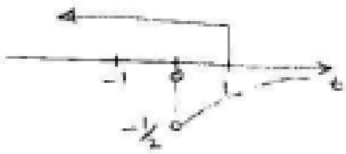
مسائل نمونه فصل دوم سیگنال ها و سیستم ها دانشگاه آزاد اسلامی – واحد تهران جنوب غفرانی

– پاسخ ضربه سیستم [illegible] زمان پیوسته داده شده است. تعیین کنید آیا سیستم علی است ؟
آیا سیستم پایدار است ؟

$$- h(t) = u(1-t) - \frac{1}{2} e^{-t} u(t)$$

$1-t \geq 0$
$t \leq 1$

$t \geq 0$



$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |$$

جمله دوم در پاسخ ضربه کراندار است در حالیکه جمله اول به ازاء $(-\infty)$ نامحدود می‌شود
پس سیستم پایدار نیست. سیستم غیر علی

$x(t) = u(t) \Rightarrow y(t) = ?$
bounded unbounded

علی - پایدار

$$- h(t) = e^{15t} \left[u(t-1) - u(t-100) \right]$$

$t \geq 1$ $t \geq 100$

Question 2:

A linear time invariant system responds the following inputs with the corresponding outputs:

If $x(t) = u(t)$ then $y(t) = (1 - e^{-2t})u(t)$

If $x(t) = \cos(2t)$ then $y(t) = 0.707\cos(2t - \pi/4)$

What will the output y(t) be for the following x(t):

(a) $x(t) = \delta(t)$

(b) $x(t) = tu(t)$

Hint: What relationship exists between the inputs?

$$u(t) \longrightarrow \boxed{LTI} \longrightarrow (1-e^{-2t})u(t)$$

$$\cos(2t) \longrightarrow \boxed{LTI} \longrightarrow 0.707\cos(2t - \tfrac{\pi}{4})$$

a) $x(t) = \delta(t)$ $\qquad h(t)\Big|_{x(t)=\delta(t)} = \dfrac{d}{dt}S(t) \qquad , \quad S(t) = y(t)\Big|_{x(t)=u(t)}$

در سیستم LTI پاسخ پله برابر انتگرال پاسخ ضربه و پاسخ ضربه برابر مشتق پاسخ پله است.

$$h(t)\Big|_{x(t)=\delta(t)} = \frac{d}{dt}(1-e^{-2t})u(t) = 2e^{-2t}u(t) + (1-e^{-2t})\delta(t) = 2e^{-2t}u(t)$$

b) $x(t) = tu(t) = r(t) = \displaystyle\int_{-\infty}^{t} u(\lambda)\,d\lambda$

$$y(t)\Big|_{x(t)=tu(t)} = \int_{-\infty}^{t} S(\lambda)\,d\lambda \qquad , \quad S(t) = y(t)\Big|_{x(t)=u(t)}$$

$$= \int_{-\infty}^{t} (1-e^{-2\lambda})u(\lambda)\,d\lambda$$

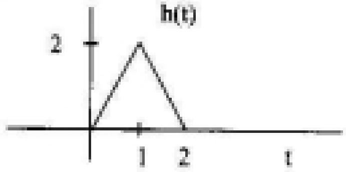
$y(t) = 0 \quad , t < 0$

$y(t) = \displaystyle\int_{0}^{t} (1-e^{-2\lambda})\,d\lambda = \lambda + \frac{1}{2}e^{-2\lambda}\Big|_{0}^{t} = t + \frac{1}{2}e^{-2t} - \frac{1}{2} \quad , t \geq 0$

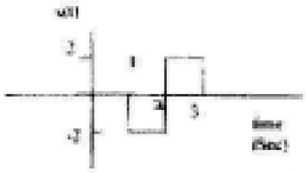
(8 marks)

$$y(t)\Big|_{x(t)=tu(t)} = \left(t + \frac{1}{2}e^{-2t} - \frac{1}{2}\right)u(t)$$

5. Determine $y(t) = x(t)*h(t)$ where $x(t) = u(t)$ and



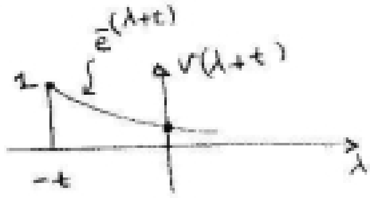
6. Compute $x(t)*v(t)$



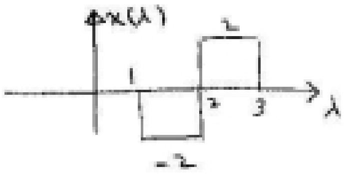
5. $y(t)=u(t)*h(t)=\int_{-\infty}^{\infty} u(\lambda)h(t-\lambda)\,d\lambda=\int_{0}^{\infty} h(t-\lambda)\,d\lambda \qquad t-\lambda=\lambda$

$$=\int_{t}^{-\infty} h(\lambda)\cdot(-d\lambda)=\int_{-\infty}^{t} h(\lambda)\,d\lambda=\begin{cases} 0 & t<0 \\ \int_0^t 2\lambda\,d\lambda & 0\leq t<1 \\ \int_0^1 2\lambda\,d\lambda+\int_1^t -2(\lambda-2)\,d\lambda & 1\leq t<2 \\ \int_0^1 2\lambda\,d\lambda+\int_1^2 -2(\lambda-2)\,d\lambda & t\geq 2 \end{cases}$$

$$y(t)=\begin{cases} 0, & t<0 \\ \lambda^2\big|_0^t=t^2 & 0\leq t<1 \\ \lambda^2\big|_0^1-(\lambda-2)^2\big|_1^t=1-(t-2)^2+(-1)^2=2-(t-2)^2=-t^2+4t-2, & 1\leq t<2 \\ \lambda^2\big|_0^1-(\lambda-2)^2\big|_1^2=1+1=2, & t\geq 2 \end{cases}$$



6. $y(t)=\int_{-\infty}^{\infty} x(\lambda)\,v(t-\lambda)\,d\lambda$



$t<1 \quad * = 0$

$1\leq t<2 \quad * = -2\int_1^t e^{\lambda-t}\,d\lambda$

$2\leq t<3 \quad * = -2\int_1^2 e^{\lambda-t}+2\int_2^t e^{\lambda-t}\,d\lambda$

$t\geq 3 \quad * = -2\int_1^2 e^{\lambda-t}\,d\lambda+2\int_2^3 e^{\lambda-t}\,d\lambda$

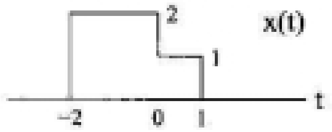
$t<1 \quad * = 0$

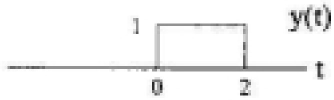
$1\leq t<2 \quad * = -2e^{\lambda-t}\big|_1^t = 2(e^{1-t}-1)$

$2\leq t<3 \quad * = -2e^{\lambda-t}\big|_1^2+2e^{\lambda-t}\big|_2^t = 2(e^{1-t}-e^{2-t})+2(1-e^{2-t}) = 2(1-2e^{2-t}+e^{1-t})$

$t\geq 3 \quad * = -2e^{\lambda-t}\big|_1^2+2e^{\lambda-t}\big|_2^3 = 2(e^{1-t}-e^{2-t}+e^{3-t}-e^{2-t}) = 2(e^{3-t}-2e^{2-t}+e^{1-t})$

3. (10 marks) Use the derivative property of convolution to find
$w(t) = x(t) * y(t)$, where





(The result from the previous question may be useful.)

$y(t) = \delta(t) \Rightarrow x(t) * y(t) = x(t)$

$$y(t) = u(t) \Rightarrow x(t) * y(t) = \int_{-\infty}^{t} x(\lambda)\, d\lambda = \begin{cases} 0, & t < -2 \\ \int_{-2}^{t} 2\, d\lambda = 2(t+2) & -2 \leq t < 0 \\ \int_{-2}^{0} 2\, d\lambda + \int_{0}^{t} d\lambda = 4 + t, & 0 \leq t < 1 \\ \int_{-2}^{0} 2\, d\lambda + \int_{0}^{1} d\lambda = 4 + 1 = 5, & t \geq 1 \end{cases}$$

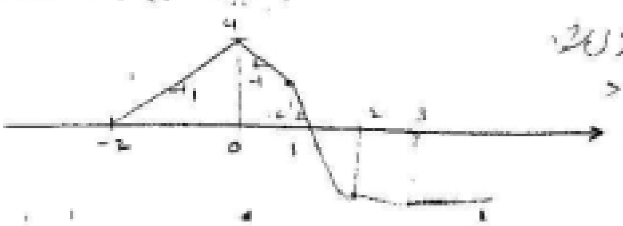
$$y(t) = u(t) \Rightarrow x(t) * y(t) = 2(t+2)\,(u(t+2) - u(t)) + (4+t)\,(u(t) - u(t-1)) + 5\,u(t-1)$$

$$\begin{aligned} y(t) = u(t) - u(t-2) \Rightarrow x(t) * y(t) = {} & 2(t+2)(u(t+2) - u(t)) \\ & + (4+t)(u(t) - u(t-1)) \\ & + 5\,u(t-1) \\ & - 2t\,(u(t) - u(t-2)) \\ & - (2+t)\,(u(t-2) - u(t-3)) \\ & - 5\,u(t-3) \end{aligned}$$

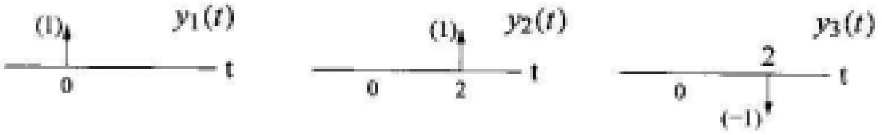
$$= 2(t+2)u(t+2) + (-2t - 4 + 4 + t - 2t)u(t) + (1-t)\,u(t-1) + (2t - 2 - t)u(t-2) + (2 + t - 5)u(t-3)$$

$$= 2(t+2)u(t+2) - 3t\,u(t) + (1-t)\,u(t-1) + (t-2)\,u(t-2) + (t-3)u(t-3)$$

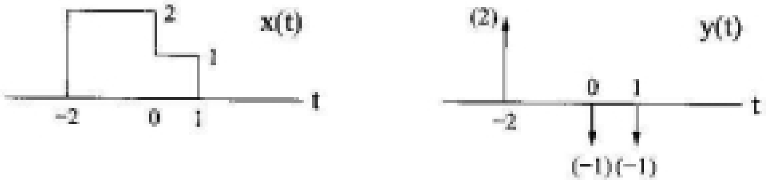
$$= 2r(t+2) - 3r(t) - r(t-1) + r(t-2) + r(t-3)$$



2. (10 marks) Suppose $y_1(t)$, $y_2(t)$ and $y_3(t)$ are as shown below:



If $x(t)$ and $y(t)$ are



then sketch

(a) $x(t) * y_1(t)$

(b) $x(t) * y_2(t)$

(c) $x(t) * y_3(t)$

(d) $y(t) * y_1(t)$

(e) $y(t) * y_2(t)$

(f) $y(t) * y_3(t)$.

$y_1(t) = \delta(t)$

$y_2(t) = \delta(t-2)$

$y_3(t) = -\delta(t-2)$

$y(t) = 2\delta(t+2) - \delta(t) - \delta(t-1)$

a) $x(t) * y_1(t) = x(t) * \delta(t) = x(t)$

b) $x(t) * y_2(t) = x(t) * \delta(t-2) = x(t-2)$

c) $x(t) * y_3(t) = x(t) * (-\delta(t-2)) = -x(t-2)$

d) $y(t) * y_1(t) = y(t) * \delta(t) = y(t)$

e) $y(t) * y_2(t) = y(t) * \delta(t-2) = y(t-2) = 2\delta(t) - \delta(t-2) - \delta(t-3)$

f) $y(t) * y_3(t) = y(t) * (-\delta(t-2)) = -y(t-2) = -2\delta(t) + \delta(t-2) + \delta(t-3)$

سیستم LTI با پاسخ ضربه $h(t) = \sin(t) \cdot u(t-2)$ را در نظر بگیرید. مطلوب است خروجی $y(t)$

اگر ورودی $x(t) = u(t) - u(t-1)$ با فرض آنکه انرژی اولیه سیستم صفر باشد.

$$s(t)\Big|_{x(t)=u(t)} = \int_{-\infty}^{t} h(\lambda)\,d\lambda = \int_{-\infty}^{t} \sin(\lambda)\, \underset{\lambda \geq 2}{u(\lambda-2)}\, d\lambda = \begin{cases} 0 , & t<2 \\ \displaystyle\int_{2}^{t} \sin(\lambda)\,d\lambda , & t \geq 2 \end{cases}$$

$$\int_{2}^{t} \sin(\lambda)\,d\lambda = -\cos(\lambda)\Big|_{2}^{t} = \cos(2) - \cos(t)$$

$$s(t)\Big|_{x(t)=u(t)} = \big(\cos(2) - \cos(t)\big)\, u(t-2)$$

$$\xrightarrow{u(t)} \boxed{LTI} \xrightarrow{s(t)}$$

$$\xrightarrow{u(t-1)} \boxed{LTI} \xrightarrow{s(t-1)}$$

$$\Rightarrow y(t)\Big|_{x(t)=u(t)-u(t-1)} = s(t) - s(t-1)$$

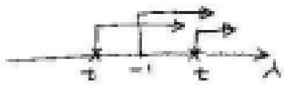
$$y(t) = \big(\cos(2) - \cos(t)\big)u(t-2) - \big(\cos(2) - \cos(t-1)\big)\, u(t-3)$$

مطلوب است $y(t) = x(t) * h(t)$

$$x(t) = e^{-t} u(t+1) \quad , \quad h(t) = e^{2t} u(-t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda = \int_{-\infty}^{\infty} e^{-\lambda} u(\lambda+1)\, e^{2(t-\lambda)} u(-(t-\lambda))\, d\lambda$$

$$y(t) = \int_{-\infty}^{\infty} e^{-\lambda} \cdot e^{2t} \cdot e^{-2\lambda} \cdot \underbrace{u(\lambda+1)}_{\lambda \geqslant -1}\, \underbrace{u(-t+\lambda)}_{\substack{-t+\lambda \geqslant 0 \\ \lambda \geqslant t}}\, d\lambda$$

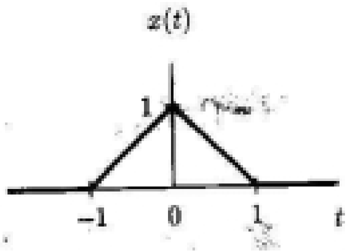


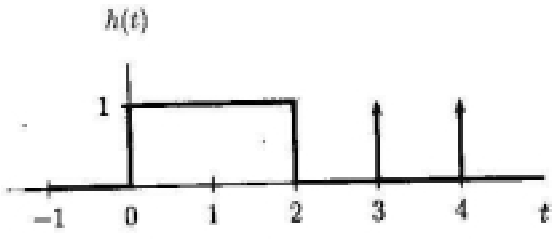
$$t < -1 \qquad * = \int_{-1}^{\infty} e^{2t} \cdot e^{-3\lambda} d\lambda = e^{2t} \cdot \frac{-1}{3} e^{-3\lambda} \Big|_{-1}^{\infty} = \frac{1}{3} e^{2t} \cdot e^{3}$$

$$t \geqslant -1 \qquad * = \int_{t}^{\infty} e^{2t} \cdot e^{-3\lambda} d\lambda = e^{2t} \cdot \frac{-1}{3} e^{-3\lambda} \Big|_{t}^{\infty} = \frac{1}{3} e^{-t}$$

$$y(t) = \begin{cases} \frac{1}{3} e^{-t} & t \geqslant -1 \\ \frac{1}{3} e^{2t+3} & t < -1 \end{cases}$$

مطلوب است $y(t)=x(t)*h(t)$



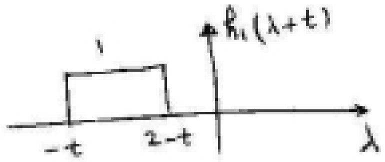


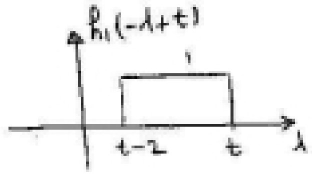
برای ساده‌تر شدن مسئله $h(t)$ را به دو بخش تفکیک می‌کنیم

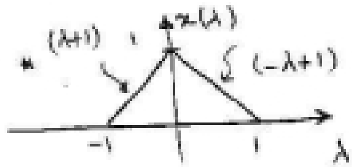
$h(t)=h_1(t)+h_2(t) \quad , \; h_2(t)=\delta(t-3)+\delta(t-4)$

$y(t)=x(t)*h_1(t)+x(t)*h_2(t)=\underbrace{x(t)*h_1(t)}_{y_1(t)}+x(t-3)+x(t-4)$

$y_1(t)=x(t)*h_1(t)=\int_{-\infty}^{\infty}x(\lambda)\,h_1(t-\lambda)\,d\lambda$







$t<-1 \qquad * = 0$

$-1\leq t<0 \qquad * = \int_{-1}^{t}(\lambda+1)\,d\lambda=\frac{1}{2}\lambda^2+\lambda\Big|_{-1}^{t}=\frac{1}{2}t^2+t-\frac{1}{2}+1=\frac{1}{2}t^2+t+\frac{1}{2}$

$0\leq t<1 \qquad * = \int_{-1}^{0}(\lambda+1)\,d\lambda+\int_{0}^{t}(-\lambda+1)\,d\lambda=(\frac{1}{2}\lambda^2+\lambda)\Big|_{-1}^{0}+(\frac{-1}{2}\lambda^2+\lambda)\Big|_{0}^{t}$

$=-(\frac{-1}{2})-\frac{1}{2}t^2+t=\frac{-1}{2}t^2+t+\frac{1}{2}$

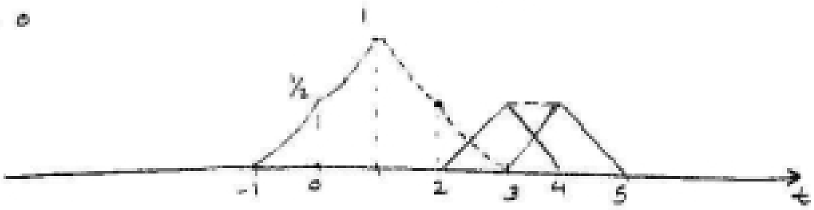
$1\leq t<2 \qquad * = \int_{t-2}^{0}(\lambda+1)\,d\lambda+\int_{0}^{1}(-\lambda+1)\,d\lambda=(\frac{1}{2}\lambda^2+\lambda)\Big|_{t-2}^{0}+(\frac{-1}{2}\lambda^2+\lambda)\Big|_{0}^{1}$

$=-\left(\frac{1}{2}(t-2)^2+(t-2)\right)+\frac{1}{2}$

$2\leq t<3 \qquad * = \int_{t-2}^{1}(-\lambda+1)\,d\lambda=\frac{-1}{2}\lambda^2+\lambda\Big|_{t-2}^{1}=\frac{1}{2}-\left(\frac{-1}{2}(t-2)^2+(t-2)\right)$

$=\frac{1}{2}+\frac{1}{2}(t-2)^2-(t-2)$

$t\geq 3 \qquad * = 0$



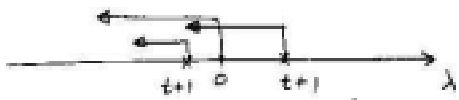
ـ مطلوبست $y(t)=x(t)*h(t)$

a) $x(t)=e^{-t}u(t+1) \quad , \quad h(t)=e^{2t}u(-t)$

$$y(t)=x(t)*h(t)=\int_{-\infty}^{\infty}h(\lambda)\,x(t-\lambda)\,d\lambda=\int_{-\infty}^{\infty}e^{2\lambda}u(-\lambda)\cdot e^{-(t-\lambda)}u(t-\lambda+1)\,d\lambda$$

$-\lambda \geq 0 \qquad\qquad t-\lambda+1 \geq 0$

$\lambda \leq 0 \qquad\qquad \lambda \leq t+1$



$$t+1<0 \Rightarrow y(t)=\int_{-\infty}^{t+1}e^{2\lambda}\cdot e^{-t}\cdot e^{\lambda}\,d\lambda=e^{-t}\cdot\frac{1}{3}e^{3\lambda}\Big|_{-\infty}^{t+1}=\frac{1}{3}e^{-t}\cdot e^{3(t+1)}$$

$$=\frac{1}{3}e^{2t+3}$$

$$t+1\geq 0 \Rightarrow y(t)=\int_{-\infty}^{0}e^{2\lambda}\cdot e^{-t}\cdot e^{\lambda}\,d\lambda=e^{-t}\cdot\frac{1}{3}e^{3\lambda}\Big|_{-\infty}^{0}=\frac{1}{3}e^{-t}$$

$$y(t)=\begin{cases}\frac{1}{3}e^{2t+3} & , \ t<-1\\ \frac{1}{3}e^{-t} & , \ t\geq -1\end{cases}$$

Differential Equations:

1. Solve the following problems for y(t). Plot your answers (either sketch them by hand or use MATLAB) for t = 0 to t = 5 sec.

۰.۱ ب ← a) $\frac{dy}{dt} + 2y(t) = 2x(t); \quad x(t) = u(t), y(0) = -1$

$\lambda + 2 = 0$

$y_h(t) = ke^{-2t}$

$y_p(t) = 1$

$\Rightarrow y(t) = ke^{-2t} + 1$

$y(0) = k + 1 = -1$

$k = -2$

$y(t) = 1 - 2e^{-2t} \quad , t \geq 0$

$y(t) = (1 - 2e^{-2t})u(t)$

۰.۱ ب ← b) $\frac{dy}{dt} - 2y(t) = 2x(t); \quad x(t) = u(t), y(0) = -1$

$\lambda - 2 = 0 \quad \lambda = 2 \quad y_h(t) = ke^{2t}$

$y_p(t) = -1$

$y(t) = ke^{2t} - 1 \quad , t \geq 0$

$y(0) = k - 1 = -1 \Rightarrow k = 0$

$\Rightarrow y(t) = -1 \quad , t \geq 0$

$y(t) = -u(t)$