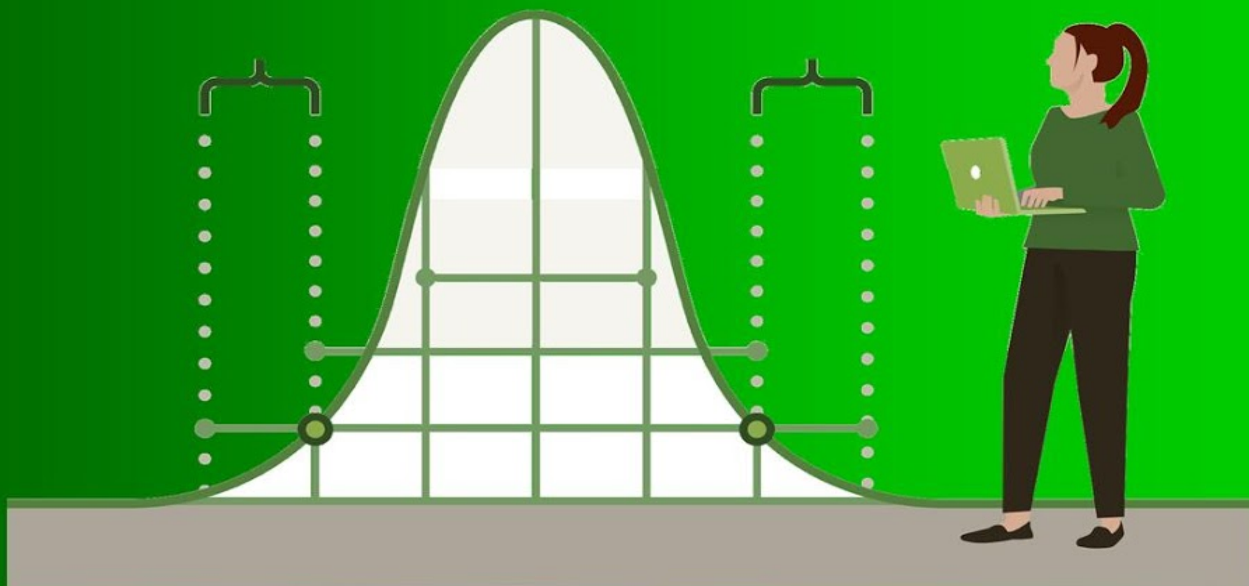
Animal Genetics & Breeding

BIOSTATISTICS AND COMPUTER APPLICATION

(UNIT - I)



Lecture Notes on

Probability distributions

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ABOUT

These lecture notes on “Probability distributions” were prepared and delivered to my undergraduate students studying Animal Genetics & Breeding course. This course was offered in the second professional year of Bachelor of Veterinary Science & Animal Husbandry degree at College of Veterinary & Animal Sciences, S.V.P.U.A.T, Meerut, Uttar Pradesh, India. This is in continuation to my lecture notes on “Probability”. I have tried to explain how probability and probability distribution are important pivots of Inferential statistics. New concepts about random variables, its associated probabilities, statistics of random variables, discrete and continuous probability distribution along with standard normal variate have been introduced and explained. Use of diagrams, explanatory boxes, examples and tables have deliberately been used throughout the notes to create an interest among the students. Once through with these lecture notes readers will be able to understand the basics and application of random variable and probability distributions. I had tried my level best to simplify the concept in easy to understand language. Further constructive suggestions to improve this lecture notes are always welcome from readers on my email and whatsapp.

Digitally signed by Kuldeep Kumar Tyagi
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Probability Distributions

1. Random variable

Random variables are related to the outcomes of a chance experiment. Such a chance experiment is also known as a random experiment.

Example 1. A candidate doesn't know the answers of 10 multiple choice questions (MCQ's) out of 100 MCQ's in a paper? He decided to go by his luck and started ticking randomly. What will be the random variable for his chances of ticking the right answer?
Solution 1.

The random variable here will be the sample space of number of answers attempted correctly in this chance experiment

Therefore, X= (0, 1, 2, 3, 4, 5, 6, 7, 8 ,9 ,10)

In the above experiment, if 4 marks are awarded for the correct answer and 1 mark is deducted for every wrong answer. What will be the outcome random variable for marks obtained by the candidate?

Correct Ans	Wrong Ans	Marks obtained	Marks deducted	Marks awarded (X)
0	10	0	-10	-10
1	9	4	-9	-5
2	8	8	-8	0
3	7	12	-7	5
4	6	16	-6	10
5	5	20	-5	15
6	4	24	-4	20
7	3	28	-3	25
8	2	32	-2	30
9	1	36	-1	35
10	0	40	0	40

Therefore, X =(-10,-5, 0, 5, 10, 15, 20, 25, 30, 35, 40)

So we have learnt that outcomes of a chance experiment decides our random variable values. Since, outcomes itself depend upon chance, the value of random variable cannot be pre decided and it also depends upon chance.

A random variable can be either discrete or continuous.

1.1. Discrete Random Variable

When the sample space of an experiment is discrete, the corresponding random variable will also be discrete, i.e., it will take certain isolated values. The random variables discussed above are examples of discrete random variables.

1.2. Continuous Random Variable

Continuous variables can take any value in an interval. Accordingly, a continuous random variable is defined when the accompanying sample space is also continuous.

2. Probability Distribution

It is defined as a statement about the possible values of a random variable along with their respective probabilities. Now let us include the probability of occurrence of each value of random variable to make it a probability distribution.

Example 2. Construct a probability distribution of obtaining the number of heads when two unbiased coins are tossed.

Solution 2.

Number of heads (X)	**Probability of Occurrence P(X)**
0	$\frac{1}{4}$
1	$\frac{2}{4}$
2	$\frac{1}{4}$
$\sum P(X)$	1

The probability mass function should satisfy the following two conditions

1. Probability of an event cannot be negative, i.e., for any value of X, p(x) ≥ 0
2. Probabilities of all possible outcomes sum to unity, i.e., $\sum_{all\ x} p(x) = 1$

Probability distribution is again further of two types, viz: Discrete and Continuous.

2.1. Discrete probability distribution

The probability distribution for a discrete random variable is defined by a function called probability mass function, denoted by p(x). This probability mass function provides the probability for each value of the discrete random variable and is known as discrete probability distribution.

Example 3. First two columns of the table given below define the litter size distribution on a goat farm. Construct the probability mass function for the random variable litter size?

Solution 3.

Litter size (x)	Number of goats (f)	Probability of occurrence p(x)
1	60	$\frac{60}{100} = 0.6$
2	25	$\frac{25}{100} = 0.25$
3	10	$\frac{10}{100} = 0.1$
4	5	$\frac{5}{100} = 0.05$
Sum	100	1

Example 4. What will be the random variable of obtaining an odd number when two dice are thrown? Write down the probability distribution for this chance experiment?

Solution 4.

Total number of possible outcomes from first dice $n_1 = 6$

Total number of possible outcomes from second dice $n_2 = 6$

Total possible outcomes = $n_1 \times n_2 = 6 \times 6 = 36$

Occurence of Odd number (x)	Favorable events for First dice (n_1)	Favorable events for second dice (n_1)	Total favourable events ($n_1 \times n_2$)	Probability of occurrence p(x)
0, (even, even)	3	3	9	$\frac{9}{36}$ =0.25
1, (even, odd)	3	3	9	$\frac{18}{36}$ =0.50
1, (odd, even)	3	3	9	
2, (even, even)	3	3	9	$\frac{9}{36}$ =0.25
		Sum	36	1

Example 5. In a chance experiment the experimenter deduced a relation between random variable and probability mass function as follows:

$$p(x)= \frac{x^2}{2}\text{ , where } x=\{-1,0,2\}$$

Justify whether experimenter has deduced the correct equation?

Solution 5.

Random Variable (x)	Probability of occurrence p(x) = $\frac{x^2}{2}$
-1	$\frac{1}{2}$ =0.5
0	$\frac{0}{2}$ =0
2	$\frac{4}{2}$ =2
Sum	2.5

Now, the probability mass function should satisfy the following two conditions

1. Probability of an event cannot be negative, i.e., for any value of X, p(x) ≥ 0
2. Probabilities of all possible outcomes sum to unity, i.e., $\sum_{all\ x} p(x) = 1$

The equation for probability mass function deduced by the experimenter fails on the second condition and hence the equation is not correct.

2.2. Continuous probability distribution

Probability distribution of a continuous random variable cannot be presented in the form of a table like that of a discrete random variable, it can nevertheless be expressed by a specific form of the probability density function p(x).

To understand this let us assume a continuous random variable milk yield. Now suppose we have a rough idea that a cow gives around 14-15 liters of milk per day.

Example 6. A cow gives around 14-15 liters of milk per day. What is the probability that she will give 14.3 liters of milk on a particular day?

Solution 6.

Now there will be infinite (∞) number of possibilities of getting different milk yields between 14 and 15 litres, Thus, $n=\infty$.

Therefore, the probability of getting exactly 14.3 litres of milk will be negligible, thus 'm' is very small.

$$\text{Therefore, } p(x) = \frac{m}{n} \sim 0$$

But at the same time the probability of getting any milk yield between 14 to 15 liters will definitely have some appreciable probability mass function.

Therefore, for a continuous random variable X, one assigns a probability to an interval and not to a particular value. Hence, continuous random variable cannot be presented in the form of a table

A probability density function is defined in such a manner that the area under its curve bounded by x-axis is equal to one when computed over the domain of X for which p(x) is defined. The probability density htion for a continuous random variable X defined over the entire set of real numbers R should satisfy the following conditions.

1. $p(x) \geq 0$ for all $x \in R$
2. $\int_{-\infty}^{+\infty} p(x).dx = 1$
3. $p(a<X<b) = \int_{a}^{b} p(x).dx$

3. Statistics of Random variable

3.1. Mean of a random variable

The mean of a random variable, also known as its mathematical expectation or expected value is defined as the sum of the products of the values of the random variable and the corresponding probabilities.

$$\text{Mean of X} = E(X) = x_1 \,.\, p_1 + x_2 \,.\, p_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ x_n \,.\, p_n = \sum_{i=1}^{n} x_i . p_i$$

Example 7. Now let us take the data in example 3 to discuss the above formula?
Solution 7.

Litter size (x)	Number of goats (f)	Probability of occurrence p(x)	fx	x.p(x)
1	60	$\frac{60}{100}$ =0.6	60	0.6
2	25	$\frac{25}{100}$ =0.25	50	0.5
3	10	$\frac{10}{100}$ =0.1	30	0.3
4	5	$\frac{5}{100}$ =0.05	20	0.2
Sum	100	1	160	1.6

Here, one can clearly see that,

$$\text{Mean of X} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \sum_{i=1}^{n} x_i . \, p_i$$

$$\frac{160}{100} = 1.6$$

For a continuous random variable, the mathematical expectation takes the form of a definite integral. Thus,

$$E(X)=\int_{a}^{b} p(x).dx$$

where, X is a continuous random variable with domain from a to b and p(x) is its probability density.

Example 8. A cow gives 6 liters extra milk when given ration A, 3 liter extra milk when given ration B and 3 liter less milk when given ration C? What will be the expected extra or less milk obtained if the cow is given rations randomly?

Solution 8.

Ration	P(x)	Extra milk (x)	E(x)
A	$\frac{1}{3}$	6	2
B	$\frac{1}{3}$	3	1
C	$\frac{1}{3}$	-3	-1
Sum	1		2

$$\text{Mean of X} = E(X) = x_1 \,.\, p_1 + x_2 \,.\, p_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ x_n \,.\, p_n = \sum_{i=1}^{n} x_i.p_i$$

$$E(X) = 2 \text{ liters}$$

3.1.1. Properties of Mathematical expectation / Mean

1. The mathematical expectation of a constant is the constant itself. If c is constant, then

   $$\mathbf{E(c) = c}$$

2. The mathematical expectation of the product of a constant and a random variable is the product of the constant and the mathematical expectation of the random variable. If c is a constant and X is a random variable, then

   $$\mathbf{E(cX) = c.E(x)}$$

3. The mathematical expectation of any function of a random variable is the sum of the products of the values of the function and the corresponding probabilities of

the values of the random variable. Thus if f(X) is a function of a random variable X that takes the values x_1 , x_2 , x_3 ,................ x_n with specific probabilities the p_1 , p_2 , p_3 ,................ p_n mathematical expectation of f(X) is

$$\mathbf{E[f(X)]} = \sum_{i=1}^{n} x_i . p_i$$

We may note here that the above summation, strictly speaking, applies to a discrete random variable. However, without any loss of generality, the theorem is valid for a continuous random variable also. There, instead of a summation over some finite values, an integration over the domain of the random variable has to be performed.

4. The mathematical expectation of the sum of a given number of random variables is the sum of their expectations. If X and Y are two random variables, the mathematical expectation of X + Y is

E(X+Y) = E(X) + E(Y)

5. The mathematical expectation of the product of a given number of independent random variables is the product of their expectations. If X and Y are two independent random variables, the mathematical expectation of XY is

E(XY)= E(X).E(Y)

3.2. Variance of a Random variable

The variance of a random variable X is given by,

$$V(X) = E[X - E(X)]^2 = E(X^2) - [E(X)]^2$$

Example 9. Now let us take the data in example 6 to discuss the above formula?

Solution 9.

X	**(f)**	**p(x)**	**X-E(X) or X-$\bar{X}$**	$[X-E(X)]^2$ **or** $(X-\bar{X})^2$	$[X-E(X)]^2 \times$ **p(x)**	$(X-\bar{X})^2 \times$ **f**	X^2	$X^2 \times$ **p(x)**
1	60	$\frac{60}{100}$ =0.6	-0.6	0.36	0.216	21.6	1	0.6
2	25	$\frac{25}{100}$ =0.25	0.4	0.16	0.040	4.0	4	1.0
3	10	$\frac{10}{100}$ =0.1	1.4	1.96	0.196	19.6	9	0.9

4	5	$\frac{5}{100}$ =0.05	2.4	5.76	0.288	28.8	16	0.8
Sum	100	1			$E[X-E(X)]^2$ =0.74	74		3.3

From example 5, Mean of X = E(X) = 1.6

Now let us compare the three formulas:

$\frac{\sum_{i=1}^{n} f(X_i-\overline{X})^2}{\sum_{i=1}^{n} f}$	$V(X) = E[X-E(X)]^2$	$V(X)= E(X^2) - [E(X)]^2$
$\frac{74}{100}$ = 0.74	0.74	$=3.3 - (1.6)^2$ = 3.3-2.56 =0.74

Hence the variance of given probability distribution is '0.74'

3.2.1. Properties of Variance

1. The variance of a constant is zero. If c is a constant, then

V(c)=0

2. The variance of the product of a constant and a random variable is the product of the square of the constant and the variance of the random variable. If c is a constant and Xis a random variable, then

V(cX)= c^2 **V(X)**

3. The variance of the sum of a given number of random variables is the sum of their variances. If X and Y are two random variables, the variance of X+ Y is

V(X+Y) = V(X) + V(Y) + 2Cov(X,Y)......................{if X & Y are dependent}

V(X+Y) = V(X) + V(Y)......................{if X & Y are independent}

Where, Cov(X,Y) = E[{X-E(X)}{Y-E(Y)}] =E(XY)-E(X)E(Y)

Example 10. The chances of getting an elite bull from a calving on a farm are $\frac{1}{20}$. What will be the expected number of elite bulls from the farm if there were 120 calvings? What will be the variance?

Solution 10.

Here, n=120, p= $\frac{1}{20}$, therefore, q= (1-p)= $\frac{19}{20}$

E(X) = np

E(X) = 120 × $\frac{1}{20}$ = 6

Therefore the expected number of elite bulls from the farm will be 10 if there were 120 calvings.

V(X)= npq

V(X)= 120 × $\frac{1}{20}$ × $\frac{19}{20}$ = 5.7

Therefore the variance will be 5.7

4. Standard normal variate

For any variable (random or otherwise) with a given mean and standard deviation, whenever the mean is subtracted from it and the result is divided by the standard deviation; the resultant variable has a mean equal to zero and a standard deviation equal to one. That is z scores for any given data has mean equal to zero and standard deviation equal to 1.

Mathematically,

$$\mathbf{E(z) = E(}\frac{(X-\mu)}{\sigma}\mathbf{) = 0}$$

$$\sigma(z) = \sqrt{V(z)} = \sqrt{V(\frac{(X-\mu)}{\sigma})} = \sqrt{1} = \mathbf{1}$$

The variable z defined in the above manner is called standard normal variate. Lateron, we shall consider how this result is used in the context of the normal distribution.

5. Binomial distribution

The binomial distribution is an example of a discrete probability distribution. James Bernoulli presented it in the year 1700. The word ‘binomial’ suggests ‘two’. It signifies two possible outcomes of an experiment, the occurrence of an event or the non occurrence of the event. A probability experiment can be termed as a Bernoulli experiment, if it satisfies the following conditions.

1. The experiment consists of a sequence of "n" repeated trials.
2. Each trial results in an outcome that may be classified either as a success or a failure.
3. The probability of a success, denoted by p, is known and remains the same in each trial. Consequently, the probability of a failure, denoted by q = (1-p) is also known and remains the same in each trial.

The probability mass function in such situations is given by:

$$p(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

Example 11. Now let us take an excerpt from example 1, A candidate doesn't know the answers of 2 multiple choice questions (MCQ's) out of 100 MCQ's in a paper? He decided to go by his luck and started ticking randomly. What will be the probability distribution for his chances of ticking the right answer?

Solution 11.

The random variable here will be the sample space of number of answers attempted correctly in this chance experiment

Therefore, X= (0, 1, 2)

Now if W represents ticking wrong option and R represents ticking right option, the pivot table for various combinations will be given as:

Ques1 / Ques 2	R	W	W	W
R	RR	RW	RW	RW
W	RW	WW	WW	WW
W	RW	WW	WW	WW
W	RW	WW	WW	WW

Now it is clear from the pivot table that total number of possible ways of ticking the options will be n=16

If p is the probability of success equals to $\frac{1}{4}$, therefore q the probability of failure will be equal to (1-p) i.e. $\frac{3}{4}$ The probability distribution in this case will be:

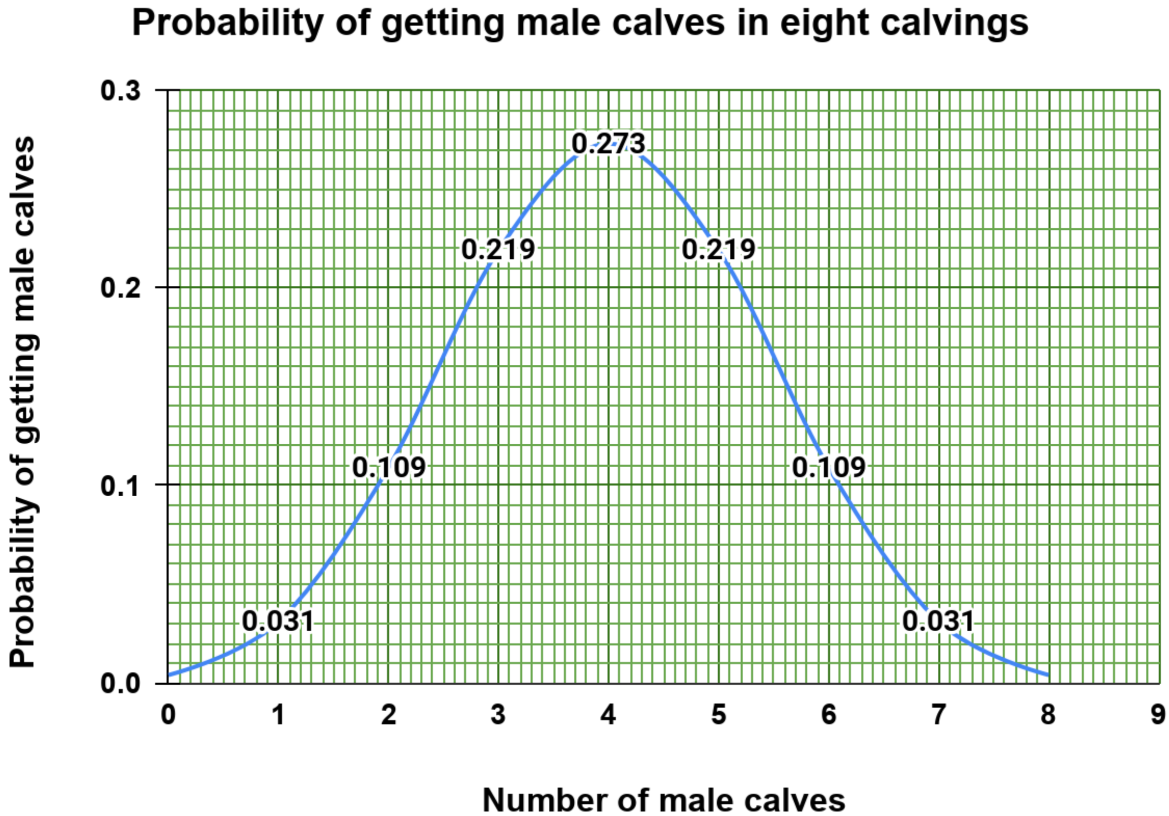
Random Variable (X)	Probability as per pivot table using classical method p(x) = $\frac{m}{n}$	Probability as per binomial distribution equation p(x) = ${}^{n}C_{x}\ p^{x}q^{n-x}$
0	$\frac{9}{16}$	${}^{2}C_{0}\ (\frac{1}{4})^{0}(\frac{3}{4})^{2-0}$ =$1\times 1\times \frac{9}{16} = \frac{9}{16}$
1	$\frac{6}{16}$	${}^{2}C_{1}\ (\frac{1}{4})^{1}(\frac{3}{4})^{2-1}$ =$2\times \frac{1}{4}\times \frac{3}{4} = \frac{6}{16}$
2	$\frac{1}{16}$	${}^{2}C_{2}\ (\frac{1}{4})^{2}(\frac{3}{4})^{2-2}$ = $1\times \frac{1}{16}\times 1 = \frac{1}{16}$

Hence probability mass function for a random variable obeying binomial distribution can be obtained using the formula:

$$p(x) = {}^{n}C_{x}\ p^{x}q^{n-x}$$

5.1 Graphical presentation of binomial distribution

Binomial distributions can be plotted graphically using a probability curve with number of events on the "X" axis and probability function on the "Y" axis. Most of the discrete distributions can be represented in a similar way. Let us understand it with an example.



Example 12. If there were 8 calvings in a farmers cattle herd. Draw binomial distributions of getting male calves?

Solution 12.

Here, n=8, p= $\frac{1}{2}$ therefore, q= (1-p)= $\frac{1}{2}$

Random Variable, Number of male calves (X)	**Probability as per binomial distribution equation** p(x) = $^{n}C_{x}\, p^{x} q^{n-x}$
0	$^{8}C_{0}\ (\frac{1}{2})^{0}(\frac{1}{2})^{8-0}$ $=1 \times 1 \times \frac{1}{256} = \frac{1}{256}$ =0.004
1	$^{8}C_{1}\ (\frac{1}{2})^{1}(\frac{1}{2})^{8-1}$ $=8 \times \frac{1}{2} \times \frac{1}{128} = \frac{8}{256}$ =0.031
2	$^{8}C_{2}\ (\frac{1}{2})^{2}(\frac{1}{2})^{8-2}$ $=28 \times \frac{1}{4} \times \frac{1}{64} = \frac{28}{256}$ =0.109
3	$^{8}C_{3}\ (\frac{1}{2})^{3}(\frac{1}{2})^{8-3}$ $=56 \times \frac{1}{8} \times \frac{1}{32} = \frac{56}{256}$ =0.219
4	$^{8}C_{4}\ (\frac{1}{2})^{4}(\frac{1}{2})^{8-4}$ $=70 \times \frac{1}{16} \times \frac{1}{16} = \frac{70}{256}$ =0.273

5	${}^{8}C_{5}\ (\frac{1}{2})^{5}(\frac{1}{2})^{8-5}$ $=56 \times \frac{1}{32} \times \frac{1}{8} = \frac{56}{256}$ $=0.219$
6	${}^{8}C_{6}\ (\frac{1}{2})^{6}(\frac{1}{2})^{8-6}$ $=28 \times \frac{1}{64} \times \frac{1}{4} = \frac{28}{256}$ $=0.109$
7	${}^{8}C_{7}\ (\frac{1}{2})^{7}(\frac{1}{2})^{8-7}$ $=8 \times \frac{1}{128} \times \frac{1}{2} = \frac{8}{256}$ $=0.031$
8	${}^{8}C_{8}\ (\frac{1}{2})^{8}(\frac{1}{2})^{8-8}$ $=1 \times 1 \times \frac{1}{256} = \frac{1}{256}$ $=0.004$

Example 13. In an organized cattle farm about 20% animals were yielding average milk less than 5 liter per day. If a person selects 5 animals randomly at that farm what is the probability of getting less than 3 animals yielding average milk less than 5 liter per day?

Solution 13.

Here, n=5, p= 0.2, therefore, q= (1-p)= 0.8, r = 2

Random Variable, Number of male calves (X)	**Probability as per binomial distribution equation** p(x) = ${}^{n}C_{x}\ p^{x}q^{n-x}$
0	${}^{5}C_{0}\ (\frac{1}{2})^{0}(\frac{1}{2})^{5-0}$ $=1 \times 1 \times \frac{1}{32} = \frac{1}{32}$ $=0.03125$
1	${}^{5}C_{1}\ (\frac{1}{2})^{1}(\frac{1}{2})^{5-1}$ $=5 \times \frac{1}{2} \times \frac{1}{16} = \frac{5}{32}$ $=0.15625$
2	${}^{5}C_{2}\ (\frac{1}{2})^{2}(\frac{1}{2})^{5-2}$ $=10 \times \frac{1}{4} \times \frac{1}{8} = \frac{10}{32}$ $=0.3125$
P(X)	0.5

Hence there are 50% chances of getting less than 3 animals out of 5 selected animals yielding average milk less than 5 liter per day.

5. Poisson distribution

The poisson distribution is another example of discrete probability distribution. It was discovered in 1837 by French mathematician Siméon Denis Poisson. This distribution is a special limiting case of binomial distribution. It expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event. The

Poisson distribution can also be used for the number of events in other specified intervals such as distance, area or volume.

When the probability of success, p, in a binomial distribution is very small and the number of trials, n, is so large that the expectation, $\mu = np$*, is a finite quantity; the binomial distribution tends to Poisson distribution.*

Now, let us understand this with an illustration:

Illustration 1. Suppose, on an average λ farmers visit a buffalo farm daily and only one farmer is allowed to visit at a time. What is the probability of getting an 'x' number of visitors on a particular day?

If farmers from a wide range of sources arrive independently of one another and arrival of a particular farmer does not affect subsequent arrival of other farmers, then arrival of farmers at buffalo farm reasonably obeys poisson distribution.

We need to apply our previous knowledge of binomial distribution to understand derivation of probability mass function for Poisson distribution.

Here λ can also be treated as mathematical expectations.

Now, how to apply our previous knowledge of binomial distribution to solve this problem. Let us divide a day into numerous sub-intervals in a way that the number of farmers visiting in a sub interval remains either 0 or 1. Now each subinterval represents a binomial distribution with only two outcomes: either there will be a farmers visit or not. The total number of subintervals are taken to be "n" equivalent to the total number of trials in binomial distribution.

Therefore if λ farmers visit a buffalo farm daily, then maximum number of farmers visiting the farm in a subinterval will be

$$p = \frac{\lambda}{n}$$

Thus, the probability of λ farmers visit a buffalo farm in a day amounts to finding the probability of x successes in n trials when n tends to infinity. This probability is given as the following limit of a binomial distribution.

$$\lim_{n \to \infty} {}^{n}C_{x} \left(\frac{\lambda}{n}\right)^{x} \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

The Poisson probability mass function can be derived from the above equation as follows

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Where,

- ➔ X is a random variable denoting the number of successes in a specified time interval or length interval.
- ➔ λ = expected value or average number of occurrences in an interval of time or length etc.
- ➔ e = a constant (base of the natural logarithm) whose value is e =2.7182

Example 14. Suppose, on an average 5 farmers visit a buffalo farm daily and only one farmer is allowed to visit at a time. What is the probability of getting an '2' number of visitors on a particular day?

Solution 14.

Now using the Poisson probability mass function

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$p(x) = \frac{5^2(2.7182)^{-5}}{2!} = \frac{25}{2\times1\times2.718\times2.718\times2.718\times2.718\times2.718} = \frac{25}{296.67}$$

P(x) = 0.084

In Poisson distribution, there is no upper limit on the random variable "x", the number of occurrences. It is a discrete random variable that can assume an infinite sequence of values (x =0, 1,2,3, .. j.) The distribution has only one parameter λ. The expectation of the Poisson distribution is given by the constant λ. It can be shown that the variance of the Poisson distribution is also given by λ.

Example 15. The average number of kiddings per year on a goat farm were 10. What is the probability of getting 8 kiddings during the current year?

Solution 15.

Here, λ =10 and x= 8

Now using the Poisson probability mass function

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$p(x) = \frac{10^8 (2.7182)^{-10}}{8!} =$$

$$\frac{100000000}{8\times7\times6\times5\times4\times3\times2\times1\times2.718\times2.718\times2.718\times2.718\times2.718\times2.718\times2.718\times2.718\times2.718\times2.718}$$

$$= \frac{100000000}{40230\times22003.64} = 0.1127$$

$$P(x) = 0.1127$$

6. Normal distribution

Normal distribution is a continuous probability distribution. This distribution is important because most of the parameters measured on a continuous scale follow this distribution. Normal distribution is perhaps the most widely used distribution in Statistics and related subjects. Normal distribution becomes rather more important because most of the traits of economic importance in livestock are continuous in nature.

6.1 History

The origin of normal distribution can be traced to a French mathematician *Abraham de Moivre.* He had scientific interest in gambling and often acted as a consultant to gamblers to determine probabilities. *De Moivre* allegedly was studying the probability distribution of coin flips. He was trying to come up with a mathematical expression such as finding a probability of 60 or more tails out of one hundred coin flips. As an answer to this question he derived a bell shaped distribution which is commonly referred to as the normal curve.

This was a crucial observation as a large number of phenomena follow approximately normal distribution. For example, such variables (phenomena) as height, weight and strength are characterized with normal distribution. That’s why it is possible to determine one’s weight or height standing compared to others using z score tables. A

Belgian astronomer - *Lambert Quetelet,* was the first one who noticed the link between weight and height distribution and the normal curve.

Initially the normal curve was used to analyze the errors of measurement in astronomical observations. These errors happened due to instrument imprecision and observers' built-in bias. It was *Galileo* who saw that errors were symmetric. He also observed that smaller errors were characterized by higher frequency. This gave an impetus to a chain of hypotheses about error distribution. However, it was only in the nineteenth century that it was noted that these errors were normally distributed. Two mathematicians *Adrian and Gauss* developed a formula for normal distribution independent of each other. Formula demonstrated that errors were well approximated by the normal curve.

It is worth noting that the same distribution was discovered by *Laplace* in the late 18th century, when he developed a very influential central limit theorem. According to this theorem, a distribution of sample means (even if drawn from a non-normal distribution) follows the normal distribution. The larger the sample, the more the distribution approximates normal.

6.2 Properties

The mathematical equation for the normal distribution was given by *Abraham de Moivre* in 1733. Normal distribution is also known as Gaussian distribution after the name of *Karl Friedrich Gauss* who independently derived its equation from a study of errors in repeated measurements of the same quantity.

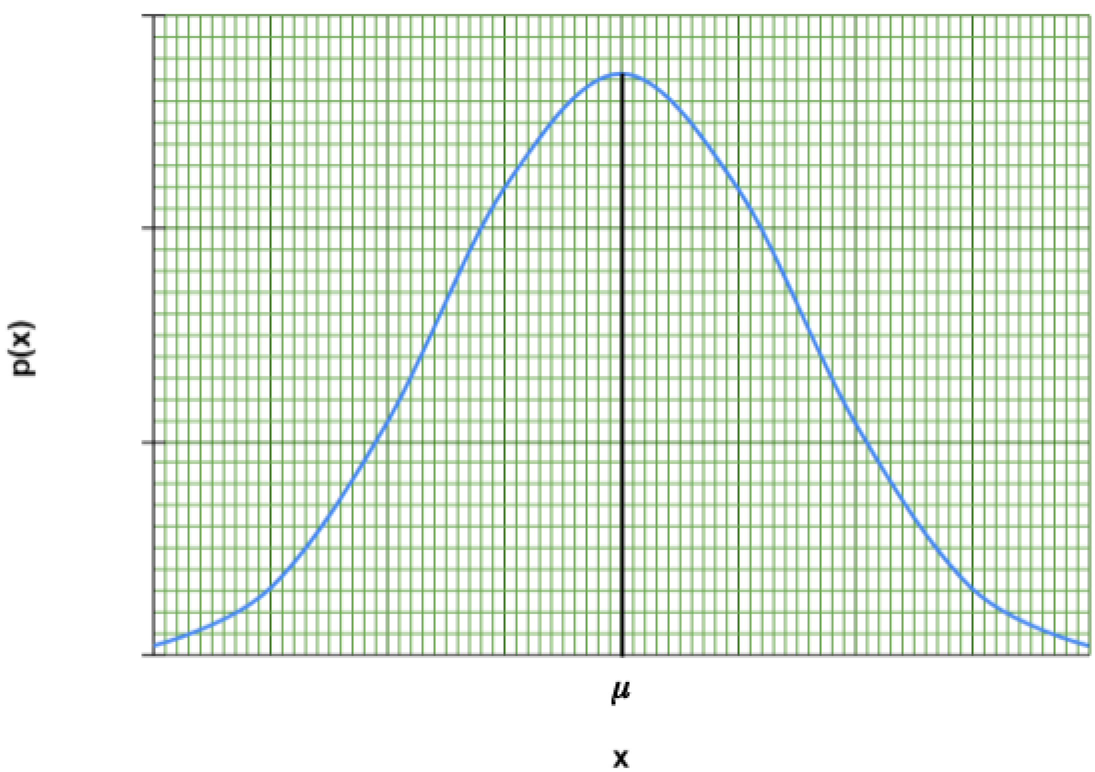
In statistical theory, the probability distributions of continuous variables can be expressed in terms of probability density functions.

$$p(x)= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where, $-\infty < x < +\infty$, $\pi = 3.17141$ (app) and e=2.71828 (app)

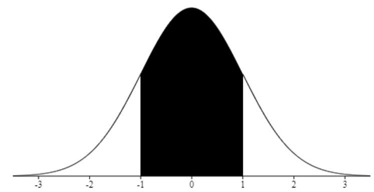
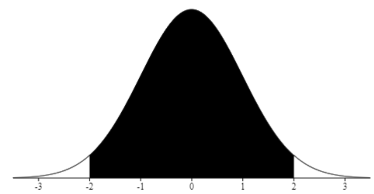
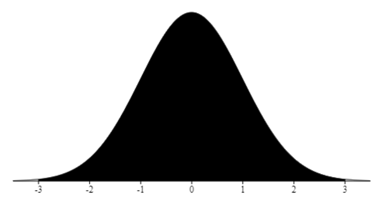
The probability mass function p(x) of a continuous variable following normal distribution solely depends upon the mean "μ" and standard deviation "σ" of the data under consideration. We write it in symbolic form like $X \sim N(\mu, \sigma)$ and read as "X follows normal distribution with mean μ and standard deviation σ. The normal curve is a symmetrical bell shaped curve with following important features.

1. The normal curve extends from $-\infty$ to $+\infty$. This means that a normal random variable (X) lies between $-\infty$ to $+\infty$.
2. The curve is symmetric about its mean, i.e.,= μ. This means that corresponding to $x = \mu + a$ and $x = \mu - a$, the values of p(x) are the same (for any arbitrarily chosen 'a').
3. The median and the mode of the distribution coincide with the mean. Thus mean = median = mode = μ.



4. The maximum of the normal curve occurs at $x = \mu$. Thus p(x) is maximum when $x = \mu$.

5. The points of inflexion of the normal curve occurs at $x = \mu + \sigma$ and $x = \mu - \sigma$. At the points of inflexion, the normal curve changes its curvature. The following area properties hold true for normal distribution.

Ordinates	**Area under the normal curve (%)**	**If, $\mu = 0$ & $\sigma = 1$**
$\mu + \sigma$ and $\mu - \sigma$	68.3	
$\mu + 2\sigma$ and $\mu - 2\sigma$	95.5	
$\mu + 3\sigma$ and $\mu - 3\sigma$	99.7	

6.3 Standard Normal Curve

We have learnt from our previous knowledge about measures of central tendencies and dispersion that for any variable with a given mean and standard deviation, whenever, the mean is subtracted from the normally distributed variable and the result is divided by the standard deviation; the resultant normal variable has a mean equal to zero and a standard deviation equal to one. This resultant variable is known as standard normal variate denoted by "z".

Example 9. Calculate the standard normal variate for the following data?

X	**X**- μ	$(X - \mu)^2$	$\mathbf{z} = \frac{X-\mu}{\sigma}$	z^2
5	-1	1	-0.707	0.50

8	2	4	1.414	2.00
6	0	0	0.000	0.00
7	1	1	0.707	0.50
4	-2	4	-1.414	2.00
$\sum X = 30$		$\sum (X - \mu)^2 = \mathbf{10}$	$\sum z = 0$	$\sum z^2 = 5$

$$\mu = \frac{\sum X}{N} \Rightarrow \frac{30}{5} \Rightarrow \mathbf{6}\,,\ \sigma = \sqrt{\frac{\sum (X-\mu)^2}{N}} \Rightarrow \sqrt{\frac{10}{5}} \Rightarrow \mathbf{1.414}$$

$$\mu_z = \mathbf{0},\ \sigma_z = \sqrt{\frac{\sum z^2}{N}} \Rightarrow \sqrt{\frac{5}{5}} = \mathbf{1}$$

Thus if X is a Variable with mean (expectation) μ and standard deviation σ then z = $\frac{X-\mu}{\sigma}$ has a mean equal to zero and standard deviation equal to one. It means that normal variables with different combinations of μ and σ can all be transformed into a unique normal variable with mean 0 and standard deviation 1.

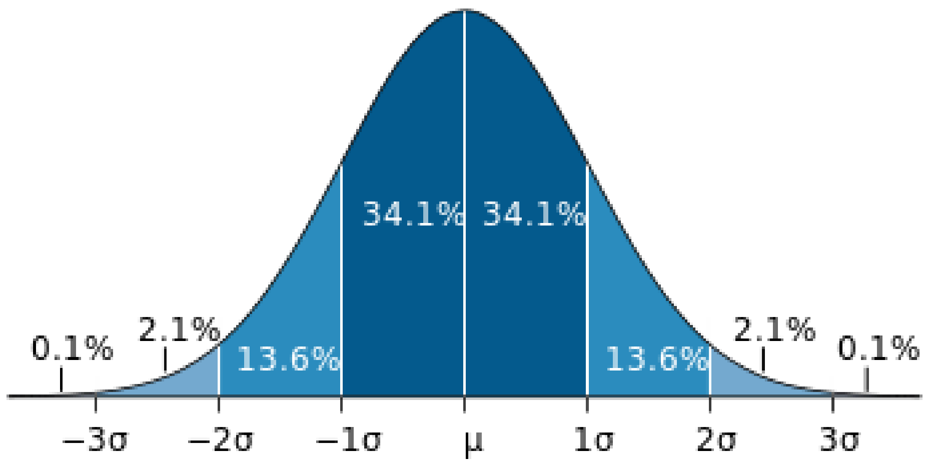
Thus if X is a normal variable with mean (expectation) μ and standard deviation σ then z = $\frac{X-\mu}{\sigma}$ for any combination of μ and σ, is always a normal variable with mean 0 and standard deviation 1.

The probability density function of a standard normal variate is given by.

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2}$$

Where, $-\infty < z < +\infty$, $\pi = 3.17141$ (app) and e=2.71828 (app)

Symbolically z~N(0,1). Thus a standard normal curve will appear like this,



We should note that a standard normal variate has a unique mean of 0 and a unique standard deviation of 1. It means, if we can construct a table for probability areas of such a unique standard normal variate, it can be used for obtaining probability for any normal variable with any combination of mean and standard deviation. The only thing is that the given normal variable is to be transformed into the standard normal variate. In fact, such a table for areas (or probability) has been compiled for a standard normal variate (Appendix 1 and 2) and is very much in use in statistics. Normal distribution and related z score calculation have a wide application in a large number of fields ranging from social sciences to medicine. Z score is a useful standardized value which allows comparison among groups of people based on weight, height, test results, income and many other variables.

Thus, for the computation of the required probability for any normal variable with some mean and standard, deviation, the upper and the lower limits say, $x = x_1$ and $x = x_2$ of the given interval are converted into the corresponding z values say, $z = z_1$ and $z = z_2$ and the relevant area is obtained from appendix 1 and 2 given at the end of this lecture notes.

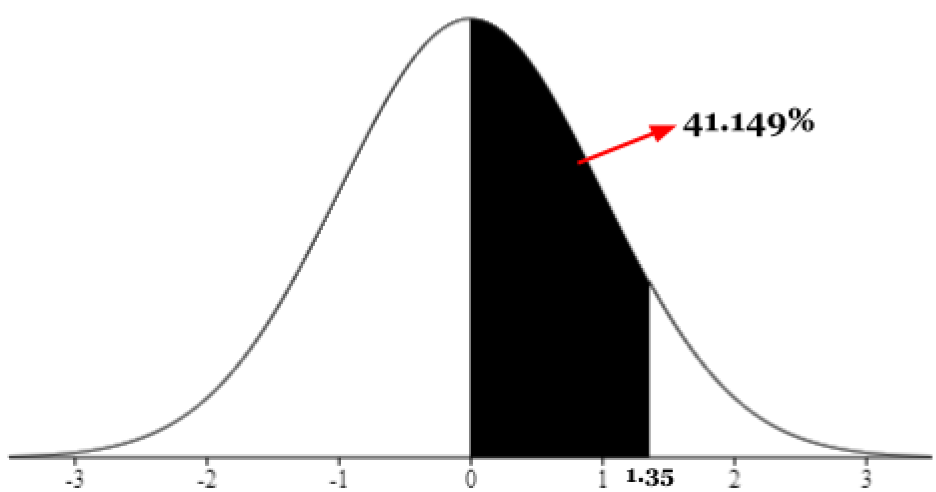
It should be noted that the standard normal curve is symmetrical and it covers an area of 1.0. Since the value of z ranges between $-\infty$ to $+\infty$, we find that the area between $0 < z < \infty$ is 0.5 (half the area under standard normal curve). Similarly, the area between $-\infty < z < 0$ is 0.5. Since the standard normal curve is symmetric we have one advantage; the area under the curve is the same on both sides. In appendix 1 the area for different positive values of z are given. If we look into column 1 of appendix 1 we find that values assumed by z ranges from 0.0 to 4.0. Corresponding to each value there are 10 columns marked .00, .01, ..., .09. These columns represent the second digit after decimal. For example, if z = 0.36, then we look for the row corresponding to 0.3. On this row we move to the right and look for the column representing .06. In appendix 1 we find that when z = 0.36 the area covered is 0.14058. Note that when z is -0.36 the area under the standard normal curve again is 0.14058. Theoretically z can assume any value between $-\infty$ to $+\infty$ However, when z = 4.09 the area covered is 0.49998. Therefore, in appendix 1 and 2 areas for z> 4.09 and z<-4.09 are not given.

Let us now consider some examples to see the applications of the normal area table.

Example 10. Find the area under the standard normal curve in the following cases?

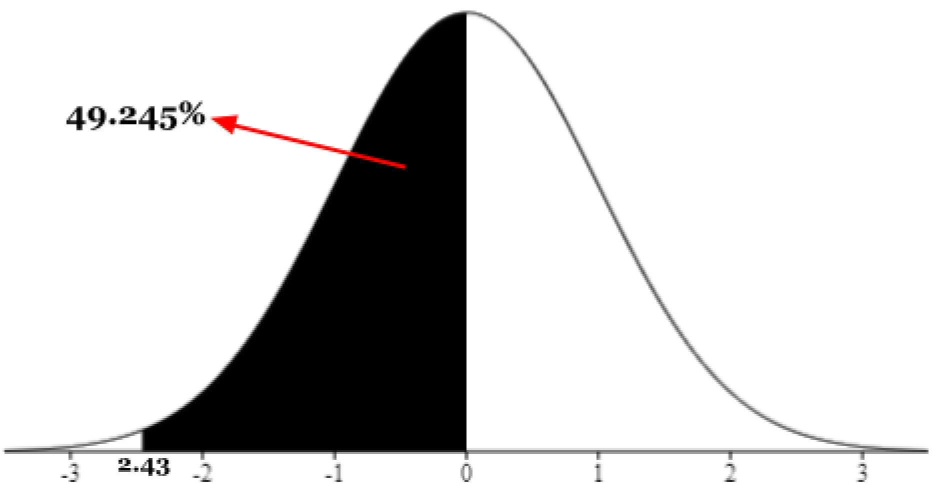
1. $z_1 = 0$ and $z_2 = 1.35$

Solution 1: In appendix 1, move downward under the first column marked "z" and look for 1.3. Now move right to the row marked 1.3 until you reach the column marked "0.05" in first row, you will get 0.41149 which is your area under the standard normal curve defined between the z_1 and z_2. Therefore, the area blackened in the figure given below constitutes 41.149%.



2. z_1 = -2.43 and z_2 = 0

Solution 2: In appendix 2, move downward under the first column marked “z” and look for -2.4. Now move right to the row marked -2.4 until you reach the column marked “0.03” in first row, you will get 0.49245 which is your area under the standard normal curve defined between the z_1 and z_2. Therefore, the area blackened in the figure given below constitutes 49.245%.



3. z_1 = -1.54 and z_2 = 1.87

Solution 3:

In appendix 2, move downward under the first column marked “z” and look for -1.5. Now move right to the row marked -1.5 until you reach the column marked “0.04” in first row, you will get 0.49245 which is your area under the standard normal curve defined between the z_1 and 0.

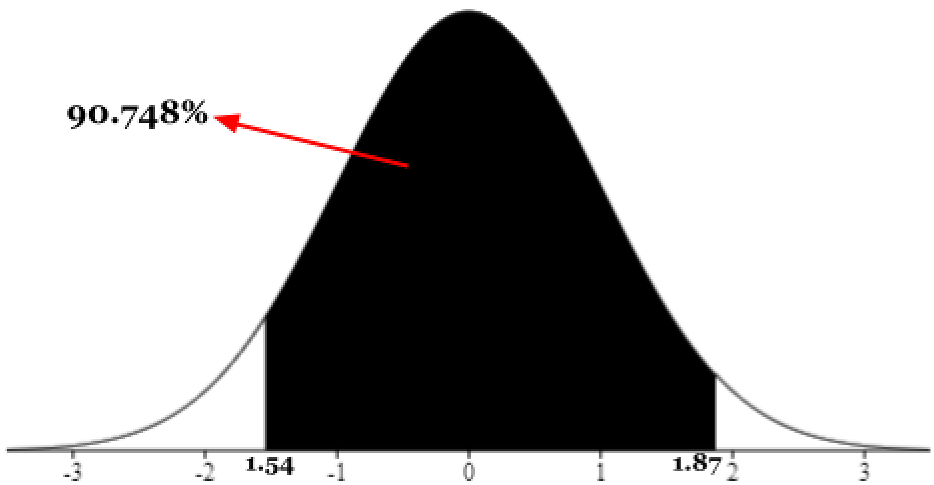
In appendix 1, move downward under the first column marked “z” and look for 1.8. Now move right to the row marked 1.8 until you reach the column marked “0.07” in first row, you will get 0.41149 which is your area under the standard normal curve defined between the 0 and z_2.

Now area between z_1 and z_2 = (Area between z_1 and 0) + (Area between 0 and z_2)

= 0.43822 + 0.46926

= 0.90748

Therefore, the area blackened in the figure given below constitutes 90.748%.



Example 11. There were 145 milking Murraha buffaloes at a farm? The average milk yield of the farm was 15 Kg with a standard deviation of 3 Kg. Now find the percentage of buffaloes giving milk between 12 to 21 kg assuming that milk production is normally distributed at the farm?

Solution:

Here, $\mu = 15,\ \sigma = 3,\ x_1 = 12,\ x_2 = 21$

Therefore, $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{-3}{3} = -1$

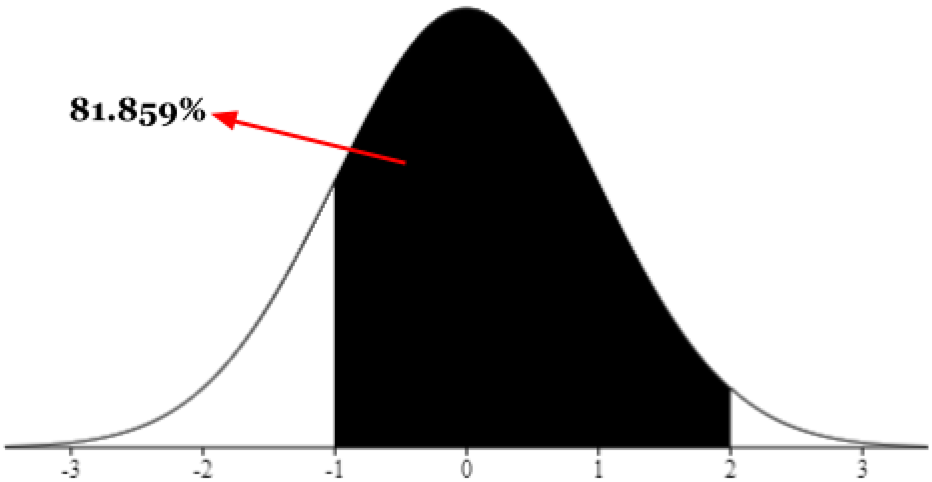
$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{6}{3} = 2$

Now area between z_1 and z_2 = (Area between z_1 and 0) + (Area between 0 and z_2)

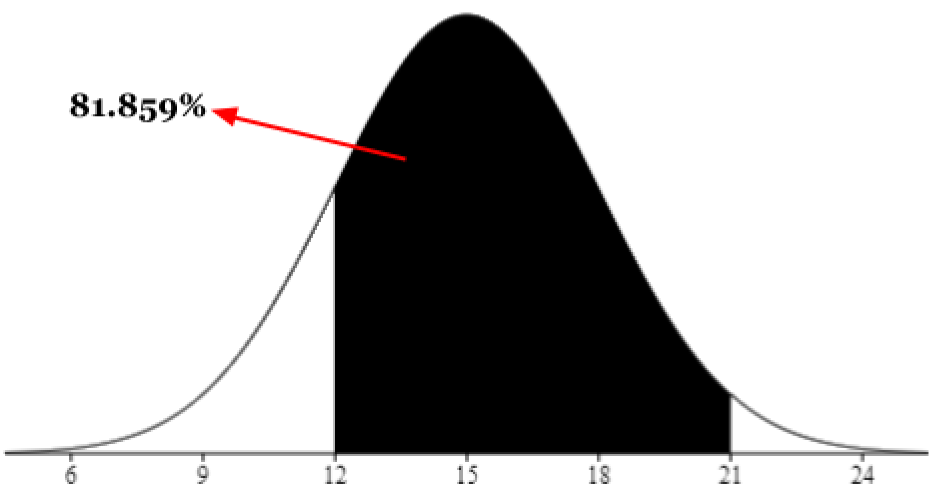
= 0.34134 + 0.47725

= 0.81859

Therefore, the area blackened in the standard normal curve given below constitutes 81.859%.



The above figure can be compared with the actual normal curve for the milk yield at the farm as follows.



In both the cases the probability density function (area) remains the same. Additionally by using standard normal curve we need not to calculate probability again and again for different values of μ and σ instead a single z table fulfills this purpose.

7. Summary

In this lecture we have learnt about random variables that are related to the outcomes of a chance experiment. A random variable can be either discrete or continuous. We understood that probability distribution is a statement about the possible values of a random variable along with their respective probabilities. Probability distribution is again further of two types, viz: Discrete and Continuous. The probability distribution for a random variable is defined by a function called probability mass function, denoted by p(x). This probability mass function provides the probability for each value of the random variable. Probability of occurrence of a random variable cannot be negative and probabilities of all possible outcomes sums to unity. The mean of a random variable, also known as its mathematical expectation or expected value is defined as the sum of the products of the values of the random variable and the corresponding probabilities. Mean of X = E(X) = $\sum_{i=1}^{n} x_i.p_i$. The variance of a random variable X is given by, V(X)= $E[X - E(X)]^2 = E(X^2) - [E(X)]^2$.

We studied about standard normal variate that for any variable (random or otherwise) with a given mean and standard deviation, whenever the mean is subtracted from it and the result is divided by the standard deviation; the resultant variable has a mean equal to zero and a standard deviation equal to one. The probability mass function of a binomial distribution is given by p(x) = ${}^{n}C_x\, p^x q^{n-x}$ whereas that of Poisson distribution is given by p(x) = $\frac{\lambda^x e^{-\lambda}}{x!}$. Binomial distribution signifies two possible outcomes of an experiment, the occurrence of an event or the non occurrence of the event. Whereas Poisson distribution expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

The probability mass function p(x) of a continuous variable following normal distribution solely depends upon the mean "μ" and standard deviation "σ" of the data under consideration and is given by equation p(x)= $\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$. The probability density function of a standard normal variate is given by the expression p(x)= $\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2}$. We learnt that normal variables can be transformed into the standard normal variate. A standard normal variate has a unique mean of 0 and a unique standard deviation of 1. Thus, probability for any normal variable with any combination of mean and standard deviation can be obtained by looking for transformed values in the standard normal table given in appendix 1 and 2.

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Appendix 1: Standard normal probabilities (area) for positive z- scores

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.10	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.20	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.30	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.40	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.50	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.60	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.70	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.80	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.90	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.00	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.10	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.20	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.30	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41309	0.41466	0.41621	0.41774
1.40	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.50	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.60	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.70	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.80	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.90	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.00	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.10	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.20	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.30	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.40	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.50	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.60	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.70	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.80	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.90	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.00	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.10	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.20	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.30	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.40	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.50	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.60	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.70	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.80	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.90	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.00	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

Appendix 2: Standard normal probabilities (area) for negative z- scores

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-4.00	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998
-3.90	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
-3.80	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
-3.70	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
-3.60	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
-3.50	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
-3.40	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
-3.30	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
-3.20	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
-3.10	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
-3.00	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
-2.90	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
-2.80	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
-2.70	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
-2.60	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
-2.50	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
-2.40	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
-2.30	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
-2.20	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
-2.10	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
-2.00	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
-1.90	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
-1.80	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
-1.70	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
-1.60	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
-1.50	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
-1.40	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
-1.30	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41309	0.41466	0.41621	0.41774
-1.20	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
-1.10	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
-1.00	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
-0.90	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
-0.80	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
-0.70	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
-0.60	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
-0.50	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
-0.40	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
-0.30	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
-0.20	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
-0.10	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
-0.00	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586

ABOUT THE AUTHOR

Dr. Kuldeep Kumar Tyagi had completed his B.V.Sc & A.H. in the year 2006 from Guru Angad Dev Veterinary and Animal Sciences University, Ludhiana, Punjab India. He got admission in a master program in the subject of Animal Genetics and Breeding at Indian Veterinary Research Institute, Bareilly, Uttar Pradesh, India after securing 6th rank in All India ICAR-JRF examination. He had completed his Masters in the year 2008 and carried out research on competent fibroblast cells used in somatic cell nuclear transfer. He qualified CSIR Net in his first attempt during the final semester of masters program itself. He got selected as Assistant Professor in the year 2009 at College of Veterinary Science & A.H. at Navsari Agricultural University, Navsari, Gujarat, India. He enriched his practical knowledge and expertise in the subject of Animal Breeding while disbursing his duties as Scheme Incharge at Livestock Research Station of the same university for 9 years. During the same tenure he also accumulated practical expertise on various aspects of field level breeding programs while heading "All India Coordinated Research Project on Goat Improvement - Surti Field Unit" as Principal Investigator. He completed his Ph.D. in the year 2016 from the same university as an inservice candidate. He had worked on gene expression studies on mammary epithelial cells of buffaloes during his Ph.D. degree program. He had been selected as Associate Professor in the department of Animal Genetics & Breeding, College of Veterinary and Animal Science, Sardar Vallbhbhai, Patel University of Agriculture & Technology, Meerut, Uttar Pradesh, India in the year 2018. Since then he has been heading the same department as Officer-Incharge. He had handled 5 externally funded and 27 institutionally funded research projects. He had coguided two masters students. He has in his credit 60 research papers, 14 research recommendations, 2 lecture notes and 4 success stories. He is a member of 4 professional societies and attended 21 conferences/ symposiums/ workshops. He has remained on a panel of experts for framing question papers for various Universities and National level examination bodies. He is hosting a google site for online teaching https://sites.google.com/view/learnagb and can be reached at drtyagivet@gmail.com for initiating a conversation.

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