Flow in Porous Media

Module 1.d
Fundamental Properties of Porous Media
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Relative Permeability

Absolute permeability: is the permeability of a porous medium saturated with a single fluid (e.g. $S_w=1$).

Except for gases at low pressure , the permeability of rock is a property of the rock and not the fluid that flows through it.

Commonly, reservoirs contain 2 or 3 fluids
- Water-oil systems
- Oil-gas systems
- Water-gas systems
- Three phase systems (water, oil, and gas)

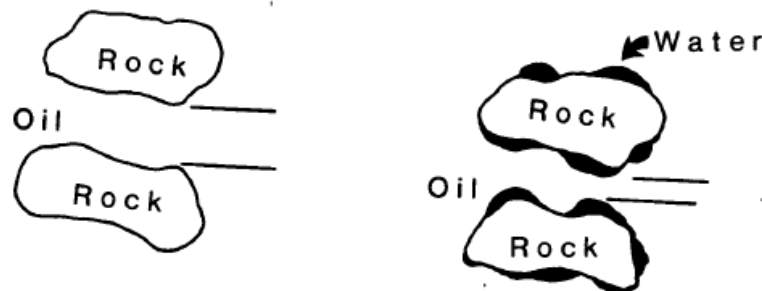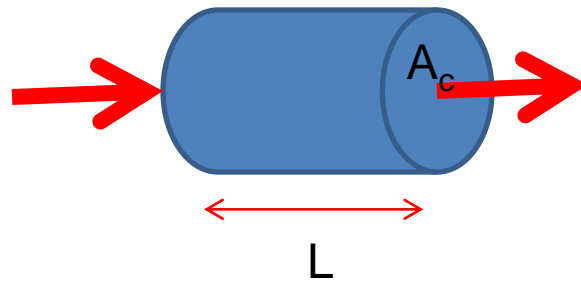To evaluate multiphase systems, we must consider the effective and relative permeability

FIGURE 2.   Oil flow reduction due to the presence of water.

L

Water Test

$$q_w = 0.3 \quad bbl / day$$

$$\mu_w = 1 \quad cp$$

$$\Delta p = 30 \quad psi$$

$$k = \frac{q_w \mu_w L}{0.001127 A_c \Delta p} = 413 \, md$$

Oil Test

$$q_o = 0.1 \quad bbl / day$$

$$\mu_o = 3 \quad cp$$

$$\Delta p = 30 \quad psi$$

$$k = \frac{q_o \mu_o L}{0.001127 A_c \Delta p} = 413 \, md$$

As the historical review showed, all efforts were made to extend the validity of the Darcy-law to multiphase filtration. If this is possible, then the following formulas may be set up:
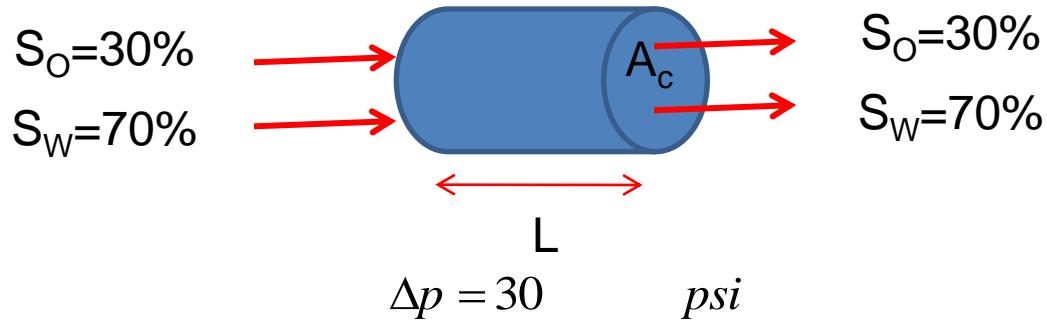
$$q_w = -A \frac{k k_{rw} \Delta p}{\mu_w L}$$

$$q_{nw} = -A \frac{k k_{rnw} \Delta p}{\mu_{nw} L}$$

where:

k_{rw} is the relative permeability of the wetting phase

k_{rnw} is the relative permeability of the nonwetting phase.

$$k_{rw}(S_w) = \frac{k_w(S_w)}{k}; \qquad 0 \le k_{rw}(S_w) \le 1$$

$S_O=30\%$ A_c $S_O=30\%$

$S_W=70\%$ $S_W=70\%$

L

$$\Delta p = 30 \qquad psi$$

$$q_w = 0.18 \qquad bbl/day$$
$$q_o = 0.012 \qquad bbl/day$$
$$\mu_w = 1 \qquad cp$$
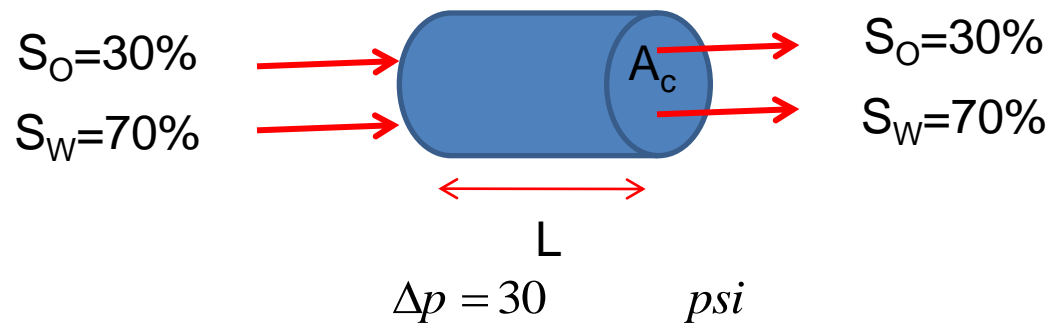$$\mu_o = 3 \qquad cp$$
$$\Delta p = 30 \qquad psi$$

$$k_w = \frac{q_w \mu_w L}{0.001127 A_c \Delta p} = 284\, md \qquad\qquad k_o = \frac{q_o \mu_o L}{0.001127 A_c \Delta p} = 50\, md$$

- <u>Effective permeability</u>: is a measure of the conductance of a porous medium for one fluid phase when the medium is saturated with more than one fluid.

- Effective permeabilities: (k_o, k_g, k_w)

$q_w = 0.18 \quad bbl/day$

$q_o = 0.012 \quad bbl/day$

$\mu_w = 1 \quad cp$

$\mu_o = 3 \quad cp$

$\Delta p = 30 \quad psi$

S_O=30%

S_W=70%

A_c

S_O=30%

S_W=70%

L

$\Delta p = 30 \quad psi$

$$k_{rw} = \frac{k_w}{k} = \frac{284\,md}{413} = 0.6$$

$$k_{ro} = \frac{k_o}{k} = \frac{50\,md}{413} = 0.12$$

• Relative Permeability is the ratio of the effective permeability of a fluid at a given saturation to some base permeability.

•The flowing water –oil ratio at reservoir conditions depends on viscosity ratio and relative permeability ratio; i.e. , at <u>MOBILITY RATIO</u>.

$$\frac{q_w}{q_o} = \left(\frac{k_w A_c \Delta p}{\mu_w L} \right) \bigg/ \left(\frac{k_o A_c \Delta p}{\mu_o L} \right)$$

$$\frac{q_w}{q_o} = \left(\frac{k_w}{\mu_w} \right) \bigg/ \left(\frac{k_o}{\mu_o} \right) = \left(\frac{k_{rw}}{k_{ro}} \right) \bigg/ \left(\frac{\mu_w}{\mu_o} \right) = \frac{\lambda_w}{\lambda_o} = M$$

$$M = \left(\frac{k_w}{\mu_w} \right) \bigg/ \left(\frac{k_o}{\mu_o} \right) = \left(\frac{248}{1} \right) \bigg/ \left(\frac{50}{3} \right)$$
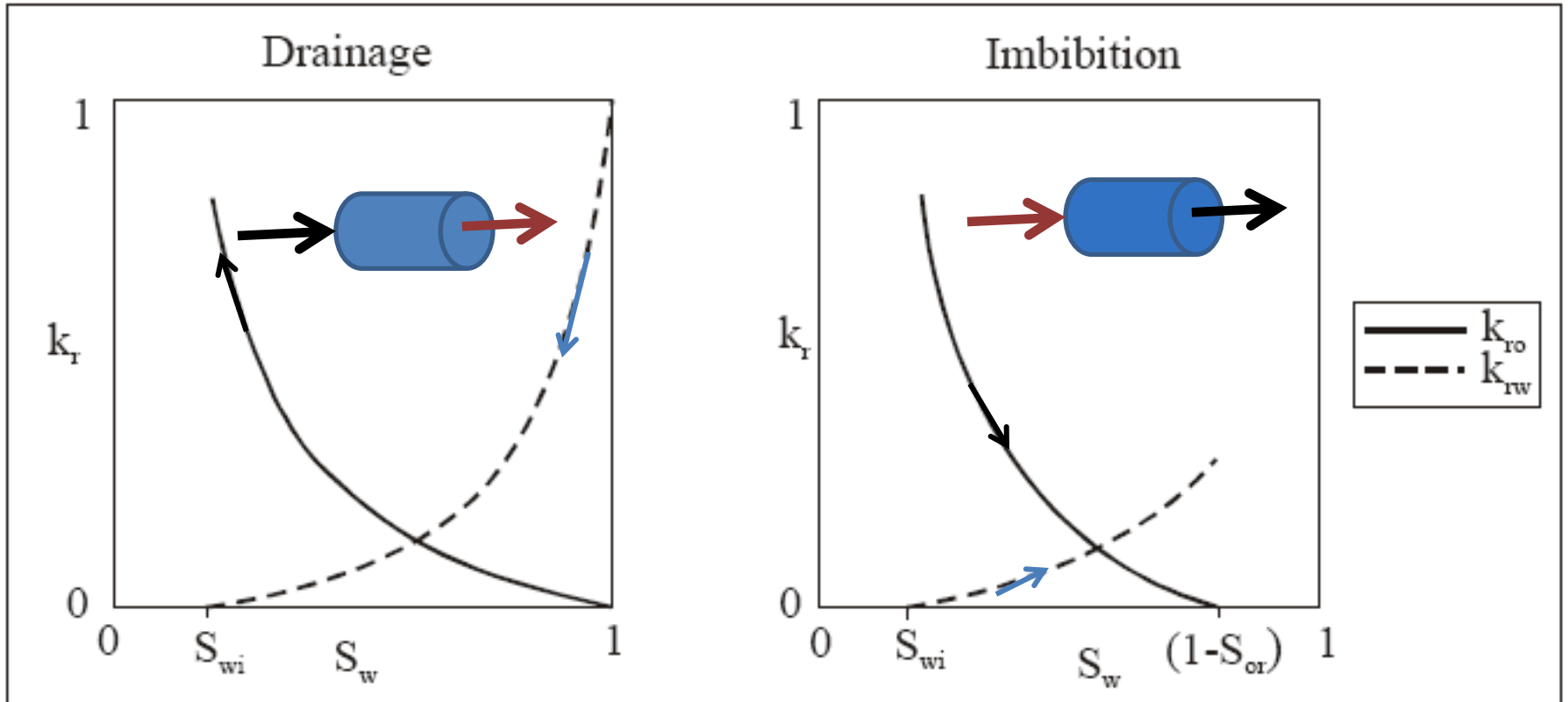
$$M = 14.9$$

•At 70% water saturation and 30% oil saturation, the water is flowing at 14.9 times the oil rate.

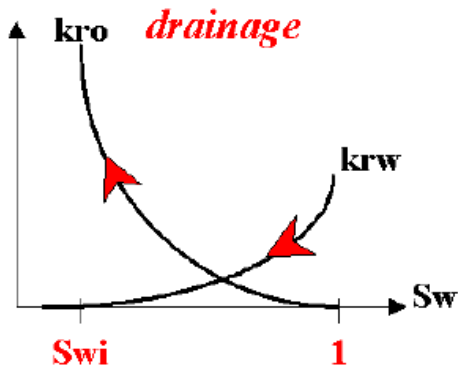# Typical Curves for Oil-water Relative Permeability at a Water-wetted System

$$k_{ro}(S_w) = \frac{k_o(S_w)}{k}; \qquad k_{rw}(S_w) = \frac{k_w(S_w)}{k}$$

The end point saturations determine the movable saturation range and is directly related to amount of recoverable oil.

Importance of Drainage and Imbibition Relative Permeability

Drainage curves are important for:
 • solution gas drive (since oil and water are generally wetting relative to gas),
 • for gravity drainage (gas displaces drained oil),
 • gas injection processes,
 • oil or gas displacing water (in tertiary recovery processes).
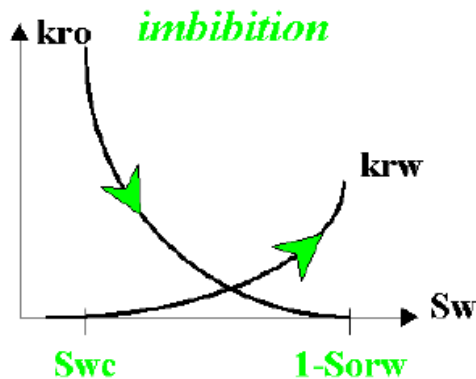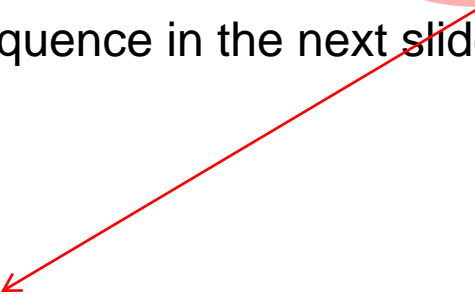

Imbibition curves are relevant to:
 • waterflood calculations,
 • water influx,
 • oil displacing gas (e.g. oil moving into a gas cap).

Oil-brine relative Permeability Theory

•It should be remembered that in water-wet systems capillary forces assist water to enter pores, whereas in the oil wet case they tend to prevent water entering pores.

•Many reservoir systems fall between the two extremes, which does nothing to make laboratory water-flood data easier to interpret. However, a knowledge of the two extreme cases allows misinterpretation of intermediate data to be minimised.

•Consideration must be given to flow rates. Close to the well bore, advance rates will be high, further away, rates can be very low. This can be modelled in laboratory tests; but in the case of oil wet systems, there is a tendency for low recoveries to be predicted due to end effects, i.e. retention of wetting phase at test plug outlet face.
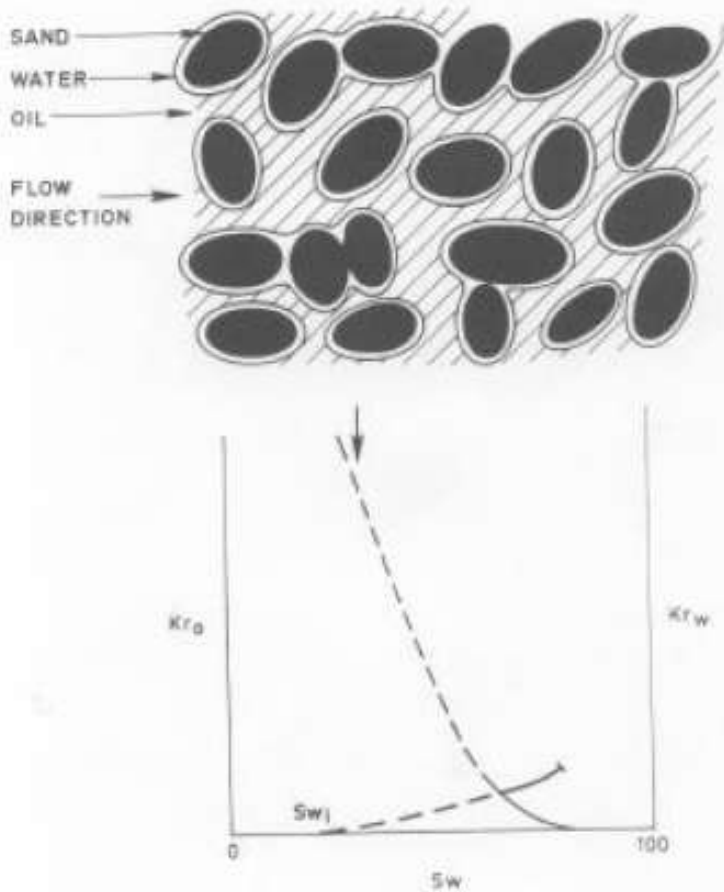
Water Wet Systems

•Consider a water-wet pore system at S_{wi} (generally 15 to 30%) some distance from well bore such that flow rates are low, typically advancing at **1 ft/day**. This is equivalent to about **4 cc/hr** in a typical laboratory waterflood. The following sequence in the next slides occurs as water migrates into the rock:

•If floods are carried out at too high a flow rate on water-wet cores the trapping mechanisms present in the reservoir are not allowed to occur. Instead of entering small pores preferentially by capillary forces, the water flows at a relatively higher velocity through larger pores, thus tending to bypass 'groups' of smaller pores containing oil. The S_{or} value obtained may then differ from the true reservoir situation.

Figure 10.5 Microsaturation and Water Flood Relperm Curve for a Water Wet System at S_{wi}
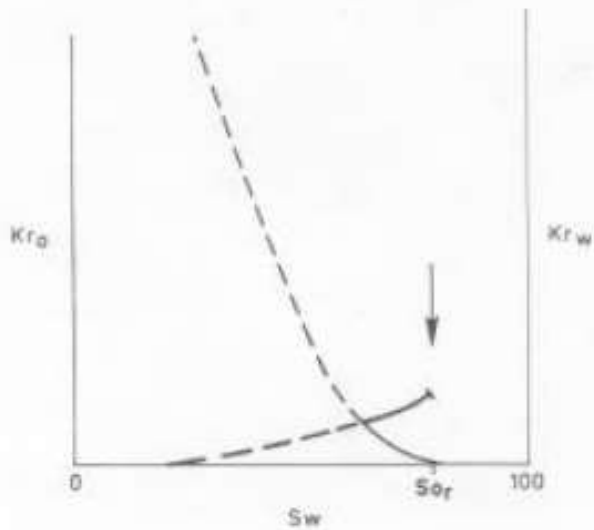

Initially at Swi, water is the wetting phase and will not flow. Kro = 1 and Krw = 0.

## Figure 10.6 Microsaturation and Water Flood Relperm Curve for a Water Wet System during Flood
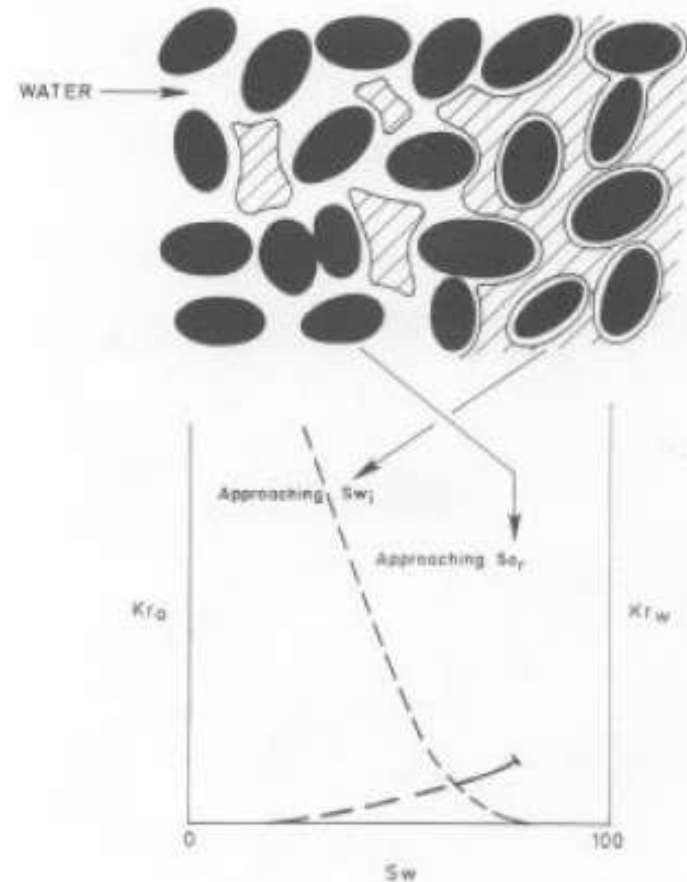
Water migrates in a piston like fashion, tending to displace most of the oil ahead of it.

Figure 10.7 Microsaturation and Water Flood Relperm Curve for a Water Wet System at S_{or}

As water saturation increases oil flow tends to cease abruptly, and Sor is reached
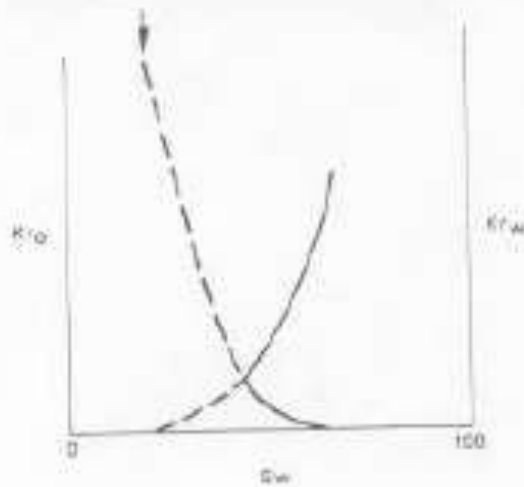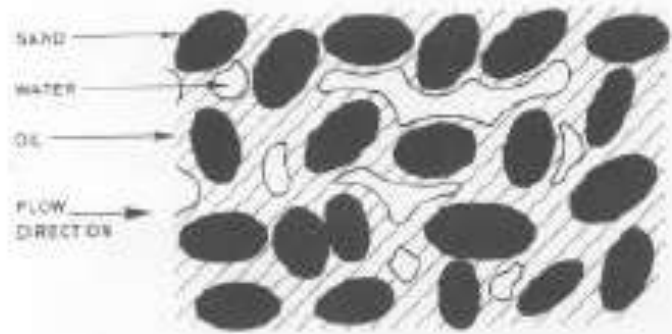

Figure 10.8 Microsaturation and Water Flood Relperm Curve for a Water Wet System after High Rate Bump
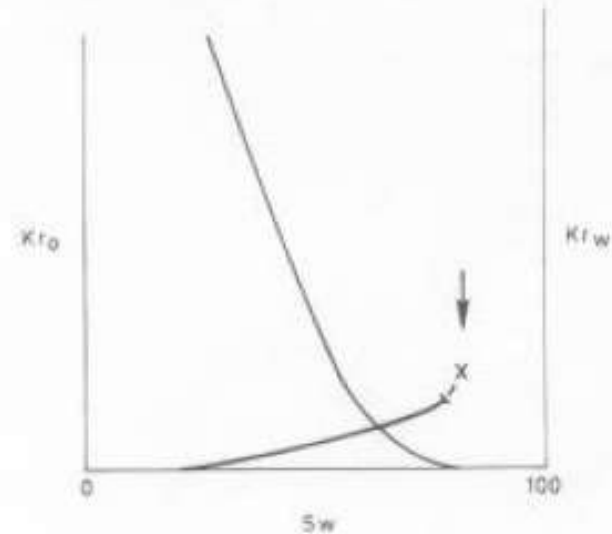
Dramatically increasing the water flow rate (bump) has very little effect on oil production or Krw. This is because capillary forces provide most of the energy required for displacement of the oil.
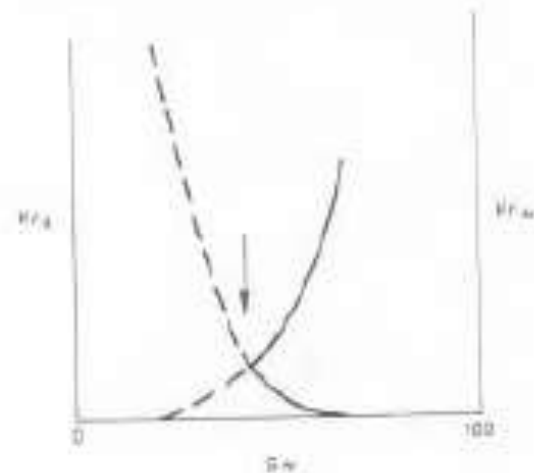
Oil Wet Systems

Figure 10.11 Microsaturation and Water Flood Relperm Curve for an Oil Wet System at S_{wi}

Figure 10.12 Microsaturation and Water Flood Relperm Curve for an Oil Wet System at Water Breakthrough

Capillary pressure considerations indicate that an applied pressure differential will be required before water will enter the largest pore.
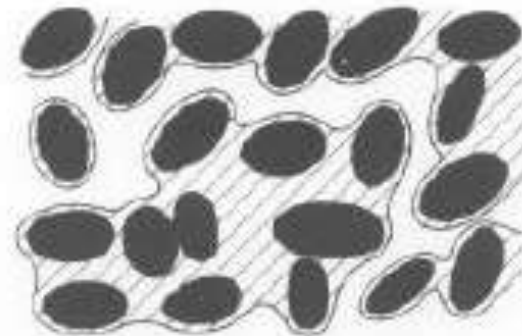
Water flows through the largest flow channels first, Kro falls and Krw rises rapidly.

Figure 10.13 Microsaturation and Water Flood Relperm Curve for an Oil Wet System at S_{or}
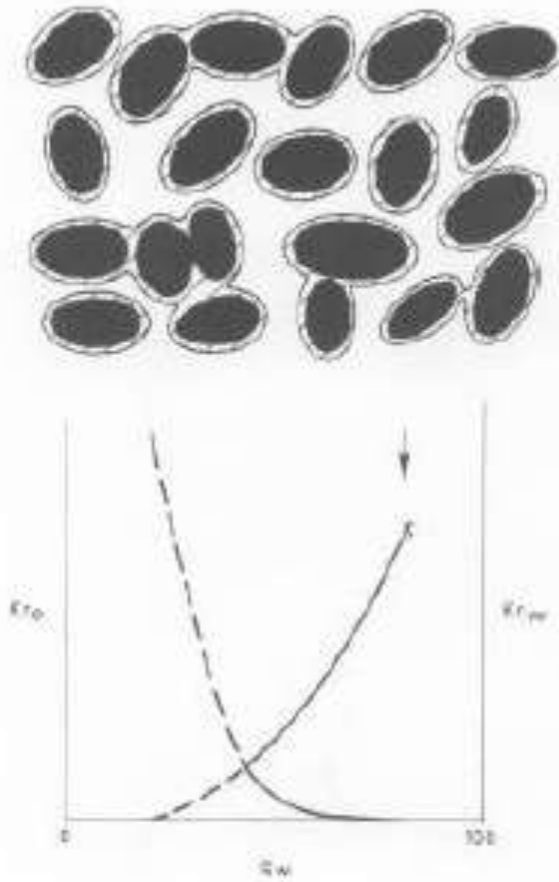
After large volumes of water have flowed through the system, Sor is reached. This equilibrium is attained slowly giving the characteristic prolonged slow production of oil after early water breakthrough.

Typical Two-phase Flow Behavior
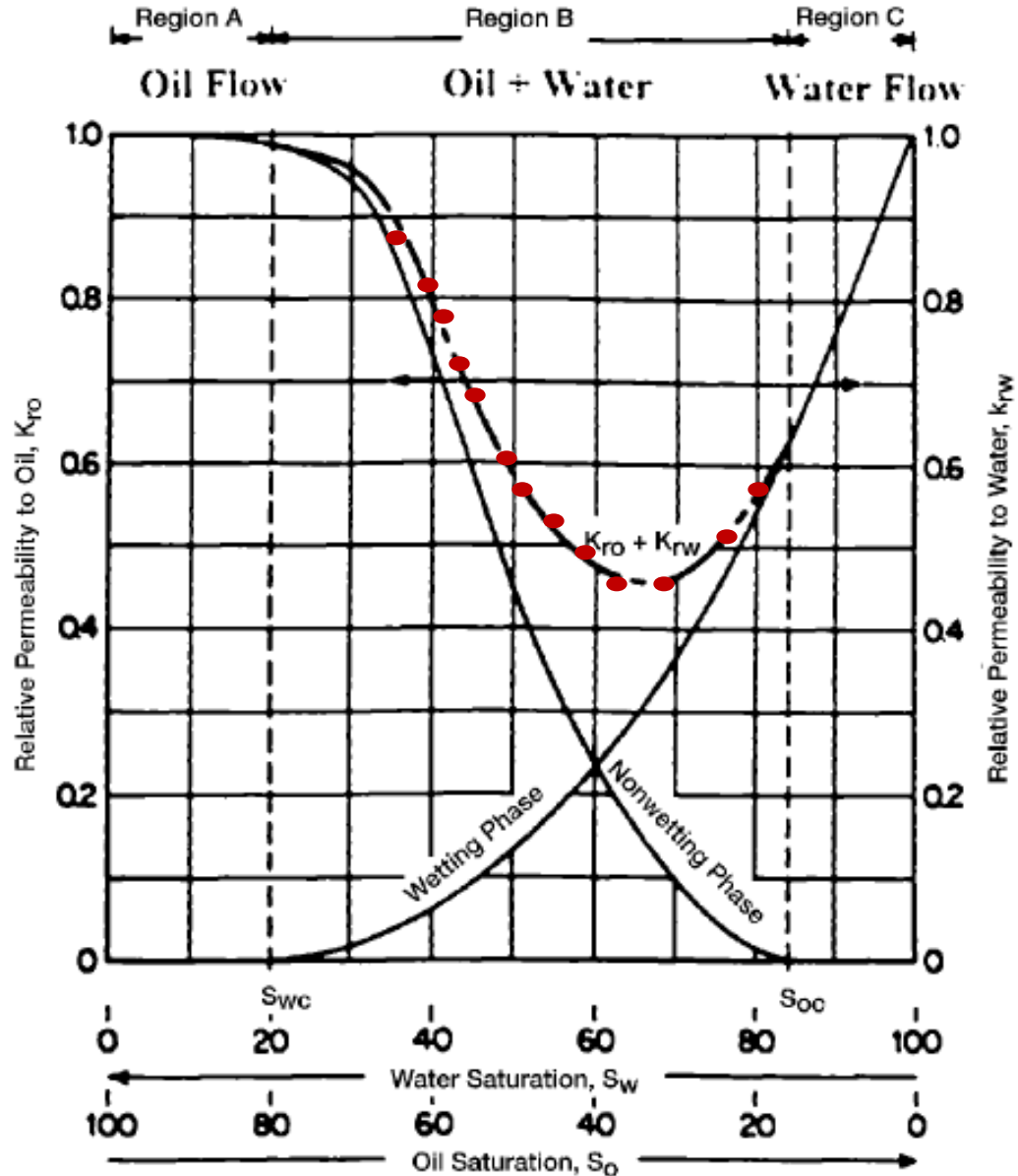
Point 1

Point 1 on the <u>wetting phase relative permeability</u> shows that a small saturation of the non-wetting phase will drastically reduce the relative permeability of the wetting phase. The reason for this is that the non-wetting phase occupies the larger pore spaces, and it is in these large pore spaces that flow occurs with the least difficulty.
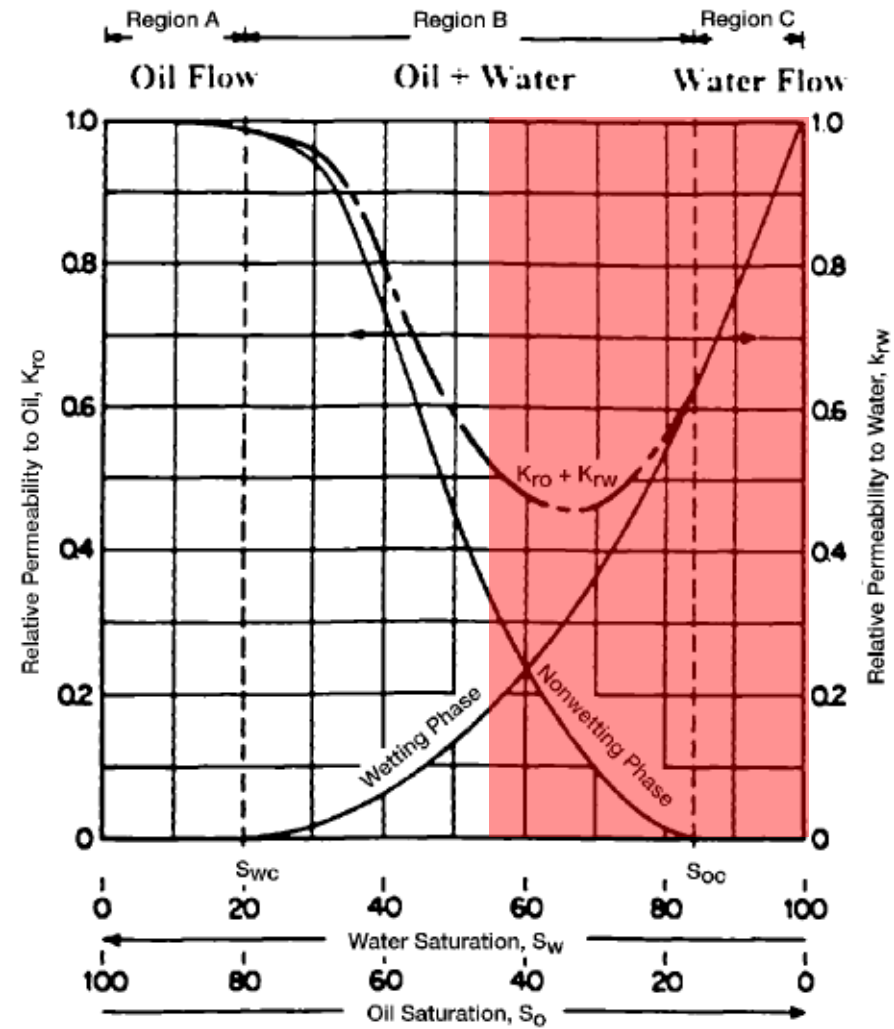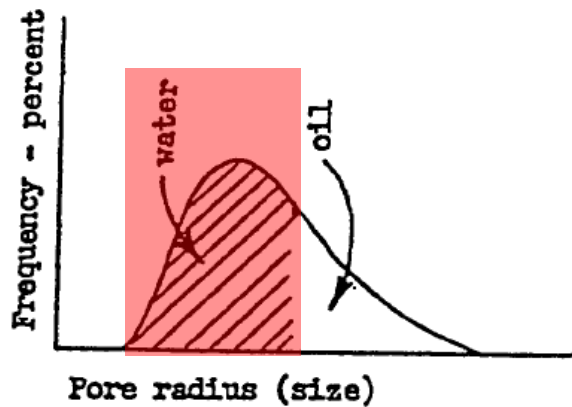
Point 2

Point 2 on the <u>non-wetting phase relative permeability curve</u> shows that the non-wetting phase begins to flow at the relatively low saturation of the non-wetting phase. The saturation of the oil at this point is called critical oil saturation S_{oc}.
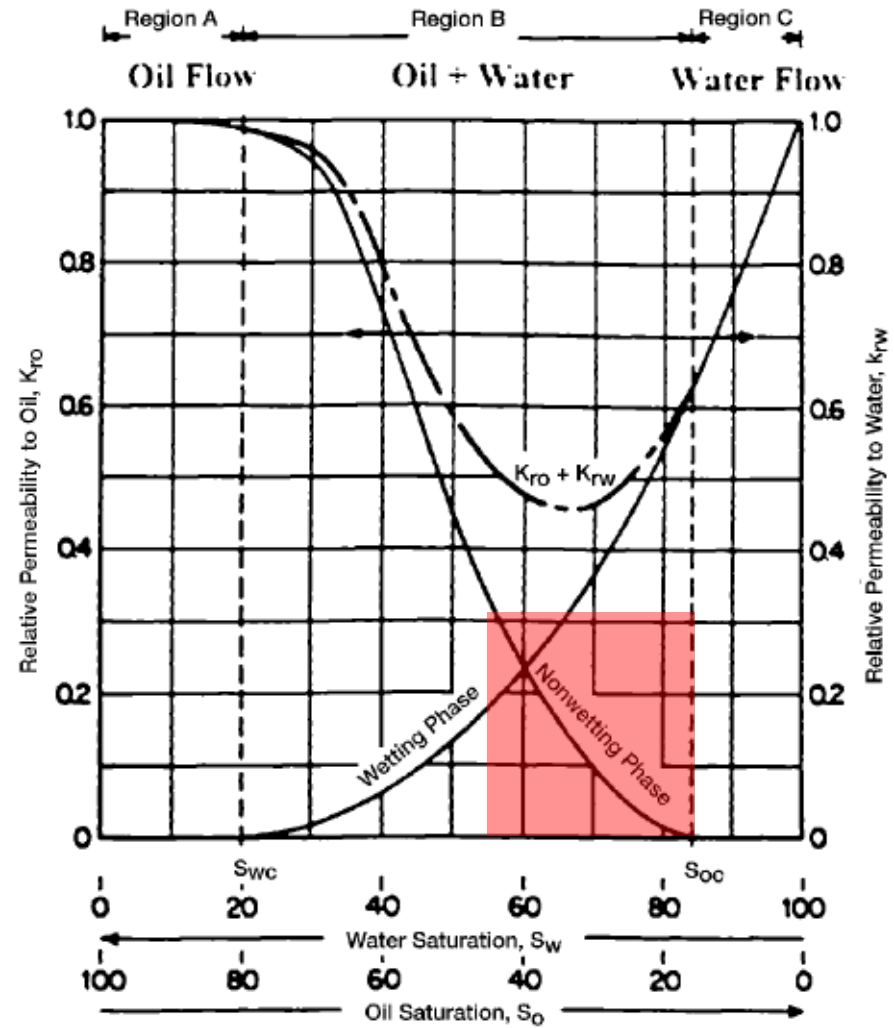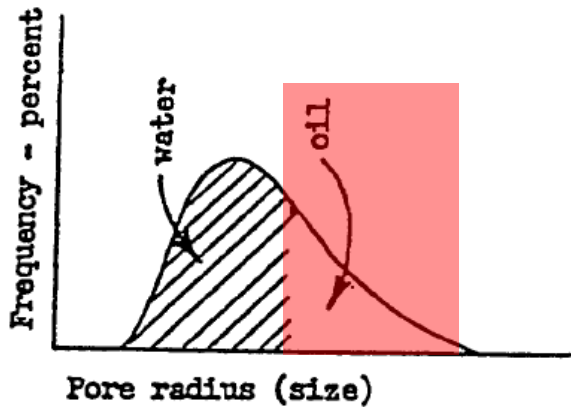
Point 3

Point 3 on the <u>wetting phase relative permeability curve</u> shows that the wetting phase will cease to flow at a relatively large saturation. This is because the wetting phase preferentially occupies the smaller pore spaces, where capillary forces are the greatest. The saturation of the water at this point is referred to as the irreducible water saturation S_{wir}
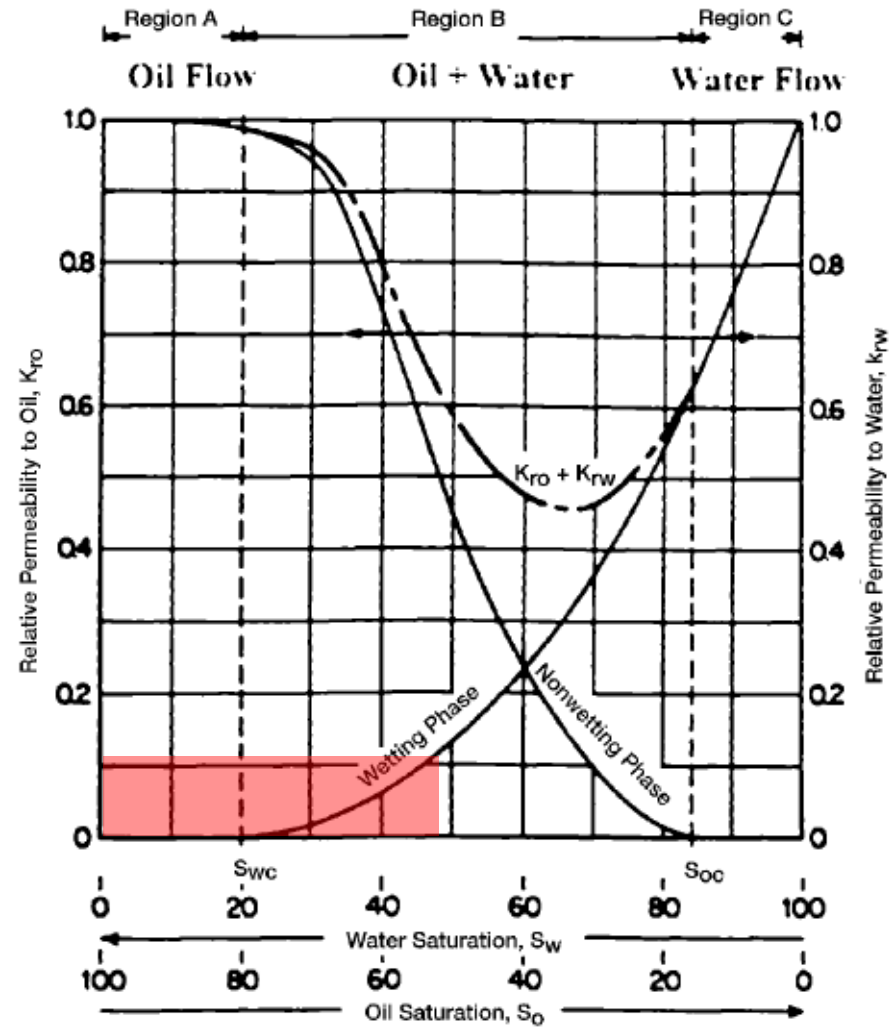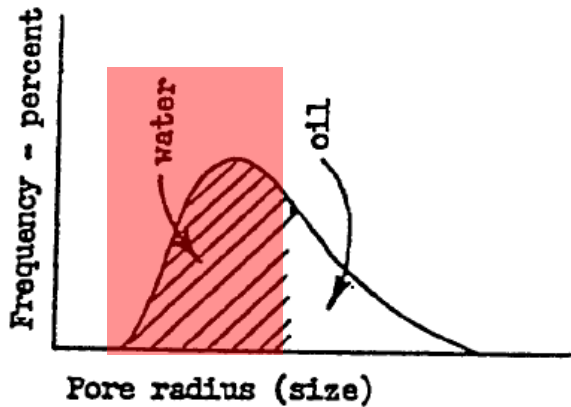
Point 4

Point 4 on the <u>non-wetting phase relative permeability curve</u> shows that, at the lower saturations of the wetting phase, changes in the wetting phase saturation have only a small effect on the magnitude of the non-wetting phase relative permeability curve. The reason for the phenomenon at Point 4 is that at the low saturations the wetting phase fluid occupies the small pore spaces which do not contribute materially to flow, and therefore changing the saturation in these small pore spaces has a relatively small effect on the flow of the non-wetting phase.
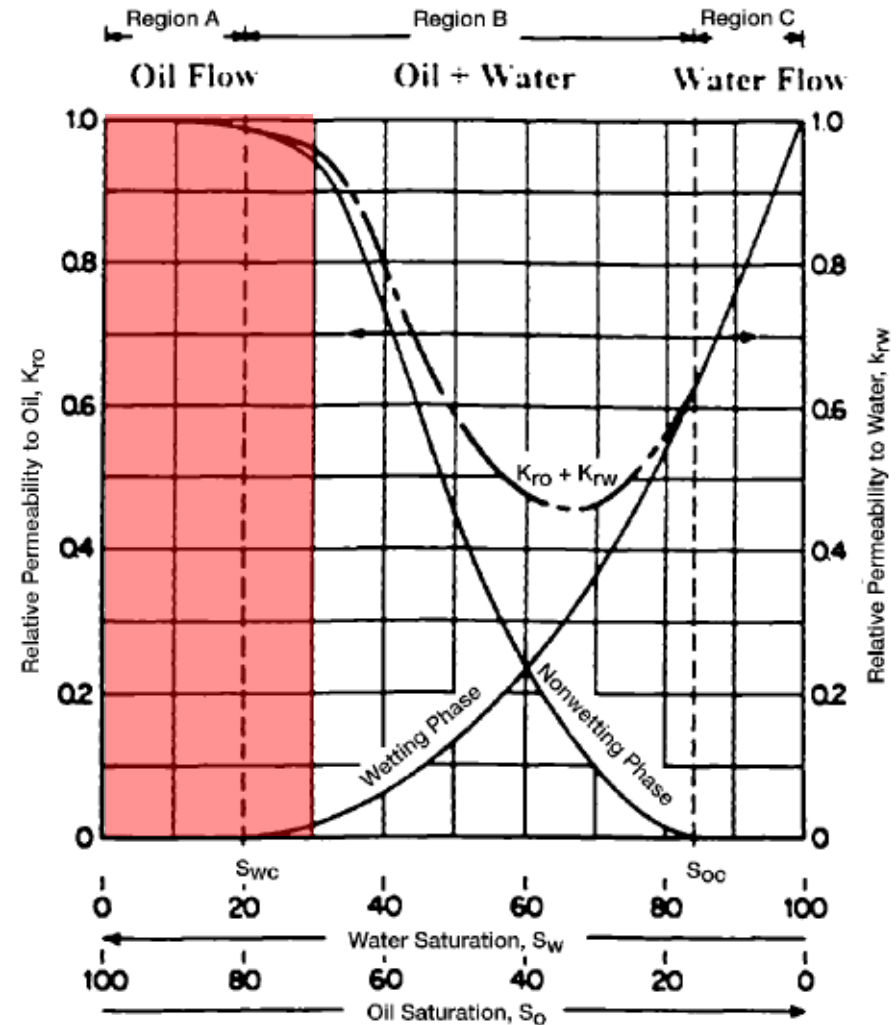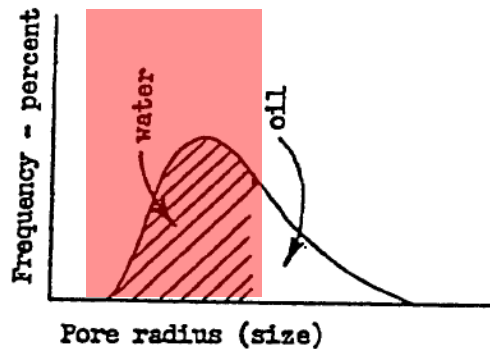
$K_{ro}+K_{rw} < 1$

The sum of the relative permeabilities for all phases is almost always less than unity <u>because of interference among phases sharing flow channels</u>. There are a number o f reasons for this interference.

1. The part of the pore channels available for flow of a fluid may be reduced in size by the other fluids present in the rock.

2. The immobilized droplets of one fluid may completely plug some constrictions in a pore channel through which another fluid would otherwise flow.

3. Some pore channels may become effectively plugged by adverse capillary forces if the pressure gradient is too low to push an interface through a constriction.

4. The trapping of a group of globules that are clustered together and cannot be moved, since the grain configuration allows fluid to flow around the trapped globules without developing a pressure gradient sufficient to move them.

Land Trapping Model

The complex physics of <u>multiphase flow</u> in porous media are usually modeled at the field scale using Darcy-type formulations. The <u>key descriptors</u> of such models are the <u>relative permeabilities</u> to each of the flowing phases. It is well known that, whenever the fluid saturations undergo a <u>cyclic process</u>, relative permeabilities display <u>hysteresis effects</u>.

The Land model (1968) is the most widely used empirical trapping model. Most relative permeability models that incorporate hysteresis (Jerauld 1997a; Killough 1976;Larsen and Skauge 1998; Lenhard and Parker 1987; Blunt 2000; Lenhard and Oostrom 1998) are based on it. It was developed to predict trapped gas saturation as a function of the initial gas saturation based on published experimental data from water-wet sandstone cores (Holmgren and Morse 1951; Kyte et al. 1956; Dardaganian 1957).

Land trapping parameter (C) is dependent on the type of rock and fluids.

$$S_{ot}(S_{oi}) = \frac{S_{oi}}{1 + CS_{oi}}$$

$$C = \frac{1}{S_{ot,max}} - \frac{1}{S_{o,max}}$$
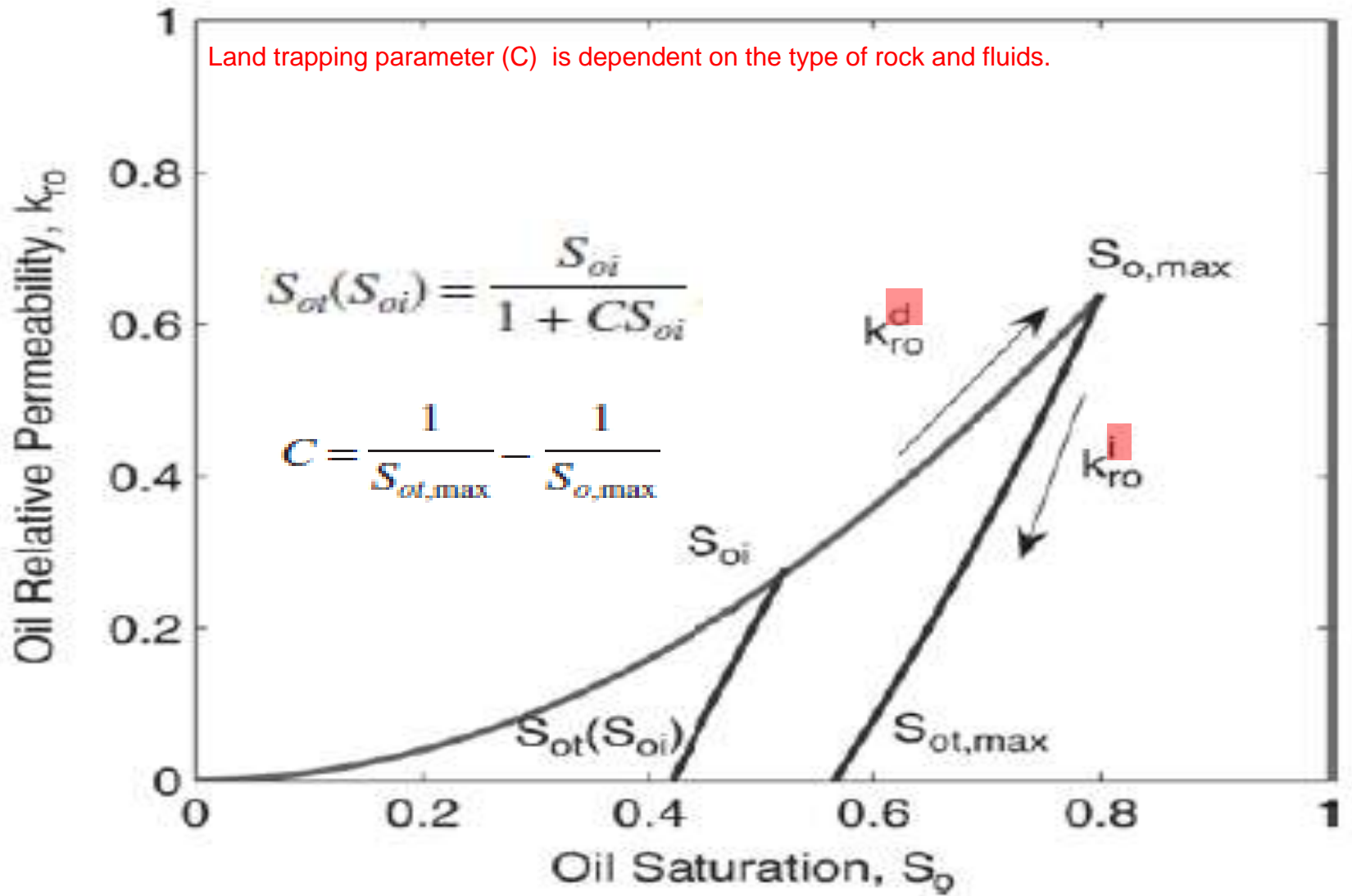
The amount of gas that is trapped in the region that has been flooded by water encroachment can be estimated by defining the following characteristic reservoir parameters :
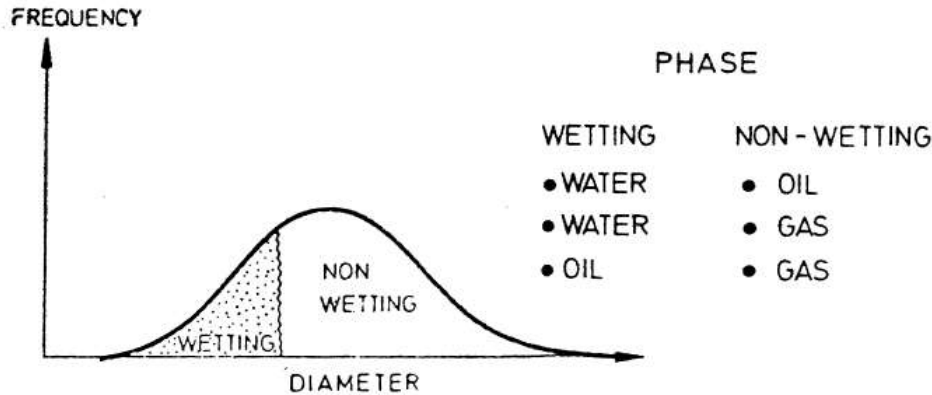
4.35 – Pore size frequency curve and fluid saturation
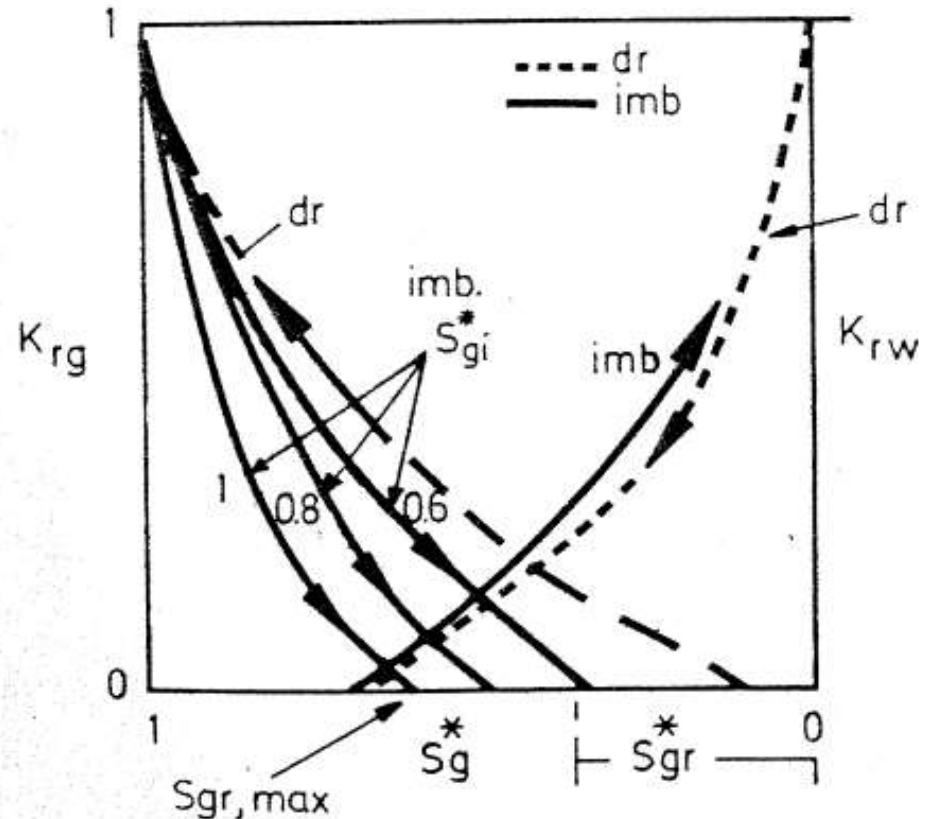
Factors Affecting Relative Permeabilities

- Fluid saturations

- Geometry of the pore spaces and pore size distribution

- Wettability

- Fluid saturation history (i.e., imbibition or drainage)

Effect of Saturation on K$_r$

At low saturations of the fluid that preferentially tends to wet the grains of a rock, the wetting phase forms doughnut-shape rings around the grain contact points. These are called pendular rings. The rings do not communicate with each other and pressure cannot be transmitted from one pendular ring to another. Sometimes such a distribution may occupy an appreciable fraction of the pore space. The amount depends upon the nature and shape of individual grains, distribution, as well as degree and type of cementation

Above the critical wetting-phase saturation, the wetting phase is mobile through a tortuous path under a pressure differential and as the wetting-phase saturation increases, the wetting phase relative permeability increasesas well. The wetting-phases saturation distribution in this region is called funicular and up to a point, the relative permeability to the wetting phase is less than the relative permeability to the non-wetting phase due to the adhesion force between the solid surface and wetting fluid, and the greater tortuosity of the flow path for the wetting phase. The non-wetting phase moves through the larger pores within this range of saturation, but as the saturation of the wetting phase further increases, the non-wetting phase breaks down and forms a discontinuous phase at the critical non-wetting phase saturation. This is called an insular state of non-wetting-phase saturation.

The effect of Pore Structure

A. Rocks with <u>large pores</u> and correspondingly <u>small specific surface</u> areas have <u>low irreducible water saturations</u>. Therefore, for such rocks, end point relative permeabilities are high and a large saturation change may occur during two phase flow.

B. With <u>small pores</u> have <u>larger specific surface areas</u> and <u>larger irreducible water saturations</u>. As a result, the end point relative permeabilities are lower and the saturation range for two phase flow is smaller than in rocks with large pores.

C. Rocks having some relatively <u>large pores connected by small pores</u> have a large surface area, resulting in high irreducible water saturation and relative permeability behavior that is similar to rocks with small pores only.
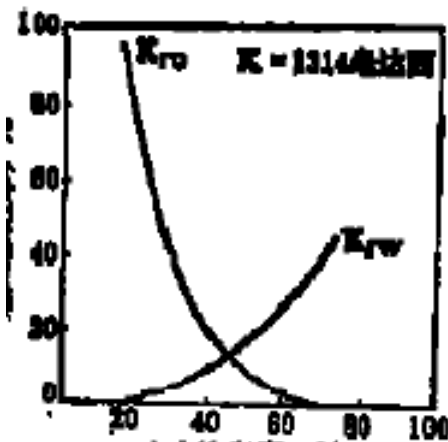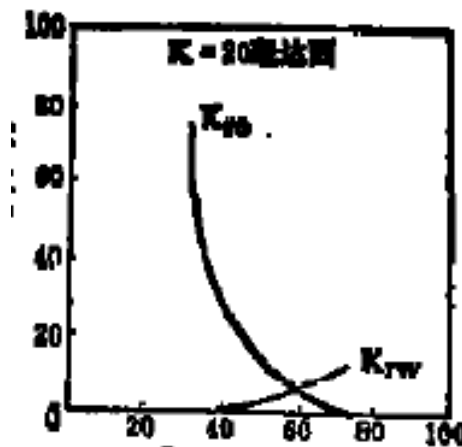
A
B
C
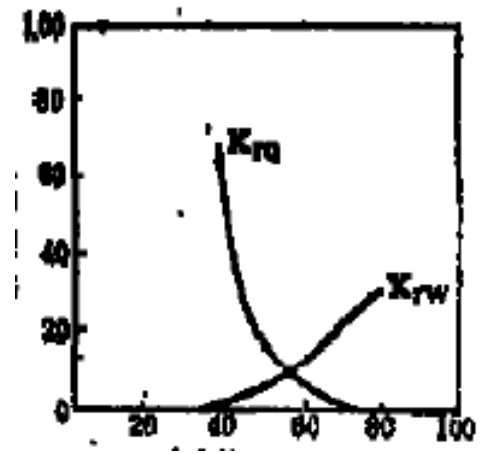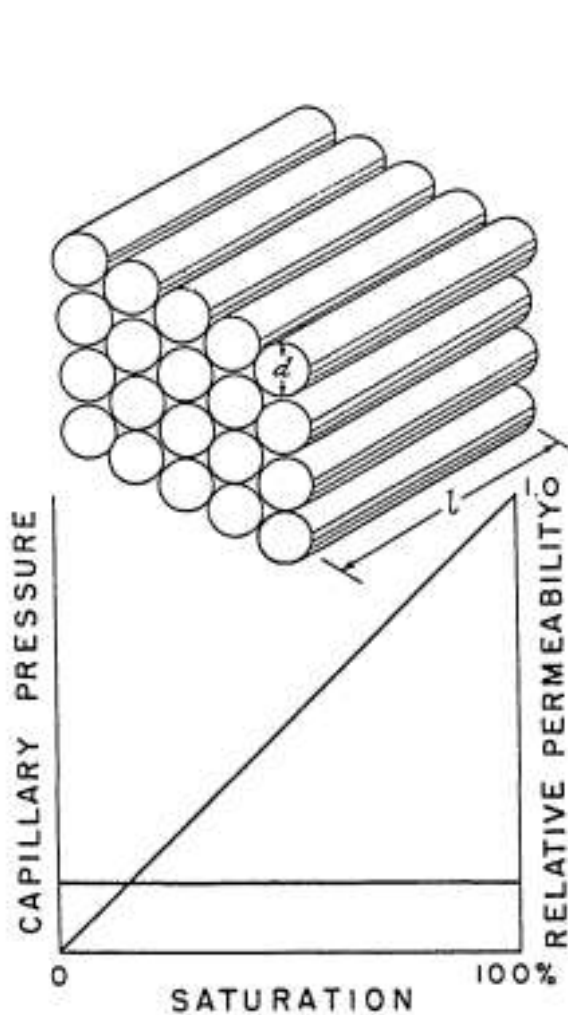
Fig. 31 – Hypothetical Capillary-pressure Curve for Bundle of Capillaries with Uniform Lengths and Uniform Diameters
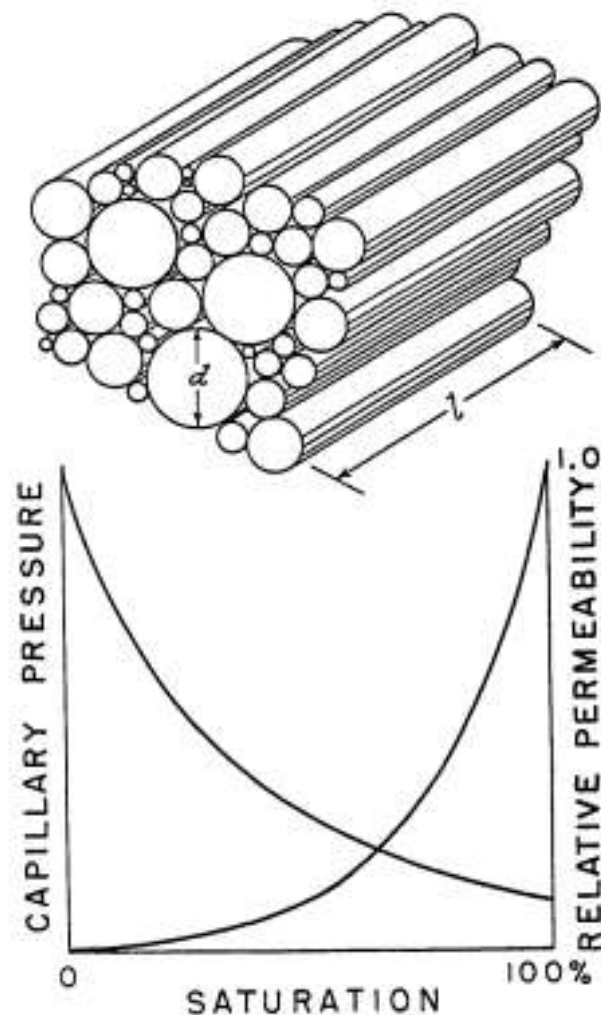
Fig. 32 – Hypothetical Capillary-pressure Curve for Bundle of Capillaries with Uniform Lengths and Wide Distribution of Diameters

From: Gates, J.I. and Templaar-Lietz, W.: "Relative Permeabilities of California Cores by the Capillary Pressure Method," API Drilling and Production Practices (1950) 285-302.

Effect of Wettability on K_r
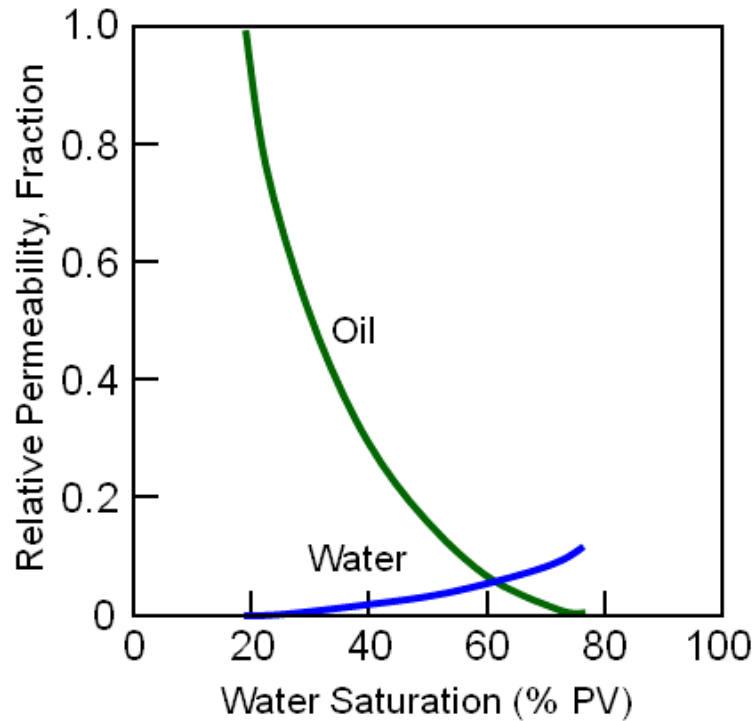
Strongly Water-Wet Rock
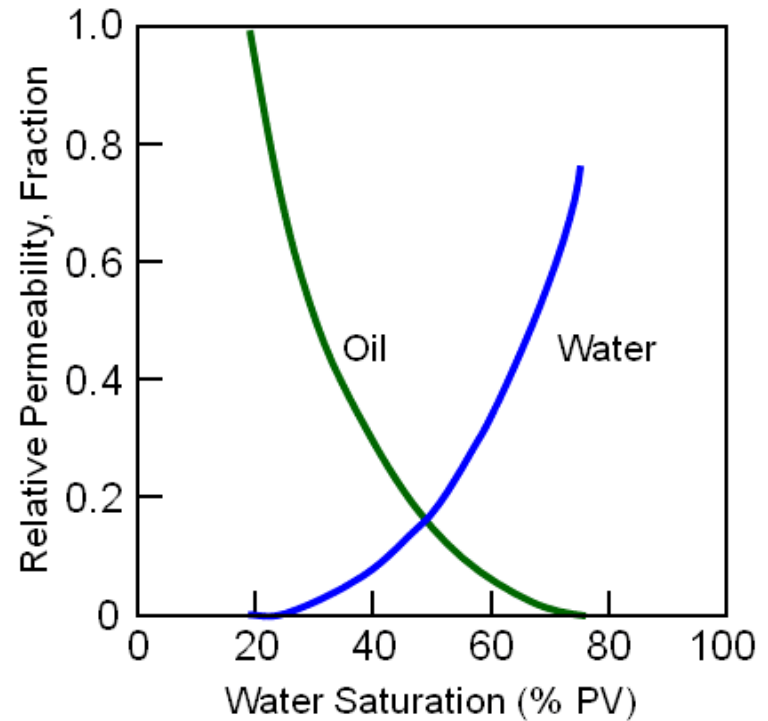
Strongly Oil-Wet Rock

- Water flows more freely
- Higher residual oil saturation

	Water-wet	**Oil-wet**
S_{wi}	>20 to 25%	<15%, usually 10%
$k_{rw} = k_{row}$	@ S_w>50%	@ S_w<50%
k_{rw} at S_{orw}	<0.3	>0.5, approaching 1.0

The effect of Saturation History
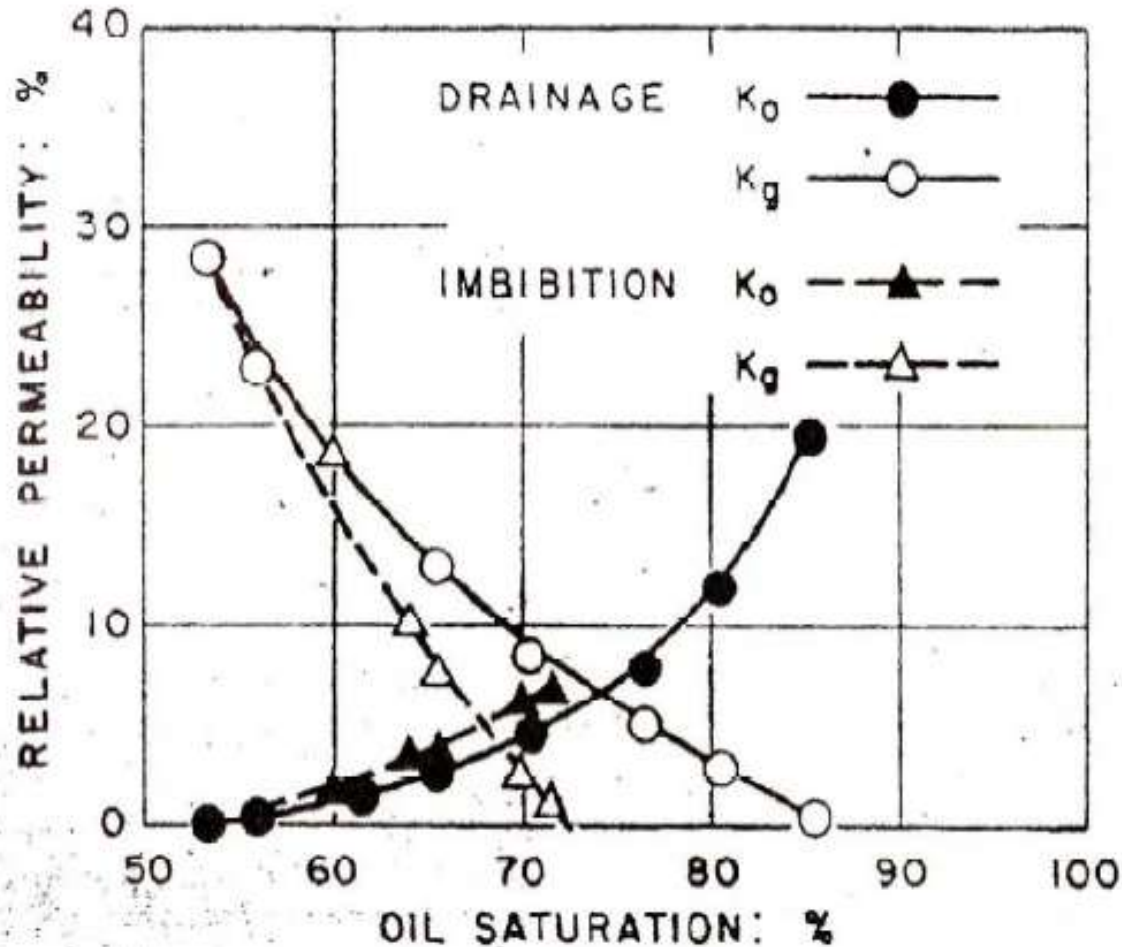
● The imbibition non-wetting phase relative permeability curve is generally lower than the drainage curve at the same saturations.
● The imbibition wetting phase relative permeability curve is slightly greater than the drainage curve.

The effect of Interfacial Tension
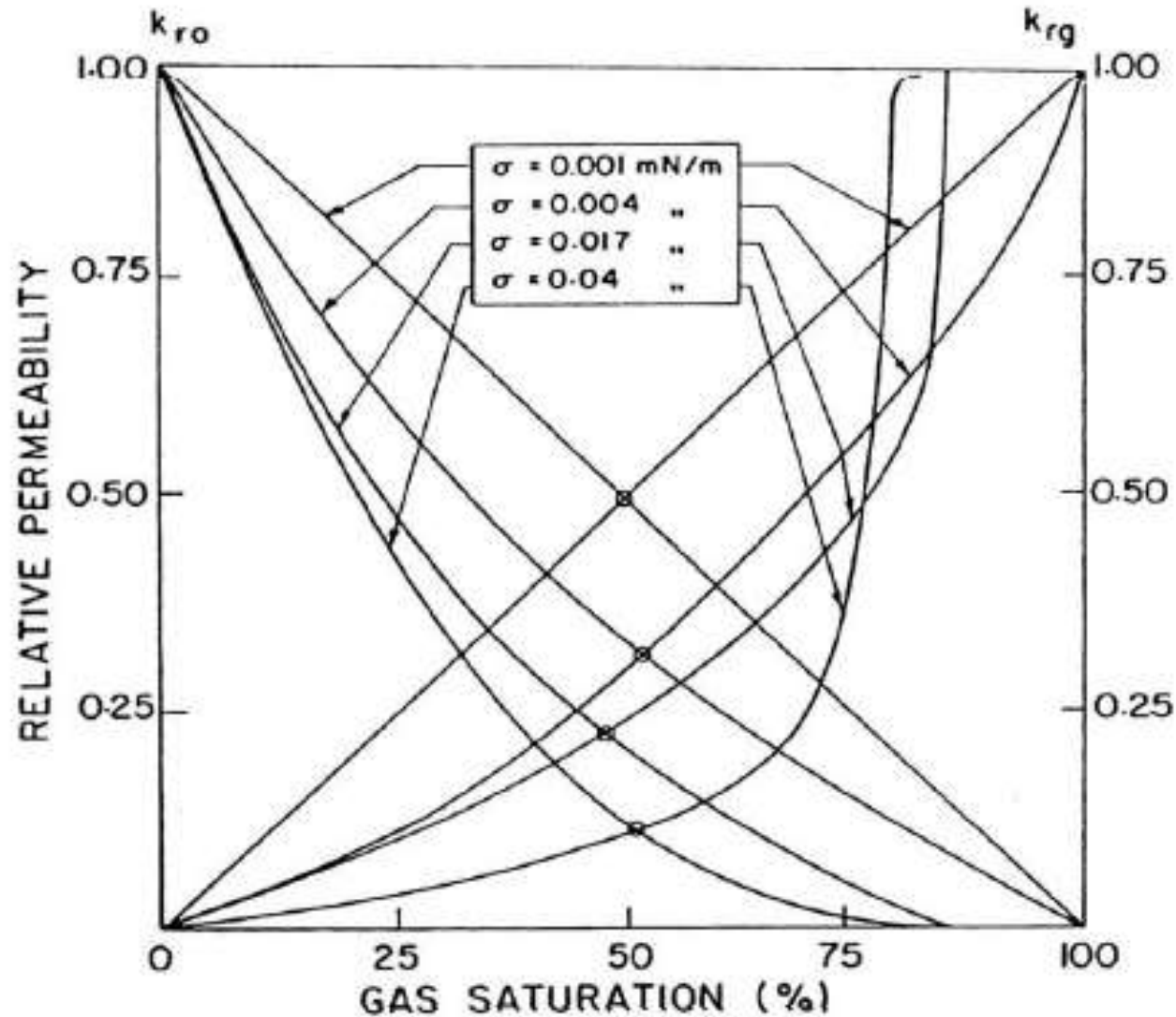
Mathematical Models of Relative Permeability

1. **Capillaric Models**: Capillary models ignore the interconnected nature of the capillaries and, therefore can not give a realistic description of two-phase flow phenomena.

2. **Statistical Models**: The statistical models, unless they take into account and make use of the interconnectedness of the pores, can not be more successful than the capillaric models.

3. **Emperical Models**: In order to bring the results of capillaric and statistical models inline with reality, empiricism had to be resorted to. At the present, the practically useful mathematical models of two-phase flow in porous media are basically empirical.

4. **Network Models**: Network models, however; have the best potential for understanding and quantitative description of two-phase flow phenomena in porous media, both because they take into account the interconnectedness of pore structure and because microscopic flow mechanisms are considered in them.

Calculate Relative Permeability Curve from Capillary Pressure Curve

Basic Theory:

By Poiseuille's Law, the flow rate in single capillary:

$$q = \frac{\pi r^4 \Delta P}{8 \mu L}$$

Volume of a single pore

$$V = \pi r^2 L, \pi r^2 = V/L$$

(From Keelan   1977)

Actual flow paths in pore spaces

(From Scheidegger   1960)

From: Freeze, R.A. and Cherry, J.A.: *Groundwater*, Prentice-Hall, Inc., Englewood Cliffs, NJ (1979).

Average linear flow path

Definition of capillary pressure

$$P_c = \frac{2\sigma \cos\theta}{r}, r^2 = \frac{4(\sigma \cos\theta)^2}{P_c^2}$$
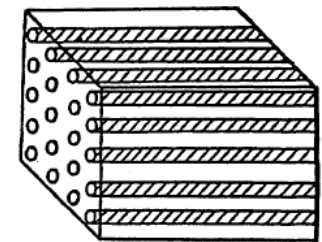
$$q = \frac{\pi r^4 \Delta P}{8 \mu L} = \frac{(\sigma \cos \theta)^2 \Delta PV}{2 \mu L^2 P_c^2}$$

If the porous medium is conceived to be comprised of n capillary tubes of unequal diameter, the total flow rate is given by:

$$Q = \frac{(\sigma \cos \theta)^2 \Delta P}{2 \mu L^2} \sum_{i=1}^{n} \frac{V_i}{(P_c)_i^2}$$

in addition: $V_i = V_{P\,I}$

$$Q = \frac{(\sigma \cos \theta)^2 \Delta P}{2 \mu L^2} \sum_{i=1}^{n} \frac{V_{pi}}{(P_c)_i^2}$$

In actual rock ,from Darcy's law

$$Q = \frac{K A \, \Delta P}{\mu L}$$

Then:

$$K = \frac{(\sigma \cos \theta)^2}{2 A L} \sum_{i=1}^{n} \frac{V_{Pi}}{(P_c)_i^2}$$

$$V_{pi} = \varphi A L S_i$$

$$K = \frac{(\sigma \cos \theta)^2}{2} \phi \sum_{i=1}^{n} \frac{S_i}{(P_C)_i^2}$$

Introducing lithology factor

$$K = 0.5(\sigma \cos \theta)^2 \phi \lambda \int_{s=0}^{s=1} \frac{dS}{P_C^2}$$

Method as follows :
Plot capillary pressure curve (Pc ~ Sw), and draw $1/P_C^2$ ~ S_w curve, obtain integral area of this curve and can calculate the relative permeability of rock.

Calculation of Effective Permeability and Relative Permeability:

$$K_W = 0.5(\sigma \cos \theta)^2 \phi \lambda \int_{s=0}^{s=s_i} \frac{dS}{P_C^2}$$

$$K_O = 0.5(\sigma \cos \theta)^2 \phi \lambda \int_{s=s_i}^{s=1} \frac{dS}{P_C^2}$$

$$K_{rw} = \frac{K_w}{K} = \frac{\int_0^{S_i} \frac{dS}{P_C^2}}{\int_0^1 \frac{dS}{P_C^2}}$$

$$K_{ro} = \frac{K_O}{K} = \frac{\int_{S_i}^1 \frac{dS}{P_C^2}}{\int_0^1 \frac{dS}{P_C^2}}$$

Drainage Relative Permeability
Burdine's Theory

Burdine's theory : $K_r = f(P_c)$
•based on hydraulic radius concepts
•based on a capillary tube bundle model
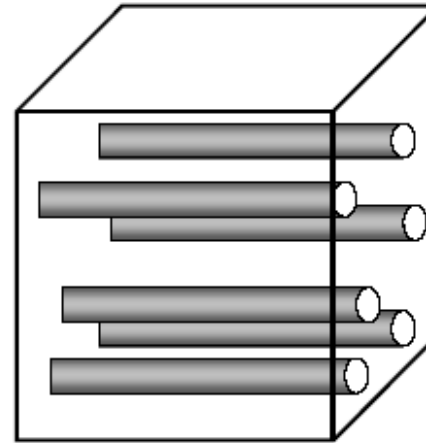
Actually, tubes are not straight

Using capillary tubes as a model for the pores, the **Carman-Kozeny** theory gives us, for each tube:

$$u_i = \frac{R_i^2 \Delta p}{8 \mu l_i}; \qquad q_i = \frac{R_i^2 \Delta p\, A_i}{8 \alpha_{si} \mu l_i} \qquad\qquad (3.5)$$

where R_i is a characteristic radius of a (noncircular) pore or cross-sectional area A_i and length L_i. α_{si} is a shape factor to account for the noncircularity of the pore.

For the drainage process, the nonwetting phase enters the largest pores first. If the nonwetting phase invades the dn_i next largest pores, the change in volume of the wetting phase is:

$$dV_w = -l_i A_i dn_i; \quad dn_i = -\frac{dV_w}{l_i A_i} \tag{3.6}$$

which decreases the water flow rate by:

$$dq_w = -\frac{R_i^2 \Delta p\, A_i}{8\alpha_{si}\mu l_i} dn_i = +\frac{R_i^2 \Delta p}{8\alpha_{si}\mu l_i^2} dV_w \tag{3.7}$$

Darcy's law for two-phase flow at the macroscopic scale gives:

$$q_w = k_w \frac{A\Delta p}{\mu L}; \quad \text{or} \quad dq_w = \frac{A\Delta p}{\mu L} dk_w \tag{3.8}$$

Equating the incremental flow rates at microscopic and macroscopic scales:

$$dk_w = \frac{R_i^2 L}{8\alpha_{si} A l_i^2} dV_w \tag{3.9}$$

However, since $dV_w = \phi A L dS_w$, at the macroscopic scale, then:

$$dk_w = \frac{R_i^2 \phi L^2}{8\alpha_{si} l_i^2} dS_w \tag{3.10}$$

$$k_w = \int_{S_{wi}}^{S_w} \frac{R_i^2 \phi L^2}{8\alpha_{si} l_i^2} dS_w \tag{3.11}$$

$$k_w = \int_{S_{wi}}^{S_w} \frac{R_i^2 \phi L^2}{8\alpha_{si} l_i^2} dS_w \qquad (3.11)$$

The lower limit of the integral is S_{wi} since none of the saturation below immobile saturation contributes to the flow. To evaluate the integral we need to know how the geometric parameters R_i, α_{si} and l_i behave as a function of saturation S_w. The hydraulic radius R_i can be related to the capillary pressure, since capillary pressure is related to the largest pore size occupied by the wetting phase. A balance of forces across a pore gives:

$$P_c A = \sigma \cos\theta \, \omega_p \qquad (3.12)$$

where ω_p is the wetted perimeter. The characteristic radius is given by:

$$\frac{A_i}{\omega_{pi}} = \frac{\pi R_i^2}{2\pi R_i} = \tfrac{1}{2} R_i \qquad (3.13)$$

So, $\qquad R_i = \dfrac{2\sigma\cos\theta}{P_c(S_w)} \qquad (3.14)$

Substituting in Eq. (3.11):

$$k_w = \int_{S_{wi}}^{S_w} \frac{(\sigma\cos\theta)^2 \phi L^2}{2\alpha_s(S_w) l^2(S_w) P_c^2(S_w)} dS_w \qquad (3.15)$$

If we define the effective saturation as:

$$S_w^* = \frac{S_w - S_{wi}}{1 - S_{wi}}$$

(3.16)

Then we can simplify Eq. (3.15) by expressing:

Average tortuosity

$$\tau = \frac{l^2(S_w^* = 1)}{L^2}$$

(3.17)

So,

$$\frac{\overline{\alpha_s(S_w)l^2(S_w)}}{L^2} \approx \overline{\alpha_s}\tau \frac{l^2(S_w^*)}{l^2(S_w^* = 1)} = \frac{\overline{\alpha_s}\tau}{(S_w^*)^2}$$

(3.18)

Notice that l approaches infinity as S_w approaches S_{wi} and the wetting phase becomes disconnected.

Substituting,

$$k_w = \frac{(\sigma\cos\theta)^2 \phi\,(S_w^*)^2}{\overline{\alpha_s}\tau} \int_{S_{wi}}^{S_w} \frac{1}{P_c^2(S_w)} dS_w$$

(3.19)

Burdine (1953), while extending the earlier work (Burdine et al., 1950) to two-phase flow, observed that the tortuosity factor depends on the extent of wetting phase saturation and could be approximated as:

$$\tau_w = \left(\frac{S_w - S_{wi}}{1 - S_{wi}}\right)^2$$

(4-13)

Them making use of the definition of relative permeability, $k_{rw} = k_w(S_w)/k_w(S_w=1)$:

$$k_{rw} = (S_w^*)^2 \frac{\int_{S_{wi}}^{S_w} \frac{1}{P_c^2(S_w)} dS_w}{\int_{S_{wi}}^{1} \frac{1}{P_c^2(S_w)} dS_w}$$

(3.20)

By a similar argument, the nonwetting phase drainage relative permeability is:

$$k_{ro} = k_{ro}(S_{wi}).(S_o^*)^2 \frac{\int_{S_w}^{1} \frac{1}{P_c^2(S_w)} dS_w}{\int_{S_{wi}}^{1} \frac{1}{P_c^2(S_w)} dS_w}$$

(3.21)

where $\quad S_o^* = \dfrac{S_o - S_{oc}}{1 - S_{oc} - S_{wi}}$

(3.22)

Notice that S_{oc} is not the same as S_{or} because we are on the primary drainage curve. S_{oc} is related to percolation.

If we make use of the Brooks-Corey relation for capillary pressure, Eq. (2.19), then:

$$P_c = \frac{P_e}{(S_w^*)^{1/\lambda}}$$

(3.23)

Then
$$\int_a^b \frac{1}{P_c^2(S)} dS = \frac{1}{p_e^2(1-S_{wi})} \int_a^b (S_w^*)^{2/\lambda} dS_w^* = \frac{1}{p_e^2(1-S_{wi})} \left[\frac{(S_w^*)^{2/\lambda+1}}{2/\lambda+1} \right]_a^b$$

(3.24)

After which we can evaluate:

$$k_{rw} = (S_w^*)^{2/\lambda+3}$$

(3.25)

and
$$k_{ro} = k_{ro}(S_{wi}).(S_o^*)^2 \left[1 - (S_w^*)^{2/\lambda+1} \right]$$

(3.26)

Normally we assume that S_{oc} is roughly zero, so:

$$S_o^* = \frac{S_o}{1-S_{wi}} = \frac{1-S_w}{1-S_{wi}} = \frac{1-S_{wi}-(S_w-S_{wi})}{1-S_{wi}} = 1-S_w^*$$

(3.27)

So
$$k_{ro} = k_{ro}(S_{wi}).(1-S_w^*)^2 \left[1 - (S_w^*)^{2/\lambda+1} \right]$$

(3.28)

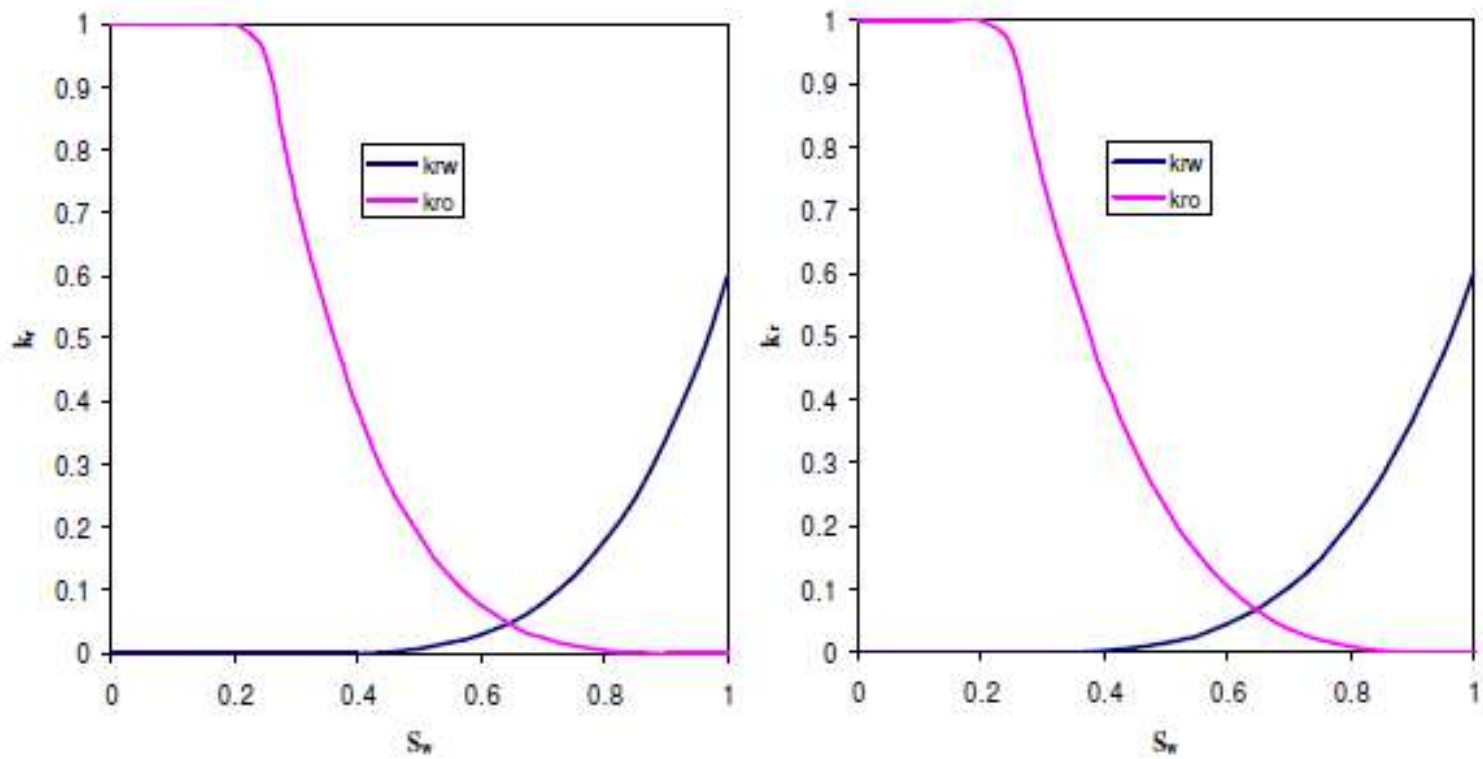These are known as the Brooks-Corey relations for relative permeability.

Figure 3.4: Brooks-Corey relative permeability curves for $\lambda=2$ (left) and $\lambda=4$ (right).

Table 4
EQUATIONS FOR THE PREDICTION OF RELATIVE PERMEABILITY IN SANDSTONE AND CONGLOMERATE

$$k_{rw} = 0.035388 \frac{(S_w - S_{wi})}{(1 - S_{wi} - S_{orw})} - 0.010874*$$

$$\left[\frac{(S_w - S_{orw})}{(1 - S_{wi} - S_{orw})}\right]^{2.9} + 0.56556(S_w)^{3.6}(S_w - S_{wi}) \quad \text{(water-wet)} \tag{67}$$

$$k_{rw} = 1.5814 \left[\frac{S_w - S_{wi}}{1 - S_{wi}}\right]^{1.91} - 0.58617 \frac{(S_w - S_{orw})}{(1 - S_{wi} - S_{orw})}*$$

$$(S_w - S_{wi}) - 1.2484\phi(1 - S_{wi})(S_w - S_{wi}) \quad \text{(intermediately wet)} \tag{68}$$

$$k_{ro,w} = 0.76067 \left[\frac{\left(\frac{S_o}{1 - S_{wi}}\right) - S_{or}}{1 - S_{orw}}\right]^{1.8} \left[\frac{S_o - S_{orw}}{1 - S_{wi} - S_{orw}}\right]^{2.0}$$

$$+ 2.6318\phi(1 - S_{orw})(S_o - S_{orw}) \quad \text{(any wettability)} \tag{69}$$

$$k_{ro,g} = 0.98372 \left(\frac{S_o}{1 - S_{wi}}\right)^4 \left[\frac{S_o - S_{org}}{1 - S_{wi} - S_{org}}\right]^2 \quad \text{(any wettability)} \tag{70}$$

$$k_{rg} = 1.1072 \left(\frac{S_g - S_{gc}}{1 - S_{wi}}\right)^2 k_{rg(Sorg)} + 2.7794*$$

$$\frac{S_{org}(S_g - S_{gc})}{(1 - S_{wi})} k_{rg\,(Sorg)} \quad \text{(any wettability)} \tag{71}$$

Table 5
EQUATIONS FOR THE PREDICTION OF RELATIVE PERMEABILITY IN LIMESTONE AND DOLOMITE

$$k_{rw} = 0.0020525 \frac{(S_w - S_{wi})}{\phi^{2.15}} - 0.051371 (S_w - S_{wi}) \left(\frac{1}{k_a}\right)^{0.43} \quad \text{(water-wet)} \tag{72}$$

$$k_{rw} = 0.29986 \left(\frac{S_w - S_{wi}}{1 - S_{wi}}\right) - 0.32797 \left(\frac{S_w - S_{orw}}{1 - S_{wi} - S_{orw}}\right)^2 *$$

$$(S_w - S_{wi}) + 0.413259 \left(\frac{S_w - S_{wi}}{1 - S_{wi} - S_{orw}}\right)^4 \quad \text{(intermediately wet)} \tag{73}$$

$$k_{ro,w} = 1.2624 \left(\frac{S_o - S_{orw}}{1 - S_{orw}}\right) \left(\frac{S_o - S_{orw}}{1 - S_{wi} - S_{orw}}\right)^2 \quad \text{(any wettability)} \tag{74}$$

$$k_{ro,g} = 0.93752 \left(\frac{S_o}{1 - S_{wi}}\right)^4 \left(\frac{S_o - S_{org}}{1 - S_{wi} - S_{org}}\right)^2 \quad \text{(any wettability)} \tag{75}$$

$$k_{rg} = 1.8655 \frac{(S_g - S_{gc}) (S_g)}{(1 - S_{wi})} k_{rg\ (S_{org})} + 8.0053 *$$

$$\frac{(S_g - S_{gc}) (S_{org})^2}{(1 - S_{wi})} - 0.025890 (S_g - S_{gc}) *$$

$$\left(\frac{1 - S_{wi} - S_{org} - S_{gc}}{1 - S_{wi}}\right)^2 *$$

$$\left(1 - \frac{1 - S_{wi} - S_{org} - S_{gc}}{1 - S_{wi}}\right)^2 \left(\frac{k_a}{\phi}\right)^{0.5} \quad \text{(any wettability)} \tag{76}$$
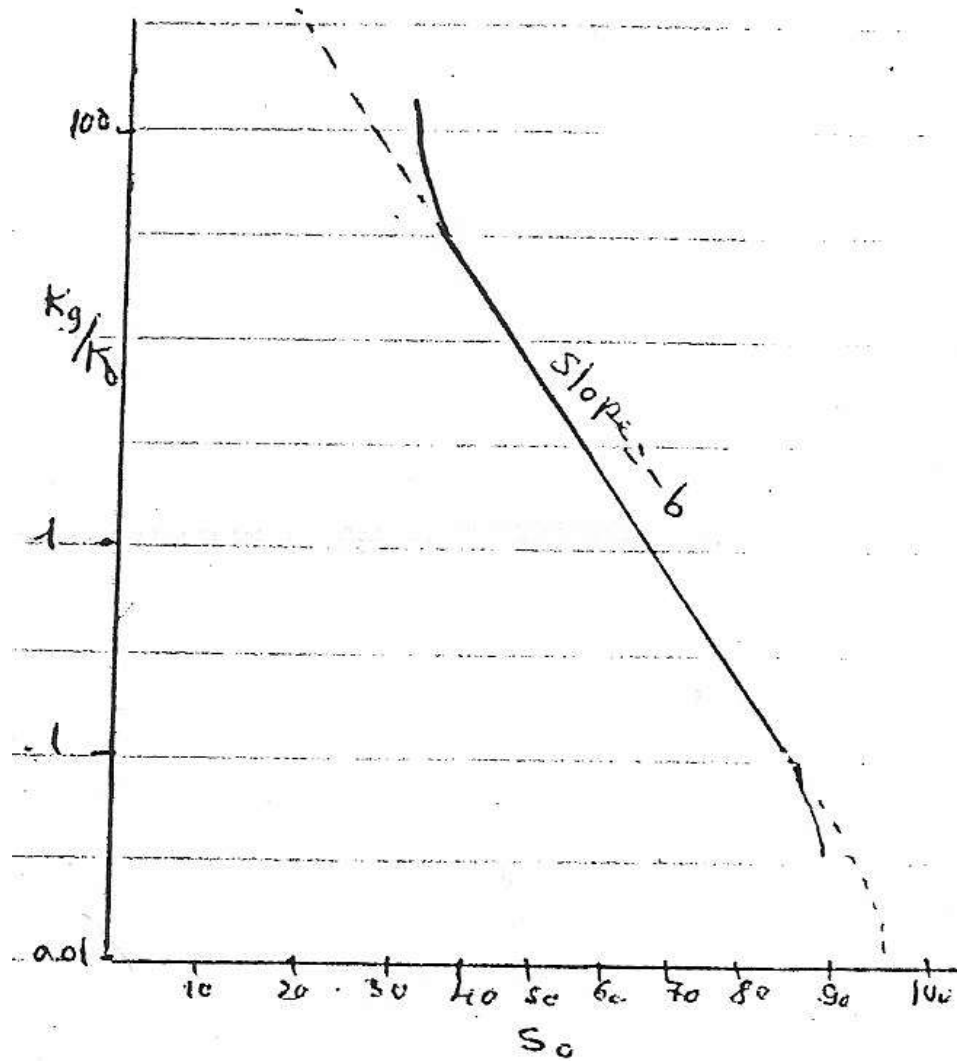
Relative permeability ratio

•This quantity lends itself more readily to analysis and to the correlation of flow performances than does relative permeability itself.

•The relative permeability ratio expresses the ability of a reservoir to permit flow of one fluid as related to its ability to permit flow of another fluid under the same circumstances.

•The two most useful permeability ratios are:

  • k_{rg}/k_{ro} the relative permeability to gas with respect to that to oil
  • k_{rw}/k_{ro} the relative permeability to water with respect to that to oil,

$$\frac{K_g}{K_o} = a \cdot e^{-b \cdot S_o}$$

a and **b** are characteristic constants of a given reservoir material and given set of fluids
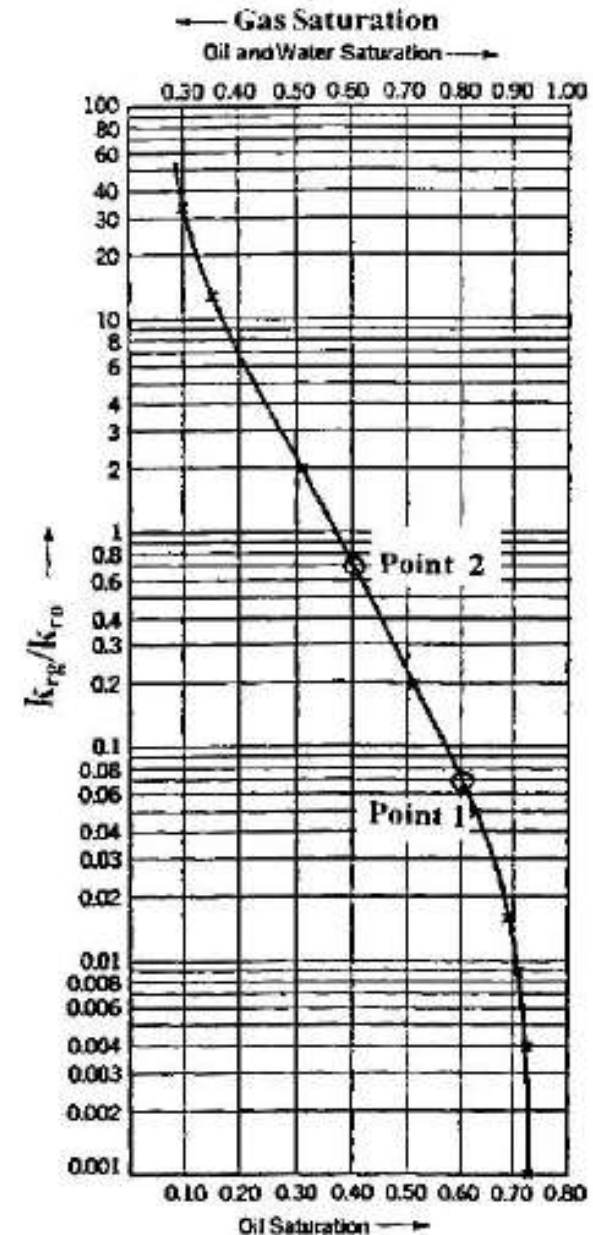
$$\log(\frac{K_g}{K_o}) = \log a - b \cdot S_o$$

The effective permeability can be expressed in terms of the relative and absolute permeability as:

$$k_o = k_{ro}k$$

$$k_w = k_{rw}k$$

$$k_g = k_{rg}k$$

Using the above concept in Darcy's equation and expressing the flow rate in standard conditions yields:

$$Q_o = 0.00708(rhk)\left(\frac{k_{ro}}{\mu_o B_o}\right)\frac{dp}{dr} \qquad [1.2.34]$$

$$Q_w = 0.00708(rhk)\left(\frac{k_{rw}}{\mu_w B_w}\right)\frac{dp}{dr} \qquad [1.2.35]$$

$$Q_g = 0.00708(rhk)\left(\frac{k_{rg}}{\mu_g B_g}\right)\frac{dp}{dr} \qquad [1.2.36]$$

where:

Q_o, Q_w = oil and water flow rates, STB/day
B_o, B_w = oil and water formation volume factor, bbl/STB
Q_g = gas flow rate, scf/day
B_g = gas formation volume factor, bbl/scf
k = absolute permeability, md

The gas formation volume factor B_g is expressed by

$$B_g = 0.005035\frac{ZT}{p} \text{ bbl/scf}$$

The instantaneous GOR, as expressed in scf/STB, is defined as the *total* gas flow rate, i.e., free gas and solution gas, divided by the oil flow rate, or:

$$GOR = \frac{Q_o R_s + Q_g}{Q_o}$$

or:

$$GOR = R_s + \frac{Q_g}{Q_o} \qquad [1.2.42]$$

where:

GOR = "instantaneous" gas–oil ratio, scf/STB
R_s = gas solubility, scf/STB
Q_g = free gas flow rate, scf/day
Q_o = oil flow rate, STB/day

Substituting Equations 1.2.34 and 1.2.36 into 1.2.42 yields:

$$GOR = R_s + \left(\frac{k_{rg}}{k_{ro}}\right)\left(\frac{\mu_o B_o}{\mu_g B_g}\right) \qquad [1.2.43]$$

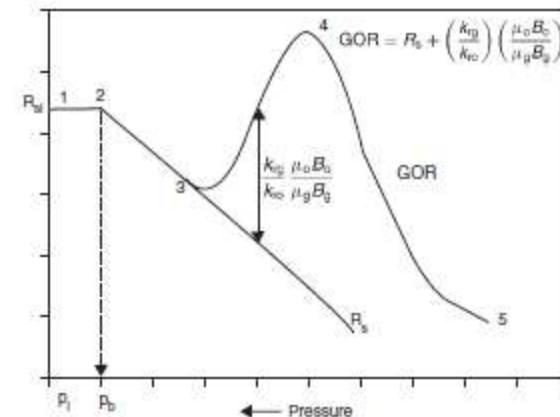where B_g is the gas formation volume factor expressed in bbl/scf.

Figure 5.2 *History of GOR and R_s for a solution gas drive reservoir.*

Relative Permeability Measurement Methods

Steady State Method

In the steady state method, two fluids are injected simultaneously at a fixed ratio until the produced ratio is equal to the injected ratio. Core saturations have to be measured at each equilibrium and new fluid ratio is applied. This is repeated until the relative permeabilities are determined. Most tests are started with the core sample saturated with 100% wetting phase, and the test is then a desaturation test (drainage).

Unsteady State Method

The procedure for performing an unsteady state test is relatively simple and fast First the core is saturated with 100% water and then the sample is desaturated by injecting oil until no more production of water is obtained. Water production is recorded and S_{wi} calculated. Effective oil permeability is then measured at S_{wi}. Oil is displaced by a predetermined constant rate of water, oil permeability and pressure drop across the core will be recorded. Alternatively, oil is displaced by keeping the differential pressure across the core constant with varying rate of fluid flow. With the recording of cumulative water injection, pressure drop and produced oil volume, it is possible to calculate relative permeabilities by theory developed by Welge (1952)

- **Steady-state**
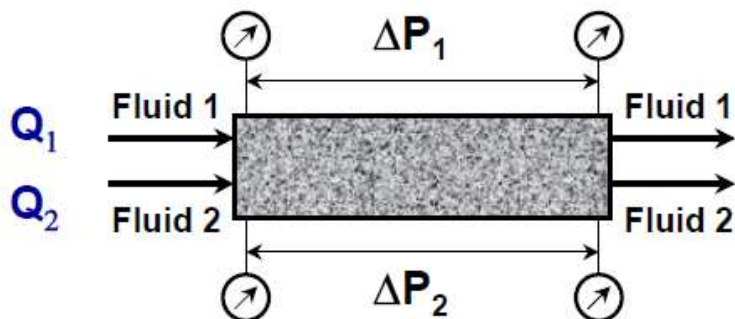
$$Q_1 = \frac{KK_{r1}A}{\mu_1}\frac{\Delta P_1}{L}$$
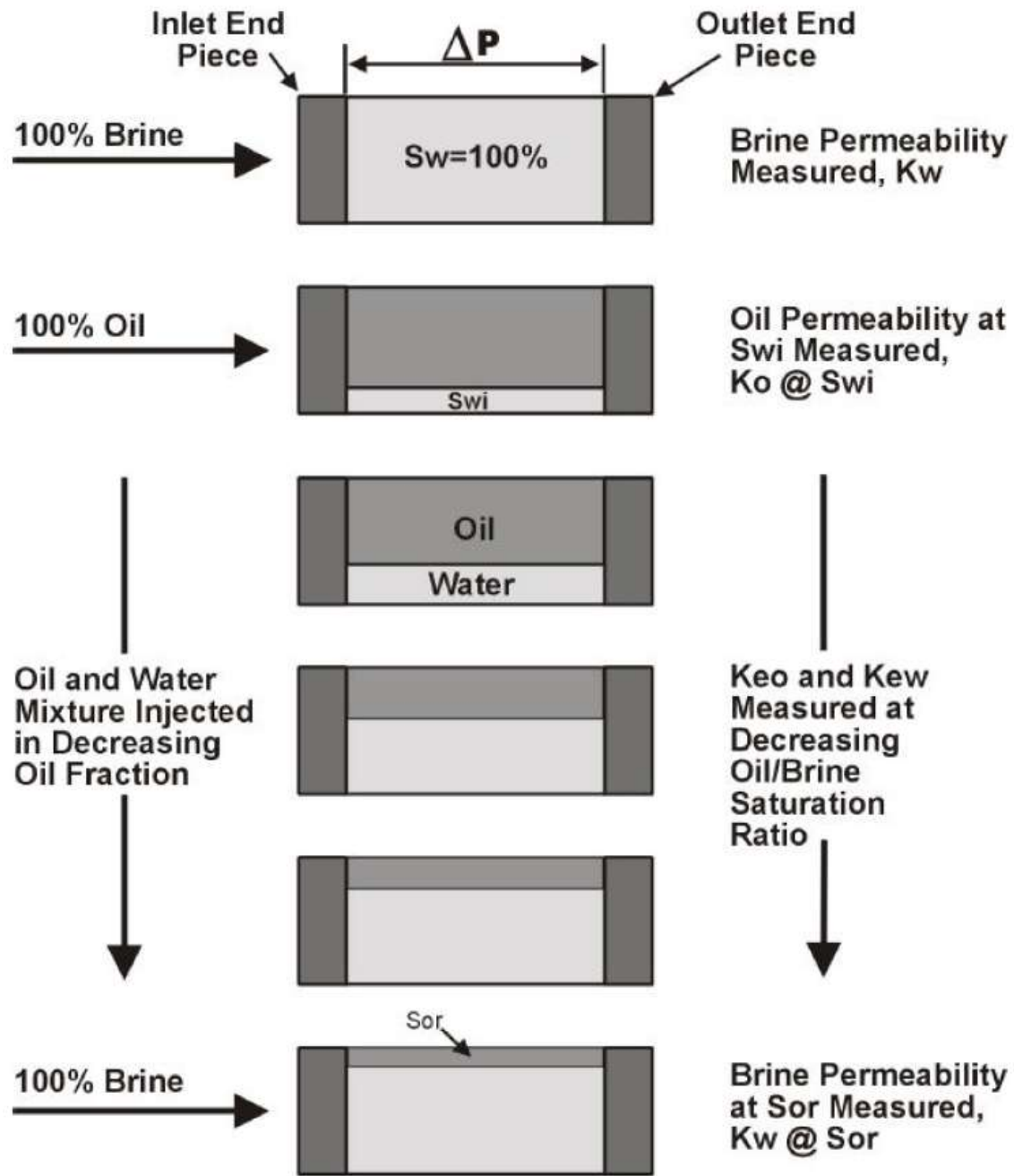
$$Q_2 = \frac{KK_{r2}A}{\mu_2}\frac{\Delta P_2}{L}$$

- **Steady state**

Advantages

- *Direct determination of Kr @ large range of S*
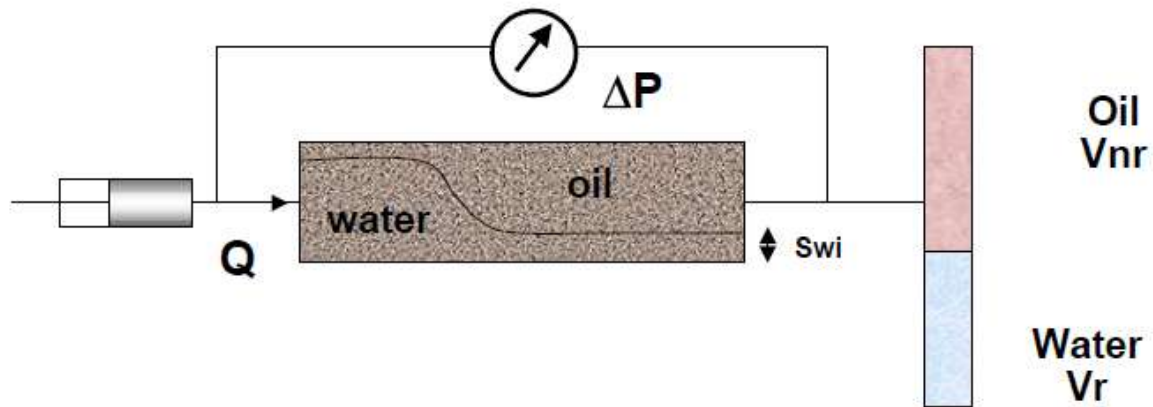- *No mathematical developments*
- *Reservoir conditions (P,T)*

Shortcomings

- *Time consuming, expensive*
- *Capillary end effects*
  - *long composite cores*
- *Representativity of flow in the reservoir questionable*

Inlet End Piece — Outlet End Piece

ΔP

100% Brine → | Sw=100% | **Brine Permeability Measured, Kw**

100% Oil → | Swi | **Oil Permeability at Swi Measured, Ko @ Swi**

| Oil / Water |

Oil and Water Mixture Injected in Decreasing Oil Fraction

Keo and Kew Measured at Decreasing Oil/Brine Saturation Ratio

100% Brine → | Sor | **Brine Permeability at Sor Measured, Kw @ Sor**

Core Saturation

• Unsteady-state method

Analytical calculation only for Pc = 0

- *Welge*   *Kro/Krw*
- *JBN*   *Kro, Krw*

Unsteady-state

Advantages

- *Fast*
- *Representativity of flow in the reservoir*
- *Reservoir conditions (P,T)*

Shortcomings

- *Calculations not straightforward*
- *Strong assumptions for analytical calculations*
  - *homogeneous samples*
  - *Pc=0*
- *Narrow range of saturations (only after BT)*

To overcome shortcomings

- *Perform in situ saturation measurements (X-ray or g-ray absorption, CT-scanner)*
- *Numerical interpretation of the displacement data*

Core Saturation

Brine Saturated Core
Sw=100%
Brine Permeability Measured, Kw

Flood Down to Swi with Oil
Oil
Swi
Oil Permeability at Swi Measured, Ko @ Swi

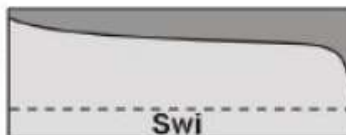**Initial Stages of Water Flood (Before Water Breakthrough) Only Oil Produced**
Swi
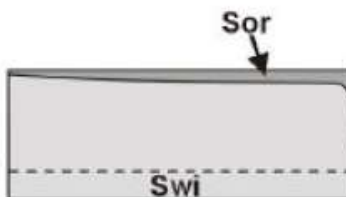**Kew and Keo Measured**
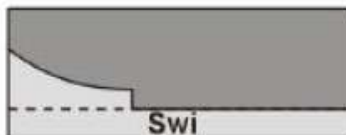
During Water Flood Water Breakthrough
Swi
Kew and Keo Measured
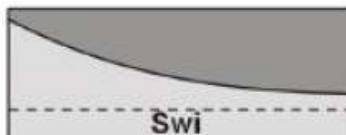
Water Flood Continues Both Oil and Water Produced
Swi
Kew and Keo Measured

**End of Water Flood Only Water Produced Residual Oil Saturation**
Sor
Swi
**Brine Permeability at Sor Measured, Kw @ Sor**

Two Phase Flow

Capillary End Effect

Definition: Before wetting phase flows to out surface, the capillary pressure is active for wetting fluid driving nonwetting fluids. But the capillary pressure is a resistance for wetting fluids to flow out when it moves to outlet surface. As a result ,the saturation of wetting phase near the outlet face is greater than the normal distribution, this phenomenon is called end effect.
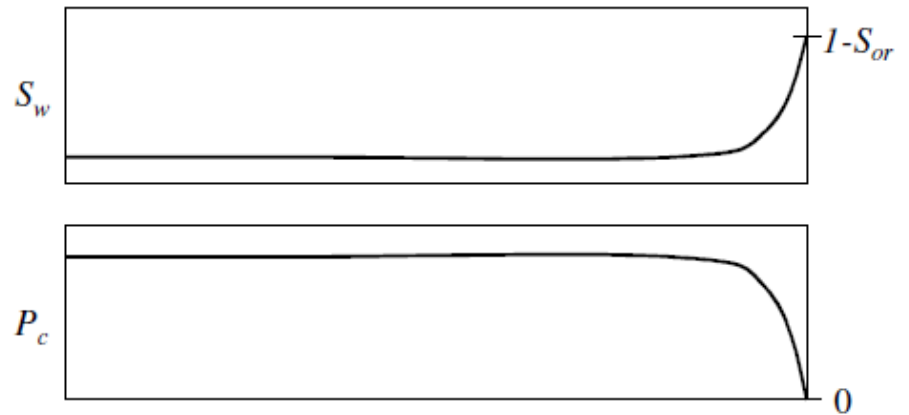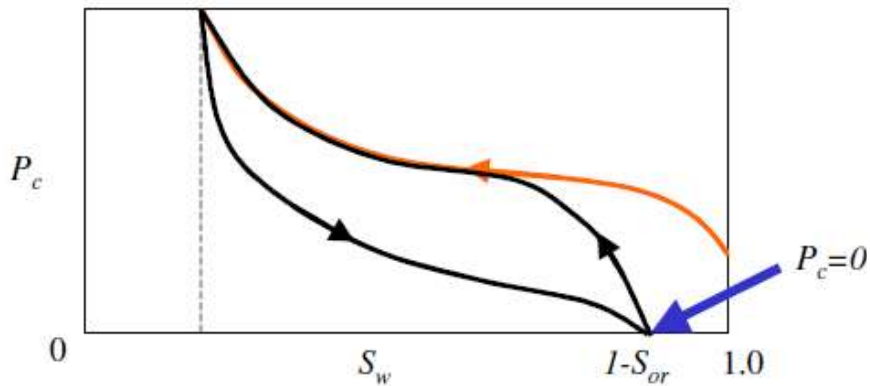
Eliminating method

1) Increase flow rate to reduce influence area of end effects;

2) Increase the experiment core length to reduce the ratio of the end effect length to the core total length;

3) Three-segment core method.
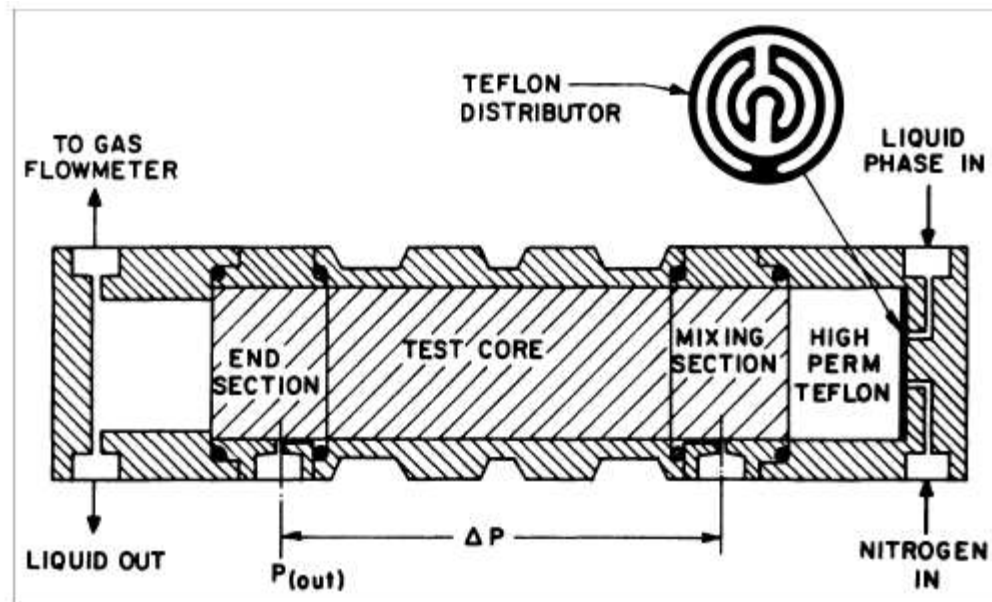
Penn State method (Figure 5.19 from Dullien).

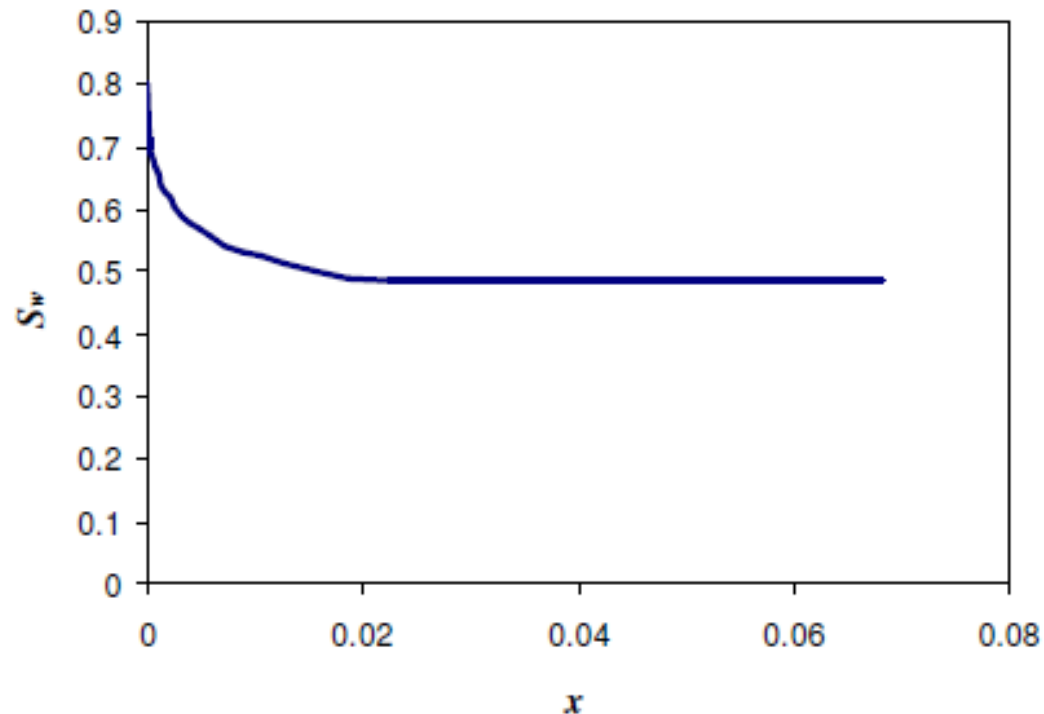The **capillary end effect** can be analyzed based on the flow equations:

$$q_o = -\frac{k\,k_{ro}}{\mu_o} A \frac{\partial p_o}{\partial x} = -\frac{k\,k_{ro}}{\mu_o} A \left(\frac{\partial p_w}{\partial x} + \frac{\partial P_c}{\partial x} \right)$$

(3.42)

$$q_w = -\frac{k\,k_{rw}}{\mu_w} A \frac{\partial p_w}{\partial x} \quad \Rightarrow \quad \frac{\partial p_w}{\partial x} = -\frac{q_w \mu_w}{k\,k_{rw} A}$$

(3.43)

$$\frac{q_o \mu_o}{k\,k_{ro} A} = \frac{q_w \mu_w}{k\,k_{rw} A} - \frac{\partial P_c}{\partial x}$$

(3.44)

So

$$\frac{\partial P_c}{\partial x} = \left(\frac{q_w \mu_w}{k_{rw}} - \frac{q_o \mu_o}{k_{ro}} \right) \frac{1}{Ak}$$

or

$$\frac{\partial S_w}{\partial x} = \frac{1}{Ak} \left(\frac{q_w \mu_w}{k_{rw}} - \frac{q_o \mu_o}{k_{ro}} \right) \frac{1}{\frac{\partial P_c}{\partial S_w}}$$

Three phase systems

Since we now have three phases flowing, we need to define the relative permeabilities and capillary pressures anew. Although the following functional relationship not always are valid in practice, we will here use the conventional definitions for a completely water wet system with no contact between gas and water phases. Thus, the parameters below are functions only of the saturations indicated:
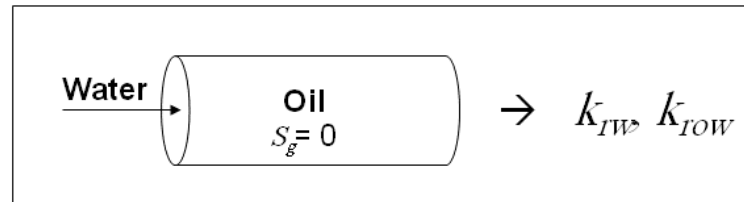
$$k_{rw}(S_w)$$
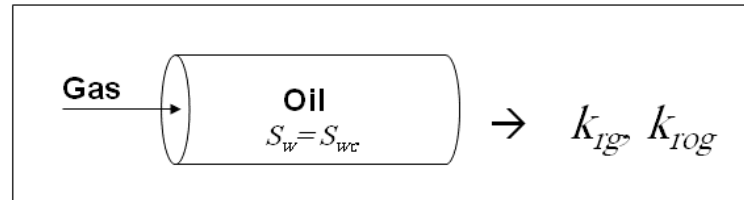$$k_{rg}(S_g)$$
$$k_{ro}(S_w, S_g)$$
$$P_{cow}(S_w)$$
$$P_{cog}(S_g)$$

• Increasing S_w

| Water | Oil $S_g = 0$ | $\rightarrow$ | k_{rw}, k_{row} |

• Increasing S_g

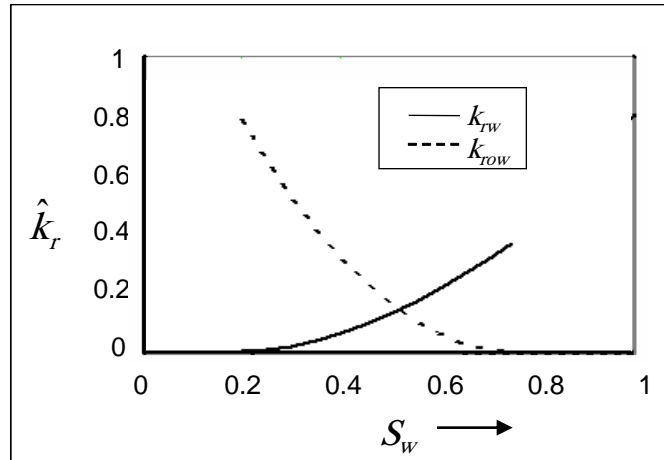| Gas | Oil $S_w = S_{wc}$ | $\rightarrow$ | k_{rg}, k_{rog} |

Except for the relative permeability to oil, k_{ro}, these parameters may be measured in two-phase measurements since they depend on one saturation only. In the discussion of three-phase relative permeability to oil, k_{ro}, we will start with typical two-phase oil-water and oil-gas relationships:
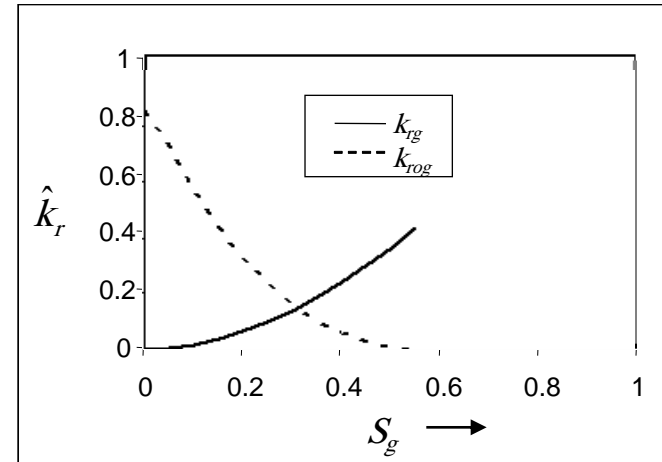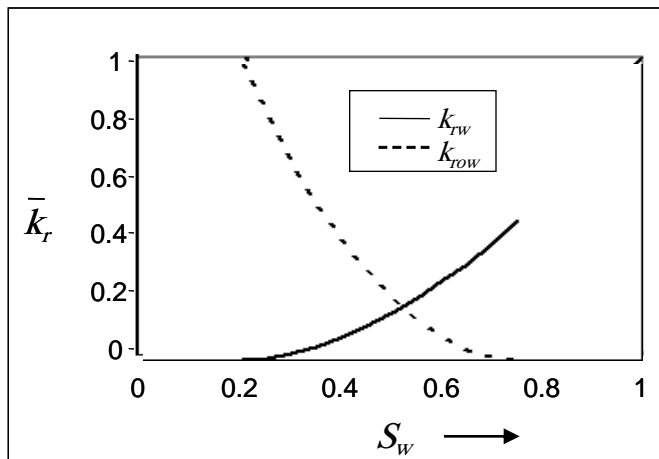
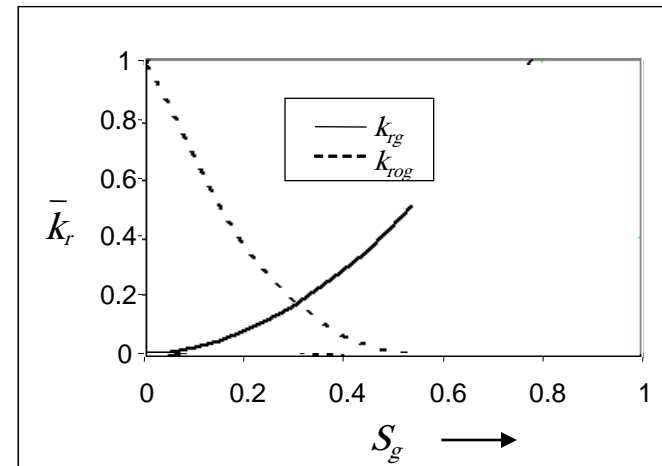# Laboratory Data

Reported in terms of k_{abs}

Water - Oil

Gas - Oil

## Reported in terms of k_{ocw}

Water - Oil

Gas - Oil

The two oil relative permeability curves are two phase curves. However, as indicated above, in a three-phase flow situation, the oil relative permeability would be a function of both water and gas saturations. Plotting it in a triangular diagram, so that each saturation is represented by one of the sides, we can define an area of mobile oil limited by the system's maximum and minimum saturations (which not necessarily are constants). Inside this area, iso- k_{ro} curves may be drawn, as illustrated below:
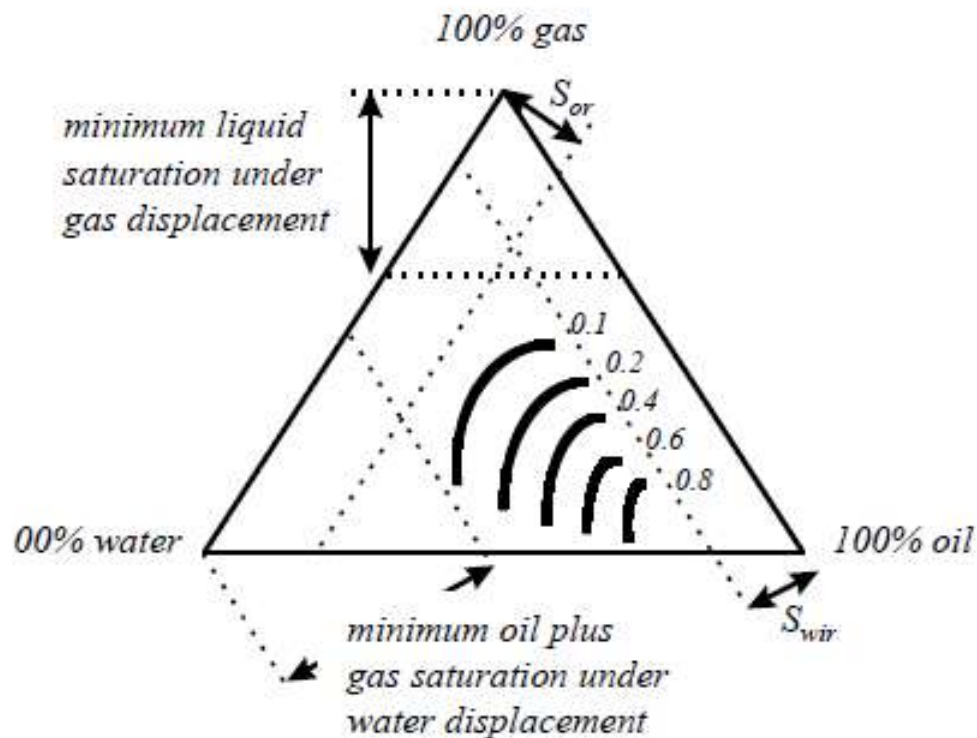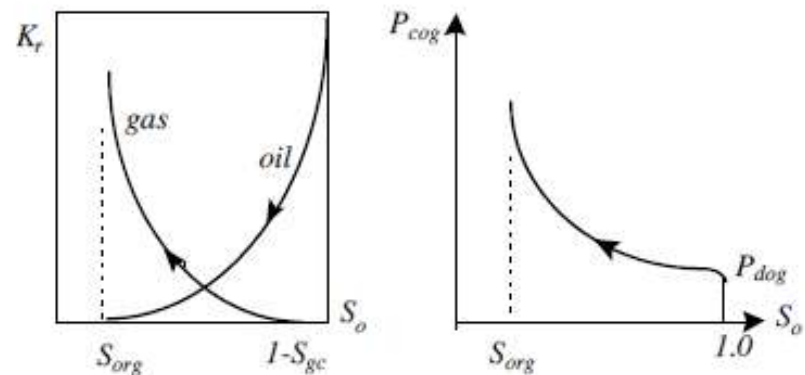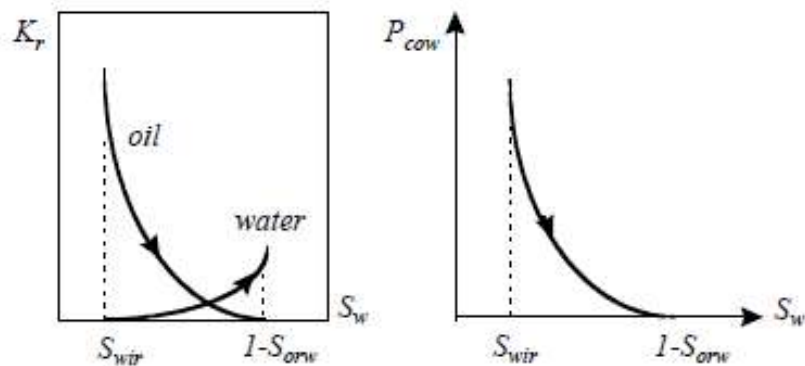
Theory and Assumptions Used in Stone's Models

In principle, k_{ro} may be measured in the laboratory. However, due to the experimental complexity of three-phase experiments, we most of the time construct it from two phase oil-water k_{row} and two phase oil-gas k_{rog} .

The so-called Stone-models may be used for construction of three-phase relative permeability curves.

Stone's method 1 and 2 are probability based models. Both methods are based on the channel flow theory, which states that in any flow channel there is at most only one mobile fluid. As a consequence, the wetting phase is located in the small pores and the nonwetting phase in the large pores, and the intermediate phase occupies the pores in between. Thus at equal water saturations the fluid distributions will be identical in a water-oil system and in a water-oil-gas system, as long as the direction of change of water saturation is the same in both. This implies that water relative permeability and water-oil capillary pressure in the three-phase system are functions of water saturations alone. Also, they vary the same way in the three-phase system as in the two-phase water oil system. The gas phase relative permeability and gas oil capillary pressure are the same functions of gas saturation in the three-phase system as in the two-phase gas oil system.

These two models require two sets of two-phase data: water oil displacement and gas oil displacement. Hysteresis can be taken into consideration by using the appropriate two-phase data.

Assumption:

$$\bar{k}_{rw} = f(S_w)$$
$$\bar{k}_{rg} = f(S_g)$$

➡ $$\bar{k}_{ro} = f(S_w, S_g)$$

Stone's model 1

$$S_{oD} = \frac{S_o - S_{or}}{1 - S_{wir} - S_{or}}$$

$$S_{wD} = \frac{S_w - S_{wir}}{1 - S_{wir} - S_{or}}$$

$$S_{gD} = \frac{S_g}{1 - S_{wir} - S_{or}}$$

$$\beta_w = \frac{k_{row}}{1 - S_{wD}}$$

$$\beta_g = \frac{k_{rog}}{1 - S_{gD}}$$

➡ $$k_{ro} = S_{oD} \, \beta_w \, \beta_g$$

Please note that the above formulas assume that end point relative permeabilities are 1. If this is not the case, the relative permeability formula must be modified accordingly.

Stone's model 2

It does not require the estimation of S_{or}, as it attempts to estimate it implicitly by its formulation.

$$k_{ro} = (k_{rog} + k_{rg})(k_{row} + k_{rw}) - (k_{rw} + k_{rg})$$