$\begin{aligned} V &= 2\frac{\lambda}{4\pi\varepsilon_0}\ \ln\left[\frac{L/2+\sqrt{(L^2/4)+d^2}}{d}\right] \\ &= 2(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.68\times10^{-12}\,\mathrm{C/m})\ln\left[\frac{(0.06\ \mathrm{m}/2)+\sqrt{(0.06\ \mathrm{m})^2/4+(0.08\ \mathrm{m})^2}}{0.08\ \mathrm{m}}\right] \\ &= 2.43\times10^{-2}\ \mathrm{V}. \end{aligned}$$

(b) The potential at $P$ is $V$ = 0 due to superposition.

24. The potential is

$$\begin{aligned} V_P &= \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}}\frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} = -\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(25.6\times10^{-12}\,\mathrm{C})}{3.71\times10^{-2}\ \mathrm{m}} \\ &= -6.20\ \mathrm{V}. \end{aligned}$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance $R$. This "coincidence" is due, in part, to the fact that $V$ is a scalar quantity.

25. (a) All the charge is the same distance $R$ from $C$, so the electric potential at $C$ is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{R} - \frac{6Q_1}{R}\right) = -\frac{5Q_1}{4\pi\varepsilon_0 R} = -\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4.20\times10^{-12}\,\mathrm{C})}{8.20\times10^{-2}\ \mathrm{m}} = -2.30\ \mathrm{V},$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from $P$. That distance is $\sqrt{R^2+D^2}$, so the electric potential at $P$ is

$$\begin{aligned} V &= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{\sqrt{R^2+D^2}} - \frac{6Q_1}{\sqrt{R^2+D^2}}\right] = -\frac{5Q_1}{4\pi\varepsilon_0\sqrt{R^2+D^2}} \\ &= -\frac{5(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(4.20\times10^{-12}\,\mathrm{C})}{\sqrt{(8.20\times10^{-2}\ \mathrm{m})^2+(6.71\times10^{-2}\ \mathrm{m})^2}} \\ &= -1.78\ \mathrm{V}. \end{aligned}$$

26. The derivation is shown in the book (Eq. 24-33 through Eq. 24-35) except for the change in the lower limit of integration (which is now $x = D$ instead of $x = 0$). The result is therefore (cf. Eq. 24-35)

$$V = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L + \sqrt{L^2 + d^2}}{D + \sqrt{D^2 + d^2}}\right) = \frac{2.0\times10^{-6}}{4\pi\varepsilon_0}\ln\left(\frac{4+\sqrt{17}}{1+\sqrt{2}}\right) = 2.18 \times 10^4 \text{ V}.$$

27. Letting $d$ denote 0.010 m, we have

$$V = \frac{Q_1}{4\pi\varepsilon_0 d} + \frac{3Q_1}{8\pi\varepsilon_0 d} - \frac{3Q_1}{16\pi\varepsilon_0 d} = \frac{Q_1}{8\pi\varepsilon_0 d} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(30\times10^{-9}\text{C})}{2(0.01 \text{ m})}$$
$$= 1.3\times10^4 \text{ V}.$$

28. Consider an infinitesimal segment of the rod, located between $x$ and $x + dx$. It has length $dx$ and contains charge $dq = \lambda\, dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from $P_1$ is $d + x$ and the potential it creates at $P_1$ is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{d+x}.$$

To find the total potential at $P_1$, we integrate over the length of the rod and obtain:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right)$$
$$= \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(56.1\times10^{-15}\text{C})}{0.12 \text{ m}}\ln\left(1+\frac{0.12 \text{ m}}{0.025 \text{ m}}\right)$$
$$= 7.39\times10^{-3} \text{ V}.$$

29. Since the charge distribution on the arc is equidistant from the point where $V$ is evaluated, its contribution is identical to that of a point charge at that distance. We assume $V \to 0$ as $r \to \infty$ and apply Eq. 24-27:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+4Q_1}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R}$$
$$= \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(7.21\times10^{-12}\text{C})}{2.00 \text{ m}}$$
$$= 3.24\times10^{-2} \text{ V}.$$

30. The dipole potential is given by Eq. 24-30 (with $\theta = 90°$ in this case)

$$V=\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}=\frac{p\cos 90°}{4\pi\varepsilon_0 r^2}=0$$

since cos(90°) = 0 . The potential due to the short arc is $q_1/4\pi\varepsilon_0 r_1$ and that caused by the long arc is $q_2/4\pi\varepsilon_0 r_2$. Since $q_1$ = +2 $\mu$C, $r_1$ = 4.0 cm, $q_2$ = –3 $\mu$C, and $r_2$ = 6.0 cm, the potentials of the arcs cancel. The result is zero.

31. **THINK** Since the disk is uniformly charged, when the full disk is present each quadrant contributes equally to the electric potential at *P*.

**EXPRESS** Electrical potential is a scalar quantity. The potential at *P* due to a single quadrant is one-fourth the potential due to the entire disk. We first find an expression for the potential at *P* due to the entire disk. To do so, consider a ring of charge with radius *r* and (infinitesimal) width *dr*. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is at a distance $\sqrt{r^2+D^2}$ from *P*, so the potential it produces at *P* is

$$dV=\frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma r dr}{\sqrt{r^2+D^2}}=\frac{\sigma r dr}{2\varepsilon_0\sqrt{r^2+D^2}}.$$

**ANALYZE** Integrating over *r*, the total potential at *P* is

$$V=\frac{\sigma}{2\varepsilon_0}\int_0^R\frac{rdr}{\sqrt{r^2+D^2}}=\frac{\sigma}{2\varepsilon_0}\sqrt{r^2+D^2}\Big|_0^R=\frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right].$$

Therefore, the potential $V_{sq}$ at *P* due to a single quadrant is

$$V_{sq}=\frac{V}{4}=\frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right]=\frac{(7.73\times10^{-15}\text{C/m}^2)}{8(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\left[\sqrt{(0.640\ \text{m})^2+(0.259\ \text{m})^2}-0.259\ \text{m}\right]$$
$$=4.71\times10^{-5}\ \text{V}.$$

**LEARN** Consider the limit $D\gg R.$ The potential becomes

$$V_{sq}=\frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+D^2}-D\right]\approx\frac{\sigma}{8\varepsilon_0}\left[D\left(1+\frac{1}{2}\frac{R^2}{D^2}+\cdots\right)-D\right]$$
$$=\frac{\sigma}{8\varepsilon_0}\frac{R^2}{2D}=\frac{\pi R^2\sigma/4}{4\pi\varepsilon_0 D}=\frac{q_{sq}}{4\pi\varepsilon_0 D}$$

where $q_{sq}=\pi R^2\sigma/4$ is the charge on the quadrant. In this limit, we see that the potential resembles that due to a point charge $q_{sq}$.

32. Equation 24-32 applies with $dq = \lambda\,dx = bx\,dx$ (along $0 \le x \le 0.20$ m).

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20}\frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0} = 36 \text{ V}.$$

(b) Now $r = \sqrt{x^2+d^2}$ where $d$ = 0.15 m, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20}\frac{bxdx}{\sqrt{x^2+d^2}} = \frac{b}{4\pi\varepsilon_0}\left(\sqrt{x^2+d^2}\right)\Big|_0^{0.20} = 18 \text{ V}.$$

33. Consider an infinitesimal segment of the rod, located between $x$ and $x + dx$. It has length $dx$ and contains charge $dq = \lambda\, dx = cx\, dx$. Its distance from $P_1$ is $d + x$ and the potential it creates at $P_1$ is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x}.$$

To find the total potential at $P_1$, we integrate over the length of the rod and obtain

$$\begin{aligned} V &= \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{xdx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\ln\left(1+\frac{L}{d}\right)\right] \\ &= (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(28.9\times10^{-12}\text{C/m}^2)\left[0.120 \text{ m} - (0.030 \text{ m})\ln\left(1+\frac{0.120 \text{ m}}{0.030 \text{ m}}\right)\right] \\ &= 1.86\times10^{-2} \text{ V}. \end{aligned}$$

34. The magnitude of the electric field is given by

$$|E| = \left|-\frac{\Delta V}{\Delta x}\right| = \frac{2(5.0\text{V})}{0.015\text{m}} = 6.7\times10^2 \text{ V/m}.$$

At any point in the region between the plates, $\vec{E}$ points away from the positively charged plate, directly toward the negatively charged one.

35. We use Eq. 24-41:

$$E_x(x,y) = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\text{V/m}^2)x^2 - 3.0\text{V/m}^2)y^2\right) = -2(2.0\text{V/m}^2)x;$$
$$E_y(x,y) = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\text{V/m}^2)x^2 - 3.0\text{V/m}^2)y^2\right) = 2(3.0\text{V/m}^2)y.$$

We evaluate at $x$ = 3.0 m and $y$ = 2.0 m to obtain

$$\vec{E} = (-12 \text{ V/m})\hat{\text{i}} + (12 \text{ V/m})\hat{\text{j}}.$$

36. We use Eq. 24-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\vec{E} = -\left(\frac{dV}{dx}\right)\hat{\text{i}} = -\frac{d}{dx}(1500x^2)\hat{\text{i}} = (-3000x)\hat{\text{i}} = (-3000 \text{ V/m}^2)(0.0130 \text{ m})\hat{\text{i}}$$
$$= (-39 \text{ V/m})\hat{\text{i}}.$$

(a) Thus, the magnitude of the electric field is $E$ = 39 V/m.

(b) The direction of $\vec{E}$ is $-\hat{\text{i}}$, or toward plate 1.

37. **THINK** The component of the electric field $\vec{E}$ in a given direction is the negative of the rate at which potential changes with distance in that direction.

**EXPRESS** With $V = 2.00xyz^2$, we apply Eq. 24-41 to calculate the *x*, *y*, and *z* components of the electric field:

$$E_x = -\frac{\partial V}{\partial x} = -2.00yz^2$$
$$E_y = -\frac{\partial V}{\partial y} = -2.00xz^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4.00xyz$$

which, at ($x$, $y$, $z$) = (3.00 m, –2.00 m, 4.00 m), gives

$$(E_x, E_y, E_z) = (64.0 \text{ V/m}, -96.0 \text{ V/m}, 96.0 \text{ V/m}).$$

**ANALYZE** The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0 \text{ V/m})^2 + (-96.0 \text{ V/m})^2 + (96.0 \text{ V/m})^2}$$
$$= 150 \text{ V/m} = 150 \text{ N/C}.$$

**LEARN** If the electric potential increases along some direction, say $x$, with $\partial V/\partial x > 0$, then there is a corresponding nonvanishing component of $\vec{E}$ in the opposite direction ($-E_x \neq 0$).

38. (a) From the result of Problem 24-28, the electric potential at a point with coordinate $x$ is given by

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x-L}{x}\right).$$

At $x = d$ we obtain

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{d+L}{d}\right) = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(43.6\times10^{-15}\text{C})}{0.135\ \text{m}}\ln\left(1+\frac{0.135\ \text{m}}{d}\right)$$
$$= (2.90\times10^{-3}\ \text{V})\ln\left(1+\frac{0.135\ \text{m}}{d}\right).$$

(b) We differentiate the potential with respect to $x$ to find the $x$ component of the electric field:

$$E_x = -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\varepsilon_0 L}\frac{\partial}{\partial x}\ln\left(\frac{x-L}{x}\right) = -\frac{Q}{4\pi\varepsilon_0 L}\frac{x}{x-L}\left(\frac{1}{x}-\frac{x-L}{x^2}\right) = -\frac{Q}{4\pi\varepsilon_0 x(x-L)}$$
$$= -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(43.6\times10^{-15}\text{C})}{x(x+0.135\ \text{m})} = -\frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{x(x+0.135\ \text{m})},$$

or

$$|E_x| = \frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{x(x+0.135\ \text{m})}.$$

(c) Since $E_x < 0$, its direction relative to the positive $x$ axis is 180°.

(d) At $x = d = 6.20$ cm, we obtain

$$|E_x| = \frac{(3.92\times10^{-4}\ \text{N}\cdot\text{m}^2/\text{C})}{(0.0620\ \text{m})(0.0620\ \text{m}+0.135\ \text{m})} = 0.0321\ \text{N/C}.$$

(e) Consider two points an equal infinitesimal distance on either side of $P_1$, along a line that is perpendicular to the $x$ axis. The difference in the electric potential divided by their separation gives the transverse component of the electric field. Since the two points are situated symmetrically with respect to the rod, their potentials are the same and the potential difference is zero. Thus, the transverse component of the electric field $E_y$ is zero.

39. The electric field (along some axis) is the (negative of the) derivative of the potential $V$ with respect to the corresponding coordinate. In this case, the derivatives can be read off of the graphs as slopes (since the graphs are of straight lines). Thus,

$$E_x = -\frac{\partial V}{\partial x} = -\left(\frac{-500\ \text{V}}{0.20\ \text{m}}\right) = 2500\ \text{V/m} = 2500\ \text{N/C}$$
$$E_y = -\frac{\partial V}{\partial y} = -\left(\frac{300\ \text{V}}{0.30\ \text{m}}\right) = -1000\ \text{V/m} = -1000\ \text{N/C}.$$

These components imply the electric field has a magnitude of 2693 N/C and a direction of –21.8º (with respect to the positive $x$ axis). The force on the electron is given by $\vec{F} = q\vec{E}$ where $q$ = –$e$. The minus sign associated with the value of $q$ has the implication that $\vec{F}$ points in the opposite direction from $\vec{E}$ (which is to say that its angle is found by adding 180º to that of $\vec{E}$ ). With $e = 1.60 \times 10^{-19}$ C, we obtain

$$\vec{F} = (-1.60\times10^{-19}\,\text{C})[(2500\ \text{N/C})\hat{\text{i}} - (1000\ \text{N/C})\hat{\text{j}}] = (-4.0\times10^{-16}\,\text{N})\hat{\text{i}} + (1.60\times10^{-16}\ \text{N})\hat{\text{j}}.$$

40. (a) Consider an infinitesimal segment of the rod from $x$ to $x + dx$. Its contribution to the potential at point $P_2$ is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}dx.$$

Thus,

$$\begin{aligned} V &= \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}dx = \frac{c}{4\pi\varepsilon_0}\left(\sqrt{L^2+y^2} - y\right) \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\,\text{C/m}^2)\left(\sqrt{(0.100\ \text{m})^2+(0.0356\ \text{m})^2} - 0.0356\ \text{m}\right) \\ &= 3.16\times10^{-2}\ \text{V}. \end{aligned}$$

(b) The $y$ component of the field there is

$$\begin{aligned} E_y &= -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}\left(\sqrt{L^2+y^2} - y\right) = \frac{c}{4\pi\varepsilon_0}\left(1 - \frac{y}{\sqrt{L^2+y^2}}\right) \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(49.9\times10^{-12}\,\text{C/m}^2)\left(1 - \frac{0.0356\ \text{m}}{\sqrt{(0.100\ \text{m})^2+(0.0356\ \text{m})^2}}\right) \\ &= 0.298\ \text{N/C}. \end{aligned}$$

(c) We obtained above the value of the potential at any point $P$ strictly on the $y$-axis. In order to obtain $E_x(x, y)$ we need to first calculate $V(x, y)$. That is, we must find the potential for an arbitrary point located at ($x$, $y$). Then $E_x(x, y)$ can be obtained from $E_x(x,y) = -\partial V(x,y)/\partial x$.

41. We apply conservation of energy for the particle with $q = 7.5 \times 10^{-6}$ C (which has zero initial kinetic energy):

$$U_0 = K_f + U_f,$$

where $U = \dfrac{q\,Q}{4\pi\varepsilon_o r}$.

(a) The initial value of $r$ is 0.60 m and the final value is (0.6 + 0.4) m = 1.0 m (since the particles repel each other). Conservation of energy, then, leads to $K_f$ = 0.90 J.

(b) Now the particles attract each other so that the final value of $r$ is 0.60 – 0.40 = 0.20 m. Use of energy conservation yields $K_f$ = 4.5 J in this case.

42. (a) We use Eq. 24-43 with $q_1 = q_2 = -e$ and $r$ = 2.00 nm:

$$U = k\frac{q_1 q_2}{r} = k\frac{e^2}{r} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(1.60\times10^{-19}\,\text{C})^2}{2.00\times10^{-9}\,\text{m}} = 1.15\times10^{-19}\,\text{J}.$$

(b) Since $U > 0$ and $U \propto r^{-1}$ the potential energy $U$ decreases as $r$ increases.

43. **THINK** The work required to set up the arrangement is equal to the potential energy of the system.

**EXPRESS** We choose the zero of electric potential to be at infinity. The initial electric potential energy $U_i$ of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$U_f = \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right) = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right).$$

Thus the amount of work required to set up the system is given by

$$W = \Delta U = U_f - U_i = U_f = \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.30\times10^{-12}\,\text{C})^2}{0.640\ \text{m}}\left(\frac{1}{\sqrt{2}}-2\right)$$
$$= -1.92\times10^{-13}\,\text{J}.$$

**LEARN** The work done in assembling the system is negative. This means that an external agent would have to supply $W_{\text{ext}} = +1.92\times10^{-13}\,\text{J}$ in order to take apart the arrangement completely.

44. The work done must equal the change in the electric potential energy. From Eq. 24-14 and Eq. 24-26, we find (with $r$ = 0.020 m)

$$W = \frac{(3e-2e+2e)(6e)}{4\pi\varepsilon_0 r} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(18)(1.60\times10^{-19}\,\text{C})^2}{0.020\,\text{m}} = 2.1\times10^{-25}\ \text{J}.$$

45. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\varepsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\varepsilon_0 r_2$,

and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where $v$ is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\varepsilon_0 r_1} = \frac{q^2}{4\pi\varepsilon_0 r_2} + \frac{1}{2}mv^2 .$$

The solution for $v$ is

$$v = \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2)(3.1\times10^{-6}\,\text{C})^2}{20\times10^{-6}\text{kg}}\left(\frac{1}{0.90\times10^{-3}\text{m}} - \frac{1}{2.5\times10^{-3}\text{m}}\right)}$$
$$= 2.5\times10^3\ \text{m/s}.$$

46. Let $r$ = 1.5 m, $x$ = 3.0 m, $q_1$ = –9.0 nC, and $q_2$ = –6.0 pC. The work done by an external agent is given by

$$W = \Delta U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2 + x^2}}\right)$$
$$= \left(-9.0\times10^{-9}\ \text{C}\right)\left(-6.0\times10^{-12}\text{C}\right)\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\cdot\left[\frac{1}{1.5\ \text{m}} - \frac{1}{\sqrt{(1.5\ \text{m})^2 + (3.0\ \text{m})^2}}\right]$$
$$= 1.8\times10^{-10}\,\text{J}.$$

47. The *escape speed* may be calculated from the requirement that the initial kinetic energy (of *launch*) be equal to the absolute value of the initial potential energy (compare with the gravitational case in Chapter 14). Thus,

$$\frac{1}{2}mv^2 = \frac{eq}{4\pi\varepsilon_0 r}$$

where $m = 9.11 \times 10^{-31}$ kg, $e = 1.60 \times 10^{-19}$ C, $q = 10000e$, and $r$ = 0.010 m. This yields $v$ = 22490 m/s $\approx 2.2\times10^4$ m/s .

48. The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2}m_e v_i^2$ we find the initial electron speed to be

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.6\times10^{-19}\text{C})(125\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 6.63\times10^6\ \text{m/s}.$$

49. We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\varepsilon_0 d$, where $d$ is half the distance between the fixed electrons. The initial

kinetic energy is $K_i = \frac{1}{2}mv^2$, where *m* is the mass of an electron and *v* is the initial speed of the moving electron. The final kinetic energy is zero. Thus,

$$K_i = U_f \quad \Rightarrow \quad \frac{1}{2}mv^2 = 2e^2/4\pi\varepsilon_0 d.$$

Hence,

$$v = \sqrt{\frac{4e^2}{4\pi\varepsilon_0 dm}} = \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4)(1.60\times10^{-19}\ \text{C})^2}{(0.010\ \text{m})(9.11\times10^{-31}\ \text{kg})}} = 3.2\times10^2\ \text{m/s}.$$

50. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1Q}{2d} + \frac{q_2Q}{d}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1Q}{2d} + \frac{(-q_1/2)Q}{d}\right) = 0.$$

51. (a) Let $\ell = 0.15\,\text{m}$ be the length of the rectangle and *w* = 0.050 m be its width. Charge $q_1$ is a distance $\ell$ from point *A* and charge $q_2$ is a distance *w*, so the electric potential at *A* is

$$V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\ell} + \frac{q_2}{w}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.15\,\text{m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.050\ \text{m}}\right)$$
$$= 6.0\times10^4\,\text{V}.$$

(b) Charge $q_1$ is a distance *w* from point *b* and charge $q_2$ is a distance $\ell$, so the electric potential at *B* is

$$V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{w} + \frac{q_2}{\ell}\right) = (8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{-5.0\times10^{-6}\,\text{C}}{0.050\ \text{m}} + \frac{2.0\times10^{-6}\,\text{C}}{0.15\,\text{m}}\right)$$
$$= -7.8\times10^5\,\text{V}.$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge $q_3$ and the electric potential. If $U_A$ is the potential energy when $q_3$ is at *A* and $U_B$ is the potential energy when $q_3$ is at *B*, then the work done in moving the charge from *B* to *A* is

$$W = U_A - U_B = q_3(V_A - V_B) = (3.0\times10^{-6}\ \text{C})(6.0\times10^4\ \text{V} + 7.8\times10^5\ \text{V}) = 2.5\ \text{J}.$$

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.

52. From Eq. 24-30 and Eq. 24-7, we have (for $\theta$ = 180º)

$$U = qV = -e\left(\frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\right) = \frac{ep}{4\pi\varepsilon_0 r^2}$$

where $r$ = 0.020 m. Using energy conservation, we set this expression equal to 100 eV and solve for $p$. The magnitude of the dipole moment is therefore $p = 4.5 \times 10^{-12}\ \text{C}\cdot\text{m}$.

53. (a) The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\ \text{C}\right)^2}{1.00\ \text{m}} = 0.225\ \text{J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-6}\ \text{C}\right)^2}{\left(1.00\ \text{m}\right)^2} = 0.225\ \text{N}.$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let $m_A$ and $m_B$ be the masses of the spheres. The acceleration of sphere $A$ is

$$a_A = \frac{F}{m_A} = \frac{0.225\ \text{N}}{5.0\times10^{-3}\ \text{kg}} = 45.0\ \text{m/s}^2$$

and the acceleration of sphere $B$ is

$$a_B = \frac{F}{m_B} = \frac{0.225\ \text{N}}{10\times10^{-3}\ \text{kg}} = 22.5\ \text{m/s}^2.$$

(c) Energy is conserved. The initial potential energy is $U$ = 0.225 J, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where $v_A$ and $v_B$ are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2.$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B.$$

These equations may be solved simultaneously for $v_A$ and $v_B$. Substituting $v_B = -(m_A / m_B) v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain

$$U = \tfrac{1}{2}(m_A / m_B)(m_A + m_B) v_A^2.$$

Thus,

$$v_A = \sqrt{\frac{2Um_B}{m_A(m_A + m_B)}} = \sqrt{\frac{2(0.225 \text{ J})(10\times10^{-3} \text{ kg})}{(5.0\times10^{-3} \text{ kg})(5.0\times10^{-3} \text{ kg} + 10\times10^{-3} \text{ kg})}} = 7.75 \text{ m/s}.$$

We thus obtain

$$v_B = -\frac{m_A}{m_B} v_A = -\left(\frac{5.0\times10^{-3} \text{ kg}}{10\times10^{-3} \text{ kg}}\right)(7.75 \text{ m/s}) = -3.87 \text{ m/s},$$

or $|v_B| = 3.87$ m/s.

54. (a) Using $U = qV$ we can "translate" the graph of voltage into a potential energy graph (in eV units). From the information in the problem, we can calculate its kinetic energy (which is its total energy at $x = 0$) in those units: $K_i = 284$ eV. This is less than the "height" of the potential energy "barrier" (500 eV high once we've translated the graph as indicated above). Thus, it must reach a turning point and then reverse its motion.

(b) Its final velocity, then, is in the negative $x$ direction with a magnitude equal to that of its initial velocity. That is, its speed (upon leaving this region) is $1.0 \times 10^7$ m/s.

55. Let the distance in question be $r$. The initial kinetic energy of the electron is $K_i = \tfrac{1}{2} m_e v_i^2$, where $v_i = 3.2 \times 10^5$ m/s. As the speed doubles, $K$ becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\varepsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2} m_e v_i^2,$$

or

$$r = \frac{2e^2}{3(4\pi\varepsilon_0) m_e v_i^2} = \frac{2\left(1.6\times10^{-19} \text{ C}\right)^2 \left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)}{3\left(9.11\times10^{-19} \text{ kg}\right)\left(3.2\times10^5 \text{ m/s}\right)^2} = 1.6\times10^{-9} \text{ m}.$$

56. When particle 3 is at $x = 0.10$ m, the total potential energy vanishes. Using Eq. 24-43, we have (with meters understood at the length unit)

$$0 = \frac{q_1 q_2}{4\pi\varepsilon_0 d} + \frac{q_1 q_3}{4\pi\varepsilon_0 (d + 0.10 \text{ m})} + \frac{q_3 q_2}{4\pi\varepsilon_0 (0.10 \text{ m})}$$

This leads to

$$q_3\left(\frac{q_1}{d+0.10\text{ m}}+\frac{q_2}{0.10\text{ m}}\right)=-\frac{q_1q_2}{d}$$

which yields $q_3 = -5.7\ \mu$C.

57. **THINK** Mechanical energy is conserved in the process.

**EXPRESS** The electric potential at (0, $y$) due to the two charges $Q$ held fixed at $(\pm x, 0)$ is

$$V=\frac{2Q}{4\pi\varepsilon_0\sqrt{x^2+y^2}}\ .$$

Thus, the potential energy of the particle of charge $q$ at (0, $y$) is

$$U=qV=\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y^2}}\ .$$

Conservation of mechanical energy ($K_i + U_i = K_f + U_f$) gives

$$K_f=K_i+U_i-U_f=K_i+\frac{2Qq}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y_i^2}}-\frac{1}{\sqrt{x^2+y_f^2}}\right),$$

where $y_i$ and $y_f$ are the initial and final coordinates of the moving charge along the $y$ axis.

**ANALYZE** (a) With $q=-15\times10^{-6}$ C, $Q = 50\times10^{-6}$ C, $x = \pm3$ m, $y_i = 4$ m, and $y_f = 0$, we obtain

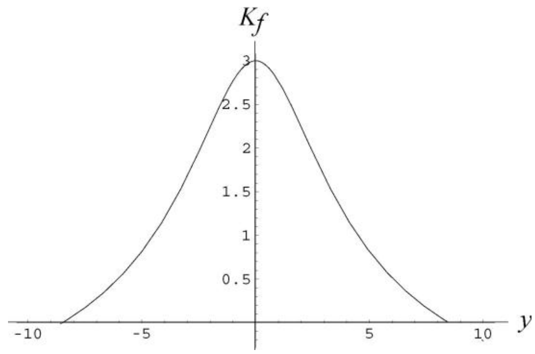
$$\begin{aligned}K_f&=1.2\text{ J}+\frac{2(50\times10^{-6}\text{ C})(-15\times10^{-6}\text{ C})}{4\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\frac{1}{\sqrt{(3.0\text{ m})^2+(4.0\text{ m})^2}}-\frac{1}{\sqrt{(3.0\text{ m})^2}}\right)\\&=3.0\text{ J}.\end{aligned}$$

(b) We set $K_f = 0$ and solve for $y_f$ (choosing the negative root, as indicated in the problem statement):

$$K_i+U_i=U_f\quad\Rightarrow\quad 1.2\text{ J}+\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y_i^2}}=\frac{2Qq}{4\pi\varepsilon_0\sqrt{x^2+y_f^2}}\ .$$

Substituting the values given, we have $U_i=-2.7$ J, and $y_f = -8.5$ m.

**LEARN** The dependence of the final kinetic energy of the particle on $y$ is plotted below. From the plot, we see that $K_f = 3.0$ J at $y = 0$, and $K_f = 0$ at $y = -8.5$ m. The particle oscillates between the two end-points $y_f = \pm 8.5$ m.




58. (a) When the proton is released, its energy is $K + U = 4.0$ eV + 3.0 eV (the latter value is inferred from the graph). This implies that if we draw a horizontal line at the 7.0 volt "height" in the graph and find where it intersects the voltage plot, then we can determine the turning point. Interpolating in the region between 1.0 cm and 3.0 cm, we find the turning point is at roughly $x = 1.7$ cm.

(b) There is no turning point toward the right, so the speed there is nonzero, and is given by energy conservation:

$$v = \sqrt{\frac{2(7.0 \text{ eV})}{m}} = \sqrt{\frac{2(7.0 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}} = 20 \text{ km/s}.$$

(c) The electric field at any point $P$ is the (negative of the) slope of the voltage graph evaluated at $P$. Once we know the electric field, the force on the proton follows immediately from $\vec{F} = q\vec{E}$, where $q = +e$ for the proton. In the region just to the left of $x = 3.0$ cm, the field is $\vec{E} = (+300 \text{ V/m})\hat{\text{i}}$ and the force is $F = +4.8 \times 10^{-17}$ N.

(d) The force $\vec{F}$ points in the $+x$ direction, as the electric field $\vec{E}$.

(e) In the region just to the right of $x = 5.0$ cm, the field is $\vec{E} = (-200 \text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 3.2 \times 10^{-17}$ N.

(f) The force $\vec{F}$ points in the $-x$ direction, as the electric field $\vec{E}$.

59. (a) The electric field between the plates is leftward in Fig, 24-59 since it points toward lower values of potential. The force (associated with the field, by Eq. 23-28) is evidently leftward, from the problem description (indicating deceleration of the rightward moving particle), so that $q > 0$ (ensuring that $\vec{F}$ is parallel to $\vec{E}$); it is a proton.

(b) We use conservation of energy:

$$K_0 + U_0 = K + U \;\;\Rightarrow\;\; \frac{1}{2}m_p v_0^2 + qV_1 = \frac{1}{2}m_p v^2 + qV_2 \;.$$

Using $q = +1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ kg, $v_0 = 90 \times 10^3$ m/s, $V_1 = -70$ V, and $V_2 = -50$ V, we obtain the final speed $v = 6.53 \times 10^4$ m/s. We note that the value of $d$ is not used in the solution.

60. (a) The work done results in a potential energy gain:

$$W = q\,\Delta V = (-e)\left(\frac{Q}{4\pi\varepsilon_0 R}\right) = +2.16 \times 10^{-13}\,\text{J}\,.$$

With $R = 0.0800$ m, we find $Q = -1.20 \times 10^{-5}$ C.

(b) The work is the same, so the increase in the potential energy is $\Delta U = +2.16 \times 10^{-13}$ J.

61. We note that for two points on a circle, separated by angle $\theta$ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where $N$ = odd and $N$ = even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all $N$ electrons are on the circle), we have

$$U_{1,N=\text{even}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta/2)} + \frac{1}{2}\right), \quad U_{1,N=\text{odd}} = \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}}\frac{1}{\sin(j\theta/2)}\right)$$

where $\theta = \dfrac{2\pi}{N}$. For configuration 2, we find

$$U_{2,N=\text{even}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1}\frac{1}{\sin(j\theta'/2)} + 2\right), \; U_{2,N=\text{odd}} = \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}}\frac{1}{\sin(j\theta'/2)} + \frac{5}{2}\right)$$

where $\theta' = \dfrac{2\pi}{N-1}$. The results are all of the form

$$U_{1\,\text{or}\,2}\;\frac{ke^2}{2R} \times \text{a pure number.}$$

In our table below we have the results for those “pure numbers” as they depend on $N$ and on which configuration we are considering. The values listed in the $U$ rows are the potential energies divided by $ke^2/2R$.

| N | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $U_1$ | 3.83 | 6.88 | 10.96 | 16.13 | 22.44 | 29.92 | 38.62 | 48.58 | 59.81 | 72.35 | 86.22 | 101.5 |
| $U_2$ | 4.73 | 7.83 | 11.88 | 16.96 | 23.13 | 30.44 | 39.92 | 48.62 | 59.58 | 71.81 | 85.35 | 100.2 |

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) $N = 12$ is the smallest value such that $U_2 < U_1$.

(b) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron $e_0$ on the circle is $R$ distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \approx 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two — one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \approx 1.1R$$

distance away from $e_0$. Thus, we see that there are only two electrons closer to $e_0$ than the one in the center.

62. (a) Since the two conductors are connected $V_1$ and $V_2$ must be equal to each other.

Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for $q_1$ and $q_2$: $q_1 = q/3$, $q_2 = 2q/3$, or

(b) $q_1/q = 1/3 = 0.333$.

(c) Similarly, $q_2/q = 2/3 = 0.667$.

(d) The ratio of surface charge densities is $\dfrac{\sigma_1}{\sigma_2} = \dfrac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\dfrac{q_1}{q_2}\right)\left(\dfrac{R_2}{R_1}\right)^2 = 2.00.$

63. **THINK** The electric potential is the sum of the contributions of the individual spheres.

**EXPRESS** Let $q_1$ be the charge on one, $q_2$ be the charge on the other, and $d$ be their separation. The point halfway between them is the same distance $d/2$ (= 1.0 m) from the center of each sphere.

For parts (b) and (c), we note that the distance from the center of one sphere to the surface of the other is $d - R$, where $R$ is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere as well as the charge on the other sphere.

**ANALYZE** (a) The potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\varepsilon_0 \, d/2} = \frac{\left(8.99\times10^9 \ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.0\times10^{-8}\,\text{C} - 3.0\times10^{-8}\,\text{C}\right)}{1.0 \text{ m}} = -1.8\times10^2\,\text{V}.$$

(b) The potential at the surface of sphere 1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}}\right] = 2.9\times10^3\,\text{V}.$$

(c) Similarly, the potential at the surface of sphere 2 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] = \left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{1.0\times10^{-8}\,\text{C}}{2.0\,\text{m} - 0.030\,\text{m}} - \frac{3.0\times10^{-8}\,\text{C}}{0.030\,\text{m}}\right] = -8.9\times10^3\,\text{V}.$$

**LEARN** In the limit where $d \to \infty$, the spheres are isolated from each other and the electric potentials at the surface of each individual sphere become

$$V_{10} = \frac{q_1}{4\pi\varepsilon_0 R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-8}\,\text{C})}{0.030\,\text{m}} = 3.0\times10^3\,\text{V},$$

and

$$V_{20} = \frac{q_2}{4\pi\varepsilon_0 R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-3.0\times10^{-8}\,\text{C})}{0.030\,\text{m}} = -8.99\times10^3\,\text{V}.$$

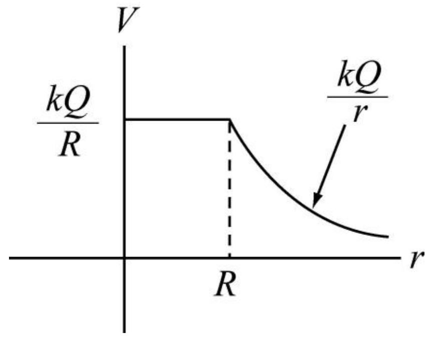
64. Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also +400 V.

65. **THINK** If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = Q/4\pi\varepsilon_0 R$, where $Q$ is the charge on the sphere and $R$ is its radius.

**EXPRESS** From $V = Q/4\pi\varepsilon_0 R$, we find the charge to be $Q = 4\pi\varepsilon_0 RV$.

**ANALYZE** With $R = 0.15$ m and $V = 1500$ V, we have

$$Q = 4\pi\varepsilon_0 RV = \frac{(0.15\text{ m})(1500\text{ V})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.5\times10^{-8}\,\text{C}.$$




**LEARN** A plot of the electric potential as a function of $r$ is shown to the right with $k = 1/4\pi\varepsilon_0$. Note that the potential is constant inside the conducting sphere.

66. Since the charge distribution is spherically symmetric we may write

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r},$$

where $q_{\text{enc}}$ is the charge enclosed in a sphere of radius $r$ centered at the origin.

(a) For $r$ = 4.00 m, $R_2$ = 1.00 m, and $R_1$ = 0.500 m, with $r > R_2 > R_1$ we have

$$E(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C} + 1.00\times10^{-6}\,\text{C})}{(4.00\ \text{m})^2} = 1.69\times10^3\ \text{V/m}.$$

(b) For $R_2 > r = 0.700$ m $> R_2$,

$$E(r) = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C})}{(0.700\ \text{m})^2} = 3.67\times10^4\ \text{V/m}.$$

(c) For $R_2 > R_1 > r$, the enclosed charge is zero. Thus, $E = 0$.

The electric potential may be obtained using Eq. 24-18:

$$V(r) - V(r') = \int_r^{r'} E(r)\,dr.$$

(d) For $r$ = 4.00 m $> R_2 > R_1$, we have

$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C} + 1.00\times10^{-6}\,\text{C})}{(4.00\ \text{m})} = 6.74\times10^3\ \text{V}.$$

(e) For $r$ = 1.00 m $= R_2 > R_1$, we have

$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\,\text{C} + 1.00\times10^{-6}\,\text{C})}{(1.00\ \text{m})} = 2.70\times10^4\ \text{V}.$$

(f) For $R_2 > r = 0.700$ m $> R_2$,

$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\text{C}}{0.700\ \text{m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\ \text{m}}\right)$$
$$=3.47\times10^4\ \text{V}.$$

(g) For $R_2 > r = 0.500$ m $= R_2$,

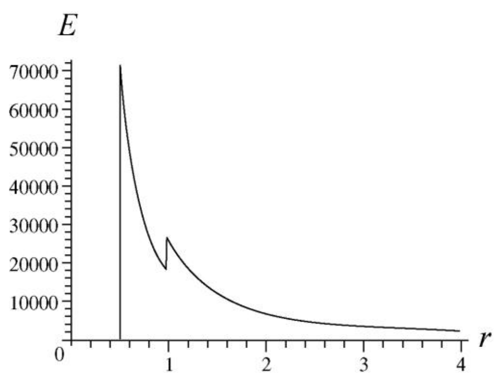
$$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\text{C}}{0.500\ \text{m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\ \text{m}}\right)$$
$$=4.50\times10^4\ \text{V}.$$

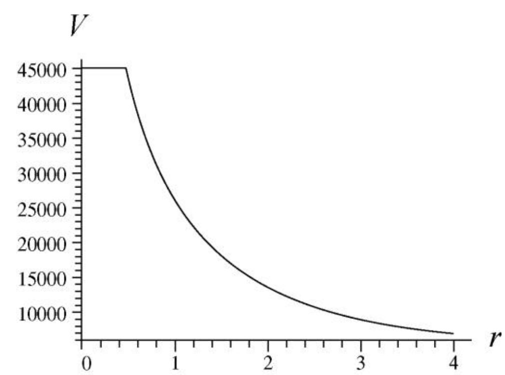
(h) For $R_2 > R_1 > r$,

$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1}+\frac{q_2}{R_2}\right)=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{2.00\times10^{-6}\text{C}}{0.500\ \text{m}}+\frac{1.00\times10^{-6}\text{C}}{1.00\ \text{m}}\right)$$
$$=4.50\times10^4\ \text{V}.$$

(i) At $r = 0$, the potential remains constant, $V=4.50\times10^4$ V.

(j) The electric field and the potential as a function of $r$ are depicted below:




67. (a) The magnitude of the electric field is

$$E=\frac{\sigma}{\varepsilon_0}=\frac{q}{4\pi\varepsilon_0 R^2}=\frac{\left(3.0\times10^{-8}\text{C}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(0.15\,\text{m}\right)^2}=1.2\times10^4\ \text{N/C}.$$

(b) $V = RE = (0.15\text{ m})(1.2\times10^4\text{ N/C}) = 1.8\times10^3$ V.

(c) Let the distance be $x$. Then

$$\Delta V=V(x)-V=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+x}-\frac{1}{R}\right)=-500\,\text{V},$$

which gives

$$x = \frac{R\Delta V}{-V-\Delta V} = \frac{(0.15\,\text{m})(-500\,\text{V})}{-1800\,\text{V}+500\,\text{V}} = 5.8\times10^{-2}\,\text{m}.$$

68. The potential energy of the two-charge system is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\,\text{C})(-4.00\times10^{-6}\,\text{C})}{\sqrt{(3.50+2.00)^2+(0.500-1.50)^2}\,\text{cm}}$$
$$= -1.93\text{ J}.$$

Thus, –1.93 J of work is needed.

69. **THINK** To calculate the potential, we first apply Gauss' law to calculate the electric field of the charged cylinder of radius *R*. The Gaussian surface is a cylindrical surface that is concentric with the cylinder.

**EXPRESS** We imagine a cylindrical Gaussian surface *A* of radius *r* and length *h* concentric with the cylinder. Then, by Gauss' law,

$$\oint_A \vec{E}\cdot d\vec{A} = 2\pi rhE = \frac{q_{\text{enc}}}{\varepsilon_0},$$

where $q_{\text{enc}}$ is the amount of charge enclosed by the Gaussian cylinder. Inside the charged cylinder ($r < R$), $q_{\text{enc}} = 0$, so the electric field is zero. On the other hand, outside the cylinder ($r > R$), $q_{\text{enc}} = \lambda h$ so the magnitude of the electric field is

$$E = \frac{q/h}{2\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

where λ is the linear charge density and *r* is the distance from the line to the point where the field is measured. The potential difference between two points 1 and 2 is

$$V(r_2)-V(r_1) = -\int_{r_1}^{r_2} E(r)\,dr.$$

**ANALYZE** (a) The radius of the cylinder (0.020 m, the same as $R_B$) is denoted *R*, and the field magnitude there (160 N/C) is denoted $E_B$. From the equation above, we see that the electric field beyond the surface of the cylinder is inversely proportional with *r*:

$$E = E_B\frac{R_B}{r}, \quad r \ge R_B.$$

Thus, if $r = R_C = 0.050$ m, we obtain

$$E_C = E_B \frac{R_B}{R_C} = (160 \text{ N/C})\left(\frac{0.020 \text{ m}}{0.050 \text{ m}}\right) = 64 \text{ N/C}.$$

(b) The potential difference between $V_B$ and $V_C$ is

$$V_B - V_C = -\int_{R_C}^{R_B} \frac{E_B R_B}{r} dr = E_B R_B \ln\left(\frac{R_C}{R_B}\right) = (160 \text{ N/C})(0.020 \text{ m}) \ln\left(\frac{0.050 \text{ m}}{0.020 \text{ m}}\right)$$
$$= 2.9 \text{ V}.$$

(c) The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged cylinder: $V_A - V_B = 0$.

**LEARN** The electric potential at a distance $r > R_B$ can be written as

$$V(r) = V_B - E_B R_B \ln\left(\frac{r}{R_B}\right).$$

We see that $V(r)$ decreases logarithmically with $r$.

70. (a) We use Eq. 24-18 to find the potential: $V_{\text{wall}} - V = -\int_r^R E dr$, or

$$0 - V = -\int_r^R \left(\frac{\rho r}{2\varepsilon_0}\right) \quad \Rightarrow \quad -V = -\frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right).$$

Consequently, $V = \rho(R^2 - r^2)/4\varepsilon_0$.

(b) The value at $r = 0$ is

$$V_{\text{center}} = \frac{-1.1 \times 10^{-3} \text{ C/m}^3}{4\left(8.85 \times 10^{-12} \text{ C/V} \cdot \text{m}\right)}\left((0.05 \text{m})^2 - 0\right) = -7.8 \times 10^4 \text{ V}.$$

Thus, the difference is $|V_{\text{center}}| = 7.8 \times 10^4$ V.

71. **THINK** The component of the electric field $\vec{E}$ in any direction is the negative of the rate at which potential changes with distance in that direction.

**EXPRESS** From Eq. 24-30, the electric potential of a dipole at a point a distance $r$ away is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

where $p$ is the magnitude of the dipole moment $\vec{p}$ and $\theta$ is the angle between $\vec{p}$ and the position vector of the point. The potential at infinity is taken to be zero.

**ANALYZE** On the dipole axis $\theta = 0$ or $\pi$, so $|\cos\theta| = 1$. Therefore, magnitude of the electric field is

$$|E(r)| = \left|-\frac{\partial V}{\partial r}\right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3}\ .$$

**LEARN** Take the $z$ axis to be the dipole axis. For $r = z > 0\,(\theta = 0)$, $E = p/2\pi\varepsilon_0 z^3$. On the other hand, for $r = -z < 0\ \ (\theta = \pi)$, $E = -p/2\pi\varepsilon_0 z^3$.

72. Using Eq. 24-18, we have

$$\Delta V = -\int_2^3 \frac{A}{r^4}\,dr = \frac{A}{3}\left(\frac{1}{2^3} - \frac{1}{3^3}\right) = A(0.029/\text{m}^3).$$

73. (a) The potential on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(4.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{0.10\,\text{m}} = 3.6\times10^5\ \text{V}\ .$$

(b) The field just outside the sphere would be

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{3.6\times10^5\ \text{V}}{0.10\,\text{m}} = 3.6\times10^6\ \text{V/m}\ ,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

74. The work done is equal to the change in the (total) electric potential energy $U$ of the system, where

$$U = \frac{q_1q_2}{4\pi\varepsilon_0 r_{12}} + \frac{q_3q_2}{4\pi\varepsilon_0 r_{23}} + \frac{q_1q_3}{4\pi\varepsilon_0 r_{13}}$$

and the notation $r_{13}$ indicates the distance between $q_1$ and $q_3$ (similar definitions apply to $r_{12}$ and $r_{23}$).

(a) We consider the difference in $U$ where initially $r_{12} = b$ and $r_{23} = a$, and finally $r_{12} = a$ and $r_{23} = b$ ($r_{13}$ doesn't change). Converting the values given in the problem to SI units (μC to C, cm to m), we obtain $\Delta U = -24$ J.

(b) Now we consider the difference in $U$ where initially $r_{23} = a$ and $r_{13} = a$, and finally $r_{23}$ is again equal to $a$ and $r_{13}$ is also again equal to $a$ (and of course, $r_{12}$ doesn't change in this case). Thus, we obtain $\Delta U = 0$.

75. Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V = q/4\pi\varepsilon_0 R$, where $q$ is the charge on Earth and $R = 6.37 \times 10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E = q/4\pi\varepsilon_0 R^2$, so

$$V = ER = (100 \text{ V/m})\,(6.37 \times 10^6 \text{ m}) = 6.4 \times 10^8 \text{ V}.$$

76. Using Gauss' law, $q = \varepsilon_0\Phi = +495.8$ nC. Consequently,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.958\times10^{-7}\text{ C})}{0.120 \text{ m}} = 3.71\times10^4 \text{ V}.$$

77. The potential difference is

$$\Delta V = E\Delta s = (1.92 \times 10^5 \text{ N/C})(0.0150 \text{ m}) = 2.90 \times 10^3 \text{ V}.$$

78. The charges are equidistant from the point where we are evaluating the potential — which is computed using Eq. 24-27 (or its integral equivalent). Equation 24-27 implicitly assumes $V \to 0$ as $r \to \infty$. Thus, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q_1}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+3Q_1}{R} = \frac{1}{4\pi\varepsilon_0}\frac{2Q_1}{R}$$
$$= \frac{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.52\times10^{-12}\text{ C})}{0.0850 \text{ m}} = 0.956 \text{ V}.$$

79. The electric potential energy in the presence of the dipole is

$$U = qV_{\text{dipole}} = \frac{qp\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{(-e)(ed)\cos\theta}{4\pi\varepsilon_0 r^2}\ .$$

Noting that $\theta_i = \theta_f = 0°$, conservation of energy leads to

$$K_f + U_f = K_i + U_i \quad \Rightarrow \quad v = \sqrt{\frac{2e^2}{4\pi\varepsilon_0 m d}\left(\frac{1}{25} - \frac{1}{49}\right)} = 7.0 \times 10^5 \text{ m/s}\ .$$

80. We treat the system as a superposition of a disk of surface charge density $\sigma$ and radius $R$ and a smaller, oppositely charged, disk of surface charge density $-\sigma$ and radius $r$. For each of these, Eq 24-37 applies (for $z > 0$)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2}-z\right)+\frac{-\sigma}{2\varepsilon_0}\left(\sqrt{z^2+r^2}-z\right).$$

This expression does vanish as $r \to \infty$, as the problem requires. Substituting $r = 0.200R$ and $z = 2.00R$ and simplifying, we obtain

$$V = \frac{\sigma R}{\varepsilon_0}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right) = \frac{(6.20\times10^{-12}\,\text{C/m}^2)(0.130\text{ m})}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\left(\frac{5\sqrt{5}-\sqrt{101}}{10}\right)$$
$$= 1.03\times10^{-2}\ \text{V}.$$

81. (a) When the electron is released, its energy is

$$K + U = 3.0\text{ eV} - 6.0\text{ eV}$$

(the latter value is inferred from the graph along with the fact that $U = qV$ and $q = -e$). Because of the minus sign (of the charge) it is convenient to imagine the graph multiplied by a minus sign so that it represents potential energy in eV. Thus, the 2 V value shown at $x = 0$ would become –2 eV, and the 6 V value at $x = 4.5$ cm becomes –6 eV, and so on. The total energy (–3.0 eV) is constant and can then be represented on our (imagined) graph as a horizontal line at –3.0 V. This intersects the potential energy plot at a point we recognize as the turning point. Interpolating in the region between 1.0 cm and 4.0 cm, we find the turning point is at $x = 1.75\text{ cm} \approx 1.8\text{ cm}$.

(b) There is no turning point toward the right, so the speed there is nonzero. Noting that the kinetic energy at $x = 7.0$ cm is

$$K = -3.0\text{ eV} - (-5.0\text{ eV}) = 2.0\text{ eV},$$

we find the speed using energy conservation:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.0\text{ eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = 8.4\times10^5\ \text{m/s}.$$

(c) The electric field at any point $P$ is the (negative of the) slope of the voltage graph evaluated at $P$. Once we know the electric field, the force on the electron follows immediately from $\vec{F} = q\vec{E}$, where $q = -e$ for the electron. In the region just to the left of $x = 4.0\text{ cm}$, the electric field is $\vec{E} = (-133\text{ V/m})\hat{\text{i}}$ and the magnitude of the force is $F = 2.1\times10^{-17}\ \text{N}$.

(d) The force points in the +*x* direction.

(e) In the region just to the right of *x* = 5.0 cm, the field is $\vec{E}$ = +100 V/m $\hat{\text{i}}$ and the force is $\vec{F}$ = ( –1.6 x $10^{-17}$ N) $\hat{\text{i}}$. Thus, the magnitude of the force is $F = 1.6\times10^{-17}$ N.

(f) The minus sign indicates that $\vec{F}$ points in the –*x* direction.

82. (a) The potential would be

$$V_e = \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2\sigma_e}{4\pi\varepsilon_0 R_e} = 4\pi R_e\sigma_e k$$
$$= 4\pi\left(6.37\times10^6\text{ m}\right)\left(1.0\,\text{electron/m}^2\right)\left(-1.6\times10^{-9}\text{ C/electron}\right)\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)$$
$$= -0.12\,\text{V}.$$

(b) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\text{V}}{6.37\times10^6\,\text{m}} = -1.8\times10^{-8}\,\text{N/C},$$

or $|E| = 1.8\times10^{-8}$ N/C.

(c) The minus sign in *E* indicates that $\vec{E}$ is radially inward.

83. (a) Using *d* = 2 m, we find the potential at *P*:

$$V_P = \frac{2e}{4\pi\varepsilon_0 d} + \frac{-2e}{4\pi\varepsilon_0(2d)} = \frac{e}{4\pi\varepsilon_0 d} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\text{C})}{2.00\text{ m}}.$$
$$= 7.192\times10^{-10}\text{ V}.$$

Note that we are implicitly assuming that $V \to 0$ as $r \to \infty$.

(b) Since *U* = *qV*, then the movable particle's contribution of the potential energy when it is at *r* = ∞ is zero, and its contribution to $U_{\text{system}}$ when it is at *P* is

$$U = qV_P = 2(1.6\times10^{-19}\text{C})(7.192\times10^{-10}\text{ V}) = 2.30\times10^{-28}\text{ J}.$$

Thus, the work done is approximately equal to $W_{\text{app}} = 2.30 \times 10^{-28}$ J.

(c) Now, combining the contribution to $U_{\text{system}}$ from part (b) and from the original pair of fixed charges

$$U_{\text{fixed}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(-2e)}{\sqrt{(4.00\text{ m})^2+(2.00\text{ m})^2}} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(4)(1.60\times10^{-19}\text{C})^2}{\sqrt{20.0}\text{ m}}$$
$$= -2.058\times10^{-28}\text{ J}$$

we obtain

$$U_{\text{system}} = W_{\text{app}} + U_{\text{fixed}} = 2.43\times10^{-29}\text{ J}.$$

84. The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged sphere:

$$V_A = V_S = \frac{q}{4\pi\varepsilon_0 R}$$

where $q = 30 \times 10^{-9}$ C and $R = 0.030$ m. For points beyond the surface of the sphere, the potential follows Eq. 24-26:

$$V_B = \frac{q}{4\pi\varepsilon_0 r}$$

where $r = 0.050$ m.

(a) We see that

$$V_S - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right) = 3.6\times10^3\text{ V}.$$

(b) Similarly,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right) = 3.6\times10^3\text{ V}.$$

85. We note that the net potential (due to the "fixed" charges) is zero at the first location ("at ∞") being considered for the movable charge $q$ (where $q = +2e$). Thus, with $D = 4.00$ m and $e = 1.60 \times 10^{-19}$ C, we obtain

$$V = \frac{+2e}{4\pi\varepsilon_0(2D)} + \frac{+e}{4\pi\varepsilon_0 D} = \frac{2e}{4\pi\varepsilon_0 D} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(1.60\times10^{-19}\text{C})}{4.00\text{ m}}$$
$$= 7.192\times10^{-10}\text{ V} .$$

The work required is equal to the potential energy in the final configuration:

$$W_{\text{app}} = qV = (2e)(7.192\times10^{-10}\text{ V}) = 2.30\times10^{-28}\text{ J}.$$

86. Since the electric potential is a scalar quantity, this calculation is far simpler than it would be for the electric field. We are able to simply take half the contribution that

would be obtained from a complete (whole) sphere. If it were a whole sphere (of the same density) then its charge would be $q_{\text{whole}} = 8.00\ \mu\text{C}$. Then

$$V = \frac{1}{2} V_{\text{whole}} = \frac{1}{2}\frac{q_{\text{whole}}}{4\pi\varepsilon_o r} = \frac{1}{2}\frac{8.00 \times 10^{-6}\,\text{C}}{4\pi\varepsilon_o(0.15\text{ m})} = 2.40 \times 10^5\text{ V}.$$

87. **THINK** The work done is equal to the change in potential energy.

**EXPRESS** The initial potential energy of the system is

$$U_i = \frac{2q^2}{4\pi\varepsilon_0 L} + U_0$$

where $q$ is the charge on each particle, $L$ is the length of the triangle side, and $U_0$ is the potential energy associated with the interaction of the two fixed charges. After moving to the midpoint of the line joining the two fixed charges, the final energy of the configuration is

$$U_f = \frac{2q^2}{4\pi\varepsilon_0 (L/2)} + U_0 .$$

Thus, the work done by the external agent is

$$W = \Delta U = U_f - U_i = \frac{2q^2}{4\pi\varepsilon_0}\left(\frac{2}{L} - \frac{1}{L}\right) = \frac{2q^2}{4\pi\varepsilon_0 L}.$$

**ANALYZE** Substituting the values given, we have

$$W = \frac{2q^2}{4\pi\varepsilon_0 L} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.12\text{ C})^2}{1.7\text{ m}} = 1.5\times10^8\ \text{J}.$$

At a rate of $P = 0.83 \times 10^3$ joules per second, it would take $W/P = 1.8 \times 10^5$ seconds or about 2.1 days to do this amount of work.

**LEARN** Since all three particles are positively charged, positive work is required by the external agent in order to bring them closer.

88. (a) The charges are equal and are the same distance from $C$. We use the Pythagorean theorem to find the distance

$$r = \sqrt{(d/2)^2 + (d/2)^2} = d/\sqrt{2}.$$

The electric potential at $C$ is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$V = \frac{2q}{4\pi\varepsilon_0}\,\frac{\sqrt{2}}{d} = \frac{2\sqrt{2q}}{4\pi\varepsilon_0 d} = \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)(2)\sqrt{2}\left(2.0\times10^{-6}\,\text{C}\right)}{0.020\text{ m}}$$
$$= 2.5\times10^6\,\text{V}.$$

(b) As you move the charge into position from far away the potential energy changes from zero to $qV$, where $V$ is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W = qV = \left(2.0\times10^{-6}\text{C}\right)\left(2.54\times10^6\,\text{V}\right) = 5.1\text{ J}.$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is $d$ so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$U = W + \frac{q^2}{4\pi\varepsilon_0 d} = 5.1\text{ J} + \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.0\times10^{-6}\,\text{C}\right)^2}{0.020\,\text{m}} = 6.9\text{ J}.$$

89. The net potential at point $P$ (the place where we are to place the third electron) due to the fixed charges is computed using Eq. 24-27 (which assumes $V \to 0$ as $r \to \infty$):

$$V_P = \frac{-e}{4\pi\varepsilon_0 d} + \frac{-e}{4\pi\varepsilon_0 d} = -\frac{2e}{4\pi\varepsilon_0 d}.$$

Thus, with $d = 2.00 \times 10^{-6}$ m and $e = 1.60 \times 10^{-19}$ C, we find

$$V_P = -\frac{2e}{4\pi\varepsilon_0 d} = -\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(1.60\times10^{-19}\text{C})}{2.00\times10^{-6}\text{ m}} = -1.438\times10^{-3}\text{ V}.$$

Then the required "applied" work is, by Eq. 24-14,

$$W_{\text{app}} = (-e)\,V_P = 2.30 \times 10^{-22}\text{ J}.$$

90. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius $r$. The charge $Q$ provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\varepsilon_0 r^2$. The acceleration of $-q$ is $v^2/r$, where $v$ is its speed. Newton's second law yields

$$\frac{Q_q}{4\pi\varepsilon_0 r^2}=\frac{mv^2}{r} \quad\Rightarrow\quad mv^2=\frac{Qq}{4\pi\varepsilon_0 r},$$

and the kinetic energy is

$$K=\frac{1}{2}mv^2=\frac{Qq}{8\pi\varepsilon_0 r}.$$

The potential energy is $U = -Qq/4\pi\varepsilon_0 r$, and the total energy is

$$E=K+U=\frac{Qq}{8\pi\varepsilon_0 r}-\frac{Qq}{4\pi\varepsilon_0 r}=-\frac{Qq}{8\pi\varepsilon_0 r}.$$

When the orbit radius is $r_1$ the energy is $E_1 = -Qq/8\pi\varepsilon_0 r_1$ and when it is $r_2$ the energy is $E_2 = -Qq/8\pi\varepsilon_0 r_2$. The difference $E_2 - E_1$ is the work $W$ done by an external agent to change the radius:

$$W=E_2-E_1=-\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

91. The initial speed $v_i$ of the electron satisfies

$$K_i=\tfrac{1}{2}m_e v_i^2=e\Delta V,$$

which gives

$$v_i=\sqrt{\frac{2e\Delta V}{m_e}}=\sqrt{\frac{2(1.60\times10^{-19}\text{ J})(625\text{ V})}{9.11\times10^{-31}\text{ kg}}}=1.48\times10^7\text{ m/s}.$$

92. The net electric potential at point $P$ is the sum of those due to the six charges:

$$\begin{aligned}V_P&=\sum_{i=1}^{6}V_{Pi}=\sum_{i=1}^{6}\frac{q_i}{4\pi\varepsilon_0 r_i}=\frac{10^{-15}}{4\pi\varepsilon_0}\left[\frac{5.00}{\sqrt{d^2+(d/2)^2}}+\frac{-2.00}{d/2}+\frac{-3.00}{\sqrt{d^2+(d/2)^2}}\right.\\&\left.\quad+\frac{3.00}{\sqrt{d^2+(d/2)^2}}+\frac{-2.00}{d/2}+\frac{+5.00}{\sqrt{d^2+(d/2)^2}}\right]=\frac{9.4\times10^{-16}}{4\pi\varepsilon_0(2.54\times10^{-2})}\\&=3.34\times10^{-4}\text{ V}.\end{aligned}$$

93. **THINK** To calculate the potential at point $B$ due to the charged ring, we note that all points on the ring are at the same distance from $B$.

**EXPRESS** Let point $B$ be at $(0, 0, z)$. The electric potential at $B$ is given by

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2 + R^2}}$$

where $q$ is the charge on the ring. The potential at infinity is taken to be zero.

**ANALYZE** With $q = 16 \times 10^{-6}$ C, $z = 0.040$ m, and $R = 0.0300$ m, we find the potential difference between points $A$ (located at the origin) and $B$ to be

$$\begin{aligned} V_B - V_A &= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{R}\right) \\ &= (8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(16.0\times10^{-6}\text{ C})\left(\frac{1}{\sqrt{(0.030\text{ m})^2+(0.040\text{ m})^2}} - \frac{1}{0.030\text{ m}}\right) \\ &= -1.92\times10^6 \text{ V}. \end{aligned}$$

**LEARN** In the limit $z \gg R,$ the potential approaches its "point-charge" limit:

$$V \approx \frac{q}{4\pi\varepsilon_0 z}.$$

94. (a) Using Eq. 24-26, we calculate the radius $r$ of the sphere representing the 30 V equipotential surface:

$$r = \frac{q}{4\pi\varepsilon_0 V} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.50\times10^{-8}\text{C})}{30 \text{ V}} = 4.5\,\text{m}.$$

(b) If the potential were a linear function of $r$ then it would have equally spaced equipotentials, but since $V \propto 1/r$ they are spaced more and more widely apart as $r$ increases.

95. **THINK** To calculate the electric potential, we first apply Gauss' law to calculate the electric field of the spherical shell. The Gaussian surface is a sphere that is concentric with the shell.

**EXPRESS** At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so the flux through the surface is given by $\Phi = \oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = q_{\text{enc}}/\varepsilon_0,$ where $r$ is the radius of the Gaussian surface and $q_{\text{enc}}$ is the charge enclosed. (i) In the region $r < r_1,$ the enclosed charge is $q_{\text{enc}} = 0$ and therefore,

$E = 0$. (ii) In the region $r_1 < r < r_2$, the volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)},$$

where $Q$ is the total charge on the spherical shell. Thus, the charge enclosed by the Gaussian surface is

$$q_{\text{enc}} = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right).$$

Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \Rightarrow E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)}.$$

(iii) In the region $r > r_2$, the charge enclosed is $q_{\text{enc}} = Q$, and the electric field is like that of a point charge:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$$

**ANALYZE** (a) For $r > r_2$ the field is like that of a point charge, and so is the potential:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r},$$

where the potential was taken to be zero at infinity.

(b) In the region $r_1 < r < r_2$, we have

$$E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)}.$$

If $V_s$ is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance $r$ from the center is given by

$$V = V_s - \int_{r_2}^{r} E\,dr = V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr$$

$$= V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right).$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\varepsilon_0 r_2$. We make this substitution and collect terms to find

$$V=\frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3-r_1^3}\left(\frac{3r_2^2}{2}-\frac{r^2}{2}-\frac{r_1^3}{r}\right).$$

Since $\rho=3Q/4\pi\left(r_2^3-r_1^3\right)$ this can also be written as

$$V(r)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{r^2}{2}-\frac{r_1^3}{r}\right).$$

(c) For $r<r_1,$ the electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. We put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V=\frac{Q}{4\pi\varepsilon_0}\frac{3\left(r_2^2-r_1^2\right)}{2\left(r_2^3-r_1^3\right)},$$

or in terms of the charge density $V=\dfrac{\rho}{2\varepsilon_0}\left(r_2^2-r_1^2\right).$

(d) Using the expression for $V(r)$ found in (b), we have

$$V(r_1)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{r_1^2}{2}-\frac{r_1^3}{r_1}\right)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{3r_1^2}{2}\right)=\frac{\rho}{2\varepsilon_0}\left(r_2^2-r_1^2\right)$$

and

$$V(r_2)=\frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2}-\frac{r_2^2}{2}-\frac{r_1^3}{r_2}\right)=\frac{\rho}{3\varepsilon_0}\left(r_2^2-\frac{r_1^3}{r_2}\right)=\frac{\rho}{3\varepsilon_0 r_2}\left(r_2^3-r_1^3\right)=\frac{3Q/4\pi}{3\varepsilon_0 r_2}=\frac{Q}{4\pi\varepsilon_0 r_2}\ .$$

So the solutions agree at $r = r_1$ and at $r = r_2$.

**LEARN** Electric potential must be continuous at the boundaries at $r = r_1$ and $r = r_2$. In the region where the electric field is zero, no work is required to move the charge around. Thus, there’s no change in potential energy and the electric potential is constant.

96. (a) We use Gauss’ law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside.

Outside the charge distribution the magnitude of the field is $E = q/4\pi\varepsilon_0 r^2$ and the potential is $V = q/4\pi\varepsilon_0 r$, where $r$ is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius $r$, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is $qr^3/R^3$. Gauss' law becomes

$$4\pi\varepsilon_0 r^2 E = \frac{qr^3}{R^3} \quad \Rightarrow \quad E = \frac{qr}{4\pi\varepsilon_0 R^3}.$$

If $V_s$ is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance $r$ from the center, is

$$V = V_s - \int_R^r E\,dr = V_s - \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r r\,dr = V_s - \frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{q}{8\pi\varepsilon_0 R}.$$

The potential at the surface can be found by replacing $r$ with $R$ in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\varepsilon_0 R$. Thus,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{q}{8\pi\varepsilon_0 R^3}\left(3R^2 - r^2\right).$$

(b) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\varepsilon_0 R} - \frac{3q}{8\pi\varepsilon_0 R} = -\frac{q}{8\pi\varepsilon_0 R},$$

or $|\Delta V| = q/8\pi\varepsilon_0 R$.

97. **THINK** The increase in electric potential at the surface of the copper sphere is proportional to the increase in electric charge.

**EXPRESS** The electric potential at the surface of a sphere of radius $R$ is given by $V = q/4\pi\varepsilon_0 R,$ where $q$ is the charge on the sphere. Thus, $q = 4\pi\varepsilon_0 RV.$ The number of electrons entering the copper sphere is $N = q/e,$ but this must be equal to $(\lambda/2)t,$ where $\lambda$ is the decay rate of the nickel.

**ANALYZE** (a) With $R$ = 0.010 m, when $V$ = 1000 V, the net charge on the sphere is

$$q = 4\pi\varepsilon_0 RV = \frac{(0.010\text{ m})(1000\text{ V})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.11\times10^{-9}\ \text{C}.$$

Dividing $q$ by $e$ yields

$$N = (1.11\times10^{-9}\ \text{C})/(1.6\times10^{-19}\ \text{C}) = 6.95\times10^{9}$$

electrons that entered the copper sphere. So the time required is

$$t = \frac{N}{\lambda/2} = \frac{6.95\times10^{9}}{(3.7\times10^{8}/\text{s})/2} = 38\ \text{s}.$$

(b) The energy deposited by each electron that enters the sphere is $E_0 = 100$ keV = 1.6 × $10^{-14}$ J. Using the given heat capacity, we note that a temperature increase of $\Delta T$ = 5.0 K = 5.0 ºC required

$$E = C\Delta T = (14\ \text{J/K})(5.0\ \text{K}) = 70\ \text{J}$$

of energy. Dividing this by $E_0$ gives the number of electrons needed to enter the sphere (in order to achieve that temperature change):

$$N' = \frac{E}{E_0} = \frac{70\ \text{J}}{1.6\times10^{-14}\ \text{J}} = 4.375\times10^{15}$$

Thus, the time needed is

$$t' = \frac{N'}{\lambda/2} = \frac{4.375\times10^{15}}{(3.7\times10^{8}/\text{s})/2} = 2.36\times10^{7}\ \text{s}$$

or roughly 270 days.

**LEARN** As more electrons get into copper, more energy is deposited, and the copper sample gets hotter.

98. (a) The potential difference is

$$\Delta V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{15\times10^{-6}\ \text{C}}{0.060\ \text{m}} - \frac{5.0\times10^{-6}\ \text{C}}{0.030\ \text{m}}\right)$$
$$= 7.49\times10^{5}\ \text{V}.$$

(b) By connecting the two metal spheres with a wire, we now have one conductor, and any excess charge must reside on the surface of the conductor. Therefore, the charge on the small sphere is zero.

(c) Since all the charges reside on the surface of the large sphere, we have

$$Q' = Q + q = 15.0\ \mu\text{C} + 5.00\ \mu\text{C} = 20.0\ \mu\text{C}.$$

99. (a) The charge on every part of the ring is the same distance from any point $P$ on the axis. This distance is $r = \sqrt{z^2 + R^2}$, where $R$ is the radius of the ring and $z$ is the distance from the center of the ring to $P$. The electric potential at $P$ is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}}.$$

(b) The electric field is along the axis and its component is given by

$$E = -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z)$$
$$= \frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}}.$$

This agrees with Eq. 23-16.

100. The distance $r$ being looked for is that where the alpha particle has (momentarily) zero kinetic energy. Thus, energy conservation leads to

$$K_0 + U_0 = K + U \;\Rightarrow\; (0.48\times 10^{-12}\text{ J}) + \frac{(2e)(92e)}{4\pi\varepsilon_0\, r_0} = 0 + \frac{(2e)(92e)}{4\pi\varepsilon_0\, r}\;.$$

If we set $r_0 = \infty$ (so $U_0 = 0$) then we obtain $r = 8.8 \times 10^{-14}$ m.

101. (a) Let the quark-quark separation be $r$. To "naturally" obtain the eV unit, we only plug in for one of the $e$ values involved in the computation:

$$U_{\text{up-up}} = \frac{1}{4\pi\varepsilon_0}\frac{(2e/3)(2e/3)}{r} = \frac{4ke}{9r}e = \frac{4(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{C})}{9(1.32\times10^{-15}\text{m})}e$$
$$= 4.84\times10^5\text{eV} = 0.484\,\text{MeV}.$$

(b) The total consists of all pair-wise terms:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(2e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r} + \frac{(-e/3)(2e/3)}{r}\right] = 0.$$

102. We imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere.

The magnitude of the electric field $E$ of the uniformly charged sphere as a function of $r$, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$.

Here $R$ is the radius of the sphere. Thus, the potential $V$ at the surface of the sphere (where $r = R$) is given by

$$V(R) = V\big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R} = \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.50\times10^8\ \text{C}\right)}{0.160\,\text{m}}$$

$$= 8.43\times10^2\ \text{V}.$$

103. Since the electric potential energy is not changed by the introduction of the third particle, we conclude that the net electric potential evaluated at $P$ caused by the original two particles must be zero:

$$\frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2} = 0\,.$$

Setting $r_1 = 5d/2$ and $r_2 = 3d/2$ we obtain $q_1 = -5q_2/3$, or $q_1/q_2 = -5/3 \approx -1.7$ .

# Chapter 25

1. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\,\text{pF}.$$

(b) The capacitance is independent of $q$; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\,\text{V}.$$

2. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus,

$$q = (25 \times 10^{-6}\text{ F})(120\text{ V}) = 3.0 \times 10^{-3}\text{ C}.$$

3. **THINK** The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where $A$ is the area of each plate and $d$ is the plate separation.

**EXPRESS** Since the plates are circular, the plate area is $A = \pi R^2$, where $R$ is the radius of a plate. The charge on the positive plate is given by $q = CV$, where $V$ is the potential difference across the plates.

**ANALYZE** (a) Substituting the values given, the capacitance is

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{\left(8.85\times10^{-12}\text{ F/m}\right)\pi\left(8.2\times10^{-2}\text{ m}\right)^2}{1.3\times10^{-3}\text{m}} = 1.44\times10^{-10}\text{ F} = 144\text{ pF}.$$

(b) Similarly, the charge on the plate when $V$ = 120 V is

$$q = (1.44 \times 10^{-10}\text{ F})(120\text{ V}) = 1.73 \times 10^{-8}\text{ C} = 17.3\text{ nC}.$$

**LEARN** Capacitance depends only on geometric factors, namely, the plate area and plate separation.

4. (a) We use Eq. 25-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm}-38.0\,\text{mm})} = 84.5\ \text{pF}.$$

(b) Let the area required be $A$. Then $C = \varepsilon_0 A/(b - a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm}-38.0\,\text{mm})}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 191\,\text{cm}^2.$$

5. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius $R'$ is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 R.$$

With $R$ = 2.00 mm, we obtain $C = 5.04\pi\left(8.85\times10^{-12}\ \text{F/m}\right)\left(2.00\times10^{-3}\,\text{m}\right) = 2.80\times10^{-13}\,\text{F}$.

6. (a) We use $C = A\varepsilon_0/d$. The distance between the plates is

$$d = \frac{A\varepsilon_0}{C} = \frac{\left(1.00\,\text{m}^2\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{1.00\,\text{F}} = 8.85\times10^{-12}\ \text{m}.$$

(b) Since $d$ is much less than the size of an atom (~ $10^{-10}$ m), this capacitor cannot be constructed.

7. For a given potential difference $V$, the charge on the surface of the plate is

$$q = Ne = (nAd)e$$

where $d$ is the depth from which the electrons come in the plate, and $n$ is the density of conduction electrons. The charge collected on the plate is related to the capacitance and the potential difference by $q = CV$ (Eq. 25-1). Combining the two expressions leads to

$$\frac{C}{A} = ne\frac{d}{V}.$$

With $d/V = d_s/V_s = 5.0\times10^{-14}$ m/V and $n = 8.49\times10^{28}/\text{m}^3$ (see, for example, Sample Problem 25.01 — "Charging the plates in a parallel-plate capacitor"), we obtain

$$\frac{C}{A} = (8.49\times10^{28}/\text{m}^3)(1.6\times10^{-19}\,\text{C})(5.0\times10-14\ \text{m/V}) = 6.79\times10^{-4}\,\text{F/m}^2\,.$$

8. The equivalent capacitance is given by $C_{eq} = q/V$, where $q$ is the total charge on all the capacitors and $V$ is the potential difference across any one of them. For $N$ identical capacitors in parallel, $C_{eq} = NC$, where $C$ is the capacitance of one of them. Thus, $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\,\text{C}}{(110\,\text{V})\,(1.00\times10^{-6}\,\text{F})} = 9.09\times10^{3}\,.$$

9. The charge that passes through meter $A$ is

$$q = C_{eq}V = 3CV = 3(25.0\,\mu\text{F})(4200\,\text{V}) = 0.315\,\text{C}.$$

10. The equivalent capacitance is

$$C_{eq} = C_3 + \frac{C_1C_2}{C_1+C_2} = 4.00\,\mu\text{F} + \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 7.33\,\mu\text{F}.$$

11. The equivalent capacitance is

$$C_{eq} = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3} = \frac{(10.0\,\mu\text{F}+5.00\,\mu\text{F})(4.00\,\mu\text{F})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}+4.00\,\mu\text{F}} = 3.16\,\mu\text{F}.$$

12. The two 6.0 $\mu$F capacitors are in parallel and are consequently equivalent to $C_{eq} = 12\ \mu\text{F}$. Thus, the total charge stored (before the squeezing) is

$$q_{total} = C_{eq}V = (12\,\mu\text{F})\,(10.0\,\text{V}) = 120\ \mu\text{C}.$$

(a) and (b) As a result of the squeezing, one of the capacitors is now 12 $\mu$F (due to the inverse proportionality between $C$ and $d$ in Eq. 25-9), which represents an increase of $6.0\ \mu\text{F}$ and thus a charge increase of

$$\Delta q_{total} = \Delta C_{eq}V = (6.0\,\mu\text{F})\,(10.0\,\text{V}) = 60\ \mu\text{C}\,.$$

13. **THINK** Charge remains conserved when a fully charged capacitor is connected to an uncharged capacitor.

**EXPRESS** The charge initially on the charged capacitor is given by $q = C_1V_0$, where $C_1$ = 100 pF is the capacitance and $V_0$ = 50 V is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1V$, where $V$ = 35 V is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where $C_2$ is the capacitance of the second capacitor.

**ANALYZE** Substituting $C_1V_0$ for $q$ and $C_1V$ for $q_1$, we obtain $q_2 = C_1(V_0 - V)$. The potential difference across the second capacitor is also $V$, so the capacitance of the second capacitor is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V} C_1 = \frac{50\,\text{V} - 35\,\text{V}}{35\,\text{V}} (100\,\text{pF}) = 42.86\text{ pF} \approx 43\,\text{pF}.$$

**LEARN** Capacitors in parallel have the same potential difference. To verify charge conservation explicitly, we note that the initial charge on the first capacitor is $q = C_1V_0 = (100\text{ pF})(50\text{ V}) = 5000\text{ pC}.$ After the connection, the charges on each capacitor are

$$q_1 = C_1V = (100\text{ pF})(35\text{ V}) = 3500\text{ pC}$$
$$q_2 = C_2V = (42.86\text{ pF})(35\text{ V}) = 1500\text{ pC}.$$

Indeed, $q = q_1 + q_2$.

14. (a) The potential difference across $C_1$ is $V_1$ = 10.0 V. Thus,

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\text{ V}) = 1.00 \times 10^{-4}\text{ C}.$$

(b) Let $C$ = 10.0 $\mu$F. We first consider the three-capacitor combination consisting of $C_2$ and its two closest neighbors, each of capacitance $C$. The equivalent capacitance of this combination is

$$C_{\text{eq}} = C + \frac{C_2C}{C + C_2} = 1.50\ C.$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C + C_{\text{eq}}} = \frac{CV_1}{C + 1.50\ C} = 0.40V_1.$$

Since this voltage difference is divided equally between $C_2$ and the one connected in series with it, the voltage difference across $C_2$ satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10.0\,\mu\text{F}) \left(\frac{10.0\,\text{V}}{5}\right) = 2.00 \times 10^{-5}\,\text{C}.$$

15. (a) First, the equivalent capacitance of the two 4.00 $\mu$F capacitors connected in series is given by 4.00 $\mu$F/2 = 2.00 $\mu$F. This combination is then connected in parallel with two other 2.00-$\mu$F capacitors (one on each side), resulting in an equivalent capacitance $C$ = 3(2.00 $\mu$F) = 6.00 $\mu$F. This is now seen to be in series with another combination, which

consists of the two 3.0-$\mu$F capacitors connected in parallel (which are themselves equivalent to $C'$ = 2(3.00 $\mu$F) = 6.00 $\mu$F). Thus, the equivalent capacitance of the circuit is

$$C_{\text{eq}} = \frac{CC'}{C+C'} = \frac{(6.00\,\mu\text{F})\,(6.00\,\mu\text{F})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 3.00\,\mu\text{F}.$$

(b) Let $V$ = 20.0 V be the potential difference supplied by the battery. Then

$$q = C_{\text{eq}}V = (3.00\ \mu\text{F})(20.0\ \text{V}) = 6.00 \times 10^{-5}\ \text{C}.$$

(c) The potential difference across $C_1$ is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.00\,\mu\text{F})\,(20.0\,\text{V})}{6.00\,\mu\text{F}+6.00\,\mu\text{F}} = 10.0\,\text{V}.$$

(d) The charge carried by $C_1$ is $q_1 = C_1V_1$= (3.00 $\mu$F)(10.0 V) = 3.00 × $10^{-5}$ C.

(e) The potential difference across $C_2$ is given by $V_2 = V - V_1$ = 20.0 V – 10.0 V = 10.0 V.

(f) The charge carried by $C_2$ is $q_2 = C_2V_2$ = (2.00 $\mu$F)(10.0 V) = 2.00 × $10^{-5}$ C.

(g) Since this voltage difference $V_2$ is divided equally between $C_3$ and the other 4.00-$\mu$F capacitors connected in series with it, the voltage difference across $C_3$ is given by $V_3$ = $V_2$/2 = 10.0 V/2 = 5.00 V.

(h) Thus, $q_3 = C_3V_3$ = (4.00 $\mu$F)(5.00 V) = 2.00 × $10^{-5}$ C.

16. We determine each capacitance from the slope of the appropriate line in the graph. Thus, $C_1$ = (12 $\mu$C)/(2.0 V) = 6.0 $\mu$F. Similarly, $C_2$ = 4.0 $\mu$F and $C_3$ = 2.0 $\mu$F. The total equivalent capacitance is given by

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2+C_3} = \frac{C_1+C_2+C_3}{C_1(C_2+C_3)},$$

or

$$C_{123} = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} = \frac{(6.0\ \mu\text{F})(4.0\ \mu\text{F}+2.0\ \mu\text{F})}{6.0\ \mu\text{F}+4.0\ \mu\text{F}+2.0\ \mu\text{F}} = \frac{36}{12}\ \mu\text{F} = 3.0\ \mu\text{F}.$$

This implies that the charge on capacitor 1 is $q_1 = (3.0\ \mu\text{F})(6.0\ \text{V})$ = 18 μC. The voltage across capacitor 1 is therefore $V_1$ = (18 $\mu$C)/(6.0 $\mu$F) = 3.0 V. From the discussion in section 25-4, we conclude that the voltage across capacitor 2 must be 6.0 V – 3.0 V = 3.0 V. Consequently, the charge on capacitor 2 is (4.0 $\mu$F)(3.0 V) = 12 $\mu$C.

17. (a) and (b) The original potential difference $V_1$ across $C_1$ is

$$V_1 = \frac{C_{eq}V}{C_1 + C_2} = \frac{(3.16\,\mu\text{F})(100.0\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 21.1\,\text{V}.$$

Thus $\Delta V_1$ = 100.0 V – 21.1 V = 78.9 V and

$$\Delta q_1 = C_1\Delta V_1 = (10.0\ \mu\text{F})(78.9\ \text{V}) = 7.89 \times 10^{-4}\ \text{C}.$$

18. We note that the voltage across $C_3$ is $V_3$ = (12 V – 2 V – 5 V) = 5 V. Thus, its charge is $q_3 = C_3 V_3 = 4\ \mu$C.

(a) Therefore, since $C_1$, $C_2$ and $C_3$ are in series (so they have the same charge), then

$$C_1 = \frac{4\ \mu\text{C}}{2\ \text{V}} = 2.0\ \mu\text{F}\ .$$

(b) Similarly, $C_2$ = 4/5 = 0.80 $\mu$F.

19. (a) and (b) We note that the charge on $C_3$ is $q_3$ = 12 $\mu$C – 8.0 $\mu$C = 4.0 $\mu$C. Since the charge on $C_4$ is $q_4$ = 8.0 $\mu$C, then the voltage across it is $q_4/C_4$ = 2.0 V. Consequently, the voltage $V_3$ across $C_3$ is 2.0 V $\Rightarrow C_3 = q_3/V_3$ = 2.0 $\mu$F.

Now $C_3$ and $C_4$ are in parallel and are thus equivalent to 6 μF capacitor which would then be in series with $C_2$ ; thus, Eq 25-20 leads to an equivalence of 2.0 μF which is to be thought of as being in series with the unknown $C_1$ . We know that the total effective capacitance of the circuit (in the sense of what the battery "sees" when it is hooked up) is (12 $\mu$C)/$V_{battery}$ = 4 $\mu$F/3. Using Eq 25-20 again, we find

$$\frac{1}{2\ \mu\text{F}} + \frac{1}{C_1} = \frac{3}{4\ \mu\text{F}} \quad\Rightarrow\quad C_1 = 4.0\ \mu\text{F}\ .$$

20. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of ($n$ – 1) identical single capacitors connected in parallel. Each capacitor has surface area $A$ and plate separation $d$ so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} = \frac{(8-1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.25\times10^{-4}\ \text{m}^2)}{3.40\times10^{-3}\ \text{m}} = 2.28\times10^{-12}\,\text{F}.$$

21. **THINK** After the switches are closed, the potential differences across the capacitors are the same and they are connected in parallel.

**EXPRESS** The potential difference from $a$ to $b$ is given by $V_{ab} = Q/C_{eq}$, where $Q$ is the net charge on the combination and $C_{eq}$ is the equivalent capacitance.

**ANALYZE** (a) The equivalent capacitance is $C_{eq} = C_1 + C_2 = 4.0 \times 10^{-6}$ F. The total charge on the combination is the net charge on either pair of connected plates. The initial charge on capacitor 1 is

$$q_1 = C_1V = \left(1.0\times10^{-6}\text{ F}\right)\left(100\text{ V}\right) = 1.0\times10^{-4}\text{ C}$$

and the initial charge on capacitor 2 is

$$q_2 = C_2V = \left(3.0\times10^{-6}\text{F}\right)\left(100\text{ V}\right) = 3.0\times10^{-4}\text{ C}.$$

With opposite polarities, the net charge on the combination is

$$Q = 3.0 \times 10^{-4}\text{ C} - 1.0 \times 10^{-4}\text{ C} = 2.0 \times 10^{-4}\text{ C}.$$

The potential difference is

$$V_{ab} = \frac{Q}{C_{eq}} = \frac{2.0\times10^{-4}\text{C}}{4.0\times10^{-6}\text{F}} = 50\text{ V}.$$

(b) The charge on capacitor 1 is now $q_1' = C_1V_{ab} = (1.0 \times 10^{-6}\text{ F})(50\text{ V}) = 5.0 \times 10^{-5}$ C.

(c) The charge on capacitor 2 is now $q_2' = C_2V_{ab} = (3.0 \times 10^{-6}\text{ F})(50\text{ V}) = 1.5 \times 10^{-4}$ C.

**LEARN** The potential difference $V_{ab} = 50$ V is half of the original $V$ ( = 100 V), so the final charges on the capacitors are also halved.

22. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1V_{bat} = 100\ \mu$C, and $q_1$, $q_2$ and $q_3$ are the charges on $C_1$, $C_2$ and $C_3$ after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3.$$

Since the parallel pair $C_2$ and $C_3$ are identical, it is clear that $q_2 = q_3$. They are in parallel with $C_1$ so that $V_1=V_3$, or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3}$$

which leads to $q_1 = q_3/2$. Therefore,

$$Q = (q_3/2) + q_3 + q_3 = 5q_3/2$$

which yields $q_3 = 2Q/5 = 2(100\ \mu\text{C})/5 = 40\ \mu\text{C}$ and consequently $q_1 = q_3/2 = 20\ \mu$C.

23. We note that the total equivalent capacitance is $C_{123} = [(C_3)^{-1} + (C_1 + C_2)^{-1}]^{-1} = 6\ \mu$F.

(a) Thus, the charge that passed point *a* is $C_{123} V_{batt}$ = (6 $\mu$F)(12 V) = 72 $\mu$C. Dividing this by the value $e = 1.60 \times 10^{-19}$ C gives the number of electrons: $4.5 \times 10^{14}$, which travel to the left, toward the positive terminal of the battery.

(b) The equivalent capacitance of the parallel pair is $C_{12} = C_1 + C_2 = 12\ \mu$F. Thus, the voltage across the pair (which is the same as the voltage across $C_1$ and $C_2$ individually) is

$$\frac{72\ \mu\text{C}}{12\ \mu\text{F}} = 6\ \text{V}.$$

Thus, the charge on $C_1$ is

$$q_1 = (4\ \mu\text{F})(6\ \text{V}) = 24\ \mu\text{C},$$

and dividing this by *e* gives $N_1 = q_1 / e = 1.5 \times 10^{14}$, the number of electrons that have passed (upward) through point *b*.

(c) Similarly, the charge on $C_2$ is $q_2$ = (8 μF)(6 V) = 48 μC, and dividing this by *e* gives $N_2 = q_2 / e = 3.0 \times 10^{14}$, the number of electrons which have passed (upward) through point *c*.

(d) Finally, since $C_3$ is in series with the battery, its charge is the same charge that passed through the battery (the same as passed through the switch). Thus, $4.5 \times 10^{14}$ electrons passed rightward though point *d*. By leaving the rightmost plate of $C_3$, that plate is then the positive plate of the fully charged capacitor, making its leftmost plate (the one closest to the negative terminal of the battery) the negative plate, as it should be.

(e) As stated in (b), the electrons travel up through point *b*.

(f) As stated in (c), the electrons travel up through point *c*.

24. Using Equation 25-14, the capacitances are

$$C_1 = \frac{2\pi\varepsilon_0 L_1}{\ln(b_1/a_1)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.050\ \text{m})}{\ln(15\ \text{mm}/5.0\ \text{mm})} = 2.53\ \text{pF}$$

$$C_2 = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.090\ \text{m})}{\ln(10\ \text{mm}/2.5\ \text{mm})} = 3.61\ \text{pF}\ .$$

Initially, the total equivalent capacitance is

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2} \Rightarrow C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2.53\ \text{pF})(3.61\ \text{pF})}{2.53\ \text{pF} + 3.61\ \text{pF}} = 1.49\ \text{pF},$$

and the charge on the positive plate of each one is (1.49 pF)(10 V) = 14.9 pC. Next, capacitor 2 is modified as described in the problem, with the effect that

$$C_2' = \frac{2\pi\varepsilon_0 L_2}{\ln(b_2'/a_2)} = \frac{2\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.090\ \text{m})}{\ln(25\ \text{mm}/2.5\ \text{mm})} = 2.17\ \text{pF}\ .$$

The new total equivalent capacitance is

$$C_{12}' = \frac{C_1 C_2'}{C_1 + C_2'} = \frac{(2.53\ \text{pF})(2.17\ \text{pF})}{2.53\ \text{pF} + 2.17\ \text{pF}} = 1.17\ \text{pF}$$

and the new charge on the positive plate of each one is (1.17 pF)(10 V) = 11.7 pC. Thus we see that the charge transferred from the battery (considered in absolute value) as a result of the modification is 14.9 pC – 11.7 pC = 3.2 pC.

(a) This charge, divided by $e$ gives the number of electrons that pass point $P$. Thus,

$$N = \frac{3.2\times10^{-12}\,\text{C}}{1.6\times10^{-19}\,\text{C}} = 2.0\times10^{7}\ .$$

(b) These electrons move rightward in the figure (that is, away from the battery) since the positive plates (the ones closest to point $P$) of the capacitors have suffered a *decrease* in their positive charges. The usual reason for a metal plate to be positive is that it has more protons than electrons. Thus, in this problem some electrons have "returned" to the positive plates (making them less positive).

25. Equation 23-14 applies to each of these capacitors. Bearing in mind that $\sigma = q/A$, we find the total charge to be

$$q_{\text{total}} = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \varepsilon_0 E_1 A_1 + \varepsilon_0 E_2 A_2 = 3.6\ \text{pC}$$

where we have been careful to convert $\text{cm}^2$ to $\text{m}^2$ by dividing by $10^4$.

26. Initially the capacitors $C_1$, $C_2$, and $C_3$ form a combination equivalent to a single capacitor which we denote $C_{123}$. This obeys the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{C_1 + C_2 + C_3}{C_1(C_2 + C_3)}\ .$$

Hence, using $q = C_{123}V$ and the fact that $q = q_1 = C_1 V_1$, we arrive at

$$V_1 = \frac{q_1}{C_1} = \frac{q}{C_1} = \frac{C_{123}}{C_1}V = \frac{C_2 + C_3}{C_1 + C_2 + C_3}V\ .$$

(a) As $C_3 \to \infty$ this expression becomes $V_1 = V$. Since the problem states that $V_1$ approaches 10 volts in this limit, so we conclude $V = 10$ V.

(b) and (c) At $C_3 = 0$, the graph indicates $V_1 = 2.0$ V. The above expression consequently implies $C_1 = 4C_2$. Next we note that the graph shows that, at $C_3 = 6.0\ \mu$F, the voltage across $C_1$ is exactly half of the battery voltage. Thus,

$$\frac{1}{2} = \frac{C_2 + 6.0\ \mu\text{F}}{C_1 + C_2 + 6.0\ \mu\text{F}} = \frac{C_2 + 6.0\ \mu\text{F}}{4C_2 + C_2 + 6.0\ \mu\text{F}}$$

which leads to $C_2 = 2.0\ \mu$F. We conclude, too, that $C_1 = 8.0\ \mu$F.

27. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1C_3V}{C_1 + C_3} = \frac{(1.00\mu\text{F})\,(3.00\mu\text{F})\,(12.0\text{V})}{1.00\mu\text{F}+3.00\mu\text{F}} = 9.00\mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2C_4V}{C_2 + C_4} = \frac{(2.00\mu\text{F})\,(4.00\mu\text{F})\,(12.0\text{V})}{2.00\mu\text{F}+4.00\mu\text{F}} = 16.0\mu\text{C}.$$

(c) $q_3 = q_1 = 9.00\mu\text{C}.$

(d) $q_4 = q_2 = 16.0\mu\text{C}.$

(e) With switch 2 also closed, the potential difference $V_1$ across $C_1$ must equal the potential difference across $C_2$ and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4}\,V = \frac{(3.00\mu\text{F}+4.00\mu\text{F})(12.0\text{V})}{1.00\mu\text{F}+2.00\mu\text{F}+3.00\mu\text{F}+4.00\mu\text{F}} = 8.40\text{ V}.$$

Thus, $q_1 = C_1V_1 = (1.00\ \mu\text{F})(8.40\text{ V}) = 8.40\ \mu\text{C}$.

(f) Similarly, $q_2 = C_2V_1 = (2.00\ \mu\text{F})(8.40\text{ V}) = 16.8\ \mu\text{C}$.

(g) $q_3 = C_3(V - V_1) = (3.00\ \mu\text{F})(12.0\text{ V} - 8.40\text{ V}) = 10.8\ \mu\text{C}$.

(h) $q_4 = C_4(V - V_1) = (4.00\ \mu\text{F})(12.0\text{ V} - 8.40\text{ V}) = 14.4\ \mu\text{C}$.

28. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_2}+\frac{1}{C_3}=\frac{C_2+C_3}{C_2C_3}.$$

Thus, $C_{\text{eq}} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination, and the potential difference across the equivalent capacitor is given by $q_2/C_{\text{eq}}$. The potential difference across capacitor 1 is $q_1/C_1$, where $q_1$ is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$.

Now, some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If $q_0$ is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where $V_0$ is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1}=\frac{q_2}{C_{\text{eq}}}$$

$$q_1+q_2=C_1V_0$$

for $q_1$ and $q_2$, we obtain

$$q_1=\frac{C_1^2V_0}{C_{\text{eq}}+C_1}=\frac{C_1^2V_0}{\dfrac{C_2C_3}{C_2+C_3}+C_1}=\frac{C_1^2\left(C_2+C_3\right)V_0}{C_1C_2+C_1C_3+C_2C_3}.$$

With $V_0$ = 12.0 V, $C_1$= 4.00 $\mu$F, $C_2$= 6.00 $\mu$F and $C_3$ =3.00 $\mu$F, we find $C_{\text{eq}}$ = 2.00 $\mu$F and $q_1$ = 32.0 $\mu$C.

(b) The charge on capacitors 2 is

$$q_2=C_1V_0-q_1=(4.00\,\mu\text{F})(12.0\text{ V})-32.0\ \mu\text{C}=16.0\ \mu\text{C}.$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3=C_1V_0-q_1=(4.00\ \mu\text{F})(12.0\text{ V})-32.0\ \mu\text{C}=16.0\ \mu\text{C}.$$

29. The energy stored by a capacitor is given by $U=\frac{1}{2}CV^2$, where $V$ is the potential difference across its plates. We convert the given value of the energy to Joules. Since $1\text{ J}=1\text{ W}\cdot\text{s}$, we multiply by ($10^3$ W/kW)(3600 s/h) to obtain $10\text{ kW}\cdot\text{h}=3.6\times10^7\text{ J}$. Thus,

$$C=\frac{2U}{V^2}=\frac{2\left(3.6\times10^7\text{ J}\right)}{\left(1000\text{ V}\right)^2}=72\text{ F}.$$

30. Let $\mathcal{V}$ = 1.00 m$^3$. Using Eq. 25-25, the energy stored is

$$U = u\mathcal{V} = \frac{1}{2}\varepsilon_0 E^2 \mathcal{V} = \frac{1}{2}\left(8.85\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(150\ \mathrm{V/m})^2\left(1.00\ \mathrm{m}^3\right) = 9.96\times10^{-8}\ \mathrm{J}.$$

31. **THINK** The total electrical energy is the sum of the energies stored in the individual capacitors.

**EXPRESS** The energy stored in a charged capacitor is

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2.$$

Since we have two capacitors that are connected in parallel, the potential difference $V$ across the capacitors is the same and the total energy is

$$U_{\mathrm{tot}} = U_1 + U_2 = \frac{1}{2}(C_1 + C_2)V^2.$$

**ANALYZE** Substituting the values given, we have

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\ \mathrm{F} + 4.0\times10^{-6}\ \mathrm{F}\right)(300\ \mathrm{V})^2 = 0.27\ \mathrm{J}.$$

**LEARN** The energy stored in a capacitor is equal to the amount of work required to charge the capacitor.

32. (a) The capacitance is

$$C = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(40\times10^{-4}\ \mathrm{m}^2\right)}{1.0\times10^{-3}\mathrm{m}} = 3.5\times10^{-11}\ \mathrm{F} = 35\ \mathrm{pF}.$$

(b) $q = CV$ = (35 pF)(600 V) = 2.1 × 10$^{-8}$ C = 21 nC.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\ \mathrm{pF})(21\ \mathrm{nC})^2 = 6.3\times10^{-6}\ \mathrm{J} = 6.3\ \mu\mathrm{J}.$

(d) $E = V/d$ = 600 V/1.0 × 10$^{-3}$ m = 6.0 × 10$^5$ V/m.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3\times10^{-6}\,\text{J}}{\left(40\times10^{-4}\text{m}^2\right)\left(1.0\times10^{-3}\text{m}\right)} = 1.6\ \text{J/m}^3.$$

33. We use $E = q/4\pi\varepsilon_0 R^2 = V/R$. Thus

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{8000\,\text{V}}{0.050\text{ m}}\right)^2 = 0.11\text{ J/m}^3.$$

34. (a) The charge $q_3$ in the figure is $q_3 = C_3V = (4.00\,\mu\text{F})(100\ \text{V}) = 4.00\times10^{-4}\ \text{C}$.

(b) $V_3 = V = 100$ V.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 2.00\times10^{-2}\,\text{J}$.

(d) From the figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1+C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\ \text{V})}{10.0\,\mu\text{F}+5.00\,\mu\text{F}} = 3.33\times10^{-4}\text{C}.$$

(e) $V_1 = q_1/C_1 = 3.33\times10^{-4}$ C/10.0 $\mu$F = 33.3 V.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 5.55\times10^{-3}\,\text{J}$.

(g) From part (d), we have $q_2 = q_1 = 3.33\times10^{-4}\text{C}$.

(h) $V_2 = V - V_1 = 100$ V – 33.3 V = 66.7 V.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 1.11\times10^{-2}\,\text{J}$.

35. The energy per unit volume is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{32\pi^2\varepsilon_0 r^4}\ .$$

(a) At $r = 1.00\times10^{-3}\text{m}$, with $e = 1.60\times10^{-19}\text{C}$ and $\varepsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$, we have $u = 9.16\times10^{-18}\ \text{J/m}^3$.

(b) Similarly, at $r = 1.00\times10^{-6}\text{m}$, $u = 9.16\times10^{-6}\ \text{J/m}^3$.

(c) At $r = 1.00\times10^{-9}\text{m}$, $u = 9.16\times10^{6}\ \text{J/m}^3$.

(d) At $r = 1.00\times10^{-12}\,\text{m}$, $u = 9.16\times10^{18}\ \text{J/m}^3$.

(e) From the expression above, $u \propto r^{-4}$. Thus, for $r \to 0$, the energy density $u \to \infty$.

36. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20\text{ m})(0.10\text{ m}) + \pi(0.20\text{ m})^2 = 0.25\text{ m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is $q = \sigma A = -0.50\ \mu\text{C}$ on the exterior surface, and consequently (according to the assumptions in the problem) that same charge $q$ is induced in the interior of the fluid.

(b) By Eq. 25-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{(5.0\times10^{-7}\ \text{C})^2}{2(35\times10^{-12}\ \text{F})} = 3.6\times10^{-3}\ \text{J}.$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

37. **THINK** The potential difference between the plates of a parallel-plate capacitor depends on their distance of separation.

**EPXRESS** Let $q$ be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $C_i = \varepsilon_0 A/d_i$, the charge is $q_i = C_i V_i = \varepsilon_0 AV_i/d_i$. After the plates are pulled apart, their separation is $d_f$ and the final potential difference is $V_f$. Thus, the final charge is $q_f = \varepsilon_0 AV_f/2d_f$. Since charge remains unchanged, $q_i = q_f$, we have

$$V_f = \frac{q_f}{C_f} = \frac{d_f}{\varepsilon_0 A}q_f = \frac{d_f}{\varepsilon_0 A}\frac{\varepsilon_0 A}{d_i}V_i = \frac{d_f}{d_i}V_i.$$

**ANALYZE** (a) With $d_i = 3.00\times10^{-3}\text{m}$, $V_i = 6.00\text{ V}$ and $d_f = 8.00\times10^{-3}\text{m}$, the final potential difference is $V_f = 16.0\text{ V}$.

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV_i^2 = \frac{\varepsilon_0 AV_i^2}{2d_i} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.50\times10^{-4}\ \text{m}^2)(6.00\text{ V})^2}{2(3.00\times10^{-3}\ \text{m})}$$
$$= 4.51\times10^{-11}\ \text{J}.$$

(c) The final energy stored is

$$U_f = \frac{1}{2}C_f V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}\left(\frac{d_f}{d_i}V_i\right)^2 = \frac{d_f}{d_i}\left(\frac{\varepsilon_0 A V_i^2}{d_i}\right) = \frac{d_f}{d_i}U_i.$$

With $d_f / d_i = 8.00/3.00$, we have $U_f = 1.20\times10^{-10}$ J.

(d) The work done to pull the plates apart is the difference in the energy:

$$W = U_f - U_i = 7.52\times10^{-11}\text{ J}.$$

**LEARN** In a parallel-plate capacitor, the energy density (energy per unit volume) is given by $u = \varepsilon_0 E^2/2$ (see Eq. 25-25), where $E$ is constant at all points between the plates. Thus, increasing the plate separation increases the volume (= $Ad$), and hence the total energy of the system.

38. (a) The potential difference across $C_1$ (the same as across $C_2$) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(15.0\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 15.0\,\mu\text{F}} = 50.0\,\text{V}.$$

Also, $V_3 = V - V_1 = V - V_2$ = 100 V – 50.0 V = 50.0 V. Thus,

$$q_1 = C_1 V_1 = (10.0\,\mu\text{F})(50.0\,\text{V}) = 5.00\times10^{-4}\,\text{C}$$
$$q_2 = C_2 V_2 = (5.00\,\mu\text{F})(50.0\,\text{V}) = 2.50\times10^{-4}\,\text{C}$$
$$q_3 = q_1 + q_2 = 5.00\times10^{-4}\,\text{C} + 2.50\times10^{-4}\,\text{C} = 7.50\times10^{-4}\,\text{C}.$$

(b) The potential difference $V_3$ was found in the course of solving for the charges in part (a). Its value is $V_3$ = 50.0 V.

(c) The energy stored in $C_3$ is $U_3 = C_3 V_3^2/2 = (15.0\,\mu\text{F})(50.0\,\text{V})^2/2 = 1.88\times10^{-2}$ J.

(d) From part (a), we have $q_1 = 5.00\times10^{-4}$ C, and

(e) $V_1$ = 50.0 V, as shown in (a).

(f) The energy stored in $C_1$ is $U_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2}(10.0\,\mu\text{F})(50.0\,\text{V})^2 = 1.25\times10^{-2}$ J.

(g) Again, from part (a), $q_2 = 2.50\times10^{-4}$ C.

(h) $V_2$ = 50.0 V, as shown in (a).

(i) The energy stored in $C_2$ is $U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(5.00\,\mu\text{F})(50.0\text{V})^2 = 6.25\times10^{-3}\,\text{J}$.

39. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 $\mu$F, then the voltage across the 20 $\mu$F capacitor is 50 V and the voltage across the 25 $\mu$F capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain $U_{\text{total}}$ = 0.095 J.

40. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0 \kappa A/2d$. Thus $C'/C = \kappa/2$ or

$$\kappa = 2C'/C = 2(2.6\text{ pF}/1.3\text{ pF}) = 4.0.$$

41. **THINK** Our system, a coaxial cable, is a cylindrical capacitor filled with polystyrene, a dielectric.

**EXPRESS** Using Eqs. 25-17 and 25-27, the capacitance of a cylindrical capacitor can be written as

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)},$$

where $C_0$ is the capacitance without the dielectric, $\kappa$ is the dielectric constant, $L$ is the length, $a$ is the inner radius, and $b$ is the outer radius.

**ANALYZE** With $\kappa$ = 2.6 for polystyrene, the capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85\times10^{-12}\text{ F/m})}{\ln[(0.60\text{ mm})/(0.10\text{ mm})]} = 8.1\times10^{-11}\text{F/m} = 81\text{ pF/m}.$$

**LEARN** When the space between the plates of a capacitor is completely filled with a dielectric material, the capacitor increases by a factor $\kappa$, the dielectric constant characteristic of the material.

42. (a) We use $C = \varepsilon_0 A/d$ to solve for $d$:

$$d = \frac{\varepsilon_0 A}{C} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.35\text{ m}^2)}{50\times10^{-12}\text{F}} = 6.2\times10^{-2}\text{ m}.$$

(b) We use $C \propto \kappa$. The new capacitance is

$$C' = C(\kappa/\kappa_{\text{air}}) = (50\text{ pf})(5.6/1.0) = 2.8\times10^{2}\text{ pF}.$$

43. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where $C_0$ is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4\times10^{-6}\text{ J})}{(7.4\times10^{-12}\text{ F})(652\text{ V})^2} = 4.7.$$

According to Table 25-1, you should use Pyrex.

44. (a) We use Eq. 25-14:

$$C = 2\pi\varepsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15\text{ m})}{2\left(8.99\times10^{9}\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8\text{ cm}/3.6\text{ cm})} = 0.73\text{ nF}.$$

(b) The breakdown potential is (14 kV/mm) (3.8 cm – 3.6 cm) = 28 kV.

45. Using Eq. 25-29, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200\times10^{3}\text{ N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6\times10^{-5}\text{ J}\ .$$

46. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know $C_1$ and $C_2$. From Eq. 25-9,

$$C_2 = \frac{\varepsilon_0 A}{d} = 2.21\times10^{-11}\text{ F}\ ,$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa\varepsilon_0 A}{d} = 6.64\times10^{-11}\text{ F}\ .$$

This leads to

$$q_1 = C_1V_1 = 8.00\times10^{-10}\text{ C},\quad q_2 = C_2V_2 = 2.66\times10^{-10}\text{ C}.$$

The addition of these gives the desired result: $q_{\text{tot}} = 1.06 \times 10^{-9}$ C. Alternatively, the circuit could be reduced to find the $q_{\text{tot}}$.

47. **THINK** Dielectric strength is the maximum value of the electric field a dielectric material can tolerate without breakdown.

**EXPRESS** The capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where $C_0$ is the capacitance without the dielectric, $\kappa$ is the dielectric constant, $A$ is the plate area, and $d$ is the plate separation. The electric field between the plates is given by $E = V/d$, where $V$ is the potential difference between the plates. Thus, $d = V/E$ and $C = \kappa\varepsilon_0 AE/V$. Therefore, we find the plate area to be

$$A = \frac{CV}{\kappa\varepsilon_0 E}.$$

**ANALYZE** For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0\times10^{-8}\,\text{F})(4.0\times10^{3}\,\text{V})}{2.8(8.85\times10^{-12}\,\text{F/m})(18\times10^{6}\ \text{V/m})} = 0.63\ \text{m}^2.$$

**LEARN** If the area is smaller than the minimum value found above, then electric breakdown occurs and the dielectric is no longer insulating and will start to conduct.

48. The capacitor can be viewed as two capacitors $C_1$ and $C_2$ in parallel, each with surface area $A/2$ and plate separation $d$, filled with dielectric materials with dielectric constants $\kappa_1$ and $\kappa_2$, respectively. Thus, (in SI units),

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1+\kappa_2}{2}\right)$$
$$= \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.56\times10^{-4}\ \text{m}^2)}{5.56\times10^{-3}\ \text{m}}\left(\frac{7.00+12.00}{2}\right) = 8.41\times10^{-12}\ \text{F}.$$

49. We assume there is charge $q$ on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1\varepsilon_0 A},$$

where $A$ is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2\varepsilon_0 A}.$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V=\frac{E_1d}{2}+\frac{E_2d}{2}=\frac{qd}{2\varepsilon_0A}\left[\frac{1}{\kappa_1}+\frac{1}{\kappa_2}\right]=\frac{qd}{2\varepsilon_0A}\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2},$$

so

$$C=\frac{q}{V}=\frac{2\varepsilon_0A}{d}\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}.$$

This expression is exactly the same as that for $C_{eq}$ of two capacitors in series, one with dielectric constant $\kappa_1$ and the other with dielectric constant $\kappa_2$. Each has plate area $A$ and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1\varepsilon_0A/d$, the correct result for a parallel-plate capacitor with plate area $A$, plate separation $d$, and dielectric constant $\kappa_1$.

With $A=7.89\times10^{-4}\,\text{m}^2$, $d=4.62\times10^{-3}\,\text{m}$, $\kappa_1=11.0$, and $\kappa_2=12.0$, the capacitance is

$$C=\frac{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(7.89\times10^{-4}\ \text{m}^2)}{4.62\times10^{-3}\ \text{m}}\frac{(11.0)(12.0)}{11.0+12.0}=1.73\times10^{-11}\,\text{F}.$$

50. Let

$$C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0A\kappa_1/4d,$$
$$C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0A\kappa_2/2d,$$
$$C_3 = \varepsilon_0A\kappa_3/2d.$$

Note that $C_2$ and $C_3$ are effectively connected in series, while $C_1$ is effectively connected in parallel with the $C_2$-$C_3$ combination. Thus,

$$C=C_1+\frac{C_2C_3}{C_2+C_3}=\frac{\varepsilon_0A\kappa_1}{4d}+\frac{(\varepsilon_0\ A/d)\ (\kappa_2/2)\ (\kappa_3/2)}{\kappa_2/2+\kappa_3/2}=\frac{\varepsilon_0A}{4d}\left(\kappa_1+\frac{2\kappa_2\kappa_3}{\kappa_2+\kappa_3}\right).$$

With $A=1.05\times10^{-3}\,\text{m}^2$, $d=3.56\times10^{-3}\,\text{m}$, $\kappa_1=21.0$, $\kappa_2=42.0$ and $\kappa_3=58.0$, we find the capacitance to be

$$C=\frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.05\times10^{-3}\ \text{m}^2)}{4(3.56\times10^{-3}\ \text{m})}\left(21.0+\frac{2(42.0)(58.0)}{42.0+58.0}\right)=4.55\times10^{-11}\,\text{F}.$$

51. **THINK** We have a parallel-plate capacitor, so the capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0A/d$, where $C_0$ is the capacitance without the dielectric, $\kappa$ is the dielectric constant, $A$ is the plate area, and $d$ is the plate separation.

**EXPRESS** The electric field in the region between the plates is given by $E = V/d$, where $V$ is the potential difference between the plates and $d$ is the plate separation. Since the separation can be written as $d = \kappa\varepsilon_0 A/C$, we have $E = VC/\kappa\varepsilon_0 A$. The free charge on the plates is $q_f = CV$.

**ANALYZE** (a) Substituting the values given, we find the magnitude of the field strength to be

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\ \text{V})(100\times10^{-12}\ \text{F})}{5.4(8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\ \text{m}^2)} = 1.0\times10^4\ \text{V/m}.$$

(b) Similarly, we have $q_f = CV = (100 \times 10^{-12}\ \text{F})(50\ \text{V}) = 5.0 \times 10^{-9}$ C.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE = 5.0\times10^{-9}\ \text{C} - (8.85\times10^{-12}\ \text{F/m})(100\times10^{-4}\ \text{m}^2)(1.0\times10^4\ \text{V/m}) \\ &= 4.1\times10^{-9}\ \text{C} = 4.1\,\text{nC}. \end{aligned}$$

**LEARN** An alternative way to calculate the induced charge is to apply Eq. 25-35:

$$q_i = q_f\left(1 - \frac{1}{\kappa}\right) = (5.0\,\text{nC})\left(1 - \frac{1}{5.4}\right) = 4.1\,\text{nC}.$$

Note that there's no induced charge ($q_i = 0$) in the absence of dielectric ($\kappa = 1$).

52. (a) The electric field $E_1$ in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\varepsilon_0 A$. Thus,

$$V_0 = E_1(d-b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d - b + \frac{b}{\kappa}\right),$$

and the capacitance is

$$C = \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(115\times10^{-4}\ \text{m}^2)(2.61)}{(2.61)(0.0124\,\text{m} - 0.00780\,\text{m}) + (0.00780\,\text{m})} = 13.4\,\text{pF}.$$

(b) $q = CV = (13.4 \times 10^{-12}$ F)(85.5 V) = 1.15 nC.

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}\ \text{C}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(115\times10^{-4}\ \text{m}^2\right)} = 1.13\times10^{4}\ \text{N/C}.$$

(d) Using Eq. 25-34, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times10^{4}\ \text{N/C}}{2.61} = 4.33\times10^{3}\ \text{N/C}.$$

53. (a) Initially, the capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.12\ \text{m}^2)}{1.2\times10^{-2}\ \text{m}} = 89\ \text{pF}.$$

(b) Working through Sample Problem 25.06 — "Dielectric partially filling the gap in a capacitor" algebraically, we find:

$$C = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.12\,\text{m}^2)(4.8)}{(4.8)(1.2-0.40)(10^{-2}\,\text{m})+(4.0\times10^{-3}\,\text{m})} = 1.2\times10^{2}\ \text{pF}.$$

(c) Before the insertion, $q = C_0V$ (89 pF)(120 V) = 11 nC.

(d) Since the battery is disconnected, $q$ will remain the same after the insertion of the slab, with $q$ = 11 nC.

(e) $E = q/\varepsilon_0 A = 11\times10^{-9}\ \text{C}/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.12\ \text{m}^2) = 10\ \text{kV/m}.$

(f) $E' = E/\kappa$ = (10 kV/m)/4.8 = 2.1 kV/m.

(g) The potential difference across the plates is

$V = E(d - b) + E'b$ = (10 kV/m)(0.012 m – 0.0040 m)+ (2.1 kV/m)(0.40 × $10^{-3}$ m) = 88 V.

(h) The work done is

$$W_{\text{ext}} = \Delta U = \frac{q^2}{2}\left(\frac{1}{C} - \frac{1}{C_0}\right) = \frac{(11\times10^{-9}\,\text{C})^2}{2}\left(\frac{1}{89\times10^{-12}\,\text{F}} - \frac{1}{120\times10^{-12}\,\text{F}}\right) = -1.7\times10^{-7}\ \text{J}.$$

54. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for $\kappa$:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\ \text{C}}{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.4\times10^{-6}\ \text{V/m}\right)\left(100\times10^{-4}\text{m}^2\right)} = 7.2.$$

(b) The charge induced is $q' = q\left(1-\frac{1}{\kappa}\right) = \left(8.9\times10^{-7}\ \text{C}\right)\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \text{C}.$

55. (a) According to Eq. 25-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right).$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant $\kappa$. Consequently, the new capacitance is

$$C = 4\pi\kappa\varepsilon_0\left(\frac{ab}{b-a}\right) = \frac{23.5}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\cdot\frac{(0.0120\ \text{m})(0.0170\ \text{m})}{0.0170\ \text{m}-0.0120\ \text{m}} = 0.107\ \text{nF}.$$

(b) The charge on the positive plate is $q = CV = (0.107\ \text{nF})(73.0\ \text{V}) = 7.79\ \text{nC}.$

(c) Let the charge on the inner conductor be $-q$. Immediately adjacent to it is the induced charge $q'$. Since the electric field is less by a factor $1/\kappa$ than the field when no dielectric is present, then $-q + q' = -q/\kappa$. Thus,

$$q' = \frac{\kappa-1}{\kappa}q = 4\pi(\kappa-1)\varepsilon_0\frac{ab}{b-a}V = \left(\frac{23.5-1.00}{23.5}\right)(7.79\ \text{nC}) = 7.45\ \text{nC}.$$

56. (a) The potential across $C_1$ is 10 V, so the charge on it is

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 100\ \mu\text{C}.$$

(b) Reducing the right portion of the circuit produces an equivalence equal to 6.00 $\mu$F, with 10.0 V across it. Thus, a charge of 60.0 μC is on it, and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60\ \mu\text{C}}{10\ \mu\text{F}} = 6.00\ \text{V}$$

which leaves 10.0 V – 6.00 V = 4.00 V across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.00 V must be equally divided by $C_2$ and the capacitor directly below it (in series with it). Therefore, with 2.00 V across $C_2$ we find

$$q_2 = C_2V_2 = (10.0\ \mu\text{F})(2.00\ \text{V}) = 20.0\ \mu\text{C}.$$

57. **THINK** Figure 25-51 depicts a system of capacitors. The pair $C_3$ and $C_4$ are in parallel.

**EXPRESS** Since $C_3$ and $C_4$ are in parallel, we replace them with an equivalent capacitance $C_{34} = C_3 + C_4 = 30\ \mu\text{F}$. Now, $C_1$, $C_2$, and $C_{34}$ are in series, and all are numerically 30 μF, we observe that each has one-third the battery voltage across it. Hence, 3.0 V is across $C_4$.

**ANALYZE** The charge on capacitor 4 is $q_4 = C_4V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C}$.

**LEARN** Alternatively, one may show that the equivalent capacitance of the arrangement is given by

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{34}} = \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} + \frac{1}{30\ \mu\text{F}} = \frac{1}{10\ \mu\text{F}}$$

or $C_{1234} = 10\ \mu\text{F}$. Thus, the charge across $C_1$, $C_2$, and $C_{34}$ are

$$q_1 = q_2 = q_{34} = q_{1234} = C_{1234}V = (10\ \mu\text{F})(9.0\ \text{V}) = 90\ \text{nC}.$$

Now, since $C_3$ and $C_4$ are in parallel, and $C_3 = C_4$, the charge on $C_4$ (as well as on $C_3$) is $q_3 = q_4 = q_{34}/2 = (90\ \mu\text{F})/2 = 45\ \mu\text{F}$.

58. (a) Here $D$ is not attached to anything, so that the $6C$ and $4C$ capacitors are in series (equivalent to $2.4C$). This is then in parallel with the $2C$ capacitor, which produces an equivalence of $4.4C$. Finally the $4.4C$ is in series with $C$ and we obtain

$$C_{\text{eq}} = \frac{(C)(4.4C)}{C + 4.4C} = 0.82\,C = 0.82\,(50\ \mu\text{F}) = 41\,\mu\text{F}$$

where we have used the fact that $C = 50\ \mu\text{F}$.

(b) Now, $B$ is the point that is not attached to anything, so that the $6C$ and $2C$ capacitors are now in series (equivalent to $1.5C$), which is then in parallel with the $4C$ capacitor (and thus equivalent to $5.5C$). The $5.5C$ is then in series with the $C$ capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)(5.5C)}{C + 5.5C} = 0.85C = 42\ \mu\text{F}\ .$$

59. The pair $C_1$ and $C_2$ are in parallel, as are the pair $C_3$ and $C_4$; they reduce to equivalent values 6.0 $\mu$F and 3.0 $\mu$F, respectively. These are now in series and reduce to 2.0 $\mu$F,

across which we have the battery voltage. Consequently, the charge on the 2.0 $\mu$F equivalence is (2.0 $\mu$F)(12 V) = 24 $\mu$C. This charge on the 3.0 $\mu$F equivalence (of $C_3$ and $C_4$) has a voltage of

$$V = \frac{q}{C} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8.0\ \text{V}.$$

Finally, this voltage on capacitor $C_4$ produces a charge (2.0 $\mu$F)(8.0 V) = 16 $\mu$C.

60. (a) Equation 25-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times10^{-12}\ \text{F}\right)\left(7.0\times10^3\ \text{V}\right)^2 = 4.9\times10^{-3}\ \text{J}.$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.

61. Initially the capacitors $C_1$, $C_2$, and $C_3$ form a series combination equivalent to a single capacitor, which we denote $C_{123}$. Solving the equation

$$\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_1C_2 + C_2C_3 + C_1C_3}{C_1C_2C_3},$$

we obtain $C_{123}$ = 2.40 $\mu$F. With $V$ = 12.0 V, we then obtain $q = C_{123}V$ = 28.8 $\mu$C. In the final situation, $C_2$ and $C_4$ are in parallel and are thus effectively equivalent to $C_{24} = 12.0\ \mu\text{F}$. Similar to the previous computation, we use

$$\frac{1}{C_{1234}} = \frac{1}{C_1} + \frac{1}{C_{24}} + \frac{1}{C_3} = \frac{C_1C_{24} + C_{24}C_3 + C_1C_3}{C_1C_{24}C_3}$$

and find $C_{1234}$ = 3.00 $\mu$F. Therefore, the final charge is $q = C_{1234}V$ = 36.0 $\mu$C.

(a) This represents a change (relative to the initial charge) of $\Delta q$ = 7.20 $\mu$C.

(b) The capacitor $C_{24}$ which we imagined to replace the parallel pair $C_2$ and $C_4$, is in series with $C_1$ and $C_3$ and thus also has the final charge $q$ =36.0 $\mu$C found above. The voltage across $C_{24}$ would be

$$V_{24} = \frac{q}{C_{24}} = \frac{36.0\ \mu\text{C}}{12.0\ \mu\text{F}} = 3.00\ \text{V}.$$

This is the same voltage across each of the parallel pairs. In particular, $V_4$ = 3.00 V implies that $q_4 = C_4 V_4$ = 18.0 $\mu$C.

(c) The battery supplies charges only to the plates where it is connected. The charges on the rest of the plates are due to electron transfers between them, in accord with the new distribution of voltages across the capacitors. So, the battery does not directly supply the charge on capacitor 4.

62. In series, their equivalent capacitance (and thus their total energy stored) is smaller than either one individually (by Eq. 25-20). In parallel, their equivalent capacitance (and thus their total energy stored) is larger than either one individually (by Eq. 25-19). Thus, the middle two values quoted in the problem must correspond to the individual capacitors. We use Eq. 25-22 and find

(a) $100\ \mu\text{J} = \frac{1}{2}C_1(10\text{ V})^2 \Rightarrow C_1 = 2.0\ \mu\text{F}$;

(b) $300\ \mu\text{J} = \frac{1}{2}C_2(10\text{ V})^2 \Rightarrow C_2 = 6.0\ \mu\text{F}$.

63. Initially, the total equivalent capacitance is $C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 3.0\ \mu\text{F}$, and the charge on the positive plate of each one is (3.0 $\mu$F)(10 V) = 30 $\mu$C. Next, the capacitor (call is $C_1$) is squeezed as described in the problem, with the effect that the new value of $C_1$ is 12 $\mu$F (see Eq. 25-9). The new total equivalent capacitance then becomes

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 4.0\ \mu\text{F},$$

and the new charge on the positive plate of each one is (4.0 $\mu$F)(10 V) = 40 $\mu$C.

(a) Thus we see that the charge transferred from the battery as a result of the squeezing is 40 $\mu$C – 30 $\mu$C = 10 $\mu$C.

(b) The total increase in positive charge (on the respective positive plates) stored on the capacitors is twice the value found in part (a) (since we are dealing with two capacitors in series): 20 $\mu$C.

64. (a) We reduce the parallel group $C_2$, $C_3$ and $C_4$, and the parallel pair $C_5$ and $C_6$, obtaining equivalent values $C' = 12\ \mu\text{F}$ and $C'' = 12\ \mu\text{F}$, respectively. We then reduce the series group $C_1$, $C'$ and $C''$ to obtain an equivalent capacitance of $C_{eq} = 3\ \mu\text{F}$ hooked to the battery. Thus, the charge stored in the system is $q_{sys} = C_{eq}V_{bat} = 36\ \mu\text{C}$.

(b) Since $q_{sys} = q_1$, then the voltage across $C_1$ is

$$V_1 = \frac{q_1}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\text{ V}.$$

The voltage across the series-pair $C'$ and $C''$ is consequently $V_{bat} - V_1 = 6.0$ V. Since $C' = C''$, we infer $V' = V'' = 6.0/2 = 3.0$ V, which, in turn, is equal to $V_4$, the potential across $C_4$. Therefore,

$$q_4 = C_4V_4 = (4.0\ \mu\text{F})(3.0\text{ V}) = 12\ \mu\text{C}.$$

65. **THINK** We may think of the arrangement as two capacitors connected in series.

**EXPRESS** Let the capacitances be $C_1$ and $C_2$, with the former filled with the $\kappa_1 = 3.00$ material and the latter with the $\kappa_2$ = 4.00 material. Upon using Eq. 25-9, Eq. 25-27, and reducing $C_1$ and $C_2$ to an equivalent capacitance, we have

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\kappa_1 \varepsilon_0 A/d} + \frac{1}{\kappa_2 \varepsilon_0 A/d} = \left(\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2}\right)\frac{d}{\varepsilon_0 A}$$

or $C_{\text{eq}} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d}$. The charge stored on the capacitor is $q = C_{\text{eq}}V$.

**ANALYZE** Substituting the values given, we find

$$C_{\text{eq}} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d} = 1.52 \times 10^{-10} \text{ F},$$

Therefore, $q = C_{\text{eq}}V = 1.06 \times 10^{-9}$ C.

**LEARN** In the limit where $\kappa_1 = \kappa_2 = \kappa$, our expression for $C_{\text{eq}}$ becomes $C_{\text{eq}} = \frac{\kappa \varepsilon_0 A}{2d}$, where $2d$ is the plate separation.

66. We first need to find an expression for the energy stored in a cylinder of radius $R$ and length $L$, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where $E$ is the magnitude of the electric field at that point. If $q$ is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance $r$ from the cylinder axis is given by (see Eq. 25-12)

$$E = \frac{q}{2\pi\varepsilon_0 Lr},$$

and the energy density at that point is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2 \varepsilon_0 L^2 r^2}.$$

The corresponding energy in the cylinder is the volume integral $U_R = \int u\, d\mathcal{V}$. Now, $d\mathcal{V} = 2\pi r L\, dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2 \varepsilon_0 L^2 r^2} 2\pi r L\, dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{R}{a}\right).$$

To find an expression for the total energy stored in the capacitor, we replace $R$ with $b$:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right).$$

We want the ratio $U_R/U_b$ to be 1/2, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln\left(\sqrt{b/a}\right)$, $\ln(R/a) = \ln\left(\sqrt{b/a}\right)$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

67. (a) The equivalent capacitance is $C_{\text{eq}} = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{(6.00\,\mu\text{F})(4.00\,\mu\text{F})}{6.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 2.40\,\mu\text{F}$.

(b) $q_1 = C_{\text{eq}}V = (2.40\,\mu\text{F})(200\text{ V}) = 4.80 \times 10^{-4}$ C.

(c) $V_1 = q_1/C_1 = 4.80 \times 10^{-4}\text{ C}/6.00\,\mu\text{F} = 80.0$ V.

(d) $q_2 = q_1 = 4.80 \times 10^{-4}$ C.

(e) $V_2 = V - V_1 = 200\text{ V} - 80.0\text{ V} = 120$ V.

68. (a) Now $C_{\text{eq}} = C_1 + C_2 = 6.00\,\mu\text{F} + 4.00\,\mu\text{F} = 10.0\,\mu\text{F}$.

(b) $q_1 = C_1V = (6.00\,\mu\text{F})(200\text{ V}) = 1.20 \times 10^{-3}$ C.

(c) $V_1 = 200$ V.

(d) $q_2 = C_2V = (4.00\,\mu\text{F})(200\text{ V}) = 8.00 \times 10^{-4}$ C.

(e) $V_2 = V_1 = 200$ V.

69. We use $U = \frac{1}{2}CV^2$. As $V$ is increased by $\Delta V$, the energy stored in the capacitor increases correspondingly from $U$ to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$. Thus,
$(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\%\ .$$

70. (a) The length $d$ is effectively shortened by $b$ so $C' = \varepsilon_0 A/(d - b) = 0.708$ pF.

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d} = \frac{d}{d-b} = \frac{5.00}{5.00-2.00} = 1.67.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{q^2}{2\varepsilon_0 A}(d-b-d) = -\frac{q^2 b}{2\varepsilon_0 A} = -5.44 \text{ J}.$$

(d) Since $W < 0$, the slab is sucked in.

71. (a) $C' = \varepsilon_0 A/(d-b) = 0.708$ pF, the same as part (a) in Problem 25-70.

(b) The ratio of the stored energy is now

$$\frac{U}{U'} = \frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2} = \frac{C}{C'} = \frac{\varepsilon_0 A/d}{\varepsilon_0 A/(d-b)} = \frac{d-b}{d} = \frac{5.00-2.00}{5.00} = 0.600.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{1}{2}(C'-C)V^2 = \frac{\varepsilon_0 A}{2}\left(\frac{1}{d-b} - \frac{1}{d}\right)V^2 = \frac{\varepsilon_0 AbV^2}{2d(d-b)} = 1.02\times10^{-9}\text{ J}.$$

(d) In Problem 25-70 where the capacitor is disconnected from the battery and the slab is sucked in, $F$ is certainly given by $-dU/dx$. However, that relation does not hold when the battery is left attached because the force on the slab is not conservative. The charge distribution in the slab causes the slab to be sucked into the gap by the charge distribution on the plates. This action causes an increase in the potential energy stored by the battery in the capacitor.

72. (a) The equivalent capacitance is $C_{eq} = C_1C_2/(C_1 + C_2)$. Thus the charge $q$ on each capacitor is

$$q = q_1 = q_2 = C_{eq}V = \frac{C_1C_2V}{C_1+C_2} = \frac{(2.00\mu\text{F})(8.00\mu\text{F})(300\text{V})}{2.00\mu\text{F}+8.00\mu\text{F}} = 4.80\times10^{-4}\text{C}.$$

(b) The potential difference is $V_1 = q/C_1 = 4.80 \times 10^{-4}$ C/2.0 $\mu$F = 240 V.

(c) As noted in part (a), $q_2 = q_1 = 4.80\times10^{-4}\text{C}.$

(d) $V_2 = V - V_1$ = 300 V – 240 V = 60.0 V.

Now we have $q'_1/C_1 = q'_2/C_2 = V'$ ($V'$ being the new potential difference across each capacitor) and $q'_1 + q'_2 = 2q$. We solve for $q'_1$, $q'_2$ and $V$:

(e) $q_1' = \dfrac{2C_1 q}{C_1 + C_2} = \dfrac{2(2.00\mu\text{F})(4.80\times10^{-4}C)}{2.00\ \mu\text{F} + 8.00\ \mu\text{F}} = 1.92\times10^{-4}\,\text{C}.$

(f) $V_1' = \dfrac{q_1'}{C_1} = \dfrac{1.92\times10^{-4}C}{2.00\,\mu\text{F}} = 96.0\,\text{V}.$

(g) $q_2' = 2q - q_1 = 7.68\times10^{-4}C.$

(h) $V_2' = V_1' = 96.0\,\text{V}.$

(i) In this circumstance, the capacitors will simply discharge themselves, leaving $q_1 = 0$,

(j) $V_1 = 0$,

(k) $q_2 = 0$,

(l) and $V_2 = V_1 = 0$.

73. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30\,\mu\text{C}}{10\,\mu\text{F}} = 3.0\ \text{V}\ .$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2 V_2 = (20\,\mu\text{F})(2\ \text{V}) = 60\,\mu\text{C}\ ,$$

which means a total of 90 $\mu$C of charge is on the pair of capacitors $C_1$ and $C_2$. This implies there is a total of 90 $\mu$C of charge also on the $C_3$ and $C_4$ pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45\ \mu$C. Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45\,\mu\text{C}}{20\,\mu\text{F}} = 2.3\,\text{V}\ .$$

Therefore, $|V_A - V_B| = V_1 + V_3 = 5.3$ V.

74. We use $C = \varepsilon_0 \kappa A/d \propto \kappa/d$. To maximize $C$ we need to choose the material with the greatest value of $\kappa/d$. It follows that the mica sheet should be chosen.

75. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system "settling down" to its final state (of having 40 V across the parallel pair of capacitors $C$ and 60 $\mu$F). We do expect charge to be conserved. Thus, if $Q$ is the charge originally stored on $C$ and $q_1$, $q_2$ are the charges on the parallel pair after "settling down," then

$$Q = q_1 + q_2 \quad \Rightarrow \quad C(100\,\text{V}) = C(40\,\text{V}) + (60\,\mu\text{F})(40\,\text{V})$$

which leads to the solution $C = 40\ \mu$F.

76. One way to approach this is to note that since they are identical, the voltage is evenly divided between them. That is, the voltage across each capacitor is $V = (10/n)$ volt. With $C = 2.0 \times 10^{-6}$ F, the electric energy stored by each capacitor is $\frac{1}{2}CV^2$. The total energy stored by the capacitors is $n$ times that value, and the problem requires the total be equal to $25 \times 10^{-6}$ J. Thus,

$$\frac{n}{2}(2.0 \times 10^{-6})\left(\frac{10}{n}\right)^2 = 25 \times 10^{-6},$$

which leads to $n = 4$.

77. **THINK** We have two parallel-plate capacitors that are connected in parallel. They both have the same plate separation and same potential difference across their plates.

**EXPRESS** The magnitude of the electric field in the region between the plates is given by $E = V/d$, where $V$ is the potential difference between the plates and $d$ is the plate separation. The surface charge density on the plate is $\sigma = q/A$.

**ANALYZE** (a) With $d = 0.00300$ m and $V = 600$ V, we have

$$E_A = \frac{V}{d} = \frac{600\ \text{V}}{3.00\times10^{-3}\ \text{m}} = 2.00\times10^5\ \text{V/m}.$$

(b) Since $d = 0.00300$ m and $V = 600$ V in capacitor $B$ as well, $E_B = 2.00\times10^5$ V/m.

(c) For the air-filled capacitor, Eq. 25-4 leads to

$$\sigma_A = \frac{q_A}{A} = \frac{C_A V}{A} = \frac{(\varepsilon_0 A/d)V}{A} = \frac{\varepsilon_0 V}{d} = \varepsilon_0 E_A = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00\times10^5\ \text{V/m})$$
$$= 1.77\times10^{-6}\ \text{C/m}^2.$$

(d) For the dielectric-filled capacitor, we use Eq. 25-29:

$$\sigma_B = \kappa\varepsilon_0 E_B = (2.60)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00\times10^5\ \text{V/m}) = 4.60\times10^{-6}\ \text{C/m}^2.$$

(e) Although the discussion in Section 25-8 of the textbook is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors that have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor $B$ has a relatively large charge but only produces the field that $A$ produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 25-34 and in the material that follows. Adapting Eq. 25-35 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma_{\text{ind}} = \sigma_A - \sigma_B = (1.77\times10^{-6}\ \text{C/m}^2) - (4.60\times10^{-6}\ \text{C/m}^2) = -2.83\times10^{-6}\ \text{C/m}^2 .$$

**LEARN** We note that the electric field in capacitor $B$ is produced by both the charge on the plates ($\sigma_B A$) and the induced charges ($\sigma_{\text{ind}} A$), while the field in capacitor A is produced by the charge on the plates alone ($\sigma_A A$). Since $E_A = E_B$, we have $\sigma_A = \sigma_B + \sigma_{\text{ind}}$, or $\sigma_{\text{ind}} = \sigma_A - \sigma_B$.

78. (a) Put five such capacitors in series. Then, the equivalent capacitance is 2.0 $\mu$F/5 = 0.40 $\mu$F. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now $C_{\text{eq}}$ = 3(0.40 $\mu$F) = 1.2 $\mu$F. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

79. (a) For a capacitor with surface area $A$ and plate separation $x$ its capacitance is given by $C_0 = \varepsilon_0 A/x$. The energy stored in the capacitor can be written as

$$U = \frac{q^2}{2C} = \frac{q^2}{2(\varepsilon_0 A/x)} = \frac{q^2 x}{2\varepsilon_0 A}.$$

The change in energy if the separation between plates increases to $x + dx$ is

$$dU = \frac{q^2}{2\varepsilon_0 A}dx.$$

Thus, the force between the plates is

$$F = -\frac{dU}{dx} = -\frac{q^2}{2\varepsilon_0 A}.$$

The negative sign means that the force between the plates is attractive.

(b) The magnitude of the electrostatic stress is

$$\frac{|F|}{A} = \frac{q^2}{2\varepsilon_0 A^2} = \frac{\sigma^2}{2\varepsilon_0} = \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

where $E = \sigma/\varepsilon_0$ is the magnitude of the electric field in the region between the plates.

80. The energy initially stored in one capacitor is $U_0 = q_0^2/2C = 4.00$ J. After a second capacitor is connected to it in parallel, with $q_1 = q_2 = q_0/2,$ the energy stored in the first capacitor becomes

$$U_1 = \frac{q_1^2}{2C} = \frac{(q_0/2)^2}{2C} = \frac{U_0}{4} = 1.00 \text{ J}$$

which is the same as that stored in the second capacitor. Thus, the total energy is

$$U = U_1 + U_2 = \frac{U_0}{2} = 2.00 \text{ J.}$$

(b) The wires connecting the capacitors have resistance, so some energy is converted to thermal energy in the wires, as well as electromagnetic radiation.

# Chapter 26

1. (a) The charge that passes through any cross section is the product of the current and time. Since $t$ = 4.0 min = (4.0 min)(60 s/min) = 240 s,

$$q = it = (5.0\text{ A})(240\text{ s}) = 1.2\times 10^3\text{ C}.$$

(b) The number of electrons $N$ is given by $q = Ne$, where $e$ is the magnitude of the charge on an electron. Thus,

$$N = q/e = (1200\text{ C})/(1.60 \times 10^{-19}\text{ C}) = 7.5 \times 10^{21}.$$

2. Suppose the charge on the sphere increases by $\Delta q$ in time $\Delta t$. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r},$$

where $r$ is the radius of the sphere. This means $\Delta q = 4\pi\varepsilon_0 r\,\Delta V$. Now, $\Delta q = (i_{\text{in}} - i_{\text{out}})\,\Delta t$, where $i_{\text{in}}$ is the current entering the sphere and $i_{\text{out}}$ is the current leaving. Thus,

$$\Delta t = \frac{\Delta q}{i_{\text{in}} - i_{\text{out}}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{\text{in}} - i_{\text{out}}} = \frac{(0.10\text{ m})(1000\text{ V})}{(8.99\times 10^9\text{ F/m})(1.0000020\text{ A} - 1.0000000\text{ A})}$$
$$= 5.6\times 10^{-3}\text{ s}.$$

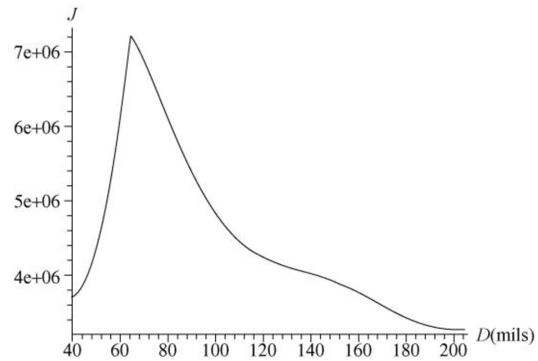
3. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using $\sigma$ for the charge per unit area and $w$ for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100\times 10^{-6}\text{ A}}{(30\text{ m/s})(50\times 10^{-2}\text{ m})} = 6.7\times 10^{-6}\text{ C/m}^2.$$

4. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2}.$$

For example, the gauge 14 wire with $D$ = 64 mil = 0.0016 m is found to have a (maximum safe) current density of $J = 7.2 \times 10^6$ A/m$^2$. In fact, this is the wire with the largest value of $J$ allowed by the given data. The values of $J$ in SI units are plotted below as a function of their diameters in mils.




5. **THINK** The magnitude of the current density is given by $J = nqv_d$, where $n$ is the number of particles per unit volume, $q$ is the charge on each particle, and $v_d$ is the drift speed of the particles.

**EXPRESS** In vector form, we have (see Eq. 26-7) $\vec{J} = nq\vec{v}_d$. Current density $\vec{J}$ is related to the current $i$ by (see Eq. 26-4): $i = \int \vec{J} \cdot d\vec{A}$.

**ANALYZE** (a) The particle concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14}\ \text{m}^{-3}$, the charge is

$$q = 2e = 2(1.60 \times 10^{-19}\ \text{C}) = 3.20 \times 10^{-19}\ \text{C},$$

and the drift speed is $1.0 \times 10^5$ m/s. Thus, we find the current density to be

$$J = \left(2 \times 10^{14}/\text{m}\right)\left(3.2 \times 10^{-19}\ \text{C}\right)\left(1.0 \times 10^5\ \text{m/s}\right) = 6.4\ \text{A/m}^2.$$

(b) Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(c) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

**LEARN** That the current density is in the direction of the motion of the *positive* charge carriers means that it is in the opposite direction of the motion of the negatively charged electrons.

6. (a) Circular area depends, of course, on $r^2$, so the horizontal axis of the graph in Fig. 26-24(b) is effectively the same as the area (enclosed at variable radius values), except for a factor of $\pi$. The fact that the current increases linearly in the graph means that $i/A = J$ = constant. Thus, the answer is “yes, the current density is uniform.”

(b) We find $i/(\pi r^2) = (0.005\text{ A})/(\pi \times 4 \times 10^{-6}\text{ m}^2) = 398 \approx 4.0 \times 10^2\text{ A/m}^2$.

7. The cross-sectional area of wire is given by $A = \pi r^2$, where $r$ is its radius (half its thickness). The magnitude of the current density vector is

$$J = i/A = i/\pi r^2,$$

so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50\text{ A}}{\pi\left(440\times10^4\text{ A/m}^2\right)}} = 1.9\times10^{-4}\text{ m}.$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4}\text{ m}) = 3.8 \times 10^{-4}\text{ m}$.

8. (a) The magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{i}{\pi d^2/4} = \frac{4\left(1.2\times10^{-10}\text{ A}\right)}{\pi\left(2.5\times10^{-3}\text{ m}\right)^2} = 2.4\times10^{-5}\text{ A/m}^2.$$

(b) The drift speed of the current-carrying electrons is

$$v_d = \frac{J}{ne} = \frac{2.4\times10^{-5}\text{ A/m}^2}{\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\text{ C}\right)} = 1.8\times10^{-15}\text{ m/s}.$$

9. We note that the radial width $\Delta r = 10$ μm is small enough (compared to $r = 1.20$ mm) that we can make the approximation

$$\int Br2\pi r dr \approx Br2\pi r\Delta r$$

Thus, the enclosed current is $2\pi Br^2\Delta r = 18.1\ \mu\text{A}$. Performing the integral gives the same answer.

10. Assuming $\vec{J}$ is directed along the wire (with no radial flow) we integrate, starting with Eq. 26-4,

$$i = \int |\vec{J}|\, dA = \int_{9R/10}^{R} (kr^2)2\pi r dr = \frac{1}{2}k\pi\left(R^4 - 0.656R^4\right)$$

where $k = 3.0 \times 10^8$ and SI units are understood. Therefore, if $R = 0.00200$ m, we obtain $i = 2.59\times10^{-3}\text{ A}$.

11. (a) The current resulting from this non-uniform current density is

$$i=\int_{\text{cylinder}} J_a dA=\frac{J_0}{R}\int_0^R r\cdot 2\pi r dr=\frac{2}{3}\pi R^2 J_0=\frac{2}{3}\pi(3.40\times10^{-3}\,\text{m})^2(5.50\times10^4\,\text{A/m}^2).$$
$$=1.33\text{ A}.$$

(b) In this case,

$$i=\int_{\text{cylinder}} J_b dA=\int_0^R J_0\left(1-\frac{r}{R}\right)2\pi r dr=\frac{1}{3}\pi R^2 J_0=\frac{1}{3}\pi(3.40\times10^{-3}\,\text{m})^2(5.50\times10^4\,\text{A/m}^2)$$
$$=0.666\text{ A}.$$

(c) The result is different from that in part (a) because $J_b$ is higher near the center of the cylinder (where the area is smaller for the same radial interval) and lower outward, resulting in a lower average current density over the cross section and consequently a lower current than that in part (a). So, $J_a$ has its maximum value near the surface of the wire.

12. (a) Since 1 cm$^3$ = 10$^{-6}$ m$^3$, the magnitude of the current density vector is

$$J=nev=\left(\frac{8.70}{10^{-6}\,\text{m}^3}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(470\times10^3\,\text{m/s}\right)=6.54\times10^{-7}\,\text{A/m}^2.$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a "target" of circular area $\pi R_E^2$. The rate of charge transport implied by the influx of protons is

$$i=AJ=\pi R_E^2 J=\pi\left(6.37\times10^6\,\text{m}\right)^2\left(6.54\times10^{-7}\,\text{A/m}^2\right)=8.34\times10^7\,\text{A}.$$

13. We use $v_d = J/ne = i/Ane$. Thus,

$$t=\frac{L}{v_d}=\frac{L}{i/Ane}=\frac{LAne}{i}=\frac{(0.85\,\text{m})\left(0.21\times10^{-14}\,\text{m}^2\right)\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)}{300\,\text{A}}$$
$$=8.1\times10^2\,\text{s}=13\,\text{min}.$$

14. Since the potential difference $V$ and current $i$ are related by $V = iR$, where $R$ is the resistance of the electrician, the fatal voltage is $V = (50\times10^{-3}\,\text{A})(2000\,\Omega) = 100\text{ V}$.

15. **THINK** The resistance of the coil is given by $R = \rho L/A$, where $L$ is the length of the wire, $\rho$ is the resistivity of copper, and $A$ is the cross-sectional area of the wire.

**EXPRESS** Since each turn of wire has length $2\pi r$, where $r$ is the radius of the coil, then

$$L = (250)2\pi r = (250)(2\pi)(0.12 \text{ m}) = 188.5 \text{ m}.$$

If $r_w$ is the radius of the wire itself, then its cross-sectional area is

$$A = \pi r_w^2 = \pi(0.65 \times 10^{-3} \text{ m})^2 = 1.33 \times 10^{-6} \text{ m}^2.$$

According to Table 26-1, the resistivity of copper is $\rho = 1.69 \times 10^{-8} \Omega \cdot \text{m}$.

**ANALYZE** Thus, the resistance of the copper coil is

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8} \Omega \cdot \text{m})(188.5 \text{ m})}{1.33 \times 10^{-6} \text{ m}^2} = 2.4 \Omega.$$

**LEARN** Resistance $R$ is the property of an object (depending on quantities such as $L$ and $A$), while resistivity is a property of the material.

16. We use $R/L = \rho/A = 0.150 \ \Omega/\text{km}$.

(a) For copper $J = i/A = (60.0 \text{ A})(0.150 \ \Omega/\text{km})/(1.69 \times 10^{-8} \ \Omega \cdot \text{m}) = 5.32 \times 10^5 \text{ A/m}^2$.

(b) We denote the mass densities as $\rho_m$. For copper,

$$(m/L)_c = (\rho_m A)_c = (8960 \text{ kg/m}^3)\,(1.69 \times 10^{-8} \ \Omega \cdot \text{m})/(0.150 \ \Omega/\text{km}) = 1.01 \text{ kg/m}.$$

(c) For aluminum $J = (60.0 \text{ A})(0.150 \ \Omega/\text{km})/(2.75 \times 10^{-8} \ \Omega \cdot \text{m}) = 3.27 \times 10^5 \text{ A/m}^2$.

(d) The mass density of aluminum is

$$(m/L)_a = (\rho_m A)_a = (2700 \text{ kg/m}^3)(2.75 \times 10^{-8} \ \Omega \cdot \text{m})/(0.150 \ \Omega/\text{km}) = 0.495 \text{ kg/m}.$$

17. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0 \text{ m})(4.0 \text{ A})}{(2.0 \text{ V})(1.0 \times 10^{-6} \text{ m}^2)} = 2.0 \times 10^6 / \Omega \cdot \text{m}.$$

18. (a) $i = V/R = 23.0 \text{ V}/15.0 \times 10^{-3} \ \Omega = 1.53 \times 10^3 \text{ A}$.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4}\pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4(1.53 \times 10^{-3} \text{ A})}{\pi(6.00 \times 10^{-3} \text{ m})^2} = 5.41 \times 10^7 \text{ A/m}^2.$$

(c) The resistivity is

$$\rho = \frac{RA}{L} = \frac{(15.0\times10^{-3}\,\Omega)\pi(6.00\times10^{-3}\text{ m})^2}{4(4.00\text{ m})} = 10.6\times10^{-8}\ \Omega\cdot\text{m}.$$

(d) The material is platinum.

19. **THINK** The resistance of the wire is given by $R = \rho L/A$, where $\rho$ is the resistivity of the material, $L$ is the length of the wire, and $A$ is its cross-sectional area.

**EXPRESS** In this case, the cross-sectional area is

$$A = \pi r^2 = \pi\left(0.50\times10^{-3}\text{ m}\right)^2 = 7.85\times10^{-7}\text{ m}^2.$$

**ANALYZE** Thus, the resistivity of the wire is

$$\rho = \frac{RA}{L} = \frac{\left(50\times10^{-3}\,\Omega\right)\left(7.85\times10^{-7}\text{ m}^2\right)}{2.0\text{ m}} = 2.0\times10^{-8}\,\Omega\cdot\text{m}.$$

**LEARN** Resistance $R$ is the property of an object (depending on quantities such as $L$ and $A$), while resistivity is a property of the material itself. The equation $R = \rho L/A$ implies that the larger the cross-sectional area $A$, the smaller the resistance $R$.

20. The thickness (diameter) of the wire is denoted by $D$. We use $R \propto L/A$ (Eq. 26-16) and note that $A = \frac{1}{4}\pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R\left(\frac{D_1}{D_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2\left(\frac{1}{2}\right) = 2R.$$

21. The resistance at operating temperature $T$ is $R = V/i$ = 2.9 V/0.30 A = 9.67 Ω. Thus, from $R - R_0 = R_0\alpha(T - T_0)$, we find

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) = 20°\text{C} + \left(\frac{1}{4.5\times10^{-3}/\text{K}}\right)\left(\frac{9.67\,\Omega}{1.1\,\Omega} - 1\right) = 1.8\times10^3\ °\text{C}.$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of $\alpha$ used in this calculation is not inconsistent with the other units involved. Table 26-1 has been used.

22. Let $r = 2.00$ mm be the radius of the kite string and $t = 0.50$ mm be the thickness of the water layer. The cross-sectional area of the layer of water is

$$A=\pi\left[(r+t)^2-r^2\right]=\pi[(2.50\times10^{-3}\text{ m})^2-(2.00\times10^{-3}\text{ m})^2]=7.07\times10^{-6}\text{ m}^2.$$

Using Eq. 26-16, the resistance of the wet string is

$$R=\frac{\rho L}{A}=\frac{(150\,\Omega\cdot\text{m})(800\text{ m})}{7.07\times10^{-6}\text{ m}^2}=1.698\times10^{10}\,\Omega.$$

The current through the water layer is

$$i=\frac{V}{R}=\frac{1.60\times10^{8}\,\text{V}}{1.698\times10^{10}\,\Omega}=9.42\times10^{-3}\,\text{A}.$$

23. We use $J = E/\rho$, where $E$ is the magnitude of the (uniform) electric field in the wire, $J$ is the magnitude of the current density, and $\rho$ is the resistivity of the material. The electric field is given by $E = V/L$, where $V$ is the potential difference along the wire and $L$ is the length of the wire. Thus $J = V/L\rho$ and

$$\rho=\frac{V}{LJ}=\frac{115\text{ V}}{(10\text{ m})(1.4\times10^{8}\text{ A/m}^2)}=8.2\times10^{-8}\,\Omega\cdot\text{m}.$$

24. (a) Since the material is the same, the resistivity ρ is the same, which implies (by Eq. 26-11) that the electric fields (in the various rods) are directly proportional to their current-densities. Thus, $J_1$: $J_2$: $J_3$ are in the ratio 2.5/4/1.5 (see Fig. 26-25). Now the currents in the rods must be the same (they are "in series") so

$$J_1 A_1 = J_3 A_3, \qquad J_2 A_2 = J_3 A_3 .$$

Since $A = \pi r^2$, this leads (in view of the aforementioned ratios) to

$$4r_2^2 = 1.5r_3^2, \qquad 2.5r_1^2 = 1.5r_3^2 .$$

Thus, with $r_3$ = 2 mm, the latter relation leads to $r_1$ = 1.55 mm.

(b) The $4r_2^2 = 1.5r_3^2$ relation leads to $r_2$ = 1.22 mm.

25. **THINK** The resistance of an object depends on its length and the cross-sectional area.

**EXPRESS** Since the mass and density of the material do not change, the volume remains the same. If $L_0$ is the original length, $L$ is the new length, $A_0$ is the original cross-sectional area, and $A$ is the new cross-sectional area, then $L_0A_0 = LA$ and

$$A = L_0A_0/L = L_0A_0/3L_0 = A_0/3.$$

**ANALYZE** The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0,$$

where $R_0$ is the original resistance. Thus, $R = 9(6.0\ \Omega) = 54\ \Omega$.

**LEARN** In general, the resistances of two objects made of the same material but different cross-sectional areas and lengths may be related by

$$R_2 = R_1\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right).$$

26. The absolute values of the slopes (for the straight-line segments shown in the graph of Fig. 26-25(b)) are equal to the respective electric field magnitudes. Thus, applying Eq. 26-5 and Eq. 26-13 to the three sections of the resistive strip, we have

$$J_1 = \frac{i}{A} = \sigma_1 E_1 = \sigma_1 (0.50 \times 10^3 \text{ V/m})$$

$$J_2 = \frac{i}{A} = \sigma_2 E_2 = \sigma_2 (4.0 \times 10^3 \text{ V/m})$$

$$J_3 = \frac{i}{A} = \sigma_3 E_3 = \sigma_3 (1.0 \times 10^3 \text{ V/m}) .$$

We note that the current densities are the same since the values of $i$ and $A$ are the same (see the problem statement) in the three sections, so $J_1 = J_2 = J_3$.

(a) Thus we see that $\sigma_1 = 2\sigma_3 = 2\,(3.00 \times 10^7 (\Omega\cdot\text{m})^{-1}) = 6.00 \times 10^7\ (\Omega\cdot\text{m})^{-1}$.

(b) Similarly, $\sigma_2 = \sigma_3/4 = (3.00 \times 10^7 (\Omega\cdot\text{m})^{-1})/4 = 7.50 \times 10^6\ (\Omega\cdot\text{m})^{-1}$.

27. **THINK** In this problem we compare the resistances of two conductors that are made of the same materials.

**EXPRESS** The resistance of conductor $A$ is given by

$$R_A = \frac{\rho L}{\pi r_A^2},$$

where $r_A$ is the radius of the conductor. If $r_o$ is the outside diameter of conductor $B$ and $r_i$ is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$, and its resistance is

$$R_B = \frac{\rho L}{\pi\left(r_o^2 - r_i^2\right)}.$$

**ANALYZE** The ratio of the resistances is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0.$$

**LEARN** The resistance $R$ of an object depends on how the electric potential is applied to the object. Also, $R$ depends on the ratio $L/A$, according to $R = \rho L / A$.

28. The cross-sectional area is $A = \pi r^2 = \pi(0.002 \text{ m})^2$. The resistivity from Table 26-1 is $\rho = 1.69 \times 10^{-8}\,\Omega \cdot \text{m}$. Thus, with $L = 3$ m, Ohm's Law leads to $V = iR = i\rho L/A$, or

$$12 \times 10^{-6}\,\text{V} = i\,(1.69 \times 10^{-8}\,\Omega \cdot \text{m})(3.0 \text{ m})/\ \pi(0.002 \text{ m})^2$$

which yields $i = 0.00297$ A or roughly 3.0 mA.

29. First we find the resistance of the copper wire to be

$$R = \frac{\rho L}{A} = \frac{(1.69\times10^{-8}\ \Omega\cdot\text{m})(0.020\text{ m})}{\pi(2.0\times10^{-3}\ \text{m})^2} = 2.69\times10^{-5}\,\Omega\,.$$

With potential difference $V = 3.00\,\text{nV}$, the current flowing through the wire is

$$i = \frac{V}{R} = \frac{3.00\times10^{-9}\,\text{V}}{2.69\times10^{-5}\,\Omega} = 1.115\times10^{-4}\,\text{A}\,.$$

Therefore, in 3.00 ms, the amount of charge drifting through a cross section is

$$\Delta Q = i\Delta t = (1.115\times10^{-4}\,\text{A})(3.00\times10^{-3}\ \text{s}) = 3.35\times10^{-7}\,\text{C}\ .$$

30. We use $R \propto L/A$. The diameter of a 22-gauge wire is 1/4 that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be

$$R = (1.00\ \Omega)(25 \text{ ft}/1000 \text{ ft})(4)^2 = 0.40\ \Omega.$$

31. (a) The current in each strand is $i = 0.750 \text{ A}/125 = 6.00 \times 10^{-3}$ A.

(b) The potential difference is $V = iR = (6.00 \times 10^{-3} \text{ A})\,(2.65 \times 10^{-6}\ \Omega) = 1.59 \times 10^{-8}$ V.

(c) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6}\ \Omega/125 = 2.12 \times 10^{-8}\ \Omega$.

32. We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 26-13 and Eq. 26-7.

(a) The magnitude of the current density is

$$J = \sigma E = (2.70 \times 10^{-14} / \Omega \cdot \text{m})\,(120 \text{ V/m}) = 3.24 \times 10^{-12} \text{ A/m}^2.$$

(b) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{\left(2.70\times10^{-14}/\Omega\cdot\text{m}\right)\left(120 \text{ V/m}\right)}{\left[\left(620+550\right)/\text{cm}^3\right]\left(1.60\times10^{-19} \text{ C}\right)} = 1.73 \text{ cm/s}.$$

33. (a) The current in the block is $i = V/R = 35.8 \text{ V}/935\ \Omega = 3.83 \times 10^{-2}$ A.

(b) The magnitude of current density is

$$J = i/A = (3.83 \times 10^{-2} \text{ A})/(3.50 \times 10^{-4} \text{ m}^2) = 109 \text{ A/m}^2.$$

(c) $v_d = J/ne = (109 \text{ A/m}^2)/[(5.33 \times 10^{22}/\text{m}^3)\,(1.60 \times 10^{-19} \text{ C})] = 1.28 \times 10^{-2}$ m/s.

(d) $E = V/L = 35.8 \text{ V}/0.158 \text{ m} = 227$ V/m.

34. The number density of conduction electrons in copper is $n = 8.49 \times 10^{28}\ /\text{m}^3$. The electric field in section 2 is (10.0 $\mu$V)/(2.00 m) = 5.00 $\mu$V/m. Since $\rho = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$ for copper (see Table 26-1) then Eq. 26-10 leads to a current density vector of magnitude

$$J_2 = (5.00\ \mu\text{V/m})/(1.69 \times 10^{-8}\ \Omega \cdot \text{m}) = 296 \text{ A/m}^2$$

in section 2. Conservation of electric current from section 1 into section 2 implies

$$J_1 A_1 = J_2 A_2 \quad \Rightarrow \quad J_1(4\pi R^2) = J_2(\pi R^2)$$

(see Eq. 26-5). This leads to $J_1 = 74$ A/m$^2$. Now, for the drift speed of conduction-electrons in section 1, Eq. 26-7 immediately yields

$$v_d = \frac{J_1}{ne} = 5.44\times10^{-9} \text{ m/s}$$

35. (a) The current *i* is shown in Fig. 26-30 entering the truncated cone at the left end and leaving at the right. This is our choice of positive *x* direction. We make the assumption that the current density *J* at each value of *x* may be found by taking the ratio *i*/*A* where $A = \pi r^2$ is the cone's cross-section area at that particular value of *x*.

The direction of $\vec{J}$ is identical to that shown in the figure for *i* (our +*x* direction). Using Eq. 26-11, we then find an expression for the electric field at each value of *x*, and next find the potential difference *V* by integrating the field along the *x* axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J=\frac{i}{\pi r^2}=\frac{E}{\rho}$$

where we must deduce how *r* depends on *x* in order to proceed. We note that the radius increases linearly with *x*, so (with $c_1$ and $c_2$ to be determined later) we may write $r=c_1+c_2x.$

Choosing the origin at the left end of the truncated cone, the coefficient $c_1$ is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient $c_2$ must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2=(b-a)/L$. Our expression, then, becomes

$$r=a+\left(\frac{b-a}{L}\right)x.$$

Substituting this into our previous statement and solving for the field, we find

$$E=\frac{i\rho}{\pi}\left(a+\frac{b-a}{L}x\right)^{-2}.$$

Consequently, the potential difference between the faces of the cone is

$$V=-\int_0^L E\,dx=-\frac{i\rho}{\pi}\int_0^L\left(a+\frac{b-a}{L}x\right)^{-2}dx=\frac{i\rho}{\pi}\frac{L}{b-a}\left(a+\frac{b-a}{L}x\right)^{-1}\Bigg|_0^L$$
$$=\frac{i\rho}{\pi}\frac{L}{b-a}\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{i\rho}{\pi}\frac{L}{b-a}\frac{b-a}{ab}=\frac{i\rho L}{\pi ab}.$$

The resistance is therefore

$$R=\frac{V}{i}=\frac{\rho L}{\pi ab}=\frac{(731\,\Omega\cdot\text{m})(1.94\times10^{-2}\text{ m})}{\pi(2.00\times10^{-3}\text{ m})(2.30\times10^{-3}\text{ m})}=9.81\times10^{5}\ \Omega$$

Note that if $b = a$, then $R = \rho L/\pi a^2 = \rho L/A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.

36. Since the current spreads uniformly over the hemisphere, the current density at any given radius *r* from the striking point is $J=I/2\pi r^2$. From Eq. 26-10, the magnitude of the electric field at a radial distance *r* is

$$E=\rho_w J=\frac{\rho_w I}{2\pi r^2},$$

where $\rho_w = 30\,\Omega\cdot\text{m}$ is the resistivity of water. The potential difference between a point at radial distance $D$ and a point at $D+\Delta r$ is

$$\Delta V = -\int_D^{D+\Delta r} E dr = -\int_D^{D+\Delta r} \frac{\rho_w I}{2\pi r^2} dr = \frac{\rho_w I}{2\pi}\left(\frac{1}{D+\Delta r} - \frac{1}{D}\right) = -\frac{\rho_w I}{2\pi}\frac{\Delta r}{D(D+\Delta r)},$$

which implies that the current across the swimmer is

$$i = \frac{|\Delta V|}{R} = \frac{\rho_w I}{2\pi R}\frac{\Delta r}{D(D+\Delta r)}.$$

Substituting the values given, we obtain

$$i = \frac{(30.0\,\Omega\cdot\text{m})(7.80\times10^4\,\text{A})}{2\pi(4.00\times10^3\,\Omega)}\frac{0.70\text{ m}}{(35.0\text{ m})(35.0\text{ m}+0.70\text{ m})} = 5.22\times10^{-2}\,\text{A}.$$

37. From Eq. 26-25, $\rho \propto \tau^{-1} \propto v_{\text{eff}}$. The connection with $v_{\text{eff}}$ is indicated in part (b) of Sample Problem 26.05 —"Mean free time and mean free distance," which contains useful insight regarding the problem we are working now. According to Chapter 20, $v_{\text{eff}} \propto \sqrt{T}$. Thus, we may conclude that $\rho \propto \sqrt{T}$.

38. The slope of the graph is $P = 5.0 \times 10^{-4}$ W. Using this in the $P = V^2/R$ relation leads to $V = 0.10$ Vs.

39. Eq. 26-26 gives the rate of thermal energy production:

$$P = iV = (10.0\,\text{A})(120\,\text{V}) = 1.20\text{ kW}.$$

Dividing this into the 180 kJ necessary to cook the three hotdogs leads to the result $t = 150$ s.

40. The resistance is $R = P/i^2 = (100\text{ W})/(3.00\text{ A})^2 = 11.1\ \Omega$.

41. **THINK** In an electrical circuit, the electrical energy is dissipated through the resistor as heat.

**EXPRESS** Electrical energy is converted to heat at a rate given by $P = V^2/R$, where $V$ is the potential difference across the heater and $R$ is the resistance of the heater.

**ANALYZE** With $V = 120$ V and $R = 14\ \Omega$, we have

$$P = \frac{(120\text{ V})^2}{14\ \Omega} = 1.0\times10^3\text{ W} = 1.0\text{ kW}.$$

(b) The cost is given by $(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW}\cdot\text{h}) = \text{US\$}0.25$.

**LEARN** The energy transferred is lost because the process is irreversible. The thermal energy causes the temperature of the resistor to rise.

42. (a) Referring to Fig. 26-33, the electric field would point down (toward the bottom of the page) in the strip, which means the current density vector would point down, too (by Eq. 26-11). This implies (since electrons are negatively charged) that the conduction electrons would be "drifting" upward in the strip.

(b) Equation 24-6 immediately gives 12 eV, or (using $e = 1.60 \times 10^{-19}$ C) $1.9 \times 10^{-18}$ J for the work done by the field (which equals, in magnitude, the potential energy change of the electron).

(c) Since the electrons don't (on average) gain kinetic energy as a result of this work done, it is generally dissipated as heat. The answer is as in part (b): 12 eV or $1.9 \times 10^{-18}$ J.

43. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50\ \text{V}}{3.00\ \text{V}}\right)^2 (0.540\ \text{W}) = 0.135\ \text{W}.$$

44. Since $P = iV$, the charge is

$$q = it = Pt/V = (7.0\ \text{W})\,(5.0\ \text{h})\,(3600\ \text{s/h})/9.0\ \text{V} = 1.4 \times 10^4\ \text{C}.$$

45. **THINK** Let $P$ be the power dissipated, $i$ be the current in the heater, and $V$ be the potential difference across the heater. The three quantities are related by $P = iV$.

**EXPRESS** The current is given by $i = P/V$. Using Ohm's law $V = iR$, the resistance of the heater can be written as

$$R = \frac{V}{i} = \frac{V}{P/V} = \frac{V^2}{P}.$$

**ANALYZE** (a) Substituting the values given, we have $i = \dfrac{P}{V} = \dfrac{1250\ \text{W}}{115\ \text{V}} = 10.9\ \text{A}.$

(b) Similarly, the resistance is

$$R = \frac{V^2}{P} = \frac{(115\ \text{V})^2}{1250\ \text{W}} = 10.6\ \Omega.$$

(c) The thermal energy $E$ generated by the heater in time $t = 1.0$ h = 3600 s is

$$E = Pt = (1250\,\text{W})(3600\,\text{s}) = 4.50\times10^{6}\ \text{J}.$$

**LEARN** Current in the heater produces a transfer of mechanical energy to thermal energy, with a rate of the transfer equal to $P = iV = V^2/R$.

46. (a) Using Table 26-1 and Eq. 26-10 (or Eq. 26-11), we have

$$|\vec{E}| = \rho|\vec{J}| = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(\frac{2.00\,\text{A}}{2.00\times10^{-6}\,\text{m}^2}\right) = 1.69\times10^{-2}\ \text{V/m}.$$

(b) Using $L$ = 4.0 m, the resistance is found from Eq. 26-16:

$$R = \rho L/A = 0.0338\ \Omega.$$

The rate of thermal energy generation is found from Eq. 26-27:

$$P = i^2 R = (2.00\ \text{A})^2(0.0338\ \Omega) = 0.135\ \text{W}.$$

Assuming a steady rate, the amount of thermal energy generated in 30 minutes is found to be (0.135 J/s)(30 × 60 s) = 2.43 × $10^2$ J.

47. (a) From $P = V^2/R = AV^2/\rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60\times10^{-6}\ \text{m}^2)(75.0\ \text{V})^2}{(5.00\times10^{-7}\ \Omega\cdot\text{m})(500\ \text{W})} = 5.85\ \text{m}.$$

(b) Since $L \propto V^2$ the new length should be $L' = L\left(\frac{V'}{V}\right)^2 = (5.85\ \text{m})\left(\frac{100\ \text{V}}{75.0\ \text{V}}\right)^2 = 10.4\ \text{m}.$

48. The mass of the water over the length is

$$m = \rho AL = (1000\ \text{kg/m}^3)(15\times10^{-5}\ \text{m}^2)(0.12\ \text{m}) = 0.018\ \text{kg},$$

and the energy required to vaporize the water is

$$Q = Lm = (2256\ \text{kJ/kg})(0.018\ \text{kg}) = 4.06\times10^4\ \text{J}.$$

The thermal energy is supplied by joule heating of the resistor:

$$Q = P\Delta t = I^2 R\Delta t.$$

Since the resistance over the length of water is

$$R=\frac{\rho_w L}{A}=\frac{(150\,\Omega\cdot\text{m})(0.120\text{ m})}{15\times10^{-5}\ \text{m}^2}=1.2\times10^5\,\Omega,$$

the average current required to vaporize water is

$$I=\sqrt{\frac{Q}{R\Delta t}}=\sqrt{\frac{4.06\times10^4\text{ J}}{(1.2\times10^5\Omega)(2.0\times10^{-3}\text{ s})}}=13.0\text{ A}.$$

49. (a) Assuming a 31-day month, the monthly cost is

$$(100\text{ W})(24\text{ h/day})(31\text{days/month})\,(6\text{ cents/kW}\cdot\text{h})=\ 446\text{ cents}=\text{US\$4.46}.$$

(b) $R = V^2/P = (120\text{ V})^2/100\text{ W} = 144\ \Omega$.

(c) $i = P/V = 100\text{ W}/120\text{ V} = 0.833\text{ A}$.

50. The slopes of the lines yield $P_1$ = 8 mW and $P_2$ = 4 mW. Their sum (by energy conservation) must be equal to that supplied by the battery: $P_{\text{batt}}$ = (8 + 4) mW = 12 mW.

51. **THINK** Our system is made up of two wires that are joined together. To calculate the electrical potential difference between two points, we first calculate their resistances.

**EXPRESS** The potential difference between points 1 and 2 is $\Delta V_{12}=iR_C$, where $R_C$ is the resistance of wire *C*. Similarly, the potential difference between points 2 and 3 is $\Delta V_{23}=iR_D$, where $R_D$ is the resistance of wire *D*. The corresponding rates of energy dissipation are $P_{12}=i^2R_C$ and $P_{23}=i^2R_D$, respectively.

**ANALYZE** (a) Using Eq. 26-16, we find the resistance of wire *C* to be

$$R_C=\rho_C\frac{L_C}{\pi r_C^2}=(2.0\times10^{-6}\,\Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00050\text{ m})^2}=2.55\,\Omega.$$

Thus, $\Delta V_{12}=iR_C=(2.0\text{ A})(2.55\,\Omega)=5.1\text{V}.$

(b) Similarly, the resistance for wire *D* is

$$R_D=\rho_D\frac{L_D}{\pi r_D^2}=(1.0\times10^{-6}\,\Omega\cdot\text{m})\frac{1.0\text{ m}}{\pi(0.00025\text{ m})^2}=5.09\ \Omega$$

and the potential difference is $\Delta V_{23}=iR_D=(2.0\text{ A})(5.09\,\Omega)=10.2\text{V}\approx10\text{V}$.

(c) The power dissipated between points 1 and 2 is $P_{12}=i^2R_C=10\text{W}.$

(d) Similarly, the power dissipated between points 2 and 3 is $P_{23} = i^2 R_D = 20\,\text{W}$.

**LEARN** The results may be summarized in terms of the following ratios:

$$\frac{P_{23}}{P_{12}} = \frac{\Delta V_{23}}{\Delta V_{12}} = \frac{R_D}{R_C} = \frac{\rho_D}{\rho_C} \cdot \frac{L_D}{L_C} \cdot \left(\frac{r_C}{r_D}\right)^2 = \frac{1}{2} \cdot 1 \cdot (2)^2 = 2.$$

52. Assuming the current is along the wire (not radial) we find the current from Eq. 26-4:

$$i = \int |\vec{J}|\, dA = \int_0^R kr^2 2\pi r dr = \frac{1}{2} k\pi R^4 = 3.50 \text{ A}$$

where $k = 2.75 \times 10^{10}$ A/m$^4$ and $R = 0.00300$ m. The rate of thermal energy generation is found from Eq. 26-26: $P = iV = 210$ W. Assuming a steady rate, the thermal energy generated in 40 s is $Q = P\Delta t =$ (210 J/s)(3600 s) = $7.56 \times 10^5$ J.

53. (a) From $P = V^2/R$ we find $R = V^2/P = (120 \text{ V})^2/500 \text{ W} = 28.8\ \Omega$.

(b) Since $i = P/V$, the rate of electron transport is

$$\frac{i}{e} = \frac{P}{eV} = \frac{500 \text{ W}}{(1.60 \times 10^{-19} \text{ C})(120 \text{ V})} = 2.60 \times 10^{19} \text{ / s}.$$

54. From $P = V^2/R$, we have

$$R = (5.0 \text{ V})^2/(200 \text{ W}) = 0.125\ \Omega.$$

To meet the conditions of the problem statement, we must therefore set

$$\int_0^L 5.00x\, dx = 0.125\ \Omega$$

Thus,

$$\frac{5}{2} L^2 = 0.125 \quad \Rightarrow \quad L = 0.224 \text{ m}.$$

55. **THINK** Since the resistivity of Nichrome varies with temperature, the power dissipated through the Nichrome wire will also depend on temperature.

**EXPRESS** Let $R_H$ be the resistance at the higher temperature (800°C) and let $R_L$ be the resistance at the lower temperature (200°C). Since the potential difference is the same for the two temperatures, the power dissipated at the lower temperature is $P_L = V^2/R_L$, and the power dissipated at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now,

$$R_H = \frac{\rho_H L}{A} = \frac{\rho_0 L}{A}\left[1+\alpha(T_H - T_0)\right]$$

$$R_L = \frac{\rho_L L}{A} = \frac{\rho_0 L}{A}\left[1+\alpha(T_L - T_0)\right]$$

so that

$$R_L = R_H + \alpha R_H \Delta T,$$

where $\Delta T$ is the temperature difference: $T_L - T_H$ = –600 C° = –600 K.

**ANALYZE** Thus, the dissipation rate at 200°C is

$$P_L = \frac{R_H}{R_H + \alpha R_H \Delta T} P_H = \frac{P_H}{1+\alpha \Delta T} = \frac{500\text{ W}}{1+(4.0\times 10^{-4}\text{ / K})(-600\text{ K})} = 660\text{ W}.$$

**LEARN** Since the power dissipated is inversely proportional to $R$, at lower temperature where $R_L < R_H$, we expect a higher rate of energy dissipation: $P_L > P_H$.

56. (a) The current is

$$i = \frac{V}{R} = \frac{V}{\rho L / A} = \frac{\pi V d^2}{4\rho L} = \frac{\pi(1.20\text{ V})[(0.0400\text{ in.})(2.54\times 10^{-2}\text{m/in.})]^2}{4(1.69\times 10^{-8}\,\Omega\cdot\text{m})(33.0\text{ m})} = 1.74\text{ A}.$$

(b) The magnitude of the current density vector is

$$|\vec{J}| = \frac{i}{A} = \frac{4i}{\pi d^2} = \frac{4(1.74\text{ A})}{\pi[(0.0400\text{ in.})(2.54\times 10^{-2}\text{ m/in.})]^2} = 2.15\times 10^6\text{ A/m}^2.$$

(c) $E = V/L$ = 1.20 V/33.0 m = $3.63 \times 10^{-2}$ V/m.

(d) $P = Vi$ = (1.20 V)(1.74 A) = 2.09 W.

57. We find the current from Eq. 26-26: $i = P/V$ = 2.00 A. Then, from Eq. 26-1 (with constant current), we obtain

$$\Delta q = i\Delta t = 2.88 \times 10^4\text{ C}.$$

58. We denote the copper rod with subscript $c$ and the aluminum rod with subscript $a$.

(a) The resistance of the aluminum rod is

$$R = \rho_a \frac{L}{A} = \frac{\left(2.75\times 10^{-8}\,\Omega\cdot\text{m}\right)\left(1.3\text{m}\right)}{\left(5.2\times 10^{-3}\text{ m}\right)^2} = 1.3\times 10^{-3}\,\Omega.$$

(b) Let $R = \rho_c L/(\pi d^2/4)$ and solve for the diameter $d$ of the copper rod:

$$d = \sqrt{\frac{4\rho_c L}{\pi R}} = \sqrt{\frac{4\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(1.3\text{ m}\right)}{\pi\left(1.3\times10^{-3}\,\Omega\right)}} = 4.6\times10^{-3}\text{ m.}$$

59. (a) Since

$$\rho = \frac{RA}{L} = \frac{R(\pi d^2/4)}{L} = \frac{(1.09\times10^{-3}\Omega)\pi(5.50\times10^{-3}\text{ m})^2/4}{1.60\text{ m}} = 1.62\times10^{-8}\ \Omega\cdot\text{m},$$

the material is silver.

(b) The resistance of the round disk is

$$R = \rho\frac{L}{A} = \frac{4\rho L}{\pi d^2} = \frac{4(1.62\times10^{-8}\,\Omega\cdot\text{m})(1.00\times10^{-3}\text{m})}{\pi(2.00\times10^{-2}\,\text{m})^2} = 5.16\times10^{-8}\,\Omega.$$

60. (a) Current is the transport of charge; here it is being transported “in bulk” due to the volume rate of flow of the powder. From Chapter 14, we recall that the volume rate of flow is the product of the cross-sectional area (of the stream) and the (average) stream velocity. Thus, $i = \rho A v$ where $\rho$ is the charge per unit volume. If the cross section is that of a circle, then $i = \rho\pi R^2 v$.

(b) Recalling that a coulomb per second is an ampere, we obtain

$$i = \left(1.1\times10^{-3}\text{ C/m}^3\right)\pi\left(0.050\,\text{m}\right)^2\left(2.0\,\text{m/s}\right) = 1.7\times10^{-5}\text{ A.}$$

(c) The motion of charge is not in the same direction as the potential difference computed in problem 70 of Chapter 24. It might be useful to think of (by analogy) Eq. 7-48; there, the scalar (dot) product in $P = \vec{F}\cdot\vec{v}$ makes it clear that $P = 0$ if $\vec{F}\perp\vec{v}$. This suggests that a radial potential difference and an axial flow of charge will not together produce the needed transfer of energy (into the form of a spark).

(d) With the assumption that there is (at least) a voltage equal to that computed in problem 70 of Chapter 24, in the proper direction to enable the transference of energy (into a spark), then we use our result from that problem in Eq. 26-26:

$$P = iV = \left(1.7\times10^{-5}\text{ A}\right)\left(7.8\times10^{4}\text{ V}\right) = 1.3\text{W.}$$

(e) Recalling that a joule per second is a watt, we obtain (1.3 W)(0.20 s) = 0.27 J for the energy that can be transferred at the exit of the pipe.

(f) This result is greater than the 0.15 J needed for a spark, so we conclude that the spark was likely to have occurred at the exit of the pipe, going into the silo.

61. **THINK** The amount of charge that strikes the surface in time $\Delta t$ is given by $\Delta q = i\,\Delta t$, where $i$ is the current.

**EXPRESS** Since each alpha particle carries charge $q = +2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\Delta t}{2e}.$$

For part (b), let $N'$ be the number of particles in a length $L$ of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where $v$ is the particle speed. The current is the charge that moves through the cross section per unit time. That is,

$$i = \frac{2eN'}{t} = \frac{2eN'v}{L}.$$

Thus $N' = iL/2ev$.

**ANALYZE** (a) Substituting the values given, we have

$$N = \frac{\Delta q}{2e} = \frac{i\Delta t}{2e} = \frac{(0.25\times10^{-6}\,\text{A})(3.0\,\text{s})}{2(1.6\times10^{-19}\,\text{C})} = 2.34\times10^{12}.$$

(b) To find the particle speed, we note the kinetic energy of a particle is

$$K = 20\,\text{MeV} = (20\times10^{6}\,\text{eV})(1.60\times10^{-19}\,\text{J/eV}) = 3.2\times10^{-12}\,\text{J}.$$

Since $K = \frac{1}{2}mv^2$, the speed is $v = \sqrt{2K/m}$. The mass of an alpha particle is (very nearly) 4 times the mass of a proton, or $m = 4(1.67\times10^{-27}\text{ kg}) = 6.68\times10^{-27}$ kg, so

$$v = \sqrt{\frac{2(3.2\times10^{-12}\text{ J})}{6.68\times10^{-27}\text{ kg}}} = 3.1\times10^{7}\text{ m/s}.$$

Therefore, the number of particles in a length $L = 20$ cm of the beam is

$$N' = \frac{iL}{2ev} = \frac{(0.25\times10^{-6})(20\times10^{-2}\text{ m})}{2(1.60\times10^{-19}\text{ C})(3.1\times10^{7}\text{ m/s})} = 5.0\times10^{3}.$$

(c) We use conservation of energy, where the initial kinetic energy is zero and the final kinetic energy is 20 MeV = $3.2\times10^{-12}$ J. We note too, that the initial potential energy is

$$U_i = qV = 2eV,$$

and the final potential energy is zero. Here $V$ is the electric potential through which the particles are accelerated. Consequently, $K_f = U_i = 2eV$, which gives

$$V = \frac{K_f}{2e} = \frac{3.2\times10^{-12}\ \text{J}}{2\left(1.60\times10^{-19}\ \text{C}\right)} = 1.0\times10^{7}\ \text{V}.$$

**LEARN** By the work-kinetic energy theorem, the work done on $2.34\times10^{12}$ such alpha particles is

$$W = (2.34\times10^{12})(20\ \text{MeV}) = (2.34\times10^{12})(3.2\times10^{-12}\ \text{J}) = 7.5\ \text{J}.$$

The same result can also be obtained from

$$W = q\Delta V = (i\Delta t)\Delta V = (0.25\times10^{-6}\,\text{A})(3.0\,\text{s})(1.0\times10^{7}\ \text{V}) = 7.5\ \text{J}.$$

62. We use Eq. 26-28: $R = \dfrac{V^2}{P} = \dfrac{(200\ \text{V})^2}{3000\ \text{W}} = 13.3\,\Omega$.

63. Combining Eq. 26-28 with Eq. 26-16 demonstrates that the power is inversely proportional to the length (when the voltage is held constant, as in this case). Thus, a new length equal to 7/8 of its original value leads to

$$P = \frac{8}{7}(2.0\ \text{kW}) = 2.4\ \text{kW}.$$

64. (a) Since $P = i^2 R = J^2 A^2 R$, the current density is

$$J = \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L/A}} = \sqrt{\frac{P}{\rho LA}} = \sqrt{\frac{1.0\,\text{W}}{\pi\left(3.5\times10^{-5}\Omega\cdot\text{m}\right)\left(2.0\times10^{-2}\ \text{m}\right)\left(5.0\times10^{-3}\ \text{m}\right)^2}}$$

$$= 1.3\times10^{5}\ \text{A/m}^2.$$

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0\,\text{W}}{\pi\left(5.0\times10^{-3}\ \text{m}\right)^2\left(1.3\times10^{5}\ \text{A/m}^2\right)} = 9.4\times10^{-2}\ \text{V}.$$

65. We use $P = i^2 R = i^2\rho L/A$, or $L/A = P/i^2\rho$.

(a) The new values of $L$ and $A$ satisfy

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2\rho}\right)_{\text{new}} = \frac{30}{4^2}\left(\frac{P}{i^2\rho}\right)_{\text{old}} = \frac{30}{16}\left(\frac{L}{A}\right)_{\text{old}}.$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$, and

$$L_{\text{new}} = \sqrt{1.875}L_{\text{old}} = 1.37L_{\text{old}} \quad \Rightarrow \quad \frac{L_{\text{new}}}{L_{\text{old}}} = 1.37.$$

(b) Similarly, we note that $(LA)_{\text{new}} = (LA)_{\text{old}}$, and

$$A_{\text{new}} = \sqrt{1/1.875}A_{\text{old}} = 0.730A_{\text{old}} \quad \Rightarrow \quad \frac{A_{\text{new}}}{A_{\text{old}}} = 0.730.$$

66. The horsepower required is $P = \frac{iV}{0.80} = \frac{(10\text{A})(12\text{ V})}{(0.80)(746\text{ W/hp})} = 0.20\text{ hp}.$

67. (a) We use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V\,\Delta V$. The percentage change is roughly

$$\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%.$$

(b) A drop in $V$ causes a drop in $P$, which in turn lowers the temperature of the resistor in the coil. At a lower temperature $R$ is also decreased. Since $P \propto R^{-1}$ a decrease in $R$ will result in an increase in $P$, which partially offsets the decrease in $P$ due to the drop in $V$. Thus, the actual drop in $P$ will be smaller when the temperature dependency of the resistance is taken into consideration.

68. We use Eq. 26-17: $\rho - \rho_0 = \rho\alpha(T - T_0)$, and solve for $T$:

$$T = T_0 + \frac{1}{\alpha}\left(\frac{\rho}{\rho_0} - 1\right) = 20°\text{C} + \frac{1}{4.3\times10^{-3}/\text{K}}\left(\frac{58\,\Omega}{50\,\Omega} - 1\right) = 57°\text{C}.$$

We are assuming that $\rho/\rho_0 = R/R_0$.

69. We find the rate of energy consumption from Eq. 26-28:

$$P = \frac{V^2}{R} = \frac{(90\text{ V})^2}{400\,\Omega} = 20.3\text{ W}$$

Assuming a steady rate, the energy consumed is $(20.3\text{ J/s})(2.00 \times 3600\text{ s}) = 1.46 \times 10^5$ J.

70. (a) The potential difference between the two ends of the caterpillar is

$$V = iR = i\rho\frac{L}{A} = \frac{(12\ \text{A})(1.69\times10^{-8}\ \Omega\cdot\text{m})(4.0\times10^{-2}\ \text{m})}{\pi(5.2\times10^{-3}\ \text{m}/2)^2} = 3.8\times10^{-4}\ \text{V}.$$

(b) Since it moves in the direction of the electron drift, which is against the direction of the current, its tail is negative compared to its head.

(c) The time of travel relates to the drift speed:

$$t = \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi L d^2 ne}{4i} = \frac{\pi(1.0\times10^{-2}\ \text{m})(5.2\times10^{-3}\ \text{m})^2(8.47\times10^{28}/\text{m}^3)(1.60\times10^{-19}\ \text{C})}{4(12\ \text{A})}$$
$$= 238\,\text{s} = 3\,\text{min}\,58\,\text{s}.$$

71. **THINK** The resistance of copper increases with temperature.

**EXPRESS** According to Eq. 26-17, the resistance of copper at temperature $T$ can be written as

$$R = \frac{\rho L}{A} = \frac{\rho_0 L}{A}[1+\alpha(T-T_0)]$$

where $T_0 = 20\ °\text{C}$ is the reference temperature. Thus, the resistance is $R_0 = \rho_0 L/A$ at $T_0 = 20\ °\text{C}.$ The temperature at which $R = 2R_0$ (or equivalently, $\rho = 2\rho_0$) can be found by solving

$$2 = \frac{R}{R_0} = 1+\alpha(T-T_0) \;\;\Rightarrow\;\; \alpha(T-T_0) = 1.$$

**ANALYZE** (a) From the above equation, we find the temperature to be

$$T = T_0 + \frac{1}{\alpha} = 20°\text{C} + \frac{1}{4.3\times10^{-3}/\text{K}} \approx 250°\text{C}.$$

(b) Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of $\alpha$ used in this calculation is not inconsistent with the other units involved.

**LEARN** It is worth noting that our result agrees well with Fig. 26-10.

72. Since 100 cm = 1 m, then $10^4\ \text{cm}^2 = 1\ \text{m}^2$. Thus,

$$R = \frac{\rho L}{A} = \frac{(3.00\times10^{-7}\,\Omega\cdot\text{m})(10.0\times10^3\ \text{m})}{56.0\times10^{-4}\ \text{m}^2} = 0.536\ \Omega.$$

73. The rate at which heat is being supplied is

$$P = iV = (5.2\text{ A})(12\text{ V}) = 62.4\text{ W}.$$

Considered on a one-second time frame, this means 62.4 J of heat are absorbed by the liquid each second. Using Eq. 18-16, we find the heat of transformation to be

$$L = \frac{Q}{m} = \frac{62.4\text{ J}}{21\times10^{-6}\text{ kg}} = 3.0\times10^{6}\text{ J/kg}.$$

74. We find the drift speed from Eq. 26-7:

$$v_d = \frac{|\vec{J}|}{ne} = \frac{2.0\times10^{6}\text{ A/m}^2}{(8.49\times10^{28}/\text{m}^3)(1.6\times10^{-19}\text{C})} = 1.47\times10^{-4}\text{ m/s}.$$

At this (average) rate, the time required to travel $L$ = 5.0 m is

$$t = \frac{L}{v_d} = \frac{5.0\text{ m}}{1.47\times10^{-4}\text{ m/s}} = 3.4\times10^{4}\text{ s}.$$

75. The power dissipated is given by the product of the current and the potential difference:

$$P = iV = (7.0\times10^{-3}\text{ A})(80\times10^{3}\text{ V}) = 560\text{ W}.$$

76. (a) The current is $4.2\times10^{18}\,e$ divided by 1 second. Using $e = 1.60\times10^{-19}$ C we obtain 0.67 A for the current.

(b) Since the electric field points away from the positive terminal (high potential) and toward the negative terminal (low potential), then the current density vector (by Eq. 26-11) must also point toward the negative terminal.

77. For the temperature of the gas to remain unchanged, the rate of the thermal energy dissipated through the resistor, $P_R = i^2R$, must be equal to the rate of increase of mechanical energy of the piston, $P_m = mg(dh/dt) = mgv$. Thus,

$$i^2R = mgv \;\;\Rightarrow\;\; v = \frac{i^2R}{mg} = \frac{(0.240\text{ A})^2(550\ \Omega)}{(12\text{ kg})(9.8\text{ m/s}^2)} = 0.27\text{ m/s}.$$

78. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using $\sigma$ for the charge per unit area and $w$ for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma vw$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100\times10^{-6}\text{ A}}{(30\text{ m/s})(50\times10^{-2}\text{ m})} = 6.7\times10^{-6}\text{ C/m}^2.$$

79. (a) The total current density is equal to the sum of the contributions from the alpha particles and the electron. Using the general expression $J = nqv,$ and noting that $n_e = 2n_\alpha$ (two electrons for each $\alpha$ particle), we have

$$\begin{aligned} J_{\text{total}} &= n_\alpha q_\alpha v_\alpha + n_e q_e v_e = n_\alpha (2e) v_\alpha + (2n_\alpha)(e) v_e = 2n_\alpha e(v_\alpha + v_e) \\ &= 2(2.80\times 10^{21}/\text{m}^3)(1.6\times 10^{-19}\text{ C})(88\text{ m/s} + 25\text{ m/s}) \\ &= 1.01\times 10^5\text{ A/m}^2 = 10.1\text{ A/cm}^2 \end{aligned}$$

(b) The direction of the current is eastward (same as the motion of the alpha particles).

80. (a) Let $\Delta T$ be the change in temperature and $\kappa$ be the coefficient of linear expansion for copper. Then $\Delta L = \kappa L\ \Delta T$ and

$$\frac{\Delta L}{L} = \kappa\Delta T = (1.7\times 10^{-5}/\text{K})(1.0°\text{C}) = 1.7\times 10^{-5}.$$

This is equivalent to 0.0017%. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of $\kappa$ used in this calculation is not inconsistent with the other units involved.

(b) Incorporating a factor of 2 for the two-dimensional nature of $A$, the fractional change in area is

$$\frac{\Delta A}{A} = 2\kappa\Delta T = 2(1.7\times 10^{-5}/\text{K})(1.0°\text{C}) = 3.4\times 10^{-5}$$

which is 0.0034%.

(c) For small changes in the resistivity $\rho$, length $L$, and area $A$ of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho}\Delta\rho + \frac{\partial R}{\partial L}\Delta L + \frac{\partial R}{\partial A}\Delta A.$$

Since $R = \rho L/A$, $\partial R/\partial\rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta\rho/\rho = \alpha\Delta T$, where $\alpha$ is the temperature coefficient of resistivity for copper ($4.3\times 10^{-3}$/K = $4.3\times 10^{-3}$/C°, according to Table 27-1). Thus,

$$\begin{aligned} \frac{\Delta R}{R} &= \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} = (\alpha + \kappa - 2\kappa)\Delta T = (\alpha - \kappa)\Delta T \\ &= (4.3\times 10^{-3}/\text{C°} - 1.7\times 10^{-5}/\text{C°})(1.0\text{ C°}) = 4.3\times 10^{-3}. \end{aligned}$$

This is 0.43%, which we note (for the purposes of the next part) is primarily determined by the $\Delta\rho/\rho$ term in the above calculation.

(d) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.

81. (a) Using $i = dq/dt = e(dN/dt)$, we obtain

$$\frac{dN}{dt} = \frac{i}{e} = \frac{15\times10^{-6}\,\text{A}}{1.6\times10^{-19}\ \text{C}} = 9.4\times10^{13}\,/\,\text{s}.$$

(b) The rate of thermal energy production is

$$P = \frac{dU}{dt} = \left(\frac{dN}{dt}\right)U_1 = (9.4\times10^{13}\,/\,\text{s})(16\ \text{MeV})\left(\frac{1.6\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = 240\ \text{W}.$$

82. (a) The charge $q$ that flows past any cross section of the beam in time $\Delta t$ is given by $q = i\Delta t$, and the number of electrons is $N = q/e = (i/e)\,\Delta t$. This is the number of electrons that are accelerated. Thus,

$$N = \frac{(0.50\ \text{A})(0.10\times10^{-6}\ \text{s})}{1.60\times10^{-19}\ \text{C}} = 3.1\times10^{11}.$$

(b) Over a long time $t$ the total charge is $Q = nqt$, where $n$ is the number of pulses per unit time and $q$ is the charge in one pulse. The average current is given by $i_{\text{avg}} = Q/t = nq$. Now $q = i\Delta t = (0.50\ \text{A})\ (0.10 \times 10^{-6}\ \text{s}) = 5.0 \times 10^{-8}$ C, so

$$i_{\text{avg}} = (500\,/\,\text{s})(5.0\times10^{-8}\ \text{C}) = 2.5\times10^{-5}\ \text{A}.$$

(c) The accelerating potential difference is $V = K/e$, where $K$ is the final kinetic energy of an electron. Since $K = 50$ MeV, the accelerating potential is $V = 50\ \text{kV} = 5.0 \times 10^{7}$ V. During a pulse the power output is

$$P = iV = (0.50\ \text{A})(5.0\times10^{7}\ \text{V}) = 2.5\times10^{7}\ \text{W}.$$

This is the peak power. The average power is

$$P_{\text{avg}} = i_{\text{avg}}V = (2.5\times10^{-5}\ \text{A})(5.0\times10^{7}\ \text{V}) = 1.3\times10^{3}\ \text{W}.$$

83. With the voltage reduced by 6.00% while resistance remains unchanged, the current through the heating element also decreases by 6.00% ($i' = 0.94i$). The power delivered is now

$$P' = i'^2R = (0.94i)^2R = 0.884i^2R = 0.884P,$$

where $P = i^2 R$ is the power delivered to the heating element under normal circumstance. Since the energy required to heat the water remains the same in both cases, $P\Delta t = P'\Delta t'$, the time required becomes

$$\Delta t' = \left(\frac{P}{P'}\right)\Delta t = \frac{100 \text{ min}}{0.884} = 113 \text{ min}.$$

84. (a) The mass of the water is $m = \rho V = (1000 \text{ kg/m}^3)(2.0 \text{ L})(10^{-3}\text{m}^3/\text{L}) = 2.00 \text{ kg}$. The energy required to raise the water temperature to the boiling point is

$$Q_1 = mc\Delta T = (2.00 \text{ kg})(4187 \text{ J/kg}\cdot\text{C°})(100\text{ °C} - 20\text{ °C}) = 6.70\times10^5 \text{ J}.$$

With $P$ = 400 W at 80% efficiency, we find the time needed to be

$$\Delta t_1 = \frac{Q_1}{P_{\text{eff}}} = \frac{6.70\times10^5 \text{ J}}{(0.80)(400 \text{ W})} = 2.09\times10^3 \text{s} \approx 35 \text{ min}.$$

(b) The energy required to vaporize half of the water is

$$Q_2 = L_V(m/2) = (2.256\times10^6 \text{ J/kg})(2.00 \text{ kg}/2) = 2.256\times10^6 \text{ J}.$$

Thus, the additional time elapsed is

$$\Delta t_2 = \frac{Q_2}{P_{\text{eff}}} = \frac{2.256\times10^6 \text{ J}}{(0.80)(400 \text{ W})} = 7.05\times10^3 \text{s} \approx 118 \text{ min},$$

or about 1.96 h.

85. (a) At $t$ = 0.500 s, the charge on the capacitor is

$$q = CV = C(6.00 + 4.00t - 2.00t^2) = (30\times10^{-6}\text{F})\left[6.00 + 4.00(0.500) - 2.00(0.500)^2\right]$$
$$= 225\times10^{-6}\text{C} = 225\ \mu\text{C}.$$

(b) The current flowing into the capacitor is

$$i = \frac{dq}{dt} = C\frac{dV}{dt} = C\frac{d}{dt}\left(6.00 + 4.00t - 2.00t^2\right) = C(4.00 - 4.00t)$$
$$= (30\times10^{-6}\text{F})\left[4.00 - 4.00(0.500)\right] = 60.0\times10^{-6}\text{A} = 60.0\ \mu\text{A}.$$

(c) The corresponding power output is

$$P = iV = (60.0\times10^{-6}\text{A})\left[6.00 + 4.00(0.500) - 2.00(0.500)^2\right] = 4.50\times10^{-4}\text{W}.$$

# Chapter 27

1. **THINK** The circuit consists of two batteries and two resistors. We apply Kirchhoff's loop rule to solve for the current.

**EXPRESS** Let $i$ be the current in the circuit and take it to be positive if it is to the left in $R_1$. Kirchhoff's loop rule gives

$$\varepsilon_1 - iR_2 - iR_1 - \varepsilon_2 = 0.$$

For parts (b) and (c), we note that if $i$ is the current in a resistor $R$, then the power dissipated by that resistor is given by $P = i^2 R$.

**ANALYZE** (a) We solve for $i$:

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2} = \frac{12\ \text{V} - 6.0\ \text{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \text{A}.$$

A positive value is obtained, so the current is counterclockwise around the circuit.

(b) For $R_1$, the dissipation rate is $P_1 = i^2 R_1 = (0.50\ \text{A})^2(4.0\ \Omega) = 1.0$ W.

(c) For $R_2$, the rate is $P_2 = i^2 R_2 = (0.50\ \text{A})^2\,(8.0\ \Omega) = 2.0$ W.

If $i$ is the current in a battery with emf $\varepsilon$, then the battery supplies energy at the rate $P = i\varepsilon$ provided the current and emf are in the same direction. On the other hand, the battery absorbs energy at the rate $P = i\varepsilon$ if the current and emf are in opposite directions.

(d) For $\varepsilon_1$, $P_1 = i\varepsilon_1 = (0.50\ \text{A})(12\ \text{V}) = 6.0$ W.

(e) For $\varepsilon_2$, $P_2 = i\varepsilon_2 = (0.50\ \text{A})(6.0\ \text{V}) = 3.0$ W.

(f) In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging.

(g) The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

**LEARN** Multiplying the equation obtained from Kirchhoff's loop rule by $idt$ leads to the "energy-method" equation discussed in Section 27-4:

$$i\varepsilon_1 dt - i^2 R_1 dt - i^2 R_2 dt - i\varepsilon_2 dt = 0.$$

The first term represents the rate of work done by battery 1, the second and third terms the thermal energies that appear in resistors $R_1$ and $R_2$, and the last term the work done *on* battery 2.

2. The current in the circuit is

$$i = (150 \text{ V} - 50 \text{ V})/(3.0\ \Omega + 2.0\ \Omega) = 20 \text{ A}.$$

So from $V_Q + 150 \text{ V} - (2.0\ \Omega)i = V_P$, we get

$$V_Q = 100 \text{ V} + (2.0\ \Omega)(20 \text{ A}) - 150 \text{ V} = -10 \text{ V}.$$

3. (a) The potential difference is $V = \varepsilon + ir = 12 \text{ V} + (50 \text{ A})(0.040\ \Omega) = 14 \text{ V}$.

(b) $P = i^2 r = (50 \text{ A})^2 (0.040\ \Omega) = 1.0\times10^2 \text{ W}$.

(c) $P' = iV = (50 \text{ A})(12 \text{ V}) = 6.0\times10^2 \text{ W}$.

(d) In this case $V = \varepsilon - ir = 12 \text{ V} - (50 \text{ A})(0.040\ \Omega) = 10 \text{ V}$.

(e) $P_r = i^2 r = (50 \text{ A})^2 (0.040\ \Omega) = 1.0\times10^2 \text{ W}$.

4. (a) The loop rule leads to a voltage-drop across resistor 3 equal to 5.0 V (since the total drop along the upper branch must be 12 V). The current there is consequently $i = (5.0 \text{ V})/(200\ \Omega) = 25 \text{ mA}$. Then the resistance of resistor 1 must be $(2.0 \text{ V})/i = 80\ \Omega$.

(b) Resistor 2 has the same voltage-drop as resistor 3; its resistance is 200 Ω.

5. The chemical energy of the battery is reduced by $\Delta E = q\varepsilon$, where $q$ is the charge that passes through in time $\Delta t = 6.0$ min, and $\varepsilon$ is the emf of the battery. If $i$ is the current, then $q = i\,\Delta t$ and

$$\Delta E = i\varepsilon\,\Delta t = (5.0 \text{ A})(6.0 \text{ V})(6.0 \text{ min})(60 \text{ s/min}) = 1.1 \times 10^4 \text{ J}.$$

We note the conversion of time from minutes to seconds.

6. (a) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/2.0 \text{ W} \cdot \text{h})(\$0.80) = \$3.2 \times 10^2$.

(b) The cost is $(100 \text{ W} \cdot 8.0 \text{ h}/10^3 \text{ W} \cdot \text{h})(\$0.06) = \$0.048 = 4.8$ cents.

7. (a) The energy transferred is

$$U = Pt = \frac{\varepsilon^2 t}{r+R} = \frac{(2.0\text{ V})^2(2.0\text{ min})(60\text{ s/min})}{1.0\Omega + 5.0\Omega} = 80\text{ J}.$$

(b) The amount of thermal energy generated is

$$U' = i^2 Rt = \left(\frac{\varepsilon}{r+R}\right)^2 Rt = \left(\frac{2.0\text{ V}}{1.0\Omega + 5.0\Omega}\right)^2 (5.0\Omega)(2.0\text{ min})(60\text{ s/min}) = 67\text{ J}.$$

(c) The difference between *U* and *U'*, which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

8. If *P* is the rate at which the battery delivers energy and $\Delta t$ is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time $\Delta t$. If *q* is the charge that passes through the battery in time $\Delta t$ and $\varepsilon$ is the emf of the battery, then $\Delta E = q\varepsilon$. Equating the two expressions for $\Delta E$ and solving for $\Delta t$, we obtain

$$\Delta t = \frac{q\varepsilon}{P} = \frac{(120\text{ A}\cdot\text{h})(12.0\text{ V})}{100\text{ W}} = 14.4\text{ h}.$$

9. (a) The work done by the battery relates to the potential energy change:

$$q\Delta V = eV = e(12.0\text{ V}) = 12.0\text{ eV}.$$

(b) $P = iV = neV = (3.40 \times 10^{18}\text{/s})(1.60 \times 10^{-19}\text{ C})(12.0\text{ V}) = 6.53\text{ W}$.

10. (a) We solve $i = (\varepsilon_2 - \varepsilon_1)/(r_1 + r_2 + R)$ for *R*:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{i} - r_1 - r_2 = \frac{3.0\text{ V} - 2.0\text{ V}}{1.0\times 10^{-3}\text{ A}} - 3.0\Omega - 3.0\Omega = 9.9\times 10^2\ \Omega.$$

(b) $P = i^2R = (1.0 \times 10^{-3}\text{ A})^2(9.9 \times 10^2\ \Omega) = 9.9 \times 10^{-4}\text{ W}$.

11. **THINK** As shown in Fig. 27-29, the circuit contains an emf device *X*. How it is connected to the rest of the circuit can be deduced from the power dissipated and the potential drop across it.

**EXPRESS** The power absorbed by a circuit element is given by $P = i\Delta V$, where *i* is the current and $\Delta V$ is the potential difference across the element. The end-to-end potential difference is given by

$$V_A - V_B = +iR + \varepsilon,$$

where $\varepsilon$ is the emf of device *X* and is taken to be positive if it is to the left in the diagram.

**ANALYZE** (a) The potential difference between *A* and *B* is

$$\Delta V = \frac{P}{i} = \frac{50\text{ W}}{1.0\text{ A}} = 50\text{ V}.$$

Since the energy of the charge decreases, point $A$ is at a higher potential than point $B$; that is, $V_A - V_B = 50$ V.

(b) From the equation above, we find the emf of device $X$ to be

$$\varepsilon = V_A - V_B - iR = 50\text{ V} - (1.0\text{ A})(2.0\ \Omega) = 48\text{ V}.$$

(c) A positive value was obtained for $\varepsilon$, so it is toward the left. The negative terminal is at $B$.

**LEARN** Writing the potential difference as $V_A - iR - \varepsilon = V_B$, we see that our result is consistent with the resistance and emf rules. Namely, starting at point $A$, the change in potential is $-iR$ for a move through a resistance $R$ in the direction of the current, and the change in potential is $-\varepsilon$ for a move through an emf device in the opposite direction of the emf arrow (which points from negative to positive terminals).

12. (a) For each wire, $R_{\text{wire}} = \rho L/A$ where $A = \pi r^2$. Consequently, we have

$$R_{\text{wire}} = (1.69 \times 10^{-8}\ \Omega\cdot\text{m})(0.200\text{ m})/\pi(0.00100\text{ m})^2 = 0.0011\ \Omega.$$

The total resistive load on the battery is therefore

$$R_{\text{tot}} = 2R_{\text{wire}} + R = 2(0.0011\ \Omega) + 6.00\ \Omega = 6.0022\ \Omega.$$

Dividing this into the battery emf gives the current

$$i = \frac{\varepsilon}{R_{\text{tot}}} = \frac{12.0\text{ V}}{6.0022\Omega} = 1.9993\text{ A}.$$

The voltage across the $R = 6.00\ \Omega$ resistor is therefore

$$V = iR = (1.9993\text{ A})(6.00\ \Omega) = 11.996\text{ V} \approx 12.0\text{ V}.$$

(b) Similarly, we find the voltage-drop across each wire to be

$$V_{\text{wire}} = iR_{\text{wire}} = (1.9993\text{ A})(0.0011\ \Omega) = 2.15\text{ mV}.$$

(c) $P = i^2R = (1.9993\text{ A})(6.00\ \Omega)^2 = 23.98\text{ W} \approx 24.0\text{ W}.$

(d) Similarly, we find the power dissipated in each wire to be 4.30 mW.

13. (a) We denote $L$ = 10 km and $\alpha$ = 13 Ω/km. Measured from the east end we have

$$R_1 = 100\ \Omega = 2\alpha(L - x) + R,$$

and measured from the west end $R_2 = 200\ \Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4(13\,\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\,\text{km}.$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - (13\,\Omega/\text{km})(10\,\text{km}) = 20\,\Omega\,.$$

14. (a) Here we denote the battery emf's as $V_1$ and $V_2$. The loop rule gives

$$V_2 - ir_2 + V_1 - ir_1 - iR = 0 \;\;\Rightarrow\;\; i = \frac{V_2 + V_1}{r_1 + r_2 + R}\;.$$

The terminal voltage of battery 1 is $V_{1T}$ and (see Fig. 27-4(a)) is easily seen to be equal to $V_1 - ir_1$; similarly for battery 2. Thus,

$$V_{1T} = V_1 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R},\;\; V_{2T} = V_2 - \frac{r_1(V_2 + V_1)}{r_1 + r_2 + R}\;.$$

The problem tells us that $V_1$ and $V_2$ each equal 1.20 V. From the graph in Fig. 27-32(b) we see that $V_{2T}$ = 0 and $V_{1T}$ = 0.40 V for $R$ = 0.10 Ω. This supplies us (in view of the above relations for terminal voltages) with simultaneous equations, which, when solved, lead to $r_1$ = 0.20 Ω.

(b) The simultaneous solution also gives $r_2$ = 0.30 Ω.

15. Let the emf be $V$. Then $V = iR = i'(R + R')$, where $i$ = 5.0 A, $i'$ = 4.0 A, and $R'$ = 2.0 Ω. We solve for $R$:

$$R = \frac{i'R'}{i - i'} = \frac{(4.0\text{ A})(2.0\,\Omega)}{5.0\text{ A} - 4.0\text{ A}} = 8.0\ \Omega.$$

16. (a) Let the emf of the solar cell be $\varepsilon$ and the output voltage be $V$. Thus,

$$V = \varepsilon - ir = \varepsilon - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get

$$0.10\text{ V} = \varepsilon - (0.10\text{ V}/500\ \Omega)r$$
$$0.15\text{ V} = \varepsilon - (0.15\text{ V}/1000\ \Omega)r.$$

We solve for $\varepsilon$ and $r$.

(a) $r = 1.0\times10^3\ \Omega$.

(b) $\varepsilon = 0.30$ V.

(c) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\text{V}}{(1000\Omega)\left(5.0\,\text{cm}^2\right)\left(2.0\times10^{-3}\,\text{W/cm}^2\right)} = 2.3\times10^{-3} = 0.23\%.$$

17. **THINK** A zero terminal-to-terminal potential difference implies that the emf of the battery is equal to the voltage drop across its internal resistance, that is, $\varepsilon = ir$.

**EXPRESS** To be as general as possible, we refer to the individual emf's as $\varepsilon_1$ and $\varepsilon_2$ and wait until the latter steps to equate them ($\varepsilon_1 = \varepsilon_2 = \varepsilon$). The batteries are placed in series in such a way that their voltages add; that is, they do not "oppose" each other. The total resistance in the circuit is therefore $R_{\text{total}} = R + r_1 + r_2$ (where the problem tells us $r_1 > r_2$), and the "net emf" in the circuit is $\varepsilon_1 + \varepsilon_2$. Since battery 1 has the higher internal resistance, it is the one capable of having a zero terminal voltage, as the computation in part (a) shows.

**ANALYZE** (a) The current in the circuit is

$$i = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R},$$

and the requirement of zero terminal voltage leads to $\varepsilon_1 = ir_1$, or

$$R = \frac{\varepsilon_2 r_1 - \varepsilon_1 r_2}{\varepsilon_1} = \frac{(12.0\text{ V})(0.016\,\Omega) - (12.0\text{ V})(0.012\,\Omega)}{12.0\text{ V}} = 0.0040\ \Omega.$$

Note that $R = r_1 - r_2$ when we set $\varepsilon_1 = \varepsilon_2$.

(b) As mentioned above, this occurs in battery 1.

**LEARN** If we assume the potential difference across battery 2 to be zero and repeat the calculation above, we would find $R = r_2 - r_1 < 0$, which is physically impossible. Thus, only the potential difference across the battery with the larger internal resistance can be made zero with suitable choice of $R$.

18. The currents $i_1$, $i_2$ and $i_3$ are obtained from Eqs. 27-18 through 27-20:

$$i_1 = \frac{\varepsilon_1(R_2+R_3)-\varepsilon_2 R_3}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\text{V})(10\ \Omega+5.0\Omega)-(1.0\text{ V})(5.0\ \Omega)}{(10\ \Omega)(10\ \Omega)+(10\ \Omega)(5.0\ \Omega)+(10\Omega)(5.0\ \Omega)} = 0.275\text{ A},$$

$$i_2 = \frac{\varepsilon_1 R_3-\varepsilon_2(R_1+R_2)}{R_1R_2+R_2R_3+R_1R_3} = \frac{(4.0\text{ V})(5.0\Omega)-(1.0\text{ V})(10\Omega+5.0\ \Omega)}{(10\Omega)(10\ \Omega)+(10\Omega)(5.0\Omega)+(10\Omega)(5.0\ \Omega)} = 0.025\text{ A},$$

$$i_3 = i_2 - i_1 = 0.025\text{A} - 0.275\text{A} = -0.250\text{A}.$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get

$$V_d - V_c = i_2R_2 = (0.0250\text{ A})\ (10\ \Omega) = +0.25\text{ V};$$

from $V_d + i_3R_3 + \varepsilon_2 = V_c$ we get

$$V_d - V_c = i_3R_3 - \varepsilon_2 = -(-0.250\text{ A})\ (5.0\ \Omega) - 1.0\text{ V} = +0.25\text{ V}.$$

19. (a) Since $R_{\text{eq}} < R$, the two resistors ($R$ = 12.0 Ω and $R_x$) must be connected in parallel:

$$R_{\text{eq}} = 3.00\,\Omega = \frac{R_x R}{R+R_x} = \frac{R_x\left(12.0\Omega\right)}{12.0\Omega + R_x}.$$

We solve for $R_x$: $R_x = R_{\text{eq}}R/(R - R_{\text{eq}})$ = (3.00 Ω)(12.0 Ω)/(12.0 Ω – 3.00 Ω) = 4.00 Ω.

(b) As stated above, the resistors must be connected in parallel.

20. Let the resistances of the two resistors be $R_1$ and $R_2$, with $R_1 < R_2$. From the statements of the problem, we have

$$R_1R_2/(R_1 + R_2) = 3.0\ \Omega \text{ and } R_1 + R_2 = 16\ \Omega.$$

So $R_1$ and $R_2$ must be 4.0 Ω and 12 Ω, respectively.

(a) The smaller resistance is $R_1$ = 4.0 Ω.

(b) The larger resistance is $R_2$ = 12 Ω.

21. The potential difference across each resistor is $V$ = 25.0 V. Since the resistors are identical, the current in each one is

$$i = V/R = (25.0\text{ V})/(18.0\ \Omega) = 1.39\text{ A}.$$

The total current through the battery is then $i_{total}$ = 4(1.39 A) = 5.56 A. One might alternatively use the idea of equivalent resistance; for four identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{eq}} = \sum \frac{1}{R} = \frac{4}{R}.$$

When a potential difference of 25.0 V is applied to the equivalent resistor, the current through it is the same as the total current through the four resistors in parallel. Thus

$$i_{total} = V/R_{eq} = 4V/R = 4(25.0 \text{ V})/(18.0\ \Omega) = 5.56 \text{ A}.$$

22. (a) $R_{eq}$ ($FH$) = (10.0 Ω)(10.0 Ω)(5.00 Ω)/[(10.0 Ω)(10.0 Ω) + 2(10.0 Ω)(5.00 Ω)] = 2.50 Ω.

(b) $R_{eq}$ ($FG$) = (5.00 Ω) $R$/($R$ + 5.00 Ω), where

$$R = 5.00\ \Omega + (5.00\ \Omega)(10.0\ \Omega)/(5.00\ \Omega + 10.0\ \Omega) = 8.33\ \Omega.$$

So $R_{eq}$ ($FG$) = (5.00 Ω)(8.33 Ω)/(5.00 Ω + 8.33 Ω) = 3.13 Ω.

23. Let $i_1$ be the current in $R_1$ and take it to be positive if it is to the right. Let $i_2$ be the current in $R_2$ and take it to be positive if it is upward.

(a) When the loop rule is applied to the lower loop, the result is

$$\varepsilon_2 - i_1 R_1 = 0\,.$$

The equation yields

$$i_1 = \frac{\varepsilon_2}{R_1} = \frac{5.0 \text{ V}}{100\,\Omega} = 0.050 \text{ A}.$$

(b) When it is applied to the upper loop, the result is

$$\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - i_2 R_2 = 0\,.$$

The equation gives

$$i_2 = \frac{\varepsilon_1 - \varepsilon_2 - \varepsilon_3}{R_2} = \frac{6.0 \text{ V} - 5.0 \text{ V} - 4.0 \text{ V}}{50\,\Omega} = -0.060 \text{ A}\,,$$

or $|\,i_2\,| = 0.060$ A. The negative sign indicates that the current in $R_2$ is actually downward.

(c) If $V_b$ is the potential at point $b$, then the potential at point $a$ is $V_a = V_b + \varepsilon_3 + \varepsilon_2$, so

$$V_a - V_b = \varepsilon_3 + \varepsilon_2 = 4.0 \text{ V} + 5.0 \text{ V} = 9.0 \text{ V}.$$

24. We note that two resistors in parallel, $R_1$ and $R_2$, are equivalent to

$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \; R_{12} = \frac{R_1 R_2}{R_1 + R_2}.$$

This situation consists of a parallel pair that are then in series with a single $R_3$ = 2.50 Ω resistor. Thus, the situation has an equivalent resistance of

$$R_{eq} = R_3 + R_{12} = 2.50\Omega + \frac{(4.00\Omega)(4.00\Omega)}{4.00\Omega + 4.00\Omega} = 4.50\Omega.$$

25. **THINK** The resistance of a copper wire varies with its cross-sectional area, or its diameter.

**EXPRESS** Let $r$ be the resistance of each of the narrow wires. Since they are in parallel the equivalent resistance $R_{eq}$ of the composite is given by

$$\frac{1}{R_{eq}} = \frac{9}{r},$$

or $R_{eq} = r/9$. Now each thin wire has a resistance $r = 4\rho\ell/\pi d^2$, where $\rho$ is the resistivity of copper, and $A = \pi d^2/4$ is the cross-sectional area of a single thin wire. On the other hand, the resistance of the thick wire of diameter $D$ is $R = 4\rho\ell/\pi D^2$, where the cross-sectional area is $\pi D^2/4$.

**ANALYZE** If the single thick wire is to have the same resistance as the composite of 9 thin wires, $R = R_{eq}$, then

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2}.$$

Solving for $D$, we obtain $D = 3d$.

**LEARN** The equivalent resistance $R_{eq}$ is smaller than $r$ by a factor of 9. Since $r \sim 1/A \sim 1/d^2$, increasing the diameter of the wire threefold will also reduce the resistance by a factor of 9.

26. The part of $R_0$ connected in parallel with $R$ is given by $R_1 = R_0 x/L$, where $L$ = 10 cm. The voltage difference across $R$ is then $V_R = \varepsilon R'/R_{eq}$, where $R' = RR_1/(R + R_1)$ and

$$R_{eq} = R_0(1 - x/L) + R'.$$

Thus,

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\varepsilon RR_1/(R+R_1)}{R_0(1-x/L)+RR_1/(R+R_1)}\right)^2 = \frac{100R(\varepsilon x/R_0)^2}{(100R/R_0 + 10x - x^2)^2},$$

where $x$ is measured in cm.

27. Since the potential differences across the two paths are the same, $V_1 = V_2$ ($V_1$ for the left path, and $V_2$ for the right path), we have $i_1 R_1 = i_2 R_2$, where $i = i_1 + i_2 = 5000\text{ A}$. With $R = \rho L / A$ (see Eq. 26-16), the above equation can be rewritten as

$$i_1 d = i_2 h \quad \Rightarrow \quad i_2 = i_1 (d/h).$$

With $d/h = 0.400$, we get $i_1 = 3571\text{ A}$ and $i_2 = 1429\text{ A}$. Thus, the current through the person is $i_1 = 3571\text{ A}$, or approximately $3.6\text{ kA}$.

28. Line 1 has slope $R_1$ = 6.0 kΩ. Line 2 has slope $R_2$ = 4.0 kΩ. Line 3 has slope $R_3$ = 2.0 kΩ. The parallel pair equivalence is $R_{12} = R_1 R_2/(R_1+R_2)$ = 2.4 kΩ. That in series with $R_3$ gives an equivalence of

$$R_{123} = R_{12} + R_3 = 2.4\text{ k}\Omega + 2.0\text{ k}\Omega = 4.4\text{ k}\Omega.$$

The current through the battery is therefore $i = \varepsilon / R_{123} =$ (6 V)/(4.4 kΩ) and the voltage drop across $R_3$ is (6 V)(2 kΩ)/(4.4 kΩ) = 2.73 V. Subtracting this (because of the loop rule) from the battery voltage leaves us with the voltage across $R_2$. Then Ohm's law gives the current through $R_2$: (6 V – 2.73 V)/(4 kΩ) = 0.82 mA.

29. (a) The parallel set of three identical $R_2$ = 18 Ω resistors reduce to $R$ = 6.0 Ω, which is now in series with the $R_1$ = 6.0 Ω resistor at the top right, so that the total resistive load across the battery is $R' = R_1 + R$ = 12 Ω. Thus, the current through $R'$ is (12V)/$R'$ = 1.0 A, which is the current through $R$. By symmetry, we see one-third of that passes through any one of those 18 Ω resistors; therefore, $i_1$ = 0.333 A.

(b) The direction of $i_1$ is clearly rightward.

(c) We use Eq. 26-27: $P = i^2 R'$ = (1.0 A)$^2$(12 Ω) = 12 W. Thus, in 60 s, the energy dissipated is (12 J/s)(60 s) = 720 J.

30. Using the junction rule ($i_3 = i_1 + i_2$) we write two loop rule equations:

$$10.0\text{ V} - i_1 R_1 - (i_1 + i_2) R_3 = 0$$

$$5.00\text{ V} - i_2 R_2 - (i_1 + i_2) R_3 = 0.$$

(a) Solving, we find $i_2$ = 0, and

(b) $i_3 = i_1 + i_2$ = 1.25 A (downward, as was assumed in writing the equations as we did).

31. **THINK** This problem involves a multi-loop circuit. We first simplify the circuit by finding the equivalent resistance. We then apply Kirchhoff's loop rule to calculate the current in the loop, and the potentials at various points in the circuit.

**EXPRESS** We first reduce the parallel pair of identical 2.0-Ω resistors (on the right side) to $R' = 1.0\ \Omega$, and we reduce the series pair of identical 2.0-Ω resistors (on the upper left side) to $R'' = 4.0\ \Omega$. With $R$ denoting the 2.0-Ω resistor at the bottom (between $V_2$ and $V_1$), we now have three resistors in series which are equivalent to

$$R_{eq} = R + R' + R'' = 7.0\ \Omega$$

across which the voltage is $\varepsilon_2 - \varepsilon_1$ = 7.0 V (by the loop rule, this is 12 V – 5.0 V), implying that the current is

$$i = \frac{\varepsilon_2 - \varepsilon_1}{R_{eq}} = \frac{7.0\text{ V}}{7.0\ \Omega} = 1.0\text{ A}.$$

The direction of $i$ is upward in the right-hand emf device. Knowing $i$ allows us to solve for $V_1$ and $V_2$.

**ANALYZE** (a) The voltage across $R'$ is (1.0 A)(1.0 Ω) = 1.0 V, which means that (examining the right side of the circuit) the voltage difference between *ground* and $V_1$ is 12 V – 1.0 V = 11 V. Noting the orientation of the battery, we conclude that $V_1 = -11\text{ V}$.

(b) The voltage across $R''$ is (1.0 A)(4.0 Ω) = 4.0 V, which means that (examining the left side of the circuit) the voltage difference between *ground* and $V_2$ is 5.0 V + 4.0 V = 9.0 V. Noting the orientation of the battery, we conclude $V_2$ = –9.0 V.

**LEARN** The potential difference between points 1 and 2 is

$$V_2 - V_1 = -9.0\text{ V} - (-11.0\text{ V}) = 2.0\text{ V},$$

which is equal to $iR = (1.0\text{ A})(2.0\ \Omega) = 2.0\text{ V}$.

32. (a) For typing convenience, we denote the emf of battery 2 as $V_2$ and the emf of battery 1 as $V_1$. The loop rule (examining the left-hand loop) gives $V_2 + i_1 R_1 - V_1 = 0$. Since $V_1$ is held constant while $V_2$ and $i_1$ vary, we see that this expression (for large enough $V_2$) will result in a negative value for $i_1$, so the downward sloping line (the line that is dashed in Fig. 27-43(b)) must represent $i_1$. It appears to be zero when $V_2$ = 6 V. With $i_1$ = 0, our loop rule gives $V_1 = V_2$, which implies that $V_1$ = 6.0 V.

(b) At $V_2$ = 2 V (in the graph) it appears that $i_1$ = 0.2 A. Now our loop rule equation (with the conclusion about $V_1$ found in part (a)) gives $R_1$ = 20 Ω.

(c) Looking at the point where the upward-sloping $i_2$ line crosses the axis (at $V_2$ = 4 V), we note that $i_1$ = 0.1 A there and that the loop rule around the right-hand loop should give

$$V_1 - i_1 R_1 = i_1 R_2$$

when $i_1$ = 0.1 A and $i_2$ = 0. This leads directly to $R_2$ = 40 Ω.

33. First, we note in $V_4$, that the voltage across $R_4$ is equal to the sum of the voltages across $R_5$ and $R_6$:

$$V_4 = i_6(R_5 + R_6) = (1.40 \text{ A})(8.00\ \Omega + 4.00\ \Omega) = 16.8 \text{ V}.$$

The current through $R_4$ is then equal to $i_4 = V_4/R_4$ = 16.8 V/(16.0 Ω) = 1.05 A.

By the junction rule, the current in $R_2$ is

$$i_2 = i_4 + i_6 = 1.05 \text{ A} + 1.40 \text{ A} = 2.45 \text{ A},$$

so its voltage is $V_2$ = (2.00 Ω)(2.45 A) = 4.90 V.

The loop rule tells us the voltage across $R_3$ is $V_3 = V_2 + V_4$ = 21.7 V (implying that the current through it is $i_3 = V_3$/(2.00 Ω) = 10.85 A).

The junction rule now gives the current in $R_1$ as

$$i_1 = i_2 + i_3 = 2.45 \text{ A} + 10.85 \text{ A} = 13.3 \text{ A},$$

implying that the voltage across it is $V_1$ = (13.3 A)(2.00 Ω) = 26.6 V. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 26.6 \text{ V} + 21.7 \text{ V} = 48.3 \text{ V}.$$

34. (a) By the loop rule, it remains the same. This question is aimed at student conceptualization of voltage; many students apparently confuse the concepts of voltage and current and speak of "voltage going through" a resistor – which would be difficult to rectify with the conclusion of this problem.

(b) The loop rule still applies, of course, but (by the junction rule and Ohm's law) the voltages across $R_1$ and $R_3$ (which were the same when the switch was open) are no longer equal. More current is now being supplied by the battery, which means more current is in $R_3$, implying its voltage drop has increased (in magnitude). Thus, by the loop rule (since the battery voltage has not changed) the voltage across $R_1$ has decreased a corresponding amount. When the switch was open, the voltage across $R_1$ was 6.0 V (easily seen from symmetry considerations). With the switch closed, $R_1$ and $R_2$ are equivalent (by Eq. 27-24) to 3.0 Ω, which means the total load on the battery is 9.0 Ω. The current therefore is 1.33 A, which implies that the voltage drop across $R_3$ is 8.0 V. The loop rule then tells us that the voltage drop across $R_1$ is 12 V – 8.0 V = 4.0 V. This is a decrease of 2.0 volts from the value it had when the switch was open.

35. (a) The symmetry of the problem allows us to use $i_2$ as the current in *both* of the $R_2$ resistors and $i_1$ for the $R_1$ resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:

$$\varepsilon - i_2 R_2 - i_1 R_1 = 0$$
$$\varepsilon - 2 i_1 R_1 - (i_1 - i_2) R_3 = 0$$

where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find $i_1$ = 0.002625 A, $i_2$ = 0.00225 A and $i_3 = i_1 - i_2$ = 0.000375 A. Therefore,

$$V_A - V_B = i_1 R_1 = 5.25 \text{ V}.$$

(b) It follows also that $V_B - V_C = i_3 R_3$ = 1.50 V.

(c) We find $V_C - V_D = i_1 R_1$ = 5.25 V.

(d) Finally, $V_A - V_C = i_2 R_2$ = 6.75 V.

36. (a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:

$$\varepsilon_1 - i_2 R_2 - (i_2 + i_3) R_1 = 0$$
$$\varepsilon_2 - i_3 R_3 - (i_2 + i_3) R_1 = 0.$$

Solving, we find $i_2$ = 0.0109 A (rightward, as was assumed in writing the equations as we did), $i_3$ = 0.0273 A (leftward), and $i_1 = i_2 + i_3$ = 0.0382 A (downward).

(b) The direction is downward. See the results in part (a).

(c) $i_2$ = 0.0109 A . See the results in part (a).

(d) The direction is rightward. See the results in part (a).

(e) $i_3$ = 0.0273 A. See the results in part (a).

(f) The direction is leftward. See the results in part (a).

(g) The voltage across $R_1$ equals $V_A$: (0.0382 A)(100 Ω) = +3.82 V.

37. The voltage difference across $R_3$ is $V_3 = \varepsilon R' /(R' + 2.00\ \Omega)$, where

$$R' = (5.00\ \Omega R)/(5.00\ \Omega + R_3).$$

Thus,

$$P_3 = \frac{V_3^2}{R_3} = \frac{1}{R_3}\left(\frac{\varepsilon R'}{R' + 2.00\,\Omega}\right)^2 = \frac{1}{R_3}\left(\frac{\varepsilon}{1 + 2.00\,\Omega/R'}\right)^2 = \frac{\varepsilon^2}{R_3}\left[1 + \frac{(2.00\,\Omega)(5.00\,\Omega + R)}{(5.00\,\Omega)R_3}\right]^{-2}$$
$$\equiv \frac{\varepsilon^2}{f(R_3)}$$

where we use the equivalence symbol ≡ to define the expression $f(R_3)$. To maximize $P_3$ we need to minimize the expression $f(R_3)$. We set

$$\frac{df(R_3)}{dR_3} = -\frac{4.00\,\Omega^2}{R_3^{\,2}} + \frac{49}{25} = 0$$

to obtain $R_3 = \sqrt{(4.00\,\Omega^2)(25)/49} = 1.43\,\Omega.$

38. (a) The voltage across $R_3 = 6.0\ \Omega$ is $V_3 = iR_3 = (6.0\text{ A})(6.0\ \Omega) = 36$ V. Now, the voltage across $R_1 = 2.0\ \Omega$ is

$$(V_A - V_B) - V_3 = 78 - 36 = 42\text{ V},$$

which implies the current is $i_1 = (42\text{ V})/(2.0\ \Omega) = 21$ A. By the junction rule, then, the current in $R_2 = 4.0\ \Omega$ is

$$i_2 = i_1 - i = 21\text{ A} - 6.0\text{ A} = 15\text{ A}.$$

The total power dissipated by the resistors is (using Eq. 26-27)

$$i_1^2(2.0\ \Omega) + i_2^2(4.0\ \Omega) + i^2(6.0\ \Omega) = 1998\text{ W} \approx 2.0\text{ kW}.$$

By contrast, the power supplied (externally) to this section is $P_A = i_A(V_A - V_B)$ where $i_A = i_1 = 21$ A. Thus, $P_A = 1638$ W. Therefore, the "Box" must be providing energy.

(b) The rate of supplying energy is $(1998 - 1638)\text{W} = 3.6\times10^2$ W.

39. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let $i$ be the current in either battery and take it to be positive to the left. According to the junction rule the current in $R$ is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and $R$ yields

$$\varepsilon - ir - 2iR = 0 \quad \Rightarrow \quad i = \frac{\varepsilon}{r + 2R}.$$

The power dissipated in $R$ is

$$P = (2i)^2 R = \frac{4\varepsilon^2 R}{(r+2R)^2}.$$

We find the maximum by setting the derivative with respect to $R$ equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\varepsilon^2}{(r+2R)^3} - \frac{16\varepsilon^2 R}{(r+2R)^3} = \frac{4\varepsilon^2 (r-2R)}{(r+2R)^3}.$$

The derivative vanishes (and $P$ is a maximum) if $R = r/2$. With $r = 0.300\ \Omega$, we have $R = 0.150\ \Omega$.

(b) We substitute $R = r/2$ into $P = 4\varepsilon^2 R/(r + 2R)^2$ to obtain

$$P_{\max} = \frac{4\varepsilon^2 (r/2)}{[r + 2(r/2)]^2} = \frac{\varepsilon^2}{2r} = \frac{(12.0\text{ V})^2}{2(0.300\ \Omega)} = 240\text{ W}.$$

40. (a) By symmetry, when the two batteries are connected in parallel the current $i$ going through either one is the same. So from $\varepsilon = ir + (2i)R$ with $r = 0.200\ \Omega$ and $R = 2.00r$, we get

$$i_R = 2i = \frac{2\varepsilon}{r+2R} = \frac{2(12.0\text{V})}{0.200\Omega + 2(0.400\Omega)} = 24.0\text{ A}.$$

(b) When connected in series $2\varepsilon - i_R r - i_R r - i_R R = 0$, or $i_R = 2\varepsilon/(2r + R)$. The result is

$$i_R = 2i = \frac{2\varepsilon}{2r+R} = \frac{2(12.0\text{V})}{2(0.200\Omega) + 0.400\Omega} = 30.0\text{ A}.$$

(c) They are in series arrangement, since $R > r$.

(d) If $R = r/2.00$, then for parallel connection,

$$i_R = 2i = \frac{2\varepsilon}{r+2R} = \frac{2(12.0\text{V})}{0.200\Omega + 2(0.100\Omega)} = 60.0\text{ A}.$$

(e) For series connection, we have

$$i_R = 2i = \frac{2\varepsilon}{2r+R} = \frac{2(12.0\text{V})}{2(0.200\Omega) + 0.100\Omega} = 48.0\text{ A}.$$

(f) They are in parallel arrangement, since $R < r$.

41. We first find the currents. Let $i_1$ be the current in $R_1$ and take it to be positive if it is to the right. Let $i_2$ be the current in $R_2$ and take it to be positive if it is to the left. Let $i_3$ be the current in $R_3$ and take it to be positive if it is upward. The junction rule produces

$$i_1 + i_2 + i_3 = 0 \ .$$

The loop rule applied to the left-hand loop produces

$$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0$$

and applied to the right-hand loop produces

$$\varepsilon_2 - i_2 R_2 + i_3 R_3 = 0 .$$

We substitute $i_3 = -i_2 - i_1$, from the first equation, into the other two to obtain

$$\varepsilon_1 - i_1 R_1 - i_2 R_3 - i_1 R_3 = 0$$

and

$$\varepsilon_2 - i_2 R_2 - i_2 R_3 - i_1 R_3 = 0 .$$

Solving the above equations yield

$$i_1 = \frac{\varepsilon_1 (R_2 + R_3) - \varepsilon_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(3.00\ \text{V})(2.00\,\Omega + 5.00\,\Omega) - (1.00\ \text{V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = 0.421\ \text{A}.$$

$$i_2 = \frac{\varepsilon_2 (R_1 + R_3) - \varepsilon_1 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(1.00\ \text{V})(4.00\,\Omega + 5.00\,\Omega) - (3.00\ \text{V})(5.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.158\ \text{A}.$$

$$i_3 = -\frac{\varepsilon_2 R_1 + \varepsilon_1 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} = -\frac{(1.00\ \text{V})(4.00\,\Omega) + (3.00\ \text{V})(2.00\,\Omega)}{(4.00\,\Omega)(2.00\,\Omega) + (4.00\,\Omega)(5.00\Omega) + (2.00\,\Omega)(5.00\,\Omega)} = -0.263\ \text{A}.$$

Note that the current $i_3$ in $R_3$ is actually downward and the current $i_2$ in $R_2$ is to the right. The current $i_1$ in $R_1$ is to the right.

(a) The power dissipated in $R_1$ is $P_1 = i_1^2 R_1 = \left(0.421\,\text{A}\right)^2 \left(4.00\,\Omega\right) = 0.709\,\text{W}.$

(b) The power dissipated in $R_2$ is $P_2 = i_2^2 R_2 = (-0.158\,\text{A})^2 (2.00\,\Omega) = 0.0499\,\text{W} \approx 0.050\ \text{W}.$

(c) The power dissipated in $R_3$ is $P_3 = i_3^2 R_3 = \left(-0.263\,\text{A}\right)^2 \left(5.00\,\Omega\right) = 0.346\,\text{W}.$

(d) The power supplied by $\varepsilon_1$ is $i_3\varepsilon_1$ = (0.421 A)(3.00 V) = 1.26 W.

(e) The power "supplied" by $\varepsilon_2$ is $i_2\varepsilon_2$ = (–0.158 A)(1.00 V) = –0.158 W. The negative sign indicates that $\varepsilon_2$ is actually absorbing energy from the circuit.

42. The equivalent resistance in Fig. 27-52 (with *n* parallel resistors) is

$$R_{\text{eq}} = R + \frac{R}{n} = \left(\frac{n+1}{n}\right)R \ .$$

The current in the battery in this case should be

$$i_n = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n}{n+1}\frac{V_{\text{battery}}}{R} \ .$$

If there were *n* +1 parallel resistors, then

$$i_{n+1} = \frac{V_{\text{battery}}}{R_{\text{eq}}} = \frac{n+1}{n+2}\frac{V_{\text{battery}}}{R} \ .$$

For the relative increase to be 0.0125 ( = 1/80 ), we require

$$\frac{i_{n+1} - i_n}{i_n} = \frac{i_{n+1}}{i_n} - 1 = \frac{(n+1)/(n+2)}{n/(n+1)} - 1 = \frac{1}{80} \ .$$

This leads to the second-degree equation $n^2 + 2n - 80 = (n + 10)(n - 8) = 0$.

Clearly the only physically interesting solution to this is $n = 8$. Thus, there are eight resistors in parallel (as well as that resistor in series shown toward the bottom) in Fig. 27-52.

43. Let the resistors be divided into groups of *n* resistors each, with all the resistors in the same group connected in series. Suppose there are *m* such groups that are connected in parallel with each other. Let *R* be the resistance of any one of the resistors. Then the equivalent resistance of any group is *nR*, and $R_{\text{eq}}$, the equivalent resistance of the whole array, satisfies

$$\frac{1}{R_{\text{eq}}} = \sum_{1}^{m} \frac{1}{nR} = \frac{m}{nR}.$$

Since the problem requires $R_{\text{eq}} = 10\ \Omega = R$, we must select $n = m$. Next we make use of Eq. 27-16. We note that the current is the same in every resistor and there are $n \cdot m = n^2$ resistors, so the maximum total power that can be dissipated is $P_{\text{total}} = n^2P$, where $P = 1.0\ \text{W}$ is the maximum power that can be dissipated by any one of the resistors. The problem demands $P_{total} \geq 5.0P$, so $n^2$ must be at least as large as 5.0. Since $n$ must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

44. (a) Resistors $R_2$, $R_3$, and $R_4$ are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50.0\Omega)(50.0\Omega)(75.0\Omega)}{(50.0\Omega)(50.0\Omega) + (50.0\Omega)(75.0\Omega) + (50.0\Omega)(75.0\Omega)}$$
$$= 18.8\Omega.$$

Thus, considering the series contribution of resistor $R_1$, the equivalent resistance for the network is $R_{eq} = R_1 + R = 100\ \Omega + 18.8\ \Omega = 118.8\ \Omega \approx 119\ \Omega$.

(b) $i_1 = \varepsilon/R_{eq} = 6.0\text{ V}/(118.8\ \Omega) = 5.05 \times 10^{-2}$ A.

(c) $i_2 = (\varepsilon - V_1)/R_2 = (\varepsilon - i_1R_1)/R_2 = [6.0\text{V} - (5.05 \times 10^{-2}\text{ A})(100\Omega)]/50\ \Omega = 1.90 \times 10^{-2}$ A.

(d) $i_3 = (\varepsilon - V_1)/R_3 = i_2R_2/R_3 = (1.90 \times 10^{-2}\text{ A})(50.0\ \Omega/50.0\ \Omega) = 1.90 \times 10^{-2}$ A.

(e) $i_4 = i_1 - i_2 - i_3 = 5.05 \times 10^{-2}\text{ A} - 2(1.90 \times 10^{-2}\text{ A}) = 1.25 \times 10^{-2}$ A.

45. (a) We note that the $R_1$ resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through $\varepsilon_2$ and $\varepsilon_3$ are the same: $i_2 = i_3 = i$. Therefore, the current through $\varepsilon_1$ is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we get

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0\text{ V} - 2.0\text{ V}}{4(1.0\ \Omega) + 2.0\ \Omega} = 0.33\text{A}.$$

Therefore, the current through $\varepsilon_1$ is $i_1 = 2i = 0.67$ A.

(b) The direction of $i_1$ is downward.

(c) The current through $\varepsilon_2$ is $i_2 = 0.33$ A.

(d) The direction of $i_2$ is upward.

(e) From part (a), we have $i_3 = i_2 = 0.33$ A.

(f) The direction of $i_3$ is also upward.

(g) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333\text{ A})(2.0\ \Omega) + 4.0\text{ V} = 3.3$ V.

46. (a) When $R_3$ = 0 all the current passes through $R_1$ and $R_3$ and avoids $R_2$ altogether. Since that value of the current (through the battery) is 0.006 A (see Fig. 27-55(b)) for $R_3$ = 0 then (using Ohm's law)

$$R_1 = (12\text{ V})/(0.006\text{ A}) = 2.0\times10^3\,\Omega.$$

(b) When $R_3 = \infty$ all the current passes through $R_1$ and $R_2$ and avoids $R_3$ altogether. Since that value of the current (through the battery) is 0.002 A (stated in problem) for $R_3 = \infty$ then (using Ohm's law)

$$R_2 = (12\text{ V})/(0.002\text{ A}) - R_1 = 4.0\times10^3\,\Omega.$$

47. **THINK** The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them.

**EXPRESS** Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where $i_C$ is the current in the copper, $i_A$ is the current in the aluminum, $R_C$ is the resistance of the copper, and $R_A$ is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where $\rho$ is the resistivity, $L$ is the length, and $A$ is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$, and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. We substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors $L$ and $\pi$ to obtain

$$\frac{i_C\rho_C}{a^2} = \frac{i_A\rho_A}{b^2 - a^2}.$$

We solve this equation simultaneously with $i = i_C + i_A$, where $i$ is the total current. We find

$$i_C = \frac{r_C^2\rho_C i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2\rho_A}$$

and

$$i_A = \frac{\left(r_A^2 - r_C^2\right)\rho_C\, i}{\left(r_A^2 - r_C^2\right)\rho_C + r_C^2\rho_A}.$$

**ANALYZE** (a) The denominators are the same and each has the value

$$\begin{aligned}\left(b^2 - a^2\right)\rho_C + a^2\rho_A &= \left[\left(0.380\times10^{-3}\text{ m}\right)^2 - \left(0.250\times10^{-3}\text{ m}\right)^2\right]\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\\ &\quad + \left(0.250\times10^{-3}\text{ m}\right)^2\left(2.75\times10^{-8}\,\Omega\cdot\text{m}\right)\\ &= 3.10\times10^{-15}\,\Omega\cdot\text{m}^3.\end{aligned}$$

Thus,

$$i_C = \frac{\left(0.250\times10^{-3}\text{ m}\right)^2\left(2.75\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\text{ A}\right)}{3.10\times10^{-15}\Omega\cdot\text{m}^3} = 1.11\text{ A}.$$

(b) Similarly,

$$i_A = \frac{\left[\left(0.380\times10^{-3}\text{m}\right)^2 - \left(0.250\times10^{-3}\text{m}\right)^2\right]\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left(2.00\,\text{A}\right)}{3.10\times10^{-15}\Omega\cdot\text{m}^3} = 0.893\text{A}.$$

(c) Consider the copper wire. If $V$ is the potential difference, then the current is given by $V = i_C R_C = i_C\rho_C L/\pi a^2$, so the length of the composite wire is

$$L = \frac{\pi a^2 V}{i_C\rho_C} = \frac{(\pi)\left(0.250\times10^{-3}\text{ m}\right)^2\left(12.0\text{ V}\right)}{\left(1.11\text{ A}\right)\left(1.69\times10^{-8}\Omega\cdot\text{m}\right)} = 126\text{ m}.$$

**LEARN** The potential difference can also be written as $V = i_A R_A = i_A\rho_A L/\pi(b^2 - a^2)$. Thus,

$$L = \frac{\pi(b^2-a^2)V}{i_A\rho_A} = \frac{\pi\left[(0.380\times10^{-3}\text{ m})^2 - (0.250\times10^{-3}\text{ m})^2\right](12.0\text{ V})}{\left(0.893\text{ A}\right)\left(2.75\times10^{-8}\Omega\cdot\text{m}\right)} = 126\text{ m},$$

in agreement with the result found in (c).

48. (a) We use $P = \varepsilon^2/R_{\text{eq}}$, where

$$R_{\text{eq}} = 7.00\,\Omega + \frac{(12.0\,\Omega)(4.00\,\Omega)R}{(12.0\,\Omega)(4.0\,\Omega)+(12.0\,\Omega)R+(4.00\,\Omega)R}.$$

Put $P$ = 60.0 W and $\varepsilon$ = 24.0 V and solve for $R$: $R$ = 19.5 Ω.

(b) Since $P \propto R_{\text{eq}}$, we must minimize $R_{\text{eq}}$, which means $R$ = 0.

(c) Now we must maximize $R_{\text{eq}}$, or set $R = \infty$.

(d) Since $R_{\text{eq, min}}$ = 7.00 Ω, $P_{\max} = \varepsilon^2/R_{\text{eq, min}}$ = (24.0 V)$^2$/7.00 Ω = 82.3 W.

(e) Since $R_{\text{eq, max}}$ = 7.00 Ω + (12.0 Ω)(4.00 Ω)/(12.0 Ω + 4.00 Ω) = 10.0 Ω,

$$P_{\min} = \varepsilon^2/R_{\text{eq, max}} = (24.0\text{ V})^2/10.0\ \Omega = 57.6\text{ W}.$$

49. (a) The current in $R_1$ is given by

$$i_1 = \frac{\varepsilon}{R_1 + R_2R_3/\left(R_2+R_3\right)} = \frac{5.0\ \text{V}}{2.0\,\Omega + (4.0\,\Omega)(6.0\,\Omega)/(4.0\,\Omega + 6.0\,\Omega)} = 1.14\ \text{A}.$$

Thus,

$$i_3 = \frac{\varepsilon - V_1}{R_3} = \frac{\varepsilon - i_1R_1}{R_3} = \frac{5.0\ \text{V} - (1.14\ \text{A})(2.0\,\Omega)}{6.0\,\Omega} = 0.45\ \text{A}.$$

(b) We simply interchange subscripts 1 and 3 in the equation above. Now

$$i_3 = \frac{\varepsilon}{R_3 + \left(R_2R_1/\left(R_2+R_1\right)\right)} = \frac{5.0\,\text{V}}{6.0\,\Omega + \left(\left(2.0\,\Omega\right)\left(4.0\,\Omega\right)/\left(2.0\,\Omega + 4.0\,\Omega\right)\right)} = 0.6818\,\text{A}$$

and

$$i_1 = \frac{5.0\ \text{V} - \left(0.6818\ \text{A}\right)\left(6.0\,\Omega\right)}{2.0\,\Omega} = 0.45\ \text{A},$$

the same as before.

50. Note that there is no voltage drop across the ammeter. Thus, the currents in the bottom resistors are the same, which we call $i$ (so the current through the battery is $2i$ and the voltage drop across each of the bottom resistors is $iR$). The resistor network can be reduced to an equivalence of

$$R_{\text{eq}} = \frac{\left(2R\right)\left(R\right)}{2R+R} + \frac{\left(R\right)\left(R\right)}{R+R} = \frac{7}{6}R$$

which means that we can determine the current through the battery (and also through each of the bottom resistors):

$$2i = \frac{\varepsilon}{R_{\text{eq}}} \Rightarrow i = \frac{\varepsilon}{2R_{\text{eq}}} = \frac{\varepsilon}{2(7R/6)} = \frac{3\varepsilon}{7R}.$$

By the loop rule (going around the left loop, which includes the battery, resistor $2R$, and one of the bottom resistors), we have

$$\varepsilon - i_{2R}\left(2R\right) - iR = 0 \Rightarrow i_{2R} = \frac{\varepsilon - iR}{2R}.$$

Substituting $i = 3\varepsilon/7R$, this gives $i_{2R} = 2\varepsilon/7R$. The difference between $i_{2R}$ and $i$ is the current through the ammeter. Thus,

$$i_{\text{ammeter}} = i - i_{2R} = \frac{3\varepsilon}{7R} - \frac{2\varepsilon}{7R} = \frac{\varepsilon}{7R} \Rightarrow \frac{i_{\text{ammeter}}}{\varepsilon/R} = \frac{1}{7} = 0.143.$$

51. Since the current in the ammeter is $i$, the voltmeter reading is

$$V' = V + i\,R_A = i\,(R + R_A),$$

or $R = V'/i - R_A = R' - R_A$, where $R' = V'/i$ is the apparent reading of the resistance. Now, from the lower loop of the circuit diagram, the current through the voltmeter is $i_V = \varepsilon/(R_{eq} + R_0)$, where

$$\frac{1}{R_{eq}} = \frac{1}{R_V} + \frac{1}{R_A + R} \quad \Rightarrow \quad R_{eq} = \frac{R_V\left(R + R_A\right)}{R_V + R + R_A} = \frac{\left(300\,\Omega\right)\left(85.0\,\Omega + 3.00\,\Omega\right)}{300\,\Omega + 85.0\,\Omega + 3.00\,\Omega} = 68.0\,\Omega.$$

The voltmeter reading is then

$$V' = i_V R_{eq} = \frac{\varepsilon R_{eq}}{R_{eq} + R_0} = \frac{(12.0\text{ V})(68.0\ \Omega)}{68.0\ \Omega + 100\Omega} = 4.86\text{ V}.$$

(a) The ammeter reading is

$$i = \frac{V'}{R + R_A} = \frac{4.86\text{ V}}{85.0\ \Omega + 3.00\Omega} = 0.0552\text{ A}.$$

(b) As shown above, the voltmeter reading is $V' = 4.86$ V.

(c) $R' = V'/i = 4.86\text{ V}/(5.52 \times 10^{-2}\text{ A}) = 88.0\ \Omega$.

(d) Since $R = R' - R_A$, if $R_A$ is decreased, the difference between $R'$ and $R$ decreases. In fact, when $R_A = 0$, $R' = R$.

52. (a) Since $i = \varepsilon/(r + R_{ext})$ and $i_{max} = \varepsilon/r$, we have $R_{ext} = R(i_{max}/i - 1)$ where $r = 1.50\text{ V}/1.00\text{ mA} = 1.50 \times 10^3\ \Omega$. Thus,

$$R_{ext} = (1.5\times10^3\ \Omega)(1/0.100 - 1) = 1.35\times10^4\ \Omega.$$

(b) $R_{ext} = (1.5\times10^3\ \Omega)(1/0.500 - 1) = 1.5\times10^3\ \Omega$.

(c) $R_{ext} = (1.5\times10^3\ \Omega)(1/0.900 - 1) = 167\ \Omega$.

(d) Since $r = 20.0\ \Omega + R$, $R = 1.50 \times 10^3\ \Omega - 20.0\ \Omega = 1.48 \times 10^3\ \Omega$.

53. The current in $R_2$ is $i$. Let $i_1$ be the current in $R_1$ and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop:

$$\varepsilon - iR_2 - i_1R_1 - ir = 0.$$

Similarly, applying the loop rule to the right-hand loop gives

$$i_1R_1 - (i - i_1)R_V = 0.$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V} i_1.$$

We substitute this into the first equation to obtain

$$\varepsilon - \frac{(R_2 + r)(R_1 + R_V)}{R_V} i_1 + R_1 i_1 = 0.$$

This has the solution

$$i_1 = \frac{\varepsilon R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V}.$$

The reading on the voltmeter is

$$i_1R_1 = \frac{\varepsilon R_V R_1}{(R_2 + r)\,(R_1 + R_V) + R_1 R_V} = \frac{(3.0\text{V})\,(5.0\times10^3\,\Omega)\,(250\Omega)}{(300\Omega + 100\Omega)(250\Omega + 5.0\times10^3\,\Omega) + (250\Omega)(5.0\times10^3\,\Omega)}$$
$$= 1.12\text{ V}.$$

The current in the absence of the voltmeter can be obtained by taking the limit as $R_V$ becomes infinitely large. Then

$$i_1R_1 = \frac{\varepsilon R_1}{R_1 + R_2 + r} = \frac{(3.0\,\text{V})(250\,\Omega)}{250\ \Omega + 300\ \Omega + 100\,\Omega} = 1.15\,\text{V}.$$

The fractional error is (1.12 – 1.15)/(1.15) = –0.030, or –3.0%.

54. (a) $\varepsilon = V + ir$ = 12 V + (10.0 A) (0.0500 Ω) = 12.5 V.

(b) Now $\varepsilon = V' + (i_{\text{motor}} + 8.00\text{ A})r$, where

$$V' = i'_A R_{\text{light}} = (8.00\text{ A})\,(12.0\text{ V}/10\text{ A}) = 9.60\text{ V}.$$

Therefore,

$$i_{\text{motor}} = \frac{\varepsilon - V'}{r} - 8.00\,\text{A} = \frac{12.5\,\text{V} - 9.60\,\text{V}}{0.0500\,\Omega} - 8.00\,\text{A} = 50.0\,\text{A}.$$

55. Let $i_1$ be the current in $R_1$ and $R_2$, and take it to be positive if it is toward point *a* in $R_1$. Let $i_2$ be the current in $R_s$ and $R_x$, and take it to be positive if it is toward *b* in $R_s$. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points *a* and *b* are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$\left(R_1 + R_2\right) i_1 = \left(R_x + R_s\right) \frac{R_1}{R_s} i_1 \quad \Rightarrow \quad R_x = \frac{R_2 R_s}{R_1}.$$

56. The currents in $R$ and $R_V$ are $i$ and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides by $V$, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R} = \frac{1}{R'} - \frac{1}{R_V} \quad \Rightarrow R' = \frac{RR_V}{R + R_V}.$$

The equivalent resistance of the circuit is $R_{\text{eq}} = R_A + R_0 + R' = R_A + R_0 + \dfrac{RR_V}{R + R_V}$.

(a) The ammeter reading is

$$i' = \frac{\varepsilon}{R_{\text{eq}}} = \frac{\varepsilon}{R_A + R_0 + R_V\, R/\left(R + R_V\right)} = \frac{12.0\,\text{V}}{3.00\,\Omega + 100\,\Omega + \left(300\,\Omega\right)\left(85.0\,\Omega\right)/\left(300\,\Omega + 85.0\,\Omega\right)}$$
$$= 7.09 \times 10^{-2}\,\text{A}.$$

(b) The voltmeter reading is

$$V = \varepsilon - i'(R_A + R_0) = 12.0\text{ V} - (0.0709\text{ A})(103.00\ \Omega) = 4.70\text{ V}.$$

(c) The apparent resistance is $R' = V/i' = 4.70\text{ V}/(7.09 \times 10^{-2}\text{ A}) = 66.3\ \Omega$.

(d) If $R_V$ is increased, the difference between $R$ and $R'$ decreases. In fact, $R' \to R$ as $R_V \to \infty$.

57. Here we denote the battery emf as $V$. Then the requirement stated in the problem that the resistor voltage be equal to the capacitor voltage becomes $iR = V_{\text{cap}}$, or

$$Ve^{-t/RC} = V(1 - e^{-t/RC})$$

where Eqs. 27-34 and 27-35 have been used. This leads to $t = RC \ln 2$, or $t = 0.208$ ms.

58. (a) $\tau = RC = (1.40 \times 10^6\ \Omega)(1.80 \times 10^{-6}\text{ F}) = 2.52$ s.

(b) $q_o = \varepsilon C = (12.0\text{ V})(1.80\ \mu\text{F}) = 21.6\ \mu\text{C}$.

(c) The time $t$ satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC \ln\left(\frac{q_0}{q_0 - q}\right) = \left(2.52\,\text{s}\right) \ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C} - 16.0\,\mu\text{C}}\right) = 3.40\,\text{s}.$$

59. **THINK** We have an $RC$ circuit that is being charged. When fully charged, the charge on the capacitor is equal to $C\varepsilon$.

**EXPRESS** During charging, the charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where $C$ is the capacitance, $\varepsilon$ is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{eq} = C\varepsilon$, so we require $q = 0.99q_{eq} = 0.99C\varepsilon$.

**ANALYZE** The time required to reach 99% of its final charge is given by

$$0.99 = 1 - e^{-t/\tau}.$$

Thus, $e^{-t/\tau} = 0.01$. Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.61$ or $t = 4.61\tau$.

**LEARN** The corresponding current in a charging capacitor is given by

$$i = \frac{dq}{dt} = \frac{\varepsilon}{R}e^{-t/\tau}.$$

The current has an initial value $\varepsilon/R$ but decays exponentially to zero as the capacitor becomes fully charged. The plots of $q(t)$ and $i(t)$ are shown in Fig. 27-16 of the text.

60. (a) We use $q = q_0 e^{-t/\tau}$, or $t = \tau \ln(q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus,

$$t_{1/3} = \tau \ln\left(\frac{q_0}{2q_0/3}\right) = \tau \ln\left(\frac{3}{2}\right) = 0.41\tau \quad \Rightarrow \quad \frac{t_{1/3}}{\tau} = 0.41.$$

(b) $t_{2/3} = \tau \ln\left(\dfrac{q_0}{q_0/3}\right) = \tau \ln 3 = 1.1\tau \quad \Rightarrow \quad \dfrac{t_{2/3}}{\tau} = 1.1.$

61. (a) The voltage difference $V$ across the capacitor is $V(t) = \varepsilon(1 - e^{-t/RC})$. At $t = 1.30\ \mu\text{s}$ we have $V(t) = 5.00$ V, so $5.00\text{ V} = (12.0\text{ V})(1 - e^{-1.30\ \mu\text{s}/RC})$, which gives

$$\tau = (1.30\ \mu\text{s})/\ln(12/7) = 2.41\ \mu\text{s}.$$

(b) The capacitance is $C = \tau/R = (2.41\ \mu\text{s})/(15.0\text{ k}\Omega) = 161\text{ pF}$.

62. The time it takes for the voltage difference across the capacitor to reach $V_L$ is given by $V_L = \varepsilon\left(1 - e^{-t/RC}\right)$. We solve for $R$:

$$R = \frac{t}{C \ln \varepsilon/(\varepsilon - V_L)} = \frac{0.500\,\text{s}}{(0.150\times10^{-6}\,\text{F}) \ln 95.0\,\text{V}/(95.0\,\text{V} - 72.0\,\text{V})} = 2.35\times10^{6}\,\Omega$$

where we used $t$ = 0.500 s given (implicitly) in the problem.

63. **THINK** We have a multi-loop circuit with a capacitor that's being charged. Since at $t$ = 0 the capacitor is completely uncharged, the current in the capacitor branch is as it would be if the capacitor were replaced by a wire.

**EXPRESS** Let $i_1$ be the current in $R_1$ and take it to be positive if it is to the right. Let $i_2$ be the current in $R_2$ and take it to be positive if it is downward. Let $i_3$ be the current in $R_3$ and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0.$$

Since the resistances are all the same we can simplify the mathematics by replacing $R_1$, $R_2$, and $R_3$ with $R$.

**ANALYZE** (a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2(1.2\times10^{3}\,\text{V})}{3(0.73\times10^{6}\,\Omega)} = 1.1\times10^{-3}\,\text{A},$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^{3}\,\text{V}}{3(0.73\times10^{6}\,\Omega)} = 5.5\times10^{-4}\,\text{A},$

(c) and $i_3 = i_2 = 5.5\times10^{-4}\,\text{A}.$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields $\varepsilon - i_1 R_1 - i_1 R_2 = 0.$

(d) The solution is $i_1 = \dfrac{\varepsilon}{2R} = \dfrac{1.2\times10^{3}\,\text{V}}{2(0.73\times10^{6}\,\Omega)} = 8.2\times10^{-4}\,\text{A}$

(e) and $i_2 = i_1 = 8.2\times10^{-4}\,\text{A}.$

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0.$$

We use the first equation to substitute for $i_1$ in the second and obtain

$$\varepsilon - 2i_2 R - i_3 R = 0.$$

Thus $i_2 = (\varepsilon - i_3 R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3 R) + (\varepsilon/2) - (i_3 R/2) = 0.$$

Now we replace $i_3$ with $dq/dt$ to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the equation for an $RC$ series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R} e^{-2t/3RC}.$$

The current in the center branch is

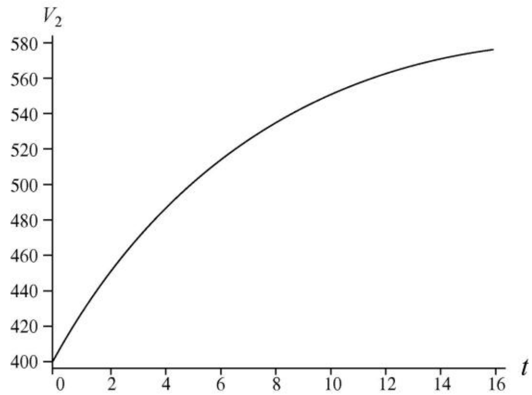
$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R} e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

and the potential difference across $R_2$ is $V_2(t) = i_2 R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right)$.

(g) For $t = 0$, $e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\,\text{V}\right)/3 = 4.0\times10^2\,\text{V}$.

(h) For $t = \infty$, $e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times20^3\,\text{V}\right)/2 = 6.0\times10^2\,\text{V}$.

(i) A plot of $V_2$ as a function of time is shown in the following graph.




**LEARN** A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current. However, a long time later after it's fully charged, it acts like a broken wire.

64. (a) The potential difference $V$ across the plates of a capacitor is related to the charge $q$ on the positive plate by $V = q/C$, where $C$ is capacitance. Since the charge on a discharging capacitor is given by $q = q_0\, e^{-t/\tau}$, this means $V = V_0\, e^{-t/\tau}$ where $V_0$ is the initial potential difference. We solve for the time constant $\tau$ by dividing by $V_0$ and taking the natural logarithm:

$$\tau = -\frac{t}{\ln(V/V_0)} = -\frac{10.0\,\text{s}}{\ln\,(1.00\,\text{V})/(100\,\text{V})} = 2.17\,\text{s}.$$

(b) At $t = 17.0$ s, $t/\tau = (17.0\text{ s})/(2.17\text{ s}) = 7.83$, so

$$V = V_0 e^{-t/\tau} = (100\,\text{V})e^{-7.83} = 3.96\times10^{-2}\text{ V}\,.$$

65. In the steady state situation, the capacitor voltage will equal the voltage across $R_2$ = 15 kΩ:

$$V_0 = R_2\frac{\varepsilon}{R_1+R_2} = (15.0\text{ k}\Omega)\left(\frac{20.0\text{V}}{10.0\text{ k}\Omega+15.0\text{ k}\Omega}\right) = 12.0\text{V}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across $R_2$ = 15.0 kΩ) after the switch is opened (at $t$ = 0). Thus, with $t$ = 0.00400 s, we obtain

$$V = (12)e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16\,\text{V}.$$

Therefore, using Ohm's law, the current through $R_2$ is 6.16/15000 = $4.11\times10^{-4}$ A.

66. We apply Eq. 27-39 to each capacitor, demand their initial charges are in a ratio of 3:2 as described in the problem, and solve for the time. With

$$\tau_1 = R_1C_1 = (20.0\ \Omega)(5.00\times10^{-6}\ \text{F}) = 1.00\times10^{-4}\ \text{s}$$
$$\tau_2 = R_2C_2 = (10.0\ \Omega)(8.00\times10^{-6}\ \text{F}) = 8.00\times10^{-5}\ \text{s}\ ,$$

we obtain

$$t = \frac{\ln(3/2)}{\tau_2^{-1} - \tau_1^{-1}} = \frac{\ln(3/2)}{1.25\times10^{4}\ \text{s}^{-1} - 1.00\times10^{4}\ \text{s}^{-1}} = 1.62\times10^{-4}\text{s}\ .$$

67. The potential difference across the capacitor varies as a function of time $t$ as $V(t) = V_0 e^{-t/RC}$. Using $V = V_0/4$ at $t = 2.0$ s, we find

$$R = \frac{t}{C\ln(V_0/V)} = \frac{2.0\,\text{s}}{\left(2.0\times10^{-6}\ \text{F}\right)\ln 4} = 7.2\times10^{5}\ \Omega.$$

68. (a) The initial energy stored in a capacitor is given by $U_C = q_0^2/2C$, where $C$ is the capacitance and $q_0$ is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2\left(1.0\times10^{-6}\ \text{F}\right)\left(0.50\ \text{J}\right)} = 1.0\times10^{-3}\ \text{C}\ .$$

(b) The charge as a function of time is given by $q = q_0 e^{-t/\tau}$, where $\tau$ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}e^{-t/\tau}\ ,$$

and the initial current is $i_0 = q_0/\tau$. The time constant is

$$\tau = RC = \left(1.0\times10^{-6}\ \text{F}\right)\left(1.0\times10^{6}\ \Omega\right) = 1.0\ \text{s}\ .$$

Thus $i_0 = \left(1.0\times10^{-3}\ \text{C}\right)\big/\left(1.0\,\text{s}\right) = 1.0\times10^{-3}\ \text{A}$ .

(c) We substitute $q = q_0 e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C}e^{-t/\tau} = \left(\frac{1.0\times10^{-3}\ \text{C}}{1.0\times10^{-6}\ \text{F}}\right)e^{-t/1.0\ \text{s}} = \left(1.0\times10^{3}\ \text{V}\right)e^{-1.0t}\ ,$$

where $t$ is measured in seconds.

(d) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau} e^{-t/\tau} = \frac{(1.0\times10^{-3}\,\text{C})(1.0\times10^{6}\,\Omega)}{1.0\,\text{s}} e^{-t/1.0\,\text{s}} = (1.0\times10^{3}\,\text{V})e^{-1.0t} \;,$$

where $t$ is measured in seconds.

(e) We substitute $i = (q_0/\tau)e^{-t/\tau}$ into $P = i^2 R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2} e^{-2t/\tau} = \frac{(1.0\times10^{-3}\,\text{C})^2(1.0\times10^{6}\,\Omega)}{(1.0\,\text{s})^2} e^{-2t/1.0\,\text{s}} = (1.0\,\text{W})e^{-2.0t} \;,$$

where $t$ is again measured in seconds.

69. (a) The charge on the positive plate of the capacitor is given by

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right),$$

where $\varepsilon$ is the emf of the battery, $C$ is the capacitance, and $\tau$ is the time constant. The value of $\tau$ is

$$\tau = RC = (3.00 \times 10^6\ \Omega)(1.00 \times 10^{-6}\ \text{F}) = 3.00\ \text{s}.$$

At $t$ = 1.00 s, $t/\tau$ = (1.00 s)/(3.00 s) = 0.333 and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\varepsilon}{\tau} e^{-t/\tau} = \frac{(1.00\times10^{-6}\ \text{F})(4.00\,\text{V})}{3.00\,\text{s}} e^{-0.333} = 9.55\times10^{-7}\ \text{C/s}.$$

(b) The energy stored in the capacitor is given by $U_C = \dfrac{q^2}{2C}$, and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt}.$$

Now

$$q = C\varepsilon\left(1 - e^{-t/\tau}\right) = \left(1.00\times10^{-6}\right)(4.00\,\text{V})\left(1 - e^{-0.333}\right) = 1.13\times10^{-6}\ \text{C},$$

so

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt} = \left(\frac{1.13\times10^{-6}\ \text{C}}{1.00\times10^{-6}\ \text{F}}\right)\left(9.55\times10^{-7}\ \text{C/s}\right) = 1.08\times10^{-6}\ \text{W}.$$

(c) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is $9.55 \times 10^{-7}$ A, so

$$P = \left(9.55 \times 10^{-7}\ \text{A}\right)^2 \left(3.00 \times 10^{6}\ \Omega\right) = 2.74 \times 10^{-6}\ \text{W}.$$

(d) The rate at which energy is delivered by the battery is

$$i\varepsilon = \left(9.55 \times 10^{-7}\ \text{A}\right)\left(4.00\ \text{V}\right) = 3.82 \times 10^{-6}\ \text{W}.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\varepsilon = (q/C)\ (dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

70. (a) From symmetry we see that the current through the top set of batteries ($i$) is the same as the current through the second set. This implies that the current through the $R$ = 4.0 Ω resistor at the bottom is $i_R = 2i$. Thus, with $r$ denoting the internal resistance of each battery (equal to 4.0 Ω) and $\varepsilon$ denoting the 20 V emf, we consider one loop equation (the outer loop), proceeding counterclockwise:

$$3\left(\varepsilon - ir\right) - \left(2i\right)R = 0.$$

This yields $i$ = 3.0 A. Consequently, $i_R$ = 6.0 A.

(b) The terminal voltage of each battery is $\varepsilon - ir$ = 8.0 V.

(c) Using Eq. 27-17, we obtain $P = i\varepsilon$ = (3)(20) = 60 W.

(d) Using Eq. 26-27, we have $P = i^2r$ = 36 W.

71. (a) If $S_1$ is closed, and $S_2$ and $S_3$ are open, then $i_a = \varepsilon/2R_1$ = 120 V/40.0 Ω = 3.00 A.

(b) If $S_3$ is open while $S_1$ and $S_2$ remain closed, then

$$R_{eq} = R_1 + R_1\,(R_1 + R_2)\,/(2R_1 + R_2) = 20.0\ \Omega + (20.0\ \Omega) \times (30.0\ \Omega)/(50.0\ \Omega) = 32.0\ \Omega,$$

so $i_a = \varepsilon/R_{eq}$ = 120 V/32.0 Ω = 3.75 A.

(c) If all three switches $S_1$, $S_2$, and $S_3$ are closed, then $R_{eq} = R_1 + R_1\,R'/(R_1 + R')$ where

$$R' = R_2 + R_1\,(R_1 + R_2)/(2R_1 + R_2) = 22.0\ \Omega,$$

that is,

$$R_{eq} = 20.0\ \Omega + (20.0\ \Omega)\,(22.0\ \Omega)/(20.0\ \Omega + 22.0\ \Omega) = 30.5\ \Omega,$$

so $i_a = \varepsilon/R_{eq}$ = 120 V/30.5 Ω = 3.94 A.

72. (a) The four resistors $R_1$, $R_2$, $R_3$, and $R_4$ on the left reduce to

$$R_{\text{eq}} = R_{12} + R_{34} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = 7.0\,\Omega + 3.0\,\Omega = 10\,\Omega .$$

With $\varepsilon = 30$ V across $R_{\text{eq}}$ the current there is $i_2 = 3.0$ A.

(b) The three resistors on the right reduce to

$$R'_{\text{eq}} = R_{56} + R_7 = \frac{R_5 R_6}{R_5 + R_6} + R_7 = \frac{(6.0\,\Omega)(2.0\,\Omega)}{6.0\,\Omega + 2.0\,\Omega} + 1.5\,\Omega = 3.0\,\Omega .$$

With $\varepsilon = 30$ V across $R'_{\text{eq}}$ the current there is $i_4 = 10$ A.

(c) By the junction rule, $i_1 = i_2 + i_4 = 13$ A.

(d) By symmetry, $i_3 = \frac{1}{2} i_2 = 1.5$ A.

(e) By the loop rule (proceeding clockwise),

$$30V - i_4(1.5\,\Omega) - i_5(2.0\,\Omega) = 0$$

readily yields $i_5 = 7.5$ A.

73. **THINK** Since the wires are connected in series, the current is the same in both wires.

**EXPRESS** Let $i$ be the current in the wires and $V$ be the applied potential difference. Using Kirchhoff's loop rule, we have $V - iR_A - iR_B = 0$. Thus, the current is $i = V/(R_A + R_B)$, and the corresponding current density is

$$J = \frac{i}{A} = \frac{V}{R_A + R_B}.$$

**ANALYZE** (a) For wire $A$, the magnitude of the current density vector is

$$J_A = \frac{i}{A} = \frac{V}{(R_A + R_B)A} = \frac{4V}{(R_1 + R_2)\pi D^2} = \frac{4(60.0\text{V})}{\pi(0.127\Omega + 0.729\Omega)(2.60\times10^{-3}\text{m})^2}$$
$$= 1.32\times10^{7}\ \text{A/m}^2 .$$

(b) The potential difference across wire $A$ is

$V_A = iR_A = V\,R_A/(R_A + R_B) = (60.0\text{ V})(0.127\ \Omega)/(0.127\ \Omega + 0.729\ \Omega) = 8.90\text{ V}.$

(c) The resistivity of wire $A$ is

$$\rho_A = \frac{R_A A}{L_A} = \frac{\pi R_A D^2}{4L_A} = \frac{\pi(0.127\,\Omega)(2.60\times10^{-3}\text{ m})^2}{4(40.0\,\text{m})} = 1.69\times10^{-8}\,\Omega\cdot\text{m}.$$

So wire $A$ is made of copper.

(d) Since wire $B$ has the same length and diameter as wire $A$, and the currents are the same, we have $J_B = J_A = 1.32\times10^7\ \text{A/m}^2$.

(e) The potential difference across wire $B$ is $V_B = V - V_A = 60.0\text{ V} - 8.9\text{ V} = 51.1\text{ V}$.

(f) The resistivity of wire $B$ is

$$\rho_B = \frac{R_B A}{L_B} = \frac{\pi R_B D^2}{4L_B} = \frac{\pi(0.729\ \Omega)(2.60\times10^{-3}\text{ m})^2}{4(40.0\,\text{m})} = 9.68\times10^{-8}\,\Omega\cdot\text{m},$$

so wire $B$ is made of iron.

**LEARN** Resistance $R$ is the property of an object (depending on quantities such as $L$ and $A$), while resistivity is a property of the material itself. Knowing the value of $\rho$ allows us to deduce what material the wire is made of.

74. The resistor by the letter $i$ is above three other resistors; together, these four resistors are equivalent to a resistor $R = 10\ \Omega$ (with current $i$). As if we were presented with a maze, we find a path through $R$ that passes through any number of batteries (10, it turns out) but no other resistors, which — as in any good maze — winds "all over the place." Some of the ten batteries are opposing each other (particularly the ones along the outside), so that their net emf is only $\varepsilon = 40$ V.

(a) The current through $R$ is then $i = \varepsilon/R = 4.0$ A.

(b) The direction is upward in the figure.

75. (a) In the process described in the problem, no charge is gained or lost. Thus, $q$ = constant. Hence,

$$q = C_1V_1 = C_2V_2 \quad\Rightarrow\quad V_2 = V_1\frac{C_1}{C_2} = (200)\left(\frac{150}{10}\right) = 3.0\times10^3\,\text{V}.$$

(b) Equation 27-39, with $\tau = RC$, describes not only the discharging of $q$ but also of $V$. Thus,

$$V = V_0 e^{-t/\tau} \;\Rightarrow\; t = RC\ln\left(\frac{V_0}{V}\right) = \left(300\times10^{9}\ \Omega\right)\left(10\times10^{-12}\ \text{F}\right)\ln\left(\frac{3000}{100}\right)$$

which yields $t$ = 10 s. This is a longer time than most people are inclined to wait before going on to their next task (such as handling the sensitive electronic equipment).

(c) We solve $V = V_0 e^{-t/RC}$ for $R$ with the new values $V_0$ = 1400 V and $t$ = 0.30 s. Thus,

$$R = \frac{t}{C\ln(V_0/V)} = \frac{0.30\,\text{s}}{\left(10\times10^{-12}\ \text{F}\right)\ln(1400/100)} = 1.1\times10^{10}\ \Omega\ .$$

76. (a) We reduce the parallel pair of resistors (at the bottom of the figure) to a single $R'$ =1.00 Ω resistor and then reduce it with its series 'partner' (at the lower left of the figure) to obtain an equivalence of $R''$ = 2.00 Ω +1.00Ω =3.00 Ω. It is clear that the current through $R''$ is the $i_1$ we are solving for. Now, we employ the loop rule, choose a path that includes $R''$ and all the batteries (proceeding clockwise). Thus, assuming $i_1$ goes leftward through $R''$, we have

$$5.00\ \text{V} + 20.0\ \text{V} - 10.0\ \text{V} - i_1 R'' = 0$$

which yields $i_1$ = 5.00 A.

(b) Since $i_1$ is positive, our assumption regarding its direction (leftward) was correct.

(c) Since the current through the $\varepsilon_1$ = 20.0 V battery is "forward", battery 1 is supplying energy.

(d) The rate is $P_1$ = (5.00 A)(20.0 V) = 100 W.

(e) Reducing the parallel pair (which are in parallel to the $\varepsilon_2$ = 10.0 V battery) to a single $R'$ = 1.00 Ω resistor (and thus with current $i'$ = (10.0 V)/(1.00 Ω) = 10.0 A downward through it), we see that the current through the battery (by the junction rule) must be $i = i' - i_1$ = 5.00 A *upward* (which is the "forward" direction for that battery). Thus, battery 2 is supplying energy.

(f) Using Eq. 27-17, we obtain $P_2$ = 50.0 W.

(g) The set of resistors that are in parallel with the $\varepsilon_3$ = 5 V battery is reduced to $R'''$ = 0.800 Ω (accounting for the fact that two of those resistors are actually reduced in series, first, before the parallel reduction is made), which has current $i'''$ = (5.00 V)/(0.800 Ω) = 6.25 A downward through it. Thus, the current through the battery (by the junction rule) must be $i = i''' + i_1$ = 11.25 A *upward* (which is the "forward" direction for that battery). Thus, battery 3 is supplying energy.

(h) Equation 27-17 leads to $P_3$ = 56.3 W.

77. **THINK** The silicon resistor and the iron resistor are connected in series. Both resistors are temperature-dependent, but we want the combination to be independent of temperature.

**EXPRESS** We denote silicon with subscript *s* and iron with *i*. Let $T_0$ = 20°. The resistances of the two resistors can be written as

$$R_s(T)=R_s(T_0)\left[1+\alpha_s(T-T_0)\right], \qquad R_i(T)=R_i(T_0)\left[1+\alpha_i(T-T_0)\right].$$

The resistors are in series connection so

$$\begin{aligned}R(T)&=R_s(T)+R_i(T)=R_s(T_0)\left[1+\alpha_s(T-T_0)\right]+R_i(T_0)\left[1+\alpha_i(T-T_0)\right]\\&=R_s(T_0)+R_i(T_0)+\left[R_s(T_0)\alpha_s+R_i(T_0)\alpha_i\right](T-T_0).\end{aligned}$$

Now, if $R(T)$ is to be temperature-independent, we must require that $R_s(T_0)\alpha_s + R_i(T_0)\alpha_i = 0$. Also note that $R_s(T_0) + R_i(T_0) = R = 1000\ \Omega$.

**ANALYZE** (a) We solve for $R_s(T_0)$ and $R_i(T_0)$ to obtain

$$R_s(T_0)=\frac{R\alpha_i}{\alpha_i-\alpha_s}=\frac{(1000\Omega)\left(6.5\times10^{-3}/\text{K}\right)}{(6.5\times10^{-3}/\text{K})-(-70\times10^{-3}/\text{K})}=85.0\Omega.$$

(b) Similarly, $R_i(T_0) = 1000\ \Omega - 85.0\ \Omega = 915\ \Omega$.

**LEARN** The temperature independence of the combined resistor was possible because $\alpha_i$ and $\alpha_s$, the temperature coefficients of resistivity of the two materials have opposite signs, so their temperature dependences can cancel.

78. The current in the ammeter is given by

$$i_A = \varepsilon/(r + R_1 + R_2 + R_A).$$

The current in $R_1$ and $R_2$ without the ammeter is $i = \varepsilon/(r + R_1 + R_2)$. The percent error is then

$$\frac{\Delta i}{i}=\frac{i-i_A}{i}=1-\frac{r+R_1+R_2}{r+R_1+R_2+R_A}=\frac{R_A}{r+R_1+R_2+R_A}=\frac{0.10\Omega}{2.0\Omega+5.0\Omega+4.0\Omega+0.10\Omega}$$
$$=0.90\%.$$

79. **THINK** As the capacitor in an *RC* circuit is being charged, some energy supplied by the emf device also goes to the resistor as thermal energy.

**EXPRESS** The charge $q$ on the capacitor as a function of time is $q(t) = (\varepsilon C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\varepsilon/R)e^{-t/RC}$. The rate at which the emf device supplies energy is $P_\varepsilon = i\varepsilon dt$.

**ANALYZE** (a) The energy supplied by the emf is then

$$U = \int_0^\infty P_\varepsilon dt = \int_0^\infty \varepsilon i\, dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-t/RC} dt = C\varepsilon^2 = 2U_C$$

where $U_C = \frac{1}{2}C\varepsilon^2$ is the energy stored in the capacitor.

(b) By directly integrating $i^2R$ we obtain

$$U_R = \int_0^\infty i^2 R dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC} dt = \frac{1}{2}C\varepsilon^2.$$

**LEARN** Half of the energy supplied by the emf device is stored in the capacitor as electrical energy, while the other half is dissipated in the resistor as thermal energy.

80. In the steady state situation, there is no current going to the capacitors, so the resistors all have the same current. By the loop rule,

$$20.0 \text{ V} = (5.00\ \Omega)i + (10.0\ \Omega)i + (15.0\ \Omega)i$$

which yields $i = \frac{2}{3}$ A. Consequently, the voltage across the $R_1$ = 5.00 Ω resistor is (5.00 Ω)(2/3 A) = 10/3 V, and is equal to the voltage $V_1$ across the $C_1$ = 5.00 μF capacitor. Using Eq. 26-22, we find the stored energy on that capacitor:

$$U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(5.00\times10^{-6}\text{ F})\left(\frac{10}{3}\text{V}\right)^2 = 2.78\times10^{-5}\text{ J}.$$

Similarly, the voltage across the $R_2$ = 10.0 Ω resistor is (10.0 Ω)(2/3 A) = 20/3 V and is equal to the voltage $V_2$ across the $C_2$ = 10.0 μF capacitor. Hence,

$$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(10.0\times10^{-6}\text{ F})\left(\frac{20}{3}\text{V}\right)^2 = 2.22\times10^{-5}\text{ J}$$

Therefore, the total capacitor energy is $U_1 + U_2 = 2.50 \times 10^{-4}$ J.

81. The potential difference across $R_2$ is

$$V_2 = iR_2 = \frac{\varepsilon R_2}{R_1 + R_2 + R_3} = \frac{(12\,\text{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\,\text{V}.$$

82. From $V_a - \varepsilon_1 = V_c - ir_1 - iR$ and $i = (\varepsilon_1 - \varepsilon_2)/(R + r_1 + r_2)$, we get

$$\begin{aligned} V_a - V_c &= \varepsilon_1 - i(r_1 + R) = \varepsilon_1 - \left(\frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2}\right)(r_1 + R) \\ &= 4.4\,\text{V} - \left(\frac{4.4\,\text{V} - 2.1\,\text{V}}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega}\right)(2.3\,\Omega + 5.5\,\Omega) \\ &= 2.5\,\text{V}. \end{aligned}$$

83. **THINK** The time constant in an $RC$ circuit is $\tau = RC$, where $R$ is the resistance and $C$ is the capacitance. A greater value of $\tau$ means a longer discharging time.

**EXPRESS** The potential difference across the capacitor varies as a function of time $t$ as $V(t) = V_0 e^{-t/\tau}$, where $\tau = RC$. Thus, $R = \dfrac{t}{C \ln(V_0/V)}$.

**ANALYZE** (a) Then, for the smaller time interval $t_{\text{min}} = 10.0\ \mu\text{s}$

$$R_{\text{min}} = \frac{10.0\,\mu\text{s}}{(0.220\,\mu\text{F})\ln(5.00/0.800)} = 24.8\,\Omega.$$

(b) Similarly, for the larger time interval $t_{\text{max}} = 6.00$ ms,

$$R_{\text{max}} = \frac{6.00 \times 10^{-3}\,\text{s}}{(0.220\,\mu\text{F})\ln(5.00\,\text{V}/0.800\,\text{V})} = 1.49 \times 10^4\ \Omega.$$

**LEARN** The two extrema of the resistances are related by

$$\frac{R_{\text{max}}}{R_{\text{min}}} = \frac{t_{\text{max}}}{t_{\text{min}}}.$$

The larger the value of $R$ for a given capacitance, the longer the discharging time.

84. (a) Since $R_{\text{tank}} = 140\,\Omega, i = 12\,\text{V}/(10\,\Omega + 140\,\Omega) = 8.0 \times 10^{-2}\,\text{A}$.

(b) Now, $R_{\text{tank}} = (140\ \Omega + 20\ \Omega)/2 = 80\ \Omega$, so $i = 12\ \text{V}/(10\ \Omega + 80\ \Omega) = 0.13$ A.

(c) When full, $R_{\text{tank}} = 20\ \Omega$ so $i = 12\ \text{V}/(10\ \Omega + 20\ \Omega) = 0.40$ A.

85. **THINK** One of the three parts could be defective: the battery, the motor, or the cable.

**EXPRESS** All three circuit elements are connected in series, so the current is the same in all of them. The battery is discharging, so the potential drop across the terminals is $V_{\text{battery}} = \varepsilon - ir,$ where $\varepsilon$ is the emf and $r$ is the internal resistance. On the other hand, the resistances in the cable and the motor are $R_{\text{cable}} = V_{\text{cable}} / i$ and $R_{\text{motor}} = V_{\text{motor}} / i,$ respectively.

**ANALYZE** The internal resistance of the battery is

$$r = \frac{\varepsilon - V_{\text{battery}}}{i} = \frac{12\ \text{V} - 11.4\ \text{V}}{50\ \text{A}} = 0.012\ \Omega$$

which is less than 0.020 Ω. So the battery is OK. For the motor, we have

$$R_{\text{motor}} = \frac{V_{\text{motor}}}{i} = \frac{11.4\ \text{V} - 3.0\ \text{V}}{50\ \text{A}} = 0.17\ \Omega$$

which is less than 0.20 Ω. So the motor is OK. Now, the resistance of the cable is

$$R_{\text{cable}} = \frac{V_{\text{cable}}}{i} = \frac{3.0\ \text{V}}{50\ \text{A}} = 0.060\ \Omega$$

which is greater than 0.040 Ω. So the cable is defective.

**LEARN** In this exercise, we see that a defective component has a resistance outside its the range of acceptance.

86. When connected in series, the rate at which electric energy dissipates is $P_s = \varepsilon^2/(R_1 + R_2)$. When connected in parallel, the corresponding rate is $P_p = \varepsilon^2(R_1 + R_2)/R_1R_2$. Letting $P_p/P_s = 5$, we get $(R_1 + R_2)^2/R_1R_2 = 5$, where $R_1 = 100\ \Omega$. We solve for $R_2$: $R_2 = 38\ \Omega$ or 260 Ω.

(a) Thus, the smaller value of $R_2$ is 38 Ω.

(b) The larger value of $R_2$ is 260 Ω.

87. When $S$ is open for a long time, the charge on $C$ is $q_i = \varepsilon_2 C$. When $S$ is closed for a long time, the current $i$ in $R_1$ and $R_2$ is

$$i = (\varepsilon_2 - \varepsilon_1)/(R_1 + R_2) = (3.0\ \text{V} - 1.0\ \text{V})/(0.20\ \Omega + 0.40\ \Omega) = 3.33\ \text{A}.$$

The voltage difference $V$ across the capacitor is then

$$V = \varepsilon_2 - iR_2 = 3.0 \text{ V} - (3.33 \text{ A})(0.40\ \Omega) = 1.67 \text{ V}.$$

Thus the final charge on $C$ is $q_f = VC$. So the change in the charge on the capacitor is

$$\Delta q = q_f - q_i = (V - \varepsilon_2)C = (1.67 \text{ V} - 3.0 \text{ V})(10\ \mu\text{F}) = -13\ \mu\text{C}.$$

88. Using the junction and the loop rules, we have

$$20.0 - i_1R_1 - i_3R_3 = 0$$
$$20.0 - i_1R_1 - i_2R_2 - 50 = 0$$
$$i_2 + i_3 = i_1$$

Requiring no current through the battery 1 means that $i_1 = 0$, or $i_2 = i_3$. Solving the above equations with $R_1 = 10.0\Omega$ and $R_2 = 20.0\Omega$, we obtain

$$i_1 = \frac{40 - 3R_3}{20 + 3R_3} = 0 \quad \Rightarrow \quad R_3 = \frac{40}{3} = 13.3\,\Omega .$$

89. The bottom two resistors are in parallel, equivalent to a 2.0$R$ resistance. This, then, is in series with resistor $R$ on the right, so that their equivalence is $R' = 3.0R$. Now, near the top left are two resistors (2.0$R$ and 4.0$R$) that are in series, equivalent to $R'' = 6.0R$. Finally, $R'$ and $R''$ are in parallel, so the net equivalence is

$$R_{\text{eq}} = \frac{(R')\,(R'')}{R' + R''} = 2.0R = 20\ \Omega$$

where in the final step we use the fact that $R = 10\ \Omega$.

90. (a) Using Eq. 27-4, we take the derivative of the power $P = i^2R$ with respect to $R$ and set the result equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left(\frac{\varepsilon^2 R}{(R+r)^2}\right) = \frac{\varepsilon^2 (r - R)}{(R+r)^3} = 0$$

which clearly has the solution $R = r$.

(b) When $R = r$, the power dissipated in the external resistor equals

$$P_{\max} = \left.\frac{\varepsilon^2 R}{(R+r)^2}\right|_{R=r} = \frac{\varepsilon^2}{4r}.$$

91. (a) We analyze the lower left loop and find

$$i_1 = \varepsilon_1/R = (12.0 \text{ V})/(4.00\ \Omega) = 3.00 \text{ A}.$$

(b) The direction of $i_1$ is downward.

(c) Letting $R$ = 4.00 Ω, we apply the loop rule to the tall rectangular loop in the center of the figure (proceeding clockwise):

$$\varepsilon_2 + (+i_1R) + (-i_2R) + \left(-\frac{i_2}{2}R\right) + (-i_2R) = 0\,.$$

Using the result from part (a), we find $i_2$ = 1.60 A.

(d) The direction of $i_2$ is downward (as was assumed in writing the equation as we did).

(e) Battery 1 is supplying this power since the current is in the "forward" direction through the battery.

(f) We apply Eq. 27-17: The current through the $\varepsilon_1$ = 12.0 V battery is, by the junction rule, 3.00 A + 1.60 A = 4.60 A and

$$P = (4.60 \text{ A})(12.0 \text{ V}) = 55.2 \text{ W}.$$

(g) Battery 2 is supplying this power since the current is in the "forward" direction through the battery.

(h) $P = i_2(4.00 \text{ V}) = 6.40$ W.

92. The equivalent resistance of the series pair of $R_3 = R_4 = 2.0\ \Omega$ is $R_{34} = 4.0\ \Omega$, and the equivalent resistance of the parallel pair of $R_1 = R_2 = 4.0\ \Omega$ is $R_{12} = 2.0\ \Omega$. Since the voltage across $R_{34}$ must equal that across $R_{12}$:

$$V_{34} = V_{12} \quad \Rightarrow \quad i_{34}R_{34} = i_{12}R_{12} \quad \Rightarrow \quad i_{34} = \frac{1}{2}i_{12}$$

This relation, plus the junction rule condition $I = i_{12} + i_{34} = 6.00$ A, leads to the solution $i_{12} = 4.0$ A. It is clear by symmetry that $i_1 = i_{12}/2 = 2.00$ A.

93. (a) From $P = V^2/R$ we find $V = \sqrt{PR} = \sqrt{(10\,\text{W})(0.10\,\Omega)} = 1.0\,\text{V}$.

(b) From $i = V/R = (\varepsilon - V)/r$ we find

$$r = R\left(\frac{\varepsilon - V}{V}\right) = (0.10\,\Omega)\left(\frac{1.5\,\text{V} - 1.0\,\text{V}}{1.0\,\text{V}}\right) = 0.050\,\Omega.$$

94. (a) $R_{eq}(AB)$ = 20.0 Ω/3 = 6.67 Ω (three 20.0 Ω resistors in parallel).

(b) $R_{eq}(AC)$ = 20.0 Ω/3 = 6.67 Ω (three 20.0 Ω resistors in parallel).

(c) $R_{eq}(BC)$ = 0 (as *B* and *C* are connected by a conducting wire).

95. The maximum power output is (120 V)(15 A) = 1800 W. Since 1800 W/500 W = 3.6, the maximum number of 500 W lamps allowed is 3.

96. Here we denote the battery emf as *V*. Eq. 27-30 leads to

$$i = \frac{\varepsilon}{R} - \frac{q}{RC} = \frac{12\text{ V}}{4.0\,\Omega} - \frac{8.0\times10^{-6}\text{ C}}{(4.0\,\Omega)(4.0\times10^{-6}\text{F})} = 2.50\text{ A}.$$

97. **THINK** To calculate the current in the resistor *R*, we first find the equivalent resistance of the *N* batteries.

**EXPRESS** When all the batteries are connected in parallel, the emf is $\varepsilon$ and the equivalent resistance is $R_{\text{parallel}} = R + r/N,$ so the current is

$$i_{\text{parallel}} = \frac{\varepsilon}{R_{\text{parallel}}} = \frac{\varepsilon}{R + r/N} = \frac{N\varepsilon}{NR + r}.$$

Similarly, when all the batteries are connected in series, the total emf is $N\varepsilon$ and the equivalent resistance is $R_{\text{series}} = R + Nr.$ Therefore,

$$i_{\text{series}} = \frac{N\varepsilon}{R_{\text{series}}} = \frac{N\varepsilon}{R + Nr}.$$

**ANALYZE** Comparing the two expressions, we see that the two currents $i_{\text{parallel}}$ and $i_{\text{series}}$ are equal if $R = r,$ with

$$i_{\text{parallel}} = i_{\text{series}} = \frac{N\varepsilon}{(N+1)r}.$$

**LEARN** In general, the current difference is

$$i_{\text{parallel}} - i_{\text{series}} = \frac{N\varepsilon}{NR + r} - \frac{N\varepsilon}{R + Nr} = \frac{N\varepsilon(N-1)(r-R)}{(NR+r)(R+Nr)}.$$

If *R* > *r*, then $i_{\text{parallel}} < i_{\text{series}}$.

98. **THINK** The rate of energy supplied by the battery is $i\varepsilon$. So we first calculate the current in the circuit.

**EXPRESS** With $R_2$ and $R_3$ in parallel, and the combination in series with $R_1$, the equivalent resistance for the circuit is

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2 + R_3}$$

and the current is

$$i = \frac{\varepsilon}{R_{eq}} = \frac{(R_2 + R_3)\varepsilon}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

The rate at which the battery supplies energy is

$$P = i\varepsilon = \frac{(R_2 + R_3)\varepsilon^2}{R_1 R_2 + R_1 R_3 + R_2 R_3}.$$

To find the value of $R_3$ that maximizes $P$, we differentiate $P$ with respect to $R_3$.

**ANALYZE** (a) With a little algebra, we find

$$\frac{dP}{dR_3} = -\frac{R_2^2 \varepsilon^2}{(R_1 R_2 + R_1 R_3 + R_2 R_3)^2}.$$

The derivative is negative for all positive value of $R_3$. Thus, we see that $P$ is maximized when $R_3 = 0$.

(b) With the value of $R_3$ set to zero, we obtain $P = \frac{\varepsilon^2}{R_1} = \frac{(12.0\text{ V})^2}{10.0\,\Omega} = 14.4\text{ W}.$

**LEARN** Mathematically speaking, the function $P$ is a monotonically decreasing function of $R_3$ (as well as $R_2$ and $R_1$), so $P$ is a maximum at $R_3 = 0$.

99. **THINK** A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current.

**EXPRESS** The capacitor is *initially* uncharged. So immediately after the switch is closed, by the Kirchhoff's loop rule, there is zero voltage (at $t = 0$) across the $R_2 = 10$ kΩ resistor, and that $\varepsilon = 30$ V is across the $R_1 = 20$ kΩ resistor.

**ANALYZE** (a) By Ohm's law, the initial current in $R_1$ is

$$i_{10} = \varepsilon / R_1 = (30\text{ V})/(20\text{ k}\Omega) = 1.5 \times 10^{-3}\text{ A}.$$

(b) Similarly, the initial current in $R_2$ is $i_{20} = 0$.

(c) As $t \to \infty$ the current to the capacitor reduces to zero and the $R_1$ = 20 kΩ and $R_2$ = 10 kΩ resistors behave more like a series pair (having the same current), equivalent to

$$R_{eq} = R_1 + R_2 = 30\ \text{k}\Omega.$$

The current through them, then, at long times, is

$$i = \varepsilon / R_{eq} = (30\ \text{V})/(30\ \text{k}\Omega) = 1.0 \times 10^{-3}\ \text{A}.$$

**LEARN** A long time later after a capacitor is being fully charged, it acts like a broken wire.

100. (a) Reducing the bottom two series resistors to a single $R'$ = 4.00 Ω (with current $i_1$ through it), we see we can make a path (for use with the loop rule) that passes through $R$, the $\varepsilon_4$ = 5.00 V battery, the $\varepsilon_1$ = 20.0 V battery, and the $\varepsilon_3$ = 5.00 V. This leads to

$$i_1 = \frac{\varepsilon_1 + \varepsilon_3 + \varepsilon_4}{R'} = \frac{20.0\ \text{V} + 5.00\ \text{V} + 5.00\ \text{V}}{40.0\ \Omega} = \frac{30.0\ \text{V}}{4.0\ \Omega} = 7.50\ \text{A}.$$

(b) The direction of $i_1$ is leftward.

(c) The voltage across the bottom series pair is $i_1 R'$ = 30.0 V. This must be the same as the voltage across the two resistors directly above them, one of which has current $i_2$ through it and the other (by symmetry) has current $\frac{1}{2} i_2$ through it. Therefore,

$$30.0\ \text{V} = i_2\,(2.00\ \Omega) + \tfrac{1}{2} i_2\,(2.00\ \Omega)$$

which leads to $i_2$ = (30.0 V)/(3.00 Ω) = 10.0 A.

(d) The direction of $i_2$ is also leftward.

(e) We use Eq. 27-17: $P_4 = (i_1 + i_2)\varepsilon_4 = (7.50\ \text{A} + 10.0\ \text{A})(5.00\ \text{V}) = 87.5\ \text{W}$.

(f) The energy is being supplied to the circuit since the current is in the "forward" direction through the battery.

101. Consider the lowest branch with the two resistors $R_4$ = 3.00 Ω and $R_5$ = 5.00 Ω. The voltage difference across $R_5$ is

$$V = i_5 R_5 = \frac{\varepsilon R_5}{R_4 + R_5} = \frac{(120\text{V})(5.00\Omega)}{3.00\Omega + 5.00\Omega} = 7.50\text{V}.$$

102. (a) Here we denote the battery emf as $V$. See Fig. 27-4(a): $V_T = V - ir$.

(b) Doing a least squares fit for the $V_T$ versus $i$ values listed, we obtain

$$V_T = 13.61 - 0.0599i$$

which implies $V = 13.6$ V.

(c) It also implies the internal resistance is 0.060 Ω.

103. (a) The loop rule (proceeding counterclockwise around the right loop) leads to $\varepsilon_2 - i_1R_1 = 0$ (where $i_1$ was assumed downward). This yields $i_1 = 0.0600$ A.

(b) The direction of $i_1$ is downward.

(c) The loop rule (counterclockwise around the left loop) gives

$$(+\varepsilon_1)+(+i_1R_1)+(-i_2R_2)=0$$

where $i_2$ has been assumed leftward. This yields $i_3 = 0.180$ A.

(d) A positive value of $i_3$ implies that our assumption on the direction is correct, i.e., it flows leftward.

(e) The junction rule tells us that the current through the 12 V battery is 0.180 + 0.0600 = 0.240 A.

(f) The direction is upward.

104. (a) Since $P = \varepsilon^2/R_{eq}$, the higher the power rating the smaller the value of $R_{eq}$. To achieve this, we can let the low position connect to the larger resistance ($R_1$), middle position connect to the smaller resistance ($R_2$), and the high position connect to both of them in parallel.

(b) For $P = 300$ W, $R_{eq} = R_1R_2/(R_1 + R_2) = (144\ \Omega)R_2/(144\ \Omega + R_2) = (120\text{ V})^2/(300\text{ W})$. We obtain $R_2 = 72\ \Omega$.

(c) For $P = 100$ W, $R_{eq} = R_1 = \varepsilon^2/P = (120\text{ V})^2/100\text{ W} = 144\ \Omega$;

105. (a) The six resistors to the left of $\varepsilon_1 = 16$ V battery can be reduced to a single resistor $R = 8.0\ \Omega$, through which the current must be $i_R = \varepsilon_1/R = 2.0$ A. Now, by the loop rule, the current through the 3.0 Ω and 1.0 Ω resistors at the upper right corner is

$$i' = \frac{16.0\text{ V} - 8.0\text{ V}}{3.0\,\Omega + 1.0\,\Omega} = 2.0\text{ A}$$

in a direction that is “backward” relative to the $\varepsilon_2$ = 8.0 V battery. Thus, by the junction rule, $i_1 = i_R + i' = 4.0\,\text{A}$.

(b) The direction of $i_1$ is upward (that is, in the “forward” direction relative to $\varepsilon_1$).

(c) The current $i_2$ derives from a succession of symmetric splittings of $i_R$ (reversing the procedure of reducing those six resistors to find $R$ in part (a)). We find

$$i_2 = \frac{1}{2}\left(\frac{1}{2}i_R\right) = 0.50\,\text{A}.$$

(d) The direction of $i_2$ is clearly downward.

(e) Using our conclusion from part (a) in Eq. 27-17, we have

$$P = i_1\varepsilon_1 = (4.0\text{ A})(16\text{ V}) = 64\text{ W}.$$

(f) Using results from part (a) in Eq. 27-17, we obtain $P = i'\varepsilon_2$ = (2.0 A)(8.0 V) = 16 W.

(g) Energy is being supplied in battery 1.

(h) Energy is being absorbed in battery 2.

# Chapter 28

1. **THINK** The magnetic force on a charged parti014cle is given by $\vec{F}_B = q\vec{v} \times \vec{B},$ where $\vec{v}$ is the velocity of the charged particle and $\vec{B}$ is the magnetic field.

**EXPRESS** The magnitude of the magnetic force on the proton (of charge +*e*) is $F_B = evB\sin\phi,$ where $\phi$ is the angle between $\vec{v}$ and $\vec{B}.$

**ANALYZE** (a) The speed of the proton is

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50\times10^{-17}\,\text{N}}{\left(1.60\times10^{-19}\,\text{C}\right)\left(2.60\times10^{-3}\,\text{T}\right)\sin 23.0°} = 4.00\times10^5\ \text{m/s}.$$

(b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)\left(4.00\times10^5\ \text{m/s}\right)^2 = 1.34\times10^{-16}\ \text{J},$$

which is equivalent to

$$K = (1.34\times10^{-16}\ \text{J}) / (1.60\times10^{-19}\ \text{J/eV}) = 835\ \text{eV}.$$

**LEARN** from the definition of $\vec{B}$ given by the expression $\vec{F}_B = q\vec{v}\times\vec{B},$ we see that the magnetic force $\vec{F}_B$ is always perpendicular to $\vec{v}$ and $\vec{B}.$

2. The force associated with the magnetic field must point in the $\hat{\text{j}}$ direction in order to cancel the force of gravity in the $-\hat{\text{j}}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{\text{k}}$ direction (since $\hat{\text{i}}\times\left(-\hat{\text{k}}\right)=\hat{\text{j}}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 28-3 (with $\phi = 90°$). Therefore, with $m = 1.0\times10^{-2}$ kg, $v = 2.0\times10^4$ m/s, and $q = 8.0\times10^{-5}\,\text{C}$, we find

$$\vec{B} = B_z\hat{\text{k}} = -\left(\frac{mg}{qv}\right)\hat{\text{k}} = (-0.061\ \text{T})\hat{\text{k}}\ .$$

3. (a) The force on the electron is

$$\vec{F}_B = q\vec{v}\times\vec{B} = q\left(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}}\right)\times\left(B_x\hat{\mathrm{i}} + B_y\vec{j}\right) = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}}$$
$$= \left(-1.6\times10^{-19}\,\mathrm{C}\right)\left[\left(2.0\times10^{6}\,\mathrm{m/s}\right)\left(-0.15\,\mathrm{T}\right) - \left(3.0\times10^{6}\,\mathrm{m/s}\right)\left(0.030\,\mathrm{T}\right)\right]$$
$$= \left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.$$

Thus, the magnitude of $\vec{F}_B$ is 6.2 × $10^{14}$ N, and $\vec{F}_B$ points in the positive *z* direction.

(b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative *z* direction, namely, $\vec{F}_B = -\left(6.2\times10^{-14}\,\mathrm{N}\right)\hat{\mathrm{k}}.$

4. (a) We use Eq. 28-3:

$$F_B = |q|\, vB \sin\phi = (+\,3.2\times10^{-19}\ \mathrm{C})\ (550\ \mathrm{m/s})\ (0.045\ \mathrm{T})\ (\sin 52°) = 6.2\times10^{-18}\ \mathrm{N}.$$

(b) The acceleration is

$$a = F_B/m = (6.2\times10^{-18}\ \mathrm{N}) / (6.6\times10^{-27}\ \mathrm{kg}) = 9.5\times10^{8}\ \mathrm{m/s^2}.$$

(c) Since it is perpendicular to $\vec{v}, \vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

5. Using Eq. 28-2 and Eq. 3-30, we obtain

$$\vec{F} = q\left(v_xB_y - v_yB_x\right)\hat{\mathrm{k}} = q\left(v_x\left(3B_x\right) - v_yB_x\right)\hat{\mathrm{k}}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\,\hat{\mathrm{k}}$ where $F_z = 6.4\times10^{-19}$ N, then we are led to the condition

$$q\left(3v_x - v_y\right)B_x = F_z \quad\Rightarrow\quad B_x = \frac{F_z}{q\left(3v_x - v_y\right)}.$$

Substituting $v_x = 2.0$ m/s, $v_y = 4.0$ m/s, and $q = -1.6\times10^{-19}$ C, we obtain

$$B_x = \frac{F_z}{q(3v_x - v_y)} = \frac{6.4\times10^{-19}\ \mathrm{N}}{(-1.6\times10^{-19}\,\mathrm{C})[3(2.0\ \mathrm{m/s}) - 4.0\ \mathrm{m}]} = -2.0\ \mathrm{T}.$$

6. The magnetic force on the proton is given by $\vec{F} = q\vec{v}\times\vec{B},$ where $q = +e$ . Using Eq. 3-30 this becomes

$(4 \times 10^{-17})\hat{\text{i}} + (2 \times 10^{-17})\hat{\text{j}} = e[(0.03v_y + 40)\hat{\text{i}} + (20 - 0.03v_x)\hat{\text{j}} - (0.02v_x + 0.01v_y)\hat{\text{k}}]$

with SI units understood. Equating corresponding components, we find

(a) $v_x = -3.5\times10^3$ m/s, and

(b) $v_y = 7.0\times10^3$ m/s.

7. We apply $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = m_e\vec{a}$ to solve for $\vec{E}$:

$$\begin{aligned}\vec{E} &= \frac{m_e\vec{a}}{q} + \vec{B}\times\vec{v} \\ &= \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.00\times10^{12}\text{ m/s}^2\right)\hat{\text{i}}}{-1.60\times10^{-19}\,\text{C}} + \left(400\mu\text{T}\right)\hat{\text{i}}\times\ \left(12.0\,\text{km/s}\right)\hat{\text{j}} + \left(15.0\,\text{km/s}\right)\hat{\text{k}} \\ &= \left(-11.4\hat{\text{i}} - 6.00\hat{\text{j}} + 4.80\hat{\text{k}}\right)\text{V/m}.\end{aligned}$$

8. Letting $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = 0$, we get $vB\sin\phi = E$. We note that (for given values of the fields) this gives a minimum value for speed whenever the sin $\phi$ factor is at its maximum value (which is 1, corresponding to $\phi = 90°$). So

$$v_{\min} = \frac{E}{B} = \frac{1.50\times10^3\text{ V/m}}{0.400\text{ T}} = 3.75\times10^3\text{ m/s}.$$

9. Straight-line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) = 0$. Note that $\vec{v}\perp\vec{B}$ so $\left|\vec{v} \times \vec{B}\right| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$B = \frac{E}{v} = \frac{E}{\sqrt{2K/m_e}} = \frac{100\text{ V}/(20\times10^{-3}\text{m})}{\sqrt{2\left(1.0\times10^3\text{V}\right)\left(1.60\times10^{-19}\text{C}\right)/\left(9.11\times10^{-31}\text{kg}\right)}} = 2.67\times10^{-4}\text{T}.$$

In unit-vector notation, $\vec{B} = -(2.67\times10^{-4}\text{ T})\hat{\text{k}}$.

10. (a) The net force on the proton is given by

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B} = \left(1.60\times10^{-19}\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{\text{k}} + \left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\times10^{-3}\text{T}\right)\hat{\text{i}}\right] \\ &= \left(1.44\times10^{-18}\,\text{N}\right)\hat{\text{k}}.\end{aligned}$$

(b) In this case, we have

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}$$
$$= \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(-4.00\,\text{V/m}\right)\hat{\text{k}} + \left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\,\text{mT}\right)\hat{\text{i}}\right]$$
$$= \left(1.60\times10^{-19}\,\text{N}\right)\hat{\text{k}}.$$

(c) In the final case, we have

$$\vec{F} = \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}$$
$$= \left(1.60\times10^{-19}\,\text{C}\right)\left[\left(4.00\,\text{V/m}\right)\hat{\text{i}} + \left(2000\,\text{m/s}\right)\hat{\text{j}}\times\left(-2.50\,\text{mT}\right)\hat{\text{i}}\right]$$
$$= \left(6.41\times10^{-19}\,\text{N}\right)\hat{\text{i}} + \left(8.01\times10^{-19}\,\text{N}\right)\hat{\text{k}}.$$

11. Since the total force given by $\vec{F} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v}\times\vec{B}$ has magnitude $vB$ and the magnitude of the electric field is given by $E = vB$. Since the particle has charge $e$ and is accelerated through a potential difference $V$, $mv^2/2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = \left(1.2\,\text{T}\right)\sqrt{\frac{2\left(1.60\times10^{-19}\,\text{C}\right)\left(10\times10^{3}\,\text{V}\right)}{\left(9.99\times10^{-27}\,\text{kg}\right)}} = 6.8\times10^{5}\,\text{V/m}.$$

12. (a) The force due to the electric field ($\vec{F} = q\vec{E}$) is distinguished from that associated with the magnetic field ($\vec{F} = q\vec{v}\times\vec{B}$) in that the latter vanishes when the speed is zero and the former is independent of speed. The graph shows that the force ($y$-component) is negative at $v = 0$ (specifically, its value is $-2.0\times10^{-19}$ N there), which (because $q = -e$) implies that the electric field points in the $+y$ direction. Its magnitude is

$$E = \frac{F_{\text{net},y}}{|q|} = \frac{2.0\times10^{-19}\,\text{N}}{1.6\times10^{-19}\,\text{C}} = 1.25\,\text{N/C} = 1.25\,\text{V/m}\,.$$

(b) We are told that the $x$ and $z$ components of the force remain zero throughout the motion, implying that the electron continues to move along the $x$ axis, even though magnetic forces generally cause the paths of charged particles to curve (Fig. 28-11). The exception to this is discussed in Section 28-3, where the forces due to the electric and magnetic fields cancel. This implies (Eq. 28-7) $B = E/v = 2.50\times10^{-2}$ T.

For $\vec{F} = q\vec{v}\times\vec{B}$ to be in the opposite direction of $\vec{F} = q\vec{E}$ we must have $\vec{v}\times\vec{B}$ in the opposite direction from $\vec{E}$, which points in the $+y$ direction, as discussed in part (a). Since the velocity is in the $+x$ direction, then (using the right-hand rule) we conclude that the magnetic field must point in the +z direction ( $\hat{\text{i}} \times \hat{\text{k}} = -\hat{\text{j}}$ ). In unit-vector notation, we have $\vec{B} = (2.50\times10^{-2}\ \text{T})\hat{\text{k}}$ .

13. We use Eq. 28-12 to solve for $V$:

$$V = \frac{iB}{nle} = \frac{(23\text{A})(0.65\,\text{T})}{(8.47\times10^{28}/\text{m}^3)(150\mu\text{m})(1.6\times10^{-19}\text{C})} = 7.4\times10^{-6}\,\text{V}.$$

14. For a free charge $q$ inside the metal strip with velocity $\vec{v}$ we have $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$. We set this force equal to zero and use the relation between (uniform) electric field and potential difference. Thus,

$$v = \frac{E}{B} = \frac{|V_x - V_y|/d_{xy}}{B} = \frac{(3.90\times10^{-9}\,\text{V})}{(1.20\times10^{-3}\text{T})(0.850\times10^{-2}\,\text{m})} = 0.382\,\text{m/s}.$$

15. (a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). With Eq. 28-10, we define the magnitude of the electric field as

$$|\vec{E}| = v|\vec{B}| = (20.0\ \text{m/s})(0.030\ \text{T}) = 0.600\ \text{V/m}\,.$$

Its direction may be inferred from Figure 28-8; its direction is opposite to that defined by $\vec{v}\times\vec{B}$. In summary,

$$\vec{E} = -(0.600\,\text{V/m})\hat{\text{k}}$$

which insures that $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$ vanishes.

(b) Equation 28-9 yields $V = Ed = (0.600\ \text{V/m})(2.00\ \text{m}) = 1.20\ \text{V}$ .

16. We note that $\vec{B}$ must be along the $x$ axis because when the velocity is along that axis there is no induced voltage. Combining Eq. 28-7 and Eq. 28-9 leads to

$$d = \frac{V}{E} = \frac{V}{vB}$$

where one must interpret the symbols carefully to ensure that $\vec{d}, \vec{v},$ and $\vec{B}$ are mutually perpendicular. Thus, when the velocity if parallel to the $y$ axis the absolute value of the voltage (which is considered in the same "direction" as $\vec{d}$ ) is 0.012 V, and

$$d = d_z = \frac{0.012\ \text{V}}{(3.0\ \text{m/s})(0.020\ \text{T})} = 0.20\ \text{m}\,.$$

On the other hand, when the velocity is parallel to the $z$ axis the absolute value of the appropriate voltage is 0.018 V, and

$$d = d_y = \frac{0.018\ \text{V}}{(3.0\ \text{m/s})(0.020\ \text{T})} = 0.30\ \text{m}.$$

Thus, our answers are

(a) $d_x$ = 25 cm (which we arrive at "by elimination," since we already have figured out $d_y$ and $d_z$ ),

(b) $d_y$ = 30 cm, and

(c) $d_z$ = 20 cm.

17. (a) Using Eq. 28-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\text{u}} = \frac{2\left(4.50\times10^{-2}\ \text{m}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(1.20\,\text{T}\right)}{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)} = 2.60\times10^{6}\ \text{m/s}\ .$$

(b) $T = 2\pi r/v = 2\pi(4.50 \times 10^{-2}\ \text{m})/(2.60 \times 10^{6}\ \text{m/s}) = 1.09 \times 10^{-7}$ s.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2}m_\alpha v^2 = \frac{\left(4.00\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)\left(2.60\times10^{6}\ \text{m/s}\right)^2}{2\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 1.40\times10^{5}\,\text{eV}\ .$$

(d) $\Delta V = K/q = 1.40 \times 10^{5}\ \text{eV}/2e = 7.00 \times 10^{4}$ V.

18. With the $\vec{B}$ pointing "out of the page," we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent toward the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 28-3 (with angle $\phi$ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e|\vec{B}|} = 4.99\times10^{6}\ \text{m/s}.$$

(b) Using either Eq. 28-14 or Eq. 28-16, we find $r$ = 0.00710 m.

(c) Using Eq. 28-17 (in either its first or last form) readily yields $T = 8.93 \times 10^{-9}$ s.

19. Let ξ stand for the ratio ($m/|q|$) we wish to solve for. Then Eq. 28-17 can be written as $T = 2\pi\xi/B$. Noting that the horizontal axis of the graph (Fig. 28-37) is inverse-field ($1/B$) then we conclude (from our previous expression) that the slope of the line in the graph must be equal to $2\pi\xi$. We estimate that slope is $7.5 \times 10^{-9}$ T·s, which implies

$$\xi = m/|q| = 1.2\times10^{-9}\ \text{kg/C}.$$

20. Combining Eq. 28-16 with energy conservation ($eV = \frac{1}{2} m_e v^2$ in this particular application) leads to the expression

$$r = \frac{m_e}{eB}\sqrt{\frac{2eV}{m_e}}$$

which suggests that the slope of the $r$ versus $\sqrt{V}$ graph should be $\sqrt{2m_e/eB^2}$. From Fig. 28-38, we estimate the slope to be $5 \times 10^{-5}$ in SI units. Setting this equal to $\sqrt{2m_e/eB^2}$ and solving, we find $B = 6.7 \times 10^{-2}$ T.

21. **THINK** The electron is in circular motion because the magnetic force acting on it points toward the center of the circle.

**EXPRESS** The kinetic energy of the electron is given by $K = \frac{1}{2}m_e v^2$, where $m_e$ is the mass of electron and $v$ is its speed. The magnitude of the magnetic force on the electron is $F_B = evB$ which is equal to the centripetal force:

$$evB = \frac{m_e v^2}{r}.$$

**ANALYZE** (a) From $K = \frac{1}{2}m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20\times10^3\,\text{eV})(1.60\times10^{-19}\ \text{eV/J})}{9.11\times10^{-31}\,\text{kg}}} = 2.05\times10^7\ \text{m/s}.$$

(b) Since $evB = m_e v^2/r$, we find the magnitude of the magnetic field to be

$$B = \frac{m_e v}{er} = \frac{(9.11\times10^{-31}\text{kg})(2.05\times10^7\ \text{m/s})}{(1.60\times10^{-19}\text{C})(25.0\times10^{-2}\,\text{m})} = 4.67\times10^{-4}\ \text{T}.$$

(c) The "orbital" frequency is

$$f = \frac{v}{2\pi r} = \frac{2.07\times10^{7}\ \text{m/s}}{2\pi\left(25.0\times10^{-2}\text{m}\right)} = 1.31\times10^{7}\ \text{Hz}.$$

(d) The period is simply equal to the reciprocal of frequency:

$$T = 1/f = (1.31 \times 10^{7}\ \text{Hz})^{-1} = 7.63 \times 10^{-8}\ \text{s}.$$

**LEARN** The period of the electron's circular motion can be written as

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|e|B} = \frac{2\pi m}{|e|B}.$$

The period is inversely proportional to $B$.

22. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $K = (rqB)^2/2m \propto q^2m^{-1}$.

(a) $K_\alpha = \left(q_\alpha/q_p\right)^2\left(m_p/m_\alpha\right)K_p = (2)^2(1/4)K_p = K_p = 1.0\text{MeV};$

(b) $K_d = \left(q_d/q_p\right)^2\left(m_p/m_d\right)K_p = (1)^2(1/2)K_p = 1.0\ \text{MeV}/2 = 0.50\text{MeV}.$

23. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(1.30\times10^{6}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(0.350\ \text{m}\right)} = 2.11\times10^{-5}\ \text{T}.$$

24. (a) The accelerating process may be seen as a conversion of potential energy $eV$ into kinetic energy. Since it starts from rest, $\frac{1}{2}m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2\left(1.60\times10^{-19}\text{C}\right)\left(350\ \text{V}\right)}{9.11\times10^{-31}\text{kg}}} = 1.11\times10^{7}\ \text{m/s}.$$

(b) Equation 28-16 gives

$$r = \frac{m_e v}{eB} = \frac{\left(9.11\times10^{-31}\text{kg}\right)\left(1.11\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{C}\right)\left(200\times10^{-3}\text{T}\right)} = 3.16\times10^{-4}\text{m.}$$

25. (a) The frequency of revolution is

$$f = \frac{Bq}{2\pi m_e} = \frac{\left(35.0\times10^{-6}\text{T}\right)\left(1.60\times10^{-19}\text{C}\right)}{2\pi\left(9.11\times10^{-31}\text{kg}\right)} = 9.78\times10^{5}\text{Hz.}$$

(b) Using Eq. 28-16, we obtain

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_e K}}{qB} = \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(100\text{eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}}{\left(1.60\times10^{-19}\text{ C}\right)\left(35.0\times10^{-6}\text{T}\right)} = 0.964\text{ m .}$$

26. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v}\times\vec{B}$ points leftward, which indeed seems to be the direction it is "pushed''; therefore, $q > 0$ (it is a proton).

(a) Equation 28-17 becomes $T = 2\pi m_p / e|\vec{B}|$, or

$$2\left(130\times10^{-9}\right) = \frac{2\pi\left(1.67\times10^{-27}\right)}{\left(1.60\times10^{-19}\right)|\vec{B}|}$$

which yields $\left|\vec{B}\right| = 0.252\text{T}$.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$ . Since the period $T$ does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$ ). Thus, $t = T/2 = 130$ ns.

27. (a) We solve for $B$ from $m = B^2qx^2/8V$ (see Sample Problem 28.04 — "Uniform circular motion of a charged particle in a magnetic field"):

$$B = \sqrt{\frac{8Vm}{qx^2}}\ .$$

We evaluate this expression using $x = 2.00$ m:

$$B = \sqrt{\frac{8\left(100\times10^{3}\text{ V}\right)\left(3.92\times10^{-25}\text{ kg}\right)}{\left(3.20\times10^{-19}\text{ C}\right)\left(2.00\text{ m}\right)^2}} = 0.495\text{T .}$$

(b) Let $N$ be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where $m$ is the mass of a single ion. $M$ has the value

$$M = \frac{100\times10^{-6}\ \text{kg}}{3600\,\text{s}} = 2.78\times10^{-8}\ \text{kg/s} .$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{\left(3.20\times10^{-19}\ \text{C}\right)\left(2.78\times10^{-8}\ \text{kg/s}\right)}{3.92\times10^{-25}\ \text{kg}} = 2.27\times10^{-2}\ \text{A} .$$

(c) Each ion deposits energy $qV$ in the cup, so the energy deposited in time $\Delta t$ is given by

$$E = NqV\,\Delta t = \frac{iqV}{q}\Delta t = iV\,\Delta t .$$

For $\Delta t$ = 1.0 h,

$$E = \left(2.27\times10^{-2}\ \text{A}\right)\left(100\times10^{3}\ \text{V}\right)\left(3600\,\text{s}\right) = 8.17\times10^{6}\ \text{J} .$$

To obtain the second expression, $i/q$ is substituted for $N$.

28. Using $F = mv^2/r$ (for the centripetal force) and $K = mv^2/2$, we can easily derive the relation

$$K = \frac{1}{2}Fr.$$

With the values given in the problem, we thus obtain $K = 2.09 \times 10^{-22}$ J.

29. Reference to Fig. 28-11 is very useful for interpreting this problem. The distance traveled parallel to $\vec{B}$ is $d_{\parallel} = v_{\parallel}T = v_{\parallel}(2\pi m_e/|q|B)$ using Eq. 28-17. Thus,

$$v_{\parallel} = \frac{d_{\parallel}eB}{2\pi m_e} = 50.3\ \text{km/s}$$

using the values given in this problem. Also, since the magnetic force is $|q|Bv_{\perp}$, then we find $v_{\perp}$ = 41.7 km/s. The speed is therefore $v = \sqrt{v_{\perp}^2 + v_{\parallel}^2}$ = 65.3 km/s.

30. Eq. 28-17 gives $T = 2\pi m_e/eB$. Thus, the total time is

$$\left(\frac{T}{2}\right)_1 + t_{\text{gap}} + \left(\frac{T}{2}\right)_2 = \frac{\pi m_e}{e}\left(\frac{1}{B_1} + \frac{1}{B_2}\right) + t_{\text{gap}} .$$

The time spent in the gap (which is where the electron is accelerating in accordance with Eq. 2-15) requires a few steps to figure out: letting $t = t_{gap}$ then we want to solve

$$d = v_0 t + \frac{1}{2}at^2 \;\Rightarrow\; 0.25\text{ m} = \sqrt{\frac{2K_0}{m_e}}\,t + \frac{1}{2}\left(\frac{e\Delta V}{m_e d}\right)t^2$$

for $t$. We find in this way that the time spent in the gap is $t \approx 6$ ns. Thus, the total time is 8.7 ns.

31. Each of the two particles will move in the same circular path, initially going in the opposite direction. After traveling half of the circular path they will collide. Therefore, using Eq. 28-17, the time is given by

$$t = \frac{T}{2} = \frac{\pi m}{Bq} = \frac{\pi\left(9.11\times10^{-31}\text{kg}\right)}{(3.53\times10^{-3}\text{T})\left(1.60\times10^{-19}\text{ C}\right)} = 5.07\times10^{-9}\text{ s}.$$

32. Let $v_{\parallel} = v\cos\theta$. The electron will proceed with a uniform speed $v_{\parallel}$ in the direction of $\vec{B}$ while undergoing uniform circular motion with frequency $f$ in the direction perpendicular to $B$: $f = eB/2\pi m_e$. The distance $d$ is then

$$d = v_{\parallel}T = \frac{v_{\parallel}}{f} = \frac{(v\cos\theta)2\pi m_e}{eB} = \frac{2\pi\left(1.5\times10^{7}\text{ m/s}\right)\left(9.11\times10^{-31}\text{kg}\right)\left(\cos10^\circ\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.0\times10^{-3}\text{T}\right)} = 0.53\text{m}.$$

33. **THINK** The path of the positron is helical because its velocity $\vec{v}$ has components parallel and perpendicular to the magnetic field $\vec{B}$.

**EXPRESS** If $v$ is the speed of the positron then $v\sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here $\phi = 89°$ is the angle between the velocity and the field. Newton's second law yields $eBv\sin\phi = m_e(v\sin\phi)^2/r$, where $r$ is the radius of the orbit. Thus $r = (m_e v/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m_e}{eB}.$$

The equation for $r$ is substituted to obtain the second expression for $T$. For part (b), the pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$.

**ANALYZE** (a) Substituting the values given, we find the period to be

$$T = \frac{2\pi m_e}{eB} = \frac{2\pi\left(9.11\times10^{-31}\,\text{kg}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(0.100\text{T}\right)} = 3.58\times10^{-10}\,\text{s}.$$

(b) We use the kinetic energy, $K = \frac{1}{2}m_e v^2$, to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2\left(2.00\times10^3\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{9.11\times10^{-31}\,\text{kg}}} = 2.65\times10^7\ \text{m/s}\,.$$

Thus, the pitch is $p = \left(2.65\times10^7\ \text{m/s}\right)\left(3.58\times10^{-10}\ \text{s}\right)\cos 89° = 1.66\times10^{-4}\,\text{m}$ .

(c) The orbit radius is

$$R = \frac{m_e v\sin\phi}{eB} = \frac{\left(9.11\times10^{-31}\,\text{kg}\right)\left(2.65\times10^7\ \text{m/s}\right)\sin 89°}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.100\ \text{T}\right)} = 1.51\times10^{-3}\,\text{m}\,.$$

**LEARN** The parallel component of the velocity, $v_{\parallel} = v\cos\phi,$ is what determines the pitch of the helix. On the other hand, the perpendicular component, $v_{\perp} = v\sin\phi,$ determines the radius of the helix.

34. (a) Equation 3-20 gives $\phi = \cos^{-1}(2/19) = 84°$.

(b) No, the magnetic field can only change the direction of motion of a free (unconstrained) particle, not its speed or its kinetic energy.

(c) No, as reference to Fig. 28-11 should make clear.

(d) We find $v_{\perp} = v \sin \phi = 61.3$ m/s, so $r = mv_{\perp}/eB =$ 5.7 nm.

35. (a) By conservation of energy (using $qV$ for the potential energy, which is converted into kinetic form) the kinetic energy gained in each pass is 200 eV.

(b) Multiplying the part (a) result by $n = 100$ gives $\Delta K = n(200\text{ eV}) = 20.0$ keV.

(c) Combining Eq. 28-16 with the kinetic energy relation ($n(200\text{ eV}) = m_p v^2/2$ in this particular application) leads to the expression

$$r = \frac{m_p}{e\,B}\sqrt{\frac{2n(200\text{ eV})}{m_p}}$$

which shows that $r$ is proportional to $\sqrt{n}$. Thus, the percent increase defined in the problem in going from $n = 100$ to $n = 101$ is $\sqrt{101/100} - 1 = 0.00499$ or 0.499%.

36. (a) The magnitude of the field required to achieve resonance is

$$B = \frac{2\pi f m_p}{q} = \frac{2\pi(12.0\times10^6 \text{ Hz})\left(1.67\times10^{-27}\text{ kg}\right)}{1.60\times10^{-19}\text{ C}} = 0.787\text{T}.$$

(b) The kinetic energy is given by

$$K = \tfrac{1}{2}mv^2 = \frac{1}{2}m\left(2\pi Rf\right)^2 = \frac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)4\pi^2(0.530 \text{ m})^2(12.0\times10^6 \text{ Hz})^2$$
$$= 1.33\times10^{-12}\text{ J} = 8.34\times10^6\text{ eV}.$$

(c) The required frequency is

$$f = \frac{qB}{2\pi m_p} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(1.57\text{T}\right)}{2\pi\left(1.67\times10^{-27}\text{ kg}\right)} = 2.39\times10^7\text{ Hz}.$$

(d) The kinetic energy is given by

$$K = \tfrac{1}{2}mv^2 = \frac{1}{2}m\left(2\pi Rf\right)^2 = \frac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)4\pi^2(0.530 \text{ m})^2(2.39\times10^7 \text{ Hz})^2$$
$$= 5.3069\times10^{-12}\text{ J} = 3.32\times10^7\text{ eV}.$$

37. We approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle, and each time it receives an energy of $qV = 80 \times 10^3$ eV. Since its final energy is 16.6 MeV, the number of revolutions it makes is

$$n = \frac{16.6\times10^6\text{ eV}}{2\left(80\times10^3\text{eV}\right)} = 104 .$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where $v$ is the deuteron's speed. Since this is given by $v = \sqrt{2K/m}$, the radius is

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} .$$

For the average energy

$$r = \frac{\sqrt{2\left(8.3\times10^6\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)\left(3.34\times10^{-27}\text{ kg}\right)}}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.57\text{T}\right)} = 0.375\text{m} .$$

The total distance traveled is about

$$n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2 \text{ m}.$$

38. (a) Using Eq. 28-23 and Eq. 28-18, we find

$$f_{\text{osc}} = \frac{qB}{2\pi m_p} = \frac{\left(1.60\times10^{-19}\,\text{C}\right)\left(1.20\text{T}\right)}{2\pi\left(1.67\times10^{-27}\,\text{kg}\right)} = 1.83\times10^{7}\,\text{Hz}.$$

(b) From $r = m_p v/qB = \sqrt{2m_P k}/qB$ we have

$$K = \frac{\left(rqB\right)^2}{2m_p} = \frac{\left[\left(0.500\text{m}\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(1.20\text{T}\right)\right]^2}{2\left(1.67\times10^{-27}\,\text{kg}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} = 1.72\times10^{7}\,\text{eV}.$$

39. **THINK** The magnetic force on a wire that carries a current *i* is given by $\vec{F}_B = i\vec{L}\times\vec{B}$, where $\vec{L}$ is the length vector of the wire and $\vec{B}$ is the magnetic field.

**EXPRESS** The magnitude of the magnetic force on the wire is given by $F_B = iLB \sin \phi$, where $\phi$ is the angle between the current and the field.

**ANALYZE** (a) With $\phi = 70°$, we have

$$F_B = \left(5000\,\text{A}\right)\left(100\,\text{m}\right)\left(60.0\times10^{-6}\,\text{T}\right)\sin 70° = 28.2\,\text{N}.$$

(b) We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L}\times\vec{B}$ to show that the force is to the west.

**LEARN** From the expression $\vec{F}_B = i\vec{L}\times\vec{B}$, we see that the magnetic force acting on a current-carrying wire is a maximum when $\vec{L}$ is perpendicular to $\vec{B}$ ($\phi = 90°$), and is zero when $\vec{L}$ is parallel to $\vec{B}$ ($\phi = 0°$).

40. The magnetic force on the (straight) wire is

$$F_B = iBL\sin\theta = \left(13.0\text{A}\right)\left(1.50\text{T}\right)\left(1.80\text{m}\right)\left(\sin 35.0°\right) = 20.1\text{N}.$$

41. (a) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force *mg* on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where $L$ is the length of the wire. Thus,

$$iLB = mg \quad \Rightarrow \quad i = \frac{mg}{LB} = \frac{(0.0130\,\text{kg})(9.8\,\text{m/s}^2)}{(0.620\,\text{m})(0.440\,\text{T})} = 0.467\,\text{A}.$$

(b) Applying the right-hand rule reveals that the current must be from left to right.

42. (a) From symmetry, we conclude that any *x*-component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the $\hat{k}$ direction produces on each part of the bent wire a *y*-component of force pointing in the $-\hat{j}$ direction; each of these components has magnitude

$$|F_y| = i\ell\,|\vec{B}|\sin 30° = (2.0\text{ A})(2.0\text{ m})(4.0\text{ T})\sin 30° = 8\,\text{N}.$$

Therefore, the force on the wire shown in the figure is $(-16\hat{j})\text{ N}$.

(b) The force exerted on the left half of the bent wire points in the $-\hat{k}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{k}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

43. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50$ cm side runs along the +*y* axis, while the $\ell_x = 120$ cm side runs along the +*x* axis. The angle made by the hypotenuse (of length 130 cm) is

$$\theta = \tan^{-1}(50/120) = 22.6°,$$

relative to the 120 cm side. If one measures the angle counterclockwise from the +*x* direction, then the angle for the hypotenuse is 180° – 22.6° = +157°. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the +*z* axis. We take $\vec{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750\,\text{T}$,

$$B_x = -B\cos\theta = -0.0692\,\text{T}\ , \quad B_y = B\sin\theta = 0.0288\,\text{T}.$$

(a) Equation 28-26 produces zero force when $\vec{L}\,\|\,\vec{B}$ so there is no force exerted on the hypotenuse of length 130 cm.

(b) On the 50 cm side, the $B_x$ component produces a force $i\ell_y B_x\hat{k}$, and there is no contribution from the $B_y$ component. Using SI units, the magnitude of the force on the $\ell_y$ side is therefore

$$(4.00\,\text{A})(0.500\,\text{m})(0.0692\,\text{T}) = 0.138\,\text{N}.$$

(c) On the 120 cm side, the $B_y$ component produces a force $i\ell_x B_y \hat{\text{k}}$, and there is no contribution from the $B_x$ component. The magnitude of the force on the $\ell_x$ side is also

$$(4.00\,\text{A})(1.20\,\text{m})(0.0288\,\text{T}) = 0.138\,\text{N}.$$

(d) The net force is

$$i\ell_y B_x \hat{\text{k}} + i\ell_x B_y \hat{\text{k}} = 0,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\vec{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$, but $B_y < 0$ and a zero net force would still be the result.

44. Consider an infinitesimal segment of the loop, of length $ds$. The magnetic field is perpendicular to the segment, so the magnetic force on it has magnitude $dF = iB\,ds$. The horizontal component of the force has magnitude

$$dF_h = (iB\cos\theta)ds$$

and points inward toward the center of the loop. The vertical component has magnitude

$$dF_y = (iB\sin\theta)ds$$

and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$\begin{aligned} F_v &= iB\sin\theta\int ds = 2\pi aiB\sin\theta = 2\pi(0.018\text{ m})(4.6\times10^{-3}\text{ A})(3.4\times10^{-3}\text{ T})\sin 20° \\ &= 6.0\times10^{-7}\text{ N}. \end{aligned}$$

We note that $i$, $B$, and $\theta$ have the same value for every segment and so can be factored from the integral.

45. The magnetic force on the wire is

$$\begin{aligned} \vec{F}_B &= i\vec{L}\times\vec{B} = iL\hat{\text{i}}\times\left(B_y\hat{\text{j}} + B_z\hat{\text{k}}\right) = iL\left(-B_z\hat{\text{j}} + B_y\hat{\text{k}}\right) \\ &= (0.500\text{A})\,(0.500\text{m})\left[-(0.0100\text{T})\,\hat{\text{j}} + (0.00300\text{T})\,\hat{\text{k}}\right] \\ &= \left(-2.50\times10^{-3}\hat{\text{j}} + 0.750\times10^{-3}\,\hat{\text{k}}\right)\text{N}. \end{aligned}$$

46. (a) The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus

$$v = at = \frac{F_B t}{m} = \frac{idBt}{m} = \frac{(9.13\times10^{-3}\,\text{A})(2.56\times10^{-2}\,\text{m})(5.63\times10^{-2}\,\text{T})(0.0611\,\text{s})}{2.41\times10^{-5}\,\text{kg}}$$
$$= 3.34\times10^{-2}\,\text{m/s}.$$

(b) The direction is to the left (away from the generator).

47. (a) The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; *mg*, the magnitude of the (downward) force of gravity; $\vec{F}_N$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value $\mu_s F_N$). Thus, $\vec{F}$ has an eastward component $F_x$ and an upward component $F_y$, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the right-hand rule, a downward component ($B_d$) of $\vec{B}$ will produce the eastward $F_x$, and a westward component ($B_w$) will produce the upward $F_y$. Specifically,

$$F_x = iLB_d, \quad F_y = iLB_w.$$

Considering forces along a vertical axis, we find

$$F_N = mg - F_y = mg - iLB_w$$

so that

$$f = f_{s,\max} = \mu_s\left(mg - iLB_w\right).$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \quad\Rightarrow\quad iLB_d = \mu_s\left(mg - iLB_w\right).$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means $\theta$ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s\left(mg - iLB\sin\theta\right) \quad\Rightarrow\quad B = \frac{\mu_s mg}{iL\left(\cos\theta + \mu_s\sin\theta\right)}$$

which we differentiate (with respect to $\theta$) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°.$$

Consequently,

$$B_{\min} = \frac{0.60(1.0\,\text{kg})(9.8\,\text{m/s}^2)}{(50\,\text{A})(1.0\,\text{m})(\cos 31° + 0.60\sin 31°)} = 0.10\,\text{T}.$$

(b) As shown above, the angle is $\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°.$

48. We use $d\vec{F}_B = i d\vec{L} \times \vec{B}$, where $d\vec{L} = dx\hat{\text{i}}$ and $\vec{B} = B_x\hat{\text{i}} + B_y\hat{\text{j}}$. Thus,

$$\vec{F}_B = \int i d\vec{L} \times \vec{B} = \int_{x_i}^{x_f} i dx\hat{\text{i}} \times (B_x\hat{\text{i}} + B_y\hat{\text{j}}) = i\int_{x_i}^{x_f} B_y dx\hat{\text{k}}$$
$$= (-5.0\text{A})\left(\int_{1.0}^{3.0}(8.0x^2 dx)\ (\text{m}\cdot\text{mT})\right)\hat{\text{k}} = (-0.35\text{N})\hat{\text{k}}.$$

49. **THINK** Magnetic forces on the loop produce a torque that rotates it about the hinge line. Our applied field has two components: $B_x > 0$ and $B_z > 0$.

**EXPRESS** Considering each straight segment of the rectangular coil, we note that Eq. 28-26 produces a nonzero force only for the component of $\vec{B}$ which is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight segment of the coil that lies along the $y$ axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight segments experience forces due to Eq. 28-26 (caused by the $B_z$ component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the $y$ axis. Consequently, the torque derives completely from the force exerted on the straight segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Fig. 28-45 carrying current in the $-y$ direction.

Now, the $B_z$ component will produce a force on this straight segment which points in the $-x$ direction (back toward the hinge) and thus will exert no torque about the hinge. However, the $B_x$ component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $F = NiLB_x$ which points (by the right-hand rule) in the $+z$ direction.

**ANALYZE** Since the action of the force $F$ is perpendicular to the plane of the coil, and is located a distance $x$ away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)(x) = NiLxB\cos\theta = (20)(0.10\text{ A})(0.10\text{ m})(0.050\text{ m})(0.50\text{ T})\cos 30°$$
$$= 0.0043\text{ N}\cdot\text{m}.$$

Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is –*y*. In unit-vector notation, the torque is $\vec{\tau} = (-4.3\times10^{-3}\ \text{N}\cdot\text{m})\hat{\text{j}}$

**LEARN** An alternative way to do this problem is through the use of Eq. 28-37: $\vec{\tau} = \vec{\mu}\times\vec{B}.$ The magnetic moment vector is

$$\vec{\mu} = -(NiA)\ \hat{\text{k}} = -(20)(0.10\,\text{A})(0.0050\,\text{m}^2)\hat{\text{k}} = -(0.01\,\text{A}\cdot\text{m}^2)\hat{\text{k}}.$$

The torque on the loop is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = (-\mu\ \hat{\text{k}})\times(B\cos\theta\ \hat{\text{i}} + B\sin\theta\ \hat{\text{k}}) = -(\mu B\cos\theta)\hat{\text{j}}\\ &= -(0.01\,\text{A}\cdot\text{m}^2)(0.50\ \text{T})\cos30°\hat{\text{j}}\\ &= (-4.3\times10^{-3}\ \text{N}\cdot\text{m})\hat{\text{j}}.\end{aligned}$$

50. We use $\tau_{\max} = |\vec{\mu}\times\vec{B}|_{\max} = \mu B = i\pi r^2 B,$ and note that $i = qf = qv/2\pi r$. So

$$\begin{aligned}\tau_{\max} &= \left(\frac{qv}{2\pi r}\right)\pi r^2 B = \frac{1}{2}qvrB = \frac{1}{2}(1.60\times10^{-19}\,\text{C})(2.19\times10^{6}\,\text{m/s})(5.29\times10^{-11}\,\text{m})(7.10\times10^{-3}\,\text{T})\\ &= 6.58\times10^{-26}\,\text{N}\cdot\text{m}.\end{aligned}$$

51. We use Eq. 28-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity *mg*, acting downward from the center of mass, the normal force of the incline $F_N$, acting perpendicularly to the incline through the center of mass, and the force of friction *f*, acting up the incline at the point of contact. We take the *x* axis to be positive down the incline. Then the *x* component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma.$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$, and the force of friction produces a torque with magnitude *fr*, where *r* is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha.$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate *f* from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length $L$ and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = Ni(2rL)$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\,\text{A}.$$

52. The insight central to this problem is that for a given length of wire (formed into a rectangle of various possible aspect ratios), the maximum possible area is enclosed when the ratio of height to width is 1 (that is, when it is a square). The maximum possible value for the width, the problem says, is $x$ = 4 cm (this is when the height is very close to zero, so the total length of wire is effectively 8 cm). Thus, when it takes the shape of a square the value of $x$ must be ¼ of 8 cm; that is, $x$ = 2 cm when it encloses maximum area (which leads to a maximum torque by Eq. 28-35 and Eq. 28-37) of $A = (0.020\text{ m})^2 = 0.00040\text{ m}^2$. Since $N = 1$ and the torque in this case is given as $4.8 \times 10^{-4}\ \text{N}\cdot\text{m}$, then the aforementioned equations lead immediately to $i$ = 0.0030 A.

53. We replace the current loop of arbitrary shape with an assembly of small adjacent rectangular loops filling the same area that was enclosed by the original loop (as nearly as possible). Each rectangular loop carries a current $i$ flowing in the same sense as the original loop. As the sizes of these rectangles shrink to infinitesimally small values, the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\vec{\tau}$ exerted by $\vec{B}$ on the $n$th rectangular loop of area $\Delta A_n$ is given by $\Delta\tau_n = NiB\sin\theta\Delta A_n$. Thus, for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB\sum_n \Delta A_n = NiAB\sin\theta.$$

54. (a) The kinetic energy gained is due to the potential energy decrease as the dipole swings from a position specified by angle $\theta$ to that of being aligned (zero angle) with the field. Thus,

$$K = U_i - U_f = -\mu B\cos\theta - (-\mu B\cos 0°).$$

Therefore, using SI units, the angle is

$$\theta = \cos^{-1}\left(1 - \frac{K}{\mu B}\right) = \cos^{-1}\left(1 - \frac{0.00080}{(0.020)(0.052)}\right) = 77°.$$

(b) Since we are making the assumption that no energy is dissipated in this process, then the dipole will continue its rotation (similar to a pendulum) until it reaches an angle $\theta$ = 77° on the other side of the alignment axis.

55. **THINK** Our system consists of two concentric current-carrying loops. The net magnetic dipole moment is the vector sum of the individual contributions.

**EXPRESS** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where $N$ is the number of turns, $i$ is the current in each turn, and $A$ is the area of a loop. Each of the loops is a circle, so the area is $A = \pi r^2$, where $r$ is the radius of the loop.

**ANALYZE** (a) Since the currents are in the same direction, the magnitude of the magnetic moment vector is

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi(7.00\,\text{A})\left[(0.200\,\text{m})^2 + (0.300\,\text{m})^2\right] = 2.86\,\text{A}\cdot\text{m}^2.$$

(b) Now, the two currents flow in the opposite directions, so the magnitude of the magnetic moment vector is

$$\mu = \pi r_2^2 i_2 - \pi r_1^2 i_1 = \pi(7.00\,\text{A})\left[(0.300\,\text{m})^2 - (0.200\,\text{m})^2\right] = 1.10\,\text{A}\cdot\text{m}^2.$$

**LEARN** In both cases, the directions of the dipole moments are into the page. The direction of $\vec{\mu}$ is that of the normal vector $\vec{n}$ to the plane of the coil, in accordance with the right-hand rule shown in Fig. 28-19(b).

56. (a) $\mu = NAi = \pi r^2 i = \pi(0.150\,\text{m})^2(2.60\,\text{A}) = 0.184\,\text{A}\cdot\text{m}^2$.

(b) The torque is

$$\tau = \left|\vec{\mu}\times\vec{B}\right| = \mu B\sin\theta = \left(0.184\,\text{A}\cdot\text{m}^2\right)(12.0\,\text{T})\sin 41.0° = 1.45\,\text{N}\cdot\text{m}.$$

57. **THINK** Magnetic forces on a current-carrying loop produce a torque that tends to align the magnetic dipole moment with the magnetic field.

**EXPRESS** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where $N$ is the number of turns, $i$ is the current in each turn, and $A$ is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where $r$ is the radius of a turn.

**ANALYZE** (a) Thus, the current is

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\,\text{A}\cdot\text{m}^2}{(160)(\pi)(0.0190\,\text{m})^2} = 12.7\,\text{A}.$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by

$$\tau_{\max} = \mu B = \left(2.30\,\text{A}\cdot\text{m}^2\right)\left(35.0\times10^{-3}\,\text{T}\right) = 8.05\times10^{-2}\,\text{N}\cdot\text{m}.$$

**LEARN** The torque on the coil can be written as $\vec{\tau} = \vec{\mu} \times \vec{B},$ with $\tau = |\vec{\tau}| = \mu B \sin\theta,$ where $\theta$ is the angle between $\vec{\mu}$ and $\vec{B}.$ Thus, $\tau$ is a maximum when $\theta = 90°,$ and zero when $\theta = 0°.$

58. From $\mu = NiA = i\pi r^2$ we get

$$i = \frac{\mu}{\pi r^2} = \frac{8.00 \times 10^{22}\ \text{J/T}}{\pi\left(3500 \times 10^3\ \text{m}\right)^2} = 2.08 \times 10^9\ \text{A}.$$

59. (a) The area of the loop is $A = \frac{1}{2}(30\,\text{cm})(40\,\text{cm}) = 6.0 \times 10^2\,\text{cm}^2$, so

$$\mu = iA = (5.0\,\text{A})\left(6.0 \times 10^{-2}\,\text{m}^2\right) = 0.30\,\text{A} \cdot \text{m}^2 .$$

(b) The torque on the loop is

$$\tau = \mu B \sin\theta = \left(0.30\,\text{A} \cdot \text{m}^2\right)\left(80 \times 10^3\,\text{T}\right)\sin 90° = 2.4 \times 10^{-2}\,\text{N} \cdot \text{m}.$$

60. Let $a$ = 30.0 cm, $b$ = 20.0 cm, and $c$ = 10.0 cm. From the given hint, we write

$$\vec{\mu} = \vec{\mu}_1 + \vec{\mu}_2 = iab\left(-\hat{\text{k}}\right) + iac\left(\hat{\text{j}}\right) = ia\left(c\hat{\text{j}} - b\hat{\text{k}}\right) = (5.00\,\text{A})(0.300\,\text{m})\left[(0.100\,\text{m})\,\hat{\text{j}} - (0.200\,\text{m})\,\hat{\text{k}}\right]$$
$$= \left(0.150\hat{\text{j}} - 0.300\hat{\text{k}}\right)\text{A} \cdot \text{m}^2.$$

61. **THINK** Magnetic forces on a current-carrying coil produce a torque that tends to align the magnetic dipole moment with the field. The magnetic energy of the dipole depends on its orientation relative to the field.

**EXPRESS** The magnetic potential energy of the dipole is given by $U = -\vec{\mu} \cdot \vec{B},$ where $\vec{\mu}$ is the magnetic dipole moment of the coil and $\vec{B}$ is the magnetic field. The magnitude of $\vec{\mu}$ is $\mu = NiA,$ where $i$ is the current in the coil, $N$ is the number of turns, $A$ is the area of the coil. On the other hand, the torque on the coil is given by the vector product $\vec{\tau} = \vec{\mu} \times \vec{B}.$

**ANALYZE** (a) By using the right-hand rule, we see that $\vec{\mu}$ is in the –$y$ direction. Thus, we have

$$\vec{\mu} = (NiA)(-\hat{\text{j}}) = -(3)(2.00\ \text{A})(4.00 \times 10^{-3}\ \text{m}^2)\hat{\text{j}} = -(0.0240\ \text{A} \cdot \text{m}^2)\hat{\text{j}}.$$

The corresponding magnetic energy is

$$U = -\vec{\mu} \cdot \vec{B} = -\mu_y B_y = -(-0.0240\ \text{A} \cdot \text{m}^2)(-3.00 \times 10^{-3}\ \text{T}) = -7.20 \times 10^{-5}\ \text{J}.$$

(b) Using the fact that $\hat{\text{j}}\cdot\hat{\text{i}}=0$, $\hat{\text{j}}\times\hat{\text{j}}=0$, and $\hat{\text{j}}\times\hat{\text{k}}=\hat{\text{i}}$, the torque on the coil is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = \mu_y B_z\hat{\text{i}} - \mu_y B_x\hat{\text{k}} \\ &= (-0.0240\ \text{A}\cdot\text{m}^2)(-4.00\times10^{-3}\ \text{T})\hat{\text{i}}\ -(-0.0240\ \text{A}\cdot\text{m}^2)(2.00\times10^{-3}\ \text{T})\hat{\text{k}} \\ &= (9.60\times10^{-5}\ \text{N}\cdot\text{m})\hat{\text{i}} + (4.80\times10^{-5}\ \text{N}\cdot\text{m})\hat{\text{k}}.\end{aligned}$$

**LEARN** The magnetic energy is highest when $\vec{\mu}$ is in the opposite direction of $\vec{B}$, and lowest when $\vec{\mu}$ lines up with $\vec{B}$.

62. Looking at the point in the graph (Fig. 28-51(b)) corresponding to $i_2$ = 0 (which means that coil 2 has no magnetic moment) we are led to conclude that the magnetic moment of coil 1 must be $\mu_1 = 2.0\times10^{-5}\ \text{A}\cdot\text{m}^2$. Looking at the point where the line crosses the axis (at $i_2$ = 5.0 mA) we conclude (since the magnetic moments cancel there) that the magnitude of coil 2's moment must also be $\mu_2 = 2.0\times10^{-5}\ \text{A}\cdot\text{m}^2$ when $i_2 = 0.0050\ \text{A}$, which means (Eq. 28-35)

$$N_2A_2 = \frac{\mu_2}{i_2} = \frac{2.0\times10^{-5}\ \text{A}\cdot\text{m}^2}{0.0050\ \text{A}} = 4.0\times10^{-3}\ \text{m}^2\,.$$

Now the problem has us consider the direction of coil 2's current changed so that the net moment is the sum of two (positive) contributions, from coil 1 and coil 2, specifically for the case that $i_2$ = 0.007 A. We find that total moment is

$$\mu = (2.0\times10^{-5}\ \text{A}\cdot\text{m}^2) + (N_2A_2\ i_2) = 4.8\times10^{-5}\ \text{A}\cdot\text{m}^2.$$

63. The magnetic dipole moment is $\vec{\mu} = \mu\left(0.60\,\hat{\text{i}} - 0.80\,\hat{\text{j}}\right)$, where

$$\mu = NiA = Ni\pi r^2 = 1(0.20\ \text{A})\pi(0.080\ \text{m})^2 = 4.02\times10^{-4}\ \text{A}\cdot\text{m}^2.$$

Here $i$ is the current in the loop, $N$ is the number of turns, $A$ is the area of the loop, and $r$ is its radius.

(a) The torque is

$$\begin{aligned}\vec{\tau} &= \vec{\mu}\times\vec{B} = \mu\left(0.60\hat{\text{i}} - 0.80\hat{\text{j}}\right)\times\left(0.25\hat{\text{i}} + 0.30\hat{\text{k}}\right) \\ &= \mu\ (0.60)(0.30)\left(\hat{\text{i}}\times\hat{\text{k}}\right) - (0.80)(0.25)\left(\hat{\text{j}}\times\hat{\text{i}}\right) - (0.80)(0.30)\left(\hat{\text{j}}\times\hat{\text{k}}\right) \\ &= \mu\ -0.18\hat{\text{j}} + 0.20\hat{\text{k}} - 0.24\hat{\text{i}}\ .\end{aligned}$$

Here $\hat{i}\times\hat{k}=-\hat{j}, \hat{j}\times\hat{i}=-\hat{k},$ and $\hat{j}\times\hat{k}=\hat{i}$ are used. We also use $\hat{i}\times\hat{i}=0$ . Now, we substitute the value for $\mu$ to obtain

$$\vec{\tau}=\left(-9.7\times10^{-4}\hat{i}-7.2\times10^{-4}\hat{j}+8.0\times10^{-4}\hat{k}\right)\text{N}\cdot\text{m}.$$

(b) The orientation energy of the dipole is given by

$$U=-\vec{\mu}\cdot\vec{B}=-\mu\left(0.60\hat{i}-0.80\hat{j}\right)\cdot\left(0.25\hat{i}+0.30\hat{k}\right)=-\mu\left(0.60\right)\left(0.25\right)=-0.15\mu=-6.0\times10^{-4}\,\text{J}.$$

Here $\hat{i}\cdot\hat{i}=1, \hat{i}\cdot\hat{k}=0, \hat{j}\cdot\hat{i}=0,$ and $\hat{j}\cdot\hat{k}=0$ are used.

64. Eq. 28-39 gives $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi$, so at $\phi = 0$ (corresponding to the lowest point on the graph in Fig. 28-52) the mechanical energy is

$$K + U = K_o + (-\mu B) = 6.7\times10^{-4}\,\text{J} + (-5\times10^{-4}\,\text{J}) = 1.7\times10^{-4}\,\text{J}.$$

The turning point occurs where $K = 0$, which implies $U_{\text{turn}} = 1.7\times10^{-4}$ J. So the angle where this takes place is given by

$$\phi=-\cos^{-1}\left(\frac{1.7\times10^{-4}\text{ J}}{\mu B}\right)=110°$$

where we have used the fact (see above) that $\mu B = 5\times10^{-4}$ J.

65. **THINK** The torque on a current-carrying coil is a maximum when its dipole moment is perpendicular to the magnetic field.

**EXPRESS** The magnitude of the torque on the coil is given by $\tau=|\vec{\tau}|=\mu B\sin\theta,$ where $\theta$ is the angle between $\vec{\mu}$ and $\vec{B}$. The magnitude of $\vec{\mu}$ is $\mu = NiA,$ where $i$ is the current in the coil, $N$ is the number of turns, $A$ is the area of the coil. Thus, if $N$ closed loops are formed from the wire of length $L$, the circumference of each loop is $L/N$, the radius of each loop is $R = L/2\pi N$, and the area of each loop is

$$A=\pi R^2=\pi\left(L/2\pi N\right)^2=L^2/4\pi N^2.$$

**ANALYZE** (a) For maximum torque, we orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular (i.e., at a $90°$ angle) to the field.

(b) The magnitude of the torque is then

$$\tau = NiAB=\left(Ni\right)\left(\frac{L^2}{4\pi N^2}\right)B=\frac{iL^2B}{4\pi N}.$$

To maximize the torque, we take the number of turns $N$ to have the smallest possible value, 1. Then $\tau = iL^2B/4\pi$.

(c) The magnitude of the maximum torque is

$$\tau = \frac{iL^2B}{4\pi} = \frac{(4.51\times10^{-3}\ \text{A})(0.250\ \text{m})^2(5.71\times10^{-3}\text{T})}{4\pi} = 1.28\times10^{-7}\ \text{N}\cdot\text{m}.$$

**LEARN** The torque tends to align $\vec{\mu}$ with $\vec{B}.$ The magnitude of the torque is a maximum when the angle between $\vec{\mu}$ and $\vec{B}$ is $\theta = 90°,$ and is zero when $\theta = 0°.$

66. The equation of motion for the proton is

$$\vec{F} = q\vec{v}\times\vec{B} = q\left(v_x\hat{\text{i}} + v_y\hat{\text{j}} + v_z\hat{\text{k}}\right)\times B\hat{\text{i}} = qB\left(v_z\hat{\text{j}} - v_y\hat{\text{k}}\right)$$
$$= m_p\vec{a} = m_p\left[\left(\frac{dv_x}{dt}\right)\hat{\text{i}} + \left(\frac{dv_y}{dt}\right)\hat{\text{j}} + \left(\frac{dv_z}{dt}\right)\hat{\text{k}}\right].$$

Thus,

$$\frac{dv_x}{dt} = 0, \quad \frac{dv_y}{dt} = \omega v_z, \quad \frac{dv_z}{dt} = -\omega v_y,$$

where $\omega = eB/m$. The solution is $v_x = v_{0x}$, $v_y = v_{0y}\cos\omega t$, and $v_z = -v_{0y}\sin\omega t$. In summary, we have

$$\vec{v}(t) = v_{0x}\hat{\text{i}} + v_{0y}\cos(\omega t)\hat{\text{j}} - v_{0y}(\sin\omega t)\hat{\text{k}}.$$

67. (a) We use $\vec{\tau} = \vec{\mu}\times\vec{B},$ where $\vec{\mu}$ points into the wall (since the current goes clockwise around the clock). Since $\vec{B}$ points toward the one-hour (or "5-minute'') mark, and (by the properties of vector cross products) $\vec{\tau}$ must be perpendicular to it, then (using the right-hand rule) we find $\vec{\tau}$ points at the 20-minute mark. So the time interval is 20 min.

(b) The torque is given by

$$\tau = |\vec{\mu}\times\vec{B}| = \mu B\sin 90° = NiAB = \pi Nir^2B = 6\pi\left(2.0\text{A}\right)\left(0.15\text{m}\right)^2\left(70\times10^{-3}\text{T}\right)$$
$$= 5.9\times10^{-2}\,\text{N}\cdot\text{m}.$$

68. The unit vector associated with the current element (of magnitude $d\ell$) is $-\hat{\text{j}}$. The (infinitesimal) force on this element is

$$d\vec{F} = i\,d\ell\left(-\hat{\text{j}}\right)\times\left(0.3y\hat{\text{i}} + 0.4y\hat{\text{j}}\right)$$

with SI units (and 3 significant figures) understood. Since $\hat{\text{j}}\times\hat{\text{i}}=-\hat{\text{k}}$ and $\hat{\text{j}}\times\hat{\text{j}}=0$, we obtain

$$d\vec{F}=0.3iy\,d\ell\ \hat{\text{k}}=\left(6.00\times10^{-4}\ \text{N/m}^2\right)y\,d\ell\ \hat{\text{k}}.$$

We integrate the force element found above, using the symbol $\xi$ to stand for the coefficient $6.00\times10^{-4}$ N/m$^2$, and obtain

$$\vec{F}=\int d\vec{F}=\xi\ \hat{\text{k}}\int_0^{0.25}ydy=\xi\ \hat{\text{k}}\left(\frac{0.25^2}{2}\right)=(1.88\times10^{-5}\ \text{N})\hat{\text{k}}\ .$$

69. From $m = B^2qx^2/8V$ we have $\Delta m = (B^2q/8V)(2x\Delta x)$. Here $x=\sqrt{8Vm/B^2q}$, which we substitute into the expression for $\Delta m$ to obtain

$$\Delta m=\left(\frac{B^2q}{8V}\right)2\sqrt{\frac{8mV}{B^2q}}\Delta x=B\sqrt{\frac{mq}{2V}}\Delta x.$$

Thus, the distance between the spots made on the photographic plate is

$$\Delta x=\frac{\Delta m}{B}\sqrt{\frac{2V}{mq}}=\frac{\left(37\,\text{u}-35\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)}{0.50\,\text{T}}\sqrt{\frac{2\left(7.3\times10^3\ \text{V}\right)}{\left(36\,\text{u}\right)\left(1.66\times10^{-27}\ \text{kg/u}\right)\left(1.60\times10^{-19}\text{C}\right)}}$$
$$=8.2\times10^{-3}\text{m}.$$

70. (a) Equating the magnitude of the electric force ($F_e = eE$) with that of the magnetic force (Eq. 28-3), we obtain $B = E / v \sin\phi$. The field is smallest when the $\sin\phi$ factor is at its largest value; that is, when $\phi = 90°$. Now, we use $K=\frac{1}{2}mv^2$ to find the speed:

$$v=\sqrt{\frac{2K}{m_e}}=\sqrt{\frac{2\left(2.5\times10^3\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{9.11\times10^{-31}\text{kg}}}=2.96\times10^7\ \text{m/s}.$$

Thus,

$$B=\frac{E}{v}=\frac{10\times10^3\ \text{V/m}}{2.96\times10^7\ \text{m/s}}=3.4\times10^{-4}\text{T}.$$

The direction of the magnetic field must be perpendicular to both the electric field ($-\hat{\text{j}}$) and the velocity of the electron ($+\hat{\text{i}}$). Since the electric force $\vec{F}_e=(-e)\vec{E}$ points in the $+\hat{\text{j}}$ direction, the magnetic force $\vec{F}_B=(-e)\vec{v}\times\vec{B}$ points in the $-\hat{\text{j}}$ direction. Hence, the direction of the magnetic field is $-\hat{\text{k}}$. In unit-vector notation, $\vec{B}=(-3.4\times10^{-4}\text{T})\hat{\text{k}}.$

71. The period of revolution for the iodine ion is

$$T = 2\pi r/v = 2\pi m/Bq,$$

which gives

$$m = \frac{BqT}{2\pi} = \frac{(45.0\times10^{-3}\text{T})(1.60\times10^{-19}\text{C})(1.29\times10^{-3}\text{s})}{(7)(2\pi)(1.66\times10^{-27}\text{ kg/u})} = 127\,\text{u}.$$

72. (a) For the magnetic field to have an effect on the moving electrons, we need a non-negligible component of $\vec{B}$ to be perpendicular to $\vec{v}$ (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$ in agreement with Eq. 28-16. The kinetic energy of the electron is $K = \frac{1}{2}m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_e K}{e^2B^2}} .$$

This must be less than $d$, so $\sqrt{\frac{2m_e K}{e^2B^2}} \le d$, or $B \ge \sqrt{\frac{2m_e K}{e^2d^2}}$ .

(b) If the electrons are to travel as shown in Fig. 28-53, the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).

73. **THINK** The electron moving in the Earth's magnetic field is being accelerated by the magnetic force acting on it.

**EXPRESS** Since the electron is moving in a line that is parallel to the horizontal component of the Earth's magnetic field, the magnetic force on the electron is due to the vertical component of the field only. The magnitude of the force acting on the electron is given by $F = evB$, where $B$ represents the downward component of Earth's field. With $F = m_e a$, the acceleration of the electron is $a = evB/m_e$.

**ANALYZE** (a) The electron speed can be found from its kinetic energy $K = \frac{1}{2}m_e v^2$ :

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(12.0\times10^3\text{eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{kg}}} = 6.49\times10^7\text{ m/s}.$$

Therefore,

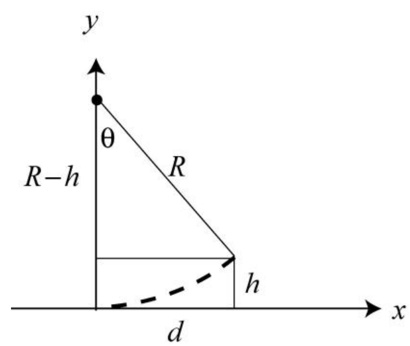
$$a = \frac{evB}{m_e} = \frac{\left(1.60\times10^{-19}\text{C}\right)\left(6.49\times10^{7}\text{ m/s}\right)\left(55.0\times10^{-6}\text{T}\right)}{9.11\times10^{-31}\text{kg}}$$
$$= 6.27\times10^{14}\text{ m/s}^2 \approx 6.3\times10^{14}\text{ m/s}^2.$$

(b) We ignore any vertical deflection of the beam that might arise due to the horizontal component of Earth's field. Then, the path of the electron is a circular arc. The radius of the path is given by $a = v^2/R$, or

$$R = \frac{v^2}{a} = \frac{(6.49\times10^{7}\text{ m/s})^2}{6.27\times10^{14}\text{ m/s}^2} = 6.72\text{ m}.$$

The dashed curve shown represents the path. Let the deflection be $h$ after the electron has traveled a distance $d$ along the $x$ axis. With $d = R\sin\theta$, we have

$$h = R(1-\cos\theta) = R\left(1-\sqrt{1-\sin^2\theta}\right)$$
$$= R\left(1-\sqrt{1-(d/R)^2}\right).$$




Substituting $R$ = 6.72 m and $d$ = 0.20 m into the expression, we obtain $h$ = 0.0030 m.

**LEARN** The deflection is so small that many of the technicalities of circular geometry may be ignored, and a calculation along the lines of projectile motion analysis (see Chapter 4) provides an adequate approximation:

$$d = vt \;\Rightarrow\; t = \frac{d}{v} = \frac{0.200\text{m}}{6.49\times10^{7}\text{ m/s}} = 3.08\times10^{-9}\text{ s}.$$

Then, with our $y$ axis oriented eastward,

$$h = \frac{1}{2}at^2 = \frac{1}{2}\left(6.27\times10^{14}\right)\left(3.08\times10^{-9}\right)^2 = 0.00298\text{m} \approx 0.0030\text{ m}.$$

74. Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using Eq. 28-2 ($\vec{F} = q\vec{v}\times\vec{B}$) and Eq. 3-30, we obtain (with SI units understood)

$$4\hat{\text{i}} - 20\hat{\text{j}} + 12\hat{\text{k}} = 2\left(\left(4B_2 - 6B_1\right)\hat{\text{i}} + \left(6B_1 - 2B_2\right)\hat{\text{j}} + \left(2B_1 - 4B_1\right)\hat{\text{k}}\right).$$

Equating like components, we find $B_1 = -3$ and $B_2 = -4$. In summary,

$$\vec{B} = \left(-3.0\hat{\text{i}} - 3.0\hat{\text{j}} - 4.0\hat{\text{k}}\right)\text{T}.$$

75. Using Eq. 28-16, the radius of the circular path is

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

where $K = mv^2/2$ is the kinetic energy of the particle. Thus, we see that $r \propto \sqrt{mK}/qB$.

(a) $\frac{r_d}{r_p} = \sqrt{\frac{m_d K_d}{m_p K_p}} \frac{q_p}{q_d} = \sqrt{\frac{2.0\text{u}}{1.0\text{u}}} \frac{e}{e} = \sqrt{2} \approx 1.4$, and

(b) $\frac{r_\alpha}{r_p} = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}} \frac{q_p}{q_\alpha} = \sqrt{\frac{4.0\text{u}}{1.0\text{u}}} \frac{e}{2e} = 1.0.$

76. Using Eq. 28-16, the charge-to-mass ratio is $\frac{q}{m} = \frac{v}{B'r}$. With the speed of the ion given by $v = E/B$ (using Eq. 28-7), the expression becomes

$$\frac{q}{m} = \frac{E/B}{B'r} = \frac{E}{BB'r}.$$

77. **THINK** Since both electric and magnetic fields are present, the net force on the electron is the vector sum of the electric force and the magnetic force.

**EXPRESS** The force on the electron is given by $\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, and $\vec{v}$ is the velocity of the electron. The fact that the fields are uniform with the feature that the charge moves in a straight line, implies that the speed is constant. Thus, the net force must vanish.

**ANALYZE** The condition $\vec{F} = 0$ implies that

$$E = vB = 500\,\text{V/m}.$$

Its direction (so that $\vec{F} = 0$) is downward, or $-\hat{\text{j}}$, in the "page" coordinates. In unit-vector notation, $\vec{E} = (-500\text{ V/m})\hat{\text{j}}$

**LEARN** Electron moves in a straight line only when the condition $E = vB$ is met. In many experiments, a velocity selector can be set up so that only electrons with a speed given by $v = E/B$ can pass through.

78. (a) In Chapter 27, the electric field (called $E_C$ in this problem) that "drives" the current through the resistive material is given by Eq. 27-11, which (in magnitude) reads $E_C = \rho J$. Combining this with Eq. 27-7, we obtain

$$E_C = \rho n e v_d .$$

Now, regarding the Hall effect, we use Eq. 28-10 to write $E = v_d B$. Dividing one equation by the other, we get $E/E_c = B/ne\rho$.

(b) Using the value of copper's resistivity given in Chapter 26, we obtain

$$\frac{E}{E_c} = \frac{B}{ne\rho} = \frac{0.65\,\text{T}}{\left(8.47\times10^{28}/\text{m}^3\right)\left(1.60\times10^{-19}\,\text{C}\right)\left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)} = 2.84\times10^{-3}.$$

79. **THINK** We have charged particles that are accelerated through an electric potential difference, and then moved through a region of uniform magnetic field. Energy is conserved in the process.

**EXPRESS** The kinetic energy of a particle is given by $K = qV$, where $q$ is the particle's charge and $V$ is the potential difference. With $K = mv^2/2,$ the speed of the particle is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2qV}{m}}.$$

In the region with uniform magnetic field, the magnetic force on a particle of charge $q$ is $qvB$, which according to Newton's second law, is equal to $mv^2/r,$ where $r$ is the radius of the orbit. Thus, we have

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{qB}.$$

**ANALYZE** (a) Since $K = qV$ we have $K_p = \frac{1}{2}K_\alpha$ $\left(\text{as } q_\alpha = 2K_p\right)$, or $K_p/K_\alpha = 0.50.$

(b) Similarly, $q_\alpha = 2K_d$, $K_d/K_\alpha = 0.50.$

(c) Since $r \propto \sqrt{mK}/q$, we have

$$r_d = \sqrt{\frac{m_d K_d}{m_p K_p}}\,\frac{q_p}{q_d}\,r_p = \sqrt{\frac{(2.00\text{ u})K_p}{(1.00\text{ u})K_p}}\,r_p = 10\sqrt{2}\text{ cm} = 14\text{ cm}.$$

(d) Similarly, for the alpha particle, we have

$$r_\alpha = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}\frac{q_p}{q_\alpha} r_p = \sqrt{\frac{(4.00\text{u})K_\alpha}{(1.00\text{u})\,(K_\alpha/2)}}\frac{e}{2e} r_p = 10\sqrt{2}\ \text{cm} = 14\ \text{cm}.$$

**LEARN** The radius of the particle's path, given by $r = \sqrt{2mK}\,/\,qB,$ depends on its mass. kinetic energy, and charge, in addition to the field strength.

80. (a) The largest value of force occurs if the velocity vector is perpendicular to the field. Using Eq. 28-3,

$$F_{B,\text{max}} = |q|\ vB \sin(90°) = ev\,B = (1.60 \times 10^{-19}\ \text{C})\ (7.20 \times 10^{6}\ \text{m/s})\ (83.0 \times 10^{-3}\ \text{T}) = 9.56 \times 10^{-14}\ \text{N}.$$

(b) The smallest value occurs if they are parallel: $F_{B,\text{min}} = |q|\ vB \sin(0) = 0$.

(c) By Newton's second law, $a = F_B/m_e = |q|\ vB \sin\theta/m_e$, so the angle $\theta$ between $\vec{v}$ and $\vec{B}$ is

$$\theta = \sin^{-1}\left(\frac{m_e a}{|q|vB}\right) = \sin^{-1}\left[\frac{(9.11\times10^{-31}\text{kg})(4.90\times10^{14}\ \text{m/s}^2)}{(1.60\times10^{-16}\text{C})(7.20\times10^{6}\ \text{m/s})(83.0\times10^{-3}\text{T})}\right] = 0.267°.$$

81. The contribution to the force by the magnetic field $\left(\vec{B} = B_x\hat{\text{i}} = (-0.020\ \text{T})\hat{\text{i}}\right)$ is given by Eq. 28-2:

$$\begin{aligned}\vec{F}_B &= q\vec{v}\times\vec{B} = q\left(\left(17000\hat{\text{i}}\times B_x\hat{\text{i}}\right) + \left(-11000\hat{\text{j}}\times B_x\hat{\text{i}}\right) + \left(7000\hat{\text{k}}\times B_x\hat{\text{i}}\right)\right)\\ &= q\left(-220\hat{\text{k}} - 140\hat{\text{j}}\right)\end{aligned}$$

in SI units. And the contribution to the force by the electric field $\left(\vec{E} = E_y\hat{\text{j}} = 300\hat{\text{j}}\ \text{V/m}\right)$ is given by Eq. 23-1: $\vec{F}_E = qE_y\hat{\text{j}}$. Using $q = 5.0 \times 10^{-6}$ C, the net force on the particle is

$$\vec{F} = (0.00080\hat{\text{j}} - 0.0011\hat{\text{k}})\ \text{N}.$$

82. (a) We use Eq. 28-10: $v_d = E/B = (10 \times 10^{-6}\text{V}/1.0 \times 10^{-2}\ \text{m})/(1.5\ \text{T}) = 6.7 \times 10^{-4}$ m/s.

(b) We rewrite Eq. 28-12 in terms of the electric field:

$$n = \frac{Bi}{V\ell e} = \frac{Bi}{(Ed)\ell e} = \frac{Bi}{EAe}$$

where we use $A = \ell d$ . In this experiment, $A$ = (0.010 m)(10 × $10^{-6}$ m) = 1.0 × $10^{-7}$ m$^2$. By Eq. 28-10, $v_d$ equals the ratio of the fields (as noted in part (a)), so we are led to

$$n = \frac{Bi}{E\ Ae} = \frac{i}{v_d Ae} = \frac{3.0\,\text{A}}{\left(6.7\times10^{-4}\ \text{m/s}\right)\left(1.0\times10^{-7}\,\text{m}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)} = 2.8\times10^{29}/\text{m}^3 .$$

(c) Since a drawing of an inherently 3-D situation can be misleading, we describe it in terms of horizontal *north*, *south*, *east*, *west* and vertical *up* and *down* directions. We assume $\vec{B}$ points up and the conductor's width of 0.010 m is along an east-west line. We take the current going northward. The conduction electrons experience a westward magnetic force (by the right-hand rule), which results in the west side of the conductor being negative and the east side being positive (with reference to the Hall voltage that becomes established).

83. **THINK** The force on the charged particle is given by $\vec{F} = q\vec{v}\times\vec{B},$ where $q$ is the charge, $\vec{B}$ is the magnetic field, and $\vec{v}$ is the velocity of the electron.

**EXPRESS** We write $\vec{B} = B\hat{\text{i}}$ and take the velocity of the particle to be $\vec{v} = v_x\hat{\text{i}} + v_y\hat{\text{j}}$. Thus,

$$\vec{F} = q\vec{v}\times\vec{B} = q(v_x\hat{\text{i}} + v_y\hat{\text{j}})\times(B\hat{\text{i}}) = -qv_yB\hat{\text{k}}.$$

For the force to point along $+\hat{\text{k}}$, we must have $q < 0$.

**ANALYZE** The charge of the particle is

$$q = -\frac{F}{v_yB} = -\frac{0.48\ \text{N}}{(4.0\times10^3\ \text{m/s})(\sin 37°)(0.0050\ \text{T})} = -4.0\times10^{-2}\ \text{C}.$$

**LEARN** The component of the velocity, $v_x$, being parallel to the magnetic field, does not contribute to the magnetic force $\vec{F}$; only $v_y$, the component of $\vec{v}$ that is perpendicular to $\vec{B}$, contributes to $\vec{F}$.

84. The current is in the $+\hat{\text{i}}$ direction. Thus, the $\hat{\text{i}}$ component of $\vec{B}$ has no effect, and (with $x$ in meters) we evaluate

$$\vec{F} = (3.00\text{A})\int_0^1 \left(-0.600\,\text{T/m}^2\right)x^2dx\left(\hat{\text{i}}\times\hat{\text{j}}\right) = \left(-1.80\frac{1^3}{3}\ \text{A}\cdot\text{T}\cdot\text{m}\right)\hat{\text{k}} = (-0.600\text{N})\hat{\text{k}}.$$

85. (a) We use Eq. 28-2 and Eq. 3-30:

$$\vec{F} = q\vec{v}\times\vec{B} = (+e)\left(\left(v_yB_z - v_zB_y\right)\hat{\text{i}} + \left(v_zB_x - v_xB_z\right)\hat{\text{j}} + \left(v_xB_y - v_yB_x\right)\hat{\text{k}}\right)$$
$$= \left(1.60\times10^{-19}\right)\left(\left((4)(0.008) - (-6)\,(-0.004)\right)\hat{\text{i}} + \right.$$
$$\left.\left((-6)(0.002) - (-2)\,(0.008)\right)\hat{\text{j}} + \left((-2)(-0.004) - (4)(0.002)\right)\hat{\text{k}}\right)$$
$$= \left(1.28\times10^{-21}\right)\hat{\text{i}} + \left(6.41\times10^{-22}\right)\hat{\text{j}}$$

with SI units understood.

(b) By definition of the cross product, $\vec{v}\perp\vec{F}$. This is easily verified by taking the dot (scalar) product of $\vec{v}$ with the result of part (a), yielding zero, provided care is taken not to introduce any round-off error.

(c) There are several ways to proceed. It may be worthwhile to note, first, that if $B_z$ were 6.00 mT instead of 8.00 mT then the two vectors would be exactly antiparallel. Hence, the angle $\theta$ between $\vec{B}$ and $\vec{v}$ is presumably "close" to 180°. Here, we use Eq. 3-20:

$$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{B}}{|\vec{v}||\vec{B}|}\right) = \cos^{-1}\left(\frac{-68}{\sqrt{56}\sqrt{84}}\right) = 173° .$$

86. (a) We are given $\vec{B} = B_x\hat{\text{i}} = (6\times10^{-5}\text{T})\hat{\text{i}}$ , so that $\vec{v}\times\vec{B} = -v_yB_x\hat{\text{k}}$ where $v_y = 4\times10^4$ m/s. We note that the magnetic force on the electron is $(-e)\left(-v_yB_x\hat{\text{k}}\right)$ and therefore points in the $+\hat{\text{k}}$ direction, at the instant the electron enters the field-filled region. In these terms, Eq. 28-16 becomes

$$r = \frac{m_ev_y}{e\,B_x} = 0.0038\,\text{m}.$$

(b) One revolution takes $T = 2\pi r/v_y = 0.60$ $\mu$s, and during that time the "drift" of the electron in the $x$ direction (which is the *pitch* of the helix) is $\Delta x = v_xT = 0.019$ m where $v_x = 32\times10^3$ m/s.

(c) Returning to our observation of force direction made in part (a), we consider how this is perceived by an observer at some point on the $-x$ axis. As the electron moves away from him, he sees it enter the region with positive $v_y$ (which he might call "upward'') but "pushed" in the +z direction (to his right). Hence, he describes the electron's spiral as clockwise.

87. (a) The magnetic force on the electrons is given by $\vec{F} = q\vec{v}\times\vec{B}$. Since the field $\vec{B}$ points to the left, and an electron (with $q = -e$) is forced to rotate clockwise (out of the page at the top of the rotor), using the right-hand-rule, the direction of the magnetic force is up the figure.

(b) The magnitude of the magnetic force can be written as $F = evB = e\omega rB$, where $\omega$ is the angular velocity and $r$ is the distance from the axis. Since $F \sim r$, the force is greater near the rim.

(c) The work per unit charge done by the force in moving the charge along the radial line from the center to the rim, or the voltage, is

$$V = \frac{W}{e} = \frac{1}{e}\int_0^R e\omega Br dr = \frac{1}{2}\omega BR^2 = \frac{1}{2}(2\pi f)BR^2 = \pi fBR^2$$
$$= \pi(4000\ /\text{s})(60\times10^{-3}\text{T})(0.250\ \text{m})^2 = 47.1\ \text{V}.$$

(d) The emf of the device is simply equal to the voltage calculated in part (c): $\varepsilon = 47.1$ V.

(e) The power produced is $P = iV = (50.0\ \text{A})(47.1\ \text{V}) = 2.36\times10^3$ W.

88. The magnetic force exerted on the U-shaped wire is given by $F = iLB$. Using the impulse-momentum theorem, we have

$$\Delta p = m\Delta v = \int F dt = \int iLBdt = LB\int idt = LBq,$$

where $q$ is the charge in the pulse. Since the wire is initially at rest, the speed at which the wire jumps is $v = LBq/m$. On the other hand, energy conservation gives $\frac{1}{2}mv^2 = mgh$. Combining the above expressions leads to

$$h = \frac{v^2}{2g} = \frac{1}{2g}\left(\frac{LBq}{m}\right)^2$$

Solving for $q$, we find

$$q = \frac{m\sqrt{2gh}}{LB} = \frac{(0.0100\ \text{kg})\sqrt{2(9.80\ \text{m/s}^2)(3.00\ \text{m})}}{(0.200\ \text{m})(0.100\ \text{T})} = 3.83\,\text{C}.$$

89. Just before striking the plate, the electric force on the electron is $F_E = eE = eV/d$, in the upward direction. Since the kinetic energy of the electron is $K = \frac{1}{2}mv^2 = eV$, $v = \sqrt{2eV/m}$. On the other hand, the magnetic force is

$$F_B = evB = eB\sqrt{\frac{2eV}{m}}$$

in the downward direction. To prevent the electron from striking the plate, we require $F_B > F_E$, or

$$eB\sqrt{\frac{2eV}{m}} > \frac{eV}{d} \quad \Rightarrow \quad B > \frac{V}{d}\sqrt{\frac{m}{2eV}} = \sqrt{\frac{mV}{2ed^2}}$$

90. The average current in the loop is $i = \frac{q}{T} = \frac{q}{2\pi r / v} = \frac{qv}{2\pi r}$ and its magnetic dipole moment is

$$\mu = iA = \left(\frac{qv}{2\pi r}\right)(\pi r^2) = \frac{1}{2}qvr.$$

With $\vec{\tau} = \vec{\mu} \times \vec{B}$, we find the maximum torque exerted on the loop by a uniform magnetic field to be

$$\tau_{\max} = \mu B = \frac{1}{2}qvrB.$$

91. When the electric and magnetic forces are in balance, $eE = ev_d B$, where $v_d$ is the drift speed of the electrons. In addition, since the current density is $J = nev_d$, we solve for $n$ and find

$$n = \frac{J}{ev_d} = \frac{J}{e(E/B)} = \frac{JB}{eE}.$$

92. With $F_z = v_z = B_x = 0$, Eq. 28-2 (and Eq. 3-30) gives

$$F_x \,\hat{\text{i}} + F_y \,\hat{\text{j}} = q\,(v_y B_z \,\hat{\text{i}} - v_x B_z \,\hat{\text{j}} + v_x B_y \,\hat{\text{k}})$$

where $q = -e$ for the electron. The last term immediately implies $B_y = 0$, and either of the other two terms (along with the values stated in the problem, bearing in mind that "fN" means femto-newtons or $10^{-15}$ N) can be used to solve for $B_z$:

$$B_z = \frac{F_x}{-ev_y} = \frac{-4.2\times10^{-15}\,\text{N}}{-(1.6\times10^{-19}\ \text{C})(35{,}000\ \text{m/s})} = 0.75\,\text{T}.$$

We therefore find that the magnetic field is given by $\vec{B} = (0.75\ \text{T})\hat{\text{k}}$.

# Chapter 29

1. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance *r* from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r}.$$

With $r$ = 20 ft = 6.10 m, we have

$$B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(100\ \text{A}\right)}{2\pi\left(6.10\ \text{m}\right)} = 3.3\times 10^{-6}\ \text{T} = 3.3\ \mu\text{T}.$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

2. Equation 29-1 is maximized (with respect to angle) by setting $\theta$ = 90º ( = $\pi$/2 rad). Its value in this case is

$$dB_{\text{max}} = \frac{\mu_0 i}{4\pi}\frac{ds}{R^2}.$$

From Fig. 29-35(b), we have $B_{\text{max}} = 60\times 10^{-12}$ T. We can relate this $B_{\text{max}}$ to our $dB_{\text{max}}$ by setting "$ds$" equal to $1 \times 10^{-6}$ m and $R$ = 0.025 m. This allows us to solve for the current: $i$ = 0.375 A. Plugging this into Eq. 29-4 (for the infinite wire) gives $B_\infty$ = 3.0 $\mu$T.

3. **THINK** The magnetic field produced by a current-carrying wire can be calculated using the Biot-Savart law.

**EXPRESS** The magnitude of the magnetic field at a distance *r* from a long straight wire carrying current *i* is, using the Biot-Savart law, $B = \mu_0 i/2\pi r$.

**ANALYZE** (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 $\mu$T and must be directed due south. Therefore,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(0.080\ \text{m}\right)\left(39\times 10^{-6}\ \text{T}\right)}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}} = 16\ \text{A}.$$

(b) The current must be from west to east to produce a field that is directed southward at points below it.

**LEARN** The direction of the current is given by the right-hand rule: grasp the element in your right hand with your thumb pointing in the direction of the current. The direction of the field due to the current-carrying element corresponds to the direction your fingers naturally curl.

4. The straight segment of the wire produces no magnetic field at $C$ (see the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current"). Also, the fields from the two semicircular loops cancel at $C$ (by symmetry). Therefore, $B_C = 0$.

5. (a) We find the field by superposing the results of two semi-infinite wires (Eq. 29-7) and a semicircular arc (Eq. 29-9 with $\phi = \pi$ rad). The direction of $\vec{B}$ is out of the page, as can be checked by referring to Fig. 29-7(c). The magnitude of $\vec{B}$ at point $a$ is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi}+\frac{1}{2}\right) = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(10\ \mathrm{A})}{2(0.0050\ \mathrm{m})}\left(\frac{1}{\pi}+\frac{1}{2}\right) = 1.0\times10^{-3}\ \mathrm{T}$$

upon substituting $i = 10$ A and $R = 0.0050$ m.

(b) The direction of this field is out of the page, as Fig. 29-7(c) makes clear.

(c) The last remark in the problem statement implies that treating $b$ as a point midway between two infinite wires is a good approximation. Thus, using Eq. 29-4,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = \frac{\mu_0 i}{\pi R} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(10\ \mathrm{A})}{\pi(0.0050\ \mathrm{m})} = 8.0\times10^{-4}\ \mathrm{T}.$$

(d) This field, too, points out of the page.

6. With the "usual" $x$ and $y$ coordinates used in Fig. 29-38, then the vector $\vec{r}$ pointing from a current element to $P$ is $\vec{r} = -s\,\hat{\mathrm{i}} + R\,\hat{\mathrm{j}}$. Since $d\vec{s} = ds\,\hat{\mathrm{i}}$, then $|d\vec{s}\times\vec{r}| = Rds$. Therefore, with $r = \sqrt{s^2+R^2}$, Eq. 29-3 gives

$$dB = \frac{\mu_0}{4\pi}\frac{iR\,ds}{(s^2+R^2)^{3/2}}.$$

(a) Clearly, considered as a function of $s$ (but thinking of "$ds$" as some finite-sized constant value), the above expression is maximum for $s = 0$. Its value in this case is $dB_{\max} = \mu_0 i\,ds/4\pi R^2$.

(b) We want to find the $s$ value such that $dB = dB_{\max}/10$. This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is $s = \sqrt{10^{2/3}-1}\,R$. If we set $R = 2.00$ cm, then we obtain $s = 3.82$ cm.

7. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with *P* do not contribute to the field at that point. Using Eq. 29-9 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius *b* contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at *P*. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at *P* is

$$B = \frac{\mu_0 i\theta}{4}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.411\text{A})(74°\cdot\pi/180°)}{4\pi}\left(\frac{1}{0.107\text{m}} - \frac{1}{0.135\text{m}}\right)$$
$$= 1.02\times10^{-7}\,\text{T}.$$

(b) The direction is out of the page.

8. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in segments *AH* and *JD* do not contribute to the field at point *C*. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc *H J* contributes $\mu_0 i/4R_1$ (into the page) to the field at *C*. Also, arc *D A* contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at *C* is

$$B = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.281\text{A})}{4}\left(\frac{1}{0.0315\text{m}} - \frac{1}{0.0780\text{m}}\right) = 1.67\times10^{-6}\,\text{T}.$$

(b) The direction of the field is into the page.

9. **THINK** The net magnetic field at a point half way between the two long straight wires is the vector sum of the magnetic fields due to the currents in the two wires.

**EXPRESS** Since the magnitude of the magnetic field at a distance *r* from a long straight wire carrying current *i* is given by $B = \mu_0 i/2\pi r$, at a point half way between the two sires, the magnetic field is $\vec{B} = \vec{B}_1 + \vec{B}_2$, where $B_1 = B_2 = \mu_0 i/2\pi r$ (assuming the two wires to be 2*r* apart). The directions of $\vec{B}_1$ and $\vec{B}_2$ are determined by the right-hand rule.

**ANALYZE** (a) The currents must be opposite or anti-parallel, so that the resulting fields are in the same direction in the region between the wires. If the currents are parallel, then the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires.

(b) The total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\text{ m})(300\times10^{-6}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 30\text{ A}.$$

**LEARN** For two parallel wires carrying currents in the opposite directions, a point that is a distance $d$ from one wire and $2r - d$ from the other, the magnitude of the magnetic field is

$$B = B_1 + B_2 = \frac{\mu_0 i}{2\pi d} + \frac{\mu_0 i}{2\pi(2r-d)} = \frac{\mu_0 i}{2\pi}\left(\frac{1}{d} + \frac{1}{2r-d}\right).$$

10. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with $C$ do not contribute to the field at that point.

Equation 29-9 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at $C$. Thus, the magnitude of the magnetic field is

$$B = \frac{\mu_0 i}{4R} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.0348\text{A})}{4(0.0926\text{m})} = 1.18\times10^{-7}\text{ T}.$$

(b) The right-hand rule shows that this field is into the page.

11. (a) $B_{P_1} = \mu_0 i_1 / 2\pi r_1$ where $i_1$ = 6.5 A and $r_1 = d_1 + d_2$ = 0.75 cm + 1.5 cm = 2.25 cm, and $B_{P_2} = \mu_0 i_2 / 2\pi r_2$ where $r_2 = d_2$ = 1.5 cm. From $B_{P1} = B_{P2}$ we get

$$i_2 = i_1\left(\frac{r_2}{r_1}\right) = (6.5\text{ A})\left(\frac{1.5\text{ cm}}{2.25\text{ cm}}\right) = 4.3\text{ A}.$$

(b) Using the right-hand rule, we see that the current $i_2$ carried by wire 2 must be out of the page.

12. (a) Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is $r$ away from the wire carrying current $i$ and is $d - r$ away from the wire carrying current 3.00$i$, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi(d-r)} \quad\Rightarrow\quad r = \frac{d}{4} = \frac{16.0\text{ cm}}{4} = 4.0\text{ cm}.$$

(b) Doubling the currents does not change the location where the magnetic field is zero.

13. Our $x$ axis is along the wire with the origin at the midpoint. The current flows in the positive $x$ direction. All segments of the wire produce magnetic fields at $P_1$ that are out of

the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at $P_1$ is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dx$$

where $\theta$ (the angle between the segment and a line drawn from the segment to $P_1$) and $r$ (the length of that line) are functions of $x$. Replacing $r$ with $\sqrt{x^2+R^2}$ and $\sin\,\theta$ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from $x = -L/2$ to $x = L/2$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}$$

$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(0.0582\text{ A}\right)}{2\pi\left(0.131\,\text{m}\right)}\frac{0.180\text{m}}{\sqrt{(0.180\text{m})^2+4(0.131\text{m})^2}} = 5.03\times10^{-8}\,\text{T}.$$

14. We consider Eq. 29-6 but with a finite upper limit ($L/2$ instead of $\infty$). This leads to

$$B = \frac{\mu_0 i}{2\pi R}\frac{L/2}{\sqrt{(L/2)^2+R^2}}.$$

In terms of this expression, the problem asks us to see how large $L$ must be (compared with $R$) such that the infinite wire expression $B_\infty$ (Eq. 29-4) can be used with no more than a 1% error. Thus we must solve

$$\frac{B_\infty - B}{B} = 0.01.$$

This is a nontrivial algebra exercise, but is nonetheless straightforward. The result is

$$L = \frac{200R}{\sqrt{201}} \approx 14.1R \quad\Rightarrow\quad \frac{L}{R}\approx 14.1.$$

15. (a) As discussed in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," the radial segments do not contribute to $\vec{B}_P$ and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0\left(0.40\,\text{A}\right)\left(\pi\,\text{rad}\right)}{4\pi\left(0.050\,\text{m}\right)}\hat{\text{k}} - \frac{\mu_0\left(0.80\,\text{A}\right)\left(2\pi/3\,\text{rad}\right)}{4\pi\left(0.040\,\text{m}\right)}\hat{\text{k}} = -(1.7\times10^{-6}\ \text{T})\hat{\text{k}}$$

or $|\vec{B}| = 1.7\times10^{-6}\,\text{T}$.

(b) The direction is $-\hat{\text{k}}$, or into the page.

(c) If the direction of $i_1$ is reversed, we then have

$$\vec{B} = -\frac{\mu_0(0.40\,\text{A})(\pi\,\text{rad})}{4\pi(0.050\,\text{m})}\hat{\text{k}} - \frac{\mu_0(0.80\,\text{A})(2\pi/3\,\text{rad})}{4\pi(0.040\,\text{m})}\hat{\text{k}} = -(6.7\times10^{-6}\text{ T})\hat{\text{k}}$$

or $|\vec{B}| = 6.7\times10^{-6}\,\text{T}$.

(d) The direction is $-\hat{\text{k}}$, or into the page.

16. Using the law of cosines and the requirement that $B$ = 100 nT, we have

$$\theta = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{-2B_1B_2}\right) = 144°,$$

where Eq. 29-10 has been used to determine $B_1$ (168 nT) and $B_2$ (151 nT).

17. **THINK** We apply the Biot-Savart law to calculate the magnetic field at point $P_2$. An integral is required since the length of the wire is finite.

**EXPRESS** We take the $x$ axis to be along the wire with the origin at the right endpoint. The current is in the $+x$ direction. All segments of the wire produce magnetic fields at $P_2$ that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at $P_2$ is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where $\theta$ (the angle between the segment and a line drawn from the segment to $P_2$) and $r$ (the length of that line) are functions of $x$. Replacing $r$ with $\sqrt{x^2+R^2}$ and sin $\theta$ with $R/r = R\big/\sqrt{x^2+R^2}$, we integrate from $x = -L$ to $x = 0$.

**ANALYZE** The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L}^{0}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}$$
$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(0.693\,\text{A})}{4\pi(0.251\,\text{m})}\frac{0.136\,\text{m}}{\sqrt{(0.136\,\text{m})^2+(0.251\,\text{m})^2}} = 1.32\times10^{-7}\,\text{T}.$$

**LEARN** In calculating $B$ at $P_2$, we could have chosen the origin to be at the left endpoint. This only changes the integration limit, but the result remains the same:

$$B=\frac{\mu_0 iR}{4\pi}\int_0^L\frac{dx}{\left(x^2+R^2\right)^{3/2}}=\frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_0^L=\frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}\ .$$

18. In the one case we have $B_{\text{small}}$ + $B_{\text{big}}$ = 47.25 $\mu$T, and the other case gives $B_{\text{small}}$ – $B_{\text{big}}$ = 15.75 $\mu$T (cautionary note about our notation: $B_{\text{small}}$ refers to the field at the center of the small-radius arc, which is actually a bigger field than $B_{\text{big}}$!). Dividing one of these equations by the other and canceling out common factors (see Eq. 29-9) we obtain

$$\frac{(1/r_{\text{small}})+(1/r_{\text{big}})}{(1/r_{\text{small}})-(1/r_{\text{big}})}=\frac{1+(r_{\text{small}}/r_{\text{big}})}{1-(r_{\text{small}}/r_{\text{big}})}=3\ .$$

The solution of this is straightforward: $r_{\text{small}}$ = $r_{\text{big}}$ /2. Using the given fact that the $r_{\text{big}}=4.00$ cm, then we conclude that the small radius is $r_{\text{small}}=2.00$ cm.

19. The contribution to $\vec{B}_{\text{net}}$ from the first wire is (using Eq. 29-4)

$$\vec{B}_1=\frac{\mu_0 i_1}{2\pi r_1}\hat{\text{k}}=\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(30\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{k}}=(3.0\times10^{-6}\ \text{T})\hat{\text{k}}.$$

The distance from the second wire to the point where we are evaluating $\vec{B}_{\text{net}}$ is $r_2$ = 4 m – 2 m = 2 m. Thus,

$$\vec{B}_2=\frac{\mu_0 i_2}{2\pi r_2}\hat{\text{i}}=\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(40\ \text{A})}{2\pi(2.0\ \text{m})}\hat{\text{i}}=(4.0\times10^{-6}\ \text{T})\hat{\text{i}}.$$

and consequently is perpendicular to $\vec{B}_1$. The magnitude of $\vec{B}_{\text{net}}$ is therefore

$$|\vec{B}_{\text{net}}|=\sqrt{(3.0\times10^{-6}\ \text{T})^2+(4.0\times10^{-6}\ \text{T})^2}=5.0\times10^{-6}\ \text{T}\ .$$

20. (a) The contribution to $B_C$ from the (infinite) straight segment of the wire is

$$B_{C1}=\frac{\mu_0 i}{2\pi R}.$$

The contribution from the circular loop is $B_{C2}=\dfrac{\mu_0 i}{2R}$. Thus,

$$B_C=B_{C1}+B_{C2}=\frac{\mu_0 i}{2R}\left(1+\frac{1}{\pi}\right)=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.78\times10^{-3}\ \text{A}\right)}{2\left(0.0189\ \text{m}\right)}\left(1+\frac{1}{\pi}\right)=2.53\times10^{-7}\text{T}.$$

$\vec{B}_C$ points out of the page, or in the $+z$ direction. In unit-vector notation, $\vec{B}_C = (2.53\times10^{-7}\,\text{T})\hat{\text{k}}$

(b) Now, $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1+\frac{1}{\pi^2}} = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(5.78\times10^{-3}\,\text{A}\right)}{2\left(0.0189\,\text{m}\right)}\sqrt{1+\frac{1}{\pi^2}} = 2.02\times10^{-7}\,\text{T}.$$

and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 17.66°.$$

In unit-vector notation,

$$\vec{B}_C = 2.02\times10^{-7}\,\text{T}(\cos17.66°\hat{\text{i}} + \sin17.66°\hat{\text{k}}) = (1.92\times10^{-7}\,\text{T})\hat{\text{i}} + (6.12\times10^{-8}\,\text{T})\hat{\text{k}}\,.$$

21. Using the right-hand rule (and symmetry), we see that $\vec{B}_{\text{net}}$ points along what we will refer to as the $y$ axis (passing through $P$), consisting of two equal magnetic field $y$-components. Using Eq. 29-17,

$$|\vec{B}_{\text{net}}| = 2\frac{\mu_0 i}{2\pi r}\sin\theta$$

where $i$ = 4.00 A, $r$ = $r = \sqrt{d_2^2 + d_1^2/4} = 5.00\text{ m},$ and

$$\theta = \tan^{-1}\left(\frac{d_2}{d_1/2}\right) = \tan^{-1}\left(\frac{4.00\text{ m}}{6.00\text{ m}/2}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1°\,.$$

Therefore,

$$|\vec{B}_{\text{net}}| = \frac{\mu_0 i}{\pi r}\sin\theta = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.00\text{ A})}{\pi(5.00\text{ m})}\sin 53.1° = 2.56\times10^{-7}\text{ T}\,.$$

22. The fact that $B_y = 0$ at $x = 10$ cm implies the currents are in opposite directions. Thus,

$$B_y = \frac{\mu_0 i_1}{2\pi(L+x)} - \frac{\mu_0 i_2}{2\pi x} = \frac{\mu_0 i_2}{2\pi}\left(\frac{4}{L+x} - \frac{1}{x}\right)$$

using Eq. 29-4 and the fact that $i_1 = 4i_2$. To get the maximum, we take the derivative with respect to $x$ and set equal to zero. This leads to $3x^2 - 2Lx - L^2 = 0$, which factors and becomes $(3x + L)(x - L) = 0$, which has the physically acceptable solution: $x = L$. This produces the maximum $B_y$: $\mu_0 i_2/2\pi L$. To proceed further, we must determine $L$.

Examination of the datum at $x$ = 10 cm in Fig. 29-50(b) leads (using our expression above for $B_y$ and setting that to zero) to $L$ = 30 cm.

(a) The maximum value of $B_y$ occurs at $x = L$ = 30 cm.

(b) With $i_2$ = 0.003 A we find $\mu_0 i_2 / 2\pi L$ = 2.0 nT.

(c) and (d) Figure 29-50(b) shows that as we get very close to wire 2 (where its field strongly dominates over that of the more distant wire 1) $B_y$ points along the $-y$ direction. The right-hand rule leads us to conclude that wire 2's current is consequently is *into the page*. We previously observed that the currents were in opposite directions, so wire 1's current is *out of the page*.

23. We assume the current flows in the $+x$ direction and the particle is at some distance $d$ in the $+y$ direction (away from the wire). Then, the magnetic field at the location of a proton with charge $q$ is $\vec{B} = (\mu_0 i / 2\pi d)\,\hat{\text{k}}$. Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 iq}{2\pi d}\left(\vec{v} \times \hat{\text{k}}\right).$$

In this situation, $\vec{v} = v\left(-\hat{\text{j}}\right)$ (where $v$ is the speed and is a positive value), and $q > 0$. Thus,

$$\vec{F} = \frac{\mu_0 iqv}{2\pi d}\left(\left(-\hat{\text{j}}\right)\times\hat{\text{k}}\right) = -\frac{\mu_0 iqv}{2\pi d}\,\hat{\text{i}} = -\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.350\text{A})(1.60\times10^{-19}\,\text{C})(200\text{m/s})}{2\pi(0.0289\text{ m})}\,\hat{\text{i}}$$
$$= (-7.75\times10^{-23}\,\text{N})\hat{\text{i}}.$$

24. Initially, we have $B_{\text{net},y} = 0$ and $B_{\text{net},x} = B_2 + B_4 = 2(\mu_0 i / 2\pi d)$ using Eq. 29-4, where $d = 0.15$ m. To obtain the 30º condition described in the problem, we must have

$$B_{\text{net},y} = B_{\text{net},x}\tan(30°) \quad \Rightarrow \quad B_1' - B_3 = 2\left(\frac{\mu_0 i}{2\pi d}\right)\tan(30°)$$

where $B_3 = \mu_0 i / 2\pi d$ and $B_1' = \mu_0 i / 2\pi d'$. Since tan(30º) = $1/\sqrt{3}$ , this leads to

$$d' = \frac{\sqrt{3}}{\sqrt{3}+2}\,d = 0.464d\ .$$

(a) With $d$ = 15.0 cm, this gives $d'$ = 7.0 cm. Being very careful about the geometry of the situation, then we conclude that we must move wire 1 to $x$ = –7.0 cm.

(b) To restore the initial symmetry, we would have to move wire 3 to $x$ = +7.0 cm.

25. **THINK** The magnetic field at the center of the circle is the vector sum of the fields of the two straight wires and the arc.

**EXPRESS** Each of the semi-infinite straight wires contributes $B_{\text{straight}} = \mu_0 i/4\pi R$ (Eq. 29-7) to the field at the center of the circle (both contributions pointing "out of the page"). The current in the arc contributes a term given by Eq. 29-9: $B_{\text{arc}} = \dfrac{\mu_0 i\phi}{4\pi R}$, pointing into the page.

**ANALYZE** The total magnetic field is

$$B = 2B_{\text{straight}} - B_{\text{arc}} = 2\left(\frac{\mu_0 i}{4\pi R}\right) - \frac{\mu_0 i\phi}{4\pi R} = \frac{\mu_0 i}{4\pi R}(2-\phi).$$

Therefore, $\phi$ = 2.00 rad would produce zero total field at the center of the circle.

**LEARN** The total contribution of the two semi-infinite wires is the same as that of an infinite wire. Note that the angle $\phi$ is in radians rather than degrees.

26. Using the Pythagorean theorem, we have

$$B^2 = B_1^2 + B_2^2 = \left(\frac{\mu_0 i_1 \phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i_2}{2\pi R}\right)^2$$

which, when thought of as the equation for a line in a $B^2$ versus $i_2^{\,2}$ graph, allows us to identify the first term as the "$y$-intercept" ($1 \times 10^{-10}$) and the part of the second term that multiplies $i_2^{\,2}$ as the "slope" ($5 \times 10^{-10}$). The latter observation leads to

$$5.00\times10^{-10}\ \text{T}^2/\text{A}^2 = \left(\frac{\mu_0}{2\pi R}\right)^2 = \left(\frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi R}\right)^2$$

or

$$R^2 = \frac{4.00\times10^{-14}\ \text{T}^2\cdot\text{m}^2/\text{A}^2}{5.00\times10^{-10}\ \text{T}^2/\text{A}^2} = 8.00\times10^{-5}\,\text{m}^2 \Rightarrow R = 8.94\times10^{-3}\ \text{m} \approx 8.9\ \text{mm}.$$

The other observation about the "$y$-intercept" determines the angle subtended by the arc:

$$1.00\times10^{-10}\ \text{T}^2 = \left(\frac{\mu_0 i_1 \phi}{4\pi R}\right)^2 = \left(\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.50\ \text{A})}{4\pi(8.94\times10^{-3}\text{m})}\right)^2 \phi^2 = (3.13\times10^{-11}\phi^2)\ \text{T}^2$$

or

$$\phi^2 = \frac{1.00\times10^{-10}\ \text{T}^2}{3.13\times10^{-11}\ \text{T}^2} = 3.19 \Rightarrow \phi = 1.79\ \text{rad} \approx 1.8\ \text{rad}.$$

27. We use Eq. 29-4 to relate the magnitudes of the magnetic fields $B_1$ and $B_2$ to the currents ($i_1$ and $i_2$, respectively) in the two long wires. The angle of their net field is

$$\theta = \tan^{-1}(B_2/B_1) = \tan^{-1}(i_2/i_1) = 53.13°.$$

The accomplish the net field rotation described in the problem, we must achieve a final angle $\theta' = 53.13° - 20° = 33.13°$. Thus, the final value for the current $i_1$ must be $i_2/\tan\theta'$ = 61.3 mA.

28. Letting "out of the page" in Fig. 29-56(a) be the positive direction, the net field is

$$B = \frac{\mu_0 i_1 \phi}{4\pi R} - \frac{\mu_0 i_2}{2\pi(R/2)}$$

from Eqs. 29-9 and 29-4. Referring to Fig. 29-56, we see that $B = 0$ when $i_2 = 0.5$ A, so (solving the above expression with $B$ set equal to zero) we must have

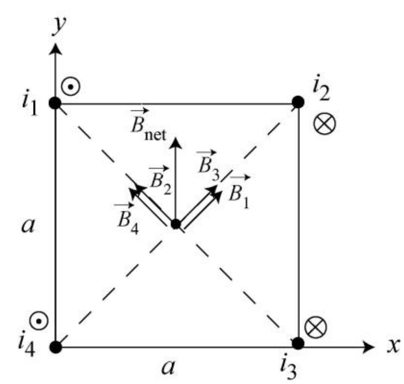
$$\phi = 4(i_2/i_1) = 4(0.5/2) = 1.00 \text{ rad (or } 57.3°).$$

29. **THINK** Our system consists of four long straight wires whose cross section form a square of length $a$. The magnetic field at the center of the square is the vector sum of the fields of the four wires.

**EXPRESS** Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where $r$ is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left (wire 1) and lower right (wire 3) corners both point toward the upper right corner of the square. The fields due to the wires at the upper right (wire 2) and lower left (wire 4) corners both point toward the upper left corner.

**ANALYZE** The horizontal components of the fields cancel and the vertical components sum to

$$B_{\text{net}} = 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45° = \frac{2\mu_0 i}{\pi a} = \frac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20\ \text{A})}{\pi(0.20\ \text{m})}$$
$$= 8.0\times10^{-5}\ \text{T}.$$




In the calculation, cos 45° was replaced with $1/\sqrt{2}$. The total field points upward, or in the $+y$ direction. Thus, $\vec{B}_{\text{net}} = (8.0\times10^{-5}\ \text{T})\hat{\text{j}}$.

**LEARN** In the figure to the right, we show the contributions from the individual wires. The directions of the fields are deduced using the right-hand rule.

30. We note that when there is no *y*-component of magnetic field from wire 1 (which, by the right-hand rule, relates to when wire 1 is at 90º = $\pi/2$ rad), the total *y*-component of magnetic field is zero (see Fig. 29-58(c)). This means wire #2 is either at $+\pi/2$ rad or $-\pi/2$ rad.

(a) We now make the assumption that wire #2 must be at $-\pi/2$ rad (–90º, the bottom of the cylinder) since it would pose an obstacle for the motion of wire #1 (which is needed to make these graphs) if it were anywhere in the top semicircle.

(b) Looking at the $\theta_1$ = 90º datum in Fig. 29-58(b)), where there is a *maximum* in $B_{\text{net }x}$ (equal to +6 $\mu$T), we are led to conclude that $B_{1x} = 6.0\ \mu\text{T} - 2.0\ \mu\text{T} = 4.0\ \mu\text{T}$ in that situation. Using Eq. 29-4, we obtain

$$i_1 = \frac{2\pi R B_{1x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(4.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 4.0\text{ A}.$$

(c) The fact that Fig. 29-58(b) increases as $\theta_1$ progresses from 0 to 90º implies that wire 1's current is *out of the page*, and this is consistent with the cancellation of $B_{\text{net }y}$ at $\theta_1 = 90°$, noted earlier (with regard to Fig. 29-58(c)).

(d) Referring now to Fig. 29-58(b) we note that there is no *x*-component of magnetic field from wire 1 when $\theta_1$ = 0, so that plot tells us that $B_{2x}$ = +2.0 $\mu$T. Using Eq. 29-4, we find the magnitudes of the current to be

$$i_2 = \frac{2\pi R B_{2x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 2.0\text{ A}.$$

(e) We can conclude (by the right-hand rule) that wire 2's current is *into the page*.

31. (a) Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with *P* do not contribute to the field at that point. We use the result of Problem 29-21 to evaluate the contributions to the field at *P*, noting that the nearest wire segments (each of length *a*) produce magnetism into the page at *P* and the further wire segments (each of length 2*a*) produce magnetism pointing out of the page at *P*. Thus, we find (into the page)

$$B_P = 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a}\right) - 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi(2a)}\right) = \frac{\sqrt{2}\mu_0 i}{8\pi a} = \frac{\sqrt{2}\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(13\text{ A})}{8\pi\left(0.047\text{ m}\right)}$$
$$= 1.96\times10^{-5}\,\text{T} \approx 2.0\times10^{-5}\,\text{T}.$$

(b) The direction of the field is into the page.

32. Initially we have

$$B_i = \frac{\mu_0 i\phi}{4\pi R} + \frac{\mu_0 i\phi}{4\pi r}$$

using Eq. 29-9. In the final situation we use Pythagorean theorem and write

$$B_f^2 = B_z^2 + B_y^2 = \left(\frac{\mu_0 i\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i\phi}{4\pi r}\right)^2.$$

If we square $B_i$ and divide by $B_f^2$, we obtain

$$\left(\frac{B_i}{B_f}\right)^2 = \frac{[(1/R)+(1/r)]^2}{(1/R)^2+(1/r)^2}\ .$$

From the graph (see Fig. 29-60(c), note the maximum and minimum values) we estimate $B_i/B_f$ = 12/10 = 1.2, and this allows us to solve for $r$ in terms of $R$:

$$r = R\,\frac{1 \pm 1.2\sqrt{2-1.2^2}}{1.2^2-1} \ = \ 2.3 \text{ cm} \quad \text{or} \quad 43.1 \text{ cm}.$$

Since we require $r < R$, then the acceptable answer is $r$ = 2.3 cm.

33. **THINK** The magnetic field at point $P$ produced by the current-carrying ribbon (shown in Fig. 29-61) can be calculated using the Biot-Savart law.

**EXPRESS** Consider a section of the ribbon of thickness $dx$ located a distance $x$ away from point $P$. The current it carries is $di = i\,dx/w$, and its contribution to $B_P$ is

$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w}.$$

**ANALYZE** Integrating over the length of the ribbon, we obtain

$$B_P = \int dB_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.61\times10^{-6}\ \text{A})}{2\pi(0.0491\ \text{m})}\ln\left(1+\frac{0.0491}{0.0216}\right)$$
$$= 2.23\times10^{-11}\,\text{T}.$$

and $\vec{B}_P$ points upward. In unit-vector notation, $\vec{B}_P = (2.23\times10^{-11}\,\text{T})\hat{\text{j}}$.

**LEARN** In the limit where $d \gg w$, using

$$\ln(1+x) = x - x^2/2 + \cdots,$$

the magnetic field becomes

$$B_P = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) \approx \frac{\mu_0 i}{2\pi w}\cdot\frac{w}{d} = \frac{\mu_0 i}{2\pi d}$$

which is the same as that due to a thin wire.

34. By the right-hand rule (which is "built-into" Eq. 29-3) the field caused by wire 1's current, evaluated at the coordinate origin, is along the $+y$ axis. Its magnitude $B_1$ is given by Eq. 29-4. The field caused by wire 2's current will generally have both an $x$ and a $y$ component, which are related to its magnitude $B_2$ (given by Eq. 29-4), and sines and cosines of some angle. A little trig (and the use of the right-hand rule) leads us to conclude that when wire 2 is at angle $\theta_2$ (shown in Fig. 29-62) then its components are

$$B_{2x} = B_2 \sin\theta_2, \quad B_{2y} = -B_2 \cos\theta_2 \, .$$

The magnitude-squared of their net field is then (by Pythagoras' theorem) the sum of the square of their net $x$-component and the square of their net $y$-component:

$$B^2 = (B_2 \sin\theta_2)^2 + (B_1 - B_2 \cos\theta_2)^2 = B_1^2 + B_2^2 - 2B_1 B_2 \cos\theta_2 \, .$$

(since $\sin^2\theta + \cos^2\theta = 1$), which we could also have gotten directly by using the law of cosines. We have

$$B_1 = \frac{\mu_0 i_1}{2\pi R} = 60 \text{ nT}, \quad B_2 = \frac{\mu_0 i_2}{2\pi R} = 40 \text{ nT}.$$

With the requirement that the net field have magnitude $B$ = 80 nT, we find

$$\theta_2 = \cos^{-1}\left(\frac{B_1^2 + B_2^2 - B^2}{2B_1 B_2}\right) = \cos^{-1}(-1/4) = 104°,$$

where the positive value has been chosen.

35. **THINK** The magnitude of the force of wire 1 on wire 2 is given by $F_{21} = \mu_0 i_1 i_2 L / 2\pi r,$ where $i_1$ is the current in wire 1, $i_2$ is the current in wire 2, and $r$ is the distance between the wires.

**EXPRESS** The distance between the wires is $r = \sqrt{d_1^2 + d_2^2}$. The $x$ component of the force is $F_{21,x} = F_{21}\cos\phi,$ where $\cos\phi = d_2 / \sqrt{d_1^2 + d_2^2}$.

**ANALYZE** Substituting the values given, the $x$ component of the force per unit length is

$$\frac{F_{21,x}}{L} = \frac{\mu_0 i_1 i_2 d_2}{2\pi(d_1^2 + d_2^2)} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\times10^{-3}\ \text{A})(6.80\times10^{-3}\ \text{A})(0.050\ \text{m})}{2\pi[(0.0240\ \text{m})^2 + (0.050\ \text{m})^2]}$$
$$= 8.84\times10^{-11}\ \text{N/m}.$$

**LEARN** Since the two currents flow in the opposite directions, the force between the wires is repulsive. Thus, the direction of $\vec{F}_{21}$ is along the line that joins the wire and is away from wire 1.

36. We label these wires 1 through 5, left to right, and use Eq. 29-13. Then,

(a) The magnetic force on wire 1 is

$$\vec{F}_1 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{4d}\right)\hat{\text{j}} = \frac{25\mu_0 i^2 l}{24\pi d}\,\hat{\text{j}} = \frac{25\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\text{A}\right)^2(10.0\text{m})}{24\pi\left(8.00\times10^{-2}\,\text{m}\right)}\,\hat{\text{j}}$$
$$= (4.69\times10^{-4}\,\text{N})\,\hat{\text{j}}.$$

(b) Similarly, for wire 2, we have

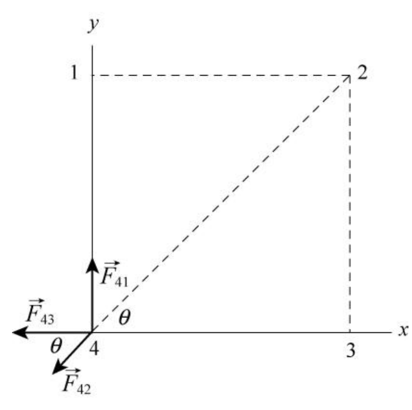
$$\vec{F}_2 = \frac{\mu_0 i^2 l}{2\pi}\left(\frac{1}{2d}+\frac{1}{3d}\right)\hat{\text{j}} = \frac{5\mu_0 i^2 l}{12\pi d}\,\hat{\text{j}} = (1.88\times10^{-4}\text{N})\,\hat{\text{j}}.$$

(c) $F_3 = 0$ (because of symmetry).

(d) $\vec{F}_4 = -\vec{F}_2 = (-1.88\times10^{-4}\text{N})\hat{\text{j}}$, and

(e) $\vec{F}_5 = -\vec{F}_1 = -(4.69\times10^{-4}\,\text{N})\hat{\text{j}}$.

37. We use Eq. 29-13 and the superposition of forces: $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$, the situation is as shown on the right.




The components of $\vec{F}_4$ are given by

$$F_{4x} = -F_{43} - F_{42}\cos\theta = -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} = -\frac{3\mu_0 i^2}{4\pi a}$$

and

$$F_{4y} = F_{41} - F_{42}\sin\theta = \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} = \frac{\mu_0 i^2}{4\pi a}.$$

Thus,

$$F_4=\left(F_{4x}^2+F_{4y}^2\right)^{1/2}=\left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2+\left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2}=\frac{\sqrt{10}\mu_0 i^2}{4\pi a}=\frac{\sqrt{10}\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(7.50\text{A}\right)^2}{4\pi\left(0.135\text{m}\right)}$$
$$=1.32\times10^{-4}\ \text{N/m}$$

and $\vec{F}_4$ makes an angle $\phi$ with the positive *x* axis, where

$$\phi=\tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right)=\tan^{-1}\left(-\frac{1}{3}\right)=162°.$$

In unit-vector notation, we have

$$\vec{F}_1=(1.32\times10^{-4}\ \text{N/m})[\cos162°\hat{\text{i}}+\sin162°\hat{\text{j}}]=(-1.25\times10^{-4}\ \text{N/m})\hat{\text{i}}+(4.17\times10^{-5}\ \text{N/m})\hat{\text{j}}$$

38. (a) The fact that the curve in Fig. 29-65(b) passes through zero implies that the currents in wires 1 and 3 exert forces in opposite directions on wire 2. Thus, current $i_1$ points *out of the page*. When wire 3 is a great distance from wire 2, the only field that affects wire 2 is that caused by the current in wire 1; in this case the force is negative according to Fig. 29-65(b). This means wire 2 is attracted to wire 1, which implies (by the discussion in Section 29-2) that wire 2's current is in the same direction as wire 1's current: *out of the page*. With wire 3 infinitely far away, the force per unit length is given (in magnitude) as $6.27 \times 10^{-7}$ N/m. We set this equal to $F_{12}=\mu_0 i_1 i_2/2\pi d$. When wire 3 is at $x$ = 0.04 m the curve passes through the zero point previously mentioned, so the force between 2 and 3 must equal $F_{12}$ there. This allows us to solve for the distance between wire 1 and wire 2:

$$d = (0.04\ \text{m})(0.750\ \text{A})/(0.250\ \text{A}) = 0.12\ \text{m}.$$

Then we solve $6.27 \times 10^{-7}$ N/m= $\mu_0 i_1 i_2/2\pi d$ and obtain $i_2$ = 0.50 A.

(b) The direction of $i_2$ is out of the page.

39. Using a magnifying glass, we see that all but $i_2$ are directed into the page. Wire 3 is therefore attracted to all but wire 2. Letting $d$ = 0.500 m, we find the net force (per meter length) using Eq. 29-13, with positive indicated a rightward force:

$$\frac{|\vec{F}|}{\ell}=\frac{\mu_0 i_3}{2\pi}\left(-\frac{i_1}{2d}+\frac{i_2}{d}+\frac{i_4}{d}+\frac{i_5}{2d}\right)$$

which yields $|\vec{F}|/\ell=8.00\times10^{-7}\ \text{N/m}$.

40. Using Eq. 29-13, the force on, say, wire 1 (the wire at the upper left of the figure) is along the diagonal (pointing toward wire 3, which is at the lower right). Only the forces

(or their components) along the diagonal direction contribute. With $\theta = 45°$, we find the force per unit meter on wire 1 to be

$$F_1 = |\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}| = 2F_{12}\cos\theta + F_{13} = 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45° + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} = \frac{3}{2\sqrt{2}\pi}\left(\frac{\mu_0 i^2}{a}\right)$$
$$= \frac{3}{2\sqrt{2}\pi}\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(15.0\,\text{A}\right)^2}{\left(8.50\times10^{-2}\,\text{m}\right)} = 1.12\times10^{-3}\,\text{N/m}.$$

The direction of $\vec{F}_1$ is along $\hat{r} = (\hat{\text{i}} - \hat{\text{j}})/\sqrt{2}$. In unit-vector notation, we have

$$\vec{F}_1 = \frac{(1.12\times10^{-3}\,\text{N/m})}{\sqrt{2}}(\hat{\text{i}} - \hat{\text{j}}) = (7.94\times10^{-4}\,\text{N/m})\hat{\text{i}} + (-7.94\times10^{-4}\,\text{N/m})\hat{\text{j}}$$

41. The magnitudes of the forces on the sides of the rectangle that are parallel to the long straight wire (with $i_1$ = 30.0 A) are computed using Eq. 29-13, but the force on each of the sides lying perpendicular to it (along our $y$ axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\,\text{sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y}\,dy.$$

Fortunately, these forces on the two perpendicular sides of length $b$ cancel out. For the remaining two (parallel) sides of length $L$, we obtain

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a\left(a+b\right)}$$
$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(30.0\,\text{A}\right)\left(20.0\,\text{A}\right)\left(8.00\,\text{cm}\right)\left(300\times10^{-2}\,\text{m}\right)}{2\pi\left(1.00\,\text{cm} + 8.00\,\text{cm}\right)} = 3.20\times10^{-3}\,\text{N},$$

and $\vec{F}$ points toward the wire, or $+\hat{\text{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\,\text{N})\hat{\text{j}}$ in unit-vector notation.

42. The area enclosed by the loop $L$ is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\oint_c \vec{B}\cdot d\vec{s} = \mu_0 i = \mu_0 jA = \left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(15\,\text{A/m}^2\right)\left(6\right)\left(0.20\,\text{m}\right)^2 = 4.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

43. We use Eq. 29-20 $B = \mu_0 ir/2\pi a^2$ for the $B$-field inside the wire ($r < a$) and Eq. 29-17 $B = \mu_0 i/2\pi r$ for that outside the wire ($r > a$).

(a) At $r = 0$, $B = 0$.

(b) At $r = 0.0100\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0100\text{m})}{2\pi(0.0200\text{m})^2} = 8.50\times10^{-4}\text{T}.$

(c) At $r = a = 0.0200\text{m}$, $B = \dfrac{\mu_0 ir}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0200\text{m})}{2\pi(0.0200\text{m})^2} = 1.70\times10^{-3}\text{T}.$

(d) At $r = 0.0400\text{m}$, $B = \dfrac{\mu_0 i}{2\pi r} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})}{2\pi(0.0400\text{m})} = 8.50\times10^{-4}\text{T}.$

44. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and $i$ is the net current through the loop.

(a) For path 1, the result is

$$\oint_1 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\text{A}+3.0\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-2.0\text{A}\right) = -2.5\times10^{-6}\text{T}\cdot\text{m}.$$

(b) For path 2, we find

$$\oint_2 \vec{B}\cdot d\vec{s} = \mu_0\left(-5.0\text{A}-5.0\text{A}-3.0\text{A}\right) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(-13.0\text{A}\right) = -1.6\times10^{-5}\text{T}\cdot\text{m}.$$

45. **THINK** The value of the line integral $\oint \vec{B}\cdot d\vec{s}$ is proportional to the net current enclosed.

**EXPRESS** By Ampere's law, we have $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where $i_{\text{enc}}$ is the current enclosed by the closed path.

**ANALYZE** (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path, or "Amperian loop" is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B}\cdot d\vec{s} = -\mu_0 i = -(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\left(2.0\,\text{A}\right) = -2.5\times10^{-6}\text{T}\cdot\text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

**LEARN** The value of $\oint \vec{B}\cdot d\vec{s}$ depends only on the current enclosed, and not the shape of the Amperian loop.

46. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\left(7i-6i+3i+i\right)=5\mu_0 i = 5\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(4.50\times10^{-3}\,\text{A}\right)=2.83\times10^{-8}\,\text{T}\cdot\text{m}.$$

47. For $r \le a$,

$$B\left(r\right)=\frac{\mu_0 i_{\text{enc}}}{2\pi r}=\frac{\mu_0}{2\pi r}\int_0^r J\left(r\right)2\pi r dr=\frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right)2\pi r dr=\frac{\mu_0 J_0 r^2}{3a}.$$

(a) At $r=0,\ B=0$.

(b) At $r=a/2$, we have

$$B\left(r\right)=\frac{\mu_0 J_0 r^2}{3a}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m}/2)^2}{3(3.1\times10^{-3}\,\text{m})}=1.0\times10^{-7}\,\text{T}.$$

(c) At $r=a$,

$$B\left(r=a\right)=\frac{\mu_0 J_0 a}{3}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(310\,\text{A/m}^2)(3.1\times10^{-3}\,\text{m})}{3}=4.0\times10^{-7}\,\text{T}.$$

48. (a) The field at the center of the pipe (point $C$) is due to the wire alone, with a magnitude of

$$B_C=\frac{\mu_0 i_{\text{wire}}}{2\pi\left(3R\right)}=\frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\text{ wire}} > B_{C,\text{ wire}}$. Thus, for $B_P = B_C = B_{C,\text{ wire}}$, $i_{\text{wire}}$ must be into the page:

$$B_P=B_{P,\text{wire}}-B_{P,\text{pipe}}=\frac{\mu_0 i_{\text{wire}}}{2\pi R}-\frac{\mu_0 i}{2\pi\left(2R\right)}.$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8 = 3(8.00\times10^{-3}\,\text{A})/8=3.00\times10^{-3}\,\text{A}$.

(b) The direction is into the page.

49. (a) We use Eq. 29-24. The inner radius is $r$ = 15.0 cm, so the field there is

$$B=\frac{\mu_0 iN}{2\pi r}=\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(0.800\,\text{A}\right)\left(500\right)}{2\pi\left(0.150\,\text{m}\right)}=5.33\times10^{-4}\,\text{T}.$$

(b) The outer radius is $r$ = 20.0 cm. The field there is

$$B=\frac{\mu_0 iN}{2\pi r}=\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.800\,\text{A}\right)\left(500\right)}{2\pi\left(0.200\text{m}\right)}=4.00\times10^{-4}\ \text{T}.$$

50. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B=\mu_0 in=\mu_0 i\left(\frac{N}{\ell}\right)$$

where $i$ = 3.60 A, $\ell=0.950$ m, and $N$ = 1200. This yields $B$ = 0.00571 T.

51. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 29-23 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

$$B=\mu_0 in=\mu_0 i\left(\frac{N}{\ell}\right)$$

where $i$ = 0.30 A, $\ell=0.25$ m, and $N$ = 200. This yields $B=3.0\times10^{-4}$ T.

52. We find $N$, the number of turns of the solenoid, from the magnetic field $B=\mu_0 in=\mu_o iN/\ell$ : $N=B\ell/\mu_0 i$. Thus, the total length of wire used in making the solenoid is

$$2\pi rN=\frac{2\pi rB\ell}{\mu_0 i}=\frac{2\pi\left(2.60\times10^{-2}\ \text{m}\right)\left(23.0\times10^{-3}\ \text{T}\right)\left(1.30\,\text{m}\right)}{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(18.0\,\text{A}\right)}=108\ \text{m}.$$

53. The orbital radius for the electron is

$$r=\frac{mv}{eB}=\frac{mv}{e\mu_0 ni}$$

which we solve for $i$:

$$i=\frac{mv}{e\mu_0 nr}=\frac{\left(9.11\times10^{-31}\text{kg}\right)\left(0.0460\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\,\text{C}\right)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(100/0.0100\text{m}\right)\left(2.30\times10^{-2}\,\text{m}\right)}$$
$$=0.272\,\text{A}.$$

54. As the problem states near the end, some idealizations are being made here to keep the calculation straightforward (but are slightly unrealistic). For circular motion (with speed, $v_\perp$, which represents the magnitude of the component of the velocity perpendicular to the magnetic field [the field is shown in Fig. 29-20]), the period is (see Eq. 28-17)

$$T = 2\pi r/v_\perp = 2\pi m/eB.$$

Now, the time to travel the length of the solenoid is $t = L/v_\parallel$ where $v_\parallel$ is the component of the velocity in the direction of the field (along the coil axis) and is equal to $v\cos\theta$ where $\theta$ = 30º. Using Eq. 29-23 ($B = \mu_0 in$) with $n = N/L$, we find the number of revolutions made is $t/T = 1.6 \times 10^6$.

55. **THINK** The net field at a point inside the solenoid is the vector sum of the fields of the solenoid and that of the long straight wire along the central axis of the solenoid.

**EXPRESS** The magnetic field at a point $P$ is given by $\vec{B} = \vec{B}_s + \vec{B}_w$, where $\vec{B}_s$ and $\vec{B}_w$ are the fields due to the solenoid and the wire, respectively. The direction of $\vec{B}_s$ is along the axis of the solenoid, and the direction of $\vec{B}_w$ is perpendicular to it, so the two fields are perpendicular to each other, $\vec{B}_s \perp \vec{B}_w$. For the net field $\vec{B}$ to be at 45° with the axis, we must have $B_s = B_w$.

**ANALYZE** (a) Thus,

$$B_s = B_w \Rightarrow \quad \mu_0 i_s n = \frac{\mu_0 i_w}{2\pi d} ,$$

which gives the separation $d$ to point $P$ on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi(20.0\times10^{-3}\,\text{A})(10\,\text{turns/cm})} = 4.77\,\text{cm} .$$

(b) The magnetic field strength is

$$B = \sqrt{2}B_s = \sqrt{2}\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20.0\times10^{-3}\,\text{A}\right)\left(10\ \text{turns}/0.0100\ \text{m}\right) = 3.55\times10^{-5}\,\text{T}.$$

**LEARN** In general, the angle $\vec{B}$ makes with the solenoid axis is give by

$$\phi = \tan^{-1}\left(\frac{B_w}{B_s}\right) = \tan^{-1}\left(\frac{\mu_0 i_w/2\pi d}{\mu_0 i_s n}\right) = \tan^{-1}\left(\frac{i_w}{2\pi d\ n i_s}\right).$$

56. We use Eq. 29-26 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 iR^2 N}{2\left[R^2 + \left(R/2\right)^2\right]^{3/2}} = \frac{8\mu_0 Ni}{5\sqrt{5}R} = \frac{8\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(200)\left(0.0122\,\text{A}\right)}{5\sqrt{5}\left(0.25\text{m}\right)} = 8.78\times10^{-6}\,\text{T}.$$

$\vec{B}_P$ is in the positive $x$ direction.

57. **THINK** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where $N$ is the number of turns, $i$ is the current, and $A$ is the area.

**EXPRESS** The cross-sectional area is a circle, so $A = \pi R^2$, where $R$ is the radius. The magnetic field on the axis of a magnetic dipole, a distance $z$ away, is given by Eq. 29-27:
$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$ .

**ANALYZE** (a) Substituting the values given, we find the magnitude of the dipole moment to be

$$\mu = Ni\pi R^2 = (300)(4.0\,\text{A})\pi(0.025\,\text{m})^2 = 2.4\,\text{A}\cdot\text{m}^2.$$

(b) Solving for $z$, we obtain

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(2.36\,\text{A}\cdot\text{m}^2\right)}{2\pi\left(5.0\times10^{-6}\,\text{T}\right)}\right)^{1/3} = 46\,\text{cm}.$$

**LEARN** Note the similarity between $B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$, the magnetic field of a magnetic dipole $\mu$ and $E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$, the electric field of an electric dipole $p$ (see Eq. 22-9).

58. (a) We set $z = 0$ in Eq. 29-26 (which is equivalent using to Eq. 29-10 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case $b$ has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4.0\,.$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2} = 0.50.$$

59. **THINK** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where $N$ is the number of turns, $i$ is the current, and $A$ is the area.

**EXPRESS** The cross-sectional area is a circle, so $A = \pi R^2$, where $R$ is the radius.

**ANALYZE** With $N$ = 200, $i$ = 0.30 A, and $R$ = 0.050 m, the magnitude of the dipole moment is

$$\mu = (200)(0.30\,\text{A})\pi(0.050\,\text{m})^2 = 0.47\,\text{A}\cdot\text{m}^2.$$

**LEARN** The direction of $\vec{\mu}$ is that of the normal vector $\vec{n}$ to the plane of the coil, in accordance with the right-hand rule shown in Fig. 28-19.

60. Using Eq. 29-26, we find that the net $y$-component field is

$$B_y = \frac{\mu_0 i_1 R^2}{2(R^2+z_1^2)^{3/2}} - \frac{\mu_0 i_2 R^2}{2(R^2+z_2^2)^{3/2}},$$

where $z_1^2 = L^2$ (see Fig. 29-74(a)) and $z_2^2 = y^2$ (because the central axis here is denoted $y$ instead of $z$). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum in Fig. 29-74(b) corresponding to $B_y$ = 0 would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).

(a) As $y \to \infty$, only the first term contributes and (with $B_y = 7.2 \times 10^{-6}$ T given in this case) we can solve for $i_1$:

$$i_1 = \frac{2(R^2+z_1^2)^{3/2} B_y}{\mu_0 R^2} = \frac{2R[1+(L/R)^2]^{3/2} B_y}{\mu_0}$$
$$= \frac{2(0.040\text{ m})[1+(0.030\text{ m}/0.040\text{ m})^2]^{3/2}(7.2\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 0.895\text{ A} \approx 0.90\text{ A}.$$

(b) With loop 2 at $y$ = 0.06 m (see Fig. 29-74(b)) we are able to determine $i_2$ from

$$\frac{\mu_0 i_1 R^2}{2(R^2+L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2+y^2)^{3/2}}.$$

We obtain $i_2 = (117\sqrt{13}/50\pi)$ A ≈ 2.7 A.

61. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(15\,\text{A})}{2(0.12\,\text{m})} = 7.9\times10^{-5}\,\text{T}.$$

(b) The torque has magnitude equal to

$$\tau = |\vec{\mu}_2 \times \vec{B}_1| = \mu_2 B_1 \sin 90° = N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1 = \pi(50)(1.3\text{A})(0.82\times10^{-2}\text{m})^2(7.9\times10^{-5}\text{T})$$
$$= 1.1\times10^{-6}\,\text{N}\cdot\text{m}.$$

62. (a) To find the magnitude of the field, we use Eq. 29-9 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.0562\,\text{A})}{4}\left(\frac{1}{0.0572\text{m}}+\frac{1}{0.0936\text{m}}\right)$$
$$= 4.97\times10^{-7}\,\text{T}.$$

(b) By the right-hand rule, $\vec{B}$ points into the paper at $P$ (see Fig. 29-7(c)).

(c) The enclosed area is $A = (\pi a^2 + \pi b^2)/2$, which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}(a^2+b^2) = \frac{\pi(0.0562\text{A})}{2}[(0.0572\text{m})^2 + (0.0936\text{m})^2] = 1.06\times10^{-3}\,\text{A}\cdot\text{m}^2.$$

(d) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper.

63. By imagining that each of the segments *bg* and *cf* (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude ($i$) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(a) The dipole moment of path *abcdefgha* is

$$\vec{\mu} = \vec{\mu}_{bcfgb} + \vec{\mu}_{abgha} + \vec{\mu}_{cdefc} = (ia^2)(\hat{\text{j}} - \hat{\text{i}} + \hat{\text{i}}) = ia^2\hat{\text{j}}$$
$$= (6.0\,\text{A})(0.10\,\text{m})^2\,\hat{\text{j}} = (6.0\times10^{-2}\,\text{A}\cdot\text{m}^2)\,\hat{\text{j}}\,.$$

(b) Since both points are far from the cube we can use the dipole approximation. For ($x$, $y$, $z$) = (0, 5.0 m, 0),

$$\vec{B}(0,\,5.0\text{ m},\,0) \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} = \frac{(1.26\times10^{-6}\ \text{T}\cdot\text{m/A})(6.0\times10^{-2}\text{m}^2\cdot\text{A})\,\hat{\text{j}}}{2\pi(5.0\text{ m})^3} = (9.6\times10^{-11}\ \text{T})\,\hat{\text{j}}\,.$$

64. (a) The radial segments do not contribute to $\vec{B}_P$, and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B}_P = \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(4.00\text{ m})}\hat{\text{k}} - \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(2.00\text{ m})}\hat{\text{k}}$$

where $i$ = 0.200 A. This yields $\vec{B} = -2.75 \times 10^{-8}\,\hat{\text{k}}$ T, or $|\vec{B}| = 2.75 \times 10^{-8}$ T.

(b) The direction is $-\hat{\text{k}}$, or into the page.

65. Using Eq. 29-20,

$$|\vec{B}| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r\,,$$

we find that $r$ = 0.00128 m gives the desired field value.

66. (a) We designate the wire along $y = r_A$ = 0.100 m wire $A$ and the wire along $y = r_B$ = 0.050 m wire $B$. Using Eq. 29-4, we have

$$\vec{B}_{\text{net}} = \vec{B}_A + \vec{B}_B = -\frac{\mu_0 i_A}{2\pi r_A}\hat{\text{k}} - \frac{\mu_0 i_B}{2\pi r_B}\hat{\text{k}} = (-52.0\times 10^{-6}\text{ T})\hat{\text{k}}.$$

(b) This will occur for some value $r_B < y < r_A$ such that

$$\frac{\mu_0 i_A}{2\pi(r_A - y)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 13/160 \approx 0.0813$ m.

(c) We eliminate the $y < r_B$ possibility due to wire $B$ carrying the larger current. We expect a solution in the region $y > r_A$ where

$$\frac{\mu_0 i_A}{2\pi(y - r_A)} = \frac{\mu_0 i_B}{2\pi(y - r_B)}.$$

Solving, we find $y = 7/40 \approx 0.0175$ m.

67. Let the length of each side of the square be $a$. The center of a square is a distance $a$/2 from the nearest side. There are four sides contributing to the field at the center. The result is

$$B_{\text{center}} = 4\left(\frac{\mu_0 i}{2\pi(a/2)}\right)\left(\frac{a}{\sqrt{a^2 + 4(a/2)^2}}\right) = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

On the other hand, the magnetic field at the center of a circular wire of radius $R$ is $\mu_0 i/2R$ (e.g., Eq. 29-10). Thus, the problem is equivalent to showing that

$$\frac{2\sqrt{2}\mu_0 i}{\pi a} > \frac{\mu_0 i}{2R} \quad \Rightarrow \quad \frac{4\sqrt{2}}{\pi a} > \frac{1}{R} \;.$$

To do this we must relate the parameters $a$ and $R$. If both wires have the same length $L$ then the geometrical relationships $4a = L$ and $2\pi R = L$ provide the necessary connection:

$$4a = 2\pi R \;\; \Rightarrow \;\; a = \frac{\pi R}{2}.$$

Thus, our proof consists of the observation that

$$\frac{4\sqrt{2}}{\pi a} = \frac{8\sqrt{2}}{\pi^2 R} > \frac{1}{R},$$

as one can check numerically (that $8\sqrt{2}/\pi^2 > 1$).

68. We take the current ($i$ = 50 A) to flow in the $+x$ direction, and the electron to be at a point $P$, which is $r$ = 0.050 m above the wire (where "up" is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at $P$. Then, combining Eq. 29-4 with Eq. 28-2, we obtain

$$\vec{F}_e = \left(-e\mu_0 i/2\pi r\right)\left(\vec{v}\times\hat{\text{k}}\right).$$

(a) The electron is moving down: $\vec{v} = -v\hat{\text{j}}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\left(-\hat{\text{i}}\right) = (3.2\times10^{-16}\text{ N})\,\hat{\text{i}}\,,$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N.

(b) In this case, the electron is in the same direction as the current: $\vec{v} = v\hat{\text{i}}$ so

$$\vec{F}_e = \frac{-e\mu_0 i v}{2\pi r}\left(-\hat{\text{j}}\right) = (3.2\times10^{-16}\text{ N})\,\hat{\text{j}}\,,$$

or $|\vec{F}_e| = 3.2\times10^{-16}$ N.

(c) Now, $\vec{v} = \pm v\hat{\text{k}}$ so $\vec{F}_e \propto \hat{\text{k}}\times\hat{\text{k}} = 0.$

69. (a) By the right-hand rule, the magnetic field $\vec{B}_1$ (evaluated at $a$) produced by wire 1 (the wire at bottom left) is at $\phi$ = 150° (measured counterclockwise from the $+x$ axis, in the $xy$ plane), and the field produced by wire 2 (the wire at bottom right) is at $\phi$ = 210°. By symmetry $\left(\vec{B}_1 = \vec{B}_2\right)$ we observe that only the $x$-components survive, yielding

$$\vec{B}=\vec{B}_1+\vec{B}_2=\left(2\frac{\mu_0 i}{2\pi\ell}\cos 150°\right)\hat{\text{i}}=(-3.46\times10^{-5}\text{T})\hat{\text{i}}$$

where $i$ = 10 A, $\ell$ = 0.10 m, and Eq. 29-4 has been used. To cancel this, wire $b$ must carry current into the page (that is, the $-\hat{\text{k}}$ direction) of value

$$i_b=B\frac{2\pi r}{\mu_0}=\left(3.46\times10^{-5}\text{ T}\right)\frac{2\pi(0.087\text{ m})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}=15\text{ A}$$

where $r=\sqrt{3}\,\ell/2=0.087$ m and Eq. 29-4 has again been used.

(b) As stated above, to cancel this, wire $b$ must carry current into the page (that is, the $-z$ direction).

70. The radial segments do not contribute to $\vec{B}$ (at the center), and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B}=\frac{\mu_0 i(\pi\text{ rad})}{4\pi(4.00\text{ m})}\hat{\text{k}}+\frac{\mu_0 i(\pi/2\text{ rad})}{4\pi(2.00\text{ m})}\hat{\text{k}}-\frac{\mu_0 i(\pi/2\text{ rad})}{4\pi(4.00\text{ m})}\hat{\text{k}}$$

where $i$ = 2.00 A. This yields $\vec{B}$ = (1.57 × $10^{-7}$ T) $\hat{\text{k}}$, or $|\vec{B}|=1.57\times10^{-7}\text{ T}$.

71. Since the radius is $R$ = 0.0013 m, then the $i$ = 50 A produces

$$B=\frac{\mu_0 i}{2\pi R}=\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(50\text{ A})}{2\pi(0.0013\text{ m})}=7.7\times10^{-3}\text{ T}$$

at the edge of the wire. The three equations, Eq. 29-4, Eq. 29-17, and Eq. 29-20, agree at this point.

72. (a) With cylindrical symmetry, we have, external to the conductors,

$$\left|\vec{B}\right|=\frac{\mu_0\, i_{\text{enc}}}{2\pi r}$$

which produces $i_{\text{enc}}$ = 25 mA from the given information. Therefore, the thin wire must carry 5.0 mA.

(b) The direction is downward, opposite to the 30 mA carried by the thin conducting surface.

73. (a) The magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and

has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current *i*, which is uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

at a distance *r* from its axis, inside the cylinder. Here *R* is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi\left(a^2 - b^2\right)},$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{i a^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance $r_1$ from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{i b^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance $r_2$ from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2\left(a^2 - b^2\right)} = \frac{\mu_0 i r_2}{2\pi\left(a^2 - b^2\right)}.$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for $B_1$ and obtain

$$B = \frac{\mu_0 i d}{2\pi\left(a^2 - b^2\right)} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})\left(5.25\,\mathrm{A}\right)(0.0200\,\mathrm{m})}{2\pi[(0.0400\,\mathrm{m})^2 - (0.0150\,\mathrm{m})^2]} = 1.53\times10^{-5}\,\mathrm{T}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes $B = \dfrac{\mu_0 id}{2\pi a^2}$. This correctly gives the field of a solid cylinder carrying a uniform current $i$, at a point inside the cylinder a distance $d$ from the axis. If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

Note: One may apply Ampere's law to show that the magnetic field in the hole is uniform. Consider a rectangular path with two long sides (side 1 and 2, each with length $L$) and two short sides (each of length less than $b$). If side 1 is directly along the axis of the hole, then side 2 would also be parallel to it and in the hole. To ensure that the short sides do not contribute significantly to the integral in Ampere's law, we might wish to make $L$ *very* long (perhaps longer than the length of the cylinder), or we might appeal to an argument regarding the angle between $\vec{B}$ and the short sides (which is 90° at the axis of the hole). In any case, the integral in Ampere's law reduces to

$$\oint_{\text{rectangle}} \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enclosed}}$$

$$\int_{side1} \vec{B}\cdot d\vec{s} + \int_{\text{side}2} \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{in hole}}$$

$$\left(B_{\text{side}1} - B_{\text{side}2}\right)L = 0$$

where $B_{\text{side 1}}$ is the field along the axis found in part (a). This shows that the field at off-axis points (where $B_{\text{side 2}}$ is evaluated) is the same as the field at the center of the hole; therefore, the field in the hole is uniform.

74. Equation 29-4 gives

$$i = \frac{2\pi RB}{\mu_0} = \frac{2\pi(0.880\,\text{m})(7.30\times10^{-6}\,\text{T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 32.1\,\text{A}.$$

75. **THINK** In this problem, we apply the Biot-Savart law to calculate the magnetic field due to a current-carrying segment at various locations.

**EXPRESS** The Biot-Savart law can be written as

$$\vec{B}(x,y,z) = \frac{\mu_0}{4\pi}\frac{i\Delta\vec{s}\times\hat{\text{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{i\Delta\vec{s}\times\vec{r}}{r^3}.$$

With $\Delta\vec{s} = \Delta s\hat{\text{j}}$ and $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, their cross product is

$$\Delta\vec{s}\times\vec{r} = (\Delta s\hat{\text{j}})\times(x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}) = \Delta s(z\hat{\text{i}} - x\hat{\text{k}})$$

where we have used $\hat{j}\times\hat{i} = -\hat{k}$, $\hat{j}\times\hat{j} = 0$, and $\hat{j}\times\hat{k} = \hat{i}$. Thus, the Biot-Savart equation becomes

$$\vec{B}(x, y, z) = \frac{\mu_0 i\,\Delta s\left(z\hat{i} - x\hat{k}\right)}{4\pi\left(x^2 + y^2 + z^2\right)^{3/2}}.$$

**ANALYZE** (a) The field on the $z$ axis (at $x = 0$, $y = 0$, and $z = 5.0$ m) is

$$\vec{B}(0, 0, 5.0\text{ m}) = \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(2.0\,\text{A})\left(3.0\times10^{-2}\,\text{m}\right)(5.0\text{ m})\hat{i}}{4\pi\left(0^2 + 0^2 + (5.0\text{ m})^2\right)^{3/2}} = (2.4\times10^{-10}\,\text{T})\hat{i}.$$

(b) Similarly, $\vec{B}$ (0, 6.0 m, 0) = 0, since $x = z = 0$.

(c) The field in the $xy$ plane, at $(x, y, z)$ = (7 m, 7 m, 0), is

$$\vec{B}(7.0\text{ m}, 7.0\text{ m}, 0) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(-7.0\,\text{m})\hat{k}}{4\pi\left((7.0\text{ m})^2 + (7.0\text{ m})^2 + 0^2\right)^{3/2}} = (-4.3\times10^{-11}\,\text{T})\hat{k}.$$

(d) The field in the $xy$ plane, at $(x, y, z)$ = (–3, –4, 0), is

$$\vec{B}(-3.0\,\text{m}, -4.0\text{m}, 0) = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times10^{-2}\,\text{m})(3.0\,\text{m})\hat{k}}{4\pi\left((-3.0\,\text{m})^2 + (-4.0\,\text{m})^2 + 0^2\right)^{3/2}} = (1.4\times10^{-10}\,\text{T})\hat{k}.$$

**LEARN** Along the $x$ and $z$ axes, the expressions for $\vec{B}$ simplify to

$$\vec{B}(x,0,0) = -\frac{\mu_0}{4\pi}\frac{i\,\Delta s}{x^2}\hat{k}, \quad \vec{B}(0,0,z) = \frac{\mu_0}{4\pi}\frac{i\,\Delta s}{z^2}\hat{i}.$$

The magnetic field at any point on the $y$ axis vanishes because the current flows in the $+y$ direction, so $d\vec{s}\times\hat{r} = 0$.

76. We note that the distance from each wire to $P$ is $r = d/\sqrt{2} = 0.071\text{m}$. In both parts, the current is $i$ = 100 A.

(a) With the currents parallel, application of the right-hand rule (to determine each of their contributions to the field at $P$) reveals that the vertical components cancel and the horizontal components add, yielding the result:

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\cos 45.0° = 4.00\times10^{-4}\,\text{T}$$

and directed in the –*x* direction. In unit-vector notation, we have $\vec{B} = (-4.00 \times 10^{-4}\ \text{T})\hat{\text{i}}$.

(b) Now, with the currents anti-parallel, application of the right-hand rule shows that the horizontal components cancel and the vertical components add. Thus,

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\sin 45.0° = 4.00 \times 10^{-4}\ \text{T}$$

and directed in the +*y* direction. In unit-vector notation, we have $\vec{B} = (4.00 \times 10^{-4}\ \text{T})\hat{\text{j}}$.

77. We refer to the center of the circle (where we are evaluating $\vec{B}$) as *C*. Recalling the *straight sections* discussion in Sample Problem 29.01 — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments that are collinear with *C* do not contribute to the field there. Eq. 29-9 (with $\phi = \pi/2$ rad) and the right-hand rule indicates that the currents in the two arcs contribute

$$\frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

to the field at *C*. Thus, the nonzero contributions come from those straight segments that are not collinear with *C*. There are two of these "semi-infinite" segments, one a vertical distance *R* above *C* and the other a horizontal distance *R* to the left of *C*. Both contribute fields pointing out of the page (see Fig. 29-7(c)). Since the magnitudes of the two contributions (governed by Eq. 29-7) add, then the result is

$$B = 2\left(\frac{\mu_0 i}{4\pi R}\right) = \frac{\mu_0 i}{2\pi R}$$

exactly what one would expect from a single infinite straight wire (see Eq. 29-4). For such a wire to produce such a field (out of the page) with a leftward current requires that the point of evaluating the field be below the wire (again, see Fig. 29-7(c)).

78. The points must be along a line parallel to the wire and a distance *r* from it, where *r* satisfies $B_{\text{wire}} = \dfrac{\mu_0 i}{2\pi r} = B_{\text{ext}}$, or

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{(1.26 \times 10^{-6}\ \text{T}\cdot\text{m/A})(100\ \text{A})}{2\pi(5.0 \times 10^{-3}\ \text{T})} = 4.0 \times 10^{-3}\,\text{m}.$$

79. (a) The field in this region is entirely due to the long wire (with, presumably, negligible thickness). Using Eq. 29-17,

$$\left|\vec{B}\right| = \frac{\mu_0\, i_w}{2\pi r} = 4.8\times 10^{-3}\ \text{T}$$

where $i_w$ = 24 A and $r$ = 0.0010 m.

(b) Now the field consists of two contributions (which are anti-parallel) — from the wire (Eq. 29-17) and from a portion of the conductor (Eq. 29-20 modified for annular area):

$$|\vec{B}| = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0 i_w}{2\pi r} - \frac{\mu_0 i_c}{2\pi r}\left(\frac{\pi r^2 - \pi R_i^2}{\pi R_0^2 - \pi R_i^2}\right)$$

where $r$ = 0.0030 m, $R_i$ = 0.0020 m, $R_o$ = 0.0040 m, and $i_c$ = 24 A. Thus, we find $|\vec{B}| = 9.3\times 10^{-4}$ T.

(c) Now, in the external region, the individual fields from the two conductors cancel completely (since $i_c = i_w$): $\vec{B} = 0$.

80. Using Eq. 29-20 and Eq. 29-17, we have

$$|\vec{B}_1| = \left(\frac{\mu_0 i}{2\pi R^2}\right) r_1 \qquad |\vec{B}_2| = \frac{\mu_0 i}{2\pi r_2}$$

where $r_1 = 0.0040\,\text{m}$, $\left|\vec{B}_1\right| = 2.8\times 10^{-4}\ \text{T}$, $r_2 = 0.010\,\text{m}$, and $|\vec{B}_2| = 2.0\times 10^{-4}$ T. Point 2 is known to be external to the wire since $|\vec{B}_2| < |\vec{B}_1|$. From the second equation, we find $i$ = 10 A. Plugging this into the first equation yields $R = 5.3\times 10^{-3}$ m.

81. **THINK** The objective of this problem is to calculate the magnetic field due to an infinite current sheet by applying Ampere's law.

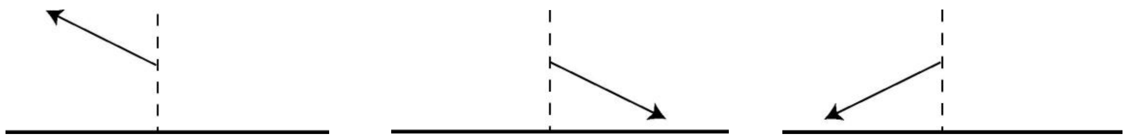
**EXPRESS** The "current per unit *x*-length" may be viewed as current density multiplied by the thickness $\Delta y$ of the sheet; thus, $\lambda = J\Delta y$. Ampere's law may be (and often is) expressed in terms of the current density vector as follows:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \int \vec{J}\cdot d\vec{A}$$

where the area integral is over the region enclosed by the path relevant to the line integral (and $\vec{J}$ is in the $+z$ direction, out of the paper). With $J$ uniform throughout the sheet, then it is clear that the right-hand side of this version of Ampere's law should reduce, in this problem, to

$$\mu_0 JA = \mu_0 J\,\Delta y\Delta x = \mu_0 \lambda\,\Delta x.$$

**ANALYZE** (a) Figure 29-84 certainly has the horizontal components of $\vec{B}$ drawn correctly at points $P$ and $P'$, so the question becomes: is it possible for $\vec{B}$ to have vertical components in the figure?



Our focus is on point *P*. Suppose the magnetic field is not parallel to the sheet, as shown in the upper left diagram. If we reverse the direction of the current, then the direction of the field will also be reversed (as shown in the upper middle diagram). Now, if we rotate the sheet by 180° about a line that is perpendicular to the sheet, the field will rotate and point in the direction shown in the diagram on the upper right. The current distribution now is exactly the same as the original; however, comparing the upper left and upper right diagrams, we see that the fields are not the same, unless the original field is parallel to the sheet and only has a horizontal component. That is, the field at *P* must be purely horizontal, as drawn in Fig. 29-84.

(b) The path used in evaluating $\oint \vec{B}\cdot d\vec{s}$ is rectangular, of horizontal length $\Delta x$ (the horizontal sides passing through points *P* and *P'*, respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown next. Ampere's law yields

$$2B\Delta x = \mu_0 \lambda \Delta x \;\;\Rightarrow\;\; B = \frac{1}{2}\mu_0 \lambda .$$

**LEARN** In order to apply Ampere's law, the system must possess certain symmetry. In the case of an infinite current sheet, the symmetry is planar.

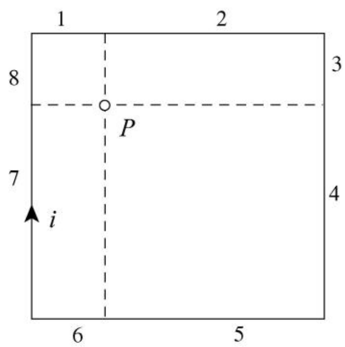
82. Equation 29-17 applies for each wire, with $r = \sqrt{R^2 + (d/2)^2}$ (by the Pythagorean theorem). The vertical components of the fields cancel, and the two (identical) horizontal components add to yield the final result

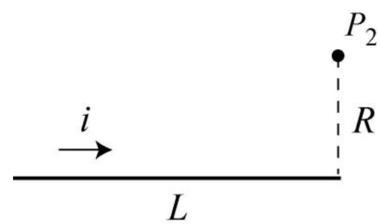
$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right) = \frac{\mu_0 i d}{2\pi\left(R^2 + (d/2)^2\right)} = 1.25\times 10^{-6}\ \text{T},$$

where $(d/2)/r$ is a trigonometric factor to select the horizontal component. It is clear that this is equivalent to the expression in the problem statement. Using the right-hand rule, we find both horizontal components point in the $+x$ direction. Thus, in unit-vector notation, we have $\vec{B} = (1.25\times 10^{-6}\ \text{T})\hat{\text{i}}$.

83. **THINK** The magnetic field at *P* is the vector sum of the fields of the individual wire segments.

**EXPRESS** The two small wire segments, each of length $a/4$, shown in Fig. 29-86 nearest to point $P$, are labeled 1 and 8 in the figure (below left). Let $-\hat{\text{k}}$ be a unit vector pointing into the page.




We use the result of Problem 29-17: namely, the magnetic field at $P_2$ (shown in Fig. 29-44 and upper right) is

$$B_{P_2} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}.$$

Therefore, the magnetic fields due to the 8 segments are

$$B_{P1} = B_{P8} = \frac{\sqrt{2}\mu_0 i}{8\pi(a/4)} = \frac{\sqrt{2}\mu_0 i}{2\pi a},$$

$$B_{P4} = B_{P5} = \frac{\sqrt{2}\mu_0 i}{8\pi(3a/4)} = \frac{\sqrt{2}\mu_0 i}{6\pi a},$$

$$B_{P2} = B_{P7} = \frac{\mu_0 i}{4\pi(a/4)}\cdot\frac{3a/4}{\left[(3a/4)^2+(a/4)^2\right]^{1/2}} = \frac{3\mu_0 i}{\sqrt{10}\pi a},$$

and

$$B_{P3} = B_{P6} = \frac{\mu_0 i}{4\pi(3a/4)}\cdot\frac{a/4}{\left[(a/4)^2+(3a/4)^2\right]^{1/2}} = \frac{\mu_0 i}{3\sqrt{10}\pi a}.$$

**ANALYZE** Adding up all the contributions, the total magnetic field at $P$ is

$$\begin{aligned}\vec{B}_P &= \sum_{n=1}^{8} B_{Pn}(-\hat{\text{k}}) = 2\frac{\mu_0 i}{\pi a}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)(-\hat{\text{k}})\\ &= \frac{2\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(10\text{A})}{\pi\left(8.0\times10^{-2}\text{m}\right)}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)(-\hat{\text{k}})\\ &= \left(2.0\times10^{-4}\text{T}\right)(-\hat{\text{k}}).\end{aligned}$$

**LEARN** If point $P$ is located at the center of the square, then each segment would contribute

$$B_{P1} = B_{P2} = \cdots B_{P8} = \frac{\sqrt{2}\mu_0 i}{4\pi a},$$

making the total field

$$B_{\text{center}} = 8B_{P1} = \frac{8\sqrt{2}\mu_0 i}{4\pi a}.$$

84. (a) All wires carry parallel currents and attract each other; thus, the "top" wire is pulled downward by the other two:

$$\left|\vec{F}\right| = \frac{\mu_0 L(5.0\,\text{A})(3.2\,\text{A})}{2\pi(0.10\,\text{m})} + \frac{\mu_0 L(5.0\,\text{A})(5.0\,\text{A})}{2\pi(0.20\,\text{m})}$$

where $L$ = 3.0 m. Thus, $\left|\vec{F}\right| = 1.7\times10^{-4}$ N.

(b) Now, the "top" wire is pushed upward by the center wire and pulled downward by the bottom wire:

$$|\vec{F}| = \frac{\mu_0 L(5.0\text{A})(3.2\text{A})}{2\pi(0.10\text{m})} - \frac{\mu_0 L(5.0\text{A})(5.0\text{A})}{2\pi(0.20\text{m})} = 2.1\times10^{-5}\,\text{N}.$$

85. **THINK** The hollow conductor has cylindrical symmetry, so Ampere's law can be applied to calculate the magnetic field due to the current distribution.

**EXPRESS** Ampere's law states that $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where $i_{\text{enc}}$ is the current enclosed by the closed path, or Amperian loop. We choose the Amperian loop to be a circle of radius $r$ and concentric with the cylindrical shell. Since the current is uniformly distributed throughout the cross section of the shell, the enclosed current is

$$i_{\text{enc}} = i\,\frac{\pi(r^2-b^2)}{\pi(a^2-b^2)} = i\left(\frac{r^2-b^2}{a^2-b^2}\right).$$

**ANALYZE** (a) Thus, in the region $b < r < a$, we have

$$\oint \vec{B}\cdot d\vec{s} = 2\pi r B = \mu_0 i_{\text{enc}} = \mu_0 i\left(\frac{r^2-b^2}{a^2-b^2}\right)$$

which gives $B = \dfrac{\mu_0 i}{2\pi(a^2-b^2)}\left(\dfrac{r^2-b^2}{r}\right)$.

(b) At $r = a$, the magnetic field strength is

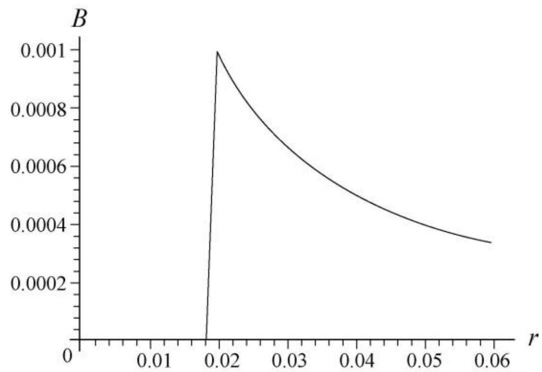
$$\frac{\mu_0 i}{2\pi(a^2-b^2)}\left(\frac{a^2-b^2}{a}\right)=\frac{\mu_0 i}{2\pi a}.$$

At $r=b, B\propto r^2-b^2=0$. Finally, for $b = 0$

$$B=\frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r}=\frac{\mu_0 ir}{2\pi a^2}$$

which agrees with Eq. 29-20.

(c) The field is zero for $r < b$ and is equal to Eq. 29-17 for $r > a$, so this along with the result of part (a) provides a determination of $B$ over the full range of values. The graph (with SI units understood) is shown below.




**LEARN** For $r < b$, the field is zero, and for $r > a$, the field decreases as $1/r$. In the region $b < r < a$, the field increases with $r$ as $r-b^2/r$.

86. We refer to the side of length $L$ as the long side and that of length $W$ as the short side. The center is a distance $W/2$ from the midpoint of each long side, and is a distance $L/2$ from the midpoint of each short side. There are two of each type of side, so the result of Problem 29-17 leads to

$$B=2\frac{\mu_0 i}{2\pi(W/2)}\frac{L}{\sqrt{L^2+4(W/2)^2}}+2\frac{\mu_0 i}{2\pi(L/2)}\frac{W}{\sqrt{W^2+4(L/2)^2}}.$$

The final form of this expression, shown in the problem statement, derives from finding the common denominator of the above result and adding them, while noting that

$$\frac{L^2+W^2}{\sqrt{W^2+L^2}}=\sqrt{W^2+L^2}.$$

87. (a) Equation 29-20 applies for $r < c$. Our sign choice is such that $i$ is positive in the smaller cylinder and negative in the larger one.

$$B = \frac{\mu_0 i r}{2\pi c^2}, \; r \le c.$$

(b) Equation 29-17 applies in the region between the conductors:

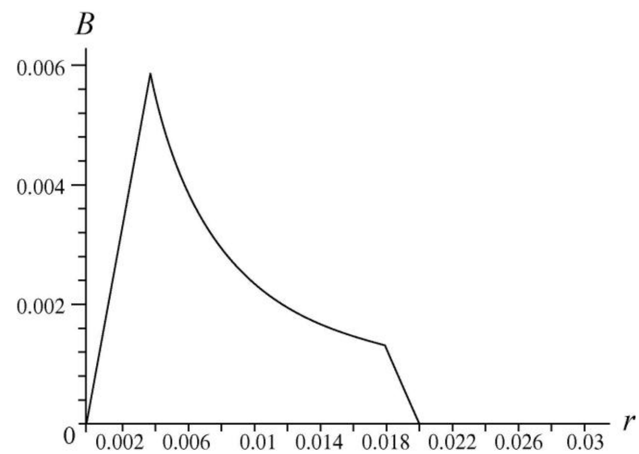
$$B = \frac{\mu_0 i}{2\pi r}, \; c \le r \le b.$$

(c) Within the larger conductor we have a superposition of the field due to the current in the inner conductor (still obeying Eq. 29-17) plus the field due to the (negative) current in that part of the outer conductor at radius less than $r$. The result is

$$B = \frac{\mu_0 i}{2\pi r} - \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - b^2}{a^2 - b^2}\right), \;\; b < r \le a.$$

If desired, this expression can be simplified to read

$$B = \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right).$$




(d) Outside the coaxial cable, the net current enclosed is zero. So $B = 0$ for $r \ge a$.

(e) We test these expressions for one case. If $a \to \infty$ and $b \to \infty$ (such that $a > b$) then we have the situation described on page 696 of the textbook.

(f) Using SI units, the graph of the field is shown to the right.

88. (a) Consider a segment of the projectile between $y$ and $y + dy$. We use Eq. 29-12 to find the magnetic force on the segment, and Eq. 29-7 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\text{i}}$ direction, and the current in rail 2 is in the $-\hat{\text{i}}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{\text{k}}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of $y$) acting on the segment of the projectile (in which the current flows in the $-\hat{\text{j}}$ direction) is given below. The coordinate origin is at the bottom of the projectile.

$$d\vec{F} = d\vec{F}_1 + d\vec{F}_2 = i dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_1 + dy\left(-\hat{\mathrm{j}}\right)\times\vec{B}_2 = i\left[B_1 + B_2\right]\hat{\mathrm{i}}\ dy$$
$$= i\left[\frac{\mu_0 i}{4\pi\left(2R+w-y\right)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\mathrm{i}}\ dy.$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R+w-y} + \frac{1}{y}\right)dy\hat{\mathrm{i}} = \frac{\mu_0 i^2}{2\pi}\ \ln\left(1+\frac{w}{R}\right)\hat{\mathrm{i}}.$$

(b) Using the work-energy theorem, we have

$$\Delta K = \tfrac{1}{2}mv_f^2 = W_{\text{ext}} = \int\vec{F}\cdot d\vec{s} = FL.$$

Thus, the final speed of the projectile is

$$v_f = \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\frac{\mu_0 i^2}{2\pi}\ln\left(1+\frac{w}{R}\right)L\right]^{1/2}$$
$$= \left[\frac{2\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(450\times10^3\ \mathrm{A}\right)^2\ \ln\left(1+1.2\ \mathrm{cm}/6.7\ \mathrm{cm}\right)\left(4.0\ \mathrm{m}\right)}{2\pi\left(10\times10^{-3}\ \mathrm{kg}\right)}\right]^{1/2}$$
$$= 2.3\times10^3\ \mathrm{m/s}.$$

# Chapter 30

1. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is zero.

2. Using Faraday's law, the induced emf is

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d\left(BA\right)}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(\pi r^2\right)}{dt} = -2\pi rB\frac{dr}{dt} \\ &= -2\pi\left(0.12\,\text{m}\right)\left(0.800\,\text{T}\right)\left(-0.750\,\text{m/s}\right) \\ &= 0.452\,\text{V}.\end{aligned}$$

3. **THINK** Changing the current in the solenoid changes the flux, and therefore, induces a current in the coil.

**EXPRESS** Using Faraday's law, the total induced emf is given by

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 ni) = -N\mu_0 nA\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt}$$

By Ohm's law, the induced current in the coil is $i_{\text{ind}} = |\varepsilon|/R,$ where $R$ is the resistance of the coil.

**ANALYZE** Substituting the values given, we obtain

$$\begin{aligned}\varepsilon &= -N\mu_0 n(\pi r^2)\frac{di}{dt} = -(120)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(22000/\text{m})\,\pi\left(0.016\,\text{m}\right)^2\left(\frac{1.5\text{ A}}{0.025\text{ s}}\right) \\ &= 0.16\,\text{V}.\end{aligned}$$

Ohm's law then yields $i_{\text{ind}} = \dfrac{|\varepsilon|}{R} = \dfrac{0.016\text{ V}}{5.3\,\Omega} = 0.030\text{ A}.$

**LEARN** The direction of the induced current can be deduced from Lenz's law, which states that the direction of the induced current is such that the magnetic field which it produces opposes the change in flux that induces the current.

4. (a) We use $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0$ s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi\left(0.12\text{m}\right)^2\left(\frac{0.5\,\text{T}}{2.0\,\text{s}}\right) = -1.1\times10^{-2}\ \text{V}.$$

(b) For 2.0 s < $t$ < 4.0 s: $\varepsilon \propto dB/dt = 0$.

(c) For 4.0 s < $t$ < 6.0 s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\text{m})^2\left(\frac{-0.5\text{T}}{6.0\text{s}-4.0\text{s}}\right) = 1.1\times10^{-2}\ \text{V}.$$

5. The field (due to the current in the straight wire) is out of the page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

6. From the datum at $t = 0$ in Fig. 30-37(b) we see 0.0015 A = $V_{\text{battery}}/R$, which implies that the resistance is

$$R = (6.00\ \mu\text{V})/(0.0015\ \text{A}) = 0.0040\ \Omega.$$

Now, the value of the current during 10 s < $t$ < 20 s leads us to equate

$$(V_{\text{battery}} + \varepsilon_{\text{induced}})/R = 0.00050\ \text{A}.$$

This shows that the induced emf is $\varepsilon_{\text{induced}} = -4.0\ \mu\text{V}$. Now we use Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -A\,a\,.$$

Plugging in $\varepsilon = -4.0\times10^{-6}$ V and $A = 5.0\times10^{-4}\ \text{m}^2$, we obtain $a = 0.0080$ T/s.

7. (a) The magnitude of the emf is

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left(6.0t^2 + 7.0t\right) = 12t + 7.0 = 12(2.0) + 7.0 = 31\,\text{mV}.$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to the left through $R$.

8. The resistance of the loop is

$$R = \rho\frac{L}{A} = \left(1.69\times10^{-8}\Omega\cdot\text{m}\right)\frac{\pi\left(0.10\,\text{m}\right)}{\pi\left(2.5\times10^{-3}\ \text{m}\right)^2/4} = 1.1\times10^{-3}\Omega.$$

We use $i = |\varepsilon|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{(10\,\text{A})(1.1\times10^{-3}\,\Omega)}{\pi(0.05\,\text{m})^2} = 1.4\ \text{T/s}.$$

9. The amplitude of the induced emf in the loop is

$$\varepsilon_m = A\mu_0 n i_0 \omega = (6.8\times10^{-6}\,\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(85400/\text{m})(1.28\ \text{A})(212\ \text{rad/s})$$
$$= 1.98\times10^{-4}\,\text{V}.$$

10. (a) The magnetic flux $\Phi_B$ through the loop is given by

$$\Phi_B = 2B\left(\pi r^2/2\right)\left(\cos 45°\right) = \pi r^2 B/\sqrt{2}\,.$$

Thus,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right) = -\frac{\pi\left(3.7\times10^{-2}\,\text{m}\right)^2}{\sqrt{2}}\left(\frac{0-76\times10^{-3}\,\text{T}}{4.5\times10^{-3}\,\text{s}}\right)$$
$$= 5.1\times10^{-2}\,\text{V}.$$

(a) The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

11. (a) It should be emphasized that the result, given in terms of sin($2\pi ft$), could as easily be given in terms of cos($2\pi ft$) or even cos($2\pi ft + \phi$) where $\phi$ is a phase constant as discussed in Chapter 15. The angular position $\theta$ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, $\theta$ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if $\theta$ is understood to be in radians (and $\omega$ would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = N\,Bab2\pi f\sin\left(2\pi ft\right)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi f\,NabB$.

(b) We solve

$$\varepsilon_0 = 150\ \text{V} = 2\pi f\,NabB$$

when $f$ = 60.0 rev/s and $B$ = 0.500 T. The three unknowns are $N$, $a$, and $b$ which occur in a product; thus, we obtain $Nab = 0.796\ \text{m}^2$.

12. To have an induced emf, the magnetic field must be perpendicular (or have a nonzero component perpendicular) to the coil, and must be changing with time.

(a) For $\vec{B}=(4.00\times10^{-2}\,\text{T/m})y\hat{\text{k}}$, $dB/dt=0$ and hence $\varepsilon = 0$.

(b) None.

(c) For $\vec{B}=(6.00\times10^{-2}\,\text{T/s})t\hat{\text{k}}$,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -(0.400\text{ m}\times 0.250\text{ m})(0.0600\text{ T/s}) = -6.00\text{ mV},$$

or $|\varepsilon| = 6.00$ mV.

(d) Clockwise.

(e) For $\vec{B}=(8.00\times10^{-2}\,\text{T/m}\cdot\text{s})yt\,\hat{\text{k}}$, $\Phi_B = (0.400)(0.0800t)\int y\,dy = 1.00\times10^{-3}t$,

in SI units. The induced emf is $\varepsilon=-d\Phi B/dt=-1.00$ mV, or $|\varepsilon| = 1.00$ mV.

(f) Clockwise.

(g) $\Phi_B=0 \Rightarrow \varepsilon=0$.

(h) None.

(i) $\Phi_B=0 \Rightarrow \varepsilon=0$.

(j) None.

13. The amount of charge is

$$q(t)=\frac{1}{R}[\Phi_B(0)-\Phi_B(t)]=\frac{A}{R}[B(0)-B(t)]=\frac{1.20\times10^{-3}\text{m}^2}{13.0\,\Omega}[1.60\,\text{T}-(-1.60\,\text{T})]$$
$$=2.95\times10^{-2}\text{C}\,.$$

14. Figure 30-42(b) demonstrates that $dB/dt$ (the slope of that line) is 0.003 T/s. Thus, in absolute value, Faraday's law becomes

$$\varepsilon=-\frac{d\Phi_B}{dt}=-\frac{d(BA)}{dt}=-A\frac{dB}{dt}$$

where $A = 8 \times 10^{-4}$ m$^2$. We related the induced emf to resistance and current using Ohm's law. The current is estimated from Fig. 30-42(c) to be $i = dq/dt = 0.002$ A (the slope of that line). Therefore, the resistance of the loop is

$$R = \frac{|\varepsilon|}{i} = \frac{A|dB/dt|}{i} = \frac{(8.0\times10^{-4}\ \text{m}^2)(0.0030\ \text{T/s})}{0.0020\ \text{A}} = 0.0012\,\Omega.$$

15. (a) Let $L$ be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\varepsilon_i = \frac{(2.00\ \text{m})^2}{2}(0.870\ \text{T/s}) = 1.74\ \text{V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0\ \text{V} + 1.74\ \text{V} = 21.7\ \text{V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

16. (a) Since the flux arises from a dot product of vectors, the result of one sign for $B_1$ and $B_2$ and of the opposite sign for $B_3$ (we choose the minus sign for the flux from $B_1$ and $B_2$, and therefore a plus sign for the flux from $B_3$). The induced emf is

$$\begin{aligned}\varepsilon &= -\sum\frac{d\Phi_B}{dt} = A\left(\frac{dB_1}{dt} + \frac{dB_2}{dt} - \frac{dB_3}{dt}\right)\\ &= (0.10\ \text{m})(0.20\ \text{m})\left(2.0\times10^{-6}\ \text{T/s} + 1.0\times10^{-6}\ \text{T/s} - 5.0\times10^{-6}\ \text{T/s}\right)\\ &= -4.0\times10^{-8}\ \text{V}.\end{aligned}$$

The minus sign means that the effect is dominated by the changes in $B_3$. Its magnitude (using Ohm's law) is $|\varepsilon|/R = 8.0\ \mu$A.

(b) Consideration of Lenz's law leads to the conclusion that the induced current is therefore counterclockwise.

17. Equation 29-10 gives the field at the center of the large loop with $R = 1.00$ m and current $i(t)$. This is approximately the field throughout the area ($A = 2.00 \times 10^{-4}$ m$^2$) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$, where $i_0 = 200$ A and

$$k = (–200\text{ A} – 200\text{ A})/1.00\text{ s} = – 400\text{ A/s},$$

we find

(a) $B(t=0)=\dfrac{\mu_0 i_0}{2R}=\dfrac{\left(4\pi\times10^{-7}\,\text{H/m}\right)\left(200\,\text{A}\right)}{2\left(1.00\,\text{m}\right)}=1.26\times10^{-4}\,\text{T},$

(b) $B(t=0.500\text{s})=\dfrac{\left(4\pi\times10^{-7}\,\text{H/m}\right)\left[200\,\text{A}-\left(400\,\text{A/s}\right)\left(0.500\,\text{s}\right)\right]}{2\left(1.00\,\text{m}\right)}=0,$ and

(c) $B(t=1.00\text{s})=\dfrac{\left(4\pi\times10^{-7}\,\text{H/m}\right)\left[200\,\text{A}-\left(400\,\text{A/s}\right)\left(1.00\,\text{s}\right)\right]}{2\left(1.00\,\text{m}\right)}=-1.26\times10^{-4}\,\text{T},$

or $|B(t=1.00\text{s})|=1.26\times10^{-4}\,\text{T}.$

(d) Yes, as indicated by the flip of sign of $B(t)$ in (c).

(e) Let the area of the small loop be $a$. Then $\Phi_B = Ba,$ and Faraday's law yields

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt}=-\frac{d(Ba)}{dt}=-a\frac{dB}{dt}=-a\left(\frac{\Delta B}{\Delta t}\right)\\ &=-(2.00\times10^{-4}\text{m}^2)\left(\frac{-1.26\times10^{-4}\text{ T}-1.26\times10^{-4}\text{ T}}{1.00\text{ s}}\right)\\ &=5.04\times10^{-8}\text{ V}.\end{aligned}$$

18. (a) The "height" of the triangular area enclosed by the rails and bar is the same as the distance traveled in time $v$: $d = vt$, where $v = 5.20$ m/s. We also note that the "base" of that triangle (the distance between the intersection points of the bar with the rails) is $2d$. Thus, the area of the triangle is

$$A=\frac{1}{2}(\text{base})(\text{height})=\frac{1}{2}(2vt)(vt)=v^2t^2\ .$$

Since the field is a uniform $B = 0.350$ T, then the magnitude of the flux (in SI units) is

$$\Phi_B = BA = (0.350)(5.20)^2t^2 = 9.46t^2.$$

At $t = 3.00$ s, we obtain $\Phi_B = 85.2$ Wb.

(b) The magnitude of the emf is the (absolute value of) Faraday's law:

$$\varepsilon = \frac{d\Phi_B}{dt} = 9.46\frac{dt^2}{dt} = 18.9t$$

in SI units. At $t$ = 3.00 s, this yields $\varepsilon$ = 56.8 V.

(c) Our calculation in part (b) shows that $n$ = 1.

19. First we write $\Phi_B = BA \cos \theta$. We note that the angular position $\theta$ of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as $BA \cos \theta$ (as opposed to, say, $BA \sin \theta$). Since the coil is rotating steadily, $\theta$ increases linearly with time. Thus, $\theta = \omega t$ if $\theta$ is understood to be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and $f$ = 1000 rev/min ≈ 16.7 rev/s is the frequency). Since the area of the rectangular coil is $A$ = (0.500 m) × (0.300 m) = 0.150 m$^2$, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = NBA2\pi f\,\sin\left(2\pi ft\right)$$

which means it has a voltage amplitude of

$$\varepsilon_{\max} = 2\pi fNAB = 2\pi\left(16.7\,\text{rev/s}\right)\left(100\,\text{turns}\right)\left(0.15\,\text{m}^2\right)\left(3.5\,\text{T}\right) = 5.50\times10^3\ \text{V}\ .$$

20. We note that 1 gauss = $10^{-4}$ T. The amount of charge is

$$q(t) = \frac{N}{R}[BA\cos 20° - (-BA\cos 20°)] = \frac{2NBA\cos 20°}{R}$$
$$= \frac{2(1000)(0.590\times10^{-4}\ \text{T})\pi(0.100\,\text{m})^2(\cos 20°)}{85.0\,\Omega + 140\,\Omega} = 1.55\times10^{-5}\,\text{C}\ .$$

Note that the axis of the coil is at 20°, not 70°, from the magnetic field of the Earth.

21. (a) The frequency is

$$f = \frac{\omega}{2\pi} = \frac{(40\ \text{rev/s})(2\pi\ \text{rad/rev})}{2\pi} = 40\ \text{Hz}\,.$$

(b) First, we define angle relative to the plane of Fig. 30-46, such that the semicircular wire is in the $\theta$ = 0 position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of $a$ above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area $A_0$, which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position).

Since the area of the semicircle is $\pi a^2/2$, then the area (as it appears to us) enclosed by the circuit, as a function of our angle $\theta$, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since $\theta$ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi ft$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + (\pi a^2/2)\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\frac{d\cos\left(2\pi ft\right)}{dt}$$

which yields $\varepsilon = B\pi^2 a^2 f \sin(2\pi ft)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude:

$$\varepsilon_m = B\pi^2 a^2 f = (0.020\text{ T})\pi^2(0.020\text{ m})^2(40/\text{s}) = 3.2\times10^{-3}\text{V}.$$

22. Since $\dfrac{d\cos\phi}{dt} = -\sin\phi\dfrac{d\phi}{dt}$, Faraday's law (with $N = 1$) becomes

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d(BA\cos\phi)}{dt} = BA\sin\phi\frac{d\phi}{dt}.$$

Substituting the values given yields $|\varepsilon| = 0.018$ V.

23. **THINK** Increasing the separation between the two loops changes the flux through the smaller loop and, therefore, induces a current in the smaller loop.

**EXPRESS** The magnetic flux through a surface is given by $\Phi_B = \int \vec{B}\cdot d\vec{A}$, where $\vec{B}$ is the magnetic field and $d\vec{A}$ is a vector of magnitude $dA$ that is normal to a differential area $dA$. In the case where $\vec{B}$ is uniform and perpendicular to the plane of the loop, $\Phi_B = BA$.

In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Equation 29-27, with $z = x$ (taken to be much greater than $R$), gives $\vec{B} = \dfrac{\mu_0 iR^2}{2x^3}\hat{\text{i}}$, where the $+x$ direction is upward in Fig. 30-47. The area of the smaller loop is $A = \pi r^2$.

**ANALYZE** (a) The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area of the smaller loop:

$$\Phi_B = BA = \frac{\pi\mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

**LEARN** The situation in this problem is like that shown in Fig. 30-5(d). The induced magnetic field is in the same direction as the initial magnetic field.

24. (a) Since $\vec{B} = B\hat{\mathrm{i}}$ uniformly, then only the area "projected" onto the *yz* plane will contribute to the flux (due to the scalar [dot] product). This "projected" area corresponds to one-fourth of a circle. Thus, the magnetic flux $\Phi_B$ through the loop is

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \frac{1}{4}\pi r^2 B \ .$$

Thus,

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left(\frac{1}{4}\pi r^2 B\right)\right| = \frac{\pi r^2}{4}\left|\frac{dB}{dt}\right| = \frac{1}{4}\pi(0.10\,\mathrm{m})^2(3.0\times10^{-3}\,\mathrm{T/s}) = 2.4\times10^{-5}\,\mathrm{V}\ .$$

(b) We have a situation analogous to that shown in Fig. 30-5(a). Thus, the current in segment *bc* flows from *c* to *b* (following Lenz's law).

25. (a) We refer to the (very large) wire length as *L* and seek to compute the flux per meter: $\Phi_B/L$. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of anti-parallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call $x = \ell/2$, where $\ell = 20\,\mathrm{mm} = 0.020\,\mathrm{m}$); the net field at any point $0 < x < \ell/2$ is the same at its "mirror image" point $\ell - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = \ell$. We make use of the symmetry by integrating over $0 < x < \ell/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{\ell/2} B\,dA = 2\int_0^{d/2} B\left(L\,dx\right) + 2\int_{d/2}^{\ell/2} B\left(L\,dx\right)$$

where $d$ = 0.0025 m is the diameter of each wire. We will use $R = d/2$, and $r$ instead of $x$ in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{L} &= 2\int_0^R\left(\frac{\mu_0 i}{2\pi R^2}r+\frac{\mu_0 i}{2\pi(\ell-r)}\right)dr + 2\int_R^{\ell/2}\left(\frac{\mu_0 i}{2\pi r}+\frac{\mu_0 i}{2\pi(\ell-r)}\right)dr\\ &= \frac{\mu_0 i}{2\pi}\left(1-2\ln\left(\frac{\ell-R}{\ell}\right)\right)+\frac{\mu_0 i}{\pi}\ln\left(\frac{\ell-R}{R}\right)\\ &= 0.23\times10^{-5}\,\mathrm{T\cdot m}+1.08\times10^{-5}\,\mathrm{T\cdot m}\end{aligned}$$

which yields $\Phi_B/L = 1.3\times10^{-5}$ T·m or $1.3\times10^{-5}$ Wb/m.

(b) The flux (per meter) existing within the regions of space occupied by one or the other wire was computed above to be $0.23\times10^{-5}$ T·m. Thus,

$$\frac{0.23\times10^{-5}\,\mathrm{T\cdot m}}{1.3\times10^{-5}\,\mathrm{T\cdot m}} = 0.17 = 17\%\,.$$

(c) What was described in part (a) as a symmetry plane at $x=\ell/2$ is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the $0<x<\ell/2$ region is now of opposite sign of the flux in the $\ell/2<x<\ell$ region, which causes the total flux (or, in this case, flux per meter) to be zero.

26. (a) First, we observe that a large portion of the figure contributes flux that "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b-a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b-a$ away from it) has exactly the equal but opposite flux that cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B\,dA = \int_{b-a}^{a}\left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr) = \frac{\mu_0 i b}{2\pi}\ln\left(\frac{a}{b-a}\right).$$

Faraday's law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 i b}{2\pi}\ln\left(\frac{a}{b-a}\right)\right] = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{di}{dt}\\ &= -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{d}{dt}\left(\frac{9}{2}t^2-10t\right)\\ &= \frac{-\mu_0 b(9t-10)}{2\pi}\ln\left(\frac{a}{b-a}\right).\end{aligned}$$

With $a$ = 0.120 m and $b$ = 0.160 m, then, at $t$ = 3.00 s, the magnitude of the emf induced in the rectangular loop is

$$|\varepsilon| = \frac{(4\pi\times10^{-7})(0.16)(9(3)-10)}{2\pi}\ln\left(\frac{0.12}{0.16-0.12}\right) = 5.98\times10^{-7}\text{ V} .$$

(b) We note that $di/dt > 0$ at $t$ = 3 s. The situation is roughly analogous to that shown in Fig. 30-5(c). From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

27. (a) Consider a (thin) strip of area of height $dy$ and width $\ell = 0.020\,\text{m}$. The strip is located at some $0 < y < \ell$. The element of flux through the strip is

$$d\Phi_B = BdA = (4t^2 y)(\ell dy)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^\ell (4t^2 y\ell)\, dy = 2t^2\ell^3 .$$

Thus, Faraday's law yields

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3 .$$

At $t$ = 2.5 s, the magnitude of the induced emf is $8.0 \times 10^{-5}$ V.

(b) Its "direction" (or "sense'') is clockwise, by Lenz's law.

28. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 29-17). We integrate according to Eq. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\,\Phi_B\,| = \int_{r-b/2}^{r+b/2}\left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 ia}{2\pi}\,\ln\left(\frac{r+b/2}{r-b/2}\right).$$

When $r = 1.5b$, we have

$$|\,\Phi_B\,| = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.7\text{A})(0.022\text{m})}{2\pi}\ln(2.0) = 1.4\times10^{-8}\,\text{Wb}.$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $dr/dt = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$i_{\text{loop}} = \left|\frac{\varepsilon}{R}\right| = -\frac{\mu_0 ia}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r+b/2}{r-b/2}\right)\right| = \frac{\mu_0 iabv}{2\pi R[r^2-(b/2)^2]}$$
$$= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.7\text{A})(0.022\text{m})(0.0080\text{m})(3.2\times10^{-3}\,\text{m/s})}{2\pi(4.0\times10^{-4}\,\Omega)[2(0.0080\text{m})^2]}$$
$$= 1.0\times10^{-5}\,\text{A}.$$

29. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (0.350\text{ T})(0.250\text{ m})(0.55\text{ m/s}) = 0.0481\text{ V}.$$

(b) By Ohm's law, the induced current is

$$i = 0.0481\text{ V}/18.0\ \Omega = 0.00267\text{ A}.$$

By Lenz's law, the current is clockwise in Fig. 30-52.

(c) Equation 26-27 leads to $P = i^2R = 0.000129$ W.

30. Equation 26-28 gives $\varepsilon^2/R$ as the rate of energy transfer into thermal forms ($dE_{\text{th}}/dt$, which, from Fig. 30-53(c), is roughly 40 nJ/s). Interpreting $\varepsilon$ as the induced emf (in absolute value) in the single-turn loop ($N$ = 1) from Faraday's law, we have

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}.$$

Equation 29-23 gives $B = \mu_0 ni$ for the solenoid (and note that the field is zero outside of the solenoid, which implies that $A = A_{\text{coil}}$), so our expression for the magnitude of the induced emf becomes

$$\varepsilon = A\frac{dB}{dt} = A_{\text{coil}}\frac{d}{dt}\left(\mu_0 ni_{\text{coil}}\right) = \mu_0 nA_{\text{coil}}\frac{di_{\text{coil}}}{dt}.$$

where Fig. 30-53(b) suggests that $di_{\text{coil}}/dt = 0.5$ A/s. With $n = 8000$ (in SI units) and $A_{\text{coil}} = \pi(0.02)^2$ (note that the loop radius does not come into the computations of this problem, just the coil's), we find V = 6.3 μV. Returning to our earlier observations, we can now solve for the resistance:

$$R = \varepsilon^2/(dE_{\text{th}}/dt) = 1.0\text{ m}\Omega.$$

31. **THINK** Thermal energy is generated at the rate given by $P = \varepsilon^2/R$ (see Eq. 27-23), where $\varepsilon$ is the emf in the wire and $R$ is the resistance of the wire.

**EXPRESS** Using Eq. 27-16, the resistance is given by $R = \rho L/A$, where the resistivity is $1.69 \times 10^{-8}\ \Omega\cdot$m (by Table 27-1) and $A = \pi d^2/4$ is the cross-sectional area of the wire ($d$ = 0.00100 m is the wire thickness). The area *enclosed* by the loop is

$$A_{\text{loop}} = \pi r_{\text{loop}}^2 = \pi\left(\frac{L}{2\pi}\right)^2$$

since the length of the wire ($L$ = 0.500 m) is the circumference of the loop. This enclosed area is used in Faraday's law to give the induced emf:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A_{\text{loop}}\frac{dB}{dt} = -\frac{L^2}{4\pi}\frac{dB}{dt}.$$

**ANALYZE** The rate of change of the field is $dB/dt$ = 0.0100 T/s. Thus, we obtain

$$P = \frac{|\varepsilon|^2}{R} = \frac{(L^2/4\pi)^2(dB/dt)^2}{\rho L/(\pi d^2/4)} = \frac{d^2L^3}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = \frac{(1.00\times10^{-3}\text{ m})^2(0.500\text{ m})^3}{64\pi(1.69\times10^{-8}\,\Omega\cdot\text{m})}\left(0.0100\text{ T/s}\right)^2$$
$$= 3.68\times10^{-6}\,\text{W}.$$

**LEARN** The rate of thermal energy generated is proportional to $(dB/dt)^2$.

32. Noting that $|\Delta B| = B$, we find the thermal energy is

$$P_{\text{thermal}}\Delta t = \frac{\varepsilon^2\Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2\Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2\Delta t = \frac{A^2B^2}{R\Delta t}$$
$$= \frac{(2.00\times10^{-4}\,\text{m}^2)^2(17.0\times10^{-6}\,\text{T})^2}{(5.21\times10^{-6}\,\Omega)(2.96\times10^{-3}\text{s})} = 7.50\times10^{-10}\,\text{J}.$$

33. (a) Letting $x$ be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 29-17, the field is $B = \mu_0 i/2\pi r$, where $r$ is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length $x$ and width $dr$, parallel to the wire and a distance $r$ from it; it has area $A = x\,dr$ and the flux is

$$d\Phi_B = B\,dA = \frac{\mu_0 i}{2\pi r}x\,dr.$$

By Eq. 30-1, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right).$$

According to Faraday's law the emf induced in the loop is

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 i v}{2\pi}\ln\left(\frac{a+L}{a}\right)$$
$$= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(100\,\text{A})(5.00\,\text{m/s})}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right) = 2.40\times10^{-4}\,\text{V}.$$

(b) By Ohm's law, the induced current is

$$i_\ell = \varepsilon / R = (2.40\times10^{-4}\,\text{V})/(0.400\,\Omega) = 6.00\times10^{-4}\,\text{A}.$$

Since the flux is increasing, the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate

$$P = i_\ell^2 R = (6.00\times10^{-4}\,\text{A})^2(0.400\,\Omega) = 1.44\times10^{-7}\,\text{W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length $dr$ at a distance $r$ from the long straight wire, is

$$dF_B = i_\ell B\,dr = (\mu_0 i_\ell i / 2\pi r)\,dr.$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$F_B = \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right)$$
$$= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(6.00\times10^{-4}\,\text{A})(100\,\text{A})}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right)$$
$$= 2.87\times10^{-8}\,\text{N}.$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of $2.87 \times 10^{-8}$ N, to the left.

(e) By Eq. 7-48, the external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\ \text{N})(5.00\ \text{m/s}) = 1.44 \times 10^{-7}\ \text{W}.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

34. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_t L}{R},$$

which yields $v_t = mgR/B^2L^2$.

35. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (1.2\ \text{T})(0.10\ \text{m})(5.0\ \text{m/s}) = 0.60\ \text{V}.$$

(b) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page.

(c) By Ohm's law, the induced current is $i = 0.60\ \text{V}/0.40\ \Omega = 1.5\ \text{A}$.

(d) The direction is clockwise.

(e) Equation 26-28 leads to $P = i^2R = 0.90\ \text{W}$.

(f) From Eq. 29-2, we find that the force on the rod associated with the uniform magnetic field is directed rightward and has magnitude

$$F = iLB = (1.5\ \text{A})(0.10\ \text{m})(1.2\ \text{T}) = 0.18\ \text{N}.$$

To keep the rod moving at constant velocity, therefore, a leftward force (due to some external agent) having that same magnitude must be continuously supplied to the rod.

(g) Using Eq. 7-48, we find the power associated with the force being exerted by the external agent:

$$P = Fv = (0.18\ \text{N})(5.0\ \text{m/s}) = 0.90\ \text{W},$$

which is the same as our result from part (e).

36. (a) For path 1, we have

$$\oint_1 \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{B1}}{dt} = \frac{d}{dt}\left(B_1A_1\right) = A_1\frac{dB_1}{dt} = \pi r_1^2\frac{dB_1}{dt} = \pi\left(0.200\text{m}\right)^2\left(-8.50\times10^{-3}\,\text{T/s}\right)$$
$$= -1.07\times10^{-3}\,\text{V}.$$

(b) For path 2, the result is

$$\oint_2 \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{B2}}{dt} = \pi r_2^2 \frac{dB_2}{dt} = \pi\left(0.300\,\text{m}\right)^2\left(-8.50\times10^{-3}\,\text{T/s}\right) = -2.40\times10^{-3}\,\text{V}\,.$$

(c) For path 3, we have

$$\oint_3 \vec{E}\cdot d\vec{s} = \oint_1 \vec{E}\cdot d\vec{s} - \oint_2 \vec{E}\cdot d\vec{s} = -1.07\times10^{-3}\,\text{V} - \left(-2.4\times10^{-3}\,\text{V}\right) = 1.33\times10^{-3}\,\text{V}\,.$$

37. **THINK** Changing magnetic field induces an electric field.

**EXPRESS** The induced electric field is given by Eq. 30-20: $\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$.

**ANALYZE** (a) The point at which we are evaluating the field is inside the solenoid, so

$$E(2\pi r) = -(\pi r^2)\frac{dB}{dt} \quad \Rightarrow \quad E = -\frac{1}{2}\frac{dB}{dt}r.$$

The magnitude of the induced electric field is

$$|E| = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}\left(6.5\times10^{-3}\,\text{T/s}\right)\left(0.0220\,\text{m}\right) = 7.15\times10^{-5}\,\text{V/m}.$$

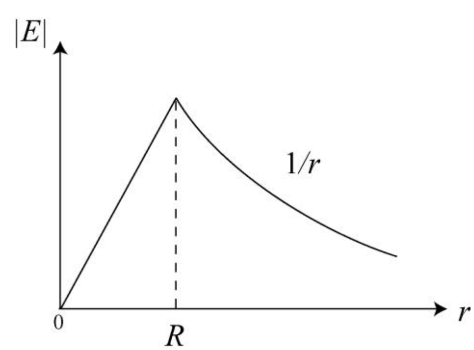
(b) Now the point at which we are evaluating the field is outside the solenoid, so

$$E(2\pi r) = -(\pi R^2)\frac{dB}{dt} \quad \Rightarrow \quad E = -\frac{1}{2}\frac{dB}{dt}\frac{R^2}{r}.$$

The magnitude of the induced field is

$$|E| = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}\left(6.5\times10^{-3}\,\text{T/s}\right)\frac{\left(0.0600\,\text{m}\right)^2}{0.0820\,\text{m}} = 1.43\times10^{-4}\ \text{V/m}.$$

**LEARN** The magnitude of the induced electric field as a function of $r$ is shown to the right. Inside the solenoid, $r < R$, the field $|E|$ is linear in $r$. However, outside the solenoid, $r > R$, $|E| \sim 1/r$.




38. From the "kink" in the graph of Fig. 30-57, we conclude that the radius of the circular region is 2.0 cm. For values of $r$ less than that, we have (from the absolute value of Eq. 30-20)

$$E(2\pi r) = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \pi r^2 a$$

which means that $E/r = a/2$. This corresponds to the slope of that graph (the linear portion for small values of $r$) which we estimate to be 0.015 (in SI units). Thus, $a = 0.030$ T/s.

39. The magnetic field $B$ can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0),$$

where $B_0$ = (30.0 T + 29.6 T)/2 = 29.8 T and $B_1$ = (30.0 T – 29.6 T)/2 = 0.200 T. Then from Eq. 30-25

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right)r = \frac{r}{2}\frac{d}{dt}\;\; B_0 + B_1 \sin(\omega t + \phi_0)\;\; = \frac{1}{2}B_1 \omega r \cos(\omega t + \phi_0).$$

We note that $\omega = 2\pi f$ and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\max} = \frac{1}{2}B_1(2\pi f)r = \frac{1}{2}(0.200\,\mathrm{T})(2\pi)(15\,\mathrm{Hz})(1.6\times10^{-2}\ \mathrm{m}) = 0.15\ \mathrm{V/m}.$$

40. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{(8.0\times10^{-3}\ \mathrm{H})(5.0\times10^{-3}\ \mathrm{A})}{400} = 1.0\times10^{-7}\,\mathrm{Wb}.$$

41. (a) We interpret the question as asking for $N$ multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB(\pi r^2) = (30.0)(2.60\times10^{-3}\ \mathrm{T})(\pi)(0.100\,\mathrm{m})^2 = 2.45\times10^{-3}\,\mathrm{Wb}.$$

(b) Equation 30-33 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45\times10^{-3}\ \mathrm{Wb}}{3.80\,\mathrm{A}} = 6.45\times10^{-4}\ \mathrm{H}.$$

42. (a) We imagine dividing the one-turn solenoid into $N$ small circular loops placed along the width $W$ of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is

$$B = \mu_0 n\Delta i = \mu_0\left(\frac{N}{W}\right)\left(\frac{i}{N}\right) = \frac{\mu_0 i}{W} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.035\mathrm{A})}{0.16\mathrm{m}} = 2.7\times10^{-7}\,\mathrm{T}.$$

(b) Equation 30-33 leads to

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2 \left(\mu_0 i / W\right)}{i} = \frac{\pi \mu_0 R^2}{W} = \frac{\pi(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(0.018\text{m})^2}{0.16\text{m}} = 8.0\times 10^{-9}\,\text{H}.$$

43. We refer to the (very large) wire length as $\ell$ and seek to compute the flux per meter: $\Phi_B / \ell$. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its "mirror image" point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{d/2} B\, dA = 2\int_0^a B\left(\ell\, dx\right) + 2\int_a^{d/2} B\left(\ell\, dx\right)$$

where $d$ = 0.0025 m is the diameter of each wire. We will use $r$ instead of $x$ in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{\ell} &= 2\int_0^a \left(\frac{\mu_0 i}{2\pi a^2} r + \frac{\mu_0 i}{2\pi(d-r)}\right) dr + 2\int_a^{d/2} \left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(d-r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{d-a}{d}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{d-a}{a}\right)\end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i \ell / \pi \ln\left((d-a)/a\right)$. Now, we use Eq. 30-33 (with $N = 1$) to obtain the inductance per unit length:

$$\frac{L}{\ell} = \frac{\Phi_B}{\ell i} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right) = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})}{\pi}\ln\left(\frac{142-1.53}{1.53}\right) = 1.81\times 10^{-6}\,\text{H/m}.$$

44. Since $\varepsilon = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\varepsilon}{L} = -\frac{60\text{V}}{12\text{H}} = -5.0\,\text{A/s},$$

or $|di/dt| = 5.0\,\text{A/s}$. We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

45. (a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then $i$ must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left|\frac{\varepsilon}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8 \times 10^{-4}\ \text{H}.$$

46. During periods of time when the current is varying linearly with time, Eq. 30-35 (in absolute values) becomes $|\varepsilon| = L\,|\Delta i/\Delta t|$. For simplicity, we omit the absolute value signs in the following.

(a) For $0 < t < 2$ ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(7.0\,\text{A} - 0)}{2.0\times10^{-3}\,\text{s}} = 1.6\times10^{4}\ \text{V}.$$

(b) For 2 ms $< t <$ 5 ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(5.0\,\text{A} - 7.0\,\text{A})}{(5.0-2.0)10^{-3}\,\text{s}} = 3.1\times10^{3}\ \text{V}.$$

(c) For 5 ms $< t <$ 6 ms,

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(0 - 5.0\,\text{A})}{(6.0-5.0)10^{-3}\,\text{s}} = 2.3\times10^{4}\ \text{V}.$$

47. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add ($V_1 + V_2$), then inductances in series must add, $L_{\text{eq}} = L_1 + L_2$, just as was the case for resistances. Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

(b) Just as with resistors, $L_{\text{eq}} = \sum_{n=1}^{N} L_n$.

48. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 27-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt}.$$

Thus, although the inductance equation Eq. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also apply to inductors. Therefore,

$$\frac{1}{L_{\text{eq}}}=\frac{1}{L_1}+\frac{1}{L_2}.$$

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in Section 30-12). The requirement is that the field of one inductor not to have significant influence (or "coupling'') in the next.

(b) Just as with resistors, $\frac{1}{L_{\text{eq}}}=\sum_{n=1}^{N}\frac{1}{L_n}$.

49. Using the results from Problems 30-47 and 30-48, the equivalent resistance is

$$L_{\text{eq}}=L_1+L_4+L_{23}=L_1+L_4+\frac{L_2L_3}{L_2+L_3}=30.0\text{ mH}+15.0\text{ mH}+\frac{(50.0\text{ mH})(20.0\text{ mH})}{50.0\text{ mH}+20.0\text{ mH}}$$
$$=59.3\text{ mH}.$$

50. The steady state value of the current is also its maximum value, $\varepsilon/R$, which we denote as $i_m$. We are told that $i = i_m/3$ at $t_0$ = 5.00 s. Equation 30-41 becomes $i=i_m\left(1-e^{-t_0/\tau_L}\right)$, which leads to

$$\tau_L=-\frac{t_0}{\ln(1-i/i_m)}=-\frac{5.00\text{ s}}{\ln(1-1/3)}=12.3\text{ s}.$$

51. The current in the circuit is given by $i=i_0e^{-t/\tau_L}$, where $i_0$ is the current at time $t$ = 0 and $\tau_L$ is the inductive time constant ($L/R$). We solve for $\tau_L$. Dividing by $i_0$ and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right)=-\frac{t}{\tau_L}.$$

This yields

$$\tau_L=-\frac{t}{\ln(i/i_0)}=-\frac{1.0\text{ s}}{\ln\left(\left(10\times10^{-3}\text{ A}\right)/\left(1.0\text{ A}\right)\right)}=0.217\text{ s}.$$

Therefore, $R = L/\tau_L$ = 10 H/0.217 s = 46 Ω.

52. (a) Immediately after the switch is closed, $\varepsilon - \varepsilon_L = iR$. But $i$ = 0 at this instant, so $\varepsilon_L = \varepsilon$, or $\varepsilon_L/\varepsilon$ = 1.00.

(b) $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L} = \varepsilon e^{-2.0\tau_L/\tau_L} = \varepsilon e^{-2.0} = 0.135\varepsilon$, or $\varepsilon_L/\varepsilon = 0.135$.

(c) From $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L}$ we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\varepsilon}{\varepsilon_L}\right) = \ln 2 \quad \Rightarrow \quad t = \tau_L \ln 2 = 0.693\tau_L \quad \Rightarrow \quad t/\tau_L = 0.693.$$

53. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it.

**EXPRESS** If the battery is switched into the circuit at $t = 0$, then the current at a later time $t$ is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$.

(a) We want to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \quad \Rightarrow \quad e^{-t/\tau_L} = 0.200.$$

Taking the natural logarithm of both sides, we obtain

$$-(t/\tau_L) = \ln(0.200) = -1.609.$$

Thus,

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\text{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\text{s}.$$

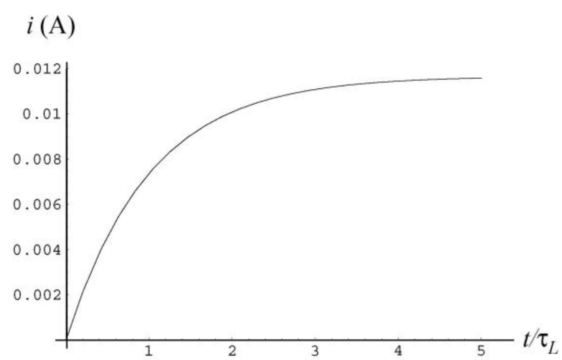
(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20\times10^{3}\Omega}\right)(1 - e^{-1.0}) = 7.37\times10^{-3}\ \text{A}.$$

**LEARN** At $t = 0$, the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is

$$i_0 = \frac{\varepsilon}{R} = \frac{14.0\,\text{V}}{1.20\times10^{3}\Omega} = 0.0117\ \text{A}.$$

The current as a function of $t/\tau_L$ is plotted to the right.



54. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{R_1 + R_2} = \frac{100\,\text{V}}{10.0\,\Omega + 20.0\,\Omega} = 3.33\,\text{A}.$$

(b) $i_2 = i_1 = 3.33\,\text{A}.$

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in $R_3$ is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0$$

$$\varepsilon - i_1 R_1 - (i_1 - i_2) R_3 = 0.$$

We solve these simultaneously for $i_1$ and $i_2$, and find

$$i_1 = \frac{\varepsilon (R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100\,\text{V})(20.0\,\Omega + 30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 4.55\,\text{A},$$

(d) and

$$i_2 = \frac{\varepsilon R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{(100\,\text{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 2.73\,\text{A}.$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in $R_3$ changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 4.55 A – 2.73 A = 1.82 A. The current in $R_2$ is the same but in the opposite direction as that in $R_3$, that is, $i_2 = -1.82$ A.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.

55. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it.

**EXPRESS** Starting with zero current at $t$ = 0 (the moment the switch is closed) the current in the circuit increases according to

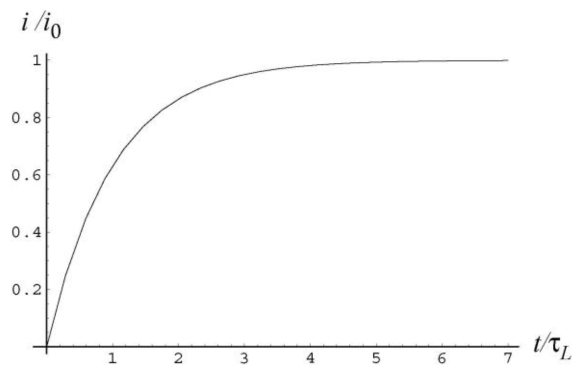
$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$ is the inductive time constant and $\varepsilon$ is the battery emf.

**ANALYZE** To calculate the time at which $i = 0.9990\varepsilon/R$, we solve for $t$:

$$0.990\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \;\Rightarrow\; \ln(0.0010) = -\frac{t}{\tau_L} \;\Rightarrow\; \frac{t}{\tau_L} = 6.91.$$

**LEARN** At $t$ = 0, the current in the circuit is zero. However, after a very long time, the inductor acts like an ordinary connecting wire, so the current is $i_0 = \varepsilon / R$. The current (in terms of $i/i_0$) as a function of $t/\tau_L$ is plotted below.




56. From the graph we get $\Phi/i = 2 \times 10^{-4}$ in SI units. Therefore, with $N$ = 25, we find the self-inductance is $L = N\,\Phi/i = 5 \times 10^{-3}$ H. From the derivative of Eq. 30-41 (or a combination of that equation and Eq. 30-39) we find (using the symbol $V$ to stand for the battery emf)

$$\frac{di}{dt} = \frac{V}{R}\frac{R}{L}\, e^{-t/\tau_L} = \frac{V}{L}e^{-t/\tau_L} = 7.1 \times 10^2 \text{ A/s} .$$

57. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\varepsilon - L\, di/dt = 0$. So $i = \varepsilon t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0$ A. Thus,

$$t_0 = \frac{i_0 L}{\varepsilon} = \frac{(3.0\,\text{A})(5.0\,\text{H})}{10\,\text{V}} = 1.5\text{ s}.$$

(b) We do not show the graph here; qualitatively, it would be similar to Fig. 30-15.

58. Applying the loop theorem,

$$\varepsilon - L\left(\frac{di}{dt}\right) = iR \ ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\varepsilon = L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0+5.0t) + (3.0+5.0t)R = (6.0)(5.0) + (3.0+5.0t)(4.0)$$
$$= (42+20t).$$

59. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it. We are interested in the currents in the resistor and the current in the inductor as a function of time.

**EXPRESS** We assume $i$ to be from left to right through the closed switch. We let $i_1$ be the current in the resistor and take it to be downward. Let $i_2$ be the current in the inductor, also assumed downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1R - L(di_2/dt) = 0$. According to the junction rule, $(di_1/dt) = -(di_2/dt)$. We substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1R = 0.$$

This equation is similar to Eq. 30-46, and its solution is the function given as Eq. 30-47: $i_1 = i_0e^{-Rt/L}$, where $i_0$ is the current through the resistor at $t = 0$, just after the switch is closed. Now just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that moment $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$.

**ANALYZE** (a) The currents in the resistor and the inductor as a function of time are:

$$i_1 = ie^{-Rt/L}, \quad i_2 = i - i_1 = i\left(1-e^{-Rt/L}\right).$$

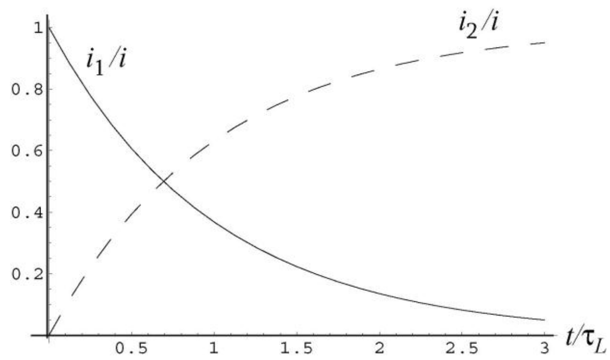
(b) When $i_2 = i_1$, we have

$$e^{-Rt/L} = 1 - e^{-Rt/L} \ \Rightarrow\ e^{-Rt/L} = \frac{1}{2}.$$

Taking the natural logarithm of both sides and using $\ln(1/2) = -\ln 2$, we obtain

$$\left(\frac{Rt}{L}\right) = \ln 2 \ \Rightarrow\ t = \frac{L}{R}\ln 2.$$

**LEARN** A plot of $i_1/i$ (solid line, for resistor) and $i_2/i$ (dashed line, for inductor) as a function of $t/\tau_L$ is shown next.




60. (a) Our notation is as follows: $h$ is the height of the toroid, $a$ its inner radius, and $b$ its outer radius. Since it has a square cross section, $h = b – a$ = 0.12 m – 0.10 m = 0.02 m. We derive the flux using Eq. 29-24 and the self-inductance using Eq. 30-33:

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi}\ln\left(\frac{b}{a}\right)$$

and

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right).$$

Now, since the inner circumference of the toroid is $l = 2\pi a = 2\pi(10\text{ cm}) \approx 62.8$ cm, the number of turns of the toroid is roughly $N \approx 62.8$ cm/1.0 mm = 628. Thus

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right) \approx \frac{\left(4\pi\times10^{-7}\text{ H/m}\right)(628)^2(0.02\text{m})}{2\pi}\ln\left(\frac{12}{10}\right) = 2.9\times10^{-4}\text{H}.$$

(b) Noting that the perimeter of a square is four times its sides, the total length $\ell$ of the wire is $\ell = (628)4(2.0\,\text{cm}) = 50\,\text{m}$, and the resistance of the wire is

$$R = (50\text{ m})(0.02\ \Omega/\text{m}) = 1.0\ \Omega.$$

Thus,

$$\tau_L = \frac{L}{R} = \frac{2.9\times10^{-4}\,\text{H}}{1.0\Omega} = 2.9\times10^{-4}\text{s}.$$

61. **THINK** Inductance $L$ is related to the inductive time constant of an $RL$ circuit by $L = \tau_L R$, where $R$ is the resistance in the circuit. The energy stored by an inductor carrying current $i$ is given by $U_B = Li^2/2$.

**EXPRESS** If the battery is applied at time $t$ = 0 the current is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\varepsilon$ is the emf of the battery, $R$ is the resistance, and $\tau_L$ is the inductive time constant ($L/R$). This leads to

$$e^{-t/\tau_L} = 1 - \frac{iR}{\varepsilon} \Rightarrow -\frac{t}{\tau_L} = \ln\left(1 - \frac{iR}{\varepsilon}\right).$$

Since

$$\ln\left(1-\frac{iR}{\varepsilon}\right) = \ln\left[1 - \frac{(2.00\times10^{-3}\,\text{A})(10.0\times10^{3}\,\Omega)}{50.0\,\text{V}}\right] = -0.5108,$$

the inductive time constant is $\tau_L = t/0.5108 = (5.00 \times 10^{-3}\text{ s})/0.5108 = 9.79 \times 10^{-3}$ s.

**ANALYZE** (a) The inductance is

$$L = \tau_L R = (9.79\times10^{-3}\,\text{s})(10.0\times10^{3}\,\Omega) = 97.9\,\text{H}.$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\text{H})(2.00\times10^{-3}\,\text{A})^2 = 1.96\times10^{-4}\,\text{J}.$$

**LEARN** Note the similarity between $U_B = \frac{1}{2}Li^2$ and $U_C = \frac{q^2}{2C}$, the electric energy stored in a capacitor.

62. (a) From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = Li\frac{di}{dt} = L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right) = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}.$$

Now,

$$\tau_L = L/R = 2.0\text{ H}/10\ \Omega = 0.20\text{ s}$$

and $\varepsilon = 100$ V, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when $t = 0.10$ s.

(b) From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9 \times 10^2 \text{ W}.$$

We note that this result could alternatively have been found from Eq. 28-14 (with Eq. 30-41).

63. From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(Li^2/2\right)}{dt} = Li\frac{di}{dt} = L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right) = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}$$

where $\tau_L = L/R$ has been used. From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

We equate this to $dU_B/dt$, and solve for the time:

$$\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2 = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L} \quad \Rightarrow \quad t = \tau_L \ln 2 = (37.0\,\text{ms})\ln 2 = 25.6\,\text{ms}.$$

64. Let $U_B(t) = \frac{1}{2}Li^2(t)$. We require the energy at time $t$ to be half of its final value: $U(t) = \frac{1}{2}U_B(t \to \infty) = \frac{1}{4}Li_f^2$. This gives $i(t) = i_f/\sqrt{2}$. But $i(t) = i_f(1-e^{-t/\tau_L})$, so

$$1-e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{t}{\tau_L} = -\ln\left(1-\frac{1}{\sqrt{2}}\right) = 1.23.$$

65. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 30-41 for the current):

$$\begin{aligned}\int_0^t P_{\text{battery}}\,dt &= \int_0^t \frac{\varepsilon^2}{R}\left(1-e^{-Rt/L}\right)dt = \frac{\varepsilon^2}{R}\left[t + \frac{L}{R}\left(e^{-Rt/L}-1\right)\right] \\ &= \frac{(10.0\,\text{V})^2}{6.70\ \Omega}\left[2.00\ \text{s} + \frac{(5.50\,\text{H})\left(e^{-(6.70\ \Omega)(2.00\ \text{s})/5.50\ \text{H}}-1\right)}{6.70\ \Omega}\right] \\ &= 18.7\ \text{J}.\end{aligned}$$

(b) The energy stored in the magnetic field is given by Eq. 30-49:

$$U_B = \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2\left(1-e^{-Rt/L}\right)^2 = \frac{1}{2}(5.50\,\text{H})\left(\frac{10.0\,\text{V}}{6.70\,\Omega}\right)^2\left[1-e^{-(6.70\,\Omega)(2.00\,\text{s})/5.50\,\text{H}}\right]^2$$
$$= 5.10\text{ J}.$$

(c) The difference of the previous two results gives the amount "lost" in the resistor: 18.7 J – 5.10 J = 13.6 J.

66. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 29-9, is

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi\times10^{-7}\text{ H/m})(100\text{ A})}{2(50\times10^{-3}\text{ m})} = 1.3\times10^{-3}\text{ T}.$$

(b) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(1.3\times10^{-3}\text{ T})^2}{2(4\pi\times10^{-7}\text{ H/m})} = 0.63\text{ J/m}^3.$$

67. **THINK** The magnetic energy density is given by $u_B = B^2/2\mu_0$, where $B$ is the magnitude of the magnetic field at that point.

**EXPRESS** Inside a solenoid, the magnitude of the magnetic field is $B = \mu_0 ni$, where

$$n = (950\text{ turns})/(0.850\text{ m}) = 1.118\times10^3\text{ m}^{-1}.$$

Thus, the energy density is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(\mu_0 ni)^2}{2\mu_0} = \frac{1}{2}\mu_0 n^2 i^2.$$

Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B\mathcal{V}$, where $\mathcal{V}$ is the volume of the solenoid.

**ANALYZE** (a) Substituting the values given, we find the magnetic energy density to be

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.118\times10^3\text{ m}^{-1}\right)^2(6.60\text{ A})^2 = 34.2\text{ J/m}^3.$$

(b) The volume $\mathcal{V}$ is calculated as the product of the cross-sectional area and the length. Thus,

$$U_B = \left(34.2\text{ J/m}^3\right)\left(17.0\times10^{-4}\text{ m}^2\right)(0.850\text{ m}) = 4.94\times10^{-2}\text{ J}.$$

**LEARN** Note the similarity between $u_B = \dfrac{B^2}{2\mu_0}$, the energy density at a point in a magnetic field, and $u_E = \dfrac{1}{2}\varepsilon_0 E^2$, the energy density at a point in an electric field. Both quantities are proportional to the square of the fields.

68. The magnetic energy stored in the toroid is given by $U_B = \frac{1}{2}Li^2$, where $L$ is its inductance and $i$ is the current. By Eq. 30-54, the energy is also given by $U_B = u_B\mathcal{V}$, where $u_B$ is the average energy density and $\mathcal{V}$ is the volume. Thus

$$i = \sqrt{\frac{2u_B\mathcal{V}}{L}} = \sqrt{\frac{2\left(70.0\,\mathrm{J/m^3}\right)\left(0.0200\,\mathrm{m^3}\right)}{90.0\times10^{-3}\ \mathrm{H}}} = 5.58\,\mathrm{A}\ .$$

69. We set $u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B = \frac{1}{2}B^2/\mu_0$ and solve for the magnitude of the electric field:

$$E = \frac{B}{\sqrt{\varepsilon_0\mu_0}} = \frac{0.50\,\mathrm{T}}{\sqrt{\left(8.85\times10^{-12}\ \mathrm{F/m}\right)\left(4\pi\times10^{-7}\ \mathrm{H/m}\right)}} = 1.5\times10^{8}\ \mathrm{V/m}\ .$$

70. It is important to note that the $x$ that is used in the graph of Fig. 30-67(b) is not the $x$ at which the energy density is being evaluated. The $x$ in Fig. 30-67(b) is the location of wire 2. The energy density (Eq. 30-54) is being evaluated at the coordinate origin throughout this problem. We note the curve in Fig. 30-67(b) has a zero; this implies that the magnetic fields (caused by the individual currents) are in opposite directions (at the origin), which further implies that the currents have the same direction. Since the magnitudes of the fields must be equal (for them to cancel) when the $x$ of Fig. 30-67(b) is equal to 0.20 m, then we have (using Eq. 29-4) $B_1 = B_2$, or

$$\frac{\mu_0 i_1}{2\pi d} = \frac{\mu_0 i_2}{2\pi(0.20\ \mathrm{m})}$$

which leads to $d = (0.20\ \mathrm{m})/3$ once we substitute $i_1 = i_2/3$ and simplify. We can also use the given fact that when the energy density is completely caused by $B_1$ (this occurs when $x$ becomes infinitely large because then $B_2 = 0$) its value is $u_B = 1.96 \times 10^{-9}$ (in SI units) in order to solve for $B_1$:

$$B_1 = \sqrt{2\mu_0\mu_B}\ .$$

(a) This combined with $B_1 = \mu_0 i_1/2\pi d$ allows us to find wire 1's current: $i_1 \approx 23$ mA.

(b) Since $i_2 = 3i_1$ then $i_2 = 70$ mA (approximately).

71. (a) The energy per unit volume associated with the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi R}\right)^2 = \frac{\mu_0 i^2}{8\pi^2 R^2} = \frac{(4\pi\times10^{-7}\ \text{H/m})(10\,\text{A})^2}{8\pi^2\left(2.5\times10^{-3}\ \text{m}/2\right)^2} = 1.0\ \text{J/m}^3\ .$$

(b) The electric energy density is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}(\rho J)^2 = \frac{\varepsilon_0}{2}\left(\frac{iR}{\ell}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\ \text{F/m}\right)\left[(10\text{A})\left(3.3\Omega/10^3\,\text{m}\right)\right]^2$$
$$= 4.8\times10^{-15}\ \text{J/m}^3\ .$$

Here we used $J = i/A$ and $R = \rho\ell/A$ to obtain $\rho J = iR/\ell$ .

72. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25\text{mH})(6.0\text{mA})}{100} = 1.5\,\mu\text{Wb}.$$

(b) The magnitude of the self-induced emf is

$$L_1\frac{di_1}{dt} = (25\text{mH})(4.0\ \text{A/s}) = 1.0\times10^2\,\text{mV}.$$

(c) In coil 2, we find

$$\Phi_{21} = \frac{Mi_1}{N_2} = \frac{(3.0\text{mH})(6.0\text{mA})}{200} = 90\text{nWb}\ .$$

(d) The mutually induced emf is

$$\varepsilon_{21} = M\frac{di_1}{dt} = (3.0\text{mH})(4.0\ \text{A/s}) = 12\text{mV}.$$

73. **THINK** If two coils are near each other, mutual induction can take place whereby a changing current in one coil can induce an emf in the other.

**EXPRESS** The mutual inductance is given by

$$\varepsilon_1 = -M\frac{di_2}{dt}$$

where $\varepsilon_1$ is the induced emf in coil 1 due to the changing current in coil 2. The flux linkage in coil 2 is $N_2\Phi_{21} = Mi_1$.

**ANALYZE** (a) From the equation above, we find the mutual inductance to be

$$M = \frac{|\varepsilon_1|}{di_2/dt} = \frac{25.0\,\text{mV}}{15.0\,\text{A/s}} = 1.67\,\text{mH}.$$

(b) Similarly, the flux linkage in coil 2 is

$$N_2\Phi_{21} = Mi_1 = (1.67\,\text{mH})(3.60\,\text{A}) = 6.00\,\text{mWb}.$$

**LEARN** The emf induced in one coil is proportional to the rate at which current in the other coil is changing:

$$\varepsilon_1 = -M_{12}\frac{di_2}{dt}, \quad \varepsilon_2 = -M_{21}\frac{di_1}{dt}.$$

The proportionality constants, $M_{12}$ and $M_{21}$, are the same, $M_{12} = M_{21} = M$, so we simply write

$$\varepsilon_1 = -M\frac{di_2}{dt}, \quad \varepsilon_2 = -M\frac{di_1}{dt}.$$

74. We use $\varepsilon_2 = -M\, di_1/dt \approx M|\Delta i/\Delta t|$ to find $M$:

$$M = \left|\frac{\varepsilon}{\Delta i_1/\Delta t}\right| = \frac{30\times10^3\text{ V}}{6.0\,\text{A}/(2.5\times10^{-3}\text{ s})} = 13\,\text{H}\ .$$

75. The flux over the loop cross section due to the current $i$ in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}}\, l\, dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r}dr = \frac{\mu_0 i l}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi}\ln\left(1+\frac{b}{a}\right).$$

From the formula for $M$ obtained above, we have

$$M = \frac{(100)(4\pi\times10^{-7}\text{ H/m})(0.30\,\text{m})}{2\pi}\ln\left(1+\frac{8.0}{1.0}\right) = 1.3\times10^{-5}\text{ H}\ .$$

76. (a) The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N\left(\mu_0 i_s n\pi R^2\right)}{i_s} = \mu_0 \pi R^2 nN \ .$$

(b) As long as the magnetic field of the solenoid is entirely contained within the cross section of the coil we have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

77. **THINK** To find the equivalent inductance, we calculate the total emf across both coils.

**EXPRESS** We assume the current to be changing at (nonzero) a rate $di/dt$. The induced emf's can take on the following form:

$$\varepsilon_1 = -\left(L_1 \pm M\right)\frac{di}{dt}, \quad \varepsilon_2 = -\left(L_2 \pm M\right)\frac{di}{dt}$$

The relative sign between $L$ and $M$ depends on how the coils are connected, as we shall see below.

**ANALYZE** (a) The connection is shown in Fig. 30-70. First consider coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emfs in the same direction. Thus, the induced emfs are

$$\varepsilon_1 = -\left(L_1 + M\right)\frac{di}{dt}, \quad \varepsilon_2 = -\left(L_2 + M\right)\frac{di}{dt} \ .$$

Therefore, the total emf across both coils is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -\left(L_1 + L_2 + 2M\right)\frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{eq} = L_1 + L_2 + 2M$.

(b) We imagine reversing the leads of coil 2 so the current enters at the back of the coil rather than the front (as pictured in Fig. 30-70). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\varepsilon_1 = -\left(L_1 - M\right)\frac{di}{dt} \ .$$

Similarly, the emf across coil 2 is

$$\varepsilon_2 = -(L_2 - M)\frac{di}{dt}\ .$$

The total emf across both coils is

$$\varepsilon = -(L_1 + L_2 - 2M)\frac{di}{dt}\ .$$

This is the same as the emf that would be produced by a single coil with inductance

$$L_{eq} = L_1 + L_2 - 2M.$$

**LEARN** The sign of the mutual inductance term is determined by the senses of the coil winding. The induced emfs can either reinforce one another (*L* + *M*), or oppose one another (*L* –*M*).

78. Taking the derivative of Eq. 30-41, we have

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{\varepsilon}{R}(1 - e^{-t/\tau_L})\right] = \frac{\varepsilon}{R\tau_L}e^{-t/\tau_L} = \frac{\varepsilon}{L}e^{-t/\tau_L}\ .$$

With $\tau_L = L/R$ (Eq. 30-42), $L = 0.023$ H and $\varepsilon = 12$ V, $t = 0.00015$ s, and $di/dt = 280$ A/s, we obtain $e^{-t/\tau_L} = 0.537$. Taking the natural log and rearranging leads to $R = 95.4\ \Omega$.

79. **THINK** The inductor in the *RL* circuit initially acts to oppose changes in current through it.

**EXPRESS** When the switch *S* is just closed, $V_1 = \varepsilon$ and no current flows through the inductor. A long time later, the currents have reached their equilibrium values and the inductor acts as an ordinary connecting wire; we can solve the multi-loop circuit problem by applying Kirchhoff's junction and loop rules.

**ANALYZE** (a) Applying the loop rule to the left loop gives $\varepsilon - i_1 R_1 = 0$, so

$$i_1 = \varepsilon/R_1 = 10\text{ V}/5.0\ \Omega = 2.0\text{ A}.$$

(b) Since now $\varepsilon_L = \varepsilon$, we have $i_2 = 0$.

(c) The junction rule gives $i_s = i_1 + i_2 = 2.0\text{ A} + 0 = 2.0$ A.

(d) Since $V_L = \varepsilon$, the potential difference across resistor 2 is $V_2 = \varepsilon - \varepsilon_L = 0$.

(e) The potential difference across the inductor is $V_L = \varepsilon = 10$ V.

(f) The rate of change of current is $\dfrac{di_2}{dt} = \dfrac{V_L}{L} = \dfrac{\varepsilon}{L} = \dfrac{10\text{ V}}{5.0\text{ H}} = 2.0\text{ A/s}$.

(g) After a long time, we still have $V_1 = \varepsilon$, so $i_1$ = 2.0 A.

(h) Since now $V_L = 0$, $i_2 = \varepsilon/R_2$ = 10 V/10 Ω = 1.0 A.

(i) The current through the switch is now $i_s = i_1 + i_2$ = 2.0 A + 1.0 A = 3.0 A.

(j) Since $V_L = 0$, $V_2 = \varepsilon - V_L = \varepsilon$ = 10 V.

(k) With the inductor acting as an ordinary connecting wire, we have $V_L = 0$.

(l) The rate of change of current in resistor 2 is $\frac{di_2}{dt} = \frac{V_L}{L} = 0$.

**LEARN** In analyzing an *RL* circuit immediately after closing the switch and a very long time after that, there is no need to solve any differential equation.

80. Using Eq. 30-41: $i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$, where $\tau_L$ = 2.0 ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - iR/\varepsilon}\right) \approx 1.0 \text{ ns}.$$

81. Using Ohm's law, we relate the induced current to the emf and (the absolute value of) Faraday's law:

$$i = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi}{dt}\right|.$$

As the loop is crossing the boundary between regions 1 and 2 (so that "$x$" amount of its length is in region 2 while "$D - x$" amount of its length remains in region 1) the flux is

$$\Phi_B = xHB_2 + (D - x)HB_1 = DHB_1 + xH(B_2 - B_1)$$

which means

$$\frac{d\Phi_B}{dt} = \frac{dx}{dt}H(B_2 - B_1) = vH(B_2 - B_1) \Rightarrow \quad i = vH(B_2 - B_1)/R.$$

Similar considerations hold (replacing "$B_1$" with 0 and "$B_2$" with $B_1$) for the loop crossing initially from the zero-field region (to the left of Fig. 30-72(a)) into region 1.

(a) In this latter case, appeal to Fig. 30-72(b) leads to

$$3.0 \times 10^{-6} \text{ A} = (0.40 \text{ m/s})(0.015 \text{ m})\, B_1/(0.020\ \Omega)$$

which yields $B_1 = 10\ \mu\text{T}$.

(b) Lenz's law considerations lead us to conclude that the direction of the region 1 field is *out of the page*.

(c) Similarly, $i = vH(B_2 - B_1)/R$ leads to $B_2 = 3.3\mu\text{T}$.

(d) The direction of $\vec{B}_2$ is out of the page.

82. Faraday's law (for a single turn, with $B$ changing in time) gives

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}.$$

In this problem, we find $\frac{dB}{dt} = -\frac{B_0}{\tau}e^{-t/\tau}$. Thus, $\varepsilon = \pi r^2 \frac{B_0}{\tau}e^{-t/\tau}$.

83. Equation 30-41 applies, and the problem requires

$$iR = L\frac{di}{dt} = \varepsilon - iR$$

at some time $t$ (where Eq. 30-39 has been used in that last step). Thus, we have $2iR = \varepsilon$, or

$$\varepsilon = 2iR = 2\left[\frac{\varepsilon}{R}(1 - e^{-t/\tau_L})\right]R = 2\varepsilon\left(1 - e^{-t/\tau_L}\right)$$

where Eq. 30-42 gives the inductive time constant as $\tau_L = L/R$. We note that the emf $\varepsilon$ cancels out of that final equation, and we are able to rearrange (and take the natural log) and solve. We obtain $t = 0.520$ ms.

84. In absolute value, Faraday's law (for a single turn, with $B$ changing in time) gives

$$\frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = \pi R^2 \frac{dB}{dt}$$

for the magnitude of the induced emf. Dividing it by $R^2$ then allows us to relate this to the slope of the graph in Fig. 30-73(b) [particularly the first part of the graph], which we estimate to be 80 $\mu$V/m$^2$.

(a) Thus, $\frac{dB_1}{dt} = (80\ \mu\text{V/m}^2)/\pi \approx 25\ \mu\text{T/s}$.

(b) Similar reasoning for region 2 (corresponding to the slope of the second part of the graph in Fig. 30-73(b)) leads to an emf equal to

$$\pi r_1^2\left(\frac{dB_1}{dt}-\frac{dB_2}{dt}\right)+\pi R^2\,\frac{dB_2}{dt}$$

which means the second slope (which we estimate to be 40 μV/m²) is equal to $\pi\,\dfrac{dB_2}{dt}$.

Therefore, $\dfrac{dB_2}{dt}$ = (40 $\mu$V/m²)/π ≈ 13 $\mu$T/s.

(c) Considerations of Lenz's law leads to the conclusion that $B_2$ is increasing.

85. **THINK** Changing magnetic field induces an electric field.

**EXPRESS** The induced electric field is given by Eq. 30-20:

$$\oint \vec{E}\cdot d\vec{s}=-\frac{d\Phi_B}{dt}.$$

The electric field lines are circles that are concentric with the cylindrical region. Thus,

$$E(2\pi r)=-(\pi r^2)\frac{dB}{dt}\quad\Rightarrow\quad E=-\frac{1}{2}\frac{dB}{dt}r.$$

The force on the electron is $\vec{F}=-e\vec{E},$ so by Newton's second law, the acceleration is $\vec{a}=-e\vec{E}/m.$

**ANALYZE** (a) At point *a*,

$$E=-\frac{r}{2}\left(\frac{dB}{dt}\right)=-\frac{1}{2}(5.0\times10^{-2}\,\text{m})(-10\times10^{-3}\,\text{T/s})=2.5\times10^{-4}\ \text{V/m}.$$

With the normal taken to be into the page, in the direction of the magnetic field, the positive direction for $\vec{E}$ is clockwise. Thus, the direction of the electric field at point *a* is to the left, that is $\vec{E}=-(2.5\times10^{-4}\ \text{V/m})\hat{\text{i}}.$ The resulting acceleration is

$$\vec{a}_a=\frac{-e\vec{E}}{m}=\frac{(-1.60\times10^{-19}\,\text{C})(-2.5\times10^{-4}\ \text{V/m})}{9.11\times10^{-31}\text{kg}}\hat{\text{i}}=(4.4\times10^{7}\ \text{m/s}^2)\hat{\text{i}}.$$

The acceleration is to the right.

(b) At point *b* we have $r_b$ = 0, so the acceleration is zero.

(c) The electric field at point *c* has the same magnitude as the field in *a*, but with its direction reversed. Thus, the acceleration of the electron released at point *c* is

$$\vec{a}_c = -\vec{a}_a = -(4.4\times 10^7\ \text{m/s}^2)\hat{\text{i}}.$$

**LEARN** Inside the cylindrical region, the induced electric field increases with *r*. Therefore, the greater the value of *r*, the greater the magnitude of acceleration.

86. Because of the decay of current (Eq. 30-45) that occurs after the switches are closed on *B*, the flux will decay according to

$$\Phi_1 = \Phi_{10} e^{-t/\tau_{L_1}}, \quad \Phi_2 = \Phi_{20} e^{-t/\tau_{L_2}}$$

where each time constant is given by Eq. 30-42. Setting the fluxes equal to each other and solving for time leads to

$$t = \frac{\ln(\Phi_{20}/\Phi_{10})}{(R_2/L_2)-(R_1/L_1)} = \frac{\ln(1.50)}{(30.0\,\Omega/0.0030\text{ H})-(25\,\Omega/0.0050\text{ H})} = 81.1\mu\text{s} .$$

87. **THINK** Changing the area of the loop changes the flux through it. An induced emf is produced to oppose this change.

**EXPRESS** The magnetic flux through the loop is $\Phi_B = BA$, where *B* is the magnitude of the magnetic field and *A* is the area of the loop. According to Faraday's law, the magnitude of the average induced emf is

$$\varepsilon_{\text{avg}} = \left|\frac{-d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{B|\Delta A|}{\Delta t}.$$

**ANALYZE** (a) substituting the values given, we obtain

$$\varepsilon_{\text{avg}} = \frac{B|\Delta A|}{\Delta t} = \frac{(2.0\text{ T})(0.20\text{ m})^2}{0.20\text{ s}} = 0.40\text{V}.$$

(b) The average induced current is $i_{\text{avg}} = \dfrac{\varepsilon_{\text{avg}}}{R} = \dfrac{0.40\text{ V}}{20\times 10^{-3}\Omega} = 20\text{A}$.

**LEARN** By Lenz's law, the more rapidly the area is changing, the greater the induced current in

88. (a) From Eq. 30-28, we have

$$L = \frac{N\Phi}{i} = \frac{(150)(50\times10^{-9}\ \text{T}\cdot\text{m}^2)}{2.00\times10^{-3}\ \text{A}} = 3.75\ \text{mH}.$$

(b) The answer for $L$ (which should be considered the *constant* of proportionality in Eq. 30-35) does not change; it is still 3.75 mH.

(c) The equations of Chapter 28 display a simple proportionality between magnetic field and the current that creates it. Thus, if the current has doubled, so has the field (and consequently the flux). The answer is 2(50) = 100 nWb.

(d) The magnitude of the induced emf is (from Eq. 30-35)

$$L\left.\frac{di}{dt}\right|_{\max} = (0.00375\ \text{H})(0.0030\ \text{A})(377\ \text{rad/s}) = 4.24\times10^{-3}\ \text{V}.$$

89. (a) $i_0 = \varepsilon/R$ = 100 V/10 Ω = 10 A.

(b) $U_B = \frac{1}{2}Li_0^2 = \frac{1}{2}(2.0\text{H})(10\text{A})^2 = 1.0\times10^2\,\text{J}$.

90. We write $i = i_0 e^{-t/\tau_L}$ and note that $i$ = 10% $i_0$. We solve for $t$:

$$t = \tau_L \ln\left(\frac{i_0}{i}\right) = \frac{L}{R}\ln\left(\frac{i_0}{i}\right) = \frac{2.00\,\text{H}}{3.00\,\Omega}\ln\left(\frac{i_0}{0.100\,i_0}\right) = 1.54\,\text{s}.$$

91. **THINK** We have an $RL$ circuit in which the inductor is in series with the battery.

**EXPRESS** As the switch closes at $t$ = 0, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit.

**ANALYZE** (a) At $t$ = 0, the current through the battery is also zero.

(b) With no current anywhere in the circuit at $t$ = 0, the loop rule requires the emf of the inductor $\varepsilon_L$ to cancel that of the battery ($\varepsilon$ = 40 V). Thus, the absolute value of Eq. 30-35 yields

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{40\ \text{V}}{0.050\ \text{H}} = 8.0\times10^2\ \text{A/s}.$$

(c) This circuit becomes equivalent to that analyzed in Section 30-9 when we replace the parallel set of 20000 Ω resistors with $R$ = 10000 Ω. Now, with $\tau_L = L/R = 5\times10^{-6}$ s, we have $t/\tau_L$ = 3/5, and we apply Eq. 30-41:

$$i_{\text{bat}} = \frac{\varepsilon}{R}\left(1-e^{-3/5}\right) \approx 1.8\times10^{-3}\,\text{A}.$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 30-35):

$$\varepsilon - i_{\text{bat}} R - |\varepsilon_L| = 0 .$$

Using the values from part (c), we obtain $|\varepsilon_L| \approx 22$ V. Then,

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{22\text{ V}}{0.050\text{ H}} \approx 4.4\times 10^2 \text{ A/s} .$$

(e) As $t \to \infty$, the circuit reaches a steady-state condition, so that $di_{\text{bat}}/dt = 0$ and $\varepsilon_L = 0$. The loop rule then leads to

$$\varepsilon - i_{\text{bat}} R - |\varepsilon_L| = 0 \;\Rightarrow\; i_{\text{bat}} = \frac{40\text{ V}}{10000\,\Omega} = 4.0\times 10^{-3}\text{ A}.$$

(f) As $t \to \infty$, the circuit reaches a steady-state condition, $di_{\text{bat}}/dt = 0$.

**LEARN** In summary, at $t = 0$ immediately after the switch is closed, the inductor opposes any change in current, and with the inductor and the battery being connected in series, the induced emf in the inductor is equal to the emf of the battery, $\varepsilon_L = \varepsilon$. A long time later after all the currents have reached their steady-state values, $\varepsilon_L = 0$, and the inductor can be treated as an ordinary connecting wire. In this limit, the circuit can be analyzed as if $L$ were not present.

92. (a) $L = \Phi/i = 26 \times 10^{-3}$ Wb/5.5 A $= 4.7 \times 10^{-3}$ H.

(b) We use Eq. 30-41 to solve for $t$:

$$t = -\tau_L \ln\left(1 - \frac{iR}{\varepsilon}\right) = -\frac{L}{R}\ln\left(1 - \frac{iR}{\varepsilon}\right) = -\frac{4.7\times 10^{-3}\text{H}}{0.75\Omega}\ln\left[1 - \frac{(2.5\text{A})(0.75\Omega)}{6.0\text{V}}\right]$$
$$= 2.4\times 10^{-3}\text{s}.$$

93. The energy stored when the current is $i$ is $U_B = \frac{1}{2}Li^2$, where $L$ is the self-inductance. The rate at which this is developed is

$$\frac{dU_B}{dt} = Li\frac{di}{dt}$$

where $i$ is given by Eq. 30-41 and $di/dt$ is obtained by taking the derivative of that equation (or by using Eq. 30-37). Thus, using the symbol $V$ to stand for the battery voltage (12.0 volts) and $R$ for the resistance (20.0 Ω), we have, at $t = 1.61\tau_L$,

$$\frac{dU_B}{dt}=\frac{V^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}=\frac{(12.0\text{ V})^2}{20.0\Omega}\left(1-e^{-1.61}\right)e^{-1.61}=1.15\text{ W}.$$

94. (a) The self-inductance per meter is

$$\frac{L}{\ell}=\mu_0 n^2 A=\left(4\pi\times10^{-7}\text{ H/m}\right)\left(100\text{ turns/cm}\right)^2\left(\pi\right)\left(1.6\text{ cm}\right)^2=0.10\text{ H/m}.$$

(b) The induced emf per meter is

$$\frac{\varepsilon}{\ell}=\frac{L}{\ell}\frac{di}{dt}=\left(0.10\text{ H/m}\right)\left(13\text{ A/s}\right)=1.3\text{ V/m}.$$

95. (a) As the switch closes at $t$ = 0, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf ($\varepsilon_{L1}$) of the $L_1$ = 0.30 H inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 30-35

$$\frac{di}{dt}=\frac{|\varepsilon_{L1}|}{L_1}=\frac{6.0}{0.30}=20\text{ A/s}.$$

(b) What is being asked for is essentially the current in the battery when the emfs of the inductors vanish (as $t\to\infty$). Applying the loop rule to the outer loop, with $R_1$ = 8.0 Ω, we have

$$\varepsilon-iR_1-|\varepsilon_{L1}|-|\varepsilon_{L2}|=0 \;\Rightarrow\; i=\frac{6.0\text{ V}}{R_1}=0.75\text{ A}.$$

96. Since $A=\ell^2$, we have $dA/dt=2\ell d\ell/dt$. Thus, Faraday's law, with $N$ = 1, becomes

$$\varepsilon=-\frac{d\Phi_B}{dt}=-\frac{d(BA)}{dt}=-B\frac{dA}{dt}=-2\ell B\frac{d\ell}{dt}$$

which yields $\varepsilon$ = 0.0029 V.

97. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem may be addressed by using Eq. 30-41. The derivative of that solution is

$$\frac{di}{dt}=\frac{d}{dt}\left[\frac{\varepsilon}{R}(1-e^{-t/\tau_L})\right]=\frac{\varepsilon}{R\tau_L}e^{-t/\tau_L}=\frac{\varepsilon}{L}e^{-t/\tau_L}$$

With $\tau_L$ = 0.28 ms (by Eq. 30-42), $L$ = 0.050 H, and $\varepsilon$ = 45 V, we obtain $di/dt$ = 12 A/s when $t$ = 1.2 ms.

98. (a) From Eq. 30-35, we find $L$ = (3.00 mV)/(5.00 A/s) = 0.600 mH.

(b) Since $N\Phi = iL$ (where $\Phi$ = 40.0 $\mu$Wb and $i$ = 8.00 A), we obtain $N$ = 120.

99. We use 1 ly = $9.46 \times 10^{15}$ m, and use the symbol $\mathcal{V}$ for volume.

$$U_B = \mathcal{V}u_B = \frac{\mathcal{V}B^2}{2\mu_0} = \frac{\left(9.46\times10^{15}\text{ m}\right)^3\left(1\times10^{-10}\text{ T}\right)^2}{2\left(4\pi\times10^{-7}\text{ H/m}\right)} = 3\times10^{36}\text{ J} .$$

100. (a) The total length of the closed loop formed by the two radii plus the arc is

$$L = 2r + r\theta = r(2+\theta),$$

where $r$ is the radius. The total resistance is

$$R = \frac{\rho L}{A} = \frac{\rho r(2+\theta)}{A} = \frac{(1.7\times10^{-8}\,\Omega\cdot\text{m})(0.24\text{ m})(2+\theta)}{1.20\times10^{-6}\,\text{m}^2}$$
$$= (3.4\times10^{-3})(2+\theta)\,\Omega.$$

(b) The area of the loop is $A = \frac{1}{2}r^2\theta$. Thus, the magnetic flux through the loop is

$$\Phi_B = BA = \frac{1}{2}Br^2\theta = \frac{1}{2}(0.150\text{ T})(0.240\text{ m})^2\theta = (4.32\times10^{-3}\theta)\text{ Wb}.$$

(c) The induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{1}{2}Br^2\theta\right) = -\frac{1}{2}Br^2\frac{d\theta}{dt} = -\frac{1}{2}Br^2\omega$$

which gives

$$i = \frac{|\varepsilon|}{R} = \frac{Br^2\omega}{2R} = \frac{Br^2\omega}{2(3.4\times10^{-3})(2+\theta)} = \frac{Br^2\alpha t}{2(3.4\times10^{-3})(2+\alpha t^2/2)}$$

as the magnitude of the induced current. Note that in the last step, we have substituted $\omega = \alpha t$ and $\theta = \frac{1}{2}\alpha t^2$, for constant angular acceleration $\alpha$. Differentiating $i$ with respect to $t$ gives

$$\frac{di}{dt} = \frac{Br^2\alpha(4-\alpha t^2)}{(3.4\times10^{-3})(4+\alpha t^2)^2}.$$

The induced current is at a maximum when $4-\alpha t^2=0$, or $t=\sqrt{4/\alpha}$. At this instant, the angle is

$$\theta=\frac{1}{2}\alpha t^2=\frac{1}{2}\alpha\left(\frac{4}{\alpha}\right)=2.0\text{ rad.}$$

(d) When current is at a maximum, $\omega=\alpha t=\alpha\sqrt{4/\alpha}=\sqrt{4\alpha}$. Thus,

$$i_{\max}=\frac{Br^2\omega}{2R}=\frac{Br^2\sqrt{4\alpha}}{2R}=\frac{Br^2\sqrt{4\alpha}}{2(3.4\times10^{-3})(2+\theta)}=\frac{(0.150\text{ T})(0.24\text{ m})^2\sqrt{4(12\text{ rad/s}^2)}}{2(3.4\times10^{-3})(2+2.0)}=2.20\text{ A.}$$

101. (a) We use $U_B=\frac{1}{2}Li^2$ to solve for the self-inductance:

$$L=\frac{2U_B}{i^2}=\frac{2\left(25.0\times10^{-3}\text{ J}\right)}{\left(60.0\times10^{-3}\text{ A}\right)^2}=13.9\text{ H}.$$

(b) Since $U_B\propto i^2$, for $U_B$ to increase by a factor of 4, $i$ must increase by a factor of 2. Therefore, $i$ should be increased to 2(60.0 mA) = 120 mA.