Flow in Porous Media

Module 1.c
Fundamental Properties of Porous Media
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Forces acting on a fluid in a reservoir

- Capillary forces
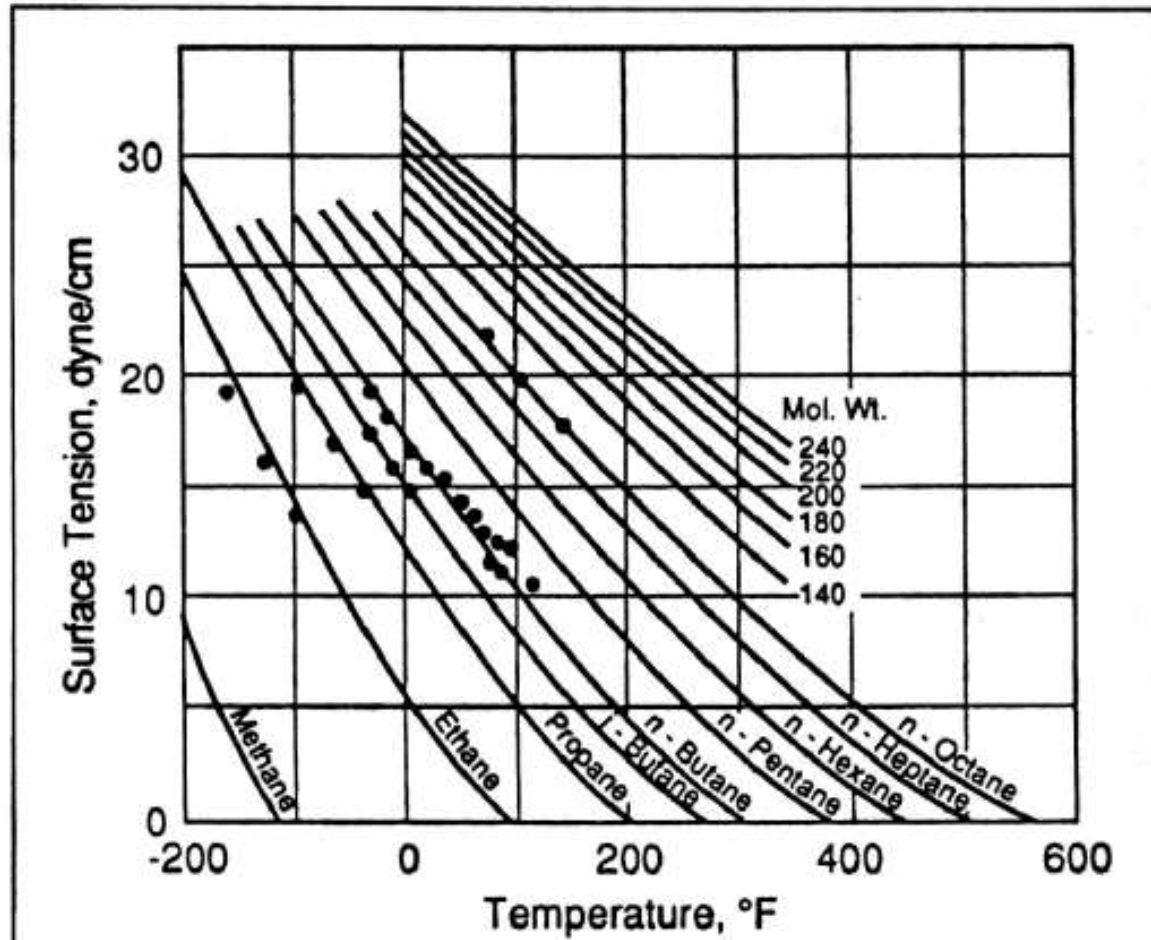- Viscous forces
- Gravitational forces

Surface Tension

Fig. 2.4—Surface tension of paraffin hydrocarbons (from Katz, D.L. and Saltman, W.: "Surface Tension of Hydrocarbons," *Ind. & Eng. Chem.* (1939)[2]; and Amyx, J.W. et *al.*: *Petroleum Reservoir Engineering* (1960) McGraw-Hill Book Co.[3] Reproduced with permission of McGraw-Hill Inc.).

Interfacial Tension

TABLE 2.1—IFT BETWEEN WATER AND VARIOUS CRUDE OILS
(after Donaldson et al. [4])

Field	Oil Formation	State	Oil Viscosity (cp)	IFT (dynes/cm)
West Delta	Offshore	Louisiana	30.4	17.9
Cayuga	Woodbine	Texas	82.9	17.9
Fairport	Lansing	Kansas	5.3	20.8
Bayou	Choctaw	Louisiana	16.1	15.6
Chase-Silica	Kansas City	Kansas	6.7	19.6
Hofra	Paleocene	Libya	6.1	27.1
Black Bay	Miocene	Louisiana	90.8	17.7
Bar-Dew	Bartlesville	Oklahoma	9.0	21.4
Bar-Dew	Bartlesville	Oklahoma	6.8	21.4
Eugene Island	Offshore	Louisiana	7.4	16.2
Cambridge	Second Berea	Ohio	15.3	14.7
Grand Isle	Offshore	Louisiana	10.3	16.1
Bastian Bay	Uvigerina	Louisiana	112.2	24.8
Oklahoma City	Wilcox	Oklahoma	6.7	20.1
Glenpool	Glen	Oklahoma	5.1	24.7
Cumberland	McLish	Oklahoma	5.8	18.5
Allen District	Allen	Oklahoma	22.0	25.9
Squirrel	Squirrel	Oklahoma	33.0	22.3
Berclair	Vicksberg	Texas	44.5	10.3
Greenwood-Waskom	Wacatoch	Louisiana	5.9	11.9
Ship Shoal	Miocene	Louisiana	22.2	17.3
Gilliland	—	Oklahoma	12.8	17.8
Clear Creek	Upper Bearhead	Louisiana	2.4	17.3
Ray	Arbuckle	Kansas	21.9	25.3
Wheeler	Ellenburger	Texas	4.5	18.2
Rio Bravo	Rio Bravo	California	3.8	17.8
Tatums	Tatums	Oklahoma	133.7	28.8
Saturday Island	Miocene	Louisiana	22.4	31.5
North Shongaloo-Red	Takio	Louisiana	5.2	17.7
Elk Hills	Shallow Zone	California	99.2	12.6
Eugene Island	Miocene	Louisiana	27.7	15.3
Fairport	Reagan	Kansas	31.8	23.4
Long Beach	Alamitos	California	114.0	30.5
Colgrade	Wilcox	Louisiana	360.0	19.9
Spivey Grabs	Mississippi	Kansas	26.4	24.5
Elk Hills	Shallow Zone	California	213.0	14.2
Trix-Liz	Woodbine A	Texas	693.8	10.6
St. Teresa	Cypress	Illinois	121.7	21.6
Bradford	Devonian	Pennsylvania	2.8	9.9
Huntington Beach	South Main Area	California	86.2	16.4
Bartlesville	Bartlesville	Oklahoma	180.0	13.0
Rhodes Pool	Mississippi Chat	Kansas	43.4	30.5
Toborg	—	Texas	153.6	18.0

Capillary Pressure

- Fluid-fluid interfaces within porous rocks are generally highly curved.
- A consequence of the presence of interfacial tension is that the pressure on both sides a curved interface cannot be equal.
- The fluid-phase on the concave side of the interface needs to be at a higher pressure to maintain the stability of the curved interface.
- This pressure difference is given by the Laplace equation.

$$\Delta P = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right),$$ where r1 and r2 are the principal radii of

curvature of the interface.

- For a fluid-fluid interface in a capillary tube, $r_1 = r_2 = \dfrac{R}{\cos \theta}$, where R is the radius of the capillary and θ is the contact angle.

- Therefore in a capillary tube, $\Delta P = P_c = \dfrac{2\sigma \cos \theta}{R}$

Capillary Pressure in a Porous Medium

• The curvature of the fluid-fluid interfaces changes with changing saturation. This makes the capillary pressure a function of saturation.

• When the wetting phase saturation is small, the interfaces occur in smaller pores and are highly curved. This makes P_C high at low wetting phase saturations.

•The capillary pressure at a given saturation is a measure of the smallest pore being entered by the nonwetting phase at that point, suggesting the curvature of the capillary pressure curve is a function of the pore size distribution. The level of the curve is determined by the mean pore size.

• When the wetting phase saturation is high, the interfaces are less curved and P_C is lower.

• P_C becomes zero at certain value of the wetting phase saturation. At this point, all non-wetting phase is in the form of trapped globules.

Viscous Force

- Viscous force is reflected in the pressure gradient generated by the flow through a porous medium.
- The pressure gradient is proportional to the viscosity and the fluid velocity and inversely proportional to the conductivity of the medium.
- For a circular tube: $\dfrac{\Delta p}{L} = \dfrac{8\mu\bar{v}}{r^2}$, where r is the radius of the capillary tube, $\bar{v}$, is the average velocity, μ is the viscosity and L is the length.
- For a porous medium, the pressure gradient is given by Darcy's law: $\dfrac{\Delta p}{L} = \dfrac{\mu v}{k}$, where v is the Darcy velocity and k is the permeability.
- The Darcy velocity is a superficial velocity obtained by dividing the flow rate by cross-sectional area of the rock.
- The actual average interstitial velocity is given by: $\bar{v} = \dfrac{v}{\phi}$, where ϕ is the porosity.

Dominance of Capillary Forces over Viscous Forces

Consider the displacement of oil by water from a capillary tube, at velocity $\bar{v}$. For simplicity let us assume that $\mu_w = \mu_o = \mu$.
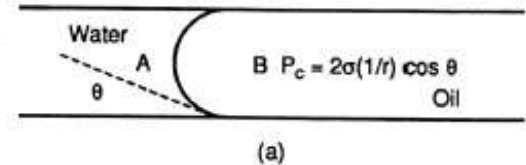
$$p_B - p_A = -\frac{8\mu L \bar{v}}{r^2} + \frac{2\sigma \cos\theta}{r}$$

Typical values are: $\mu = 1.0$ cp and $\sigma = 30$ mN/m and velocity $= 0.3$ m/d. Calculated vales of $p_B - p_A$ are listed in Table 2.2 for different r's .

Pore Radii	$-8\mu L\bar{v}/r^2$		ΔP_c		$p_B - p_A$	
(μm)	dynes/cm^2	psi × 10^4	dynes/cm^2	psi	dynes/cm^2	psi
2.5	− 22.6	− 3.25	240,000	3.45	+ 239,977	3.45
5	− 5.6	− 0.81	120,000	1.73	+ 119,994	1.73
10	− 1.41	− 0.20	60,000	0.86	+ 59,999	0.86
25	− 0.23	− 0.033	24,000	0.35	+ 24,000	0.35
50	− 0.056	− 0.008	12,000	0.17	+ 12,000	0.17
100	− 0.014	− 0.002	6,000	0.086	+ 6,000	0.086

TABLE 2.2—CAPILLARY AND VISCOUS FORCES FOR DIFFERENT SIZES OF PORE RADII, EXAMPLE 2.5

Note that the capillary force is much higher than the viscous force and the downstream pressure is higher.

Capillary Number

We can define the capillary number as a dimensionless number that relates the magnitudes of the capillary and viscous forces. The capillary number allows us to understand whether capillary forces dominate at the pore scale.

$$Capillary\ Number \propto \frac{Viscous\ force}{Capillary\ force}$$

$$N_{ca} = \frac{V\mu_w}{\sigma_{ow}}$$

Where:

V = interstitial velocity,

μ_w = water viscosity,

σ_{ow} = Interfacial tension

Some authors use the Darcy velocity instead of the interstitial velocity

$$N_{ca}^* = \frac{V_{darcy}\mu_w}{\sigma_{ow}} = \frac{k_{rw}k\Delta P}{\sigma_{ow}} \; or \; \frac{k\Delta P}{\sigma_{ow}}$$

$$Since \; V_{darcy} = \phi V$$

$$N_{ca}^* = \phi N_{ca}$$

Some authors use the below relation to account for variations in the contact angle.

$$N_{ca} = \frac{V\mu_w}{\sigma_{ow}\cos\theta}$$

The pressure difference within a pore of radius r due to capillarity is given by:

$$P_c = \frac{2\sigma}{r}$$

(2.24)

The viscous pressure drop across the same pore, assuming it has a length L and permeability k is:

$$u = -\frac{k}{\mu}\frac{\Delta p_{visc}}{L}, \quad so \quad \Delta p_{visc} = \frac{u\mu L}{k}$$

(2.25)

Hence the ratio of viscous pressure drop to capillary pressure is of order:

$$\frac{Viscous\ pressure\ drop}{Capillary\ pressure} \approx \frac{u\mu L}{k}\frac{r}{2\sigma}$$

(2.26)

Recalling the Carman-Kozeny relationship (Eq. (1.11)) between permeability and pore size:

$$k = \frac{1}{72\tau}\frac{\phi^3 D_p^2}{(1-\phi)^2} \approx 10^{-3}r^2$$

(2.27)

For a pack of spheres, typically L/r is around 10, so we can write:

$$\frac{Viscous\ pressure\ drop}{Capillary\ pressure} \approx \frac{u\mu L}{k}\frac{r}{2\sigma} = \frac{u\mu}{\sigma}\frac{10r^2}{10^{-3}r^2} = 10^2\frac{u\mu}{\sigma} = 10^2 N_c \tag{2.28}$$

Here we have defined the capillary number $u\mu/\sigma$. For a more realistic porous material:

$$\frac{Viscous\ pressure\ drop}{Capillary\ pressure} \approx 10^3 N_c \tag{2.29}$$

For a capillary number of around 10^{-3}, the capillary and viscous forces would be about the same.

Considering typical values of the porous medium parameters in common types of problems, for groundwater flow (where air and water are the nonwetting and wetting phases), u is of order 1 m/day to 1 m/year ($\sim 10^{-5}$ to 10^{-8} m/sec), μ is 1 cp $= 10^{-3}$ Pa.s, and σ is around 60×10^{-3} N/m, hence N_c is around 10^{-6} to 10^{-9}. In this case capillary forces dominate. In infiltration into soil, u may be much larger, for example 1 m/hr, so N_c may be as large as around 10^{-4} in which case viscous forces may become equally important.

For oil reservoirs, u is of order 10^{-5} m/sec at most (except very close to gas wells), μ is of order 1 cp $= 10^{-3}$ Pa.s, and σ is around 50×10^{-3} N/m, hence N_c is around 10^{-7}. Again, capillary forces dominate.

Bond Number

In some cases we may also find it useful to relate the magnitudes of capillary and buoyancy forces to understand the effects of gravity. For this we define the *Bond number*. The buoyancy force due to a density change over length L can be written:

$$\Delta p_{grav} = \Delta \rho g L \qquad (2.30)$$

$$\frac{Buoyancy\ pressure}{Capillary\ pressure} \approx \frac{\Delta \rho g L r}{2\sigma} \approx 10^{-1}\frac{\Delta \rho g L^2}{\sigma} = 10^{-1}N_B \qquad (2.31)$$

The Bond number is $\Delta \rho g L^2 / \sigma$. If the Bond number were to be around 10, then the two forces would be of similar significance.

Typical values of the Bond number for hydrology problems, in which the density difference between air and water gives rise to $\Delta \rho g$ values around 10^4 N/m^2 and L is around 10^{-3} m (grain size) would be around 0.2. This means the ratio of buoyancy to capillary forces is around 0.02. In oil reservoirs, the density difference is smaller, and the pore size is also smaller, so Bond number is of order 4×10^{-4}. In both cases, capillary forces dominate over buoyancy forces.

Phase Trapping

• When oil is displaced from a rock, the process is never perfect and a part of the oil is left behind in the form of globules or ganglia.

• Developing EOR techniques requires understanding how the trapping occurs and how can the trapped oil be mobilized.

• The actual mechanism in real rocks is quite complex and difficult to describe mathematically.

• A number of simplified models provide insight into the mechanisms involved.
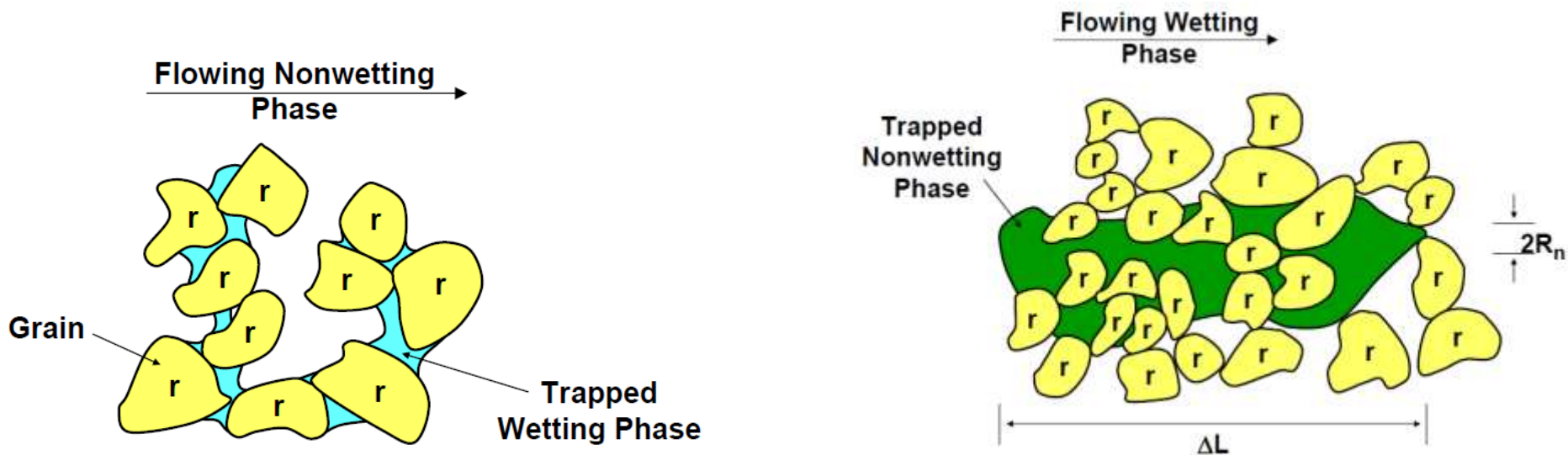
Requirement for Phase Trapping

•Nonzero interfacial tension

•Local heterogeneity

•Poor connectivity

Initial state
Oil Sat. = S_{oi}

Intermediate

Final state
Oil Sat. = S_{orw}
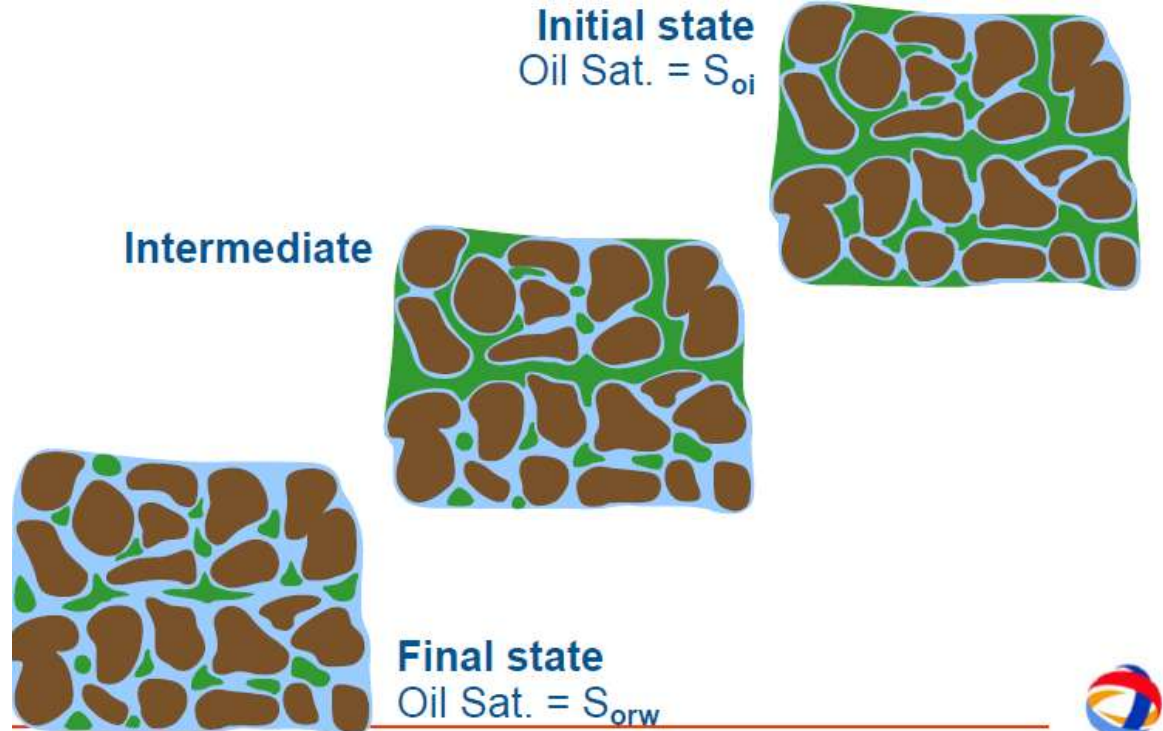
Residual Saturations

There is a residual saturation at which the capillary pressure appears to head towards infinity or zero. In the case of the wetting phase we often refer to this as the immobile saturation. No matter how much pressure we apply, we cannot reduce the wetting phase saturation any further (actually this is an oversimplification, since what we mean is that we cannot reduce it in any reasonable amount of time).

- **Bubbles of oil are trapped behind the water f**
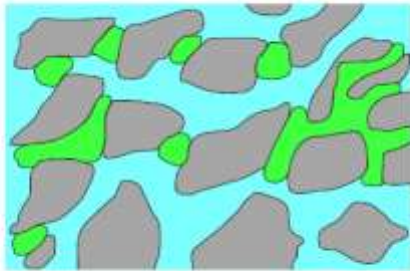
Grains
Oil
Water

Flowing Nonwetting Phase

Grain

Trapped Wetting Phase

immobile wetting phase saturation

Residual nonwetting phase saturation

Berea sandstone

Capillary pressure (psi Hg)

Mercury saturation
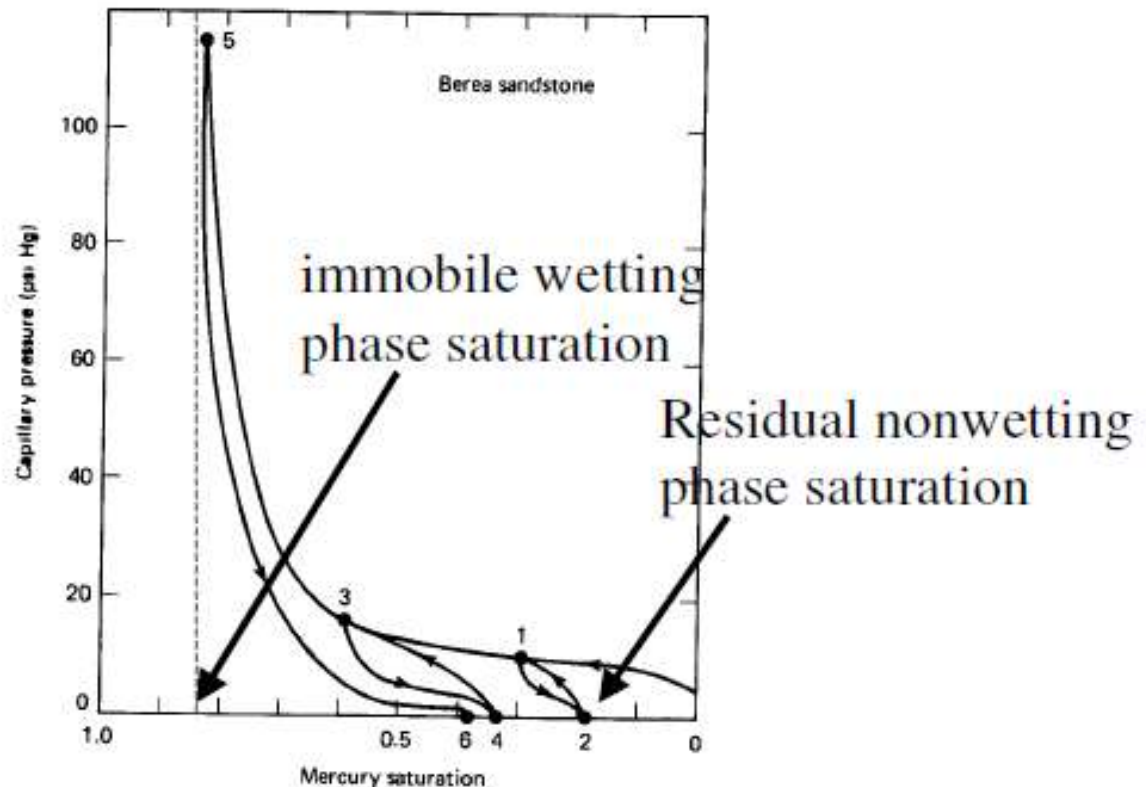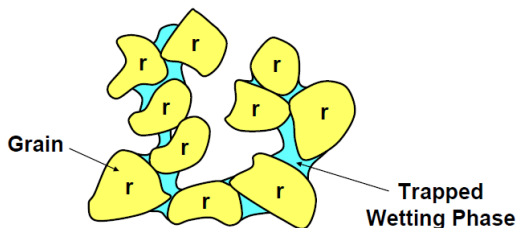
RESIDUAL OIL SATURATION (S$_{or}$)

For each initial saturation of nonwetting phase, there is a certain residual saturation that would remain after flooding with the wetting phase. For example, in Figure 2.31, if a rock had the initial nonwetting phase saturation represented by point A, then were it to be saturated with wetting phase it would have the residual saturation represented by A'.
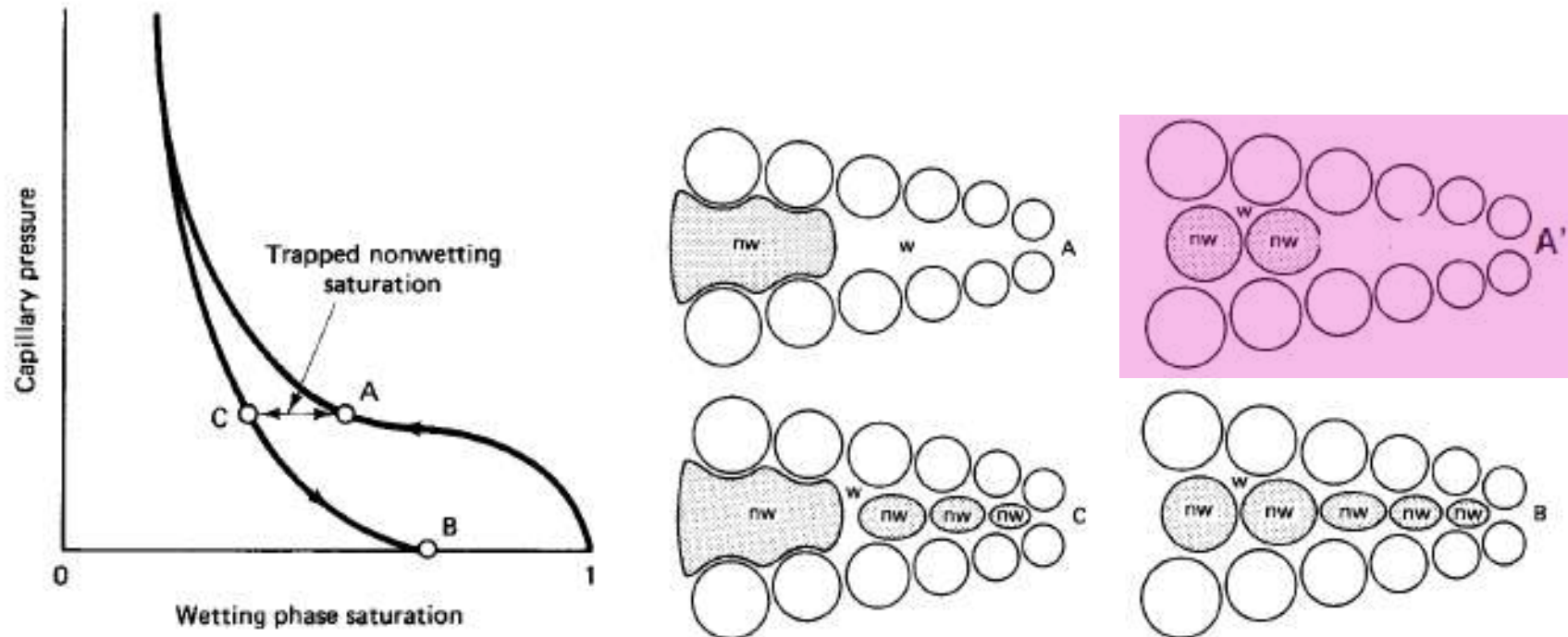
Figure 2.31: Initial and residual nonwetting phase saturation (based on Figure 3-6 in Lake).

Capillary pressure vs. Wetting phase saturation

Trapped nonwetting saturation

free | trapped

Points A, B, C on capillary pressure curves
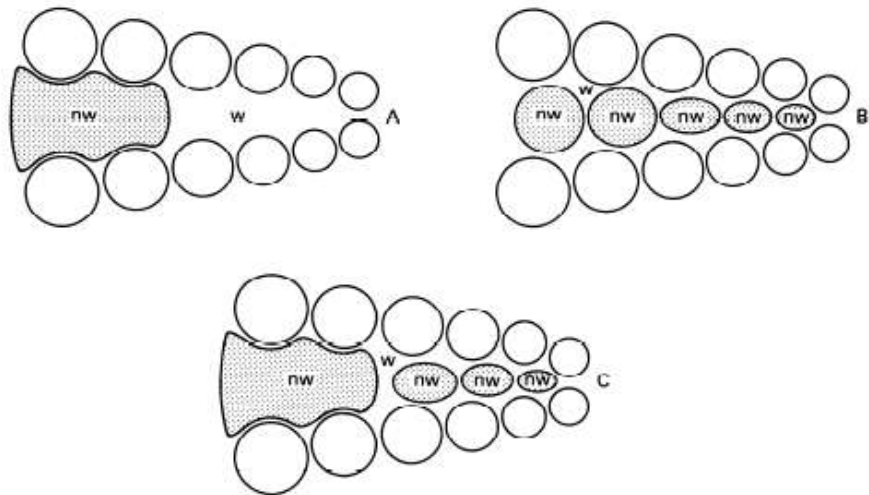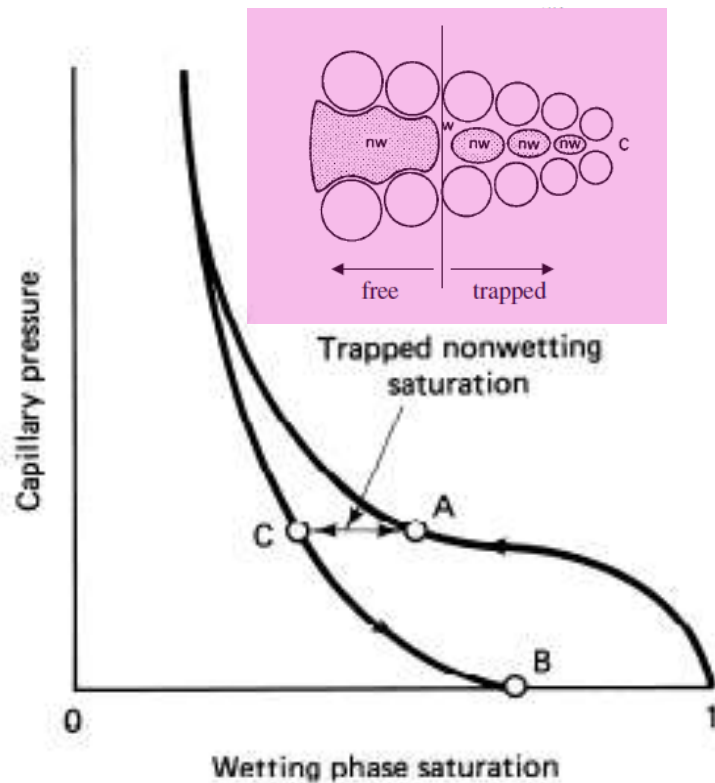
- Point B represents the maximum trapped nonwetting phase saturation
- Points A and C have equal capillary pressures but on different capillary pressure curves.
- The difference between the x coordinates of points A and C is the disconnected nonwetting phase saturation at point C.
- The connected nonwetting phase configuration is identical in configurations A and C; hence the trapped nonwetting phase saturation corresponding to an initial saturation at point A is the difference between the nonwetting phase saturation at point B minus the difference between the nonwetting phase saturations at points C and A (i.e., B – [C – A]).
- This procedure yields one point on the IR curve, but the whole curve may be traced by picking several points along the two P_c curves.

Residual Non-wetting Phase Saturation

The residual nonwetting phase saturation occurs because small blobs of nonwetting phase become trapped in the pores, and once disconnected from each other they can no longer flow. In a sequence of injections of non-wetting followed by wetting phase fluids (figure below) the capillary pressure is governed by the connected nonwetting phase saturation.

For each initial saturation of nonwetting phase, there is a certain residual saturation that would remain after flooding with the wetting phase.
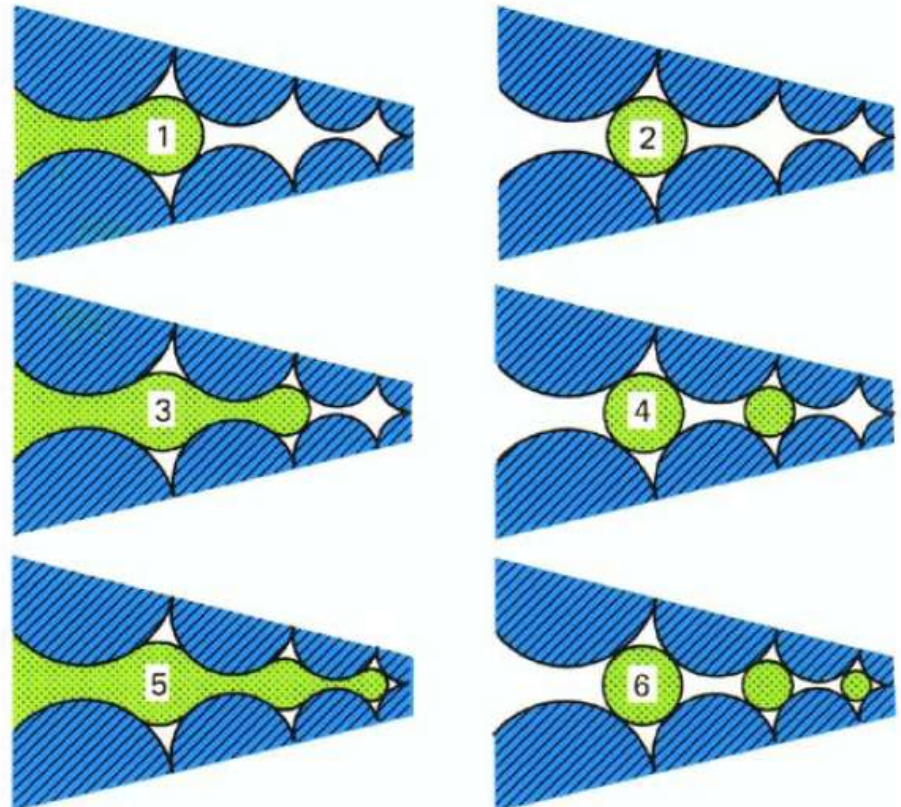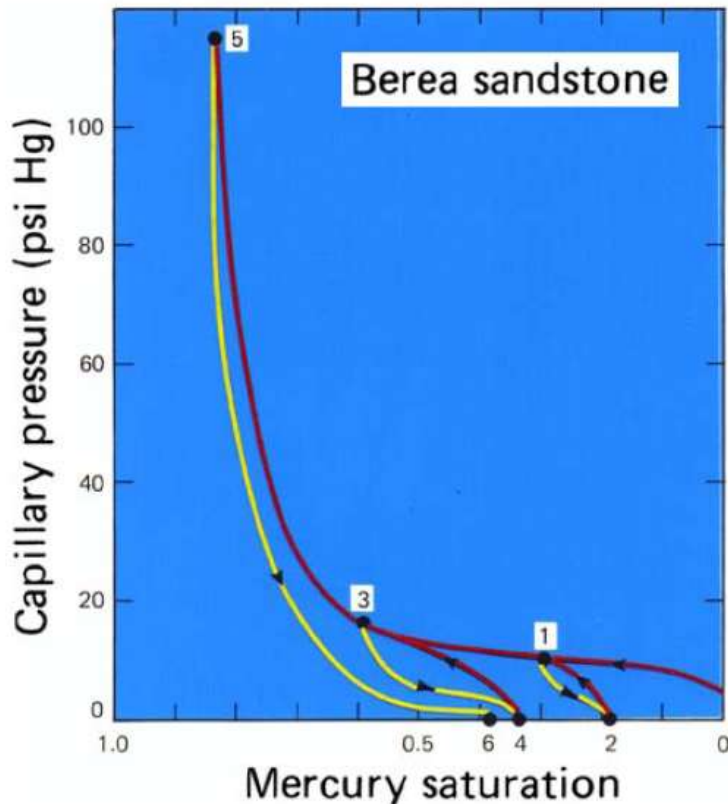
The relationship between the initial and residual nonwetting phase saturation is of great importance in determining the effectiveness of oil recovery during waterflooding. We can plot initial vs. residual saturation in an IR curve, such as below figure.
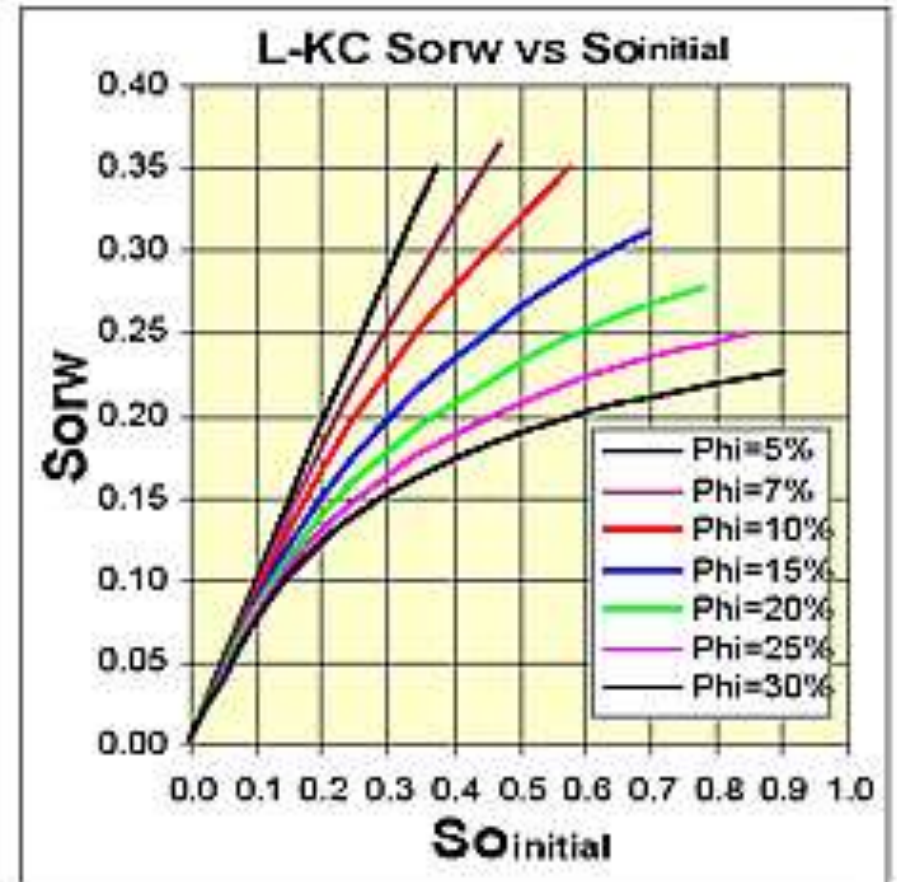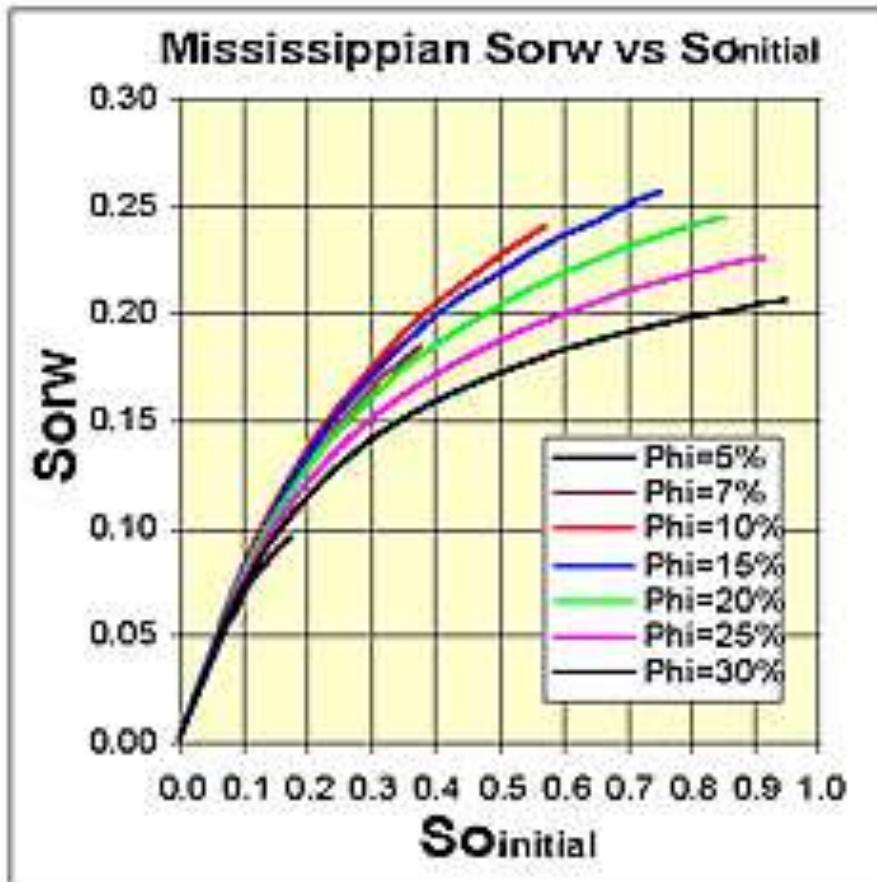
Land Trapping Model

The curve can be characterized using the Land trapping coefficient, C, defined as:
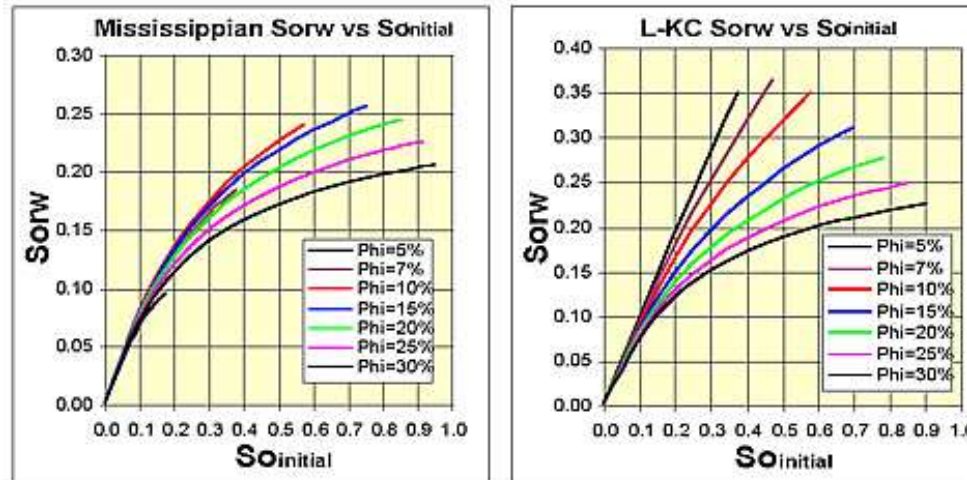
$$\frac{1}{S^*_{nwr}} - \frac{1}{S^*_{nwi}} = C \qquad (2.40)$$

Where the normalized saturations S* are defined:

$$S^*_{nwi} = \frac{S_{nwi}}{1 - S_{wi}}; \quad S^*_{nwr} = \frac{S_{nwr}}{1 - S_{wi}} \qquad (2.41)$$

We can also write:

$$S^*_{nwr} = \frac{S^*_{nwi}}{1 + CS^*_{nwi}} \qquad (2.42)$$

C varies from zero (complete trapping) to infinity (no trapping).

Typical IR Nonwetting Phase Saturations Curves

$$\frac{1}{S_{nwr}} - \frac{1}{S_{nwi}} = C$$

$$S_{nwr} = \frac{S_{nwi}}{1 + CS_{nwi}}$$
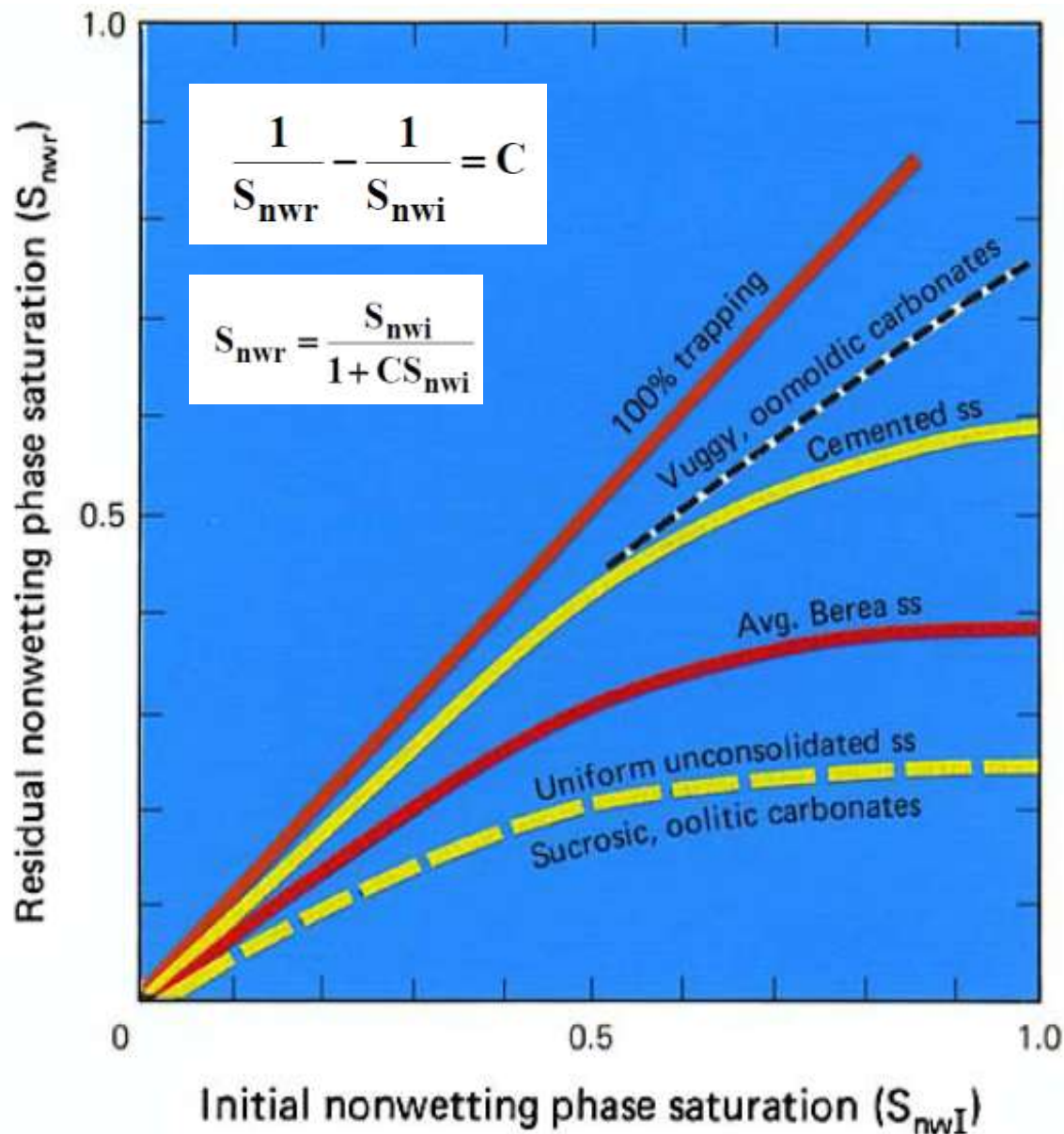
- where C is positive constant and a function of rock type, porosity

The maximum amount of trapped nonwetting phase is given by:

$$\frac{1}{S^*_{nwr}} - \frac{1}{S^*_{nwi}} = C = \frac{1}{\left(S^*_{nwr}\right)_{max}} - 1$$

(2.43)

This provides a useful experimental procedure to determine C as follows:
1. Saturate the medium 100% with wetting phase.
2. Conduct primary drainage to $S_w = S_{wi}$ ($S^*_w = 0$, $S^*_{nw} = 1$).
3. Conduct imbibition to $\left(S^*_{nwr}\right)_{max}$.
4. Calculate C.

In real reservoirs, the starting condition is usually $S_w = S_{wi}$, that is the maximum S_{nw}, which leads to the maximum S_{nwr}. The phase permeability for the nonwetting phase is due only to the "free" saturation (connected saturation) S_{nwf} and not the trapped saturation S_{nwr}.
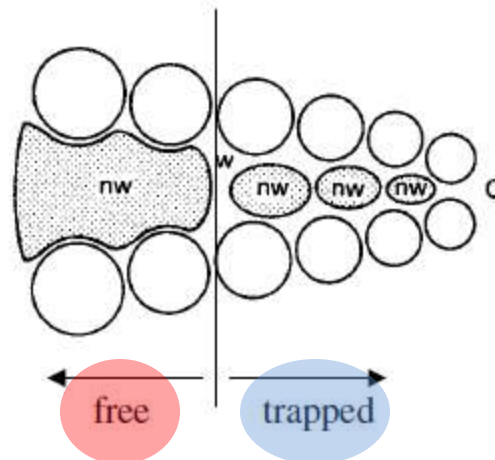
Figure 2.33: Free and trapped nonwetting phase saturation (from Figure 3-6 in Lake).

At intermediate saturations:

$$S_{nw} = S_{nwf} + S_{nwt} \; ; \qquad S_{nw}^{*} = S_{nwf}^{*} + S_{nwt}^{*} \tag{2.44}$$

We can calculate S_{nwt}^{*} from Land's relation and the knowledge that S_{nwt}^{*} equals $\left(S_{nwr}^{*}\right)_{\max}$ less the amount of S_{nwf}^{*} that will be trapped as the imbibition proceeds to $\left(S_{nwr}^{*}\right)_{\max}$, namely:

$$S_{nwt}^{*} = \left(S_{nwr}^{*}\right)_{\max} - \Delta S_{nwt}^{*} = S_{nwr}^{*} - \Delta S_{nwt}^{*} \tag{2.45}$$

This can be obtained from Land's relation starting with $S_{nwf}^{*} = S_{nwi}^{*}$:

$$\frac{1}{\Delta S_{nwt}^{*}} - \frac{1}{S_{nwf}^{*}} = C \tag{2.46}$$

$$\frac{S_{nwf}^{*}}{\Delta S_{nwt}^{*}} - 1 = C S_{nwf}^{*} \; ; \qquad \Delta S_{nwt}^{*} = \frac{S_{nwf}^{*}}{1 + C S_{nwf}^{*}} \tag{2.47}$$

So
$$S_{nwt}^{*} = S_{nwr}^{*} - \Delta S_{nwt}^{*} = S_{nwr}^{*} - \frac{S_{nwf}^{*}}{1 + C S_{nwf}^{*}} \tag{2.48}$$

The free saturation that contributes to flow is:

$$S^*_{nwf} = S^*_{nw} - S^*_{nwt} = S^*_{nw} - S^*_{nwr} + \frac{S^*_{nwf}}{1 + CS^*_{nwf}}$$

(2.49)

$$S^*_{nwf} + C(S^*_{nwf})^2 = S^*_{nw} + CS^*_{nw}S^*_{nwf} - S^*_{nwr} - CS^*_{nwr}S^*_{nwf} + S^*_{nwf}$$

(2.50)

$$C(S^*_{nwf})^2 + S^*_{nwf}(-CS^*_{nw} + CS^*_{nwr}) - (S^*_{nw} - S^*_{nwr})$$

(2.51)

This is a quadratic of the form $ax^2 + bx + c = 0$, where:

$$x = S^*_{nwf}; \qquad a = C; \qquad b = -C(S^*_{nw} - S^*_{nwr}); \qquad c = -(S^*_{nw} - S^*_{nwr})$$

Hence

$$S^*_{nwf} = \tfrac{1}{2}\left[(S^*_{nw} - S^*_{nwr}) + \sqrt{(S^*_{nw} - S^*_{nwr})^2 + \frac{4}{C}(S^*_{nw} - S^*_{nwr})} \right]$$

(2.52)

Non wetting phase blocking

$$PA - PB = - \frac{2\sigma}{R} \cos\theta$$

$$PC - PD = \frac{2\sigma}{r} \cos\theta$$
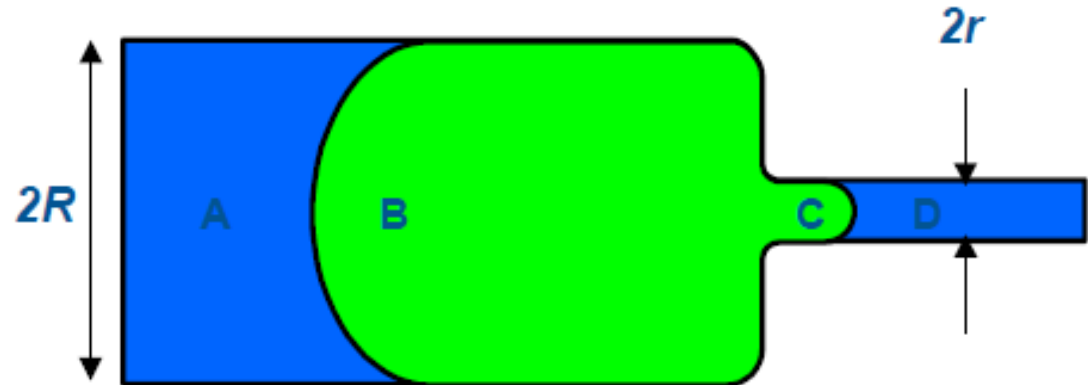
$$PA - PD = 2\sigma \left(\frac{1}{r} - \frac{1}{R} \right) \cos\theta$$

Suppose solid is perfectly water-wet   ($\theta = 0$)

$r = 0,5\,\mu m$ $R = 5\mu m$ et $\sigma = 30$ dynes / cm

$PA - PD =$ 1 bar

Trapping in a straight Capillary tube

- Experiments show that the pressure gradient required for moving a slug of a non-wetting phase through a capillary tube is higher than what would be predicted by the pipe flow equation.
- The reason is the pressure discontinuity at the oil/water interfaces.
- If the discontinuity was the same on both sides of the drop, the effect should cancel out.
- In reality, the discontinuity is not of the same magnitude on both sides of the drop, due to the contact angle hysteresis.
- At the downstream side, the contact angle is water receding, while at the upstream side it is water advancing.
- Water advancing angle can be considerably larger than water receding contact angle.
- It makes the radius of curvature smaller at the downstream interface, making that pressure discontinuity larger.
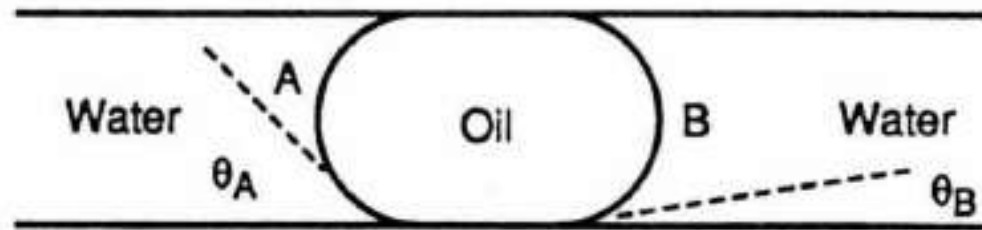
After the flow has stopped:

$$p_A - p_B = -\left(\frac{2\sigma_{o/w}\cos\theta}{r}\right)_A + \left(\frac{2\sigma_{o/w}\cos\theta}{r}\right)_B$$

$$= \frac{2\sigma_{ow}}{r}(\cos\theta_B - \cos\theta_A)$$

This pressure difference is positive, since angle θ_B is smaller than θ_A. So a positive pressure difference exists at zero velocity and to move the oil drop requires this pressure difference to be exceeded.

Trapping at Pore Throats

- As an oil drop is pushed through a pore throat, it downstream end gets squeezed into a much narrower segment, making its radius of curvature much smaller than the upstream part.
- This again generates a pressure drop due to the difference in capillary pressure at the leading and the trailing interfaces.

$$\Delta p = \left(\frac{2\sigma_{o/w} \cos\theta_A}{r_A} \right) - \left(\frac{2\sigma_{o/w} \cos\theta_B}{r_B} \right)$$

Using typical values: $\sigma_{ow} = 25$ mN/m, $r_A = 15$ microns, $r_B = 6.0$ microns, $\theta_A = \theta_B = 0$, L = 0.01 cm.

$\Delta P = 4.8$ kPa and $\Delta P/L = 47.3$ MPa/m = 2,073 psi/ft.

Conceptual Models for Trapping

Trapping in Pore Doublet
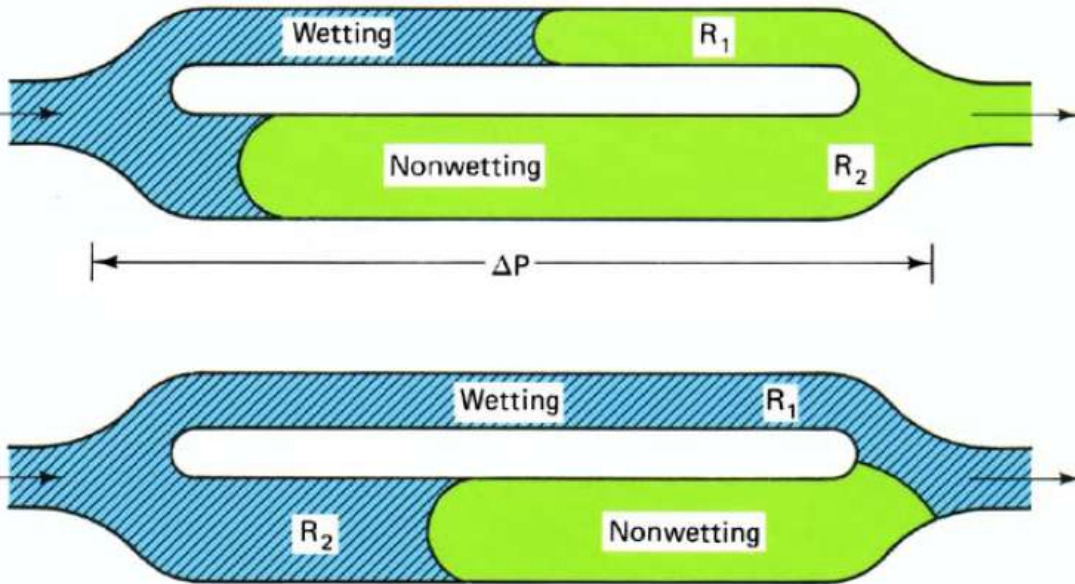
We are fundamentally interested in trapping of the nonwetting phase. In petroleum recovery the nonwetting phase is usually the oil, which we would earnestly like not to be trapped in the ground.

Trapping in Snap-Off Model

The forming of the discontinuous blobs through a running-ahead wetting phase is known as snap off. The consequences of snap off is described in more depth below but it arises largely because of the difference in the sizes between the pore throat and body, the larger the disparity, the more the trapping.
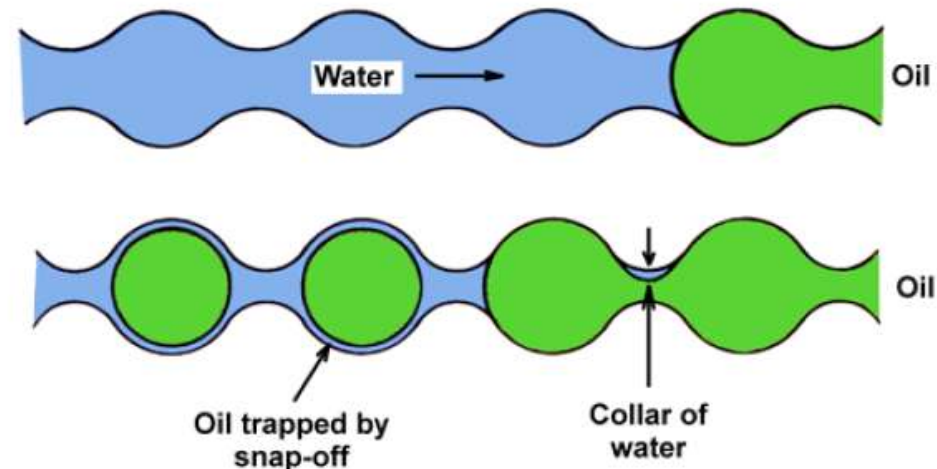
Trapping Mechanisms-Pore Doublet Model

(a) Pore Doublet Model

[Dullien, p. 426-429] [Lake, p. 63-67]

Consider two adjacent pores of different sizes initially filled with nonwetting fluid, as in Figure 2.34. Wetting fluid is introduced to displace the nonwetting fluid (imbibition).
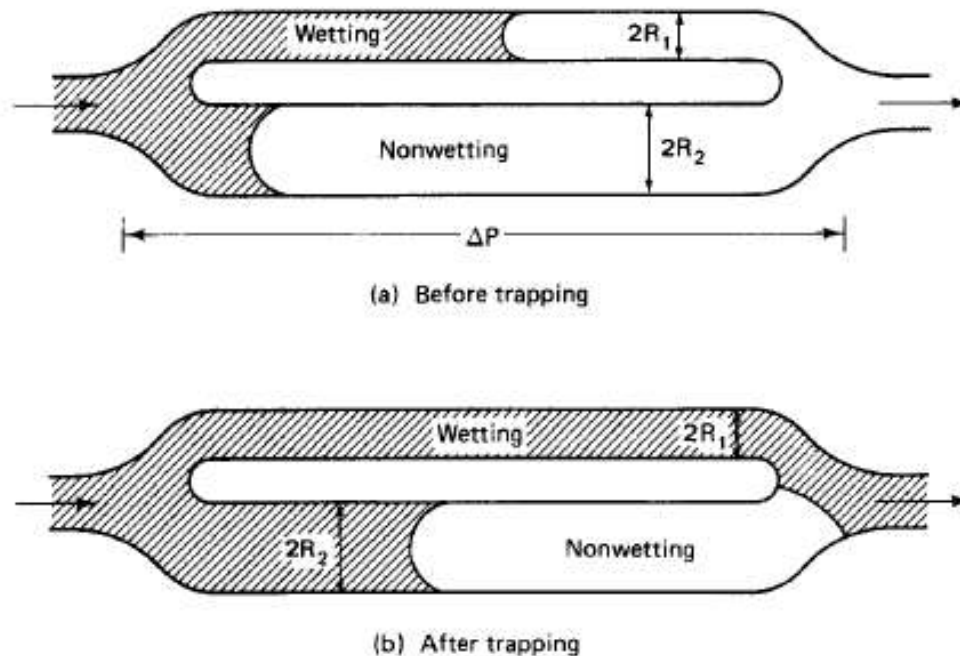

Figure 2.34: Pore doublet model (Figure 3-14 in Lake).

If we assume that: (a) Poiseuille flow describes the behavior of the fluids in the tubes (no effect due to the interface), and (b) that viscosities are equal, then:

$$q = q_1 + q_2 = \frac{\pi}{8\mu L}(R_1^4 \Delta p_1 + R_2^4 \Delta p_2)$$

(2.53)

The driving force for the two channels must be the same:

$$\Delta p = \Delta p_1 - P_{c1} = \Delta p_2 - P_{c2}$$

(2.54)

While the interfaces are in the tubes:

$$P_{c1} - P_{c2} = 2\sigma \cos\theta \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

(2.55)

We can combine Eqs. 2.54 and 2.55 to eliminate Δp_1, then eliminate Δp_2 in favor of q:

$$q_1 = \frac{q - \dfrac{\pi R_2^4 \sigma \cos\theta}{4\mu L}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2} \right)}{1 + \left(\dfrac{R_2}{R_1} \right)^4}$$

(2.56)

$$q_2 = \frac{q\left(\dfrac{R_2}{R_1}\right)^4 + \dfrac{\pi R_2^4 \sigma \cos\theta}{4\mu L}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}{1 + \left(\dfrac{R_2}{R_1}\right)^4}$$

(2.57)

To investigate the trapping behavior, we can investigate the ratio of the average velocities ($v = q/\pi r^2$) for each tube:

$$\frac{v_2}{v_1} = \frac{4N_{cap} + \left(\dfrac{1}{\beta} - 1\right)}{\dfrac{4N_{cap}}{\beta^2} - \beta^2\left(\dfrac{1}{\beta} - 1\right)}$$

(2.58)

where $\beta = R_2/R_1$ is a *heterogeneity factor*, and the local capillary number Ncap is:

$$N_{cap} = \frac{\mu L q}{\pi R_1^3 \sigma \cos\theta}$$

(2.59)

$$\frac{v_2}{v_1} = \frac{4N_{cap} + \left(\frac{1}{\beta} - 1\right)}{\frac{4N_{cap}}{\beta^2} - \beta^2 \left(\frac{1}{\beta} - 1\right)}$$

$$N_{cap} = \frac{\mu L q}{\pi R_1^3 \sigma \cos\theta}$$

$$\beta = R_2/R_1$$

When q is large, capillary forces are negligible (capillary number approaches infinity) and the velocity ratio is:

$$\frac{v_2}{v_1} \approx \beta^2 = \left(\frac{R_2}{R_1}\right)^2 \tag{2.60}$$

In this case the interface advances faster in the larger tube and traps nonwetting fluid in the small tube. When q is small and capillary forces dominate, then $v_2/v_1 < 1$ and the interface moves faster in the small tube and traps nonwetting phase in the large tube, as in Figure 2.34(b). This condition is typical for realistic values of q, R_1 and R_2.

Note that:
- Nonwetting fluid is trapped in the large pores and the wetting phase flows past it in the small pores.
- Lowering capillary forces (increasing capillary number) decreases the trapping phenomenon.
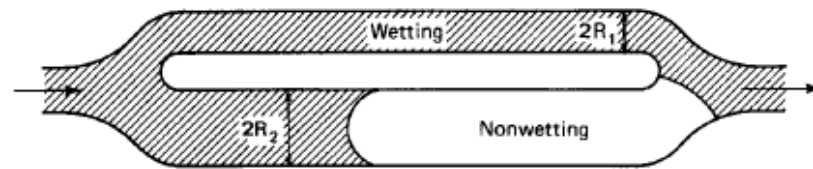- There is no trapping without local heterogeneity.

In general this model overestimates the amount of trapping in real porous media.

Pore Doublet Explains

•Nonwetting phase trapping
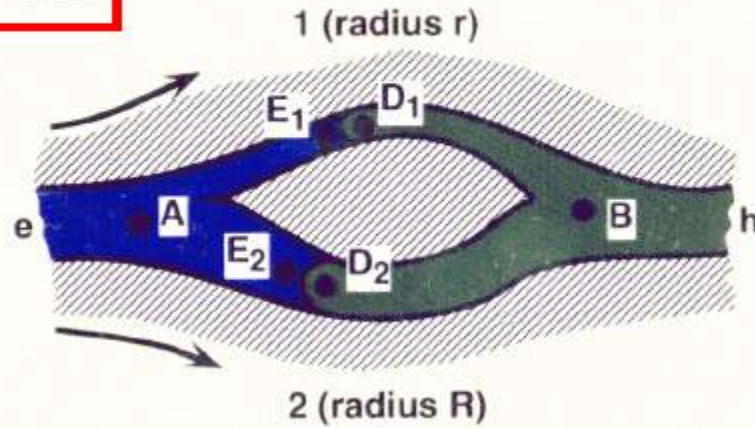•Trapping controlled by viscous/capillary ratio
•Local heterogeneity required
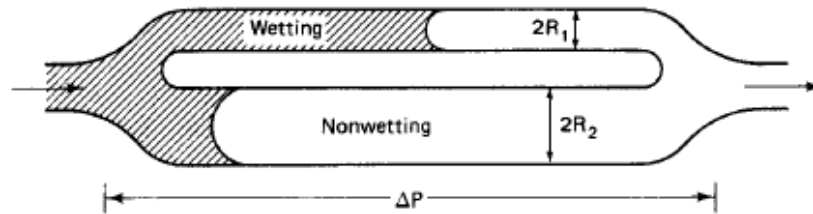
(a) Before trapping

(b) After trapping

Water wet

1 (radius r)

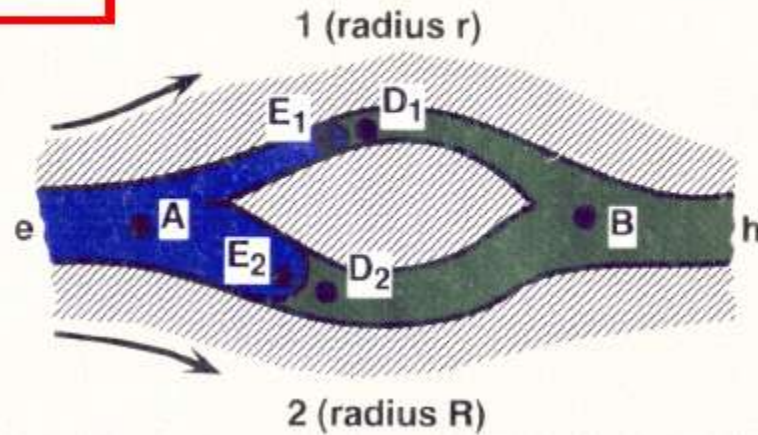E_1 D_1

e A B h

E_2 D_2

2 (radius R)

Oil trapped in the greatest pores

Oil wet

1 (radius r)

E₁ D₁

e A E₂ D₂ B h

2 (radius R)

Oil trapped in the smallest pores

Oil wetting the surface

Trapping Snap-off Model

(b) Snap-Off Model

[Dullien, p. 429-436] [Lake, p. 67-68]

The snap-off model is a better description of the trapping phenomenon in real porous media, and can account for 80% of the trapped nonwetting phase. In this model, the pore is envisaged of a tube of varying cross section. With a low aspect ratio of area variation, the wetting phase can effectively displace the oil in a piston-like fashion as in Figure 2.35(a). For a higher aspect-ratio channel, there is a higher gradient of capillary pressure in the nonwetting phase than in the (continuous) wetting phase, so the nonwetting phase wants to flow backwards locally and the collar of wetting phase "snaps off" as in Figure 2.35(b).

Oil

Water →

(a)
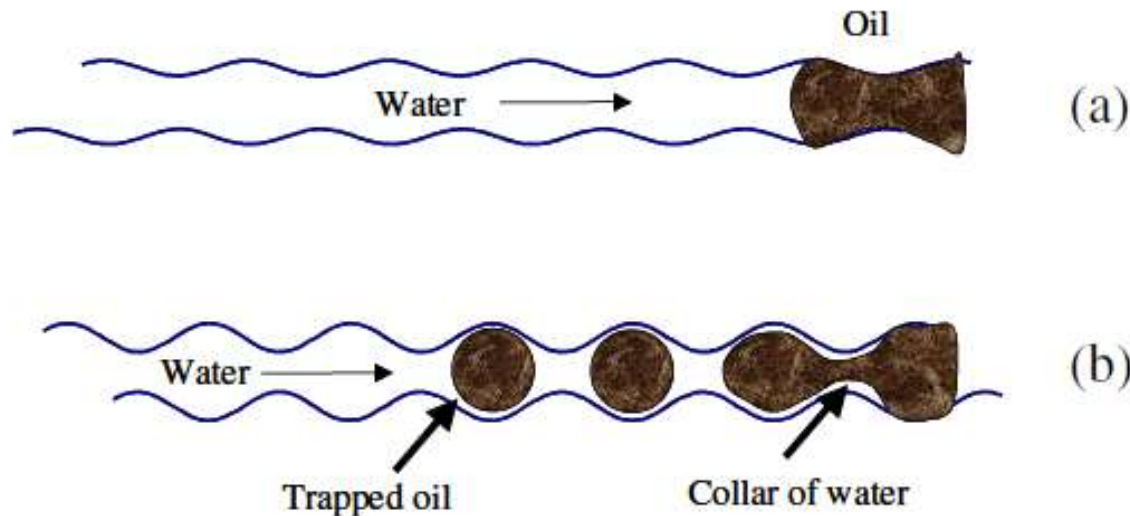
Water →

Trapped oil   Collar of water

(b)

Figure 2.35: (a) Piston-like flow in low aspect-ratio channel; (b) Snap-off in high aspect-ratio channel.
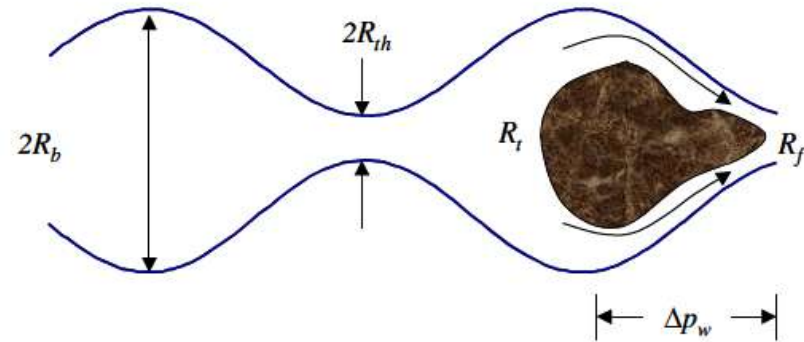
Figure 2.36: Pressures around a trapped ganglion of nonwetting phase.

Looking at the pressures at the upstream and downstream ends of the trapped ganglion (Figure 2.36), if p is the pressure in the wetting phase at the upstream point, then the nonwetting phase pressure is:

$$p_1 = p + \frac{2\sigma}{R_t}$$

(2.61)

The pressure in the ganglion at the downstream end is:

$$p_2 = p - \Delta p_w + \frac{2\sigma}{R_f}$$

(2.62)

The ganglion can only move downstream if $p_1 > p_2$, hence we can say that the ganglion becomes trapped unless:

$$\Delta p_w > 2\sigma \left(\frac{1}{R_f} - \frac{1}{R_t} \right) \approx 2\sigma \left(\frac{1}{R_{th}} - \frac{1}{R_b} \right)$$

(2.63)

This explains why the trapping occurs in high aspect-ratio pores, and also trapping is a rate-dependent

Immobile Wetting Phase Saturation

In a drainage process the invading nonwetting phase flows through the center of the pores while the wetting phase forms a film on the walls.

In the formation of most oil reservoirs, oil migrates into sedimentary rocks that are initially filled with water as the wetting phase. As oil saturation increases, the water is forced out of the largest pores first and occupies smaller and smaller spaces in the rock. When the water saturation becomes low enough, the water becomes disconnected and forms pendular rings. Once disconnected, the wetting phase can no longer flow and hence the remaining fluid is at *immobile wetting phase saturation*.
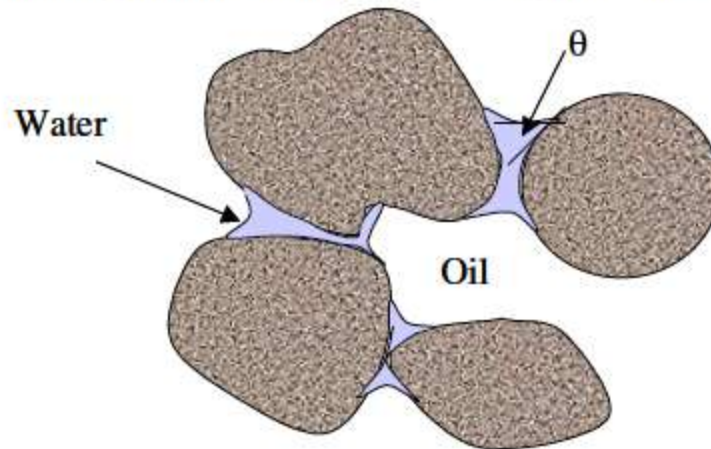

Figure 2.38: Immobile wetting phase saturation in the form of pendular rings.

The capillary pressure at wetting phase saturation is a function of the two principal radii of the donut-shaped ring (the "waist" radius and the "groove" radius):

$$P_c = p_{nw} - p_w = \sigma \cos\theta \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

(2.64)

(a): At low wetting phase saturation, the pendular rings are isolated and do not form a continuous water phase except for the very thin film of adsorbed water on the solid surfaces, a few tens of molecules in thickness. We shall discuss the effect of this film at a later stage. Thus, when all the water is in the form of pendular rings, no flow is possible, as there is no continuous pathway of water. Water pressure applied at some point cannot be transmitted to other points.

(c): As water saturation increases, the pendular rings expand and coalesce, until continuous water phase is formed. Above this critical saturation, the 'bulk' water forms a continuous phase, and its saturation is called funicular; flow of 'bulk' water is possible.

(a)

(c)

R

t

Air

Water

(b)

(d)
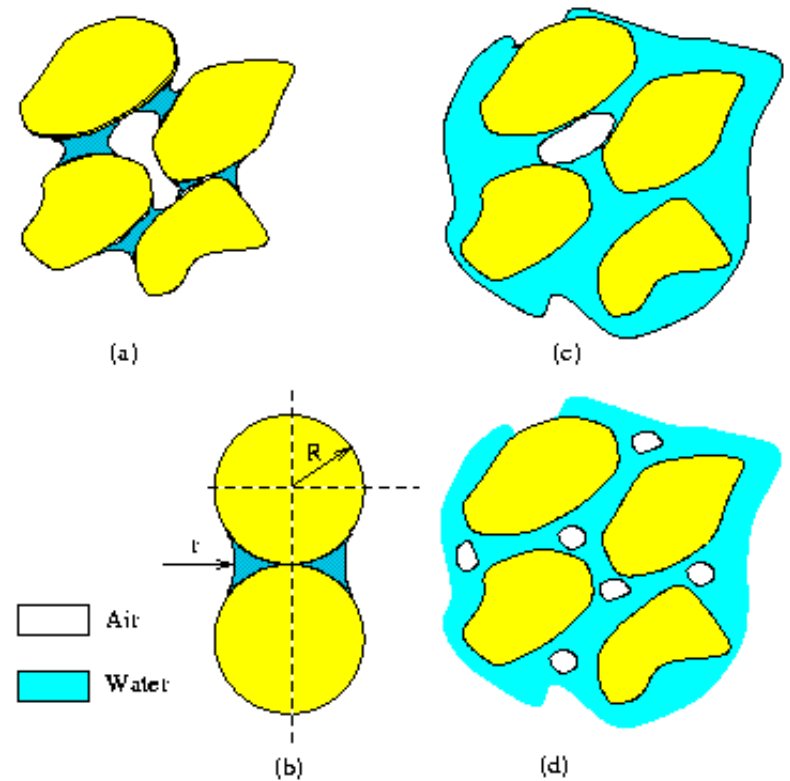
Capillary Desaturation Curve (CDC) for Pore Doublet

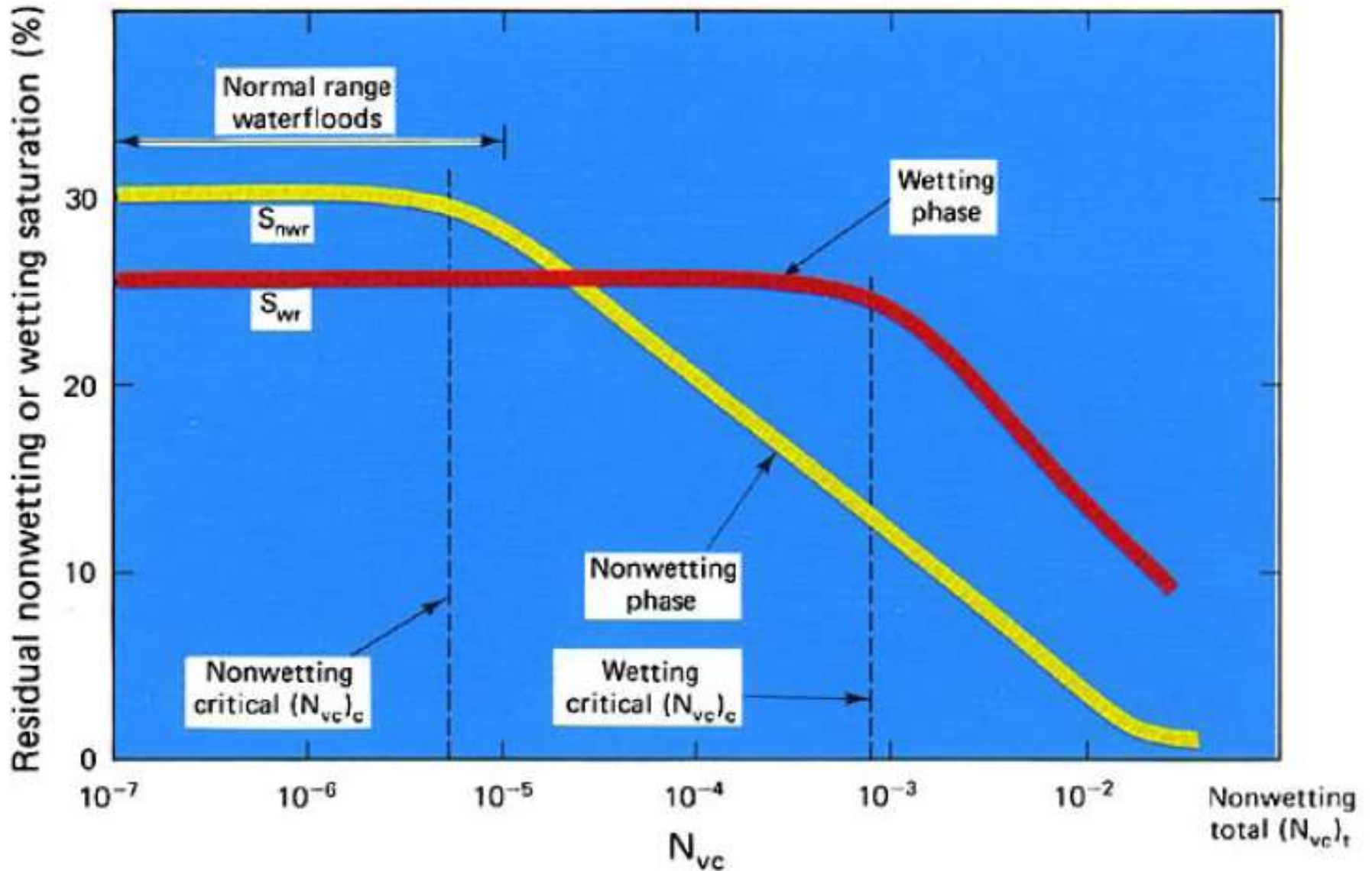The CDC relates the amount of trapped nonwetting or wetting phase as a function of capillary number.
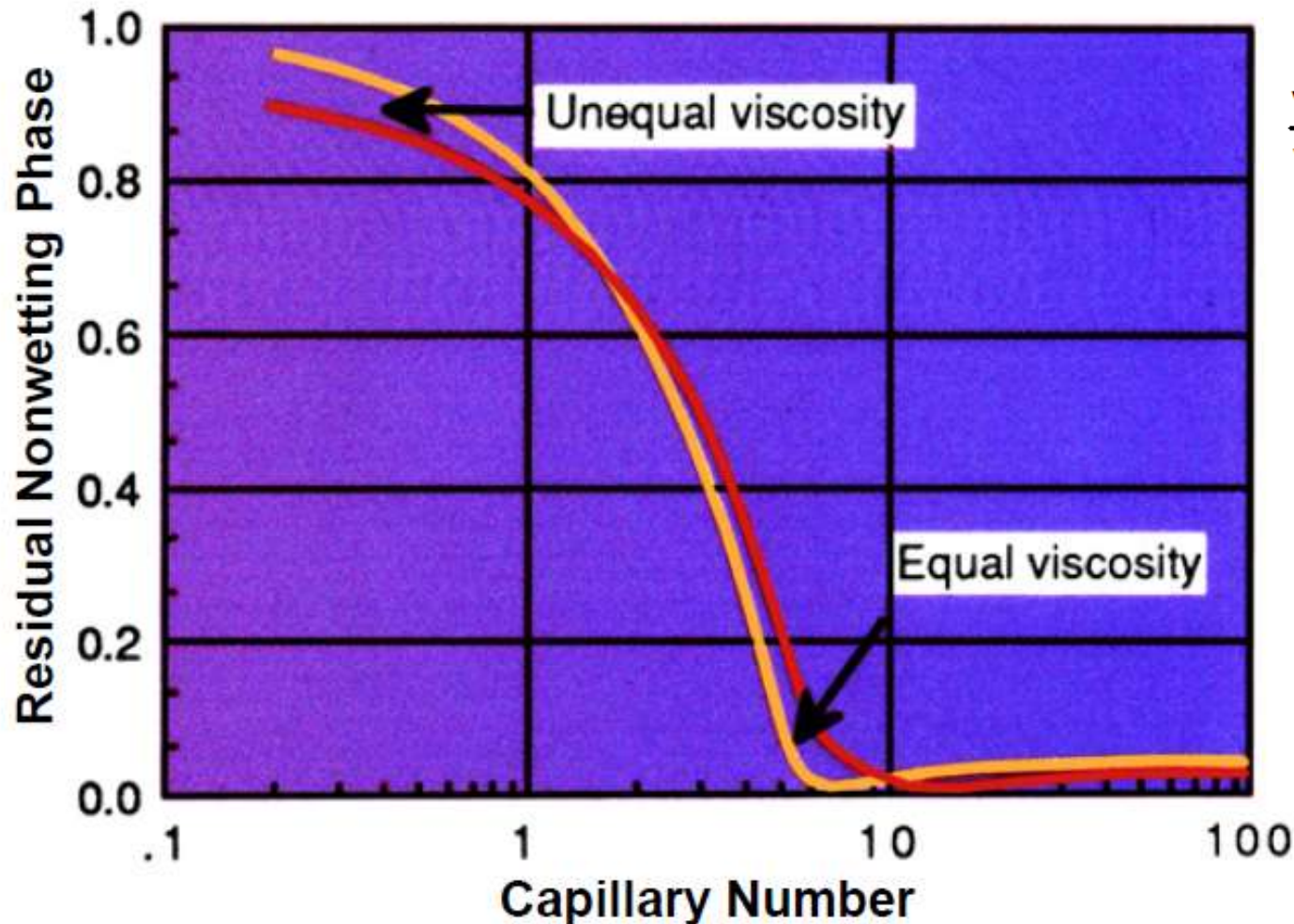
$$\frac{V_2}{V_1} = \frac{4N_{vc} + \left(\frac{1}{\gamma} - 1\right)}{\frac{4N_{vc}}{\gamma^2} - \gamma^2\left(\frac{1}{\gamma} - 1\right)}$$

$$\gamma = \frac{R_2}{R_1}$$
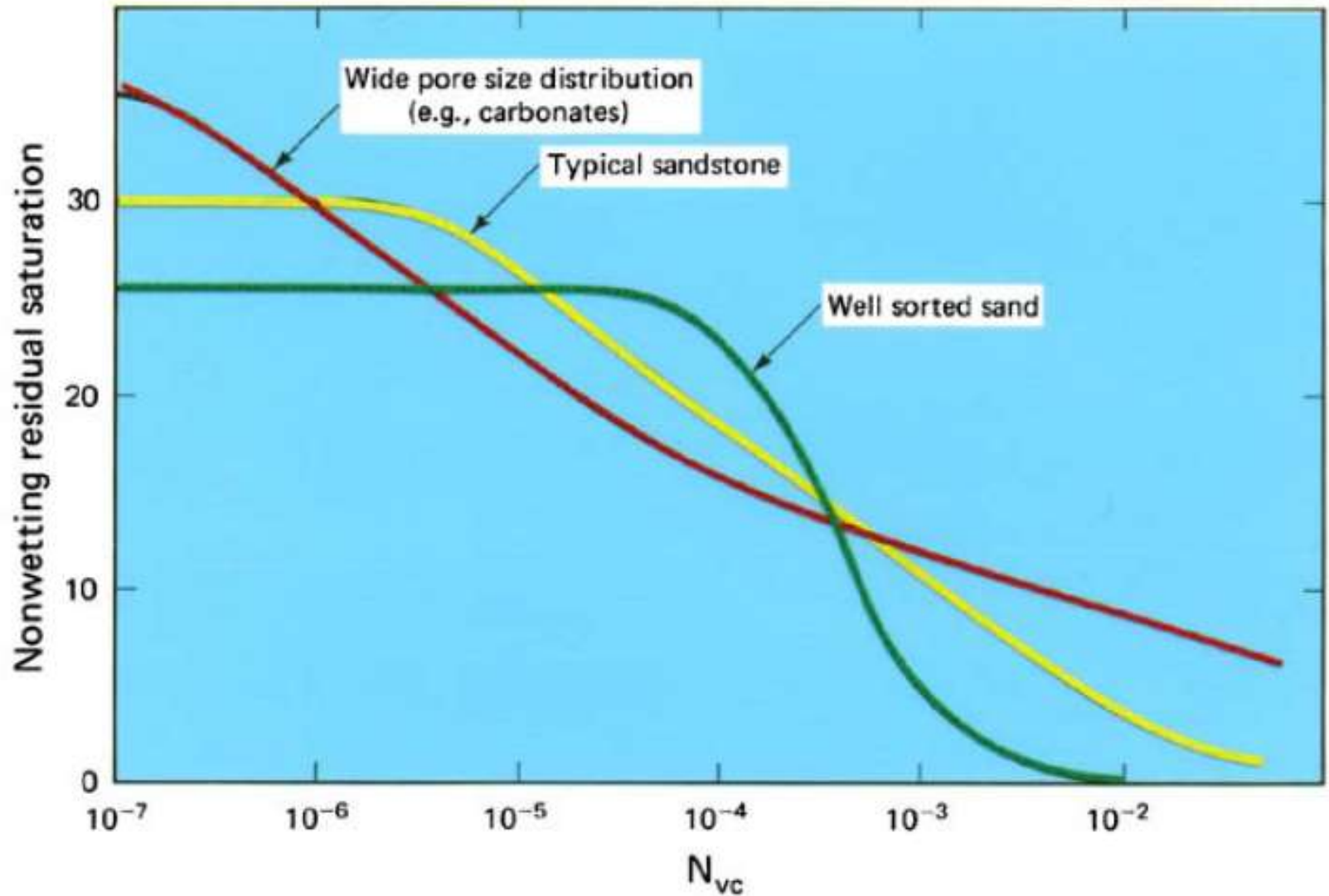
$$N_{vc} = \frac{\mu L_t q}{\pi R_1^3 \sigma \cos\theta}$$
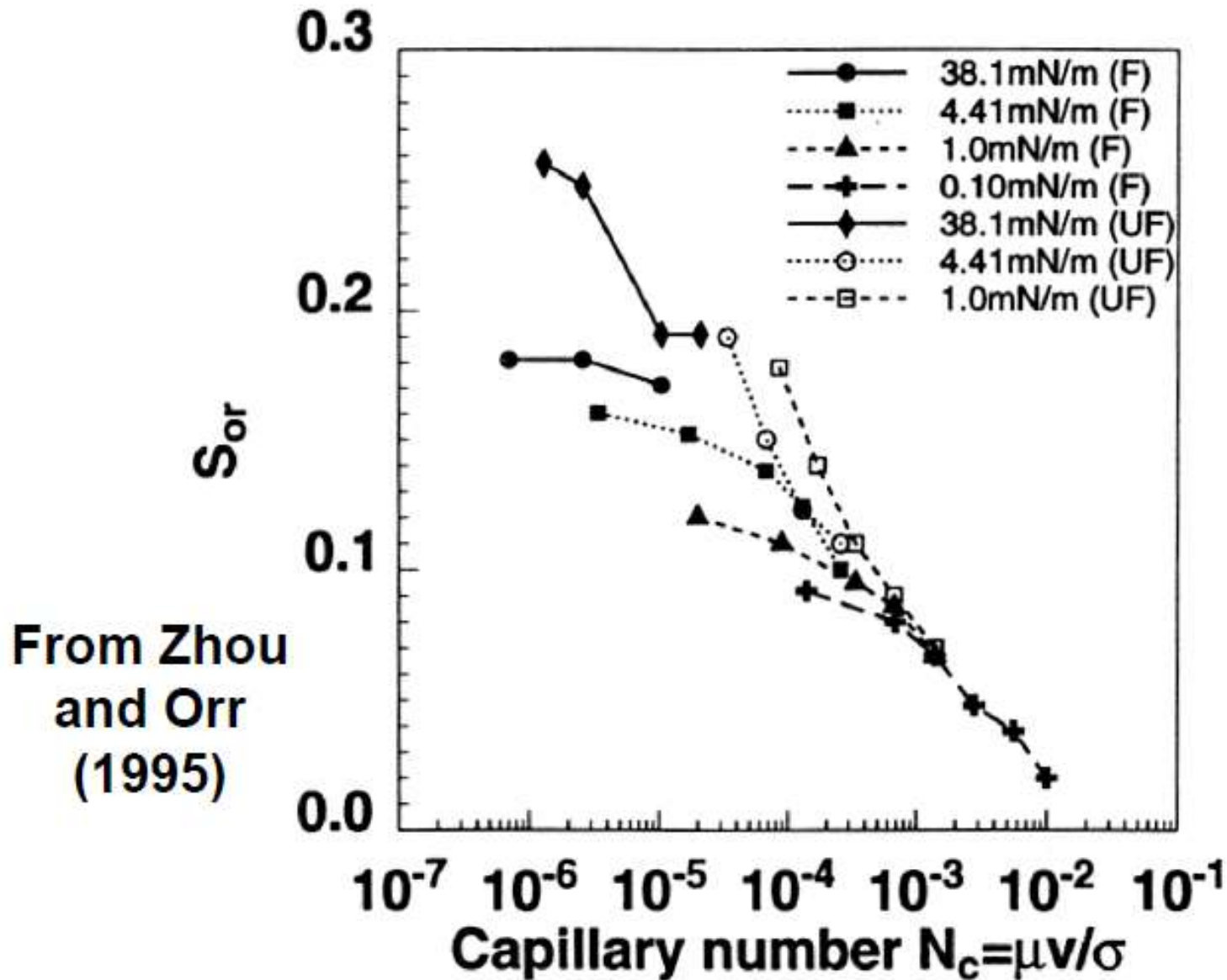
Capillary Desaturation Curve

Critical Capillary Number

•In most cases the nonwetting phase has a higher residual (more trapping) than the wetting phase.

• Both phases tend to have a *critical Capillary number at which the trapped phase begins to mobilize. The* critical Capillary number for wetting phase is often higher than that for nonwetting phase, hence the target for enhanced oil recovery is to modify the Capillary number to lie between the two critical values.

•In practice it is difficult to raise either the viscosity or velocity, so the most accessible way to increase Capillary number is to reduce the interfacial tension s, for example by adding surfactant.

Pore Size Distribution Broadens CDC

Figure showing Nonwetting residual saturation versus N_{vc} with three curves labeled: Wide pore size distribution (e.g., carbonates), Typical sandstone, and Well sorted sand.

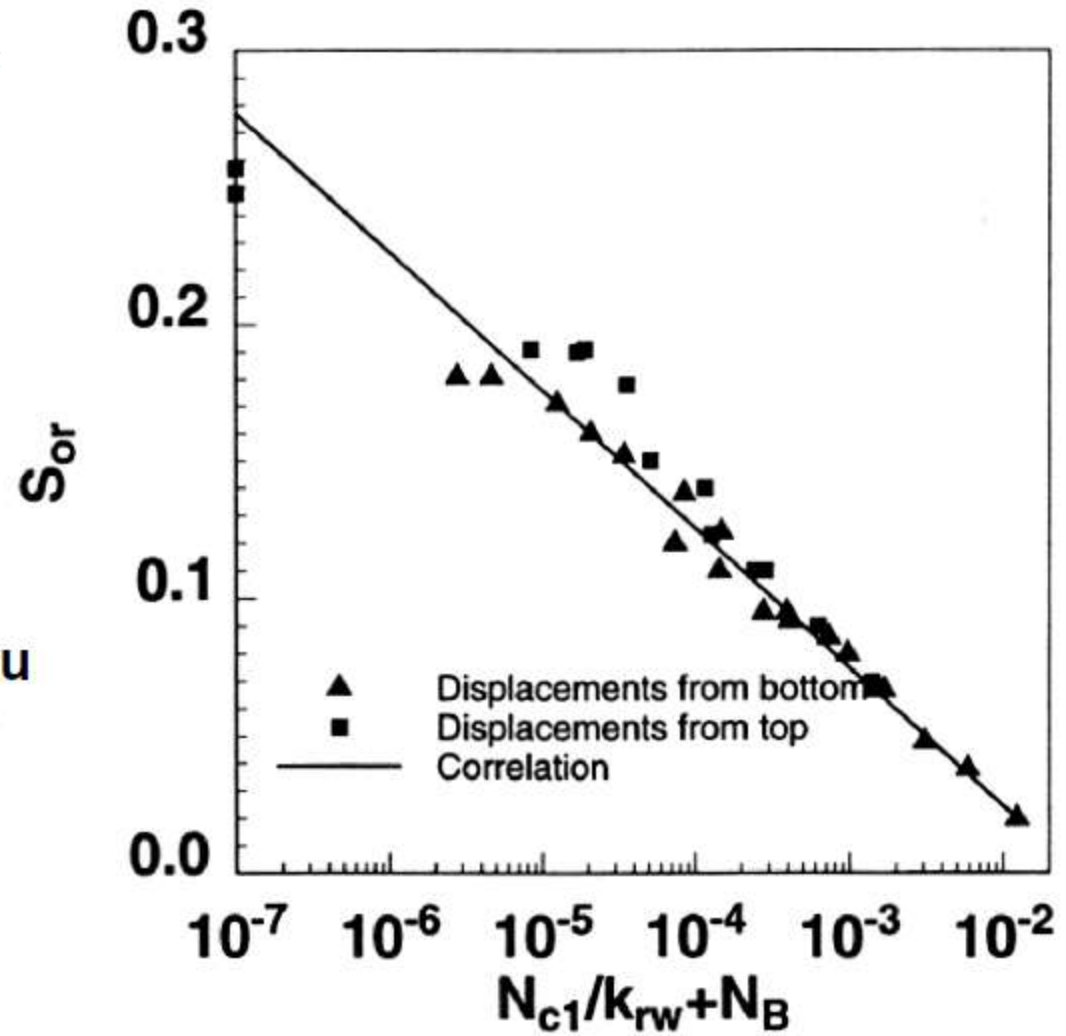# Trapping and Capillary Number

From Zhou and Orr (1995)

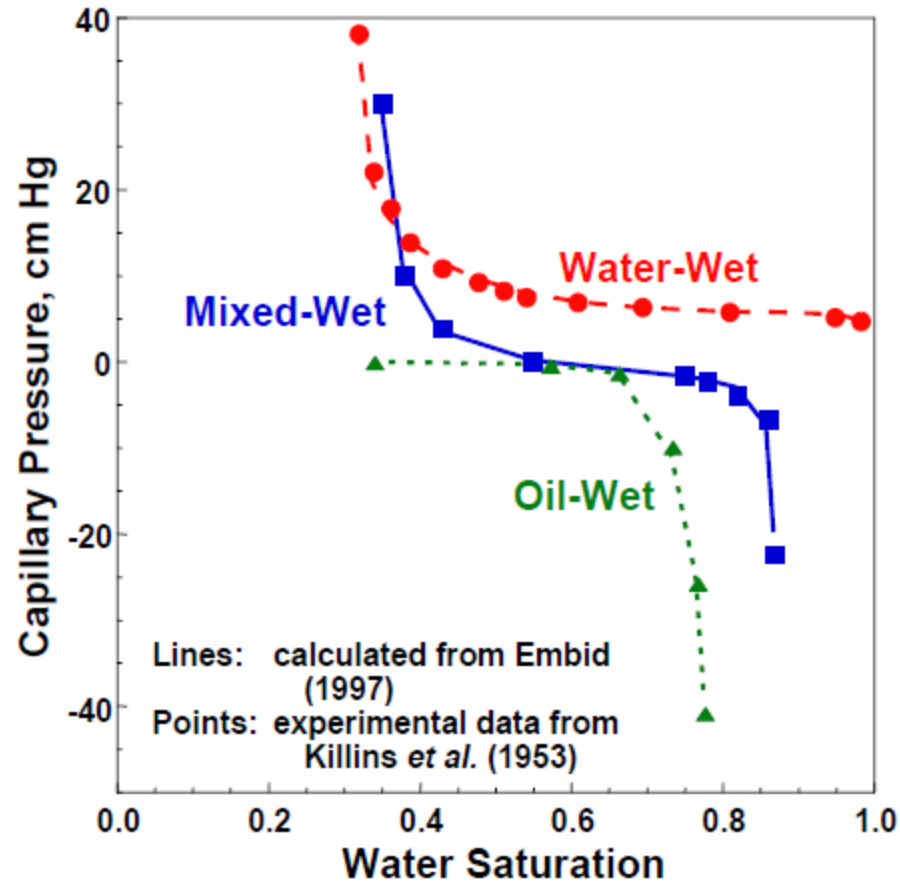# Trapping and Gravity

$$N_B = \frac{\Delta\rho g_X k}{\sigma}$$

$\Delta\rho$ = density difference (displacing – displaced)

g_X = gravitation acceleration component in direction of flow

From Zhou and Orr (1995)

Effect of Wettability on Residual Oil Saturation

Effect of Wettability on Residual Oil Saturation