# CHAPTER 11

11.1. The parameters of a certain transmission line operating at $6 \times 10^8$ rad/s are $L = 0.4\ \mu$H/m, $C = 40$ pF/m, $G = 80\ \mu$S/m, and $R = 20\ \Omega$/m.

a) Find $\gamma$, $\alpha$, $\beta$, $\lambda$, and $Z_0$: We use

$$\begin{aligned}\gamma &= \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{[20 + j(6 \times 10^8)(0.4 \times 10^{-6})][80 \times 10^{-6} + j(6 \times 10^8)(40 \times 10^{-12})]} \\ &= \underline{0.10 + j2.4\ \text{m}^{-1}} = \alpha + j\beta\end{aligned}$$

Therefore, $\alpha = \underline{0.10\ \text{Np/m}}$, $\beta = \underline{2.4\ \text{rad/m}}$, and $\lambda = 2\pi/\beta = \underline{2.6\ \text{m}}$. Finally,

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{20 + j2.4 \times 10^2}{80 \times 10^{-6} + j2.4 \times 10^{-2}}} = \underline{100 - j4.0\ \Omega}$$

b) If a voltage wave travels 20 m down the line, what percentage of the original amplitude remains, and by how many degrees is it phase shifted? First,

$$\frac{V_{20}}{V_0} = e^{-\alpha L} = e^{-(0.10)(20)} = 0.13 \text{ or } \underline{13 \text{ percent}}$$

Then the phase shift is given by $\beta L$, which in degrees becomes

$$\phi = \beta L \left(\frac{360}{2\pi}\right) = (2.4)(20)\left(\frac{360}{2\pi}\right) = \underline{2.7 \times 10^3 \text{ degrees}}$$

11.2. A lossless transmission line with $Z_0 = 60\ \Omega$ is being operated at 60 MHz. The velocity on the line is $3 \times 10^8$ m/s. If the line is short-circuited at $z = 0$, find $Z_{in}$ at:

a) $z = -1$m: We use the expression for input impedance (Eq. 12), under the conditions $Z_2 = 60$ and $Z_3 = 0$:

$$Z_{in} = Z_2 \left[\frac{Z_3 \cos(\beta l) + jZ_2 \sin(\beta l)}{Z_2 \cos(\beta l) + jZ_3 \sin(\beta l)}\right] = j60 \tan(\beta l)$$

where $l = -z$, and where the phase constant is $\beta = 2\pi c/f = 2\pi(3 \times 10^8)/(6 \times 10^7) = (2/5)\pi$ rad/m. Now, with $z = -1$ ($l = 1$), we find $Z_{in} = j60 \tan(2\pi/5) = \underline{j184.6\ \Omega}$.

b) $z = -2$ m: $Z_{in} = j60 \tan(4\pi/5) = \underline{-j43.6\ \Omega}$

c) $z = -2.5$ m: $Z_{in} = j60 \tan(5\pi/5) = \underline{0}$

d) $z = -1.25$ m: $Z_{in} = j60 \tan(\pi/2) = \underline{j\infty\ \Omega}$ (open circuit)

11.3. The characteristic impedance of a certain lossless transmission line is $72\ \Omega$. If $L = 0.5\,\mu$H/m, find:

a) $C$: Use $Z_0 = \sqrt{L/C}$, or

$$C = \frac{L}{Z_0^2} = \frac{5 \times 10^{-7}}{(72)^2} = 9.6 \times 10^{-11}\ \text{F/m} = \underline{96\ \text{pF/m}}$$

11.3b) $v_p$:

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5\times 10^{-7})(9.6\times 10^{-11})}} = \underline{1.44\times 10^8 \text{ m/s}}$$

c) $\beta$ if $f = 80$ MHz:

$$\beta = \omega\sqrt{LC} = \frac{2\pi\times 80\times 10^6}{1.44\times 10^8} = \underline{3.5 \text{ rad/m}}$$

d) The line is terminated with a load of 60 $\Omega$. Find $\Gamma$ and $s$:

$$\Gamma = \frac{60-72}{60+72} = \underline{-0.09} \qquad\qquad s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+.09}{1-.09} = \underline{1.2}$$

11.4. A lossless transmission line having $Z_0 = 120\Omega$ is operating at $\omega = 5\times 10^8$ rad/s. If the velocity on the line is $2.4\times 10^8$ m/s, find:

a) $L$: With $Z_0 = \sqrt{L/C}$ and $v = 1/\sqrt{LC}$, we find $L = Z_0/v = 120/2.4\times 10^8 = \underline{0.50\ \mu\text{H/m}}$.

b) $C$: Use $Z_0 v = \sqrt{L/C}/\sqrt{LC} \Rightarrow C = 1/(Z_0 v) = [120(2.4\times 10^8)]^{-1} = \underline{35 \text{ pF/m}}$.

c) Let $Z_L$ be represented by an inductance of $0.6\,\mu$H in series with a 100-$\Omega$ resistance. Find $\Gamma$ and $s$: The inductive impedance is $j\omega L = j(5\times 10^8)(0.6\times 10^{-6}) = j300$. So the load impedance is $Z_L = 100 + j300\ \Omega$. Now

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j300 - 120}{100 + j300 + 120} = 0.62 + j0.52 = \underline{0.808\angle 40^\circ}$$

Then

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.808}{1-0.808} = \underline{9.4}$$

11.5. Two characteristics of a certain lossless transmission line are $Z_0 = 50\ \Omega$ and $\gamma = 0 + j0.2\pi\ \text{m}^{-1}$ at $f = 60$ MHz.

a) Find $L$ and $C$ for the line: We have $\beta = 0.2\pi = \omega\sqrt{LC}$ and $Z_0 = 50 = \sqrt{L/C}$. Thus

$$\frac{\beta}{Z_0} = \omega C \quad\Rightarrow\quad C = \frac{\beta}{\omega Z_0} = \frac{0.2\pi}{(2\pi\times 60\times 10^6)(50)} = \frac{1}{3}\times 10^{10} = \underline{33.3 \text{ pF/m}}$$

Then $L = CZ_0^2 = (33.3\times 10^{-12})(50)^2 = 8.33\times 10^{-8}$ H/m $= \underline{83.3 \text{ nH/m}}$.

b) A load, $Z_L = 60 + j80\ \Omega$ is located at $z = 0$. What is the shortest distance from the load to a point at which $Z_{in} = R_{in} + j0$? I will do this using two different methods:

*The Hard Way*: We use the general expression

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right]$$

We can then normalize the impedances with respect to $Z_0$ and write

$$z_{in} = \frac{Z_{in}}{Z_0} = \left[\frac{(Z_L/Z_0) + j\tan(\beta l)}{1 + j(Z_L/Z_0)\tan(\beta l)}\right] = \left[\frac{z_L + j\tan(\beta l)}{1 + jz_L\tan(\beta l)}\right]$$

where $z_L = (60 + j80)/50 = 1.2 + j1.6$.

11.5b. (continued) Using this, and defining $x = \tan(\beta l)$, we find

$$z_{in} = \left[ \frac{1.2 + j(1.6 + x)}{(1 - 1.6x) + j1.2x} \right] \left[ \frac{(1 - 1.6x) - j1.2x}{(1 - 1.6x) - j1.2x} \right]$$

The second bracketed term is a factor of one, composed of the complex conjugate of the denominator of the first term, divided by itself. Carrying out this product, we find

$$z_{in} = \left[ \frac{1.2(1 - 1.6x) + 1.2x(1.6 + x) - j[(1.2)^2 x - (1.6 + x)(1 - 1.6x)]}{(1 - 1.6x)^2 + (1.2)^2 x^2} \right]$$

We require the imaginary part to be zero. Thus

$$(1.2)^2 x - (1.6 + x)(1 - 1.6x) = 0 \quad \Rightarrow \quad 1.6x^2 + 3x - 1.6 = 0$$

So

$$x = \tan(\beta l) = \frac{-3 \pm \sqrt{9 + 4(1.6)^2}}{2(1.6)} = (.433, -2.31)$$

We take the positive root, and find

$$\beta l = \tan^{-1}(.433) = 0.409 \quad \Rightarrow \quad l = \frac{0.409}{0.2\pi} = 0.65 \text{ m} = \underline{65 \text{ cm}}$$

*The Easy Way*: We find

$$\Gamma = \frac{60 + j80 - 50}{60 + j80 + 50} = 0.405 + j0.432 = 0.59\angle 0.818$$

Thus $\phi = 0.818$ rad, and we use the fact that the input impedance will be purely real at the location of a voltage minimum or maximum. The first voltage maximum will occur at a distance in front of the load given by

$$z_{max} = \frac{\phi}{2\beta} = \frac{0.818}{2(0.2\pi)} = 0.65 \text{ m}$$

11.6. The propagation constant of a lossy transmission line is $1 + j2$ m$^{-1}$, and its characteristic impedance is $20 + j0\,\Omega$ at $\omega = 1$ Mrad/s. Find $L$, $C$, $R$, and $G$ for the line: Begin with

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega L}} = 20 \quad \Rightarrow \quad R + j\omega L = 400(G + j\omega C) \tag{1}$$

Then

$$\gamma^2 = (R + j\omega L)(G + j\omega C) = (1 + j2)^2 \quad \Rightarrow \quad 400(G + j\omega C)^2 = (1 + j2)^2 \tag{2}$$

where (1) has been used. Eq. 2 now becomes $G + j\omega C = (1 + j2)/20$. Equating real and imaginary parts leads to $G = \underline{.05 \text{ S/m}}$ and $C = 1/(10\omega) = 10^{-7} = \underline{0.1\,\mu\text{F/m}}$.

11.6. (continued) Now, (1) becomes

$$20 = \sqrt{\frac{R + j\omega L}{1 + j2}}\sqrt{20} \quad \Rightarrow \quad 20 = \frac{R + j\omega L}{1 + j2} \quad \Rightarrow \quad 20 + j40 = R + j\omega L$$

Again, equating real and imaginary parts leads to $R = \underline{20\,\Omega/\text{m}}$ and $L = 40/\omega = \underline{40\,\mu\text{H/m}}$.

11.7. A transmitter and receiver are connected using a cascaded pair of transmission lines. At the operating frequency, Line 1 has a measured loss of 0.1 dB/m, and Line 2 is rated at 0.2 dB/m. The link is composed of 40m of Line 1, joined to 25m of Line 2. At the joint, a splice loss of 2 dB is measured. If the transmitted power is 100mW, what is the received power?

The total loss in the link in dB is $40(0.1) + 25(0.2) + 2 = 11$ dB. Then the received power is $P_r = 100\text{mW} \times 10^{-0.1(11)} = \underline{7.9 \text{ mW}}$.

11.8. A measure of absolute power is the dBm scale, in which power is specified in decibels relative to 1 milliwatt. Specifically, $P(\text{dBm}) = 10\log_{10}[P(\text{mW})/1\text{mW}]$. Suppose a receiver is rated as having a *sensitivity* of -5 dBm – indicating the *minimum* power that it must receive in order to adequately interpret the transmitted data. Consider a transmitter having an output of 100 mW connected to this receiver through a length of transmission line whose loss is 0.1 dB/m. What is the maximum length of line that can be used?

First we find the transmitted power in dBm: $P_t(\text{dBm}) = 10\log_{10}(100/1) = 20$ dBm. From this result, we subtract the maximum dB loss to obtain the receiver sensitivity:

$$20\,\text{dBm} - \text{loss}\,(\text{dB}) = -5\,\text{dBm} \quad \Rightarrow \quad \text{loss}\,(\text{dB}) = 0.1L_{max} = 25\,\text{dB}$$

Therefore, the maximum distance is $L_{max} = \underline{250\,\text{m}}$.

11.9. A sinusoidal voltage source drives the series combination of an impedance, $Z_g = 50 - j50\ \Omega$, and a lossless transmission line of length $L$, shorted at the load end. The line characteristic impedance is 50 Ω, and wavelength $\lambda$ is measured on the line.

a) Determine, in terms of wavelength, the shortest line length that will result in the voltage source driving a total impedance of 50 Ω: Using Eq. (98), with $Z_L = 0$, we find the input impedance, $Z_{in} = jZ_0\tan(\beta L)$, where $Z_0 = 50$ ohms. This input inpedance is in series with the generator impedance, giving a total of $Z_{tot} = 50 - j50 + j50\tan(\beta L)$. For this impedance to equal 50 ohms, the imaginary parts must cancel. Therefore, $\tan(\beta L) = 1$, or $\beta L = \pi/4$, at minimum. So $L = \pi/(4\beta) = \pi/(4 \times 2\pi/\lambda) = \underline{\lambda/8}$.

b) Will other line lengths meet the requirements of part *a*? If so what are they? Yes, the requirement being $\beta L = \pi/4 + m\pi$, where $m$ is an integer. Therefore

$$L = \frac{\pi/4 + m\pi}{\beta} = \frac{\pi(1 + 4m)}{4 \times 2\pi/\lambda} = \frac{\lambda}{8} + m\frac{\lambda}{2}$$

11.10. A 100 MHz voltage source drives the series combination of an impedance, $Z_g = 25 + j25\ \Omega$ and a lossless transmission line of length $\lambda/4$, terminated by a load impedance, $Z_L$. The line characteristic impedance is 50 $\Omega$.

a) Determine the load impedance value required to achieve a net impedance (seen by the voltage source) of 50 $\Omega$: From Eq. (98), the input impedance for a quarter-wave line is $Z_{in} = Z_0^2/Z_L$, and the net impedance seen by the voltage source is now

$$Z_{tot} = 25 + j25 + \frac{(50)^2}{Z_L} = 50 \text{ as requested}$$

Solving for $Z_L$, obtain

$$Z_L = \frac{(50)^2}{25 - j25} = \underline{50 + j50 \text{ ohms}}$$

b) If the inductance of the line is $L = 1\,\mu$H/m, determine the line length in meters: We know that $Z_0 = \sqrt{L/C} = 50$, so that $C = L/(50)^2 = 10^{-6}/2500 = 4.0 \times 10^{-10}$ F. Next, the line phase velocity is $v_p = 1/\sqrt{LC} = 1/\sqrt{(10^{-6})(4.0 \times 10^{-10})} = 5.0 \times 10^7$ m/s. Then the wavelength in the line is $\lambda = v_p/f = 5.0 \times 10^7/10^8 = 0.5$ m. Finally the line length is $L = \lambda/4 = \underline{0.125 \text{ m}}$.

11.11. A transmission line having primary constants $L$, $C$, $R$, and $G$, has length $\ell$ and is terminated by a load having complex impedance $R_L + jX_L$. At the input end of the line, a *DC* voltage source, $V_0$, is connected. Assuming all parameters are known at zero frequency, find the steady state power dissipated by the load if

a) $R = G = 0$: Here, the line just acts as a pair of lossless leads to the impedance. At zero frequency, the dissipated power is just $P_d = \underline{V_0^2/R_L}$.

b) $R \neq 0$, $G = 0$: In this case, the load is effectively in series with a resistance of value $R\ell$. The voltage at the load is therefore $V_L = V_0 R_L/(R\ell + R_L)$, and the dissipated power is $P_d = V_L^2/R_L = \underline{V_0^2 R_L/(R\ell + R_L)^2}$.

c) $R = 0$, $G \neq 0$: Now, the load is in parallel with a resistance, $1/(G\ell)$, but the voltage at the load is still $V_0$. Dissipated power by the load is $P_d = \underline{V_0^2/R_L}$.

d) $R \neq 0$, $G \neq 0$: One way to approach this problem is to think of the power at the load as arising from an incident voltage wave of vanishingly small frequency, and to assume that losses in the line are sufficient to allow steady state conditions to be reached after a single reflection from the load. The "forward-traveling" voltage as a function of $z$ is given by $V(z) = V_0 \exp(-\gamma z)$, where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \rightarrow \sqrt{RG}$ as frequency approaches zero. Considering a single reflection only, the voltage at the load is then $V_L = (1 + \Gamma)V_0 \exp\left(-\sqrt{RG}\,\ell\right)$. The reflection coefficient requires the line characteristic impedance, given by $Z_0 = [(R + j\omega L)/(G + j\omega C)]^{1/2} \rightarrow \sqrt{R/G}$ as $\omega \rightarrow 0$. The reflection coefficient is then $\Gamma = (R_L - \sqrt{R/G})/(R_L + \sqrt{R/G})$, and so the load voltage becomes:

$$V_L = \frac{2R_L}{R_L + \sqrt{R/G}} \exp\left(-\sqrt{RG}\,\ell\right)$$

The dissipated power is then

$$P_d = \frac{V_L^2}{R_L} = \frac{4R_L V_0^2}{\left(R_L + \sqrt{R/G}\right)^2} \exp\left(-2\sqrt{RG}\,\ell\right) \text{ W}$$

11.12. In a circuit in which a sinusoidal voltage source drives its internal impedance in series with a load impedance, it is known that maximum power transfer to the load occurs when the source and load impedances form a complex conjugate pair. Suppose the source (with its internal impedance) now drives a complex load of impedance $Z_L = R_L + jX_L$ that has been moved to the end of a lossless transmission line of length $\ell$ having characteristic impedance $Z_0$. If the source impedance is $Z_g = R_g + jX_g$, write an equation that can be solved for the required line length, $\ell$, such that the displaced load will receive the maximum power.

The condition of maximum power transfer will be met if the *input impedance* to the line is the conjugate of the internal impedance. Using Eq. (98), we write

$$Z_{in} = Z_0 \left[ \frac{(R_L + jX_L)\cos(\beta\ell) + jZ_0 \sin(\beta\ell)}{Z_0 \cos(\beta\ell) + j(R_L + jX_L)\sin(\beta\ell)} \right] = R_g - jX_g$$

This is the equation that we have to solve for $\ell$ – assuming that such a solution exists. To find out, we need to work with the equation a little. Multiplying both sides by the denominator of the left side gives

$$Z_0(R_L + jX_L)\cos(\beta\ell) + jZ_0^2 \sin(\beta\ell) = (R_g - jX_g)[Z_0 \cos(\beta\ell) + j(R_L + jX_L)\sin(\beta\ell)]$$

We next separate the equation by equating the real parts of both sides and the imaginary parts of both sides, giving

$$(R_L - R_g)\cos(\beta\ell) = \frac{X_L X_g}{Z_0} \sin(\beta\ell) \quad \text{(real parts)}$$

and

$$(X_L + X_g)\cos(\beta\ell) = \frac{R_g R_L - Z_0^2}{Z_0} \sin(\beta\ell) \quad \text{(imaginary parts)}$$

Using the two equations, we find two conditions on the tangent of $\beta\ell$:

$$\tan(\beta\ell) = \frac{Z_0(R_L - R_g)}{X_g X_L} = \frac{Z_0(X_L + X_g)}{R_g R_L - Z_0^2}$$

For a viable solution to exist for $\ell$, both equalities must be satisfied, thus limiting the possible choices of the two impedances.

11.13. The incident voltage wave on a certain lossless transmission line for which $Z_0 = 50\,\Omega$ and $v_p = 2 \times 10^8$ m/s is $V^+(z,t) = 200\cos(\omega t - \pi z)$ V.

a) Find $\omega$: We know $\beta = \pi = \omega/v_p$, so $\omega = \pi(2 \times 10^8) = \underline{6.28 \times 10^8 \text{ rad/s}}$.

b) Find $I^+(z,t)$: Since $Z_0$ is real, we may write

$$I^+(z,t) = \frac{V^+(z,t)}{Z_0} = \underline{4\cos(\omega t - \pi z) \text{ A}}$$

The section of line for which $z > 0$ is replaced by a load $Z_L = 50 + j30\ \Omega$ at $z = 0$. Find

c) $\Gamma_L$: This will be

$$\Gamma_L = \frac{50 + j30 - 50}{50 + j30 + 50} = .0825 + j0.275 = \underline{0.287\angle 1.28 \text{ rad}}$$

d) $V_s^-(z) = \Gamma_L V_s^+(z) e^{j2\beta z} = 0.287(200)e^{j\pi z}e^{j1.28} = \underline{57.5 e^{j(\pi z + 1.28)}}$

e) $V_s$ at $z = -2.2$ m:

$$V_s(-2.2) = V_s^+(-2.2) + V_s^-(-2.2) = 200e^{j2.2\pi} + 57.5e^{-j(2.2\pi - 1.28)} = 257.5e^{j0.63}$$
$$= \underline{257.5\angle 36^\circ}$$

11.14. A 50-$\Omega$ lossless line is terminated with 60- and 30-$\Omega$ resistors in parallel. The voltage at the input to the line is $\mathcal{V}(t) = 100\cos(5\times 10^9 t)$ and the line is three-eighths of a wavelength long. What average power is delivered to each load resistor?

First, we need the input impedance. The parallel resistors give a net load impedance of 20 ohms. The line length of $3\lambda/8$ gives $\beta\ell = (2\pi/\lambda)(3\lambda/8) = (3/4)\pi$. Eq. (98) then yields:

$$Z_{in} = 50\left[\frac{20\cos(3\pi/4) + j50\sin(3\pi/4)}{50\cos(3\pi/4) + j20\sin(3\pi/4)}\right] = 50\left[\frac{-20/\sqrt{2} + j50/\sqrt{2}}{-50/\sqrt{2} + j20/\sqrt{2}}\right] = 34.5 - j36.2\ \Omega$$

Now, the power delivered to the load is the power delivered to the input impedance. This is

$$P = \frac{1}{2}\mathcal{R}e\left\{\frac{|V|^2}{Z_{in}^*}\right\} = \frac{1}{2}\mathcal{R}e\left\{\frac{10^4}{34.5 + j36.2}\right\} = 69\ \text{W}$$

The load resistors, 30 and 60 ohms, will divide the power, with the 30-ohm resistor dissipating twice the power of the 60-ohm. Therefore, the power divides as $\underline{23\,\text{W}\ (60\Omega)}$ and $\underline{46\,\text{W}\ (30\Omega)}$.

11.15. For the transmission line represented in Fig. 11.29, find $V_{s,out}$ if $f =$:

a) 60 Hz: At this frequency,

$$\beta = \frac{\omega}{v_p} = \frac{2\pi\times 60}{(2/3)(3\times 10^8)} = 1.9\times 10^{-6}\ \text{rad/m}\ \ \text{So}\ \beta l = (1.9\times 10^{-6})(80) = 1.5\times 10^{-4} << 1$$

The line is thus essentially a lumped circuit, where $Z_{in} \doteq Z_L = 80\ \Omega$. Therefore

$$V_{s,out} = 120\left[\frac{80}{12+80}\right] = \underline{104\ \text{V}}$$

b) 500 kHz: In this case

$$\beta = \frac{2\pi\times 5\times 10^5}{2\times 10^8} = 1.57\times 10^{-2}\ \text{rad/s}\ \ \text{So}\ \beta l = 1.57\times 10^{-2}(80) = 1.26\ \text{rad}$$

Now

$$Z_{in} = 50\left[\frac{80\cos(1.26) + j50\sin(1.26)}{50\cos(1.26) + j80\sin(1.26)}\right] = 33.17 - j9.57 = 34.5\angle - .28$$

The equivalent circuit is now the voltage source driving the series combination of $Z_{in}$ and the 12 ohm resistor. The voltage across $Z_{in}$ is thus

$$V_{in} = 120\left[\frac{Z_{in}}{12 + Z_{in}}\right] = 120\left[\frac{33.17 - j9.57}{12 + 33.17 - j9.57}\right] = 89.5 - j6.46 = 89.7\angle - .071$$

11.15. (continued) The voltage at the line input is now the sum of the forward and backward-propagating waves just to the right of the input. We reference the load at $z = 0$, and so the input is located at $z = -80$ m. In general we write $V_{in} = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$, where

$$V_0^- = \Gamma_L V_0^+ = \frac{80-50}{80+50} V_0^+ = \frac{3}{13} V_0^+$$

At $z = -80$ m we thus have

$$V_{in} = V_0^+ \left[ e^{j1.26} + \frac{3}{13} e^{-j1.26} \right] \quad \Rightarrow \quad V_0^+ = \frac{89.5 - j6.46}{e^{j1.26} + (3/13)e^{-j1.26}} = 42.7 - j100 \text{ V}$$

Now

$$V_{s,out} = V_0^+(1+\Gamma_L) = (42.7 - j100)(1 + 3/(13)) = 134\angle - 1.17\,\text{rad} = \underline{52.6 - j123 \text{ V}}$$

As a check, we can evaluate the average power reaching the load:

$$P_{avg,L} = \frac{1}{2}\frac{|V_{s,out}|^2}{R_L} = \frac{1}{2}\frac{(134)^2}{80} = 112 \text{ W}$$

This must be the same power that occurs at the input impedance:

$$P_{avg,in} = \frac{1}{2}\text{Re}\left\{V_{in} I_{in}^*\right\} = \frac{1}{2}\text{Re}\left\{(89.5 - j6.46)(2.54 + j0.54)\right\} = 112 \text{ W}$$

where $I_{in} = V_{in}/Z_{in} = (89.5 - j6.46)/(33.17 - j9.57) = 2.54 + j0.54$.

11.16. A 300 ohm transmission line is 0.8 m long and is terminated with a short circuit. The line is operating in air with a wavelength of 0.3 m and is lossless.

a) If the input voltage amplitude is 10V, what is the maximum voltage amplitude at any point on the line? The net voltage anywhere on the line is the sum of the forward and backward wave voltages, and is written as $V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$. Since the line is short-circuited at the load end ($z = 0$), we have $V_0^- = -V_0^+$, and so

$$V(z) = V_0^+ \left(e^{-j\beta z} - e^{j\beta z}\right) = -2jV_0^+ \sin(j\beta z)$$

We now evaluate the voltage at the input, where $z = -0.8$m, and $\lambda = 0.3$m.

$$V_{in} = -2jV_0^+ \sin\left(\frac{2\pi(-0.8)}{0.3}\right) = -j1.73 V_0^+$$

The magnitude of $V_{in}$ is given as 10V, so we find $V_0^+ = 10/1.73 = 5.78$V. The maximum voltage amplitude on the line will be twice this value (where the sine function is unity), so $|V|_{max} = 2(5.78) = \underline{11.56 \text{ V}}$.

b) What is the current amplitude in the short circuit? At the shorted end, the current will be

$$I_L = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = \frac{2V_0^+}{Z_0} = \frac{11.56}{300} = 0.039A = \underline{39\,\text{mA}}$$

11.17. Determine the average power absorbed by each resistor in Fig. 11.30: The problem is made easier by first converting the current source/100 ohm resistor combination to its Thevenin equivalent. This is a $50\angle 0$ V voltage source in series with the 100 ohm resistor. The next step is to determine the input impedance of the $2.6\lambda$ length line, terminated by the 25 ohm resistor: We use $\beta l = (2\pi/\lambda)(2.6\lambda) = 16.33$ rad. This value, modulo $2\pi$ is (by subtracting $2\pi$ twice) 3.77 rad. Now

$$Z_{in} = 50\left[\frac{25\cos(3.77) + j50\sin(3.77)}{50\cos(3.77) + j25\sin(3.77)}\right] = 33.7 + j24.0$$

The equivalent circuit now consists of the series combination of 50 V source, 100 ohm resistor, and $Z_{in}$, as calculated above. The current in this circuit will be

$$I = \frac{50}{100 + 33.7 + j24.0} = 0.368\angle - .178$$

The power dissipated by the 25 ohm resistor is the same as the power dissipated by the real part of $Z_{in}$, or

$$P_{25} = P_{33.7} = \frac{1}{2}|I|^2 R = \frac{1}{2}(.368)^2(33.7) = \underline{2.28\ \text{W}}$$

To find the power dissipated by the 100 ohm resistor, we need to return to the Norton configuration, with the original current source in parallel with the 100 ohm resistor, and in parallel with $Z_{in}$. The voltage across the 100 ohm resistor will be the same as that across $Z_{in}$, or $V = IZ_{in} = (.368\angle - .178)(33.7 + j24.0) = 15.2\angle 0.44$. The power dissipated by the 100 ohm resistor is now

$$P_{100} = \frac{1}{2}\frac{|V|^2}{R} = \frac{1}{2}\frac{(15.2)^2}{100} = \underline{1.16\ \text{W}}$$

11.18 The line shown in Fig. 11.31 is lossless. Find $s$ on both sections 1 and 2: For section 2, we consider the propagation of one forward and one backward wave, comprising the superposition of all reflected waves from both ends of the section. The ratio of the backward to the forward wave amplitude is given by the reflection coefficient at the load, which is

$$\Gamma_L = \frac{50 - j100 - 50}{50 - j100 + 50} = \frac{-j}{1 - j} = \frac{1}{2}(1 - j)$$

Then $|\Gamma_L| = (1/2)\sqrt{(1 - j)(1 + j)} = 1/\sqrt{2}$. Finally

$$s_2 = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 1/\sqrt{2}}{1 - 1/\sqrt{2}} = \underline{5.83}$$

For section 1, we need the reflection coefficient at the junction (location of the 100 Ω resistor) seen by waves incident from section 1: We first need the input impedance of the $.2\lambda$ length of section 2:

$$Z_{in2} = 50\left[\frac{(50 - j100)\cos(\beta_2 l) + j50\sin(\beta_2 l)}{50\cos(\beta_2 l) + j(50 - j100)\sin(\beta_2 l)}\right] = 50\left[\frac{(1 - j2)(0.309) + j0.951}{0.309 + j(1 - j2)(0.951)}\right]$$
$$= 8.63 + j3.82 = 9.44\angle 0.42\,\text{rad}$$

11.18. (continued) Now, this impedance is in parallel with the 100Ω resistor, leading to a net junction impedance found by

$$\frac{1}{Z_{inT}} = \frac{1}{100} + \frac{1}{8.63 + j3.82} \quad\Rightarrow\quad Z_{inT} = 8.06 + j3.23 = 8.69\angle 0.38\,\text{rad}$$

The reflection coefficient will be

$$\Gamma_j = \frac{Z_{inT} - 50}{Z_{inT} + 50} = -0.717 + j0.096 = 0.723\angle 3.0\,\mathrm{rad}$$

and the standing wave ratio is $s_1 = (1 + 0.723)/(1 - 0.723) = \underline{6.22}$.

11.19. A lossless transmission line is 50 cm in length and operating at a frequency of 100 MHz. The line parameters are $L = 0.2\ \mu$H/m and $C = 80$ pF/m. The line is terminated by a short circuit at $z = 0$, and there is a load, $Z_L = 50 + j20$ ohms across the line at location $z = -20$ cm. What average power is delivered to $Z_L$ if the input voltage is $100\angle 0$ V? With the given capacitance and inductance, we find

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2 \times 10^{-7}}{8 \times 10^{-11}}} = 50\ \Omega$$

and

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2 \times 10^{-7})(9 \times 10^{-11})}} = 2.5 \times 10^8\ \mathrm{m/s}$$

Now $\beta = \omega/v_p = (2\pi \times 10^8)/(2.5 \times 10^8) = 2.5$ rad/s. We then find the input impedance to the shorted line section of length 20 cm (putting this impedance at the location of $Z_L$, so we can combine them): We have $\beta l = (2.5)(0.2) = 0.50$, and so, using the input impedance formula with a zero load impedance, we find $Z_{in1} = j50\tan(0.50) = j27.4$ ohms. Now, at the location of $Z_L$, the net impedance there is the parallel combination of $Z_L$ and $Z_{in1}$: $Z_{net} = (50 + j20)||(j27.4) = 7.93 + j19.9$. We now transform this impedance to the line input, 30 cm to the left, obtaining (with $\beta l = (2.5)(.3) = 0.75$):

$$Z_{in2} = 50\left[\frac{(7.93 + j19.9)\cos(.75) + j50\sin(.75)}{50\cos(.75) + j(7.93 + j19.9)\sin(.75)}\right] = 35.9 + j98.0 = 104.3\angle 1.22$$

The power delivered to $Z_L$ is the same as the power delivered to $Z_{in2}$: The current magnitude is $|I| = (100)/(104.3) = 0.96$ A. So finally,

$$P = \frac{1}{2}|I|^2 R = \frac{1}{2}(0.96)^2(35.9) = \underline{16.5\ \mathrm{W}}$$

11.20a. Determine $s$ on the transmission line of Fig. 11.32. Note that the dielectric is air: The reflection coefficient at the load is

$$\Gamma_L = \frac{40+j30-50}{40+j30+50} = j0.333 = 0.333\angle 1.57\,\text{rad} \qquad \text{Then} \quad s = \frac{1+.333}{1-.333} = \underline{2.0}$$

b) Find the input impedance: With the length of the line at $2.7\lambda$, we have $\beta l = (2\pi)(2.7) = 16.96$ rad. The input impedance is then

$$Z_{in} = 50\left[\frac{(40+j30)\cos(16.96)+j50\sin(16.96)}{50\cos(16.96)+j(40+j30)\sin(16.96)}\right] = 50\left[\frac{-1.236-j5.682}{1.308-j3.804}\right] = \underline{61.8-j37.5\ \Omega}$$

c) If $\omega L = 10\,\Omega$, find $I_s$: The source drives a total impedance given by $Z_{net} = 20 + j\omega L + Z_{in} = 20 + j10 + 61.8 - j37.5 = 81.8 - j27.5$. The current is now $I_s = 100/(81.8 - j27.5) = \underline{1.10 + j0.37\,\text{A}}$.

d) What value of $L$ will produce a maximum value for $|I_s|$ at $\omega = 1$ Grad/s? To achieve this, the imaginary part of the total impedance of part $c$ must be reduced to zero (so we need an inductor). The inductor impedance must be equal to negative the imaginary part of the line input impedance, or $\omega L = 37.5$, so that $L = 37.5/\omega = \underline{37.5\,\text{nH}}$. Continuing, for this value of $L$, calculate the average power:

e) supplied by the source: $P_s = (1/2)\text{Re}\{V_s I_s^*\} = (1/2)(100)(1.10) = \underline{55.0\,\text{W}}$.

f) delivered to $Z_L = 40 + j30\,\Omega$: The power delivered to the load will be the same as the power delivered to the input impedance. We write

$$P_L = \frac{1}{2}\text{Re}\{Z_{in}\}|I_s|^2 = \frac{1}{2}(61.8)[(1.10+j.37)(1.10-j.37)] = \underline{41.6\,\text{W}}$$

11.21. A lossless line having an air dielectric has a characteristic impedance of 400 Ω. The line is operating at 200 MHz and $Z_{in} = 200 - j200\ \Omega$. Use analytic methods or the Smith chart (or both) to find: (a) $s$; (b) $Z_L$ if the line is 1 m long; (c) the distance from the load to the nearest voltage maximum: I will first use the analytic approach. Using normalized impedances, Eq. (13) becomes

$$z_{in} = \frac{Z_{in}}{Z_0} = \left[\frac{z_L\cos(\beta L)+j\sin(\beta L)}{\cos(\beta L)+jz_L\sin(\beta L)}\right] = \left[\frac{z_L+j\tan(\beta L)}{1+jz_L\tan(\beta L)}\right]$$

Solve for $z_L$:

$$z_L = \left[\frac{z_{in}-j\tan(\beta L)}{1-jz_{in}\tan(\beta L)}\right]$$

where, with $\lambda = c/f = 3\times10^8/2\times10^8 = 1.50$ m, we find $\beta L = (2\pi)(1)/(1.50) = 4.19$, and so $\tan(\beta L) = 1.73$. Also, $z_{in} = (200 - j200)/400 = 0.5 - j0.5$. So

$$z_L = \frac{0.5-j0.5-j1.73}{1-j(0.5-j0.5)(1.73)} = 2.61 + j0.174$$

Finally, $Z_L = z_L(400) = \underline{1.04\times10^3 + j69.8\ \Omega}$. Next

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{6.42\times10^2 + j69.8}{1.44\times10^3 + j69.8} = .446 + j2.68\times10^{-2} = .447\angle 6.0\times10^{-2}\ \text{rad}$$

11.21. (continued) Now

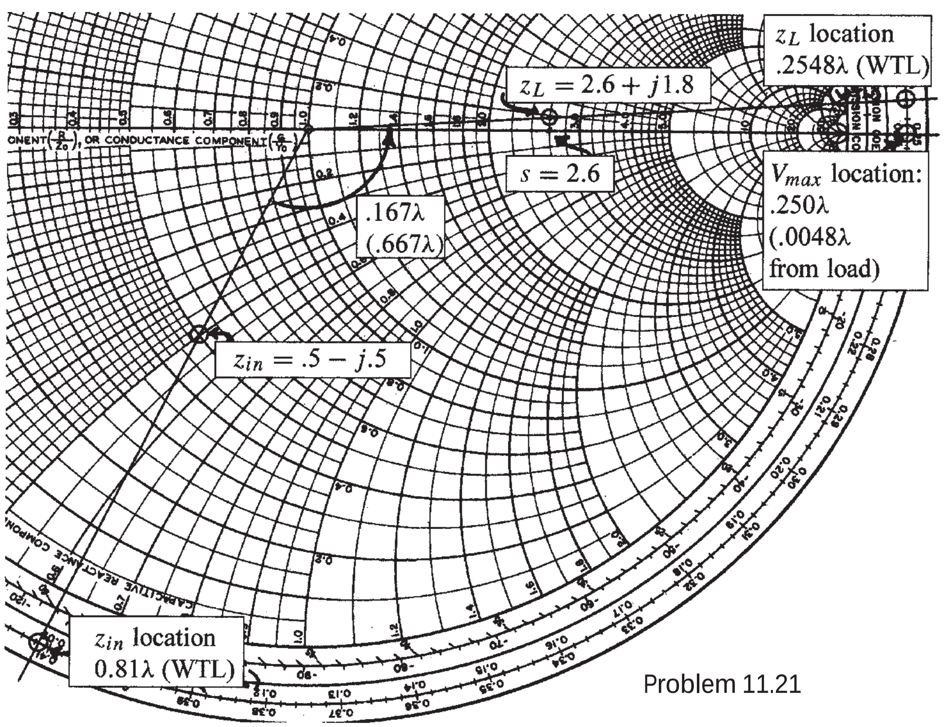
$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + .447}{1 - .447} = \underline{2.62}$$

Finally

$$z_{max} = -\frac{\phi}{2\beta} = -\frac{\lambda\phi}{4\pi} = -\frac{(6.0 \times 10^{-2})(1.50)}{4\pi} = -7.2 \times 10^{-3}\ \text{m} = \underline{-7.2\ \text{mm}}$$

We next solve the problem using the Smith chart. Referring to the figure below, we first locate and mark the normalized input impedance, $z_{in} = 0.5 - j0.5$. A line drawn from the origin through this point intersects the outer chart boundary at the position $0.0881\,\lambda$ on the wavelengths toward load (WTL) scale. With a wavelength of 1.5 m, the 1 meter line is 0.6667 wavelengths long. On the WTL scale, we add $0.6667\lambda$, or equivalently, $0.1667\lambda$ (since $0.5\lambda$ is once around the chart), obtaining $(0.0881 + 0.1667)\lambda) = 0.2548\lambda$, which is the position of the load. A straight line is now drawn from the origin though the $0.2548\lambda$ position. A compass is then used to measure the distance between the origin and $z_{in}$. With this distance set, the compass is then used to scribe off the same distance from the origin to the load impedance, along the line between the origin and the $0.2548\lambda$ position. That point is the normalized load impedance, which is read to be $z_L = 2.6 + j0.18$. Thus $Z_L = z_L(400) = 1040 + j72$. This is in reasonable agreement with the analytic result of $1040 + j69.8$. The difference in imaginary parts arises from uncertainty in reading the chart in that region.

In transforming from the input to the load positions, we cross the $r > 1$ real axis of the chart at r=2.6. This is close to the value of the VSWR, as we found earlier. We also see that the $r > 1$ real axis (at which the first $V_{max}$ occurs) is a distance of $0.0048\lambda$ (marked as $.005\lambda$ on the chart) in front of the load. The actual distance is $z_{max} = -0.0048(1.5)\,\text{m} = -0.0072\,\text{m} = -7.2\,\text{mm}$.



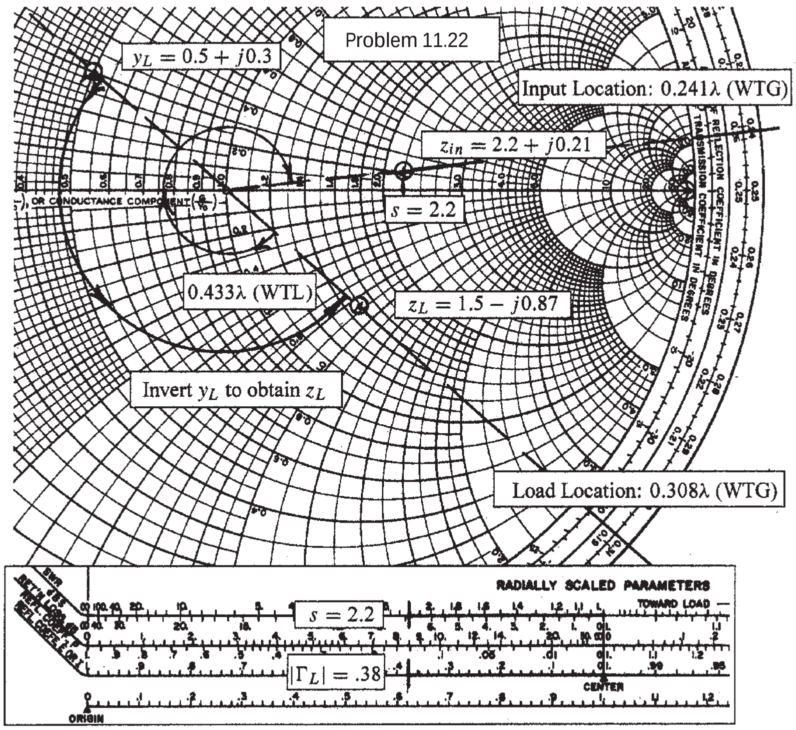

Problem 11.21

11.22. A lossless two-wire line has a characteristic impedance of 300 Ω and a capacitance of 15 pF/m. The load at $z = 0$ consists of a 600-Ω resistor in parallel with a 10-pF capacitor. If $\omega = 10^8$ rad/s and the line is 20m long, use the Smith chart to find a) $|\Gamma_L|$; b) $s$; c) $Z_{in}$. First, the wavelength on the line is found using $\lambda = 2\pi v_p/\omega$, where $v_p = 1/(CZ_0)$. Assuming higher accuracy in the given values than originally stated, we obtain

$$\lambda = \frac{2\pi}{\omega C Z_0} = \frac{2\pi}{(10^8)(15 \times 10^{-12})(300)} = 13.96\,\mathrm{m}$$

The line length in wavelengths is therefore $20/13.96 = 1.433\lambda$. The normalized load admittance is now

$$y_L = Y_L Z_0 = Z_0\left[\frac{1}{R_L} + j\omega C\right] = 300\left[\frac{1}{600} + j(10^8)(10^{-11})\right] = 0.50 + j0.30$$

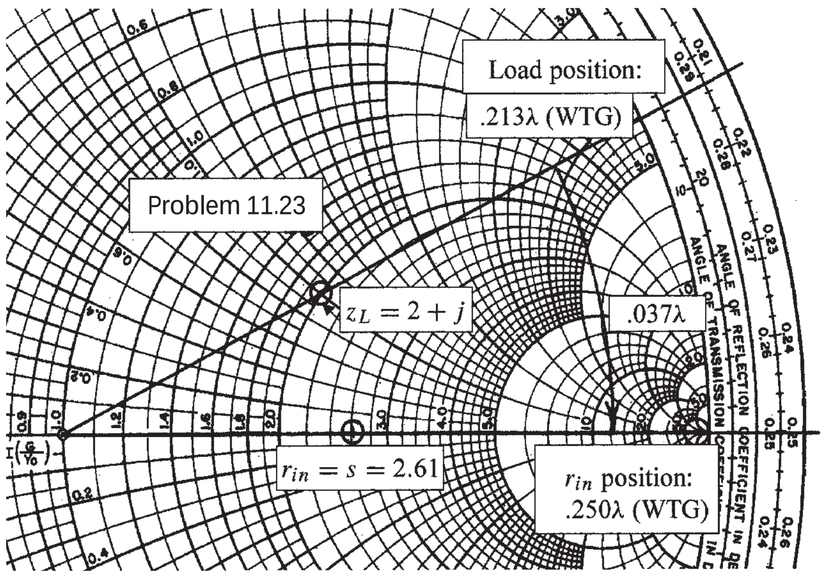



The $y_L$ value is plotted on the chart and labeled as $y_L$. Next, $y_L$ is inverted to find $z_L$ by transforming the point halfway around the chart, using the compass and a straight edge. The result, labeled $z_L$ on the chart is read to be $z_L = 1.5 - j0.87$. This is close to the computed inverse of $y_L$, which is $1.47 - j0.88$. Scribing the compass arc length along the bottom scale for reflection coefficient yields $|\Gamma_L| = \underline{0.38}$. The VSWR is found by scribing the compass arc length either along the bottom SWR scale or along the positive real axis of the chart, both methods yielding $s = \underline{2.2}$. Now, the position of $z_L$ is read on the outer edge of the chart as $0.308\lambda$ on the WTG scale. The point is now transformed through the line length distance of $1.433\lambda$ toward the generator (the net chart distance will be $0.433\lambda$, since a full wavelength is two complete revolutions). The final reading on the WTG scale after the transformation is found through $(0.308 + 0.433 - 0.500)\lambda = 0.241\lambda$. Drawing a line between this mark on the WTG scale and the chart center, and scribing the compass arc length on this line, yields the normalized input impedance. This is read as $z_{in} = 2.2 + j0.21$ (the computed value found through the analytic solution is $z_{in} = 2.21 + j0.219$. The input impedance is now found by multiplying the chart reading by 300, or $Z_{in} = \underline{660 + j63\,\Omega}$.

11.23. The normalized load on a lossless transmission line is $z_L = 2 + j1$. Let $\lambda = 20$ m Make use of the Smith chart to find:

a) the shortest distance from the load to the point at which $z_{in} = r_{in} + j0$, where $r_{in} > 1$ (not greater than 0 as stated): Referring to the figure below, we start by marking the given $z_L$ on the chart and drawing a line from the origin through this point to the outer boundary. On the WTG scale, we read the $z_L$ location as $0.213\lambda$. Moving from here toward the generator, we cross the positive $\Gamma_R$ axis (at which the impedance is purely real and greater than 1) at $0.250\lambda$. The distance is then $(0.250 - 0.213)\lambda = \underline{0.037\lambda}$ from the load. With $\lambda = 20$ m, the actual distance is $20(0.037) = 0.74$ m.

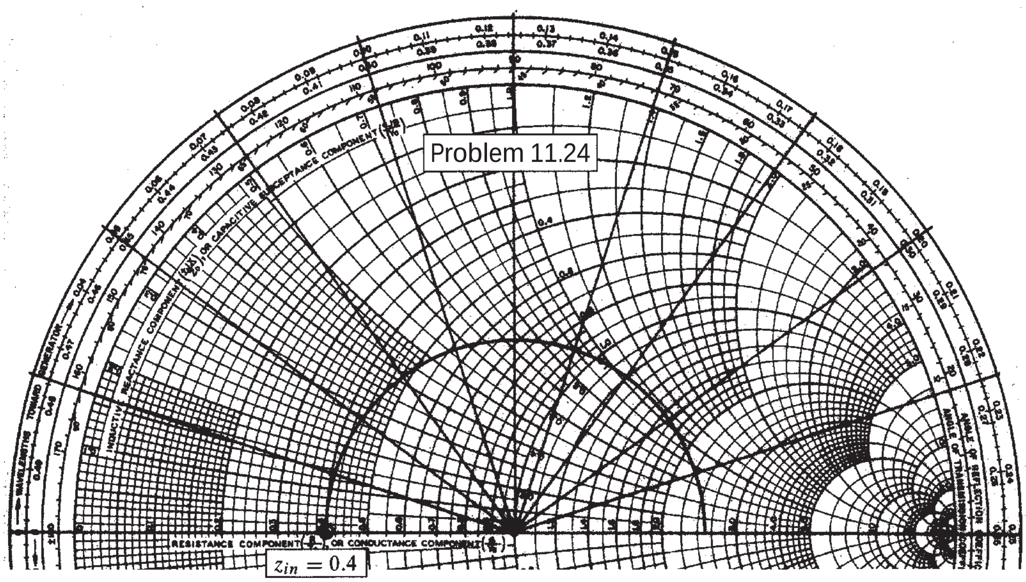
b) Find $z_{in}$ at the point found in part $a$: Using a compass, we set its radius at the distance between the origin and $z_L$. We then scribe this distance along the real axis to find $z_{in} = r_{in} = \underline{2.61}$.




c) The line is cut at this point and the portion containing $z_L$ is thrown away. A resistor $r = r_{in}$ of part $a$ is connected across the line. What is $s$ on the remainder of the line? This will be just $s$ for the line as it was before. As we know, $s$ will be the positive real axis value of the normalized impedance, or $s = \underline{2.61}$.

d) What is the shortest distance from this resistor to a point at which $z_{in} = 2 + j1$? This would return us to the original point, requiring a complete circle around the chart (one-half wavelength distance). The distance from the resistor will therefore be: $d = 0.500\,\lambda - 0.037\,\lambda = \underline{0.463\,\lambda}$. With $\lambda = 20$ m, the actual distance would be $20(0.463) = 9.26$ m.

11.24. With the aid of the Smith chart, plot a curve of $|Z_{in}|$ vs. $l$ for the transmission line shown in Fig. 11.33. Cover the range $0 < l/\lambda < 0.25$. The required input impedance is that at the actual line input (to the left of the two 20Ω resistors. The input to the line section occurs just to the right of the 20Ω resistors, and the input impedance there we first find with the Smith chart. This impedance is in series with the two 20Ω resistors, so we add 40Ω to the calculated impedance from the Smith chart to find the net line input impedance. To begin, the 20Ω load resistor represents a normalized impedance of $z_l = 0.4$, which we mark on the chart (see below). Then, using a compass, draw a circle beginning at $z_L$ and progressing clockwise to the positive real axis. The circle traces the locus of $z_{in}$ values for line lengths over the range $0 < l < \lambda/4$.




On the chart, radial lines are drawn at positions corresponding to $.025\lambda$ increments on the WTG scale. The intersections of the lines and the circle give a total of 11 $z_{in}$ values. To these we add normalized impedance of $40/50 = 0.8$ to add the effect of the 40Ω resistors and obtain the normalized impedance at the line input. The magnitudes of these values are then found, and the results are multiplied by 50Ω. The table below summarizes the results.

| $l/\lambda$ | $z_{inl}$ (to right of 40Ω) | $z_{in} = z_{inl} + 0.8$ | $\|Z_{in}\| = 50\|z_{in}\|$ |
|---|---|---|---|
| 0 | 0.40 | 1.20 | 60 |
| .025 | 0.41 + j.13 | 1.21 + j.13 | 61 |
| .050 | 0.43 + j.27 | 1.23 + j.27 | 63 |
| .075 | 0.48 + j.41 | 1.28 + j.41 | 67 |
| .100 | 0.56 + j.57 | 1.36 + j.57 | 74 |
| .125 | 0.68 + j.73 | 1.48 + j.73 | 83 |
| .150 | 0.90 + j.90 | 1.70 + j.90 | 96 |
| .175 | 1.20 + j1.05 | 2.00 + j1.05 | 113 |
| .200 | 1.65 + j1.05 | 2.45 + j1.05 | 134 |
| .225 | 2.2 + j.7 | 3.0 + j.7 | 154 |
| .250 | 2.5 | 3.3 | 165 |

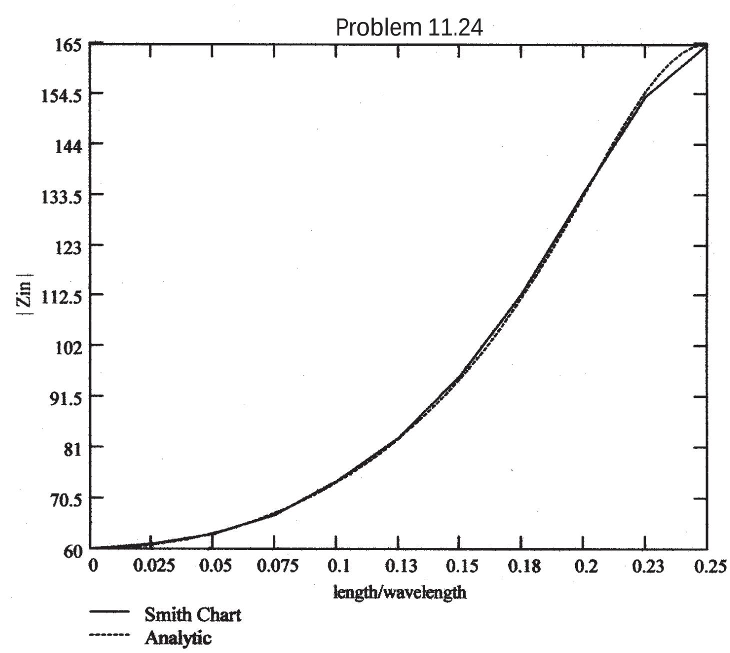
11.24. (continued) As a check, the line input input impedance can be found analytically through

$$Z_{in} = 40 + 50 \left[ \frac{20\cos(2\pi l/\lambda) + j50\sin(2\pi l/\lambda)}{50\cos(2\pi l/\lambda) + j20\sin(2\pi l/\lambda)} \right] = 50 \left[ \frac{60\cos(2\pi l/\lambda) + j66\sin(2\pi l/\lambda)}{50\cos(2\pi l/\lambda) + j20\sin(2\pi l/\lambda)} \right]$$

from which

$$|Z_{in}| = 50 \left[ \frac{36\cos^2(2\pi l/\lambda) + 43.6\sin^2(2\pi l/\lambda)}{25\cos^2(2\pi l/\lambda) + 4\sin^2(2\pi l/\lambda)} \right]^{1/2}$$

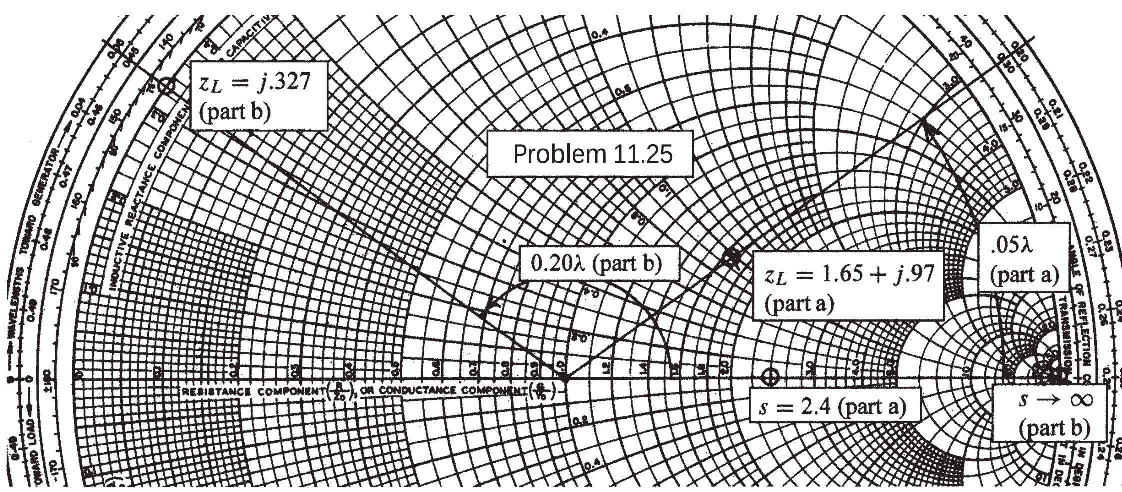
This function is plotted below along with the results obtained from the Smith chart. A fairly good comparison is obtained.



11.25. A 300-ohm transmission line is short-circuited at $z = 0$. A voltage maximum, $|V|_{max} = 10$ V, is found at $z = -25$ cm, and the minimum voltage, $|V|_{min} = 0$, is found at $z = -50$ cm. Use the Smith chart to find $Z_L$ (with the short circuit replaced by the load) if the voltage readings are:

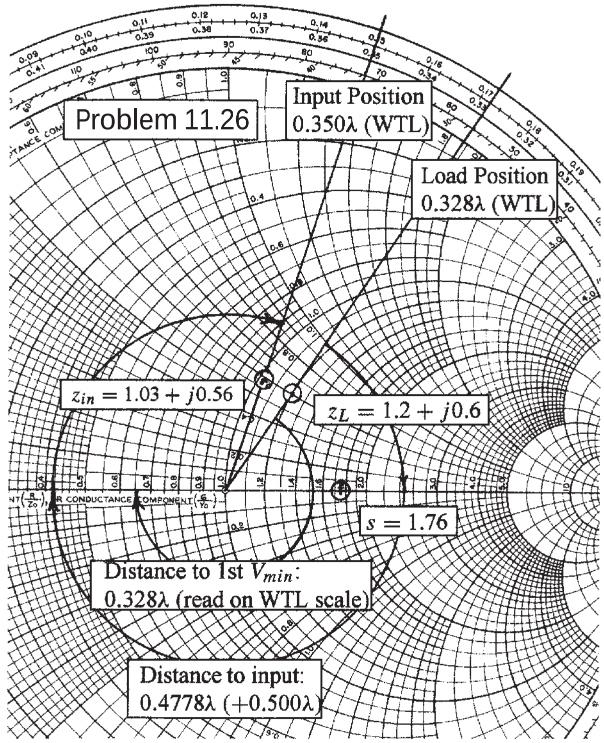
a) $|V|_{max} = 12$ V at $z = -5$ cm, and $vertV|_{min} = 5$ V: First, we know that the maximum and minimum voltages are spaced by $\lambda/4$. Since this distance is given as 25 cm, we see that $\lambda = 100$ cm $= 1$ m. Thus the maximum voltage location is $5/100 = 0.05\lambda$ in front of the load. The standing wave ratio is $s = |V|_{max}/|V|_{min} = 12/5 = 2.4$. We mark this on the positive real axis of the chart (see next page). The load position is now 0.05 wavelengths toward the load from the $|V|_{max}$ position, or at 0.30 $\lambda$ on the WTL scale. A line is drawn from the origin through this point on the chart, as shown. We next set the compass to the distance between the origin and the $z = r = 2.4$ point on the real axis. We then scribe this same distance along the line drawn through the .30 $\lambda$ position. The intersection is the value of $z_L$, which we read as $z_L = 1.65 + j.97$. The actual load impedance is then $Z_L = 300z_L = \underline{495 + j290\ \Omega}$.

b) $|V|_{max} = 17$ V at $z = -20$ cm, and $|V|_{min} = 0$. In this case the standing wave ratio is infinite, which puts the starting point on the $r \to \infty$ point on the chart. The distance of 20 cm corresponds to $20/100 = 0.20\,\lambda$, placing the load position at $0.45\,\lambda$ on the WTL scale. A line is drawn from the origin through this location on the chart. An infinite standing wave ratio places us on the outer boundary of the chart, so we read $z_L = j0.327$ at the $0.45\,\lambda$ WTL position. Thus $Z_L = j300(0.327) \doteq \underline{j98\,\Omega}$.



11.26. A lossless 50Ω transmission line operates with a velocity that is $3/4c$. A load, $Z_L = 60 + j30\,\Omega$ is located at $z = 0$. Use the Smith chart to find:

a) $s$: First we find the normalized load impedance, $z_L = (60 + j30)/50 = 1.2 + j0.6$, which is then marked on the chart (see below). Drawing a line from the chart center through this point yields its location at $0.328\lambda$ on the WTL scale. The distance from the origin to the load impedance point is now set on the compass; the standing wave ratio is then found by scribing this distance along the positive real axis, yielding $s = \underline{1.76}$, as shown. Alternately, use the $s$ scale at the bottom of the chart, setting the compass point at the center, and scribing the distance on the scale to the left.




b) the distance from the load to the nearest voltage minimum if $f = 300$ MHz: This distance is found by transforming the load impedance clockwise around the chart until the negative real axis is reached. This distance in wavelengths is just the load position on the WTL scale, since the starting point for this scale is the negative real axis. So the distance is $0.328\lambda$. The wavelength is

$$\lambda = \frac{v}{f} = \frac{(3/4)c}{300\text{MHz}} = \frac{3(3 \times 10^8)}{4(3 \times 10^8)} = 0.75\,\text{m}$$

So the actual distance to the first voltage minimum is $d_{min} = 0.328(0.75)\,\text{m} = \underline{24.6\,\text{cm}}$.

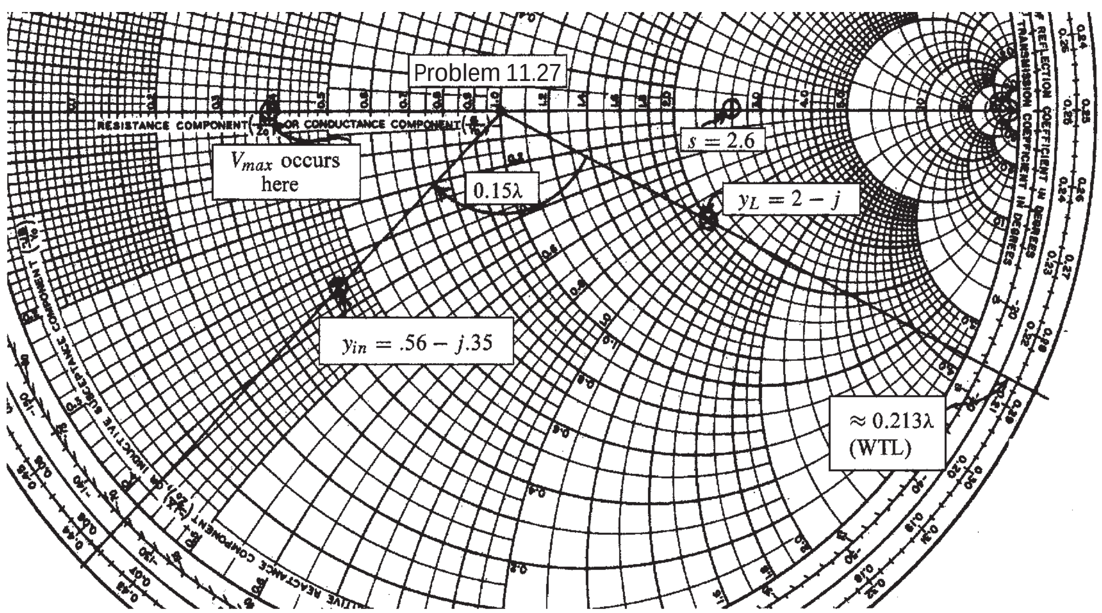
c) the input impedance if $f = 200$ MHz and the input is at $z = -110$cm: The wavelength at this frequency is $\lambda = (3/4)(3 \times 10^8)/(2 \times 10^8) = 1.125$ m. The distance to the input in wavelengths is then $d_{in} = (1.10)/(1.125) = 0.9778\lambda$. Transforming the load through this distance toward the generator involves revolution once around the chart ($0.500\lambda$) plus the remainder of $0.4778\lambda$, which leads to a final position of $0.1498\lambda \doteq 0.150\lambda$ on the WTG scale, or $0.350\lambda$ on the WTL scale. A line is drawn between this point and the chart center. Scribing the compass arc length through this line yields the normalized input impedance, read as $z_{in} = 1.03 + j0.56$. The actual input impedance is $Z_{in} = z_{in} \times 50 = \underline{51.5 + j28.0\,\Omega}$.

11.27. The characteristic admittance ($Y_0 = 1/Z_0$) of a lossless transmission line is 20 mS. The line is terminated in a load $Y_L = 40 - j20$ mS. Make use of the Smith chart to find:

a) $s$: We first find the normalized load admittance, which is $y_L = Y_L/Y_0 = 2 - j1$. This is plotted on the Smith chart below. We then set on the compass the distance between $y_L$ and the origin. The same distance is then scribed along the positive real axis, and the value of $s$ is read as 2.6.

b) $Y_{in}$ if $l = 0.15\,\lambda$: First we draw a line from the origin through $z_L$ and note its intersection with the WTG scale on the chart outer boundary. We note a reading on that scale of about 0.287 $\lambda$. To this we add 0.15 $\lambda$, obtaining about 0.437 $\lambda$, which we then mark on the chart (0.287 $\lambda$ is not the precise value, but I have added 0.15 $\lambda$ to that mark to obtain the point shown on the chart that is near to 0.437 $\lambda$. This "eyeballing" method increases the accuracy a little). A line drawn from the 0.437 $\lambda$ position on the WTG scale to the origin passes through the input admittance. Using the compass, we scribe the distance found in part $a$ across this line to find $y_{in} = 0.56 - j0.35$, or $Y_{in} = 20y_{in} = 11 - j7.0$ mS.

c) the distance in wavelengths from $Y_L$ to the nearest voltage maximum: On the admittance chart, the $V_{max}$ position is on the negative $\Gamma_r$ axis. This is at the zero position on the WTL scale. The load is at the approximate $0.213\,\lambda$ point on the WTL scale, so this distance is the one we want.

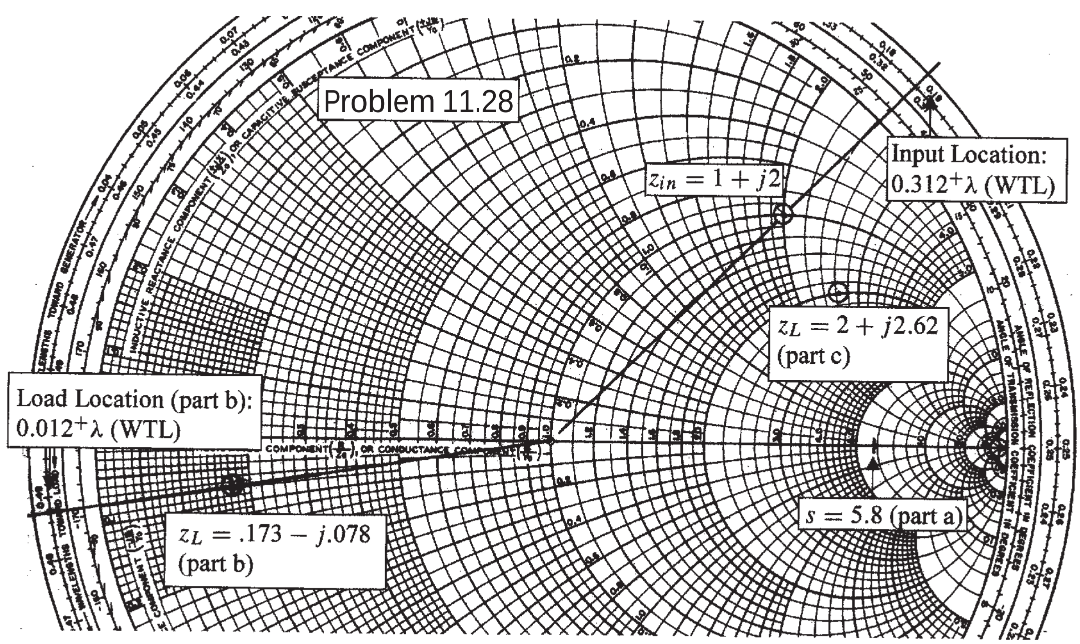


11.28. The wavelength on a certain lossless line is 10cm. If the normalized input impedance is $z_{in} = 1 + j2$, use the Smith chart to determine:

a) $s$: We begin by marking $z_{in}$ on the chart (see below), and setting the compass at its distance from the origin. We then use the compass at that setting to scribe a mark on the positive real axis, noting the value there of $s = \underline{5.8}$.

b) $z_L$, if the length of the line is 12 cm: First, use a straight edge to draw a line from the origin through $z_{in}$, and through the outer scale. We read the input location as slightly more than $0.312\lambda$ on the WTL scale (this additional distance beyond the .312 mark is not measured, but is instead used to add a similar distance when the impedance is transformed). The line length of 12cm corresponds to 1.2 wavelengths. Thus, to transform to the load, we go counter-clockwise twice around the chart, plus $0.2\lambda$, finally arriving at (again) slightly more than $0.012\lambda$ on the WTL scale. A line is drawn to the origin from that position, and the compass (with its previous setting) is scribed through the line. The intersection is the normalized load impedance, which we read as $z_L = \underline{0.173 - j0.078}$.

c) $x_L$, if $z_L = 2 + jx_L$, where $x_L > 0$. For this, use the compass at its original setting to scribe through the $r = 2$ circle in the upper half plane. At that point we read $x_L = \underline{2.62}$.



11.29. A standing wave ratio of 2.5 exists on a lossless 60 Ω line. Probe measurements locate a voltage minimum on the line whose location is marked by a small scratch on the line. When the load is replaced by a short circuit, the minima are 25 cm apart, and one minimum is located at a point 7 cm toward the source from the scratch. Find $Z_L$: We note first that the 25 cm separation between minima imply a wavelength of twice that, or $\lambda = 50$ cm. Suppose that the scratch locates the first voltage minimum. With the short in place, the first minimum occurs at the load, and the second at 25 cm in front of the load. The effect of replacing the short with the load is to move the minimum at 25 cm to a new location 7 cm toward the load, or at 18 cm. This is a possible location for the scratch, which would otherwise occur at multiples of a half-wavelength farther away from that point, toward the generator. Our assumed scratch position will be 18 cm or $18/50 = 0.36$ wavelengths from the load. Using the Smith chart (see below) we first draw a line from the origin through the $0.36\lambda$ point on the wavelengths toward load scale. We set the compass to the length corresponding to the $s = r = 2.5$ point on the chart, and then scribe this distance through the straight line. We read $z_L = 0.79 + j0.825$, from which $Z_L = 47.4 + j49.5\ \Omega$. As a check, I will do the problem analytically. First, we use

$$z_{min} = -18\,\text{cm} = -\frac{1}{2\beta}(\phi + \pi) \quad \Rightarrow \quad \phi = \left[\frac{4(18)}{50} - 1\right]\pi = 1.382\,\text{rad} = 79.2^\circ$$

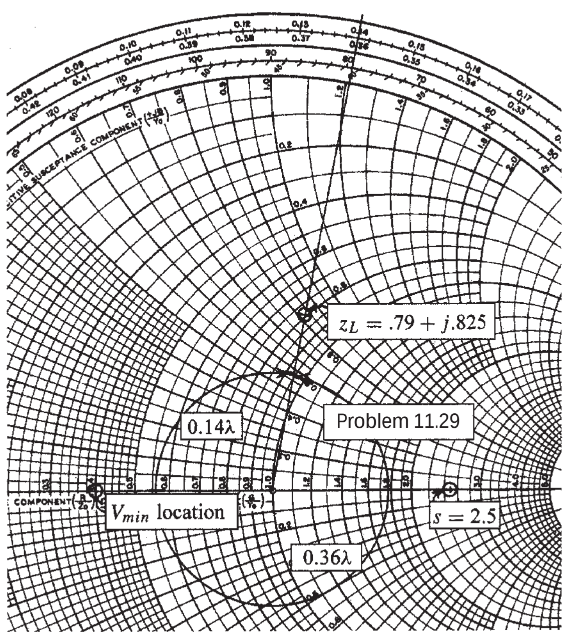
Now

$$|\Gamma_L| = \frac{s-1}{s+1} = \frac{2.5-1}{2.5+1} = 0.4286$$

and so $\Gamma_L = 0.4286\angle 1.382$. Using this, we find

$$z_L = \frac{1+\Gamma_L}{1-\Gamma_L} = 0.798 + j0.823$$

and thus $Z_L = z_L(60) = \underline{47.8 + j49.3\ \Omega}$.

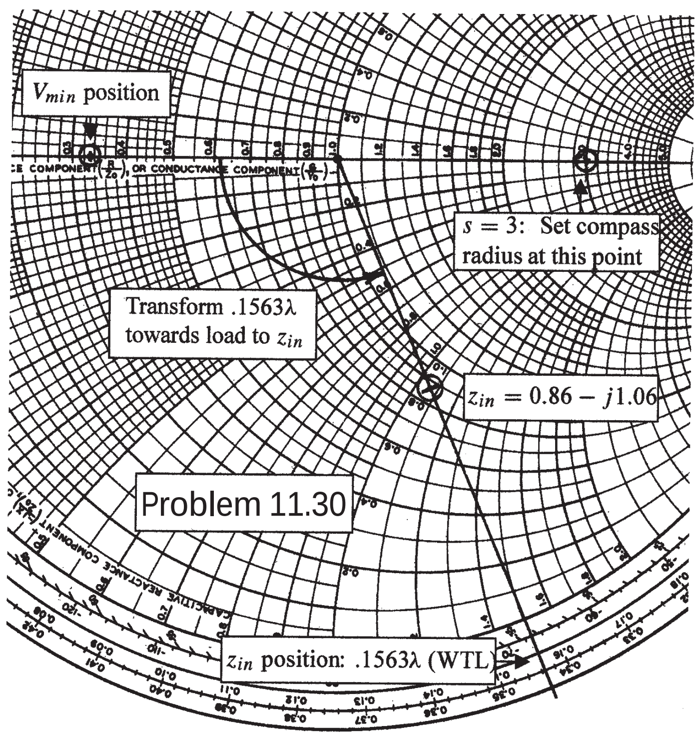


11.30. A 2-wire line, constructed of lossless wire of circular cross-section is gradually flared into a coupling loop that looks like an egg beater. At the point $X$, indicated by the arrow in Fig. 11.34, a short circuit is placed across the line. A probe is moved along the line and indicates that the first voltage minimum to the left of $X$ is 16cm from $X$. With the short circuit removed, a voltage minimum is found 5cm to the left of $X$, and a voltage maximum is located that is 3 times voltage of the minimum. Use the Smith chart to determine:

a) $f$: No Smith chart is needed to find $f$, since we know that the first voltage minimum in front of a short circuit is one-half wavelength away. Therefore, $\lambda = 2(16) = 32$cm, and (assuming an air-filled line), $f = c/\lambda = 3 \times 10^8/0.32 = \underline{0.938\,\text{GHz}}$.

b) $s$: Again, no Smith chart is needed, since $s$ is the ratio of the maximum to the minimum voltage amplitudes. Since we are given that $V_{max} = 3V_{min}$, we find $s = \underline{3}$.

c) the normalized input impedance of the egg beater as seen looking the right at point $X$: Now we need the chart. From the figure below, $s = 3$ is marked on the positive real axis, which determines the compass radius setting. This point is then transformed, using the compass, to the negative real axis, which corresponds to the location of a voltage minimum. Since the first $V_{min}$ is 5cm in front of $X$, this corresponds to $(5/32)\lambda = 0.1563\lambda$ to the left of $X$. On the chart, we now move this distance from the $V_{min}$ location toward the load, using the WTL scale. A line is drawn from the origin through the $0.1563\lambda$ mark on the WTL scale, and the compass is used to scribe the original radius through this line. The intersection is the normalized input impedance, which is read as $z_{in} = \underline{0.86 - j1.06}$.



11.31. In order to compare the relative sharpness of the maxima and minima of a standing wave, assume a load $z_L = 4 + j0$ is located at $z = 0$. Let $|V|_{min} = 1$ and $\lambda = 1$ m. Determine the width of the

a) minimum, where $|V| < 1.1$: We begin with the general phasor voltage in the line:

$$V(z) = V^+(e^{-j\beta z} + \Gamma e^{j\beta z})$$

With $z_L = 4 + j0$, we recognize the real part as the standing wave ratio. Since the load impedance is real, the reflection coefficient is also real, and so we write

$$\Gamma = |\Gamma| = \frac{s-1}{s+1} = \frac{4-1}{4+1} = 0.6$$

The voltage magnitude is then

$$\begin{aligned} |V(z)| &= \sqrt{V(z)V^*(z)} = V^+ \left[(e^{-j\beta z} + \Gamma e^{j\beta z})(e^{j\beta z} + \Gamma e^{-j\beta z})\right]^{1/2} \\ &= V^+ \left[1 + 2\Gamma\cos(2\beta z) + \Gamma^2\right]^{1/2} \end{aligned}$$

Note that with $\cos(2\beta z) = \pm 1$, we obtain $|V| = V^+(1 \pm \Gamma)$ as expected. With $s = 4$ and with $|V|_{min} = 1$, we find $|V|_{max} = 4$. Then with $\Gamma = 0.6$, it follows that $V^+ = 2.5$. The net expression for $|V(z)|$ is then

$$V(z) = 2.5\sqrt{1.36 + 1.2\cos(2\beta z)}$$

To find the width in $z$ of the voltage minimum, defined as $|V| < 1.1$, we set $|V(z)| = 1.1$ and solve for $z$: We find

$$\left(\frac{1.1}{2.5}\right)^2 = 1.36 + 1.2\cos(2\beta z) \quad \Rightarrow \quad 2\beta z = \cos^{-1}(-0.9726)$$

Thus $2\beta z = 2.904$. At this stage, we note the the $|V|_{min}$ point will occur at $2\beta z = \pi$. We therefore compute the range, $\Delta z$, over which $|V| < 1.1$ through the equation:

$$2\beta(\Delta z) = 2(\pi - 2.904) \quad \Rightarrow \quad \Delta z = \frac{\pi - 2.904}{2\pi/1} = 0.0378\,\text{m} = \underline{3.8\,\text{cm}}$$

where $\lambda = 1$ m has been used.

b) Determine the width of the maximum, where $|V| > 4/1.1$: We use the same equation for $|V(z)|$, which in this case reads:

$$4/1.1 = 2.5\sqrt{1.36 + 1.2\cos(2\beta z)} \quad \Rightarrow \quad \cos(2\beta z) = 0.6298$$

Since the maximum corresponds to $2\beta z = 0$, we find the range through

$$2\beta\Delta z = 2\cos^{-1}(0.6298) \quad \Rightarrow \quad \Delta z = \frac{0.8896}{2\pi/1} = 0.142\,\text{m} = \underline{14.2\,\text{cm}}$$

11.32. A lossless line is operating with $Z_0 = 40\,\Omega$, $f = 20$ MHz, and $\beta = 7.5\pi$ rad/m. With a short circuit replacing the load, a minimum is found at a point on the line marked by a small spot of puce paint. With the load installed, it is found that $s = 1.5$ and a voltage minimum is located 1m toward the source from the puce dot.

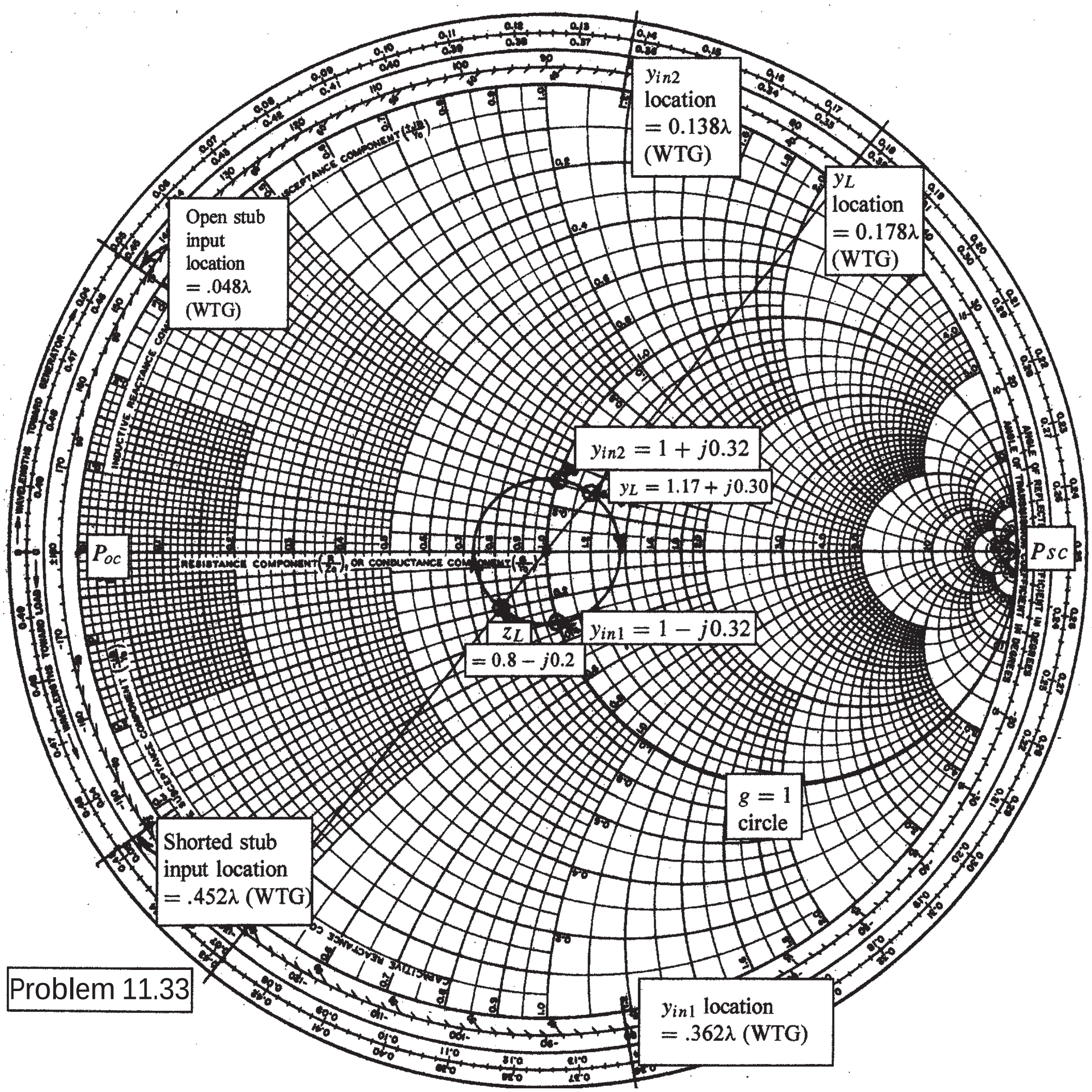
a) Find $Z_L$: First, the wavelength is given by $\lambda = 2\pi/\beta = 2/7.5 = 0.2667$m. The 1m distance is therefore $3.75\lambda$. With the short installed, the $V_{min}$ positions will be at multiples of $\lambda/2$ to the left of the short. Therefore, with the actual load installed, the $V_{min}$ position as stated would be $3.75\lambda + n\lambda/2$, *which means that a maximum voltage occurs at the load.* This being the case, the normalized load impedance will lie on the positive real axis of the Smith chart, and will be equal to the standing wave ratio. Therefore, $Z_L = 40(1.5) = \underline{60\,\Omega}$.

b) What load would produce $s = 1.5$ with $|V|_{max}$ at the paint spot? With $|V|_{max}$ at the paint spot and with the spot an integer multiple of $\lambda/2$ to the left of the load, $|V|_{max}$ must also occur at the load. The answer is therefore the same as part a, or $Z_L = \underline{60\,\Omega}$.

11.33. In Fig. 11.17, let $Z_L = 40 - j10\ \Omega$, $Z_0 = 50\ \Omega$, $f = 800$ MHz, and $v = c$.

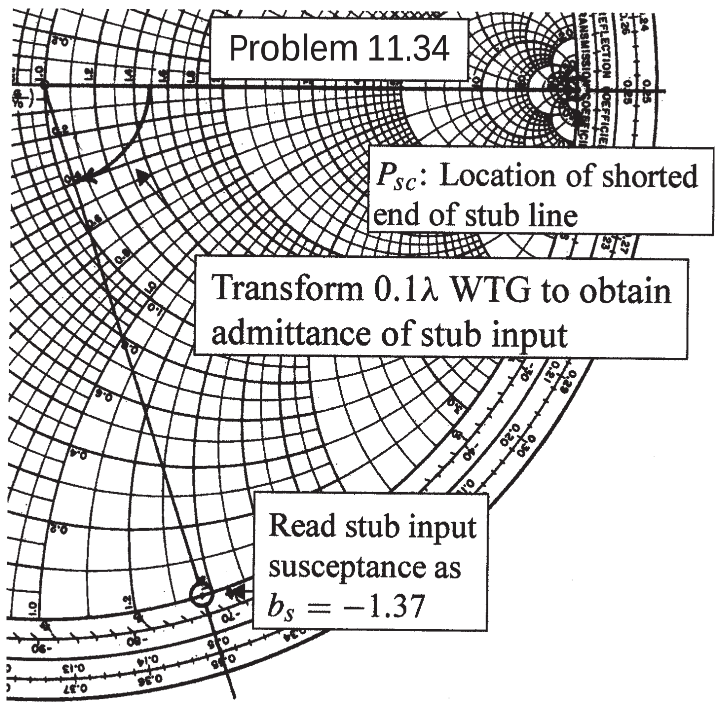
a) Find the shortest length, $d_1$, of a short-circuited stub, and the shortest distance $d$ that it may be located from the load to provide a perfect match on the main line to the left of the stub: The Smith chart construction is shown on the next page. First we find $z_L = (40 - j10)/50 = 0.8 - j0.2$ and plot it on the chart. Next, we find $y_L = 1/z_L$ by transforming this point halfway around the chart, where we read $y_L = 1.17 + j0.30$. This point is to be transformed to a location at which the real part of the normalized admittance is unity. The $g = 1$ circle is highlighted on the chart; $y_L$ transforms to two locations on it: $y_{in1} = 1 - j0.32$ and $y_{in2} = 1 + j0.32$. The stub is connected at either of these two points. The stub input admittance must cancel the imaginary part of the line admittance at that point. If $y_{in2}$ is chosen, the stub must have input admittance of $-j0.32$. This point is marked on the outer circle and occurs at $0.452\,\lambda$ on the WTG scale. The length of the stub is found by computing the distance between its input, found above, and the short-circuit position (stub load end), marked as $P_{sc}$. This distance is $d_1 = (0.452 - 0.250)\lambda = 0.202\,\lambda$. With $f = 800$ MHz and $v = c$, the wavelength is $\lambda = (3 \times 10^8)/(8 \times 10^8) = 0.375$ m. The distance is thus $d_1 = (0.202)(0.375) = 0.758\,\text{m} = \underline{7.6\,\text{cm}}$. This is the shortest of the two possible stub lengths, since if we had used $y_{in1}$, we would have needed a stub input admittance of $+j0.32$, which would have required a longer stub length to realize. The length of the main line between its load and the stub attachment point is found on the chart by measuring the distance between $y_L$ and $y_{in2}$, in moving clockwise (toward generator). This distance will be $d = [0.500 - (0.178 - 0.138)]\,\lambda = 0.46\,\lambda$. The actual length is then $d = (0.46)(0.375) = 0.173\text{m} = \underline{17.3\,\text{cm}}$.

11.33b) Repeat for an open-circuited stub: In this case, everything is the same, except for the load-end position of the stub, which now occurs at the $P_{oc}$ point on the chart. To use the shortest possible stub, we need to use $y_{in1} = 1 - j0.32$, requiring $y_s = +j0.32$. We find the stub length by moving from $P_{oc}$ to the point at which the admittance is $j0.32$. This occurs at $0.048\,\lambda$ on the WTG scale, which thus determines the required stub length. Now $d_1 = (0.048)(0.375) = 0.18\,\text{m} = \underline{1.8\,\text{cm}}$. The attachment point is found by transforming $y_L$ to $y_{in1}$, where the former point is located at $0.178\,\lambda$ on the WTG scale, and the latter is at $0.362\,\lambda$ on the same scale. The distance is then $d = (0.362 - 0.178)\lambda = 0.184\lambda$. The actual length is $d = (0.184)(0.375) = 0.069\,\text{m} = \underline{6.9\,\text{cm}}$.



Problem 11.33

11.34. The lossless line shown in Fig. 11.35 is operating with $\lambda = 100$cm. If $d_1 = 10$cm, $d = 25$cm, and the line is matched to the left of the stub, what is $Z_L$? For the line to be matched, it is required that the sum of the normalized input admittances of the shorted stub and the main line at the point where the stub is connected be unity. So the input susceptances of the two lines must cancel. To find the stub input susceptance, use the Smith chart to transform the short circuit point $0.1\lambda$ toward the generator, and read the input value as $b_s = -1.37$ (note that the stub length is one-tenth of a wavelength). The main line input admittance must now be $y_{in} = 1 + j1.37$. This line is one-quarter wavelength long, so the normalized load impedance is equal to the normalized input admittance. Thus $z_L = 1 + j1.37$, so that $Z_L = 300z_L = \underline{300 + j411\ \Omega}$.

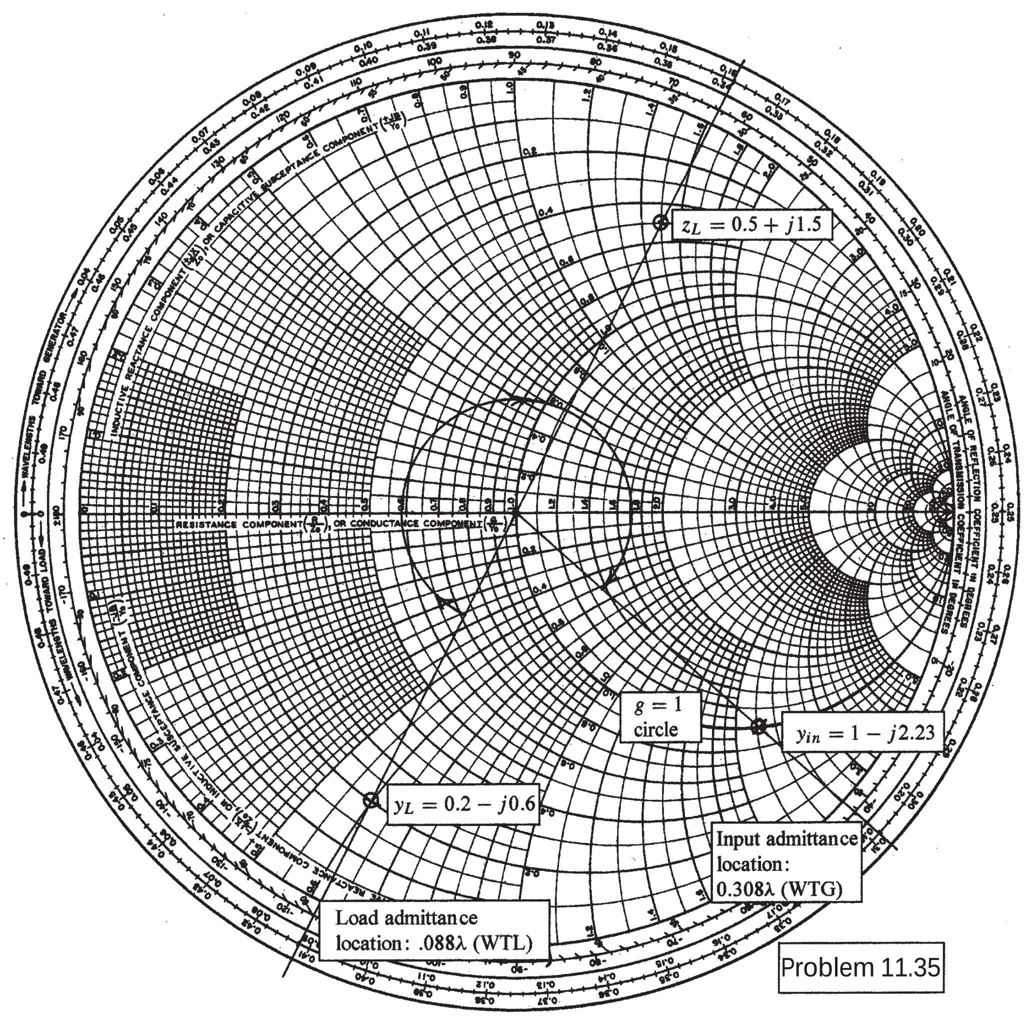


11.35. A load, $Z_L = 25 + j75\ \Omega$, is located at $z = 0$ on a lossless two-wire line for which $Z_0 = 50\ \Omega$ and $v = c$.

a) If $f = 300$ MHz, find the shortest distance $d$ ($z = -d$) at which the input impedance has a real part equal to $1/Z_0$ and a negative imaginary part: The Smith chart construction is shown below. We begin by calculating $z_L = (25 + j75)/50 = 0.5 + j1.5$, which we then locate on the chart. Next, this point is transformed by rotation halfway around the chart to find $y_L = 1/z_L = 0.20 - j0.60$, which is located at 0.088 $\lambda$ on the WTL scale. This point is then transformed toward the generator until it intersects the $g = 1$ circle (shown highlighted) with a negative imaginary part. This occurs at point $y_{in} = 1.0 - j2.23$, located at 0.308 $\lambda$ on the WTG scale. The total distance between load and input is then $d = (0.088 + 0.308)\lambda = 0.396\lambda$. At 300 MHz, and with $v = c$, the wavelength is $\lambda = 1$ m. Thus the distance is $d = 0.396\,\text{m} = \underline{39.6\,\text{cm}}$.

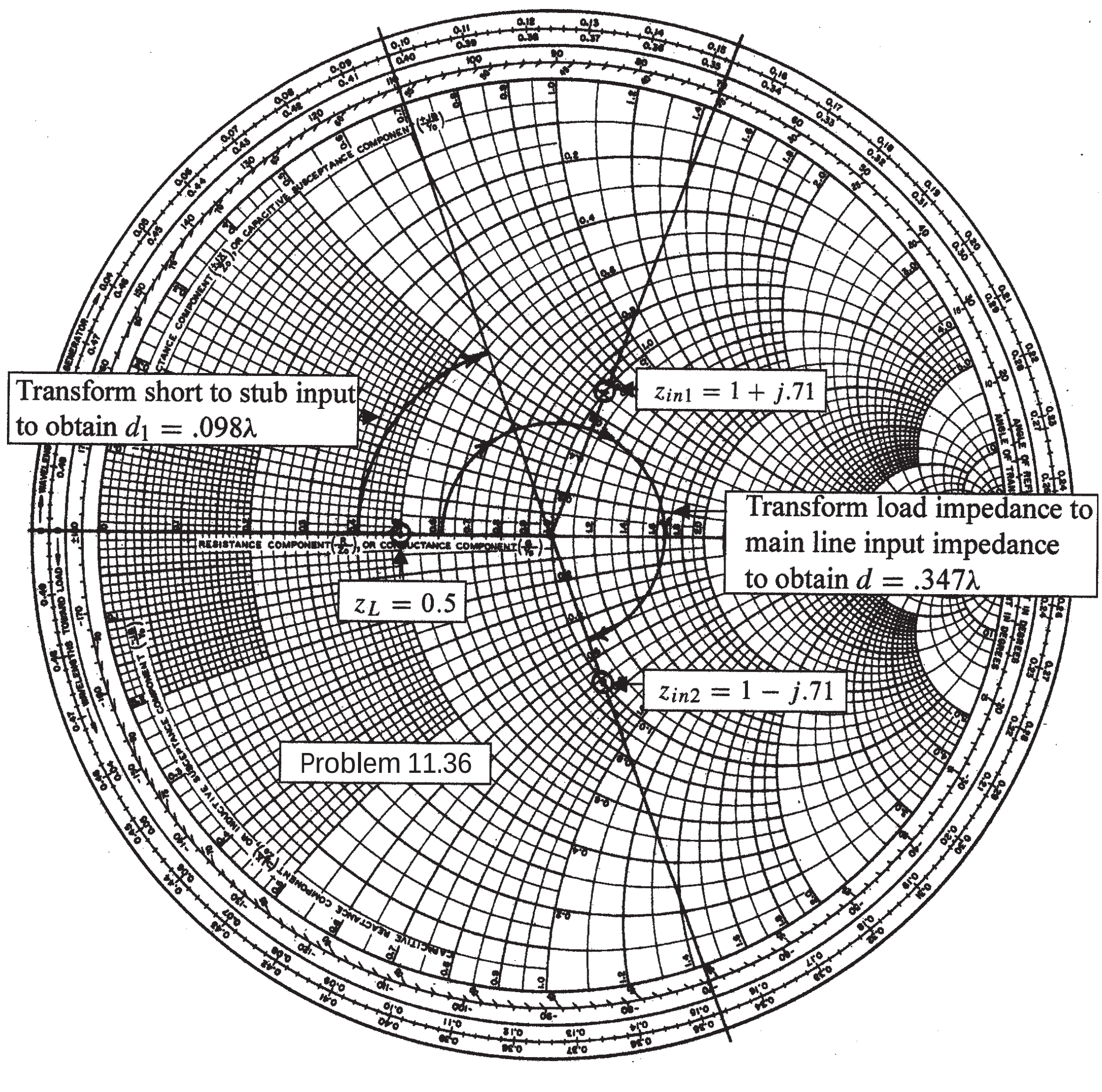
b) What value of capacitance $C$ should be connected across the line at that point to provide unity standing wave ratio on the remaining portion of the line? To cancel the input normalized susceptance of -2.23, we need a capacitive normalized susceptance of +2.23. We therefore write

$$\omega C = \frac{2.23}{Z_0} \quad \Rightarrow \quad C = \frac{2.23}{(50)(2\pi \times 3 \times 10^8)} = 2.4 \times 10^{-11}\ \text{F} = \underline{24\ \text{pF}}$$




Problem 11.35

11.36. The two-wire lines shown in Fig. 11.36 are all lossless and have $Z_0 = 200\,\Omega$. Find $d$ and the shortest possible value for $d_1$ to provide a matched load if $\lambda = 100$cm. In this case, we have a series combination of the loaded line section and the shorted stub, so we use impedances and the Smith chart as an impedance diagram. The requirement for matching is that the total normalized impedance at the junction (consisting of the sum of the input impedances to the stub and main loaded section) is unity. First, we find $z_L = 100/200 = 0.5$ and mark this on the chart (see below). We then transform this point toward the generator until we reach the $r = 1$ circle. This happens at two possible points, indicated as $z_{in1} = 1 + j.71$ and $z_{in2} = 1 - j.71$. The stub input impedance must cancel the imaginary part of the loaded section input impedance, or $z_{ins} = \pm j.71$. The shortest stub length that accomplishes this is found by transforming the short circuit point on the chart to the point $z_{ins} = +j0.71$, which yields a stub length of $d_1 = .098\lambda = \underline{9.8\,\text{cm}}$. The length of the loaded section is then found by transforming $z_L = 0.5$ to the point $z_{in2} = 1 - j.71$, so that $z_{ins} + z_{in2} = 1$, as required. This transformation distance is $d = 0.347\lambda = \underline{37.7\,\text{cm}}$.



11.37. In the transmission line of Fig. 11.20, $R_L = Z_0 = 50\ \Omega$. Determine and plot the voltage at the load resistor and the current in the battery as functions of time by constructing appropriate voltage and current reflection diagrams: Referring to the figure, closing the switch launches a voltage wave whose value is given by Eq. (50):

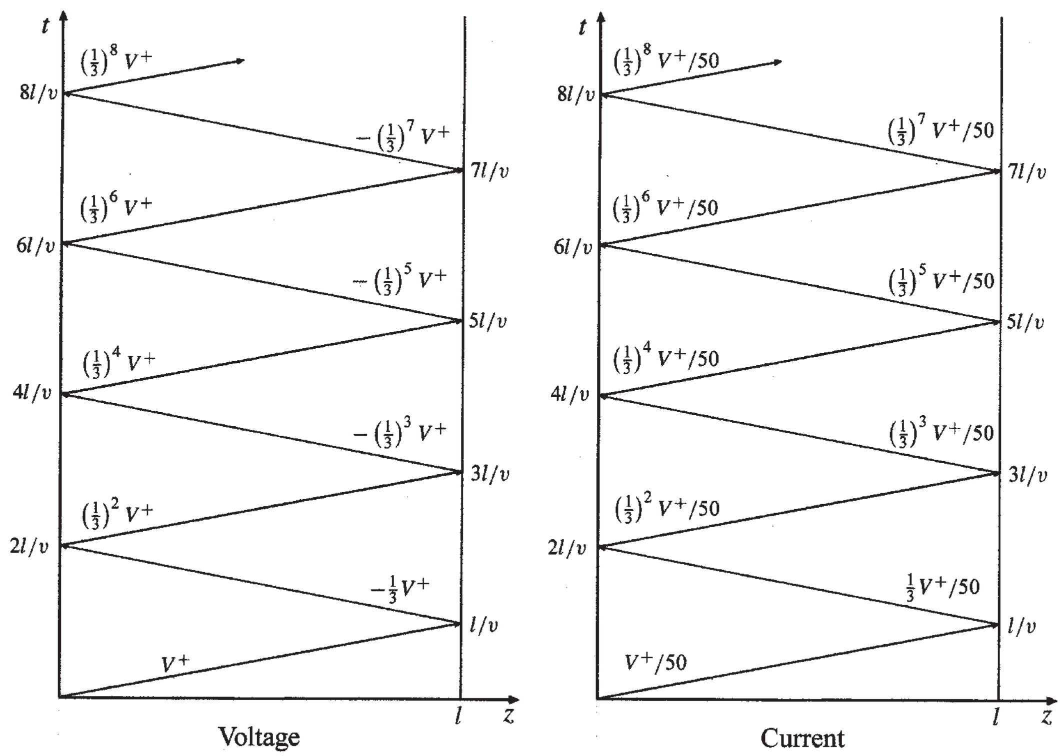
$$V_1^+ = \frac{V_0 Z_0}{R_g + Z_0} = \frac{50}{75} V_0 = \frac{2}{3} V_0$$

We note that $\Gamma_L = 0$, since the load impedance is matched to that of the line. So the voltage wave traverses the line and does not reflect. The voltage reflection diagram would be that shown in Fig. 11.21a, except that no waves are present after time $t = l/v$. Likewise, the current reflection diagram is that of Fig. 11.22a, except, again, no waves exist after $t = l/v$. The voltage at the load will be just $V_1^+ = (2/3)V_0$ for times beyond $l/v$. The current through the battery is found through

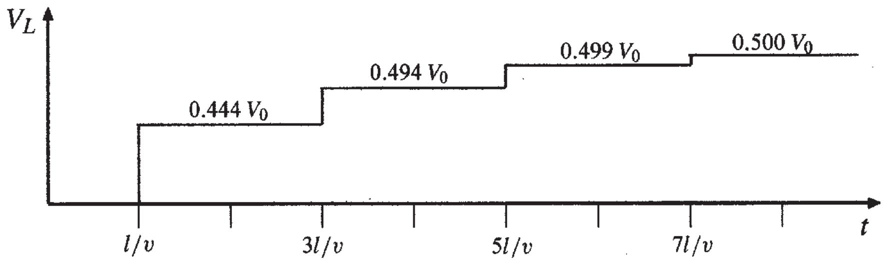
$$I_1^+ = \frac{V_1^+}{Z_0} = \frac{V_0}{75}\ \text{A}$$

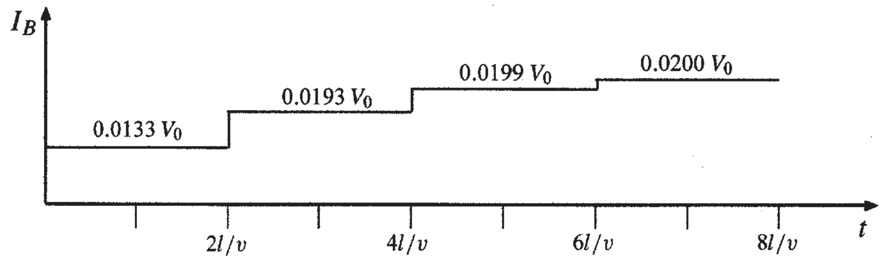
This current initiates at $t = 0$, and continues indefinitely.

11.38. Repeat Problem 37, with $Z_0 = 50\Omega$, and $R_L = R_g = 25\Omega$. Carry out the analysis for the time period $0 < t < 8l/v$. At the generator end, we have $\Gamma_g = -1/3$, as before. The difference is at the load end, where $\Gamma_L = -1/3$, whereas in Problem 37, the load was matched. The initial wave, as in the last problem, is of magnitude $V^+ = (2/3)V_0$. Using these values, voltage and current reflection diagrams are constructed, and are shown below:



11.38. (continued) From the diagrams, voltage and current plots are constructed. First, the load voltage is found by adding voltages along the right side of the voltage diagram at the indicated times. Second, the current through the battery is found by adding currents along the left side of the current reflection diagram. Both plots are shown below, where currents and voltages are expressed to three significant figures. The steady state values, $V_L = 0.5$V and $I_B = 0.02$A, are expected as $t \to \infty$.



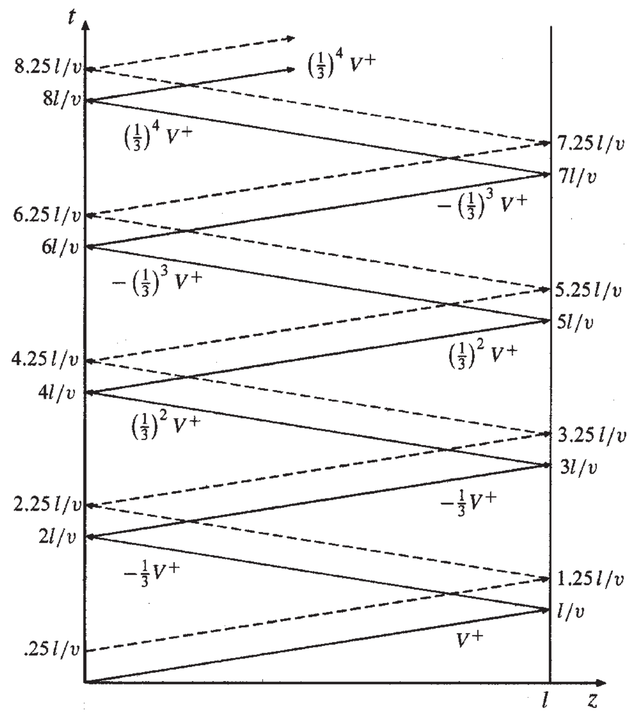

11.39. In the transmission line of Fig. 11.20, $Z_0 = 50\,\Omega$ and $R_L = R_g = 25\,\Omega$. The switch is closed at $t = 0$ and *is opened again* at time $t = l/4v$, thus creating a rectangular voltage *pulse* in the line. Construct an appropriate voltage reflection diagram for this case and use it to make a plot of the voltage at the load resistor as a function of time for $0 < t < 8l/v$ (note that the effect of opening the switch is to initiate a second voltage wave, whose value is such that it leaves a net current of zero in its wake): The value of the initial voltage wave, formed by closing the switch, will be

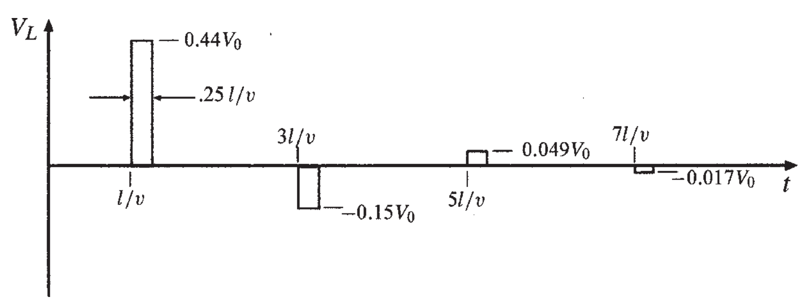
$$V^+ = \frac{Z_0}{R_g + Z_0} V_0 = \frac{50}{25 + 50} V_0 = \frac{2}{3} V_0$$

On opening the switch, a second wave, $V^{+\prime}$, is generated which leaves a net current behind it of zero. This means that $V^{+\prime} = -V^+ = -(2/3)V_0$. Note also that when the switch is opened, the reflection coefficient at the generator end of the line becomes unity. The reflection coefficient at the load end is $\Gamma_L = (25 - 50)/(25 + 50) = -(1/3)$. The reflection diagram is now constructed in the usual manner, and is shown on the next page. The path of the second wave as it reflects from either end is shown in dashed lines, and is a replica of the first wave path, displaced later in time by $l/(4v)$.a All values for the second wave after each reflection are equal but of opposite sign to the immediately preceding first wave values. The load voltage as a function of time is found by accumulating voltage values as they are read moving up along the right hand boundary of the chart. The resulting function, plotted just below the reflection diagram, is found to be a sequence of pulses that alternate signs. The pulse amplitudes are calculated as follows:

11.39. (continued)

$$
\begin{aligned}
\frac{l}{v} < t < \frac{5l}{4v}: &\quad V_1 = \left(1 - \frac{1}{3}\right) V^+ = 0.44\,V_0 \\
\frac{3l}{v} < t < \frac{13l}{4v}: &\quad V_2 = -\frac{1}{3}\left(1 - \frac{1}{3}\right) V^+ = -0.15\,V_0 \\
\frac{5l}{v} < t < \frac{21l}{4v}: &\quad V_3 = \left(\frac{1}{3}\right)^2 \left(1 - \frac{1}{3}\right) V^+ = 0.049\,V_0 \\
\frac{7l}{v} < t < \frac{29l}{4v}: &\quad V_4 = -\left(\frac{1}{3}\right)^3 \left(1 - \frac{1}{3}\right) V^+ = -0.017\,V_0
\end{aligned}
$$

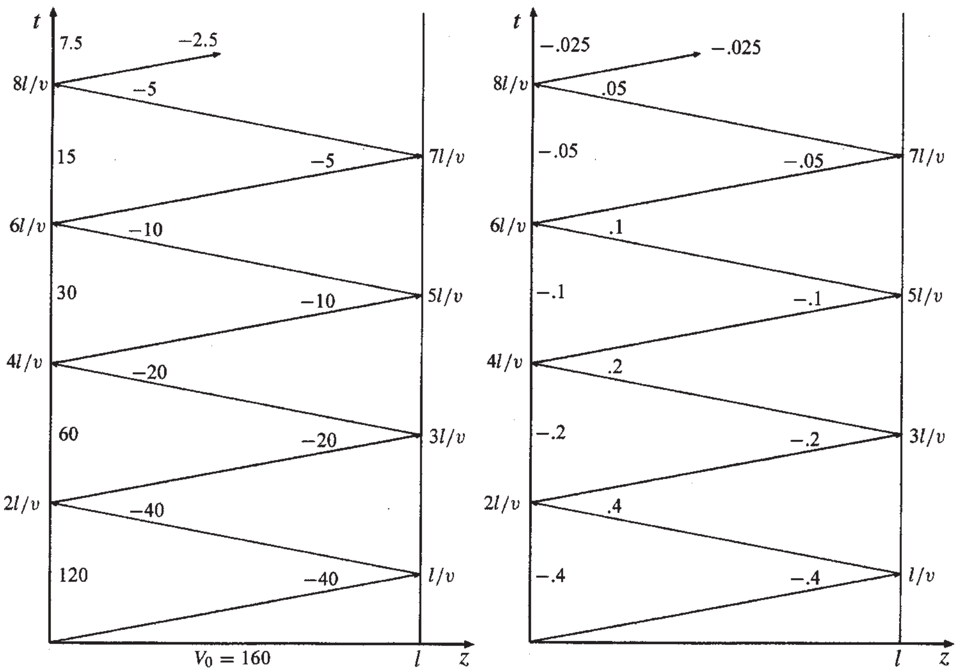


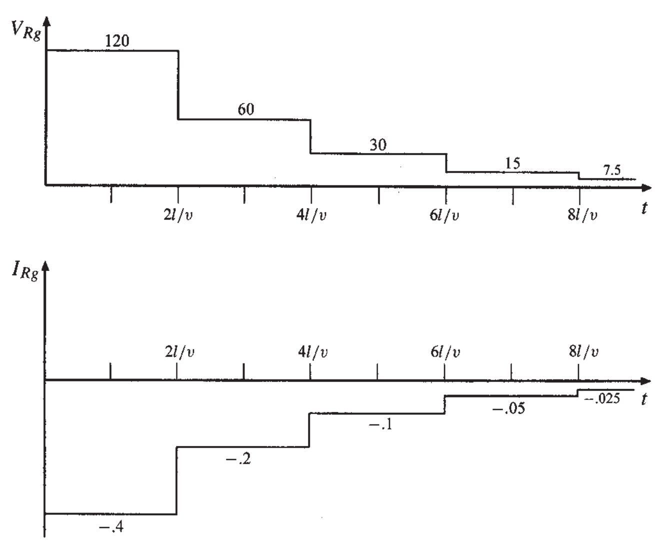
11.40. In the charged line of Fig. 11.25, the characteristic impedance is $Z_0 = 100\Omega$, and $R_g = 300\Omega$. The line is charged to initial voltage $V_0 = 160$ V, and the switch is closed at $t = 0$. Determine and plot the voltage and current through the resistor for time $0 < t < 8l/v$ (four round trips). This problem accompanies Example 13.6 as the other special case of the basic charged line problem, in which now $R_g > Z_0$. On closing the switch, the initial voltage wave is

$$V^+ = -V_0 \frac{Z_0}{R_g + Z_0} = -160 \frac{100}{400} = -40\,\text{V}$$

Now, with $\Gamma_g = 1/2$ and $\Gamma_L = 1$, the voltage and current reflection diagrams are constructed as shown below. Plots of the voltage and current at the resistor are then found by accumulating values from the left sides of the two charts, producing the plots as shown.

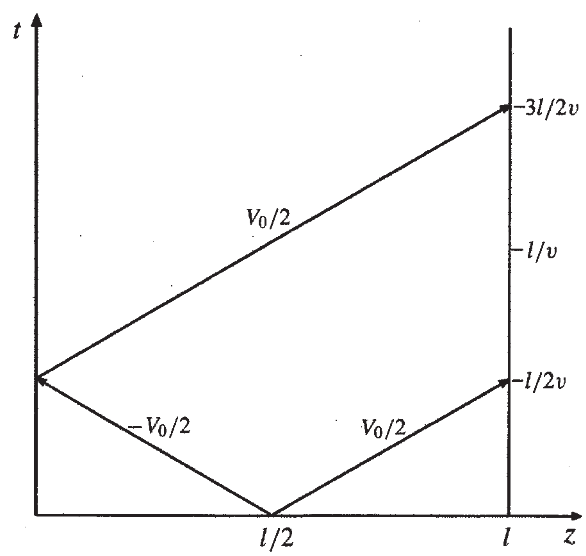


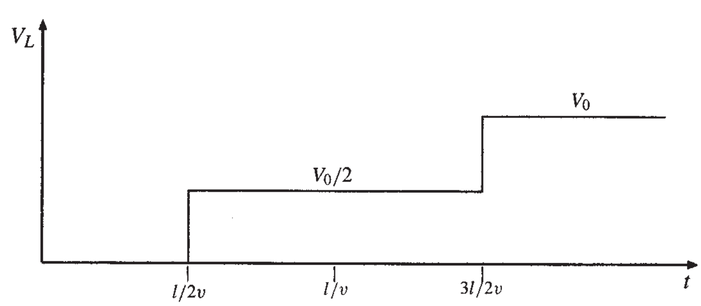

Voltage          Current



11.41. In the transmission line of Fig. 11.37, the switch is located *midway* down the line, and is closed at $t = 0$. Construct a voltage reflection diagram for this case, where $R_L = Z_0$. Plot the load resistor voltage as a function of time: With the left half of the line charged to $V_0$, closing the switch initiates (at the switch location) *two* voltage waves: The first is of value $-V_0/2$ and propagates toward the left; the second is of value $V_0/2$ and propagates toward the right. The backward wave reflects at the battery with $\Gamma_g = -1$. No reflection occurs at the load end, since the load is matched to the line. The reflection diagram and load voltage plot are shown below. The results are summarized as follows:

$$
\begin{aligned}
0 < t < \frac{l}{2v} &: \quad V_L = 0 \\
\frac{l}{2v} < t < \frac{3l}{2v} &: \quad V_L = \frac{V_0}{2} \\
t > \frac{3l}{2v} &: \quad V_L = V_0
\end{aligned}
$$



11.42. A simple *frozen wave generator* is shown in Fig. 11.38. Both switches are closed simultaneously at $t = 0$. Construct an appropriate voltage reflection diagram for the case in which $R_L = Z_0$. Determine and plot the load voltage as a function of time: Closing the switches sets up a total of four voltage waves as shown in the diagram below. Note that the first and second waves from the left are of magnitude $V_0$, since in fact we are superimposing voltage waves from the $-V_0$ and $+V_0$ charged sections acting alone. The reflection diagram is drawn and is used to construct the load voltage with time by accumulating voltages up the right hand vertical axis.