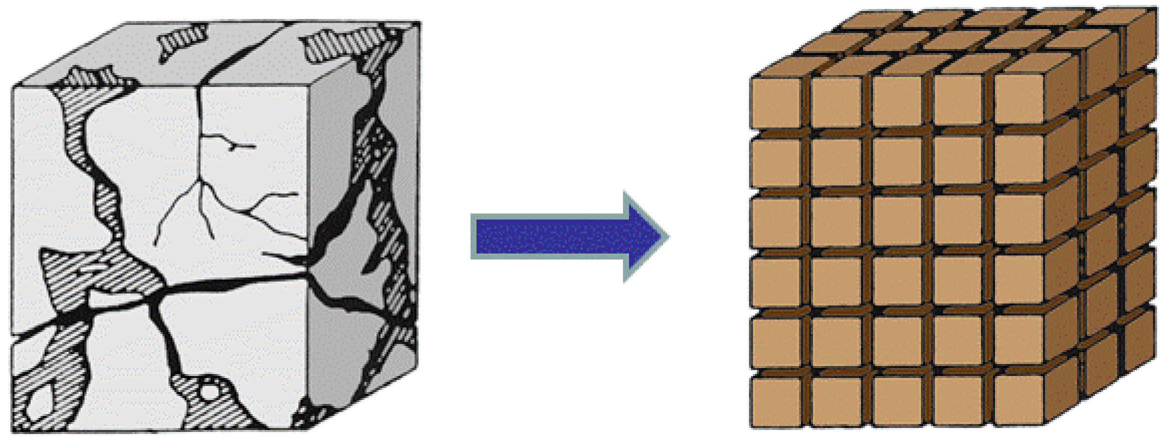
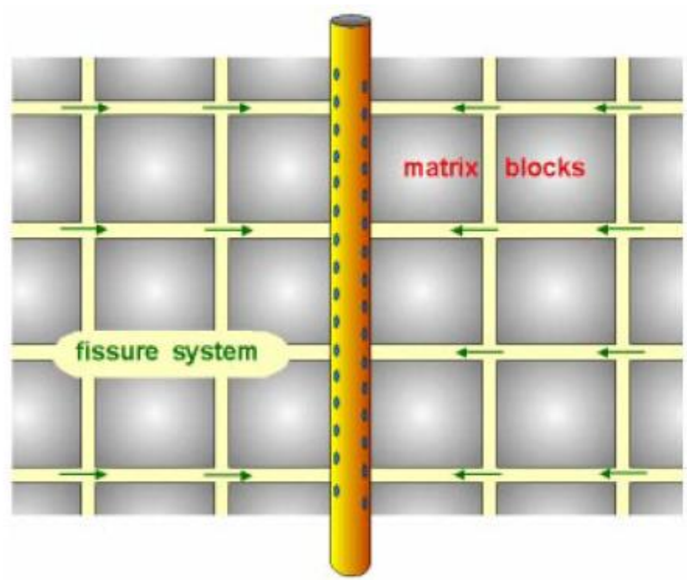
Naturally Fractured Reservoirs

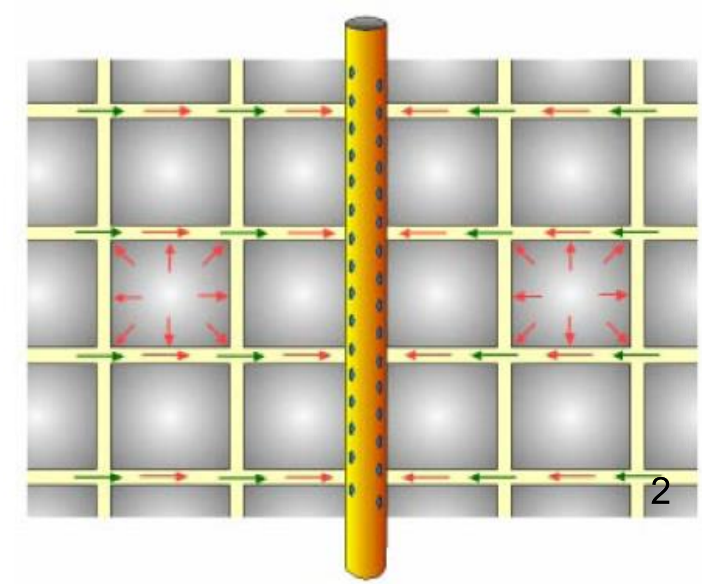
Well Testing of Naturally Fractured Reservoirs (b)

Shahab Gerami, Fall 2010

Well Test Analysis on the Basis of Warren & Root Model







Dual Porosity Parameters Storativity Ratio

$$\omega = \frac{\phi_2 c_2}{\left(\phi_1 c_1 + \phi_2 c_2\right)}$$

The storativity ratio is a measure of the pore space in the fracture system relative to the total pore space.

For naturally fractured reservoirs, ω will normally be in the range of 10^{-2} to 10^{-5}. For layered reservoirs, ω may be as high as 0.1.

Values higher than 0.1 usually do not exhibit dual porosity behavior.

Nomenclature:

ω - storativity ratio, dimensionless

ϕ_m - matrix porosity, dimensionless

ϕ_f - fracture porosity ($\cong$1.0), dimensionless

c_{tm} - total compressibility of matrix porosity and fluids, psi^{-1}

c_{tf} - total compressibility of fracture porosity and fluids, psi^{-1}

Dual Porosity Parameters Interporosity Flow Coefficient

$$\lambda = \frac{\alpha k_m r_w^2}{k_f}$$

$$\alpha = \frac{4n(n+2)}{L_m^2}$$

The interporosity flow coefficient λ is a measure of the ability of fluids to flow from the matrix to the natural fracture system, relative to the ability of fluids to flow from the fracture system to the wellbore.

The interporosity flow coefficient λ is not a pure property of the reservoir rock, because it includes the wellbore radius.

For naturally fractured reservoirs, λ will usually be in the range of 10^{-3} to 10^{-8}. Larger values of λ cause the effects of dual porosity behavior to end very quickly. In this case, the dual porosity behavior is often obscured by wellbore storage. Smaller values of λ will cause the dual porosity behavior to occur much later in time, and may not be apparent before the end of the test.

Nomenclature

λ - interporosity flow coefficient, dimensionless

k_m - matrix permeability, md

k_f - bulk fracture permeability, md

r_w - wellbore radius, ft

α - shape factor, ft^{-2}

n - number of sets of mutually orthogonal fractures

L_m - characteristic size of matrix blocks

Interporosity Flow Coefficient

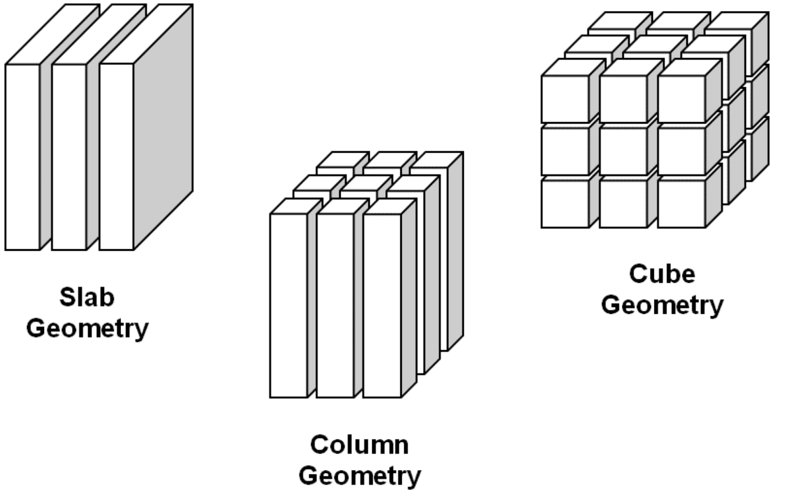
The "characteristic matrix dimension" in the definition of λ is simply the width of a matrix block, if the blocks are the same dimensions in each direction. If the blocks are different sizes in different directions, then L_m is given by the expressions in the third column of this table.

Nomenclature:

a - width of matrix block

b - length of matrix block

c - height of matrix block



Geometry	n	L_m	λ
Slabs	1	a	$\frac{12k_m r_w^2}{k_f L_m^2}$
Columns	2	$\frac{2ab}{(a+b)}$	$\frac{32k_m r_w^2}{k_f L_m^2}$
Cubes	3	$\frac{3abc}{(ab+bc+ca)}$	$\frac{60k_m r_w^2}{k_f L_m^2}$

Interprosity Flow Parameter

The factor α is the block-shape parameter that depends on the geometry and the characteristic shape of the matrix–fissures system and has the dimension of a reciprocal of the area.

$$\alpha = \frac{A}{Vx}$$

where:

A = surface area of the matrix block, ft^2
V = volume of the matrix block
x = characteristic length of the matrix block, ft

Most of the proposed models assume that the matrix–fissures system can be represented by one the following four geometries:

(a) *Cubic* matrix blocks separated by fractures with λ as given by:

$$\lambda = \frac{60}{l_m^2}\left(\frac{k_m}{k_f}\right)r_w^2$$

where l_m is the length of a block side.

(b) *Spherical* matrix blocks separated by fractures with λ as given by:

$$\lambda = \frac{15}{r_m^2}\left(\frac{k_m}{k_f}\right)r_w^2$$

where r_m is the radius of the sphere.

(c) *Horizontal strata* (rectangular slab) matrix blocks separated by fractures with λ as given by:

$$\lambda = \frac{12}{h_f^2}\left(\frac{k_m}{k_f}\right)r_w^2$$

where h_f is the thickness of an individual fracture or high-permeability layer.

(d) *Vertical cylinder* matrix blocks separated by fractures with λ as given by:

$$\lambda = \frac{8}{r_m^2}\left(\frac{k_m}{k_f}\right)r_w^2$$

where r_m is the radius of the each cylinder

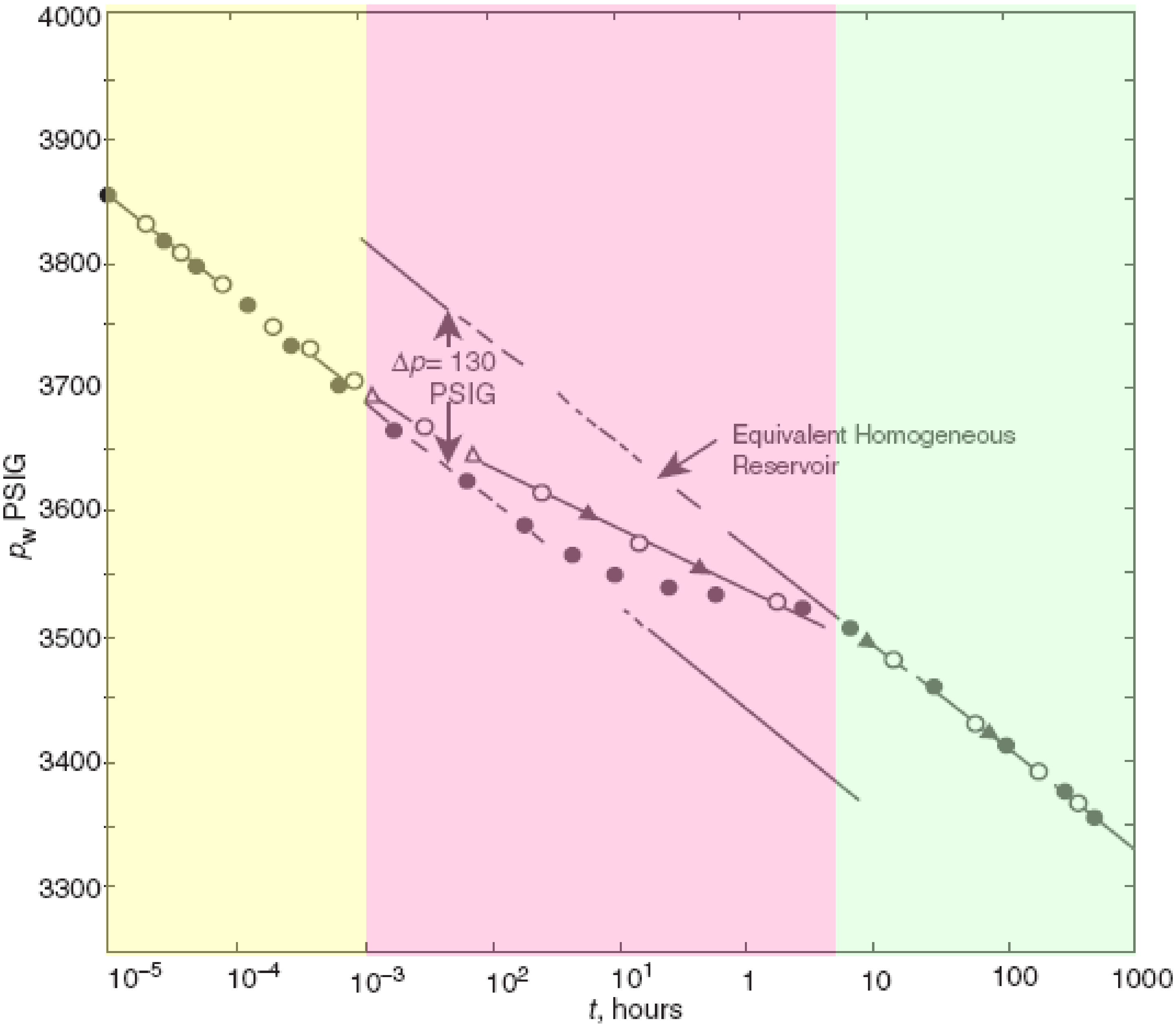


Figure 1.61 *Pressure drawdown according to the model by Warren and Root (Copyright ©1969 SPE, Kazemi,[7] SPEJ, Dec. 1969).*

- The curve is characterized by *two parallel straight lines* due to the two separate porosities in the reservoir.
- Because the secondary porosity (fissures) has the greater transmissivity and is connected to the wellbore, it responds first as described by the first semilog straight line.
- The primary porosity (matrix), having a much lower transmissivity, responds much later. The combined effect of the two porosities gives rise to the second semilog straight line.
- The two straight lines are separated by a transition period during which the pressure tends to stabilize.

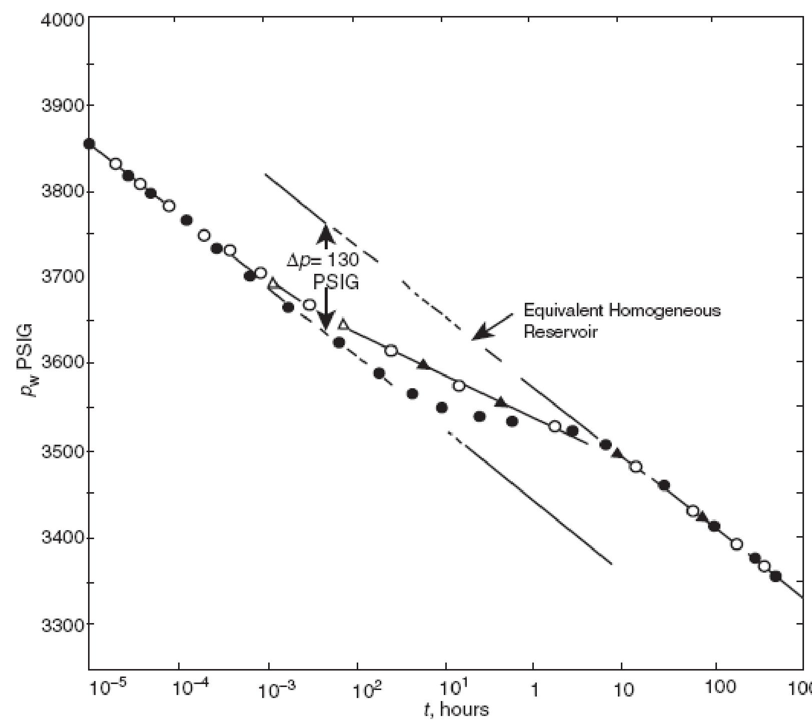
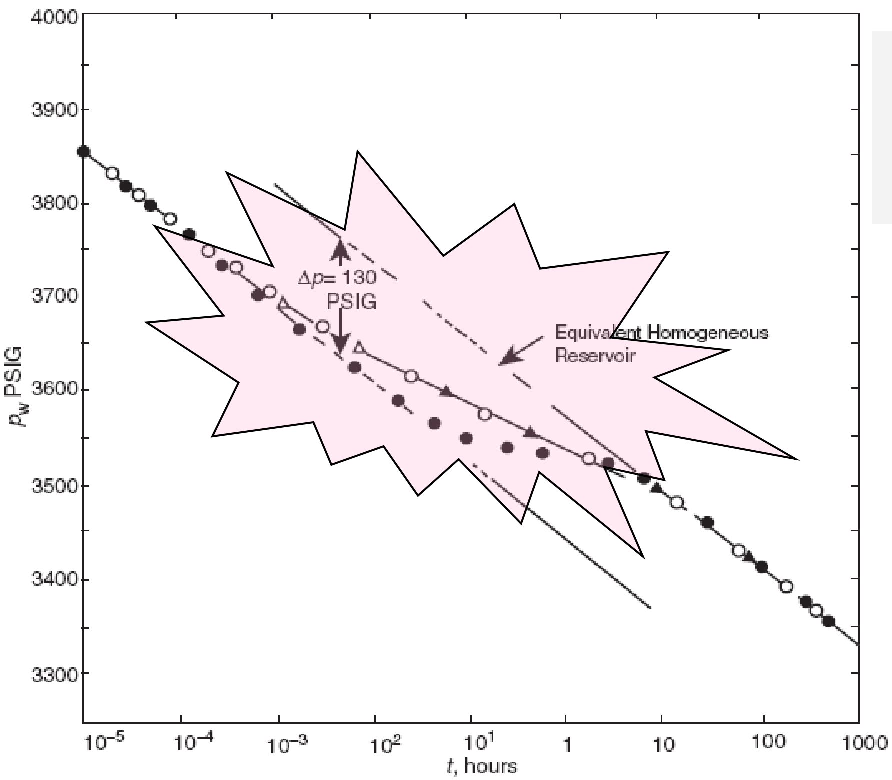


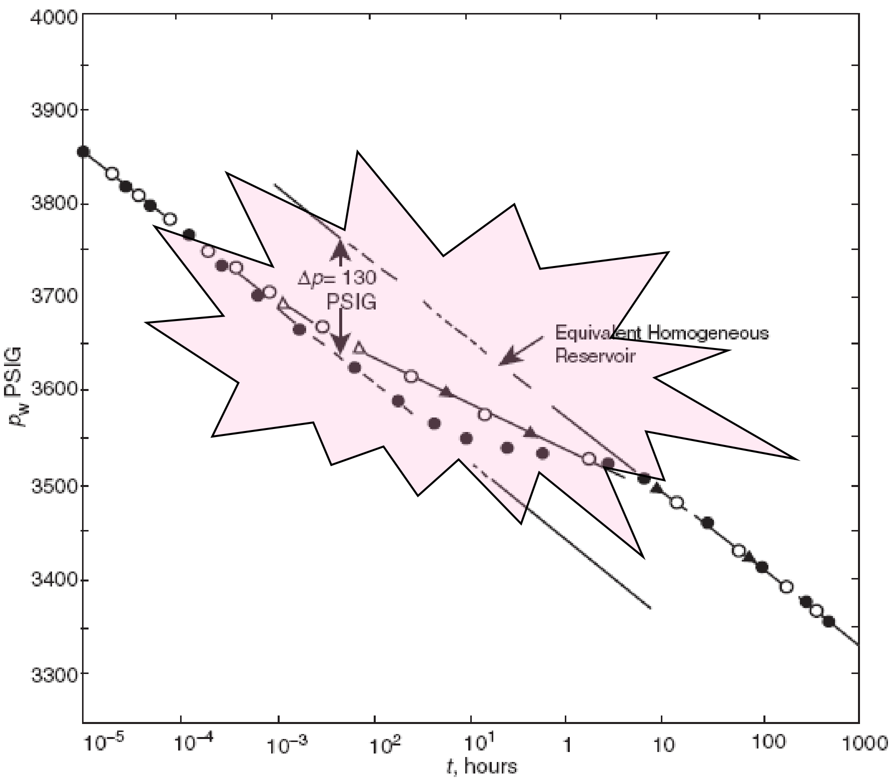
Figure 1.61 *Pressure drawdown according to the model by Warren and Root (Copyright ©1969 SPE, Kazemi, SPEJ, Dec. 1969).*

- In theory, double-porosity behavior yields two parallel straight lines on a semi-log plot, provided there is no wellbore nor outer boundary effects.
- The semi-log plot consists of three sections:
  - (i) the first straight line, which represents the homogeneous behavior of the naturally fractured medium before the matrix medium starts to respond (transient radial flow) — the slope of this line gives the fracture permeability;
  - (ii) a transition section (between two straight lines), which corresponds to the onset of inter-porosity flow;
  - (iii) the second semi-log straight line, which represents the homogeneous behavior of composite media (fracture permeability with the sum of matrix and fracture storages) when recharge from the matrix medium is fully established.

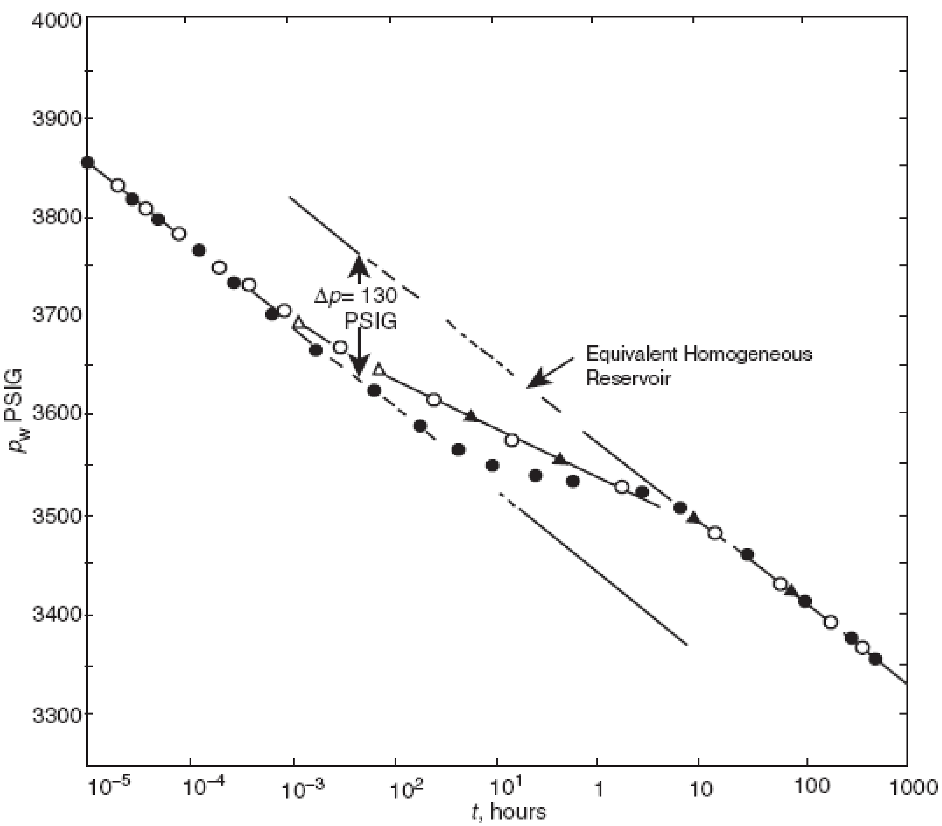


- The nature of matrix and fracture interaction is manifested during the transitional period of matrix-to-fracture fluid transfer.

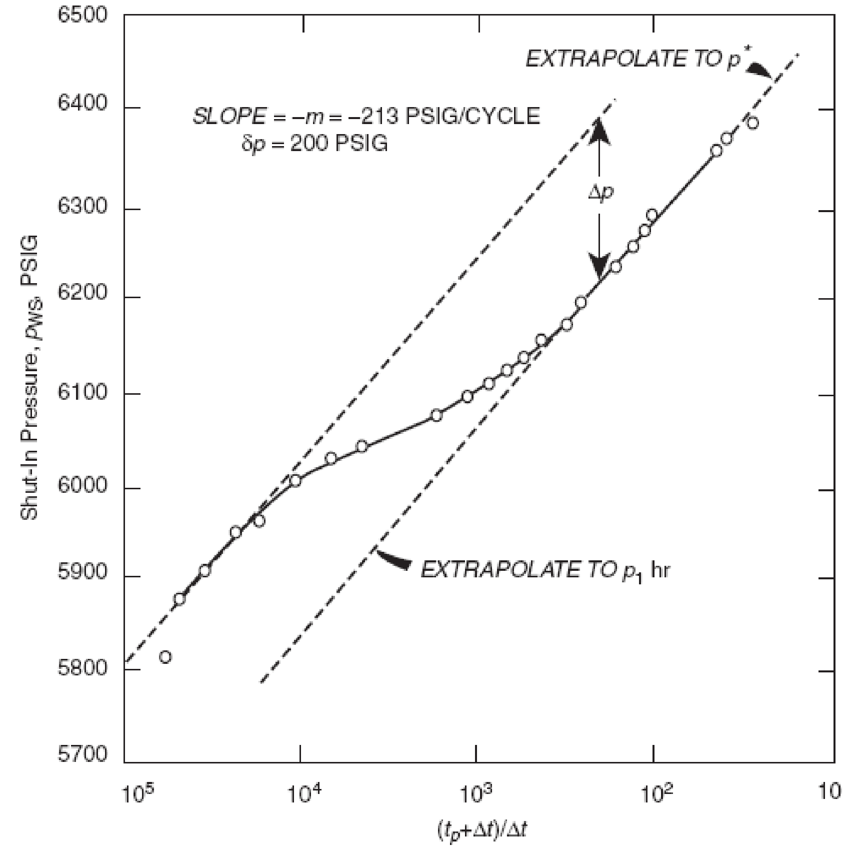
- The characteristics of the transitional segment are determined by the way the matrix and fracture interact.
- Flow from matrix to fractures takes place according to the assumptions used in the available double porosity models:
  1. The flow rate is proportional to the pressure difference between matrix and fracture (Warren and Root, 1963)
  2. The flow rate is proportional to the averaged pressure gradient through the matrix (Streltsova, 1983)
  3. The flow rate is an unsteady state function of pressure drop across the matrix(Kazemi, 1969; deSwaan, 1976, and Najurrieta, 1980)



Drawdown



Buildup



For a well producing at a constant rate from an infinite, naturally fractured reservoir with the assumption that matrix-to-fracture flow occurs under instantaneously established pseudosteady state conditions, Warren and Root derived drawdown and buildup equations. Useful forms of their equations were presented by Kazemi (1969):

For drawdown:

$$P_i - P_{wf} = \frac{162.6\ q\ B\ \mu}{k_f\ (h_f + h_m)} \Bigg[\log t + \log \frac{k}{(\phi_m C_m + \phi_f C_f)\ \mu\ r_w^2} - 3.23$$

$$+ 0.435\ \mathrm{Ei}\left[-\lambda t_D/\omega\ (1-\omega) \right]$$

$$- 0.435\ \mathrm{Ei}\left[-\lambda t_D/(1-\omega) \right] + 0.87s \Bigg] \qquad (6\text{-}1a)$$

For buildup:

$$P_i - P_{ws} = \frac{162.6\ q\ B\ \mu}{k_f\ (h_f + h_m)} \Bigg[\log \frac{t_p + \Delta t}{\Delta t} - 0.435\ \mathrm{Ei}\left[-\lambda\ \Delta t_D/\omega(1-\omega) \right]$$

$$+ 0.435\ \mathrm{Ei}\left[-\lambda\ \Delta t_D/(1-\omega) \right] \Bigg] \qquad (6\text{-}1b)$$

where $t_D = \dfrac{2.64 \times 10^{-4}\ k_f\ t}{(\phi_f C_f + \phi_m C_m)\ \mu\ r_w^2}$

and, $\Delta t_D = \dfrac{2.64 \times 10^{-4}\ k_f\ \Delta t}{(\phi_f C_f + \phi_m C_m)\ \mu\ r_w^2}$

When t or Δt is small, the value of the first Ei function in Equations 6-1a and 6-1b will be close to zero, whereas the second Ei function will be a constant. Thus a plot of P_{wf} versus t or P_{ws} versus $[(t_p + \Delta t)/\Delta t]$ on semi-log graph paper should yield a straight line of slope m. From the slope of the straight line, one can determine $k_f(h_m + h_f)$. As t or Δt increases, however, the Ei functions exert their influence and we get the transitional segment of the curve shown in Figure 6-3.

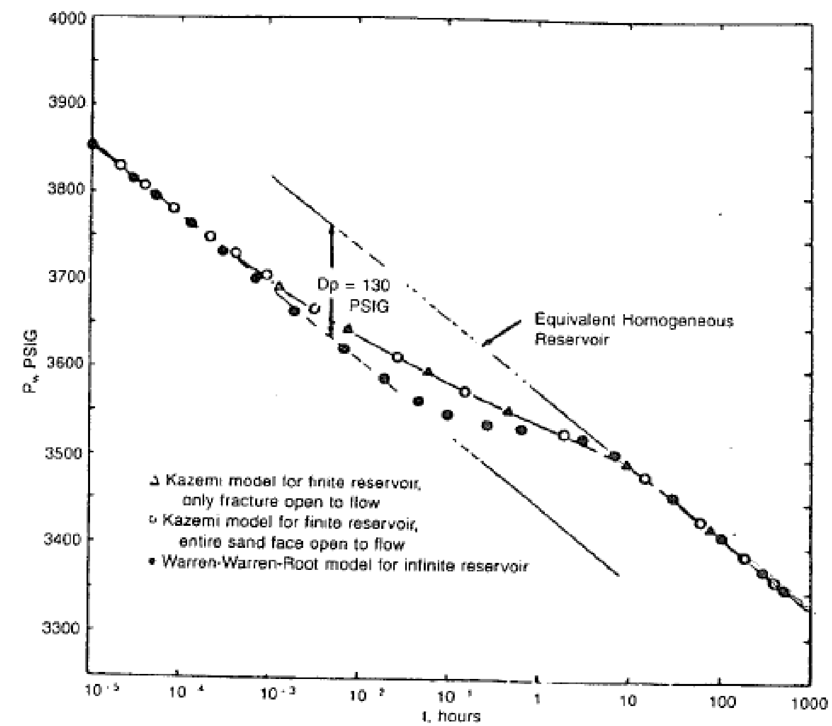


Figure 6-3. Pressure drawdown according to the model by Warren and Root. Copyright © 1969 SPE, Kazemi, *SPEJ*, Dec. 1969 [8].

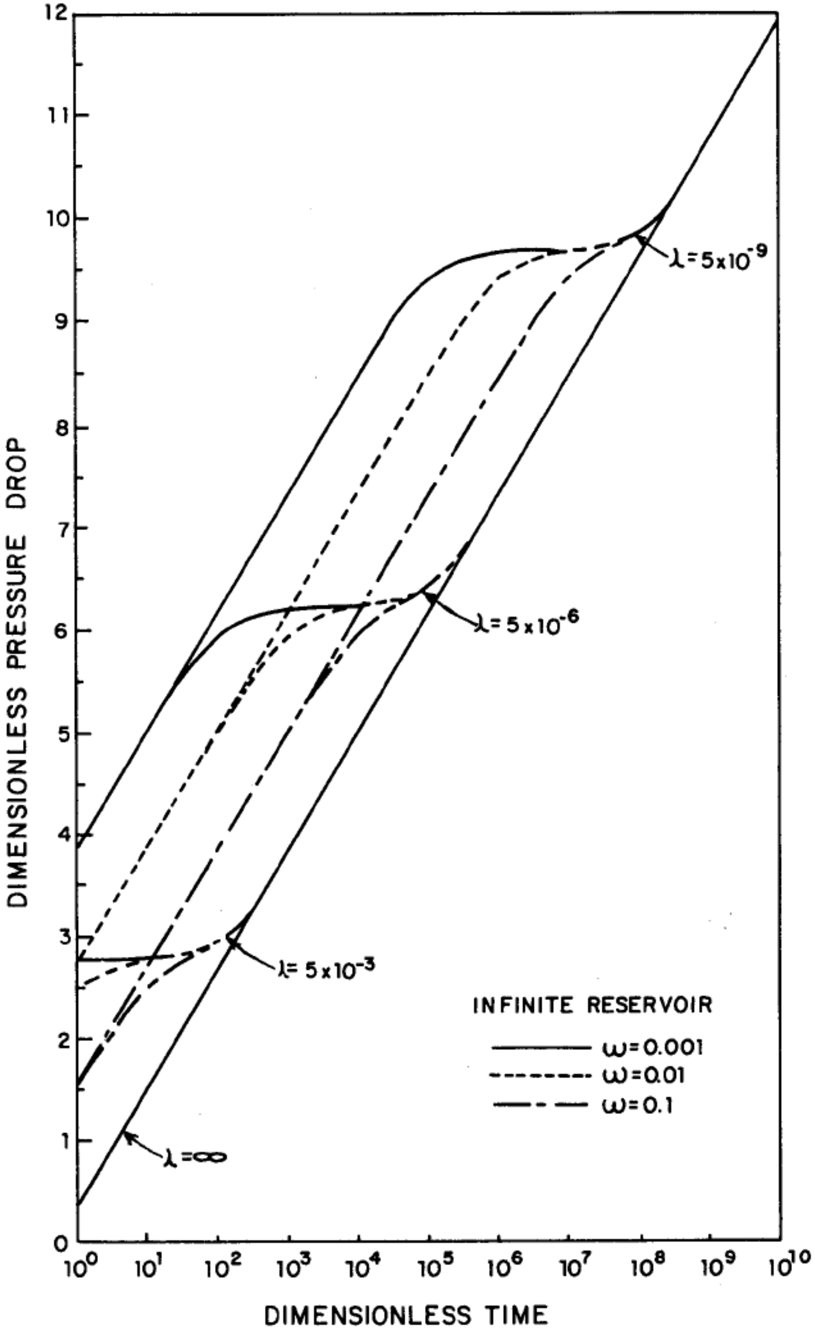


Fig. 2-5. Semi-log plot showing the dimensionless pressure solutions as a function of time for several values of ω and λ (after Warren and Root, 1963). Drawdown case; infinite reservoir. Courtesy of SPE-AIME.

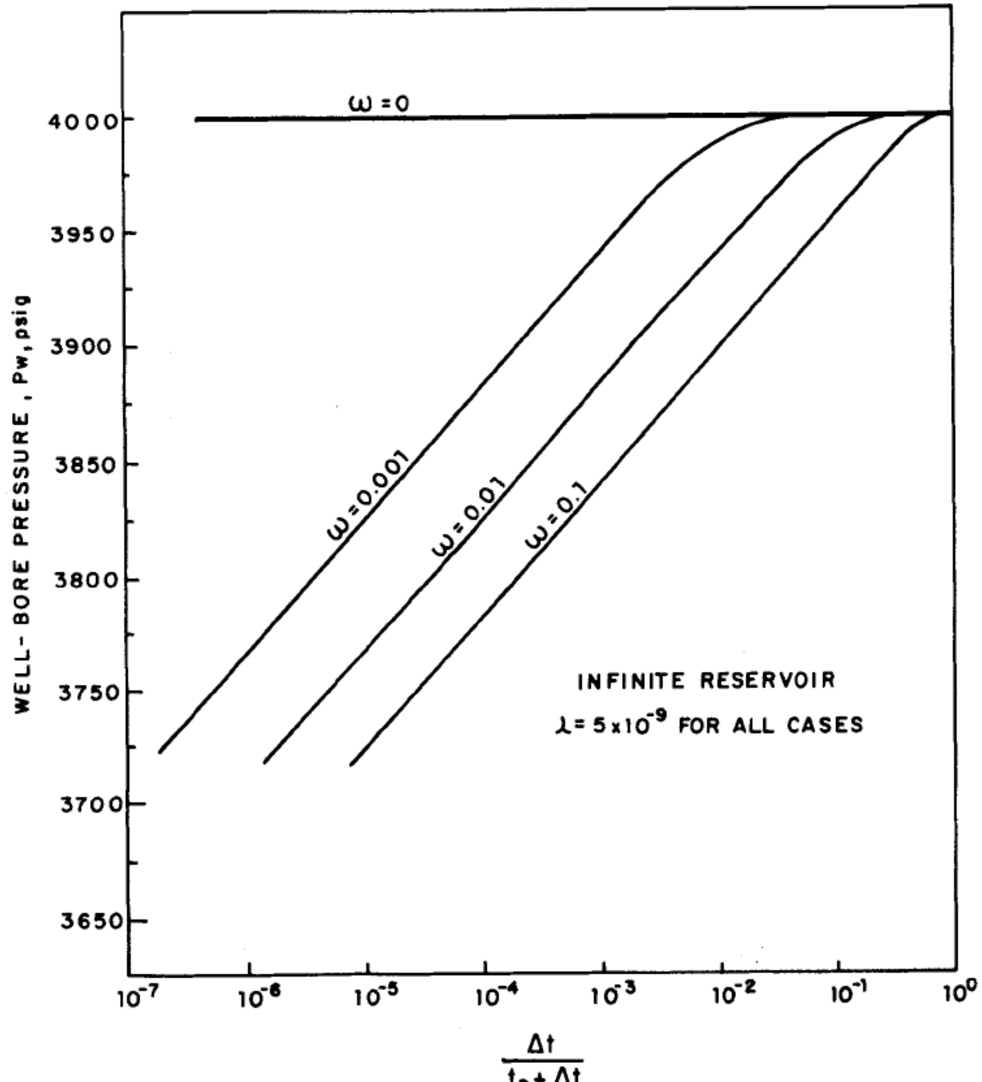
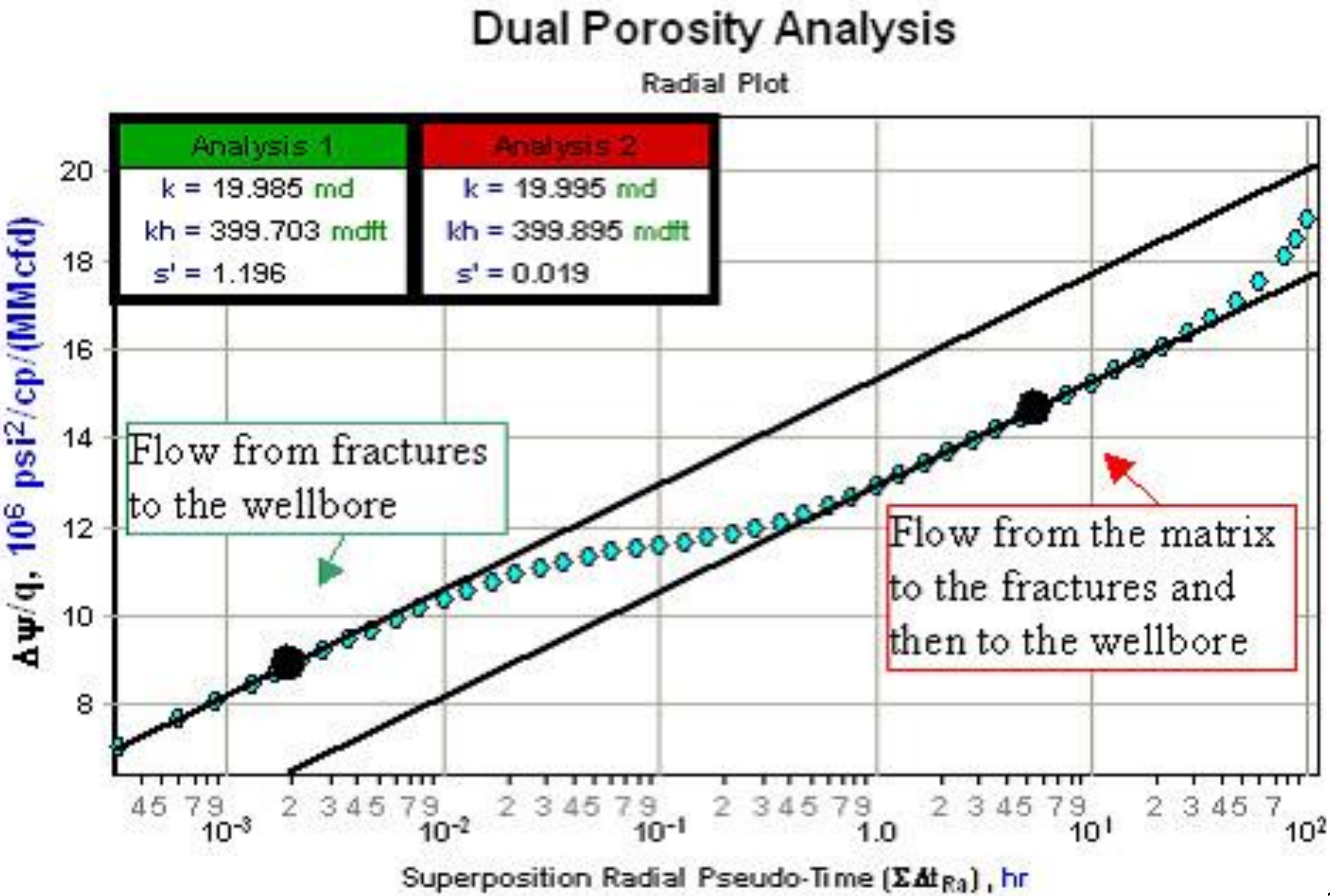
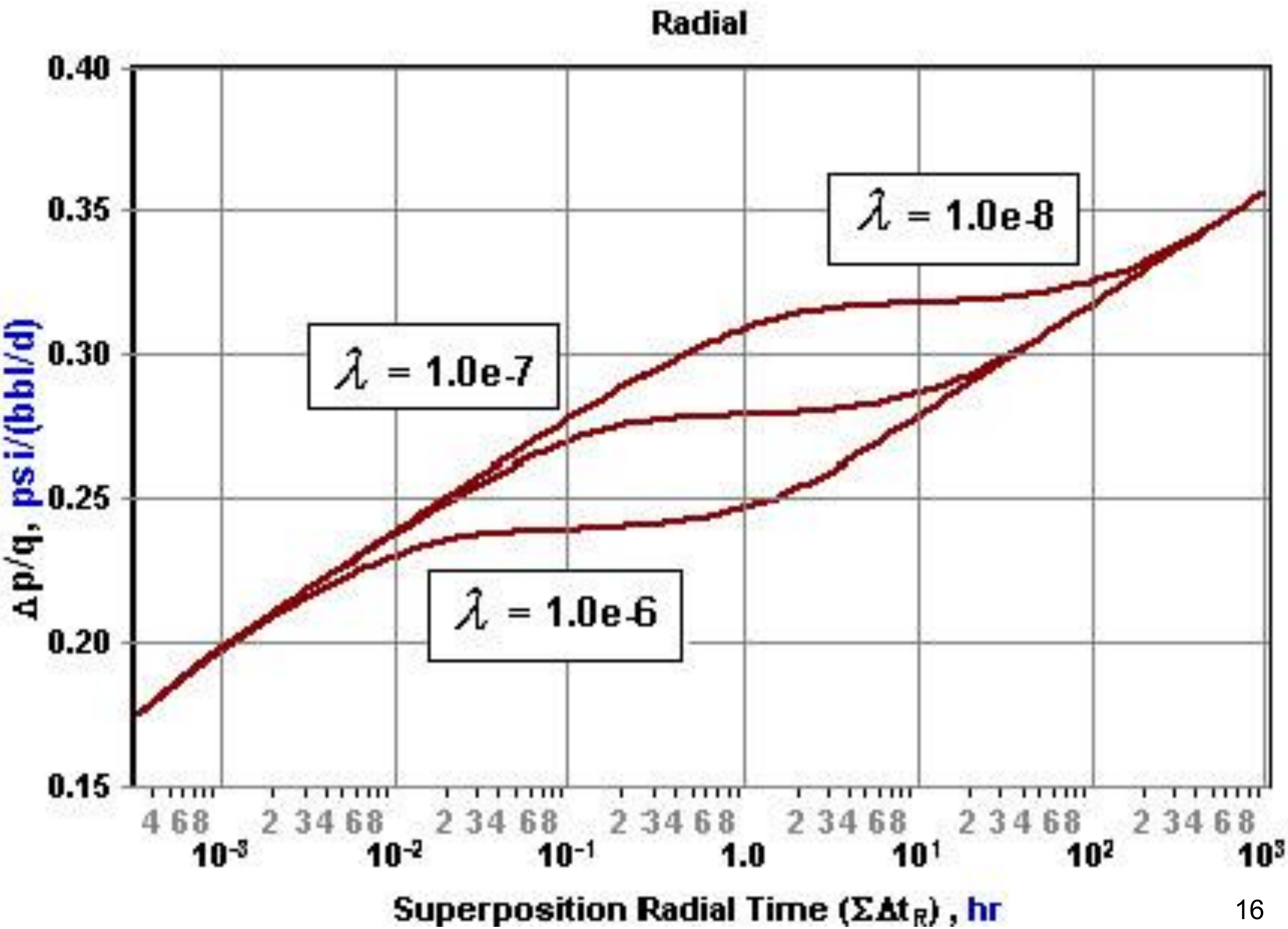
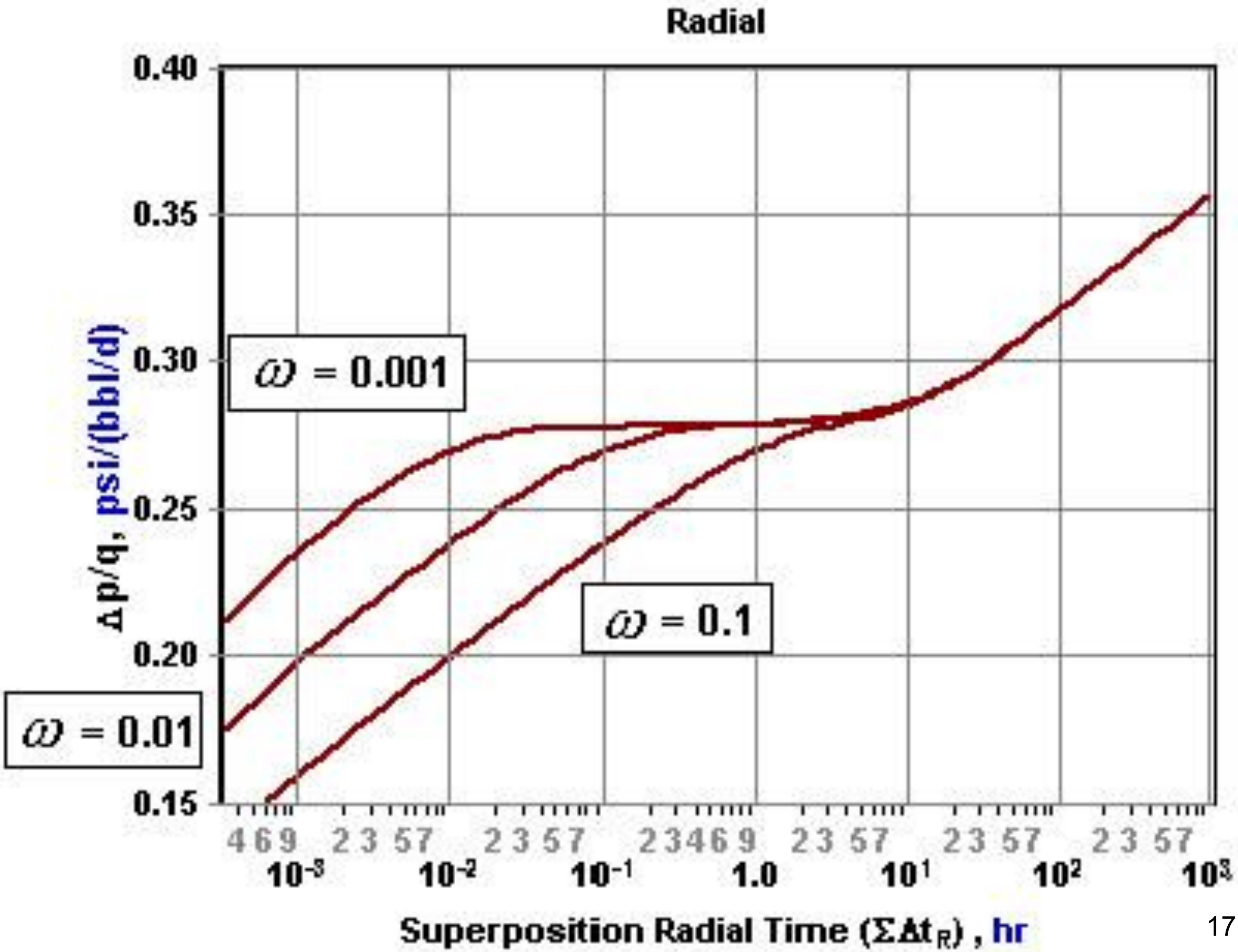


Fig. 2-6. Horner-type plot showing several pressure solutions depending on the value of ω (after Warren and Root, 1963). Fixed λ, equal to 5×10^{-6}; infinite reservoir. In this case, two semi-log straight lines are present for all values of ω. Courtesy of SPE-AIME.

The signature of dual porosity systems on a semi-log plot is two parallel lines as shown below.

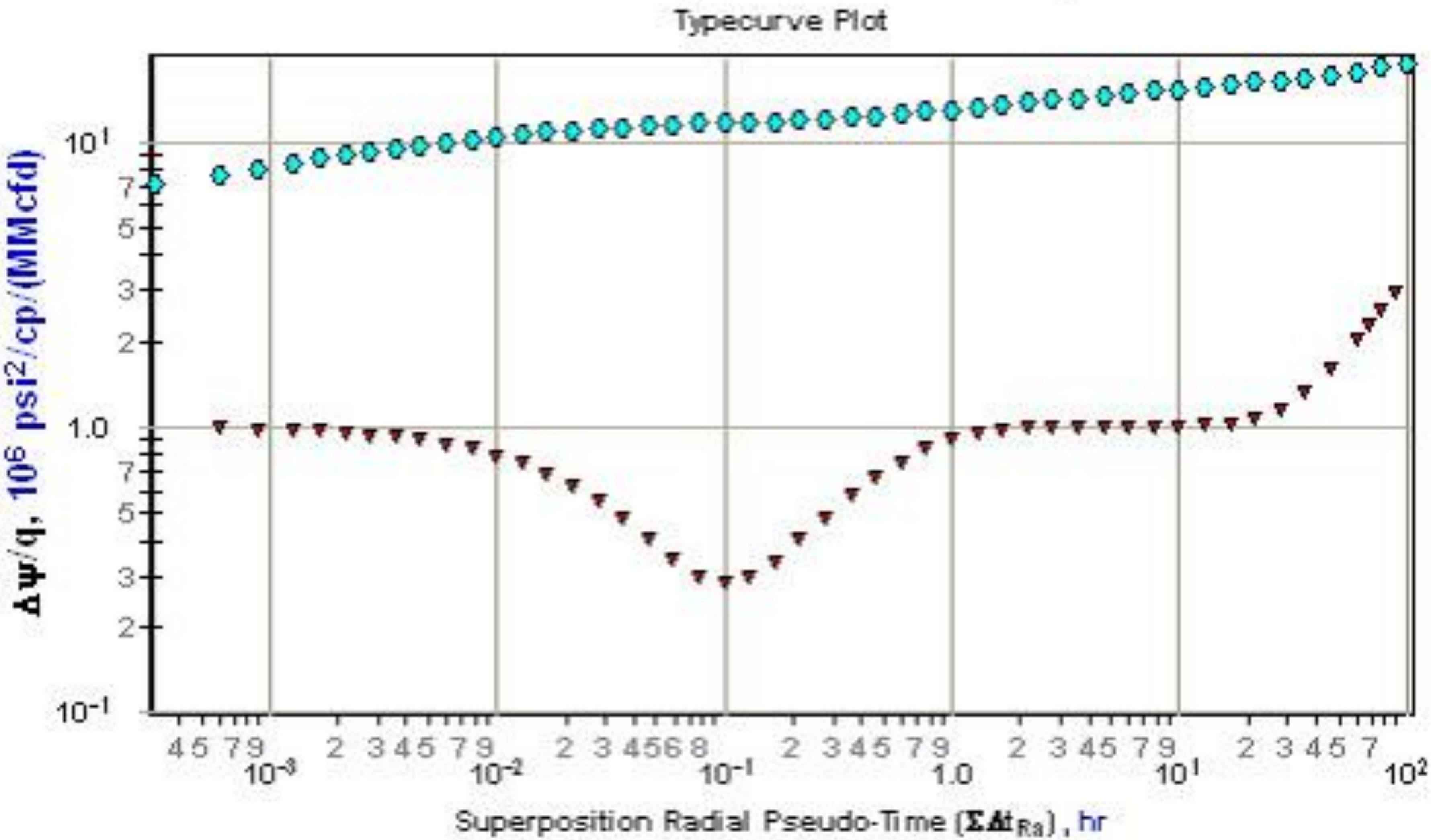


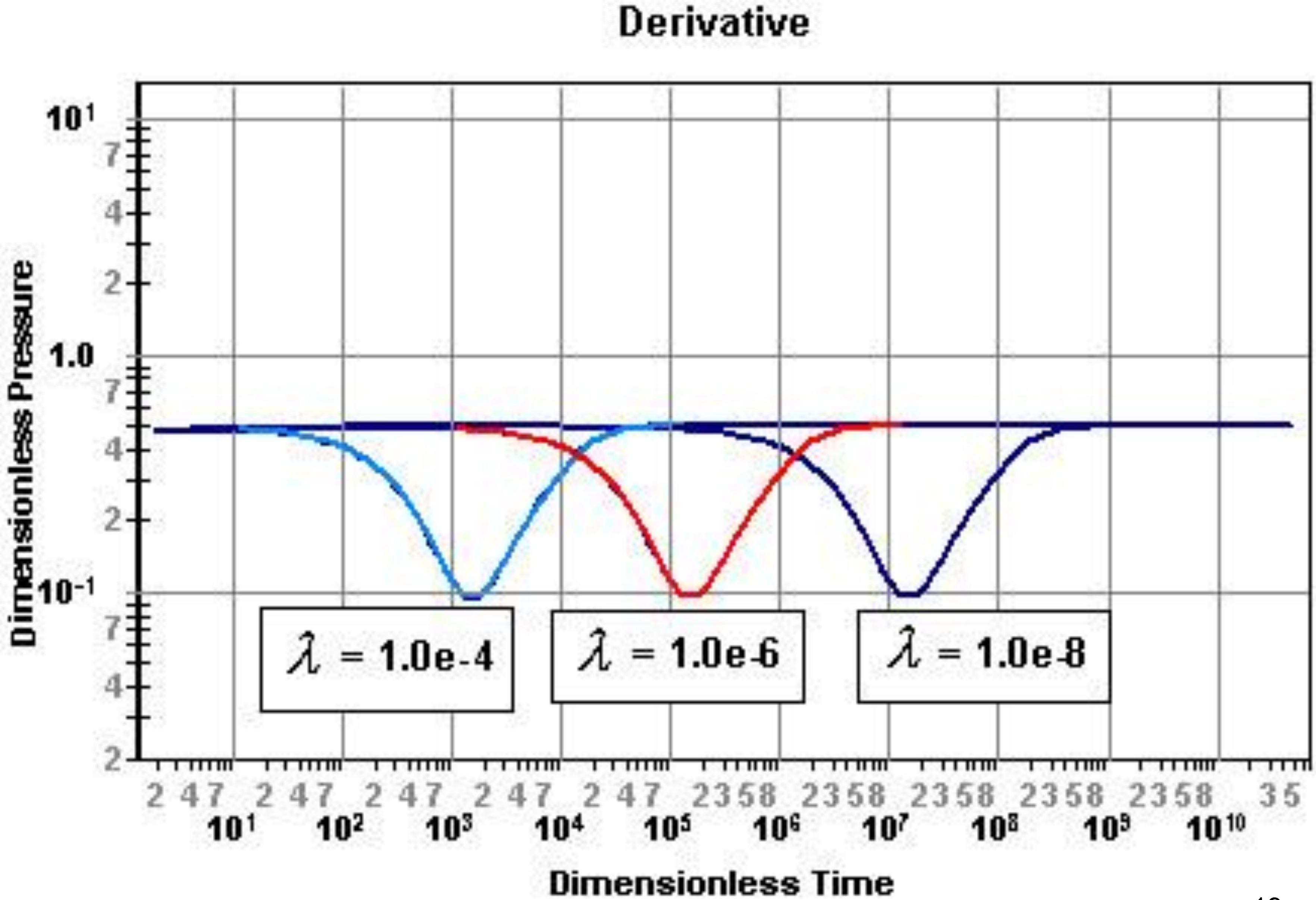

Radial
0.40
0.35
0.30
0.25
0.20
0.15
$\hat{\lambda}$ = 1.0e-8
$\hat{\lambda}$ = 1.0e-7
$\hat{\lambda}$ = 1.0e-6
Δp/q, psi/(bbl/d)
4 6 8 2 3 4 6 8 2 3 4 6 8 2 3 4 6 8 2 3 4 6 8 2 3 4 6 8 2 3 4 6 8
10^{-3} 10^{-2} 10^{-1} 1.0 10^{1} 10^{2} 10^{3}
Superposition Radial Time ($\Sigma\Delta t_R$), hr



Radial
0.40
0.35
0.30
0.25
0.20
0.15
Δp/q, psi/(bbl/d)
ω = 0.001
ω = 0.1
ω = 0.01
4 6 9 2 3 5 7 2 3 5 7 2 3 4 6 9 2 3 5 7 2 3 5 7 2 3 5 7
10⁻³ 10⁻² 10⁻¹ 1.0 10¹ 10² 10³
Superposition Radial Time (ΣΔt_R) , hr

Dual Porosity Derivative Analysis




Derivative
Dimensionless Pressure
Dimensionless Time
10^1
1.0
10^{-1}
10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10}
$\hat{\lambda}$ = 1.0e-4
$\hat{\lambda}$ = 1.0e-6
$\hat{\lambda}$ = 1.0e-8

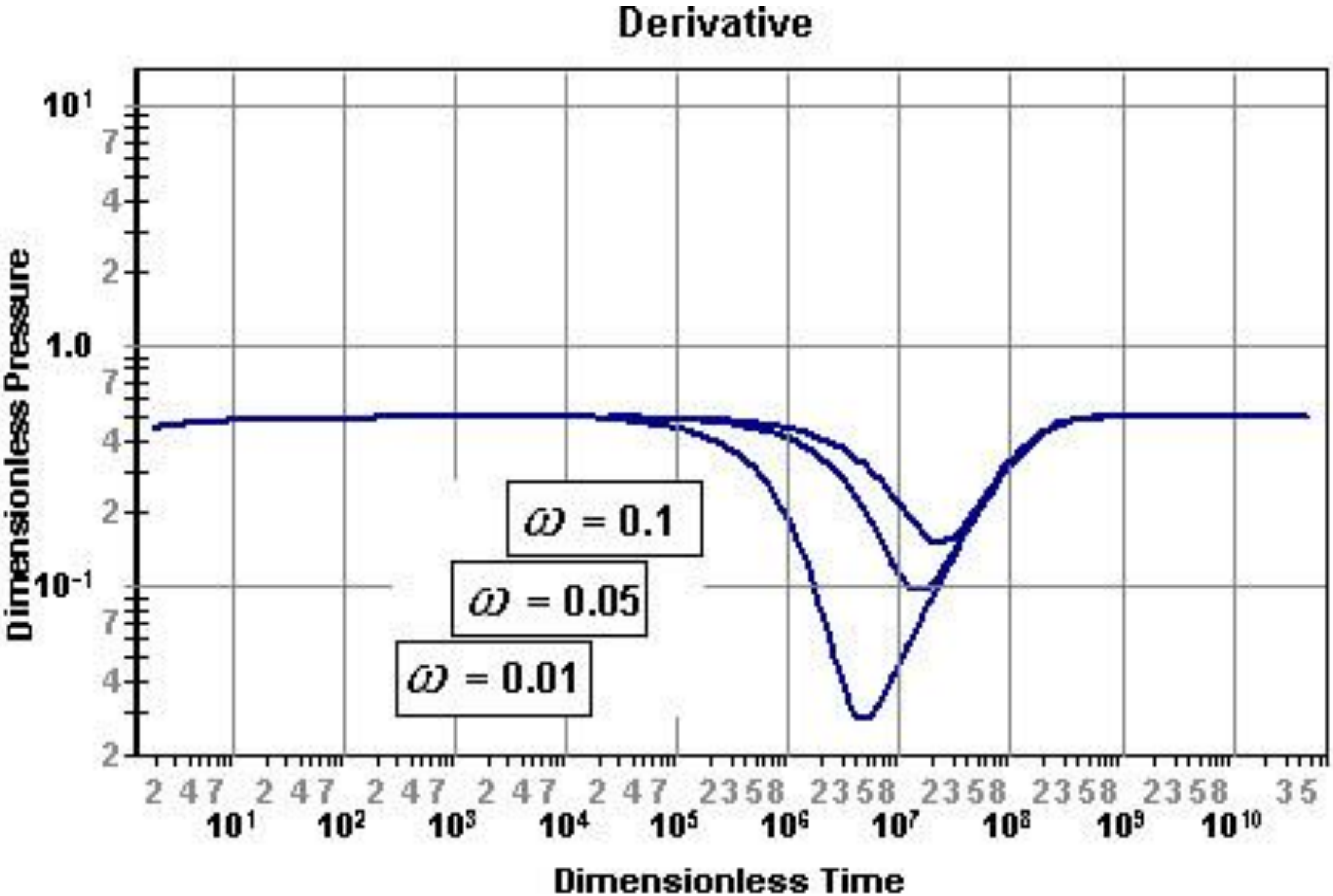

Derivative
Dimensionless Pressure
Dimensionless Time
ω = 0.1
ω = 0.05
ω = 0.01
10^1
1.0
10^-1
10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^10

Figure 1.62 shows the pressure buildup data for a naturally fractured reservoir. As for the drawdown, wellbore storage effects may obscure the first semilog straight line. If both semilog straight lines develop, analysis of the total permeability–thickness product is estimated from the slope m of either straight line and the use of Equation 1.3.8, or:

$$(k_f h) = \frac{162.6QB\mu}{m}$$

Bourdet and Gringarten (1980) indicated that by drawing a horizontal line through the *middle* of the transition curve to intersect with both semilog straight lines, as shown in Figures 1.61 and 1.62, the interporosity flow coefficient λ can be determined by reading the corresponding time at the *intersection* of either of the two straight lines, e.g. t_1 or t_2, and applying the following relationships:

In drawdown tests:

$$\lambda = \left[\frac{\omega}{1-\omega}\right]\left[\frac{(\phi h c_t)_m \mu r_w^2}{1.781 k_f t_1}\right] = \left[\frac{1}{1-\omega}\right]\left[\frac{(\phi h c_t)_m \mu r_w^2}{1.781 k_f t_2}\right] \quad [1.5.11]$$

In buildup tests:

$$\lambda = \left[\frac{\omega}{1-\omega}\right]\left[\frac{(\phi h c_t)_m \mu r_w^2}{1.781 k_f t_p}\right]\left(\frac{t_p + \Delta t}{\Delta t}\right)_1$$

or:

$$\lambda = \left[\frac{1}{1-\omega}\right]\left[\frac{(\phi h c_t)_m \mu r_w^2}{1.781 k_f t_p}\right]\left(\frac{t_p + \Delta t}{\Delta t}\right)_2 \quad [1.5.12]$$

where:

k_f = permeability of the fracture, md
t_p = producing time before shut-in, hours
r_w = wellbore radius, ft
μ = viscosity, cp

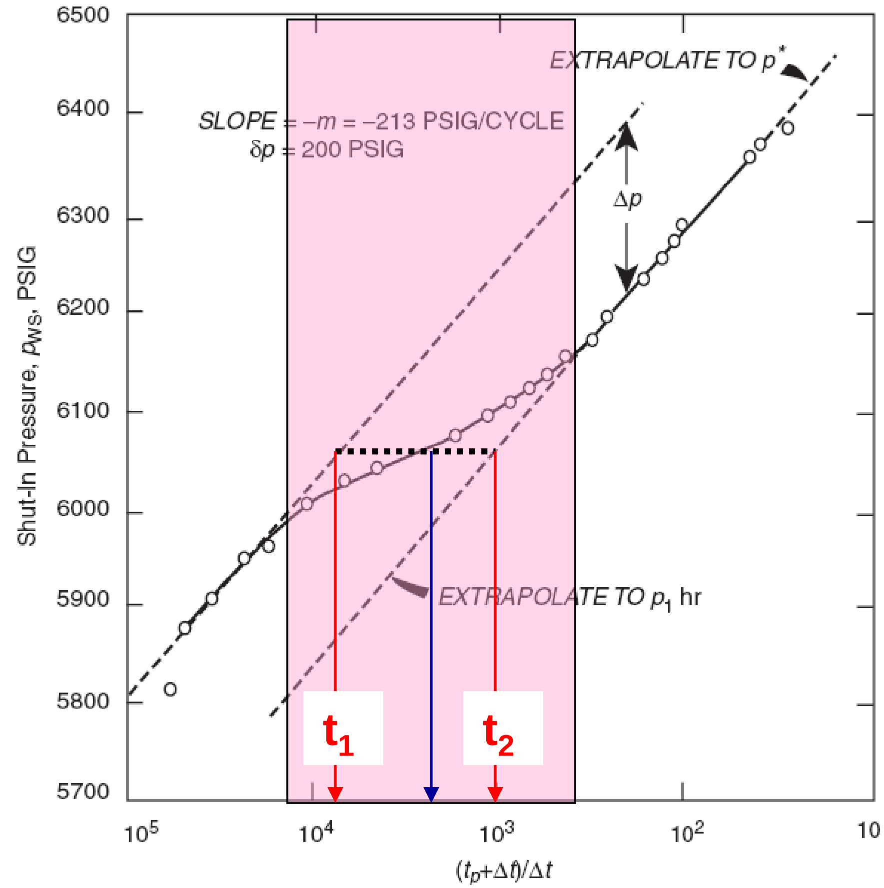


Figure 1.62 Buildup curve from a fractured reservoir (After Warren and Root, 1963).

The skin factor s and the false pressure p^* are calculated as described by using the *second straight line*. Warren and Root indicated that the storativity ratio ω can be determined from the vertical displacement between the two straight lines, identified as Δp in Figures 1.61 and 1.62, by the following expression:

$$\omega = 10^{(-\Delta p/m)} \quad [1.5.10]$$

The subscripts 1 and 2 (e.g., t_1) refer to the first and second line time intersection with the horizontal line drawn through the middle of the transition region pressure response during drawdown or buildup tests.

The above relationships indicate that the value of λ is dependent on the value of ω. Since ω is the ratio of fracture to matrix storage, as defined in terms of the *total* isothermal compressibility coefficients of the matrix and fissures by Equation 1.5.8, thus:

$$\omega = \frac{1}{1 + \left[\dfrac{(\phi h)_m}{(\phi h)_f} \dfrac{(c_t)_m}{(c_t)_f}\right]}$$

it suggests that ω is also dependent on the *PVT* properties of the fluid. It is quite possible for the oil contained in the fracture to be below the bubble point while the oil contained in the matrix is above the bubble point. Thus, ω is pressure dependent and, therefore, λ is greater than 10, so the level of heterogeneity is insufficient for dual porosity effects to be of importance and the reservoir can be treated with a single porosity.

Example 1.34 The pressure buildup data as presented by Najurieta (1980) and Sabet (1991) for a double-porosity system is tabulated below:

Δt (hr)	p_{ws} (psi)	$\frac{t_p+\Delta t}{\Delta t}$
0.003	6617	31 000 000
0.017	6632	516 668
0.033	6644	358 334
0.067	6650	129 168
0.133	6654	64 544
0.267	6661	32 293
0.533	6666	16 147
1.067	6669	8 074
2.133	6678	4 038
4.267	6685	2 019
8.533	6697	1 010
17.067	6704	506
34.133	6712	253

The following additional reservoir and fluid properties are available:

$$p_i = 6789.5 \text{ psi}, \; p_{wf \text{ at } \Delta t=0} = 6352 \text{ psi},$$

$$Q_o = 2554 \text{ STB/day}, \; B_o = 2.3 \text{ bbl/STB},$$

$$\mu_o = 1 \text{ cp}, \; t_p = 8611 \text{ hours}$$

$$r_w = 0.375 \text{ ft}, \; c_t = 8.17 \times 10^{-6} \text{ psi}^{-1}, \; \phi_m = 0.21$$

$$k_m = 0.1 \text{ md}, \; h_m = 17 \text{ ft}$$

Estimate ω and λ.

Solution

Step 1. Plot p_{ws} vs. $(t_p + \Delta t)/\Delta t$ on a semilog scale as shown in Figure 1.63.

Step 2. Figure 1.63 shows two parallel semilog straight lines with a slope of $m = 32$ psi/cycle.

Step 3. Calculate $(k_f h)$ from the slope m:

$$(k_f h) = \frac{162.6 Q_o B_o \mu_o}{m} = \frac{162.6(2556)(2.3)(1.0)}{32}$$

$$= 29\,848.3 \text{ md ft}$$

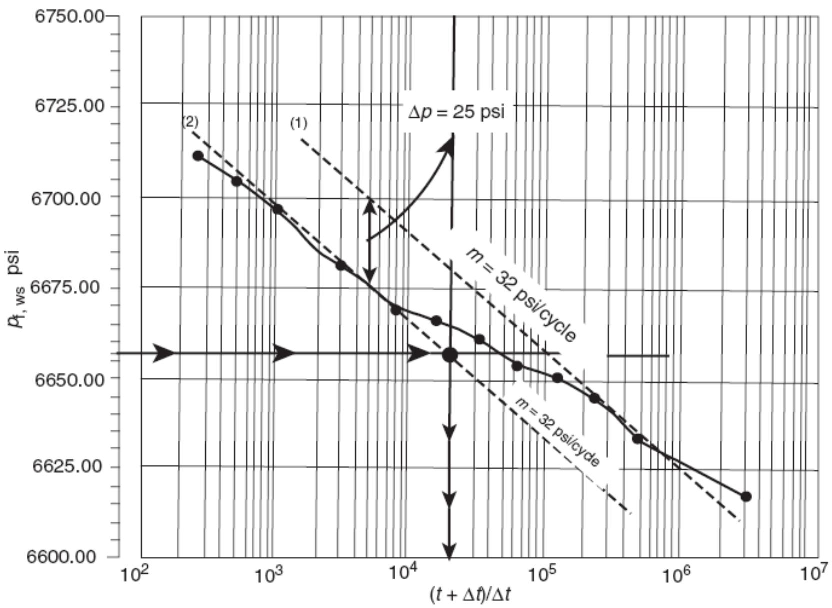


Figure 1.63 Semilog plot of the buildup test data (After Sabet, M. A. Well Test Analysis *1991, Gulf Publishing Company).*

and:

$$k_f = \frac{29848.3}{17} = 1756 \text{ md}$$

Step 4. Determine the vertical distance Δp between the two straight lines:

$$\Delta p = 25 \text{ psi}$$

Step 5. Calculate the storativity ratio ω from Equation 1.5.10:

$$\omega = 10^{-(\Delta p/m)} = 10^{-(25/32)} = 0.165$$

Step 6. Draw a horizontal line through the middle of the transition region to intersect with the two semilog straight lines. Read the corresponding time at the second intersection, to give:

$$\left(\frac{t_p + \Delta t}{\Delta t}\right)_2 = 20000$$

Step 7. Calculate λ from Equation 1.5.12:

$$\lambda = \left[\frac{1}{1-\omega}\right]\left[\frac{(\phi h c_t)_m \mu r_w^2}{1.781 k_f t_p}\right]\left(\frac{t_p + \Delta t}{\Delta t}\right)_2$$

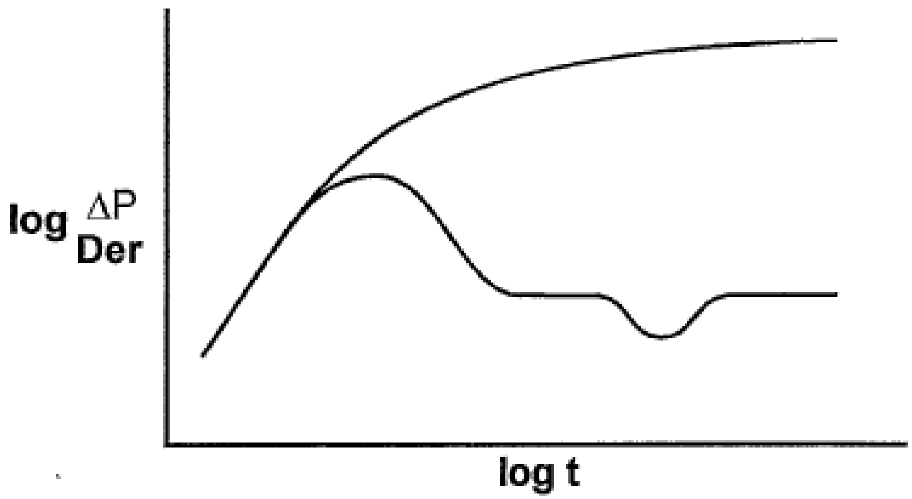
$$= \left[\frac{1}{1 - 0.165}\right]$$

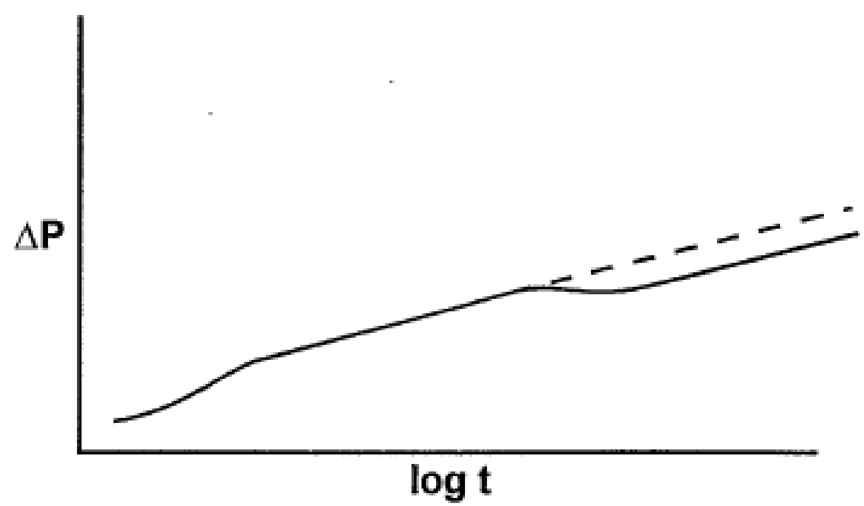
$$\times \left[\frac{(0.21)(17)(8.17 \times 10^{-6})(1)(0.375)^2}{1.781(1756)(8611)}\right](20000)$$

$$= 3.64 \times 10^{-9}$$

It should be noted that pressure behavior in a naturally fractured reservoir is similar to that obtained in a *layered reservoir with no crossflow*. In fact, in any reservoir system with two predominant rock types, the pressure buildup behavior is similar to that of Figure 1.62.

Naturally Fractured Reservoir
(dual porosity)





Gringarten (1987) pointed out that the two straight lines on the semilog plot may or may not be present depending on the condition of the well and duration of the test. He concluded that the semilog plot is not an efficient or sufficient tool for identifying double-porosity behavior. In the log–log plot, as shown in Figure 1.62, the double-porosity behavior yields an S-shaped curve. The *initial portion* of the curve represents the homogeneous behavior resulting from depletion in the most permeable medium, e.g., fissures. A *transition period* follows and corresponds to the interporosity flow. Finally, the *last portion* represents the homogeneous behavior of both media when recharge from the least permeable medium (matrix) is fully established and pressure is equalized. The log–log analysis represents a significant improvement over conventional semilog analysis for identifying double-porosity behavior. However, S-shape behavior is difficult to see in highly damaged wells and well behavior can then be erroneously diagnosed as homogeneous.

Furthermore, a similar S-shape behavior may be found in irregularly bounded well drainage systems.

Perhaps the most efficient means for identifying double-porosity systems is the use of the pressure derivative plot. It allows unambiguous identification of the system, provided that the quality of the pressure data is adequate and, more importantly, an accurate methodology is used in calculating pressure derivatives. As discussed previously, the pressure derivative analysis involves a log–log plot of the derivative of the pressure with respect to time versus elapsed time. Figure 1.64 shows the combined log–log plot of pressure and derivative versus time for a dual-porosity system. The derivative plot shows a "minimum" or a "dip" on the pressure derivative curve caused by the interporosity flow during the transition period. The "minimum" is between two horizontal lines; the first represents the radial flow controlled by the fissures and the second describes the combined behavior of the double-porosity system. Figure 1.64 shows, at early time, the typical behavior of wellbore storage effects with the deviation from the 45° straight line to a maximum representing a wellbore damage. Gringarten (1987) suggested that the shape of the minimum depends on the double-porosity behavior. For a restricted interporosity flow, the minimum takes a V-shape, whereas unrestricted interporosity yields an open U-shaped minimum.

Based on Warren and Root's double-porosity theory and the work of Mavor and Cinco (1979), Bourdet and Gringarten (1980) developed specialized pressure type curves that can be used for analyzing well test data in dual-porosity systems. They showed that double-porosity behavior is controlled by the following independent variables:

- p_D
- t_D/C_D
- $C_D e^{2s}$
- ω
- λe^{-2s}

with the dimensionless pressure p_D and time t_D as defined below:

$$p_D = \left[\frac{k_f h}{141.2 Q B \mu}\right] \Delta p$$

$$t_D = \frac{0.0002637 k_f t}{[(\phi\mu c_t)_f + (\phi\mu c_t)_m]\mu r_w^2} = \frac{0.0002637 k_f t}{(\phi\mu c_t)_{f+m}\mu r_w^2}$$

where:

k = permeability, md
t = time, hours
μ = viscosity, cp
r_w = wellbore radius, ft

and subscripts:

f = fissure
m = matrix
f + m = total system
D = dimensionless

Bourdet et al. (1984) extended the practical applications of these curves and enhanced their use by introducing the pressure derivative type curves to the solution. They developed two sets of pressure derivative type curves as shown in Figures 1.65 and 1.66. The first set, i.e., Figure 1.65, is based on the assumption that the interporosity flow obeys the pseudosteady-state flowing condition and the other set (Figure 1.66) assumes transient interporosity flow. The use of either set involves plotting the pressure difference Δp and the derivative function, as defined by Equation 1.5.4 for drawdown tests or Equation 1.5.5 for buildup tests, versus time with same size log cycles as the type curve. The controlling variables in each of the two type curve sets are given below.

Drawdown

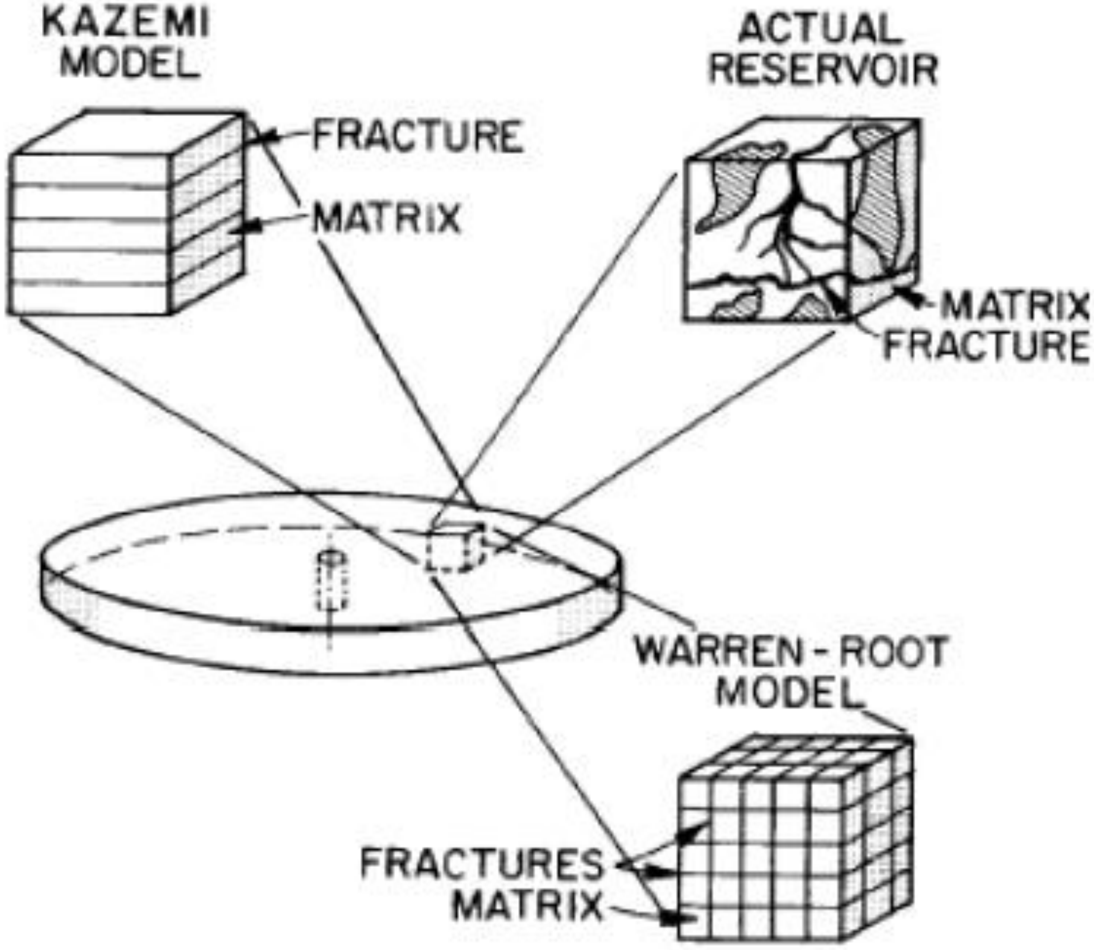
$$t\Delta p' = -t\left(\frac{d(\Delta p)}{d(t)}\right) \quad [1.5.4]$$

Buildup

$$\Delta t_e \Delta p' = \Delta t\left(\frac{t_p + \Delta t}{\Delta t}\right)\left[\frac{d(\Delta p)}{d(\Delta t)}\right] \quad [1.5.5]$$

Identification Of Double Porosity System from Well Tests

- In practice the two parallel straight line may or may not be present (It depends on the condition of the well, composition of the reservoir fluid, duration of the test,…) thus a semilog plot is not an efficient tool for identifying double porosity behavior.
- Double porosity behavior yields an S-shaped log-log pressure curve on a log-log plot.
- Double porosity behavior is characterized by the existence of a minimum on the pressure derivative.
  - The P.S.S. model shows a V-shaped minimum
  - The Transient models show an open U-shaped minimum.


KAZEMI
MODEL
FRACTURE
MATRIX
ACTUAL
RESERVOIR
MATRIX
FRACTURE
WARREN - ROOT
MODEL
FRACTURES
MATRIX

Important Features of Some Mathematical Models Describing Flow from Matrix to Fracture

Warren and Root (1965)

- Analytical model.
- Pseudo-steady state model.
- Matrix flux is independent of a spatial position and is proportional to the pressure difference between matrix and fracture.
- Simplifying the mathematical analysis of the flow problem.
- S-shaped transitional curve with an inflection point.
- The separation of the two parallel lines allows calculation of the storativity ratio.

Kazemi (1969)

- Numerical model.
- Unsteady state model.
- Linear transitional curve with no inflection point.

de Swaan (1976)

- Analytical model.
- Unsteady state model.
- A convolution theorem gives the relationship between the source term and the pressure in the fracture medium.
- Linear transitional curve with no inflection point.

Najurieta (1980)

- Analytical model.
- Unsteady state model.
- Approximate solution to de Swaan model.
- Only applicable for transient period (no boundary dominated period).
- Linear transitional curve with no inflection point.

Streltsova (1983)

- Analytical model.
- Pressure gradient model.
- Matrix flux is proportional to the averaged pressure gradient throughout the matrix block.
- Linear transitional curve with no inflection point.

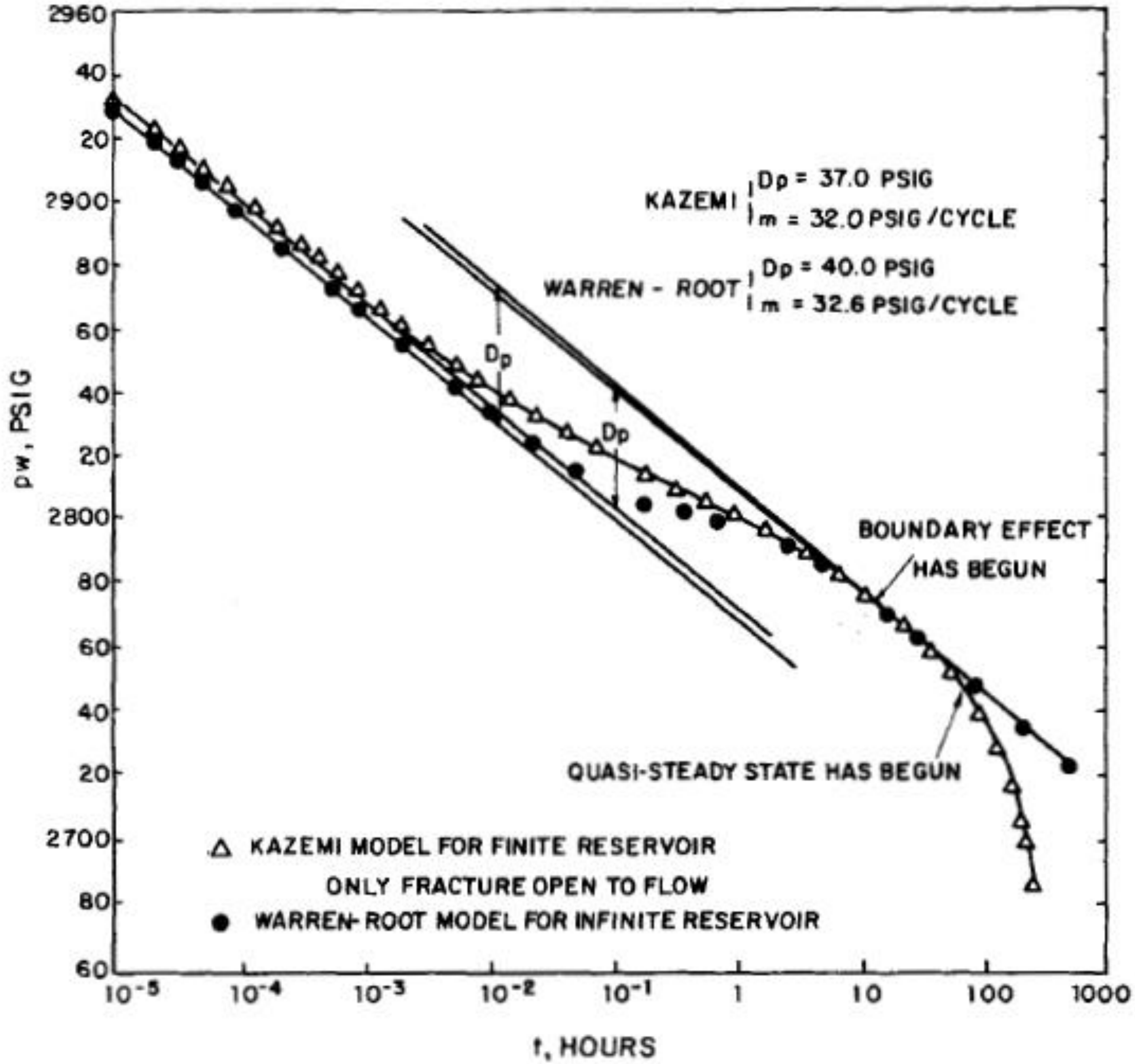


Fig. 2-9. Wellbore pressure solution as given by Kazemi's model and comparison with Warren and Root's model (after Kazemi, 1969). Duration of the first semi-log straight line is shorter as predicted by Kazemi's model. Courtesy of SPE-AIME.