Flow in Porous Media

Module 4.b
Dispersion During Miscible Displacement

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- **In immiscible displacement in porous media:**
    - The immiscible fluids flow in separate channels.
    - An interface separates the fluids.
    - There will be irreducible or residual saturations after the displacement has reached steady state.
    - The interfacial tensions associated with the immiscible fluid interfaces play a significant role in determining the fluid distributions within the porous medium.
    - The displacement front in an immiscible displacement will be sharper at higher flow rates, or when capillary forces are neglected.

- **In a miscible displacement in porous media:**
    - No interface exists between miscible fluids of different composition.
    - A mixing zone is established, where the composition of the in-situ fluid changes from the composition of one pure fluid to that of the other pure fluid.
    - In the absence of fluid/fluid interfaces, capillary forces are absent.
    - Fluid composition is determined by mixing, which occurs by molecular diffusion and convective dispersion.
    - Steady state is reached when one fluid has completely displaced the other fluid; the concept of irreducible or residual saturations does not apply.
    - The mixing zone will be short at low flow rates, resulting in a steep concentration profile.

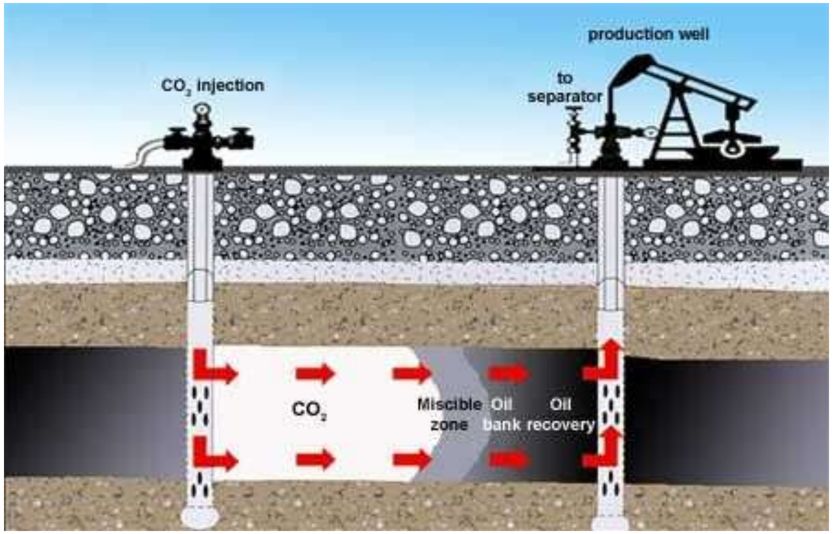
Miscible Displacement in Porous Media

❑The oil recovery in a miscible displacement process depends on the **size of the mixing zone** between the injected fluid and the reservoir oil.

❑For maximum oil recovery at breakthrough to occur, the **mixing zone should remain small** compared to the reservoir volume so that the oil produced is not diluted by the injected fluid.

❑Ideally in a reservoir with a small mixing zone, for complete oil recovery, slightly more than one reservoir pore volume of injection fluid is required. However, **if the mixing zone is large**, several reservoir pore volumes of injection fluid may be needed to achieve complete recovery.

❑The mixing due to diffusion and dispersion can **dampen out viscous fingers** in an unstable displacement, leading to **increased sweep efficiency**.



- **Dispersive mixing** is caused by molecular diffusion and mechanical dispersion and is the main part of the mixing in miscible displacements.

- **Molecular diffusion** is a phenomenon whereby the transport of mass of a species (component) occurs within a single fluid phase from one point to the other in the direction of decreasing concentration.
    - Diffusion is a consequence of the random motion of molecules and can also take place in the absence of bulk movement or agitation.

- Spreading of a component in a phase due to microscopic variations in the velocity field is called **convective or mechanical-dispersion**. A number of mechanisms are responsible for variations in both magnitude and direction of the velocity in permeable media:
    - parabolic velocity distribution associated with viscous flow through a pore,
    - existence of different pore geometries, and
    - fluctuations in the stream lines with respect to the mean flow direction.

The Equation of Continuity

Application of the principle of mass conservation of species i in a multi-component fluid mixture to an arbitrary control volume of the fluid yields the well-known equation of continuity, which, in it's most general form, can be written as follows (Bird et al., 1960):

$$\frac{\partial c_i}{\partial t} + \nabla \cdot \vec{n}_i = r_i \qquad (1)$$

c_i: Concentration of species i (mass per unit volume),
n_i: Mass flux vector (mass of species i per unit area per unit time)
r_i: A source or sink term (mass of i per unit volume per unit time)

$$\vec{n}_i = c_i \vec{u} - \rho D_0 \nabla x_i \qquad (2)$$

u_i: the mass average velocity vector (length per unit time),
ρ: fluid (mixture) mass density (mass per unit volume)
D_o *:* molecular diffusion coefficient (length squared per unit time)
X_i: mass fraction of species i
($x_i = c_i/\rho$)

$$\frac{\partial c_i}{\partial t} + \nabla \cdot c_i \vec{u} = \nabla \cdot \rho D_0 \nabla x_i + r_i \qquad (3)$$

For constant ρ and D_o

$$\frac{\partial c_i}{\partial t} + \vec{u} \cdot \nabla c_i = D_0 \nabla^2 c_i + r_i \tag{4}$$

For one-dimensional flow

$$\frac{\partial c_i}{\partial t} + u \frac{\partial c_i}{\partial z} = D_0 \frac{\partial^2 c_i}{\partial z^2} + r_i \tag{5}$$

Some examples of the magnitude of the molecular diffusion coefficients are as follows (Bird et al., 1960):

Pairs of dilute gases	0.1 – 0.7 cm^2/s
Liquids	of the order of 10^{-5} cm^2/s
Solids	10^{-8} – 10^{-30} cm^2/s

The Equation of Continuity in Porous Media

- ❑The molecular diffusion coefficient (D_o), is replaced by the dispersion coefficient
    - ▪Dispersion coefficient: a measure of mixing during flow
- ❑ *The velocity “u* “ is replaced by “*v*”

$$\vec{v} = \frac{\vec{Q}}{A\phi}$$

$$\frac{\partial c_i}{\partial t} + \nabla \cdot c_i \vec{v} = \nabla \cdot \rho \underline{D} \nabla x_i + r_i \qquad (6)$$

The dispersion of a substance during flow in porous media results from:

- ❑ molecular diffusion in the direction of flow,
- ❑coupled with transverse molecular diffusion due to the velocity profiles, and
- ❑mechanical mixing arising from velocity variations due to the complex nature of the pore structure

1-D Constant Density and Constant Dispersion Coefficient

$$\frac{\partial c_i}{\partial t} + v \frac{\partial c_i}{\partial z} = D \frac{\partial^2 c_i}{\partial z^2} + r_i \qquad (7)$$

Solutions to the 1-D Convection-Dispersion Model

Infinite System

For a single solute and no source/sink, Equation 7 becomes:

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial z^2} \qquad (20)$$

where c is solute concentration and D represents longitudinal dispersion.

The following assumptions are implicit in Equation 20:

☐homogeneous porous medium of constant cross-section;
☐bulk flow in the axial direction at constant interstitial velocity;
☐constant fluid density;
☐constant dispersion coefficient;
☐incompressible porous medium;
☐uniform concentration distribution in the direction perpendicular to flow (i.e., time is "long" enough for the convection-dispersion model to hold);
☐no solute sources or sinks.

Equation 20 is a second order partial differential equation, requiring two boundary conditions and an initial condition for its solution.

When the space variable z is translated such that the transformed distance **z' becomes the distance from the flood** front rather than the distance from the inlet of the porous medium, Equation 20 reduces to the one-dimensional unsteady diffusion or heat conduction equation, also known as Fick's second law (Taylor, 1953; Nunge and Gill, 1970):

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial z'^2} \qquad (21)$$

$z' = z - vt$

vt : distance of the flood front from the porous medium entrance

This equation has been solved analytically for a variety of initial and boundary conditions (see, for example, Carslaw and Jaeger, 1959; Özişik, 1980).

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial z'^2} \qquad (21)$$

- **Assumptions:**
    - An infinite porous medium
    - Zero initial concentration,
    - A step change in inlet concentration (c_0)

- **IC & BCs:**

$t = 0$ $\quad z' \geq 0 \quad c = 0$
$\qquad\quad\; z' < 0 \quad c = c_0$

(22)

$t > 0$ $\quad z' \rightarrow -\infty \quad c \rightarrow c_0$
$\qquad\quad\; z' \rightarrow +\infty \quad c \rightarrow 0$

The solution to Equation 21 with initial and boundary conditions 22 is (Danckwerts, 1953; Brigham et al., 1961)

$$\frac{c}{c_0} = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{z'}{2\sqrt{Dt}}\right)\right] \qquad (23)$$

where c/c_0 is normalized concentration and erf(r) is the error function of a variable r,

$$\frac{c}{c_0} = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{z'}{2\sqrt{Dt}}\right)\right] \tag{23}$$

$$\operatorname{erf}(r) = \frac{2}{\sqrt{\pi}}\int_0^r e^{-s^2}\,ds \tag{24}$$

Some properties of the error function are

$\operatorname{erf}(0) = 0$ $\operatorname{erf}(\infty) = 1$ $\operatorname{erf}(-r) = -\operatorname{erf}(r)$

Then, from Equation (23),

$z' = 0$ $c/c_0 = 1/2$
$z' \rightarrow -\infty$ $c/c_0 = 1$
$z' \rightarrow +\infty$ $c/c_0 = 0$

Typical concentration profiles, obtained from Equation 23, are shown in Figure 6. The curves are symmetrical about the point ($z'=0$, $c/c_0=1/2$). The profiles become more dispersed when D increases or as time progresses.

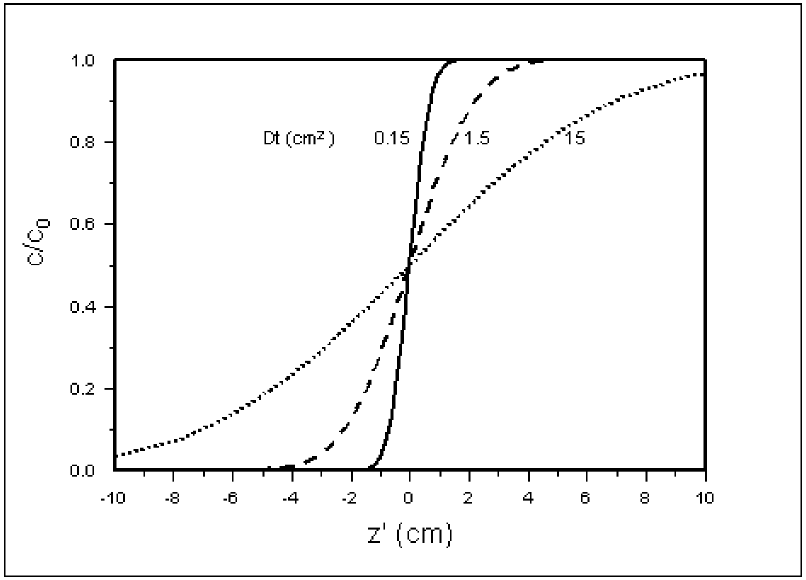


Figure 6. Concentration as a function of transformed distance for different values of dispersion coefficient or time, calculated from Equation 23, infinite system.

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial z} = D\frac{\partial^2 c}{\partial z^2} \tag{20}$$

Dimensionless form of Equation 20 becomes

$$\frac{\partial c_D}{\partial t_D} + \frac{\partial c_D}{\partial z_D} = \frac{1}{Pe}\frac{\partial^2 c_D}{\partial z_D{}^2} \tag{25}$$

$$z_D = \frac{z}{L}$$

$$t_D = \frac{vt}{L} = \frac{V}{V_p} \tag{26}$$

$$c_D = \frac{c}{c_0}$$

and V is volume injected, V_p is pore volume, and P_e is the porous medium Peclet number,

$$Pe = \frac{Lv}{D} \tag{27}$$

The **Peclet number** represents the ratio of characteristic times for mass transfer by flow and by dispersion, and is a measure of the length of the mixing zone relative to the length of the porous medium.

Determination of the Dispersion Coefficient

- Because of its simplicity, Equation 23, although representing infinite systems, is often used to approximate concentration distributions in finite systems.

- Typical laboratory core flood experiment:
  - A porous medium is fully saturated with a fluid.
  - A miscible fluid or tracer, such as a salt solution or dye or radioactive material, is then injected at a known, constant flow rate.
  - Effluent samples are collected at the core exit and analyzed for tracer concentration.
  - In such an experiment, Equation 23 represents the effluent concentration, i.e., the concentration at a fixed distance (z=L, or z'=L-vt).

$$\frac{c}{c_0} = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{z'}{2\sqrt{Dt}}\right)\right] \qquad (23)$$

In an experiment such as this one, the dispersion coefficient can be determined using a simple procedure described by Brigham et al. (1961). When Equation 23 is used to calculate effluent concentrations from a finite porous medium (z=L or z'=L-vt), it is convenient to transform variables such that time is replaced by throughput in pore volumes (Brigham et al., 1961; Brigham, 1974). Change variables such that

$$t = \frac{V}{V_p}\frac{L}{v} \tag{28}$$

and

$$z' = L - vt = L\,\frac{V_p - V}{V_p} \tag{29}$$

Equation 23 then becomes

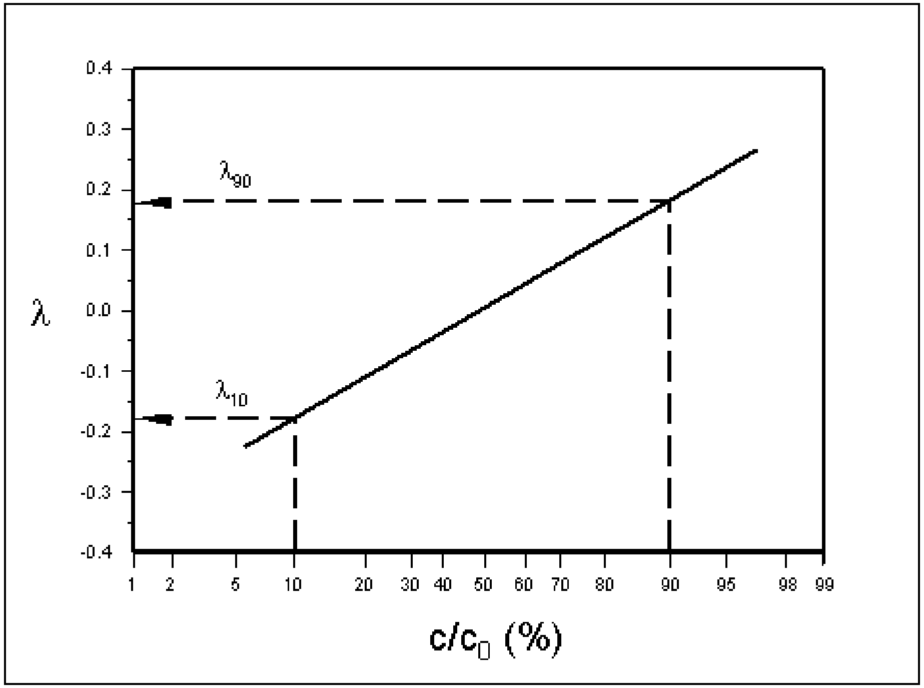
$$\boxed{\frac{c}{c_0} = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\lambda}{2\sqrt{D/vL}}\right)\right] = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{\lambda}{2\sqrt{1/Pe}}\right)\right]} \tag{30}$$

where

$$\lambda = \frac{V/V_p - 1}{\sqrt{V/V_p}} \tag{31}$$

When λ is plotted against the percentage of displacing fluid in the effluent on arithmetic probability coordinates (Figure 7), and provided the convection-dispersion model holds, a straight line results from which the longitudinal dispersion coefficient can be obtained using the following equation (Brigham et al., 1961; Perkins and Johnston, 1963):

$$D = vL\left[\frac{\lambda_{90} - \lambda_{10}}{3.625}\right]^2 \qquad (32)$$

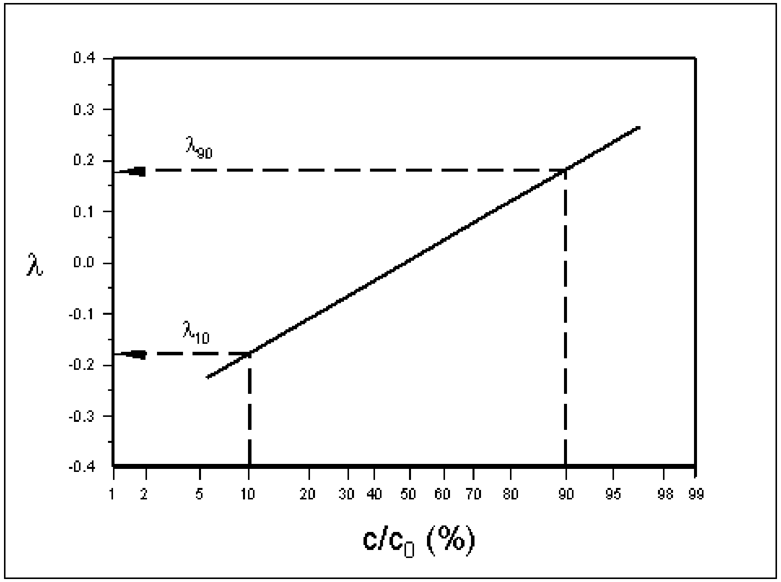


- ➢λ_{10} and λ_{90} are the values of λ when the effluent concentration is 10% and 90% of the injected value, respectively, and v and L are known experimental parameters.
- ➢The constant 3.625 arises from the error function and the choice of 10% and 90% concentrations.

Figure 7. Typical probability plot for determination of longitudinal dispersion coefficient.

$$D = vL\left[\frac{\lambda_{80} - \lambda_{20}}{2.380}\right]^2$$

$$D = vL\left[\frac{\lambda_{95} - \lambda_{5}}{4.653}\right]^2$$



Alternatively, the dispersion coefficient can be determined by plotting experimental effluent concentration profiles against volume injected and adjusting D until Equation 30 fits the experimental data. This is commonly done with effluent concentrations, since they are easily measured through chemical analysis.

$$\frac{c}{c_0} = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\lambda}{2\sqrt{D/vL}}\right)\right] = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\lambda}{2\sqrt{1/Pe}}\right)\right] \qquad (30)$$

1D Pure Diffusion Equation

Governing Equation

$$D_0 \frac{\partial^2 C}{\partial x^2} = \frac{\partial C}{\partial t} \quad \ldots\ldots (9)$$

IC:

$$C(x,0) = C_0 \qquad x < 0 \quad \ldots\ldots (10)$$

$$C(x,0) = 0 \qquad x > 0 \quad \ldots\ldots (11)$$

BCs:

$$\frac{\partial C}{\partial x} = 0, \qquad x = L \quad \ldots\ldots (12)$$

Solution:

$$C(x,t) = \frac{1}{2} C_0 \sum_{n=-\infty}^{\infty} \left[\operatorname{erf}\left\{ \frac{h + 2nL - x}{2\sqrt{tD_0}} \right\} + \operatorname{erf}\left\{ \frac{h - 2nL + x}{2\sqrt{tD_0}} \right\} \right] \quad \ldots\ldots (13)$$

The Analytical Solution to 1D Convection-Diffusion Equation

- The flow of a component in a phase can be described by the convection-diffusion (C-D) equation in 1D, dimensionless form

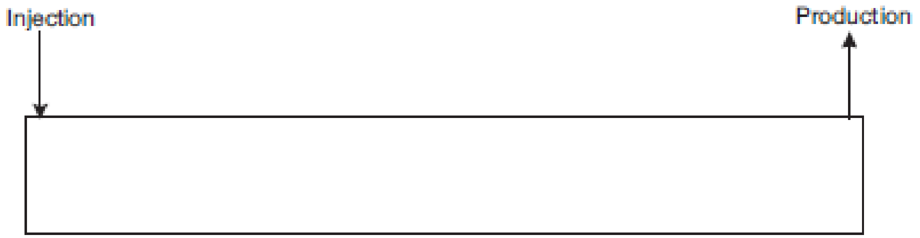


FIGURE 1: Schematic of a 1D model used for comparison with the analytical solution.

Assumptions:

- flow of a single phase incompressible fluid in permeable media
- ideal mixing
- miscible displacement of an initially
- uniform fluid distribution in a finite length *L*

Governing Equation
1D Convection-Diffusion Equation

- The flow of a component in a phase can be described by the convection-diffusion (C-D) equation in 1D, dimensionless form

$$\frac{\partial C_D}{\partial t_D} + \frac{\partial C_D}{\partial x_D} = \frac{1}{N_{Pe}} \frac{\partial^2 C_D}{\partial x_D^2} \qquad \text{..........} \qquad (1)$$

$$C_D = \frac{C - C_I}{C_J - C_I} \qquad \text{..........} \qquad (2)$$

$$x_D = \frac{x}{L} \qquad \text{..........} \qquad (3)$$

$$t_D = \frac{ut}{\phi L} \qquad \text{..........} \qquad (4)$$

$$N_{Pe} = \frac{uL}{\phi D_\ell} \qquad \text{..........} \qquad (5)$$

NOMENCLATURE

C = concentration (mol L^{-3})
D_0 = molecular diffusivity ($L^2 t^{-1}$)
D_ℓ = longitudinal dispersion coefficient ($L^2 t^{-1}$)
L = model length (L)
num = numerical
N_{Pe} = Peclet number, dimensionless
t = time (t)
u = Darcy velocity ($L\, t^{-1}$)
x = distance (L)
α = dispersivity (L)
ϕ = porosity, fraction

Initial & Boundary Conditions

IC:

$$C_D(x_D,0)=0 \qquad x_D>0 \quad \text{...}(6a)$$

BCs:

$$C_D(0,t_D)=1 \qquad t_D>0 \quad \text{...}(6b)$$

$$\frac{\partial C_D}{\partial x_D}=0 \qquad x_D=1 \quad \text{...}(6c)$$

- C, C_I, and C_J indicate current, initial, and injected concentrations.
- L denotes the distance over which longitudinal dispersion, D_l, is measured.
- The dimensionless time, t_D, corresponds physically to the number of cumulative pore volumes injected into the medium.

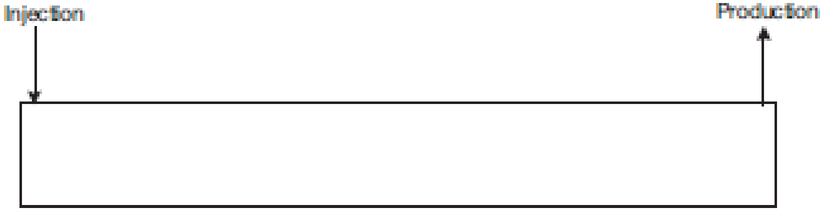
Analytical Solution

$$\begin{aligned}
C_D(x_D,t_D) = \frac{1}{2}\Bigg[&\operatorname{erfc}\left\{\frac{1}{2}\sqrt{\frac{N_{Pe}}{t_D}}(x_D - t_D)\right\} + (\exp(N_{Pe}x_D))\operatorname{erfc}\left\{\frac{1}{2}\sqrt{\frac{N_{Pe}}{t_D}}(x_D + t_D)\right\}\Bigg]\Bigg] \\
&+\Bigg[\left\{\frac{N_{Pe}}{2}(2 - x_D + t_D) + 1\right\}(\exp(N_{Pe}))\operatorname{erfc}\left\{\frac{1}{2}\sqrt{\frac{N_{Pe}}{t_D}}(2 - x_D + t_D)\right\} \\
&-\sqrt{\frac{N_{Pe}t_D}{\pi}}\exp\left\{\frac{-N_{Pe}}{4}\left(\frac{(2 - x_D)^2}{t_D} - 2x_D + t_D\right)\right\}\Bigg]\Bigg] \\
&-\Bigg[\left\{\frac{N_{Pe}}{2}(2 + x_D + t_D) + 1\right\}(\exp(N_{Pe}))\operatorname{erfc}\left\{\frac{1}{2}\sqrt{\frac{N_{Pe}}{t_D}}(2 + x_D + t_D)\right\} \\
&-\sqrt{\frac{N_{Pe}t_D}{\pi}}\exp\left\{\frac{-N_{Pe}}{4}\left(\frac{(2 + x_D)^2}{t_D} - 2x_D + t_D\right)\right\}\Bigg]\Bigg]
\end{aligned} \tag{7}$$

The solution has the form of an infinite series of error functions where successive terms arise from the superimposed reflections at the outlet. The series converges rapidly for Peclet numbers of interest and can be accurately approximated by the first three terms.

Assignment No. 4: Reproduce the analytical parts of the paper

Reservoir Models



The reservoir models are set up as shown in Figure 1, wherein solvent is injected at one end and production is obtained from the other end of the model, with the production well(s) kept at a constant bottom-hole pressure. Table 1 provides model details.

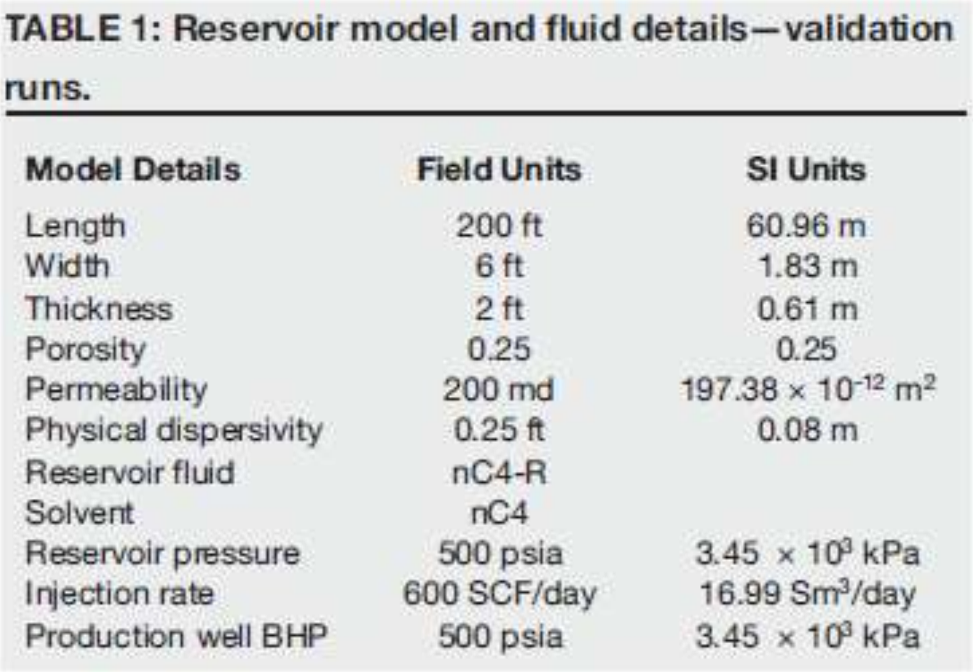


TABLE 1: Reservoir model and fluid details—validation runs.

Model Details	Field Units	SI Units
Length	200 ft	60.96 m
Width	6 ft	1.83 m
Thickness	2 ft	0.61 m
Porosity	0.25	0.25
Permeability	200 md	197.38×10^{-12} m^2
Physical dispersivity	0.25 ft	0.08 m
Reservoir fluid	nC4-R	
Solvent	nC4	
Reservoir pressure	500 psia	3.45×10^3 kPa
Injection rate	600 SCF/day	16.99 Sm^3/day
Production well BHP	500 psia	3.45×10^3 kPa

The reservoir and injected fluids have exactly the same thermodynamic properties and consist of a single component, which is named as nC4-R in the reservoir and nC4 as the only component in the injected solvent.

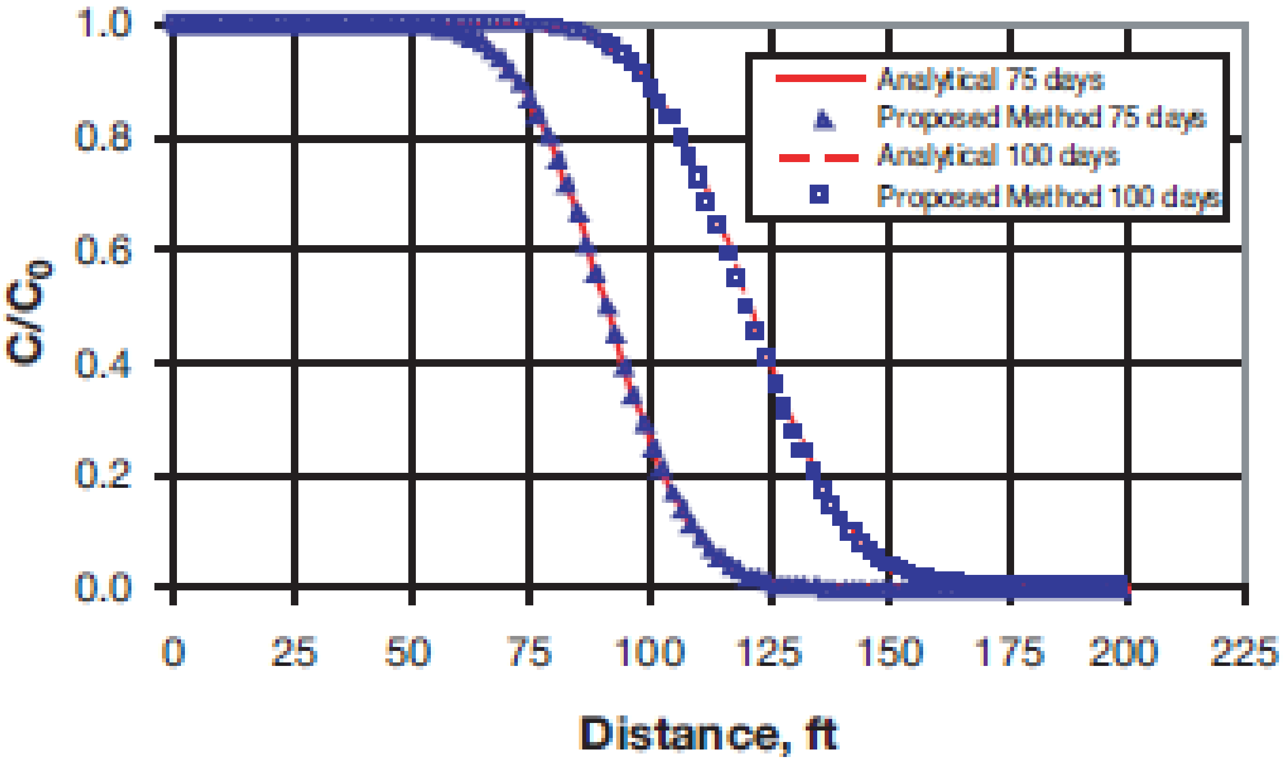


FIGURE 2: Comparison of analytical solution with 1D model using single point upstream weighting. The analytical dispersivity = Lantz dispersivity + physical dispersivity.

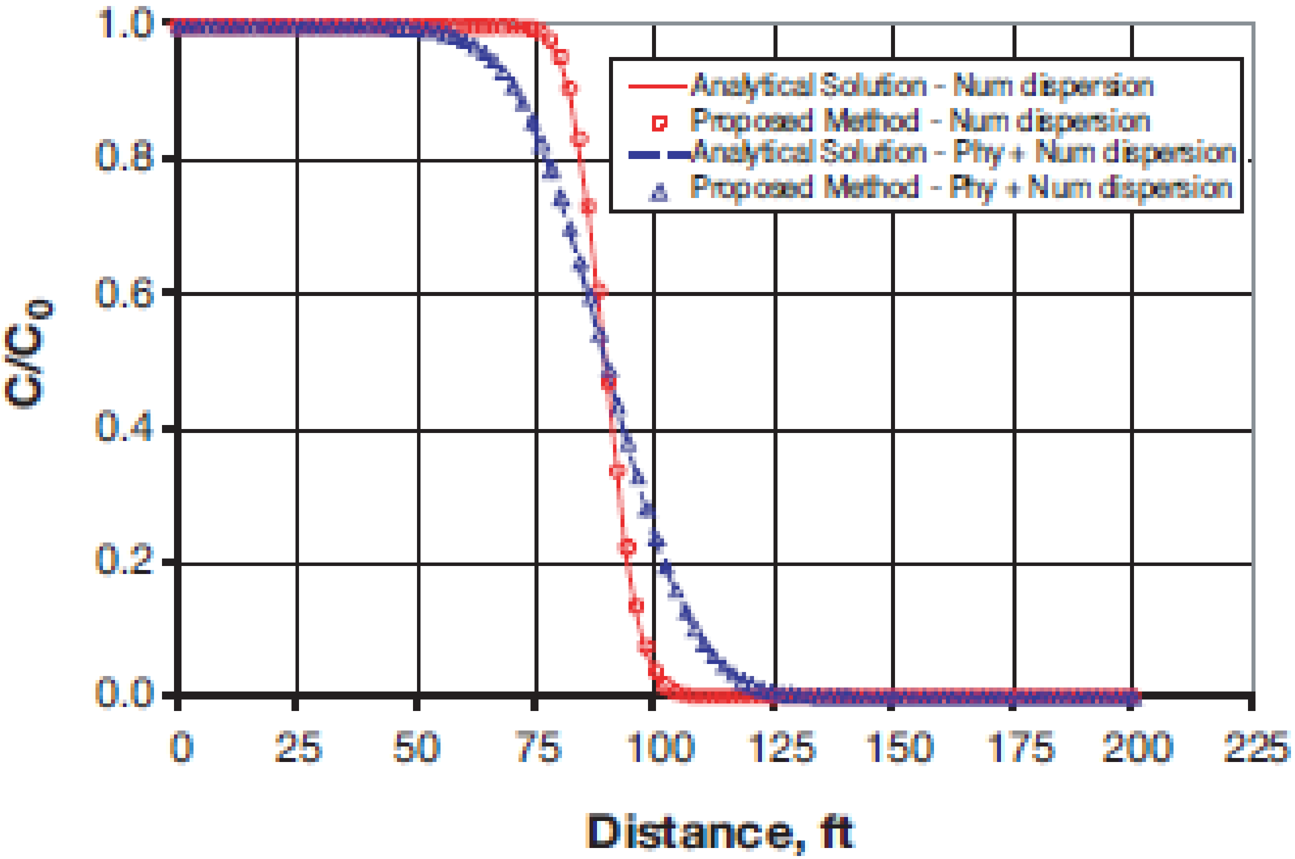
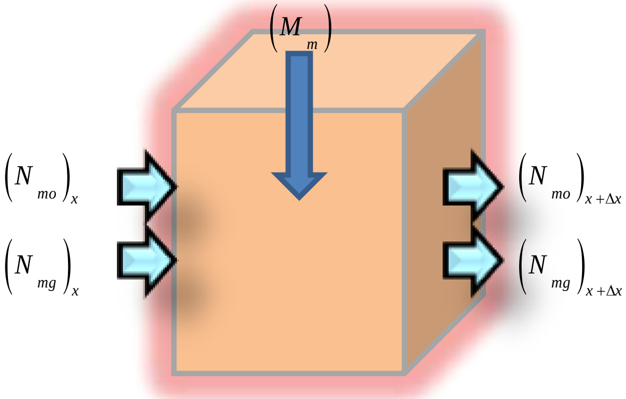


FIGURE 3: Comparison of analytical solution with 1D model for numerical and physical dispersion. The analytical dispersivity = Lantz dispersivity for solid curve, and analytical dispersivity = Lantz + physical dispersivity for the dashed curve.

Mass Balance

Moles Of component “m” entering the CV − Moles Of component “m” leaving the CV + Molar Source/sink Component “m” =

Accumulation Of component “m” from t to t+Δt



$$-\frac{\partial}{\partial x}\left(x_m\bar{\rho}_o v_{mo}+y_m\bar{\rho}_g v_{mg}\right)+\frac{Q_m}{V_b}=\frac{\partial}{\partial t}\left[\phi\left(S_o x_m\bar{\rho}_o+S_g y_m\bar{\rho}_g\right)\right]$$

Multi-Mechanistic Flow Consideration

$$v_{mo} = v_o^D + v_{mo}^F$$

diffusion phenomena in the liquid phase can be about hundred times less significant than diffusive effects within a gas phase, therefore,

$$v_{mg} = v_g^D + v_{mg}^F$$

In this work we are going to consider the combined action of the diffusion of all chemical composition that results in a net diffusive motion of the phase as a whole, rather than diffusion of each of the species alone. Therefore,

$$v_{mo} = v_o^D$$

$$v_{mg} = v_g^D + v_g^F$$

$$-\frac{\partial}{\partial x}\left(x_m \bar{\rho}_o v_{mo} + y_m \bar{\rho}_g v_{mg}\right) + \frac{Q_m}{V_b} = \frac{\partial}{\partial t}\left[\phi\left(S_o x_m \bar{\rho}_o + S_g y_m \bar{\rho}_g\right)\right]$$

Darcy's Law

$$\begin{cases} v_g = \dfrac{k\,k_{rg}}{\mu_g}\dfrac{\partial p_g}{\partial x} \\[2ex] v_o = \dfrac{k\,k_{ro}}{\mu_o}\dfrac{\partial p_o}{\partial x} \end{cases}$$

Fick's Law

$$v_g^{eff} = -\phi\, S_g \frac{D_{eff}}{\bar{\rho}_g}\frac{\partial \bar{\rho}_g}{\partial x}$$

$$\frac{\partial}{\partial x}\left(x_m \bar{\rho}_o A \frac{k\,k_{ro}}{\mu_o}\frac{\partial p_o}{\partial x} + y_m \bar{\rho}_g A\, \frac{k\,k_{rg}}{\mu_g}\frac{\partial p_g}{\partial x} + y_m S_g A\, \frac{D_{eff}}{5.615}\frac{\partial \bar{\rho}_g}{\partial x}\right)\Delta x$$

$$+M_m = \frac{V_b}{5.615}\frac{\partial}{\partial t}\left[\phi\left(S_o x_m \bar{\rho}_o + S_g y_m \bar{\rho}_g\right)\right]$$

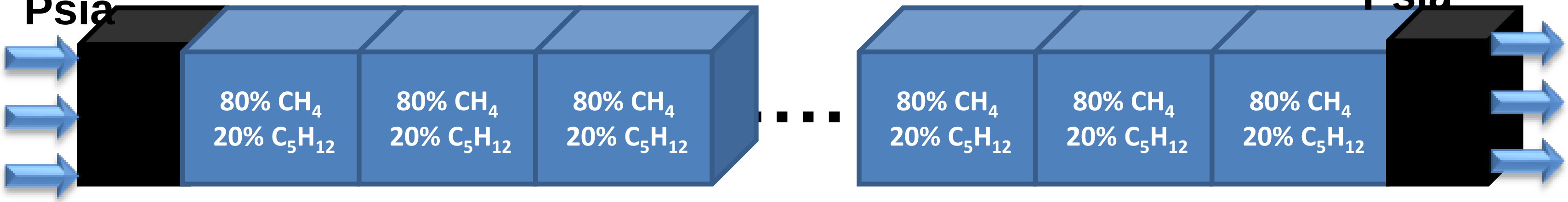
$$m = 1, 2, \ldots, n_c$$

Two-component, Two-phase

$$\phi = 0.13,\ L = 1004\ ft$$

P=3500 Psia

P=1200 Psia



K=5000 md

K=5000 md

$$\frac{\partial}{\partial x}\left(x_m \bar{\rho}_o A_x \frac{k_x k_{ro}}{\mu_o}\frac{\partial p}{\partial x} + y_m \bar{\rho}_g A_x \frac{k_x k_{rg}}{\mu_g}\frac{\partial p}{\partial x} + y_m S_g A_x \frac{D_{eff}}{5.615}\frac{\partial \bar{\rho}_g}{\partial x}\right)\Delta x$$

$$= \frac{V_b}{5.615}\frac{\partial}{\partial t}\left[\phi\left(S_o x_m \bar{\rho}_o + S_g y_m \bar{\rho}_g\right)\right]$$

$$\begin{cases} p_R = 3500 \\ p_L = 1200 \end{cases}$$

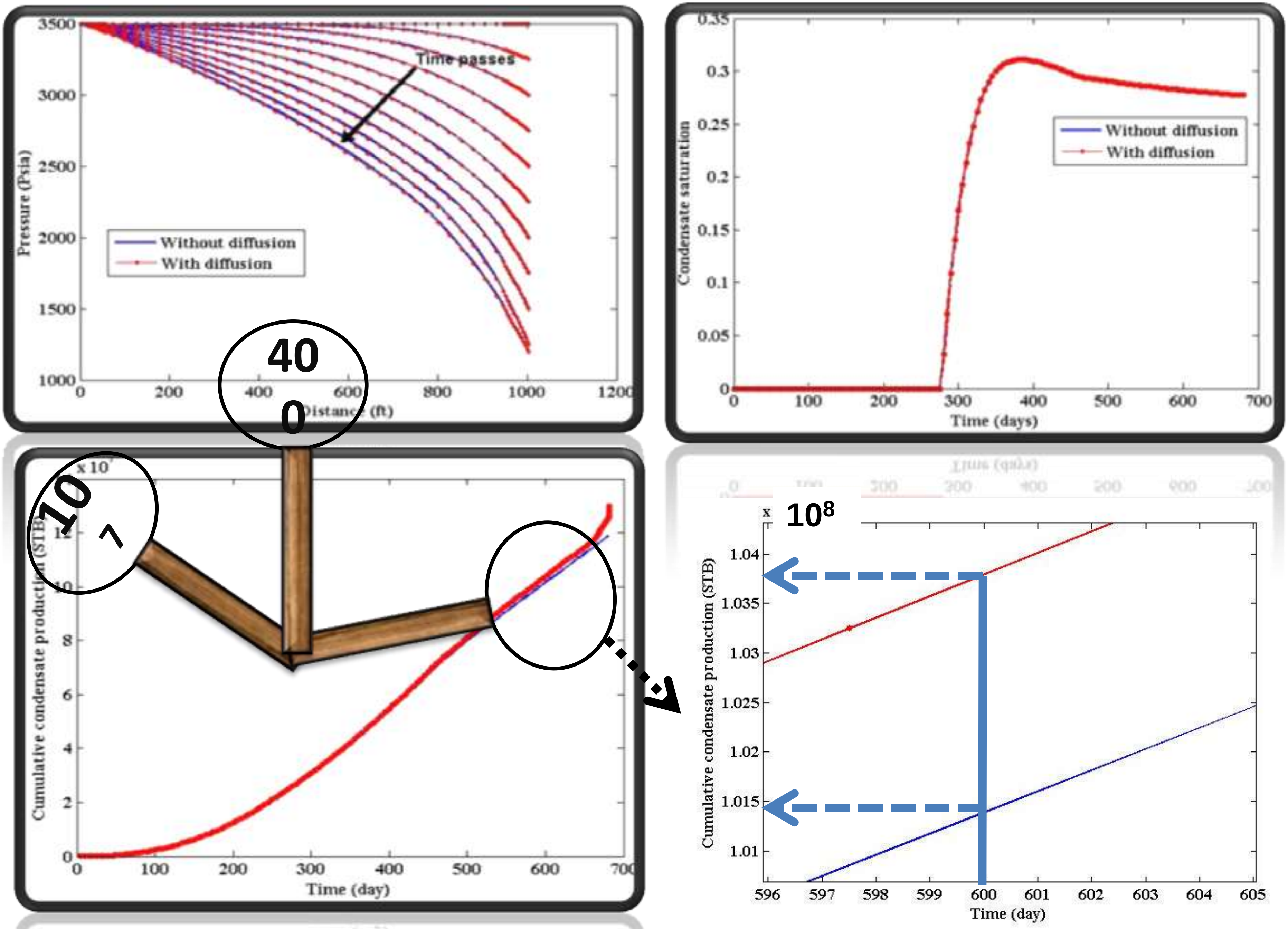
$$p_{initial} = 3500$$

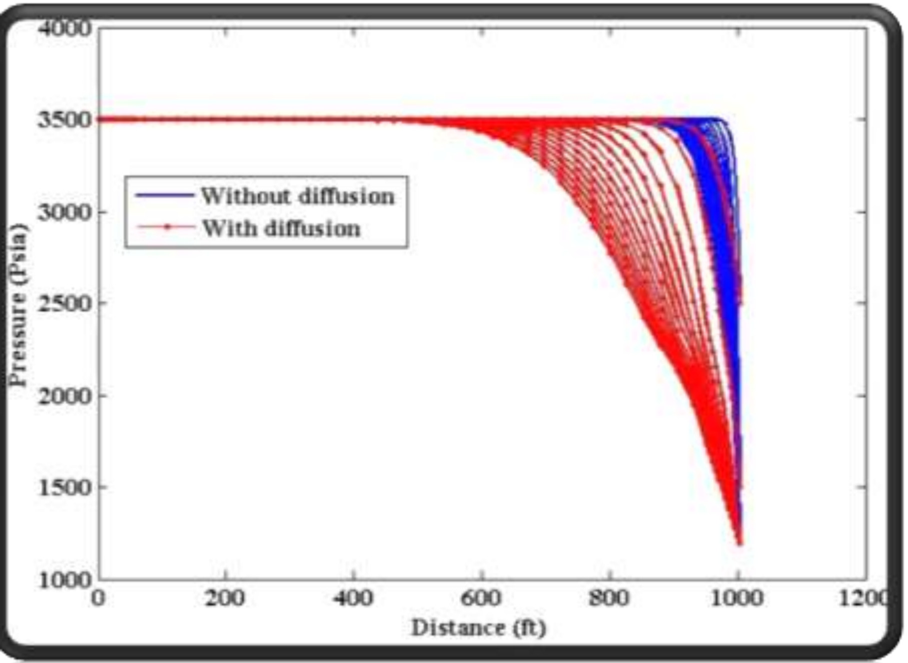
$$p = \frac{RT}{v-b} - \frac{a_c \alpha}{v(v+b)+b(v-b)}$$

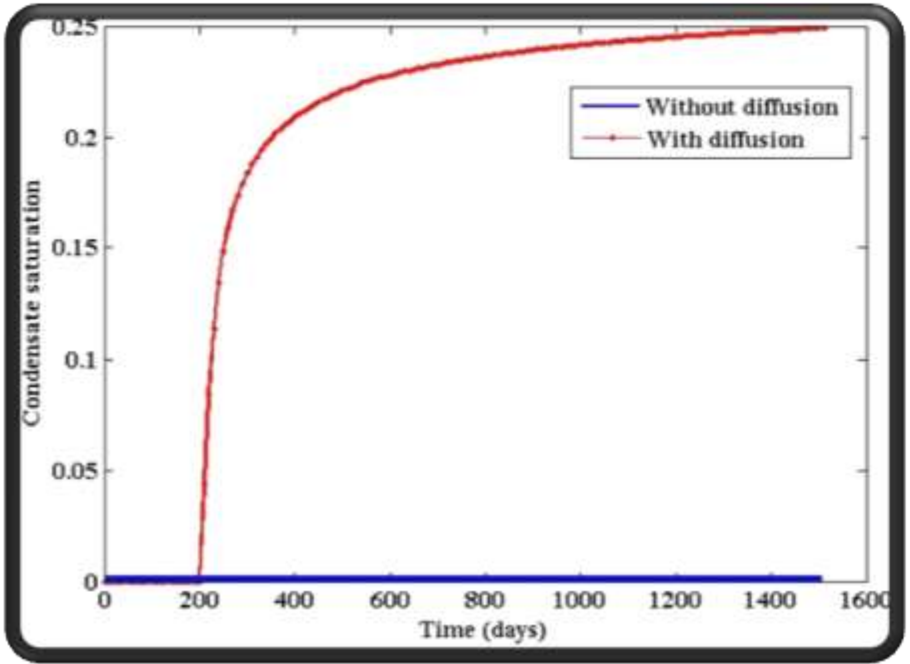
Newton – Raphson's linearization method

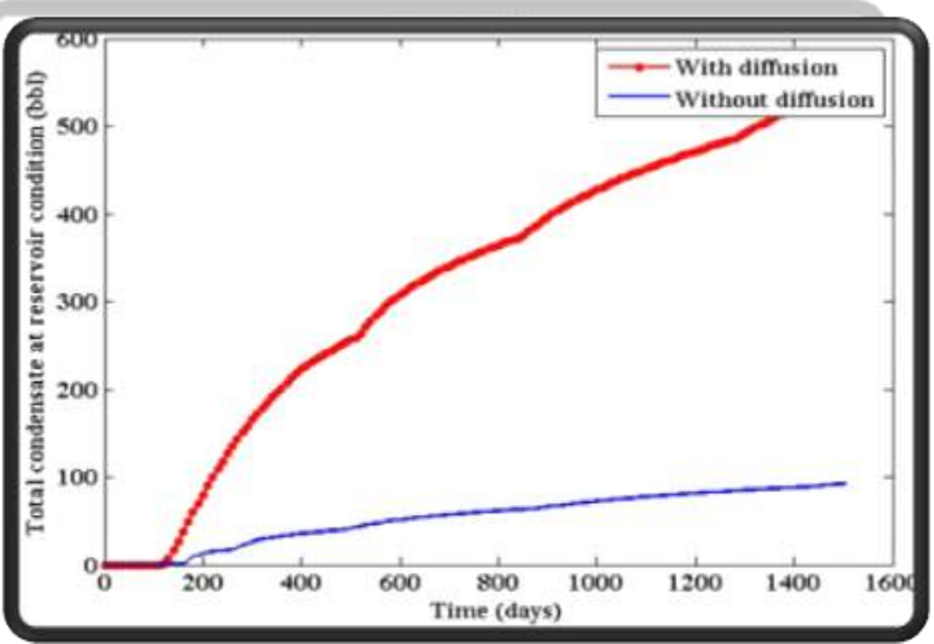
Broyden's method

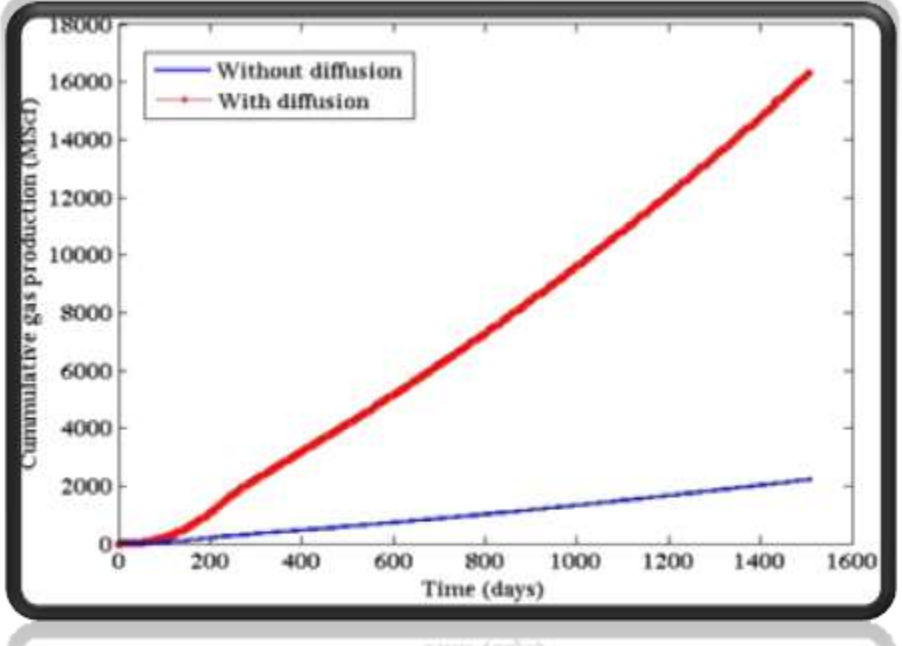
Special equation ordering

3500
3000
2500
2000
1500
1000
Pressure (Psia)
0
200
400
600
800
1000
1200
Distance (ft)
Time passes
Without diffusion
With diffusion
40
0
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Condensate saturation
0
100
200
300
400
500
600
700
Time (days)
Without diffusion
With diffusion
x 10^7
10
7
12
10
8
6
4
2
0
Cumulative condensate production (STB)
0
100
200
300
400
500
600
700
Time (day)
x 10^8
1.04
1.035
1.03
1.025
1.02
1.015
1.01
Cumulative condensate production (STB)
596
597
598
599
600
601
602
603
604
605
Time (day)


Pressure (Psia)
Distance (ft)
4000
3500
3000
2500
2000
1500
1000
0
200
400
600
800
1000
1200
Without diffusion
With diffusion


Condensate saturation
Time (days)
0.25
0.2
0.15
0.1
0.05
0
0
200
400
600
800
1000
1200
1400
1600
Without diffusion
With diffusion


Total condensate at reservoir condition (bbl)
Time (days)
600
500
400
300
200
100
0
0
200
400
600
800
1000
1200
1400
1600
With diffusion
Without diffusion


Cummulative gas production (MScf)
Time (days)
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0
200
400
600
800
1000
1200
1400
1600
Without diffusion
With diffusion

With diffusion

Without diffusion

500 days

1000 days

1500 days