```
> #asadi_80216931az
```

```
> restart;
```

```
> eq1 := diff(f(x), x$4) + alpha·(x·diff(f(x), x, x, x) + 3·diff(f(x), x, x)) + R·(f(x)
       ·diff(f(x), x, x, x) − diff(f(x), x)·diff(f(x), x, x)) = 0;
```

$$eq1 := \frac{d^4}{dx^4} f(x) + \alpha \left(x \left(\frac{d^3}{dx^3} f(x) \right) + 3 \left(\frac{d^2}{dx^2} f(x) \right) \right) + R \left(f(x) \left(\frac{d^3}{dx^3} f(x) \right) \right.$$

$$\left. - \left(\frac{d}{dx} f(x) \right) \left(\frac{d^2}{dx^2} f(x) \right) \right) = 0 \tag{1}$$

```
> H(x, p) := (1 − p)·(diff(f(x), x$4)) + p·(diff(f(x), x$4) + alpha·(x·diff(f(x), x, x, x) + 3
       ·diff(f(x), x, x)) + R·(f(x)·diff(f(x), x, x, x) − diff(f(x), x)·diff(f(x), x, x)));
```

$$H(x, p) := (1 - p) \left(\frac{d^4}{dx^4} f(x) \right) + p \left(\frac{d^4}{dx^4} f(x) + \alpha \left(x \left(\frac{d^3}{dx^3} f(x) \right) + 3 \left(\frac{d^2}{dx^2} f(x) \right) \right) \right.$$

$$\left. + R \left(f(x) \left(\frac{d^3}{dx^3} f(x) \right) - \left(\frac{d}{dx} f(x) \right) \left(\frac{d^2}{dx^2} f(x) \right) \right) \right) \tag{2}$$

```
> f(x) := sum(p^i·f[i](x), i = 0..3);
```

$$f(x) := f_0(x) + p f_1(x) + p^2 f_2(x) + p^3 f_3(x) \tag{3}$$

```
> #ves
```

```
> H(x, p) := (1 − p)·(diff(f(x), x$4)) + p·(diff(f(x), x$4) + alpha·(x·diff(f(x), x, x, x) + 3
       ·diff(f(x), x, x)) + R·(f(x)·diff(f(x), x, x, x) − diff(f(x), x)·diff(f(x), x, x)));
```

$$H(x, p) := (1 - p) \left(\frac{d^4}{dx^4} f_0(x) + p \left(\frac{d^4}{dx^4} f_1(x) \right) + p^2 \left(\frac{d^4}{dx^4} f_2(x) \right) + p^3 \left(\frac{d^4}{dx^4} f_3(x) \right) \right) \tag{4}$$

$$+ p \left(\frac{d^4}{dx^4} f_0(x) + p \left(\frac{d^4}{dx^4} f_1(x) \right) + p^2 \left(\frac{d^4}{dx^4} f_2(x) \right) + p^3 \left(\frac{d^4}{dx^4} f_3(x) \right) \right.$$

$$+ \alpha \left(x \left(\frac{d^3}{dx^3} f_0(x) + p \left(\frac{d^3}{dx^3} f_1(x) \right) + p^2 \left(\frac{d^3}{dx^3} f_2(x) \right) + p^3 \left(\frac{d^3}{dx^3} f_3(x) \right) \right) \right.$$

$$\left. + 3 \left(\frac{d^2}{dx^2} f_0(x) \right) + 3 p \left(\frac{d^2}{dx^2} f_1(x) \right) + 3 p^2 \left(\frac{d^2}{dx^2} f_2(x) \right) + 3 p^3 \left(\frac{d^2}{dx^2} f_3(x) \right) \right)$$

$$+ R \left(\left(f_0(x) + p f_1(x) + p^2 f_2(x) + p^3 f_3(x) \right) \left(\frac{d^3}{dx^3} f_0(x) + p \left(\frac{d^3}{dx^3} f_1(x) \right) \right. \right.$$

$$+ p^2 \left(\frac{d^3}{dx^3} f_2(x) \right) + p^3 \left(\frac{d^3}{dx^3} f_3(x) \right) \right) - \left(\frac{d}{dx} f_0(x) + p \left(\frac{d}{dx} f_1(x) \right) + p^2 \left(\frac{d}{dx} f_2(x) \right) \right.$$

$$+ p^3 \left(\frac{d}{dx} f_3(x) \right) \right) \left(\frac{d^2}{dx^2} f_0(x) + p \left(\frac{d^2}{dx^2} f_1(x) \right) + p^2 \left(\frac{d^2}{dx^2} f_2(x) \right) \right.$$

$$\left. \left. \left. + p^3 \left(\frac{d^2}{dx^2} f_3(x) \right) \right) \right) \right)$$

```
> eq2 := simplify(%);
```

$$eq2 := (1 - p) \left(\frac{d^4}{dx^4} f_0(x) + p \left(\frac{d^4}{dx^4} f_1(x) \right) + p^2 \left(\frac{d^4}{dx^4} f_2(x) \right) + p^3 \left(\frac{d^4}{dx^4} f_3(x) \right) \right) \tag{5}$$

$$+ p\left(\frac{d^4}{dx^4}f_0(x) + p\left(\frac{d^4}{dx^4}f_1(x)\right) + p^2\left(\frac{d^4}{dx^4}f_2(x)\right) + p^3\left(\frac{d^4}{dx^4}f_3(x)\right)\right.$$

$$+ \alpha\left(x\left(\frac{d^3}{dx^3}f_0(x) + p\left(\frac{d^3}{dx^3}f_1(x)\right) + p^2\left(\frac{d^3}{dx^3}f_2(x)\right) + p^3\left(\frac{d^3}{dx^3}f_3(x)\right)\right)\right.$$

$$+ 3\left(\frac{d^2}{dx^2}f_0(x)\right) + 3\,p\left(\frac{d^2}{dx^2}f_1(x)\right) + 3\,p^2\left(\frac{d^2}{dx^2}f_2(x)\right) + 3\,p^3\left(\frac{d^2}{dx^2}f_3(x)\right)\right)$$

$$+ R\left(\left(f_0(x) + p\,f_1(x) + p^2 f_2(x) + p^3 f_3(x)\right)\left(\frac{d^3}{dx^3}f_0(x) + p\left(\frac{d^3}{dx^3}f_1(x)\right)\right.\right.$$

$$+ p^2\left(\frac{d^3}{dx^3}f_2(x)\right) + p^3\left(\frac{d^3}{dx^3}f_3(x)\right)\right) - \left(\frac{d}{dx}f_0(x) + p\left(\frac{d}{dx}f_1(x)\right) + p^2\left(\frac{d}{dx}f_2(x)\right)\right.$$

$$+ p^3\left(\frac{d}{dx}f_3(x)\right)\right)\left(\frac{d^2}{dx^2}f_0(x) + p\left(\frac{d^2}{dx^2}f_1(x)\right) + p^2\left(\frac{d^2}{dx^2}f_2(x)\right)\right.$$

$$\left.\left.\left.\left.+ p^3\left(\frac{d^2}{dx^2}f_3(x)\right)\right)\right)\right)\right)$$

> $eq3 := collect(expand(eq2), p);$

$$eq3 := \left(\left(\frac{d^3}{dx^3}f_3(x)\right)f_3(x)\,R - \left(\frac{d^2}{dx^2}f_3(x)\right)\left(\frac{d}{dx}f_3(x)\right)R\right)p^7 + \left(\left(\frac{d^3}{dx^3}f_2(x)\right)f_3(x)\,R \qquad\qquad (6)\right.$$

$$+ \left(\frac{d^3}{dx^3}f_3(x)\right)f_2(x)\,R - \left(\frac{d^2}{dx^2}f_2(x)\right)\left(\frac{d}{dx}f_3(x)\right)R - \left(\frac{d^2}{dx^2}f_3(x)\right)\left(\frac{d}{dx}f_2(x)\right)R\right)$$

$$p^6 + \left(\left(\frac{d^3}{dx^3}f_1(x)\right)f_3(x)\,R + \left(\frac{d^3}{dx^3}f_2(x)\right)f_2(x)\,R + \left(\frac{d^3}{dx^3}f_3(x)\right)f_1(x)\,R\right.$$

$$- \left(\frac{d^2}{dx^2}f_1(x)\right)\left(\frac{d}{dx}f_3(x)\right)R - \left(\frac{d^2}{dx^2}f_2(x)\right)\left(\frac{d}{dx}f_2(x)\right)R$$

$$- \left(\frac{d^2}{dx^2}f_3(x)\right)\left(\frac{d}{dx}f_1(x)\right)R\right)p^5 + \left(\left(\frac{d^3}{dx^3}f_0(x)\right)f_3(x)\,R + \left(\frac{d^3}{dx^3}f_1(x)\right)f_2(x)\,R\right.$$

$$+ \left(\frac{d^3}{dx^3}f_2(x)\right)f_1(x)\,R + \left(\frac{d^3}{dx^3}f_3(x)\right)f_0(x)\,R + \left(\frac{d^3}{dx^3}f_3(x)\right)\alpha\,x$$

$$- \left(\frac{d^2}{dx^2}f_0(x)\right)\left(\frac{d}{dx}f_3(x)\right)R - \left(\frac{d^2}{dx^2}f_1(x)\right)\left(\frac{d}{dx}f_2(x)\right)R$$

$$- \left(\frac{d^2}{dx^2}f_2(x)\right)\left(\frac{d}{dx}f_1(x)\right)R - \left(\frac{d^2}{dx^2}f_3(x)\right)\left(\frac{d}{dx}f_0(x)\right)R + 3\left(\frac{d^2}{dx^2}f_3(x)\right)\alpha\right)p^4$$

$$+ \left(\left(\frac{d^3}{dx^3}f_0(x)\right)f_2(x)\,R + \left(\frac{d^3}{dx^3}f_1(x)\right)f_1(x)\,R + \left(\frac{d^3}{dx^3}f_2(x)\right)f_0(x)\,R\right.$$

$$+ \left(\frac{d^3}{dx^3}f_2(x)\right)\alpha\,x - \left(\frac{d^2}{dx^2}f_0(x)\right)\left(\frac{d}{dx}f_2(x)\right)R - \left(\frac{d^2}{dx^2}f_1(x)\right)\left(\frac{d}{dx}f_1(x)\right)R$$

$$- \left(\frac{d^2}{dx^2}f_2(x)\right)\left(\frac{d}{dx}f_0(x)\right)R + 3\left(\frac{d^2}{dx^2}f_2(x)\right)\alpha + \frac{d^4}{dx^4}f_3(x)\right)p^3$$

$$+ \left(\left(\frac{d^3}{dx^3} f_0(x) \right) f_1(x)\, R + \left(\frac{d^3}{dx^3} f_1(x) \right) f_0(x)\, R + \left(\frac{d^3}{dx^3} f_1(x) \right) \alpha\, x \right.$$

$$- \left(\frac{d^2}{dx^2} f_0(x) \right) \left(\frac{d}{dx} f_1(x) \right) R - \left(\frac{d^2}{dx^2} f_1(x) \right) \left(\frac{d}{dx} f_0(x) \right) R + 3 \left(\frac{d^2}{dx^2} f_1(x) \right) \alpha$$

$$\left. + \frac{d^4}{dx^4} f_2(x) \right) p^2 + \left(R f_0(x) \left(\frac{d^3}{dx^3} f_0(x) \right) + \alpha\, x \left(\frac{d^3}{dx^3} f_0(x) \right) \right.$$

$$\left. - R \left(\frac{d}{dx} f_0(x) \right) \left(\frac{d^2}{dx^2} f_0(x) \right) + 3\, \alpha \left(\frac{d^2}{dx^2} f_0(x) \right) + \frac{d^4}{dx^4} f_1(x) \right) p + \frac{d^4}{dx^4} f_0(x)$$

`> s[0] :=` $\dfrac{d^4}{dx^4} f_0(x) = 0;$

$$s_0 := \frac{d^4}{dx^4} f_0(x) = 0 \tag{7}$$

`> ics[0] := f[0](0) = 0, f[0](1) = 1, ` $D^{(2)}(f[0])(0) = 0, D(f[0])(1) = 0;$

$$ics_0 := f_0(0) = 0, f_0(1) = 1, D^{(2)}(f_0)(0) = 0, D(f_0)(1) = 0 \tag{8}$$

`> dsolve({s[0], ics[0]});`

$$f_0(x) = -\frac{1}{2} x^3 + \frac{3}{2} x \tag{9}$$

`> assign(%)`

`> for i from 1 to 3 do`
`    s[i] :=` $coeff(eq3, p^i) = 0;$
`    ics[i] := f[i](0) = 0, f[i](1) = 0,` $D^{(2)}(f[i])(0) = 0, D(f[i])(1) = 0;$
`    dsolve({s[i], ics[i]});`
`    f[i](x) :=` $rhs(\%);$
`    end do;`

$$s_1 := -3R \left(-\frac{1}{2} x^3 + \frac{3}{2} x \right) - 12\, \alpha\, x + 3R \left(-\frac{3}{2} x^2 + \frac{3}{2} \right) x + \frac{d^4}{dx^4} f_1(x) = 0$$

$$ics_1 := f_1(0) = 0, f_1(1) = 0, D^{(2)}(f_1)(0) = 0, D(f_1)(1) = 0$$

$$f_1(x) = \frac{1}{280} R x^7 + \frac{1}{10} \alpha x^5 + \frac{1}{6} \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x^3 + \left(\frac{1}{140} R + \frac{1}{10} \alpha \right) x$$

$$f_1(x) := \frac{1}{280} R x^7 + \frac{1}{10} \alpha x^5 + \frac{1}{6} \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x^3 + \left(\frac{1}{140} R + \frac{1}{10} \alpha \right) x$$

$$s_2 := -3 \left(\frac{1}{280} R x^7 + \frac{1}{10} \alpha x^5 + \frac{1}{6} \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x^3 + \left(\frac{1}{140} R + \frac{1}{10} \alpha \right) x \right) R$$

$$+ \left(\frac{3}{4} R x^4 + 6 \alpha x^2 - \frac{9}{140} R - \frac{6}{5} \alpha \right) \left(-\frac{1}{2} x^3 + \frac{3}{2} x \right) R + \left(\frac{3}{4} R x^4 + 6 \alpha x^2 \right.$$

$$\left. - \frac{9}{140} R - \frac{6}{5} \alpha \right) \alpha x + 3 x \left(\frac{1}{40} R x^6 + \frac{1}{2} \alpha x^4 + \frac{1}{2} \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x^2 + \frac{1}{140} R \right.$$

$$\left. + \frac{1}{10} \alpha \right) R - \left(\frac{3}{20} R x^5 + 2 \alpha x^3 + \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x \right) \left(-\frac{3}{2} x^2 + \frac{3}{2} \right) R$$

$$+ 3 \left(\frac{3}{20} R x^5 + 2 \alpha x^3 + \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x \right) \alpha + \frac{d^4}{dx^4} f_2(x) = 0$$

$$ics_2 := f_2(0) = 0, f_2(1) = 0, D^{(2)}(f_2)(0) = 0, D(f_2)(1) = 0$$

$$f_2(x) = \frac{1}{92400} R^2 x^{11} + \frac{1}{70560} \left(-21 R^2 - 56 R \alpha \right) x^9 + \frac{1}{19600} \left(3 R^2 - 84 R \alpha - 280 \alpha^2 \right) x^7$$

$$+ \frac{1}{2800} \left(6 R \alpha + 112 \alpha^2 \right) x^5 + \frac{1}{6} \left(\frac{219}{53900} R^2 + \frac{37}{525} R \alpha - \frac{39}{175} \alpha^2 \right) x^3 + \left(\right.$$

$$\left. -\frac{703}{1293600} R^2 - \frac{37}{4200} R \alpha + \frac{2}{175} \alpha^2 \right) x$$

$$f_2(x) := \frac{1}{92400} R^2 x^{11} + \frac{1}{70560} \left(-21 R^2 - 56 R \alpha \right) x^9 + \frac{1}{19600} \left(3 R^2 - 84 R \alpha \right.$$

$$\left. - 280 \alpha^2 \right) x^7 + \frac{1}{2800} \left(6 R \alpha + 112 \alpha^2 \right) x^5 + \frac{1}{6} \left(\frac{219}{53900} R^2 + \frac{37}{525} R \alpha - \frac{39}{175} \alpha^2 \right) x^3$$

$$+ \left(-\frac{703}{1293600} R^2 - \frac{37}{4200} R \alpha + \frac{2}{175} \alpha^2 \right) x$$

$$s_3 := -3 \left(\frac{1}{92400} R^2 x^{11} + \frac{1}{70560} \left(-21 R^2 - 56 R \alpha \right) x^9 + \frac{1}{19600} \left(3 R^2 - 84 R \alpha \right.\right.$$

$$\left. - 280 \alpha^2 \right) x^7 + \frac{1}{2800} \left(6 R \alpha + 112 \alpha^2 \right) x^5 + \frac{1}{6} \left(\frac{219}{53900} R^2 + \frac{37}{525} R \alpha - \frac{39}{175} \alpha^2 \right) x^3$$

$$+ \left(-\frac{703}{1293600} R^2 - \frac{37}{4200} R \alpha + \frac{2}{175} \alpha^2 \right) x \right) R + \left(\frac{3}{4} R x^4 + 6 \alpha x^2 - \frac{9}{140} R \right.$$

$$\left. - \frac{6}{5} \alpha \right) \left(\frac{1}{280} R x^7 + \frac{1}{10} \alpha x^5 + \frac{1}{6} \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x^3 + \left(\frac{1}{140} R + \frac{1}{10} \alpha \right) x \right) R$$

$$+ \left(\frac{3}{280} R^2 x^8 + \frac{1}{140} \left(-21 R^2 - 56 R \alpha \right) x^6 + \frac{3}{280} \left(3 R^2 - 84 R \alpha - 280 \alpha^2 \right) x^4 \right.$$

$$+ \frac{3}{140} \left(6 R \alpha + 112 \alpha^2 \right) x^2 + \frac{219}{53900} R^2 + \frac{37}{525} R \alpha - \frac{39}{175} \alpha^2 \right) \left(-\frac{1}{2} x^3 + \frac{3}{2} x \right) R$$

$$+ \left(\frac{3}{280} R^2 x^8 + \frac{1}{140} \left(-21 R^2 - 56 R \alpha \right) x^6 + \frac{3}{280} \left(3 R^2 - 84 R \alpha - 280 \alpha^2 \right) x^4 \right.$$

$$+ \frac{3}{140} \left(6 R \alpha + 112 \alpha^2 \right) x^2 + \frac{219}{53900} R^2 + \frac{37}{525} R \alpha - \frac{39}{175} \alpha^2 \right) \alpha x$$

$$+ 3 x \left(\frac{1}{8400} R^2 x^{10} + \frac{1}{7840} \left(-21 R^2 - 56 R \alpha \right) x^8 + \frac{1}{2800} \left(3 R^2 - 84 R \alpha \right.\right.$$

$$\left. - 280 \alpha^2 \right) x^6 + \frac{1}{560} \left(6 R \alpha + 112 \alpha^2 \right) x^4 + \frac{1}{2} \left(\frac{219}{53900} R^2 + \frac{37}{525} R \alpha - \frac{39}{175} \alpha^2 \right) x^2$$

$$\left. - \frac{703}{1293600} R^2 - \frac{37}{4200} R \alpha + \frac{2}{175} \alpha^2 \right) R - \left(\frac{3}{20} R x^5 + 2 \alpha x^3 + \left(-\frac{9}{140} R \right.\right.$$

$$\left. - \frac{6}{5} \alpha \right) x \right) \left(\frac{1}{40} R x^6 + \frac{1}{2} \alpha x^4 + \frac{1}{2} \left(-\frac{9}{140} R - \frac{6}{5} \alpha \right) x^2 + \frac{1}{140} R + \frac{1}{10} \alpha \right) R$$

$$- \left(\frac{1}{840} R^2 x^9 + \frac{1}{980} \left(-21 R^2 - 56 R \alpha \right) x^7 + \frac{3}{1400} \left(3 R^2 - 84 R \alpha - 280 \alpha^2 \right) x^5 \right.$$

$$+ \frac{1}{140} \left(6 R \alpha + 112 \alpha^2 \right) x^3 + \left(\frac{219}{53900} R^2 + \frac{37}{525} R \alpha - \frac{39}{175} \alpha^2 \right) x \right) \left(-\frac{3}{2} x^2 \right.$$

$$+ \frac{3}{2}\Bigg) R + 3\left(\frac{1}{840} R^2 x^9 + \frac{1}{980}\left(-21 R^2 - 56 R\alpha\right) x^7 + \frac{3}{1400}\left(3 R^2 - 84 R\alpha\right.\right.$$

$$\left.- 280\,\alpha^2\right) x^5 + \frac{1}{140}\left(6 R\alpha + 112\,\alpha^2\right) x^3 + \left(\frac{219}{53900} R^2 + \frac{37}{525} R\alpha - \frac{39}{175}\,\alpha^2\right) x\Bigg)\,\alpha$$

$$+ \frac{d^4}{dx^4} f_3(x) = 0$$

$$ics_3 := f_3(0) = 0, f_3(1) = 0, D^{(2)}\big(f_3\big)(0) = 0, D\big(f_3\big)(1) = 0$$

$$f_3(x) = \frac{19}{144144000} R^3 x^{15} + \frac{1}{11099088000}\left(-32340 R^3 - 52360 R^2\alpha\right) x^{13}$$

$$+ \frac{1}{5122656000}\left(128304 R^3 + 426888 R^2\alpha + 406560 R\alpha^2\right) x^{11} + \frac{1}{1955923200}\Big($$

$$-27720 R^3 + 593208 R^2\alpha + 2638944 R\alpha^2 + 3104640\,\alpha^3\Big) x^9 + \frac{1}{543312000}\Big(-4365 R^3$$

$$- 159544 R^2\alpha - 1378608 R\alpha^2 - 3104640\,\alpha^3\Big) x^7 + \frac{1}{77616000}\Big(-10512 R^2\alpha$$

$$- 182336 R\alpha^2 + 576576\,\alpha^3\Big) x^5 + \frac{1}{6}\left(-\frac{8951}{53508000} R^3 - \frac{463}{171600} R^2\alpha + \frac{479}{12250} R\alpha^2\right.$$

$$\left.- \frac{64}{2625}\,\alpha^3\right) x^3 + \left(\frac{2047}{73573500} R^3 + \frac{41779}{84084000} R^2\alpha - \frac{1349}{441000} R\alpha^2 + \frac{2}{2625}\,\alpha^3\right) x$$

$$f_3(x) := \frac{19}{144144000} R^3 x^{15} + \frac{1}{11099088000}\left(-32340 R^3 - 52360 R^2\alpha\right) x^{13} \qquad \textbf{(10)}$$

$$+ \frac{1}{5122656000}\left(128304 R^3 + 426888 R^2\alpha + 406560 R\alpha^2\right) x^{11} + \frac{1}{1955923200}\Big($$

$$-27720 R^3 + 593208 R^2\alpha + 2638944 R\alpha^2 + 3104640\,\alpha^3\Big) x^9 + \frac{1}{543312000}\Big(-4365 R^3$$

$$- 159544 R^2\alpha - 1378608 R\alpha^2 - 3104640\,\alpha^3\Big) x^7 + \frac{1}{77616000}\Big(-10512 R^2\alpha$$

$$- 182336 R\alpha^2 + 576576\,\alpha^3\Big) x^5 + \frac{1}{6}\left(-\frac{8951}{53508000} R^3 - \frac{463}{171600} R^2\alpha + \frac{479}{12250} R\alpha^2\right.$$

$$\left.- \frac{64}{2625}\,\alpha^3\right) x^3 + \left(\frac{2047}{73573500} R^3 + \frac{41779}{84084000} R^2\alpha - \frac{1349}{441000} R\alpha^2 + \frac{2}{2625}\,\alpha^3\right) x$$

```
> f(x) := sum(f[n](x), n = 0 .. 3);
```

$$f(x) := -\frac{1}{2} x^3 + \frac{3}{2} x + \frac{1}{280} R x^7 + \frac{1}{10}\,\alpha x^5 + \frac{1}{6}\left(-\frac{9}{140} R - \frac{6}{5}\,\alpha\right) x^3 + \left(\frac{1}{140} R\right. \qquad \textbf{(11)}$$

$$\left.+ \frac{1}{10}\,\alpha\right) x + \frac{1}{92400} R^2 x^{11} + \frac{1}{70560}\left(-21 R^2 - 56 R\alpha\right) x^9 + \frac{1}{19600}\left(3 R^2 - 84 R\alpha\right.$$

$$\left.- 280\,\alpha^2\right) x^7 + \frac{1}{2800}\left(6 R\alpha + 112\,\alpha^2\right) x^5 + \frac{1}{6}\left(\frac{219}{53900} R^2 + \frac{37}{525} R\alpha - \frac{39}{175}\,\alpha^2\right) x^3$$

$$+ \left(-\frac{703}{1293600} R^2 - \frac{37}{4200} R\alpha + \frac{2}{175}\,\alpha^2\right) x + \frac{19}{144144000} R^3 x^{15} + \frac{1}{11099088000}\Big($$

$$-32340 R^3 - 52360 R^2\alpha\Big) x^{13} + \frac{1}{5122656000}\left(128304 R^3 + 426888 R^2\alpha\right.$$

$$+ 406560\, R\, \alpha^2 \big)\, x^{11} + \frac{1}{1955923200} \big(-27720\, R^3 + 593208\, R^2\, \alpha + 2638944\, R\, \alpha^2$$

$$+ 3104640\, \alpha^3 \big)\, x^9 + \frac{1}{543312000} \big(-4365\, R^3 - 159544\, R^2\, \alpha - 1378608\, R\, \alpha^2$$

$$- 3104640\, \alpha^3 \big)\, x^7 + \frac{1}{77616000} \big(-10512\, R^2\, \alpha - 182336\, R\, \alpha^2 + 576576\, \alpha^3 \big)\, x^5 + \frac{1}{6} \bigg($$

$$- \frac{8951}{53508000}\, R^3 - \frac{463}{171600}\, R^2\, \alpha + \frac{479}{12250}\, R\, \alpha^2 - \frac{64}{2625}\, \alpha^3 \bigg)\, x^3 + \bigg(\frac{2047}{73573500}\, R^3$$

$$+ \frac{41779}{84084000}\, R^2\, \alpha - \frac{1349}{441000}\, R\, \alpha^2 + \frac{2}{2625}\, \alpha^3 \bigg)\, x$$

> $f(x) := subs(\{R=5, \text{alpha}=0.2\}, f(x));$

$$f(x) := -0.5708442354\, x^3 + 1.539132268\, x + 0.01380082214\, x^7 + 0.02265526448\, x^5 \qquad \textbf{(12)}$$

$$+ 0.003833900227\, x^{11} - 0.008206689342\, x^9 + \frac{19}{1153152}\, x^{15} - 0.0003878066378\, x^{13}$$

> $f[HPM](x) := evalf(f(x));$

$$f_{HPM}(x) := -0.5708442354\, x^3 + 1.539132268\, x + 0.01380082214\, x^7 + 0.02265526448\, x^5 \qquad \textbf{(13)}$$

$$+ 0.003833900227\, x^{11} - 0.008206689342\, x^9 + 0.00001647657898\, x^{15}$$

$$- 0.0003878066378\, x^{13}$$

> $plot(f[HPM](x), x=0..1);$