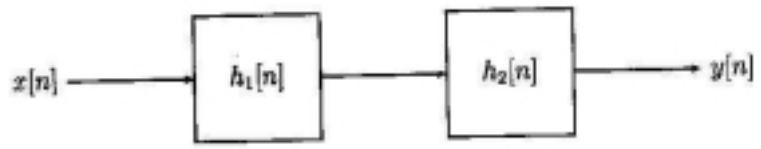
Problem 5 Consider the cascade of LTI systems with unit sample responses $h_1[n]$ and $h_2[n]$ depicted below:

کلاس

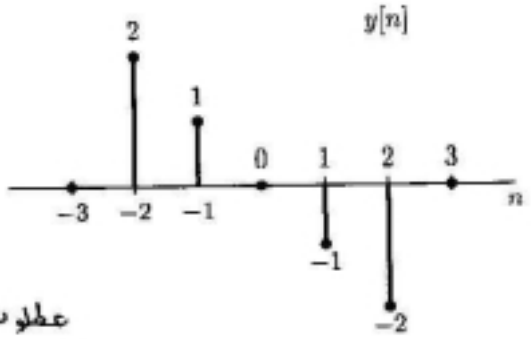


Suppose we are given the following information:

- $h_2[n] = \delta[n] - \delta[n-1]$
- If the input is

$$x[n] = u[n] - u[n-2]$$

then the output is as depicted below



• $h_1[n]$ علاوبت

$h[n] = h_1[n] * h_2[n] = h_1[n] * (\delta[n] - \delta[n-1]) = h_1[n] - h_1[n-1]$

$x[n] = u[n] - u[n-2] = \delta[n] + \delta[n-1]$

$y[n] = h[n] * x[n] = (h_1[n] - h_1[n-1]) * (\delta[n] + \delta[n-1])$

$= h_1[n] + h_1[n-1] - h_1[n-1] - h_1[n-2]$

$= h_1[n] - h_1[n-2]$

$y[-2] = h_1[-2] - \underbrace{h_1[-4]}_{0} \Rightarrow h_1[-2] = 2$

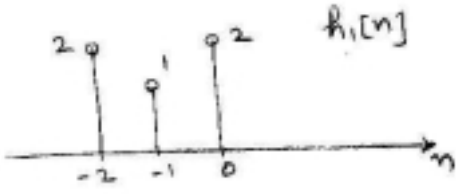
$y[-1] = h_1[-1] - \underbrace{h_1[-3]}_{0} \Rightarrow h_1[-1] = 1$

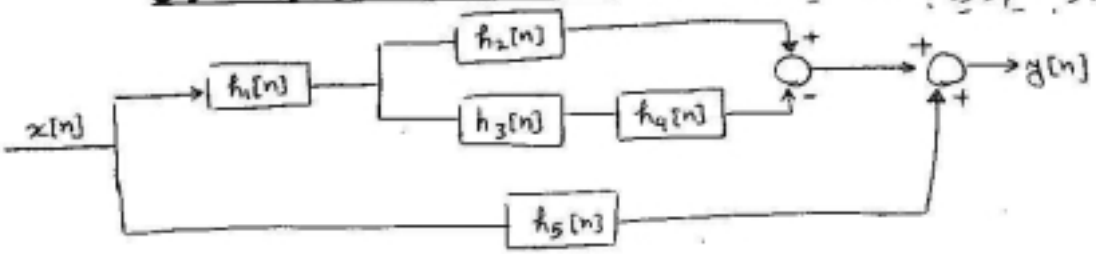
$y[0] = h_1[0] - h_1[-2] = 0 \Rightarrow h_1[0] = 2$

$y[1] = h_1[1] - h_1[-1] = -1 \Rightarrow h_1[1] = 0$

$y[2] = h_1[2] - h_1[0] = -2 \Rightarrow h_1[2] = 0$

$y[3] = h_1[3] - h_1[1] = 0 \Rightarrow h_1[3] = 0$





$h_1[n] = 4\left(\frac{1}{2}\right)^n \left(u[n] - u[n-3]\right)$ $\quad h_4[n] = \delta[n-1]$

$h_5[n] = \delta[n] - 4\delta[n-3]$

$h_2[n] = h_3[n] = (n+1)u[n]$

$$h[n] = h_5[n] + \left\{ h_1[n] * \left[h_2[n] - h_3[n] * h_4[n] \right] \right\}$$

$$h_2[n] - h_3[n] * h_4[n] = h_3[n] - h_3[n] * \delta[n-1] = h_3[n] - h_3[n-1]$$

$$= (n+1)u[n] - (n)u[n-1]$$

$$= (n+1)\left(\delta[n] + u[n-1]\right) - (n)u[n-1]$$

$$= \delta[n] + (n+1-n)u[n-1] = \delta[n] + u[n-1] = u[n]$$

$$h[n] = h_5[n] + \underline{h_1[n] * u[n]}$$

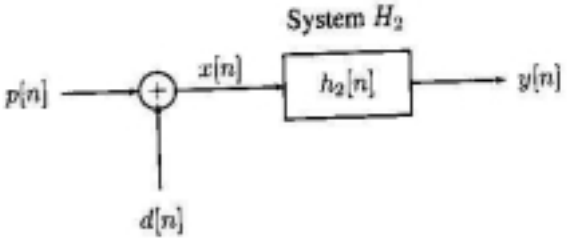
$$h_1[n] = 4\left(\tfrac{1}{2}\right)^n \left(u[n] - u[n-3]\right) = 4\left(\tfrac{1}{2}\right)^n \left(\delta[n] + \delta[n-1] + \delta[n-2]\right)$$

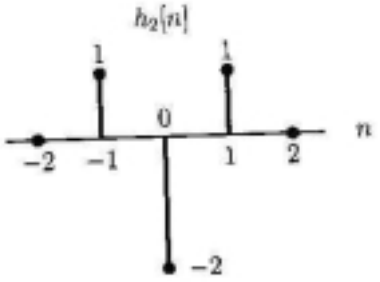
$$= 4\left(\delta[n] + \tfrac{1}{2}\delta[n-1] + \tfrac{1}{4}\delta[n-2]\right)$$

$$h_1[n] * u[n] = 4\left(u[n] + \tfrac{1}{2}u[n-1] + \tfrac{1}{4}u[n-2]\right)$$

$$h[n] = \delta[n] - 4\delta[n-3] + 4u[n] + 2u[n-1] + u[n-2]$$ ✓✓

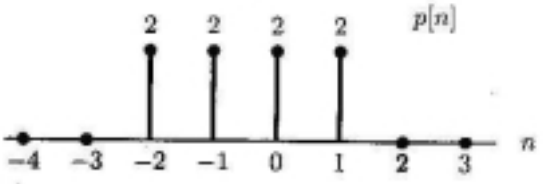
- سیستم



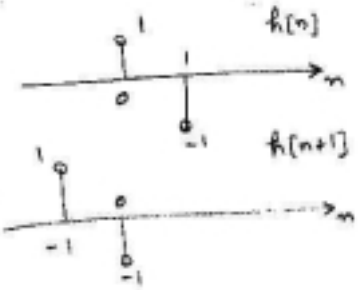


$h[n] = \delta[n] - \delta[n-1]$ و $h_2[n] = h[n] * h[n+1]$ ک

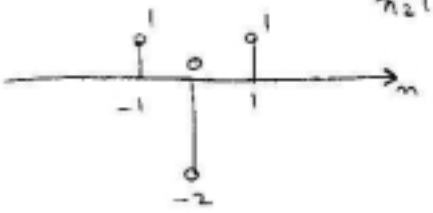
الف) با فرض آنکه $p[n]$ داده شده و نویز $d[n] = 0$ باشد. مطلوبست خروجی $y[n]$



$h_2[n] = h[n] * h[n+1] = (\delta[n] - \delta[n-1]) * h[n+1] = h[n+1] - h[n]$



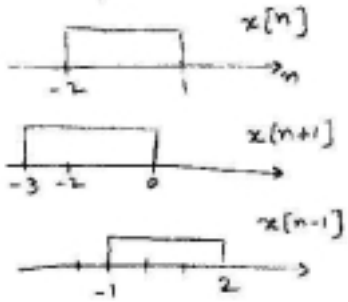
$\Rightarrow$

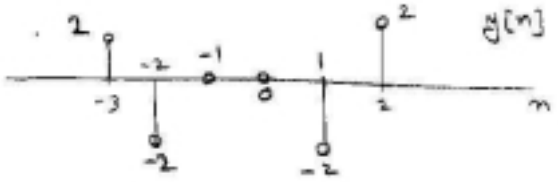


$h_2[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$

$d[n]=0 \Rightarrow x[n]=p[n] \Rightarrow y[n] = x[n] * h_2[n] = p[n] * (\delta[n+1] - 2\delta[n] + \delta[n-1])$

$y[n] = p[n+1] - 2p[n] + p[n-1]$





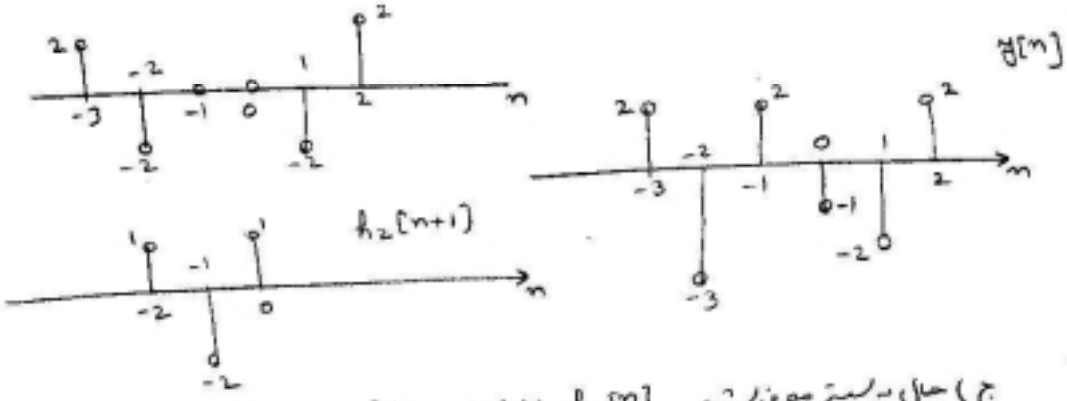
ب) برای قسمت الف) داریم $p[n]$ داده شده و $d[n] = -\delta[n+1]$.

خروجی $y[n]$ را به دست آورید.

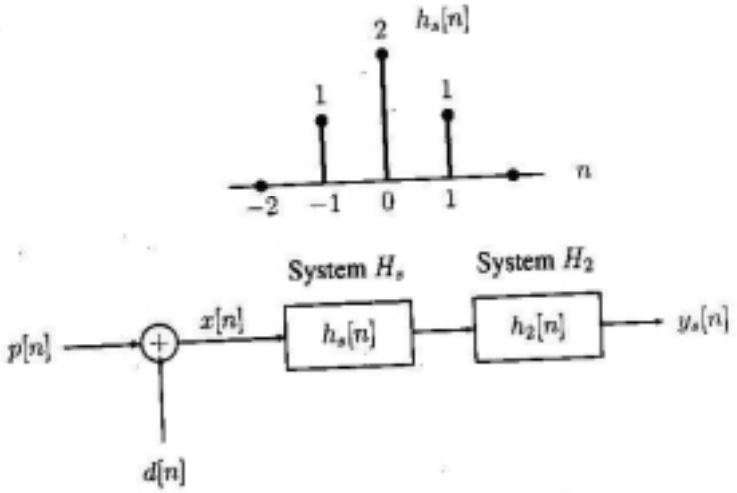
$$x[n] = p[n] + d[n]$$

$$y[n] = h_2[n] * (p[n] + d[n])$$

$$= \underbrace{p[n+1] - 2p[n] + p[n-1]}_{\text{در قسمت قبل به دست آوردیم}} + \underbrace{h_2[n] * (-\delta[n+1])}_{-h_2[n+1]}$$



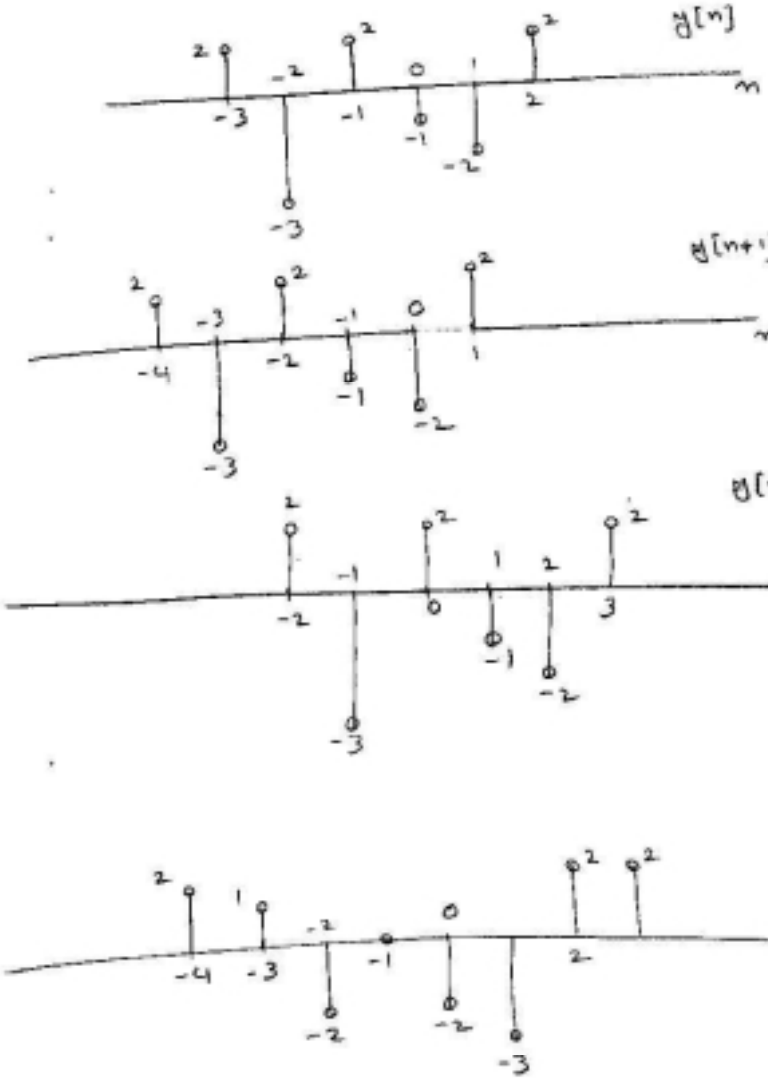
ج) حال به سیستم معرفی شده $h_s[n]$ را اضافه می نمائیم. مطلوب است خروجی به ازاء $p[n]$, $d[n]$ داده شده در دو قسمت "الف" و "ب".



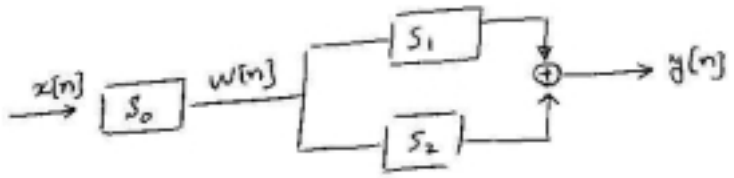
از آنجایی که سیستم های LTI جابجا پذیر هستند و ما در قسمت قبل خروجی $h_2[n]$ را به دست آوردیم حال آن خروجی را در ورودی $h_s[n]$ قرار می دهیم تا $y_s[n]$ محاسبه گردد.

$h_s[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$

$y_s[n] = y[n] * h_s[n] = y[n+1] + 2y[n] + y[n-1]$



بلوک دیاگرام سیستم LTI داده شده است



معادله تفاضلی ارتباط دهنده $w[n]$ و $x[n]$ به صورت $w[n]=x[n-3]-x[n-4]$ است.

پاسخ ضربه سیستم S_1 برابر $h_1[n]=(n+2)\big(u[n+3]-u[n-4]\big)$ است.

پاسخ ضربه سیستم S_2 برابر $h_2[n]=h_1[-n]$ است

معلوم است.

الف) پاسخ ضربه زیرسیستم S_0 و زیرسیستم S_2

ب) پاسخ ضربه کل سیستم $h[n]$

ج) پاسخ کل سیستم $y[n]$ اگر ورودی $x[n]=(0.99)^n\cos\left(\frac{2}{10}n\right)u[n]$ باشد.

$$w[n]=x[n-3]-x[n-4] \Rightarrow h_0[n]=\delta[n-3]-\delta[n-4]$$

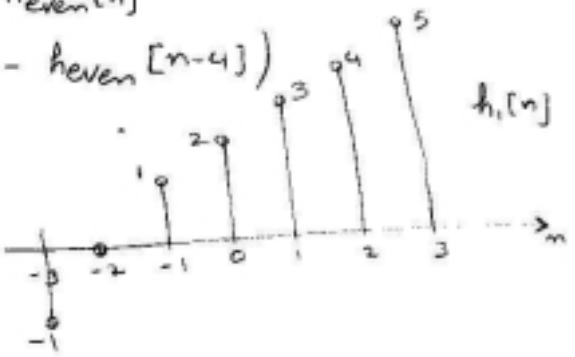
$$h_2[n]=h_1[-n] \quad , h_1[n]=(n+2)\big(u[n+3]-u[n-4]\big)$$

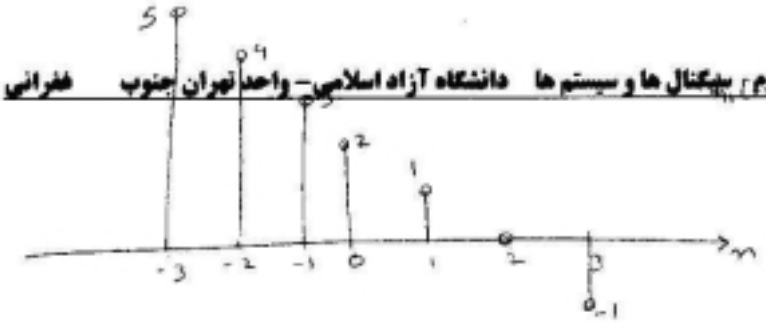
$$\Rightarrow h_2[n]=(-n+2)\big(u[-n+3]-u[-n-4]\big)$$

$$h[n]=h_0[n]*\big(h_1[n]+h_2[n]\big)=h_0[n]*\big(h_1[n]+h_1[-n]\big)$$

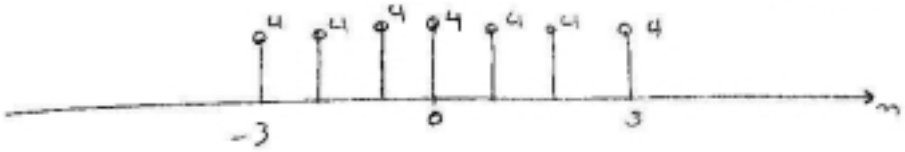
$$=2\,h_0[n]*\big(\underbrace{Even(h_1[n])}_{h_{even}[n]}\big)=2\big(\delta[n-3]-\delta[n-4]\big)*h_{even}[n]$$

$$h[n]=2\big(h_{even}[n-3]-h_{even}[n-4]\big)$$

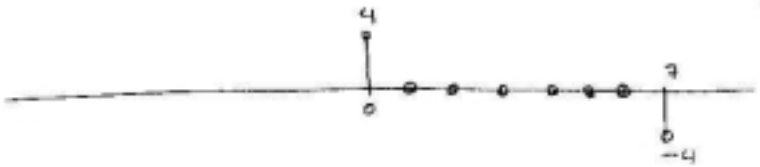




$h_1[n] + h_1[-n]$



$h[n]$



$h[n] = 4(\delta[n] + \delta[n-7])$ $g[n] = x[n] * h[n]$

$= 4(x[n] + x[n-7])$

$x[n] = (0.99)^n \cos(\frac{\pi}{10} n)\, u[n] \Rightarrow y[n] = 4(0.99)^n \cos(\frac{\pi}{10} n)\, u[n]$

$+ 4(0.99)^{n-7} \cos(\frac{\pi}{10}(n-7))\, u[n-7]$