SOLUTIONS TO THE CLASS 1 AND CLASS 2 PROBLEMS IN

TRANSPORT PHENOMENA

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CHAPTER 1 – Checked by T. J. Sadowski

1.A Computation of Viscosities of Gases at Low Density

Two methods of solution are given in the text: the kinetic theory method based on Eq. 1.4-18, and the corresponding states method based on Fig. 1.3-1. The kinetic theory method is more accurate. Calculations by both methods are summarized here.

Kinetic Theory Method:

Gas	Constants from Table B-1			$KT/\epsilon = \frac{293.2}{\epsilon/K}$	Ω_μ from Table B-2	Predicted viscosity (cp) $\mu = 2.6693\times10^{-3}\frac{\sqrt{MT}}{\sigma^2\Omega_\mu}$
	M	σ (Å)	ϵ/K (°K)			
O_2	32.00	3.433	113.	2.60	1.081	0.02029
N_2	28.02	3.681	91.5	3.20	1.022	0.01747
CH_4	16.04	3.822	137.	2.14	1.149	0.01091

Corresponding States Method:

Gas	Constants from Table B-1			$T_r = \frac{293.2}{T_c}$	$p_r = \frac{1.00}{p_c}$	μ/μ_c (Fig. 1.3-1)	Predicted Viscosity μ (cp)
	T_c (°K)	p_c (atm)	μ_c (cp)				
O_2	154.4	49.7	0.0250	1.90	0.020	0.82	0.0205
N_2	126.2	33.5	0.0180	2.32	0.030	0.97	0.0175
CH_4	191.0	45.8	0.0159	1.54	0.022	0.69	0.0110

The observed values are: O_2 (0.0203 cp), N_2 (0.0175 cp), CH_4 (0.0109 cp), as given in Table 1.1-2.

1.B Calculation of Viscosities of Gas Mixtures at Low Density

For the binary system being considered

i	Species	$\mu_i \times 10^6$	M_i
1	H_2	88.4	2.016
2	CCl_2F_2	124.0	120.9

According to Eq. 1.4-20, $\Phi_{11} = 1$, $\Phi_{22} = 1$, and

$$\Phi_{12} = \frac{1}{\sqrt{8}}\left[1 + \frac{2.016}{120.9}\right]^{-1/2}\left[1 + \left(\frac{88.4}{124.0}\right)^{1/2}\left(\frac{120.9}{2.016}\right)^{1/4}\right]^2 = 3.934$$

$$\Phi_{21} = \frac{1}{\sqrt{8}}\left[1 + \frac{120.9}{2.016}\right]^{-1/2}\left[1 + \left(\frac{124.0}{88.4}\right)^{1/2}\left(\frac{2.016}{120.9}\right)^{1/4}\right]^2 = 0.0920$$

Application of Eq. 1.4-19 then gives:

x_i	$\Sigma_1 = \Sigma x_j \Phi_{1j}$	$\Sigma_2 = \Sigma x_j \Phi_{2j}$	$A = x_1\mu_1/\Sigma_1$	$B = x_2\mu_2/\Sigma_2$	$A+B = \mu_{calc} \times 10^6$	$\mu_{obs} \times 10^6$
0.00	3.934	1.000	0.0	124.0	(124.0)	124.0
0.25	3.200	0.773	6.9	120.3	127.1	128.1
0.50	2.467	0.546	17.9	113.6	131.5	131.9
0.75	1.734	0.319	38.2	97.2	135.5	135.1
1.00	1.000	0.092	88.4	0.0	(88.4)	88.4

1.C Estimation of Viscosity of a Gas at High Density

(a.) From Table B-1 we find that $T_c = 126.2\,^\circ K$, $p_c = 33.5$ atm., and $\mu_c = 180 \times 10^{-6}$ g cm⁻¹ sec⁻¹. Hence:

$$\left.\begin{aligned} p_r &= p/p_c = (1000 + 14.7)/(33.5)(14.7) = 1.40 \\ T_r &= T/T_c = (460 + 68)/(126.2)(1.8) = 2.32 \end{aligned}\right\} \text{From Fig. 1.3-1, } \mu_r = 1.07$$

Hence the predicted viscosity is $\mu = \mu_r \mu_c = (1.07)(180 \times 10^{-6}) = 1.93 \times 10^{-4}$ g cm⁻¹ sec⁻¹

(b.) From Table 1.1-2, at 20°C (68°F), $\mu^0 = 1.76 \times 10^{-4}$ g cm⁻¹ sec⁻¹. From Fig. 1.3-2 for the values of p_r and T_r calculated above, $\mu^\# = 1.1$. Hence the predicted viscosity is:

$$\mu = \mu^\# \mu^0 = (1.1)(1.75 \times 10^{-4}) = 1.93 \times 10^{-4} \text{ g cm}^{-1}\text{sec}^{-1}$$

1.D Estimation of Liquid Viscosity

(a) Equation 1.5-10 may be used with values of $\tilde{N}$, h, R from Table C.2, and

T (°K)	273.2	373.2
ρ (g cm⁻³)	0.9998	0.9584
$\tilde{V} = M/\rho$ (cm³ g-mole⁻¹)	18.02	18.80
$\Delta\hat{U}_{vap}$ (cal g⁻¹) at normal boiling pt.	4929	498.9
$\Delta\tilde{U}_{vap} = M\,\Delta\hat{U}_{vap}$ (cal g-mole⁻¹)	8,988	8,988
exp $0.408\,\Delta\tilde{U}_{vap}/RT$	8.60×10^2	1.40×10^2
$\tilde{N}h/\tilde{V}$ (g cm⁻¹ sec⁻¹)	2.21×10^{-4}	2.12×10^{-4}
Predicted Viscosity (g cm⁻¹ sec⁻¹)	0.19	0.0297

(b) Use Eq. 1.5-12 to get: (Here $T_b = 373.2°K$)

At 273.2°K $\mu = (2.21 \times 10^{-4}) \exp (3.8)(373.2)/(273.2) = 4.0 \times 10^{-2} \, g \, cm^{-1} sec^{-1}$

At 373.2°K $\mu = (2.12 \times 10^{-4}) \exp (3.8) = 9.5 \times 10^{-3} \, g \, cm^{-1} sec^{-1}$

Summary of results:

	0°C	100°C
Observed viscosity (cp)	1.787	0.2821
Predicted by Eq. 1.5-10	48.4	2.97
Predicted by Eq. 1.5-12	4.0	0.95

Both equations give poor predictions. This is not surprising inasmuch as the empirical formula in Eqs. 1.5-9 *et seq.* do not hold for water, nor for most associated substances.

1.E Molecular Velocity and Mean Free Path

From Eq. 1.4-1 the mean speed of a molecule is

$$\bar{u} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8(8.314 \times 10^7)(273.2)}{\pi (32.00)}} = 4.25 \times 10^4 \text{ cm sec}^{-1}$$

From Eq. 1.4-3 the mean free path is:

$$\lambda = \frac{RT}{\sqrt{2}\,\pi d^2 p \tilde{N}} = \frac{(82.057)(273.2)}{\sqrt{2}\,\pi (3 \times 10^{-8})^2 (1)(6.023 \times 10^{23})} = 9.3 \times 10^{-6} \text{ cm}$$

Hence the mean free path is $(9.3 \times 10^{-6})/(3 \times 10^{-8}) = 310$ molecular diameters under these conditions. In the liquid state, on the other hand, the corresponding ratio would be of the order of magnitude of, or even less than, unity.

1.F Comparison of the Uyehara-Watson Chart with Kinetic Theory

(a) Combination of Eqs. 1.4-11 and 18 gives.

$$\frac{\mu \sigma^2}{\sqrt{M T_c}} = 2.6693 \times 10^{-5} \frac{\sqrt{T_r}}{\Omega_\mu}$$

In addition, from Eq. 1.4-11 we know that $T_r = 0.77 \, \kappa T/\epsilon$
The calculations for the kinetic theory plot are summarized below:

$\kappa T/\epsilon$	T_r	Ω_μ from Table B-2	Ordinate = $2.6693\sqrt{T_r}/\Omega_\mu$	μ_r from Fig. 1-3-1
0.3	0.231	2.785	0.461	—
0.4	0.308	2.492	0.595	—
0.5	0.385	2.257	0.735	—
0.8	0.616	1.780	1.178	0.275
1.0	0.770	1.587	1.492	0.342
1.5	1.155	1.314	2.185	0.515
2.0	1.54	1.175	2.82	0.69
3.0	2.31	1.039	3.90	0.96
5.0	3.85	0.927	5.65	1.38
8.0	6.16	0.854	7.76	1.88
10.0	7.70	0.824	9.00	2.16

As may be seen in the figure both the kinetic theory plot and the Uyehara-Watson plot predict essentially the same temperature dependence of viscosity, as is seen by the similar shapes of the two curves.

(b) Using an average value for the ratio of the ordinates of the two curves, we get:

$$\frac{\mu\sigma^2 M^{-1/2} T_c^{-1/2}}{\mu/\mu_c} \doteq 4.2 \times 10^{-5}$$

where μ [=] g cm^{-1} sec^{-1}, σ [=] Å, and T_c [=] °K. Solving for μ_c in micropoises we get:

$$\mu_c \doteq 42 \ M^{1/2} T_c^{1/2} \sigma^{-2}$$

(c) Estimation of σ by Eq. 1.4-12 gives:

$$\mu_c \doteq 42 \ M^{1/2} \ T_c^{1/2} \ [0.841 \ \tilde{V}_c^{1/3}]^{-2} = 59.4 \ M^{1/2} T_c^{1/2} \tilde{V}_c^{-2/3}$$

Eq. 1.3-2 gives a coefficent of 61.6 but is otherwise identical.

Estimation of σ by Eq. 1.4-13 gives:

$$\mu_c = 42 \ M^{1/2} \ T_c^{1/2} \ [2.44 \ (T_c/p_c)^{1/3}]^{-2}$$

$$= 7.06 \ M^{1/2} \ p_c^{2/3} \ T_c^{-1/6}$$

This should be compared with Eq. 1.3-3 in which the numerical coefficient is 7.70. In view of the approximate nature of Eqs. 1.4-11, 12, 13, Uyehara and Watson's relations are to be preferred.

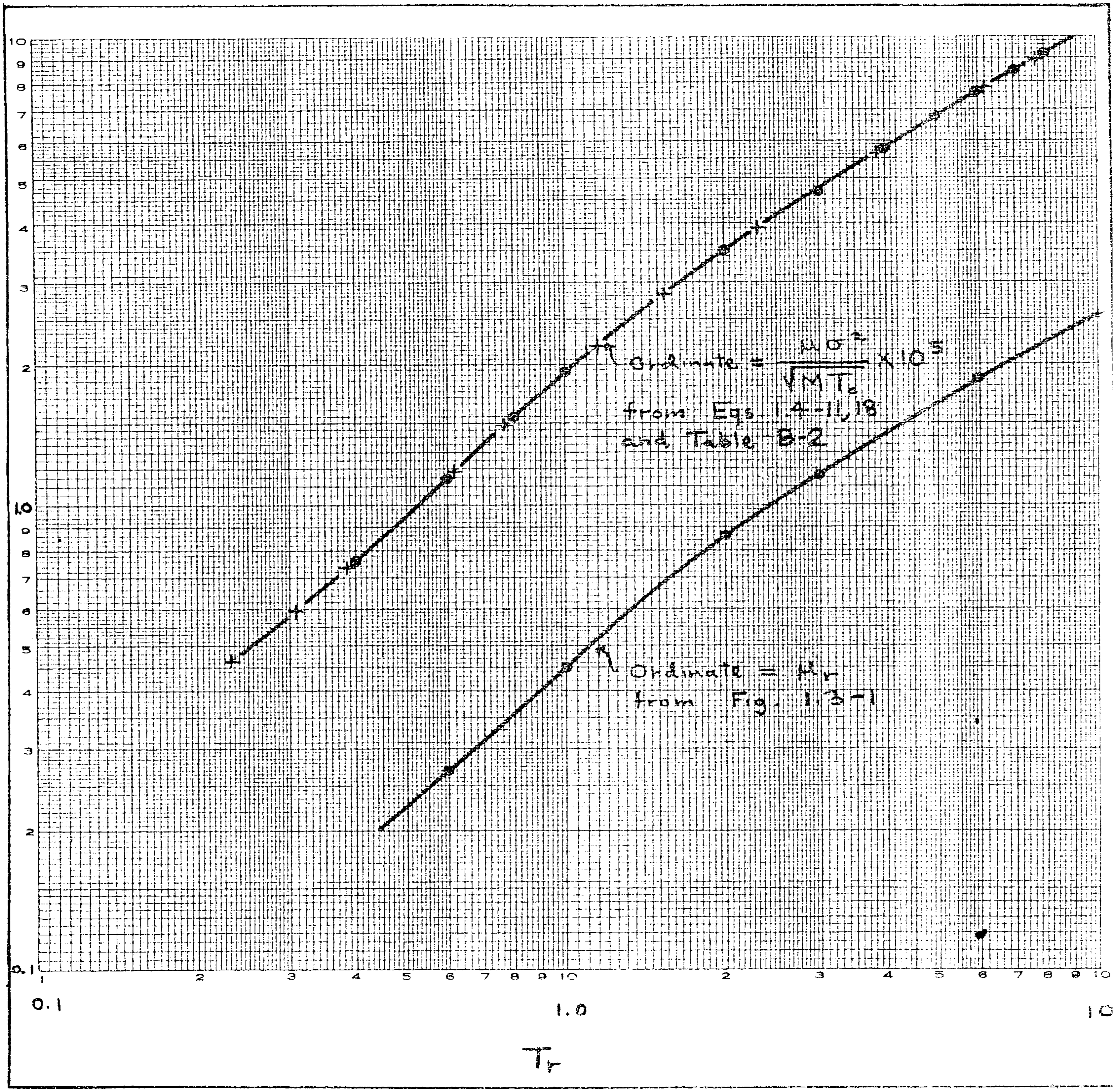


Figure for Problem 1.F

1.G Comparison of the Simple Kinetic Theory with the Exact Theory for Rigid Spheres

When Eq. 1.4-9 is written in terms of the units used in Eq. 1.4-18, and d is assumed to be the same as σ we get:

$$\mu = \frac{2}{3\pi^{3/2}} \frac{\sqrt{(M/\tilde{N})\kappa T}}{d^2}$$

$$= \frac{2}{3\pi^{3/2}} \left(\frac{1.3805 \times 10^{-16}}{6.023 \times 10^{23}} \right)^{1/2} \frac{\sqrt{MT}}{(10^{-8}\ d)^2}$$

$$= 1.813 \times 10^{-5} \frac{\sqrt{MT}}{\sigma^2}$$

Eq. 1.4-18 reduces to the same form as that above (by setting Ω_μ equal to unity) but the numerical constant is 2.6693×10^{-5}. Hence the simple kinetic theory is low by about 32% for rigid spheres.

CHAPTER 2 — Checked by V. D Shah

2.A Determination of Capillary Radius by Flow Measurements

Solve the Hagen-Poiseuille formula for R to get

$$R = \sqrt[4]{\frac{8\mu L Q}{\pi \Delta p}} = \sqrt[4]{\frac{8\nu L w}{\pi \Delta p}}$$

in which $\nu = \mu/\rho$ and $w = \rho Q$. Using mks units we substitute into this formula to get:

$$R = \sqrt[4]{\frac{8(4.03 \times 10^{-5})(0.5002)(2.997 \times 10^{-3})}{(3.1416)(4.829 \times 10^{5})}}$$

$$= \sqrt[4]{3.185 \times 10^{-13}} = \underline{7.51 \times 10^{-4}\ \text{m}} \quad \text{or} \quad \underline{0.751\ \text{mm}}.$$

The Reynolds number for the system is: (mks units used here)

$$Re = \frac{D\langle v\rangle\rho}{\mu} = \frac{2}{\pi}\left(\frac{w}{R\nu\rho}\right) = \frac{2}{\pi} \frac{(2.997 \times 10^{-3})}{(7.512 \times 10^{-4})(4.03 \times 10^{-5})(0.9552 \times 10^{3})}$$

Hence Re = 66.0, and the flow is laminar (this justifies use of the Hagen-Poiseuille law. Inasmuch as $L_e = 0.035\ D\,Re = 0.35$ cm., allowance for the end effects would not change R by more than a factor of:

$$\sqrt[4]{\frac{L - L_e}{L}} = 0.998$$

2.B Volume Rate of Flow through an Annulus

We use Eq. 2.4-16 in which

$$\kappa = \frac{0.495}{1.1} = 0.45$$

$$\mu = (136.8\ \mathrm{lb_m\ ft^{-1}\ hr^{-1}})\left(\frac{1}{3600}\ \frac{\mathrm{hr}}{\mathrm{sec}}\right) = 3.80 \times 10^{-2}\ \mathrm{lb_m\ ft^{-1}\ sec^{-1}}$$

$$(\mathcal{P}_0 - \mathcal{P}_L) = (5.37\ \mathrm{lb_f\ in^{-2}})(32.17\ \mathrm{poundals\ lb_f^{-1}})(144\ \mathrm{in^2\ ft^{-2}}) = 2.44 \times 10^4\ \mathrm{poundals\ ft^{-2}}$$

$$R = (1.1\ \mathrm{in})\left(\tfrac{1}{12}\ \mathrm{ft\ in^{-1}}\right) = 0.0917\ \mathrm{ft}$$

Then substitution into Eq. 2.4-16 gives:

$$Q = \frac{\pi\ (2.44 \times 10^4\ \mathrm{lb_m\ ft^{-1}\ sec^{-2}})(0.0917\ \mathrm{ft})^4\ (0.163)}{8\ (3.8 \times 10^{-2}\ \mathrm{lb_m\ ft^{-1}\ sec^{-1}})\ (27\ \mathrm{ft})} = 0.108\ \mathrm{ft^3\ sec^{-1}}$$

To verify that it was proper to use the laminar flow formula of Eq. 2.4-16 we compute the Reynolds number:

$$Re = \frac{2R\,(1-\kappa)\,\langle v_z \rangle \rho}{\mu} = \frac{2}{\pi}\left(\frac{Q\rho}{R\mu}\right)\left(\frac{1}{1+\kappa}\right)$$

$$= \frac{2}{\pi}\ \frac{(0.108)(80.3)}{(0.0917)(3.8 \times 10^{-2})(1.45)} = 1090$$

2.C Loss of Catalyst Particles in a Stack Gas

(a) Rearrangement of Eq. 2.6-15 gives:

$$v_t = D^2(\rho_s - \rho)\,g\,/\,18\,\mu$$

in which D is the sphere diameter. When v_t is larger than 1.0 ft sec^{-1} (vertically downward) then the particle will not go up the stack. Hence we want to find that value of D for which v_t = 1.0 ft sec^{-1}. This will be the maximum diameter of particles that can be lost.

Using cgs units we get:

$$v_t = (1\ \text{ft sec}^{-1})(12\ \text{in ft}^{-1})(2.54\ \text{cm in}^{-1}) = 30.5\ \text{cm sec}^{-1}$$

$$\rho = (0.045\ \text{lb}_m\ \text{ft}^{-3})(454\ \text{g lb}_m^{-1})(12 \times 2.54)^{-3}(\text{ft}^3\ \text{cm}^{-3})$$

$$= 7.2 \times 10^{-4}\ \text{g cm}^{-3}$$

Therefore:

$$D_{max} = \sqrt{\frac{18\,\mu\, v_t}{(\rho_s - \rho)\,g}} = \sqrt{\frac{(18)(0.00026)(30.5)}{(1.2 - 7.2\times10^{-4})(981)}}$$

$$= \sqrt{1.21 \times 10^{-4}} = 1.1 \times 10^{-2}\ \text{cm} = 110\ \text{microns}$$

(b) Eq. 2.6–16 is of course valid only for $Re < 0.1$ although it may be applied approximately up to about $Re = 1$. For the system at hand

$$Re = \frac{D v_t \rho}{\mu} = \frac{(1.1 \times 10^{-2})(30.5)(7.2 \times 10^{-4})}{(0.00026)} = 0.93$$

In Chapter 6, methods are given for handling flow around spheres for $Re > 1$.

2.D Flow of Falling Film--- Alternate Derivations

(a) Set up a momentum balance as before and obtain the differential equation:

$$\frac{d\tau_{\bar{x}z}}{d\bar{x}} = \rho g \cos\beta$$

which may be integrated to give:

$$\tau_{\bar{x}z} = \rho g \bar{x} \cos\beta + C_1$$

Inasmuch as no momentum is transferred at $\bar{x} = \delta$, then at that plane we have $\tau_{\bar{x}z} = 0$. This boundary condition enables us to determine C_1 to be:

$$C_1 = -\rho g \delta \cos\beta$$

and hence the momentum flux distribution is:

$$\tau_{\bar{x}z} = -\rho g \delta \cos\beta\ [1 - (\bar{x}/\delta)]$$

Note that the momentum flux is in the negative $\bar{x}$-direction.

Insertion of Newton's law of viscosity $\tau_{\bar{x}z} = -\mu\,(dv_z/d\bar{x})$ into the above expression then gives the differential equation for the velocity distribution:

$$\frac{dv_z}{d\bar{x}} = \left(\frac{\rho g \delta}{\mu}\right) \cos\beta \left(1 - \frac{\bar{x}}{\delta}\right)$$

This first-order differential equation is easily integrated to give:

$$v_z = \left(\frac{\rho g \delta}{\mu}\right)^2 \cos\beta \left(\frac{\bar{x}}{\delta} - \frac{1}{2}\left(\frac{\bar{x}}{\delta}\right)^2\right)$$

the constant of integration, C_2, being zero because $v_z = 0$ at $\bar{x} = 0$.

Now we note that $\bar{x}$ and x are related thus:

$$\left(\frac{\bar{x}}{\delta}\right) = 1 - \left(\frac{x}{\delta}\right)$$

where x is the coordinate used in § 2.2. If the above relation is substituted into the velocity distribution we get:

$$v_z = \left(\frac{\rho g \delta}{\mu}\right)^2 \cos\beta \left[1 - \frac{x}{\delta} - \frac{1}{2}\left\{1 - 2\frac{x}{\delta} + \left(\frac{x}{\delta}\right)^2\right\}\right]$$

which, upon simplification, becomes:

$$v_z = \left(\frac{\rho g \delta}{2\mu}\right)^2 \cos\beta \left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

which is the same as Eq. 2.2-16. This just illustrates that the choice of coordinate system makes no difference in the final answer.

(b) Substitution of Eq. 2.2-12 into Eq. 2.2-8 gives <u>for constant μ</u>

$$\frac{d^2 v_z}{dx^2} = \left(\frac{\rho g}{\mu}\right) \cos\beta$$

Integration twice gives:

$$v_z = \left(\frac{\rho g}{\mu}\right) \cos\beta \quad \frac{1}{2}x^2 + C_1 x + C_2$$

Application of the boundary conditions

B.C. 1 At $x = 0$, $dv_z/dx = 0$
B.C 2 At $x = \delta$, $v_z = 0$

gives two simultaneous equations for the integration constants C_1 and C_2:

$$\begin{cases} 0 = (\rho g/\mu)(\cos\beta)\cdot 0 + C_1 \\ 0 = (\rho g/\mu)(\cos\beta)\cdot \frac{1}{2}\delta^2 + C_1\delta + C_2 \end{cases}$$

whence $C_1 = 0$ and $C_2 = \frac{1}{2}(\rho g \delta^2/\mu)\cos\beta$. Insertion of these values into the last expression for v_z then gives finally:

$$v_z = (\rho g \delta^2/2\mu)(\cos\beta)\left[1 - (x/\delta)^2\right]$$

which is the same as Eq. 2.2-16.

2.E Laminar Flow in a Narrow Slit

A differential momentum balance leads to:

$$\frac{d\tau_{xz}}{dx} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L}$$

Then, insertion of Newton's law gives

$$\frac{d^2 v_z}{dx^2} = -\frac{(\mathcal{P}_0 - \mathcal{P}_L)}{\mu L}$$

Two successive integrations then yield:

$$v_z = -\frac{(\mathcal{P}_0 - \mathcal{P}_L)}{2\mu L}x^2 + C_1 x + C_2$$

Use of the boundary conditions that $v_z = 0$ at $x = \pm B$ then gives the values of C_1 and C_2; when these are inserted into the last equation we get for the velocity distribution:

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L}\left[1 - \left(\frac{x}{B}\right)^2\right]$$

Once the velocity profiles are known we can get various derived quantities:

The momentum flux distribution is: $\tau_{xz} = -\mu\frac{dv_z}{dx} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L}x$

The average velocity is:

$$\langle v_z\rangle = \frac{\int_0^W\int_0^B v_z\,dx\,dz}{\int_0^W\int_0^B dx\,dz} = \frac{1}{B}\int_0^B v_z\,dx$$

$$= \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L}\int_0^1\left[1 - \left(\frac{x}{B}\right)^2\right]d\left(\frac{x}{B}\right)$$

$$= (\mathcal{P}_0 - \mathcal{P}_L)B^2/3\mu L$$

The maximum velocity is: $v_{z,max} = \dfrac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L}$

Ratio of avg. to max. velocity: $\langle v_z \rangle / v_{z,max} = \dfrac{2}{3}$

Volume rate of flow: $Q = BW\langle v_z \rangle = \dfrac{2}{3}\dfrac{(\mathcal{P}_0 - \mathcal{P}_L)B^3 W}{\mu L}$

The latter result is the analog of the Hagen-Poiseuille law.

2.F Interrelation of Slit and Annulus Formulas

Substitution of $\kappa = 1-\epsilon$ into Eq. 2.4-16 gives:

$$Q = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R^4}{8\mu L}\left[\begin{array}{l}(1 - 1 + 4\epsilon - 6\epsilon^2 + 4\epsilon^3 - \epsilon^4) \\ + \dfrac{(1-1+2\epsilon-\epsilon^2)^2}{-\epsilon - \frac{1}{2}\epsilon^2 - \frac{1}{3}\epsilon^3 - \frac{1}{4}\epsilon^4 - \cdots}\end{array}\right]$$

$$= \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R^4}{8\mu L}\left[\begin{array}{l}4\epsilon - 6\epsilon^2 + 4\epsilon^3 - \epsilon^4 \\ - \dfrac{4\epsilon^2 - 4\epsilon^3 + \epsilon^4}{\epsilon + \frac{1}{2}\epsilon^2 + \frac{1}{3}\epsilon^3 + \cdots}\end{array}\right]$$

When the indicated long division is performed we then get:

$$Q = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R^4}{8\mu L}\left[\begin{array}{l}\cancel{4\epsilon} - \cancel{6\epsilon^2} + 4\epsilon^3 - \epsilon^4 \\ - \cancel{4\epsilon} + \cancel{6\epsilon^2} - \frac{8}{3}\epsilon^3 - \frac{2}{3}\epsilon^4 + \cdots\end{array}\right]$$

or

$$Q = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R^4\epsilon^3}{6\mu L}\left[1 - \frac{5}{4}\epsilon + \cdots\right]$$

If $\epsilon \ll 1$, then the term $\frac{5}{4}\epsilon$ and higher terms may be neglected.

2.G Laminar Flow of a Falling Film on Outside of a Circular Tube

(a) We set up a momentum balance over a thin shell of thickness Δr:

$$\{\text{MOMENTUM IN}\} - \{\text{MOMENTUM OUT}\} + \{\text{FORCE}\} = 0$$

$$(2\pi r L \tau_{rz})\Big|_r - (2\pi r L \tau_{rz})\Big|_{r+\Delta r} + 2\pi L \Delta r \rho g = 0$$

Divide by $2\pi L \Delta r$ and take the limit as $\Delta r \to 0$

$$-\frac{d}{dr}(r\tau_{rz}) + r\rho g = 0$$

Substitution of Newton's law into this equation gives: (for constant μ)

$$+\mu \frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = -\rho g r$$

Two integrations then give:

$$v_z = -\frac{\rho g r^2}{4\mu} + C_1 \ln r + C_2$$

Application of the boundary conditions:

B.C. 1: At $r = R$, $v_z = 0$

B.C. 2: At $r = aR$, $dv_z/dr = 0$

gives the following two equations for C_1 and C_2

B.C 1: $0 = -\frac{\rho g R^2}{4\mu} + C_1 \ln R + C_2$

B.C. 2: $0 = -\frac{\rho g a R}{2\mu} + \frac{C_1}{aR}$

These may be solved for C_1 and C_2, and the results substituted into the expression for v_z above; hence we finally get:

$$v_z = \frac{\rho g R^2}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2 + 2a^2 \ln \frac{r}{R}\right]$$

(b) The volume rate of flow is:

$$Q = \int_0^{2\pi}\int_R^{aR} v_z \, r \, dr \, d\theta = 2\pi R^2 \int_1^a v_z \, \xi \, d\xi \quad \text{where } \xi = \frac{r}{R}$$

Substitution of the velocity profile into this integral gives:

$$Q = \frac{\pi \rho g R^4}{2\mu} \int_1^a (\xi - \xi^3 + 2a^2 \xi \ln \xi)\, d\xi$$

$$= \frac{\pi \rho g R^4}{2\mu} \left(\frac{\xi^2}{2} - \frac{\xi^4}{4} + 2a^2 \left[-\frac{1}{4}\xi^2 + \frac{1}{2}\xi^2 \ln \xi \right] \right) \Bigg|_1^a$$

$$= \frac{\pi \rho g R^4}{8\mu} \left(-1 + 4a^2 - 3a^4 + 4a^4 \ln a \right)$$

(c) If we set $a = 1 + \epsilon$ (where ϵ is small) and expand in powers of ϵ, we get:

$$Q = \frac{\pi \rho g R^4}{8\mu} \left(\frac{16}{3} \epsilon^3 + \text{higher powers of } \epsilon \right)$$

or

$$Q \approx \frac{2\pi \rho g R^4 \epsilon^3}{3\mu}$$

This is in agreement with Eq. 2.2-19 with $W = 2\pi R$, $\delta = \epsilon R$, and $\cos\beta = 1$.

2.H Non-Newtonian Flow in a Tube

(a) Combination of Eq. 2.H-1 and Eq. 2.3-12 gives:

$$m\left(-\frac{dv_z}{dr}\right)^n = \frac{(\mathcal{P}_0 - \mathcal{P}_L)\, r}{2L}$$

Let $s = 1/n$; then:

$$-\frac{dv_z}{dr} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2mL} \right)^s r^s$$

Integration gives:

$$-v_z = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2mL} \right)^s \left[\frac{r^{s+1}}{s+1} + C \right]$$

The boundary condition of zero velocity at the wall allows one to evaluate C, with the result that:

$$v_z = \left(\frac{(\mathcal{P}_0 - \mathcal{P}_L) R}{2mL} \right)^s \frac{R}{s+1} \left[1 - \left(\frac{r}{R}\right)^{s+1} \right]$$

The volume rate of flow is then:

$$Q = 2\pi \left(\frac{(\mathcal{P}_0 - \mathcal{P}_L) R}{2mL} \right)^s \frac{R^3}{s+1} \int_0^1 \left[1 - \left(\frac{r}{R}\right)^{s+1} \right] \left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

Hence, when the integration is performed:

$$Q = \pi \left[\frac{(\mathcal{P}_0 - \mathcal{P}_L) R}{2mL} \right]^s \frac{R^3}{s+3}$$

Note that when $s = 1$ and $m = \mu$, this becomes the Hagen-Poiseuille formula

(b) For the Ellis fluid the velocity distribution is:

$$v_z = \frac{\varphi_0 (\mathcal{P}_0 - \mathcal{P}_L) R^2}{4L} \left[1 - \left(\frac{r}{R}\right)^2 \right] + \varphi_1 \left[\frac{(\mathcal{P}_0 - \mathcal{P}_L) R}{2L} \right]^\alpha \frac{R}{\alpha+1} \left[1 - \left(\frac{r}{R}\right)^{\alpha+1} \right]$$

and the volume rate of flow is:

$$Q = \frac{\pi (\mathcal{P}_0 - \mathcal{P}_L) R^4 \varphi_0}{8L} + \pi \left[\frac{(\mathcal{P}_0 - \mathcal{P}_L) R}{2L} \right]^\alpha \frac{R^3 \varphi_1}{\alpha+3}$$

These results may be obtained by the same process described in (a).

2.I Flow of a Bingham Fluid from a Circular Tube

There will be flow only if the greatest value of the momentum flux τ_{rz} exceeds the value of τ_0 characterizing the Bingham fluid. The greatest value of τ_{rz} occurs at the wall and is $\rho g R/2$. Hence:

If $\rho g R/2 < \tau_0$ fluid <u>will not</u> flow

If $\rho g R/2 > \tau_0$ fluid <u>will</u> flow

2.J. Annular Flow with Inner Cylinder Moving Axially

A shell momentum balance leads to the differential equation

$$\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) = 0$$

This has to be integrated with the boundary conditions

B.C. 1 : At $r = \kappa R$, $v_z = V$

B.C. 2 : At $r = R$, $v_z = 0$

When the integration is performed and the two integration constants are evaluated from the above two boundary conditions, we get:

$$\frac{v_z}{V} = \frac{\ln(r/R)}{\ln \kappa}$$

Then the volume rate of flow is found to be:

$$Q = 2\pi \int_{\kappa R}^{R} v_z \, r \, dr$$

$$= \frac{2\pi R^2 V}{\ln \kappa} \int_{\kappa}^{1} \ln\left(\frac{r}{R}\right)\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

$$= \frac{2\pi R^2 V}{\ln \kappa} \left[-\frac{1}{4}\xi^2 + \frac{1}{2}\xi^2 \ln \xi\right]\Big|_{\kappa}^{1}$$

$$= \frac{\pi R^2 V}{2}\left[\frac{(1-\kappa^2)}{\ln(1/\kappa)} - 2\kappa^2\right]$$

2.K Non-Newtonian Film Flow

According to §2.2 for any fluid $\tau_{xz} = \rho g x$. When the rheological equation for the Bingham fluid is inserted into this, we get:

$$\tau_0 - \mu_0 \frac{dv_z}{dx} = \rho g x$$

When this is integrated we get:

$$v_z = -\frac{\rho g}{2\mu_0}x^2 + \frac{\tau_0}{\mu_0}x + C_2$$

At $x = \delta$, $v_z = 0$ so that

$$0 = -\frac{\rho g}{2\mu_0}\delta^2 + \frac{\tau_0}{\mu}\delta + C_2$$

Subtracting these last two equations gives:

$$v_z = \frac{\rho g \delta^2}{2\mu_0}\left[1 - \left(\frac{x}{\delta}\right)^2\right] - \frac{\tau_0 \delta}{\mu_0}\left[1 - \left(\frac{x}{\delta}\right)\right] \qquad (x_0 \leq x \leq \delta)$$

for the velocity distribution in the range $x_0 \leq x \leq \delta$. The velocity in the "plug flow region" $(0 \leq x \leq x_0)$ is:

$$v_z = \frac{\rho g \delta^2}{2\mu_0}\left[1 - \frac{x_0}{\delta}\right]^2 \qquad (0 \leq x \leq x_0)$$

where $\tau_0 = \rho g x_0$ is the defining equation for x_0.

Next, we get the mass rate of flow (with W = width of film, Q = volume rate of flow):

$$\Gamma = \frac{\rho Q}{W} = \rho \int_0^\delta v_z \, dx$$

Integration by parts

$$= \rho\left[v_z x \Big|_0^\delta - \int_0^\delta x\left(\frac{dv_z}{dx}\right) dx\right]$$

$$= \rho \int_{x_0}^{\delta} x\left(-\frac{dv_z}{dx}\right) dx$$

$$= \rho \int_{x_0}^{\delta} x\left[\frac{\rho g}{\mu_0}x - \frac{\tau_0}{\mu_0}\right] dx$$

Finally

$$\Gamma = \frac{\rho^2 g \delta^3}{3\mu_0}\left[1 - \frac{3}{2}\left(\frac{x_0}{\delta}\right) + \frac{1}{2}\left(\frac{x_0}{\delta}\right)^3\right]$$

Hence δ would have to be obtained from a graphical solution, by plotting

$$\frac{3\Gamma\mu_0}{\rho^2 g \delta^3} \quad \underline{\text{vs.}} \quad \frac{x_0}{\delta}$$

CHAPTER 3 — Checked by V. D. Shah

3.A Torque Required to Turn a Friction Bearing

In Eq. 3.5-13 is given an expression for calculating the torque needed to turn an outer rotating cylinder at an angular velocity Ω_o. For the problem described in Prob. 3.A we need a similar formula for the torque need to turn an inner rotating cylinder at an angular velocity Ω_i. It should be obvious that the two expressions must be the same. If it is not one can easily show that when the inner cylinder is rotated at Ω_i and the outer held stationary, then the velocity distribution is *

$$v_\theta = \frac{\kappa R \Omega_i}{\left(\frac{1}{\kappa} - \kappa\right)} \left[\left(\frac{R}{r}\right) - \left(\frac{r}{R}\right) \right]$$

and the torque is then:

$$\mathcal{T} = (2\pi\kappa R L)\left(+ \tau_{r\theta}\Big|_{r=\kappa R} \right)(\kappa R)$$

or

$$\mathcal{T} = 4\pi L \mu \Omega_i R^2 \left(\frac{\kappa^2}{1-\kappa^2}\right)$$

which is the same in form as Eq. 3.5-13

For the situation described in Problem 3.A we evaluate the various quantities needed:

$$\kappa = \frac{\text{inner radius}}{\text{outer radius}} = \frac{1.000''}{1.002''} = 0.998 \;; \qquad \kappa^2 = 0.996$$

$$\left(\frac{\kappa^2}{1-\kappa^2}\right) = \frac{0.996}{0.004} = 249$$

These conversion factors come from Table C.3-1

$$\mu = 200\ \text{cp} = (200)(6.72\times10^{-4})\ \text{lb}_m\,\text{ft}^{-1}\,\text{sec}^{-1} = 0.1344\ \text{lb}_m\,\text{ft}^{-1}\,\text{sec}^{-1}$$

$$= (200)(2.09\times10^{-5})\ \text{lb}_f\ \text{sec}\ \text{ft}^{-2} = 4.18\times10^{-3}\ \text{lb}_f\ \text{sec}\ \text{ft}^{-}$$

$$\Omega_i = (200\ \text{rpm})\left(\frac{1}{60}\,\frac{\text{min}}{\text{sec}}\right)\left(2\pi\,\frac{\text{radians}}{\text{revolution}}\right) = \frac{20}{3}\pi\ \frac{\text{radians}}{\text{sec}}$$

$$R^2 = (1\ \text{in})^2 = \left(\tfrac{1}{12}\right)^2 \text{ft}^2 = \frac{1}{144}\ \text{ft}^2$$

$$L = 2\ \text{in} = \tfrac{1}{6}\ \text{ft}$$

---

* See Problem 3.F

Hence the torque is

$$\mathcal{T} = (4\pi)\left(\frac{1}{6}\right)\left(4.18 \times 10^{-3}\right)\left(\frac{20\pi}{3}\right)\left(\frac{1}{144}\right)(249)$$

$$= 0.32 \ \text{lb}_f \ \text{ft}$$

And the power is

$$P = \mathcal{T}\Omega = (0.32 \ \text{lb}_f \ \text{ft})\left(\frac{20}{3}\pi \ \text{sec}^{-1}\right)\left(\frac{1}{550} \ \frac{\text{hp}}{\text{ft lb}_f \ \text{sec}^{-1}}\right)$$

$$= 0.012 \ \text{hp}$$

In using the above formulae it has been tacitly assumed that the flow is stable and laminar. We must verify that this is indeed so by using Eq. 3.5-14. Hence we must check to be sure that:

$$\left(\frac{\Omega_i \kappa R^2 \rho}{\mu}\right)(1-\kappa)^{3/2} < 41.3$$

Inserting the values from this example we have:

$$\frac{\left(\frac{20\pi}{3}\right)(0.998)\left(\frac{1}{144}\right)(50)}{(0.1344)} (0.002)^{3/2} \approx 10^{-2}$$

which is well below the critical limit of 41.3.

3.B. The Cone-and-Plate Viscometer

Throughout this problem we recognize that in the conical slit $\theta \approx \pi/2$ and that $\sin\theta$ can everywhere be set equal to unity to a very good approximation; hence the expressions for torque, momentum flux, and velocity profile may be written as follows:

From Eq. 3.5-32: $\mathcal{T} \doteq \frac{2}{3}\pi R^3 \tau_{\theta\phi}$

From Eq. 3.5-33: $\tau_{\theta\phi} \doteq -\mu \frac{d}{d\theta}\left(\frac{v_\phi}{r}\right)$

From Eq. 3.5-37: $\frac{v_\phi}{r} \doteq \Omega \frac{\left(\frac{\pi}{2} - \theta\right)}{\left(\frac{\pi}{2} - \theta_1\right)}$

When these three relations are combined we get

$$\mathcal{T} \doteq \frac{2}{3}\pi R^3 \mu \Omega \Big/ \left(\frac{\pi}{2} - \theta_1\right)$$

which is a very good approximation for very tiny cone angles. [This same result is given by S.Oka in Vol. III of Eirich's "Rheology" (Academic Press, 1960)—see p. 62.]

For the situation at hand

$$R = 10 \text{ cm}$$

$$\mu = 100 \text{ cp} = 1 \text{ g cm}^{-1} \text{sec}^{-1}$$

$$\Omega = 10 \text{ radians min}^{-1} = \frac{10}{60} \frac{\text{radians}}{\text{sec}} = \frac{1}{6} \frac{\text{radians}}{\text{sec}}$$

$$\frac{\pi}{2} - \theta_1 = 0.5^\circ = \frac{\pi}{360} \text{ radians}$$

Hence the torque is:

$$\mathcal{T} \doteq \frac{2}{3}\pi (10)^3 (1)\left(\frac{1}{6}\right) \Big/ \frac{\pi}{360} = 4 \times 10^4 \text{ dyn-cm}$$

3.C The Effect of Altitude on Air Pressure

If we assume a stationary atmosphere—i.e., no wind currents— then the equation of motion is:

$$\frac{dp}{dz} = \rho g \qquad (z \text{ is measured from level of Lake Superior})$$

If it be assumed that the ideal gas law is applicable, then

$$\rho = \frac{pM}{RT}$$

From the given temperature data the temperature in °R is:

$$T(z) = 530 - (3 \times 10^{-3})z$$

Then the equation of motion may be integrated:

$$\int_{750}^{p} dp = \int_{0}^{1421} \rho g \, dz = \frac{Mg}{R} \int_{0}^{1421} \frac{dz}{530 - (3\times10^{-3})z}$$

When the integrations are performed, we get:

$$\ln(p/750) = \left(\frac{Mg}{R}\right)\left(\frac{1}{3\times10^{-3}}\right) \ln \frac{530 - (3\times10^{-3})(0)}{530 - (3\times10^{-3})(1421)}$$

$$= \left(\frac{Mg}{R}\right)\left(\frac{-1}{3\times10^{-3}}\right) \ln \left[1 - \frac{(3)(1.421)}{530}\right]$$

$$\doteq \left(\frac{Mg}{R}\right)\left(\frac{1}{3\times10^{-3}}\right)\left(\frac{(3\times10^{-3})(1421)}{530}\right)$$

use series $\ln(1-x) = -(x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots)$

$$= \left(\frac{Mg}{R}\right)\left(\frac{1421}{530}\right)$$

Now insert the values

$g = -32.19 \text{ ft sec}^{-2}$

$M = 29 \text{ lb}_m \text{ (lb-mole)}^{-1}$

$R = 1545 \text{ ft lb}_f \text{ (lb-mole)}^{-1} (^\circ R)^{-1}$

and obtain:

$$\frac{p}{750} = 0.951$$

whence $p = \underline{712 \text{ mm Hg}}$ at $z = 1421$ ft above Lake Superior

3.D. Viscosity Determination with a Couette-Hatschek Viscometer

Here it will be desirable to use a sufficiently high torque that accuracy of viscosity determinations is primarily limited by the error in measuring angular velocity. A torque of 10^4 dyn-cm, corresponding to a torque uncertainty of 1 %, appears reasonable, if this corresponds to a Reynolds number in the stable laminar range.

The angular velocity corresponding to this torque is:

$$\Omega_0 = \frac{\mathcal{T}(1-\kappa^2)}{4\pi\kappa^2\mu L R^2} = \frac{(10^4)(1-0.79)}{4\pi(0.79)(0.57)(4)(20.25)} = 4.58 \ \frac{\text{radians}}{\text{sec}}$$

The Reynolds number for this case is:

$$Re = \frac{\Omega_0 R^2 \rho}{\mu} = \frac{(4.58)(20.25)(1.28)}{(0.57)} = 208$$

According to Fig. 3.5-2 this is well within the stable, laminar range, and therefore a torque of 10^4 dyne-cm is acceptable.

3.E Use of Navier-Stokes Equations to Set-Up Simple Problems

Problem	Start with equations given in:	Assumptions	Results
(a) Flow of isothermal film	Continuity: Table 3.4-1 (A) Motion: Table 3.4-2 (D,E,F)	$\frac{\partial}{\partial t}=0$ $v_x=v_y=0$ $v_z=v_z(x)$ $p=$ const. $g_z=g\cos\beta$	$0=\mu\frac{d^2v_z}{dx^2}+\rho g\cos\beta$
(b) Two-phase flow in a horizontal slit	Continuity: Table 3.4-1 (A) Motion: Table 3.4-2 (D,E,F)	$\frac{\partial}{\partial t}=0$ $v_x=v_y=0$ $v_z=v_z(x)$ $p=$ constant $\vec{g}=0$	$0=\mu\frac{d^2v_z}{dx^2}$ [in both phases]
(c) Axial annular flow	Continuity: Table 3.4-1 (A) Motion: Table 3.4-3 (D,E,F)	$\frac{\partial}{\partial t}=0$ $v_\theta=v_r=0$ $v_z=v_z(r)$ $p=$ constant $\vec{g}=0$	$0=\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)$

Note that in each of these cases the equation of continuity is automatically satisfied by the assumptions in the 3rd column.

3.F Velocity Distribution in a Stormer Viscometer

From Table 3.4-3 (D, E, F), assuming that $v_\theta=v_\theta(r)$ and $v_r=v_z=0$, the θ-component of the equation of motion becomes:

$$0=\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right)$$

When this is integrated we get (by "peeling off" the $\frac{d}{dr}$'s one at a time):

$$v_\theta = C_1 r + \frac{C_2}{r}$$

When the boundary conditions

$$\begin{cases} \text{At } r = \kappa R, & v_\theta = \kappa R \Omega_i \\ \text{At } r = R, & v_\theta = 0 \end{cases}$$

are used, C_1 and C_2 are evaluated to give:

$$v_\theta = \frac{\kappa R \Omega_i}{\left(\frac{1}{\kappa} - \kappa\right)} \left[\frac{R}{r} - \frac{r}{R} \right]$$

or

$$\left(\frac{v_\theta}{r}\right) = \frac{\kappa R^2 \Omega_i}{\left(\frac{1}{\kappa} - \kappa\right)} \left[\frac{1}{r^2} - \frac{1}{R^2} \right]$$

We may now use Eq.(D) of Table 3.4-6 to get the torque as follows:

$$\mathcal{T} = \underset{[\text{area}]}{(2\pi \kappa R L)} \cdot \underset{[\text{force per unit area}]}{\left(+\tau_{r\theta}\right)\Big|_{r=\kappa R}} \cdot \underset{[\text{lever arm}]}{(\kappa R)}$$

$$= \left(2\pi \kappa^2 R^2 L\right) \left(- \mu r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r}\right) \Big|_{r=\kappa R} \right)$$

Then use of the velocity distribution above gives:

$$\mathcal{T} = 4\pi L \mu \Omega_i R^2 \left(\frac{\kappa^2}{1-\kappa^2}\right)$$

When this is solved for Ω_i and inserted into the velocity distribution we get:

$$\left(\frac{v_\theta}{r}\right) = \frac{\mathcal{T}}{4\pi\mu L} \left(\frac{1}{r^2} - \frac{1}{R^2} \right)$$

which is the expression given in the text. Note that v_θ / r is the angular velocity distribution.

An alternate solution may be effected by starting with Eq.(B) of Table 3.4-3 and integrating once, getting $\mathcal{T}$ into the momentum flux expression by application of the boundary condition at $r = \kappa R$.

3.G Equation of Motion for an Incompressible Fluid

(i) First compute the various derivatives of the components of $\underset{\sim}{\tau}$:

$$\frac{\partial}{\partial x}(\tau_{xx}) = -2\mu \frac{\partial}{\partial x}\frac{\partial v_x}{\partial x} + 0 = -\mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial}{\partial x}\frac{\partial v_x}{\partial x}\right)$$

$$\frac{\partial}{\partial y}(\tau_{yx}) = -\mu \frac{\partial}{\partial y}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) = -\mu\left(\frac{\partial^2 v_x}{\partial y^2} + \frac{\partial}{\partial x}\frac{\partial v_y}{\partial y}\right)$$

$$\frac{\partial}{\partial z}(\tau_{zx}) = -\mu \frac{\partial}{\partial z}\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right) = -\mu\left(\frac{\partial^2 v_x}{\partial z^2} + \frac{\partial}{\partial x}\frac{\partial v_z}{\partial z}\right)$$

(ii) Then add the various contributions

$$\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx} = -\mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right)$$

$$-\mu\frac{\partial}{\partial x}\left(\underbrace{\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}}\right)$$

This collection of derivatives is just $(\nabla \cdot \vec{v})$ and is zero for a fluid of constant ρ.

3.H Velocity Distribution between Two Rotating Cylinders

This problem is set up exactly as 3.F. The velocity distribution is $v_\theta = C_1 r + C_2 r^{-1}$ as before, but now the constants C_1 and C_2 are determined from the boundary conditions:

At $r = \kappa R$, $v_\theta = \kappa R \Omega_i$ [at inner wall]
At $r = R$, $v_\theta = R\Omega_o$ [at outer wall]

Both Ω_i and Ω_o are considered to be positive in the counterclockwise (positive θ) direction. Solution for the constants gives:

$$C_1 = \frac{(\kappa R)^2\Omega_i - R^2\Omega_o}{(\kappa R)^2 - R^2} \qquad C_2 = \frac{\kappa^2 R^4 (\Omega_o - \Omega_i)}{(\kappa R)^2 - R^2}$$

and hence, finally:

$$v_\theta = \frac{1}{R^2(1-\kappa^2)}\left[R^2 r\left(\Omega_o - \kappa^2\Omega_i\right) - \frac{\kappa^2 R^4}{r}\left(\Omega_o - \Omega_i\right)\right]$$

3.I Changing the Form of the Equation of Motion

(i) The equivalence of Eqs. 3.2-8 and 3.2-10 may easily be shown by showing that their x-, y-, and z-components are the same. For the x-components this corresponds to demonstrating the equivalence of Eqs. 3.2-5 and 3.2-9.

We begin by performing the differentiations of products in Eq. 3.2-5:

$$\rho \frac{\partial}{\partial t} v_x + v_x \frac{\partial \rho}{\partial t} = -\left[\begin{array}{ll} \underline{v_x \frac{\partial}{\partial x} \rho v_x} & + \underset{\sim\sim\sim}{\rho v_x \frac{\partial v_x}{\partial x}} \\ + \underline{v_x \frac{\partial}{\partial y} \rho v_y} & + \underset{\sim\sim\sim}{\rho v_y \frac{\partial v_x}{\partial y}} \\ + \underline{v_x \frac{\partial}{\partial z} \rho v_z} & + \underset{\sim\sim\sim}{\rho v_z \frac{\partial v_x}{\partial z}} \end{array}\right] + \cdots$$

where $\cdots$ means the additional terms which are common to both equations. Note that the ---- underlined terms are:

$$v_x \left(\frac{\partial}{\partial x} \rho v_x + \frac{\partial}{\partial y} \rho v_y + \frac{\partial}{\partial z} \rho v_z \right) = v_x \left(- \frac{\partial \rho}{\partial t} \right)$$

where the equation of continuity has been used to get the right side. Note that the $\sim\sim\sim$ underlined terms are:

$$\left(\rho v_x \frac{\partial}{\partial x} + \rho v_y \frac{\partial}{\partial y} + \rho v_z \frac{\partial}{\partial z} \right) v_x = \rho \frac{D}{Dt} v_x - \rho \frac{\partial}{\partial t} v_x$$

where the definition of the substantial derivative has been used. Substitution of these expressions into the equation of motion above gives:

$$\rho \frac{D v_x}{Dt} = \cdots$$

which is Eq. 3.2-9.

(ii) Alternatively one can use some of the vector-tensor formulae of the Appendix A. We note that Eq. A.4-30 allows us to write (by setting $\underset{\sim}{w}$ equal to $\underset{\sim}{v}$ and $\underset{\sim}{v}$ equal to $\rho \underset{\sim}{v}$)

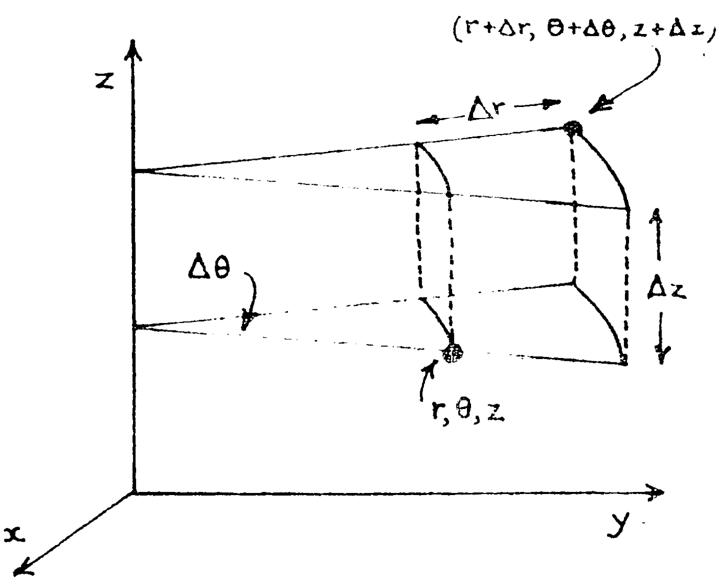
$$[\nabla \cdot \rho \underset{\sim}{v}\underset{\sim}{v}] = [\rho \underset{\sim}{v} \cdot \nabla \underset{\sim}{v}] + \underset{\sim}{v} (\nabla \cdot \rho \underset{\sim}{v})$$

Then using the equation of continuity we get:

$$\begin{aligned} [\nabla \cdot \rho \underset{\sim}{v}\underset{\sim}{v}] &= \rho [\underset{\sim}{v} \cdot \nabla \underset{\sim}{v}] - \underset{\sim}{v} \frac{\partial \rho}{\partial t} \\ &= \rho [\underset{\sim}{v} \cdot \nabla \underset{\sim}{v}] - \frac{\partial}{\partial t} \rho \underset{\sim}{v} + \rho \frac{\partial \underset{\sim}{v}}{\partial t} \Big) \swarrow \quad \text{Eq. A-3-17} \\ &= \rho \frac{D \underset{\sim}{v}}{Dt} - \frac{\partial}{\partial t} \rho \underset{\sim}{v} \end{aligned}$$

Insertion of this result into Eq. 3.2-8 then gives Eq. 3.2-10.

3. J. The Equation of Continuity in Cylindrical Coördinates



(a) Consider a volume element fixed in space as shown in the accompanying diagram. The volume of this element is approximately equal to $r\,\Delta\theta\,\Delta r\,\Delta z$.

The rate of increase of mass within this volume is given very nearly by:

$$r\,\Delta\theta\,\Delta r\,\Delta z\;\frac{\partial\rho}{\partial t}$$

The net rate of input of mass is:

$$(\rho v_r\cdot r\Delta\theta\Delta z)\Big|_r-(\rho v_r\cdot r\Delta\theta\Delta z)\Big|_{r+\Delta r}$$

$$+(\rho v_\theta\cdot\Delta r\Delta z)\Big|_\theta-(\rho v_\theta\cdot\Delta r\Delta z)\Big|_{\theta+\Delta\theta}$$

$$+(\rho v_z\cdot r\Delta\theta\,\Delta z)\Big|_z-(\rho v_z\cdot r\Delta\theta\,\Delta z)\Big|_{z+\Delta z}$$

When the rate of increase of mass is equated to the net rate of input, we then have a statement of the law of conservation of mass. It is not quite exact inasmuch as one really should specify an appropriate average value for r and for ρ in the volume element. If now we divide by $\Delta r\Delta\theta\Delta z$ and let Δr, $\Delta\theta$, Δz all go to zero, then the above mentioned errors become unimportant and we get:

$$r\frac{\partial\rho}{\partial t}=\lim_{\Delta r\to 0}\frac{(\rho r v_r)|_r-(\rho r v_r)|_{r+\Delta r}}{\Delta r}+\lim_{\Delta\theta\to 0}\frac{(\rho v_\theta)|_\theta-(\rho v_\theta)_{\theta+\Delta\theta}}{\Delta\theta}$$

$$+\lim_{\Delta z\to 0} r\,\frac{(\rho v_z)|_z-(\rho v_z)|_{z+\Delta z}}{\Delta z}$$

This yields finally, after division by r:

$$\frac{\partial\rho}{\partial t}=-\left[\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)\right]$$

This is the same as Eq. (b) in Table 3.4-1.

(b) Using the chain rule of partial differentiation:

$$\frac{\partial}{\partial x}(\rho v_x) = \left(\frac{\partial \rho v_x}{\partial r}\right)_{\theta,z}\left(\frac{\partial r}{\partial x}\right)_{y,z} + \left(\frac{\partial \rho v_x}{\partial \theta}\right)_{r,z}\left(\frac{\partial \theta}{\partial x}\right)_{y,z} + \left(\frac{\partial \rho v_x}{\partial z}\right)\overset{=0}{\left(\frac{\partial z}{\partial x}\right)_{y,z}}$$

$$= \frac{\partial}{\partial r}\left[\rho v_r \cos\theta - \rho v_\theta \sin\theta\right]\cdot\cos\theta$$

$$+ \frac{\partial}{\partial \theta}\left[\rho v_r \cos\theta - \rho v_\theta \sin\theta\right]\cdot\left(-\frac{\sin\theta}{r}\right) \quad + 0$$

$$\frac{\partial}{\partial y}(\rho v_y) = \left(\frac{\partial \rho v_y}{\partial r}\right)_{\theta,z}\left(\frac{\partial r}{\partial y}\right)_{x,z} + \left(\frac{\partial \rho v_y}{\partial \theta}\right)_{r,z}\left(\frac{\partial \theta}{\partial y}\right)_{x,z} + \left(\frac{\partial \rho v_y}{\partial z}\right)\overset{=0}{\left(\frac{\partial z}{\partial y}\right)_{z,x}}$$

$$= \frac{\partial}{\partial r}\left[\rho v_r \sin\theta + \rho v_\theta \cos\theta\right]\cdot\sin\theta$$

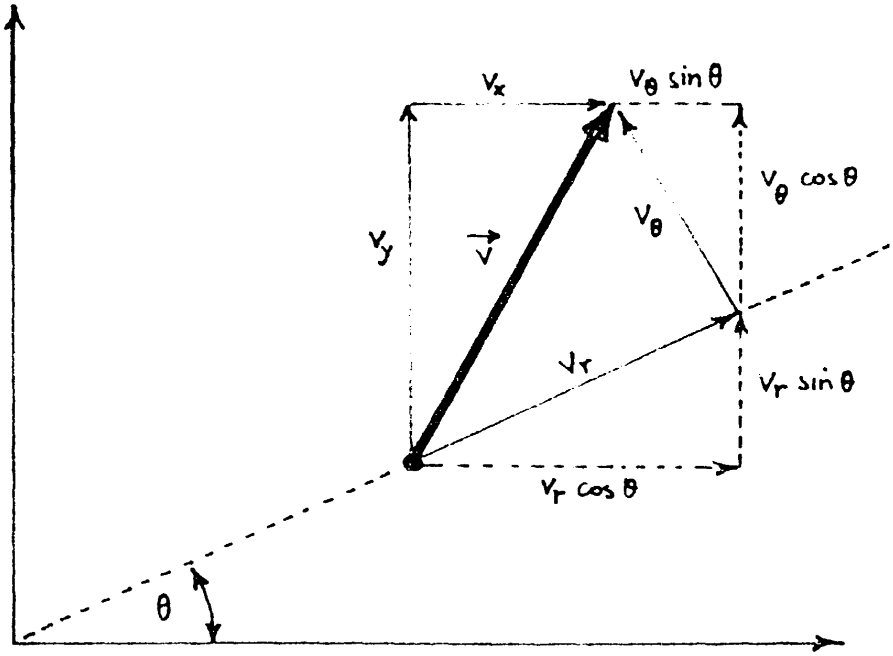
$$+ \frac{\partial}{\partial \theta}\left[\rho v_r \sin\theta + \rho v_\theta \cos\theta\right]\cdot\left(\frac{\cos\theta}{r}\right) \quad + 0$$

When these two expressions are added, considerable cancellation occurs and one obtains:

$$\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) = \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)$$

Insertion of this into Eq. (A) of Table 3.4-1 gives Eq (B).

(c) This may be done easily by using simply trigonometrical and geometrical arguments as seen at the right.



3.K Radial Flow between Two Parallel Disks

(a) Equation of continuity: $\frac{1}{r}\frac{\partial}{\partial r}(rv_r) = 0$

Equation of motion: $\rho v_r \frac{\partial v_r}{\partial r} = -\frac{dp}{dr} + \mu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \mu \frac{\partial^2 v_r}{\partial z^2}$

From the continuity equation it follows that $rv_r = \phi(z)$, that is, some function of z. Because of symmetry we know that ϕ does not depend on θ.

(b) Because of the result in (a), the second term on the rightside of the equation of motion is identically zero. And because $v_r = \phi(z)/r$ we may rewrite the equation of motion thus:

$$\rho \frac{\phi}{r} \cdot \left(-\frac{\phi}{r^2}\right) = -\frac{dp}{dr} + \mu \cdot \frac{1}{r}\frac{d^2\phi}{dz^2}$$

or

$$-\rho \frac{\phi^2}{r^3} = -\frac{dp}{dr} + \frac{\mu}{r}\frac{d^2\phi}{dz^2}$$

(c) We now integrate this equation with respect to r to get:

$$+\frac{1}{2}\rho\phi^2\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right) = \Delta p + \left(\mu \ln \frac{r_2}{r_1}\right)\frac{d^2\phi}{dz^2}$$

(d) When the left side is small we get:

$$\frac{d^2\phi}{dz^2} = -\frac{\Delta p}{\mu \ln \frac{r_2}{r_1}}$$

And integrating we get:

$$\phi = -\frac{\Delta p}{2\mu \ln \frac{r_2}{r_1}} z^2 + C_1 z + C_2$$

Now ϕ must be zero at $z = +b$ and $z = -b$. Hence C_1 must be zero and

$$C_2 = +\frac{\Delta p}{2\mu \ln \frac{r_2}{r_1}} b^2$$

So that finally:

$$v_r = \frac{\phi(z)}{r} = \frac{\Delta p\, b^2}{2\mu r \ln \frac{r_2}{r_1}}\left[1 - \left(\frac{z}{b}\right)^2\right]$$

(e) The volume rate of flow is given by:

$$Q = 2\pi \int_{-b}^{+b} r v_r \, dz$$

$$= 2\pi \cdot \frac{\Delta p \, b^3}{2\mu \ln (r_2/r_1)} \int_{-1}^{+1} (1-\zeta^2) \, d\zeta$$

$$= 2\pi \cdot \frac{\Delta p \, b^3}{2\mu \ln (r_2/r_1)} \cdot 2 \left[1 - \frac{1}{3} \right]$$

$$= \frac{4}{3} \frac{\pi \, \Delta p \, b^3}{\mu \ln (r_2/r_1)}$$

3.L Symmetry of the Tensor $\underset{\approx}{\tau}$

The moment of inertia of a rectangular body, of dimensions Δx by Δy and mass M, with respect to an axis through its center of gravity and perpendicular to the surface $\Delta x \Delta y$ is:

$$I = M \left(\frac{(\Delta x)^2 + (\Delta y)^2}{12} \right)$$

For the volume element under consideration, Newton's 2d law of motion becomes:

$$\Delta x \, \Delta z \cdot \frac{\Delta y}{2} \left(\tau_{yx}\big|_y + \tau_{yx}\big|_{y+\Delta y} \right) - \Delta y \, \Delta z \cdot \frac{\Delta x}{2} \left(\tau_{xy}\big|_x + \tau_{xy}\big|_{x+\Delta x} \right)$$

$$= \rho \, \Delta x \, \Delta y \, \Delta z \;\; \Omega \left(\frac{(\Delta x)^2 + (\Delta y)^2}{12} \right)$$

where Ω is the instantaneous angular velocity.

Divide by $\Delta x \Delta y \Delta z$ and let $\Delta x \Delta y \Delta z$ go to zero. The term containing Ω is clearly of higher order and drops out. We are then left with

$$\tau_{xy} = \tau_{yx}$$

3.M. Air Entrainment in a Draining Tank

As this system is too complex to permit analytic treatment, we use dimensional analysis. We must establish operating conditions so that the differential equations and boundary conditions describing the system are both the same. This means that the large and small tanks must be geometrically similar, and that the Froude and Reynolds numbers must be equal.

Choose D (tank diameter) as a characteristic length, and $(4Q/\pi D^2)$ as the characteristic velocity, where Q is the volumetric flow rate out of the tank. Then :

$$Re = \frac{4Q\rho}{\pi D \mu} \quad ; \quad Fr = \frac{16 Q^2}{\pi^2 D^5 g}$$

Denote quantities associated with the large and small tanks by subscripts L and S respectively. We exclude the possibility of altering the gravitational field. Then the requirement of equal Reynolds and Froude numbers is:

Re: $$\left(\frac{Q_s}{Q_L}\right)\left(\frac{D_L}{D_s}\right) = \left(\frac{\rho_L}{\mu_L}\right)\left(\frac{\mu_s}{\rho_s}\right) = \left(\frac{1.286}{56.7}\right)\left(\frac{1.00}{1.00}\right) = 0.0227$$

Fr: $$\left(\frac{Q_s}{Q_L}\right) = \left(\frac{D_s}{D_L}\right)^{5/2}$$

Then: from the above two statements:

$$\left(\frac{D_s}{D_L}\right) = (0.0227)^{2/3} = 0.080$$

Consequently:

$$D_s = (0.080)(60 \text{ ft}) = \underline{4.8 \text{ ft}}$$

$$Q_s = (0.080)^{5/2}(800 \text{ gal min}^{-1}) = \underline{1.46 \text{ gal min}^{-1}}$$

Therefore:

a. The model tank should be 4.8 ft in diameter

b. The drawoff tube should be 0.080 ft in diameter, and 0.080 ft high

c. The drawoff tube should be placed 0.32 feet from the side of the tank.

If in this tank water is withdrawn at 1.46 gal min^{-1}, air will be entrained when the liquid level is (4.8 / 60) of the level producing entrainment in the large tank at a withdrawal rate of 800 gal min^{-1}.

CHAPTER 4 – Checked by T. J. Sadowski

4.A Time for Attainment of Steady State in Tube Flow

(a) From Figure 4.1-3 the velocity will be within 10% of its final value when $\nu t/R^2 = 0.45$. Hence:

$$t = (0.45)R^2/\nu = (0.45)\frac{(0.7\times10^{-2})^2}{(3.45\times10^{-4})} = 6.39\times10^{-2}\text{ sec.}$$

(b) For water $\nu = 10^{-6}$ m² sec⁻¹; hence:

$$t = (0.45)\frac{(0.7\times10^{-2})}{(10^{-6})} = 22 \text{ sec.}$$

4.B Velocity near a Moving Sphere

From Eq. 4.2–13 for $\theta = \pi/2$

$$-0.99 = -\left[1-\frac{3}{4}\left(\frac{R}{r}\right)-\frac{1}{4}\left(\frac{R}{r}\right)^3\right]$$

where $R/r < 1$. If $R/r \ll 1$ then the cubic term will be unimportant. Then we get:

$$0.01 = \frac{3}{4}\left(\frac{R}{r}\right)$$

or

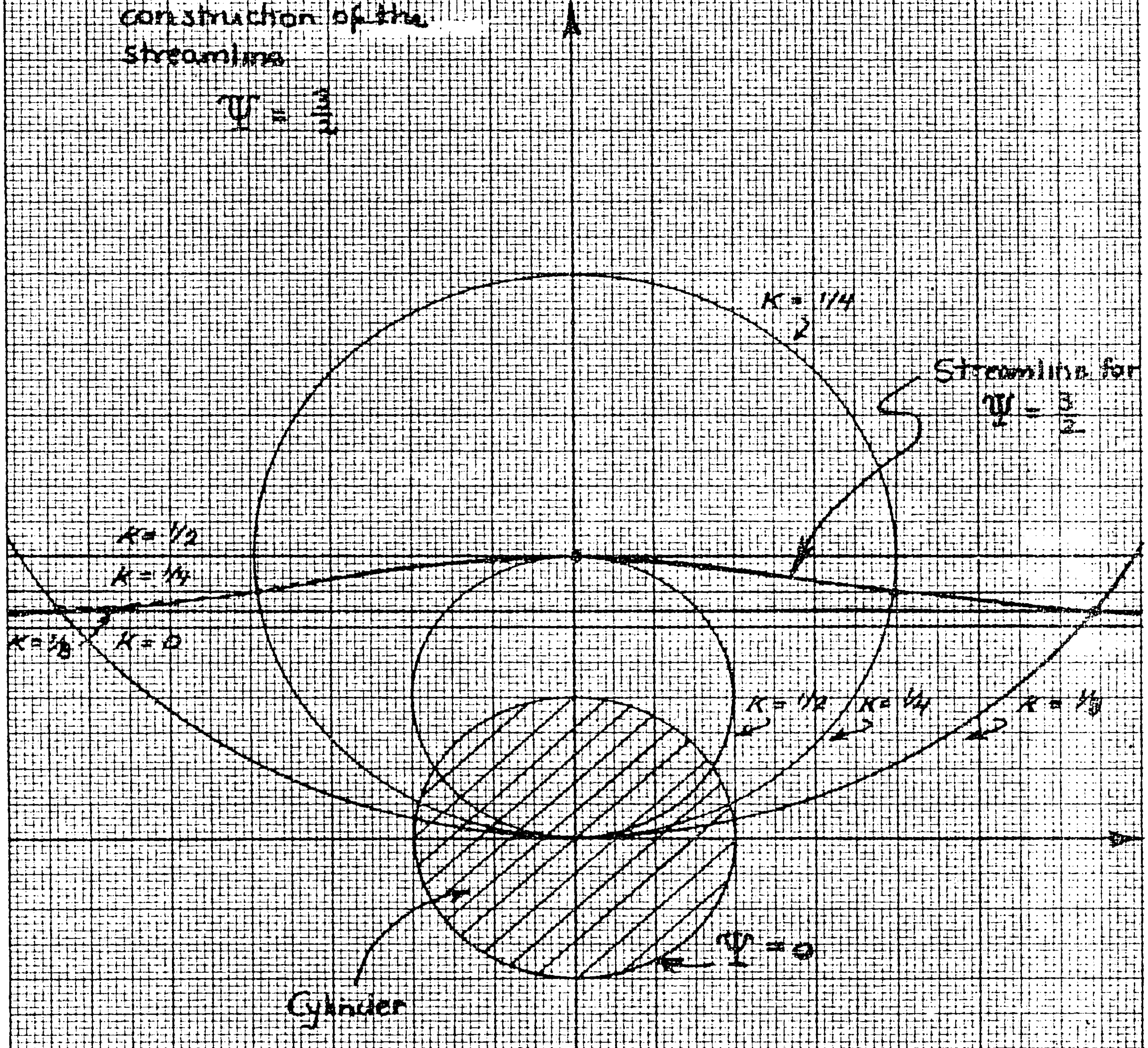
$$\therefore r = \frac{3}{0.04}R = 75\ R$$

[Clearly the neglect of the cubic tarm was justifiable.]

4.C Construction of Streamlines for Ideal Flow around a Cylinder

We show here the construction of the streamline

$$\Psi = \frac{3}{2}$$



4.D Comparison of Exact Result and Boundary-Layer Result for Flow Near Suddenly Moved Wall

For the exact solution, we simply look up the appropriate values of the error function:

(a) 0.777
(b) 0.480
(c) 0.157

From Eq. 4.4-10 we get for the boundary-layer solution:

$$(a)\quad 1 - \frac{3}{2}\frac{1}{\sqrt{2}}(0.2) + \frac{1}{4}\frac{1}{\sqrt{2}}(0.2)^3 = 1 - 0.212 + 0.001 = 0.789$$

$$(b)\quad 1 - \frac{3}{2}\frac{1}{\sqrt{2}}(0.5) + \frac{1}{4}\frac{1}{\sqrt{2}}(0.5)^3 = 1 - 0.530 + 0.022 = 0.492$$

$$(c)\quad 1 - \frac{3}{2}\frac{1}{\sqrt{2}}(1.0) + \frac{1}{4}\frac{1}{\sqrt{2}}(1.0)^3 = 1 - 1.061 + 0.177 = 0.116$$

Hence the percent error in the boundary-layer treatment is:

(a) + 1.5% (b) + 2.5% (c) − 26%

4.E Unsteady Pseudoplastic Flow near a Moving Wall

The partial differential equation to be solved is:

$$\frac{\partial v_x}{\partial t} = -\left(\frac{m}{\rho}\right)\frac{\partial}{\partial y}\left(-\frac{\partial v_x}{\partial y}\right)^n = \frac{mn}{\rho}\left(-\frac{\partial v_x}{\partial y}\right)^{n-1}\left(\frac{\partial^2 v_x}{\partial y^2}\right)$$

We now assume that:

$$\frac{v_x}{V} = \phi\left(\frac{y}{\delta(t)}\right) = \phi(\eta)$$

Substitute these "similar profiles" into the original differential equation and integrate from $\eta=0$ to $\eta=1$

$$\underbrace{\left[\int_0^1 \eta\phi'\,d\eta\right]}_{\text{call this } -B}\delta^n\dot{\delta} + \underbrace{\left[\frac{mnV^{n-1}}{\rho}\right]}_{\text{call this } \left(\frac{\sigma}{1+n}\right)}\underbrace{\left[\int_0^1(-\phi')^{n-1}\phi''\,d\eta\right]}_{\text{call this } +A} = 0$$

Then

$$\delta^n\dot{\delta} = \frac{A}{B}\frac{\sigma}{1+n}$$

or

$$\delta = \sqrt[n+1]{\left(\frac{A}{B}\right)\sigma t}$$

When we take $\phi(\eta)$ to be $\phi(\eta) = 1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3$ we find that $B = 3/8$ and $A = (3/2)^n(1/n)$. Hence:

$$\left(\frac{A}{B}\sigma\right) = \frac{8}{3}\left(\frac{3}{2}\right)^n \frac{1}{n} \cdot \frac{mnV^{n-1}}{\rho} \cdot (n+1)$$

whence:

$$\delta(t) = \left[\frac{8}{3}\left(\frac{3}{2}\right)^n \frac{m(n+1)V^{n-1}t}{\rho}\right]^{\frac{1}{n+1}}$$

describes how the boundary layer thickness advances with time.

4.F Use of the von Kármán Momentum Balance

(a) For flow past a flat plate V_∞ is a constant and:

$$\frac{1}{\rho}\tau_{xy}\Big|_{y=0} = V_\infty^2 \frac{d}{dx}\int_0^\infty \frac{v_x}{V_\infty}\left(1-\frac{v_x}{V_\infty}\right)dy$$

We assume the following similar profiles:

$$\left(\frac{v_x}{V_\infty}\right) = \phi(\eta) \qquad \text{where} \qquad \eta = y/\delta(x)$$

Then:

$$-\frac{1}{\rho}\tau_{xy} = \frac{\mu}{\rho}\frac{dv_x}{dy} = \frac{\mu}{\rho}\frac{d\phi}{d\eta}V_\infty\frac{1}{\delta(x)} = \frac{\nu V_\infty}{\delta}\phi'$$

$$\frac{d}{dx}\int_0^\infty \phi(1-\phi)\,d\eta \cdot \delta(x) = \int_0^\infty \phi(1-\phi)\,d\eta \cdot \frac{d\delta}{dx}$$

Now we let $\phi(\eta) = \frac{3}{2}\eta - \frac{1}{2}\eta^3$

Then: $\phi'(\eta) = \frac{3}{2} - \frac{3}{2}\eta^2$ and $\phi'(0) = \frac{3}{2}$

$$\int_0^\infty \phi(1-\phi)\,d\eta = \int_0^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right)d\eta = \frac{39}{280}$$

Hence the boundary layer momentum balance gives us:

$$\frac{\nu V_\infty}{\delta}\cdot\frac{3}{2} = V_\infty^2\frac{d\delta}{dx}\cdot\frac{39}{280}$$

or

$$\delta\frac{d\delta}{dx} = \frac{140}{13}\cdot\frac{\nu}{V_\infty}$$

Integration then gives

$$\delta(x) = \sqrt{\frac{280}{13}}\sqrt{\frac{\nu x}{v_\infty}}$$

(b) The drag force on a plate with dimensions $W \times L$ wetted on both sides would be:

$$\begin{aligned} 2\left(-\tau_{xy}\big|_{y=0}\right) WL &= 2\int_0^L \frac{\rho\nu v_\infty \phi'(0)}{\delta(x)}\, dx \cdot W \\ &= 3\rho\nu W v_\infty \sqrt{\frac{13}{280}}\sqrt{\frac{v_\infty}{\nu}}\int_0^L x^{-1/2}\, dx \\ &= 6\rho\nu^{1/2} W v_\infty^{3/2}\sqrt{\frac{13}{280}} \\ &= 1.292\sqrt{\rho\mu L W^2 v_\infty^3} \end{aligned}$$

4.G. Ideal Flow near a Stagnation Point

(a) Complex potential is

$$\begin{aligned} \frac{w}{v_0} &= -z^2 \\ &= -(x+iy)^2 \\ &= -(x^2-y^2) - i(2xy) \end{aligned}$$

Hence the velocity potential and stream function are:

$$\begin{aligned} \phi(x,y) &= -(x^2-y^2)v_0 \\ \psi(x,y) &= -2xy\,v_0 \end{aligned}$$

The curves of $\psi = C$ are streamlines:

$$y = -\frac{1}{2}\frac{C v_0}{x}$$

Hence, for positive values of C we get hyperbolas in the second quadrant as shown in Fig 4.G. For negative values of C we get hyperbolas in the first quadrant. When $C=0$ we get the x-axis as a streamline. This streamline may be replaced by a solid surface.

(b) The velocity components are:

$$\begin{aligned} v_x &= -\frac{\partial\phi}{\partial x} = +2v_0 x \\ v_y &= -\frac{\partial\phi}{\partial y} = -2v_0 y \end{aligned}$$

(c) $|v_x| = |v_y| = v_0$ when $x = \pm\frac{1}{2}$ and $y = \frac{1}{2}$. That is, v_0 specifies the "speed level" of the flow net.

4.H Ideal Flow around a Sphere

(a) From Eq. 4.H-1

$$v_r = -\frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta}$$

$$= -\frac{v_\infty R^3}{2r^3 \sin\theta}\cdot 2\sin\theta\cos\theta + \frac{v_\infty r^2}{2r^2 \sin\theta}\cdot 2\sin\theta\cos\theta$$

$$= -v_\infty \frac{R^3}{r^3}\cos\theta + v_\infty\cos\theta$$

$$= v_\infty \cos\theta\left(1 - \frac{R^3}{r^3}\right)$$

$$v_\theta = +\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}$$

$$= \frac{v_\infty R^3}{2r\sin\theta}\sin^2\theta\left(-\frac{1}{r^2}\right) - \frac{v_\infty}{2}\sin^2\theta\,\frac{1}{r\sin\theta}\,(2r)$$

$$= -v_\infty\sin\theta\left(1 + \frac{R^3}{2r^3}\right)$$

Then $v_z = v_\infty\cos^2\theta\left(1 - \frac{R^3}{r^3}\right) + v_\infty\sin^2\theta\left(1 + \frac{R^3}{2r^3}\right)$

When $r \to \infty$, the terms with r^3 in the denominator get small and

$$v_z = v_\infty\cos^2\theta + v_\infty\sin^2\theta = v_\infty$$

(b) At the surface $r = R$, from part (a) we get:

$$v_\theta = -v_\infty\sin\theta\cdot\left(1 + \tfrac{1}{2}\right) = -\tfrac{3}{2}v_\infty\sin\theta$$

(c) For ideal, irrotational flow:

$$p + \tfrac{1}{2}\rho v^2 = p_\infty + \tfrac{1}{2}\rho v_\infty^2$$

and hence:

$$p - p_\infty = \tfrac{1}{2}\rho v_\infty^2\left[1 - \left(\frac{v_\theta}{v_\infty}\right)^2\right] = \tfrac{1}{2}\rho v_\infty^2\left(1 - \tfrac{9}{4}\sin^2\theta\right)$$

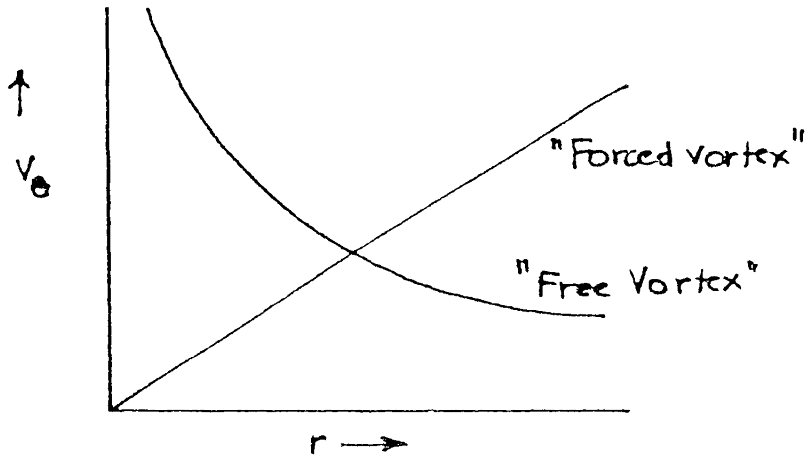
4.I. Vortex Flow

(a) $w = i(\Gamma/2\pi)\ln z = -(\Gamma/2\pi)\theta + i(\Gamma/2\pi)\ln r$

Hence $\begin{cases}\phi = -(\Gamma/2\pi)\theta \\ \psi = +(\Gamma/2\pi)\ln r\end{cases}$

and $v_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = +\frac{\mu}{r}$

(b) For a "free vortex" $v_\theta = \mu/r$ [from (a)]
For a "forced vortex" $v_\theta = \Omega r$ [from Ex. 3.4-2]



CHAPTER 5 – Checked by T. J. Sadowski

5.A Pressure Drop Required for Laminar-Turbulent Transition

At the transition point

$$Re = D\langle v\rangle\rho/\mu \doteq 2.1 \times 10^3$$

Now from a rearrangement of Poiseuille's law, we can get $\Delta p/L$ in terms of the Reynolds number, thus:

$$Q = \frac{\pi \Delta p R^4}{8\mu L} = \pi R^2 \langle v\rangle$$

Solving for the pressure gradient, we get:

$$\frac{\Delta p}{L} = \frac{8\mu\langle v\rangle}{R^2} = \frac{8\mu^2}{\rho R^2 D}\frac{D\langle v\rangle\rho}{\mu} = \frac{4\mu^2}{\rho R^3}\cdot Re$$

If we now insert the critical value of Re given above, and the following values for μ, ρ, and R:

$\mu = 0.183$ poise ; $\rho = 1.32 \text{ g cm}^{-3}$; $R = 0.21$ in.

then we get:

$$\frac{\Delta p}{L} = \frac{4\,(0.183)^2}{(1.32)(0.21 \times 2.54)^3} \cdot (2.10 \times 10^{+3}) \cdot \left[(2.54)(12)(1.4504 \times 10^{-5})\right]$$

$$\therefore \quad \frac{\Delta p}{L} = 0.62 \text{ psi}/\text{ft}$$

5.B Velocity Distribution in Turbulent Pipe Flow

(a) $$\tau_0 = \frac{\Delta p R}{2L} = \frac{R}{2}\left(\frac{\Delta p}{L}\right) = \frac{(0.5)}{2}\left(1.0 \times \frac{1}{5280}\right) = 4.73 \times 10^{-5} \text{ psi}$$

(b) For the situation described we use the following values:

$$\rho = 1.0 \text{ g cm}^{-3} = 62.4 \text{ lb ft}^{-3}$$

$$\mu = 0.01 \text{ g cm}^{-1} \text{sec}^{-1}$$

$$\nu = 0.01 \text{ cm}^2 \text{sec}^{-1} = 1.1 \times 10^{-5} \text{ ft}^2 \text{sec}^{-1}$$

Hence:

$$\frac{\sqrt{\tau_0/\rho}}{\mu/\rho} = \frac{\sqrt{\dfrac{(4.73 \times 10^{-5} \text{ lb}_f \text{ in}^{-2})(144 \text{ in}^2 \text{ft}^{-2})}{62.4 \text{ lb}_m \text{ ft}^{-3}} \cdot (32.2) \dfrac{\text{lb}_m \text{ ft}}{\text{lb}_f \text{ sec}^2}}}{1.1 \times 10^{-5} \text{ ft}^2 \text{ sec}^{-1}}$$

$$= 5390 \text{ ft}^{-1}$$

And

$$\sqrt{\tau_0/\rho} = 5.93 \times 10^{-2} \text{ ft sec}^{-1}$$

Thus $$v^+ = \frac{\bar{v}}{5.93 \times 10^{-2}} \quad ; \quad s^+ = 5390 \; s$$

At the tube center, $r = 0$, $s = R = 0.5$ ft, so that

$$s^+\Big|_{s=R} = (5390)(0.5) = 2695$$

which enables us to get from Fig 5.3-1 the following value for the maximum velocity:

$$v^+\Big|_{s=R} = 25.8$$

Consequently

$$v^+ = 25.8 \frac{\bar{v}}{\bar{v}_{max}} \quad ; \quad s^+ = 5390 \; s$$

We are now in a position to calculate the following table:

$\bar{v}/\bar{v}_{max}$	v^+ (calc)	s^+ (Fig. 5.3-1)	s (ft)	s (in)
0.0	0.0	0.0	0.0	0
0.1	2.58	2.58	4.79×10^{-4}	0.00575
0.2	5.16	5.6	1.04×10^{-3}	0.01250
0.4	10.32	15.2	2.83×10^{-3}	0.0340
0.7	18.06	178	3.31×10^{-2}	0.397
0.85	21.95	810	1.50×10^{-1}	1.800
1.0	25.8	2695	0.50	6.000

(c) See Graph on next page

just a reasonable number picked out of thin air

(d) If we assume that $\langle \bar{v}_z \rangle / \bar{v}_{z,max} = 0.83$ then we can calculate an approximate value for the Reynolds number.

$$\langle \bar{v}_z \rangle = 0.83\ \bar{v}_{max}$$

$$= 0.83\ (5.93 \times 10^{-2}\ v^+_{max})$$

$$= 0.83\ (5.93 \times 10^{-2})(25.8) = 1.270 \text{ ft sec}^{-1}$$

Then the Reynolds number is estimated to be:

$$Re = \frac{D \langle \bar{v}_z \rangle \rho}{\mu} = \frac{(0.5)(1.270)}{(1.1 \times 10^{-5})} = 5.76 \times 10^{4}$$

Hence the flow is certainly turbulent

(e) The volume rate of flow is obtained by integrating the velocity profile over the cross-section.

$$\frac{\langle \bar{v}_z \rangle}{\bar{v}_{z,max}} = \frac{2\pi \int_0^R (\bar{v}_z / \bar{v}_{z,max})\, r dr}{\pi R^2}$$

$$\doteq \frac{2}{(6 \text{ in})^2} \cdot \frac{\frac{1}{2} \text{ in}}{3} \Big[\left(\frac{v_z}{v_{max}} r\right)_{r=0} + 4\left(\frac{v_z}{v_{max}} r\right)_{r=\frac{1}{2}''} + 2\left(\frac{v_z}{v_{max}} r\right)_{r=1''} + 4\left(\frac{v_z}{v_{max}} r\right)_{r=1\frac{1}{2}''} + \cdots \Big]$$

That is, we evaluate the integral using "Simpson's Rule". The terms in the bracket are computed below

$(1)(1.000)(0.0) = 0.000$
$(4)(0.992)(0.5) = 1.984$
$(2)(0.977)(1.0) = 1.954$
$(4)(0.965)(1.5) = 5.790$
$(2)(0.950)(2.0) = 3.800$
$(4)(0.933)(2.5) = 9.330$
$(2)(0.913)(3.0) = 5.478$
$(4)(0.891)(3.5) = 12.474$
$(2)(0.865)(4.0) = 6.920$
$(4)(0.830)(4.5) = 14.940$
$(2)(0.784)(5.0) \doteq 7.840$
$(4)(0.717)(5.5) = 15.774$
$(1)(0.000)(6.0) = 0.000$
$\Sigma = 86.284$

Therefore

$$\frac{\langle \bar{v}_z \rangle}{\bar{v}_{z,max}} = \frac{2}{36} \cdot \frac{\frac{1}{2}}{3} \cdot 86.3 = 0.80$$

and

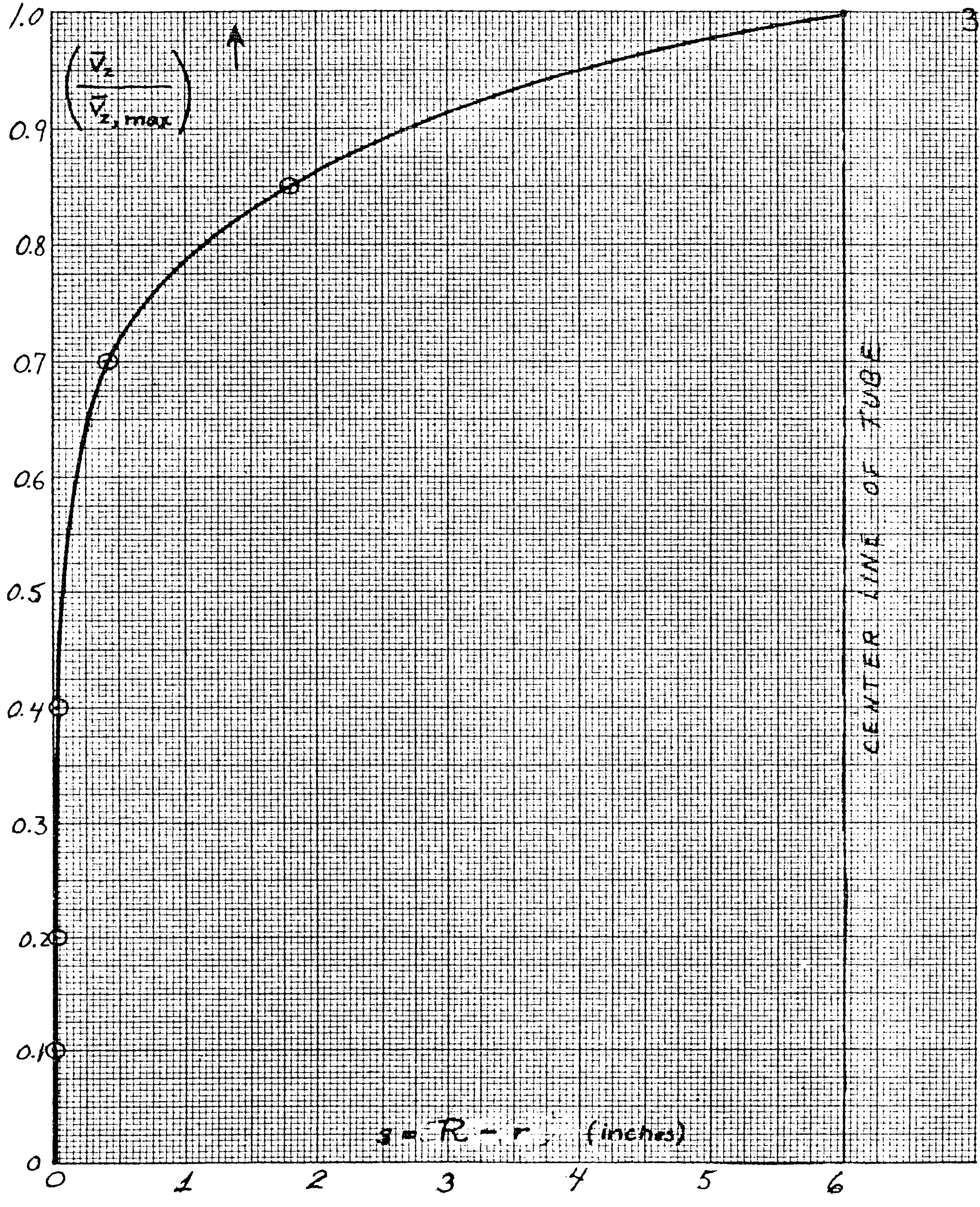
$$\langle \bar{v}_z \rangle = (0.80)(5.93 \times 10^{-2})(25.8)$$

$$= 1.223 \text{ ft sec}^{-1}$$

and

$$Q = \pi (0.5)^2 (1.223)$$

$$= 0.963 \text{ ft}^3 \text{ sec}^{-1}$$


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$\left(\frac{\overline{V}_z}{\overline{V}_{z,\,max}}\right)$
CENTER LINE OF TUBE
$s = R - r$ (inches)
0
1
2
3
4
5
6

5.C Average Flow Velocity in Turbulent Tube Flow

(a)
$$\frac{\langle \bar{v}_z \rangle}{\bar{v}_{z,max}} = \frac{\int_0^R (\bar{v}_z/\bar{v}_{z,max})\, r\,dr}{\int_0^R r\,dr} = \frac{2}{R^2}\int_0^R \left(1-\frac{r}{R}\right)^{\frac{1}{n}} r\,dr$$

$$= 2\int_0^1 (1-\xi)^{\frac{1}{n}}\,\xi\,d\xi \qquad \text{where } \xi = r/R$$

$$= 2\int_0^1 \zeta^{\frac{1}{n}}(1-\zeta)\,d\zeta \qquad \text{where } \zeta = 1-\xi$$

$$= 2\left[\frac{\zeta^{\frac{1}{n}+1}}{\frac{1}{n}+1} - \frac{\zeta^{\frac{1}{n}+2}}{\frac{1}{n}+2}\right]\Bigg|_0^1$$

$$= 2\,\frac{n\left[(2n+1)-(n+1)\right]}{(n+1)(n+2)} = \frac{2n^2}{(n+1)(2n+1)}$$

(b)
$$\frac{\langle \bar{v}_z \rangle}{\bar{v}_{z,max}} = \frac{2}{R^2}\frac{v_*}{\bar{v}_{z,max}}\int_0^R \left(\frac{1}{0.36}\ln s^+ + 3.8\right) r\,dr$$

$$= 2\,\frac{v_*}{\bar{v}_{z,max}}\left(\frac{\mu}{Rv^*\rho}\right)^2 \int_0^{\frac{Rv^*\rho}{\mu}} \left(\frac{1}{0.36}\ln s^+ + 3.8\right)\left(\frac{Rv_*\rho}{\mu} - s^+\right) ds^+$$

$$= 2\,\frac{v_*}{\bar{v}_{z,max}}\left(\frac{\mu}{Rv^*\rho}\right)^2 \left[\frac{1}{0.36}\left(\frac{Rv_*\rho}{\mu}\right)\left(s^+\ln s^+ - s^+\right) - \frac{1}{0.36}\left(\frac{s^{+2}}{2}\ln s^+ - \frac{s^{+2}}{4}\right) + 3.8\left(\frac{Rv_*\rho}{\mu}\right)s^+ - 3.8\left(\frac{s^{+2}}{2}\right)\right]_{s^+=0}^{s^+=\frac{Rv_*\rho}{\mu}}$$

When the limits are inserted, we get

$$\frac{\langle \bar{v}_z \rangle}{\bar{v}_{z,max}} = \left(\frac{v_*}{\bar{v}_{z,max}}\right)\left(2.78\ln\frac{Rv_*\rho}{\mu} - 0.38\right)$$

Note that this result shows a dependence on a Reynolds number $Rv_*\rho/\mu$, whereas the result in (a) showed no dependence of this sort.

5.D Velocity Distribution in a Channel

(a) The momentum flux distribution in the channel is:

$$\tau_{yz} = \tau_0 \, (y/h)$$

where τ_0 is the wall shear stress. Hence according to the von Kármán similarity hypothesis:

$$\tau_0 \cdot (y/h) = \rho \kappa_2 \frac{(d\bar{v}_z/dy)^4}{(d^2\bar{v}_z/dy^2)^2}$$

(b) Take the square root of both sides to get:

$$\frac{1}{\kappa_2} v_* \sqrt{\eta} = \frac{(d\bar{v}_z/d\eta)^2}{(d^2\bar{v}_z/d\eta^2)} \qquad \text{where } v_* = \sqrt{\frac{\tau_0}{\rho}} \; ; \; \eta = \frac{y}{h}$$

Let $p = d\bar{v}_z/d\eta$, so that

$$\frac{1}{\kappa_2} v_* \sqrt{\eta} = \frac{p^2}{p'} \qquad \text{where } p' = dp/d\eta$$

or

$$\int_p^{-\infty} \frac{dp}{p^2} \left(\frac{v_*}{\kappa_2}\right) = \int_\eta^1 \frac{d\eta}{\sqrt{\eta}}$$

which gives:

$$\left(\frac{v_*}{\kappa_2}\right)\left(\frac{1}{p}\right) = 2\left(1 - \sqrt{\eta}\right)$$

Setting $p = d\bar{v}_z/d\eta$ we then get a first-order equation for $\bar{v}_z$ which can be integrated:

$$\left(\frac{v_*}{\kappa_2}\right) \int_0^\eta \frac{d\eta}{1-\sqrt{\eta}} = 2 \int_{\bar{v}_{z,\max}}^{\bar{v}_z} d\bar{v}_z$$

Integration is easily performed by letting $\sqrt{\eta} = \beta$; one gets finally

$$\frac{\bar{v}_{z,\max} - \bar{v}_z}{v_*/\kappa_2} = \sqrt{\frac{y}{h}} + \ln\left(1 - \sqrt{\frac{y}{h}}\right)$$

(c) If the result in (b) is expanded in a Taylor's series, we get:

$$\frac{\bar{v}_{z,\max} - \bar{v}_z}{v_*/\kappa_2} = \sqrt{\frac{y}{h}} + \left[-\sqrt{\frac{y}{h}} - \frac{1}{2}\left(\frac{y}{h}\right) - \frac{1}{3}\left(\frac{y}{h}\right)^{3/2} - \cdots\right] = -\frac{1}{2}\left(\frac{y}{h}\right) - \cdots$$

Hence $d\bar{v}_z/dy\big|_{y=0} = (v_*/\kappa)(1/2h)$, which shows that the velocity distribution (incorrectly) has a non-zero slope at $y=0$.

Chapter 6 — Checked by V. D. Shah

6.A Pressure Drop for Given Flow Rate with Fittings

$$\text{Average velocity} = \langle v \rangle = \frac{Q}{A} = \frac{1.97 \times 10^6 \text{ cm}^3\text{sec}^{-1}}{\pi\,(25/2)^2 \text{ cm}^2} = 4020 \text{ cm sec}^{-1}$$

$$\text{Reynolds number} = \frac{D\langle v \rangle \rho}{\mu} = \frac{(25)(4020)(1.0)}{(0.01)} = 1.005 \times 10^7$$

From Fig. 6.2-1 for smooth pipes $f = 0.0020$

The required pressure drop is then:

$$(p_0 - p_L) = 2 \cdot \frac{L}{D} \cdot \rho \langle v \rangle^2 f$$

$$= 2\left(\frac{123400 + 4(32)(25) + 2(15)(25)}{25}\right)(1.0)(4020)^2(0.0020)$$

$$= 3.29 \times 10^8 \text{ g cm}^{-1}\text{sec}^{-2} = 4.63 \times 10^3 \text{ psi}$$

6.B. Pressure Drop Required for a Given Flow Rate with Elevation Change

The Reynolds number is

$$\boxed{\times \frac{4}{\pi\,(3.068 \times 2.54)^2}\,\frac{1}{\text{cm}^2}}$$

$$Re = \frac{D\langle v \rangle \rho}{\mu}$$

$$= \frac{\left(3.068 \text{ in} \times 2.54 \frac{\text{cm}}{\text{in}}\right)\left(18 \frac{\text{gal}}{\text{min}} \times 3785 \frac{\text{cm}^3}{\text{gal}} \times \frac{1}{60} \frac{\text{min}}{\text{sec}}\right)\left(0.9982 \frac{\text{g}}{\text{cm}^3}\right)}{(0.01005 \text{ g cm}^{-1}\text{sec}^{-1})}$$

$$= 1.84 \times 10^4$$

In Fig. 6.2-1 we find that for this Reynolds number $f = 0.0064$ for <u>smooth</u> tubes. Therefore:

$$p_0 - p_L = -\rho g (h_0 - h_L) + 4 \frac{L}{D} \cdot \frac{1}{2} \rho \langle v \rangle^2 \cdot f$$

$$= \left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)(0.9982)\left(32.16 \frac{\text{ft}}{\text{sec}^2}\right)\left(\frac{50}{\sqrt{2}} \text{ ft}\right)\left(2.16 \times 10^{-4} \frac{\text{psia}}{\text{poundal}}\right)$$

$$+ 4\left[\frac{(95)(12) + 2(15)(3.068)}{3.068}\right]\frac{1}{2}\left(62.4 \frac{\text{lbm}}{\text{ft}^3}\right)(0.9982)$$

$$\times \left[\frac{\left(18 \frac{\text{gal}}{\text{min}} \cdot 0.002228 \frac{\text{min}}{\text{gal}} \frac{\text{ft}^3}{\text{sec}}\right)}{\pi\,(3.068/12)^2/4 \text{ ft}^2}\right]^2 \cdot (0.0064)\left(2.16 \times 10^{-4} \frac{\text{psia}}{\text{poundal}}\right)$$

$$= 15.2 + 0.04 \doteq 15.2 \text{ psia}$$

6.C Flow Rate for a Given Pressure Drop

The quantities needed for the calculation are:

$$p_0 - p_L = (0.251 \text{ lb}_f \text{ in}^{-2})(32.17 \text{ lb}_m \text{ ft sec}^{-2} \text{ lb}_f^{-1})(144 \text{ in}^2 \text{ ft}^{-2})$$

$$= 1.16 \times 10^3 \text{ lb}_m \text{ ft}^{-1} \text{ sec}^{-2}$$

$D = 0.5$ ft. $\qquad \rho = 62.4 \text{ lb}_m \text{ ft}^{-3}$

$L = 1320$ ft $\qquad \mu = 6.72 \times 10^{-4} \text{ lb}_m \text{ ft}^{-1} \text{ sec}^{-1}$

Hence

$$Re\sqrt{f} = \frac{D\rho}{\mu}\sqrt{\frac{(p_0 - p_L)D}{2L\rho}}$$

$$= \frac{(0.5)(62.4)}{(6.72 \times 10^{-4})}\sqrt{\frac{(1.16 \times 10^3)(0.5)}{2(1320)(62.4)}}$$

$$= (4.64 \times 10^4)(5.92 \times 10^{-2}) = 27.4 \times 10^2 = 2.74 \times 10^3$$

A straight line of slope -2 on the $\log_{10} f$ vs. $\log_{10} Re$ plot through $f = 1$, $Re = 2.74 \times 10^3$ intersects the f vs. Re curve at $Re = 3.6 \times 10^4$. Hence the average velocity is:

$$\langle v \rangle = \frac{Re\,\mu}{D\rho} = \frac{(3.6 \times 10^4)(6.72 \times 10^{-4})}{(0.5)(62.4)} = 0.775 \text{ ft sec}^{-1}$$

The volume rate of flow is:

$$Q = \frac{\pi D^2}{4}\langle v \rangle = \frac{(3.142)(0.5)^2}{4}(0.775) = 0.152 \text{ ft}^3 \text{ sec}^{-1} = 68 \frac{\text{gal}}{\text{min}}$$

6.D Motion of a Sphere in a Liquid

(a) Force of gravity on sphere is: $F_{grav} = mg = (0.0500)(980.7) = 49.0$ dynes

Force of buoyancy is: $F_{buoy} = \frac{4}{3}\pi(0.25)^3(0.900)(980.7) = 57.8$ dynes

Hence net upward force is:

$$F_{buoy} - F_{grav} = 8.8 \text{ dynes}$$

Hence drag force in the <u>downward</u> direction is $F_k = 8.8$ dynes

(b) The friction factor is defined by:

$$F_k = (\pi R^2)(\tfrac{1}{2}\rho v_\infty^2)f$$

Then: $f = \dfrac{F_k}{\left(\dfrac{\pi D^2}{4}\right)\left(\frac{1}{2}\rho v_\infty^2\right)} = \dfrac{8 F_k}{\pi D^2 \rho v_\infty^2}$

$$= \frac{8\ (8.8 \text{ dynes})}{\pi\ (0.500 \text{ cm})^2\ (0.900 \text{ g cm}^{-3})\ (0.500 \text{ cm sec}^{-1})^2}$$

$$= 398$$

(c) Since $f = 398$ is in the Stokes's law region we can write

$$398 = \frac{24}{Re}$$

whence $Re = 0.0602$. Hence

$$\mu = \frac{D v_\infty \rho}{\mu} = \frac{(0.5)(0.5)(0.90)}{(0.0602)} = 3.74 \text{ g cm}^{-1}\text{sec} \doteq 370 \text{ cp.}$$

6.E Drag Calculations when Sphere Diameter is Unknown

(a) Method A: Plot f/Re vs. Re. Since f/Re does not contain D, hence from this curve we can read off the value of Re for a calculable value of f/Re.

Method B: On the log-log plot of $f = f(Re)$, plot also the curve $f = \left(\frac{f}{Re}\right)Re$, which, on the log-log plot, will be a line of slope +1. This method avoids any necessity of preparing a separate plot.

(b) First calculate f/Re:

$$\frac{f}{Re} = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{sphere} - \rho}{\rho}\right) \cdot \frac{\mu}{D v_\infty \rho}$$

$$= \frac{4}{3}\frac{g\mu}{v_\infty^3 \rho}\left(\frac{\rho_{sphere} - \rho}{\rho}\right)$$

$$= \frac{4}{3}\frac{(980)(2.6 \times 10^{-4})(1.2)}{(30.5)^3\ (7.2 \times 10^{-4})^2}$$

$$= 27.8$$

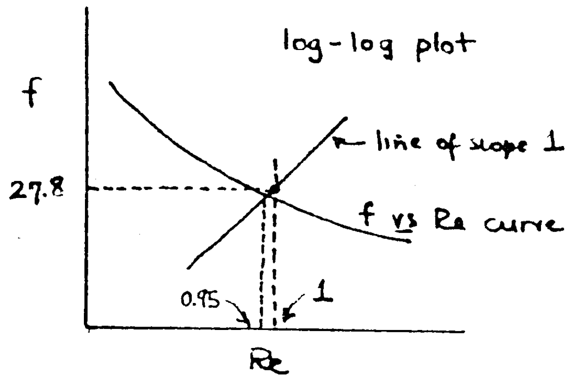
$v_\infty = 30.5 \text{ cm sec}^{-1}$

$\rho = 7.2 \times 10^{-4} \text{ g cm}^{-3}$

$\rho_{sph} = 1.2 \text{ g cm}^{-3}$

$\mu = 2.6 \times 10^{-4} \text{ g cm}^{-1}\text{sec}^{-1}$

We now draw a line of slope 1 through $Re = 1$, $f = 27.8$ on Fig 6.3-1. It intersects the f vs. Re curve at about 0.95



Then we can get the particle diameter from the known Reynolds number:

$$D = \frac{Re\,\mu}{v_\infty \rho} = \frac{(0.95)(2.6 \times 10^{-4})}{(30.5)(7.2 \times 10^{-4})} = 112 \text{ microns}$$

(c) When v_∞ is increased by a factor of 10 then $f/Re = 10^{-3} \cdot 27.8$ and the plotting procedure described earlier gives $Re = 75$. Hence:

$$D = \frac{(75)(2.6 \times 10^{-4})}{(305)(7.2 \times 10^{-4})} = 890 \text{ microns}$$

6.F Estimation of Void Fraction of a Packed Bed

The superficial velocity is:

$$v_0 = \frac{(244 \frac{\text{lb}}{\text{min}})(\frac{1}{60} \frac{\text{min}}{\text{sec}})(454 \frac{\text{g}}{\text{lb}})(\frac{1}{1.2865} \frac{\text{cm}^3}{\text{g}})}{(146 \text{ in}^2)(2.54 \frac{\text{cm}}{\text{in}})^2} = 1.53 \text{ cm sec}^{-1}$$

According to the Blake-Kozeny equation:

$$\frac{\epsilon^3}{(1-\epsilon)^2} = \frac{150 \mu L v_0}{D_p^2 \Delta p}$$

$$= \frac{150 \; (0.565 \text{ g cm}^{-1} \text{sec}^{-1})(73 \times 2.54 \text{ cm})(1.53 \text{ cm sec}^{-1})}{(0.2 \text{ cm})^2 \; (158 \times 68947 \text{ dynes cm}^{-2})}$$

$$= 0.0553$$

Solving the above equation for ϵ, we find: $\epsilon = 0.30$

The quantity $\frac{D_p v_0 \rho}{\mu} \frac{1}{1-\epsilon} = \frac{(0.2)(1.53)(1.287)}{(0.565)} \frac{1}{(1-0.30)} = 0.99$

and hence it was legitimate to use the Blake-Kozeny equation (see Figure 6.4-1).

6.G Friction Factor for Flow around a Flat Plate

Define $K = \frac{1}{2} \rho v_\infty^2$; $A = 2WL$ = total wetted surface; $Re = \frac{L v_\infty \rho}{\mu}$

Then f is defined as:

$$f = \frac{F}{WL\rho\, v_\infty^2}$$

(a) Laminar flow: $$f = \frac{1.328\sqrt{\rho\mu L W^2 v_\infty^3}}{WL\rho v_\infty^2} = \frac{1.328}{\sqrt{Re}}$$

(b) Turbulent flow: $$f = \frac{0.72\,\rho v_\infty^2 WL\,(Lv_\infty\rho/\mu)^{-1/5}}{WL\rho v_\infty^2} = \frac{0.072}{\sqrt[5]{Re}}$$

6.H Friction Factor for Laminar Slit Flow

For a slit of width W the quantities in Eq. 6.1-1 are defined as follows:

$$F_k = (p_0 - p_L)\cdot 2BW$$

$$K = \tfrac{1}{2}\rho\langle v\rangle^2$$

$$A = 2WL + 4BL \approx 2WL$$

$$Re = 2B\langle v\rangle\rho/\mu$$

Hence:

$$f = \frac{(p_0 - p_L)\,2BW}{WL\rho\langle v\rangle^2}$$

From Problem 2.E, since $Q = 2BW\langle v\rangle$,

$$(p_0 - p_L) = \frac{3}{2}\frac{\mu LQ}{B^3W} = \frac{3}{2}\frac{\mu L\,(2BW\langle v\rangle)}{B^3W}$$

Combination of these last two results gives:

$$f = 6\frac{\mu}{B\langle v\rangle\rho} = \frac{12}{Re}$$

6.I Friction Factor for a Rotating Disk

(a) Laminar Flow: $$f = \frac{\mathcal{T}}{KAR} = \frac{0.616\pi\rho R^4(\mu\Omega^3/\rho)^{1/2}}{\frac{1}{2}\rho R^2\Omega^2\cdot 2\pi R^2\cdot R} = \frac{0.616}{\sqrt{Re}}$$

(b) Turbulent Flow: $$f = \frac{\mathcal{T}}{KAR} = \frac{0.073\rho\Omega^2R^5(\mu/R^2\Omega\rho)^{1/5}}{\frac{1}{2}\rho R^2\Omega^2\cdot 2\pi R^2\cdot R} = \frac{0.023}{\sqrt[5]{Re}}$$

6.J Friction Factor for Turbulent Flow in Smooth Tubes

According to the logarithmic velocity distribution given in Eq. 5.3-12,

$$v^+ = 2.5 \ln s^+ + 5.5$$

where $v^+ = \bar{v}/v_*$ and $s^+ = s v_* \rho/\mu$. The average velocity is given by

$$\frac{\langle v\rangle}{v_*} = \frac{2}{v_* R^2}\int_0^R \bar{v}\, r\, dr = \frac{2\mu}{v_* R\rho}\int_0^{R v_* \rho/\mu} \frac{\bar{v}}{v_*}\left(1-\frac{s}{R}\right) d\left(\frac{s v_* \rho}{\mu}\right)$$

$$= \frac{2}{Re_*}\int_0^{Re_*} \frac{\bar{v}}{v_*}\left(1 - \frac{s v_* \rho/\mu}{R v_* \rho/\mu}\right) d\left(\frac{s v_* \rho}{\mu}\right)$$

$$= \frac{2}{Re_*}\int_0^{Re_*} (2.5 \ln s^+ + 5.5)\left(1-\frac{s^+}{Re_*}\right) ds^+$$

$$= \frac{2}{Re_*}\left[\begin{array}{l} 5.5\, Re_* - \frac{1}{2}(5.5) Re_* + 2.5\left(s^+ \ln s^+ - s^+\right)\Big|_0^{Re_*} \\ \quad - 2.5\left(\frac{1}{2} s^{+2} \ln s^+ - \frac{s^{+2}}{4}\right)\Big|_0^{Re_*} \end{array}\right]$$

$$= 1.75 + 2.5 \ln Re_*$$

Hence $f = 2\left(\frac{v_*}{\langle v\rangle}\right)^2$

or

$$\frac{1}{\sqrt{f}} = \frac{1}{\sqrt{2}}\frac{\langle v\rangle}{v_*} = \frac{1}{\sqrt{2}}\left(2.5 \ln Re_* + 1.75\right)$$

$$= 4.07 \log_{10} Re\sqrt{f} - 0.60$$

the latter step making use of the following:

$$Re_* = \frac{R v_* \rho}{\mu} = \frac{D\langle v\rangle\rho}{\mu}\,\frac{1}{2}\frac{v_*}{\langle v\rangle} = Re\,\frac{1}{2}\frac{v_*}{\langle v\rangle} = Re\,\frac{1}{2}\sqrt{\frac{f}{2}}$$

6.K Friction Factor for Power-Law Non-Newtonian Flow in Tubes

For tubes we know from Eq. 6.1-4

$$f = \frac{1}{4}\frac{D}{L}\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle v_z\rangle^2}\right)$$

And from Problem 2.H:

$$Q = \pi R^2 \langle v_z\rangle = \pi\left[\frac{(\mathcal{P}_0-\mathcal{P}_L)R}{2mL}\right]^s \frac{R^3}{s+3}$$

When the second relation is solved to get $(\mathcal{P}_0-\mathcal{P}_L)$ in terms of $\langle v\rangle$ and substituted into the expression for f, we get:

$$f = \frac{\langle v_z\rangle^{n-2}\, m\, R^{-n}\,(3n+1)^n}{\frac{1}{2}\rho\, n^n}$$

$$= \underbrace{\left[\!\!\left[\frac{m}{D^n\langle v_z\rangle^{2-n}\rho}\right]\!\!\right]}_{\text{call this } Re_n} 2^{n+1}\,\frac{(3n+1)^n}{n^n} = \frac{16}{Re_n'}.$$

when

$$Re_n' = \frac{Re_n}{2^{n-3}\left(3+\frac{1}{n}\right)^n}$$

The only advantage to defining Re_n' is that the result for f has the same form as the expression for f for Newtonian fluids.

6.L Inadequacy of Mean Hydraulic Radius for Laminar Flow

(a) For the annulus with radii κR and R respectively the mean hydraulic radius is:

$$R_h = \frac{S}{Z} = \frac{\pi R^2(1-\kappa^2)}{2\pi R(1+\kappa)} = \frac{R}{2}(1-\kappa)$$

Then Eq. 6.2-16 with $f = 16/Re$ becomes:

$$\frac{p_0 - p_L}{\frac{1}{2}\rho\langle v\rangle^2} = \frac{L}{\frac{R}{2}(1-\kappa)} = \frac{16}{\frac{2R(1-\kappa)\langle v\rangle\rho}{\mu}}$$

Solving for $\langle v \rangle$ we get:

$$\langle v \rangle = \frac{(p_0 - p_L)R^2}{8\mu L}\left[(1-\kappa)^2\right]$$

(b) At $\kappa = \frac{1}{2}$, the bracketed quantity in Eq. 2.4-5 is:

$$\left[(1+\kappa^2) - \frac{(1-\kappa^2)}{\ln\frac{1}{\kappa}}\right] = \left(\frac{5}{4}\right) - \frac{\left(\frac{3}{4}\right)}{0.69} = 1.25 - 1.08 = 0.17$$

At $\kappa = \frac{1}{2}$, the bracketed quantity in the equation for $\langle v \rangle$ at the top of this page is:

$$\left[(1-\kappa)^2\right] = 0.25$$

Hence, the error in using the mean hydraulic radius is

$$\frac{0.25 - 0.17}{0.17} = 0.47 \quad \text{or} \quad 47\%$$

6.M Falling Sphere in Newton's Law Region

Newton's law of motion is for a sphere of fixed mass m

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

The force $\vec{F}$ is made up of

$$F_g = \text{gravitational force} = mg$$

$$F_k = \text{drag force} = (\pi R^2)\left(\tfrac{1}{2}\rho v^2\right)(0.44) = mgc^2v^2$$

where c is a constant defined by the above. Then Newton's second law of motion becomes for the z-direction:

$$\frac{dv}{dt} = g\left(1 - c^2v^2\right)$$

Integration for $v=0$ at $t=0$ then yields:

$$\int_0^v \frac{dv}{1-c^2v^2} = g\int_0^t dt$$

or

$$\frac{1}{c}\tanh^{-1} cv = gt$$

and hence $\frac{dz}{dt} \equiv v = \frac{1}{c} \tanh cgt$

Note that as $t \to \infty$, $\tanh cgt \to 1$. Hence the "terminal velocity" is $1/c$. A second integration then gives (for $z=0$ at $t=0$):

$$\int_0^z dz = \frac{1}{c}\int_0^t \tanh cgt \ dt$$

or

$$z = \frac{1}{c^2 g}\int_0^{cgt} \tanh \beta \ d\beta$$

$$= \frac{1}{c^2 g} \log_e \cosh cgt$$

But $mgc^2 = 0.22 \pi R^2 \rho$, so that

$$c = \sqrt{0.22 \pi R^2 \rho / mg}$$

$$= \sqrt{\frac{0.22 \pi R^2 \rho}{\frac{4}{3}\pi R^3 \rho_{sph} g}}$$

$$= \sqrt{\left(\frac{3}{4}\right)(0.22)\left(\frac{\rho}{\rho_{sph}}\right)\left(\frac{1}{Rg}\right)}$$

Note that this solution assumes that the particle is always in the Newton's law region during its entire trajectory. We have used the initial condition that $v=0$ at $t=0$, which is clearly outside the Newton's law region.

CHAPTER 7 — Checked by V. D. Shah

7.A Pressure Rise in a Sudden Expansion

According to Eq. 7.5-6

$$p_2 - p_1 = \rho v_2^2 \left(\frac{1}{\beta} - 1\right) \qquad \text{where } \beta = \frac{S_1}{S_2}$$

We now compute β and v_2 thus:

$$\beta = \frac{S_1}{S_2} = \left(\frac{D_1}{D_2}\right)^2 = \left(\frac{5}{9}\right)^2 = 0.31$$

$$v_2 = \frac{Q}{S_2} = \frac{(450 \ \frac{\text{gals}}{\text{min}})(0.00222 \ \frac{\text{ft}^3\,\text{sec}^{-1}}{\text{gals min}^{-1}})}{\pi \ (3/8 \ \text{ft})^2} = 2.27 \ \text{ft sec}^{-1}$$

Hence

$$p_2 - p_1 = \left(63 \ \frac{\text{lb}_m}{\text{ft}^3}\right)\left(2.27 \ \frac{\text{ft}}{\text{sec}}\right)^2 \left(\frac{1}{0.31} - 1\right)\left(\frac{1}{144} \ \frac{\text{ft}^2}{\text{in}^2}\right)\left(\frac{1}{32.2} \ \frac{\text{lb}_f}{\text{poundals}}\right)$$

$$= 0.157 \ \text{psi}$$

7.B Compressible Gas Flow in Cylindrical Pipes

From Eq. 7.3-2

$$\hat{E}_v = -\int_{p_1}^{p_2} \frac{1}{\rho} dp - \left(\tfrac{1}{2} v_2^2 - \tfrac{1}{2} v_1^2\right)$$

$$= \frac{RT}{M} \ln \frac{p_1}{p_2} + \frac{1}{2}\left(\frac{wRT}{SM}\right)^2 \left(\frac{1}{p_1^2} - \frac{1}{p_2^2}\right)$$

$$= \frac{(1.987)(537.4)}{(28)} \ln 2$$

$$+ \frac{1}{2}\left(\frac{(0.28)(0.7302)(5.364)}{\frac{\pi(1/4)^2}{4} \cdot 28}\right)^2 \left(\frac{1}{2^2} - \frac{1}{1^2}\right) \frac{1}{32.2} \quad \frac{1}{778}$$

$$= 26.4 - 0.1 = 26.3 \ \text{Btu lb}_m^{-1}$$

Here we have used $R = 1.987 \ \frac{\text{Btu}}{\text{lb mole °R}} = 0.7302 \ \frac{\text{ft}^3\,\text{atm}}{\text{lb mole °R}}$

and

$1 \ \text{lb}_f = 32.2$ poundals
$1 \ \text{Btu} = 778 \ \text{lb}_f \ \text{ft}$

7.C Incompressible Flow in an Annulus

(i) The mean hydraulic radius is (see Eq. 6.2-14)

$$R_h = \frac{S}{Z} = \frac{\pi(R_2^2 - R_1^2)}{2\pi(R_2+R_1)} = \frac{1}{2}(R_2-R_1)$$

and the average flow velocity is

$$\langle v \rangle = \frac{Q}{S} = \frac{Q}{\pi(R_2^2-R_1^2)}$$

(ii) The Reynolds number is (see Eq. 6.2-16)

$$Re = \frac{4R_h\langle v\rangle\rho}{\mu} = \frac{2(R_2-R_1)Q\rho}{\pi(R_2^2-R_1^2)\mu} = \frac{2Q\rho}{\pi(R_2+R_1)\mu}$$

$$= \frac{2\left(240\,\frac{\text{gal}}{\text{min}}\right)\left(3785\,\frac{\text{cm}^3}{\text{gal}}\right)\left(\frac{1}{60}\,\frac{\text{min}}{\text{sec}}\right)\left(1\,\frac{\text{gm}}{\text{cm}^3}\right)}{\pi\,(5\text{ in.})\left(2.54\,\frac{\text{cm}}{\text{in}}\right)\left(0.001\,\frac{\text{g}}{\text{cm sec}}\right)} = 7.56\times10^4$$

Hence flow is turbulent and $f = 0.0047$

(iii) From Eq. 7.4-10 we determine $-\hat{W}\rho Q$ (rate of doing work in foot-poundals per second):

$$-\rho\hat{W}Q = g(h_2-h_1)\rho Q + \frac{1}{2}Q\rho\langle v\rangle^2\frac{L}{R_h}f$$

$$= g(h_2-h_1)\rho Q + \cancel{\frac{1}{2}}\frac{Q^3\rho}{\pi^2(R_2^2-R_1^2)^2}\cdot\frac{L}{\cancel{\frac{1}{2}}(R_2-R_1)}\cdot f$$

$$= \left(32.2\,\frac{\text{ft}}{\text{sec}^2}\right)(5\text{ ft})\left(62.4\,\frac{\text{lb}_m}{\text{ft}^3}\right)\left(240\,\frac{\text{gal}}{\text{min}}\times0.134\,\frac{\text{ft}^3}{\text{gal}}\times\frac{1}{60}\,\frac{\text{min}}{\text{sec}}\right)$$

$$+ \frac{\left(240\times0.134\times\frac{1}{60}\right)^3(62.4)}{\pi^2\left[\left(\frac{7}{24}\right)^2-\left(\frac{3}{24}\right)^2\right]}\cdot\frac{20.3}{\frac{1}{12}\left(\frac{7}{2}-\frac{3}{2}\right)}\cdot(0.0047)$$

$$= 5360 + 114 = 5470\text{ ft-poundals sec}^{-1} = 0.31\text{ hp}$$

Keep in mind that the mean hydraulic radius is an empiricism. A recent analysis of available experimental data (D.M. Meter and R.B. Bird, AIChE Journal (1961)) shows that for annuli the mean hydraulic radius gives friction factors which are off by as much as 15 or 20%, in the turbulent regime. Keep in mind further that R_h should not be used for laminar flow (see Problem 6.L).

7.D Force on a U-bend in a Pipe

The net force on the U-bend will be the sum of the force of the fluid and the force of the surrounding atmosphere. The force of the fluid in the x-direction is:

$$F = \frac{\rho\langle v_1\rangle^2 S_1^2}{S_1} \underset{\uparrow}{+} \frac{\rho\langle v_2\rangle^2 S_2^2}{S_2} + p_1 S_1 \underset{\uparrow}{+} p_2 S_2$$

(Signs marked with an arrow are "+" because the exit control surface — at plane "2" — is oriented in the $-x$-direction.) Insertion of numerical values gives:

$$F = 2 \cdot \left[\frac{\left(62.4 \frac{\text{lb}_m}{\text{ft}^3}\right)(3 \text{ ft}^3 \text{ sec}^{-1})^2}{\pi \left(\frac{1}{6} \text{ft}\right)^2} \right]$$

$$+ \{21 + 14.7\} \, \pi \, (2 \text{ in})^2 \cdot (32.2)$$

$$+ \{19 + 14.7\} \, \pi \, (2 \text{ in})^2 \cdot (32.2)$$

$$= 1.29 \times 10^4 + 2.81 \times 10^4 = 4.10 \times 10^4 \text{ poundals} = 1.27 \times 10^3 \text{ lb}_f$$

Now to get the net force, omit the two 14.7's and get:

$$F_{net} = (1.29 \times 10^4) + (1.62 \times 10^4) = 2.91 \times 10^4 \text{ poundals} = 905 \text{ lb}_f$$

7.E Disintegration of Wood Chips

(i) From Eq. 7.4-10 taking "1" at the top of the slurry suspension and "2" at the outlet to the digestor:

$$\left[\tfrac{1}{2}\langle v_2\rangle^2 - 0\right] + \left[0 - gh_1\right] + \left(\frac{p_2 - p_1}{\rho}\right) = 0$$

Hence
$$\langle v \rangle = \sqrt{2\left[\left(\frac{p_1 - p_2}{\rho}\right) + gh_1\right]}$$

$$= \sqrt{2\left[\frac{100 \frac{\text{lb}_f}{\text{in}^2} \times 144 \frac{\text{in}^2}{\text{ft}^2} \times 32.2 \frac{\text{poundals}}{\text{lb}_f}}{65 \text{ lb}_m \text{ ft}^{-3}} + 32.2 \frac{\text{ft}}{\text{sec}^2} \times 20 \text{ft}\right]}$$

$$= \sqrt{15550} = 124 \text{ ft sec}^{-1}$$

And $w = \rho \langle v \rangle S = (65 \frac{lb_m}{ft^3})(124 \frac{ft}{sec})(\frac{\pi}{9} ft^2) = 2810 \, lb_m \, sec^{-1}$

To get the initial impact force we apply Eq. 7.2-3 between plane "2" and plane "3" (the impact plane):

$$F = v_2 w_2 = (124 \frac{ft}{sec})(2810 \frac{lb_m}{sec})(\frac{1}{32} \frac{lb_f}{poundals}) = 10{,}900 \, lb_f$$

7.F Calculation of Flow Rate

Eq. 7.4-10 for this system is (H = depth of liquid in tank):

$$-gh_1 - \frac{1}{\rho}(\rho g H) + \frac{v^2}{2}\frac{L}{D}4f + \Sigma_i \left(\tfrac{1}{2}v^2 e_v\right)_i = 0$$

or

$$2\left(\frac{Re\,\mu}{D\rho}\right)^2 \frac{L}{D} f + \frac{1}{2}\left(\frac{Re\,\mu}{D\rho}\right)^2 \Sigma (e_v)_i = g(H + h_1)$$

For the situation described:

$$\mu = 6.72 \times 10^{-4} \, lb_m \, ft^{-1} \, sec^{-1}$$
$$\rho = 62.4 \, lb_m \, ft^{-3}$$
$$D = (5/12) \, ft$$
$$L = 27 + 11 + 14 = 52 \, ft$$
$$\Sigma (e_v)_i = 0.5 + 2(0.7) = 1.9$$
$$H + h_1 = 12 + 27 + 14 = 53 \, ft$$

Hence

$$2\left(\frac{6.72 \times 10^{-4}}{(\frac{5}{12})(62.4)}\right)^2 \left(\frac{52}{(\frac{5}{12})}\right) Re^2 f + \frac{1}{2}\left(\frac{6.72 \times 10^{-4}}{(\frac{5}{12})(62.4)}\right)^2 (1.9) Re^2 = (32.2)(53)$$

or:

$$Re^2 f + 3.8 \times 10^{-3} Re^2 = 1.03 \times 10^{10}$$

Trial and error solution:

Re	f	$Re^2 f$	$[3.8 \times 10^{-3} Re^2 + Re^2 f]$
2×10^6	0.0025	1×10^{10}	2.5×10^{10}
1×10^6	0.0029	0.29×10^{10}	0.67×10^{10}
1.5×10^6	0.0021	0.61×10^{10}	1.41×10^{10}

Graphical interpolation gives $Re = 1.25 \times 10^6$.

Then, knowing the Reynolds number we can get the average flow velocity:

$$\langle v \rangle = Re \times \left(\frac{\mu}{D\rho}\right) = (1.23 \times 10^{6})(2.58 \times 10^{-5}) = 31.8 \text{ ft sec}^{-1}$$

The volume rate of flow is:

$$Q = \langle v \rangle S = \left(31.8 \ \frac{\text{ft}}{\text{sec}}\right)\left(\pi \left(\frac{5}{24}\right)^2 \text{ft}^2\right)\left(449 \ \frac{\text{gal/min}}{\text{ft}^3/\text{sec}}\right) = 1930 \ \frac{\text{gal.}}{\text{min}}$$

7.G. Evaluation of Various Averages of Velocity from Pitot Tube Data

Use of Simpson's rule gives:

$$\frac{\langle \bar{v} \rangle}{\bar{v}_{max}} = 0.84 \ ; \qquad \frac{\langle \bar{v}^2 \rangle}{\bar{v}^2_{max}} = 0.74 \ ; \qquad \frac{\langle \bar{v}^3 \rangle}{\bar{v}^3_{max}} = 0.67$$

7.H Velocity Averages in Turbulent Flow

(a) For the turbulent velocity distribution $\dfrac{\bar{v}}{\bar{v}_{max}} = \left(1 - \dfrac{r}{R}\right)^{\frac{1}{7}}$

we have:

$$\frac{\langle \bar{v}^2 \rangle}{\langle v \rangle^2} = \frac{\int_0^{2\pi}\int_0^R \left(1 - \frac{r}{R}\right)^{\frac{2}{7}} r\,dr\,d\theta}{\int_0^{2\pi}\int_0^R r\,dr\,d\theta} \left[\frac{\int_0^{2\pi}\int_0^R r\,dr\,d\theta}{\int_0^{2\pi}\int_0^R \left(1 - \frac{r}{R}\right)^{1/7} r\,dr\,d\theta}\right]^2$$

$$= \frac{\int_0^R \left(1 - \frac{r}{R}\right)^{\frac{2}{7}} r\,dr}{\frac{R^2}{2}} \left[\frac{\frac{R^2}{2}}{\int_0^R \left(1 - \frac{r}{R}\right)^{\frac{1}{7}} r\,dr}\right]^2$$

$$[\eta = 1 - \xi] \qquad = \frac{2\int_0^1 (1-\xi)^{2/7}\,\xi\,d\xi}{\left[2\int_0^1 (1-\xi)^{1/7}\,\xi\,d\xi\right]^2} = \frac{2\int_0^1 \eta^{2/7}(1-\eta)\,d\eta}{\left[2\int_0^1 \eta^{1/7}(1-\eta)\,d\eta\right]^2}$$

$$= \frac{2\left[\frac{7}{9} - \frac{7}{16}\right]}{4\left[\frac{7}{8} - \frac{7}{15}\right]^2} = 1.02 \qquad \therefore \text{ Error } = \frac{1.02 - 1.00}{1.02} \doteq 2\%$$

(b) Similarly:

$$\frac{\langle \bar{v}^3 \rangle}{\langle \bar{v} \rangle^3} = \frac{2\int_0^1 \eta^{3/7}(1-\eta)\,d\eta}{\left[2\int_0^1 \eta^{1/7}(1-\eta)\,d\eta\right]^3} = 1.06 \qquad \therefore \text{Error} \doteq 6\%$$

7.I Multiple Discharge into a Common Conduit

(a) As plane "1" we select the collection of cross-sections of all the small tubes leading into the big tube; plane "2" is taken to be a plane far enough down the tube that a distinct velocity profile has been established. Over the system thus defined we make three balances:

MASS BALANCE: At steady state $W_1 = W_2$ or $\langle v_1\rangle S_1 = \langle v_2\rangle S_2$ so that

$$⊛ \quad \beta = \frac{S_1}{S_2}$$

MOMENTUM BALANCE: At steady state with no external forces:

$$\rho\langle v_1^2\rangle S_1 - \rho\langle v_2^2\rangle S_2 + p_1 S_1 - p_2 S_2 - F = 0$$

The force F of the fluid on the solid will consist of the viscous forces acting tangentially at the walls (which we neglect) and the normal force $p_1(S_2 - S_1)$ acting in the direction <u>opposite</u> to the direction of flow. Hence:

$$\rho\langle v_1^2\rangle S_1 - \rho\langle v_2^2\rangle S_2 + p_1 S_1 - p_2 S_2 \underset{\uparrow}{+} p_1(S_2 - S_1) = 0$$

The result may be rewritten by using the mass balance and the definition of a set of quantities $K_i^{(j)}$ defined in Eq. 7.I-3:

$$⊛ \quad (p_2 - p_1) = \rho\langle v_1\rangle^2\left[\beta K_1^{(2)} - \beta^2 K_2^{(2)}\right]$$

MECHANICAL ENERGY BALANCE: At steady-state the mechanical energy balance gives:

$$\hat{E}_v = \frac{p_1 - p_2}{\rho} + \frac{1}{2}\left(\frac{\langle v_1^3\rangle}{\langle v_1\rangle} - \frac{\langle v_2^3\rangle}{\langle v_2\rangle}\right)$$

We insert into this the expression obtained from the momentum balance for $(p_2 - p_1)$; thereby we obtain:

$$⊛ \quad \hat{E}_v = \tfrac{1}{2}K_1^{(3)}\langle v_1\rangle^2\left[1 - 2\beta\frac{K_1^{(2)}}{K_1^{(3)}} + \beta^2\left(2\frac{K_2^{(2)}}{K_1^{(3)}} - \frac{K_2^{(3)}}{K_1^{(3)}}\right)\right]$$

The equations marked with ⊛ are the results of applying the macroscopic balances.

(b) The calculation of the $K_i^{(j)}$ has been described in the solution to Problem 7.H.

(c) For $\beta = 0$ (i.e. for discharge into a very large tank) the kinetic energy of the entering stream is completely dissipated, regardless of the flow regime in the small tube. For laminar flow in small tubes and highly turbulent flow in the large tube, and with $\beta \approx 1$, we have $K_1^{(3)} = 2$ and $K_2^{(3)} \doteq 1$ so that the [ ] in the expression for $\hat{E}_v$ is 1/6.

7.J Evaluation of Velocity Averages for Laminar Flow of Non-Newtonian Fluids in Circular Tubes

(a) The method is the same as in Problem 7.H. ; we set

$$\frac{v_z}{v_{max}} = 1 - \left(\frac{r}{R}\right)^{s+1} \equiv 1 - \xi^{s+1}$$

where $s = 1/n$, and perform the same integrations as in 17.H. For example:

$$\frac{\langle v^3 \rangle}{v^3_{max}} = \frac{\int_0^1 (1-\xi^{s+1})^3 \, \xi \, d\xi}{\int_0^1 \xi \, d\xi}$$

$$= 2\int_0^1 \left(1 - 3\xi^{s+1} + 3\xi^{2s+2} - \xi^{3s+3}\right) \xi d\xi$$

$$= \frac{3(s+1)^3}{(s+3)(s+2)(3s+5)}$$

And:

$$\frac{\langle v \rangle}{v_{max}} = \frac{\int_0^1 (1-\xi^{s+1}) \xi \, d\xi}{\int_0^1 \xi \, d\xi} = \frac{s+1}{s+3}$$

Hence:

$$\frac{\langle v^3 \rangle}{\langle v \rangle^3} = \frac{3(s+3)^2}{(s+2)(3s+5)} = \frac{3(3n+1)^2}{(2n+1)(5n+3)}$$

The expression for the $\langle v^2 \rangle / \langle v \rangle^2$ is obtained similarly

(b) For the Bingham fluid:

$$\langle v \rangle = K\left(1 - \tfrac{4}{3}\xi_0 + \tfrac{1}{3}\xi_0^4\right)$$

$$\langle v^2 \rangle = 4K^2\left(\tfrac{1}{15}\xi_0^6 - \tfrac{1}{6}\xi_0^5 + \tfrac{1}{12}\xi_0^4 + \tfrac{1}{6}\xi_0^2 - \tfrac{7}{30}\xi_0 + \tfrac{1}{12}\right)$$

$$\langle v^3 \rangle = 6K^3\left(\tfrac{47}{840}\xi_0^8 - \tfrac{4}{15}\xi_0^7 + \tfrac{7}{15}\xi_0^6 - \tfrac{1}{3}\xi_0^5 + \tfrac{1}{12}\xi_0^4 - \tfrac{2}{15}\xi_0^3 + \tfrac{4}{15}\xi_0^2 - \tfrac{76}{420}\xi_0 + \tfrac{1}{24}\right)$$

In which $\xi_0 = \frac{r_0}{R}$ and $K = \frac{\Delta p R^2}{4\mu_0 L}$

7.K Friction Losses in Non-Newtonian Flow

(a) From the solution to Problem 2.H:

$$v_z = \left[\frac{(\mathcal{P}_0 - \mathcal{P}_L)R}{2mL}\right]^s \frac{R}{s+1}\left[1 - \left(\frac{r}{R}\right)^{s+1}\right]$$

$$\langle v_z \rangle = \left[\frac{(\mathcal{P}_0 - \mathcal{P}_L)R}{2mL}\right]^s \frac{R}{s+3}$$

Dividing gives:

$$\frac{v_z}{\langle v_z \rangle} = \left(\frac{s+3}{s+1}\right)\left(1 - \left(\frac{r}{R}\right)^{s+1}\right) \qquad \text{where } s = \frac{1}{n}$$

(b)
$$E_v = -\int (\underset{\sim}{\tau} : \nabla \underset{\sim}{v})\, dV$$

$$= 2\pi \int_0^L \int_0^R m \left|\frac{dv_z}{dr}\right|^{n-1} \frac{dv_z}{dr}\frac{dv_z}{dr}\, r\,dr\,dz$$

$$= 2\pi m L \int_0^R \left(-\frac{dv_z}{dr}\right)^{n+1} r\,dr$$

$$= \frac{2\pi m L \langle v_z \rangle^{n+1}}{R^{n-1}} (s+3)^{n+1} \int_0^1 \left(\xi^{\frac{1}{n}}\right)^{n+1} \xi\, d\xi$$

$$= \frac{2\pi m L \langle v_z \rangle^{n+1}}{R^{n-1}} \left(3 + \frac{1}{n}\right)^n$$

(c)
$$E_v = \langle v_z \rangle \cdot F \qquad \text{(Evaluate } F \text{ from Problem 6.K)}$$

$$= \langle v_z \rangle \cdot \tfrac{1}{2}\rho \langle v \rangle^2 \cdot 2\pi R L \cdot f$$

$$= \langle v \rangle \cdot \tfrac{1}{2}\rho \langle v \rangle^2 \cdot 2\pi R L \cdot \frac{16}{\left(\dfrac{D^n \langle v_z \rangle^{2-n} \rho / m}{2^{n-3}\left(3 + \frac{1}{n}\right)^n}\right)}$$

$$= \frac{2\pi m L \langle v_z \rangle^{n+1}}{R^{n-1}} \left(3 + \frac{1}{n}\right)^n \qquad \text{which is same as result in (b).}$$

7.L Inventory Reservations in a Gas Reservoir

(a) $(W_2)_{max} = A + B$

$(W_2)_{min} = A - B$

$$(W_2)_{avg} = \int_{\omega t=0}^{2\pi} (A + B \cos \omega t)\, dt \Big/ \int_0^{2\pi} \cos \omega t \, dt = A$$

(b) A mass balance over a 24-hour period gives:

$$\int_0^{24} \frac{dm}{dt}\, dt = 0 = \int_0^{24} w_1 \, dt - \int_0^{24} w_2 dt = 24\, w_1 - 24\, A$$

whence $w_1 = A$

(c) $$m_{tot} = m_o + \int_0^t (w_1 + w_2)\, dt$$

$$= m_o + \int_0^t (A - A - B \cos \omega t)\, dt$$

$$= m_o - \frac{B}{\omega} \sin \omega t$$

(d) The minimum reservoir capacity which will permit steady operation is just the difference between the maximum and minimum values of m_{tot} :

$$\rho V = \left(m_o + \frac{B}{\omega}\right) - \left(m_o - \frac{B}{\omega}\right) = \frac{2B}{\omega}$$

$$\therefore V = \frac{2B}{\rho\omega} = \frac{(2)(2000)}{(0.044)(2\pi/24)} = 3.47 \times 10^5 \text{ ft}^3$$

(e) Add a three-day supply to the amount found above:

$$\rho V = \frac{2B}{\omega} + (3 \times 24A)$$

$$V = \frac{2B}{\rho\omega} + \frac{72A}{\rho} = (3.47 \times 10^5) + \frac{(72)(5{,}000)}{(0.044)}$$

$$= (3.47 \times 10^5) + (8.18 \times 10^6)$$

$$= (8.53 \times 10^6) \text{ ft}^3$$

CHAPTER 8 - Checked by T. J. Sadowski

8.A Prediction of Thermal Conductivities of Gases at Low Density.

a. For argon we may write, from Table B-1: $M = 39.944$, $\sigma = 3.418$ Å, and $\epsilon/\kappa = 124$°K. Then at 100°C:

$$\kappa T/\epsilon = 373.16/124 = 3.009 \quad ; \quad \Omega_K = 1.038$$

Substitution of the above values into Eq. 8.3-13 gives:

$$k = 1.9891 \times 10^{-4}\sqrt{373.16/39.944}/(3.418)^2(1.038)$$
$$= 502. \times 10^{-7} \text{ cal sec}^{-1}\text{ cm}^{-1}\text{ °K}^{-1}$$

b. For NO: substitution into the Eucken formula, Eq. 8.3-15, gives:

$$k = (7.15 + \tfrac{5}{4}\cdot 1.987)(1929 \times 10^{-7})/30.01$$
$$= 619. \times 10^{-7} \text{ cal sec}^{-1}\text{ cm}^{-1}\text{ °K}^{-1}$$

This calculation checks the measured value exactly.

For CH_4:

$$k = (8.55 + \tfrac{5}{4}\cdot 1.987)(1116 \times 10^{-7})/16.04$$
$$= 768 \times 10^{-7} \text{ cal sec}^{-1}\text{ cm}^{-1}\text{ °K}^{-1}$$

The experimental value is 6.6% higher. Such a discrepancy is not unusual for polyatomic molecules.

8.B Computation of the Prandtl Number for Gases at Low Density

	Prandtl Number	
Gas	a. From Eq. 8.3-13	b. From observed properties
He	0.667	0.697
Ar	0.667	0.671
H_2	0.735	0.721
Air	0.737	0.740
CO_2	0.782	0.789
H_2O	0.764	1.003

8.C Prediction of the Thermal Conductivity of a Dense Gas.

a. The critical properties of methane are: $T_c = 191.0\ ^\circ K$, $p_c = 45.8$ atm, and $k_c = 158 \times 10^{-6}$ cal $\text{sec}^{-1}\ \text{cm}^{-1}\ ^\circ K^{-1}$.

For the conditions of the problem: $T_r = \frac{460 + 127}{1.8 \times 190.7} = 1.71$

$p_r = \frac{110.4}{45.8} = 2.41$. From Fig. 8.2-1 we find $k_r = 0.77$. Hence:

$$\begin{aligned} k = k_r k_c &= 0.77 \times 158 \times 10^{-6} \\ &= 1.22 \times 10^{-4} \text{ cal sec}^{-1} \text{ cm}^{-1}\ ^\circ K^{-1} \\ &= 2.94 \times 10^{-2} \text{ Btu hr}^{-1} \text{ ft}^{-1}\ ^\circ F^{-1} \end{aligned}$$

These calculated results are 4 % above the observed value.

b. We first compute the viscosity of methane at 127 °F ($326^\circ K$) from Eq. 1.4-18:

$$\kappa T/\epsilon = 326/137 = 2.38 \;;\; \Omega_\mu = 1.110$$

$$\mu = 2.6693 \times 10^{-5} \frac{\sqrt{(16.04)(326)}}{(3.822)^2(1.110)}$$

$$= 1191 \times 10^{-7} \text{ g sec}^{-1} \text{ cm}^{-1}$$

We next use the Eucken equation, Eq. 8.3-15 to calculate the thermal conductivity at low pressure:

$$k = (8.86 + 2.484) \frac{1191 \times 10^{-7}}{16.04}$$

$$= 842 \times 10^{-7} \text{ cal sec}^{-1} \text{ cm}^{-1}\ ^\circ K^{-1}$$

This value is to be used as k° in determining the thermal conductivity at 110.4 atm from Fig. 8.2-2. From this figure we find $k^{\#} = 1.4$. Hence:

$$\begin{aligned} k = k^{\#} k^\circ &= (1.4)(842 \times 10^{-7}) \\ &= 1180 \times 10^{-7} \text{ cal sec}^{-1} \text{ cm}^{-1}\ ^\circ K^{-1} \\ &= 0.028 \text{ Btu hr}^{-1} \text{ ft}^{-1}\ ^\circ F^{-1} \end{aligned}$$

Note that this result is in very good agreement with the observed value.

8.D Prediction of the Thermal Conductivity of a Gas Mixture.

Data for this problem are summarized in the table immediately below:

Component	M	$\mu \times 10^7$, g sec^{-1} cm^{-1}	$k \times 10^7$, cal sec^{-1} cm^{-1} $°K^{-1}$	Mole fraction
1 (H_2)	2.016	896	4250	0.80
2 (CO_2)	44.010	1495	383	0.20

Insertion of these data into Eq. 8.3-18 (or Eq. 1.4-20) gives:

$$\Phi_{11} = \Phi_{22} = 1.0$$

$$\Phi_{12} = \frac{1}{\sqrt{8}}\left[1 + \frac{2.016}{44.010}\right]^{-1/2}\left[1 + \left(\frac{896}{1495}\right)^{1/2}\left(\frac{44.010}{2.016}\right)^{1/4}\right]^2$$

$$= 2.47$$

$$\Phi_{21} = \frac{1}{\sqrt{8}}\left[1 + \frac{44.010}{2.016}\right]^{-1/2}\left[1 + \left(\frac{1495}{896}\right)^{1/2}\left(\frac{2.016}{44.010}\right)^{1/4}\right]^2$$

$$= 0.189$$

Substitution of these calculated Φ_{ij} into Eq. 8.3-17 gives:

$$k_{mix} = \frac{(0.80)(4250 \times 10^{-7})}{(0.80)(1.0) + (0.20)(2.47)} + \frac{(0.20)(383 \times 10^{-7})}{(0.80)(0.189) + (0.20)(1.0)}$$

$$= 2630 \times 10^{-7} + 218 \times 10^{-7}$$

$$= 2850 \times 10^{-7} \text{ cal sec}^{-1} \text{ cm}^{-1} \text{ °K}^{-1}$$

8.E Prediction of the Thermal Conductivity of a Pure Liquid.

First we compute $(\partial p/\partial \rho)_T = \rho^{-1}[\rho^{-1}(\partial \rho/\partial p)_T]^{-1} = \frac{10^6}{(0.9938)(38)}$

$$= 2.648 \times 10^4 \text{ megabar cm}^3 \text{ g}^{-1}$$

$$= 2.648 \times 10^{10} \text{ cm}^2 \text{ sec}^{-2}$$

We now substitute this value of $(\partial p/\partial \rho)_T$ into Eq. 8.4-4. By assuming $C_p = C_v$ we thus obtain:

$$v_s = \sqrt{(1.0)(2.648 \times 10^{10})} = 1.627 \times 10^5 \text{ cm sec}^{-1}$$

We next use Eq. 8.4-3 to obtain the thermal conductivity:

$$k = 2.80\,(\tilde{N}\rho/M)^{2/3}$$

$$= 2.80\left[\frac{(6.023\times10^{23})(0.9938)}{18.02}\right]^{2/3}\times(1.3805\times10^{-16})(1.627\times10^{5})$$

$$= 2.80\,(10.33\times10^{14})(1.3805\times10^{-16})(1.627\times10^{5})$$

$$= 6.50\times10^{4}\ \text{erg sec}^{-1}\ \text{cm}^{-1}\ {}^\circ\text{K}^{-1}$$

$$= 1.55\times10^{-3}\ \text{cal sec}^{-1}\ \text{cm}^{-1}\ {}^\circ\text{K}^{-1}$$

$$= 0.375\ \text{Btu hr}^{-1}\ \text{ft}^{-1}\ {}^\circ\text{F}^{-1}$$

8.F Calculation of Molecular Diameters from Transport Properties.

a. We may rewrite Eq. 1.4-9 to obtain:

$$d = (2/3\mu)^{1/2}\,(M\kappa T/\tilde{N}\,\pi^{3})^{1/4}$$

On substition of numerical values into this equation we get, using c.g.s. units:

$$d = (2/3\times2270\times10^{-7})^{1/2}\left(\frac{39.944\times1.3805\times10^{-16}\times300}{6.023\times10^{23}\times\pi^{3}}\right)^{1/4}$$

$$= 2.95\times10^{-8}\,\text{cm} = 2.95\ \text{Å}$$

b. Rearrangement of Eq. 8.3-12 gives:

$$d = \sqrt{1/k}\,(\kappa^{3}T\tilde{N}/M\pi^{3})^{1/4}$$

In terms of c.g.s. units we may then write:

$$d = \sqrt{1/1761}\left(\frac{2.631\times10^{-48}\times300\times6.023\times10^{23}}{39.944\times\pi^{3}}\right)$$

$$= 1.88\times10^{-8}\,\text{cm} = 1.88\ \text{Å}$$

Note that this result is considerably lower than that obtained from fluid viscosity in part a. Note also that k is expressed in terms of ergs rather than calories in making this calculation.

c. Equation 1.4-18 may be rearranged to give:

$$\sigma = (2.6693\times10^{-5}/\mu\,\Omega_{\mu})^{1/2}\,(MT)^{1/4}$$

Substituting numerical values as above we get, with $\kappa T/\epsilon = 2.419$

$$\Omega_{\mu} = \Omega_{k} = 1.104$$

$$\sigma = \left(\frac{2.6693\times10^{-5}}{2270\times10^{-7}\times1.104}\right)^{1/2}(39.944\times300)^{1/4} = 3.415\ \text{Å}$$

Equation 8.3-13 gives:

$$\sigma = \left(\frac{1.9891\times10^{-4}}{421\times10^{-7}\times1.104}\right)^{1/2}\left(\frac{300}{39.944}\right)^{1/4} = 3.425\ \text{Å}$$

d. Both values of σ agree closely with the value of 3.418 Å given in Table B-1. This shows that the Chapman-Enskog theory can be used to predict k from measurements of μ, whereas the simple kinetic theory cannot.

CHAPTER 9 - Checked by V.D. Shah

9.A Heat Loss from an Insulated Pipe.

We make use of the notation in Fig. 9.6-2. When the wall temperatures are known (i.e. T_0 and T_3), Eq. 9.6-29 may be simplified to give:

$$\frac{Q_0}{L} = \frac{2\pi(T_0 - T_3)}{\frac{\ln(r_1/r_0)}{k^{01}} + \frac{\ln(r_2/r_1)}{k^{12}} + \frac{\ln(r_3/r_2)}{k^{23}}}$$

The r_i for this problem are:

$r_0 = 2.067/2 = 1.03''$

$r_1 = 1.03 + 0.15 = 1.18''$

$r_2 = 1.18 + 2.0 = 3.18''$

$r_3 = 3.18 + 2.0 = 5.18''$

Substitution of numerical values into the above formula gives:

$$\frac{Q_0}{L} = \frac{2\pi(250°F - 90°F)}{2.303\left[\frac{\log(1.18/1.03)}{26.1} + \frac{\log(3.18/1.18)}{0.35} + \frac{\log(5.18/3.18)}{0.03}\right]}$$

$$= \frac{320\pi}{2.303\left[\frac{0.059}{26.1} + \frac{0.431}{0.35} + \frac{0.212}{0.03}\right]}$$

$$= 1005/(2.303)(8.26)$$

$= 53$ Btu hr^{-1} (per foot of pipe)

9.B Heat Loss from a Rectangular Fin.

From Eq. 9.7-14 we may write:

$$Q = 2WLh(T_w - T_a)\cdot\eta$$

where η is given by Eq. 9.7-16 as:

$$\eta = \tanh(hL^2/kB)^{1/2}/(hL^2/kB)^{1/2}$$

For the conditions of this problem

$$(hL^2/kB)^{1/2} = \sqrt{\frac{(120)(0.04)}{(60)(0.08/12)}} = 2\sqrt{3}$$

We may then write:

$$Q = (2.0)(1.0)(0.2)(120)(150)\tanh(2\sqrt{3})/2\sqrt{3}$$

$$= 2080 \text{ Btu hr}^{-1}$$

9.C Maximum Temperature in a Lubricant.

We begin by multiplying both sides of Eq. 9.4-11 by $(T_b - T_o)$. We then set $(T_b - T_o)$ equal to zero to obtain:

$$T - T_o = \frac{1}{2}\frac{\mu V^2}{k}(x/b)[1-(x/b)]$$

The maximum temperature occurs at $(x/b) = 1/2$, and $V = \Omega R$. We may therefore write:

$$T_{max} - T_o = \frac{1}{8}(\mu\Omega^2R^2/k)$$

$$= \frac{1}{8}\left[\frac{(0.923)(2\pi\times7908/60)^2(5.06)^2}{(0.0065\times4.18\times10^7)}\right]$$

$$= 9°C$$

Hence $T_{max} = 158 + 9\times1.8 = 174°F$

The Reynolds number for this system is

$$Re = Rb\Omega\rho/\mu = \frac{(5.06)(0.027)(7908\times0.105)(1.22)}{(0.923)}$$

$$= 150$$

This is below the critical Reynolds number* of about 900; hence the flow is laminar as assumed in the above calculations.

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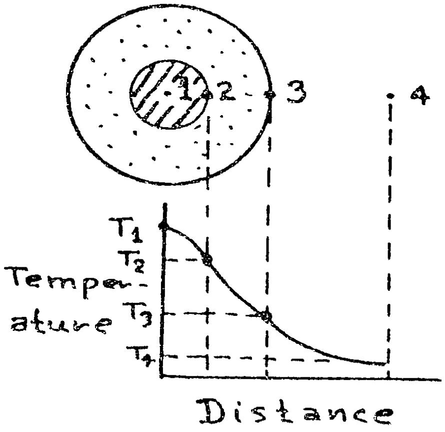
* H.L. Dryden, Sr., F.D. Murnaghan, and H. Bateman, "Hydrodynamics", Dover (New York) --- p. 205

9.D Current-carrying Capacity of a Wire.



From the temperature profile sketched at the left it is clear the maximum temperature in the plastic is T_2, the temperature at the copper-plastic interface. Therefore, for the conditions of this problem, the limiting current is reached when $T_2 = 200°F$.

We begin the solution of this problem by determining the rate of heat loss per foot of wire corresponding to a T_2 of 200°C.

From Eq. 9.6-29 the rate of heat loss is:

$$Q/L = 2\pi(T_2 - T_4)\Big/\left[\frac{\ln(r_3/r_2)}{k} + \frac{1}{r_3 h_3}\right]$$

$$= 2\pi(100)\Big/\left[\frac{\ln(0.06/0.02)}{0.20} + \frac{1}{(0.06/12)1.5}\right]$$

$$= 200\pi/[5.5 + 133] = 4.53 \text{ Btu hr}^{-1}\text{ ft}^{-1}$$

$$= 0.0435 \text{ watts cm}^{-1}$$

We next equate the heat loss to the heat generated by dissipation of electrical energy:

$$Q/L = i^2 R/L$$

where

$R = (\pi r_2^2 k_e)^{-1}$ = wire resistance per unit length

i = total current carried by the wire.

Solving for current we then get:

$$i = \sqrt{(Q/L)/(R/L)}$$

The maximum allowable current is then obtained by substituting numerical values into this equation:

$$i_{max} = (0.0435 \text{ watts cm}^{-1}/2.4\times10^{-4}\text{ ohms cm}^{-1})^{1/2}$$

$$= 13.7 \text{ amperes}$$

9.E Free-convection Velocity.

From Eq. 9.9-15 we may write:

$$\langle v_z\rangle = \frac{\bar\rho\bar\beta g b^2 \Delta T}{12\mu}\int_{-1}^{0}(\eta^3 - \eta)\,d\eta = \bar\rho\bar\beta g b^2\Delta T/48\mu$$

For the conditions of this problem:

$$\bar{\beta} = 1/\bar{T} = (1/333°K) \quad \text{(ideal-gas behavior assumed)}$$
$$\nu = 0.192 \quad cm^2\ sec^{-1}$$

Hence:

$$\langle v_z \rangle = \frac{(0.003\,°K^{-1})(980\ cm\ sec^{-2})(0.3\ cm)^2(80\,°K)}{(48)(0.192\ cm^2 sec^{-1})}$$

$$= 2.3 \quad cm \quad sec^{-1}$$

9.F Evaporation Loss from an Oxygen Tank.

a. A thermal energy balance on a shell of thickness Δr gives:

$$-4\pi\left(r^2 k \frac{dT}{dr}\right)_r + 4\pi\left(r^2 k \frac{dT}{dr}\right)_{r+\Delta r} = 0$$

Hence

$$\frac{d}{dr}\left(k\, r^2 \frac{dT}{dr}\right) = 0$$

and

$$k\, r^2 \frac{dT}{dr} = C_1$$

where C_1 is a constant of integration. In this problem we may write

$$k(T) = k_o\left[1 + \left(\frac{k_1 - k_o}{k_o}\right)\left(\frac{T - T_o}{T_1 - T_o}\right)\right.$$

where k_o is the the thermal conductivity at T_o and k_1 is the thermal conductivity at T_1. We now define a reduced temperature $\Theta = (T-T_o)/(T_1-T_o)$. Our differential equation may then be re-arranged to give:

$$k_o(T_1-T_o)\left[1 + \left(\frac{k_1-k_o}{k_o}\right)\Theta\right] d\Theta = C_1\, dr/r^2$$

This expression may be integrated to give:

$$k_o(T_1-T_o)\left[1 + \left(\frac{k_1-k_o}{k_o}\right)\frac{\Theta}{2}\right]\Theta = -\frac{C_1}{r} + C_2$$

We now make use of the boundary conditions:

At $r = r_o$, $\Theta = 0$

At $r = r_1$, $\Theta = 1$

Hence:
$$0 = -\frac{C_1}{r_o} + C_2$$

$$(1/2)(T_1-T_o)(k_1+k_o) = -\frac{C_1}{r_1} + C_2$$

Eliminating C_2 between these two equations:

$$C_1 = \frac{1}{2}(T_1 - T_0)(K_0 + K_1)/\left(\frac{1}{r_0} - \frac{1}{r_1}\right)$$

We may now write for the heat-transfer rate at $r = r_0$:

$$Q_0 = -4\pi r_0^2 K_0 \frac{dT}{dr}\bigg|_{r=r_0} = -4\pi C_1$$

Then

$$Q_0 = -4\pi (T_1 - T_0)\left(\frac{K_1 + K_0}{2}\right)/\left(\frac{1}{r_0} + \frac{1}{r_1}\right)$$

This result shows that the use of an arithmetic mean thermal conductivity is justified if thermal conductivity varies linearly with temperature.

b. The desired heat flow is in the minus-r direction and hence equal to

$$-Q_0 = \frac{4\pi(183)(0.081)(4.137\times10^{-3})}{\left(\frac{1}{3} - \frac{1}{4}\right)\left(\frac{1}{2.54\times12}\right)}$$

$$= 282 \text{ cal sec}^{-1}$$

Hence the evaporation rate is

$$282 \text{ cal sec}^{-1}/1636 \text{ cal gm-mol}^{-1}$$

$$= 0.172 \text{ gm-mol sec}^{-1} \text{ or } 19.8 \text{ Kg hr}^{-1}$$

9.G Alternate Methods of Setting Up the Heated-wire Problem.

a. Heat produced within cylinder = $\pi r^2 L S_e$

Heat entering cylinder = 0

Heat leaving cylinder = $2\pi r L q_r$

Substitution into a heat balance gives:

$$\pi r^2 L S_e = 2\pi r L q_r$$

or

$$q_r = \frac{1}{2} S_e r$$

b. Substitution of Fourier's law into Eq. 9.2-6 gives, for constant thermal conductivity:

$$-k \frac{d}{dr}\left(r \frac{dT}{dr}\right) = S_e r$$

Integration twice with respect to r gives:

$$T = -\frac{S_e r^2}{4k} + C_1 \ln r + C_2$$

The two integration constants may be evaluated through use of the boundary conditions:

At $r = 0$, $T =$ finite

At $r = R$, $T = T_0$

Application of these boundary conditions leads to Eq. 9.2-13.

9.H Heat Conduction from a Sphere to a Stagnant Fluid.

a. The differential equation is the same as that derived in Prob. 9.F. For constant thermal conductivity this becomes:

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

b. Integration twice with respect to r gives:

$$T = -\frac{C_1}{r} + C_2$$

The boundary conditions are:

At $r = R$ $\quad T = T_R$
At $r = \infty$ $\quad T = T_\infty$

Therefore the temperature distribution is:

$$\frac{T - T_\infty}{T_R - T_\infty} = \frac{R}{r}$$

c. The heat flux at the wall is then:

$$q_r\Big|_{r=R} = -k \frac{dT}{dr}\Big|_{r=R} = +k \frac{(T_R - T_\infty)}{R}$$

We now compare this expression with Newton's law of cooling:

$$q_r\Big|_{r=R} = h\,(T_R - T_\infty)$$

which defines h. Equating these last two expressions we get:

$$h = k/R$$

or

$$\frac{hD}{k} = 2$$

where D is the diameter of the sphere.

9.I Heat Conduction in a Nuclear Fuel-rod Assembly.

Within the fuel element the differential equation is similar to that developed in the text for the current-carrying wire. Hence we write immediately, for constant k_f and the given heat-source:

$$-k_f \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_f}{dr}\right) = S_{no}\left[1 - b\left(\frac{r}{R_f}\right)^2\right]$$

The differential equation for the cladding is similar but contains no source term:

$$-k_c \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right) = 0$$

These two second-order differential equations are to be solved with the aid of these boundary conditions:

B.C. 1: At $r=0$ T_f is finite

B.C. 2: At $r=R_f$ $T_f = T_c$

B.C. 3: At $r=R_f$ $-k_f\left(\frac{dT_f}{dr}\right) = -k_c\left(\frac{dT_c}{dr}\right)$

B.C. 4: At $r=R_c$ $-k_c\left(\frac{dT_c}{dr}\right) = h_L(T_c - T_L)$

Integration of the above differential equations gives:

$$T_f = \frac{S_{no}R_f^2}{4k_f}\left[\left(\frac{r}{R_f}\right)^2 - \frac{b}{4}\left(\frac{r}{R_f}\right)^4\right] + F_1 \ln r + F_2$$

$$T_c = C_1 \ln r + C_2$$

Application of the boundary conditions then gives, after considerable manipulation:

$$T_f - T_L = \frac{S_{no}R_f^2}{4k_f}\left\{\left[1-\left(\frac{r}{R_f}\right)^2\right] - \frac{b}{4}\left[1-\left(\frac{r}{R_f}\right)^4\right]\right\}$$
$$+ \frac{S_{no}R_f^2}{2k_c}\left(1-\frac{b}{2}\right)\left(\frac{k_c}{R_c h_L} + \ln\frac{R_c}{R_f}\right)$$

The desired answer is obtained from this expression by setting $r=0$.

9.J Heat Conduction in an Annulus.

a. A heat balance on a shell of thickness Δr gives:

$$2\pi r L q_r\Big|_r - 2\pi r L q_r\Big|_{r+\Delta r} = 0$$

Hence:

$$\frac{d}{dr}(r q_r) = 0$$

Then, through use of Fourier's law we may write:

$$\frac{d}{dr}\left(-k r \frac{dT}{dr}\right) = 0 \quad ; \quad k r \frac{dT}{dr} = C_1$$

We now write $k(\Theta)$ where $\Theta = (T-T_0)/(T_1-T_0)$ just as in Prob. 9.F to obtain:

$$[k_0 + (k_1 - k_0)\Theta]\,dT = C_1\,dr/r$$

or

$$(T_1-T_0)\left[k_0 + \tfrac{1}{2}(k_1-k_0)\Theta\right]\Theta = C_1 \ln r + C_2$$

The boundary conditions for this problem are:

At $r = r_0$ $\quad \Theta = 0$

At $r = r_1$ $\quad \Theta = 1$

We may then write:

$$0 = C_1 \ln r_0 + C_2$$

$$\tfrac{1}{2}(T_1-T_0)(k_0+k_1) = C_1 \ln r_1 + C_2$$

The constant C_1 is then:

$$C_1 = (1/2)(T_1-T_0)(k_0+k_1)/\ln r_1/r_0$$

Hence the heat-transfer rate at $r = r_0$ is:

$$Q_0 = -2\pi r_0 L k_0 \frac{dT}{dr}\Big|_{r=r_0} = -2\pi L C_1$$

$$= (+2\pi L)\frac{(1/2)(k_0+k_1)(T_0-T_1)}{\ln(r_1/r_0)}$$

b. We begin by defining ϵ, the wall thickness, for thin annuli, by

$$\frac{r_1}{r_0} = 1 + \frac{\epsilon}{r_0}$$

We then expand $\ln(r_1/r_0)$ in a Taylor series to obtain:

$$\ln(r_1/r_0) = \ln\left(1 + \frac{\epsilon}{r_0}\right) = \frac{\epsilon}{r_0} - \frac{1}{2}\left(\frac{\epsilon}{r_0}\right)^2 + ---$$

If we consider all terms but the first negligible, we get:

$$Q_o = 2\pi r_o L\left(\frac{k_o+k_1}{2}\right)\left(\frac{T_o-T_1}{\epsilon}\right)$$

This just says: heat flow = area × average thermal conductivity × temperature gradient.

9.K Heat Generation in a Non-Newtonian Fluid.

a. For the Ostwald-de Waele (power-law) fluid:

$$\tau_{xz} = -m\left|\frac{dv_z}{dx}\right|^{n-1}\frac{dv_z}{dx}$$

In this problem dv_z/dx is positive so that

$$\tau_{xz} = -m\,(dv_z/dx)^n$$

and hence

$$S_v = -\tau_{xz}\frac{dv_z}{dx} = +m\left(\frac{dv_z}{dx}\right)^{n+1}$$

b. The velocity profile may be determined from the equation of motion. Thus

$$d\tau_{xz}/dx = 0 \;;\; \frac{d}{dx}(dv_z/dx)^n = 0$$

Therefore:

$$(dv_z/dx)^n = C_1^n \;;\; dv_z/dx = C_1$$

Integrating with respect to x we get:

$$v_z = C_1 x + C_2$$

Determination of the constants gives:

$$\frac{v_z}{V} = \frac{x}{b}$$

just as for the Newtonian case. Then the volumetric heat generation rate is:

$$S_v = m\left(\frac{V}{b}\right)^{n+1}$$

c. The analog of Eq. 9.4-8 is

$$T = -(m/k)(V/b)^2(x^2/2) + C_1 x/K + C_2$$

and the analog of Eq. 9.4-11 is

$$\frac{T-T_0}{T_b-T_0} = \frac{x}{b} + \frac{1}{2}Br_n\left(\frac{x}{b}\right)\left[1-\left(\frac{x}{b}\right)\right]$$

where $Br_n = mV^{n+1}/k\,b^{n-1}(T_b-T_0)$

9.L Calculation of Insulation Thickness for a Furnace Wall.

The minimum wall thickness will occur when $T_1 = 2000°F$. Then in the region "01" the thickness must be:

$$x_1 - x_0 = \left(\frac{k_1 + k_0}{2}\right)(T_0 - T_1) / q_0$$

$$= \left(\frac{4.1 + 3.6}{2}\right)(500) / 5000 = 0.39 \text{ ft}$$

For the remaining two regions:

$$q_0 = \frac{T_1 - T_3}{\dfrac{x_2 - x_1}{k_{avg}^{12}} + \dfrac{x_3 - x_2}{k_{avg}^{23}}} \; ; \; 5000 = \frac{2000 - 100}{\dfrac{(x_2 - x_1)}{\frac{1}{2}(0.9 + 1.8)} + \dfrac{1/48}{26.1}}$$

$$x_2 - x_1 = 0.513 \text{ ft}$$

9.M Radial Temperature Gradients in a Chemical Reactor.

a. The behavior of this system can be described by the following differential equation and boundary conditions:

$$k_{eff} \frac{1}{r} \frac{d}{dr}\left(r \frac{dT}{dr}\right) + S_c = 0$$

B.C.1: At $r = R_0$, $T = T_0$

B.C.2: At $r = R_0$, $dT/dr = 0$

b. In terms of the reduced variables this description is:

$$\frac{1}{\xi} \frac{d}{d\xi}\left(\xi \frac{d\Theta}{d\xi}\right) = -4$$

B.C.1: At $\xi = 1$ $\Theta = 0$

B.C.2: At $\xi = 1$ $d\Theta / d\xi = 0$

c. Integrating twice with respect to ξ we get:

$$\Theta = -\xi^2 + C_1 \ln \xi + C_2$$

Application of the boundary conditions gives:

$C_2 = 1$

$C_1 = 2$

Hence: $\Theta = 1 - \xi^2 + 2 \ln \xi$

d. The reduced temperature of the outer wall is simply:

$$\Theta(a) = 1 - a^2 + 2 \ln a$$

where $a = R_1/R_0$. The volume-average reduced temperature is:

$$\langle \Theta \rangle = \frac{\int_0^{2\pi}\int_1^{a} (1-\xi^2+2\ln\xi)\,\xi\, d\xi\, d\theta}{\int_0^{2\pi}\int_1^{a} \xi\, d\xi\, d\theta}$$

$$= \frac{\left(\frac{1}{2}\xi^2 - \frac{1}{4}\xi^4 + \xi^2\ln\xi - \frac{1}{2}\xi^2\right)\Big|_1^a}{\left(\frac{1}{2}\xi^2\right)\Big|_1^a}$$

$$= -\frac{1}{2}(a^2+1) + 2\left(\frac{a^2}{a^2-1}\right)\ln a$$

In terms of the original variables we may write:

$$\frac{\langle T\rangle - T_0}{S_c R_0^2/4k_{eff}} = -\frac{1}{2}\left(\frac{R_1^2+R_0^2}{R_0^2}\right) + 2\left(\frac{R_1^2}{R_1^2-R_0^2}\right)\ln\frac{R_1}{R_0}$$

e. For the stated conditions:

$$a = 0.50/0.45 = 1.11$$

$$S_c = \left(480\,\frac{\text{cal}}{\text{hr, cm}^3}\right)\left(3.97\times10^{-3}\,\frac{\text{Btu}}{\text{cal}}\right)\left(2.54\times12\,\frac{\text{cm}}{\text{ft}}\right)^3$$

$$= 53{,}600\ \text{Btu hr}^{-1}\,\text{ft}^{-3}$$

Then $$T_1 = 900 + \frac{(53{,}600)(0.45/12)^2}{4\,(0.3)}\left[1 - 1.23 + 0.21\right]$$

$$= 900 - 63\,[0.02] \doteq 899\ °\text{F}$$

f. The temperature difference between inner and outer walls would be four times as great.

9.N Heat Transfer to a Non-Newtonian Fluid.

For tube flow we may write the analog of Eq. 1.2-5 as:

$$-\frac{dv_z}{dr} = \varphi_0\tau_{rz} + \varphi\tau_{rz}^{s}$$

The corresponding velocity distribution is:

$$v_z = \left(\frac{\varphi_0 R\tau_R}{2}\right)\left[1-\left(\frac{r}{R}\right)^2\right] + \left(\frac{\varphi_1 R\tau_R^{s}}{s+1}\right)\left[1-\left(\frac{r}{R}\right)^{s+1}\right]$$

where τ_R is the wall shear stress. This expression may be written in dimensionless form as:

$$v_z/v_{max} = \frac{K}{K+1}(1-\xi^2) + \frac{1}{K+1}(1-\xi^{s+1})$$

where: $v_{max} = (\varphi_0 R \tau_R/2) + (\varphi_1 R \tau_R^{s+1})/(s+1)$

$\xi = r/R$

$K = \varphi_0(s+1)/2\varphi_1 \tau_R^{s-1}$

This velocity distribution is then substituted into Eq. 9.8-9 in place of the Poiseuille distribution used in Sect. 9.8. An exactly analogous development, differing only in the form of $v_z(r)$ then yields the temperature distribution:

$$\Theta_E = C_0\left\{\zeta + \frac{Pe'}{K+1}\left[K\left(\frac{\xi^2}{4} - \frac{\xi^4}{16}\right) + \left(\frac{\xi^2}{4} - \frac{\xi^{s+3}}{(s+3)^2}\right) - C_2\right]\right\}$$

where

$\Theta_E = (T-T_0)k/Rq_1$ = reduced temperature in Ellis fluid.

$C_0 = (K+1)/\left\{Pe'\left[\frac{1}{4}K + \frac{s+1}{2(s+3)}\right]\right\}$

$Pe' = \rho \hat{C}_p R v_{max}/k$ = Péclet number

$\zeta = z/R$ = reduced distance along tube axis.

$$C_2 = \left[\frac{7K^2}{384} + F_0(s) + K F_1(s)\right]\left[\frac{K}{4} + \frac{s+1}{2(s+3)}\right]^{-1}$$

$$F_0(s) = \frac{s^4+10s^3+36s^2+46s+19}{16(s+5)(s+3)^3}$$

$$F_1(s) = \frac{7s^4+108s^3+566s^2+1236s+771}{96(s+3)^2(s+5)(s+7)}$$

Note that this solution, like that in §9.8, is only exact in the limit of very large ζ.

CHAPTER 10 – Checked by R.H. Weaver

10.A Temperature in a Friction Bearing.

We start by multiplying both numerator and denominator of Eq. 10.5-15 by κ^2/N. Then, remembering that N is very large for the conditions of this problem, we may write:

$$\xi_{max} = \left[\frac{2\kappa^2 \ln \kappa}{(\kappa^2-1)}\right]^{1/2}$$

Expanding $\ln \kappa$ in a Taylor series we obtain:

$$\xi_{max} = \left[\frac{2\kappa^2\left(1 - \frac{\kappa-1}{2} + \frac{(\kappa-1)^2}{3\cdot 2} + \cdots\right)}{(\kappa+1)}\right]$$

We may now evaluate this expression for the given conditions $\kappa = 1/1.002 = 0.998$:

$$\xi_{max} = \left[\frac{2(0.996)\left(1 + \frac{0.002}{2} + \cdots\right)}{1.998}\right] \doteq 0.999$$

We next multiply Eq. 10.5-14 by $(T_1 - T_\kappa)$ to get:

$$T_{max} - T_\kappa = \left[\frac{\mu\Omega^2R^2}{k}\cdot\frac{\kappa^4}{(1-\kappa^2)^2}\right]\left[\left(1-\frac{1}{\xi^2_{max}}\right) - \frac{\ln \xi_{max}}{\ln \kappa}\left(1-\frac{1}{\kappa^2}\right)\right]$$

$$= \left[\frac{\mu\Omega^2R^2}{k}\cdot\frac{0.992}{16\times 10^{-6}}\right]\left[-2.004\times 10^{-6} + \frac{1.005}{2.002}\cdot 4.016\times 10^{-3}\right]$$

$$= 0.184\,\mu\Omega^2R^2/k$$

For the conditions of this problem:

$\mu = 2$ g sec^{-1} cm^{-1}

$\Omega = 400\times 2\pi/60 = 419$ rad sec^{-1}

$R \doteq 2.54$ cm

$k = 4\times 10^{-4}$ cal sec^{-1} cm^{-1} °C^{-1}

$T_\kappa = 200$ °C

We may then write:

$$T_{max} = 200 + 0.184\left[\frac{2(419\times 2.54)^2}{4\times 10^{-4}}\cdot 2.39\times 10^{-8}\right]$$

$$= 200 + 24.8$$

$$\doteq 225\ ^\circ\mathrm{C}$$

Note that a conversion factor was included to transform calories to dyne-centimeters.

10.B Viscosity Variation and Velocity Gradients in a Non-isothermal Film.

a. We begin by determing the temperature at which the discrepancy between the two methods of calculation is greatest. To do this we define a fractional discrepancy as

$$\Delta = \frac{\mu^* - \mu_a^*}{\mu_a^*}$$

where $\mu^* = \mu/\mu_0$ as calaulated from Eq. 10.5-19a, considered as correct for our present purposes.

$\mu_a^* = \mu/\mu_0$ as calculated from Eq. 10.5-19.

We may use Eq. 1.5-12 to write Δ as:

$$\Delta = \exp\left\{ \left[\frac{B}{TT_0}(T_0-T_\delta)\frac{x}{\delta}\right] - \left[\frac{B}{T_\delta T_0}(T_0-T_\delta)\frac{x}{\delta}\right]\right\} - 1$$

We next eliminate x with the aid of Eq. 10.5-18:

$$\Delta = \exp\left\{-\frac{B}{T_0}\left[\left(\frac{T-T_0}{T}\right) - \left(\frac{T-T_0}{T_\delta}\right)\right]\right\} - 1$$

We now differentiate Δ with respect to T and set the derivative equal to zero:

$$\frac{\partial \Delta}{\partial T} = 0 = \frac{B}{T_0}\left[\frac{1}{T} - \left(\frac{T-T_0}{T}\right) - \frac{1}{T_\delta}\right]\exp\left\{-\frac{B}{T_0}\left[\left(\frac{T-T_0}{T}\right) - \left(\frac{T-T_0}{T_\delta}\right)\right]\right\}$$

The temperature of maximum discrepancy is then

$$T_d = \sqrt{T_0 T_\delta}$$

b. The data needed to obtain the desired solution are:

$\mu\,(80°C) = 0.357$ cp.

$\mu\,(100°c) = 0.284$ cp.

$T_d = \sqrt{373 \cdot 353}\ °K \doteq 363\ °K = 90\ °C$

The temperature of maximum discrepancy thus occurs very nearly at $x = 0.05$ mm. Then using Eq. 10.5-19 we calculate the viscosity here to be:

$$\mu(90°C) = (0.357 \times 0.284)^{1/2} = 0.319 \text{ cp.}$$

The observed viscosity of water at 90° = 0.317 cp.

The maximum discrepancies in μ and dv_z/dx are then both equal to

$$100 \times \frac{0.002}{0.317} = 0.63\,\%$$

10.C Transpiration Cooling.

a. Define $(T-T_1)/(T_\kappa-T_1)$ as Θ^o in the absence of mass transfer and as Θ^w in the presence of mass transfer.

(1) $w_r = 0$:

$$\Theta^o = \frac{(100/r) - 0.200}{1.000 - 0.200}$$ where is in microns.

We then find:

$\underline{r}$:	100	200	300	400	500
$\underline{\Theta^o}$:	unity	0.375	0.1667	0.0625	zero

(2) $w_r = 10^{-5}$ g sec^{-1}

$R_o = 10^{-5}/(4)(4\pi)(6.13\times10^{-5}) = 3.25\times10^{-3}$ sec

$= 32.5$ microns

$$\Theta^w = (e^{-32.5/r} - 0.937)/(0.722 - 0.937)$$

we then find

$\underline{r}$:	100	200	300	400	500
$\underline{\Theta^w}$:	unity	0.404	0.186	0.070	zero
$\underline{\Theta^o/\Theta^w}$:	unity	0.93	0.89	0.89	—

b. $\phi = (32.5/500)(0.8/0.2) = 0.260$

$$\epsilon = 1 - \frac{0.26}{1.297-1.000} = 1-0.875 = 0.125$$

10.D Free Convection from a Vertical Surface.

For the conditions of this problem the average temperature is 110°F or 570°R.

$g = 980$ cm/sec^2

$\rho = 1.25\times10^{-3}$ g cm^{-3} (at $\bar{T} = 530$°R)

$\mu = 1.95\times10^{-4}$ g sec^{-1} cm^{-1} (at 110°F)

$\nu = 0.156$ cm^2 sec^{-1}

$k = 6.37\times10^{-5}$ cal cm^{-1} sec^{-1} °C^{-1} (at 110°F)

$\hat{C}_p = 0.24$ cal g^{-1} °C^{-1} (at 110°F)

$\beta = 1/\bar{T} = 3.40\times10^{-3}$ °K (at $\bar{T} = 530$°R)

$\Delta T = 80°F = 44.4\ °C$

$H = 30\ cm$

$Pr = (0.24)(1.95\times10^{-4})/(6.37\times10^{-5}) = 0.735$

$Gr = (980)(30)^3(3.40\times10^{-3})(44.4)/(0.156)^2$

$= 1.64 \times 10^8$

$(Gr\,Pr)^{1/4} = 1.05 \times 10^2$

By the Lorenz formula:

$Q = (0.548)(50)(6.37\times10^{-5})(44.4)(105)$

$= 8.14\ cal\ sec^{-1}$

According to Schmidt and Beckmann

$Q = 8.14\,(.517/.548) = 7.68\ cal\ sec^{-1}$

10.E Velocity, Temperature, and Pressure, Changes in a Shock Wave.

a. $v_0 = 2\sqrt{\gamma R T_0} = 2\sqrt{1.4\,(530)(1545)(32.16)/(29)}$

$= 2250\ ft\ sec^{-1}$

b. (1) Velocity: Setting $\xi = +\infty$ in Eq. 10.5-69 we find

$\phi = \alpha = \frac{0.4}{2.4} + \frac{2}{4\times2.4} = 0.375$

$v_\infty = 0.375 \times 2250 = 844\ ft\ sec^{-1}$

(2) Temperature:

$\frac{1}{2}v_0^2 = 2.53 \times 10^6\ ft^2/sec^2$

$\hat{C}_{p_0}T_0 = (0.242)(530)(778)(32.16) = 3.22\left\{\begin{array}{l}\times 10^{6}\\ ft^{2}\ sec^{-2}\end{array}\right.$

$\frac{1}{2}v_0^2 + \hat{C}_{p_0}T_0 = 5.75 \times 10^6\ ft^2\ sec^{-2}$

$\hat{C}_{p_\infty}T_\infty = (5.75 - 0.356)\times10^6 = 5.39\times10^6\ ft^2\ sec^{-2}$

$T_\infty = (5.39/3.22)(530) = 888\ °R$

(3) Pressure: The velocity gradient well downstream of the shock will be negligible. We may then write:

$$\rho_0 v_0 v_\infty + p_\infty = \rho_0 v_0^2 + p_0$$

$$p_\infty - p_0 = (5.07-1.90)(\times 10^6)(29)/(1545)(530)(32.16)$$

$$= 3.38 \text{ atm}$$

$$p_\infty = 4.48 \text{ atm}$$

c. (1) l_0: $\mu_0/\rho_0 = 0.15 \text{ cm}^2 \text{ sec}^{-1}$

$$RT_0 = 1125/\sqrt{1.4} = 950 \text{ ft sec}^{-1} = 2.9 \times 10^4 \text{ cm sec}^{-1}$$

$$l_0 = (15 \times 10^{-2})(3)(\sqrt{\pi/8})/(2.9 \times 10^{-4})$$

$$= 9.75 \times 10^{-6} \text{ cm}$$

$$\alpha = 0.375$$

$$\beta = (9/8)(2.4)\sqrt{\pi/(8 \times 1.4)} = 1.43$$

(2) $\xi(\phi) = \{\ln[(1-\phi)/(\phi-\alpha)^\alpha] / 2\beta(1-\alpha)\} + \xi_0$

(3) $x - x_0 = \{[(9.75 \times 10^{-6})/(2)(1.43)(0.625)] \ln\left[\frac{1-\phi}{(\phi-\alpha)^\alpha}\right]\}$

$$= (5.45 \times 10^{-6} \text{ cm}) \ln\left[\frac{1-\phi}{(\phi-\alpha)^\alpha}\right]$$

This relationship is shown graphically in Fig. 10.5-5

d. (1) $\Delta\hat{U} = (\hat{C}_p - \frac{R}{M})\Delta T = (0.24 - \frac{1.987}{29})(888-530)$

$$= +61.4 \text{ Btu lb}_m^{-1}$$

(2) $\Delta\hat{K} = -\frac{1}{2}(5.07-0.71) \times 10^6 \text{ ft}^2 \text{ sec}^{-2} = -2.18 \times 10^6 \text{ ft}^2 \text{ sec}^{-2}$

$$= -87 \text{ Btu lb}_m^{-1}$$

10.F Adiabatic Frictionless Compression of an Ideal Gas.

For the conditions of this problem we may write:

$$p_1 \hat{V}_1^{1.4} = p_2 \hat{V}_2^{1.4}$$

We rewrite this expression with the aid of the ideal-gas law to get:

$$T_2 = T_1 (\hat{V}_1/\hat{V}_2)^{0.4}$$

$$= (460+100)(10)^{0.4}$$

$$= 1406°\text{R or } 946°\text{F}$$

10.G Use of the Energy Equation to Set Up Problems.

a. Start with Eq. B. of Table 10.2-2. Discard all of left side of equation because of restriction to steady state, and discard all terms containing velocities or viscous stresses on the right side. Discard terms containing q_θ or q_z on grounds of problem symmetry. Equation B then becomes:

$$0 = -\frac{1}{r}\frac{d}{dr}(r q_r)$$

Now remember that electrical energy was not considered in deriving Eq. B. Therefore add a source term S_e to the right side to get:

$$0 = -\frac{1}{r}\frac{d}{dr}(r q_r) + S_e$$

Since Eq. B describes rate of change of energy content per unit volume, S_e is the rate of "production" of thermal energy per unit volume, e.g. cal cm^{-3} sec^{-1}.

b. Begin now with Eq. C of Table 10.2-2 and make simplifications analogous to those just described. Again add a source term to obtain:

$$0 = -\frac{1}{r^2}\frac{d}{dr}\left(r^2 q_r^{(F)}\right) + S_n$$

To go further one needs information as to the dependence of the source term on r, as given in §9.3.

c. The desired result is obtained directly from Eq. A of Table 10.2-2 on the assumptions of steady state, no motion, and negligible heat flow in the y- and z-directions.

d. Begin with Eq. A of Table 10.2-2. For a steady-state stationary system with no heat flow in the y-direction this equation becomes:

$$0 = +k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

Next integrate this expression across the half-width B of the fin:

$$0 = +k\int_0^B \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right)dx$$

Assume, just as was done in §9.7, that (d^2T/dz^2) is independent of x so that:

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=B} = B \frac{\partial^2 T}{\partial z^2}$$

(Why is $\partial T/\partial x|_{x=0}$ zero?) The left side of this equation is just the surface heat flux:

$$\left. q_x \right|_{x=B} = h(T - T_a)$$

Then

$$\frac{\partial^2 T}{\partial y^2} = \frac{h}{B}(T - T_a)$$

which is the desired relation.

10.H Viscous Heating in Laminar Slit Flow.

From Eq. A of Table 10.2-3 we may write:

$$0 = k\frac{d^2T}{dx^2} + \mu\left(\frac{dv_z}{dx}\right)^2$$

It may readily be shown that the velocity profile is:

$$v_z = v_{max}\left[1 - (x/B)^2\right]$$

and therefore that

$$(dv_z/dx)^2 = v_{max}^2\,(4x^2/B^4)$$

The energy equation is then:

$$(d^2T/dx^2) = -4\mu v_{max}^2 x^2/kB^4$$

The boundary conditions are:

At $x = B$, $T = T_0$

At $x = -B$, $T = T_0$

Solving the energy equation with these boundary conditions we find:

$$T - T_0 = \tfrac{1}{3}(\mu v_{max}^2/k)\left[1 - (x/B)^4\right]$$

10.I Velocity Distribution in a Non-isothermal Film.

(1) At $x = \delta$

$$v_z = \frac{\rho g}{\mu}\left[\frac{\delta}{\ln(\mu_\delta/\mu_0)}\right]^2\left[\frac{1+\ln\frac{\mu_\delta}{\mu_0}}{(\mu_\delta/\mu_0)} - \frac{1+\ln\frac{\mu_\delta}{\mu_0}}{(\mu_\delta/\mu_0)}\right] = 0$$

(2) At $x = 0$

$$\partial v_z/\partial x = \frac{\rho g}{\mu}\left[\frac{\delta}{\ln(\mu_\delta/\mu_0)}\right]^2\left\{\frac{0+\frac{1}{\delta}\ln\frac{\mu_\delta}{\mu_0}+[1+0]\left[-\frac{1}{\delta}\ln\frac{\mu_\delta}{\mu_0}\right]}{(\mu_\delta/\mu_0)^0}\right\}$$

$$= 0$$

(3) For $\mu_\delta \rightarrow \mu_0$

$$v_z = \frac{\rho g \delta^2}{\mu} \left\{ \left[\frac{1+ay}{e^{ay}} - \frac{1+a}{e^{a}} \right] \Big/ a^2 \right\}$$

Where $a = \ln(\mu_\delta / \mu_0)$
$y = x/\delta$

Using l'Hôpital's rule twice we obtain:

$$v_z = \frac{\rho g \delta^2}{\mu} \lim_{a \to 0} \left\{ \frac{e^{-a} - y^2 e^{-ay} - a e^{-a} + a y^3 e^{-ay}}{2} \right\}$$

$$= \frac{\rho g \delta^2}{2\mu} (1 - y^2)$$

Which is the desired relation.

10.J Transpiration Cooling in a Planar System.

Assuming ideal-gas behavior and ignoring pressure changes we may write:

$$\rho \hat{C}_p v_y \frac{dT}{dy} = k \frac{d^2T}{dy^2} \quad ; \quad \frac{dT}{dy} = Y_0 \frac{d^2T}{dy^2}$$

where $Y_0 = k/\rho \hat{C}_p v_y$ = a constant, as shown by the equation of continuity.

We now integrate the above differential equation to get:

$$T = A \exp(y/Y_0) + B$$

where A and B are constants of integration. We next use the given boundary conditions to evaluate these constants. We may then write:

$$\frac{T - T_0}{T_L - T_0} = \frac{e^{y/Y_0} - 1}{e^{L/Y_0} - 1}$$

The rate of heat removal may be calculated from Fourier's law:

$$q_0 = +k \left.\frac{dT}{dy}\right|_{y=0} = +\hat{C}_p \rho v_y (T_L - T_0) / (e^{L/Y_0} - 1)$$

$$= k \left(\frac{T_L - T_0}{L} \right) \left[\frac{\phi}{e^{\phi} - 1} \right]$$

where $\phi = L/Y_0$.

10.K Use of Transpiration to Reduce Heat Losses.

(1) Without transpiration:

$$Q_r = -4\pi k r^2 \frac{dT}{dr}$$

Integrating over the shell thickness we get:

$$Q_r = +4\pi k (T_2 - T_1)/\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$= 4\pi(0.02)(30+362)/\left(\frac{1}{1} - \frac{1}{0.5}\right)$$

$$-Q_r = 99 \text{ Btu hr}^{-1}$$

(2) With transpiration:

From Eq. 10.5-30:

$$Q_r = 4\pi k R_0 (T_1 - T_\kappa)/\left(e^{(R_0/\kappa R)(1-\kappa)} - 1\right)$$

From an energy balance:

$$-Q_r = w_r \lambda = 4\pi k \lambda R_0 / \hat{C}_p$$

We now combine these two expressions and rearrange the result to obtain:

$$-Q_r = \frac{\kappa R}{(1-\kappa)}\left(\frac{4\pi k \lambda}{\hat{C}_p}\right)\ln\left[\frac{(T_1 - T_\kappa)\hat{C}_p}{\lambda} + 1\right]$$

$$= \left(\frac{0.5 \times 1}{0.5}\right)\left(\frac{4\pi \times 0.02 \times 91.7}{0.22}\right)\ln\left[\frac{(392)(0.22)}{91.7} + 1\right]$$

$$= 69.5 \text{ Btu hr}^{-1}$$

10.L Frictionless Processes in an Ideal Gas.

a. Substituting $\hat{C}_p - p\hat{V}/T$ for $\hat{C}_v$ we may write:

$$\rho \hat{C}_p (\mathbf{v} \cdot \nabla T) = k \nabla^2 T - p(\nabla \cdot \mathbf{v}) + \frac{p}{T}(\mathbf{v} \cdot \nabla T)$$

The last term of this expression may be rearranged with the aid of the ideal-gas law to give:

$$(p/T)(\mathbf{v} \cdot \nabla T) = \rho(\mathbf{v} \cdot \nabla(p/\rho))$$

$$= \rho\left(\frac{1}{\rho}(\mathbf{v} \cdot \nabla p) - \frac{p}{\rho^2}(\mathbf{v} \cdot \nabla \rho)\right)$$

$$= (\mathbf{v} \cdot \nabla p) - \frac{p}{\rho}\left((\nabla \cdot \rho \mathbf{v}) - \rho(\nabla \cdot \mathbf{v})\right)$$

For a steady-state system $(\nabla\cdot\rho\underset{\sim}{v})=0$ by the equation of continuity. Then:

$$\frac{p}{T}(\underset{\sim}{v}\cdot\nabla T) = (\underset{\sim}{v}\cdot\nabla p) + p(\nabla\cdot\underset{\sim}{v})$$

Substituting this expression into the first of the above equations we achieve the desired result.

b. Performing the indicated operation we obtain:

$$\frac{\rho\hat{C}_p V T_o}{D}(\underset{\sim}{v}^*\cdot\nabla^* T^*) = \frac{kT_o}{D^2}\nabla^{*2}T^* - \frac{\rho V^3}{D}(\underset{\sim}{v}^*\cdot\nabla^* p^*)$$

We now divide both sides of this equation by $\rho\hat{C}_p V T_o/D$ to obtained the desired expression:

$$(\underset{\sim}{v}^*\cdot\nabla^* T^*) = \frac{1}{Re\,Pr}\nabla^{*2}T^* - Ec\,(\underset{\sim}{v}^*\cdot\nabla^* p^*)$$

c. $v_s = \sqrt{\gamma\,(\partial p/\partial\rho)_T} = \sqrt{(C_p/C_v)\,RT/M}$

$$= \sqrt{\hat{C}_p T\,(\hat{C}_p - \hat{C}_v)/\hat{C}_v}$$

$$= \sqrt{\hat{C}_p T(\gamma-1)}$$

d. $E_c = V^2/\hat{C}_p T_o = V^2(\gamma-1)/v_s^2 = (\gamma-1)Ma^2$

10.M Dimensional Analysis of Forced-convection Heat Transfer in an Agitated Tank.

If the same Reynolds number, $D^2N\rho/\mu$, is used in the two tanks then $T^*(t^*, x^*, y^*, z^*)$ will be the same in each. This is because the Prandtl number and the initial and boundary conditions are automatically the same in each tank.

Constancy of Reynolds number requires that the rate of rotation N of the impeller in the small tank be four times that in the larger. Under these conditions temperatures at any pair of corresponding positions will be equal at the same reduced time $t^* = Nt$.

10.N Equivalence of Different Forms of the Energy Equation.

a. Here we start with the expression

$$\rho D\hat{U}/Dt = -(\nabla\cdot\underset{\sim}{q}) - p(\nabla\cdot\underset{\sim}{v}) - (\tau:\nabla\underset{\sim}{v})$$

Since $\hat{H} = \hat{U} + p\hat{V}$ we may write:

$$\rho\frac{D\hat{U}}{Dt} = \rho\frac{D\hat{H}}{Dt} - \rho\frac{D(p/\rho)}{Dt}$$

But:

$$\frac{D(p/\rho)}{Dt} = \frac{1}{\rho}\frac{Dp}{Dt} - \frac{p}{\rho^2}\frac{D\rho}{Dt}$$

$$= \frac{1}{\rho}\frac{Dp}{Dt} - \frac{p}{\rho^2}\left[\underline{\frac{\partial\rho}{\partial t} + (\nabla\cdot\rho\underset{\sim}{v})} - \rho(\nabla\cdot\underset{\sim}{v})\right]$$

From the equation of continuity it can be seen that the the underlined terms above sum to zero. Therefore:

$$\rho\frac{D}{Dt}(p/\rho) = \frac{Dp}{Dt} + p(\nabla\cdot\underset{\sim}{v})$$

Combining this result with the second of the above equations gives us our desired relation.

b. Here we must show that:

$$\rho\hat{C}_p\frac{DT}{Dt} = \frac{\partial}{\partial t}(\rho\hat{C}_pT) + (\nabla\cdot\rho\hat{C}_pT\underset{\sim}{v}) - \rho T\frac{D\hat{C}_p}{Dt}$$

$$= \rho\hat{C}_p\frac{\partial T}{\partial t} + T\frac{\partial}{\partial t}(\rho\hat{C}_p)$$

$$+\rho\hat{C}_p(\nabla\cdot T\underset{\sim}{v}) + T(\underset{\sim}{v}\cdot\nabla\rho\hat{C}_p)$$

$$-\rho T\frac{D\hat{C}_p}{Dt}$$

$$= \underline{\rho\hat{C}_p\left[\frac{\partial T}{\partial t} + (\underset{\sim}{v}\cdot\nabla T)\right]} + \rho\hat{C}_pT(\nabla\cdot\underset{\sim}{v})$$

$$+T\frac{\partial}{\partial t}\rho\hat{C}_p + T(\underset{\sim}{v}\cdot\nabla\rho\hat{C}_p) - \rho T\frac{D\hat{C}_p}{Dt}$$

The term underlined with dashes is just $\rho\hat{C}_p DT/Dt$ and can thus be cancelled against the left side of the equation. The remaining terms may be rewritten to give:

$$\rho T \frac{D\hat{C}_p}{Dt} = \underline{\rho T\left[\frac{\partial \hat{C}_p}{\partial t} + (\underset{\sim}{v} \cdot \nabla \hat{C}_p)\right]}$$

$$+ \hat{C}_p T\left[\underline{\frac{\partial \rho}{\partial t} + (\underset{\sim}{v} \cdot \nabla \rho) + \rho(\nabla \cdot \underset{\sim}{v})}\right]$$

The term underlined with dashes cancels with the left side of the equation, and the terms underlined with dots sums to zero by the equation of continuity.

CHAPTER 11 - Checked by V.D. Shah

11.A Unsteady-state Heat Conduction in an Iron Sphere.

a. $\alpha = k/\rho\hat{C}_p = 30/(436)(0.12) = 0.574$ ft^2 hr^{-1}

b. The center temperature is to be 128°F; hence:

$$\frac{T_{ctr} - T_0}{T_1 - T_0} = \frac{128-70}{270-70} = 0.29$$

Then, according to Fig. 11.1-3, $\alpha t/R^2 = 0.1$, and:

$$t = 0.1\, R^2/\alpha$$

$$= (0.1)(1/24)^2/(0.574) = 3.02 \times 10^{-4} \text{ hrs}$$

$$= 1.1 \text{ sec}$$

c. $\alpha_1 t_1 / R_1^2 = \alpha_2 t_2 / R_2^2$

$$\alpha_2 = \alpha_1 (t_1/t_2) = (0.574)(1/2)$$

$$= 0.287 \text{ ft}^2 \text{ hr}^{-1}$$

d. $\partial T/\partial t = \alpha \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$

11.B Comparison of the Two Slab Solutions for Short Times.

According to Fig. 11.1-1, at $\alpha t/b^2 = 0.01$ and $y/b = 0.9$,

$$\frac{T-T_0}{T_1-T_0} = 0.46$$

where y is distance from the mid-plane. Next we use Fig. 4.1-1, which can be interpreted as $(T-T_0)/(T_1-T_0)$ vs.

$y'/\sqrt{4\alpha t}$ where $y' = b - y$ is distance from the wall. We may then write:

$$\frac{y'}{\sqrt{4\alpha t}} = \frac{1}{2}\frac{(1-0.9)}{(\alpha t/b^2)^{1/2}} = 1/2$$

Then from Fig. 4.1-1,

$$\frac{T-T_0}{T_1-T_0} = 0.48$$

Hence the use of this solution introduces an error of about four per cent. Smaller errors occur at smaller values of $\alpha t/b^2$.

11.C Bonding with Thermoplastic Adhesive.

The reduced center at the time of bonding is:

$$\frac{T_{ctr}-T_0}{T_1-T_0} = \frac{160-20}{220-20} = 0.70$$

This occurs very nearly at $\alpha t/b^2 = 0.6$. Hence

$$t = 0.6\, b^2/\alpha = \frac{(0.6)(0.77)^2}{4.2\times 10^{-3}}$$

$$= 85 \text{ sec}$$

11.D Quenching of a Steel Billet.

Thermal diffusivity of the billet is:

$$\alpha = k/\hat{C}_p\rho = (25)(4.1365\times10^{-3})/(7.7)(0.12)$$

$$= 0.111 \text{ cm}^2 \text{ sec}^{-1}$$

The reduced time (or Fourier number) is then:

$$\alpha t/R^2 = (0.111)(5\times 60)/(6\times 2.54)^2$$

$$= 0.144$$

From Fig. 11.1-2 the reduced center-line temperature is:

$$0.31 = (T-T_0)/(T_1-T_0)$$

Then:

$$T = 0.31(1000-200)+200$$

$$= 448°F$$

11.E Temperature in a Slab with Heat Production.

a. From page 130 of Carslaw and Jaeger we may write:

$$\frac{k(T-T_0)}{Q_0 b^2} = \frac{1}{2}\left[1-\eta^2 - \frac{4}{\pi^3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+\frac{1}{2})^3}\cdot\cos(n+\tfrac{1}{2})\pi\eta\cdot e^{-(n+\frac{1}{2})^2\pi^2\tau}\right]$$

b. The maximum centerline temperature is approached at very large τ and is

$$T_{max} = T_0 + Q_0 b^2/2k$$

c. According to the graph on page 131 of Carslaw and Jaeger 90% of the temperature rise occurs when $\tau = 1.0$.

11.F Heating of a Semi-infinite Slab: Constant Wall Flux.

Eq. 11.1-2 becomes:

$$\partial T/\partial t = \alpha\, \partial^2 T/\partial y^2$$

On differentiation with respect to y and multiplication by k we get:

$$k\frac{\partial^2 T}{\partial y \partial t} = \alpha k \frac{\partial^3 T}{\partial y^3}\ ;\ \frac{\partial q_y}{\partial t} = \alpha\frac{\partial^2 q_y}{\partial y^2}$$

Here use has been made of the relation $q_y = -k\frac{\partial T}{\partial y}$. We now define a dimensionless heat flux $Q = q_y/q_0$ and rewrite our differential equation as

$$\frac{\partial Q}{\partial t} = \alpha\frac{\partial^2 Q}{\partial y^2}$$

This is to be solved with the initial and boundary conditions:

At $t=0$ $Q=0$
At $y=\infty$ $Q=0$
At $y=0$ $Q=1$

We may then write, by analogy with Ex. 11.1-1:

$$Q = q_y/q_0 = \text{erfc}\left[y/\sqrt{4\alpha t}\right]$$

We must now integrate once more with respect to y to obtain the temperature distribution:

$$T-T_0 = -\frac{q_0}{k}\int_{\infty}^{y} \mathrm{erfc}[y/\sqrt{4\alpha t}]\,dy$$

$$= -\frac{q_0}{k}\sqrt{4\alpha t}\int_{\infty}^{y/\sqrt{4\alpha t}} \mathrm{erfc}\,\beta\,d\beta$$

$$= +(q_0/k)\sqrt{4\alpha t}\;\mathrm{ierfc}[y/\sqrt{4\alpha t}]$$

11.G Dimensional Analysis of the Heat-conduction Equation.

In rectangular co-ordinates the heat-conduction equation is

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

On introduction of the reduced variables and multiplication by $L^2/\alpha(T_1-T_0)$ we get:

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\partial^2 \Theta}{\partial \eta^2} + \frac{\partial^2 \Theta}{\partial \zeta^2}$$

The initial and boundary conditions may also be given in terms of the reduced variables:

I.C.: At $\tau = 0$ $\quad \Theta = 0$ over $V^*(\xi,\eta,\zeta)$

B.C.: At $\tau > 0$ $\quad \Theta = 1$ over $S^*(\xi,\eta,\zeta)$

Here V^* and S^* represent the (dimensionless) space occupied by the solid and its (dimensionless) surface respectively. For any given solid V^* and S^* will contain only pure numbers.

11.H Time-table for Roasting Turkey.

For a given shape the weight of the turkey

$$W = C\rho L^3$$

where C is a constant, ρ the volume-average density, and L some characteristic length. Hence we may write the reduced time as

$$\tau = \alpha t/L^2 = C'\alpha t/(W/\rho)^{2/3}$$

where $c' = c^{2/3}$.

We now tentatively assume the following:

(1) All turkeys have the same physical properties at corresponding points, as well as the same shape, irrespective of size.

(2) Different turkeys are cooked to the same degree when their reduced temperature distributions are equal.

(3) Reduced initial and boundary conditions do not vary with turkey size.

If these assumptions are justified we should expect cooking time to vary with the two-thirds power of turkey weight. This appears to be the case:

Average mass, w_{avg}	Average time, t_{avg}	$\frac{t_{avg}}{(w_{avg})^{2/3}}$
8 lb_m	180 min	45.0
13	247	44.8
21.5	355	46.0

It may be seen that the figures in the third column are remarkably near to equal considering the wide weight and time ranges in the given data--- not to speak of the analytical problems involved in experimental determination of the "end point" of the cooking process.

11.I Mean Temperature in a Slab.

The desired answer is obtained through multiplication of each term of Eq. 11.1-31 by $(1/b)dy$ and then integrating each term from $y=0$ to $y=b$. (What is the physical significance of this process?)

CHAPTER 12 - Checked by V.D. Shah

12.A Temperature Profile in Turbulent Tube Flow.

a. $s^+ = s v_* \rho/\mu = (R-r)\sqrt{\frac{1}{2}\rho\langle v\rangle^2 f/\rho}\ \rho/\mu$

$= (1/2\sqrt{2})(1-\frac{r}{R})(D\langle v\rangle\rho/\mu)\sqrt{f}$

b. $T^+ = \rho\hat{C}_p v_* (\bar{T}-T_0)/q_0 = \rho\hat{C}_p \frac{1}{\sqrt{2}}\langle v\rangle\sqrt{f}\,(\bar{T}-T_0)/q_0$

$= (D\langle v\rangle\rho/\mu)(\hat{C}_p\mu/k)(f/2)^{1/2}\,[k(\bar{T}-T_0)/Dq_0]$

c. (1) $[k(\bar{T}-T_0)/Dq_0] = T^+/\,Re\,Pr\,(f/2)^{1/2}$

$f \doteq 0.079\,Re^{-1/4} = 25\times10^{-4}$

$[k(\bar{T}-T_0)/q_0 D] = T^+(1.414)/(10^2)(10^6)(5\times10^{-2})$

$= 2.828\,T^+\times10^{-7}$

(2) $1-\frac{r}{R} = s^+\,2\sqrt{2}\,/\,Re\sqrt{f}$

$= 4\sqrt{2}\,s^+\times10^{-7}$

$\frac{r}{R} = 1 - 5.66\,s^+\times10^{-7}$

(3) Since Fig. 12.3-2 gives T^+ as a function of s^+ for various Pr it can be used to calculate the desired relation using the expressions developed in (1) and (2) just above. Some representative approximate values are:

$\log s^+$	0.1	0.5	1.0	4.0
$\log T^+$	0.1	0.4	0.5	0.5
$[k(\bar{T}-T_0)/q_0D]\times10^7$	3.1	7.1	8.9	8.9
r/R	0.99993	0.99982	0.99944	0.44

12.B Asymptotic Expressions for Cup-mixing Temperature at Very High Prandtl Number.

We begin by writing Eq. 12.3-22 for $s^+ = \infty$:

$$T^+(\infty) = \int_0^\infty \frac{ds^+}{\frac{1}{Pr} + n^2 s^{+2}(1 - 1 + n^2 s^{+2} + \cdots)}$$

We use this equation beyond its normal range of $0<s^+26$ because at high Pr $T^+(26)$ is very nearly equal to $T^+(\infty)$. (Why?) Neglecting the higher terms of the Taylor series as suggested we obtain:

$$T^+(\infty) = \int_0^\infty \frac{ds^+}{\frac{1}{Pr}+n^4 s^{+4}} = \int_0^\infty \frac{ds^+}{a+s^{+4}} \cdot \frac{1}{n^4} \quad ; \; a = \frac{1}{Pr\, n^4}$$

Performing the indicated integration we obtain:

$$T^+(\infty) = \frac{1}{n^4} \cdot \frac{\pi}{2\sqrt{2}\, a^3}$$

$$T^+(\infty) = \pi Pr^{3/4} / (n\, 2\sqrt{2})$$

CHAPTER 13 - Checked by V.D. Shah

13.A Average Heat-transfer Coefficients.

The total heat-transfer rate is:

$$Q = w \hat{C}_p (T_{b2} - T_{b1})$$
$$= (10,000)(0.6)(200-100) = 600,000 \text{ Btu hr}^{-1}$$

The total inside surface area of the tubes is:

$$A = \pi D L_{tot} = \pi(1.00 - 2 \times 0.065)(300)/12 = 68.4 \text{ ft}^2$$

The various temperature differences between the water and the oil are:

$$(T_0 - T_b)_1 = 213 - 100 = 113\ °F$$

$$(T_0 - T_b)_a = (113 + 13)/2 = 63\ °F$$

$$(T_0 - T_b)_{ln} = (113 - 13)/\ln\left(\frac{113}{13}\right) = 46.2\ °F$$

Substitution into the defining equations for these h's gives:

$$h_1 = (600,000)/(68.4)(113) = 78 \text{ Btu hr}^{-1} \text{ ft}^{-2}\ °F^{-1}$$

$$h_a = (600,000)/(68.4)(63) = 139$$

$$h_{ln} = (600,000)/(68.4)(46.2) = 190$$

13.B Heat Transfer in Laminar Tube Flow.

a. $Re = D\langle v\rangle\rho/\mu = 4w/D\mu\pi = (4)(100)(1/12)(1.42)\pi$
$= 1075$

b. $Pr = \hat{C}_p\mu/k = (0.49)(1.42)/(0.0825)$

$= 8.44$

c. From Fig 13.2-1, at $L/D = 240$:

$$\frac{(T_{b2}-T_{b1})}{(T_0-T_b)_{ln}}\cdot\left(\frac{D}{4L}\right)\cdot\left(Pr_b\right)^{2/3}\left(\frac{\mu}{\mu_0}\right)^{-0.14} = 0.0028$$

Now for constant T_0

$$\frac{(T_{b2}-T_{b1})}{(T_0-T_b)_{ln}} = \ln\left(\frac{T_0-T_{b1}}{T_0-T_{b2}}\right)$$

Then for this problem:

$$\ln\left(\frac{T_0-T_{b1}}{T_0-T_{b2}}\right) = (0.0028)(960)(8.44)^{-2/3}(1.0) = 0.646$$

$$\frac{T_0-T_{b1}}{T_0-T_{b2}} = e^{0.646} = 1/0.524$$

Insertion of $T_0 = 215$ °F and $T_{b1} = 100$ °F gives:

$$T_{b2} = 215 - 0.524(215-100) = 155\ °F$$

13.C Effect of Flow Rate on Exit Temperature from a Heat Exchanger.

a. From the solution of Prob. 13.B we find that $Re = 1.075\,w$ and that

$$\frac{T_0-T_{b2}}{T_0-T_{b1}} = \exp\left(-\frac{0.646}{0.0028}Y\right) = e^{-231Y}$$

where w is rate of flow in pounds per hour and Y is the ordinate of Fig. 13.2-1 at the prevailing Reynolds number. The outlet temperature is then

$$T_{b2} = T_{b1} + (T_0-T_{b1})(1-e^{-231Y})$$

b. The total heat flow through the tube wall to the oil is given by:

$$Q = w\hat{C}_p(T_{b2}-T_{b1})$$

Calculations for a. and b. are summarized below:

w	Re	Y	$1-e^{-231Y}$	$(T_{b2}-T_{b1})$, °F	T_{b2}, °F	Q, Btu hr^{-1}
100	1075	0.0028	0.476	54.8	155	3300
200	2150	0.00185	0.348	40.0	140	4800
400	4300	0.0036	0.565	65.0	165	15,600
800	8600	0.0040	0.603	69.4	169	33,000
1,600	17,200	0.0037	0.574	66.0	166	63,000
3,200	34,400	0.0033	0.534	61.4	161	118,000

13.D Local Heat-transfer Coefficient for Turbulent Forced Convection in a Tube.

The physical properties of interest at $T_b = 60°F$ and $T_0 = 160°F$ are:

$$\hat{C}_p = 1.00 \text{ Btu lb}_m^{-1} \text{ °F}^{-1}$$

$$\mu_b = 2.76 \text{ lb}_m \text{ hr}^{-1} \text{ ft}^{-1}$$

$$\mu_0 = 0.969 \text{ lb}_m \text{ hr}^{-1} \text{ ft}^{-1}$$

$$k_b = 0.343 \text{ Btu hr}^{-1} \text{ ft}^{-1} \text{ °F}^{-1}$$

The Reynolds and Prandtl numbers are:

$$Re = (4w/\pi D\mu)_b = (4)(15{,}000)/\pi(2/12)(2.76) = 41{,}500$$

$$Pr = (\hat{C}_p\mu/k)_b = (1.00)(2.76)/(0.343) = 8.05$$

At this Reynolds number we may write, from Fig. 13.2-1:

$$\frac{h_{loc}D}{k_b} = 0.00315 \, Re_b \, Pr_b^{+1/3} \, (\mu_b/\mu_0)^{0.14}$$

$$h_{loc} = (0.00315)\left(\frac{0.343}{2/12}\right)(41{,}500)(2.005)(1.158)$$

$$= 625 \text{ Btu hr}^{-1} \text{ ft}^{-2} \text{ °F}^{-1}$$

The local radial heat flux at the pipe wall is then

$$q_r|_{r=R} = h_{loc}(T_b - T_0) = (625)(60-160)$$

$$= -6.25 \times 10^4 \text{ Btu hr}^{-1} \text{ ft}^{-2}$$

13.E Heat Transfer from Condensing Vapors.

a. The steam saturation temperature is 212°F, and the film temperature is $(1/2)(190+212) = 201°F$. The following physical properties apply (see Ex. 13.6-1):

$\Delta \hat{H}_{vap} = 970$ Btu lb_m^{-1} at 212°F (Sub-cooling of condensate is neglected.)

$k_f = 0.393$ Btu hr^{-1} ft^{-1} $°F^{-1}$

$\rho_f = 60.1$ lb_m ft^{-3}

$\mu_f = 0.738$ lb_m hr^{-1} ft^{-1}

The abscissa of Fig. 13.6-2 then is:

$$\frac{k_f \rho_f^{2/3} g^{1/3} (T_d - T_0) L}{\mu_f^{5/3} \Delta \hat{H}_{vap}} = \frac{(0.393)(60.1)^{2/3}(4.17\times10^{-8})^{1/3}(22)(1.0)}{(0.738)^{5/3}(970)}$$

$$= 168$$

This falls in the laminar region of the figure; extrapolation of the laminar line with a slope of 3/4 (see Eq. 13.6-5) gives:

$\Gamma/\mu_f = 170\,(0.168)^{3/4} = 45$

The heat transfer rate, neglecting sub-cooling, is:

$$Q = \pi D \Gamma \Delta \hat{H}_{vap} = \pi(1/12)(45 \times 0.738)(970)$$

$$= 8400 \text{ Btu/hr.}$$

A similar result could have been obtained from Eq. 13.6-5.

b. Comparison of Eqs. 13.6-3 and 5 at constant T_0 and T_d gives, for laminar condensate flow:

$$\frac{Q_{hor.}}{Q_{ver.}} = \frac{0.725}{0.943}\left(\frac{L}{D}\right)^{1/4}$$

Hence if the tube were horizontal the heat transfer rate would be:

$$Q_{hor} = 8400(0.725/0.943)(12)^{1/4} = 12,000 \text{ Btu hr}^{-1}$$

The value of Γ/μ_f is clearly less here than for the vertical tube; hence laminar flow is to be expected.

13.F Forced-convection Heat Transfer from an Isolated Sphere.

For this flow system Eq. 13.3-1 or Fig. 13.3-2 may be used. The physical properties at 1 atm and $T_f = \frac{1}{2}(T_0 + T_\infty) = 150°F$ are:

$\rho_f = pM/RT = 0.0651$ lb_m ft^{-3}

$$\mu_f = 0.0490 \ \text{lb}_m \ \text{ft}^{-1} \ \text{hr}^{-1} = 1.36 \times 10^{-5} \ \text{lb}_m \ \text{ft}^{-1} \text{sec}^{-1}$$

$$\hat{C}_f = 0.241 \ \text{Btu} \ \text{lb}_m^{-1} \ {}^\circ\text{F}^{-1}$$

$$k_f = 0.0169 \ \text{Btu} \ \text{hr}^{-1} \ \text{ft}^{-1} \ {}^\circ\text{F}^{-1}$$

The Reynolds and Prandtl numbers are:

$$\begin{aligned} Re &= (1/12)(100)(0.0651)/(1.36 \times 10^{-5}) \\ &= 3.99 \times 10^{4} \end{aligned}$$

$$Pr = (0.241)(0.0490)/(0.0169) = 0.70$$

Substitution of these values into Eq. 13.3-1 gives:

$$\begin{aligned} Nu_f &= 2.0 + 0.60(3.99 \times 10^4)^{1/2}(0.70)^{1/3} \\ &= 108 \end{aligned}$$

$$h_m = 108\left(\frac{0.0169}{1/12}\right) = 21.9 \ \text{Btu} \ \text{hr}^{-1} \ \text{ft}^{-2} \ {}^\circ\text{F}^{-1}$$

The heat loss from the sphere is:

$$\begin{aligned} Q &= (21.9)(\pi)(1/12)^2(200-100) = 47.8 \ \text{Btu} \ \text{hr}^{-1} \\ &= 3.35 \ \text{cal} \ \text{sec}^{-1} \end{aligned}$$

The heat loss by radiation is about 1/20 as much and can safely be neglected.

13.G Free-Convection Heat Loss from an Isolated Sphere.

For the conditions of this problem the thermal expansion coefficient $\beta = 1/T_f$ is $(1/610°R)$. Other physical properties are the same as in Prob. 13.F. (Note that for the correlations of §13.5 β and ρ are evaluated at T_f, rather than $\overline{T}$ as in Chap. 10, for calculating Gr.) We may then write from Eq. 13.5-2:

$$\begin{aligned} Nu_m &= 2.0 + 0.6\left(\frac{(\frac{1}{12})^3(0.0651)^2(32.2)(1/610)(100)}{(1.36 \times 10^{-5})^2}\right)^{1/4}(0.7)^{1/3} \\ &= 2.0 + 0.6(14.3) \\ &= 10.7 \end{aligned}$$

Since $Gr^{1/4}Pr^{1/3} < 200$ this expression is applicable.

$$h_m = 10.7\left(\frac{0.0169}{1/12}\right) = 2.17 \ \text{Btu} \ \text{hr}^{-1} \ \text{ft}^{-2} \ {}^\circ\text{F}^{-1}$$

The heat loss is

$$\begin{aligned} Q &= (2.17)(\pi/144)(200-100) = 4.74 \ \text{Btu} \ \text{hr}^{-1} \\ &= 0.332 \ \text{cal} \ \text{sec}^{-1} \end{aligned}$$

Allowance for radiation increases the calculated heat loss by about 50% if the sphere acts as a black body.

13.H Limiting Local Nusselt Number for Laminar Pipe Flow with Constant Wall Flux.

We may write

$$h_{loc} = q_1/(T_b - T_0) = k/R(\Theta_b - \Theta_0)$$

where Θ is the reduced temperature introduced in §9.8. Then the local Nusselt number is

$$Nu_{loc} = h_{loc} D/k = 2/(\Theta_b - \Theta_0)$$

Next we write:

$$\begin{aligned}\Theta(\xi, \zeta) - \Theta(1, \zeta) &= \Theta - \Theta_0 \\ &= -4\zeta - \xi^2 + \xi^4/4 + 7/24 \\ &\quad + 4\zeta + 1 - 1/4 - 7/24 \\ &= 3/4 - \xi^2 + \xi^4/4\end{aligned}$$

The reduced thermal driving force is then

$$\begin{aligned}\Theta_b - \Theta_0 &= \int_0^1 (3/4 - \xi^2 + \xi^4/4)(1-\xi^2)\xi d\xi \Big/ \int_0^1 (1-\xi^2)\xi d\xi \\ &= \int_0^1 (3/4 - \alpha + \alpha^2/4)(1-\alpha) d\alpha \Big/ \int_0^1 (1-\alpha) d\alpha \\ &= \left[\frac{3}{4} - \frac{3}{8} - \frac{1}{2} + \frac{1}{3} + \frac{1}{12} - \frac{1}{16}\right] / \left(\frac{1}{2}\right) \\ &= 11/24\end{aligned}$$

Then

$$Nu_{loc} = 2/(11/24) = 48/11.$$

13.I Local Over-all Heat-transfer Coefficients.

We first estimate a steam-film coefficient assuming a heat flux of some reasonable magnitude, using Eq. 13.6-2:

$$h_m = 0.954 \left(\frac{k_f^3 \rho_f^2 g L}{\mu_f \Delta \hat{H}_{vap} Q} \right)^{1/3}$$

As a first trial we assume $Q/L = 2500$ Btu hr^{-1} per foot of pipe. Physical properties are assumed to be the as in Prob. 13.E. Then

$$h_m = 0.954\left[\frac{(0.393)^3(60.1)^2(4.17\times10^8)}{(0.738)(970)(2500)}\right]^{1/3}$$

$$= 1610 \text{ Btu hr}^{-1}\text{ ft}^{-2}\ {}^\circ\text{F}^{-1}$$

We now use this trial value of h_m to calculate an overall heat-transfer coefficient according to Eq. 13.1-9:

$$\frac{1}{D_oU_o} = \frac{12}{U_o} = \left[\frac{12}{1610} + \frac{\ln(1/0.87)}{2\times218} + \frac{1.2}{0.87\times190}\right]$$

$$U_o = 149 \text{ Btu hr}^{-1}\text{ ft}^{-2}\ {}^\circ\text{F}^{-1}$$

$$Q = 149\left(\frac{\pi}{12}\right)(63) = 2460 \text{ Btu hr}^{-1}$$

This is close enough that our trial value of h_m may be accepted.

To determine the temperature drop across the oil we may write

$$U_o\,\Delta T_{oa} = h_{oil}\ \Delta T_{oil}$$

where ΔT_{oa} is the overall temperature drop and ΔT_{oil} is the temperature drop in the oil. Then:

$$\Delta T_{oil}/\Delta T_{oa} = 149/190 = 0.78.5$$

Then temperature at the inner pipe surface is

$$150^\circ\text{F} + (0.785)(63)^\circ\text{F} = 199.5\,^\circ\text{F}$$

13.J The Hot-wire Anemometer.

a Fig. 13.3-1 will be used. The physical properties of interest at $p_f = 1$ atm and $T_f = \frac{1}{2}(70+600) = 335$°F are:

$\rho_f = 0.0499 \text{ lb}_m \text{ ft}^{-3}$

$\mu_f = 0.0594 \text{ lb}_m \text{ hr}^{-1}\text{ft}^{-1} = 1.64\times10^{-5} \text{ lb}_m \text{ ft}^{-1}\text{sec}^{-1}$

(as calculated from Eq. 1.4-18)

$\hat{C}_{pf} = 0.242 \text{ Btu lb}_m^{-1}\ {}^\circ\text{F}^{-1}$

From Eq. 8.3-16, Pr = 0.74

The Reynolds number is

$$Re = (0.01/12)(100)(0.0499)/(1.64 \times 10^{-5}) = 254$$

From Fig. 13.3-1 $j_H = 0.035$ and

$$h_m = (0.035)(0.0499)(0.242)(3.6 \times 10^5)(0.74)^{-2/3}$$
$$= 186 \text{ Btu hr}^{-1} \text{ ft}^{-1} \, {}^{\circ}F^{-1}$$

The heat loss is then:

$$Q = h_m \pi D L (T_0 - T_\infty)$$
$$= (186 \pi)(0.01/12)(0.5/12)(600-70) = 10.75 \text{ Btu hr}^{-1}$$
$$= 3.15 \text{ watts}$$

b. As velocity increases from 100 to 1000 ft sec^{-1} j_H drops from 0.035 to about 0.012. Then

$$Q(1000)/Q(100) = \frac{0.012}{0.035}(10) = 3.43$$

The predicted value is $\sqrt{10}$ or about 3.16. King's relation is therefore not very reliable in this range of Re. The constant C represents heat loss at zero velocity, that is for steady heat conduction to infinitely large quiescent surrounding fluid. Since no steady-state solution of this type exists for an infinitely long cylinder, C must depend on the ratio of wire length to diameter.

13.K Extension of Dimensional Analysis.

a. According to Eq. 9.8-33:

$$T_{b2} - T_0 = \int_0^{2\pi}\int_0^R v_z(r)\left[T_i(H) - T_0\right] r\,dr\,d\theta \Big/ \int_0^{2\pi}\int_0^R v_z(r)\, r\,dr\,d\theta$$

In terms of reduced variables this equation may be written:

$$\frac{T_{b2}-T_0}{T_{b1}-T_0} = T_{b2}^* = \int_0^{2\pi}\int_0^{1/2} v_z^* T^* r^* dr^* d\theta \Big/ \int_0^{2\pi}\int_0^{1/2} v_z^* r^* dr^* d\theta$$

$$= T_{b2}^*(Re, Pr, Br, L/D)$$

$$\frac{T_{b2}-T_{b1}}{T_0 - T_{b1}} = \frac{T_{b2}-T_0}{T_0 - T_{b1}} + \frac{T_0 - T_{b1}}{T_0 - T_{b1}} = 1 - T_{b2}^*(Re, Pr, Br, L/D)$$

b. Eq. 13.2-12: We may write analogously to Eq. 13.2-2

$$h_a = \frac{2\ I}{\pi D L\left[(T_0 - T_{b1}) + (T_0 - T_{b2})\right]}$$

where I is the integral in Eq. 13.2-2. The analog to

Eq. 13.2-3 is then:

$$Nu_a = DI^*/(1 + T_{b2}^*)\pi L$$

Since T_{b2}^* depends only upon Re, Pr, Br, and L/D the suggested functional relation is valid.

Similir manipulations of the expressions for h_{ln} and h_{loc} yields:

$$Nu_{ln} = (DI^*/2\pi L)\ln[1/T_{b2}^*]/(1 - T_{b2}^*)$$

$$Nu_{loc} = (DI^*/2\pi L)/T_{b2}^* \quad (Nu_{loc} \text{ evaluated at } z = L/D)$$

13.L Relation between h_{loc} and h_{ln}.

a. To obtain the right side of Eq. 13.L-1: simply equate the rate of heat transfer though the wall in section dz with the rate of gain of energy by the fluid in passing through this differential section. Assume $\rho\hat{C}_p$ constant and neglect both kinetic energy changes and heat conduction in the direction of flow. (See the definition of T_b, Eq. 9.8-33.) Eq. 13.L-2 is obtained by a simple integration.

b. Again equate rate of heat transfer with rate of energy gain. For the length L of the pipe:

$$Q = w\hat{C}_p(T_b(L) - T_b(0)) = h_{ln} A (T_0 - T_b)_{ln}$$

$$w = (\pi D^2/4)\rho\langle v\rangle$$

$$A = \pi DL$$

It then follows that

$$(1/4)\rho\hat{C}_p\langle v\rangle(T_b(L) - T_b(0)) = L h_{ln}$$

The desired answer then follows directly by substituting the above expression into Eq. 13.L-2 and dividing the result by L.

13.M Heat Loss by Free Convection from a Pipe.

$$Nu_m = 0.525(Gr\,Pr)^{1/4}$$

$$h_1/h_2 = (k_1/k_2)^{3/4}(T_2/T_1)^{1/4}(\hat{C}_{p1}/\hat{C}_{p2})^{1/4}(\mu_2/\mu_1)^{1/4}$$

At the conditions of this problem:

$\mu =$ 0.0216 cp.
$k =$ 0.0173 Btu hr^{-1} ft^{-1} °F^{-1}
$\beta =$ 1/650 °R^{-1}
$\hat{C}_p =$ 0.242 Btu lb$_m^{-1}$ °F^{-1}

Then, using subscripts 1 & 2 for the previous and present conditions, respectively:

$$h_1/h_2 = \left(\frac{0.0152}{0.0173}\right)^{3/4}\left(\frac{650}{550}\cdot\frac{0.241}{0.242}\cdot\frac{0.0216}{0.0190}\right)^{1/4}$$

$$= (1.18 \times 0.995 \times 1.137/1.14^3)^{1/4} = 0.90^{1/4}$$

$$= 0.975$$

Thus the heat transfer rate is slightly increased. Note that if it had been assumed that k and μ varied with the ½-power of T, according to the simplified kinetic theories of Chaps. 8 and 1, then almost no change would have been predicted.

CHAPTER 14 - Checked by V. D. Shah

14.A Approximation of a Black Body by a Hole in a Sphere.

According to Eq. 14.2-12:

$$e' = \frac{e}{e + f(1-e)} \quad \text{or} \quad f = \frac{e(1-e')}{e'(1-e)}$$

Here $e = 0.57$ and $e' = 0.99$. Then

$$f = (0.57)(0.01)/(0.99)(0.43) = 0.0134$$

$$= (\text{area of hole})/(\text{area of sphere}) = \pi R^2/4\pi(3)^2$$

The radius of the hole must then be:

$$R = \sqrt{6} = 6\sqrt{0.0134} = 0.695 \text{ in.}$$

14.B Efficiency of a Solar Engine.

Area of mirror = 25π ft^2.

Solar constant = 428 Btu hr^{-1} ft^{-2}

Energy input to device is:

$$(428)(25\pi)(3.93\times10^{-4}) = 13.2 \text{ hp}$$

Efficiency = 2 / 13.2 = 15.2 %

14.C Radiant Heating Requirement.

The heat required is the sum of the heat-transfer rates between the floor and each of the other surfaces. Since no pertinent data are supplied for its estimation, the convective heat transfer will be neglected. It can be expected to be appreciable, however.

We write for the total heat-transfer rate:

$$Q_{rad} = \sigma A_{floor} (T_{floor}^4 - T_{wall}^4) \Sigma F_i$$

Floor to ceiling: $F_1 = 0.49$

Floor to large wall: $F_2 = F_3 = 0.17$

Floor to small wall: $F_4 = F_5 = 0.075$

$$\Sigma F_i = 0.49 + 2\times 0.17 + 2\times 0.075 = 0.98$$

Now, alternately, we may consider the floor to be completely surrounded by black surroundings. We know then $\Sigma F_i = 1.000$. An accumulated error of 2% has thus resulted from considering each of the cold surfaces separately. Then

$$Q_{rad} = 1712 \times 450 (0.535^4 - 0.450^4) = 31,800 \text{ Btu hr}^{-1}$$

14.D Steady-state Temperature of a Roof.

Assume the angle of incidence of the sun's rays to be 21.5°. Equate energy input by radiation to rate of heat loss by convection:

$$a \cdot 430 \cdot \cos 21.5° = h (T_{roof} - T_{air}) + \sigma e T_{roof}^4$$

$$a.\ (1.00)(430)(0.9304) = 2(T_{roof} - 560) + (1712)(1.00)\left(\frac{T_{roof}}{1000}\right)^4$$

By trial and error the roof temperature is found to be about 625°R or 165°F.

b. $(0.3)(430)(0.9304) = 2(T_{roof} - 560) + (1712)(0.07)\left(\frac{T_{roof}}{1000}\right)^4$

Again using trial and error the roof temperature is found to be about 610°R or 150°F.

14.E Radiation Errors in Temperature Measurements.

Assume the thermocouple to act as a grey body in large black surroundings, and equate the net radiation loss with the convective input:

$$e\sigma(T_{t.c.}^4 - T_w^4) = h(T_g - T_{t.c.})$$

For the conditions of this problem:

$$(0.8)(1712)(0.96^4 - 0.76^4) = 50(T_g - 960)$$

$$T_g = 960°R + (705/50) = 974.1°R$$
$$= 514.1°F$$

There is thus a 14.1°F discrepancy between the calculated gas temperature and the thermocouple reading.

14.F Mean Temperature for Effective Emissivity.

By the assumption of linear variation of emissivity with temperature:

$$e = a + bT$$

where a and b are constants. Then

$$e_1 - e^{\circ} = b(T_1 - T^{\circ})\;;\quad e_2 - e^{\circ} = b(T_2 - T^{\circ})$$

$$(e_1 - e^{\circ})T_1^4 = b(T_1 - \dot{T})T_1^4$$
$$(e_2 - e^{\circ})T_2^4 = b(T_2 - T^{\circ})T_2^4$$

But $(e_1 - e^{\circ})T_1^4 = (e_2 - e^{\circ})T_2^4$

and therefore

$$T_1^5 - T_1^4 T^{\circ} = T_2^5 - T_2^4 T^{\circ}$$

The desired answer is now obtained by rearrangement.

14.G Radiation across an Annulus.

We proceed by following the history of a single primary ray emitted from the inner cylinder (area A_1). However, before we can do this we need to know the fraction of any beam from either surface directly impinging on the other. Clearly the fraction of any beam from the inner surface directly impinging on the outer surface is unity, since the inner surface cannot "see" itself. The fraction of a beam from the outer surface impinging directly on the inner surface is:

$$f = \int_0^{\arcsin(R_1/R_2)} \cos\theta \, d\theta \Big/ \int_0^{\arcsin(1)} \cos\theta \, d\theta$$

$$= R_1/R_2 = A_1/A_2$$

where the subscript 1 refers to the inner surface and 2 to the outer one. This result is obtained by considering the view which any differential element of the outer surface has of the inner surface.

We are now in a position to follow the course of a primary beam emitted from the inner surface:

(1) The rate of radiation from A_1 toward A_2 is:

$$e_1 A_1 \sigma T_1^4$$

Of this $e_2 e_1 A_1 \sigma T_1^4$ is absorbed directly, and the remainder, $(1-e_2) e_1 A_1 \sigma T_1^4$, is reflected, in part directly to A_2 and in part to A_1.

(2) The secondary beam, that portion of the primary beam reflected back to surface 2 (both directly and via surface 1) is:

$$(1-f)(1-e_2)\, e_1 \sigma A_1 T_1^4 \quad \text{(direct reflection)}$$
$$+ \underline{f(1-e_1)(1-e_2)\, e_1 \sigma A_1 T_1^4} \quad \text{(reflection via } A_1\text{)}$$
$$= (e_1 \sigma A_1 T_1^4)[f(1-e_1)(1-e_2) + (1-f)(1-e_2)] = e_1 \sigma A_1 T_4 \cdot \alpha$$

Here α is the ratio of the intensity of the secondary beam to that of the primary beam. The amount of the secondary beam directly absorbed is

$$e_1 e_2 \sigma A_1 T_1^4 \cdot \alpha$$

The remainder is reflected in the same way as the primary beam.

(3) The strength of the tertiary beam is clearly:

$$e_1 \sigma A_1 T^4 \alpha^2$$

The amount of this beam which is absorbed directly is:

$$e_1 e_2 \sigma A_1 T_1^4 \alpha^2$$

(n) We may now generalize the above development and write, for the n_{th} reflection of the original beam, that the amount absorbed is:

$$e_1 e_2 \sigma A_1 T_1^4 \alpha^{n-1}$$

(Σ) The total amount of the original beam absorbed is the sum of the amounts absorbed during each reflection. Then

$$Q_{1\to 2} = e_1 e_2 A_1 \sigma T_1^4 \sum_{i=0}^{\infty} \alpha^i = \frac{e_1 e_2 \sigma A_1 T_1^4}{(1-\alpha)}$$

and

$$Q_{12} = e_1 e_2 \sigma A_1 (T_1^4 - T_2^4)/(1-\alpha)$$

where $\alpha = (A_1/A_2)(1-e_1)(1-e_2) + (1-\frac{A_1}{A_2})(1-e_2)$

This result may be easily re-arranged to give the desired answer.

14.H Multiple Radiation Shields.

For a system of n gray plates we may write:

$$q_{rad}/\sigma = \frac{T_1^4 - T_2^4}{1/F_{12}} = \frac{T_2^4 - T_3^4}{1/F_{23}} = \cdots = \frac{T_{n-1}^4 - T_n^4}{1/F_{n-1,n}} = \frac{T_1^4 - T_n^4}{1/F_{1n}}$$

The F_{ij} in these expressions are obtained from Eq. 14.5-8:

$$1/F_{ij} = \left(\frac{1}{e_i} + \frac{1}{e_j} - 1\right)$$

It may be noted that the expression is closely analogous to that developed in Chap. 9 for conductive resistances in series. Clearly

$$\frac{1}{F_{1n}} = \frac{1}{F_{12}} + \frac{1}{F_{23}} + \frac{1}{F_{34}} + \cdots \frac{1}{F_{n-1,n}}$$

$$= \sum_{i=1}^{n-1} \frac{1}{F_{i,i+1}}$$

For n identical sheets we may write:

$$Q_n = Q_2/(n-1)$$

where Q_n is the rate of heat transfer for n sheets. Thus $Q_3 = Q_2/2$ in accordance with the results of Ex. 14.5-1.

14.I Radiation and Conduction Through Absorbing Media.

a. Begin with Eqs. 14.6-5 and 6 to determine $\mathscr{A}(z)$. These equations can be combined and integrated to give (see Eq. 14.6-8):

$$q_z^{(r)} = q_0^{(r)} e^{-m_a z}$$

and: $$\mathscr{A} = m_a q_0^{(r)} e^{-m_a z}$$

Next substitute the above expression for $\mathscr{A}$ into Eq. 14.6-4 to get (for constant k):

$$0 = +k \frac{d^2T}{dz^2} + m_a q_0^{(r)} e^{-m_a z}$$

Integrate this expression twice with respect to z to obtain:

$$T = -(q_0^{(r)}/k\,m_a)\, e^{-m_a z} + C_1 z + C_2$$

The boundary conditions for this problem are:

At $z = 0$ $\quad T = T_0$

At $z = \delta$ $\quad T = T_\delta$

The final solution is then:

$$(T-T_0) = (q_0^{(r)}/k\,m_a)(1 - e^{-m_a z})$$
$$+ (z/\delta)\left[(T_\delta - T_0) - (q_0^{(r)}/k\,m_a)(1 - e^{-m_a \delta}) \right]$$

b. At very high m_a the radiation terms in the above expression both become negligible. Here all of the radiation is absorbed very near the surface of the slab, and the effect of radiation is important only at very small z. In this region q_z is large and negative. At very low m_a the radiation are small and nearly equal and opposite. Again radiation effects are unimportant, this time because most of the radiation passes through the slab. At intermediate m_a, where appreciable rates of absorption occur throughout the slab, there may be a temperature maximum. In this case

q_z will be negative for small z/S and become positive as z/S approaches unity.

CHAPTER 15 - Checked by R.H. Weaver

15.A Rates of Heat Transfer in a Double-pipe Exchanger.

a. $w_c \hat{C}_{pc} \Delta T_c = w_h \hat{C}_{ph} \Delta T_h$

$(5000)(1.00)(T_{c2} - 60) = (10,000)(0.60)(200-100)$

$T_{c2} = 60 + 120 = 180°F$

$Q_c = (5,000)(1.0)(180-60) = 600,000 \text{ Btu hr}^{-1}$

$A_o = Q_c / U_o (T_h - T_c)_{ln}$

$(T_h - T_c)_{ln} = (20-40)/\ln(20/40) = 20/\ln 2$
$= 28.9 °F$

$A_o = (600,000)/(200)(28.9) = 103.7 \text{ ft}^2$

b. $(U\Delta T)_{ln} = [(50)(20)-(350)(40)] / \ln\left[\frac{(50)(20)}{(350)(40)}\right] = \frac{13,000}{\ln 14}$
$= 4,930$

$A_o = 600,000/4,930 = 121.7 \text{ ft}^2$

c. (1) $w_c = (10,000)(0.6)(100)/(1.0)(200-60) = 4290 \text{ lb}_m \text{ hr}^{-1}$

(2) $w_c = (10,000)(0.6)(100)/(1.0)(40) = 15,000 \text{ lb}_m \text{ hr}^{-1}$

d. Exit temperature $= 60 + (10,000)(0.6)(100)/(15,500)(1.0)$
$= 60 + 38.7$
$= 98.7°F$

$\Delta T_{ln} = (140 - 1.3)/\ln\left(\frac{140}{1.3}\right) = 138.7/\ln(107.9)$
$= 29.8 °F$

$A = (3,000)/(29.8) = 101 \text{ ft}^2$

15.B Adiabatic Flow of Natural Gas in a Pipeline.

$\hat{V}_1 = RT_1/Mp_1 = (530)(10.73)/(16)(100)$
$= 3.55 \text{ ft}^3 \text{ lb}_m^{-1}$

$G = 80/3.55 = 22.5 \text{ lb}_m \text{ sec}^{-1} \text{ ft}^{-2}$

$N = (4)(0.0018)(52,800)/(2) = 190$

$p_1/\hat{V}_1 = (100)(44)(32.2)/(3.55) = 130,700 \text{ lb}_m^2 \text{ sec}^{-2} \text{ ft}^{-4}$

$$22.5 = \left\{ \frac{130,700}{\left[\frac{190 - \frac{2.3}{2.6}\ln r}{1-r}\right] - \frac{0.3}{2(0.3)}} \right\} \qquad (r = (\hat{V}_1/\hat{V}_2)^2)$$

$r = 0.258 \; ; \quad \hat{V}_2 = 3.55/0.507$

$\rho_2 = 1/\hat{V}_2 = 0.143 \text{ lb}_m \text{ ft}^{-3}$

b. $$p_2/p_1 = 0.507\left[\frac{(-2.88)(507)(3.55)}{(100)(144)(32.2)} \cdot \frac{(0.3)}{(2.6)} + 1\right] = 0.506$$

$p_2 = 50.6 \text{ lb}_f \text{ in}^{-2}$

c. $v_2^2 = v_1^2(\hat{V}_2/\hat{V}_1)^2 = 3.88 v_1^2$

$$\begin{aligned} -W' &= (6400)(1-3.88)/(2)(32.2) \\ &\quad + (50.6)(144)(7.00)(1.3/0.3)[(1.98)^{(0.3/1.3)} - 1] \\ &= -286 + 37,800 = 37,500 \text{ ft-lb}_f \text{ lb}_m^{-1} \end{aligned}$$

$P = 4850$ hp.

15.C Mixing of Two Ideal-gas Streams.

a. $$\begin{aligned} P &= (1000/3600)\{1000 + (1545)(540)(32.2)/(29)(10^3)\} \\ &\quad + (10,000/3600)\{100 + (1545)(540)(32.2)/(29)(100)\} \\ &= 535 + 25,800 = 26,335 \text{ ft-lb}_m \text{ sec}^{-2} \end{aligned}$$

$P/w = 8620 \text{ ft sec}^{-1}$

$$\begin{aligned} E &= \left(\frac{1}{3.6} + \frac{1}{0.36}\right)[(7.0/29)(540)(778)(32.2)] \\ &\quad + (1/3.6)(\tfrac{1}{2} \times 10^6) + (1/0.36)(\tfrac{1}{2} \times 10^4) \\ &= (3.06)(3.27 \times 10^6) + 1.29 \times 10^5 + 1.29 \times 10^4 \\ &= 101.3 \times 10^5 \text{ lb}_m \text{ ft}^2 \text{ sec}^{-2} \end{aligned}$$

$E/w = 3.31 \times 10^6 \text{ ft}^2 \text{ sec}^{-2}$

$$v_3 = (8620)(1.4/2.4)\left\{1 \pm \sqrt{1 - 2(\tfrac{0.96}{1.96})(3.31\times10^6)/(7.43\times10^7)}\right\}$$

$$= 5030\{1 \pm \sqrt{1 - 0.0437}\} = 5030[1 \pm 0.978]$$

$$= 110 \text{ ft sec}^{-1},\ 9940 \text{ ft sec}^{-1}$$

$$T_3 = [(3.31\times10^6) - \tfrac{1}{2}(1.21\times10^4)]/\hat{C}_p$$

$$= (3.31\times10^6)/(0.241)(778)(32.2) = 548\ {}^\circ R$$

$$= 88^\circ F$$

For the (hypothetical) supersonic case:

$$T_3 = (3.31\times10^6 - 9.87\times10^7)(1.655\times10^{-4})$$

$$= -1083^\circ R$$

$$S_3 = S_1 + S_2 = \frac{w_1}{v_1\rho_1} + \frac{w_2}{v_2\rho_2} = \frac{RT_1}{M p_1}\left(\frac{w_1}{v_1} + \frac{w_2}{v_2}\right)$$

$$= \frac{(0.7302)(540)}{(29)}\left(\frac{0.278}{1000} + 0.0278\right) = 0.381 \text{ ft}^2$$

$$p_3 = \left(26{,}335 - \frac{11{,}000}{3{,}600}110\right)/(0.381)(32.2) = 2120 \text{ lb}_f \text{ ft}^{-2}$$

$$= 1.00 \text{ atm}$$

b. $v_3 = (v_1 S_1 + v_2 S_2)/S_3$

$$= (2.78\times10^{-2}\times100 + 2.78\times10^{-4}\times1000)/(1.01)(2.78\times10^{-2})$$

$$= 109 \text{ ft sec}^{-1}$$

c. $\hat{E}_v \doteq -\Delta\hat{K}$

$$= (10^3\times\tfrac{1}{2}\times10^6 + 10^4\cdot\tfrac{1}{2}\times10^4 - 1.1\times10^4\times1.21\times10^4)/(1.1\times10^4)$$

$$= 4.4\times10^4 \text{ ft}^2 \text{ sec}^{-2}$$

$$= 1.366\times10^3 \text{ ft-lb}_f \text{ lb}_m^{-1}$$

15.D Flow through a Venturi Tube.

a. $\rho_2 = \rho_1 (0.75)^{1/1.4} = (0.0799)(0.814) = 0.0609$

$S_2 = \pi(0.25)^2/4 = 0.049 \text{ ft}^2$; $S_o/S_1 = 1/16$

$p_1/\rho_1 = (1.0)(14.7)(144)(32.2)/(0.0749) = 910{,}000 \text{ ft}^2 \text{ sec}^{-2}$

$$w = (0.98)(0.0609)(0.049)\left\{\frac{2(910{,}000)(\frac{1.4}{0.4})\left[1-(0.75)^{(0.4/1.4)}\right]}{1-(\frac{1}{16})^2(0.75)^{(2/1.4)}}\right\}^{1/2}$$

$$= 2.07 \text{ lb}_m \text{ sec}^{-1}$$

b. $\rho_2 = (0.75)(29)/(0.7302)(530) = 0.0563$

$$w = (0.98)(0.0563)(0.049)\left\{\frac{-2(0.73)(530)(144)(32.2)\ln 0.75}{(29)\left[1-(\frac{0.0563}{0.0749}\cdot\frac{1}{16})^2\right]}\right\}^{1/2}$$

$$= 1.96 \text{ lb}_m \text{ sec}^{-1}$$

c. $$w = (0.98)(0.0749)(0.049)\left\{\frac{-2(1/0.0749)(0.75-1.0)(14.7)(144)(32.2)}{[1-(1/16)^2]}\right\}^{1/2}$$

$$= 2.43 \text{ lb}_m \text{ sec}^{-1}$$

15.E Performance of a Double-pipe Heat Exchanger with Variable Over-all Heat-transfer Coefficient.

Our starting equation is:

$$-\frac{d(T_h-T_c)}{(T_h-T_c)} = -\frac{d\,\Delta T}{\Delta T} = U\left(\frac{1}{w_c\hat{C}_p} + \frac{1}{w_h\hat{C}_{ph}}\right)2\pi r_0\,dz = U\,d\alpha$$

For the conditions of this problem:

$$\Delta T = \Delta T_1 + \beta(U-U_1)\ ; \quad \beta = (\Delta T_2 - \Delta T_1)/(U_2-U_1)$$

$$d(\Delta T) = \beta\,dU$$

Then we may write our original equation as:

$$\frac{-\beta\,dU}{U(\Delta T_1 - \beta U_1 + \beta U)} = d\alpha$$

This equation in turn may be integrated to give:

$$\alpha = \beta\left[\frac{1}{\Delta T_1 - \beta U_1}\right]\ln\left[\frac{\Delta T_1 - \beta U_1 + \beta U}{U}\right]\Bigg|_{U_1}^{U_2}$$

We may now simplify the terms in this expression:

$$\beta\left[\frac{1}{\Delta T_1 - \beta U_1}\right] = \frac{\Delta T_2 - \Delta T_1}{U_2\Delta T_1 - U_1\Delta T_1 - U_1\Delta T_2 + U_1\Delta T_1} = \frac{\Delta T_2 - \Delta T_1}{U_2\Delta T_1 - U_1\Delta T_2}$$

$$= \frac{(T_{h2} - T_{h1}) - (T_{c2} - T_{c1})}{U_2\Delta T_1 - U_1\Delta T_2}$$

$$\ln\left[\frac{\Delta T_1 + \beta(U - U_1)}{U}\right]\Bigg|_{U_1}^{U_2} = \ln\left[\frac{(\Delta T_1 + \Delta T_2 - \Delta T_1)/U_2}{(\Delta T_1 + 0)/U_1}\right]$$

$$= \ln(U_1\Delta T_2/U_2\Delta T_1)$$

We may then rewrite our integrated expression as:

$$\left(\frac{1}{w_c\hat{C}_{pc}} + \frac{1}{w_h\hat{C}_{ph}}\right)A = \frac{(T_{h2} - T_{h1}) - (T_{c2} - T_{c1}) \quad \ln(U_1\Delta T_2/U_2\Delta T_1)}{U_2\Delta T_1 - U_1\Delta T_2}$$

or:

$$\frac{(T_{h2} - T_{h1}) - (T_{c2} - T_{c1})}{\left(\frac{1}{w_c\hat{C}_{pc}} + \frac{1}{w_h\hat{C}_{ph}}\right)} = -A \cdot \frac{(U_1\Delta T_2 - U_2\Delta T_1)}{\ln(U_1\Delta T_2/U_2\Delta T_1)}$$

$$= -A(U\Delta T)_{ln}$$

But

$$Q_h = w_h\hat{C}_p(T_{h2} - T_{h1}) = -w_c\hat{C}_{pc}(T_{c2} - T_{c1})$$

Therefore we may write:

$$Q_h = -A(U\Delta T)_{ln}$$

Note that the minus sign is needed because $\Delta T = T_h - T_c$.

15.F Steady-state Flow of Ideal Gases in Ducts of Constant Cross-section.

b. The Bernoulli equation may be rewritten as:

$$v\,dv + d(p\hat{V}) - p\hat{V}\frac{d\hat{V}}{\hat{V}} + \frac{1}{2}v^2\,de_v = 0$$

(c) For isothermal flow of ideal gases $p\hat{V} = RT$ is a constant, and $d(p\hat{V}) = 0$. The equation of continuity may be written in the form $G = \rho v = v/\hat{V}$ = a constant. Then $dv/d\hat{V} = v/\hat{V}$, and the Bernoulli equation may be rewritten to give:

$$v\,dv - RT\,dv/v + \tfrac{1}{2}v^2\,de_v = 0$$

The variables v and e_v may now be separated to obtain Eq. 15.F-3:

$$de_v = 2RT\frac{dv}{v^3} - 2\,dv/v$$

This expression may be readily integrated to give:

$$e_v = RT\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right) - 2\ln v_2/v_1$$

where e_v is the total dimensionless resistance between control surfaces "1" and "2". Again taking advantage of the equation of continuity and the ideal gas law we may write:

$$e_v = (p_1/G^2\hat{V}_1)[1 - (p_2/p_1)^2] - \ln(p_1/p_2)^2$$

We may then easily solve for G. Designating $(p_2/p_1)^2$ as r we write:

$$G = \frac{v_1}{\hat{V}_1} = [(p_1/\hat{V}_1)(1-r)/(e_v - \ln r)]^{1/2}$$

The maximum flow rate occurs at $\partial G^2/\partial r = 0$ where:

$$0 = -(p_1/\hat{V}_1)/(e_v - \ln r) + \frac{(p_1/\hat{V}_1)(1-r)}{(e_v - \ln r)^2}(-1)\left(-\frac{1}{r^2}\right)$$

This expression can be rearranged simply to give the desired answer.

d. For the conditions of this part of the problem Eq. 15.1-4 may be written as:

$$d\hat{U} + d(p\hat{V}) + d\hat{K} = 0$$

For an ideal gas $d\hat{U} = \hat{C}_v\,dT = (M\hat{C}_v/R)\,d(p\hat{V})$. We may then write:

$$0 = [(M\hat{C}_v/R) + 1]\,d(p\hat{V}) + d(\tfrac{1}{2}v^2) = \left(\frac{\gamma}{\gamma-1}\right)d(p\hat{V}) + d(\tfrac{1}{2}v^2)$$

It then follows that

$$p\hat{V} + \left(\frac{\gamma-1}{\gamma}\right)(\tfrac{1}{2}v^2) = \text{a constant.}$$

Substituting $\left(-\frac{\gamma-1}{\gamma}\right)d(\frac{1}{2}v^2)$ for $d(p\hat{V})$ in the Bernoulli equation (part b of this problem) we obtain:

$$-\left(\frac{\gamma+1}{\gamma}\right)\frac{dv}{v} - 2\left[p_1\hat{V}_1 + \left(\frac{\gamma-1}{2\gamma}\right)v_1^2\right]\frac{dv}{v^3} = de_v$$

On integration of this expression we get:

$$e_v = -\left(\frac{\gamma+1}{\gamma}\right)\ln(v_2/v_1) - \left[p_1\hat{V}_1 + \frac{(\gamma-1)}{2\gamma}v_1^2\right]\left(\frac{1}{v_2^2} - \frac{1}{v_1^2}\right)$$

$$= \left[\frac{p_1\hat{V}_1}{v_1^2} + \left(\frac{\gamma-1}{2\gamma}\right)\right]\left(1 - \frac{v_1^2}{v_2^2}\right) + \left(\frac{\gamma+1}{\gamma}\right)\ln(v_1/v_2)$$

$$= \left[\frac{p_1}{\hat{V}_1 G^2} + \left(\frac{\gamma-1}{2\gamma}\right)\right]\left[1 - \left(\frac{\hat{V}_1}{\hat{V}_2}\right)^2\right] + \left(\frac{\gamma+1}{2\gamma}\right)\ln(\hat{V}_1/\hat{V}_2)^2$$

The desired answer is obtained by solving this equation for G.

e. For an ideal gas of constant heat capacity we may write Eq. 15.1-4 as:

$$\hat{C}_p(T_2 - T_1) + \frac{1}{2}(v_2^2 - v_1^2) = 0$$

But $(T_2 - T_1) = (p_2\hat{V}_2 - p_1\hat{V}_1)(M/R)$, and $G = v/\hat{V}$; therefore we may write:

$$(\hat{C}_p/R)(p_2\hat{V}_2 - p_1\hat{V}_1) + \frac{1}{2}G^2(\hat{V}_2^2 - \hat{V}_1^2) = 0$$

We may re-arrange this equation to get:

$$(p_2/p_1) = (\hat{V}_1/\hat{V}_2)\left\{\frac{[1 - (\hat{V}_2/\hat{V}_1)^2]\,G^2\,\hat{V}_1\left(\frac{\gamma-1}{2\gamma}\right)}{p_1} + 1\right\}$$

15.G The Mach Number in the Mixing of Two Fluid Streams.

a. For the conditions of this problem we may combine Eqs. 15.4-30 and 32 to obtain:

$$v_3 = \left(v_3 + \frac{RT_3}{Mv_3}\right)\left(\frac{\gamma}{\gamma+1}\right)$$

On rearrangement this expression yields:

$$v_3 = \sqrt{\gamma R T_3 / M}$$

It then follows that v_3 is sonic velocity and that the Mach number is unity. (See Probs. 10.L and Q.)

b. Set w_1 or w_2 equal to zero.

15.H Limiting Discharge Rates for Venturi Meters.

a. Since under the conditions of this part of the problem $(p_2/p_1)^{1/\gamma} = \rho_2/\rho_1$ we may write:

$$w^2 = C_D^2 \rho_1^2 S_0^2 \left(\frac{\gamma}{\gamma-1}\right) 2 (p_1/\rho_1) \left\{ \frac{r^{2/\gamma}\left[1 - r^{\left(\frac{\gamma-1}{\gamma}\right)}\right]}{1 - \beta r^{2/\gamma}} \right\} \; ; \; \beta = (S_0/S_1)^2$$

The maximum discharge rate occurs at:

$$\frac{\partial w^2}{\partial r} = 0 = \frac{\partial}{\partial r}\left[\frac{r^{2/\gamma} - r^{\left(\frac{\gamma+1}{\gamma}\right)}}{1 - \beta r^{2/\gamma}}\right]$$

$$= \frac{\left(\frac{2}{\gamma}\cdot r^{2/\gamma} - \left(\frac{\gamma+1}{\gamma}\right) r^{\left(\frac{\gamma+1}{\gamma}\right)}\right)\frac{1}{r}}{(1 - \beta r^{2/\gamma})}$$

$$+ \frac{(r^{2/\gamma} - r^{\frac{\gamma+1}{\gamma}})}{(1 - \beta r^{2/\gamma})^2}(-1)\left(0 - \frac{2\beta}{\gamma}\cdot r^{2/\gamma}\right)$$

$$= \frac{2}{\gamma} r^{2/\gamma} - \left(\frac{\gamma+1}{\gamma}\right) r^{\frac{\gamma+1}{\gamma}} + \frac{\frac{2\beta}{\gamma}(r^{2/\gamma} - r^{\frac{\gamma+1}{\gamma}})\, r^{2/\gamma}}{(1 - \beta r^{2/\gamma})}$$

Multiplying through by $(1 - \beta r^{2/\gamma})/r^{2/\gamma}$ we obtain:

$$\frac{2}{\gamma} - \frac{\gamma+1}{\gamma} r^{\left(\frac{\gamma-1}{\gamma}\right)} - \cancel{\frac{2\beta}{\gamma} r^{2/\gamma}} + \beta\left(\frac{\gamma+1}{\gamma}\right) r^{\frac{\gamma+1}{\gamma}} = \frac{2\beta}{\gamma} r^{\frac{\gamma+1}{\gamma}} - \cancel{\frac{2\beta}{\gamma} r^{2/\gamma}}$$

Collecting terms we get:

$$\frac{2}{\gamma} - \frac{\gamma+1}{\gamma} r^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma+1}{\gamma}}\left[\frac{2\beta}{\gamma} - \frac{\beta + \beta\gamma}{\gamma}\right] = \beta r^{\frac{\gamma+1}{\gamma}}\left(\frac{1-\gamma}{\gamma}\right)$$

Multiplying through by $[\gamma / r^{\frac{\gamma+1}{\gamma}}]$ we may write

$$\frac{1-\gamma}{1/\beta} + \frac{\gamma+1}{r^{2/\gamma}} - \frac{2}{r^{\left(\frac{\gamma+1}{\gamma}\right)}} = 0$$

This is the desired answer.

b. This answer is obtained by straightforward substitution.

c. It may readily be shown that, for the conditions of this part of the problem, the discharge rate is:

$$w = C_D \rho_2 S_2 \sqrt{\frac{(2RT/M)\ln(p_1/p_2)}{1-(S_0/S_1)^2}}$$

Since $\rho_2 = \rho_1 (p_2/p_1)$ we may rewrite this expression as:

$$w = C_D \rho_1 S_2\, r \sqrt{\frac{(2RT/M)\ln(1/r)}{1-\beta}}$$

where $r = p_2/p_1$ and $\beta = (S_0/S_1)^2$ as before. Again the maximum discharge rate occurs at

$$\frac{\partial w^2}{\partial r} = 0 = \frac{\partial}{\partial r}(r^2 \ln r) = 2r\ln r + r$$

Then maximum discharge occurs when

$$r = p_2/p_1 = 1/\sqrt{e}$$

Then for small β the maximum discharge rate may be expressed as:

$$w_{max} = \rho_1 S_2 C_D \left(\frac{1}{\sqrt{e}}\right)\sqrt{\frac{2RT}{M}\cdot\frac{1}{2}} = \rho_1 S_2 C_D \sqrt{RT/eM}$$

But for our present conditions $\rho_1 = p_1 (M/RT)$ so that we may write:

$$w_{max} = p_1 S_2 C_D \sqrt{M/eRT}$$

A careful comparison of the adiabatic and isothermal parts of this problem shows that isothermal behavior of an ideal gas corresponds to the adiabatic behavior of an ideal gas of $\gamma = 1$ $(\hat{C}_p = \hat{C}_v)$. In other words a gas with γ of unity neither heats nor cools in expanding through a frictionless nozzle (or when under going a frictionless compression). Furthermore the difference of calculated discharge rates based on assumptions of adiabatic and isothermal conditions is almost never greater than 25%. Smaller differences are observed at smaller discharge rates or when there are appreciable friction losses. (For a representative comparison see Prob. 15.D) Then the simpler isothermal expressions are frequently useful for preliminary estimates of flow behavior.

15.I Flow of a Compressible Fluid Through a Convergent-divergent Nozzle.

a. We may write the Bernoulli equation, Eq. 15.2-3, for the conditions of this problem as:

$$\frac{1}{2}(v_2^2 - v_1^2) + \int_1^2 \frac{1}{\rho}\,dp = 0$$

The desired solution is obtained by setting v_1 equal to zero and by replacing the integral in the above equation by the right side of Eq. 15.2-6.

b. We may start with Eq. 15.4-37 with S_1 considered very large, and S_2 substituted for S_0:

$$w = C_D \rho_2 S_2 \sqrt{\frac{2(p_1/\rho_1)\left(\frac{\gamma}{\gamma-1}\right)\left[1-(p_2/p_1)^{\frac{\gamma-1}{\gamma}}\right]}{1-0}}$$

We next substitute $\rho_1 (p_2/p_1)^{1/\gamma}$ for ρ_2 and rearrange to solve for S_2, assuming C_D to be unity:

$$S_2 = w \Big/ \left\{ \rho_1 \sqrt{(2p_1/\rho_1)\left(\frac{\gamma}{\gamma-1}\right)\left(r^{2/\gamma} - r^{\frac{\gamma+1}{\gamma}}\right)} \right\}$$

The minimum cross-section, for any w and (p_1, ρ_1) will occur when

$$\frac{\partial S_2^2}{\partial r} = 0 = \frac{\partial}{\partial r}\left[r^{2/\gamma} - r^{\left(\frac{\gamma+1}{\gamma}\right)}\right]^{-1}$$

$$= \frac{(-1)\left[\frac{2}{\gamma} r^{2/\gamma} - \frac{\gamma+1}{\gamma} r^{\left(\frac{\gamma+1}{\gamma}\right)}\right]}{r\left[r^{2/\gamma} - r^{\left(\frac{\gamma+1}{\gamma}\right)}\right]^2}$$

$$0 = \frac{2}{\gamma} r^{2/\gamma} - \frac{\gamma+1}{\gamma} r^{\left(\frac{\gamma+1}{\gamma}\right)}$$

Then, collecting r terms:

$$2/(\gamma+1) = r^{\left(\frac{\gamma-1}{\gamma}\right)} \ ; \qquad r = \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

This is the desired answer.

c. Substituting (p_2/p_1) from Eq. 15.I-2 into Eq. 15.I-1 we obtain:

$$\frac{1}{2} v_2^2 = \frac{RT_1}{M}\left(\frac{\gamma}{\gamma-1}\right)\left[1 - \left(\frac{2}{\gamma+1}\right)\right]$$

$$= \frac{RT_1}{M}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\gamma-1}{\gamma+1}\right) = \frac{RT_1}{M}\left(\frac{\gamma}{\gamma+1}\right)$$

From Eq. 10.5-74:

$$T_1 = T_2\ (p_1/p_2)^{\left(\frac{\gamma-1}{\gamma}\right)} = T_2\left(\frac{\gamma+1}{2}\right)$$

We may then write:

$$\frac{1}{2}v_2^2 = \frac{RT_2}{M}\cdot\frac{\gamma}{2}$$

$$v_2 = \sqrt{\gamma RT_2/M}$$

Thus the velocity at control surface "2" is sonic. (See also the solution to Prob. 15.G.) It may be seen that the above development closely parallels that of Prob. 15.H. Here, however, we are considering the effect of varying S_2 at constant w, rather than the reverse as in 15.H.

d. Calculation of v:

$R = 1545 \times 32.2$ ft^2 sec^{-2} lb$_m$ lb-mol^{-1} °R^{-1}

$RT_1/M = (1545)(32.2)(562)/(29) = 967{,}000$ ft^2 sec^{-2}

$$[2\gamma RT_1/(\gamma-1)M]^{1/2} = [(9.67\times10^5)(2.8/0.4)]^{1/2}$$
$$= 2575 \text{ ft sec}^{-1}$$

$$v = 2575(1 - r^{(0.4/1.4)})^{(1/2)}$$

$$= 2575\sqrt{1 - r^{0.286}} \text{ ft sec}^{-1}$$

We may then summarize the calculation of v:

p, atm	r	$r^{0.286}$	$1 - r^{0.286}$	$\sqrt{1-r^{0.286}}$	v, ft sec^{-1}
10.	1.0	1.0	0	0	0
9.	0.9	0.9705	0.0295	0.1713	442
8.	0.8	0.9384	0.0616	0.248	638
7.	0.7	0.903	0.097	0.309	795
6.	0.6	0.864	0.156	0.395	1020
5.3	0.53	0.834	0.166	0.407	1050
5.	0.5	0.820	0.180	0.424	1092
4.	0.4	0.769_5	0.230_5	0.480	1237
3.	0.3	0.709	0.291	0.539	1390
2.	0.2	0.631	0.369	0.607	1560
1.	0.1	0.518	0.481	0.694	1790
0.	0.0	0	1.000	1.000	2575

Calculation of T:

$$T_2 = T_1 (p_2 / p_1)^{(\gamma-1)/\gamma} = 562°R \cdot r^{0.286}$$

We may then summarize the calculations of T:

r :	1.0	0.9	0.8	0.7	0.6	0.53	0.5	0.4	0.3	0.2	0.1	0
T :	562	546	525	506	485	468	461	433	399	354	292	0

Calculation of S:

From the results obtained in part b:

$$w = S_2 \rho_1 (p_2/p_1)^{1/\gamma} \sqrt{(2RT_1/M)(\frac{\gamma}{\gamma-1})(1-r^{(\frac{\gamma-1}{\gamma})})}$$

$$w = 29. \text{ lb}_m \text{ sec}^{-1}$$

$$\rho_1 = (29/359)(10)(492/562) = 0.707 \text{ lb}_m \text{ ft}^{-3}$$

$$S_2 = (29/0.707) \Big/ \left\{ \sqrt{\frac{2.8}{1.4} \times 9.67 \times 10^5} \cdot \sqrt{r^{1.43} - r^{1.715}} \right\}$$

$$= 0.158 / \sqrt{r^{1.43} - r^{1.715}}$$

We may then summarize the calculations for S:

r	$r^{1.43}$	$r^{1.715}$	$r^{1.43} - r^{1.715}$	$\sqrt{r^{1.43} - r^{1.715}}$	S
1.0	1.0	1.0	0	0	∞
0.9	0.8602	0.8347	0.0255	0.1595	0.99
0.8	0.727	0.682	0.045	0.212	0.745
0.7	0.600	0.542	0.058	0.241	0.656
0.6	0.4815	0.416	0.065	0.255	0.620
0.5	0.370	0.305	0.065	0.255	0.620
0.4	0.270	0.208	0.062	0.249	0.635
0.3	0.1785	0.1265	0.052	0.228	0.693
0.2	0.1000	0.0660	0.034	0.184	0.860
0.1	0.0372	0.0198	0.0179	0.134	1.182
0	0	0	0	0	∞
→ 0.53	0.403	0.337	0.066	0.258	0.612

└ r of minimum S.

Chapter 16 - Checked by T. J. Sadowski

16.A Prediction of Mass Diffusivity for a Gas-Pair at Low Density

Equation 16.4-13 may be used. Preliminary calculations follow:

$$T = (104 + 459.7)/1.8 = 313\ ^\circ K$$

$$\left.\begin{aligned} \left(\frac{1}{M_A} + \frac{1}{M_B}\right) &= \left(\frac{1}{16.04} + \frac{1}{30.07}\right) = 0.0956 \\ \sigma_{AB} &= \tfrac{1}{2}(3.822 + 4.418) = 4.120\ \mathring{A} \\ \epsilon_{AB}/K &= \sqrt{(137)(230)} = 177.5^\circ K \end{aligned}\right\} \text{See Table B-1 and Equations 16.4 - 15, 16}$$

$$\frac{\kappa T}{\epsilon_{AB}} = \frac{313}{177.5} = 1.763$$

$$\Omega_{\mathcal{D},AB} = 1.125 \quad \text{from Table B-2}$$

Substitution of numerical values into Eq. 16.4-13 gives:

$$\mathcal{D}_{AB} = 0.0018583 \frac{\sqrt{T^3\left(\frac{1}{M_A} + \frac{1}{M_B}\right)}}{p\, \sigma_{AB}^2\, \Omega_{\mathcal{D},AB}} = 0.0018583 \frac{\sqrt{(313)^3(0.0956)}}{(1)(4.120)^2(1.125)}$$

Or $\mathcal{D}_{AB} = 0.166\ cm^2\ sec^{-1}$

16.B Prediction of Mass Diffusivity at Low Density from Critical Properties

(a) We designate methane as "A" and ethane as "B" and obtain the critical properties from Table B-1. Then

$$(p_{cA}\,p_{cB})^{1/3} = (45.8 \times 48.2)^{1/3} = 13.02$$

$$(T_{cA}\,T_{cB})^{5/12} = (190.7)^{5/12}(305.4)^{5/12} = 96.72$$

$$\left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2} = \left(\frac{1}{16.04} + \frac{1}{30.07}\right)^{1/2} = 0.3092$$

$$\sqrt{T_{cA}\,T_{cB}} = \sqrt{190.7 \times 305.4} = 241.3$$

$$\left(T/\sqrt{T_{cA}\,T_{cB}}\right)^{1.823} = (313/241.3)^{1.823} = 1.607$$

Substitution of these quantities into Eq. 16.3-1 gives:

$$\mathcal{D}_{AB} = \frac{a}{p}\left(\frac{T}{\sqrt{T_{cA}\,T_{cB}}}\right)^b (p_{cA}\,p_{cB})^{1/3}(T_{cA}\,T_{cB})^{5/12}\left(\frac{1}{M_A} + \frac{1}{M_B}\right)^{1/2}$$

$$= \frac{2.745 \times 10^{-4}}{1}(1.607)(13.02)(96.72)(0.3092)$$

$$\mathcal{D}_{AB} = 0.172\ cm^2\ sec^{-1}$$

(b) Using Eqs. 1.4-11,13, Eqs. 16.4-15,16, and the same critical properties we find:

$\epsilon_A/\kappa = 0.77 \times 190.7 = 147°K$ $\qquad \sigma_A = 2.44(190.7/45.8)^{1/3} = 3.93 \text{ Å}$

$\epsilon_B/\kappa = 0.77 \times 305.4 = 235°K$ $\qquad \sigma_B = 2.44(305.4/48.2)^{1/3} = 4.52 \text{ Å}$

$\epsilon_{AB}/\kappa = \sqrt{147 \times 235} = 186°K$ $\qquad \sigma_{AB} = \frac{1}{2}(3.93 + 4.52) = 4.22 \text{ Å}$

Then $\kappa T/\epsilon_{AB} = 313/186 = 1.68$ and, from Table B-2, $\Omega_{\mathcal{D},AB} = 1.145$. Substitution of numerical values in Eq. 16.4-13 gives

$$\mathcal{D}_{AB} = \frac{\sqrt{(313)^3(0.0956)}}{(1)(4.22)^2(1.145)} = 0.156 \text{ cm}^2 \text{ sec}^{-1}$$

16.C Correction of Mass Diffusivity for Temperature at Low Density

(a) Equation 16.3-1 gives

$$\mathcal{D}_{AB}\Big|_{1500°K} = \mathcal{D}_{AB}\Big|_{293°K}\left(\frac{1500}{293}\right)^{1.823} = 2.96 \text{ cm}^2 \text{ sec}^{-1}$$

(b) Equation 16.4-11 gives

$$\mathcal{D}_{AB}\Big|_{1500°K} = \mathcal{D}_{AB}\Big|_{293°K}\left(\frac{1500}{293}\right)^{3/2} = 1.75 \text{ cm}^2 \text{ sec}^{-1}$$

(c) Equation 16.4-13 gives

$$\mathcal{D}_{AB} \propto T^{3/2}/\Omega_{\mathcal{D},AB}$$

Using Tables B-1 and B-2 we find:

$\epsilon_{AB}/\kappa = \sqrt{97.0 \times 190} = 135.8°K$ (see Eq. 16.4-16)

T	$\kappa T/\epsilon_{AB}$	$\Omega_{\mathcal{D},AB}$
293	2.16	1.047
1500	11.05	0.734 (linear interpolation)

Then

$$\mathcal{D}_{AB}\Big|_{1500} = 0.151\left(\frac{1500}{293}\right)^{3/2}\frac{1.047}{0.734} = 2.49 \text{ cm}^2 \text{ sec}^{-1}$$

Method (c) is in best agreement with the observed value of 2.45 cm² sec⁻¹.

16.D Prediction of Mass Diffusivity for a Gas Mixture at High Density

(a) Preliminary calculations for use of Fig. 16.3-1 follow:

$p_c' = (0.50 \times 72.9) + (0.50 \times 33.5)$ $\qquad p_r = 40/53.2 = 0.75$

$= 53.2$ atm

$T_c' = (0.50 \times 304.2) + (0.50 \times 126.2)$ $\qquad T_r = 288.2/215.2 = 1.34$

$= 215.2\,°K$

Fig. 16.3-1 gives $p\mathscr{D}_{AB}/(p\mathscr{D}_{AB})^{\circ} = 0.83$, and Table 16.2-2 gives $(p\mathscr{D}_{AB})^{\circ} \doteq 0.158$. Therefore

$$\mathscr{D}_{AB} = (p\mathscr{D}_{AB})^{\circ} \cdot \frac{p\mathscr{D}_{AB}}{(p\mathscr{D}_{AB})^{\circ}} \cdot \frac{1}{p} = (0.158)(0.83)\left(\frac{1}{40}\right)$$

or $\qquad \mathscr{D}_{AB} = 0.0033 \text{ cm}^2\,\text{sec}^{-1}$

(b) Equation 16.4-13 may be used to predict $(p\mathscr{D}_{AB})^{\circ}$. With the aid of Tables B-1 and B-2 we find:

$\sigma_{AB} = \frac{1}{2}(3.996 + 3.681)$ $\qquad \epsilon_{AB}/\kappa = \sqrt{190 \times 91.5} = 131.9\,°K$

$= 3.838$ Å (See Eq. 16.4-15) $\qquad$ (See Eq. 16.4-16)

$\left(\frac{1}{M_A} + \frac{1}{M_B}\right) = \frac{1}{44.01} + \frac{1}{28.02}$ $\qquad \kappa T/\epsilon_{AB} = 288.2/131.9 = 2.185$

$= 0.0584$ $\qquad \Omega_{\mathscr{D},AB} = 1.043$

Substitution of numerical values into Eq. 16.4-13 gives

$$(p\mathscr{D}_{AB})^{\circ} = 0.0018583\,\frac{\sqrt{(288.2)^3(0.0584)}}{(3.838)^2(1.043)} = 0.143 \text{ cm}^2\,\text{sec}^{-1}$$

Replacing $(p\mathscr{D}_{AB})^{\circ}$ in part (a) by this value gives

$$\mathscr{D}_{AB} = (0.143)(0.83)\left(\frac{1}{40}\right) = 0.0030 \text{ cm}^2\,\text{sec}^{-1}$$

16.E Estimation of Mass Diffusivity for a Binary Liquid Mixture

The data needed for use of Eq. 16.5-9 are:

$\psi_B = 2.6$ (B is water) $\qquad \tilde{V}_A = M_A/\rho_A = 60.05/0.937$

$M_B = 18.02$ $\qquad = 64.1 \text{ cm}^3\,\text{g-mole}^{-1}$

$\mu = 1.22$ cp. $\qquad T = 12.5 + 273.2 = 285.7\,°K$

Substitution of numerical values into Eq. 16.5-9 gives:

$$\mathscr{D}_{AB} = 7.4 \times 10^{-8}\,\frac{(2.6 \times 18.02)^{1/2}(285.7)}{(1.22)(64.1)^{0.6}} = 9.8 \times 10^{-6} \text{ cm}^2\,\text{sec}^{-1}$$

16.F Correction of Mass Diffusivity for Temperature for a Binary Liquid Mixture

Equation 16.5-9 gives

$$\mathcal{D}_{AB} \propto \frac{T}{\mu}$$

from which

$$\mathcal{D}_{AB}\Big|_{373.2°K} = (1.28 \times 10^{-5})\left(\frac{373.2}{273.2}\right)\left(\frac{1.14}{0.284}\right)$$
$$= 6.7 \times 10^{-5} \ \mathrm{cm^2\ sec^{-1}}$$

16.G Correction of Mass Diffusivity for Temperature for a Dense Gas Mixture

Figure 16.3-1 gives the pressure dependence of $\mathcal{D}_{AB}$ at constant temperature. Since the temperature dependence of $\mathcal{D}_{AB}$ is predictable at low pressures, we calculate from p_1, T_1 to p_2, T_2 along the following path:

$$\begin{tikzcd}
p_1, T_1 \arrow[d] & & p_2, T_2 \\
0, T_1 \arrow[rr] & & 0, T_2 \arrow[u]
\end{tikzcd}$$

This gives

$$\mathcal{D}_{AB}\Big|_{p_2,T_2} = \mathcal{D}_{AB}\Big|_{p_1,T_1}\left[\frac{(p\mathcal{D}_{AB})^\circ}{p\mathcal{D}_{AB}}\right]_{p_1,T_1}\frac{(p\mathcal{D}_{AB})^\circ_{T_2}}{(p\mathcal{D}_{AB})^\circ_{T_1}}\left[\frac{p\mathcal{D}_{AB}}{(p\mathcal{D}_{AB})^\circ}\right]_{p_2,T_2}\left(\frac{p_1}{p_2}\right)$$

The initial and final <u>pseudo</u>-reduced conditions are:

$$p_{r1} = p_{r2} = \frac{(2000/14.7)}{(0.8 \times 45.8) + (0.2 \times 48.2)} = 2.94$$

$$T_{r1} = \frac{(459.7 + 104)/1.8}{(0.8 \times 190.7) + (0.2 \times 305.4)} = 1.47$$

$$T_{r2} = \frac{(459.7 + 171)/1.8}{(0.8 \times 190.7) + (0.2 \times 305.4)} = 1.64$$

From Fig. 16.3-1 we get $p\mathcal{D}_{AB}/(p\mathcal{D}_{AB})^\circ = 0.73$ at p_1, T_1 and 0.84 at p_2, T_2.

(a) According to Eq. 16.3-1,

$$\frac{(p\mathcal{D}_{AB})^\circ_{T_2}}{(p\mathcal{D}_{AB})^\circ_{T_1}} = \left(\frac{459.7+171}{459.7+104}\right)^{1.823} = 1.227$$

Combining the pressure and temperature corrections, we obtain

$$\mathcal{D}_{AB}\Big|_{p_2,T_2} = (8.4\times10^{-4})\left[\frac{1}{0.73}\right](1.227)\left[0.84\right]\left(\frac{2000}{2000}\right)$$

$$= 1.19\times10^{-3}\ \text{cm}^2\ \text{sec}^{-1}$$

(b) From Appendix B and Eqs. 16.4-15,16 we find

$$\epsilon_{AB}/\kappa = \sqrt{137\times230} = 177.5^\circ\text{K}$$

$$\frac{\kappa T_1}{\epsilon_{AB}} = \frac{459.7+104}{1.8\times177.5} = 1.765 \qquad \Omega_{\mathcal{D},AB} = 1.124 \text{ at } T_1$$

$$\frac{\kappa T_2}{\epsilon_{AB}} = \frac{459.7+171}{1.8\times177.5} = 1.975 \qquad \Omega_{\mathcal{D},AB} = 1.080 \text{ at } T_2$$

Then from Eq. 16.4-13 we get

$$\frac{(p\mathcal{D}_{AB})^\circ_{T_2}}{(p\mathcal{D}_{AB})^\circ_{T_1}} = \frac{(459.7+171)^{3/2}}{1.080}\ \frac{1.124}{(459.7+104)^{3/2}} = 1.232$$

This is almost identical with the temperature correction found in part (a). Hence we again obtain

$$\mathcal{D}_{AB}\Big|_{p_2,T_2} = 1.19\times10^{-3}\ \text{cm}^2\ \text{sec}^{-1}$$

16.H Relations Among the Fluxes in a Binary System

(K) From the definitions of $\underset{\sim}{j}_A$ and $\underset{\sim}{j}_B$,

$$\underset{\sim}{j}_A + \underset{\sim}{j}_B = \rho_A(\underset{\sim}{v}_A - \underset{\sim}{v}) + \rho_B(\underset{\sim}{v}_B - \underset{\sim}{v})$$

But from Eq.16.1-1, $\rho\underset{\sim}{v} = \rho_A\underset{\sim}{v}_A + \rho_B\underset{\sim}{v}_B$

and from Table 16.1-1, $\rho = \rho_A + \rho_B$

Hence, $\underset{\sim}{j}_A + \underset{\sim}{j}_B = (\rho_A\underset{\sim}{v}_A + \rho_B\underset{\sim}{v}_B) - (\rho_A+\rho_B)\underset{\sim}{v} = \rho\underset{\sim}{v} - \rho\underset{\sim}{v}$

and $\underset{\sim}{j}_A + \underset{\sim}{j}_B = 0$ Q. E. D.

(Z) By definition, $\underset{\sim}{J}_A = c_A(\underset{\sim}{v}_A - \underset{\sim}{v}) = c_A(\underset{\sim}{v}_A - \underset{\sim}{v}^{\star}) + c_A(\underset{\sim}{v}^{\star} - \underset{\sim}{v})$

$$= \underset{\sim}{J}_A^{\star} + \frac{c_A}{c}(\underset{\sim}{J}_A + \underset{\sim}{J}_B) = \underset{\sim}{J}^{\star}_A + x_A(\underset{\sim}{J}_A + \underset{\sim}{J}_B)$$

But, from K, $\underset{\sim}{j}_B = -\underset{\sim}{j}_A$

Hence $M_B\underset{\sim}{J}_B = M_A\underset{\sim}{J}_A$

Substituting this above, we get

$$\underset{\sim}{J}^{\star}_A = \underset{\sim}{J}_A - x_A\left(\underset{\sim}{J}_A - \frac{M_A}{M_B}\underset{\sim}{J}_A\right) = (1-x_A)\frac{M_B}{M_B}\underset{\sim}{J}_A + x_A\frac{M_A}{M_B}\underset{\sim}{J}_A$$

$$= \frac{M}{M_B}\underset{\sim}{J}_A \qquad \text{Q.E.D.}$$

(o) In a binary mixture, according to Eqs. H and M of Table 16.1-1,

$$\omega_A = \frac{x_A M_A}{x_A M_A + (1-x_A) M_B}$$

Taking the total differential of both sides we get:

$$d\omega_A = \frac{[x_A M_A + (1-x_A)M_B] M_A dx_A - x_A M_A (M_A - M_B) dx_A}{[x_A M_A + (1-x_A) M_B]^2}$$

$$\equiv \frac{M_A M_B dx_A}{[x_A M_A + (1-x_A) M_B]^2}$$

16.I Equivalence of Various Forms of Fick's Law

(a) We will show that Equation D is equivalent to all the others.

(A) and (C) are equivalent, since $\boldsymbol{n}_A - \omega_A(\boldsymbol{n}_A + \boldsymbol{n}_B) = \boldsymbol{j}_A$

(B) and (D) are equivalent, since $\boldsymbol{N}_A - x_A(\boldsymbol{N}_A + \boldsymbol{N}_B) = \boldsymbol{J}_A^*$

(C): From Table 16.1-3, $\boldsymbol{J}_A^* = \frac{M}{M_B}\boldsymbol{J}_A = \frac{M}{M_A M_B}\boldsymbol{j}_A$

From Table 16.1-1, $dx_A = \frac{M^2}{M_A M_B} d\omega_A$

With these substitutions, Eq. (D) becomes:

$$\frac{M}{M_A M_B}\boldsymbol{j}_A = -c\mathcal{D}_{AB}\frac{M^2}{M_A M_B}\nabla\omega_A$$

And since $cM = \rho$ (see Eq. G, Table 16.1-1) we get:

$$\boldsymbol{j}_A = -\rho\mathcal{D}_{AB}\nabla\omega_A \qquad \text{(C)}$$

Hence Equation D is equivalent to A and C.

(D): No proof required.

(E): Substitution of $d\omega_A = \frac{M_A M_B}{M^2} dx_A = \frac{c^2}{\rho^2} M_A M_B dx_A$ into the result of (C) gives:

$$\boldsymbol{j}_A = -\frac{c^2}{\rho} M_A M_B \mathcal{D}_{AB} \nabla x_A \qquad \text{(E)}$$

(F): Substitution of $dx_A = \frac{\rho^2}{c^2 M_A M_B} d\omega_A$ into Eq. D gives:

$$\boldsymbol{J}_A^* = -\left(\frac{\rho^2}{c M_A M_B}\right)\mathcal{D}_{AB}\nabla\omega_A \qquad \text{(F)}$$

(G) The definitions of $\underset{\sim}{J}_A^\star$ and x_A are first inserted in Eq. D to get:

$$c(\underset{\sim}{v}_A - \underset{\sim}{v}^\star) = -\frac{1}{x_A} c\mathcal{D}_{AB} \nabla x_A$$

The corresponding result for species B is:

$$c(\underset{\sim}{v}_B - \underset{\sim}{v}^\star) = -\frac{1}{x_B} c\mathcal{D}_{AB} \nabla x_B$$

$$= +\frac{1}{x_B} c\mathcal{D}_{AB} \nabla x_A$$

Subtraction of the second equation from the first gives:

$$c(\underset{\sim}{v}_A - \underset{\sim}{v}_B) = -(c\mathcal{D}_{AB} \nabla x_A)\left[\frac{1}{x_A} + \frac{1}{x_B}\right]$$

$$= -(c\mathcal{D}_{AB} \nabla x_A)\left[\frac{x_B + x_A}{x_A x_B}\right]$$

Hence $$c(\underset{\sim}{v}_A - \underset{\sim}{v}_B) = -\frac{c\mathcal{D}_{AB}}{x_A x_B} \nabla x_A$$

(b) First interchange the labels A and B in Equation 16.2-1:

$$\underset{\sim}{J}_B^\star = -c\mathcal{D}_{BA} \nabla x_B$$

From Eq. 16.1-4, $\underset{\sim}{J}_B^\star = -\underset{\sim}{J}_A^\star$

Since $x_A + x_B = 1$, $\nabla x_B = -\nabla x_A$

With these substitutions in the first equation we get:

$$-\underset{\sim}{J}_A^\star = -c\mathcal{D}_{BA}(-\nabla x_A)$$

Comparison with Eq. 16.2-1 shows that $\mathcal{D}_{AB} = \mathcal{D}_{BA}$.

16.J Determination of Collision Parameters from Diffusivity Data

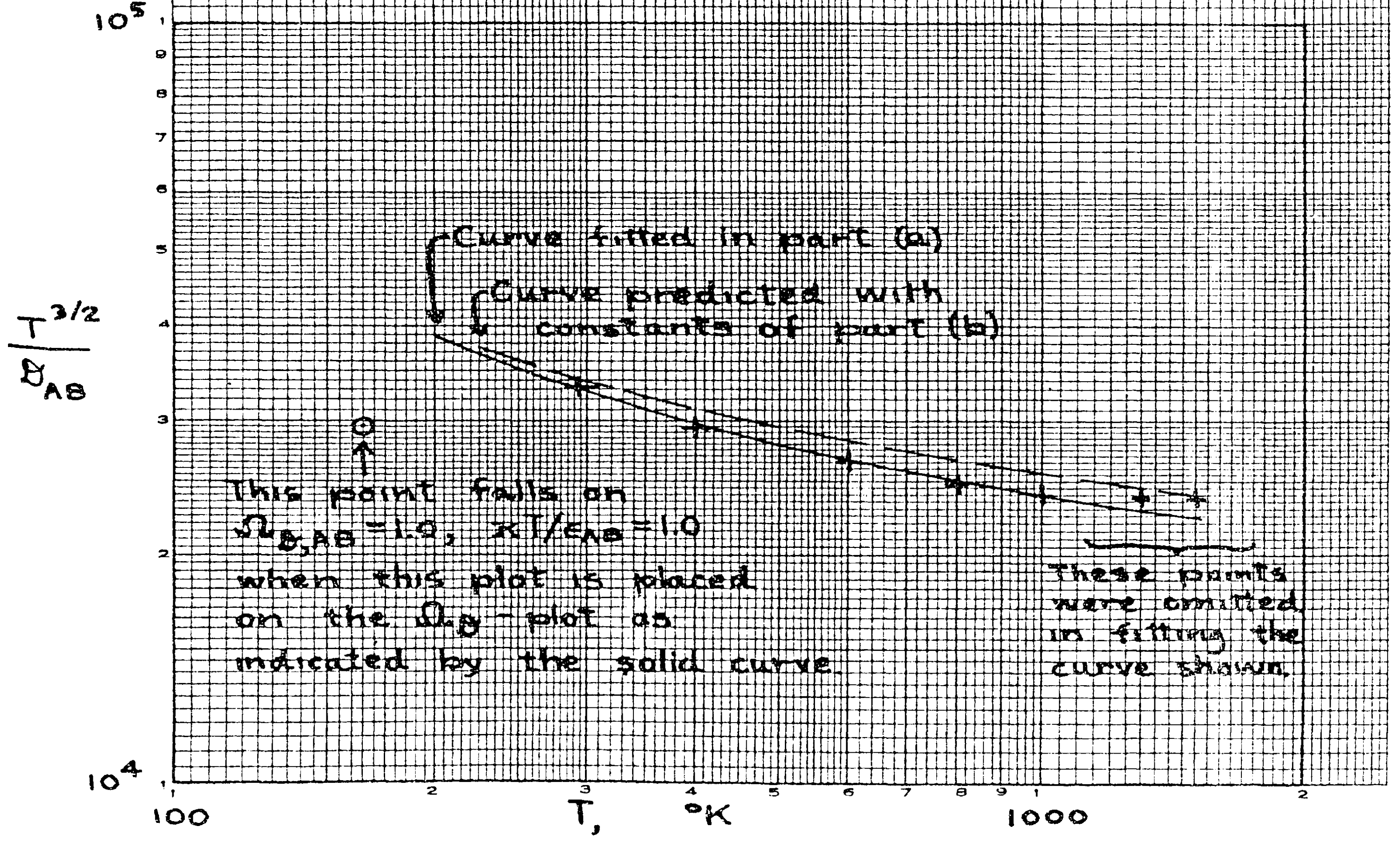
(a) The transformed data are as follows:

T, °K	293	400	600	800	1000	1300	1500
$(T^{3/2}\mathcal{D}_{AB}^{-1}) \times 10^{-4}$	3.32	2.93	2.65	2.48	2.40	2.38	2.37

These data were plotted on logarithmic graph paper and then superimposed on a logarithmic plot of the function $\Omega_{\mathcal{D},AB}$ versus $\kappa T/\epsilon_{AB}$, obtained from Table B-2. The resulting curve-fit is shown in the adjoining graph. Comparison of corresponding abscissae on the two plots gives

$$\epsilon_{AB}/\kappa = T/(\kappa T/\epsilon_{AB}) = 165/1.0 = 165\,°K$$

Figure for Problem 16.J



Comparison of corresponding ordinates, and use of Eq. 16.4-13, gives

$$\frac{T^{3/2}\mathcal{D}_{AB}^{-1}}{\Omega_{\mathcal{D},AB}} = \frac{p\,\sigma_{AB}^{2}}{0.0018583\sqrt{\frac{1}{M_A}+\frac{1}{M_B}}} = \frac{2.95\times10^{4}}{1.0}$$

Here $p = 1$ atm and $\sqrt{M_A^{-1}+M_B^{-1}} = \sqrt{\frac{1}{44.01}+\frac{1}{28.97}} = 0.2392.$

Hence $$\sigma_{AB} = \left[(2.95\times10^{4})(0.0018583)(0.2392)(1)^{-1}\right]^{1/2}$$
$$= 3.62\ \text{Å}$$

Other sets of values will give nearly as good a fit; a few such sets follow:

ϵ_{AB}/κ, °K	160	170	180
σ_{AB}, Å	3.63	3.60	3.57

The data at 1300°K and 1500°K were omitted in the text because they appear to be out of line; at any rate, they cannot be fitted very well by the theory for a Lennard-Jones potential.

(b) From Table B-1 and Eqs. 16.4-15,16 one gets the following predicted values:

$$\epsilon_{AB}/\kappa = \sqrt{97.0\times190} = 136\ \text{°K}$$
$$\sigma_{AB} = \tfrac{1}{2}(3.617+3.996) = 3.81\ \text{Å}$$

The behavior predicted with these constants is shown by the broken line in the attached figure. The predicted values of $\mathcal{D}_{AB}$ are systematically low but the deviations are less than 6 percent.

Chapter 17 - Checked by R. H. Weaver

17.A Rate of Evaporation

Let A = chloropicrin and B = air ; then Eq. 17.2-15a gives:

$$N_{Az} = \frac{p\mathcal{D}_{AB}/RT}{(z_2 - z_1)} \ln \frac{p_{B2}}{p_{B1}} = \frac{\left(\frac{770}{760} \times 0.088\right) / \left(82.06 \times 298.2\right)}{(11.14)} \ln \frac{(770)}{\left(\begin{matrix}770\\-23.81\end{matrix}\right)}$$

$$= 1.03 \times 10^{-8} \text{ g-mole cm}^{-2} \text{ sec}^{-1}$$

Finally, the evaporation rate in g/hr is:

$$N_{Az} M_A S_z = \left(1.03 \times 10^{-8} \frac{\text{g-mole}}{\text{cm}^2 \text{sec}} \times 3600 \frac{\text{sec}}{\text{hr}}\right)\left(164.39 \frac{\text{g}}{\text{g-mole}}\right)\left(2.29 \text{ cm}^2\right)$$

$$= 1.39 \times 10^{-2} \text{ g/hr}$$

17.B Error in Calculating Absorption Rate

From Eq. 17.5-17

$$\mathcal{W}_A = K c_{A0} \sqrt{\mathcal{D}_{AB}}$$

where K is a collection of known quantities.
Then the differential of $\mathcal{W}_A$ is

$$d\mathcal{W}_A = \left(\frac{\partial \mathcal{W}_A}{\partial c_{A0}}\right) dc_{A0} + \left(\frac{\partial \mathcal{W}_A}{\partial \mathcal{D}_{AB}}\right) d\mathcal{D}_{AB}$$

and for small but finite errors Δc_{A0} and $\Delta \mathcal{D}_{AB}$, the error in $\mathcal{W}_A$ is:

$$\Delta \mathcal{W}_A \doteq K\sqrt{\mathcal{D}_{AB}}\, \Delta c_{A0} + \frac{K c_{A0}}{2\sqrt{\mathcal{D}_{AB}}} \Delta \mathcal{D}_{AB}$$

Division by $\mathcal{W}_A$ then gives:

$$\frac{\Delta \mathcal{W}_A}{\mathcal{W}_A} \doteq \frac{\Delta c_{A0}}{c_{A0}} + \frac{1}{2} \frac{\Delta \mathcal{D}_{AB}}{\mathcal{D}_{AB}}$$

Hence the maximum percentage error in absorption rate is

$$\left(100 \frac{\Delta \mathcal{W}_A}{\mathcal{W}_A}\right)_{\text{Max}} \doteq 5\% + \tfrac{1}{2}(10\%) = 10\%$$

17.C Rate of Absorption in a Falling Film

The absorption rate is given by Eq. 17.5-17, which may be rewritten in terms of the average film velocity by using the results of § 2.2:

$$\mathcal{W}_A = 2\pi R L c_{A0} \sqrt{\frac{6 \mathcal{D}_{AB} \langle v_z \rangle}{\pi L}}$$

The solubility may be converted to molar units and expressed as 0.116 g-moles / liter $\doteq 0.116 \times 10^{-3}$ g-moles cm^{-3}. Then, in cgs units,

$$\mathcal{W}_A = 2\pi (1.4)(13.0)(0.116 \times 10^{-3}) \sqrt{\frac{6(1.26 \times 10^{-5})(17.7)}{\pi (13.0)}}$$

$$= 7.58 \times 10^{-5} \text{ g-moles sec}^{-1}$$

Hence, multiplying by 3600 sec/hr, we get

$$\mathcal{W}_A = 0.273 \text{ g-moles hr}^{-1}$$

The Reynolds number for the film is of the order of 100; hence rippling will occur. This should not affect the value of $\mathcal{W}_A$ very much.

17.D Diffusion Method for Separating Helium from Natural Gas

A differential mass balance gives, for steady state,

$$\frac{d}{dr}(r N_{He,r}) = 0$$

Insertion of Eq. 17.0-1 with $N_{Py,r} = 0$ and $x_{He} << 1$ gives, for constant diffusivity:

$$\frac{d}{dr}\left(r \frac{dc_{He}}{dr}\right) = 0$$

Integrating twice we get:

$$c_{He} = K_1 \ln r + K_2$$

The boundary conditions are:

$$c_{He} = c_{He,1} \quad \text{at } r = R_1$$

$$c_{He} = 0 \quad \text{at } r = R_2$$

Evaluation of K_1 and K_2 from these boundary conditions gives:

$$\frac{c_{He}}{c_{He,1}} = \frac{\ln R_2/r}{\ln R_2/R_1}$$

Then $$N_{He,r} = -\mathscr{D}_{He-Pyr}\frac{dc_{He}}{dr} = +\mathscr{D}_{He-Pyr}\frac{c_{He,1}}{r\ln(R_2/R_1)}$$

and
$$\mathscr{W}_{He} = (N_{He,r})2\pi rL \qquad \text{for} \quad R_1 < r < R_2$$
$$= 2\pi L\,\frac{\mathscr{D}_{He-Pyr}\,c_{He,1}}{\ln(R_2/R_1)}$$

This result can be derived without finding the profiles if one first integrates the differential mass balance:
$$rN_{He,r} = K_1$$

Substitution of Eq. 17.0-1 as before gives:
$$-r\mathscr{D}_{He-Pyr}\frac{dc_{He}}{dr} = K_1$$

Integration ∫ from R_1 to R_2 gives:
$$-\mathscr{D}_{He-Pyr}(c_{He,2} - c_{He,1}) = K_1\ln\frac{R_2}{R_1} = rN_{He,r}\ln\frac{R_2}{R_1}$$

From which we again get:
$$N_{He,r} = +\mathscr{D}_{He-Pyr}\frac{c_{He,1}}{r\ln(R_2/R_1)}$$

17.E Diffusion Through a Stagnant Film - Alternate Derivation

From 17.0-1, when $N_{Bz} = 0$,
$$N_{Az} = -c\mathscr{D}_{AB}\frac{dx_A}{dz} + x_A N_{Az}$$

or
$$N_{Az} = -\frac{c\mathscr{D}_{AB}}{(1-x_A)}\frac{dx_A}{dz}$$

Integration from z_1 to z_2 gives, for constant N_{Az} and $c\mathscr{D}_{AB}$,
$$\int_{z_1}^{z_2} N_{Az}\,dz = N_{Az}(z_2 - z_1) = -c\mathscr{D}_{AB}\int_{x_{A1}}^{x_{A2}}\frac{dx_A}{(1-x_A)}$$
$$N_{Az}(z_2 - z_1) = +c\mathscr{D}_{AB}\ln(1-x_A)\Big|_{x_{A1}}^{x_{A2}}$$

Hence
$$N_{Az} = \frac{c\mathscr{D}_{AB}}{(z_2 - z_1)}\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$$

or
$$N_{Az} = \frac{c\mathscr{D}_{AB}}{(z_2 - z_1)}\ln\frac{x_{B2}}{x_{B1}}$$

17.F Diffusion Through a Stagnant Liquid Film

(a) The mass balance leads to the differential equation

$$\mathcal{D}_{AB} \frac{d^2 c_A}{dz^2} = 0$$

(b) Integration gives

$$c_A = C_1 z + C_2$$

Application of the boundary conditions gives

$$\frac{c_A - c_{A\delta}}{c_{A0} - c_{A\delta}} = 1 - \frac{z}{\delta}$$

(c) The mass flux of A anywhere in the film will be

$$N_{Az} = - \mathcal{D}_{AB} \frac{dc_A}{dz} \qquad \left(\text{from } 17.0\text{-}1 \text{ for } x_A << 1\right)$$

$$= + \mathcal{D}_{AB} \frac{c_{A0} - c_{A\delta}}{\delta}$$

17.G Diffusion from a Droplet into a Quiescent Gas

(a) A shell mass balance on A gives:

$$4\pi r^2 N_{Ar}\big|_r - 4\pi r^2 N_{Ar}\big|_{r+\Delta r} = 0$$

This result is correct for finite Δr, hence both terms are constant, and

$$r^2 N_{Ar} = \text{Constant} = r_1^2 N_{Ar1}, \text{ say.}$$

(b) From Eq. 17.0-1, with $N_B = 0$,

$$N_{Ar} = - \frac{c\mathcal{D}_{AB}}{(1-x_A)} \frac{dx_A}{dr}$$

Combining with the result of (a) we get:

$$r_1^2 N_{Ar1} = - \frac{c\mathcal{D}_{AB}}{(1-x_A)} r^2 \frac{dx_A}{dr}$$

(c) Integration then gives, for constant $\mathcal{D}_{AB}$,

$$r_1^2 N_{Ar1} \int_{r_1}^{r_2} \frac{dr}{r^2} = - c\mathcal{D}_{AB} \int_{x_{A1}}^{x_{A2}} \frac{dx_A}{(1-x_A)}$$

from which

$$N_{Ar1} = \frac{c\mathcal{D}_{AB}}{(r_2 - r_1)} \left(\frac{r_2}{r_1}\right) \ln \frac{x_{B2}}{x_{B1}}$$

(d) In the limit as $r_2/r_1 \rightarrow \infty$, the result of (c) reduces to

$$N_{Ar1} = \frac{c\mathcal{D}_{AB}}{r_1} \ln \frac{x_{B2}}{x_{B1}} = \frac{c\mathcal{D}_{AB}}{r_1} \ln \frac{p_{B2}}{p_{B1}}$$

Inserting this expression into the definition of k_p we get:

$$k_p = \frac{N_{Ar1}}{(p_{A1} - p_{A2})} = \frac{N_{Ar1}}{(p_{B2} - p_{B1})}$$

$$= \frac{c\mathcal{D}_{AB}}{r_1} \cdot \frac{\ln(p_{B2}/p_{B1})}{(p_{B2} - p_{B1})}$$

$$= \frac{c\mathcal{D}_{AB}}{(D/2)} \cdot \frac{1}{(p_B)_{ln}}$$

$$= \frac{2c\mathcal{D}_{AB}/D}{(p_B)_{ln}}$$

17.H Diffusion with Catalytic Polymerization

(a) For every n moles of A converted there will be one mole of A_n produced. Therefore

$$-N_{Az} = n\, N_{A_nz}$$

Therefore Eq. 17.0-1 becomes:

$$N_{Az} = -c\mathcal{D}_{A-An} \frac{dx_A}{dz} + x_A\left(N_{Az} - \frac{1}{n} N_{Az}\right)$$

Solving for N_{Az} we get:

$$N_{Az} = -\frac{c\mathcal{D}_{AAn}}{1 - x_A\left(1 - \frac{1}{n}\right)} \frac{dx_A}{dz}$$

(b) Integrating the last expression we get (since N_{Az} is constant)

$$N_{Az} \int_0^{\delta} dz = -c\mathcal{D}_{AAn} \int_{x_{A0}}^{0} \frac{dx_A}{1 - x_A\left(1 - \frac{1}{n}\right)}$$

or

$$N_{Az}\delta = + \frac{c\mathcal{D}_{AAn}}{\left(1 - \frac{1}{n}\right)} \ln \frac{1}{1 - x_{A0}\left(1 - \frac{1}{n}\right)}$$

and finally

$$N_{Az} = \frac{n\, c\mathcal{D}_{AAn}}{(n-1)} \ln \frac{1}{1 - x_{A0}\left(\frac{n-1}{n}\right)}$$

17.I Effectiveness Factors for Thin Disks

A shell balance gives (on the assumption that the catalyst is a continuum):

$$\frac{dN_{Az}}{dz} = R_A$$

Inserting the expression for N_{Az} analogous to Eq. 17.6-4, and assuming an irreversible first-order reaction, we get

$$\frac{d}{dz}\left(-\mathfrak{D}_A \frac{dc_A}{dz}\right) = -k_1''a\,c_A$$

For constant $\mathfrak{D}_A$ this becomes:

$$\mathfrak{D}_A \frac{d^2c_A}{dz^2} = k_1''a\,c_A$$

This may be solved with the boundary conditions that $c_A = c_{AS}$ at $z = \pm b$ to get:

$$\frac{c_A}{c_{AS}} = \frac{\cosh \lambda z}{\cosh \lambda b}$$

where

$$\lambda = \sqrt{k_1''a/\mathfrak{D}_A}$$

The reaction rate in moles/unit time for the disk of thickness b is then:

$$|\mathcal{R}_A| = 2\cdot\pi R^2\,|N_{Az}|_{z=b} = 2\pi R^2\left|-\mathfrak{D}_A\frac{dc_A}{dz}\right|_{z=b}$$

$$= 2\pi R^2\mathfrak{D}_A c_{AS}\lambda \tanh \lambda b$$

For n disks of thickness b/n, the above result becomes

$$|\mathcal{R}_A^{(n)}| = n\left(2\pi R^2\mathfrak{D}_A c_{AS}\lambda\left[\tanh\frac{\lambda b}{n}\right]\right)$$

Now

$$\lim_{n\to\infty}|\mathcal{R}_A^{(n)}| = n\left(2\pi R^2\mathfrak{D}_A c_{AS}\lambda\left[\frac{\lambda b}{n} + \cdots\right]\right)$$

$$= 2\pi R^2\mathfrak{D}_A c_{AS}\lambda^2 b$$

$$= 2\pi R^2\mathfrak{D}_A c_{AS}\frac{k_1''a}{\mathfrak{D}_{AB}}b$$

$$= 2\pi R^2 b\,k_1''a\,c_{AS}$$

Hence

$$\eta_A = \lim_{n\to\infty}\frac{|\mathcal{R}_A|}{|\mathcal{R}_A^{(n)}|} = \frac{2\pi R^2\mathfrak{D}_A c_{AS}\lambda\tanh\lambda b}{2\pi R^2\mathfrak{D}_A c_{AS}\lambda^2 b}$$

$$= \frac{\tanh\lambda b}{\lambda b}$$

Chapter 18 - Checked by V. D. Shah

18.A Dehumidification of Air

Let A = H_2O and B = air, to correspond to Ex. 18.5-1. The gas composition at the interface is estimated to be:

$$x_{A0} \doteq \frac{p_{H_2O,\,vap}|_{50°F}}{p} = \frac{0.178 \text{ psia}}{14.70 \text{ psia}} = 0.0121$$

The average gas temperature in the film is $\frac{1}{2}(50+80) = 65°F$ or 291.5 °K. The average gas properties evaluated at $x_B \doteq 1.0$ are:

$$\mathcal{D}_{AB} = 0.246 \text{ cm}^2 \text{ sec}^{-1} \text{ from Eq. 16.3-1}$$
$$c \doteq p/RT = 4.18 \times 10^{-5} \text{ g-moles cm}^{-3}$$
$$k = 5.9 \times 10^{-5} \text{ cal cm}^{-1} \text{ sec}^{-1} \text{ °K}^{-1} \text{ from Eq. 8.3-15}$$

The mean heat capacity of the transferred vapor is (see Problem 8.B)

$$\tilde{C}_{pA} = 8.02 \text{ cal g-mole}^{-1} \text{ °K}^{-1}$$

With the aid of Eq. 18.5-5 we get:

$$\frac{N_{Az}\tilde{C}_{pA}\delta}{k} = \left(\frac{N_{Az}\delta}{c\mathcal{D}_{AB}}\right)\left(\frac{c\mathcal{D}_{AB}\tilde{C}_{pA}}{k}\right) = \left(\ln\frac{1-x_{A\delta}}{1-x_{A0}}\right)\left(\frac{c\mathcal{D}_{AB}\tilde{C}_{pA}}{k}\right)$$
$$= \left(\ln\frac{1-0.0180}{1-0.0121}\right)\left(\frac{(4.18\times10^{-5})(0.246)(8.02)}{5.9\times10^{-5}}\right)$$
$$= (-0.0060)(1.40) = -0.0084$$

Substitution in Eq. 18.A-1 then gives:

(a)
$$\frac{-(N_{Az}\tilde{C}_{pA}/k)\delta}{1-\exp[(N_{Az}\tilde{C}_{pA}/k)\delta]} = \frac{0.0084}{1-e^{-0.0084}} = \frac{0.0084}{1-\left[1-0.0084+\frac{1}{2!}(0.0084)^2-\cdots\right]}$$
$$= \frac{1}{1-\frac{1}{2!}(0.0084)+\cdots} = 1.0042$$

From Eq. 18.5-9 we see that this is the ratio of the heat conduction rates with and without mass transfer; clearly the distinction between the two rates is unimportant in the present problem.

(b) From the definitions on p. 566 we get

$$q_z^{(d)} = \bar{H}_A J_{Az} + \bar{H}_B J_{Bz}$$

and

$$q_z^{(c)} = -k\frac{dT}{dz}$$

for the present problem. Assuming ideal gas behavior and using the reference states selected in Ex. 18.5-1, we get

$$(\bar{H}_A|_{z=0}) = (H_A|_{T=T_0}) = 0$$
$$(\bar{H}_B|_{z=0}) = (H_B|_{T=T_0}) = 0$$

and since the conductive heat flux at $z=0$ is non-zero, we get

$$\left(\frac{q_z^{(d)}}{q_z^{(c)}} \Bigg|_{z=0} \right) = 0$$

This result is due solely to the choice of reference states, and does not in itself indicate that diffusion is unimportant here. One can easily obtain much larger values of $q^{(d)}$ by referring $\bar{H}_A$ and $\bar{H}_B$ to more remote states, e.g., liquid water and gaseous air at 32°F. Note that $q_z^{(c)}$ and $(q^{(d)}|_z - q^{(d)}|_{z=0})$ are independent of the choice of reference states (see Eqs. 18.5-2,7).

18.B Thermal Diffusion

(a) From Table 18.4-1 we see that, for positive k_T and the conditions shown, we must designate H_2 as component A. Then Eq. 18.5-15 gives

$$x_{A2} - x_{A1} = -k_T \ln \frac{T_2}{T_1} = -0.0166 \ln \frac{600}{200} = -0.0183$$

Thus there is a higher concentration of A (H_2) in the bulb at T_1, the mole fraction difference being 0.0183.

(b) From Eq. 18.5-16 we get the recommended mean temperature:

$$T_m = \frac{T_1 T_2}{T_2 - T_1} \ln \frac{T_2}{T_1} = \frac{(200)(600)}{400} \ln \frac{600}{200} = 330\ °K$$

18.C Ultracentrifuging of Proteins

Setting $x_B \doteq x_{B0}$ in Eq. 18.5-18 and taking the $1/\bar{V}_B$ power on both sides, we get

$$\frac{x_A}{x_{A0}} = \exp\left\{ \left[\frac{\bar{V}_A}{\bar{V}_B} M_B - M_A \right] \frac{g_\Omega z}{RT} \right\}$$

Here $M_A = 45{,}000 \frac{g}{g\text{-mole}}$; $\bar{V}_A = M_A/(M_A/\bar{V}_A) = 45{,}000/1.34 = 33{,}582 \frac{cm^3}{g\text{-mole}}$

$M_B/\bar{V}_B = 1.00\ g\ cm^{-3}$

$g_\Omega = 50{,}000 \times 980.665 = 4.903 \times 10^7\ cm^2\ sec^{-1}$

$R = 8.314 \times 10^7\ g\ cm^2\ sec^{-2}\ g\text{-mole}^{-1}\ °K^{-1}$

$T = 75°F = 297\ °K$

$z\ [=]\ cm$

Substitution in the above equation gives:

$$\frac{x_A}{x_{A0}} = \exp\left\{\left[(33{,}582 \times 1.00) - 45{,}000\right]\frac{(4.903 \times 10^7)z}{(8.314 \times 10^7)(297)}\right\} = e^{-22.7z}$$

A somewhat more accurate expression can be obtained by including the variation of x_B. Taking the $1/\bar{V}_B$ power of Eq. 18.5-18 in its entirety we get

$$\frac{x_A}{x_{A0}} = \left(\frac{1-x_A}{1-x_{A0}}\right)^{\bar{V}_A/\bar{V}_B} \exp\left\{\left[\frac{\bar{V}_A}{\bar{V}_B}M_B - M_A\right]\frac{g\Omega z}{RT}\right\}$$

This differs from the preceding solution by the factor $\left(\frac{1-x_A}{1-x_{A0}}\right)^{\bar{V}_A/\bar{V}_B}$; for positive z the two solutions agree within 0.93 percent for the conditions given.

18.D Electrode Polarization

From Eq. 18.5-26 we get:

$$I_{max} = \frac{4\,\mathcal{D}_{M^+w}\,(c\,x_{avg})}{L} = \frac{4\,(10^{-5})(1 \times 10^{-4})}{0.1} = 4 \times 10^{-8}\ \frac{\text{g-eqt.}}{\text{cm}^2\,\text{sec}^{-1}}$$

where $cx_{avg} = 1 \times 10^{-4}$ is the average cation concentration in g-eqts. cm^{-3}. Multiplication by the Faraday constant gives

$$I_{max} = (4 \times 10^{-8})(9.652 \times 10^{4}) = 3.86 \times 10^{-3}\ \frac{\text{abs-coulombs}}{\text{cm}^2\ \text{sec}}$$

$$= 3.86 \times 10^{-3}\ \text{amperes}\ \text{cm}^{-2}$$

18.E Effective Binary Diffusivities in a Multicomponent Gaseous Mixture

We assume that the point in question is so near the catalyst surface that the mass fluxes are determined entirely by the reaction rate within the catalyst. Then from the reaction stoichiometry (Eq. 18.E-1) we can express the fluxes as follows:

$$\underset{\sim}{N}_2 = -3\underset{\sim}{N}_1 \quad ; \quad \underset{\sim}{N}_3 = -\underset{\sim}{N}_1 \quad ; \quad \underset{\sim}{N}_4 = 0 \quad ; \quad \sum_{j=1}^{4}\underset{\sim}{N}_j = 3\underset{\sim}{N}_1$$

When these substitutions are made in Eq. 18.4-22, $\underset{\sim}{N}_1$ can be factored out of every term, leaving only scalar quantities. The results are:

$$\frac{1}{c\mathcal{D}_{1m}} = \frac{1}{1-3x_1}\left[\frac{x_1-x_1}{c\mathcal{D}_{11}} + \frac{x_2-3x_1}{c\mathcal{D}_{12}} + \frac{x_3-(-1)x_1}{c\mathcal{D}_{13}} + \frac{x_4-0}{c\mathcal{D}_{14}}\right]$$

$$= \frac{10^6}{1-3(0.1)}\left[0 + \frac{0.80-3(0.10)}{24.2} + \frac{0.05+0.10}{1.95} + \frac{0.05}{6.21}\right]$$

$$= \frac{10^6}{0.7}\left[0 + 0.0207 + 0.0769 + 0.0080\right] = \frac{0.1056 \times 10^6}{0.7}$$

$$c\mathcal{D}_{1m} = \frac{0.7}{0.1056} \times 10^{-6} = 6.63 \times 10^{-6}\ \text{g-mole cm}^{-1}\ \text{sec}^{-1}$$

$$\frac{1}{c\mathcal{D}_{2m}} = \frac{1}{3-3x_2}\left[\frac{3x_1 - x_2}{c\mathcal{D}_{12}} + \frac{3x_2 - 3x_2}{c\mathcal{D}_{22}} + \frac{3x_3 - (-1)x_2}{c\mathcal{D}_{23}} + \frac{3x_4 - 0}{c\mathcal{D}_{44}}\right]$$

$$= \frac{10^6}{3-3(0.8)}\left[\frac{3(0.10) - 0.80}{24.2} + 0 + \frac{3(0.05) + 0.80}{20.7} + \frac{3(0.05)}{41.3}\right]$$

$$= \frac{10^6}{0.6}\left[-0.02066 + 0 + 0.04589 + 0.00363\right] = \frac{0.02885}{0.6 \times 10^{-6}}$$

$$c\mathcal{D}_{2m} = \frac{0.6 \times 10^{-6}}{0.02885} = \left(\begin{matrix}20.8\\ \times 10^{-6}\end{matrix}\right) \text{g-mole cm}^{-1}\text{ sec}^{-1}$$

$$\frac{1}{c\mathcal{D}_{3m}} = \frac{1}{-1-3(0.05)}\left[\frac{-x_1 - x_3}{c\mathcal{D}_{13}} + \frac{-x_2 - 3x_3}{c\mathcal{D}_{23}} + \frac{-x_3 - (-1)x_3}{c\mathcal{D}_{33}} + \frac{-x_4}{c\mathcal{D}_{34}}\right]$$

$$= \frac{10^6}{-1.15}\left[\frac{-0.10 - 0.05}{1.95} + \frac{-0.80 - 3(0.05)}{20.7} + 0 + \frac{-0.05}{5.45}\right]$$

$$= \frac{10^6}{-1.15}\left[-0.07692 - 0.04589 + 0 - 0.00917\right] = \frac{-0.13198}{-1.15 \times 10^{-6}}$$

$$c\mathcal{D}_{3m} = \frac{-1.15 \times 10^{-6}}{-0.13198} = 8.71 \times 10^{-6} \text{ g-mole cm}^{-1}\text{ sec}^{-1}$$

$$\frac{1}{c\mathcal{D}_{4m}} = \frac{1}{0-3x_4}\left[\frac{0 - x_4}{c\mathcal{D}_{14}} + \frac{0 - 3x_4}{c\mathcal{D}_{24}} + \frac{0 - (-1)x_4}{c\mathcal{D}_{34}} + \frac{0 - (0)x_4}{c\mathcal{D}_{44}}\right]$$

$$= \frac{10^6}{0-3(0.05)}\left[\frac{-0.05}{6.21} + \frac{-3(0.05)}{41.3} + \frac{0.05}{5.45} + 0\right]$$

$$= \frac{10^6}{-0.15}\left[-0.00805 - 0.00363 + 0.00917\right] = \frac{-0.00251}{-0.15 \times 10^{-6}}$$

$$c\mathcal{D}_{4m} = \frac{0.15 \times 10^{-6}}{0.00251} = 60 \times 10^{-6} \text{ g-mole cm}^{-1}\text{ sec}^{-1}$$

18.F Setting Up Diffusion Problems

Problem	Starting Equations	Assumptions	Results
17.D Radial diffusion through a tube wall	Table 18.2-2, Eq. B	$\partial/\partial t = 0$ $\rho, \mathcal{D}_{AB} = \text{const.}$ $\underset{\sim}{v} = 0$ $c_A = c_A(r)$ $R_A = 0$	$0 = \mathcal{D}_{AB}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_A}{\partial r}\right)\right]$
17.E Diffusion through a stagnant film	Table 16.2-1, Eq. B Table 18.2-1, Eq. A	$N_B = 0$ $x_A = x_A(z)$ $\partial/\partial t = 0$ $N_{Ax} = N_{Ay} = 0$ $R_A = 0$	$N_{Az} = -\frac{c\mathcal{D}_{AB}}{1-x_A}\frac{dx_A}{dz}$ $N_{Az} = \text{constant}$
17.F Diffusion through a stagnant liquid film	Table 18.2-2, Eq. A	$\partial/\partial t = 0$ $\rho, \mathcal{D}_{AB} = \text{const.}$ $\underset{\sim}{v} \doteq 0$ $c_A = c_A(z)$ $R_A = 0$	$0 = \mathcal{D}_{AB}\frac{d^2 c_A}{dz^2}$
17.G Diffusion from a droplet into a quiescent gas	Table 18.2-1, Eq. C Table 16.2-1, Eq. B	$\partial/\partial t = 0$ $N_{A\theta} = N_{A\phi} = 0$ $R_A = 0$ $N_B = 0$ $x_A = x_A(r)$	$r^2 N_{Ar} = \text{constant}$ $N_{Ar} = -\frac{c\mathcal{D}_{AB}}{(1-x_A)}\frac{dx_A}{dr}$
17.H Diffusion with catalytic polymerization	Table 16.2-1, Eq. B Table 18.2-1 Eq. A	$N_{A_nz} = -\frac{1}{n}N_{Az}$ $x_A = x_A(z)$ $\partial/\partial t = 0$ $N_{Ax} = N_{Ay} = 0$ $R_A = 0$	$N_{Az} = -\frac{c\mathcal{D}_{AB}}{1-x_A(1-n^{-1})}\frac{dx_A}{dz}$ $N_{Az} = \text{constant}$
17.J Solid dissolution into a falling film	Table 18.2-1 Eq. A	$\partial/\partial t = 0$ $\rho, \mathcal{D}_{AB} = \text{const.}$ $c_A = c_A(y, z)$ $\frac{\partial^2 c_A}{\partial z^2} << \frac{\partial^2 c_A}{\partial y^2}$ $v_x = v_y = 0$ $R_A = 0$	$v_z\frac{\partial c_A}{\partial z} = \mathcal{D}_{AB}\frac{\partial^2 c_A}{\partial y^2}$ or for small y, $ay\frac{\partial c_A}{\partial z} = \mathcal{D}_{AB}\frac{\partial^2 c_A}{\partial y^2}$
17.K Diffusion from a point source in a moving stream	Table 18.2-2, Eq. B	$\partial/\partial t = 0$ $\rho, \mathcal{D}_{AB} = \text{const.}$ $v_r = v_\theta = 0$ $c_A = c_A(r, z)$ $R_A = 0$	$v_z\frac{\partial c_A}{\partial z} = \mathcal{D}_{AB}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_A}{\partial r}\right) + \frac{\partial^2 c_A}{\partial z^2}\right]$

18.F (cont'd.)

Problem	Starting Equations	Assumptions	Results
17.L Gas absorption in a falling film with chemical reaction	Velocity profile: Eq. 17.5-1 Continuity of A: Table 18.2-2, Eq. A	See solution to Problem 3.E $\partial/\partial t = 0$ $\rho, \mathcal{D}_{AB} = \text{const.}$ $v_x = v_y = 0$ $c_A = c_A(x, z)$ $\frac{\partial^2 c_A}{\partial z^2} << \frac{\partial^2 c_A}{\partial x^2}$ $R_A = -k_1''' c_A$	$v_z \frac{\partial c_A}{\partial z} = \mathcal{D}_{AB} \frac{\partial^2 c_A}{\partial x^2} - k_1''' c_A$ or, for very small values of x, $v_{max} \frac{\partial c_A}{\partial z} = \mathcal{D}_{AB} \frac{\partial^2 c_A}{\partial x^2} - k_1''' c_A$

18.G Alternate Forms of the Equation of Continuity

(a) The definition of $\mathbf{n}_A$ may be rearranged as follows:

$$\mathbf{n}_A = \rho_A \mathbf{v}_A = \rho_A \mathbf{v} + \rho_A(\mathbf{v}_A - \mathbf{v}) = \rho_A \mathbf{v} + \mathbf{j}_A$$

Inserting this in Eq. 18.1-6 we get:

$$\frac{\partial \rho_A}{\partial t} + (\nabla \cdot \rho_A \mathbf{v}) + (\nabla \cdot \mathbf{j}_A) = r_A \qquad (18.G-1)$$

(b) The definition of $\mathbf{N}_A$ may be similarly rearranged to give:

$$\mathbf{N}_A = c_A \mathbf{v}_A = c_A \mathbf{v}^\star + c_A(\mathbf{v}_A - \mathbf{v}^\star) = c_A \mathbf{v}^\star + \mathbf{J}_A^\star$$

Dividing Eq. 18.1-6 by M_A and substituting this expression for $\mathbf{N}_A$ we get:

$$\frac{\partial c_A}{\partial t} + (\nabla \cdot c_A \mathbf{v}^\star) + (\nabla \cdot \mathbf{J}_A^\star) = R_A \qquad (18.G-2)$$

For a small volume element δV, the terms in 18.G-2 have the following significance:

$\frac{\partial c_A}{\partial t}$ = (Rate of increase of moles of A in the element)/δV

$(\nabla \cdot c_A v^\star) = \left\{\begin{matrix}\text{Rate of efflux of A due to the molar average} \\ \text{velocity } \mathbf{v}^\star\end{matrix}\right\}/\delta V$

$(\nabla \cdot \mathbf{J}_A^\star)$ = (Rate of efflux of A from the element by diffusion)/δV

$R_A = \left(\begin{matrix}\text{Rate of production of A in the element} \\ \text{by chemical reaction}\end{matrix}\right)/\delta V$

The terms in Eq. 18.G-1 have analogous significance except that the quantities of A are in mass units and the reference velocity is $\mathbf{v}$ rather than $\mathbf{v}^\star$.

18.H Simplification of the Multicomponent Mass Flux Expressions for Use in a Binary System

For a binary mixture Eqs. 18.4-12,13 give:

$$\mathcal{D}_{AA} = \mathcal{D}_{BB} = 0 \quad , \qquad \mathcal{D}_{AB} = \mathcal{D}_{BA}$$

In addition, Table 16.1-3 gives $\underset{\sim}{j}_A + \underset{\sim}{j}_B = 0$. Inserting these relations in Eq. 18.4-8 we get:

$$\underset{\sim}{j}_A^{(x)} = -\underset{\sim}{j}_B^{(x)} = -\frac{c^2}{\rho RT} M_B M_A \mathcal{D}_{BA} x_A \left(\frac{\partial \bar{G}_A}{\partial x_B}\right)_{p,T} \nabla x_B \longleftarrow \left\{\begin{array}{l}\text{This is the}\\ \text{term with } i=B\\ \text{and } j=A\text{; the}\\ \text{term with } i=j=B\\ \text{vanishes since}\\ \mathcal{D}_{BB}=0.\end{array}\right.$$

$$= -\frac{c^2}{\rho RT} M_A^2 M_B \mathcal{D}_{AB} x_A \left[\left(\frac{\partial}{\partial x_A}\frac{\bar{G}_A}{M_A}\right)_{p,T} \nabla x_A\right]$$

Making the same substitutions in Eq. 18.4-9 we get:

$$\underset{\sim}{j}_A^{(p)} = -\underset{\sim}{j}_B^{(p)} = -\frac{c^2}{\rho RT} M_B M_A \mathcal{D}_{BA} x_A M_A \left(\frac{\bar{V}_A}{M_A} - \frac{1}{\rho}\right)\nabla p$$

$$= -\frac{c^2}{\rho RT} M_A^2 M_B \mathcal{D}_{AB} x_A \left[\left(\frac{\bar{V}_A}{M_A} - \frac{1}{\rho}\right)\nabla p\right]$$

In the same way Eq. 18.4-10 becomes

$$\underset{\sim}{j}_A^{(g)} = -\underset{\sim}{j}_B^{(g)} = +\frac{c^2}{\rho RT} M_B M_A \mathcal{D}_{BA} x_A M_A \left(\underset{\sim}{g}_A - \frac{\rho_A \underset{\sim}{g}_A + \rho_B \underset{\sim}{g}_B}{\rho}\right)$$

$$= +\frac{c^2}{\rho RT} M_A^2 M_B \mathcal{D}_{AB} x_A \left[\frac{(\rho_A+\rho_B)\underset{\sim}{g}_A - \rho_A \underset{\sim}{g}_A - \rho_B \underset{\sim}{g}_B}{\rho}\right]$$

$$= +\frac{c^2}{\rho RT} M_A^2 M_B \mathcal{D}_{AB} x_A \left[\frac{\rho_B}{\rho}\left(\underset{\sim}{g}_A - \underset{\sim}{g}_B\right)\right]$$

and Eq. 18.4-11 gives directly

$$\underset{\sim}{j}_A^{(T)} = -D_A^{(T)} \nabla \ln T$$

Adding these flux expressions as indicated in Eq. 18.4-7, we get:

$$\underset{\sim}{j}_A = \underset{\sim}{j}_A^{(x)} + \underset{\sim}{j}_A^{(p)} + \underset{\sim}{j}_A^{(g)} + \underset{\sim}{j}_A^{(T)}$$

$$\underset{\sim}{j}_A = -\frac{c^2}{\rho RT} M_A^2 M_B \mathcal{D}_{AB} x_A \left[\begin{array}{r}\left(\frac{\partial}{\partial x_A}\frac{\bar{G}_A}{M_A}\right)_{T,p}\nabla x_A + \left(\frac{\bar{V}_A}{M_A} - \frac{1}{\rho}\right)\nabla p \\ -\frac{\rho_B}{\rho}\left(\underset{\sim}{g}_A - \underset{\sim}{g}_B\right)\end{array}\right]$$

$$-D_A^{(T)} \nabla \ln T$$

18.I Pressure Diffusion

(a) Multiplication of Eq. 18.5-17 by $\overline{V}_B$ gives:

$$\overline{V}_B \frac{dx_A}{x_A} = - \overline{V}_B \frac{g_\Omega}{RT}(M_A - \rho \overline{V}_A)dz$$

Interchanging the labels A and B gives:

$$\overline{V}_A \frac{dx_B}{x_B} = - \overline{V}_A \frac{g_\Omega}{RT}(M_B - \rho \overline{V}_B)dz$$

Subtracting the second equation from the first, we get

$$\overline{V}_B \frac{dx_A}{x_A} - \overline{V}_A \frac{dx_B}{x_B} = \frac{g_\Omega}{RT}(M_B \overline{V}_A - M_A \overline{V}_B)dz$$

Let x_{A0} and x_{B0} be the compositions at $z=0$, and assume that T, $\overline{V}_A$ and $\overline{V}_B$ are independent of z; then integration gives:

$$\overline{V}_B \int_{x_{A0}}^{x_A} \frac{dx_A}{x_A} - \overline{V}_A \int_{x_{B0}}^{x_B} \frac{dx_B}{x_B} = \frac{M_B \overline{V}_A - M_A \overline{V}_B}{RT} \int_0^z g_\Omega dz$$

and for constant g_Ω we get:

$$\overline{V}_B \ln \frac{x_A}{x_{A0}} - \overline{V}_A \ln \frac{x_B}{x_{B0}} = \frac{M_B \overline{V}_A - M_A \overline{V}_B}{RT} g_\Omega z$$

Taking the exponential of both sides gives:

$$\left(\frac{x_A}{x_{A0}}\right)^{\overline{V}_B} \left(\frac{x_{B0}}{x_B}\right)^{\overline{V}_A} = \exp\left[(\overline{V}_A M_B - \overline{V}_B M_A) \frac{g_\Omega z}{RT}\right]$$

(b) The exact expression for g_Ω is:

$$g_\Omega = \Omega^2 r$$

Substituting this in the first integral of (a) and setting $z = R_1 - r$, we get:

$$\int_0^z g_\Omega dz = \int_{R_1}^r \Omega^2 r(-dr) = \frac{1}{2}\Omega^2(R_1^2 - r^2)$$

where R_1 is the distance from the axis of rotation when $z=0$. Substituting this expression into the result of (a) in place of $g_\Omega z$ we get:

$$\left(\frac{x_A}{x_{A0}}\right)^{\overline{V}_B} \left(\frac{x_{B0}}{x_B}\right)^{\overline{V}_A} = \exp\left[(\overline{V}_A M_B - \overline{V}_B M_A)\frac{\Omega^2}{2RT}(R_1^2 - r^2)\right]$$

(c) If x_A is <u>very</u> small, then $x_B \doteq x_{B0}$ and $\overline{V}_B \doteq V_B$. With these simplifications Eq. 18.5-18 becomes:

$$\left(\frac{x_A}{x_{A0}}\right)^{V_B} = \exp\left[(\overline{V}_A M_B - V_B M_A) \frac{g_\Omega z}{RT}\right]$$

or

$$\frac{x_A}{x_{A0}} = \exp\left[\left(\frac{\overline{V}_A}{\overline{V}_B} M_B - M_A\right)\frac{\gamma\Omega^2}{RT}\right]$$

In most ultracentrifuge applications the partial molar volumes differ by several orders of magnitude. The simplification made here is then valid only for exceedingly dilute solutions if A is the high-molecular-weight component. (See solution to Problem 18.C).

18.J Mobility

From Eq. 18.4-14a we get

$$\tilde{\underset{\sim}{F}}_A = \frac{\underset{\sim}{v}_A - \underset{\sim}{v}^*}{m_A} = \frac{(\underset{\sim}{v}_A - \underset{\sim}{v}^*)RT}{\mathcal{D}_{AB}}$$

Insertion of the data in c.g.s. units gives:

$$\tilde{F}_A = \frac{(1.0 \text{ cm sec}^{-1})(8.314\times10^{7} \text{ dyne cm g-mole}^{-1}\ {}^\circ\text{K}^{-1})(298.2^\circ\text{K})}{(10^{-5} \text{ cm}^2 \text{ sec}^{-1})}$$

$$= 2.48 \times 10^{15} \text{ dynes/g-equivalent}$$

Now

$$1 \text{ kg force} = (10^3 \text{ g mass})(980.665 \text{ cm sec}^{-2}) = 980,665 \text{ dynes}$$

Hence

$$\tilde{F}_A = \frac{2.48\times10^{15}}{0.980665\times10^{6}} = 2.53\times10^{9} \text{ kg force}$$

Chapter 19 - Checked by T. J. Sadowski

19.A Estimation of Point Concentration in Binary Diffusional Evaporation

According to Eq. 19.1-16 and Table 19.1-1,

$$\mathcal{X} = \frac{1 - \text{erf}\,(0.50 - 0.66)}{1 + \text{erf}\ 0.66} = \frac{1 - \text{erf}\,(-0.16)}{1 + \text{erf}\ 0.66} = \frac{1 + \text{erf}\ 0.16}{1 + \text{erf}\ 0.66} = \frac{1.18}{1.65}$$

$$= 0.715$$

Hence

$$x_A = \mathcal{X}\, x_{A0} = (0.715)(0.75) = 0.54$$

This agrees closely with the result estimated from Fig. 19.1-1.

19.B Rate of Evaporation of n-Octane

The volume of vapor A produced in time t is:

$$V_A = \psi^{(2)} S x_{A0} \sqrt{4 \mathcal{D}_{AB} t/\pi} \qquad (19.1-1)$$

According to the ideal gas law the mass of this vapor is:

$$m_A = M_A \frac{p V_A}{RT}$$

Hence the mass of A evaporated is:

$$m_A = M_A \psi^{(2)} \frac{p S x_{A0}}{RT} \sqrt{\frac{4 \mathcal{D}_{AB} t}{\pi}}$$

Assuming interfacial equilibrium and ideal gas behavior, we get

$$x_{A0} = \frac{p_{A,vap.}}{p} = \frac{10.45}{760} = 0.01375$$

From Table 19.1-1, then, $\psi^{(2)} = 1.006$.

$\mathcal{D}_{AB}$ may be calculated as in Example 16.4-1:

$$\sigma_{AB} = \tfrac{1}{2}(7.451 + 3.681) = 5.566\ \text{Å} \qquad \epsilon_{AB}/\kappa = \sqrt{320 \times 91.5} = 171.1\ °K$$

$$\frac{1}{M_A} + \frac{1}{M_B} = \frac{1}{114.22} + \frac{1}{28.02} = 0.04443 \qquad \kappa T/\epsilon_{AB} = 293.2/171.1 = 1.714$$

$$\Omega_{\mathcal{D},AB} = 1.137$$

$$\mathcal{D}_{AB} = 0.0018583 \frac{\sqrt{(293.2)^3 (0.04443)}}{(1)(5.566)^2 (1.137)} = 0.0558\ \text{cm}^2\ \text{sec}^{-1}$$

With these numerical values we get:

$$m_A = (114.22)(1.006) \frac{(1)(1.29)(0.01375)}{(82.06)(293.2)} \sqrt{\frac{(4)(0.0558)(24.5 \times 3600)}{\pi}}$$

$$= (8.47 \times 10^{-5}) \sqrt{6266} = 6.71 \times 10^{-3}\ \text{g.} = 6.71\ \text{mg.}$$

19.C Boundary-Layer Results for Binary Diffusional Evaporation

By assumption, $f = 1 - 2\zeta + 2\zeta^3 - \zeta^4$

Hence $f' = -2 + 6\zeta^2 - 4\zeta^3$

and $f'_0 = -2$. Furthermore,

$$\int_0^1 f\,d\zeta = \left(\zeta - \zeta^2 + \tfrac{1}{2}\zeta^4 - \tfrac{1}{5}\zeta^5\right)\Big|_0^1 = \frac{3}{10}$$

Substitution in Eq. 19.2-10 then gives:

$$\psi^{(2)}_{\text{bdry. lyr.}} = \sqrt{\frac{\left(\int_0^1 f\,d\zeta\right)(-f'_0)\,\pi}{2(1 - x_{A0})}} = \sqrt{\frac{3\pi}{10(1 - x_{A0})}}$$

Chapter 20 - Checked by T. J. Sadowski

20.A Determination of Eddy Diffusivity

(a) From Eq. 17.K-2 we get, after multiplication by s,

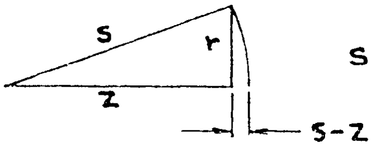
$$sc_A = \frac{\mathcal{W}_A}{4\pi \mathfrak{D}_{AB}^{(t)}} \exp\left[-\frac{v_0}{2\mathfrak{D}_{AB}^{(t)}}(s-z)\right]$$

Taking the natural logarithm of both sides we get:

$$\ln sc_A = \ln \frac{\mathcal{W}_A}{4\pi \mathfrak{D}_{AB}^{(t)}} - \frac{v_0}{2\mathfrak{D}_{AB}^{(t)}}(s-z)$$

Since $\mathcal{W}_A$, v_0 and $\mathfrak{D}_{AB}^{(t)}$ are considered independent of position, this equation predicts that a plot of $\ln sc_A$ versus $(s-z)$ should be linear with slope $(-v_0/2\mathfrak{D}_{AB}^{(t)})$.

(b) The quantity s-z is a small difference of large numbers; hence the following development leads to simpler calculations:



$$s-z = \frac{s^2-z^2}{s+z} = \frac{r^2}{s+z} \doteq \frac{r^2}{2z}$$

For the data of Fig. 20.A, the approximation $s+z \doteq 2z$ is good to four significant figures. The calculations of sc_A/c and $s-z$ are summarized below.

Data from Fig. 20.A		$r^2 = \left(7.62\frac{r}{R}\right)^2$	$s = \sqrt{z^2+r^2}$	$s-z = \frac{r^2}{s+z} \doteq 0.2581\left(\frac{r}{R}\right)^2$	$sc_A/c \doteq 112.5\frac{c_A}{c}$
r/R	c_A/c				
0.00	0.0105	0.00	112.50	0.0000	1.18
0.06	0.0103	0.21		0.0009	1.16
0.07	0.0100	0.28		0.0013	1.12
0.13	0.0087	0.98	112.50	0.0044	0.98
0.14	0.0082	1.14		0.0051	0.92
0.20	0.0069	2.32		0.0103	0.78
0.21	0.0064	2.56	112.51	0.0114	0.72
0.26	0.0050	3.93		0.0174	0.56
0.27	0.0048	4.23		0.0188	0.54
0.33	0.0034	6.32		0.028	0.38
0.34	0.0033	6.71		0.030	0.37
0.45	0.0014	11.76	112.55	0.052	0.16

The rest of the calculations are shown on the adjoining plot.

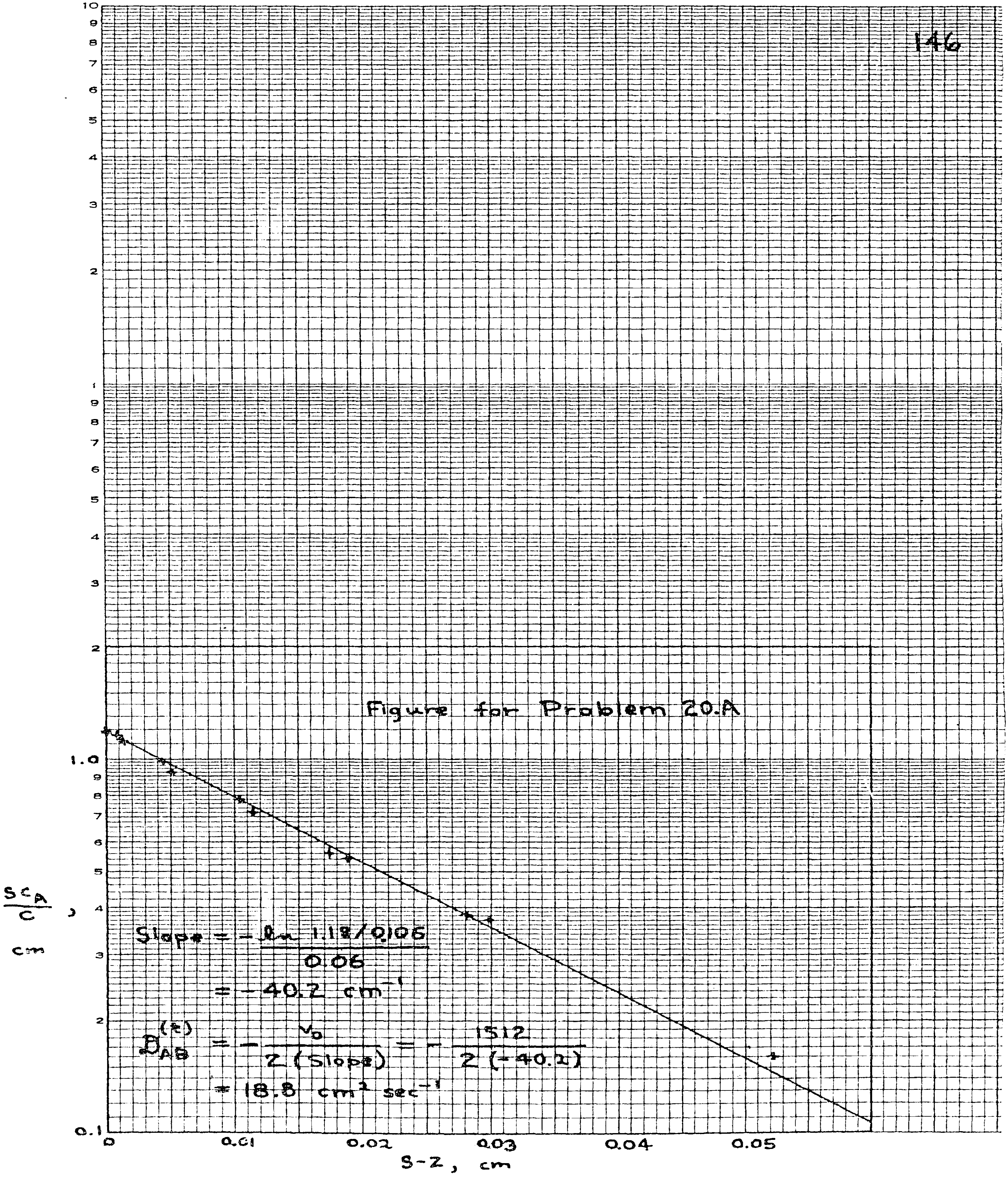


Figure for Problem 20.A

$$\text{Slope} = -\frac{\ln 1.18/0.106}{0.06}$$

$$= -40.2 \text{ cm}^{-1}$$

$$\mathcal{D}_{AB}^{(t)} = -\frac{v_0}{2\,(\text{Slope})} = -\frac{1512}{2\,(-40.2)}$$

$$= 18.8 \text{ cm}^2 \text{ sec}^{-1}$$

Chapter 21 - Checked by G. F. Kuether

21.A Prediction of Mass Transfer Coefficients in Closed Channels

With the aid of Table 1.1-1, Appendix C and the ideal gas law we get:

$$\mu = 0.0184 \text{ cp.} = 0.0444 \quad \text{lb}_m \text{ ft}^{-1} \text{ hr}^{-1}$$

$$c = p/RT = 5.11 \times 10^{-3} \quad \text{lb-moles ft}^{-3}$$

$$\rho = c M_{air} = 0.148 \quad \text{lb}_m \text{ ft}^{-3}$$

$$\mathcal{D}_{AB} = 0.130 \times 3.8750 = 0.504 \quad \text{ft}^2 \text{ hr}^{-1}$$

$$\frac{\mu}{\rho \mathcal{D}_{AB}} = \frac{0.0444}{(0.148)(0.504)} = 0.596 \text{, dimensionless}$$

(a) The Reynolds number for this system is:

$$\frac{DG}{\mu} = \frac{4w}{\pi D \mu} = \frac{(4)(1570)}{\pi(6/12)(0.0444)} = 9.00 \times 10^4$$

Equation 21.A-1 is therefore applicable, and

$$k_{x,loc} = 0.023 \; \frac{c\mathcal{D}_{AB}}{D} \left(\frac{DG}{\mu}\right)^{0.8} \left(\frac{\mu}{\rho \mathcal{D}_{AB}}\right)^{0.44}$$

$$= 0.023 \frac{(5.11 \times 10^{-3})(0.504)}{(6/12)} (1.307 \times 10^4) \quad (0.796)$$

$$= 1.23 \quad \text{lb-moles hr}^{-1} \text{ ft}^{-2}$$

(b) For packed beds of spheres the shape factor Ψ is unity. The Reynolds number here is (see Eq. 13.4-5):

$$Re = \frac{G_0}{a \mu_f \Psi} = \frac{w}{S a \mu_f \Psi} = \frac{1570}{(\pi/16)(100)(0.0444)(1.0)} = 1.80 \times 10^3$$

From Table 21.2-1 and Eq. 13.4-3 we get:

$$j_D = j_H = 0.61 (Re)^{-0.41} \Psi = (0.61)(1800)^{-0.41}(1.00) = 0.0282$$

By definition, for this flow system the characteristic velocity is G_0/ρ_b, hence

$$j_D = \frac{k_{x,loc} M_b}{G_0} \left(\frac{\mu}{\rho \mathcal{D}_{AB}}\right)^{2/3}$$

Rearrangement and substitution of numerical values gives:

$$k_{x,loc} = \frac{j_D w}{S M_b} \left(\frac{\mu}{\rho \mathcal{D}_{AB}}\right)^{-2/3} = \frac{(0.0282)(1570)}{(\pi/16)(28.97)} \left(\frac{1}{0.708}\right)$$

or

$$k_{x,loc} = 11.0 \quad \text{lb-moles hr}^{-1} \text{ ft}^{-2}$$ based on particle surface

21.B Calculation of Gas Composition from Psychrometric Data

The solution is obtained from Eqs. 21.2-35 and 37. Here $T_f = \frac{1}{2}(80+130) = 105°F$ and the mole fraction of water is small, hence the film properties from Ex. 21.2-2 may be used.

$$\tilde{C}_{pf} = 6.98 \frac{\text{Btu}}{\text{lb-mole °F}} \quad , \quad Sc_f = 0.58 \quad , \quad Pr_f = 0.74$$

For water at 80° F,

$$p_{A,vap} = 26.2 \text{ mm Hg} \quad \text{and} \quad \Delta\tilde{H}_{A,vap} = 18,870 \text{ Btu/lb-mole}$$

From Eq. 21.2-37 we get

$$x_{A0} = p_{A,vap}/p = 26.2/800 = 0.0328$$

Substitution in Eq. 21.2-35 gives:

$$\frac{x_{A0} - x_{A\infty}}{(T_\infty - T_0)(1-x_{A0})} = \frac{\tilde{C}_{pf}}{\Delta\tilde{H}_{A,vap}}\left(\frac{Sc}{Pr}\right)_f^{2/3}$$

$$\frac{0.0328 - x_{A\infty}}{(130-80)(1-0.0328)} = \frac{6.98}{18,870}\left(\frac{0.58}{0.74}\right)^{2/3} = 3.14 \times 10^{-4}$$

From which the mole fraction of water in the approaching air stream is found to be:

$$x_{A\infty} = 0.0328 - 0.0152 = 0.0176$$

21.C Calculation of Air Temperature for Drying in a Fixed Bed

An energy balance on the solids contained in unit volume of the packed bed gives, for constant solids temperature,

$$Q_A = \mathcal{W}_A^{(m)} \Delta\tilde{H}_{A,vap}$$

and evaluation of Q_A and $\mathcal{W}_A^{(m)}$ in terms of local transfer coefficients gives (compare Eq. 21.2-31):

$$\frac{x_{A0} - x_{Ab}}{(T_b - T_0)(1-x_{A0})} = \frac{h_{loc}}{k_{x,loc}\,\Delta\tilde{H}_{A,vap}}$$

From Eq. 13.4-4 and Table 21.2-1 we get:

$$\frac{h_{loc}}{\hat{C}_{pb}G_0}(Pr)_f^{2/3} = \frac{k_{x,loc}M_b}{G_0}(Sc)_f^{2/3}$$

Inserting this result above we get (compare Eq. 21.2-35):

$$\frac{(x_{A0} - x_{Ab})}{(T_b - T_0)(1-x_{A0})} = \frac{\tilde{C}_{pb}}{\Delta\tilde{H}_{A,vap}}\left(\frac{Sc}{Pr}\right)^{2/3}$$

The following numerical data may then be inserted:

$T_0 = 60°F$

$x_{A0} = \frac{0.2562 \text{ psia}}{(1.1)(14.70 \text{ psia})} = 0.0159$ from Eq. 21.2-37

$x_{Ab} = 0$

$\Delta \tilde{H}_{A,vap} = 19{,}080$ Btu / lb-mole

$\tilde{C}_{pb} = 6.98$ Btu lb-mole^{-1} °F^{-1}

$Sc_f = 0.58$

$Pr_f = 0.74$

The last three properties are not very sensitive to temperature; we have used the values from Ex. 21.2-1 in which $T_f = 105°F$. Solving for the gas stream temperature we get:

$$T_b = T_0 + \left(\frac{x_{A0} - x_{Ab}}{1 - x_{A0}}\right)\left(\frac{\Delta \tilde{H}_{A,vap}}{\tilde{C}_{pb}}\right)\left(\frac{Pr}{Sc}\right)_f^{2/3}$$

$$= 60 + \left(\frac{0.0159 - 0}{1 - 0.0159}\right)\left(\frac{19{,}080}{6.98}\right)\left(\frac{0.74}{0.58}\right)^{2/3}$$

$$= 60 + 52 = 112°F.$$

{This gives a corrected T_f of 86°F, and for precise work a recalculation would be advisable.

21.D Rate of Drying of Granular Solids in a Fixed Bed

The rate of water removal in moles hr^{-1} ft^{-3} of packed bed is, from Eq. 21.1-11,

$$a N_{A0} = \frac{(k_{x,loc})(a)(x_{A0} - x_{Ab})}{(1 - x_{A0})}$$

Here we have set $N_{B0} = 0$ and assumed a small mass transfer rate. To estimate $k_{x,loc}$ we use the mass-transfer analog of Eq. 13.4-4:

$$j_D = \frac{k_{x,loc} M_b}{G_0}(Sc)_f^{2/3} = \frac{k_{x,loc} M_b}{\rho_b v_0}(Sc)_f^{2/3}$$

In order to determine j_D we first calculate the Reynolds number:

$$Re = \frac{G_0}{a \mu_f \psi} = \frac{\rho_b v_0}{a \mu_f \psi}$$

Here

$a = 180$ ft^{-1}

$v_0 = 15$ ft sec^{-1}

$\psi = 0.86$ from Table 13.4-1

$T_f = \frac{1}{2}(60 + 112) = 86°F = 30°C$

$\mu_f = 0.01862$ cp $= 1.252 \times 10^{-5}$ lb$_m$ ft^{-1} sec^{-1}

$\rho_b = p/RT_b = 0.0764$ lb$_m$ ft^{-3}

Hence

$$Re = \frac{(0.0764)(15)}{(180)(1.252 \times 10^{-5})(0.86)} = 592$$

Then from Eq. 13.4-3 and Table 21.2-1,

$$j_H = j_D = 0.61\ Re^{-0.41}\ \psi \qquad (\text{since } Re > 50)$$

$$= (0.61)(592)^{-0.41}(0.86) = 0.0382$$

From the definition of j_D we then get:

$$k_{x,loc} = \frac{j_D \rho_b v_0}{M_b}(Sc)_f^{-2/3} = \frac{(0.0382)(0.0764)(15)}{28.97}(0.58)^{2/3}$$

$$= 2.17 \times 10^{-3} \text{ lb-mole sec}^{-1}\text{ ft}^{-2}$$

$$= 7.82 \text{ lb-mole hr}^{-1}\text{ ft}^{-2}$$

The rate of evaporation is then found to be:

$$aN_{A0} = \frac{(7.82)(180)(0.0159)}{(1-0.0159)} = 22.7 \text{ lb-moles hr}^{-1}\text{ ft}^{-3}$$

The same result can be obtained by a calculation based on the heat transfer rate (see Eq. 21.2-29).

21.E Evaporation of a Freely Falling Drop

The properties of the air-water system at 1 atm, an estimated T_f of 80°F, and $x_{Af} \doteq 0.0$ mole fraction water vapor are:

$$c_f \doteq p/RT_f = 4.06 \times 10^{-4} \text{ lb-}$$

$$\rho_f = M_f c_f = 1.18 \times 10^{-3} \text{ g cm}^{-3}$$

$$\mu_f = 0.0184 \text{ cp} = 1.84 \times 10^{-4} \text{ g cm}^{-1}\text{ sec}^{-1}$$

$$\tilde{C}_{pf} = 6.98 \text{ cal g-mole}^{-1}\ {}^\circ\text{K}^{-1}$$

$$\mathcal{D}_{ABf} = 0.292\left(\frac{540}{565}\right)^{2.334} \quad \text{from Ex. 21.1-2 and Eq. 16.3-1}$$

$$\left(\frac{\tilde{C}_p \mu}{k}\right)_f = 0.737$$

$$\left(\frac{\mu}{\rho \mathcal{D}_{AB}}\right)_f = 0.605$$

The densities of the drop (at 60°F) and surrounding air (at 100°F) are:

$$\rho_{sph} = 0.998 \text{ g cm}^{-3} \quad (\text{we consider the drop a rigid sphere})$$

$$\rho_\infty = 1.14 \times 10^{-3} \text{ g cm}^{-3}$$

(a) A momentum balance on the drop, in the direction opposed to gravity, gives the following result under pseudo-steady-state conditions:

$$0 = F_{z,\text{buoyant}} + F_{z,\text{drag}} + m_{tot}\, g_z$$

$$= \rho_\infty \tfrac{4}{3}\pi R^3 g + \pi R^2\left(\tfrac{1}{2}\rho_f v_\infty^2\right) f + \rho_{sph}\tfrac{4}{3}\pi R^3(-g)$$

We use ρ_∞ to compute the buoyant force because the pressure gradient, which results in the buoyant force, is governed by ρ_∞. We use ρ_f in computing $F_{z,\text{drag}} = F_K$ (see Fig. 13.3-1); this choice is arbitrary and has little effect here. Hence

$$v_\infty = \sqrt{\frac{4D(\rho_{sph}-\rho)g}{3\rho_f f}}$$ is the air velocity relative to the sphere, at $r = \infty$

Here f may be regarded as a function of the unknown quantity $Re = Dv_\infty \rho_f/\mu_f$, or, more conveniently, a function of the known quantity (compare Ex. 6.2-2 and Problem 6.E):

$$Re\sqrt{f} = \frac{D\rho_f}{\mu_f}\sqrt{\frac{4D(\rho_{sph}-\rho)}{3\rho_f}}$$

$$= \frac{(0.1)(1.18\times10^{-3})}{1.84\times10^{-4}}\sqrt{\frac{4(0.1)(0.998-0.00114)(980.7)}{3(1.18\times10^{-3})}}$$

$$=(0.638)\sqrt{1.11\times10^{5}} = 212.5$$

Plotting the equation $Re\sqrt{f} = 212.5$ on Fig. 6.2-1 we get a straight line of slope -2 which intersects the curve $f = f(Re)$ at $Re = 250$. Hence

$$v_\infty = \frac{Re\,\mu_f}{D\rho_f} = \frac{250}{0.638} = 390 \text{ cm sec}^{-1}$$

(b) From Eq. 13.3-1 and its mass-transfer analog we get (see also Eqs. 21.2-24 and 25):

$$\frac{h_m D}{k_f} = 2.0 + 0.60(250)^{1/2}(0.737)^{1/3} = 10.56$$

$$\frac{k_{xm} D}{(c\mathcal{D}_{AB})_f} = 2.0 + 0.60(250)^{1/2}(0.605)^{1/3} = 10.03$$

$$\frac{h_m}{k_{xm}} = \frac{10.56}{10.03}\frac{k_f}{(c\mathcal{D}_{AB})_f} = \frac{10.56}{10.03}\left(\frac{\tilde{C}_p Sc}{Pr}\right)_f = 6.03 \text{ cal g-mole}^{-1}\,{}^\circ K^{-1}$$

or

Btu lb-mole^{-1} °F^{-1}

Substitution in Eq. 21.2-31 gives, if $\Delta\tilde{H}_{A,vap}$ is evaluated at 60°F,

$$\frac{x_{A0} - 0}{(100-T_0)(1-x_{A0})} = \frac{6.03}{19{,}080} = 3.16\times10^{-4}\ {}^\circ F^{-1}$$

from which

$$x_{A0} = \frac{(3.16\times10^{-4})(100-T_0)}{1+(3.16\times10^{-4})(100-T_0)}$$

Equation 21.2-37 gives a second relation between x_{A0} and T_0 since $p_{A,vap}$ is a known function of the temperature. A tabular trial-and-error solution of these two simultaneous equations is convenient:

Assumed T_0	$(x_{A0})_1$ from $(100-T_0)$	$(x_{A0})_2 = p_{A,vap}/p$
60	0.0125	0.0174
50	0.0156	0.0121
54	0.0143	0.0140

The last trial gives sufficiently close agreement between $(x_{A0})_1$ and $(x_{A0})_2$, in view of the uncertainties of the various correlations and properties involved. Hence the droplet temperature T_0 is rounded off to 54°F.

(c) An unsteady-state mass balance on the drop gives:

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3 c_{A,liq.}\right) = \mathcal{W}_A^{(m)}$$

Combining this with Eq. 21.2-26 we get, for evaporation of a drop of pure A with constant density:

$$4\pi R^2 c_{A,liq.}\frac{dR}{dt} = k_{xm}\pi D^2 \frac{x_{A0}-x_{A\infty}}{1-x_{A0}}$$

and since $R = D/2$,

$$2\frac{dR}{dt} = \frac{dD}{dt} = \frac{2k_{xm}}{c_{A,liq.}}\ \frac{x_{A0}-x_{A\infty}}{1-x_{A0}}$$

From parts (a) and (b) we find

$$k_{xm} = 10.03\frac{(c\mathcal{D}_{AB})_f}{D} = \frac{(10.03)(4.06\times10^{-5})(0.259)}{(0.1)}$$

$$= 1.06\times10^{-3}\ \text{g-mole sec}^{-1}\text{cm}^{-2}$$

$$c_{A,liq.} = 0.998/18.02 = 0.0554\ \text{g-mole cm}^{-3}$$

Hence

$$\frac{dD}{dt} = \frac{(2)(1.06\times10^{-3})}{(0.0554)}\ \frac{(0.0143-0)}{(1-0.0143)} = 5.55\times10^{-4}\ \text{cm sec}^{-1}$$

21.F Effect of Radiation on Psychrometric Measurements

(a) An energy balance on unit area of the dry bulb gives, for steady state:

$$h_{db}(T_\infty - T_{db}) + \sigma a_{db}T_s^4 - \sigma e_{db}T_{db}^4 = 0$$

from which

$$T_\infty = T_{db} + \frac{\sigma}{h_{db}}\left[e_{db}T_{db}^4 - a_{db}T_s^4\right]$$

We consider here that h_{db} and h_{wb} are mean values for the two thermometers, rather than point values

(b) An energy balance on unit area of the wet bulb gives, for steady state and small mass transfer rate:

$$h_{wb}(T_\infty - T_{wb}) + \sigma(a_{wb}T_s^4 - e_{wb}T_{wb}^4) - N_{A0}\Delta\tilde{H}_{A,vap} = 0$$

In which N_{A0} is an average value for the wet bulb. Thus

$$N_{A0} = \frac{h_{wb}}{\Delta\tilde{H}_{A,vap}}(T_\infty - T_{wb}) + \frac{\sigma}{\Delta\tilde{H}_{A,vap}}\left[a_{wb}T_s^4 - e_{wb}T_{wb}^4\right]$$

(c) Combining the result of (b) with Eq. 21.2-30 gives:

$$\frac{x_{A0} - x_{A\infty}}{1 - x_{A0}} = \frac{h_{wb}}{k_{xm}\Delta\tilde{H}_{A,vap}}\left\{\begin{matrix}(T_\infty - T_{wb}) \\ + \frac{\sigma}{h_{wb}}\left[a_{wb}T_s^4 - e_{wb}T_{wb}^4\right]\end{matrix}\right\}$$

We first determine T_∞ from the result of (a). Assuming $T_f = 140°F$ around the dry bulb, we get

$$\rho_f = 0.0663 \ lb_m \ ft^{-3}$$

$$\mu_f = 0.01999 \ cp = 0.0483 \ lb_m ft^{-1} hr^{-1}$$

$$\hat{C}_{pf} = 0.241 \ Btu \ lb_m^{-1} \ °F^{-1}$$

$$(Pr)_f = 0.74 \quad \text{from Eq. 8.3-16}$$

$$Re = \frac{D v_\infty \rho_f}{\mu_f} = \frac{(0.1/12)(15\times 3600)(0.0663)}{(0.0483)} = 618$$

From Fig. 13.4-1, $j_H = 0.0225$; hence

$$h_{db} = j_H \rho_f \hat{C}_{pf} v_\infty (Pr)_f^{-2/3}$$

$$= (0.0225)(0.0663)(0.241)(15\times3600)(0.74)^{-2/3}$$

$$= 23.7 \ Btu \ hr^{-1} \ ft^{-2} \ °F^{-1}$$

Substitution in the result of (a) gives:

$$T_\infty = 140 + \frac{0.1712}{23.7}\left[0.93\left(\frac{600}{100}\right)^4 - 0.93\left(\frac{590}{100}\right)^4\right]$$

$$= 140 + \frac{(0.1712)(0.93)}{23.7}\left[1296 - 1212\right]$$

$$= 140.6 \ °F$$

Hence the assumed T_f for the dry bulb was satisfactory. For the wet-bulb thermometer the film properties given in Ex. 21.2-2 are still good because T_0 is the same and T_∞ is only slightly different. Thus, in c.g.s. units,

$$Re = \frac{(0.15\times2.54)(15\times30.48)(1.12\times10^{-3})}{(1.91\times10^{-4})} = 1020$$

$$j_H = 0.0175$$

$$h_{wb} = (0.0175)(1.12\times10^{-3})(0.241)(15\times30.48)(0.74)^{-2/3}$$
$$= 2.64 \times 10^{-3} \text{ cal sec}^{-1}\text{ cm}^{-2}\ {}^{o}\text{K}^{-1}$$
$$= 19.5 \text{ Btu hr}^{-1}\text{ ft}^{-2}\ {}^{o}\text{F}^{-1}$$

From Ex. 21.2-2 we get $x_{A0} = 0.0247$ and from Eq. 21.2-34,

$$\frac{h_{wb}}{k_{xm}\Delta\tilde{H}_{A,vap}} = \frac{\tilde{C}_{pA}}{\Delta\tilde{H}_{A,vap}}\left(\frac{Sc}{Pr}\right)_f^{2/3} = \frac{6.98}{18,900}\left(\frac{0.58}{0.74}\right)^{2/3}$$
$$= 3.14 \times 10^{-4}$$

Inserting these numerical values in the first equation of (c) gives:

$$\frac{0.0247 - x_{A\infty}}{1 - 0.0247} = (3.14\times10^{-4})\left\{\begin{array}{l}(140.6-70)\\ +\dfrac{(0.1712)}{19.5}\left[(0.93)\left(\dfrac{590}{100}\right)^4-(0.93)\left(\dfrac{530}{100}\right)^4\right]\end{array}\right\}$$

$$\frac{0.0247 - x_{A\infty}}{0.9753} = (3.14\times10^{-4})\{70.6 + 3.4\} = 0.0232$$

$$x_{A\infty} = 0.0247 - (0.9753)(0.0232)$$
$$= 0.0021 \quad \text{vs. } 0.0033 \text{ in Ex. 21.2-2}$$

The simplified method of Ex. 21.2-2 is noticeably in error under the conditions stated here.

21.6 Transfer of A and B across a Fluid-Fluid Interface

(a) For compactness we make the substitutions $N_{A\ell0} = d\mathcal{W}_{A\ell}^{(m)}/dA$, etc; then from Eq. 21.1-11 we get:

$$\frac{N_{A\ell0} - x_{A0}(N_{A\ell0} + N_{B\ell0})}{k_x} = x_{A0} - x_{Ab}$$

$$\frac{N_{Ag0} - y_{A0}(N_{Ag0} + N_{Bg0})}{k_y} = y_{A0} - y_{Ab}$$

Combining these equations and noting that $N_{A\ell0} = -N_{Ag0}$, $N_{B\ell0} = -N_{Bg0}$, we get

$$\frac{y_{Ab} - y_{Ab}}{x_{Ab} - x_{A0}} = -\frac{k_x}{k_y}\,\frac{N_{Ag0} - y_{A0}(N_{Ag0} + N_{Bg0})}{N_{Ag0} - x_{A0}(N_{Ag0} + N_{Bg0})}$$

$$= -\frac{k_x}{k_y}\,\frac{\left(\dfrac{N_{A0}}{N_{A0}+N_{B0}}\right) - y_{A0}}{\left(\dfrac{N_{A0}}{N_{A0}+N_{B0}}\right) - x_{A0}}$$

in which $\dfrac{N_{A0}}{N_{A0}+N_{B0}} = \dfrac{N_{Ag0}}{N_{Ag0}+N_{Bg0}} = \dfrac{N_{A\ell 0}}{N_{A\ell 0}+N_{B\ell 0}}$

(b) We first rearrange the definition of K_y to get:

$$\frac{N_{Ag0} - y_{Ae}(N_{Ag0}+N_{Bg0})}{K_y} = y_{Ae} - y_{Ab}$$

Assuming interfacial equilibrium as in § 21.3, we write

$$\begin{aligned} y_{Ae}-y_{Ab} &= (y_{Ae}-y_{A0}) + (y_{A0}-y_{Ab}) \\ &= m_y(x_{Ab}-x_{A0}) + (y_{A0}-y_{Ab}) \end{aligned}$$

Inserting the definitions of K_y, k_x and k_y, we get

$$\frac{N_{Ag0} - y_{Ae}(N_{Ag0}+N_{Bg0})}{K_y} = m_y \frac{N_{Ag0} - x_{A0}(N_{Ag0}+N_{Bg0})}{k_x} + \frac{N_{Ag0} - y_{A0}(N_{Ag0}+N_{Bg0})}{k_y}$$

If $N_{Ag0}+N_{Bg0}$ is non-zero, we can rewrite this in the neater form

$$\frac{\left(\dfrac{N_{A0}}{N_{A0}+N_{B0}}\right) - y_{Ae}}{K_y} = m_y \frac{\left(\dfrac{N_{A0}}{N_{A0}+N_{B0}}\right) - x_{A0}}{k_x} + \frac{\left(\dfrac{N_{A0}}{N_{A0}+N_{B0}}\right) - y_{A0}}{k_y}$$

(c) If $N_{Ag0} = -N_{Bg0}$, the last form in (b) is indeterminate; the general solution in (b) reduces to

$$\frac{1}{K_x} = \frac{m_y}{k_x} + \frac{1}{k_y}$$

21.H Interdependence of Mass Transfer Coefficients in a Multicomponent System

(a) Summation of Eq. 21.8-4 from $i=1$ to $i=n$ gives:

$$\sum_{i=1}^{n} N_{i0} - \left(\sum_{i=1}^{n} x_{i0}\right)\sum_{j=1}^{n} N_{j0} = \sum_{i=1}^{n} \overset{\bullet}{k}_{xi,loc}\,\Delta x_i$$

or

$$\sum_{i=1}^{n} \overset{\bullet}{k}_{xi,loc}\,\Delta x_i = \sum_{i=1}^{n} N_{i0} - \sum_{j=1}^{n} N_{j0} = 0$$

which verifies Eq. 21.H-1.

From Eq. 21.8-7, setting $x_{i0} - x_{i\infty} = \Delta x_i$, we get

$$\frac{\Delta x_i}{R_{im}} = \frac{\overset{\bullet}{k}_{xi,loc}\,\Delta x_i}{\sum_{j=1}^{n} N_{j0}}$$

Summation of this equation from $i=1$ to $i=n$, and use of Eq. 21.H-1, gives

$$\sum_{i=1}^{n} \frac{\Delta x_i}{R_{im}} = \frac{\sum_{i=1}^{n} \overset{\bullet}{k}_{xi,loc}\,\Delta x_i}{\sum_{j=1}^{n} N_{j0}} = 0$$

which verifies Eq. 21.H-2.

From Eq. 21.8-10 we get:

$$\phi_{im} k_{xi,loc} = \sum_{j=1}^{n} N_{j0}$$

Since the right-hand side is clearly independent of i, Eq. 21.H-3 follows directly.

(b) From Table 21.8-1 we get the following check values to compare with the above identities:

$$\sum_{i=1}^{n} \overset{\bullet}{k}_{xi,loc}\,\Delta x_i = \sum_{i=1}^{4} k_{xi,loc} \frac{\phi_{im}}{R_{im}} \Delta x_i$$

$$= 3.58\left(\frac{0.123}{0.116}\right)(0.069 - 0.100)$$

$$+ 7.66\left(\frac{0.058}{0.056}\right)(0.788 - 0.800)$$

$$+ 4.30\left(\frac{0.103}{0.098}\right)(0.092 - 0.050)$$

$$+ 15.5\left(\frac{0.028}{0.028}\right)(0.051 - 0.050)$$

$$= -0.118 - 0.095 + 0.190 + 0.016$$

$$= -0.0007$$

$$\sum_{i=1}^{4} \frac{\Delta x_i}{R_{im}} = \frac{0.069 - 0.100}{-0.116} + \frac{0.788 - 0.800}{-0.056} + \frac{0.092 - 0.050}{-0.098} + \frac{0.051 - 0.050}{-0.028}$$

$$= 0.267 + 0.214 - 0.429 - 0.036$$

$$= 0.016$$

$\phi_{im} k_{xi}$ = 0.440 for species 1
0.444 for species 2
0.442 for species 3
0.43 for species 4

All of these checks are within the expected roundoff error. Equation 21.H-3 is automatically fulfilled here, but may provide a more informative check in problems where the mass transfer rates are unknown.

21.I Diffusion-Controlled Combustion in a Fixed Bed

(a) The fluxes of the various species into the gas at the external surface of a catalyst particle are:

$$N_{O_2} = N_{10} \quad \text{(a negative quantity ; see below)}$$
$$N_{N_2} = 0$$
$$N_{C_{10}H_8} = 0$$
$$N_{CO_2} = -\frac{10}{12} N_{10}$$
$$N_{H_2O} = -\frac{4}{12} N_{10}$$
$$\sum_{j=1}^{5} N_{j0} = -\frac{2}{12} N_{10}$$

The maximum possible mass flux of O_2 occurs when the mole fraction O_2 at the outer particle surface is zero. Under these conditions Eq. 21.8-7 gives:

$$R_{1m} = \frac{x_{10} - x_{1b}}{\frac{N_{10}}{\sum_j N_{j0}} - x_{10}} = \frac{0 - 0.010}{\frac{N_{10}}{\left(-\frac{2}{12} N_{10}\right)} - 0} = \frac{-0.010}{-6} = 0.00167$$

This small value of R_{1m} justifies the neglect of the mass-transfer correction θ_{1m} in this problem. Hence we set $\overset{\bullet}{k}_{xi,loc} \doteq k_{xi,loc}$ and Eq. 21.8-4 becomes:

$$N_{10} - x_{10}\left(-\frac{2}{12} N_{10}\right) \doteq k_{x1,loc} \Delta x_1$$

Or

$$(N_{10})_{max} \doteq \frac{k_{x1,loc}(x_{10} - x_{1b})}{1 + \frac{2}{12} x_{10}} = -k_{x1,loc} x_{1b}$$

The negative value of N_{10} indicates that this flux is directed out of the gas phase.

(b) Equating the rate of heat liberation per unit particle surface by the chemical reaction to the rate of conductive heat loss to the gas phase, we get

$$q_0^{(c)} = N_{10} \Delta \tilde{H}_1 = \overset{\bullet}{h}_{loc}(T_0 - T_b)$$

or

$$N_{10} \doteq \frac{h_{loc}(T_0 - T_b)}{\Delta \tilde{H}_1}$$

(c) Combining the results of (a) and (b) gives

$$(T_0 - T_b)_{max} \doteq \frac{-\Delta\tilde{H}_1 \, k_{x1,loc}}{h_{loc}} \, x_{1b}$$

Using the analogy $j_H = j_D$, we get

$$\frac{k_{x1,loc}}{h_{loc}} = \frac{j_D \, c_b v_0}{j_H \, \rho_b \hat{C}_{pb} v_0} \, \frac{(Sc)_f^{-2/3}}{(Pr)_f^{-2/3}} = \frac{1}{\tilde{C}_{pb}} \left(\frac{Sc_{1m}}{Pr} \right)_f^{-2/3}$$

Hence

$$(T_0 - T_b)_{max} = - \frac{\Delta\tilde{H}_1}{\tilde{C}_{pb}} \left(\frac{Pr}{Sc_{1m}} \right)_f^{2/3} x_{1b}$$

(d) Numerical substitution gives

$$(T_0 - T_b)_{max} = - \frac{-100{,}000}{7.54} \left(\frac{0.7}{0.8} \right)^{2/3} (0.01)$$

$$= 121\,°K = 220\,°F$$

$$T_{0,max} = 1000 + 220 = 1220\,°F$$

21.J Catalyst Temperature in a Multicomponent Reaction

An energy balance on unit area of catalyst surface gives:

$$N_{30} \, \Delta\tilde{H}_3 = h^{\circ} (T_b - T_0)$$

The uncorrected heat transfer coefficient, h, is given by Eq. 13.4-4, with numerical values from Ex. 21.8-1:

$$h = j_H \hat{C}_p G_0 (Pr)_f^{-2/3}$$

$$= (0.111)(11.7/14.43)(600)(0.60)^{-2/3} = 75.8 \text{ Btu hr}^{-1}\text{ft}^{-2}\,°\text{F}^{-1}$$

If h is used for h°, then

$$(T_b - T_0) \doteq \frac{N_{30} \, \Delta\tilde{H}_3}{h} = \frac{(0.147)(-51{,}000 \times 1.8)}{75.8} = -178\,°F$$

$$= -99\,°K$$

Hence $T_0 \doteq 500 + 100 = 600\,°K$.

A more definitive answer can be obtained by determining $\Delta\tilde{H}_3$ at 600°K and correcting the heat transfer coefficient for mass transfer:

$$(\Delta\tilde{H}_3 \text{ at } 600\,°K) = -52{,}281 \text{ cal g-mole}^{-1}$$

$$\phi_T = \frac{\Sigma_j N_{j0}\tilde{C}_{pi}}{h} = \frac{(-0.147)(35.27)+(-0.441)(7.000)+(0.147)(49.65)}{75.8} = -0.0128$$

Here the $\tilde{C}_{pj}$ have been evaluated at 650°K. The correction factor θ_T is estimated from Eq. 21.5-48:

$$\theta_T = \frac{\phi}{e^{\phi}-1} = \frac{-0.0128}{e^{-0.0128}-1} = 1.0064$$

Then the second approximation to the catalyst temperature is:

$$(T_b - T_o) = \frac{N_{30}\,\Delta\tilde{H}_3}{\theta_T h} = \frac{(0.147)(-52{,}281\times 1.8)}{(1.0064)(75.8)} = -181\,°F$$
$$= -101\,°K$$

$$T_o = 500 + 101 = 601\,°K$$

This is the temperature of the exterior surface of the catalyst particles; higher temperatures will prevail inside the particles.

21.K Film Theory for Spheres

We consider a sphere of radius R surrounded by a film of thickness δ_T for heat transfer, and a film of thickness δ_{AB} for diffusion. The following boundary conditions are assumed:

$$\text{at } r = R,\quad T = T_o \qquad\qquad \text{at } r \geq R+\delta_T,\quad T = T_\infty$$
$$x_A = x_{Ao} \qquad\qquad \text{at } r \geq R+\delta_{AB},\quad x_A = x_{A\infty}$$
$$N_{Ar}/N_{Br} = N_{Ao}/N_{BO}$$

Using Eq. 18.3-12, with $e_\theta = e_\phi = 0$ and $\underset{\sim}{g}_i$ neglected, we get:

$$(\nabla\cdot\underset{\sim}{e}) = \frac{1}{r^2}\frac{d}{dr}(r^2 e_r) = 0$$

Using Eq. 18.1-10, with $N_{i\theta} = N_{i\phi} = 0$ and $R_i = 0$, we get:

$$(\nabla\cdot\underset{\sim}{N}_A) = \frac{1}{r^2}\frac{d}{dr}(r^2 N_{Ar}) = 0$$
$$(\nabla\cdot\underset{\sim}{N}_B) = \frac{1}{r^2}\frac{d}{dr}(r^2 N_{Br}) = 0$$

Integration, and use of Eqs. 18.4-6 and 16.2-2, gives

$$r^2 e_r = r^2 N_{Ar}\tilde{H}_A + r^2 N_{Br}\tilde{H}_B - r^2 k\frac{dT}{dr} = C_1$$
$$r^2 N_{Ar} = r^2 N_{Ar} x_A + r^2 N_{Br} x_A - r^2 c\mathcal{D}_{AB}\frac{dx_A}{dr} = C_2$$
$$r^2 N_{Br} = C_3$$

from which

$$r^2 e_r\Big|_R^r = r^2 N_{Ar}(\tilde{H}_A - \tilde{H}_{Ao}) + r^2 N_{Br}(\tilde{N}_B - \tilde{H}_{BO}) - \left(r^2 k\frac{dT}{dr}\right)\Big|_R^r = 0$$
$$r^2 N_{Ar}\Big|_R^r = r^2 N_{Ar}(x_A - x_{Ao}) + r^2 N_{Br}(x_A - x_{AO}) - \left(r^2 c\mathcal{D}_{AB}\frac{dx_A}{dr}\right)\Big|_R^r = 0$$

Setting $\tilde{H}_A - \tilde{H}_{A0} = \tilde{C}_{pA}(T-T_0)$ and $\tilde{H}_B - \tilde{H}_{B0} = \tilde{C}_{pB}(T-T_0)$ we get

$$(T-T_0)\left[C_2\tilde{C}_{pA} + C_3\tilde{C}_{pB}\right] - r^2 k \frac{dT}{dr} = R^2 q_0$$

$$(x_A - x_{A0})\left[C_2 + C_3\right] \qquad - r^2 c\mathcal{D}_{AB}\frac{dx_A}{dr} = R^2 J_{A0}^{\star}$$

Separation of variables and integration gives:

$$\int_{T_0}^{T} \frac{dT}{\left[C_2\tilde{C}_{pA} + C_3\tilde{C}_{pB}\right](T-T_0) - R^2 q_0} = \int_R^r \frac{dr}{kr^2}$$

$$\int_{x_{A0}}^{x_A} \frac{dx_A}{\left[C_2 + C_3\right](x_A - x_{A0}) - R_1^2 J_{A0}^{\star}} = \int_R^r \frac{dr}{r^2 c\mathcal{D}_{AB}}$$

in which two of the boundary conditions have been used. For constant k, $c\mathcal{D}_{AB}$, $\tilde{C}_{pA}$ and $\tilde{C}_{pB}$, we get:

$$\ln\left\{\frac{\left[C_2\tilde{C}_{pA} + C_3\tilde{C}_{pB}\right](T-T_0) - R^2 q_0}{-R^2 q_0}\right\} = \frac{\left[C_2\tilde{C}_{pA} + C_3\tilde{C}_{pB}\right]}{k}\left(\frac{1}{R} - \frac{1}{r}\right)$$

$$\ln\left\{\frac{\left[C_2 + C_3\right](x_A - x_{A0}) - R^2 J_{A0}^{\star}}{-R^2 J_{A0}^{\star}}\right\} = \frac{C_2 + C_3}{c\mathcal{D}_{AB}}\left(\frac{1}{R} - \frac{1}{r}\right)$$

Rearranging and evaluating C_2 and C_3 in terms of the mass fluxes at $r=R$, we get:

$$1 - \frac{(T-T_0)(N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB})}{q_0} = \exp\left[\frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{k}\left(R - \frac{R^2}{r}\right)\right]$$

$$1 - \frac{(x_A - x_{A0})(N_{A0} + N_{B0})}{N_{A0} - x_{A0}(N_{A0} + N_{B0})} = \exp\left[\frac{N_{A0} + N_{B0}}{c\mathcal{D}_{AB}}\left(R - \frac{R^2}{r}\right)\right]$$

in place of Eqs. 21.5-20 and 21.

Applying these equations at $r = R + \delta$ and using the boundary conditions there we get:

$$1 + \frac{(T_0 - T_\infty)(N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB})}{q_0} = \exp\left[\frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{k}\left(R - \frac{R^2}{R+\delta_T}\right)\right]$$

$$1 + \frac{(x_{A0} - x_{A\infty})(N_{A0} + N_{B0})}{N_{A0} - x_{A0}(N_{A0} + N_{B0})} = \exp\left[\frac{N_{A0} + N_{B0}}{c\mathcal{D}_{AB}}\left(R^2 - \frac{R^2}{R+\delta_{AB}}\right)\right]$$

in place of Eqs. 21.5-23 and 24. Inserting local transfer coefficients we get:

$$1 + \frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{h^{\bullet}_{loc}} = \exp\left[\frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{k}\left(R - \frac{R^2}{R+\delta_T}\right)\right]$$

$$1 + \frac{N_{A0} + N_{B0}}{k^{\bullet}_{x,loc}} = \exp\left[\frac{N_{A0} + N_{B0}}{c\mathcal{D}_{AB}}\left(R - \frac{R^2}{R+\delta_{AB}}\right)\right]$$

In the limit as N_{A0} and N_{B0} approach zero, these equations yield the following expressions for the film thicknesses:

$$\frac{1}{h_{loc}} = \frac{1}{k}\left(R - \frac{R^2}{R+\delta_T}\right)$$

$$\frac{1}{k_{x,loc}} = \frac{1}{c\mathcal{D}_{AB}}\left(R - \frac{R^2}{R+\delta_{AB}}\right)$$

Inserting these results in the solutions for the fluxes given earlier, and assuming δ_T and δ_{AB} independent of mass transfer rate, we get:

$$1 + \frac{(T_0 - T_\infty)(N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB})}{q_0} = \exp\left[\frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{h_{loc}}\right]$$

$$1 + \frac{x_{A0} - x_{A\infty}}{\left(\frac{N_{A0}}{N_{A0}+N_{B0}}\right) - x_{A0}} = \exp\left[\frac{N_{A0}+N_{B0}}{k_{x,loc}}\right]$$

and these results are identical with Eqs. 21.5-35 and 36. In the special case of no flow past the sphere ($v_\infty = 0$ and $v_\theta = v_\phi = 0$) these solutions are exact and the films extend to $r = \infty$; if there is flow past the sphere then the profiles of T and x_A are distorted and these solutions are only approximate.

Extension of the film theory to calculate drag forces on spheres would involve many difficulties; in particular the velocity could not be treated as a function of r alone (especially when there is flow separation).

21.L Film Theory for Cylinders

We consider a cylinder of radius R and infinite length, placed either parallel or perpendicular to the approaching stream which is otherwise unbounded. The cylinder is assumed to be surrounded by a film of thickness δ_T for heat transfer, and a film of thickness δ_{AB} for diffusion. The boundary conditions of Problem 21.K are used (note, however, the difference in the meaning of r and R).

From Eqs. 18.3-12 and 18.1-10, we obtain (for radially-directed fluxes and $\underset{\sim}{g}_i = R_i = 0$):

$$(\nabla \cdot \underset{\sim}{e}) = \frac{1}{r}\frac{d}{dr}(re_r) = 0$$

$$(\nabla \cdot \underset{\sim}{N}_A) = \frac{1}{r}\frac{d}{dr}(rN_{Ar}) = 0$$

$$(\nabla \cdot \underset{\sim}{N}_B) = \frac{1}{r}\frac{d}{dr}(rN_{Br}) = 0$$

Integration, and use of Eqs. 18.4-6 and 16.2-2, gives

$$re_r = rN_{Ar}\tilde{H}_A + rN_{Br}\tilde{H}_B - rk\frac{dT}{dr} = C_1$$

$$rN_{Ar} = rN_{Ar}x_A + rN_{Br}x_A - rc\mathcal{D}_{AB}\frac{dx_A}{dr} = C_2$$

$$rN_{Br} = C_3$$

from which

$$re_r\Big|_R^r = rN_{Ar}(\tilde{H}_A-\tilde{H}_{A0}) + rN_{Br}(\tilde{H}_B-\tilde{H}_{B0}) - \left(rk\frac{dT}{dr}\right)\Big|_R^r = 0$$

$$rN_{Ar}\Big|_R^r = rN_{Ar}(x_A-x_{A0}) + rN_{Br}(x_A-x_{A0}) - \left(rc\mathcal{D}_{AB}\frac{dx_A}{dr}\right)\Big|_R^r = 0$$

Setting $\tilde{H}_A-\tilde{H}_{A0}=\tilde{C}_{pA}(T-T_0)$ and $\tilde{H}_B-\tilde{H}_{B0}=\tilde{C}_{pB}(T-T_0)$ (for assumptions see § 21.5) we get:

$$(T-T_0)\left[C_2\tilde{C}_{pA} + C_3\tilde{C}_{pB}\right] - rk\frac{dT}{dr} = Rq_0$$

$$(x_A-x_{A0})\left[\,C_2 + C_3\,\right] - rc\mathcal{D}_{AB}\frac{dx_A}{dr} = RJ^*_{A0}$$

Integration, and use of the boundary conditions on T and x_A at the surface $r=R$, gives:

$$\int_{T_0}^{T}\frac{dT}{[C_2\tilde{C}_{pA}+C_3\tilde{C}_{pB}](T-T_0)-Rq_0} = \int_R^r\frac{dr}{kr}$$

$$\int_{x_{A0}}^{x_A}\frac{dx_A}{[C_2+C_3](x_A-x_{A0})-RJ^*_{A0}} = \int_R^r\frac{dr}{rc\mathcal{D}_{AB}}$$

For constant k, $c\mathcal{D}_{AB}$, $\tilde{C}_{pA}$ and $\tilde{C}_{pB}$ we then get:

$$\ln\left\{\frac{[C_2\tilde{C}_{pA}+C_3\tilde{C}_{pB}](T-T_0)-Rq_0}{-Rq_0}\right\} = \frac{C_2\tilde{C}_{pA}+C_3\tilde{C}_{pB}}{k}\ln\frac{r}{R}$$

$$\ln\left\{\frac{[C_2+C_3](x_A-x_{A0})-RJ^*_{A0}}{-RJ^*_{A0}}\right\} = \frac{C_2+C_3}{c\mathcal{D}_{AB}}\ln\frac{r}{R}$$

Rearranging and evaluating the constants in terms of N_{A0} and N_{B0}, we get:

$$1-\frac{(T-T_0)[N_{A0}\tilde{C}_{pA}+N_{B0}\tilde{C}_{pB}]}{q_0} = \exp\left[\frac{RN_{A0}\tilde{C}_{pA}+RN_{B0}\tilde{C}_{pB}}{k}\ln\frac{r}{R}\right]$$

$$1-\frac{(x_A-x_{A0})(N_{A0}+N_{B0})}{N_{A0}-x_{A0}(N_{A0}+N_{B0})} = \exp\left[\frac{N_{A0}+N_{B0}}{c\mathcal{D}_{AB}}\ln\frac{r}{R}\right]$$

in place of Eqs. 21.5-20 and 21. Application of these equations at $r=R+\delta$ and insertion of the boundary conditions then gives:

$$1 + \frac{(T_0 - T_\infty)(N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB})}{q_0} = \exp\left[\frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{k} R \ln \frac{R+\delta_T}{R}\right]$$

$$1 + \frac{(x_{A0} - x_{A\infty})(N_{A0} + N_{B0})}{N_{A0} - x_{A0}(N_{A0} + N_{B0})} = \exp\left[\frac{N_{A0} + N_{B0}}{c\mathcal{D}_{AB}} R \ln \frac{R+\delta_{AB}}{R}\right]$$

in place of Eqs. 21.5-23 and 24. Inserting the local transfer coefficients defined in Eqs. 21.5-26 and 27, we get:

$$1 + \frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{h^{\bullet}_{loc}} = \exp\left[\frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{k} R \ln \frac{R+\delta_T}{R}\right]$$

$$1 + \frac{N_{A0} + N_{B0}}{k^{\bullet}_{x,loc}} = \exp\left[\frac{N_{A0} + N_{B0}}{c\mathcal{D}_{AB}} R \ln \frac{R+\delta_{AB}}{R}\right]$$

In the limit as N_{A0} and N_{B0} approach zero, these equations yield the following expressions for the film thicknesses:

$$\frac{1}{h_{loc}} = \frac{R}{k} \ln \frac{R+\delta_T}{R}$$

$$\frac{1}{k_{x,loc}} = \frac{R}{c\mathcal{D}_{AB}} \ln \frac{R+\delta_{AB}}{R}$$

Inserting these results in the above solutions for the fluxes, and assuming δ_T and δ_{AB} independent of mass transfer rate, we get:

$$1 + \frac{(T_0 - T_\infty)(N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB})}{q_0} = \exp\left[\frac{N_{A0}\tilde{C}_{pA} + N_{B0}\tilde{C}_{pB}}{h_{loc}}\right]$$

$$1 + \frac{x_{A0} - x_{A\infty}}{\left(\dfrac{N_{A0}}{N_{A0} + N_{B0}}\right) - x_{A0}} = \exp\left[\frac{N_{A0} + N_{B0}}{k_{x,loc}}\right]$$

and these results are identical with Eqs. 21.5-35 and 36. These solutions are presumed to be more accurate for parallel flow than for perpendicular flow. Note that the no-flow solution here, obtained by setting $\delta_T = \delta_{AB} = \infty$, is trivial: the fluxes vanish. For spheres there is a more interesting solution with $v_{A0} = 0$ (see Solution 21.K).

Chapter 22 - Checked by R.H. Weaver

22.A Expansion of a Gas Mixture: Very Slow Reaction Rate

We first compute the gas composition from Eqs. 21.5-37 through 40:

$$K_x = \frac{(x_{H_2O})(x_{CO})}{(x_{H_2})(x_{CO_2})} = \frac{x_{CO}^2}{x_{CO_2}^2} = \left(\frac{x_{CO}}{0.5-x_{CO}}\right)^2 \;; \quad x_{CO} = \frac{0.5\sqrt{K}}{1+\sqrt{K}}$$

Here $\sqrt{K} = 10^{-(0.15/2)} = 0.841$, hence $x_{CO} = x_{H_2O} = \frac{(0.5)(0.841)}{1.841} = 0.228$

$$x_{H_2} = x_{CO_2} = 0.5 - x_{CO} = 0.272$$

Then

$$\sum_{i=1}^{4} x_i M_i = 0.228\,(28.01) + 0.228\,(18.02) + 0.272\,(2.016) + 0.272\,(44.01) = 23.01$$

$$\sum_{i=1}^{4} x_i \tilde{C}_{pi} = 0.228\,(7.932) + 0.228\,(9.861) + 0.272\,(2.016) + 0.272\,(44.01) = 9.554\ \frac{\text{cal}}{\text{g-mole}\ {}^\circ K} = 9.554 \times 4.1840 \times 10^7\ \frac{\text{ergs}}{\text{g-mole}\ {}^\circ K}$$

Substitution in Eq. 22.5-35 gives:

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(R/\Sigma_i x_i \tilde{C}_{pi})} = (1000)\left(\frac{1.0}{1.5}\right)^{(1.987/9.554)} = 920^\circ K$$

Substitution in Eq. 22.5-36 gives:

$$v_2^2 = \left\{2T_1\left[1-\left(\frac{p_2}{p_1}\right)^{(R/\Sigma_i x_i \tilde{C}_{pi})}\right]\frac{\Sigma_i x_i \tilde{C}_{pi}}{\Sigma_i x_i M_i}\right\} = \left\{2\,(T_1 - T_2)\,\frac{\Sigma_i x_i \tilde{C}_{pi}}{\Sigma_i x_i M_i}\right\}$$

$$= \left\{2\,(1000-920)\,\frac{9.554 \times 4.1840 \times 10^7}{23.01}\right\} = 2.78 \times 10^9\ \text{cm}^2\,\text{sec}^{-2}$$

$$v_2 = \sqrt{27.8 \times 10^8} = 5.27 \times 10^4\ \text{cm sec}^{-1} = 1726\ \text{ft sec}^{-1}$$

The velocity of sound at exit temperature is (see Eq. 10.L-4):

$$v_{s2} = \sqrt{\gamma R T_2 / M} = \sqrt{\frac{\tilde{C}_p}{\tilde{C}_p - R}\,\frac{RT_2}{M}} = \sqrt{\frac{9.554}{9.554-1.987}\,\frac{(1.987)(920)}{23.01}\times 4.184 \times 10^7}$$

$$= \sqrt{42.0 \times 10^8} = 6.48 \times 10^4\ \text{cm sec}^{-1} = 2126\ \text{ft sec}^{-1}$$

The velocity v_2 is therefore subsonic, and the assumption that p_2 is equal to the ambient pressure is correct.

22.B Height of a Packed-Tower Absorber

(a) The data for calculation of the operating line are:

$$Y_{A1} = \frac{0.001}{1-0.001} = 0.00100 \quad Y_{A2} = \frac{0.010}{1-0.010} = 0.01010 \quad X_{A1} = \frac{0.003}{1-0.003} = 0.00301$$

$$\mathcal{W}_L = 20(1-0.003) = 19.94 \text{ lb-moles hr}^{-1}$$

$$\mathcal{W}_G = -\frac{(363)(60)}{359}(1.05)\left(\frac{273}{303}\right)(1-0.010) = -56.8 \text{ lb-moles hr}^{-1}$$

An overall cyclohexane balance on the tower gives (see Eq. 22.5-17)

$$X_{A2} = X_{A1} - (\mathcal{W}_G/\mathcal{W}_L)(Y_{A2}-Y_{A1})$$

$$= 0.00301 - (-56.8/19.94)(0.00010-0.00100) = 0.0289$$

The operating line (see Eq. 22.5-18) can now be plotted as a straight line from Y_{A1}, X_{A1} to Y_{A2}, X_{A2}. This line is shown in the adjoining graph.

(b) According to Raoult's law and the assumption of interfacial equilibrium,

$$y_{A0} = \frac{p_{A,vap}}{p} x_{A0} = \frac{121}{(760)(1.05)} x_{A0} = 0.1516\, x_{A0}$$

Coordinates computed from this equation are as follows:

Y_{A0}	$y_{A0} = \frac{Y_{A0}}{1+Y_{A0}}$	$x_{A0} = \frac{y_{A0}}{0.1516}$	$X_{A0} = \frac{x_{A0}}{1-x_{A0}}$
0	0	0	0
0.001	0.000999	0.00659	0.00663
0.002	0.001996	0.01316	0.01334
0.004	0.003984	0.02628	0.02699
0.006	0.005964	0.03933	0.04094
0.008	0.007937	0.05234	0.05523
0.010	0.009901	0.06530	0.06986

(c) From Eq. 22.5-21 (for dilute solutions) we get

$$\frac{Y_A - Y_{A0}}{X_A - X_{A0}} \doteq -\left(\frac{k_x a}{k_y a}\right) = -\frac{0.32}{14.2} = -0.0225$$

From this equation and the adjoining plot, the terminal interface compositions are estimated to be:

	Y_{A0}	X_{A0}
Top ("1")	0.00093	0.0061
Bottom ("2")	0.00930	0.0646

(d) From Eq. 22.5-24 the required tower height is

$$Z = -\frac{w_G}{S(k_y a)}\int_{Y_{A1}}^{Y_{A2}} \frac{dY}{(Y_A - Y_{A0})} = -\frac{(-56.8)}{(2.00)(14.2)}\left[31.1\right] = 62 \text{ ft.}$$

The calculation of the integral is summarized below.

Y_A	$X_{A0}-X_A$ (from graph)	$\frac{1}{Y_A - Y_{A0}}$ $= \frac{1}{0.0225(X_A - X_{A0})}$	Contributions to $\int_{Y_{A1}}^{Y_{A2}} \frac{dY_A}{Y_A - Y_{A0}}$
0.001	0.00313 (calc'd)	14,200	29.7 by Simpson's rule
0.002	0.0065	6,800	
0.003	0.0099	4,500	
0.004	0.0134	3,320	
0.005	0.0168	2,640	
0.006	0.0204	2,180	
0.007	0.0240	1,850	
0.008	0.0277	1,600	
0.009	0.0316	1,400	1.4 by to trap. rule
0.0101	0.0357	1,244	
			Total = 31.1

(e) From part (d) we get: $(Y_A - Y_{A0})_1 = 0.0225\,(0.00313) = 0.0000704$

$(Y_A - Y_{A0})_2 = 0.0225\,(0.0357) = 0.000803$

Hence $$(Y_A - Y_{A0})_{ln} = \frac{0.000803 - 0.0000704}{\ln \frac{0.000803}{0.0000704}} = \frac{0.000733}{\ln 11.1} = 0.000301$$

Then Eq. 22.5-25 gives

$$Z = \frac{w_G}{S(k_y a)}\,\frac{Y_{A2} - Y_{A1}}{(Y_{A0} - Y_A)_{ln}} = \frac{(-56.8)}{(2.00)(14.2)}\,\frac{(0.01010 - 0.00100)}{(-0.000301)}$$

$$= 60 \text{ ft.}$$

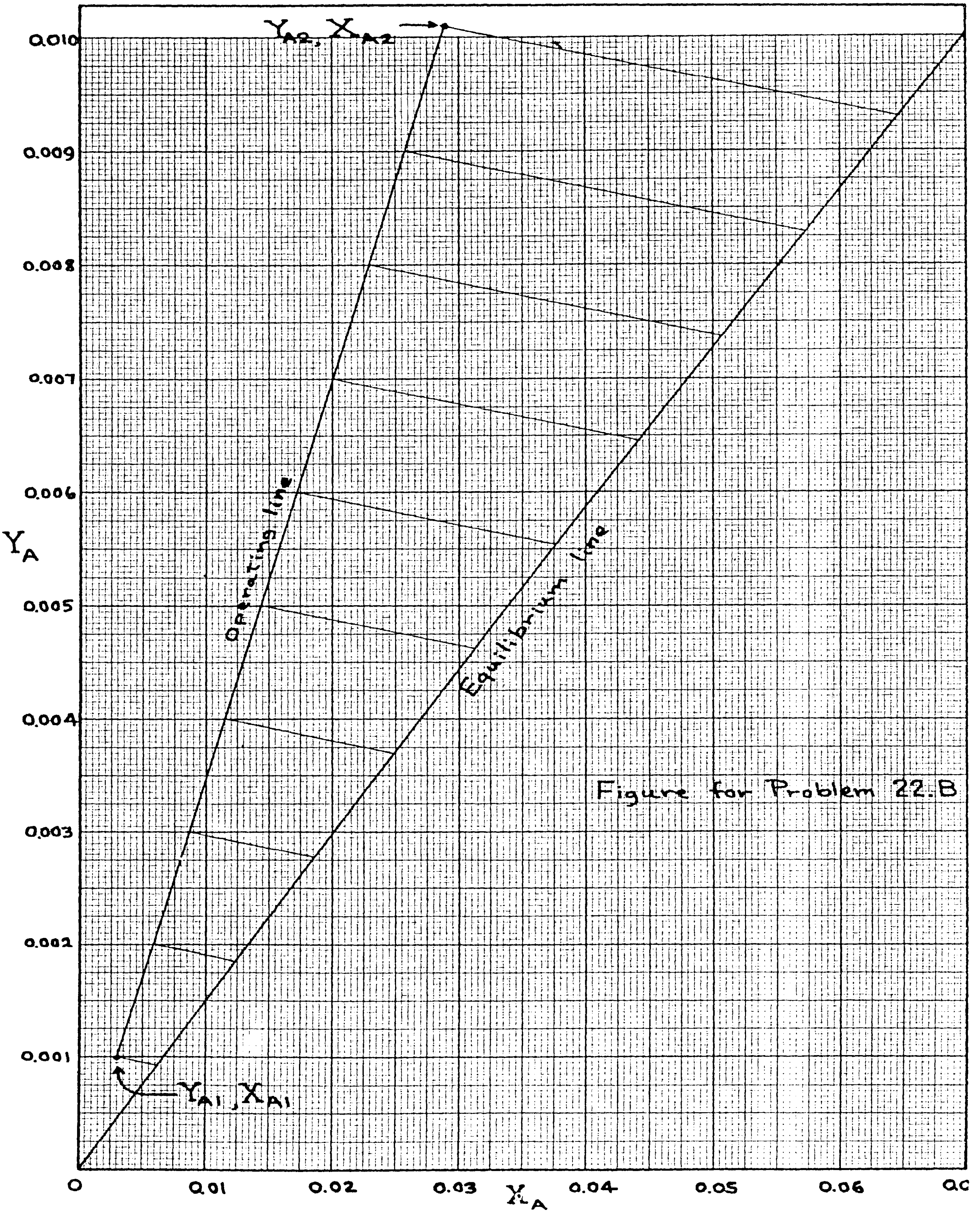


Figure for Problem 22.B

22.C Expansion of a Gas Mixture: Very Fast Reaction Rate

(a) The temperature dependence of $\tilde{H}$ under equilibrium conditions is computed as follows:

	900°K	950°K	1000°K
$\sqrt{K}$	0.676	0.754	0.841
$x_{H_2} = x_{CO_2} = 0.5/(1+\sqrt{K})$	0.298	0.285	0.272
$x_{CO} = x_{H_2O} = 0.5 - x_{H_2}$	0.202	0.215	0.228
$\tilde{H}_{H_2} = 6340 + 7.217(T-900)$	6340	6701	7062
$\tilde{H}_{CO_2} = -83{,}242 + 12.995(T-900)$	-83,242	-82,592	-81,942
$\tilde{H}_{CO} = -16{,}636 + 7.932(T-900)$	-16,636	-16,239	-15,843
$\tilde{H}_{H_2O} = -49{,}378 + 9.861(T-900)$	-49,378	-48,885	-48,392
$\tilde{H} = \sum_{i=1}^{4} x_i \tilde{H}_i$, cal g-mole^{-1}	-36,252	-35,631	-35,013
$\Delta\tilde{H}/\Delta T$, cal g-mole^{-1} °K^{-1}	12.42		12.36

The proposed expression $(d\tilde{H}/dT)_{avg} = 12.40$ is clearly adequate.

(b) From Eq. 22.5-41, which holds for constant $\Sigma_i x_i M_i$, we get

$$\ln \frac{T_2}{T_1} = \frac{1.987}{12.40} \ln \frac{1.0}{1.5} = -0.065$$

$$T_2 = 1000\, e^{-0.065} = 937\ \text{°K}$$

(c) To be consistent with (b) we do not interpolate for $\tilde{H}_2$, but set

$$\tilde{H}_2 - \tilde{H}_1 = 12.40\,(937 - 1000) = -781 \text{ cal g-mole}^{-1}$$

Hence

$$\hat{H}_1 - \hat{H}_2 = \frac{\tilde{H}_1 - \tilde{H}_2}{\Sigma_i x_i M_i} = \frac{(781)(4.1840 \times 10^7)}{(23.01)} = 1.42 \times 10^9 \ \frac{\text{ergs}}{\text{g}}$$
$$= 1.420 \times 10^9 \text{ cm}^2 \text{ sec}^{-2}$$

Substitution in Eq. 22.5-29 gives:

$$v_2 = \sqrt{2(\hat{H}_1 - \hat{H}_2)} = \sqrt{2 \times 1.420 \times 10^9} = 5.33 \times 10^4 \text{ cm sec}^{-1}$$
$$= 1748 \text{ ft sec}^{-1}$$

22.D Disposal of an Unstable Waste Material

We consider first the period $0 \le t \le V/Q$ during which the tank is filled. A molar balance on species A in the tank then gives

$$\begin{aligned}\frac{d}{dt} m_{A,tot} &= \mathcal{W}_{A1} - \mathcal{W}_{A2} + \mathcal{W}_A^{(m)} + R_{A,tot} \\ &= Qc_{A0} - 0 + 0 + (-k_1''' c_A Q t) \\ &= Qc_{A0} \qquad\qquad - k_1''' m_{A,tot}\end{aligned}$$

Integrating, with the initial condition that $m_{A,tot} = 0$ at $t=0$, we get:

$$\int_0^{m_{A,tot}} \frac{dm_{A,tot}}{Qc_{A0} - k_1''' m_{A,tot}} = \int_0^t dt$$

or

$$-\frac{1}{k_1'''} \ln\left[\frac{Qc_{A0} - k_1''' m_{A,tot}}{Qc_{A0}}\right] = t$$

or

$$m_{A,tot} = \frac{Qc_{A0}}{k_1'''}\left(1 - e^{-k_1''' t}\right)$$

Setting $m_{A,tot} = Qtc_A$, we get:

$$\frac{c_A}{c_{A0}} = \frac{1 - e^{-k_1''' t}}{k_1''' t} \qquad \left(t \le \frac{V}{Q}\right)$$

and the concentration at the instant the tank becomes full (here denoted as c_{Af}) is given by:

$$\frac{c_{Af}}{c_{A0}} = \frac{1 - e^{-k_1''' V/Q}}{(k_1''' V/Q)}$$

A molar balance on A after the tank is filled gives:

$$\frac{d}{dt}(c_A V) = Qc_{A0} - Qc_A + 0 - k_1''' c_A V$$

or

$$V\frac{dc_A}{dt} = Qc_{A0} - (Q + k_1''' V)c_A$$

Integrating, with the initial condition that $c_A = c_{Af}$ at $t = V/Q$, we get:

$$\int_{c_{Af}}^{c_A} \frac{V\,dc_A}{Qc_{A0} - (Q + k_1''' V)c_A} = \int_{V/Q}^{t} dt$$

or

$$-\left(\frac{V}{Q + k_1''' V}\right) \ln\left[\frac{Qc_{A0} - (Q + k_1''' V)c_A}{Qc_{A0} - (Q + k_1''' V)c_{Af}}\right] = t - \frac{V}{Q}$$

or

$$\frac{c_A - c_{A\infty}}{c_{Af} - c_{A\infty}} = \exp\left[-\left(\frac{Q + k_1''' V}{V}\right)\left(t - \frac{V}{Q}\right)\right]$$

in which $c_{A\infty} = \dfrac{Qc_{A0}}{Q + k_1''' V}$ is the asymptotic value of c_A as $t \to \infty$.

22.E Irreversible First-Order Reaction in a Continuous Reactor

A balance on species A in the reactor gives:

$$\frac{d}{dt}\mathcal{M}_{A,tot} = \mathcal{W}_{A1} - \mathcal{W}_{A2} - \mathcal{W}_A^{(m)} + \mathcal{R}_{A,tot}$$

for $t > 0$ the terms in this equation become:

$$\frac{d}{dt}(c_A V) = V\frac{dc_A}{dt} = Qc_{A0} - Qc_A - 0 - k_1''' c_A V$$

Integration of this equation, with the initial condition that $c_A = c_{A0}$ at $t = 0$, gives (compare Problem 22.D):

$$\int_{c_{A0}}^{c_A} \frac{V\,dc_A}{Qc_{A0} - (Q + k_1'''V)c_A} = \int_0^t dt$$

from which we get

$$\left(c_A - \frac{Qc_{A0}}{Q + k_1'''V}\right) = \left(c_{A0} - \frac{Qc_{A0}}{Q + k_1'''V}\right)\exp\left[-\left(\frac{Q + k_1'''V}{V}\right)t\right]$$

or

$$\frac{c_A}{c_{A0}} = \frac{Q}{Q + k_1'''V} + \left(1 - \frac{Q}{Q + k_1'''V}\right)\exp\left[-\left(\frac{Q + k_1'''V}{V}\right)t\right]$$

22.F Effective Average Driving Forces in a Gas Absorber

(a) The assumptions of a straight equilibrium line and a straight operating line give:

$$Y_{Ae} = m_1 X_A + b_1 \quad (1)$$

$$Y_A = m_2 X_A + b_2 \quad (2)$$

Hence

$$(Y_A - Y_{Ae}) = (m_2 - m_1)X_A + b_2 - b_1$$

$$= \left(\frac{m_2 - m_1}{m_2}\right)(Y_A - b_2) + (b_2 - b_1)$$

$$= C_1 Y_A + C_2 \quad (3)$$

where $C_1 = \frac{m_2 - m_1}{m_2}$ and $C_2 = b_2 - b_1 - b_2 C_1$. Q.E.D.

(b) Assuming interfacial equilibrium and using Eq. (1) we get:

$$Y_{A0} = m_1 X_{A0} + b_1 \quad (4)$$

Assuming $(k_x a / k_y a)$ = constant and using Eq. 22.5-21 (for dilute solutions) we get:

$$Y_A - Y_{A0} = m_3(X_A - X_{A0}) \quad (5)$$

where $m_3 = -k_x a / k_y a$.

Inserting Eqs. (2) and (4) on the right side of (5) we get:

$$Y_A - Y_{A0} = m_3\left[\frac{Y_A - b_2}{m_2} - \frac{Y_{A0} - b_1}{m_1}\right] \qquad (6)$$

from which

$$Y_A\left(1 - \frac{m_3}{m_2}\right) = Y_{A0}\left(1 - \frac{m_3}{m_1}\right) + m_3\left[\frac{b_1}{m_1} - \frac{b_2}{m_2}\right] \qquad (7)$$

and

$$Y_{A0} = Y_A \frac{\left(1 - \frac{m_3}{m_2}\right)}{\left(1 - \frac{m_3}{m_1}\right)} + \frac{m_3}{\left(1 - \frac{m_3}{m_1}\right)}\left[\frac{b_1}{m_1} - \frac{b_2}{m_2}\right] \qquad (8)$$

Hence

$$(Y_A - Y_{A0}) = Y_A\left[1 - \frac{\left(1 - \frac{m_3}{m_2}\right)}{\left(1 - \frac{m_3}{m_1}\right)}\right] - \frac{m_3}{\left(1 - \frac{m_3}{m_1}\right)}\left[\frac{b_1}{m_1} - \frac{b_2}{m_2}\right]$$

$$= C_3 Y_A + C_4 \qquad (9)$$

(c) Insertion of Eq. (9) in Eq. 22.5-4 gives:

$$Z = -\frac{\mathcal{W}_G}{S(k_y a)}\int_{Y_{A1}}^{Y_{A2}} \frac{dY_A}{C_3 Y_A + C_4}$$

If $C_3 \neq 0$ and if $C_3 Y_A + C_4$ is non-zero between Y_{A1} and Y_{A2}, this may be integrated to give:

$$Z = -\frac{1}{C_3}\frac{\mathcal{W}_G}{S(k_y a)} \ln \frac{C_3 Y_{A2} + C_4}{C_3 Y_{A1} + C_4}$$

Now, from Eq. (9),

$$C_3 Y_{A2} + C_4 = (Y_A - Y_{A0})_2$$

$$C_3 Y_{A1} + C_4 = (Y_A - Y_{A0})_1$$

$$C_3 = \frac{(Y_A - Y_{A0})_2 - (Y_A - Y_{A0})_1}{(Y_{A2} - Y_{A1})}$$

With these substitutions we get:

$$Z = -\frac{\mathcal{W}_G}{S(k_y a)} \frac{(Y_{A2} - Y_{A1})}{(Y_A - Y_{A0})_2 - (Y_A - Y_{A0})_1} \ln \frac{(Y_A - Y_{A0})_2}{(Y_A - Y_{A0})_1}$$

$$= -\frac{\mathcal{W}_G}{S(k_y a)} \frac{(Y_{A2} - Y_{A1})}{(Y_A - Y_{A0})_{\ln}}$$

(If $C_3 = 0$, then $Y_A - Y_{A0}$ is constant and the result here follows by direct integration of Eq. 22.5-4.) Setting

$\mathcal{W}_{Ag}^{(m)} = \mathcal{W}_G(Y_{A2}-Y_{A1})$ and rearranging, we get:

$$\mathcal{W}_{Ag}^{(m)} = (k_y a) Z S (Y_{A0}-Y_A)_{ln} \qquad (22.F-1)$$

For dilute mixtures Eq. 21.3-6 may be written (with $dA = a S dZ$):

$$d\mathcal{W}_{Ag}^{(m)} = (K_y a)(Y_{Ae}-Y_A) S dZ$$

Inserting this in Eq. 22.5-2 and integrating gives, for constant $(\mathcal{W}_G/SK_ya)$:

$$Z = -\frac{\mathcal{W}_G}{S(K_y a)}\int_{Y_{A1}}^{Y_{A2}} \frac{dY_A}{(Y_A - Y_{Ae})}$$

One can now insert the result of part (a) and derive Eq. 22.F-2 by parallelling the derivation of Eq. 22.F-1.

22.G Effect of Initial Solute Concentration on the Unsteady-State Operation of an Adsorption Column

In terms of the new dependent variables x_A' and c_A', Eqs. 22.6-15, 16, 17 and 18 become:

$$\left(\frac{\partial x_A'}{\partial z}\right)_{t'} = -\frac{(k_x a)S}{\mathcal{W}_B}(x_A' - x_{A0}') \qquad (22.6\text{-}15')$$

$$\left(\frac{\partial c_{AS}'}{\partial t'}\right)_{z} = \frac{(k_x a)}{(1-\epsilon)}(x_A' - x_{A0}') \qquad (22.6\text{-}16')$$

B.C. 1: at $t'=0$, $c_{AS}'=0$ for all $z>0$ $\qquad (22.6\text{-}17')$

B.C. 2: at $z=0$, $\left.\begin{array}{l} x_A' = x_{A1} - x_{Ai} \\ \quad = x_{A1}' \end{array}\right\}$ for all $t'>0$ $\qquad (22.6\text{-}18')$

The interphase equilibrium distribution function becomes

$$x_{A0}' = m c_{AS}'$$

These transformations are formally equivalent to a mere substitution of symbols: x_A' for x_A, c_{AS}' for c_{AS}, and similarly for subscripted values of these variables. Hence the remainder of the solution in Ex. 22.6-2 can be taken over to the present problem by the same substitutions.

Corrigenda to TRANSPORT PHENOMENA, first Printing (Sept 1960)

In reporting line: 12 a = line 12 from above (including equations)
12 b = line 12 from below (including footnotes)

Page	Location	Text incorrecty states:	Text should state
28	Eq. 1.5-6	dv_x/dy	$-dv_x/dy$
29	Line 14 a	n-$C_{20}H_{342}$	n-$C_{20}H_{42}$
31	Prob. 1.D	---	Delete "1021.6 ... and"
246	Line 13 a	heat	heat-transfer
261	Line 19 a	$cm^2 sec^{-1}$	$cm^2 sec^{-2}$
275	Eq. 9.3-2	$\vert_a$	$\vert_r$
344	Prob 10.D-soln	7.67	8.14
348	Eq. 10.Q-5	$= v_s^2 \nabla^2 \rho$	$= -v_s^2 \nabla \rho$
387	Prob. 12.A	Eq. 6.2-12	the definition of v_z following Eq. 6.2-12
422	Prob. 13.D-soln	6.25	−6.25
423	Prob. 13.J, b	10 to 1000	100 to 1000
485	Prob. 15.H, a, b	S_2	S_0
487	Prob. 15.I, d	1 lb-mole	10 lb-moles (also, see solution manual for corrected values of [illegible], T, and S
516	Qu. 4	J_i^* (light face)	J_i^* (bold face)
518	Prob. 16.K	---	All superscript [illegible] should be superscript [illegible]
577	Line 9b	N_{M+} (bold face)	N_{M+} (light face)
583	[illegible]	---	Should be added that p= 1 atm
		$\sum_{m=0}^{\infty} c_m p^m$	$\left(\sum_{m=0}^{\infty} c_m p^m\right)^{-1}$
		7.15	6.71
		1177 cm sec^{-1}	1512 cm sec^{-1}
		15 cm^2 sec^{-1}	19 cm^2 sec^{-1}
		64 ft	62 ft
		57 ft	60 ft
		−0.34	−0.34 [illegible]

Page	Location	Text incorrectly states:	Text should state:
81	Line 6a	$-\nabla p$	∇p
95	Eq. 3.5-12	$1 - \frac{1}{\kappa}$	$\kappa - \frac{1}{\kappa}$
115	Eq. 3.L-1	(Equation is missing)	$\tau_{xy} = \tau_{yx}$
133	Eq. 4.2-15	$r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}$	$\left[r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2$
302	Prob. 9.D_1	1.5 Btu hr^{-1} ft^{-1} $°F^{-1}$	1.5 Btu hr^{-1} ft^{-1} $°F^{-2}$
371	Prob. 11.C_1	0.77 cmt	0.77 cm
603	Line 2a	resect	respect