الزکات العلم نشرهُ

Sharif University of Technology
SPE Student Chapter

حفاری ۲

نام استاد: دکتر داوود خوزانی

نام دانشگاه: دانشگاه تهران

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مهندسی حفاری ۲ - جلسه اول

drilling Engineering 2

drilling Hydraulics

casing Design

midterm ← سه‌شنبه ۲۰ آبان

۵ تا ۷ نمره

final ← ۱۰ تا ۱۲ نمره

project ← ۳ نمره

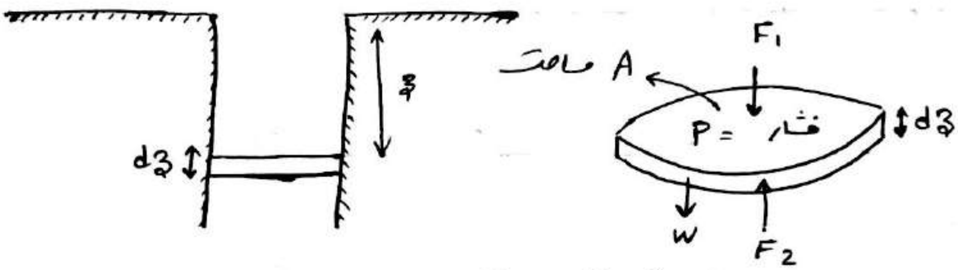
Applied drilling Engineering

Fundamental of drilling engineering

sci_hub.ac/ec

B_ok.org

Hydrostatic



$$F_1 = P \cdot A$$

$$F_2 = (P + \Delta P) \cdot A$$

$$\Delta P = \frac{dP}{dz} dz$$

$$W = \rho g \cdot A \cdot dz$$

$$F_1 + W = F_2$$

$$P \cdot \cancel{A} + \rho \cdot g \cdot \cancel{A} \cdot dz = \left(P + \frac{dP}{dz} dz\right) \cdot \cancel{A}$$

$$\cancel{P} + \rho \cdot g \cdot dz = \cancel{P} + \frac{dP}{dz} dz$$

$$\rho g \cdot \cancel{dz} = \frac{dP}{dz} \cancel{dz} \longrightarrow \boxed{\frac{dP}{dz} = \rho \cdot g}$$

$$\frac{dP}{dz} = 0.052 \rho$$

P : Psi

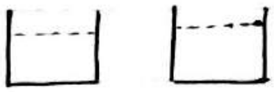
dz = ft

ρ = lbm/gal (PPg)

in Compressible fluid

$$\frac{dP}{dz} = 0.052\,\rho \rightarrow dP = 0.052\,\rho . dz \rightarrow \int_{P_0}^{P} dP = \int_{z_0}^{z} 0.052\,\rho\, dz$$

$$\rightarrow P - P_0 = 0.052\,\rho(z - z_0) \rightarrow P = \underbrace{P_0}_{14.7\ \text{Psia}} + 0.052\,\rho\underbrace{(z - z_0)}_{\Delta z}$$



با فضای آزاد ارتباط داشته باشد

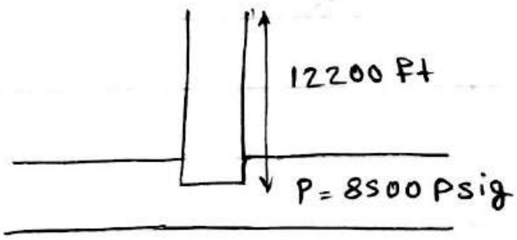
mud pit

(Psig) gauge فشار $\leftarrow P = 0.052\,\rho\,\Delta z$

(Psia) absolute فشار $\leftarrow P = 14.7 + 0.052\,\rho\,\Delta z$

در گازها از Psia استفاده می‌کنیم و Psig معنایی ندارد.

Example ← چگالی گل را به گونه‌ای به دست بیاورید که چاه Kick نکند؟



هدف اصلی گل حفاری:
جلوگیری از Kick کردن چاه

$\rho_{mud} = ?$ to prevent Kick

$P_{hh} \geq 8500$ Psig

$$P_{hh} = 0.052\,\rho.\,12200 \geq 8500 \text{ Psig} \rightarrow \rho \geq \frac{8500}{0.052 \times 12200}$$

$\rho_{mud} \geq 13.4$ PPg

چگالی زیاد شود مخزن را می‌شکند چون از فشار شکست
لایه‌ها بیشتر می‌شود

چگالی کم باشد چاه Kick می‌کند.

2/

Compressible fluid (gas)

$$P \cdot V = z n R T$$

$$P \cdot V = z \frac{m}{M} R T$$

$$\frac{V}{m} = \frac{z R T}{P M} \longrightarrow \frac{1}{\rho} = \frac{z R T}{P M}$$

$$R = 1545 \ \frac{lbf/ft^2 \ ft^3}{lbm.mol \ . \ R}$$

$$R = 80.3 \ \frac{Psia.gal}{lbmmol \cdot R} = 8.3144 \ \frac{Pa \cdot m^3}{mol \cdot K} = 8314.4 \ \frac{Pa \ m^3}{Kmol \cdot K}$$

Field: $\rho = \dfrac{PM}{80.3 \ zT}$, ρ: ppg , P: Psia , T: R

$$dP/dz = \rho \cdot g = \frac{PMg}{zRT} \longrightarrow \frac{dP}{P} = \frac{Mg}{zRT} dz \rightarrow \int_{P_0}^{P} \frac{dP}{P} = \int_{z_0}^{z} \frac{Mg}{zRT} dz$$

$$\rightarrow \ln P/P_0 = \frac{Mg}{R} \int_{z_0}^{z} dz/zT$$

فرض می‌کنیم تغییرات T ناچیز باشد.

فرض می‌کنیم تغییرات z ناچیز بوده و میانگین آن $\bar{z}$ باشد.

$$\ln \frac{P}{P_0} = \frac{Mg}{\bar{z} R T} (z - z_0)$$

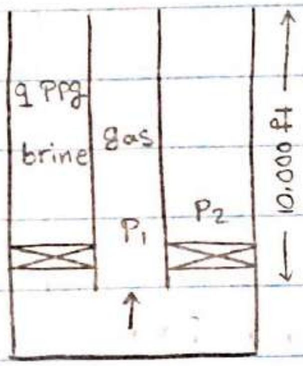
$$P = P_0 \exp\left[\frac{Mg}{\bar{z} R T}(z - z_0)\right] : SI \rightarrow P = P_0 \exp\left(\frac{M \Delta z}{1554 \ \bar{z} T}\right) : \text{Fluid unit}$$

Psia, lbm/lbm.mol, ft, R

Example: - tubing fluid with methane gas @ 10,000 ft

- annular space filled with 9 ppg brine
- surface tubing pressure = 1,000 psia
- assume ideal gas ($z = 1$)
- gas temprature is 140°F

if the collapse resistance of the tubing is 8,330 psia : will the tubing collapse ?



@ 1,000 ft

$$P_2 = P_0 + 0.052\,\rho\,\Delta z = 14.7 + 0.052 \times 9 \times 10{,}000$$

$$= 4695 \text{ Psia}$$

هنوز فشار اتمسفر نیست
فشاری است که نسبت به آن
محاسبات انجام می‌شود. (در اینجا سطح)

$$P = P_0 \exp\left(\frac{M\,\Delta z}{1554\,z\,T}\right)$$

$$= 1{,}000 \exp\left(\frac{16 \times 10{,}000}{1544 \times 1\,(460+140)}\right) = 1188 \text{ Psia}$$

$$\Delta P = P_2 - P_1 = 4695 - 1188 = 3507 \text{ Psia} < 8330 \text{ Psia}$$

tubing will not collapse

فرض استفاده از معادله incompressible

$$\rho_{@\,surface} = \frac{PM}{80.3\,zT} = \frac{1{,}000 \times 16}{80.3 \times 1\,(460+140)} = 0.331 \text{ ppg}$$

$$P_1 = 0.052 \times \rho \times \Delta z + 1{,}000 = 0.052 \times 0.331 \times 10{,}000 + 1{,}000$$

$$= 1172 \text{ Psia}$$

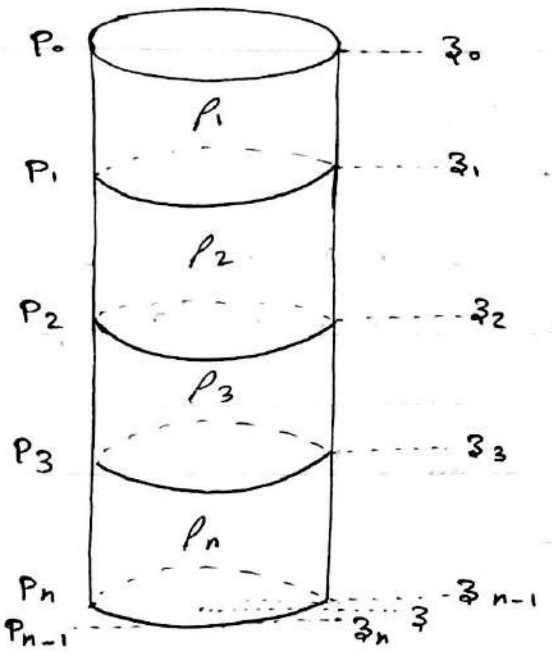
صرفاً جهت اطلاع:

۱- اگر فشار بالای 1000 Psia باشد
۲- تغییرات عمق شدید باشد

⇐ می‌توان از معادله‌ی فشار سیال تراکم ناپذیر در گازها استفاده نمود.

3/

Hydrostatic pressure in complex fluid system



$$P_1 = 0.052\,\rho_1\,(z_1 - z_0) + P_0$$

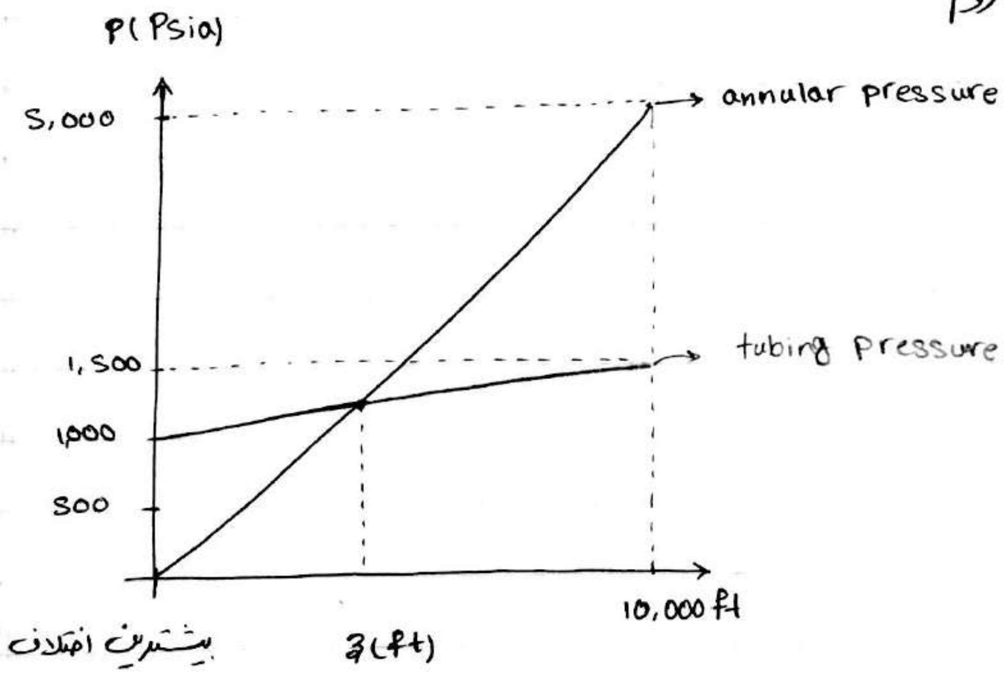
$$P_2 = 0.052\,\rho_2\,(z_2 - z_1) + P_1$$

$$P_2 = 0.052\,\rho_2\,(z_2 - z_1) + 0.052\,\rho_1\,(z_1 - z_0) + P_0$$

$$z_{n-1} < z < z_n$$

$$P = P_0 + 0.052 \sum_{i=1}^{n-1} \rho_i\,(z_i - z_{i-1}) + 0.052\,\rho_n\,(z - z_{n-1})$$

صفحه ۲. جلسه دوم



بیشترین اختلاف
فشار در سطح

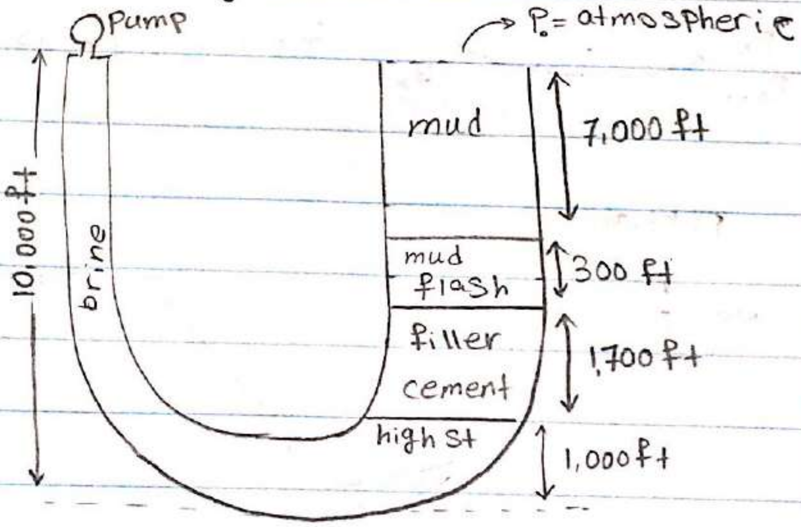
example: cementing operation @ 10,000 ft

well contains 10.5 ppg mud. mud willbe displaced from annulus by: برای ایجاد فاصله بین گل و سیمان

1. 300 ft of 8.5 ppg mud flash
2. 1,700 ft of 12.7 ppg filler cement
3. 1,000 ft of 16.7 ppg high strength cement

the high strength cement will be displaced from casing by 9 ppg brine

Calculate the pump pressure required to completely remove cement from casing.



② فشار از بالا به پایین +
از پایین به بالا −

① $P_{pump,g} + 0.052\, \overset{9}{\rho_{brine}} \times 10{,}000 = 0 + 0.052\,(\overset{10.5}{\rho_{mud}} \times 7{,}000 + \overset{8.5}{\rho_{mf}} \times 300 + \underset{12.7}{\rho_{fc}} \times 1{,}700 + \underset{16.7}{\rho_{hs}} \times 1{,}000)$

② $0 + 0.052(\rho_{mud} \times 7{,}000 + \rho_{mf}\, 300 + \rho_{fc}\, 1700 + \rho_{hs}\, 1000 - \rho_{brine}\, 10{,}000) = P_{pump,g}$

4/

equivalent density

$$\rho_e = \frac{P}{0.052\, z}$$

درمثال قبل $P = P_{pump} + 0.052 \times 9 \times 10{,}000 = 5946$ Psig

$$\rho_e = \frac{5946}{0.052 \times 10{,}000} = 11.4 \text{ ppg}$$

effect of entrained solids, liquids and gases in drilling fluid

ideal mixture

$$\rho = \frac{\Sigma m_i}{\Sigma v_i} = \frac{\Sigma \rho_i v_i}{\underbrace{\Sigma v_i}_{v_t}} = \Sigma \rho_i \frac{v_i}{v_t} = \Sigma \rho_i f_i$$

if N_v moles of gas are displaced in $1\,m^3$ of drilling fluid:

$$\rho = \Sigma \rho_i f_i \qquad f_g = \frac{V_g}{V_g + V_m} = \frac{z N_v RT/P}{z N_v RT/P + 1} = \frac{z N_v RT}{P + z N_v RT}$$

$$f_g + f_m = 1$$

$$\rho_g = \frac{PM}{zRT} \longrightarrow \bar{\rho} = \rho_m f_m + \rho_g f_g = \rho_m (1 - f_g) + \rho_g f_g$$

$$= \rho_m \left(1 - \frac{z N_v RT}{P + z N_v RT}\right) + \frac{PM}{zRT} \times \frac{z N_v RT}{P + z N_v RT}$$

$$\bar{\rho} = \frac{P(\rho_m + z N_v)}{P + z N_v RT}$$

$$\frac{dP}{dz} = \rho g \qquad \int_{z_1}^{z_2} dz = \int_{P_1}^{P_2} \frac{dP}{\underbrace{dz}_{\rho g}} = \int_{P_1}^{P_2} \frac{P + z N_v RT}{g(\rho_f + M N_v)} \times \frac{dP}{P}$$

$$\rightarrow z_2 - z_1 = \frac{P_2 - P_1}{a} + \frac{b}{a} \ln \frac{P_2}{P_1}$$ معادلات مهم:

$a = g(\rho_m + M N_v)$: SI

$a = 0.052(\rho_m + M N_v)$: Feild

$b = \bar{z} . N_v . R\bar{T}$: SI , Feild

اگر گاز روی drilling fluid ← در اثر شکستن سازند گاز موجود در آن
وارد گل حفاری می‌شود.

گلی که داخل آن گاز دارد. ← gas cut mud

جلسه سوم

یادآوری جلسه قبل:

$$\bar{\rho} = \rho_m f_m + \rho_g f_g = \rho_m (1 - f_g) + \rho_g f_g$$

$$f_g = \frac{V_g}{V_g + V_m} \xrightarrow[\text{is dispered in } 1m^3 \text{ of drilling fluid}]{\text{if } N_v \text{ moles of gas}} = \frac{z N_v RT/P}{1 + z N_v RT/P}$$

$$\Rightarrow \rho_g = \frac{PM}{zRT} \Rightarrow \boxed{\bar{\rho} = \frac{P(\rho_m + M N_v)}{P + z N_v RT}}$$

حال برای این که تاثیر اختلاف چگالی روی فشار ته چاه را بفهمیم به معادلات زیر رجوع می‌کنیم

$$\frac{dP}{dz} = \rho g = \frac{P(\rho_m + M N_v)}{P + z N_v RT} \times g \rightarrow \frac{P + z N_v RT}{g P(\rho_m + M N_v)} dP = dz \xrightarrow{\int}$$

$$z_2 - z_1 = \frac{P_2 - P_1}{a} + b/a \ln \frac{P_2}{P_1}$$ معادلات بالای همین صفحه

S/

Eample: Sandstone: $\Phi = 0.2$, $\rho_s = 21.9$ ppg cuttings چگالی

$S_w = 0.3$: $\rho_w = 9$ ppg

سرعت حفاری

$S_g = 0.7$: gas in methane

ROP $= 50\ ft/h$ with a 9.875 in. bit @ 12,000 ft

$\rho_m = 14$ ppg , $Q_m = 350$ gal/min .

$\bar{T} = 620\ R$ چگالی گل حفاری

assume ideal gas behavoir

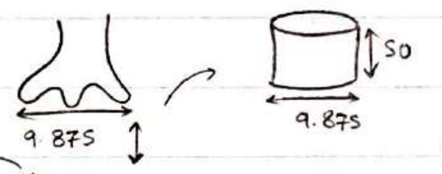
" both the gas and the rock cuttings move at the same annular velocity as the mud. مخلوط ایده‌آل باشد درست

calculate the bottom hole pressure?

estimate the pressure at the bottom $\rightarrow P = 0.052\,\rho g + P_0$

$= 0.052(14)(12{,}000) + 14.7$

$= 8751$ psia



سرعت

اول باید ببینیم نرخ حجم حفاری چقدر است:

(در یک دقیقه)

$$Q = \dot{V} = \text{ROP} \times A = \text{ROP} \times \frac{\pi}{4} d^2$$

$$= 50\ \frac{ft}{h} * \frac{\pi * 9.875^2\ in^2}{4} * \frac{1\ ft^2}{144\ in^2} * \frac{7.48\ gal}{1\ ft^3} * \frac{1\ hr}{60\ min} = 3.31\ gal/min$$

cutting نرخ حفاری

Q_s = volumetric rat of drilling * V سنگ $\rightarrow 1-\Phi$

$= 3.31 * (1-0.2) = 2.65$ gal/min

$Q_w = \dot{V}_{drilling} * \Phi * S_w = 3.31 * 0.2 * 0.3 = 0.2$ gal/min

$$\bar{\rho} = \frac{\sum \rho_i v_i}{\sum v_i} \xrightarrow{\div t} \bar{\rho} = \frac{\sum \rho_i\, v_i/t}{\sum v_i/t_i} = \frac{\sum \rho_i Q_i}{Q_i}$$

$$\bar{\rho} = \frac{\rho_m Q_m + \rho_w Q_w + \rho_s Q_s}{Q_m + Q_w + Q_s} = \frac{14*350 + 9*0.2 + 21.9*2.65}{350 + 0.2 + 2.65} = 14.057\ \text{ppg}$$

حال می‌خواهیم اثر گاز را مشاهده کنیم:

$$Q_g = \dot{V}_{drilling} * \Phi * S_g = 3.31 * 0.2 * 0.7 = 0.464 \text{ gal/min}$$

فرض می‌کنیم فشار ته چاه بدون اثر خورده است (فقط ناشی از گل) ← P اول مسئله

گاز متان است

$$\rho_g = \frac{PM}{80.3\,zT} = \frac{8751 * 16}{80.3 * 1 * 620} = 2.8 \text{ ppg}$$

حال باید N_v را بیابیم:

نرخ مولی ورود گاز

$$\dot{n}_g = \frac{\rho_g * Q_g}{M} = \frac{2.8 \text{ lb/gal} * 0.464 \text{ gal/min}}{16} = 0.081 \text{ mol/min}$$

$$N_v = \frac{n_g}{v_m} \xrightarrow{\div t} N_v = \frac{n_g/t}{v_m/t} = \frac{\dot{n}_g}{Q_m} = \frac{0.081}{350} = 0.000231 \text{ mol/gal}$$

در اثر گاز دو پارامتر داشتیم:

$$* \; a = 0.052 * (\bar{\rho} + M N_v)$$

$$= 0.052 * (14.057 + 16 * 0.000231) = 0.7312$$

$$* \; b = z N_v R T = 1 * 0.000231 * 80.3 * 620 = 11.5$$

$$z_2 - z_1 = \frac{P_2 - P_1}{a} + b/a \ln P_2/P_1$$

$$\underset{12{,}000}{z} = \frac{P_2 - 14.7}{0.7312} + \underbrace{\frac{11.5}{0.7312}}_{15.72} \ln P_2/14.7$$

معادله بالا به روش آزمون و خطا حل می‌شود.

$$f(n) = 0$$

$$\text{Newton-Raphson}: \; n_{new} = n_{old} - \frac{f(n_{old})}{f'(n_{old})}$$

$$f(P_2) = \frac{P_2 - 14.7}{0.7312} + 15.72 \ln \frac{P_2}{14.7} - 12000 \quad , \quad f'(P_2) = \frac{1}{0.7312} + 15.72 \, 1/P_2$$

$$n_{old} = 8750 \rightarrow n_{new} = 8750 - \frac{46.91}{1.3694} = 8715.7$$

$$n_{old} = 8715 \rightarrow n_{new} = 8715.7 - \frac{-0.0102}{1.3694} = \boxed{8715.7 \text{ psia} \checkmark = P_2}$$

$$\Delta P = P_2 - P_1 = 8715.7 - 8751 = -35.3$$

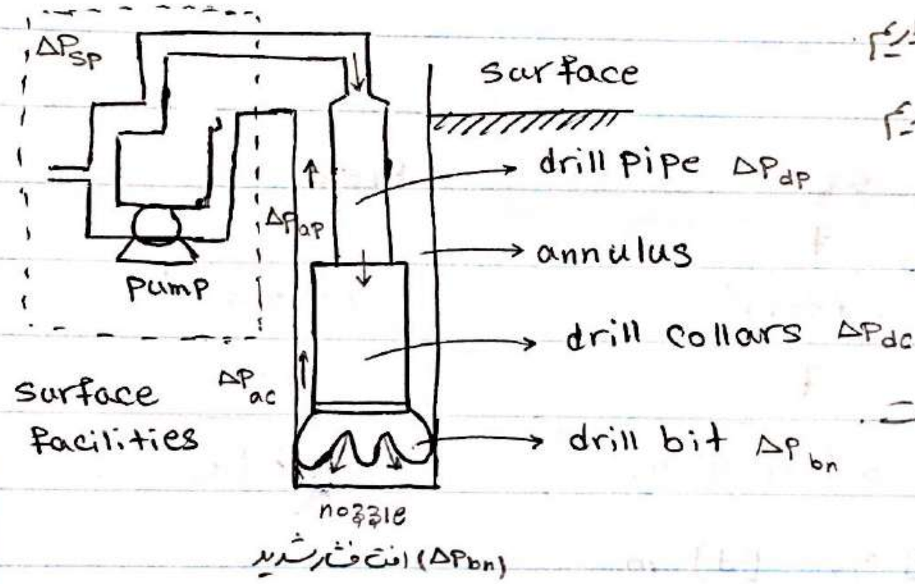
حال می‌خواهیم بفهمیم چگالی گل وقتی به سطح می‌رسد چه تغییری می‌کند؟

$$\bar{\rho} = \frac{P(\rho_f + M N_v)}{P + z N_v R T} = \frac{14.7\,(14.057 + 16 * 0.000231)}{14.7 + 0.000231 * 80.3 * 620} = 7.9 \text{ ppg}$$

کاهش چگالی چشمگیر است اما الزاماً به معنای Kick نیست.

جلسه چهارم

Steady flow of drilling fluid



هرجا قطر تغییر کرد ← ΔP داریم

در جهت جریان که پیش می‌رویم فشار کم می‌شود.

ΔP به Q وابسته است

جریان در حالت S.S. است.

$$\Delta P_{pump} = \Delta P_{sp} + \Delta P_{dp} + \Delta P_{dc} + \Delta P_{bn} + \Delta P_{ac} + \Delta P_{ab}$$

پارامترهای وابسته به محاسبه ΔP

۱- نوع جریان ← laminar , turbolent

۲- نوع رئولوژی

mass balance

هنگامی که جریان داریم:

$$\dot{m} = \rho \cdot \overset{\text{سرعت}}{v} \cdot A = cte$$

$$\underbrace{v \cdot A}_{\dot{V} = q}$$

incompressible fluid $\rightarrow \rho = cte$

$$\dot{m} = \cancel{\rho} \cdot v \cdot A \rightarrow vA = cte = q$$

محاسبه ی سرعت برای ما اهمیت دارد.

	SI	Field
pipe flow	$v = \dfrac{q}{0.7854 d^2}$	$v = \dfrac{q}{2.448 d^2}$
annular flow	$v = \dfrac{q}{0.7854 (d_2^2 - d_1^2)}$	$v = \dfrac{q}{2.448 (d_2^2 - d_1^2)}$

SI: $[q] = m^3/s$ $\quad [v]\ m/s \quad [d] = m$

field: $[q]$: gal/min (gpm) $\quad [v] = ft/s \quad [d] = in$

دبی تزریق گل همان دبی جریان گل است ← یعنی دبی در طول مسیر ثابت است.

example: $\rho_{mud} = 12\,ppg$

$Q_{pump} = 400\ gpm$

$OD_{dp} = 5\ in \qquad ID_{dp} = 4.33\ in$

$ID_{dc} = 2.5\ in$

$d_{bit} = 9.875\ in$

$V_{dp},\ V_{dc},\ V_{ap} = ?$

$$V_{dp} = \frac{q}{2.448\,d_{dp}^2} = \frac{400}{2.448\,(4.33)^2} = 8.715\ ft/s$$

$$V_{dc} = \frac{q}{2.448\,d_{dc}^2} = \frac{400}{2.448\,(2.5)^2} = 26.143\ ft/s$$

$$V_{ap} = \frac{q}{2.448\,(d_2^2 - d_1^2)} = \frac{400}{2.448\,(9.875^2 - 5^2)} = 2.253\ ft/s$$

قطر مته — قطر خارجی لوله حفاری

Energy Balance

closed system: $\Delta E - g\Delta z + \Delta V^2/2 + \Delta(PV) = W + Q$

$\Delta F_{friction} . \Delta E + \int_1^2 p\,dv - q$

$\int_1^2 v\,dp = \Delta P/\rho$

$\Delta P/\rho - g\Delta z + \Delta V^2/2 = W - \Delta F_{friction} \xrightarrow{\times\rho}$

$\Delta P - \rho g \Delta z + \rho/2\, V^2 = \rho W - \rho\,\Delta F_{friction}$

$P_1 - P_2 + 9.81\rho(z_2 - z_1) - 1/2\,\rho(V_2^2 - V_1^2) + \Delta P_{pump} - \Delta P_{friction} = 0$

$$\begin{cases} SI: & P_1 + 9.81\rho(z_2 - z_1) - 1/2\,\rho(V_2^2 - V_1^2) + \Delta P_{pump} - \Delta P_f = P_2 \\ field: & P_1 + 0.052\,\rho(z_2 - z_1) - 8.074\times10^{-4}\rho(V_2^2 - V_1^2) + \Delta P_{pump} - \Delta P_f = P_2 \end{cases}$$

حلب ينعم

example: ΔP_f of drillstring = 1400 psia

$q_m = 400$ gpm

$\rho_m = 12$ ppg

well depth = 10,000 ft

ID of drillcollars at the bottom of drillstring = 2.5 in

pressure increase of pump: 3000 psi

calculate pressure at the bottom of drilling

$$P_2 = P_1 + 0.052\, \rho (z_2 - z_1) - 8.074 \times 10^{-4} \rho (V_2^2 - V_1^2) + \Delta P_p - \Delta P_f$$

① mud pit ضخ

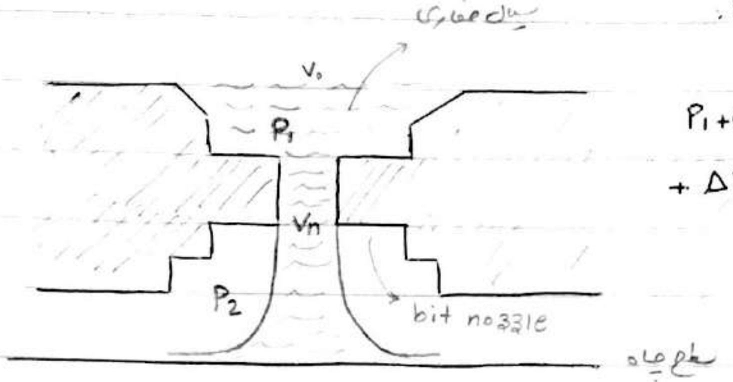
$P_1 = 0 \qquad V_1 = 0$

② $V_2 = \dfrac{q}{2.448\, d^2} = \dfrac{400}{2.448 \times 2.5^2} = 26.14$ ft/s

$$P = 0 + 0.052 \times 12 \times (10{,}000 - 0) - 8.074 \times 10^{-4} \times 12 \times (26.14^2 - 0)$$

$$+ 3{,}000 - 1400 = 7833 \text{ psig}$$

Flow through jet bits:



معادله انرژی بین دو نقطه P_1 ، P_2 :

$$P_1 + 0.052\,\rho(z_2 - z_1) - 8.074\times10^{-4}\rho(V_2^2 - V_1^2) + \Delta P_P - \Delta P_f = P_2$$

① سرعت V_0 در برابر V_n قابل صرف نظر کردن است.

② افت فشار ناشی از اصطکاک در داخل مسیر نازل بیت ناچیز است.

③ تغییرات عمق در مسیر نازل بیت ناچیز است.

پس معادله افت فشار به صورت زیر در می آید.

$$P_1 - 8.074\times10^{-4}\rho\, V_n^2 = P_2$$

$$\Delta P_b = P_1 - P_2 = 8.074\times10^{-4}\rho V_n^2 \Rightarrow V_n = \sqrt{\frac{\Delta P_b}{8.074\times10^{-4}\rho}}$$

در آزمایشات انجام شده مشخص شد که V_n دقیقاً با عبارت فوق یکی نیست

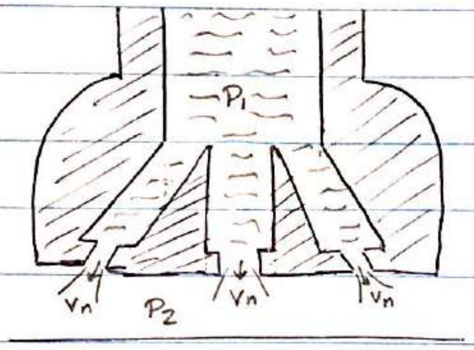
پس ضریب discharge (C_d) تعریف می شود (آزمایشگاهی)

C_d = discharge coefficient

$$V_n = C_d\sqrt{\frac{\Delta P_b}{8.074\times10^{-4}\rho}} \quad \text{(I)}$$

in field : $C_d \cong 0.95$
recomended value

موارد فوق برای افت فشار در یک نازل بود ولی اصولاً بیت هیچ وقت یک نازل ندارد و دارای چندین نازل است:



for each nozzle: $\Delta P_b = P_2 - P_1$

$\Rightarrow V_n$ is equal in all nozzle

$$V_n = \frac{q_1}{A_1} = \frac{q_2}{A_2} = \frac{q_3}{A_3}$$

$$q = q_t = q_1 + q_2 + q_3$$
$$= V_n A_1 + V_n A_2 + V_n A_3$$
$$= V_n (A_1 + A_2 + A_3)$$
$$= V_n A_t$$
$$\Rightarrow V_n = q / A_t \quad \text{SI :}$$

$$\text{II} \quad \underset{ft/s}{V_n} = \frac{\overset{gal/min}{q}}{3.117 \underset{in^2}{A_t}} \quad \text{: field}$$

$$\text{I} \; \text{II} \Rightarrow \boxed{\Delta P_b = \frac{8.311 \times 10^{-5} \rho q^2}{C_d^2 \, A_t^2}}$$ مستقل از نوع سیال و جریان

قطر نازل بیت معمولاً براساس $1/32$ in بیان می‌گردد.

example: $\rho_m = 12$ ppg

$q_m = 400$ gal/min

bit contains three $13/32$ in nozzle (13-13-13)

$\Delta P_b = ?$

$$\Delta P_b = \frac{8.311 \times 10^{-5} \overset{12\ ppg}{\rho} \overset{(400)^2}{q^2}}{\underset{(0.95)^2}{C_d^2} \; \underset{(0.3889)^2}{A_t^2}} = 1169 \text{ Psi}$$

$$A_t = \pi/4 \left(d_1^2 + d_2^2 + d_3^2 \right) = \pi/4 \left[(13/32)^2 + (13/32)^2 + (13/32)^2 \right] = 0.3889 \text{ in}^2$$

Bit hydroulic Power

توان مصرفی (هدر رفت) مته

$$(P_H)_b = \frac{\Delta P_b\, q}{1714}$$

$[P_H]$: hp

$[\Delta P_b]$: Psi

$[q]$: gpm

example: $\Delta P_p = 3000$ Psi

$\Delta P_f = 1400$ Psi

$q = 400$ gpm

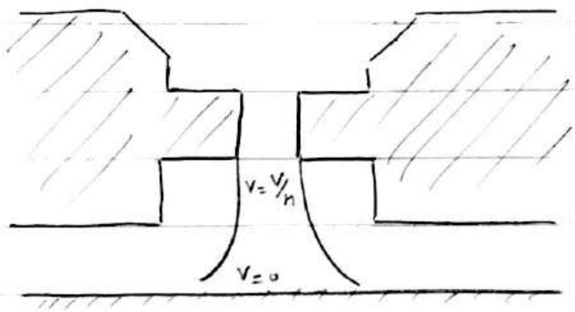
توان تولیدی پمپ

$$(PH)_{Pump} = \frac{\Delta P_p\, q}{1714} = 700 \text{ hp}$$ توان کل

$$(PH)_f = \frac{\Delta P_f\, q}{1714} = 327 \text{ hp}$$ مصرفی اصطکاک

Bit hydrolic impact force

جلسه ششم



$$F_j = \frac{\Delta(mv)}{\Delta t} = \frac{m}{\Delta t}\Delta v = \frac{\rho V}{\Delta t}\Delta v = \rho q \Delta v = \rho q v_n \quad : \text{SI}$$

$v_n - 0 = v_n$

نیروی که کل به کف چاه وارد می کند

$$F_j = \frac{\rho q v_n}{1930.2} \quad : \text{Feild}$$

$$v_n = C_d \sqrt{\frac{\Delta P_b}{8.074 \times 10^{-4} \rho}}$$

$$\Rightarrow F_j = 0.01823\, C_d\, q \sqrt{\rho\, \Delta P_b}$$

ppg

Psi

lbf

gpm

example: Calculate F_j in previous example

$$F_j = 0.01823\, \underset{0.95}{C_d}\, \overset{400}{q} \sqrt{\underset{12}{\rho}\, \Delta P_b^{\,1169}} = 820 \text{ lbf}$$

$F_j = \rho q v_n$ اگر چند نازل داشته باشیم:

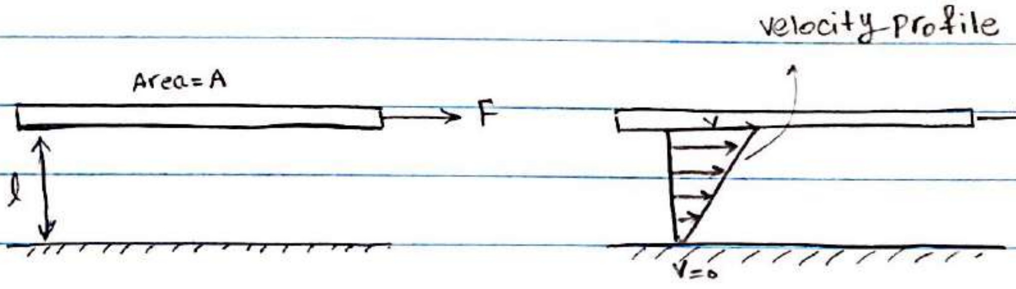
$F_{j_1} = \rho q_1 v_{n_1}$

$F_{j_2} = \rho q_2 v_{n_2} \Rightarrow F_j = F_{j_1} + F_{j_2} + F_{j_3}$

$F_{j_3} = \rho q_3 v_{n_3} \qquad = \rho (q_1 + q_2 + q_3) v_n$

Rheological models of drilling fluids

1- Newton Fluid model

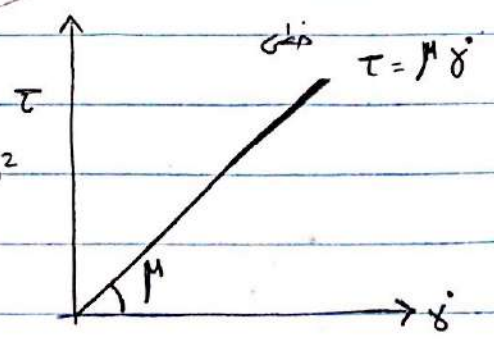


$$F/A = \mu \frac{V}{l} \Rightarrow \tau = F/A \quad \text{shear stress}$$

$$\dot{\gamma} = \frac{dV}{dl} \simeq \frac{V}{l} \quad \text{shear rate}$$

$\Rightarrow \tau = \mu \dot{\gamma}$ for newton fluid, laminar flow

μ → viscosity



cgs: 1 poise = 1 dynes.s/cm²

SI: Pa.s = 10 P

Feild: CP = 0.01 P

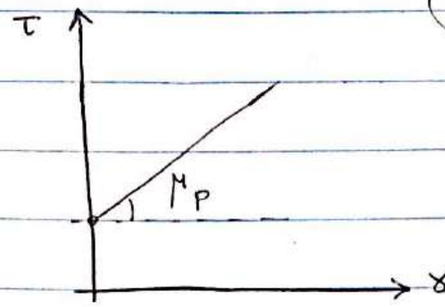
$$\frac{\text{lbf.s}}{\text{ft}^2} = 47.883 \text{ Pa.s} = 47\,883 \text{ cp}$$

2- Bingham plastic fluid

$$\tau = \tau_y + \mu_p \dot{\gamma} \qquad \tau > \tau_y$$

Plastic viscosity

yield point, yield stress — laminar flow



تفاوت bingham با newton این است که یک تنش برشی آستانه دارد و برای حرکت سیال باید به آن آستانه برسیم

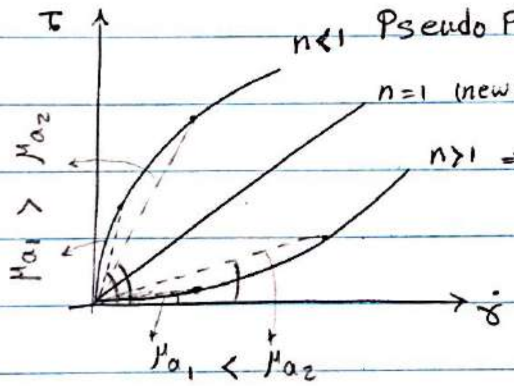
$[\mu_p]$: Feild: cp SI: Pa.s

$[\tau_y]$ Feild: $\frac{lbf}{100\,ft^2} = 0.4788\ Pa$ SI: Pa

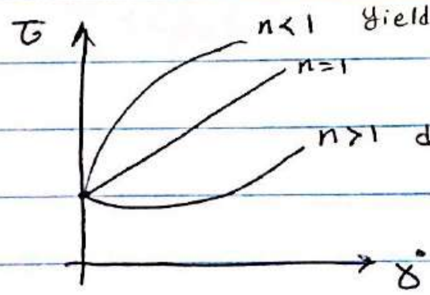
cgs: $\frac{dyne \cdot s^n}{cm^2}$ = eq.cp = [Consistency index] = $\frac{lbf \cdot s^n}{ft^2}$

3- Powel law fluid $\tau = K \dot{\gamma}^n$ → n: Flow behavier index

laminar flow



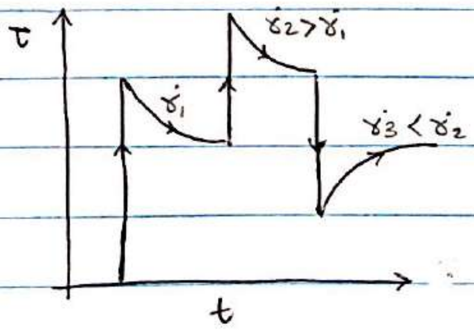
4. Hershel - Bulkley fluid (general model) $\tau = \tau_y + K \dot{\gamma}^n$



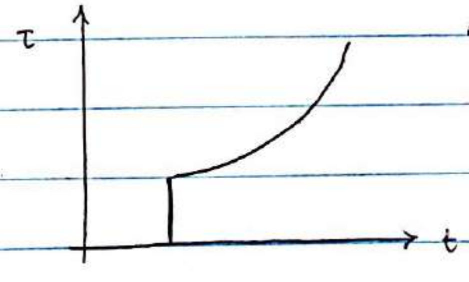
S- time varying fluid   درصنعت زیاد استفاده نمی‌شود

if apprent viscosity (μ_a) decreases with time after the shear rate is increased $\rightarrow$ thixotropic

if μ_a increases " $\rightarrow$ rheopectic



thixotropic $\rightarrow$ معمولاً سیالات حفاری

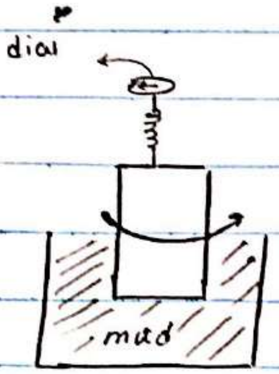


rheopectic

جلسه هفتم

Rotational viscometer

وسیله ای برای اندازه گیری ویسکوزیته که معروف ترین آن Fann V-G metter است.



ابعاد این دستگاه به گونه ای تنظیم می شود که میزان ویسکوزیته ظاهری بر حسب CP برابر با میزان اندازه گیری شده dial در دور 300 rpm باشد.

(θ) (θ_{300})

A) محاسبه ویسکوزیته برای سیالات نیوتنی:

$$\mu = \frac{300}{N}\,\theta_N$$

ویسکوزیته (CP)، N: دور (rpm)

$$N = 300 \Rightarrow \mu = \theta_{300}$$

B) برای سیال Bingham:

$$\mu_p = \frac{300}{N_2 - N_1}\left(\theta_{N_2} - \theta_{N_1}\right)$$

$$\tau_y = \theta_{N_1} - \mu_p \frac{N_1}{300}$$

$$N_1 = 300\ \text{rpm},\quad N_2 = 600\ \text{rpm} \Rightarrow \mu_p = \theta_{600} - \theta_{300},\quad \tau_y = \theta_{300} - \mu_p$$

C) برای سیال power law:

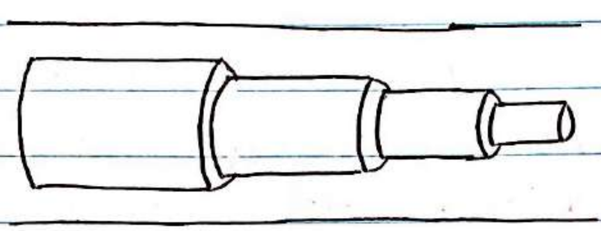
$$n = \frac{\log\left(\frac{\theta_{N_2}}{\theta_{N_1}}\right)}{\log\left(\frac{N_2}{N_1}\right)},\quad K = \frac{510\,\theta_N}{(1.703N)^n}$$

$$N_1 = 300\ \text{rpm},\quad N_2 = 600\ \text{rpm} \rightarrow n = 3.322 \log\left(\frac{\theta_{600}}{\theta_{300}}\right)$$

$$K = \frac{510\,\theta_{300}}{(511)^n}$$

laminar flow in pipe and annulus:

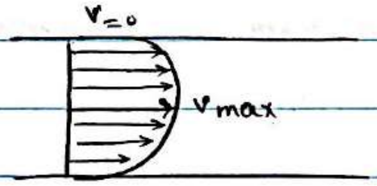
لایه های سیال روی هم می لغزند.



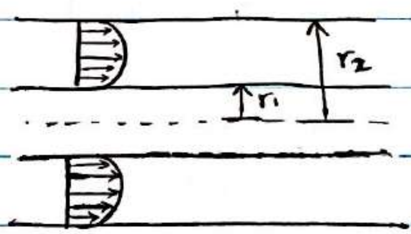
فرضیات:

1- No ecentricity
2- No rotation
3- fluid is incompressible
4- flouid is isothermal ($T = cte$)

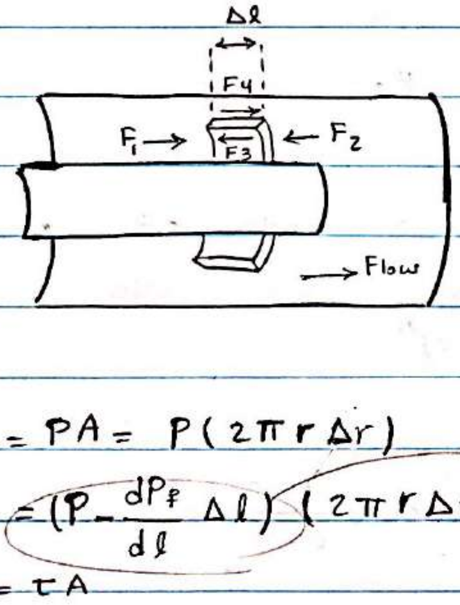
Volecity profile

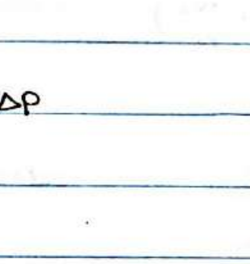


pipe



annulus





$$F_1 = PA = P(2\pi r \Delta r) \longrightarrow P_2 = P_1 - \Delta P$$

$$F_2 = \left(P - \frac{dP_f}{dl}\Delta l\right)(2\pi r \Delta r)$$

$$F_3 = \tau A$$

$$F_3 = \tau (2\pi r)\Delta l \longrightarrow \tau_2 = \tau_1 + \Delta \tau$$

$$F_4 = \left(\tau + \frac{d\tau}{dr}\Delta r\right)(2\pi (r + \Delta r)\Delta l)$$

$$r_2 = r_1 + \Delta r$$

حال برای اینکه المان در تعادل باشد باید برآیند نیروها صفر شود:

$$\Sigma F=0 \Rightarrow F_1+F_4-F_3-F_2=0 \Rightarrow F_1-F_2+F_3+F_4=0$$

$$P(2\pi \cancel{r}\Delta r)-\left(P+\frac{dP_f}{dl}\Delta l\right)2\pi \cancel{r}\Delta r-\tau\, 2\pi r\Delta l+\left(\tau+\frac{d\tau}{dr}\Delta r\right)2\pi(r+\Delta r)\Delta l=0$$

طرفین معادله را بر $2\pi r\Delta r\Delta l$ تقسیم می‌کنیم.

$$\frac{dP_f}{dl}+\frac{1}{r}\frac{d(\tau r)}{dr}=0$$

جلسه هشتم:

معادله فوق را می‌توان به صورت زیر نوشت:

$$\int d(\tau r)=-\frac{dP_f}{dl}\int r\,dr \xrightarrow{\int}$$

$$\tau r=-\frac{dP_f}{dl}\frac{r^2}{2}+C_1 \Rightarrow$$

$$\tau=-\frac{dP_f}{dl}\,r/2+\frac{C_1}{r} \quad (*)$$

برای سیالات نیوتنی داریم:

$$\tau=\mu\dot{\gamma}=\mu\frac{dv}{dr}$$

از معادله‌ی فوق و معادله‌ی $(*)$ داریم ←

$$\mu\frac{dv}{dr}=-\frac{dP_f}{dl}\frac{r}{2}+\frac{C_1}{r}$$

$$\frac{dv}{dr}=-\frac{1}{\mu}\frac{dP_f}{dl}\frac{r}{2}+\frac{C_1}{\mu r} \xrightarrow{\int}$$

$$v=-\frac{1}{4\mu}\frac{dP_f}{dl}r^2+\frac{C_1}{\mu}\ln r+C_2 \qquad \text{(general)}$$

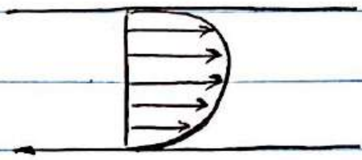
حال برای pipe flow داریم ←

at $r=0 \rightarrow$ تعریف نشده است $\tau \Rightarrow C_1=0$

at $r=R,\ v=0 \Rightarrow 0=-\frac{1}{4\mu}\frac{dP_f}{dl}R^2+C_2 \Rightarrow C_2=\frac{1}{4\mu}\frac{dP_f}{dl}R^2$

سرعت

$$\Rightarrow \overset{\text{سرعت}}{v}=\frac{1}{4\mu}\frac{dP_f}{dl}(R^2-r^2)$$



$$q=\int_0^R v\,(2\pi r)\,dr=\frac{\pi R^4}{8\mu}\frac{dP_f}{dl} \quad (1)$$

$$q=\pi R^2\,\overline{V} \quad (2)$$

$$(1),(2) \Rightarrow \pi R^2 V_{avg}=\frac{\pi R^4}{8\mu}\frac{dP_f}{dl}$$

$$\Rightarrow \frac{dP_f}{dl}=\frac{8\mu V_{avg}}{R^2}=\frac{32\mu V_{avg}}{d^2}$$

معادله افت فشار لوله برای سیال نیوتنی

$$\underbrace{\overset{psi}{dP_f}}_{ft}\big/ dl=\frac{\overset{cp}{\mu}\ \overset{ft/s}{V_{avg}}}{1500\,\underbrace{d^2}_{in}}$$

: Feild

حال برای annular flow داریم:

$$v=-\frac{r^2}{4\mu}\frac{dP_f}{dl}+\frac{C_1}{\mu}\ln r+C_2$$

at $r=r_1 \rightarrow v=0 \overset{\text{معادله}}{\Longrightarrow}$

at $r=r_2 \rightarrow v=0 \overset{\text{معادله}}{\Longrightarrow}$

حل دو معادله دو مجهول

$C_1=$

$C_2=$

در نهایت v به فرم زیر به دست می‌آید:

$$v = \frac{1}{4\mu}\frac{dP_f}{dl}\left[(r_2^2 - r^2) - (r_2^2 - r_1^2)\frac{\ln r_2/r}{\ln r_2/r_1}\right]$$

$$q = \int_{r_1}^{r_2} v(2\pi r)dr = \frac{\pi}{8\mu}\frac{dP_f}{dl}\left[r_2^4 - r_1^4 \frac{(r_2^2 - r_1^2)^2}{\ln r_2/r_1}\right] \quad ①$$

$$q = \pi(r_2^2 - r_1^2)\bar{v}$$

$$①, ② \Rightarrow \frac{dP_f}{dl} = \frac{8\mu\bar{v}}{\left(r_2^2 + r_1^2 - \frac{r_2^2 - r_1^2}{\ln r_2/r_1}\right)}$$

$$\underbrace{\frac{dP_f}{dl}}_{\text{psi/ft}} = \frac{\underset{\text{cp}}{\mu}\ \underset{\text{ft/s}}{V_{avg}}}{1500\left(\underset{\text{in}}{d_2^2} + \underset{\text{in}}{d_1^2} - \frac{d_2^2 - d_1^2}{\ln d_2/d_1}\right)} \quad \text{: Feild}$$

example:

Newton fluid: $\mu = 15$ cp , $\rho = 9$. ppg

at $z = 10{,}000$ ft casing: ID = 7 in

drillstring ID = 4.5 in OD: 5 in

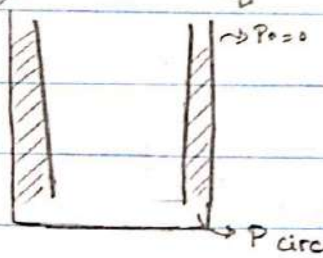
$q = 80$ gpm

calculate static & circulating bottom hole pressure?

for static:

$P_{static} = 0.052\,\rho z + P_0 = 0.052 * 9 * 10{,}000 = 4680$ psi

ادامه در صفحه‌ی بعد ←



$$P_{circulate} - 0.052\,\rho\, z - \Delta P_f = 0$$

$$P_{circ} = 0.052\,\rho\, z + \Delta P_f$$

$$= 0.052 \times 9 \times 10{,}000 + 51 = 4731 \text{ Psig}$$

$$\frac{dP_f}{dl} = \frac{\mu \bar{V}}{1500\left(d_2^2 + d_1^2 - \dfrac{d_2^2 - d_1^2}{\ln d_2/d_1}\right)} = \frac{15 \times 1.362}{1500\left(7^2 + 5^2 - \dfrac{7^2 - 5^2}{\ln 7/5}\right)} = 0.0051 \text{ Psi/ft}$$

$$\bar{V} = \frac{q}{2.448\,(d_2^2 - d_1^2)} = \frac{80}{2.448\,(7^2 - 5^2)} = 1.362 \text{ ft/s}$$

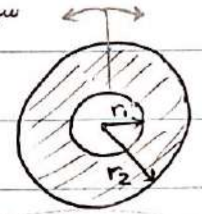
$$\Delta P_f = \frac{dP_f}{dl} \times \Delta l = 0.0051 \times 10{,}000 = 51 \text{ Psi}$$

* ECD: equivalent circulating Density $= \dfrac{P_{bh\,(circ)}}{0.052\, z}$

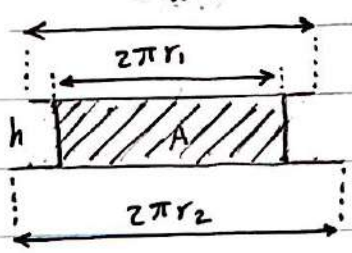
جلسه نهم

Representing annulus as a slot (Slot flow approximation)

چون حل معادلات در مختصات قطبی سخت است



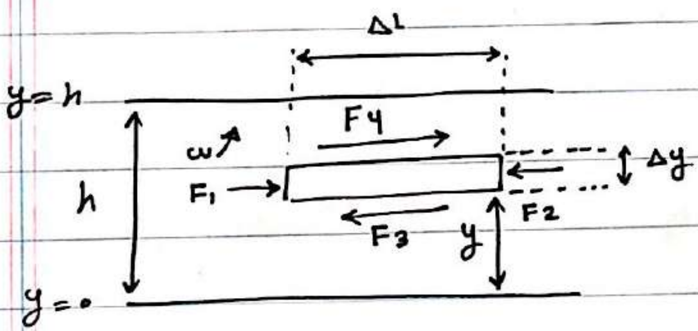
if $\frac{d_2}{d_1} > 0.3$



if $\frac{d_1}{d_2} > 0.3$

$$h = r_2 - r_1$$

$$A = wh = \pi (r_2^2 - r_1^2) \rightarrow w = 2\pi \frac{r_2 + r_1}{2}$$



ضخامت

$$F_1 = P w \Delta y$$

$$F_2 = (P - \frac{dP_f}{dL} \Delta L) w \Delta y$$

$$F_3 = \tau w \Delta L$$

$$F_4 = (\tau + \frac{d\tau}{dy} \Delta y) w \Delta L$$

$$F_1 - F_2 - F_3 + F_4 = 0$$

$$P w \Delta y - (P - \frac{dP_f}{dL} \Delta L) w \Delta y - \tau w \Delta L + (\tau + \frac{d\tau}{dy} \Delta y) w \Delta L = 0$$

$$\cancel{P w \Delta y} - \cancel{P w \Delta y} + \frac{dP_f}{dL} w \Delta y - \cancel{\tau w \Delta L} + \cancel{\tau w \Delta L} + \frac{d\tau}{dy} \Delta y w \Delta L = 0$$

$$\left(\frac{dP_f}{dL} \Delta L w \Delta y + \frac{d\tau}{dy} \Delta y w \Delta L = 0 \right) \times \frac{1}{w \Delta y \Delta L}$$

$$\frac{dP_f}{dL} + \frac{d\tau}{dy} = 0 \rightarrow \frac{dP_f}{dL} = - \frac{d\tau}{dy}$$

$$\tau = -y \frac{dP_f}{dL} + \tau_0 \qquad (1)$$

A) Newton fluid:

$$(2)\quad \tau = \mu \dot{\gamma} = \mu \frac{dv}{dy} \xrightarrow[(1),(2)]{\text{توجه به معادله قبلی}} \mu \frac{dv}{dy} = -y \frac{dP_f}{dl} + \tau_0$$

$$\Rightarrow V = -\frac{y^2}{2\mu}\frac{dP_f}{dl} + \frac{\tau_0 y}{\mu} + V_0 \qquad : \text{velocity profile}$$

شرایط مرزی:

$$y = 0 \rightarrow V = 0 \Rightarrow 0 = -0 + 0 + V_0 \Rightarrow V_0 = 0 \quad (1)$$

$$y = h,\ V = 0 \Rightarrow 0 = -\frac{h^2}{2\mu}\frac{dP_f}{dl} + \frac{\tau_0 h}{\mu} + V_0 \quad (2) \xrightarrow{V_0 = 0} \tau_0 = -h/2 \frac{dP_f}{dl}$$

$$1,2 \Rightarrow V = \frac{1}{2\mu}\frac{dP_f}{dl}(hy - y^2)$$

$$dq = v\,dA = V\,w\,dy$$

$$q = \frac{w}{2\mu}\frac{dP_f}{dl}\int_0^h (hy - y^2)\,dy \Rightarrow q = \frac{wh^3}{12\mu}\frac{dP_f}{dl}$$

$$q = \frac{\pi}{12\mu}\frac{dP_f}{dl}(r_2^2 - r_1^2)(r_2 - r_1)^2$$

$$q = \bar{V}\pi(r_2^2 - r_1^2)$$

$$\Rightarrow \frac{dP_f}{dl} = \frac{12\mu\bar{V}}{(r_2 - r_1)^2} \quad : \text{SI}$$

CP, ft/s

$$\frac{dP_f}{dl} = \frac{\mu V_{avg}}{1000(d_2 - d_1)^2} \quad : \text{Field}$$

psi, ft, in

example: previous example: $\mu = 15\ cp$ $\ell = 10{,}000\ ft$

محاسبه افت فشار؟ $d_2 = 7\ in$ $d_1 = 5\ in$

$d_1/d_2 = 0.714 > 0.3$ $V = 1.362\ ft/s$

$$\Delta P_f = \frac{dP_f}{d\ell}\Delta \ell = \frac{\mu \bar{V} \Delta L}{1000(d_2-d_1)^2} = \frac{15 \times 1.362 \times 1000}{1000(7-5)^2} = 51\ psi$$

Shear rate ($\dot{\gamma}$): حداکثر نرخ برش در لوله ← روی دیواره

Shear rate معادل Rotation در viscometer است.

$$\tau_w = \frac{r_w}{2}\frac{dP_f}{d\ell} \quad \text{(circular pipe) ①}$$

$$= \frac{r_w}{2}\left(\frac{8\mu \bar{V}}{r_w^2}\right) = 4\frac{\mu \bar{V}}{r_w}$$

$$\dot{\gamma}_w = \frac{\tau_w}{\mu} = \frac{4\bar{V}}{r_w} \ : SI \qquad \dot{\gamma}_w = \frac{96\bar{V}}{d} \ : Field$$

② annulus flow (slot)

$$\tau_w = \frac{h}{2}\frac{dP_f}{d\ell} = \frac{(r_2-r_1)}{2}\frac{dP_f}{d\ell} = \frac{(r_2-r_1)}{2}\left[\frac{12\mu\bar{V}}{(r_2-r_1)^2}\right]$$

$$\dot{\gamma}_w = \frac{\tau_w}{\mu} = \frac{6\bar{V}}{r_2-r_1} \ : SI \qquad \dot{\gamma}_w = \frac{144\bar{V}}{d_2-d_1} \ : Field$$

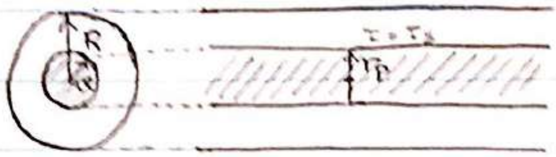
s^{-1} ft/s in

example: Previous example → محاسبه نرخ برش؟

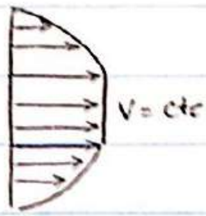
$$\dot{\gamma}_w = \frac{144\bar{V}}{d_2-d_1} = \frac{144 \times 1.362}{7-5} = 98\ s^{-1}$$

(B) Bingham fluid:

(1) Pipe flow:



Velocity profile:



✓ $v = \frac{1}{4\mu_p}\frac{dP_f}{dl}(R^2 - r^2) - \frac{\tau_y}{\mu_p}(R - r) \qquad r \geq r_p$

$\tau_y = \tau_p = \frac{r_p}{2}\frac{dP_f}{dl} \Rightarrow r_p = 2\tau_y \frac{1}{dP_f/dl}$

$\tau|_{r=r_p}$

✓ $v_p = \frac{1}{4\mu_p}\frac{dP_f}{dl}(R - r_p)^2 \qquad 0 \leq r \leq r_p$

$\frac{dP_f}{dl} = \frac{32\mu_p \bar{v}}{d^2} + \frac{16\tau_y}{3d}$: SI

psi, cP, ft/s, $\frac{lbf}{100\,ft^2}$

$\frac{dP_f}{dl} = \frac{\mu_p \bar{v}}{1500\, d^2} + \frac{\tau_y}{255\, d}$: Feild

ft, in

$\dot{\gamma}_w = \frac{8\bar{v}}{d} + \frac{\tau_y}{3\mu_p}$: SI

s^{-1}, ft/s, $\frac{lbf}{100\,ft^2}$

$\dot{\gamma}_w = \frac{96\bar{v}}{d} + 1597\frac{\tau_y}{\mu_p}$: Field

in, cP

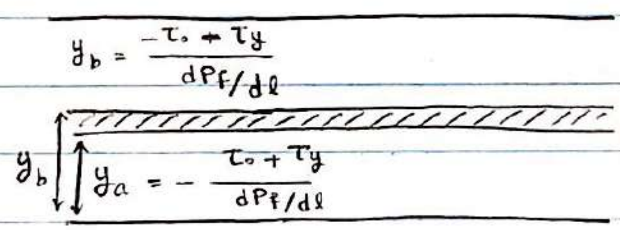
② annular flow (slot)

$$\frac{dP_f}{dl} = \frac{12 \mu_p \bar{V}}{(r_2 - r_1)^2} + \frac{3 \tau_y}{(r_2 - r_1)} \quad : \text{SI}$$

$$\frac{dP_f}{dl} = \frac{\mu_p \bar{V}}{1000 (d_2 - d_1)^2} + \frac{\tau_y}{200 (d_2 - d_1)} \quad : \text{Field}$$

$$\dot{\gamma}_w = \frac{144 \bar{V}}{d_2 - d_1} + 239.5 \frac{\tau_y}{\mu_p} \quad : \text{Field}$$

برای علاقه‌مندان :



$$V = \frac{1}{2\mu_p} \frac{dP_f}{dl} (-y^2 + 2 y_a y) \quad y \leq y_a$$

$$V = \frac{1}{2\mu_p} \frac{dP_f}{dl} (h^2 - y^2 + 2 y_b y - 2 y_b h) \quad y \geq y_b$$

$$V_p = \frac{y_a^2}{2\mu_p} \frac{dP_f}{dl} \quad y_a \leq y \leq y_b$$

جلسه دهم

example: well depth : 10,000 ft

hole size = $8\frac{7}{8}$ in

drill pipe: OD = 5 in ID = 4 in

drill collars: OD = 0.7 in ID = 4 in length: 1000 ft

min allowed average fluid velocity = 100 ft/min

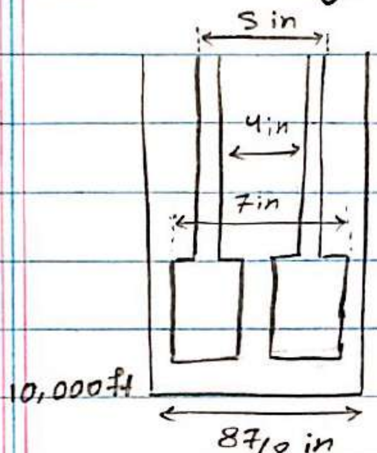
mud is Bingham plastic fluid, $\theta_{600} = 55$ $\theta_{300} = 35$, $\rho_m = 5$ ppg

assume surface facilities to be equivalent to 500 ft of drillpipe.

assume laminar flow

a) determine the hydrolic horsepower that will be lost due to friction while drilling at 10,000 ft and a flow rate corresponding to the min allowed annular velocity

b) determine whether formation fracturing will occure if the flow rate is 400 gpm assume the formation fracture gradiant is 9 ppg



$\mu_p = \theta_{600} - \theta_{300} = 55 - 35 = 20 \text{ cp}$

$\tau_y = \theta_{300} - \mu_p = 35 - 20 = 15 \text{ lbf}/100 \text{ ft}^2$

$$\bar{V} = \frac{q}{2.448(d_2^2 - d_1^2)} \Rightarrow q = 2.448\,\bar{V}(d_2^2 - d_1^2)$$

$$\rightarrow q = 2.448\,(100 \text{ ft/min}) \times \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \times (8.875^2 - 5^2)$$

$= 220 \text{ gpm}$

$$\Delta P_f = \Delta P_{f_{sp}} + \Delta P_{f_{dp}} + \Delta P_{f_{dc}} + \Delta P_{f_{ac}} + \Delta P_{f_{ap}}$$

$$= L_{sp}\frac{dP_{f_{sp}}}{dL} + L_{dp}\frac{dP_{f_{dp}}}{dL} + L_{dc}\frac{dP_{f_{dc}}}{dL} + L_{ac}\frac{dP_{f_{ac}}}{dL} + L_{ap}\frac{dP_{f_{ap}}}{dL}$$

$d_{is} = d_{ip} + d_{ic} \rightarrow \bar{V}_s = \bar{V}_{dp} + \bar{V}_{dc}$

$$\frac{dP_{f\,dp}}{dL} = \frac{\mu_p\,\bar{V}_{dp}}{1500\,d_{i_{dp}}^2} + \frac{\tau_y}{225\,d_{i_{dp}}} = \frac{20 \times 5.617}{1500 \times 4^2} + \frac{15}{225 \times 4} = 0.0213 \ \frac{\text{psi}}{\text{ft}}$$

$$\bar{V}_{dp} = \frac{q}{2.448\,d_{i_{dp}}^2} = \frac{220}{2.448 \times 4^2} = 5.617 \text{ ft/s}$$

$$\bar{V}_{ac} = \frac{q}{2.448(d_h^2 - d_{o_{dc}}^2)} = \frac{220}{2.448(8.875^2 - 7^2)} = 3.019 \text{ ft/s}$$

$$\bar{V}_{ap} = \frac{q}{2.448(d_h^2 - d_{o_{dp}}^2)} = \frac{220}{2.448(8.875^2 - 5^2)} = 1.667 \ ft/s$$

$$\frac{dP_{f_{ac}}}{dL} = \frac{\mu_p \bar{V}_{ac}}{1000(d_h - d_{o_{dc}})^2} + \frac{\tau_y}{200(d_h - d_{o_{dc}})} = 0.0572 \ Psi/ft$$

$\mu_p \to 20$, $\bar{V}_{ac} \to 3.019$, $\tau_y \to 15$, $d_h \to 8.875$, $d_{o_{dc}} \to 7$

$$\frac{dP_{f_{ap}}}{dL} = \frac{20 \times 1.667}{1000(d_h - d_{o_{dp}})^2} + \frac{\tau_y}{200(8.875-5)} = 0.0216 \ Psi/ft$$

$1.667 \to \bar{V}_{ap}$, $\tau_y \to 15$, $d_h \to 8.875$, $d_{o_{dp}} \to 5$

$$\Delta P_f = (500 \times 0.0213) + (9000 \times 0.0213) + (1000 \times 0.0572) + (9000 \times 0.0216)$$
$$= 475.25 \ psi$$

$$P_{H_f} = \frac{\Delta P_f \ q}{1714} = \frac{475.25(220)}{1714} = 61 \ hP$$

b) $$ECD = \frac{P_{bh}}{0.052 \ Z} = \rho_m + \frac{\Delta P_{f_{ac}} + \Delta P_{f_{ap}}}{0.052 \ Z}$$

$$\bar{V}_{ac} = \frac{400 \to q}{2.448(8.875^2 - 7^2)} = 5.496 \ ft/s$$

$$\bar{V}_{ap} = \frac{400}{2.448(8.875^2 - 5^2)} = 3.039 \ ft/s$$

$$\frac{dP_{f_{ac}}}{dL} = \frac{20 \times 5.496}{1000(8.875-7)^2} + \frac{15}{200(8.875-7)} = 0.0712 \ Psi/ft$$

$$\frac{dP_{f_{ap}}}{dL} = \frac{20 \times 3.039}{1000(8.875-5)^2} + \frac{15}{200(8.875-5)} = 0.0234 \ Psi/ft$$

جلسه یازدهم

Power law model

A) pipe flow $\tau = K\dot{\gamma}^n$

$$V = \frac{\left(\frac{1}{2K}\frac{dP_f}{dl}\right)^{1/n}}{1+1/n}\left(R^{1+1/n} - r^{1+1/n}\right)$$

$$\frac{dP_f}{dl} = \frac{2K\,V_{avg}^{+n}(3+1/n)^n}{R^{n+1}} \quad : SI$$

$$\frac{dP_f}{dl} = \frac{K\,V_{avg}^{+n}}{144{,}000\,d^{n+1}}\left(\frac{3+1/n}{0.0416}\right)^n \quad : \text{Field}$$

$$\dot{\gamma}_w = \frac{24\,\bar{V}\,(3+1/n)}{d} \quad , \text{Field}$$

B) annular flow (slot flow)

$$\frac{dP_f}{dl} = \frac{2K(4+2/n)^n\,V_{avg}^n}{(r_2-r_1)^{n+1}} \quad : SI$$

$$\frac{dP_f}{dl} = \frac{K\,\bar{V}^n}{144{,}000\,(d_2-d_1)^{1+n}}\left(\frac{2+1/n}{0.0208}\right)^n \quad : \text{Field}$$

$$\dot{\gamma}_w = \frac{48\,\bar{V}\,(2+1/n)}{d_2-d_1} \quad : \text{Field}$$

example: A cement slurry has a flow behavior index (n) of 0.3 and consistency index (K) of 9400 eq CP is being pumped in an 8.097 x 4.5 in annulus at a rate of 200 gpm.

power law

OD, ID

- Assume flow pattern is laminar
- calculate frictional pressure loss per 1000 ft of annulus
- calculate the shear rate of the wall pipe

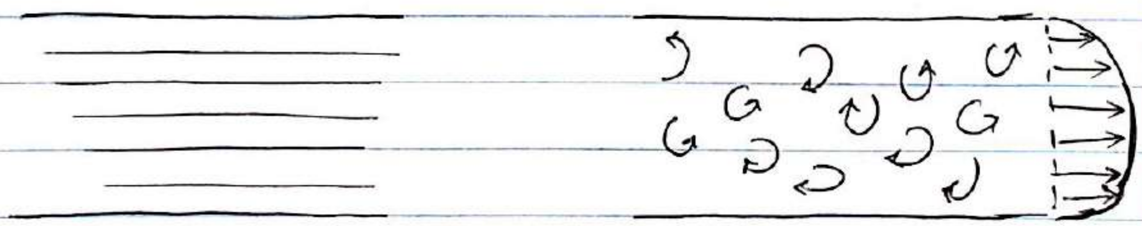
$$\bar{V} = \frac{q}{2.448(d_2^2 - d_1^2)} = \frac{200}{2.448(8.097^2 - 4.5^2)} = 1.803 \text{ ft/s}$$

$$\frac{dP_f}{dL} = \frac{K\bar{V}^n}{144{,}000(d_2-d_1)^{1+n}}\left(\frac{2+1/n}{0.0208}\right)^n = \frac{9400 \times 1.803^{0.3}}{144000(8.097-4.5)^{1+0.3}}\left(\frac{2+1/0.3}{0.0208}\right)^{0.3}$$

$$= 0.0779 \text{ psi/ft} \rightarrow \Delta P_f = 0.0779 \times 1000 = 77.9 \text{ psi}$$

$$\dot{\gamma}_w = \frac{48\bar{V}(2+1/n)}{d_2-d_1} = \frac{48 \times 1.803 \times (2+1/0.3)}{8.097-4.5} = 128\ s^{-1}$$

Turbulent Flow



laminar flow　　　　turbulent flow

Reynolds no: $N_{Re} = \frac{\rho \bar{V} d}{\mu}$: SI

$$N_{Re} = \frac{928\,\underbrace{\rho}_{PPg}\,\underbrace{\bar{v}}_{ft/s}\,\underbrace{d}_{in}}{\underbrace{\mu}_{cp}}$$: in Field

برای سیالات نیوتنی

$N_{Re} < 2100 \rightarrow$ laminar

$N_{Re} > 2100 \rightarrow$ turbulent $\begin{cases} 2100 < N_{Re} < 4000 & \text{transition} \\ N_{Re} > 4000 & \text{turbulent} \end{cases}$

example: 9 lbm/gal brine

$\mu = 1\,cp$ $\qquad ID_{dp} = 4.276$ in

$q = 600$ gpm

Laminar or turbulent ????

$$N_{Re} = \frac{928 \rho \bar{v} D}{\mu} = \frac{928(9)(13.4)(4.276)}{1} = 478{,}556$$

$$\bar{v} = \frac{q}{2.448(d^2)} = \frac{600}{2.448 \times 4.287^2} = 13.4 \ ft/s$$

$N_{Re} > 2100 \rightarrow$ turbulent flow

turbulent flow

① Newtonian fluid

A) pipe flow

رابطه ای مشخص برای افت فشار در جریان آشفته وجود ندارد پس
از نتایج آزمایشگاهی استفاده می کنیم:

Friction factor:

$$f = \frac{F_K}{A\,E_K}$$

F_K → نیرویی که سیال به دیواره ی لوله وارد می کند که باعث حرکت سیال می شود

A → مساحت مقطع لوله

E_K → انرژی جنبشی سیال در واحد حجم

$$\tau_w = \frac{r_w}{2}\frac{dP_f}{dL} = \frac{d}{4}\frac{dP_f}{dL}$$

$$F_K = (2\pi r_w \times \Delta L)\,\tau_w = \frac{\pi d^2}{4}\frac{dP_f}{dL}\Delta L$$

$$E_K = \tfrac{1}{2}\rho v^2$$

$$f = \frac{\pi d^2/4 \; dP_f/dL \; \Delta L}{A \times \tfrac{1}{2}\rho v^2} \quad , \; A = 2\pi r_w \Delta L$$

$$f = \frac{d}{2\rho v^2}\frac{dP_f}{dL} \leftarrow \text{Fanning Eq}$$

$f \rightarrow$ Fanning Friction Factor

$$\frac{dP_f}{dL} = \frac{2 f \rho V_{avg}^2}{d} \quad : SI$$

$$\frac{dP_f}{dL} = \frac{f \rho V_{avg}^2}{25.8\, d} \quad : \text{Field}$$

$$f = f(N_{Re}, \varepsilon_e / d)$$

$\varepsilon_e \rightarrow$ absolute roughness

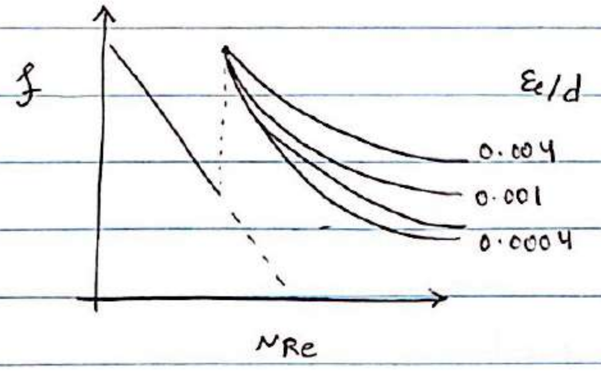
$d \rightarrow$ Pip flow

ε_e / D : relative roughness

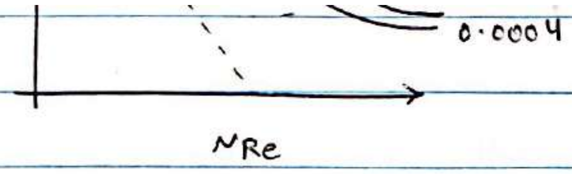
Type of pipe	ε_e (in)
Commercial Steel	0.000013
Concrete	0.000083 – 0.00083

Colebrook function

$$\frac{1}{\sqrt{f}} = -4 \log \left(0.269\, \varepsilon_e/d + \frac{1.255}{N_{Re}\sqrt{f}}\right)$$



Stanton Chart



جلسه دوازدهم

$$\varepsilon/d = 0 \rightarrow \frac{1}{\sqrt{f}} = 4 \log (N_{Re}\sqrt{f}) - 0.395$$

$$\left.\begin{array}{l} \varepsilon/d = 0 \\ N_{Re} < 100{,}000 \end{array}\right\} \text{Blasius: } f = \frac{0.0791}{N_{Re}^{0.25}}$$

$$\frac{dP_f}{dL} = \frac{0.0791 \times \rho \bar{V}^2}{\left(\frac{928 \rho \bar{V} d}{\mu}\right)^{0.25} \times 25.8 d} \Rightarrow \frac{dP_f}{dL} = \frac{\rho^{0.75} \bar{V}^{1.75} \mu^{0.25}}{1800\, d^{1.25}}$$

example: determine ΔP_f in 10,000 ft of 4.5 in. drill pipe having ID: 3.826. Drilling fluid is newtonian with $\mu = 20$ cp and $\rho = 9$ ppg, $q = 400$ gpm

Assume the drill pipe is commerical steel $\rightarrow \varepsilon_0 = 0.000013$

$$\bar{V} = \frac{q \rightarrow 400}{2.448\, d^2 \rightarrow 3.826^2} = 11.16 \text{ ft/s}$$

$$N_{Re} = \frac{928 \rho \bar{V} d}{\mu} = \frac{928(9)(11.16)(3.826)}{20} = 17831 > 2100$$

$\varepsilon/d = \frac{0.000013}{3.826} = 0.0000034 \Rightarrow f = 0.00666$

$$\Delta P_f = \frac{dP_f}{dL} \times \Delta L = \frac{f \rho \bar{V}^2 \times \Delta L}{25.8\, d} = \frac{0.00666(9)(11.16)^2(10,000)}{25.8\,(3.826)} = 756 \text{ Psi}$$

$$\Delta P_f = \frac{\rho^{0.75}\, \bar{V}^{1.75}\, \mu^{0.25}}{1800\, d^{1.25}} \times \Delta L = \frac{(9)^{0.75}(11.16)^{1.75}(20)^{0.25}(10,000)}{1800(3.826)^{1.25}} = 777 \text{ Psi}$$

Annular flow

معادل سازی جریان در لوله و فضای حلقوی

equivalent diameter (d_e)

1. hydraulic radius (r_H) $= \frac{\text{cross sectional area}}{\text{wetted perimeter of the channel}} = \frac{\pi (r_2^2 - r_1^2)}{2\pi (r_2 - r_1)} = \frac{r_2 - r_1}{2} = \frac{d_2 - d_1}{4}$

(محیط فضای خیس شده (داخلی و خارجی فضای حلقوی))

$$\boxed{d_e = 4 \times r_H = 4\,\frac{d_2 - d_1}{4} = d_2 - d_1}$$

2. Pressure loss eq. in laminar flow

Pipe: $\frac{dP_f}{dL} = \frac{\mu \bar{V}}{1500\, d^2}$ (I)

annulus: $\frac{dP_f}{dL} = \frac{\mu \bar{V}}{1500\left(d_2^2 + d_1^2 - \frac{d_2^2 - d_1^2}{\ln d_2/d_1}\right)}$ (II)

$$\text{I}, \text{II} \rightarrow d_e^2 = d_2^2 + d_1^2 - \frac{d_2^2 - d_1^2}{\ln d_2/d_1} \Rightarrow \boxed{d_e = \sqrt{d_2^2 + d_1^2 - \frac{d_2^2 - d_1^2}{\ln d_2/d_1}}}$$

3. Pressure loss eq. in laminar flow (slot flow)

$$\left.\begin{aligned} dP_f/dL &= \mu\bar{V}/1500\, d^2 \\ dP_f/dL &= \mu\bar{V}/1000\,(d_2 - d_1)^2 \end{aligned}\right\} \rightarrow 1500\, d^2 = 1000\,(d_2 - d_1)^2 \rightarrow d_e = 0.816\,(d_2 - d_1)$$

ساده ترین و پرکاربردترین معادله

4. $d_e = \dfrac{\sqrt[4]{d_2^4 - d_1^4 - (d_2^2 - d_1^2)^2 / \ln d_2/d_1} + \sqrt{d_2^2 - d_1^2}}{2}$ Crittedon

در این فرمول برخلاف ۳ فرمول قبل که از v در d_1, d_2 استفاده می کردیم از v ای استفاده می کنیم که در آن de وارد شده باشد

$$\bar{v} = \frac{q}{2.448\, d_e^2}$$

example: 9 ppg brine having $\mu = 1\,cp$

$q = 200\ gpm$ annulus opposite drillpipe with OD = 5 in. and had diameter of 10 in.

- calculate pressure loss gradient using 4 criterica

$d_e = d_2 - d_1 = 10 - 5 = 5\ in$

$$d_e = \sqrt{d_2^2 + d_1^2 - \frac{d_2^2 - d_1^2}{\ln \frac{d_2}{d_1}}} = \sqrt{10^2 + 5^2 - \frac{10^2 - 5^2}{\ln(\frac{10}{5})}} = 4.099\ in.$$

$d_e = 0.816\,(d_2 - d_1) = 0.816\,(10 - 5) = 4.080\ in$

$d_e = 7.309\ in$ (crittedon)

$$\bar{v} = \frac{q}{2.448(d_2^2 - d_1^2)} = \frac{200}{2.448(10^2 - 5^2)} = 1.089\ ft/s$$

$$\bar{v} = \frac{q}{2.448\, d_e^2} = \frac{200}{2.448 \times 7.309^2} = 1.529\ ft/s$$

$$N_{Re} = \frac{928 \rho \bar{v} d_e}{\mu} = \frac{928 \times 9 \times \bar{v} \times d_e}{1} = 8352\, \bar{v} d_e$$

de	$\bar{v}$	N_{Re}	dP_f/dl
5	1.089	45476	4.48×10^{-4}
4.099	1.089	37285	5.75×10^{-4}
4.080	1.089	37109	5.78×10^{-4} پرکاربردترین
7.309	1.529	93337	4.97×10^{-4}

افت // در جریان turbulent

$$\frac{dP_f}{dl} = \frac{\rho^{0.75} \times \bar{v}^{1.75} \times \mu^{0.25}}{1800\, d_e^{1.25}} = 0.0002887 \frac{\bar{v}^{1.75}}{d_e^{1.25}}$$

جلسه سیزدهم

2. Bingham plastic Fluid

a) Turbulent flow

در جریان توربولنت چون τ_y ها تأثیر چندانی ندارند و اثری روی افت فشار نمی‌گذارند برای bingham از همان معادلات نیوتنی استفاده می‌کنیم.

$$N_{Re} = \frac{928 \rho \bar{v} d_e}{\mu_p}$$

$$\begin{cases} \dfrac{dP_f}{dl} = \dfrac{f \rho \bar{v}^2}{25.8 d_e} \\ \dfrac{1}{\sqrt{f}} = -4 \log\left(0.269 \, \varepsilon/d_e + \dfrac{1.255}{N_{Re}\sqrt{f}}\right) \end{cases}$$

$$\frac{dP_f}{dl} = \frac{\rho^{0.75} \, \bar{v}^{1.75} \, \mu_p^{0.25}}{1800 \, d_e^{1.25}} \qquad \begin{cases} Re < 100,1000 \\ \varepsilon = 0 \end{cases}$$

برای فهمیدن اینکه جریان توربولنت است یا خیر از روش زیر استفاده می‌کنیم:

روش اول: Apprant Viscosity:

① Pipe flow:

laminar newtonian: $$\frac{dP_f}{dl} = \frac{\mu \bar{v}}{1500 d^2} \quad (1)$$

Laminar Bingham: $$\frac{dP_f}{dl} = \frac{\mu_p \bar{v}}{1500 d^2} + \frac{\tau_y}{225 d} \quad (2)$$

$$1,2 \rightarrow \frac{\mu_a \bar{v}}{1500 d^2} = \frac{\mu_p \bar{v}}{1500 d^2} + \frac{\tau_y}{225 d} \rightarrow$$

$$\mu_a = \mu_p + \frac{6.66 \, \tau_y d}{\bar{v}}$$

$$N_{Re} = \frac{928 \rho \bar{v} d}{\mu_a} > 2100 \rightarrow \text{turbulent flow}$$

② annular flow (Slot flow)

Laminar newtonian: $\dfrac{\mu \bar{v}}{1000(d_2-d_1)^2}$ ①

Laminar bingham: $\dfrac{\mu_p \bar{v}}{1000(d_2-d_1)^2} + \dfrac{\tau_y}{200(d_2-d_1)}$ ②

$$1,2 \rightarrow \frac{\mu_a \bar{v}}{1000(d_2-d_1)^2} = \frac{\mu_p \bar{v}}{1000(d_2-d_1)^2} + \frac{\tau_y}{200(d_2-d_1)} \rightarrow \mu_a = \mu_p + \frac{5\tau_y(d_2-d_1)}{\bar{v}}$$

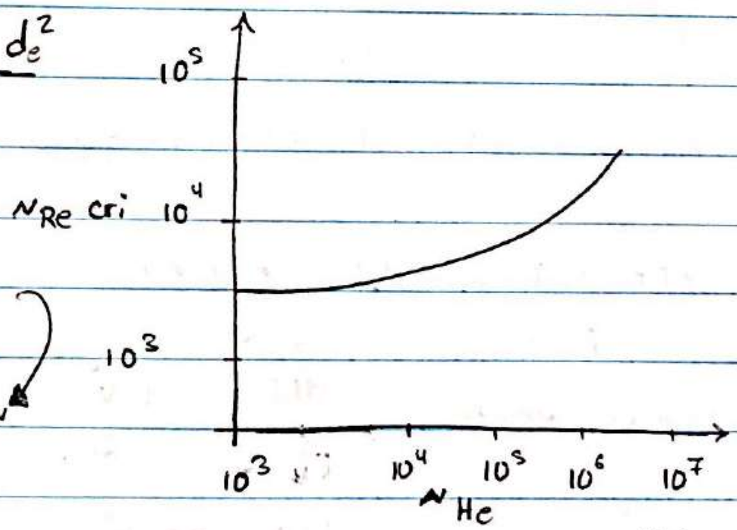
$$N_{Re} = \frac{928 \rho \bar{v} d_e}{\mu_a} > 2100 \longrightarrow \text{turbulent flow}$$

روش دوم: Hanks $\longrightarrow$ Hedstorm number

$$N_{He} = \frac{37100 \rho \tau_y d_e^2}{\mu_p^2}$$

$$N_{Re} = \frac{928 \rho \bar{v} d}{\mu_p} > N_{Re_c}$$

turbulent flow



example: A 10 ppg mud having $\mu_p = 40\,cp$, $\tau_y = 15\ lbf/100ft^2$

$q = 600\,gpm$, 6.5 in. hole

drill Collar OD = 4.5 in , L = 1000 ft

- Check for turbulency using both approaches. (in the annulus opposite drill collars)
- calculate friction pressure loss
- calculate friction pressure loss if the flow was assumed laminar

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حل:

$$\bar{V} = \frac{q}{2.448(d_2^2 - d_1^2)} = \frac{600}{2.448(6.5^2 - 4.5^2)} = 11.14 \text{ ft/s}$$

روش اول $\Rightarrow$ $$\mu_a = \mu_p + \frac{5\tau_y(d_2 - d_1)}{\bar{V}} = 40 + \frac{5\times15(6.5-4.5)}{11.14} = 53.5 \text{ cp}$$

$$d_e = 0.816(d_2 - d_1) = 0.816(6.5 - 4.5) = 1.632 \text{ in}$$

$$N_{Re} = \frac{928\rho\bar{V}d_e}{\mu_a} = \frac{928(10)(11.14)(1.632)}{53.5} = 3154 > 2100 \rightarrow \text{turbulent}$$

روش دوم $\rightarrow$ $$N_{He} = \frac{37100\rho\tau_y d_e^2}{\mu_p^2} = \frac{37100(10)(15)(1.632)^2}{40^2} = 9263$$

از نمودار استفاده می کنیم $\rightarrow$ $N_{Re_c} = 3300$

$$N_{Re} = \frac{928\rho\bar{v}d_e}{\mu_p} = \frac{928(10)(11.14)(1.632)}{40} = 4218 > 3300 \rightarrow \text{turbulent}$$

$$\Delta P_f = \frac{\rho^{0.75}\bar{V}^{1.75}\mu_p^{0.25}}{1800\, d_e^{1.25}}\Delta L = \frac{(10)^{0.75}(11.14)^{1.75}(40)^{0.25}}{1800(1.632)^{1.25}} \times 1000 = 289 \text{ psi}$$

if laminar flow was assumed:

$$\Delta P_f = \frac{\mu_p\bar{V}\Delta L}{1000(d_2 - d_1)^2} + \frac{\tau_y\Delta L}{200(d_2 - d_1)} = \frac{40\times11.14\times1000}{1000(6.5-4.5)^2} + \frac{15\times1000}{200(6.5-4.5)} = 149 \text{ psi}$$

جلسه چهاردهم

example: - mud: Binghom → $\rho = 10.2$ ppg

$\theta_{600} = 65 \quad \theta_{300} = 40$

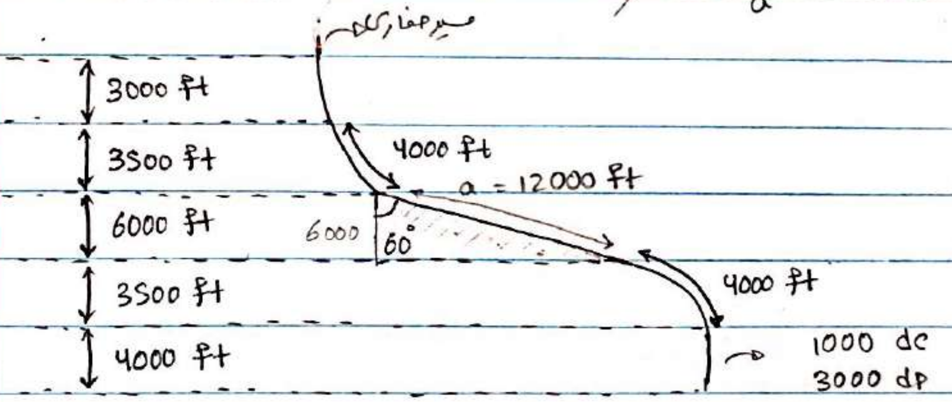
- Bit → tri-cone roller-cone jet bit (12-12-12)
- drillpipe: OD = 4.5 in ID: 4 in
- drillcollar: OD = 7.5 in ID: 4 in $l_{dc} = 1000$ ft
- surface connections equivalent to 500 ft of drillpipe
- hole: Last intermediate casing set at 16000 ft TVD (true vertical depth), ID: 8.5 in

open hole washed out to 8.5 in using a 7.5 in bit.

- Determine the pump hydrolic horsepower required.

Assume pump Volumetric efficiency of 85% and a flow rate of 400 gpm.

$\cos 60 = \frac{6000}{a}$ ($\cos 60 = 1/2$) $\rightarrow a = 12000$ ft



solution:

$\mu_P = \theta_{600} - \theta_{300} = 65 - 40 = 25$ cp

$\tau_y = \theta_{300} - \mu_P = 40 - 25 = 15 \ \text{lbf}/100\ \text{ft}^2$

$$\Delta P_f = \Delta P_{f\,dp} + \Delta P_{f\,dc} + \Delta P_{f\,ap} + \Delta P_{f\,ac} + \Delta P_{f\,s} + \Delta P_{f\,b} = \Delta P_{pump}$$

drill pipe → $\bar{V}_{dp} = \frac{q}{2.448\, d_{i\,dp}^2} = \frac{400}{2.448 \times 4^2} = 10.2$ ft/s

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$$\mu_{a\,dp} = \mu_p + \frac{6.66\,\tau_y\,d_{i\,dp}}{\bar{V}_{dp}} = 25 + \frac{6.66 \times 15 \times 4}{10.2} = 64.2\ CP$$

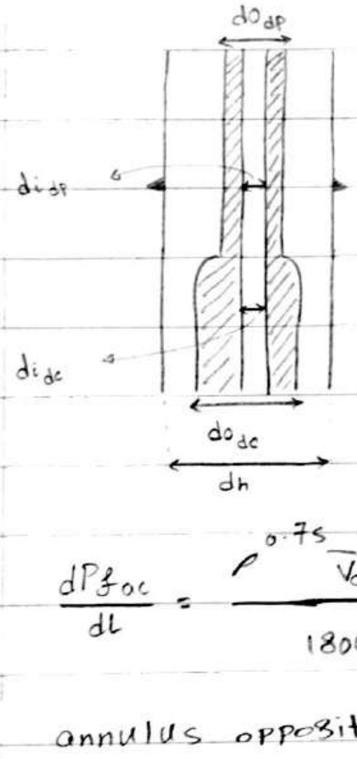
$$N_{Re_{dp}} = \frac{928\,\rho\,\bar{V}_{dp}\,d_{i_{dp}}}{\mu_a} = \frac{928 \times 10.2 \times 10.2 \times 4}{64.2} = 6016 > 2100 \rightarrow \text{turbulent}$$

$$\frac{dP_{f_{dp}}}{dL} = \frac{\rho^{0.75}\,\bar{V}^{1.75}\,\mu_p^{0.25}}{1800\,d_{i_{dp}}^{1.25}} = \frac{(10.2)^{0.75}\,(10.2)^{1.75}\,(25)^{0.25}}{1800\,(4)^{1.25}} = 0.073\ Psi/ft$$

drill collar: $d_{i_{dc}} = 4\,in \rightarrow \bar{V}_{dc} = \bar{V}_{dp} \Rightarrow \dfrac{dP_{f_{dc}}}{dL} = 0.073\ Psi/ft$

surface connection. $\dfrac{dP_f}{dL} = 0.073\ Psi/ft$

annulus opposite drill collar:



$$\bar{V}_{ac} = \frac{q}{2.448\,(d_h^2 - d_{o_{dc}}^2)} = \frac{400}{2.448\,(8.5^2 - 7.5^2)} = 10.2\ ft/s$$

$$\mu_{a\,ac} = \mu_p = \frac{5\,\tau_y\,(d_h - d_{o_{dc}})}{\bar{V}_{ac}} = 25 + \frac{5 \times 15\,(8.5 - 7.5)}{10.2} = 32.4\ CP$$

چون در فضای حلقوی هستیم باید de حساب کنیم.

$$d_e = 0.816\,(d_h - d_{o_{dc}}) = 0.816\,(8.5 - 7.5) = 0.816\ in$$

$$N_{Re_{ac}} = \frac{928\,\rho\,\bar{V}_{ac}\,d_{e_{ac}}}{\mu_{a_{ac}}} = \frac{928 \times 10.2 \times 10.2 \times 0.816}{32.4} = 2431 \rightarrow \text{turbo}$$

$$\frac{dP_{f_{ac}}}{dL} = \frac{\rho^{0.75}\,\bar{V}_{ac}^{1.75}\,\mu_p^{0.25}}{1800\,d_{e_{ac}}^{1.25}} = \frac{(10.2)^{0.75}\,(10.2)^{1.75}\,(25)^{0.25}}{1800 \times 0.816^{1.25}} = 0.532\ Psi/ft$$

annulus opposite drillpipe

$$\bar{V}_{ap} = \frac{q}{2.448\,(d_h^2 - d_{o_{dp}}^2)} = \frac{400}{2.448\,(8.5^2 - 4.5^2)} = 3.14\ ft/s$$

$$\mu_{a_{ap}} = \mu_p + \frac{5\,\tau_y\,(d_h - d_{o_{dp}})}{\bar{V}_{ap}} = 25 + \frac{5\,(15)\,(8.5 - 4.5)}{3.14} = 120.5\ CP$$

$$de_{ap} = 0.816(d_h - do_{dp}) = 0.816(8.5 - 4.5) = 3.264\ in$$

$$N_{Re_{ap}} = \frac{928\,\rho\,\overline{V}_{ap}\,de_{ap}}{\mu_{a\,ap}} = \frac{928\,(10.2)(3.14)(3.264)}{120.5} = 805 < 2100 \rightarrow \text{laminar}$$

$$\frac{dP_{f\,ap}}{dL} = \frac{\mu_p\,\overline{V}_{ap}}{1000\,(d_h - do_{dp})^2} + \frac{\tau_y}{200\,(d_h - do_{dp})} = \frac{25 \times 3.14}{1000\,(8.5-4.5)^2} + \frac{15}{200\,(8.5-4.5)}$$

$$= 0.0237\ psi/ft$$

drill bit: $A_t = \pi/4 \times 32^2\,(12^2 + 12^2 + 12^2) = 0.3313\ in^2$

$$\Delta P_{bit} = \frac{8.311 \times 10^{-5} \times \rho \times q^2}{Cd^2\ \ A_t^2} = \frac{8.311 \times 10^{-5} \times 10.2 \times 400^2}{0.95^2 \times 0.3313^2} = 1369\ psi$$

$$\Delta P_{pump} = \Delta P_{f\,dp} + \Delta P_{f\,dc} + \Delta P_{f\,ap} + \Delta P_{f\,ac} + \Delta P_{f\,s} + \Delta P_b$$

$$= 0.073\,(26000) + 0.073\,(1000) + 0.0237\,(26000) + 0.523\,(1000)$$

$$+ 0.073\,(500) + 1369$$

$$\Delta P_{pump} = 4524.7$$

$$P_{H\,pump} = \frac{\Delta P_{pump} \times q}{1714 \times 0.85} = \frac{4524.7 \times 400}{1714 \times 0.85} = \boxed{1242\ hp}$$ min مقدار

η

power law model

newtonian, laminar : $$\frac{dP_f}{dL} = \frac{\mu \bar{V}}{1500\, d^2}$$

Power law, laminar: $$\frac{dP_f}{dL} = \frac{K \bar{V}^n}{144{,}000\, d^{n+1}} \left(\frac{3 + 1/n}{0.0416}\right)^n$$

$$\frac{\mu_a \bar{V}}{1500\, d^2} = \frac{K \bar{V}^n}{144{,}000\, d^{n+1}} \left(\frac{3 + 1/n}{0.0416}\right)^n \Rightarrow \mu_a = \frac{K d^{1-n}}{96\, \bar{V}^{1-n}} \left(\frac{3 + 1/n}{0.0416}\right)^n$$

A) Pipe flow:

$$N_{Re} = \frac{928 \rho \bar{V} d}{\mu_a} \Rightarrow N_{Re} = \frac{89100 \rho \bar{V}^{2-n}}{K} \left(\frac{0.0416\, d}{3 + 1/n}\right)^n$$

$N_{Re_c} = 3470 - 1370\, n$ $\qquad n = 1 \longrightarrow$ newtonian

$$\sqrt{\frac{1}{f}} = \frac{4}{n^{0.75}} \log\left(N_{Re}\, f^{1-n/2}\right) - \frac{0.395}{n^{1.2}}$$

B) Annular flow:

Newtonian, laminar : $$\frac{dP_f}{dL} = \frac{\mu_a \bar{V}}{1000\, (d_2 - d_1)^2}$$

Power law, laminar: $$\frac{dP_f}{dL} = \frac{K \bar{V}^n}{144{,}000\, (d_2 - d_1)^{1+n}} \left(\frac{2 + 1/n}{0.0208}\right)^n$$

$$\mu_a = \frac{K (d_2 - d_1)^{1-n}}{144\, \bar{V}^{1-n}} \left(\frac{2 + 1/n}{0.0208}\right)^n$$

$$N_{Re} = \frac{928\,\rho\,\bar{v}\,de}{\mu_a} \qquad de = 0.816\,(d_2 - d_1)$$

$$N_{Re} = \frac{109{,}000\,\rho\,\bar{V}^{2-n}}{K}\left(\frac{0.0208\,(d_2-d_1)}{2+1/n}\right)^n$$

example: 15.6 ppg cement slurry

$K = 335$ eq cp

$n = 0.67$

$q = 672$ gpm

9.625 hole and a 7.0 in. casing

ΔP_f for 100 ft = ?

$$\bar{V} = \frac{q}{2.448\,(d_2^2 - d_1^2)} = \frac{672}{2.448\,(9.625^2 - 7^2)} = 6.29 \text{ ft/s}$$

$$N_{Re} = \frac{109000\,\rho\,\bar{V}^{2-n}}{K}\left[\frac{0.0208\,(d_2-d_1)}{2+1/n}\right]^n$$

$$= \frac{109000 \times 15.6 \times (6.29)^{2-0.67}}{335}\left(\frac{0.0208\,(9.625-7)}{2+\frac{1}{0.67}}\right)^{0.67} = 3612$$

$$N_{Re_c} = 3470 - 1370\,(n)^{0.67} = 2552$$

$3612 > 2552 \rightarrow$ Turbolent

$$\left.\begin{array}{l} n = 0.67 \\ N_{Re} = 3612 \end{array}\right\} \rightarrow f = 0.00815$$

$$\Delta P_f = \frac{f\,\rho\,\bar{V}^2}{858\,de}\,\Delta L = \frac{0.00815 \times 15.6 \times 6.29^2}{258 \times 0.816\,(9.625-7)} \times 100 = \boxed{9.1 \text{ Psi}}$$

mud hydrolic optimization jet bit nozzle selection

maximize ROP $\longrightarrow$ hole cleaning $\begin{cases} \text{Bit hydrolic horsepower} \\ \text{Jet impact Force} \\ \text{Jet nozzle velosity} \end{cases}$

pump pressure requirement (Stand pipe pressure)

$$P_s = \Delta P_{f_s} + \Delta P_{f_{dp}} + \Delta P_{f_{dc}} + \Delta P_{f_{ac}} + \Delta P_{f_{ap}} + \Delta P_{bit}$$

$$P_s = \Delta P_f + \Delta P_{bit}$$

ΔP_f $\hookrightarrow$ total Friction pressure loss or parasitic pressure loss

hydrolic Power requirement

$$P_s = \Delta P_f + \Delta P_{bit}$$

$$\frac{P_s\, q}{1714} = \frac{\Delta P_f\, q}{1714} + \frac{\Delta P_b\, q}{1714} \Rightarrow P_{Hs} = P_{Hf} + P_{Hb}$$

Flow exponent:

$$\Delta P_f = C\, q^{(\alpha)}$$

C: وابسته به ρ، μ، ...

$1 < \alpha < 2$: (Flow exponent) α: نوع جریان

if α not available: $\alpha = 1.86$

$$\log \Delta P_f = \log (c q^{\alpha}) = \underbrace{\log c}_{\text{عرض از مبداء}} + \alpha \log q$$

$i = \Delta P_{f_i}, q_i$

$j = \Delta P_{f_j}, q_j$

$$\alpha = \frac{\log \Delta P_{f_j} - \log \Delta P_{f_i}}{\log q_j - \log q_i} \Rightarrow \alpha = \frac{\log (\Delta P_{f_j}/\Delta P_{f_i})}{\log (q_j/q_i)}$$

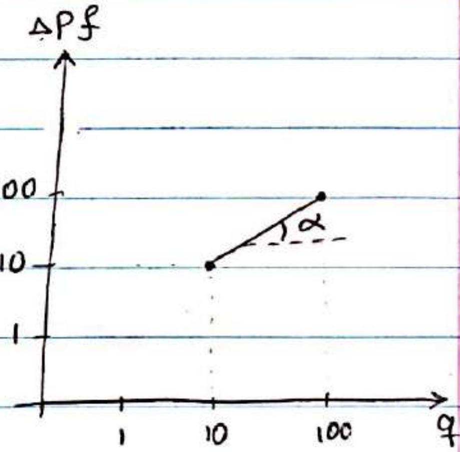
$P_{s_j} = \Delta P_{f_j} + \Delta P_{b_j}$

$P_{s_i} = \Delta P_{f_i} + \Delta P_{b_i}$

$$\Delta P_{i,j} = \frac{8.311 \times 10^{-5} \rho q_{i,j}^2}{A_t^2 \; C_d^2}$$

$$\Delta P_{f_j} = P_{s_j} - \frac{8.311 \times 10^{-5} \rho q_j^2}{A_t^2 \; C_d^2}$$

$$\Delta P_{f_i} = P_{s_i} - \frac{8.311 \times 10^{-5} \rho q_i^2}{A_t^2 \; C_d^2}$$



example: drilling at 12,000 ft

$\rho_m = 15.5$ ppg

14-14-14 drill bit

for $q = 400$ gpm stand pipe pressure was measured 4,622 psi

for $q = 300$ gpm → stand pipe pressure = 2,800 psi

determine $\alpha = ?$

$$\Delta P_f = P_s - \frac{8.311 \times 10^{5} \times \rho \times q^2}{A_t^2 \; C_d^2}$$

$$\Delta P_{f_j} = 4622 - \frac{8.311 \times 10^{5} \times 15.5 \times 400^2}{\frac{\pi}{4}(14/32)^2 \times 3 \times 0.95^2} = 1472 \text{ Psi}$$

$$\alpha = \frac{\log \left(\frac{1472}{1000}\right)}{\log \left(\frac{400}{300}\right)} = 1.34$$

$$\Delta P_{f_i} = 2800 - \frac{8.311 \times 10^{5} \times 15.5 \times 300^2}{\frac{\pi}{4}(14/32)^2 \times 3 \times 0.95^2} = 1000 \text{ Psi}$$

maximum drill bit horse power

$$P_{Hb} = \Delta P_b \, q$$

$$\left.\begin{aligned} \Delta P_b &= P_s - \Delta P_f \\ \Delta P_f &= C q^{\alpha} \end{aligned}\right\} \Delta P_b = P_s - C q^{\alpha}$$

$$\Rightarrow P_{Hb} = (P_s - C q^{\alpha})\, q = P_s q - C q^{\alpha-1}$$

$$\Rightarrow \frac{dP_{Hb}}{dq} = P_s - (\alpha+1)\, C q^{\alpha} = 0$$

فشار پمپ

$$\Rightarrow P_s - (\alpha+1)\, \Delta P_f = 0 \Rightarrow \Delta P_f = \frac{1}{\alpha+1} P_s$$

$$\frac{d^2 P_{Hb}}{dq^2} < 0 \longrightarrow \text{maximum}$$

$$\Delta P_{f\,opt} = \frac{1}{\alpha+1} P_{s_{max}} \Rightarrow \Delta P_{b\,opt} = P_{s_{max}} - \frac{1}{\alpha+1} P_{s_{max}}$$

$$\Rightarrow \Delta P_{b_{opt}} = \frac{\alpha}{\alpha+1} P_{s_{max}}$$

$$\alpha = \frac{\log \left(\Delta P_{f\,opt} / \Delta P_{f\,qa} \right)}{\log \left(q_{opt} / q_a \right)} \Rightarrow q_{opt} = q_a \ \text{antilog}\left(\frac{1}{\alpha} \log \left(\frac{\Delta P_{f\,opt}}{\Delta P_{f\,qa}} \right) \right)$$

$$\Delta P_{b_{opt}} = \frac{8.311 \times 10^{-5} \rho \, q_{opt}}{A t_{opt}^2 \; C d^2} \Rightarrow A t_{opt} = \sqrt{\frac{8.311 \times 10^{-5} \rho \, q_{opt}}{C d^2 \; \Delta P_{b_{opt}}}}$$

$$A t_{opt} = \frac{\pi}{4} \left(d_{1\,opt}^2 + d_{2\,opt}^2 + d_{3\,opt}^3 + \dots + d_{n\,opt}^? \right) \quad ①$$

$$d_{1\,opt} = d_{2\,opt} = \dots = d_{n\,opt} = d_{opt} \quad ②$$

$$1,2 \Rightarrow A t_{opt} = \frac{\pi}{4} n \, d_{opt}^2 \Rightarrow d_{opt} = 2 \sqrt{\frac{A t_{opt}}{n \pi}}$$

به سمت عدد کوچکتر رند می‌کنیم

$q_{min} < q_{opt} < q_{max}$

$\downarrow$

minimum annular velocity

$q_{min} = V_{a_{min}} \times A_{a_{min}}$

اگر q_{opt} از q_{min} کمتر شود همان q_{min} را در نظر می‌گیریم.

$$q_{max} = \frac{H_{hb} \times \eta_v}{P_{s_{max}}} \quad (1)$$

$\eta_v \rightarrow$ Volumetric efficiency

Potential Erosion Problems (2)

ECD: $P_{fracture} > \Delta P_{hydrostatic} + \Delta P_{f_{annulus}}$

اگر q_{opt} از q_{max} بیشتر شود، همان q_{max} را در نظر می‌گیریم

example: drill pipe: OD: 4.5 in

ID: 3.64 in

drill collar: OD = 7 in.

ID = 2 in

length: 1000 ft

mud: bingham, $\theta_{600} = 29$, $\theta_{300} = 21$, $\rho = 15.5$ ppg

pump: max allowed operating pressure = 5440 psi

hydrolic horse power = 1600 hp

volumetric efficiency = 80 %

min. req. Annular velocity = 85 ft/min

drill bit: 12 7/8 in. Tricon with three 14 nozzles

to 12,000 ft TVD next bit to be used is 8 7/8 in.

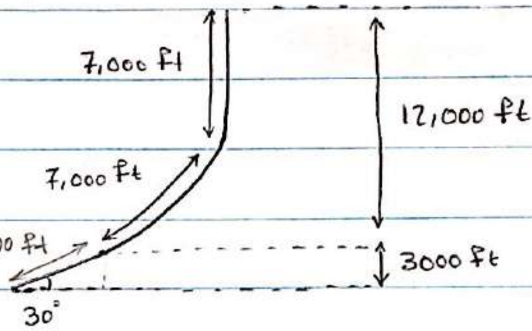
- assume hole is washed out to 9 7/8 in.
- lost intermediate casing is 9 7/8 in at 12,000 ft TVD

Feild data: At 12,000 ft while using the 8 7/8 in. Tricon 14-14-14 nozzles.

$q_1 = 300$ gpm $\quad P_{p_1} = 2966$ psi $\quad P_{pump} = P_{stand\,pipe}$

$q_2 = 400$ gpm $\quad P_{p_2} = 4883$ psi

determine the optimum nozzle size to be used using bit hydrolic horsepower criterion.



solontion :

$$\Delta P_b = \frac{8.311 \times 10^{-5} \times \rho \times q^2}{A_t^2 \; cd^2}$$

$$A_t = \pi/4 \times 3 \, (14/32)^2 = 0.45099 = 0.45099 \text{ in}^2$$

$$\Delta P_{b_1} = \frac{8.311 \times 10^{-5} \times 15.5 \times 300^2}{0.45099^2 \times 0.95^2} = 631.6 \text{ Psi}$$

$$\Delta P_{b_2} = \frac{8.311 \times 10^{-5} \times 15.5 \times 400^2}{0.45099^2 \times 0.95^2} = 1122.8 \text{ Psi}$$

$$\Delta P_{f_1} = \Delta P_{p_1} - \Delta P_{b_1} = 2966 - 631.6 = 2334.4 \text{ Psi}$$

$$\Delta P_{f_2} = \Delta P_{p_2} - \Delta P_{b_2} = 4883 - 1122.8 = 3760.2 \text{ Psi}$$

$$\alpha = \frac{\log\left(\Delta P_{f_2}/\Delta P_{f_1}\right)}{\log\left(q_2/q_1\right)} = 1.66$$

$$q_{max} = 1714\,\eta\,\frac{P_{HP}}{P_{P_{max}}} = 1714 \times 0.8 \times 1600/5440 = 403.3 \text{ gpm}$$

$$q_{min} = 2.448\,(d_h^2 - d_{o_{dp}}^2) \times V_{min} = 2.448(9.857^2 - 4.5^2)\left(\frac{85}{60}\right) = 267.9$$

$$\simeq 268 \text{ gpm}$$

$$\Delta P_{f\,opt} = \frac{1}{\alpha+1} P_{P_{max}} = \frac{1}{1.66+1}(5440) = 2047 \text{ Psi}$$

$$\Delta P_{b_{opt}} = P_{P_{max}} - \Delta P_{f_{opt}} = 5440 - 2047 = 3393 \text{ Psi}$$

$$q_{opt} = q_a \text{ antilog}\left(\frac{1}{\alpha}\log\left(\frac{\Delta P_{f\,opt}}{\Delta P_{f\,q_a}}\right)\right)$$

$$= 300 \text{ antilog}\left(\frac{1}{1.66}\log\left(\frac{2047}{2334.4}\right)\right) = 277 \text{ gpm}$$

$$q_{min} < q_{opt} < q_{max}$$

$$A_{t\,opt} = \sqrt{\frac{8.311 \times 10^{-5} \times \rho \times q_{opt}^2}{Cd^2 \times \Delta P_{b_{opt}}}} = \sqrt{\frac{8.311 \times 10^{-5} \times 15.5 \times 277^2}{0.95^2 \times 3393}} = 0.18 \text{ in}^2$$

$$d_{opt} = 2\sqrt{\frac{0.18}{3\pi}} = 0.28 \text{ in} \simeq 9/32 \text{ in}$$

maximum jet impact Force:

$$F_j = BQ\sqrt{\Delta P_b} \Rightarrow F_j = BQ\sqrt{P_s - CQ^{\alpha}}$$

$$F_j = BQ = \sqrt{P_{s_{max}} - CQ^{\alpha}}$$

$$B = 0.01823\, C_d \sqrt{\rho}$$

$$\Delta P_b = P_s - \Delta P_f$$

$$\Delta P_b = P_s - CQ^{\alpha}$$

برای اینکه F نسبت به Q ماکسیمم شود از آن مشتق می‌گیریم و برابر صفر قرار می‌دهیم

$$\frac{dF_j}{dQ} = \frac{\frac{1}{2} B\left[2P_{s_{max}} Q - (\alpha+2) CQ^{\alpha-1}\right]}{\sqrt{P_{s_{max}} Q^2 - CQ^{\alpha+2}}}$$

$$\frac{dF_j}{dQ} = 0 \rightarrow \frac{1}{2} B\left[2P_{s_{max}} Q - (\alpha+2) CQ^{\alpha-1}\right] = 0$$

$$2P_{s_{max}} Q - (\alpha+2) CQ^{\alpha+1} = 0$$

$$2P_{s_{max}} - (\alpha+2) \overbrace{CQ^{\alpha}}^{\Delta P_f} = 0$$

$$2P_{s_{max}} - (\alpha+2)\Delta P_f = 0$$

در نقطه بهینه

$$\star\ \Delta P_{f_{opt}} = \frac{2}{\alpha+2} P_{s_{max}} \rightarrow \Delta P_{b_{opt}} = P_{s_{max}} - \frac{2}{\alpha+2} P_{s_{max}}$$

$$\star\ \Delta P_{b_{opt}} = \frac{\alpha}{\alpha+2} P_{s_{max}}$$

$$\frac{d^2F_j}{dQ^2} < 0 \rightarrow \text{maximum point}$$

$$\star\ Q_{opt} = Q_a \text{ antilog}\left[\frac{1}{\alpha}\log\left(\frac{\Delta P_{f_{opt}}}{\Delta P_{f_{q_a}}}\right)\right]$$

$$\star\ A_{t_{opt}} = \sqrt{8.311\times10^{-5}\rho Q_{opt}^2 / C_d^2\, \Delta P_{b_{opt}}}$$

$$\star\ d_{opt} = 2\sqrt{A_{t_{opt}}/n\pi}$$

example: drill pipe: OD = 4.5 in ID: 3.64 in

drill collar: OD = 7 in ID: 2 in length: 1000 ft

mud: Binghum, $\theta_{300} = 21$, $\theta_{600} = 29$ $\rho = 15.5$ ppg

Pump: maximum allowed operating pressure = 5440 psi

hydrolic horse power: 1600 hp

Volumetric efficiency: 80 %.

Bit: 12 7/8 in tri-core with 14-14-14 nozzles.

to 12000 ft, next bit to be used is 8 7/8 in

well capacity: last intermediate casing is 9 7/8 in

at 12000 ft + TVD rate is washed

out to 9 7/8 in.

maximum required anulur velocity = 85 ft/min

Field data at 12000 ft while using the 8 7/8 in 14-14-14 bit

$Q_1 = 300$ gpm, $P_{P_1} = 2966$ Psi

$Q_2 = 400$ gpm $P_{P_2} = 4883$ psi

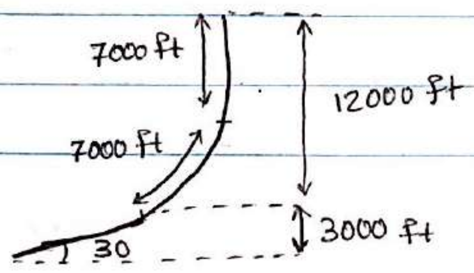
a) using the impact force Criterion, determine the optimum nozzle sizes to be used to drill the next intral starting at 12000 ft.

b) determine the maximum total hydrolic horse power that will be utilized under the drilling conditions in part(a)

Assume a bit life of 10 h and a drilling rate of 50 ft/h

Friction pressure gradients at the flow rate found in part(a) are: $dP_{f\,dc}/dl = 1.587$ psi/ft ; $\frac{dP_{f\,ac}}{dl} = 0.0126$ Psi/ft

$dP_{f\,ap}/dl = 0.0254$ Psi/ft



حل: از جلسه قبل داریم:

a) $\alpha = 1.66$

$Q_{min} = 268\ gpm$

$Q_{max} = 403\ gpm$

جلسه قبل محاسبه شده

$$\Delta P_{f\,opt} = \frac{2}{\alpha+2} P_{s\,max} = \frac{2}{1.66+2} 5440 = 2975\ Psi$$

$$\Delta P_{b\,opt} = P_{s\,max} - \Delta P_{f\,opt} = 5440 - 2975 = 2465\ Psi$$

$$Q_{opt} = Q_a \text{ antilog}\left[\frac{1}{\alpha} \log \frac{\Delta P_{f\,opt}}{\Delta P_{f\,q_a}}\right]$$

$$= 300 \text{ antilog}\left[\frac{1}{1.66} \log \frac{2975}{2334}\right]$$ جلسه قبل محاسبه شد

$$= 347\ gpm$$

$$Q_{min} < Q_{opt} < Q_{max} \checkmark$$

پس با همین مقدار مسئله را ادامه می‌دهیم:

$$A_{t\,opt} = \sqrt{\frac{8.311 \times 10^{-5} \times \underbrace{\rho}_{15.5} \times \underbrace{Q_{opt}^2}_{(347)^2}}{\underbrace{C_d^2}_{(0.95)^2}\ \underbrace{\Delta P_{b\,opt}}_{2465}}} = 0.2639\ in = 3 \times \frac{\pi}{4} \times (d_{opt})^2$$

$$d_{opt} \times 32 = 10.71$$

$$d_{opt} = \frac{10.71}{32}\ in \longrightarrow d_{opt} = 11\ in \rightarrow 11-11-11$$

b) $L = ROP \times t = 50\ ft/h \times 10h = 500\ ft$

$$MD = 7000 + 7000 + 500 = 14500\ ft$$

برای محاسبه $\Delta P_{f\,dp}$ داریم:

$\mu_p = \theta_{600} - \theta_{300} = 8\ CP$ $\qquad \tau_y = \theta_{300} - \mu_p = 13\ lbf/100ft^2$

$$\bar{V}_{dp} = \frac{Q}{2.448\, d_{i_{dp}}^2} = \frac{347}{2.448 \times 3.64^2} = 10.7\ ft/s$$

$$\mu_{a_{dp}} = \mu_p + \frac{6.66\, \tau_y\, d_{i_{dp}}}{\bar{V}_{dp}} = 8 + \frac{6.66 \times 13 \times 3.64}{10.7} = 37.5\ CP$$

$$N_{Re_{dp}} = \frac{928\,\rho\,\bar{v}\,d_{i_{dp}}}{\mu_{a_{dp}}} = \frac{928 \times 15.5 \times 10.7 \times 3.64}{37.5} = 14939 \rightarrow \text{turbo}$$

$$\frac{dP_{f_{dp}}}{dl} = \frac{\rho^{0.75}\,\bar{v}^{1.75}\,\mu_p^{0.25}}{1800\,d_{i_{dp}}^{1.25}} = \frac{15.5^{0.75} \times 10.7^{1.75} \times 8^{0.25}}{1800 \times 3.64^{1.25}} = 0.0919\ \frac{psi}{ft}$$

حالا باید ΔP_{bit} را حساب کنیم

$$\Delta P_b = \frac{8.311 \times 10^{-5} \times \overset{15.5}{\rho} \times \overset{347^2}{Q^2}}{\underset{0.95^2}{C_d^2}\,\underset{0.2789^2}{A_t^2}} = 2217.5\ \text{in}$$

$$A_t = \frac{\pi}{4 \times 32^2}\left(11^2 + 11^2 + 11^2\right) = 0.2789\ in^2$$

$$P_p = \Delta P_{f_{dp}} + \Delta P_{f_{dc}} + \Delta P_{f_{ap}} + \Delta P_{f_{ac}} + \Delta P_b$$

$$= 0.0919 \times 13500 + 1000 \times 1.587 + 13500 \times 0.0254 + 1000 \times 0.0126$$

$$+ 2217.5 = 5240.7\ psi$$

$$P_{HP} = \frac{\overset{P_p}{5240.7} \times \overset{Q}{347}}{1714 \times \underset{0.8}{\eta_v}} = 1326\ hp$$

۲۰ مفارس ۲۰. جلسه هجدهم

Swab / surge Pressure calculations:

surge pressure ← افزایش فشار مربوط به حرکت رو به پایین لوله

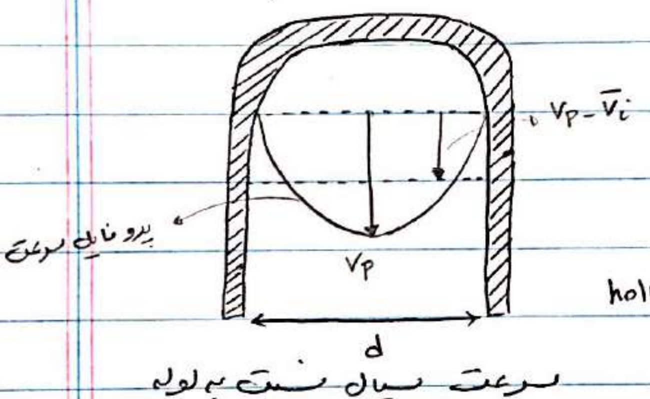
swab pressure ← افت فشار مربوط به حرکت رو به بالای لوله

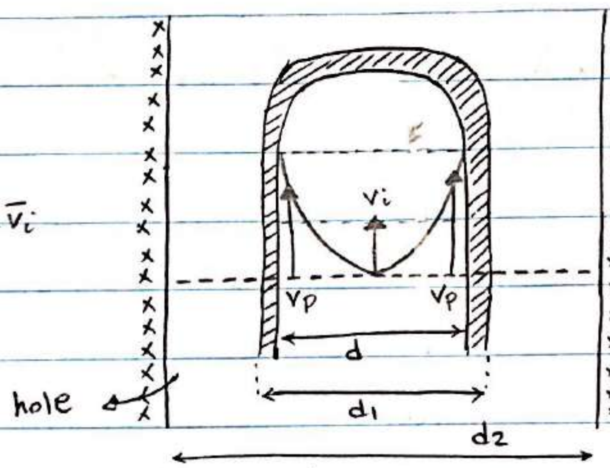
surge pressure می تواند باعث شکست سازند شود

swab pressure می تواند باعث kick شدن شود.

Newtonian fluid:

- laminar flow:



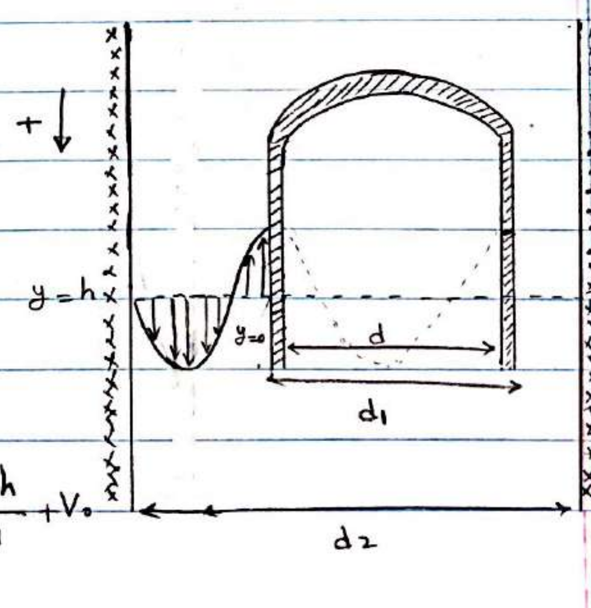


سرعت سیال نسبت به لوله   سرعت سیال نسبت به دیواره چاه

وقتی لوله را به سمت بالا می کشیم، فشار ته چاه کم می شود.

$$\frac{dP_f}{dL} = \frac{v_p - \bar{v}_i}{1500\, d^2}$$

- slot flow:



$$v = -\frac{y^2}{2\mu}\frac{dP_f}{dL} + \tau_0 \frac{y}{\mu} + v_0$$

$$y = 0 \rightarrow v = -v_p \rightarrow -v_p = v_0$$

$$y = h \rightarrow v = 0 \rightarrow 0 = -\frac{h^2}{2\mu}\frac{dP_f}{dL} + \frac{\tau_0 h}{\mu} + v_0$$

$$\Rightarrow \begin{cases} V_0 = -V_p \\ \tau_0 = \frac{h}{2} \frac{dP_f}{dL} + \frac{V_p \mu}{h} \end{cases}$$

$$\Rightarrow V = \frac{1}{2\mu} \frac{dP_f}{dL} (yh - y^2) - V_p (1 - y/h)$$

$$\Rightarrow q = \int_0^h v w \, dy = \frac{w}{2\mu} \frac{dP_f}{dL} \int_0^h (hy - y^2) dy - V_p w \int_0^h (1 - y/h) dy$$

$$\Rightarrow q = -\frac{wh^2}{12\mu} \frac{dP_f}{dL} - \frac{V_p w h}{2}$$

$$\begin{cases} A = wh = \pi (r_2^2 - r_1^2) \rightarrow w \checkmark \\ h = r_2 - r_1 \end{cases}$$

$$q = \frac{\pi}{12\mu} \frac{dP_f}{dL} (r_2^2 - r_1^2)(r_2 - r_1)^2 - \frac{\pi V_p}{2} (r_2^2 - r_1^2)$$

$$q = \pi (r_2^2 - r_1^2) \bar{V}_a$$ → annulus

$$\frac{dP_f}{dL} = \frac{12\mu (\bar{V}_a + V_p/2)}{(r_2^2 - r_1^2)}$$: SI

$$\frac{dP_f}{dL} = \frac{\mu (\bar{V}_a + V_p/2)}{1000 (d_2 - d_1)^2}$$: Field

closed pipe:

$$q_t = \frac{\pi}{4} d_1^2 * v_p$$

$$\bar{V}_a = \frac{q_t}{\frac{\pi}{4}(d_2^2 - d_1^2)} = \frac{\cancel{\pi/4}\, d_1^2\, V_p}{\cancel{\pi/4}\,(d_2^2 - d_1^2)} \Rightarrow \bar{V}_a = \frac{d_1^2}{d_2^2 - d_1^2} V_p$$

opened pipe:

فشار سر چاه و فشار ته چاه نسبت به pipe و هم نسبت به annulus یکی است.

$$\left(\frac{dP_f}{dL}\right)_{pipe} = \left(\frac{dP_f}{dL}\right)_{annulus}$$

$$\Rightarrow \frac{\mu (V_p - \bar{V}_i)}{1500\, d^2} = \frac{\mu(\bar{V}_a + V_p/2)}{1000\,(d_2 - d_1)^2} \quad \text{I}$$

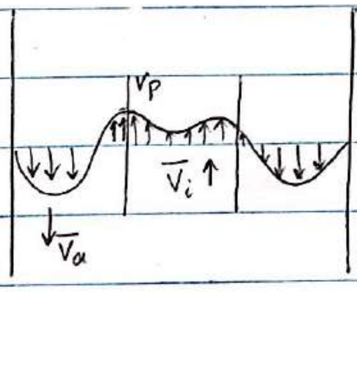
$$\overset{\uparrow}{q_p} + \overset{\uparrow}{q_i} = \overset{\downarrow}{q_a} \Rightarrow \frac{\pi}{4}(d_1^2 - d^2)\, V_p + \frac{\pi}{4} d^2 \bar{V}_i = \frac{\pi}{4}(d_2^2 - d_1^2)\, \bar{V}_a \quad \text{II}$$

گل با زور

به سمت بالا می آید

$$\text{I}, \text{II} \rightarrow \bar{V}_a = \frac{-3d^4 + 4d_1^2 (d_2 - d_1)^2}{6d^4 + 4(d_2 - d_1)^2 (d_2^2 - d_1^2)} V_p$$

اگر $\bar{V}_a < 0$ شود یعنی با سرعت V_p مذکور نمی‌توان لوله را بالا کشید



$$q_a = q_i + q_p$$

example : Calculate equivalent density below the bottom joint of a 10.75 in casing having a 10 in ID at 4000 ft. The casing is being lowered at a rate of 1 ft/s in a 12 in. hole containing a 9 PPg brine having a viscosity of 2.0 cp

- perform the calculations for

1) Casing that is open

2) casing with a closed bottom end.

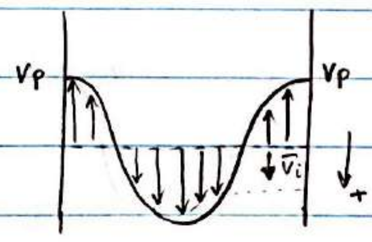
$$\bar{V}_a = \frac{-3d^4 + 4d_1^2\,(d_2 - d_1)^2}{6d^4 + 4(d_2 - d_1)^2\,(d_2^2 - d_1^2)}\,V_p$$

$$= \frac{3(10^4) + 4(10.75)^2(12 - 10.75)^2}{6(10^4) + 4(12 - 10.75)^2(12^2 - 10.75^2)} \times 1 = -0.4865 \text{ ft/s}$$ طبق کتاب

$$\bar{V}_i : \bar{V}_a\,\pi(d_2^2 - d_1^2) = \bar{V}_i\,\pi d^2 + V_p\,\pi(d_1^2 - d_2)$$

$$\rightarrow \bar{V}_i = \frac{d_2^2 - d_1^2}{d^2}\,\bar{V}_a - \frac{d_1^2 - d^2}{d^2}\,V_p$$

$$= \frac{12^2 - 10.75^2}{10^2}(-0.4865) - \frac{10.75^2 - 10^2}{10^2} \times 1 = -0.2940 \text{ ft/s}$$



شکل داخل کتاب غلط است. شکل درست: داخل pipe

جهت را به سمت پایین در نظر می‌گیریم:

$$V = -\frac{r^2}{4\mu}\,\frac{dP_f}{dL} + \frac{C_1}{\mu}\ln r + C_2$$

at $r=0$ → shear stress is limited → $C_1 = 0$ → $V = -\frac{r^2}{4\mu}\,\frac{dP_f}{dL} + C_2$

at $r=R$ → $V = -V_p$ → $-V_p = \frac{-R^2}{4\mu}\,\frac{dP_f}{dL} + C_2$ → $C_2 = \frac{R^2}{4\mu}\,\frac{dP_f}{dL} - V_p$

$$\Rightarrow V = \frac{1}{4\mu}\frac{dP_f}{dL}(R^2 - r^2) - V_p$$

$$q = \int_0^R v(2\pi r)\,dr = \frac{\pi R^4}{8\mu}\frac{dP_f}{dL} - \pi R^2 V_p$$

$$q = \pi R^2 \bar{V}_i$$

$$\Rightarrow \frac{dP_f}{dL} = \frac{8\mu(\bar{V}_i + V_p)}{R^2}$$ SI

$$\boxed{\frac{dP_f}{dL} = \frac{\mu(\bar{V}_i + V_p)}{1500\, d^2}}$$ feild.

$$\left(\frac{dP_f}{dL}\right)_a = \left(\frac{dP_f}{dL}\right)_p$$

$$\frac{\mu(\bar{V}_a + V_p/2)}{1000(d_2 - d_1)^2} = \frac{\mu(\bar{V}_i + V_p)}{1500\, d^2} \quad (1)$$

$$q_p = q_i + q_a \quad (2) \rightarrow V_p(d_2^2 - d_1^2) = \bar{V}_i d^2 + \bar{V}_a(d_2^2 - d_1^2)$$

$$1,2 \rightarrow \boxed{\bar{V}_a = V_p \frac{-3d^4 + 4d_1^2(d_2 - d_1)^2}{6d^4 + 4(d_2 - d_1)^2(d_2^2 - d_1^2)}} \rightarrow$$ معادله صحیح

$$\bar{V}_a = -0.4865$$

$$\frac{dP_f}{dL} = \frac{\mu(\bar{V}_a + V_p/2)}{1000(d_2 - d_1)^2} = \frac{2(0.4865 + 1/2)}{1000(12 - 10.75)^2} = 0.0001728 \text{ Psi/ft}$$

2) closed casing:

$$\bar{V}_a = \frac{d_1^2}{d_2^2 - d_1^2} V_p = \frac{10.75^2}{12^2 - 10.75^2} \times 1 = 4.06 \text{ ft/s}$$

$$\frac{dP_f}{dL} = \frac{\mu(\bar{V_a} + V_P/2)}{1000\,(d_2 - d_1)^2} = \frac{2(4.06 + 1/2)}{1000(12 - 10.75)^2} = 0.00584 \ \text{Psi/ft}$$

$$\Delta P_f = \Delta P_{surge} = \frac{dP_f}{dL} \times \Delta L = 0.00584 \times 4000 = 23 \ \text{Psi}$$

$$\Rightarrow \rho_e = \rho + \frac{\Delta P_f}{0.052\,D} = 9 + \frac{23}{0.052 \times 4000} = 9.11 \ \text{ppg}$$

non - Newtonian fluid:

برای سیال نیوتنی داریم

$$\frac{dP_f}{dL} = \frac{\mu \bar{V}}{1000\,(d_2 - d_1)^2}$$

$$\frac{dP_f}{dL} = \frac{\mu\,(\bar{V}_a + V_p/2)}{1000\,(d_2 - d_1)^2}$$

$$\bar{V}_{ae} = \bar{V}_a + 0.5\,V_p$$

swab, surge (newtonian)

A) Bingham fluid :

mud clinging constant

Burkhardt : $\bar{V}_{ae} = \bar{V}_a + K\,V_p$

for laminar

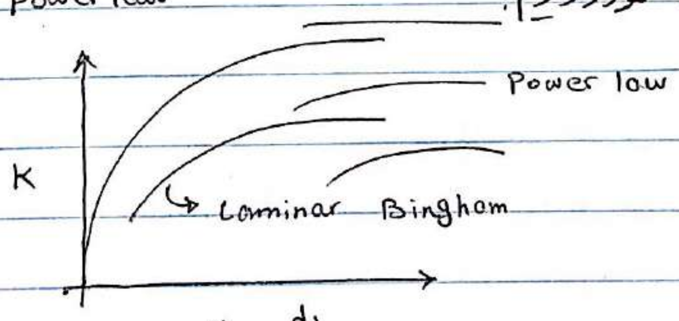
$$\Rightarrow \quad K = \frac{\alpha^2 - 2\alpha^2 (\ln\alpha) - 1}{2\,(1-\alpha^2)\,\ln\alpha} \quad : \quad \alpha = \frac{d_1}{d_2}$$

for turbulent : $$K = \frac{\sqrt{\frac{\alpha^4 + \alpha}{1+\alpha}} - \alpha^2}{1-\alpha^2} \qquad \alpha = \frac{d_1}{d_2}$$

در این جا شکل ما این است که معادله ای برای فهم جریان laminar و turbulent نداریم. پس باید یک بار محاسبات را برای laminar و یک بار برای turbulent انجام می دهیم. هر کدام Vp کمتری داشت آن را به عنوان سرعت مجاز در نظر می گیریم.

B) Powerlaw

برای سیال power law فرمول داریم:



$\alpha = \frac{d_1}{d_2}$

example: Current well depth : 10,000 ft

mud: Binghom-plastic : $\rho = 10$ ppg $\theta_{600} = 60$ $\theta_{300} = 35$

Hole size: 7 7/8 in.

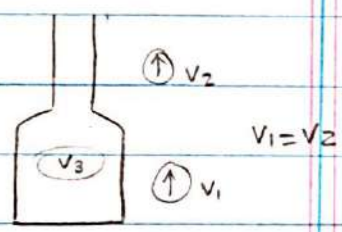
drill pipe: OD = 4 in , ID: 3 1/4 in

drill collar: OD = 6 in ID: 3 1/4 in length: 1000 ft

Formation: pressure gradiant : 0.5 Psi/ft

Fracture gradiant : 0.56 Psi/ft

- determine the max pipe speed at wich trapping maybe performed.
- Assume the flow regime around pipe and drill collar can be both considered laminar or turbalent.
- Assume closed pipe trapping.



Solution:

$\mu_p = \theta_{600} - \theta_{300} =$ ۶۰ − ۳۵ = ۲۵ cp

$\tau_y = \theta_{300} - \mu_p =$ ۳۵ − ۲۵ = ۱۰ lbf/۱۰۰ ft۲

tripping concerns:

$$\underbrace{P_{hydrostatic} - \Delta P_{swob}}_{\text{فشار ته چاه (بالا کشیدن)}} \geq P_{formation}$$

$$\underbrace{P_{hydrostatic} + \Delta P_{surge}}_{\text{فشار ته چاه (پایین کشیدن)}} \leq P_{fracture}$$

drill pipe: $\bar{V}_{a_{dp}} = \dfrac{d_{o_{dp}}^2}{d_h^2 - d_{o_{dp}}^2} \times V_p = \dfrac{4^2}{7.875^2 - 4^2} V_p = 0.348 V_p$

$\alpha_{dp} = \dfrac{d_{o_{dp}}}{d_h} = \dfrac{4}{7.875} = 0.508$

یک بار با فرض جریان laminar و یکبار با فرض turbulent حل می‌کنیم:

$$K_{dp,lam} = \frac{\alpha_{dp}^2 - 2\alpha_{dp}^2 \ln \alpha_{dp} - 1}{2(1-\alpha_{dp}^2)\ln \alpha_{dp}} = \frac{0.508^2 - 2(0.508)^2 \ln(0.508) - 1}{2(1-0.508^2)\ln(0.508)}$$

$$= 0.390$$

$$K_{dp,turbo} = \frac{\sqrt{\frac{\alpha_{dp}^4 + \alpha_{dp}}{1+\alpha_{dp}}} - \alpha_{dp}^2}{1-\alpha_{dp}^2} = 0.484$$

$$\overline{V}_{ae_{dp,lam}} = \overline{V}_{a_{dp}} + K_{dp,lam} V_p = 0.348 V_p + 0.390 V_p = 0.738 V_p$$

$$\overline{V}_{ae_{dp,tur}} = \overline{V}_{a_{dp}} + K_{dp,tur} V_p = 0.348 V_p + 0.484 V_p = 0.832 V_p \ ft/s$$

laminar: $$\frac{dP_{f_{dp}}}{dL} = \frac{\mu_p \overline{V}_{ae_{dp,lam}}}{1000(d_h - d_{o_{dp}})^2} + \frac{\tau_y}{200(d_h - d_{o_{dp}})} \quad psi/ft$$

$$= \frac{25 \times 0.738 V_p}{1000(7.875-4)^2} + \frac{10}{200(7.875-4)} = 0.001229 V_p + 0.01290$$

turbolent: $$d_{ea_{dp}} = 0.816(d_h - d_{o_{dp}}) = 0.816(7.875-4) = 3.162 \ in$$

$$\frac{dP_{f_{dp}}}{dL} = \frac{\rho^{0.75}\, \overline{V}_{ae_{dp,tur}}^{1.75}\, \mu_p^{0.25}}{1800\, d_{ea_{dp}}^{1.25}} = \frac{10^{0.75} \times (0.832 V_p)^{1.75} \times 25^{0.25}}{1800\,(3.162)^{1.25}} = 0.00120 V_p^{1.75}$$

برای drill collar نیز به روش مشابه عمل می‌کنیم و برای هر دو جریان laminar, turbolent افت فشار را محاسبه می‌کنیم:

جواب‌های نهایی برای drill collar ←

$$\overline{V}_{a_{dc}} = \frac{d_{o_{dc}}^2}{d_h^2 - d_{o_{dc}}^2} = 1.384 V_p \qquad \alpha_{dc} = 0.782$$

$$K_{dc,lam} = 0.448 \qquad K_{dc,tur} = 0.498$$

$\bar{V}_{ae_{dc}, lam} = 1.832\ V_p$ $\qquad$ $\bar{V}_{ae_{dc}, tur} = 1.882\ V_p\ ft/s$

$$\left(\frac{dP_{f_{adc}}}{dl}\right)_{lam} = 0.01303\ V_p + 0.02667\ Psi/ft$$

$$\left(\frac{dP_{f_{adc}}}{dl}\right)_{tur} = 0.01242\ V_p^{1.75}\ Psi/ft$$

swob , surge محاسبه

$$\Delta P_{surge, swob} = \left(\frac{dP_{f_{adP}}}{dl}\right) \times L_{dp} + \left(\frac{dP_{f_{adc}}}{dl}\right) \times L_{dc}$$

laminar $\rightarrow \Delta P_{surge, swob} = 9000(0.001229\ V_p + 0.01290) +$

$1000\ (0.01303\ V_p + 0.02667)$

$= 24.091\ V_p + 142.77$

turbo $\rightarrow \Delta P_{surge, swob} = 23.22\ V_p^{1.75}$

swob tripping out :

$$P_{hydrostatic} - \Delta P_{swob} \geq P_{formation}$$

laminar: $0.052 \times 10 \times 1000 - (24.091\ V_p + 142.77) \geq 0.5 \times 1000$

$5200 - 24.091\ V_p + 142.77 \geq 5000$

$24.91\ V_p + 142.77 \leq 200$

$\Rightarrow V_p \leq 2.38\ ft/s \rightarrow$ سرعت مجاز

turbo: $5200 - (23.22\ V_p^{1.75}) \geq 5000$

$\rightarrow V_p \leq 3.42\ ft/s$

surge tripping in:

$P_{hydro} + \Delta P_{surge} \leqslant P_{fracture}$

lam: $5200 + (24.091 V_p + 142.77) \leqslant 0.56 \times \overset{5600}{10000}$

$24.091 V_p + 142.77 \leqslant 400$

$\Rightarrow \quad V_p \leqslant 10.68 \ ft/s$

turb: $5200 + 23.22 V_p^{1.75} \leqslant 5600$

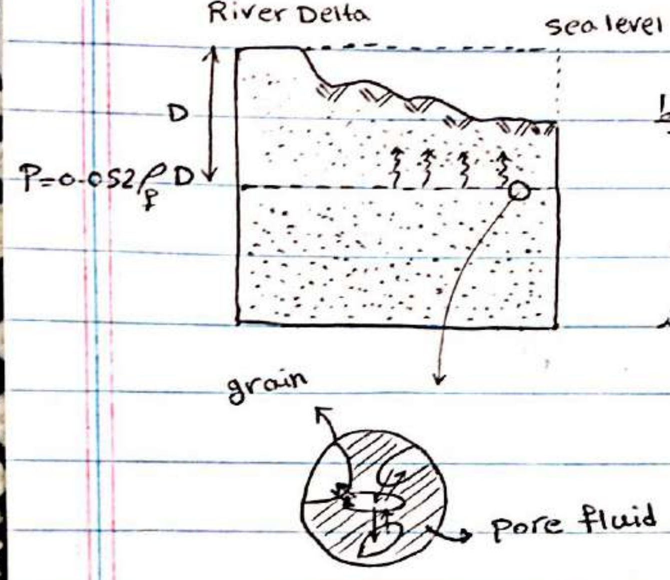
$V_p \leqslant 5.09 \ ft/s$ ✓ → سرعت مجاز

در حل این مثال $\bar{V}_{adp}$ ، $\bar{V}_{adc}$ مستقل از هم نوشته شده است:

علت این امر چیست؟

Pore pressure:

فشار سیالات محبوس داخل سنگ های «sedimentary»



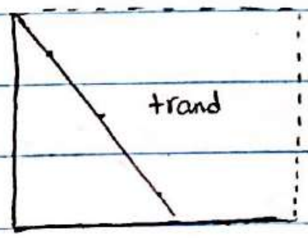
هنگامی که لایه ها روی هم دفن می شوند چون محیط دریایی است لابه لای ذرات آب نیز هست وقتی به سمت پایین حرکت می کنیم فشار لایه ها بیشتر می شود و آب به سمت بالا حرکت می کند

→ (D) فشار داخل سیال تنها به عمق وابسته است. چون میزان وزن سنگ ها را خود grain ها و ارتباطی که باهم دارند تحمل می کنند.

$$P = 0.052\, \rho_{fluid}\, D$$

Region (country)	pressure gradiant (Psi/ft)	equivalent water density (Kg/m³)	
1- west Texas	0.433	1,000	
2- Gulf of Mexico coast line	0.465	1,074	normal Pressure
3- north sea	0.452	1,044	

normal pressure ← اگر عملکرد مخزن ما از trend ها پیروی کند.



example) Compute Normal pressure at 6000 ft in Louisiania gulf coast.

$P_f = 0.052\ \rho_f\ D$ ← solution

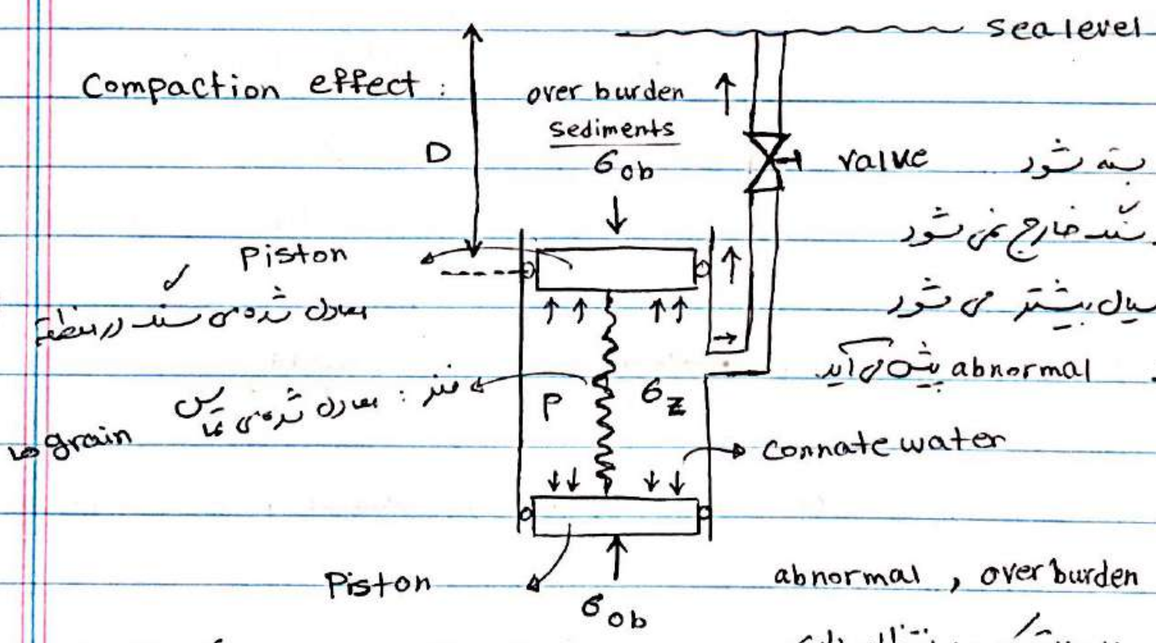
$P_f = 0.465\ psi/_{ft} \times 6000\ ft = 2790\ psi$

Formation pressure > normal pressure → Abnormal formation Pressure

" " < " " → Subnormal " "

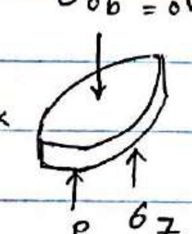
Abnormal formation pressure → 1- Compaction effect
2- diffrential density effect
3- fluid migration effect

Compaction effect :



اگر شیر بسته شود
سیال از سنگ خارج نمی شود
و فشار سیال بیشتر می شود
در حالت abnormal پیش می آید

σ_{ob} = over burdon stress



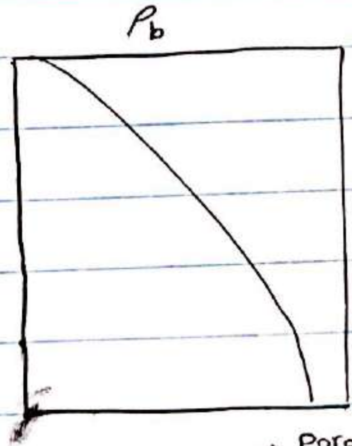
P σ_Z = matrix stress

P → Pore fluid pressure

$$\sigma_{ob} = \sigma_z + P$$

در حالت abnormal , overburden
چون سنگ از حالتی که ما انتظار داریم
بیشتر می شود که یکی از راه های
تشخیص حالت abnormal pressure است.

$$\sigma_{ob} = \int_0^D \rho_b \, g \, dD$$



$$\rho_b = \rho_g (1-\phi) + \rho_{fl} \phi$$

$$\Rightarrow \phi = \frac{\rho_g - \rho_b}{\rho_g - \rho_{fl}}$$

plot(ϕ - depth) semi_log paper → stright line ⇒ $\phi = \phi_0 e^{-K D_s}$

تابع نمایی

Porosity decline constant (K)

depth below the surfa of sediments (D_s)

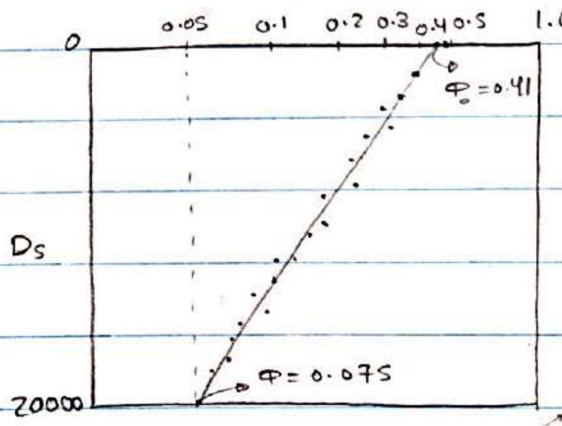
surface porosity (ϕ_0)

example: assume average grain density of 2.6 gr/cm³
" pore fluid " " 1.074 gr/cm³

determine values for ϕ_0 and K

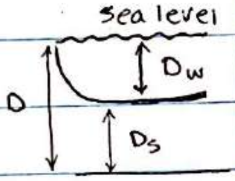
$$\phi = \frac{\rho_g - \rho_b}{\rho_g - \rho_f} = \frac{2.6 - \rho_b}{2.6 - 1.074}$$

Sediment thickness (DS)	Bulk density (gr/cm³)	ϕ
0	1.95	0.43
1000	2.02	0.38
2000	2.06	0.35
3000	2.11	0.32
4000	2.16	0.29
5000	2.19	0.27
6000	2.24	0.24
7000	2.27	0.22
8000	2.29	0.2
⋮	⋮	⋮
20,000	2.48	0.079



$$K = \frac{\ln \phi_0 - \ln \phi}{D_s} = \frac{\ln \phi_0/\phi}{D_s} = 0.000085 \ ft^{-1}$$

($\phi_0 = 0.41$, $\phi = 0.075$, $D_s = 20000$)



$$\sigma_{ob} = \sigma_z + P$$

$$\sigma_{ob} = \int_0^D \rho_b\, g\, dD = \int_0^D (\rho_g(1-\phi) + \rho_{fl}\,\phi)\, g\, dD$$

$$= g\int_0^{D_w} \rho_{sw} + dD + g\int_{D_w}^{D} (\rho_g - (\rho_g - \rho_{fl})\,\phi_0\, e^{-KD})\, dD$$

$$\Rightarrow \sigma_{ob} = \underbrace{\rho_{sw}\, g\, D_w}_{\text{در محیط های خشکی} = 0} + \rho_g\, g\, D_s - \frac{(\rho_g - \rho_{fl})\, g\, \phi_0}{K}(1 - e^{-KD_s}) \quad ; D_s = D - D_w$$

example: compute the vertical overburden pressure at a depth of 10 000 ft for the previous example

$\rho_g = 2.6 \ g/cm^3$

$\rho_{fl} = 1.079 \ g/cm^3$

$K = 0.000085 \ ft^{-1}$

$\phi_0 = 0.41$

محیط خشکی

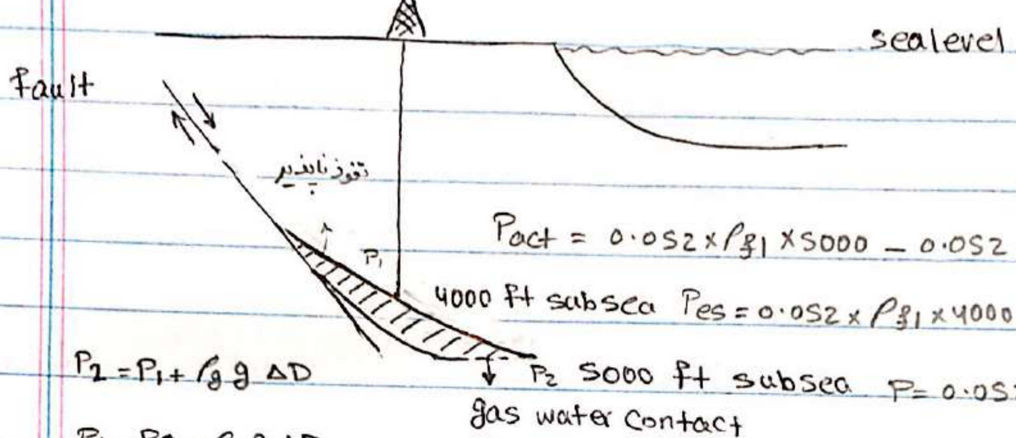
$$\sigma_{ob} = \rho_g\, g\, D - \frac{(\rho_g - \rho_{fl})\, g\, \phi_0}{K}(1 - e^{-KD})$$

$$= 0.052 \times 2.6 \times 8.33 \times 10000 - \frac{0.052(2.6 - 1.074) \times 8.33 \times 0.41}{0.000085}(1 - e^{0.000085 \times 10000})$$

Field: $\rho_g = 0.052 \times 2.6$ *

$$= 11262 - 1826 = 9436 \text{ Psi}$$

Differential Density Effects:



$P_{act} = 0.052 \times \rho_{g1} \times 5000 - 0.052 \times \rho_g \times 1000$

4000 ft subsea $P_{es} = 0.052 \times \rho_{g1} \times 4000$

5000 ft subsea $P = 0.052 \times \rho_{g1} \times 5000$

$P_2 = P_1 + \rho_g g \Delta D$

$P_1 = P_2 - \rho_g g \Delta D$

abnormal pressure $\leftarrow$ $P_{act} > P_{es}$

example: Normal pressure gradiant: 0.465 psi/ft

$\rho_g = 0.8$ ppg

- Calculate the safe mud weight to drill the formation at 4000 ft:

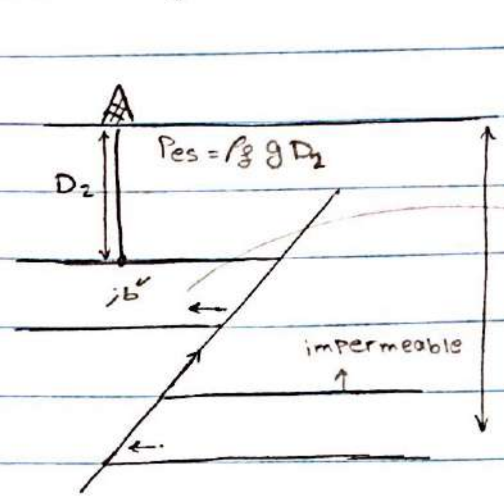
$P_{at\ 5000\ ft} = 0.465 \times 5000 = 2325$

$P_{@\ 4000\ ft} = P_{@\ 5000\ ft} - 0.052 \times \rho_g \times 1000$

$= 2325 - 0.052 \times 0.8 \times 1000 = 2283$ psi

$0.052 \times \rho_m \times 4000 > 2283 \rightarrow \rho_m > \dfrac{2283}{0.052 \times 4000} \Rightarrow \rho_m = 11$ ppg

Fluid migration effect

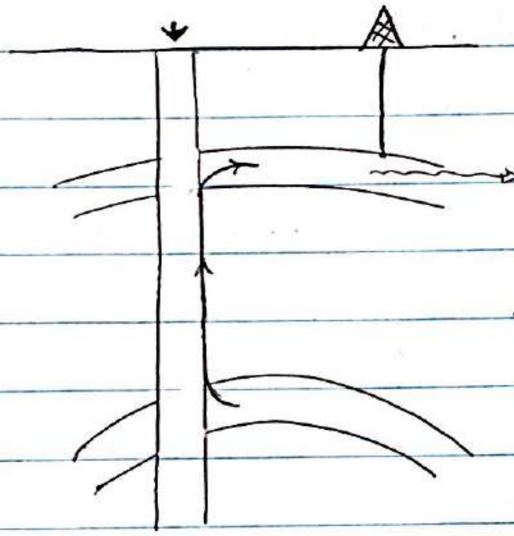


abnormal Pressure ← $P_{act} > P_{es}$

فشار واقعی در این ناحیه تقریباً برابر با فشار در عمق D_1 است.

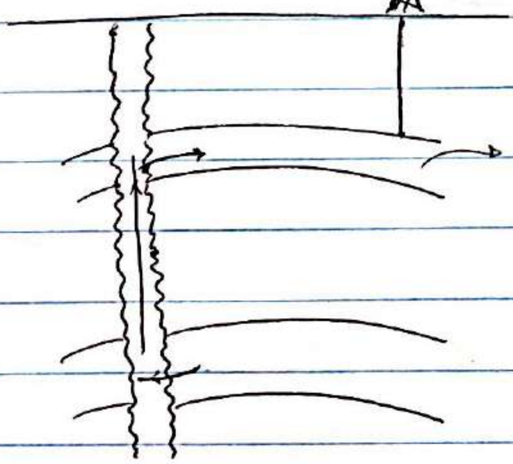
$D_1 \rightarrow P = \rho_g g D$

leaky fault



در این ناحیه با گازی روبرو می‌شویم که در عمق بالاتر فشاری تقریباً برابر با عمق خیلی پایین‌تر دارد و در نتیجه blow out پیش می‌آید.

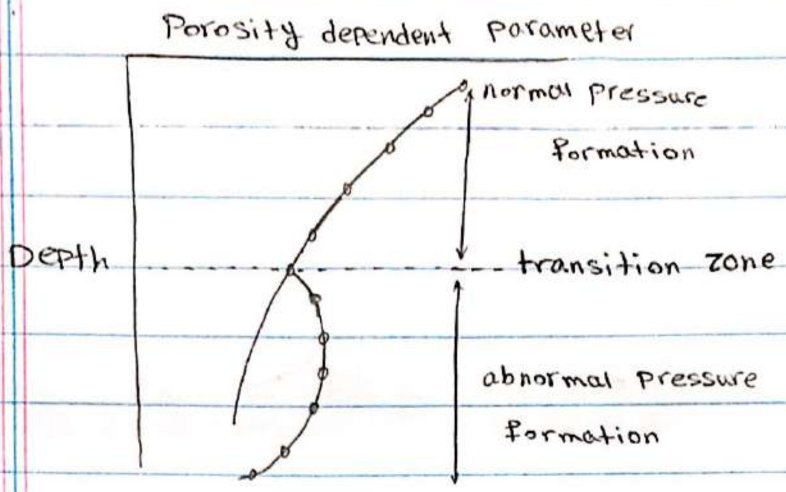
leaky cement
or casing



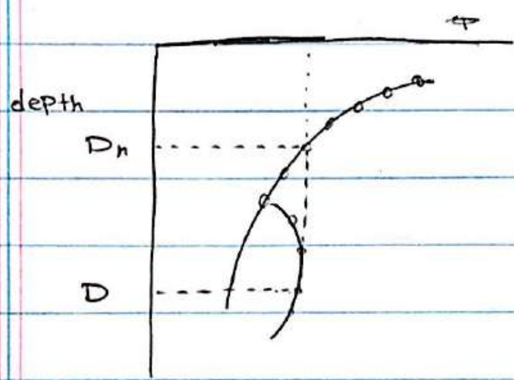
در این ناحیه یک blowout باعث ایجاد blowout زیرین می‌شود پس باید چاهی که حفاری شده را با سیمان پر کرد و بعد آن را رها کرد.

inproperly abonded Blow out

Methods for estimating pore pressure



1st method: Similar formation having the same value of the porosity dependent variable are under the same effective matrix stress σ_z



$\sigma_{z(D)} = \sigma_{z(D_n)}$ رابط 0.052

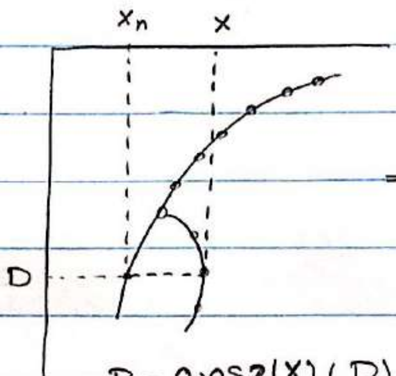
? $\sigma_{zn} = \sigma_{ob} - P_n$ @ D_n

$\sigma_{ob} = \sigma_z + P$ @ D

? $P = \sigma_{ob} - \sigma_z$

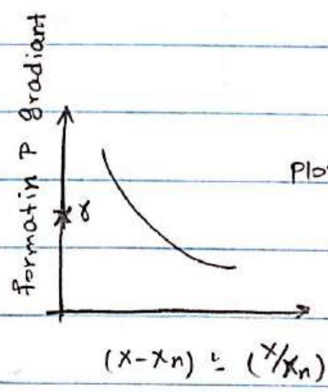
σ_{zn}

2nd method: Emprical correlations



$P = 0.052(\gamma)(D)$

⇒



Plot ها از روی correlation بدست می‌آید.

۴.

(1) predictive method (before drilling)

(2) while drilling

(3) verification (After drilling)

predictive methode

(1) correlation of available data from wells

(2) seismic data

+ interval transit time

باید trend t ها را داشته باشیم

$$t = t_{ma}(1-\Phi) + t_f \Phi$$

t_{ma} → rock matrix transit time

t_f → pore fluid transit time

فرض: تغییرات t_f , t_m خیلی شدید نیست

matrix material	matrix transit time (10^{-6} s/ft)
dolomit	44
calcite	46
limestone	48
shale	62-167
sandstone	53-59
pore fluid:	
water (distilled)	218
100,000 ppm NaCl	208
200,000 " "	189

example: water salinity of formation: 90,000 ppm

use K and ϕ_0 from previous ex.

Depth interval (ft)	Average transit interval time (10^{-6} s/ft)
① 1500 - 2500 → 2000 representative depth در این بازه باید نظر بگیریم	153
2500 - 3500	140
3500 - 4500	132
4500 - 5500	126
5500 - 6500	118
6500 - 7500	120
7500 - 8500	112
8500 - 9500	106
9500 - 10500	102
10500 - 11500	103
11500 - 12500	93

The above inform. are normally pressured seismic data

$$\phi = \phi_0 e^{-KD} = 0.41\, e^{-0.000085 D}$$

$$90{,}000 \text{ ppm}: \quad t_f = 209$$

$$t_{ma} = \frac{t - t_f\,\phi}{1-\phi} = \frac{t - 209\,\phi}{1-\phi}$$

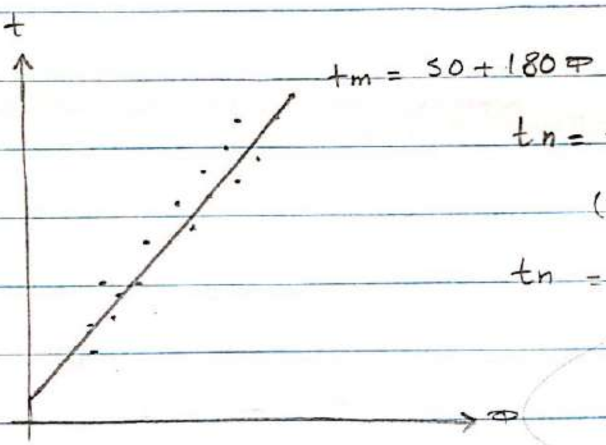
① $D = 2000$ ft, $\phi = 0.41\, e^{-0.000085 \times 2000} = 0.346$

$$t_m = \frac{153 - 209\,(0.346)}{1 - 0.346} = 122\ \mu s/ft$$

برای بقیه موارد نیز به همین منوال است.

F1

Average depth	Avg Porosity	APP matrix transit time
2000	0.346	122
3000	0.318	128
4000	0.292	100
5000	0.268	96
6000	0.246	88
7000	0.226	94
8000	0.208	87
9000	0.191	82
10000	0.176	79
11000	0.161	83
12000		



$$t_n = t_m(1-\phi) + t_f\,\phi$$

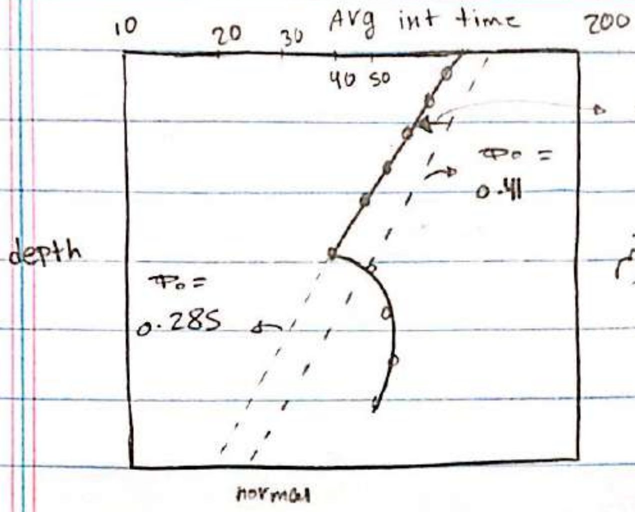
$$(50 + 180\phi)(1-\phi) + 209\phi$$

$$t_n = 50 + 339\phi - 180\phi^2$$

$$t_n = 50 + 339\phi_0\, e^{-0.000085 D} - 180\phi_0^2\, e^{-0.00017 D}$$

ex: Seismic data for a propored well location estimate formation pressure at 9000 ft using Both methods.

depth internal	Avg internal transit time
1500 - 2500	137
2500 - 3500	122
3500 - 4500	107
4500 - 5500	104
5500 - 6500	98
6500 - 7500	95
7500 - 8500	93
8500 - 9500	125
9500 - 10500	132
10500 - 11500	130
11500 - 12500	126



نمودار semi-log

: Shift عامل

ϕ_o

پس در عمق های مختلف ϕ_o را پیدا می کنیم

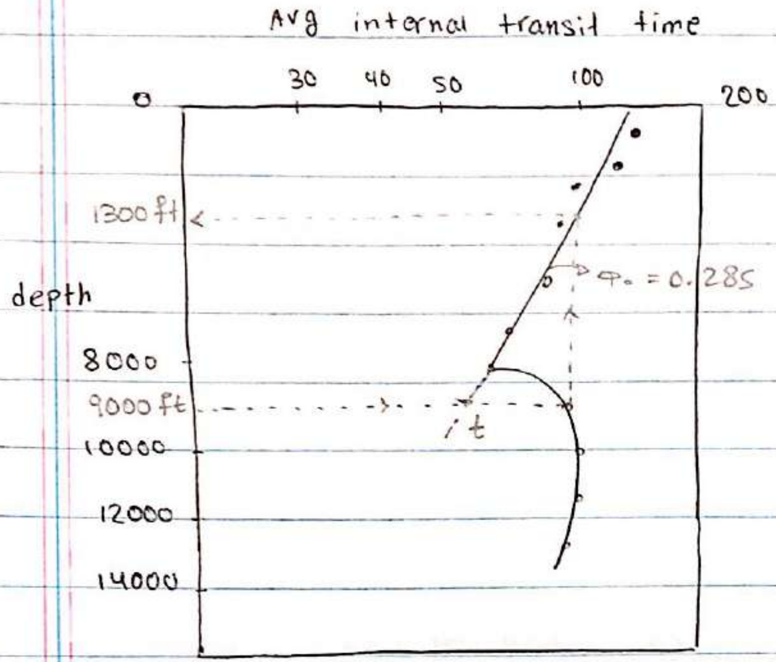
و avg می گیریم

$$\phi_o = \left[\frac{339}{e^{0.000085D}} - \sqrt{\left(\frac{339}{e^{0.000085D}}\right)^2 - \left(\frac{720\,(t_n - 50)}{e^{0.00017D}}\right)} \right] \div \frac{360}{e^{0.00017D}}$$

Depth interval	D	t_n	ϕ_o
1500 - 2500	2000	137	0.346
2500 - 3500	3000	122	0.315
3500 - 4500	4000	107	0.262
4500 - 5500	5000	104	0.269
5500 - 6500	6000	98	0.257
6500 - 7500	7000	95	0.261
7500 - 8500	8000	93	0.270

Avg = 0.285

ادامه مثال قبل:



$$t_n = 50 + 96.6\,e^{-0.000085D} - 14.6\,e^{-0.00017D}$$

روش اول ① $\sigma_{ob} = \sigma_z + P_f$

$D = 9000\ ft \rightarrow P_f = ?$

ⓐ $1300\ ft:\ \sigma_{ob} = 0.052\,\rho_g D_s - \dfrac{0.052(\rho_g - \rho_f)\,\phi_0}{K}\left(1 - e^{-KD_s}\right)$

$= 0.052(2.6 \times 8.33)(1300) - \dfrac{0.052(2.6 - 1.074) \times 8.33 \times 0.285}{0.000085}\left(1 - e^{-0.000085 \times 1300}\right)$

$= 1464 - 232 = 1232\ psig$

$\sigma_z = \sigma_{ob} - P_f = 1232 - (1300 \times 0.465) = 627\ psig$

ⓑ $9000\ ft:\ \sigma_{ob} = \sigma_z + P_f \Rightarrow P_f = \sigma_{ob} - \sigma_z$

$\sigma_{ob} = 0.052 \times 2.6 \times 8.33 \times 9000 - \dfrac{0.052(2.6 - 1.074) \times 8.33 \times 0.285}{0.000085} \times \left(1 - e^{(0.000085 \times 9000)}\right)$

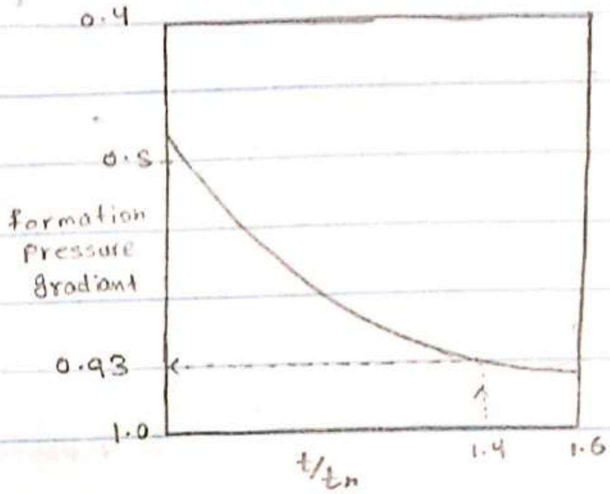
$= 10316 - 1185 = 8951\ Psig$

$P_f = 8951 - 627 = 8324\ Psig$

روش ① $t_n = 92$

$t = 129$

$$\frac{t}{t_n} = \frac{129}{92} = 1.4$$



$P = 0.93 \times 9000 = 8370$ Psig

estimation of pore pressure while drilling :

ROP → اگر جنس سنگ از یک خانواده باشد . هرچه پایین تر می‌رویم ROP کم‌تر می‌شود.

Bingham : $$ROP = K \left(\frac{W}{d_b}\right)^{d_{exp}} N$$

weight on bit

Rotary speed

proportionity Constant

bit diameter

$$d_{exp} = \frac{\log\left(\frac{R}{60N}\right)}{\log\left(\frac{12W}{1000\, d_b}\right)}$$

ft/hr

rpm

Klbf

ft

dexp با عمق زیاد می‌شود وقتی به abnormal pressure می‌رسد کم می‌شود.

$$d_{mod} = d_{exp} \frac{\rho_n}{\rho_e}$$

Rehm and Mc clendon

ρ_n: mud density equivalent to normal formation Pressure

ρ_e: equivalent mud density on bit

Rehm and Mc Clendon:

$$(d_{mod})_n = (d_{mod})_o + m \underbrace{D}_{\text{depth}} \qquad m>0 \quad = 0.000038\ ft^{-1}$$

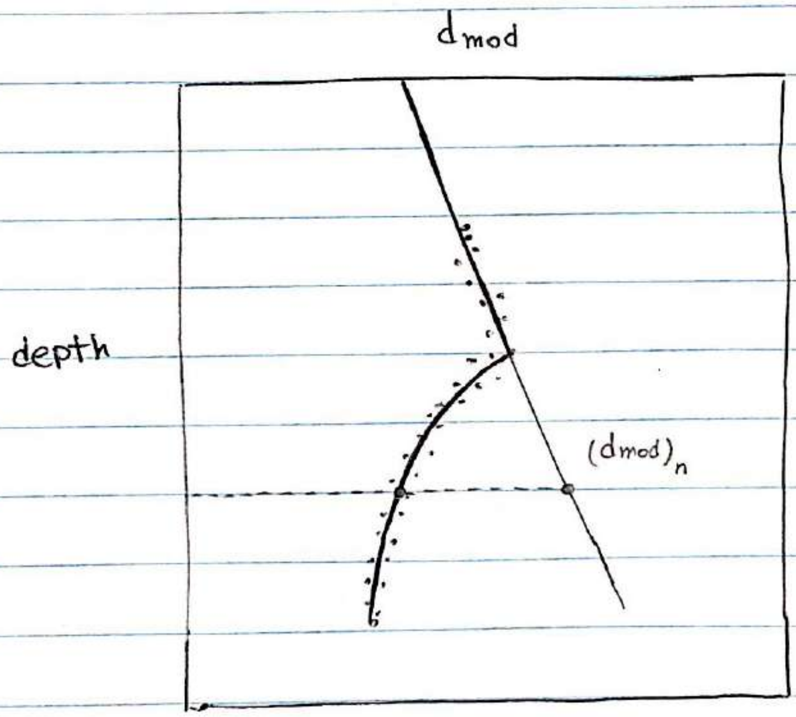
$$g_p = 7.65 \log\left[(d_{mod})_n - (d_{mod})_o\right] + 16.5$$

formation pressure gradiant

zamora: $(d_{mod})_n = (d_{mod})_o\, e^{mD} \qquad m = 0.000039\ ft^{-1}$

$$g_p = g_n \frac{(d_{mod})_n}{d_{mod}}$$

Normal formation pressure gradiant



مباحث میان ترم:

surge, swab

optimization

casing design , formation / fracture

example:

Depth (ft)	modified - dexponent
8100	1.52
9000	1.55
9600	1.57
10100	1.49
⋮	
16200	
16800	0.65

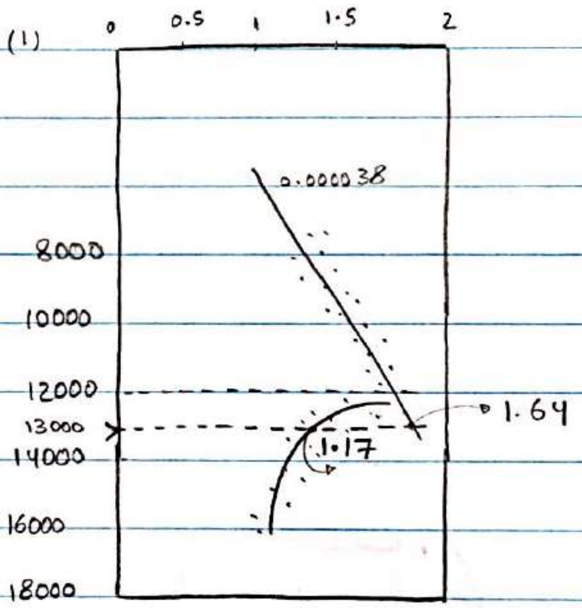
estimate formation pore pressure at 13000 ft

using (1) Rehm and Mcclendon

(2) zamura methode

$g_n = 0.465$ psi/ft

solution: (1)

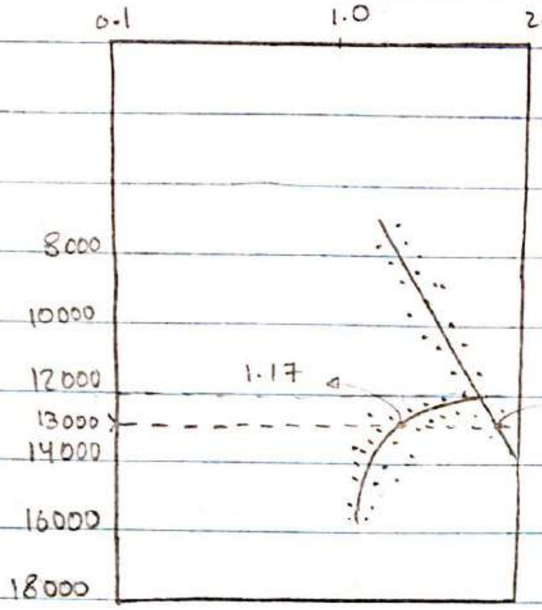


$$g_p = 7.65 \log\left((d_{mod})_n - (d_{mod})\right) + 16.5$$

$$= 7.65 \log(1.64 - 1.17) + 16.5 = 14 \ \boxed{\text{lbm/gal}}$$

$$P = 0.052\, g_p D = 0.052 \times 14 \times 13000 = 9464 \text{ psig}$$

(۲)



$$g_p = \boxed{g_n}\frac{(d_{mod})_n}{d_{mod}}$$

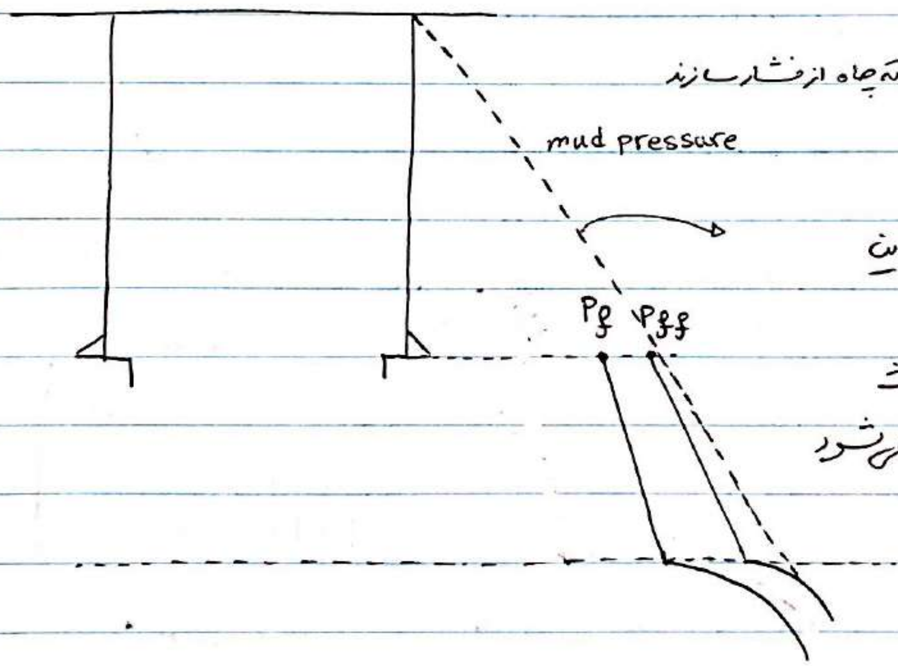
وابسته به این

$$= 0.465 \times \frac{1.64}{1.17} = 0.625\ \boxed{Psi/ft}$$

$$P = g_p \times D = 0.625 \times 13000$$

$$= 8476\ Psig$$

Fracture pressure (Formation Fracture Resistance)



همیشه باید فشار ته چاه از فشار سازند بیشتر باشد.

اگر فشاری را با این mud pressure ادامه دهیم باعث شکست سازند می شود

methods $\begin{cases} (1)\ \text{predictive} \\ (2)\ \text{verification} \end{cases}$

(1) Predictive methode

قبل از حفاری است و برای محاسبات آن نیاز به pore pressure داریم:

① Hubbert and willis Eq

$$P_{ff} = \sigma_{min} + P_f$$ رابطه اصلی

$$P_{ff} = \sigma_{min} + P_f \xrightarrow{\sigma_{min} = 1/3\, \sigma_z} P_{ff} = 1/3\, \sigma_z + P_f \quad ①$$

در اکثر موارد همان تنش افقی است.

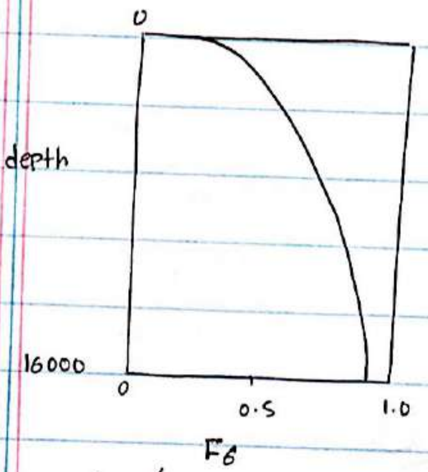
$$\sigma_z = \sigma_{ob} - P_f \quad ②$$

$$1,2 \longrightarrow P_{ff} = 1/3\, (\sigma_{ob} + 2 P_f)$$

② Pennebaker correlation:

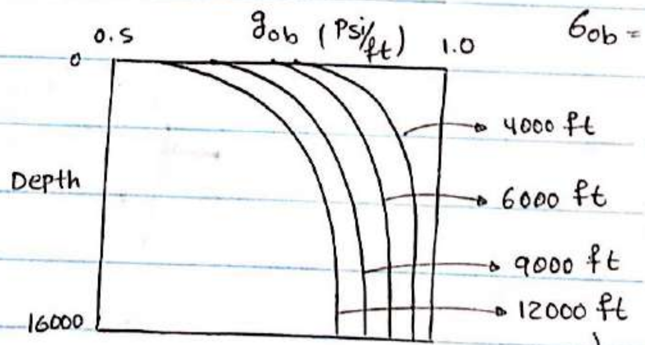
$$\sigma_{min} = F_\sigma\, \sigma_z$$

F_σ → effective stress ratio



→ $\sigma_{min} \simeq \sigma_z$ در عمق های بالا (زیاد)

$\sigma_{ob} = g_{ob} \times D$



100 میکرو ثانیه

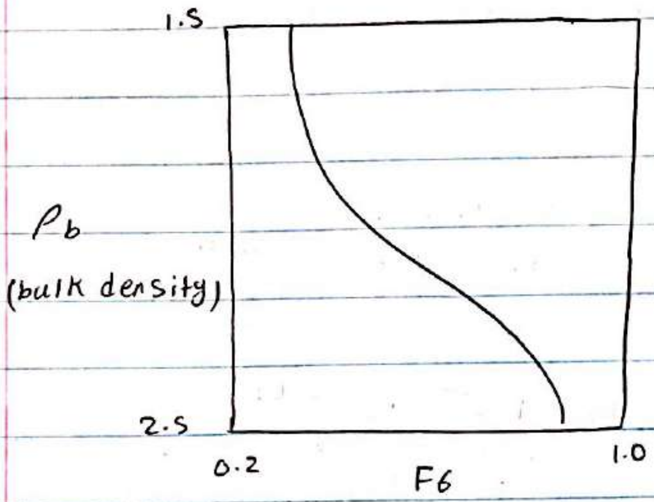
برای استفاده همراه با داده های بر مبنای seismic

$$\sigma_{ob} = g_{ob} \times D$$

$$\sigma_z = \sigma_{ob} - P_f$$

$$\sigma_{min} = F_\sigma\, \sigma_z \longrightarrow P_{ff} = \sigma_{min} + P_f$$

③ Christman correlation:



$$\rho_b = (1-\phi)\rho_g + \phi \rho_f$$

ϕ از داده های seismic بدست می آید.

آنالیز آن مانند روش ② است.

example: P_{ff} @ 10,000 ft = ?

(g/cm^3)

$\phi_0 = 0.45$, $K = 0.000085$, $\rho_{fl} = 1.074$, $\rho_g = 2.6$

$P_f = 6500$ $\quad \sigma_{ob} = 8167$

Sol:

$$\phi = \phi_0 e^{-KD} = 0.45\, e^{-0.000085 \times 10000} = 0.192$$

$$\rho_b = (1-\phi)\rho_g + \phi \rho_f = (1-0.192)(2.6) + (0.192)(1.074)$$
$$= 2.31\ g/cm^3$$

$F6 = 0.8$ از روی نمودار با توجه به ρ_b ، F6 را می خوانیم ←

$$\sigma_{min} = F6\ \sigma_z = 0.8\,(8167 - 6500) = 1334$$

(σ_z = $\sigma_{ob} - P_f$)

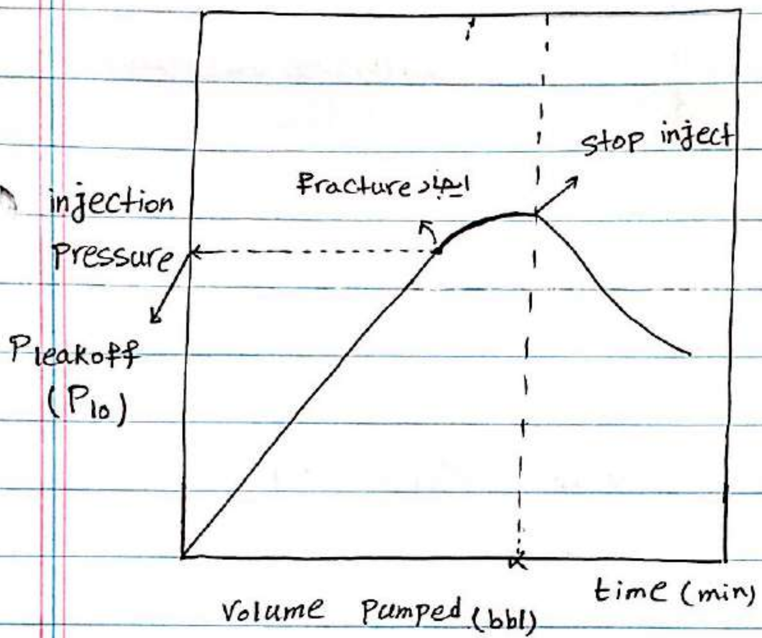
$$P_{ff} = \sigma_{min} + P_f = 1334 + 6500 = 7834\ \text{psi}$$

verification methode

در این روش ۲ بار تست leakoff انجام می‌دهیم:

این تست پس از casing گذاری و سیمان کاری و حفاری مجدد ۲۰ الی ۳۰ متر و بستن چاه انجام می‌گیرد. یک بار قبل از و یک بار بعد آن

نرخ تزریق: $0.25 - 1.5 \ bbl/min$



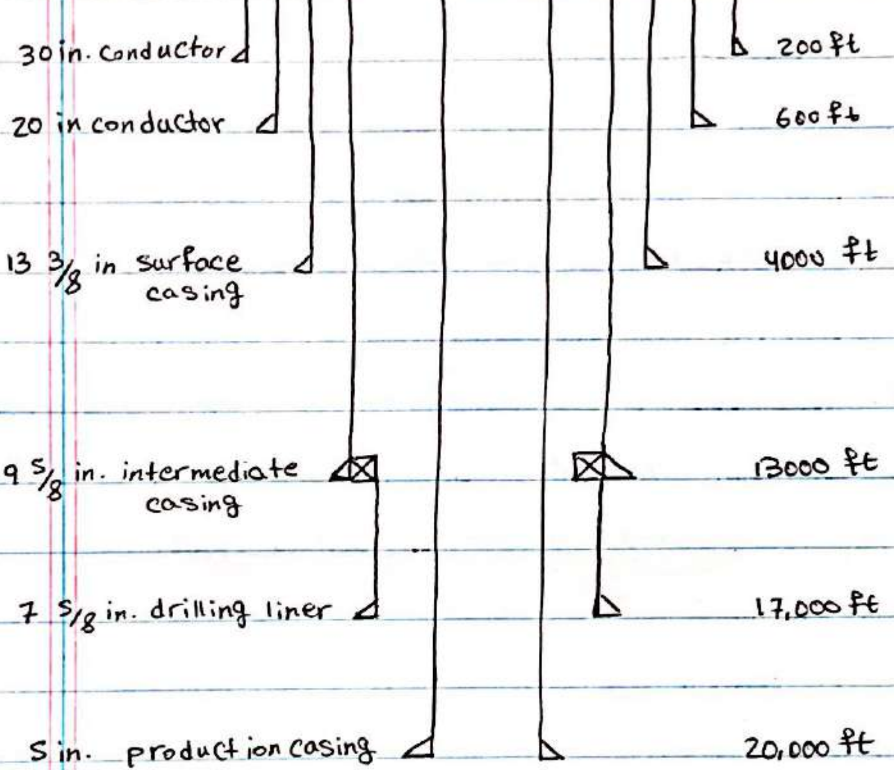
نمودار ?

$$P_{lo} = P_{ff} - 0.052 \rho_m D + \Delta P_f$$

$$\frac{dP_f}{dL} = \frac{\tau_g}{300d}$$

τ_g → gel strength

Casing Design



عملیات casing گذاری برای ایزوله کردن محیط حفاری جهت جلوگیری از تخریب چاه و تخریب عملیات است.
در واقع section ای را که مشکل دارد را برای ادامه حفاری کنار می‌گذاریم.

Conductor pipe: در عمق های پایین گل و لای به کار می رود قبل از عملیات casing گذاری.

surface casing: اولین casing ای که می‌گذاریم و اگر به مشکل بر بخورد تا آخر عملیات ادامه می‌یابد.

intermediate casing: اگر surface casing به مشکل بخورد بین آن و production casing می توان یک یا چند intermediate casing قرار داد.

Production casing → آخرین casing ای که می‌گذاریم
به عنوان casing تولیدی

liner: برای محیط‌های خاص که نیازی نیست تا سرچاه
casing گذاری کنیم.
هزینه‌ی کمی دارد ولی ممکن است باعث leak سیمان شود.

Standardization of casing

(API) American Petroleum institute این استاندارد سازی توسط
انجام شده است.

i) size (OD)

ii) range of length

iii) casing grade:

iv) type of Coupling

casing size	4	4 1/2	5	5 1/2		
bit size	4 3/4	5 7/8	6 1/8	6 1/2	7 7/8	
casing size	6 5/8	7	7 5/8	8 5/8	9 5/8	
bit size	7 7/8	8 1/2	8 3/4	9 1/2	10 5/8	12 1/4
casing size						
bit size						

→ مربوط به آخرین casing
(production casing)

+ picture

Range of length		average
R-1	16-25 ft	22 ft
R-2	25-34 ft	31 ft
R-3	>34 ft	42 ft

بیشترین استفاده

طراحی روی این مورد

تا این درصد طول آن اضافه شود، نمی شکند

Casing Grade → توان تحمل casing در مقابل فشارها

API Grade	yield strength (Psi) *min	max	ultimate Strength (Psi)	minimum elongation (%)
H-40	40000	80000	60000	29.5
J-55	55000	80000	75000	24.0
K-55	55000	80000	95000	19.5
C-75	75000	90000	95000	19.5
L-80	80000	95000	95000	19.5

نشان دهنده minimum yield strength

Casing Weight

برای انتخاب مته بعدی

weight ($\frac{lbf}{ft}$)	outer diameter (in)	inner diameter	well thick	drift diameter
53.5	9.625	8.535	0.545	8.379
47	9.625	8.681	0.472	8.525
43.5	9.625	8.755	0.435	8.599
40	9.625	8.835	0.395	8.679

type of coupling

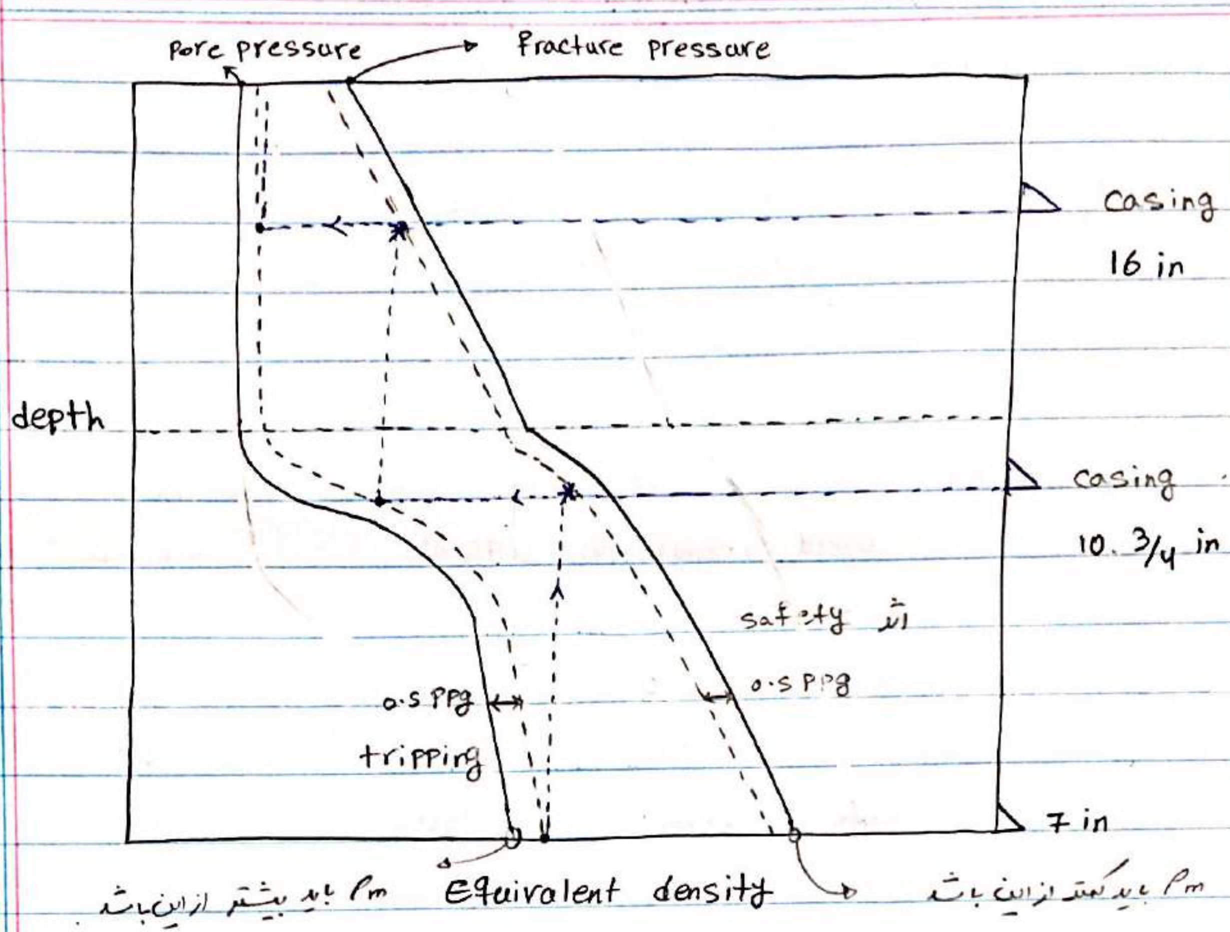
۲ نوع شکست در casing: Pipe body failure

نیروی کششی Axial tension

Joint Failure → بیشتر اتفاق می افتد

Burst pressure → فشار داخلی که منجر به ترکیدن casing می شود

Collapse pressure → فشار خارجی که منجر به جمع شدن casing می شود



example: production casing : 7 in

↓

bit : 8.625 in

↓

casing OD : 9.625 in