

Flow in Porous Media

Module 1.a

Fundamental Properties of Porous Media

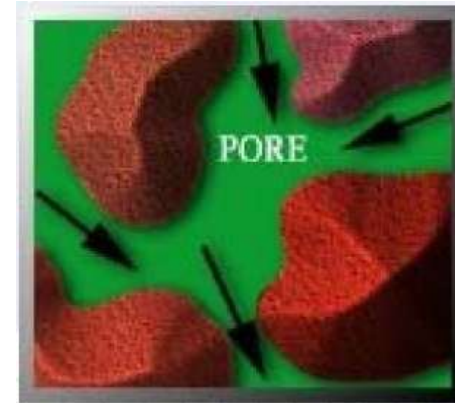
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Outline

- Porosity
 - Classification of Porosity
- Rock Compressibility
- Saturation
- Permeability
 - Non-Darcy Flow - High Flow Rates
 - Klinkenberg effect
 - Carman-Kozeny Equation

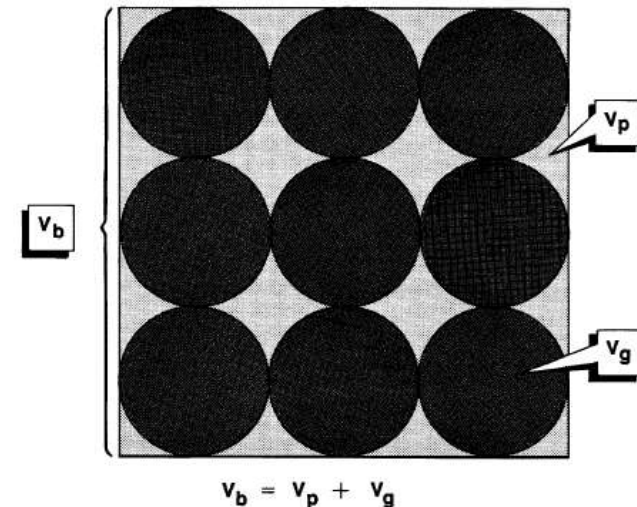
Two Essential Characteristics for a Commercial Reservoir of Hydrocarbons

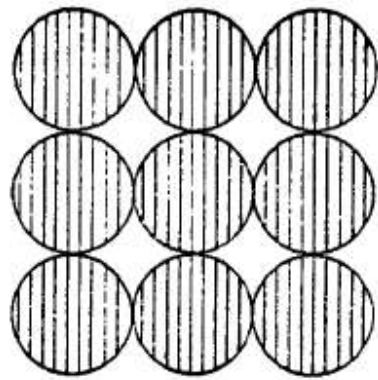


1. A capacity for storage (requires void spaces within the rock)
2. Transmissibility to the fluid (requires that there should be continuity of those void spaces)

Porosity: The porosity of a rock is a measure of the storage capacity (pore volume) that is capable of holding fluids.

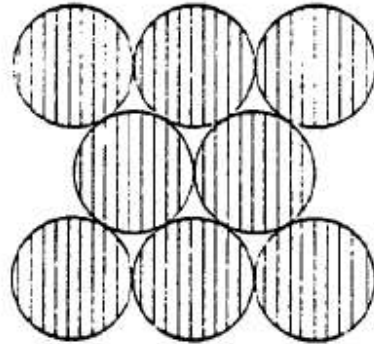
$$\phi = \frac{\text{pore volume}}{\text{bulk volume}} = \frac{\text{bulk volume} - \text{grain volume}}{\text{bulk volume}}$$





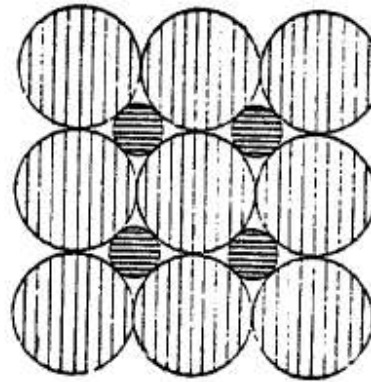
48% POROSITY

(a)



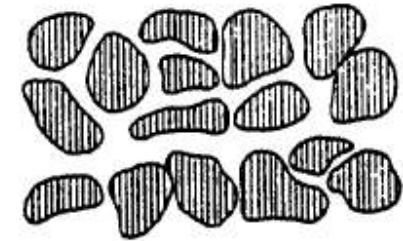
26% POROSITY

(b)



14% POROSITY

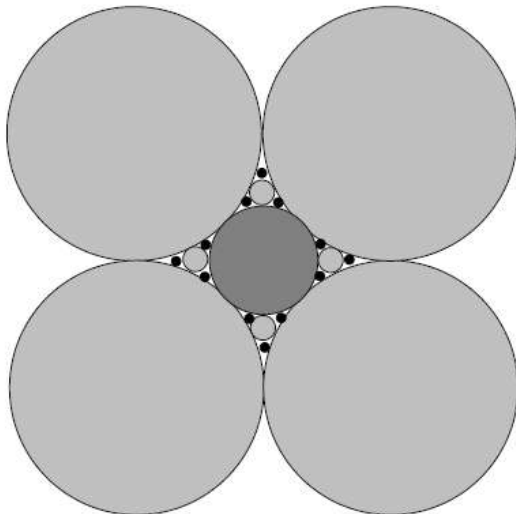
(c)



(d)

Fig. 5.1: Cubic packing (a), rhombohedral (b), cubic packing with two grain sizes (c), and typical sand with irregular grain shape (d).

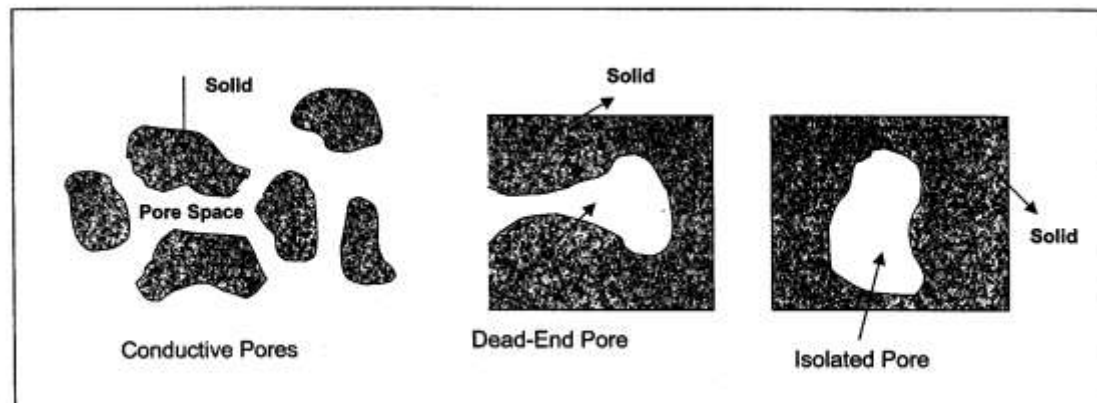
Grain size distribution: Sorting



Porosity increases as the **range of particle size** decreases.

Classification of Porosity According to Its Measurement (Engineering)

1. **Total or absolute porosity:** *total porosity* is the ratio of all the pore spaces in a rock to the bulk volume of the rock.
 2. **Effective porosity:** *effective porosity* is the ratio of interconnected void spaces to the bulk volume.
- **Only** the effective porosity contains fluids that can be produced from wells.
 - For granular materials such as sandstone, the effective porosity may approach the total porosity, however, for shales and for highly cemented or vugular rocks such as some limestones, large variations may exist between effective and total porosity.

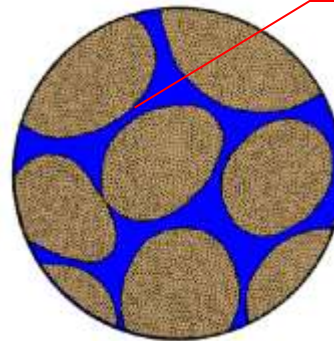


Classification of Porosity According to Its Origin

1. **Primary or original porosity** is developed during deposition of the sediment.

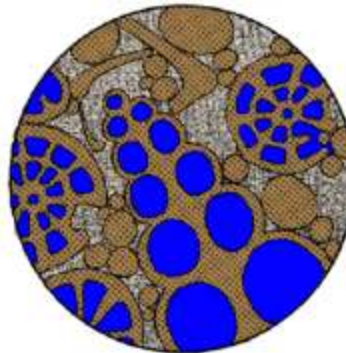
- intergranular or interparticle porosity, which occurs between the grains of a sediment (more typical of sandstones)

Effective porosity in these reservoirs is equivalent to total porosity.



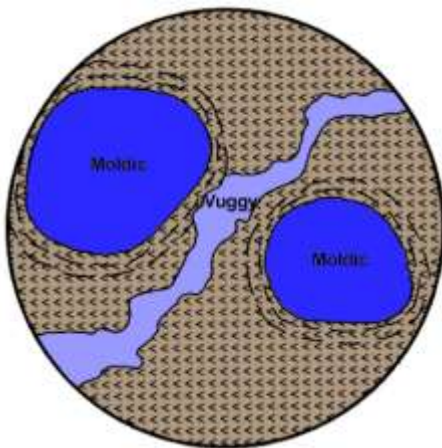
Throat passages

- intragranular or intraparticle porosity, this actually occurs within the sediment grains themselves

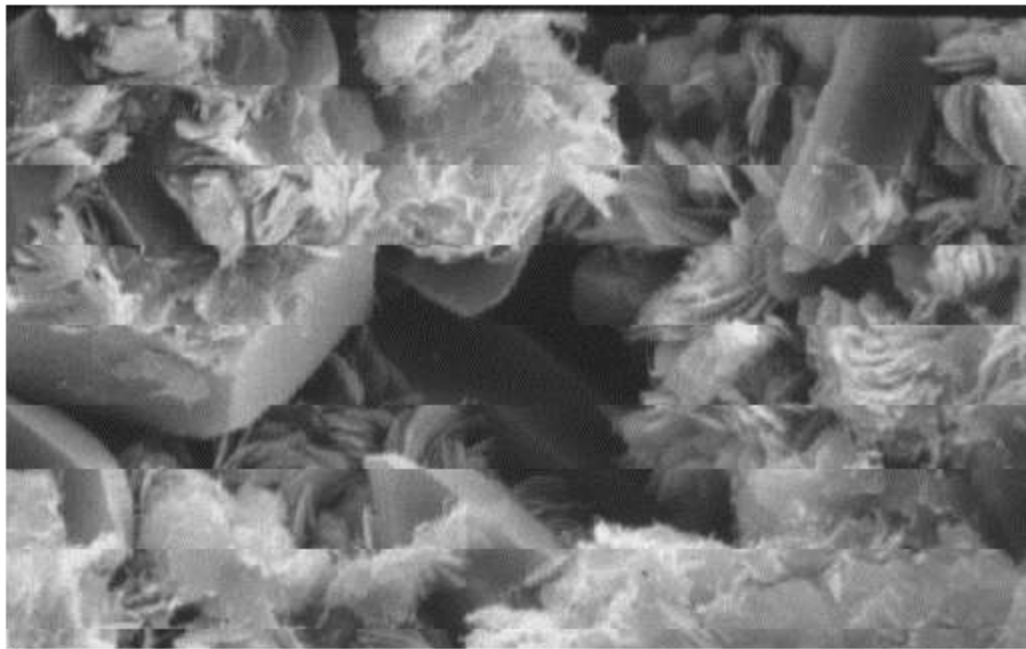


Classification of Porosity According to Its Origin

2. **Secondary porosity:** Secondary porosity is porosity formed within a reservoir after deposition.
- Secondary porosity is caused by the action of the formation fluids or tectonic forces on the rock matrix after deposition.
 - These changes in the original pore spaces may be created by ground stresses, water movement, or various types of geological activities after the original sediments were deposited.
 - Fracturing or formation of solution cavities often will increase the original porosity of the rock.



Pore throats in sandstone may be lined with variety of cement minerals that affect petrophysical properties.



Scanning Electron Micrograph

Specific Surface Area

- ❑ Specific surface area (specific surface): The total internal surface area obtained in a unit volume (or unit mass) of the rock.
- ❑ Rocks containing smaller and flatter or irregular shaped grains will have larger specific surface compared rocks containing large and well rounded grains.
- ❑ Specific surface area is an important property of the rock that has a direct influence on many reservoir processes. For example, the amount of wetting fluid that is held immobile in the form of a thin wetting layer would increase with increasing specific surface area.

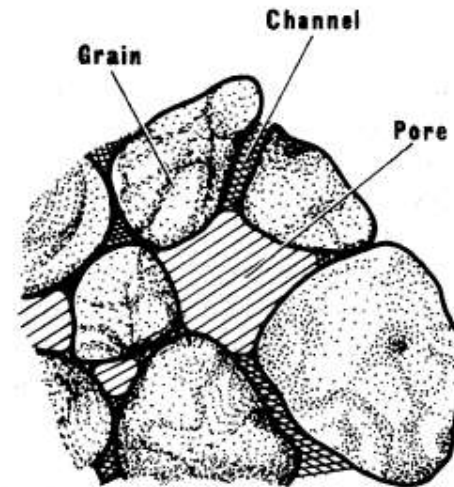


Fig. 1-2. Interstitial spaces in a clastic rock.

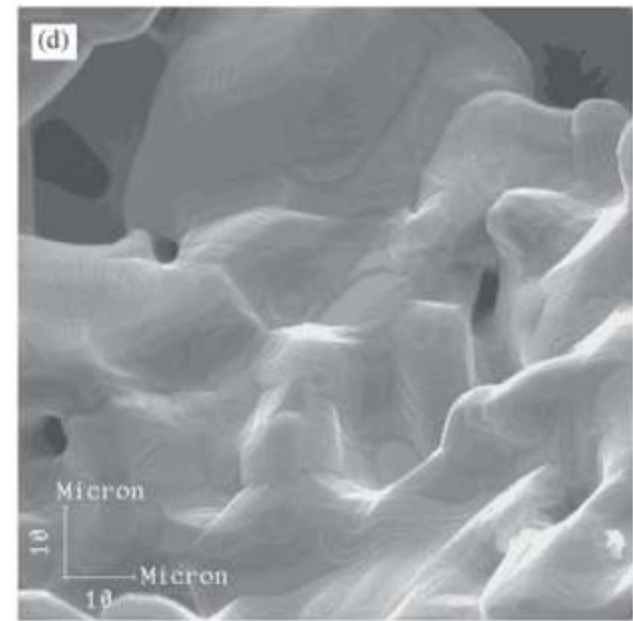
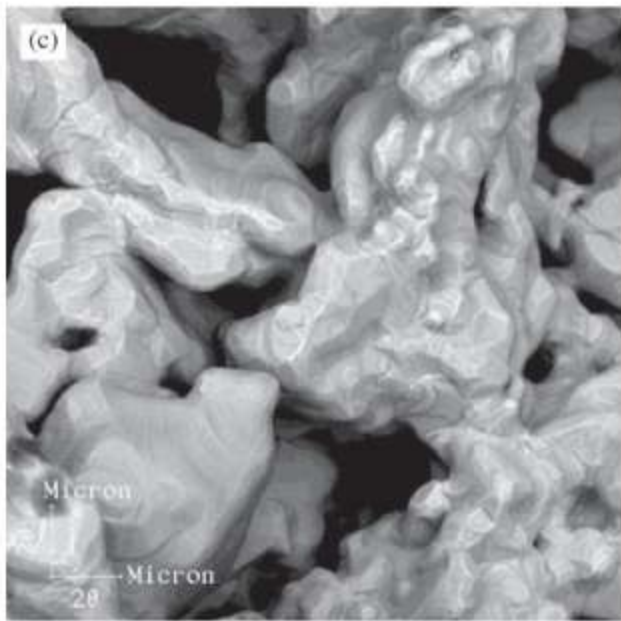
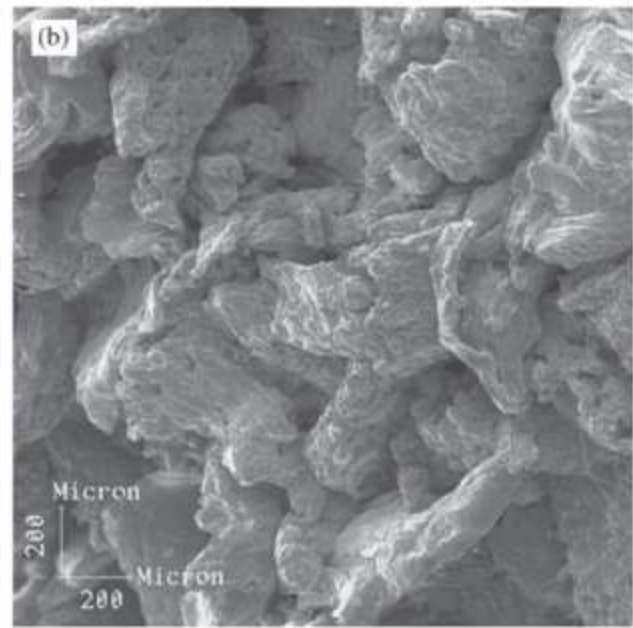
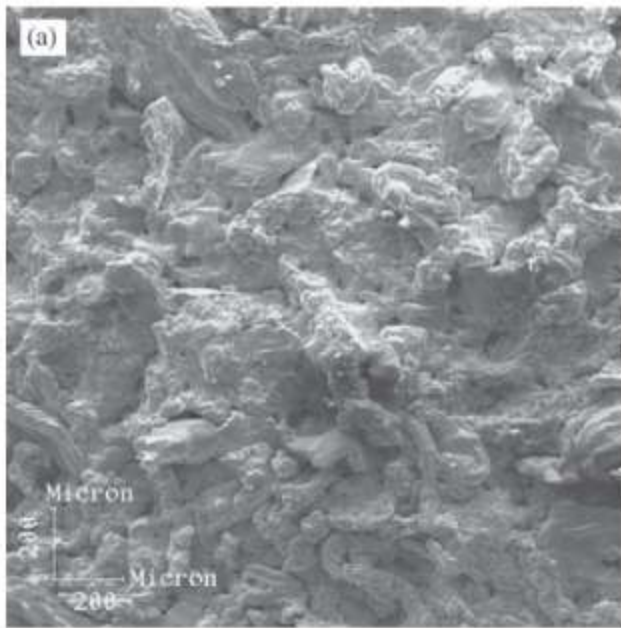
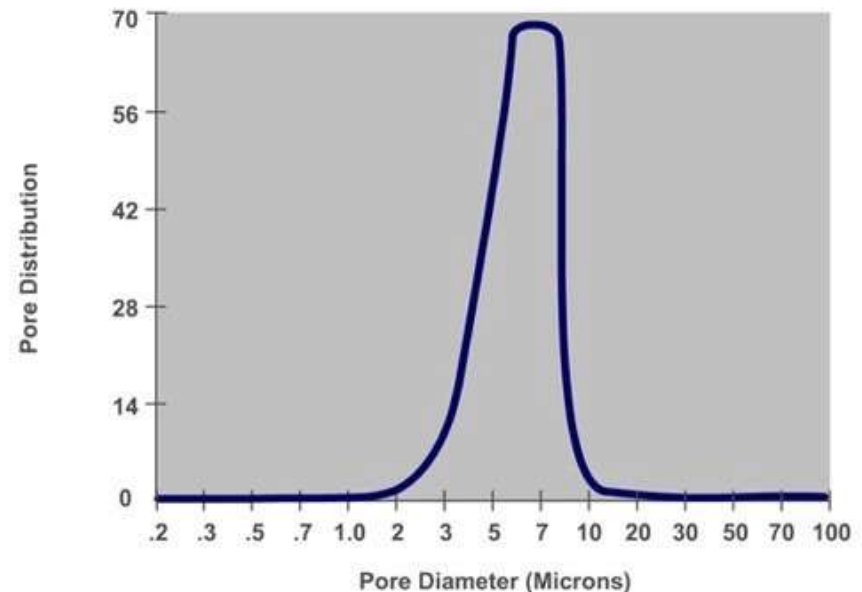
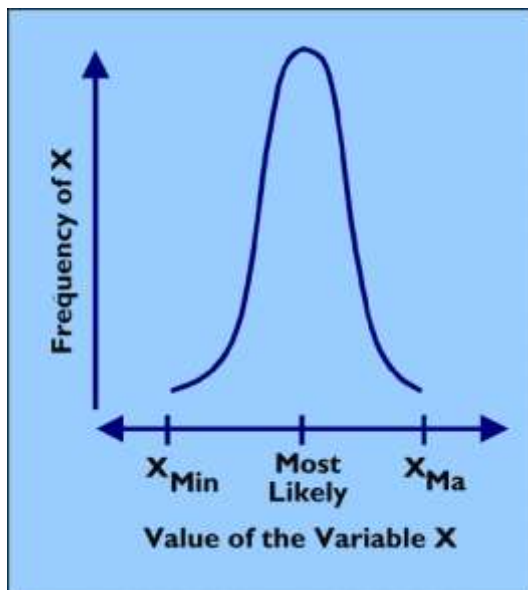


Figure 2. SEM surface topographical images of the specimens compacted at 200 MPa: type 2 in (a) (c); type 4 in (b), type 1 in (d).

Pore Size Distribution

• Individual pores in a reservoir rock vary considerably in their sizes. The degree of size variation is described by pore size distribution. It is often useful to know the fraction of pore space represented by pore of various size ranges. To some extent, pore size distribution is related to grain size distribution. A rock containing uniform grain size would naturally possess a narrow pore size distribution.

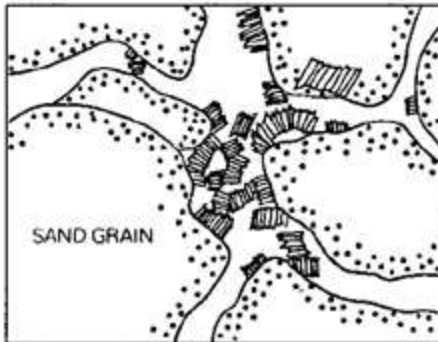


Probability Distribution Function

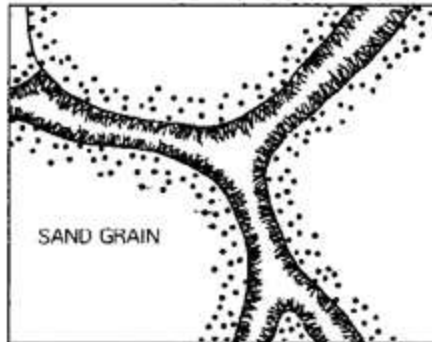
Pore Geometry

The geometrical shape of the pores and the degree to which various pores are interconnected has significant influence on multiphase flow behavior. The pore geometry is one of the important factors influencing fluid flow in porous media.

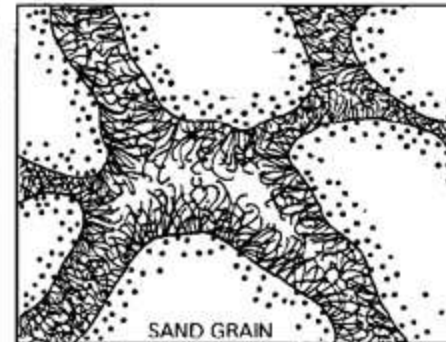
Discrete Particle
- Kaolinite and Chlorite



Pore Lining
- Illite, Chlorite, Smectite,
& Mixed Layer



Pore Bridging
- Illite and Chlorite



Types of clay-mineral occurrences and pore geometry. After Neasham, 1977; courtesy SPE.

Average Pore Size:

The ratio of porosity and specific surface provides a measure of the average pore size of the medium. It has the dimension of length and is physically analogous to hydraulic radius of an irregularly shaped flow channel in hydraulics. The hydraulic Radius for a tube is:

$$R_h = \frac{\text{Cross - sectional area}}{\text{Wetted Perimeter}} \qquad R_h = \frac{A}{P} \qquad (1-1)$$

The hydraulic radius is *not* half the [hydraulic diameter](#) as the name may suggest.

By multiplying both numerator and denominator by L this can be written as:

$$R_h = \frac{\textit{Volume open to flow}}{\textit{Wetted surface area}} \qquad (1-2)$$

In one unit volume of rock:

Volume open to flow = Porosity

Wetted surface area = Specific Surface

$$R_h = \frac{\textit{Porosity}}{\textit{SpecificSurface}} \qquad (1-3)$$

Thus, the ratio of porosity to specific surface provides a measure of the average pore size. Clearly, rocks containing small grains will have large specific surface and small average pore-size.

Representative Control Volume for Porosity Measurement

Porosity is a statistical quantity which depends on the magnitude of the total volume taken into consideration.

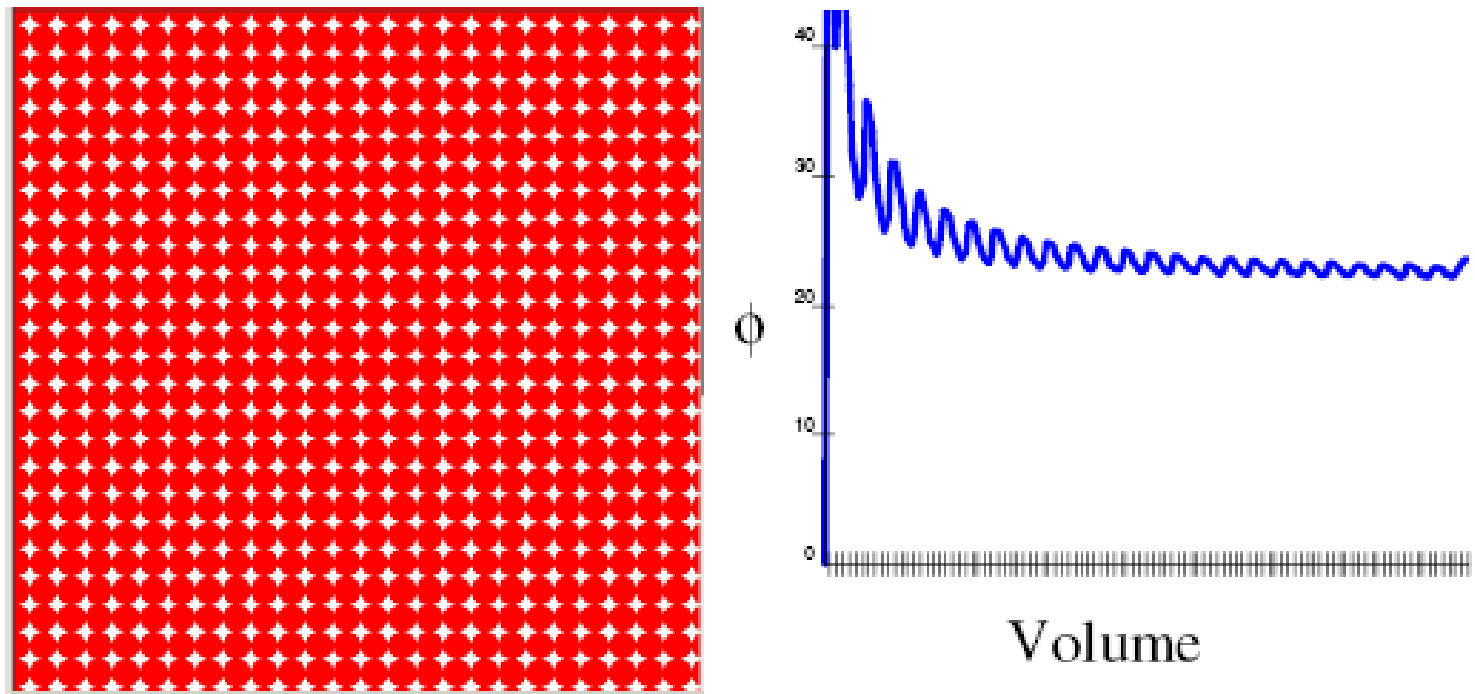


Figure 1.3: Computed porosities as a function of volume considered.

- If the selected volume is too small (e.g.: $V_T = 10^{-9} \text{m}^3$) the calculated porosity can deviate greatly from the true value, therefore the volume V_T should be large enough in order to obtain the statistical average of porosity.
- On the other side if the volume is too large the porosity may deviate from the real value due to the influence of heterogeneity.

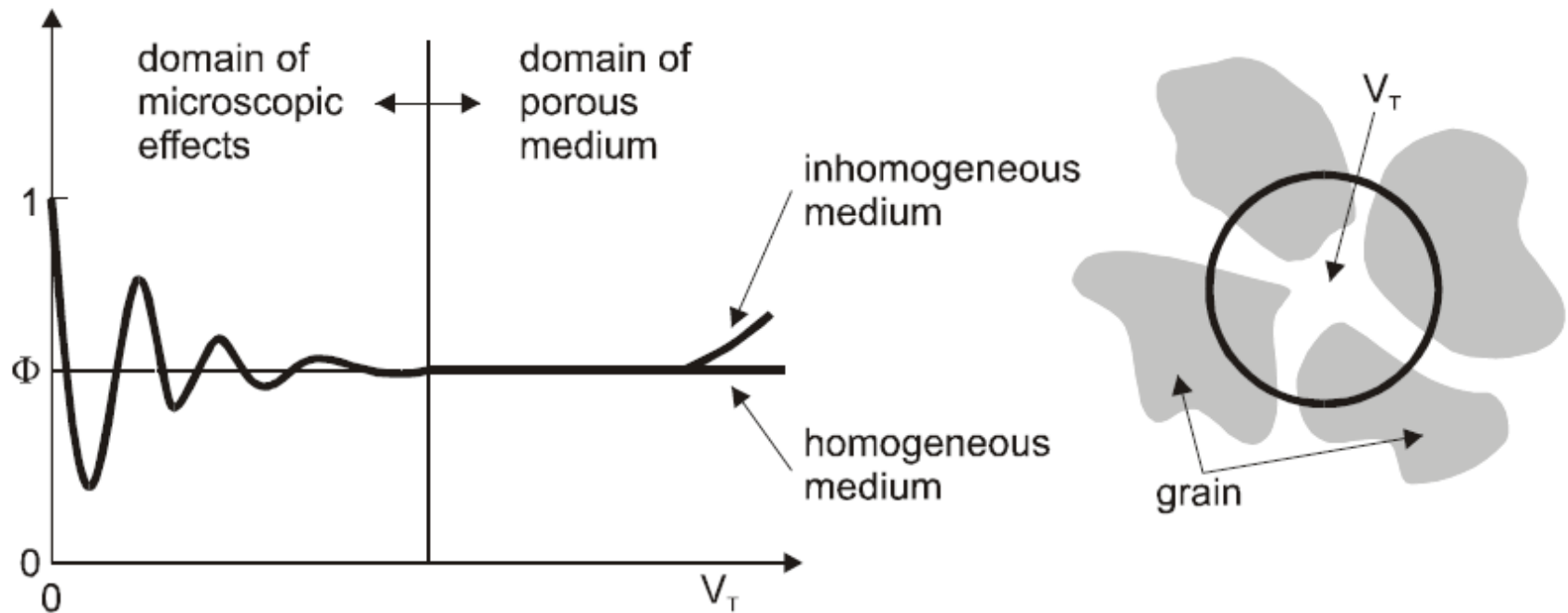
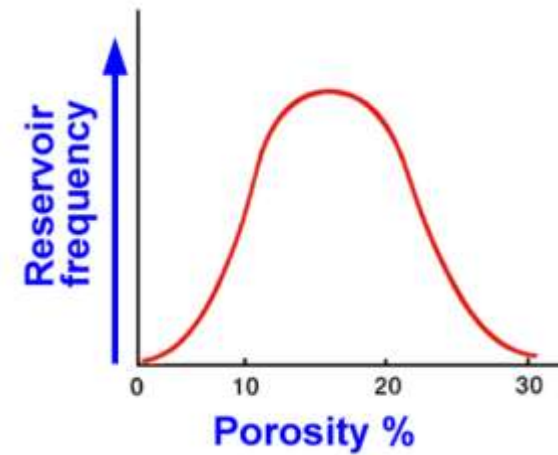


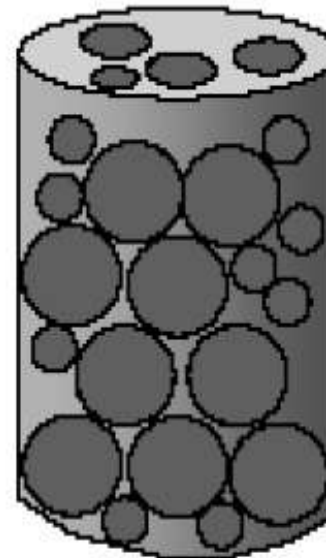
Figure 1.1: Definition of representative control volume for porosity measurements

The porosities of petroleum reservoirs range from **5%** to **40%** but most frequently are between **10%** and **20%**.



Porosity Ranges

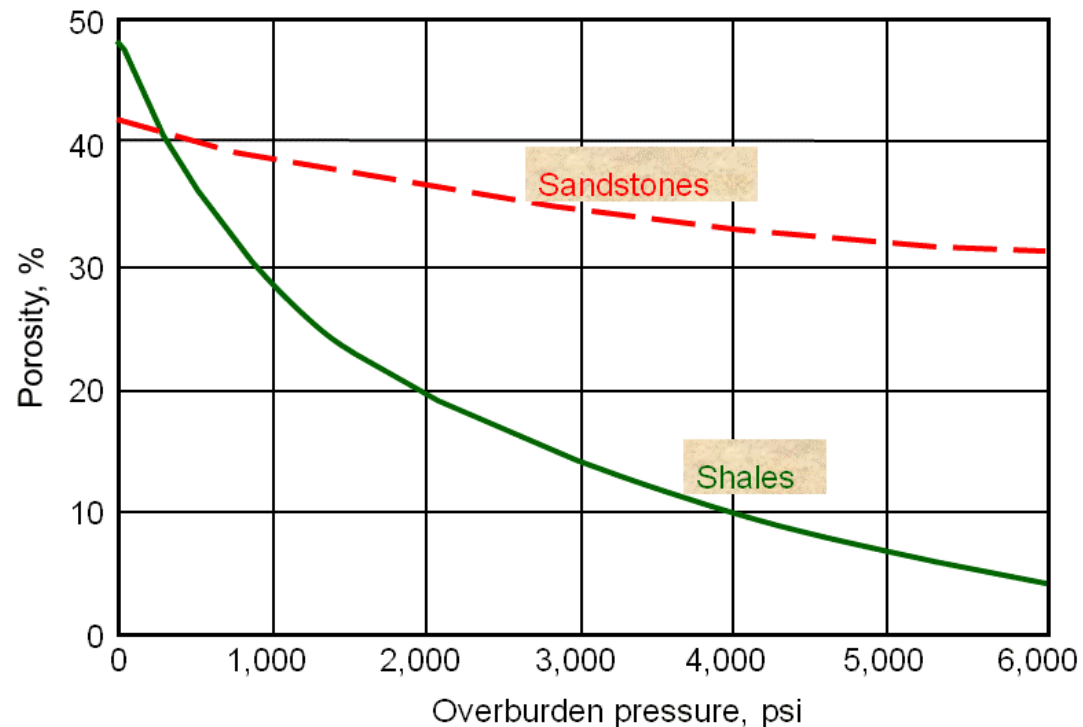
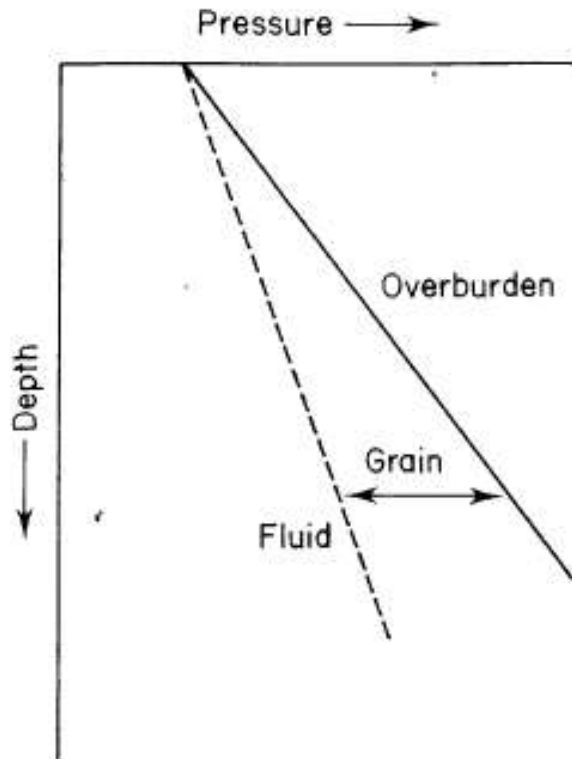
Sand and gravel	20-50 %
Till	10-20 %
Silt	35-50 %
Clay	33-60 %
Clastic sediments	typically 3-30 %
Limestone	<1 to 30 %
Basalt	1-12 %
Tuff	14-40 %
Pumice	- 87 %
Fractured crystalline rock	1-5 %
Unfractured crystalline rock	~0.1 %





Rock Compressibility

- The pressure difference between overburden and internal pore pressure is referred to as the effective overburden pressure. During pressure depletion operations, the internal pore pressure decreases and, therefore, the effective overburden pressure increases. This increase causes the following effects:
 - The bulk volume of the reservoir rock is reduced.
 - Sand grains within the pore spaces expand.



Isothermal Compressibility

- General Definition
 - The relative volume change of matter per unit pressure change under conditions of constant temperature
 - Usually, petroleum reservoirs can be considered isothermal (an exception: thermal stimulation)
 - Increasing pressure causes volume of material to decrease (compression) - e.g. reservoir fluids
 - Decreasing pressure causes volume of material to increase (expansion) - e.g. reservoir fluids

C: Coefficient of Isothermal Compressibility

ALWAYS positive value; oilfield units: 1/psia

V: Volume; oilfield units: ft³

p: Pressure exerted on material ; oilfield units: psia

• Negative sign in equation determined by $\frac{\partial V}{\partial p}$ term, to force the coefficient C to be positive

• Volume is a function of pressure only (temperature is constant, and amount of material is constant)

$$C = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T$$

Subsidence and Bulk Compressibility

- Process of subsidence
 - Bulk volume decreases as fluids are produced
 - Area is constant
 - Formation thickness decreases (causing subsidence of strata above)
- Porosity: $\phi = V_p/V_b = 1-(V_m/V_b)$; where $V_b=V_p+V_m$
- Net compaction pressure: $p_m = p_o - p$
- Overburden (p_o) is constant $\therefore dp_m = -dp$
- As net compaction pressure increases
 - Bulk volume decreases; $C_b = -1/V_b (\partial V_b/\partial p_m)$
 - Pore volume decreases; $C_f = -1/V_p (\partial V_p/\partial p_m)$
 - Matrix volume decreases; $C_m = -1/V_m (\partial V_m/\partial p_m)$
- Substituting from definitions above
 - $C_b = (-1/V_b) [(\partial V_p/\partial p_m) + (\partial V_m/\partial p_m)]$
 - $C_b = (-1/V_b) [(-C_f V_p) + (-C_m V_m)]$
 - $C_b = \phi C_f + (1-\phi)C_m$; usually $C_m \ll C_f$

It should be pointed out that the total reservoir compressibility c_t is extensively used in the transient flow equation and the material balance equation as defined by the following expression:

$$c_t = S_o c_o + S_w c_s + S_g c_g + c_f \quad (4-68)$$

where S_o, S_w, S_g = oil, water, and gas saturation

c_o = oil compressibility, psi^{-1}

c_w = water compressibility, psi^{-1}

c_g = gas compressibility, psi^{-1}

c_t = total reservoir compressibility

For undersaturated oil reservoirs, the reservoir pressure is above the bubble-point pressure, i.e., no initial gas cap, which reduces Equation 4-68 to:

$$c_t = S_o c_o + S_w c_w + c_f$$

In general, the formation compressibility c_f is the same order of magnitude as the compressibility of the oil and water and, therefore, cannot be regulated.

Several authors have attempted to correlate the pore compressibility with various parameters including the formation porosity. Hall (1953) correlated the pore compressibility with porosity as given by the following relationship:

$$c_f = (1.782/\phi^{0.438}) 10^{-6} \quad (4-69)$$

where c_f = formation compressibility, psi^{-1}
 ϕ = porosity, fraction

Newman (1973) used 79 samples for consolidated sandstones and limestones to develop a correlation between the formation compressibility and porosity. The proposed generalized hyperbolic form of the equation is:

$$c_f = \frac{a}{[1 + cb\phi]}$$

where

For consolidated sandstones

$$a = 97.32 \times 10^{-6}$$

$$b = 0.699993$$

$$c = 79.8181$$

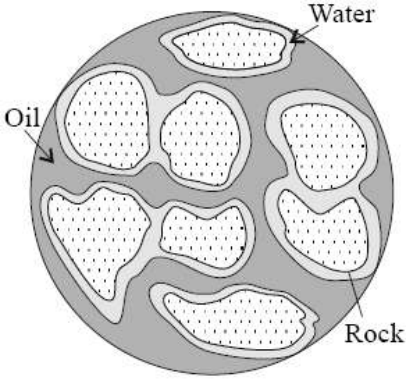
For limestones

$$a = 0.8535$$

$$b = 1.075$$

$$c = 2.202 \times 10^6$$

Pore Compressibility Correlations



Saturation

Saturation is defined as that fraction, or percent, of the pore volume occupied by a particular fluid (oil, gas, or water). This property is expressed mathematically by the following relationship:

- Equations: $S_o = V_o / V_p$, $S_w = V_w / V_p$, $S_g = V_g / V_p$
- Applying the above mathematical concept of saturation to each reservoir fluid gives. The saturation of each individual phase ranges between zero to 100 percent.
- These fluid volumes are measured under specific conditions of pressure and temperature (e.g. reservoir, or laboratory)
 - reservoir conditions are often noted as “*in situ*”
- By definition, the sum of the saturations is 100%, therefore

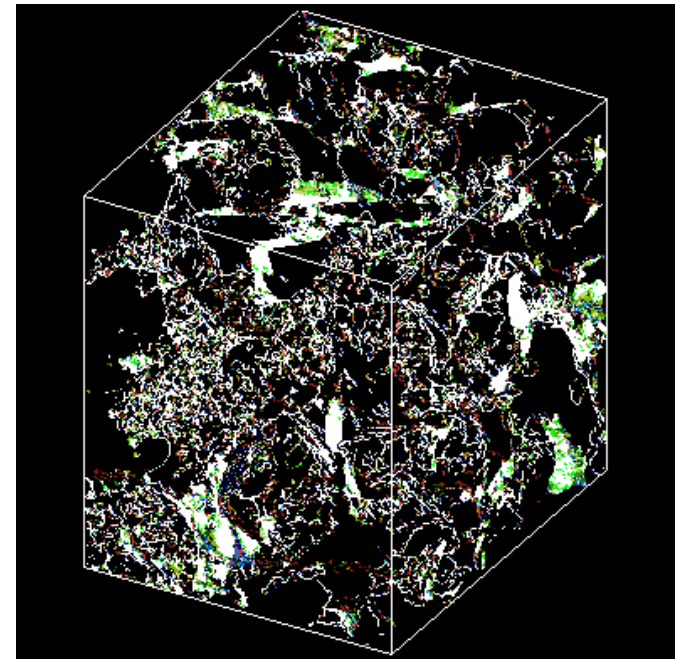
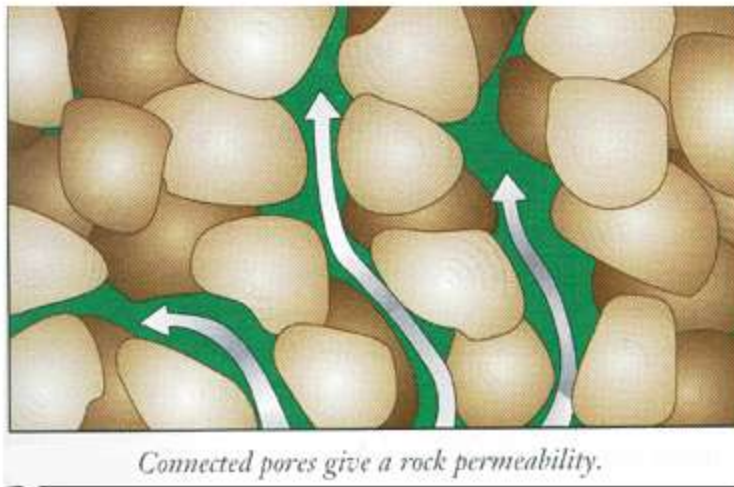
$$S_g + S_o + S_w = 1.0$$

Permeability

- **Definition:**

-The **permeability** of a rock is a measure of how easily a fluid may flow through the pore channels in a rock. It depends on the size, shape, tortuosity and number of the pore channels in the porous medium.

-[Absolute permeability](#) is the permeability of the porous medium if a single fluid is flowing. [Effective permeability](#) is the permeability of a fluid if another fluid is present. [Relative permeability](#) is the effective permeability divided by the absolute permeability.



Units of Permeability

$$k = \frac{\mu q}{A} \frac{\Delta P}{L} = \frac{[cp][cm^3/s]}{[cm^2]} \frac{[atm]}{[cm]} = 1 \text{ Darcy}.$$

$$\frac{[Nsm^{-2}][m^3s^{-1}]}{[m^2]} \frac{[Nm^{-2}]}{[m]} = [m^2]$$

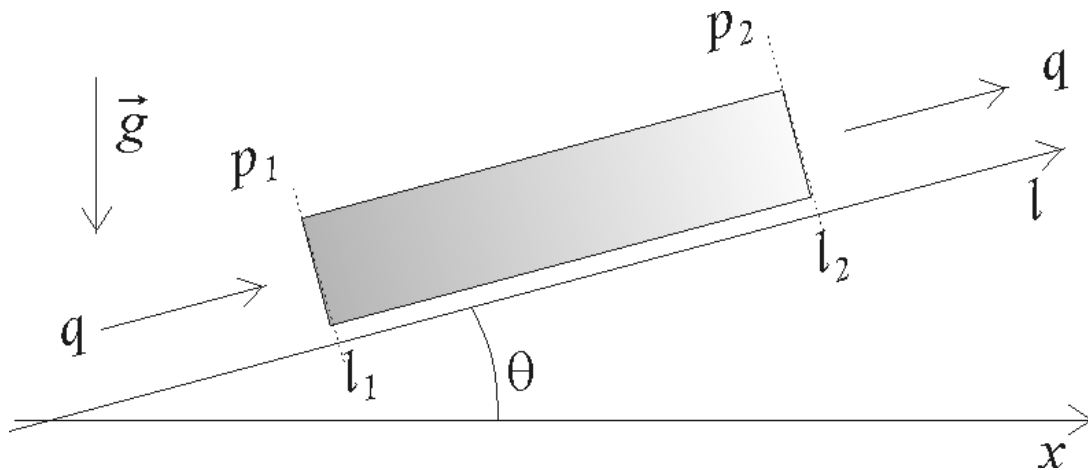
The relationship between the two units is:

$$1 \text{ Darcy} = 0.987 \times 10^{-12} [m^2]$$

Generalised Form of Darcy's Law

$$u = \frac{q}{A} = -\frac{k}{\mu} \left(\frac{dp}{dl} + \frac{\rho g}{1.0133 \times 10^6} \frac{dz}{dl} \right)$$

u – Darcy velocity, cm/s
 q – fluid flow rate, cm³/s
 A – cross-sectional area, cm²
 k – permeability, D
 μ – viscosity, cp
 ρ – density, cp
 g – gravity, cm/s²
 dp/dl – pressure gradient, atm/cm
 dz – elevation, cm



- The purpose of the gravitational term is to cancel out the pressure gradient for a column of fluid in equilibrium. Then, the fluid velocity will be zero, as it should be in equilibrium.

Darcy's law

$$q = A \frac{k \Delta p}{\mu L} \quad (1.39)$$

Ohm's law of electrical current:

$$J = A \frac{1 U}{\rho L} \quad (1.40)$$

Fourier's law of heat conduction:

$$Q = A \lambda \frac{\Delta T}{L}, \quad (1.41)$$

where:

- A is the cross section [m²]
- J is the amperage [A]
- k is the permeability [m²]
- L is the length [m]
- Q is the rate of heat [Js⁻¹]
- q is the flow rate [m³s⁻¹]
- U is the voltage [V]
- Δp is the difference in pressure [Pa]
- ΔT is the difference in temperature [°C]
- μ is the dynamic viscosity [Pa.s]
- λ is the thermal conductivity [Wm⁻¹ °K]
- ρ is the electrical resistance [Ωm]

The form of Eq. 1.39, Eq. 1.40 and Eq. 1.41 is similar.

- Permeability has direct analogies with thermal conductivity, electrical conductivity and diffusivity, and is defined similarly by a transport equation

$$\frac{Q}{A} = U = - \text{constant} \frac{d\Phi'}{dL}$$

i.e. a rate of transfer is proportional to a potential gradient.

Table 1.6: Comparison between the laws of *Darcy*, *Ohm*, and *Fourier*

Darcy	Ohm	Fourier
Flow Rate $u[m^3 s^{-1}]$	Amperage $J[A]$	Heat conduction rate $Q[Js^{-1}]$
Permeability coefficient $K = \frac{k}{\mu}$ $[m^2 Pa^{-1} s^{-1}]$	Electrical conductivity $\gamma = \frac{1}{\rho}$ $[\frac{1}{\Omega m}]$	Thermal conductivity λ $[\frac{W}{mK}]$
Pressure difference $\Delta p[Pa]$	Voltage $U[V]$	Temperature difference $\Delta T[K]$

The Klinkenberg Effect

- Klinkenberg (1941) discovered that permeability measurements made with air as the flowing fluid showed different results from permeability measurements made with a liquid as the flowing fluid.
- The permeability of a core sample measured by flowing air is always greater than the permeability obtained when a liquid is the flowing fluid.
- Klinkenberg postulated, on the basis of his laboratory experiments, that liquids had a zero velocity at the sand grain surface, while gases exhibited some finite velocity at the sand grain surface. In other words, the gases exhibited *slippage at the sand grain surface*.
- This slippage resulted in a higher flow* rate for the gas at a given pressure differential.
- Klinkenberg also found that for a given porous medium as the mean pressure increased the calculated permeability decreased.

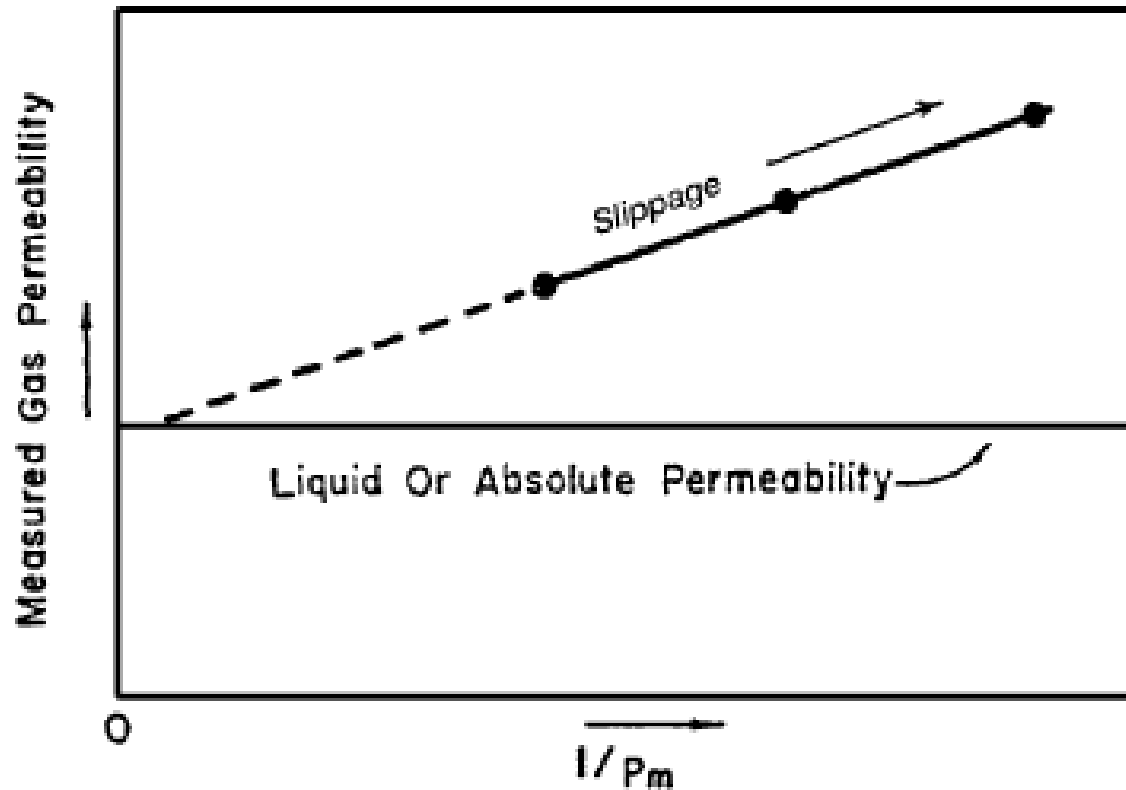


Figure 4-21. The Klinkenberg effect in gas permeability measurements.

$$k_g = k_L + c \left[\frac{1}{P_m} \right] \quad (4-43)$$

where k_g = measured gas permeability

P_m = mean pressure

k_L = equivalent liquid permeability, i.e., absolute permeability, k

c = slope of the line

The magnitude of the Klinkenberg effect varies with the core permeability and the type of the gas used in the experiment as shown in Figures 4-22 and 4-23.

Klinkenberg suggested that the slope is a function of the following factors:

- Absolute permeability k , i.e., permeability of medium to a single phase completely filling the pores of the medium k_L .
- Type of the gas used in measuring the permeability, e.g., air.
- Average radius of the rock capillaries.

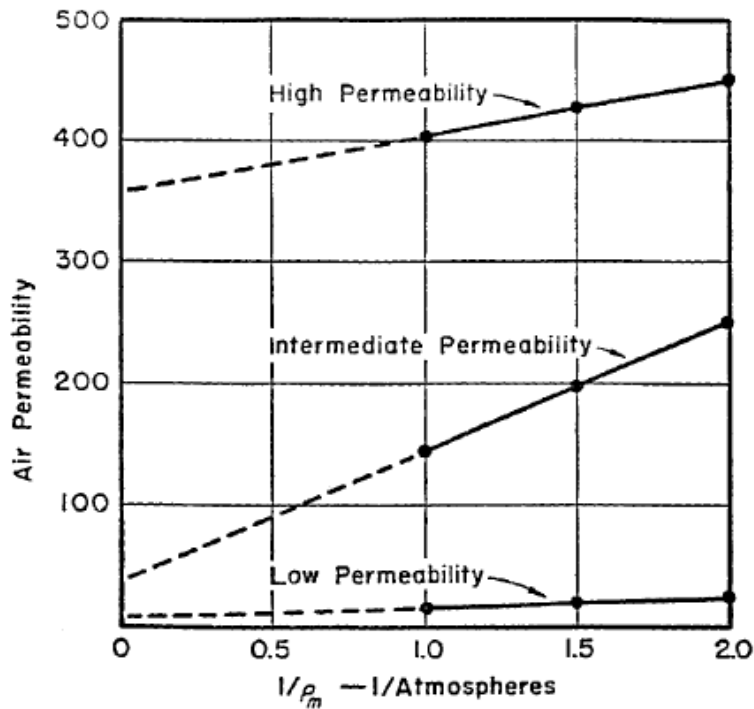


Figure 4-22. Effect of permeability on the magnitude of the Klinkenberg effect. (After Cole, F., 1969.)

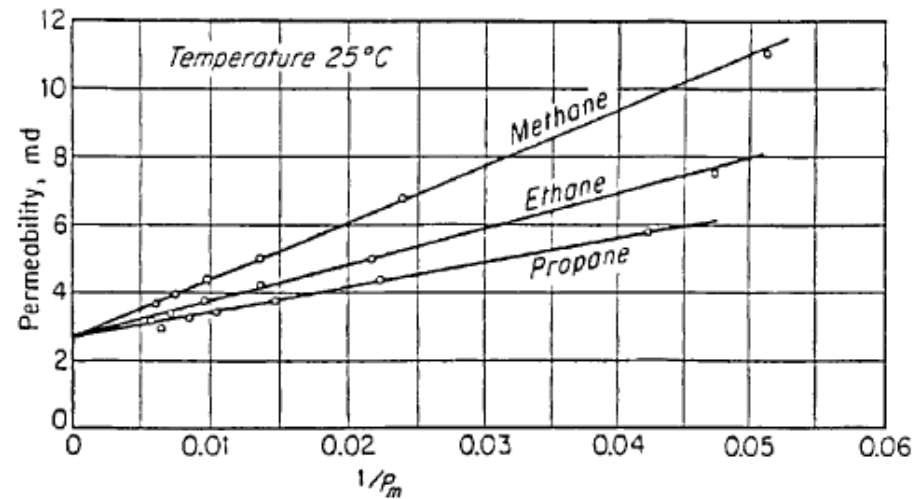
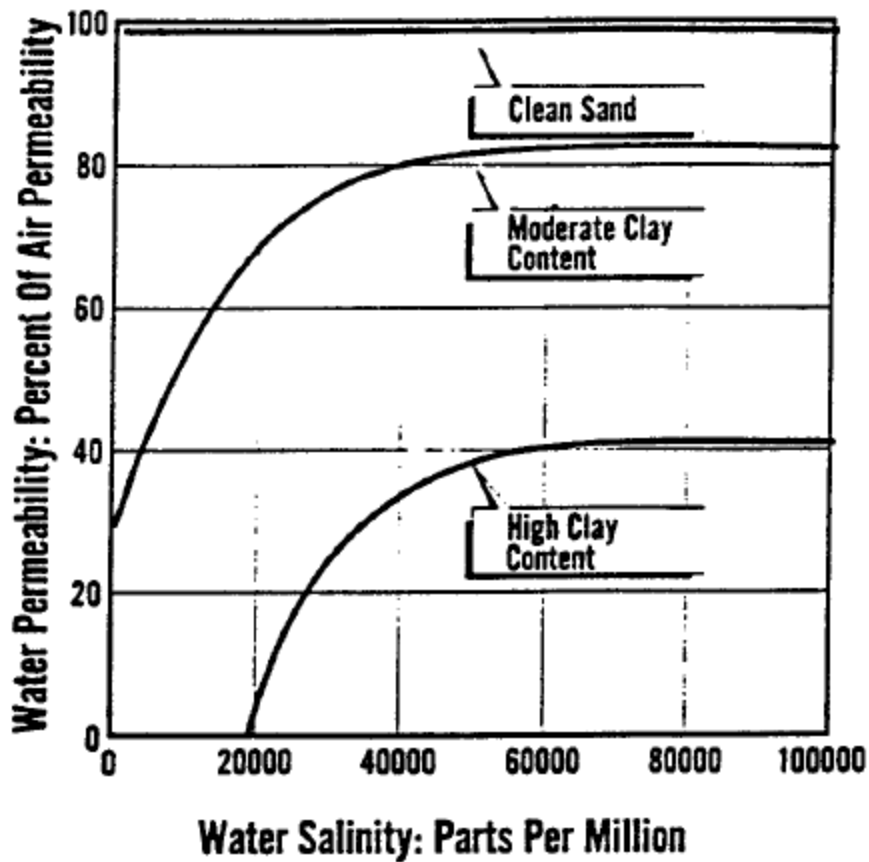
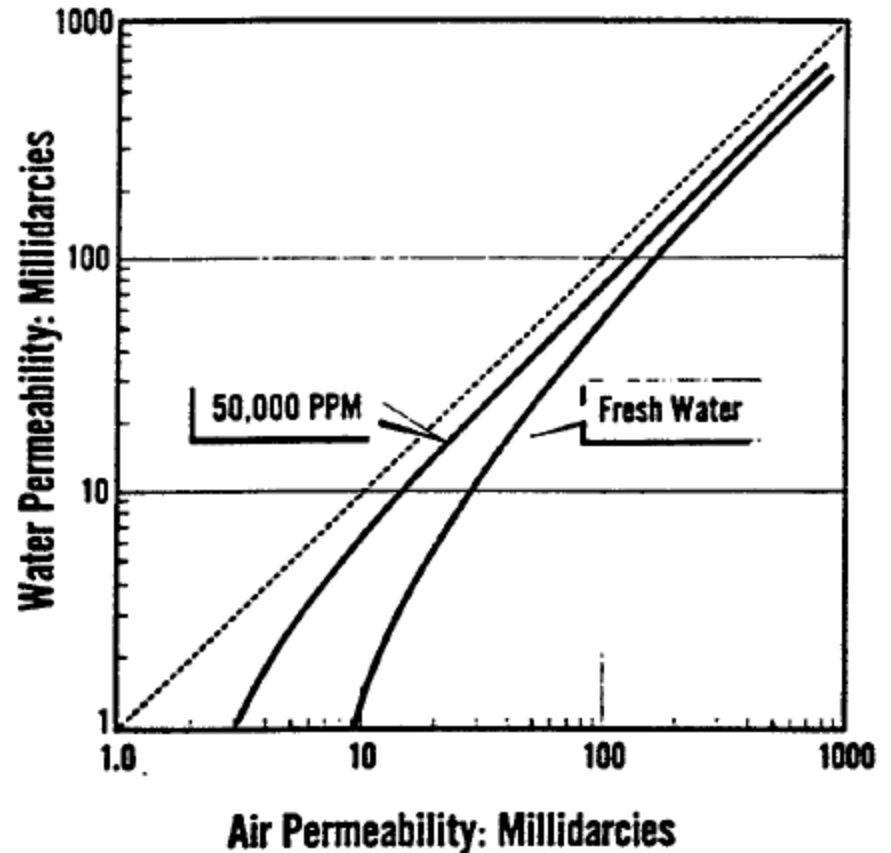


Figure 4-23. Effect of gas pressure on measured permeability for various gases. (After Calhoun, J., 1976.)

Permeability Reduction Due to Clay Swelling



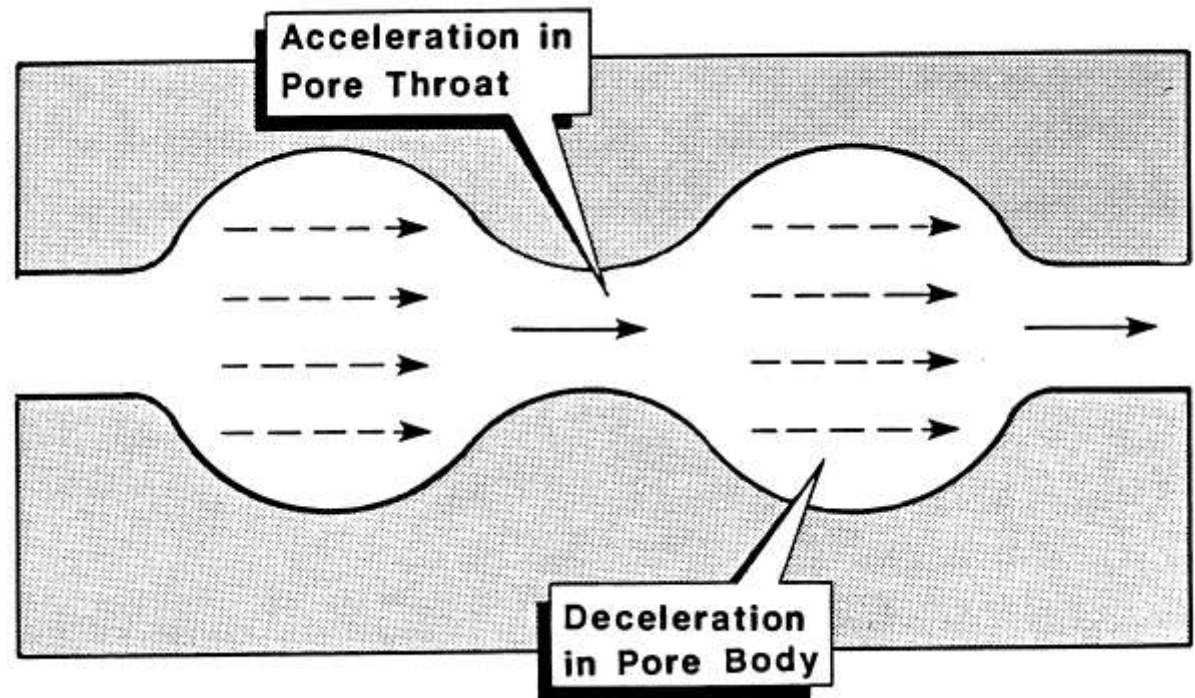
Variation in Water Permeability With Salinity and Clay Content



Variation In Water Permeability With Salinity and Air Permeability

Non-Darcy Flow - High Flow Rates

- In the field, gas wells exhibit non-Darcy flow at high flow rates
- At high flow velocity, inertial effects and turbulence become important, and cause non-Darcy flow
 - inertial effect



Non-Darcy Flow - Turbulence

- Recalling Darcy's equation for gas flow, $(z\mu_g)=\text{Constant}$

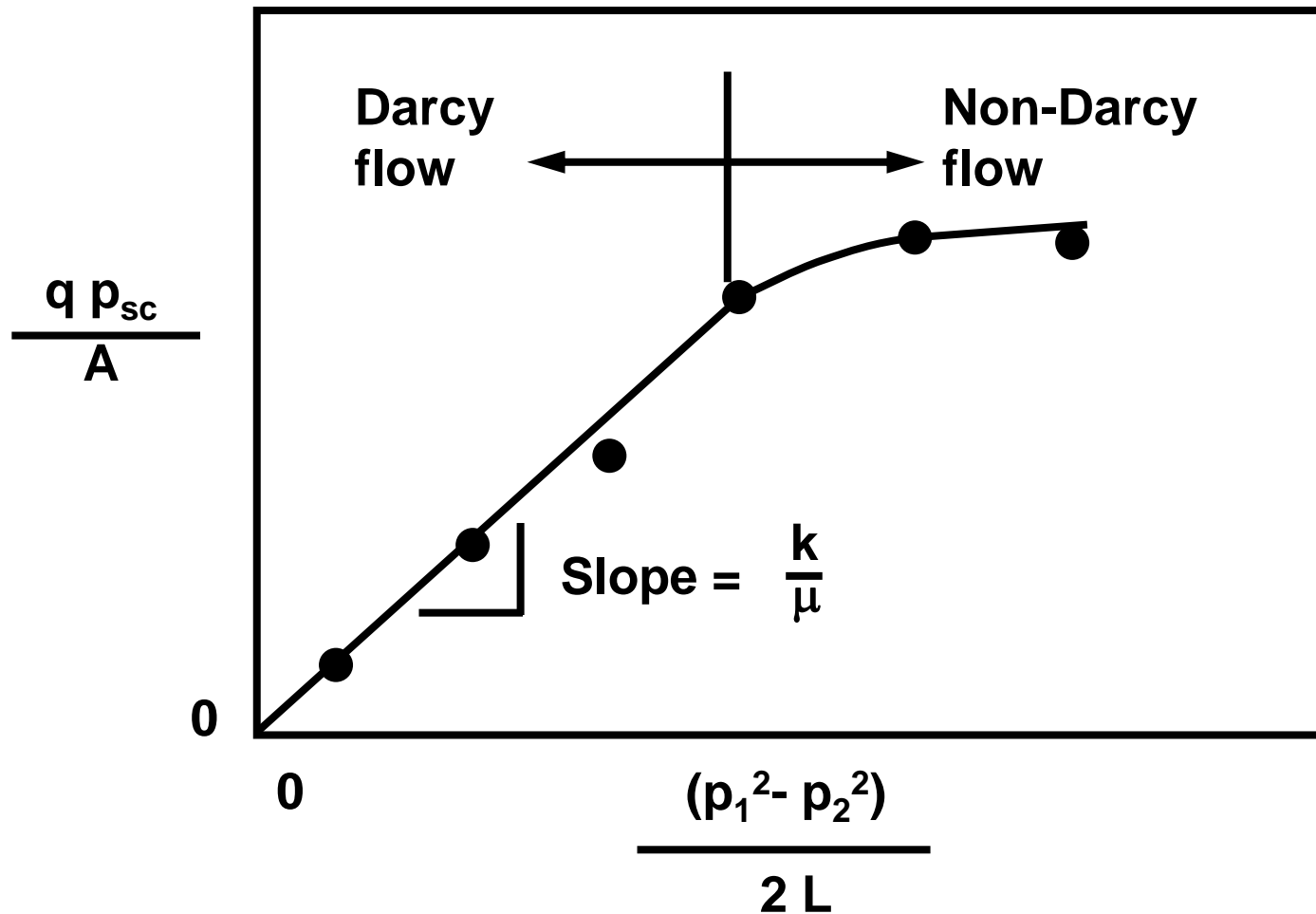
$$q_{g,sc} = \frac{k A}{L} \left(\frac{T_{sc}}{T p_{sc}} \right) \left(\frac{1}{2 z \mu_g} \right) (p_1^2 - p_2^2)$$

- For laboratory flow experiments we can assume $T=T_{sc}$ and $z=1$

$$q_{g,sc} = \frac{k}{\mu_g} \left(\frac{A}{p_{sc}} \right) \frac{(p_1^2 - p_2^2)}{2L}$$

- For Darcy flow, plotting $(q_{g,sc} p_{sc})/A$ vs. $(p_1^2 - p_2^2)/(2L)$ results in straight line.
 - line passes through origin [when $q_{g,sc}=0$, then $(p_1^2 - p_2^2)=0$]
 - slope = k/μ_g
 - behavior departs from straight line under turbulent flow conditions (high flow velocity)

Non-Darcy Flow - Turbulence



Non-Darcy Flow - Forchheimer Equation

- Forchheimer proposed a flow equation to account for the non-linear effect of turbulence by adding a second order term

$$\frac{-dp}{ds} = \frac{\mu_g}{k} \left(\frac{q_g}{A} \right) + \beta \rho_g \left(\frac{q_g}{A} \right)^2$$

- Note that unit corrections factors would be required for non-coherent unit systems.
- As flow rate decreases, we approach Darcy's Law (2nd order term approaches zero)

Controls on Permeability and the Range of Permeability Values in Nature

Intuitively, it is clear that permeability will depend on porosity; the higher the porosity the higher the permeability. However, permeability also depends upon the connectivity of the pore spaces, in order that a pathway for fluid flow is possible. The connectivity of the pores depends upon many factors including the size and shape of grains, the grain size distribution, and other factors such as the operation of capillary forces that depend upon the wetting properties of the rock. However, we can make some generalizations if all other factors are held constant:

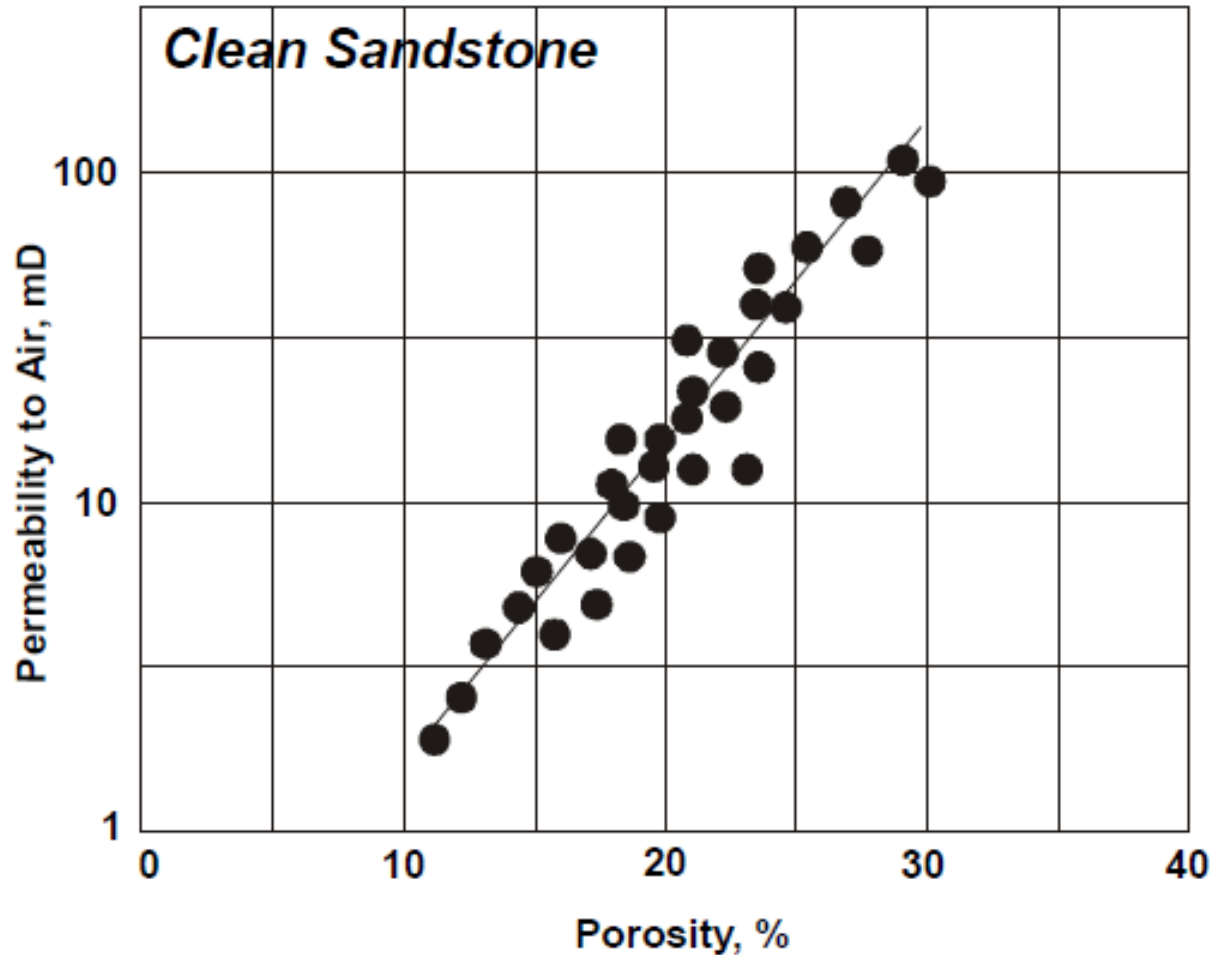
- The higher the porosity, the higher the permeability.
- The smaller the grains, the smaller the pores and pore throats, the lower the permeability.
- The smaller the grain size, the larger the exposed surface area to the flowing fluid, which leads to larger friction between the fluid and the rock, and hence lower permeability.

Reservoir Permeability Classification

Permeability Value (mD)	Classification
<10 10 – 100 100 – 1000 >1000	Fair High Very High Exceptional

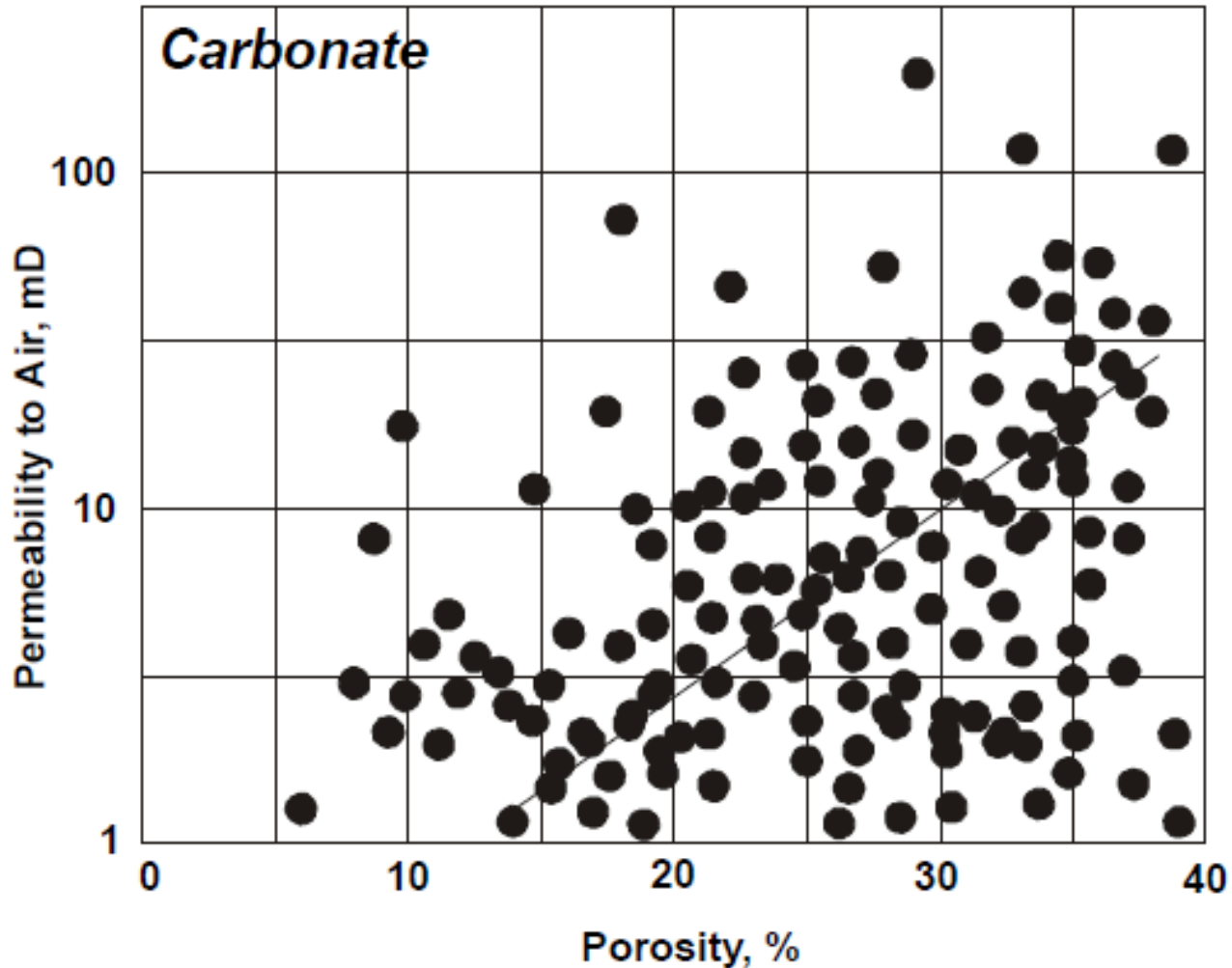
PorPerm Relationships

The permeability of the sandstone is extremely well controlled by the porosity

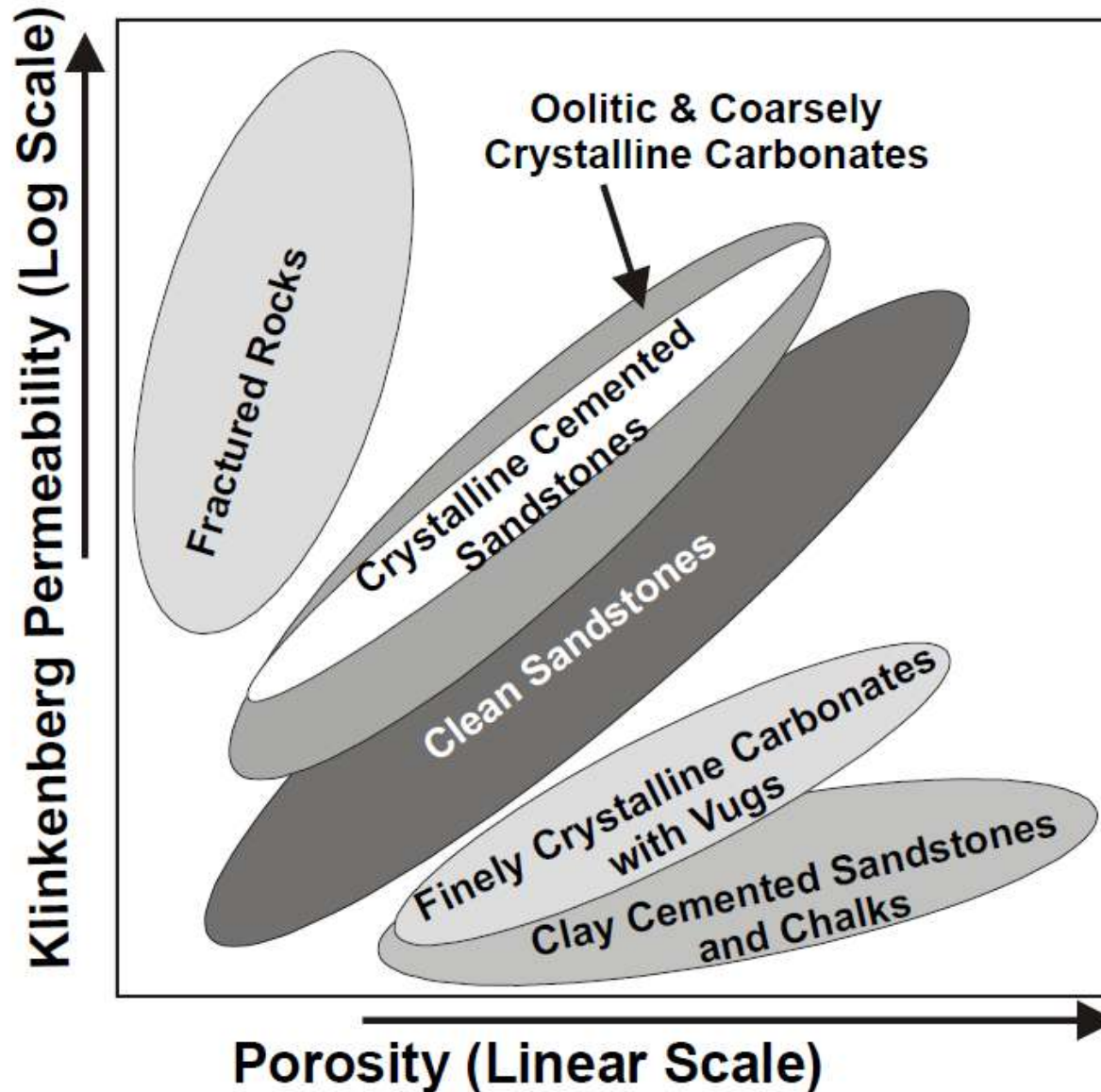


PorPerm Relationships

the carbonate has a more diffuse cloud indicating that porosity has an influence, but there are other major factors controlling the permeability.

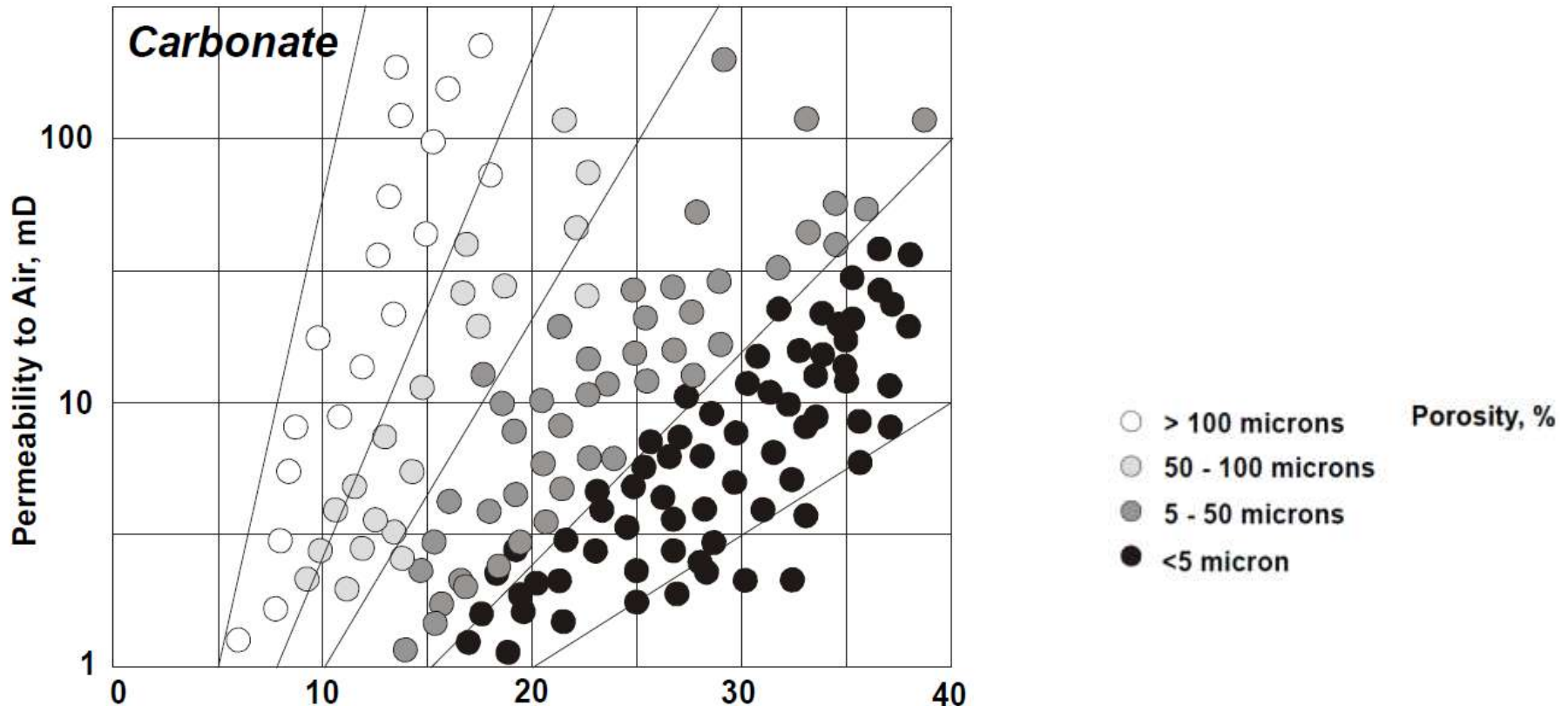


PorPerm Relationships



Poroperm Cross-plots and the Influence of Grain Size

smaller grain sizes produce smaller pores, and rather more importantly, smaller pore throats, which constrain the fluid flow more than larger grains which produce larger pore throats.



It is important to distinguish between **mass flow** and **filtration**: In the case of *mass flow* all particles in the field of flow are in motion, whereas in the case of *filtration*, only a portion of the mass particles flows and the remaining part forms the flooded framework.

Filtration Velocity

The velocity of filtration is defined as a fluid volume q flowing through the surface A of a porous medium within unit time:

$$u = \frac{q}{A} [ms^{-1}]$$

flux velocity

Darcy velocity

superficial velocity

Displacement Velocity

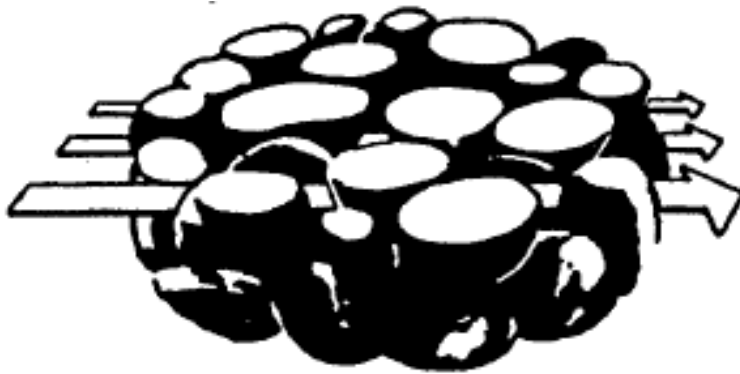
$$v = \frac{q}{A\phi} = \frac{u}{\phi}$$

interstitial velocity

If a fluid at a velocity of $u = 1$ [m/day] is injected into a porous medium with a porosity of 0.1, a specific fluid particle will be transported within a distance of 10 [m] in one day.

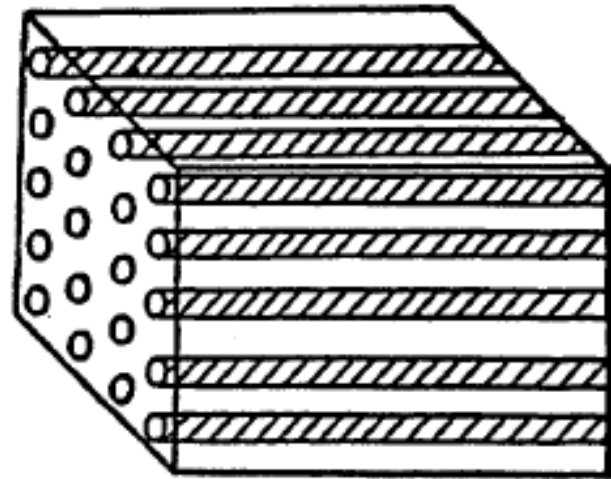
Conceptualization of Flow In a Porous Medium

Physical model

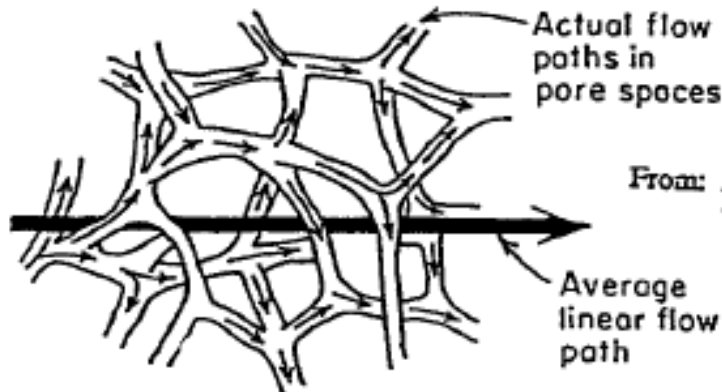


(From Keelan 1977)

Conceptual model



(From Scheidegger 1960)



Actual flow paths in pore spaces

Average linear flow path

From: Freeze, R.A. and Cherry, J.A.: *Groundwater*, Prentice-Hall, Inc., Englewood Cliffs, NJ (1979).

For a single tube Poiseuille's law for viscous (Laminar) flow is given by:

$$v = \frac{q}{A} = \frac{d^2}{32} \frac{1}{\mu} \frac{\Delta p}{L} \quad (1)$$

Written in terms of flow rate:

$$q = \frac{\pi r^4}{8} \frac{1}{\mu} \frac{\Delta p}{L} \quad (2)$$

For a single tube we have:

$$q_i = \frac{\pi r_i^4}{8} \frac{1}{\mu} \frac{\Delta p}{L} \quad (3)$$

As we consider a bundle of tube in parallel, the total flow rate is given by:

$$q_t = q_1 + q_2 + \dots + q_n \quad (4)$$

Substituting Equation (3) in Equation (4) gives:

$$q_i = \frac{1}{\mu} \frac{\Delta p}{L} \left[\frac{\pi}{8} \sum r_i^4 \right] \quad (5)$$

Defining:

$$C = \left[\frac{\pi}{8} \sum r_i^4 \right] \quad (6)$$

This gives:

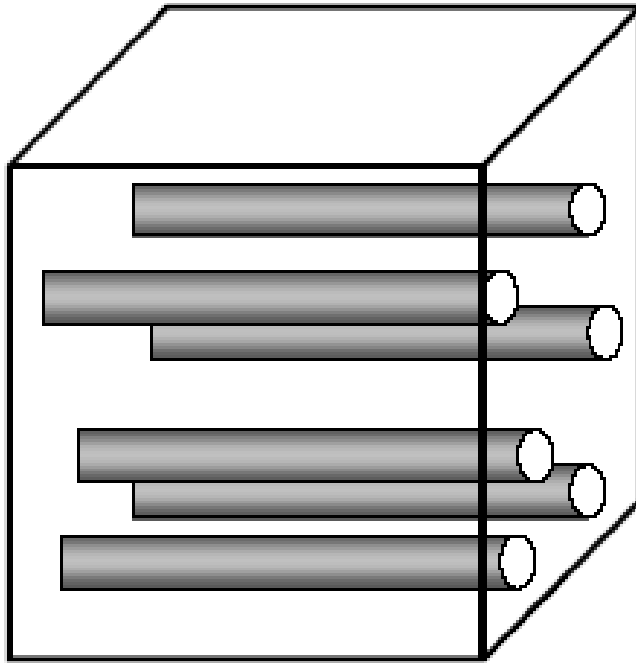
$$q_t = C \frac{1}{\mu} \frac{\Delta p}{L} \quad (7)$$

$$q_t = C \frac{1}{\mu} \frac{\Delta p}{L}$$

$$C = \left[\frac{\pi}{8} \sum r_i^4 \right]$$

At least in principal Equation (7) describes the analogy of flow in porous media as modeled by a bundle of tubes. Obviously, a porous medium is not generally thought of as a bundle of tubes, but rather as a collection of interconnected tortuous paths through which fluid flows through a pressure gradient. We will use Equation (7) as our starting point and the “C” constant of proportionality will become known as “permeability”.

Permeability (Carman-Kozeny Equation)



Actually, tubes are not straight



Conceptual model of porous medium -- capillary tube bundle.

The Hagen-Poiseuille equation can be derived for flow of a Newtonian fluid in a single tube of radius R and length L_t :

$$q_t = \frac{\pi R^4}{8\mu} \frac{\Delta p}{L_t} \quad (1.2)$$

In steady laminar flow, the velocity distribution across the tube is parabolic. The average velocity is given by:

$$\bar{v} = \frac{q_t}{\pi R^2} = \frac{R^2}{8\mu} \frac{\Delta p}{L_t} \quad (1.3)$$

Since we know that in a true porous medium the tubes are not straight, we need to define a representative volume (REV) over which we can define the representative length and flow velocity. The time taken for the fluid to pass through the tortuous path will be the same as the time to pass through the REV:

$$t = \left(\frac{L_t}{\bar{v}} \right)_{tube} = \left(\frac{L}{v} \right)_{REV} ; \quad \text{where } v = \frac{q}{A\phi} \quad (1.4)$$

Notice the distinction between the *flux velocity*, $u = q/A$, also known as the *Darcy velocity* or the *superficial velocity*, and the *interstitial velocity*, $v = u/\phi$, which is the speed at which the fluid actually moves in the pore space. The *interstitial velocity* is the speed at which a tracer front would move in the medium.

Combining with Darcy's Law:

$$\left(\frac{L_t}{\bar{v}}\right)_{tube} = \frac{8\mu L_t^2}{R^2 \Delta p} = \left(\frac{L}{v}\right)_{REV} = \frac{\phi \mu L^2}{k \Delta p} \quad (1.5)$$

$$k = \frac{\phi R^2}{8} \frac{L^2}{L_t^2} = \frac{\phi R^2}{8\tau}, \quad \text{where } \tau = \left(\frac{L_t}{L}\right)^2 \quad (1.6)$$

The *tortuosity*, ζ , is a variable that defines the “straightness” of the flow paths. A straight tube has a tortuosity of 1, whereas common porous materials have tortuosity values between 2 and 5.

Tortuosity can be determined experimentally from resistivity measurements. In an actual porous medium, the radius of the “tubes” is not going to be uniform. We can define the *hydraulic radius* for a noncircular tube as:

$$R_h = 2 \times \frac{\text{Cross-sectional area open to flow}}{\text{Wetted perimeter}} \quad (1.7)$$

This definition differs from that in Lake, who leaves out the 2. With the definition of Eq. (1.7), the hydraulic radius of a cylindrical tube would be R , the radius of the tube – which makes more sense.

Since a porous medium is not really made up of actual tubes, we can define the hydraulic radius instead as:

$$R_h = 2 \times \frac{\text{Volume open to flow}}{\text{Wetted surface area}} = \frac{2\phi}{a_v(1-\phi)} \quad (1.8)$$

where a_v is the specific surface area, the surface area of the pore space per unit volume of solid.

Specific surface area is an intrinsic property of the porous medium.
Substituting in Eq. (1.6):

$$k = \frac{\phi^3}{2\tau(1-\phi)^2 a_v^2} \quad (1.9)$$

For a porous medium made up of uniform spheres of radius R (diameter D_p):

$$a_v = \frac{4\pi R^2}{\frac{4}{3}\pi R^3} = \frac{3}{R} = \frac{6}{D_p} \quad (1.10)$$

Hence:
$$k = \frac{1}{72\tau} \frac{\phi^3 D_p^2}{(1-\phi)^2} \quad (1.11)$$

This is what is known as the Carman-Kozeny equation and defines the Carman-Kozeny permeability.

The Carman-Kozeny equation provides good estimates of the permeability of packs of uniform spheres, for real porous media it reveals the dependence on pore size (D_p), tortuosity (τ) and of packing (through porosity ϕ).

Permeability Relationships

The complexity of the relationship between permeability and pore geometry has resulted in much research. No fundamental law linking the two has been found. Instead, we have a plethora of empirical approximations for calculating permeability

Name	Equation	Notes
Solution Channel	$k = 0.2 \times 10^8 \times d^2$	k = permeability (D) d = channel diameter (inches)
Fractures	$k = \frac{0.544 \times 10^8 \times w^3}{h}$	k = permeability (D) h = fracture width (inches) w = fracture aperture (inches)
Wyllie and Rose equations I	$k = \left(\frac{100 \phi^{2.25}}{S_{wi}} \right)^2$	k = permeability (mD) ϕ = porosity (fraction) S_{wi} = irreducible water saturation (fraction)
Wyllie and Rose equations II	$k = \left(\frac{100 \phi^2 [1 - S_{wi}]}{S_{wi}} \right)^2$	k = permeability (mD) ϕ = porosity (fraction) S_{wi} = irreducible water saturation (fraction)
Timur equation	$k = \frac{0.136 \phi^{4.4}}{S_{wi}^2}$	k = permeability (mD) ϕ = porosity (%) S_{wi} = irreducible water saturation (%)
Morris and Biggs equation	$k = \left(\frac{C \phi^3}{S_{wi}^2} \right)$	k = permeability (mD) ϕ = porosity (fraction) S_{wi} = irreducible water saturation (fraction) C = constant; oil=250; gas=80

Permeability Relationships

Slichter equation	$k = \frac{10.2 d^2}{K_s}$	<p>k = permeability (mD) d = median grain size (microns) K_s = packing correction; slope of line when plotting median grain size vs. permeability.</p>
Kozeny-Carman equation	$k = \frac{c d^2 \phi^3}{(1 - \phi)^2}$	<p>k = permeability (mD) ϕ = porosity (fraction) c = constant d = median grain size (microns)</p>
Berg equation	$k = 8.4 \times 10^{-2} \times d^2 \phi^{5.1}$	<p>k = permeability (mD) ϕ = porosity (fraction) d = median grain size (microns)</p>
Van Baaren equation	$k = 10 D_d^2 \phi^{(3.64+m)} C^{-3.64}$	<p>k = permeability (mD) ϕ = porosity (fraction) D_d = modal grain size (microns) C = sorting index m = Archie cementation exponent.</p>
RGPZ equation	$k = \frac{1000 d^2 \phi^{3m}}{4 a m^2}$	<p>k = permeability (mD) d = weighted geometric mean grain size (microns) ϕ = porosity (fraction) m = Archie cementation exponent. a = grain packing constant</p>