Naturally Fractured Reservoirs

Physical Properties of Fractures and Matrix

Shahab Gerami, Fall 2010

Outline

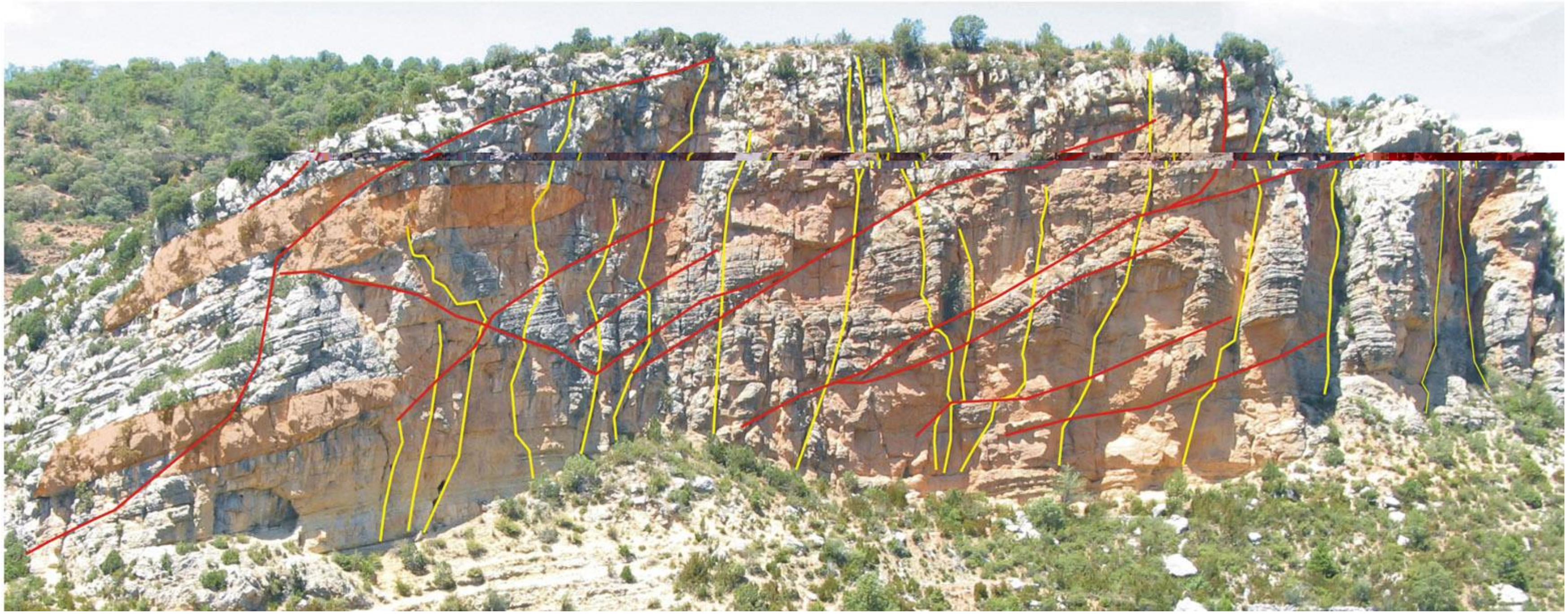
- **Evaluating Fractures and Fields**
- **Geological Condition of Fracturing**

- Physical properties of fractures and matrix
  - Porosity and permeability
  - Rock compressibility in fractured reservoirs

Evaluating Fractures and Fields

Characterization of natural fractures:

- **Dynamic methods** seek to characterize the effects of fractures by measuring or directly describing the movement of fluids through fractures and matrix.
  - **Medium-scale interval**: pressure-transient testing, which provides information on fractures and fracture-related flow, and estimates of fracture conductivity. These tests can be obtained with the MDT Modular Formation Dynamics Tester.
  - **Medium- to large-scale interval**: this dynamic method uses injected tracers and water-composition analysis to determine direct communication attributed to fractures between zones and between wells.
- **Geometric methods** measure specific attributes to identify and characterize natural fractures and assess their potential impact on production or injection.
- **Traditional logging** measurements, such as caliper and microresistivity logs, can allude to the presence of natural fractures, they are generally not quantitative.
- The most common small-scale, log-based fracture-evaluation techniques use **ultrasonic and resistivity borehole imaging technologies** that can be deployed by wireline method.



Carbonate reefs in the Spanish Pyrenees

Geological Condition of Fracturing (1)

➢From a geo-mechanical point of view, fractures in a fractured carbonate reservoir correspond to a solid surface in which a loss of cohesion has taken place and a rupture with no noticeable displacement is observed.

➢Under the same stresses, fracturing resulting from tectonic events will be different in different types of rocks.

❖Fracturing will be more efficient, in brittle reservoir rocks of low porosity and low permeability, where the fractures are relatively extended and have large openings. These are called macrofractures.

❖ In less brittle rocks of high porosity, the fractures are of limited extent and have relatively small openings. These are called "microfractures“ or "fissures".

Geological Condition of Fracturing(2)

Fractures which are generated as a result of the stress that reduces rock cohesion can be attributed to various geological events, such as:

(1) diastrophism in the case of folding and/or faulting;

(2) expansion of upper part of sediments as a result of consistent erosion associated with the removal of the overburden, which causes a differential stress on the rock through the planes of weakness;

(3) rock volume shrinkage as a result of water loss in shale or shaly sands;

(4) rock volume shrinkage in the case of temperature variation in igneous rocks

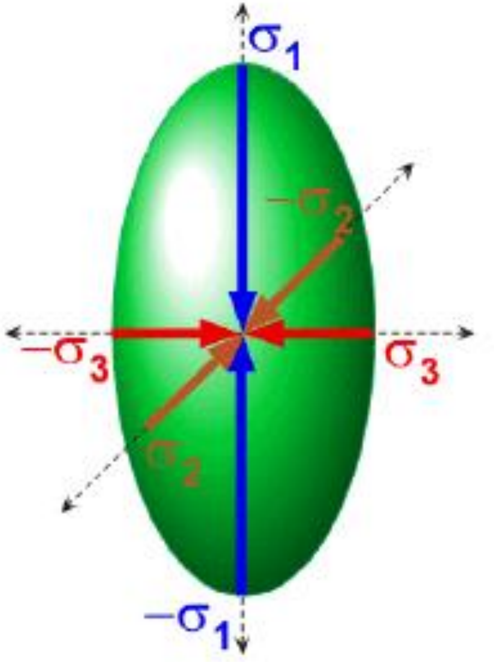
Fractures and Stress State

- Stress is defined as the force per unit area acting on a given plane.

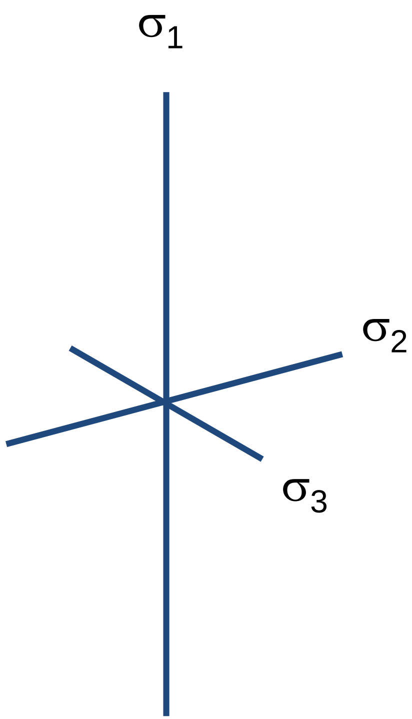
- Any stress state at a point in a solid body can be described completely by the orientations and magnitudes of three stresses called principal stresses and oriented perpendicular to each other.

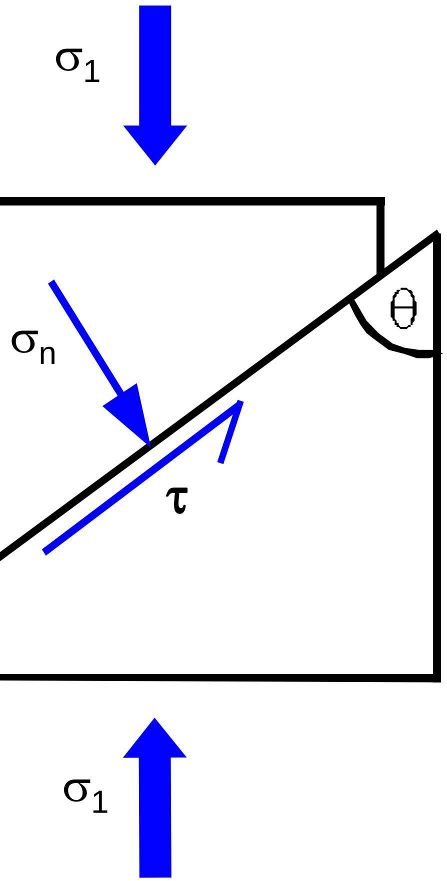
- The principal stresses are defined: $\sigma_1 > \sigma_2 > \sigma_3$

- No shear stress acts along a principal stress direction



- Stresses acting on a surface resolve into two components:
  - s_n - a normal (closing) stress acting perpendicular to the surface
  - τ - a shear stress acting parallel to the surface





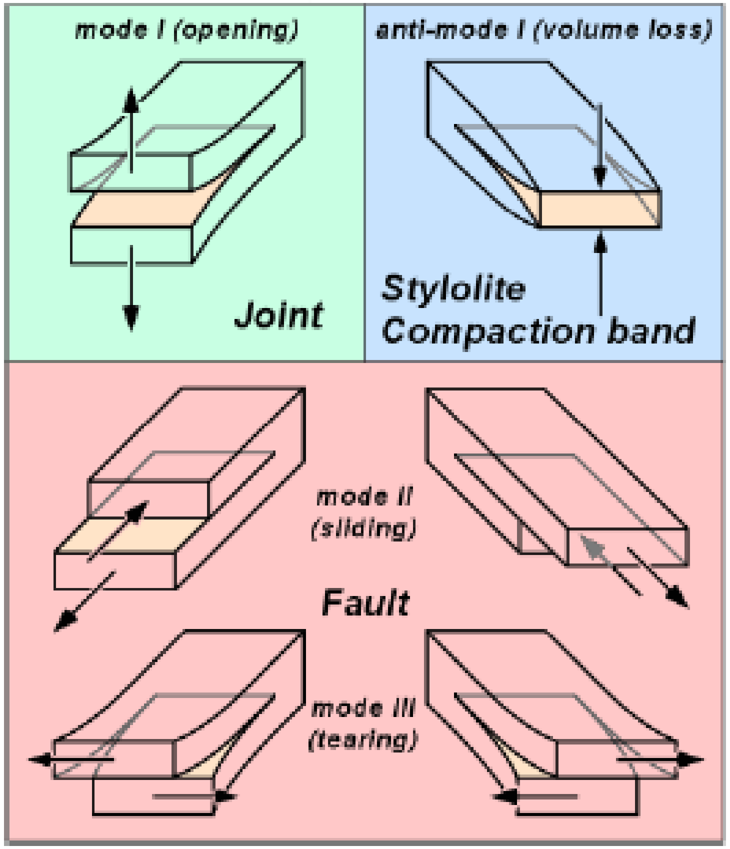
Fracture Propagation Modes

•Mode I fractures form in pure extension normal to the fracture surface. They are termed *joints.*

•Mode II fracture form in shear parallel to the plane of the fractures and with sliding in the direction perpendicular to the fracture front.

•Mode III fractures form in shear parallel to the plane of the fractures and with sliding in the direction parallel to the fracture front.

Mode II and Mode III fractures are termed *faults.*



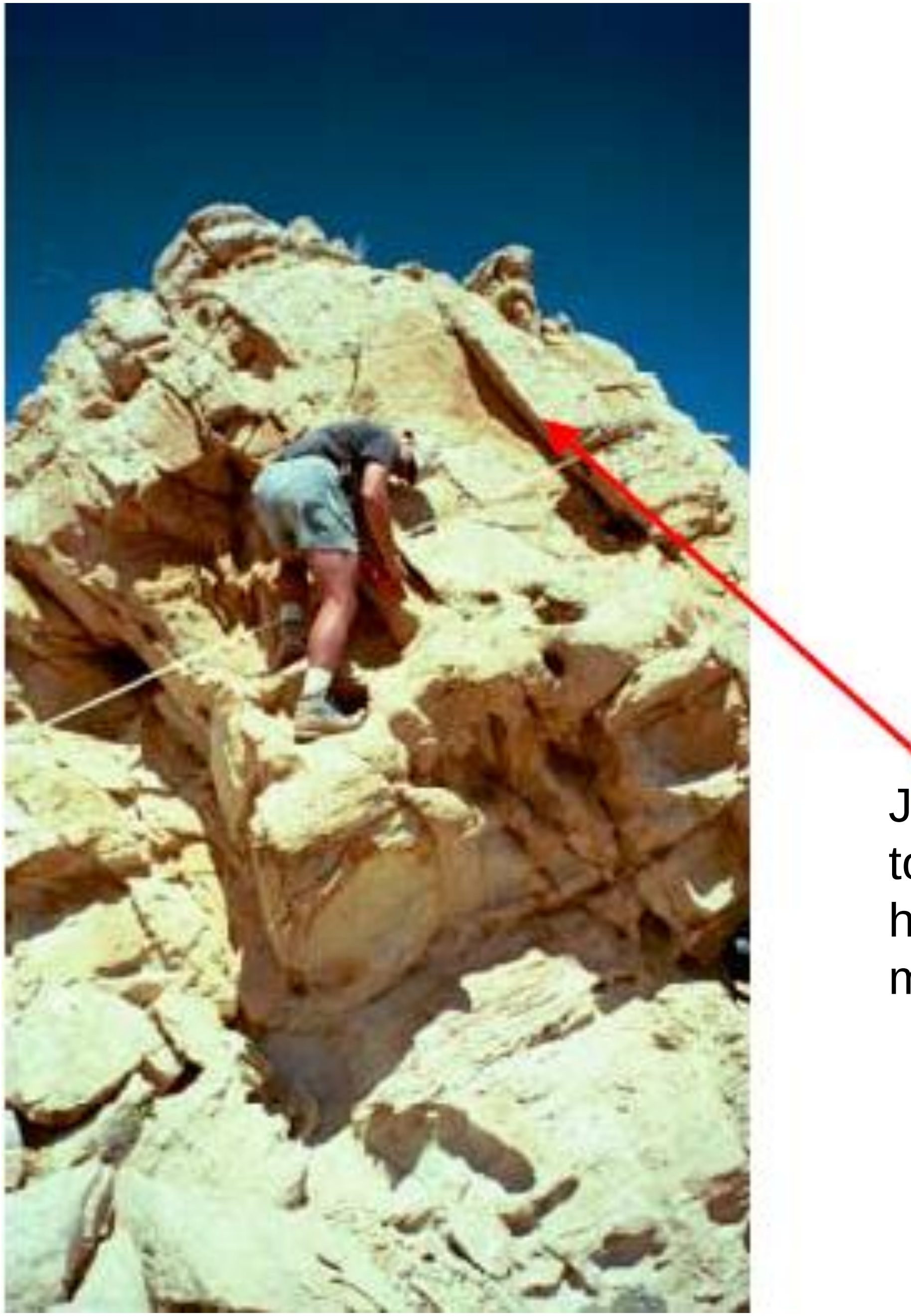
TYPES OF FRACTURES

- Joint - a break in the rock with opening displacement only. A Mode I fracture in fracture mechanics terminology.

- Vein - a break containing precipitated minerals

- Fault - a break with shear displacement. A Mode II fracture in fracture mechanics terminology.

Why are they divided this way? Because each group typically has very different geometry and fluid flow properties.



Joints, or Mode I fractures, tend to be smaller than faults and may have characteristics surface morphologies.

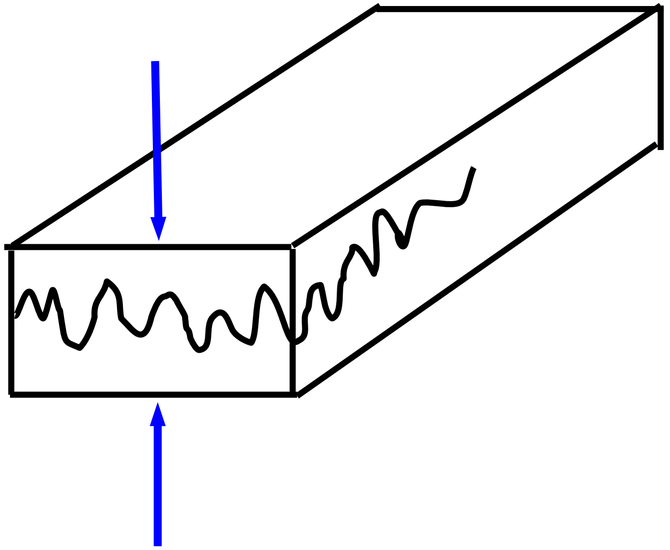
VEINS & DIKES

Mineralized fractures can form barriers to matrix and fracture flow

Faults

Stylolites

A **stylolite** is an irregular discontinuity or non-structural fracture in limestone and other sedimentary rocks. Stylolites result from compaction and pressure solution during digenesis and may be enlarged by subsequent groundwater flow.



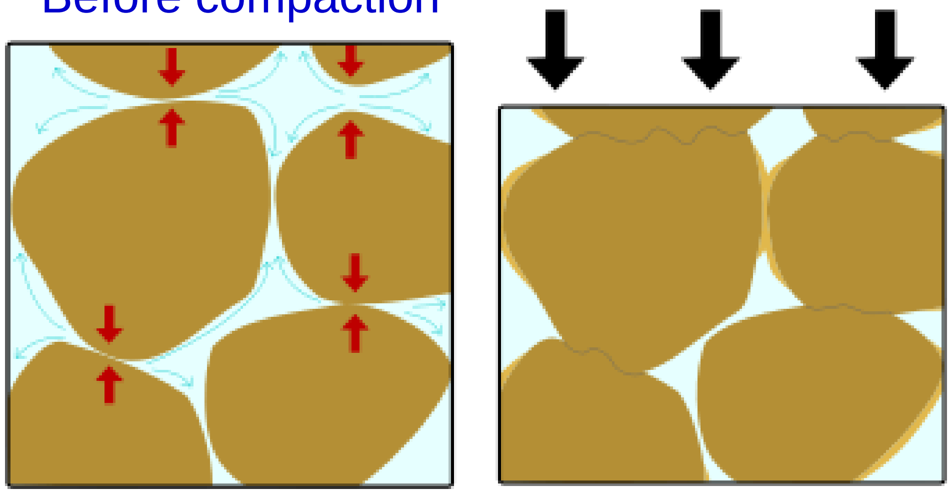
Contractional displacements (**stylolites**)

Stylolites

Schematic diagram of pressure solution accommodating compression/ compaction in a clastic rock.

After compaction

Before compaction



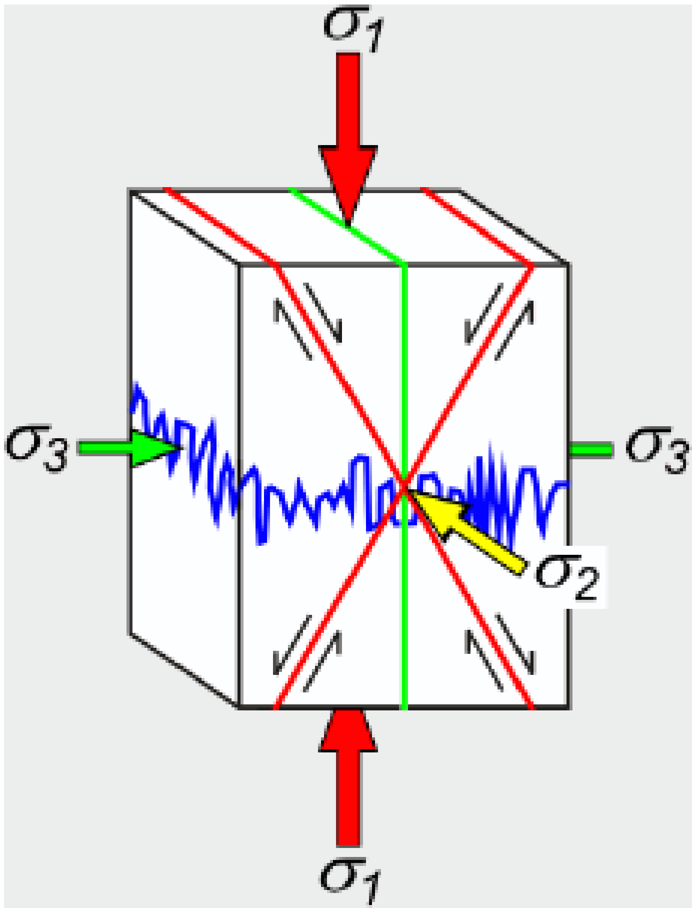
Blue arrows indicate the flow of particles in solution. Red arrows indicate areas of maximum stress (= grain contacts).

In light colored areas new mineral growth has reduced pore space.

Stylolites

Cross section of a stylolite. Stylolites are diagenetic features commonly found in low-permeability carbonate rocks. They form as irregular surfaces between two layers and are generally thought to be the result of pressure solution under a state of differential stress. Stylolites normally inhibit subsurface fluid flow, but are often associated with small fractures called tension gashes, which sometimes appear permeable on core tests.

Example Fractures and Stress State



Joints (mode I) in green
Shear fractures/faults (mode II) in red
Stylolites in blue

FRACTURE GENESIS

Cooling

Gravitational Unloading

Primarily important for Basement reservoirs.

Regional Stress

Folding

Faulting

Primarily important for sandstone and carbonate reservoirs

Basement rocks are considered as any metamorphic or igneous rocks (regardless of age) which are unconformably overlain by a sedimentary sequence.

FAULTING & FAULT HYDROLOGY

Fracture Evaluation

➢Evaluation of fractures in fractured carbonates are carried out continuously, starting during the exploration phase and continuing during the production phase.

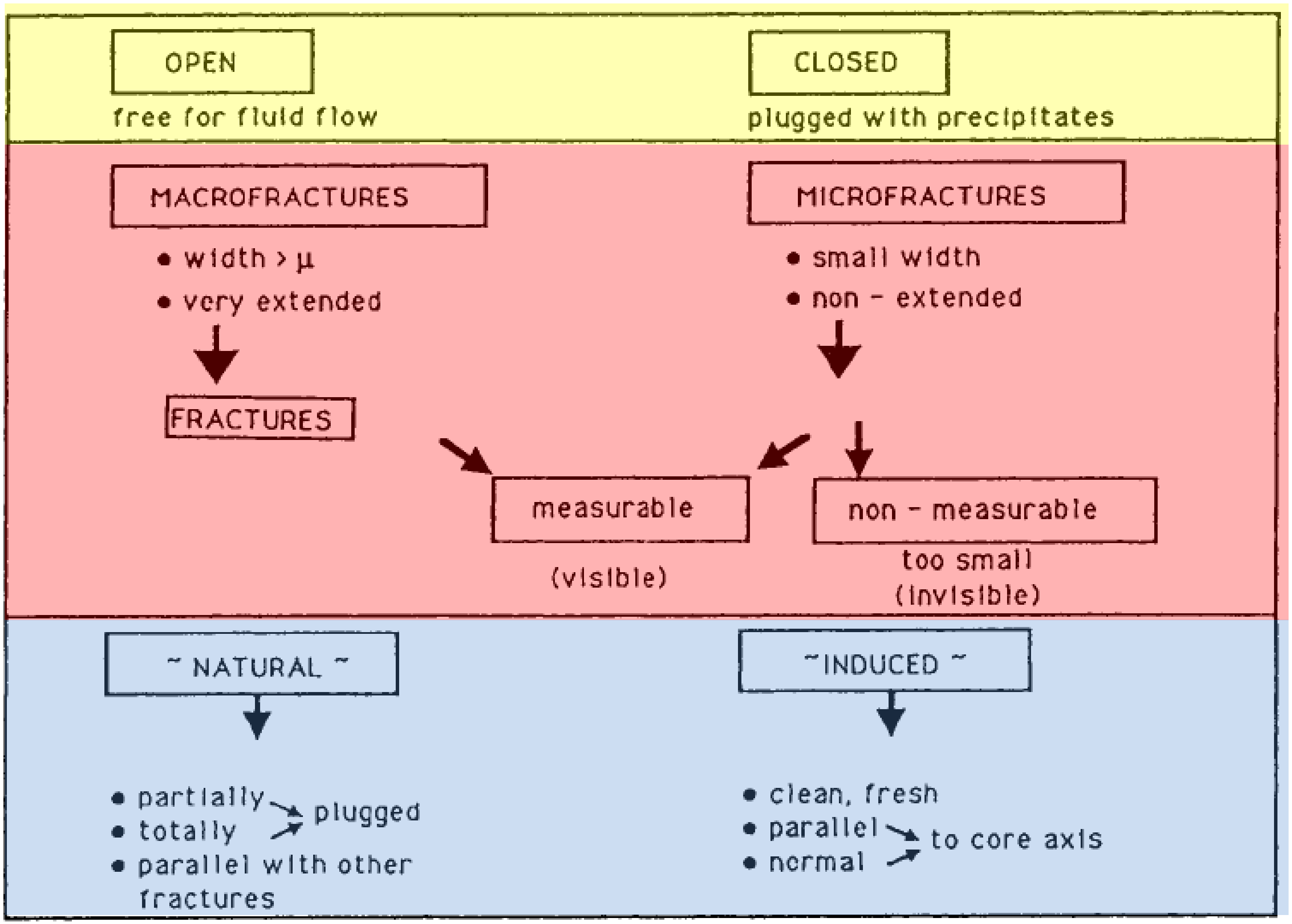
➢The material for observation is in general provided by:

- ❖Outcrops (whenever such is the case)
- ❖Cores obtained from drilling

➢The examination of fractures requires a certain definition and classification in relation to purely descriptive criteria, and with the relationship of fracturing to geological history.

- ❖open/closed fractures
- ❖macro/micro fractures
- ❖natural/induced fractures

Classification of Fractures Based on Descriptive Criteria



1- Open and closed fractures

Based on direct examination, there are two categories of fractures - open fractures and closed fractures; these depend mainly on circulating water and precipitation, which is capable of plugging the fractures with anhydrite, minerals, etc. On the other hand, fractures which are closed in surface conditions may often be open or partially open in reservoir conditions where pore pressure acts on fracture walls.

2.A- Macrofractures and microfractures

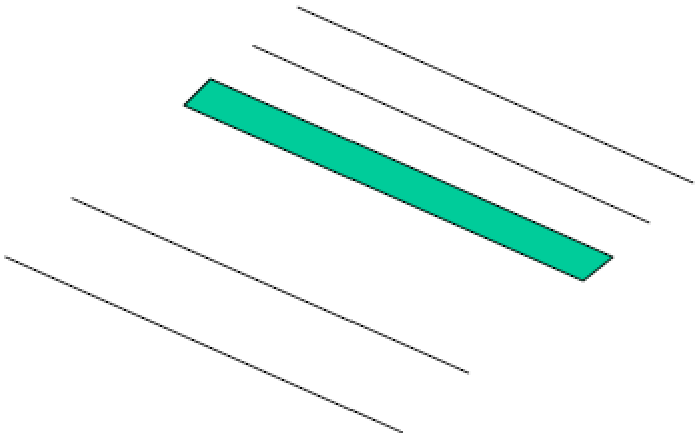
The difference between these two categories mainly concerns the dimensions of the fractures. In general, macrofracture corresponds to a fracture with a large width (over 100 microns) and considerable length, while microfracture applies to a fracture of limited length and width (sometimes microfractures form a continuous network which is hydrodynamically very similar to a porous medium). In the literature it is often possible to come across terms such as, macrofractures = fractures while microfractures = fissures

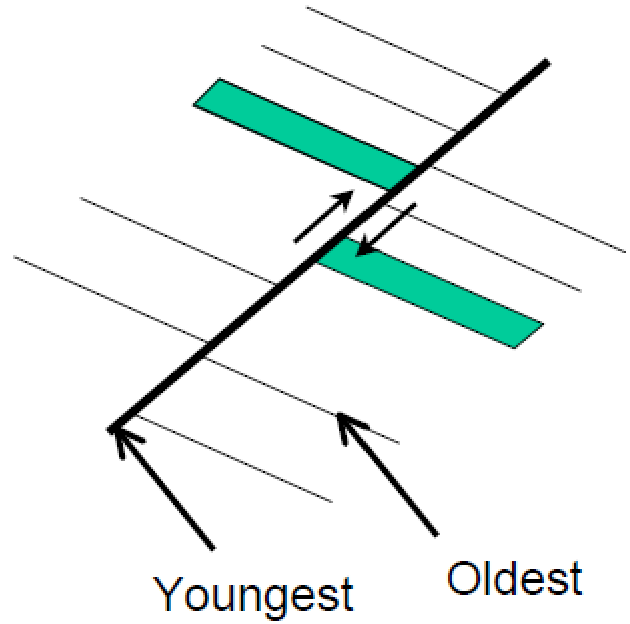
2.B-Measurable and non-measurable fractures

Measurable fractures are visible fractures which may be defined by width, length, orientation (dip and strike angle), while non-measurable fractures are only traces across the core which end within the core. Any reference to fracture density or intensity, etc., should refer to the visible natural fracture which indicates a certain dip angle and direction. In addition, there are broken cores with fractures which are not measurable either because they are too dense and irregular or because there is no criterion of evaluation.

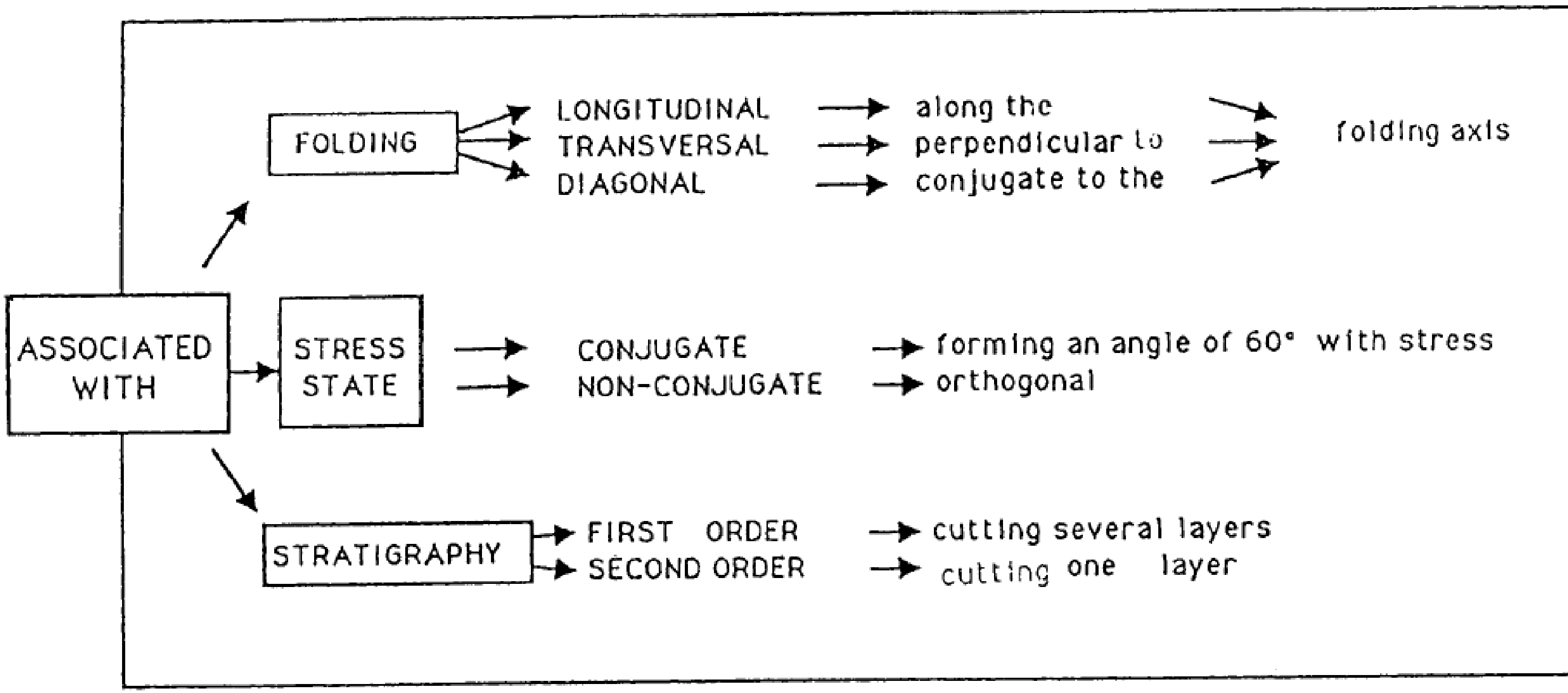
3-Natural fracture vs. induced fracture:

A natural fracture is any‘ break or crack, occurring in the rock, including those cracks which can be identified by the presence of mineralization. On the other hand, induced fractures are all those cracks which result during coring (such as breaks along the bedding plane) or from mishandling of cores.





Classification of Fractures Based on Geological Criteria



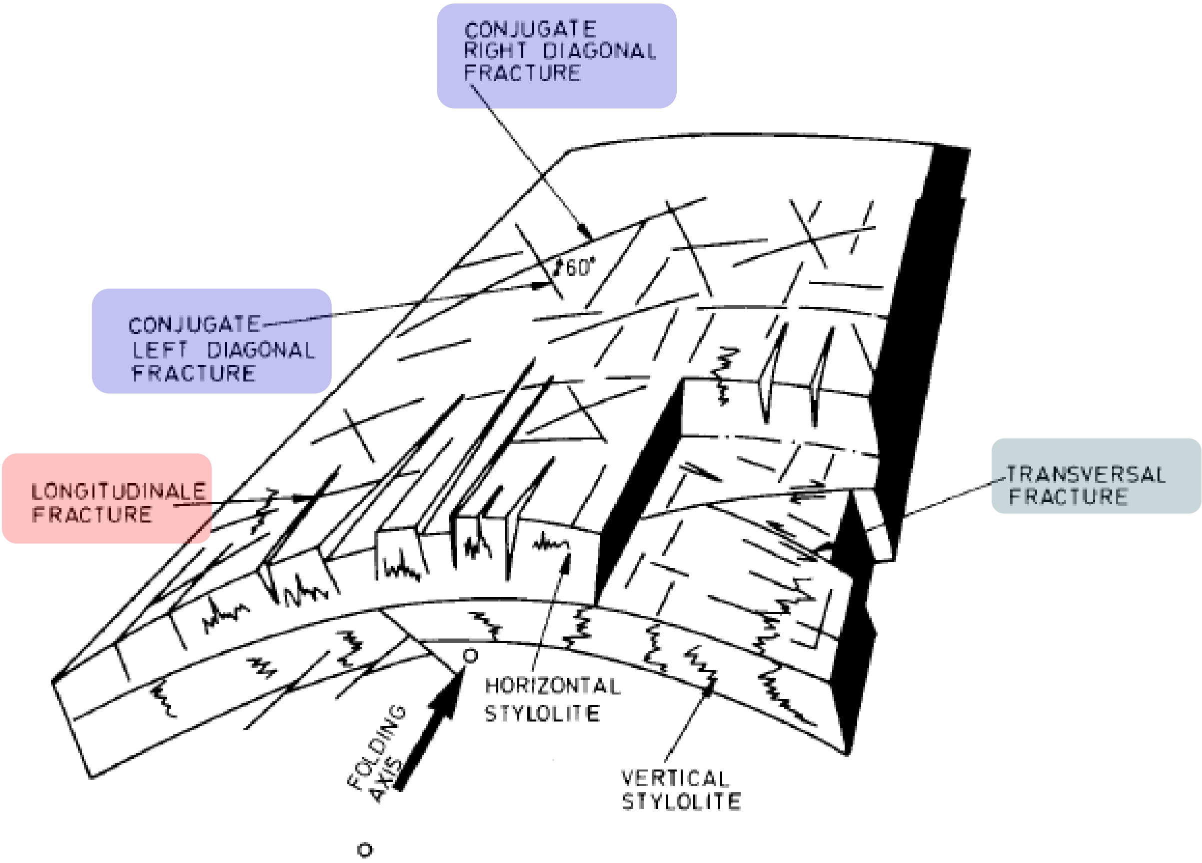
Fractures associated with folding

In general, fractures are related to the folding axis (figure 2.1) and therefore are denominated:

a. longitudinal fractures - along the folding axis

b. transversal fractures - perpendicular to the folding axis

c .diagonal fractures - in relation with the folding axis



2.1 – Various types of fractures generated by folding (courtesy of Leroy[2]).

Fractures and the stress state

If fractures are associated to one or more states of stress they are divided into two groups:

a. conjugate fractures

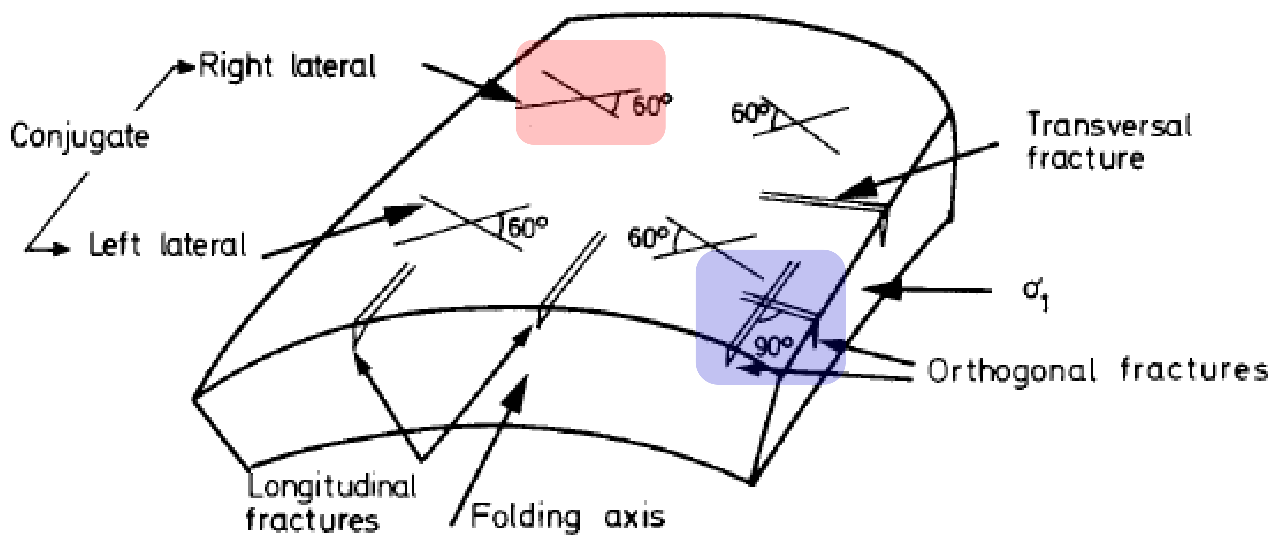
b. non-conjugate (orthogonal) fractures

where conjugate fractures are those which have been developed from a unique state of stress (figure 2.1).

The totality of the fractures could be associated with their direction and therefore:

a. The fracture system is formed by all fractures having the same mutually

b. the fracture network is the result of various fracture systems.



1.11 – Conjugate and orthogonal fractures referred to the folding axis.

Fractures associated with stratigraphy

The variations of dimensions and density of fractures depend on lithology and thickness of the layer in which the fractures are developed. The results obtained will divide the fractures into two categories:

a. first-order fractures

b. second-order fractures

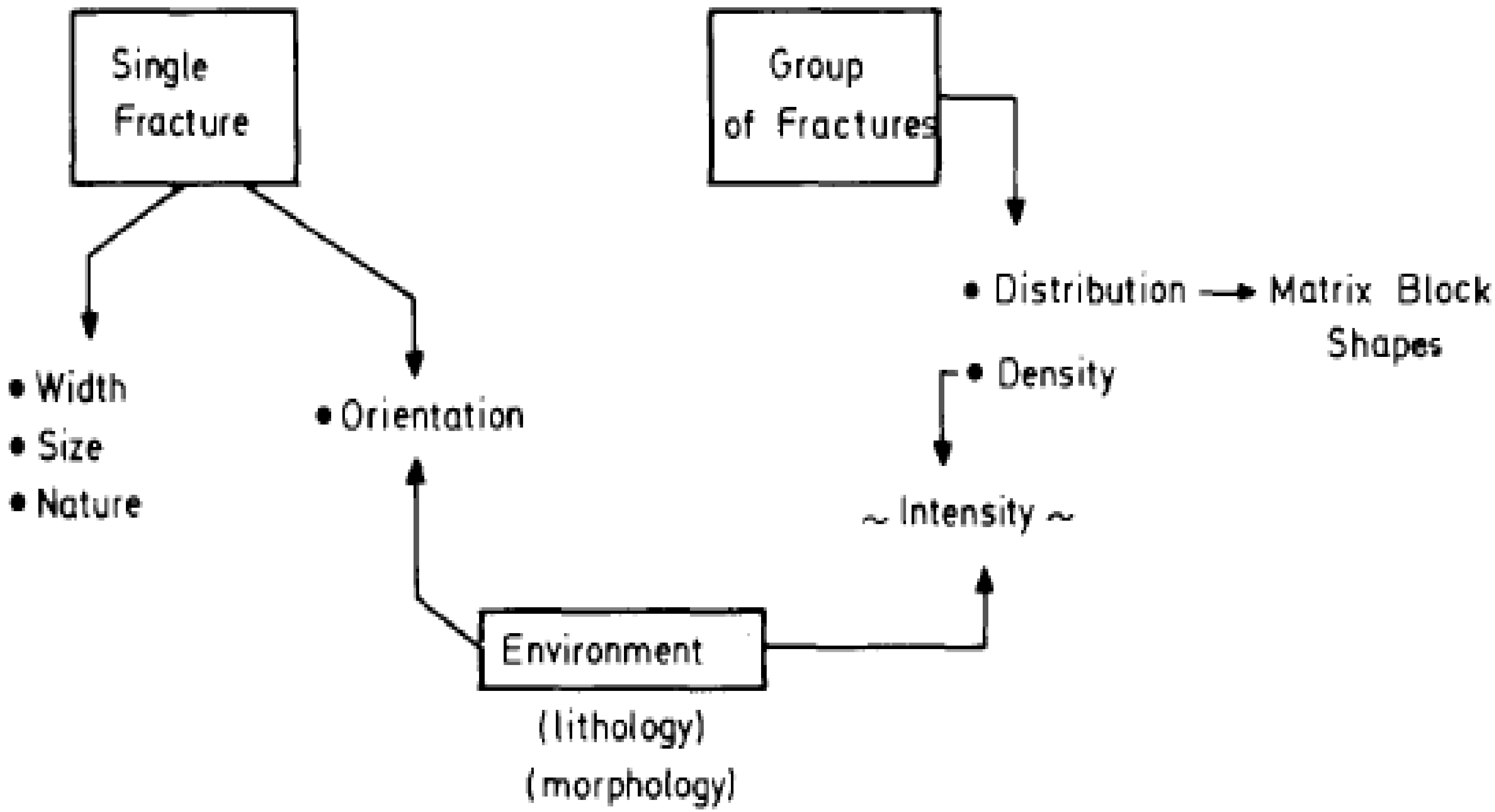
First-order fractures are those which cut through several layers of rock, while second-order fractures are limited to a single layer of rock.

BASIC PARAMETERS OF FRACTURES

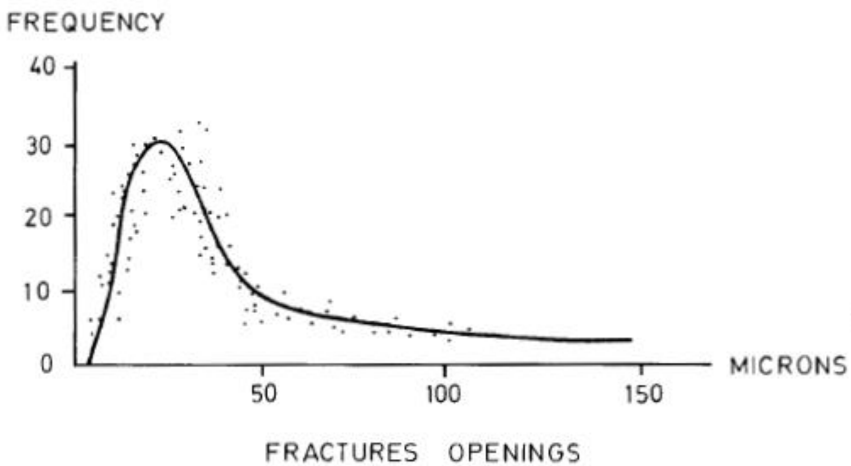
- **Single fracture** parameters refer to the intrinsic characteristics, such as opening (width), size and nature of fracture. If the single fracture is associated with the reservoir environment, another essential characteristic, the fracture orientation, will result.

- The **multi-fracture** parameters refer to the fracture arrangement (geometry) which further generates the bulk unit, called the matrix block. The number of fractures and their orientation are directly related to fracture distribution and density. When fracture density is related to lithology, another parameter of particular interest, called fracture intensity, is obtained.

BASIC PARAMETERS OF FRACTURES

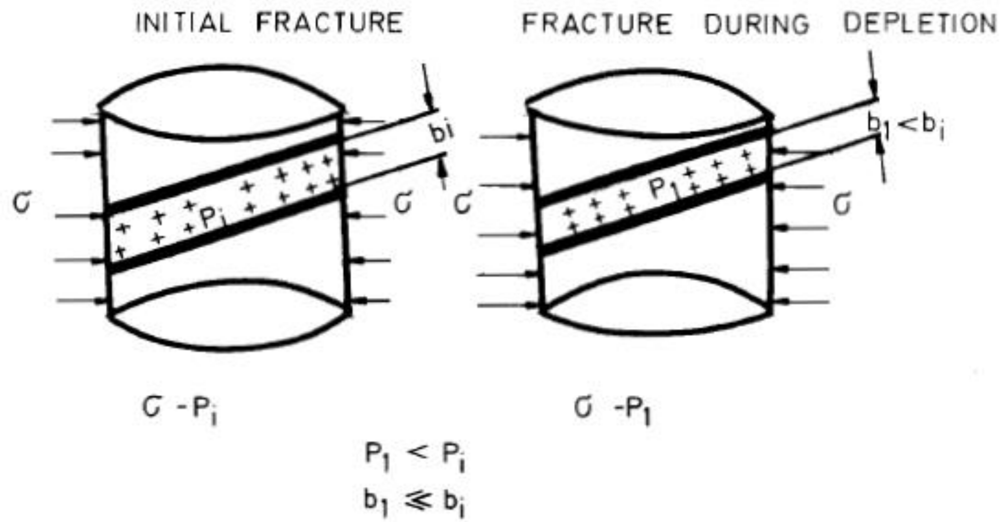


Fracture opening or fracture width is represented by the distance between the fracture walls. The width of the opening may depend (in reservoir conditions) on depth, pore pressure and type of rock.



2.2 – Statistical frequency curve of opening width

The fracture width varies between 10-200 microns, but statistics have shown that the most frequent range is between 10-40 microns.



2.3 – Reduction of fracture width as an effect of reservoir pressure depletion

In reservoir conditions where the confined pressure u (overburden pressure) remains constant, but the initial pore pressure Pi is reduced (during reservoir depletion) to P,, the width **b** will become smaller (figure 2.3), as effect of rock expansion.

The **nature of fractures** mainly concerns the state of fractures under observation with reference to opening, filling and wall characteristics, and is generally discussed in the following terms:

a. opening - open, joint, closed
b. filling - mineral, various minerals
c. closed by - homogeneous or diffused filling material
d. fracture walls - smooth, polished, creeping

Fracture orientation is the parameter which connects the single fracture to the environment. The fracturing plane can be defined (as in classic geological practice) by two angles, azimuth **δ** and dip angle **ω**.

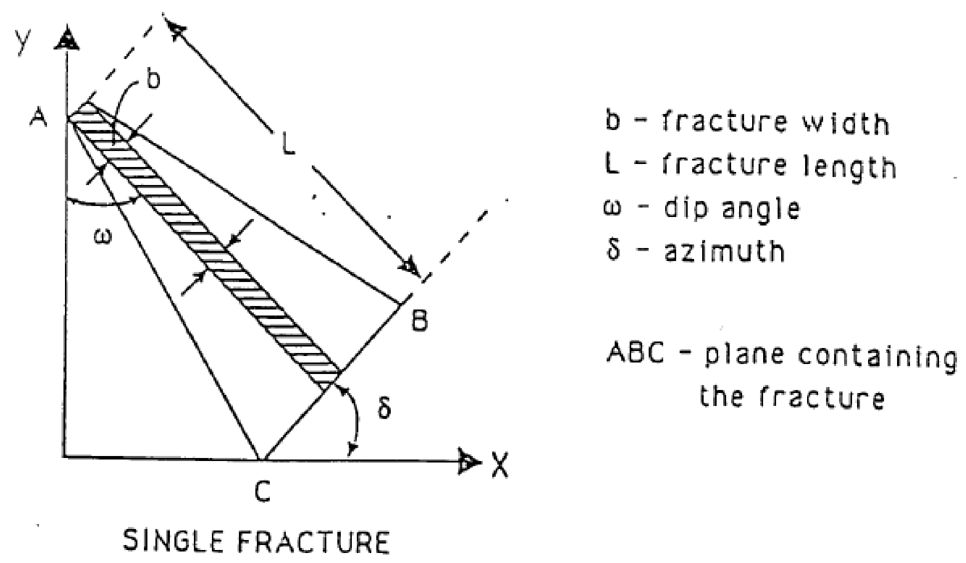
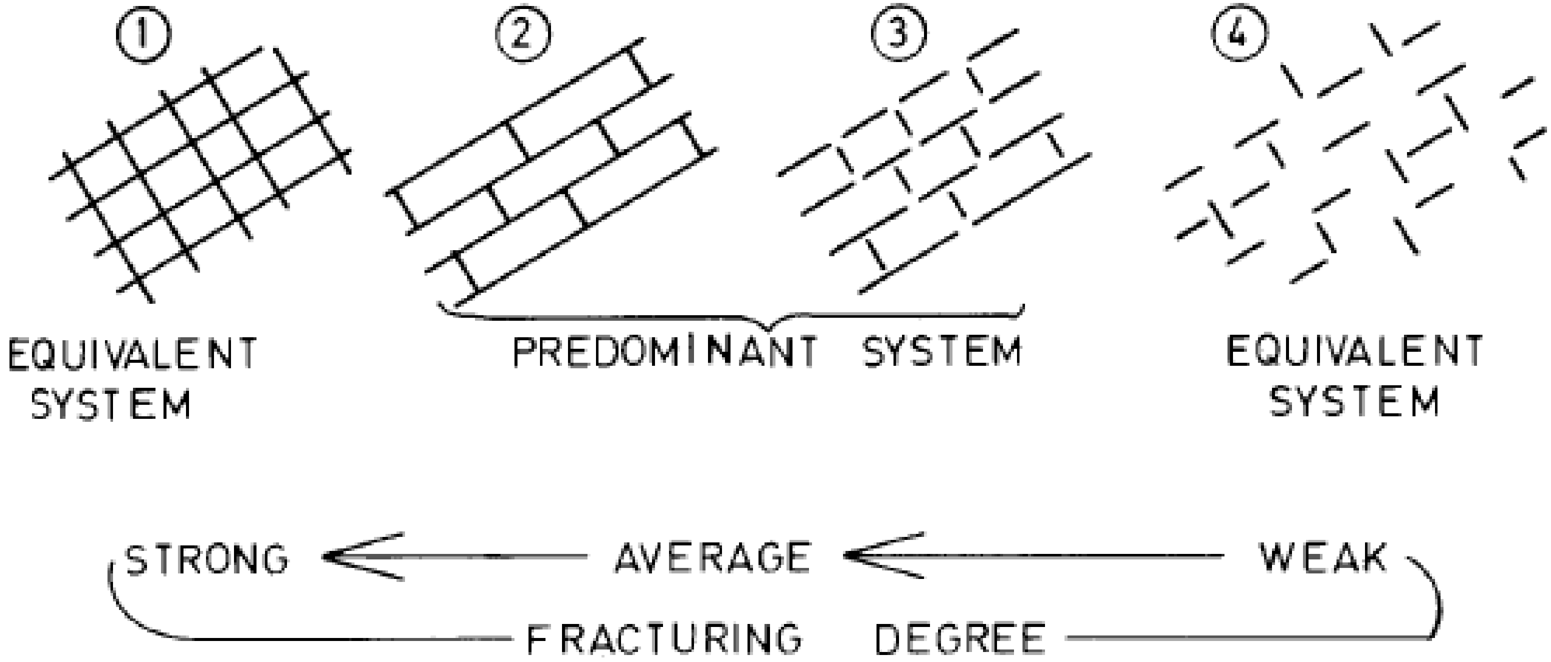


Fig. -20 Single fracture orientation.

- From comparison of the orientation of the various single fractures it follows that all parallel fractures belong to a **fracture system**.
- If more intercommunicating fracture systems are recognized in a reservoir, those systems will form the **fractured reservoir network**.

Group of fractures-Fracture distribution : In a fracture network which contains two or more fracture systems, each fracture system will generally be generated by a certain state of stress. Pairs of conjugate fractures which have been generated by the same stress state are an exception. Fracture distribution is then expressed by :

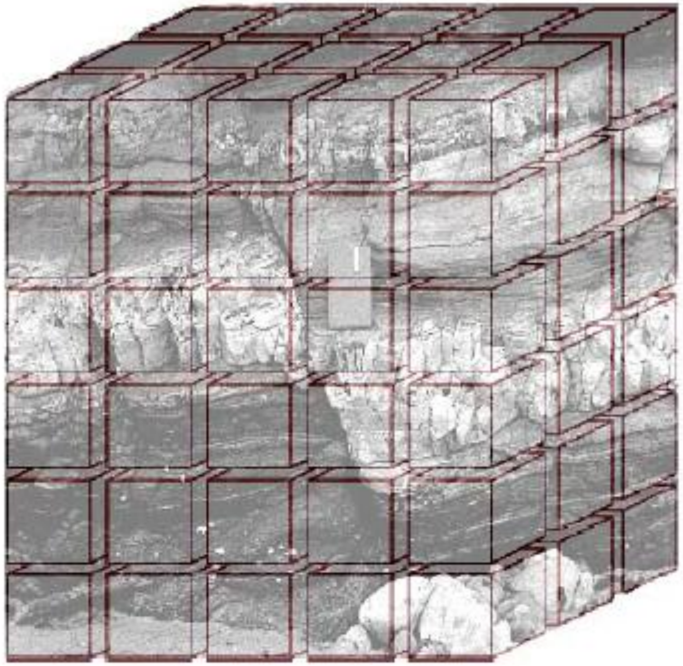


2.5 – Various combinations of orthogonal fracture systems and the qualitative evaluation of the degree of fracturing (Ruhland[3])

In case **1** the fracture densities of the two systems are equivalent and are continuously intercommunicating. This corresponds to a strong degree of fracturing.

•The fractures which cut the reservoir rock in various directions, delineate a bulk unit referred to as the **matrix block** unit or simply the matrix block.

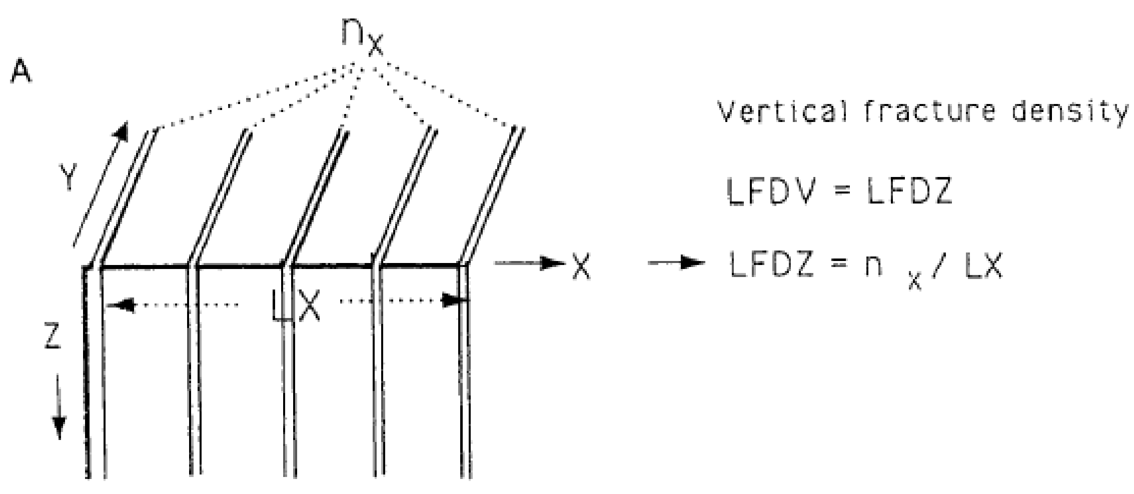
•The shape of the matrix block is irregular, but for practical work the block units are reduced to simplified geometrical volumes, such as cubes or as elongated or flat parallelepipeds.



Fracture Density

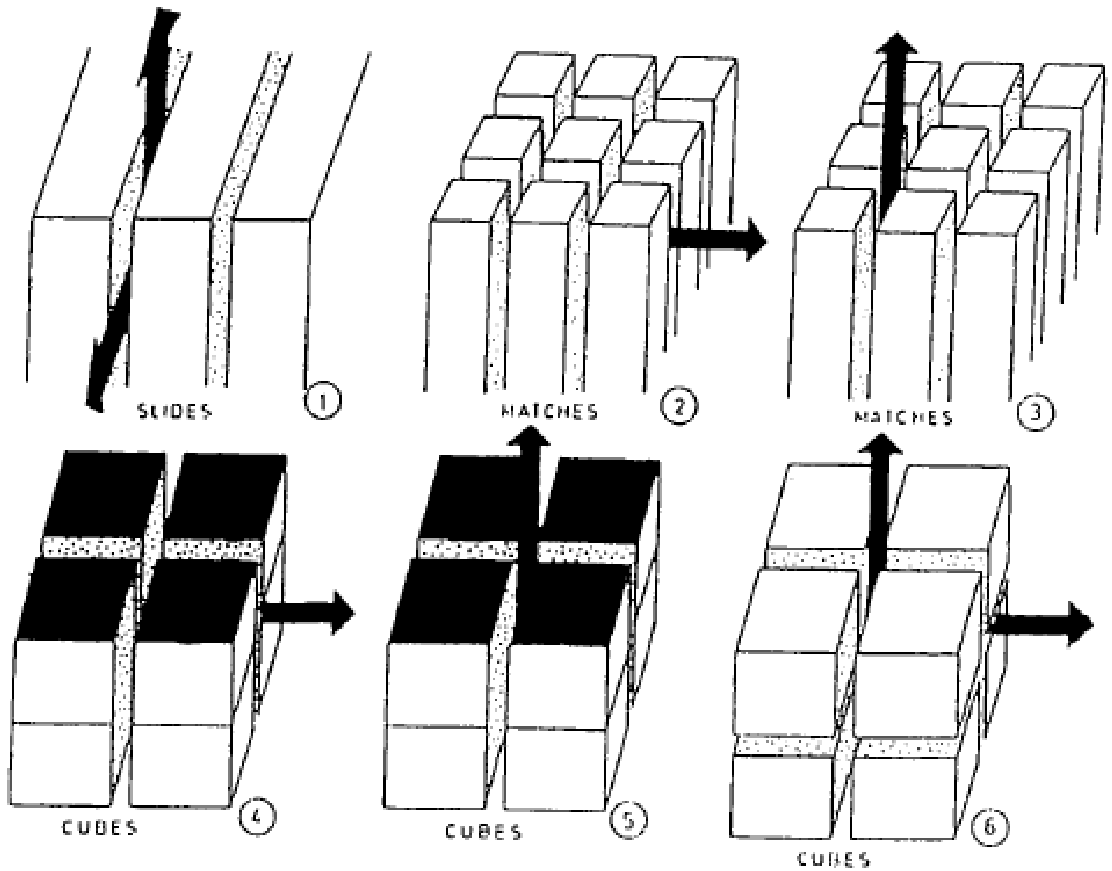
Fracture density expresses the degree of rock fracturing through various relative ratios. If the ratio refers to the bulk volume the fracture density is called volumetric fracture density.

$$[LFD]_x = \text{number of fractures / length (along a certain direction)} = n_f / L_x$$



Linear fracture density; the ratio between the number (n) of fractures intersecting a straight line (normal on flowing direction) and the length of the straight line:

Based on this approach, some idealized block shapes (Reiss, 1966) resulting from various distributions of fractures in an orthogonal fracture network gives several fracture density values.



Shape :	No :	L.F.D.
Slides	1	1/a
Matches	2	1/a
Matches	3	2/a
Cube	4	2/a
Cube	5	1/a
Cube	6	2/a

The blocks can be structured as elongated slides (No.1) or matches offering only one permeability direction (Nos. 2 and 3), and finally, cubes having one flowing direction (Nos. 4 and 5) or two flowing directions (No. 6)

Simplified geometrical matrix blocks. (From Reiss, 1966.)

Fracture Intensity

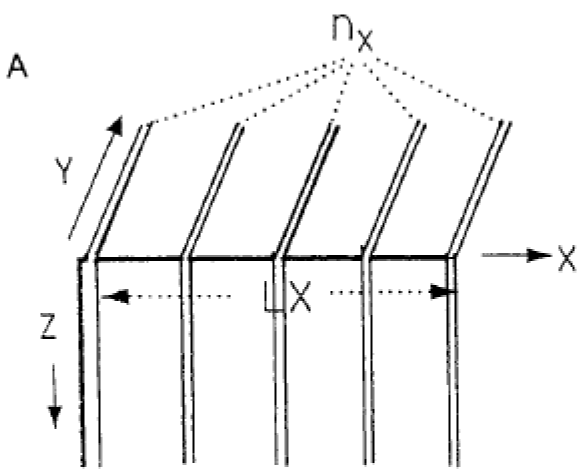
When a single-layer productive zone is small, in order to discern the tectonic effect vs. lithology it is necessary to refer all fractures (vertical and subvertical) to the single-layer pay. If the pay is larger and fractures are vertical (or sub-vertical) and horizontal (or sub-horizontal), the notion of fracture intensity can be introduced as the ratio between the vertical and horizontal fracture densities:

$$FINT = (LFDV / LFDH) =$$
$$= \text{Linear fracture density (vertical)} / \text{Linear fracture density (horizontal)}$$

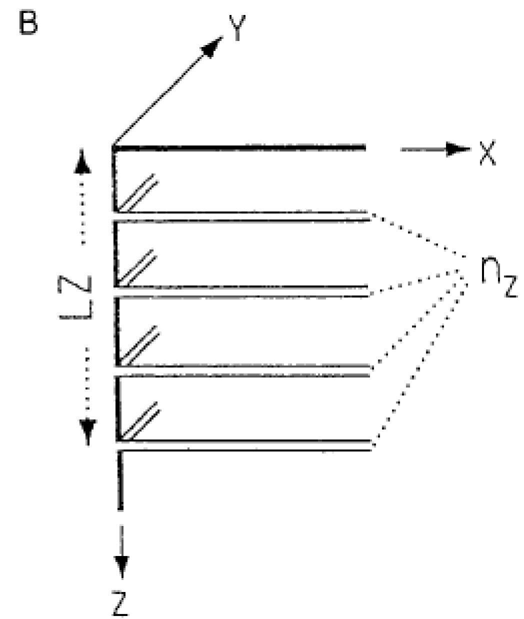
where vertical and horizontal fractures may in certain cases be interpreted also as fractures that are normal and parallel to the stratification.

Fracture Intensity

In an orthogonal fracture network oriented along the three orthogonal axes, the fracture intensity will be the ratio of vertical fracture density to the horizontal fracture density.



Vertical fracture density = $LFDV = LFDZ = n_x / L_x$



Horizontal fracture density = $LFDH = LFDX = n_z / L_z$

$$FINT = LFDZ / LFDX = (n_x / L_x) / (n_z / L_z)$$

Fracture Intensity

The matrix block dimensions:

$$Z_{bl} = L_z / n_z = 1 / LFDX = 1 / LFDH$$

$$X_{bl} = L_x / n_x = 1 / LFDZ = 1 / LFDV$$

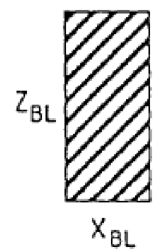
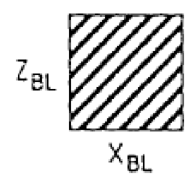
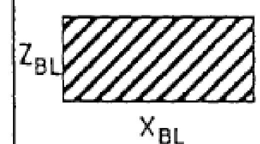
or expressed as a ratio of matrix dimensions:

$$FINT = LFDZ / LFDH = (1 / X_{bl}) / (1 / Z_{bl}) = Z_{bl} / X_{bl}$$

FINT values show a relationship between vertical and horizontal fracture distributions and also give an indication about the matrix block shape as presented on Table 7-III: vertically elongated ("match" shape), horizontally elongated ("slab" shape), and/or cubes.

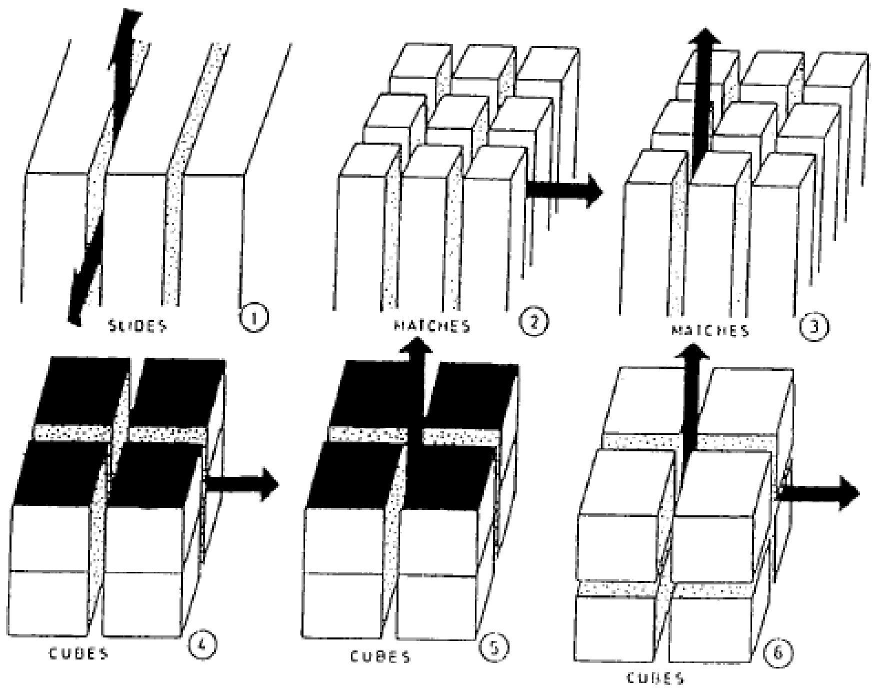
TABLE 7-11I

Relationship between vertical/ horizontal fracture density and matrix block shape.

CASE 1	CASE 2	CASE 3
Vertical Density > Horizontal Density	Vertical Density = Horizontal Density	Vertical Density < Horizontal Density
LFDZ > LFDX	LFDZ = LFDX	LFDZ < LFDX
FINT > 1	FINT = 1	FINT < 1
$\frac{Z_{BL}}{X_{BL}} > 1$	$\frac{Z_{BL}}{X_{BL}} = 1$	$\frac{Z_{BL}}{X_{BL}} < 1$
 VERTICALLY ELONGATED **MATCH**	 **CUBE**	 ELONGATED **SLAB**

Simplified Correlation and Procedures

A complex fracture-matrix structure geometry could be modified to a simplified geometrical shape of matrix block (parallelepipeds, cubes, spheres, etc.), which is evidently surrounded by uniform fractures.



Shape and Block Magnitude

The dimension of a matrix is directly related to fracture density and intensity because an increase of fracture density in one direction represents a reduction of block dimensions along the same direction. The block shape vs. fracture intensity is expressed through a comprehensive diagram (Fig. 24) where the basic relationship in two directions is expressed as follows:

$$LFDV = LFDZ = 1/X_{bl}$$

Block extension

$$LFDH = LFDX = 1/Z_{bl}$$

Block height

$$FINT = LFDZ / LFDX = Z_{bl} / X_{bl}$$

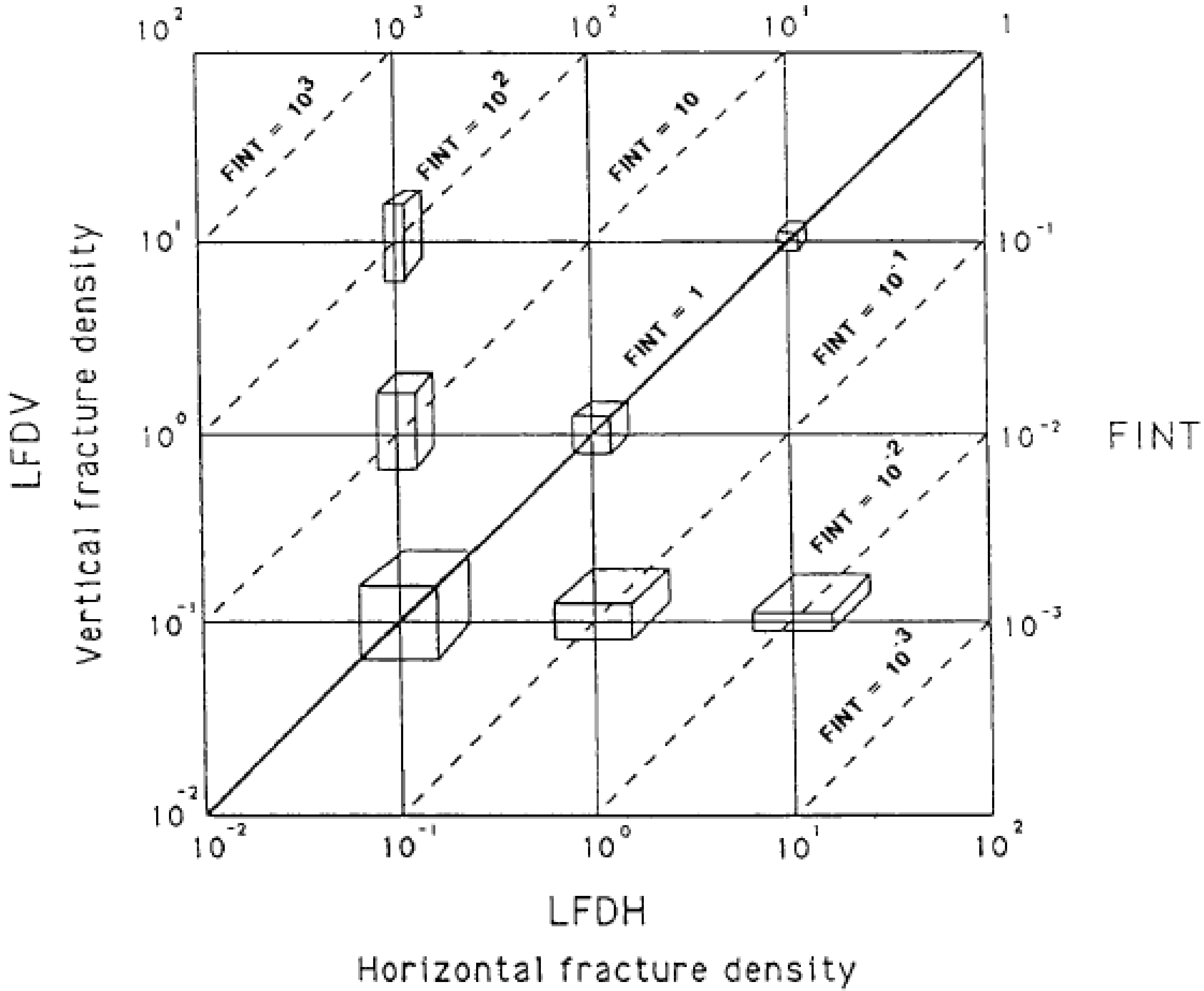


Fig. 24- Block of matrix resulting from the intersection of an orthogonal fracture system.

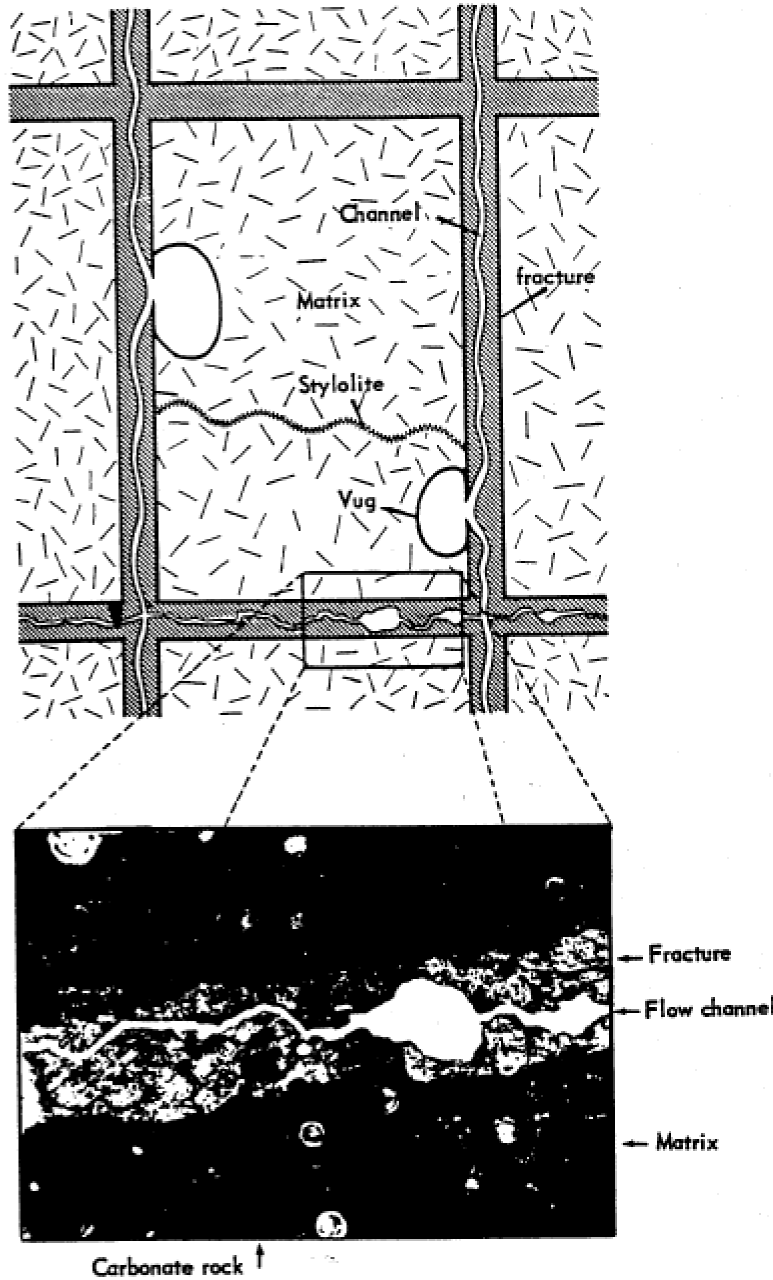
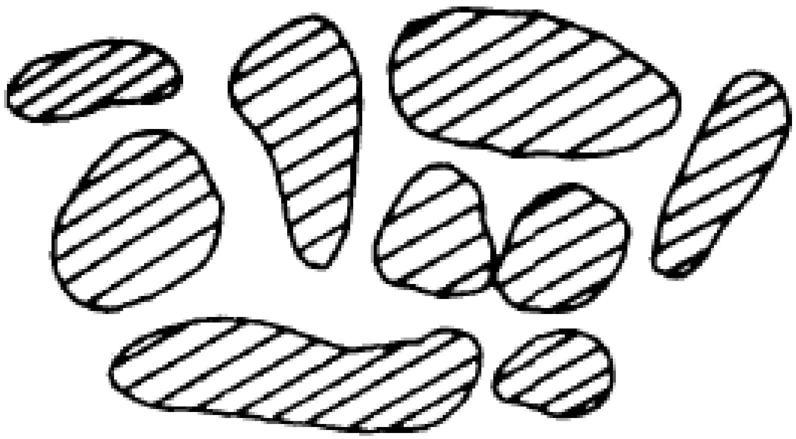
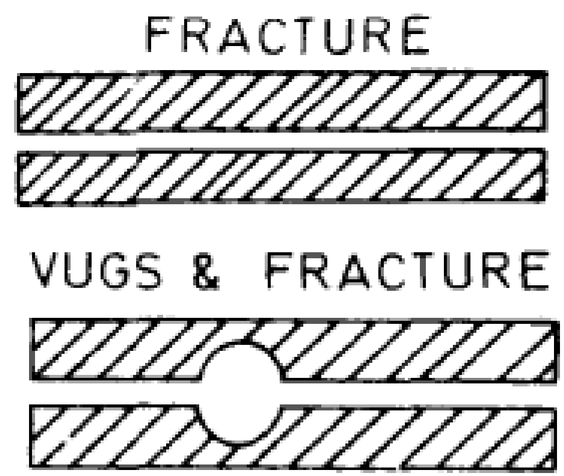


Fig. 1. Idealized matrix element and illustration of secondary porosity within a fracture.

Primary Porosity & Secondary Porosity



4.1 – Consolidated grain void space (matrix).



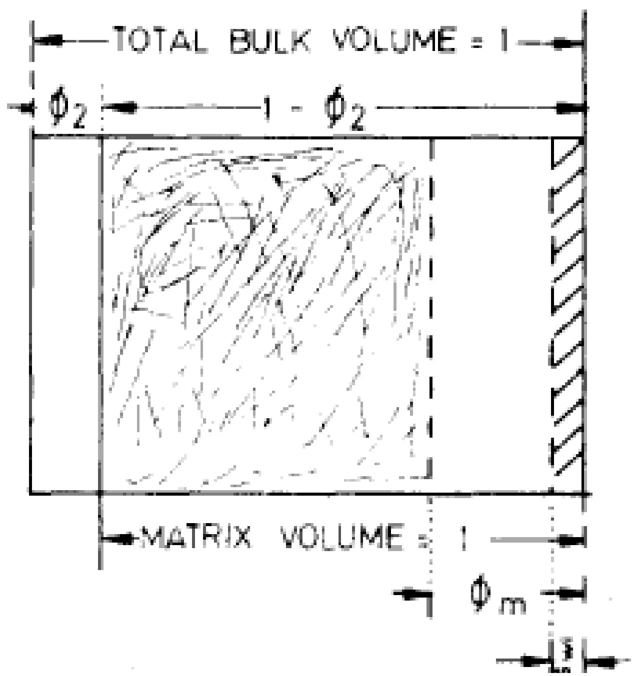
4.2 – Simplified representation of vugs and fracture void space.

Secondary porosity is generally found in compact, brittle rock of relatively low intergranular porosity, such as compact limestones, shales, shaly sandstones, siltstones, schists, etc. Secondary porosity is normally caused by rock fracturing, jointing and dissolution by circulating water.

Definition of Double Porosity

$$\Phi_t = \Phi_1 + \Phi_2 \quad (4.1)$$

$$\left.\begin{aligned}\Phi_1 &= \text{matrix void volume/total bulk volume}\\ \Phi_2 &= \text{fracture void volume/total bulk volume}\end{aligned}\right\} \quad (4.2)$$



$$\Phi_m = \frac{\text{volume voids of the matrix}}{\text{matrix bulk volume}}$$

4.3 – Schematization of double-porosity.

$$\Phi_2 \approx \Phi_f \quad (4.3)$$

$$\Phi_2 \approx \Phi_f \tag{4.3}$$

In this case the primary porosity, as a function of matrix porosity, is expressed by,

$$\Phi_1 = (1-\Phi_2)\Phi_m \tag{4.4}$$

and the effective primary porosity, containing the oil phase, is,

$$\Phi_{1,eff} = (1-\Phi_2)\Phi_m(1-S_{wi}) \tag{4.4'}$$

Relationship between the fracture parameters

A typical illustration of a fractured reservoir is shown in Fig. A.3.1. It represents one of the many networks that can result from geological evaluation. The parameters such as matrix size and fracture width are not well defined by direct observation, but a good estimate of permeability can be made from well tests. Fracture permeability and porosity (k_f and ϕ_f), matrix size a and fracture width b are related, and our objective will be to derive the relationships between these parameters for different simple geometric schemes encountered in practice, shown in Fig. A.3.2.:

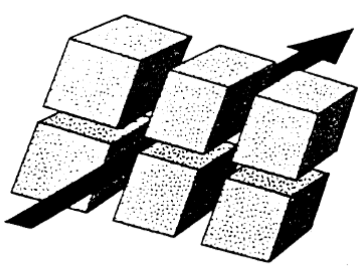


Fig. A.3.1. Simplified fracture network.

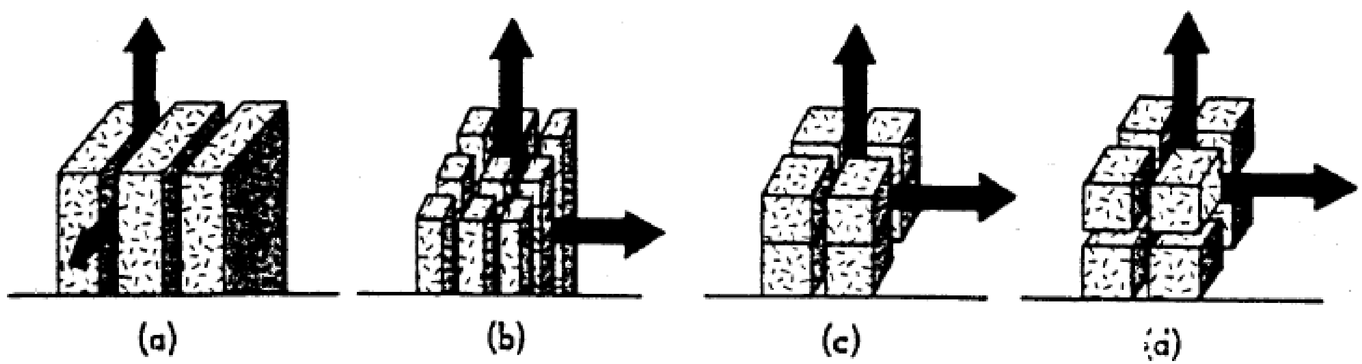


Fig. A.3.2. Typical fracture networks – Arrows indicate possible directions of flow.

The rectangular element with sides a_1, a_2, a_3 is shown in Fig. A.3.3. The fracture porosity is given by:

$$\phi_f = \frac{(a_1 + b)(a_2 + b)(a_3 + b) - a_1 a_2 a_3}{(a_1 + b)(a_2 + b)(a_3 + b)} \tag{A.3.1}$$

$$\simeq b\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right) \qquad \text{since } b \ll a_1, a_2, a_3 \tag{A.3.2}$$

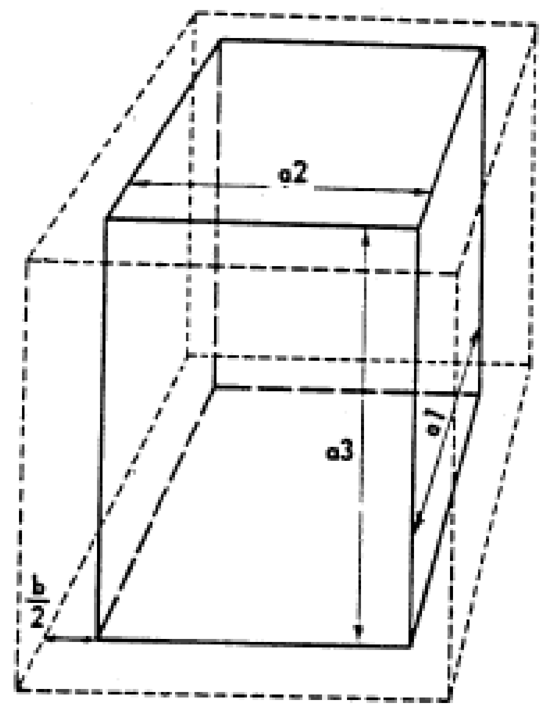


Fig. A.3.3. Definition of fracture porosity.

For the four schemes in Fig. A.3.2.:

Sheets .... $\phi_f = b/a$
Match-sticks .... $\phi_f = 2\,b/a$
Cubes with two effective fracture planes .... $\phi_f = 2\,b/a$
Cubes .... $\phi_f = 3\,b/a$

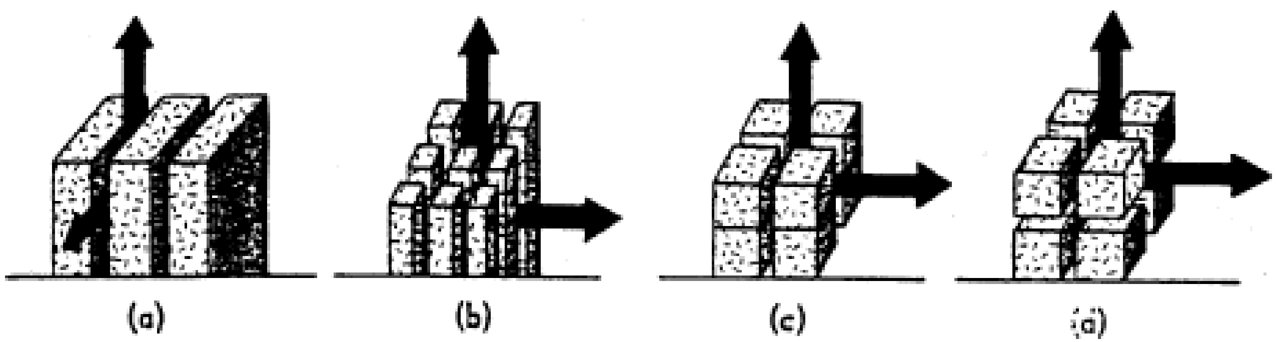


Fig. A.3.2. Typical fracture networks – Arrows indicate possible directions of flow.

(a) "Sheets" of matrix separated by parallel fracture planes – matrix size a is represented by the width of the sheets.

(b) "Match-sticks" separated by two orthogonal fracture planes – matrix size a is represented by the side of the square cross section.

(c) "Cubes" separated by three orthogonal fracture planes: two cases are illustrated in Fig. A.3.2. In case c the horizontal fracture is replaced by a thin stratification, a case frequently encountered in nature. In case d the three fracture systems are of equal importance. Matrix size a is represented by the side of a cubic element.

Fracture porosity from direct measurements.

A direct measurement of fracture porosity requires : (1) fracture width [b] from cores, (2) fracture density [LFD] from core examination, so that in idealized condition as shown in Fig. 27, will result

Porosity = Void Fracture Surface / Total Surface

$$\phi_f = n_f * b * X_{bl} / X_{bl} * Z_{bl} = b * LFD = n_f * b / Z_{bl} \qquad (8)$$

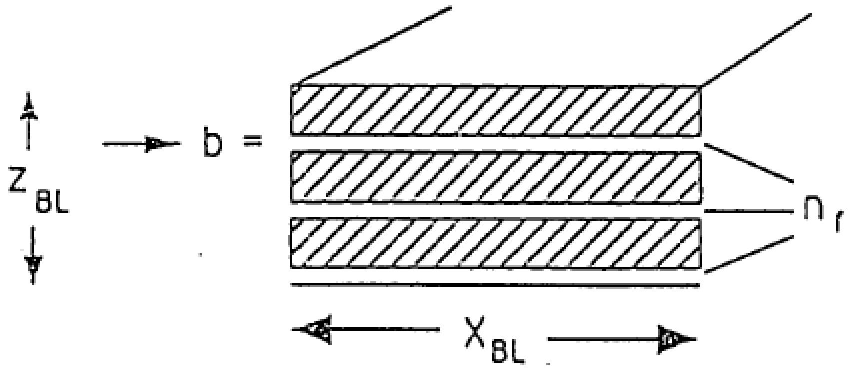


Fig. -27 Idealized matrix / fracture unit .

Fracture density.

Fracture density expresses the frequency of fractures along a given direction and reciprocally the extension of the matrix delimited by fractures encountered .

The intersection of several orthogonal fracture systems results in single matrix blocks of different sizes and shapes.

In fact, along the direction *X* the linear fracture density (LFD) is

$$[LFD]_x = \text{number of fracture / length (along a certain direction)} = n_f / L_x \qquad (1)$$

Storage capacity of matrix and fractures.

In transient flowing conditions the term which plays an important role is not the single porosity (ϕ_m or ϕ_f) but rather the storage capacity expressed by the association of porosity and compressibility. In this case the product ϕ*C becomes:

ϕ_m* C_m ====>> for the matrix storage capacity
ϕ_f * C_f ====>> " " fracture " "

Order of magnitude of fracture porosity

In general, fracture porosity is very small compared with matrix porosity . As a general rule it could be stated that fracture porosity is below 1% and in only very exceptional cases may reach a value of 1% . However , in very tight rocks having a primary porosity ϕ_m <10% and a very extended network of macrofractures and microfractures ,a fracture porosity between 1% and 3-4% may occur. As consequence, for reservoirs with high matrix porosity and thus very small fracture porosity it is practically impossible by conventional logging tools to evaluate the fracture porosity. Representative fracture (Ruhland, 1975) porosity values can be obtained only from observations and direct measurements on cores .

Fracture Permeability

Permeability depends on the direction of flow which we shall assume to be parallel to the fracture planes as shown in Fig. A.3.4.

Poiseuille's Equation gives the rate q_1 in terms of the pressure drop ΔP for laminar flow along a single fracture whose length and cross section are L and Lb respectively:

$$q_1 = \frac{b^3 l}{12\,\mu} \frac{\Delta P}{L} \tag{A.3.3}$$

For n fractures, flow across a section A can be written:

$$q_n = n\,\frac{b^3 l}{12\,\mu} \frac{\Delta P}{L} \tag{A.3.4}$$

Darcy's law would be written:

$$q = \frac{A k_f}{\mu} \frac{\Delta P}{L} \tag{A.3.5}$$

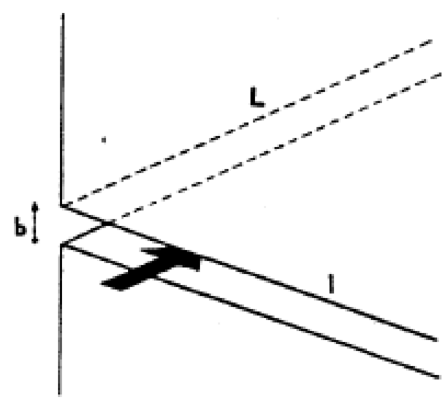


Fig. A.3.4. Definition of fracture permeability.

so that for $q = q_n$:

$$k_f = \frac{n}{A}\frac{b^3 l}{12} = f_s \frac{b^3}{12} \qquad \text{(A.3.6)}$$

where $f_s = nl/A$ represents the total fracture length per unit cross section.

For our four simplified models:

(a) Sheets $f_s = 1/a$.

(b) Match-sticks $f_s = 1/a$ or $2/a$ depending on the direction of flow: the smaller value corresponds to flow perpendicular to the axis of the matches, the larger value represents flow parallel to the axis.

(c) Cubes with two effective fracture planes: as for match-sticks.

(d) Cubes $f_s = 2/a$ for flow parallel to a fracture plane.

Note that for flow in intermediate directions, f_s would take on different values: it represents the anisotropy implied by the idealized representations of Fig. A.3.2.

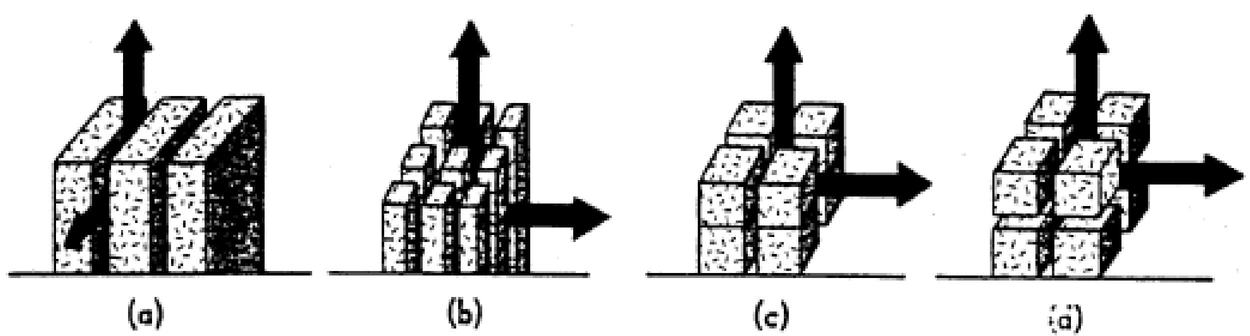
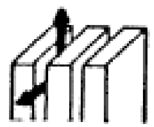
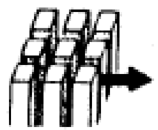
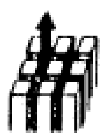
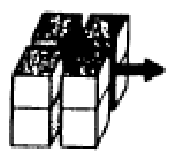
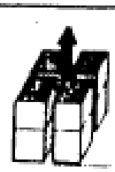
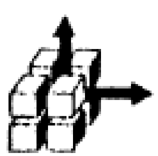


Fig. A.3.2. Typical fracture networks – Arrows indicate possible directions of flow.

TABLE A.3.1

Fracture Structure	Fracture network	Dimensionless				Practical Units (1)			
		f_s	ϕ_f	$k_f(\phi_f, a)$	$k_f(\phi_f, b)$	f_s (cm^{-1})	ϕ_f (%)	$k_f(\phi_f, a)$ (darcy)	$k_f(\phi_f, b)$ (darcy)
Sheets		$\frac{1}{a}$	$\frac{b}{a}$	$\frac{1}{12} a^2 \phi_f^3$	$\frac{1}{12} b^2 \phi_f$	$\frac{1}{a}$	$\frac{1}{100} \frac{b}{a}$	$8{,}33\, a^2 \phi_f^3$ (A. 3.7)	$8{,}33 . 10^{-4} b^2 \phi_f$ (A. 3.8)
Match-sticks		$\frac{1}{a}$	$\frac{2b}{a}$	$\frac{1}{96} a^2 \phi_f^3$	$\frac{1}{24} b^2 \phi_f$	$\frac{1}{a}$	$\frac{1}{100} \frac{2b}{a}$	$1{,}04\, a^2 \phi_f^3$ (A. 3.9)	$4{,}16 \times 10^{-4} b^2 \phi_f$ (A. 3.10).
		$\frac{2}{a}$	$\frac{2b}{a}$	$\frac{1}{48} a^2 \phi_f^3$	$\frac{1}{12} b^2 \phi_f$	$\frac{2}{a}$	$\frac{1}{100} \frac{2b}{a}$	$2{,}08\, a^2 \phi_f^3$ (A. 3.11)	$8{,}33 \times 10^{-4} b^2 \phi_f$ (A. 3.8)
Cubes with one fracture plane impermeable		$\frac{1}{a}$	$\frac{2b}{a}$	$\frac{1}{96} a^2 \phi_f^3$	$\frac{1}{12} b^2 \phi_f$	$\frac{1}{a}$	$\frac{1}{100} \frac{2b}{a}$	$1{,}04\, a^2 \phi_f^3$ (A. 3.9)	$4{,}16 \times 10^{-4} b^2 \phi_f$ (A. 3.8)
		$\frac{2}{a}$	$\frac{2b}{a}$	$\frac{1}{48} a^2 \phi_f^3$	$\frac{1}{12} b^2 \phi_f$	$\frac{2}{a}$	$\frac{1}{100} \frac{2b}{a}$	$2{,}08\, a^2 \phi_f^3$ (A. 3.11)	$8{,}33 \times 10^{-4} b^2 \phi_f$ (A. 3.8)
Cubes		$\frac{2}{a}$	$\frac{3b}{a}$	$\frac{1}{162} a^2 \phi_f^3$	$\frac{1}{18} b^2 \phi_f$	$\frac{2}{a}$	$\frac{1}{100} \frac{3b}{a}$	$0{,}62\, a^2 \phi_f^3$ (A. 3.12)	$5{,}55 \times 10^{-4} b^2 \phi_f$ (A. 3.8)

(1) a in cm, b in microns ($1\ \mu = 10^{-4}$ cm), ϕ_f in percent, k_f in darcys

The analogy and similarity between the two laws makes it possible to calculate the relationship between permeability of porous material and its equivalent average pore diameter. If a porous sample is composed of a bundle of capillary tubes of equal radius, the permeability of such a bundle of capillaries is given by solving equation (1) for K, i.e. :

$$K = \frac{r^2}{8} \qquad (3)$$

Permeability K calculated from equation (3) is defined as (cc/sec) (poise)/bar/cm. When using the API definition of permeability which is given by (cc/sec) (poise)/(atmosphere/cm) the right hand side of equation (3) should be multiplied by 1.01325. This gives the following relationship :

$$r = 8.886 \times 10^{-2} (K)^{\frac{1}{2}} \qquad (4)$$

where r is in microns and K is in millidarcys.

The corresponding permeability in terms of the spacing between parallel planes is :

$$T = 10.811 \times 10^{-2} (K)^{\frac{1}{2}} \qquad (7)$$

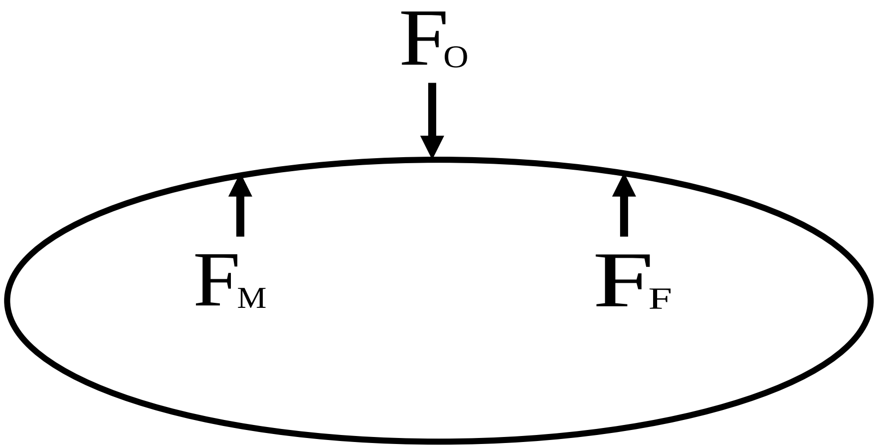
Compressibility of Fractured Reservoirs
Review of single porosity compressibility

Defn: the change ΔV per unit volume V for an applied pressure ΔP

$$C = -\frac{1}{V}\frac{\Delta V}{\Delta P}$$

- **Property of a certain rock volume**
  - **bulk volume** V_B
  - **pore volume** V_p
  - **fluid volume** V_f

FORMATION COMPRESSIBILITY



1. Under static conditions, downward overburden force must be balanced by upward forces of the matrix and fluid in pores

2. Thus:

$$F_o = F_m + F_f$$

AND

$$p_o = p_m + p$$

3. Pressure Gradients, Normal Reservoirs:

$dp_o/dZ = 1.0$ psia/ft

$dp/dZ = 0.465$ psia/ft

4. As fluids are produced from reservoir, fluid pressure (p) usually decreases while overburden is constant, and:
   (a) force on matrix increases (“net compaction pressure”, $p_m=p_o-p$)
   (b) bulk volume decreases, and
   (c) pore volume decreases.

The compressibility of rock in a conventional reservoir refers to bulk, rock and pore volume,

$$\left.\begin{aligned} V_B &= V_r + V_p \\ V_p &= V_B \times \Phi \end{aligned}\right\} \quad (4.57)$$

and is expressed through a basic definition[20],

$$\left.\begin{aligned} &-\ \text{Bulk compressibility } C_B = \frac{1}{V_B} \cdot (dV_B/dP)_{\sigma=\text{constant}} \\ &-\ \text{Matrix compressibility } C_r = \frac{1}{V_r} \cdot (dV_r/dP)_{\sigma=\text{constant}} \\ &-\ \text{Pore compressibility } C_P = \frac{1}{V_p} \cdot (dV_p/dP)_{\sigma=\text{constant}} \end{aligned}\right\} \quad (4.58)$$

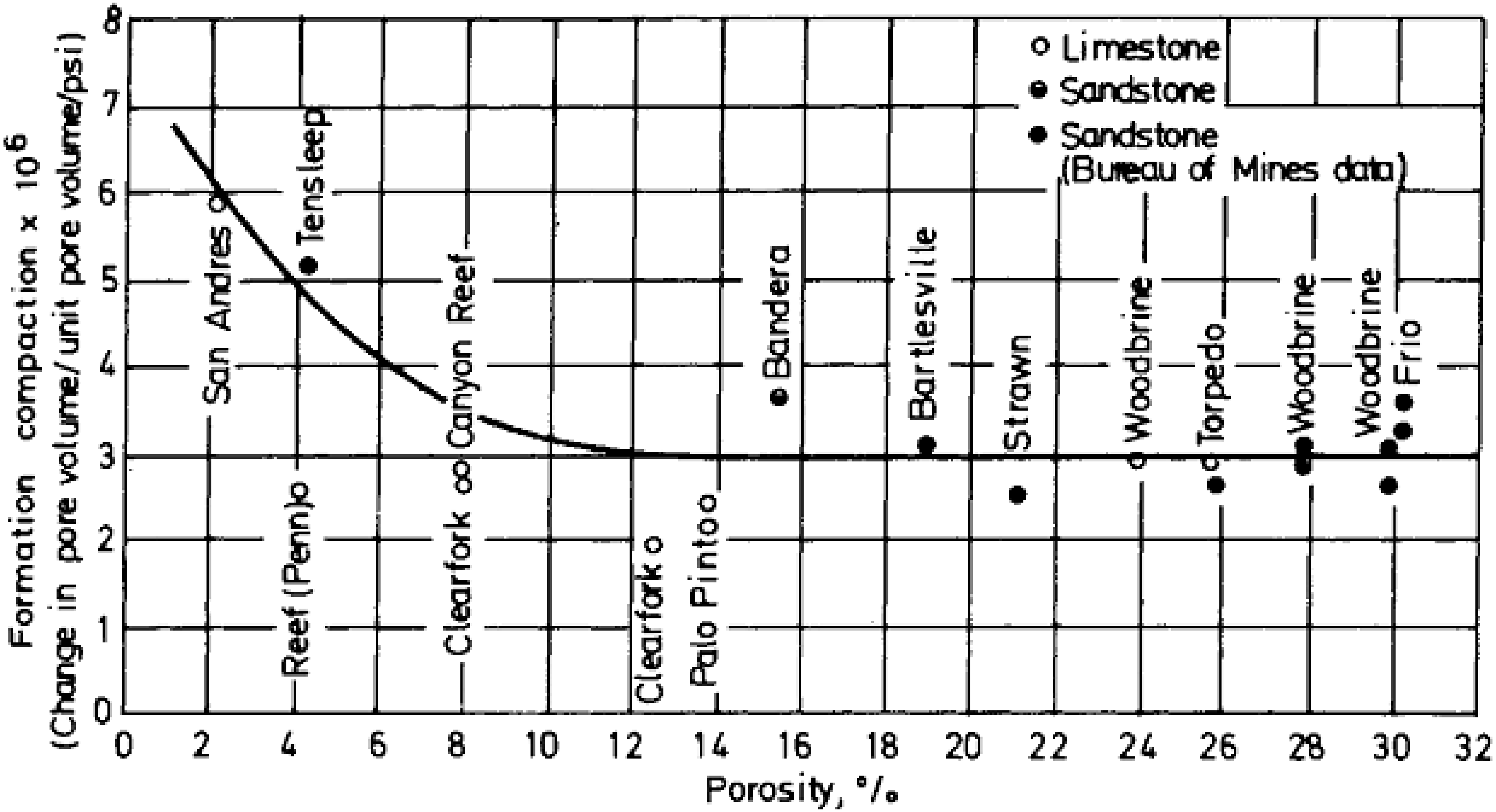
Based on equations 4.57 and 4.58,

$$\frac{dV_P}{V_p} \cong \frac{1}{\Phi} \times \frac{dV_B}{V_B} \tag{4.59}$$

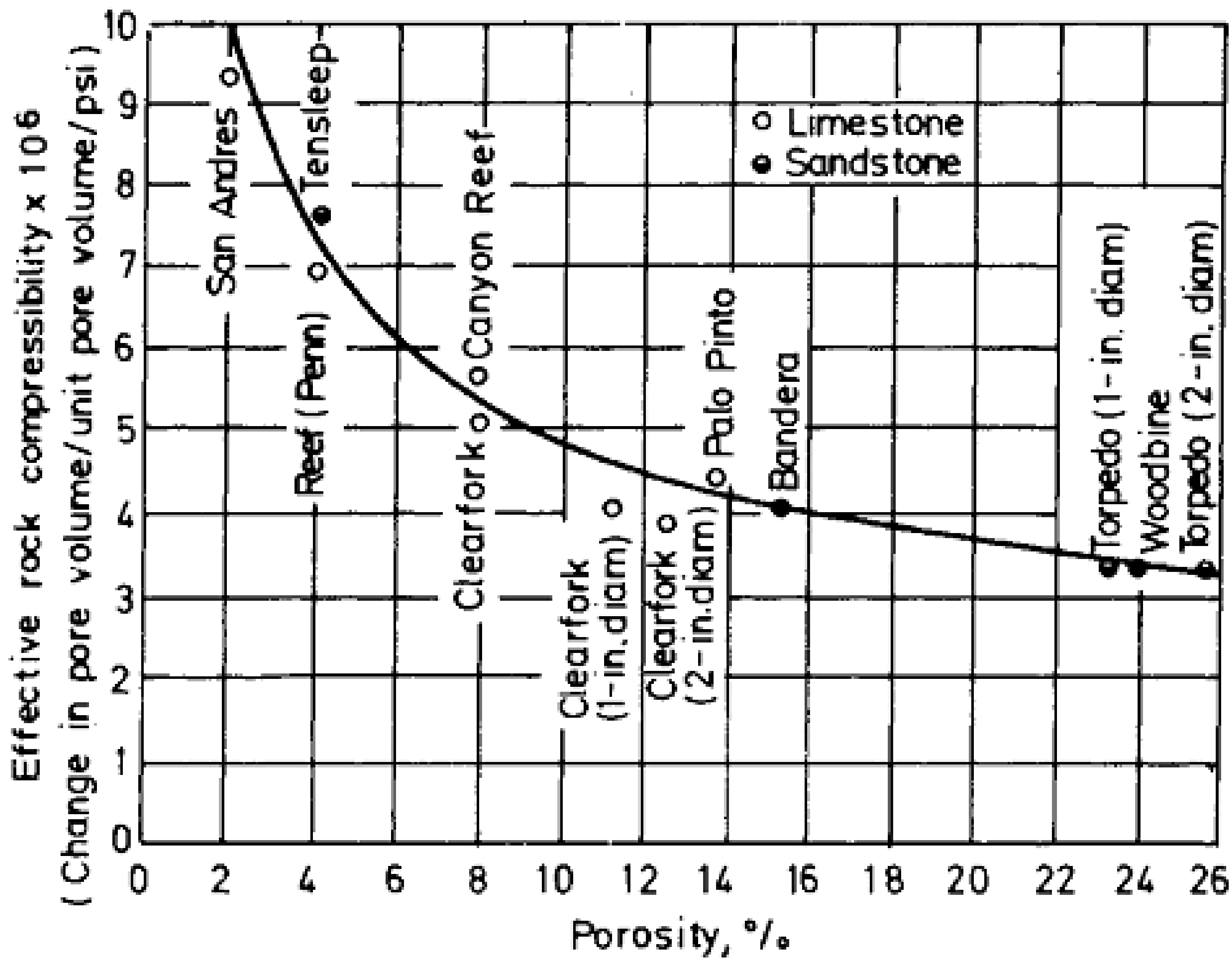
$$C_p = \frac{1}{\Phi} C_B \tag{4.60}$$

$$C_r = \frac{1}{1-\Phi} C_B \tag{4.60'}$$

$$C_p = \frac{1-\Phi}{\Phi} C_r \tag{4.60''}$$



4.29 – Formation compaction vs. porosity (Hall[25], courtesy AIME).



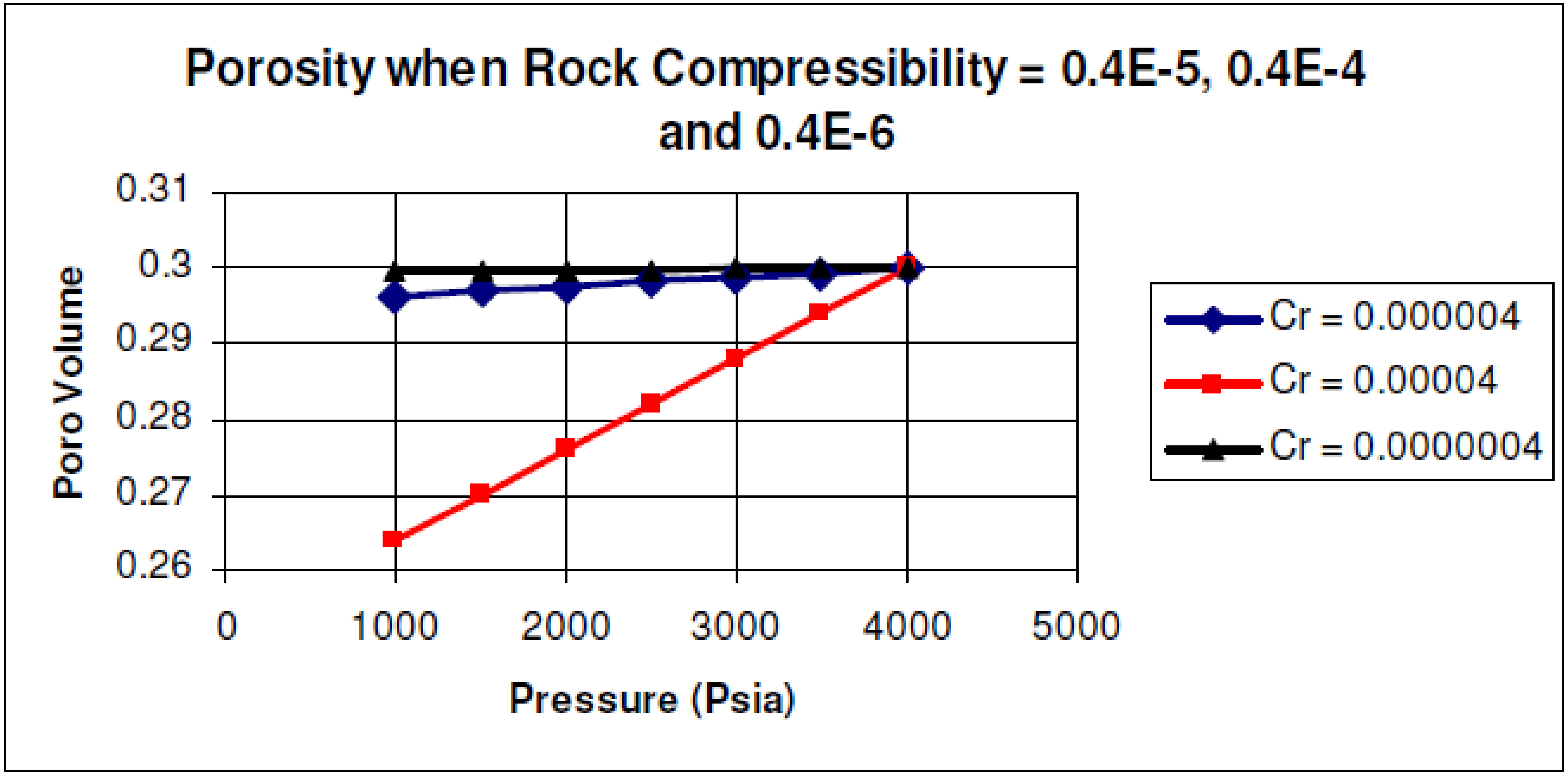
4.30 – Effective rock compressibility vs. porosity (Hall[25], courtesy AIME).

Bulk compressibility $C_B = 1/V_B \cdot (dV_B/dP)_{\sigma=constant}$

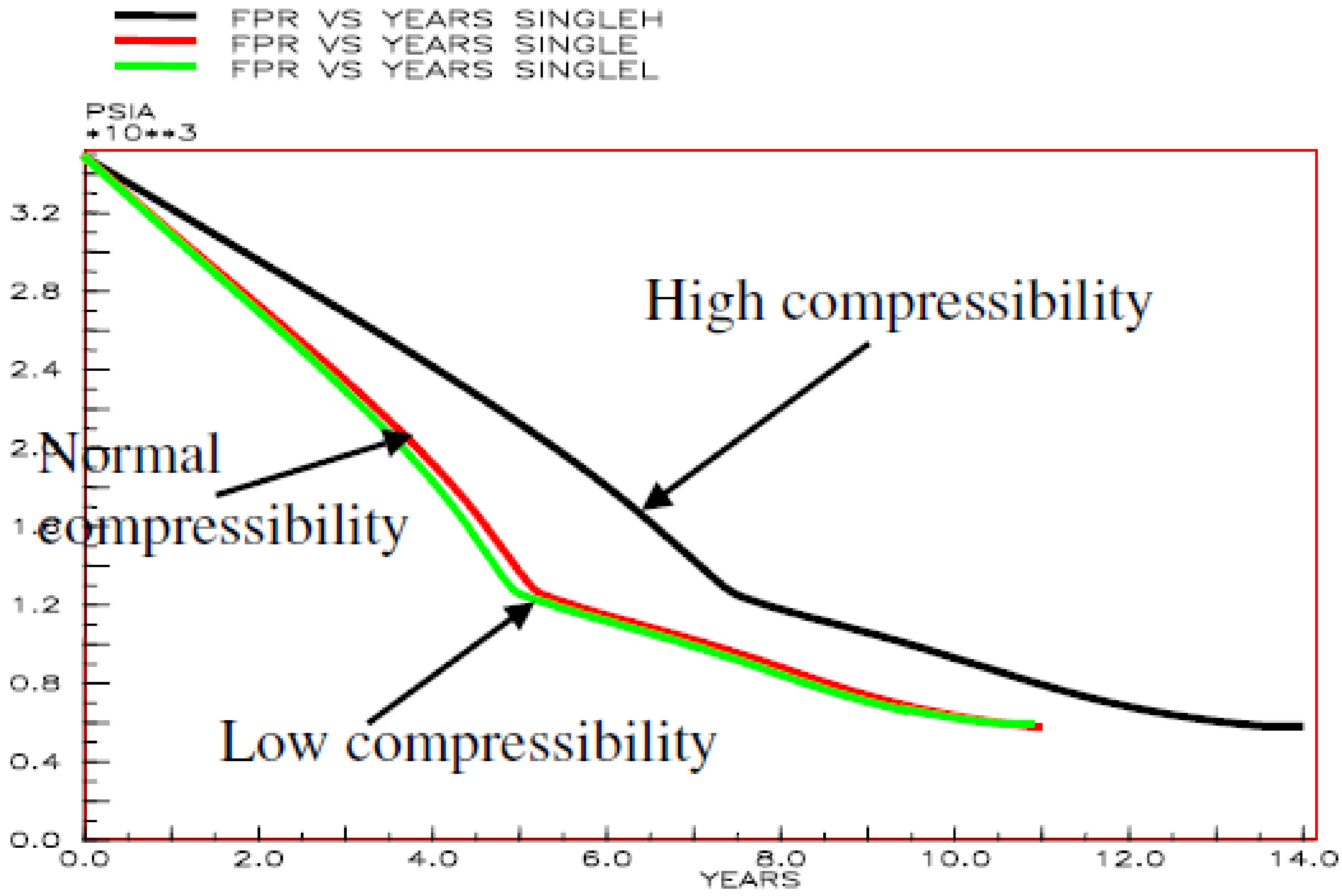
Matrix compressibility $C_r = 1/V_r \cdot (dV_r/dP)_{\sigma=constant}$

Pore compressibility $C_p = 1/V_p \cdot (dV_p/dP)_{\sigma=constant}$

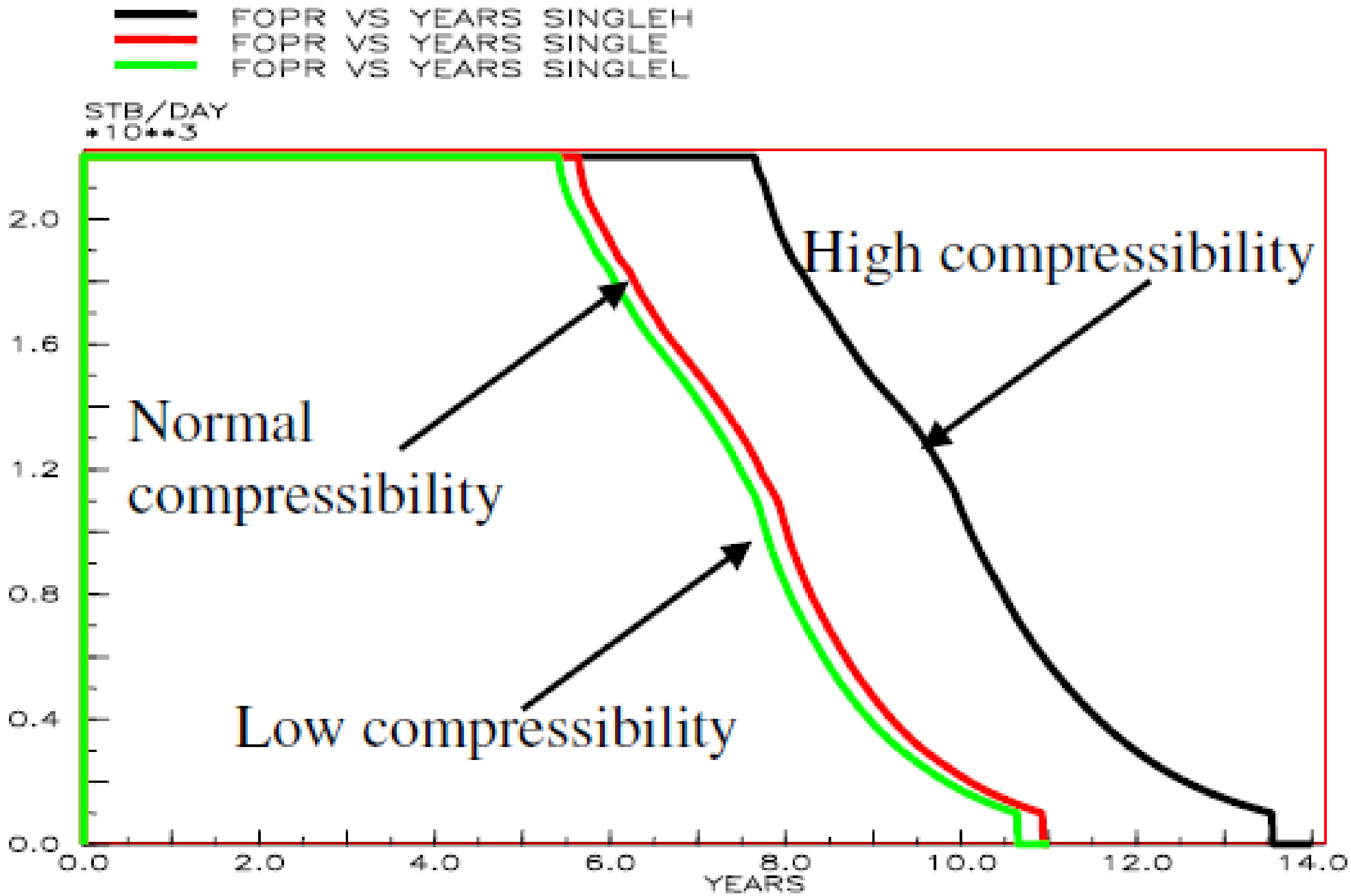
$$\phi = \phi^o\left[1 + c_r(p - p^o)\right]$$



Field Pressure



Oil Production Rate



Fractured and Triple Reservoirs Compressibility

In addition to primary porosity of matrix (m) must consider secondary porosity of fractures (f) and triple porosity of vugs (v)

$$C_r = C_m + \Phi_f C_f + \Phi_v C_v$$

Experimental results have shown $C_f = C_v \approx 3C_m$

Relationship between K_f and C_{pf}

We shall base this analysis on a system of (side a) separated by fractures (width b).

The fracture permeability and porosity are:

$$k_f = f_s \frac{b^3}{12} \qquad \text{where} \quad f_s = \frac{1}{a} \tag{A.3.14}$$

$$\phi_f = \frac{b}{a} \tag{A.3.15}$$

The fracture compressibility is given by:

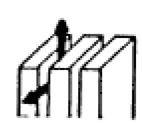
$$C_{pf} = -\frac{1}{\phi_f}\Delta\phi_f = -\frac{1}{b}\frac{\Delta b}{\Delta P} \tag{A.3.16}$$

$$b = b_i(1 - C_{pf}\,\Delta P) \tag{A.3.17}$$

Note that this fracture pore compressibility is defined with respect to the fracture volume ϕ_f.

These equations can be combined to give the following relationship between permeability and compressibility:

$$\frac{k_f}{k_{fi}} = (1 - C_{pf}\,\Delta P)^3 \tag{A.3.18}$$

Fracture Structure	Fracture network	f_s	ϕ_f	$k_f(\phi_f, a)$	$k_f(\phi_f, b)$
		← Dimensionless			→
Sheets		$\frac{1}{a}$	$\frac{b}{a}$	$\frac{1}{12}a^2\phi_f^3$	$\frac{1}{12}b^2\phi_f$