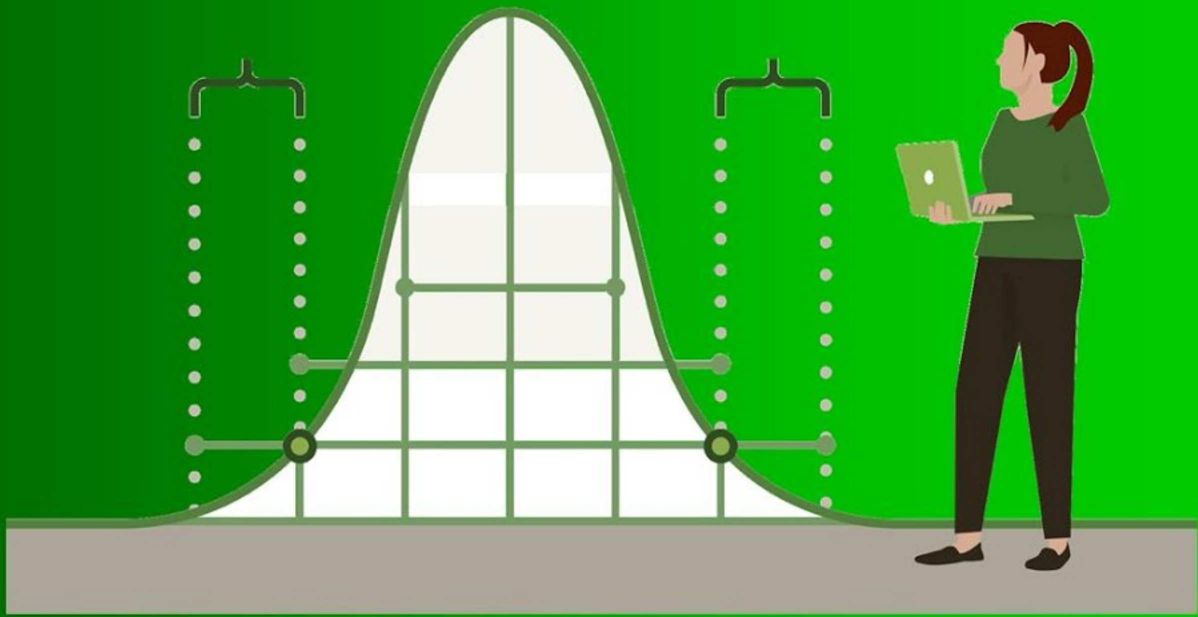
Animal Genetics & Breeding

BIOSTATISTICS AND COMPUTER APPLICATION

(UNIT - I)



Lecture Notes on

Probability

Second revised eprint

Dr. Kuldeep Kumar Tyagi

Department of Animal Genetics & Breeding
College of Veterinary & Animal Sciences
Sardar Vallabhbhai Patel University of Agriculture & Technology
Modipuram, Meerut- 250 110

Published Online:

First eprint: 2020 , Total pages: 17
Second revised eprint: 2020 , Total pages: 24
2020.11.25 12:12:42 +05'30'

Published by:

Department of Animal Genetics & Breeding
College of Veterinary & Animal Sciences
Sardar Vallabhbhai Patel University of Agriculture and Technology
Meerut- 250 110, Uttar Pradesh, India

Publication No. *(for official use)*

First eprint: SVP/2020/06/02/196, Dt. 20.08.2020
Second revised eprint: SVP/2020/06/02/205, Dt. 25.11.2020

How to cite this Lecture notes

Tyagi, K 2020, *Probability*, lecture notes, Biostatistics and Computer Application AGB UNIT I, Sardar Vallabhbhai Patel University of Agriculture & Technology, Meerut, India, Delivered 24th November 2020. Retrieved online from https://www.vepub.com

Address Correspondence to:

Dr. Kuldeep Kumar Tyagi
Associate Professor & OIC
Department of Animal Genetics & Breeding
COVAS, SVPUAT, Meerut- 250110 (U.P.) India
drtyagivet@gmail.com
+91 9601283365 (M)

ABOUT

This is the second revised eprint of my lecture notes on “Probability” delivered to my undergraduate students studying Animal Genetics & Breeding course. This course was offered during the academic year 2020-21 in the second professional year of Bachelor of Veterinary Science & Animal Husbandry degree at College of Veterinary & Animal Sciences, S.V.P.U.A.T, Meerut, Uttar Pradesh, India. This lecture provides basic terminology and concept on probability. Various basic laws and theorems related to probability were also introduced along with explanatory examples. This lecture paves the foundation and linkage between descriptive and inferential statistics thus making it valuable for students to understand forthcoming lectures. In this revised second eprint more examples and figures have been included. Additional topic on rules of counting has also been incorporated. I had tried my level best to simplify the concept in easy to understand language. Further constructive suggestions to improve this lecture note are always welcome from its users on my email: drtyagivet@gmail.com and whatsapp +91 9601283365.

Digitally signed by Kuldeep Kumar Tyagi
Date: 2020.11.25 12:13:35 +05'30'

TABLE OF CONTENTS

Probability

---

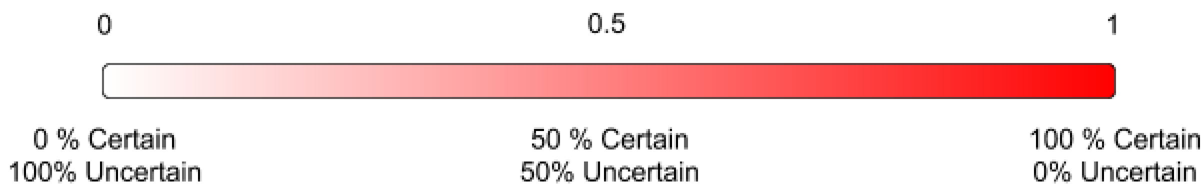
1. Introduction

1.1. Probability statement

Any statement, in which there is an element of uncertainty about the occurrence of some event, is called a probability statement. For example-

1. India may remain number one milk producer in the world for the next 50 years.
2. There may be drought in Western India next year.

1.2. Scale / Limits of Probability



Probability Scale

The magnitude of probability varies from 0 (0%) to 1(100%). That is from 100% uncertain to 100% certain.

1.3. Probability of an Event

The relative measure of the degree of certainty with which an event can occur can be termed as the probability of the event.

2. Definition of Probability

There are three approaches in defining probability

1. The classical or mathematical definition
2. The relative frequency definition or the statistical definition
3. The modern definition or the axiomatic approach to probability

2.1. Classical or mathematical definition

We need to understand a few basic terminologies before we can define probability in mathematical terms.

2.1.1 Terminologies

2.1.1.1. Statistical experiment

An experiment having more than one possible outcome is called a statistical experiment. A statistical experiment is also known as a trial. e.g. Sun rises in the east cannot have more than one outcome hence it cannot be set and considered as a statistical experiment.

2.1.1.2. Event

A possible outcome of a trial is called an event. e.g. Birth of a male calf is a possible event at the time of parturition at a farm.

2.1.1.3. Exhaustive events

A set of events is said to be exhaustive, if it includes all possible outcomes of a trial. e.g. All milking or non milking buffaloes will constitute inclusion of all female buffaloes at a farm.Therefore set of milking and non milking buffaloes are exhaustive with respect to the number of female buffaloes at a farm.

2.1.1.4. Favourable events

Such cases that support the occurrence of an event are said to be cases favourable to that event.

2.1.1.5. Equally likely events

If in a trial, the chance of the occurrence of any possible event is the same, the events are said to be equally likely. e.g. Birth of either a male or female calf at parturition are equally likely events.

2.1.1.6. Mutually exclusive events

If in a trial, the occurrence of an event rules out the simultaneous occurrence of any other possible event, the events are said to be mutually exclusive. e.g. Birth of male calf is mutually exclusive to the birth of a female calf in uniparous animals.

2.1.2. Definition

If a trial can result in 'n' mutually exclusive, equally likely and exhaustive outcomes and out of which 'm' outcomes are favourable to an event A, the probability of A, denoted by P(A), is then

$$\mathbf{P(A)} = \frac{m}{n}$$

Example 1. A fair dice is thrown. What is the probability that either 1 or 6 will show up? In this example discuss various terms involved in defining mathematical probability?

Solution 1.
Statistical Experiment: Here statistical experiment (trial) consists of throwing a fair dice and observing the outcomes.
Event: When we throw the dice, the possibility of occurrence of various outcomes like 1, 2, 3, ... are said to be events.
Exhaustive events: The set {1,2,3,4,5,6} consist of exhaustive events and includes all possible outcomes of a trial. It means, n=6
Favourable events: The set {1,6} consist of favourable events that support the occurrence of an event as per the condition given in trial. It means, m=2.
Equally likely events: Each event in the set {1,2,3,4,5,6} has equal chances of occurrence and hence known as equally likely events.
Mutually exclusive events: Each event in the set {1,2,3,4,5,6} does not affect the occurrence of another event and hence known as mutually exclusive events.

Hence as per the mathematical definition:

$$\mathbf{P(A)} = \frac{m}{n}$$

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

2.2. Relative frequency or statistical definition

The probability of getting a favourable event in a repeated trial tends to stabilize after a particular number of repetitions. Now, the number of times an event occurs is its frequency and when this frequency is divided by the total number of trials, we get the relative frequency of the event.

According to the relative frequency definition, when the number of trials becomes sufficiently large, the relative frequency of an event tends to a limit. This limiting value is the probability of the event under consideration.

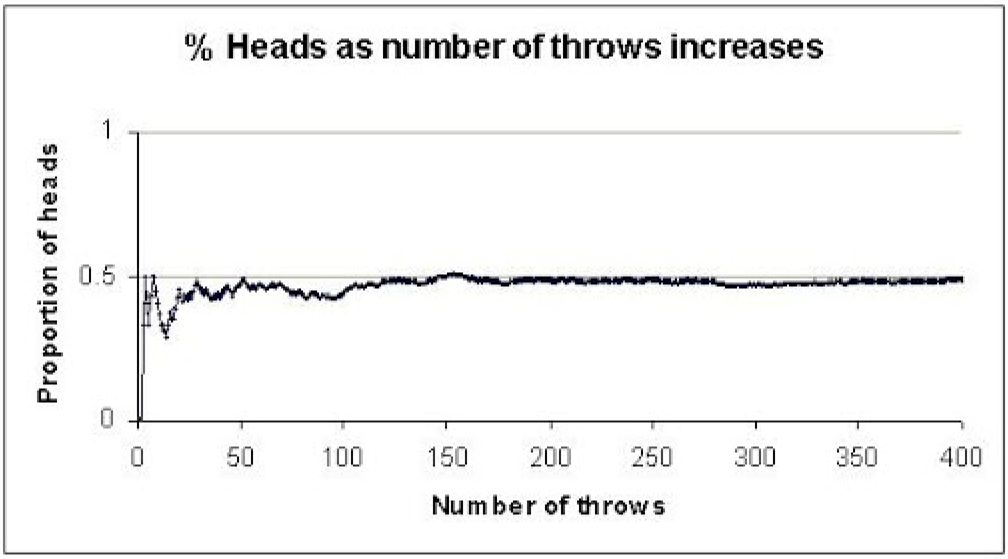
Example 2.
Suppose, we repeat the experiment of parturition in cows and observe the number of times a female calf is born. We shall find that as we increase the number of observations on parturition from say, 10 to 100 to 1000 to 10000 and so on, the relative frequency of getting a female calf will gradually stabilise at $\frac{1}{2}$.

Trial No.	Sex of Calf born	Cumulative freq of favourable event	Probability of favourable event
1	Male	0	$\frac{0}{1} = 0$
2	Female	1	$\frac{1}{2} = 0.5$
50	Female	29	$\frac{29}{50} = 0.58$
100	Male	41	$\frac{41}{100} = 0.41$
5500	Female	2710	$\frac{2710}{5500} = 0.49$
9998	Male	5002	$\frac{5002}{9998} = 0.5$
9999	Male	5002	$\frac{5002}{9999} =0.5$
10000	Female	5003	$\frac{5003}{10000} =0.5$

Mathematically, if n is the total number of trials out of which, an event A occurs m times, the probability of A

$$P(A)= \lim_{n\to\infty} \frac{m}{n}$$

Example 3.
Example 2, can be compared with the probability of getting a head. As we increase the number of throws, the probability of getting heads out of total throws tends to 0.5. Same can be appraised from the figure given below.



2.3. Modern definition | Axiomatic approach to probability

The axiomatic approach to probability can better be understood after going through following concepts.

2.3.1 Terminologies

2.3.1.1. Sample space

It is the set of all possible (or exhaustive) outcomes of a trial. The sample space of a trial can be denoted by S and is given by $S = \{e_1, e_2, ... e_n\}$, where, $e_1, e_2, ... e_n$ are n elementary events. The sample space can further be of

1. Ordered pairs
2. Finite or infinite depending upon whether it consists of finite or infinite number of elements

2.3.1.2. Event

An event is any subset of the sample space.

2.3.1.3. Occurence of an Event

An event is said to have occurred whenever the outcome of a trial belongs to the relevant event-subset.

2.3.2. Definition

According to the modem definition, the probability of an event A, denoted by P(A) is a real valued set function that associates a real value P(A) corresponding to any subset A of the sample space S.

Restrictions | Axioms of probability theory

1. The probability of an event A, in a sample space S, is a non-negative real number less than or equal to unity, i.e., $0 \le P(A) \le 1$.
2. The probability of an event that is certain to occur, is unity. Since S is certain to occur, this implies that P(S) = 1.
3. If A_1, A_2 *and* A_3 , are mutually exclusive events in a sample space S, then $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$

   The above relation can be generalised to any number of events.

   If a sample space S = $\{e_1, e_2, ... e_n\}$ and elementary events $e_1, e_2, ... e_n$ are mutually exclusive

$$P(S)= \sum_{i=1}^{n} P(e_i) = 1$$

Example :4

In a game of cards, where a pack contains 52 cards, 4 categories exist namely spade, club, diamond, and heart. If you are asked to draw a card from this pack, what is the probability that the card drawn belongs to spade?

Solution:

Events	**Notations**	**Spade (A)**
Favourable events	m	13
Total possible events	n	52
Probability	$P= \frac{m}{n}$	$\frac{13}{52} = \frac{1}{4}$

Example 5: In above example 4, what is the probability that the card drawn belongs to either spade or club category?

Solution:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) \quad \text{(Mutually exclusive events)}$$

Events	**Notations**	**A**	**B**
Favourable events	m	13	13
Total possible events	n	52	52
Probability	P= $\frac{m}{n}$	$\frac{13}{52} = \frac{1}{4}$	$\frac{13}{52} = \frac{1}{4}$

$$P(A_1 \cup A_2) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

3. Probability Laws

There are certain notations which are used to understand various probability laws. So let us first understand them-

1. If A and B are two events, then P(A ∪ B) or P(A+ B) denotes the probability that either, A occurs or B occurs or both occur simultaneously. It can also be interpreted as the probability of the occurrence of at least one of the two events A and B. The symbol ∪ above represents 'union' between two events. (Read A ∪ B as 'A union B').
2. P(A ∩ B) or P(AB) denotes the probability of the simultaneous occurrence of both A and B. (Read A ∩ B as 'A intersection B').
3. P(A|B) denotes the conditional probability of the occurrence of A given that B has already occurred.

3.1. Multiplication law

This law states that the probability of the simultaneous occurrence of the two events A and B is equal to the product of

1. The probability of A and the conditional probability of B given that A has already occurred

or

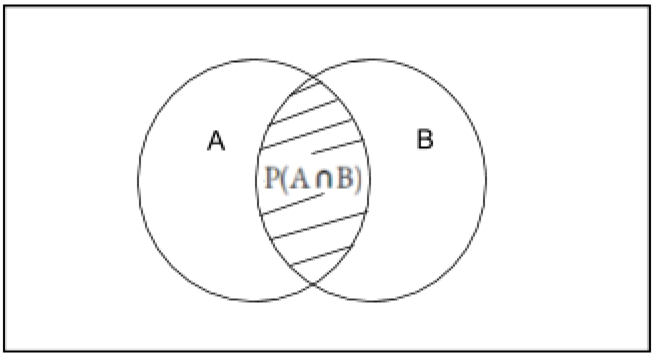
2. The probability of B and the conditional probability of A given that B has already occurred.

Using notations: $P(A \cap B) = P(A). P(B|A) = P(B) . P(A|B)$

i.e. $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Using Venn diagram:



Example :6

One card is drawn from a pack of 52 cards. The card is not replaced in the pack and another card is drawn. What is the probability that both the cards are spade?

Solution:

Event 1: One card is drawn, find the probability that it is spade

Events	A	$\bar{A}$	Total
Possibilities	13	39	52
Probability	$\frac{13}{52} = \frac{1}{4}$	$\frac{39}{52} = \frac{3}{4}$	$\frac{52}{52} = 1$

Thus, P(A)= $\frac{1}{4}$

Event 2: One card is drawn, find the probability that it is spade when the previous one has not been replaced

Events	B|A	$\bar{B}\|A$	Total
Possibilities	12	39	51
Probability	$\frac{12}{51}$	$\frac{39}{51}$	$\frac{51}{51} = 1$

Thus, P(B|A)= $\frac{12}{51}$

Therefore, the probability that both the cards drawn are spade

$$P(A \cap B) = P(A). P(B|A)$$

$$P(A \cap B) = \frac{1}{4} \times \frac{12}{51} = \frac{3}{51}$$

If two events are mutually exclusive, $P(B|A) = P(B)$ and $P(A|B) = P(A)$

Therefore, $P(A \cap B)$ or $P(AB) = P(A).P(B)$

Example :7

One card is drawn from a pack of 52 cards. The card is replaced in the pack and another card is drawn. What is the probability that both the cards are spade?

Solution: Here, now two events are mutually exclusive

Event 1: One card is drawn, find the probability that it is spade

Events	A	$\bar{A}$	Total
Possibilities	13	39	52
Probability	$\frac{13}{52} = \frac{1}{4}$	$\frac{39}{52} = \frac{3}{4}$	$\frac{52}{52} = 1$

Thus, $P(A) = \frac{1}{4}$

Event 2: One card is drawn, find the probability that it is spade when the previous one has not been replaced

Events	$B|A=B$	$\bar{B}|A = \bar{B}$	Total
Possibilities	13	39	52
Probability	$\frac{13}{52} = \frac{1}{4}$	$\frac{39}{52} = \frac{3}{4}$	$\frac{52}{52} = 1$

Thus, $P(B|A) = P(B) = \frac{12}{51}$

Therefore, the probability that both the cards drawn are spade

$$P(A \cap B) = P(A).\ P(B|A)$$

$$P(A \cap B) = P(A).\ P(B)$$

$$P(A \cap B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

Example :8

A card is drawn at random from the deck of 52 cards, what is the probability of drawing an ace of diamonds?

Solution:

$$P(A \cap B) = P(A).\ P(B|A)$$

Events	**Notations**	**A**	**B|A**
Favourable events	m	1	1
Total possible events	n	4	13
Probability	$P = \frac{m}{n}$	$\frac{1}{4}$	$\frac{1}{13}$

$$P(A \cap B) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

Example :9

Two cards are drawn with replacement from a deck of 52 cards, what is the probability that the first card drawn is a spade and the second card drawn is a diamond?

Solution:

$$P(A \cap B) = P(A).\ P(B) \qquad \text{(Mutually exclusive events)}$$

Events	**Notations**	**A**	**B**
Favourable events	m	13	13
Total possible events	n	52	52
Probability	$P = \frac{m}{n}$	$\frac{1}{4}$	$\frac{1}{4}$

$$P(A \cap B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

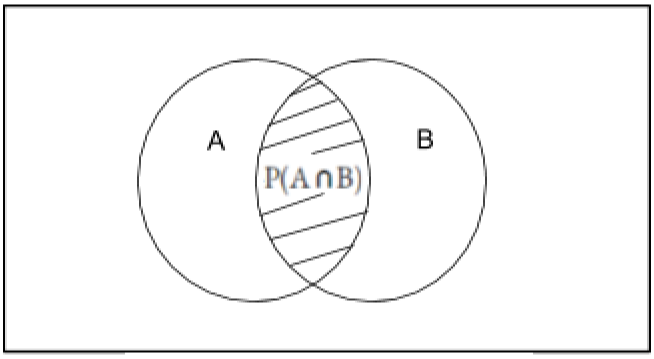
3.2. Addition law

This law states that the probability of the occurrence of at least one of the two events (i.e., either A or B or both) is equal to the probability of A plus the probability of B minus the probability of both A and B.

Using notations: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Also $P(A \cup B) = 1 - P(\bar{A}).P(\bar{B})$

Using Venn diagram:



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

One should note from venn diagram that P(A ∩ B) is counted both in Set A and Set B. Hence, in the formula for P(A ∪ B) it must be subtracted once so that it is not double counted.

Example :10

A dice is thrown. What is the probability of getting a number less than 5 or an odd number?

Solution: The two events are not mutually exclusive,

Event : Let A be the event of getting a number less than five and B is the event of getting an odd number.

Events	A	B	B|A
Possibilities	4	3	2
Probability	$\frac{4}{6} = \frac{2}{3}$	$\frac{3}{6} = \frac{1}{2}$	$\frac{2}{4} = \frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - [P(A).P(B|A)]$ .... Law of multiplication

$= \frac{2}{3} + \frac{1}{2} - [\frac{2}{3} \times \frac{1}{2}]$

$= \frac{7}{6} - \frac{1}{3}$

$= \frac{7-2}{6}$

$= \frac{5}{6}$

Also, $P(A \cup B) = 1 - P(\bar{A}).P(\bar{B})$

Events	A	B	$\bar{A}$	$\bar{B}$
Possibilities	4	3	2	3
Probability	$\frac{4}{6} = \frac{2}{3}$	$\frac{3}{6} = \frac{1}{2}$	$\frac{2}{6} = \frac{1}{3}$	$\frac{3}{6} = \frac{1}{2}$

$P(A \cup B) = 1 - (\frac{1}{3} \times \frac{1}{2}) = 1 - \frac{1}{6} = \frac{5}{6}$

Example :11

The chances of getting more than 15 liters of milk per day from cow A & B are 25% and 50% respectively. What will be the probability of getting more than 15 liters milk from either of them on a particular day?

Solution:

Events	**Notations**	**A**	**B**	$\bar{A}$	$\bar{B}$
Favourable events	m	25	50	75	50
Total possible events	n	100	100	100	100
Probability	$P = \frac{m}{n}$	0.25	0.5	0.75	0.5

$P(A \cup B) = 1 - P(\bar{A}).P(\bar{B})$

$P(A \cup B) = 1 - (0.75 \times 0.5) = 1 - 0.375 = 0.625$

Example :12

The breed of cows and their average milk yield per day maintained by a farmer are given below

Cow No.	Breed	Notation	Average milk yield(Kg) per day
1.	Sahiwal	S_1	10
2.	Gir	G_2	8
3.	Gir	G_3	15
4.	Sahiwal	S_4	12
5.	Gir	G_5	9
6.	Sahiwal	S_6	11

What will be the probability that a selection committee for purchase of animals will select either a Gir cow or a cow with milk yield more than 9 kg per day?

Solution:

$$P(A \cup B) = P(A) + P(B) - P(A).P(B|A)$$

Events	**Notations**	**A**	**B**	**B|A**
Favourable events	m	3	4	1
Total possible events	n	6	6	3
Probability	$P= \frac{m}{n}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$

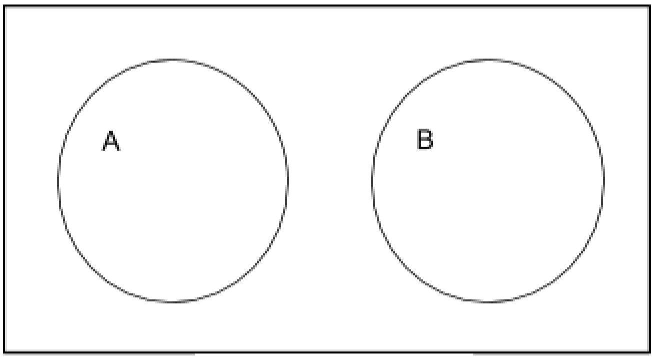
$$P(A \cup B) = \frac{1}{2} + \frac{2}{3} - \left(\frac{1}{2} \times \frac{1}{3}\right) = \frac{1}{2} + \frac{2}{3} - \left(\frac{1}{6}\right) = \frac{3+4-1}{6} = 1$$

3.2.1. Mutually exclusive events

A and B are mutually exclusive, that is, the occurrence of A precludes the occurrence of B and vice versa; then, the two events cannot occur simultaneously and $P(A \cap B) = 0$. Thus, for two mutually exclusive events A and B, the probability of the occurrence of either A or B is equal to the probability of A plus the probability of B.

Using notations: $P(A \cup B) = P(A) + P(B)$

Using Venn diagram:



$P(A \cup B) = P(A) + P(B)$

Example :13

In a game of cards, where a pack contains 52 cards, 4 categories exist namely spade, club, diamond, and heart. If you are asked to draw a card from this pack, what is the probability that the card drawn belongs to either spade or club category.

Solution: The two events are mutually exclusive, let A be the event of getting a spade and B is the event of getting a club.

Events	A	B
Possibilities	4	4
Probability	$\frac{4}{52} = \frac{1}{13}$	$\frac{4}{52} = \frac{1}{13}$

$P(A \cup B) = P(A) + P(B)$ ...(Since $P(A \cap B)=0$)

$= \frac{1}{13} + \frac{1}{13}$

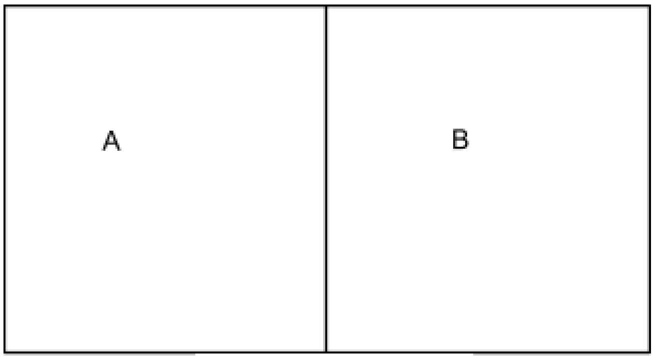
$= \frac{2}{13}$

3.2.2. Exhaustive events

Again, if A and B are the only possible outcomes of a trial (i.e., A and B are exhaustive events), then the occurrence of either A or B is a certainty. We know that the probability of an event that is certain to occur is 1.

Using notations: $P(A \cup B) = P(A) + P(B)$

Using Venn diagram:



$P(A \cup B) = P(A) + P(B)$

Example :14

Twenty (25) male and 35 female calves are born at a farm. A calf is selected at random, what is the probability that it will either be a male or female?

Solution: The two events are exhaustive events, hence $P(A \cup B) = 1$

Proof:

Events	A	B
Possibilities	25	35
Probability	$\frac{25}{60} = \frac{5}{12}$	$\frac{35}{60} = \frac{7}{12}$

$P(A \cup B) = P(A) + P(B)$ ...(Since $P(A \cap B)=0$)

$$= \frac{5}{12} + \frac{7}{12}$$

$$= \frac{12}{12}$$

$$= 1$$

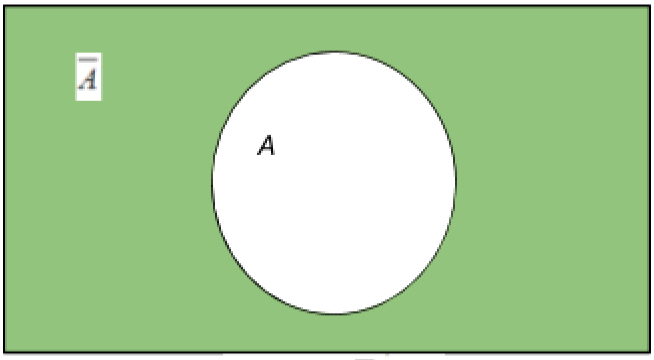
3.2.3. Complementary events

Suppose A is a possible outcome of some trial. It is clear that the trial either results in the occurrence of A or the nonoccurrence of A. Thus, A and 'not A' exhaust all the possible outcomes of any trial. So, if $\bar{A}$ (Read as 'A-bar') denotes 'not A', we have,

Using notations: $P(A) + P(\bar{A}) = 1$

i.e. $P(\bar{A}) = 1 - P(A)$

Using Venn diagram:



$P(A) + P(\bar{A}) = 1$

Here, $\bar{A}$ is called complement to the event A. Thus, the sum of probabilities of any event and its complement is always equal to 1

4. Rules of Counting

The solution to many statistical experiments involves being able to count the number of points in a sample space. Counting points can be hard, tedious, or both. Fortunately, there are ways to make the counting task easier. There are three rules of counting that can save both time and effort - event multiples, combinations and permutations.

4.1 Event Multiples

An event multiple occurs when two or more independent events are grouped together. This rule of counting helps us determine how many ways an event multiple can occur.

Suppose we have k independent events. Event 1 can be performed in n_1 ways; Event 2, in n_2 ways; and so on up to Event k (which can be performed in n_k ways). The number of ways that these events can be performed together is equal to $n_1 n_2 \ldots n_k$ ways.

Example :15

There were 10 sires and 60 dams on a farm. In how many ways these can be paired and mated?

Solution: Here $n_1 = 10$ and $n_2 = 60$

Total number of pairs = $n_1 \times n_2 = 10 \times 60 = 600$ pairs

4.2 Combinations

Sometimes, we want to count all of the possible ways that a single set of objects can be selected - without regard to the order in which they are selected.

Total combinations (when sequence don't matters) denoted by ($^{n}C_{r}$)

$$^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$$

(Where ! is factorial)

Example :16
In example 12, if two cows are selected at random, In how many ways can this be done without giving weightage to sequence?
Solution:
Combinations (When sequence doesn't matters)

Cows	S_1	G_2	G_3	S_4	G_5	S_6
S_1	--	$S_1\ G_2$	$S_1\ G_3$	$S_1\ S_4$	$S_1\ G_5$	$S_1\ S_6$
G_2	$G_2\ S_1$	--	$G_2\ G_3$	$G_2\ S_4$	$G_2\ G_5$	$G_2\ S_6$
G_3	$G_3\ S_1$	$G_3\ G_2$	--	$G_3\ S_4$	$G_3\ G_5$	$G_3\ S_6$
S_4	$S_4\ S_1$	$S_4\ G_2$	$S_4\ G_3$	--	$S_4\ G_5$	$S_4\ S_6$
G_5	$G_5\ S_1$	$G_5\ G_2$	$G_5\ G_3$	$G_5\ S_4$	--	$G_5\ S_6$
S_6	$S_6\ S_1$	$S_6\ G_2$	$S_6\ G_3$	$S_6\ S_4$	$S_6\ G_5$	--

According to formula:

$$^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$$

Here, n = 6, r =2

$$^{6}C_{2} = \frac{6!}{(6-2)! \times 2!} = \frac{6 \times 5 \times 4!}{4! \times 2 \times 1} = 15$$

4.2 Permutations

Sometimes, we want to count all of the possible ways that a single set of objects can be selected - with regard to the order in which they are selected. That is we want to count all of the possible ways that a single set of objects can be arranged.

Total permutations (when sequence matters) denoted by (${}^{n}P_{r}$)

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

(Where ! is factorial)

Example :17

In example 12, if two cows are selected at random, In how many ways can this be done with due weightage to sequence?

Solution:

Permutations (When sequence matters)

Cows	S_1	G_2	G_3	S_4	G_5	S_6
S_1	--	$S_1\ G_2$	$S_1\ G_3$	$S_1\ S_4$	$S_1\ G_5$	$S_1\ S_6$
G_2	$G_2\ S_1$	--	$G_2\ G_3$	$G_2\ S_4$	$G_2\ G_5$	$G_2\ S_6$
G_3	$G_3\ S_1$	$G_3\ G_2$	--	$G_3\ S_4$	$G_3\ G_5$	$G_3\ S_6$
S_4	$S_4\ S_1$	$S_4\ G_2$	$S_4\ G_3$	--	$S_4\ G_5$	$S_4\ S_6$
G_5	$G_5\ S_1$	$G_5\ G_2$	$G_5\ G_3$	$G_5\ S_4$	--	$G_5\ S_6$
S_6	$S_6\ S_1$	$S_6\ G_2$	$S_6\ G_3$	$S_6\ S_4$	$S_6\ G_5$	--

According to formula:

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

Here, n = 6, r =2

$${}^{6}P_{2} = \frac{6!}{(6-2)!} = \frac{6 \times 5 \times 4!}{4!} = 30$$

Example :18

In example 12, what is the probability of selecting Gir and Sahiwal cow together without due weightage to sequence?

Solution:

According to event multiple counting rule:

Number of Sahiwal cows n_1=3

Number of Gir Cows n_2 = 3

Total number of possible pairs = $n_1 \times n_2 = 3 \times 3 = 9$

Now total number of combinations possible without giving weightage to sequence will be

Cows	S_1	G_2	G_3	S_4	G_5	S_6
S_1	--	$S_1\ G_2$	$S_1\ G_3$	$S_1\ S_4$	$S_1\ G_5$	$S_1\ S_6$
G_2	$G_2\ S_1$	--	$G_2\ G_3$	$G_2\ S_4$	$G_2\ G_5$	$G_2\ S_6$
G_3	$G_3\ S_1$	$G_3\ G_2$	--	$G_3\ S_4$	$G_3\ G_5$	$G_3\ S_6$
S_4	$S_4\ S_1$	$S_4\ G_2$	$S_4\ G_3$	--	$S_4\ G_5$	$S_4\ S_6$
G_5	$G_5\ S_1$	$G_5\ G_2$	$G_5\ G_3$	$G_5\ S_4$	--	$G_5\ S_6$
S_6	$S_6\ S_1$	$S_6\ G_2$	$S_6\ G_3$	$S_6\ S_4$	$S_6\ G_5$	--

According to formula:

$$^{n}C_r = \frac{n!}{(n-r)! \times r!}$$

Here, n = 6, r =2

$$^{6}C_2 = \frac{6!}{(6-2)! \times 2!} = \frac{6 \times 5 \times 4!}{4! \times 2 \times 1} = 15$$

Therefore probability of selecting Gir and Sahiwal cow together = $\frac{9}{15} = \frac{3}{5}$

5. Bayes Theorem

Let $A_1, A_2, \ldots\ldots A_n$, be 'n' mutually exclusive and exhaustive events and there is event 'D' which can occur in conjunction with any of them. If D actually happens, then the conditional probability of the occurrence of A, (i = 1,2,3,........n) given D, is given by

$$\mathrm{P}(A_i|D) = \frac{P(A_i \cap D)}{P(D)}$$

$$\text{Where, P(D)} = \sum_{i=n}^{n} (A_i \cap D)$$

Example :19

The share of total calves born at three farms A_1, A_2, and A_3 are 25%, 35% and 40% respectively. of total output respectively. If total 100 calves were born during the year out of which 1.25%, 1.4% and 0.8% of calves born at respective farms were found to be congenitally deformed. A calf is drawn at random from total congenitally deformed calves born during the year. What is the probability that it was from Farm A_1, (ii) Farm A_2, and (iii) Farm A_3 ?

Solution:

Events	A_1	A_2	A_3	$D\vert A_1$	$D\vert A_2$	$D\vert A_3$
Possibilities	25	35	40	1.25	1.4	0.8
Probability	$\frac{25}{100}=\frac{1}{4}$	$\frac{35}{100}=\frac{7}{20}$	$\frac{40}{100}=\frac{2}{5}$	$\frac{1.25}{25}=\frac{1}{20}$	$\frac{1.4}{35}=\frac{1}{25}$	$\frac{0.8}{40}=\frac{1}{50}$

$$\text{Therefore, } P(D) = \sum_{i=n}^{n}(A_i \cap D)$$

P(D) = P(A_1).P(D|A_1)+P(A_2).P(D|A_2)+P(A_3).P(D|A_3)

$$= [\frac{1}{4} \times \frac{1}{20}]+[\frac{7}{20} \times \frac{1}{25}]+[\frac{2}{5} \times \frac{1}{50}]$$

$$= \frac{1}{80} + \frac{7}{500} + \frac{1}{125}$$

$$= \frac{125+140+80}{10000}$$

$$= \frac{345}{10000}$$

Now,

$$P(A_i|D) = \frac{P(A_i \cap D)}{P(D)}$$

$$P(A_1|D) = \frac{P(A_1 \cap D)}{P(D)} = \frac{P(A_1).P(D|A_1)}{P(D)}$$

$$= \frac{\frac{1}{4} \times \frac{1}{20}}{\frac{345}{10000}}$$

$$= \frac{1}{80} \times \frac{10000}{345}$$

$$= \frac{10000}{27600}$$

$$= \frac{1}{276}$$

Similarly, P($A_2|D$) and P($A_3|D$) can also be calculated

****

ABOUT THE AUTHOR

Dr. Kuldeep Kumar Tyagi had completed his B.V.Sc & A.H. in the year 2006 from Guru Angad Dev Veterinary and Animal Sciences University, Ludhiana, Punjab India. He got admission in a master program in the subject of Animal Genetics and Breeding at Indian Veterinary Research Institute, Bareilly, Uttar Pradesh, India after securing 6th rank in All India ICAR-JRF examination. He had completed his Masters in the year 2008 and carried out research on competent fibroblast cells used in somatic cell nuclear transfer. He qualified CSIR Net in his first attempt during the final semester of masters program itself. He got selected as Assistant Professor in the year 2009 at College of Veterinary Science & A.H. at Navsari Agricultural University, Navsari, Gujarat, India. He enriched his practical knowledge and expertise in the subject of Animal Breeding while disbursing his duties as Scheme Incharge at Livestock Research Station of the same university for 9 years. During the same tenure he also accumulated practical expertise on various aspects of field level breeding programs while heading "All India Coordinated Research Project on Goat Improvement - Surti Field Unit" as Principal Investigator. He completed his Ph.D. in the year 2016 from the same university as an inservice candidate. He had worked on gene expression studies on mammary epithelial cells of buffaloes during his Ph.D. degree program. He had been selected as Associate Professor in the department of Animal Genetics & Breeding, College of Veterinary and Animal Science, Sardar Vallbhbhai, Patel University of Agriculture & Technology, Meerut, Uttar Pradesh, India in the year 2018. Since then he has been heading the same department as Officer-Incharge. He had handled 5 externally funded and 27 institutionally funded research projects. He had coguided two masters students. He has in his credit 60 research papers, 14 research recommendations, 2 lecture notes and 4 success stories. He is a member of 4 professional societies and attended 21 conferences/ symposiums/ workshops. He has remained on a panel of experts for framing question papers for various Universities and National level examination bodies. He is hosting a google site for online teaching https://sites.google.com/view/learnagb and can be reached at drtyagivet@gmail.com for initiating a conversation.

Published by:

Department of Animal Genetics & Breeding
College of Veterinary & Animal Sciences
Sardar Vallabhbhai Patel University of Agriculture and Technology
Meerut- 250 110, Uttar Pradesh, India

To cite this lecture notes:

Tyagi, K 2020, Probability, lecture notes, Biostatistics and Computer Application AGB UNIT I, Sardar Vallabhbhai Patel University of Agriculture & Technology, Meerut, India, Delivered 24th November 2020. Retrieved online from https://www.vepub.com