Flow in Porous Media

Module 3.b
Fundamental of Two Phase Flow in Porous Media
Imbibition process and Modeling

Determination of Effective Capillary Pressures for Porous Media from Imbibition Data
SPE 1361

Capillary forces: f(properties of water-oil-solid surfaces)

DERIVATION OF DIFFUSION-TYPE EQUATION

The soil scientists have derived an equation to describe the movement of water into dry soils. The basic assumptions in the derivation are (1) both the water and air are continuous phases behind the imbibing water front, (2) the pressure gradient in the gas phase is negligible both ahead and behind the imbibing water front, and (3) the capillary pressure gradient over any increment of length provides the driving force for overcoming viscous forces in that same incremental length.

With these assumptions one can combine Darcy's equation, the capillary pressure equation and the equation of continuity to obtain the following equation.

$$\phi \frac{\partial S_w}{\partial t} = - \frac{\partial}{\partial x}\left[\left(\frac{k_w}{\mu_w}\frac{\partial P_c}{\partial S_w}\right)\frac{\partial S_w}{\partial x}\right] \quad . \quad . \quad . \quad (1)$$

where

ϕ = fractional porosity,
S_w = fractional water content of the pore spaces,
k_w = effective water permeability in darcies,
μ_w = water viscosity in centipoises,
P_c = capillary pressure,
t = time in seconds, and
x = distance in centimeters

The similarity to the diffusion equation is noted if a saturation-dependent diffusion term, D, is substituted for $\frac{k_w}{\mu_w}\frac{\partial P_c}{\partial S_w}$.

$$\phi \frac{\partial S_w}{\partial t} = -\frac{\partial}{\partial x}\left(D \frac{\partial S_w}{\partial x}\right) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

DERIVATION OF EQUATION FOR PISTON-LIKE DISPLACEMENT

Many of the assumptions in the development of the diffusion-type equation can be questioned. First, most of the gas is probably trapped as a discontinuous phase behind the imbibing water front in an unrestricted imbibition. Second, if the gas phase is discontinuous, the pressure gradient in the gas phase is a meaningless quantity. Almost certainly, however, gas trapped at the upstream end will be at a pressure greater than that trapped farther downstream; furthermore, the pressure in any particular isolated island of gas will increase as imbibition continues. Gas trapped at ambient pressure will end up at a pressure greater than the ambient by an amount equal to the capillary pressure. Third, for a region in which the gas is immobile, the capillary pressure gradient cannot provide the pressure gradient for flow.

With different assumptions, an alternative equation can be derived which leads to the same dependence of volume imbibed on the square root of time as that predicted by the diffusion equation. However, the proportionality constant is more easily interpreted in terms of capillary pressure and, hence, in terms of the wettability of the rock surfaces.

In many respects, the rate of imbibition in porous media is analogous to the rate of capillary rise in capillaries. From this analogy, the assumptions in the derivation of the alternative equation are (1) the water imbibes in a piston-like manner and (2) the pressure gradient in the gas phase ahead of the water front can be neglected. If imbibition occurs vertically upward, the flow equation is

$$v_w = \frac{k_w}{\mu_w}\left(\frac{P_c}{x} - \Delta\rho g\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where v_w = flow rate ($cm^3/cm^2/sec$),
$\Delta\rho$ = density difference for water and air,
g = acceleration due to gravity, and
x = position of front.

In Eq. 3, P_c is a constant.

For a piston displacement,

$$v_w = \phi S_w \frac{\partial x}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Substituting Eq. 4 in Eq. 3, one obtains

$$\frac{dx}{dt} = \frac{k_w}{\phi \mu_w S_w}\left(\frac{P_c}{x} - \Delta\rho g\right). \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The integration of Eq. 5 gives a result analogous to that for rate of capillary rise in a capillary tube.

$$x + \frac{P_c}{\Delta\rho g} \ln\left(1 - \frac{\Delta\rho g x}{P_c}\right) = -\frac{k_w \Delta\rho g}{\phi S_w \mu_w} t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

For $\frac{\Delta\rho g x}{P_c} \ll 1$ (i.e.. when the gravity forces are much less than capillary forces), Eq. 10 reduces to

$$x^2 = \left(\frac{2P_c k_w}{\phi S_w \mu_w}\right) t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since $x = \frac{Q_w}{\phi A S_w}$ when Q_w equals total volume of water imbibed,

$$Q_w^2 = \left(\frac{2P_c k_w \phi A^2 S_w}{\mu_w}\right) t \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where A = cross-sectional area of sample.

$$Q_w^2 = \left(\frac{2 P_c k_w \phi A^2 S_w}{\mu_w} \right) t$$

Thus, $P_c k_w$ can be obtained from the slope of the plot of the square of the volume of imbibed water vs time. Since the effective water permeability, k_w, can be measured independently, an effective capillary pressure can be derived from an imbibition experiment. This effective capillary pressure is a measure of the wettability properties of the rock surfaces.

Two important differences can be noted between the diffusion equation and Eq. 8. First, the diffusion equation (at least when steady-state permeability and capillary-pressure data are used) predicts that the small capillaries fill first and the larger capillaries later. In piston-like displacement, all capillaries fill at the same time leaving a residual gas saturation behind. Second, the diffusion equation is based on the assumption that, over any increment of length in the direction of flow, the potential generated by capillary forces is dissipated in viscous flow within that same incremental length. In the derivation of the piston displacement equation, the capillary forces at the front were assumed to provide the driving force to overcome viscous flow throughout the porous medium in which water is flowing.

In capillary tubes, imbibition proceeds more rapidly in larger capillaries. It is reasonable to expect similar behavior in porous media. If this is correct, the larger capillaries will imbibe first, and the water will move through them into the smaller capillaries. Also in capillary tubes, the capillary pressure is the pressure difference across the interface. That pressure difference furnishes the driving force for viscous flow throughout the water-filled tube. For porous media, the capillary-tube analogy is good if only a residual gas saturation is left behind the imbibing front.

$$\phi \frac{\partial S_w}{\partial t} = -\frac{\partial}{\partial x}\left[\left(\frac{k_w}{\mu_w}\frac{\partial P_c}{\partial S_w}\right)\frac{\partial S_w}{\partial x}\right] \quad . \quad . \quad . \quad (1)$$

$$Q_w^2 = \left(\frac{2P_c k_w \phi A^2 S_w}{\mu_w}\right) t \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Apparatus to Measure the Volume of Water Imbibed as a Function of Time

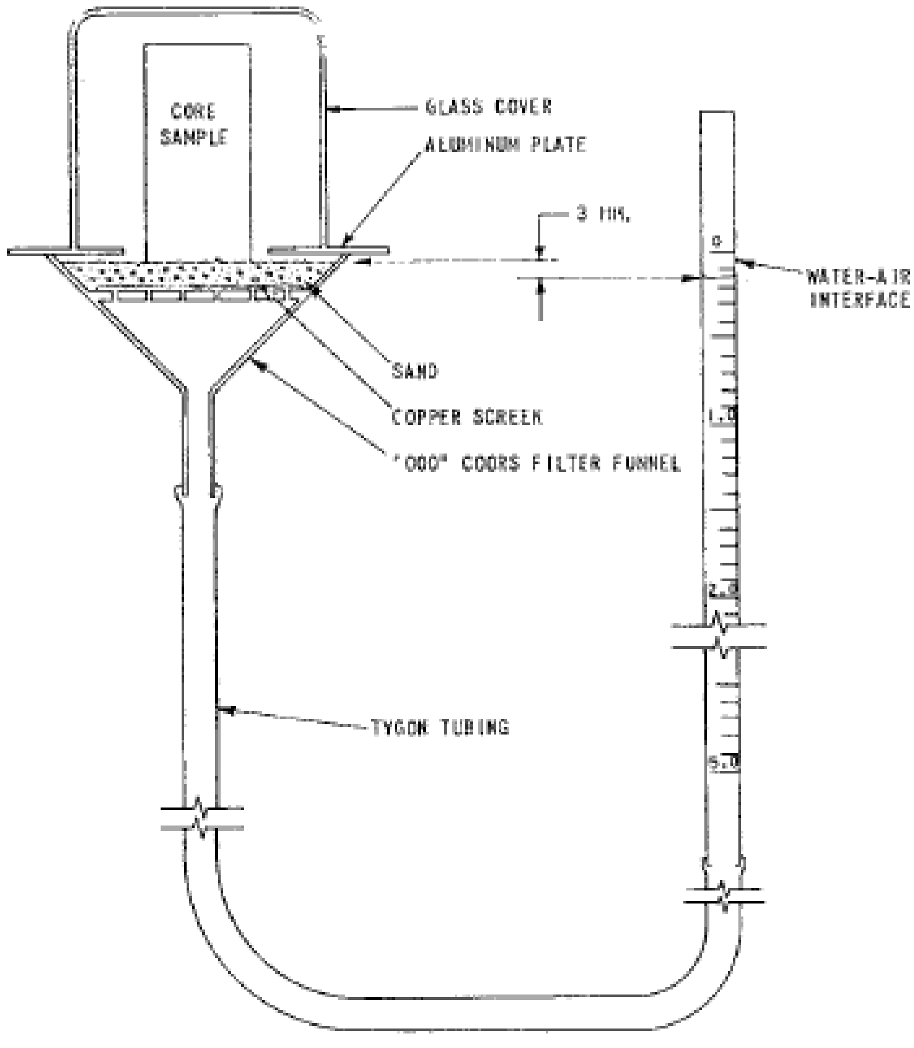

Fig. 1—Schematic Diagram of Water Imbibition Apparatus.

TABLE 1 — SUMMARY OF RESULTS OF IMBIBITION EXPERIMENTS FOR FIRED CORES

	Core Number			
	1	2	3	4
Type*	ss	ss	ss	ls
Permeability (darcies)	.528	1.224	1.100	.318
Porosity	.279	.297	.299	.272
Cross-sectional Area (cm^2)	4.85	4.85	4.85	4.90
Water Saturation (PV)**	.665	.691	.675	.719
Water Permeability (darcies)	.064	.15	.14	.028
Capillary Pressure (atm)	.070	.050	.060	.074

*ss = sandstone
ls = limestone
**PV = Pore Volume

1. An equation has been derived for predicting water-imbibition behavior in porous media. The principal assumption is that water displaces air in a piston-like manner. Although the resulting equation is comparatively simple, the experimental observations are more in agreement with this equation than with more elegant equations which assume phase continuity behind the front.

2. From the slope of a plot of Q^2 vs t, one can compute an effective capillary pressure which, in turn, can be used as a measure of rock wettability in the water-air-solid system.

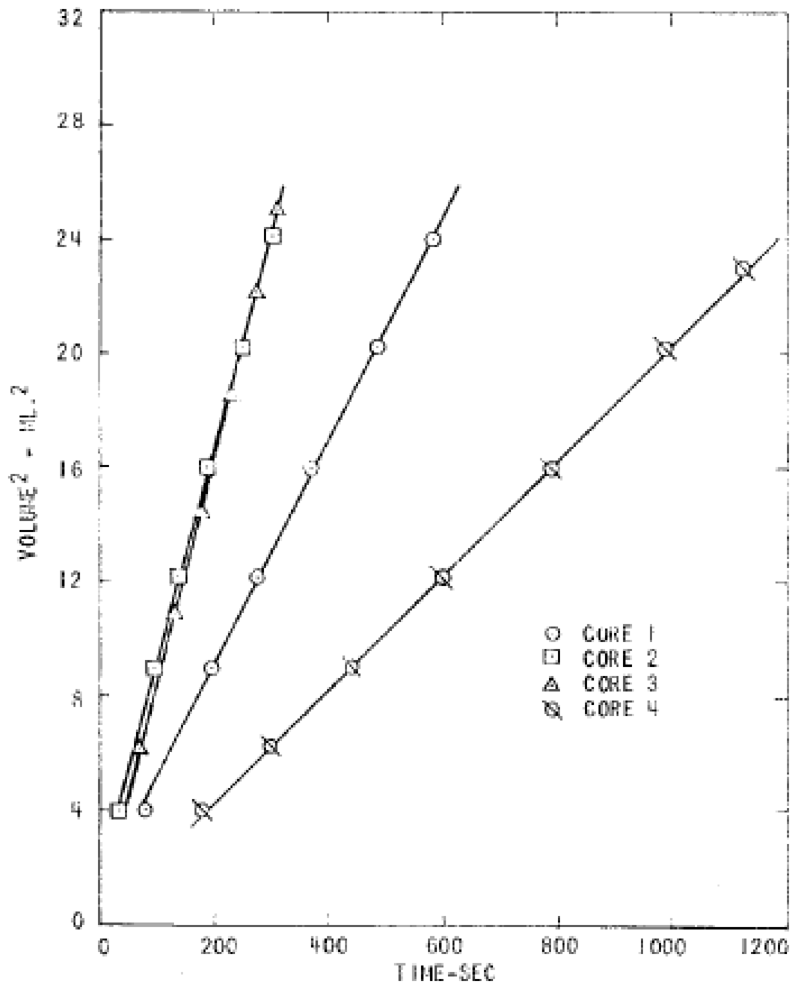

FIG. 2—IMBIBITION RATE AS A FUNCTION OF TIME FOR FIRED CORES.

Imbibition in Naturally Fractured Reservoir

•The basic characteristic of fractured reservoirs is that they are composed of two media, fractures and matrix blocks with the following characteristics:

1. Fractures have permeability values order of magnitude higher than that in matrix
2. Most of the oil is in low permeability matrix

•**Insignificance of viscous forces**: Because of high fracture permeability, pressure drops are low in the fracture. Since the pressure drop in the fracture is what is being exposed on the matrix, the viscous deriving force for oil recovery from matrix is low. This characteristic separates naturally fractured reservoirs from conventional reservoirs in which viscous deriving force is the dominant deriving force.

•**Segregation**: Since the permeability of fractures is high, gravity forces are likely to be higher than the viscous forces and segregation would occur, especially for the cases of aquifer expansion.

•**Imbibition**: As water is moving in the fractures, it displaces the oil in the fracture, and comes in contact with the blocks. If blocks are water wet, water is imbibed into the block by capillary forces.

•**Matrix Transfer Rate**: It is clear that for proper modeling of water advance through the fracture, water breakthrough, and rate of oil recovery, one needs to properly predict the rate of oil production by imbibition. This is called matrix transfer rate.

Modeling of Counter-current Imbibition in Naturally Fractured Reservoir; SPE 38443

Objective: Prediction of Matrix Transfer Rate

- It is clear that for proper modeling of water advance through the fracture, water breakthrough and rate of oil recovery one needs to properly predict the rate of oil production by imbibition. This is called matrix transfer rate.

- **Modeling**
  - Physical process
  - Simplifying assumptions
  - Mathematical formulation
    - Governing equation
    - IC & BCs
    - Solution
    - Application

- **Physical process**:
  - Traditionally, one of the blocks that is totally surrounded by water is considered. For such a block, water goes in from all direction and oil is produced counter current to the direction of water flow. The process is called counter-current imbibition.

- **Simplifying assumptions**
  - 1-D assumptions: In a real reservoir, this process is 3-D. But it seems that the main characteristics of the process do not change if one considers a 1-D representation. We simplify our physical system to a 1-D problem.
  - Incompressible oil and water: considering the pressure changes are occurring due to capillary pressure, which is small in magnitude, and are unlikely to cause change in density of the fluids.
  - Porosity does not change

Mathematical formulation

Continuity equation:

Water flow: $\frac{\partial}{\partial x}(v_w \rho_w) = -\frac{\partial}{\partial x}(\varphi \rho_w S_w)$ (1)

Oil flow: $\frac{\partial}{\partial x}(v_o \rho_o) = -\frac{\partial}{\partial x}(\varphi \rho_o S_o)$ (2)

Darcy's Law

$$v_w = -\frac{k k_{rw}}{\mu_w} \frac{\partial p_w}{\partial x}$$

$$v_o = -\frac{k k_{ro}}{\mu_o} \frac{\partial p_o}{\partial x}$$

Constitutive equation

$$p_o - p_w = p_{c(S_w)}$$

$$S_o - S_w = 1$$

Unknowns:

p_w, v_w, S_w

p_o, v_o, S_o

Continuity equation:

$$\frac{\partial}{\partial x}\left(\frac{kk_{rw}}{\mu_w}\frac{\partial p_w}{\partial x}\right) = \varphi\frac{\partial}{\partial x}(S_w)$$

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o}\frac{\partial p_o}{\partial x}\right) = \varphi\frac{\partial}{\partial x}(S_o)$$

Constitutive equation

$$p_o - p_w = p_{c(S_w)}$$

$$S_o - S_w = 1$$

Unknowns:

$p_w\ , S_w$

p_o, S_o

Counter-Current Imbibition:

$$v_t = v_w + v_o = 0$$

$$-\frac{kk_{rw}}{\mu_w}\frac{\partial p_w}{\partial x} - \frac{kk_{ro}}{\mu_o}\frac{\partial p_o}{\partial x} = 0$$

Co-Current Imbibition:

$$v_t = v_w + v_o$$

$$-\frac{kk_{rw}}{\mu_w}\frac{\partial p_w}{\partial x} - \frac{kk_{ro}}{\mu_o}\frac{\partial p_o}{\partial x} = v_t$$

$$\frac{\partial p_o}{\partial x} = \frac{\partial p_w}{\partial x} + \frac{\partial p_c}{\partial S_w}\frac{\partial S_w}{\partial x}$$

Combining Equations:

$$-\frac{kk_{rw}}{\mu_w}\frac{\partial p_w}{\partial x}-\frac{kk_{ro}}{\mu_o}[\frac{\partial p_w}{\partial x}+\frac{\partial p_c}{\partial S_w}\frac{\partial S_w}{\partial x}]=0$$

$$\frac{\partial p_w}{\partial x}=-\frac{\frac{k_{ro}}{\mu_o}\frac{\partial p_c}{\partial S_w}}{\frac{k_{ro}}{\mu_o}+\frac{k_{rw}}{\mu_w}}\frac{\partial S_w}{\partial x}$$

$$\frac{\partial}{\partial x}\left(D(S_w)\frac{\partial S_w}{\partial x}\right)=\frac{\partial}{\partial x}(S_w)$$

Non-linear diffusivity equation
Because diffusion coefficient $D(S_w)$ is strong function of S_w

Where

$D(S_w)$ exhibit a bell-shape behavior

$$D(S_w)=-\frac{k}{\varphi}\frac{k_{ro}}{\mu_o}f(S_w)\frac{\partial p_c}{\partial S_w}$$

$$f(S_w)=\frac{1}{1+\frac{k_{ro}}{k_{rw}}\frac{\mu_o}{\mu_w}}$$

Rate in a diffusive process

$$q_w=-\varphi D(S_w)\frac{\partial S_w}{\partial x}$$

Physical Interpretation: It is interesting to note that the process of water imbibition into a water wet rock can be considered as diffusion of water saturation into that block. In other wors, in a capillary dominated flow regime, flow is caused by saturation gradient in the same as saying flow is caused by pressure gradients.

Governing equation, IC & BCs

$$\frac{\partial}{\partial x}\left(D(S_w)\frac{\partial S_w}{\partial x}\right)=\frac{\partial S_w}{\partial t}, \quad (1)$$

where

$$D(S_w)=-\frac{k}{\phi}\frac{k_{ro}}{\mu_o}f(S_w)\frac{\mathrm{d}P_c}{\mathrm{d}S_w} \quad (2)$$

and

$$f(S_w)=\frac{1}{1+\frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}}. \quad (3)$$

The initial and boundary conditions are

$$S_w=S_{wi}, \quad t=0, \quad 0\leqslant x\leqslant L, \quad (4)$$

$$S_w=1-S_{or}, \quad t=0^+, \quad x=0, \quad (5)$$

$$q_w=0, \quad t=0^+, \quad x=L. \quad (6)$$

•The diffusion coefficient of Eq. 2 is bell shaped with respect to water saturation, attaining a value of zero at S_{wi} and S_{or}

• Eq. 5 expresses the continuity of capillary pressure at the inlet face (P_c=0).

•Eq. 6 is the no-flow boundary condition at the outlet.

Neglecting Oil Phase Pressure Gradient in the Oil Phase

This assumption is based on the common practice in hydrology, where the mathematical formulation of unsaturated water flow ignores the air pressure gradient.

$$\frac{\partial}{\partial x}\left(D(S_w)\frac{\partial S_w}{\partial x}\right)=\frac{\partial S_w}{\partial t}, \qquad (1)$$

where

$$D(S_w)=-\frac{k}{\phi}\frac{k_{rw}}{\mu_w}\frac{\mathrm{d}P_c}{\mathrm{d}S_w}. \qquad (8)$$

$D(S_w)$ does NOT exhibit a bell-shape behavior

The initial and boundary conditions are

$$S_w=S_{wi}, \quad t=0, \quad 0\leqslant x\leqslant L, \qquad (4)$$

$$S_w=1-S_{or}, \quad t=0^+, \quad x=0, \qquad (5)$$

$$q_w=0, \quad t=0^+, \quad x=L. \qquad (6)$$

Numerical Model

1-D finite-difference models to study counter- and co-current imbibition in finite-size porous media.

Peaceman and co-workers' approach is used where the continuity equation is coupled with the generalized form of Darcy's law for two-phase flow to obtain

$$\nabla\left(k\frac{k_{ro}}{\mu_o}\nabla p_o\right) = -\phi\frac{\mathrm{d}S_w}{\mathrm{d}P_c}\left(\frac{\partial p_o}{\partial t} - \frac{\partial p_w}{\partial t}\right), \qquad (9)$$

$$\nabla\left(k\frac{k_{rw}}{\mu_w}\nabla p_w\right) = \phi\frac{\mathrm{d}S_w}{\mathrm{d}P_c}\left(\frac{\partial p_o}{\partial t} - \frac{\partial p_w}{\partial t}\right). \qquad (10)$$

Counter-current Imbibition

- The initial and boundary conditions are considered for a physical problem in which the viscous forces have no effect and flow is purely capillary-driven.
- First, consider countercurrent imbibition, in which the only open end is initially in contact with the oil at the ambient pressure, say, zero pressure
- The water pressure in the core is given by the capillary pressure relationship, which at **t=0** leads to $\mathbf{p_w=p_o-P_c(S_{wi})=-P_c(S_{wi})}$.
- The imbibition begins when the oil outside the core in the open end is replaced by water at the ambient pressure.

$$p_o=0, \quad t=0, \quad 0\leq x\leq L, \qquad (11)$$

$$p_w=-P_c(S_{wi}), \quad t=0, \quad 0\leq x\leq L, \qquad (12)$$

$$p_w=0, \quad t=0^+, \quad x=0, \qquad (13)$$

continuity of the capillary pressure at the imbibition face

$$q_w=0, \quad t=0^+, \quad x=L, \qquad (14)$$

$$p_o=0, \quad t=0^+, \quad x=0, \qquad (15)$$

$$q_o=0, \quad t=0^+, \quad x=L. \qquad (16)$$

Co-current Imbibition,

•For co-current imbibition, consider a situation in which the oil pressure at both ends of the core is initially fixed at zero, for example, by exposing it to oil at the ambient pressure.

•Water at the ambient pressure is then introduced at one end. Water will be imbibed into the core and oil might be produced from one or two end faces depending on the inlet boundary condition.

- •zero-capillary pressure,
- •zero-oil flow,

$$p_o = 0, \quad t = 0, \quad 0 \leq x \leq L, \tag{11}$$

$$p_w = -P_c(S_{wi}), \quad t = 0, \quad 0 \leq x \leq L, \tag{12}$$

$$p_w = 0, \quad t = 0^+, \quad x = 0, \tag{13}$$

$$q_w = 0, \quad t = 0^+, \quad x = L, \tag{14}$$

$$p_o = 0, \quad t = 0^+, \quad x = 0, \tag{17a}$$

$$q_o = 0, \quad t = 0^+, \quad x = 0, \tag{17b}$$

$$p_o = 0, \quad t = 0^+, \quad x = L. \tag{18}$$

Note that the only difference in the mathematical description of co- and counter-current imbibition, i.e., Eqs. 9–18, is related to the inlet and outlet boundary conditions of the oil phase. This difference leads to the absence or presence of the convective term in Eqs. 1 and 7, respectively

$$\frac{\partial}{\partial x}\left(D(S_w)\frac{\partial S_w}{\partial x}\right)=\frac{\partial S_w}{\partial t}, \qquad (1)$$

$$\frac{\partial}{\partial x}\left(D(S_w)\frac{\partial S_w}{\partial x}-q_t f(S_w)\right)=\frac{\partial S_w}{\partial t}. \qquad (7)$$

$$q_t = q_o + q_w$$

The convective term

TABLE 1–DATA FOR THE BASE CASE EXAMPLE			
L	20 cm (0.2 m)	n_o	4.0
k	20 md (0.02 μm^2)	n_w	4.0
μ_o	1 cp (1 mPa·s)	k^0_{ro}	0.75
μ_w	1 cp (1 mPa·s)	k^0_{rw}	0.2
ϕ	0.3	B	1.45 psi (10 kPa)
S_i	0.001		

$$k_{ro} = k^0_{ro}(1-S)^{n_o}, \quad k_{rw} = k^0_{rw} S^{n_w}, \tag{19}$$

$$P_c(S) = -B \ln(S), \tag{20}$$

$$S = \frac{S_w - S_{iw}}{1 - S_{or} - S_{iw}}$$

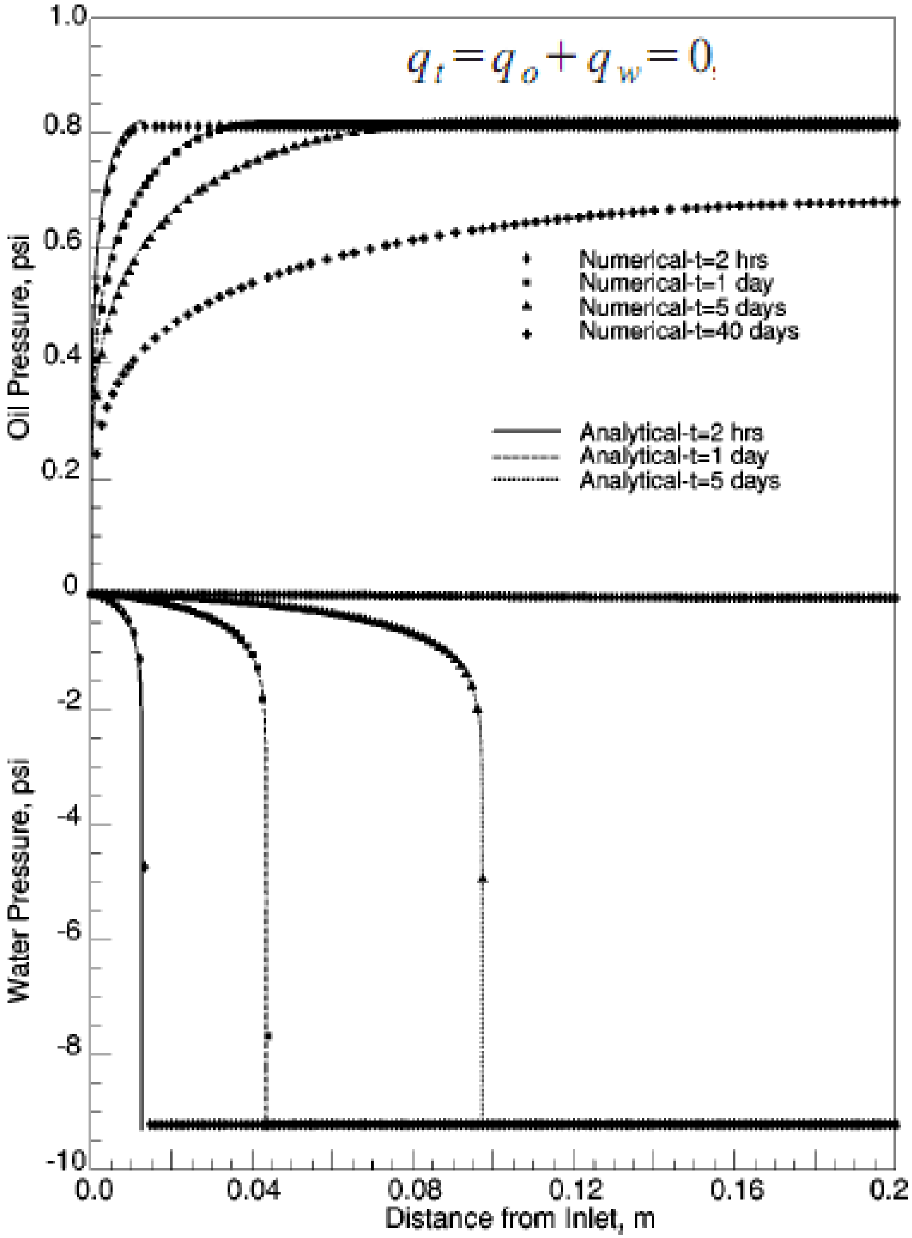

Fig. 1–Oil and water pressure vs. distance for 1D counter-current imbibition.

- The oil pressure inside the core is high. As time progresses, the oil pressure decreases approaching zero at very long times.
- Water pressure follows the reverse trend.
- Fig. 1 indicates that oil and water pressures are constant beyond the saturation front and are independent of position. One may conclude that countercurrent imbibition exhibits an infinite acting behavior, and the solution does not depend on length *L before the saturation front reaches the far boundary.*
- oil exhibits very sharp pressure gradients at the inlet end.
- water exhibit very sharp pressure gradients at the front.
- Fig. 1 indicates that oil-phase pressure drop is smaller than the water phase, but the steep oil pressure profile at the inlet suggests that neglecting the local oil pressure gradient might not be appropriate.

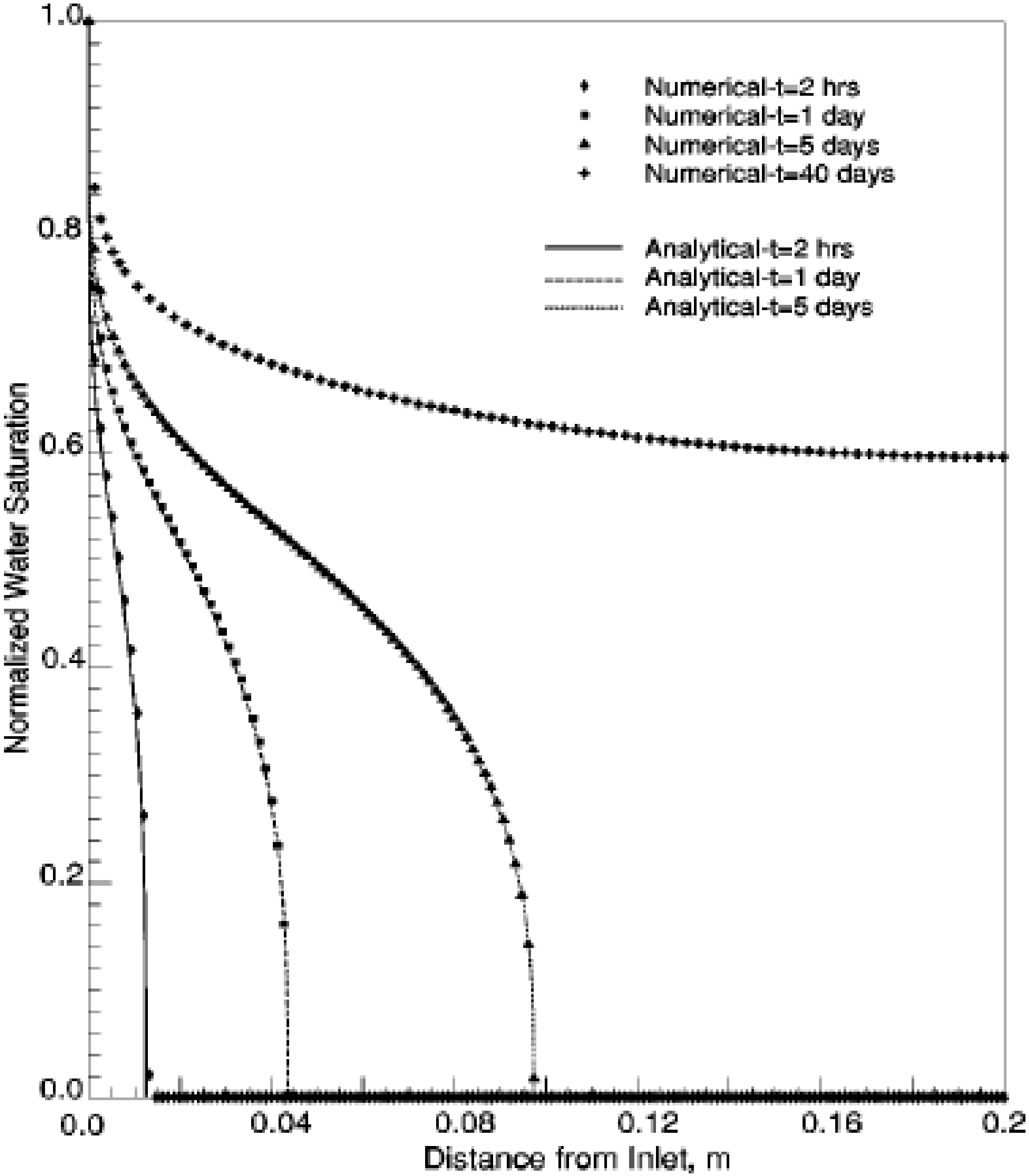

Fig. 2–Saturation distribution for 1D countercurrent imbibition.

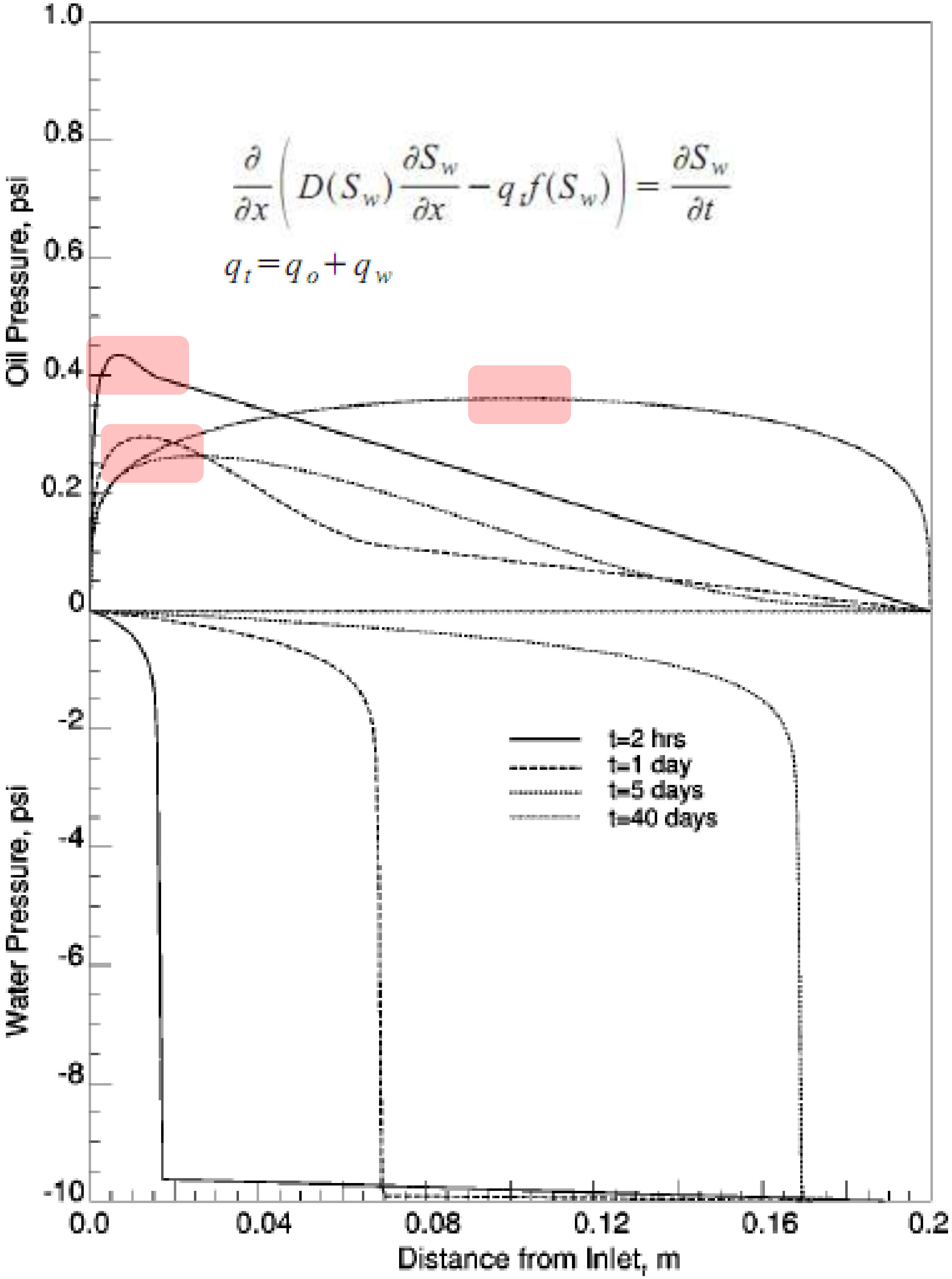

Fig. 3–Oil and water pressure vs. distance for 1D cocurrent imbibition.

- Oil and water pressures are not constant downstream of the saturation front; they vary with time and position.
- Contrary to counter-current imbibition, co-current imbibition does not show an infinite-acting behavior, because oil pressure feels the effect of the far boundary even before the saturation front reaches the far boundary.
- Oil pressure is not monotonic; it passes through a maximum in the two-phase region. Behind the maximum, oil flows in the opposite direction of water.
- Co-current imbibition takes advantage of oil pressure gradient downstream of the front in the single-phase region, and results in an increased recovery rate.
- There is a contribution of a convective term for co-current imbibition in Eq. 7.
- Physically, in co-current imbibition, oil is produced downstream of the water front through the single-phase region, whereas in counter-current imbibition, oil flows through the two-phase region, reducing oil recovery efficiency.

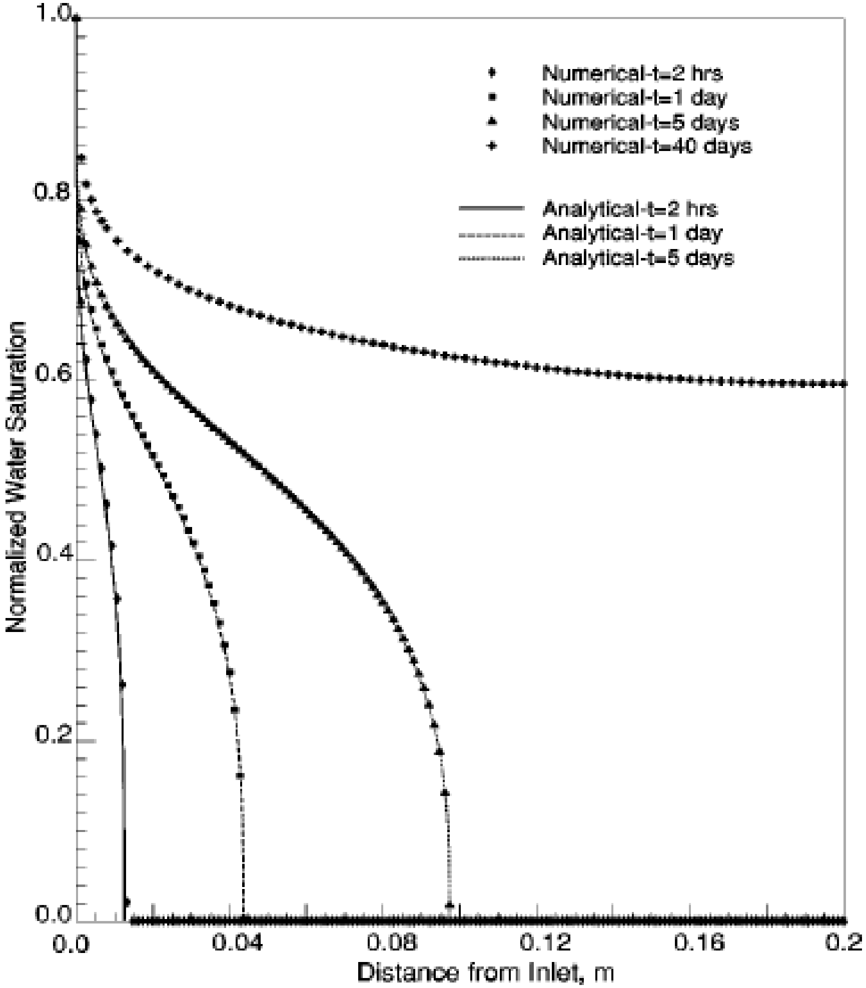

Fig. 2–Saturation distribution for 1D countercurrent imbibition.

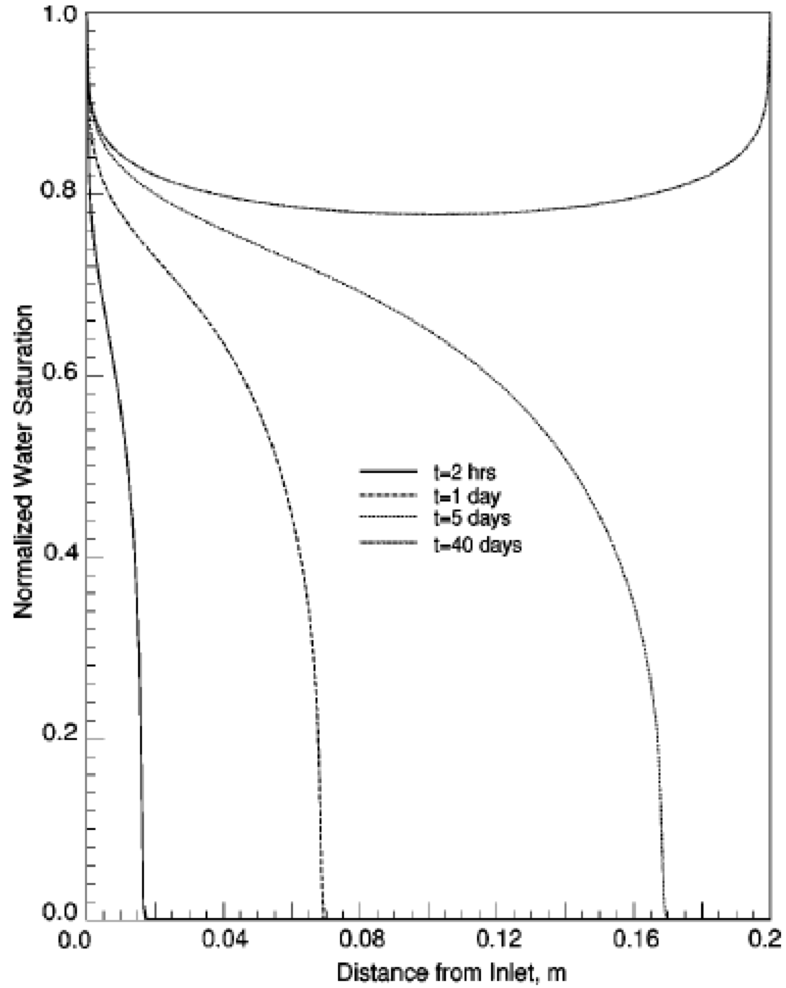

Fig. 4–Saturation distribution for 1D cocurrent imbibition.

Comparison of Figs 2 and 4 suggests that saturation profiles advance further in co-current imbibition compared with that in counter-current imbibition, which shows the superiority of co-current over counter-current imbibition.

Fig. 5 shows that recovery is significantly overestimated when the oil phase pressure gradient is neglected.

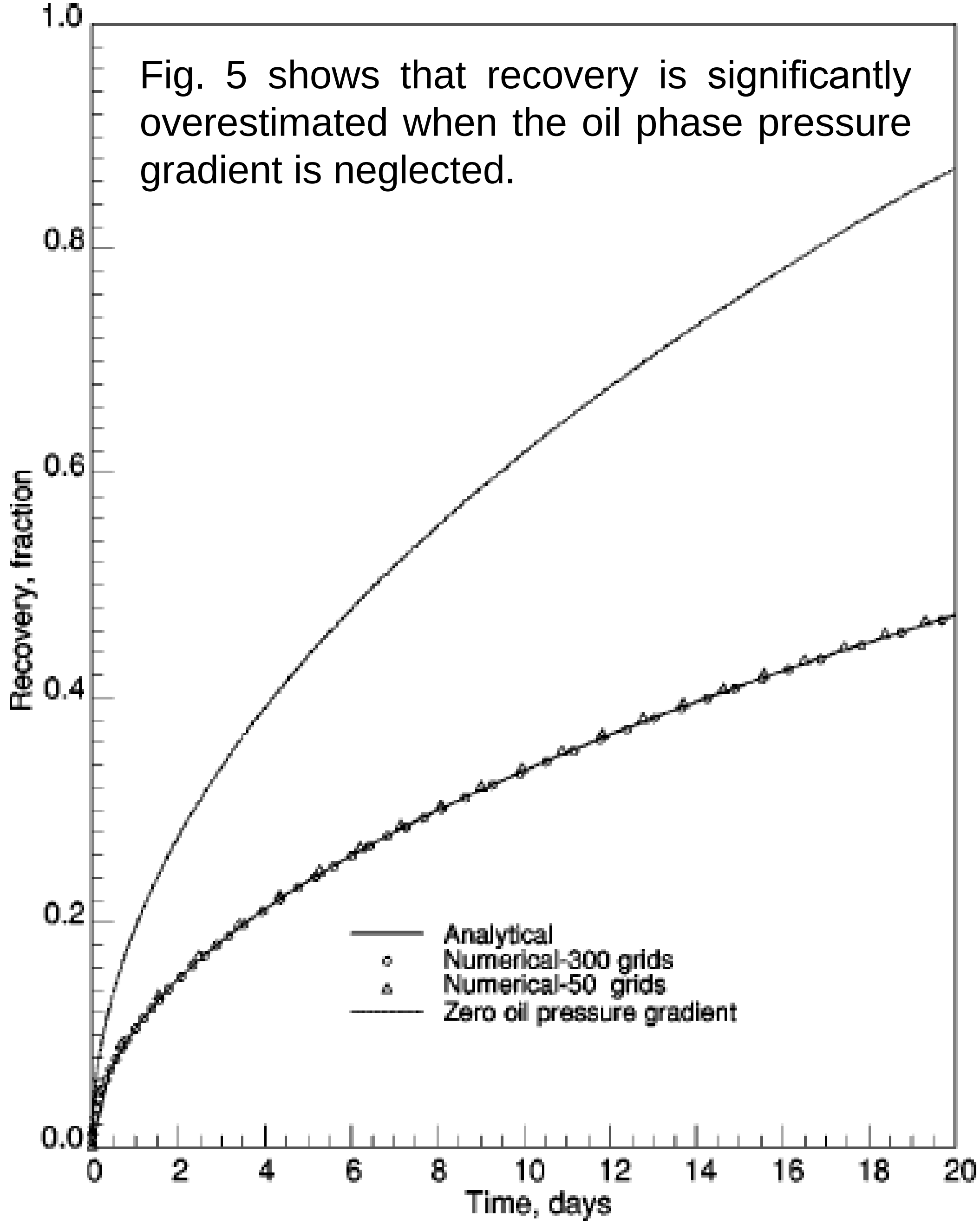

Fig. 5–Recovery for 1D countercurrent imbibition.

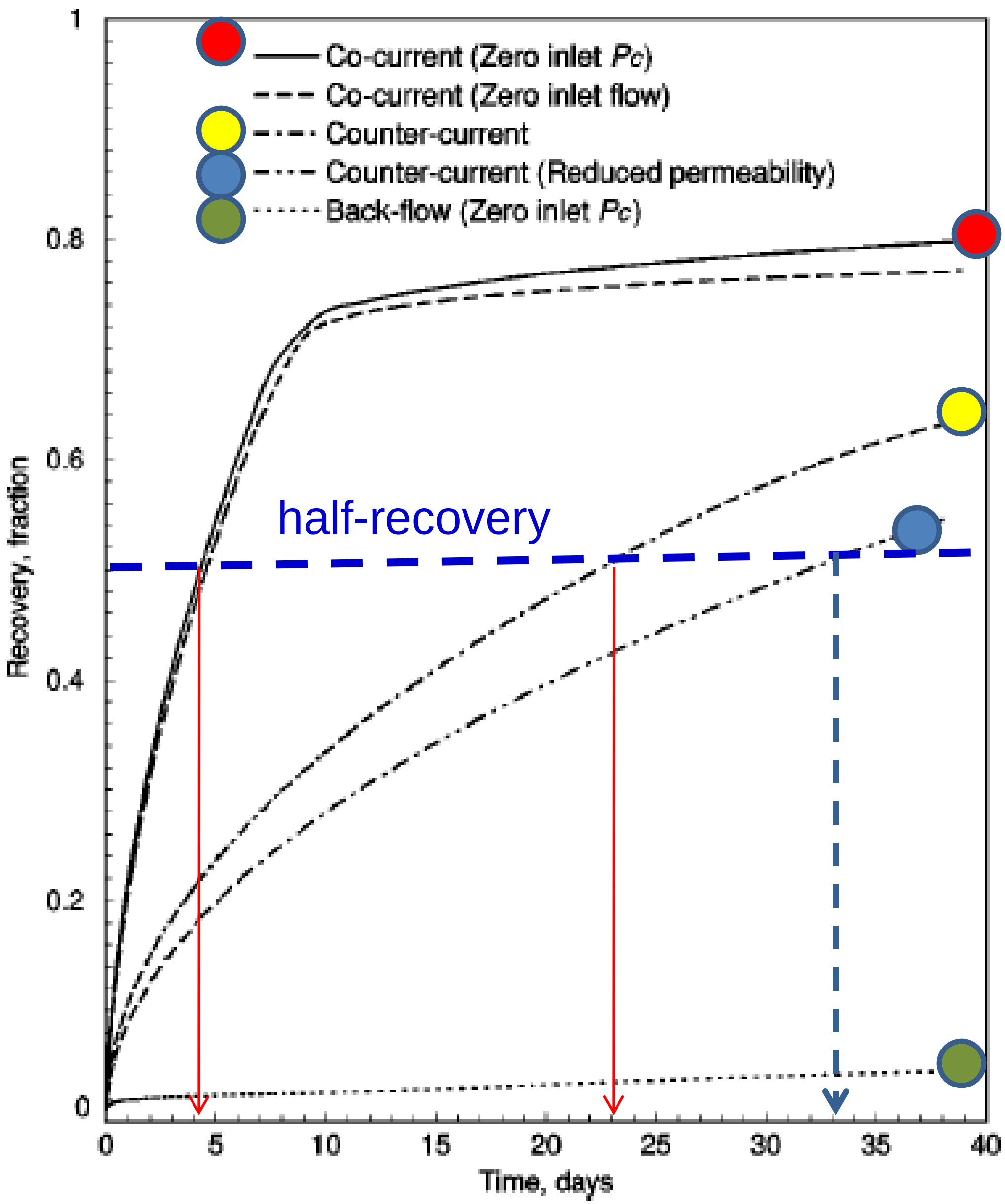

Fig. 6–Recovery for 1D co- and countercurrent imbibition.

$$p_o = 0, \quad t = 0^+, \quad x = 0, \qquad (17a)$$

$$q_o = 0, \quad t = 0^+, \quad x = 0, \qquad (17b)$$

- If the residual oil saturations for co- and counter-current imbibition are equal, as assumed here, recovery curves at very late time approach the same value—1.
- At earlier times, however, especially before the saturation front reaches the far boundary, there is a substantial difference between the two curves.
- Several studies suggest that due to viscous coupling between the flowing phases, relative permeability curves for the two processes could be different. Fig.7 shows the recovery curve for countercurrent imbibition when relative permeability curves are reduced by 30%.
- Fig. 6 also shows the contribution of oil recovery from the face in contact with water (the other face is in contact with oil). The contribution of the back-flow production at a recovery of 80% is less than 5%, a large portion of the backflow recovery is obtained at a very early time.
- The small recovery from the inlet face suggests that if a no-oil flow boundary condition is imposed (Eq. 17b), there will be only a small change in the recovery performance.

Scaling Studies

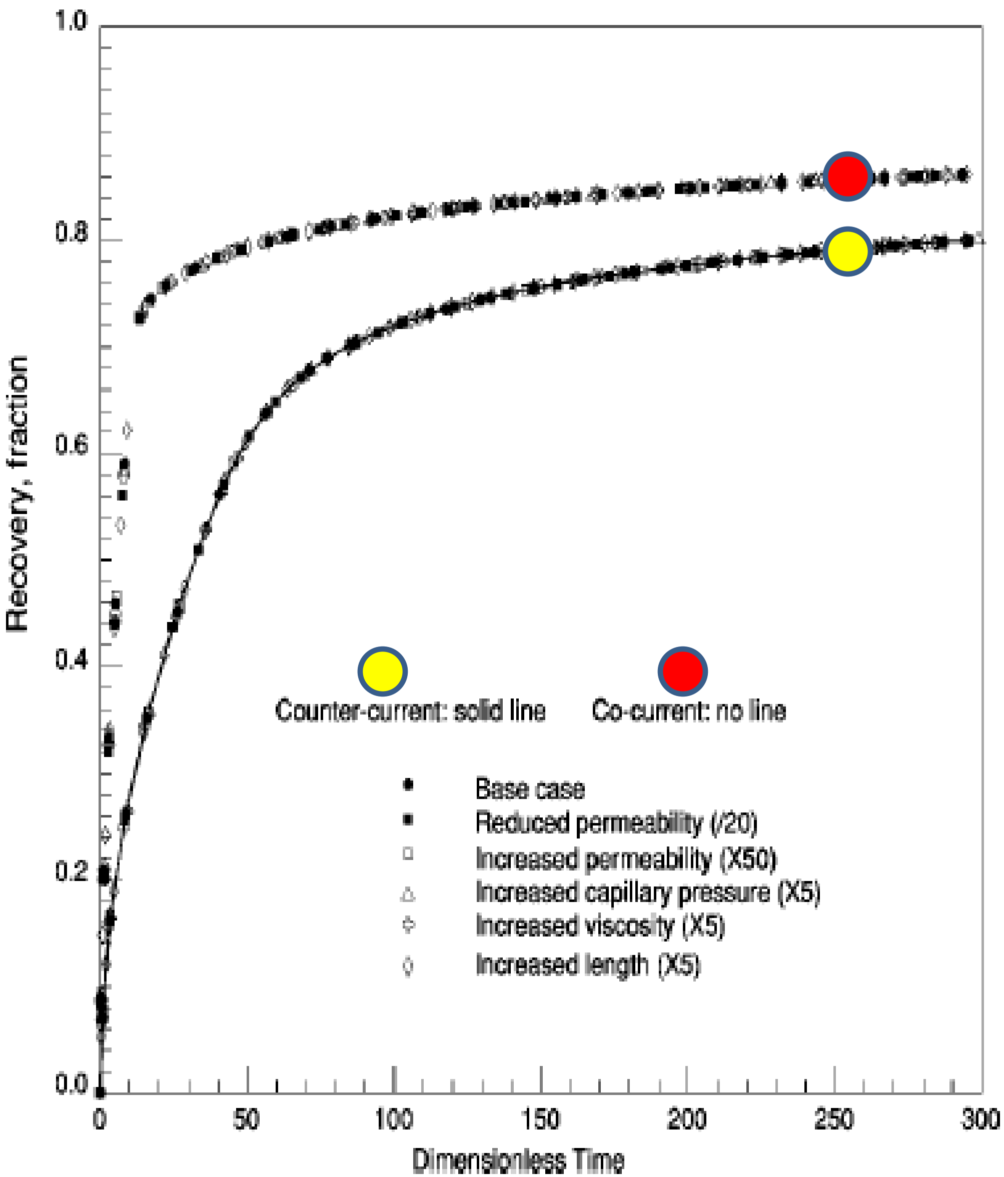

Fig. 7–Scaling for 1D co- and countercurrent imbibition.

- **Rapoport** presented the scaling criteria for two-phase incompressible flow through porous media.
- Using inspectional analysis of the differential equations of water/oil flow through porous media, he found that saturation distribution is a function of dimensionless time provided certain similarities are present.

$$t_D = \frac{kt}{\phi \mu_w L^2} \left. \frac{dP_c}{dS_w} \right|_{S_w^*}.$$

$$S_w^* = 1 - S_{or}$$

$$dP_c / dS_w \big|_{S_w^*} = B$$

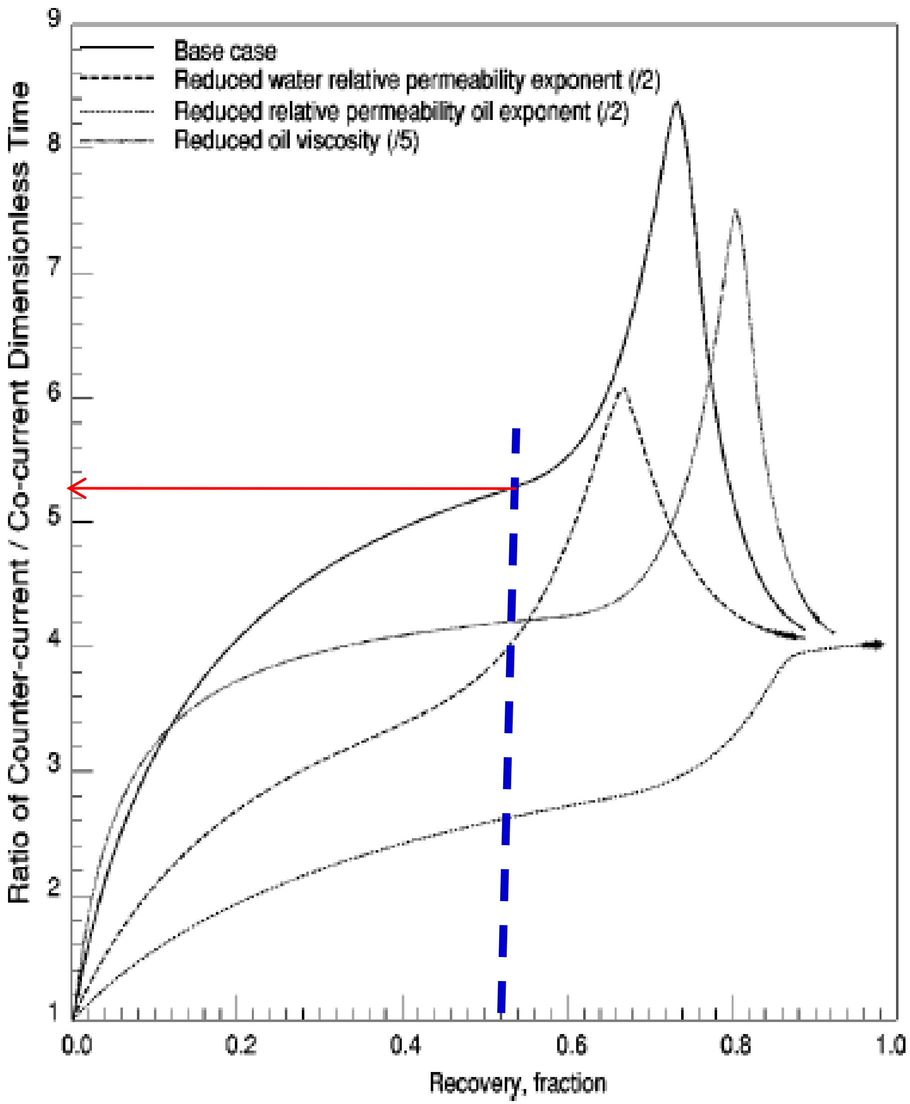

Fig. 8–Comparison between 1D co- and countercurrent imbibition.

- To compare the time scales of the two imbibition processes, a dimensionless time ratio is defined as the time ratio of counter-current to co-current imbibition to achieve a specific recovery.
- At the very early time, the recovery performance of co- and counter-current imbibition is similar, and the time ratio is equal to 1.
- The time ratio increases rapidly such that half-recovery time for countercurrent imbibition is more than five times that of co-current imbibition.

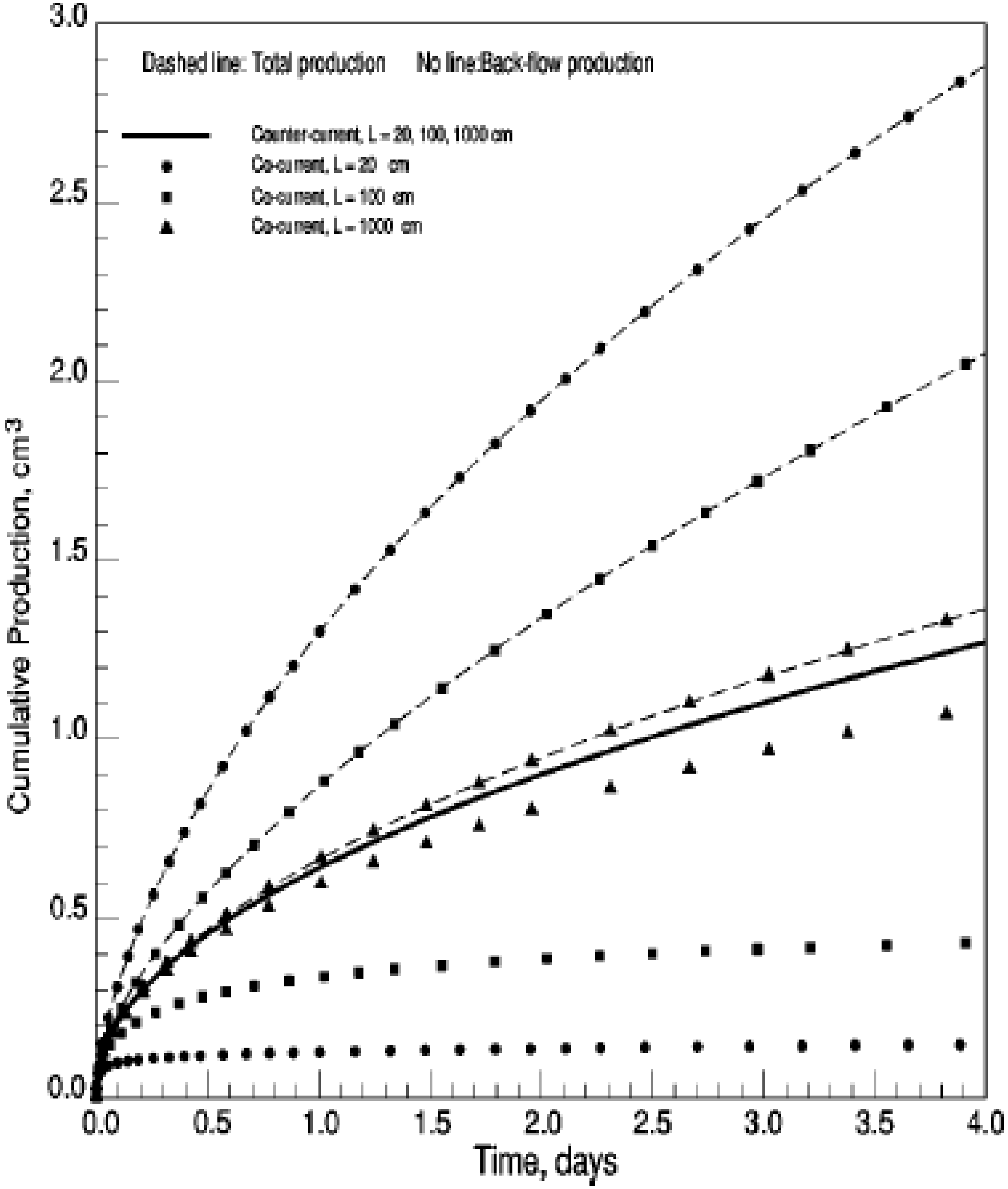

Fig. 9–Effect of length on 1D cocurrent imbibition.

Assignment No.3.b

Assume that the diffusion coefficient in a 1-D counter-current imbibition is constant and solve the problem as a linear diffusion equation. Use the following relation to define constant diffusion coefficient.

$$\bar{D} = \frac{\int_{Swc}^{1-Sor} D(Sw)\,dSw}{1 - Sor - Swc}$$