Solving Difference Equations:

1. Solve the following difference equations using recursion first by hand (for n=0 to n=4), then using MATLAB (for n=0 to n=30). Plot the output computed by MATLAB on a stem plot.

مستقل
a) y[n] + 0.5y[n-1] = 2x[n-1]; x[n] = δ[n], y[-1] = 0
b) y[n] + 2y[n-1] = 2x[n-1]; x[n] = δ[n], y[-1] = 0

[illegible] → c) y[n] + 1.2y[n-1] + 0.32y[n-2] = x[n]-x[n-1]; x[n] = u[n], y[-2] = 1, y[-1]=2

a) $y[n] = -0.5\,y[n-1] + 2x[n-1]$

$y[n] = -\frac{1}{2}y[n-1] + 2\delta[n-1]$

$y[0] = 0$

$y[1] = \frac{-1}{2}y[0] + 2 = 2$

$y[2] = \frac{-1}{2}y[1] = \frac{-1}{2}(2) \quad \Rightarrow y[n] = \left(\frac{-1}{2}\right)^{n-1}(2)\,u[n-1]$

$y[3] = \frac{-1}{2}y[2] = \left(\frac{-1}{2}\right)^2(2)$

$y[4] = \frac{-1}{2}y[3] = \left(\frac{-1}{2}\right)^3(2)$

b) $y[n] = -2y[n-1] + 2x[n-1]$

$y[n] = -2y[n-1] + 2\delta[n-1]$

$y[0] = 0$

$y[1] = -2y[0] + 2 = 2$

$y[2] = -2y[1] + 0 = -2(2) \quad \Rightarrow y[n] = 2(-2)^{n-1}\,u[n-1]$

$y[3] = -2y[2] = (-2)^2(2)$

$y[4] = -2y[3] = (-2)^3(2)$

c) $y[n] = -1.2y[n-1] - 0.32y[n-2] + \underbrace{u[n] - u[n-1]}_{\delta[n]}$

$y[0] = -1.2(2) - 0.32(1) + 1 = -1.72$

$y[1] = -1.2y[0] - 0.32(2) = 1.424$

$y[2] = -1.2y[1] - 0.32y[0] = -1.1584$

$y[3] = -1.2y[2] - 0.32y[1] = 0.9344$

$y[4] \qquad = -0.750592$

$y[k] = -1.2y[k-1] - 0.32y[k-2]$

علی

1. Find the impulse response for each of the following discrete-time systems:

a) y[n] + 0.2y[n-1] = x[n] - x[n-1]

b) y[n] + 1.2y[n-1] = 2x[n-1]

c) y[n] = 0.24(x[n]+x[n-1]+x[n-2]+x[n-3])

d) y[n] = x[n] + 0.5x[n-1] + x[n-2]

a) $h[n] = -0.2\,h[n-1] + \delta[n] - \delta[n-1]$

$h[0] = 1$

$h[1] = -0.2\,h[0] - 1 = -1.2$

$h[2] = -0.2\,h[1] = (-0.2)(-1.2)$

$h[3] = -0.2\,h[2] = (-0.2)^2(-1.2)$

$h[k] = -0.2\,h[k-1] = (-0.2)^{k-1}(-1.2)$

$$h[n] = \begin{cases} 1, & n=0 \\ (-0.2)^{n-1}(-1.2), & n \geq 1 \end{cases}$$

$(-0.2)^n \cdot (-0.2)^{-1}(-1.2)$

$(-0.2)^n \cdot \left(\frac{1}{-0.2}\right)(-1.2)$

$6(-0.2)^n$

$$h[n] = \begin{cases} 1, & n=0 \\ 6(-0.2)^n, & n \geq 1 \end{cases}$$

b) $h[n] = -1.2\,h[n-1] + 2\,\delta[n-1]$

$h[1] = 0 + 2$

$h[2] = -1.2\,h[1] = (-1.2)(2)$

$h[3] = -1.2\,h[2] = (-1.2)^2(2)$

$h[k] = -1.2\,h[k-1] = (-1.2)^{k-1}(2)$

$h[n] = (-1.2)^{n-1}(2), \quad n \geq 1$

c) $h[n] = 0.24(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])$

d) $h[n] = \delta[n] + 0.5\,\delta[n-1] + \delta[n-2]$

1. (5 marks) Determine the impulse response of the LTI system described by the difference equation

$$y[n] - 0.2y[n-1] = x[n] + 0.5x[n-1]$$

under the assumption that it is causal. Is the system stable?

$h[n] = 0.2\,h[n-1] + \delta[n] + \frac{1}{2}\delta[n-1]$

[illegible]: $x[n]=\delta[n]$

$\delta[-1]=0$

$h[0]=1$

$h[1] = 0.2\,h[0] + \frac{1}{2} = \frac{2}{10} + \frac{1}{2} = \frac{7}{10}\left(\frac{2}{10}\right)^0$

$h[2] = 0.2\,h[1] = \frac{2}{10}\left(\frac{7}{10}\right)$

$h[3] = 0.2\,h[2] = \left(\frac{2}{10}\right)^2\left(\frac{7}{10}\right)$

$\vdots$

$h[k] = 0.2\,h[k-1] = \left(\frac{2}{10}\right)^{k-1}\left(\frac{7}{10}\right)$

$\Rightarrow h[0]=1$

$h[n] = \left(\frac{2}{10}\right)^{n-1}\left(\frac{7}{10}\right) \quad n \geq 1$

$h[n] = \delta[n] + \left(\frac{7}{10}\right)\left(\frac{2}{10}\right)^{n-1} u[n-1]$

$\sum_{n=-\infty}^{\infty} |h[n]|^2 < \infty \Rightarrow \text{stable}$

معادله تفاضلی سیستم به ازاء ورودی $x[n]=3^n$ داده شده است مطلوبست خروجی $y[n]$ با فرض شرایط اولیه $y[-1]=1$, $y[-2]=2$.

$$y[n]-\frac{5}{6}y[n-1]-\frac{1}{6}y[n-2]=3^n$$

همگن $\quad y_h[n]=r^n \Rightarrow r^n-\frac{5}{6}r^{n-1}-\frac{1}{6}r^{n-2}=0$

$$r^{n-2}\left(r^2-\frac{5}{6}r-\frac{1}{6}\right)=0 \qquad r^2-\frac{5}{6}r-\frac{1}{6}=0$$

$$r=\frac{5}{12}\pm\sqrt{\left(\frac{5}{12}\right)^2+\frac{1}{6}}=\frac{5}{12}\pm\sqrt{\frac{25+24}{(12)^2}}=\frac{5}{12}\pm\frac{7}{12}=\begin{cases}1\\-\frac{1}{6}\end{cases}$$

$$y_h[n]=K_1(1)^n+K_2\left(\frac{-1}{6}\right)^n=K_1+K_2\left(\frac{-1}{6}\right)^n$$

خاص : $\quad y[n]-\frac{5}{6}y[n-1]-\frac{1}{6}y[n-2]=3^n \qquad y_p[n]=A(3^n)$

$$\Rightarrow A(3^n)-\frac{5}{6}A(3^{n-1})-\frac{1}{6}A(3^{n-2})=3^n$$

$$A\left(1-\frac{5}{6}\times\frac{1}{3}-\frac{1}{6}\times\frac{1}{9}\right)=1 \Rightarrow A\left(\frac{54-15-1}{54}\right)=1 \Rightarrow A=\frac{54}{38}$$

کامل : $\quad y[n]=K_1+K_2\left(\frac{-1}{6}\right)^n+\frac{54}{38}(3^n)$

$$\left.\begin{aligned}y[-1]&=K_1+K_2\left(\frac{-1}{6}\right)^{-1}+\frac{54}{38}(3^{-1})=1\\ y[-2]&=K_1+K_2\left(\frac{-1}{6}\right)^{-2}+\frac{54}{38}(3^{-2})=2\end{aligned}\right\} \Rightarrow \begin{cases}K_1-6K_2=\frac{10}{19}\\ K_1+36K_2=\frac{35}{19}\end{cases}$$

$$\Rightarrow 42K_2=\frac{35}{19}-\frac{10}{19} \Rightarrow K_2=\frac{25}{19}\times\frac{1}{42}$$

$$K_1=6\times\frac{25}{19}\times\frac{1}{42}+\frac{10}{19}=\frac{25}{19}\times\frac{1}{7}+\frac{10}{19}=\frac{25+70}{19\times7}=\frac{95}{133}$$

$$\Rightarrow y[n]=\frac{95}{133}+\frac{25}{798}\left(-\frac{1}{6}\right)^n+\frac{54}{38}(3^n)$$

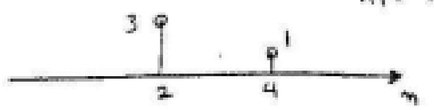
2. (5 marks) Which of the impulse responses

$$h_1[n] = 3\delta[n-2] + \delta[n-4]$$
$$h_2[n] = u[n-3] - u[n+5]$$

describe causal, stable, LTI processors? Give reasons for your answers. Sketch the step response of each system.

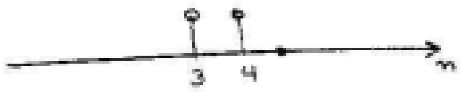
$h_1[n]$

علّی - پایدار

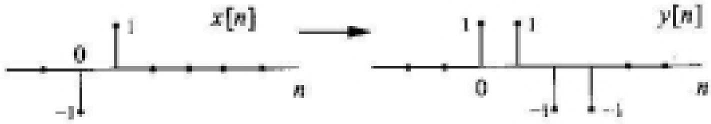


$h_2[n]$

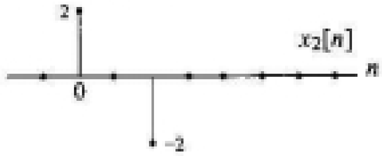
علّی - پایدار



4. (5 marks) Suppose $y[n]$ is the output of an LTI system when $x[n]$ is the input:



Find the response of the system to the input



$$x[n] = -\delta[n] + \delta[n-1] \Rightarrow y[n] = -h[n] + h[n-1]$$

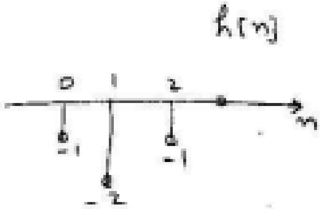
$n=0 \quad h[0] = -1$

$n=1 \quad -h[1] + h[0] = 1 \Rightarrow h[1] = -2$

$n=2 \quad -h[2] + h[1] = -1 \Rightarrow h[2] = -1$

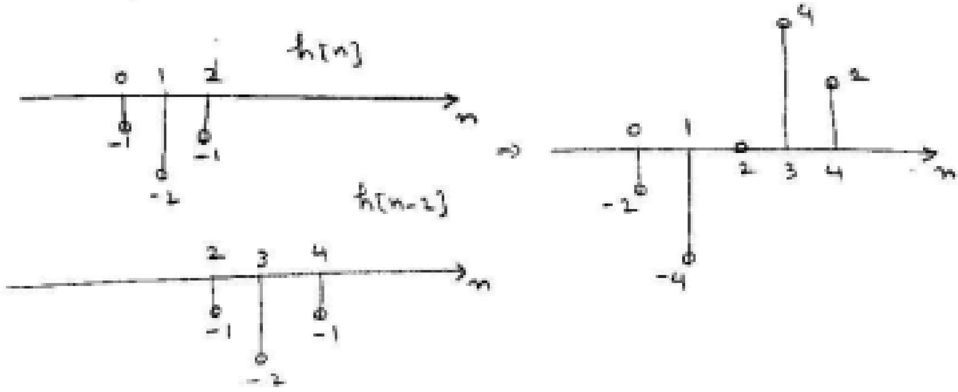
$n=3 \quad -h[3] + h[2] = -1 \Rightarrow h[3] = 0$

$n=4 \quad -h[4] + h[3] = 0 \Rightarrow h[4] = 0$



$$y_2[n] = x_2[n] * h[n] = (2\delta[n] - 2\delta[n-2]) * h[n]$$

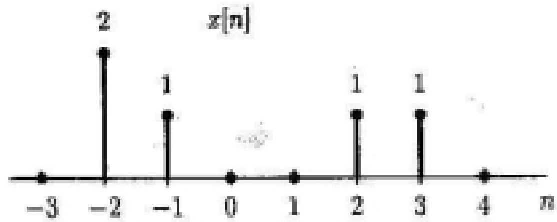
$$= 2(h[n] - h[n-2])$$



- سیستم LTI در حالت سکون اولیه با معادله دیفرانسیل زیر را در نظر بگیرید.

$$y[n] - \frac{1}{2}y[n-1] = 2x[n] - x[n-2]$$

پاسخ سیستم را به ورودی زیر بدست آورید.



$$x[n] = 2\delta[n+2] + \delta[n+1] + \delta[n-2] + \delta[n-3]$$

با توجه به شرط سکون $y[n]=0$ ، $n<-2$

$$y[n] = \frac{1}{2}y[n-1] + 2\left(2\cancel{\delta[n+2]} + \cancel{\delta[n+1]} + \delta[n-2] + \delta[n-3]\right)$$
$$-\left(2\delta[n] + \delta[n-1] + \delta[n-4] + \delta[n-5]\right)$$

$$y[n] = \frac{1}{2}y[n-1] + 4\delta[n+2] + 2\delta[n+1] - 2\delta[n] - \delta[n-1] + 2\delta[n-2] + 2\delta[n-3]$$
$$-\delta[n-4]$$
$$-\delta[n-5]$$

$$y[-2] = 0 + 4 = 4$$
$$y[-1] = \frac{1}{2}y[-2] + 2 = 4$$
$$y[0] = \frac{1}{2}y[-1] - 2 = 0$$
$$y[1] = \frac{1}{2}y[0] - 1 = -1$$
$$y[2] = \frac{1}{2}y[1] + 2 = \frac{3}{2}$$
$$y[3] = \frac{1}{2}y[2] + 2 = \frac{11}{4}$$
$$y[4] = \frac{1}{2}y[3] - 1 = \frac{3}{8}$$
$$y[5] = \frac{1}{2}y[4] - 1 = -13/16$$
$$y[6] = \frac{1}{2}y[5] = \frac{1}{2}\left(\frac{-13}{16}\right) = -13\left(\frac{1}{2}\right)^5$$
$$y[7] = \frac{1}{2}y[6] = \left(\frac{1}{2}\right)^2\left(\frac{-13}{16}\right) = -13\left(\frac{1}{2}\right)^6$$
$$\vdots$$
$$y[k] = \frac{1}{2}y[k-1] = -13\left(\frac{1}{2}\right)^{k-1}$$

$$y[n] = \begin{cases} 4, & n=-2 \\ 4, & n=-1 \\ 0, & n=0 \\ -1, & n=1 \\ \frac{3}{2}, & n=2 \\ \frac{11}{4}, & n=3 \\ \frac{3}{8}, & n=4 \\ -13\left(\frac{1}{2}\right)^{n-1}, & n\geq 5 \end{cases}$$

ـ معادله تفاضلی سیستم

① $$y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{2^2}x[n-2] + \cdots + \left(\frac{1}{2}\right)^K x[n-K] + \cdots$$

$$h[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \frac{1}{2^2}\delta[n-2] + \cdots + \left(\frac{1}{2^n}\right)\delta[n-K] + \cdots$$

$$h[n] = \frac{1}{2^n}u[n]$$

ـ معادله تفاضلی $y[n] - \frac{1}{2}y[n-1] = x[n]$ ② را در نظر بگیرید. چه ارتباطی بین معادله تفاضلی ① و ② وجود دارد.

$$y[n] = x[n] + \quad y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{2^2}x[n-2] + \cdots$$

$$y[n] = \sum_{K=0}^{\infty} \frac{1}{2^K} x[n-K]$$

$$y[n] = x[n] + \sum_{K=1}^{\infty} \frac{1}{2^K} x[n-K]$$

$$K-1=m$$

$$= x[n] + \sum_{m=0}^{\infty} \frac{1}{2^{m+1}} x[n-m-1]$$

$$y[n] = x[n] + \frac{1}{2}\underbrace{\sum_{K=0}^{\infty} \frac{1}{2^K} \cdot x[n-1-K]}_{y[n-1]}$$

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$