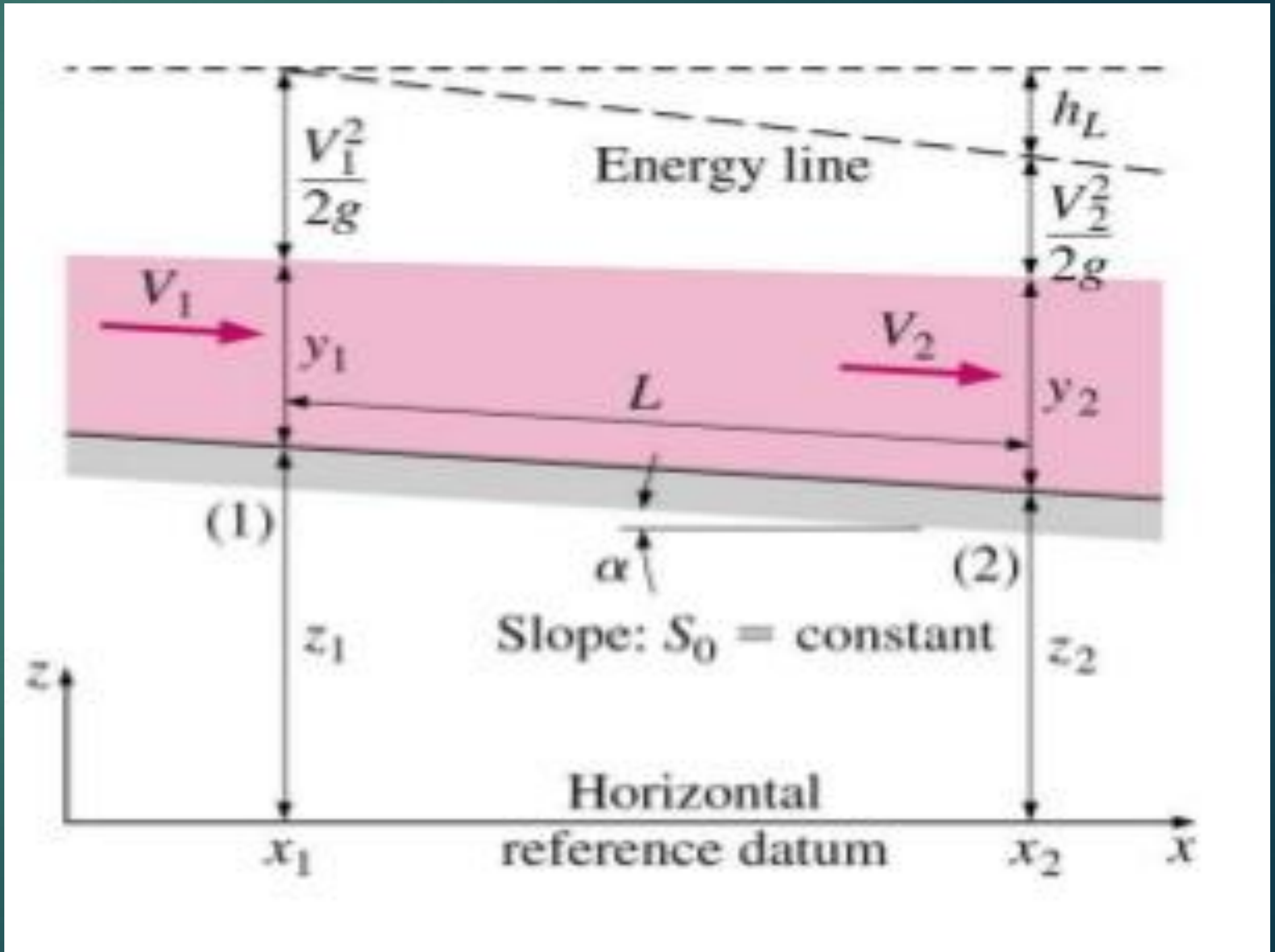
CHAPTER 2

Fundamental Equations

Equations in Open Channel Flow (Energy equation)

Energy equation

$$\frac{V_1^2}{2g} + y_1 + Z_1 = \frac{V_2^2}{2g} + y_2 + Z_2 + h_L$$



$\frac{V^2}{2g}$ is the velocity head

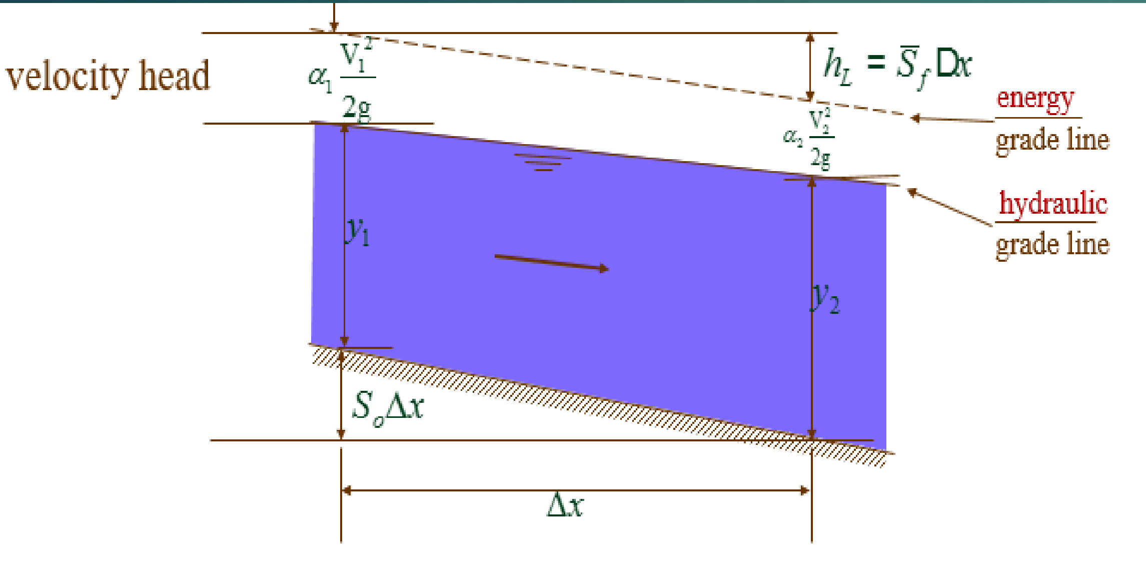
h_L is the head loss

Z is the static head

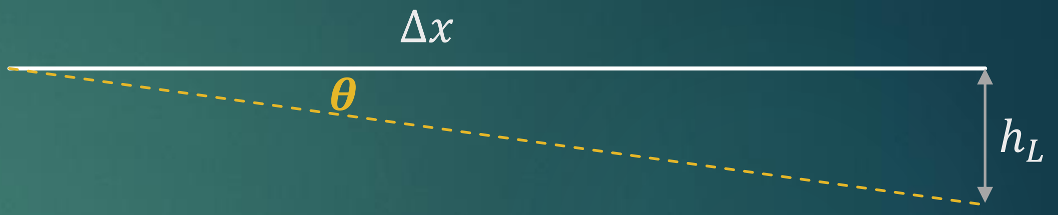
Equations in Open Channel Flow (Energy equation)

Energy equation

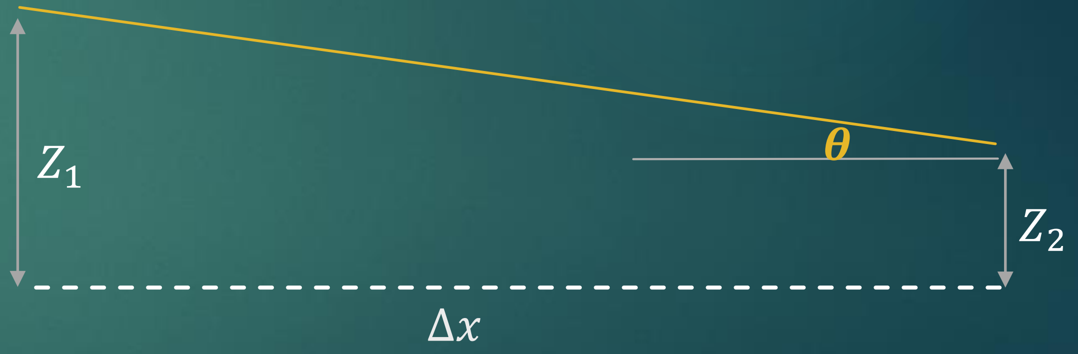
$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + (S_f - S_0)\Delta x$$



Bottom slope (S_o) not necessarily equal to EGL slope (S_f)



$$S_f = Tan\theta = \frac{h_L}{\Delta x} \rightarrow \boldsymbol{h_L = S_f \Delta x}$$

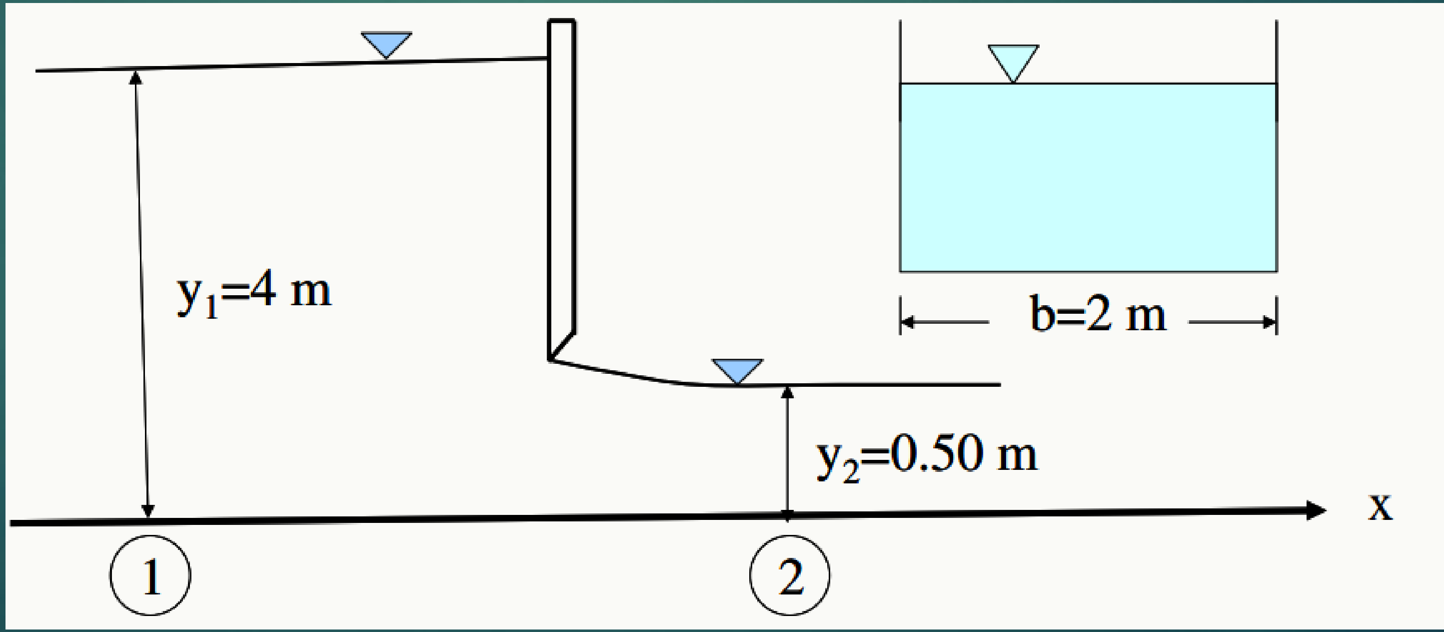


$$Tan\theta = \frac{Z_1 - Z_2}{\Delta x} = S_0 \rightarrow \boldsymbol{Z_1 - Z_2 = S_0 \Delta x}$$

Equations in Open Channel Flow (Energy equation)

Example 2-1

Water flows under a sluice gate in a horizontal rectangular channel of 2 m wide. If the depths of flow before and after the gate are 4 m, and 0.50 m, compute the discharge in the channel (no head loss).



Equations in Open Channel Flow (Energy equation)

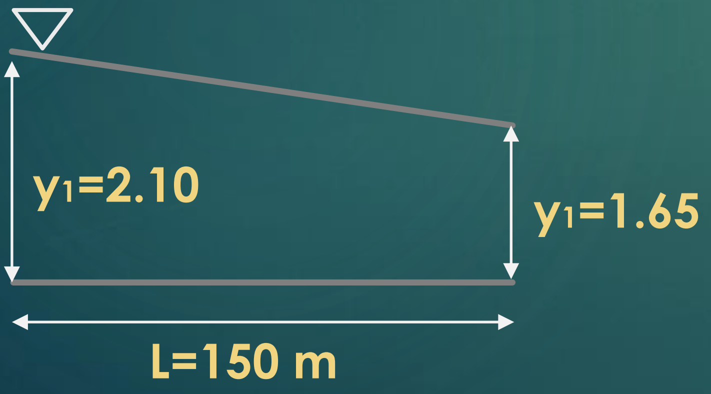
Example 2-2

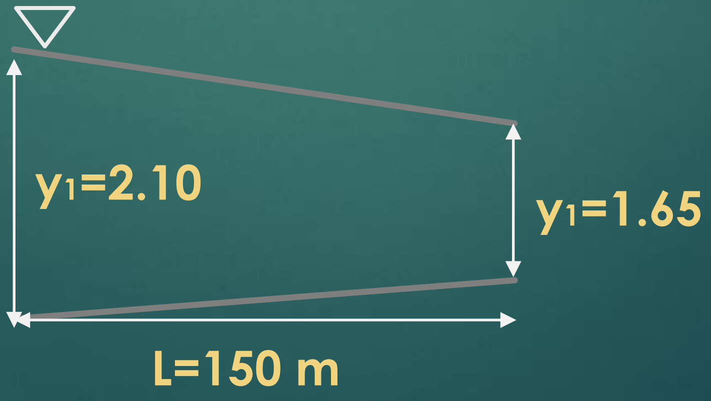
Determine the head loss in a rectangular open channel with the width of 3 (m) and flow rate of 8.5 (m3/s) if:

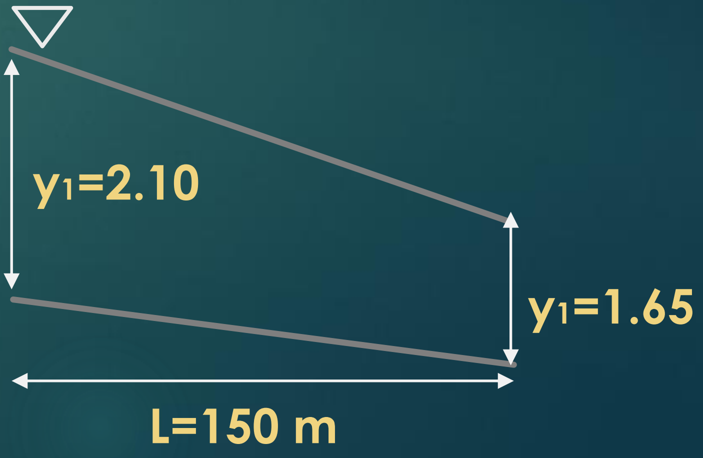
a. The bed slope is 0.0

b. The bed slope increasing in flow direction and it is 0.2 m in 100 m

c. The bed slope decreasing in flow direction and it is 0.2 m in 100 m







Specific Energy

$$\mathrm{E} = \frac{V^2}{2g} + y$$

E known as **specific energy**, (total energy per unit weight measured above bed level)

For a given discharge *Q*, the velocity is *Q/A*. Then:

$$E = \frac{Q^2}{2gA^2} + y$$

For a given specific energy, we have:

$$\underbrace{\frac{V_1^2}{2g} + y_1}_{E_1} + Z_1 = \underbrace{\frac{V_2^2}{2g} + y_2}_{E_2} + Z_2$$

$$E_1 - E_2 = \underbrace{Z_2 - Z_1}_{\Delta Z}$$

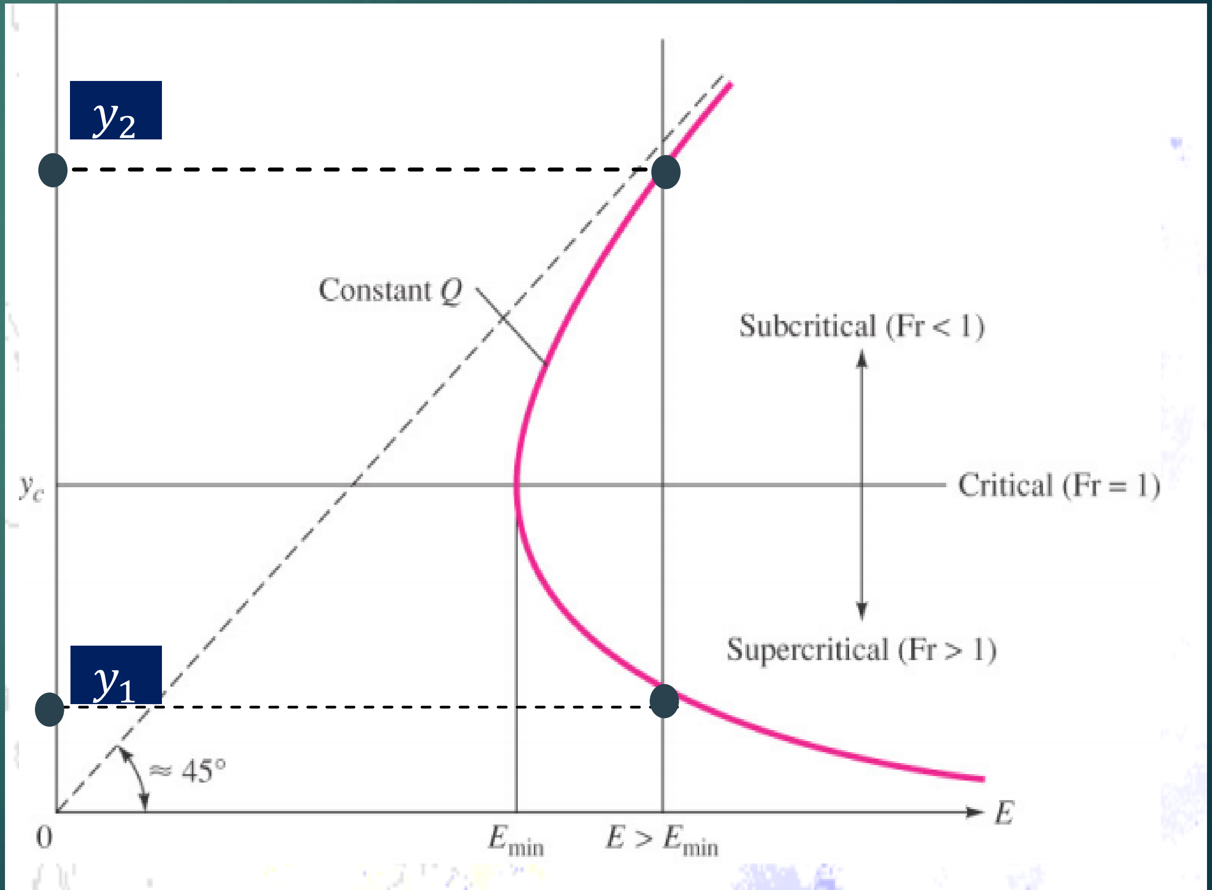
Specific Energy

- How many depths given a specific energy? **3** (in which one of them is negative)
- How many possible depths given a specific energy? **2**

$$E = \frac{Q^2}{2gA^2} + y$$

- The specific energy reaches a minimum value Es, called the critical point, characterized by the critical depth y_c and critical velocity V_c.

- y_1 and y_2 are **Alternate** depths (same specific energy)



Specific Energy

The specific energy diagram reveals that the flow needs the minimum specific energy, E_{min}, to pass a channel section at critical depth.

We can show this mathematically, by taking the derivative of following Equation with respect to y, noting $T = \frac{dA}{dy}$ and setting the derivative equal to zero as:

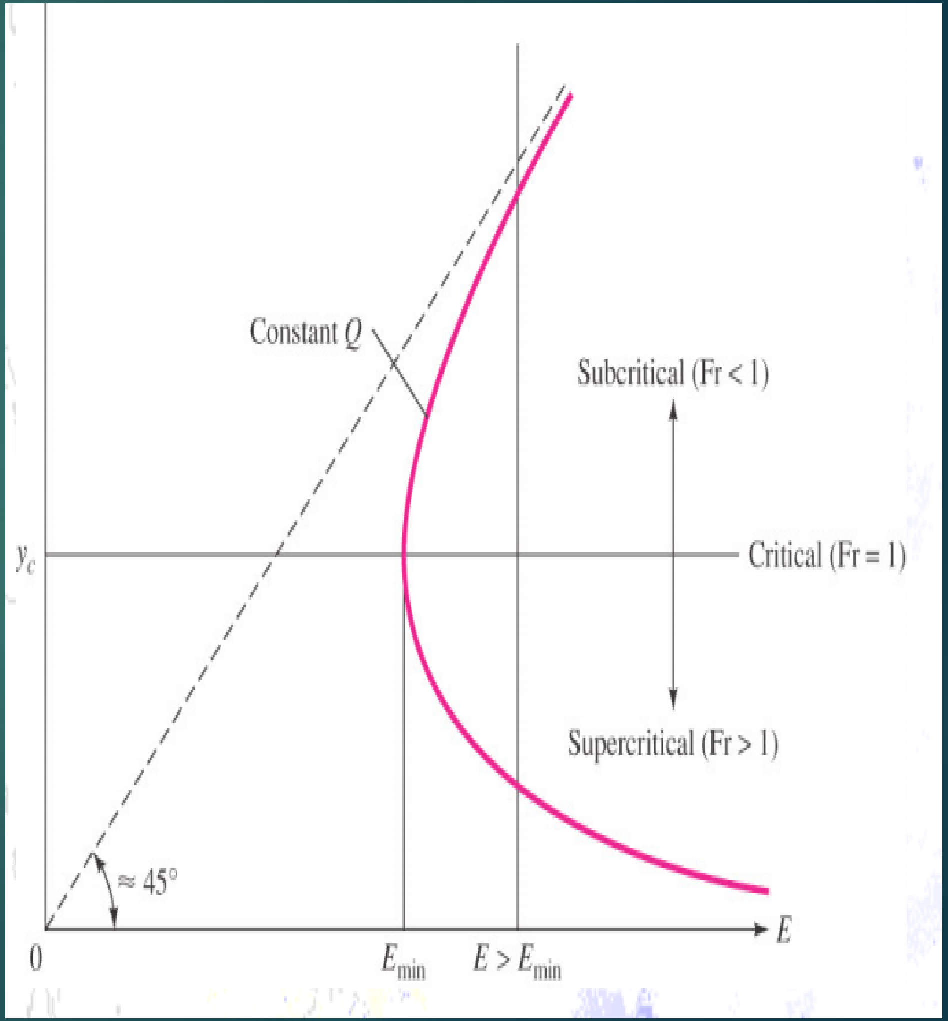
$$\frac{dE}{dy} = 1 - \frac{Q^2}{2g}\frac{2(dA/dy)}{A^3} = 1 - \frac{Q^2 T}{gA^3} = 1 - \frac{V^2 T}{gA} = \frac{V^2}{gD} = 1 - Fr^2 = 0$$

Thus, when the specific energy is minimum, the Froude number is equal to unity, and the flow depth is equal to the critical depth.

Specific Energy

Upper part of the curve

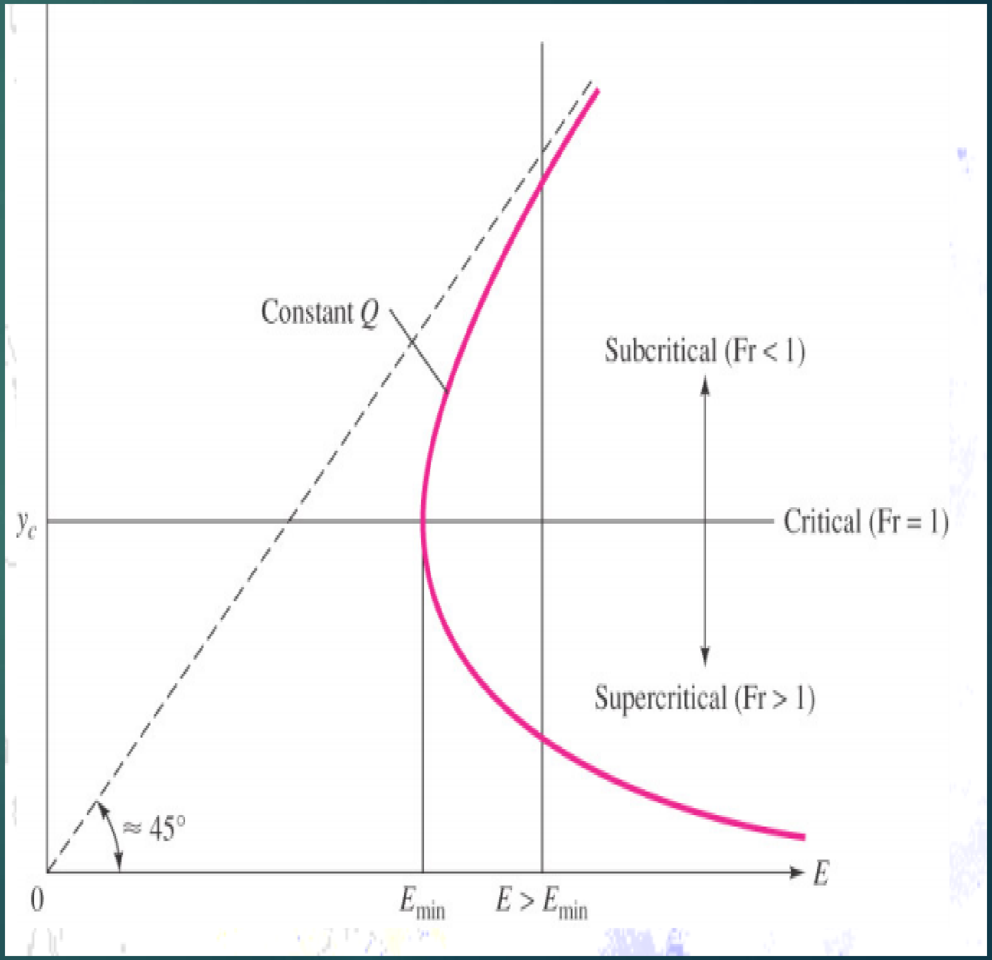
- If the value of "E" **increases** on the upper part of the curve, then "y" **increases**.
- For upper part, "y" is <u>greater</u> than "yc".
- For upper part of the curve, <u>velocity</u> is less than <u>critical velocity</u>.
- The flow in this portion is termed as *sub-critical flow.*
- The channel is called as deep channel for sub-critical flow.



Specific Energy

Lower part of the curve

- If the value of "E" **increases**, we can see that the value of "y" **decreases** in lower part of the curve.
- For lower part of the curve, "y" is <u>less</u> than "yc".
- For lower part of the curve, <u>velocity</u> is greater than <u>critical velocity</u>.
- For lower part of the curve, the flow is termed as *super critical flow*.
- The channel is called as shallow channel for super critical flow.



Specific Energy

By manipulating the resulting expression of $\frac{dE}{dy} = 1 - Fr^2$ we obtain:

$$\boxed{\frac{dy}{dx} = \frac{dE/dx}{1 - Fr^2}}$$

This relationship shows that, for subcritical flow ($Fr < 1.0$), the flow depth **increases** in the flow direction with increasing specific energy.

However, for supercritical flow ($Fr > 1$), the flow depth **decreases** in the flow direction with increasing specific energy.

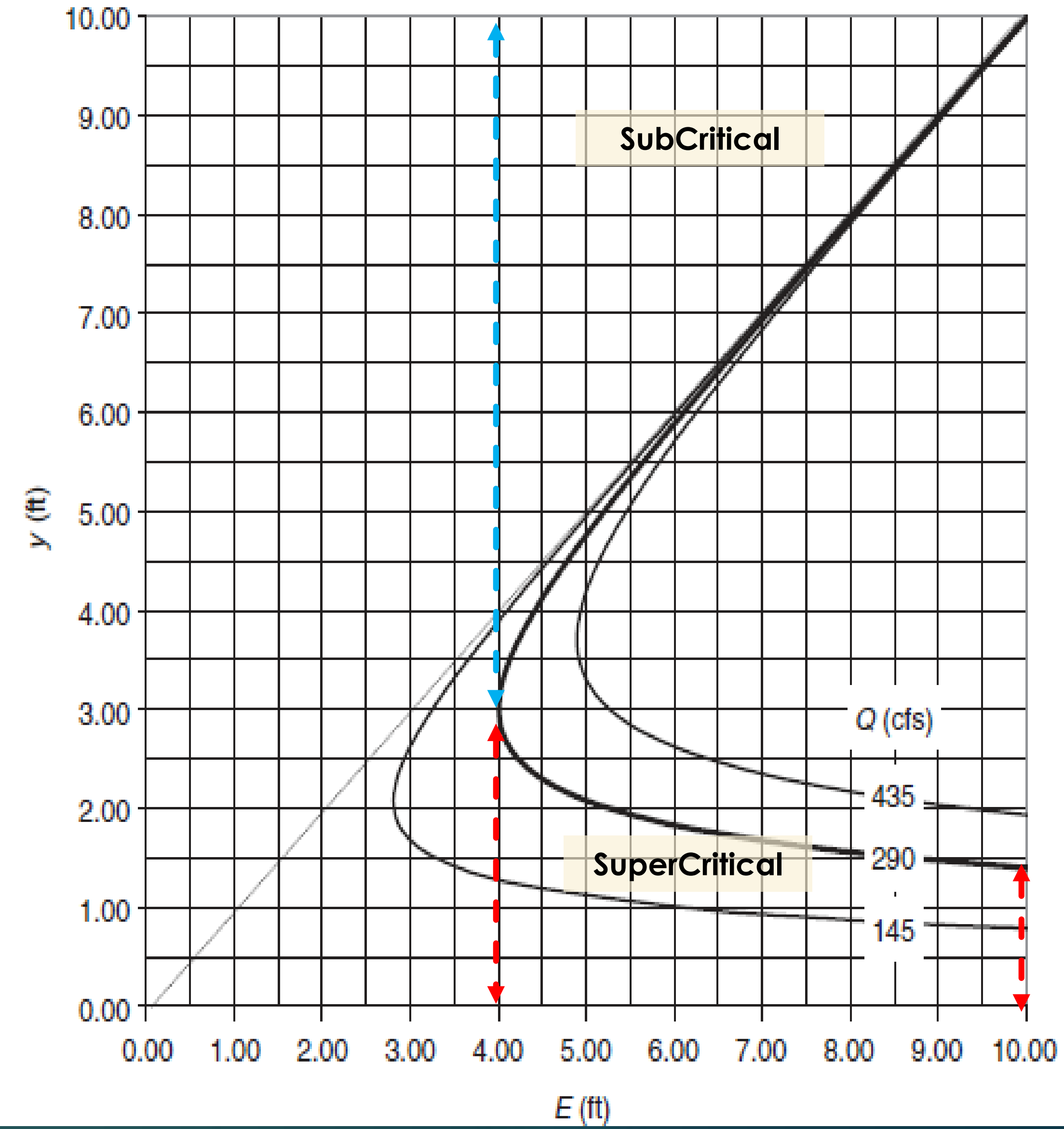
Specific Energy

Example 2-3

A trapezoidal channel has bottom width of $6\ ft$ and the side slope of $m = 2\ (1V{:}\,2H)$ and it carries a discharge of $Q = 290\ cfs$. Calculate and plot the specific energy diagram for this channel. Also, calculate and plot the specific energy for $Q = 135\ ft$ and $Q = 435\ cfs$.

Specific Energy

y (ft)	A (ft^2)	E (ft)	T (ft)	F_r
1.00	8.00	21.40	10.00	7.14
1.25	10.63	12.82	11.00	4.89
1.50	13.50	8.67	12.00	3.57
1.75	16.63	6.47	13.00	2.72
2.00	20.00	5.26	14.00	2.14
2.25	23.63	4.59	15.00	1.72
2.50	27.50	4.23	16.00	1.42
2.75	31.63	4.06	17.00	1.18
3.00	36.00	4.01	18.00	1.00
3.25	40.63	4.04	19.00	0.86
3.50	45.50	4.13	20.00	0.74
3.75	50.63	4.26	21.00	0.65
4.00	56.00	4.42	22.00	0.57
4.25	61.63	4.59	23.00	0.51
4.50	67.50	4.79	24.00	0.45
4.75	73.63	4.99	25.00	0.40
5.00	80.00	5.20	26.00	0.36
5.25	86.63	5.42	27.00	0.33
5.50	93.50	5.65	28.00	0.30
6.00	108.00	6.11	30.00	0.25
7.00	140.00	7.07	34.00	0.18
8.00	176.00	8.04	38.00	0.13
9.00	216.00	9.03	42.00	0.10
10.00	260.00	10.02	46.00	0.08



Specific Energy

The relationship between energy at any point and minimum energy is:

$$E_1 - E_c = \Delta Z_c$$

In a rectangular channel, we would have $Q = bq$ and $A = by$

$$E = \frac{q^2}{2gy^2} + y$$

In a rectangular channel when there is critical flow, we have:

$$y_c = \frac{2}{3}E_c = \frac{2}{3}E_{min}$$

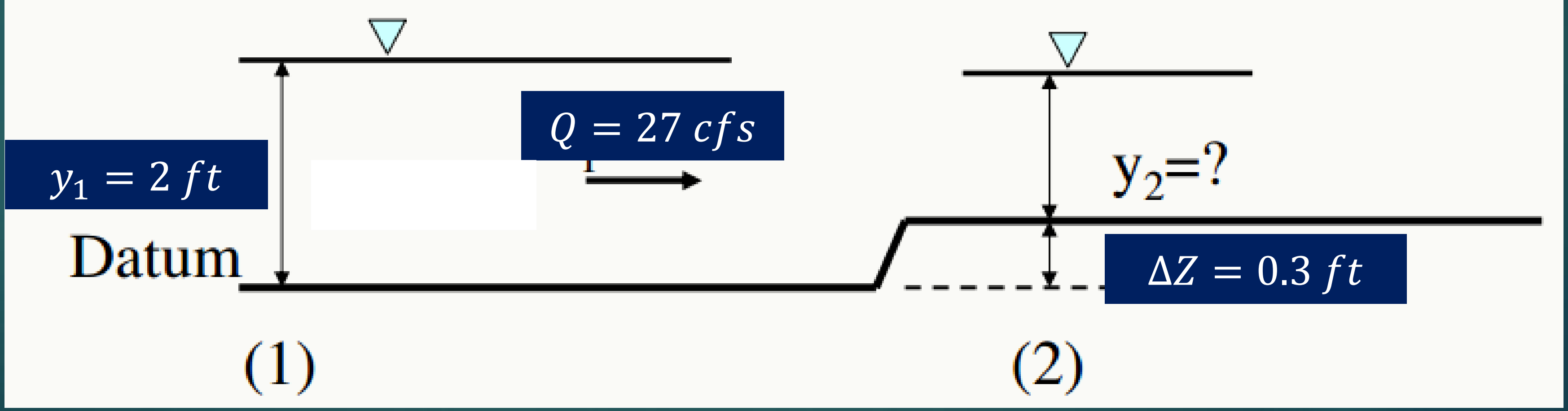
Specific Energy

Example 2-4

Water flow in a rectangular channel with the flow rate of 27 ft3/s, depth of 2 ft, and width of 4 ft.

a. Determine the flow condition (sub or super critical flow)

b. If there is an upward step of 0.3 ft (as shown in figure below), what would be the water depth on the upward step (assume there is no head loss)?



Momentum Principals

MASS TRANSFER

The mass (m) of an object is the quantity of matter contained in the object.

The volume (V) of an object is the space it occupies.

The density (ρ) is the mass per unit volume.

The mass transfer rate or mass flux in open-channel flow is the rate with which the mass is transferred through a channel section.

Momentum Principals

- In words, the momentum equation states that the change in momentum per unit time in any body is equal to the resultant of all the external forces acting on the body in unit time.

$$Ft = m(v_2 - v_1)$$

$$F = ma = \frac{m(v_2 - v_1)}{t}$$

$$\rho(density) = \frac{m\ (mass)}{V\ (volume)}$$

$$F = \frac{\rho V(v_2 - v_1)}{t}$$

$$F = \rho Q(v_2 - v_1)$$

or

$$\sum F = \rho Q(v_2 - v_1)$$

$$Q = \frac{V(volume)}{t}$$

$Q = discharge$ is the volume transfer rate

Momentum Principals

$$\sum F = \rho Q(v_2 - v_1) = F_1 - F_2 - F_B = \rho Q\left(\frac{Q}{A_2} - \frac{Q}{A_1}\right)$$

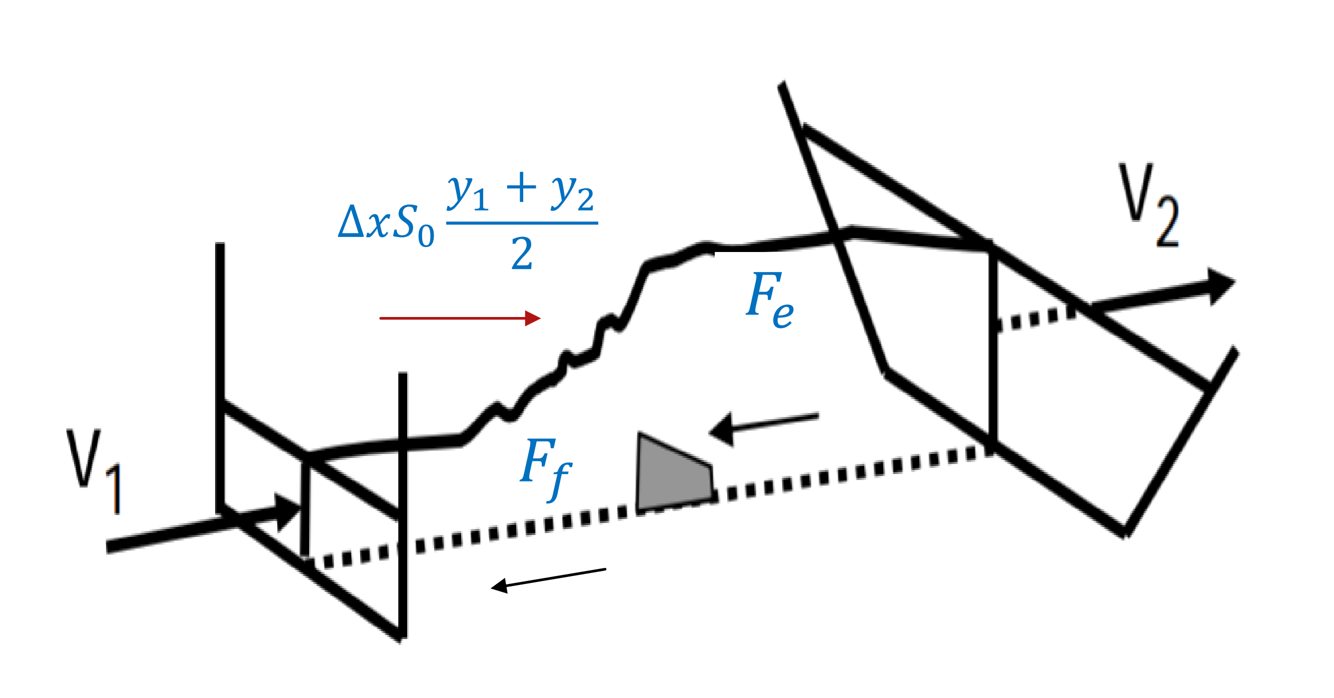
$$F_B = \frac{F_f}{\gamma} + \frac{F_e}{\gamma} - \Delta x S_0 \frac{y_1 + y_2}{2}$$

F_B is all forces

F_f is friction force resisting to flow, F_e is sum of all external forces (other than hydrostatic pressure, friction, and gravity forces).

Δx is distance between the two sections, and, S_0 is longitudinal bottom slope of the channel.

$\Delta x S_0 \frac{y_1 + y_2}{2}$ represents the component of weight of water between the two sections in the flow direction



Momentum Principals

$$\sum F = F_1 - F_2 - F_B = \rho Q(\frac{Q}{A_2} - \frac{Q}{A_1})$$

$$F = P.A = \gamma Y_C A$$

F is hydrostatic pressure force.

Y_C is distance from free surface to centroid of the flow section.

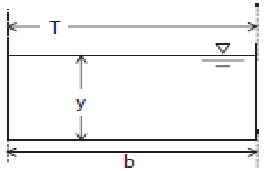
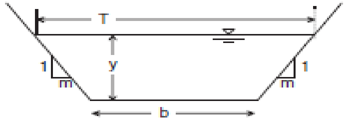
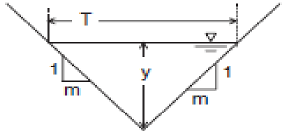
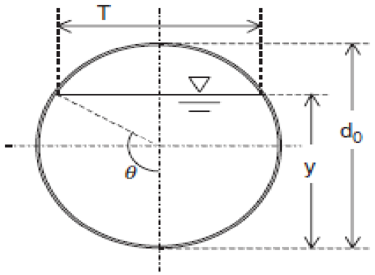
$$\sum F = \gamma Y_{C1} A_1 - \gamma Y_{C2} A_2 - F_B = \rho Q^2(\frac{1}{A_2} - \frac{1}{A_1})$$

Proof is required

M is Specific momentum

$$\begin{cases} M = Y_C A + \dfrac{Q^2}{gA} \\ M_1 = M_2 + \dfrac{F_B}{\gamma} \end{cases}$$

Relationships between y, A, and Y_C for various channel sections

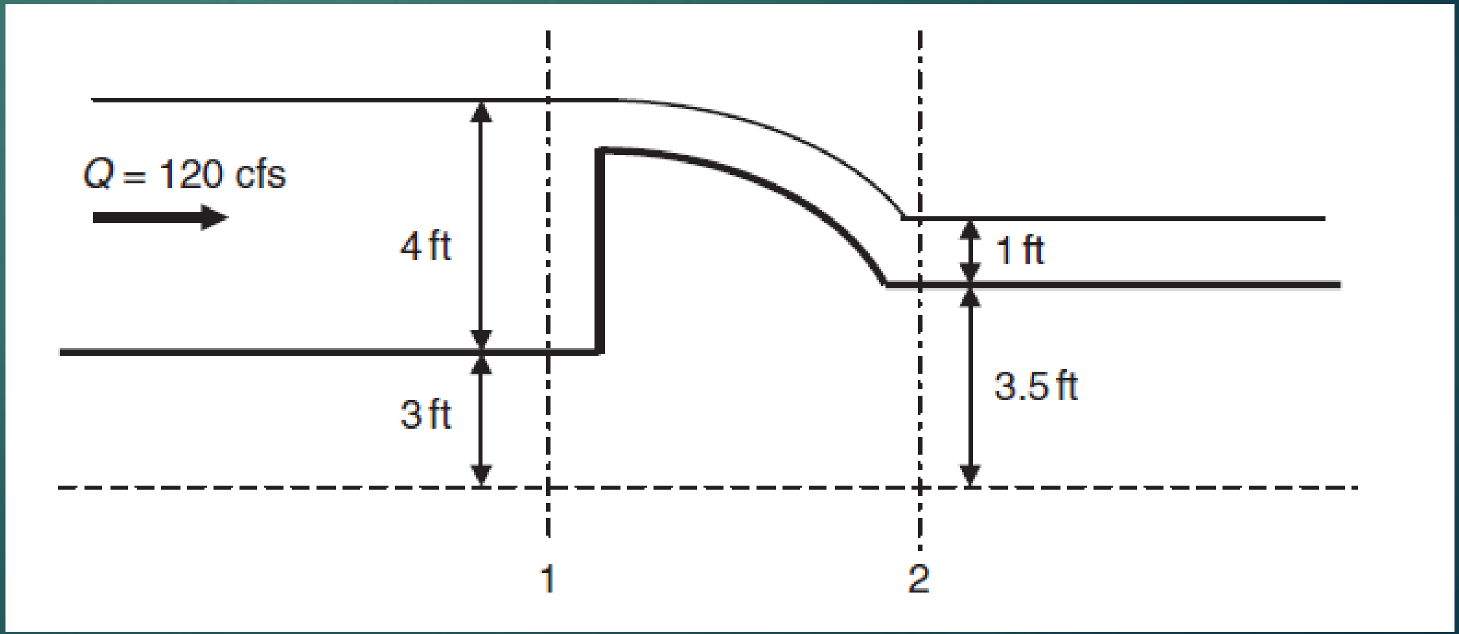
Section type	Area A	Product AY_C
Rectangular 	by	$b\dfrac{y^2}{2}$
Trapezoidal 	$(b+my)y$	$\dfrac{y^2}{6}(2my+3b)$
Triangular 	my^2	$\dfrac{my^3}{3}$
Circular 	$\dfrac{1}{8}(2\theta - \sin 2\theta)d_0^2$ $\theta = \pi - \text{arc}\ \cos[(y-d_0/2)/(d_0/2)]$	$\dfrac{d_0^3}{24}(3\sin\theta - \sin^3\theta - 3\theta\cos\theta)$ $\theta = \pi - \text{arc}\ \cos[(y-d_0/2)/(d_0/2)]$

Y_C = vertical distance from the free surface to centroid of flow section (see Chapter 1).

Momentum Principals

Example 2-5

The channel shown in figure is rectangular in cross-section, and it is 10 ft wide. Suppose the friction forces and the weight component in the flow direction are negligible. Determine the magnitude and the direction of the force exerted by flow on the spillway.

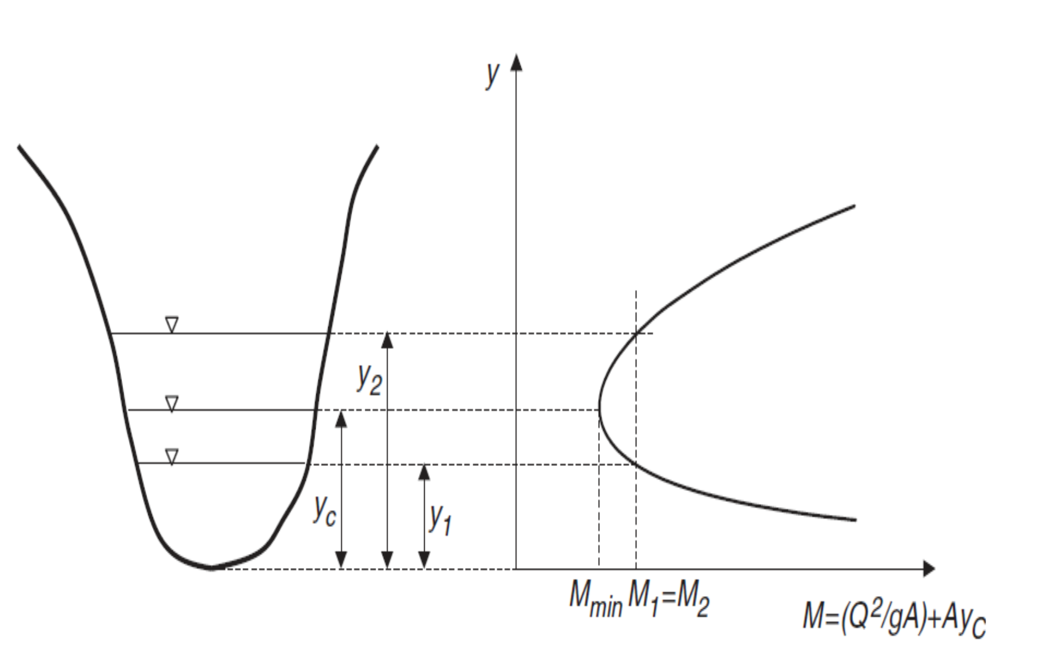


Specific Momentum Diagram for Constant Discharge

Specific momentum is defined as:

$$M = \left(\frac{Q^2}{gA} + Y_C A\right)$$

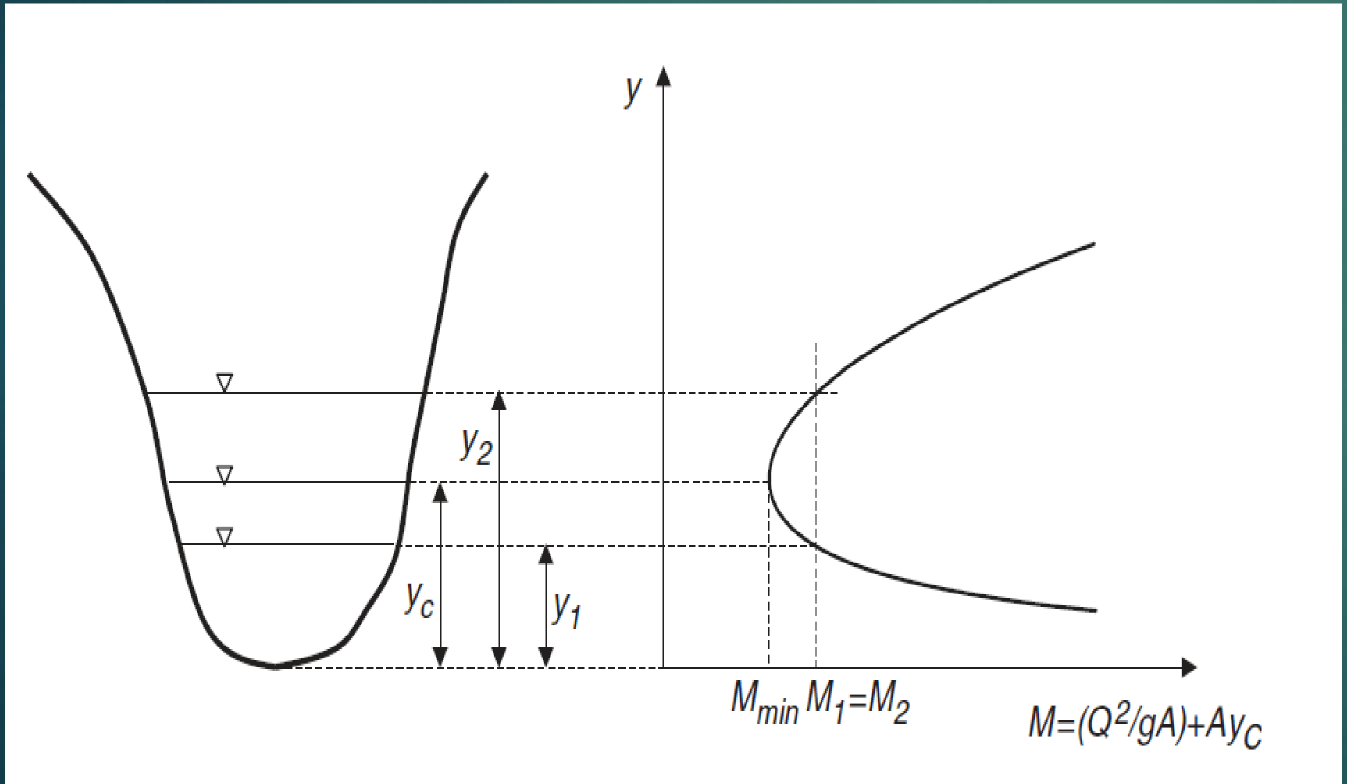
and a plot of flow depth versus the specific momentum for a constant discharge is called the specific momentum diagram.



A specific momentum diagram indicates that the same discharge can pass through a channel section at two different flow depths corresponding to the same specific momentum.

Specific Momentum Diagram for Constant Discharge

- These depths, marked as y_1 (supercritical) and y_2 (subcritical) in the figure, are called the conjugate depths (Henderson 1966).



- The minimum momentum required to pass a given discharge through the section occurs at critical depth.
- The upper limb of the diagram is for subcritical flow, and the lower limb represents supercritical flow.

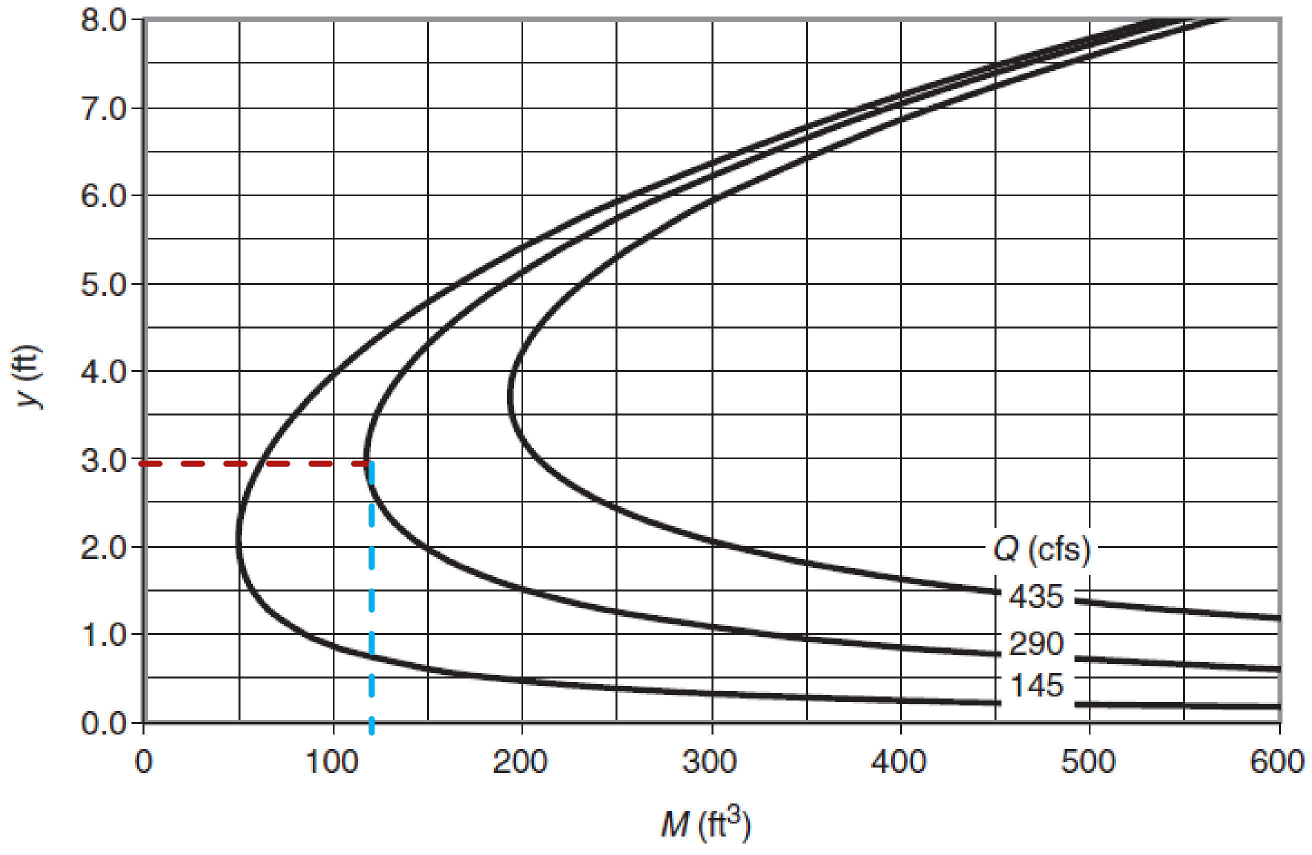
Specific Momentum Diagram for Constant Discharge

Example 2-6

A trapezoidal channel has a bottom width of $b = 6ft$ ft and side slopes of $m = 2$ $(1V\colon 2H)$, and it carries a discharge of $Q = 290\ cfs$. Calculate and plot the specific momentum diagram for this channel. Also, calculate and plot the specific energy diagrams for the same channel for $Q = 135cfs$ and $Q = 435\ cfs$.

Specific Momentum Diagram for Constant Discharge

y (ft)	A (ft^2)	AY_C (ft^3)	M (ft^3)
0.50	3.50	0.83	747.06
0.60	4.32	1.22	605.81
0.80	6.08	2.26	431.83
0.90	7.02	2.92	374.97
1.00	8.00	3.67	330.14
1.25	10.63	5.99	251.81
1.50	13.50	9.00	202.47
1.75	16.63	12.76	169.86
2.00	20.00	17.33	147.92
2.25	23.63	22.78	133.33
2.50	27.50	29.17	124.14
2.75	31.63	36.55	119.14
3.00	36.00	45.00	117.55
3.25	40.63	54.57	118.86
3.50	45.50	65.33	122.74
3.75	50.63	77.34	128.93
4.00	56.00	90.67	137.31
4.25	61.63	105.36	147.75
4.50	67.50	121.50	160.19
4.75	73.63	139.14	174.61
5.00	80.00	158.33	190.98
5.25	86.63	179.16	209.31
5.50	93.50	201.67	229.60
6.00	108.00	252.00	276.18
7.00	140.00	375.67	394.32
8.00	176.00	533.33	548.17

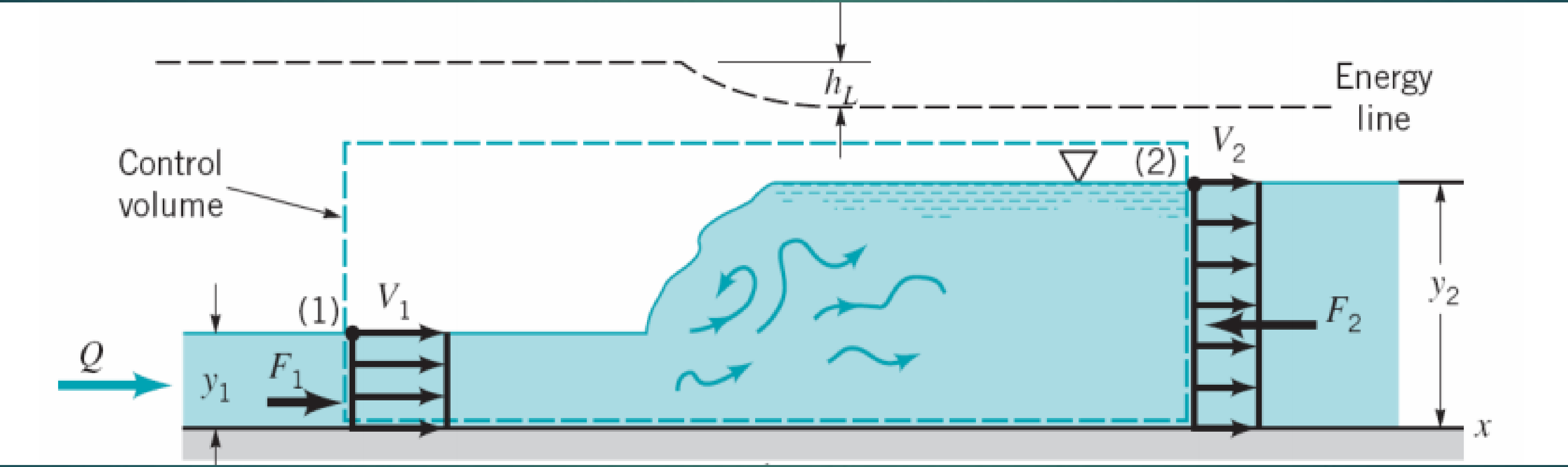


Hydraulic Jump

- When subcritical flow accelerates into the supercritical state the transition often is smooth with gradually increasing velocity and decreasing depth bringing about a smooth drop in the water surface until the alternate depth is achieved.
- When supercritical flow changes to subcritical flow, however, there is no smoothing of the water surface upstream of the transition.
- As a result the transition to subcritical flow is sudden and marked by an abrupt discontinuity, or hydraulic jump, in the water.
- The greater the difference between the alternate depths the more severe the hydraulic jump.

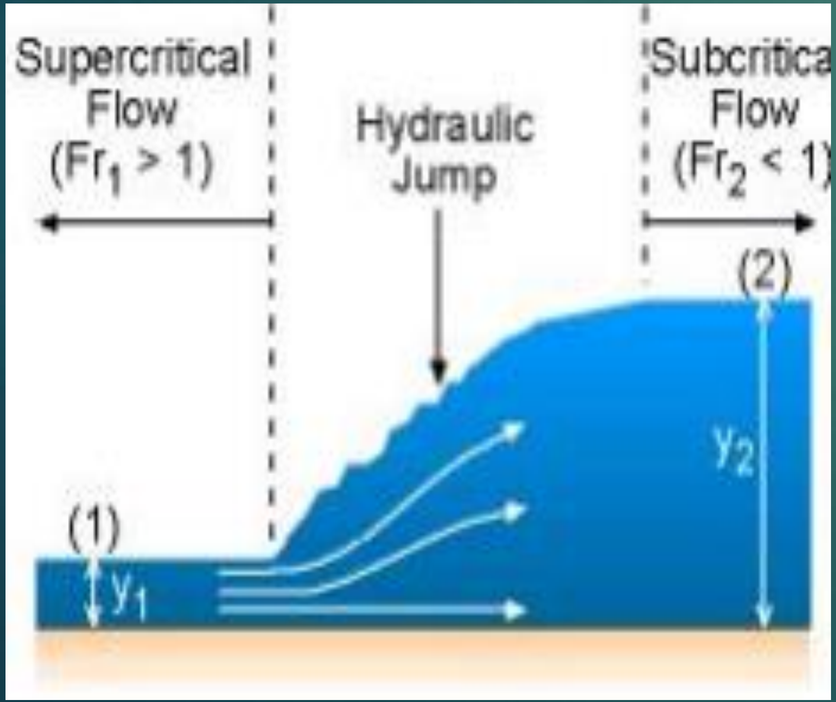
Hydraulic Jump

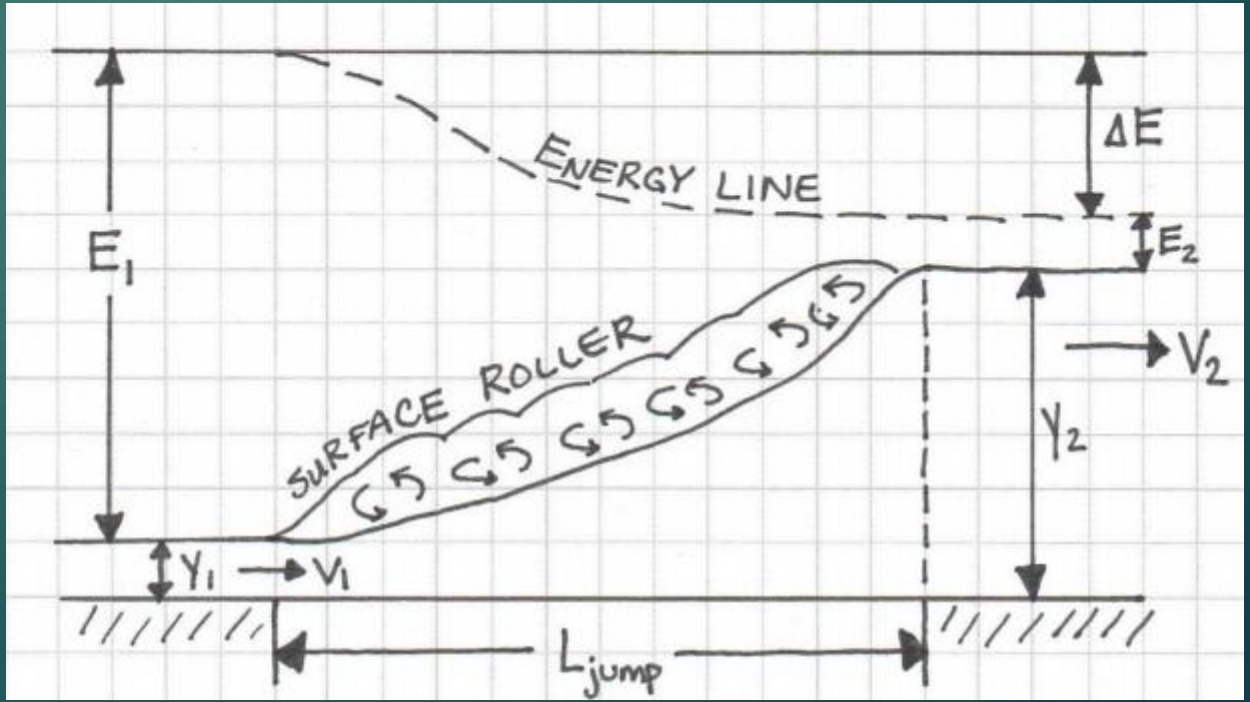
- When flow is supercritical in a upstream section of a channel and is then forced to become subcritical in a downstream section, the Hydraulic Jump occurs.



- Conjugate depths refer to the depth (y_1) upstream and the depth (y_2) downstream of the hydraulic jump.

Hydraulic Jump

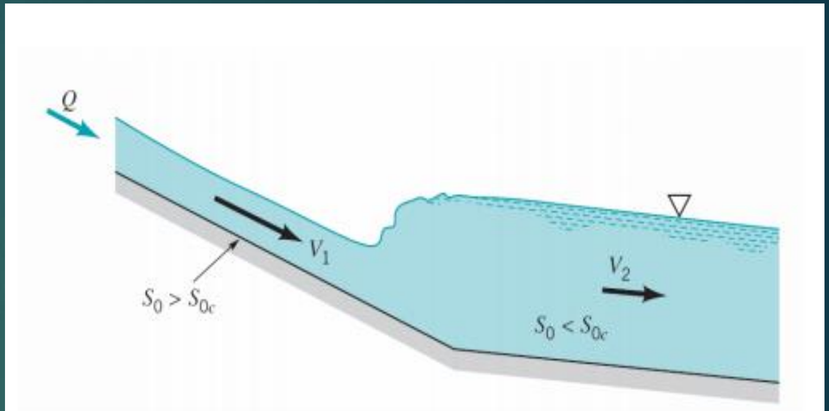


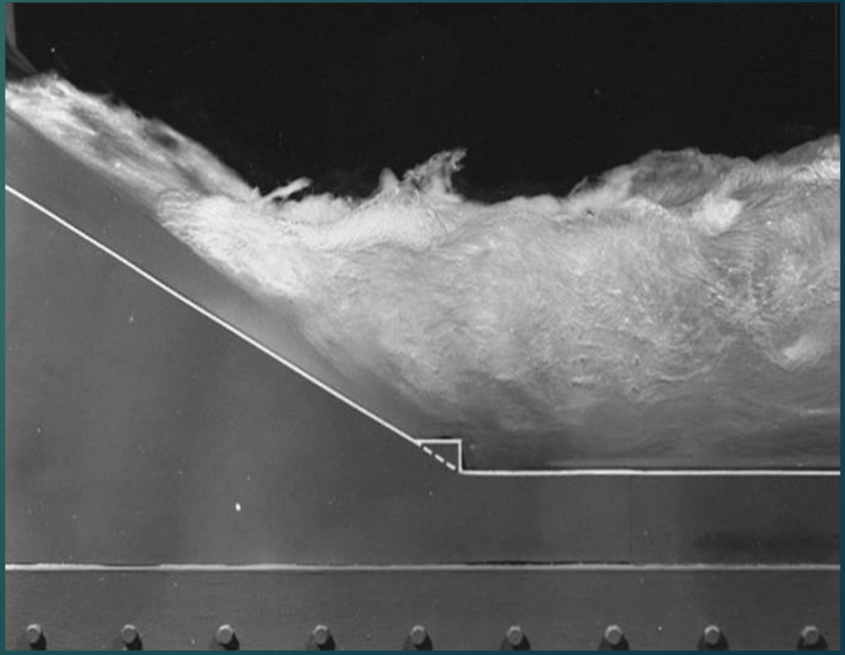


Hydraulic Jump

Hydraulic Jump

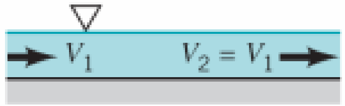
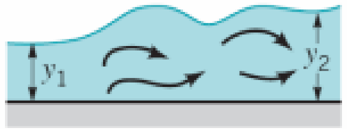
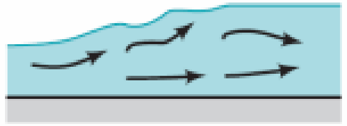
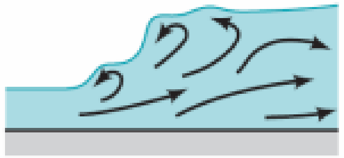
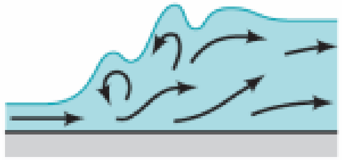
- Jump caused by a change in channel slope.
- Jump caused by a hydraulic structure





Hydraulic Jump

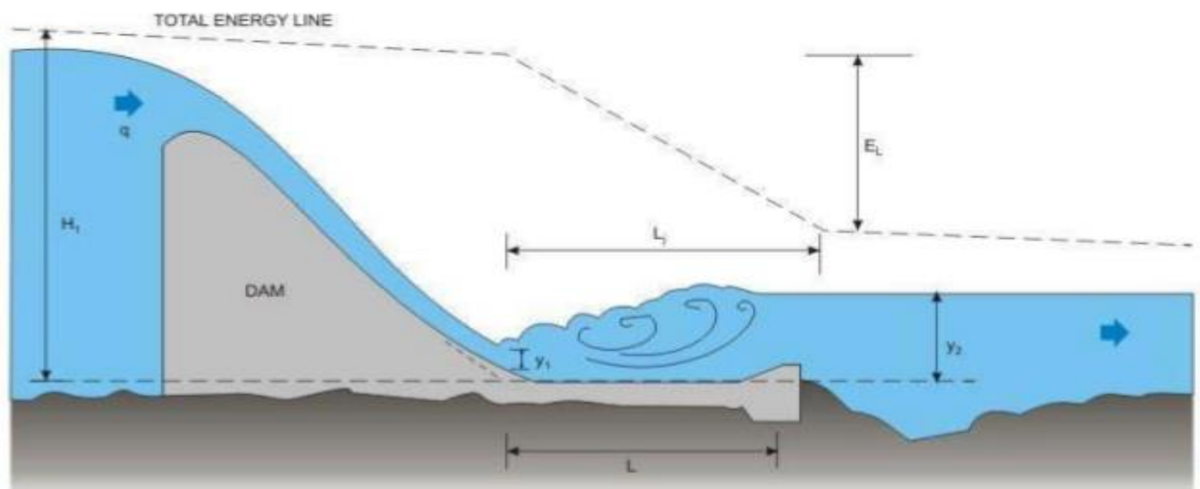
- Jump classification

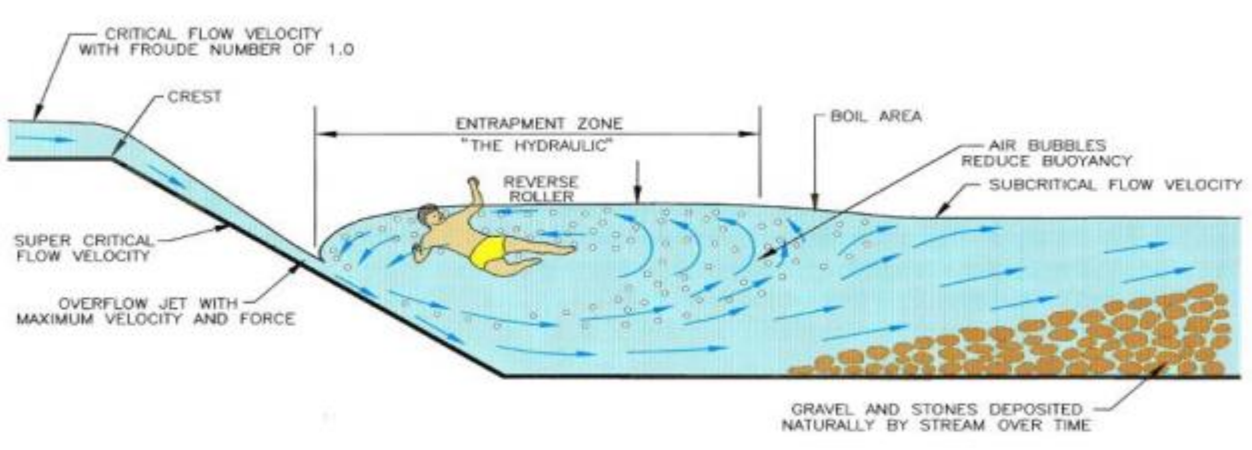
Fr_1	y_2/y_1	Classification	Sketch
<1	1	Jump impossible	
1 to 1.7	1 to 2.0	Standing wave or undulant jump	
1.7 to 2.5	2.0 to 3.1	Weak jump	
2.5 to 4.5	3.1 to 5.9	Oscillating jump	
4.5 to 9.0	5.9 to 12	Stable, well-balanced steady jump; insensitive to downstream conditions	
>9.0	>12	Rough, somewhat intermittent strong jump	

Hydraulic Jump

Facts

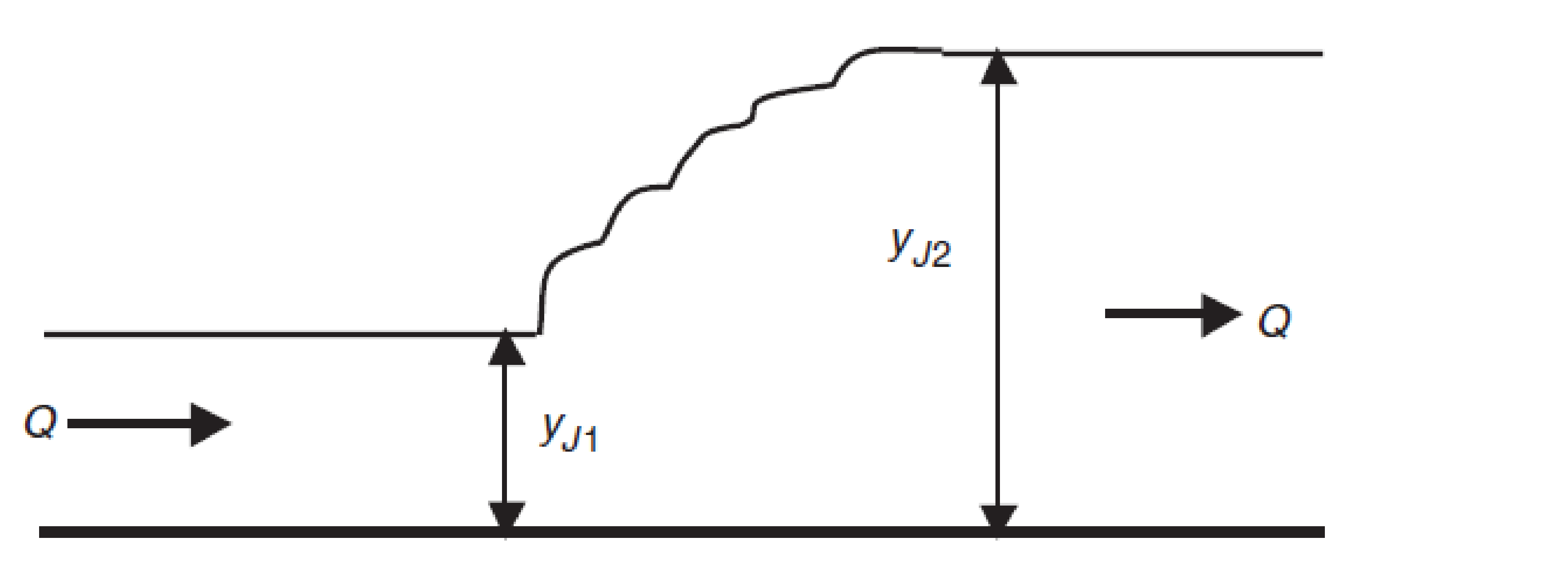
- Dissipates the energy of water over a spillway to reduce the erosion issue.
- Traps air in the water that could be useful for removing wastes and pollution in the water
- Reverses the flow of water, it can be used to mix chemicals for water purification
- Maintains a high water level on the downstream side that is useful for irrigation purposes





Hydraulic Jump

- The nature of the hydraulic jump cannot be accounted for by use of the energy equation, because there is a substantial dissipation of energy.
- As Energy loss due the hydraulic jump is usually significant and Unknown, we need to use conservation of momentum.

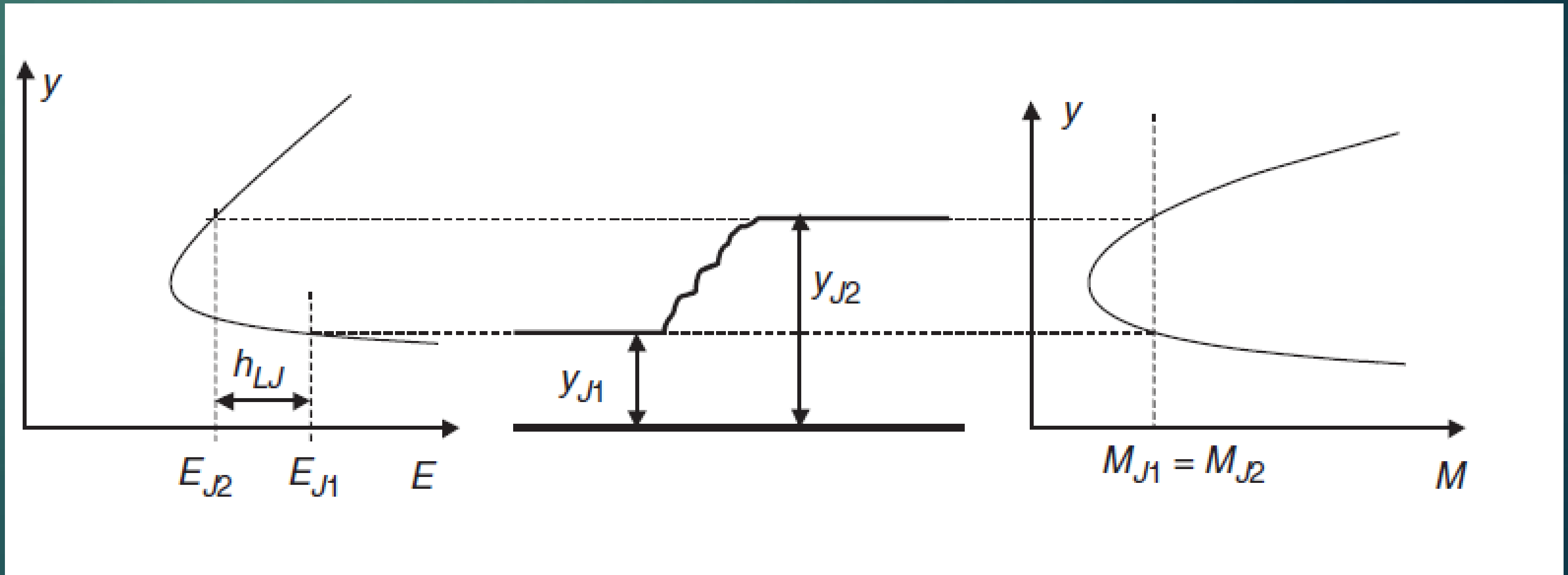


Hydraulic Jump

- However, usually the friction force between sections J1 (before jump) and J2 (after jump) is negligible.
- Also, if the channel is nearly horizontal, the component of the weight in the flow direction is negligible.
- Then, in the absence of any other external forces (other than pressure forces), we have:

$$M_1 = M_2 + \frac{F_B}{\gamma} \rightarrow 0$$

$$M_1 = M_2$$



Hydraulic Jump

- In most open-channel flow problems involving hydraulic jumps, one of the two depths $y_{before\ jump}$ or $y_{after\ jump}$ would be known, and we would need to calculate the second one.
- This equation is valid for any cross-sectional shape. $M_1 = M_2$
- Once this equation is solved for the unknown depth, the energy equation can be used to calculate the head loss due to the hydraulic jump.
- For rectangular channels, an explicit solution is available for the equation.
- For most other types, the solution requires either a trial and error procedure.

Example 2-7

A trapezoidal channel has a bottom width of $b = 6ft$ ft and side slopes of $m = 2$ $(1V\!: 2H)$, and it carries a discharge of $Q = 290\,cfs$. A hydraulic jump occurs in this channel. The flow depth just before the jump is $y_{j1} = 0.9\,f$t. Determine the depth after the jump.

Hydraulic Jump

A momentum equation for sequent depths of a hydraulic jump on a level floor in a rectangular channel can be derived by applying momentum equation between sections 1 and 2 as given below.

$$M_1 = M_2$$

$$M = \left(\frac{Q^2}{gA} + Y_C A\right)$$

$$\left(\frac{Q^2}{gA_1} + Y_{C1} A_1\right) = \left(\frac{Q^2}{gA_2} + Y_{C2} A_2\right)$$

Proof: In a rectangular channel, the relation between depth of flow before and after the jump is:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$$

Momentum and Hydraulic Jump

- In a rectangular channel, the relation between depth of flow before and after the jump is:

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$$

- The height of hydraulic jump is:

$$h_j = y_2 - y_1$$

- The head loss or energy dissipation during a jump is:

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_2y_1}$$

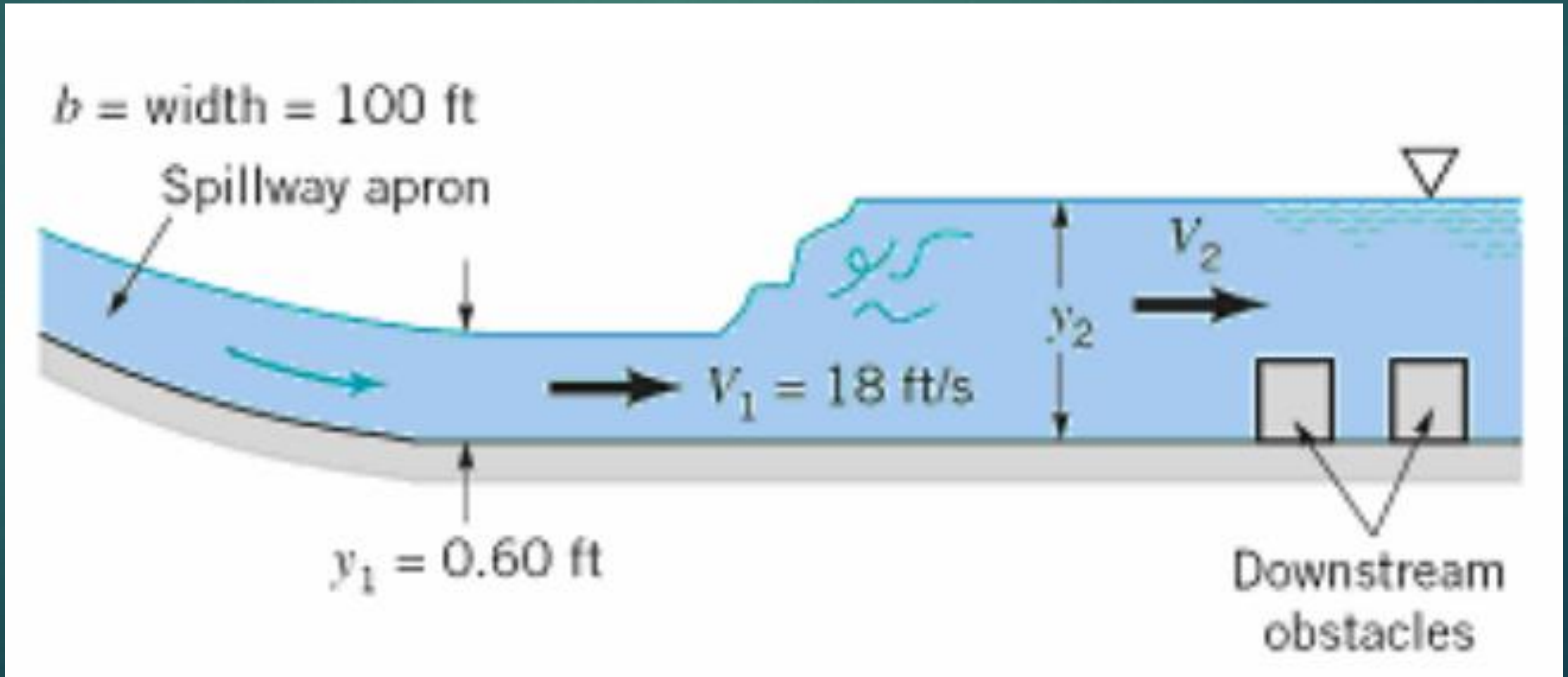
- The power lost by hydraulic jump is:

$$P = \gamma Q \Delta E \rightarrow \begin{cases} \gamma = \text{Specific weight of water} \\ Q = \text{Discharge} \end{cases}$$

Hydraulic Jump

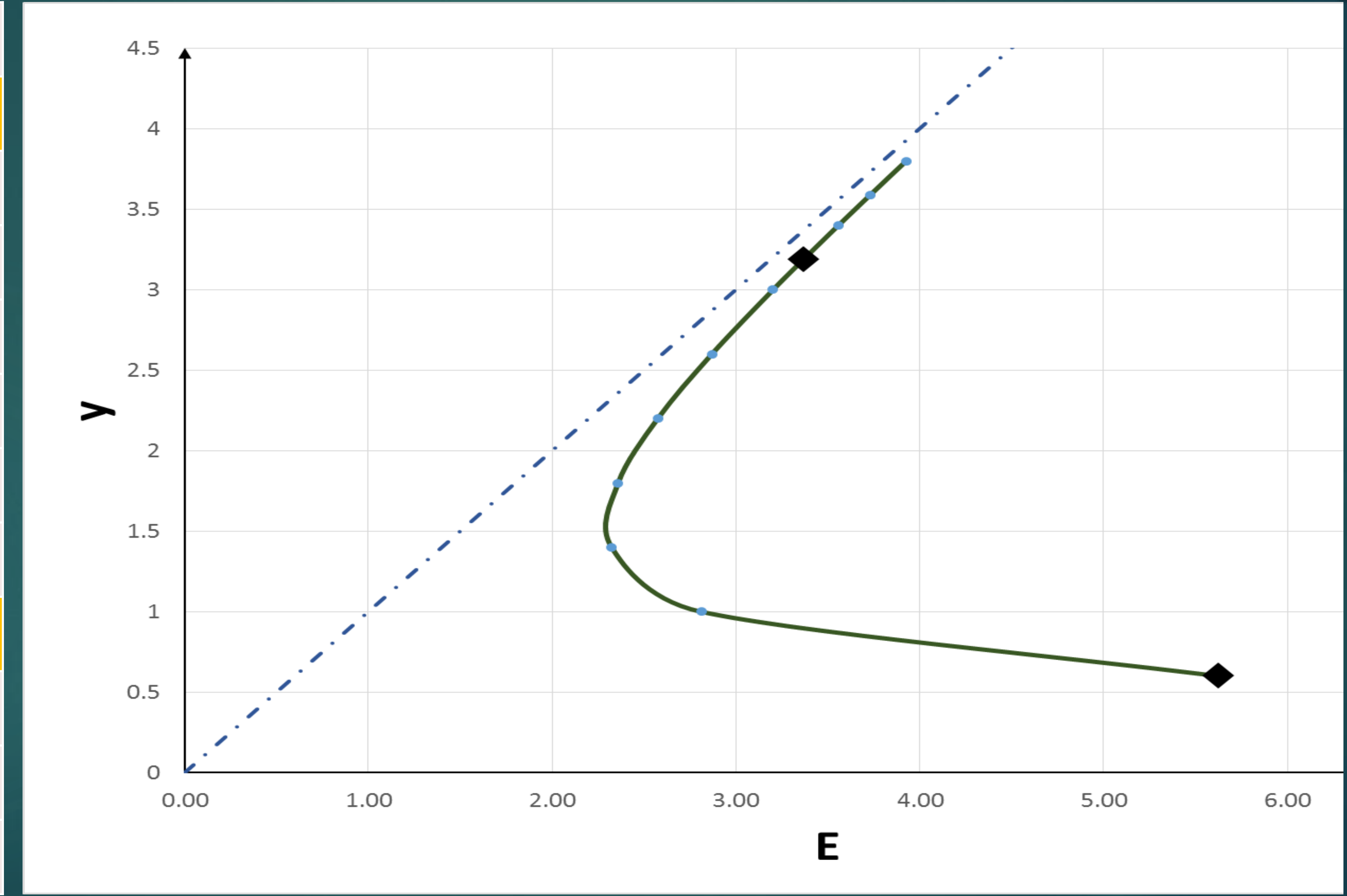
Example 2-8

Water downstream of an spillway flows in a 100 ft wide rectangular channel with the depth of 0.6 ft and velocity of 18 ft/s. Determine the depth after the jump, the Froude numbers before and after the jump, height of the jump, the head loss and power dissipated during the jump. And plot the y vs specific energy.



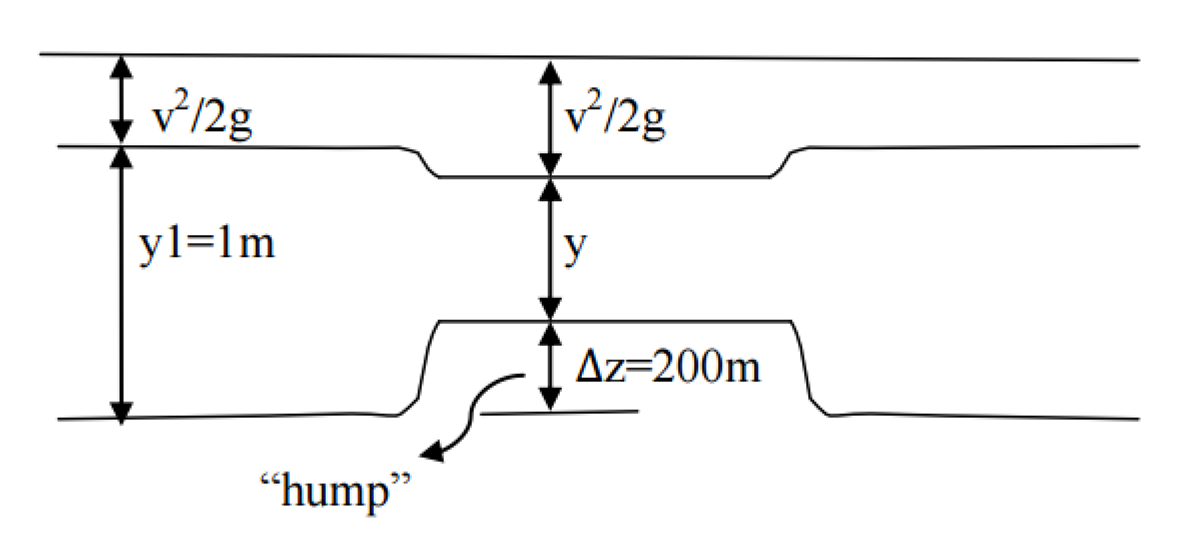
Hydraulic Jump

y	E
0.6	5.63
1	2.81
1.4	**2.32**
1.8	2.36
2.2	2.57
2.6	2.87
3	3.20
3.19	3.37
3.4	3.56
3.59	3.73
3.8	3.93



Homework 2

Q2-1. A rectangular channel b= 1.5m , Q= 900L/s, the depth of flow before the hump is 1m and Δz=200mm , compute the depth of flow above the hump.



Homework 2

Q2-2. A rectangular channel b= 2.0 m , Q= 2 m3/s, the depth of uniform flow before the hump is 0.8 m. What should be the height of the hump (Δz) to have critical flow over it (no head loss).

Homework 2

Q2-3 A 36-inch storm sewer (d_0= 3.0 ft) carries a discharge of Q = 30 cfs.

a. Determine the critical depth.
b. Calculate and plot the specific energy diagram for this channel.
c. Also calculate the specific energy diagrams for Q = 10 cfs and Q = 20 cfs.

Homework 2

Q2-4. Hydraulic jump occurs in a rectangular channel with the flow rate of 500 ft3/s and width of 10 ft. If the depth of flow before the jump is 3.1 ft, what would be the depth after the jump, head loss during the jump, and velocity after and before the jump.

Homework 2

Q2-5. Hydraulic jump occurs in a rectangular channel with the width of 9 m. If the depths of flow before and after the jump are 1.55 m and 3.08 m, respectively, what would be the flow rate in the channel (Assume $R_h=y$)?