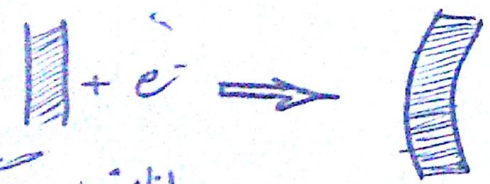
Chapter 1 Introduction

Processing → Structure → Properties → Performance

Advanced Material
- Semiconductors
- Bio Materials
- Smart Materials
- Nano Materials → دیگر ترکیب شیمیایی تعیین شده نیست و سطح و ساختار تعیین شده است.

Smart Material
1. shape alloy memory → در دماهای مختلف به شکل سابق خود بر می‌گردند (استنت‌های قلبی + سیم‌های انبساطی)
2. Piezoelectric → در اختلاف پتانسیل مختلف، با اعمال جریان، به شکل دیگری در می‌آیند (مثال: بلندگو + میکروفن)
3. Magneto strictive Material
4. Electro rheological fluids
5. Magneto rheological fluids → با اعمال میدان مغناطیسی گرانروی مختلفی پیدا می‌کنند. (مثال: [illegible] کمک فنر خودرو)

→ Sensor + actuators → پالس الکتریکی → حرکت مکانیکی



(نانوتیوب + گرافن)

Chapter 2 Atomic Structure and Interatomic Bonding

The attractive bonding forces are **coulombic**

$$E_A = -\frac{A}{r}$$

Theoretically, the constant A is equal to

$$A = \frac{1}{4\pi\epsilon_0}(|Z_1|e)(|Z_2|e)$$

An analogous equation for the repulsive energy is[5]

$$E_R = \frac{B}{r^n}$$

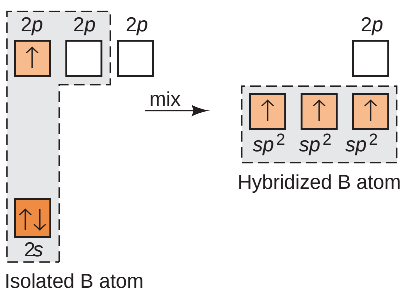
$$\begin{aligned} E_N &= \int_r^{\infty} F_N\,dr \\ &= \int_r^{\infty} F_A\,dr + \int_r^{\infty} F_R\,dr \\ &= E_A + E_R \end{aligned}$$

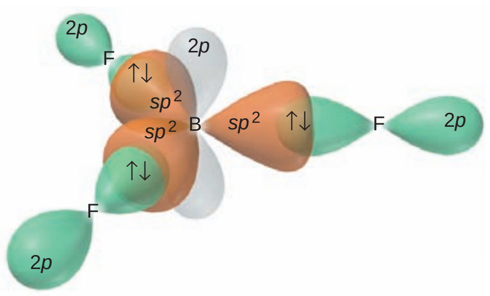
$$F = \frac{dE}{dr}$$

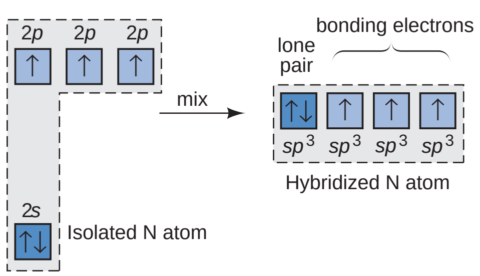
$$\begin{aligned} F_N &= F_A + F_R \\ &= \frac{dE_A}{dr} + \frac{dE_R}{dr} \end{aligned}$$

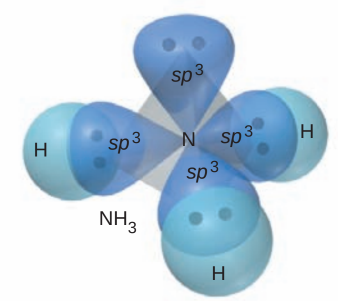
Percent ionic character(%IC) of a bond between elements A and B :

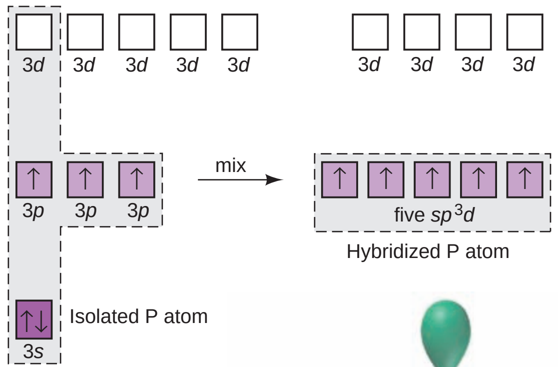
$$\%IC = \left\{1 - e^{-\left(\frac{X_A - X_B}{2}\right)^2}\right\} \times 100$$

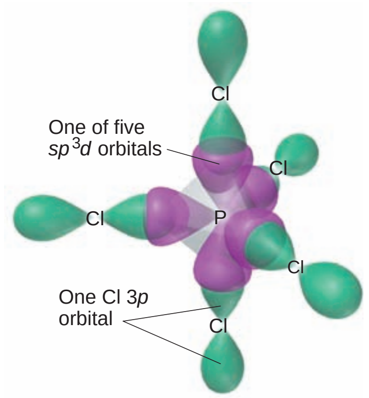












EXAMPLE PROBLEM 2.2

Computation of Attractive and Repulsive Forces between Two Ions

The atomic radii of K^+ and Br^- ions are 0.138 and 0.196 nm, respectively.

(a) Using Equations 2.9 and 2.10, calculate the force of attraction between these two ions at their equilibrium interionic separation (i.e., when the ions just touch one another).

(b) What is the force of repulsion at this same separation distance?

Solution

(a) From Equation 2.5b, the force of attraction between two ions is

$$F_A = \frac{dE_A}{dr}$$

Whereas, according to Equation 2.9,

$$E_A = -\frac{A}{r}$$

Now, taking the derivation of E_A with respect to r yields the following expression for the force of attraction F_A:

$$F_A = \frac{dE_A}{dr} = \frac{d\left(-\frac{A}{r}\right)}{dr} = -\left(\frac{-A}{r^2}\right) = \frac{A}{r^2} \tag{2.12}$$

Now substitution into this equation the expression for A (Eq. 2.10) gives

$$F_A = \frac{1}{4\pi\epsilon_0 r^2}(|Z_1|e)(|Z_2|e) \tag{2.13}$$

Incorporation into this equation values for e and ϵ_0 leads to

$$\begin{aligned} F_A &= \frac{1}{4\pi(8.85 \times 10^{-12}\ \text{F/m})(r^2)}[|Z_1|(1.602 \times 10^{-19}\text{C})][|Z_2|(1.602 \times 10^{-19}\text{C})] \\ &= \frac{(2.31 \times 10^{-28}\ \text{N}\cdot\text{m}^2)(|Z_1|)(|Z_2|)}{r^2} \end{aligned} \tag{2.14}$$

For this problem, r is taken as the interionic separation r_0 for KBr, which is equal to the sum of the K^+ and Br^- ionic radii inasmuch as the ions touch one another—that is,

$$\begin{aligned} r_0 &= r_{K^+} + r_{Br^-} \\ &= 0.138\ \text{nm} + 0.196\ \text{nm} \\ &= 0.334\ \text{nm} \\ &= 0.334 \times 10^{-9}\ \text{m} \end{aligned} \tag{2.15}$$

When we substitute this value for r into Equation 2.14, and taking ion 1 to be K^+ and ion 2 as Br^- (i.e., $Z_1 = +1$ and $Z_2 = -1$), then the force of attraction is equal to

$$F_A = \frac{(2.31 \times 10^{-28}\ \text{N}\cdot\text{m}^2)(|+1|)(|-1|)}{(0.334 \times 10^{-9}\ \text{m})^2} = 2.07 \times 10^{-9}\ \text{N}$$

(b) At the equilibrium separation distance the sum of attractive and repulsive forces is zero according to Equation 2.4. This means that

$$F_R = -F_A = -(2.07 \times 10^{-9}\ \text{N}) = -2.07 \times 10^{-9}\ \text{N}$$

EXAMPLE PROBLEM 2.3

Calculation of the Percent Ionic Character for the C-H Bond

Compute the percent ionic character (%IC) of the interatomic bond that forms between carbon and hydrogen.

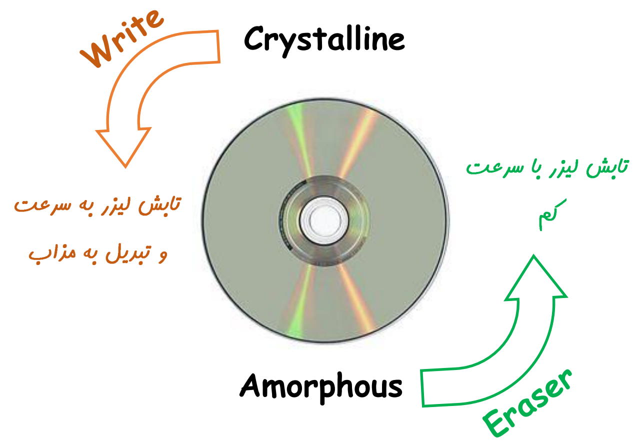
Solution

The %IC of a bond between two atoms/ions, A and B (A being the more electronegative) is a function of their electronegativities X_A and X_B, according to Equation 2.16. The electronegativities for C and H (see Figure 2.9) are $X_C = 2.5$ and $X_H = 2.1$. Therefore, the %IC is

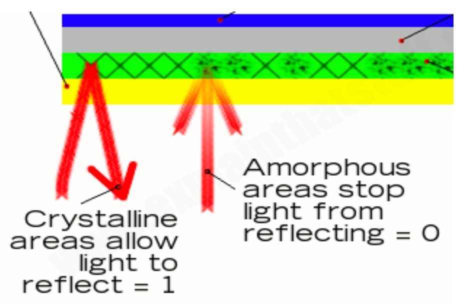
$$\begin{aligned} \%\text{IC} &= \{1 - \exp[-(0.25)(X_C - X_H)^2]\} \times 100 \\ &= \{1 - \exp[-(0.25)(2.5 - 2.1)^2]\} \times 100 \\ &= 3.9\% \end{aligned}$$

Thus the C—H atomic bond is primarily covalent (96.1%).

Chapter 3 The Structure of Crystalline Solids

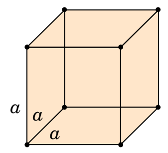
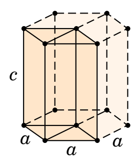
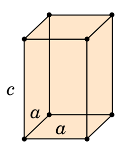
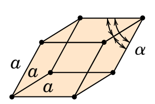
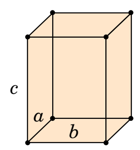
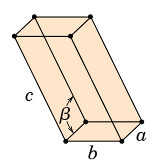
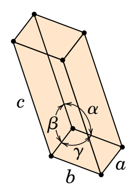


A



B

Crystal Structures

Crystal System	Axial Relationships	Interaxial Angles	Unit Cell Geometry
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90°$	
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90°, \gamma = 120°$	
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90°$	
Rhombohedral (Trigonal)	$a = b = c$	$\alpha = \beta = \gamma \neq 90°$	
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90°$	
Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90° \neq \beta$	
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90°$	

	BCC	FCC	HCP
	$A = \frac{4R}{\sqrt{3}}$	$A = \frac{4R}{\sqrt{2}}$	a=2R $V_C = \frac{3\sqrt{3}}{2}a^2C$
Packing Factor	68%	74%	74%
N	2	4	6
Coordination	8	12	12

	Simple	Body center	Face center	Base center
Cubic	✓	✓	✓	✗
Hexagonal	✓	✗	✗	✗
Tetragonal	✓	✗	✗	✗
Rhombohedral	✓	✗	✗	✗
Orthorhombic	✓	✓	✓	✓
Monoclinic	✓	✓	✗	✗
Triclinic	✓	✗	✗	✗

Simple

Body center : مرکز پر

Face center : مرکز وجوه پر

Base center : مرکز پایه ها (وجوه بالا و پایینی) پر

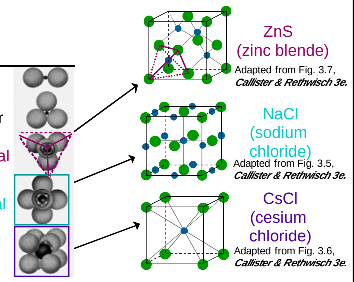
$$\rho = \frac{N\frac{M}{N_A}}{V_c}$$

Coordination # and Ionic Radii

- Coordination # increases with $\frac{r_{cation}}{r_{anion}}$

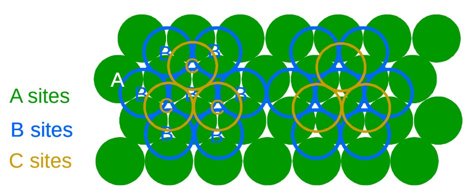
$\frac{r_{cation}}{r_{anion}}$	Coord #	
< 0.155	2	linear
0.155 - 0.225	3	triangular
0.225 - 0.414	4	tetrahedral
0.414 - 0.732	6	octahedral
0.732 - 1.0	8	cubic

Adapted from Table 3.3, *Callister & Rethwisch 3e.*

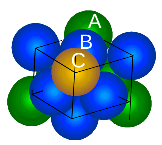


FCC Stacking Sequence

- ABCABC... Stacking Sequence

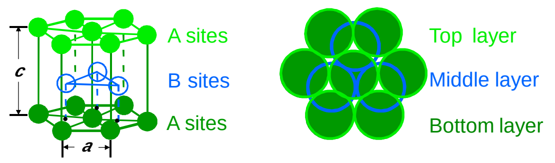


- FCC Unit Cell



Hexagonal Close-Packed Structure (HCP)

- ABAB... Stacking Sequence



$$LD = \frac{\text{number of atoms centered on direction vector}}{\text{length of direction vector}}$$

$$PD = \frac{\text{number of atoms centered on a plane}}{\text{area of plane}}$$

isotropic : خواص در همه جهات یکسان

anisotropic : خواص در همه جهات غیریکسان

اگر ذرات بصورت کاملا رندوم قرار گرفته باشند ،ماده ایزوتوپ است و اگرذرات بصورت خاص آرایش گرفته باشند ماده **anisotropic** است . به عنوان مثال خود تک کریستال **BCC** ، **anisotropic** است .

x و y و z : مختصات نقطه /// a و b و c : ابعاد مکعب

A و B و C : نقاط برخورد صفحه با محورهای مختصات

نقطه

$$q = \frac{x}{a} \quad r = \frac{y}{b} \quad s = \frac{z}{c}$$

بردار مکعبی

$$u = n\left(\frac{x_2 - x_1}{a}\right)$$
$$v = n\left(\frac{y_2 - y_1}{b}\right)$$
$$w = n\left(\frac{z_2 - z_1}{c}\right)$$

بردار هگزاگونال

$$[U\ V\ W] \rightarrow [u\ v\ t\ z]$$

$$\frac{1}{3}(2U - V) = u = 3n\left(\frac{\ddot{a}_1 - \dot{a}_1}{a}\right)$$
$$\frac{1}{3}(2V - U) = v = 3n\left(\frac{\ddot{a}_2 - \dot{a}_2}{a}\right)$$
$$-(u + v) = t = 3n\left(\frac{\ddot{a}_3 - \dot{a}_3}{a}\right)$$
$$W = z = 3n\left(\frac{\ddot{z} - \dot{z}}{c}\right)$$

صفحه‌ی مکعبی

$$h = \frac{na}{A} \quad k = \frac{nb}{B} \quad l = \frac{nc}{C}$$

صفحه‌ی هگزاگونال

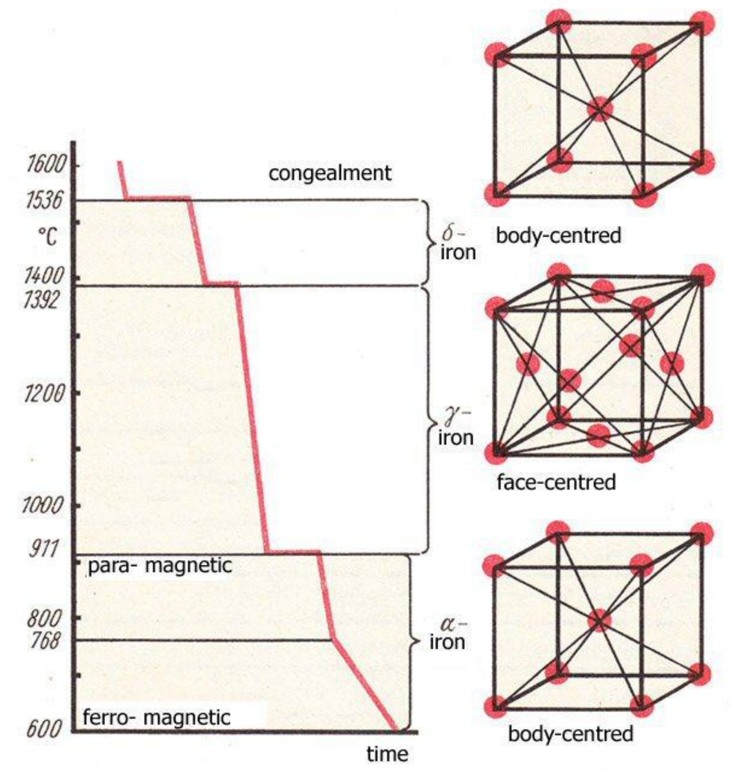
$$(h\ k\ i\ l)$$

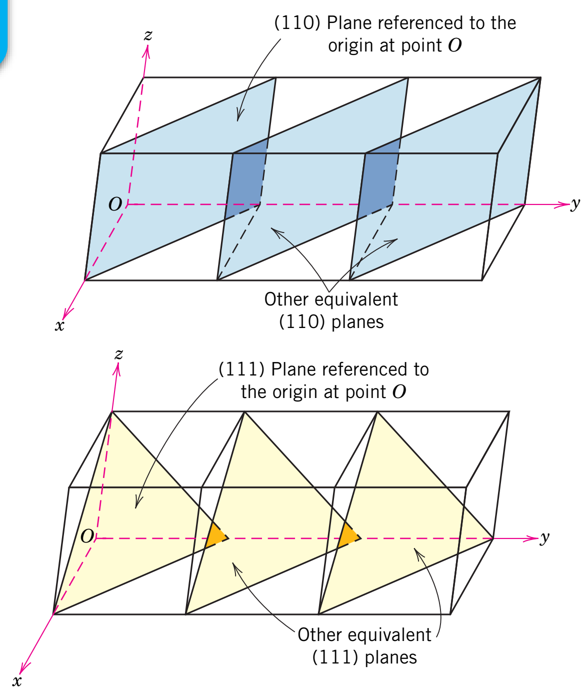
$$h = \frac{na}{A} \quad k = \frac{nb}{B} \quad l = \frac{nc}{C}$$
$$i = -(h + k)$$

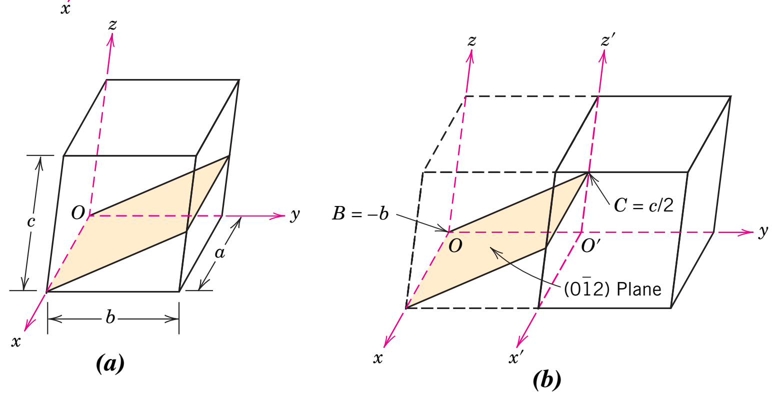
$q\ r\ s$: نقط : می‌تواند صحیح نباشد.

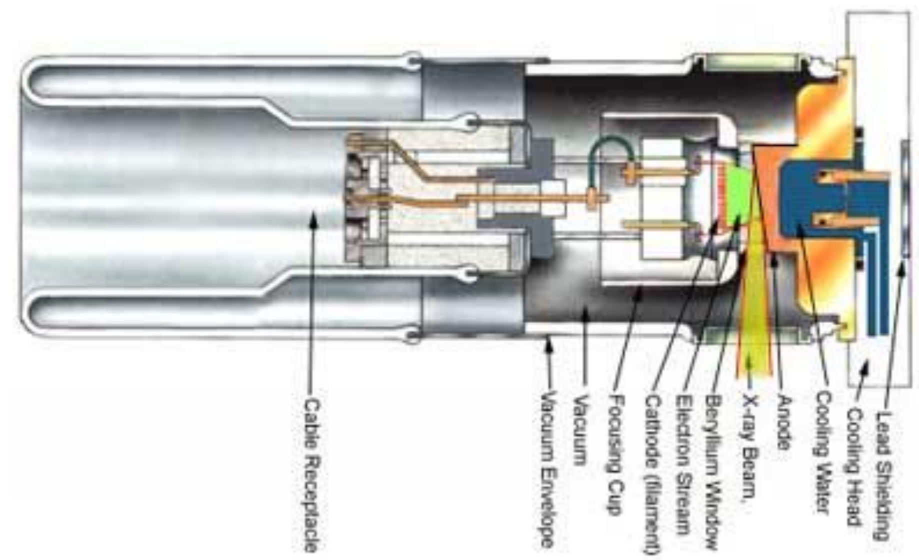
$[u\ v\ w]$: بردار : باید صحیح باشد.

$(h\ k\ l)$: صفحه : باید صحیح باشد.









X-Ray Generation : between Gun & target $\rightarrow\ \Delta V \Rightarrow$ تابش e $\Rightarrow$(یا 1 لایه یا 2 لایه) جابجایی الکترونهای تارگت $\Rightarrow$ ($k\alpha$ و $k\beta$)نشر اشعه ی ایکس در دو طول موج

ما فقط یک طول میخواهیم پس از یک فیلتر استفاده میکنیم . عناصر موج هایی با طول موج های معینی را جذب میکنند و در جامدات آن ها بصورت پیوسته است ؛ یعنی از طول موجی به قبل را در بازه ای جذب میکنند و از آن به بعد را جذب نمی کنند و این چنین ما فقط $k\alpha$ را جدا می کنیم .

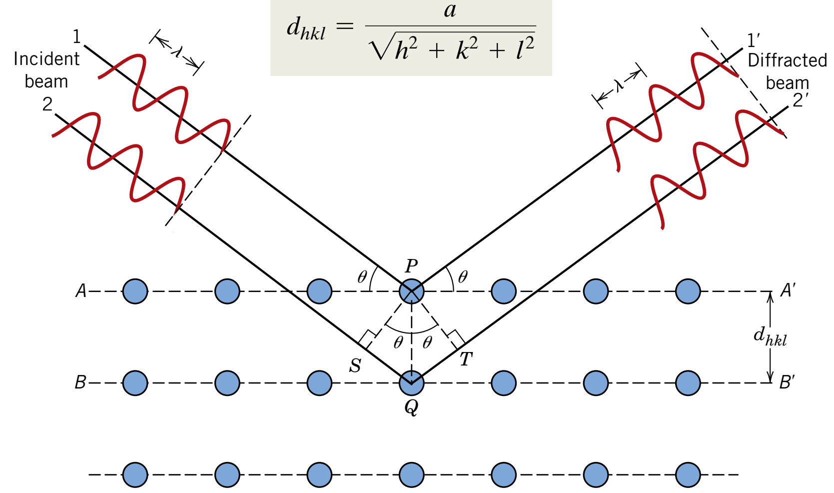
X – Ray ⇒ Fillter ⇒ تابش به بلور ⇒ $Diffraction$ ⇒ چرخش صفحه ⇒ ثبت یک اثر انگشت برای ماده

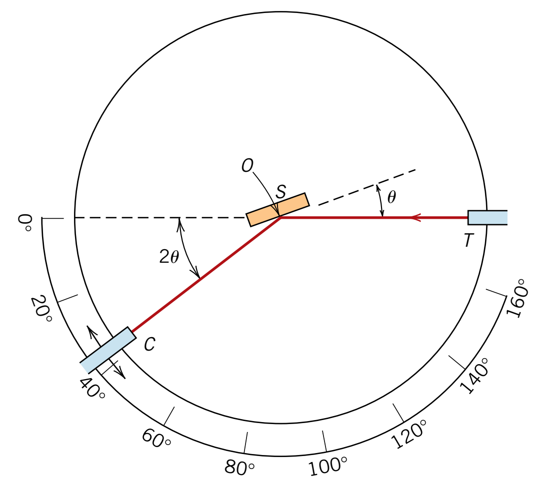
X-Ray Diffraction and Bragg's Law

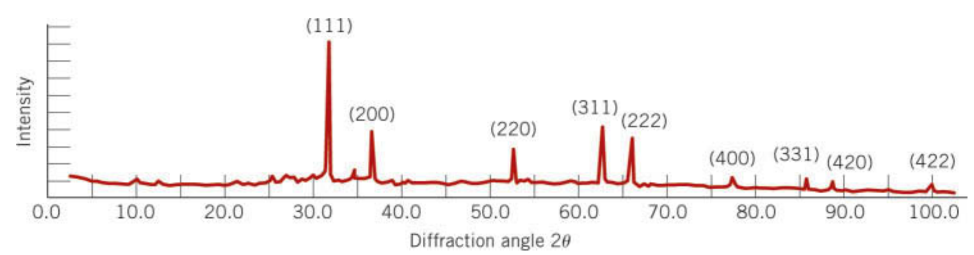
$$n\lambda = \overline{SQ} + \overline{QT}$$

$$n\lambda = 2d_{hkl}\sin\theta$$

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$



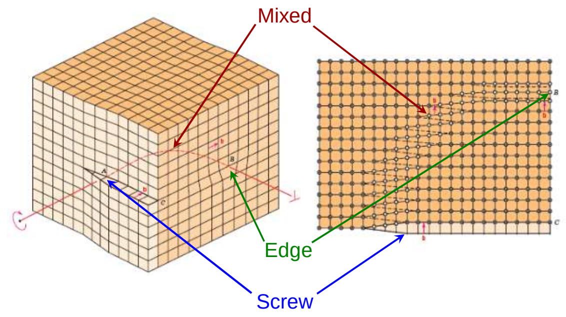




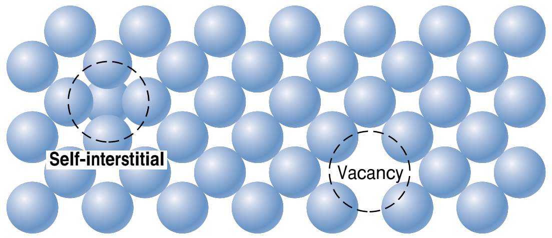
Chapter 4 Imperfections in Solids

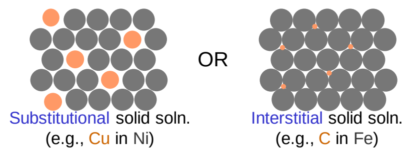
crystal imperfection
- Point defects
- Line defects
- Surface defects
- Volume defects
- **Atomic vibration**

Line defects
- Edge → exist or not exist a plan
- Screw → a shear stress
- Mixed → both of them

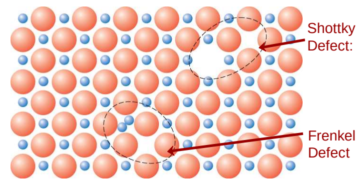


Point defects
- Vacancy → یک اتم سر جای خودش نباشد .
- Self-interstitial → یک اتم اضافه نشسته باشد .
- Sub situational → یک اتم از جنسی دیگر در حفره ها می نشیند .
- Interstitial → در فضاهای خالی بین اتمی می نشیند.
(مثلا BCC ، 68 درصد فضای اشغال شده دارد.)

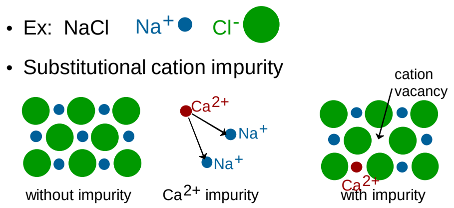




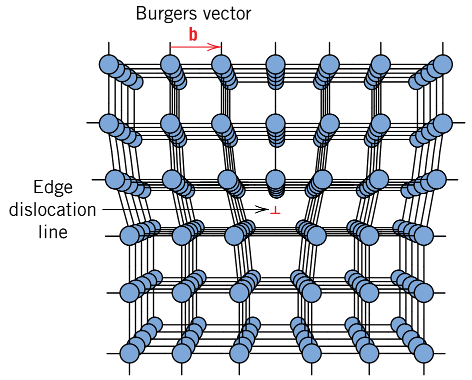
Point defects in Ceramics
- schottky defect → یک جفت کاتیون و آنیون سر جای خود نباشند .
- Frenkel defect → یک کاتیون جابجا شده باشد و سر جای خودش نباشد .

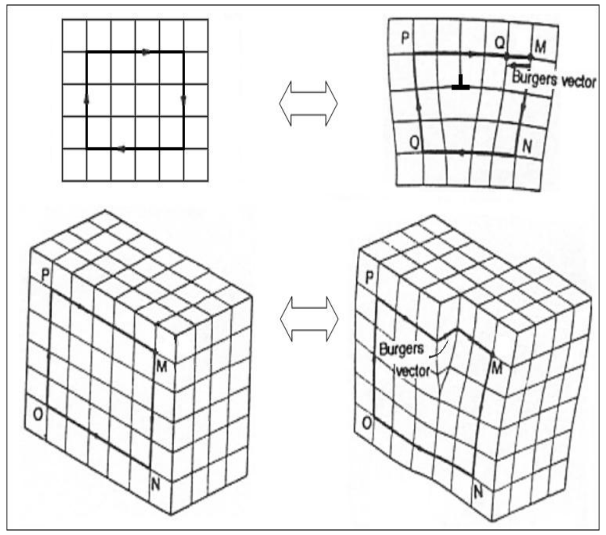


- Electroneutrality (charge balance) must be maintained when impurities are present
- Ex: NaCl Na^+ Cl^-
- Substitutional cation impurity



Conditions for substitutional solid solution (S.S.)
- W. Hume – Rothery rule
  - 1. Δr(atomic radius) < 15%
  - 2. Proximity in periodic table
    - i.e., similar electronegativities
  - 3. Same crystal structure for pure metals
  - 4. Valency
    - All else being equal, a metal will have a greater tendency to dissolve a metal of higher valency than one of lower valency





Volume defects
- Pores + Voids (حفره)
- Cracks (ترک)
- Precipitate (رسوب)

قفل شدن ناجابجایی ها موجب افزایش مقاومت ماده میشور .

ناجابجایی ها در یکدیگر ، پشت مرز دانه ها و در رسوبات قفل میشونر .

Equilibrium Concentration: Point Defects

- Equilibrium concentration varies with temperature!

No. of defects

No. of potential defect sites

$$\frac{N_v}{N} = \exp\left(\frac{-Q_v}{kT}\right)$$

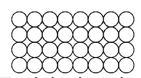
Activation energy

Temperature

Boltzmann's constant

(1.38×10^{-23} J/atom-K)

(8.62×10^{-5} eV/atom-K)



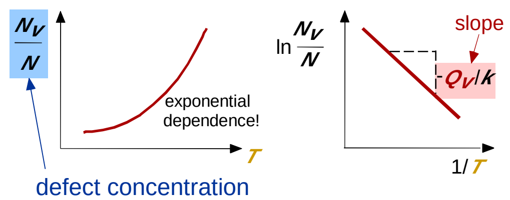
Each lattice site is a potential vacancy site

Measuring Activation Energy

- We can get Q_v from an experiment.

$$\frac{N_v}{N} = \exp\left(\frac{-Q_v}{kT}\right)$$

- Measure this...
- Replot it...



defect concentration

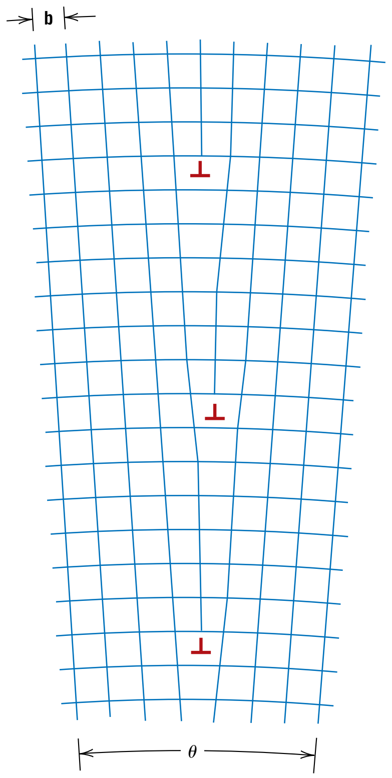


Figure 4.9 Demonstration of how a tilt boundary having an angle of misorientation θ results from an alignment of edge dislocations.

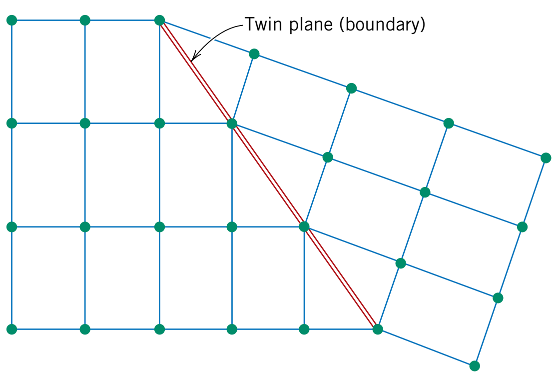
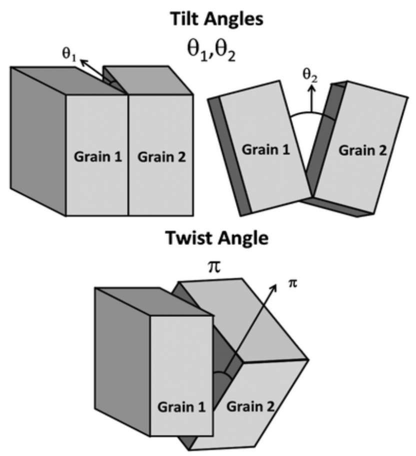
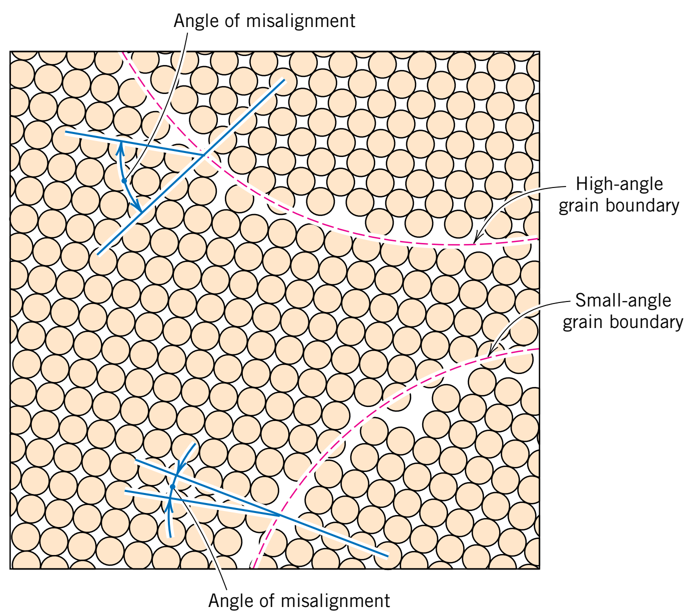


Figure 4.10 Schematic diagram showing a twin plane or boundary and the adjacent atom positions (colored circles).

Surface defects
- External Surface
- Grain Boundaries
  - Tilt Boundaries
  - twist Boundaries
- Twins Plan
- Stacking Faults





EXAMPLE PROBLEM 4.1

Number-of-Vacancies Computation at a Specified Temperature

Calculate the equilibrium number of vacancies per cubic meter for copper at 1000°C. The energy for vacancy formation is 0.9 eV/atom; the atomic weight and density (at 1000°C) for copper are 63.5 g/mol and 8.4 g/cm³, respectively.

Solution

This problem may be solved by using Equation 4.1; it is first necessary, however, to determine the value of N—the number of atomic sites per cubic meter for copper, from its atomic weight A_{Cu}, its density ρ, and Avogadro's number N_A, according to

$$N = \frac{N_A \rho}{A_{Cu}} \tag{4.2}$$

$$= \frac{(6.022 \times 10^{23} \text{ atoms/mol})(8.4 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)}{63.5 \text{ g/mol}}$$

$$= 8.0 \times 10^{28} \text{ atoms/m}^3$$

Thus, the number of vacancies at 1000°C (1273 K) is equal to

$$N_v = N \exp\left(-\frac{Q_v}{kT}\right)$$

$$= (8.0 \times 10^{28} \text{ atoms/m}^3) \exp\left[-\frac{(0.9 \text{ eV})}{(8.62 \times 10^{-5} \text{ eV/K})(1273 \text{ K})}\right]$$

$$= 2.2 \times 10^{25} \text{ vacancies/m}^3$$

$$C_1 = \frac{m_1}{m_1 + m_2} \times 100$$ Composition in weight percent

$$C_1' = \frac{n_{m1}}{n_{m1} + n_{m2}} \times 100$$ Composition in atom percent

$$C_1' = \frac{C_1 A_2}{C_1 A_2 + C_2 A_1} \times 100$$ Conversion from weight percent to atom percent

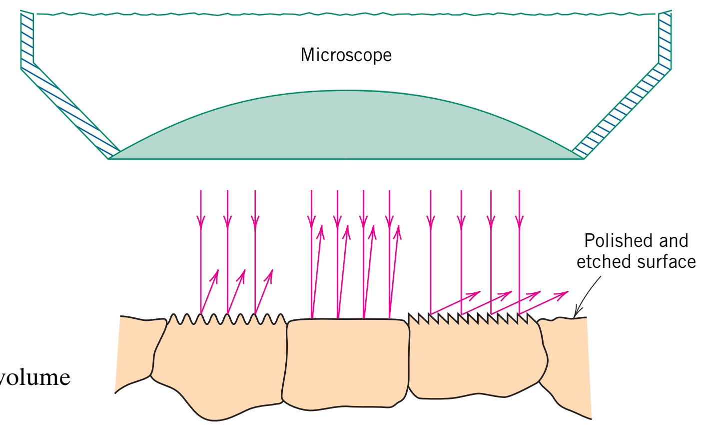
$$C_1 = \frac{C_1' A_1}{C_1' A_1 + C_2' A_2} \times 100$$ Conversion from atom percent to weight percent

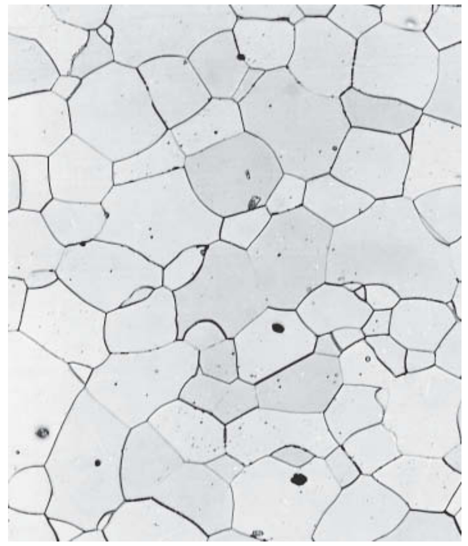
$$C_1'' = \left(\frac{C_1}{\frac{C_1}{\rho_1} + \frac{C_2}{\rho_2}} \right) \times 10^3$$ Conversion from weight percent to mass per unit volume

$$\rho_{\text{ave}} = \frac{100}{\frac{C_1}{\rho_1} + \frac{C_2}{\rho_2}}$$ Average density of a two-component alloy

$$A_{\text{ave}} = \frac{100}{\frac{C_1}{A_1} + \frac{C_2}{A_2}}$$ Average atomic weight of a two-component alloy

Optical Microscopy





چون انرژی شبکه به علت پایداری کمتر در مرزها بیشتر است ؛ مرزها هستند که در واکنش ها اول وارد میشوند .

پس اگر نمونه را در اسید بزنیم و سپس زیر میکروسکوپ بگذاریم ، مرزها مشخص میشوند .

Metallography : Sectioning ⇒ *Mounting* ⇒ *Grinding* ⇒ Polishing ⇒ Etching

نمونه برداری

Hot Cold

سمباده زنی

(نوعی پلیمر برای صاف نگه داشتن و
ایجاد یک قالب نگهدارنده ی نمونه)

(خمیر الماس یا پودر آلومینا روی نمد
می ریزیم و با دستگاه پولیش میکنیم .)

grain size

$$\bar{\ell} = \frac{L_T}{PM}$$ Mean intercept length (measure of average grain diameter)

$$n = 2^{G-1}$$ Number of grains per square inch at a magnification of 100×

$$n_M = (2^{G-1})\left(\frac{M}{100}\right)^2$$ Number of grains per square inch at a magnification other than 100×

M Magnification

G ASTM grain-size number

$$M = \frac{\text{measured scale length (converted to microns)}}{\text{the number appearing by the scale bar (in microns)}}$$

$$G = -6.6457 \log \bar{\ell} - 3.298 \quad (\text{for } \bar{\ell} \text{ in mm})$$
$$G = -6.6353 \log \bar{\ell} - 12.6 \quad (\text{for } \bar{\ell} \text{ in in.})$$

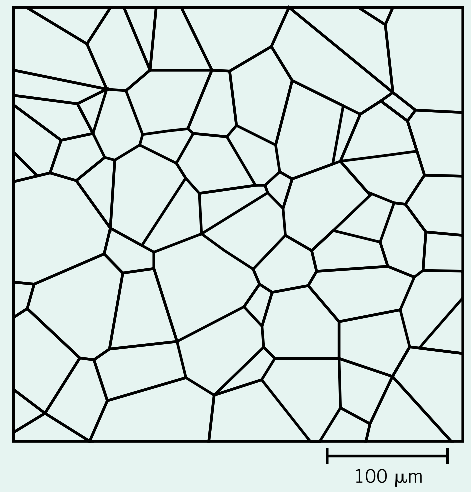
EXAMPLE PROBLEM 4.5

Grain-Size Computations Using ASTM and Intercept Methods

The following is a schematic micrograph that represents the microstructure of some hypothetical metal.

Determine the following:

(a) Mean intercept length
(b) ASTM grain-size number, G using Equation 4.19a

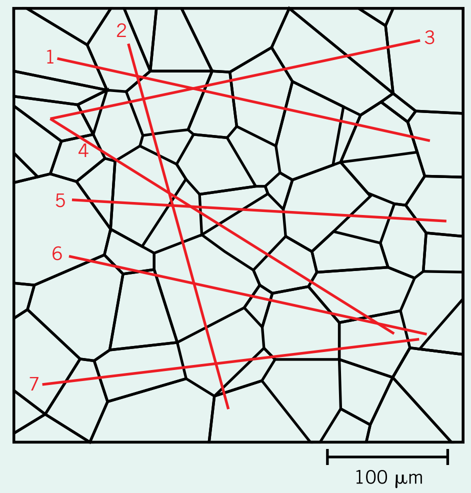


Solution

(a) We first determine the magnification of the micrograph using Equation 4.20. The scale bar length is measured and found to be 16 mm, which is equal to 16,000 µm; and because the scale bar number is 100 µm, the magnification is

$$M = \frac{16{,}000\ \mu\text{m}}{100\ \mu\text{m}} = 160\times$$

The following sketch is the same micrograph on which have been drawn seven straight lines (in red), which have been numbered.



The length of each line is 50 mm, and thus the total line length (L_T in Equation 4.16) is

$$(7 \text{ lines})(50 \text{ mm/line}) = 350 \text{ mm}$$

Tabulated next is the number of grain-boundary intersections for each line:

Line Number	*Number of Grain-Boundary Intersections*
1	8
2	8
3	8
4	9
5	9
6	9
7	7
Total	58

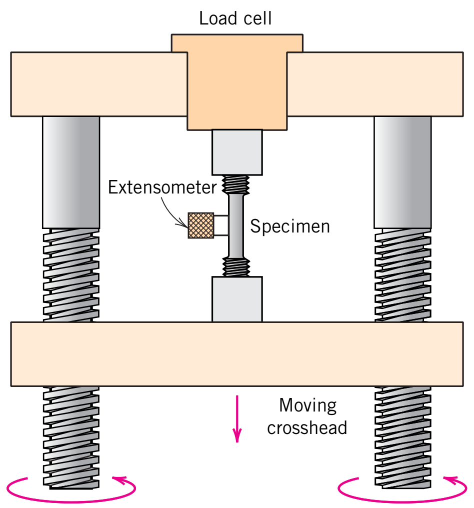
Thus, inasmuch as $L_T = 350$ mm, $P = 58$ grain-boundary intersections, and the magnification $M = 160\times$, the mean intercept length $\bar{\ell}$ (in millimeters in real space), Equation 4.16, is equal to

$$\bar{\ell} = \frac{L_T}{PM}$$
$$= \frac{350 \text{ mm}}{(58 \text{ grain boundary intersections})(160\times)} = 0.0377 \text{ mm}$$

(b) The value of G is determined by substitution of this value for $\bar{\ell}$ into Equation 4.19a; therefore,

$$G = -6.6457 \log \bar{\ell} - 3.298$$
$$= (-6.6457)\log(0.0377) - 3.298$$
$$= 6.16$$

Chapter 6 Mechanical Properties of Metals



$$\sigma = \frac{F}{A_0} \qquad \epsilon = \frac{\Delta l}{l_0}$$

$$\sigma = E\epsilon$$

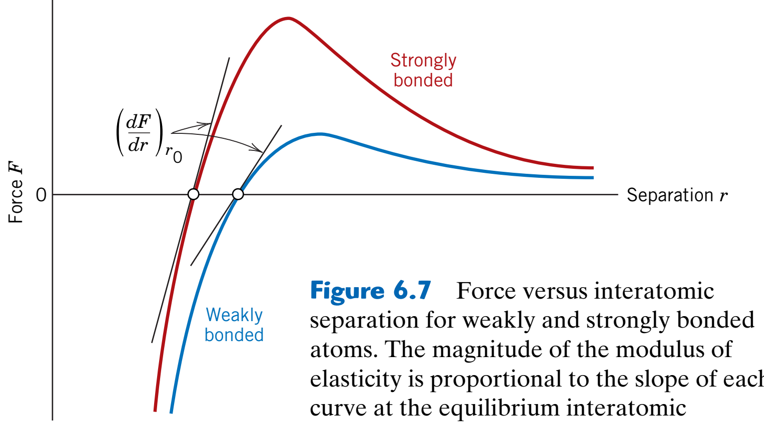
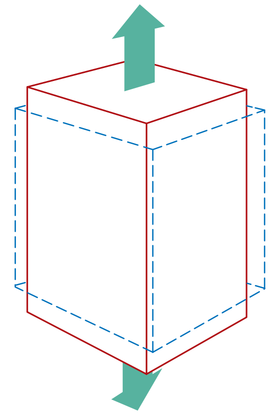


Figure 6.7 Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation r_0.

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$



$$E = 2G(1+\nu)$$

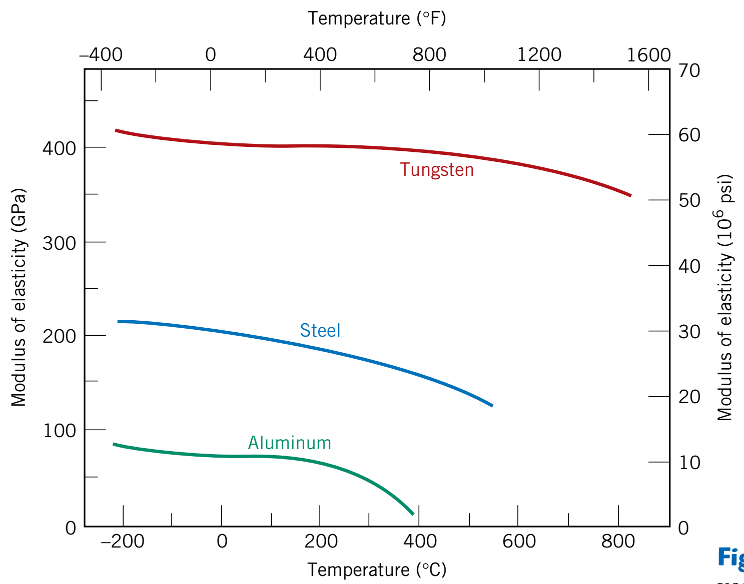
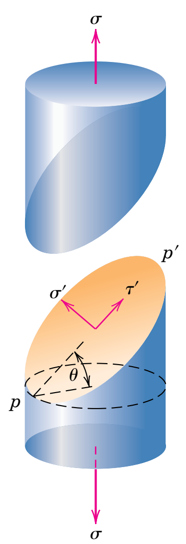


Figure 6.8 Plot of modulus of elasticity versus temperature for tungsten, steel, and aluminum.

$$\tau = G\gamma$$

Definition of shear stress



$$\tau = \frac{F}{A_0}$$

$$\sigma' = \sigma \cos^2\theta$$

$$\tau' = \sigma \sin\theta \cos\theta$$

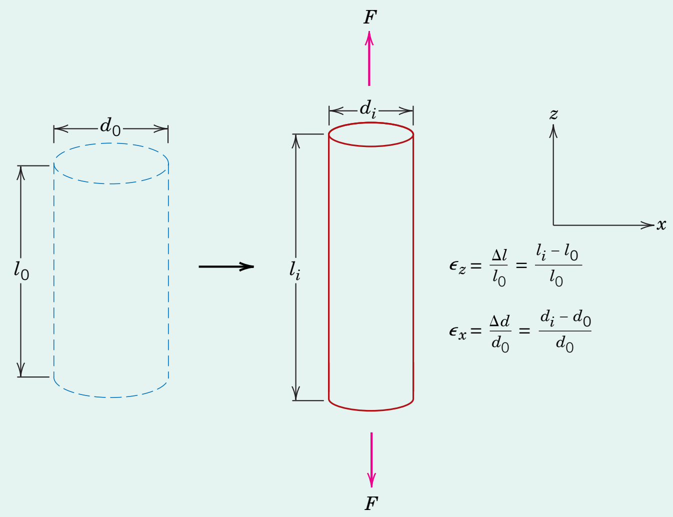
$$\%EL = \left(\frac{l_f - l_0}{l_0}\right) \times 100$$

$$\%RA = \left(\frac{A_0 - A_f}{A_0}\right) \times 100$$

EXAMPLE PROBLEM 6.2

Computation of Load to Produce Specified Diameter Change

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a 2.5×10^{-3}-mm (10^{-4}-in.) change in diameter if the deformation is entirely elastic.



Solution

This deformation situation is represented in the accompanying drawing.

When the force F is applied, the specimen will elongate in the z direction and at the same time experience a reduction in diameter, Δd, of 2.5×10^{-3} mm in the x direction. For the strain in the x direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3}\text{ mm}}{10\text{ mm}} = -2.5 \times 10^{-4}$$

which is negative because the diameter is reduced.

It next becomes necessary to calculate the strain in the z direction using Equation 6.8. The value for Poisson's ratio for brass is 0.34 (Table 6.1), and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

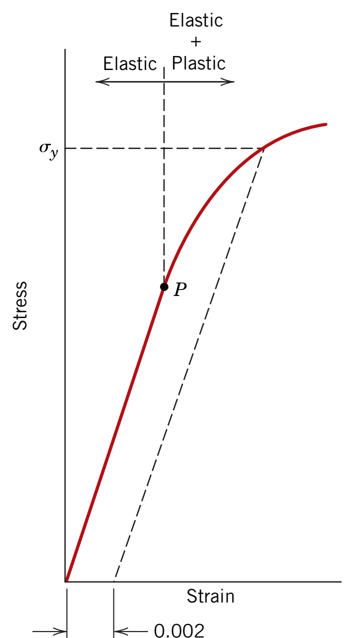
The applied stress may now be computed using Equation 6.5 and the modulus of elasticity, given in Table 6.1 as 97 GPa (14×10^6 psi), as

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3\text{ MPa}) = 71.3\text{ MPa}$$

Finally, from Equation 6.1, the applied force may be determined as

$$F = \sigma A_0 = \sigma\left(\frac{d_0}{2}\right)^2 \pi$$

$$= (71.3 \times 10^6\text{ N/m}^2)\left(\frac{10 \times 10^{-3}\text{ m}}{2}\right)^2 \pi = 5600\text{ N } (1293\text{ lb}_f)$$



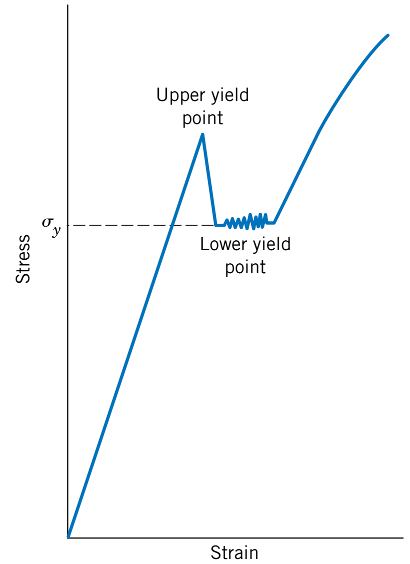
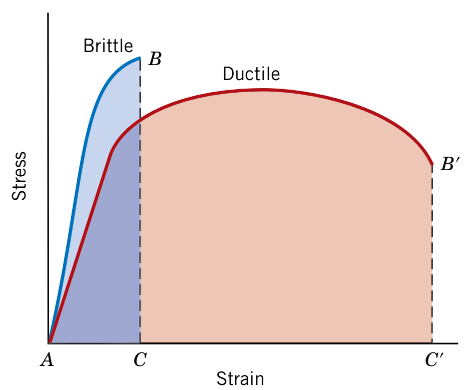
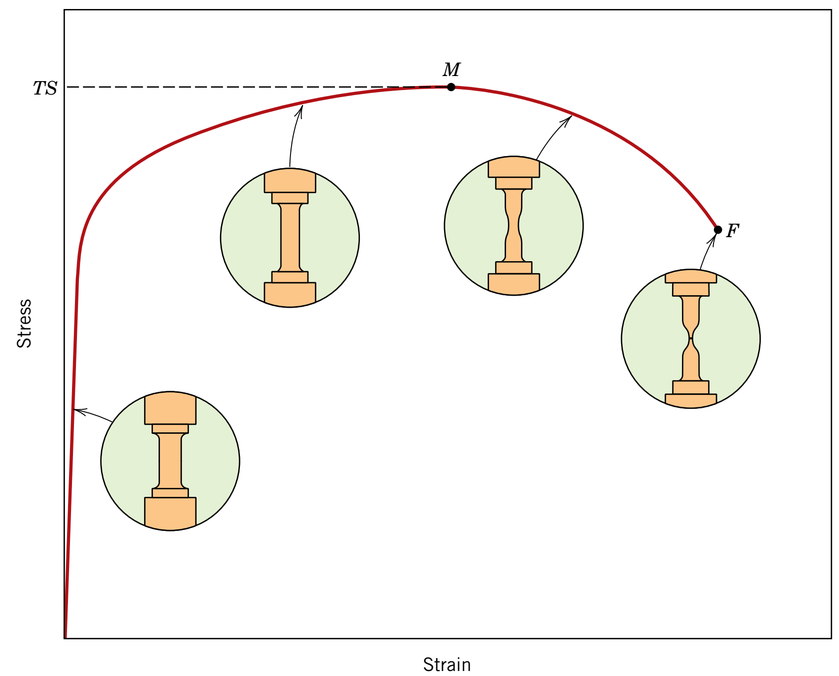


Figure 6.10 (*a*) Typical stress–strain behavior for a metal showing elastic and plastic deformations, the proportional limit P, and the yield strength σ_y, as determined using the 0.002 strain offset method. (*b*) Representative stress–strain behavior found for some steels demonstrating the yield point phenomenon.





Resilience

$$U_r = \frac{1}{2}\sigma_y \epsilon_y = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y^2}{2E}$$

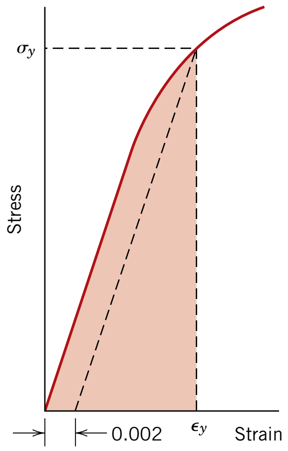


Figure 6.15 Schematic representation showing how modulus of resilience (corresponding to the shaded area) is determined from the tensile stress–strain behavior of a material.

$$\epsilon_T = \ln\frac{l_i}{l_0}$$

$$\sigma_T = \sigma(1 + \epsilon)$$

$$\epsilon_T = \ln(1 + \epsilon)$$

$$\sigma_T = K\epsilon_T^n$$

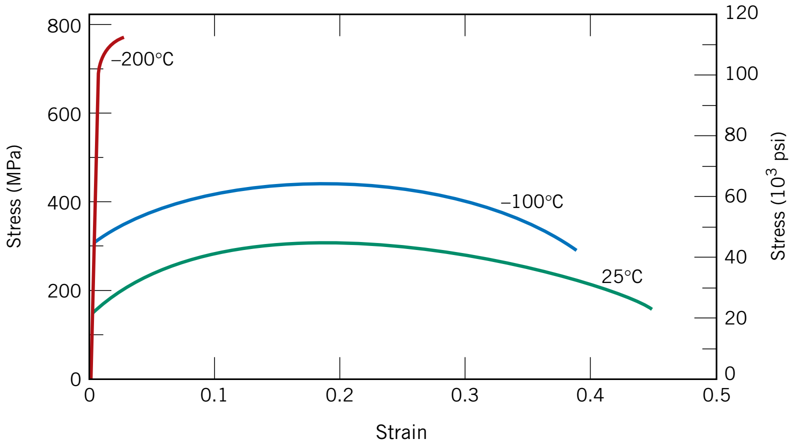


Figure 6.14 Engineering stress–strain behavior for iron at three temperatures.

TRUE STRESS AND STRAIN

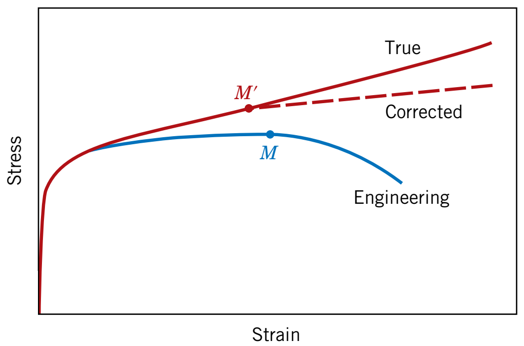
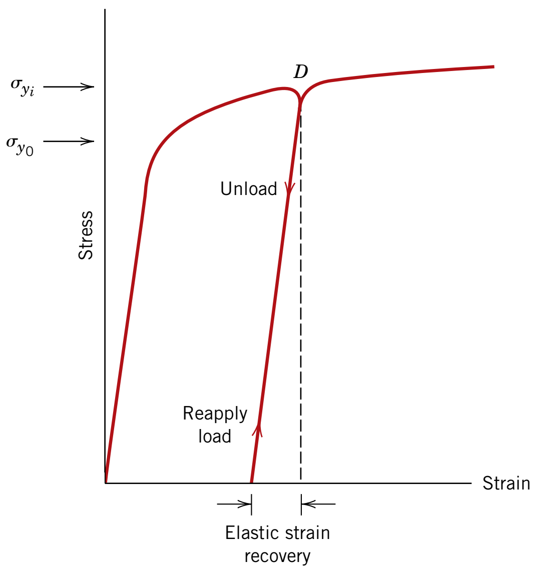
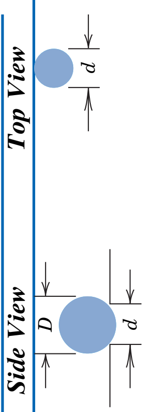
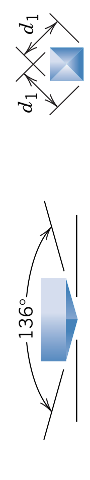
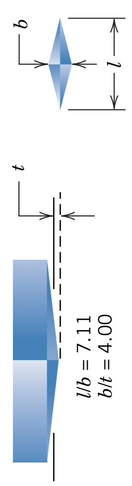
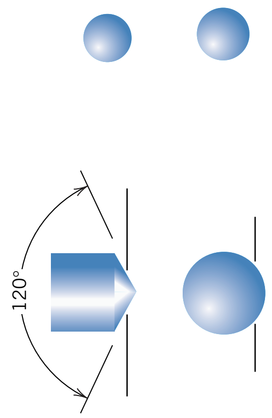


Figure 6.16 A comparison of typical tensile engineering stress–strain and true stress–strain behaviors. Necking begins at point M on the engineering curve, which corresponds to M' on the true curve. The "corrected" true stress–strain curve takes into account the complex stress state within the neck region.



HARDNESS

Table 6.5 Hardness-Testing Techniques

Test	Indenter	Shape of Indentation: Side View	Shape of Indentation: Top View	Load	Formula for Hardness Number[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			P	$HK = 14.2P/l^2$
Rockwell and superficial Rockwell	Diamond cone: $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres			60 kg, 100 kg, 150 kg: Rockwell; 15 kg, 30 kg, 45 kg: Superficial Rockwell	

[a] For the hardness formulas given, P (the applied load) is in kg, and D, d, d_1, and l are all in millimeters.

Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York.

EXAMPLE PROBLEM 6.4

Ductility and True-Stress-at-Fracture Computations

A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile-tested to fracture and found to have an engineering fracture strength σ_f of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine

(a) The ductility in terms of percentage reduction in area

(b) The true stress at fracture

Solution

(a) Ductility is computed using Equation 6.12, as

$$\%\,RA = \frac{\left(\dfrac{12.8\text{ mm}}{2}\right)^2\pi - \left(\dfrac{10.7\text{ mm}}{2}\right)^2\pi}{\left(\dfrac{12.8\text{ mm}}{2}\right)^2\pi} \times 100$$

$$= \frac{128.7\text{ mm}^2 - 89.9\text{ mm}^2}{128.7\text{ mm}^2} \times 100 = 30\%$$

(b) True stress is defined by Equation 6.15, where, in this case, the area is taken as the fracture area A_f. However, the load at fracture must first be computed from the fracture strength as

$$F = \sigma_f A_0 = (460 \times 10^6\text{ N/m}^2)(128.7\text{ mm}^2)\left(\frac{1\text{ m}^2}{10^6\text{ mm}^2}\right) = 59{,}200\text{ N}$$

Thus, the true stress is calculated as

$$\sigma_T = \frac{F}{A_f} = \frac{59{,}200\text{ N}}{(89.9\text{ mm}^2)\left(\dfrac{1\text{ m}^2}{10^6\text{ mm}^2}\right)}$$

$$= 6.6 \times 10^8\text{ N/m}^2 = 660\text{ MPa (95,700 psi)}$$

EXAMPLE PROBLEM 6.5

Calculation of Strain-Hardening Exponent

Compute the strain-hardening exponent n in Equation 6.19 for an alloy in which a true stress of 415 MPa (60,000 psi) produces a true strain of 0.10; assume a value of 1035 MPa (150,000 psi) for K.

Solution

This requires some algebraic manipulation of Equation 6.19 so that n becomes the dependent parameter. This is accomplished by taking logarithms and rearranging. Solving for n yields

$$n = \frac{\log\sigma_T - \log K}{\log\epsilon_T}$$

$$= \frac{\log(415\text{ MPa}) - \log(1035\text{ MPa})}{\log(0.1)} = 0.40$$

EXAMPLE PROBLEM 6.6

Average and Standard Deviation Computations

The following tensile strengths were measured for four specimens of the same steel alloy:

Sample Number	*Tensile Strength (MPa)*
1	520
2	512
3	515
4	522

(a) Compute the average tensile strength.
(b) Determine the standard deviation.

Solution

(a) The average tensile strength ($\overline{TS}$) is computed using Equation 6.21 with $n = 4$:

$$\overline{TS} = \frac{\sum_{i=1}^{4} (TS)_i}{4}$$

$$= \frac{520 + 512 + 515 + 522}{4}$$

$$= 517 \text{ MPa}$$

(b) For the standard deviation, using Equation 6.22, we obtain

$$s = \left[\frac{\sum_{i=1}^{4} \{(TS)_i - \overline{TS}\}^2}{4 - 1}\right]^{1/2}$$

$$= \left[\frac{(520 - 517)^2 + (512 - 517)^2 + (515 - 517)^2 + (522 - 517)^2}{4 - 1}\right]^{1/2}$$

$$= 4.6 \text{ MPa}$$

Figure 6.20 presents the tensile strength by specimen number for this example problem and also how the data may be represented in graphical form. The tensile strength data point (Figure 6.20*b*) corresponds to the average value $\overline{TS}$, and scatter is depicted by error bars (short horizontal lines) situated above and below the data point symbol and connected to this symbol by vertical lines. The upper error bar is positioned at a value of the average value plus the standard deviation ($\overline{TS} + s$), and the lower error bar corresponds to the average minus the standard deviation ($\overline{TS} - s$).

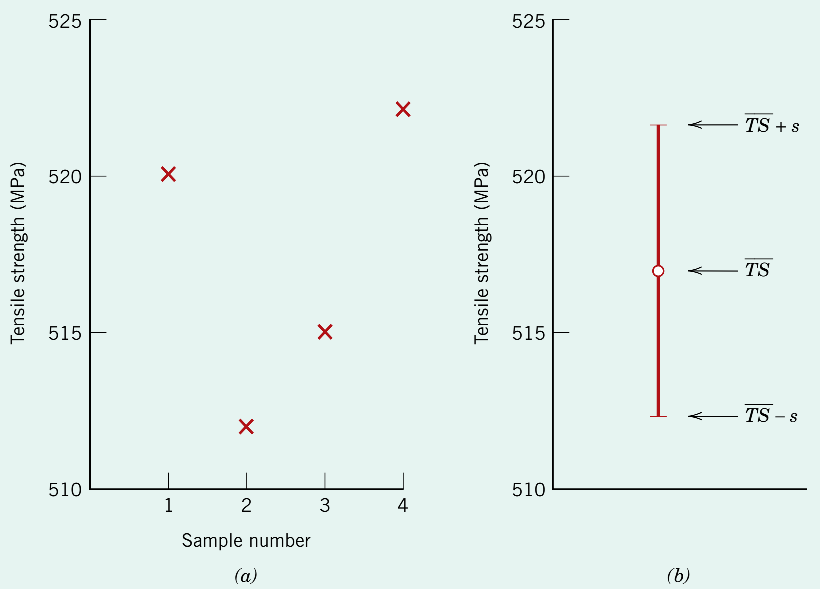


Figure 6.20 (*a*) Tensile strength data associated with Example Problem 6.6. (*b*) The manner in which these data could be plotted. The data point corresponds to the average value of the tensile strength ($\overline{TS}$); error bars that indicate the degree of scatter correspond to the average value plus and minus the standard deviation ($\overline{TS} \pm s$).

Correlation between Hardness and Tensile Strength

$$TS\,(\text{MPa}) = 3.45 \times \text{HB}$$

$$TS\,(\text{psi}) = 500 \times \text{HB}$$

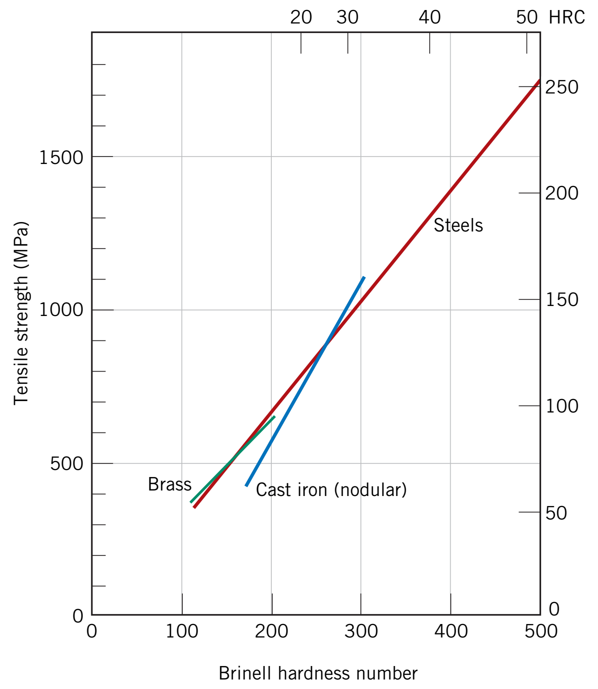


Figure 6.19 Relationships between hardness and tensile strength for steel, brass, and cast iron.

Computation of Average and Standard Deviation Values

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad s = \left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}\right]^{1/2}$$

DESIGN/SAFETY FACTORS

design stress

$$\sigma_d = N'\sigma_c$$

safe stress

$$\sigma_w = \frac{\sigma_y}{N}$$

DESIGN EXAMPLE 6.1

Specification of Support-Post Diameter

A tensile-testing apparatus is to be constructed that must withstand a maximum load of 220,000 N (50,000 lb_f). The design calls for two cylindrical support posts, each of which is to support half of the maximum load. Furthermore, plain-carbon (1045) steel ground and polished shafting rounds are to be used; the minimum yield and tensile strengths of this alloy are 310 MPa (45,000 psi) and 565 MPa (82,000 psi), respectively. Specify a suitable diameter for these support posts.

Solution

The first step in this design process is to decide on a factor of safety, N, which then allows determination of a working stress according to Equation 6.24. In addition, to ensure that the apparatus will be safe to operate, we also want to minimize any elastic deflection of the rods during testing; therefore, a relatively conservative factor of safety is to be used, say $N = 5$. Thus, the working stress σ_w is just

$$\sigma_w = \frac{\sigma_y}{N}$$

$$= \frac{310 \text{ MPa}}{5} = 62 \text{ MPa (9000 psi)}$$

From the definition of stress, Equation 6.1,

$$A_0 = \left(\frac{d}{2}\right)^2 \pi = \frac{F}{\sigma_w}$$

where d is the rod diameter and F is the applied force; furthermore, each of the two rods must support half of the total force, or 110,000 N (25,000 psi). Solving for d leads to

$$d = 2\sqrt{\frac{F}{\pi \sigma_w}}$$

$$= \sqrt{\frac{110{,}000 \text{ N}}{\pi(62 \times 10^6 \text{ N/m}^2)}}$$

$$= 4.75 \times 10^{-2} \text{ m} = 47.5 \text{ mm (1.87 in.)}$$

Therefore, the diameter of each of the two rods should be 47.5 mm, or 1.87 in.

DESIGN EXAMPLE 6.2

Materials Specification for a Pressurized Cylindrical Tube

(a) Consider a thin-walled cylindrical tube having a radius of 50 mm and wall thickness 2 mm that is to be used to transport pressurized gas. If inside and outside tube pressures are 20 and 0.5 atm (2.027 and 0.057 MPa), respectively, which of the metals and alloys listed in Table 6.8 are suitable candidates? Assume a factor of safety of 4.0.

For a thin-walled cylinder, the circumferential (or "hoop") stress (σ) depends on pressure difference (Δp), cylinder radius (r_i), and tube wall thickness (t) as follows:

$$\sigma = \frac{r_i \Delta p}{t} \tag{6.25}$$

These parameters are noted on the schematic sketch of a cylinder presented in Figure 6.21.

(b) Determine which of the alloys that satisfy the criterion of part (**a**) can be used to produce a tube with the lowest cost.

Solution

(a) In order for this tube to transport the gas in a satisfactory and safe manner, we want to minimize the likelihood of plastic deformation. To accomplish this, we replace the circumferential stress in Equation 6.25 with the yield strength of the tube material divided by the factor of safety, N—that is,

$$\frac{\sigma_y}{N} = \frac{r_i \Delta p}{t}$$

And solving this expression for σ_y leads to

$$\sigma_y = \frac{N r_i \Delta p}{t} \tag{6.26}$$

Table 6.8 Yield Strengths, Densities, and Costs per Unit Mass for Metal Alloys That Are the Subjects of Design Example 6.2

Alloy	*Yield Strength, σ_y (MPa)*	*Density, ρ (g/cm^3)*	*Unit mass cost, $\bar{c}$ (\$US/kg)*
Steel	325	7.8	1.75
Aluminum	125	2.7	5.00
Copper	225	8.9	7.50
Brass	275	8.5	10.00
Magnesium	175	1.8	12.00
Titanium	700	4.5	85.00

We now incorporate into this equation values of N, r_i, Δp, and t given in the problem statement and solve for σ_y. Alloys in Table 6.8 that have yield strengths greater than this value are suitable candidates for the tubing. Therefore,

$$\sigma_y = \frac{(4.0)(50 \times 10^{-3}\ \text{m})(2.027\ \text{MPa} - 0.057\ \text{MPa})}{(2 \times 10^{-3}\ \text{m})} = 197\ \text{MPa}$$

Four of the six alloys in Table 6.8 have yield strengths greater than 197 MPa and satisfy the design criterion for this tube—that is, steel, copper, brass, and titanium.

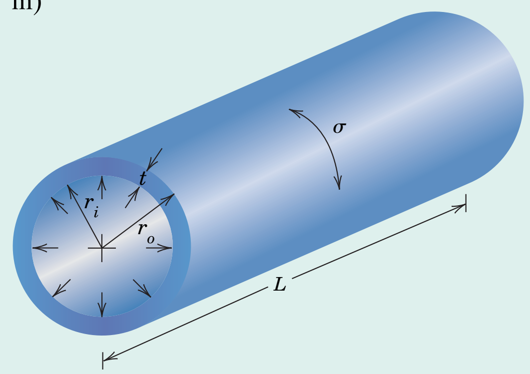


Figure 6.21 Schematic representation of a cylindrical tube, the subject of Design Example 6.2.

(b) To determine the tube cost for each alloy, it is first necessary to compute the tube volume V, which is equal to the product of cross-sectional area A and length L—that is,

$$\begin{aligned} V &= AL \\ &= \pi(r_o^2 - r_i^2)L \end{aligned} \tag{6.27}$$

Here, r_o and r_i are, respectively, the tube inside and inside radii. From Figure 6.21, it may be observed that $r_o = r_i + t$, or that

$$\begin{aligned} V &= \pi(r_o^2 - r_i^2)L = \pi[(r_i + t)^2 - r_i^2]L \\ &= \pi(r_i^2 + 2r_it + t^2 - r_i^2)L \\ &= \pi(2r_it + t^2)L \end{aligned} \tag{6.28}$$

Because the tube length L has not been specified, for the sake of convenience, we assume a value of 1.0 m. Incorporating values for r_i and t, provided in the problem statement leads to the following value for V:

$$\begin{aligned} V &= \pi[(2)(50 \times 10^{-3}\ \text{m})(2 \times 10^{-3}\ \text{m}) + (2 \times 10^{-3}\ \text{m})^2](1\ \text{m}) \\ &= 6.28 \times 10^{-4}\ \text{m}^3 = 628\ \text{cm}^3 \end{aligned}$$

Next, it is necessary to determine the mass of each alloy (in kilograms) by multiplying this value of V by the alloy's density, ρ (Table 6.8) and then dividing by 1000, which is a unit-conversion factor because 1000 mm = 1 m. Finally, cost of each alloy (in \$US) is computed from the product of this mass and the unit mass cost ($\bar{c}$) (Table 6.8). This procedure is expressed in equation form as follows:

$$\text{Cost} = \left(\frac{V\rho}{1000}\right)(\bar{c}) \tag{6.29}$$

For example, for steel,

$$\text{Cost (steel)} = \left[\frac{(628\ \text{cm}^3)(7.8\ \text{g/cm}^3)}{(1000\ \text{g/kg})}\right](1.75\ \$\text{US/kg}) = \$8.60$$

Cost values for steel and the other three alloys, as determined in the same manner are tabulated below.

Alloy	*Cost (\$US)*
Steel	8.60
Copper	41.90
Brass	53.40
Titanium	240.20

Hence, steel is by far the least expensive alloy to use for the pressurized tube.