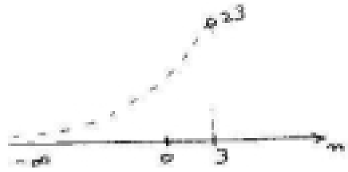
- پاسخ ضربه سیستم [illegible] زمان گسسته داده شده در هر مورد تعیین کنید آیا سیستم علی است

آیا سیستم پایدار است.

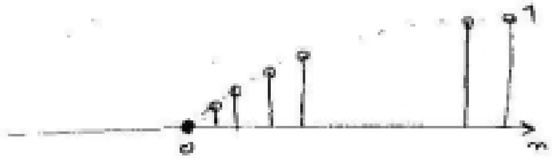


- $h[n] = 2^n u[3-n]$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |2^n u[3-n]| = \sum_{n=-\infty}^{3} 2^n = \frac{2^3}{1-\frac{1}{2}} = 16 < \infty$$

$3-n \geq 0 ,\ n \leq 3$

غیرعلی - پایدار



- $h[n] = [1-(0.99)^n] u[n]$

$n \geq 0$

علی - ناپایدار

- $h[n] = 2^n u[4-n]$

$4-n \geq 0$

$n \leq 4 \rightarrow$ غیرعلی

$$\sum_{n=-\infty}^{\infty} |2^n u[4-n]| = \sum_{n=-\infty}^{4} 2^n = \frac{2^4}{1-\frac{1}{2}} = 2^5 < \infty \rightarrow$$ پایدار

تحلیل - بررسی مسائل

2. Perform the following convolutions, $x[n]*v[n]$

a) $x[n] = u[n] - u[n-4]$, $v[n] = 0.5^n u[n]$

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b) $x[n] = [1\ 4\ 8\ 2]$; $v[n] = [0\ 1\ 2\ 3\ 4]$ (the sequences both start at n=0)

c) $x[n] = u[n]$, $v[n] = 2(0.8)^n u[n]$

d) $x[n] = u[n-1]$, $v[n] = 2(0.5)^n u[n]$

$$a)\quad y[n] = \sum_{k=-\infty}^{\infty} 0.5^k \underbrace{u[k]}_{k\geq 0} \cdot \big(\underbrace{u[n-k]}_{\substack{n-k\geq 0\\ k\leq n}} - \underbrace{u[n-k-4]}_{k\leq n-4}\big) = \overbrace{\sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^k u[k]\,u[n-k]}^{1} - \overbrace{\sum_{k=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^k u[k]\,u[n-k-4]}^{2}$$

$$1 \Rightarrow \begin{cases} * = 0 & n<0 \\ * = \displaystyle\sum_{k=0}^{n} \left(\tfrac{1}{2}\right)^k = \frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} & , n\geq 0 \end{cases}$$

$$2 \Rightarrow \begin{cases} * = 0 & n-4<0 \\ * = \displaystyle\sum_{0}^{n-4} \left(\tfrac{1}{2}\right)^k = \frac{1-\left(\frac{1}{2}\right)^{n-3}}{1-\frac{1}{2}} & , n-4\geq 0 \end{cases}$$

$$y[n] = \begin{cases} 0 & n<0 \\ \dfrac{1-\left(\frac{1}{2}\right)^{n+1}}{1/2} & 0\leq n<4 \\ & n\geq 4 \\ \dfrac{1-\left(\frac{1}{2}\right)^{n+1}}{1/2} - \dfrac{1-\left(\frac{1}{2}\right)^{n-3}}{\frac{1}{2}} = \left(\tfrac{1}{2}\right)^{-1}\left(\tfrac{1}{2}\right)^{n-3} - \left(\tfrac{1}{2}\right)^{-1}\left(\tfrac{1}{2}\right)^{n+1} & \\ = \left(\tfrac{1}{2}\right)^{n-4} - \left(\tfrac{1}{2}\right)^{n} = \left(\tfrac{1}{2}\right)^{n}(16-1) = 15\left(\tfrac{1}{2}\right)^{n} & \end{cases}$$

$$y[n] = \begin{cases} 0 & , n<0 \\ 2-\left(\frac{1}{2}\right)^n & 0\leq n<4 \\ 15\left(\frac{1}{2}\right)^n & n\geq 4 \end{cases}$$

c) $x[n] = u[n]$, $V[n] = 2(0.8)^n u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} 2(0.8)^k \underbrace{u[k]}_{k\geq 0}\,\underbrace{u[n-k]}_{k\leq n} = \begin{cases} 0 & , n<0 \\ \displaystyle\sum_{k=0}^{n} 2(0.8)^k = \frac{2\left(1-(0.8)^{n+1}\right)}{1-0.8} & , n\geq 0 \end{cases}$$

$$y[n] = 10\left(1-(0.8)^{n+1}\right)u[n]$$

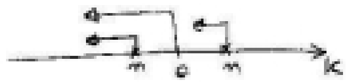
d) $x[n] = u[n-1]$, $v[n] = 2(0.5)^n u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} 2(0.5)^k \underbrace{u[k]}_{k\geq 0} \cdot \underbrace{u[n-k-1]}_{\substack{n-k-1\geq 0\\ k\leq n-1}}$$

$$y[n] = \begin{cases} 0, & n-1<0 \\ \displaystyle\sum_{k=0}^{n-1} 2(0.5)^k = \frac{2\left(1-(0.5)^n\right)}{1-0.5} & , n-1\geq 0 \end{cases} \Rightarrow y[n] = 4\left(1-(0.5)^n\right)u[n-1]$$

- مطلوب $y[n]=x[n]*h[n]$

$$x[n] = \left(\tfrac{1}{2}\right)^n u[n] \quad , \; h[n]=u[-n]$$



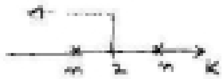
$$y[n] = \sum_{k=-\infty}^{\infty} \underbrace{u[-k]}_{\substack{-k\geq 0 \\ k\leq 0}} \left(\tfrac{1}{2}\right)^{n-k} \underbrace{u[n-k]}_{\substack{n-k\geq 0 \\ k\leq n}}$$

$$\begin{cases} n<0 & * = \displaystyle\sum_{k=-\infty}^{n} \left(\tfrac{1}{2}\right)^{n-k} \underset{k=-k}{=} \sum_{k=-n}^{\infty} \left(\tfrac{1}{2}\right)^{n} \cdot \left(\tfrac{1}{2}\right)^{k} = \left(\tfrac{1}{2}\right)^{n} \cdot \left(\tfrac{1}{2}\right)^{-n} \cdot \frac{1}{1-\frac{1}{2}} = 2 \\ n\geq 0 & * = \displaystyle\sum_{k=-\infty}^{0} \left(\tfrac{1}{2}\right)^{n-k} = \sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^{n} \left(\tfrac{1}{2}\right)^{k} = \left(\tfrac{1}{2}\right)^{n} \frac{1}{1-\frac{1}{2}} = 2\left(\tfrac{1}{2}\right)^{n} \end{cases}$$

$$y[n] = \begin{cases} 2 & , \; n<0 \\ 2\left(\tfrac{1}{2}\right)^{n} & , \; n\geq 0 \end{cases}$$

- مطلوب است $y[n]=x[n]*h[n]$

$$x[n] = u[n+4] - u[n-1] \qquad h[n] = 2^n u[2-n]$$



$$s[n]\Big|_{x[n]=u[n]} = \sum_{k=-\infty}^{n} 2^k u[2-k] \qquad \begin{matrix} 2-k \geq 0 \\ k \leq 2 \end{matrix}$$

$$n<2 \qquad * = \sum_{k=-\infty}^{n} 2^k = \frac{2^n}{1-\frac{1}{2}} = 2^{n+1}$$

$$n \geq 2 \qquad * = \sum_{k=-\infty}^{2} 2^k = \frac{2^2}{1-\frac{1}{2}} = 2^3$$

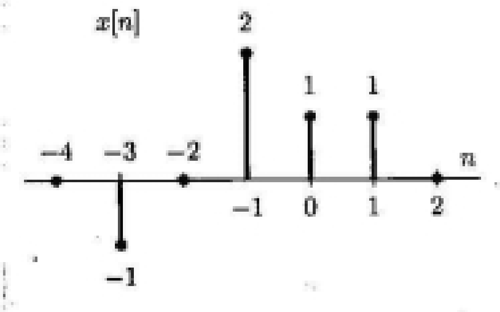
$$s[n]\Big|_{x[n]=u[n]} = 2^{n+1} u[3-n] + 2^3 u[n-2]$$

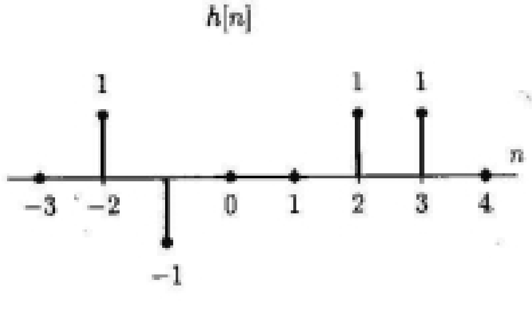
$$y[n]\Big|_{x[n]=u[n+4]-u[n-1]} = 2^{n+5} u[3-(n+4)] + 2^3 u[n+4-2] - 2^n u[3-(n-1)] - 2^3 u[n-3]$$

$$y[n] = 2^{n+5} u[-n-1] + 2^3 \big(u[n+2] - u[n-3] \big) - 2^n u[4-n]$$

کنترل شود

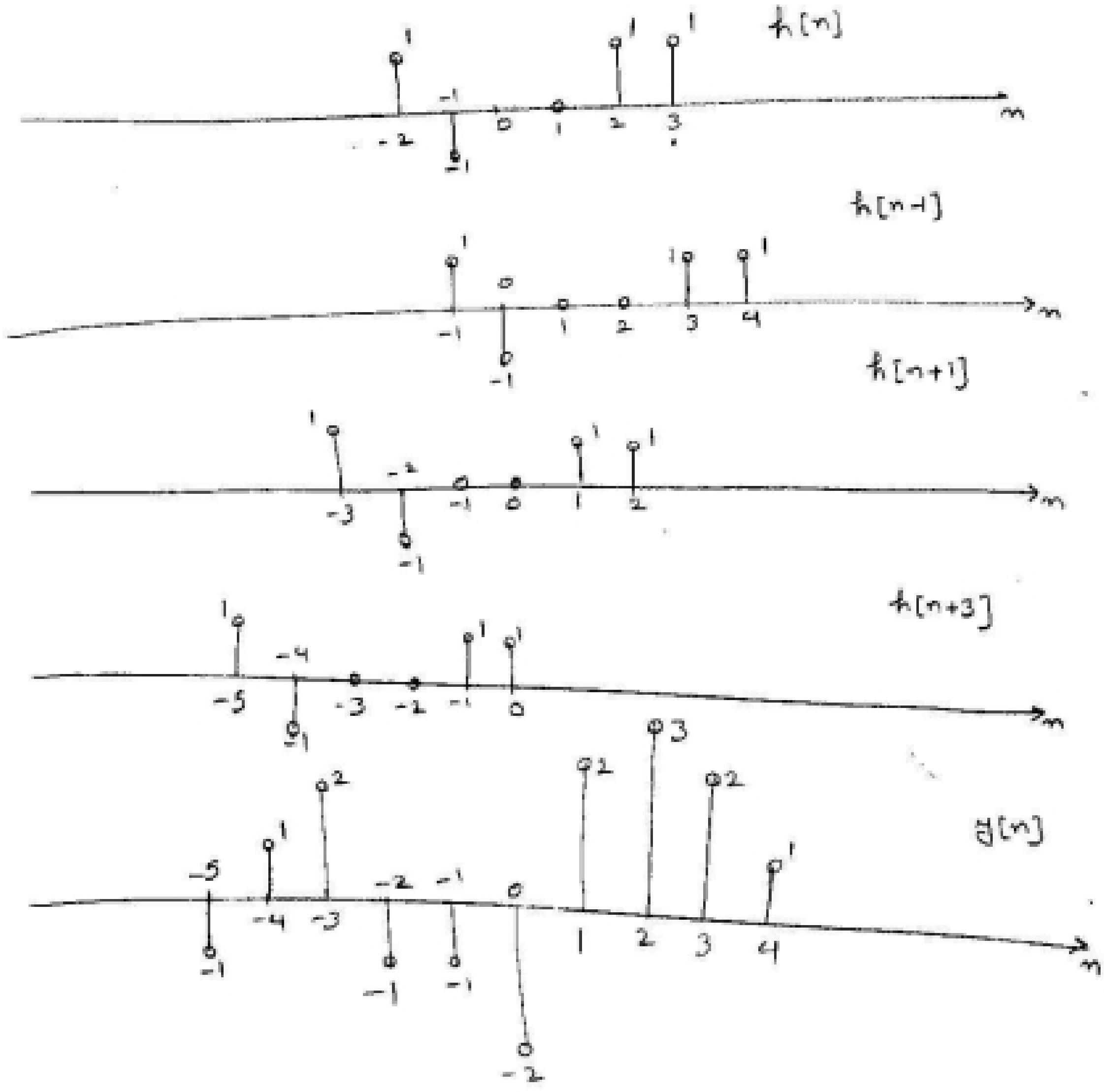
مطلوبست محاسبه $y[n] = x[n] * h[n]$





$x[n] = -\delta[n+3] + 2\delta[n+1] + \delta[n] + \delta[n-1]$

$y[n] = h[n] * x[n] = -h[n+3] + 2h[n+1] + h[n] + h[n-1]$



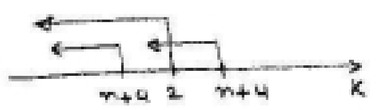
- مطلوبست $y[n]=x[n]*h[n]$

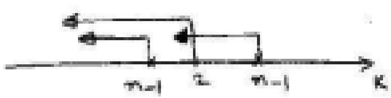
$-\ x[n]=u[n+4]-u[n-1] \quad , \quad h[n]=2^n u[2-n]$

$$y[n]=\sum_{K=-\infty}^{\infty} h[K]\,x[n-K] = \sum_{K=-\infty}^{\infty} 2^K u[2-K]\big(u[n-K+4]-u[n-K-1]\big)$$

$$2-K\geq 0 \qquad n-K+4\geq 0 \qquad n-K-1\geq 0$$
$$K\leq 2 \qquad K\leq n+4 \qquad K\leq n-1$$

$$=\sum_{K=-\infty}^{\infty} 2^K\cdot \underset{K\leq 2}{u[2-K]}\ \underset{K\leq n+4}{u[n-K+4]} - \sum_{K=-\infty}^{\infty} 2^K \underset{K\leq 2}{u[2-K]}\ \underset{K\leq n-1}{u[n-K-1]}$$



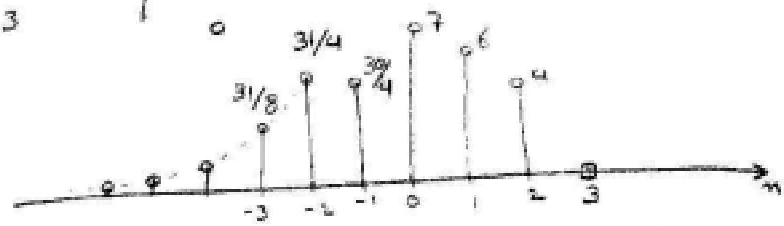


$$① \begin{cases} \sum_{K=-\infty}^{n+4} 2^K & , n+4<2 \ , \ n<-2 \\ \sum_{K=-\infty}^{2} 2^K & , n+4\geq 2 \ , \ n\geq -2 \end{cases} \qquad ② \begin{cases} -\sum_{K=-\infty}^{n-1} 2^K & , n-1<2 \ , \ n<3 \\ -\sum_{K=-\infty}^{2} 2^K & , n-1\geq 2 \ , \ n\geq 3 \end{cases}$$

$$y[n]=\begin{cases} \sum_{K=-\infty}^{n+4} 2^K - \sum_{K=-\infty}^{n-1} 2^K & , n<-2 \\ \sum_{K=-\infty}^{2} 2^K - \sum_{K=-\infty}^{n-1} 2^K & , -2\leq n<3 \\ \sum_{K=-\infty}^{2} 2^K - \sum_{K=-\infty}^{2} 2^K & , n\geq 3 \end{cases}$$

$$y[n]=\begin{cases} \sum_{K=n}^{n+4} 2^K & , n<-2 \\ \sum_{K=m}^{2} 2^K & , -2\leq m<3 \\ 0 & , n\geq 3 \end{cases} = \begin{cases} \dfrac{2^n(1-2^5)}{1-2} = (31)(2^n) & , n<-2 \\ \dfrac{2^n(1-2^{2-n+1})}{1-2} = 2^n(2^3\cdot 2^{-n}-1) & , -2\leq n<3 \\ 0 & n\geq 3 \end{cases}$$

$$y[n]=\begin{cases} 31(2^n) & , n<-2 \\ 8-2^n & , -2\leq n<3 \\ 0 & n\geq 3 \end{cases}$$



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