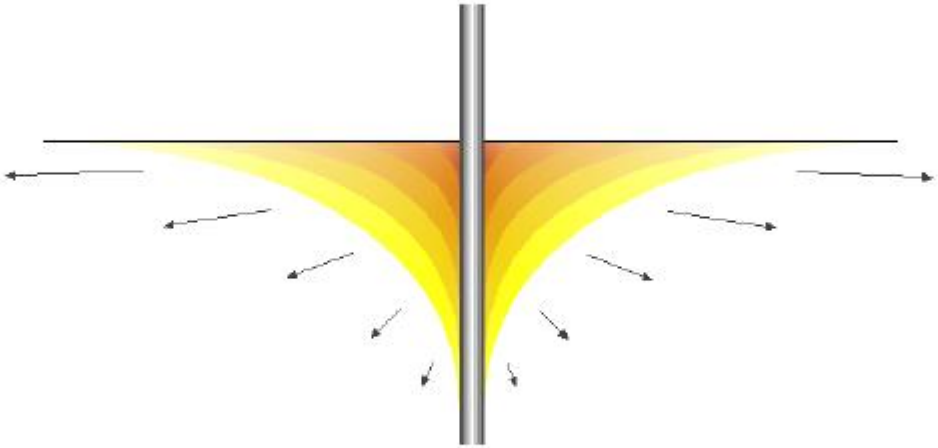
Advanced Pressure Transient Analysis



A Quick Review on Well Testing

By: Shahab Gerami

Outline

- Introduction
- Basic definitions and concepts
- Components of well test models
- Inverse and direct solutions
- Input data required for well test analysis

Introduction

What is a test?

Measurement of rate, time and pressure under controlled conditions.

Why test?

- Reservoir pressure
- Permeability
- Wellbore damage
- Deliverability
- Reservoir management
- Reservoir description
- Fluid samples
- Regulations

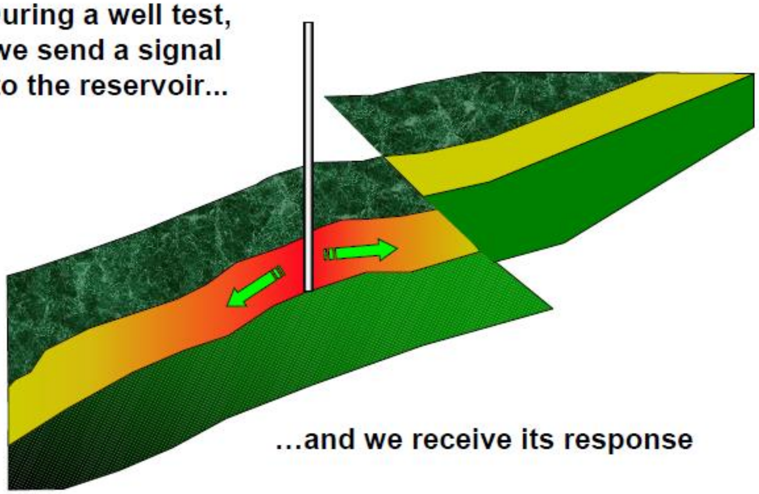
+ Well testing theory is based on constant rate Drawdown tests.
Drawdown tests are not very practical (due to poor data quality).
Buildup tests are more common.

Formation Evaluation

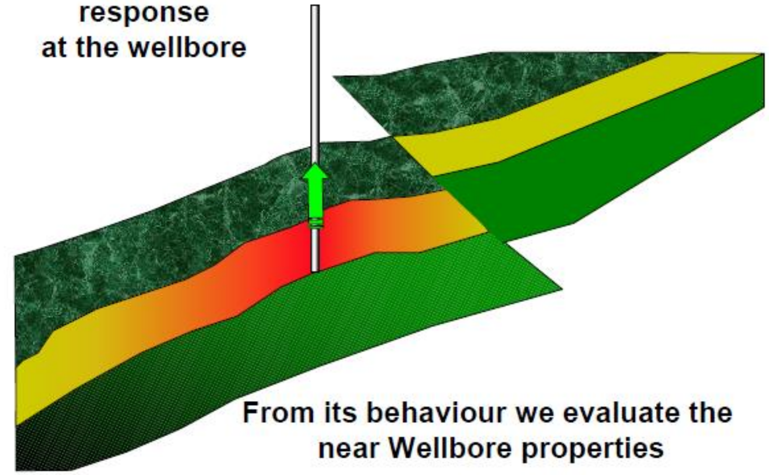
	APPROXIMATE DEPTH OF INVESTIGATION
1. CORING	10 cm
2. LOGGING	50 cm
3. DST / RFT	1 - 10 metres
4. WELL TESTING	50 - 500 metres
5. PRODUCTION	whole reservoir

The Well Test Concept

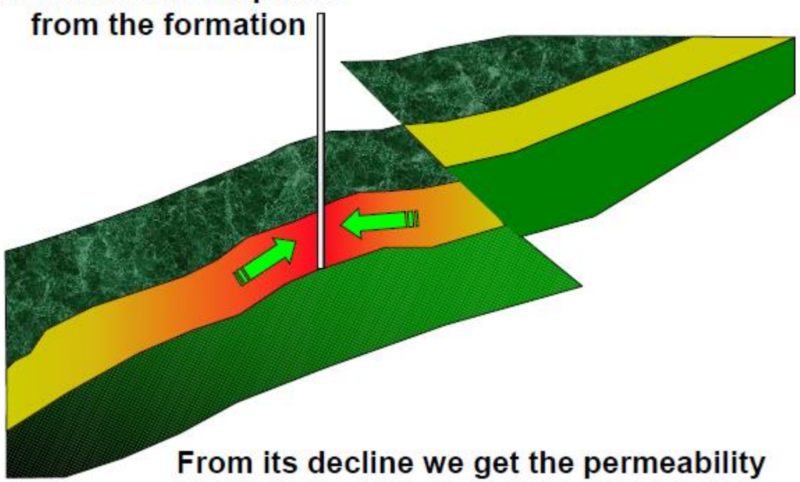
During a well test,
we send a signal
to the reservoir...



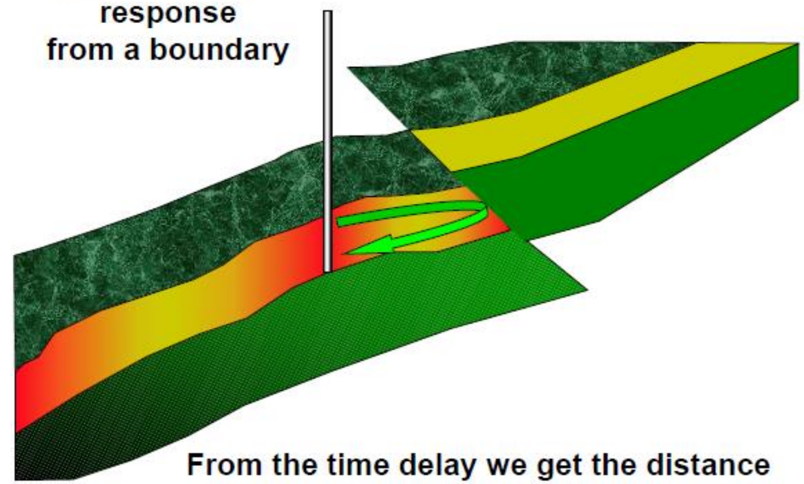
we receive the
response
at the wellbore



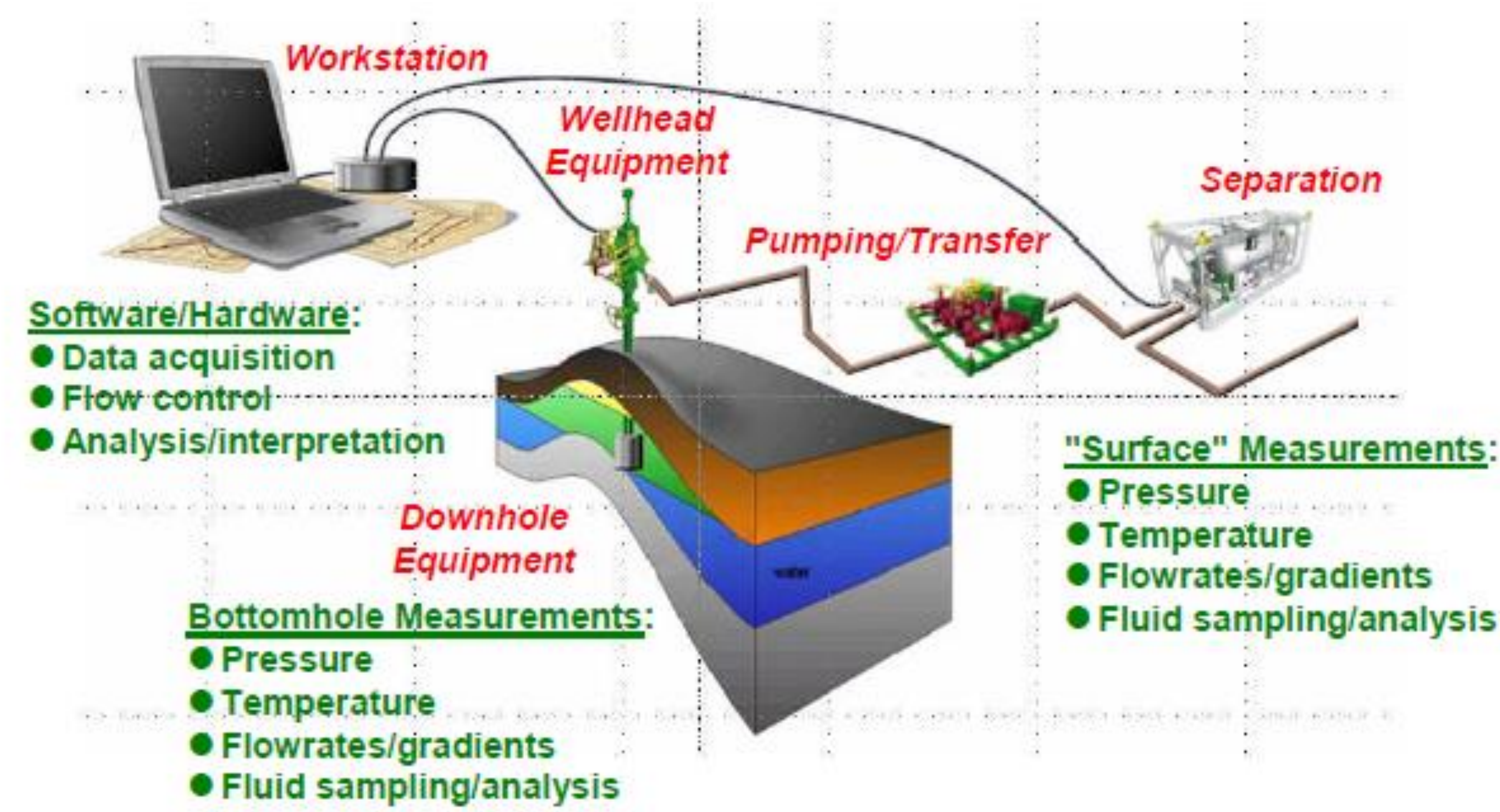
we receive the response
from the formation



we receive the
response
from a boundary

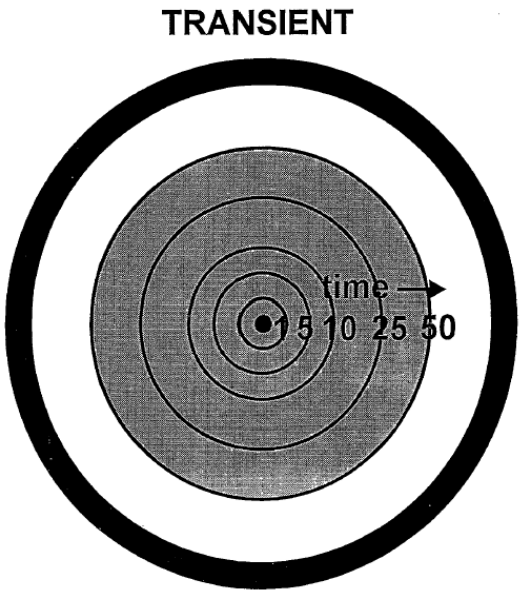


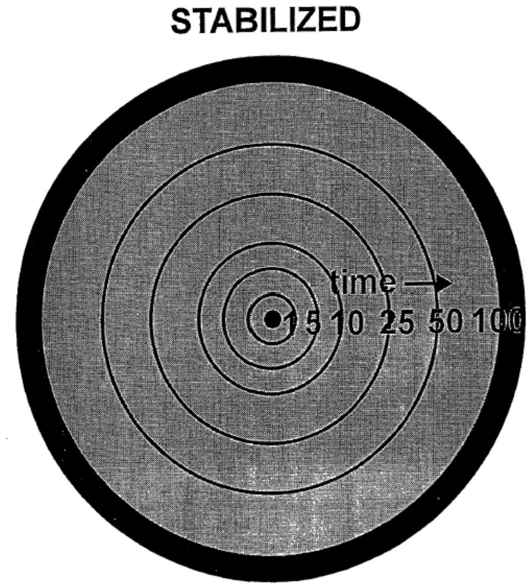
Standard Well Test Set-up

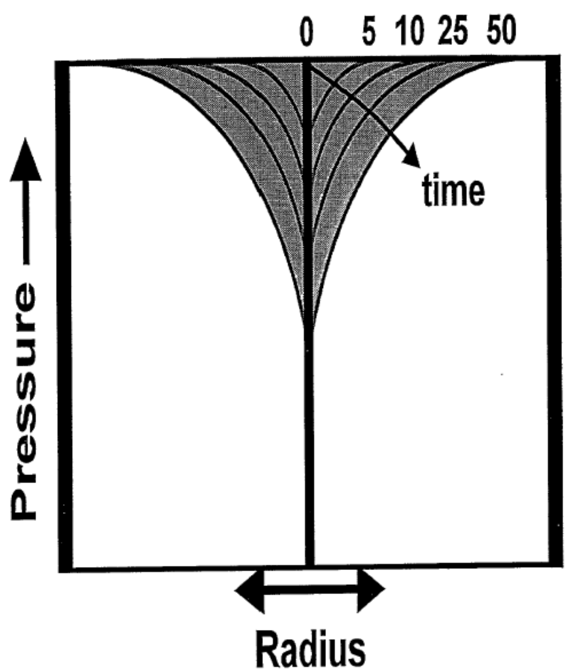


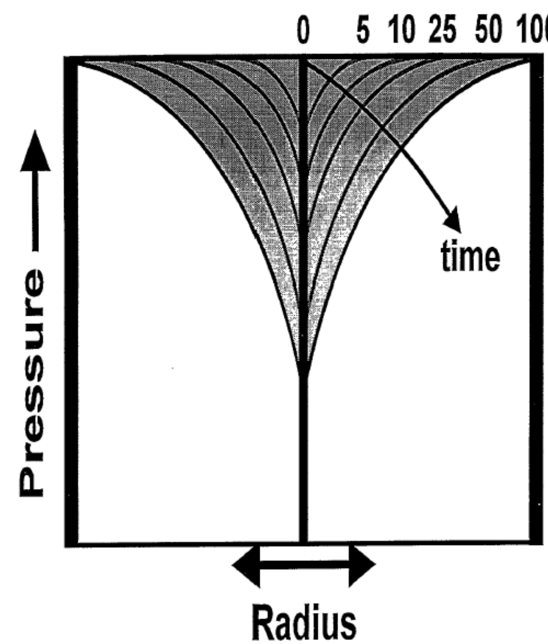
Transient & Stabilized Tests

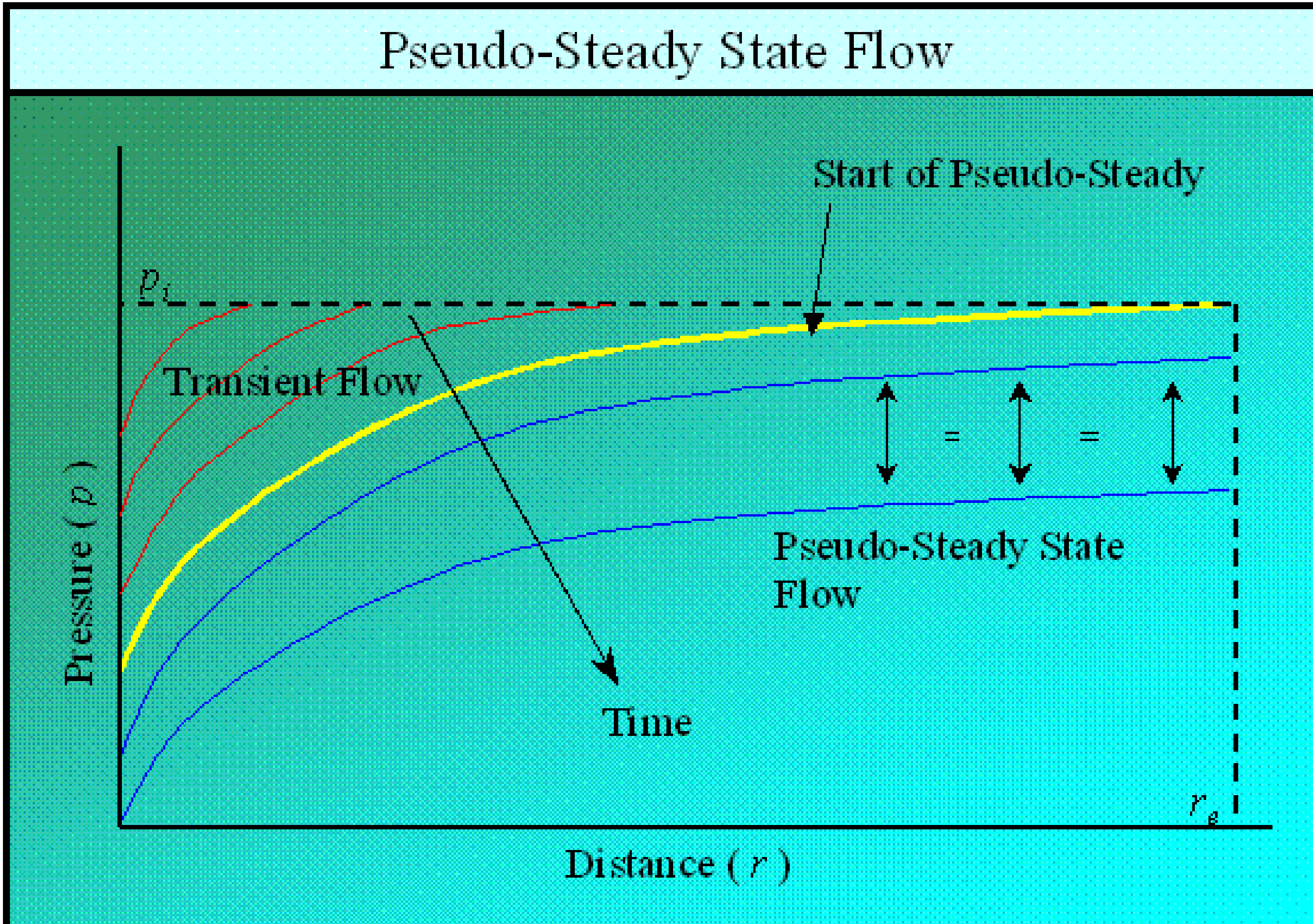
Transient Tests →	Reservoir Characterization
RFT®, WFT®, MDT® …	p_i, k, fluid samples
DST	p_i, k, fluid samples
Drawdown+ / Injection	k, s (often un-interpretable)
Buildup+ / Falloff	k, s, $\overline{p}_R$
Interference/Pulse	k, ϕc_t, lateral/vertical continuity
PITA, PID, Minifrac, CCT	p_i, k
Stabilized Tests →	**Deliverability Forecasting**
IPR	q_{stab}
AOF	q_{stab}


TRANSIENT
time
1 5 10 25 50


STABILIZED
time
1 5 10 25 50 100


0 5 10 25 50
time
Pressure
Radius

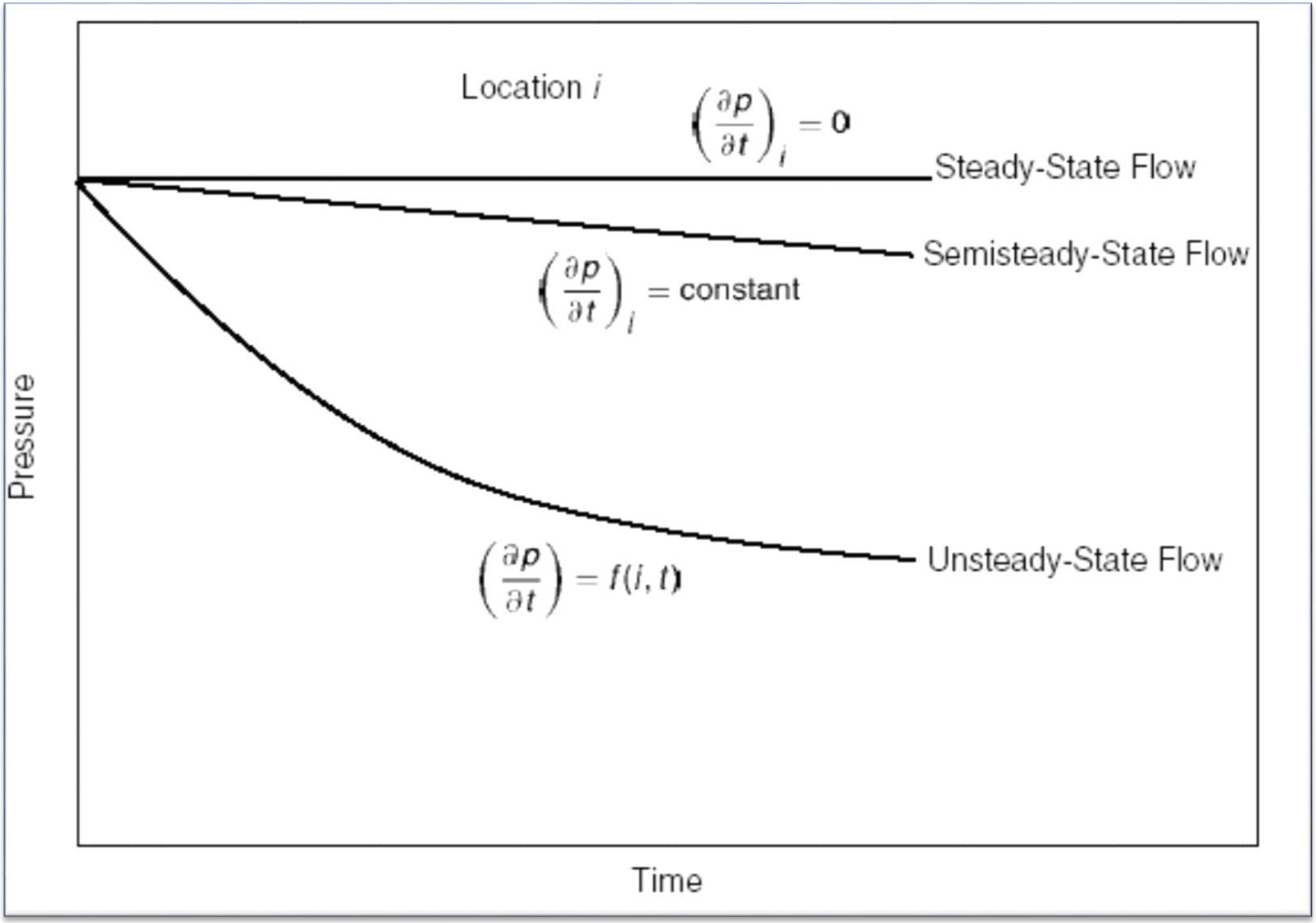

0 5 10 25 50 100
time
Pressure
Radius


Pseudo-Steady State Flow
Start of Pseudo-Steady
p_i
Transient Flow
=
=
Pseudo-Steady State Flow
Time
Pressure (p)
r_e
Distance (r)

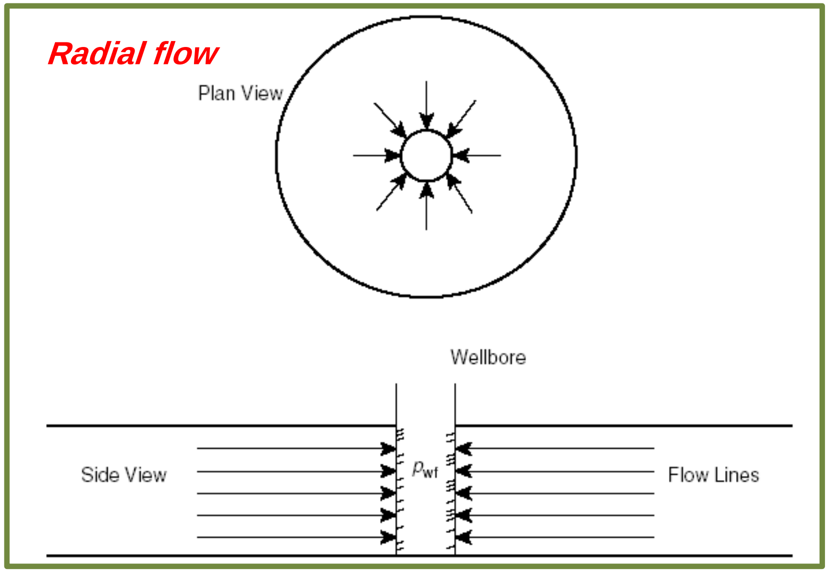
Primary Reservoir Characteristics

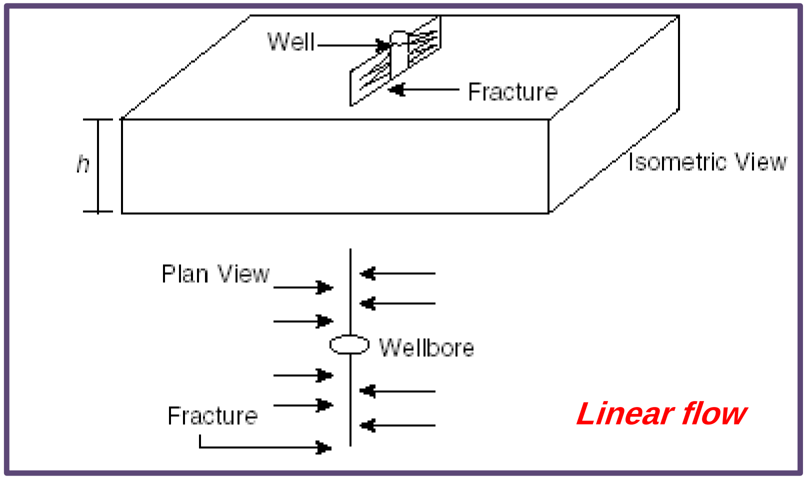
- Types of fluids in the reservoir
  - Incompressible fluids
  - Slightly compressible fluids
  - Compressible fluids
- Flow regimes
  - Steady-state flow
  - Unsteady-state flow
  - Pseudosteady-state flow
- Reservoir geometry
  - Radial flow
  - Linear flow
  - Spherical and hemispherical flow
- Number of flowing fluids in the reservoir.
  - Single-phase flow (oil, water, or gas)
  - Two-phase flow (oil–water, oil–gas, or gas–water)
  - Three-phase flow (oil, water, and gas)

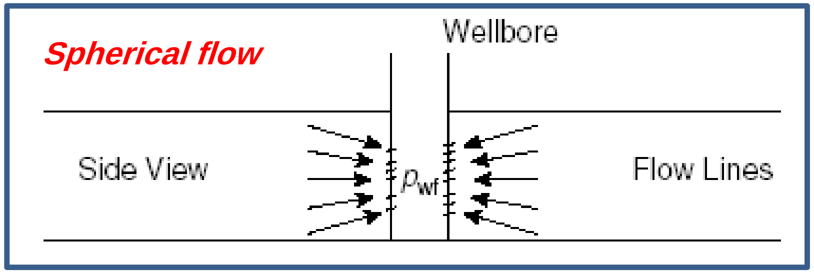
Flow Regimes

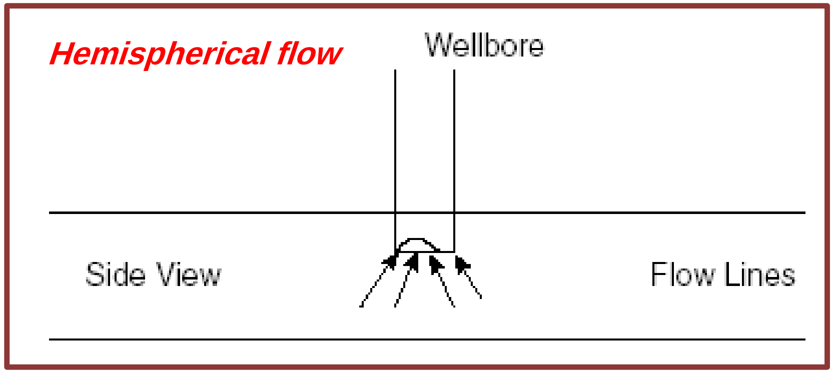


Reservoir Flow Geometry







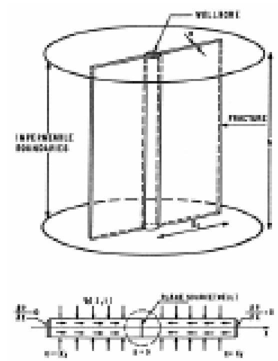


Components of Well Test Models

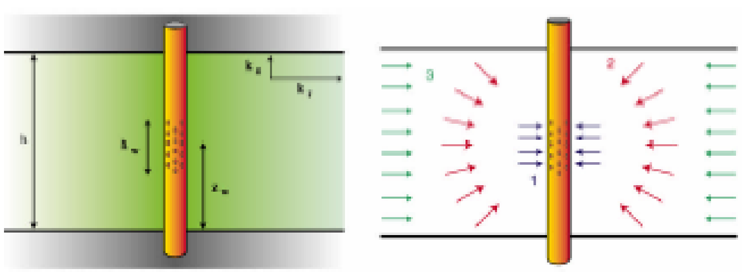
- **Well**
  - Direction (Vertical, Horizontal)
  - Storage (Constant, Changing)
  - Completion (Damaged, Fractured and Acidized)
- **Reservoir**
  - Homogeneous
  - Heterogeneous
  - Composite
  - Multilayer
  - Dual porosity
- **Boundaries**
  - Flow boundaries (No flow, Constant pressure, infinite)
  - Geometrical boundaries (Circular, Rectangular)

Well Models

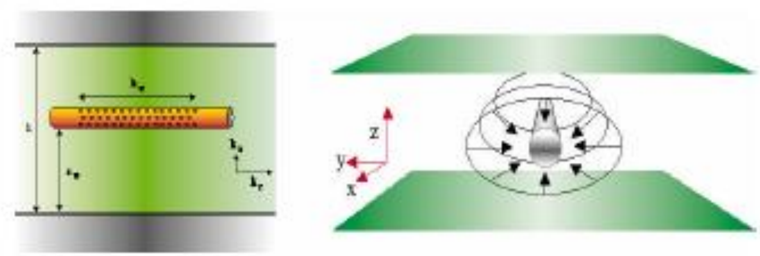
- ➢Wellbore Storage and Skin
- ➢Uniform Flux Vertical Fracture
- ➢Infinite Conductivity Vertical Fracture
- ➢Finite Conductivity Vertical Fracture
- ➢Horizontal Well
- ➢Limited Entry



b. Vertically Fractured Well: Uniform flux; infinite or finite fracture conductivity.



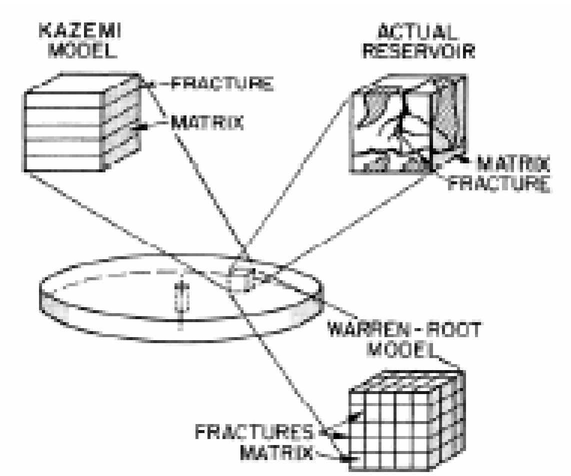
a. Vertical Well: Full or partial penetration, note that this model must also include permeability anistropy.

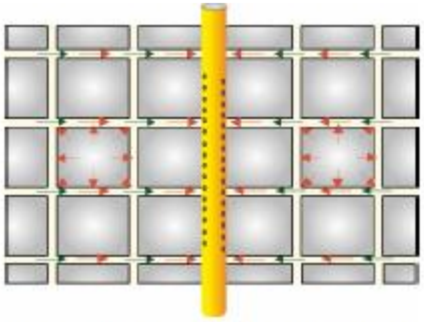


c. Horizontal Well: Full or partial penetra-tion — this model includes permeability anistropy and vertical position.

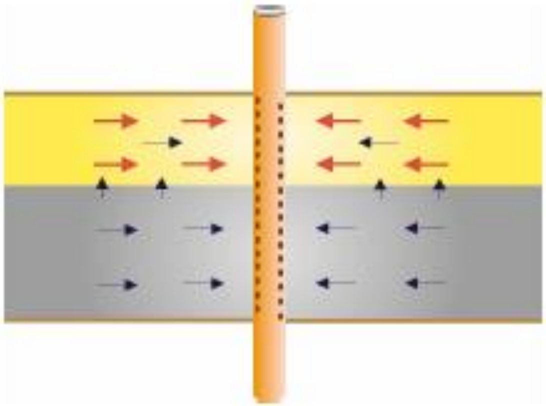
Reservoir Models

- ➢Homogeneous
- ➢Two-Layer
- ➢Radial Composite
- ➢Linear Composite
- ➢Dual Porosity/Naturally Fractured
  - — Pseudosteady-state Interporosity Flow
  - — Transient Interporosity Flow (Slab/Sphere)

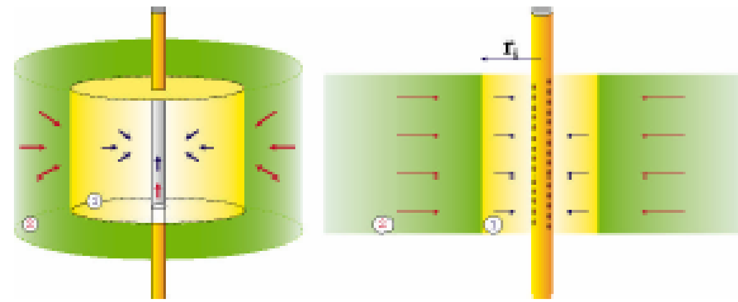




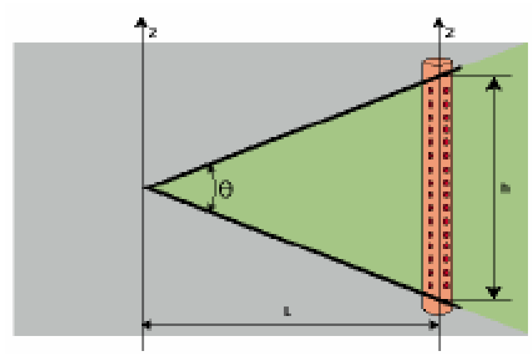
a. Vertical Well: Naturally fractured/ dual porosity reservoir system.



b. Vertical Well: Two-layer reservoir, with crossflow in the reservoir.



c. Vertical Well: Radial composite reservoir system.



d. Vertical Well: "Wedge" or pinch-out reservoir system.

Boundary Models

- Infinite-Acting
- Circle
- Rectangle
- Faults:
  - Single Fault
  - Parallel Faults
  - Multiple Intersecting Faults

Mathematical Model-Governing Equation

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)\right]=\frac{\varphi\mu c_t}{0.000264\ k}\frac{\partial p}{\partial t}$$

Units: p — psia; r — ft; μ — cp; c_t — 1/psia; k — md; t — hr

$$c_t = c_f + c_o S_o + c_w S_w$$

Initial And Boundary Conditions
Radial Flow In a Circular Reservoir

$$Initial \quad Condition: \quad p = p_i , \qquad t = 0, \quad r \ge r_w$$

Well production	Flow regime	Inner Boundary Condition	Outer Boundary conditions
Constant rate	Infinite acting	$\left(\frac{\partial p}{\partial r}\right)_{r_w} = -\frac{\mu q B_o}{2\pi r_w hk}$	$(p)_{r\to\infty} = p_i$
Constant rate	Finite acting (Bounded)	$\left(\frac{\partial p}{\partial r}\right)_{r_w} = -\frac{\mu q B_o}{2\pi r_w hk}$	$\left(\frac{\partial p}{\partial r}\right)_{r\to re} = 0$
Constant pressure	Infinite acting	$(p)_{r_w} = p_{wf}$	$(p)_{r\to\infty} = p_i$
Constant pressure	Finite acting (Bounded)	$(p)_{r_w} = p_{wf}$	$\left(\frac{\partial p}{\partial r}\right)_{r\to re} = 0$

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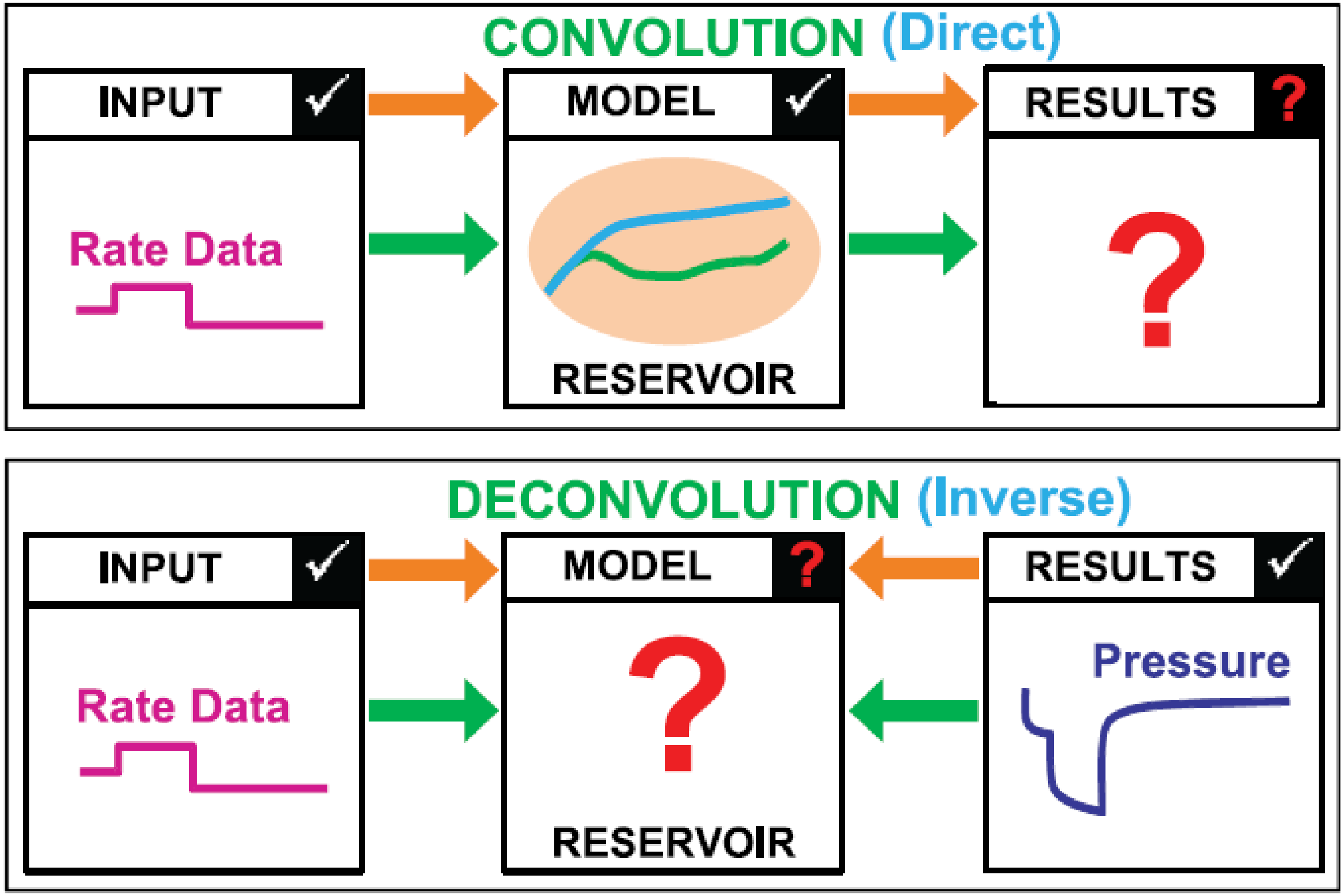
Direct and Inverse Processes

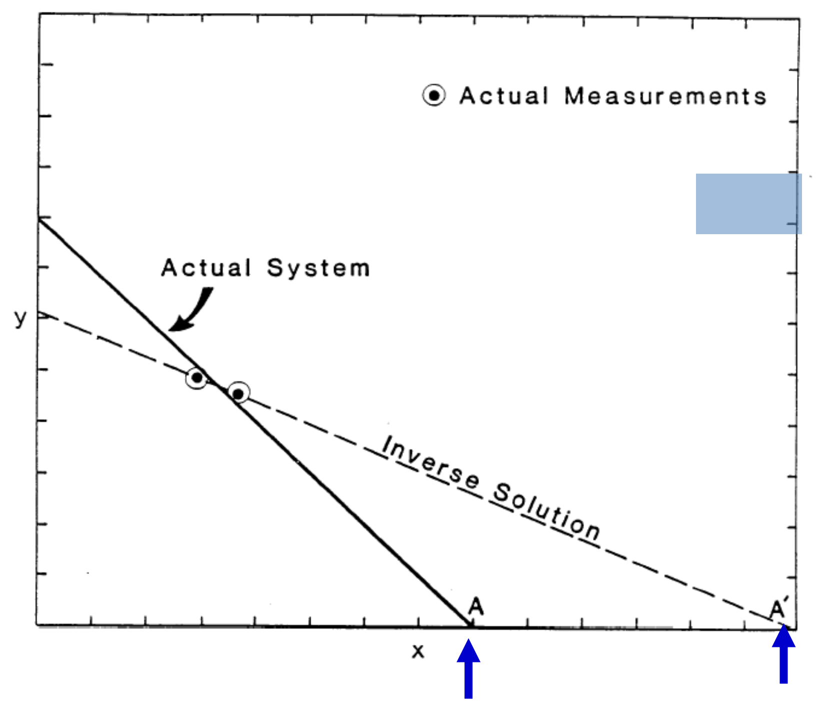


Inverse Solution Compared to Actual System

- Inverse solution can be used for the identification of system characteristics.

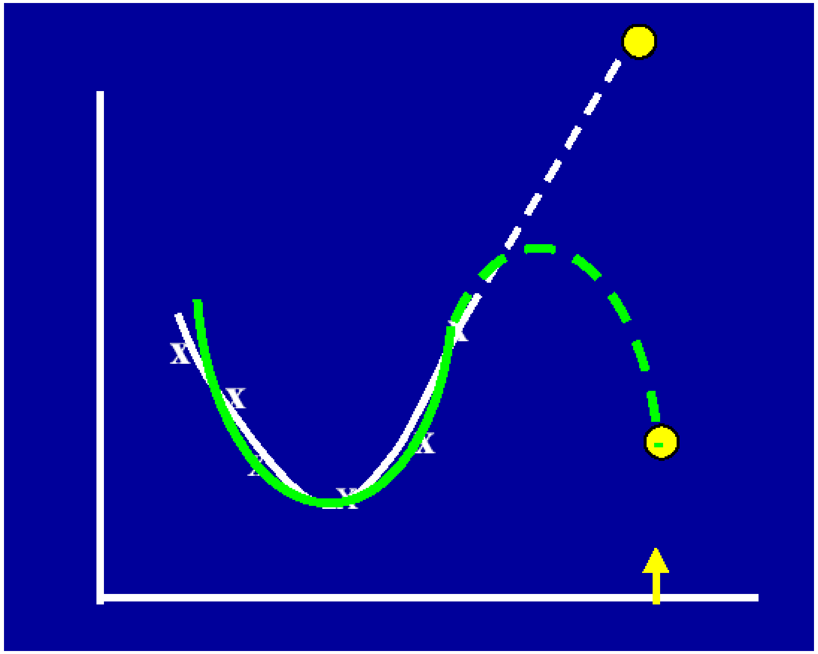
- Inverse solution can result in grossly erroneous answers.

- Whereas the mathematics is correct, the utility of the results derived from this mathematically process is questionable.

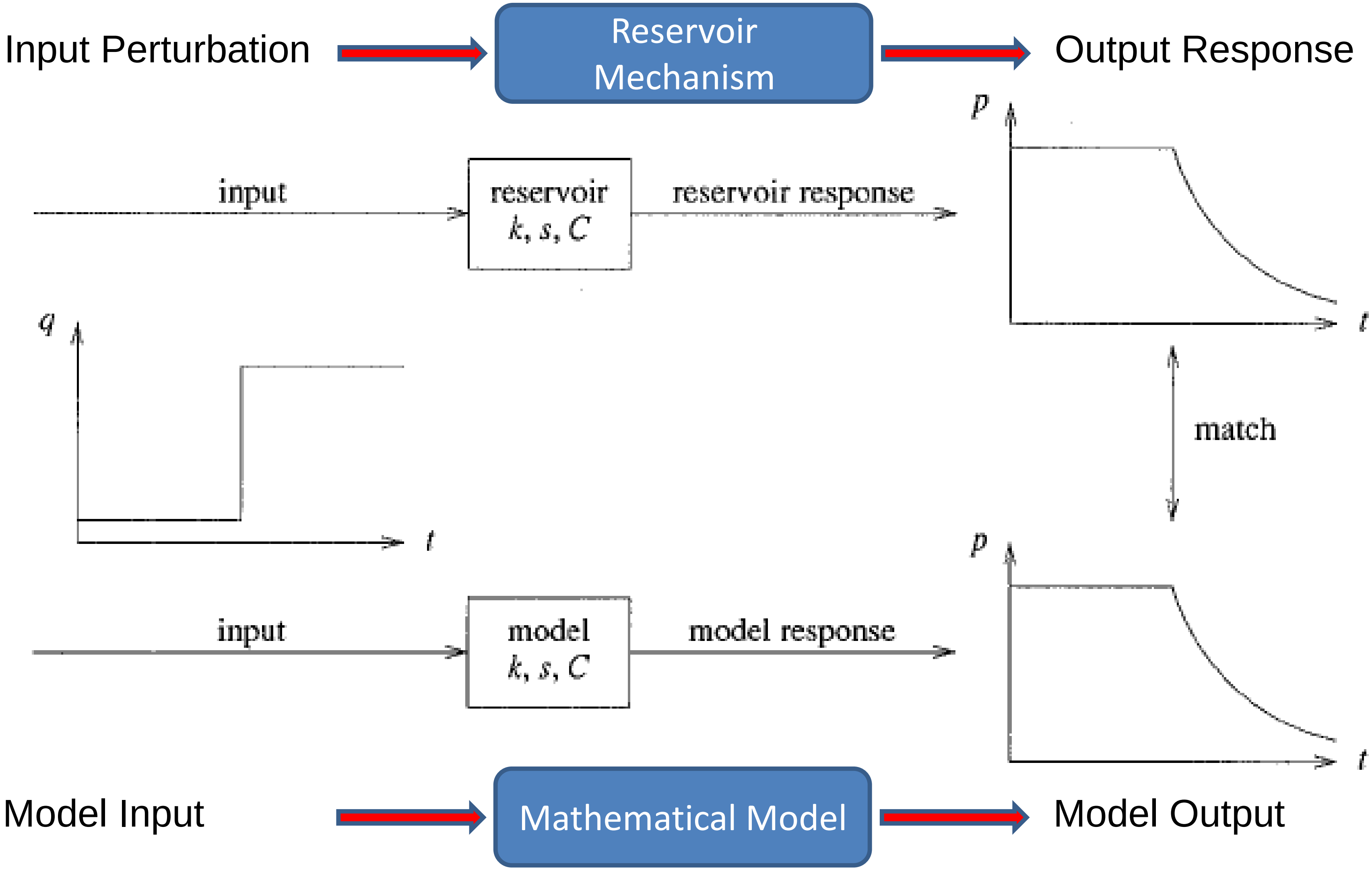


Characteristic of Inverse Solution

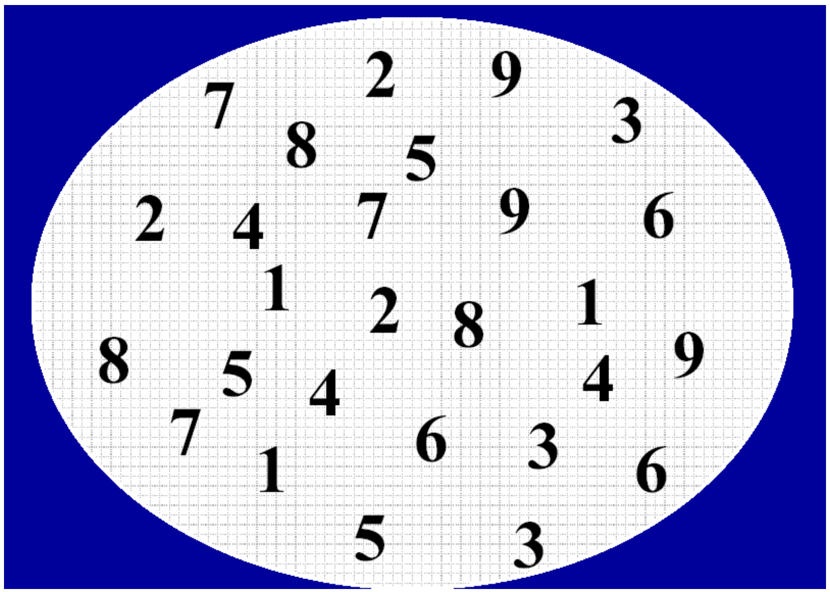
- Non-unique solution (the inverse solution has its limitation)
- A good looking history match is not a good enough answer



Input-System-Response



“Average” permeability in a region
Not
Permeability at a “fixed radius”



Drainage area: The reservoir area or volume drained by the well .

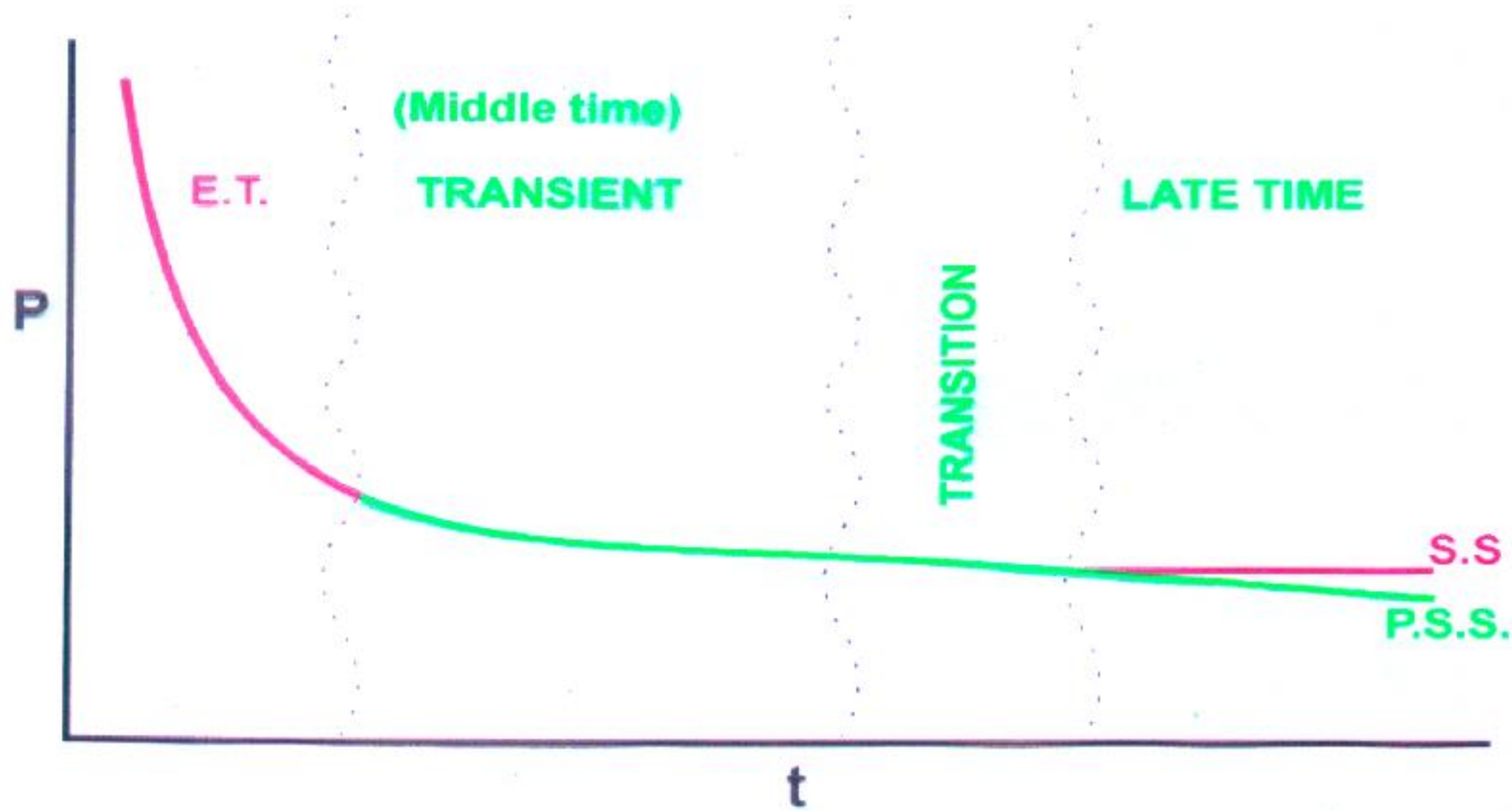
Input Data Required for Well Test Analysis

- **Test data:**
  - ❑ flow rate and bottom hole pressure as a function of time.
  - ❑ the test sequence of events must be detailed, including any operational problems that may affect the well response.

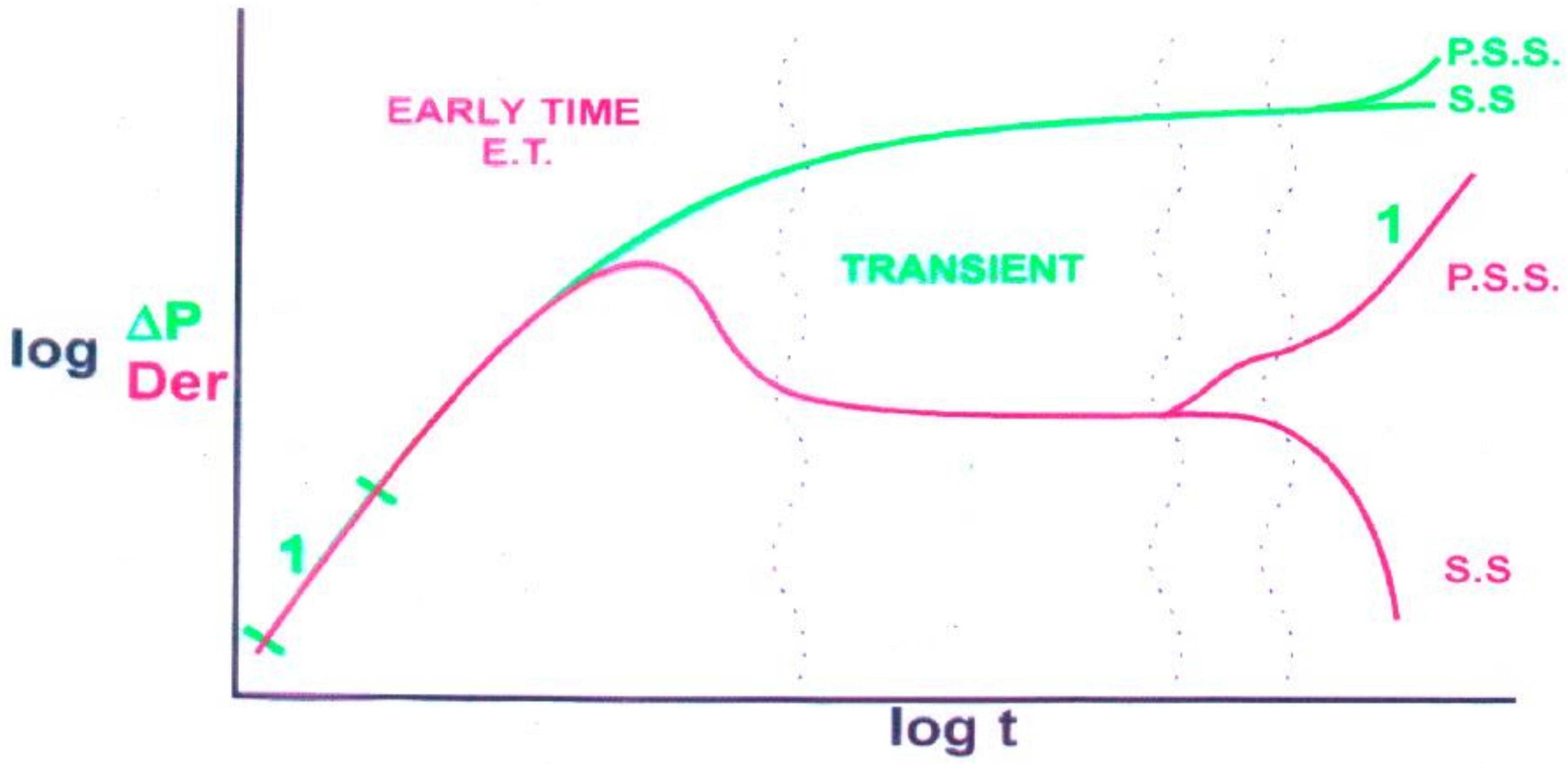
- **Well data:**
  - ❑ wellbore radius r_w
  - ❑ well geometry (such as inclined or horizontal well)
  - ❑ depths (formation, gauges)

- **Reservoir and fluid parameters:**
  - ❑ formation thickness h (net),
  - ❑ porosity Φ,
  - ❑ compressibility of oil $c_{o,}$ water c_w, and formation c_f
  - ❑ water saturation S_w,
  - ❑ oil viscosity μ
  - ❑ formation volume factor B

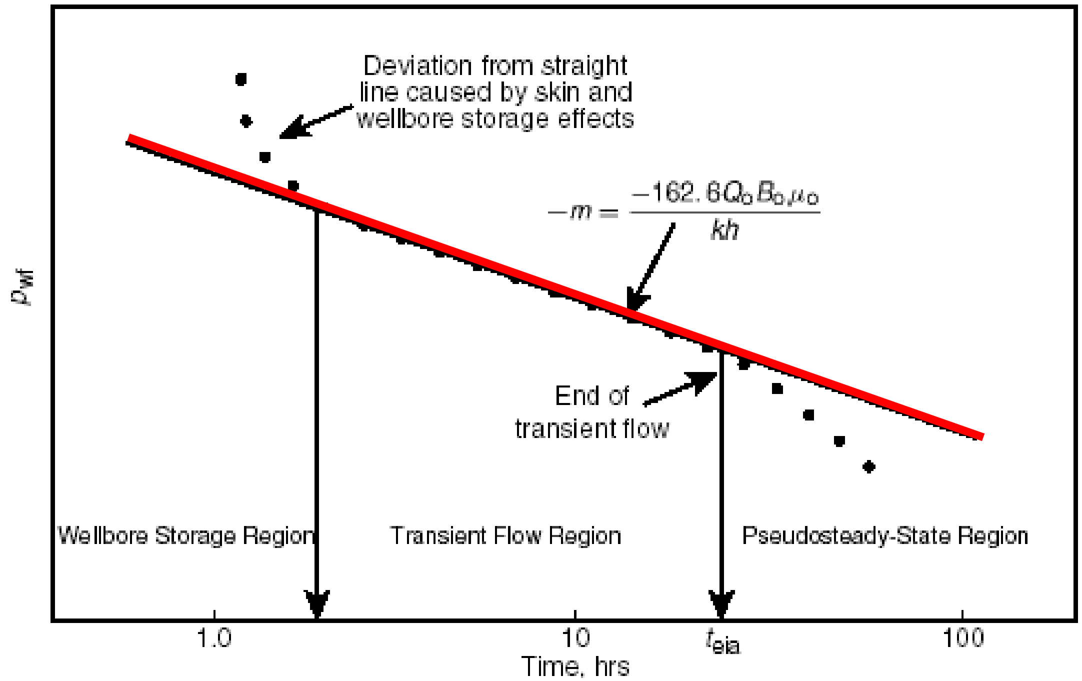
Drawdown Analysis



Drawdown Analysis



Drawdown Analysis



$$m = \frac{p_{wf} - p_{1\,hr}}{\log(t) - \log(1)} = \frac{p_{wf} - p_{1\,hr}}{\log(t) - 0}$$

$$p_{wf} = m \log(t) + p_{1\,hr}$$

$$k = \frac{162.6 Q_o B_o \mu_o}{|m|\, k}$$

$$s = 1.151 \left[\frac{p_i - p_{1\,hr}}{|m|} - \log\left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right]$$

$$\Delta p_{skin} = 0.87\, |m|\, s$$

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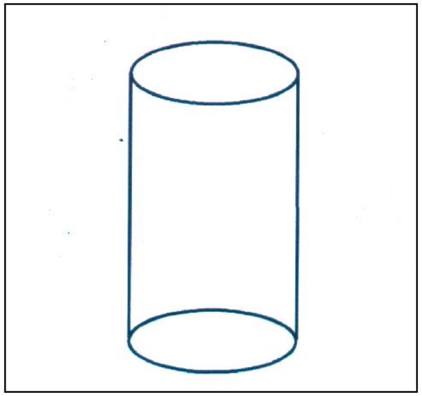
Drawdown Analysis

Wellbore Storage

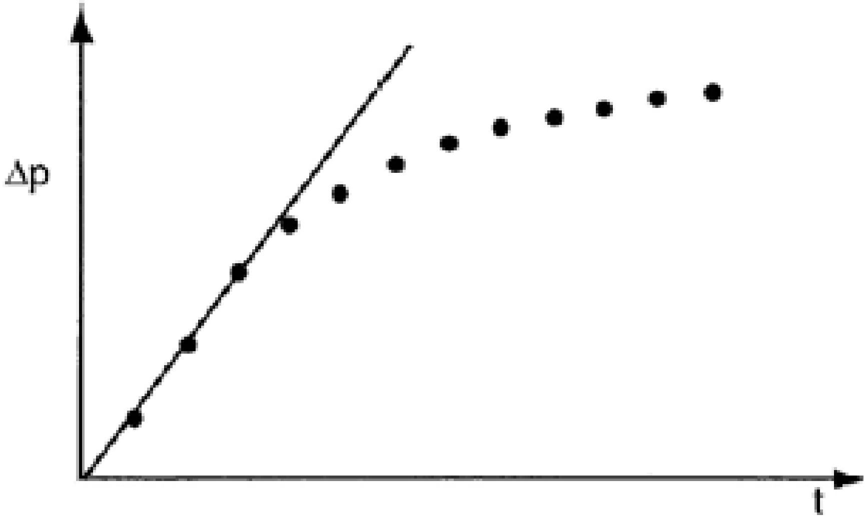


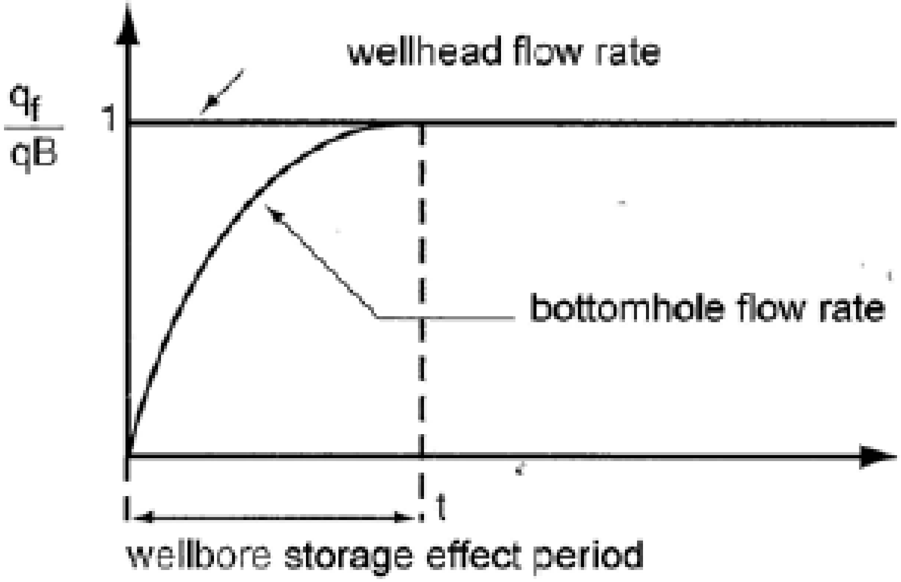
Wellbore is a tank

ΔP = constant x time

Log ΔP vs log t - Data UNIT SLOPE

- Derivative is UNIT SLOPE





•Due to wellbore storage at early times a deviation from constant rate solution to the diffusivity equation is observed. After a certain period of time, $\mathbf{t_{ws}}$, this deviation becomes negligible.

•Ramey(1965) has shown that for various values of $\mathbf{C_{sD}}$, the time for which wellbore storage effects are significant, is given by:

$$\begin{cases} t_{wsD} = 60C_{sD} \\ t_{wsD} = \dfrac{0.000264kt}{\phi\mu c_t r_w^2} \end{cases}$$

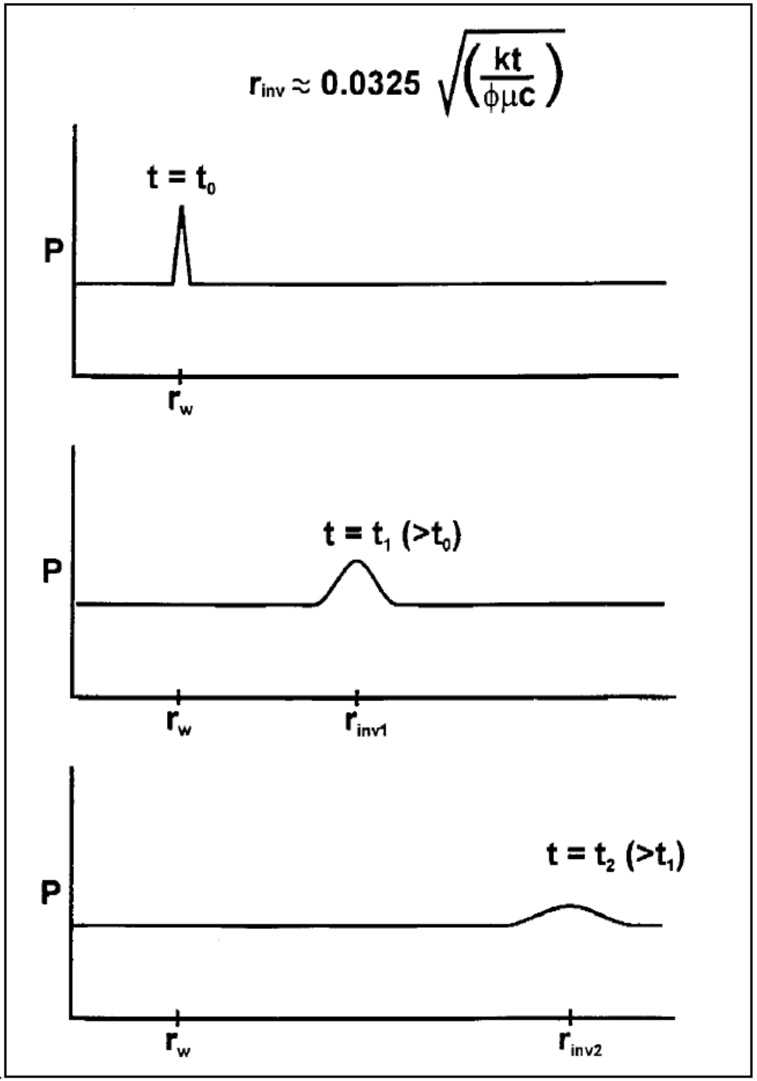
$$t_{ws} = \frac{60 \times 0.894\mu V_{ws} c_s}{0.000264k\,h} = 203182\frac{V_{ws} c_{ws}\mu}{k\,h}$$

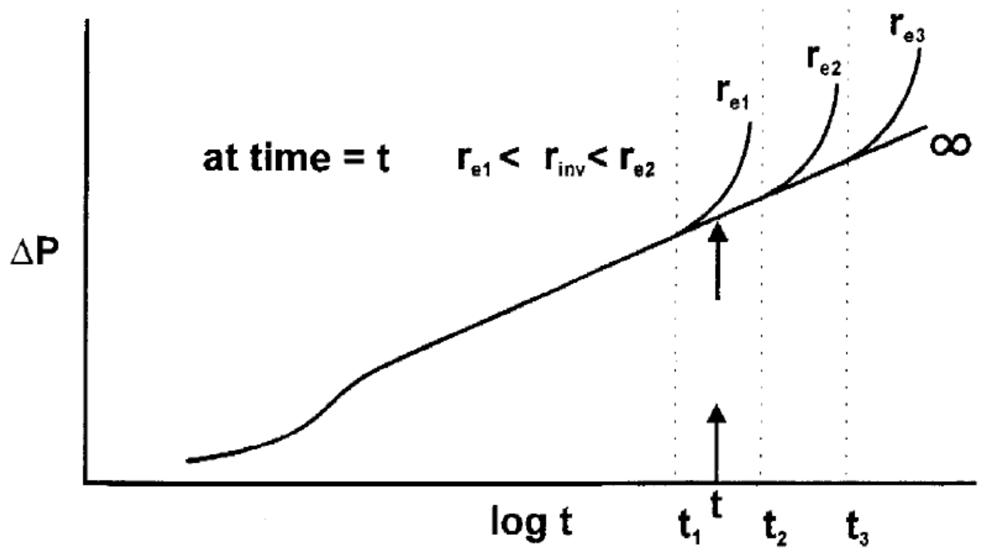
•Two important trends:

- •Wellbore storage effects increase directly with well depth (V_{ws}) and inversly with formation flow capacity (kh).
- •Wellbore storage effects decrease with increasing pressure level (c_{ws}).

Radius of Investigation

How far into the reservoir have we investigated





Drawdown Analysis

$$\Delta P_D = \frac{(P_i - P_f)kh}{141.2\, qB\mu}$$

$$t_D = \frac{2.637 \times 10^{-4} kt}{\phi \mu c r^2}$$

$$r_{eD} = \frac{r_e}{r_w}$$

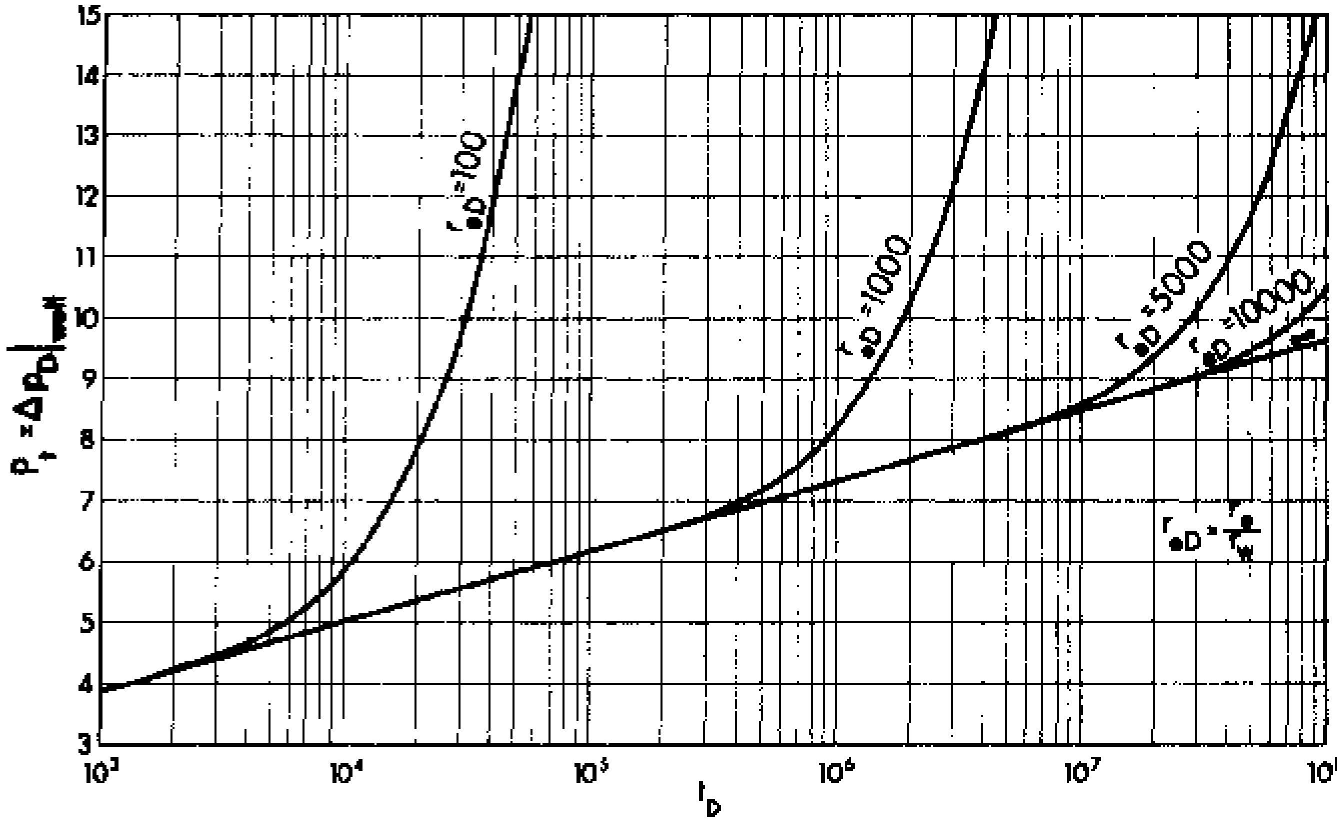


FIGURE 2-8. VALUES OF $P_t (= \Delta p_D|_{well})$ FOR VARIOUS FINITE CIRCULAR RESERVOIRS WITH NO FLOW AT THE EXTERNAL BOUNDARY - EQUATION (2-82)

$$P_t = \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4} \qquad \text{for } \frac{t_D}{r_{eD}^2} > 0.25 \qquad (2\text{-}82)$$

Transient approximate solution

$$P_t = \frac{1}{2}(\ln t_D + 0.809) \quad \text{for} \quad t_D > 25 \qquad (2\text{-}75)$$

P.S.S approximate solution

$$P_t = \frac{2t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4} \qquad \text{for} \quad \frac{t_D}{r_{eD}^2} > 0.25 \qquad (2\text{-}82)$$

For a closed outer boundary reservoir, Katz and Coats (1968) defined the time to stabilization as that time at which the slope of the P_t curves for the infinite-acting and finite-acting reservoirs are equal.

$$\frac{\partial}{\partial t_D}(P_t)_{\text{Equation (2-75)}} = \frac{\partial}{\partial t_D}(P_t)_{\text{Equation (2-82)}}$$

$$\frac{\partial}{\partial t_D}\left[\frac{1}{2}(\ln t_D + 0.809)\right] = \frac{\partial}{\partial t_D}\left[\frac{2\, t_D}{r_{eD}^2} + \ln r_{eD} - \frac{3}{4}\right]$$

$$\frac{1}{2\, t_D} = \frac{2}{r_{eD}^2}$$

from which

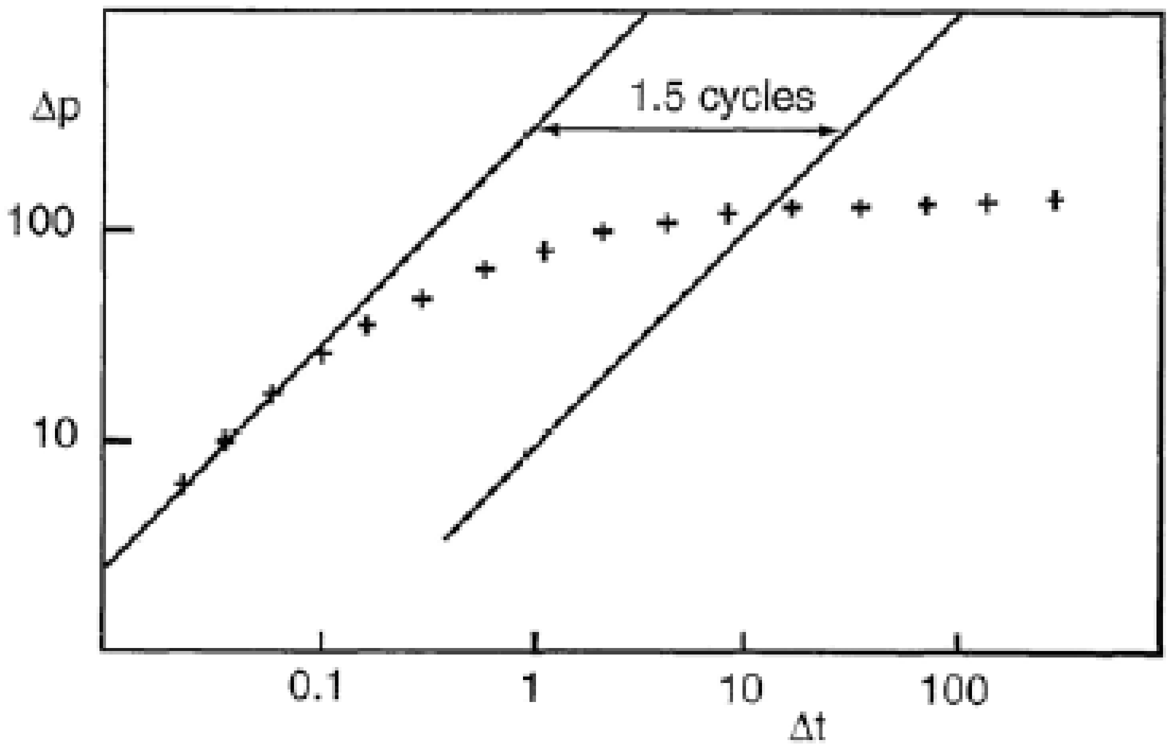
$$t_D = \frac{1}{4} r_{eD}^2$$

which corresponds to Equation (2-106) above. This results in a time to stabilization t_s, given by

$$t_s = \frac{1}{4}\frac{\phi\, \mu\, c\, r_e^2}{\lambda\, k} \qquad (2\text{-}107)$$

Transition Time-Drawdown Analysis

A "rule of thumb, " developed from the fundamental solutions of the diffusivity equation including wellbore storage and skin effect (Agarwal et al., 1970), suggests that the "transition" period lasts 1.5 log cycles from the cessation of predominant wellbore storage effects (unit slope line). Points beyond that time fall on a semi-log straight line.



Accounting for Non-circular Drainage Area

Equation 1.2.123 can be slightly rearranged as:

$$p_{wf} = \left[p_i - \frac{162.6QB\mu}{kh}\log\left(\frac{2.2458A}{C_A r_w^2}\right)\right] - \left(\frac{0.23396QB}{Ah\phi c_t}\right)t$$

The above expression indicates that under semisteady-state flow and constant flow rate, it can be expressed as an equation of a straight line:

$$p_{wf} = a_{pss} + m_{pss}t$$

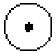
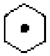
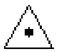
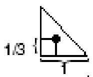
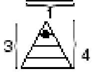
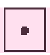
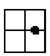
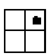
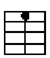
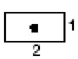
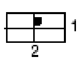
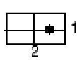
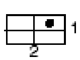
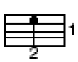
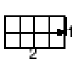
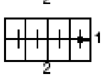
with a_{pss} and m_{pss} as defined by:

$$a_{pss} = \left[p_i - \frac{162.6QB\mu}{kh}\log\left(\frac{2.2458A}{C_A r_w^2}\right)\right]$$

$$m_{pss} = -\left(\frac{0.23396QB}{c_t(Ah\phi)}\right) = -\left(\frac{0.23396QB}{c_t(\text{pore volume})}\right)$$

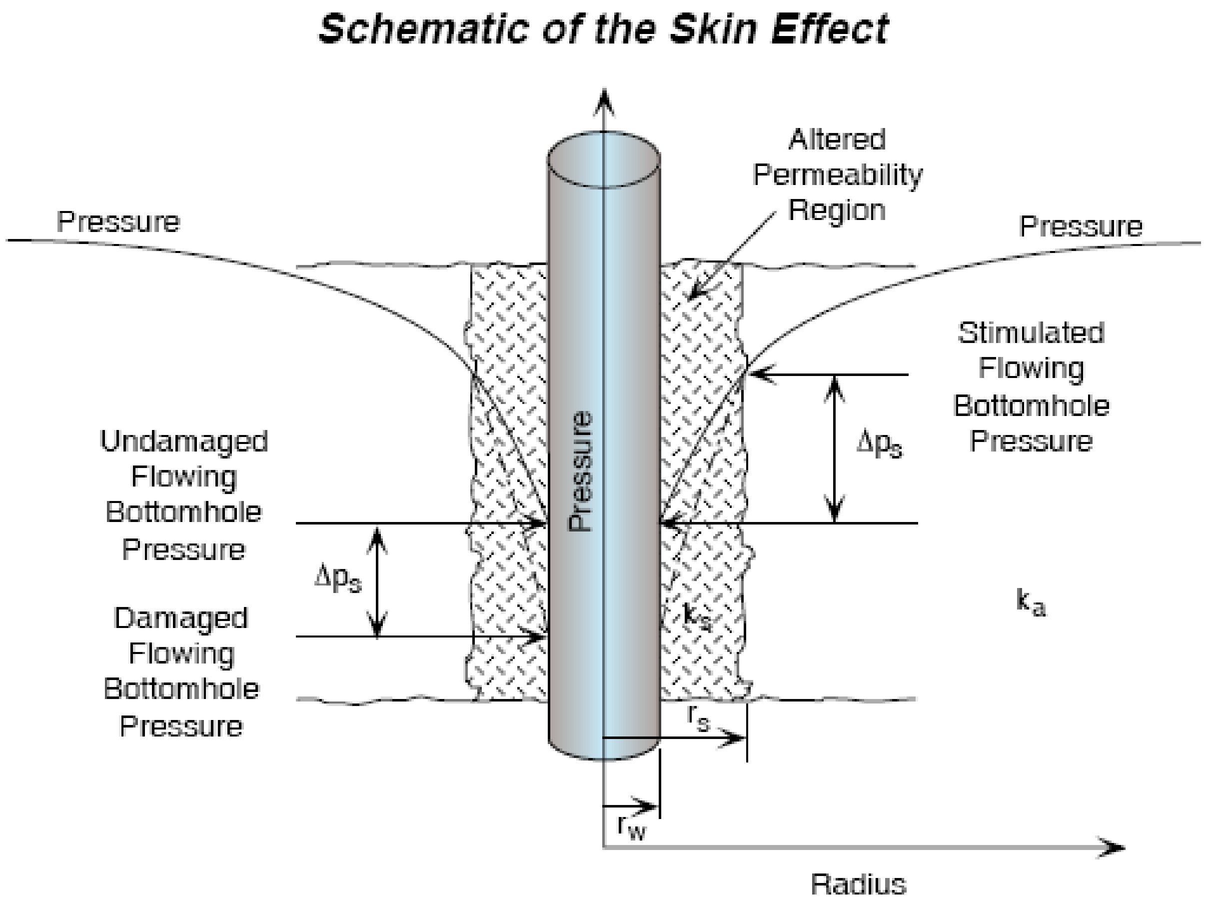
It is obvious that during the pseudosteady (semisteady)-state flow condition, a plot of the bottom-hole flowing pressure p_{wf} versus time t would produce a straight line with a negative slope of m_{pss} and intercept of a_{pss}.

Table 1.4 *Shape factors for various single-well drainage areas (After Earlougher, R, Advances in Well Test Ana permission to publish by the SPE, copyright SPE, 1977)*

In bounded reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	*Exact for* t_{DA} >	*Less than 1% error for* t_{DA} >	*Use infinite system solution with less than 1% error for* t_{DA} >
	31.62	3.4538	−1.3224	0.1	0.06	0.10
	31.6	3.4532	−1.3220	0.1	0.06	0.10
	27.6	3.3178	−1.2544	0.2	0.07	0.09
	27.1	3.2995	−1.2452	0.2	0.07	0.09
	21.9	3.0865	−1.1387	0.4	0.12	0.08
	0.098	−2.3227	+1.5659	0.9	0.60	0.015
	30.8828	3.4302	−1.3106	0.1	0.05	0.09
	12.9851	2.5638	−0.8774	0.7	0.25	0.03
	10132	1.5070	−0.3490	0.6	0.30	0.025
	3.3351	1.2045	−0.1977	0.7	0.25	0.01
	21.8369	3.0836	−1.1373	0.3	0.15	0.025
	10.8374	2.3830	−0.7870	0.4	0.15	0.025
	10141	1.5072	−0.3491	1.5	0.50	0.06
	2.0769	0.7309	−0.0391	1.7	0.50	0.02
	3.1573	1.1497	−0.1703	0.4	0.15	0.005
	0.5813	−0.5425	+0.6758	2.0	0.60	0.02
	0.1109	−2.1991	+1.5041	3.0	0.60	0.005

Skin

- The skin effect, first introduced by van Everdingen and Hurst (1949) defines a steady-state pressure difference around the wellbore.



Skin

Hawkins (1956) suggested that the permeability in the skin zone, i.e., skin, is uniform and the pressure drop across the zone can be approximated by Darcy's equation. Hawkins proposed the following approach:

$$\Delta p_{\text{skin}} = \begin{bmatrix} \Delta p \text{ in skin zone} \\ \text{due to } k_{\text{skin}} \end{bmatrix} - \begin{bmatrix} \Delta p \text{ in the skin zone} \\ \text{due to } k \end{bmatrix}$$

Applying Darcy's equation gives:

$$(\Delta p)_{\text{skin}} = \left(\frac{Q_o B_o \mu_o}{0.00708 h k_{\text{skin}}}\right) \ln\left(\frac{r_{\text{skin}}}{r_w}\right) - \left(\frac{Q_o B_o \mu_o}{0.00708 h k}\right) \ln\left(\frac{r_{\text{skin}}}{r_w}\right)$$

or:

$$\Delta p_{\text{skin}} = \left(\frac{Q_o B_o \mu_o}{0.00708 k h}\right) \left[\frac{k}{k_{\text{skin}}} - 1\right] \ln\left(\frac{r_{\text{skin}}}{r_w}\right)$$

where:

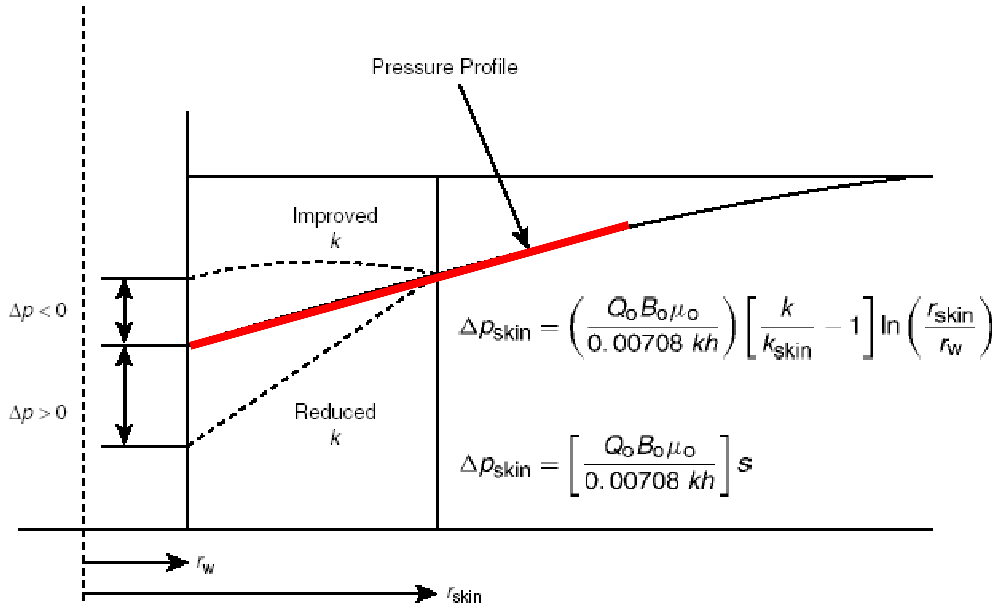
k = permeability of the formation, md
k_{skin} = permeability of the skin zone, md

The above expression for determining the additional pressure drop in the skin zone is commonly expressed in the following form:

$$\Delta p_{\text{skin}} = \left(\frac{Q_o B_o \mu_o}{0.00708 k h}\right) s = 141.2 \left(\frac{Q_o B_o \mu_o}{k h}\right) s \qquad [1.2.130]$$

where s is called the skin factor and defined as:

$$s = \left[\frac{k}{k_{\text{skin}}} - 1\right] \ln\left(\frac{r_{\text{skin}}}{r_w}\right) \qquad [1.2.131]$$



$$(\Delta p)_{\text{actual}} = (\Delta p)_{\text{ideal}} + (\Delta p)_{\text{skin}}$$

or:

$$(p_i - p_{wf})_{\text{actual}} = (p_i - p_{wf})_{\text{ideal}} + \Delta p_{\text{skin}}$$

Steady state radial flow (accounting for the skin factor)

Substituting Equations 1.2.15 and 1.2.130 into Equation 1.2.132, gives:

$$(\Delta p)_{\text{actual}} = (\Delta p)_{\text{ideal}} + (\Delta p)_{\text{skin}}$$

$$(p_i - p_{wf})_{\text{actual}} = \left(\frac{Q_o B_o \mu_o}{0.00708kh}\right) \ln\left(\frac{r_e}{r_w}\right) + \left(\frac{Q_o B_o \mu_o}{0.00708kh}\right) s$$

Solving for the flow rate gives:

$$Q_o = \frac{0.00708kh\,(p_i - p_{wf})}{\mu_o B_o \left[\ln \frac{r_e}{r_w} + s\right]} \quad [1.2.133]$$

where:

Q_o = oil flow rate, STB/day
k = permeability, md
h = thickness, ft
s = skin factor
B_o = oil formation volume factor, bbl/STB
μ_o = oil viscosity, cp
p_i = initial reservoir pressure, psi
p_{wf} = bottom-hole flowing pressure, psi

Unsteady-state radial flow (accounting for the skin factor)

For slightly compressible fluids Combining Equations 1.2.71 and 1.2.130 with that of 1.2.132 yields:

$$(\Delta p)_{\text{actual}} = (\Delta p)_{\text{ideal}} + (\Delta p)_{\text{skin}}$$

$$p_i - p_{wf} = 162.6\left(\frac{Q_o B_o \mu_o}{kh}\right)\left[\log \frac{kt}{\phi \mu c_t r_w^2} - 3.23\right] + 141.2\left(\frac{Q_o B_o \mu_o}{kh}\right) s$$

or:

$$p_i - p_{wf} = 162.6\left(\frac{Q_o B_o \mu_o}{kh}\right)\left[\log \frac{kt}{\phi \mu c_t r_w^2} - 3.23 + 0.87s\right] \quad [1.2.134]$$

Pseudosteady-state flow (accounting for the skin factor)

For slightly compressible fluids Introducing the skin factor into Equation 1.2.123 gives:

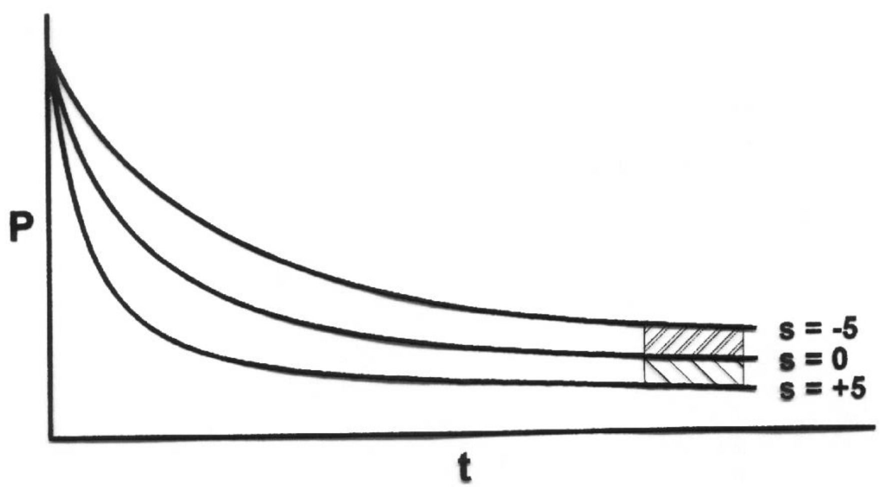
$$Q_o = \frac{0.00708kh\,(\bar{p}_r - p_{wf})}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \quad [1.2.137]$$

Effective Wellbore Radius

- If the permeability in the altered zone k_a is much larger than the formation permeability k, then the wellbore will act like a well having an apparent wellbore radius r_{wa}.

- The apparent wellbore radius may be calculated from the actual wellbore radius and the skin factor.

$$s = -\ln\left(\frac{r_{wa}}{r_w}\right)$$



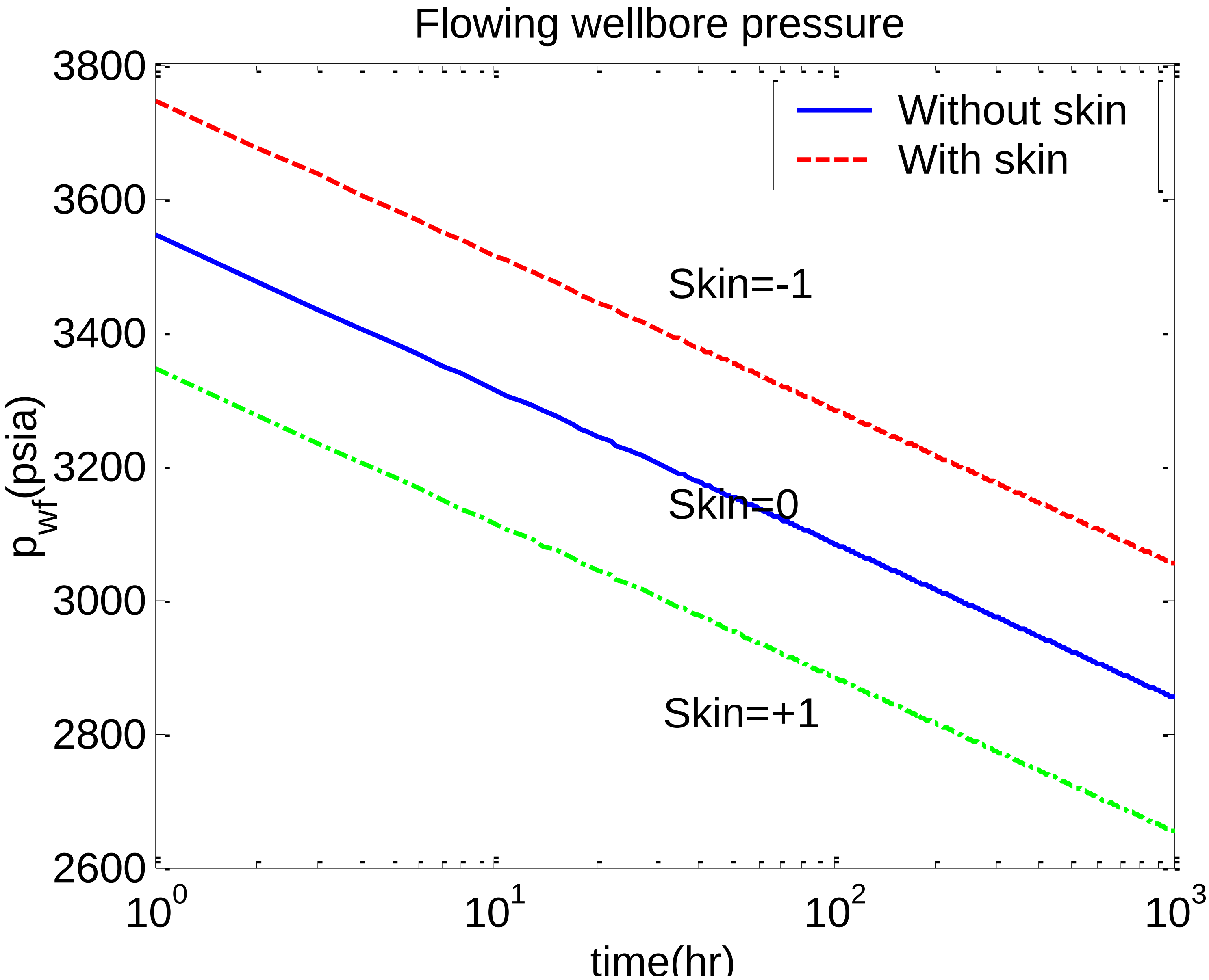
Minimum Skin Factor

The minimum skin factor possible (most negative skin factor) would occur when the apparent wellbore radius r_{wa} is equal to the drainage radius r_e of the well.

$$s_{min} = -\ln\left(\frac{r_e}{r_w}\right)$$

For a circular drainage area of 40 acres (r_e = 745 feet) and a wellbore radius of 0.5 feet, this gives a minimum skin factor (maximum stimulation) of -7.3.

$$s_{min} = -\ln\left(\frac{r_e}{r_w}\right) = -\ln\left(\frac{745}{0.5}\right) = -7.3$$



Flowing wellbore pressure
3800
3600
3400
3200
3000
2800
2600
p_{wf}(psia)
10^0
10^1
10^2
10^3
time(hr)
Without skin
With skin
Skin=-1
Skin=0
Skin=+1

Well Test Analyses with Straight-line Methods

y = b + mx

$$y \equiv p_{wf} \qquad x \equiv \log t \qquad m \equiv \left| \frac{162.6 q B \mu}{kh} \right|$$

$$b \equiv p_i - \frac{162.6 q B \mu}{kh} \left[\log \frac{k}{\phi \mu c_t r_w^2} - 3.23 + 0.8691 s \right]$$

$$p_{wf} = p_i - \frac{162.6 q B \mu}{kh} \left[\log t + \log \frac{k}{\phi \mu c_t r_w^2} - 3.23 + 0.8691 s \right]$$

$$s = 1.151 \left[\frac{p_i - p_{wf}}{m} - \log \frac{kt}{\phi \mu c_t r_w^2} + 3.23 \right]$$

$$k = \frac{162.6 q B \mu}{mh}$$

$$s = 1.151 \left[\frac{p_i - p_{1hr}}{m} - \log \frac{k}{\phi \mu c_t r_w^2} + 3.23 \right]$$

Superposition Principle

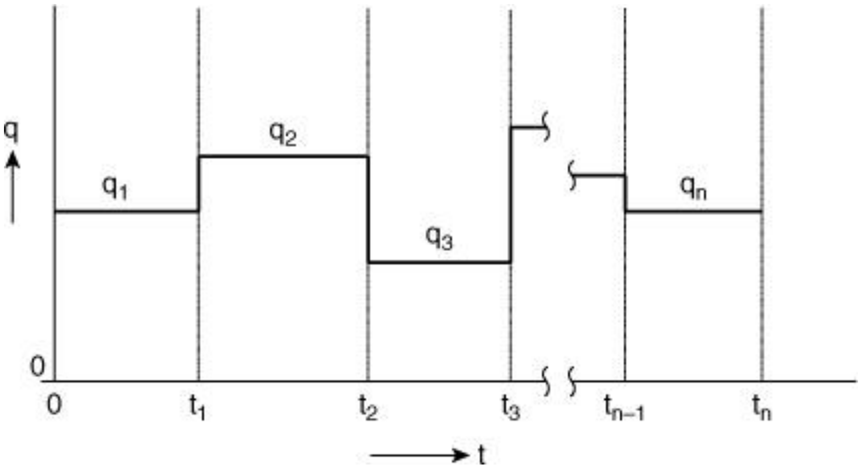
Linear diffusivity equation

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{k}\frac{\partial p}{\partial t}$$

- Mathematically the superposition theorem states that any sum of individual solutions to the diffusivity equation is also a solution to that equation. This concept can be applied to account for the following effects on the transient flow solution:

- **Superposition in time**
  - Effects of rate change
- **Super position in space**
  - Effects of multiple wells
  - Effects of the boundary

Multi-Rate Drawdown Tests



$$p_i - p_{wf} = m_m q[\log(t) + b_m]$$

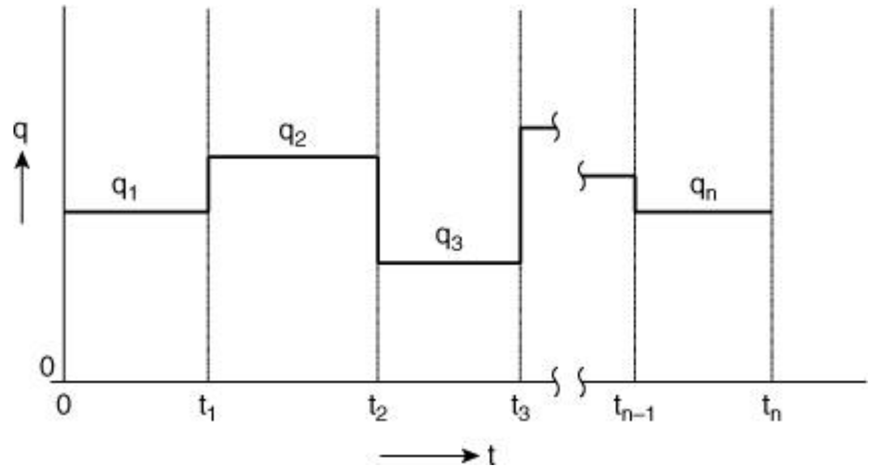
$$m_m = \frac{162.6B\mu}{kh}$$

$$b_m = \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.8691s$$

$$\begin{aligned} p_i - p_{wf} = {} & m_m q_1[\log(t) + b_m] + m_m(q_2 - q_1)[\log(t - t_1) + b_m] \\ & + m_m(q_3 - q_2)[\log(t - t_2) + b_m] + \cdots\cdots \\ & \cdots\cdots + m_m(q_n - q_{n-1})[\log(t - t_{n-1}) + b_m] \end{aligned}$$

$$\frac{p_i - p_{wf}}{q_n} = m_m \sum_{j=1}^{n}\left[\frac{(q_j - q_{j-1})}{q_n}\log(t_n - t_{j-1})\right] + m_m b_m \quad \text{for} \quad q_n \neq 0$$

Multi-Rate Drawdown Tests



$$X_F = \sum_{j=1}^{n} \left[\frac{(q_j - q_{j-1})}{q_n} \log(t_n - t_{j-1}) \right]$$

$$Y_F = \frac{p_i - p_{wf}}{q_n}$$

$$slope, m_m = \frac{162.6 B\mu}{kh}$$

$$Y_{intercept} = b_I = m_m \left[\log\left(\frac{k}{\phi \mu c_t r_w^2} \right) - 3.23 + 0.8691 s \right]$$

$$s = 1.151 \left[\frac{b_I}{m_m} - \log\left(\frac{k}{\phi \mu c_t r_w^2} \right) + 3.23 \right]$$

Example

A multi-rate test was conducted on an oil well at production rates and flowing bottom-hole pressures shown in Table .2.1. Other reservoir properties and well data are listed as follows:

Initial reservoir pressure, p_i	2906 psia
Formation thickness, h	40 ft
Formation porosity, ϕ	0.24
Total compressibility, c_t	5×10^{-5} psi^{-1}
Oil viscosity, μ_o	0.60 cp
Oil FVF, B_o	1.27 RB/STB
Wellbore radius, r_w	0.5 ft

n	t (hrs.)	q_n (STB/D)	P_{wf} (psi)
1	0	0	2906
1	1.00	1580	2023
1	1.50	1580	1968
1	1.89	1580	1941
1	2.40	1580	—
2	3.00	1490	1892
2	3.45	1490	1882
2	3.98	1490	1873
2	4.50	1490	1867
2	4.80	1490	—
3	5.50	1440	1853
3	6.05	1440	1843
3	6.55	1440	1834
3	7.00	1440	1830
3	7.20	1440	—
4	7.50	1370	1827
4	8.95	1370	1821
4	9.60	1370	—
5	10.00	1300	1815
5	12.00	1300	1797
6	14.40	1260	—
7	15.00	1190	1775
7	18.00	1190	1771
7	19.20	1190	—
8	20.00	1160	1772
8	21.60	1160	—
9	24.00	1137	1756
10	28.80	1106	—
11	30.00	1080	1751
11	33.60	1080	—
12	36.00	1000	—
13	36.20	983	1756
13	48.00	983	1743

Solution

Step 1: Calculate the Y_F function.

$$Y_F = \frac{p_i - p_{wf}}{q_n}$$

As example, at t = 1.89 hrs, p_i = 2906 psi; p_{wf} = 1941 psi; q_n = 1580 STB/D

$$Y_F = \frac{2906 - 1941}{1580} = 0.6108 \text{ psi/STB-D}$$

Similarly, at t = 20 hrs, p_{wf} = 1772 psi; q_n = 1160 STB/D

$$Y_F = \frac{2906 - 1772}{1160} = 0.9776 \text{ psi/STB-D}$$

Solution

Step 2: Calculate the X_F function.

$$X_F = \sum_{j=1}^{n}\left[\frac{(q_j - q_{j-1})}{q_n}\log(t_n - t_{j-1})\right]$$

As example, at t = 1.89 hrs,

$$X_F = \left(\frac{1580 - 0}{1580}\right)\log(1.89 - 0) = 0.2765$$

At t = 20 hrs,

$$X_F = \frac{1}{1160} \times \left[\begin{array}{l} (1580 - 0)\log(20 - 0) + (1490 - 1580)\log(20 - 2.4) \\ + (1440 - 1490)\log(20 - 4.8) + (1370 - 1440)\log(20 - 7.2) \\ + (1300 - 1370)\log(20 - 9.6) + (1260 - 1300)\log(20 - 12) \\ + (1190 - 1260)\log(20 - 14.4) + (1160 - 1190)\log(20 - 19.2) \end{array}\right] = 1.4225$$

Solution

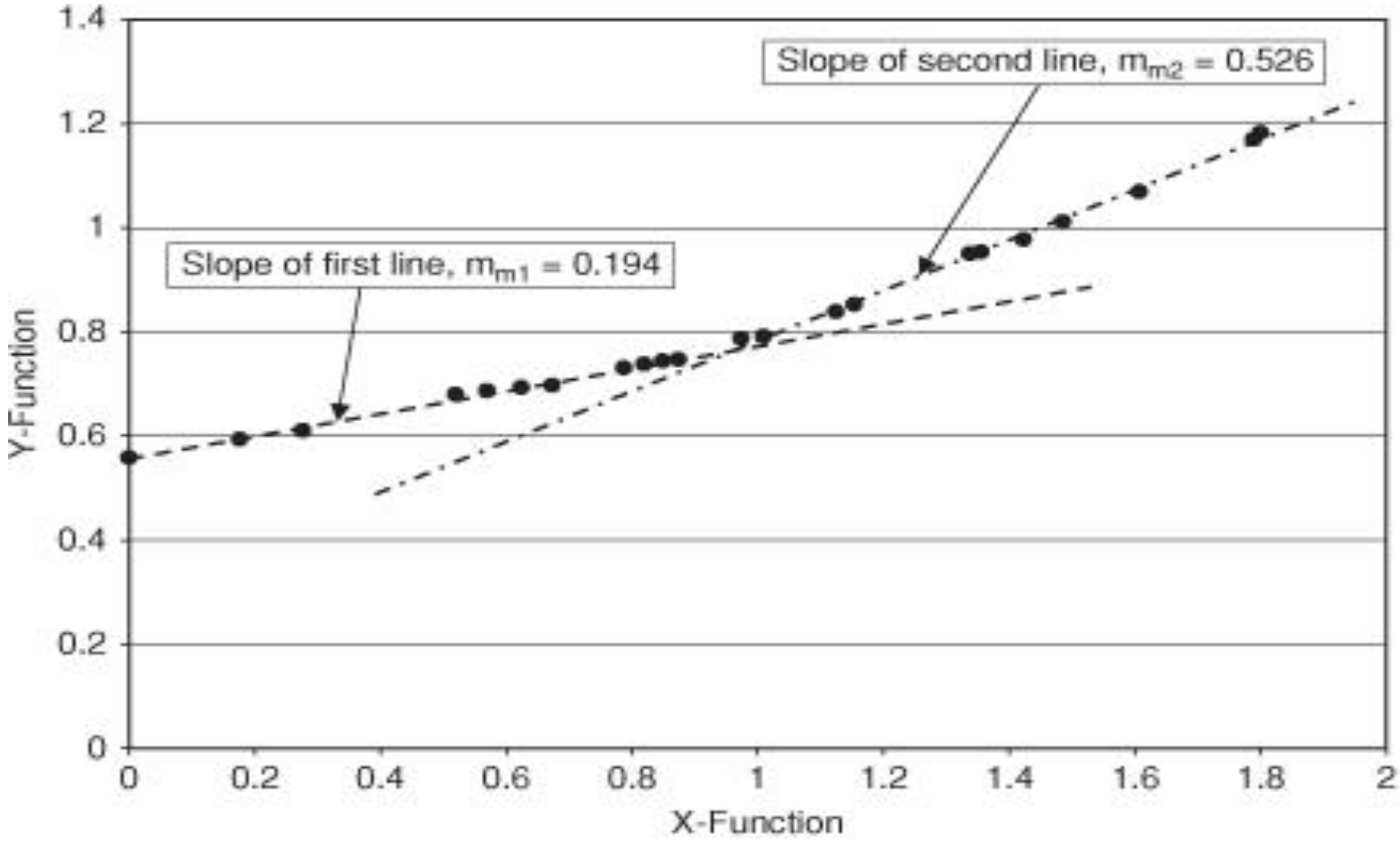
n	t (hrs.)	q_n (STB/D)	P_{wf} (psi)	Y_F (psi/STB-D)	X_F
1	0	0	2906	—	—
1	1.00	1580	2023	0.5589	0.0000
1	1.50	1580	1968	0.5937	0.1761
1	1.89	1580	1941	0.6108	0.2765
1	2.40	1580	—	—	—
2	3.00	1490	1892	0.6805	0.5193
2	3.45	1490	1882	0.6872	0.5690
2	3.98	1490	1873	0.6933	0.6241
2	4.50	1490	1867	0.6973	0.6732
2	4.80	1490	—	—	—
3	5.50	1440	1853	0.7313	0.7870
3	6.05	1440	1843	0.7382	0.8193
3	6.55	1440	1834	0.7444	0.8485
3	7.00	1440	1830	0.7472	0.8739
3	7.20	1440	—	—	—
4	7.50	1370	1827	0.7876	0.9737

Solution

4	8.95	1370	1821	0.7920	1.0091
4	9.60	1370	—	—	—
5	10.00	1300	1815	0.8392	1.1242
5	12.00	1300	1797	0.8531	1.1535
6	14.40	1260	—	—	—
7	15.00	1190	1775	0.9504	1.3374
7	18.00	1190	1771	0.9538	1.3553
7	19.20	1190	—	—	—
8	20.00	1160	1772	0.9776	1.4225
8	21.60	1160	—	—	—
9	24.00	1137	1756	1.0114	1.4851
10	28.80	1106	—	—	—
11	30.00	1080	1751	1.0694	1.6067
11	33.60	1080	—	—	—
12	36.00	1000	—	—	—
13	36.20	983	1756	1.1699	1.7883
13	48.00	983	1743	1.1831	1.7995

Solution

Step 3: Plot X_F vs. Y_F on a Cartesian graph.



Solution

Note that Figure has two straight lines. The slope of the first straight line is determined thus:

$$m_m = \frac{Y_{F2} - Y_{F1}}{X_{F2} - X_{F2}}$$

Using the values for Y_F and X_F at t = 6.55 hrs and t = 3.00 hrs give:

$$m_m = \frac{0.7444 - 0.6805}{0.8485 - 0.5193} = 0.194 \text{ psi/(STB/D-cycle)}$$

Similarly the slope of the second line is determined to be 0.526 psi/(STB/D-cycle). It is apparent that this second straight line represents late time region data that may have been affected by boundary effects. The slope of this line is **not used** in the calculations of permeability and skin factor since it is influenced by boundary effects.

Solution

Step 5: Calculate formation permeability.

$$k = \frac{162.6 B\mu}{m_m h}$$

$$= \frac{162.6 \times 1.27 \times 0.6}{0.194 \times 40} = 15.97 \text{ md.}$$

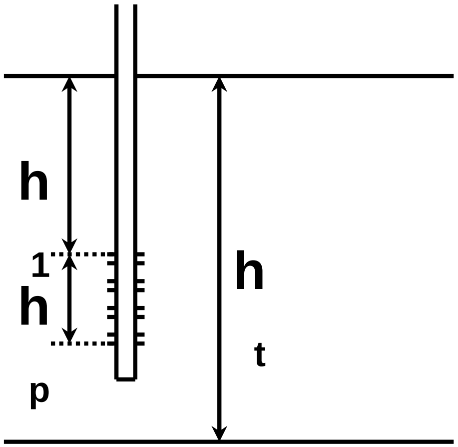
Step 6: Calculate skin factor.

From Figure, the intercept on the *y*-axis is $b_l = 0.5589$. Substituting gives:

$$s = 1.151\left[\frac{0.5589}{0.194} - \log\left(\frac{15.97}{0.24 \times 0.6 \times 5 \times 10^{-5} \times (0.5)^2}\right) + 3.23\right]$$

$$= -0.96$$

Partial Penetration Skin



$$s = \left(\frac{h}{h_p}\right) s_d + s_p$$

Saidikowski estimated the skin factor due to partial penetration from the following expression:

$$s_p = \left(\frac{h}{h_p} - 1\right)\left[\ln\left(\frac{h}{r_w}\sqrt{\frac{k_h}{k_v}}\right) - 2\right]$$

where:

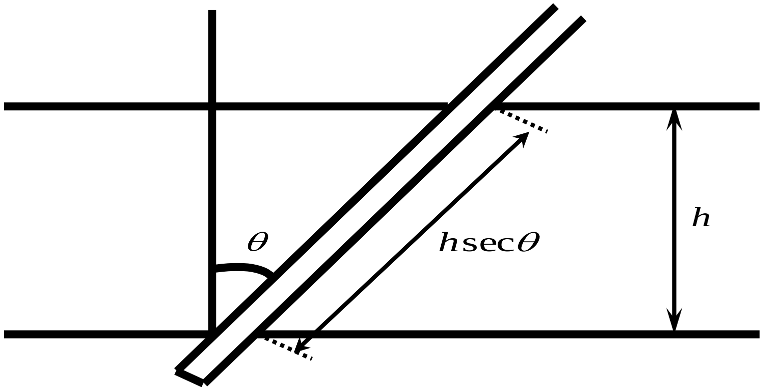
r_w = wellbore radius, ft
h_p = perforated interval, ft
h = total thickness, ft
k_h = horizontal permeability, md
k_v = vertical permeability, md

Geometric Skin - Deviated Wellbore

When a well penetrates the formation at an angle other than 90 degrees, there is more surface area in contact with the formation. This results in a negative apparent skin factor. This effect decreases as the vertical permeability decreases, and increases as the angle from the vertical increases.

$$s = s_d + s_\theta$$

$$h_D = \frac{h}{r_w}\sqrt{\frac{k_h}{k_v}}$$



$$\theta_w' = \tan^{-1}\left(\sqrt{\frac{k_v}{k_h}}\tan\theta_w\right)$$

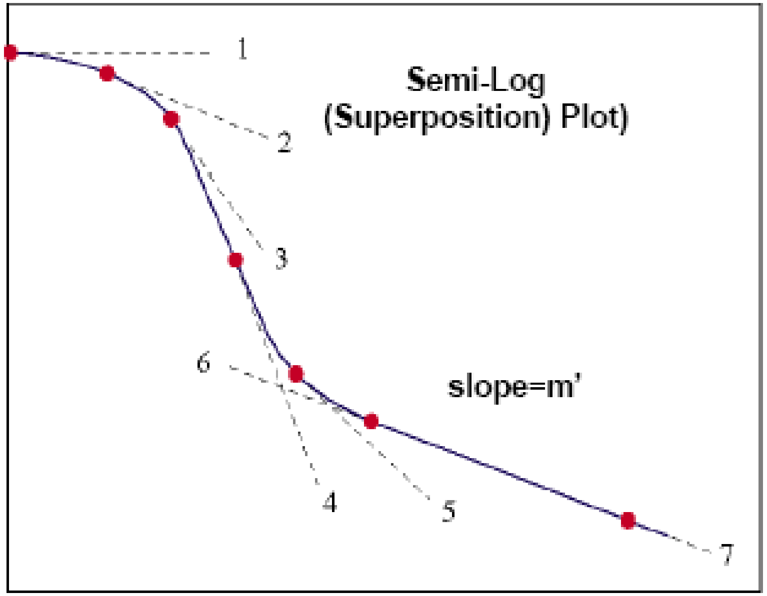
$$s_\theta = -\left(\frac{\theta_w'}{41}\right)^{2.06} - \left(\frac{\theta_w'}{56}\right)^{1.865}\log\left(\frac{h_D}{100}\right)$$

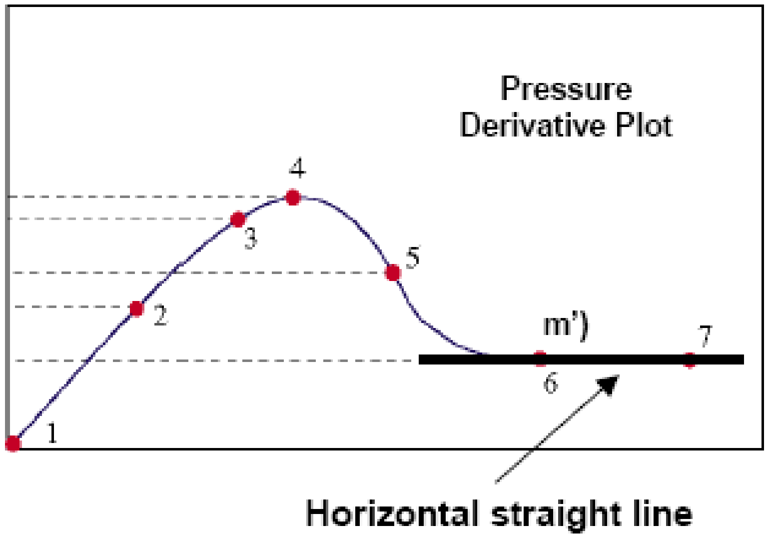
Derivative Analysis

Semi-log approximation $$p_D(t_D) = \frac{1}{2}\left[\ln t_D + 0.80907 + 2S\right]$$

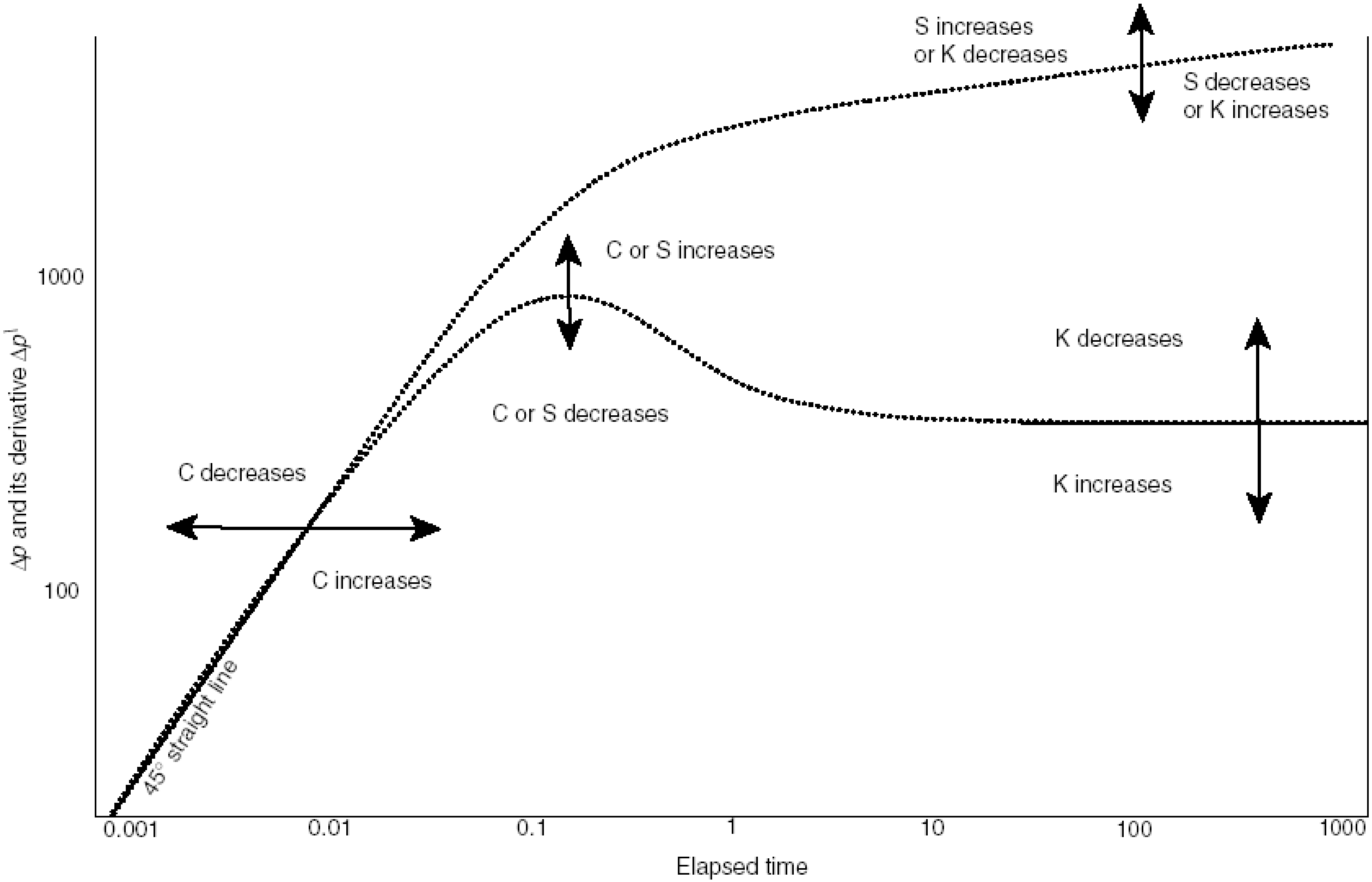
Derivative $$p_D{}' = \frac{dp_D}{d\ln t_D} = t_D\frac{dp_D}{dt_D} = \frac{1}{2}$$

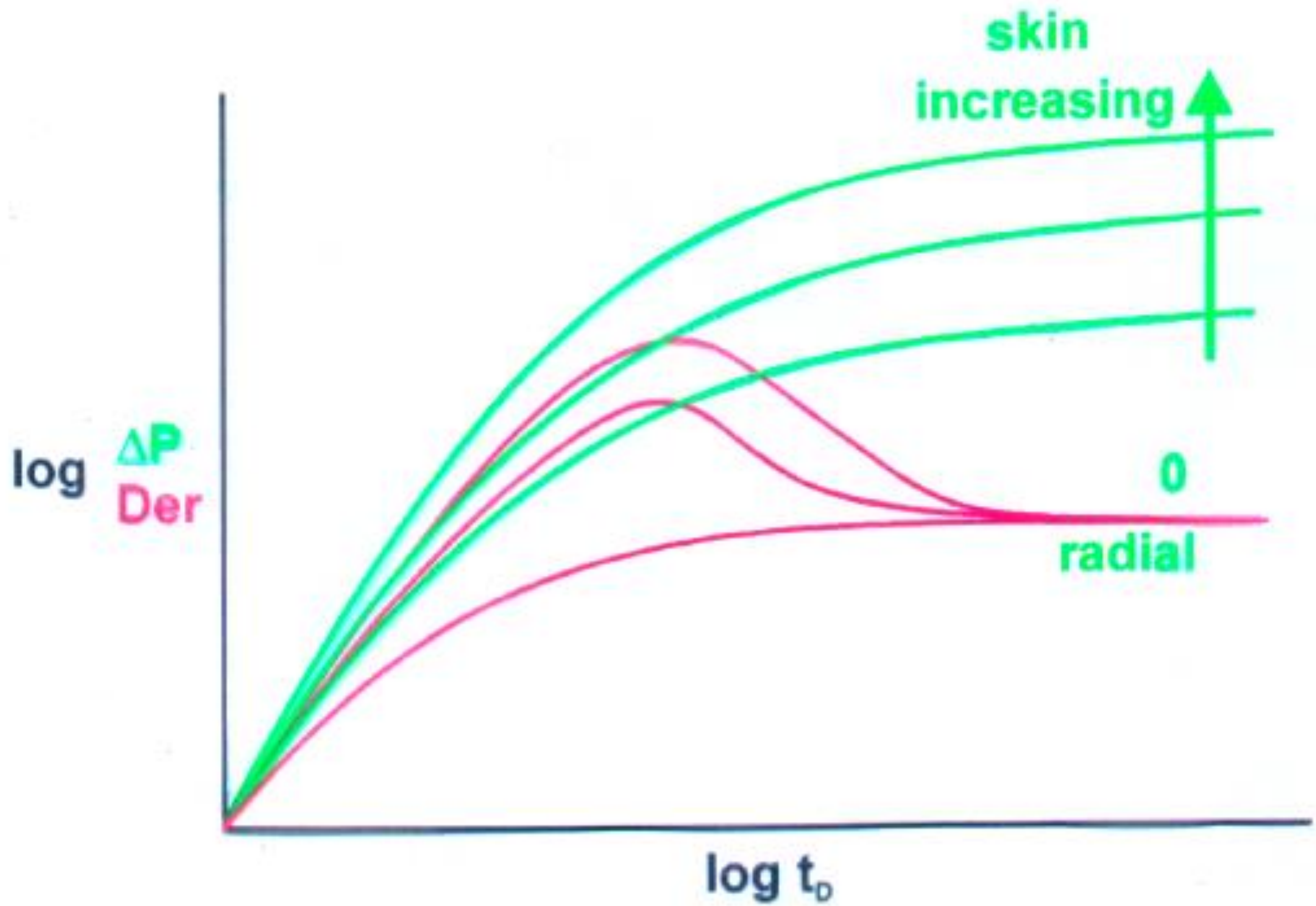
In infinite acting radial flow, the derivative stabilizes at 0.5



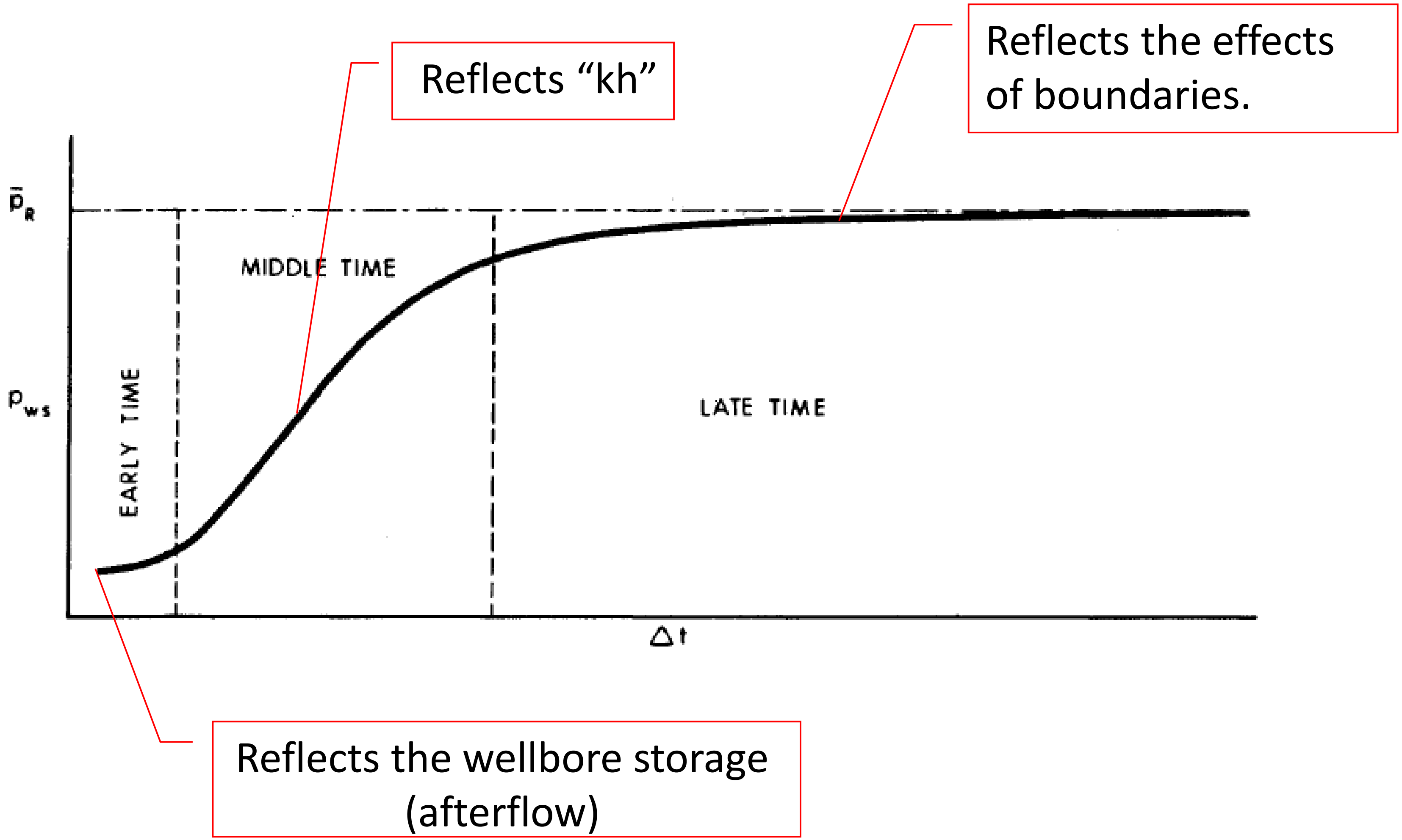


Derivative Analysis



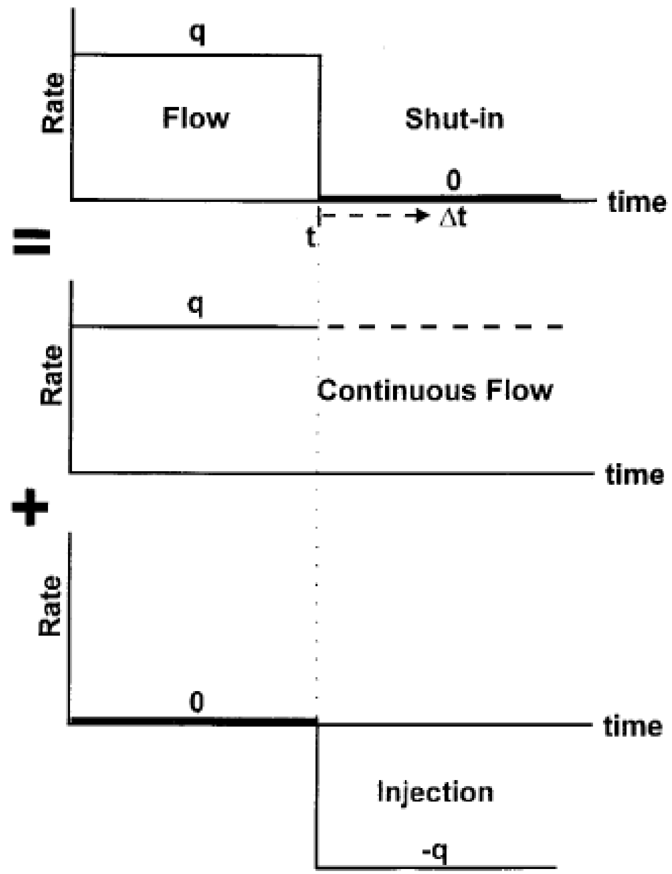

skin
increasing
log ΔP
Der
0
radial
log t_D

Behavior of Static Sandface Pressure Upon Shut-in of a Well

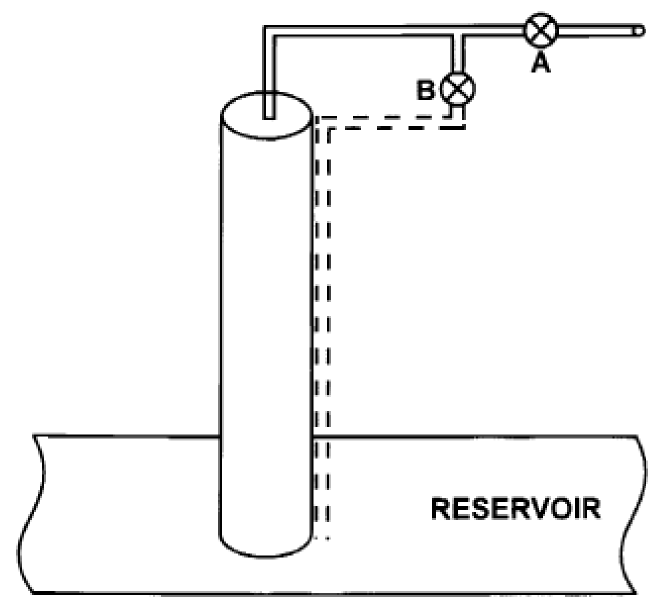


Buildup Test Analysis

Buildup as Superposition (Rates)



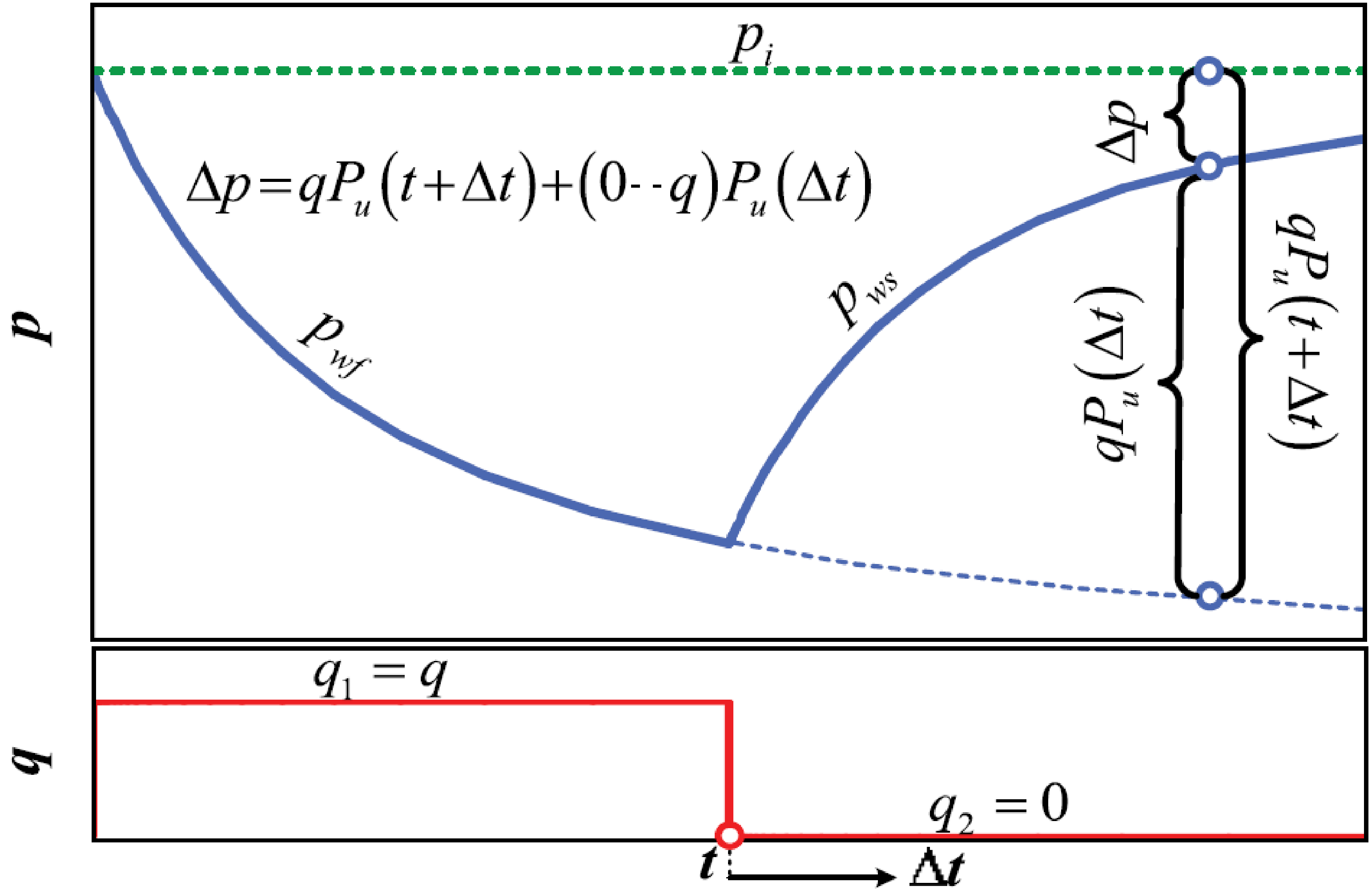
MATHEMATICAL MODELLING of SHUT-IN



FLOW	-	VALVE A	OPEN
	-	VALVE B	CLOSED
BUILDUP	-	VALVE A	CLOSED
	-	VALVE B	OPEN

≡
- NO production at surface
- Continuous production and injection at reservoir

Buildup as Example of Superposition



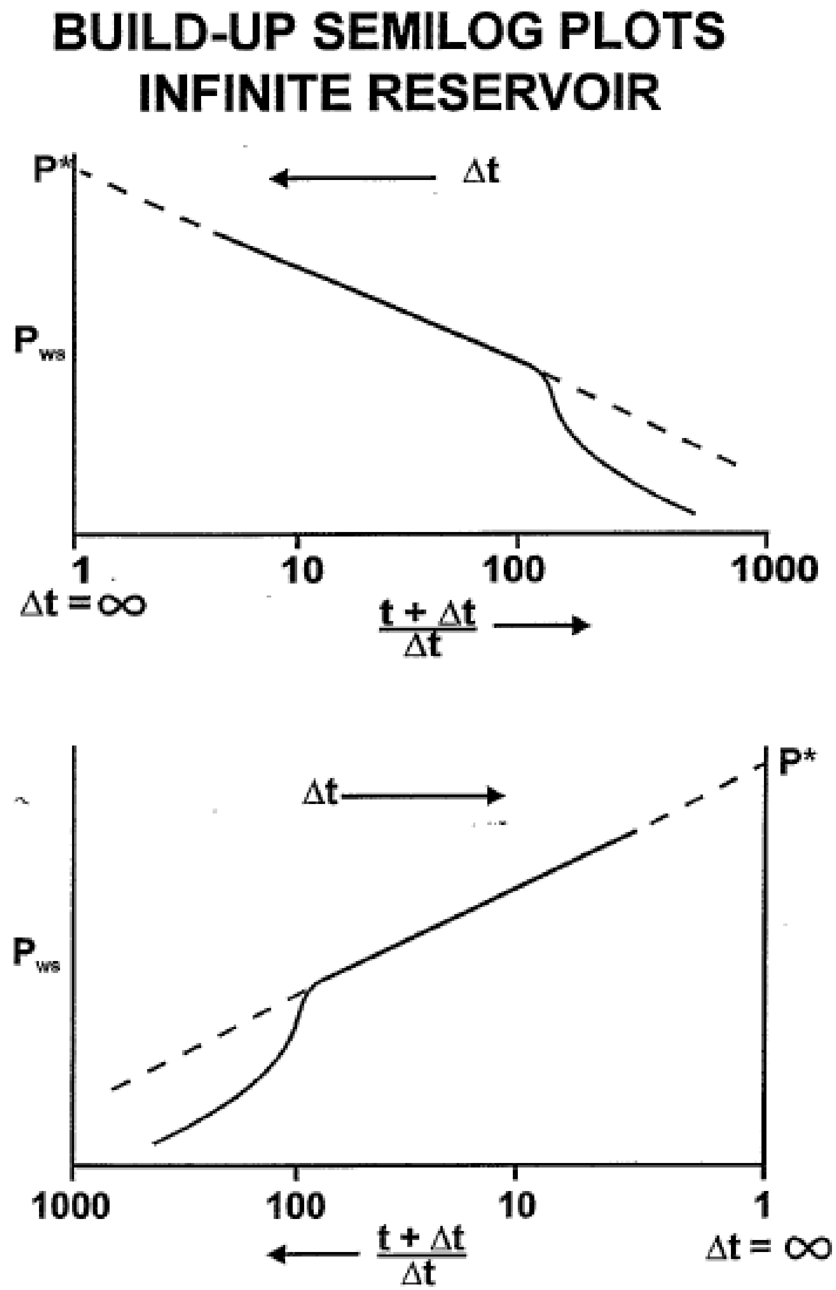
Horner plot relationship

$$p_i - p_{ws}(t) = \frac{162.6 q B_o \mu}{kh} \log\left(\frac{t+\Delta t}{\Delta t}\right)$$

$$Horner\ time = \left(\frac{t+\Delta t}{\Delta t}\right)$$

Slope of semilog straight line same as drawdown – used to calculate permeability.

$$m = \frac{162.6 q B_o \mu}{kh}$$



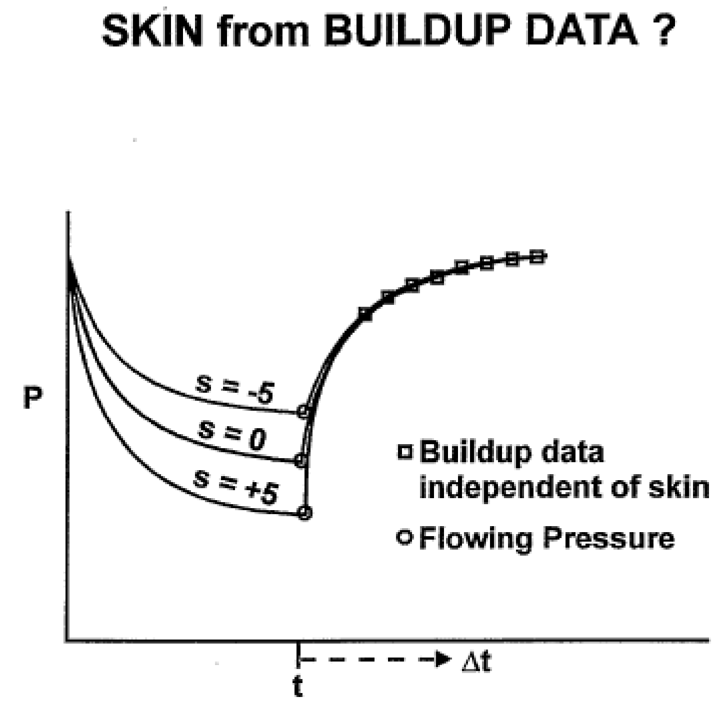
Buildup test does NOT allow for skin calculation. Skin is obtained from FLOWING pressure before shut-in.

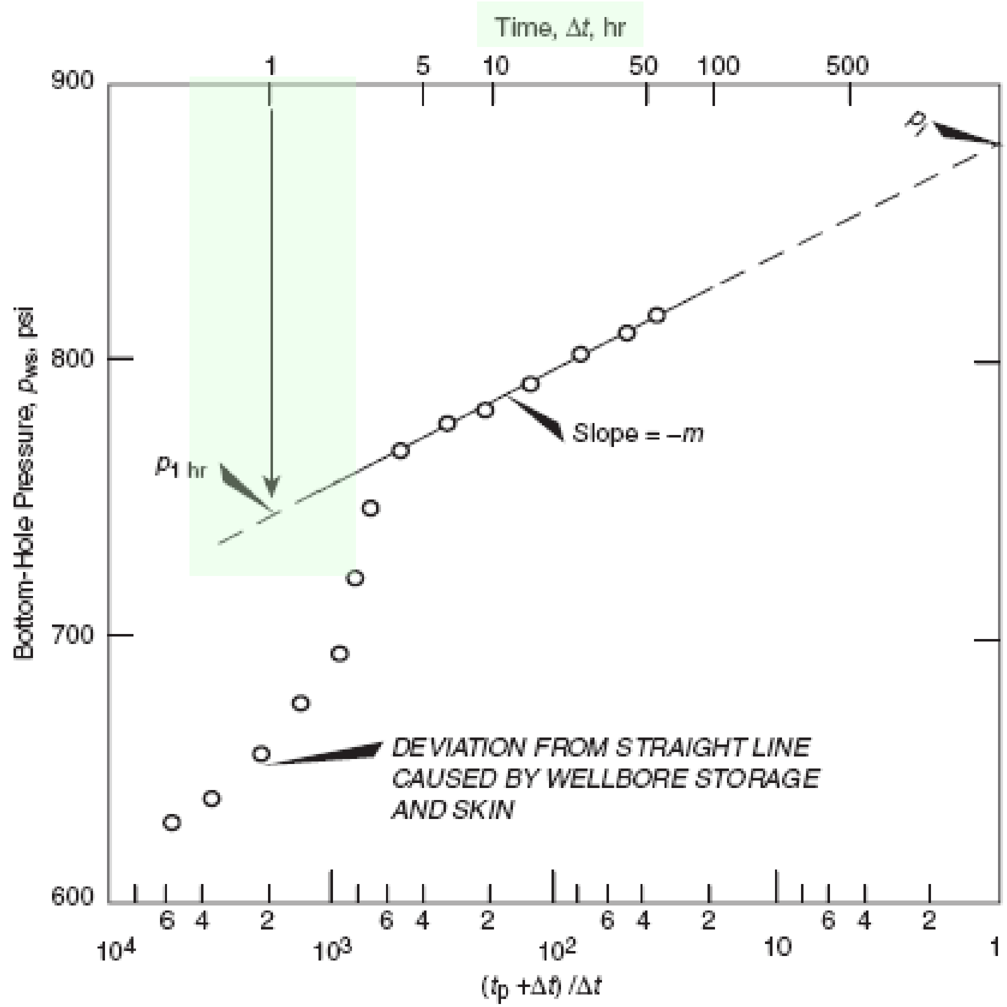
$$p_{ws}(t_p+\Delta t)-p_{wf}(t_p)=\frac{162.6qB_o\mu}{kh}\left[\log(t_p)+\log\left(\frac{k}{\phi\mu c_t r_w^2}\right)-3.23+0.87S\right]-\frac{162.6qB_o\mu}{kh}\log\left(\frac{t_p+\Delta t}{\Delta t}\right)$$

$$p_{ws}(t_p+\Delta t)-p_{wf}(t_p)=\frac{162.6qB_o\mu}{kh}\left[\log\left(\frac{t_p\Delta t}{t_p+\Delta t}\right)+\log\left(\frac{k}{\phi\mu c_t r_w^2}\right)-3.23+0.87S\right]$$

$$\Delta t=1\ hr$$

$$S=1.151\left\{\frac{p_{1hr}-p_{wf}}{m}-\log\left[\frac{k}{\phi\mu c_t r_w^2}\frac{t_p}{t_p+1}\right]+3.23\right\}$$





$$S = 1.151\left[\frac{p_{1hr} - p_{wf}}{m} - \log\left(\frac{k}{\phi\mu\, c_t r_w^2}\right) + 3.23\right]$$

Equivalent Time

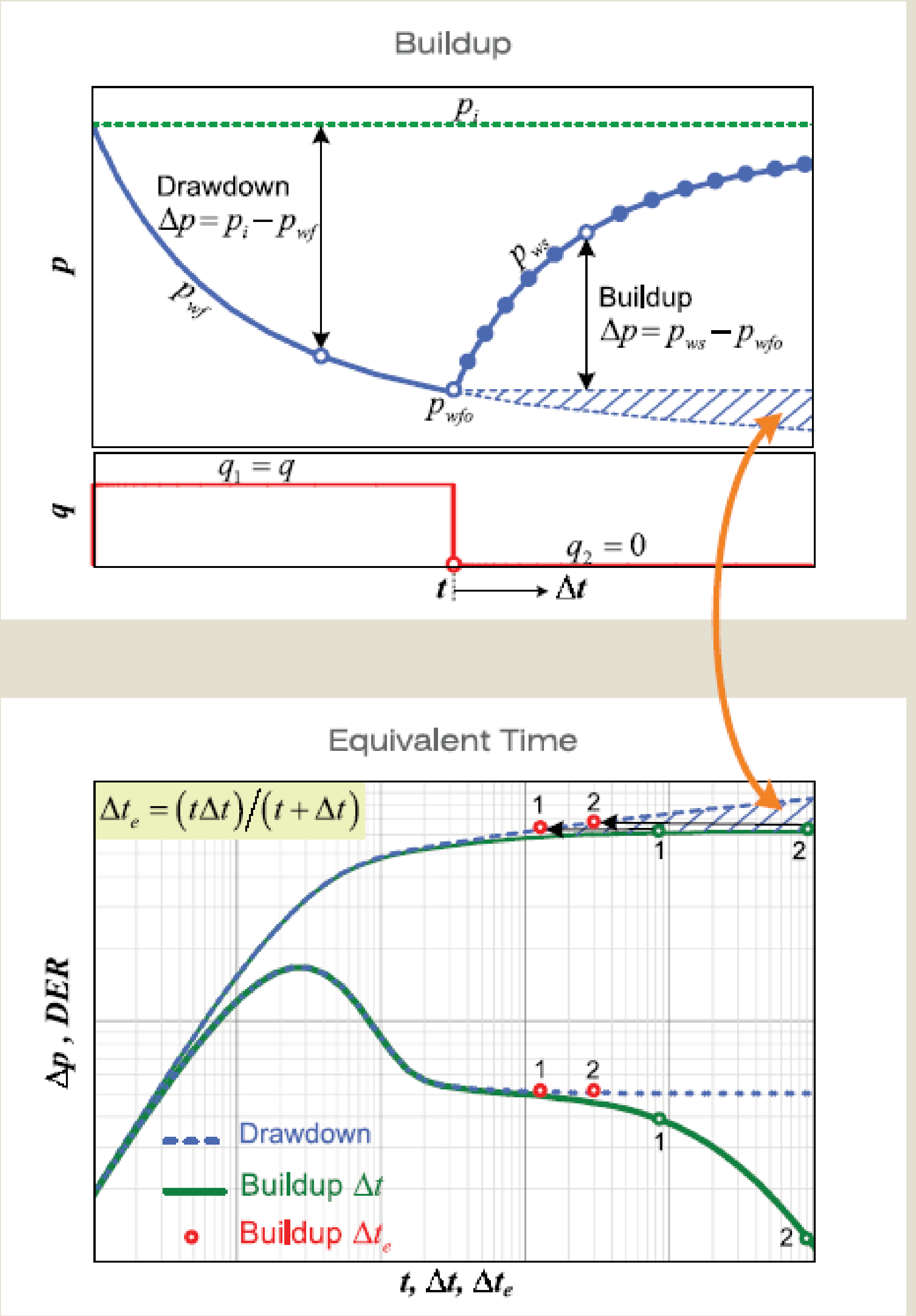
Measurable Pressure Difference

$$\left\lfloor p_{ws}(\Delta t) - p_{wf}(t_p) \right\rfloor$$

Correct Pressure Difference

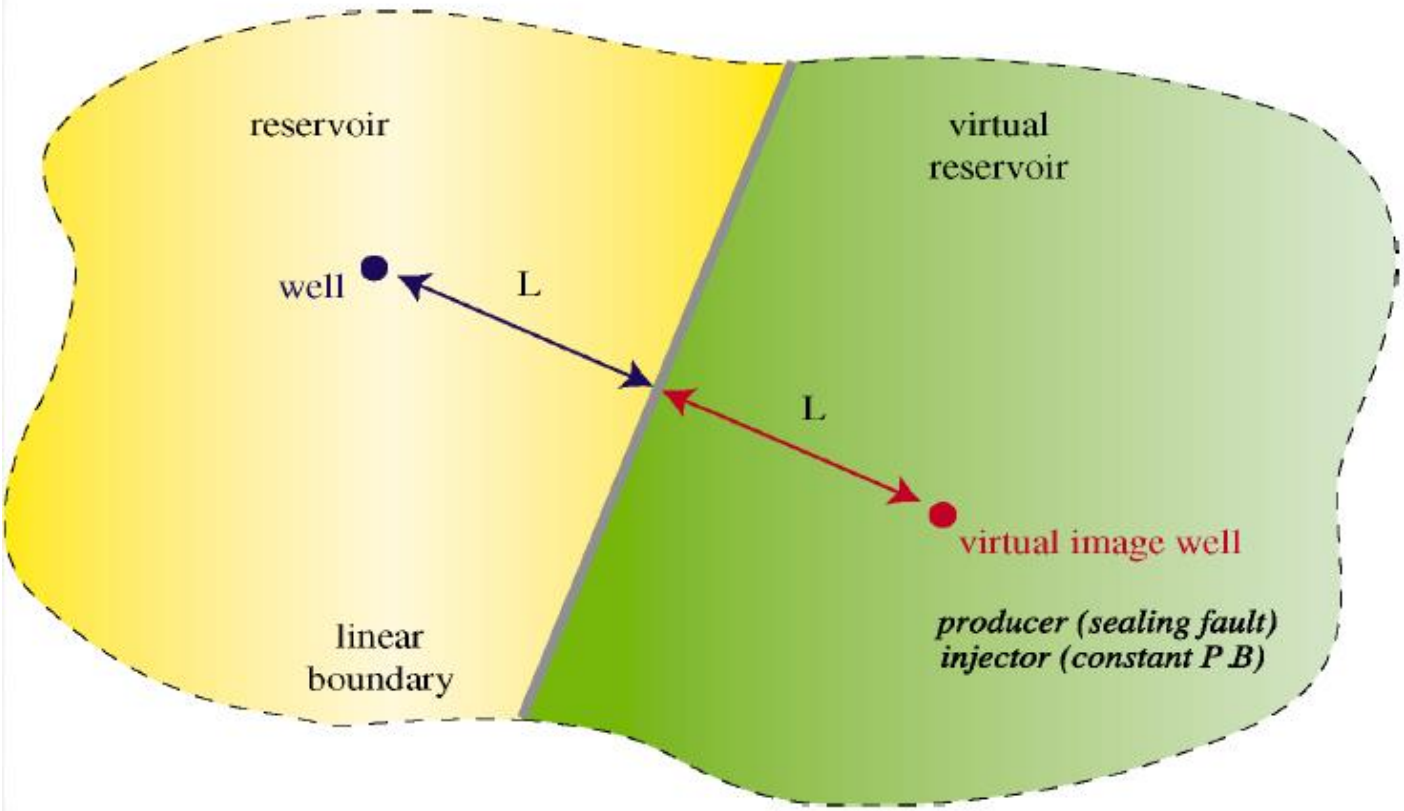
$$\left\lfloor p_{ws}(\Delta t) - p_{wf}(t_p + \Delta t) \right\rfloor$$

$$\Delta t_e \equiv \frac{t_p \Delta t}{t_p + \Delta t}$$



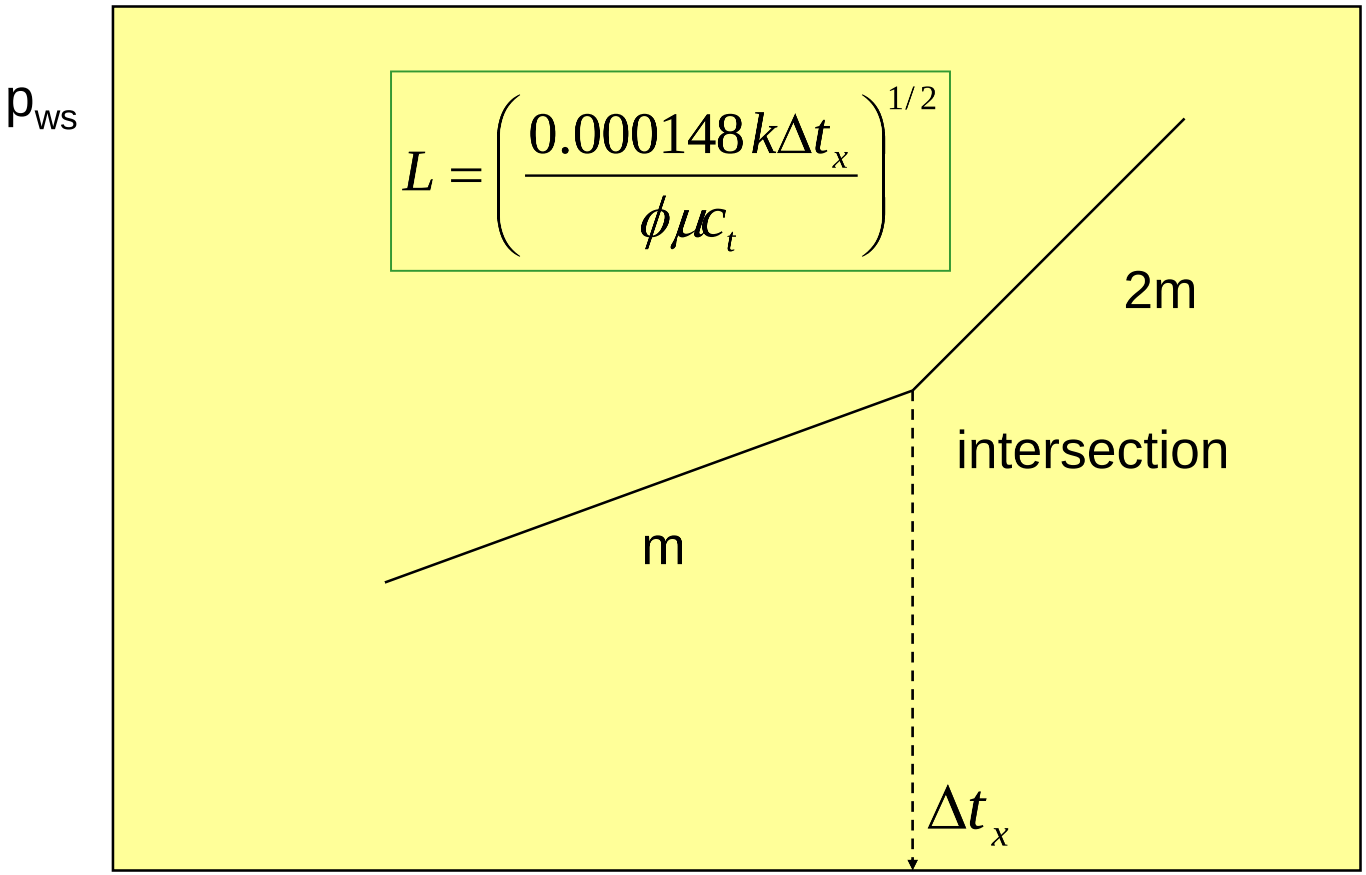
Distance to Fault - Image Well

Mathematically, the above boundary condition can be met by placing an image well, identical to that of the actual well, on the other side of the fault at exactly distance L. Consequently, the effect of the boundary on the pressure behavior of a well would be the same as the effect from an image well located a distance 2L from the actual well.



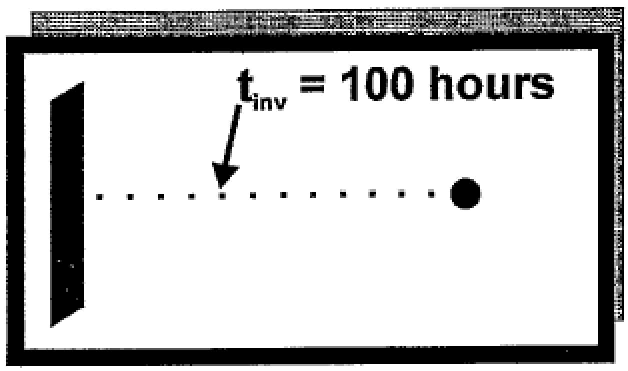
Estimating Distance to a Fault from Horner Plot

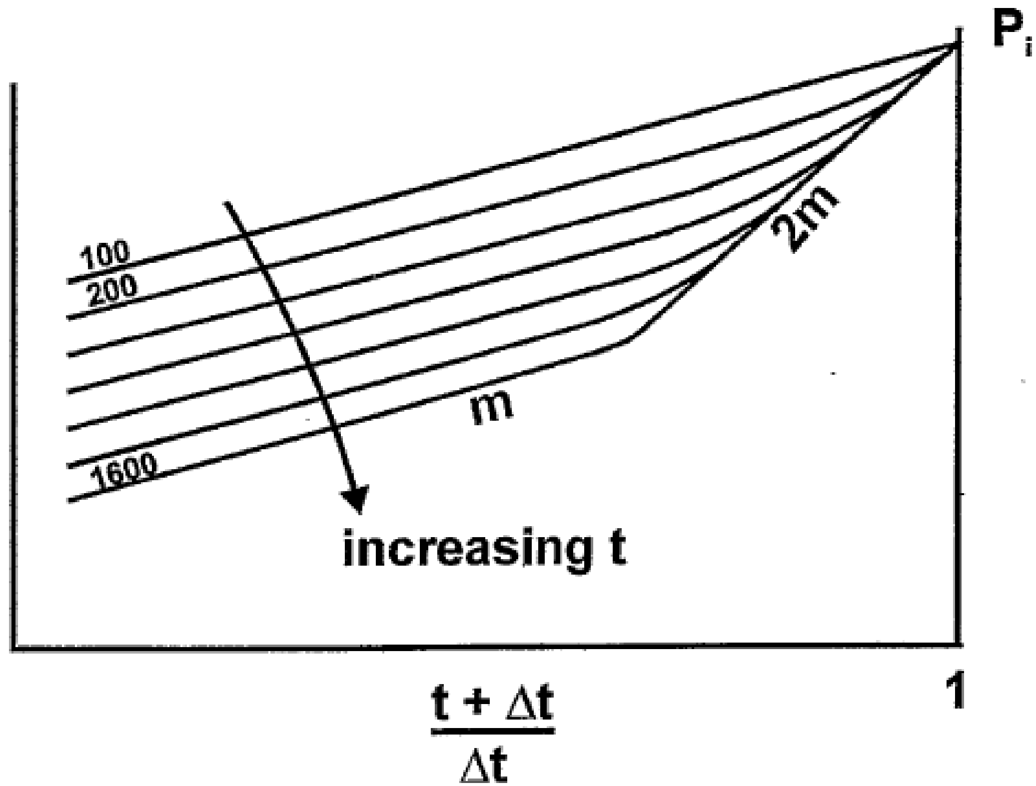
$$L = \left(\frac{0.000148\, k \Delta t_x}{\phi \mu c_t} \right)^{1/2}$$



Log of Horner Time Ratio $\frac{t_p + \Delta t}{\Delta t}$

Time to Doubling the Slope





Doubling of slope only seen clearly when t >> 16 t_{inv}

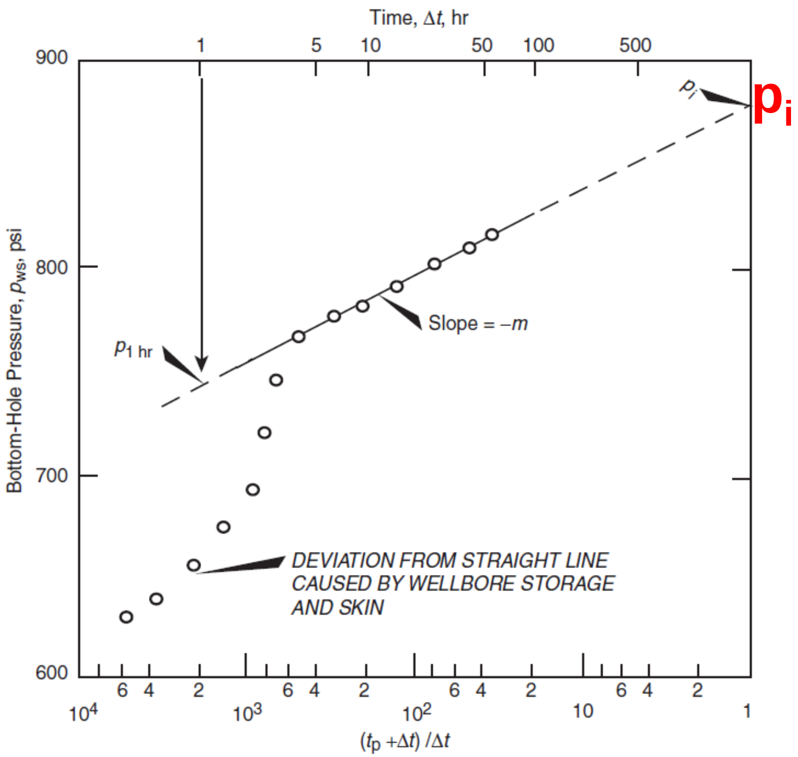
Horner Plot

It should be pointed out that Equation 1.3.6 assumes the reservoir to be <u>infinite in size</u>, i.e., $r_e = \infty$, which implies that at some point in the reservoir the pressure would be always equal to the initial reservoir pressure p_i and the Horner straight-line plot will <u>always extrapolate to p_i</u>.

$$p_{ws} = p_i - \frac{162.6 Q_o \mu_o B_o}{kh}\left[\log\left(\frac{t_p + \Delta t}{\Delta t}\right)\right] \qquad [1.3.6]$$

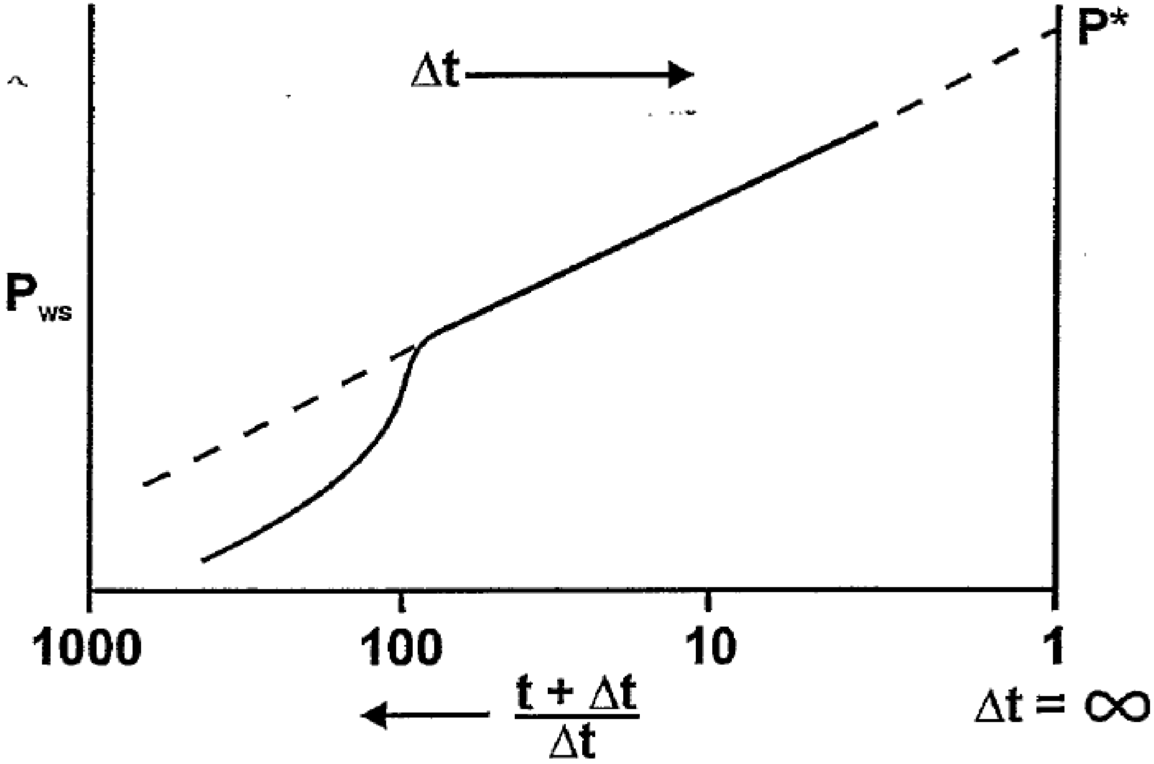
where:

p_i = initial reservoir pressure, psi
p_{ws} = sand face pressure during pressure buildup, psi
t_p = flowing time before shut-in, hours
Q_o = stabilized well flow rate before shut-in, STB/day
Δt = shut-in time, hours



False Pressure (1)

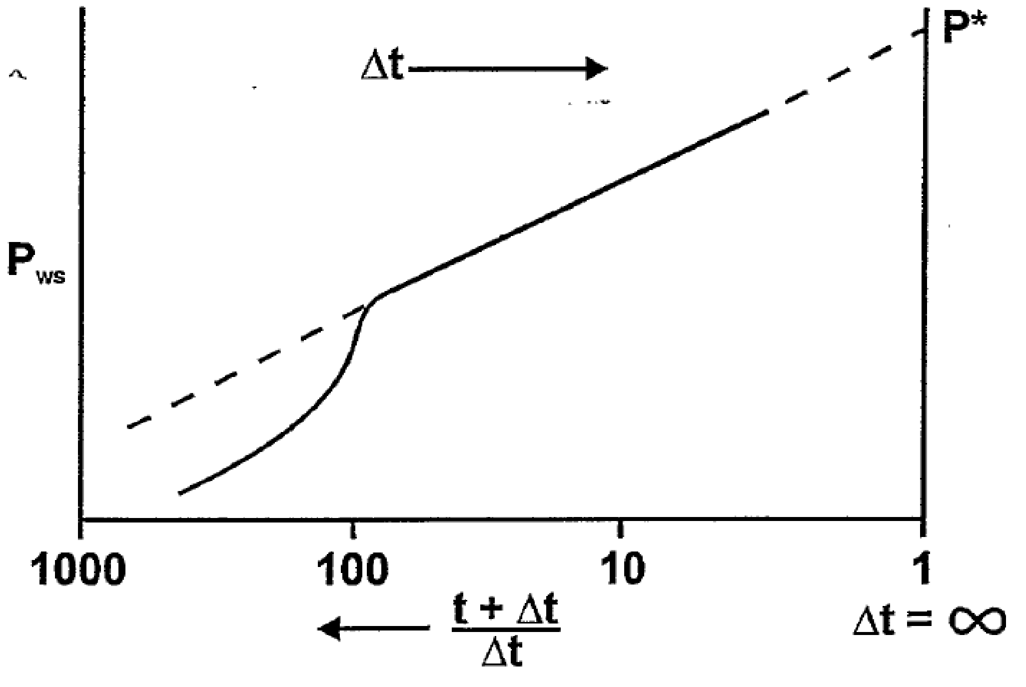
However, reservoirs are finite and soon after production begins, fluid removal will cause a pressure decline everywhere in the reservoir system. Under these conditions, the straight line will not extrapolate to the initial reservoir pressure p_i but, instead, the pressure obtained will be a false pressure as denoted by p^*.



False Pressure (2)

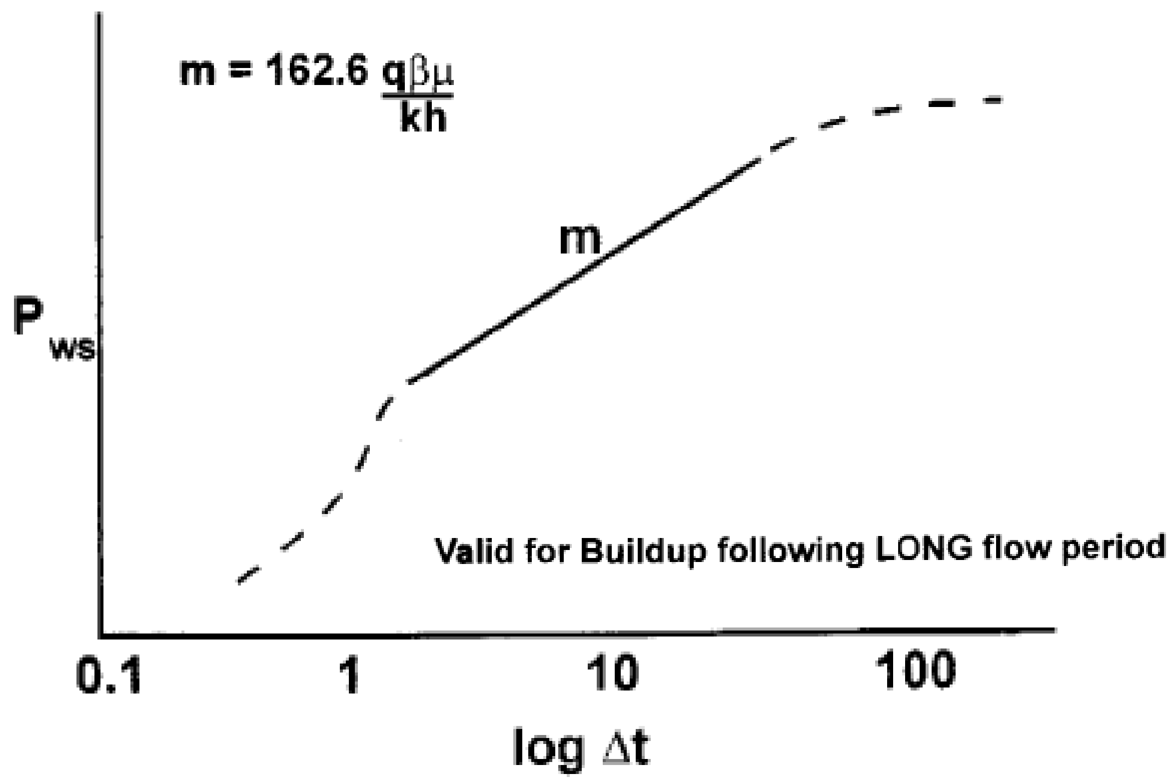
The false pressure, as illustrated by Matthews and Russell (1967), has **no** physical meaning but it is usually used to determine the average reservoir pressure **p**. It is clear that **p*** will only equal the initial (original) reservoir pressure $\mathbf{p_i}$ when a new well in a newly discovered field is tested.

$$p_{ws} = p^* - m\left[\log\left(\frac{t_p + \Delta t}{\Delta t}\right)\right] \quad [1.3.10]$$



Miller–Dyes–Hutchinson Method (MDH)

M.D.H. Plot



Miller–Dyes–Hutchinson Method (MDH)

The observed pressure behavior of the test well following the end of the transient flow will depend on:

- shape and geometry of the test well drainage area;
- the position of the well relative to the drainage boundaries;
- length of the producing time $\mathbf{t_p}$ before shut-in.

(1) Choose any convenient time on the semilog straight line Δt and read the corresponding pressure p_{ws}.

(2) Calculate the dimensionless shut-in time based on the drainage area A from:

$$\Delta t_{DA} = \frac{0.0002637 k \Delta t}{\phi \mu c_t a}$$

(3) Enter Figure 1.41 with the dimensionless time $\Delta t_D A$ and determine an MDH dimensionless pressure p_{DMDH} from the upper curve of Figure 1.41.

(4) Estimate the average reservoir pressure in the closed drainage region from:

$$\bar{p}_r = p_{ws} + \frac{m p_{DMDH}}{1.1513}$$

where m is the semilog straight line of the MDH plot.

MDH

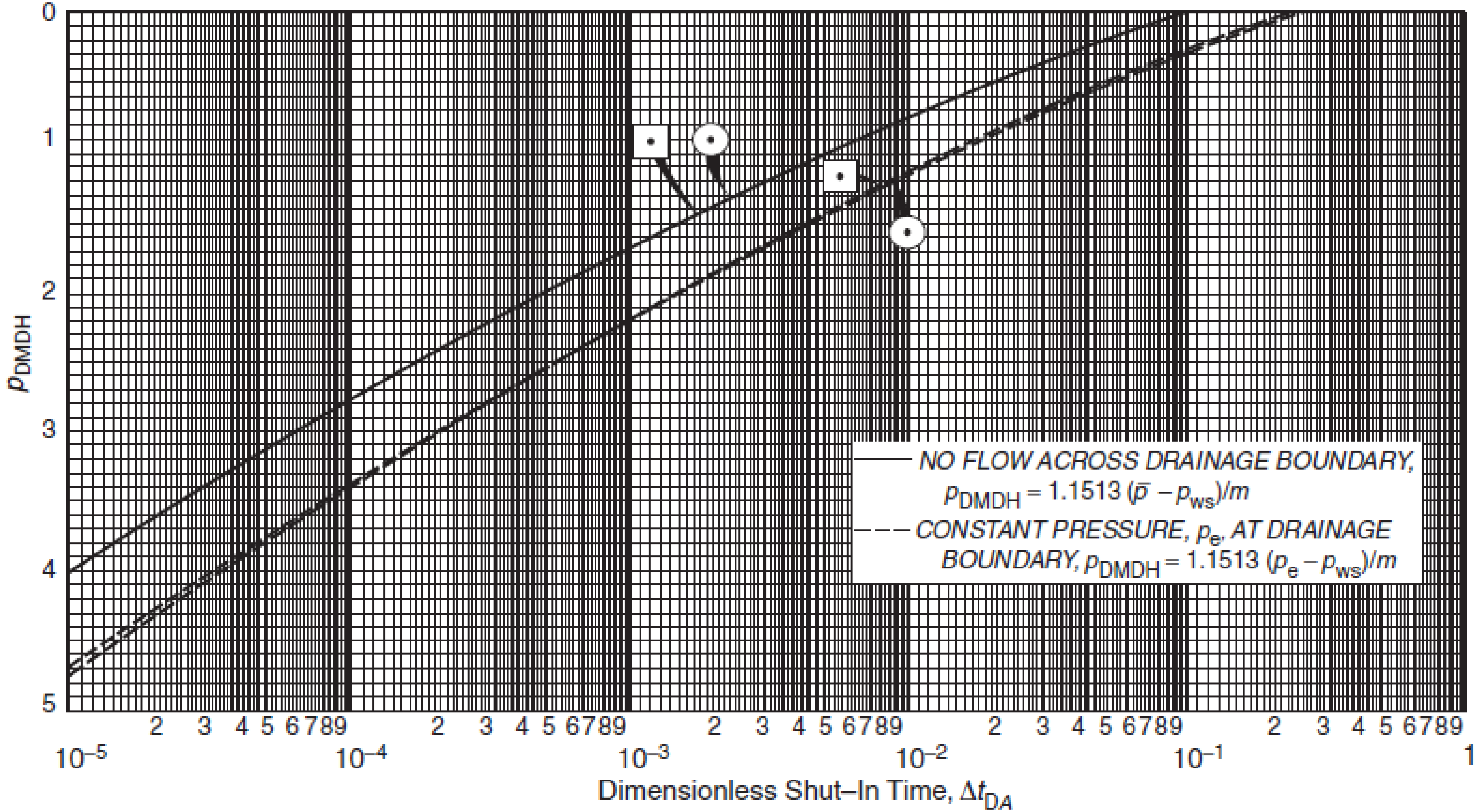


Figure 1.41 *Miller–Dyes–Hutchinson dimensionless pressure for circular and square drainage areas (After Earlougher,* R. Advances in Well Test Analysis) *(Permission to publish by the SPE, copyright SPE, 1977).*

Matthews *et al* (MBH) Method

- A methodology for estimating average pressure from buildup tests in bounded drainage regions.

- The MBH method is based on theoretical correlations between the extrapolated semi-log straight line to the false pressure **p*** and current average drainage area pressure $\mathbf{p_{avg}}$.

- The authors point out that the average pressure in the drainage area of each well can be related to **p*** if the geometry, shape, and location of the well relative to the drainage boundaries are known.

- They developed a set of correction charts, as shown in Figures 1.42 through 1.45, for various drainage geometries.

MBH Method

The y axis of these figures represents the MBH dimensionless pressure p_{DMBH} that is defined by:

$$p_{DMBH} = \frac{2.303(p^* - \overline{p})}{|m|}$$

or:

$$\overline{p} = p^* - \left(\frac{|m|}{2.303}\right) p_{DMBH} \qquad [1.3.13]$$

where m is the *absolute* value of the slope obtained from the Horner semilog straight-line plot. The MBH dimensionless pressure is determined at the dimensionless producing time t_{pDA} that corresponds to the flowing time t_p. That is:

$$t_{pDA} = \left[\frac{0.0002637k}{\phi \mu c_t A}\right] t_p \qquad [1.3.14]$$

where:

t_p = flowing time before shut-in, hours
A = drainage area, ft^2
k = permeability, md
c_t = total compressibility, psi^{-1}

MBH Method

The following steps summarize the procedure for applying the MBH method:

Step 1. Make a Horner plot.
Step 2. Extrapolate the semilog straight line to the value of p^* at $(t_p + \Delta t)/\Delta t = 1.0$.
Step 3. Evaluate the slope of the semilog straight line m.
Step 4. Calculate the MBH dimensionless producing time t_{pDA} from Equation 1.3.14:

$$t_{pDA} = \left[\frac{0.0002637k}{\phi \mu c_t A}\right] t_p$$

Step 5. Find the closest approximation to the shape of the well drainage area in Figures 1.41 through 1.44 and identify the correction curve.

Step 6. Read the value of p_{DMBH} from the correction curve at t_{PDA}
Step 7. Calculate the value of $\bar{p}$ from Equation 1.3.13:

$$\bar{p} = p^* - \left(\frac{|m|}{2.303}\right) p_{DMBH}$$

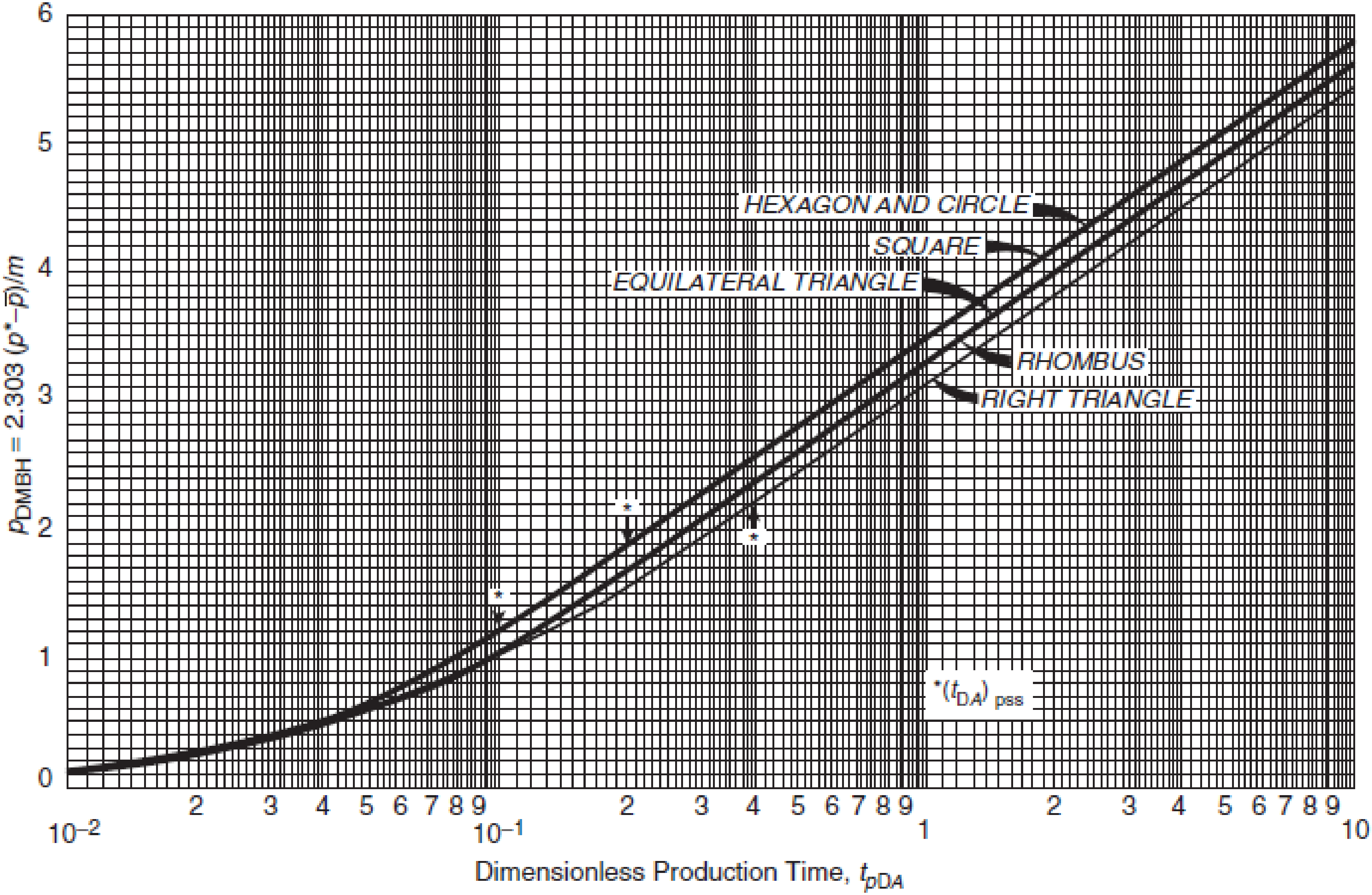


Figure 1.42 *Matthews–Brons–Hazebroek dimensionless pressure for a well in the center of equilateral drainage areas (After Earlougher, R.* Advances in Well Test Analysis*) (Permission to publish by the SPE, copyright SPE, 1977).*

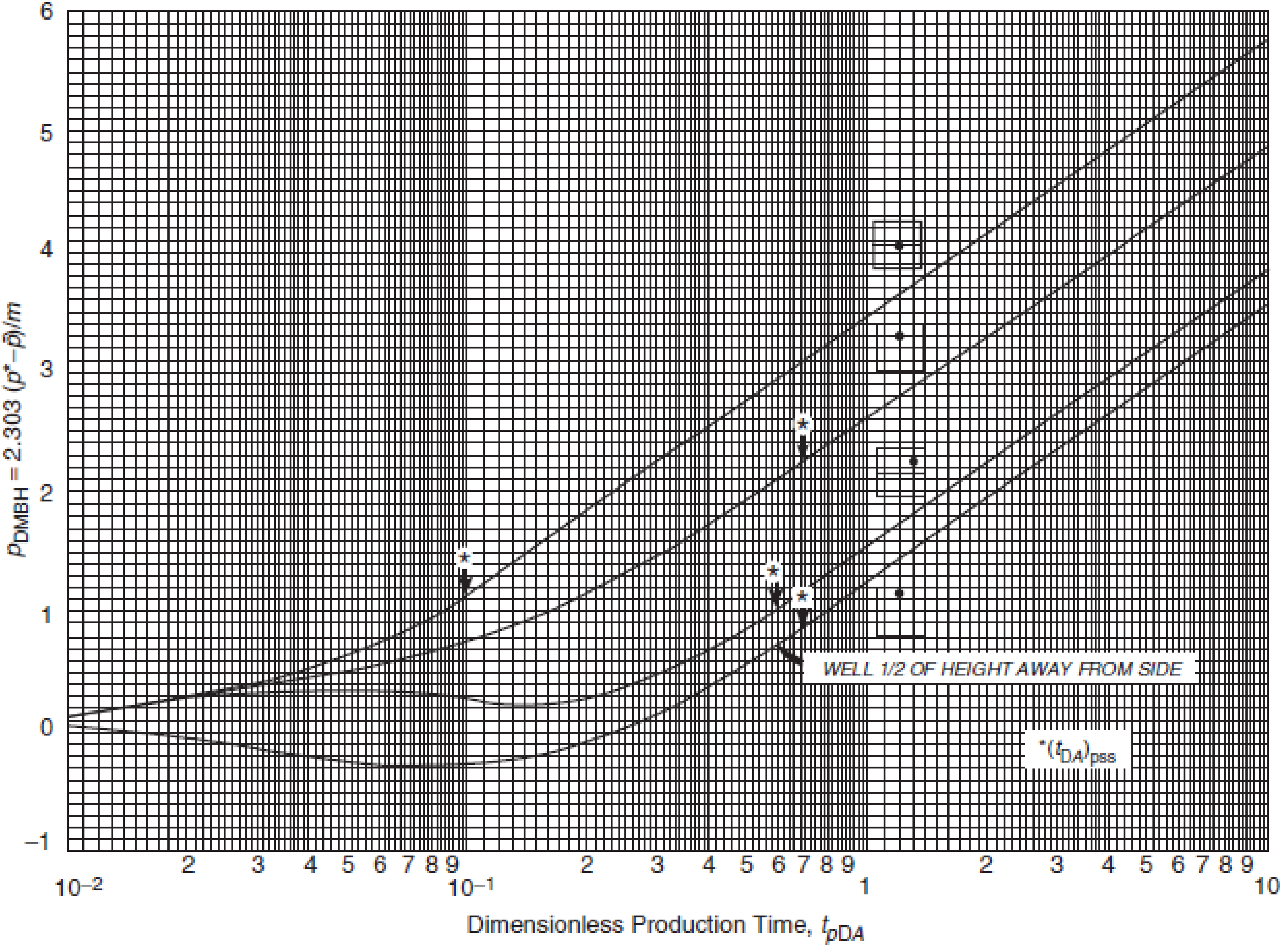


Figure 1.43 *Matthews–Brons–Hazebroek dimensionless pressure for different well locations in a square drainage area. (After Earlougher, R.* Advances in Well Test Analysis*) (Permission to publish by the SPE, copyright SPE, 1977).*

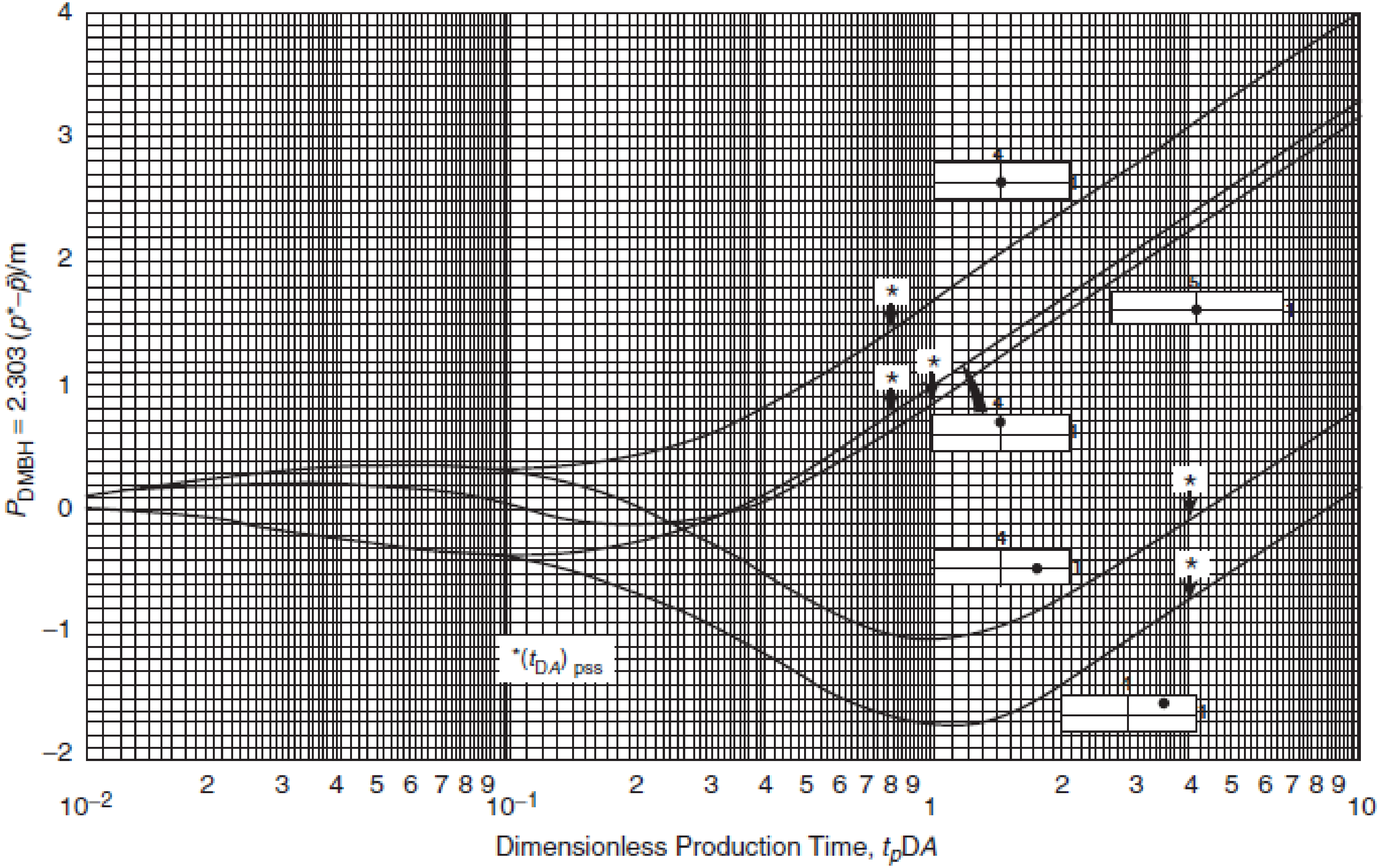
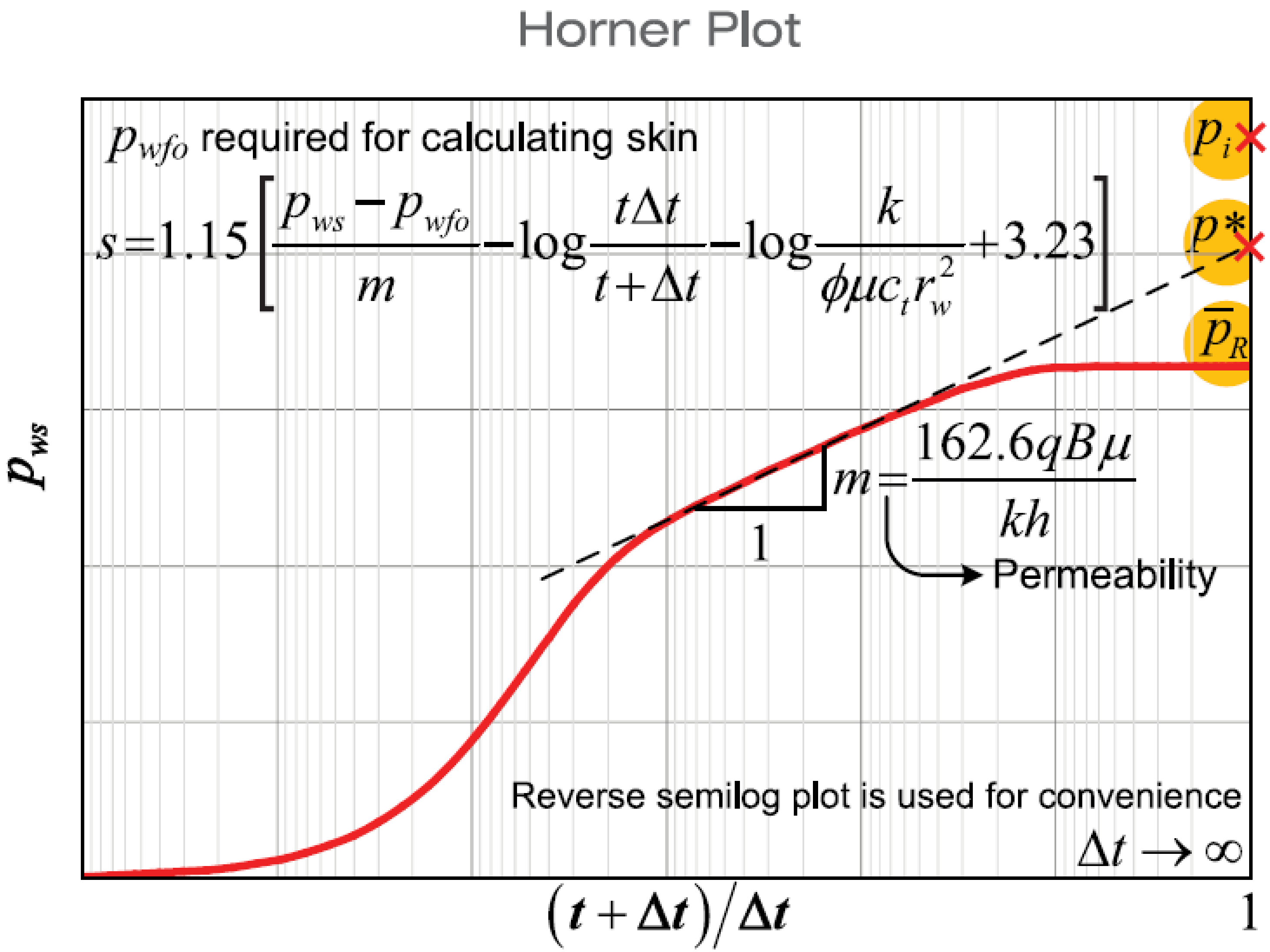


Figure 1.45 *Matthews–Brons–Hazebroek dimensionless pressure for different well locations in 4:1 and 5:1 rectangular drainage areas (After Earlougher, R.* Advances in Well Test Analysis*) (Permission to publish by the SPE, copyright SPE, 1977).*

Summary



Horner Plot
p_{wfo} required for calculating skin
$s = 1.15\left[\frac{p_{ws} - p_{wfo}}{m} - \log\frac{t\Delta t}{t+\Delta t} - \log\frac{k}{\phi\mu c_t r_w^2} + 3.23\right]$
p_i
p^*
$\bar{p}_R$
$m = \frac{162.6qB\mu}{kh}$
1
Permeability
Reverse semilog plot is used for convenience
$\Delta t \rightarrow \infty$
p_{ws}
$(t+\Delta t)/\Delta t$
1

Summary

- **Horner plot**: graph of p_{ws} vs. $\log\left((t+\Delta t)/\Delta t\right)$
  - Extrapolation to infinite shut-in time yields p^*
  - Infinite reservoir: $p^* = p_i$
  - Finite reservoir: $p^* \neq p_i \neq \overline{p}_R$
  - If $p^* > p_i$, the Horner semi-log line is wrong, p_i is wrong, or there is a constant pressure boundary

- **MDH plot**: graph of p_{ws} vs. $\log(\Delta t)$
  - Useful when producing time (t) is long
  - Analysis only valid when Δt <10% of t

Type Curves

- The type curve analysis approach was introduced in the petroleum industry by Agarwal et al. (1970) as a valuable tool when used in conjunction with conventional semilog plots.
- A type curve is a graphical representation of the theoretical solutions to flow equations.
- The type curve analysis consists of finding the theoretical type curve that "matches" the actual response from a test well and the reservoir when subjected to changes in production rates or pressures.
- The match can be found graphically by physically superposing a graph of actual test data with a similar graph of type curve(s) and searching for the type curve that provides the best match.
- Since type curves are plots of theoretical solutions to transient and pseudosteady-state flow equations, they are usually presented in terms of dimensionless variables (e.g., pD, Td, rD, and CD) rather than real variables (e.g., p, t, r, and C).
- The reservoir and well parameters, such as permeability and skin, can then be calculated from the dimensionless parameters defining that type curve.

Dimensionless Variables
Radial Flow With WBS And Skin

$$p_D \equiv \frac{kh\left(p_i - p\right)}{141.2qB\mu}$$

$$t_D \equiv \frac{0.0002637kt}{\phi\mu c_t r_w^2}$$

$$r_D \equiv \frac{r}{r_w}$$

$$s \equiv \frac{kh\Delta p_s}{141.2qB\mu}$$

$$C_D \equiv \frac{0.8936C}{\phi c_t h r_w^2}$$

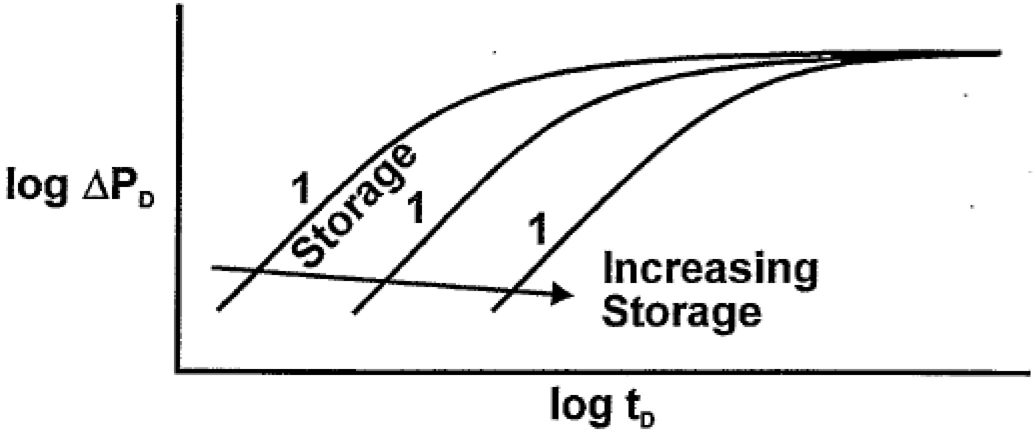
Gringarten Type Curve

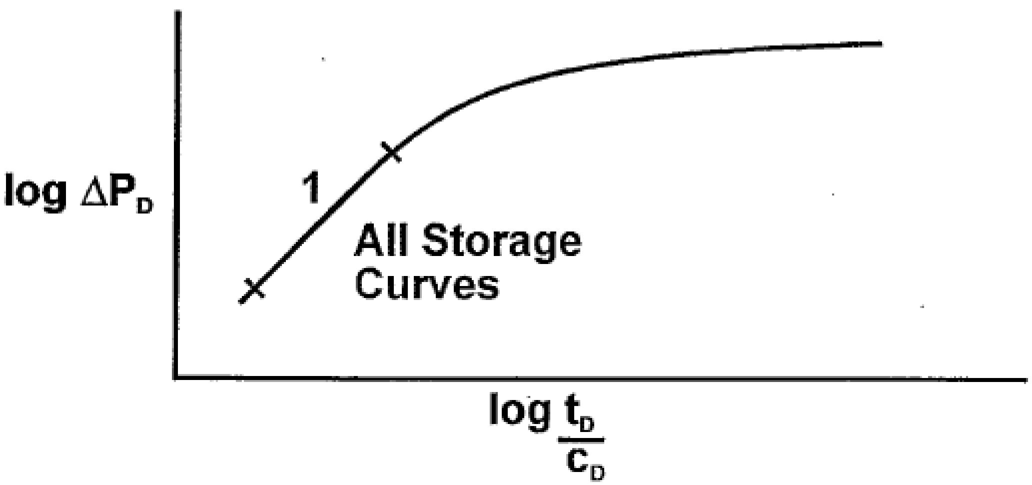
The Gringarten type curve describes the pressure response under the following assumptions:

1. Constant rate production
2. Vertical wellbore
3. Infinite-acting homogeneous reservoir
4. Single phase liquid of small and constant compressibility
5. Infinitesimal skin that may be modeled with an apparent wellbore radius
6. Constant wellbore storage coefficient

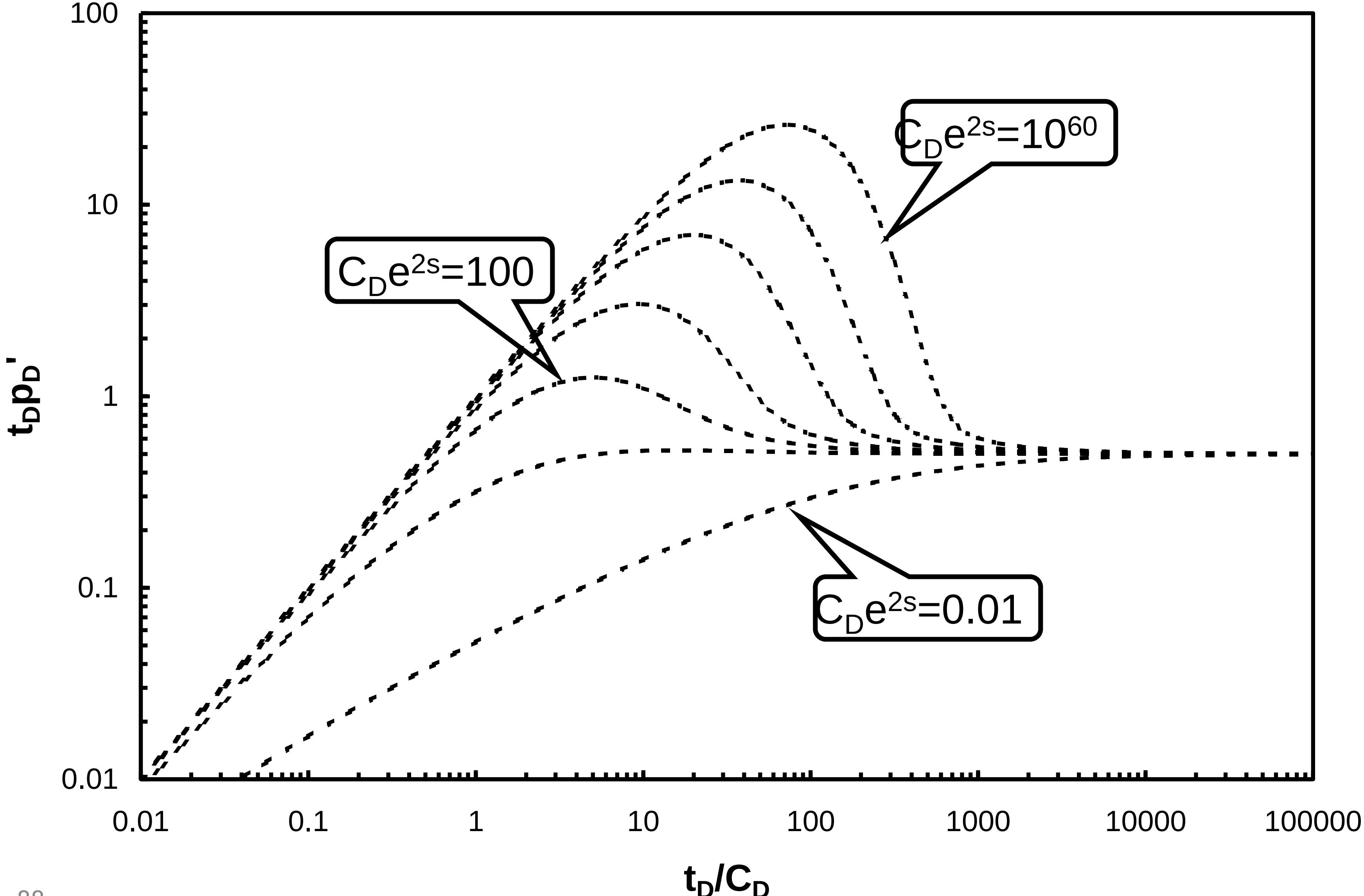
The Gringarten type curve was specifically developed for drawdown tests in oil wells. We will see that we may use it (with some limitations) to analyze pressure buildup tests in addition to drawdown tests, and to analyze gas well tests as well as oil well tests.

t_D vs t_D/C_D

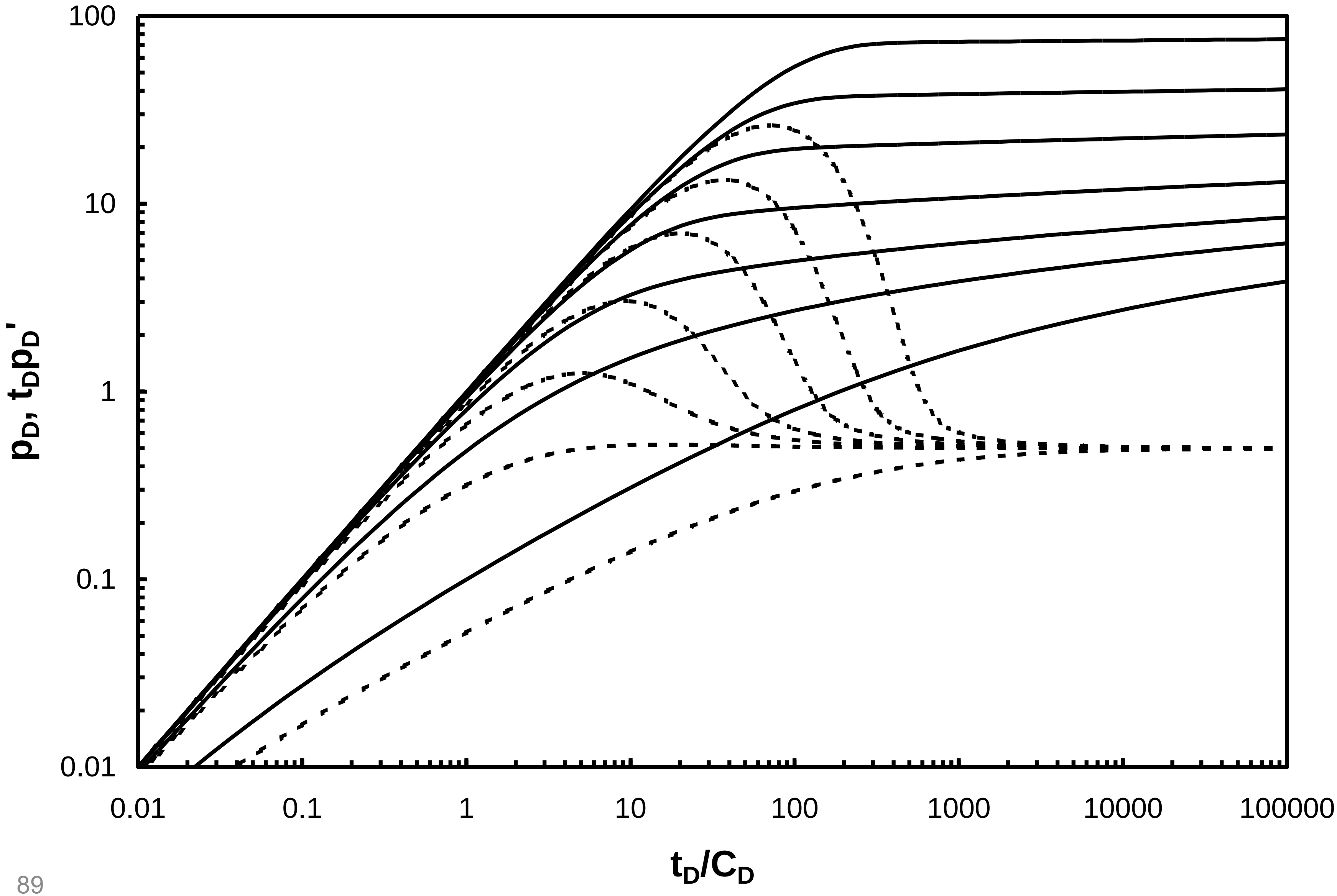

log ΔP_D
1
Storage
1
1
Increasing
Storage
log t_D


log ΔP_D
1
All Storage
Curves
log t_D/C_D

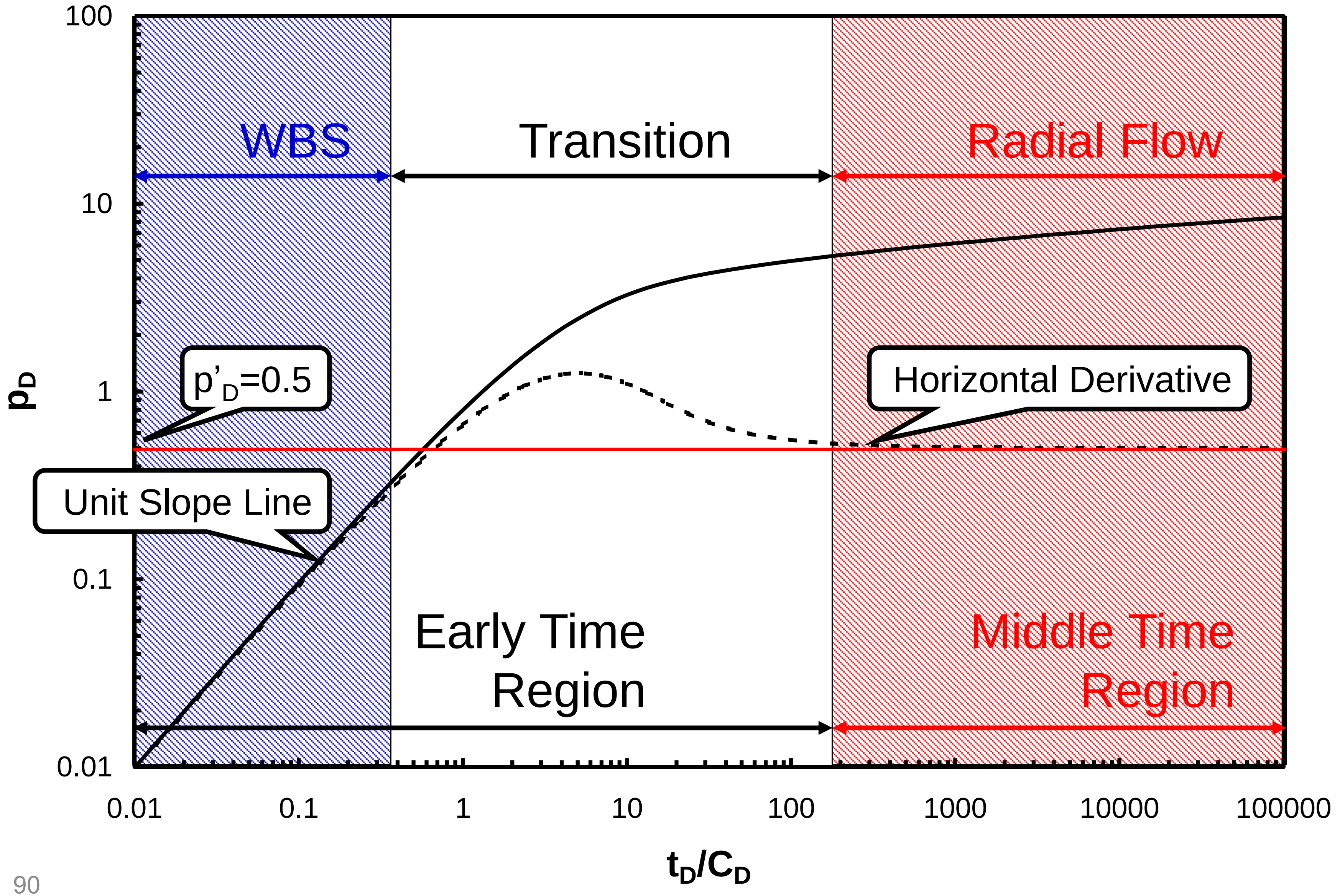
Derivative Type Curve



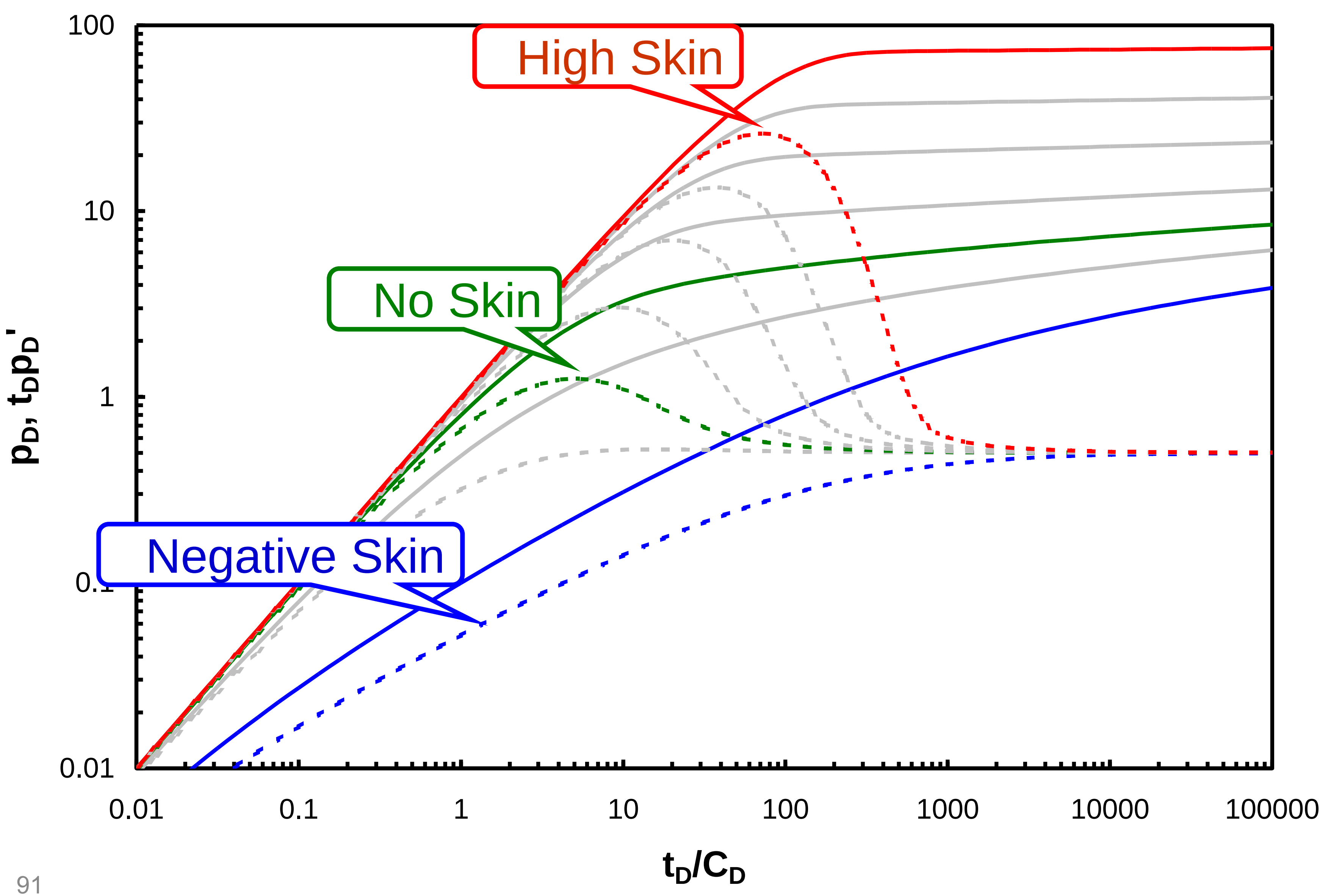
Pressure And Derivative Type Curves



Time Regions On The Type Curve



Estimating Skin Factor



Calculation of Reservoir Parameters from Buildup Test Using the Gringarten-Bourdet Type Curves

A well in a reservoir above its bubble point pressure was producing oil at a constant rate of 185 BOPD before it was shut-in for a buildup test. The buildup test data are given in Table 1 showing elapsed shut-in time, Δt, and shut-in pressure, p_{ws}. Other reservoir and well data are given below:

Formation thickness, h	114 ft
Formation porosity, ϕ	0.28
Total compressibility, c_t	4.1×10^{-6} psi^{-1}
Oil viscosity, μ_o	2.2 cp
Oil FVF, B_o	1.1 RB/STB
Wellbore radius, r_w	0.50 ft
Flowing BHP, p_{wf} at $\Delta t = 0$	2820 psia
Producing time before shut-in, t_p	540 hrs

Calculate dimensionless wellbore storage, C_D; wellbore storage coefficient, C; formation permeability, k; and skin factor, s. Use the technique of type curve matching with the combined Gringarten-Bourdet type curve. Compare the results from type curves to the results obtained from using straightline methods based on the Horner plot.

Buildup Test Data

Δt (hrs)	p_{ws} (psia)	Δt (hrs)	p_{ws} (psia)	Δt (hrs)	p_{ws} (psia)	Δt (hrs)	p_{ws} (psia)
0.0000	2820.00	0.3188	3006.01	2.5500	3344.09	12.9625	3398.19
0.0018	2822.15	0.3542	3022.46	2.7625	3350.80	13.6000	3398.75
0.0035	2823.18	0.3896	3036.49	2.9750	3355.10	14.2375	3399.31
0.0071	2825.61	0.4250	3051.08	3.1875	3362.38	14.8750	3399.86
0.0106	2827.67	0.4604	3065.68	3.4000	3367.23	15.5125	3400.25
0.0142	2830.28	0.4958	3080.63	3.6125	3370.40	16.1500	3400.80
0.0177	2832.71	0.5313	3091.29	3.8250	3371.72	16.7875	3401.10
0.0213	2835.33	0.5667	3104.95	4.0375	3373.03	17.4250	3401.55
0.0248	2837.76	0.6021	3116.73	4.2500	3375.09	18.0625	3401.83
0.0283	2840.19	0.6375	3125.52	4.4625	3377.52	18.9125	3402.39
0.0319	2842.62	0.6906	3144.23	4.6750	3379.73	19.7625	3402.71
0.0390	2844.86	0.7438	3158.26	4.8875	3381.28	20.6125	3403.21
0.0425	2847.11	0.7969	3170.42	5.1000	3383.15	21.4625	3403.51
0.0496	2851.97	0.8500	3184.07	5.3125	3384.46	22.3125	3403.96
0.0567	2857.19	0.9031	3194.93	5.7375	3384.80	23.1625	3404.24
0.0638	2861.13	0.9563	3207.47	6.1625	3387.00	24.2250	3404.78
0.0708	2865.98	1.0094	3218.31	6.5875	3387.51	25.5000	3405.13
0.0815	2876.72	1.0625	3228.62	7.0125	3388.64	27.6250	3406.22
0.0921	2883.26	1.1156	3235.90	7.4375	3389.96	29.7500	3406.62
0.1027	2889.05	1.1688	3244.89	7.8625	3390.89	31.8750	3407.52
0.1133	2895.59	1.2219	3253.49	8.2875	3391.64	34.0000	3407.90
0.1240	2900.93	1.2750	3260.61	8.7125	3392.76	38.2500	3409.71
0.1381	2909.72	1.3813	3274.46	9.1375	3393.32	42.5000	3410.10
0.1523	2917.76	1.4875	3284.39	9.5625	3394.07	46.7500	3411.42
0.1665	2926.17	1.5938	3294.70	9.9875	3394.63	51.0000	3411.83
0.1806	2934.03	1.7000	3297.68	10.4125	3395.39	55.2500	3412.86
0.1948	2942.06	1.9125	3317.54	10.8375	3395.94	59.5000	3413.25
0.2125	2951.60	2.0188	3323.15	11.2625	3396.56	—	—
0.2479	2970.66	2.1250	3327.83	11.6875	3396.88	—	—
0.2833	2987.87	2.3375	3336.24	12.3250	3397.63	—	—

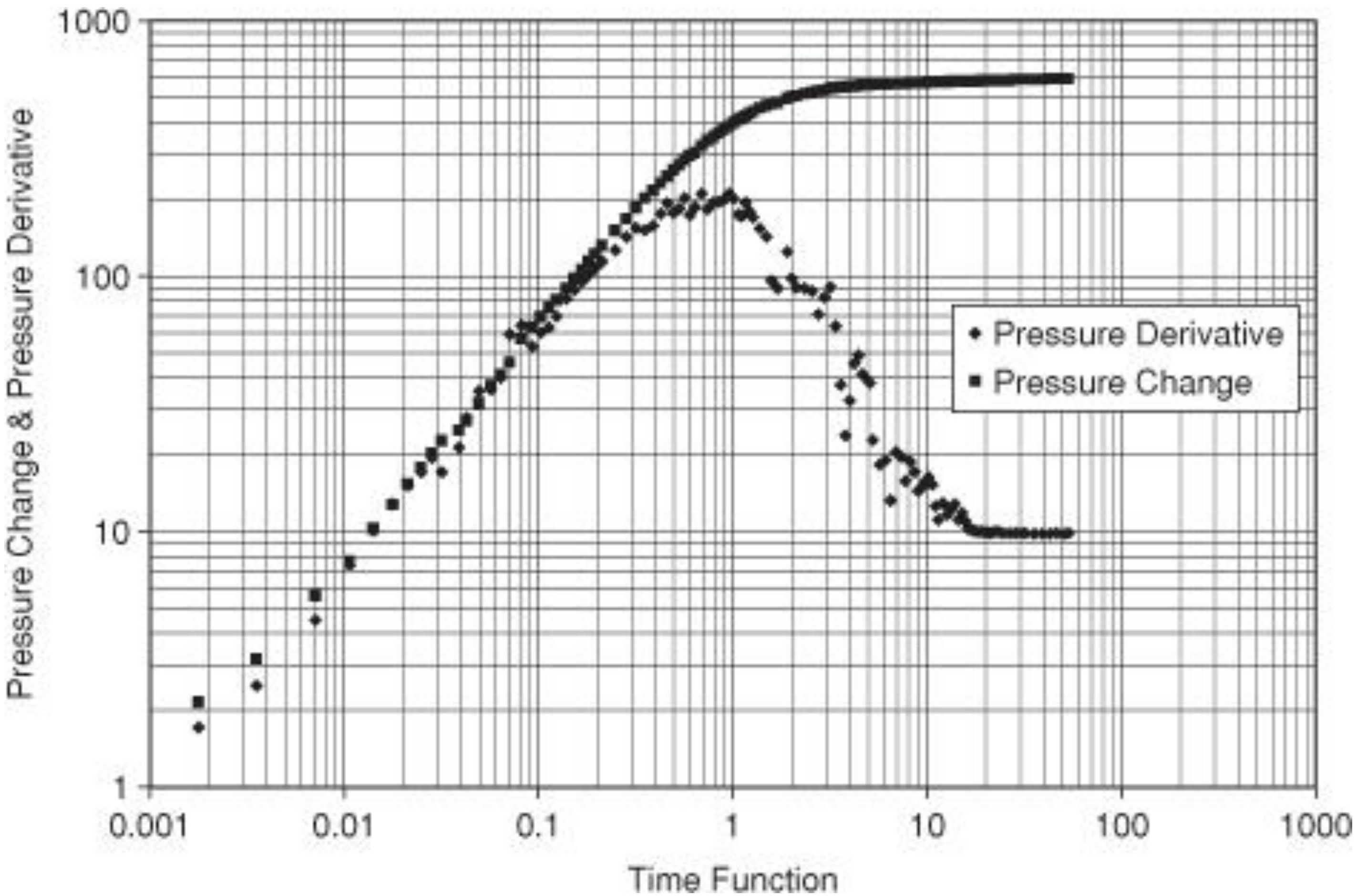
Step 1: Prepare the test data for analysis.

Calculate Agarwal equivalent shut-in time, Δt_e, and pressure change, $\Delta p = (p_{ws} - p_{wf@\Delta t=0})$. The calculated data are Table 1. Plot Δp vs. Δt_e on a log-log scale as shown in Figure 1.

Δt_e	Δp (psi)	$\Delta t_e \Delta p'$	Δt_e	Δp (psi)	$\Delta t_e \Delta p'$	Δt_e	Δp (psi)	$\Delta t_e \Delta p'$
0.0018	2.15	1.7098	0.7427	338.26	182.7992	8.5742	572.76	17.1367
0.0035	3.18	2.4958	0.7957	350.42	194.7352	8.9855	573.32	14.4583
0.0071	5.61	4.4916	0.8487	364.07	195.1176	9.3961	574.07	14.9275
0.0106	7.67	7.4107	0.9016	374.93	200.1915	9.8061	574.63	15.8172
0.0142	10.28	10.0963	0.9546	387.47	210.0963	10.2155	575.39	16.2879
0.0177	12.71	12.8207	1.0075	398.31	201.1232	10.6243	575.94	15.2523
0.0212	15.33	15.1382	1.0604	408.62	174.7776	11.0324	576.56	12.5318
0.0248	17.76	17.0691	1.1133	415.90	172.1645	11.4399	576.88	11.1707
0.0283	20.19	19.4921	1.1662	424.89	193.8354	12.0500	577.63	12.8853
0.0319	22.62	17.1346	1.2191	433.49	180.5007	12.6586	578.19	11.6352
0.0390	24.86	21.4190	1.2720	440.61	169.6267	13.2659	578.75	12.2291
0.0425	27.11	27.9062	1.3777	454.46	153.1716	13.8718	579.31	12.8085
0.0496	31.97	35.5838	1.4834	464.39	142.3950	14.4762	579.86	11.1516
0.0567	37.19	36.0873	1.5891	474.70	96.3010	15.0793	580.25	11.8288

Step 2: Calculate pressure derivative with respect to natural logarithm of equivalent time.

Plot of Δp & $\Delta p'$ vs. Δt_e



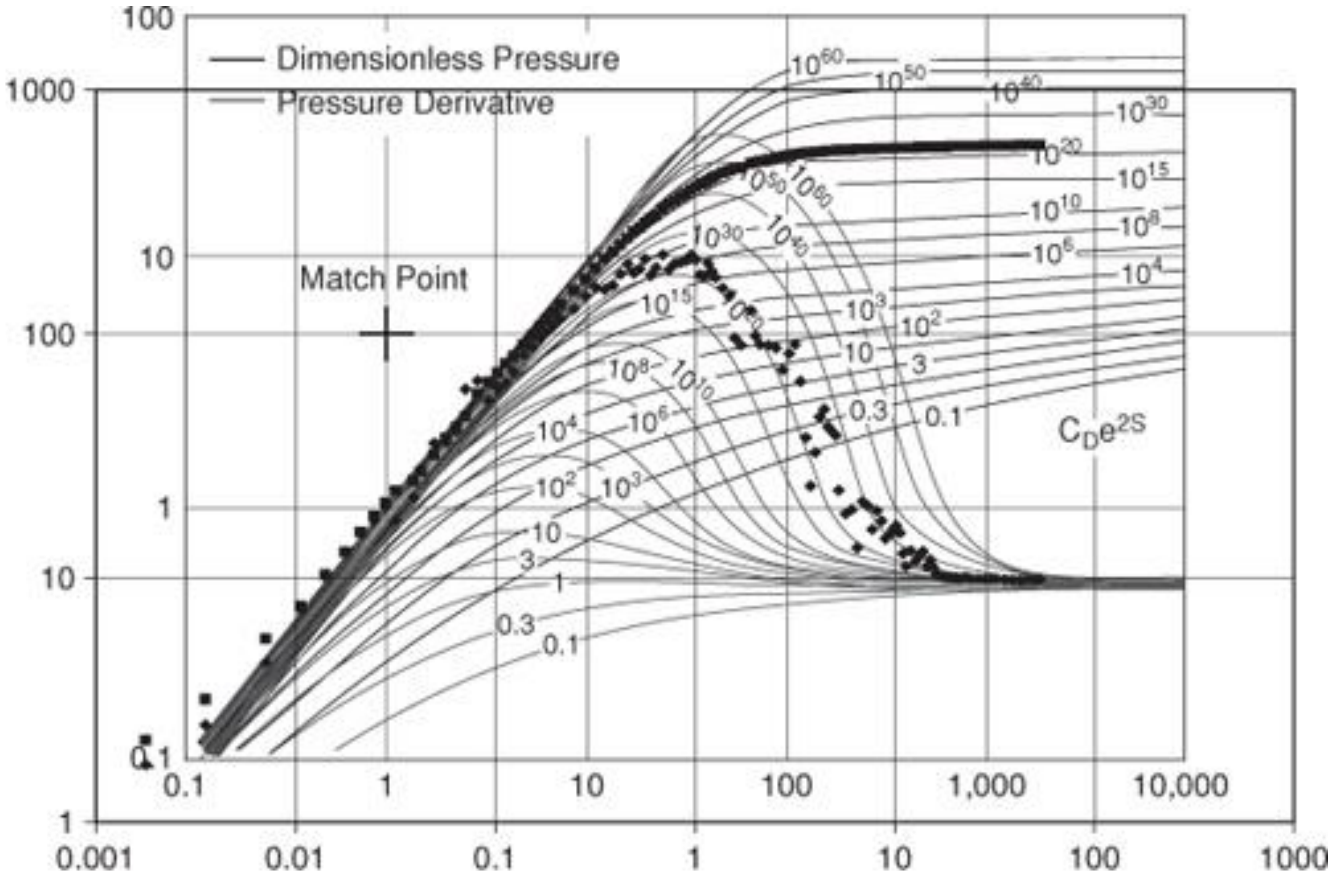
Step 3: Calculate C_D and C from the unit-slope line.

Select any point on the unit-slope line from Figure. One such point on the unit-slope line is $(\Delta t_e/\Delta p)_{USL}$ = (0.06374/41.13).

$$C_D = \frac{0.03723qB}{\phi c_t h r_w^2}\left(\frac{\Delta t_e}{\Delta p}\right)_{USL}$$
$$= \frac{0.03723 \times 185 \times 1.1}{0.28 \times 4.1 \times 10^{-6} \times 114 \times (0.5)^2}\left[\frac{0.06374}{41.13}\right]$$
$$= 358.86$$

$$C = 0.04165qB\left(\frac{\Delta t_e}{\Delta p}\right)_{USL}$$
$$= 0.04165 \times 185 \times 1.1 \times \left(\frac{0.06374}{41.13}\right)$$
$$= 0.013 \text{ RB/psi}$$

Step 4: Perform type-curve matching using the Figure and Gringarten-Bourdet type curve.



$$\left(\frac{p_D}{\Delta p}\right)_{PMP} = \left(\frac{4.5}{100}\right)$$

$$k = \frac{141.2qB\mu}{h}\left[\frac{p_D}{\Delta p}\right]_{PMP}$$

$$= \frac{141.2 \times 185 \times 1.1 \times 2.2}{114}\left(\frac{4.5}{100}\right)$$

$$= 24.95 \text{ md}.$$

Step 5: Calculate C_D from time match point (TMP).

$$\left(\frac{\Delta t_e}{t_D/C_D}\right)_{TMP} = \left(\frac{0.03}{1}\right)$$

$$C_D = \frac{0.0002637k}{\phi\mu c_t r_w^2}\left[\frac{\Delta t_e}{t_D/C_D}\right]_{TMP}$$

$$= \frac{0.0002637 \times 24.95}{0.28 \times 2.2 \times 4.1 \times 10^{-6} \times (0.5)^2}\left(\frac{0.03}{1}\right)$$

$$= 312.61$$

Step 6: Calculate skin factor, *s*.

$$s = 0.5 \ln\left(\frac{C_D e^{2s}}{C_D}\right)$$

$$= 0.5 \ln\left(\frac{10^{21}}{312.61}\right), \text{ where } C_D e^{2s} = 10^{21} \text{ from Figure 12.7.}$$

$$= 22.46$$

Treatment of Gas Flow Equations in a Porous Medium

$$-\frac{1}{r}\left[\frac{\partial\left(r\rho_g v_{gr}\right)}{\partial r}\right]=\frac{\partial\left(\rho_g\phi\right)}{\partial t}$$

$$\psi(p)=2\int_{p_b}^{p}\frac{p}{\mu Z}dp$$

$$t_a\left[\bar{p}(t)\right]=\int_{t_b}^{t}\frac{dt}{\mu(\bar{p})\,c_t(\bar{p})}$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)\right]=\frac{\phi\mu_i c_{ti}}{k}\frac{\partial\psi}{\partial t_a}$$

Al-Hussainy and Ramey (1966)- Pseudo-pressure Transformation
(1) Variable compressibility factor
(2) Variable viscosity

Fraim and Wattenbarger (1987)- Pseudo-time Transformation
(1) Variable compressibility
(2) Variable viscosity

Non-Darcy Flow

- Darcy law $$-\frac{\partial p}{\partial x} = \frac{\mu}{k} v$$

- At high velocities pressure drop is more than what is predicted by Darcy's law

$$-\frac{\partial p}{\partial x} = \frac{\mu}{k} v + \rho \beta v^2$$

Forchheimer equation

$$\beta = 1.88 \times 10^{10} k^{-1.47} \phi^{-0.53}$$