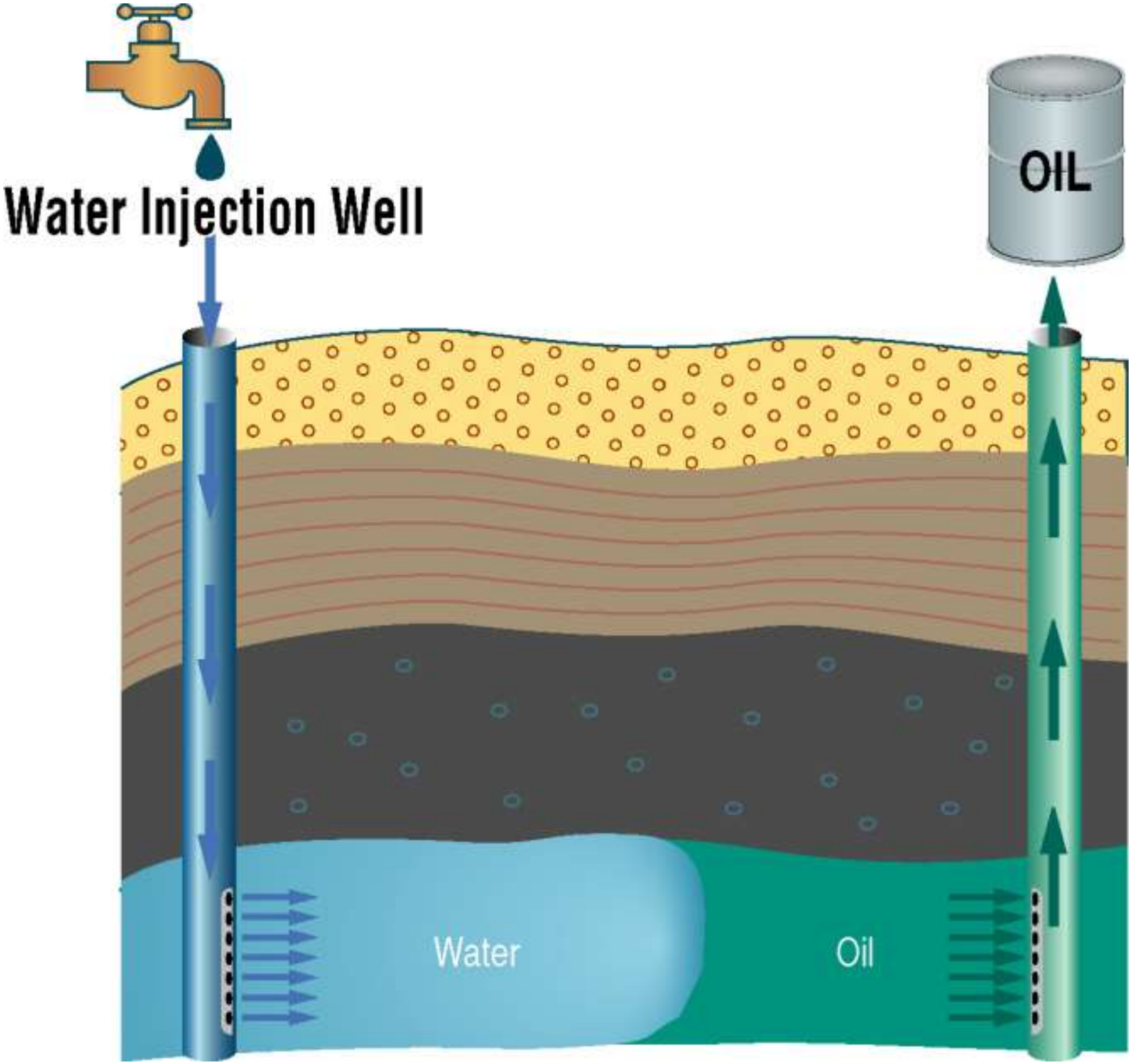
Flow in Porous Media

Module 3.a
Fundamental of Two Phase Flow in Porous Media
1-D Immiscible Displacement


Water Injection Well
OIL
Water
Oil

THE FRACTIONAL FLOW EQUATION

In this lecture oil displacement will be assumed to take place under the so-called diffuse flow condition. This means that fluid saturations at any point in the linear displacement path are uniformly distributed with respect to thickness. The sole reason for making this assumption is that it permits the displacement to be described, mathematically, in one dimension and this provides the simplest possible model of the displacement process.

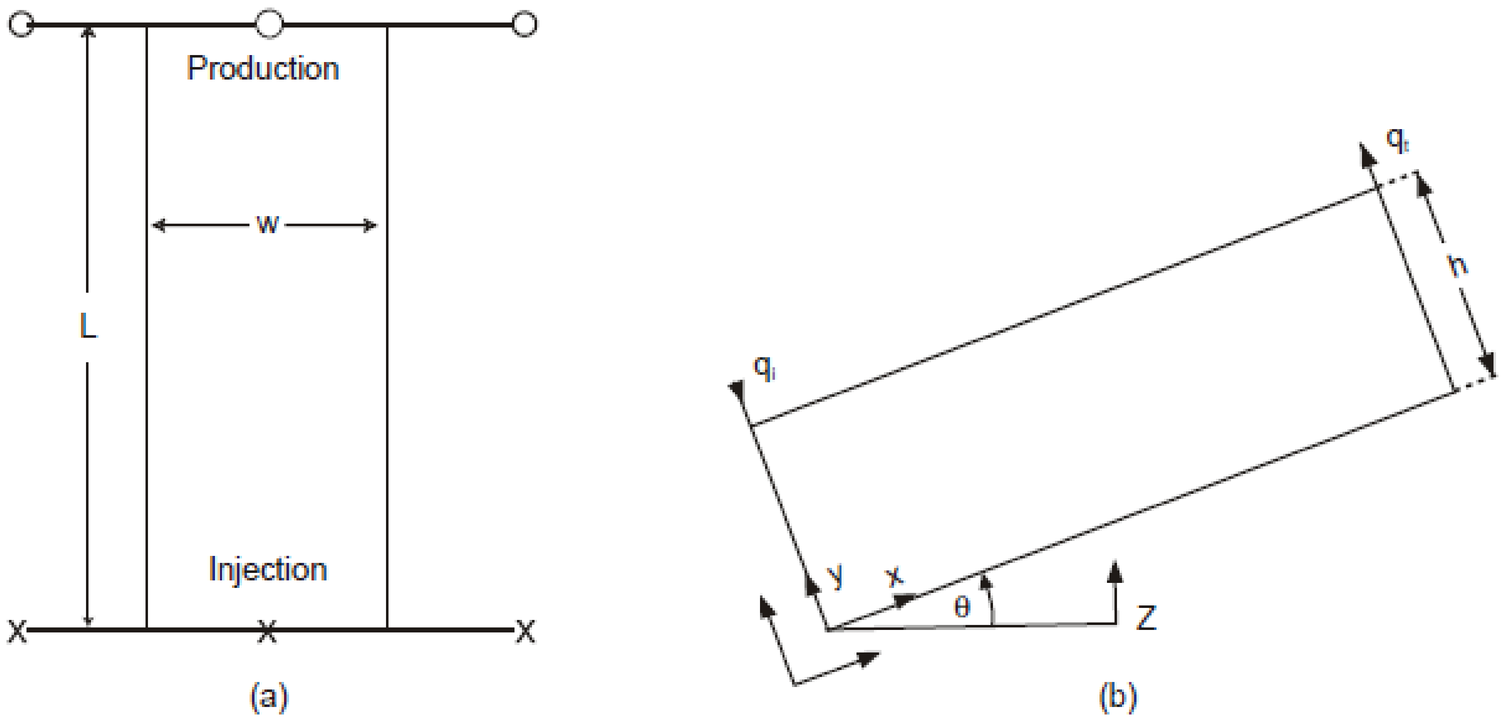


Fig. 10.6 Linear prototype reservoir model, (a) plan view; (b) cross section

$$q_o = -\frac{kk_{ro}A\rho_o}{\mu_o}\frac{\partial\Phi_o}{\partial x} = -\frac{kk_{ro}A}{\mu_o}\left(\frac{\partial p_o}{\partial x} + \frac{\rho_o g \ \sin\theta}{1.0133\times10^6}\right)$$

and

$$q_w = -\frac{kk_{rw}A\rho_w}{\mu_w}\frac{\partial\Phi_w}{\partial x} = -\frac{kk_{rw}A}{\mu_w}\left(\frac{\partial p_w}{\partial x} + \frac{\rho_w g \ \sin\theta}{1.0133\times10^6}\right)$$

$$q_o = q_t - q_w$$

$$f_w = \frac{q_w}{q_o + q_w} = \frac{q_w}{q_t}$$

$$f_w = \frac{1 + \frac{kk_{ro}A}{q_t\mu_o}\left(\frac{\partial P_c}{\partial x} - \frac{\Delta\rho \ g \ \sin\theta}{1.0133\times10^6}\right)}{1 + \frac{\mu_w}{k_{rw}}\cdot\frac{k_{ro}}{\mu_o}} \qquad (10.9)$$

$$\frac{\partial P_c}{\partial x} = \frac{dP_c}{dS_w} \cdot \frac{\partial S_w}{\partial x} \tag{10.11}$$

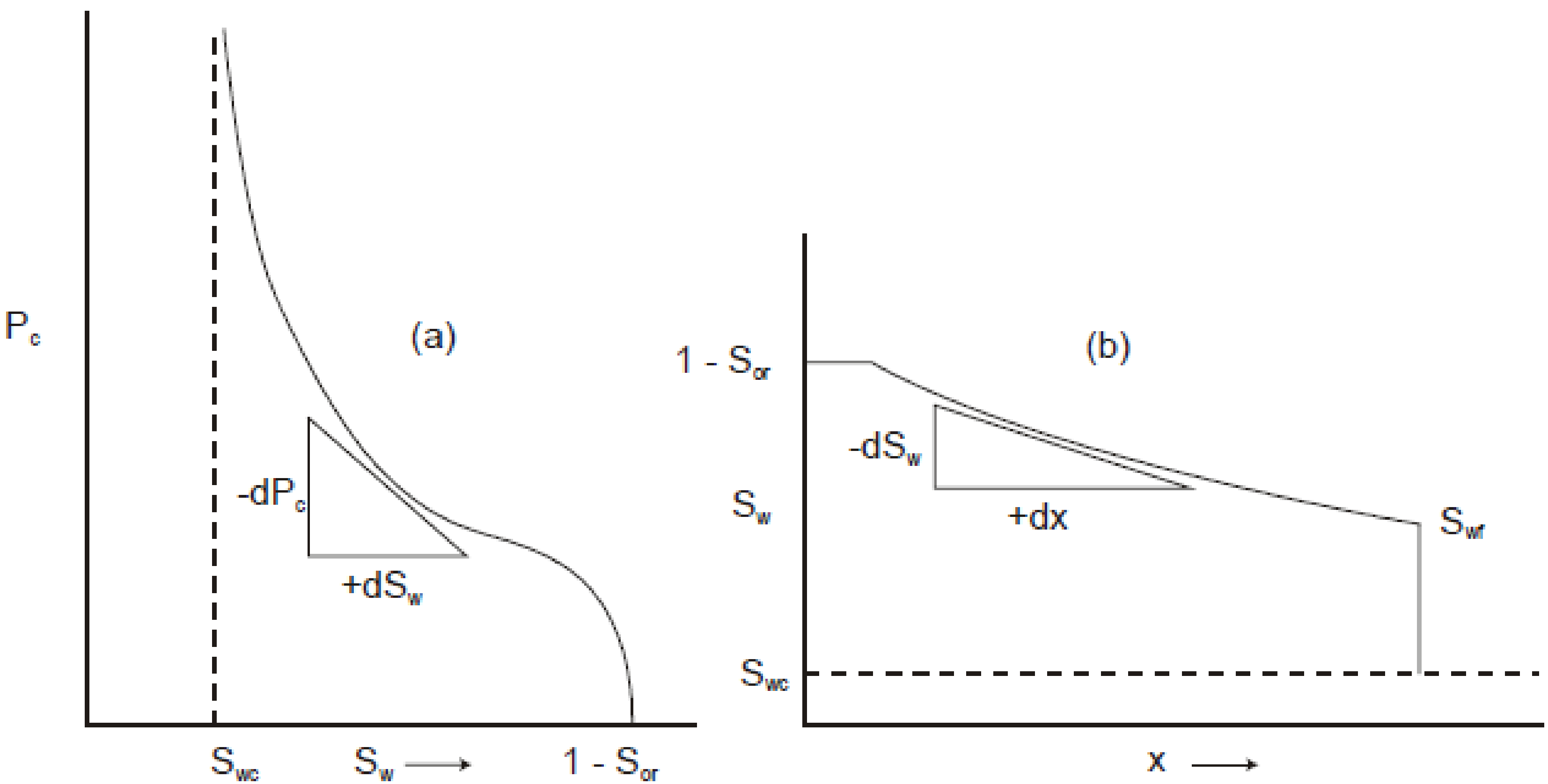
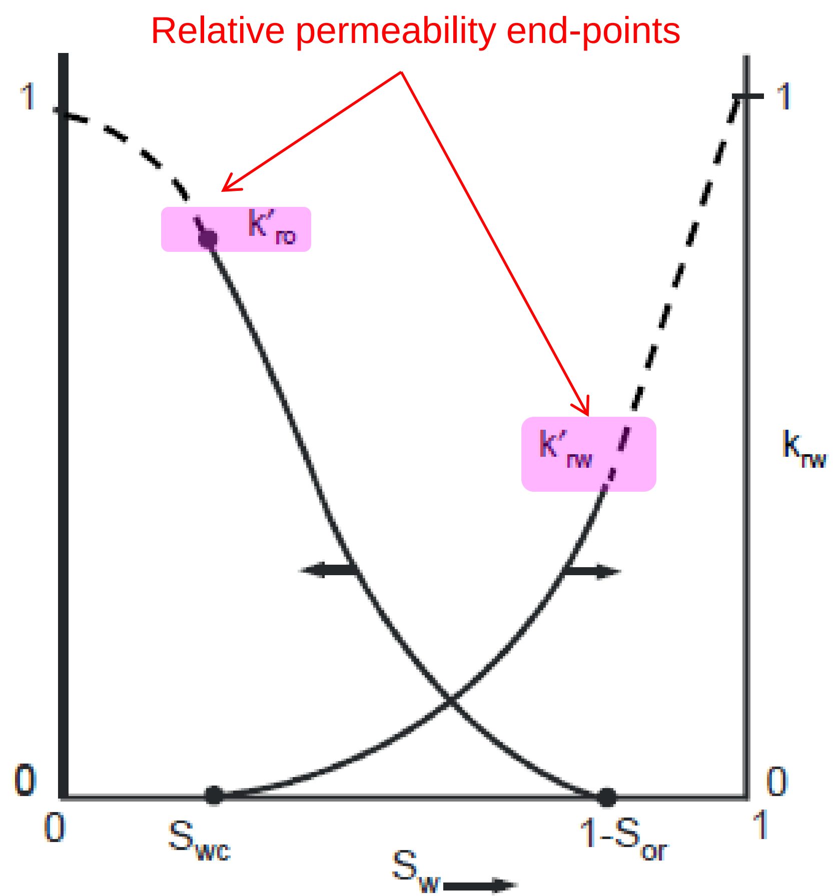
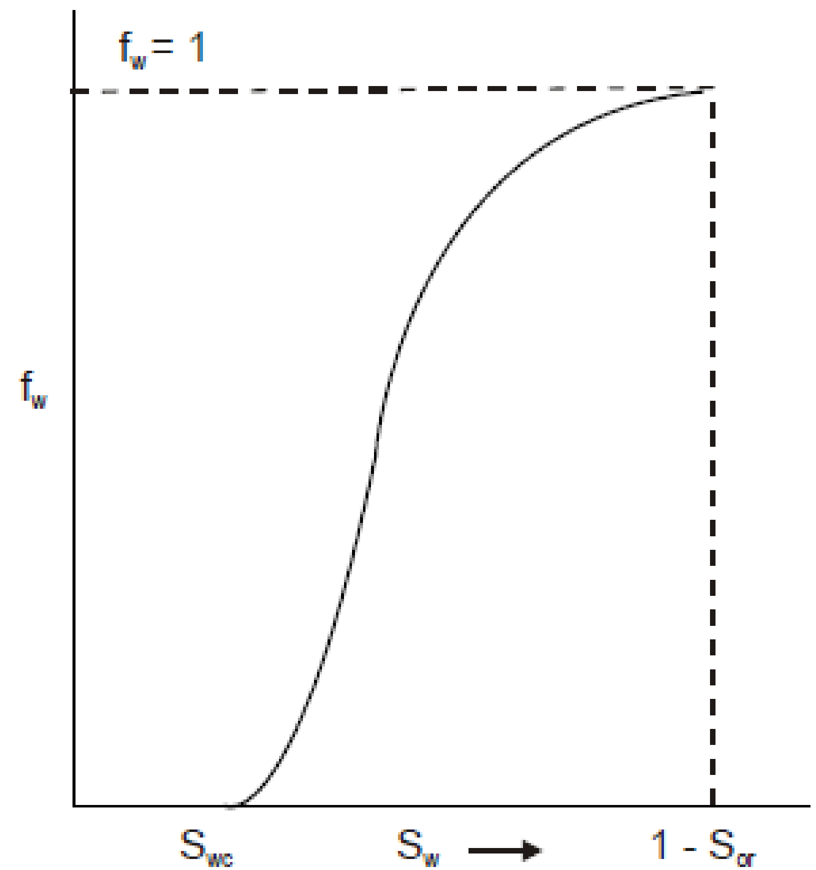


Fig. 10.8 (a) Capillary pressure function and; (b) water saturation distribution as a function of distance in the displacement path

For displacement in a horizontal reservoir (sin θ = 0), and neglecting, for the moment, the capillary pressure gradient, the fractional flow equation is reduced to

$$f_w = \frac{1}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \qquad (10.12)$$





BUCKLEY-LEVERETT ONE DIMENSIONAL DISPLACEMENT

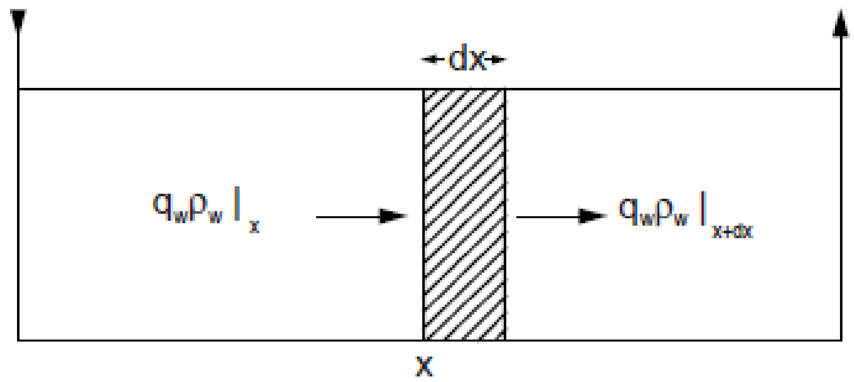


Fig. 10.10 Mass flow rate of water through a linear volume element $A\phi\, dx$

Mass flow rate In - Out	=	Rate of increase of mass in the volume element

$$q_w \rho_w \Big|_x - q_w \rho_w \Big|_{x+dx} = A\phi \; dx \frac{\partial}{\partial t}(\rho_w S_w) \qquad (10.13)$$

$$q_w \rho_w\Big|_x - \left(q_w \rho_w\Big|_x + \frac{\partial}{\partial x}(q_w \rho_w) dx \right) = A\phi \; dx \frac{\partial}{\partial t}(\rho_w S_w)$$

which can be reduced to

$$\frac{\partial}{\partial x}(q_w \rho_w) = -A\phi \frac{\partial}{\partial t}(\rho_w S_w) \qquad (10.14)$$

and for the assumption of incompressible displacement ($\rho_w \approx$ constant)

$$\left.\frac{\partial q_w}{\partial x}\right|_t = -A\phi \left.\frac{\partial S_w}{\partial t}\right|_x \qquad (10.15)$$

The full differential of the water saturation is **$S_w = S_w(x,t)$**

$$dS_w = \left.\frac{\partial S_w}{\partial x}\right|_t dx + \left.\frac{\partial S_w}{\partial t}\right|_x dt$$

and since it is the intention to study the movement of a plane of constant water saturation, that is, $dS_w = 0$, then

$$\left.\frac{\partial S_w}{\partial t}\right|_x = -\left.\frac{\partial S_w}{\partial x}\right|_t \left.\frac{dx}{dt}\right|_{S_w} \qquad (10.16)$$

3rd term in R.H.S OF Eq. 10.15

Furthermore,

$$\left.\frac{\partial q_w}{\partial x}\right|_t = \left(\frac{\partial q_w}{\partial S_w} \cdot \frac{\partial S_w}{\partial x}\right)_t \qquad (10.17)$$

L.H.S OF Eq. 10.15

and substituting equs. (10.16) and (10.17) in equ. (10.15) gives

$$\left.\frac{\partial q_w}{\partial S_w}\right|_t = A\phi \left.\frac{dx}{dt}\right|_{S_w} \qquad (10.18)$$

Again, for incompressible displacement, q_t is constant and, since $q_w = q_t f_w$, equ. (10.18) may be expressed as

$$v_{S_w} = \left.\frac{dx}{dt}\right|_{S_w} = \frac{q_t}{A\phi} \left.\frac{df_w}{dS_w}\right|_{S_w} \qquad (10.19)$$

This is the equation of Buckley-Leverett which implies that, for a constant rate of water injection ($q_t = q_i$), the velocity of a plane of constant water saturation is directly proportional to the derivative of the fractional flow equation evaluated for that saturation.

Integrating for the total time since the start of injection gives

$$x_{S_w} = \frac{1}{A\phi} \frac{df_w}{dS_w} \int_0^t q_t dt$$

or

$$x_{S_w} = \frac{W_i}{A\phi} \frac{df_w}{dS_w}\bigg|_{S_w} \qquad (10.20)$$

W_i : the cumulative water injected

Therefore, at a given time after the start of injection (W_i = constant) the position of different water saturation planes can be plotted, using eq. (10.20), merely by determining the slope of the fractional flow curve for the particular value of each saturation.

$$f_w = \frac{1}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}}$$

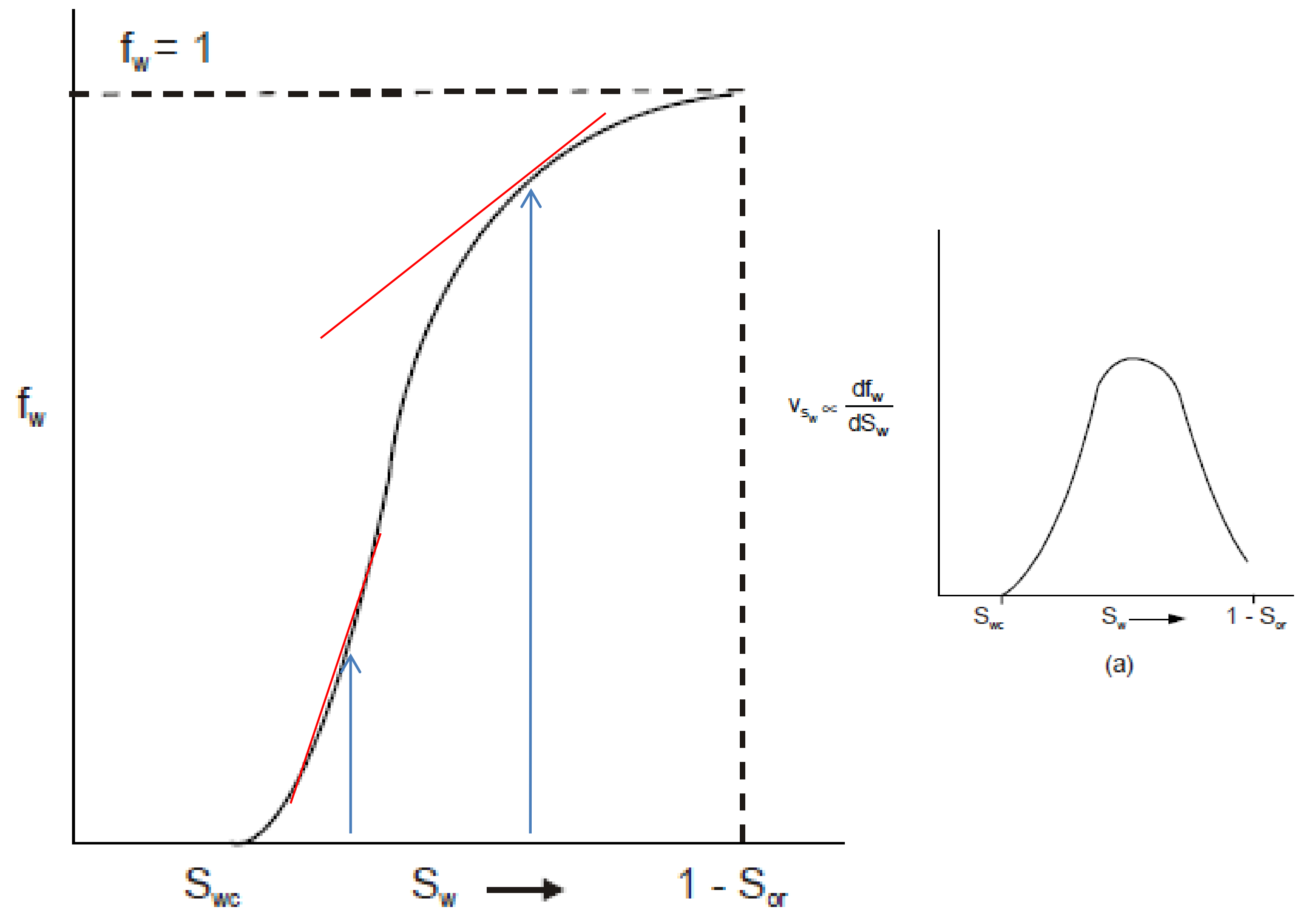


Fig. 10.9 Typical fractional flow curve as a function of water saturation, equ. (10.12)

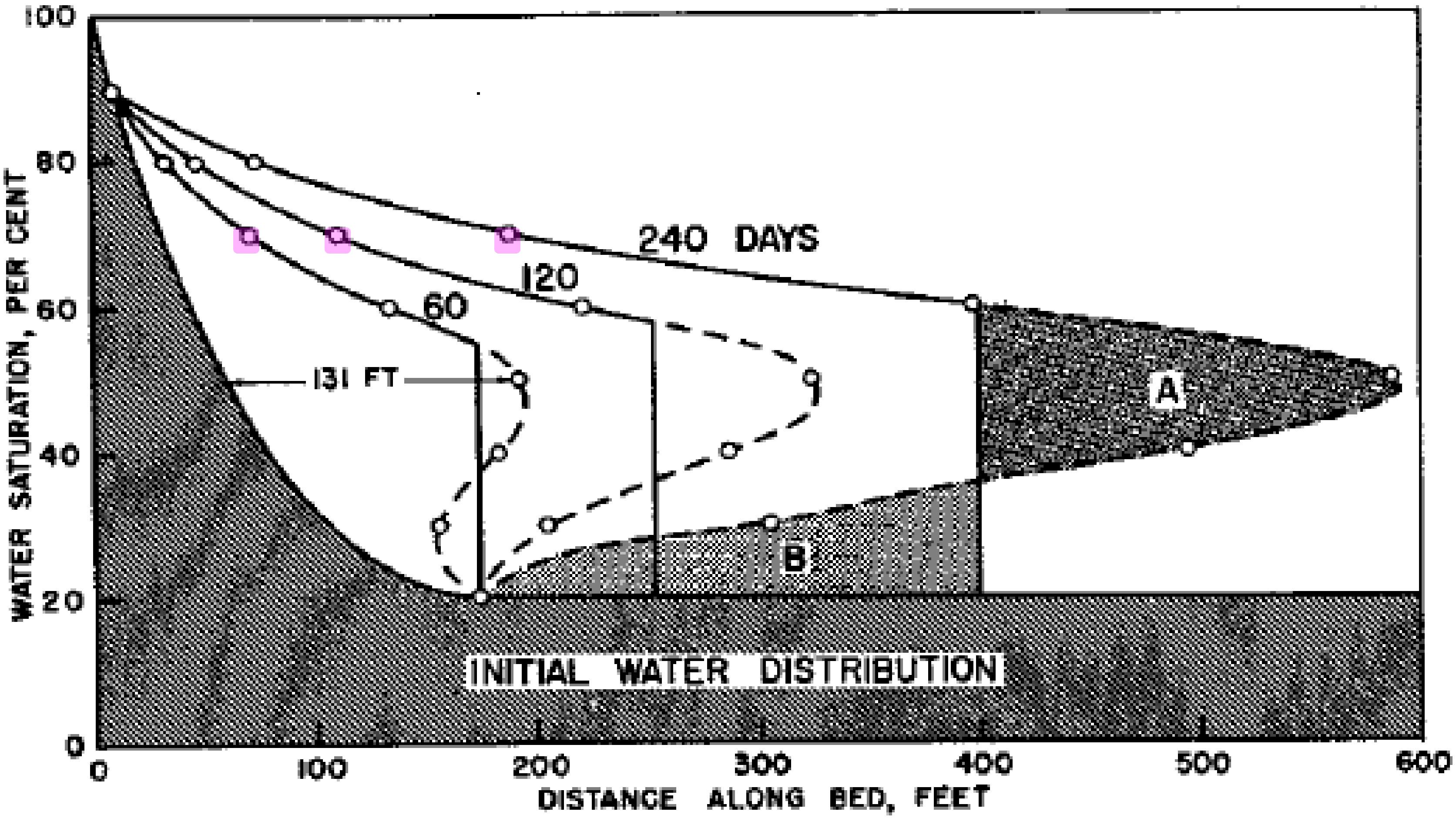
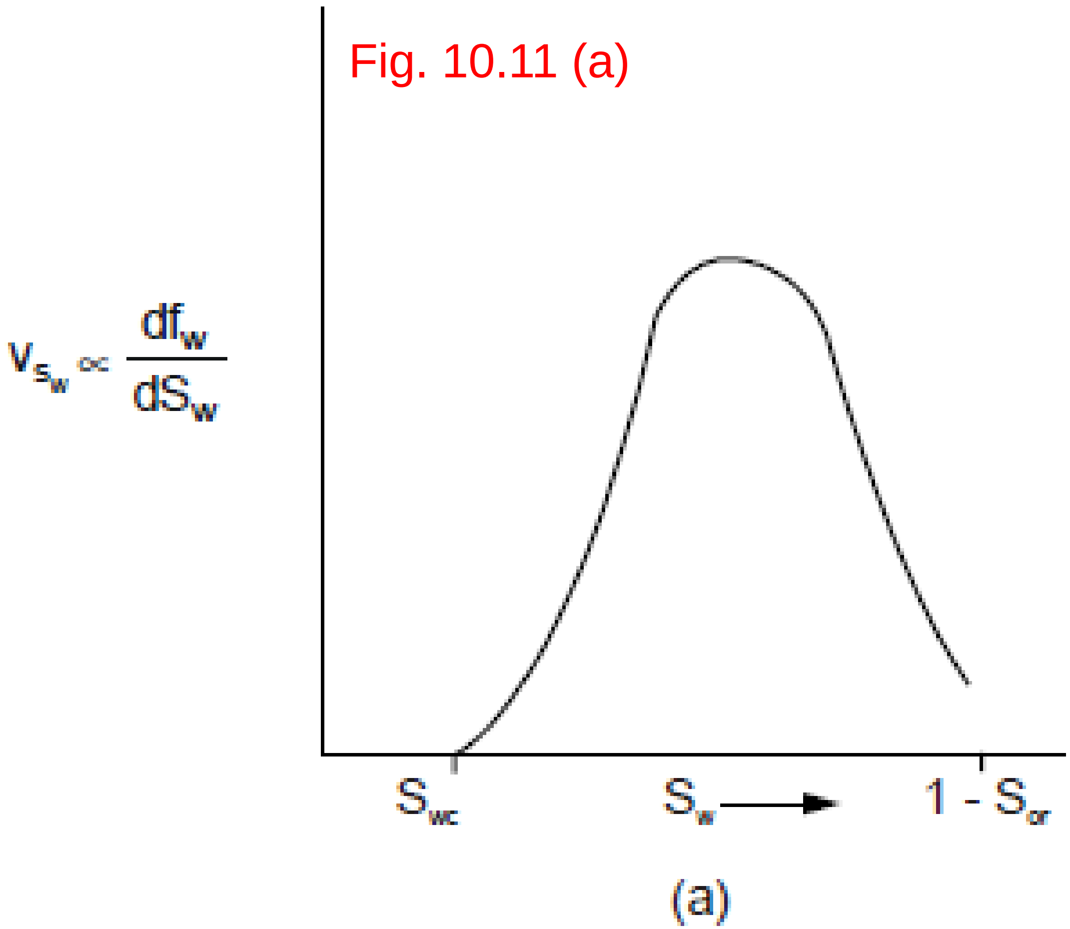


Fig. 9.10. Fluid distributions at initial conditions and at 60, 120, and 240 days.

There is a mathematical difficulty encountered in applying this technique which can be appreciated by considering the typical fractional flow curve shown in fig. 10.9 in conjunction with eq. (10.20). Since there is frequently a point of inflexion in the fractional flow curve then the plot of df_w/dS_w versus S_w will have a maximum point, as shown in fig. 10.11 (a).

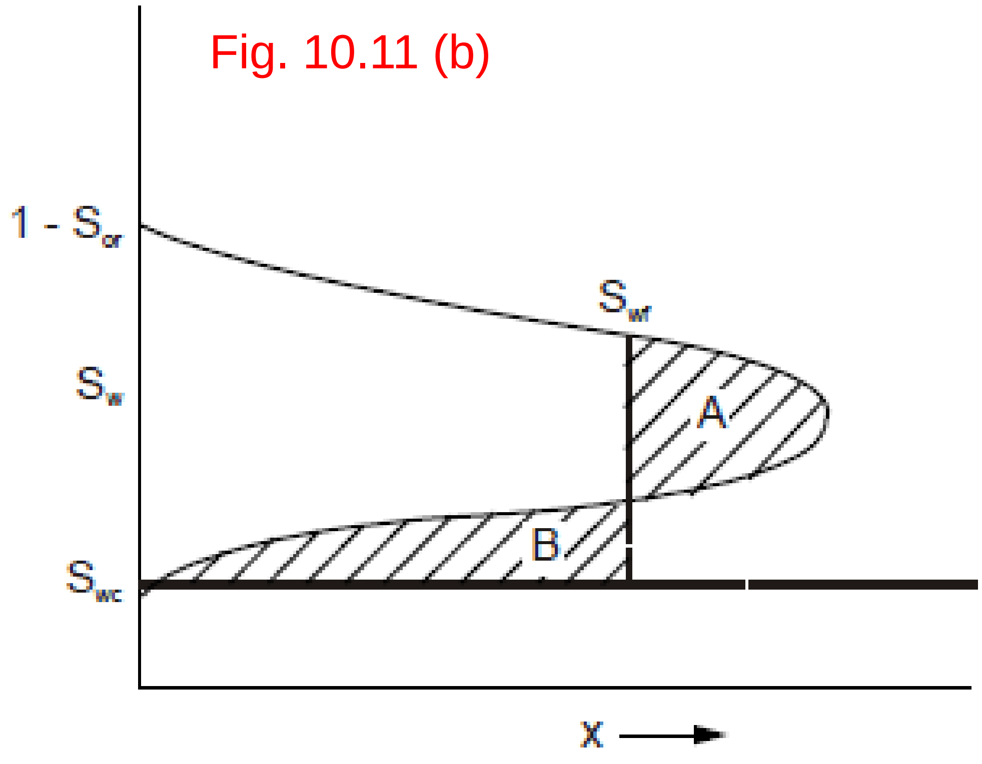
$$x_{S_w} = \frac{W_i}{A\phi}\left.\frac{df_w}{dS_w}\right|_{S_w} \qquad (10.20)$$



Using eq. (10.20) to plot the saturation distribution at a particular time will therefore result in the solid line shown in fig. 10.11(b). This bulbous saturation profile is physically impossible since it indicates that multiple water saturations can co-exist at a given point in the reservoir.

$$x_{S_w} = \frac{W_i}{A\phi} \left. \frac{df_w}{dS_w} \right|_{S_w} \qquad (10.20)$$

Fig. 10.11 (b)



•Behind the front in the saturation range $\mathbf{S_{wf} < S_w < 1-S_{or}}$ where S_{wf} is the shock front saturation, equs. (10.19) and (10.20) can be applied to determine the water saturation velocity and position. Furthermore, in this saturation range the capillary pressure gradient is usually negligible and the fractional flow equation to be used in equs. (10.19) and (10.20) is simply:

$$x_{S_w} = \frac{W_i}{A\phi} \left.\frac{df_w}{dS_w}\right|_{S_w} \tag{10.20}$$

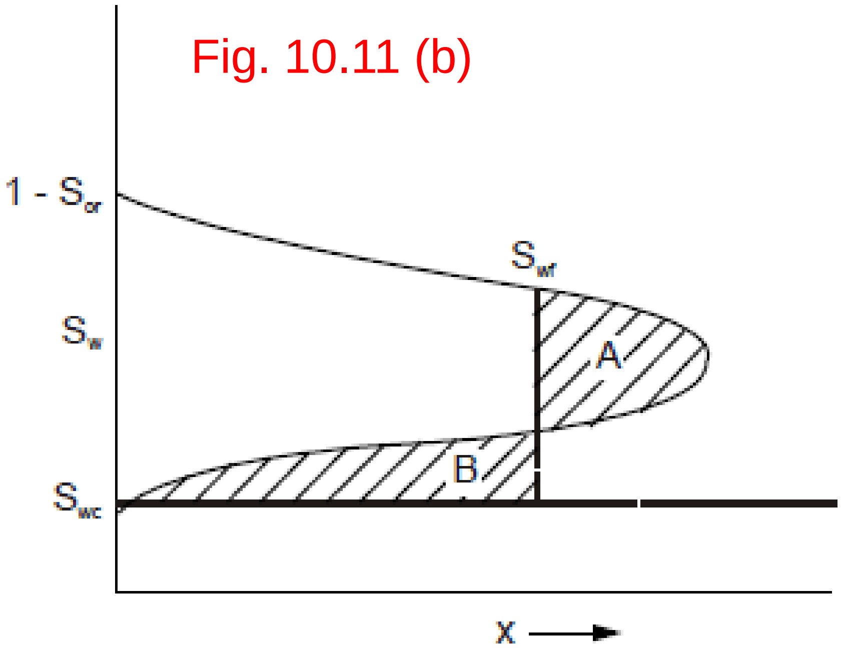
For a horizontal reservoir

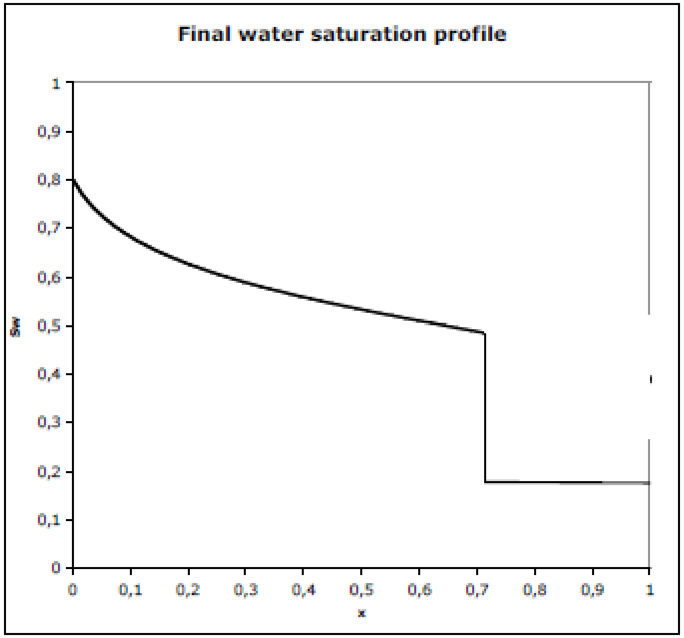
$$f_w = \frac{1}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \tag{10.12}$$

For a dipping reservoir

$$f_w = \frac{1 - \frac{kk_{ro}A}{q_t\mu_o} \frac{\Delta\rho \ g \ \sin\theta}{1.0133\times10^6}}{1 + \frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \tag{10.21}$$

•To draw the correct water saturation profile using the Buckley- Leverett technique requires the determination of the vertical dashed line, shown in fig. 10.11(b), such that the shaded areas A and B are equal. The dashed line then represents the shock front saturation discontinuity.





As a consequence of the conservation of mass the location of the discontinuity must be fixed in a way so that the areas on both sides of the discontinuity are equal in size.

•A more elegant method of achieving the same result was presented by **Welge** in 1952, This consists of integrating the saturation distribution over the distance from the injection point to the front, thus obtaining the average water saturation behind the front $\overline{\overline{S}}_w$ as shown in fig. 10.12.

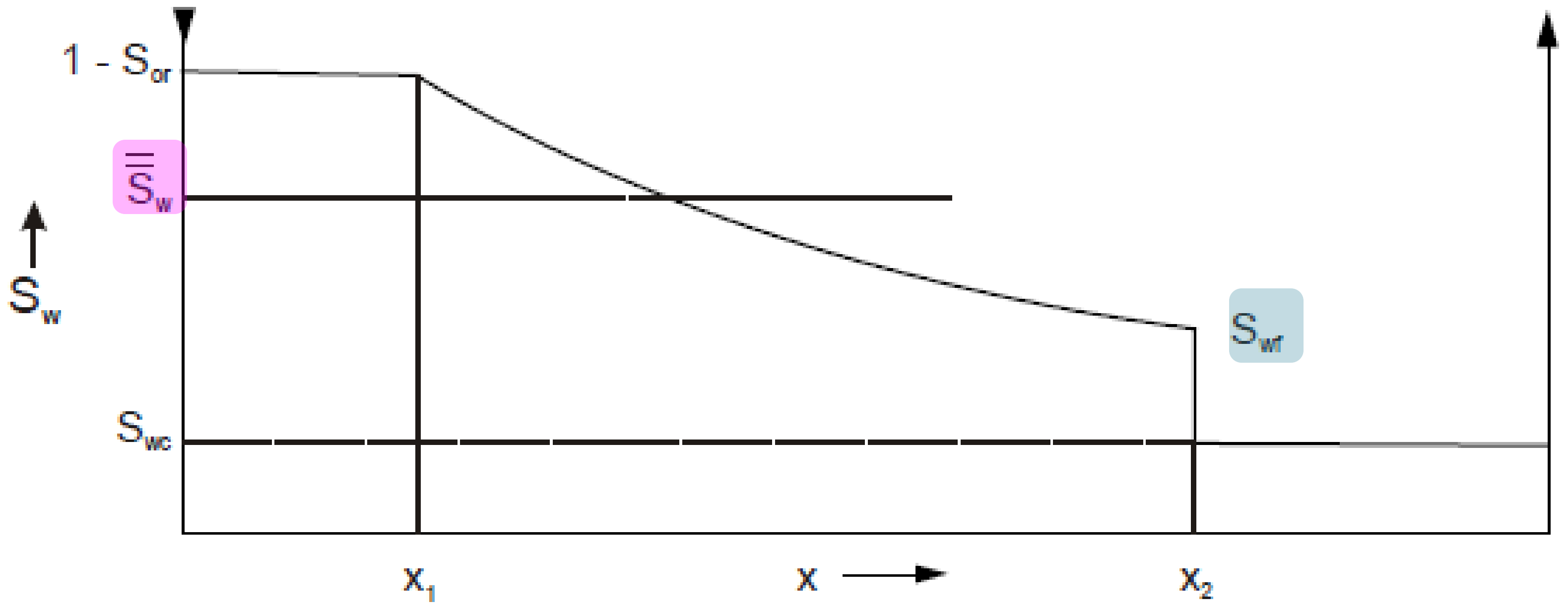


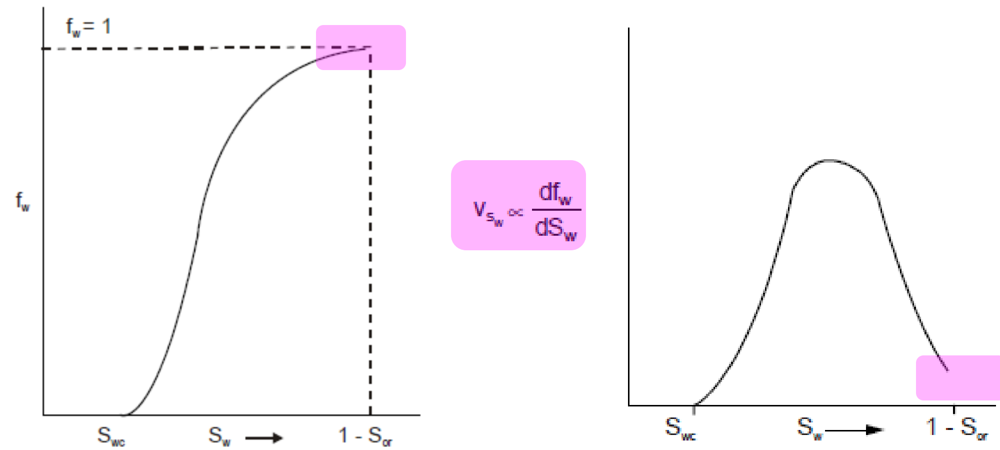
Fig. 10.12 Water saturation distribution as a function of distance, prior to breakthrough in the producing well

The situation depicted is at a fixed time, before water breakthrough in the producing well, corresponding to an amount of water injection W_i. At this time the maximum water saturation, $S_w = 1 - S_{or}$, has moved a distance x_1, its velocity being proportional to the slope of the fractional flow curve evaluated for the maximum saturation which, as shown in figs. 10.9 and 10.11 (a), is small but finite. The flood front saturation S_{wf} is located at position x_2 measured from the injection point. Applying the simple material balance

$$W_i = x_2 A\phi \, (\overline{\overline{S}}_w - S_{wc})$$

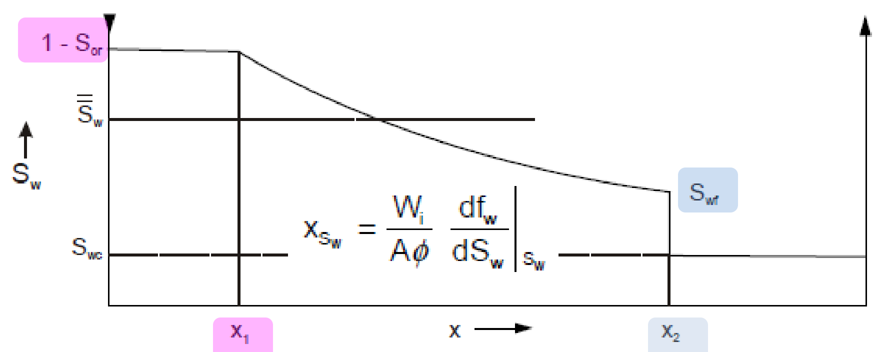
or

$$\overline{\overline{S}}_w - S_{wc} = \frac{W_i}{x_2 A\phi}$$



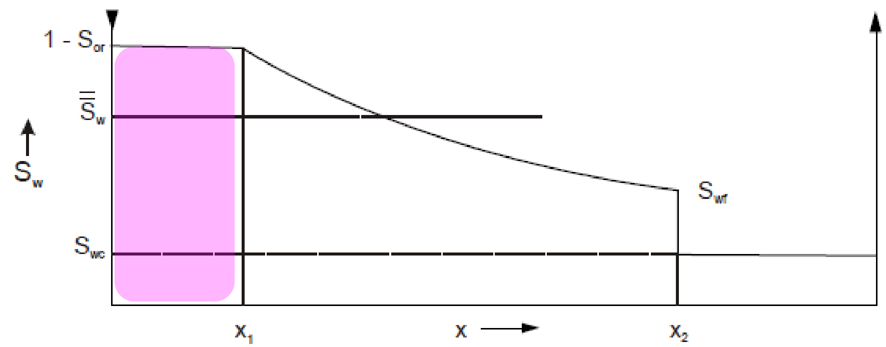
and using equ. (10.20) which is applicable up to the flood front at x_2, then

$$\overline{\overline{S}}_w - S_{wc} = \frac{W_i}{x_2 A\phi} = \frac{1}{\left.\dfrac{df_w}{dS_w}\right|_{S_{wf}}} \qquad (10.22)$$



An expression for the average water saturation behind the front can also be obtained by direct integration of the saturation profile as

$$\overline{\overline{S}}_w = \frac{(1-S_{or})x_1 + \int_{x_1}^{x_2} S_w \, dx}{x_2} \qquad (10.23)$$



and again since

$$x_{S_w} \propto \left.\frac{df_w}{dS_w}\right|_{S_w}$$

for a given volume of injected water, and for $S_w \geq S_{wf}$, then equ. (10.23) can be expressed as

$$\overline{\overline{S}}_w = \frac{(1-S_{or})\left.\dfrac{df_w}{dS_w}\right|_{1-S_{or}} + \displaystyle\int_{1-S_{or}}^{S_{wf}} S_w \, d\left(\dfrac{df_w}{dS_w}\right)}{\left.\dfrac{df_w}{dS_w}\right|_{S_{wf}}} \qquad (10.24)$$

The integral in the numerator of this equation can be evaluated using the method of integration by parts, i.e.

$$\int udv = uv - \int vdu$$

to give

$$\int_{1-S_{or}}^{S_{wf}} S_w \, d\left(\frac{df_w}{dS_w}\right) = \left[S_w \frac{df_w}{dS_w}\right]_{1-S_{or}}^{S_{wf}} - [f_w]_{1-S_{or}}^{S_{wf}}$$

and substituting this in equ. (10.24) and cancelling terms gives

$$\overline{\overline{S}}_w = S_{wf} + \left(1 - f_w\Big|_{S_{wf}}\right) \Big/ \frac{df_w}{dS_w}\Bigg|_{S_{wf}} \qquad (10.25)$$

in which both f_w and its derivative are evaluated for the shock front saturation S_{wf}. Finally, equating (10.22) and (10.25) gives

$$\overline{\overline{S}}_w - S_{wc} = \frac{W_i}{x_2 A\phi} = \frac{1}{\dfrac{df_w}{dS_w}\Big|_{S_{wf}}} \qquad (10.22)$$

$$\frac{df_w}{dS_w}\Bigg|_{S_{wf}} = \frac{(1 - f_w\big|_{S_{wf}})}{\overline{\overline{S}}_w - S_{wf}} = \frac{1}{\overline{\overline{S}}_w - S_{we}} \qquad (10.26)$$

The significance of this result is illustrated in fig.10.13.

To satisfy equ. (10.26) the tangent to the fractional flow curve, from the point $S_w = S_{wc}$; $f_w = 0$, must have a point of tangency with co-ordinates $S_w = S_{wf}; f_w = f_w\big|_{S_{wf}}$, and the extrapolated tangent must intercept the line $f_w = 1$ at the point $S_w = \bar{\bar{S}}_w; f_w = 1$.

$$\left.\frac{df_w}{dS_w}\right|_{S_{wf}} = \frac{(1 - f_w\big|_{S_{wf}})}{\bar{\bar{S}}_w - S_{wf}} = \frac{1}{\bar{\bar{S}}_w - S_{we}}$$

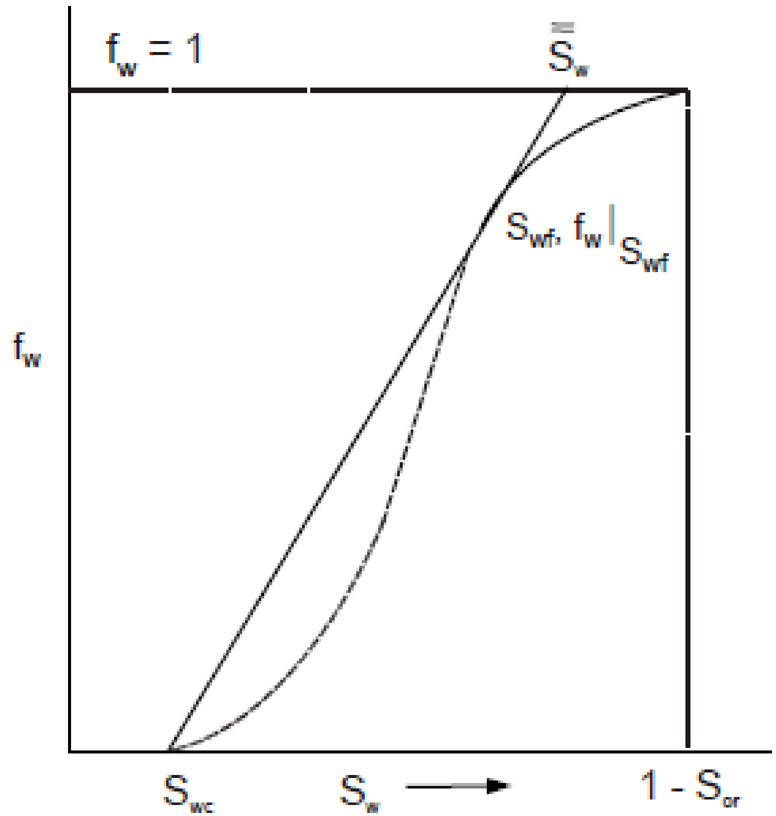


Fig. 10.13 Tangent to the fractional flow curve from $S_w = S_{wc}$

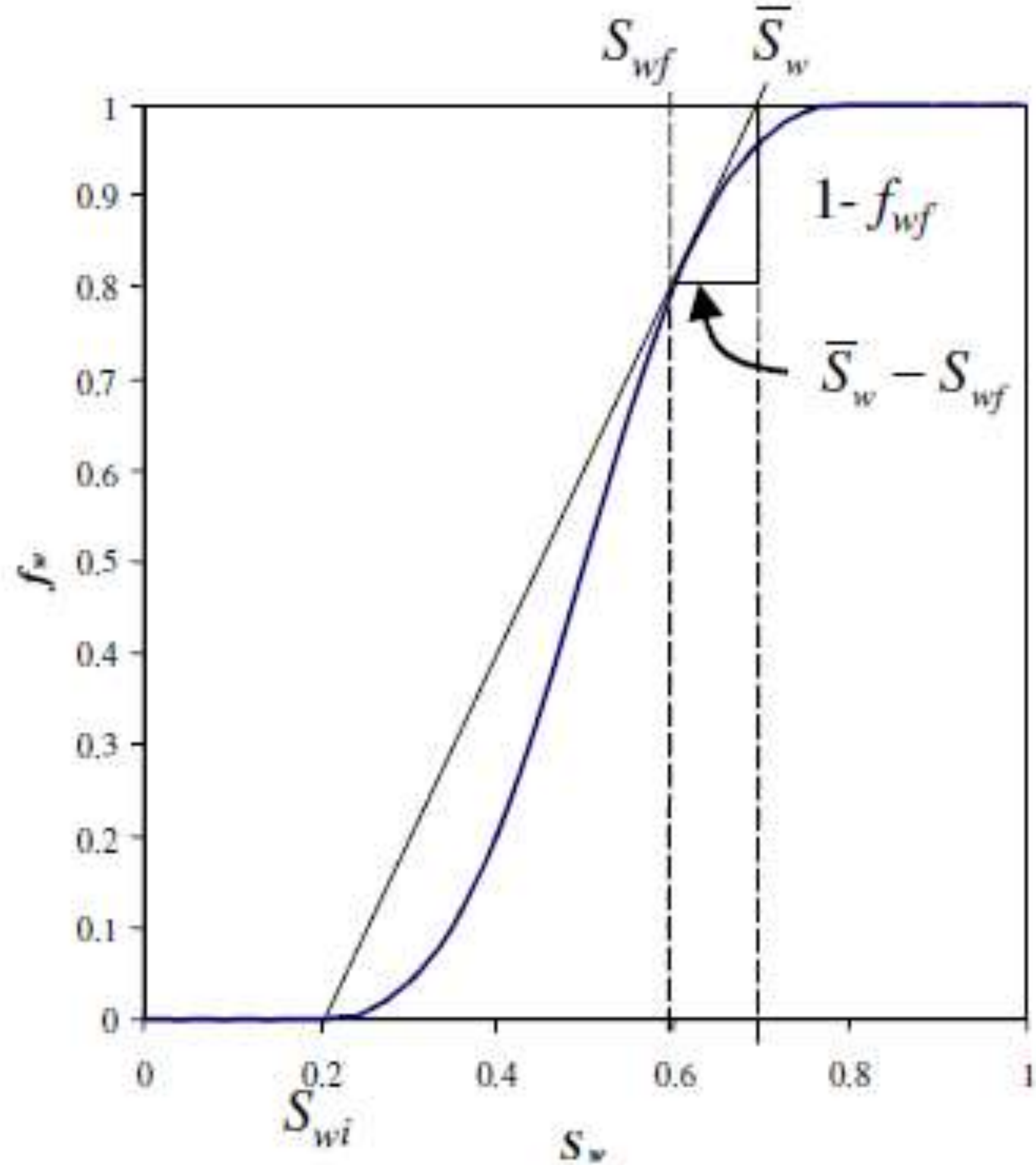
This method of determining S_{wf}, $f_w\big|_{S_{wf}}$ and $\bar{\bar{S}}_w$, requires that the fractional flow curve be plotted, using either equ. (10.12) or equ. (10.21), for the entire water saturation range

$$S_{wc} < S_w < 1 - S_{or}$$

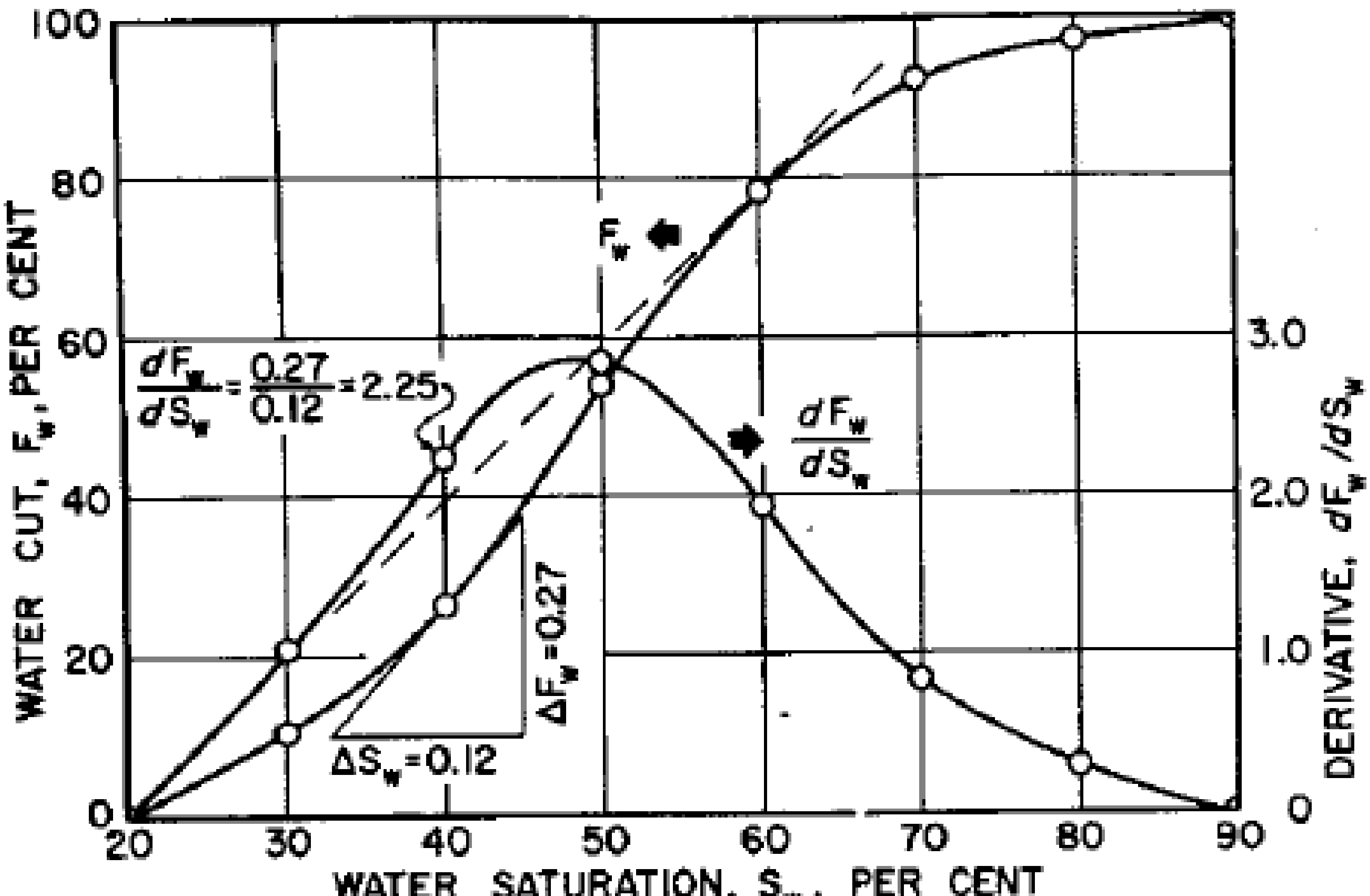
As noted previously, the use of either of these equations ignores the effect of the capillary pressure gradient, $\partial P_c/\partial x$. This neglect, however, is only admissible behind the flood front for

$$S_{wf} < S_w < 1 - S_{or}$$

The part of the fractional flow curve for saturations less than S_{wf} is, therefore, virtual and the first real point on the curve has the co-ordinates S_{wf}, $f_w\big|_{S_{wf}}$, corresponding to the shock front. This simple graphical technique of Welge has much wider application in the field of oil recovery calculations which will be described in the following section.



S_{wf}
$\overline{S}_w$
$1-f_{wf}$
$\overline{S}_w - S_{wf}$
S_{wi}
f_w
S_w
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1



100
80
60
40
20
0
20
30
40
50
60
70
80
90
3.0
2.0
1.0
0
WATER CUT, F_w, PER CENT
WATER SATURATION, S_w, PER CENT
DERIVATIVE, dF_w/dS_w
F_w
$\frac{dF_w}{dS_w}$
$\frac{dF_w}{dS_w} = \frac{0.27}{0.12} = 2.25$
$\Delta F_w = 0.27$
$\Delta S_w = 0.12$

Fig. 9.9.

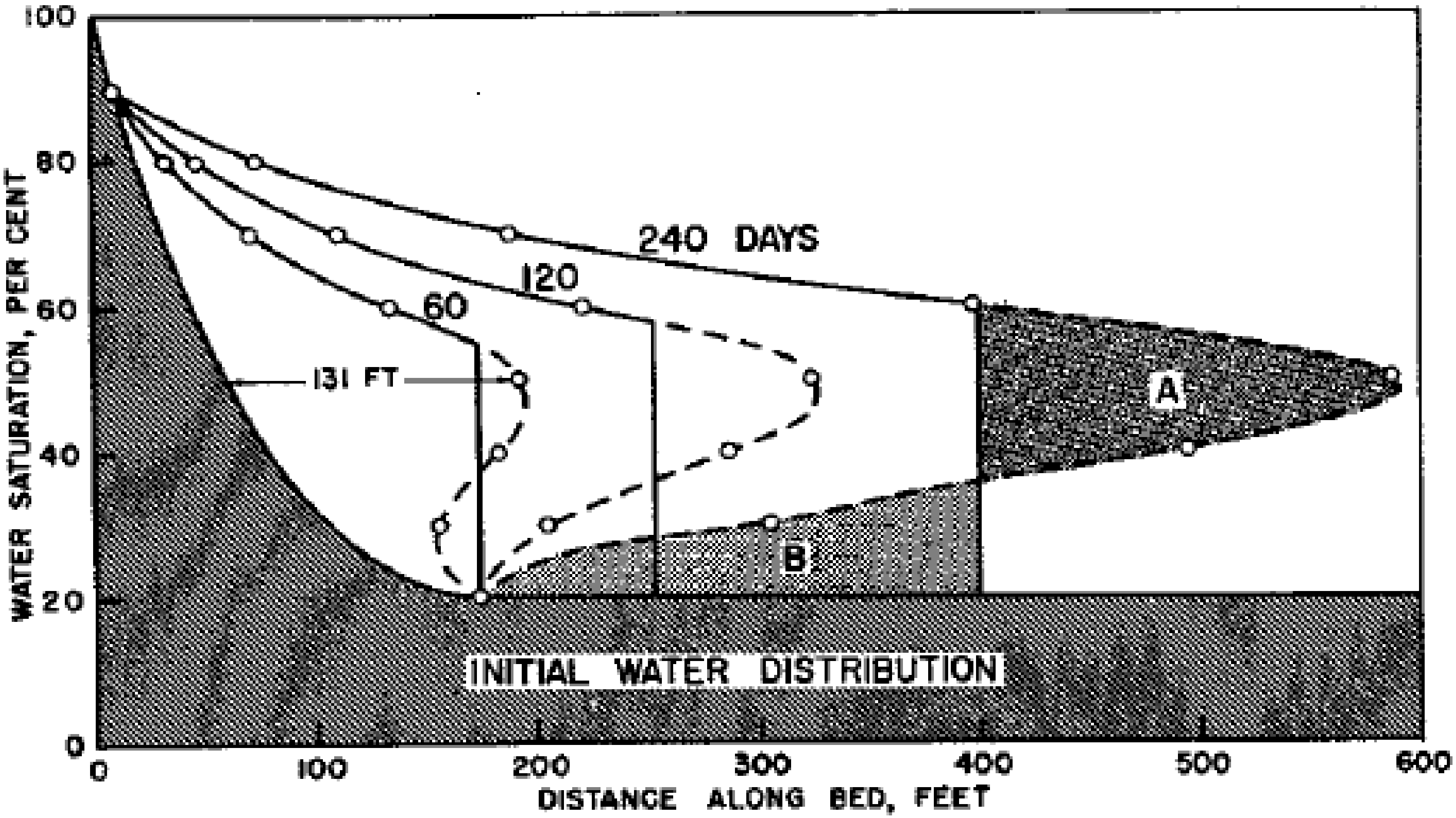
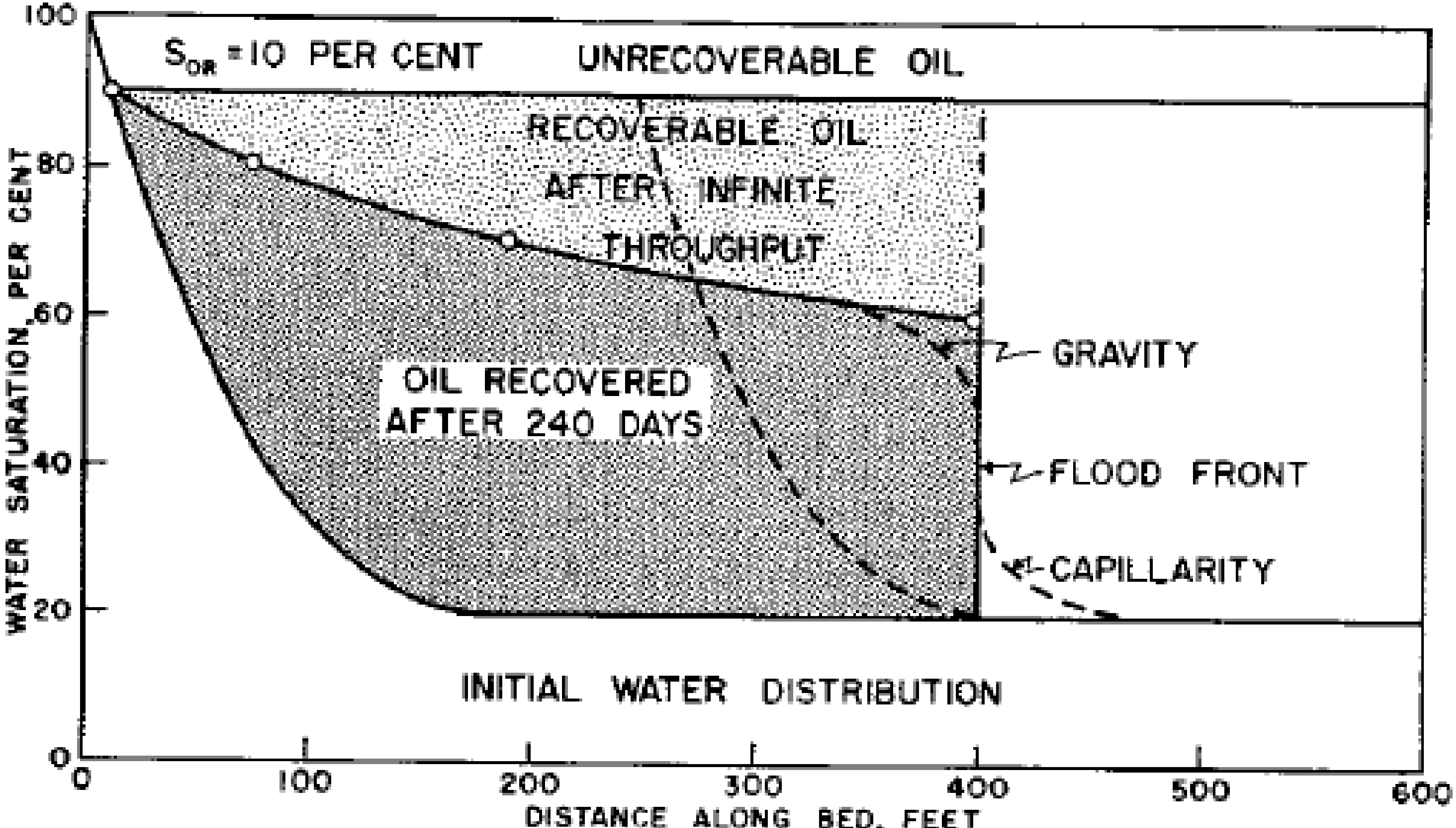


Fig. 9.10. Fluid distributions at initial conditions and at 60, 120, and 240 days.



S_{OR} = 10 PER CENT
UNRECOVERABLE OIL
RECOVERABLE OIL
AFTER INFINITE
THROUGHPUT
OIL RECOVERED
AFTER 240 DAYS
GRAVITY
FLOOD FRONT
CAPILLARITY
INITIAL WATER DISTRIBUTION
WATER SATURATION, PER CENT
100
80
60
40
20
0
0
100
200
300
400
500
600
DISTANCE ALONG BED, FEET

Fig. 9.11.

OIL RECOVERY CALCULATIONS

Before water breakthrough (bt) in the producing well, equ. (10.20) can be applied to determine the positions of planes of constant water saturation, for $S_{wf} < S_w < 1 - S_{or}$, as the flood moves through the reservoir, and hence the water saturation profile. At the time of breakthrough and subsequently, this equation is used in a different manner, to study the effect of increasing the water saturation at the producing well. In this case $x = L$, the length of the reservoir block, which is a constant, and equ. (10.20) can be expressed as

$$\frac{W_i}{LA\phi} = \frac{1}{\left.\frac{df_w}{dS_w}\right|_{S_{we}}} = W_{id} \qquad (10.27)$$

in which S_{we} is the current value of the water saturation in the producing well, fig. 10.14, and W_{id} the dimensionless number of pore volumes of water injected ($1PV = LA\phi$).

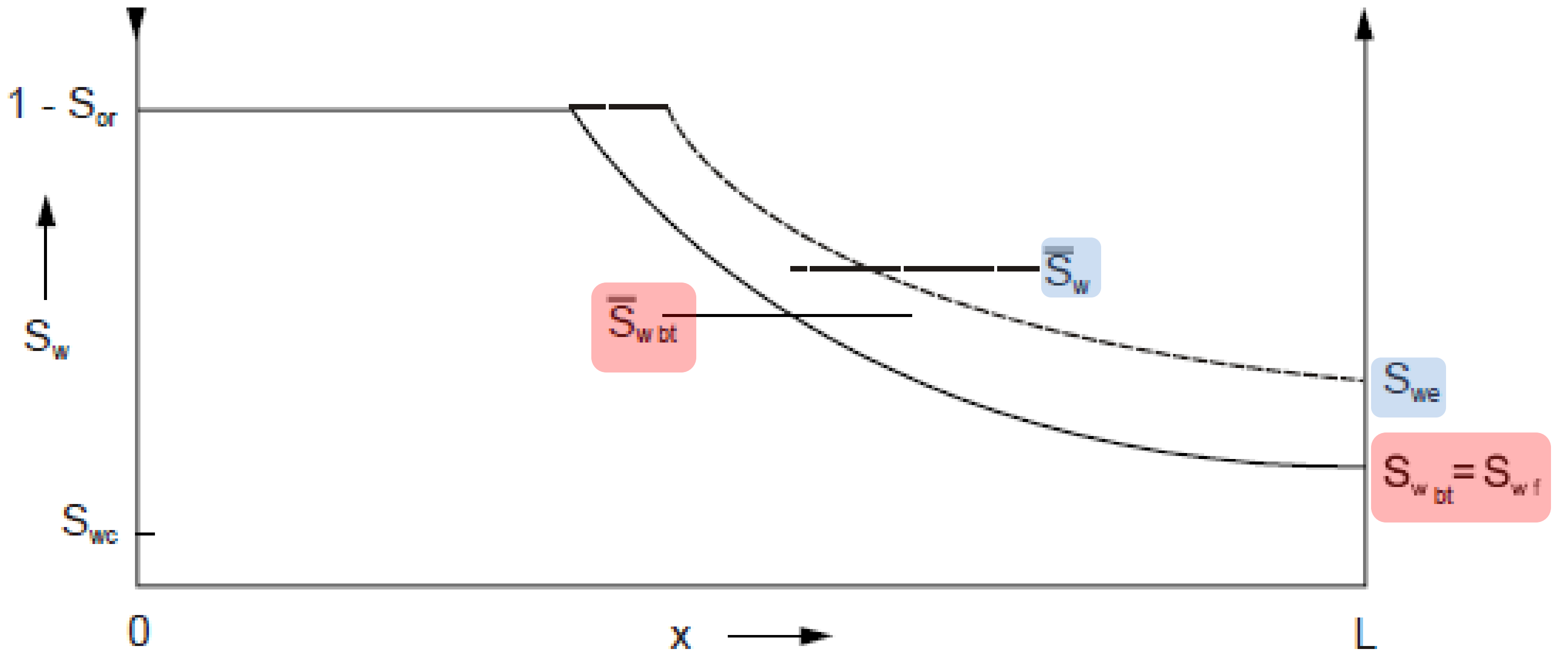


Fig. 10.14 Water saturation distributions at breakthrough and subsequently in a linear waterflood

Before breakthrough occurs the oil recovery calculations are trivial. For incompressible displacement the oil recovered is simply equal to the volume of water injected, there being no water production during this phase. At the time of breakthrough the flood front saturation, $S_{wf} = S_{w_{bt}}$, reaches the producing well and the reservoir watercut increases suddenly from zero to $f_{w_{bt}} = f_w\big|_{S_{wf}}$ a phenomenon frequently observed in the field and one which confirms the existence of a shock front. At this time equ. (10.22) can be interpreted in terms of equ. (10.27) to give

$$N_{pd_{bt}} = W_{id_{bt}} = q_{id} t_{bt} = \left(\overline{\overline{S}}_{w_{bt}} - S_{wc}\right) = \frac{1}{\left.\dfrac{df_w}{dS_w}\right|_{S_{w_{bt}}}} \qquad (10.28)$$

in which all volumes are expressed, for convenience, as dimensionless pore volumes.

q_{id} the dimensionless injection rate is $q_i/(LA\phi)$ (PV/unit of time)

$$\overline{\overline{S}}_w - S_{wc} = \frac{W_i}{x_2 A\phi} = \frac{1}{\left.\dfrac{df_w}{dS_w}\right|_{S_{wf}}} \qquad (10.22)$$

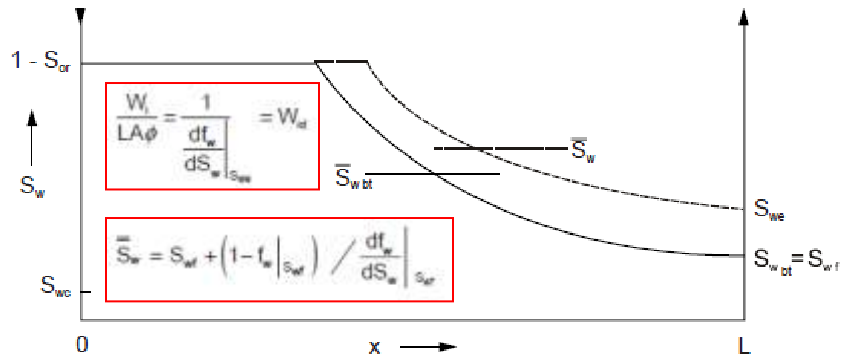
$$\frac{W_i}{LA\phi} = \frac{1}{\left.\dfrac{df_w}{dS_w}\right|_{S_{we}}} = W_{id} \qquad (10.27)$$

In particular, the dimensionless injection rate is $q_i/(LA\phi)$ (PV/unit of time) which facilitates the calculation of the time at which breakthrough occurs as

$$t_{bt} = \frac{W_{id_{bt}}}{q_{id}} \tag{10.29}$$

After breakthrough, L remains constant in equ. (10.27) and S_{we} and f_{we}, the water saturation and fractional flow at the producing well, gradually increase as the flood moves through the reservoir, as shown in fig. 10.14. During this phase the calculation of the oil recovery is somewhat more complex and requires application of the Welge equation, (10.25), as

$$\bar{\bar{S}}_w = S_{we} + (1 - f_{we}) \frac{1}{\left.\dfrac{df_w}{dS_w}\right|_{S_{we}}} \tag{10.30}$$



which, using equ. (10.27), can also be expressed as

$$\bar{\bar{S}}_w = S_{we} + (1 - f_{we}) W_{id} \tag{10.31}$$

Finally, subtracting S_{wc} from both sides of equ. (10.31) gives the oil recovery equation

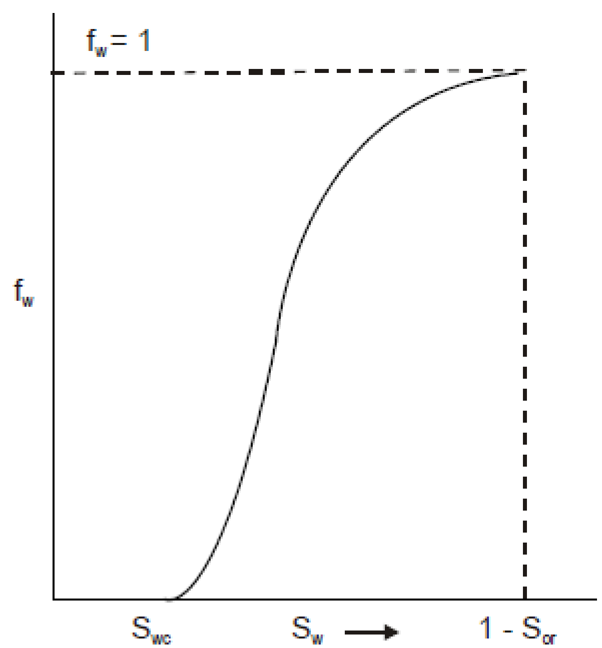
$$N_{pd} = \bar{\bar{S}}_w - S_{wc} = (S_{we} - S_{wc}) + (1 - f_{we})\, W_{id}(PV) \tag{10.32}$$

The manner in which equs. (10.28) and (10.32) can be used in practice is described below.

a) Draw the fractional flow curve, equ. (10.12) or (10.21), allowing for gravity effects, if necessary, but neglecting the capillary pressure gradient $\partial P_c/\partial x$.

$$f_w = \frac{1}{1+\frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \tag{10.12}$$

$$f_w = \frac{1-\frac{kk_{ro}A}{q_t\mu_o}\,\frac{\Delta\rho\ g\ \sin\theta}{1.0133\times10^6}}{1+\frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \tag{10.21}$$

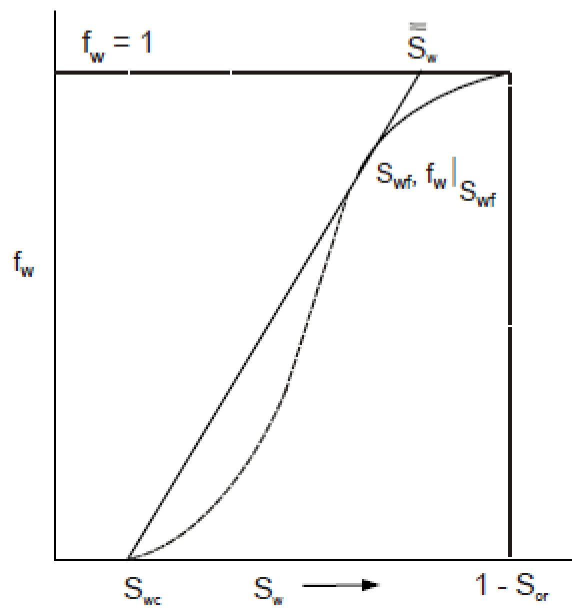


b) Draw the tangent to this curve from the point $S_w = S_{wc}$, $f_w = 0$. As described in the previous section, the point of tangency has the co-ordinates $S_w = S_{wf} = S_{w_{bt}}$, $f_w = f_w\big|_{S_{wf}} = f_{w_{bt}}$ and the extrapolation of this line to $f_w = 1$ gives the value of the average saturation behind the front at breakthrough $\overline{\overline{S}}_w = \overline{\overline{S}}_{w_{bt}}$.

Equations (10.28) and (10.29) can then be applied to calculate the oil recovery and time at which breakthrough occurs.

$$N_{pd_{bt}} = W_{id_{bt}} = q_{id} t_{bt} = \left(\overline{\overline{S}}_{w_{bt}} - S_{wc}\right) = \frac{1}{\left.\frac{df_w}{dS_w}\right|_{S_{w_{bt}}}} \qquad (10.28)$$

$$t_{bt} = \frac{W_{id_{bt}}}{q_{id}} \qquad (10.29)$$



c) Choosing S_{we} as the independent variable; allow its value to increase in increments of, say, 5% above the saturation at breakthrough. Each point on the fractional flow curve, for $S_{we} > S_{w_{bt}}$, has co-ordinates $S_w = S_{we}$, $f_w = f_{we}$ and, applying equ. (10.30), fig. 10.15 demonstrates that the tangent to fractional flow curve intersects the line $f_w = 1$ to give the current value of the average water saturation in the reservoir block, $\overline{\overline{S}}_w$.

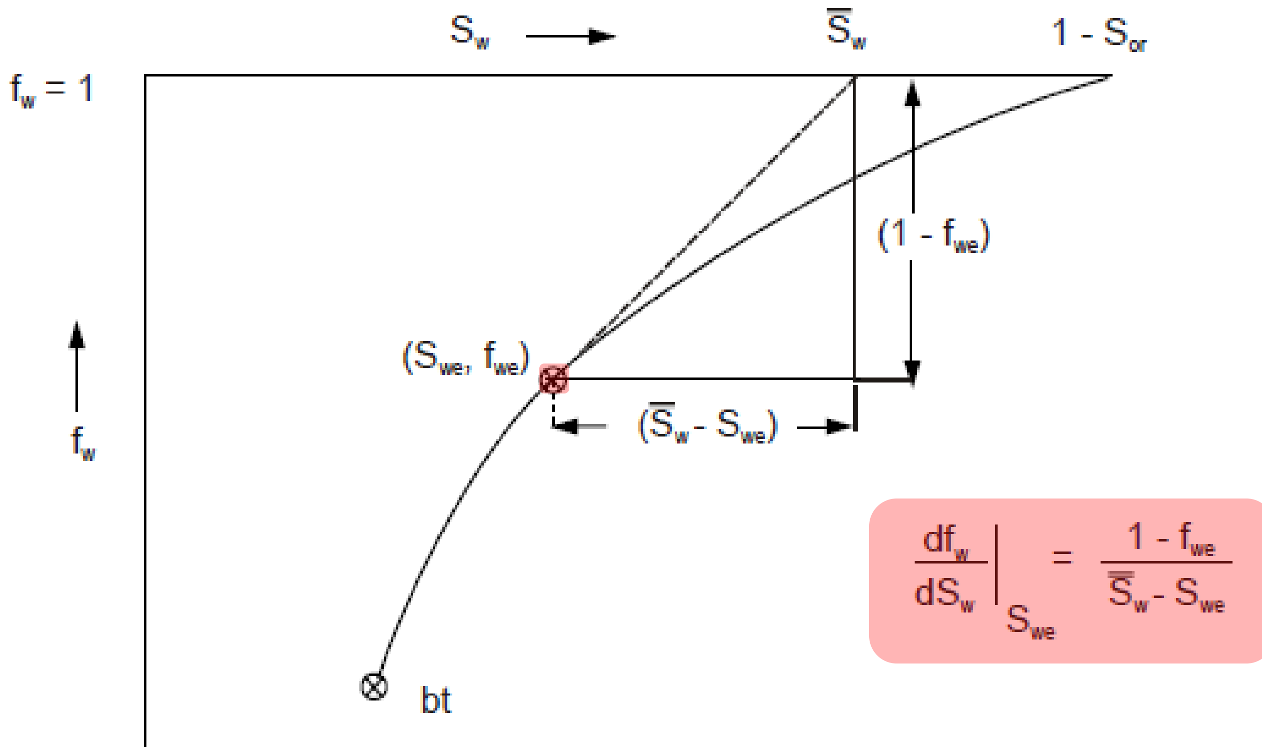


Fig. 10.15 Application of the Welge graphical technique to determine the oil recovery after water breakthrough

For each new value of S_{we} the corresponding value of $\bar{\bar{S}}_w$ is determined graphically and the oil recovery calculated as

$$N_{pd} = \bar{\bar{S}}_w - S_{wc} \qquad \text{(PV)}$$

The reciprocal of the slope of the fractional flow curve, for each value of S_{we}, gives W_{id}, the number of pore volumes of water injected, equ. (10.27). This allows a time scale to be attached to the recovery since

$$W_{id} = q_{id}\, t$$

Alternatively, equ. (10.32) can be used directly to calculate the oil recovery by determining f_{we} and W_{id} from the fractional flow curve for each chosen value of S_{we}. This latter method is illustrated in exercise (10.2) in which N_{pd} and W_{id} are evaluated numerically.

$$\frac{W_i}{LA\phi} = \frac{1}{\left.\frac{df_w}{dS_w}\right|_{S_{we}}} = W_{id} \qquad (10.27)$$

$$N_{pd} = \bar{\bar{S}}_w - S_{wc} = (S_{we} - S_{wc}) + (1 - f_{we})\, W_{id} \text{(PV)} \qquad (10.32)$$

EXERCISE 10.1 FRACTIONAL FLOW

Oil is being displaced by water in a horizontal, direct line drive under the diffuse flow condition. The rock relative permeability functions for water and oil are listed in table 10.1.

S_w	k_{rw}	k_{ro}	S_w	k_{rw}	k_{ro}
.20	0	.800	.50	.075	.163
.25	.002	.610	.55	.100	.120
.30	.009	.470	.60	.132	.081
.35	.020	.370	.65	.170	.050
.40	.033	.285	.70	.208	.027
.45	.051	.220	.75	.251	.010
			.80	.300	0

TABLE 10.1

Pressure is being maintained at its initial value for which

B_o = 1.3 rb/stb and B_w = 1.0 rb/stb

Compare the values of the producing watercut (at surface conditions) and the cumulative oil recovery at breakthrough for the following fluid combinations.

Case	oil viscosity	water viscosity
1	50 cp	.5 cp
2	5 "	.5 "
3	.4 "	1.0 "

Assume that the relative permeability and PVT data are relevant for all three cases.

EXERCISE 10.1 SOLUTION

1) For horizontal flow the fractional flow in the reservoir is

$$f_w = \frac{1}{1+\frac{\mu_w}{k_{rw}} \cdot \frac{k_{ro}}{\mu_o}} \qquad (10.12)$$

while the producing watercut at the surface, f_{ws}, is

$$f_{ws} = \frac{q_w / B_w}{q_w / B_w + q_o / B_o}$$

where the rates are expressed in rb/d. Combining the above two equations leads to an expression for the surface watercut as

$$f_{ws} = \frac{1}{1+\frac{B_w}{B_o}\left(\frac{1}{f_w}-1\right)} \qquad (10.33)$$

				Fractional Flow (f_w)		
				Case 1	Case 2	Case3
S_w	k_{rw}	k_{ro}	k_{ro}/k_{rw}	μ_w/μ_o = .01	μ_w/μ_o = .1	μ_w/μ_o = 2.5
.2	0	.800	∞	0	0	0
.25	.002	.610	305.000	.247	.032	.001
.30	.009	.470	52.222	.657	.161	.008
.35	.020	.370	18.500	.844	.351	.021
.40	.033	.285	8.636	.921	.537	.044
.45	.051	.220	4.314	.959	.699	.085
.50	.075	.163	2.173	.979	.821	.155
.55	.100	.120	1.200	.988	.893	.250
.60	.132	.081	.614	.994	.942	.394
.65	.170	.050	.294	.997	.971	.576
.70	.208	.027	.130	.999	.987	.755
.75	.251	.010	.040	.999	.996	.909
.80	.300	0	0	1.000	1.000	1.000

TABLE 10.2

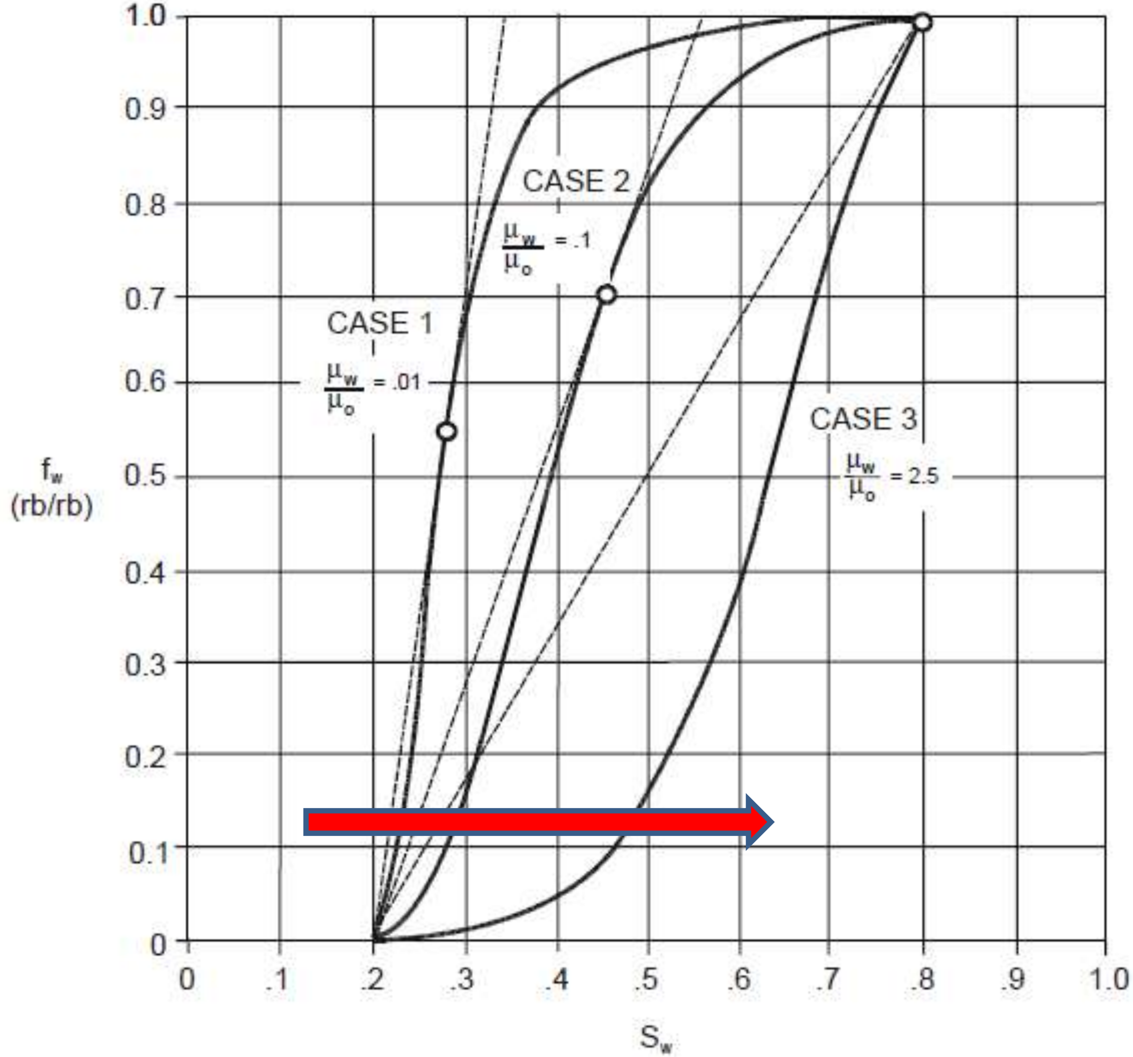


Fig. 10.16 Fractional flow plots for different oil-water viscosity ratios (table 10.2)

Fractional flow plots for the three cases are shown in fig. 10.16, and the results obtained by applying Welge's graphical technique, at breakthrough, are listed in table 10.3.

Case	S_{wbt}	f_{wbt} (reservoir)	$f_{ws_{bt}}$ (surface)	$\overline{\overline{S}}_{wbt}$	$N_{pd_{bt}}$ (PV)
1	.28	.55	.61	.34	.14
2	.45	.70	.75	.55	.35
3	.80	1.00	1.00	.80	.60

TABLE 10.3

Influence of the Capillary Force

The *Buckley-Leverett* solution neglecting the capillary force is illustrated in Figure 4.7 which has been discussed previously. The other profiles were calculated by applying Eq. 4.30 for various rates of filtration with the help of the method of finite differences.

In case of a slow displacement the capillary force is larger than the viscous forces. This is expressed in a rather flat saturation profile. In the case of a fast displacement the profile becomes steeper and tends to the *Buckley-Leverett* solution. It can be observed that when the displacing phase reaches the end of the medium the displacing efficiency is larger at a fast displacement than at a slow displacement.

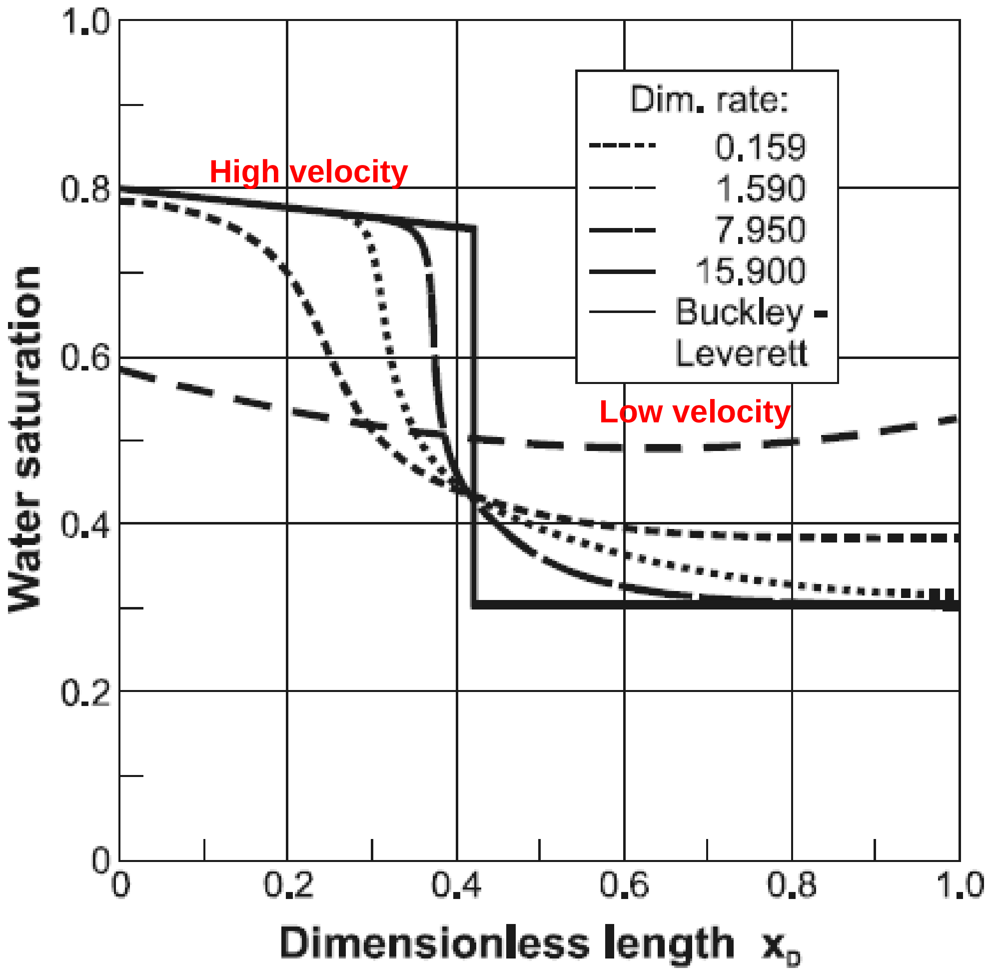


Figure 4.7: Influence of the velocity of displacement on the distribution of saturation regarding the capillary force (by *Douglas et al* 1958)

Figure 4.8 shows the oil recovery versus the rate factor for different core lengths and for a strong water-wet system. Two things are of importance: First the efficiency of displacement is at certain values independent of velocity. Second the time period between the arrival and breakthrough of the displacing phase at a small displacing speed is large.

The point of breakthrough is defined as the moment of first outflow of the displacing phase. The deviation is effected by the capillary end-effect.

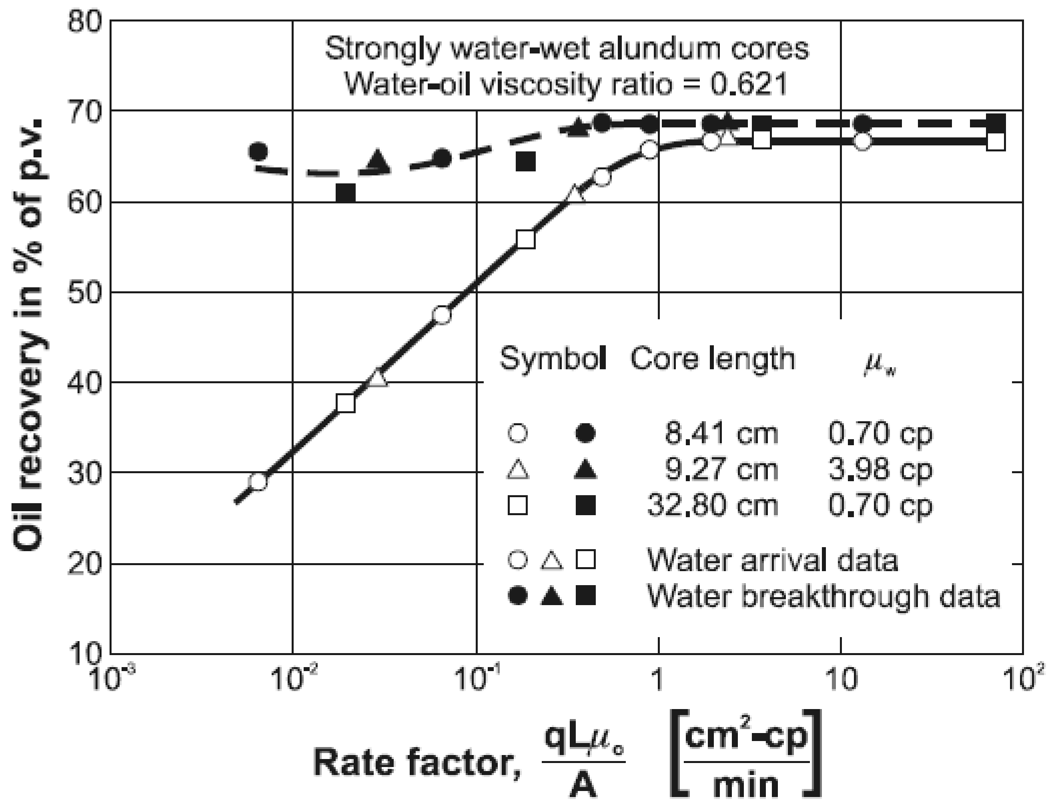
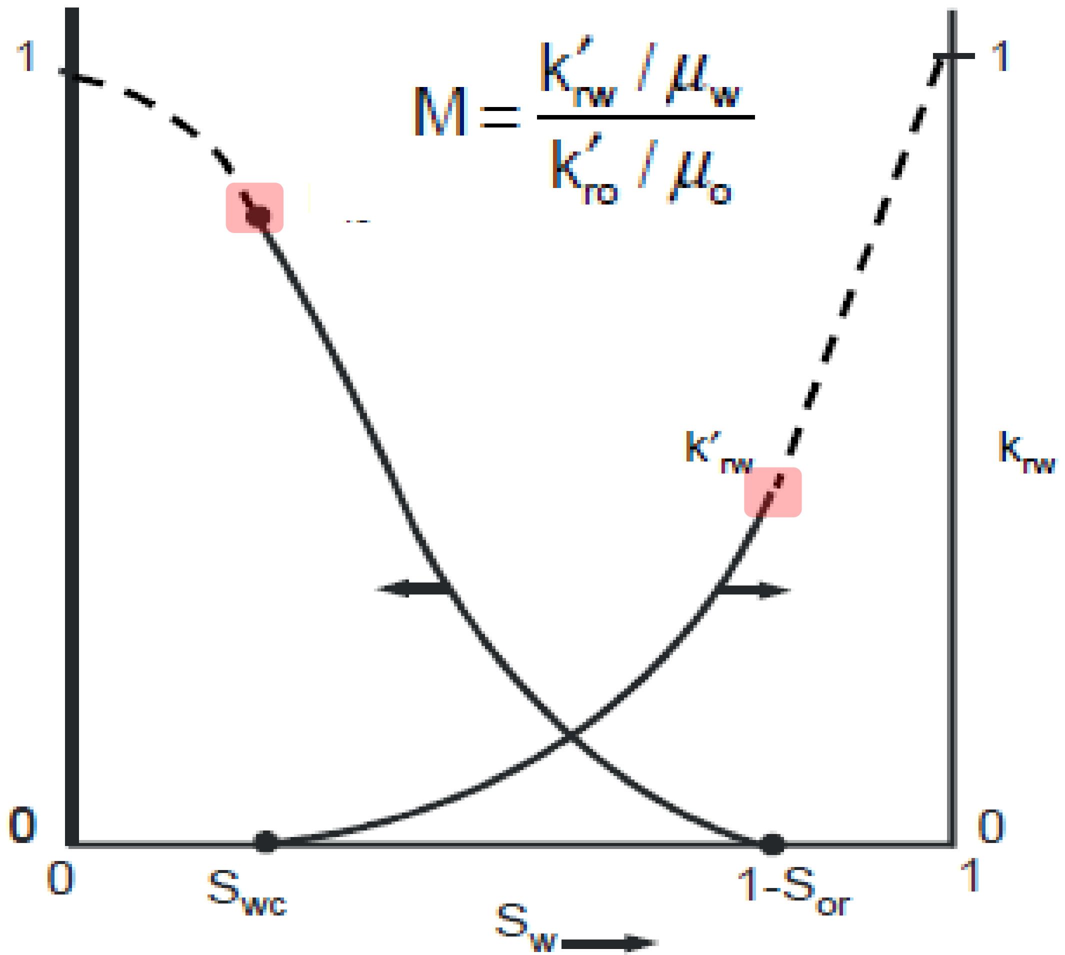


Figure 4.8: The displacing efficiency as a function of velocity (by *Kyte, Rappoport* 1958)

Effect of End-Point Mobility Ratio

An important parameter in determining the effectiveness of a waterflood is the end point mobility ratio.



•For horizontal flow, stable, piston-like displacement will occur for $M \leq 1$

•The more significant parameter for characterizing the stability of Buckley Leverett displacement is the shock front mobility ratio, M_s,

$$M_s = \frac{k_{ro}(S_{wf})/\mu_o + k_{rw}(S_{wf})/\mu_w}{k'_{ro}/\mu_o} \quad (10.34)$$

•If this condition is not satisfied there will be severe viscous channelling of water through the oil and breakthrough will occur even earlier than predicted using the Welge technique.

Values of M and M_s for the three cases defined in exercise 10.1 are listed in table 10.3(a). Using these data

Case No. (exercise 10.1)	$\frac{\mu_o}{\mu_w}$	S_{wf}	$k_{rw}(S_{wf})$	$k_{ro}(S_{wf})$	M_s	M
1	100	.28	.006	.520	1.40	37.50
2	10	.45	.051	.220	.91	3.75
3	.4	.80	.300	0	.15	0.15

TABLE 10.3(a)

Values of the shock front and end point relative permeabilities calculated using the data of exercise 10.1

the results of exercise 10.1 can be analysed as follows:

a) Case 1 - this displacement is unstable due to the very high value of the oil/water viscosity ratio. This results in the by-passing of oil and consequently the premature breakthrough of water. The oil recovery at breakthrough is very small and a great many pore volumes of water will have to be injected to recover all the movable oil. Under these circumstances oil recovery by water injection is hardly feasible and consideration should be given to the application of thermal recovery methods with the aim of reducing the viscosity ratio.

b) Case 2 - the oil/water viscosity ratio is an order of magnitude lower than in case 1 which leads to a stable and much more favourable type of displacement ($M_s < 1$). This case will be analysed in greater detail in exercise 10.2, in which the oil recovery after breakthrough is determined as a function of the cumulative water injected and time.

c) Case 3 - for the displacement of this very low viscosity oil (μ_o = .4 cp) both the end point and shock front mobility ratios are less than unity and piston-like displacement occurs. The tangent to the fractional flow curve, from $S_w = S_{wc}$, $f_w = 0$, meets the curve at the point $S_{w_{bt}} = 1 - S_{or}$, $f_{w_{bt}} = 1$ and therefore $S_{w_{bt}} = \overline{S}_{w_{bt}} = 1 - S_{or}$. The total oil recovery at breakthrough is $\overline{\overline{S}}_{w_{bt}} = S_{wc} = 1 - S_{or} - S_{wc}$, which is the total movable oil volume.

DISPLACEMENT UNDER SEGREGATED FLOW CONDITIONS

Previously, a one dimensional displacement theory was presented which relied on the assumption of diffuse flow. Now, precisely the opposite will be assumed, namely, that displacement occurs under the segregated flow condition shown in fig.10.18.

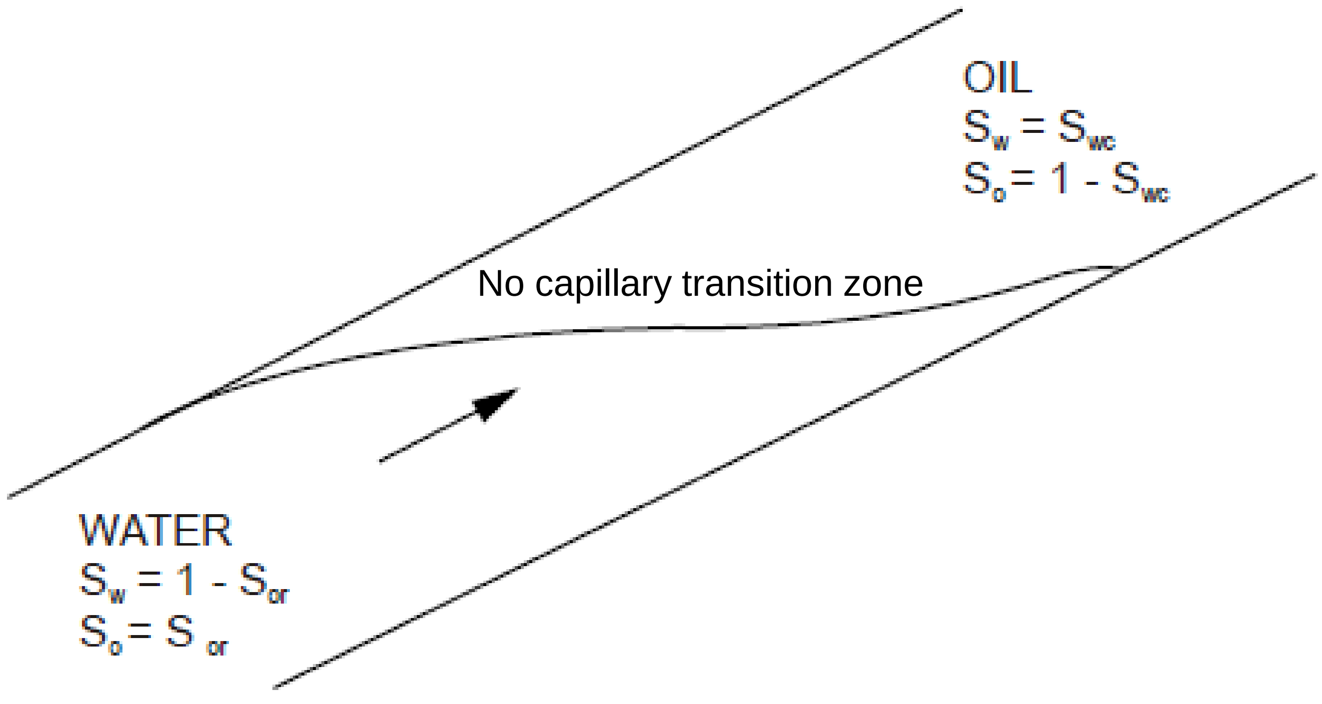


Fig. 10.18 Displacement of oil by water under segregated flow conditions

$$G = \frac{kk'_{rw} \; A \; \Delta\rho \; g \; \sin\theta}{1.0133\times10^{6} \; q_t \mu_w}$$

$$M - 1 = G\left(\frac{dy}{dx}\;\frac{1}{\tan\theta} + 1\right)$$

$$\frac{dy}{dx} = -\tan\beta = \left(\frac{M-1-G}{G}\right)\;\tan\theta$$

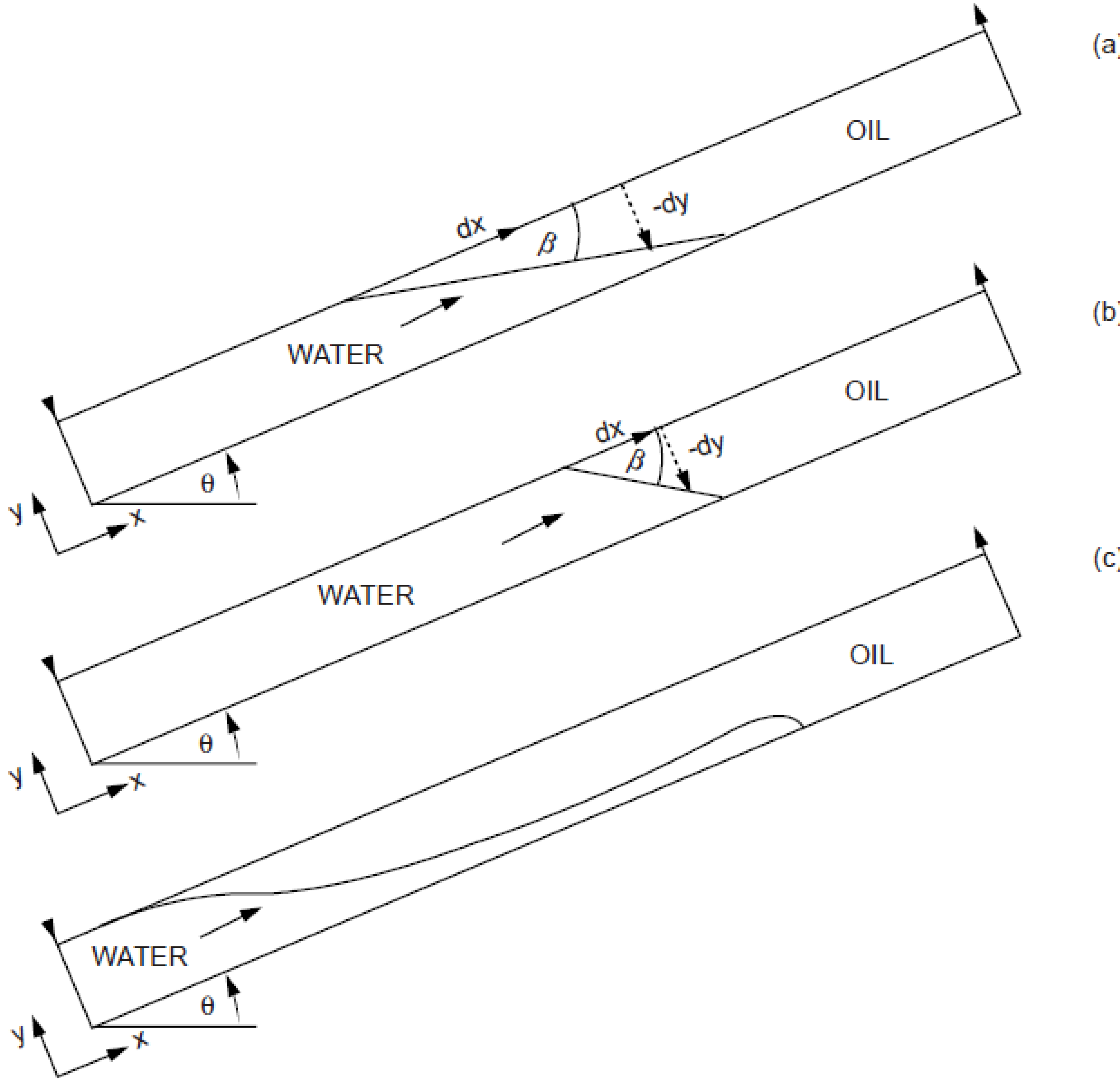


Fig. 10.19 Illustrating the difference between stable and unstable displacement, under segregated flow conditions, in a dipping reservoir; (a) stable: G > M–1; M > 1; *β* < *θ*. (b) stable: G > M–1; M < 1; *β* > *θ*. (c) unstable: G < M–1.

Assignment No. 3.a:

Determination of relative permeability from **unsteady-state** experiments

1. JBN Method
2. Jones and Roszelle Method

Your report should include both of the theory (derivation) and experimental procedure