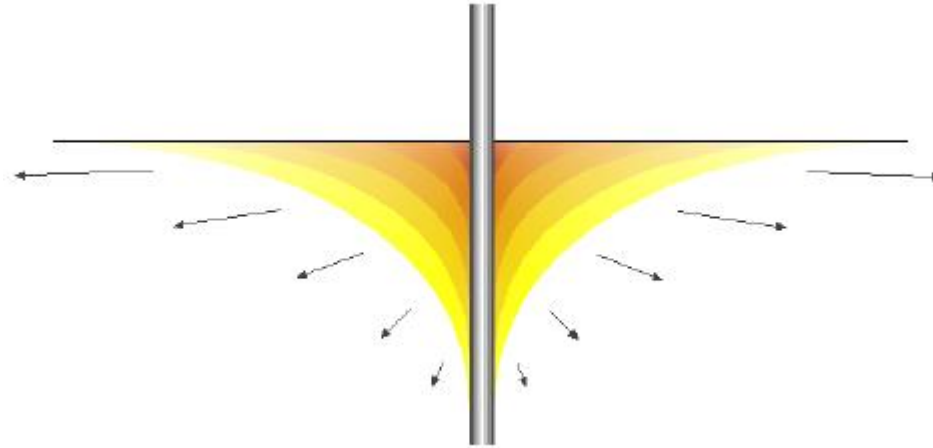


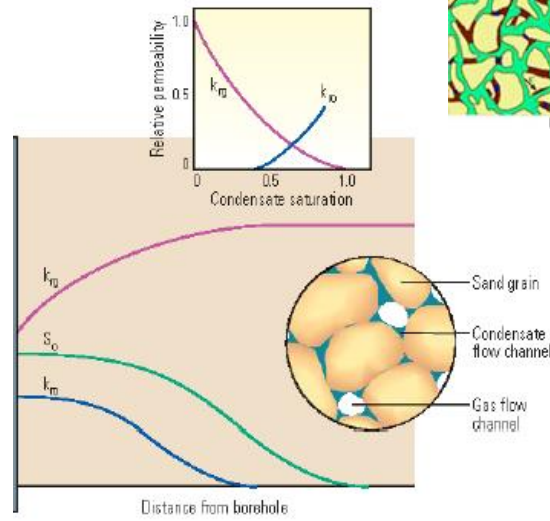
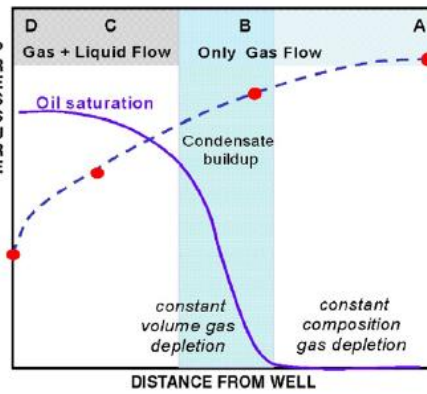
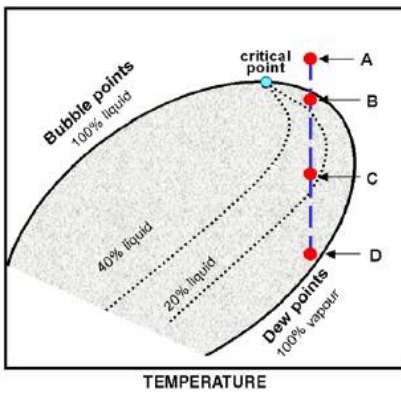
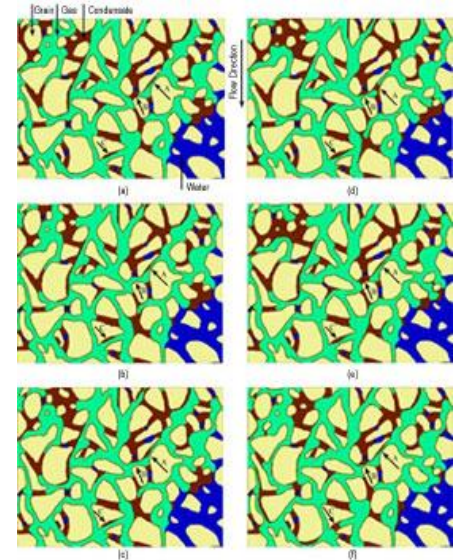
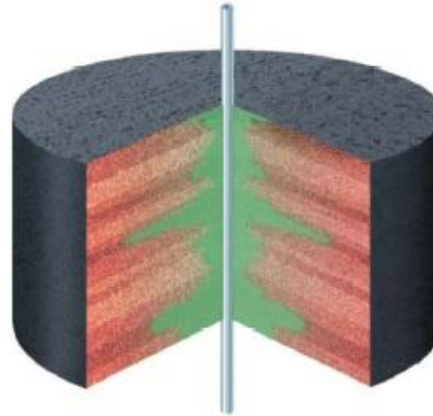
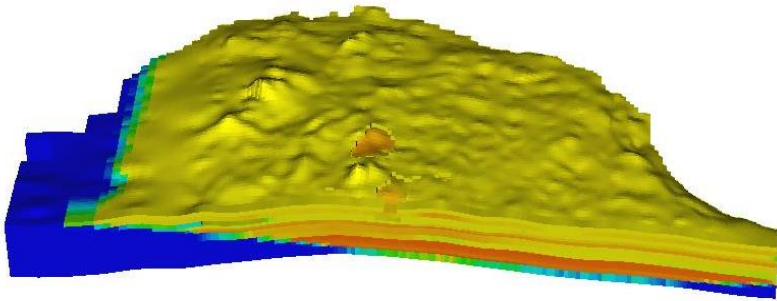
Advanced WellTest Analysis



Well Test Analysis of Gas Condensate Reservoir

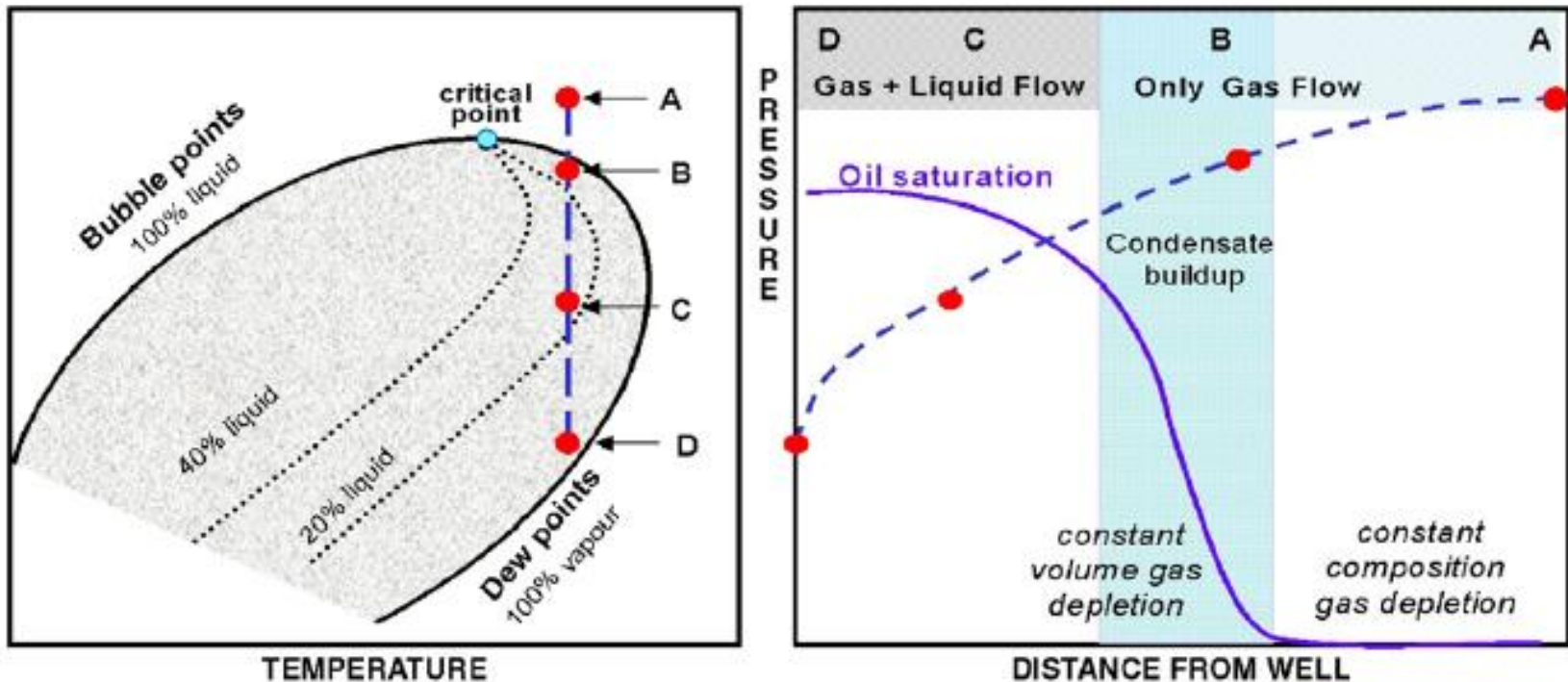
By: Shahab Gerami

Introduction



Gas-condensate Flow and Saturation Distribution

Fevang and Whitson (1996)



Single Phase Gas Flow Equations

$$-\frac{1}{r} \left[\frac{\partial (r \rho_g v_{gr})}{\partial r} \right] = \frac{\partial (\rho_g \phi)}{\partial t}$$

Pseudo-pressure Transformation; Al-Hussainy and Ramey (1966)

- (1) Variable compressibility factor
- (2) Variable viscosity

$$\psi(p) = 2 \int_{p_b}^p \frac{p}{\mu Z} dp$$

Pseudo-time Transformation; Fraim and Wattenbarger (1987)

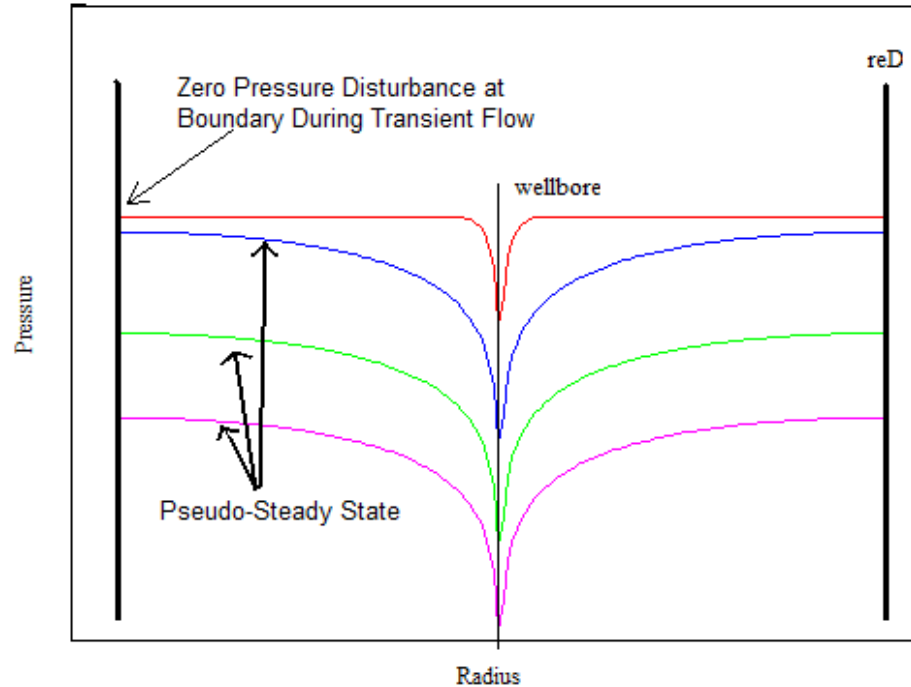
- (1) Variable compressibility
- (2) Variable viscosity

$$t_a [\bar{p}(t)] = \int_{t_b}^t \frac{dt}{\bar{\mu} \bar{c}_t}$$

$$\frac{1}{r} \left[\frac{\partial}{\partial r} \left(r \frac{\partial \psi}{\partial r} \right) \right] = \frac{\phi \mu_i c_{ti}}{k} \frac{\partial \psi}{\partial t_a}$$

Deliverability Equation

Single Phase GAS



$$q_g = \frac{kh(p_p(\bar{p}) - p_p(p_{wf}))}{1422T \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + Dq_g \right]}$$

$$q_g = C(p_p(\bar{p}) - p_p(p_{wf}))$$

Well Test Analysis

1. Darcy's Law applies,
2. Single-phase flow,
3. Porosity, permeabilities, viscosity and compressibility are constant,
4. Fluid compressibility is small,
5. Pressure gradients in the reservoir are small,
6. Gravity and thermal effects are negligible.

$$\nabla^2 p - \frac{\phi \mu c_t}{k} \frac{\partial p}{\partial t} = 0 \quad (3.1)$$

Dry gas reservoir modification

$$\nabla^2 m(p) - \frac{1}{\eta} \frac{\partial m(p)}{\partial t} = 0 \quad (3.2)$$

For gas condensate reservoirs, liquid may condense in the reservoir where gas and condensate will be present together. Not only are the fluid properties strong functions of pressure but multiphase flow may also occur in the reservoir.

Flow Governing Equation

The partial differential equation that models the radial flow of component i in a reservoir that contains N components is:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[\left(k_{ro} \frac{\rho_o}{\mu_o} x_i + k_{rg} \frac{\rho_g}{\mu_g} y_i \right) kr \frac{\partial p}{\partial r} \right] = \phi \frac{\partial}{\partial t} (\rho_o S_o x_i + \rho_g S_g y_i) , \quad i = 1, 2, 3, \dots, N$$

$$\rho_o = \rho_o(p, T, x_1, x_2, \dots, x_{N-1})$$

$$\rho_g = \rho_g(p, T, y_1, y_2, \dots, y_{N-1})$$

$$\mu_o = \mu_o(p, T, x_1, x_2, \dots, x_{N-1})$$

$$\mu_g = \mu_g(p, T, y_1, y_2, \dots, y_{N-1})$$

Flow Governing Equation

The reservoir is assumed to be initially in the single gas phase. The initial condition is expressed as:

$$p(r, t = 0) = p_{init.} > P_{dew}$$

$$y_i(r, t = 0) = z_i$$

$$x_i(r, t = 0) = 0$$

Because the well is fully penetrating and producing at constant molar rate q_t , the inner boundary condition can be expressed as

$$\frac{2\pi kh}{q_t} \left[\left(k_{ro} \frac{\rho_o}{\mu_o} + k_{rg} \frac{\rho_g}{\mu_g} \right) r \frac{dp}{dr} \right]_{r_w} = 1$$

Considering an infinite reservoir, we describe the outer boundary condition as

$$p(r = \infty, t) = p_i$$

Flow Governing Equation

The constraints for saturation, gas, and liquid phase component mole fractions are, respectively,

$$S_o + S_g = 1$$

$$\sum_{i=1}^N x_i = 1$$

$$\sum_{i=1}^N y_i = 1$$

The constraint for thermodynamic equilibrium is expressed as the equivalence in fugacities of each component in each phase. In equation form,

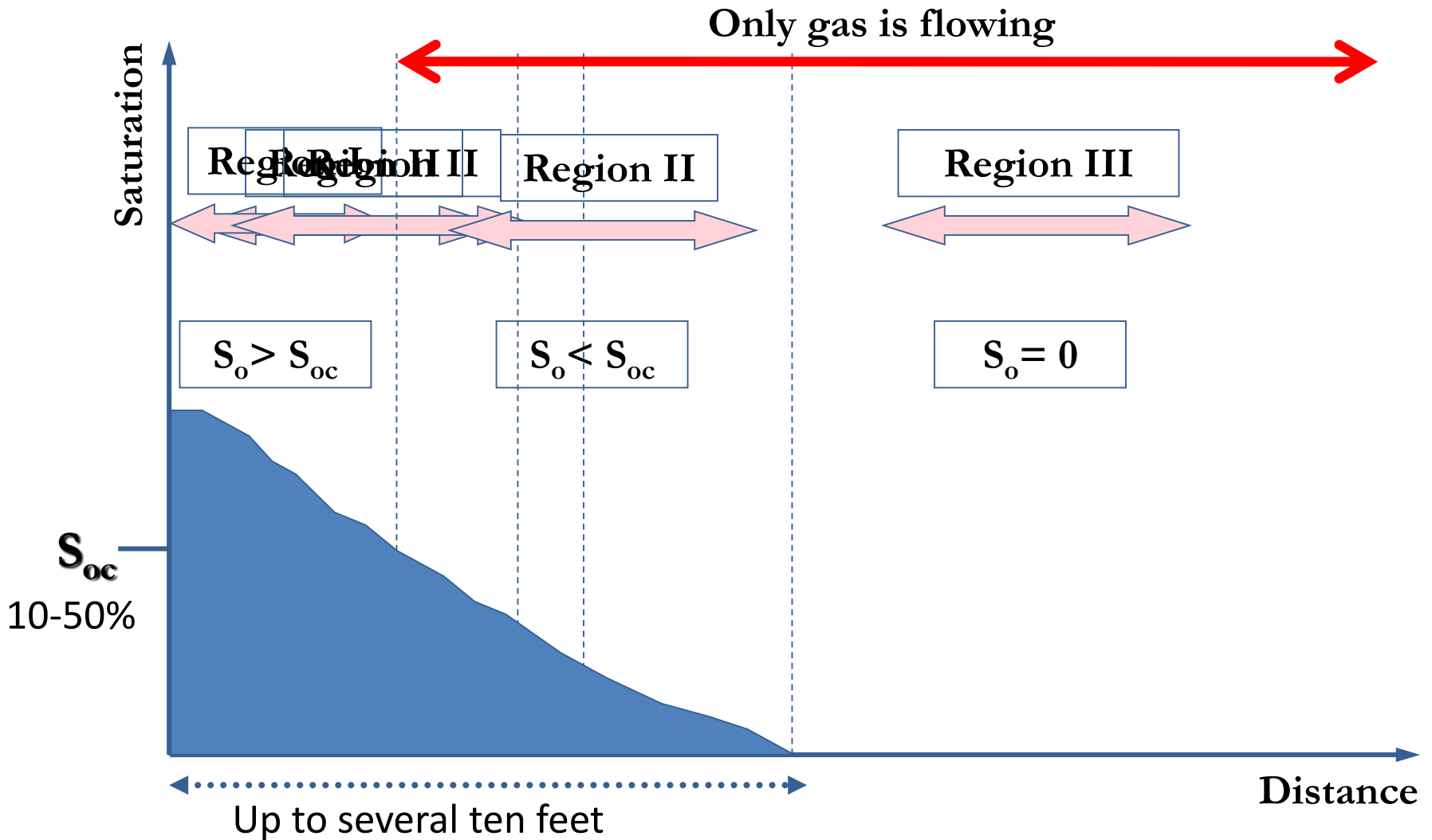
$$f_i^L = f_i^V, \quad i = 1, 2, \dots, N.$$

Flow Governing Equation

There are $(2N + 3)$ unknown variables $(\rho, S_o, S_g, x, y, i=1,2,\dots, N)$, except initial and boundary conditions, where there are $(2N + 3)$ equations. Therefore they are solvable.

$$\frac{1}{r} \frac{\partial}{\partial r} \left[\left(k_{ro} \frac{\rho_o}{\mu_o} x_i + k_{rg} \frac{\rho_g}{\mu_g} y_i \right) kr \frac{\partial p}{\partial r} \right] = \phi \frac{\partial}{\partial t} (\rho_o S_o x_i + \rho_g S_g y_i) , \quad i = 1, 2, 3, \dots, N$$

Gas Condensate Reservoirs- Flow region 3-Zones Approaches (Fevang et al., 1996)



Region 1

➤ **Region 1:** An inner near-wellbore region saturated with oil and gas which both are flowing simultaneously.

➤ **Size:** The size of this region varies with time and richness of the initial reservoir fluid. The size is typical 50-100 feet (15-30 meter) for leaner gas condensates and 200-500 feet (50-150 meter) for richer gas condensates.

➤ **Saturation:** The saturation in Region 1 is usually in the range of 40-60%. The oil saturation is established such that the oil drooping of the reservoir gas has enough mobility to move together with the reservoir gas (no accumulation).

➤ **Relative permeability:** The gas permeability in this area might be reduced to 10% of the initial permeability.

➤ **Composition:** The flowing composition (GOR) within Region 1 is constant throughout. That means that the single phase gas entering Region 1 has the same composition as the produced well stream mixture.

➤ **Deliverability loss:** The main source of flow resistance, and thus deliverability loss in gas condensate wells. The deliverability loss depends mainly on gas relative permeability in Region 1, and the size of Region 1.

➤ **$K_{rg}/K_{ro} < 1$, High velocity, Moderate to high IFT**

Region 2,3

➤ **Region 2:** A region of condensate buildup where the liquid condensate is (practically) immobile and only gas is flowing

➤ **Size:** The size of this region depends on size of region 1 and reservoir pressure (relative to dew point pressure).

➤ **Oil mobility:** The pressure in this region is lower than the dew point pressure of the initial reservoir fluid. The oil condensing from the reservoir gas has no or negligible mobility.

➤ **Deliverability loss:** The deliverability loss in region 2 is usually limited as the gas relative permeability is usually high (often S-shaped gas relative permeability) at low oil saturations.

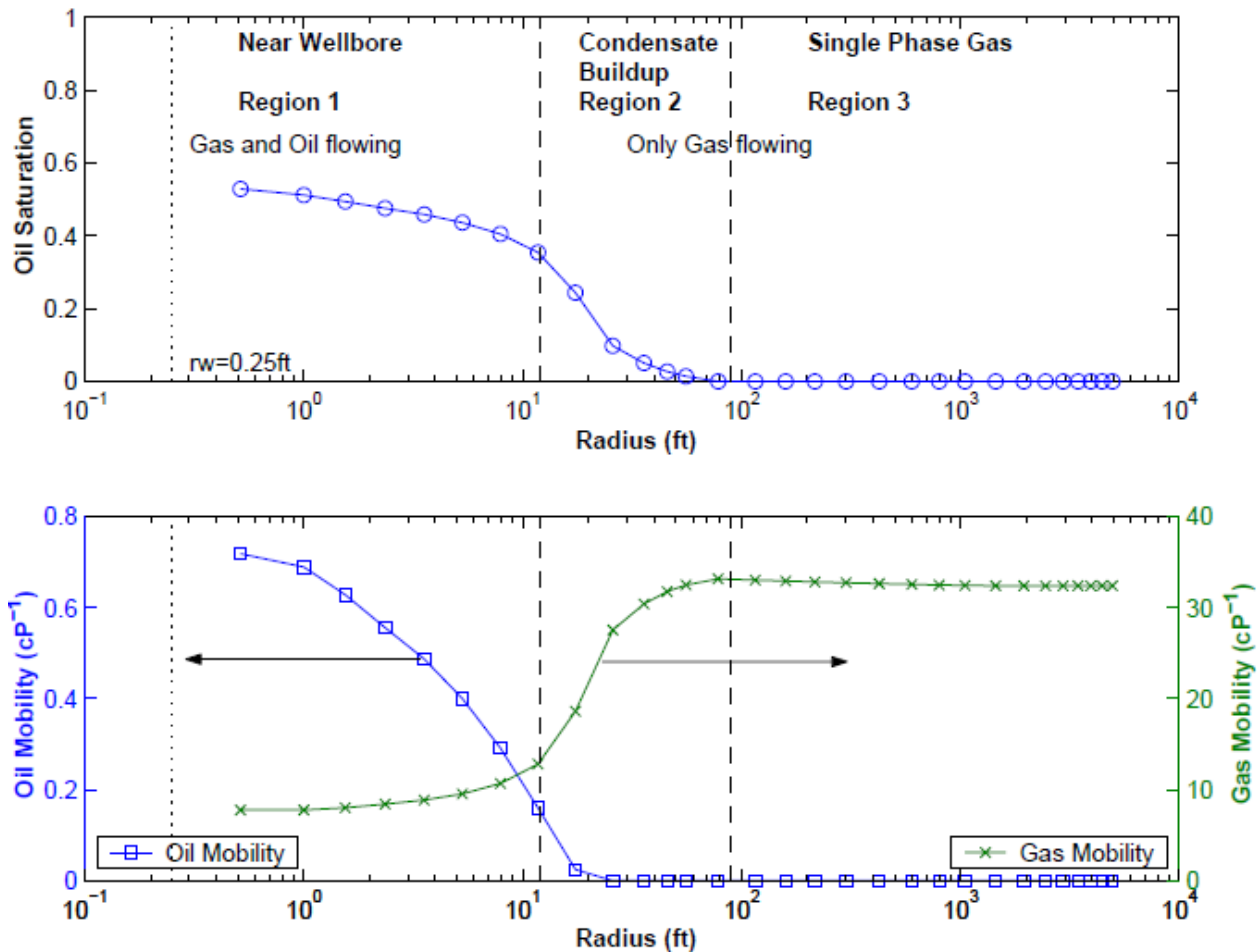
➤ **Experimental simulation of region 2:** For well deliverability calculations, the condensate saturations in Region 2 can be approximated by the liquid dropout curve from a CVD experiment, corrected for S_w .

➤ **GOR:** The important consequence of Region 2 is that producing wellstream composition (GOR) is leaner than calculated by a simple volumetric material balance (e.g. CVD measurements).

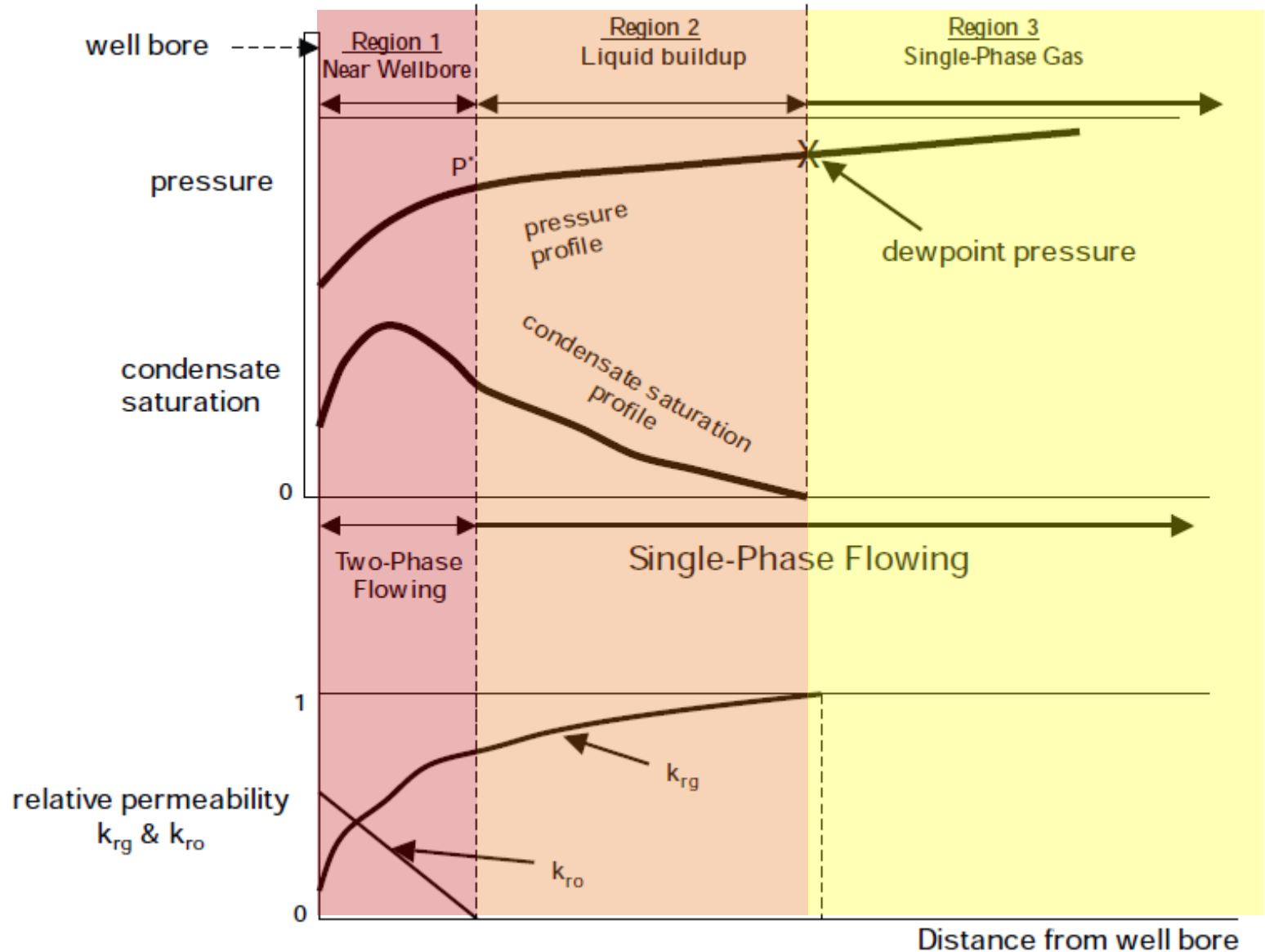
➤ **Region 3:** A region containing single phase (original) reservoir gas

Drawdown Behavior

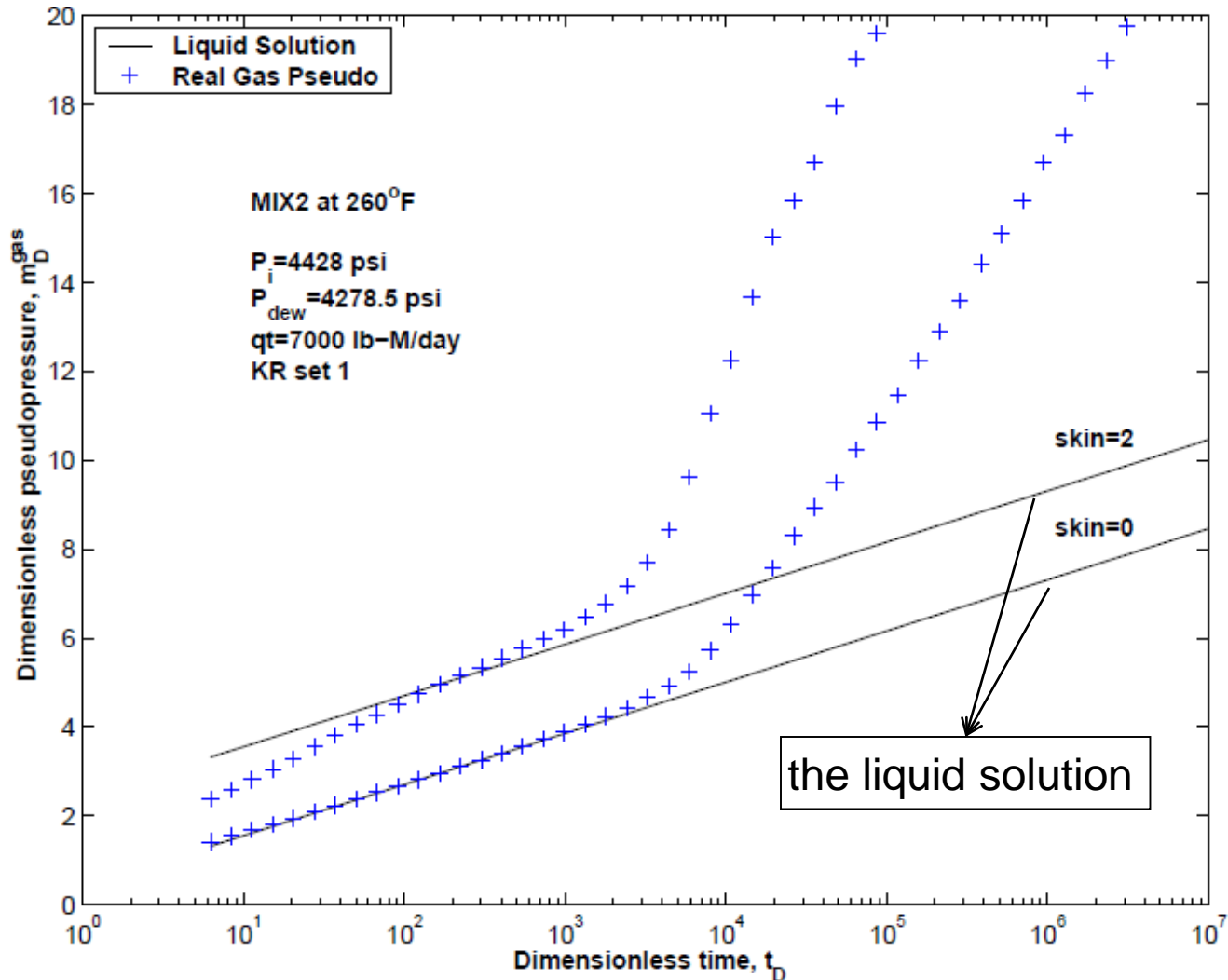
Fluid flow towards the well in a gas condensate reservoir during depletion can be divided into three concentric main flow regions, from the wellbore to the reservoir (Fevang, 1995):



Drawdown Behavior

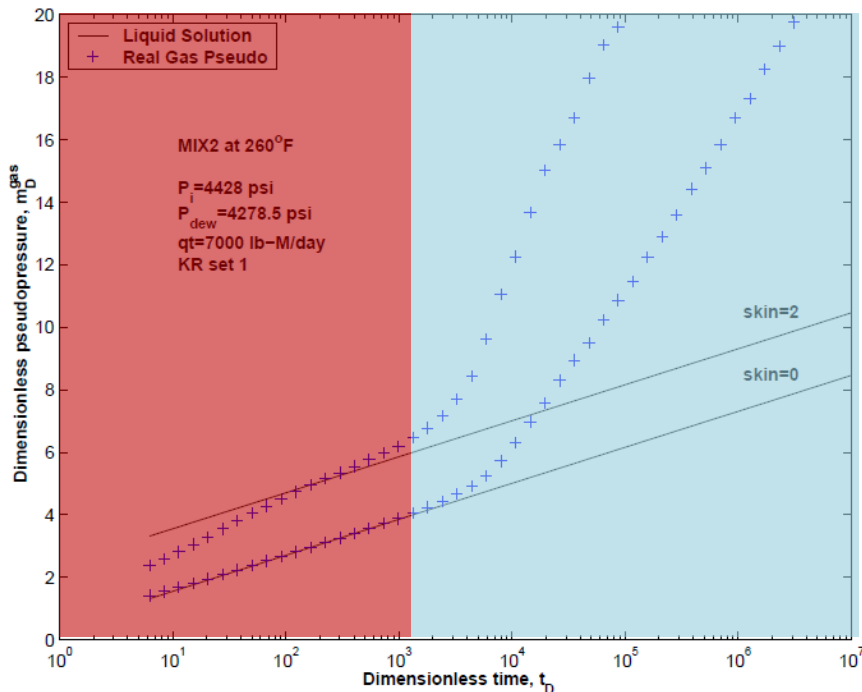


Drawdown Response Interpreted with the Classical Real Gas Pseudopressure



Well Test Analysis

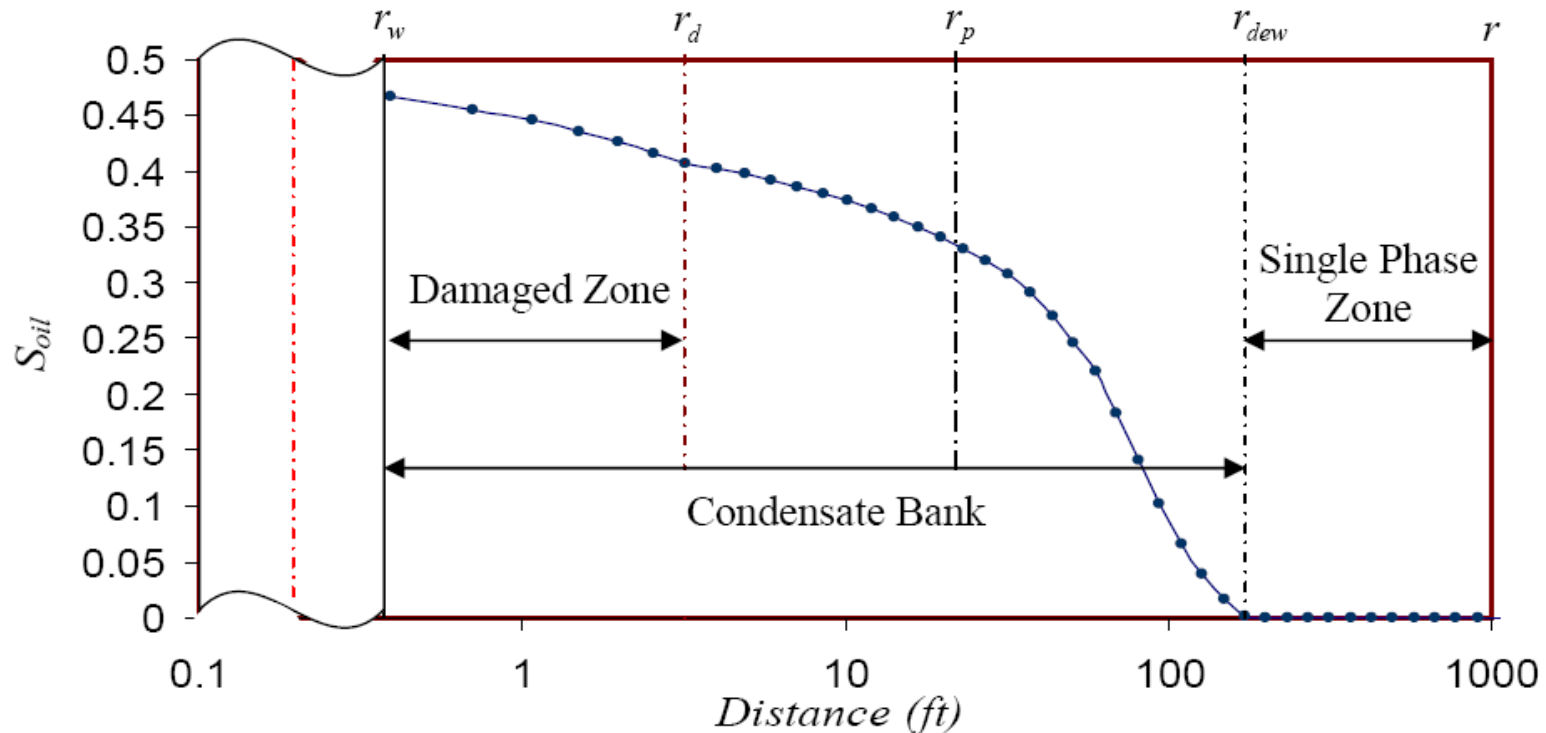
At early times, the bottom hole flowing pressure (BHFP) is greater than the dew point pressure of the original reservoir gas (p_{dew}), therefore only gas is present in the reservoir and the real gas pseudopressure matches the liquid solution.



When skin is included in the model, the real gas pseudopressure first shows a transition region until the compressible zone goes beyond the damaged zone. As soon as the BHFP drops below p_{dew} , the gas relative permeability drops below unity and the crosses deviate from their corresponding liquid solution. The deviation is more pronounced for a positive skin.

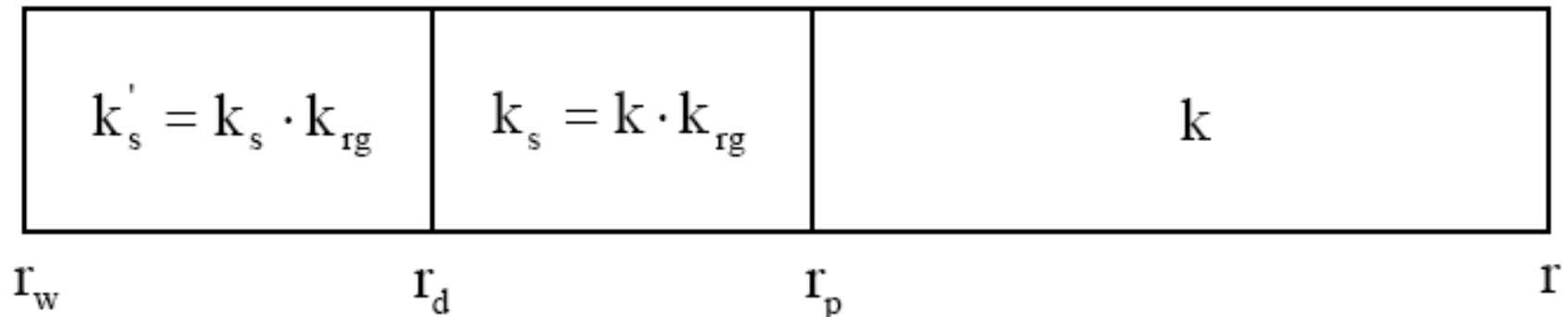
Single-Phase Pseudo-pressure Approach

- we can make a single phase model by assuming that **gas is the only mobile phase** in the reservoir and the **condensate bank accounts as a formation damage skin**. This method is well known as **single-phase method**, which is similar to the conventional model for dry gas flow.



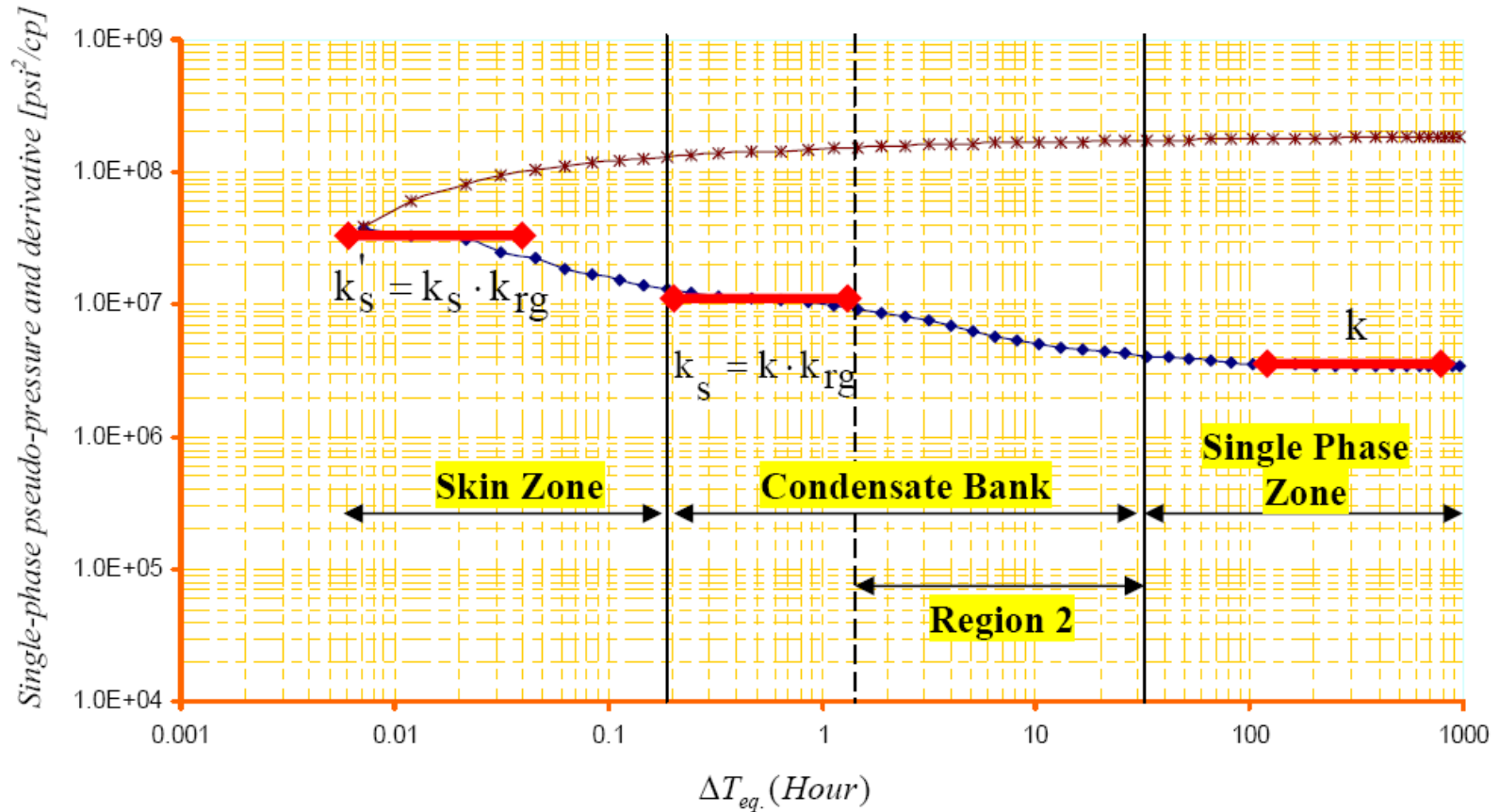
Three Zone Reservoir Model, Single Phase Analogy

The effective permeability to gas in the reservoir changes in the radial direction. Thus, to use single-phase analogy, in terms of gas effective permeability variation a radially composite reservoir model should be established. The three zone reservoir model can be obtained by assuming uniform oil saturation distribution in the reservoir for the single-phase model. The permeability in third zone is the effective permeability to gas at that zone, if liquid has dropped out in the entire reservoir.



3- Zone Reservoir Model, Single Phase

Build-up analysis, Single-phase Pseudo-pressure and Derivative

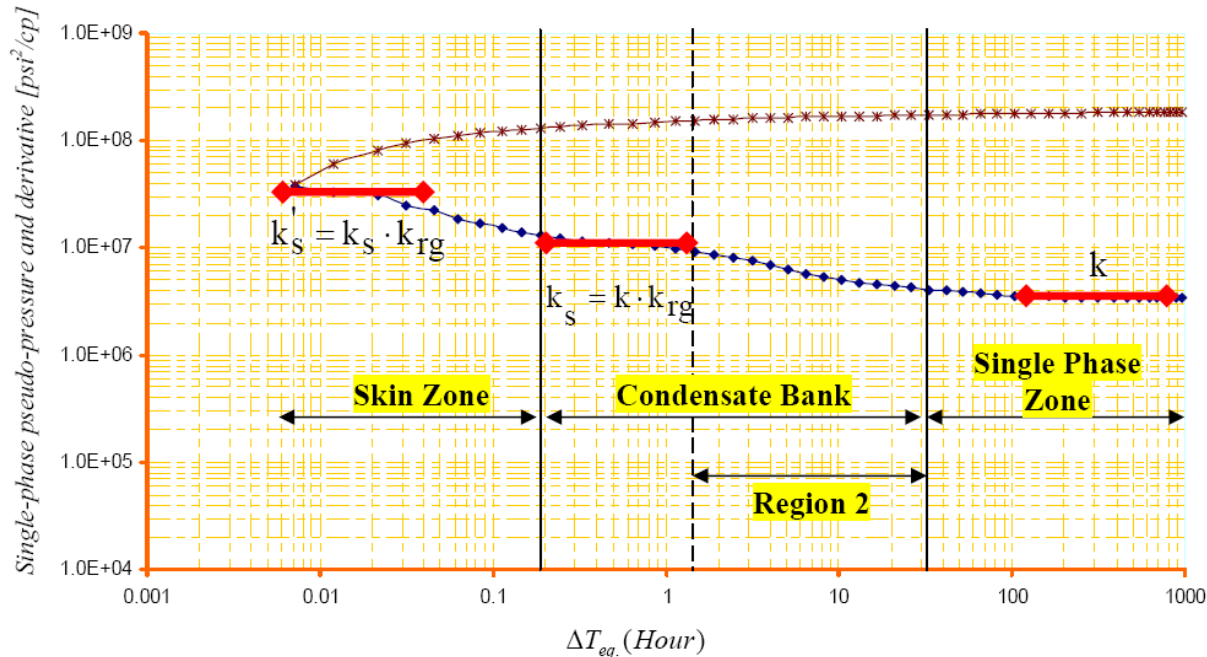


These three horizontal lines reflect the effective permeabilities of gas in the damaged zone, condensate bank and the reservoir.

3- Zone Reservoir Model, Single Phase

Build-up analysis, Single-phase Pseudo-pressure and Derivative

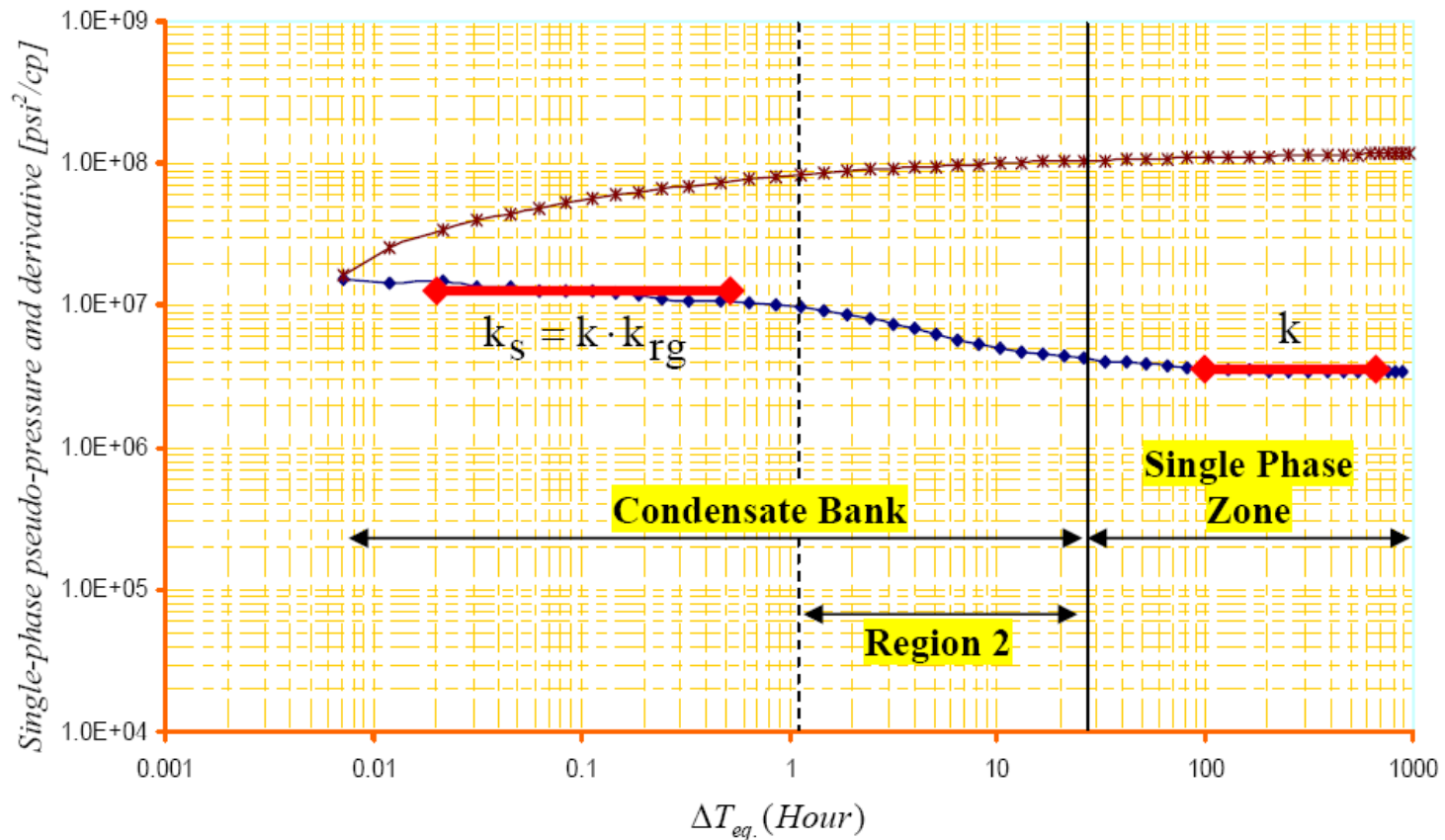
- This type of pressure response indicates that a **large condensate bank has developed** and **oil has not condensed in the entire reservoir**. It means the average reservoir pressure is above the dew point pressure of original gas in place.
- If the offset between the second and third horizontal segments is reduced, it shows the condensation occurs in all over the reservoir.



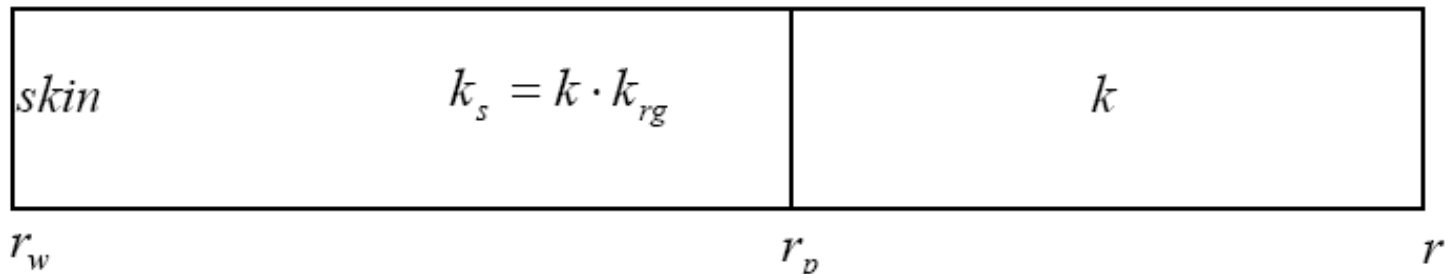
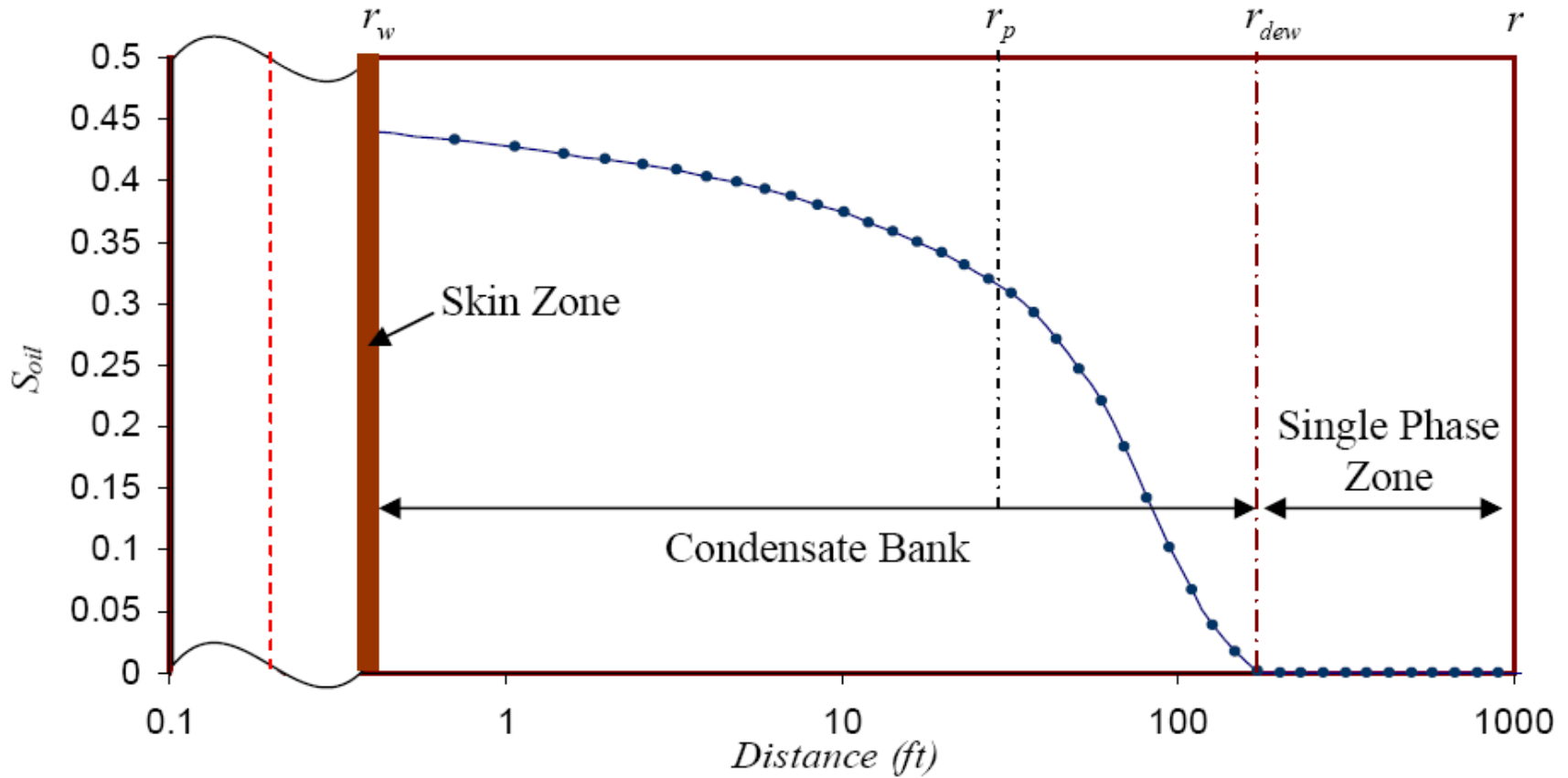
2- Zone Reservoir Model, Single Phase

Build-up analysis, Single-phase Pseudo-pressure and Derivative

The pressure build up test for second model shows **two horizontal segments** on its pressure derivative plot. When the **damage zone is too small** to recognize from the condensate bank, this type of pressure response can be obtained.



Two Zone Reservoir Model, Large Condensate Bank



Two-Phase Pseudo-Pressure Approach

Fevang's Approach, 3-Zone method

Using dry gas pseudo-pressure diffusivity equation for condensate and gas are not linear; the real gas pseudo-pressure only improves the compressibility effect which causes the deviation from the liquid solution. Due to the presence of condensate phase, the effective permeability to gas reduces significantly. The real gas pseudo-pressure does not consider this effective permeability reduction.

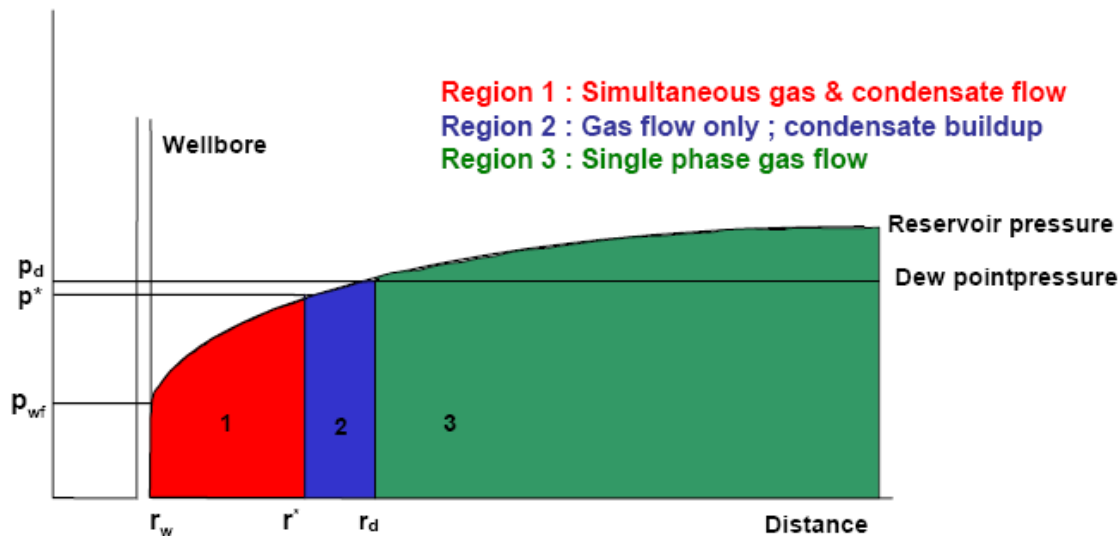
$$\Psi^{Total}(p) = 2 \int_{p_{ws}}^{p_w} \left(\frac{k_{ro}}{\mu_o \cdot z_o} + \frac{k_{rg}}{\mu_g \cdot z_g} \right) p \cdot dp$$

Gas and oil flow are taken into account in two phase pseudo-pressure method. This equation considers the compressibility of fluid as well as the relative permeability effect due to multiphase flow.

Two-Phase Pseudo-Pressure Approach

Fevang's Approach, 3-Zone method

- ❑ To calculate pseudo-pressure equation, the producing gas-oil ratio GOR is also required for each flowing bottomhole pressure p_{wf} along with PVT compositional properties and gas oil relative permeability.
- ❑ At any time of depletion, three flow regions may exist depending on the values of flowing bottomhole pressure and reservoir pressure.



Two-Phase Pseudo-Pressure Approach

Fevang's Approach, 3-Zone method

$$\Psi^{Total}(p) = 2 \int_{p_{ws}}^{p_w} \left(\frac{k_{ro}}{\mu_o \cdot z_o} + \frac{k_{rg}}{\mu_g \cdot z_g} \right) p \cdot dp$$

$$\Psi^{Total}(p) = \Psi^{R-1}(p) + \Psi^{R-2}(p) + \Psi^{R-3}(p)$$

Two-Phase Pseudo-Pressure Approach

Fevang's Approach, 3-Zone method

$$\Psi^{Total}(p) = \Psi^{R-1}(p) + \Psi^{R-2}(p) + \Psi^{R-3}(p)$$

$$\Psi^{R-1}(p) = 2 \int_{p_{ws}}^{p^*} \left(\frac{k_{ro}}{\mu_o \cdot z_o} + \frac{k_{rg}}{\mu_g \cdot z_g} \right) p \cdot dp$$

$$\Psi^{R-2}(p) = 2 \int_{p^*}^{p_{dew}} \left(\frac{k_{ro}}{\mu_o \cdot z_o} + \frac{k_{rg}}{\mu_g \cdot z_g} \right) p \cdot dp$$

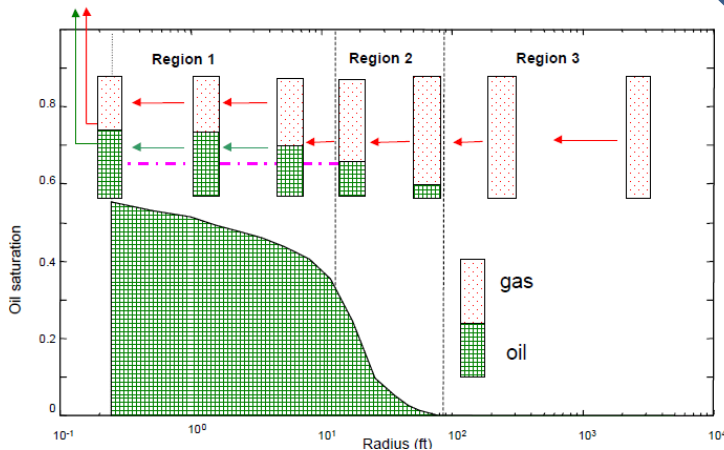
$$\Psi^{R-3}(p) = 2 \cdot k_{rg}(s_{wi}) \cdot \int_{p_{dew}}^{p_w} \left(\frac{p}{\mu_g \cdot z_g} \right) dp$$

Region 1

In Region 1, based on the assumptions that the oil and gas flow steadily and GOR is constant, the oil saturation for pressure range (p_{wf} , p^*) can be obtained by the following equation:

$$\frac{k_{ro}}{k_{rg}}(p) = \left(\frac{1 - r_s R_p}{R_p - R_s} \right) \frac{\mu_o B_o}{\mu_g B_{gd}}$$

Fetkovich et al. (1986)



$$R_p = \frac{q_g}{q_o} = \frac{q_{gstd} + q_{gdissolved}}{q_{ostd} + q_{odissolved}} = \frac{q_{gstd} + q_{ostd} \cdot R_s}{q_{ostd} + q_{gstd} \cdot r_s}$$

$$R_p = \frac{\frac{q_{gstd}}{q_{ostd}} + R_s}{1 + \left(\frac{q_{gstd}}{q_{ostd}} \right) \cdot r_s}$$

Or

$$R_p + \left(\frac{q_{gstd}}{q_{ostd}} \right) (R_p \cdot r_s - 1) = R_s$$

Therefore

$$\frac{q_{gstd}}{q_{ostd}} = \frac{R_s - R_p}{R_p \cdot r_s - 1}$$

k_{rg}/k_{ro} - Compositional Model

The saturation-pressure relationship corresponding to the steady-state flow shown by Chopra and Carter (1985) and also Jones and Raghavan (1988)

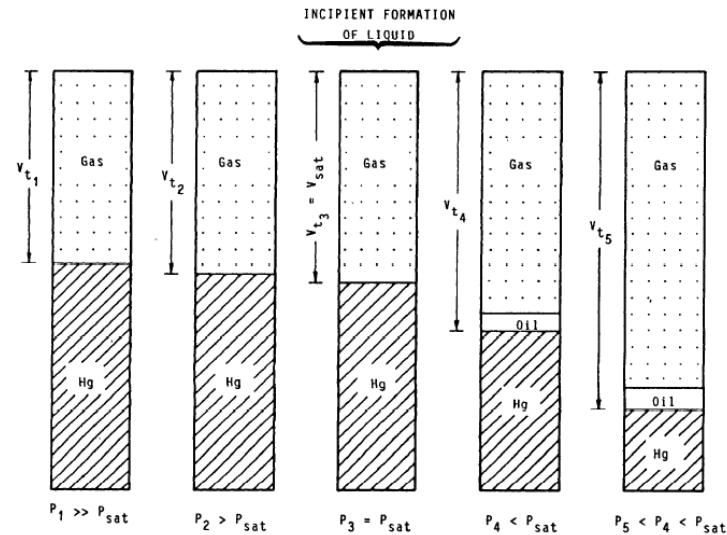
$$\frac{k_{ro}}{k_{rg}} = \frac{\rho_g L \mu_o}{\rho_o V \mu_g}$$

where L and V are the molar fraction of liquid and vapor calculated from flash equations. The left hand side of Eq. 3.19 is a function of saturation only and the right hand side is given by a CCE experiment and is only function of pressure.

k_{rg}/k_{ro} - Compositional Model

$$\frac{k_{rg}}{k_{ro}}(p) = \left(\frac{1}{V_{roCCE}} - 1 \right) \frac{\mu_g}{\mu_o}$$

$$V_{roCCE} = V_o / (V_g + V_o)$$



Data

P_s

Z

$$V_{ro} = V_o / V_t$$

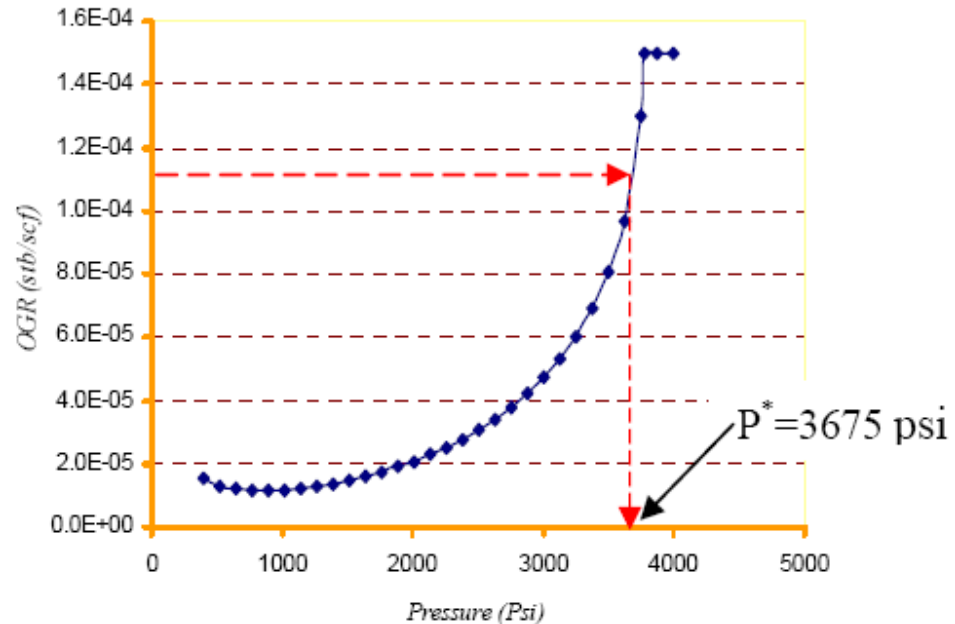
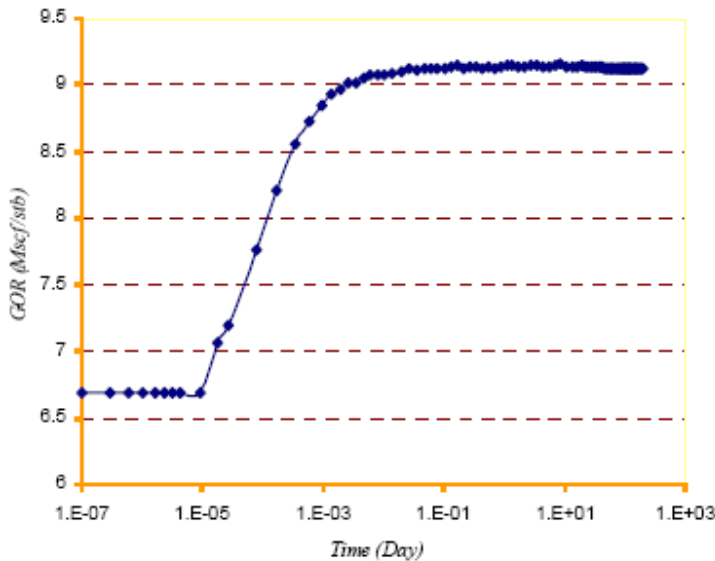
or

$$V_{ro} = V_o / V_d$$

V_{roCCE} : Relative oil volume in CCE experiment for each pressure step
 V_o : Oil volume in an experimental cell in each pressure step, ft³
 V_g : Gas volume in an experimental cell in each pressure step, ft³

Procedure to Calculate Relative Permeability as a Function of Pressure

1) Determine p^* p^* is the pressure where $r_s=1/R_p$



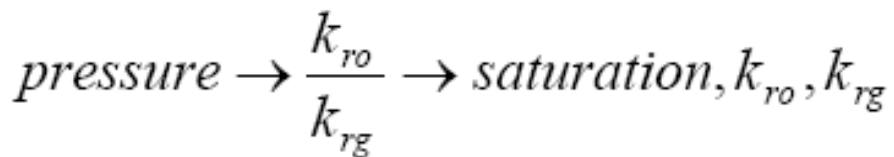
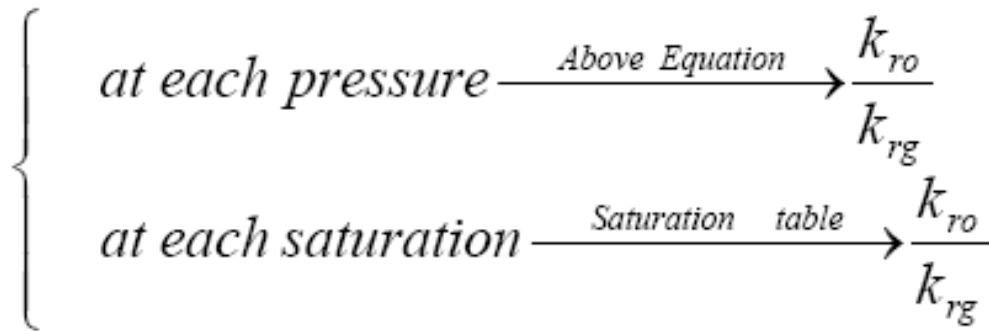
$$R_p = GOR @ \text{shut-in time} = 9.12 \frac{Mscf}{stb}$$

$$r_s = \frac{1}{R_p} = \frac{1}{9120} = 0.00011 \frac{stb}{scf}$$

Procedure to Calculate Relative Permeability as a Function of Pressure

2) if $p < p^*$ then use following equation,

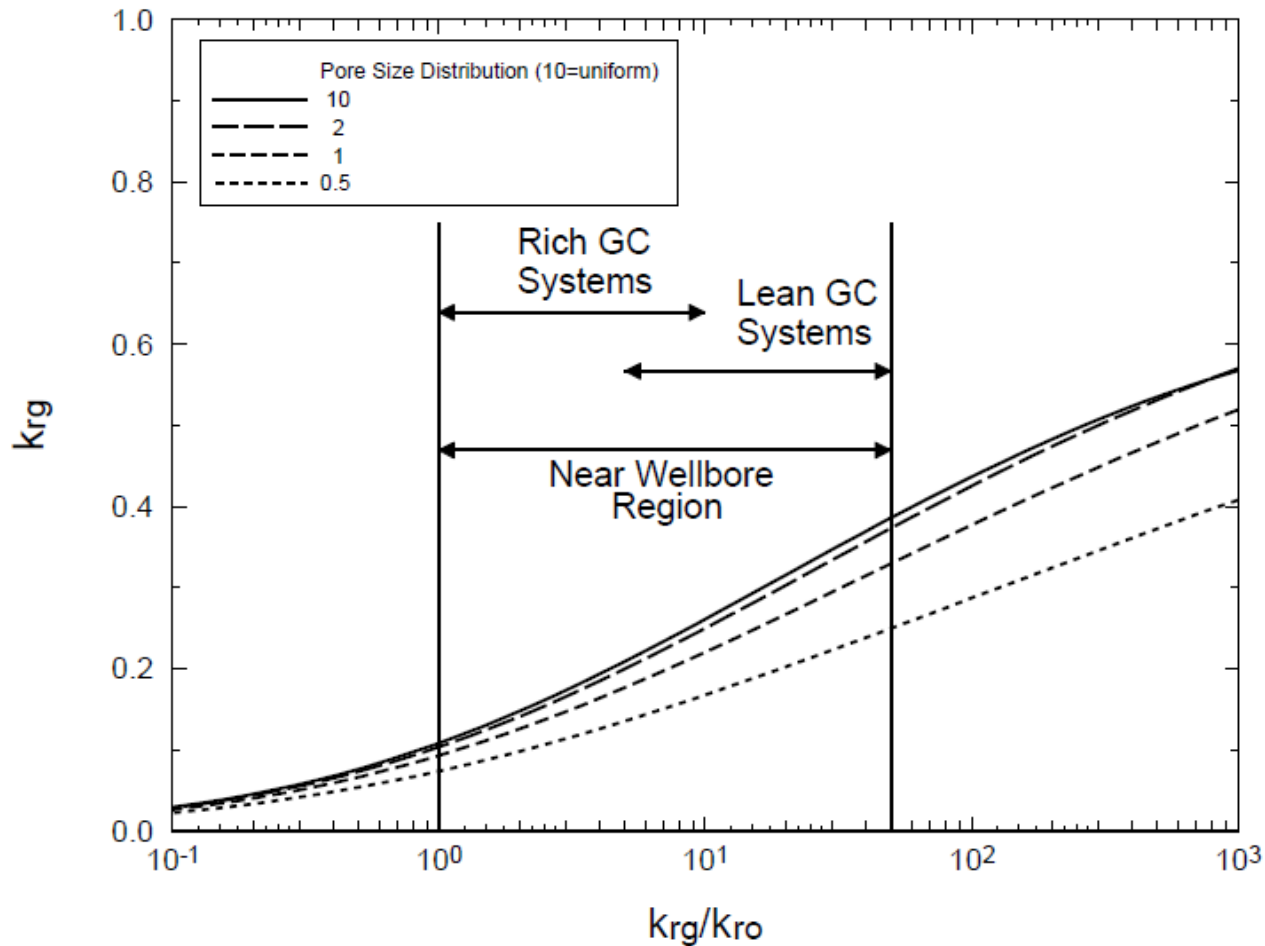
$$\frac{k_{ro}}{k_{rg}}(p) = \left(\frac{1 - r_s R_p}{R_p - R_s} \right) \frac{\mu_o B_o}{\mu_g B_{gd}}$$



$$\frac{k_{rg}}{k_{ro}}(p) = \left(\frac{R_p - R_s}{1 - r_s R_p} \right) \frac{\mu_g B_{gd}}{\mu_o B_o}$$

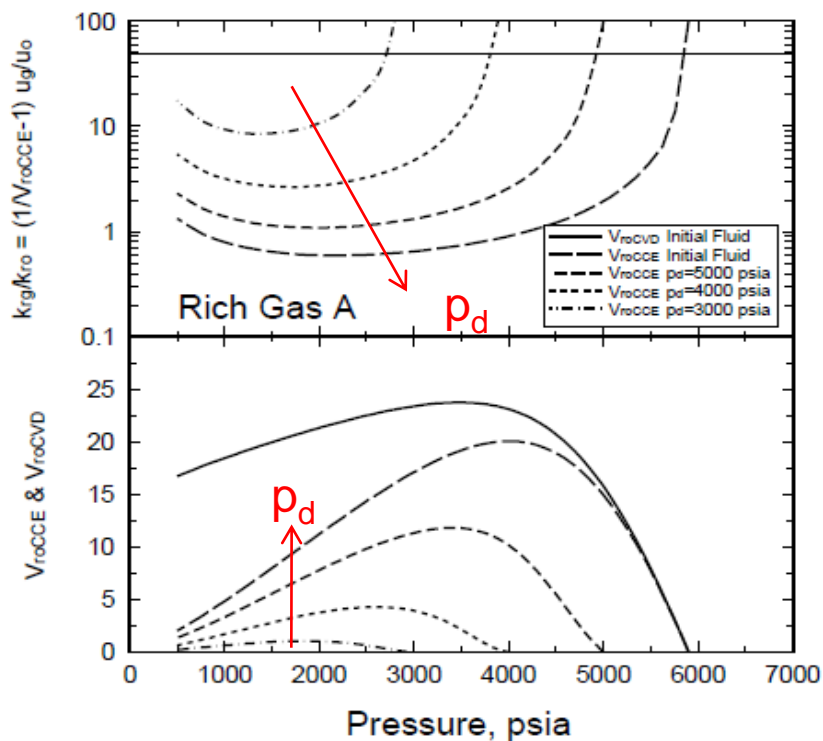
so

$$k_{rg}(p) = f(k_{rg}/k_{ro}(p))$$

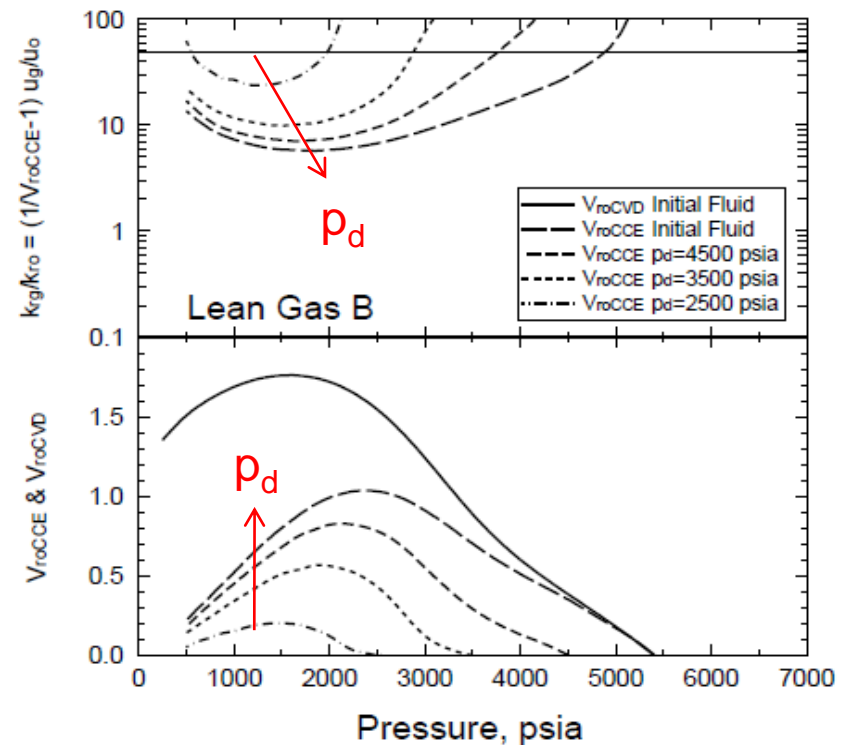


- k_{rg} and k_{ro} can each be expressed directly as a function of the ratio k_{rg}/k_{ro} when both phases are mobile. (Evinger and Muskat).

- If the ratio of k_{rg}/k_{ro} is known, the values of k_{rg} and k_{ro} can be calculated from the relative permeability curves, and the pseudo-pressure integral evaluated.

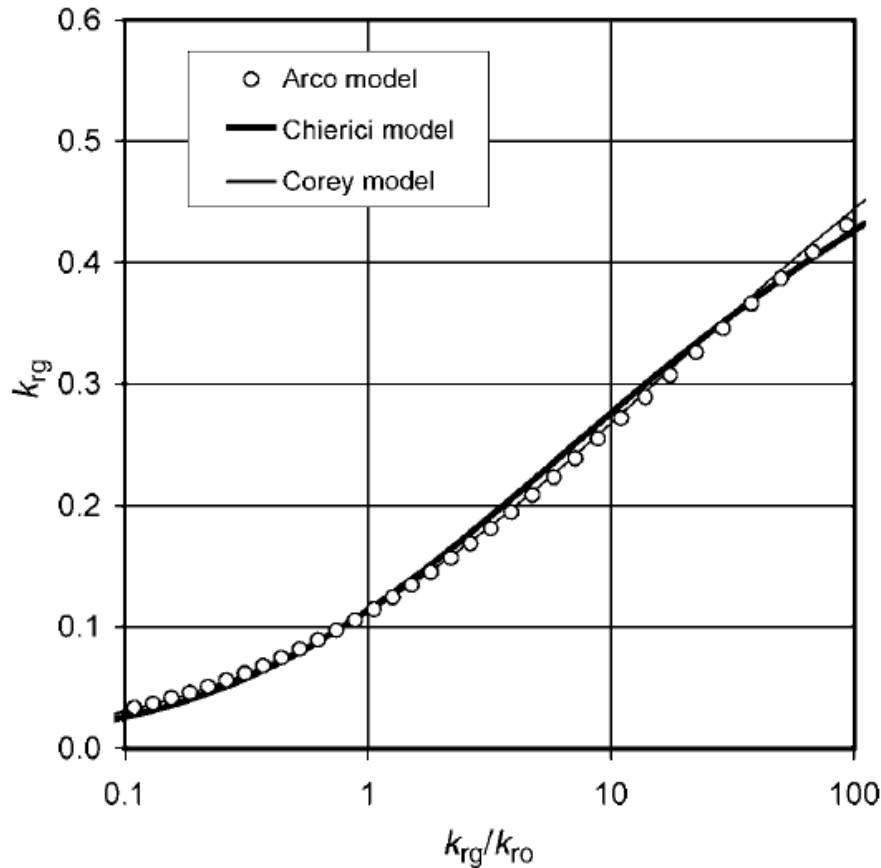


(Fevang, 1995)

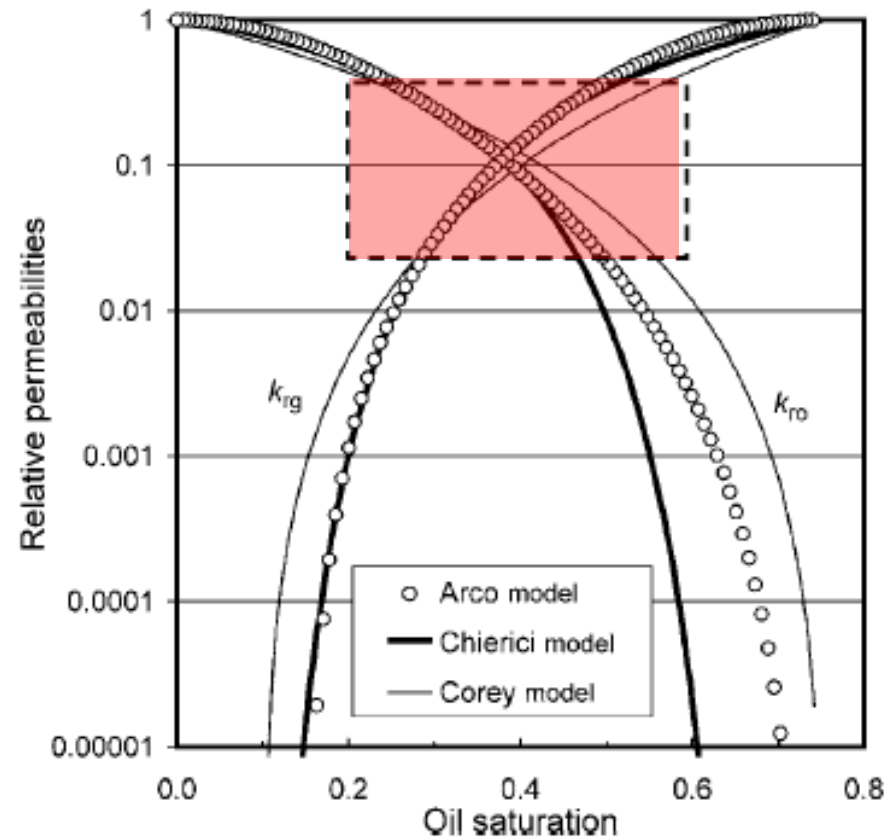


Diagnostic plots for Rich Gas A and Lean Gas B showing the variation of k_{rg}/k_{ro} (in Region 1) and CCE oil relative volume as a function of pressure during depletion.

$K_{rg}(k_{rg}/k_{ro})$ is the fundamental relationship controlling the reduction in gas relative permeability in the near well bore region.



(Whitson et al., 2003)



Saturation dependent relative permeability curves for three different immiscible correlations, showing in particular the region that affect flow behavior in the near-well region (boxed area). The three correlations have the 'same' $k_{rg} = f(k_{rg}/k_{ro})$ relationship in the boxed region.

Procedure to Calculate Relative Permeability as a Function of Pressure

3) if dew $p^* < p < p_{\text{dew}}$ then use following equation,

$$S_o(p) = (1 - S_{wi}) \cdot S_{o,CVD}(p)$$

Region 2

The oil saturation in Region 2 for pressure range p_{dew} , p^* can be obtained from the following equation,

$$S_o(p) = (1 - S_{wi}) \cdot S_{o,CVD}$$

□ p^* must be equal to the dew-point of the producing wellstream, since the flowing mixture composition and the producing GOR is constant in Region 1 and equal to R_p .

The Required Data and Information for 3-Zone Method

1. Measurements of the well pressure during the test.
2. Standard requirements for well tests (flow rate, production time, average porosity).
3. Representative original reservoir gas characterization (tuned equation of state together with gas composition).
4. Representative relative permeability curves.
5. A correct measure of the producing GOR (R_p).
6. Black-oil representation of the reservoir fluid. (This can be deduced directly from the reservoir fluid composition and a tuned equation of state).

Main Equations for Gas Condensate Reservoirs

$$\Psi^{Total}(p) = 2 \int_{p_{ws}}^{p_w} \left(\frac{k_{ro}}{\mu_o \cdot z_o} + \frac{k_{rg}}{\mu_g \cdot z_g} \right) p \cdot dp$$

$$\Psi^{Total}(p) = \Psi^{R-1}(p) + \Psi^{R-2}(p) + \Psi^{R-3}(p)$$

$$\Psi^{R-1}(p) = 2 \int_{p_{ws}}^{p^*} \left(\frac{k_{ro}}{\mu_o \cdot z_o} + \frac{k_{rg}}{\mu_g \cdot z_g} \right) p \cdot dp$$

$$\Psi^{R-2}(p) = 2 \int_{p^*}^{p_{dew}} \left(\frac{k_{ro}}{\mu_o \cdot z_o} + \frac{k_{rg}}{\mu_g \cdot z_g} \right) p \cdot dp$$

$$\Psi^{R-3}(p) = 2 \cdot k_{rg}(s_{wi}) \cdot \int_{p_{dew}}^{p_w} \left(\frac{p}{\mu_g \cdot z_g} \right) dp$$

Main Equations for Gas Condensate Reservoirs

$$\Psi_i - \Psi_{wf}(t) = 1.632 \times 10^6 \frac{q_g T}{kh} \left[\log \frac{k \Delta t_{eq}}{\phi \cdot \mu_{gi} \cdot c_{ti} \cdot r_w^2} - 3.23 + 0.87s \right]$$

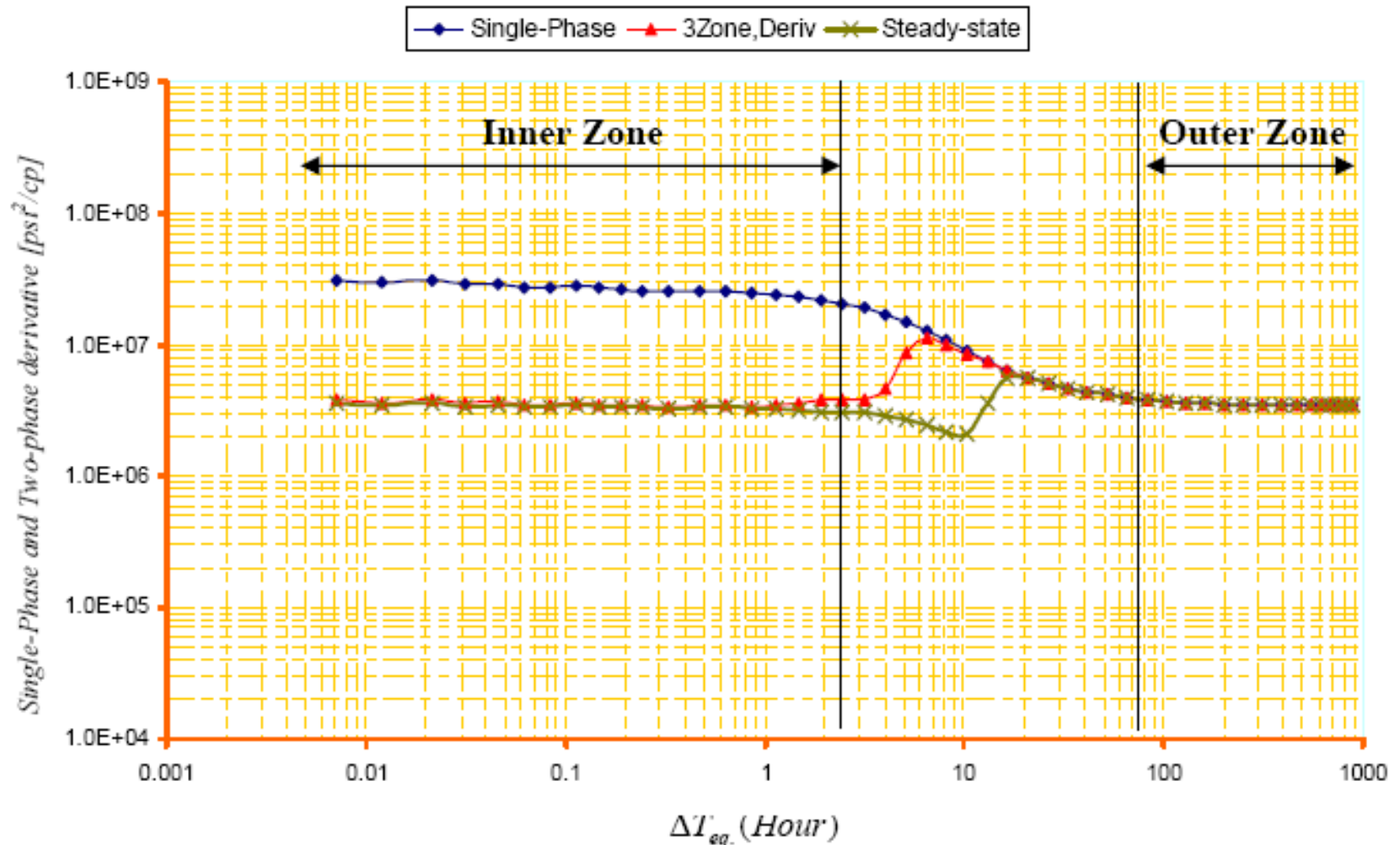
$$\Delta t_{eq} = \frac{t_p \cdot \Delta t}{t_p + \Delta t}$$

$$\text{Der} = \frac{d\Delta\Psi}{d \ln(\Delta t_{eq})}$$

$$k = 7.088 * 10^5 \frac{q_g T}{\text{Der} \cdot h}$$

$$s = 1.151 \left[\frac{\Psi_{ws,1hour} - \Psi_{wf0}}{2.303 \cdot \text{Der}} - \log \left(\frac{k}{\phi \cdot \mu_{gi} \cdot c_{ti} \cdot r_w^2} \right) + 3.23 \right]$$

Comparison of (1) 3Zone and Steady-state Two-phase Pseudo-pressure Derivative with (2) Single phase Pseudo-pressure Derivative



Gas Condensate Reservoirs- Fluid Behavior Near-wellbore Phenomena (Danesh, *et al.*, 1994)

Positive Coupling ➤

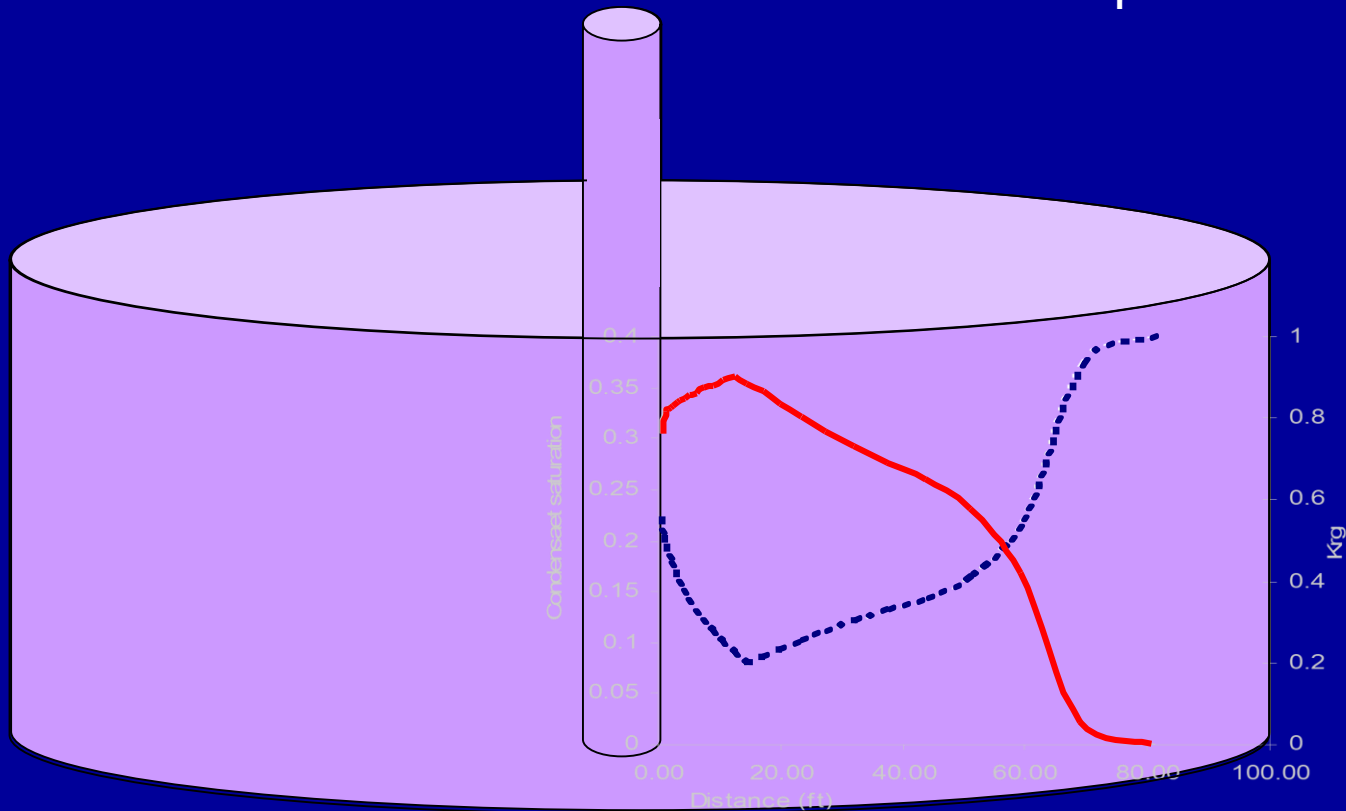


K_r increase

Negative Inertia ➤



K_r decrease



Capillary Number

- Most gas-condensate reservoirs are found at near-critical conditions where the interfacial tension between gas and condensate is low.
- Experimental studies have shown that as interfacial tensions (IFT) decrease, the relative permeability curves become progressively straighter (miscible) whereas the residual fluid saturations decrease.
- An increase in relative permeability with velocity has been demonstrated in numerous laboratory core-flood experiments and actual field data. Danesh et al were the first to report laboratory experiments results showing improvements of relative permeability in condensate systems with increases in velocity or decreases in interfacial tension.
- High velocities and low IFT's both increase the ratio of viscous to capillary forces and can be represented by a single parameter, called the capillary number (N_c)
- A combination of IFT and velocity is called Capillary Number.
$$N_c = \frac{\mu_g v_s}{\sigma}$$

Effect of IFT & Rate on Darcy K_r

Until 1995 it was assumed that relative permeability of a fluid is mainly a function of:

$$k_r = f \left\{ \begin{array}{l} \textit{Pore size distribution} \\ \textit{Wettability} \\ \textit{Saturation} \\ \textit{Saturation history} \\ \textit{IFT} \end{array} \right.$$

$$N_c = \frac{\mu_g v_s}{\sigma}$$

In 1995 both Shell and Heriot-Watt (Henderson) published results showing the effect of capillary number (N_c) on relative permeability.

- Capillary number describes the relative balance of viscous and capillary forces

- $(N_c = \Delta p_{\text{viscous}}/P_c, \text{ or } N_c = v_{pg}\mu_g/\sigma_{go})$

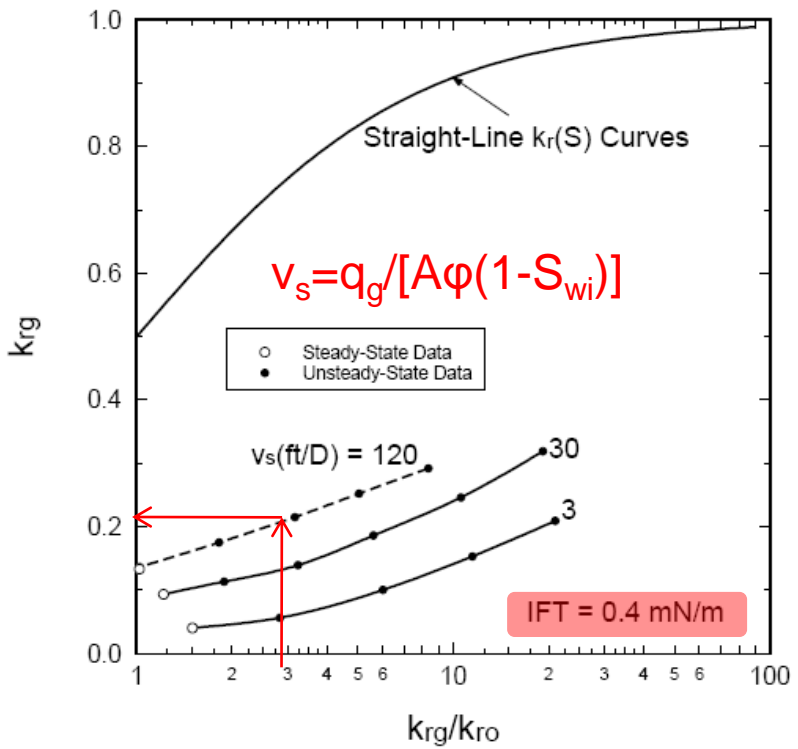
- For small N_c , capillary forces dominate and traditional ('immiscible') relative permeability behavior is found.

- For large N_c , viscous forces dominate and relative permeabilities tend to approach straight lines or 'miscible-like' behavior.

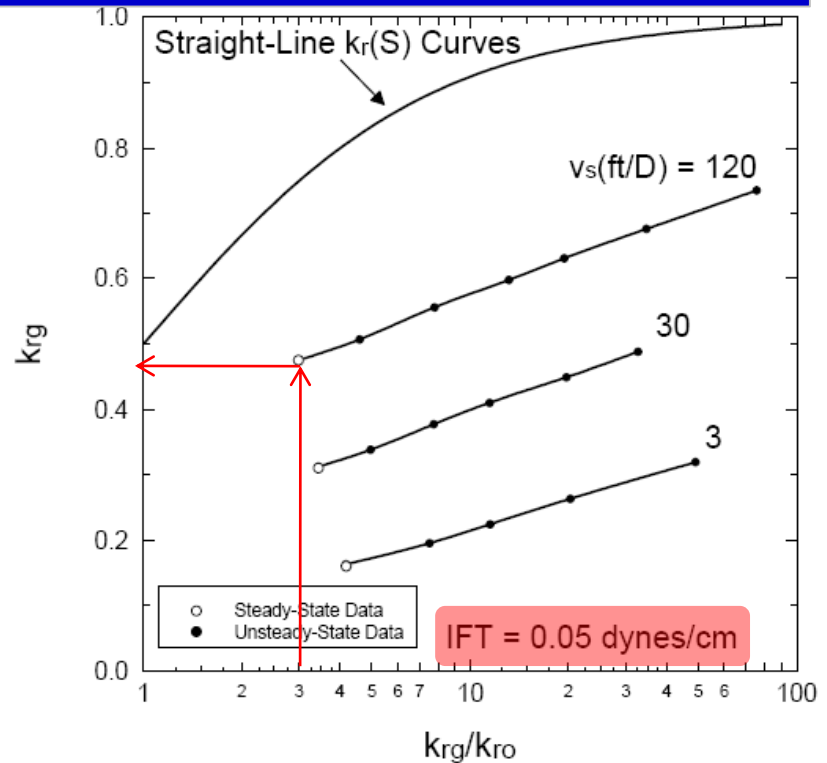
- IFT change the shape of the curve

- v_s shifts the curve to higher k_{rg}

Velocity effect on $k_{rg} = f(k_{rg}/k_{ro})$ relationship for a Berea sandstone and a synthetic gas condensate mixture Data taken from Henderson et al.

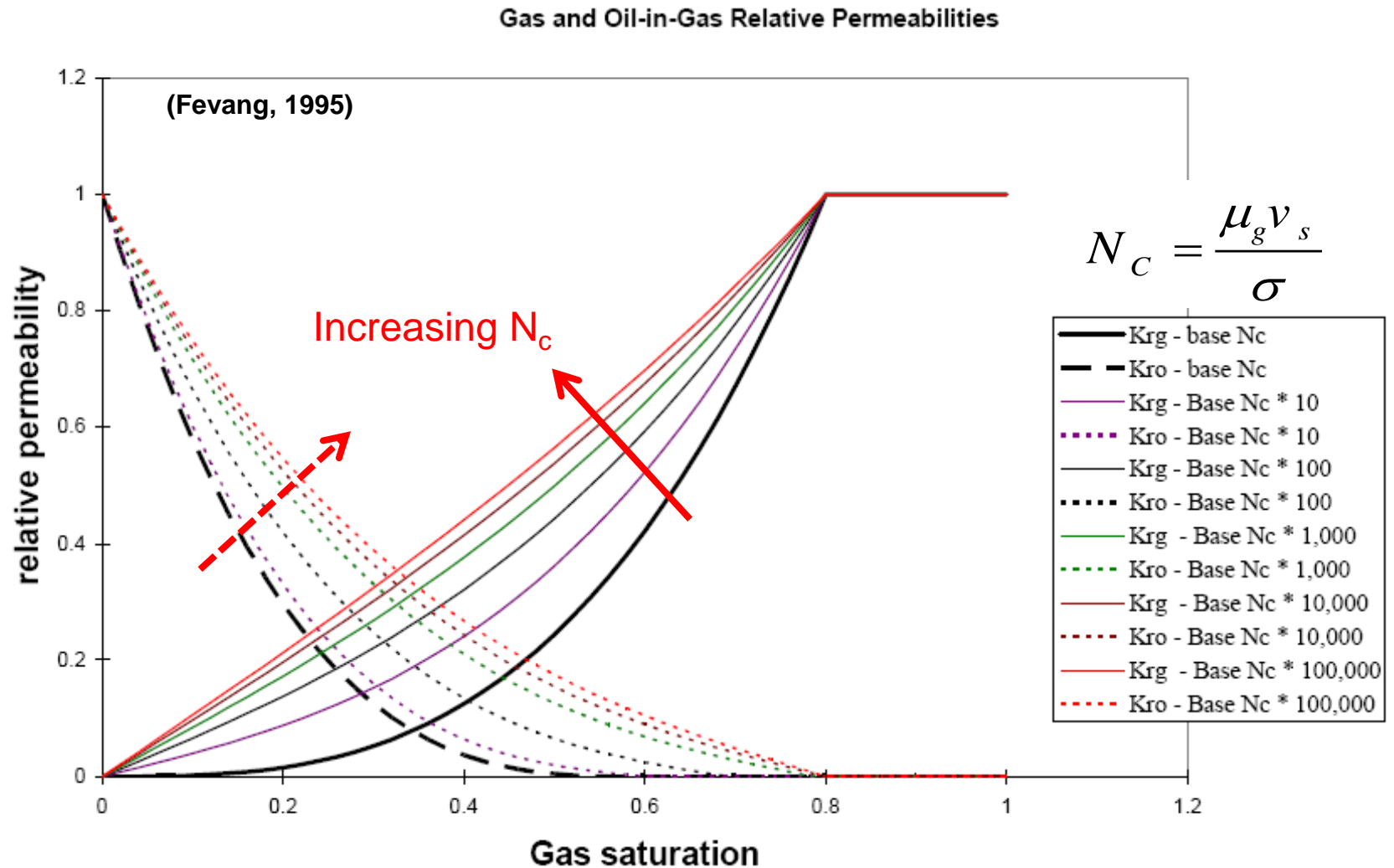


(Fevang, 1995)



Miscible and Immiscible K_r

As IFT approaches zero the relative permeabilities approach straight lines with zero residual saturations



Whitson and Fevang (1997)

$$k_{rg} = f_I \cdot k_{rgI} + (1 - f_I) \cdot k_{rgM}$$

$$k_{rgM} = k_{rg}^o \cdot \frac{1}{1 + (k_{rg}/k_{ro})^{-1}}$$

Transition function $f_I = \frac{1}{(\alpha \cdot N_c)^n + 1}$

α is a constant dependent only on rock properties $\alpha = \frac{2 \cdot 10^4}{\sqrt{k\phi}}, \quad n = 0.7$

k_{rg}^o gas relative permeability at S_{wi}

k_{rgI} and k_{rgM} are evaluated at the same value of k_{rg}/k_{ro} – not at the same saturation.

It only can be used for the steady-state region where both gas and oil are flowing.

Effect of Non-Darcy Flow on K_r

In near the wellbore region where the velocity is highest, any positive effect that high N_c has on "Darcy" relative permeability may be reduced by non-Darcy flow effects.

Non-Darcy Flow: Forchheimer equation

To quantify the effect of non-Darcy pressure loss, an effective gas relative permeability $k_{rg,eff}$ is defined.

$$-\frac{\partial P}{\partial x} = \frac{\mu}{k} V + \beta \rho V^2$$



$$-\frac{\partial P}{\partial x} = \frac{\mu}{k_{HVF}} V$$

$$\beta = ak^b \phi^c \quad a \sim 10^9 - 10^{10}, \quad b \sim 0.5 - 1.5, \quad \text{and} \quad c \sim 0 - 5.5.$$

The correction of β for relative permeability effect (Blom and Hagoort ;2003b)

$$\beta_{eff} = \beta \cdot k_{rg}^{-b'}$$

$k_{rg, HVF}$ is defined such that the pressure drop using only two-phase Darcy's law with $k_{rg, HVF}$ is equal to the pressure drop using two-phase Forchheimer equation.

$$\frac{k_{rgHVF}}{k_{rg}} = \left[1 + \frac{k \cdot k_{rg}}{\mu_g} \cdot \beta_{eff} \cdot \rho_g \cdot V_g \right]^{-1}$$

$$k_{ro} = k_{rgHVF} / (k_{ro} / k_{rg})$$