# Naturally Fractured Reservoirs

Physical Properties of Fractures and Matrix

Shahab Gerami, Fall 2010

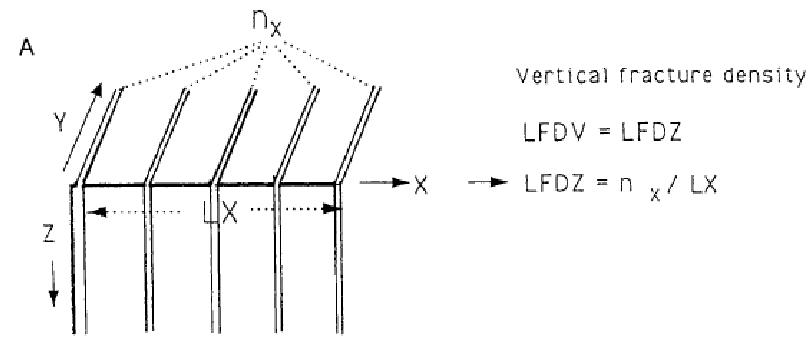
# Outline

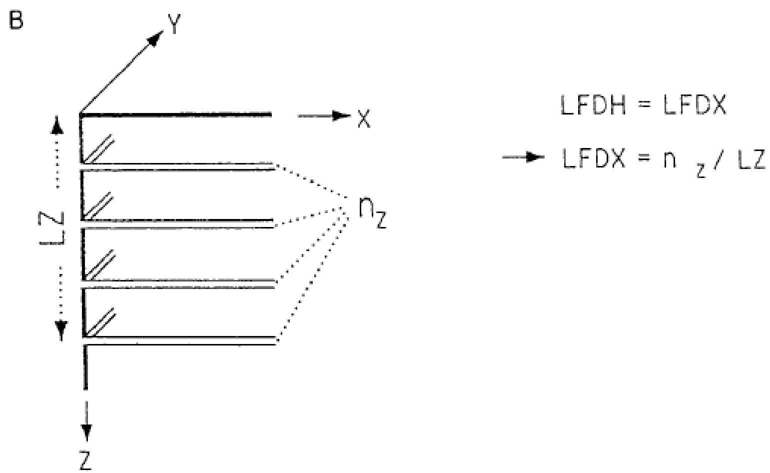
- **Evaluating Fractures and Fields**
- **Geological Condition of Fracturing**

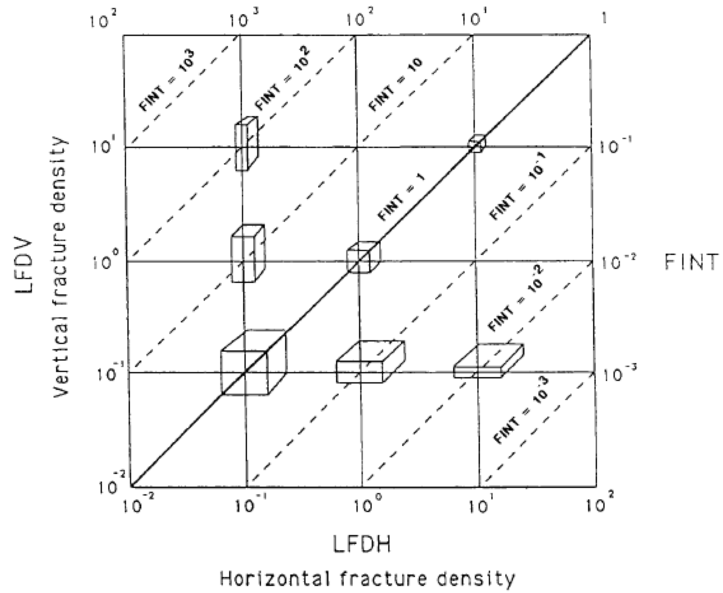
- Physical properties of fractures and matrix
  - Porosity and permeability
  - Rock compressibility in fractured reservoirs

# Qualitative Fracture Evaluation through FINT

Based on FINT definition, a qualitative interpretation could be made for shape and fracturing:



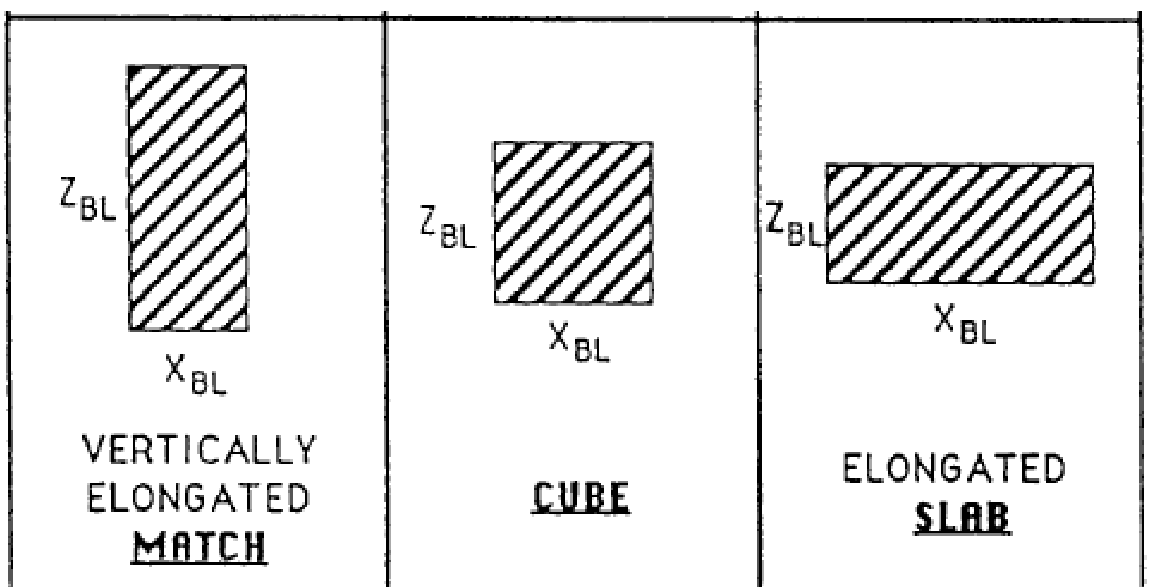

$$FINT = LFDZ / LFDX = (n_x / L_x) / (n_z / L_z)$$

*Shape*

| | | |
|---|---|---|
| FINT > 1 | FINT = 1 | FINT < 1 |
| [Matches] | [Cubes] | [Slabs] |

*Degree of fracturing*

| | | |
|---|---|---|
| FINT > 0.05 | ==> | Fractured zone |
| FINT = 0.1 | ==> | Average fractured zone |
| FINT = 5 – 10 | ==> | Strongly fractured zone |
| FINT = 20 – 50 | ==> | Very fractured zone |
| FINT = > 100 | ==> | Breccia |



# Data Processing of Fractures

The observations of fractures gathered from cores are tabulated and then processed through various criteria. The characteristics to be gathered are:

| | | | |
|---|---|---|---|
| Lithology vs. hardness | ===> rock hardness | => | Soft |
| | | => | Medium-hard |
| | | => | Hard |
| | | => | Very-hard |

| | | |
|---|---|---|
| Lithology vs. | ===> | Presence of shales |
| | ===> | Presence of stylolites |
| | ===> | Orientation of the bedding planes |

| | |
|---|---|
| Fracture Characteristics | ==> fracture opening, size |
| | ==> fracture orientation (dip, azimuth, angle) |
| | ==> fracture density |
| | ==> fracture intensity |
| | ==> matrix block dimensions |

# Porosity and Permeability in Fractured Carbonate Reservoirs

Total porosity = Matrix porosity + Fracture porosity

(Primary porosity) + (Secondary porosity)

Total voids/Total bulk =

Matrix voids/Total bulk + Fracture voids/Total bulk (7-11)

$$\phi_T = \phi_1 + \phi_2 \quad (7\text{-}12')$$

Inasmuch as secondary porosity $\phi_2 = \phi_f << \phi_m$

Total bulk volume = matrix bulk volume

$$V_B = V_{B\,m} \quad (7\text{-}12'')$$

or

$$\phi_T \approx \phi_m \quad (7\text{-}13)$$

# Storage Capacity of Matrix and Fractures

In transient flowing conditions the term which plays an important role is not the single porosity ($\Phi_m$ or $\Phi_f$ ), but rather, the storage capacity expressed by the association of porosity and compressibility. In this case the product r becomes:

$\phi_m * C_m$ ====>> for the matrix storage capacity

$\phi_f * C_f$ ====>> for the fracture storage capacity

**Order of Magnitude of Fracture Porosity**

➢As a general rule it could be stated that fracture porosity is below 1% and in only very exceptional cases may reach a value of 1%.

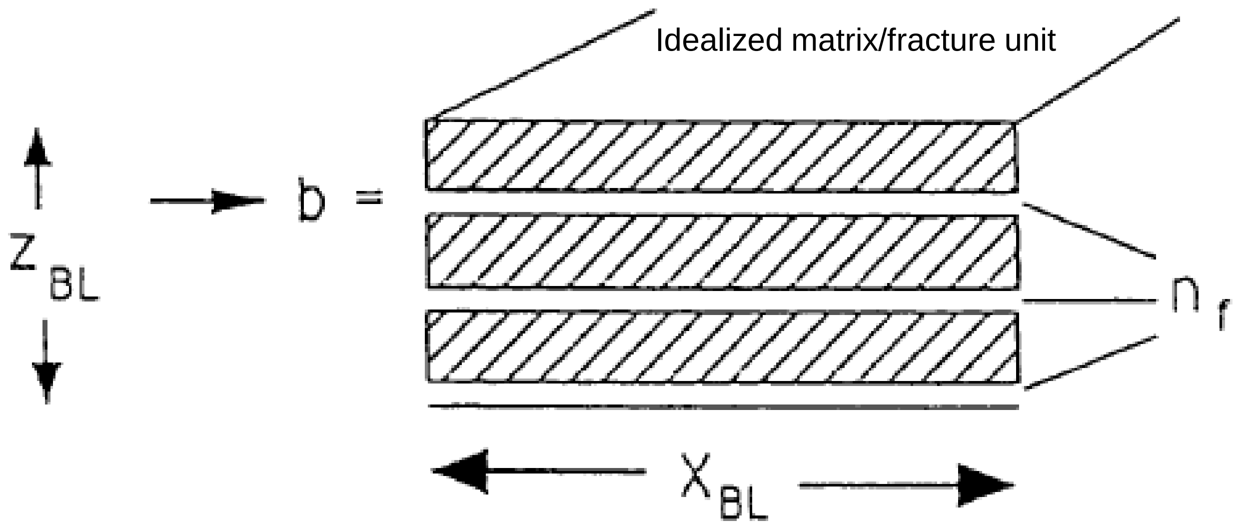
➢However, in very tight rocks having a primary porosity $\Phi_m$ <10% and a very extended network of macrofractures and microfractures, a fracture porosity between 0.5% and 2% may occur.

# Fracture Porosity from Direct Measurements

A direct measurement of fracture porosity requires:

(1) fracture width [b] from cores; and

(2) fracture density [LFD] from core examination



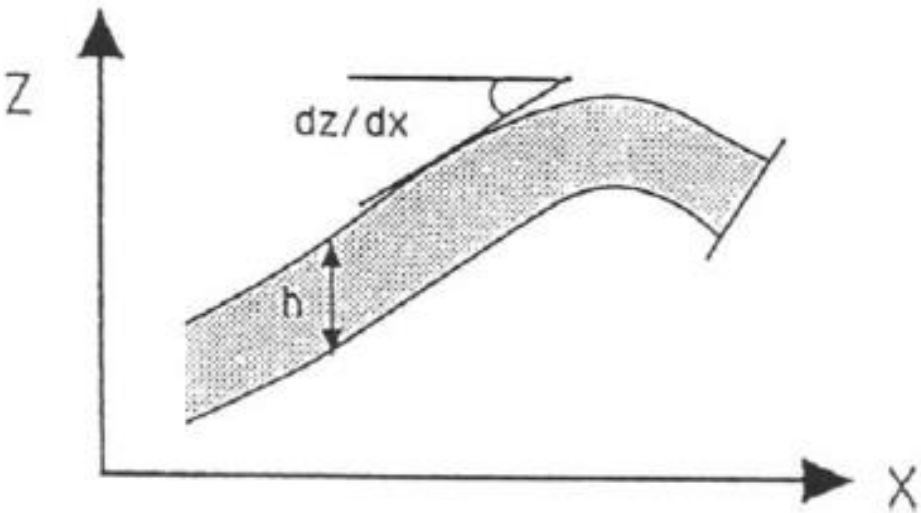

Porosity = Void fracture surface / Total surface

$$\phi_f = n_f * b * X_{bl} / X_{bl} * Z_{bl} = b * LFD = n_f * b / Z_{bl}$$

# Fracture Porosity from Structural Geological Data (Murray, 1977):

The presence of fractures in the case of a folded structure could be related to the bed thickness (h) and structural curvature expressed by $[d^2z/dx^2]$ for the cross-section shown in the below-figure. Fracture porosity in this case is approximated by the equation:

$$\phi_f = h\,[d^2z/dx^2]$$



# Permeability

In principle, the permeability established in the case of a conventional porous media remains valid in the case of a fractured reservoir. But in the presence of two systems (matrix and fractures), permeability has to be redefined in relation:

- to matrix ("matrix“ permeability),
- to fractures ("fracture" permeability) and
- to the fracture-matrix system ("fracture-matrix" permeability).

The matrix permeability remains the same as in a conventional reservoir, but the fracture permeability requires a review of its basic definition

1. Single-fracture case
2. Multi-fracture case

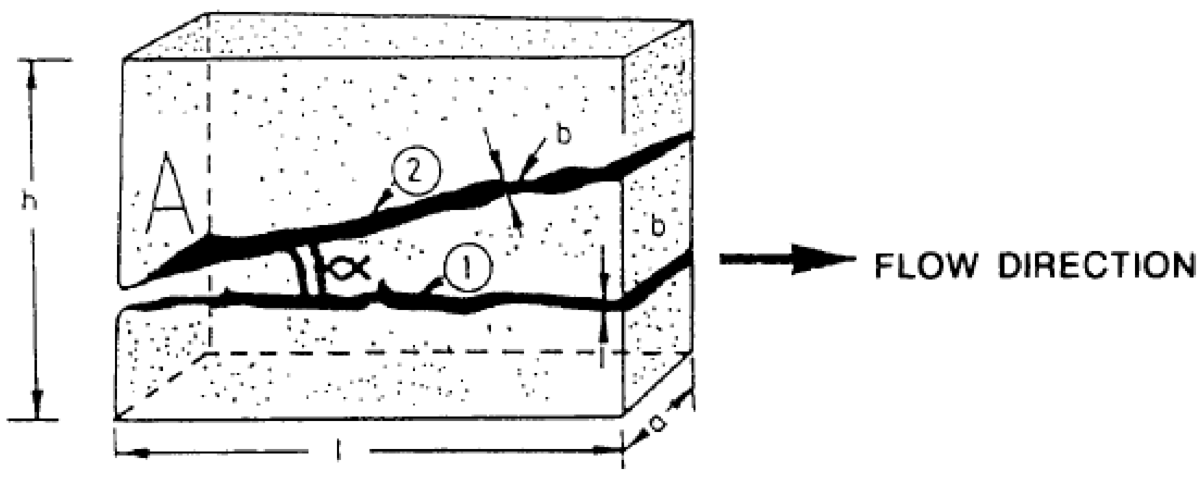
# Single-Fracture Case

The difference resulting from the flowing cross-section could be:
The effective "real flow cross-section" ($S_{effective}$ of a single fracture based on the below-figure is represented by:

$$S_{effective} = a*b$$

The "pseudo-cross flow section" based on the Darcy concept, which includes matrix and fractures, will result as:

$$S_{Darcy} = a*h = A$$




Matrix block containing two fractures. Fracture 1 (α =0). Fracture 2 (α >0).

The flow along the length l, through parallel plates (very close to each other):

$$q_f = a * b\ (b^2 / 12 * \mu) * (\Delta p / \Delta l)$$

The flow in a porous media based on Darcy law:

$$q_f = a * h * .\ (k_f / \mu) * (\Delta p / \Delta l)$$

$$b^3 / 12 = h * k_f$$

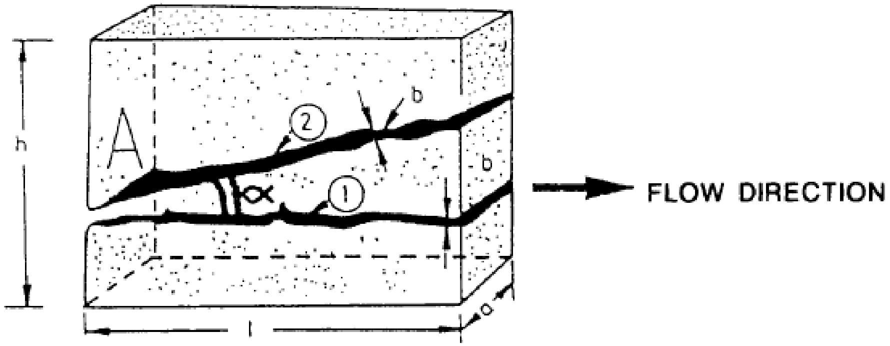
$$(b / h) * b^2 / 12 = k_f$$

The term ($b^2/12$) could be considered as a "pseudo-permeability", which physically represents the "intrinsic permeability" ($k_{ff}$) of the fracture, while the term (b / h ) represents the fracture porosity ($\Phi_f$). In this case a number of basic correlations can be expressed as:

$$\phi_f \cdot k_{ff} = k_f$$

$$\phi_f = b / h\ 12 * k_f / b^2$$

$$b = (12 \cdot k_f * h)^{0.33} = (12 * k_f / \phi_f)^{0.5}$$



# Multi-fracture Case(1)

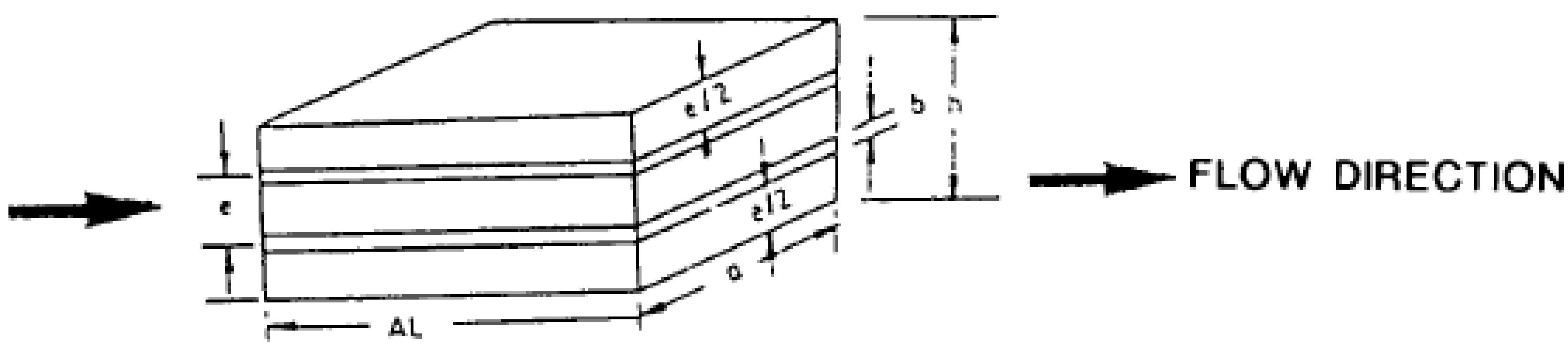
If, instead of a single fracture, the flow is examined through a fracture system formed by several parallel fractures (n), separated by matrix of height "e", then the flowing equation (similar to the case of single fracture) will give"

$$Q = n{*}ab{*}(b^2 / 12\,\mu)\,(\Delta p / \Delta l) = ah{*}\,(k_f / \mu)\,(\Delta p / \Delta l)$$

or

$$nb{*}b^2 / 12 = h{*}k_f \quad or \quad (n{*}b / h) * (b^2 / 12) = k_f$$

$$nb / h = LFD\, {*}b = \phi_f$$



# Multi-fracture Case(2)

Thus:

$$k_f \phi_f * b^2 / 12 = k_{ff} * \phi_f = k_{ff} * b * LFD = (b^3 / 12) * LFD$$

$$\phi_f = 12 * k_f / b^2 = (12 * k_f * LFD^2)^{0.333}$$

$$b = [12 * k_f / \phi_f]^{0.5} = [12 k_f / LFD]^{0.333}$$

For a random distribution of fractures, a correction factor for porosity could be written through **(Π/2)** $^{2}$ as follows:

$$\phi_f = [12 * k_f * (\pi / 2)^2 * LFD^2]^{0.333} = [29.6 * k_f * LFD^2]^{0.333}$$

# Fracture Permeability Measurements and Evaluation(1)

The fracture permeability can be measured as follows:

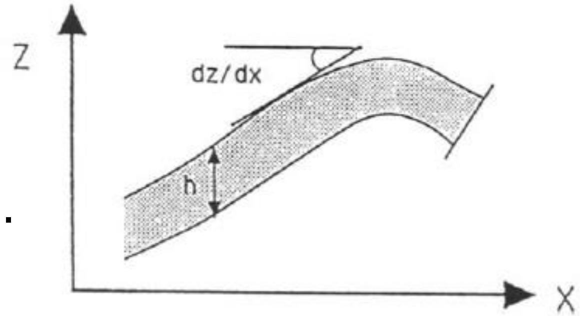
(1) by special equipment (Kelton), where the core is oriented so that the flow takes place along the fracturing direction, between the two ends of fracture contained in the lateral cylindrical surface of the core;

(2) by measuring the fracture opening, **b**, and counting the number **n** of the fractures for estimating of **LFD**; thus:

$$k_f = b^3 / 12 * LFD = (1 / 12) * (b^2 * \phi_f)$$

# Fracture Permeability Measurements and Evaluation(2)

(3) if structural geologic data are available (Murray, 1977), then when reservoir fracturing occurs as a result of structural folding for a layer having a pay "**h**" (Fig. 7-29), the fracture permeability $\mathbf{k_f}$(in mD) can be estimated through the equation:




$$k_f = (0.2) * 10^9 * e^2 * [h * (d^2z / dx^2)]^3$$

where the distance between the two fractures **e** is in **cm**.

(4) from well testing in conditions of steady-state flow:

$$k_f = PI * \{\mu_o * B_o . [ln(r_e / r_w) + S]\} / [2 * \pi * h]$$

because the flow toward the wellbore is taking place through the fracture network.

$$\phi_f = [29.6 * k_f * LFD^2]^{0.33} = 0.00173\ [PI \frac{\mu_o B_o \ln r_e/r_w}{h} LFD^2]^{0.333}$$

A random distribution of fractures

where: $PI$ is in STM³/D/atm, $\mu_o$ is in cP.; h is in m; and LFD is in 1/cm; and $\phi_f$ is fractional.

# Correlation between Field Data and Idealized Fracture/Matrix System(1)

(1) During stabilized flow toward a well in a fractured reservoir, the productivity index is directly correlated to fracture permeability:

$$k_f = f(PI) ==>> k_f$$

$$k_f = PI * \{\mu_o * B_o. [ln(r_e / r_w) + S]\} / [2 * \pi * h]$$

(2) If the observation of the cores has been carried out and processed, the estimation of fracture density **LFD** from core observations makes possible the evaluation of the fracture porosity as a function of productivity index:

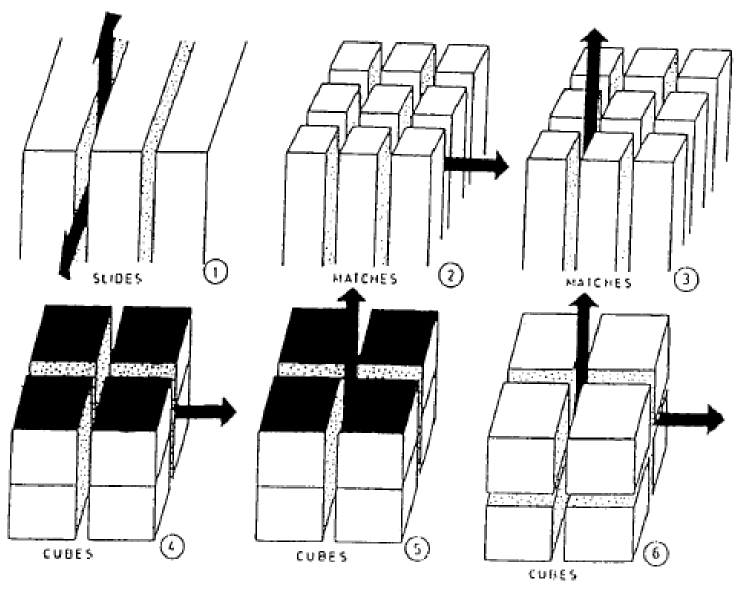
$$\phi_f = f(PI, LFD) ===>> \phi_f$$

$$\phi_f = [29.6 * k_f * LFD^2]^{0.33} = 0.00173 \left[PI \frac{\mu_o B_o \ln r_e / r_w}{h} LFD^2\right]^{0.333}$$

# Correlation between Field Data and Idealized Fracture/Matrix System(2)

(3) Assuming the six simplified and idealized models of matrix blocks as shown in Figure, it is possible to correlate the basic data of idealized blocks as: **a** – block dimension; **b** - fracture width; $\mathbf{k_f}$ - fracture permeability; $\mathbf{\Phi_f}$ - fracture porosity; and **LFD**- fracture density.

The theoretical correlations are given in **Table 7-IV** for various idealized block shapes. The block dimensions (**a**) and fracture opening (**b**) can be estimated if permeability ($\mathbf{k_f}$) and porosity ($\mathbf{\Phi_f}$) have been evaluated from well testing results.




| Shape : | No : | L.F.D. |
|---|---|---|
| Slides | 1 | 1/a |
| Matches | 2 | 1/a |
| Matches | 3 | 2/a |
| Cube | 4 | 2/a |
| Cube | 5 | 1/a |
| Cube | 6 | 2/a |

TABLE 7-IV

Correlation of parameters for idealized matrix blocks (Reiss, 1966).

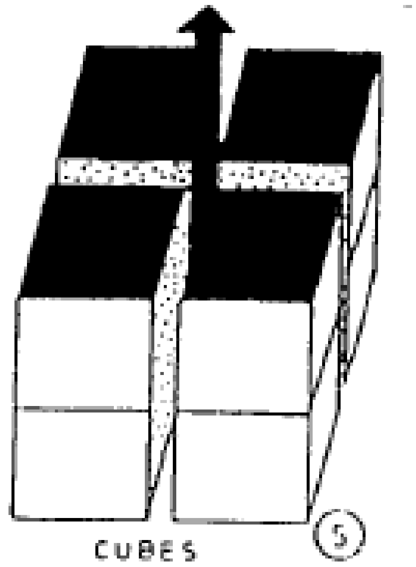
| MODEL | | DIMENSIONLESS EQUATIONS | | | | DIMENSIONAL EQUATIONS | | | |
|---|---|---|---|---|---|---|---|---|---|
| Nº | TYPE | L.F.D | $\phi_f$ | $k_f(\phi_f, a)$ | $k_f(\phi_f, b)$ | L.F.D | $\phi_f$ % | $k_f(\phi_f, a)$ darcy | $k_f(\phi_f, b)$ darcy |
| 1 | SLIDES | $\frac{1}{a}$ | $\frac{b}{a}$ | $\frac{1}{12} a^2 \phi_f^3$ | $\frac{1}{12} b^2 \phi_f$ | $\frac{1}{a}$ | $\frac{1}{100} \frac{b}{a}$ | $8{,}33\, a^2 \phi_f^3$ | $8{,}33 \times 10^{-4} b^2 \phi_f$ |
| 2 | MATCHES | $\frac{1}{a}$ | $\frac{2b}{a}$ | $\frac{1}{96} a^2 \phi_f^3$ | $\frac{1}{24} b^2 \phi_f$ | $\frac{1}{a}$ | $\frac{1}{100} \frac{2b}{a}$ | $1{,}04\, a^2 \phi_f^3$ | $4{,}16 \times 10^{-4} b^2 \phi_f$ |
| 3 | MATCHES | $\frac{2}{a}$ | $\frac{2b}{a}$ | $\frac{1}{48} a^2 \phi_f^3$ | $\frac{1}{12} b^2 \phi_f$ | $\frac{2}{a}$ | $\frac{1}{100} \frac{2b}{a}$ | $2{,}08\, a^2 \phi_f^3$ | $8{,}33 \times 10^{-4} b^2 \phi_f$ |
| 4 | CUBES | $\frac{1}{a}$ | $\frac{2b}{a}$ | $\frac{1}{96} a^2 \phi_f^3$ | $\frac{1}{12} b^2 \phi_f$ | $\frac{1}{a}$ | $\frac{1}{100} \frac{2b}{a}$ | $1{,}04\, a^2 \phi_f^3$ | $4{,}16 \times 10^{-4} b^2 \phi_f$ |
| 5 | CUBES | $\frac{2}{a}$ | $\frac{2b}{a}$ | $\frac{1}{48} a^2 \phi_f^3$ | $\frac{1}{12} b^2 \phi_f$ | $\frac{2}{a}$ | $\frac{1}{100} \frac{2b}{a}$ | $2{,}08\, a^2 \phi_f^3$ | $8{,}33 \times 10^{-4} b^2 \phi_f$ |
| 6 | CUBES | $\frac{2}{a}$ | $\frac{3b}{a}$ | $\frac{1}{162} a^2 \phi_f^3$ | $\frac{1}{18} b^2 \phi_f$ | $\frac{2}{a}$ | $\frac{1}{100} \frac{3b}{a}$ | $0{,}62\, a^2 \phi_f^3$ | $5{,}55 \times 10^{-4} b^2 \phi_f$ |

UNITS : K (Darcy), a (cm) ; b (microns)

# Example 7-1: Evaluation of the Matrix Block from Production Data

In a fractured reservoir, from production testing data a rate of 12,260 STB/D was measured, and the formation pressure drop was 68 psi. Other reservoir data are: oil viscosity $\mu_o = 1.1$ cP; oil volume factor $B_o = 1.36$; total reservoir pay $h = 86$ m; drainage radius $r = 1200$ m; and well radius $r = 10$ cm. From core examination an average fracture density LFD was estimated to be 2/m.

**Question:** Assuming the model 5 of Fig. (cube with 1 flowing directions), evaluate:
(1) the fracture permeability;
(2) the fracture porosity; and
(3) the block size (a) and the fracture opening (b) by using the field production data.



# Solution(1)

Evaluation of $\mathbf{k_f}$ and $\mathbf{\Phi_f}$ from field data.

$$k_f = PI * \{\mu_o * B_o. [ln(r_e / r_w) + S]\} / [2 * \pi * h]$$

$$\phi_f = [29.6 * k_f * LFD^2]^{0.33} = 0.00173\ [PI \frac{\mu_o B_o \ln r_e/r_w}{h} LFD^2]^{0.333}$$

(1) The productivity index PI (STM³/D/atm) is given by

$PI = \Delta Q / \Delta p$ = 12660 (STB/D) / 68 (psi) = 186.17 (STB/D/psi) =

435 (STm³/ D /atm) = 5034 (STcm³ / D/atm)

(2) The fracture permeability is equal to (Eq. 7-20):

$k_f = PI * B_o * \mu_o. (\ln r_e / r_w) / 6.28\ h$ = 5034 (STcm³ / D/atm)*

* 1.36 * 1.1 * ln 12000 / 6.28 * 8600 = 1.31 D

The porosity (Eq. 7-27) is:

$\phi_f = 1.73.10^{-3}\ [PI.\ \mu_o * B_o * \ln (r_e / r_w) * LFD^2 / h\ ]^{0.333}$

$\phi_f = 1.73.\ 10^{-3}\ [435 * 1.36 * 1.1 * \ln (12000) * 0.02^2 / 86]^{0.3333} = 0.00025 = 0.025\%$

# Solution(2)

Evaluation of " block size **a** and fracture opening **b**. Based on Table 7-IV for cube-shaped  matrix blocks

(3) The cube dimension is equal to:

$$a = [\, k_f / 2.08^* \, \phi_f^{\ 3}]^{0.5} = [1.31 / 2.08^* 0.025^{\ 3}]^{0.5}$$

$$= 200 \text{ cm.} = 2 \text{ m}$$

(4) The fracture width is equal to:

$$b\,(\mu\text{m}) = 100^* a(\text{cm}) \times \phi(\%)/2 = 100^*200^*\, 0.025/2 = 250\ \mu\text{m}$$

| MODEL | | DIMENSIONLESS EQUATIONS | | | | DIMENSIONAL EQUATIONS | | | |
|---|---|---|---|---|---|---|---|---|---|
| Nº | TYPE | L.F.D | $\phi_f$ | $k_f(\phi_f, a)$ | $k_f(\phi_f, b)$ | L.F.D | $\phi_f$ % | $k_f(\phi_f, a)$ darcy | $k_f(\phi_f, b)$ darcy |
| 4 | | $\frac{1}{a}$ | $\frac{2b}{a}$ | $\frac{1}{96} a^2 \phi_f^3$ | $\frac{1}{12} b^2 \phi_f$ | $\frac{1}{a}$ | $\frac{1}{100} \frac{2b}{a}$ | $104\ a^2 \phi_f^3$ | $4{,}16 \times 10^{-4} b^2 \phi_f$ |
| 5 | CUBES | $\frac{2}{a}$ | $\frac{2b}{a}$ | $\frac{1}{48} a^2 \phi_f^3$ | $\frac{1}{12} b^2 \phi_f$ | $\frac{2}{a}$ | $\frac{1}{100} \frac{2b}{a}$ | $2{,}08\ a^2 \phi_f^3$ | $8{,}33 \times 10^{-4} b^2 \phi_f$ |
| 6 | | $\frac{2}{a}$ | $\frac{3b}{a}$ | $\frac{1}{162} a^2 \phi_f^3$ | $\frac{1}{18} b^2 \phi_f$ | $\frac{2}{a}$ | $\frac{1}{100} \frac{3b}{a}$ | $0{,}62\ a^2 \phi_f^3$ | $5{,}55 \times 10^{-4} b^2 \phi_f$ |

UNITS : K (Darcy) , a (cm) ; b (microns)

# Relative Permeability and Capillary Pressure Curves in Fractured Carbonate Reservoirs

Relative permeabilities in a conventional reservoir are obtained from special core analysis. In a fractured reservoir, the evaluation of relative permeability curves is complicated because of the nature of double-porosity system, where the fracturing plane between two matrix units develops a discontinuity in the multi-phase flowing process.

**Matrix Relperm Curves**: The relative permeability of the matrix for two or three phases is evaluated by the procedure used for any intergranular rock sample. The results have to be representative in relation to the shape of Relperm curves and the magnitude of their endpoints (irreducible saturation in the wetting and non-wetting phases and the respective relative permeability values at these critical saturations).

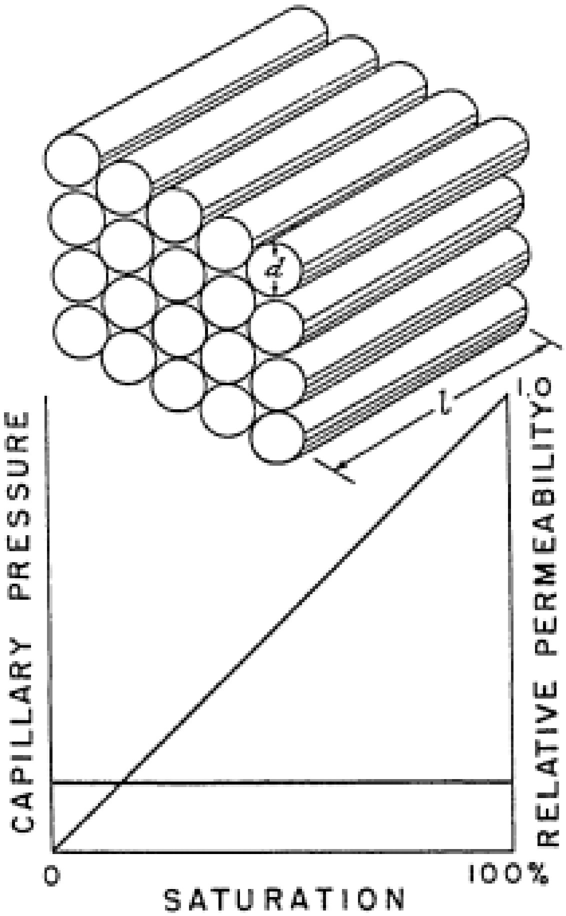



**Fig. 31 – Hypothetical Capillary-pressure Curve for Bundle of Capillaries with Uniform Lengths and Uniform Diameters**

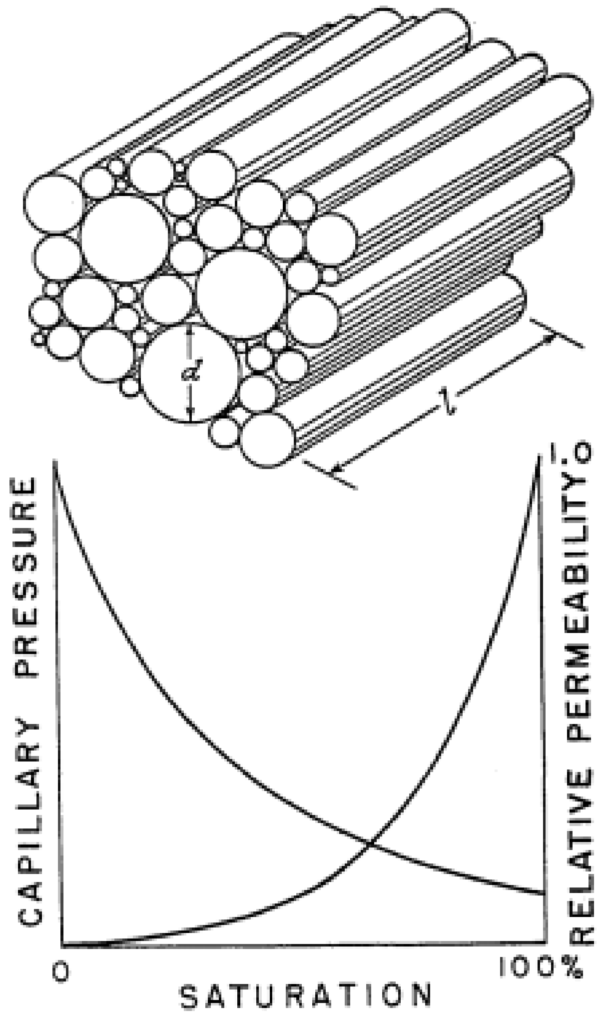
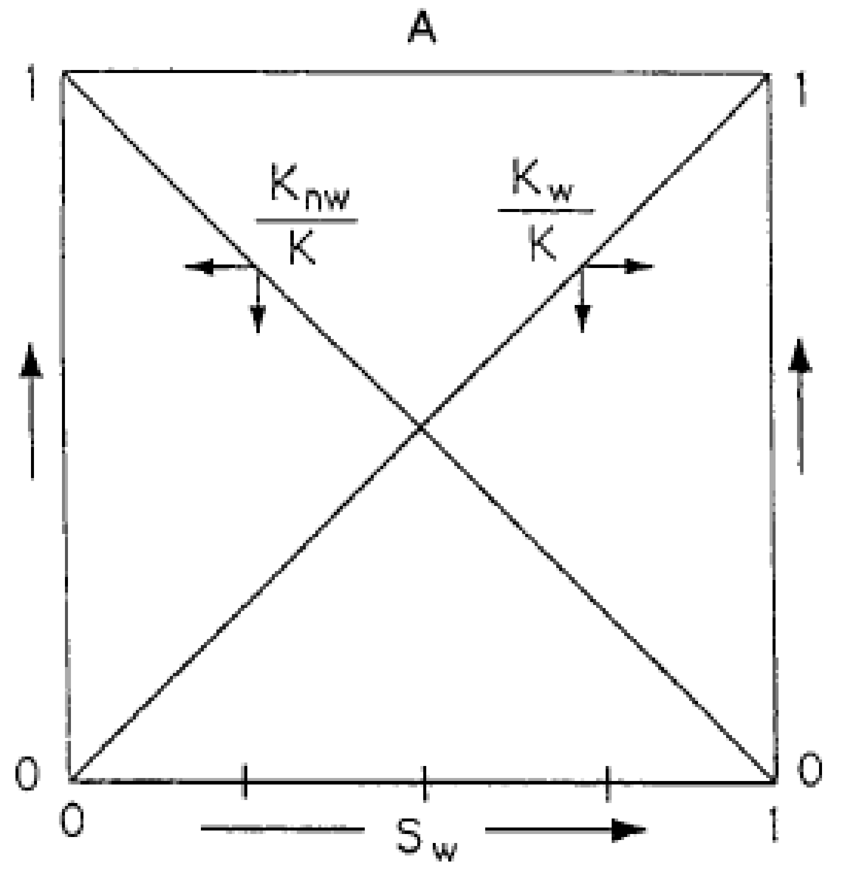


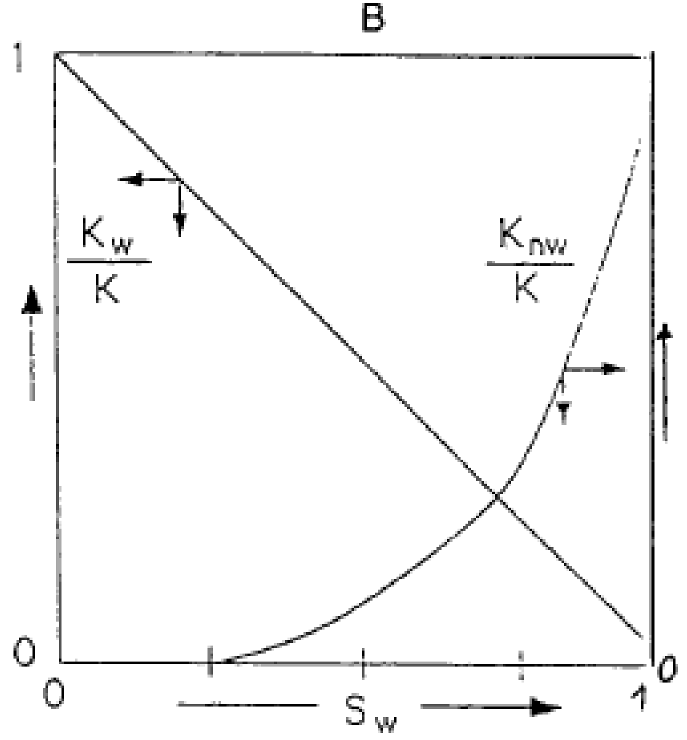

**Fig. 32 – Hypothetical Capillary-pressure Curve for Bundle of Capillaries with Uniform Lengths and Wide Distribution of Diameters**

From: Gates, J.I. and Templaar-Lietz, W.: "Relative Permeabilities of California Cores by the Capillary Pressure Method," API Drilling and Production Practices (1950) 285-302.

**Fracture Relperm curves**. The fracture network Relperm curves are basically different from matrix Relperm curves as a consequence of the very high intrinsic permeability of fractures. This very high permeability will have as a main consequence the predominant control of gravity forces in multiphase flow in fractures. As a result of gravity equilibrium, the relative permeability curves will essentially be reduced to two straight lines (diagonals) as shown in Fig A.



**Fracture Relperm curves**: At certain conditions, especially when drops of oil are moving in the fracture saturated with water, it is more correct to adjust the wetting phase Relperm curve (Fig. B) by a different relationship:




$$k_{wetting} = [S_{wett}]^n$$

where often $n = 3$ and dimensionless saturation, $S_{wett}$, is equal to:

$$S_{wett} = (S_{wett} - S_{wett,i}) / (1 - S_{wi})$$

# Flow through Single Fractures
# Transition from Laminar to Turbulent Flow in Fractures

From the experience of flow in pipes it is known that :

- Turbulent flow depends on pipe roughness, the magnitude of which is directly related to the friction factor.
- The transition from the laminar state to turbulent state of flow is controlled by the Reynolds number which is the ratio of the inertia to viscous forces:

$$Re = 2V * D * \rho / \mu = 2VD / \nu \qquad (7\text{-}48)$$

where $V$ is the average velocity in the pipe, $D$ is the length of the pipe, $\rho$ and $\mu$ are, respectively, the density and viscosity of the fluid transported, and $\nu$ is the kinematic viscosity.

- For flow between parallel walls delimiting a fracture (Snow, 1965), the Reynolds number is given by:

$$Re = 2 * \rho * V * b / \mu = 2 * V * b / \nu \qquad (7\text{-}49)$$

"Relative roughness" values for an average fracture opening of 20 μm are around 0.0015 - 0.025 for limestones and 0.004 - 0.007 for dolomites.

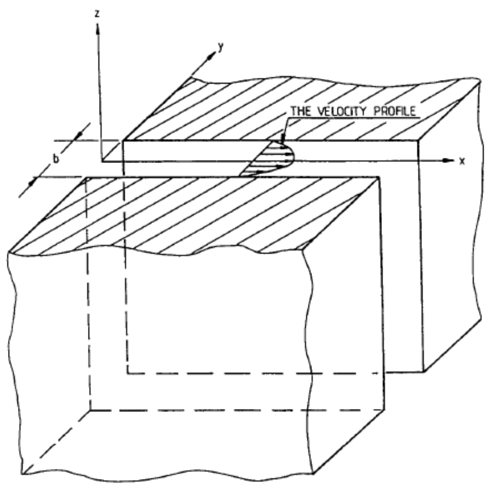
In the event the fracture network is treated similar to a porous medium, it is necessary to introduce permeability and, thus, the critical Reynolds number is:

$$Re_{cr} = 1$$

$$Re = 10 * \rho * V * k^{0.5} / \mu * \phi^{2.3} \tag{7-50}$$

Thus, the analogy between a conventional reservoir and a fracture network is based on the similarity between:

1. the parameters of the fracture system ($k_f$, $k_{ff}$, $\Phi_f$ b, n, LFD) and
2. the parameters of a conventional reservoir (k, Φ, h)



# Basic Equations Describing Flow in Fractures

If the flowing process in a fractured limestone network is considered analogous to the flow in an intergranular limestone, then the relationship between pressure drop Δp and rate Q will be:

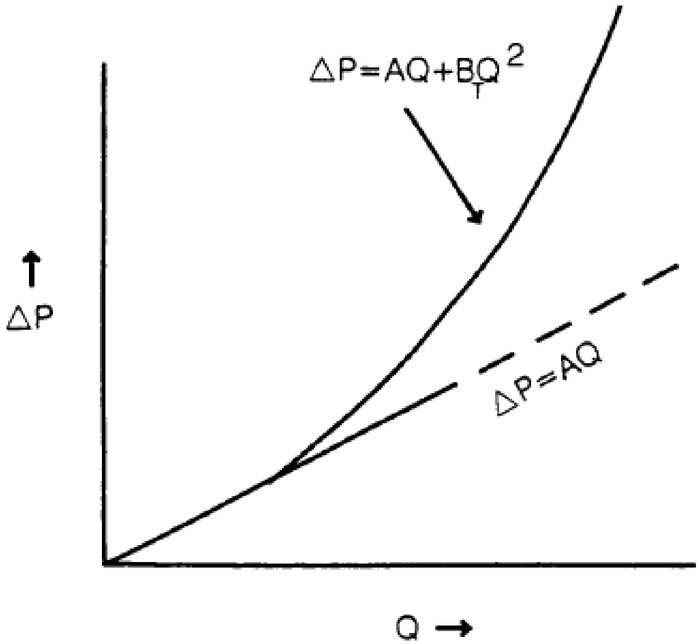
$$\Delta p = A * Q + B_T * Q^2$$

**for low rates**

$$B_T \cdot Q^2 << A.Q$$

**for high rates**

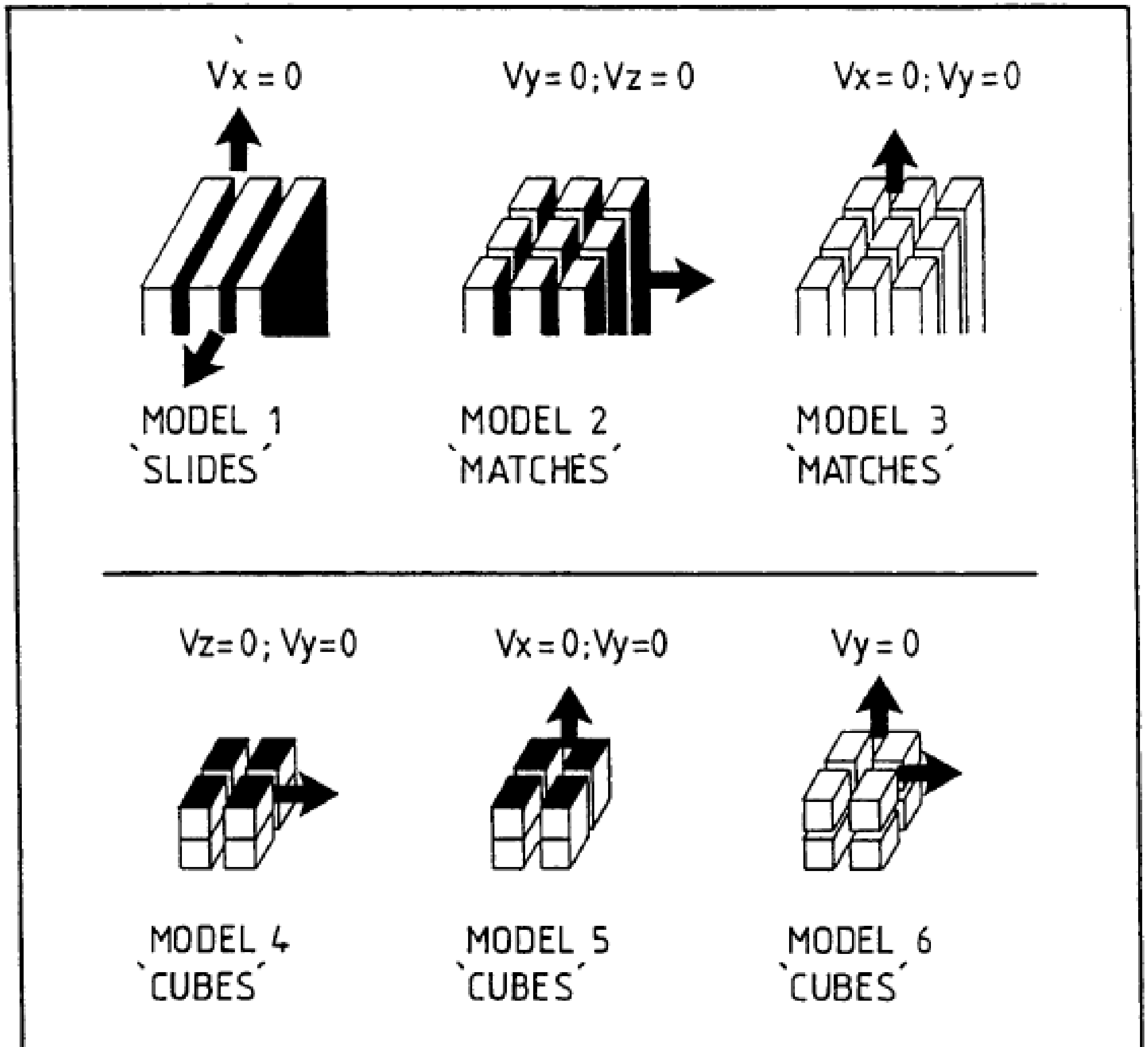
$$B_T \cdot Q^2 > A.Q$$




Both constants A and $B_T$ in these cases depend on <u>flow geometry</u> and <u>physical properties</u> of rock and fluids.

TABLE 7-VI

Idealized geometrical models of fractured reservoirs



| Model TYPE | Velocity | L.F.D. | $\phi_f$ | $\phi_f^3$ | a | b |
|---|---|---|---|---|---|---|
| 1. Slides | $V_x = 0$ | 1/a | b/a | $12*K_f*LFD^2$ | 1/LFD | $(12*K^f/\phi_f)^{0.5}$ |
| 2. Matches | $V_y = 0; V_z = 0$ | 1/a | 2b/a | $96*K_f*LFD^2$ | 1/LFD | $(24*K_f/\phi_f)^{0.5}$ |
| 3. Matches | $V_x = 0; V_y = 0$ | 2/a | 2b/a | $12*K_f*LFD^2$ | 2/LFD | $(12*K_f/\phi_f)^{0.5}$ |
| 4. Cubes | $V_z = 0; V_y = 0$ | 1/a | 2b/a | $96*K_f*LFD^2$ | 2/LFD | $(12*K_f/\phi_f)^{0.5}$ |
| 5. Cubes | $V_x = V_y = 0$ | 2/a | 2b/a | $12*K_f*LFD^2$ | 1/LFD | $(12*K_f/\phi_f)^{05}$ |
| 6. Cubes | $V_y = 0$ | 2/a | 3b/a | $40.5*K_f*LFD^2$ | 2/LFD | $(18*K_f/\phi_f)^{0.5}$ |

TABLE 7-VII

Evaluation of permeability $k_f$ and rate $Q$ for idealized geometrical models of fractured reservoirs.

| MODELS* | $k_f$ | Rate |
|---|---|---|
| 1 | $\phi_f^3/12\ LFD^2$ | $Q = (A^*\ \phi_f^3 / 12\ LFD^2)\ (dp/dl)$ |
| 2 | $\phi_f^3/96\ LFD^2$ | $Q = (A^*\ \phi_f^3 / 96\ LFD^2)\ (dp/dl)$ |
| 3 | $\phi_f^3/48\ LFD^2$ | $Q = (A^*\ \phi_f^3 / 48\ LFD^2)\ (dp/dl)$ |
| 4 | $\phi_f^3/96\ LFD^2$ | $Q = (A^*\ \phi_f^3 / 96\ LFD^2)\ (dp/dl)$ |
| 5 | $\phi_f^3/162\ LFD^2$ | $Q = (A^*\ \phi_f^3 / 162\ LFD^2)\ (dp/dl)$ |

* Similar to the models of Table 7-VI

# Steady-state Radial Symmetrical Flow

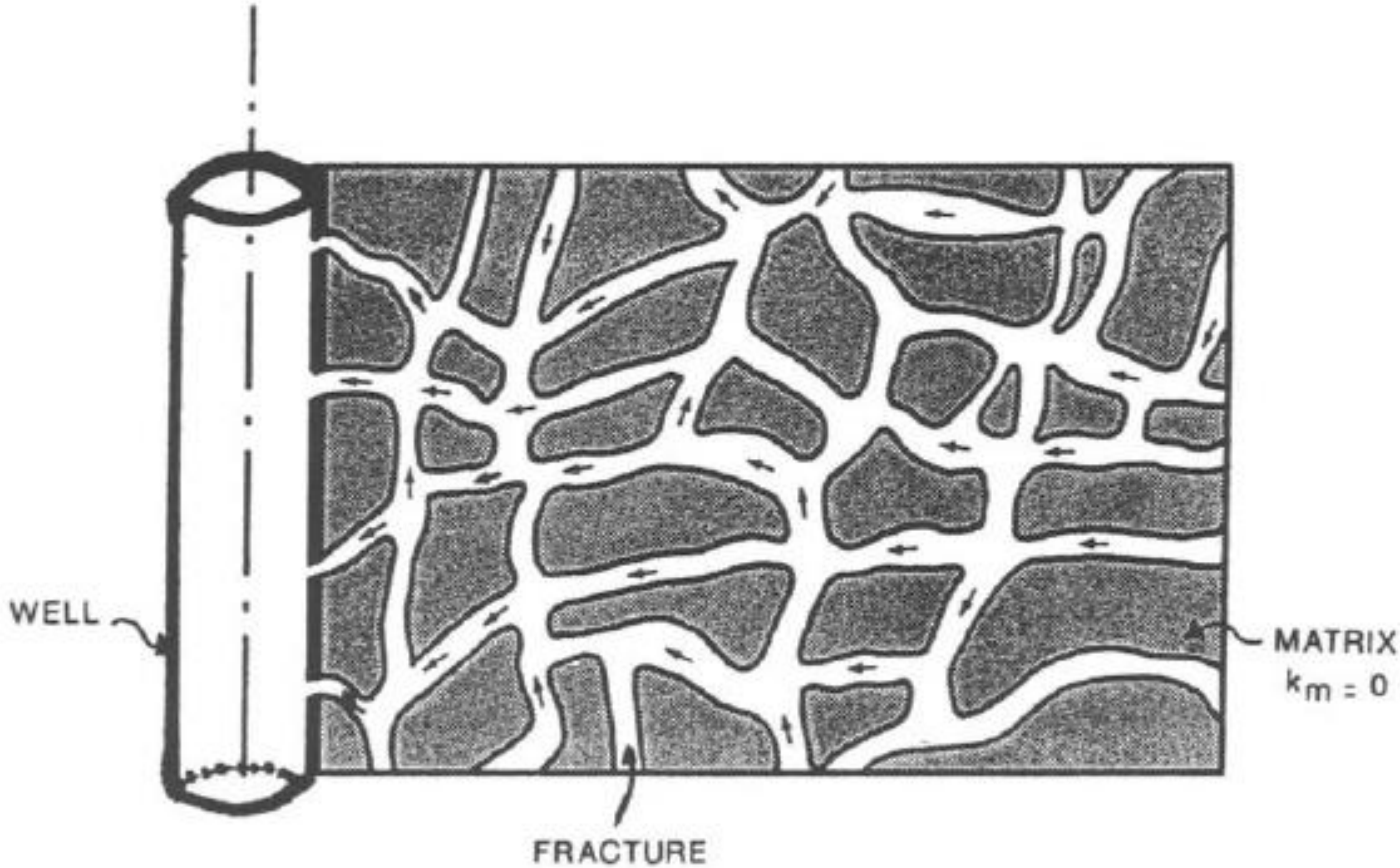



Fig. 7-38. Flow toward a well through fracture network.

**Radial flow analogy:** In case of a radial symmetrical flow and based on the analogy discussed (Snow, 1965), the constants A and B are equal to:

$$A = [\mu_o * B_o (\ln r_e / r_w + S)] / 2 * \pi * k_f * h \quad (7\text{-}52)$$

$$B_T = \beta * \mu_o * B_o (1 / r_w - 1 / r_e) / 4 * \pi^2 * h^2 \quad (7\text{-}53)$$

where $\beta$ is expressed as a function of permeability and porosity:

$$\beta\ (1/\text{ft}) = 2.23.10^9 / [k_f .\ \phi_f\ (\text{mD.fraction})]^{1.085} \quad (7\text{-}54)$$

The use of the above equations can help two objectives:

(1) to express the flowing equation:

$$\Delta p = A * Q + B_T * Q^2$$

This is possible if the physical parameters ($\phi_f$, $k_f$, $\mu$, $h$, $B_o$) and geometrical data ($r_w$, $r_e$) are available, in order to estimate the parameters $A$ and $B_T$ through Eqs. 7-52 and 7-53.

(2) To estimate the reservoir characteristics:

$k_f$, $\phi_f$, $\beta$, $a$ and $b$.

This is possible if the production data ($Q$ and $p$) recorded during well testing are available.

# Procedure for Field Parameters Evaluation

By using the well stabilized rate [Q] and pressure difference [Δp] during the steady-state conditions of flow, a linear relationship Δp / Q vs. Q is obtained when data are plotted as in Fig. 7-40. From the straight-line relationship Δp / Q vs. Q, the parameters A and B are directly obtained:

$A$ = as the value $\Delta p / Q$ @ $Q = 0$ ;

$B_T$ = as the slope of straight-line $\Delta p / Q$ vs. Q.

The constants A and B can be further used for evaluation of reservoir characteristics.

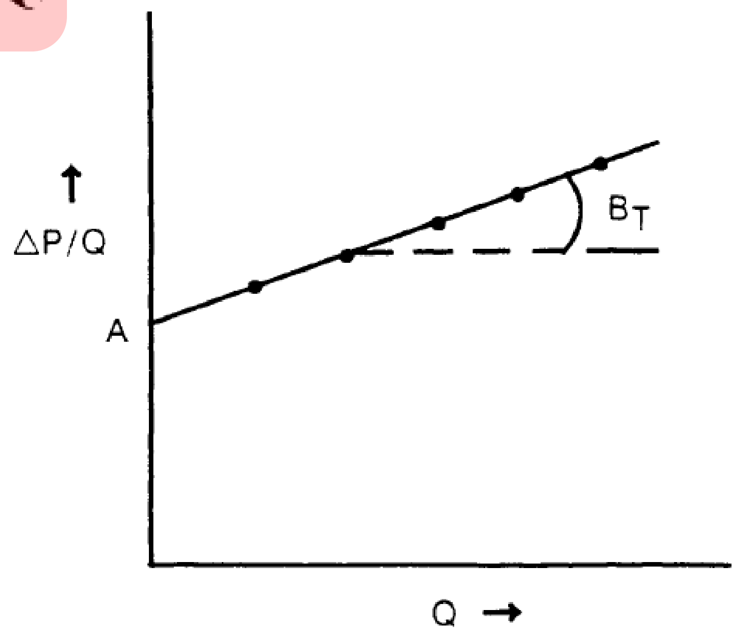



Fig. 7-40. Relationship between $\Delta P$ and $Q$ for the evaluation of parameters $A$ and $B_T$.

# Permeability &Porosity & Turbulence

Permeability $k_f$ from Eq. 7-26 and using parameter **A** equal to **1/PI**, gives:

$$k_f = \{\mu_o * B_o * [\ln (r_e / r_w) + S]\} / \; 6.28 * h * A \qquad (7\text{-}55)$$

Porosity $\mathbf{\Phi_f}$ can be estimated only if the fracture density **LFD** is known from core observations. Equation 7-27 requires the knowledge of productivity index (**PI**) and of fracture density **LFD**:

$$\phi_f = 1.73.10^{-3} \{PI*B_o * \mu_o [\ln (r_e / r_w) + S)] * LFD^2 / h]^{1/3} \qquad (7\text{-}56)$$

where productivity index (PI) and parameter A are related by the expression PI = 1 / A.

Turbulence factor **β**, could be obtained from Eq. 7-53 by using the parameter $\mathbf{B_T}$:

$$\beta = 4 * \pi^2 h^2 * r_w * B_T / \rho_o * B_o \qquad (7\text{-}57)$$

Permeability k: can be obtained from the turbulence factor fl by using Eq. 7-54:

$$k_f \,(\text{mD}) = (1 / \phi_f) * [(2.2.10^9) / \beta \,(1 / \text{ft})]^{0.922} \qquad (7\text{-}54')$$

# Idealized Model Characterization

*Idealized model characterization.* Based on parameters as $k_f$, $\phi_f$ and *LFD* for a given idealized block shape, the block size $a$ and fracture opening $b$ can be obtained from Table 7-VI:

$a = \mathrm{f}\,(LFD)$

$b = \mathrm{f}\,(k, \phi_f)$

*Example 7-2*

*Fractured reservoir characterization from well testing data*

In a fractured limestone reservoir, from well production testing the following rates and pressure drop data rates were obtained:

TEST 1 ==> $Q$ = 5600 STB/D $\Delta p$ = 127.7 psi

TEST 2 ==> $Q$ = 11300 STB/D $\Delta p$ = 318.5 psi

TEST 3 ==> $Q$ = 16700 STB/D $\Delta p$ = 556.0 psi

In addition, the following data are known:

| | | |
|---|---|---|
| Oil viscosity | $\mu_o$ | = 0.4 cP |
| Oil gravity | $\gamma_o$ | = $0.814.10^{-3}$ kg/cm$^3$ |
| Oil volume factor | $B_o$ | = 1.41 |
| Fracture density | $LFD$ | = 2/m |
| Average pay | $h$ | = 96 m |
| Drainage radius | $r_e$ | = 350 m |
| Well radius | $r_w$ | = 10 cm |

Determine:

(1) flowing parameters $A$ and $B_{\text{T}}$ by plotting $\Delta p / Q$ vs. $Q$ obtained by the well testing;

(2) the flowing equation $\Delta p$ vs $Q$;

(3) the limestone fracture permeability, $k_f$;

(4) the limestone fracture porosity, $\phi_f$;

(5) the turbulence factor, $\beta$;

(6) a simplified / idealized matrix block, assuming a cubic matrix block where the flow takes place in all three cartesian directions (model 6 of Fig. 7-22).

*Solution*

(1) Based on well testing data:

| | | | | |
|---|---|---|---|---|
| $Q$ | STB/D | 5600 | 11,300 | 16,700 |
| $\Delta p$ | psi | 127.7 | 318.5 | 556 |
| $\Delta p / Q$ | psi/STB/D | 0.0228 | 0.02818 | 0.0332 |

These data are plotted in Fig. 7-41 as a straight-line ($\Delta p / Q$ vs. $Q$), from which constants $A$ and $B_T$ can be obtained:

If $Q = 0$

$A = 1.75 * 10^{-2}$ psi/STBD $= 6.49.10^{-4}$ atm / STcm$^3$ / sec

$= 7.51.\ 10^{-3}$ atm / STm$^3$ / D

The slope $B_T = [(\Delta p / Q)] / \Delta Q = 9.4.10^{-7}$ psi / STBD$^2$

$= 2 * 10^{-8}$ atm / (STcm$^3$ / sec)$^2$

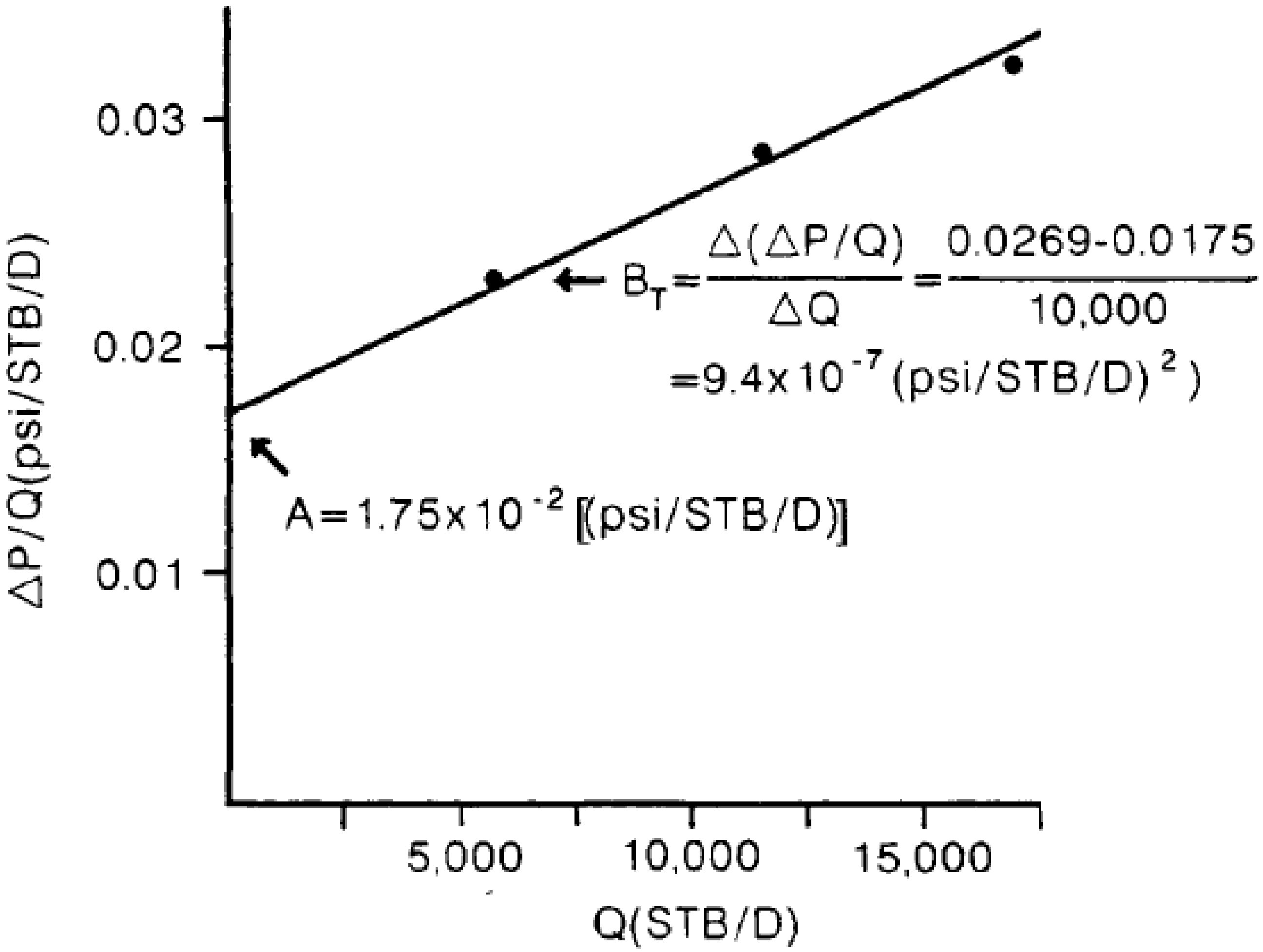



Fig. 7-41. Relationship between $\Delta P / Q$ and $Q$ obtained from well testing data in a fractured reservoir.

(2) The flowing equation is:

$$\Delta P \text{ (psi)} = 1.75.10^{-2} * Q \text{ (STBD)} + 9.4.10^{-7} Q^2 \text{ (STBD)}^2$$

(3) The permeability $k_f$ is determined from linear relationship $\Delta p$ vs. $Q$ (Eq. 7-55)

$$k_f = [\mu_o * B_o * \ln (r_e / r_w)] / 6.28 * h * A =$$
$$0.4 * 1.41 * \ln (3500) / 6.28 * 96.10^2 * 6.49 . 10^{-4} = 0.117 \text{ D} = 117 \text{ mD}$$

where: $h$ (cm); $\mu_o$ (cP); $A$ (atm / STcm$^3$ /sec).

(4) Fracture porosity $\phi_f$ is obtained from Eq. 7-56, assuming a random distribution of the fractures:

$$\phi_f = 1.73.10^{-3} [PI * B_o * \mu_o * \ln (r_e / r_w) * LDF^2 / h]^{1/3}$$

where: $PI = 1 / A$ (STm$^3$ / D / atm); $\phi_f$ fraction; $\mu_o$ is in cP; $h$ is in m and $LFD$ is in 1 / cm.

$$\phi_f = 1.73 . 10^{-3} * [(1 / 7.51 . 10^{-3}) * 1.41 * 0.4 * \ln (3500) * 0.02^2 / 96]^{1/3}$$

$$\phi_f = 0.00023 = 0.023 \%$$

(5) The turbulence factor can be obtained from Eq. 7-57″:

$\beta = B_T\,[4\,\pi^2\,h^2\,r_w / \rho_o B_o] = 2 * 10^{-8}$ .
$*[4 * (3.14 * 9600)^2\,10 / 0.83.10^{-6} * 1.4] = 0.625 * 10^9$ (1/cm) $= 19 * 10^9$ (1/ft)

where: $h$ is in cm; $r$ is in cm; $\rho_o$ (kg * sec$^2$ /cm$^4$); $B_T$ is in atm / (cm $^3$ / sec)$^2$.

(6) Evaluation of the idealized matrix block

Using the matrix block having a cube shape and where the flow takes place in all three flowing directions, from Table 7-IV one obtains the matrix block size:

$a = [k_f / 0.62 * \phi_f^3]^{0.5} = [0.117(D) / 0.62 * 0.018\,^3]^{0.5} = 179$ cm $= 1.79$ m

The fracture opening

$b = [100\,\phi_f / 3a(cm)] = [100 * 0.018 / 3 * 179]^{0.5} = 0.0033$ cm $= 33$ μm

# Coning in Fractured Reservoirs

The general considerations concerning the formation and development of coning in a conventional porous reservoir will not change in the case of a fractured reservoir, but the flowing conditions must be reviewed with regard to the specific conditions which govern flow in fractures. The basic equations are almost the same and can be extended to fractured reservoirs, so long as the continuity of the fracture network is developed throughout the oil and water zones, or oil and gas-cap zones.

The fractured reservoir, producing either through an open-hole well or through a cased and perforated well, will have a certain producing pay delimited by two, upper and lower, boundaries (Fig. 7-42):

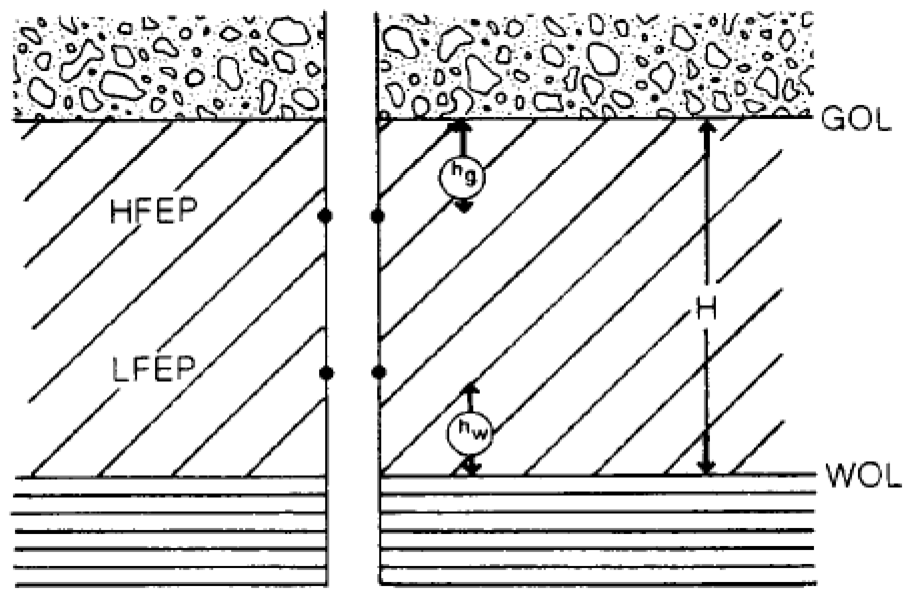
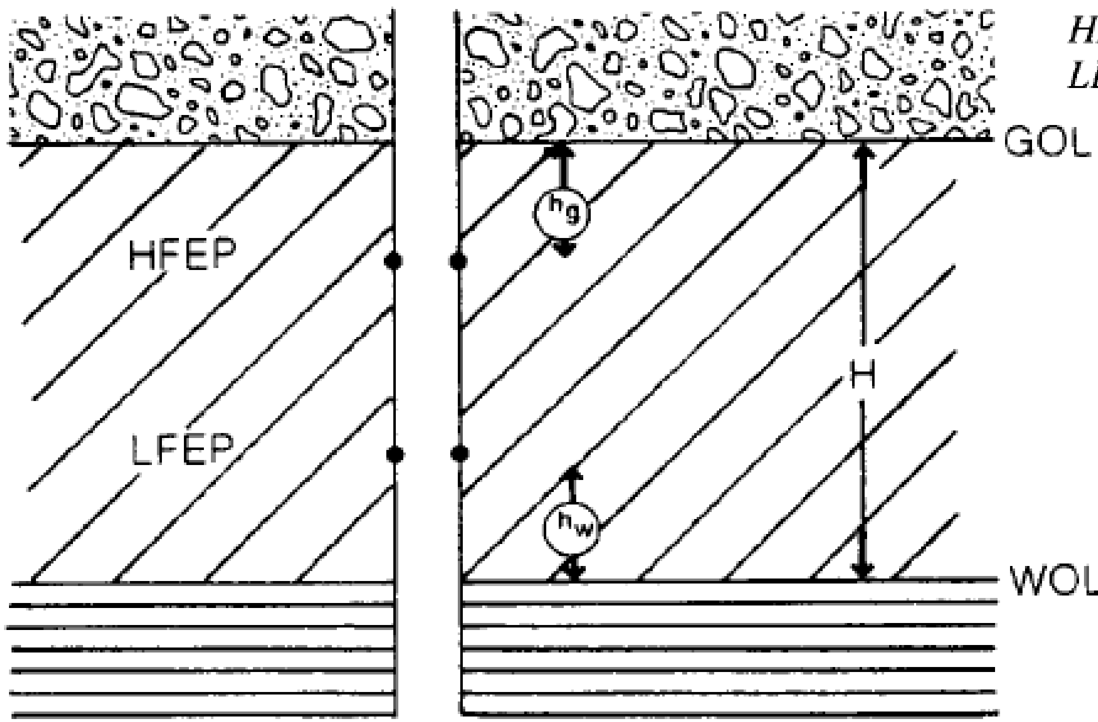



Fig. 7-42. Sketch of water-oil and gas-oil contacts vs. *LFEP* and *HFEP*.

# Coning in Fractured Reservoirs

The evaluation of HFEP and LFEP and respective gas-oil and water-oil contacts in the fracture network (GOL and WOL) will indicate the "non-completed" height ($h_g$ and $h_w$, Fig. 7-42) equivalent to the gas-coning and water-coning heights, respectively.




*HFEP*: the highest fluid entry point, and
*LFEP*: the lowest fluid entry point

Fig. 7-42. Sketch of water-oil and gas-oil contacts vs. *LFEP* and *HFEP*.

# Coning in Fractured Reservoirs

The coning is thus associated with a certain critical radius $r_{cr}$' which will correspond to the limit over which the water will arrive toward the well. Inasmuch as during production of oil through a fractured reservoir the flow toward the well is radial-symmetrical, the pressure distribution will follow a logarithmic variation along the flowing streamline. Around the wellbore, a critical zone of radius $r_{cr}$ is associated with the possibility of coning as an effect of high pressure gradient (as shown in Fig. 7-43).

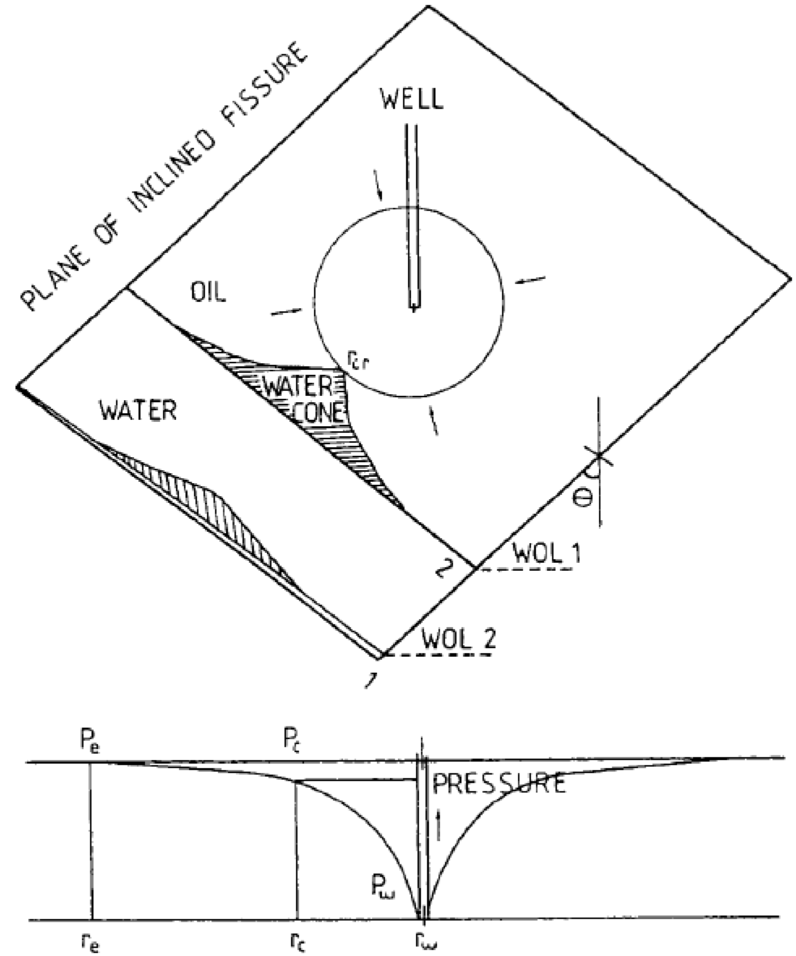



Fig. 7-43. Water coning, function of movable water-oil contact position.

# Coning in Fractured Reservoirs

As a consequence, for both contacts (gas-oil and water-oil) the coning criteria values will always be represented by a critical and a safe coning value. Critical coning is obtained in relation to the laminar flow (B T = 0) and thus is expressed by:

$$h_{cr} = \Delta p_{laminar} / \Delta\gamma * \ln (r_e / r_w) = A * Q / \Delta\gamma * 6.9 \quad (7\text{-}58)$$

where it is assumed that $(r_e / r_w) = 1000$.

Safe coning height in both flowing states, i.e., laminar and turbulent, is equal to

$$h_{SAFE} = \Delta p_{total}\ 6.9 * \Delta\gamma = (A * Q + B_T * Q^2) / 6.9 * \Delta\gamma \quad (7\text{-}59)$$

As a general rule, the coning criteria have to be associated with the reservoir pay thickness.

For pay 200 ft < $h$ < 1000 ft Eq. 7-58 is used for coning evaluation.

For pay $h$ < 200 ft, Eq. 7-59 is used for coning evaluation.

# Coning in Fractured Reservoirs

The explicit coning evaluations for gas-oil and water-oil are:

Water-oil coning $h_{wo-cr} = A*Q/6.9*\Delta\gamma_{wo}$ (7-60a)

Water-oil coning $h_{wo-SAFE} = [A*Q+B_T*Q^2]/6.9*\Delta\gamma_{wo}$ (7-60b)

Gas-oil coning $h_{go-cr} = A*Q/6.9*\Delta\gamma_{go}$ (7-60c)

Gas-oil coning $h_{go-SAFE} = [A*Q+B_T*Q^2]/6.9*\Delta\gamma_{go}$ (7-60d)