Chapter 3

Analytical Water & Chemical Flooding Performance

Analytical Performance of water & Chemical Flooding

- The objective is to introduce fundamental concepts of EOR processes using simple mathematical models that retain important features of more complex models.
- The application of frontal advance theory to predict
  - Water flooding performance
  - Polymer flooding performance
  - Surfactant flooding performance

Fractional Flow Theory

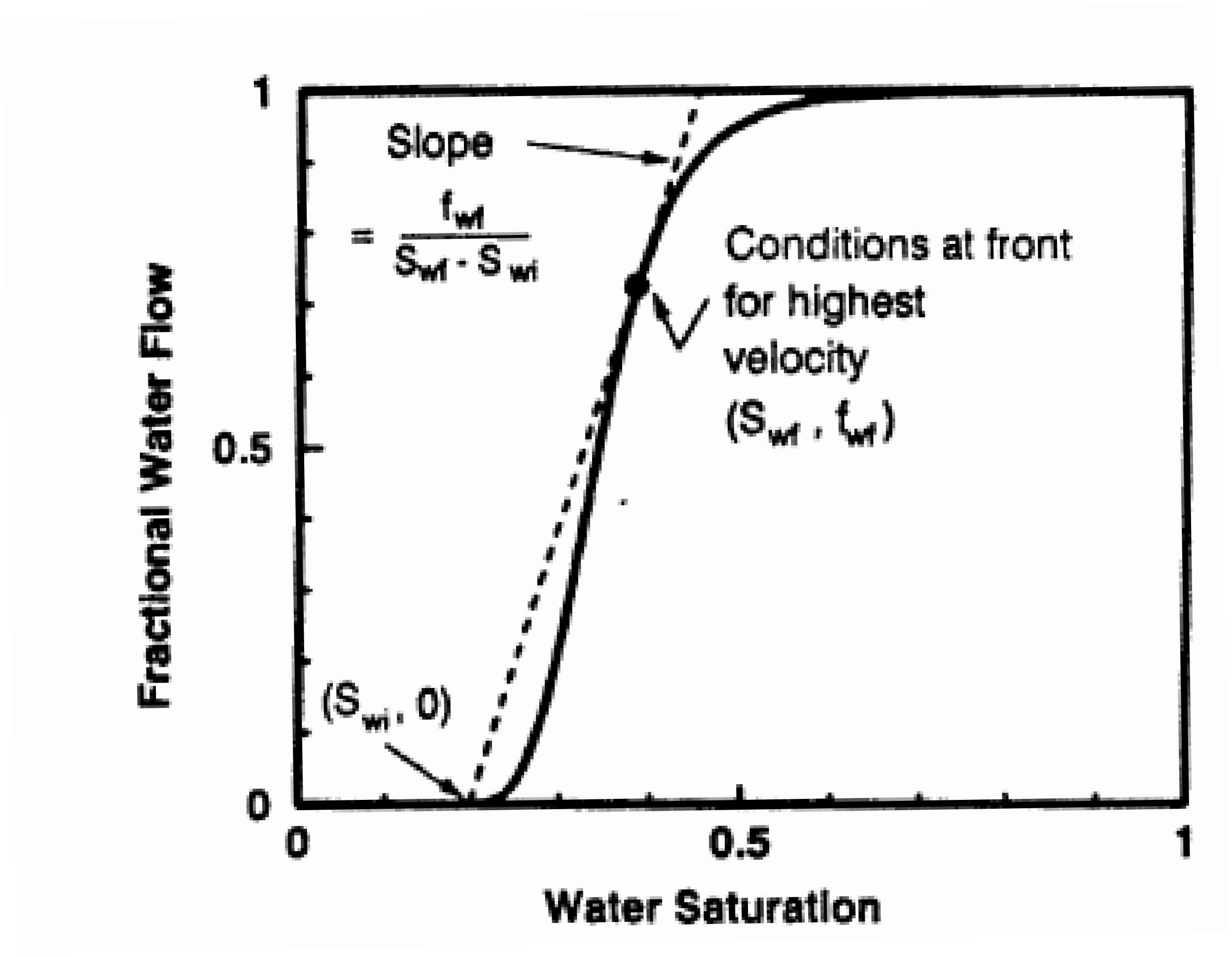
- **Objectives:**
  - Estimate oil recoveries using fractional flow theory and Buckley-Leveret 1-D displacement
  - Estimate the front location at any time
  - Estimate production ratios

Fractional flow of water

- In the absence of gravity & capillary effect the fractional flow can be shown as :

$$f_w = \frac{k_w / \mu_w}{k_o / \mu_o + k_w / \mu_w}$$

Typical Fractional flow vs. saturation



Frontal Advanced Equation

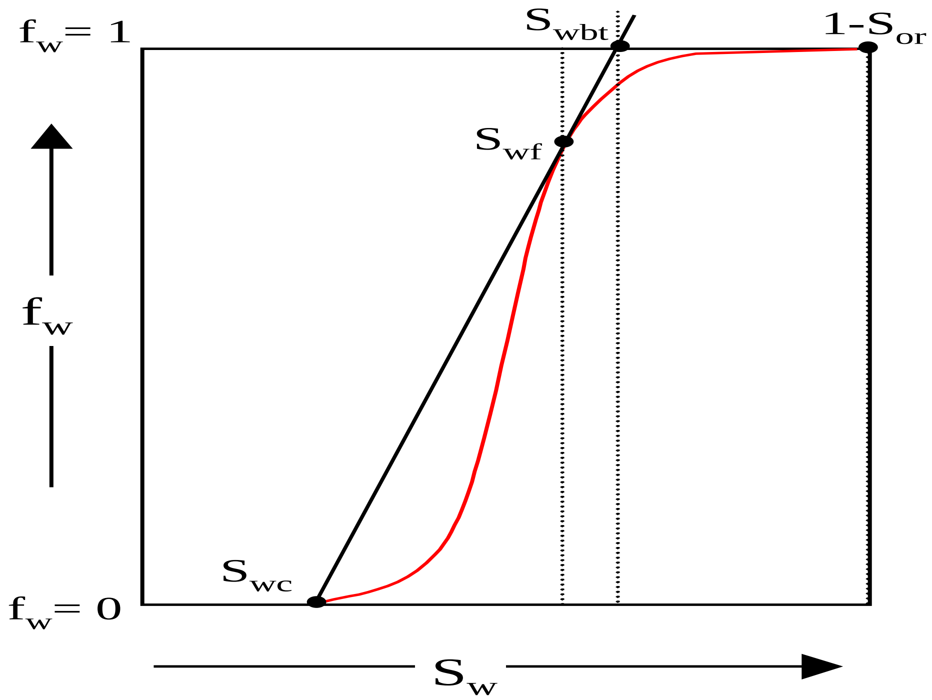
- When water is injected into a linear system as sufficient rate for the front advance assumptions to apply, each water saturation, S_w, travels at a constant velocity through the system given:

$$\frac{dx_{sw}}{dt} = \frac{q_t}{A\phi}\left(\frac{\partial f_{wf}}{\partial s_{wf}}\right)_{s=sw}$$

- This is called the frontal advance, or Buckley Leverett equation
- Where x_{sw}= location of water saturation
- S_w, measured from x=0
- t=time from the beginning of injection

General Shape of the Fractional Flow Equation

Since the viscosities and densities are assumed constant, the fractional flow equation depends only upon saturation through the relative permeabilities.



How to apply the Buckley-Leverett-Welge Method

1. Draw the fractional flow curve
2. Draw the tangent line
3. Calculate the oil recovery at breakthrough
4. Calculate the time of breakthrough
5. Calculate the oil production after breakthrough

Frontal-Advanced Equation

- The slope of flood-front saturation:

$$f'_{wf} = \frac{f_{wf} - f_{iw}}{s_{wf} - s_{iw}} = \frac{f_{wf}}{s_{wf} - s_{iw}}$$

- The velocity of the flood front or saturation discontinuity,

$$v_{wf} = \frac{q_t}{A\phi}\left(\frac{\partial f_w}{\partial s_w}\right)_{s_{wf}} = \frac{q_t}{A\phi}\frac{f_{wf}}{s_{wf} - s_{iw}}$$

- All saturation less that S_{wf} travel at the flood front velocity

Dimensionless Variables

- The location of a particular saturation is found by integrating $\frac{dx_{sw}}{dt}$ respect to time,

$$x_{sw} = \frac{q_t t}{A\phi}\left(\frac{\partial f_w}{\partial s_w}\right)_{s_w}$$

- Introducing dimensionless variables

$$x_D = \frac{x}{L}$$

$$t_D = \frac{qt}{A\phi L}$$

$$x_{Ds_w} = t_D f_w^{'}$$

Performance Calculation

- At breakthrough,

$$x_D = 1$$

$$t_{Dbt} = \frac{1}{f'_{wf}}$$

- The oil displaced before BT= q_t
- The oil displaced after BT=Np

$$N_p = \frac{\left(\overline{s_w} - s_{iw}\right) A \phi L}{B_o}$$

Average Saturation Calculation

- Where $\overline{s_w}$ is computed from the Welge equation,

$$\overline{s_w} = s_{w2} + t_{D2}\left(1 - f_{w2}\right)$$

$where;$

$s_{w2} = water\ saturation\ at\ x_D = 1$

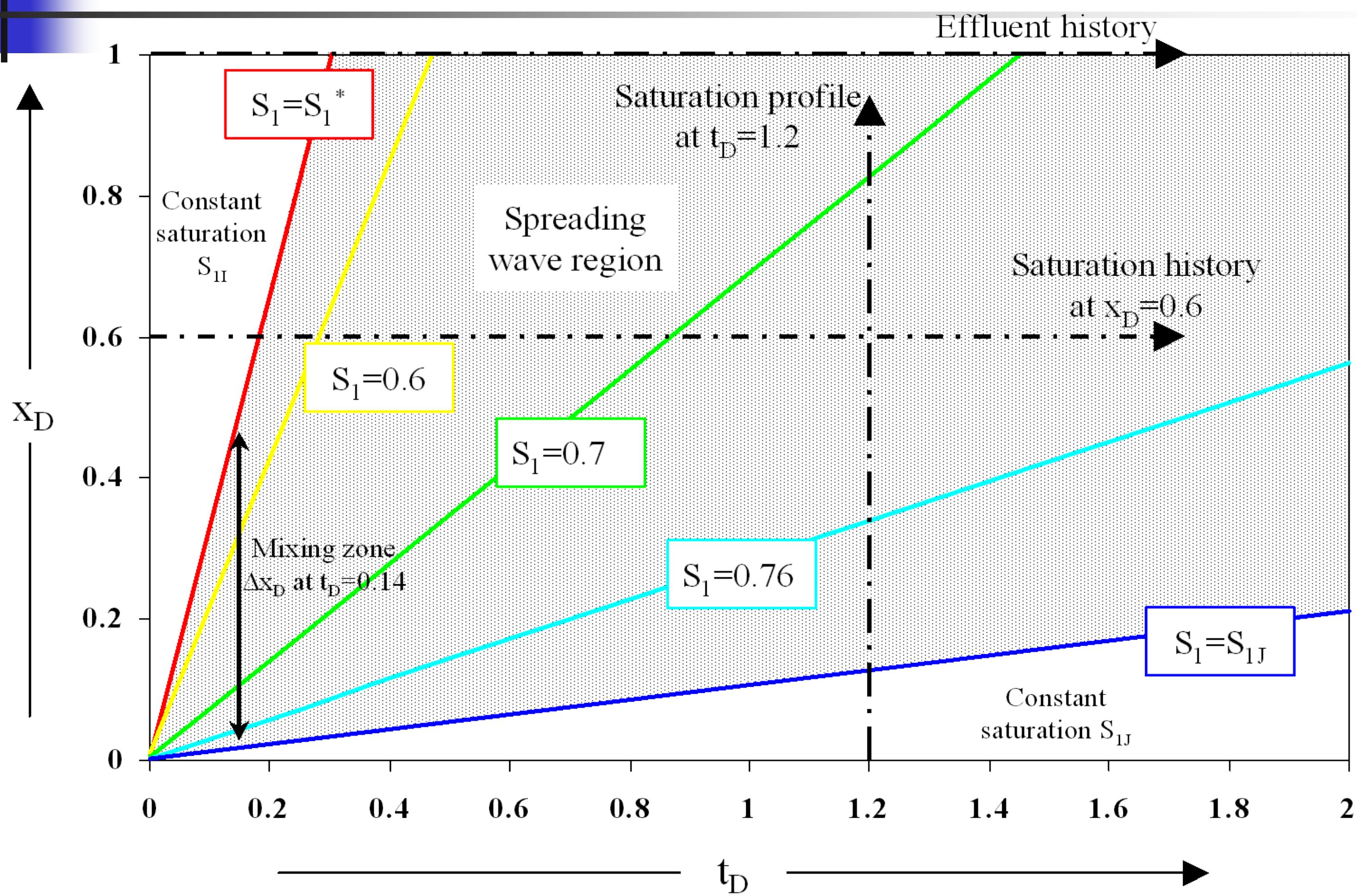
$f_{w2} = fractional\ flow\ of\ water\ at\ x_D = 1$

$t_{D2} = Dimentionless\ time\ required\ to\ propagate\ saturation\ S_{w2}$

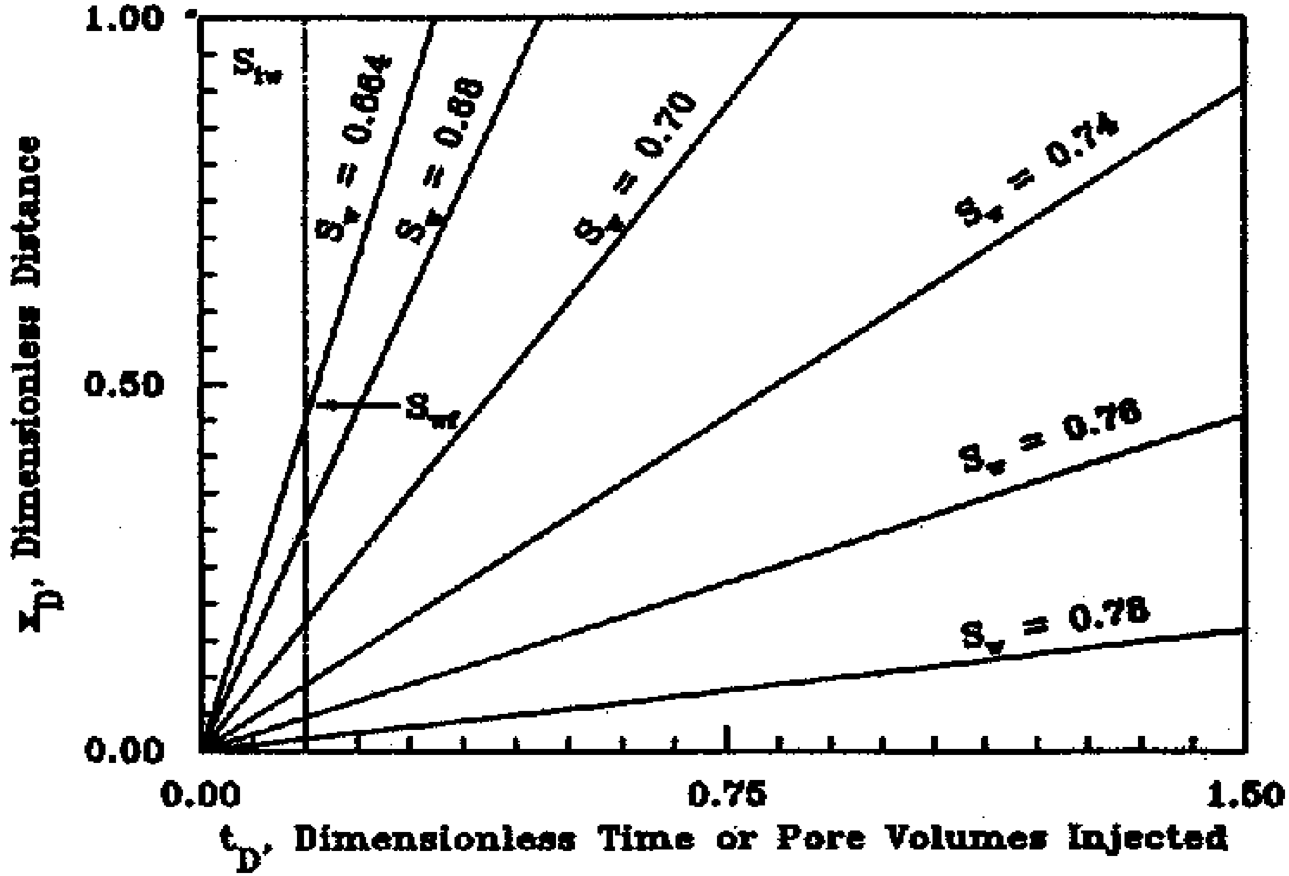
$from\ the\ inlet\ of\ the\ system\ \left(x_D = 0\right)\ to\ the\ end\ of\ the\ system\ \left(x_D = 1\right)$

$$t_{D2} = \frac{1}{f'_{w2}}$$

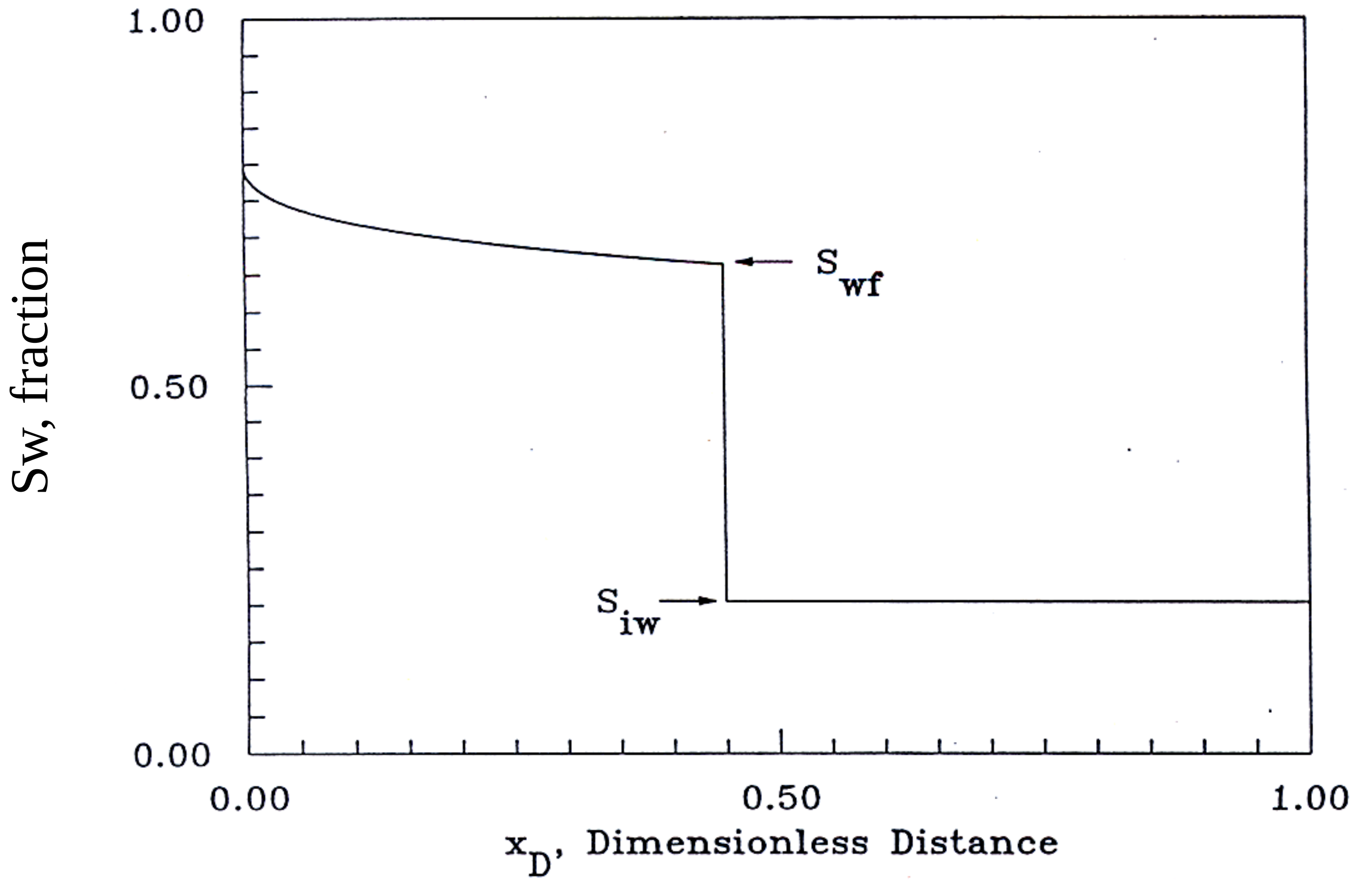
Dimensionless time



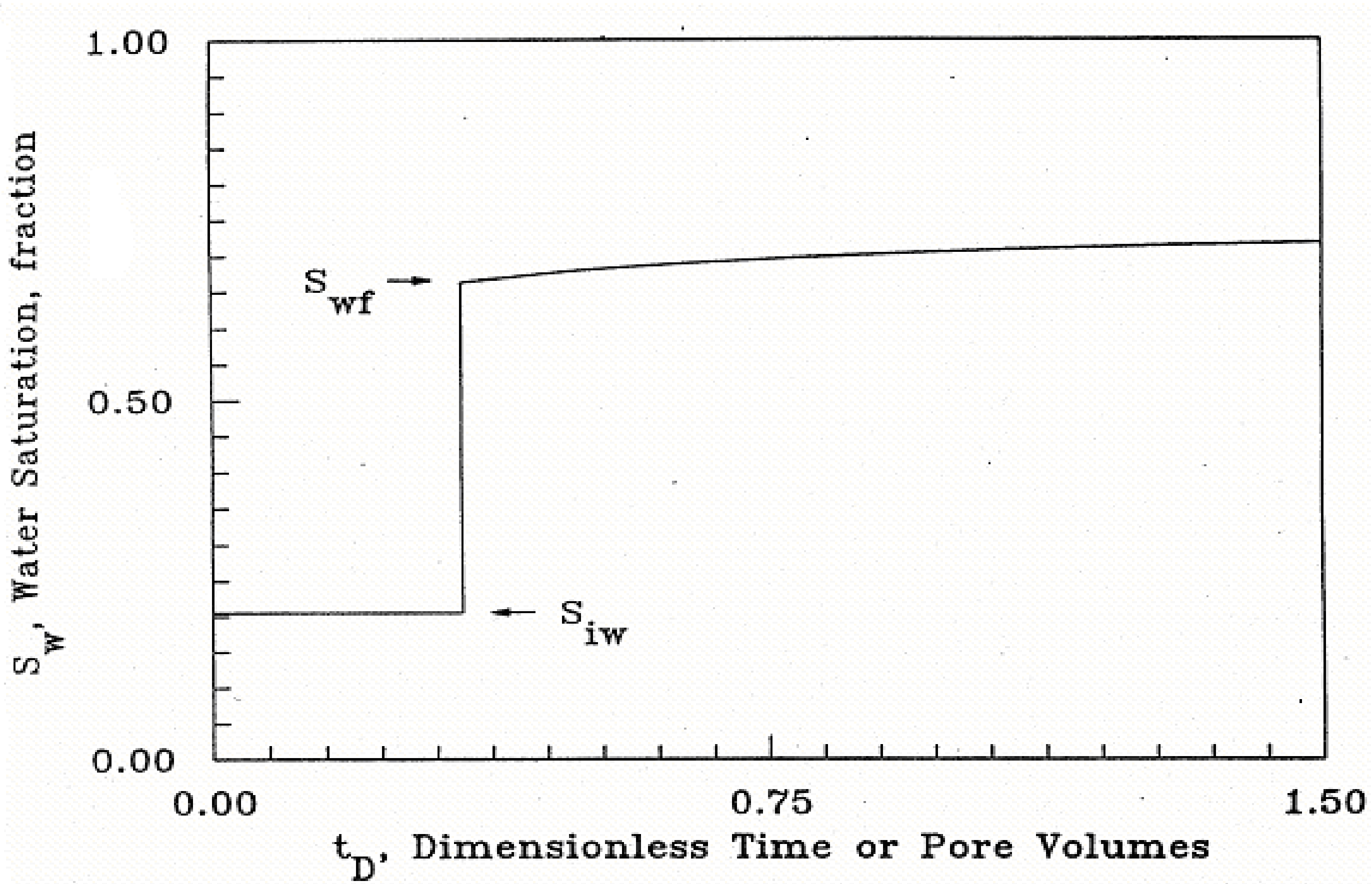
X_D VS. t_D Diagram



Saturation Profile- Specified t_D



Saturation History Graph-Specified X_D



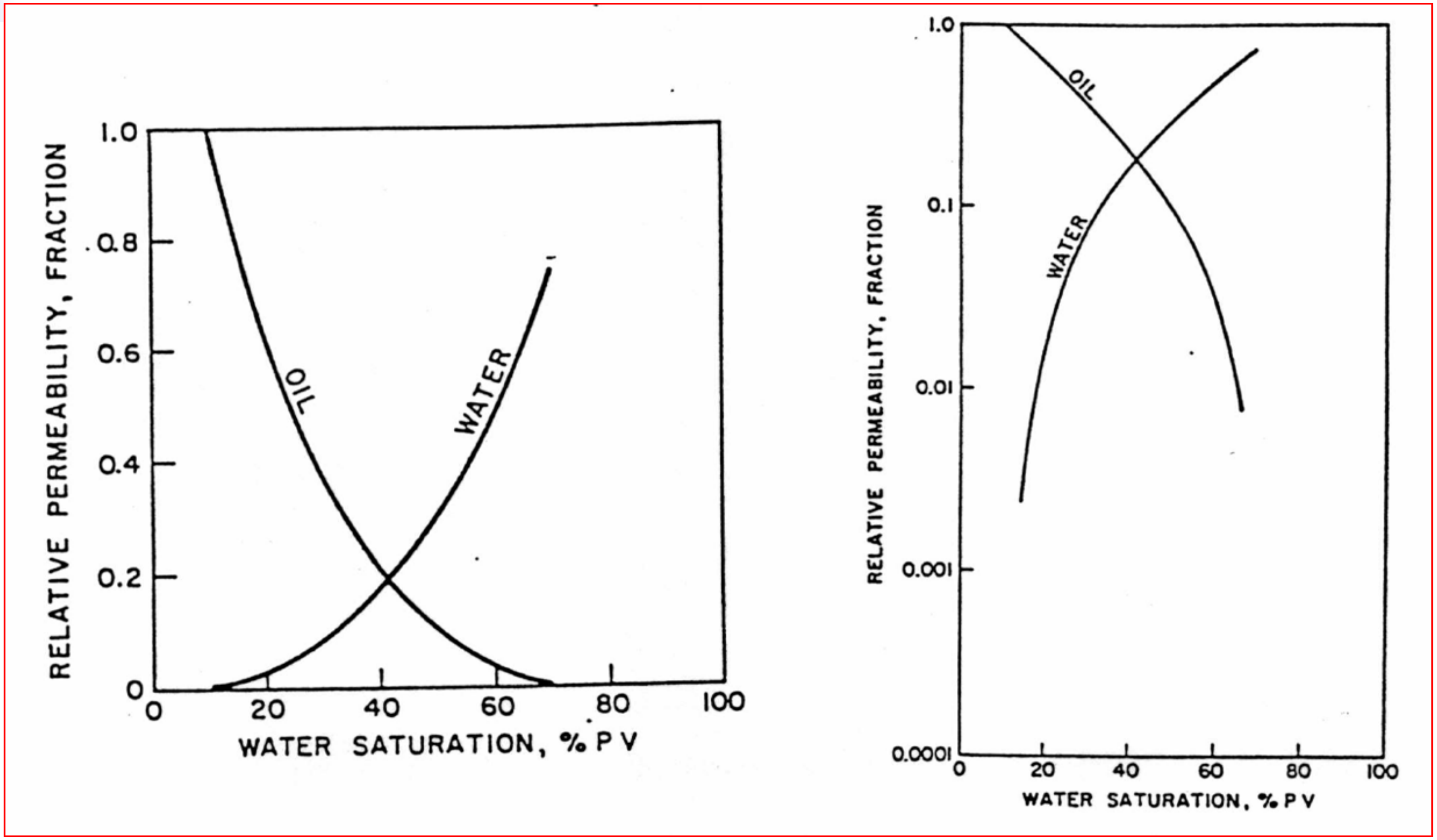
Example 1

A line-drive water flood is being considered for a reservoir having the characteristics given in the following Table.

Find the breakthrough time & Oil production at WOR OF 49.

Data

Distance between injection wells and production wells, m	200
Average reservoir width between injection and production wells, m	75
Average formation height, between injection and production wells, m	20
Formation dip angle, deg.	0
Porosity, percent	20
Average permeability, md	50
Proposed water injection rate, m3/d	150
Connate water saturation, percent	10
Oil viscosity at current reservoir temperature & pressure, mPa.s	1.0
Water viscosity at current reservoir conditions, mPa.s	0.5
Oil formation volume factor at current conditions	1.2
Water formation volume factor,	1.0



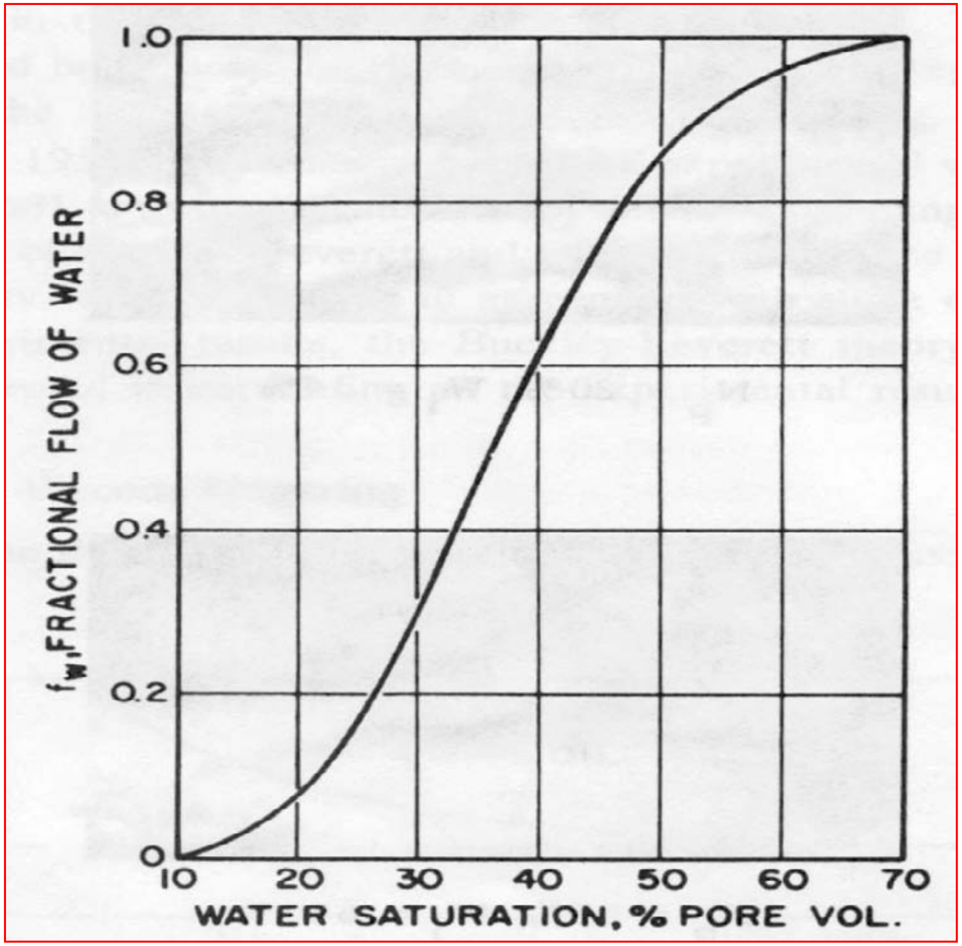
RELATIVE PERMEABILITY, FRACTION
1.0
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
OIL
WATER
WATER SATURATION, % PV
RELATIVE PERMEABILITY, FRACTION
1.0
0.1
0.01
0.001
0.0001
0
20
40
60
80
100
OIL
WATER
WATER SATURATION, % PV

Calculate the fractional flow curve

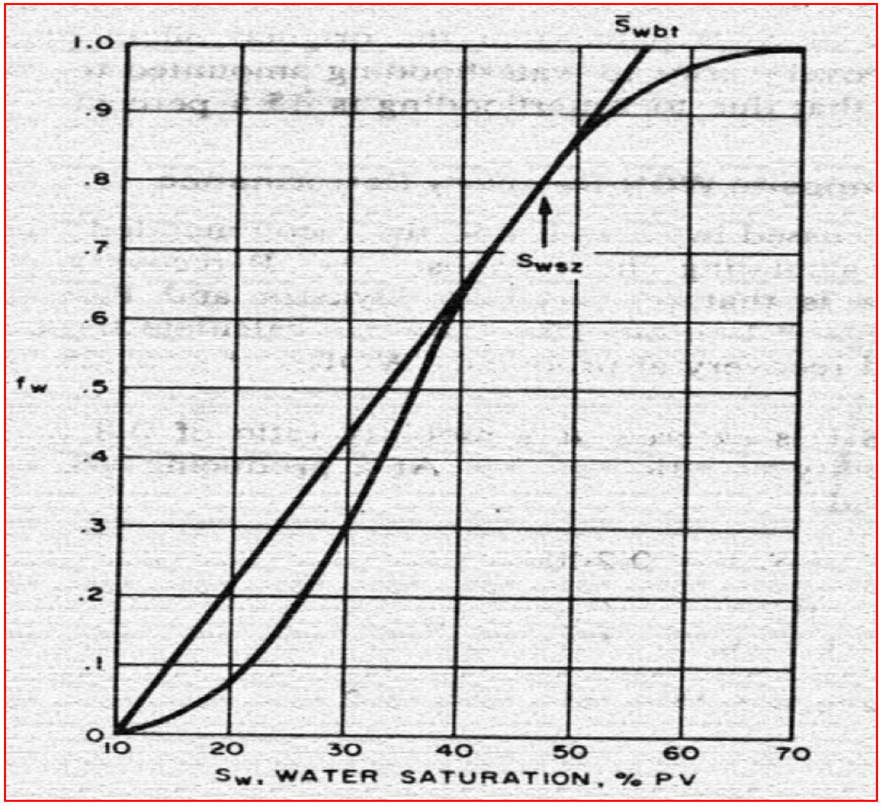
$$f_w = \frac{\frac{k_w}{\mu_w}}{\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}} = \frac{1}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$

Water Saturation S_w (fraction)	Relative Permeability		Fractional Flow of Water f_w
	Oil, k_{ro} (fraction)	Water, k_{rw} (Fraction)	
0.10	1.000	0.000	0.0000
0.30	0.373	0.070	0.2729
0.40	0.210	0.169	0.6168
0.45	0.148	0.226	0.7533
0.50	0.100	0.300	0.8571
0.55	0.061	0.376	0.9250
0.60	0.033	0.476	0.9665
0.65	0.012	0.600	0.9901
0.70	0.000	0.740	1.0000

Plot the fractional flow curve



Draw a tangent to determine the water saturation at the front and the average water saturation at the time of breakthrough.



$S_{wf} = 0.469$

$f_{wf} = 0.798$

$\bar{S}_{wbt} = 0.563$

Calculate the volume of water injected and oil produced up to breakthrough and the time of water breakthrough using:

$$W_i = L\,A\,\phi\,(\overline{S}_w - S_{wc}) = 21780\ \mathrm{m}^3$$

$$t_b = \frac{AL\phi(\overline{S}_w - S_{wc})}{q_i} = 145\ \mathrm{days}$$

Calculate the post breakthrough performance using:

Pore Volumes Injected $= \frac{W_i}{L A \phi} = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{S_{w2}}}$

$$\overline{S}_w = S_{w2} + \frac{(1 - f_{w2})}{f'_{w2}}$$

and

$$(N_p)_{surface} = \frac{A \phi L(\overline{S}_w - S_{wc})}{B_o}$$

$$WOR = \frac{f_{w2}}{1 - f_{w2}}$$

S_{w2} Exit-end Water Saturation (Fraction PV)	f_{w2} Exit-end Flowing Fraction of Water	df_w/dS_w Slope of Fractional Flow Curve	Q_i Cumulative Water Injected (PV)	$\overline{S_w}$ Average Water Saturation (fraction PV)	Cumulative Oil Produced $(N_p)_{surface}$ m^3	WOR
0.469	0.798	2.16	0.463	0.563	23150	3.95
0.495	0.848	1.75	0.472	0.582	24100	5.58
0.520	0.888	1.41	0.711	0.600	25000	7.93
0.546	0.920	1.13	0.887	0.617	25850	11.50
0.572	0.946	0.851	1.176	0.636	26800	17.52
0.597	0.965	0.649	1.540	0.652	27600	27.57
0.622	0.980	0.477	2.100	0.666	28300	49.00
0.649	0.990	0.317	3.157	0.681	29050	99.00
0.674	0.996	0.195	5.13	0.694	29700	249.00
0.700	1.000	0.102	9.80	0.700	30000	

Example 2

A core is saturated with oil and water at interstitial water saturation. The properties are given as below. Relative permeability relationships is also given by correlation.

Property	Value
ϕ	0.20
S_{iw}	0.30
S_{or}	0.30
μ_o, cp	40
μ_w, cp	1
B_o, bbl/STB	1.0

Relative Permeability Correlation

$$k_{ro} = \alpha_1 (1 - S_{wD})^m$$

$$k_{rw} = \alpha_2 S_{wD}^n$$

Where $\alpha_1 = 0.8, \alpha_2 = 0.2, m = 2, n = 2, and$

$$S_{wD} = (S_w - S_{iw}) / (1 - S_{or} - S_{iw})$$

Where $S_{or} =$ waterflood residual oil saturation (ROS). Assume that

$$B_o = B_w = 1.0$$

Required

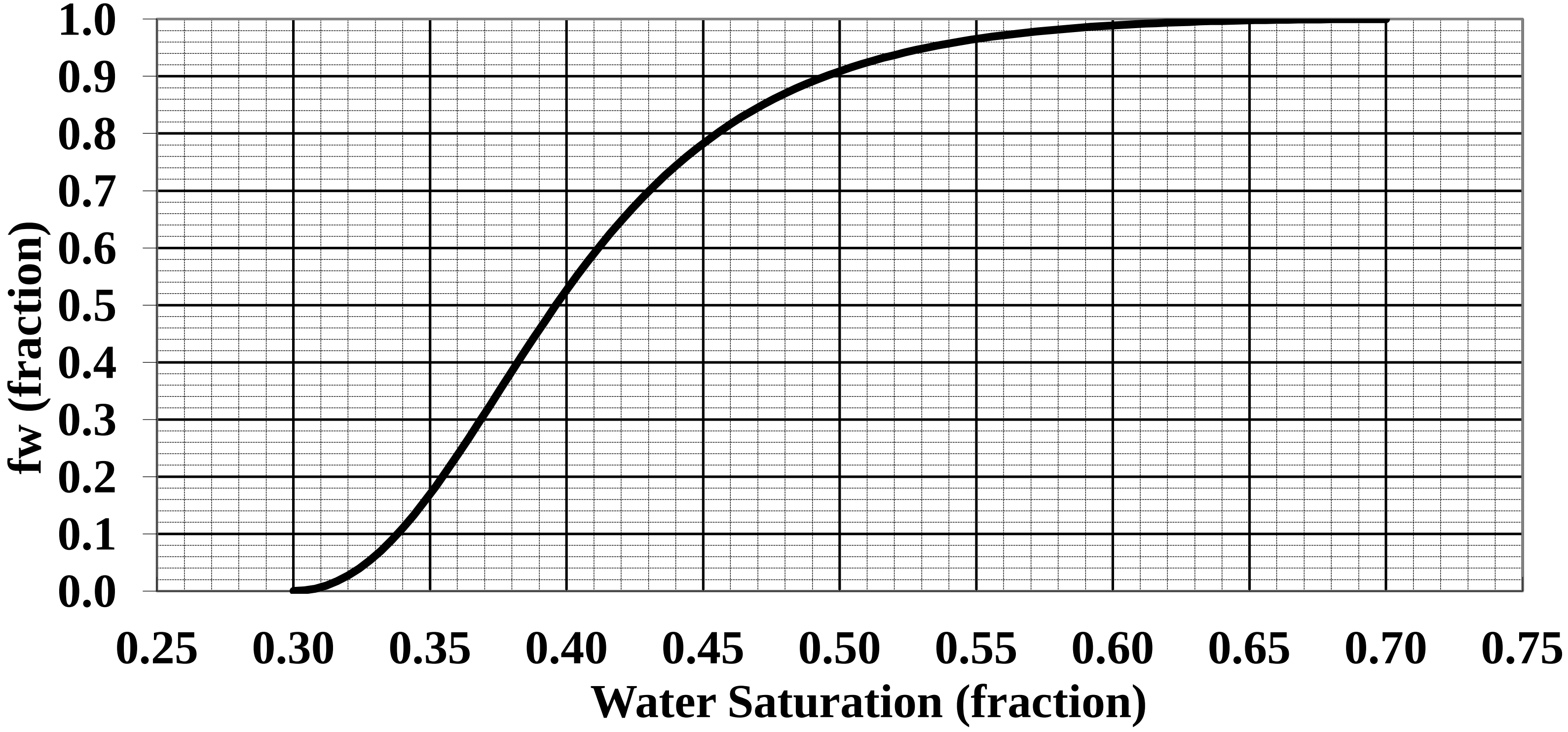
Prepare the following:

- A dimensionless distance, dimensionless time x_D/t_D graph showing the displacement until WOR=50 is reached at the end of the system.
- A saturation profile when the flood front is located at x_D=0.75.
- the volume of oil displaced from the beginning of the waterflood to WOR=50

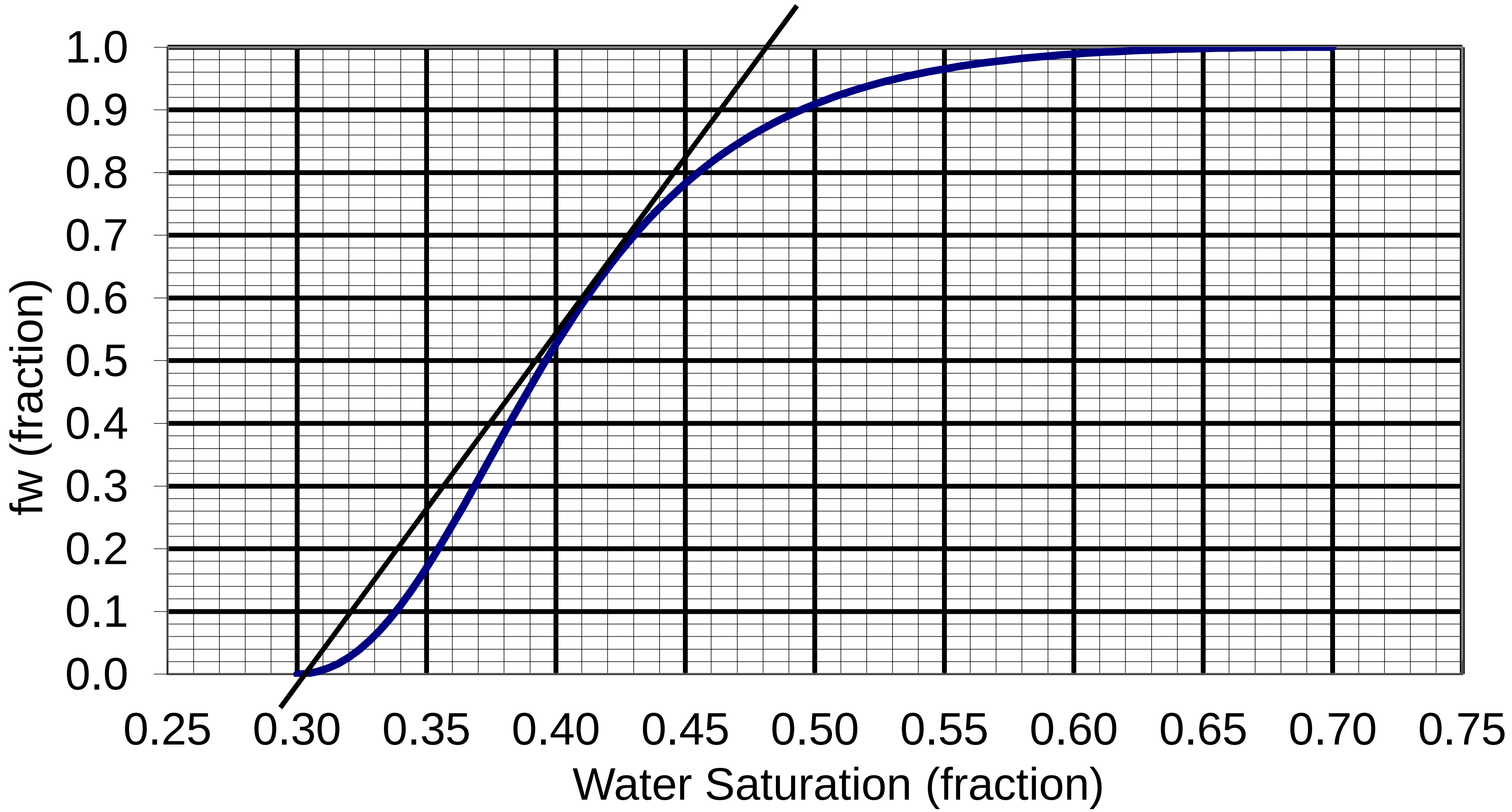
Construct the fractional curve

S_w, S_w^*	f_w $\mu_w = 1.0$ cp
0.30	0.00000
0.32	0.02695
0.34	0.10989
0.36	0.23747
0.38	0.38462
0.40	0.52632
0.42	0.64748
0.44	0.74355
0.46	0.81633
0.48	0.87003
0.50	0.90909
0.52	0.93726
0.54	0.95745
0.56	0.97182
0.58	0.98196
0.60	0.98901
0.62	0.99379
0.64	0.99690
0.66	0.99877
0.68	0.99972
0.70	1.00000

Fractional Curve



Fractional Curve



Solution

- Construction of distance/time diagram, saturation profile, and displacement of performance requires determination of the flood-front saturation and derivatives of the fractional-flow curve at various saturation values.

- For this example $S_{wf}=0.4206$ and $f_{wf}=0.65076$. The slope of the tangent to the fractional flow curve from $f_w=0$, $S_{iw}=0.3$ is found with:

Solution

$$f'_{wf} = \left(f_{wf} - f_{iw}\right) / \left(S_{wf} - S_{iw}\right)$$

$$= \left(0.65076 - 0.0\right) / \left(0.4206 - 0.3\right)$$

$$= 5.396$$

When the flood reaches the end of the linear system (x_D=1)

$$S_{w2} = S_{wf} \ ,$$

$$t_{D2} = 1 / \ f'_w$$

$$= 1 / 5.396$$

$$= 0.185$$

- The average water saturation at t_{D2}=0.185 is computed with:

$$\bar{S}_w = S_{w2} + t_{D2}(1 - f_{w2}) =$$

$$0.4206 + 0.185(1 - 0.6508)$$

$$= 0.485$$

- So at S_{w2}=0.4206, f_{w2}=0.6508, f'_{w2}=5.39578, t_{D2}=0.185 and $\acute{S}_{w=}$0.485
- The same procedure is followed for other saturations
- The following table shows other saturations and computed parameters.(with an increment of ΔS_{w2}=0.0006)

Summary of averaged water saturation calculation

S_{w2}	f_{w2}	f'_{w2}	t_{D2}	$\bar{S}_w$
0.4206	0.65076	5.39578	0.18533	0.4853
0.4262	0.67991	5.03890	0.19846	0.4897
0.4318	0.70708	4.68777	0.21332	0.4943
0.4374	0.73232	4.34687	0.23005	0.4989
0.4430	0.75569	4.01949	0.24879	0.5037
0.4485	0.77727	3.70789	0.26970	0.5086
0.4541	0.79716	3.41350	0.29295	0.5136
0.4597	0.81545	3.13708	0.31877	0.5185
0.4653	0.83225	2.87883	0.34736	0.5236
0.4709	0.84766	2.63857	0.37899	0.5286
0.4765	0.86177	2.41585	0.41393	0.5337
0.4821	0.87469	2.20995	0.45250	0.5388
0.4877	0.88650	2.02007	0.49503	0.5438
0.4932	0.89729	1.84530	0.54192	0.5489
0.4988	0.90715	1.68468	0.59358	0.5540
0.5044	0.91614	1.53725	0.65051	0.5590
0.5100	0.92435	1.40505	0.71324	0.5640
0.5156	0.93183	1.27817	0.78237	0.5689
0.5212	0.93865	1.16472	0.85857	0.5739
0.5268	0.94487	1.06086	0.94263	0.5787
0.5324	0.95053	0.96581	1.03541	0.5836
0.5380	0.95568	0.87882	1.13789	0.5884
0.5435	0.96036	0.79922	1.25123	0.5931
0.5491	0.96462	0.72637	1.37671	0.5978
0.5547	0.96849	0.65970	1.51585	0.6025
0.5603	0.97200	0.59866	1.67039	0.6071
0.5659	0.97519	0.54278	1.84237	0.6116
0.5715	0.97808	0.49160	2.03418	0.6161
0.5771	0.98069	0.44471	2.24865	0.6205

Distance time graph construction

- The distance/time graph is constructed by determining the time that each saturation arrives at the end of the system (X_D=1.0)
- Constructing a straight line from that point to the origin

$$t_{D2} = 1 / f'_{w2}$$

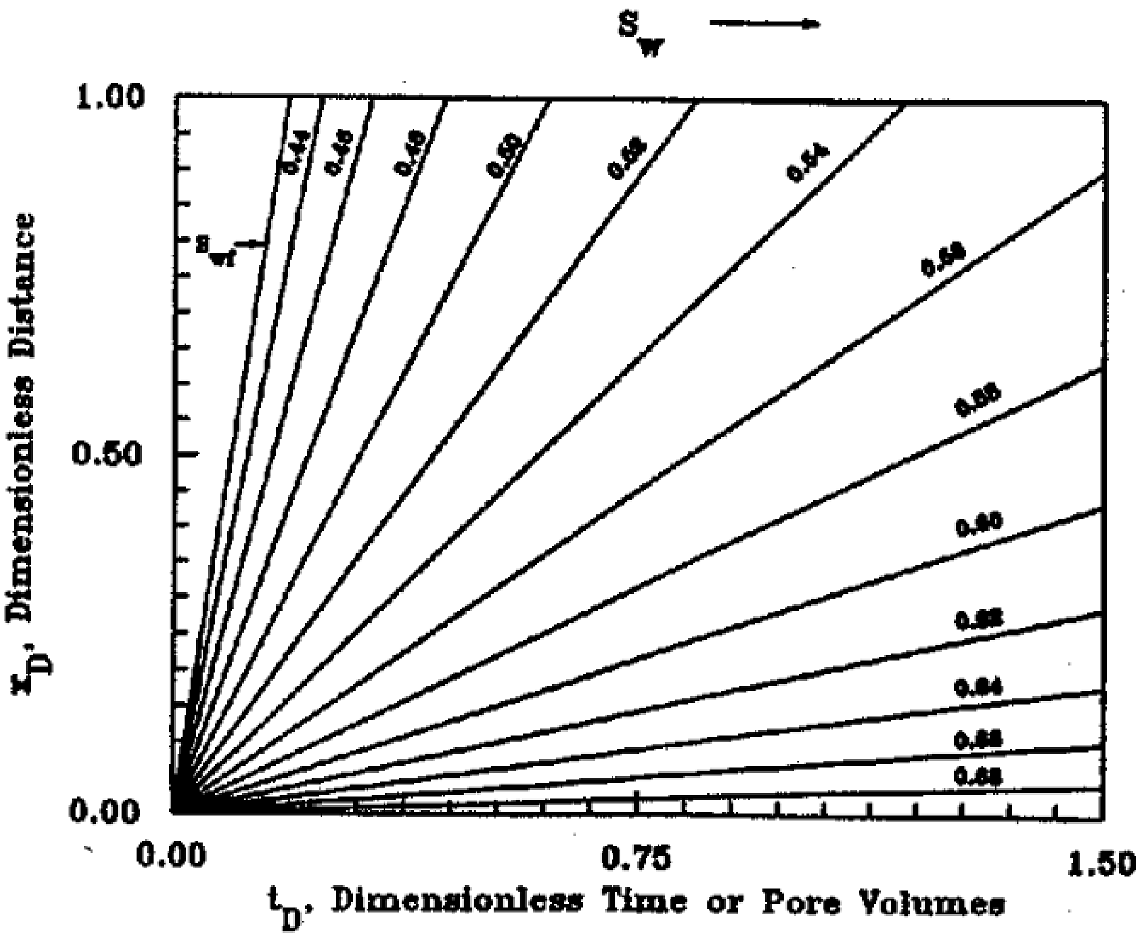
$$At \quad S_{w2} = 0.538, \quad f'_{w2} = 0.87882$$

$$thus, \quad t_{D2} = 1 / 0.87882$$

$$= 1.13789$$

- The distance/time graph shown in the following graph
- The graph is prepared by drawing a line from the orgin (X_D=0, t_D=0) to the end of the system (x_D=1) for the arrival time selected water saturations (t_D=t_{D2}).

x_D/t_D graph for parameters of example 1



Saturation Profile when X_D=.75

- The saturation profile when the flood front is at x_D=0.75 is the locus of all saturations at the corresponding value of t_D. From the frontal advance equation,

$$t_D = x_{Df} / f'_{wf}$$

$$= 0.75 / 5.39578$$

$$= 0.139$$

Calculation of other saturations

- Locations of other saturations ($S_{wf}<S_w<S_{or}$) at t_D=0.139 are found from

$$x_{DS_w}=t_D f'_w$$

At S_w=0.560, f'_w =0.60183

Thus, x_{DSw}=(0.139)(0.60183)

=0.0837

The computed saturation profile is given in next Table & Figure

The computed saturation profile is given the following Table and presented in the next Figure. Distance were determined at increments 0f 0.02 saturation units from $S_w = 1 - S_{or}$ to S_{wf}. ($t_D = .139$)

S_w	f'_w	x_D
0.70	0.00000	0.000
0.68	0.02914	0.00405
0.66	0.06842	0.00951
0.64	0.12137	0.01687
0.62	0.19289	0.02681
0.60	0.28982	0.04028
0.58	0.42169	0.05861
0.56	0.60183	0.08365
0.54	0.84880	0.11798
0.52	1.18799	0.16513
0.50	1.65289	0.22975
0.48	2.28437	0.31752
0.46	3.12370	0.43419
0.44	4.19083	0.58251
0.4206*	5.3958	0.75
0.30		0.75
0.30		1.00

*S_{wf}.

Saturation Profile at x_D=.75

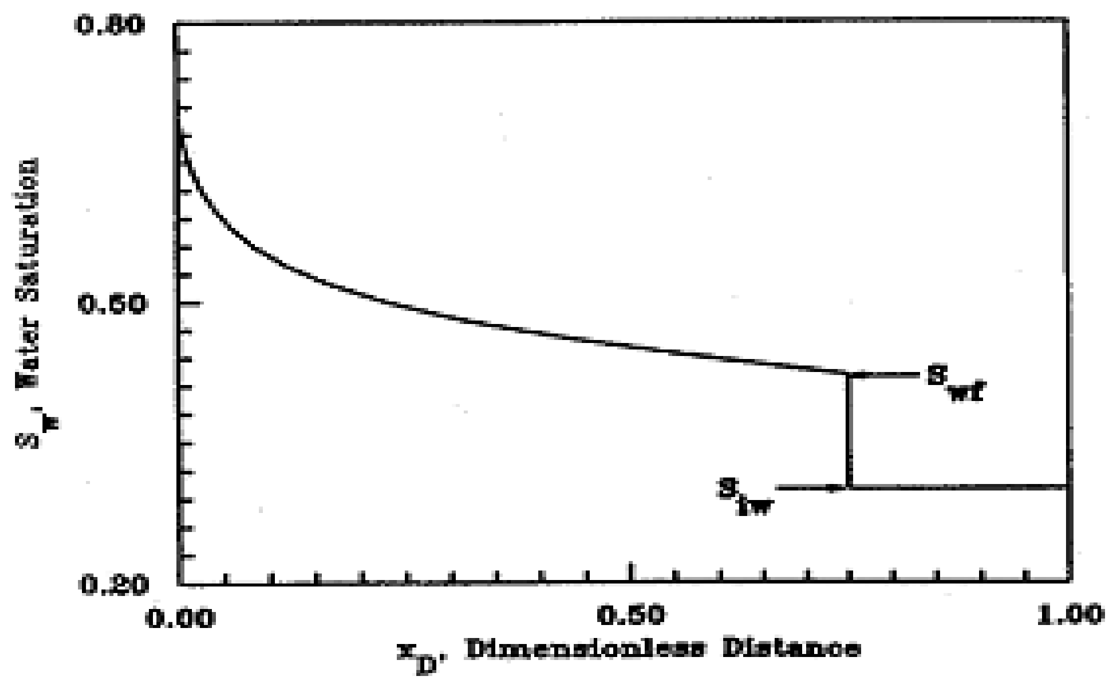


Fig. 3.6—Saturation profile when x_D = 0.75, Example 3.1.

Displacement Performance Calculation

Displacement performance is obtained by computing the average water saturation with:

$$N_p = \left\lfloor \left(\bar{S}_{iw} - S_{iw}\right) / B_o \right\rfloor A\phi L$$

$$\textit{Thus} \quad N_p / A\phi L = \left(\bar{S}_w - S_{iw}\right) / B_o$$

$$\textit{From the developed table at } S_{w2} = 0.5547,\ \bar{S}_w = 0.6025$$

$$\textit{and} \quad N_p / A\phi L = 0.6025 - 0.3$$

$$= 0.3025 \quad \left(B_o = 1\right)$$

F_{wo} Calculation

The producing WOR is calculated by rearrangement of the fractional flow equation.

Since

$$f_w = \frac{q_w}{q_w + q_o}$$

$$F_{wo} = (q_w / q_o)(B_o / B_w)$$

So $$F_{wo} = \left[f_w / (1 - f_w) \right] \left(B_o / B_w \right)$$

Estimated displacement performance

S_{w2}	t_D	F_{wo} (bbl/bbl)	$N_p/A\phi L$ (PV)
0.30	0.00000	0.00000	0.00000
0.30	0.01853	0.00000	0.01853
0.30	0.03707	0.00000	0.03707
0.30	0.05560	0.00000	0.05560
0.30	0.07413	0.00000	0.07413
0.30	0.09266	0.00000	0.09266
0.30	0.11120	0.00000	0.11120
0.30	0.12973	0.00000	0.12973
0.30	0.14826	0.00000	0.14826
0.4206	0.18533	1.86332	0.18533
0.4262	0.19846	2.12410	0.18972
0.4318	0.21332	2.41390	0.19427
0.4374	0.23005	2.73576	0.19895
0.4430	0.24879	3.09308	0.20374
0.4485	0.26970	3.48969	0.20861
0.4541	0.29295	3.92990	0.21356
0.4597	0.31877	4.41858	0.21855
0.4653	0.34736	4.96123	0.22358
0.4709	0.37899	5.56411	0.22863
0.4765	0.41393	6.23434	0.23370

Estimated displacement performance

S_{w2}	t_D	F_{wo} (bbl/bbl)	$N_p/A\phi L$ (PV)
0.4765	0.41393	6.23434	0.23370
0.4821	0.45250	6.98001	0.23878
0.4877	0.49503	7.81043	0.24385
0.4932	0.54192	8.73625	0.24891
0.4988	0.59358	9.76974	0.25395
0.5044	0.65051	10.92510	0.25897
0.5100	0.71324	12.21878	0.26397
0.5156	0.78237	13.66995	0.26893
0.5212	0.85857	15.30101	0.27386
0.5268	0.94263	17.13829	0.27874
0.5324	1.03541	19.21285	0.28359
0.5380	1.13789	21.56150	0.28839
0.5435	1.25123	24.22817	0.29313
0.5491	1.37671	27.26550	0.29783
0.5547	1.51585	30.73700	0.30248
0.5603	1.67039	34.71980	0.30707
0.5659	1.84237	39.30822	0.31160
0.5715	2.03418	44.61852	0.31607
0.5771	2.24865	50.79517	0.32048

Example 3: Viscous Flooding

- Injection of a viscous fluid is an attractive possibility to improve water flood displacement efficiency.
- Assume that the viscosity of water has been increased to 4 cp (both interstitial and injected water) rather than 1 cp.
- The effect of viscosity should be considered only for example 2.

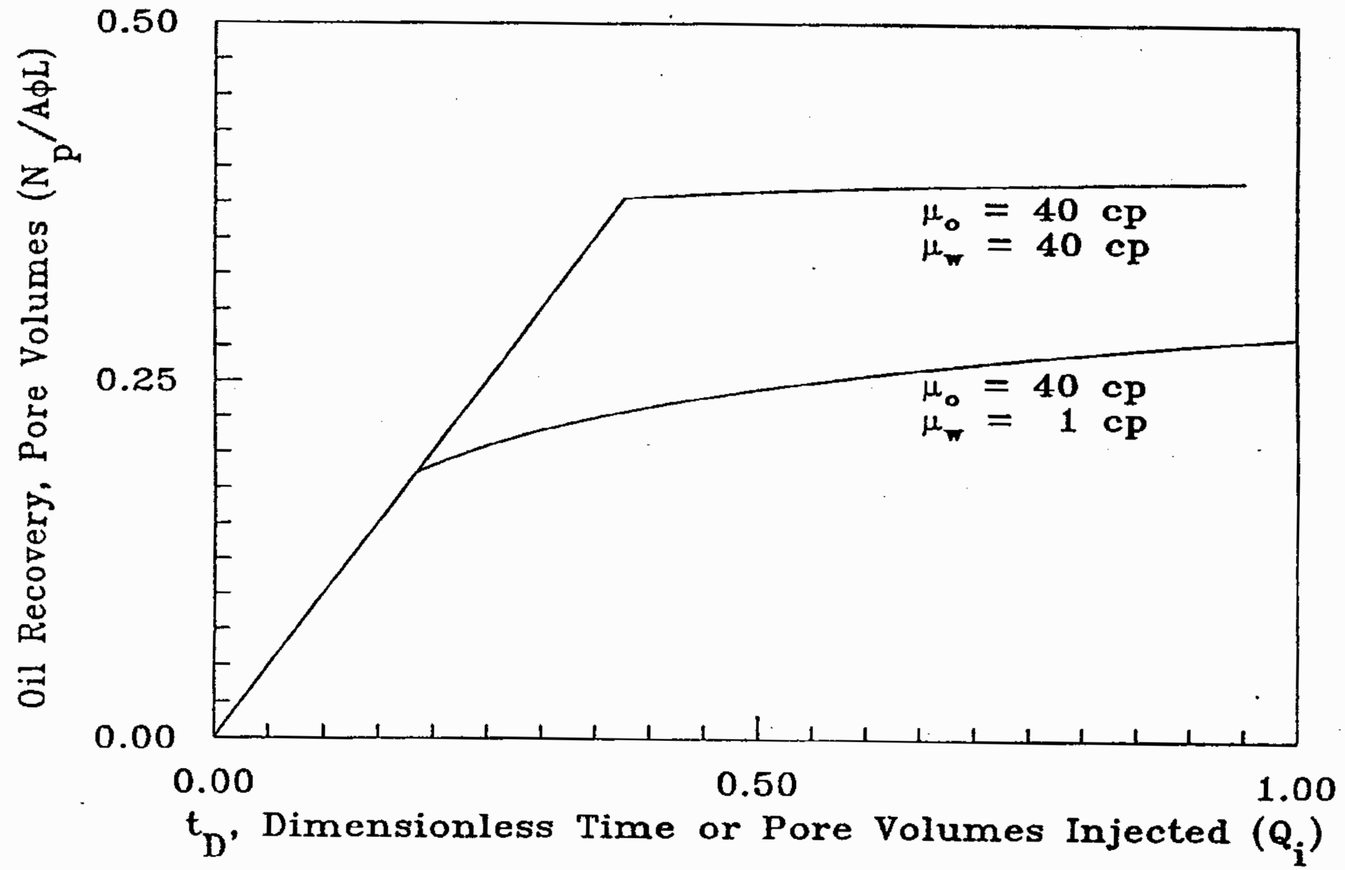
Solution

$$f_w = \frac{\frac{k_w}{\mu_w}}{\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}} = \frac{1}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$

Fractional flow curves for the viscous and nonviscous waterfloods are shown in the next figure. The following table contains values of fractional flow and saturations used to prepare the figure.

S_w, S_w^*	f_w^* $\mu_w^* = 4.0$ cp	f_w $\mu_w = 1.0$ cp
0.30	0.00000	0.00000
0.32	0.00688	0.02695
0.34	0.02994	0.10989
0.36	0.07223	0.23747
0.38	0.13514	0.38462
0.40	0.21739	0.52632
0.42	0.31469	0.64748
0.44	0.42024	0.74355
0.46	0.52632	0.81633
0.48	0.62597	0.87003
0.50	0.71429	0.90909
0.52	0.78879	0.93726
0.54	0.84906	0.95745
0.56	0.89608	0.97182
0.58	0.93156	0.98196
0.60	0.95745	0.98901
0.62	0.97561	0.99379
0.64	0.98770	0.99690
0.66	0.99509	0.99877
0.68	0.99889	0.99972
0.70	1.00000	1.00000

Viscous Water flood in a Linear System



Viscous Flood in Linear System

- Frontal advance techniques are used to estimate waterflood performance when the injected fluid is viscous but still miscible with the interstitial or previously injected water, which has a low viscosity.
- No adsorption on the rock is assumed
- No mixing occurs between the viscous fluid and the low viscosity resident water
- A boundary exists between the viscous and displaced water where there is a step change, or jump, in viscosity from μ_w to μ_w^*

Viscous Flood

- The displacement process can be described as a waterflood in which a viscous fluid displaces both oil and low-viscosity resident water.
- The resident water is miscibly displaced by the injected fluid.
- It is the resident water, rather than the injected viscous fluid, that forms the leading flood front
- Because there is a discontinuity in the viscosity between the viscous and resident fluids, a second discontinuity in saturation, or shock front, must form at the viscous water/resident-water boundary.

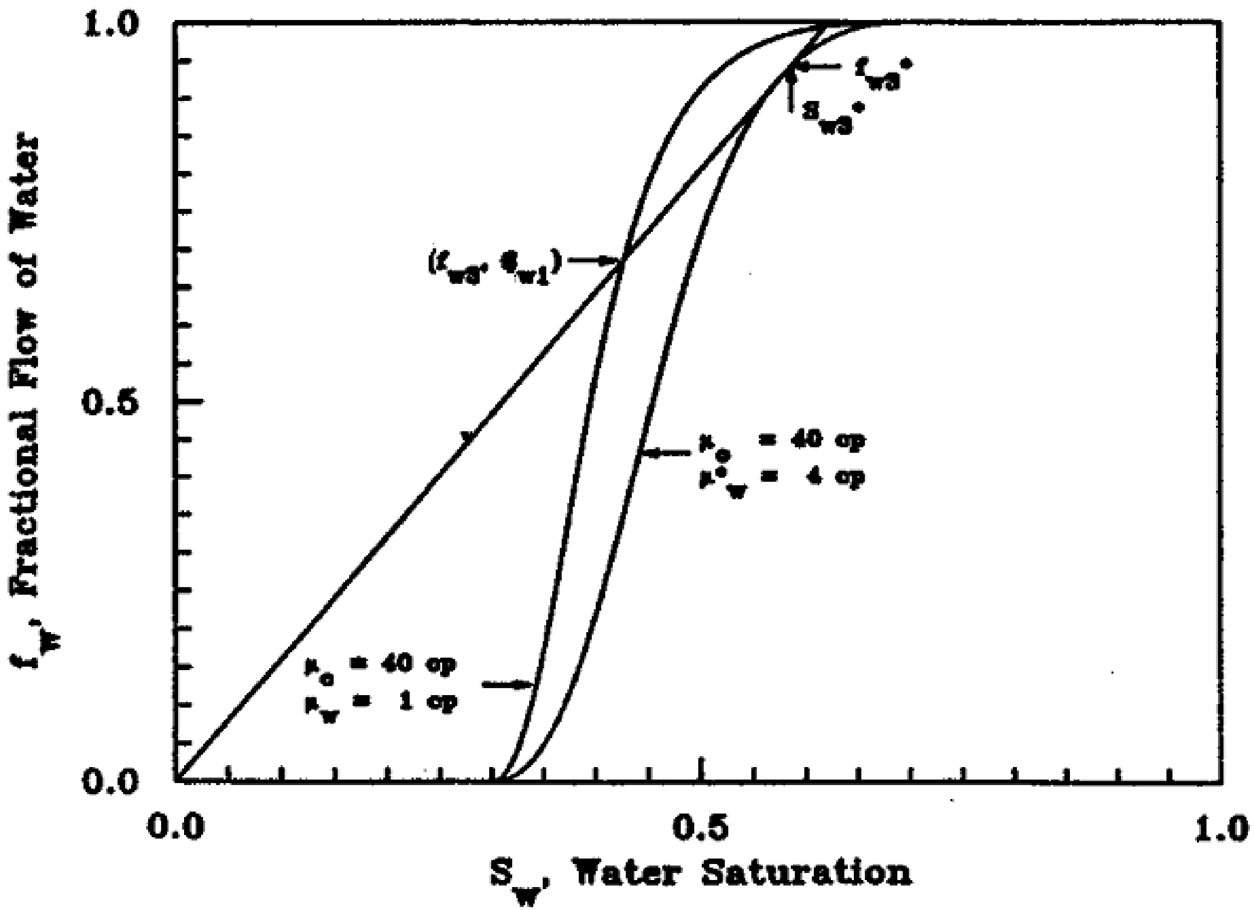
Determination of Velocity & Saturation of Viscous Shock

- Based on material balance and the application of frontal advance theory we get the following (see the text):

$$\left(\frac{\partial f_w^*}{\partial S_w^*}\right)_{S_w^*} = \frac{f_{w3}^* - f_{w1}}{S_{w3}^* - S_{w1}} = \frac{f_{w3}^*}{S_{w3}^*} = \frac{f_{w1}}{S_{w1}}$$

- Inspection of this equation shows that f_{w3}* and S_{w3}* can be found by constructing a tangent from the origin to the f_w*-S_w* curve for µw, as shown next Figure.

Construction of tangent to find f_{w3}^*, S_{w3}^*, f_{w1}, and S_{w1}



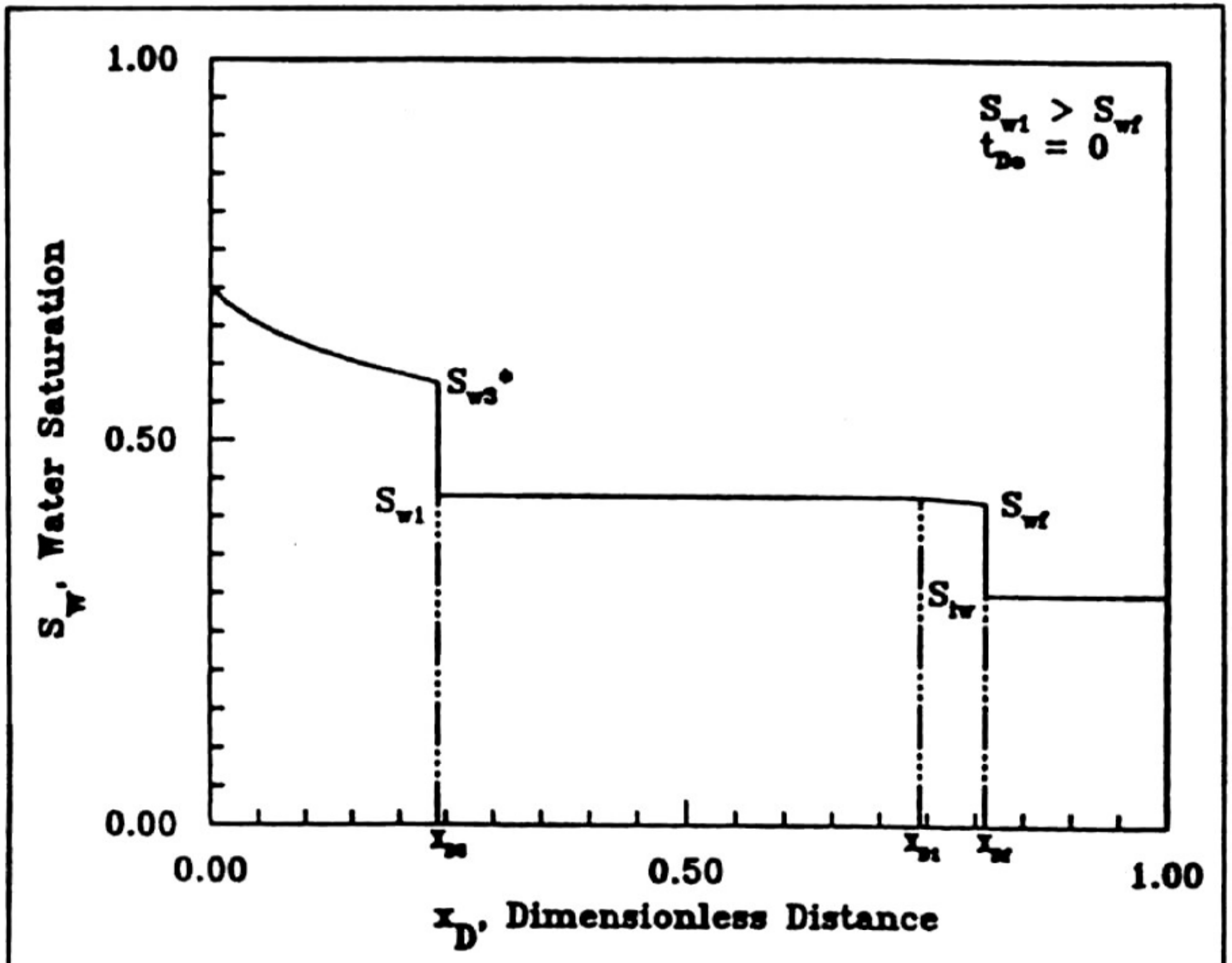
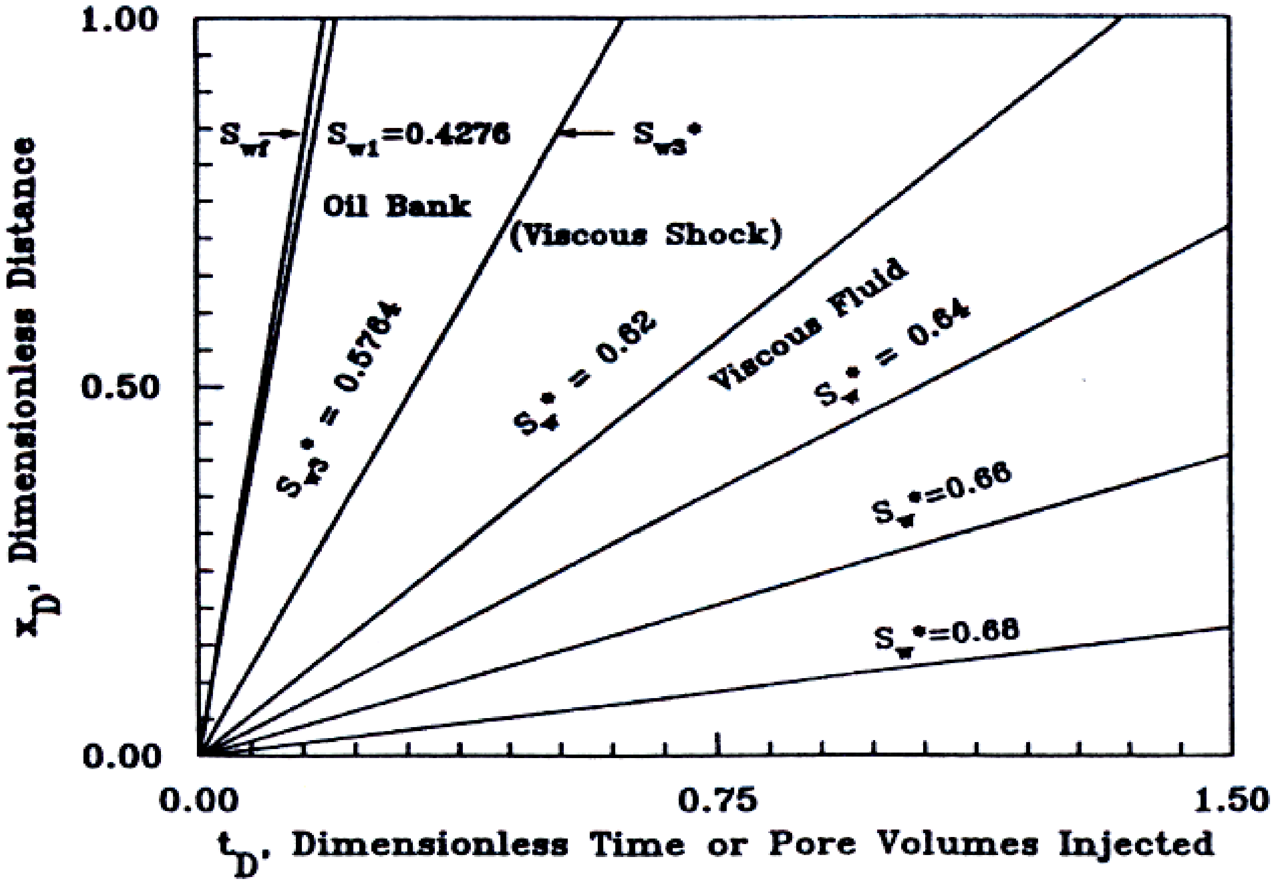


Fig. 3.13—Saturation profile during a viscous waterflood at interstitial water saturation when $S_{w1} > S_{wf}$.

Distance/time diagram



Oil Recovery during a viscous waterflood

- Because the saturation profile for a viscous waterflood may have several discontinuities, the average water saturation must be determined by integrating the saturation distribution at discrete times.
- Displacement efficiency and the viscous flood generates an oil bank of much lower water saturation.
- In such cases the water saturation in the oil bank can be lower than the waterflood front saturation.
- This saturation will move faster than the waterflood front and will overtake that front.
- Now the saturation distribution will look like that shown in the next slide.

Methodology for Analyzing Viscous Waterfloods in Linear Systems

Objective of Analysis

Determine the flood performance in terms of pore volumes of oil displaced as a function of the pore volumes of viscous water injected

Minimum Require Data

Relative permeability curves
Viscosity of oil
Viscosity of connate water
Viscosity of viscous water

Example 4: Viscous waterflood

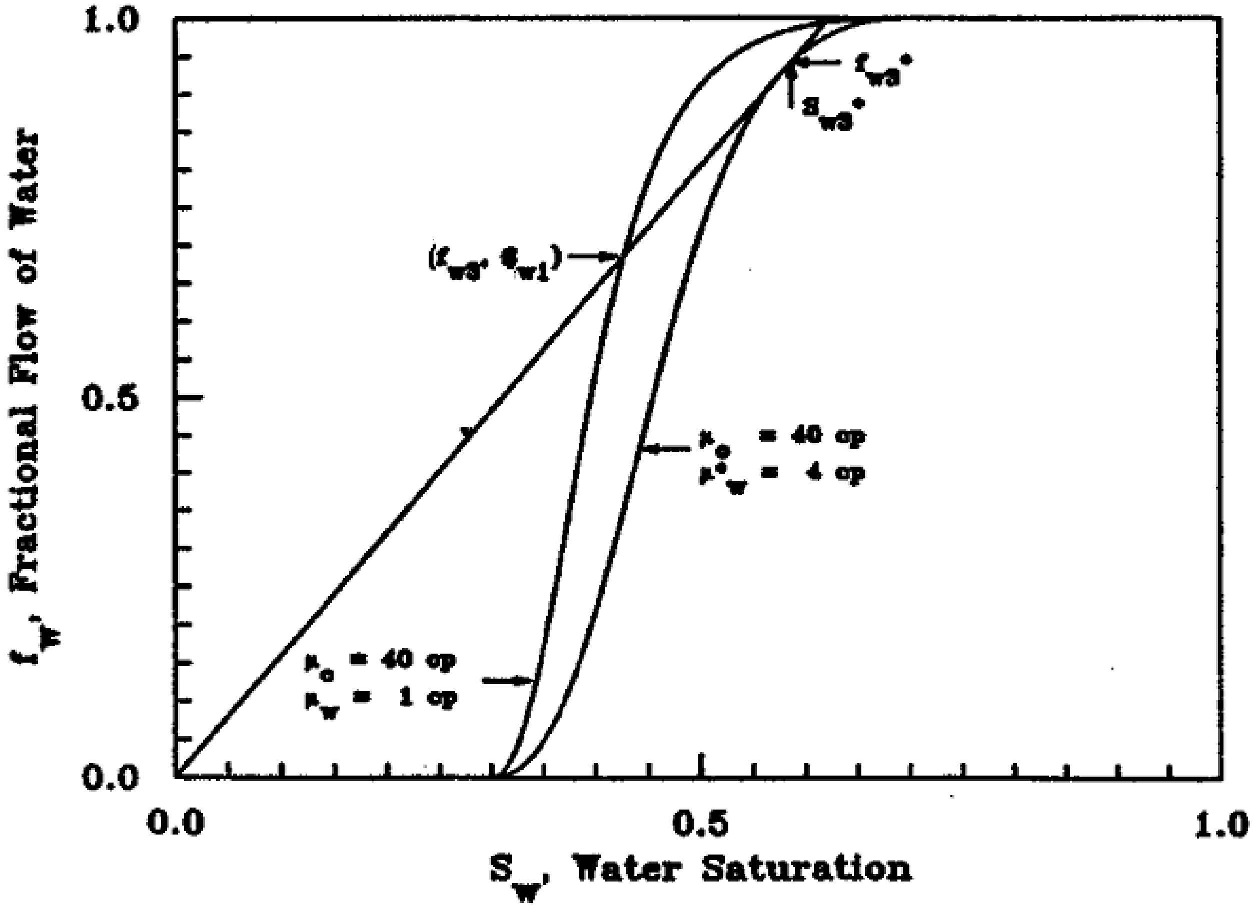
The linear reservoir in example 2 is to be flooded with a viscous solution that does not absorb. The viscosity of the solution is 4 cp, and no mixing occurs between the viscous solution and the interstitial water. All other parameters used in example 2 remain the same. Find the flood-front saturations.

Solution

Fractional flow curves for the viscous and nonviscous waterfloods are shown in the next figure. The following table contains values of fractional flow and saturations used to prepare the figure.

S_w, S_w^*	f_w^* $\mu_w^* = 4.0$ cp	f_w $\mu_w = 1.0$ cp
0.30	0.00000	0.00000
0.32	0.00688	0.02695
0.34	0.02994	0.10989
0.36	0.07223	0.23747
0.38	0.13514	0.38462
0.40	0.21739	0.52632
0.42	0.31469	0.64748
0.44	0.42024	0.74355
0.46	0.52632	0.81633
0.48	0.62597	0.87003
0.50	0.71429	0.90909
0.52	0.78879	0.93726
0.54	0.84906	0.95745
0.56	0.89608	0.97182
0.58	0.93156	0.98196
0.60	0.95745	0.98901
0.62	0.97561	0.99379
0.64	0.98770	0.99690
0.66	0.99509	0.99877
0.68	0.99889	0.99972
0.70	1.00000	1.00000

Construction of tangent to find f_{w3}^*, S_{w3}^*, f_{w1}, and S_{w1}

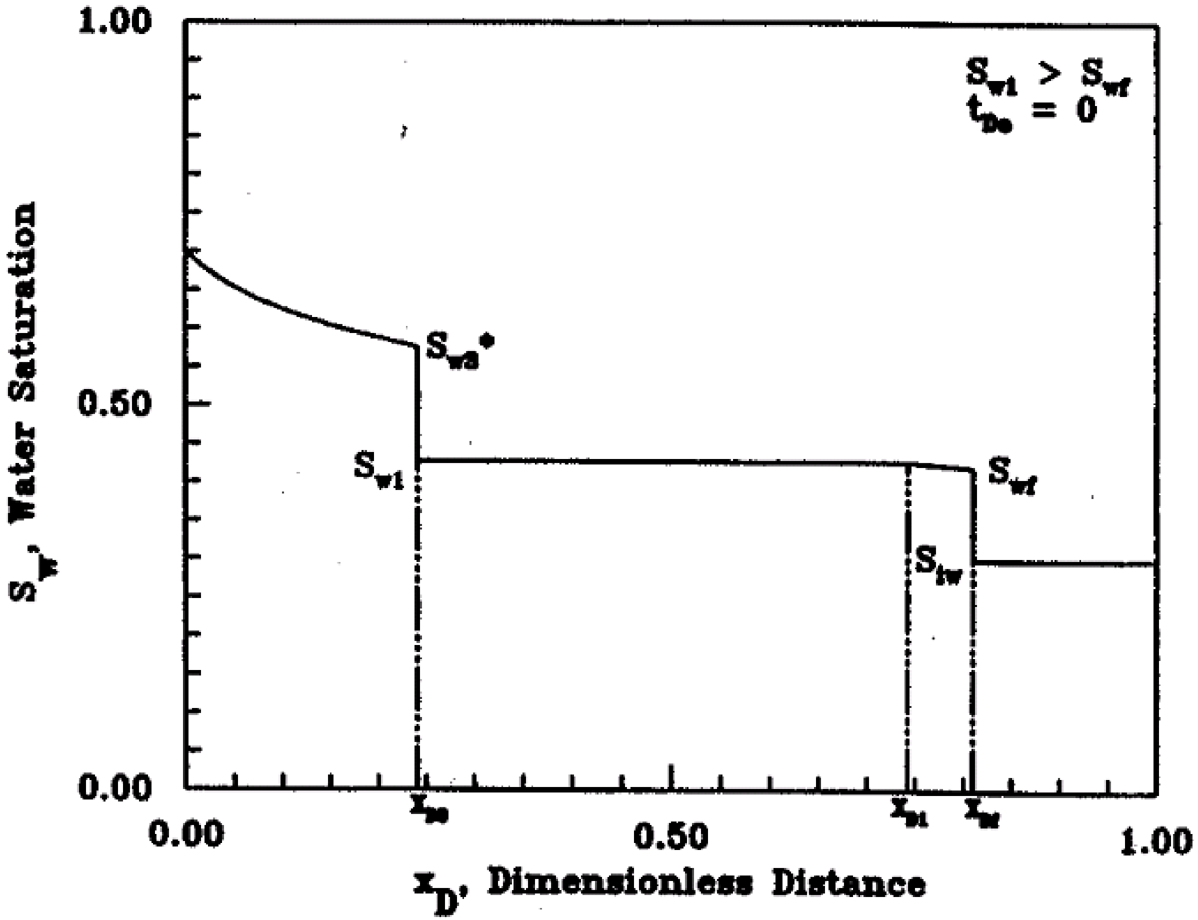


- A tangent was constructed to μ_w* factional curve to determine f_{w3}*(0.926) and S_{w3}*(0.576).

- The intersection of the tangent with the fractional flow curve is at f_{w1}=0.687 and S_{w1}=0.428.

- Note that S_{wf} from example 1 is 0.4206 and that $S_{w1}>S_{wf}$.

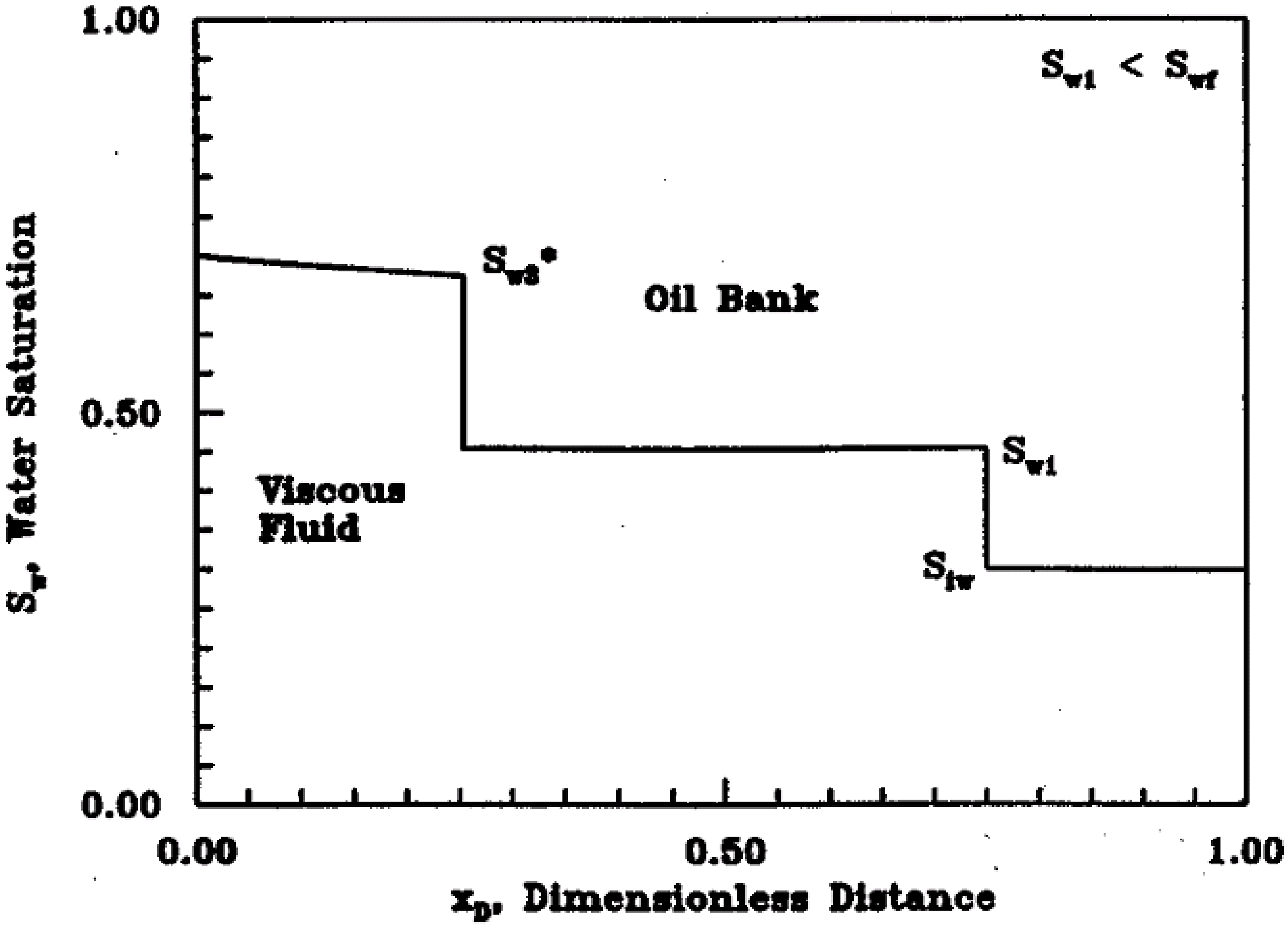
Saturation profile during a viscous waterflood at interstitial water saturation when $S_{w1} > S_{wf}$.



- Water bank
- Arrival of Oil bank
- Arrival of viscous shock

Saturation profile during a viscous waterflood at interstitial water saturation when $S_{w1}<S_{wf.}$

.



The oil bank forms immediately, overtakes S_{wf}, and has a uniform water saturation, S_{w1}.

Example 5: performance of a viscous waterflood-Initial Interstitial water

Estimate the performance of a viscous water flood when the viscous waterflood stats with reservoir at interstitial water saturation. Use the same parameters and properties of the previous examples.

Solution

When the reservoir is at interstitial water saturation, the saturation profile develops immediately and propagates though the system.

Step 1: Prepare a plot of f_w versus S_w and f^*_w versus S^*_w using the relative permeability and viscosity data.

Step 2: Draw a tangent from S_{wi} on to the f_w-S_w curve to determine the saturation and the fractional flow at the ordinary waterflood front (S_{wf} and f_{wf}).

Step 3: Draw a tangent on to the f^*_w versus S^*_w curve from the origin to determine the values of f^*_w , S^*_w and f_{w1}. Determine the slope of the curves at these points. Make a table as follows.

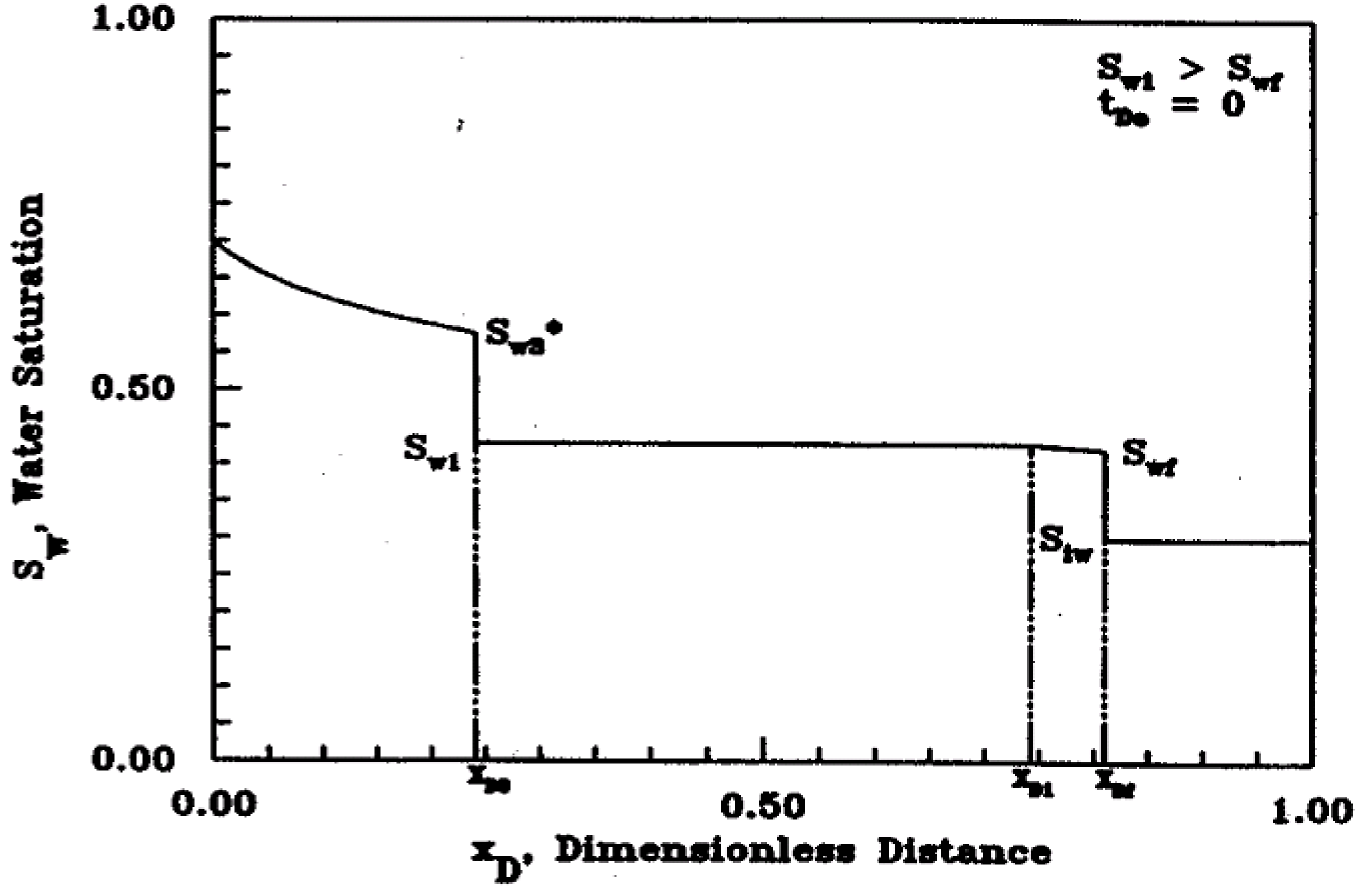
Waterflood	oil bank	Viscose shock
S_{wf} =0.421	S_{w1} = 0.428	S^*_{w3} =0.576
F_{wf} =0.651	f_{w1} = 0.687	f^*_{w3} =0.926
f'_{wf} =5.396	f'_{w1} = 4.950	f'^*_{w3} =1.606

Saturation for different zones & location of saturation behind the viscous shock when t_D=1.5

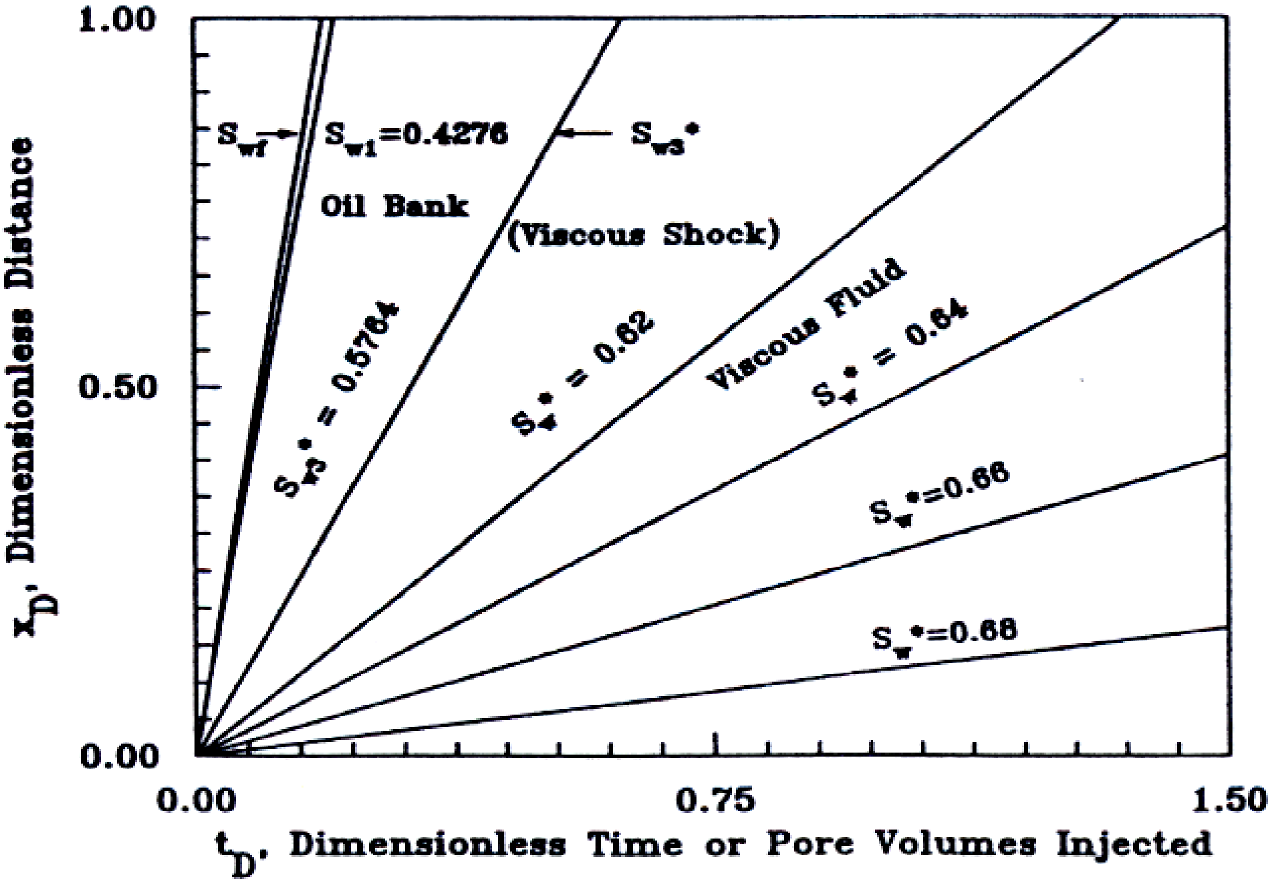
Waterflood	Front of Oil Bank	Viscous Shock
$S_{wf} = 0.4206$	$S_{w1} = 0.4276$	$S^*_{w3} = 0.5764$
$f_{wf} = 0.6508$	$F_{w1} = 0.6869$	$f^*_{w3} = 0.9259$
$f'_{wf} = 5.3958$	$f'_{w1} = 4.9499$	$f'_{w3} = 1.6064$

S^*_w	f'^*_w	x_{Dw} at $t_D = 1.5$	t_D at $x_{Dw} = 1.0$
0.5764	1.6064	2.410	0.622
0.60	1.0865	1.630	0.920
0.62	0.7436	1.115	1.345
0.64	0.4766	0.715	2.098
0.66	0.2717	0.408	3.680
0.68	0.1164	0.175	8.591
0.70	0	0	∞

Saturation profile during a viscous waterflood at interstitial water saturation when $S_{w1}>S_{wf}$.



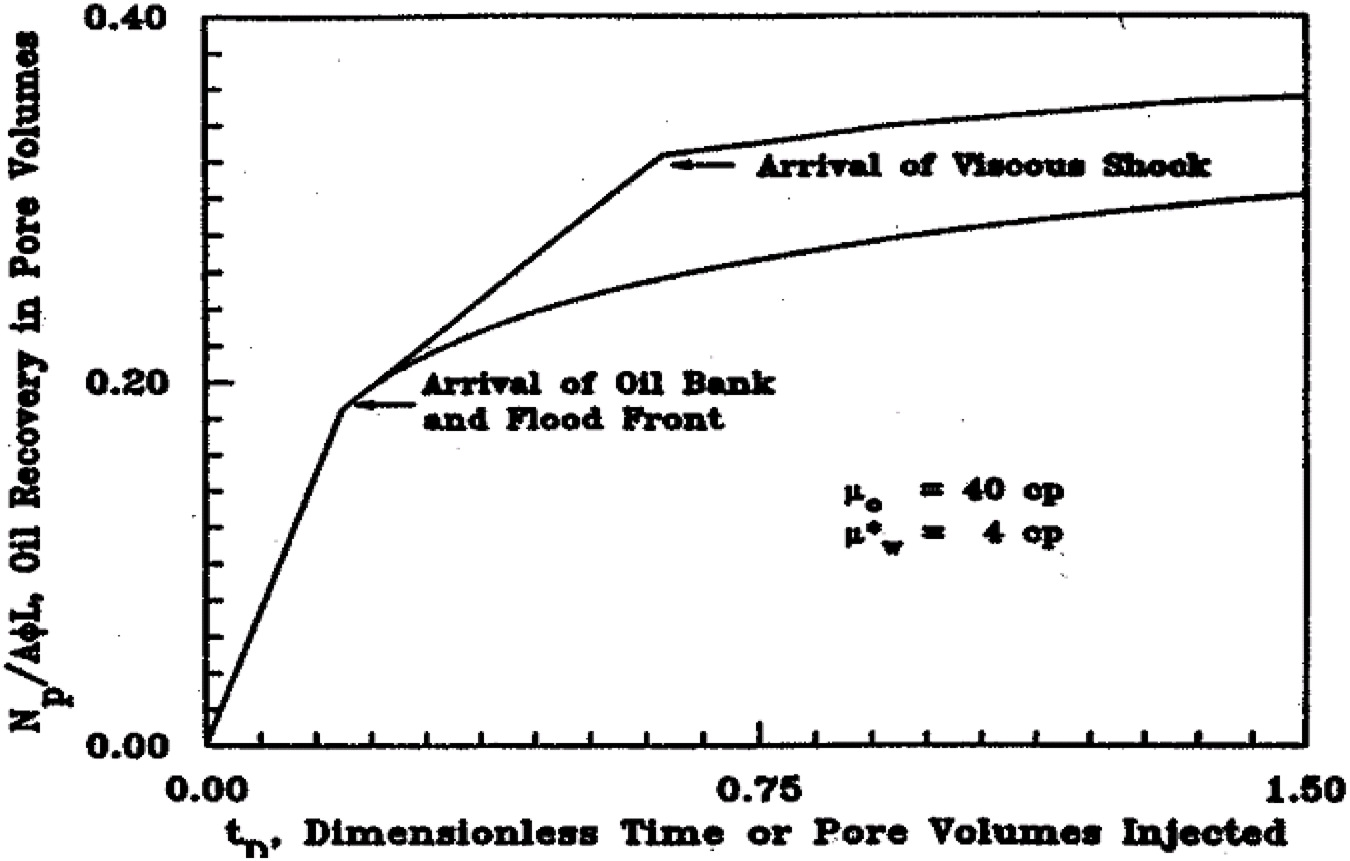
Distance/time diagram



Summary of recovery calculation

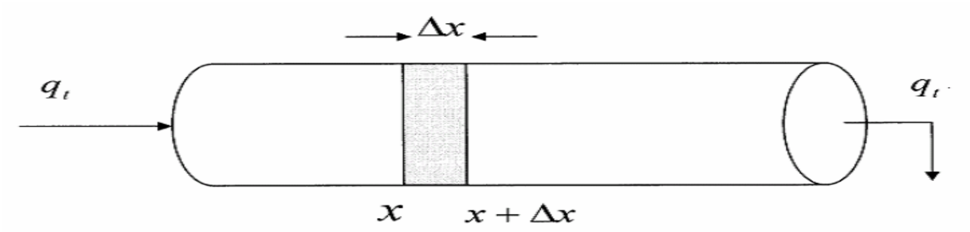
S_w	f_w	Event at End of System	t_D	$\bar{S}_w$	$\bar{S}_w - S_{iw}$
0.4206	0.6508	←Water bank	0.1853	0.4853	0.1853
0.4276	0.6869	←Arrival of oil bank	0.202	0.4909	0.1909
0.5764	0.9259	←Arrival of viscous shock	0.623	0.623	0.323
0.60	0.9575		0.920	0.639	0.339
0.62	0.9756	Region behind viscous shock	1.345	0.653	0.353
0.64	0.9877		2.098	0.666	0.366
0.66	0.9951		3.681	0.678	0.378
0.68	0.9989		8.591	0.689	0.389

Comparison of displacement performance for a viscous waterflood at interstitial water saturation with a normal water flood



Chemical Flooding in a Linear System

- The viscous waterflood discussed earlier assumed that the properties of the viscous water were not subject to change during the flood.
- In real life, this is not a valid assumption.
- Injected fluid properties change due to adsorption of the added chemical on rock surfaces, due to mixing with connate water, due to partitioning into the oil, and due to degradation of the chemical.

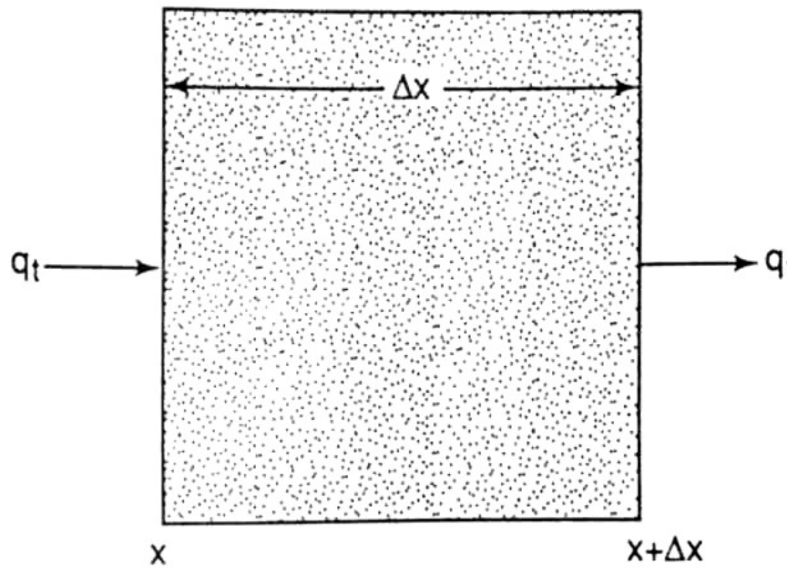


Assumptions

- Only single chemical specie is added to the injected water.
- Flow is 1D, isothermal, with uniform properties of the medium.
- There is no dispersion or fluid mixing.
- No gravity or capillary effects are involved.
- Fluids are incompressible.
- There is no mass transfer between oil and water phases.
- The retention of the chemical by adsorption is assumed to reach equilibrium instantaneously and is irreversible

Based on the flow of oil and water through an elemental volume , a material balance equation can be developed that includes adsorption of the chemical on rock surfaces.

Material balance and fractional advanced theory Application



The material balance can be written as:
Net accumulation = inflow – outflow

$$(C_i S_w|_{t+\Delta t} - C_i S_w|_t) A\Delta x\phi + (A_i \rho_{gr}(1-\phi)\Delta x A)|_{t+\Delta t} - (A_i \rho_{gr}(1-\phi)\Delta x A)|_t =$$
$$q_t f_w C_i|_x \Delta t - q_t f_w C_i|_{x+\Delta x} \Delta t$$

Dividing both sides by $A\Delta x\Delta t$, gives

$$-\frac{[(q_t f_w C_i)|_{x+\Delta x} - (q_t f_w C_i)|_x]}{A\Delta x} = \frac{[C_i S_w|_{t+\Delta t} - C_i S_w|_t]\phi}{\Delta t} +$$

$$\frac{[A_i \rho_{gr}(1-\phi)]_{t+\Delta t} - A_i \rho_{gr}(1-\phi)]_{t+\Delta t}}{\Delta t}$$

Now taking the limit as Δt and Δx tend to zero,

$$-\frac{q_t}{A\phi}\frac{\partial(f_w C_i)}{\partial x} = \frac{\partial(C_i S_w)}{\partial t} + \frac{1}{\phi}\frac{\partial[A_i \rho_{gr}(1-\phi)]}{\partial t}$$

We now define a new variable, $\hat{C}_i$, that represents the specie 'i' per unit pore volume of the rock.

$$\hat{C}_i = \frac{A_i \rho_{gr}(1-\phi)}{\phi}$$

Based on material balance and fractional advanced theory we get:

$$f_w \frac{\partial C_i}{\partial x_D} + S_w \frac{\partial C_i}{\partial t_D} + \frac{\partial \hat{C}_i}{\partial t_D} = 0$$

$$D_i = \frac{\partial \hat{C}_i}{\partial C_i}$$

or

$$f_w \frac{\partial C_i}{\partial x_D} + S_w \frac{\partial C_i}{\partial t_D} + D_i \left(\frac{\partial C_i}{\partial t_D} \right) = 0$$

or

$$(S_w + D_i) \frac{\partial C_i}{\partial t_D} + f_w \frac{\partial C_i}{\partial x_D} = 0$$

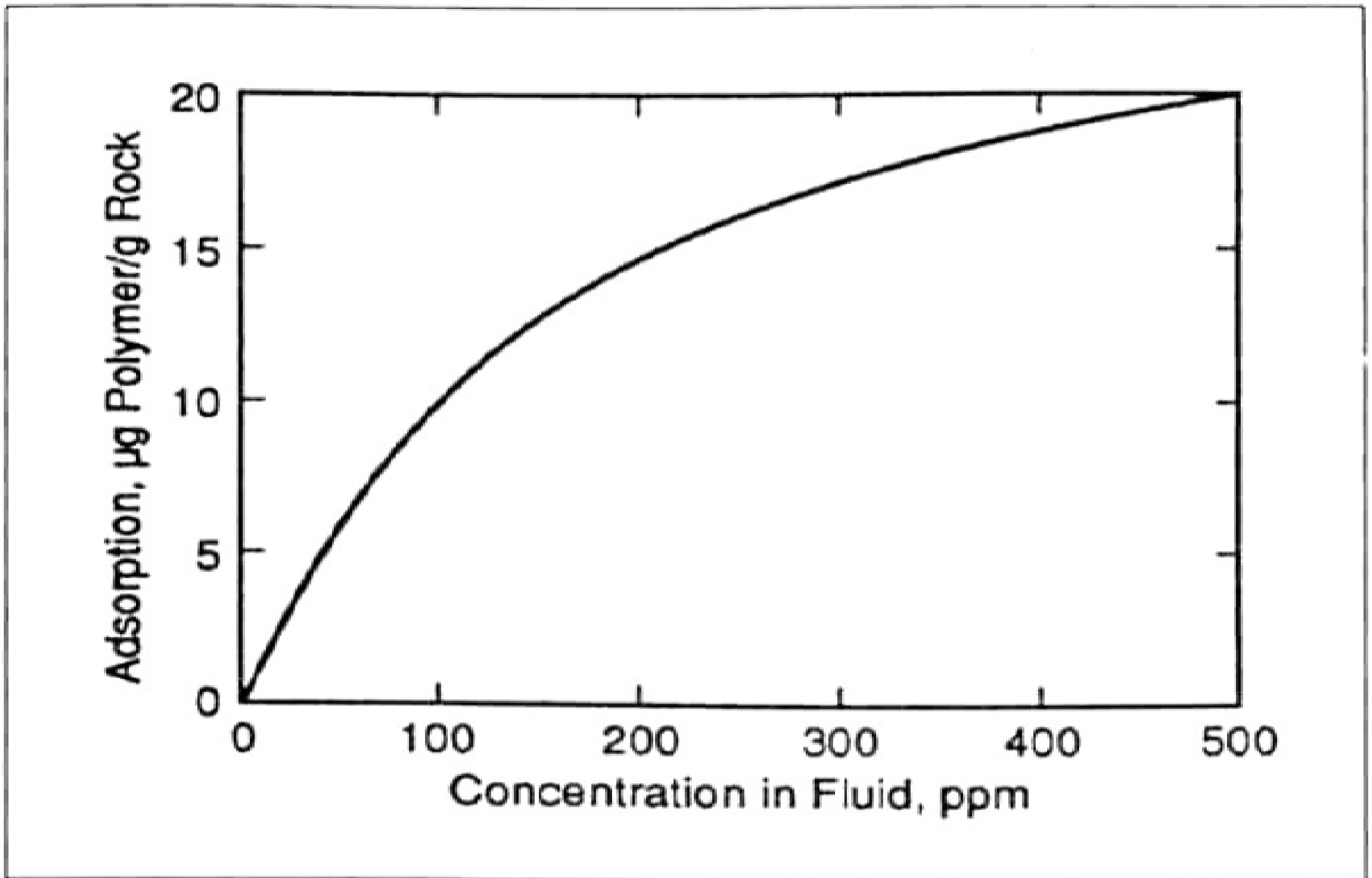
$$\left(\frac{dx_D}{dt_D} \right)_{S_{w3}} = \left(\frac{dx_D}{dt_D} \right)_{C_i} = \left(\frac{f^*_{w3} - f_{w1}}{S^*_{w3} - S_{w1}} \right) = \frac{f_{w1}}{S_{w1} + D_i} = \frac{f^*_{w3}}{S^*_{w3} + D_i}$$

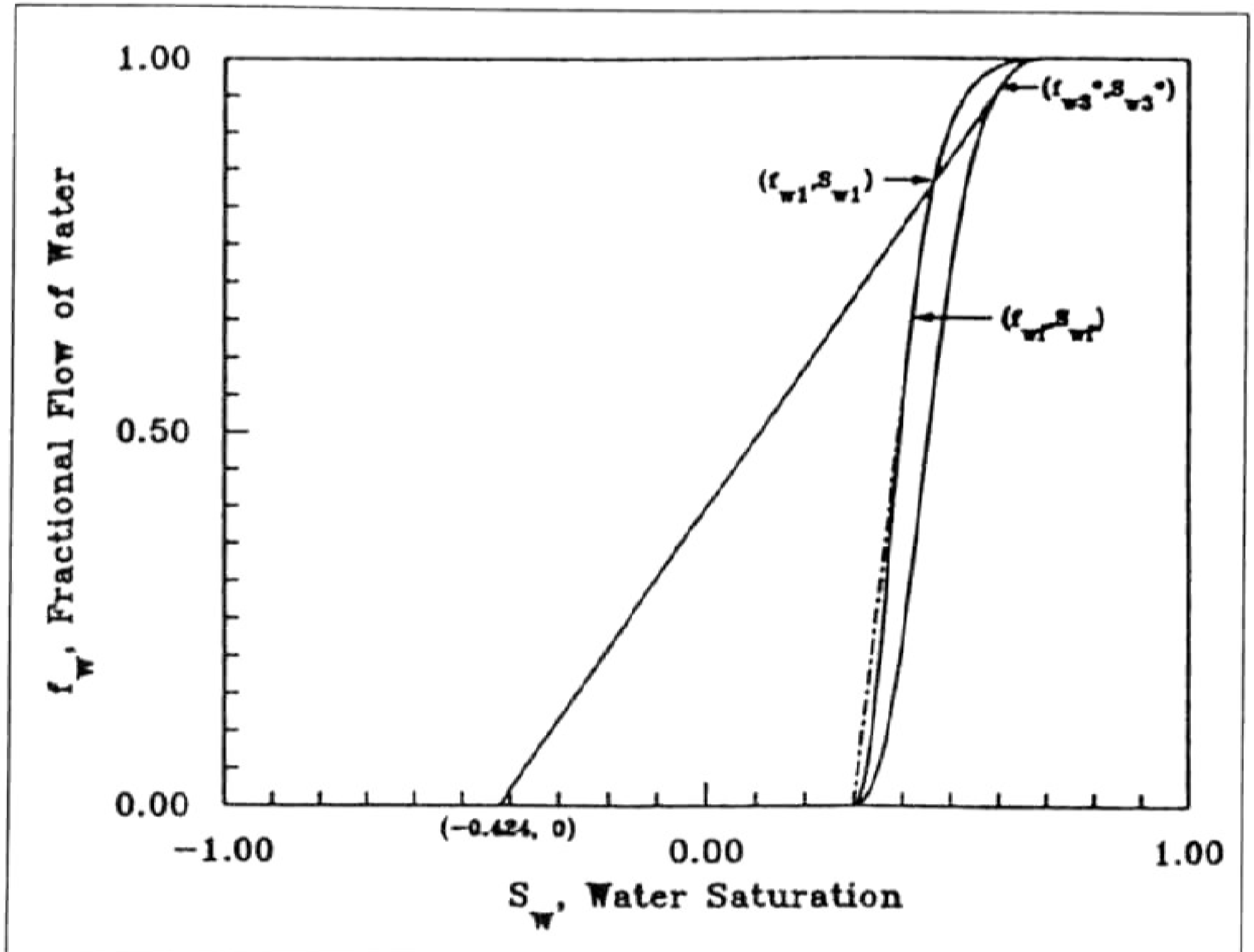
A tangent is drawn from (–Di, 0) to the fractional flow curve

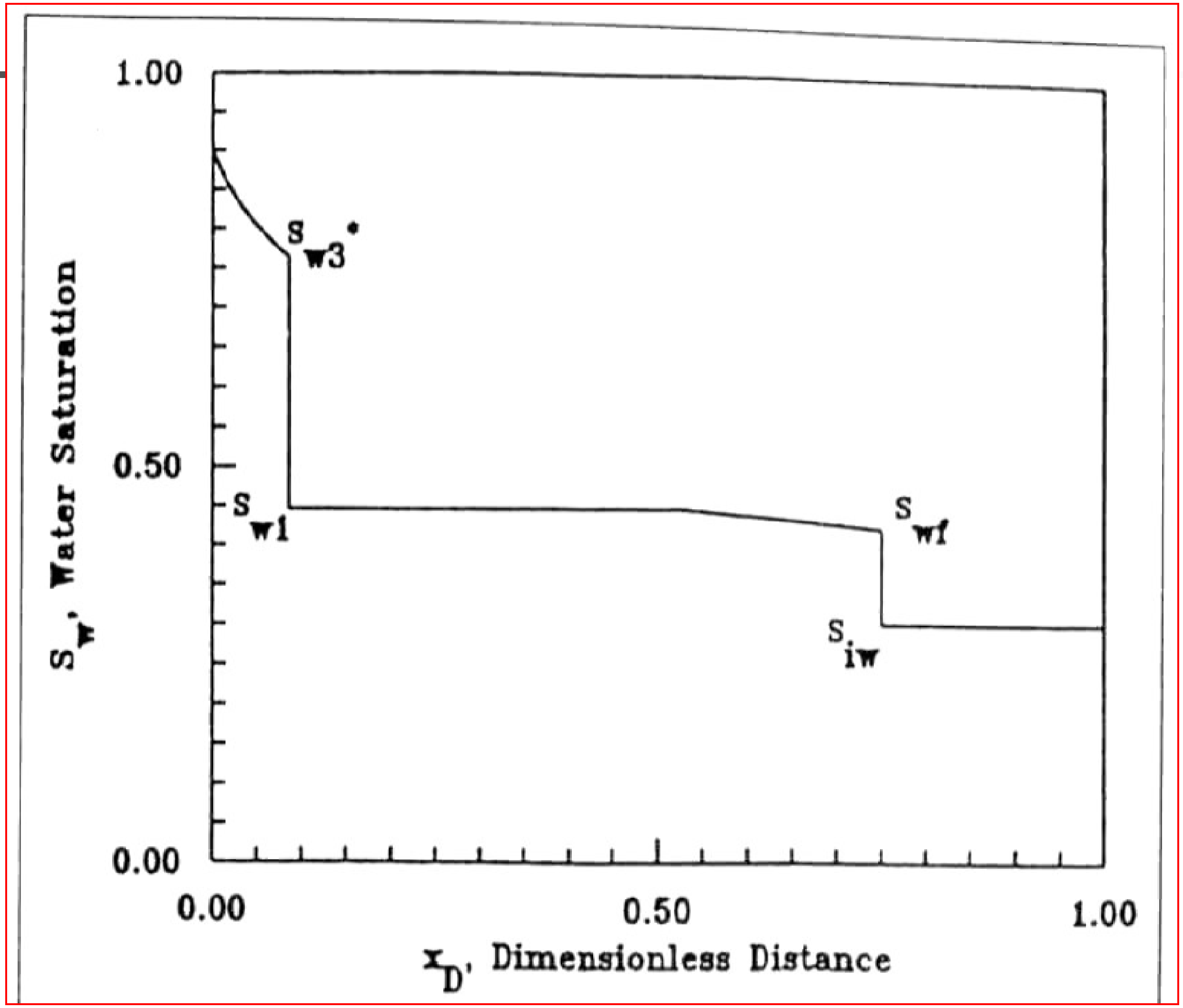
f^*_w.

- The point of tangency gives f^*_{w3} *and* S^*_{w3}
- The intersection of this tangent with the original fractional flow curve gives f_{w1} and S_{w1}
- An ordinary waterflood front will also be present and its saturation and velocity can be determined using the normal waterflooding theory.
- The saturation profile before the arrival of this ordinary front at the producing end is shown in the next slide.

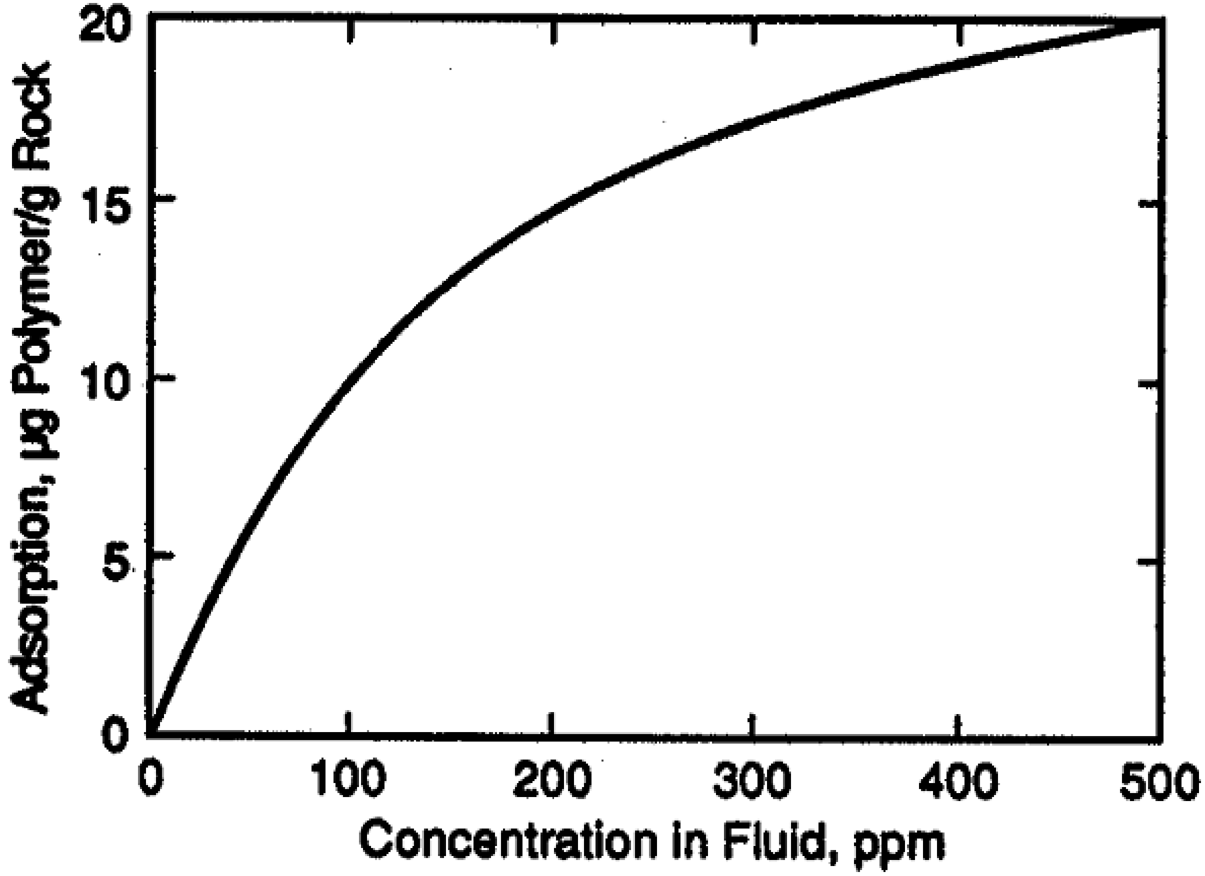
Typical adsorption isotherm




1.00
0.50
0.00
f_w, Fractional Flow of Water
(f_{w3}^*, S_{w3}^*)
(f_{w1}, S_{w1})
(f_{wi}, S_{wi})
(−0.424, 0)
−1.00
0.00
1.00
S_w, Water Saturation


1.00
0.50
0.00
S_w, Water Saturation
S_{w3}^*
S_{w1}
S_{wf}
S_{iw}
0.00
0.50
1.00
x_D, Dimensionless Distance

Typical adsorption isotherm for chemical species on porous media



Application of the chemical flooding model

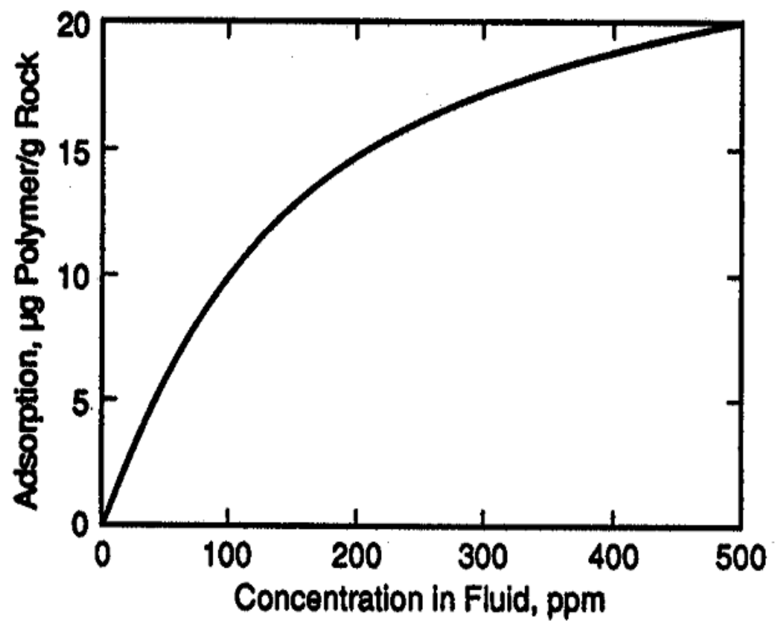
- The chemical flooding model developed is useful for understanding basic displacement process for both polymer and surfactant floods
- Certain high molecular weight polymers increase the viscosity of water significantly when concentrations on the order of a few hundred parts per million are dissolved in the water
- Addition of certain chemicals to the injected water can reduce the IFT between the injected fluid and the oil.
- A low tension flood is one in which the IFT is on the order of 10^{-3} dynes/cm o lower.

Example 6 : polymer flood in a linear system

- A polymer flood is to be conducted in a linear system of example 2. The oil viscosity is 40 cp. A concentration of 300 ppm polymer is used to raise the viscosity of the injected water to4 cp, and the adsorption isotherm is given. The density of the rock is 2.65 g/cc, and porosity is 0.267
- Estimate the oil recovery as a function of pore volume injected.

Solution

- The value of D_i must be estimated from the adsorption isotherm before (fw3*,Sw3*) and (fw1,Sw1) can be determined. From the Figure, A_i=17.5 µg polymer/g rock.



$$C_{ii} = 300 \text{ ppm}$$
$$= 300 \times 10^{-6} \text{ g/cm}^3.$$

From Eq. 3.89,

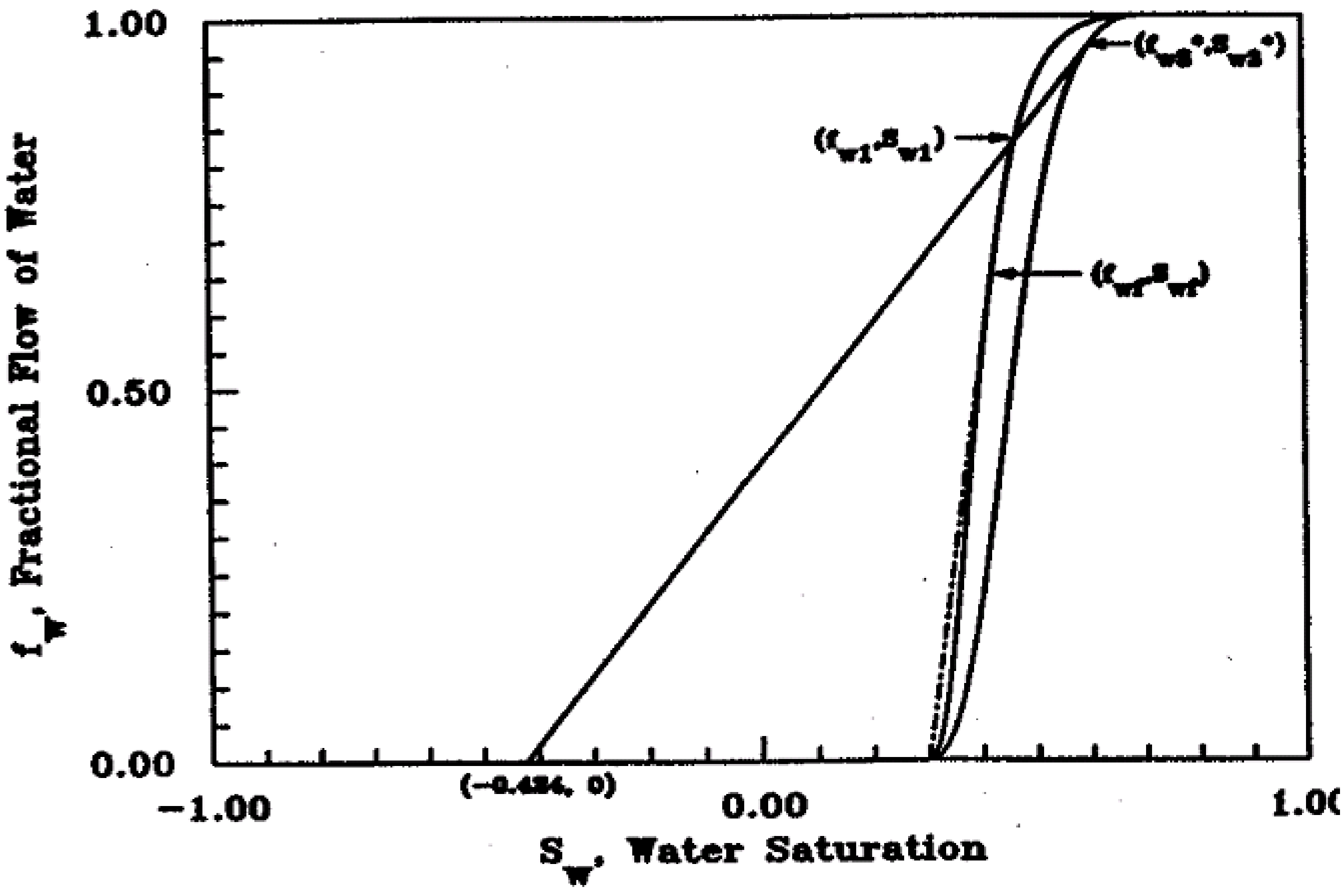
$$\hat{C}_{ii} = \left(\frac{17.5 \times 10^{-6} \text{ g}}{\text{g rock}}\right)\left(\frac{2.65 \text{ g rock}}{\text{cm}^3 \text{ rock volume}}\right)$$
$$\times \left[\frac{(1-0.267) \text{ cm}^3 \text{ rock volume}}{0.267 \text{ cm}^3 \text{ PV}}\right]$$
$$= 1.27 \times 10^{-4} \text{ g/cm}^3 \text{ PV}.$$

Thus, D_i is computed with Eq. 3.88:

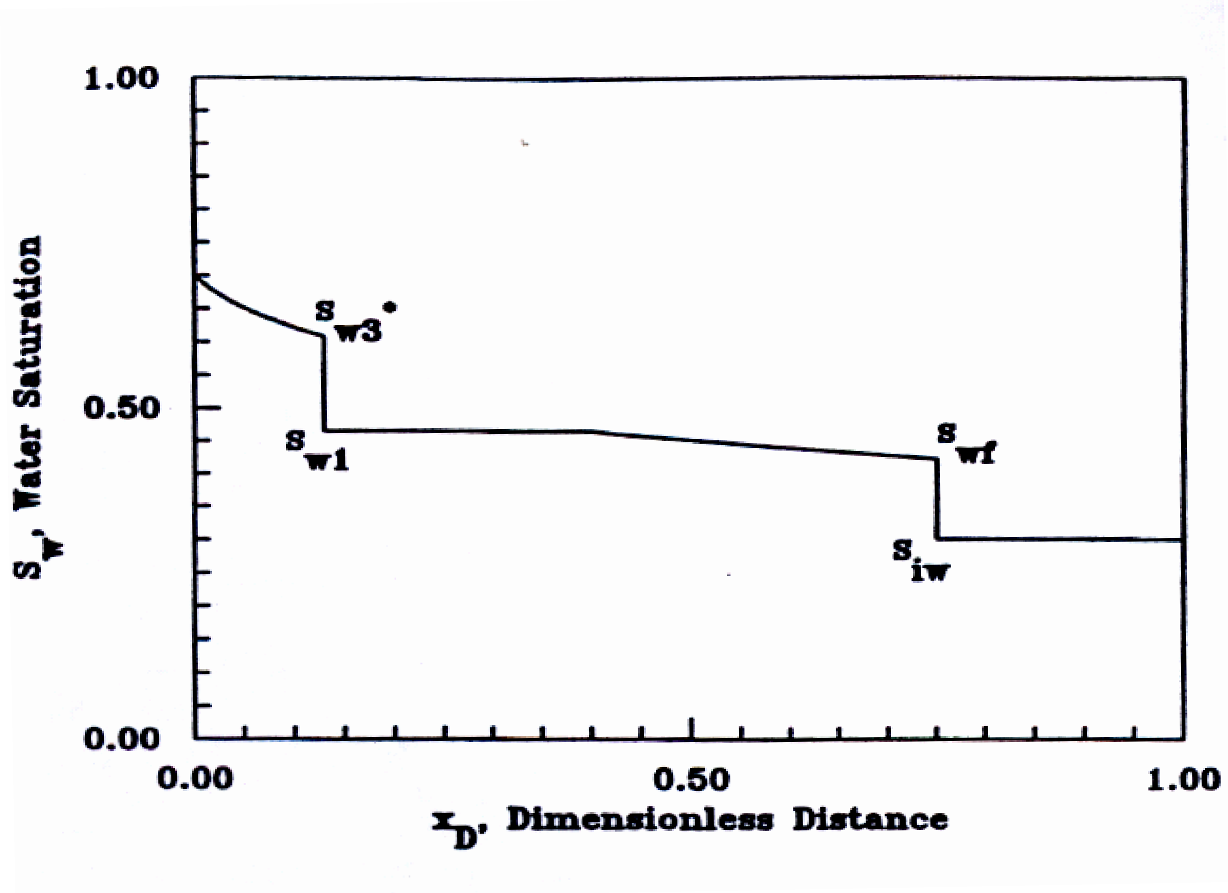
$$D_i = \hat{C}_{ii} / C_{ii}$$
$$= (1.275 \times 10^{-4})/(300 \times 10^{-6})$$
$$= 0.424.$$

- The graphs of f_w^*-S_w^* and f_w-S_w are shown next slide.
- From the construction,
- $f_{w3}^*=0.9657$, $f_{w1}=0.831$
- $S_{w3}^*=0.6082$, $S_{w1}=0.462$
- $S_{w1}>S_{wf}$

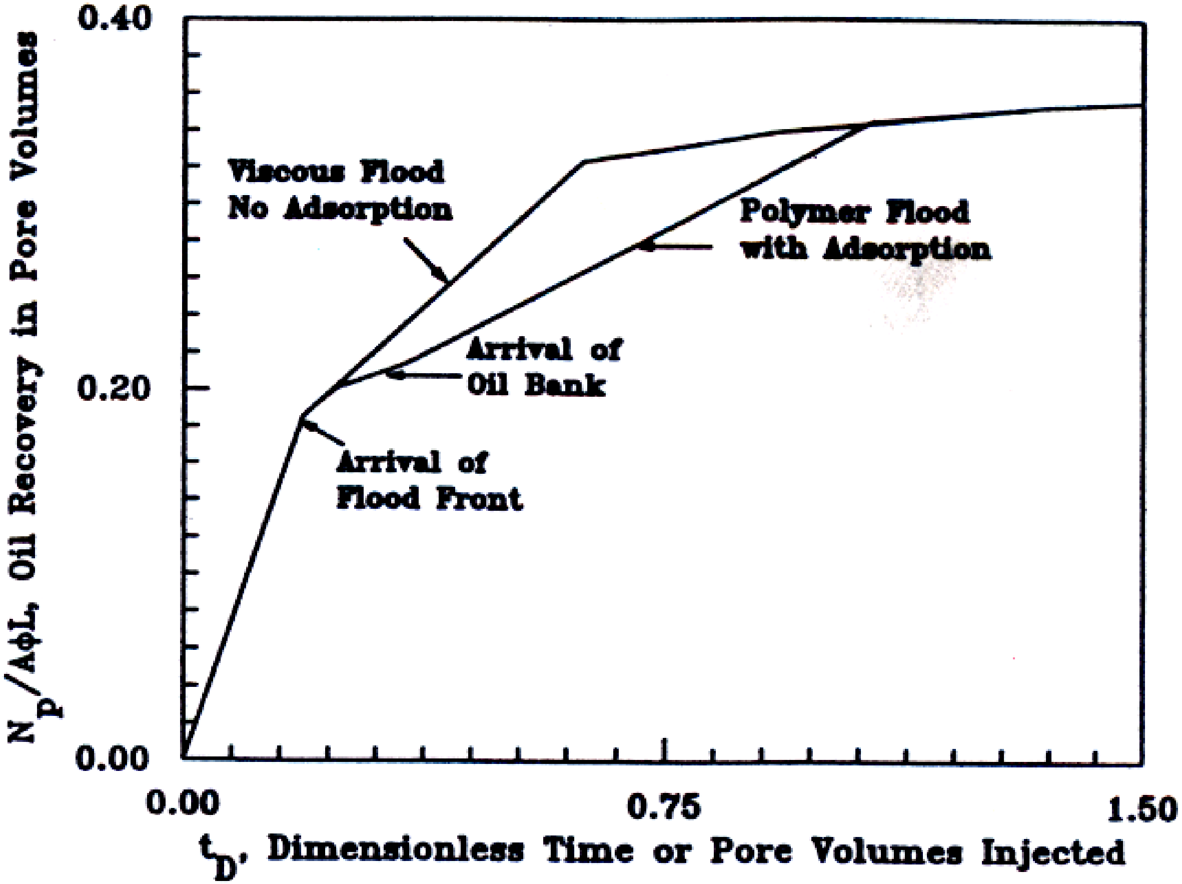
Construction procedure to determine f^*_{w3} and S^*_{w3} when adsorption occurs



Saturation profile when x_{Df}=0.75



Comparison of oil recovery from viscous water flood and polymer flood initiated at interstitial water saturation

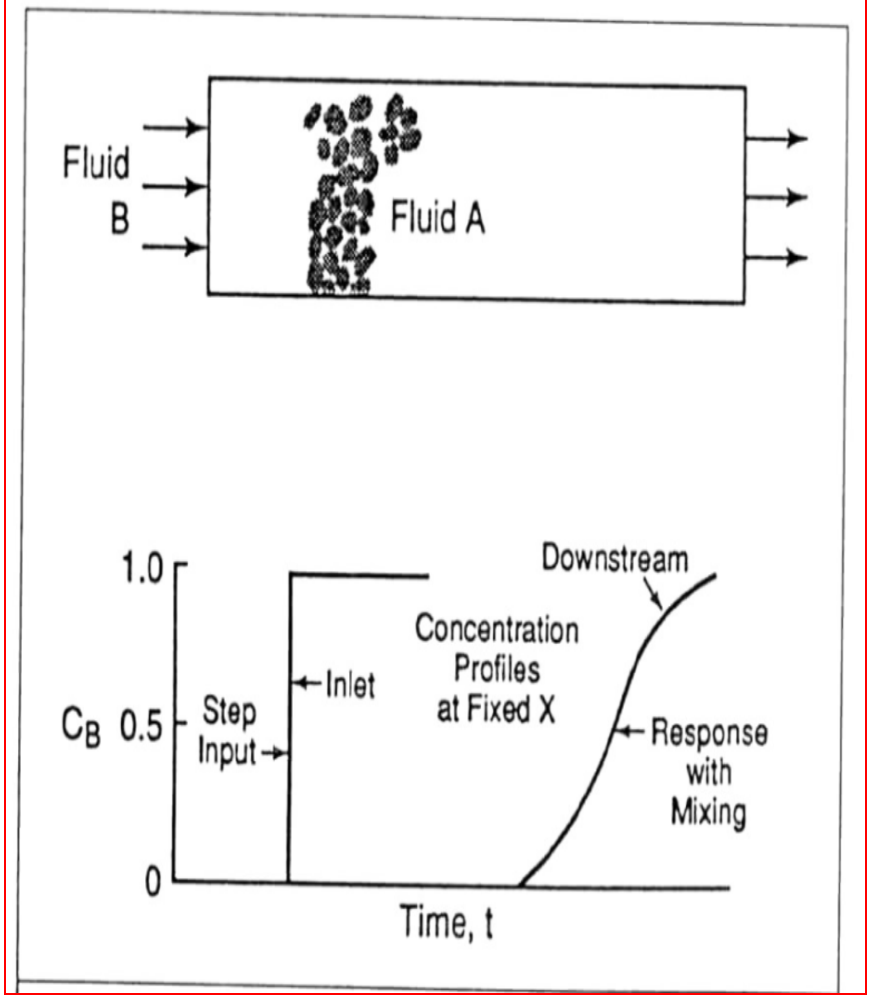


- So far we have assumed no mixing at the boundary between two miscible fluids.
- This is obviously an approximation.
- There will always be some mass transfer by diffusion and there may be other mixing mechanisms present.
- These other mechanisms that cause mixing, taken together, are called “Dispersion.”
- Dispersion has a profound impact on the performance of EOR processes.
- Mixing will occur at both ends of the chemical slug.
- In general, dispersion will increase the size of slug required in a process.

Dispersion during miscible displacement

- In all miscible displacement, mixing occurs between the displacement and displaced fluids. That is, there is dispersion between the different fluids.
- This dispersion dilutes the displacing fluid with the displaced fluid and thereby affects the phase behavior.
- Understanding the dispersion phenomenon is important in the design of miscible displacement processes.

When fluid B is injected to displace fluid A, two theoretical possibilities exist for the change of concentration with time at the producing end.



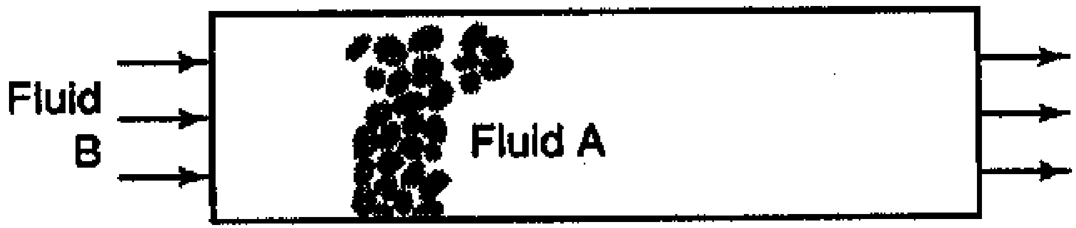
- If there is no mixing, the concentration will remain 100% A, and then jump to 100% B at one pore volumes of injection.

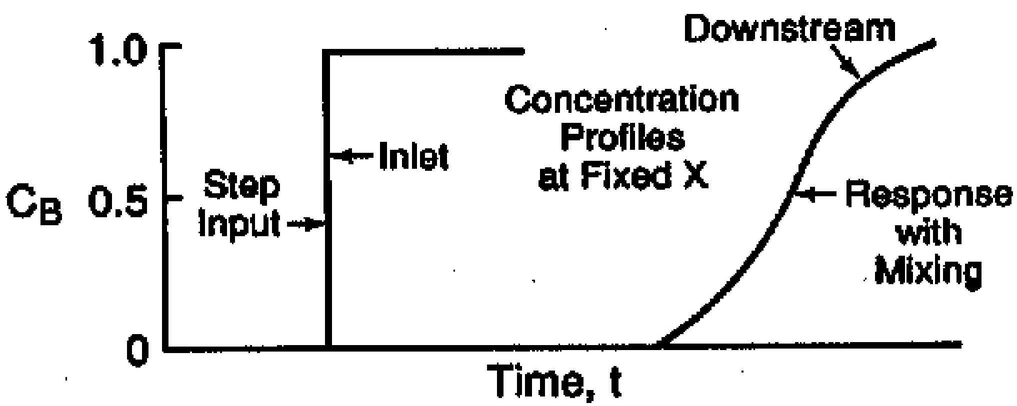
- If there is mixing, the fluid B will start appearing in the produced fluid before one pore volume has been injected and its concentration will increase gradually.

- This second scenario is what we see in reality.

Miscible displacement of fluid A by fluid B

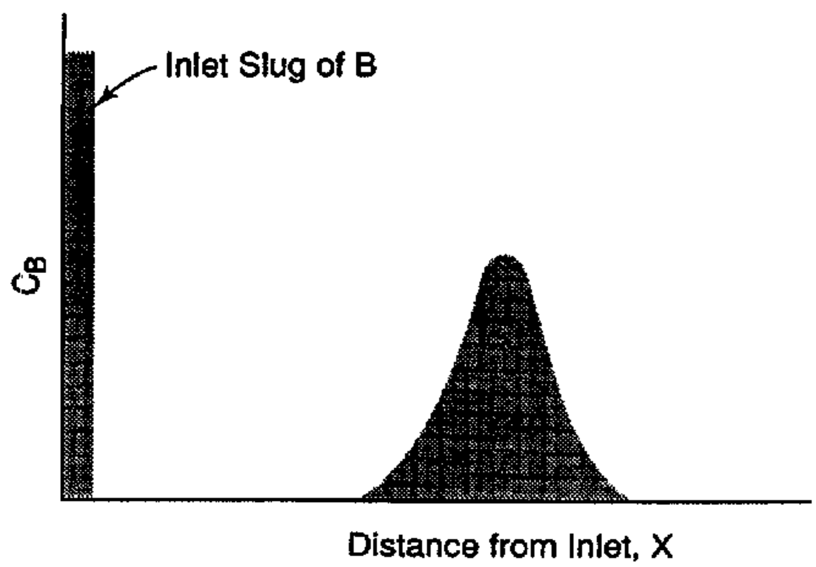
Consider a linear porous medium such as shown





Concentration profiles for injection of a slug of fluid B to displace fluid A.

- A small slug of fluid B followed again by fluid A, rather than changing totally to fluid B at the inlet. If this fluid B slug were followed as it moved down stream and concentration profiles were measured at different fixed times, the profile would appear as shown:



Dispersion

- **Longitudinal dispersion**
  - It occurs in the direction of primary flow-i.e., along the axis of flow
- **Transverse dispersion**
  - It occurs in a direction perpendicular to flow

Mechanisms & Models of Longitudinal Dispersion Phenomena

- The dispersion of miscible fluids may be attributed to several different physical phenomena
  - Molecular diffusion
  - Velocity profile (Taylor) effect
  - Series of Mixing cells
  - Stagnant pockets
  - Variation in flow paths

Molecular Diffusion

- Molecular diffusion is present in all systems in which miscible fluids are brought into physical contact
- Fick's law apply

$$m_{Bx} = - D_{BA}A(\partial C_B/\partial\partial x)$$

For the porous media

$$Da_{BA}/D_{BA} = 0.707$$

Where Da_{BA} is the apparent diffusion coefficient

- Better approach
- $Da_{BA}/D_{BA} = 1/FR$

  $FR = R/R'$

Where:

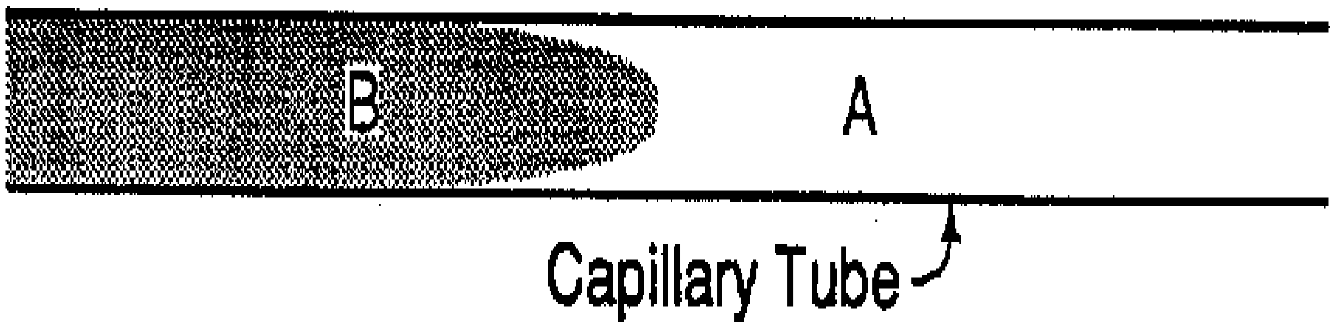
FR= formation factor

R= Electrical resistivity of rock saturated with brine

R'=Electrical resistivity of brine

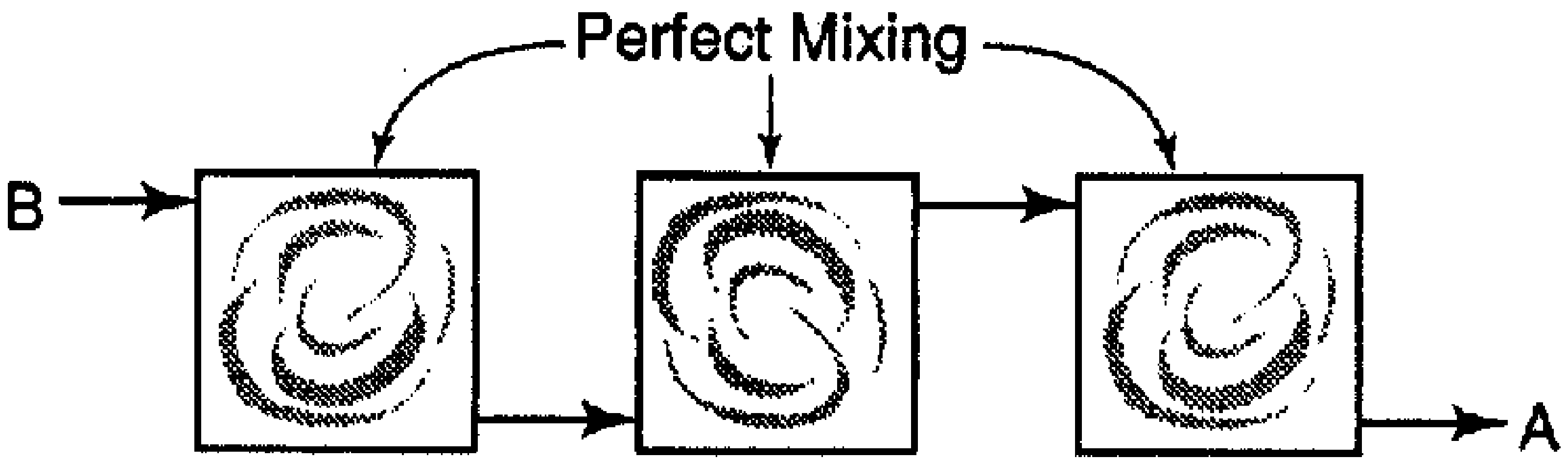
Velocity profile (Taylor) effect

$$V_{average} = \frac{1}{2} v_{max}$$



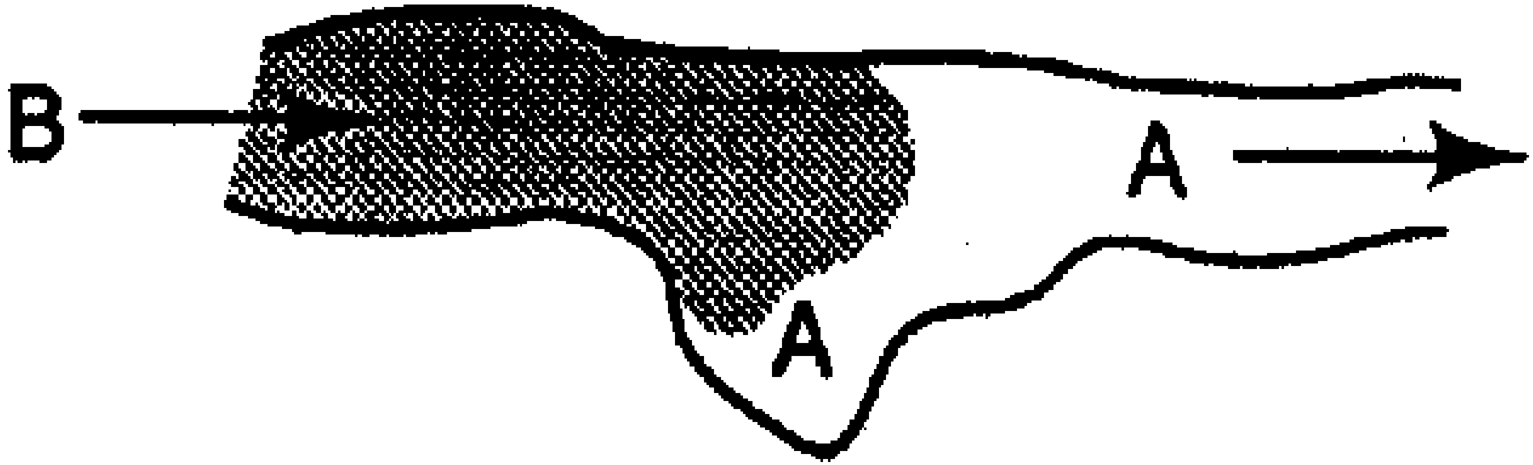
- The velocity profile for laminar flow in tubes is parabolic, with the velocity at the axis being twice the average velocity.
- The streamline at the center moves twice as fast as the average velocity.
- The distribution of velocity causes spreading of the sharp interface as it travels through the tube.

Series of mixing cells

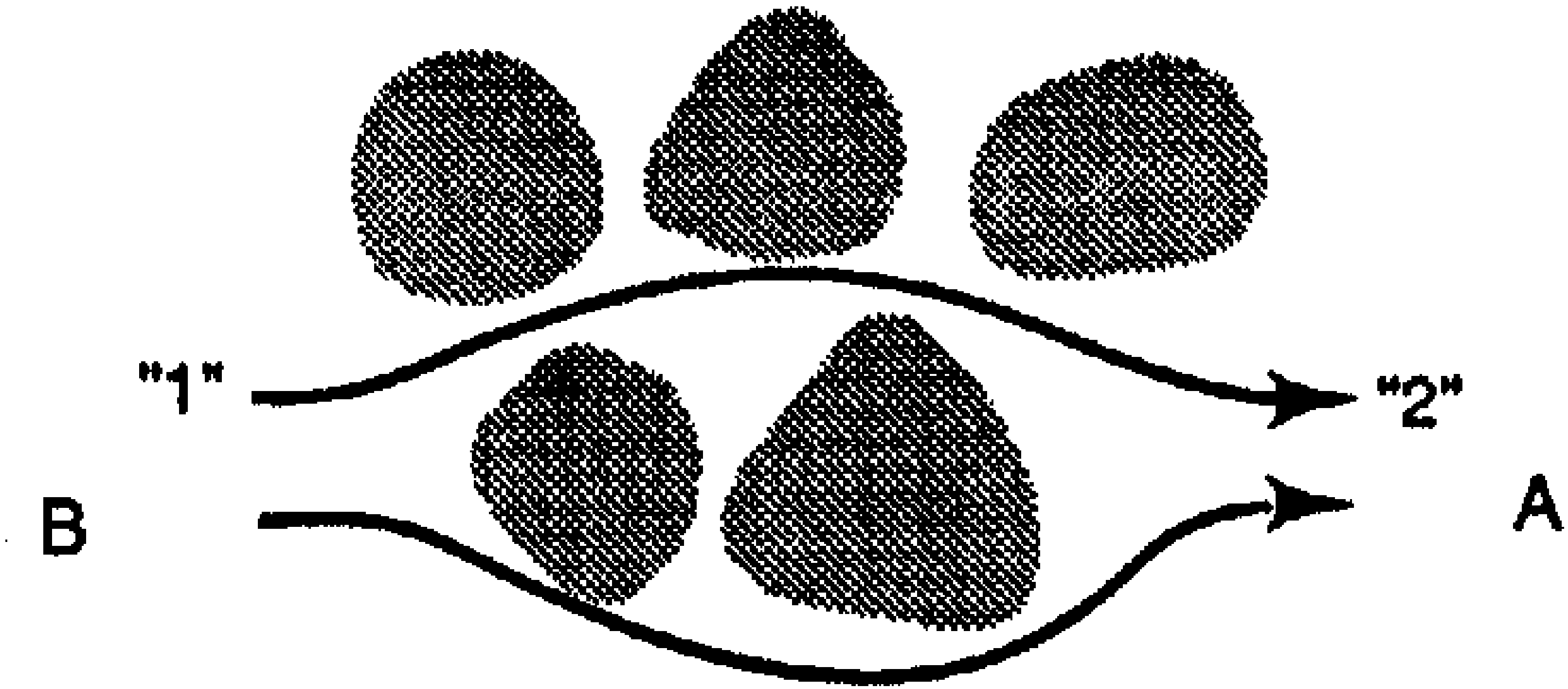


Dispersion based on porous medium being viewed as a series of mixing tanks.

Stagnant Pockets: Dispersion from bypassing of fluid trapped in stagnant pockets



Dispersion caused by variation of paths in a porous medium



Overall Effect of All Mechanisms

- The net effect of all dispersion mechanisms is to enhance the mixing of the displacing and displaced fluids.
- The dispersion process can be described by an equation similar to the Fick's law of molecular diffusion

Dispersive flux of B in x direction $= m_{Bx} = -K_l A \dfrac{\partial C_B}{\partial x}$, where

K_l = longitudinal dispersion coefficient,

A = area open for flow,

$\dfrac{\partial C_B}{\partial x}$ = concentration gradient in x direction.

Mathematical Description of Longitudinal Dispersion

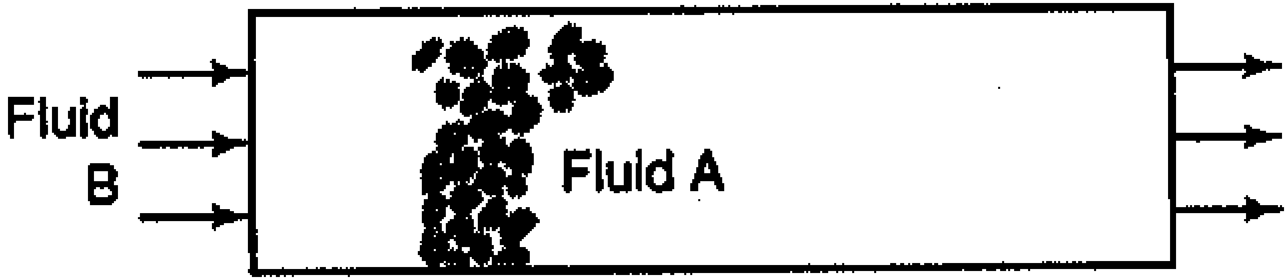
- The following primary assumptions are made in the derivation of the model:
  - Fluid B is displacing fluid A, and the two fluids are miscible
  - Flow is single phase
  - The fluids are incompressible
  - The mobility ratio is unity (no viscous fingering)
  - Fluids density are equal
  - Flow is only in the x direction
  - Fluid viscosity is constant
  - Flow is through a porous medium of constant porosity and of constant cross sectional area.

Model Development

Consider a mass balance on fluid B over the volume element $A\Delta x$ and over a time period Δt.

$$\text{Accumulation} = \text{In} - \text{Out}$$

$$M(t+\Delta t) - M(t) = m(x) - m(x+\Delta x)$$



$$C_B|_{t+\Delta t} - C_B|_t \Delta x = vC_B|_x - vC_B|_{x+\Delta x}\Delta t$$
$$+[-K_\ell(\partial C_B/\partial x)|_x + K_\ell(\partial C_B/\partial x)|_{x+\Delta x}]\Delta t.$$

Dividing through by Δx and Δt gives

$$\frac{C_B|_{t+\Delta t} - C_B|_t}{\Delta t} = \frac{-(vC_B|_{x+\Delta x} - vC_B|_x)}{\Delta x}$$

$$+\frac{K_\ell(\partial C_B/\partial x)|_{x+\Delta x} - K_\ell(\partial C_B/\partial x)|_x}{\Delta x}.$$

Now let $\Delta t \to 0$ and $\Delta x \to 0$, and let v and K_ℓ be constant:

$$\frac{\partial C_B}{\partial t} = -v\frac{\partial C_B}{\partial x} + K_\ell \frac{\partial^2 C_B}{\partial x^2}.$$

$$C_B = \tfrac{1}{2}\{1 - \mathrm{erf}[(x - vt)/2\sqrt{K_\ell\, t}]\},$$

with boundary and initial conditions given by $C_B=0$, $0<x$, $t=0$; $C_B=1.0$, $x=0$, $t>0$; and $C_B=0$, $x\rightarrow\infty$, $t>0$.

Defining C_B as having values between 0 and 1.0 is analogous to defining a normalized concentration

$$C_B = (C_B^* - C_{B0})/(C_{Bi} - C_{B0}),$$

where C_B^* = actual concentration of Fluid B, C_{B0} = initial concentration of Fluid B in the system, and C_{Bi} = injected or maximum concentration of Fluid B. The error function (erf) is a tabulated function defined as

$$\mathrm{erf}(\zeta) = \frac{2}{\sqrt{\pi}} \int_0^{\zeta} e^{-\zeta^2}\, d\zeta$$

or $d\,\mathrm{erf}(\zeta)/d\zeta = 2/\sqrt{\pi}\, e^{-\zeta^2}$

Calculation of concentration profile in a linear miscible displacement

- A core is 0.165 ft in diameter and 4.01 ft long
- Porosity is 0.206
- The core is initially saturated with brine of concentration 30000 ppm of NaCl
- Brine flow rate is 2.12×10^{-4} ft^3/hr.
- Flow interstitial velocity is 1.155 ft/D
- The concentration of the brine injected is suddenly changed to 20000 ppm NaCl
- Calculate the concentration (normalized) at the effluent end of the core at x=L=4.01 ft
- Assuming the dispersion coefficient is given by $K_l = 3.46 \times 10^{-4}$ ft^2/hr.

Solution

- The C_B is calculated & Results are shown below:
- Notice that the time of 83.4 hrs corresponds to the time required to inject 1 PV of brine and that C_B=0.5 at this time.

$$C_B = (C_B^* - C_{B0})/(C_{Bi} - C_{B0})$$

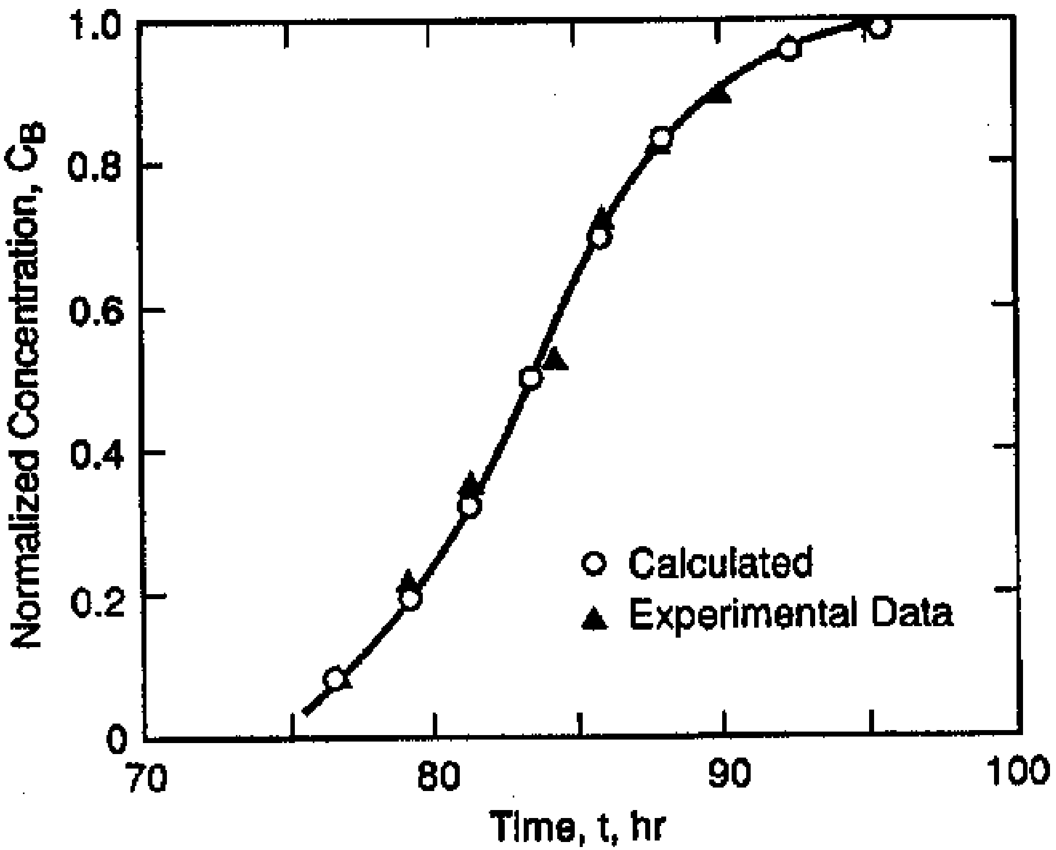
$$or\ C_B = (C_B^* - 30{,}000)/(20{,}000 - 30{,}000)$$

$$where\ C_B^* = concentration\ in\ parts\ per\ million\ of\ NaCl$$

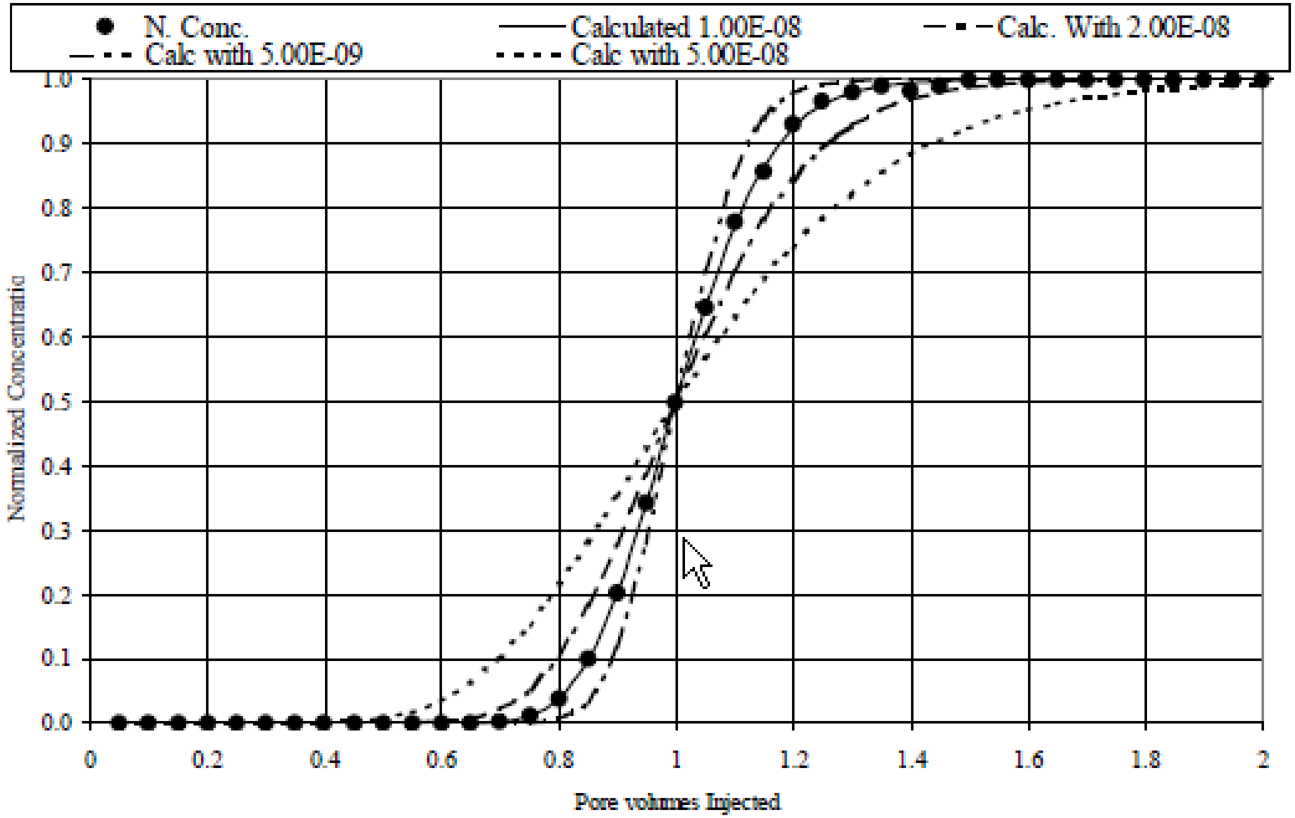
t (hours)	Error Function Argument	erf Argument	C_B^*
76.7	0.9851	0.8364	0.082
79.2	0.6062	0.6087	0.196
81.3	0.2971	0.3256	0.337
83.4	0.000	0.0000	0.500
85.9	−0.3527	−0.3821	0.692
88.0	−0.6379	−0.6330	0.817
92.6	−1.2399	−0.9205	0.960
95.4	−1.5923	−0.9757	0.988

*C_B is a normalized concentration.

Calculated and experimental concentration profiles in a linear system



Effect of viscosity ratio on the effluent concentration curve



Example- plotting Experimental Dispersion Data

- The nature of the solution to the describing partial differential equation for the case of a step concentration is such that a plot of C_B vs. yields a straight line on probability paper.
- Experimental miscible displacement data were taken for a system having the same properties as described in last example. The data are given in the following table.
- Plot the data on probability paper

$$\frac{x-vt}{2\sqrt{K_\ell}\sqrt{t}} = \frac{1}{2\sqrt{K_\ell}}\left(\frac{x-vt}{\sqrt{t}}\right)$$

$$\frac{1}{2\sqrt{K_\ell}}\left(\frac{L-vt^*}{\sqrt{t^*}}\right) = \frac{1}{2\sqrt{K_\ell}}\left(\frac{LA\phi-vA\phi t^*}{\sqrt{A\phi}\sqrt{A\phi t^*}}\right)$$

$$C_B = \frac{1}{2}\left[1-\mathrm{erf}\left(\frac{UL}{2\sqrt{K_\ell}\sqrt{V_p t^*}}\right)\right],$$

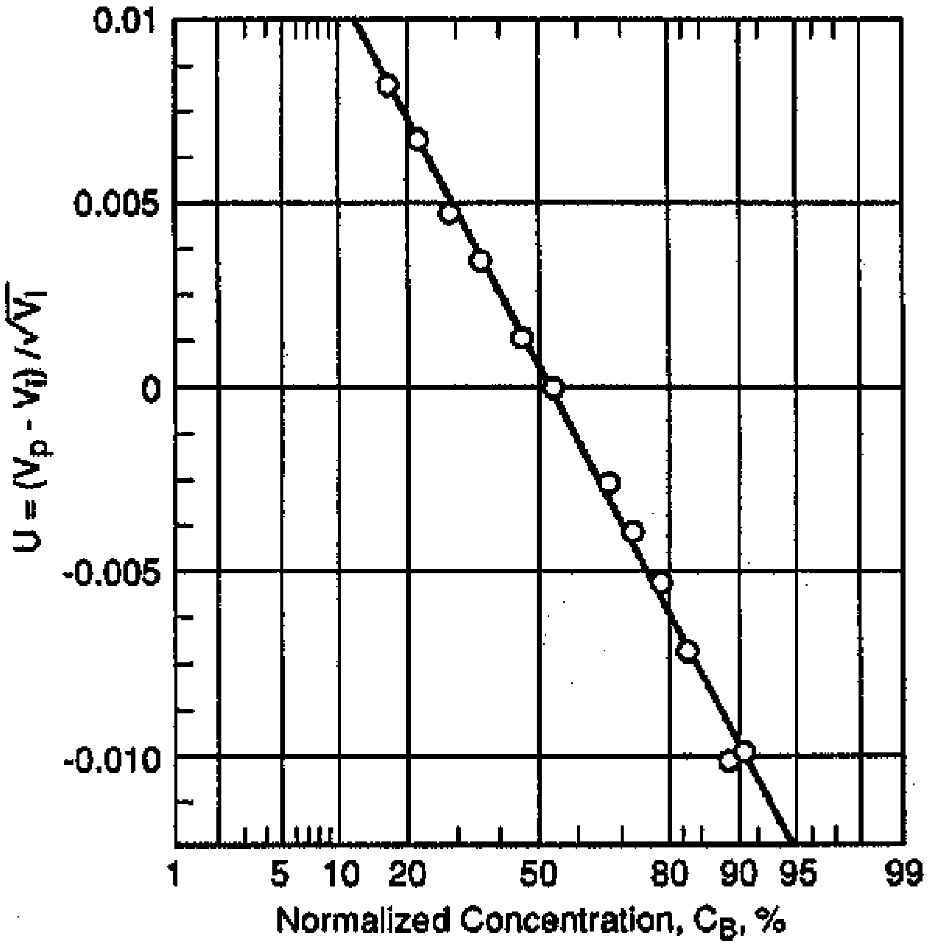
where $U=(V_p-V_i)/\sqrt{V_i}$.

Calculated concentration profile in a dispersion process

L, ft	4.01
V_p, ft^3	0.01766
v, ft/hr	0.0481

t	C_B	V_i/V_p	U
76.7	0.085	0.920	0.0111
78.4	0.160	0.940	0.00822
79.2	0.205	0.950	0.00682
80.5	0.275	0.965	0.00474
81.3	0.350	0.975	0.00336
82.6	0.445	0.990	0.00134
84.3	0.525	1.000	0.0
85.1	0.650	1.020	−0.00263
85.9	0.725	1.030	−0.00393
87.2	0.770	1.045	−0.00585
88.0	0.820	1.055	−0.00712
90.1	0.890	1.080	−0.0102
92.6	0.960	1.110	−0.0139

Solution



0.01
0.005
0
-0.005
-0.010
U = (Vp - Vi) / √Vi
1
5
10
20
50
80
90
95
99
Normalized Concentration, CB, %

Calculation of K_l

$$0.9 = \frac{1}{2}\left[1 - \mathrm{erf}\left(U_{90}\frac{1}{2\sqrt{K_\ell}}\frac{L}{\sqrt{V_p t^*}}\right)\right],$$

where U_{90} = value of U evaluated when $C_B = 0.9$.

$$0.90622 = -U_{90}\frac{1}{2\sqrt{K_\ell}}\frac{L}{\sqrt{V_p t^*}}$$

At $C_B = 0.1$, where $U = U_{10}$,

$$0.10 = \frac{1}{2}\left[1 - \operatorname{erf}\left(U_{10}\frac{1}{2\sqrt{K_\ell}}\frac{L}{\sqrt{V_p t^*}}\right)\right]$$

$$\text{and } 0.90622 = U_{10}\frac{1}{2\sqrt{K_\ell}}\frac{L}{\sqrt{V_p t^*}}$$

$$1.8124 = \frac{1}{2\sqrt{K_\ell}}\frac{L}{\sqrt{V_p t^*}}(U_{10} - U_{90})$$

Now, squaring both sides and solving for K_ℓ gives

$$k_l = \left[\frac{L\left(U_{10} - U_{90}\right)}{3.625} \right]^2 \frac{1}{V_p t^*}$$

Measurement of K_l

K_l may be calculated from the probability paper plot with U values at 10% and 90% concentrations. The following procedure can be used to calculate K_l from dispersion data in a linear displacement experiment:

1. Measure C_B at the exit of the core. Measure the PV's of fluid displaced.
2. Plot C_B vs. U on probability paper
3. Place the best straight line through the data
4. Select U_{10} and U_{90} from the plot
5. Calculate K_l

Example: Calculation of K_l from experimental data

- From the probability plot of the experimental data, we calculate the value of K_l.
- U_{10}=0.0105 & U_{90}=-0.0099

$$k_l = \left[\frac{L\left(U_{10} - U_{90}\right)}{3.625} \right]^2 \frac{1}{V_{p^{t*}}}$$

- K_l= 3.46 x 10^{-4} ft^2/hr

Dispersion zone width

- The width of the dispersion zone when one fluid displaces a second, miscible fluid is an important parameter to examine because it is a reflection of the amount of mixing between the displaced and displacing fluids.
- In a miscible displacement process, the mixing zone width directly relates to the miscible slug size that must be injected
- If the width is arbitrarily defined as the distance between positions at which the dimensionless concentration is 10% and 90%, it can be calculated from:

Mixing Zone Width

- The mixing zone can be defined as the zone over which the concentration increases from 10% to 90%. The lateral extent of this mixing zone increases with square root of time. It can be shown that,

$$x_{10} - x_{90} = 3.625\sqrt{K_l t}$$

- Since the width is proportional to *t and the distance traveled* is proportional to t, the ratio between the two decreases with distance traveled.
- Because of this decreasing ratio, when the effluent profile is plotted against pore volumes injected, it appears to be sharper for a longer core.

example

The objective of this example is to calculate the dispersion zone width after the front has traveled different distances through the porous medium. Width is defined as the distance between positions at which normalized concentration are 10% and 90%. The data from the last example.

Solution

- Given from last example $v=0.0481$ ft/hr, $K_l=3.46\times10^{-4}$ ft^2/hr, and $x_{10}-x_{90}=3.625\sqrt{K_l t}$

Let x=4.0 ft, so $x_{10}-x_{90}= 0.61$ ft, relative width is (0.61/4.0)x100=15%

For x=100 ft, so $x_{10}-x_{90}= 3.07$ ft, relative width is (3.07/100)x100=3.1%

For x=400 ft, so $x_{10}-x_{90}= 6.15$ ft, relative width is (6.15/400)x100=1.5%

- This example indicates that relative width decreases as the interface moves through the porous medium.
- For normal reservoir well spacing, calculated dispersion zone widths are relatively small fractions of the distances between wells

Empirical correlations for longitudinal dispersion

Perkins and Johnston (1963) reviewed the early work and found that K_l/D correlates well with Peclet number multiplied by the inhomogeneity factor.

$$\frac{K_l}{D} = f\left(N_{Pe}, F_i\right) = f\left(\frac{vd_p}{D} F_i\right)$$

where,

$N_{Pe} = vd_p/D$ = Peclet number,

F_I = Inhomogeneity factor

D = Molecular diffusivity

d_p = particle diameter

Empirical correlations for longitudinal dispersion

- For moderate values of the Peclet Number, the correlation can be written as:

$$\frac{K_l}{D} = \frac{1}{F_R \phi} + 0.5 \frac{v F_I d_p}{D}$$

$$\text{for } \frac{v F_I d_p}{D} < 50$$

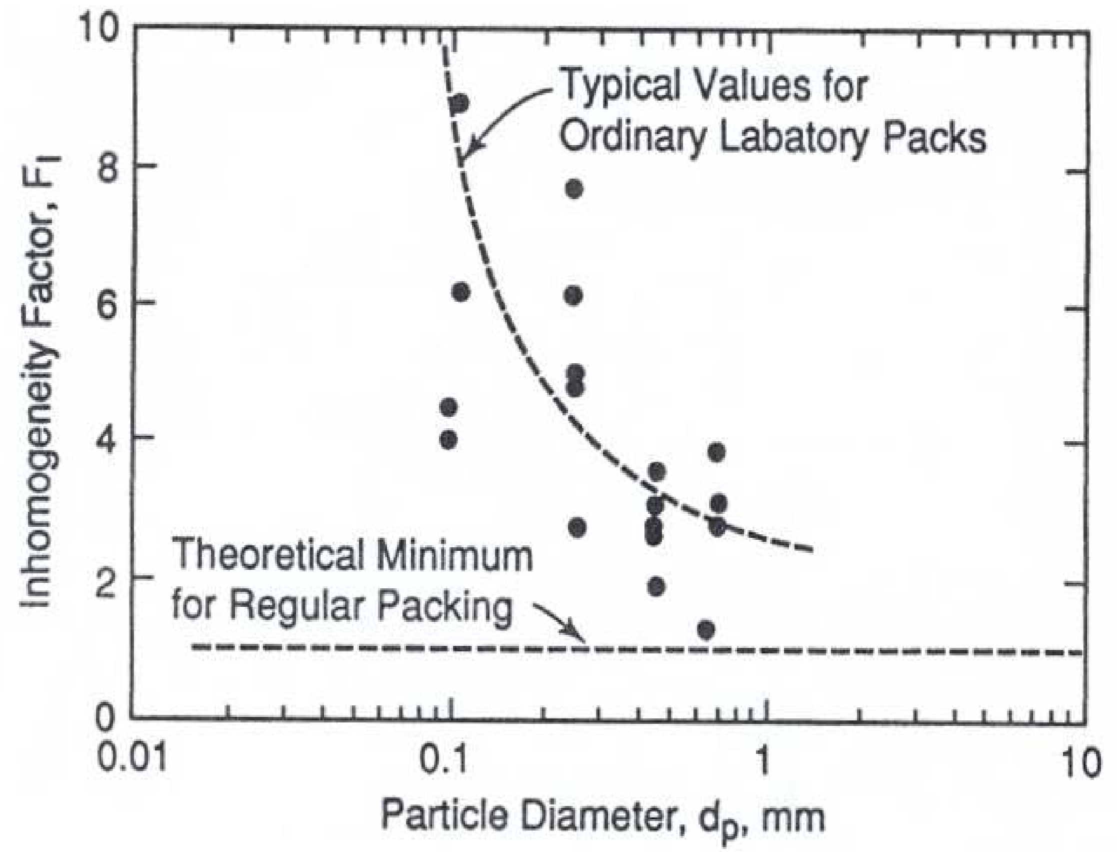
where, F_R is the formation resistivity factor.

- Use Fig 3.50 for higher values of the Peclet number.
- At high values of Peclet number, the second term is much larger than the first; dispersion much larger than diffusion.
- The formation inhomogeneity factor, F_I is included with Peclet number to reconcile results obtained with different packs. Its value must be measured independently.
- For consolidated rocks, $F_I.d_p$ is measured together

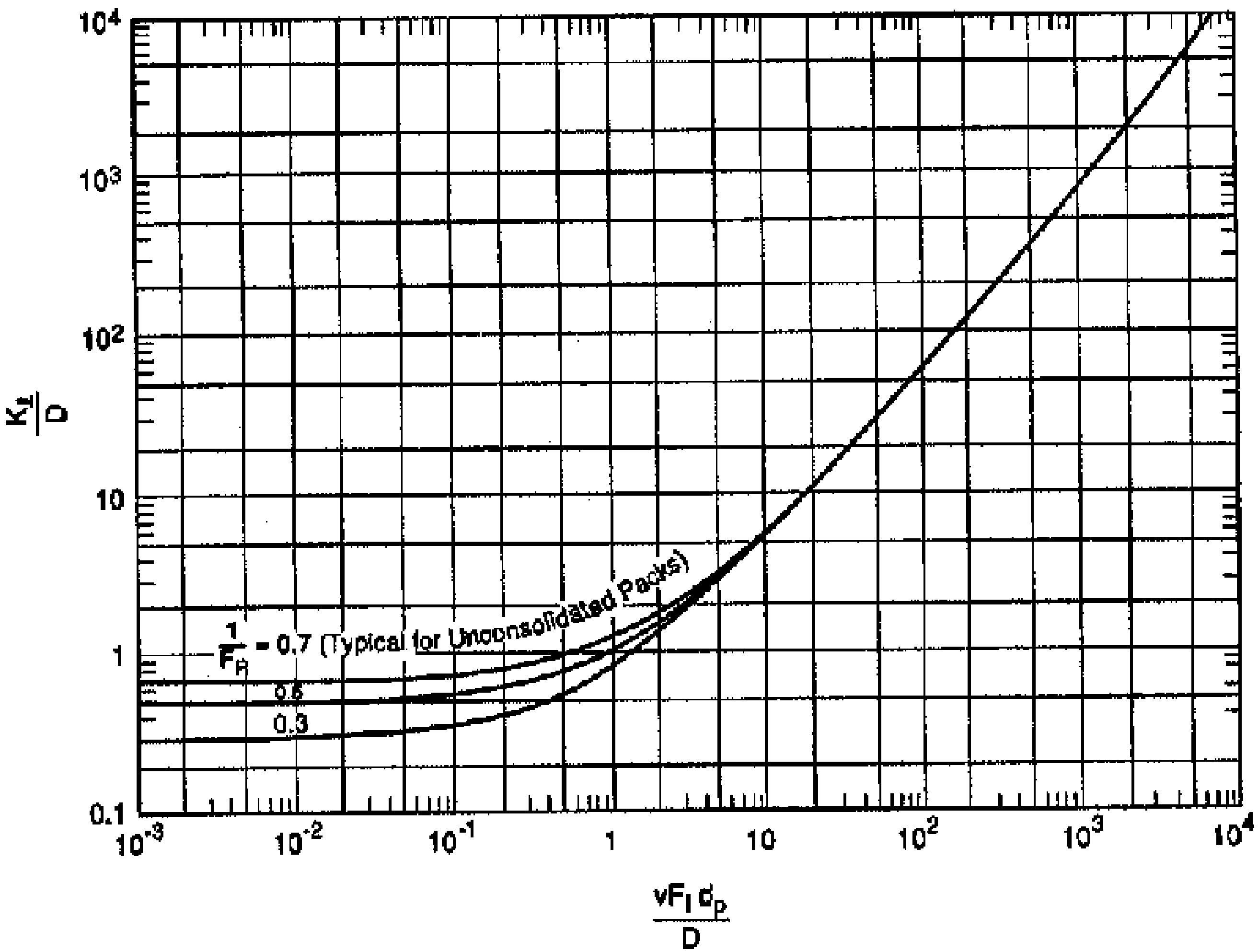
Values of $F_i d_p$ for outcrop sandstones

Source	Dispersion	Rock	$F_l d_p$ (in.)
Crane and Gardner[16]	Transverse	Berea	0.098
Brigham *et al.*[9]	Longitudinal	Berea	0.154
	Longitudinal	Torpedo	0.067
Raimondi *et al.*[14]	Longitudinal	Berea	0.181
Handy[17]	Longitudinal	Boise	0.217
		Average	0.143

Inhomogeneity factor for random packs of spheres



Longitudinal dispersion coefficient for porous medium



Example

Use the correlation of Perkins and Johnston to estimate K_l for experimental given before.

Solution

Use the correlation

$$K_l / D = 1 / F_R\phi + 0.5(vF_I d_p / D)$$

In Berea core, v=0.481 ft/hr, $F_I d_p$=0.146 in=0.0121 ft(average value of Berea cores)

$1/F_R\Phi$=0.6 (estimated for relatively homogenous rock)

D=3.87x10^{-5} ft^2/hr (assumed for brine system)

$$K_l / D = 0.6 + [0.5(0.0481\ ft / hr)(0.0121\ ft) / 3.87 x10^{-5}\ ft^2 / hr]$$

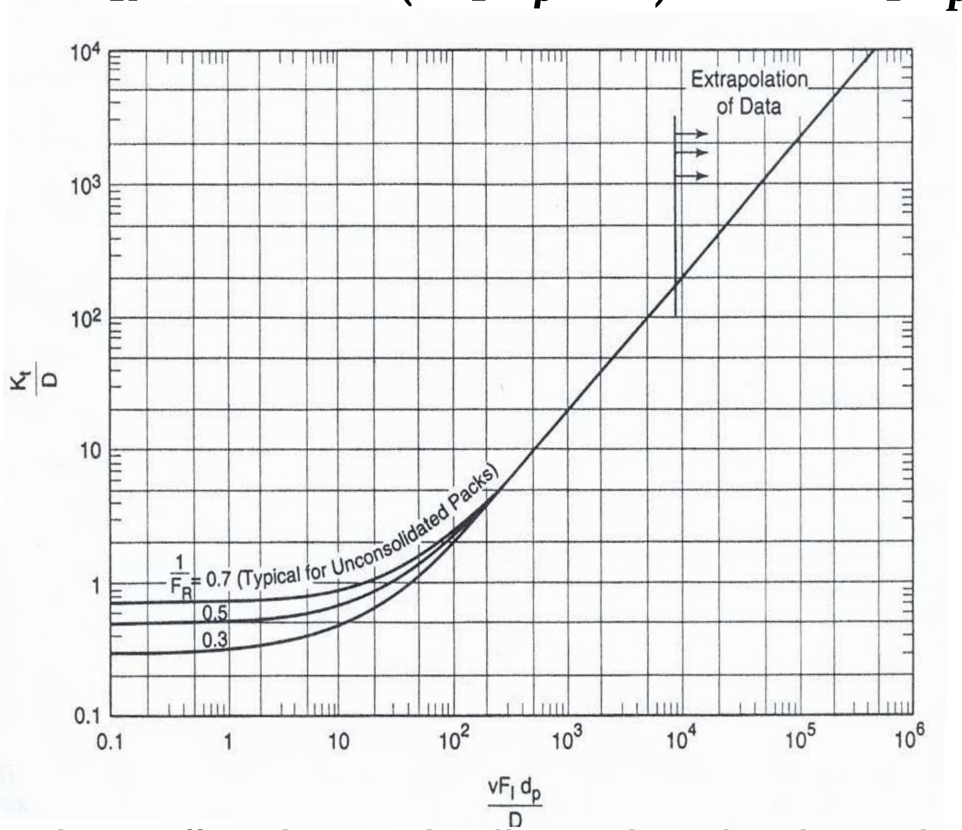
$$K_l = 3.14x10^{-4}\ ft^2/hr$$

Transverse Dispersion

- Diffusion and dispersion also occur in perpendicular direction to the direction of flow.
- If a tracer is released at a point in the flow stream, it will spread not only in the direction of flow but also in perpendicular directions.
- This spreading in perpendicular direction is described by a transverse dispersion coefficient.
- A correlation is available for estimating the values of transverse dispersion coefficient.

Empirical correlations for the transverse dispersion coefficient, K_t

$$K_t / D = 1 / F_R\phi + 0.0157\left(vF_I d_p / D\right) \quad for\ vF_I d_p / D < 10^4$$



Note that K_t is much smaller than K_l in dispersion dominated zone